MATHESEOS ET GEOMETRIAE DIVISIO.

Mathematicæ disciplinæ, quæ omnes circa quantitatem versantur, nomen acceperunt à Græca dictione μάθημα, seu μάθησις, quæ disciplinam, & doctrinam significat; eò quod tum gradatim ascendendo doceantur, & addiscantur; tùm solæ semper ex præcognitis quibusdam, concessis, & probatis, principijs, (Haud ex hypothesibus nondum explicatis) ad cóclusiones demonstrandas, quod proprium est doctrinarum & disciplinarum officium, teste Aristotele lib. 1. Poster. procedant.

Pythagoras, & Mathematici vniuersas Mathematicas disciplinas in quatuor partes principes distribuũt, nempe in Arithmeticam, Musicam, Geometriam, & Astronomiam. Cum enim omnis quantitas, circa quam hæ disciplinæ versantur, sit vel discreta, sub qua omnes numeri continentur; vel continua, sub qua omnes magnitudines comprehenduntur; & vtraque tam secundum se, quam comparatione alterius possit considerari; Visum est illis has quatuor partes instituere, quæ vtramque quantitatem pro duplici consideratione, diligenter contemplarentur. Itaque Arithmetica agit de quantitate discreta secundum se; omnesque numerorum proprietates ac passiones inquirit, & accuratè explicat. Musica tractat eandẽ quantitatẽ discretã, seu numerum cum alio comparatum; quatenus sonorum Harmoniam, & concentum respicit. Geometria de magnitudine, seu quantitate continua, secundum se quoque, vt immobilis existit, disputat. Astronomia dẽnique magnitudinẽ, vt est mobilis, cõsiderat; qualia sũt corpora cœlestia cõtinuè mota. Ad has aũt quatuor scientias Mathematicas, tum puras, tũ mixtas, omnes alię de quantitate tractantes, vti Perspectiua Geographia Stereomatria, & cæteræ, facili negocio, tanquã ad sua capita, & fontes, ex quib. emanant, reducũtur.

Geometria apud Euclidem diuiditur in planorum (superficierum planarum.) contemplationem, seu Geometriam propriè dictam; quæ libris sex primis absoluitur; Et in solidorum (corporum solidorum:) speculationem, seu Stereomatriam; quæ libris postremis pertractatur. Prior quidem pars, nempe Geometria, in tres partes subdiuiditur. Nam in prioribus quatuor libris agitur de planis absolutè, ita vt eorum æqualitas, & inæqualitas inueſtigetur. In quinto verò libro de rationalibus magnitudinum proportionibus in genere diſputatur. In ſexto denique libro proportiones figurarum planarum diſcutiuntur. Poſterior verò pars, nempe Etereometrica, in tres quoque partes ſubdiuitur. In quarum prima, videlicet libris tribus, ſeptimo, octauo, & nono, agitur de numerorum proprietatibus, paſſionibuſq́ue ad linearum (& aliarum magnitudinum:) ſymetrarum ſeu commenſurabilium, & aſymetrarum, ſeu incommenſurabilium tractationem neceſſarijs. In ſecunda, nimirum libro decimo, ſatis prolixo, de lineis commenſurabilibus, & incommenſurabilibus, ſine quarum notitia, corpora illa quinque ſolida, regularia, ſeu platonica perfectè tractari nequeunt, diſſeritur. In tertia denique, nempe libris ſex poſtremis (qui & ipſi ſubdiuidi poterũt) de ſolidis illis acutiſſimè diſputatur, eorumque proprietates inueſtigantur. De punctis autem, & lineis in hoc opere nulla ex profeſſo extat contemplatio: quoniam Geometria potiſſimum circa figuras, quibus plana, & ſolida duntaxat (non puncta, & lineæ:) afficiuntur, verſatur.

Demonſtratio omnis Mathematicorum ab antiquis ſcriptoribus diuiditur in Problema, & Theorema. Problema quidem vocatur ea demonstratio, quæ iubet atque docet aliquid constituere, facere, inuenire, describere, &c. vt super data linea recta terminata triangulum æquilaterum constituere. Theorema verò appellatur ea demonstratio, quæ solum aliquam proprietatem, seu passionem vnius, vel plurium simul quantitatum (multarum vel magnarum iam inuentarum:) perscrutatur, vt in omni triangulo tres angulos esse æquales duobus rectis. Cæterùm tàm problema, quàm Theorema dicitur apud Mathematicos propositio; eò quod vtrumque aliquid nobis proponit, vti in exemplis adductis constat. Demonstrationes problematum concluduntur his ferè verbis; Quod faciendum erat: Theorematum verò Demonstrationes, his verbis; Quod ostendendum, vel demonstrandum erat; habitâ nimirum ratione vtriusque. Adhæc problemata per modum Infinitum, sed Theoremata per modum finitum ferè proferuntur. Lemma (sumptio, vel assumptũ latinè) appellatur ea minus principalis, & aliqua tantum declaratione indigens, demonstratio, quæ ad demonstrationem alicuius problematis, vel Theorematis principalis assumitur, vt illa demonstratio expeditior, ac breuior fiat; vt videre licet lib. 6 propos. 22. & passim. Corollarium, seu porisma, denique quæ est, quod protenus ex facta demonstratione tanquam lucrum aliquod additum, seu cõsectarium accipitur; vt videre est lib. 2. propos. 4. & passim.

Cùm omnis autem doctrina, omnisque disciplina ex præexistente cognitione gignatur, atq; ex assumptis, & concessis quibusdam principijs suas demonstrat conclusiones: Nulla autem scientia, teste Aristotele, sua principia demonstret; habebunt & Mathematicæ disciplinæ sua principia, ex quibus positis, & concessis sua problemata, & Theoremata confirment. Horum autem tria solum genera apud Mathematicos reperiuntur. Quorum primum genus continet definitiones, quas nonnulli Hypotheses appellant, vt punctum est, cuius pars nulla est. Secundum genus complectitur petitiones, seu postulata, quæ per se adeò perspicua sunt, vt nulla confirmatione indigeant, sed auditoris duntaxat assensum exposcant, vt postuletur, vt à quouis puncto in quoduis punctum, rectam lineam ducere concedatur. Tertium denique genus comprehendit Axiomata, seu communes animi notiones, quæ non solum in scientia proposita sed etiam in alijs omnibus vsqueadeo euidentes sunt, vt ab eis nulla ratione dissentire queat is, qui ipsa vocabula rectè perceperit, vt: Quæ eidem æqualia, & inter se sunt æqualia.

Verum secundum nonnullos principium formale duplex existit, nempe indemonstrabile seu facile, vt definitio, postulatum, & axioma; demonstrabile, vt Problema, Theorema, & omnis propositio. Principium vero materiale est punctum, linea, &c. Vid. & P. Christophorus Clauius, nobilissimus Elementorum Euclidis Interpres.

Coloniæ Agrippinæ, anno 1607.
Iunij 8.

EVCLIDIS ELEMENTVM PRIMVM.

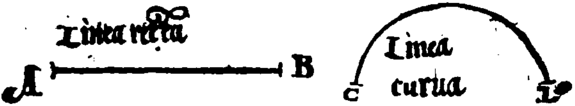
DEFINITIONES.

1.

Punctum eſt, cuius pars Punctum nulla eſt.

2.

Linea verò, longitudo latitudinis expers.



3.

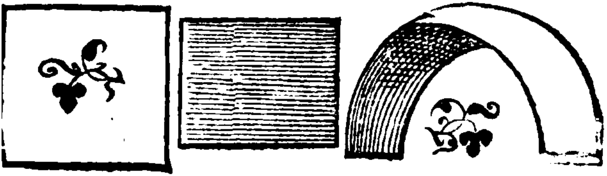
Lineæ autem termini, ſunt puncta.

4.

Recta linea eſt, quæ ex æquo ſua interiacet puncta.

5.

Superficies eſt, quæ longitudinem latitudinemq́ue tantùm habet



6.

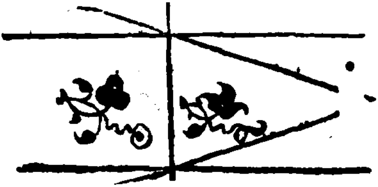
Superficiei extrema sunt lineæ.

7.

Plana superficies est, quæ ex æquo suas interiacet lineas.

8.

Planus angulus est duarum lineaũ in plano se mutuò tangentium, & non in directum



iacētium alterius ad alteram inclinatio.

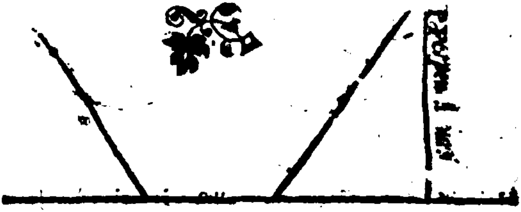
9.

Cùm autem quę angulum continent lineę, rectæ fuerint, rectilineus ille angulus appellatur.

10.

Cùm verò recta linea super rectam consistens lineam, eos, qui sunt deinceps, angulos æquales inter se fecerit; rectus est vterque æqua-

æqualium angulorum: quæ insistit recta linea; perpendicularis vocatur eius, cui insistit.



11.

Obtusus angulus est, qui recto maior est.

12.

Acutus verò, qui minor est recto.

13.

Terminus est, quod alicuius extremum est.

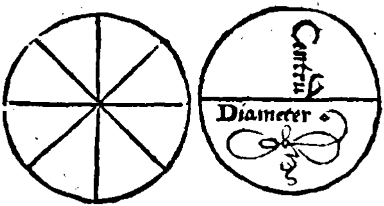
14.

Figura est, quæ sub aliquo, vt aliquibus terminis comprehenditur.

15.

Circulus est, figura plana sub vna linea cōprehensa, quę peripheria appellatur; ad quā ab vno puncto eorum, quæ intra figuram sunt

ſunt poſita, cadentes omnes rectæ lineæ inter ſe ſunt æquales.



16.

Hoc verò punctum, centrum circuli appellatur.

17.

Diameter autem circuli, eſt recta quædam linea per centrum ducta, & ex vtraque parte in circuli peripheriam terminata, quæ circulum bifariam ſecat.

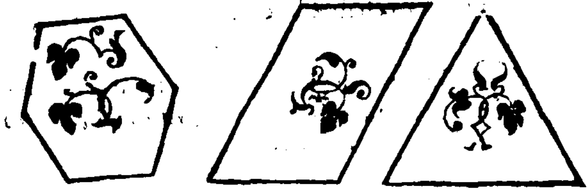
18.

Semicirculus verò eſt figura, quæ continetur ſub diametro, & ſub ea linea, quæ de circuli peripheria aufertur.

19.

Recti lineæ figuræ ſunt, quę ſub rectis lineis continentur.

20. Tri-



20. Trilateræ quidem, quæ ſub tribus.

21.

Quadrilateræ, quæ ſub quatuor.

22.

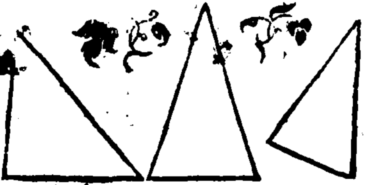
Multilateræ verò, quæ ſub pluribus, quàm quatuor, rectis lineis comprehenditur.

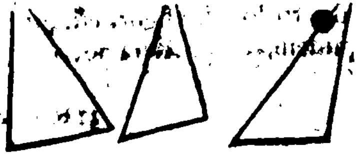
23.

Trilaterarum autem figurarum, æquilaterum eſt triangulum, quod tria latera habet æqualia.

24.

Iſoſceles autem eſt, quod duo tantùm æqualia habet latera.





25.

Scalenum verò eſt, quod tria inæ-

inæqualia habet latera.

26.

Ad hæc etiam, trilaterarum figurarum, re-ctangulum quidem triangulum eſt, quod rectum angulum habet.

27.

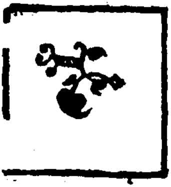
Amblygonium autem, quod obtuſum angulum habet.

28.

Oxygonium verò, quod tres habet acutos angulos.

29.

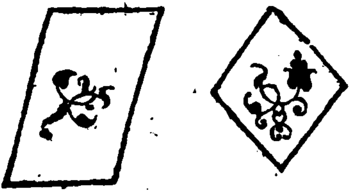
Quadrilaterarum autem figurarum, quadratum quidem eſt, quod & æquilaterum, & rectangulum eſt.



30.

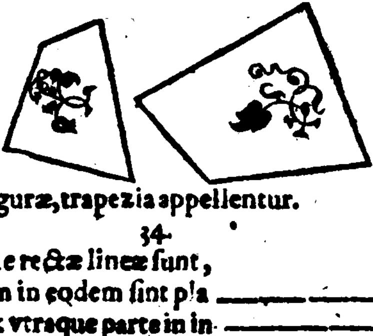
Altera verò parte longior figura eſt, quę rectangula quidem, at æquilatera non eſt.

31.
Rhóbus autem, quę ęquilatera, sed rectangula non est.



32.
Rhomboides verò, quæ aduersa & latera, & angulos habens inter se æquales, neque æquilatera est, neque rectangula.

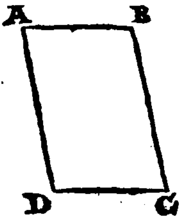
33.
Pręter hęs autē, reliquæ quadrilaterę figuræ, trapezia appellentur.



34.
Parallelę rectæ lineæ sunt, quæ cùm in eodem sint plano & ex vtraque parte in infinitum producantur, in neutram sibi mutuò incidunt.

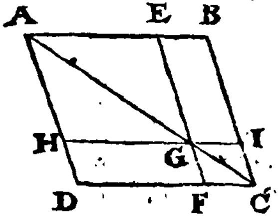
35.
Parallelo grammum est figura, quadrilatera, cuius bina apposita latera sunt parallela, seu æqui distantia.

 36. Cùm



36.

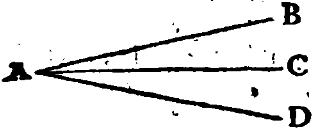
Cùm verò in parallelogrammo diameter ducta fuerit, duæq; lineæ lateribus parallelæ secantes diametrum in vno eodemque puncto, ita vt parallelogrammum ab hisce parallelis in quatuor distribuatur parallelogramma; appellantur duo illa, per quæ diameter non transit, complementa; duo verò reliqua, per quæ diameter incedit, circa diametrum consistere dicentur.



Petitiones siue Postulata.

1.

Postuletur, vt à quouis puncto in quoduis punctum,



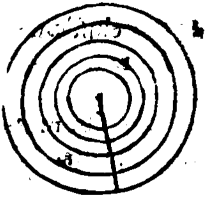
punctum, rectam lineam ducere cōcedatur.

2.

Et rectam lineam terminatam in continuū recta producere.

3.

Item quouis centro, & interuallo circulum describere.



4.

Item quacunque magnitudine data, sumi posse aliam magnitudinem vel maiorem, vel minorem.

Communes notiones siue axiomata.

1.

Quæ eidem æqualia, & inter se sunt æqualia.

2.

Et, si æqualibus æqualia adiecta sint, tota sunt æqualia.

3.

Et, si æqualibus æqualia ablata sint, quæ relinquuntur sunt æqualia.

4.

Et, si inæqualibus æqualia adiecta sint, tota sunt inæqualia.

5.

Et, si ab inæqualibus æqualia ablata sint, reliqua sunt inæqualia.

6.

Et, quæ eiusdem duplicia sunt, inter se sunt æqualia.

7.

Et quæ eiusdem sunt dimidia, inter se æqualia sunt.

8.

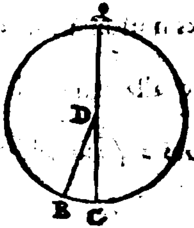
Et quæ sibi mutuò congruunt, ea inter se sunt æqualia.

9.

Et totum sua parte maius est.

10.

Duæ lineæ rectæ non habent vnum, & idem segmentum commune.



11.

Duæ lineæ rectæ in vno puncto concurrentes,

rentes, si producantur ambæ, necessariò se mutuò in eo puncto intersecabunt.

12.

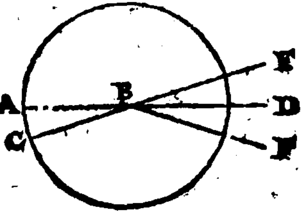
Item, omnes recti anguli sunt inter se æquales.

13.

Et, si in duas rectas lineas altera recta incidens, internos, ad easdemque partes angulos, duobus rectis minores faciat; duæ illæ rectæ lineæ in infinitum productę sibi mutuò incident ad eas partes, vbi sunt anguli duobus rectis minores.

14.

Duæ rectæ lineæ spatium non comprehendunt.



15.

Si æqualibus inæqualia adiiciantur, erit totorum excessus, adiectorũ excessui æqualis.

16.

Si inæqualibus æqualia adiungantur, erit totorum excessus, excessui eorum, quæ à principio erant, æqualis.

17.

Si ab æqualibus inæqualia demantur, erit reſiduorum exceſſus, exceſſui ablatorum æqualis.

18.

Si ab inæqualibus æqualia demantur, erit reſiduorum exceſſus, exceſſui totorum æqualis.

19.

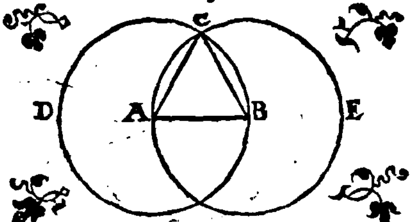
Omne totum æquale eſt omnibus ſuis partibus ſimul ſumptis.

20.

Si totum totius eſt duplum, & ablatum ablati; erit & reliquum reliqui duplum.

Problema 1. Propoſitio 1.

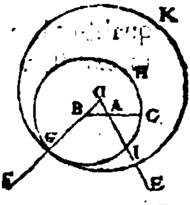
Super data recta linea termi nata, trian-



gulum æquilaterum conſtituere.

Problema 2. Propoſitio 2.

Ad datum punctum, datæ rectæ lineæ, æqualem rectam lineam ponere.



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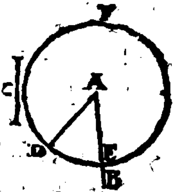
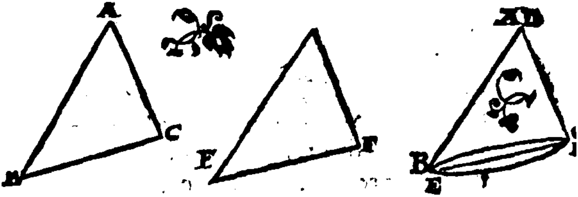
Problema 3. Propositio 3.



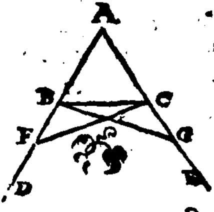
Duabus datis rectis lineis inæqualibus, de maiore æqualem minori rectam lineam detrahere.

Theorema 1. Propositio 4.

Si duo triangula duo latera duobus lateribus æqualia habeant, vtrunq; vtrique; habeant verò & angulum angulo æqualem sub æqualibus rectis lineis contentum: & basin basi æqualem habebunt; eritq; triangulum triangulo æquale, ac reliqui anguli reliquis angulis æquales erunt, vterque vtrique, sub quibus æqualia latera subtenduntur.



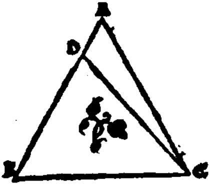
Theorema 2. Propositio 5.



Isosceleum triangulorum, qui ab basim sunt anguli, inter se sunt æquales; & si vlterius productæ sint æquales illæ rectæ lineæ, qui ſub baſi ſunt anguli, inter ſe æquales erunt.

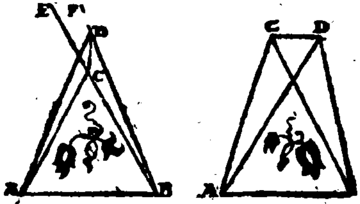
Problema 3. Propoſitio 6.

Si trianguli duo anguli æquales inter ſe fuerint; & ſub æqualibus angulis ſubtenſa latera æqualia inter ſe erunt.



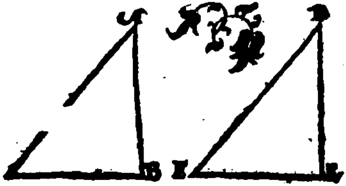
Theorema 4. Propoſitio 7.

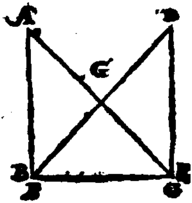
Super eadem recta linea, duabus eiſdem rectis lineis aliæ duæ rectæ lineæ æquales, vtraque vtrique, non conſtituentur, ad aliud atque aliud punctū, ad eaſdem partes, eoſdemque terminos cum duabus initio ductis rectis lineis habentes.



Theorema 5. Propoſitio. 8.

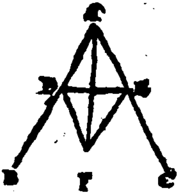
Si duo triangula duo latera habuerint duobus lateribus, vtrunque vtrique, æqualia: habuerint verò & baſim baſi æqualem: angulum quoque ſub æqualibus rectis lineis contentum angulo æqualem habebunt.





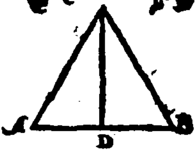
Problema 4. Propositio 9.

Datum angulum rectilineum bifariam secare.



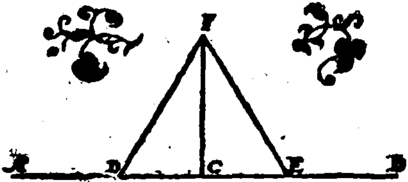
Problema 5. Propositio 10.

Datam rectam lineam finitam bifariam secare.



Problema 6. Propositio 11.

Data recta linea, à puncto in ea dato, rectam lineam ad angulos rectos excitare.



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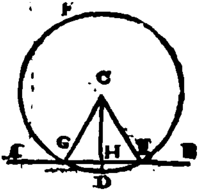
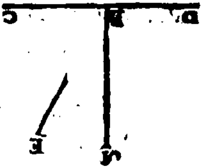
Problema 7. Propoſitio 12.

Super datam rectam lineam infinitam, à dato puncto, quod in ea non eſt, perpendicularem rectam deducere.



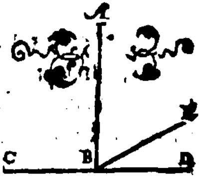
Theorema 6. Propoſitio 13.

Cùm recta linea ſuper rectam cõſiſtens lineam angulos facit, aut duos rectos, aut duobus rectis æquales efficiet.



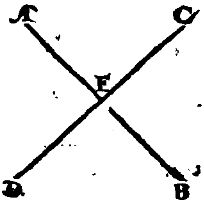
Theorema 7. Propoſitio 14.

Si ad aliquam rectam lineam, atq; ad eius punctum, duæ rectæ lineæ non ad eaſdem partes ductæ, eos, qui ſunt deinceps, angulos duobus rectis æquales fecerint, indirectũ erunt inter ſe ipſe rectæ lineæ.



Theorema 8. Propoſitio 15.

Si duæ rectæ lineæ ſe mutuò ſecuerint, angulos, qui ad verticem ſunt, æquales inter ſe efficient.



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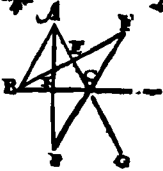
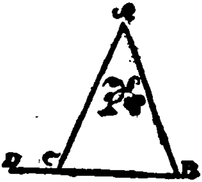
Theorema 9. Propositio 16.



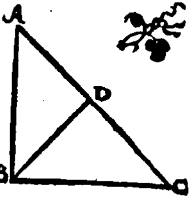
Cuiuſcunque trianguli vno latere producto, externus angulus vtroque interno & oppoſito maior eſt.

Theorema 10. Propositio 17.



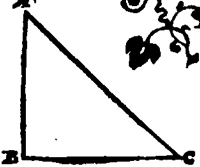
Cuiuſcunque trianguli duo anguli duobus rectis ſunt minores, omnifariam ſumpti.

Theorema 11. Propoſitio.



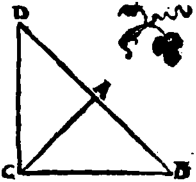
Omnis trianguli maius latus maiorem angulũ ſubtendit.

Theorema 12. Propoſitio 19.



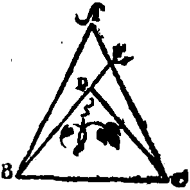
Omnis trianguli maior angulus maiori lateri ſubtenditur.

Theorema 13. Propositio 20.



Omnis trianguli duo latera reliquo sunt maiora, quomodocunque assumpta.

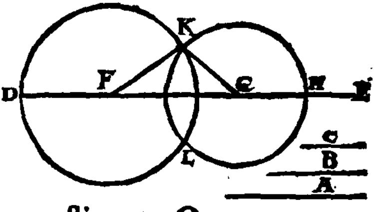
Theorema 14. Propositio 21.



Si super trianguli vno latere, ab extremitatibus duæ rectæ lineæ, interius constitutæ fuerint; hæ constitutæ reliquis trianguli duobus lateribus minores quidem erunt; minorem verò angulũ continebunt.

Problema 8. Propositio 22.

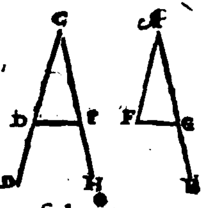
Ex tribus rectis lineis, quæ sunt trib. datis rectis lineis æquales,



triangulum constituere. Ooportet autem duas reliqua esse maiores omnifariam sumptas: quoniam vnuscuiusque trianguli duo latera omnifariam sumpta, reliquo sunt maiora.

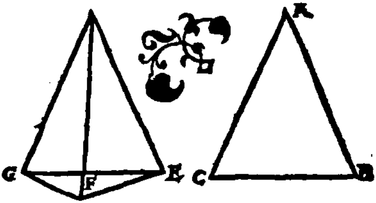
Problema 9. Propositio 23.

Ad datam rectam lineã, datumq; in ea punctum, dato angulo rectilineo æqualem angulum rectilineum constituere.



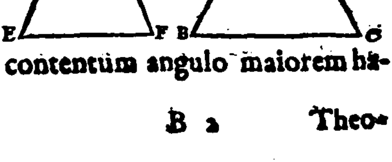
Theorema 15. Propositio 24.

Si duo triangula duo latera duobus lateribus æqualia habuerint, vtrũq; vtrique, angulum verò angulo maiorẽ sub æqualib. rectislineis cõtentum: & basin basi maiorẽ habebunt.



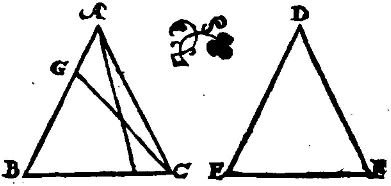
Theorema 16. Propositio. 25.

Si duo triangula duo latera duobus lateribus æqualia habuerint, vtrunque vtrique: basin verò basi maiorẽ; & angulum sub æqualibus rectislineis contentum angulo maiorem habebunt.



Theorema 17. Propositio 26.

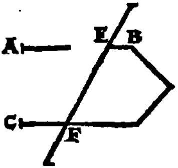
Si duo triangula duos angulos duobus angulis æquales habuerint, vtrunque vtrique; vnumque latus vni lateri æquale, siue quod æqualibus adiacet angulis, seu quod vni æqualium angulorum subtenditur: & reliqua latera reliquis lateribus æqualia vtrunq; vtriq;



& reliquum angulum reliquo angulo æqualem habebunt.

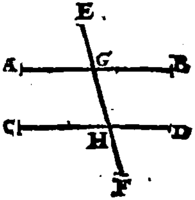
Theorema 18. Propositio 27.

Si in duas rectas lineas recta incidens linea alternatim angulos æquales inter se fecerit: parallelæ erunt inter se illæ rectæ lineæ



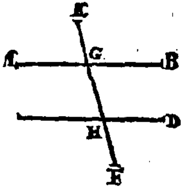
Theorema 19. Propositio 28.

Si in duas rectas lineas recta incidens linea, externum angulum interno, & opposito, & ad easdem partes, æqualem fecerit,



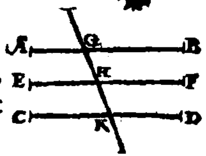
cerit, aut internos, & ad easdem partes duobus rectis æquales: parallelæ erunt inter se ipsæ rectæ lineæ.

Theorema 20. Propositio 29.



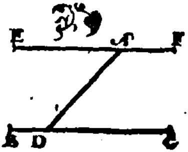
In parallelas rectas lineas recta incidens linea: & alternatim angulos inter se æquales efficit, & externum interno & opposito, & ad easdem partes æqualem, & internos & ad easdem partes duobus rectis æquales facit.

Theorema 21. Propositio 30.



Quæ eidem rectæ lineæ, parallelæ, & inter se sunt parallelæ.

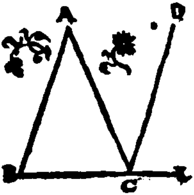
Problema 10. Propositio 31.



A dato puncto datæ rectæ lineæ parallelam rectam lineam ducere.

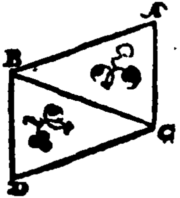
Theorema 22. Propositio 31.

Cuiuscunque trianguli vno latere alterius producto: externus angelus duobus internis & oppositis est æqualis. Et trianguli tres interni anguli duobus sunt rectis æquales.



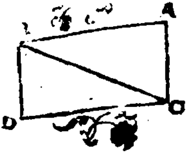
Theorema 23 Propositio 33.

Rectæ lineæ quæ æquales & parallelas lineas ad partes easdem coniungunt, & ipsæ æquales & parallelæ sunt.



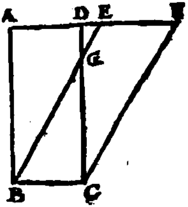
Theorema 24. Propositio 34.

Parallelogrammorum spatiorum æqualia sunt inter se, quæ ex aduerso, & latera, & anguli: atque illa bifariam secat diameter.



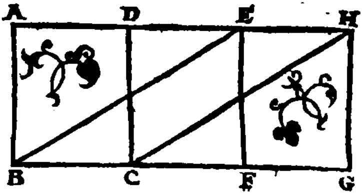
Theorema 25. Propositio 35.

Parallelogramma super eadem basi, & in eisdem parallelis constituta inter se sunt æqualia.



Theorema 26. Propoſitio 36.

Parallelogramma ſuper æqualibus baſibus, in eiſdē parallelis cōſtituta inter ſe ſunt æqualia.

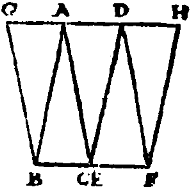


Theorema 27. Propoſitio 37.

Triangula ſuper eadem baſi conſtituta, & in eiſdem parallelis, inter ſe ſunt æqualia.

