EVCLIDIS ELEMENTORVM LIBRI XV.

ACCESSIT LIBER XVI. DE QVINQVE SOLIDORVM REGVLARIVM inter se comparatione.

AD EXEMPLARIA R. P. CHRISTOphori Clauij è Societ. IESV, & aliorum collati, emendati & aucti.

COLONIAE
Apud Gosuinum Cholinum
M. D. CVII.
Cum Gratia & Priuilegio.

MATHESEOS ET GEOMETRIAE DIVISIO.

MAthematicæ diſciplinæ, quæ omnes circa quantitatem verſantur, nomen acceperunt à Græca dictione μάθημα, ſeu μάθησις, quæ diſciplinam, & doctrinam ſignificat; eò quod tum gradatim aſcendendo doceantur, & addiſcantur; tùm ſolæ ſemper ex præcognitis quibuſdam, conceſſis, & probatis, principijs, (Haud ex hypotheſibus nondum explicatis) ad cõclusiones demonſtrandas, quod proprium eſt doctrinarum & diſciplinarum officium, teſte Ariſtotele lib. 1. Poſter. procedant.

Pythagoras, & Mathematici vniuerſas Mathematicas diſciplinas in quatuor partes principes diſtribuũt, nempe in Arithmeticam, Muſicam, Geometriam, & Aſtronomiam. Cum enim omnis quantitas, circa quam hæ diſciplinæ verſantur, ſit vel diſcreta, ſub qua omnes numeri continentur; vel continua, ſub qua omnes magnitudines comprehenduntur; & vtraque tam ſecundum se, quam comparatione alterius possit considerari; Visum est illis has quatuor partes instituere, quæ vtramque quantitatem pro duplici consideratione, diligenter contemplarentur. Itaque Arithmetica agit de quantitate discreta secundum se; omnesque numerorum proprietates ac passiones inquirit, & accuratè explicat. Musica tractat eandé quantitaté discretã, seu numerum cum alio comparatum; quatenus sonorum Harmoniam, & concentum respicit. Geometria de magnitudine, seu quantitate continua, secundum se quoque, vt immobilis existit, disputat. Astronomia denique magnitudiné, vt est mobilis, cósiderat; qualia sũt corpora cœlestia cótinuè mota. Ad has aũt quatuor scientias Mathematicas, tum puras, tũ mixtas, omnes alię de quantitate tractan- vti Perspectiua Geographia Stereomatria, & cæteræ, facili negocio, tanquã ad sua capita, & fontes, ex quib. emanant, reducũtur.

Geometria apud Euclidem diuiditur in planorum (superficierum planarum.) contemplationem, seu Geometriam propriedictam; quæ libris sex primis absoluitur; Et in solidorum (corporum solidorum:) speculationem, seu Stereomatriam; quæ libris postremis pertractatur. Prior quidem pars, nempe Geometria, in tres partes subdiuiditur. Nam in prioribus quatuor libris agitur de planis absolutè, ita vt eorum æqualitas, & inæqualitas inuestigetur. In quinto verò libro de rationalibus magnitudinum proportionibus in genere disputatur. In sexto denique libro proportiones figurarum planarum discutiuntur. Posterior verò pars, nempe Stereometrica, in tres quoque partes subdiuitur. In quarum prima, videlicet libris tribus, septimo, octauo, & nono, agitur de numerorum proprietatibus, passionibusque ad linearum (& aliarum magnitudinum;) symetrarum seu commensurabilium, & asymetrarum, seu incommensurabilium tractationem necessarijs. In secunda, nimirum libro decimo, satis prolixo, de lineis commensurabilibus, & incommensurabilibus, sine quarum notitia, corpora illa quinque solida, regularia, seu platonica perfectè tractari nequeunt, disseritur. In tertia denique, nempe libris sex postremis (qui & ipsi subdiuidi poterũt) de solidis illis acutissimè disputatur, eorumque proprietates inuestigantur. De punctis autem, & lineis in hoc opere nulla ex professo extat contemplatio: quoniam Geometria potissimum circa figuras, quibus plana, & solida duntaxat (non puncta, & lineæ:) afficiuntur, versatur.

Demonstratio omnis Mathematicorum ab antiquis scriptoribus diuiditur in Problema, & Theorema. Problema quidem vocatur ea demonſtratio, quæ iubet atque docet aliquid conſtituere, facere, inuenire, deſcribere, &c. vt ſuper data linei recta terminata triangulum æquilaterum conſtituere. Theorema verò appellatur ea demonſtratio, quæ ſolum aliquam proprietatem, ſeu paſſionem vnius, vel plurium ſimul quantitatum (multarum vel magnarum iam inuentarum:) perſcrutatur, vt in omni triangulo tres angulos eſſe æquales duobus rectis. Cæterùm tàm problema, quàm Theorema dicitur apùd Mathematicos propoſitio; eò quod vtrumque aliquid nobis proponit, vti in exemplis adductis conſtat, Demonſtrationes problematum concluduntur his ferè verbis; Quod faciendum erat: Theorematum verò Demonſtrationes, his verbis; Quod oſtendendum, vel demonſtrandum erat; habitâ nimirum ratione vtriuſque. Adhæc problemata per modum infinitum, ſed Theoremata per modum finitum ferè proferuntur. Lemma (ſumptio, vel aſſumptũ latinè) appellatur ea minus principalis, & aliqua tantum declaratione indigens, demonſtratio, quæ ad demonſtrationem alicuius problematis, vel Theorematis principalis aſſumitur, vt illa demonſtratio expeditior, ac breuior fiat; vt videre licet lib 6 propoſ. 22. & paſſim Corollarium, ſeu porîſma, denique

que est, quod protenus ex facta demonstratione tanquam lucrum aliquod additum, seu cõsectarium accipitur; vt videre est lib. 2. propos. 4. & passim.

Cùm omnis autem doctrina, omnisque disciplina ex præexistente cognitione gignatur, atq; ex assumptis, & concessis quibusdam principijs suas demonstrat conclusiones: Nulla autem scientia, teste Aristotele, sua principia demonstret; habebunt & Mathematicæ disciplinæ sua principia, ex quibus positis, & concessis sua problemata, & Theoremata confirment. Horum autem tria solum genera apud Mathematicos reperiuntur. Quorum primum genus continet definitiones, quas nonnulli Hypotheses appellant, vt punctum est, cuius pars nulla est. Secundum genus complectitur petitiones, seu postulata, quæ per se adeò perspicua sunt, vt nulla confirmatione indigeant, sed auditoris duntaxat assensum exposcant, vt postuletur, vt à quouis puncto in quoduis punctum, rectam lineam ducere concedatur. Tertium denique genus comprehendit Axiomata, seu communes animi notiones, quæ non solum in scientia proposita sed etiam in alijs omnibus vsqueadeo euidentes sunt, vt ab eis nulla ratione dissentire queat is, qui ipsa vocabula rectè perceperit, vt: Quæ eidem æqualia, & inter se sunt æqualia.

Verum

erum secundum nonnullos principium
rmale duplex existit, nempe indemon-
rabile seu facile, vt definitio, postulatum,
: axioma; demonstrabile, vt Problema,
Theorema, & omnis propositio. Principi-
m vero materiale est punctum, linea, &c.
Vid. & P. Christophorus Clauius, nobilissi-
mus Elementorum Euclidis Interpres.
Coloniæ Agrippinæ, anno 1607.
Iunij 8.

EVCLI-

EVCLIDIS ELEMENTVM PRIMVM.

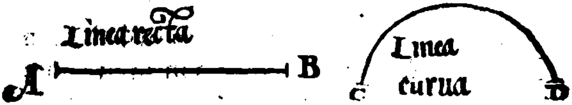
DEFINITIONES.

1.

PVnctum eſt, cuius pars Punctum nulla eſt.

2.

Linea verò, longitudo latitudinis expers.



3.

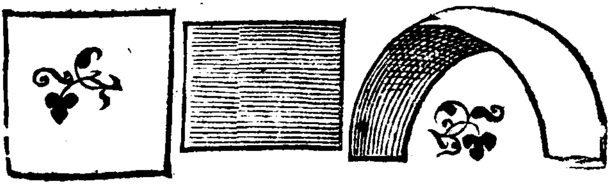
Lineæ autem termini, ſunt puncta.

4.

Recta linea eſt, quæ ex æquo ſua interiacet puncta.

5.

Superficies eſt, quæ longitudinem latitudinemq́ue tantùm habet

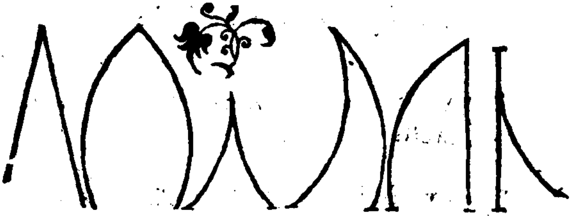


6.

Superficiei extrema sunt lineæ.

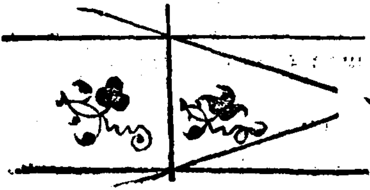
7.

Plana superficies est, quæ ex æquo suas interiacet lineas.



8.

Planus angulus est duarum linearũ in plano se mutuò tangentium, & non indirectum



iacẽtium alterius ad alteram inclinatio.

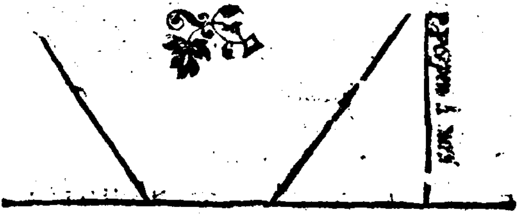
9.

Cùm autem quę angulum continent lineę, rectæ fuerint, rectilineus ille angulus appellatur.

10.

Cùm verò recta linea super rectam consistens lineam, eos, qui sunt deinceps, angulos æquales inter se fecerit; rectus est vterque æqua-

æqualium angulorum: quæ insistit recta linea, perpendicularis vocatur eius, cui insistit.



11.

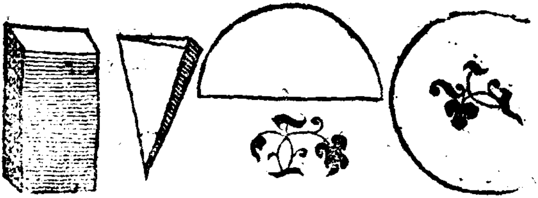
Obtusus angulus est, qui recto maior est.

12.

Acutus verò, qui minor est recto.

13.

Terminus est, quod alicuius extremum est.

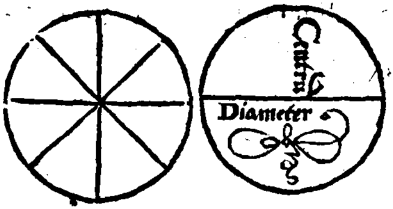


14.

Figura est, quæ sub aliquo, vt aliquibus terminis comprehenditur.

15.

Circulus est, figura plana sub vna linea cõprehensa, quę peripheria appellatur; ad quã ab vno puncto eorum, quæ intra figuram ſunt poſita, cadentes omnes rectæ lineæ inter ſe ſunt æquales.



16.

Hoc verò punctum, centrum circuli appellatur.

17.

Diameter autem circuli, eſt recta quædam linea per centrum ducta, & ex vtraque parte in circuli peripheriam terminata, quæ circulum biſariam ſecat.

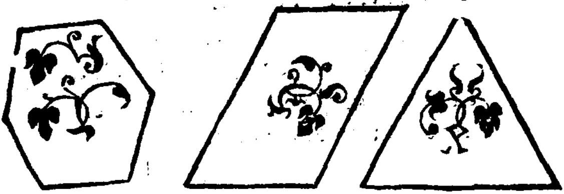
18.

Semicirculus verò eſt figura, quæ continetur ſub diametro, & ſub ea linea, quæ de circuli peripheria auſertur.

19.

Recti lineæ figuræ ſunt, quę ſub rectis lineis continentur.

20. Tri-



20. Trilateræ quidem, quæ sub tribus.

21.

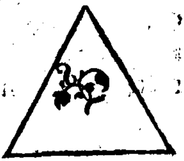
Quadrilateræ, quæ sub quatuor.

22.

Multilateræ verò, quæ sub pluribus, quàm quatuor, rectis lineis comprehenditur.

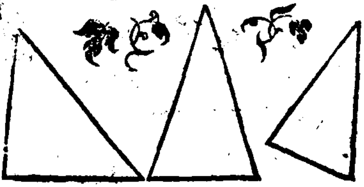
23.

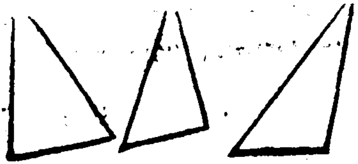
Trilaterarum autem figurarum, æquilaterum est triangulum, quod tria latera habet æqualia.



24.

Isosceles autem est, quod duo tantùm æqualia habet latera.





25.

Scalenum verò est, quod tria inæ-

inæqualia habet latera.

26.

Ad hæc etiam, trilaterarum figurarum, rectangulum quidem triangulum est, quod rectum angulum habet.

27.

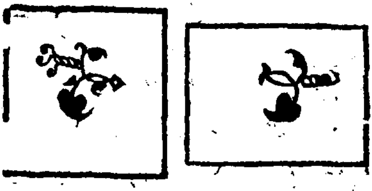
Amblygonium autem, quod obtusum angulum habet.

28.

Oxygonium verò, quod tres habet acutos angulos.

29.

Quadrilaterarum autem figurarum, quadratum quidem est, quod & æquilaterum, & rectangulum est.

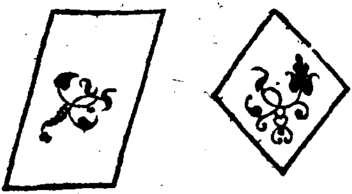


30.

Altera verò parte longior figura est, quę rectangula quidem, at æquilatera non est.

31. Rhom-

31.

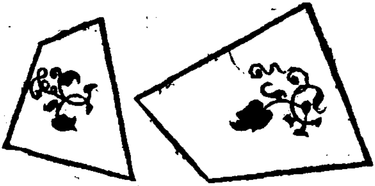


Rhombus autem, quę ęquilatera, sed rectangula non est.

32.

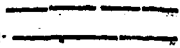
Rhomboides verò, quæ aduersa & latera, & angulos habens inter se æquales, neque æquilatera est, neque rectangula.

33.



Preter hęs autē, reliquæ quadrilaterę figuræ, trapezia appellentur.

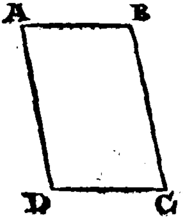
34.



Parallele rectæ lineæ sunt, quæ cùm in eodem sint plano & ex vtraque parte in infinitum producantur, in neutram sibi mutuò incidunt.

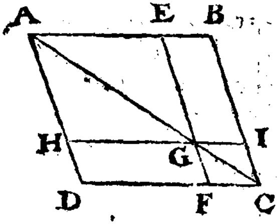
35.

Parallelogrammum est figura, quadrilatera, cuius bina apposita latera sunt parallela, seu æqui distantia.



36.

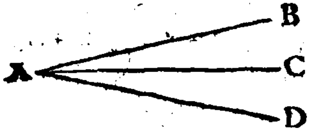
Cùm verò in parallelogrammo diameter ducta fuerit, duæq; lineæ lateribus parallelæ secantes diametrum in vno eodemque puncto, ita vt parallelogrammum ab hisce parallesis in quatuor distribuatur parallelogramma; appellantur duo illa, per quæ diameter non transit, complementa; duo verò reliqua, per quæ diameter incedit, circa diametrum consistere dicentur.



Petitiones siue Postulata.

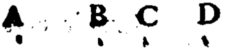
I.

Postuletur, vt à quouis puncto in quoduis punctum,



punctum, rectam lineam ducere cōcedatur.

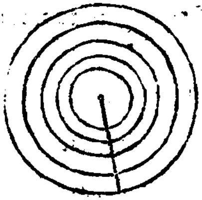
2.



Et rectam lineam terminatam in continuū recta producere.

3.

Item quouis centro, & interuallo circulum deſcribere.



4.

Item quacunque magnitudine data, ſumi poſſe aliam magnitudinem vel maiorem, vel minorem.

Communes notiones ſiue axiomata.

1.

Quæ eidem æqualia, & inter ſe ſunt æqualia.

2.

Et, ſi æqualibus æqualia adiecta ſint, tota ſunt æqualia.

3.

Et, ſi æqualibus æqualia ablata ſint, quæ relinquuntur ſunt æqualia.

4.

Et, si inæqualibus æqualia adiecta sint, tota sunt inæqualia.

5.

Et, si ab inæqualibus æqualia ablata sint, reliqua sunt inæqualia.

6.

Et, quæ eiusdem duplicia sunt, inter se sunt æqualia.

7.

Et quæ eiusdem sunt dimidia, inter se æqualia sunt.

8.

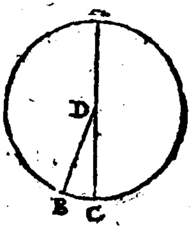
Et quæ sibi mutuò congruunt, ea inter se sunt æqualia.

9.

Et totum sua parte maius est.

10.

Duæ lineæ rectæ non habent vnum, & idem segmentum commune.



11.

Duæ lineæ rectæ in vno puncto concurrentes,

rentes, si producantur ambæ, necessariò se mutuò in eo puncto intersecabunt.

12.

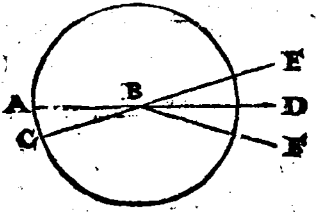
Item, omnes recti anguli sunt inter se æquales.

13.

Et, si in duas rectas lineas altera recta incidens, internos, ad easdemque partes angulos, duobus rectis minores faciat; duæ illæ rectæ lineæ in infinitum productę sibi mutuò incident ad eas partes, vbi sunt anguli duobus rectis minores.

14.

Duæ rectæ lineæ spatium non comprehendunt.



15.

Si æqualibus inæqualia adiiciantur, erit totorum excessus, adiectorũ excessui æqualis.

16.

Si inæqualibus æqualia adiungantur, erit totorum excessus, excessui eorum, quæ à principio erant, æqualis.

17. Si

17.

Si ab æqualibus inæqualia demantur, erit residuorum excessus, excessui ablatorum æqualis.

18.

Si ab inæqualibus æqualia demantur, erit residuorum excessus, excessui totorum æqualis.

19.

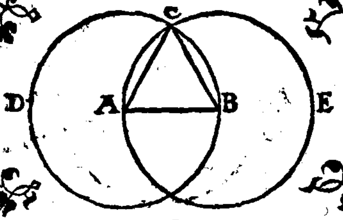
Omne totum æquale est omnibus suis partibus simul sumptis.

20.

Si totum totius est duplum, & ablatum ablati; erit & reliquum reliqui duplum.

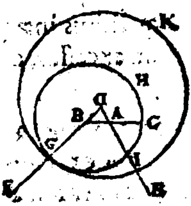
Problema 1. Propositio 1.

Super data recta linea terminata, triangulum æquilaterum constituere.

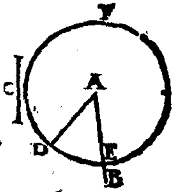


Problema 2. Propositio 2.

Ad datum punctum, datæ rectæ lineæ, æqualem rectam lineam ponere.



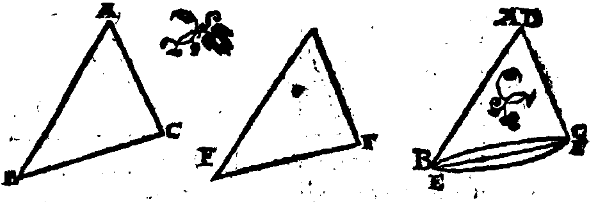
Problema 3. Propositio 3.



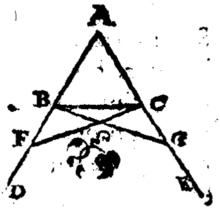
Duabus datis rectis lineis inæqualibus, de maiore æqualem minori rectam lineam detrahere.

Theorema 1. Propositio 4.

Si duo triangula duo latera duobus lateribus æqualia habeant, vtrunq; vtrique; habeant verò & angulum angulo ęqualem sub æqualibus rectis lineis contentum: & basin basi æqualem habebunt; eritq; triangulum triangulo æquale, ac reliqui anguli reliquis angulis æquales erunt, vterque vtrique, sub quibus æqualia latera subtenduntur.

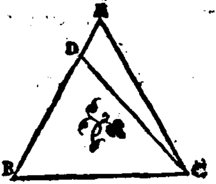


Theorema 2. Propositio 5.