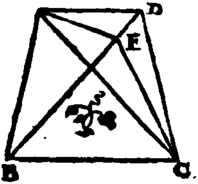
Theorema 28. Propoſitio 38.

Triangula ſuper æqualibus baſibus conſtituta, & in eiſdem parallelis, inter ſe ſunt æqualia.

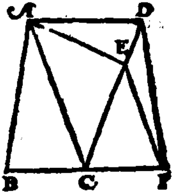


Theorema 29. Propoſitio 39.

Triangula ęqualia ſuper eadem baſi, & ad eaſdem partes conſtituta: & in eiſdem ſunt parallelis.



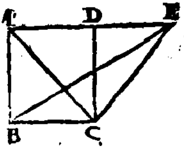
Theorema 30. Propo-
poſitio 40.



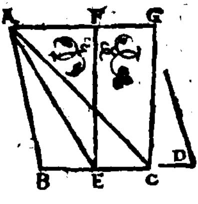
Triangula æqualia ſuper æqualibus baſibus, & ad eaſdem partes conſtituta, & in eiſdem ſunt parallelis.

Theorema 31. Propo-
ſitio 41.

Si parallelogrammum cum triangulo eandem baſin habuerit, in eiſdemque fuerit parallelis, duplum erit parallelogrammum ipſius trianguli.



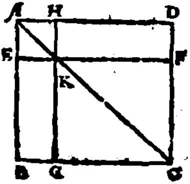
Problema 11. Pro-
ſitio 42.



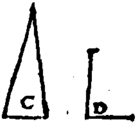
Dato triangulo æquale parallelogrammum cõſtituere in dato angulo rectilineo.

Theorema 32. Propo-
ſitio 34.

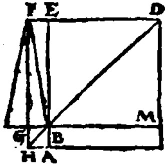
In omni parallelo grammo, complementa eorum, eõrum, quæ circa diametrum sunt, parallelogrã-morum, inter se sunt æqualia.



Problema 12. Propositio 44.

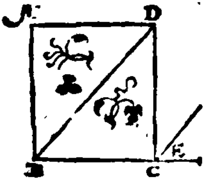


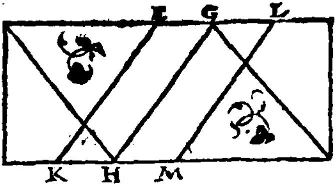
Ad datam rectam lineam, dato, triangulo, æquale parallelogrammum applicare, in dato angulo rectilineo.



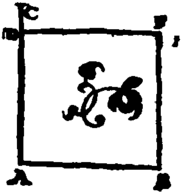
Problema 13. Propositio 45.

Dato rectilineo, æquale parallelogrammũ constituere in dato angulo rectilineo.





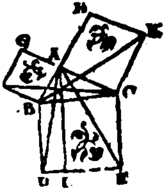
Problema 14. Propoſitio 46.



A data recta linea quadratum deſcribere.

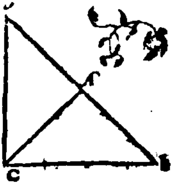
Theorema 33. Propoſitio 47.

In rectangulis triangulis, quadratum, quod à latere rectum angulum ſubtendente deſcribitur, æquale eſt eis, quæ à lateribus rectum angulum continentibus deſcribuntur, quadratis.



Theorema 34. Propoſitio 48.

Si quadratum, quod ab vno laterum trianguli deſcribitur, æquale ſit eis, qui à reliquis trianguli lateribus deſcribuntur, quadratis; angulus comprehenſus ſub reliquis duobus trianguli lateribus, rectus eſt.



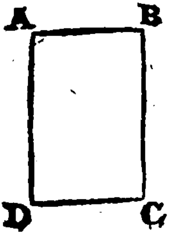
FINIS ELEMENTI I.

EVCLIDIS ELEMENTVM SECVNDVM.

DEFINITIONES.

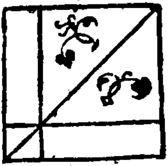
1.

Omne parallelogrammum rectangulum cõtineri dicitur ſub rectis duabus lineis, quæ rectum comprehendunt angulum.



2.

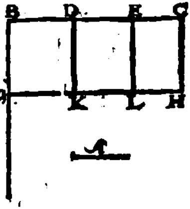
In omni parallelogrammo ſpatio, vnum quodlibet eorum, quæ circa diametrum illius ſunt, parallelogrammorum, cum duobus complementis, Gnomon vocetur.



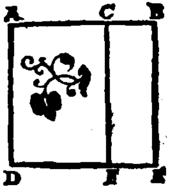
Theorema 1. Propoſitio 1.

Si fuerint duæ rectæ lineæ, ſeceturq; ipſarũ altera in quotcunque ſegmenta rectangulũ com-

comprehenſum ſub illis duabus rectis lineis, æquale eſt eis quæ ſub inſecta & quolibet ſegmẽtorum comprehenduntur, rectangulis.



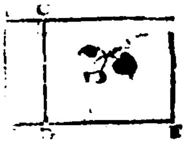
Theorema 2. Propoſitio 2.



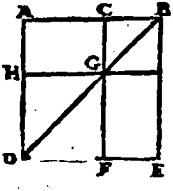
Si recta linea ſecta ſit vtcunque: rectangula, quæ ſub tota, & quolibet ſegmentorum compræhenduntur, æqualia ſunt ei, quod à toto fit, quadrato.

Theorema 3. Propoſitio. 3.

Si recta linea ſecta ſit vtcunque rectangulum ſub tota, & vno ſegmentorum comprę henſum, æquale eſt illi, quod ſub ſegmentis comprehenditur rectangulo, & illi, quod à prædicto ſegmẽto deſcribitur, quadrato.



Theorema 4. Propoſitio 4.



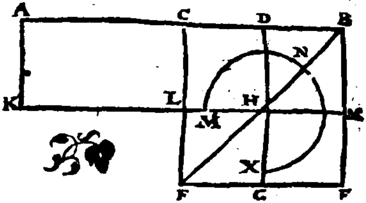
Si recta linea ſecta ſit vtcunque quadratum, quod à tota deſcribitur, æquale eſt & illis, quæ à ſegmen-

tis

tis describuntur, quadratis; & ei quod bis sub segmentis comprehenditur, rectangulo.

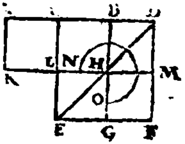
Theorema 5. Propositio 5.

Si recta linea secetur in æqualia, & non æqualia: rectangulum sub inæqualibus segmentis totius comprehensum, vnà cũ quadrato, quod ab intermedia sectionum, æquale est ei, quod à dimidia describitur, quadrato.



Theorema 6. Propositio 6.

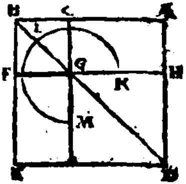
Si recta linea bifariam secetur, & illi recta quædam linea in rectum adijciatur; rectangulum comprehensum sub tota cum adiecta, & adiecta, vnà cum quadrato à dimidia, æquale est quadrato à linea, quæ tum ex dimidia, tum ex adiecta componitur, tanquam ab vna, descripto.



Theorema 7. Propositio 7.

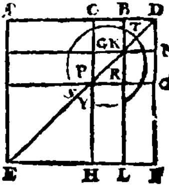
Si recta linea secetur vtcunque; quod à tota, quod-

quodq́ue ab vno segmentorum, vtraq; simul quadrata, æqualia sunt & illi, quod bis sub tota, & dicto segmento comprehenditur, rectãgulo; & illi quod à reliquo segmento fit, quadrato.



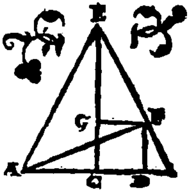
Theorema 8. Propositio 8.

Si recta linea secetur vtcunque; rectangulũ quater comprehensum sub tota, & vno segmentorum, cum eo, quod à reliquo segmẽto fit, quadrato, ęquale est ei, quod à tota, & dicto segmẽto, tanquam ab vna linea describitur, quadrato.



Theorema 9. Propositio 9.

Si recta linea secetur in æqualia & non æqualia: quadrata, quæ ab inæqualibus totius segmentis fiunt, simul duplicia sunt, & eius, quod à dimidia, & eius, quod ab intermedia sectionum fit, quadratorum.

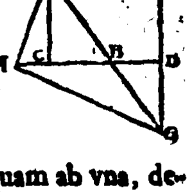


Theorema 10. Propositio 10.

Si recta linea secetur bifariã, adijciatur aũt

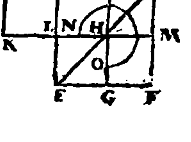
ei in

ei in rectũ quæpiam recta linea: quod à tota cum adiuncta, & quod ab adiuncta, vtraque simul quadrata; duplicia sunt, & eius, quod à dimidia, & eius, quod à composita ex dimidia & adiuncta, tanquam ab vna, descriptum sit, quadratorum.



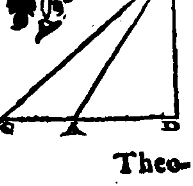
Problema I. Propositio II,

Datam rectam lineã secare, vt comprehensum sub tóta, & altero segmẽtorum rectangulum, æquale sit ei, quod à reliquo segmento fit, quadrato.



Theorema II. Propositiõ 12.

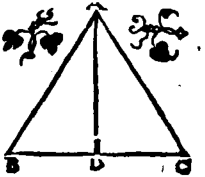
In amblygonijs triãgulis, quadratum, quod fit à latere angulum obtusum subtendente, maius est quadratis, quæ fiunt à lateribus obtusum angulum comprehendentibus; rectangulo bis comprehenso, & ab vno laterum, quæ sunt circa obtusum angulum, in quod cũ protractum fuerit, cadit perpendicularis, & ab assumpta exterius linea sub perpendiculari prope angulum obtusum.



Theo-

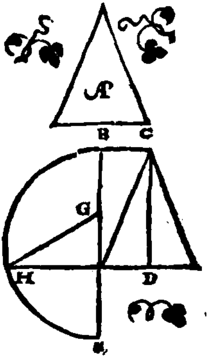
Theorema 12. Propositio 28.

In oxygonijs triangulis, quadratum à latere angulum acutum subtendente, minus est quadratis, quæ fiunt à lateribus acutum angulum comprehendentibus; rectangulo bis comprehenso; & vno laterum, quæ sunt circa acutum angulum, in quod perpédicularis cadit, & ab assumpta interius linea sub perpendiculari prope acutum angulum.



Problema 2. Propositio 14.

Dato rectilineo æquale quadratum constituere.



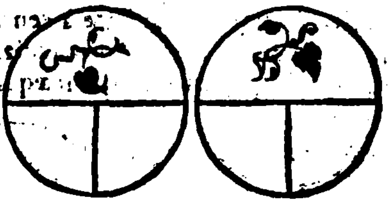
FINIS ELEMENTI II.

EVCLIDIS ELEMENTVM TERTIVM.

DEFINITIONES.

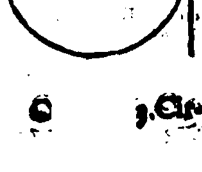
1.

Aequales circuli sunt, quorum diametri sunt aequales; vel quorum, quae ex centris, rectae lineae sunt aequales.



2.

Recta linea circulum tangere dicitur, quae cùm circulum tangat; si producatur, circulum non secat.

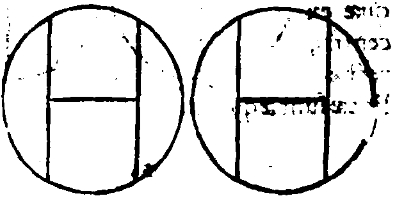


3.

Circuli ſeſe mutuò tangere dicuntur: qui ſeſe mutuò tangentes, ſeſe mutuò non ſecant.

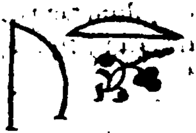
4.

In circulo æqualiter diſtare à centro rectæ lineæ dicuntur, cùm perpendiculares, quæ à centro in ipſis ducuntur, ſunt æquales. Longius autem abeſſe illa dicitur, in quã maior perpendicularis cadit.



5.

Segmentum circuli eſt figura, quæ ſub recta linea, & circuli peripheria comprehenditur.



6.

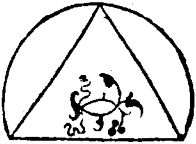
Segmenti autem angulus eſt, qui ſub recta

linea

linea, & circuli peripheria comprehenditur.

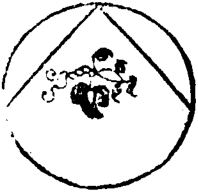
7.

In segmento autem angulus est, cùm in segmenti peripheria sumptum fuerit quodpiam punctum, & ab illo in terminos rectæ eius lineæ quæ segmenti basis est, adiunctæ fuerint, rectæ lineæ: is, inquam, angulus ab adiunctis illis lineis comprehensus.



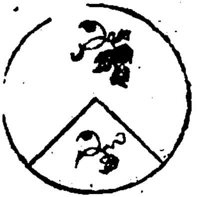
8.

Cùm verò comprehendentes angulum rectæ lineę aliquam assumant peripheriam, illi angulus insistere dicitur.



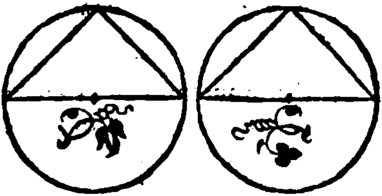
9.

Sector autem circuli est, cùm ad ipsius circuli cētrum constitutus fuerit angulus, comprehensa nimirum figura, & à rectis lineis angulum continentibus, & à peripheria ab illis assumpta.



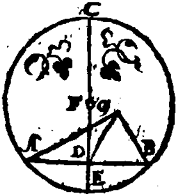
10.

Similia circuli segmenta sunt, quæ angulos capiunt æquales aut in quibus anguli inter se sunt æquales.



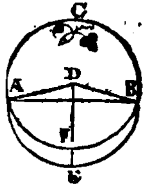
Problema 1. Propositio 1.

Dati circuli centrum reperire.



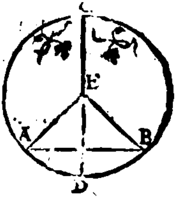
Theorema 1. Propositio 2.

Si in circuli peripheria duo quælibet puncta accepta fuerint; recta linea, quæ ad ipsa puncta adiungitur, intra circulum cadet.



Theorema 2. Propositio. 3.

Si in circulo recta quædam linea per centrum extensa quandam non per centrum extensam bifariam secet: & ad angulos rectos ipsam secabit: Et si ad angulos rectos eam secet, bifariam quoque eam secabit.



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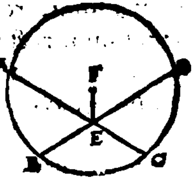
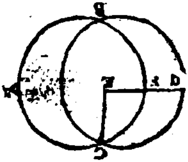
Theorema 3. Propositio 4.



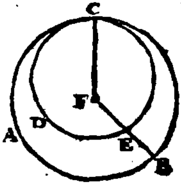
Si in circulo duæ rectæ lineæ sese mutuò secent, nó per centrum extensæ; sese mutuò bifariam non secabunt.

Theorema 4. Propositio 5.



Si duo circuli sese mutuò secent; non erit illorum idem centrum.

Theorema 5. Propositio 6.



Si duo circuli sese mutuò interius tangent, eorum non erit idem centrum.

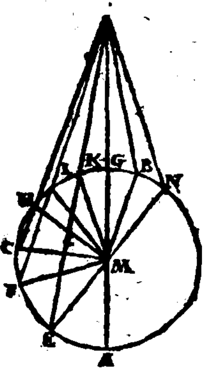
Theorema 6. Propositio 7.

Si in diametro circuli quodpiam sumatur punctum, quod circuli centrum non sit; ab eoq; puncto in circulum quædam rectæ lineæ cadant; maxima quidem erit ea in qua centrũ; minima verò reliqua; aliarum verò propinquior illi, quæ per centrum du-

citur, remotiore semper maior est. Duæ autem solùm rectæ lineæ æquales ab eodem puncto in circulum cadunt, ad vtrasq; partes minimæ, vel maximæ.

Theorema 7. Propositio 8.

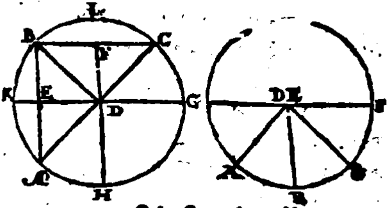
Si extra circulum sumatur punctum quodpiam, ab eoque puncto ad circulum deducantur rectæ quædam lineæ, quarum vna quidem per centrum protendatur, reliquæ verò vt libet: in cauam peripheriam cadentium rectarum linearum minima quidem est illa, quæ per centrum ducitur: aliarum autem propinquior ei, quæ per centrum transit, remotiore semper maior est: in conuexam verò peripheriã cadentium rectarum linearũ minima quidem est illa, quæ inter punctum, & diametrum interponitur: aliarum autem, ea, quæ propinquior est minimæ, remotiore semper minor est. Duæ autem tantùm rectæ lineæ æquales ab eo puncto in ipsum circulum cadunt, ad vtrasque partes minimæ, vel maximæ.



Theo-

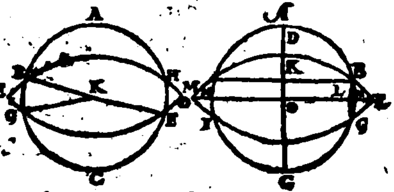
Theorema 8. Propoſitio 9.

Si in circulo acceptum fuerit punctum aliquod, & ab eo puncto ad circulum cadant plures, quam duæ, rectæ lineæ æquales; acceptũ punctum centrum eſt ipſius circuli.



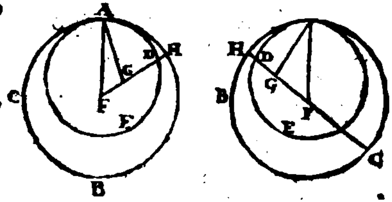
Theorema 9. Propoſitio 10.

Circulus circulum in pluribus, quàm duobꝰ punctis non ſecat.



Theorema 10. Propoſitio 11.

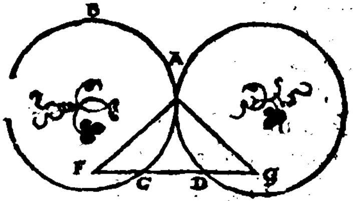
Si duo circuli ſeſe intus cõtingãt, atque accepta



ſuerint eorum centra; ad eorum centra adiuncta recta linea, & producta, in cōtactum circulorum cadet.

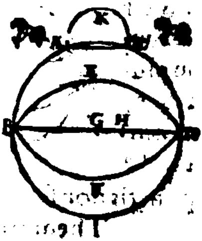
Theorema 11. Propoſitio 12.

Si duo circuli ſeſe exterius contingant, linea recta, quæ ad centra eorum adiungitur, per contactum illum traſibit.



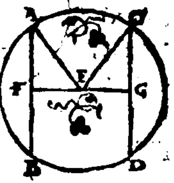
Theorema 12. Propoſitio 13.

Circulum circulum non tangit in pluribus punctis, quam vno, ſiue intus ſiue extra tangat.



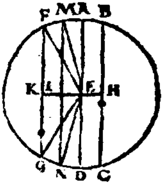
Theorema 13. Propoſitio 14.

In circulo æquales rectæ lineæ æqualiter diſtant à centro. Et quæ æqualiter diſtant à centro, æquales ſunt inter ſe.



Theo-

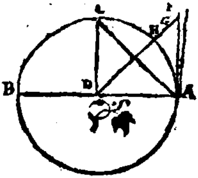
Theorema 14. Propositio 15.



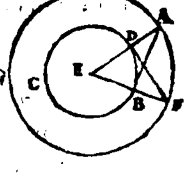
In circulo maxima quidem linea est diameter; aliarum autem propinquior centro, remotiore semper maior.

Theorema 15. Propositio 16.

Quæ ab extremitate diametri cuiusque circuli ad angulos rectos ducitur, extra ipsum circulum cadet; & in locum inter ipsam rectam lineam, & peripheriam comprehensum, altera recta linea non cadet. Et semicirculi quidem angulus quouis angulo acuto rectilineo maior est, reliquus autē minor.



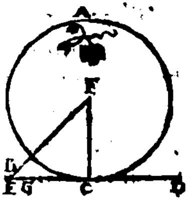
Problema 12. Propositio 17.



A dato puncto rectam lineam ducere, quę datum tangat circulum.

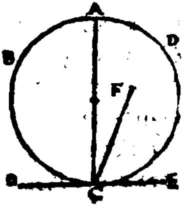
Theorema 16. Propositio 18.

Si circulum tangat recta quæpiam linea, à centro autem ad conta-ctum adiungatur recta quædam linea: quæ ad-iuncta fuerit, ad ipſam contingentem, perpen-dicularis erit.



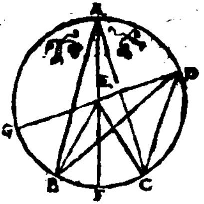
Theorema 17. Propo-ſitio 19.

Si circulum tetigerit re-cta quæpiam linea, à con-tactu autem recta linea ad angulos rectos ipſi tã-genti excitetur; in excita-ta erit centrum circuli.



Theorema 18. Pro-poſitio 20.

In circulo, angulus ad centrum duplex eſt an-guli ad peripheriam, cũ fuerit eadem peripheria baſis angulorum.



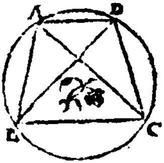
Theorema 19. Pro-poſitio 21.

In circulo, qui in eodem ſegmẽto ſunt, anguli, ſunt inter ſe æquales.

Theo-

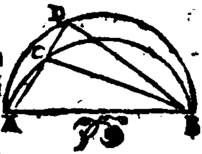
Theorema 20. Propoſitio 22.

Quadrilaterorum in circulis deſcriptorum anguli, qui ex aduerſo, duobꝰ rectis ſunt æquales.

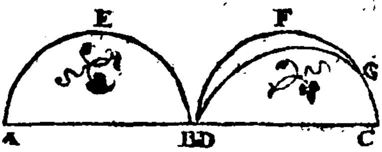


Theorema 21. Propoſitio 23.

Super eadem recta linea duo ſegmenta circulorũ ſimilia, & inæqualia non conſtituentur ad eaſdem partes.



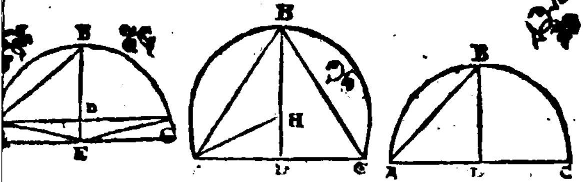
Super æqualibus rectis lineis, ſimilia circulorum ſegmenta, ſunt inter ſe æqualia.



Problema 3. Propoſitio 25.

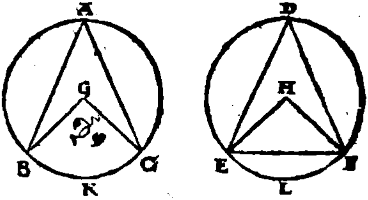
Circuli ſegmento dato, deſcribere circulũ, cuius

cuius eſt ſegmentum.



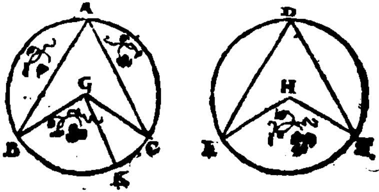
Theorema 23. Propoſitio 26.

In æqualibus circulis, æquales anguli æqualibus peripherijs inſiſtunt, ſiue ad cẽtra, ſiue ad peripherias conſtituti, inſiſtant.



Theorem 24. Propoſitio 27.

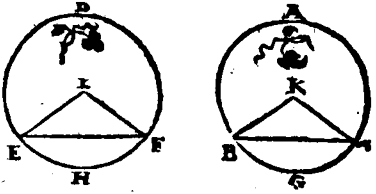
In æqualibus circulis, anguli, qui æqualibus peripherijs inſiſtunt, ſunt inter ſe æquales, ſiue ad centra, ſiue ad peripherias conſtituti, inſiſtant.



Theo-

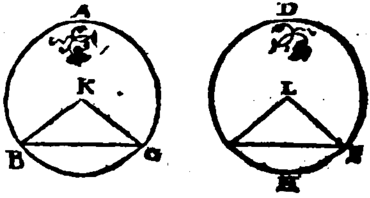
Theorēma 25. Propoſitio 28.

In æqualibus circulis, æquales rectæ lineæ, æquales peripherias auferunt; maiorē quidē maiori, minorem autem minori.



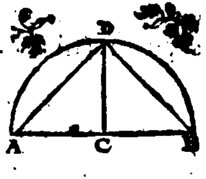
Theorema 26. Propoſitio 29.

In æqualibus circulis æquales peripherias, æquales rectæ lineæ ſubtendunt.



Problema 4. Propoſitio 30.

Datam peripheriam bifariam ſecare.

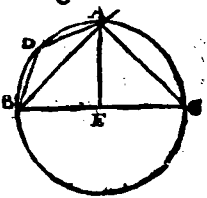


Theorema 27. Propoſitio 31.

In circulo angulus, qui in ſemicirculo, re-

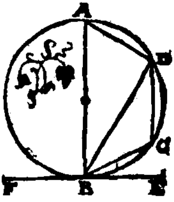
ctus

ctus est: qui autem in maiore segmento, minor recto: qui verò in minore segmento, maior est recto. Et insuper angulus maioris segmenti, recto quidem maior est, minoris autem segmenti angulus, minor est recto.



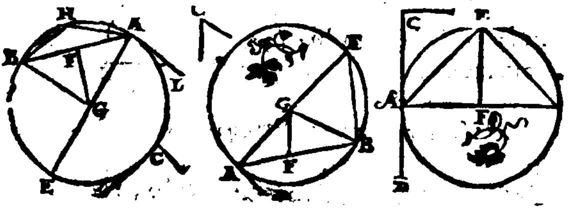
Theorema 28. Propositio 32.

Si circulum tetigerit aliqua recta linea, à contactu autē producatur quædam recta linea circulum secans: anguli, quos ad contingentem facit, ęquales sunt ijs, qui in alternis circuli segmētis consistunt, angulis.



Problema 5. Propositio 33.

Super data recta linea describere segmentū circuli, quod capiat angulum æqualem dato angulo rectilineo.

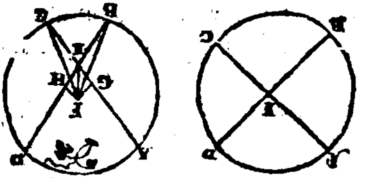


Problema 6. Propositio 34.

A dato circulo segmentum abscindere, capiens angulum æqualem dato angulo rectilineo.

Theorema 29. Propositio 35.

Si in circulo duæ rectæ lineæ sese mutuò secuerint; rectangulum comprehensum sub segmentis vnius, æquale est ei, quod sub segmentis alterius comprehenditur, rectangulo.



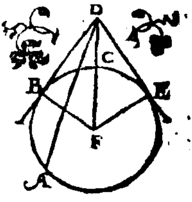
Theorema 30. Propositio 36.

Si extra circulũ sumatur pũctum aliquod, ab eoque in circulũ cadant duæ rectæ lineæ; quarum altera quidem circulũ secet, altera verò

verò tangat: quod sub tota secante, & exterius inter punctum & conuexam peripheriam assumpta comprehenditur, rectangulum; æquale erit ei, quod à tangente describitur, quadrato.

Theorema 31. Propositio 37.

Si extra circulum sumatur punctum aliquod ab eoque puncto in circulum cadant duæ rectæ lineæ, quarum altera circulum secet, altera in eum incidat; sit autem, quod sub tota secante, & exterius inter punctum, & conuexam peripheriam assumpta, comprehenditur, rectangulum, æquale ei, quod ab incidente describitur, quadrato; incidens ipsa circulum tanget.



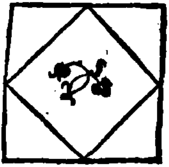
FINIS ELEMENTI III.

EVCLIDIS ELEMENTVM QVARTVM.

DEFINITIONES.

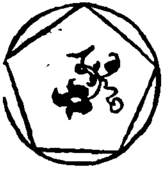
1.

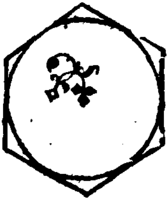
Figura rectilinea in figura rectilinea inſcribi dicitur, cum ſinguli eius figurę; quæ inſcribitur, anguli, ſingula latera eius, in qua inſcribitur, tangunt.



2.

Similiter & figura circum figuram deſcribi dicitur, quum ſingula eius, quæ circumſcribitur, latera ſingulos eius figuræ angulos tetigerint, circum quam illa deſcribitur.





3.

Figura rectilinea in circulo inſcribi dicitur, quum ſinguli eius figuræ, quæ inſcribitur, ſinguli tetigerint circuli peripheriam.

4.

Figura verò rectilinea circa circulum deſcribi dicitur, quùm ſingula latera eius, quæ circum ſcribitur, circuli peripheriam tangunt.

5.

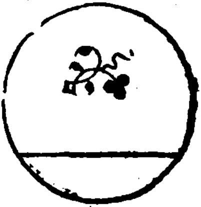
Similiter & circulus in figura rectilinea inſcribi dicitur, quũ circuli peripheria ſingula latera tangit eius figuræ, cui inſcribitur.

6.

Circulus autem circum figuræ deſcribi dicitur, quum circuli peripheria ſingulos tangit eius figuræ, quam circumſcribit, angulos.

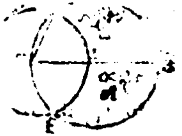
7.

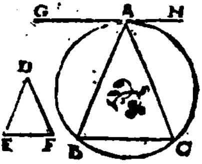
Recta linea in circulo accommodari, ſeu coaptari dicitur, quum eius extrema in circuli peripheria fuerint.



Problema 1. Propoſitio 1.

In dato circulo, rectam lineam accommodare æqualem datæ rectæ lineę, quæ circuli diametro nõ ſit maior.

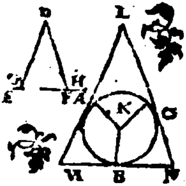




Problema 2. Propositio 2.

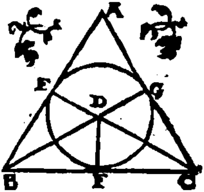
In dato circulo, triangulum describere, dato triangulo æquiangulum.

Problema 3. Propositio 3.



Circa datum circulum triangulum describere, dato triangulo æquiangulum.

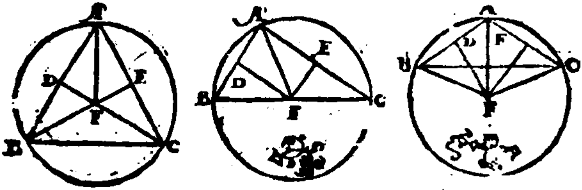
Problema 4 Propositio 4.



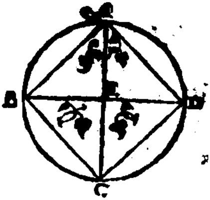
In dato triangulo circulum inscribere.

Problema 5. Propositio 5.

Circa datum triangulū, circulū describere.

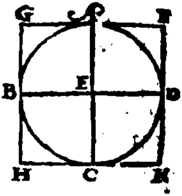


Problema 6. Propositio 6.



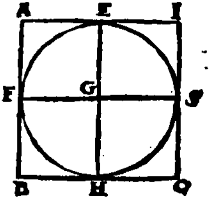
In dato circulo quadratum describere.

Problema 7. Propositio. 7.



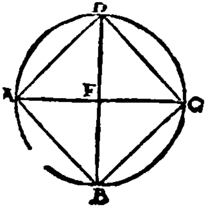
Circa datum circulum, quadratum describere.

Problema 8. Propositio 8.



In dato quadrato circulum inscribere.

Problema 9. Propositio 9.

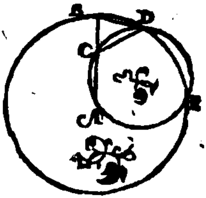


Circa datum quadratũ, circulum describere.

Pro-

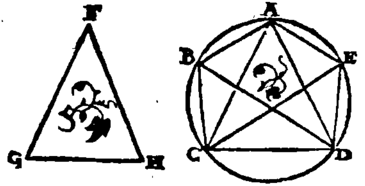
Problema 10. Propoſitio. 10.

Iſoſceles triangulum cõſtituere; quod habeat vtrunque eorum, qui ad baſin ſunt, angulorum, duplum reliqui.



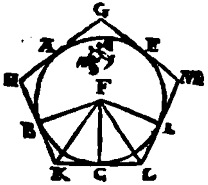
Problema 11. Propoſitio 11.

In dato circulo, pentagonũ æquilaterum, & æquiangulum inſcribere.



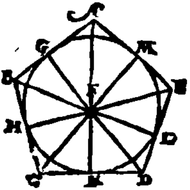
Problema 12. Propoſitio 12.

Circa datum circulum, pentagonum, æquilaterum & æquiangulum deſcribere.

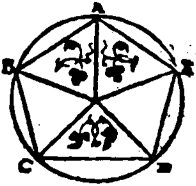


Problema 13. Propoſitio 13.

In dato pentagono æquilatero, & æquiangulo circulum inſcribere.



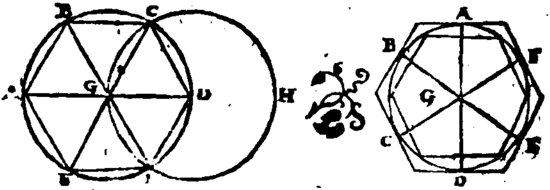
Problema 14. Propositio 14.



Circa datum pentagonũ æquilaterum & æquiangulum, circulum describere.

Problema 15. Propositio 15.

In dato circulo hexagonum & æquilaterũ, & æquiangulum inscribere.



Problema 16. Propositio 16.

In dato circulo quintidecagonum & æquilaterũ, & æquiangulum describere.

FINIS ELEMENTI IV.

EVCLIDIS ELEMENTVM QVINTVM.

DEFINITIONES.

1.

Pars est magnitudo magnitudinis minor maioris, quum minor metitur maiorem.

2.

Multiplex autem est maior minoris, cùm minor metitur maiorem.

3.

Proportio, est duarum magnitudinum eiusdem generis mutua quædam secundum quanti talem habitudo.

4.

Proportionalitas verò est proportionũ similitudo.

5.

Proportionem habere inter se magnitudines dicuntur, quæ possunt multiplicatæ sese mutuò superare.

6.

In eadẽ proportione magnitudines dicũtur esse, prima ad secundam, & tertia ad quartam, cùm primæ & tertiæ æquè multiplicia, à secundæ & quartæ æquè multiplicibus, qualiſcunque ſit hæc multiplicatio, vtrunque ab vtroque; vel vnà deficiunt, vel vnà æqualia ſunt, vel vnà excedunt; ſi ea ſumantur, quæ inter ſe reſpondent.

7.

Eandem autem proportionem habentes magnitudines, proportionales vocentur.

8.

Cùm verò æquè multiplicium, multiplex primæ magnitudinis excesserit multiplicé ſecundæ; at multiplex tertiæ non excesserit multiplicem quartæ: tunc prima ad ſecundam, maiorem proportionem habere dicetur, quàm tertia ad quartam.

9.

Proportinoalitas autem in tribus terminis paucißimis conſiſtit.

10.

Cùm autem tres magnitudines proportionales, fuerint, prima ad tertiam, duplicatam proportionem habere dicitur eius, quam habet ad ſecundam. At cùm quatuor magnitudines proportionales fuerint, prima ad quartam, triplicatam proportionem habere dicitur eius, quam habet ad ſecundam: & ſemper deinceps, vno amplius, quamdiu proportio extiterit.

11.

Homologæ, ſeu ſimiles proportione magnitudines dicuntur, antecedentes quidem ante-

antecedentibus, consequentes verò consequentibus.

12.

Alterna proportio, est sumptio antecedentis ad antecedentem, & consequentis ad consequenrem.

13.

Inuersa proportio, est sumptio consequentis, ceu antecedentis, ad antecedentem, velut ad ipsam consequentem.

14.

Compositio proportionis, est sumptio antecedentis cum consequente, ceu vnius ad ipsam consequentem.

15.

Diuisio proportionis, est sumptio excessus, quo consequentem superat antecedens, ad ipsam consequentem.

16.

Conuersio proportionis, est sumptio antecedentis ad excessum, quo superat antecedens ipsam consequentem.

17.

Ex æqualitate proportio est, si plures duabus sint magnitudines, & his aliæ multitudine pares, quæ binæ sumantur, & in eadem proportione: quum vt in primis magnitudinibus prima ad vltimam, sic & in secundis magnitudinibus prima ad vltimam sese habuerit.

Vel aliter, sumptio extremorum per subductionem mediorum.

18.

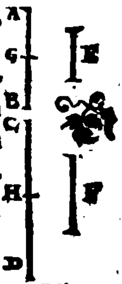
Ordinata proportio est, cùm fuerit, quemadmodum antecedens ad consequentem, ita antecedens ad consequentem: fuerit etiam vt consequens ad aliud quidpiam, ita consequens ad aliud quidpiam.

19.

Perturbata autem proportio est, cùm trib' positis magnitudinibus, & alijs quæ sint his multitudine pares, vt in primis quidem magnitudinibus se habet antecedens ad consequentem, ita in secundis magnitudinibus antecedens ad consequentem: vt autem aliud quidpiam sic in secundis magnitudinibus aliud quidpiam ad antecedentem.

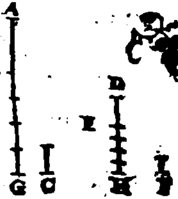
Theorema 1. Propositio 1.

Si sint quotcunq; magnitudines, quotcunque magnitudinum æqualium numero, singulæ singularum, æquè multiplices; quàm multiplex est vnius vna magnitudo, tam multipleces erunt & omnes omnium.



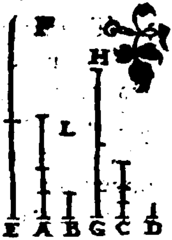
Theorema 2. Propopositio 2.

Si prima secundæ æquè fuerit multiplex, atque tertia quartæ; fuerit autem & quinta secundæ æquè multiplex, atque sexta quartæ: erit & composita prima cũ quinta, secundæ æquè multiplex; atq; tertia cum sexta, quartæ.



Theorema 3. Propositio 3.

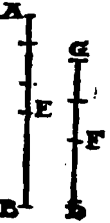
Si sit prima secundæ æquè multiplex, atque tertia quartæ; sumantur autem æquè multiplices primæ, & tertiæ; erit & ex æquo, sumptarum vtraque vtriusque æquè multiplex; altera quidem secundæ, altera autem quartæ.



Theorema 4. Propositio 4.

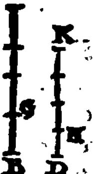
Si prima ad secundã, eandem habuerit proportionẽ, & tertia ad quartam; etiam æquè multiplices primę & tertię, ad ęquè multiplices secundæ & quartæ, iuxta quamuis multiplicationẽ, eandem

dem habebunt proportionem; ſi, prout inter ſe reſpondent, ita ſumptæ fuerint.



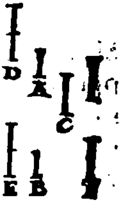
Theorema 5. Propoſitio 5.

Si magnitudo magnitudinis æquè fuerit multiplex, atque ablata ablatæ: etiam reliqua reliquæ ita multiplex erit, vt tota totius.



Theorema 6. Propoſitio 6.

Si duæ magnitudines, duarum magnitudinum ſint æquè multiplices, & detractæ quædam ſint earundem æquè multiplices: & reliquæ eiſdem aut æquales ſunt, aut æquè ipſarum multiplices.



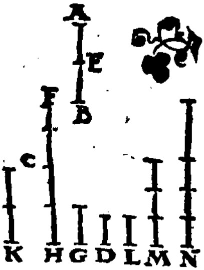
Theorema 7. Propoſitio 7.

Aequales ad eandem, eandem habent rationem: & eadem ad æquales.

Theorema 8. Propoſitio 8.

Inæqualium magnitudinum maior ad eandem

dem, maiorem proportionem habet: quàm minor: & eadem ad minorem, maiorem proportionem habet, quàm ad maiorem.



Theorema 9. Propositio 9.

Quæ ad eandem, eandem habent proportionem, æquales sunt inter se: & ad quas eadem, eandem habet proportionem, eæ quoq; sunt inter se æquales.

Theorema 10. Propositio, 10.

Ad eandem magnitudinem proportionem habentium, quę maiorem proportionem habet, illa maior est, ad quam autem eadem maiorem proportionem habet, illa minor est.

Theorema 11. Propositio 11.

Quæ eidem sunt eædem proportiones, & inter se sunt eædem.

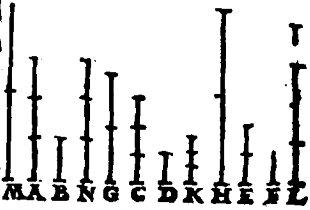
Theo-

Theorema 12. Propositio. 12.

Si sint magnitudines quotcunque proportionales; quemadmodũ se habuerit vna antecedentium ad vnã consequentium; ita se habebunt omnes antecedentes ad omnes consequentes.

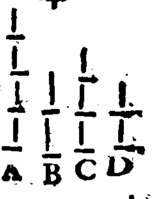
Theorema 13. Propositio 13.

Si prima ad secundã, eandem habuerit proportionẽ, quam tertia ad quartã; tertia verò ad quartam maiorẽ proportionẽ habuerit, quàm quinta ad sextam: prima quoq; ad secundã maiorẽ proportionẽ habebit, quàm quinta ad sextam.



Theorema 14. Propositio 14.

Si prima ad secundam eandem habuerit proportionẽ, quã tertia ad quartã: prima verò quàm tertia maior fuerit, erit & secunda maior, quàm quarta. Quòd si prima fuerit æqualis tertiæ,

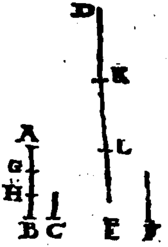


erit

erit ſecunda æqualis quartæ: ſi verò minor, & minor erit.

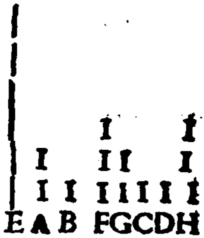
Theorema 15. Propoſitio 15.

Partes cum pariter multiplicibus in eadem ſunt proportione, ſi prout ſibi mutuò reſpondent, ita ſumantur.



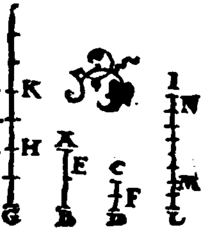
Theorema 16. Propoſitio 16.

Si quatuor magnitudines proportionales fuerint, & viciſsim proportionales erunt.



Theorema 17. Propoſitio 17.

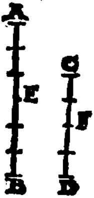
Si compoſitæ magnitudines proportionales fuerint, hæ quoq; diuiſæ proportionales erunt.



Theorema 18. Propositio 18.

Si diuisæ magnitudines sint proportionales, hæ quoque compositæ proportionales erunt.

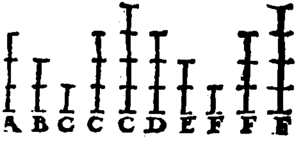
Theorema 19. Propositio 19.



Si quemadmodum totum ad totum, ita ablatum se habuerit ad ablatum: & reliquum ad reliquum, vt totum ad totum, se habebit.

Theorema 20. Propositio 20.

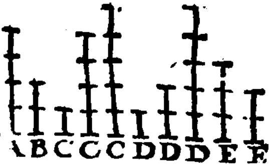
Si sint tres magnitudines, & aliæ ipsis æquales numero, quæ binæ, & in eadem proportione sumantur; ex æquo autem prima quàm tertia maior fuerit; erit & quarta, quam sexta, maior. Quod si prima tertiæ fuerit æqualis, erit & quarta æqualis sextæ: sin illa minor, quoque minor erit.



Theo-

Theorema 21. Propoſitio 21.

Si ſint tres magnitudines, & aliæ ipſis æquales numero, quæ binæ, & in eadem proportione ſumantur, fueritq́ue perturbata earum proportio: ex æquo autē prima, quā tertia, maior fuerit; erit & quarta, quàm ſexta, maior: quod ſi prima tertiæ fuerit æqualis, erit & quarta æqualis ſextæ: ſin illa minor, hæc quoque minor erit.

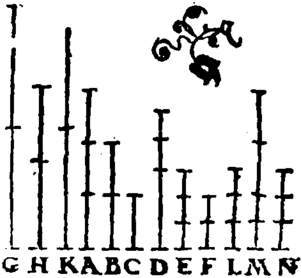


Theorema 22. Propoſitio 22.

Si ſint quodcunque magnitudines, & aliæ ipſis æquales numero, quæ binæ in eadē proportione ſumantur; & ex æqualitate in eadem proportione erunt.

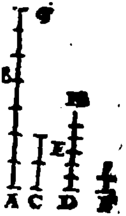
Theorema 23. Propoſitio 23.

Si ſint tres magnitudines, aliæq́ue ipſis æqua les numero, quæ binæ in eadem proportione sumantur; fuerit auté perturbata earum proportio: Etiam ex æqualitate in eadem proportione erunt.



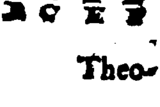
Theorema 24. Propositio 24.

Si prima ad secundam, eandem habuerit proportionem, quam tertia ad quartam; habuerit autem & quinta ad secundam, eandem proportionem, quàm sexta ad quartam: Etiam composita prima cum quinta, ad secundam eandem habebit proportionem, quam tertia cum sexta, ad quartam.



Theorema 25. Propositio 25.

Si quatuor magnitudines proportionales fuerint; maxima, & minima reliquis duabus maiores erunt.



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Theorema 26. Propopositio 26.

Si prima ad secundam, maiorem habuerit proportionem, quàm tertia ad quartam: habebit couertendo secunda ad primam, minorem proportionem, quàm quarta ad tertiam.

Theorema 27. Propositio 27.

Si prima ad secundam, habuerit maiorem proportionem; quàm tertia ad quartam: habebit quoq; vicissim prima ad tertiam, maiorem proportionem, quàm secunda ad quartam.

Theorema 28. Propositio. 28.

Si prima ad secundam habuerit maiorem proportionem, quàm tertia ad quartam: habebit quoque composita prima cum secunda, ad secundam, maiorem proportionem, quàm composita tertia cũ quarta, ad quartam.

Theorema 29. Propositio 29.

Si composita prima cum secunda, ad secundam, maiorem habuerit proportionem quã composita tertia cum quarta, ad quartam: Habebit quoq; diuidando prima ad secundam, maiorem proportionem, quam tertia, ad quartam.

Theorema 30. Propositio 30.

Si composita prima cum secunda, ad secundam habuerit maiorem proportionem, quàm composita tertia cum quarta, ad quartã: Habebit quoq; per conuersionem proportionis, prima cum secunda, ad primam, minorem proportionem, quàm tertia cum quarta, ad tertiam.

Theorema 31. Propositio 31.

Si sint tres magnitudines, & aliæ ipsis æquales numero; sitque maior proportio primæ priorum ad secundam, quàm primæ posteriorum ad secundam; Item secundæ priorem ad tertiam maior quàm secundæ posteriorum ad tertiam: Erit quoque ex æqualitate maior proportio primæ priorum ad tertiam, quàm primæ posteriorum ad tertiam.

Theorema 32. Propositio 32.

Si sint tres magnitudines, & aliæ ipsis æquales numero; sitque maior proportio primæ priorum ad secundam, quàm secundę posteriorum

riorum ad tertiam; Item secundæ priorum ad tertiam maior, quàm primæ posteriorũ ad secundam: Erit quoque ex æqualitate, maior proportio primæ priorum ad tertiam, quàm primæ posteriorum ad tertiam.

Theorema 33. Propositio 33.

Si fuerit maior proportio totius ad totum, quàm ablati ad ablatum: Erit & reliqui ad reliquum, maior proportio, quàm totius ad totum.

Theorema 34. Propositio 34.

Si sint quotcunque magnitudines, & aliæ ipsis æquales numero; sitque maior proportio primæ priorum ad primam posteriorum, quàm secúdę ad secundam; & hæc maior, quàm tertiæ ad quartam, & sic deinceps: Habebunt omnes priores simul, ad omnes posteriores simul, maiorem proportionem quàm omnes priores, relictâ primâ, ad omnes posteriores, relictâ quoque primâ; minorem autem, quàm prima priorum ad primam posteriorum; maiorem denique etiã, quàm vltima priorum ad vltimam posteriorum.

FINIS ELEMENTI V.

EVCLIDIS ELEMENTVM SEXTVM.

DEFINITIONES.

1.

Similes figuræ rectilineæ sunt, quæ & angulos singulos singulis æquales habent; atque etiam latera, quæ circum angulos æquales, proportionalia.

2.

Reciprocæ autem figuræ sunt, cùm in vtraque figura, antecedentes, & consequentes proportionum termini fuerint.

3.

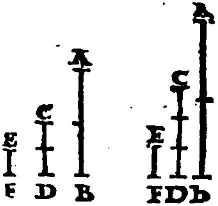
Secundum extremam, & mediam proportionem recta linea secta esse dicitur, cùm vt tota ad maius segmentum, ita maius ad minus se habuerit.

4.

Altitudo cuiusque figuræ est linea perpendicularis à vertice ad basin deducta.

5.

Proportio proportionibus componi dicitur, cũ proportionum quãtitates inter se multiplicatæ, aliquã effecerint proportionem.

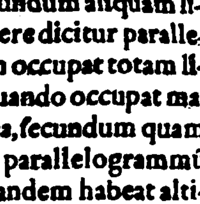


6.

Parallelogrammum secundum aliquam lineam applicatum, deficere dicitur parallelogrammo, quando non occupat totam lineam: Excedere vero, quando occupat maiorem lineam, quàm sit ea, secundum quam applicatur: Ita tamen, vt parallelogrammũ deficiens, aut excadens, eandem habeat altitudinem cum parallelogrammo applicato, constituatque cum eo totum vnum parallelogrammum.

Theorema 1. Propositio 1.

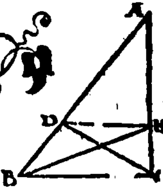
Triangula, & parallelogramma, quorum eadem fuerit altitudo, ita se habent inter se, vt bases.



Theorema 2. Propositio 2.

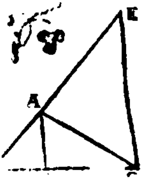
Si ad vnum trianguli latus parallela ducta fuerit recta quædam linea:

Hæc proportionaliter secabit ipsius trianguli latera. Et ſi trianguli latera proportionaliter ſecta fuerint; quæ ad ſectiones adiuncta fuerit recta linea, erit ad reliquum ipſius trianguli latus parallela.



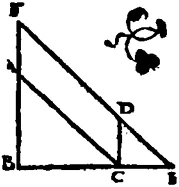
Theorema 3. Propoſitio 3.

Si trianguli angulus bifariam ſectus ſit; ſecans autem angulum recta linea ſecuerit & baſin: baſis ſegmenta, eandē habebunt proportionē, quā reliqua ipſius trianguli latera: Et ſi baſis ſegmenta eandem habeant proportionem quam reliqua ipſius trianguli latera; recta linea, quæ à vertice ad ſectionem producitur, bifariam ſecat trianguli ipſius angulum.



Theorema 4. Propoſitio 4.

Aequiangulorum triangulorum proportionalia ſunt latera, quæ circum ęquales angulos, & homologa ſunt latera, quæ ęqualibus angulis ſubtenduntur.



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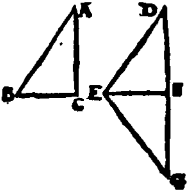
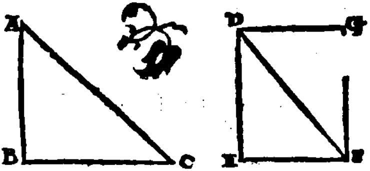
Theorema 5. Propositio. 5.

Si duo triangulo, latera proporrionalia habeanr, æquiangula erunt triangu la, & æquales habebunt eos angulos, ſub quibus homologa latera ſubtenduntur.



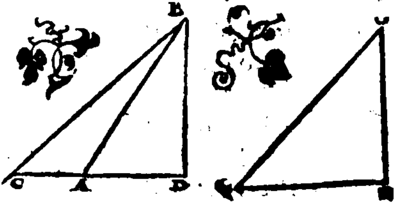
Theorema 6. Propoſitio 6.

Si duo triangula vnum angulum vni angulo æqua'é, & circum æquales angulos latera proportionalia habuerint: æquiangula erunt triangula, æqualeſque habebunt angulos, ſub quibus homologa latera ſubtenduntur.



Theoreme 7. Propoſitio 7.

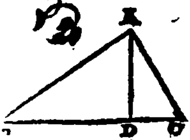
Si duo triangula vnum angulum vni angulo æqualē circū autē alios angulos latera proportionalia habent; reliquorum



vero simul vtrunque aut minorem, aut non minorem recto: æquiangula erunt triangula, & æquales habebunt eos angulos, circum quos proportionalia sunt latera.

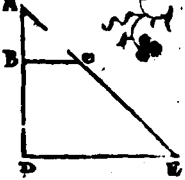
Theorema 8. Propositio 8.

Si in triangulo rectangulo, ab angulo recto in basin perpendicularis ducta sit; quæ ad perpendicularem triangula, tum toti triangulo, tum ipsa inter se similia sunt.



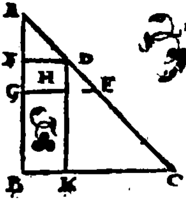
Problema 1. Propositio. 9.

A data recta linea imperatam partem auferre.



Problema 2. Propositio 10.

Datam rectam lineam insectam similiter secare, vt data altera recta secta fuerit.



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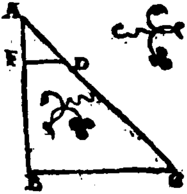
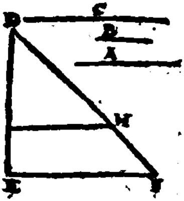
Problema 3. Propoſitio. 11.

Duabus datis rectis lineis, tertium proportionalem adinuenire.



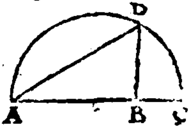
Problema 4. Propoſitio 12.



Tribus datis rectis lineis quartam proportionalē adinuenire.

Problema 5. Propoſitio 13.

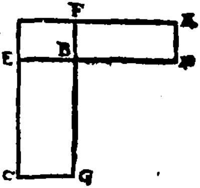
Duabus datis rectis lineis, mediam proportionalem adinuenire.



Theorema 9. Propoſitio 14.

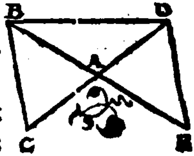
Aequalium, & vnum vni æqualem habentium angulum, parallelogrammorum reciproca ſunt latera, quæ circum æquales angulos; Et quorum parallelogrammorū vnum angu-

angulum vni angulo æqualem habentium reciproca ſunt latera, quæ circum æquales angulos, illa ſunt æqualia.



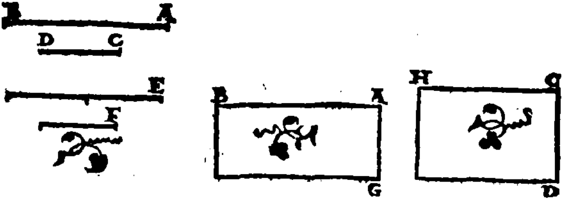
Theorema 10. Propoſitio 15.

Aequalium, & vnum angulum vni ęqualem habentium triangulorum reciproca ſunt latera, quæ circum æquales angulos: Et quorum triangulorum vnum angulum vni æqualem habentium, reciproca ſunt latera, quæ circũ æquales angulos, illa ſunt ęquales.



Theorema 11. Propoſitio 16.

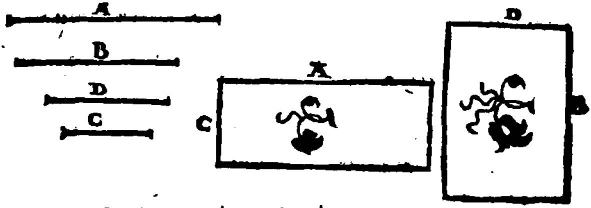
Si quatuor rectæ lineæ proportionales fuerint, quod ſub extremis comprehenditur rectangulum, ęquale eſt ei, quod ſub medijs com-



comprehenditur, rectangulo. Et sub extremis comprehensum rectangulum æquale fuerit ei, quod sub medijs continetur, rectã-gulo, illæ quatuor rectæ lineę proportionales erunt.

Theorema 12. Propositio 17.

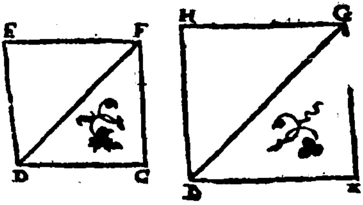
Si tres rectæ lineę sint proportionales; quod sub extremis comprehẽditur rectangulum



æquale fit ei, quod à media describitur, quadrato: Et si sub extremis comprehẽsum rectangulum æquale fit ei, quòd à media describitur, quadrato; illę tres rectę lineæ proportionales erant.

Theorema 6. Propositio 16.

A data recta linea, dato recti lineo simile, similiterq; positum rectilineum describere.



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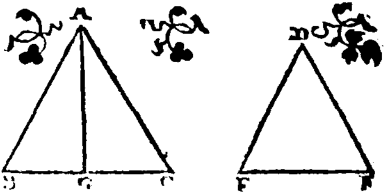
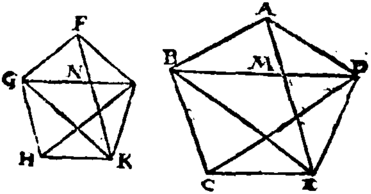
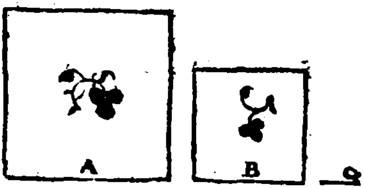
Theorema 13. Propositio 19.

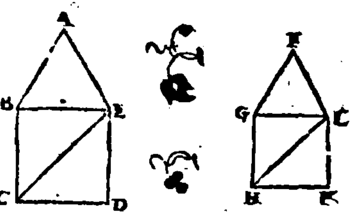


Similia triangula inter se sunt in duplicata proportione laterum homologorum.

Theorema 14. Propositio 20.







Similia polygona in similia triangula diuiduntur, & numéro ęqualia, & homologa totis. Et polygona duplicatam habét eam inter se proportionem, quam latus homologum ad homologum latus.

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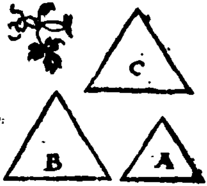
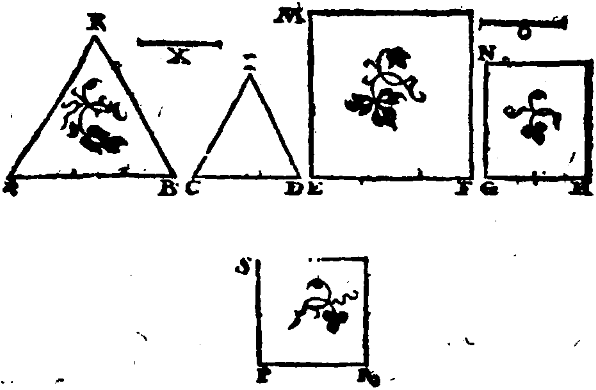
Theorema 15. Propositio 21.

Quæ eidem rectilineo sunt similia, & inter se sunt similia.



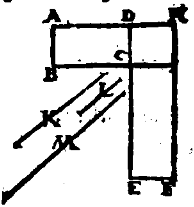
Theorema 16. Propositio 22.

Si quatuor rectæ lineæ proportionales fuerint: & ab eis rectilinea similia similiterque descripta proportionalia erunt. Et si à rectis lineis similia similiterque descripta rectilinea proportionalia fuerint; ipsæ etiam rectæ lineæ proportionales erunt.



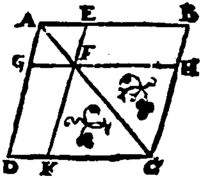
Theorema 17. Propoſitio 23.

Aequiangula parallelogramma inter ſe proportionem habent eam, quæ ex lateribus componitur.

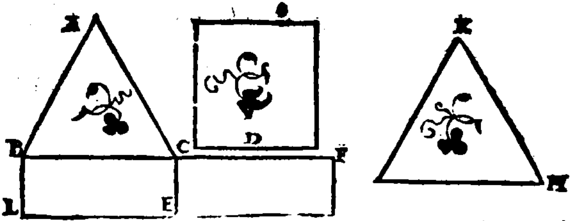


Theorema 18. Propoſitio 24.

In omni parallelogrammo, quæ circa diametrũ ſunt parallelogramma, & toti, & inter ſe ſunt ſimilia.



Problema 7. Propoſitio 25.



Dato recto lineo ſimile: ſimiliterque poſitum; & alteri dato æquale idem conſtituere.

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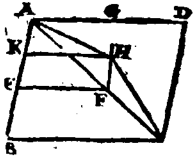
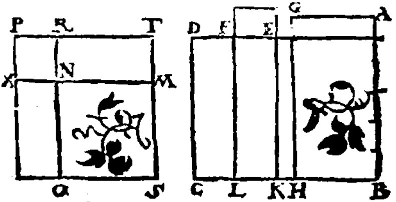
Theorema 19. Propoſitio 26.