Isosceleum triangulorum, qui ad basim sunt anguli, inter se sunt æquales; & si vlterius productæ sint æquales illæ rectæ lineæ, qui ſub baſi ſunt anguli, inter ſe æquales erunt.

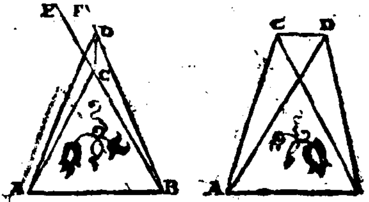
Problema 3. Propoſitio 6.



Si trianguli duo anguli æquales inter ſe fuerint; & ſub æqualibus angulis ſubtenſa latera æqualia inter ſe erunt.

Theorema 4. Propoſitio 7.

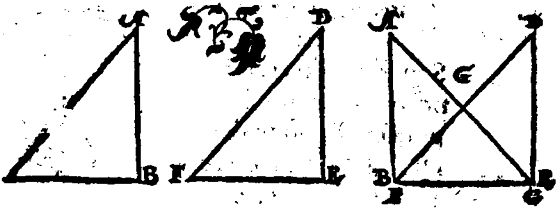
Super eadem recta linea, duabus eiſdem rectis lineis aliæ duæ rectæ lineæ æquales, vtraque vtrique, non conſtituentur, ad aliud atq; aliud punctũ, ad eaſdem partes, eoſdemque terminos cum duabus initio ductis rectis lineis habentes.



Theorema 5. Propoſitio. 8.

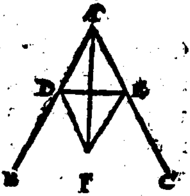
Si duo triangula duo latera habuerint duobus lateribus, vtrunque vtrique, æqualiá: habuerint verò & baſim baſi æqualem: angulum quoque ſub æqualibus rectis lineis contentum angulo æqualem habebunt.

Pro-



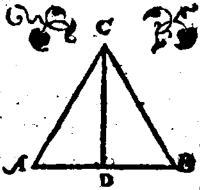
Problema 4. Propositio 9.

Datum angulum rectilineum bifariam secare.



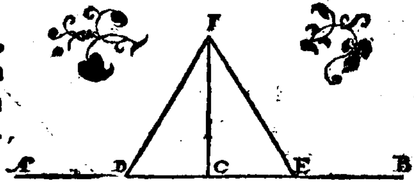
Problema 5. Propositio 10.

Datam rectam lineam finitam bifariam secare.



Problema 6. Propositio 11.

Data recta linea, à puncto in ea dato, rectam lineam ad angulos rectos excitare.



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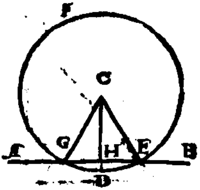
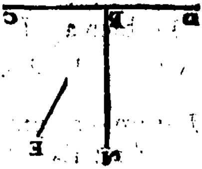
Problema 7. Propositio 12.



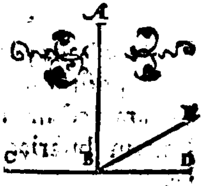
Super datam rectam lineam infinitam, à dato puncto, quod in ea non est, perpendicularem rectam deducere.

Theorema 6. Propositio 13.



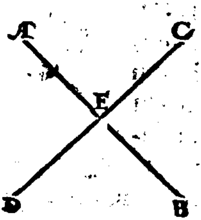
Cùm recta linea super rectam cōsistens lineam angulos facit, aut duos rectos, aut duobus rectis æquales efficiet.

Theorema 7. Propositio 14.



Si ad aliquam rectam lineam, atq; ad eius punctum, duæ rectæ lineæ non ad easdem partes ductæ, eos, qui sunt deinceps, angulos duobus rectis æquales fecerint, in directū erunt inter se ipse rectæ lineæ.

Theorema 8. Propositio 15.



Si duæ rectæ lineæ se mutuò secuerint, angulos, qui ad verticem sunt, æquales inter se efficient.

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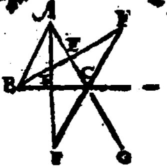
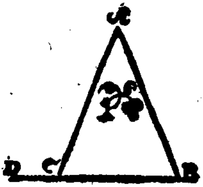
Theorema 9. Propositio 16.

Cuiuscunque trianguli vno latere producto, externus angulus vtroque interno & opposito maior est.



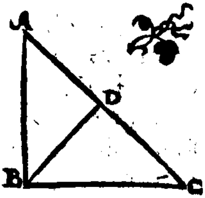
Theorema 10. Propositio 17.

Cuiuscunque trianguli duo anguli duobus rectis sunt minores, omnifariam sumpti.



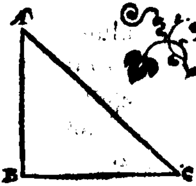
Theorema 11. Propositio.

Omnis trianguli maius latus maiorem angulũ subtendit.



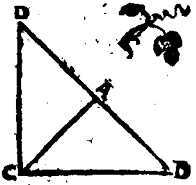
Theorema 12. Propositio 19.

Omnis trianguli maior angulus maiori lateri subtenditur.



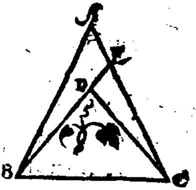
Theorema 13. Propoſitio 20.

Omnis trianguli duo latera reliquo ſunt maiora, quomodocunque aſſumpta.



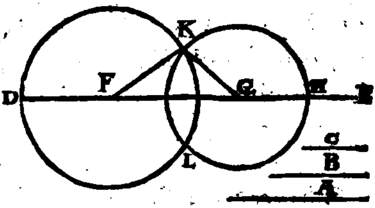
Theorema 14. Propoſitio 21.

Si ſuper trianguli vno latere, ab extremitatibus duæ rectæ lineæ, interius conſtitutæ fuerint; hæ conſtitutæ reliquis trianguli duobus lateribus minores quidem erunt; minorem verò angulũ continebunt.



Problema 8. Propoſitio 22.

Ex tribus rectis lineis, quæ ſunt trib. datis rectis lineis æquales, triangulum conſtituere. Oportet autem duas reliqua eſſe maiores omnifariam ſumptas: quoniam vnuſcuiuſque trianguli duo latera omnifariam ſumpta, reliquo ſunt maiora.



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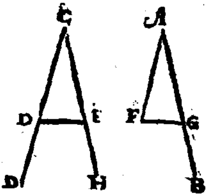
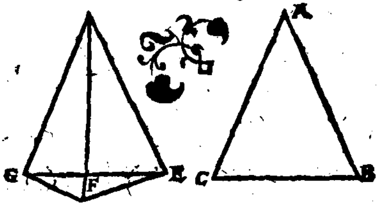
Problema 9. Propositio 23.

Ad datam rectam lineá, datumq; in ea punctum, dato angulo rectilineo æqualem angulum rectilineum constituere.



Theorema 15. Propositio 24.

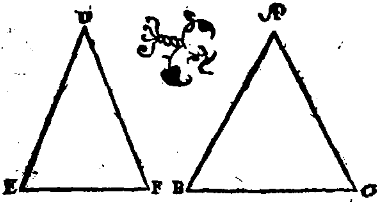
Si duo triangula duo latera duobus lateribus



çqualia habuerint, vtrūq; vtrique, angulum verò angulo maiorē sub æqualib. rectis lineis cōtentum: & basin basi maiorē habebunt.

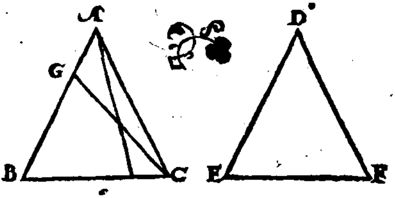
Theorema 16. Propositio. 25.

Si duo triangula duo latera duobus lateribus æqualia habuerint, vtrunque vtrique: basin verò basi maiorē: & angulum sub æqualibus rectislineis contentum angulo maiorem habebunt.



Theorema 17. Propositio 26.

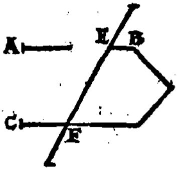
Si duo triangula duos angulos duobus angulis æquales habuerint, vtrunque vtrique; vnumque latus vni lateri æquale, siue quod æqualibus adiacet angulis, seu quod vni æqualium angulorum subtenditur: & reliqua latera reliquis lateribus æqualia vtrunq; vtriq;



& reliquum angulum reliquo angulo æqualem habebunt.

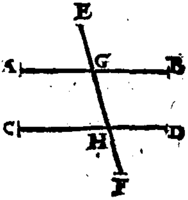
Theorema 18. Propositio 27.

Si in duas rectas lineas recta incidens linea alternatim angulos æquales inter se fecerit: parallelæ erunt inter se illæ rectæ lineæ



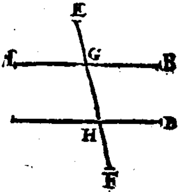
Theorema 19. Propositio 28.

Si in duas rectas lineas recta incidens linea, externum angulum interno, & opposito, & ad easdem partes, æqualem fecerit,



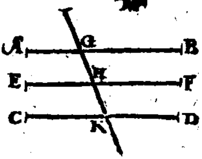
terit, aut internos, & ad eaſdem partes duobus rectis æquales: parallelæ erunt inter ſe ipſæ rectæ lineæ.

Theorema 20. Propoſitio 29.



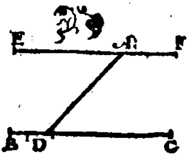
In parallelas rectas lineas recta incidens linea: & alternatim angulos inter ſe æquales efficit, & externum interno & oppoſito, & ad eaſdem partes æqualem, & internos & ad eaſdem partes duobus rectis æquales facit.

Theorema 21. Propoſitio 30.



Quæ eidem rectæ lineæ, parallelæ, & inter ſe ſunt parallelæ.

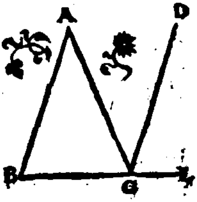
Problema 10. Propoſitio 31.



A dato puncto datæ rectæ lineæ parallelam rectam lineam ducere.

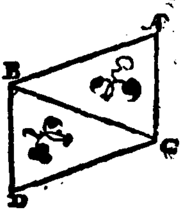
Theorema 22. Propoſitio 31.

Cuiuſcunque trianguli vno latere alterius producto: externus angelus duobus internis & oppositis est æqualis. Et trianguli tres interni anguli duobus sunt rectis æquales.



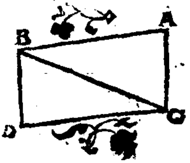
Theorema 23 Propositio 33.

Rectæ lineæ quæ æquales & parallelas lineas ad partes easdem coniungunt, & ipsæ æquales & parallelæ sunt.



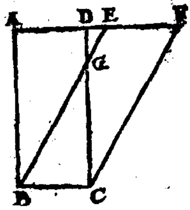
Theorema 24. Propositio 34.

Parallelogrammorum spatiorum æqualia sunt inter se, quæ ex aduerso, & latera, & anguli: atque illa bifariam secat diameter.



Theorema 25. Propositio 35.

Parallelogramma super eadem basi, & in eisdem parallelis constituta inter se sunt æqualia.



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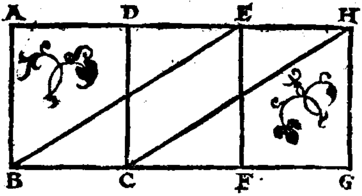
Theorema 26. Propoſitio 36.

Parallelogramma ſuper æqualibus baſibus, in eiſdē parallelis cōſtituta inter ſe ſunt æqualia.

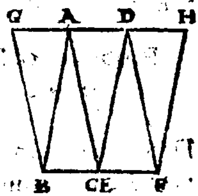


Theorema 27. Propoſitio 37.

Triangula ſuper eadem baſi cōſtituta, & in eiſdem parallelis, inter ſe ſunt æqualia.

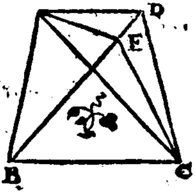
Theorema 28. Propoſitio 38.

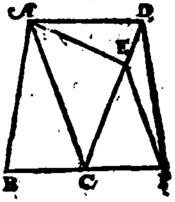
Triangula ſuper æqualibus baſibus conſtituta, & in eiſdem parallelis, inter ſe ſunt æqualia.



Theorema 29. Propoſitio 39.

Triangula ęqualia ſuper eadem baſi, & ad eaſdem partes conſtituta: & in eiſdem ſunt parallelis.



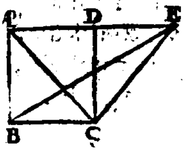


Theorema 30. Propositio 40.

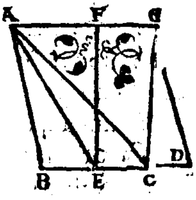
Triangula æqualia super æqualibus basibus, & ad easdem partes constituta, & in eisdem sunt parallelis.

Theorema 31. Propositio 41.

Si parallelogrammum cum triangulo eandem basin habuerit, in eisdemque fuerit parallelis, duplum erit parallelogrammum ipsius trianguli.



Problema 11. Propositio 42.

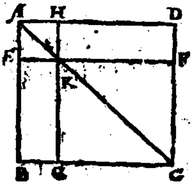


Dato triangulo æquale parallelogrammum cõstituere in dato angulo rectilineo.

Theorema 32. Propositio 34.

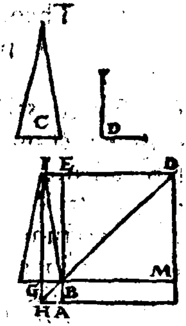
In omni parallelogrammo, complementa eorum,

eorum, quæ circa diametrum sunt, parallelográmorum, inter se sunt æqualia.



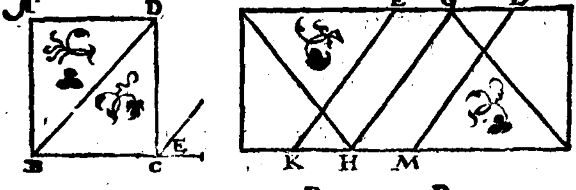
Problema 12. Propositio 44.

Ad datam rectam lineam, dato triangulo, æquale parallelogrammum applicare, in dato angulo rectilineo.

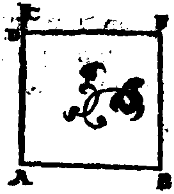


Problema 13. Propositio 45.

Dato rectilineo, æquale parallelogrammũ constituere in dato angulo rectilineo.



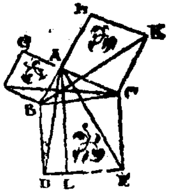
Problema 14. Propositio 46.



A data recta linea quadratum describere.

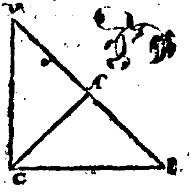
Theorema 33. Propositio 47.

In rectangulis triangulis, quádratum, quod à latere rectum angulum subtendente describitur, æquale est eis, quæ à lateribus rectum angulum continentibus describuntur, quadratis.



Theorema 34. Propositio 48.

Si quadratum, quod ab vno laterum trianguli describitur, æquale sit eis, qui à reliquis trianguli lateribus describuntur, quadratis; angulus comprehensus sub reliquis duobus trianguli lateribus, rectus est.



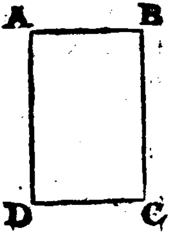
FINIS ELEMENTI I.

EVCLIDIS ELEMENTVM SECVNDVM.

DEFINITIONES.

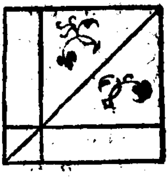
1.

OMne parallelogrammum rectangulum cótineri dicitur sub rectis duabus lineis, quæ rectum comprehendunt angulum.



2.

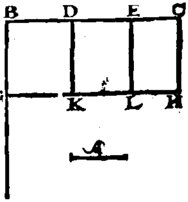
In omni parallelogrammo spatio, vnum quodlibet eorum, quæ circa diametrum illius sunt, parallelogrammorum, cum duobus complementis, Gnomon vocetur.



Theorema 1. Propositio 1.

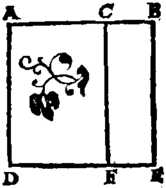
Si fuerint duæ rectæ lineæ, seceturq; ipsarũ altera in quotcunque segmenta rectangulũ com-

comprehenſum ſub illis duabus rectis lineis, æquale eſt eis quæ ſub inſecta & quolibet ſegmẽtorum comprehenduntur, rectangulis.



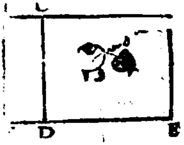
Theorema 2. Propoſitio 2.

Si recta linea ſecta ſit vtcunque: rectangula, quæ ſub tota, & quolibet ſegmentorum comprætenduntur, æqualia ſunt ei, quod à toto fit, quadrato.



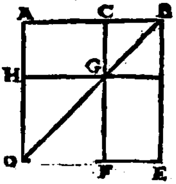
Theorema 3. Propoſitio. 3.

Si recta linea ſecta ſit vtcunque rectangulum ſub tota, & vno ſegmentorum comprę henſum, æquale eſt illi, quod ſub ſegmentis comprehenditur rectangulo, & illi, quod à prædicto ſegmẽto deſcribitur, quadrato.



Theorema 4. Propoſitio 4.

Si recta linea ſecta ſit vtcunque quadratum, quod à tota deſcribitur, æquale eſt & illis, quæ à ſegmen-

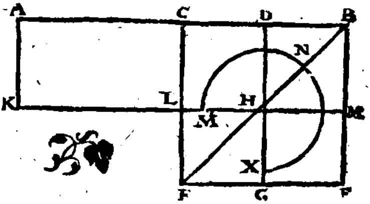


tis

tis describuntur, quadratis; & ei quod bis sub segmentis comprehenditur, rectangulo.

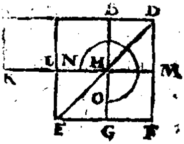
Theorema 5. Propositio 5.

Si recta linea secetur in æqualia, & non æqualia: rectangulum sub inæqualibus segmentis toto tius comprehensum, vnà cũ quadrato, quod ab intermedia sectionum, æquale est ei, quod à dimidia describitur, quadrato.



Theorema 6. Propositio 6.

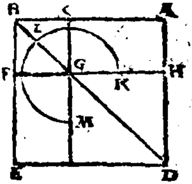
Si recta linea bifariam secetur, & illi recta quædam linea in rectum adijciatur; rectangulum comprehensum sub tota cum adiecta, & adiecta, vnà cum quadrato à dimidia, æquale est quadrato à linea, quæ tum ex dimidia, tum ex adiecta componitur, tanquam ab vna, descripto.



Theorema 7. Propositio 7.

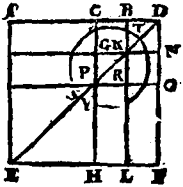
Si recta linea secetur vtcunque; quod à tota, quod-

quodque ab vno segmentorum, vtraq; simul quadrata, æqualia sunt & illi, quod bis sub tota, & dicto segmento comprehenditur, rectãgulo; & illi quod à reliquo segmento fit, quadrato.

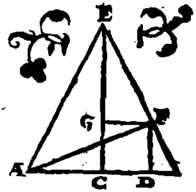


Theorema 8. Propositio 8.

Si recta linea secetur vtcunque; rectangulũ quater comprehensum sub tota, & vno segmentorum, cum eo, quod à reliquo segmẽto fit, quadrato, æquale est ei, quod à tota, & dicto segmẽto, tanquam ab vna linea describitur, quadrato.



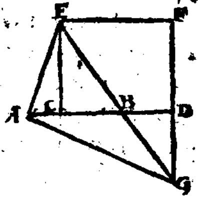
Theorema 9. Propositio 9.



Si recta linea secetur in æqualia & non æqualia: quadrata, quæ ab inæqualibus totius segmentis fiunt, simul duplicia sunt, & eius, quod à dimidia, & eius, quod ab intermedia sectionum fit, quadratorum.

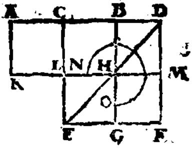
Theorema 10. Propositio 10.

Si recta linea secetur bifariã, adijciatur aũt ei in ti in rectū quæpiam recta linea: quod à tota cum ad iuncta, & quod ab adiuncta, vtraque simul quadrata, duplicia sunt, & eius, quod à dimidia, & eius, quod à composita ex dimidia & adiuncta, tanquam ab vna, descriptum sit, quadratorum.



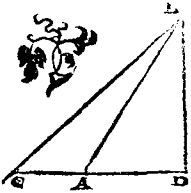
Problema I. Propositio 11.

Datam rectam lineā secare, vt comprehensum sub tota, & altero segmentorum rectangulum, æquale sit ei, quod à reliquo segmento fit, quadrato.



Theorema 11. Propositio 12.

In amblygonijs triāgulis, quadratum, quod fit à latere angulum obtusum subtendente, maius est quadratis, quæ fiunt à lateribus obtusum angulum comprehendentibus; rectangulo bis comprehenso, & ab vno laterum, quæ sunt circa obtusum angulum, in quod cū protractum fuerit, cadit perpendicularis, & ab assumpta exterius linea sub perpendiculari prope angulum obtusum.



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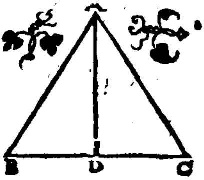
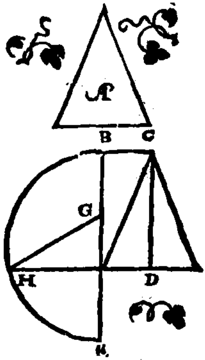
Theorema 12. Propositio 28.

In oxygonijs triangulis, quadratum à latere angulum acutum subtendente, minus est quadratis, quæ fiunt à lateribus acutum angulum comprehendentibus; rectangulo bis comprehenso; & vno laterum, quæ sunt circa acutum angulum, in quod perpẽdicularis cadit, & ab assumpta interius linea sub perpendiculari prope acutum angulum.



Problema 2. Propositio 14.



Dato rectilineo æquale quadratum constituere.

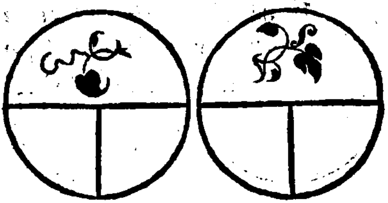
FINIS ELEMENTI II.

EVCLIDIS ELEMENTVM TERTIVM.

DEFINITIONES.

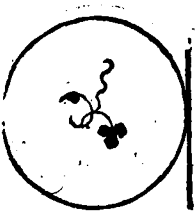
I.

Aequales circuli sunt, quorum diametri sunt æquales; vel quorum, quæ ex centris, rectæ lineæ sunt æquales.



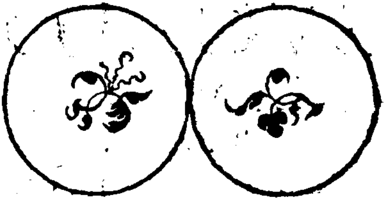
2.

Recta linea circulum tangere dicitur, quæ cùm circulum tangat; si producatur, circulum non secat.



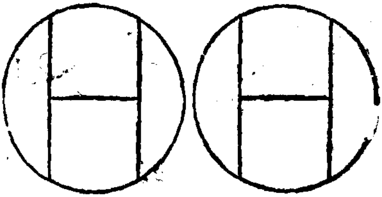
3.

Circuli ſeſe mutuò tangere dicuntur: qui ſeſe mutuò tangentes, ſeſe mutuò non ſecant.



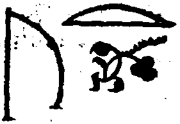
4.

In circulo æqualiter diſtare à centro rectæ lineæ dicuntur, cùm perpendiculares, quæ à centro in ipſis ducuntur, ſunt æquales. Longius autem abeſſe illa dicitur, in quã maior perpendicularis cadit.



5.

Segmentum circuli eſt figura, quæ ſub recta linea, & circuli peripheria comprehenditur.



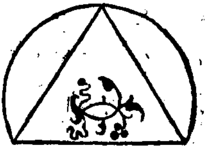
6.

Segmenti autem angulus eſt, qui ſub recta linea

linea, & circuli peripheria comprehenditur.

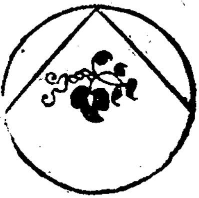
7.

In ſegmento autem angulus eſt, cùm in ſegmenti peripheria ſumptum fuerit quodpiam punctum, & ab illo in terminos rectæ eius lineæ quæ ſegmenti baſis eſt, adiunctæ fuerint, rectæ lineæ: is, inquam, angulus ab adiunctis illis lineis comprehenſus.



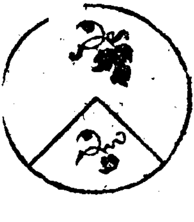
8.

Cùm verò comprehendentes angulum rectæ lineę aliquam aſſumant peripheriam, illi angulus inſiſtere dicitur.



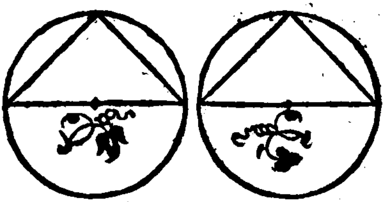
9.

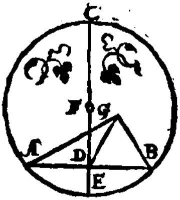
Sector autem cirouli eſt, cùm ad ipſius circuli cētrum conſtitutus fuerit angulus, compræhenſa nimirum figura, & à rectis lineis angulum continentibus, & à peripheria ab illis aſſumpta.



10.

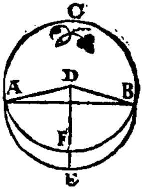
Similia circuli ſegmenta ſunt, quæ angulos capiunt æquales aut in quibus anguli inter ſe ſunt æquales.





Problema 1. Propoſitio 1.

Dati circuli centrum reperire.

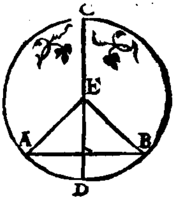


Theorema 1. Propoſitio 2.

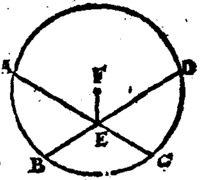
Si in circuli peripheria duo quælibet puncta accepta fuerint; recta linea, quæ ad ipſa puncta adiungitur, intra circulum cadet.

Theorema 2. Propoſitio. 3.

Si in circulo recta quædam linea per centrum extenſa quandam non per centrum extenſam bifariam ſecet: & ad angulos rectos ipſam ſecabit: Et ſi ad angulos rectos eam ſecet, bifariam quoque eam ſecabit.

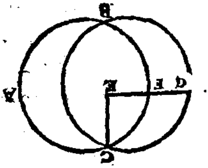


Theorema 3. Propositio 4.



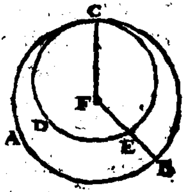
Si in circulo duæ rectæ lineæ sese mutuò secent, nõ per centrum extensæ; sese mutuò bifariam non secabunt.

Theorema 4. Propositio 5.



Si duo circuli sese mutuò secent; non erit illorum idem centrum.

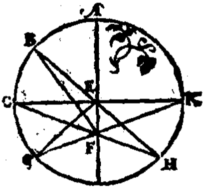
Theorema 5. Propositio 6.



Si duo circuli sese mutuò interius tangent, eorum non erit idem centrum.

Theorema 6. Propositio 7.

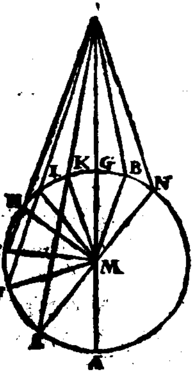
Si in diametro circuli quodpiam sumatur punctum, quod circuli centrum non sit, ab eoq; puncto in circulum quædam rectæ lineæ cadant; maxima quidem erit ea in qua centrũ; minima verò reliqua; aliarum verò propinquior illi, quæ per centrum du-



citur, remotiore ſemper maior eſt. Duæ autem ſolùm rectæ lineæ æquales ab eodem puncto in circulum cadunt, ad vtraſq; partes minimæ, vel maximæ.

Theorema 7. Propoſitio 8.

Si extra circulum ſumatur punctum quodpiam, ab eoque puncto ad circulum deducantur rectæ quædam lineæ, quarum vna quidem per centrum protendatur, reliquæ verò vt libet: in cauam peripheriam cadentium rectarum linearum minima quidem eſt illa, quæ per centrum ducitur: aliarum autem propinquior ei, quæ per centrum tranſit, remotiore ſemper maior eſt: in conuexam verò peripheriã cadentium rectarum linearũ minima quidem eſt illa, quæ inter punctum, & diametrum interponitur: aliarum autem, ea, quæ propinquior eſt minimæ, remotiore ſemper minor eſt. Duæ

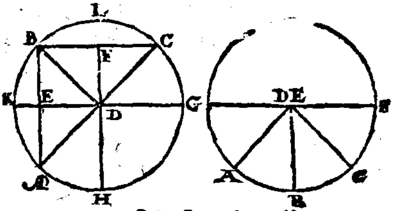


autem tantùm rectæ lineæ æquales ab eo puncto in ipſum circulum cadunt, ad vtraſque partes minimæ, vel maximæ.

Theo-

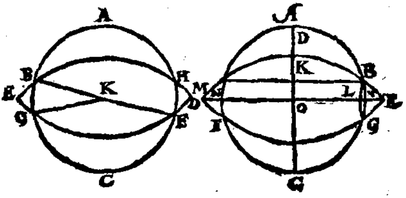
Theorema 8. Propositio 9.

Si in circulo acceptum fuerit punctum aliquod, & ab eo puncto ad circulum cadant plures, quam duæ, rectæ lineæ æquales; acceptũ punctum centrum est ipsius circuli.



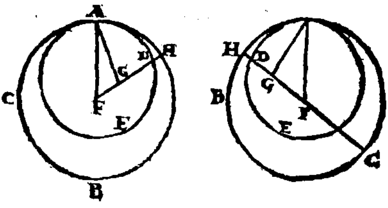
Theorema 9. Propositio 10.

Circulus circulum in pluribus, quàm duobus punctis non secat.



Theorema 10. Propositio 11.

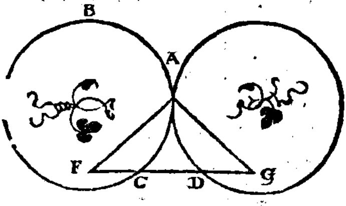
Si duo circuli sese intus cõtingãt, atque accepta



ſuerint eorum centra; ad eorum centra adiuncta recta linea, & producta, in cótactum circulorum cadet.

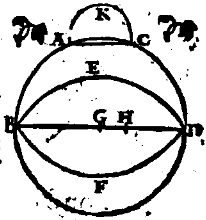
Theorema 11. Propoſitio 12.

Si duo circuli ſeſe exterius contingant, linea recta, quæ ad centra eorum adiungitur, per contactum illum traſibit.



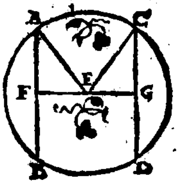
Theorema 12. Propoſitio 13.

Circulum circulum non tangit in pluribus punctis, quam vno, ſiue intus ſiue extra tangat.



Theorema 13. Propoſitio 14.

In circulo æquales rectæ lineæ æqualiter diſtant à centro. Et quæ æqualiter diſtant à centro, æquales ſunt inter ſe.



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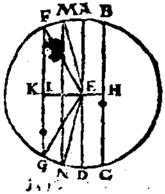
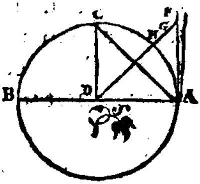
Theorema 14. Propositio 15.



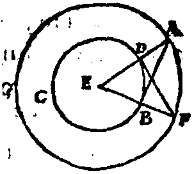
In circulo maxima quidem linea est diameter: aliarum autem propinquior centro, remotiore semper maior.

Theorema 15. Propositio 16.

Quæ ab extremitate diametri cuiusque circuli ad angulos rectos ducitur, extra ipsum circulum cadet; & in locum inter ipsam rectam lineam, & peripheriam comprehensum, altera recta linea non cadet. Et semicirculi quidem angulus quouis angulo acuto rectilineo maior est, reliquus autē minor.



Problema 12. Propositio 17.

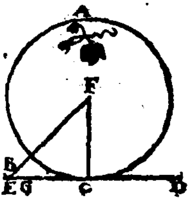


A dato puncto rectam lineam ducere, quę datum tangat circulum.

Theorema 16. Propositio 18.

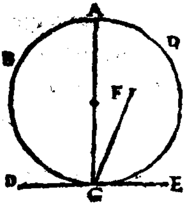
Si circulum tangat recta quæpiam linea, à

centro autem ad conta-
ctum adiungatur recta
quædam linea: quæ ad-
iuncta fuerit, ad ipsam
contingentem, perpen-
dicularis erit.



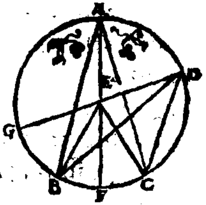
Theorema 17. Propositio 19.

Si circulum tetigerit recta quæpiam linea, à contactu autem recta linea ad angulos rectos ipsi tã genti excitetur; in excitata erit centrum circuli.



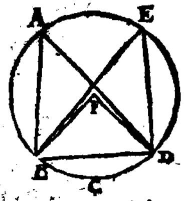
Theorema 18. Propositio 20.

In circulo, angulus ad centrum duplex est anguli ad peripheriam, cũ fuerit eadem peripheria basis angulorum.



Theorema 19. Propositio 21.

In circulo, qui in eodem segmẽto sunt, anguli, sunt inter se æquales.



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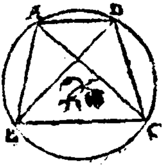
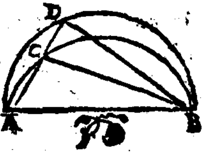
Theorema 20. Propoſitio 22.

Quadrilaterorum in circulis deſcriptorum anguli, qui ex aduerſo, duob' rectis ſunt æquales.

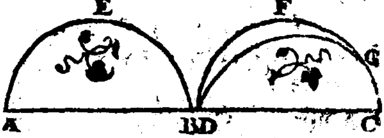


Theorema 21. Propoſitio 23.

Super eadem recta linea duo ſegmenta circulorũ ſimilia, & inæqualia non conſtituentur ad eaſdem partes.



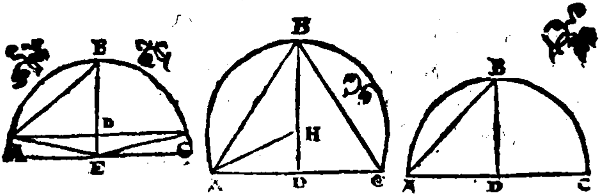
Super æqualibus rectis lineis, ſimilia circulorum ſegmenta, ſunt inter ſe æqualia.



Problema 3. Propoſitio 25.

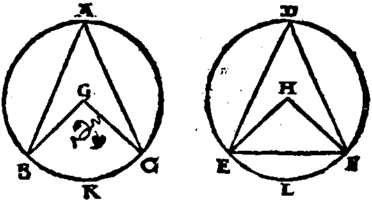
Circuli ſegmento dato, deſcribere circulũ, cuius

cuius est segmentum.



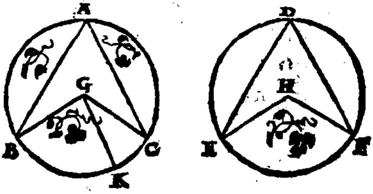
Theorema 23. Propositio 26.

In æqualibus circulis, æquales anguli æqualibus peripheriis insistunt, siue ad cētra, siue ad peripherias constituti, insistant.



Theorema 24. Propositio 27.

In æqualibus circulis, anguli, qui æqualibus peripherijs insistunt, sunt inter se æquales, siue ad centra, siue ad peripherias constituti, insistant.



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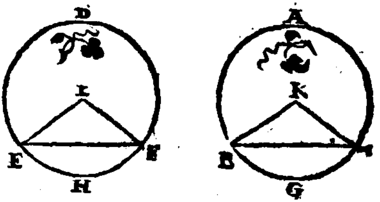
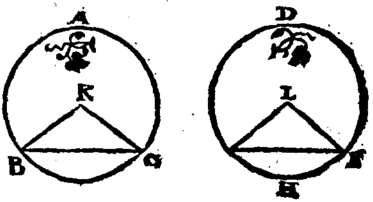
Theorema 25. Propositio 28.

In æqualibus circulis, æquales rectæ lineæ, æquales peripherias auferunt; maiorē quidē maiori, minorem autem minori.



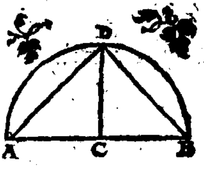
Theorema 26. Propositio 29.

In æqualibus circulis æquales peripherias, æquales rectæ lineæ subtendunt.



Problema 4. Propositio 30.

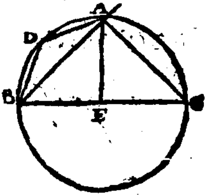
Datam peripheriam bifariam secare.



Theorema 27. Propopositio 31.

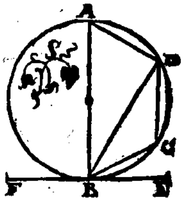
In circulo angulus, qui in semicirculo, rectus

ctus est: qui autem in maiore segmento, minor recto: qui verò in minore segmento, maior est recto. Et insuper angulus maioris segmenti, recto quidem maior est, minoris autem segmenti angulus, minor est recto.



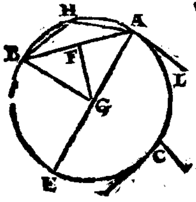
Theorema 28. Propositio 32.

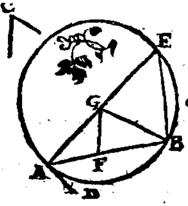
Si circulum tetigerit aliqua recta linea, à contactu autẽ producatur quædam recta linea circulum secans: anguli, quos ad contingentem facit, ęquales sunt ijs, qui in alternis circuli segmẽtis consistunt, angulis.

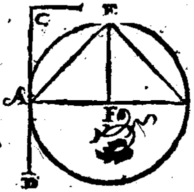


Problema 5. Propositio 33.

Super data recta linea describere segmentũ circuli, quod capiat angulum æqualem dato angulo rectilineo.







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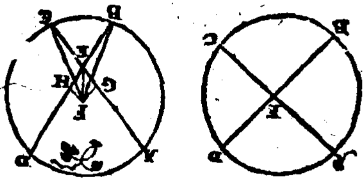
Problema 6. Propositio 34.

A dato circulo segmentum abscindere, capiens angulum æqualem dato angulo rectilineo.

Theorema 29. Propositio 35.

Si in circulo duæ rectæ lineæ sese mutuò secuerint; rectangulum comprehensum sub segmentis vnius, æquale est ei, quod sub segmentis alterius comprehenditur, rectangulo.



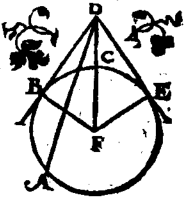
Theorema 30. Propositio 36.

Si extra circulũ sumatur pũctũ aliquod, ab eoque in circulũ cadant duæ rectæ lineæ; quarum altera quidem circulũ secet, altera verò

verò tangat: quod sub tota secante, & exterius inter punctum & conuexam peripheriam assumpta comprehenditur, rectangulum; æquale erit ei, quod à tangente describitur, quadrato.

Theorema 31. Propositio 37.

Si extra circulum sumatur punctum aliquod ab eoque puncto in circulum cadant duæ rectæ lineæ, quarum altera circulum secet, altera in eum incidat; sit autem, quod sub tota secante, & exterius inter punctum, & conuexam peripheriam assumpta, comprehenditur, rectangulum, æquale ei, quod ab incidente describitur, quadrato; incidens ipsa circulum tanget.



FINIS ELEMENTI III.

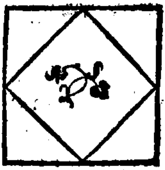
EVCLI-

EVCLIDIS ELEMENTVM QVARTVM.

DEFINITIONES.

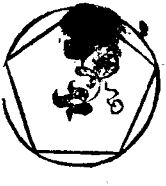
1.

Figura rectilinea in figura rectilinea inscribi dicitur, cum singuli eius figurę; quæ inscribitur, anguli, singula latera eius, in qua inscribitur, tangunt.



2.

Similiter & figura circum figuram describi dicitur, quum singula eius, quæ circumscribitur, latera singulos eius figuræ angulos tetigerint, circum quam illa describitur.



3.

Figura rectilinea in circulo inscribi dicitur, quum singuli eius figuræ, quæ inscribitur, anguli tetigerint circuli peripheriam.

4.

Figura verò rectilinea circa circulum describi dicitur, quùm singula latera eius, quæ circum scribitur, circuli peripheriam tangunt.

5.

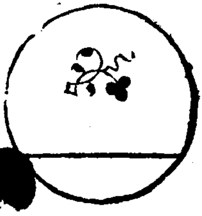
Similiter & circulus in figura rectilinea inscribi dicitur, quũ circuli peripheria singula latera tangit eius figuræ, cui inscribitur.

6.