Si à parallelogrammo parallelogrammum ablatum ſit, & ſimile toti, & ſimiliter poſitum, communem cum eo habens angulum; hoc circum eandem cum toto diametrum conſiſtit.

Theorema 20. Propoſitio 27.

Omnium parallelogrammorum ſecũdum eandem rectam lineam applicatorum, deficientiumq́; ſi guris parallelo grãmis ſimilibus, ſimiliterq́ue poſitis, ei, quod à dimidia deſcribitur; maximum, id eſt, quod ad dimidiam applicatur, parallelogrammum ſimile exiſtens defectui.

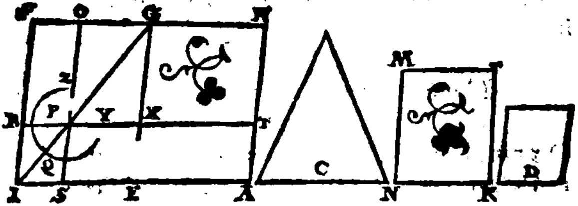


Problema 8. Propoſitio 28.

Ad datam lineam rectam, dato rectilineo æquale parallelogrammum applicare, deficiens figura parallelogramma, quæ ſimilis

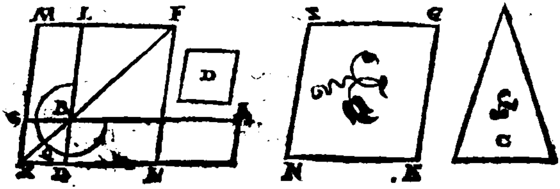
 ſit

ſit alteri rectilineo dato. Oportet autem datum rectilineum, cui æquale applicandum eſt, non maius eſſe eo, quod ad dimidiam applicatur, cùm ſimiles ſint defectus & eius, quod à dimidia deſcribitur, & eius, cui ſimile deeſſe debet.



Problema 9. Propoſitio 29.

Ad datam rectam lineam, dato rectilineo æquale parallelogrammum applicare, excedens figura parallelogramma, quę ſimilis ſit parallelogrammo alteri dato.



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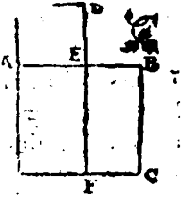
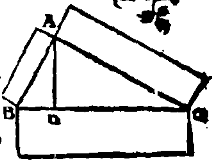
Problema 10. Propositio 30.



Propositam rectam lineam terminatam, extrema ac media ratione (proportione:) secare.

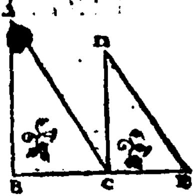
Theorema 21. Propositio 31.

In rectangulis triangulis, figura quævis à latere rectum angulum subtendente descripta, æqualis est figuris, quæ priori illi similes, & similiter positæ, à lateribus rectum angulum cõtinentibus describũtur.



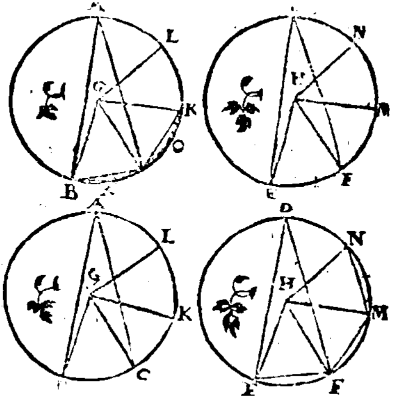
Theorema 22. Propositio 32.

Si duo triangula, quæ duo latera duobus lateribus proportionalia habeant, secundum vnum angulum composita fuerint, ita vt homologa eorum latera sint etiam parallela; tum reliqua illorum triangulorum latera in rectam lineam collocata reperientur.



Theorema 25. Propositio 33.

In æqualibus circulis, anguli eandem habent rationem, cum ipsis peripherijs, in quibus insistũt, siue ad centra, siue ad peripherias cõstituti, illis insistant peripherijs. Insuper verò & sectores, quippe qui ad centra consistunt.



FINIS ELEMENTI VI.

EVCLI-

EVCLIDIS ELEMENTVM SEPTIMVM.

DEFINITIONES.

1.

VNitas est, secundum quam vnum, quodque eorum, quæ sunt, vnum dicitur.

2.

Numerus autem, ex vnitatibus composita multitudo.

3.

Pars est numerus numeri, minor maioris, cum minor metitur maiorem.

4.

Partes autem, cùm non metitur.

5.

Multiplex verò, maior minoris, cùm maiorem metitur minor.

6.

Par numerus est, qui bifariam diuiditur.

7.

Impar verò, qui bifariam non diuiditur: vel, qui vnitate differt à pari.

8

Pariter par numerus est, quem par numerus metitur per numerum parem.

9.

Pariter autem impar est, quem par numero metitur per numerum imparem.

10.

Impariter verò impar numerus est, quem impar numerus metitur per numerum imparem.

11.

Primus numerus est, quem vnitas sola metitur.

12.

Primi inter se numeri sunt, quos sola vnitas, mensura communis, metitur.

13.

Compositus numerus est, quem numerus quispiam metitur.

14.

Compositi autẽ inter se numeri sunt, quos numerus aliquis, mensura communis, metitur.

15.

Numerus numerum multiplicare dicitur, cùm toties compositus fuerit is, qui multiplicatur, quot sunt in ipso multiplicante vnitates; & procreatus fuerit aliquis.

16.

Cùm autem duo numeri mutuò sese multiplicantes quempiam faciunt, qui factus erit, planus appellabitur. Qui verò numeri mutuò sese multiplicarint, illius latera dicentur.

Cùm

17.

Cùm verò tres numeri mutuò ſeſe multiplicantes quempiam faciunt, qui procreatus erit, ſolidus appellabitur, qui autem numeri mutuò ſeſe multiplicarint, illius latera dicentur.

18.

Quadratus numerus eſt, qui æqualiter æqualis: vel, qui à duabus æqualibus numeris cõtinetur.

19.

Cubus verò, qui æqualiter æquali æqualiter: vel, qui à tribus æqualibus numeris continetur.

20.

Numeri proportionales ſunt, cùm primus ſecundi, & tertius quarti, æquè multiplex eſt; vel eadem pars, vel eædem partes.

21.

Similes plani, & ſolidi numeri ſunt, qui proportionalia habent latera.

22.

Perfectus numerus eſt, qui ſuis ipſius partibus eſt æqualis.

23.

Numerus numerum metiri dicitur per illum numerum, quem multiplicans, vel à quo multiplicatus, illum producit.

24.

Proportio numerorum eſt habitudo quædam vnus numeri & alterum, ſecundum quod illius eſt multiplex, vel pars, partesue.

25.

Termini, ſiue radices proportionis dicuntur duo numero, quibus in eadem proportione minores ſumi nequeunt.

26.

Cùm tres numeri proportionales fuerint; Primus ad tertium, duplicatam proportionem habere dicitur eius, quam habet ad ſecundum. At cùm quatuor numeri proportionales fuerint; primus ad quartum, triplicatam proportionem habere dicitur eius, quam habet ad ſecundum. Et ſuper deinceps vno amplius, quamdiu proportio extiterit.

27.

Quotlibet numerus ordine poſitis, proportio, primi ad vltimum componi dicitur ex proportionibus primi ad ſecundum, & ſecundi ad tertium, & tertij ad quartum, & ita deinceps, donec extiterit proportio.

Poſtulata, ſiue petitiones.

1.

Poſtuletur, cuilibet numero, quotlibet poſſe ſumi æquales, vel multiplices.

2.

Quolibet numero, ſumi poſſe maiorem.

Axio-

Axiomata, siue pronunciata.

1.

Qui numeri æqualium numerorum, vel eiusdem, æquè multiplices sunt, inter se sunt æquales,

2.

Quorum idem numerus æquè multiplex est, vel æquè multiplices sunt æquales, inter se æquales sunt.

3.

Qui numeri æqualium numerorum, vel eiusdem, eadem pars, partes fuerint, inter se æquales sunt.

4.

Quorum idem numerus, vel æquales, eadē pars, vel eædem partes fuerint, æquales inter se sunt.

5.

Vnitas omnem numerum per vnitates, quæ in ipso sunt, hoc est, per ipsummet numerū, metitur.

6.

Omnis numerus seipsum metitur per vnitatem.

7.

Si numerus numerum multiplicans, aliquem produxerit, metietur multiplicans productum per multiplicatum; multiplicatus autem eundem per multiplicantem.

8.

Si numerus numerum metiatur, vt ille, per quem metitur, eundem metietur per eas, quæ in metiente sunt, vnitates, hoc est, per ipsum numerum metientem.

9.

Si numerus numerum metiens, multiplicet eum, per quem metitur; vel ab eo multiplicatur, illum, quem metitur, producat.

10.

Numerus quotcunque numeros metiens, compositum quoque ex ipsis metitur.

11.

Numerus quemcunq; numerum metiens, metitur quoque omnem numerum, quem ille metitur.

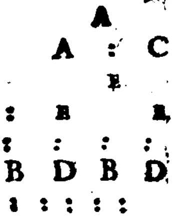
12.

Numerus metiens totum, & ablatum, metitur & reliquum.

Theorema 1. Propositio 1.

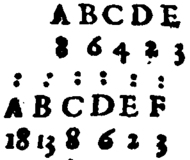
Si Duobus numeris inæqualibus propositis, detrahatur semper minor de maiore, alterna quadam detractione; neque reliquus vnquam metiatur præcedentem, quo ad assumpta sit vnitas: qui principiò propositi sunt numeri, primi inter se erunt.

A
H
: C
F :
: G
: : :
B D E



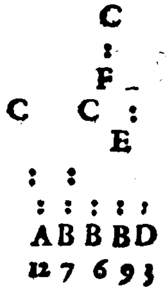
Problema 1. Propositio 2.

Duobus numeris datis non primis inter se, maximam eorum communem mensuram reperire.



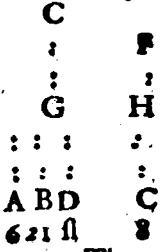
Problema 2. Propositio 3.

Tribus numeris datis non primis inter se, maximam eorum communem mensuram reperire.



Theorema 2. Propositio 4.

Omnis numerus cuiusq; numeri, minor maioris, aut pars est, aut partes.

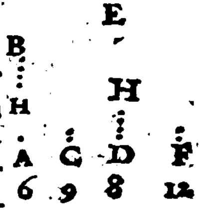


Theorema 3. Propositio 5.

Si numerus numeri pars fuerit, & alter alterius eadẽ pars; & simul vterque vtriusque simul eadem pars erit, quæ vnus est vnius.

Theo-

Theorema 4 Propositio 6.



Si numerus sit numeri partes, & alter alterius eædem partes; & simul vterque vtriusque simul eædem partes erunt, quæ sunt vnus vnius.

Theorema 5. Propositio 7.

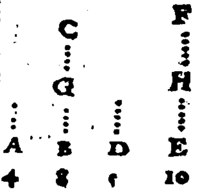
Si numerus numeri eadem sit pars quæ detractus detracti; & reliquus reliqui eadem pars erit, quæ totus est totius.

Theorema 6. Propositio 8.

Si numerus numeri eædem sint partes, quæ detractus detracti; & reliquus reliqui eædem partes erunt, quæ sunt totus totius.

G M.K.N.H.

Theorema 7. Propositio 9.



Si numerus numeri pars sit, & alter alterius eadem pars: & vicissim, quæ pars est, vel partes primus tertij, eadem pars erit vel eædem partes, & secundus quarti.

Theo-

Theorema 8. Propositio 10.

Si numerus numeri partes sint, & alter alterius eadem partes: etiam vicissim, quæ sunt partes, aut pars primus tertij, eędē partes erunt, vel pars & secundus quarti.

H		E	
G		H	
A	C	D	F
4	6	10	18

Theorema 9. Propositio 11.

Si quemadmodum se habet totus ad totum, ita detractus ad detractum: & reliquis ad reliquum ita se habebit, vt totus ad totum.

B	D
E	F
A	C
6	8

Theorema 10. Propositio 12.

Si sint quotcunque numeri proportionales; quemadmodum se habet vnus antecedentium ad vnum consequentium, ita se habebunt omnes antecedentes ad omnes consequentes.

A	B	C	D
9	6	3	2

Theorema 11. Propositio 13.

Si quatuor numeri sint proportionales; & vicissim proportionales erũt.

A	B	C	D
12	4	9	3

Theorema 12. Propositio 14.

Si sint quotcunque numeri, & alij illis ęquales multitudi-

A	B	C	D	E	F
12	6	3	8	4	2

ne,

ne, qui bini sumantur, & in eadem proportione, etiam ex æqualitate in eadem proportione erunt.

Theorema 13. Propositio 15.

Si vnitas numerum quempiam metiatur, alter verò numerus alium quendam numerum æquè metiatur; & vicissim vnitas tertium numerum æquè metietur, atque secúdus quartum.

	G H G		F L K
A	B	D	E
1	3		6

Theorema 14. Propositio 16.

Si duo numeri mutuò sese multiplicãtes faciant aliquos; qui ex illis geniti fuerint, inter se æquales erunt.

E	A	B	C	D
1	2	4	8	8

Theorema 15. Propositio 17.

Si numerus duos numeros multiplicans faciat aliquos; qui ex illis procreati erunt, eandẽ proportionem inter se habebunt, quam multiplicati.

I	A	B	C	D	E
1	3	4	5	12	15

Theorema 16. Propositio 18.

Si duo numeri numerum quempiam multiplicantes, faciant aliquos; geniti ex illis eandem habebunt proportionem, quam qui illum multiplicarunt.

A	B	C	D	E
4	5	3	12	15

Theo-

Theorema 17. Propositio 19.

Si quatuor numeri sint proportionales; quod ex primo, & quarto fit, numerus, æqualis erit ei, qui ex secundo & tertio, fit, numero: Et si, qui ex primo & quarto fit numerus, æqualis sit ei, qui ex secundo & tertio, fit, numero; illi quatuor numeri proportionales erunt.

A B C D E F G
6 4 3 2 12 12 18

Theorema 18. Propositio. 20.

Si tres numeri sint proportionales; qui ab extremis continetur, æqualis est ei, qui à medio efficitur: Et si, qui ab extremis continetur, æqualis sit ei, qui à medio describitur, illi tres numeri proportionales erunt.

A B C
9 6 4
D
6

Theorema 19. Propositio 21.

Minimi numeri omnium qui eandem cum eis proportionem habent, æqualiter metiuntur numeros eandem cum eis proportionem habentes; maior quidem maiorem, minor verò minorem.

D L
G H
C E A B
4 3 8 6

Theo-

Theorema 20. Propopositio 22.

Si tres sint numeri, & alij multitudine illis æquales, qui bini sumantur & in eadem ratione; sit autem perturbata eorum proportio; etiam ex æqualitate in eadem proportione erunt.

A	B	C	D	E	F
6	4	3	12	8	6

Theorema 21. Propositio 23.

Primi inter se numeri, minimi sunt omniũ eandem cum eis proportionem habentium.

A	B	E	C	D
5	6	2	4	3

Theorema 22. Propositio 24.

Minimi numeri omnium eandem cum eis proportionem habentium, primi sunt inter se.

A	B	C	D	E
7	6	4	3	2

Theorema 23. Propositio. 25.

Si duo numeri sint primi inter se, qui alterutrum eorum metitur, numerus, is ad reliquũ primus erit.

A	B	C	D
6	7	3	4

Theorema 24. Propositio. 26.

Si duo numeri ad quẽpiam numerum primi sint, ad eundem primus is quoque futurus est, qui ab illis productus fuerit.

A	C	D	E	F	B
5	5	5	3	2	3

Theorema 25. Propositio 27.

Si duo numeri primi ſint inter ſe, qui ab vno eorum gignitur, ad reliquum primus erit.

B		
A	C	D
7	6	3

Theorema 26. Propoſitio 28.

Si duo numeri ad duos numeros ambo ad vtrunque primi ſint; & qui ex eis gignentur, primi inter ſe erunt.

A	B	E	C	D	F
3	5	15	2	4	8

Theorema 27. Propoſitio 29.

Si duo numeri primi ſint inter ſe, & multiplicans vterque ſeipſum procreet aliquem; qui ex ijs producti fuerint, primi inter ſe erunt. Quòd ſi numeri initiò propoſiti multiplicantes eos, qui producti ſunt, effecerint aliquos; hi quoque inter ſe primi erunt; & circa extremos idem hoc ſemper eueniet.

A	C	E	B	D	F
3	6	27	4	16	63

Theorema 28. Propoſitio 30.

Si duo numeri primi ſint inter ſe, etiam ſimul vterque ad vtrunque illorum primus erit. Et ſimul vterque ad vnum aliquem eorum primus ſit, etiam qui initio poſiti ſunt numeri, primi inter ſe erunt.

	C	
A	B	D
7	5	4

Theorema 29. Propositio 31.

Omnis primus numerus ad omnem numerum, quem non metitur, primus est.

:	:	:
A	B	C
7	10	5

Theorema 30. Propositio 32.

Si duo numeri sese mutuò multiplicantes faciant aliquem; hunc autem ex ipsis productum metiatur primus quidam numerus: is alterum etiam eorum, qui initiò positi erant, metietur.

:	:	:	:	:
A	B	C	D	E
3	6	12	3	4

Theorema 31. Propositio 33.

Omnem compositum numerum aliquis primus metietur.

:	:	:
A	B	C
27	9	3

Theorema 32. Propositio 34.

Omnis numerus aut primus est, aut eum aliquis primus metitur.

:	:	:
A	A	
3	6	3

Problema 3. Propositio 35.

Numeris datis quotcunque, reperire minimos omnium, qui eandem cum illis proportionem habeant.

:	:	:	:	:	:	:	:	:	:	:
A	B	C	D	E	F	G	H	K	I	M
6	8	12	2	3	4	6	2	3	4	3

Pro-

Problema 4. Propositio 36.

B
A C D E F
7 12 8 4 5

Duobus[a] numeris datis, reperire, quē illi minimum metiantur, numerum

A
F E C D G H
6 9 12 0 2 3

Theorema 33. Propositio 37.

Si duo numeri numerum quempiam metiantur; & minimus, quem illi metiuntur, eundem metietur.

F
A B E C
2 3 6 12

Problema 5. Propositio. 32.

Tribus numeris datis, reperire quēm minimum numerum illi metiantur.

A B C D E
3 4 6 12 8

A B C D E F
3 6 8 12 24 16

Theorema 34. Propositio 39.

Si numerum quispiam numerus metiatur, mensus partem habebit metienti cognominem.

A B C D
12 4 3 1

Theorema 35. Propositio 40.

Si numerus partem habuerit quamlibet, illum metietur numerus parti cognominis.

A	B	C	D
8	4	2	1

Problema 6. Propositio. 41.

Numerum reperire, qui minimus cùm sit, datas habeat partes.

A	B	C	G	H
2	3	4	12	10

FINIS ELEMENTI VII.

EVCLI-

EVCLIDIS ELEMENTVM OCTAVVM.

DEFINITIONES.

Theorema 1. Propositio 1.

SI sint quotcunque numeri deinceps proportionales, quorum extremi sint inter se, primi;
ipsi minimi
sunt omniũ
eandem cum eis proportionem habentium.

A	B	C	D	E	F	G	H
8	12	18	27	6	9	12	18

Problema 1. Propositio 2.

Numeros reperire deinceps proportionales minimos, quotcunque iusserit quispiam in data proportione.

A	B	C	D	E	F	G	H	K
3	4	9	12	16	27	36	49	64

Theorema 2. Propositio 3. Conuersa primæ.

Si sint quotcunque numeri deinceps proportionales minimi habentium eandem cũ eis proportionem; illorum extremi sunt inter se primi.

A	B	C	D	E	F	G	H	K	L	M	N	O
27	16	48	64	3	4	9	12	16	27	36	48	64

Problema 2. Propositio 4.

Proportionibus datis quotcunque in minimis numeris, reperire numeros deinceps minimos in datis proportionibus.

A	B	C	D	E	F	H	G	K	L	N	X	M	O
3	4	2	7	4	5	6	8	12	15	4	6	10	12

Theorema 3. Propositio 5.

Plani numeri proportionem inter se habent ex lateribus compositam.

A	L	B	C	D	E	F	G	H	K
18	22	34	3	6	4	8	9	12	16

Theorema 4. Propositio 6.

Si sint quotlibet numeri de-

A	B	C	D	E	F	G	H
16	24	36	54	82	4	6	9

inceps,

inceps, proportionales; primus autem secũdum non metiatur; neq; alius quispiam vllum metietur.

Theorema 5. Propositio 7.

Si sint quotcunque numeri deinceps proportionales; primus autem extremum metiatur; is etiam secundum metietur.

A	B	C	D
4	6	12	24

Theorema 6. Propositio 8.

Si inter duos numeros medij continua proportione indicant numeri; quot inter eos medij continua proportione incidunt numeri, totidem & inter alios eandem cum illis habentes proportionem medij continua proportione incident.

A	C	D	B	G	H	K	L	C	M	N	F
4	9	27	81	1	3	9	27	2	6	18	54

Theorema 7. Propositio 9.

Si duo numeri sunt inter se primi, & inter eos medij continua proportione incidant numeri, quot inter eos medij continua proportione incidunt numeri, totidem & inter vtrunque eorum, ac vnitatem deinceps medij continua proportione incident.

 Theo-

A M H E F N C K X G D L O B
27 27 9 36 3 36 1 12 48 4 48 16 64 64

Theorema 8. Propositio 10.

Si inter duos numeros, & vnitatem, continuè proportionales incidant numeri; quot inter vtrunque ipsorum, & vnitatem, deinceps medij continua proportione incidunt numeri; totidē & inter illos medij continua proportione incident.

A 27 — E 9 — K 36 — D 3 — H 12 — C 1 — L 48 — F 4 — G 16 — B 64

Theorema 9. Propositio 11.

Duorum quadratorum numerorum vnus medius proportionalis est numerus: & quadratus ad quadratum duplicatam habet lateris ad latus proportionem.

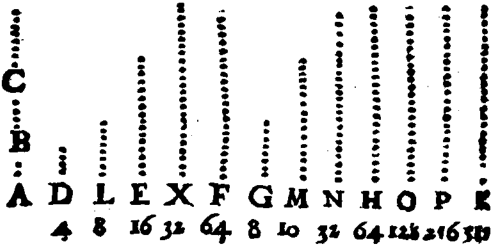
A C E D B
9 3 12 4 16

Theorema 10. Propositio 12.

Duorum cuborum numerorum duo medij proportionales sunt numerorum; Et cubus ad ad cubum triplicatam habet lateris ad latus proportionem.

Theorema 11. Propositio 13.

Si sint quotlibet numeri deinceps proportionales, & multiplicans quisque seipsum faciat aliquos; qui ab illis producti fuerint, proportionales erunt. Et, si numeri primú positi, ex suo id procreatos ductu, faciant aliquos; ipsi quoque proportionales erunt.



Theorema 12. Propositio 14.

Si quadratus numerus quadratum numerū metiatur, & latus vnius metietur latus alterius.

rius. Et si vnius quadrati latus metiatur latus alterius, & quadratus quadratum metietur.

A	E	B	C	D
9	12	16	8	4

Theorema 13. Propositio 15.

Si cubus numerus cubum numerus metiatur, & latus vnius metietur alterius latus. Et si latus vnius cubi latus alterius metiatur, tum cubus cubum metietur.

A	H	K	B	C	D	E	F	G
8	16	28	64	2	4	4	8	16

Theorema 14. Propositio 16.

Si quadratus numerus quadratum numerũ non metiatur, neque latus vnius metietur alterius latus. Et si latus vnius quadrati non metiatur latus alterius, neque quadratus quadratum metietur.

A	B	C	D
9	16	3	4

Theorema 15. Propositio 17.

Si cubus numerus cubum numerum nó metiatur; neque latus vnius latus alterius metietur. Et si latus cubi vnius latus alterius non metiatur, neque cubus cubum metietur.

A	B	C	D
8	27	9	11

Theo-

Theorema 16. Propoſitio 18.

Duorum ſimilium planorum numerorum vnus medius proportionalis eſt numerus; & planus ad planum duplicatam habet lateris homologi ad latus homologum proportionem.

A	G	B	C	D	E	F
12	18	27	2	6	3	9

Theorema 17. Propoſitio 19.

Duorum ſimilium numerorum ſolidorum duo medij proportionales ſunt numeri: Et ſolidus ad ſimilem ſolidum triplicatam, habet lateris homologi ad latus homologum proportionem,

A	N	X	B	C	D	E	F	G	H	K	M	L
8	12	18	27	2	2	2	3	3	3	4	6	9

Theorema 18. Propoſitio 20.

Si inter duos numeros vnus medius proportionalis incidat numerus; ſimiles plani erunt illi numeri,

A	C	B	D	E	F	G
18	24	32	3	4	6	8

Theo-

Theorema 19. Propositio 21.

Si inter duos numeros duo medij proportionales incidãt numerĩ; similes solidi sunt illi numeri.

A	C	D	B	E	F	G	H	K	L	M
27	36	44	64	9	12	16	3	3	3	4

Theorema 20. Propositio 22.

Si tres numeri deinceps sint proportionales; primus autem sit quadratus, & tertius quadratus erit.

A	B	D
9	15	25

Theorema 21. Propositio 23.

Si quatuor numeri deinceps sint proportionales; primus autem si cubus, & quartus cubus erit.

A	B	C	D
8	12	18	27

Theorema 22. Propositio 24.

Si duo numeri eã proportionẽ habeãt inter se, quam quadratus numerus ad quadratum numerum, primus autem sit quadratus, & secundus quadratus erit.

A		B	C		D
4	9	9	16	34	36

Theorema 23. Propositio 25.

Si numeri duo proportionem inter se habeant,

beant, quam cubus numerus ad cubum numerum, primus autem cubus sit, & secũdus cubus erit.

A	E	F	B	C			D
8	12	18	2,	64	95	140	216

Theorema 24. Propositio 26.

Similes plani numeri proportionem inter se habent, quam quadratus numerus ad quadratum numerum.

A	C	B	D	E	F
18	24	32	9	12	16

Theorema 25. Propositio 27.

Similes solidi numeri proportionem habẽt inter se, quam cubus numerus ad cubum numerum.

A	C	D	B	E	F	G	H
36	24	26	54	8	12	18	27

FINIS ELEMENTI VIII.

EVCLI.

EVCLIDIS ELEMENTVM NONVM.

Theorema 1. Propositio 1.

Si duo similes plani numeri mutuò sese multiplicantes, quendam procreent, productus quadratus erit.

A	E	B	D		C
4	6	9	16	24	36

Theorema 2. Propositio 2.

Si duo numeri mutuò sese multiplicantes, quadratum faciant, illi similes sunt plani.

A		B	D		C
4	6		9	18	36

Theorema 3. Propositio 3.

Si cubus numerus seipsum multiplicãs procreet aliquem, productus cubus erit.

	:	:	:	:	:	:
Vnitas	D	D	A			B
	5	4	8	16	32	64

Theorema 4. Propositio 4.

Si cubus numerus cubũ numerum multiplicans quendam procreet, procreatus cubus erit.

:	:	:	:
A	B	D	C
8	27	64	116

Theorema 5. Propositio. 5.

Si cubus numerus numerum quendam multiplicans cubum procreet, & multiplicatus cubus erit.

:	:	:	:
A	B	C	D
27	64	729	2718

Theorema 6. Propositio 6.

Si numerus seipsum multiplicans cubum procreet; & ipse cubus erit.

:	:	:
A	B	C
27	729	19683

Theorema 7. Propositio 7.

Si compositus numerus quẽdam numerum multiplicans, quempiam procreet, productus solidus erit.

:	:	:	:	:
A	B	C	D	E
6	8	48	2	3

Theorema 8. Propositio 8.

Si ab vnitate quotlibet numeri deinceps proportionales sint: Tertius ab vnitate quadratus est, & vnum intermittentes omnes: Quartus aũt cubus est, & duobus intermissis omnes:

nes: Septimus verò cubus simul & æquadratus est, & quinque intermissis omnes

•	:	⋮	⋮	⋮	⋮	⋮
Vnitas	A	B	C	D	E	F
	3	9	27	81	243	[illegible]

Theorema 9. Propositio 9.

Si ab vnitate sint quotcunque numeri deinceps proportionales; sit autem quadratus is, qui vnitatem sequitur, & reliqui oẽs quadrati erũt. Quòd si, qui vnitatem sequitur, cubus sit; & reliqui omnes cubi erunt.

quadrati		cubi
531441	F	732969
59040	E	53441
6561	D	6561
723	C	6561
81	B	729
9	A	81
	o	
	vnitas.	

Theorema 10. Propositio 10.

Si ab vnitate numeri quotcunque proportionales sint; non sit autem quadratus is, qui vnitatem sequitur, neque alius vllus quadratus

o	:	:	:	:	:	:
Vnitas.	•	:	:	:	:	:
	A	B	C	D	E	F
	3	9	36	81	243	729

tus

tus erit, demptis, tertio ab vnitate ac omnibus vnum intermittentibus. Quòd si, qui vnitatem sequitur, cubus non sit: neque alius vllus cubus erit; demptis, quarto ab vnitate, ac omnibus duos intermittentibus.

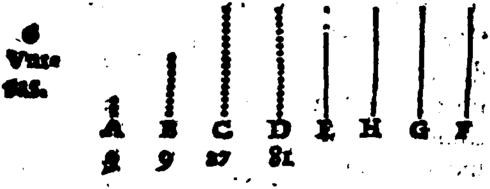
Theorema 11. Propositio 11.

Si ab vnitate numeri quotlibet deinceps proportionales sint; minor maiorem metitur per quempiam eorum, qui in proportionalibus sunt numeris.

	A	B	C	D	E
1	1	2	4	8	16

Theorema 12. Propositio 12.

Si ab vnitate quotlibet numeri sint proportionales; quot primorum numerorum vltimum metiuntur, totidem & eum, qui vnitati proximus est, metientur.

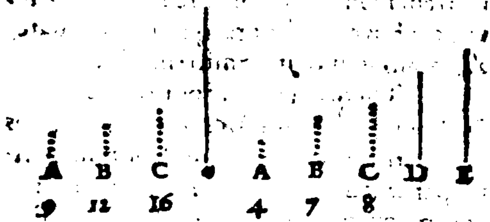


Theorema 13. Propositio 13.

Si ab vnitate sint quotcunque numeri deinceps proportionales; primus autem sit, qui vnitatem sequitur; maximum nullus alius

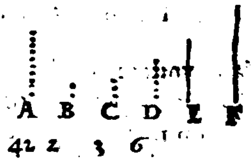
H metie-

metietur; ijs exceptis, qui in proportionalibus sunt, numeris.