Circulus autem circum figuræ describi dicitur, quum circuli peripheria singulos tangit eius figuræ, quam circumscribit, angulos.

7.

Recta linea in circulo accommodari, seu coaptari dicitur, qu[illegible] eius extrema in circuli peripheria fuerint.

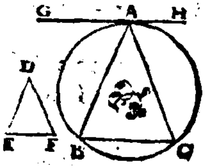


Problema 1. Propositio 1.

In dato circulo, rectam lineam accommodare æqualem datæ rectæ lineę, quæ circuli diametro nõ sit maior.

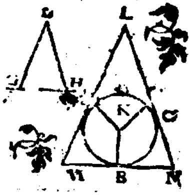
Problema 2. Propoſitio 2.

In dato circulo, triangulum deſcribere, dato triangulo æquiangulum.



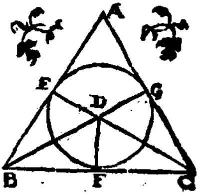
Problema 3. Propoſitio 3.

Circa datum circulum triangulum deſcribere, dato triangulo æquiangulum.



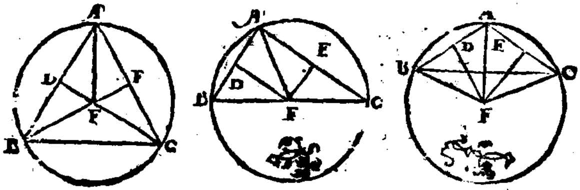
Problema 4. Propoſitio 4.

In dato triangulo circulum inſcribere.



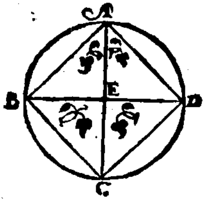
Problema 5. Propoſitio 5.

Circa datum triangulũ, circulũ deſcribere.



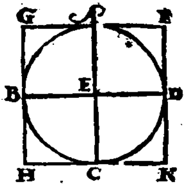
Problema 6. Propositio 6.

In dato circulo quadratum describere.



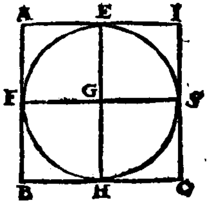
Problema 7. Propositio. 7.

Circa datum circulum, quadratum describere.



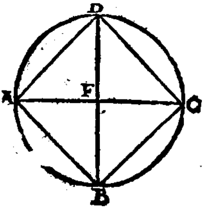
Problema 8. Propositio 8.

In dato quadrato circulum inscribere.



Problema 9. Propositio 9.

Circa datum quadratũ, circulum describere.



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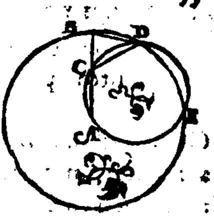
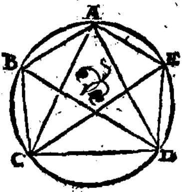
Problema 10. Propositio. 10.

Isosceles triangulum cõstituere; quod habeat vtrunque eorum, qui ad basin sunt, angulorum, duplum reliqui.



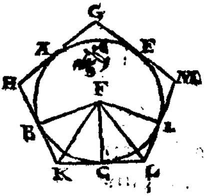
Problema 11. Propositio 11.

In dato circulo, pentagonũ æquilaterum, & æquiangulum inscribere.



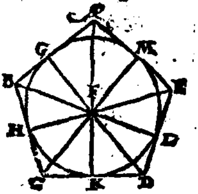
Problema 12. Propositio 12.

Circa datum circulum, pentagonum, æquilaterum & æquiangulum describere.



Problema 13. Propositio 13.

In dato pentagono æquilatero, & æquiangulo circulum inscribere.

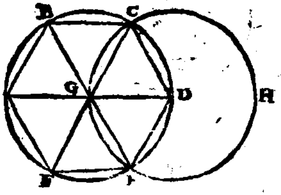


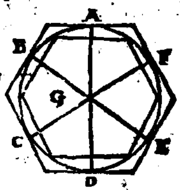
Problema 14. Propositio 14.

Circa datum pentagonũ æquilaterum & æquiangulum, circulum describere.

Problema 15. Propositio 15.

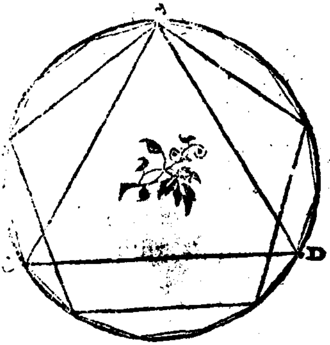
In dato circulo hexagonum & æquilaterũ, & æquiangulum inscribere.





Problema 16. Propositio 16.

In dato circulo quinti decagonum & æquilaterũ, & æquiangulum describere.



FINIS ELEMENTI IV.

EVCLIDIS ELEMENTVM QVINTVM.

DEFINITIONES.

1.

Pars eſt magnitudo magnitudinis minor maioris, quum minor metitur maiorem.

2.

Multiplex autem eſt maior minoris, cùm minor metitur maiorem.

3.

Proportio, eſt duarum magnitudinum eiuſdem generis mutua quædam ſecundum quãtitatem habitudo.

4.

Proportionalitas verò eſt proportionũ ſimilitudo.

5.

Proportionem habere inter ſe magnitudines dicuntur, quæ poſſunt multiplicatæ ſeſe mutuò ſuperare.

6.

In eadẽ proportione magnitudines dicũtur eſſe, prima ad ſecundam, & tertia ad quartam, cùm primæ & tertiæ æquè multiplicia, à ſecundæ & quartæ æquè multiplicibus, qua-

qualiſcunque ſit hæc multiplicatio, vtrunque ab vtroque; vel vnà deficiunt, vel vnà æqualia ſunt, vel vnà excedunt; ſi ea ſumantur, quæ inter ſe reſpondent.

7.

Eandem autem proportionem habentes magnitudines, proportionales vocentur.

8.

Cùm verò æquè multiplicium, multiplex primæ magnitudinis exceſſerit multiplicem ſecundæ; at multiplex tertiæ non exceſſerit multiplicem quartæ: tunc prima ad ſecundam, maiorem proportionem habere dicetur, quàm tertia ad quartam.

9.

Proportinoalitas autem in tribus terminis paucisſimis conſiſtit.

10.

Cùm autem tres magnitudines proportionales, fuerint, prima ad tertiam, duplicatam proportionem habere dicitur eius, quam habet ad ſecundam. At cùm quatuor magnitudines proportionales fuerint, prima ad quartam, triplicatam proportionem habere dicitur eius, quam habet ad ſecundam: & ſemper deinceps, vno amplius, quamdiu proportio extiterit.

11.

Homologæ, ſeu ſimiles proportione magnitudines dicuntur, antecedentes quidem ante-

antecedentibus, consequentes verò consequentibus.

12.

Alterna proportio, est sumptio antecedentis ad antecedentem, & consequentis ad consequentem.

13.

Inuersa proportio, est sumptio consequentis, ceu antecedentis, ad antecedentem, velut ad ipsam consequentem.

14.

Compositio proportionis, est sumptio antecedentis cum consequente, ceu vnius ad ipsam consequentem.

15.

Diuisio proportionis, est sumptio excessus, quo consequentem superat antecedens, ad ipsam consequentem.

16.

Conuersio proportionis, est sumptio antecedentis ad excessum, quo superat antecedens ipsam consequentem.

17.

Ex æqualitate proportio est, si plures duabus sint magnitudines, & his aliæ multitudine pares, quæ binæ sumantur, & in eadem proportione: quum vt in primis magnitudinibus prima ad vltimam, sic & in secundis magnitudinibus prima ad vltimam sese habuerit.

Vel aliter, ſumptio extremorum per ſubductionem mediorum.

18.

Ordinata proportio eſt, cùm fuerit, quemadmodum antecedens ad conſequentem, ita antecedens ad conſeqnentem: fuerit etiam vt conſequens ad aliud quidpiam, ita conſequens ad aliud quidpiam.

19.

Perturbata autem proportio eſt, cùm trib9 poſitis magnitudinibus, & alijs quæ ſint his multitudine pares, vt in primis quidem magnitudinibus ſe habet antecedens ad conſequentem, ita in ſecundis magnitudinibus antecedens ad conſequentem: vt autem aliud quidpiam ſic in ſecundis magnitudinibus aliud quidpiam ad antecedentem.

Theorema 1. Propoſitio 1.

Si ſint quotcunq; magnitudines, quotcunque magnitudinum æqualium numero, ſingulæ ſingularum, æquè multiplices; quàm multiplex eſt vnius vna magnitudo, tam multipleces erunt & omnes omnium.

Theorema 2. Propopositio 2.

Si prima secundæ æquè fuerit multiplex, atque tertia quartæ; fuerit autem & quinta secundæ æquè multiplex, atque sexta quartæ: erit & composita prima cũ quinta, secundæ æquè multiplex; atq; tertia cum sexta, quartæ.

Theorema 3. Propositio 3

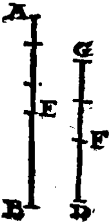
Si sit prima secundæ æquè multiplex, atque tertia quartæ; sumantur autem æquè multiplices primæ, & tertiæ; erit & ex æquo, sumptarum vtraque vtriusque æquè multiplex; altera quidem secundæ, altera autem quartæ.

Theorema 4. Propositio 4.

Si prima ad secundã, eandem habuerit proportioné, & tertia ad quartam: etiam æquè multiplices primę & tertię, ad ęquè multiplices secundæ & quartæ, iuxta quãmuis multiplicationé, ean

dem

dem habebunt propositionem; si, prout inter se respondent, ita sumptæ fuerint.

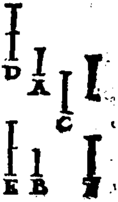


Theorema 5. Propositio 5.

Si magnitudo magnitudinis æquè fuerit multiplex, atque ablata ablatæ: etiam reliqua reliquæ ita multiplex erit, vt tota totius.

Theorema 6. Propositio 6.

Si duæ magnitudines, duarum magnitudinum sint æquè multiplices, & detractæ quædam sint earundem æquè multiplices: & reliquæ eisdem aut æquales sunt, aut æquè ipsarum multiplices.



Theorema 7. Propositio 7.

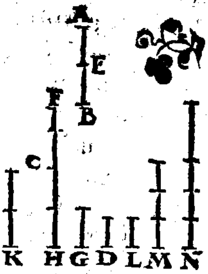
Aequales ad eandem, eandem habent rationem: & eadem ad æquales.

Theorema 8. Propositio 8.

Inæqualium magnitudinum maior ad eandem

dem

dem, maiorem proportionem habet: quàm minor: & eadem ad minorem, maiorem proportionem habet, quàm ad maiorem.



Theorema 9. Propositio 9.

Quæ ad eandem, eandem habent proportionem, æquales sunt inter se: & ad quas eadem, eandem habet proportionem, eæ quoque sunt inter se æquales.

Theorema 10. Propositio 10.

Ad eandem magnitudinem proportionem habentium, quę maiorem proportionem habet, illa maior est, ad quam autem eadem maiorem proportionem habet, illa minor est.

Theorema 11. Propositio 11.

Quæ eidem sunt eædem proportiones, & inter se sunt eædem.

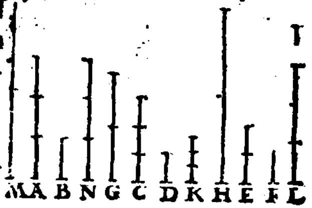
Theo-

Theorema 12. Propositio 12.

Si sint magnitudines quotcunque proportionales; quemadmodũ se habuerit vna antecedentium ad vnã consequentium; ita se habebunt omnes antecedentes ad omnes consequentes.

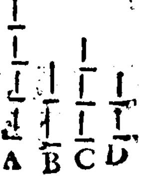
Theorema 13. Propositio 13.

Si prima ad secundã, eandem habuerit proportionẽ, quam tertia ad quartã; tertia verò ad quartam maiorem proportionẽ habuerit, quàm quinta ad sextam: prima quoq; ad secundã maiorẽ proportionẽ habebit, quàm quinta ad sextam.



Theorema 14. Propositio 14.

Si prima ad secundam eandem habuerit proportionẽ, quã tertia ad quartã: prima verò quàm tertia maior fuerit, erit & secunda maior quàm quarta. Quòd si prima fuerit æqualis tertiæ,

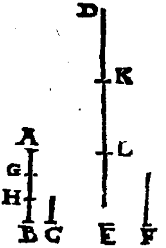


erit

erit secunda æqualis quartæ: si verò minor, & minor erit.

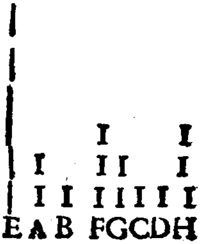
Theorema 15. Propositio 15.

Partes cum pariter multiplicibus in eadem sunt proportione, si prout sibi mutuò respondent, ita sumantur.



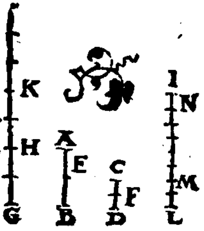
Theorema 16. Propositio 16.

Si quatuor magnitudines proportionales fuerint, & vicissim proportionales erunt.



Theorema 17. Propositio 17.

Si compositæ magnitudines proportionales fuerint, hæ quoq; diuisæ proportionales erunt.



Theo-

Theorema 18. Propoſitio 18.

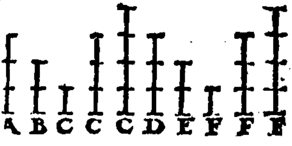
Si diuiſæ magnitudines ſint proportionales, hæ quoque compoſitæ proportionales erunt.

Theorema 19. Propoſitio 19.

Si quemadmodum totum ad totum, ita ablatum ſe habuerit ad ablatum: & reliquum ad reliquum, vt totum ad totum, ſe habebit.

Theorema 20. Propoſitio 20.

Si ſint tres magnitudines, & aliæ ipſis æquales numero, quæ binæ, & in eadem proportione ſumantur; ex æquo autem prima quàm tertia maior fuerit; erit & quarta, quam ſexta, maior.

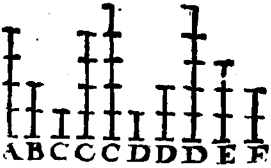


Quod ſi prima tertiæ fuerit æqualis, erit & quarta æqualis ſextæ: ſin illa minor, quoque minor erit.

Theo-

Theorema 21. Propoſitio 21.

Si ſint tres magnitudines, & aliæ ipſis æquales numero, quæ binæ, & in eadem proportione ſumantur, fuéritque perturbata earum proportio: ex æquo autẽ prima, quã tertia, maior fuerit; erit & quarta, quàm ſexta, maior: quod ſi prima tertiæ fuerit æqualis, erit & quarta æqualis ſextæ: ſin illa minor, hæc quoque minor erit.

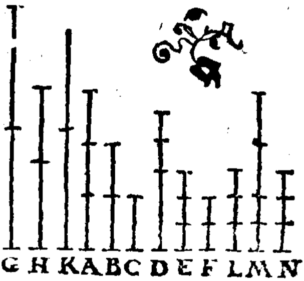


Theorema 22. Propoſitio 22.

Si ſint quodcunque magnitudines, & aliæ ipſis æquales numero, quæ binæ in eadẽ proportione ſumantur: & ex æqualitate in eadem proportione erunt.

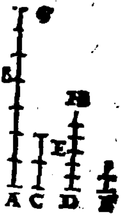
Theorema 23. Propoſitio 23.

Si ſint tres magnitudines, aliæque ipſis æqua les numero, quæ binæ in eadem proporti-one ſumantur; fuerit autẽ perturbata earum proportio: Etiam ex æqualitate in eadem proportione erunt.



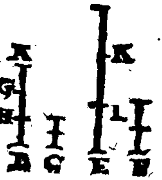
Theorema 24. Propoſitio 24.

Si prima ad ſecundam, eandem habuerit proportionem, quam tertia ad quartam; habuerit autem & quinta ad ſecundam, eandem proportionem, quàm ſexta ad quartam: Etiam compoſita prima cum quinta, ad ſecundam eandem habebit proportionem, quam tertia cum ſexta, ad quartam.



Theorema 25. Propoſitio 25.

Si quatuor magnitudines proportionales fuerint; maxima, & minima reliquis duabus maiores erunt.



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Theorema 26. Propopositio 26.

Si prima ad secundam, maiorem habuerit proportionem, quàm tertia ad quartam: habebit couertendo secunda ad primam, minorem proportionem, quàm quarta ad tertiam.

Theorema 27. Propositio 27.

Si prima ad secundam, habuerit maiorem proportionem, quàm tertia ad quartam: habebit quoq; vicissim prima ad tertiam, maiorem proportionem, quàm secunda ad quartam.

Theorema 28. Propositio. 28.

Si prima ad secundam habuerit maiorem proportionem, quàm tertia ad quartam: habebit quoque composita prima cum secunda, ad secundam, maiorem proportionem, quàm composita tertia cũ quarta, ad quartam.

Theorema 29. Propositio 29.

Si composita prima cum secunda, ad secundam, maiorem habuerit proportionem quã com-

composita tertia cum quarta, ad quartam: Habebit quoq; diuidando prima ad secundam, maiorem proportionem, quam tertia, ad quartam.

Theorema 30. Propositio 30.

Si composita prima cum secunda, ad secundam habuerit maiorem proportionem, quàm composita tertia cum quarta, ad quartá; Habebit quoq; per conuersionem proportionis, prima cum secunda, ad primam, minorem proportionem, quàm tertia cum quarta, ad tertiam.

Theorema 31. Propositio 31.

Si sint tres magnitudines, & aliæ ipsis æquales numero; sitque maior proportio primæ priorum ad secundam, quàm primæ posteriorum ad secundam; Item secundæ priorem ad tertiam maior quàm secundæ posteriorum ad tertiam: Erit quoque ex æqualitate maior proportio primæ priorum ad tertiam, quàm primæ posteriorum ad tertiam.

Theorema 32. Propositio 32.

Si sint tres magnitudines, & aliæ ipsis æquales numero; sitque maior proportio primæ priorum ad secundam, quàm secundę posteriorum

riorum ad tertiam; Item secundæ priorum ad tertiam maior, quàm primæ posteriori ad secundam: Erit quoque ex æqualitate, maior proportio primæ priorum ad tertiam, quàm primæ posteriorum ad tertiam.

Theorema 33. Propositio 33.

Si fuerit maior proportio totius ad totum, quàm ablati ad ablatum: Erit & reliqui ad reliquum, maior proportio, quàm totius ad totum.

Theorema 34. Propositio 34.

Si sint quotcunque magnitudines, & aliæ ipsis æquales numero; sitque maior proportio primæ priorum ad primam posteriorum, quàm secũdę ad secundam; & hæc maior, quàm tertiæ ad quartam, & sic deinceps: Habebunt omnes priores simul, ad omnes posteriores simul, maiorem proportionem quàm omnes priores, relictâ primâ, ad omnes posteriores, relictâ quoque primâ; minorem autem, quàm prima priorum ad primam posteriorum; maiorem denique etiã, quàm vltima priorum ad vltimam posteriorum.

FINIS ELEMENTI V.

EVCLI-

EVCLIDIS ELEMENTVM SEXTVM.

DEFINITIONES.

1.

Similes figuræ rectilineæ sunt, quæ & angulos singulos singulis æquales habent; atque etiam latera, quæ circum angulos æquales, proportionalia.

2.

Reciprocæ autem figuræ sunt, cùm in vtraque figura, antecedentes, & consequentes proportionum termini fuerint.

3.

Secundum extremam, & mediam proportionem recta linea secta esse dicitur, cùm vt tota ad maius segmentum, ita maius ad minus se habuerit.

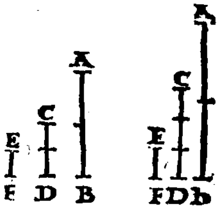
4.

Altitudo cuiusque figuræ est linea perpendicularis à vertice ad basin deducta.

5. Ra-

5.

Proportio proportionibus componi dicitur, cũ proportionum quátitates inter se multiplicatæ, aliquá effecerint proportionem.



6.

Parallelogrammum secundum aliquam lineam applicatum, deficere dicitur parallelogrammo, quando non occupat totam lineam: Excedere vero, quando occupat maiorem lineam, quàm sit ea, secundum quam applicatur: Ita tamen, vt parallelogrammũ deficiens, aut excadens, eandem habeat altitudinem cum parallelogrammo applicato, constituatque cum eo totum vnum parallelogrammum.

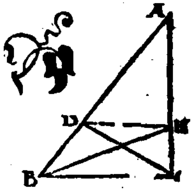
Theorema 1. Propositio 1.

Triangula, & parallelogramma, quorum eadem fuerit altitudo, ita se habent inter se, vt bases.

Theorema 2. Propositio 2.

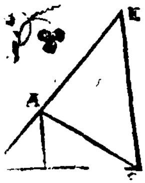
Si ad vnum trianguli latus parallela ducta fuerit recta quædam linea:

Hæc proportionaliter secabit ipsius trianguli latera. Et si trianguli latera proportionaliter secta fuerint; quæ ad sectiones adiuncta fuerit recta linea, erit ad reliquum ipsius trianguli latus parallela.



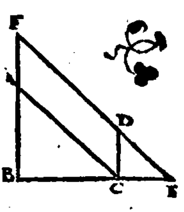
Theorema 3. Propositio 3.

Si trianguli angulus bifariam sectus sit; secans autem angulum recta linea secuerit & basin; basis segmenta, eandē habebunt proportionē, quā reliqua ipsius trianguli latera: Et si basis segmenta eandem habeant proportionem quam reliqua ipsius triangu li latera; recta linea, quæ à vertice ad sectionem producitur, bifariam secat trianguli ipsius angulum.



Theorema 4 Propositio 4.

Aequiangulorum triangulorum proportionalia sunt latera, quæ circum ęquales angulos, & homologa sunt latera, quæ ęqualibus angulis subtenduntur.



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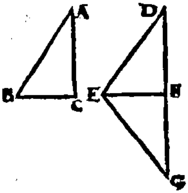
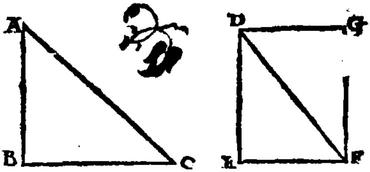
Theorema 5. Propositio. 5.

Si duo triangulo, latera proportionalia habeant, æquiangula erunt triangula, & æquales habebunt eos angulos, sub quibus homologa latera subtenduntur.



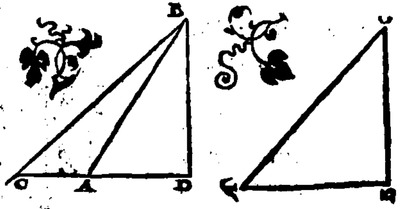
Theorema 6. Propositio 6.

Si duo triangula vnum angulum vni angulo æqualẽ, & circum æquales angulos latera proportionalia habuerint; æquiangula erunt triangula, æqualesque habebunt angulos, sub quibus homologa latera subtenduntur.



Theorema 7. Propositio 7.

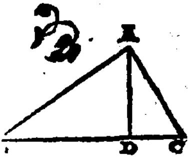
Si duo triangula vnum angulum vni angulo æqualẽ circũ autẽ alios angulos latera proportionalia habent; reliquorum



vero ſimul vtrunque aut minorem, aut non minorem recto: æquiangula erunt triangula, & æquales habebunt eos angulos, circum quos proportionelia ſunt latera.

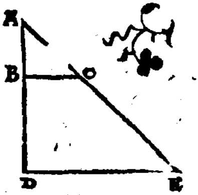
Theorema 8. Propoſitio 8.

Si in triangulo rectangulo, ab angulo recto in baſin perpendicularis ducta ſit; quæ ad perpendicularem triangula, tum toti triangulo, tum ipſa inter ſe ſimilia ſunt.



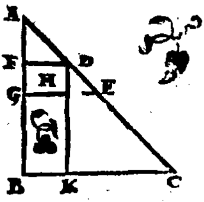
Problema 1. Propoſitio. 9.

A data recta linea imperatam partem auferre.



Problema 2. Propoſitio 10.

Datam rectam lineam inſectam ſimiliter ſecare, vt data altera recta ſecta fuerit.



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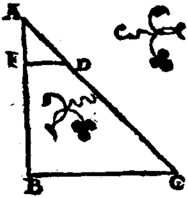
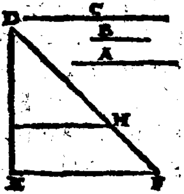
Problema 3. Propositio. 11.

Duabus datis rectis lineis, tertiam proportionalem adinuenire.



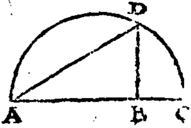
Problema 4. Propositio 12.



Tribus datis rectis lineis quartam proportionalē adinuenire.

Problema 5. Propositio 13.

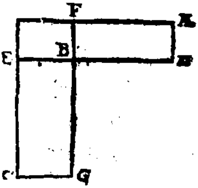
Duabus datis rectis lineis, mediam proportionalem adinuenire.



Theorema 9. Propositio 14.

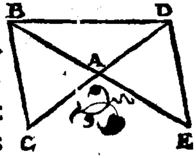
Aequalium, & vnum vni æqualem habentium angulum, parallelogrammorum reciproca sunt latera, quæ circum æquales angulos; Et quorum parallelogrammorū vnum angu-

angulum vni angulo æqualem habentium reciproca sunt latera, quæ circum æquales angulos, illa sunt æqualia.



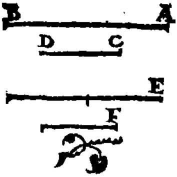
Theorema 10. Propositio 15.

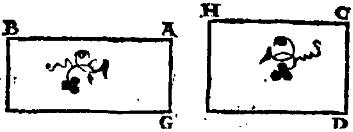
Aequalium, & vnum angulum vni ęqualem habentium triangulorum reciproca sunt latera, quæ circum æquales angulos: Et quorum triangulorum vnum angulum vni æqualem habentium, reciproca sunt latera, quæ circū æquales angulos, illa sunt ęquales.



Theorema 11. Propositio 16.

Si quatuor rectæ lineæ proportionales fuerint, quod sub extremis compræhenditur rectangulum, ęquale est ei, quod sub medijs com-

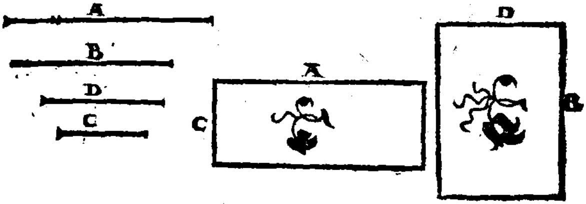




comprehenditur, rectangulo. Et sub extremis comprehensum rectangulum æquale fuerit ei, quod sub medijs continetur, rectãgulo, illæ quatuor rectæ lineę proportionales erunt.

Theorema 12. Propositio 17.

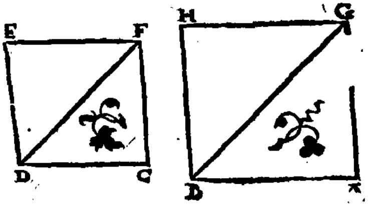
Si tres rectæ lineę sint proportionales; quod sub extremis comprehẽditur rectangulum



æquale fit ei, quod à media describitur, quadrato: Et si sub extremis comprehẽsum rectangulum æquale fit ei, quòd à media describitur, quadrato; illę tres rectę lineæ proportionales erunt.

Theorema 6. Propositio 16.

A data recta linea, dato rectilineo simile, similiterq́; positum rectilineum describere.



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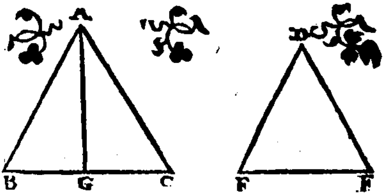
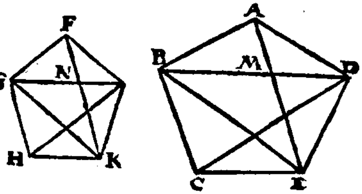
Theorema 13. Propositio 19.

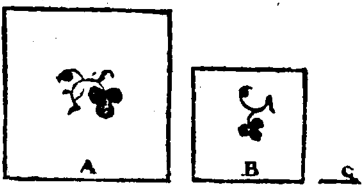
Similia triangula inter ſe ſunt in duplicata proportione laterum homologorum.

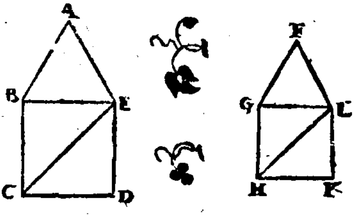


Theorema 14. Propoſitio 20.

Similia polygona in ſimilia triangula diuiduntur, & numero ęqualia, & homologa totis. Et polygona duplicatam habẽt eam inter ſe proportionem, quam lat' homologum ad homologum latus.







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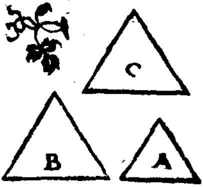
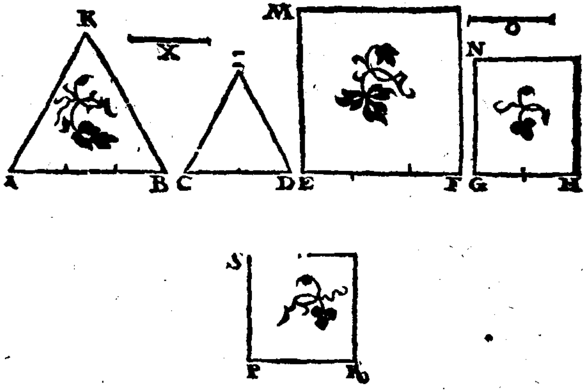
Theorema 15. Propoſitio 21.

Quæ eidem rectilineo ſunt ſimilia, & inter ſe ſunt ſimilia.



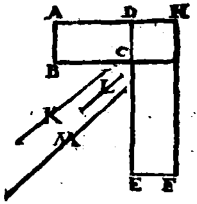
Theorema 16. Propoſitio 22.

Si quatuor rectæ lineæ proportionales fuerint: & ab eis rectilinea ſimilia ſimiliterq́ue deſcripta proportionalia erunt. Et ſi à rectis lineis ſimilia ſimiliterq́ue deſcripta rectilinea proportionalia fuerint; ipſæ etiam rectæ lineæ proportionales erunt.



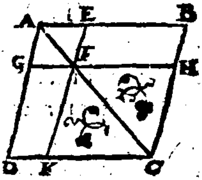
Theorema 17. Propositio 23.

Aequiangula parallelogramma inter se proportionem habent eam, quæ ex lateribus componitur.

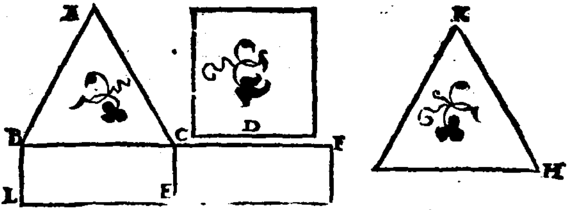


Theorema 18. Propositio 24.

In omni parallelogrammo, quæ circa diametrū sunt parallelogramma, & toti, & inter se sunt similia.



Problema 7. Propositio 25.



Dato recto lineo simile: similiterque positum; & alteri dato æquale idem constituere.

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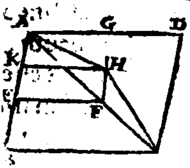
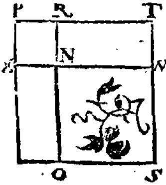
Theorema 19. Propositio 26.

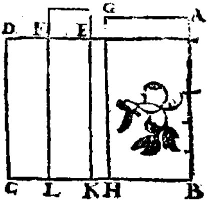


Si à parallelogrammo parallelogrammum ablatum sit; & simile toti, & similiter positum, communem cum eo habens angulum; hoc circum eandem cum toto diametrum consistit.

Theorema 20. Propositio 27.

Omnium parallelogrammorum secũdum eandem rectam lineam applicatorum, deficientiumq́; si guris parallelo grãmis similibus, si-





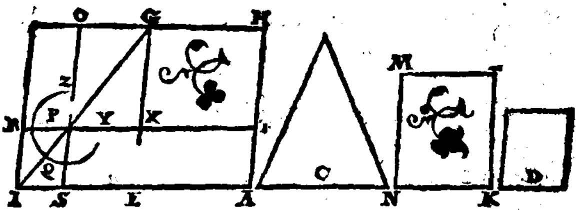
militerque positis, ei, quod à dimidia describitur; maximum, id est, quod ad dimidiam applicatur, parallelogrammum simile existens defectui.

Problema 8. Propositio 28.

Ad datam lineam rectam, dato rectilineo æquale parallelogrammum applicare, deficiens figura parallelogramma, quæ similis

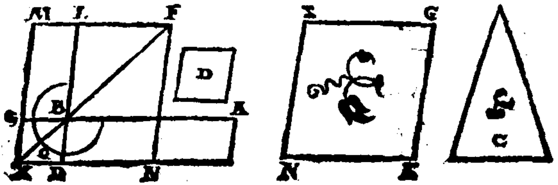
F sit

sit alteri rectilineo dato. Oportet autem datum rectilineum, cui æquale applicandum est, non maius esse eo, quod ad dimidiam applicatur, cùm similes sint defectus & eius, quod à dimidia describitur, & eius, cui simile deesse debet.



Problema 9. Propositio 29.

Ad datam rectam lineam, dato rectilineo æquale parallelogrammum applicare, excedens figura parallelogramma, quę similis sit parallelogrammo alteri dato.



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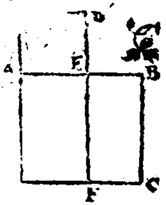
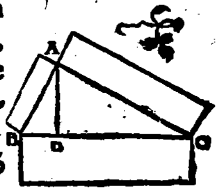
Problema 10. Propoſitio 30.



Propoſitam rectam lineam terminatam, extrema ac media ratione (proportione:) ſecare.

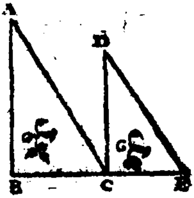
Theorema 21. Propoſitio 31.

In rectangulis triangulis, figura quæuis à latere rectum angulum ſubtendente deſcripta, æqualis eſt figuris, quæ priori illi ſimiles, & ſimiliter poſitæ, à lateribus rectum angulum cõtinentibus deſcribũtur.



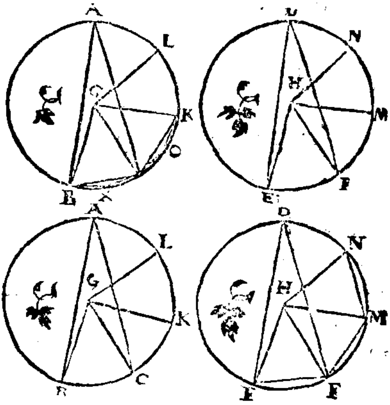
Theorema 22. Propoſitio 32.

Si duo triangula, quæ duo latera duobus lateribus proportionalia habeant, ſecundum vnum angulum compoſita fuerint, ita vt homologa eorum latera ſint etiam parallela; tum reliqua illorum triangulorum latera in rectam lineam collocata reperientur.



Theorema 23. Propositio 33.

In æqualibus circulis, anguli eandem habent rationem, cum ipsis peripherijs, in quibus insistũt, siue ad centra, siue ad peripherias cõstituti, illis insistant peripherijs. Insuper verò & sectores, quippe qui ad centra consistunt.



FINIS ELEMENTI VI.

EVCLI-

EVCLIDIS ELEMENTVM SEPTIMVM.

DEFINITIONES.

1.

VNitas est, secundum quam vnum, quodque eorum, quæ sunt, vnum dicitur.

2.

Numerus autem, ex vnitatibus composita multitudo.

3.

Pars est numerus numeri, minor maioris, cum minor metitur maiorem.

4.

Partes autem, cùm non metitur.

5.

Multiplex verò, maior minoris, cùm maiorem metitur minor.

6.

Par numerus est, qui bifariam diuiditur.

7.

Impar verò, qui bifariam non diuiditur: vel, qui vnitate differt à pari.

8

Pariter par numerus est, quem par numerus metitur per numerum parem.

9.

Pariter autem impar est, quem par numerus metitur per numerum imparem.

10.

Impariter verò impar numerus est, quem impar numerus metitur per numerum imparem.

11.

Primus numerus est, quem vnitas sola metitur.

12.

Primi inter se numeri sunt, quos sola vnitas, mensura communis, metitur.

13.

Compositus numerus est, quem numerus quispiam metitur.

14.

Compositi autē inter se numeri sunt, quos numerus aliquis, mensura communis, metitur.

15.

Númerus numerum multiplicare dicitur, cùm toties compositus fuerit is, qui multiplicatur, quot sunt in ipso multiplicante vnitate; & procreatus fuerit aliquis.

16.

Cùm autem duo numeri mutuò sese multiplicantes quempiam faciunt, qui factus erit, planus appellabitur. Qui verò numeri mutuò sese multiplicarint, illius latera dicentur.

Cùm

17.

Cùm verò tres numeri mutuò sese multiplicantes quempiam faciunt, qui procreatus erit, solidus appellabitur, qui autem numeri mutuò sese multiplicarint, illius latera dicentur.

18.

Quadratus numerus est, qui æqualiter ęqualis; vel, qui à duabus æqualibus numeris cõtinetur.

19.

Cubus verò, qui æqualiter æquali æqualiter: vel, qui à tribus æqualibus numeris continetur.

20.

Numeri proportionales sunt, cùm primus secundi, & tertius quarti, æquè multiplex est; vel eadem pars, vel eædem partes.

21.

Similes plani, & solidi numeri sunt, qui proportionalia habent latera.

22.

Perfectus numerus est, qui suis ipsius partibus est æqualis.

23.

Numerus numerum metiri dicitur per illum numerum, quem multiplicans, vel à quo multiplicatus, illum producit.

24.

Proportio numerorum est habitudo quædam vnius numeri & alterum, secundum quod illius est multiplex, vel pars, partesue.

25.

Termini, siue radices proportionis dicuntur duo numero, quibus in eadem proportione minores sumi nequeunt.

26.

Cùm tres numeri proportionales fuerint; Primus ad tertium, duplicatam proportionem habere dicitur eius, quam habet ad secundum. At cùm quatuor numeri proportionales fuerint, primus ad quartum, triplicatam proportionem habere dicitur eius, quam habet ad secundum. Et super deinceps vno amplius, quamdiu proportio extiterit.

27.

Quotlibet numerus ordine positis, proportio, primi ad vltimum composita dicitur ex proportionibus primi ad secundum, & secundi ad tertium, & tertij ad quartum, & ita deinceps, donec extiterit proportio.

Postulata, siue petitiones.

1.

Postuletur, cuilibet numero quotlibet posse sumi æquales, vel multiplices.

2.

Quolibet numero, sumi posse maiorem.

Axio-

Axiomata, siue pronunciata.

1.

Qui numeri æqualium numerorum, vel eiusdem, æquè multiplices sunt, inter se sunt æquales.

2.

Quorum idem numerus æquè multiplex est, vel æquè multiplices sunt æquales, inter se æquales sunt.

3.

Qui numeri æqualium numerorum, vel eiusdem, eadem pars, partes fuerint, inter se æquales sunt.

4.

Quorum idem numerus, vel æquales, eadē pars, vel eædem partes fuerint, æquales inter se sunt.

5.

Vnitas omnem numerum per vnitates, quæ in ipso sunt, hoc est, per ipsummet numerū, metitur.

6.

Omnis numerus seipsum metitur per vnitatem.

7.

Si numerus numerum multiplicans, aliquem produxerit, metietur multiplicans productum per multiplicatum, multiplicatus autem eundem per multiplicantem.

8.

Si numerus numerum metiatur, vt ille, per quem metitur, eundem metietur per eas, quæ in metiente sunt, vnitates, hoc est, per ipsum numerum metientem.

9.

Si numerus numerum metiens, multiplicet eum, per quem metitur; vel ab eo multiplicatur, illum, quem metitur, producat.

10.

Numerus quotcunque numeros metiens, compositum quoque ex ipsis metitur.

11.

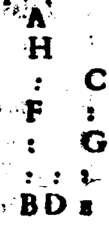
Numerus quemcunq; numerum metiens, metitur quoque omnem numerum, quem ille metitur.

12.

Numerus metiens totum, & ablatum, metitur & reliquum.

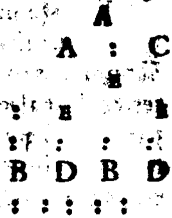
Theorema 1. Propositio 1.

Si Duobus numeris inæqualibus propositis, detrahatur semper minor de maiore, alterna quadam detractione; neque reliquus vnquam metiatur præcedentem, quo ad assumpta sit vnitas: qui principio propositi sunt numeri, primi inter se erunt.



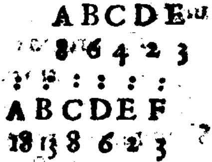
Problema 1. Propositio 2.

Duobus numeris datis non primis inter se, maximam eorum communem mensuram reperire.



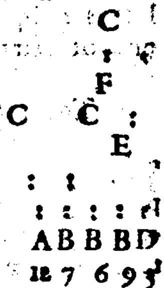
Problema 2. Propositio 3.