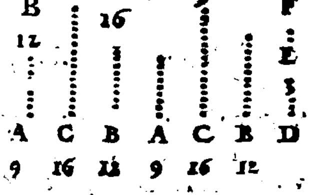
Theorema 14. Propositio 14.

Si minimum numerum primi aliquot numeri metiantur; nullus alius numerus primus illum metietur; ijs exceptis, qui primò metiuntur.



Theorema 15. Propositio 15.

Si tres numeri deinceps proportionales sint minimi omnium, eandem cum ipsis proportionem habentium, duo quilibet compositi, ad tertium primi erũt.



Theo-

Theorema 16. Propositio 16.

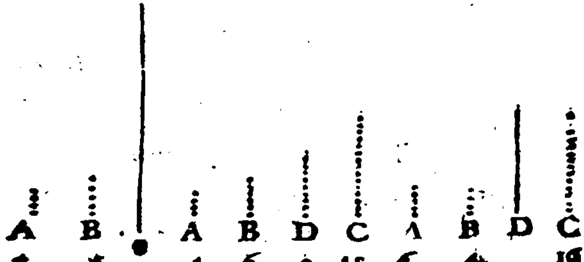
Si duo numeri sint inter se primi; non se habebit quemadmodum primus ad secundum, ita secundus ad quempiam alium.

Theorema 17. Propositio 17.

Si sint quotlibet numeri deinceps proportionales, quorum extremi sint inter se primi; non erit quẽadmodum primus ad secundum, ita vltimus ad quempiam alium.

Problema 1. Propositio 18.

Duobus numeris datis, considerare an possit ipsis tertius inueniri proportionalis.

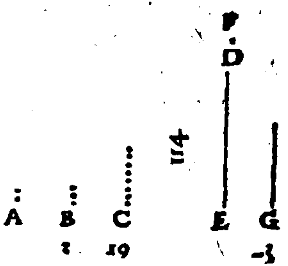


Problema 12. Propoſitio 19.

Tribus numeris datis, conſiderare, an poſſit ipſis quartus reperiri proportionalis.

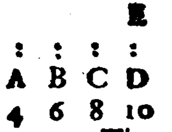
Theorema 18. Propoſitio 20.

Primi numeri plures ſunt, quacunque propoſita multitudine primorum numerorum.



Theorema 19. Propoſitio 21.

Si pares numeri quotlibet compoſiti ſint, totus eſt par.



Theo-

Theorema 20. Propositio 22.

Si impares numeri quotlibet compositi sint; sit autem par illorum multitudo; totus par erit.

E
A B C D
1 9 7 5

Theorema 21. Propositio 23.

Si impares numeri quotcunque compositi sint; sit autem impar illorum multitudo; & totus impar erit.

A B C E
5 7 8 1

Theorema 22. Propositio 24.

Si à pari numero par detractus sit; & reliquus par erit.

B
A C
6 4

Theorema 23. Propositio 25.

Si à pari numero impar detractus sit; & reliquus impar erit.

A C D
8 1 4

Theorema 24. Propositio 26.

Si ab impari numero impar detractus sit; & reliquus par erit.

B
A C D
4 6

Theorema 25. Propositio 27.

Si ab impari numero par ablatus sit; reliquus impar erit.

B
A D C
1 4 4

Theorema 26. Propositio 28.

Si impar numerus parem multiplicans, procreet quempiam; procreatus par erit.

A B C
3 4 12

Theorema 27. Propositio 29.

Si impar numerus imparem numerum multiplicans, quendam procreet; procreatus impar erit.

A B C
3 5 15

Theorema 28. Propositio 30.

Si impar numerus parem numerum metiatur; & illius dimidium metietur.

A C B
3 6 18

Theorema 29. Propositio 31.

Si impar numerus ad numerum quempiam primus sit: & illius duplum primus erit.

A B C D
7 8 16

Theo-

Theorema 30. Propositio 32.

Numerorum, qui à binario dupli sunt, *Vnivnusquisque* pariter tantùm par est tantùm.

A B C D
2 4 8 16

Theorema 31. Propositio 33.

Si numerus dimidium habeat imparem: pariter impar est tantùm.

A
20

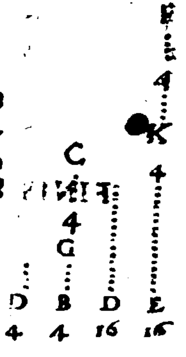
Theorema 32. Propositio 34.

Si par numerus neque à binario duplus sit, neque dimidium habeat imparem: pariter par est, & pariter impar.

A
20

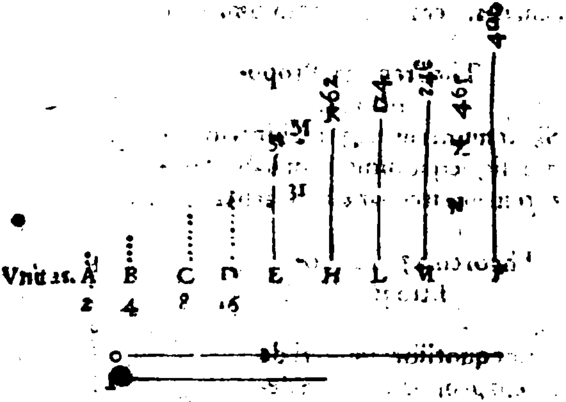
Theorema 33. Propositio 35.

Si sint quotlibet numeri deinceps proportionales; detrahantur autem à secundo & vltimo æquales ipsi primo: Erit quemadmodum secundi excessus ad primũ, ita vltimi excessus ad omnes, qui vltimum antecedunt.



Theorema 34. Propositio 36.

Si ab vnitate numeri quotlibet deinceps expositi sint in dupla proportione, quoad totus compositus primus factus sit; isque totus in vltimum multiplicatus, quempiam procreatus perfectus erit.



FINIS ELEMENTI IX.

EVCLIDIS ELEMENTVM DECIMVM.

DEFINITIONES.

PRIMAE, NEMPE MAGNITVdinem symmetrarum.

1.

Commensurabiles magnitudines dicũtur illæ, quas eadem mensura metitur.

2.

Incommensurabiles verò magnitudines dicuntur, quarum nullam mensuram communem contingit reperiri.

3.

Lineæ rectæ potentia commensurabiles sunt, quarum quadrata vna eadem superficies, siue area metitur.

4.

Incommensurabiles verò lineæ sunt, quarum quadrata, quæ metiatur area communis, reperiri nulla potest.

5.

Hæc cùm ita sint, ostendi potest, quòd quætacunque linea recta nobis proponantur, existunt etiam aliæ lineæ innumerabiles eidé commensurabiles, aliæ autem incommensurabiles; hę quidem longitudine & potentia, illæ verò potentia tantùm. Vocetur igitur linea recta, quantacunq; proponatur ῥητὴ, id est, rationalis.

6.

Lineæ quoque illi ῥητῇ commensurabiles siue longitudine & pontentia, siue potentia tantùm, vocentur & ipsæ ῥηταὶ, id est, rationales.

7.

Quæ verò lineæ sunt incommensurabiles illi τῇ ῥητῇ, id est, primo loco rationali, vocentur ἄλογοι, id est, irrationales.

8.

Et quadratum, quod à linea proposita describitur, quam ῥητὴν vocari volumus, vocetur ῥητὸν, id est, rationale.

9.

Et, quæ sunt huic commensurabilia, vocentur ῥητά, id est, rationalia.

10.

Quæ verò sunt illi quadrato, ῥητῷ scilicet, incommensurabilia, vocentur, ἄλογα, id est, surda, siue irrationalia.

11.

Et lineæ, quæ illa incommensurabilia describunt, vocentur ἄλογοι. Et quidam si illa incommensurabilia fuerint quadrata, ipsa eorum altera vocabuntur ἄλογοι lineæ, quòd

si qua-si quadrata quidem non fuerint, verum aliæ quæpiam superficies siue figuræ rectilineæ, tunc vero lineæ illæ quæ describunt quadrata æqualia figuris rectilineis, vocentur ἄλογοι.

Postulatum, siue petitio.

Postuletur quamlibet magnitudinem toties posse multiplicari, donec quamlibet magnitudinem eiusdem generis excedat.

Axiomata, siue pronunciata.

1.

Magnitudo quotcunque magnitudines metiens, compositam quoque ex ipsis metitur.

2.

Magnitudo quamcunq; magnitudinem metiens, metitur quoque omnem magnitudinem, quam illa metitur.

3.

Magnitudo metiens totam magnitudinem, & ablatam; metitur, & reliquam.

Problema 1. Propositio 1.

Duabus magnitudinibus inæqualibus propositis, si de maiore detrahatur plus dimidio; & rursus de residuo iterum detrahatur plus dimidio; idque semper fiat: relinquetur tandem quædam magnitudo minor altera minore ex duabus propositis.

Theorema 2. Propositio 2.

Duabus magnitudinibus propositis inæqualibus, si detrahatur semper minor de maiore, alterna quadam detractione; neque residuum vnquam metiatur id, quod ante se metiebatur; incommensurabiles sunt illæ magnitudines.

Problema 1. Propositio 3.

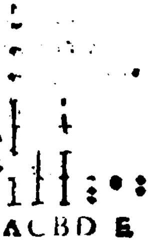
Duabus magnitudinibus commensurabilibus datis, maximam ipsarum communem mensuram reperire.

Problema 2. Propositio 4.

Tribus magnitudinibus commensurabilibus datis, maximam ipsarum communem mensuram reperire.

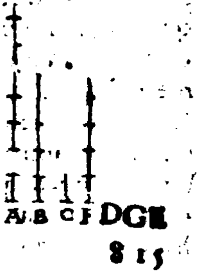
Theorema 3. Propositio 5.

Commensurabiles magnitudines inter se proportionem eam habent, quam habet numerus ad numerum.



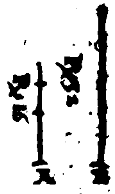
Theorema 4. Propositio 6.

Si duæ magnitudines proportionem eam habent inter se, quam numerus ad numerum: commensurabiles sunt illæ magnitudines.



Theorema 5. Propositio 7.

Incommensurabiles magnitudines inter se proportionem non habent, quam numerus ad numerum.

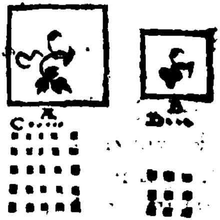


Theorema 6. Propositio 8.

Si duæ magnitudines inter se proportionem non habent, quam numerus ad numerum: incommensurabiles illæ sunt magnitudines.

Theorema 7. Propositio 9.

Quadrata, quæ describitur à rectis lineis longitudine commensurabilibus, inter se proportiportionem habent, quam numerus quadratus ad alium numerum quadratum. Et quadrata habentia proportionem inter se, quam quadratus numerus ad numerum quadratum, habent quoque latera longitudine commensurabilia. Quadrata verò quæ describũtur à lineis longitudine incommensurabilibus; proportionem non habent inter se, quã quadratus numerus ad numerũ alium quadratum. Et quadrata non habentia proportionem inter se, quam numerus quadratus ad numerum quadratum, neque latera habebunt longitudine commensurabilia.



Theorema 8. Propositio 10.

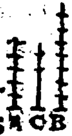
Si quatuor magnitudines fuerint proportionales; prima verò secundæ fuerit commẽsurabilis; tertia quoque quartæ commẽsurabilis erit; quòd si prima secũdæ fuerit incommensurabilis; tertia quoque quartæ incommensurabilis erit.

Theorema 3. Propositio 11.

Propositæ lineæ rectæ (quam ῥητὴν vocari dixi-

diximus) reperire duas lineas rectas incommenſurabiles, alteram quidẽ longitudine tantùm, alteram verò non longitudine tantũ, ſed etiam potentia incõmẽſurabilem.

Theorema 9. Propoſitio 12.

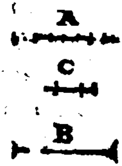


Magnitudines, quæ eidem magnitudini ſunt cõmẽſurabiles; inter ſe quoque ſunt commenſurabiles.

6 D... 4 F..
4 E..... 8 G..
3 H...
2 K...
4 L...

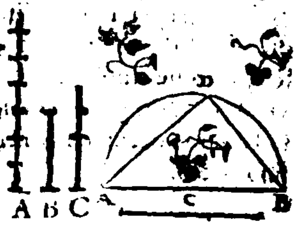
Theorema 10. Propoſitio 13.

Si ex duabus magnitudinibus hæc quidem commenſurabilis ſit tertiæ magnitudini, illa verò eidem incommenſurabilis, incõmẽſurabiles ſunt illæ duæ magnitudines.



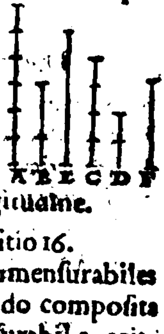
Theorema 11. Propoſitio 14.

Si duarum magnitudinum commenſurabilium, altera fuerit commenſurabilis magnitunitudini alteri cupiam tertiæ; reliqua quoque magnitudo eidem tertiæ incommensurabilis erit.



Theorema 12. Propositio 15.

Si quatuor rectæ lineæ proportionales fuerint; posit autem prima plusquam secunda tanto, quantum est quadratum rectæ lineæ sibi commensurabilis longitudine: tertia quoque poterit plusquam quarta tanto, quantum est quadratum rectæ lineæ sibi commensurabilis longitudine. Quòd si prima posit plusquam secũda quadrato rectæ lineæ sibi longitudine incommensurabilis: tertia quoque poterit plusquam quarta quadrato rectæ lineæ sibi incommensurabilis longitudine.

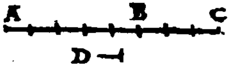


Theorema 13 Propositio 16.

Si duæ magnitudines commensurabiles componantur; tota magnitudo composita singulis partibus commensurabilis erit; Quòd si tota magnitudo composita alterutri parti commensurabilis fuerit; illæ duæ

quo-

quoque partes commē-
ſurabiles erunt.

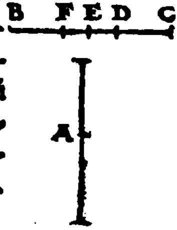


Theorema 14. Propoſitio 17.

Si duæ magnitudines incommenſurabiles componantur, ipſa quoque tota magnitudo ſingulis partibus componentibus incómenſurabilis erit. Quòd ſi tota alteri parti incómenſurabilis fuerit illæ quoque primæ magnitudines inter ſe incommenſurabiles erunt.

Theorema 15. Propoſitio 18.

Si fuerint duæ rectæ lineæ inæquales, & quartæ parti quadrati, quod deſcribitur à minore, æquale parallelogrammum applicetur ad maiorem, ex qua maiore tantum excurrat extra latus parallelogrammi, quantum eſt alterum latus ipſius parallelogrammi: ſi præterea parallelogrammum ſui applicatione diuidat lineam illam in partes inter ſe commenſurabiles longitudine, illa maior linea tanto plus poteſt quam minor, quantum eſt quadratum lineæ ſibi commenſurabilis longitudine. Quòd ſi maior plus poſſit quam minor, tanto, quantum eſt quadratum lineæ ſibi commenſurabilis longitudine; & præterea quartæ parti quadrati lineæ minoris, æquale parallelogrammum applicetur ad maiorem, ex qua maiore tantum excurrat extra latus parallelogrammi, quantum est alterum latus ipsius parallelogrammi: parallelogrammũ sui applicatione diuidit maiorem in partes inter se longitudine commensurabiles.

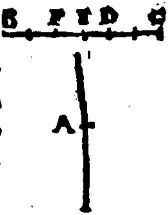


Theorema 16. Propositio 19.

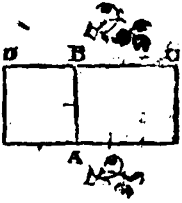
Si fuerint duæ rectæ lineæ inæquales; quartæ autem parti quadrati lineæ minoris, æquale parallelogrammorum ad lineam maiorem applicetur, ex qua linea tantum excurrat extra latus parallelogrammi, quantum est alterum latus eiusdem parallelogrammi: si parallelogrammum præterea sui applicatione diuidat lineam in partes inter se longitudine incommensurabiles, maior illa linea tantò plus potest quàm minor quantum est quadratum lineæ sibi minori commensurabiles longitudine. Quòd si maior linea tantò plus possit quàm minor, quantum est quadratum lineæ incommensurabilis sibi longitudine: & præterea quartæ parti quadrati lineæ minoris æquale parallelogrammum applicetur ad maiorem

ex

ex qua tantum excurrat extra latus parallelogrammi, quantum est alterum latus ipsius: parallelogrammum sui applicatione diuidit maiorem in partes inter se incōmensurabiles longitudine.

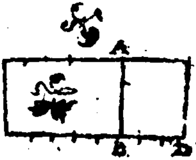


Theorema 17. Propositio 20.



Superficies rectanguli contenta ex lineis rectis rationalibus longitudine commensurabilibus secundum vnum aliquem modum ex antedictis, rationalis est.

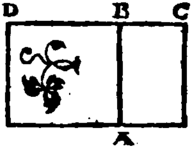
Theorema 18. Propositio 21.



Si rationale ad lineam rationalem applicetur; habebit alterum latus lineam rationalem & commensurabilem longitudine lineæ cui rationale parallelogrammum applicatur.

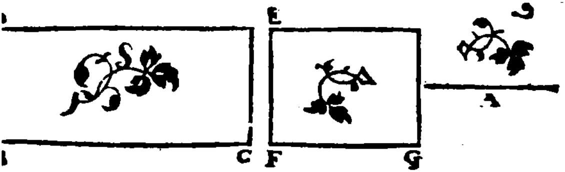
Theorema 19. Propositio 22.

Superficies rectangula contenta duabus lineis rectis rationalibus potētia tantū cōmensurabilibus, irrationalis est. Linea autem quæ illam superficiem potest, irrationalis & ipsa est; vocetur verò media.



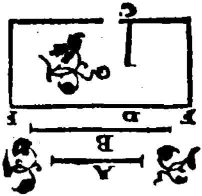
Theorema 20. Propositio 23.

Quadrati lineæ mediæ ad lineam rationalem applicāti, alterum latus est linea rationalis, & incommensurabilis longitudine lineæ, ad quam applicatur.



Theorema 21. Propositio 24.

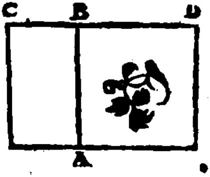
Linea recta mediæ commēsurabilis, est ipsa quoque media.



Theo-

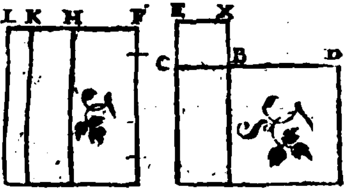
Theorema 22. Propoſitio 25.

Parallelogrammum rectangulum contentum ſub rectis lineis medijs longitudine commenſurabilibus, medium eſt.



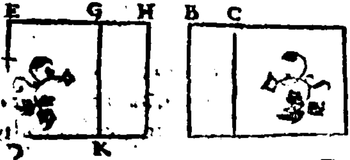
Theorema 23. Propoſitio 26.

Parallelogrammum rectangulum comprehenſum ſub duabus lineis medijs potentia tantum commenſurabilibus; NMG vel rationale eſt, vel medium.

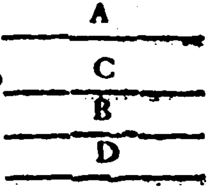


Theorema 24. Propoſitio 27.

Medium non eſt maius, quàm medium ſuperficie rationali.

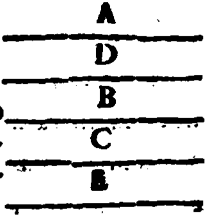


Próblema 4. Propositio 28



Medias lineas inuenire potentia tantùm commensurabiles rationale comprehendentes.

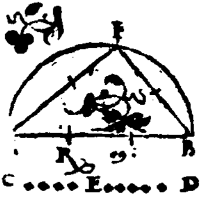
Problema 5. Propositio 29



Medias lineas inuenire potentia tantùm commensurabiles, medium comprehendentes.

Problema 6. Propositio 30.

Reperire duas rationales potentia tantum commẽsurabiles huiusmodi, vt maior ex illis possit plus, quàm minor, quadrato rectæ lineæ sibi commensurabilis longitudine.



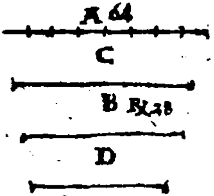
Problema 7. propositio 31.

Inuenire duas rationales potentia tantùm commensurabiles; ita vt maior, quam minor plus possit, quadrato rectæ lineæ sibi longitudine incommensurabilis.

pro-

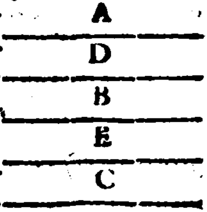
Problema 8. propositio 32.

Reperire duas lineas medias potentia tantùm commensurabiles rationalem superficiem continentes tales inquam, vt maior possit plus, quàm minor, quadrato rectæ lineæ sibi commensurabilis longitudine.



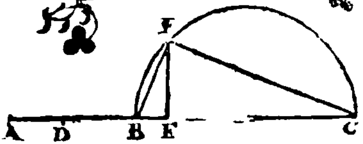
Problema 9. Propopositio 33.

Inuenire duas lineas medias potentia tantùm cõmensurabiles, quæ mediam superficiem contineant, ita vt maior plus possit, quam minor, quadrato rectæ lineæ sibi longitudine commensurabilis.



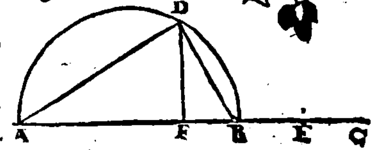
Problema 10. Propositio 34.

Inuenire duas rectas lineas potentia incommensurabiles; quarum quadrata simul composita faciant superficiem rationalem: Rectangulum vero sub ipsis contentum, faciant medium.



Problema 11. Propoſitio. 35.

Reperire lineas duas rectas potentia incommenſurabiles, conficientes compoſitum ex ipſarum quadratis medium parallelogrammũ verò ex ipſis contentum rationale.



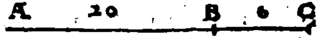
Problema 12. Propoſitio 36.

Reperire duas lineas rectas potentia incommenſurabiles, conficientes id, quod ex ipſarum quadratis componitur, medium, parallelogrãmum ex ipſis contentum, mediũ; quod præterea parallelogrammum ſit incommẽſurabile compoſito ex quadratis ipſarum.

PRINCIPIVM SENARIOrum per compositionem, & Synthesin.

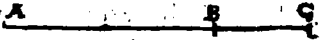
Theorema 25. Propositio 37.

Si duæ rationales potentia tantùm commésurabilis componantur; tota irrationalis erit. Vocetur autem Binomium, vel ex binis nominibus.



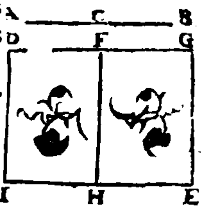
Theorema 26. Propositio 38.

Si duæ mediæ potentia tantùm commensurabiles, rationale continentes, componantur; tota linea est irrationalis, vocetur autem ex binis medijs prima.



Theorema 27, Propositio 39.

Si duæ mediæ potentia tantùm commésurabiles medium continentes cóponantur; tota linea est irrationalis: vocetur auté ex binis medijs secunda.



Theorema 28. Propositio 40.

Si duæ rectæ lineę potentia incommensurabiles componantur; conficientes compositum ex quadratis ipsarum, rationale paral-

 lelo-

lelogrammum verò ex ipsis contentum, medium; tota li

nea recta est irrationalis. Vocetur autem linea maior.

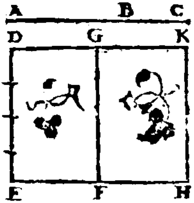
Theorema 29. Propositio 41.

Si duæ rectæ lineæ potentia incommensurabiles componantur, conficientes compositum ex ipsarum quadratis, medium: id verò, quod fit ex ipsis, rationale; tota recta linea est irrationalis erit.

Vocetur autem potens rationale & medium.

Theorema 30. Propositio 42.

Si duæ rectæ lineę potentia incommensurabiles componantur, conficientes compositum ex ipsarum quadratis medium; & quod continetur ex ipsis, medium; & præterea incommensurabile composito ex quadratis ipsarum: tota recta linea est irrationalis. Vocetur autem bina media potens.



Theorema 31. Propositio 43.

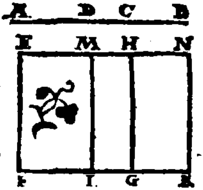
Quæ linea ex binis nominibus vocata, in vnico tantum puncto diuiditur in sua nomina.

Theo-

Theorema 32. Propositio 44.

Quæ ex binis medijs prima, in vnica tantū puncto diuiditur in sua nomina.

Theorema 33. Propositio 45.



Quæ ex binis medijs secunda, in vnico tantum puncto diuiditur in sua nomina.

Theorema 34. Propositio 46.

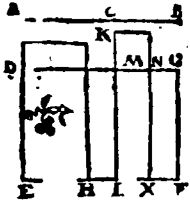
Linea maior in vnico tantùm puncto diuiditur in sua nomina.

Problema 35. Propositio 47.

Linea potens rationale & medium, in vnico tantùm puncto, diuiditur in sua nomina.

Theorema 36. Propositio 48.

Linea potens duo media in vnico tantum puncto diuiditur in sua nomina.



DE-

DEFINITIONES secundæ, nempe binorum nominum.

Proposita linea rationali; & linea ex binis nominibus vocata, diuisa, in sua nomina, cuius maius nomen, id est, maior portio, possit plusquam minus nomen; quadrato lineæ sibi, maiori inquam nomini, commensurabilis longitudine.

1.

Si quidem maius nomen fuerit commensurabile longitudine propositæ lineæ rationali; Vocetur tota linea composita ex binis nominibus, prima.

2.

Si verò minus nomen, id est, minor portio, fuerit commensurabile longitudine propositæ lineæ rationali; Vocetur tota linea ex binis nominibus secunda.

3.

Si verò neutrum ipsorum nominum fuerit commensurabile longitudine propositæ lineæ rationali; Vocetur tota ex binis nominibus tertia.

Rursus si maius nomen possit plusquam minus nomen, quadrato lineæ sibi incommensurabiles longitudine.

4.

Si quidem maius nomen sit commensurabi-

rabi-

rabile longitudine propositæ lineæ rationali; Vocetur tota linea ex binis nominibus quarta.

5.

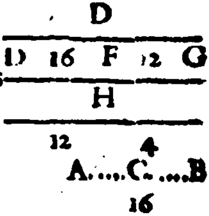
Si verò minus nomen fuerit commensurabile longitudine lineæ rationali; Vocetur tota ex binis nominibus quinta.

6.

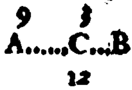
Si verò neutrum ipsorum nominum fuerit longitudine commensurabile: propositæ lineæ rationali; Vocetur tota ex binis nominibus sexta.

Problema 13. Propositio 49.

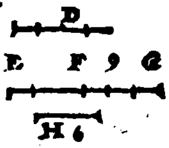
Reperire lineam ex binis nominibus primam.



Problema 14. Propositio. 50.



Reperire ex binis nominibus secundam.



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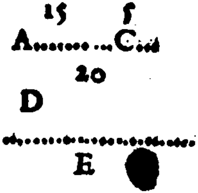
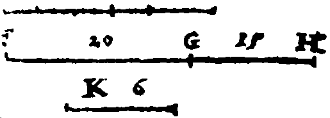
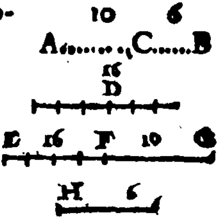
Problema 15. Propositio 51.



reperire ex binis nominibus tertiã

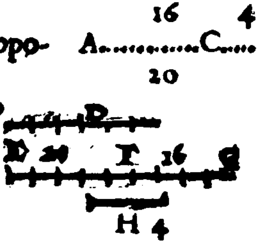


Problema 16. Propositio 52.



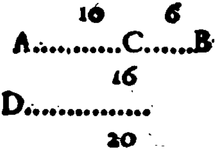
Reperire ex binis nominibus quartam.

Problema 17. Propositio 53.



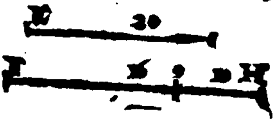
Reperire ex binis nominibus quintam.

Problema 18. Propositio 54.



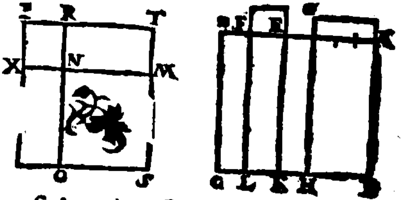
Reper-

Reperire ex binis nominibus sextam.



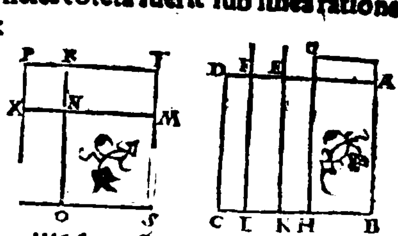
Theorema 37. Propositio 55.

Si superficies contenta fuerit sub rationali, & ex binis nominibus prima recta linea, quæ illam superficiem potest, est irrationalis; quæ ex binis nominibus vocatur.



Theorema 38. Propositio 56.

Si superficies cõtẽta fuerit sub linea rationali, & ex binis nominibus secunda; Recta linea potens illã superficiem, est irrationalis; quæ ex binis medijs prima vocatur.



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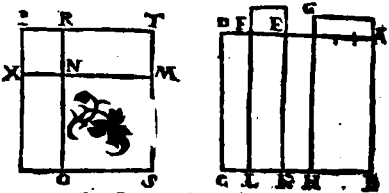
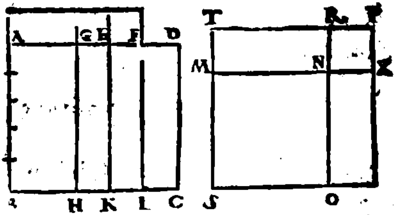
Theorema 39. Propoſitio 57.

Si ſuperficies contineatur ſub rationali, & ex binis nominibus tertia; recta linea, qua illam ſuperficiem poteſt, eſt irrationalis; quæ ex binis medijs dicitur tertia.



Problema 40. Propoſitio 58.

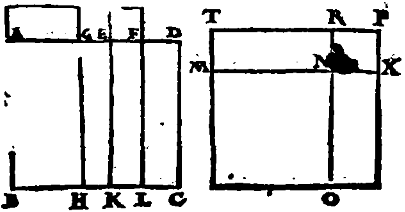


Si ſuperficies contineatur ſub rationali, & ex binis nominibus quarta; recta linea potens, ſuperficiem illam, eſt irrationalis; quæ dicitur maior.

Theorema 41. Propoſitio 59.