Tribus numeris datis non primis inter se, maximam eorum communem mensuram reperire.



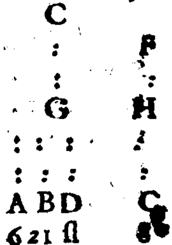
Theorema 2. Propositio 4.

Omnis numerus cuiusq; numeri, minor maioris, aut pars est, aut partes.



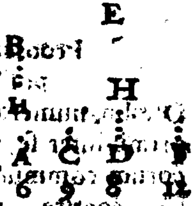
Theorema 3. Propositio 5.

Si numerus numeri pars fuerit, & alter alterius eadē pars; & simul vterque vtriusque simul eadem pars erit, quæ vnus est vnius.



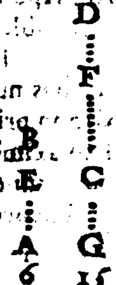
Theo-

Theorema 4 Propositio 6.



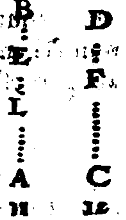
Si numerus sit numeri partes, & alter alterius eædem partes; & simul vterque vtriusque simul eædem partes erunt, quæ sunt vnus vnius.

Theorema 5. Propositio 7.



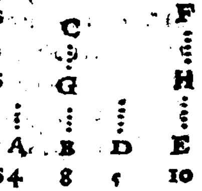
Si numerus numeri eadem sit pars quæ detractus detracti; & reliquus reliqui eadem pars erit, quæ totus est totius.

Theorema 6. Propositio 8.



Si numerus numeri eædem sint partes, quæ detractus detracti; & reliquus reliqui eædem partes erunt, quæ sunt totus totius.

Theorema 7. Propositio 9.



Si numerus numeri pars sit, & alter alterius eadem pars: & vicissim, quæ pars est, vel partes primus tertij, eadem pars erit vel eædem partes, & secundus quarti.

Theo-

Theorema 8. Propositio 10.

Si numerus numeri partes sint, & alter alterius eadem partes: etiam vicissim, quæ sunt partes, aut pars primus tertij, eędẽ partes erunt, vel pars & secundus quarti.

H G A C D E H F

4 6 10 18

Theorema 9. Propositio 11.

Si quemadmodum se habet totus ad totum, ita detractus ad detractum: & reliquis ad reliquum ita se habebit, vt totus ad totum.

B E A D F C

6 8

Theorema 10. Propositio 12.

Si sint quotcunque numeri proportionales; quemadmodum se habet vnus antecedentium ad vnum consequentium, ita se habebunt omnes antecedentes ad omnes consequentes.

A B C D

9 6 3 2

Theorema 11. Propositio 13.

Si quatuor numeri sint proportionales; & vicissim proportionales erũt.

A B C D

12 4 9 3

Theorema 12. Propositio 14.

Si sint quotcunque numeri, & alij illis ęquales multitudi-

A B C D E F

12 6 3 8 4 2

ne,

nè, qui bini sumantur, & in eadem proportione, etiam ex æqualitate in eadem proportione erunt.

Theorema 13. Propositio 15.

Si vnitas numerum quẽpiam metiatur, alter verò numerus alium quendam numerum æquè metiatur; & vicissim vnitas tertium numerum æquè metietur, atq; secũdus quartum.

A	B	D	E
	G		K
	H		L
	C		F
1	3	2	6

Theorema 14. Propositio 16.

Si duo numeri mutuò sese multiplicãtes faciant aliquos; qui ex illis geniti fuerint, inter se æquales erunt.

E	A	B	C	D
1	2	4	8	8

Theorema 15. Propositio 17.

Si numerus duos numeros multiplicans faciat aliquos; qui ex illis procreati erunt, eandẽ proportionem inter se habebunt, quam multiplicati.

I	A	B	C	D	E
1	3	4	5	12	15

Theorema 16. Propositio 18.

Si duo numeri numerum quempiam multiplicantes, faciant aliquos; geniti ex illis eandem habebunt proportionem, quam qui illum multiplicarunt.

A	B	C	D	E
4	5	3	12	15

Theo-

Theorema 17. Propositio 19.

Si quatuor numeri sint ꝓportionales; quod ex primo, & quarto fit, numerus, æqualis erit ei, qui ex secundo & tertio, fit, numero: Et si, qui ex primo & quarto fit numerus, æqualis sit ei, qui ex secundo & tertio, fit, numero; illi quatuor numeri proportionales erunt.

A	B	C	D	E	F	G
6	4	3	2	12	12	18

Theorema 18 Propositio. 20.

Si tres numeri sint proportionales; qui ab extremis continetur, æqualis est ei, qui à medio efficitur: Et si, qui ab extremis continetur, æqualis sit ei, qui à medio describitur, illi tres numeri proportionales erunt.

A	B	C
9	6	4
D		
6		

Theorema 19. Propositio 21.

Minimi numeri omnium qui eandem cum eis proportionem habent, æqualiter metiuntur numeros eandem cum eis proportionem habentes; maior quidem maiorem, minor verò minorem.

D	L		
G	H		
C	E	A	B
4	3	8	6

Theo-

Theorema 20. Propopositio 22.

Si tres sint numeri, & alij multitudine illis æquales, qui bini sumantur & in eadem ratione; sit autem perturbata eorum proportio; etiam ex æqualitate in eadem proportione erunt.

A	B	C	D	E	F
6	4	3	12	8	6

Theorema 21. Propositio 23.

Primi inter se numeri, minimi sunt omniũ eandem cum eis proportionem habentium.

A	B	E	C	D
5	6	2	4	3

Theorema 22. Propositio 24.

Minimi numeri omnium eandem cum eis proportionem habentium, primi sunt inter se.

A	B	C	D	E
7	6	4	3	2

Theorema 23. Propositio. 25.

Si duo numeri sint primi inter se, qui alterutrum eorum metitur, numerus, is ad reliquũ primus erit.

A	B	C	D
6	7	3	4

Theorema 24. Propositio. 26.

Si duo numeri ad quẽpiam numerum primi sint, ad eundem primus is quoque futurus est, qui ab illis productus fuerit.

3				
B				
A	C	D	E	F
5	5	5	3	2

Theo-

Theorema 25. Propositio 27.

Si duo numeri primi sint inter se, qui ab vno eorum gignitur, ad reliquum primus erit.

B	C	D
A		
7	6	3

Theorema 26. Propositio 28.

Si duo numeri ad duos numeros ambo ad vtrunque primi sint; & qui ex eis gignentur, primi inter se erunt.

A	B	E	C	D	F
3	5	15	2	4	8

Theorema 27. Propositio 29.

Si duo numeri primi sint inter se, & multiplicans vterque seipsum procreet aliquem; qui ex ijs producti fuerint, primi inter se erunt. Quòd si numeri initiò propositi multiplicantes eos, qui producti sunt, effecerint aliquos; hi quoque inter se primi erunt; & circa extremos idem hoc semper eueniet.

A	C	E	B	D	F
3	6	27	4	16	63

Theorema 28. Propositio 30.

Si duo numeri primi sint inter se, etiam simul vterque ad vtrunque illorum primus erit. Et simul vterque ad vnum aliquem eorum primus sit, etiam qui initio positi sunt numeri, primi inter se erunt.

C		
A	B	D
7	5	4

Theorema 29. Propositio 31.

Omnis primus numerus ad omnem numerum, quem non metitur, primus est.

:	:	:
A	B	C
7	10	5

Theorema 30. Propositio 32.

Si duo numeri sese mutuò multiplicantes faciant aliquem; hunc autem ex ipsis productum metiatur primus quidam numerus: is alterum etiam eorum, qui initiò positi erant, metietur.

:	:	:	:	:
A	B	C	D	E
3	6	12	3	4

Theorema 31. Propositio 33.

Omnem compositum numerum aliquis primus metietur.

:	:	:
A	B	C
27	9	3

Theorema 32. Propositio 34.

Omnis numerus aut primus est, aut eum aliquis primus metitur.

:	:	:
A	A	
3	6	3

Problema 3. Propositio 35.

Numeris datis quotcunque, reperire minimos omnium, qui eandem cum illis proportionem habeant.

:	:	:	:	:	:	:	:	:	:	:
A	B	C	D	E	F	G	H	K	I	M
6	8	12	2	3	4	6	2	3	4	3

Pro-

B

Problema 4. Propositio 36.

A C D E F

7 12 8 4 3

Duobus numeris datis, reperire, quē illi minimum metiantur, numerum

A

F E C D G H

6 9 12 0 2 3

Theorema 33. Propositio 37.

Si duo numeri numerum quempiam metiantur; & minimus, quem illi metiuntur, eundem metietur.

F

A B E C

2 3 6 12

Problema 5. Propositio. 32.

A B C D E

3 4 6 12 8

Tribus numeris datis, reperire quem minimum numerum illi metiantur.

A B C D E F

3 6 8 12 24 16

Theorema 34. Propositio 39.

Si numerum quispiam numerus metiatur, mensus partem habebit metienti cognominem.

A B C D

12 4 3 1

Theorema 35. Propositio 40.

Si numerus partem habuerit quamlibet, illum metietur numerus parti cognominis.

A	B	C	D
8	4	2	1

Problema 6. Propositio. 41.

Numerum reperire, qui minimus cùm sit, datas habeat partes.

A	B	C	G	H
2	3	4	12	10

FINIS ELEMENTI VII.

EVCLI-

EVCLIDIS ELEMENTVM OCTAVVM.

DEFINITIONES.

Theorema 1. Propositio 1.

SI sint quotcunque numeri deinceps proportionales, quorum extremi sint inter se, primi; ipsi minimi sunt omniū eandem cum eis proportionem habentium.

A	B	C	D	E	F	G	H
8	12	18	27	6	8	12	18

Problema 1. Propositio 2.

Numeros reperire deinceps proportionales minimos, quotcunque iusserit quispiam in data proportione.

A	B	C	D	E	F	G	H	K
3	4	9	12	16	27	36	49	64

Theorema 2. Propositio 3. Conuersa primæ.

Si sint quotcunque numeri deinceps proportionales minimi habentium eandem cū eis proportionem; illorum extremi sunt inter se primi.

A	B	C	D	E	F	G	H	K	L	M	N	O
27	16	48	64	1	4	9	12	16	27	36	48	64

Problema 2. Propositio 4.

Proportionibus datis quotcunque in minimis numeris, reperire numeros deinceps minimos in datis proportionibus.

A	B	C	D	E	F	H	G	K	L	N	X	M	O
5	4	2	2	4	5	6	8	12	15	4	6	10	12

Theorema 3. Propositio 5.

Plani numeri proportionem inter se habent ex lateribus compositam.

A	L	B	C	D	E	F	G	H	K
18	22	32	3	6	4	8	9	12	16

Theorema 4. Propositio 6.

Si sint quotlibet numeri de-

A	B	C	D	E	F	G	H
6	24	36	54	82	4	6	9

inceps,

ſinceps, proportionales; primus autem ſecũdum non metiatur; neq; alius quiſpiam vllum metietur.

Theorema 5. Propoſitio 7.

Si ſint quotcunque numeri deinceps proportionales; primus autem extremum metiatur; is etiam ſecundum metietur.

A	B	C	D
4	6	12	24

Theorema 6. Propoſitio 8.

Si inter duos numeros medij continua proportione incidant numeri; quot inter eos medij continua proportione incidunt numeri, totidem & inter alios eandem cum illis habentes proportionem medij continua proportione incident.

A	C	D	B	G	H	K	L	C	M	N	P
4	9	27	81	1	3	9	27	2	6	18	54

Theorema 7. Propoſitio 9.

Si duo numeri ſunt inter ſe primi, & inter eos medij continua proportione incidant numeri, quot inter eos medij continua proportione incidunt numeri, totidem & inter vtrunque eorum, ac vnitatem deinceps medij continua proportione incident.

A	M	H	E	F	N	C	K	X	G	D	L	O	B
27	27	9	36	3	36	1	12	48	4	48	16	64	64

Theorema 8. Propositio 10.

Si inter duos numeros, & vnitatem, continuè proportionales incidant numeri; quot inter vtrunque ipsorum, & vnitatem, deinceps medij continua proportione incidunt numeri; totidẽ & inter illos medij continua proportione incident.

A 27 E 9 D 3 K 36 H 12 C 1 L 48 F 4 G 16 B 64

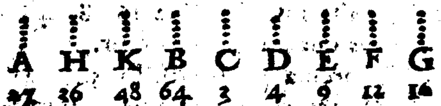
Theorema 9. Propositio 11.

Duorum quadratorum numerorum vnus medius proportionalis est numerus: & quadratus ad quadratum duplicatam habet lateris ad latus proportionem.

A	C	E	D	B
9	3	12	4	16

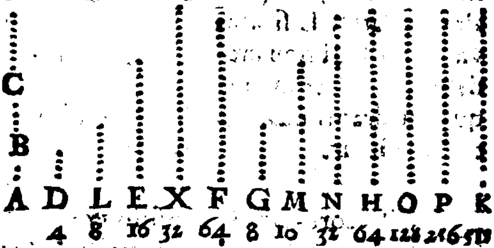
Theorema 10. Propositio 12.

Duorum cuborum numerorum duo medij proportionales sunt numerorum; Et cubus ad ad cubum triplicatam habet lateris ad latus proportionem.



Theorema 11, Propositio 13.

Si sint quotlibet numeri deinceps proportionales, & multiplicans quisque seipsum faciat aliquos; qui ab illis producti fuerint, proportionales erunt. Et, si numeri primum positi, ex suo in procreatos ductu, faciant aliquos, ipsi quoque proportionales erunt.



Theorema 12. Propositio 14.

Si quadratus numerus quadratum numerum metiatur, & latus vnius metietur latus alterius.

rius. Et si vnius quadrati latus metiatur latus alterius, & quadratus quadratum metietur.

A	E	B	C	D
9	12	16	8	4

Theorema 13. Propositio 15.

Si cubus numerus cubum numerus metiatur, & latus vnius metietur alterius latus. Et si latus vnius cubi latus alterius metiatur, tum cubus cubum metietur.

A	H	K	B	C	D	E	F	G
8	16	28	64	2	4	4	8	16

Theorema 14. Propositio 16.

Si quadratus numerus quadratum numerũ non metiatur, neque latus vnius metietur alterius latus. Et si latus vnius quadrati non metiatur latus alterius, neque quadratus quadratum metietur.

A	B	C	D
9	16	3	4

Theorema 15. Propositio 17.

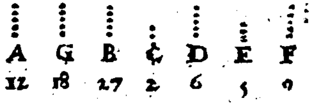
Si cubus numerus cubum numerum nõ metiatur; neque latus vnius latus alterius metietur. Et si latus cubi vnius latus alterius non metiatur, neque cubus cubum metietur.

A	B	C	D
8	27	9	11

Theo-

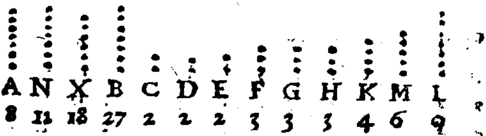
Theorema 16. Propositio 18.

Duorum similium planorum numerorum, vnus medius proportionalis est numerus; & planus ad planum duplicatam habet lateris homologi ad latus homologum proportionem.



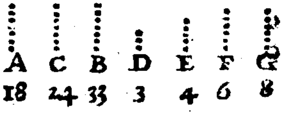
Theorema 17. Propositio 19.

Duorum similium numerorum solidorum duo medij proportionales sunt numeri: Et solidus ad similem solidum triplicatam, habet lateris homologi ad latus homologum proportionem.



Theorema 18. Propositio 20.

Si inter duos numeros vnus medius proportionalis incidat numerus; similes plani erunt illi numeri.



Theo-

Theorema 19. Propositio 21.

Si inter duos numeros duo medij proportionales incidãt numeri; similes solidi sunt illi numeri.

A	C	D	B	E	F	G	H	K	L	M
27	36	44	64	9	12	16	3	3	3	4

Theorema 20. Propositio 22.

Si tres numeri deinceps sint proportionales; primus autem sit quadratus, & tertius quadratus erit.

A	B	D
9	15	25

Theorema 21. Propositio 23.

Si quatuor numeri deinceps sint proportionales; primus autem si cubus, & quartus cubus erit.

A	B	C	D
8	12	18	27

Theorema 22. Propositio 24.

Si duo numeri eã proportioné habeãt inter se, quam quadratus numerus ad quadratum numerum, primus autem sit quadratus, & secundus quadratus erit.

A		B	C	D	
4	9	9	16	34	36

Theorema 23. Propositio 25.

Si numeri duo proportionem inter se habeant,

beant, quam cubus numerus ad cubum numerum; primus autem cubus sit, & secūdus cubus erit.

A	E	F	B	C			D
8	12	18	27	64	95	140	216

Theorema 24. Propositio 26.

Similes plani numeri proportionem inter se habent, quam quadratus numerus ad quadratum numerum.

A	C	B	D	E	F
18	24	32	9	12	16

Theorema 25. Propositio 27.

Similes solidi numeri proportionem habēt inter se, quam cubus numerus ad cubum numerum.

A	C	D	B	E	F	G	H
16	24	36	54	8	12	18	27

FINIS ELEMENTI VIII.

EVCLI-

EVCLIDIS ELEMENTVM NONVM.

Theorema 1. Propositio 1.

Si duo similes plani numeri mutuò sese multiplicantes, quendam procreent, productus quadratus erit.

A	E	B	D		C
4	6	9	16	24	36

Theorema 2. Propositio 2.

Si duo numeri mutuò sese multiplicantes, quadratum faciant, illi similes sunt plani.

A		B	D		C
4	6		9	18	36

Theorema 3. Propositio 3.

Si cubus numerus seipsum multiplicans procreet

creet aliquem, productus cubus erit.

Vnitas	D	D	A			B
	3	4	8	16	32	64

Theorema 4. Propositio 4.

Si cubus numerus cubũ numerum multiplicans quendam procreet, procreatus cubus erit.

A	B	D	C
8	27	64	116

Theorema 5. Propositio. 5.

Si cubus numerus numerum quendam multiplicans cubum procreet, & multiplicatus cubus erit.

A	B	C	D
27	64	729	2718

Theorema 6. Propositio 6.

Si numerus seipsum multiplicans cubum procreet; & ipse cubus erit.

A	B	C
27	729	19683

Theorema 7. Propositio 7.

Si compositus numerus quẽdam numerum multiplicans, quempiam procreet, productus solidus erit.

A	B	C	D	E
6	8	48	2	3

Theorema 8. Propositio 8.

Si ab vnitate quotlibet numeri deinceps proportionales sint: Tertius ab vnitate quadratus est, & vnum intermittentes omnes: Quartus aũt cubus est, & duobus intermissis omnes:

nes: Septimus verò cubus simul & æquadratus est, & quinque intermissis omnes

Vnitas	A	B	C	D	E	F
	3	9	27	81	243	729

Theorema 9. Propositio 9.

Si ab vnitate sint quotcunque numeri deinceps proportionales; sit autem quadratus is, qui vnitatem sequitur, & reliqui oēs quadrati erūt. Quòd si, qui vnitatem sequitur, cubus sit; & reliqui omnes cubi erunt.

quadrati		cubi
531441	F	753969
59040	E	53144I
6561	D	6561
723	C	6561
81	B	729
9	A	81
	o	
	vnitas.	

Theorema 10. Propositio 10.

Si ab vnitate numeri quotcunque proportionales sint; non sit autem quadratus is, qui vnitatem sequitur, neque alius vllus quadratus

o Vnitas.	A	B	C	D	E	F
	3	9	36	81	243	729

tus erit; demptis, tertio ab vnitate ac omnibus vnum intermittentibus. Quòd si, qui vnitatem sequitur, cubus non sit: neque alius vllus cubus erit; demptis, quarto ab vnitate, ac omnibus duos intermittentibus.

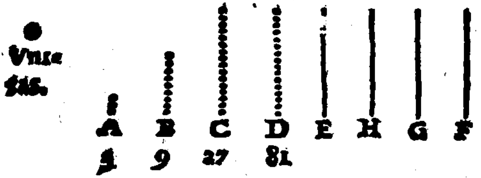
Theorema 11. Propositio 11.

Si ab vnitate numeri quotlibet deinceps proportionales sint; minor maiorem metitur per quempiam eorum, qui in proportionalibus sunt numeris.

:	:	:	:	:
A	B	C	D	E
1	2	4	8	16

Theorema 12. Propositio 12.

Si ab vnitate quotlibet numeri sint proportionales; quot primorum numerorum vltimum metiuntur, totidem & eum, qui vnitati proximus est, metientur.

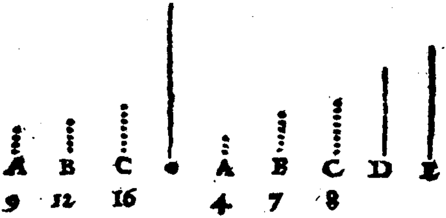


Theorema 13. Propositio 13.

Si ab vnitate sint quotcunque numeri deinceps proportionales; primus autem sit, qui vnitatem sequitur; maximum nullus alius

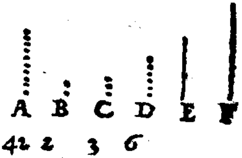
H metie-

metietur; ijs exceptis, qui in proportionalibus sunt, numeris.



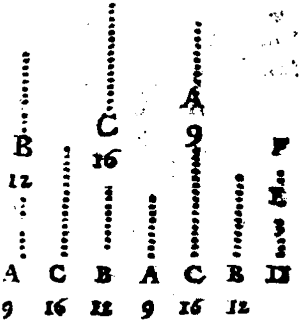
Theorema 14. Propositio 14.

Si minimum numerum primi aliquot numeri metiantur; nullus alius numerus primus illum metietur; ijs exceptis, qui primò metiuntur.



Theorema 15. Propositio 15.

Si tres numeri deinceps proportionales sint minimi omnium, eandem cum ipsis proportionem habentium, duo quilibet compositi, ad tertium primi erût.



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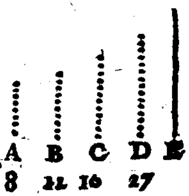
Theorema 16. Propoſitio 16.

Si duo numeri ſint inter ſe primi; non ſe habebit quemadmodum primus ad ſecundum, ita ſecundus ad quempiam alium.

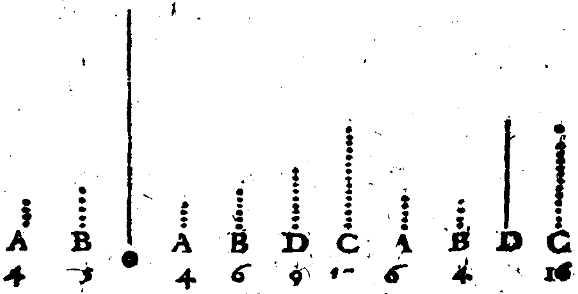
Theorema 17. Propoſitio 17.

Si ſint quotlibet numeri deinceps proportionales, quorum extremi ſint inter ſe primi; non erit quẽadmodum primus ad ſecundum, ita vltimus ad quempiam alium.



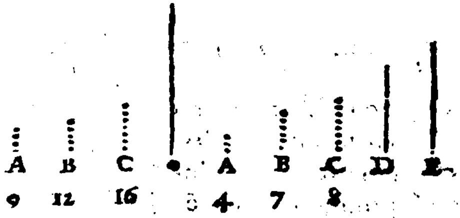
Problema 1. Propoſitio 18.

Duobus numeris datis, conſiderare an poſſit ipſis tertius inueniri proportionalis.



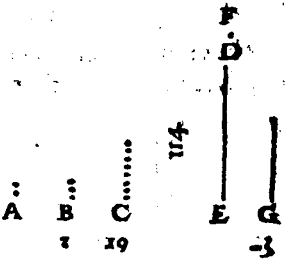
Problema 12. Propositio 19.

Tribus numeris datis, considerare, an possit ipsis quartus reperiri proportionalis.



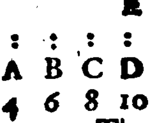
Theorema 18. Propositio 20.

Primi numeri plures sunt, quacunque proposita multitudine primorum numerorum.



Theorema 19. Propositio 21.

Si pares numeri quotlibet compositi sint, totus est par.



Theo-

Theorema 20. Propositio 22.

Si impares numeri quotlibet compositi sint; sit autem par illorum multitudo; totus par erit.

A B C D E
5 9 7 5

Theorema 21. Propositio 23.

Si impares numeri quotcunque compositi sint; sit autem impar illorum multitudo; & totus impar erit.

A B C E
5 7 8 F

Theorema 22. Propositio 24.

Si à pari numero par detractus sit; & reliquus par erit.

B
A C
6 4

Theorema 23. Propositio 25.

Si à pari numero impar detractus sit; & reliquus impar erit.

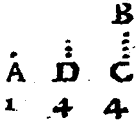
A C D
8 1 4

Theorema 24. Propositio 26.

Si ab impari numero impar detractus sit; & reliquus par erit.

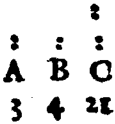
B
A C D
4 6

Theorema 25. Propositio 27.



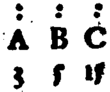
Si ab impari numero par ablatus sit; reliquus impar erit.

Theorema 26. Propositio 28.



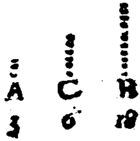
Si impar numerus parem multiplicans, procreet quempiam; procreatus par erit.

Theorema 27. Propositio 29.



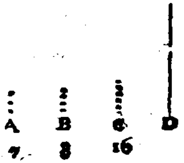
Si impar numerus imparem numerum multiplicans, quędam procreet; procreatus impar erit.

Theorema 28. Propositio 30.



Si impar numerus parem numerum metiatur; & illius dimidium metietur.

Theorema 29. Propositio 31.

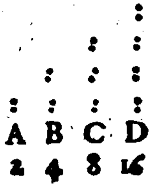


Si impar numerus ad numerum quempiam primus sit: & illius duplum primus erit.

Theo-

Theorema 30. Propositio 32.

Numerorum, qui à binario dupli sunt, *Vnusquisque* pariter *tan.* par est tantùm.



Theorema 31. Propositio 33.

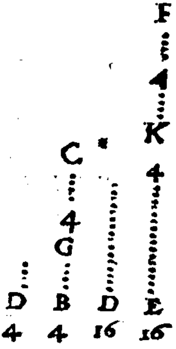
Si numerus dimidium habeat imparem; pariter impar est tantùm.

Theorema 32. Propositio 34.

Si par numerus neque à binario duplus sit, neque dimidium habeat imparem: pariter par est, & pariter impar.

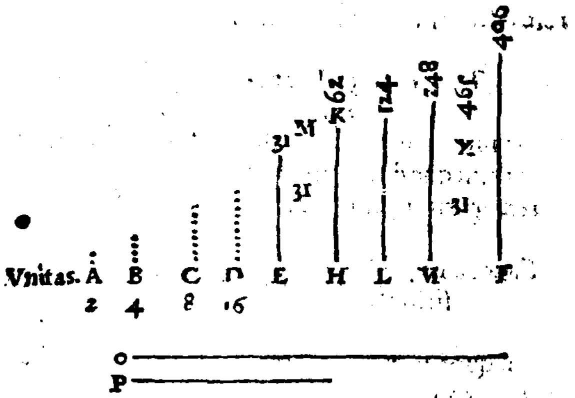
Theorema 33. Propositio 35.

Si sint quotlibet numeri deinceps proportionales; detrahantur autem à secundo & vltimo æquales ipsi primo: Erit quemadmodum secundi excessus ad primū, ita vltimi excessus ad omnes, qui vltimum antecedunt.



Theorema 34. Propositio 36.

Si ab vnitate numeri quotlibet deinceps expositi sint in dupla proportione, quoad totus compositus primus factus sit; isque totus in vltimum multiplicatus, quempiam procreatus perfectus erit.



FINIS ELEMENTI IX.

EVCLI-

EVCLIDIS ELEMENTVM DECIMVM.

DEFINITIONES.

PRIMAE, NEMPE MAGNITVdinem *symmetrarum.*

I.

COmmensurabiles magnitudines dicũtur illæ, quas eadem mensura metitur.

2.

Incommensurabiles verò magnitudines dicuntur, quarum nullam mensuram communem contingit reperiri.

3.

Lineæ rectæ potentia commensurabiles sunt, quarum quadrata vna eadem superficies, siue area metitur.

4.

Incommensurabiles verò lineæ sunt, quarum quadrata, quæ metiatur area communis, reperiri nulla potest.

5.

Hæc cùm ita sint, ostendi potest, quòd quãtacunque linea recta nobis proponantur, existunt etiam aliæ lineæ innumerabiles eidẽ commensurabiles, aliæ item incommensurabiles; hę quidem longitudine & potentia; illæ verò potentia tantùm. Vocetur igitur linea recta, quantacunq; proponatur, ῥητὴ, id est, rationalis.

6.

Lineæ quoque illi ῥητῇ commensurabiles siue longitudine & pontentia, siue potentia tantùm, vocentur & ipsę ῥηταὶ, id est, rationales.

7.

Quæ verò lineæ sunt incommensurabiles illi τῇ ῥητῇ, id est, primo loco rationali, vocentur ἄλογοι, id est, irrationales.

8.

Et quadratum, quod à linea proposita describitur, quam ῥητὴν vocari volumus, vocetur ῥητὸν, id est, rationale.

9.

Et, quæ sunt huic commensurabilia, vocentur ῥητά, id est, rationalia.

10.

Quæ verò sunt illi quadrato, ῥητῷ scilicet, incommensurabilia, vocentur, ἄλογα, id est, surda, siue irrationalia.

11.

Et lineæ, quæ illa incommensurabilia describunt, vocentur ἄλογοι. Et quidam si illa incommensurabilia fuerint quadrata, ipsa eorum altera vocabũtur ἄλογοι lineæ, quòd si qua-ſi quadrata quidem non fuerint, verum aliæ quæpiam ſuperficies ſiue figuræ rectilineæ, tunc vero lineæ illæ quæ deſcribunt quadrata æqualia figuris rectilineis, vocentur ἄλογοι.

Poſtulatum, ſiue petitio.

Poſtulatur quemlibet magnitudinē totius poſſe multiplicari, donec quamlibet magnitudinem eiuſdem generis excedat.

Axiomata, ſiue pronunciata.

1.

Magnitudo quotcunque magnitudines metiens, compoſitam quoque ex ipſis metitur.

2.

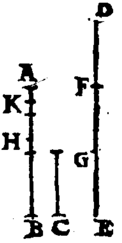
Magnitudo quamcunq; magnitudinem metiens, metitur quoque omnem magnitudinem, quam illa metitur.

3.

Magnitudo metiens totam magnitudinem, & ablatam; metitur, & reliquam.

Problema 1. Propoſitio 1.

Duabus magnitudinibus inæqualibus propoſitis, ſi de maiore detrahatur plus dimidio; & rurſus de reſiduo iterum detrahatur plus dimidio; idque ſemper fiat: relinquetur tādem quædam magnitudo minor altera minore ex duabus propoſitis.



Theo-

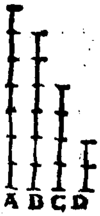
Theorema 2. Propositio 2.

Duabus magnitudinibus propositis inæqualibus, si detrahatur semper minor de maiore, alterna quadam detractione; neque residuum vnquam metiatur id, quod ante se metiebatur; incommensurabiles sunt illæ magnitudines.

Problema 1. Propositio 3.

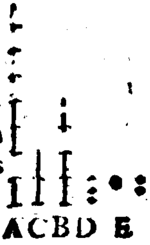
Duabus magnitudinibus commensurabilibus datis, maximam ipsarum communem mensuram reperire.

Problema 2. Propositio 4.



Tribus magnitudinibus commensurabilibus datis, maximam ipsarum communnem mensuram reperire.

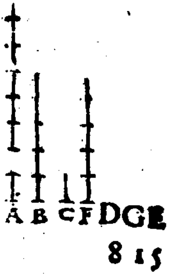
Theorema 3. Propositio 5.



Commensurabiles magnitudines inter se proportionem eam habent, quam habet numerus ad numerum.

Theo-

Theorema 4. Propo-
sitio 6.



Si duæ magnitudines proportionem eam habent inter se, quam numerus ad numerum: commensurabiles sunt illæ magnitudines.

Theorema 5. Propo-
sitio 7.

Incommensurabiles magnitudines inter se proportionem non habent, quam numerus ad numerum.

Theorema 6. Propo-
sitio 8.

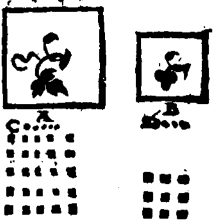
Si duæ magnitudines inter se proportionem non habent, quam numerus ad numerum: incommensurabiles illæ sunt magnitudines.

Theorema 7. Propo-
sitio 9.

Quadrata, quæ describitur à rectis lineis lógitudine commensurabilibus, inter se pro-

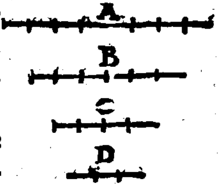
porti-

portionem habent, quam numerus quadratus ad alium numerum quadratum. Et quadrata habentia proportionem inter se, quam quadratus numerus ad numerum quadratum; habent quoque latera longitudine commensurabilia. Quadrata verò quæ describũtur à lineis longitudine incommensurabilibus; proportionem non habent inter se, quã quadratus numerus ad numerũ alium quadratum. Et quadrata non habentia proportionem inter se, quam numerus quadratus ad numerum quadratum, neque latera habebunt longitudine commensurabilia.



Theorema 8. Propositio 10.

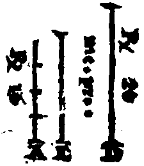
Si quatuor magnitudines fuerint proportionales; prima verò secundæ fuerit commẽsurabilis, tertia quoque quartæ commẽsurabilis erit; quòd si prima secũdæ fuerit incommensurabilis; tertia quoque quartæ incommensurabilis erit.



Theorema 3. Propositio 11.

Propositæ lineæ rectæ (quam ῥητὴν vocari dixi-

diximus) reperire duas lineas rectas incommensurabiles, alteram quidē longitudine tantùm, alteram verò non longitudine tantũ, sed etiam potentia incommẽsurabilem.



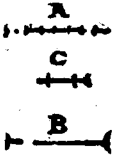
Theorema 9. Propositio 12.

Magnitudines, quæ eidem magnitudini sunt commẽsurabiles; inter se quoque sunt commensurabiles.

6 D.... 4 F..
4 E..... 8 G..
3 H...
2 K..
4 L....

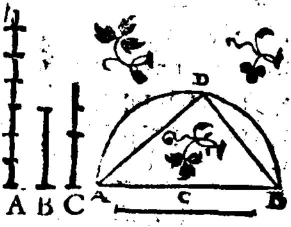
Theorema 10. Propositio 13.

Si ex duabus magnitudinibus hæc quidem commensurabilis sit tertiæ magnitudini, illa verò eidem incommensurabilis, incommẽsurabiles sunt illæ duæ magnitudines.



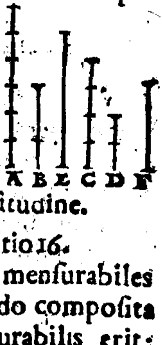
Theorema 11. Propositio 14.

Si duarum magnitudinum commensurabilium, altera fuerit commensurabilis magnitunitudini alteri cupiam tertiæ; reliqua quoq; magnitudo eidem tertiæ incommensurabilis erit.



Theorema 12. Propositio 15.

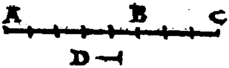
Si quatuor rectæ lineæ proportionales fuerint; posſit autem prima pluſquam ſecunda tanto, quantum eſt quadratum rectæ lineæ ſibi commenſurabilis longitudine: tertia quoque potuerit pluſquam quarta tanto, quantum eſt quadratum rectæ lineæ ſibi commenſurabilis longitudine. Quòd ſi prima poſsit pluſquam ſecũda quadrato rectæ lineæ ſibi longitudine incommenſurabilis: tertia quoque poterit pluſquã quarta quadrato rectæ lineæ ſibi incommenſurabilis longitudine.



Theorema 13. Propositio 16.

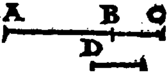
Si duæ magnitudines commenſurabiles componantur; tota magnitudo compoſita ſingulis partibus commenſurabilis erit; Quòd ſi tota magnitudo compoſita alterutri parti commenſurabilis fuerit; illæ duæ quo-

quoque partes commẽſurabiles erunt.



Theorema 14. Propoſitio 17.

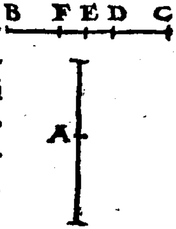
Si duæ magnitudines incommenſurabiles componantur, ipſa quoque tota magnitudo ſingulis partibus componentibus incõmenſurabilis erit. Quòd ſi tota alteri parti incõmenſurabilis fuerit illæ quoque primæ magnitudines inter ſe incommenſurabiles erunt.



Theorema 15. Propoſitio 18.

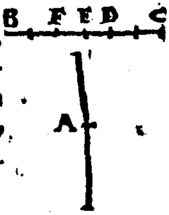
Si fuerint duæ rectæ lineæ inæquales, & quartæ parti quadrati, quod deſcribitur à minore, æquale parallelogrammum applicetur ad maiorem, ex qua maiore tantum excurrat extra latus parallelogrammi, quãtum eſt alterum latus ipſius parallelogrammi: ſi præterea parallelogrammum ſui applicatione diuidat lineam illam in partes inter ſe commenſurabiles longitudine, illa maior linea tanto plus poteſt quam minor, quantum eſt quadratum lineæ ſibi commẽſurabilis longitudine. Quòd ſi maior plus poſſit quam minor, tanto, quantum eſt quadratum lineæ ſibi commenſurabilis longitudine; & præterea quartæ parti quadrati li-

neæ minoris, æquale parallelogrammum ap-
plicetur ad maiorem, ex qua maiore tantū
excurrat extra latus parallelogrammi, quantum eſt alterum latus ipſius parallelogrammi: parallelogrammū ſui applicatione diuidit maiorem in partes inter ſe longitudine commenſurabiles.

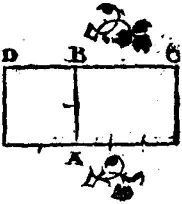


Theorema 16. Propoſitio 19.

Si fuerint duæ rectæ lineæ inæquales; quartæ autem parti quadrati lineæ minoris, æquale parallelogrammorum ad lineam maiorem applicetur, ex qua linea tantum excurrat extra latus parallelogrammi, quantum eſt alterum latus eiuſdem parallelogrammi: ſi parallelogrammum prçterea ſui applicatione diuidat lineam in partes inter ſe longitudine incommenſurabiles, maior illa linea tantò plus poteſt quàm minor quantum eſt quadratum lineæ ſibi minori commenſurabiles longitudine. Quòd ſi maior linea tantò plus poſſit quàm minor, quantum eſt quadratum lineæ incommenſurabilis ſibi longitudine: & præterea quartæ parti quadrati lineæ minoris æquale parallelogrammum applicetur ad maiorem ex qua tantum excurrat extra latus parallelogrammi, quantum est alterum latus ipsius: parallelogrammum sui applicatione diuidit maiorem in partes inter se incōmensurabiles longitudine.

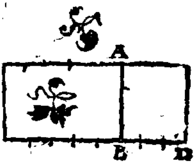


Theorema 17. Propositio 20.



Superficies rectanguli contenta ex lineis rectis rationalibus longitudine commensurabilibus secundum vnum aliquem modum ex antedictis, rationalis est.

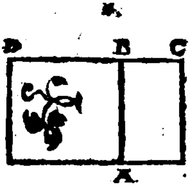
Theorema 18. Propositio 21.



Si rationale ad lineam rationalem applicetur; habebit alterum latus lineam rationalem & commensurabilem longitudine lineæ cui rationale parallelogrammum applicatur.

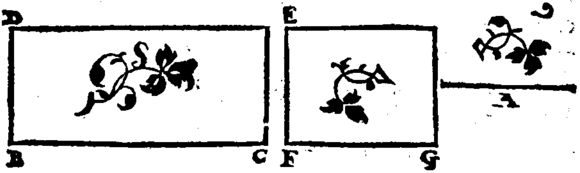
Theorema 19. Propositio 22.

Superficies rectangula contenta duabus lineis rectis rationalibus potētia tantū cōmensuraſurabilibus, irrationalis eſt. Linea autem quæ illam ſuperficiem poteſt, irrationalis & ipſa eſt; vocetur verò media.



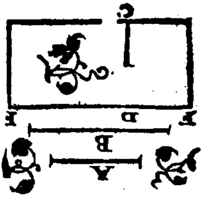
Theorema 20. Propoſitio 23.

Quadrati lineæ mediæ ad lineam rationalem applicāti, alterum latus eſt linea rationalis, & incommenſurabilis longitudine lineæ, ad quam applicatur.



Theorema 21. Propoſitio 24.

Linea recta mediæ commēſurabilis, eſt ipſa quoque media.



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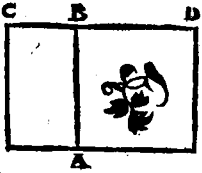
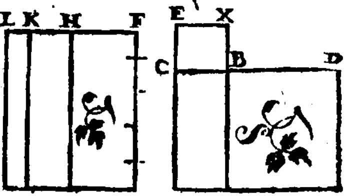
Theorema 22. Propositio 25.