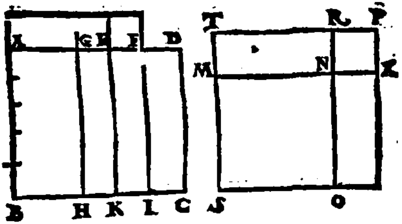
Si ſuperficies contineatur ſub rationali, & ex binis

binis nominibus quarta; recta linea, quæ illam superficiem potest, est irrationalis; quæ dicitur potens rationale, & medium.



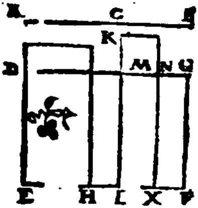
Theorema 42. Propositio 60.

Si superficies contineatur sub rationali, & ex binis nominibus sexta; Recta linea, quæ illam superficiem potest, est irrationalis; quæ dicitur potens, bina media.

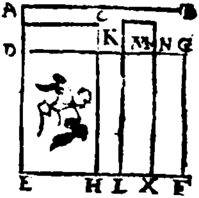


Theorema 43. Propositio 61.

Quadratum eius lineæ, quæ est ex binis nomini-

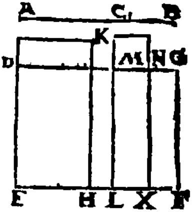


minibus, ad lineam rationalē applicatum, facit latitudinem ex binis nominibus primam.



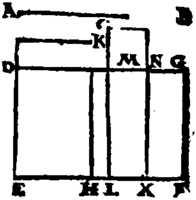
Theorema 44. Propositio 62.

Quadratum, eius quæ est ex binis medijs prima, ad lineam rationalem lineam applicatum, facit latitudinem ex binis nominibus secundam.



Theorema 45. Propositio 63.

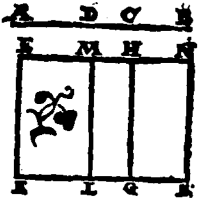
Quadratum eius, quæ est ex binis medijs secunda, ad lineam rationalem applicatum, facit latitudinē ex binis nominibus tertiam.



Theorema 46. Propositio 64.

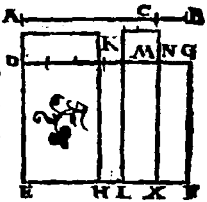
Quadratum lineæ maioris secundum lineam rationalem applicatum, facit latitudinem ex binis nominibus quartam.

Theorema 47. Propositio 65.



Quadratum lineæ poté tis rationale, & medium secundum rationalem applicatum, facit latitudinem ex binis nominibus quintam.

Theorema 48. Propositio 66.

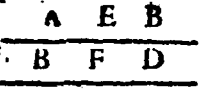


Quadratum lineæ poten tis duo media, secundum rationalem applicatum, facit latitudinem ex binis nominibus sextam.

Theorema 49. Propositio 67.

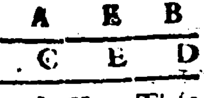
Linea longitudine commensurabilis, ei lineę quę est ex binis nominbus; & ipsa ex binis nominibus est, atque in ordine eadem.

Theorema 50. Propositio 68.



Linea longitudine commensurabilis alteri lineæ quæ est ex binis Medijs; & ipsa ex binis medijs est, atque in ordine eadem.

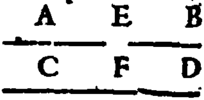
Theorema 51. Propositio 69.



Linea commensurabilis lineæ maiori, & ipsa maior est. K 2 Theo

Theorema 52. Propoſitio 70.

Linea commenſurabilis lineæ potenti rationale & medium, eſt & ipſa linea potens rationale & medium.

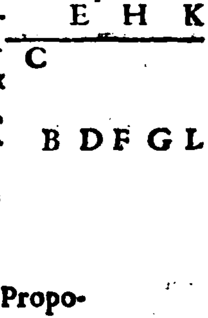


Theorema 53. Propoſitio 71.

Linea commenſurabilis lineæ potenti duo media, eſt & ipſa linea potens duo media.

Theorema 54. Propoſitio 72.

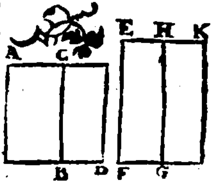
Si duæ ſuperficies, rationalis, & media ſimul componantur, linea quę totam ſuperficiem compoſitam poteſt, eſt vna ex quatuor irrationalibus; vel ea, quæ dicitur ex binis nominibus, vel ea, quæ ex binis medijs prima, vel linea maior, vel linea potens, rationale & medium.



Theorema 55. Propoſitio 73.

Si duæ ſuperficies mediæ inter ſe incommē ſura-

surabiles simul compônantur; fiunt reliquę duę lineæ irrationales; & ex binis medijs secunda, vt bina media potens.



SCHOLIVM EX THEONE, ZAMberto, Campano, & P. Christ. Clauio.

Ex his omnibus facilè colligitur, quod lineæ ea, quæ est ex binis nominibus, & cætera ipsam subsequentes, lineæ irrationales, neque sunt eadem cum linea media, neque ipsæ inter se sunt eadem.

Nam quadratum lineæ mediæ, ad lineam rationalem comparatum & applicatum, efficit alterum latus, lineam rationalem, seu latitudinem rationalem, ipsi lineæ rationali (hoc est, lineæ, ad quã applicatur) longitudine incommensurabilem: per propos. 23. libri decimi.

Quadratum verò eius lineæ, quæ est ex binis nominibus, ad rationalem, applicatum, efficit alterum latus, & lineam, seu latitudinem ex binis nominibus primam: per 61.

Quadratum verò eius, quæ est ex binis mediis prima ad Rationalem applicatum, latitudinem efficit ex binis nominibus secundam: per 62.

Quadratum verò eius, quæ est ex binis mediis secunda, ad rationalem applicatum, latitudinem ef-

ficit ex binis nominibus tertiam: per 63.

Quadratum lineæ maioris, ad rationalem applicatum, latitudinem efficit ex binis nominibus quartam: per 64.

Quadratum vero eius, quæ rationale, & medium potest, ad rationalem applicatum; efficit alterum latus, seu latitudinem ex binis nominibus quintam: per 65.

Quadratum denique lineæ eius, quæ bina media potest, ad rationalem applicatum, efficit alterum latus, seu latitudinem ex binis nominibus sextam: per 66.

Cùm igitur hæ latitudines (quæ à nonnullis lateræ dicuntur.) differant, & à latitudine media & inter se, à latitudine quidem media, quod hæc rationalis sit, illæ verò irrationales, inter se autem, quod in ordine non sint eædem cum iis ex binis nominibus: manifestum est omnes ipsas irrationales lineas, de quibus hactenus dictum est, inter se differentes esse.

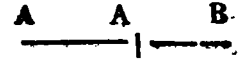
PRINCIPIVM SENARIORVM per detractionem, & aphæresin.

Theorema 56. Propositio 74.

Si de linea rationali detrahatur rationalis, potentia tantùm commensurabilis ipsi toti: Residua est irrationalis: Vocetur autem residuum, hoc est, apotome.

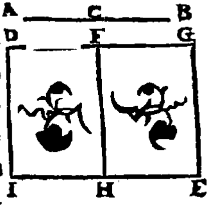
Theorema 57. Propositio 75.

Si de linea media detrahatur media, potentia tantum commensurabilis toti lineæ; quæ verò detracta est, cum tota contineat superficiem rationalem; Residua est irrationalis. Vocetur autem Residuum mediũ primum: hoc est, mediæ apotome prima.



Theorema 58. Propositio 76.

Si de linea media detrahatur media, potentia tantùm commensurabilis toti; quæ verò detracta est, cum tota cõtineat superficiem mediam: Reliqua est irrationalis. Vocetur autem Residuum medium secundum, hoc est, mediæ apotome secunda.



Theorema 59. Propositio 77.

Si de linea recta detrahatur recta, potentia incommensurabilis toti; compositum autem ex quadratis totius lineæ, & lineæ detractæ, sit rationale; parallelogrammum verò ex

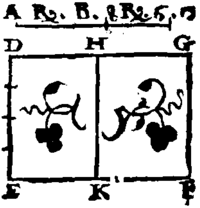
rò ex eiſdem contentum, ſit medium: Reliqua linea erit irrationalis. A C B
Vocetur autem linea minor.

Theorema 60. Propoſitio. 78.

Si de linea recta detrahatur recta, potentia incõmenſurabilis toti lineæ; compoſitum autem ex quadratis totius, & lineæ detractæ ſit medium; parallelogrammum verò bis ex eiſdem contentum, ſit rationale; Reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie rationali totam ſuperficiem mediam. A C B

Theorema 61. Propoſitio 79.

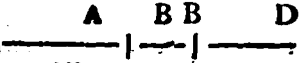
Si de linea recta detrahatur recta potentia incommenſurabilis toti lineæ: compoſitũ autem ex quadratis totius, & lineæ detractæ, ſit medium: Parallelogrammum verò bis ex ijſdem ſit etiam medium: præterea ſint quadrata ipſarum incommenſurabilia parallelogrammo bis ex ijſdem contento, Reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie media totam ſuperficiem mediam.



Theo-

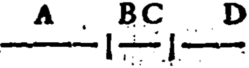
Theorema 62. Propoſitio 80.

Reſiduo vnica tantùm linea recta coniungitur rationalis, potentia tantùm commenſurabilis toti lineæ.



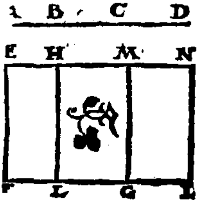
Theorema 63. Propoſitio 81.

Reſiduo medio primo vnica tantùm linea coniũgitur media, potentia tantùm commenſurabilis toti, ipſa cum tota rationale continens.



Theorema 64. Propoſitio 82.

Reſiduo medio ſecundo vnica tantùm coniũgitur recta linea media, potẽtia tantùm commenſurabilis toti, ipſa cum tota medium continens.



Theorema 65. Propoſitio 83.

Lineæ minori vnica tantum recta linea coniungitur, potentia incommenſurabilis toti, faciens cum tota compoſitum ex quadratis ipſa-

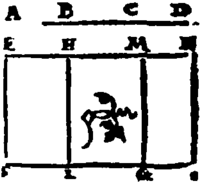
ipſarum rationale;id verò parallelogrammum, quod bis ex ipſis fit, medium.

Theorema 66. Propoſitio 84.

Lineæ facienti cum ſuperficie rationali totam ſuperficiem mediam, vnica tantùm cõiungitur linea recta, potentia incommenſurabilis toti; faciens autem cum tota compoſitum ex quadratis ipſarum, medium; Id verò, quod fit bis ex ipſis, rationale.

Theorema 67. Propoſitio 85.

Lineæ cum media ſuperficie facienti totam ſuperficiem mediã, vnica tantum coniungitur linea, potentia toti incõmenſurabilis, faciens cũ tota compoſitum ex quadratis ipſarum, medium, id verò, quod bis ex ipſis etiam medium: & præterea faciens compoſitum ex quadratis ipſarum incommenſurabile ei, quod fit bis ex ipſis



DE·

DEFINITIONES.

TERTIAE, NEMPE APOTOMARUM, seu Residuorum.

Proposita linea rationali, & Residuo, si tota népe composita ex ipso Residuo, & linea illi coniuncta, seu congruente, plus possit, quá coniuncta, quadrato rectæ lineæ sibi longitudine commensurabilis;

I.

Si quidé tota lineæ propositæ rationali sit longitudine commensurabilis; Vocetur residuum primum, seu Apotome prima.

2.

Si verò coniuncta fuerit longitudine commensurabilis propositæ rationali; ipsa autem tota plus possit, quam coniuncta, quadrato lineæ, sibi longitudine commensurabilis; Vocetur Residuum secundú, seu Apotome secunda.

3.

Si verò neutra linearum fuerit longitudine commensurabilis propositæ rationali; possit autem ipsa tota plusquam coniuncta, quadrato lineæ sibi longitudine commensurabilis; Vocetur Residuum tertium, seu Apotome tertia.

Rursus si tota possit plus, quàm coniuncta seu congruens, quadrato rectę lineæ sibi longitudine incommensurabilis;

4. Et

4.

Et quidem ſi tota fuerit longitudine commenſurabilis ipſi rationali; Vocetur Reſiduum quartum, ſeu Apotome quarta.

5.

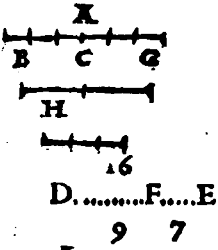
Si verò coniuncta fuerit longitudine cõmenſurabilis ipſi rationali; & tota plus poſſit, quam coniuncta, quadrato lineæ ſibi lõgitudine incommenſurabilis, Vocetur Reſiduum quintum, ſeu Apotome quinta.

6.

Si denique neutra linearum fuerit commenſurabilis longitudine ipſi rationali; fueritque tota potentior, quam coniuncta; quadrato lineæ ſibi longitudine incommenſurabilis; Vocetur Reſiduum ſextum, ſeu Apotome ſexta.

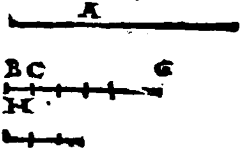
Problema 19. Propoſitio 86.

Reperire primum Reſiduum.



Problema 20. Propoſitio 87.

Reperire ſecundum Reſiduum.



Pro

D........F....E

27 9

E..........

Problema 21. Propositio 88.

12

B.........E....C

9 7

Reperire tertium Residuum.

F H G

K

Problema 22. Propositio 89.

A

B C G

H

Reperire quartum Residuum.

D...........F....E

16 4

Problema 23. Propositio 90.

A

B C G

H

Reperire quintum Residuum.

D...........F.....E

25 7

Problema 24. Propositio 91.

F H G

Reperire sextum Residuum.

K

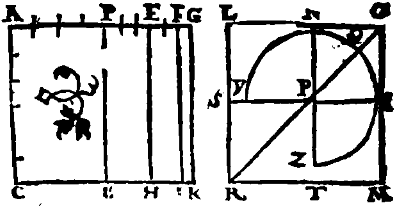
E.......... 13

B......D.....C

Theo-

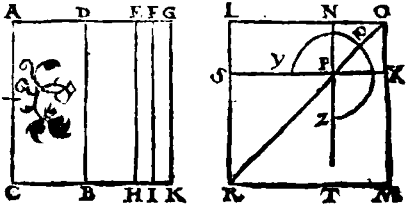
Theorema 68. Propositio 92.

Si superficies contineatur sub linea rationali, & residuo primo; recta linea, quæ illam superficiem potest, est residuum.



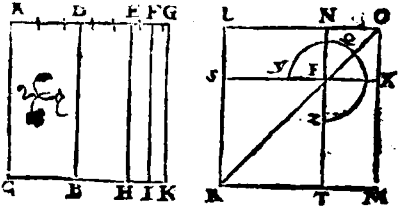
Theorema 69. Propositio 93.

Si superficies contineatur sub linea rationali, & residuo secundo; recta lineæ, quæ illam superficiem potest, est residuum mediale primum, seu mediæ Apotome prima.



Theorema 70. Propositio 94.

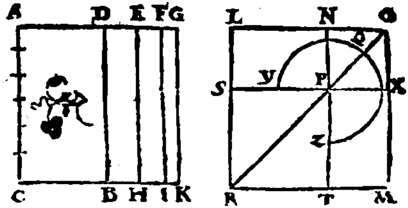
Si superficies cõtineatur sub linea rationali,



tionali, & reſiduo tertio; recta linea, quæ illam ſuperficiem poteſt, eſt reſiduum mediale ſecundum, ſeu mediæ eſt Apotome ſecunda.

Theorema 71. Propoſitio 95.

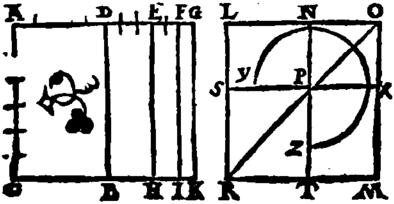
Si ſuperficies contineatur ſub linea rationali, & reſiduo quarto; recta linea, quæ



illam ſuperficiem poteſt, eſt linea minor.

Theorema 72. Propoſitio 96.

Si ſuperficies contineatur ſub linea rationali, & reſiduo quinto; recta linea, quæ illam ſuperficiem poteſt, eſt ea, quæ dicitur cum

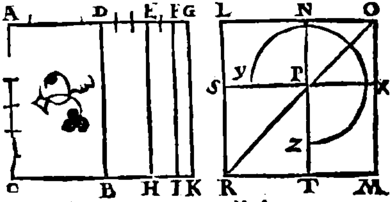


rationali ſuperficie faciẽs totam medialem.

Theorema 73. Propoſitio 97.

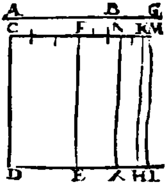
Si ſuperficies cõtineatur ſub linea rationali, &

& Residuo sexto; recta linea, quæ illam superficiē potest, est ea, quæ dicitur faciens cum



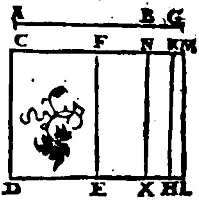
mediali superficie totam medialem.

Theorema 74. Propositio 98.



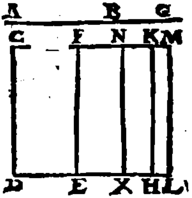
Quadratum residui ad lineam rationalem applicatum, facit alterum latus residuum primum.

Theorema 75. Propositio 99.



Quadratum residui medialis primi ad rationalem applicatum, facit alterum latus, residuum secundum.

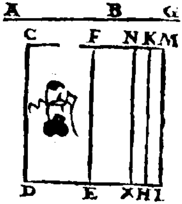
Theorema 76. Propositio 100.



Quadratum residui medialis secundi ad rationalem applicatum, facit alterum latus residuum tertium.

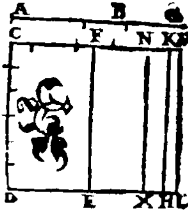
Theo-

Theorema 77. Propositio 101.



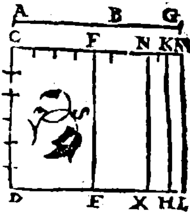
Quadratum lineę minoris ad rationalem applicatum, facit alterum latus residuum quartum.

Theorema 78. Propositio 102.



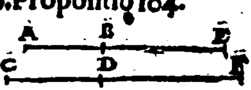
Quadratum lineæ cum rationali superficie facientis totam medialem, ad rationalem applicatum, facit alterum latus residuum quintum.

Theorema 79. Propositio 103.



Quadratum lineæ cum mediali superficie faciẽtis totam medialem, ad rationalem applicatum, facit alterum latus residuum sextum.

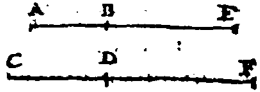
Theorema 80. Propositio 104.



Recta linea residuo commensurabilis longitudine; est & ipsa residuum, seu in ordine eadem.

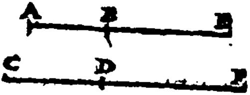
Theorema 81. Propositio 105.

Recta linea commensurabilis residuo mediali, est & ipsa residuum mediale,& eiusdem ordinis; seu in ordine eadem.



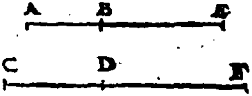
Theorema 82. Propositio 106.

Recta linea cómensurabilis lineæ minori: est& ipsa linea minor.



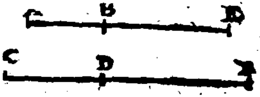
Theorema 83. Propositio 107.

Recta linea commensurabilis lineæ cum rationali superficie facienti totam medialem; est& ipsa linea cum rationali superficie faciens totam medialem.



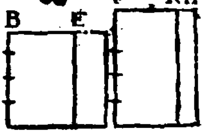
Theorema 84. Propositio 108.

Recta linea commensurabilis lineæ cum mediali superficie facienti totam medialem; est & ipsa cum mediali superficie faciens totam medialem.



Theorema 85. Propositio 109.

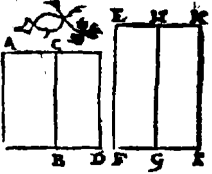
Si de superficie rationali detrahatur superficies medialis; recta linea, quæ reliquam superficiem potest, est alterutra ex duabus irrationalibus, aut residuum, aut linea minor.



Theo-

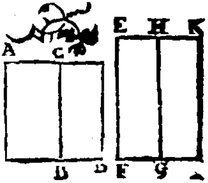
Theorema 86. Propositio 110.

Si de superficie mediali detrahatur superficies rationalis; aliæ duæ irrationales fiunt, aut residuum mediale primum, aut cum rationali superficies totam medialem.



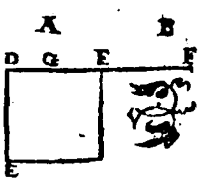
Theorema 87. Propositio 111.

Si de superficie mediali detrahatur superficies medialis, quæ sit incommẽsurabilis toti; reliquæ duæ fiunt irrationales, aut residuum mediale secũdum, aut cum mediali superficie faciẽs totam medialem.



Theorema 88. Propositio 112.

Linea, quæ residuum dicitur, non est eadem cum ea, quæ dicitur Binomium.



SCHOLIVM EX THEONE, ZAMberto, Campano, & P. Clauio.

Ex his demonstratis facilè intelligitur, quod recta linea, quæ residuum dicitur, & cætera quinque eam consequētes irrationales, neque lineæ mediali, neque sibi ipsis inter se sunt eadem.

Nam quadratum lineæ medialis secundum rationalem applicatum, facit alterum latus, rationalem lineam, longitudine incommensurabilem ei, seu ad quam applicatur, per propos. 23. libri decimi.

Quadratum, verò residui secundum rationalem applicatum, facit alterum latus, residuum primum per 98.

Quadratum verò residui medialis primi secundum rationalem applicatum, facit alterum latus, residuum secundum per 99.

Quadratum verò residui medialis secundi, facit alterum latus, residuum tertium, per 100.

Quadratum verò lineæ minoris, facit alterum latus residuum quartum, per 101.

Quadratum verò lineæ cum rationali superficie facientis totam medialem. facit alterum latus residuum quintum, per 102.

Quadratum verò lineæ cum mediali superficie facientis totam medialem, secundum rationalem applicatum, facit alterum latus residuum sextum, per 103.

Cùm

Cùm igitur dicta latera, quæ sunt latitudines cuiusque parallelogrammi vnicuique quadrato æqualis, & ad rationalem, applicati, differant & à primo latere, & ipsa inter se, (nam à primo differunt: quoniam sunt residua non eiusdem ordinis) constat ipsas quoque lineas irrationales inter se differentes esse. Et quoniam demonstratum est, residuum non esse idem quod Binomium: quadrata autem residui, & quinque linearum irrationalium illud consequentium, ad rationalem applicata, faciunt altera latera ex residuis eiusdem ordinis, cuius sunt & residua, quorum quadrata applicantur rationali, similiter & quadrata Binomij, & quinque linearum irrationalium illud consequentium, ad rationalem applicata, faciunt altera latera ex Binomijs eiusdem ordinis, cuius sunt & Binomia, quorum quadrata applicantur rationali. Ergo lineæ irrationales, quæ consequuntur Binomium, & quæ consequuntur residuum, sunt inter se differentes. Quare dictæ lineæ omnes irrationales sunt numero, 13. subsequentes.

1. Media linea; quæ vulgò medialis appellatur: propos. 22.
2. Linea ex binis nominibus (vulgò Binomium;) cuius sex sunt species inuentæ: propos. 37.
3. Ex binis medijs prima; vulgò Bimediale primum: propos. 38.
4. Ex binis medijs secunda; vulgò Bimediale secundum: propos. 39.
5. Maior. propos. 40.

6. Linea

6. Linea rationale, ac medium potens: propos. 41.

7. Bina media potens: propos. 42

8. Apotome (vulgò residuum:) cuius etiam species sex sunt reperta: propos. 74

9. Media Apotome prima; vulgò residuum: propos. 75

10. Media Apotome secunda, vulgò residuum mediale secundum: propos. 76

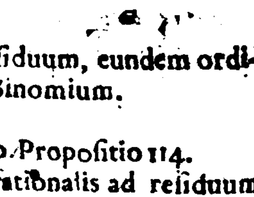
11. Minor: propos. 77

12. Linea cum rationali medium totum efficiens; vulgò linea cum rationali superficie totam medialem faciens: propos. 78

13. Cum medio medium totum efficiens; vulgò linea cum mediali superficie totam medialem faciens: propos. 79

Theorema 89. Propositio 113.

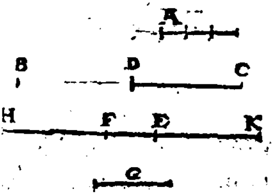
Quadratum lineæ rationalis ad Binomium applicatum, facit alterum latus residuum, cuius nomina sunt commensurabilia Binomij nominibus, & in eadem proportione præterea id, quod fit residuum, eundem ordinem retinet, quem Binomium.



Theorema 90. Propositio 114.

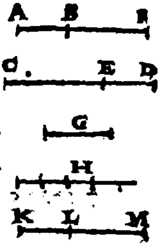
Quadratum lineæ rationalis ad residuum applicatum, facit alterum latus Binomium, cuius

cuius nomina sunt commensurabilia nominibus residui & in eadem proportione; præterea id, quod fit Binomium est eiusdem ordinis, cuius & residuum.



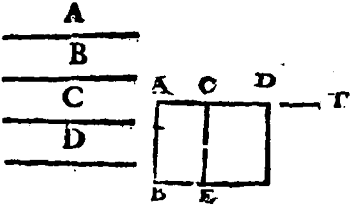
Theorema 91. Propositio 115.

Si parallelogrammum contineatur ex residuo, & Binomio, cuius nomina sunt commensurabilia nominibus residui, & in eadem proportione; recta linea, quæ illam superficiem potest, est rationalis.



Theorema 92. Propositio. 116.

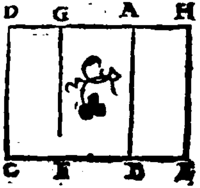
Ex linea media nascuntur lineæ irrationales innumerabiles, quarum nulla vlli antedictarum eadem sit,



Theorema 103. Propositio 117.

E...H...E
E....

Propositum sit nobis demonstrare, in figuris quadratis diametrum esse lógitudine incommensurabilem ipsi lateri.



FINIS ELEMENTI X.

EVCLIDIS ELEMENTVM VNDECIMVM ET SOLIDORVM primum.

DEFINITIONES.

1.

Solidum, est quod longitudinem, latitudinem, & crassitudinem habet.

2.

Solidi autem extremum est superficies.

3.

Linea recta est ad planum recta, cùm ad rectas omnes lineas, à quibus illa tãgitur, quæque in proposito sunt plano, rectos angulos efficit.

4.

Planũ ad planũ rectum est, cùm rectæ lineæ, quæ cõmuni planorũ sectioni ad rectos angulos in vno planorũ ducũtur, alteri plano ad recto sunt angulos.

5.

Rectæ lineæ ad planum inclinatio est angulus acutus, ipsa insistente linea, & adiuncta altera comprehensus, cùm à sublimi rectæ illius lineæ termino deducta fuerit perpendicularis, atque à puncto, quod perpendicularis in ipso plano fecerit, ad propositæ illius lineæ extremum, quod in eodem est plano, altera recta linea fuerit adiuncta.

 6. Plani

6.

Plani ad planum inclinatio, est angulus acutus rectis lineis contentus, quæ in vtroque planorum ad idem communis sectionis púctum ductæ, rectos ipsi sectioni angulos efficiunt.

7.

Planum similiter inclinatum esse ad planum, atq; alterum ad alterum dicitur, cùm dicti inclinationum anguli inter se sunt æquales.

8.

Parallela plana sunt, quæ inter se non incidunt, nec concurrunt.

9.

Similes figuræ solidæ sunt, quæ similibus planis, multitudine æqualibus continentur.

10.

Aequales, & similes figuræ solidæ sunt, quæ similibus planis, multitudine & magnitudine æqualibus continentur.

11.

Solidus angulus est plurimum, quàm duarum linearum, quæ se mutuò contingant, nec in eadem sint superficie, ad omnes lineas inclinatio.

Aliter.

Solidus angulus est, qui pluribus, quam duobus planis angulis, in eodem plano non consistentibus, sed ad vnum punctum collectis continetur.

12. Pyra-

12.

Pyramis est figura solida, quæ planis continetur, ab vno plano ad vnum punctum collecta.

13.

Prisma figura est solida, quæ planis continetur, quorum aduersa duo sunt, & æqualia, & similia, & parallela; alia verò parallelogramma.

14.

Sphæra est figura, quæ conuerso circum quiescentem diametrum semicirculo continetur, cùm in eundem rursus locum restitutus fuerit, vnde moueri cœperat.

Aliter ex Theodosio.

Sphæra est figura solida, sub vna superficie comprehensa, ad quam ab vno puncto eorum, quæ intra figuram sunt posita, cadentes omnes rectæ lineæ, inter se sunt æquales.

15.

Axis autem sphæræ est, quiescens illa linea recta, per centrum ducta, circum quam semicirculus conuertitur.

16

Centrum verò Sphæræ est idem, quod & semicirculi.

17.

Diameter autem sphæræ est, recta quædam linea per centrum ducta, & vtrinque à sphæræ superficie terminata.

18. Conus

18.

Conus est figura, quæ sub conuerso circum quiescens alterum latus eorum, quę rectum angulum continent, orthogonio triangulo continetur, cum in eundem rursus locum illud triangulum restitutum fuerit, vnde moueri cœperat.

Atque si quiescens recta linea æqualis sit alteri, quæ circum rectum angulum conuertitur, orthogonius erit Conus: sin minor, amblygonius: si verò maior, oxygonius.

19.

Axis autem Coni est, quiescens illa recta linea, circum quam triangulum vertitur.

20.

Basis verò Coni est, circulus qui à circumducta linea recta describitur.

21.

Cylindrus est figura, quæ sub conuerso circumquiescens alterum latus eorum, quæ rectum angulum continent, parallelogrammo orthogonio compræhenditur, cum in eundem rusus locum restitutum fuerit illud parallelogrammum, vnde moueri cœperat.

22.

Axis autem Cylindri est quiescens illa recta linea, circum quam parallelogrammum vertitur.

23. Bases

23.

Bases verò cylindri sunt circuli, à duobus aduersus lateribus, quæ circum aguntur, descripti.

24.

Similes coli, & cylindri sunt, quorum & axes, & basium diametri proportionales sunt.

25.

Cubus hexandrum est figura solida, quæ sub sex quadratis æqualibus continetur.

26.

Tetraedrum est figura solida, quæ sub triangulis quatuor æqualibus, & æquilateris continetur.

27.

Octaedrum figura est solida, quæ sub octo triangulis æqualibus, & æquilateris continetur.

28.

Dedecaedrum figura est solida, quæ sub duodecim pentagonis æqualibus, æquilateris, & æquiangulis continetur.

29.

Eicosaedrum figura est solida, quæ sub triangulis viginti æqualibus, & ęquilateris cótinetur.

30.

Parallelepipedum est figura solida, quæ sub sex figuris quadrilateris, quarum quæ ex aduerso, parallelæ sunt, continetur.

31. Soli-

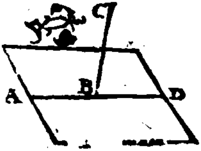
31.

Solida figura in ſolida dicitur inſcribi, quando omnes anguli figuræ inſcriptæ conſtituuntur, vel in angulis, vel in lateribus, vel deniq; in planis figuræ, cui inſcribitur.

32.

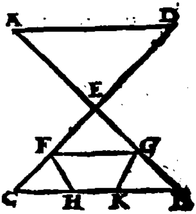
Solida figura ſolidæ figuræ viciſſim circumſcribi dicitur, quando vel anguli, vel latera, vel deniq; plana figuræ circumſcriptæ tangunt omnes angulos figuræ, circum quam deſcribitur.

Theorema 1. Propoſitio 1.



Quædã rectæ lineæ pars in ſubiecto quidem non eſt plano, quædam verò in ſublimi.

Theorema 2. Propoſitio 2.



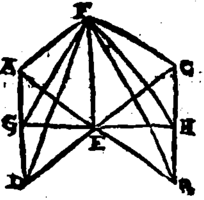
Si duæ rectæ lineæ ſe mutuo ſecent, in vno ſunt plano: atque triangulum omne in vno eſt plano.

Theorema 3. Propoſitio 3.

Si duo plana ſe mutuò ſecent, communis eorum ſectio eſt recta linea.

Theo-

Theorema 4. Propoſitio 4.

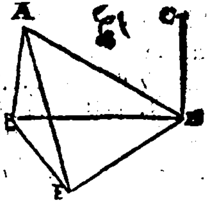


Si recta linea, rectis duabus lineis ſe mutuò ſecantibus, in communi ſectione ad rectos angulos inſiſtat: illa, ducto etiam per ipſas plano, ad angulos rectos erit.

Theorema 5. Propoſitio 5.

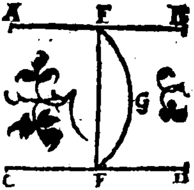
Si recta linea, rectis tribus lineis ſe mutuò tangentibus, in communi ſectione ad rectos angulos inſiſtat: illæ tres rectæ in vno ſunt plano.

Theorema 6. Propoſitio 6.



Si duæ rectę lineę eidem plano ad rectos ſint angulos: parallelæ erunt illæ rectæ lineæ.

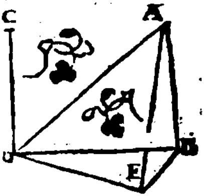
Theorema 7. Propoſitio 7.



Si duæ ſint parallelæ rectæ lineæ, in quarum vtraq; ſumpta ſint quælibet punctà: illa linea, quę ad hæc puncta adiungitur, in eodem eſt cum parallelis plano.

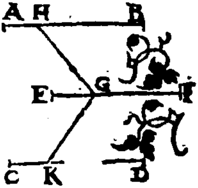
Theo-

Theorema 8. Propositio 8.



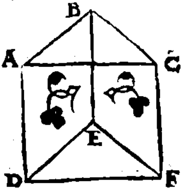
Si duæ sint parallelæ rectæ lineæ, quarum altera ad rectos cuidam plano sit angulos: & reliqua eidem plano ad rectos angulos erit.

Theorema 9. Propositio 9.



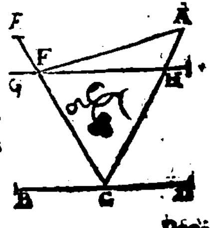
Quæ eidem rectæ lineæ sunt parallelæ, sed non in eodem cum illa plano: hę quoque sunt inter se parallelæ.

Theorema 10. Propositio 10.



Si duæ rectæ lineę se mutuò tangentes ad duas rectas se mutuò tangentes sint parrallelæ, non autē in eodem plano; illæ angulos æquales comprehendent.

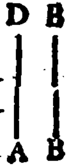
Problema 1. Propositio 11.



A dato puncto in sublimi, ad subiectum planū perpendicularem rectā lineam ducere.

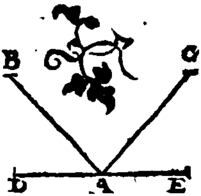
Pro-

Problema 2. Propositio. 12.



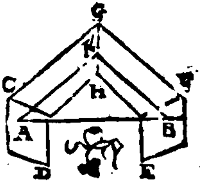
Dato plano, à puncto, quod in illo datum est, ad rectos angulos rectam lineam excitare.

Theorema 11. Propositio 13.



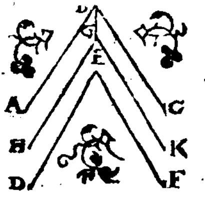
Dato plano, à pũcto quod in illo datum est, duæ rectæ lineæ ad rectos angulos non excitabuntur, ad easdem partes.

Theorema 12. Propositio 14.



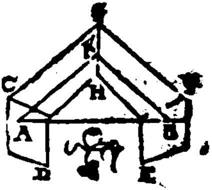
Ad quæ plana, eadem recta linea recta est; illa sunt parallela.

Theorema 13. Propositio 15.



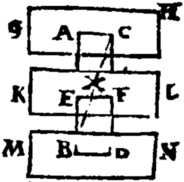
Si duæ rectæ lineæ se mutuo tangentes, ad duas rectas se mutuò tangentes sint parallelæ, non in eodem consistentes plano: parrallela sunt, quæ per illas ducuntur, plana.

Theorema 14. Propo-
ſitio 16.



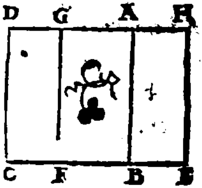
Si duo plana parallela plano quopiam ſecentur; communes illorum ſectiones ſunt parallelæ.

Theorema 15. Propo-
ſitio 17.



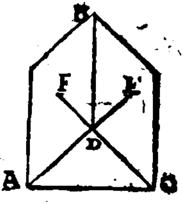
Si duæ rectæ lineæ parallelis planis ſecentur; in eaſdem proportiones ſecabuntur.

Theorema 16 Propo-
ſitio 18.



Si recta linea plano cuipiam ad rectos ſit angulos; illa etiam omnia, quæ per ipſam plana, ad rectos eidem plano angulos erunt.

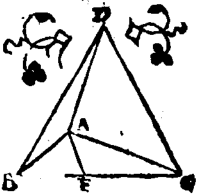
Theorema 17. Propo-
ſitio 19.



Si duo plana ſe mutuò ſecantia, plano cuidam ad rectos ſint angulos; communis etiam illorum ſectio ad rectos eidem plano angulos erit.

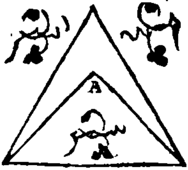
Theo-

Theorema 18 Propositio 20.



Si angulus solidus sub planis tribus angulis contineatur: ex his duo quilibet, vtut assumpti, tertio sunt maiores.

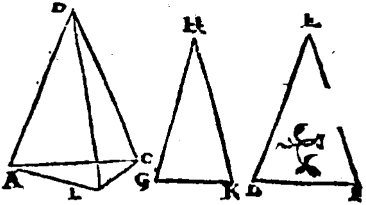
Theorema 19. Propositio 21.



Solidus omnis angulus sub minoribus quàm rectis quatuor angulis planis, continetur.

Theorema 20. Propositio 22.

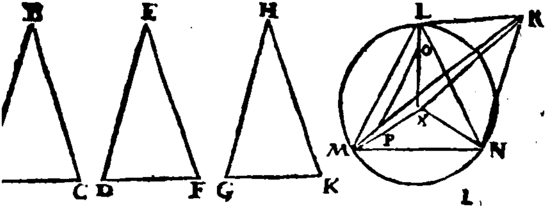
Si plani tres anguli æqualibus rectis contineantur lineis, quorum duo vt libet assumpti, tertio sint maiores; triangulum constitui potest ex lineis æquales. illas rectas coniungentibus.

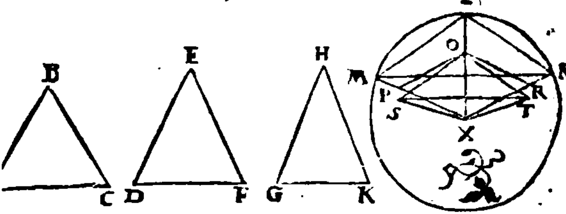


Problema 3. Propositio 23.

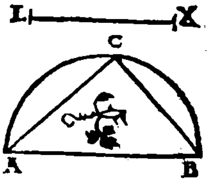
Ex planis tribus angulis, quorum duo, vt libet assumpti, tertio sint maiores, solidum angu-

angulum constituere. Oportet autem illos tres angulos rectis quatuor esse minores.

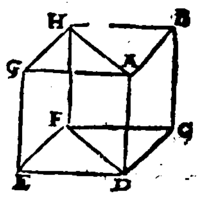




Theorema 21. Propositio 24.



Si solidum sub parallelis planis contineatur; aduersa illius plana, sunt parallelogramma, similia, & æqualia.



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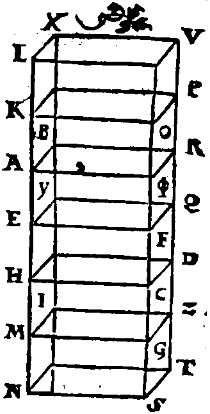
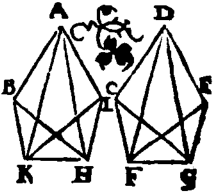
Theorema 22. Propositio 25.



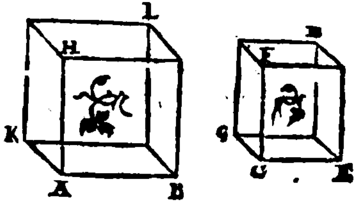
Si solidum parrallelopipedum, seu parallelis planis contentum plano secetur, aduersis planis parallelo: erit quemadmodum basis ad basim, ita solidum ad solidum.

Problema 4. propositio 26.



Ad datam rectam lineã, eiusque punctum, angulum solidum constituere, solido angulo dato æqualem.

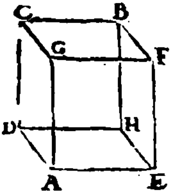
Problema 5. Propopositio 27.



A data recta linea, dato solido parallelis planis comprehenso, simile, & similiter positum solidũ parallelis planis contentũ describere.

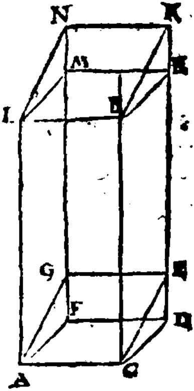
Theorema 23. Propoſitio 28.

Si ſolidum parallelis planis comprehenſum ducto per aduerſorum planorum diagonios plano, ſectum ſit: illud ſolidũ ab hoc plano bifariam ſecabitur.



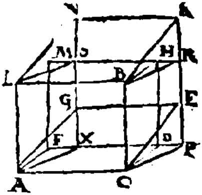
Theorema 24. Propoſitio 29.

Solida parallelapipeda, ſeu parallelis planis cõprehenſa, quæ ſuper eandem baſim, & in eadem ſunt altitudine, quorum inſiſtentes lineæ in iiſdem collocantur rectis lineis: illa ſunt inter ſe æqualia.



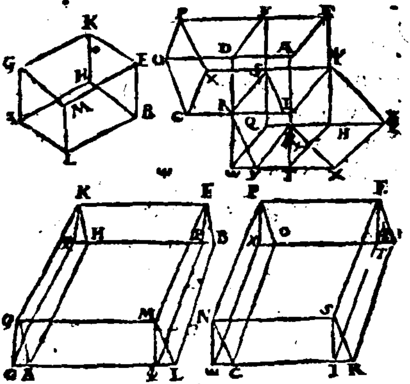
Theorema 25. Propositio 30.

Solida parallelis planis circumscripta, quæ super eandem basim, & in eadem sunt altitudine, quorum insistentes lineæ non in ijsdem reperiuntur rectis lineis; illa sunt inter se æqualia.



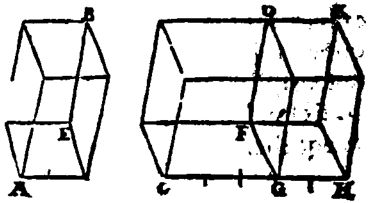
Theorema 26. Propositio 31.

Solida parallelelis planis, circumscripta, quæ super æqualibus basibus, & in eadem sunt altitudine: æqualia sunt inter se.



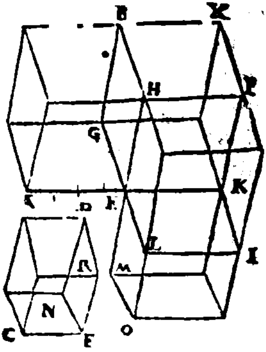
Theorema 27. Propositio 32.

Solida parallelis planis circumscripta, quæ eiusdem sunt altitudinis; eam habent inter se proportionẽ, quam bases.



Theorema 28. Propositio 33.

Similia solida parallelis planis circumscripta habent inter se proportionem homologorum laterum triplicatam.



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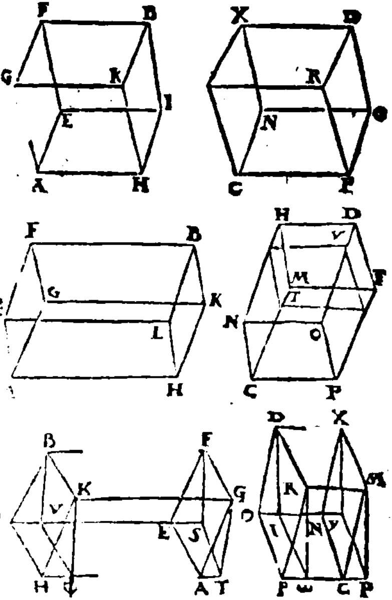
Theorema 29. Propoſitio 34.



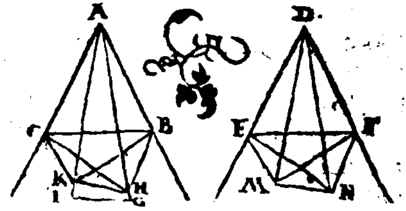
Aequaliũm ſolidorũ parallelis planis contentorum baſes, cum altitudinibus reciprocantur. Et ſolida parallelis planis contenta, quorum baſes cum altitudinibus reciprocantur; illa ſunt æqualia.

Theorema 30. Propoſitio 35.

Si duo plani ſint anguli æquales, quorum

 ver-

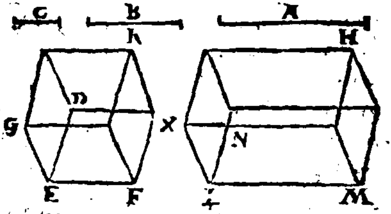
vertibus sublimes rectæ lineæ insistant, quæ cum lineis primò positis angulos contineant æquales, vtrunque vtrique: in sublimibus autem lineis quælibet sumpta sint puncta, & ab his ad plana, in quibus consistunt anguli primum positi, ductæ sint perpenpendiculares; ab earum verò punctis, quæ in planis signata fuerint, ad angulos primum positos ad iunctæ sint rectæ lineæ;



hæ cum sublimibus æquales angulos comprehendent.

Theorema 31. Propositio 36.

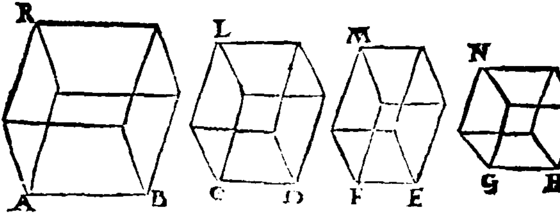
Si rectæ tres lineæ sint proportionales; quod ex his tribus fit solidum parallelis planis contentum, æquale est descripto à media linea solido parallelis planis comprehenso, quod æquilaterum quidem sit, sed antedicto æquiangulum.



Theo-

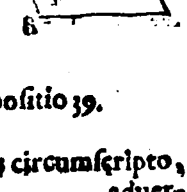
Theorema 32, Propositio 37.

Si rectæ quatuor lineæ sint proportionales, illa quoque solida parallelis planis contenta, quæ ab ipsis lineis & similia & similiter describuntur, proportionalia erunt. Et si solida parallelis planis comprehensa, quæ & similia & similiter describuntur, sint proportionalia; illæ quoque rectæ lineæ proportionales erunt.



Theorema 33, Propositio 38.

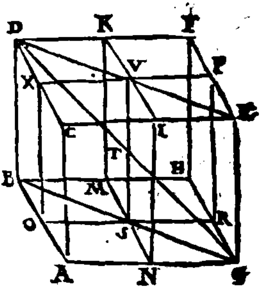
Si planum ad planum rectum sit; & à quodam puncto eorum, quæ in vno sunt planorum, perpendicularis ad alterum planum ducta sit: illa, quæ ducitur perpendicularis, in communem cadet ipsorum planorum sectionem.



Theorema 34. Propositio 39.

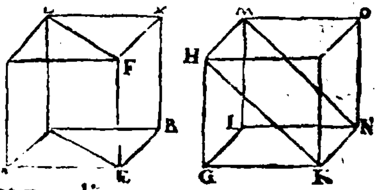
Si in solido parallelis planis circumscripto, aduer-

aduerſorum planorum lateribus biſariam ſectis, educta ſint per ſectiones plana; communis illorum planorum ſectio, & ſolidi parallelis planis circumſcripti diameter, ſe mutuò biſariam ſecabũt.



Theorema 35. Propoſitio 40.

Si duo ſint æqualis altitudinis priſmata; quorum hoc quidem baſim habeat parallelogrammum; illud verò triangulum ſit autem parallelogrammũ trianguli duplum: illa priſmata erunt æqualia.



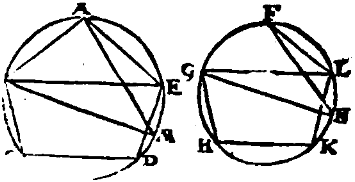
FINIS ELEMENTI XI.

EVCLIDIS ELEMENTVM DVODECIMVM

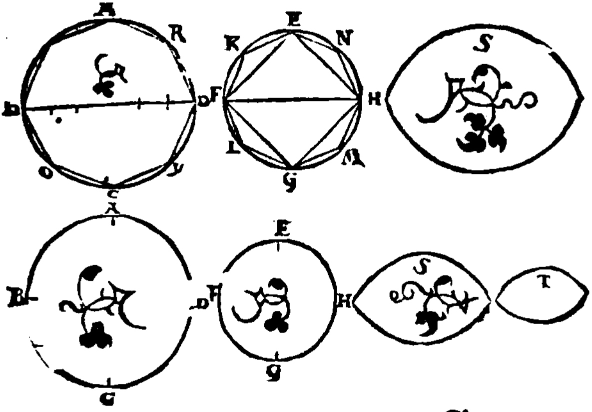
ET SOLIDORVM seucndum.

Theorema 1. Propositio 1.

Similia, quæ sunt in circulis polygona, proportioné habent inter se, quã descripta à diametris quadrata.



Theorema 2. Propositio 2.

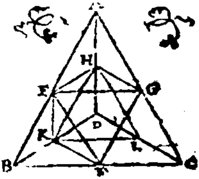


Cir-

Circuli eam inter se proportionem habent, quàm descripta à diametris quadrata.

Theorema 3. Propositio 3.

Omnis pyramis trigonam habens basim, in duas diuiditur pyramides non tantùm æquales & similes inter se, sed toti etiam pyramidi similes, quarum trigonæ sunt bases; atque in duo prismata æqualia, quæ duo prismata dimidio pyramidis totius sunt maiora.



Theorema 4. Propositio 4.

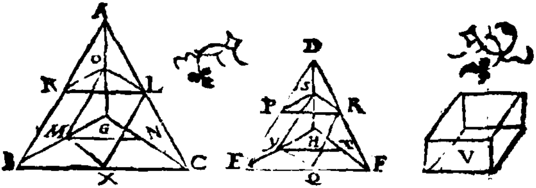
Si duæ eiusdem altitudinis pyramides trigonas habeant bases; sit autem illarum vtraque diuisa & in duas pyramides inter se æquales, totique similes; & in duo prismata æqualia; Ac eodem modo diuidatur vtraque pyramidum, quæ ex superiore diuisione natæ sunt; idque perpetuo fiat: quemadmodũ

se habet vnius pyramidis basis ad alterius pyramidis basim; ita & omnia, quæ in vna pyra-

pyramide, prismata ad omnia, quæ in altera pyramide, prismata multitudine æqualia.

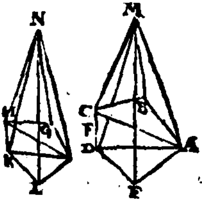
Theorema 5. Propositio 5.

Pyramides eiusdem altitudinis, quarum trigonæ sunt bases; eam inter se proportionem habent, quàm ipsæ bases.



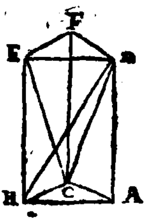
Theorema 6 Propositio 6.

Pyramides eiusdem altitudinis, quarum polygonæ sunt bases, eam inter se proportionem habent quam ipsæ bases.



Theorema 7. Propositio 7.

Omne prisma trigonam habens basim, diuiditur in tres pyramides inter se æquales, quarum trigonæ sunt bases.



Theo-

Theorema 8. Propositio 8.

Similes pyramides, quæ trigonas habent ba-
ses in
tripli-
cata
sunt
homo
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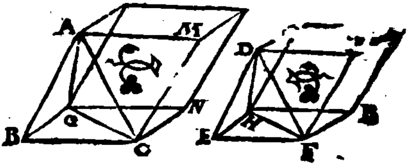
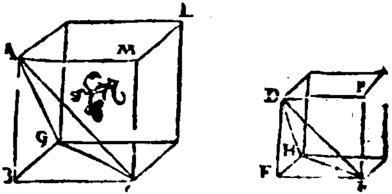
rum
laterum proportione.



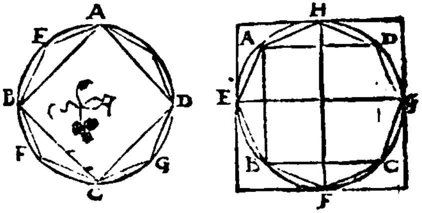
Theorema 9. Propositio 9.

Aequalium pyramidum, & trigonas bases
habentium, reciprocantur bases cum altitu-
dinibus. Et quarum pyramidum trigonas
bases
habẽ-
tium
reci-
pro-
cãtur
bases
cum altitudinibus; illæ sunt æquales.



Theorema 10. Propositio 10.

Om-
nis co
nꝰ ter-
tia
pars
est cy-
lindri

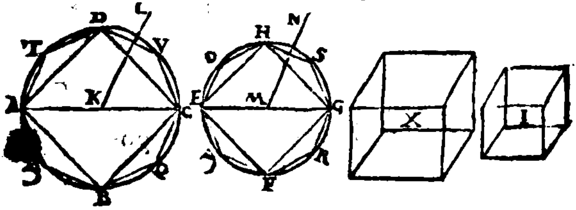


eandem

eandem cum ipſo cono baſim habentis, & altitudinem æqualem.

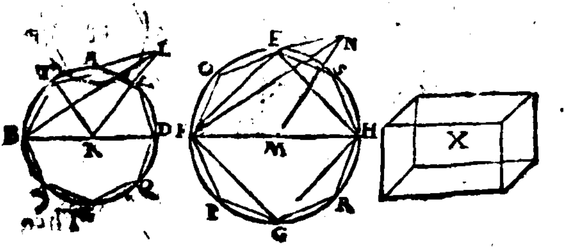
Theorema 11. Propoſitio 11.

Coni, & cylindri eiuſdem altitudinis, eam inter ſe proportionem habent, quam baſes.



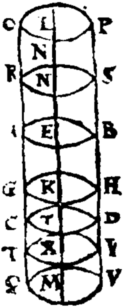
Theorema 12. Propoſitio 12.

Similes coni, & cylindri, triplicatam habẽt, inter ſe proportionem diametrorum, quæ ſunt in baſibus.



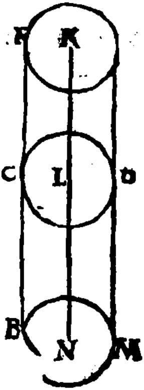
Theorema 13. Propositio 13.

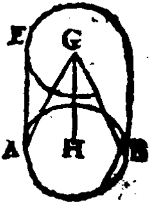
Si cylindrus plano sectus sit aduersis planis parallelo: Erit quéadmodum cylindrus ad cylindrum, ita axis ad axem.



Theorema 14. Propositio 14.

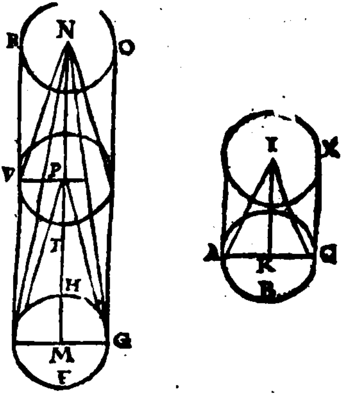
Coni, & cylindri, qui in æqualibus sunt basibus; eam habent inter se proportioné, quam altitudines.





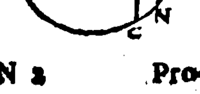
Theorema 15. Propositio 15.

Aequalium conorum, & cylindrorum bases cum altitudinibus reciprocãtur. Et quorum conorum & cylindrorum rum bases cum altitudinibus reciprocãtur; illi sunt æquales.



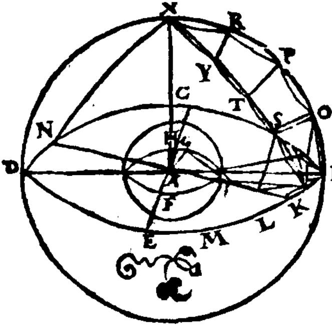
Problema 1. Propositio 16.

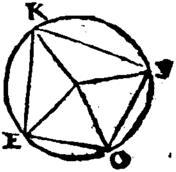
Duobus circulis circa idem centrum existentibus, in maiore circulo polygonum æqualium, pariumque laterum inscribere, quod minorem circulum non tãgat.



Problema 2. Propositio 17.

Duabus sphæris circa idem centrum exsistentibus, in maiori sphæra solidum polyedrum inscribere, quod minoris sphæræ superficiem non tangat.





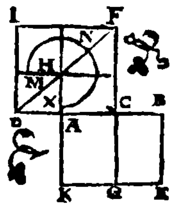
Theorema 16. Propositio 18.

Sphæræ inter se proportionem habent suorum diametrorum triplicatam.

FINIS ELEMENTI XII.

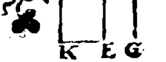
EVCLIDIS ELEMENTVM DECIMVM TERTIVM, ET SOLIDORVM tertium.

Theorema 1. Propositio 1.



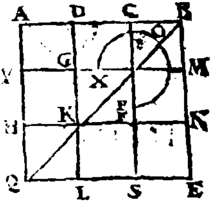
Si recta linea per extremam & mediam proportionem secta sit; maius segmentū, quod totius lineæ dimidium assumpserit, quintuplum potest eius, quod à totius dimidia describitur, quadrati.

Theorema 2. Propositio 2.



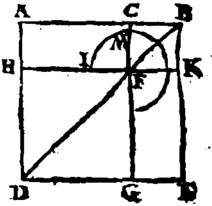
Si recta linea sui ipsius segmenti quintuplum possit, & dupla segmenti huius linea per extremam & mediam proportionem secetur, maius segmentum reliqua pars est liniæ primùm positæ.

Theorema 3. Propoſitio 3.



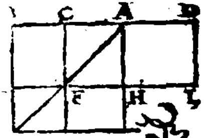
Si recta linea per extremam & mediam proportionem ſecta ſit; minus ſegmentum, quod maioris ſegmenti dimidium aſſumpſerit, quintuplum poteſt eius, quòd à maioris ſegmenti dimidio deſcribitur quadrati.

Theorema 4. Propoſitio 4.



Si recta linea per extremam & mediam proportionem ſecta ſit: quòd à tota, quodque à minore ſegmento, ſimul vtraque quadrata, tripla ſunt eius, quod à maiore ſegmento deſcribitur, quadrati.

Theorema 5. Propoſitio 5.



Si ad rectam lineam, quæ per extremam & mediam proportionẽ ſecatur, adiuncta ſit altera ſegmento maiori æqualis: tota hæc linea recta per extremam & mediam proportionem

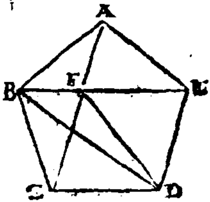
nem secta; estéque maius segmentum recta linea primùm posita.

Theorema 6. Propositio 6.

Si recta linea ῥητὴ, siue rationalis, per extremam & mediam proportionem secta sit; vtrunque segmentorum ἄλογος, siue irrationalis, est linea, quæ dicitur Residuum, seu Apotome.

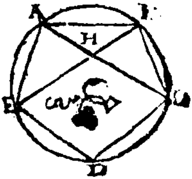
Theorema 7. Propositio 7.

Si pentagoni æquilateri tres sint æquales anguli, siue qui deinceps, siue qui non deinceps sequuntur: illud pentagonum erit æquiangulum.

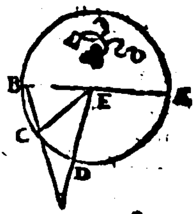


Theorema 8. Propositio 8.

Si pentagoni æquilateri, & æquianguli duos qui deinceps sequuntur, angulos rectæ subtendãt lineæ; illę per extremam & mediam proportionem se mutuò secant; earumque maiora segmenta, ipsius pentagoni lateri sunt æqualia.

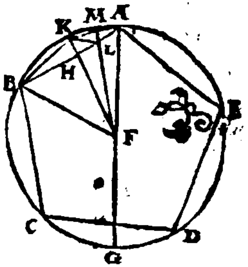


Theorema 9. Propositio 9.



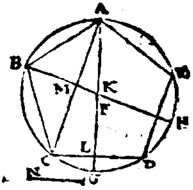
Si latus hexagoni, & latus decagoni eidem circulo inscriptorum composita sint; tota recta linea per extremam & mediam proportionem secta est; eiusque segmentum maius, est hexagoni latus.

Theorema 10. Propositio 10.



Si in circulo pentagonum æquilaterū inscriptum sit; pentagoni latus potest & latus hexagoni, & latus decagoni, eidem circulo inscriptorum.

Theorema 11. Propositio 11.

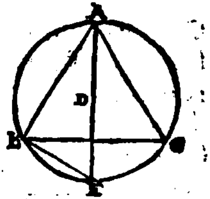


Si In circulo ῥητὸν, seu rationalem diametrum habente, inscriptum sit pentagonum æquilaterum; pentagoni latus irrationalis est linea, quæ vocatur minor.

Theo-

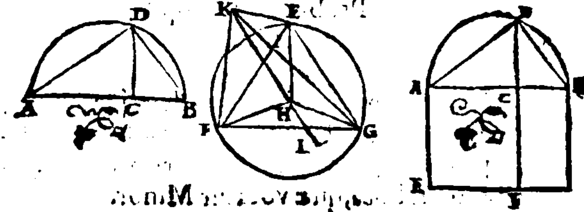
Theorema 12. Propoſitio 12.

Si in circulo inſcriptum ſit triangulum æquilaterum; huius trianguli latus potentia triplum eſt eius lineæ, quæ ex circuli centro ducitur.



Problema 1. Propoſitio 13.

Pyramidem conſtituere; & data ſphæra cõplecti: atque docere quod illius ſphæræ diameter potentia ſeſquialtera ſit lateris ipſius pyramidis.

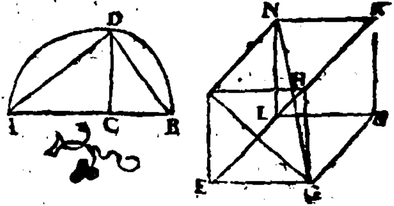


Theorema 2. Propoſitio. 14.

Octaedrum conſtituere, eaq; ſphęra, qua pyramidem, complecti; atq; probarequod illius ſphærę diameter potentia dupla ſit lateris ipſius octaedri.

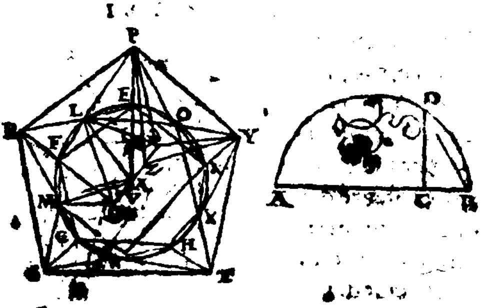
Problema 3. Propositio 15.

Cubum constituere; eaque sphæra, qua & superiores figuras complecti; atque docere quod illius sphæræ diameter potentia tripla sit lateris ipsius cubi.

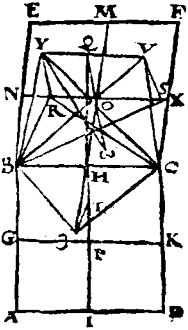


Problema 4. Propositio 16.

Icosaedrum constituere; eademque sphæra, qua & antedictas figuras, complecti; atque probare, quod illius Icosaedri latus irrationalis sit linea, quæ vocatur Minor.

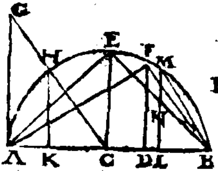


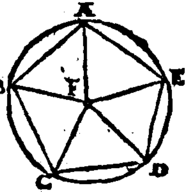
Problema 5. Propoſitio 17.



Dodecaedrum conſtituere; eademq́ue ſphæra, qua & antedictas figuras, complecti; atq; probare quod illius dodecaedri latus irrationalis ſit linea, quæ vocatur Reſiduum, ſeu Apotome.

Problema 6. Propoſitio 18.





quinque figurarũ latera proponere, & inter ſe comparare.

SCHOLIVM.

Interpretes hoc in loco demonſtrant, præter dictas quinque figuras, non poſſe aliam conſtitui figuram ſolidam, quæ ex planis & æquilateris & æquiangulis contineatur, inter ſe æqualibus.

Non enim ex duobus triangulis, neque ex aliis duabus figuris, solidus constituitur angulus; cùm saltem tres anguli plani requirantur ad solidi anguli constitutionem.

Ex tribus autem triangulis æquilateris, constat pyramidis angulus.

Ex quatuor angulus, Octaedri.

Ex quinque angulus, Icosaedri.

Nam ex triangulis, sex & æquilateris & æquiangulis ad idem punctum coeuntibus, non fiet angulus solidus: cum enim trianguli æquilateri angulus, recti vnius bessem (hoc est duas tertias partes:) cōtineat, erunt eiusmodi sex anguli rectis quatuor æquales. Quod fieri non potest. Nam solidus omnis angulus minoribus, quam rectis quatuor angulis, continetur; per propos. 21 lib. 11.

Multò ergo minus ex pluribus, quam sex planis eiusmodi angulis, solidus angulus constabit.

Sed ex tribus quadratis, Cubi angulus continetur:

Ex quatuor autem quadratis, nullus angulus solidus constitui potest. Rursus enim recti quatuor erunt. Multò ergo minus ex pluribus, quàm quatuor eiusmodi angulis solidus angulus constabit.

Ex tribus autem pentagonis æquilateris, & æquiangulis, Dodecaedri angulus continetur.

Sed ex quatuor huiusmodi angulis, nullus solidus angulus confici potest. Cùm enim pentagoni æquilateri angulus rectus sit, & quinta recti pars erunt quatuor anguli rectis quatuor maiores. Quod *fieri nequit. Nec sanè ex alijs polygonis figuris solidus angulus continebitur, quod hinc quoque absurdum sequatur.*

Quamobrem perspicuum est, praeter dictas quinque figuras aliam figuram solidam non posse constitui, quae ex planis aequilateris, & aequiangulis inter se aequalibus, contineatur.

Vid. Theon p.244. Et P. Clavius p.277.

FINIS ELEMENTI XIII.

EVCL.

EVCLIDIS ELEMENTVM DECIMVMQVARTVM ET SOLIDORVM QVARTVM, VT QVIdam arbitrantur; Vt alij verò, Hypsiclis Alexandrini, de quinque corporibus.

LIBER PRIMVS.

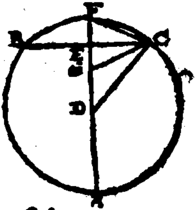
Proœmium Hypsiclis Alexandrini ad Protarchum.

Basilides Tyrius, Protarche, Alexandriam profectus, patriq; nostro ob disciplinæ societatem commendatus, longissimo peregrinationis tempore cum eo versatus est. Cūq; differerent aliquando de scripta ab Apollonio comparatione Dodecaedri, & Icosaedri eidem sphæra inscriptorum; quam hæc inter se habeant rationem, censuerunt ea non rectè tradidisse Apollonium; quæ à se emendata, vt de patre audire erat, literis prodiderunt. Ego autem postea incidi in alterum librum ab Apollonio editum, qui demonstrationem accuratè complecteretur de re proposita, ex eiusq; problematis indagatione magnam equidem cœpi voluptatem. Illud certè ab omnibus perspici potest, quod scripsit Apollonius, cùm sit in omnium manibus. Quod autem diligenti, quantum coniicere licet, studio nos postea scripsisse videmur, id mo-

id monumentis consignatum tibi dedicandum duximus; vt qui feliciter cum in omnibus disciplinis, tum vel maximè in Geometria versatus, scitè ac prudenter iudices ea, quæ dicturi sumus: Ob eam verò, quæ tibi cum patre fuit, vitæ consuetudinem, quáq; nos complecteris, beneuolentiam, tractationem ipsam libenter audias. Sed iam tempus est, vt prooemium finem facientes, hanc syntaxim aggrediamur.

Theorema 1. Propositio 1.

Perpendicularis linea, quæ ex circuli cuiuspiam centro in latus pentagoni ipsi circulo inscripti ducitur; dimidia est vtriusque simul lineæ, & eius, quæ ex centro, & lateris decagoni in eodem circulo inscripti.



Theorema 2. Propositio 2.

Si binæ rectæ lineæ extrema, & media proportione secentur; ipsæ similiter secabuntur, in easdem scilicet proportiones.

Theorema 3. Propositio 3.

Si in circulo pentagonum æquilaterum inscribatur; quod ex latere pentagoni; & quod ex ea, quæ binis lateribus pentagonis subtenditur, recta linea; vtraque simul quadrata, quintupla sunt eius, quod ex se midiametro describitur, quadrati.

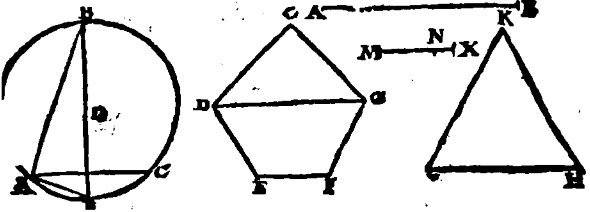
Theo-

Theorema 4. Propositio 4.

Si latus hexagoni alicuius circuli secetur extrema,& media proportione; maius illius segmentum erit latus decagoni eiusdem circuli.

Theorema 5. Propositio 5.

Idem circulus comprehendit, & dodecaedri pentagonum, & icosaedri triangulum, eidem sphæræ inscriptorum.



Theorema 6. Propositio 6.

Si pentagono,& æquilatero, & æquiangulo circumscribatur circulu.; ex cuius centro ad vnum pentagoni latus ducatur linea perpédicularis: Erit,quod sub dicto latere,& perpendiculari continetur,rectangulum trigesies sumptum, dodecaedri superficiei æquale.

Theorema 7. Propositio 7.

Si ex centro circuli triangulum icosaedri

cir-

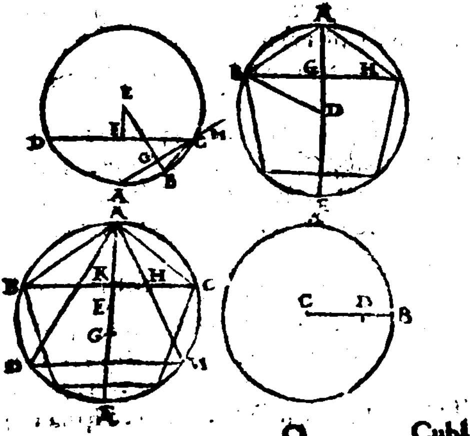
circumſcribentis, linea perpendicularis ducatur ad vnum latus trianguli: Erit, quod ſub dicto latere, & perpendiculari comprehenditur, rectangulum trigeſies ſumptum, Icoſaedri ſuperficiei æquale.

Theorema 8. Propoſitio 8.

Rectangulum contentum ſub tribus quartis partibus diametri alicuius circuli; & ſub quinque ſextis partibus lineæ ſubtendentis angulum pentagoni æquilateri in eodem circulo deſcripti; æquale eſt dicto pētagono.

Theorema 9. Propoſitio 9.

Superficies Dodecaedri ad ſuperficiem Icoſaedri, in eadem ſphæra deſcripti, eandem proportionem habet, quàm latus cubi ad latus Icoſaedri.



Cubi latus.

E————————

Dodecaedri, latus.

F—— ————

Icosaedri, latus.

G————————

Theorema 10. Propositio 10.

Si recta linea secetur extrema, & media proportione; Erit, vt recta potens id, quod à tota; & id, quod à maiori segmento, ad rectam potentem id, quod à tota, & id, quod à minori segmento; Ita latus cubi ad latus icosaedri, in eadem sphæra cum cubo inscripti.

Theorema 11. Propositio 11.

Dodecaedrum, ad Icosaedrum in eadem cum ipso sphæra inscriptum, est, vt cubi latus, ad Icosaedri latus, in vna eademq; sphæra.

Theorema 12. Propositio 12.

Latus trianguli æquilateri potentia sesquitertium est lineæ perpendicularis ab vno angulo ad latus oppositum, seu basim, deductæ.

Theorema 13. Propositio 13.

Si sphæra (duo solida corpora, Tetraedrum & Octaedrum circumscribentis:) diameter fuerit Rationalis: Erit tam superficies Tetraedri, quàm Octaedri in ea sphæra media.

Theo-

Theorema 14. Propositio 14.

SI Tetraedrum, atque octaedrum in eadem sphæra inscribantur: Erit basis Tetraedri sesquitertia baseos Octaedri: Superficies autem Octaedri sesquialtera superficiei Tetraedri.

Theorema 15. Propositio 15.

Recta linea ex angulo quouis tetraedri in sphæra inscripti, per centrum sphæræ ducta; cadit in baseos oppositæ; estq; perpendicularis ad dictam basin.

Theorema 16. Propositio 16.

Octaedrum in sphæra inscriptum, diuiditur in duas pyramides æquales, & similes, æqualium altitudinum, basis vero vtriusque pyramidis est quadratum subduplum quadrati quadrati diametri sphæræ.

Theorema 17. Propositio 17.

Tetraedrum sphæræ impositum ad Octaedrum in eadem sphæra descriptum, se habet, vt rectangulum sub linea potente viginti septem sexagesimas quartas partes quadrati lateris tetraedri; & sub linea continente octo nonas partes eiusdem lateris, comprehensum, ad quadratum diametri sphæræ.

Theorema 18. Propositio 18.

Linea perpendicularis ex quolibet angulo triangulo æquilateri ad basin oppositam demissa; tripla est eius perpendicularis, quæ ex centro trianguli ad eandem basin deducitur.

Theorema 19. Propositio 19.

Si Octaedrum sphæræ inscribatur; erit semidiameter sphæræ potentia tripla eius perpendicularis, quæ ex centro sphæræ ad basin quamcunque Octaedri ducitur.

Theorema 20. Propositio 20.

Duplum quadrati ex diametro cuiuslibet sphæræ descripti, æquale est superficiei cubi in illa sphæra collocat: perpendicularis autem à centro sphæræ in aliquam basin cubi demissa, æqualis est dimidio lateris cubi.

Theorema 21. Propositio 21.

Idem circulus comprehendit, & cubi quadratum, & Octaedri triangulum, eiusdem sphæræ.

Theorema 22. Propositio 22.

Si Octaedrum, atque Tetraedrum eidem sphæræ inscribantur; Erit Octaedrum ad triplum Tetraedri, vt latus Octaedri ad latus Tetraedri.

Theorema 23. Propositio 23.

Si recta linea proposita potuerit totam aliquam lineam sectam extrema, & media ratione, & maius eius segmentum; Item totam aliam similiter sectam, & minus eius segmentum: Erit maius segmentum prioris lineæ latus Icosaedri; minus autem segmentum posterioris lineæ latus Dodecaedri, eius sphæræ cuius recta linea proposita diameter existit.

Theo-

Theorema 24. Propositio 24.

Si latus Octaedri potuerit maius, & minus segmentum rectæ lineæ extrema, ac media ratione sectæ: poterit latus Icosaedri in eadem sphæra descripti, duplum minoris segmenti.

Theorema 25 Propositio 25.

Si recta linea diuisa extrema ac media ratione cum minore segmento angulum rectũ constituat, cui recta subtendatur: Erit recta linea, quæ potentia sit subdupla ipsius rectæ subtensæ, latus Octaedri eius sphæræ, in qua dictum minus segmentum latus existit dodecaedri.

Theorema 26. Propositio 26.

Si latus Tetraedri possit maius, & minus segmentum lineæ rectæ extrema ac media ratione sectæ: latus Icosaedri eidem sphæræ inscripti potentia sesquialterum est minoris segmenti.

Theorema 27. Propositio 27.

Cubus ad Octaedrum in eadem cum ipso sphæra descriptum, est, vt superficies cubi ad Octaedri superficiem: Item vt latus cubi ad semidiametrum sphæræ.

Theorema 28 Propositio 28.

Si sint quatuor lineæ rectæ continuæ proportionalis, nec non & aliæ quatuor, ita vt sit eadem antecedens omnium: Erit proportio tertiæ ad tertiam proportionis secundæ ad ſecundam duplicata; & proportio quartę ad quartam eiuſdem proportionis ſecundæ ad ſecundam triplicata.

Theorema 29. Propoſitio 29.

Quadratum lateris trianguli æquilateri ad ipſum triangulum habet proportionem duplicatam proportionis lateris trianguli ad lineam medio loco proportionalem inter perpendicularem ab vno angulo ad latus oppoſitum ductam, & dimidium ipſius lateris.

Theorema 30. Propoſitio 30.

Si cubus, & Tetraedrum in eadem ſphæra deſcribantur; Erit quadratum cubi ad triangulum Tetraedri, vt latus Tetraedri ad lineam perpendicularem, quę ex vno angulo trianguli Tetraedri ad latus oppoſitum protrahatur.

Theorema 31. Propoſitio 31.

Latus Tetraedri potentia ſeſquialterum eſt axis, ſeu altitudinis ipſius; Axis verò, ſiue altitudo Tetraedri potentia ſeſquialtera eſt lateris cubi in eadem ſphæra deſcripti.

Theorema 32. Propoſitio 32.

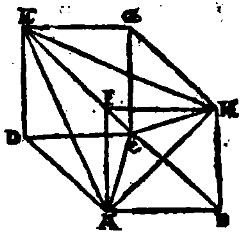
Cubus triplus eſt Tetraedri eidem ſphæra inſcripti.

FINIS ELEMENTI XIV.

EVCLI-

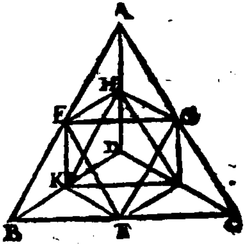
EVCLIDIS ELEMENTVM DECIMVMQVINTVM ET SOLIDORVM QVINTVM, VT NONnulli putant; Vt autem alij, Hypsiclis Alexandrini, de quinque corporibus.

LIBER II.



Problema 1. Propositio 1.

In dato cubo pyramidem (Tetraedrum inscribere.

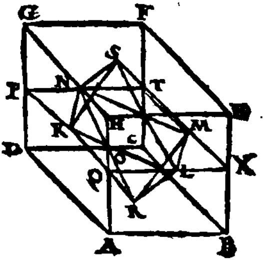


Problema 2. Propositio 2.

In data pyramide Octaedrum inscribere.

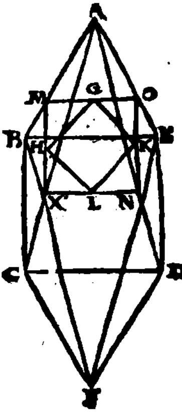
Problema 3. Propositio 3.

In dato cubo (hexaedro) Octaedrũ inscribere.



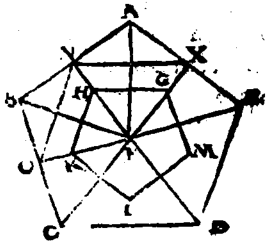
Problema 4. Propositio 4.

In dato octaedro cubũ inscribere.

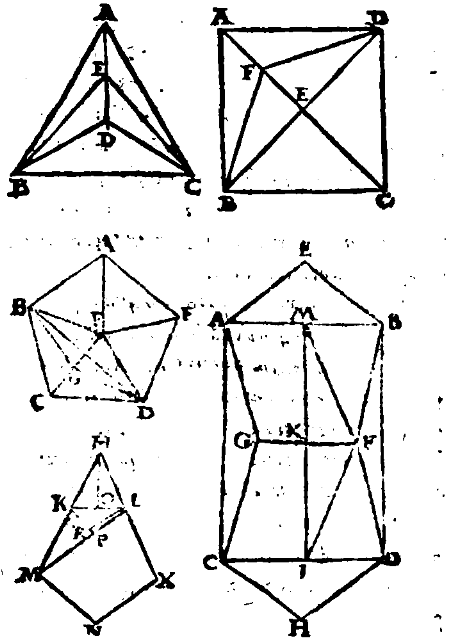


Problema 5. Propositio 5.

In dato Icosaedro dodecaedrum inscribere.



SCHOLIVM EX ZAMBERTO lib. 15. propos. 5. p. 260.



MEminisse decet, si quis nos roget, quot Icosaedrum habeat latera, ita respondendum esse: Patet Icosaedrum viginti contineri triangulis quodlibet verò triangulum rectis tribus con-

constare lineis. Quare multiplicanda sunt nobis viginti triangula in trianguli vnius latera fiuntq; sexaginta, quorum dimidium est triginta. Ad eundem modum & in dodecaedro. Cum enim rursus duodecim pentagona dodecaedrum comprehendant, itemq; pentagonum quoduis rectis quinque constet lineis; quinque duodecies multiplicamus, fiunt sexaginta, quorum rursus dimidium est triginta. Sed cur dimidium capimus? Quoniam vnumquodque latus siue sit trianguli. siue pentagoni, siue quadrati vt in cubo, iterato sumitur. Similiter autem eadem via & in cubo, & in tetraedro & in octaedro latera inuenies.

va figura inuenien

Quòd si item velis singularum quoque figurarum angulos reperire, facta eadem multiplicatione, numerum procreatum partire in numerum planorum, quæ vnum solidum angulum includunt: vt quoniam triangula quinque vnum Icosaedri angulum continet, partire 60. in quinque nascuntur duodecim anguli Icosaedri. In dodecaedro autem tria pentagona angulum comprehendunt, partire ergo 60. in tria. & habebis dodecaedri angulos viginti. Atque simili ratione in reliquis figuris angulos reperies, &c.

li figura inuenien

Problema 6. Propositio 6.

In dato octaedro pyramidem, seu tetraedrum describere.

pro-

Problema 7. Propositio 7.

In dato Dodecaedro Icosaedrum describere.

Problema 8. Propositio 8.

In dato Dodecaedro Cubum describere.

Problema 9. Propositio 9.

In dato Dodecaedro Octaedrum describere.

Problema 10. Propositio 10.

In dato Dodecaedro pyramidem describere.

Problema 11. Propositio 11.

In dato Icosaedro Cubum describere.

Problema 12. Propositio 12.

In dato Icosaedro pyramidem describere.

Problema 13. Propositio 13.

In dato cubo Dodecaedrum describere.

Problema 14. Propositio 14.

In dato cubo Icosaedrum describere.

Problema 15. Propositio 15.

In dato Icosaedro Octaedrum describere.

Problema 16. Propositio 16.

In dato Octaedro Icosaedrum describere.

pro-

Problema 17. propositio 17.

In dato Octaedro Dodecaedrum describere.

Problema 18. Propositio 18.

In data pyramide Cubum describere: hoc est, in proposito Tetraedro hexaedrum delineare.

Problema 19 Propositio 19.

In data pyramide Icosaedrum describere.

Problema 20. Propositio 20.

In data pyramide Dodecaedrum describere.

Problema 21. Propositio 21.

In dato solido regulari sphęram describere: hoc est, Interprete Campano; In fabricato quouis quinq; corporum regularium, sphęram fabricare.

FINIS ELEMENTI XV.

EVCLI-

EVCLIDIS ELEMENTVM DECIMVM SEXTVM, ET SOLIDORVM sextum.

Quo varia solidorum regularium sibi mutuò inscriptorum, & laterum eorundem comparationes explicantur, à Francisco Flussate Candalla, & P. Christophoro Clauio adiectam, & de quinque corporibus,

LIBER TERTIVS.

Theorema 1. Propositio 1.

SI in Dodecaedro Cubus describatur, & in hoc Cubo aliud Dodecaedrum: Erit proportio Dodecaedri exterioris ad Dodecaedrum interius proportionis eius, quam habet maius segmentum ad minus rectæ lineę diuisæ extrema, ac media ratione triplicata.

Theorema 2. Propositio 2.

Linea perpendicularis ex quouis angulo pentagoni æquilateri, & æquianguli in latus oppositum demissa; secatur à linea recta illum angulum subtendente, extrema ac media ratione.

Theo-

descripti, proportionem habet, quam minus segmentum ad maius, eiusdem rectæ lineæ.

Theorema 14. Propositio 14.

Latus octaedri sesquialterum est lateris pyramidis sibi inscriptæ.

Theorema 15. Propositio 15.

Si ex quadrato diametri Icosaedri auferatur triplum quadrati lateris cubi in eo descripti; relinquitur quadratum sesquitertium quadrati lateris Icosaedri.

Theorema 16. Propositio 16.

Latus Dodecaedri minus segmentum est rectæ lineæ extrema ac media ratione diuisæ, quę duplum potest lateris octaedri in eo descripti.

Theorema 17. Propositio 17.

Diameter Icosaedri potest, & sui ipsius lateris sesquitertium, & lateris pyramidis in eo descriptæ sesquialterum.

Theorema 18. Propositio 18.

Latus Dodecaedri ad Icosaedri sibi inscripti latus, se habet, vt minus segmentum lineæ perpendicularis ab vno angulo pentagoni ad latus oppositum ductæ, atque extrema ac media ratione diuisæ, ad partem eiusdem lineæ inter centrum pentagoni, & latus eiusdem positæ.

Problema 19. Propositio 19.

Si dimidium lateris Icosaedri extrema ac media ratione sectum fuerit, minusque eius segmentum à toto latere Icosaedri sublatum; à reliqua quoque recta linea pars rursum tertia detracta: Relinquetur latus Dodecaedri in Icosaedro descripti.

Theorema 20. Propositio. 20.

Cubus sibi inscriptæ pyramidis triplus est.

Theorema 21. Propositio 21.

Pyramis sibi inscripti Octaedri dupla est.

Theorema 22. Propositio 22.

Cubus sibi inscripti Octaedri sextuplus est.

Theorema 23. Propositio 23.

Octaedrum sibi inscripti Cubi quadruplum sesquialterum est.

Theorema 24. Propositio 24.

Octaedrum sibi inscriptæ pyramidis tredecuplum sesquialterum est.

Theorema 25. Propositio 25.

Pyramis sibi inscripti Cubi noncupla est.

Theorema 26. Propositio 26.

Octaedrum ad Icosaedrum sibi inscriptum proportionem habet, quam duæ bases octaedri ad quinque bases Icosaedri.

Theorema 27. Propositio 27.

Icosaedrum ad Dodecaedrum sibi inscriptum, proportionem habet compositam ex proportione lateris Icosaedri ad latus cubi in eadem cum Icosaedro sphæra descriscripti; & ex proportione triplicata eius quam habet diameter Icosaedri ad rectam lineam contra basium Icosaedri oppositarum coniungentem.

Theorema 28. Propositio 28.

Dodecaedrum excedit Cubum sibi inscriptum parallelepipedo, cuius quidem basis à quadrato Cubi deficit rectangulo contento sub latere Cubi, tertiaque parte, minoris segmenti eiusdem lateris Cubi: At verò altitudo ab altitudine, siue latere Cubi deficit, minore segmento eius lineæ, quæ dimidiati lateris Cubi segmentum minus existit.

Theorema 29. Propositio 29.

Dodecaedrum ad Icosaedrum sibi inscriptum, proportionem habet compositam ex proportione triplicata eius, quam habet diameter Dodecaedri ad rectam lineam contra basium Dodecaedri oppositarum copulantem; & ex proportione lateris Bubi ad latus Icosaadri in eadem sphæra cum Cubo descripti.

Theorema 30. Propositio 30.

Dodecaedrum pyramidis, in qua describitur, duas nonas partes continet, minus duobus parallelepipedis; quorum vnius longitudo lateri Cubi in eadem pyramide descripti, æqualis est; latitudo verò tertiæ parti miti minoris segmenti lateris eiusdem Cubi; altitudo denique à latere eiusdem Cubi deficit minore segmento eius lineæ, quæ dimidiati lateris Cubi eiusdem minus segmẽtum existit: Alterius autem & longitudo, & latitudo lateri Cubi prædicti, est æqualis; altitudo vero minus segmentum eius lineæ quæ dimidiati lateris Cubi eiusdem segmentum minus existit; ita vt amborum altitudines simul altitudini, siue lateri eiusdem Cubi sint æquales.

Theorema 31. Propositio 31.

Octaedrum excedit sibi inscriptum Icosaedrum parallelepipedo, cuius basis est quadratum lateris Icosaedri; altitudo vero maius segmentum semidiametri Octaedri extrema ac media ratione sectæ.

DE QVINQVE CORPORVM regularium descriptione in data sphæra, ex Pappo Alexandrino.

Lemma I.

Datis duobus circulis in sphæra parallelis, dataque in vno eorum linea recta; ducere in altero diametrum huius rectæ datæ parallelam.

Lemma II.

Si in planis parallelis deſcripti ſint duo circuli, in quibus duæ rectæ parallelæ abſcindant arcus ſimiles: Erunt duæ rectæ coniungentes extrema vnius rectæ cum centro parallelæ duabus rectis, quæ extrema alterius rectæ cum centro coniungant.

Lemma III.

Si in ſphæra ſint duo circuli paralleli, & æquales, in quibus ductæ ſint duæ rectæ parallelæ, & æquales, ad eaſdem partes centrorum: Rectæ harum parallelarum extrema puncta ad eaſdem partes coniungentes, æquales quoque, & parallelæ ſunt, & ad plana circulorum perpendiculares.

Lemma IV.

Si in ſphæra ſint duo circuli paralleli, & æquales, in quibus ductæ ſint duæ rectæ parallelæ, & æquales, non ad eaſdem partes cētrorum: Rectæ lineæ harum parallelarum extrema puncta non ad eaſdem partes coniungentes, in centro ſphæræ ſeſe interſecant; ac proinde diametri ſphæræ erunt, & inter ſe æquales: Rectæ verò lineæ earundem parallelarum extrema puncta ad eaſdem partes connectentes, & æquales, & parallelæ inter ſe ter se sunt, & cum parallelis rectos angulos constituunt.

Lemma V.

Si in sphæra sint duæ rectæ parallelæ; rectæ earum puncta extrema ad easdem partes coniungentes, æquales inter se erunt: Et si parallelæ sint æquales, coniungentes non solùm æquales, sed & parallelæ erunt, rectosque cum ipsis angulos conficient.

Lemma VI.

In data sphæra duos circulos æquales, ac parallelos describere; ita vt diameter sphæræ sit vtriúsque diametri potentia sesquialtera.

Problema 1. Propositio 1.

In data sphæra pyramidem trigonam describere.

Problema 2. Propositio 2.

In data sphæra Octaedrum describere.

Problema 3. Propositio 3.

In data sphæra Cubum describere.

Problema 4. propositio 4.

In data sphæra Icosaedrum describere.

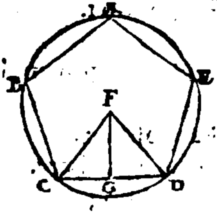
Problema 5 Propositio 5.

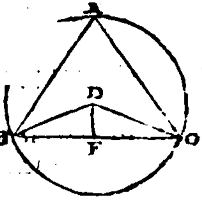
In data sphæra Dodecaedrum describere.

SCHOLIVM EX P. CLAVIO.

Ex his, quæ hoc in loco & lib. 13. & lib. 14. demonstrata sunt, facile etiam ostenditur, si omnia quinque corpora regularia in vna eademque sphæra describantur, maximum omnium esse Dodecaëdrum: Deinde Icosaëdrum maius reliquis tribus: Tertio Cubum maiorem reliquis duobus: Octaëdrum denique Tetraëdro esse maius. Ex quo euidenter constabit, Euclidem recto ordine quinque hæc corpora construxisse, cum post Tetraëdrum statim Octaëdrum non autem Cubum constituerit. Ita enim à minoribus ad maiora progressus est.

FINIS ELEMENTI XVI. ET Vltimi.





Pagina 208. Theoremate 6. desunt hæ duæ Figuræ.