Parallelogrammum rectangulum contentum sub rectis lineis medijs longitudine commensurabilibus, medium est.



Theorema 23. Propositio 26.

Parallelogrammum rectangulum comprehensum sub duabus lineis medijs potentia tantum commensurabilibus;



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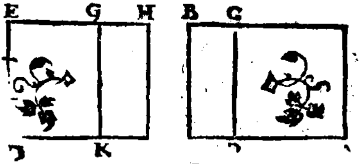
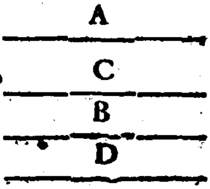
vel rationale est, vel medium.

Theorema 24. Propositio 27.

Medium non est maius, quàm medium superficie rationali.

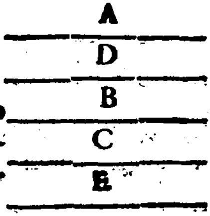


Problema 4. Propoſitio 28.



Medias lineas inuenire potentia tantùm commenſurabiles rationale comprehendentes.

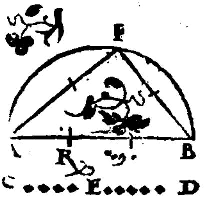
Problema 5. Propoſitio 29.



Medias lineas inuenire potentia tantùm commenſurabiles, medium comprehendentes.

Problema 6. Propoſitio 30.

Reperire duas rationales potentia tantum commẽſurabiles huiuſmodi, vt maior ex illis poſſit plus, quàm minor, quadrato rectæ lineæ ſibi commenſurabilis longitudine.



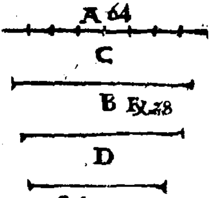
Problema 7. propoſitio 31.

Inuenire duas rationales potentia tantùm commenſurabiles; ita vt maior, quam minor plus poſſit, quadrato rectæ lineæ ſibi longitudine incommenſurabilis.

pro-

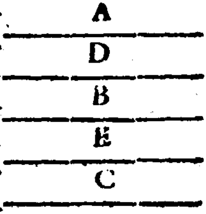
Problema 8. propositio 32.

Reperire duas lineas medias potentia tantùm commensurabiles rationalem superficiem continentes tales inquam, vt maior possit plus, quàm minor, quadrato rectæ lineæ sibi commensurabilis longitudine.



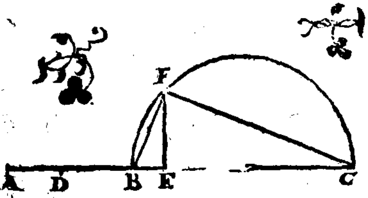
Problema 9. Propopositio 33.

Inuenire duas lineas medias potentia tantùm cõmensurabiles, quæ mediam superficiem contineant, ita vt maior plus possit, quam minor, quadrato rectæ lineæ sibi longitudine commensurabilis.



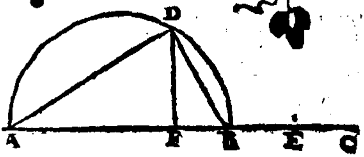
Problema 10. Propositio 34.

Inuenire duas rectas lineas potentia incommensurabiles, quarum quadrata simul composita faciant superficiem rationalem: Rectangulum vero sub ipsis contentum, faciant medium.



Problema 11. Propositio. 35.

Reperire lineas duas rectas potentia incommensurabiles, conficientes compositum ex ipsarum quadratis medium parallelogrammũ verò ex ipsis contentum rationale,



Problema 12. Propositio 36.

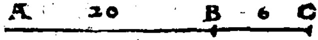
Reperire duas lineas rectas potentia incommensurabiles, conficientes id, quod ex ipsarum quadratis componitur, medium, parallelográmum ex ipsis contentum, mediũ; quod præterea parallelogrammum sit incommésurabile composito ex quadratis ipsarum.

PRIN-

PRINCIPIVM SENARIO-rum per compositionem, & Synthesin.

Theorema 25. Propositio 37.

Si duæ rationales potentia tantùm commẽsurabilis componantur; tota



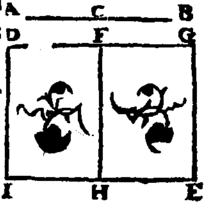
irrationalis erit. Vocetur autem Binomium, vel ex binis nominibus.

Theorema 26. Propositio 38.

Si duæ mediæ potentia tantùm commensurabiles, rationale continentes, componantur; tota linea est irrationalis, vocetur autem ex binis medijs prima.

Theorema 27. Propositio 39.

Si duæ mediæ potentia tantùm commẽsurabiles medium continentes cõponantur; tota linea est irrationalis: vocetur autẽ ex binis medijs secunda.

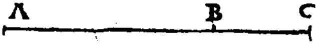


Theorema 28. Propositio 40.

Si duæ rectæ lineę potentia incommensurabiles componantur; conficientes compositum ex quadratis ipsarum, rationale parallelolelogrammum verò ex ipsis contentum, medium: tota linea recta est irrationalis. Vocetur autem linea maior.

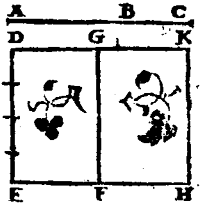
Theorema 29. Propositio 41.

Si duæ rectæ lineæ potentia incommensurabiles componantur, conficientes compositum ex ipsarum quadratis, medium: id verò, quod fit ex ipsis, rationale; tota recta linea est irrationalis erit. Vocetur autem potens rationale & medium.



Theorema 30. Propositio 42.

Si duæ rectæ lineę potentia incommensurabiles componantur, conficientes compositum ex ipsarum quadratis medium; & quod continetur ex ipsis, medium; & præterea incommensurabile composito ex quadratis ipsarum: tota recta linea est irrationalis. Vocetur autem bina media potens.



Theorema 31. Propositio 42.

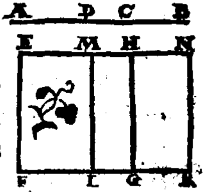
Quæ linea ex binis nominibus vocata, in vnico tantum puncto diuiditur in sua nomina.

Theo-

Theorema 32. Propositio 44.

Quæ ex binis medijs prima, in vnica tantũ puncto diuiditur in sua nomina.

Theorema 33. Propositio 45.



Quæ ex binis medijs secunda, in vnico tantùm puncto diuiditur in sua nomina.

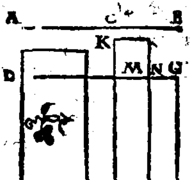
Theorema 34. Propositio 46.

Linea maior in vnico tantùm puncto diuiditur in sua nomina.

Problema 35. Propositio 47.

Linea potens rationale & medium, in vnico tantùm puncto, diuiditur in sua nomina.

Theorema 36. Propositio 48.



Linea potens duo media in vnico tantum puncto diuiditur in sua nomina.

DE-

DEFINITIONES secundæ, nempe binorum nominum.

Proposita linea rationali; & linea ex binis nominibus vocata, diuisa, in sua nomina, cuius maius nomen, id est, maior portio, possit plusquam minus nomen; quadrato lineæ sibi, maiori inquam nomini, commensurabilis longitudine.

1.

Si quidem maius nomen fuerit commensurabile longitudine propositæ lineæ rationali; Vocetur tota linea composita ex binis nominibus, prima.

2.

Si verò minus nomen, id est, minor portio, fuerit commensurabile longitudine propositæ lineæ rationali; Vocetur tota linea ex binis nominibus secunda.

3.

Si verò neutrum ipsorum nominum fuerit commensurabile longitudine propositæ lineæ rationali; Vocetur tota ex binis nominibus tertia.

Rursus si maius nomen possit plusquam minus nomen, quadrato lineæ sibi incommensurabiles longitudine.

4.

Si quidem maius nomen sit commensura-

rabi-

rabile longitudine propositæ lineæ rationali; Vocetur tota linea ex binis nominibus quarta.

5.

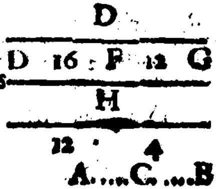
Si verò minus nomen fuerit commensurabile longitudine lineæ rationali; Vocetur tota ex binis nominibus quinta.

6.

Si verò neutrum ipsorum nominum fuerit longitudine commensurabile: propositæ lineæ rationali; Vocetur tota ex binis nominibus sexta.

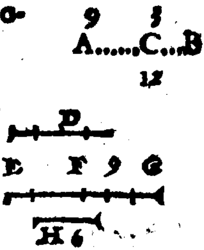
Problema 13. Propositio 49.

Reperire lineam ex binis nominibus primam.



Problema 14. Propositio. 50.

Reperire ex binis nominibus secundam.



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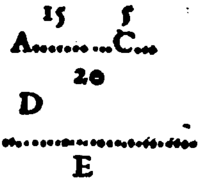
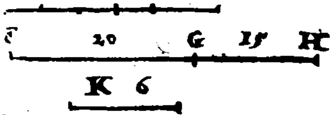
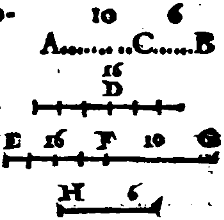
Problema 15. Propositio 51.



Reperire ex binis nominibus tertiā

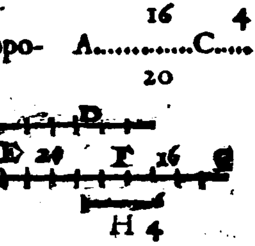


Problema 16. Propositio 52.



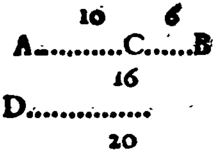
Reperire ex binis nominibus quartam.

Problema 17. Propositio 53.



Reperire ex binis nominibus quintam.

Problema 18. Propositio 54.



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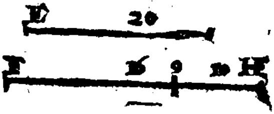
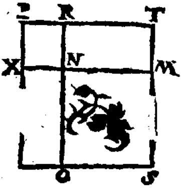
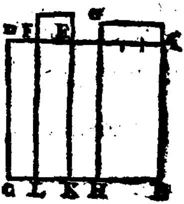
Reperire ex binis nominibus sextam.



Theorema 37. Propositio 55.

Si superficies contenta fuerit sub rationali, & ex binis nominibus prima recta linea, quæ il

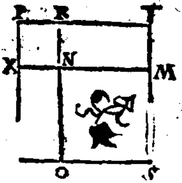


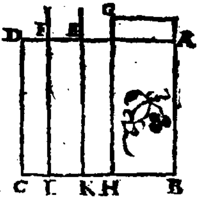


lam superficiem potest, est irrationalis; quæ ex binis nominibus vocatur.

Theorema 38. Propositio 56.

Si superficies cõtẽta fuerit sub linea rationali, & ex binis nominibus secunda; Recta li



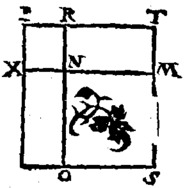


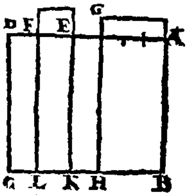
nea potens illã superficiem, est irrationalis; quæ ex binis medijs prima vocatur.

Theo-

Theorema 39. Propoſitio 57.

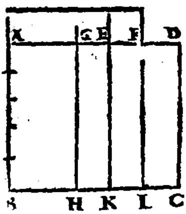
Si ſuperficies contineatur ſub rationali, & ex binis nominibus tertia; recta linea, qua illam ſuperficiem poteſt, eſt irrationalis; quæ ex binis medijs dicitur tertia.

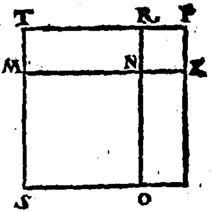




Problema 40. Propoſitio 58.

Si ſuperficies contineatur ſub rationali, & ex binis nominibus quarta; recta linea potens, ſuperficiem illam, eſt irrationalis; quæ dicitur maior.

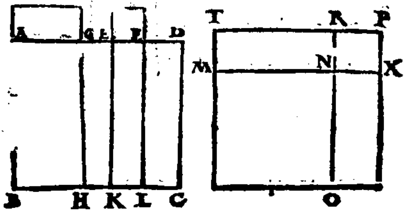




Theorema 41. Propoſitio 59.

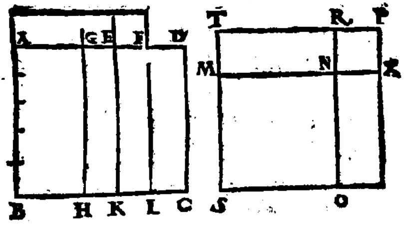
Si ſuperficies contineatur ſub rationali, & ex binis

binis nominibus quarta; recta linea, quæ illam superficiem potest, est irrationalis; quæ dicitur potens rationale, & medium.



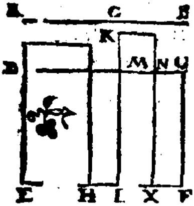
Theorema 42. Propositio 60.

Si superficies contineatur sub rationali, & ex binis nominibus sexta; Recta linea, quæ illam superficiem potest, est irrationalis; quæ dicitur potens, bina media.



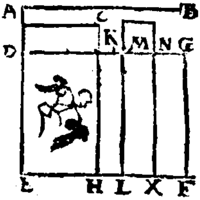
Theorema 43. Propositio 61.

Quadratum eius lineæ, quæ est ex binis nominiminibus, ad lineam rationalẽ applicatum, facit latitudinem ex binis nominibus primam.



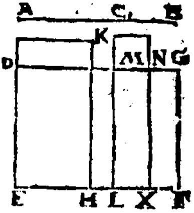
Theorema 44. Propoſitio 62.

Quadratum, eius quæ eſt ex binis medijs prima, ad lineam rationalem lineam applicatum, facit latitudinem ex binis nominibus ſecundam.



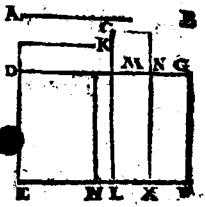
Theorema 45. Propoſitio 63.

Quadratum eius, quæ eſt ex binis medijs ſecunda, ad lineam rationalem applicatum, facit latitudinẽ ex binis nominibus tertiam.



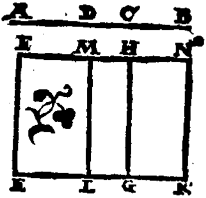
Theorema 46. Propoſitio 64.

Quadratum lineæ maioris ſecundum lineam rationalem applicatum, facit latitudinem ex binis nominibus quartam.



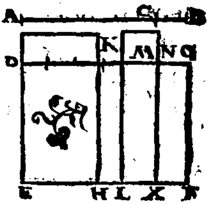
Theo-

Theorema 47. Propositio 65.



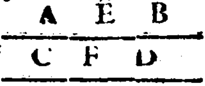
Quadratum lineæ potẽtis rationale, & medium secundum rationalem applicatum, facit latitudinem ex binis nominibus quintam.

Theorema 48. Propositio 66.



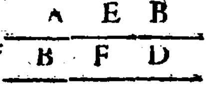
Quadratum lineæ potentis duo media, secundum rationalem applicatum, facit latitudinem ex binis nominibus sextam.

Theorema 49. Propositio 67.



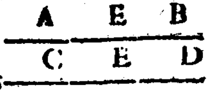
Linea longitudine commensurabilis, ei lineę quę est ex binis nominbus; & ipsa ex binis nominibus est, atque in ordine eadem.

Theorema 50. Propositio 68.



Linea longitudine commensurabilis alteri lineæ quæ est ex binis Medijs; & ipsa ex binis medijs est, atque in ordine eadem.

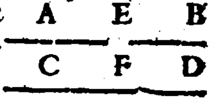
Theorema 51. Propositio 69.



Linea commensurabilis lineæ maiori, & ipsa maior est.

Theorema 52. Propositio 70.

Linea commensurabilis lineæ potenti rationale & medium, est & ipsa linea potens rationale & medium.

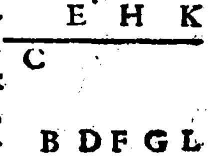


Theorema 53. Propositio 71.

Linea commensurabilis lineæ potenti duo media, est & ipsa linea potens duo media.

Theorema 54. Propositio 72.

Si duæ superficies, rationalis, & media simul componantur, linea quę totam superficiem compositam potest, est vna ex quatuor irrationalibus; vel ea, quæ dicitur ex binis nominibus, vel ea, quæ ex binis medijs prima, vel linea maior, vel linea potens, rationale & medium.

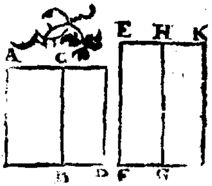


Theorema 55. Propositio 73.

Si duæ superficies mediæ inter se incommen-

sura-

surabiles simul componantur; fiunt reliquę duę lineæ irrationales; & ex binis medijs secunda, vt bina media potens.



SCHOLIVM EX THEONE, ZAMberto, Campano, & P. Christ. Clauio.

Ex his omnibus facilè colligitur, quod linea ea, quæ est ex binis nominibus, & cætera ipsam subsequentes, lineæ irrationales, neque sunt eadem cum linea media, neque ipsæ inter se sunt eadem.

Nam quadratum lineæ mediæ, ad lineam rationalem comparatum & applicatum, efficit alterum latus, lineam rationalem, seu latitudinem rationalem, ipsi lineæ rationali (hoc est, lineæ, ad quã applicatur) longitudine incommensurabilem; per propos. 23. libri decimi.

Quadratum verò eius lineæ, quæ est ex binis nominibus, ad rationalem, applicatum, efficit alterum latus, & lineam, seu latitudinem ex binis nominibus primam; per 61.

Quadratum verò eius, quæ est ex binis mediis prima ad Rationalem applicatum, latitudinem efficit ex binis nominibus secundam: per 62.

Quadratum verò eius, quæ est ex binis mediis secunda, ad rationalem applicatum, latitudinem ef-

ficit ex binis nominibus tertiam: per 63.

Quadratum lineæ maioris, ad rationalem applicatum, latitudinem efficit ex binis nominibus quartam: per 64.

Quadratum vero eius, quæ rationale, & medium potest, ad rationalem applicatum; efficit alterum latus, seu latitudinem ex binis nominibus quintam: per 65.

Quadratum denique lineæ eius, quæ bina media potest, ad rationalem applicatum, efficit alterum latus, seu latitudinem ex binis nominibus sextam: per 66.

Cùm igitur hæ latitudines (quæ à nonnullis lateræ dicuntur.) differant, & à latitudine media & inter se, à latitudine quidem media, quod hæc rationalis sit, illæ verò irrationales, inter se autem, quod in ordine non sint eædem cum iis ex binis nominibus: manifestum est omnes ipsas irrationales lineas, de quibus hactenus dictum est, inter se differentes esse.

PRINCIPIVM SENARIORVM per detractionem, & aphæresin.

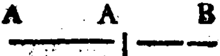
Theorema 56. Propositio 74.

Si de linea rationali detrahatur rationalis, potentia tantùm commensurabilis ipsi toti: Residua est irrationalis: Vocetur autem Residuum, hoc est, apotome.

Theo-

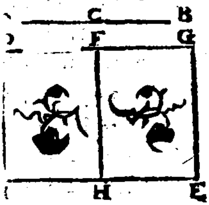
Theorema 57. Propoſitio 75.

Si de linea media detrahatur media, potentia tantum commenſurabilis toti lineæ; quæ verò detracta eſt, cum tota contineat ſuperficiem rationalem; Reſidua eſt irrationalis. Vocetur autem Reſiduum mediũ primum: hoc eſt, mediæ apotome prima.



Theorema 58. Propoſitio 76.

Si de linea media detrahatur media, potentia tantùm commenſurabilis toti; quæ verò detracta eſt, cum tota cõtineat ſuperficiem mediam: Reliqua eſt irrationalis. Vocetur autem Reſiduum medium ſecundum, hoc eſt, mediæ apotome ſecunda.



Theorema 59. Propoſitio 77.

Si de linea recta detrahatur recta, potentia incommenſurabilis toti; compoſitum autem ex quadratis totius lineæ, & lineæ detractæ, ſit rationale; parallelogrammum verò ex

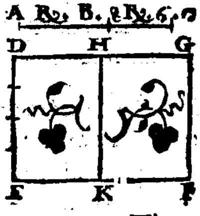
rò ex eiſdem contentum, ſit medium: Reliqua linea erit irrationalis. A C B
Vocetur autem linea minor.

Theorema 60. Propoſitio. 78.

Si de linea recta detrahatur recta, potentia incõmenſurabilis toti lineæ; compoſitum autem ex quadratis totius, & lineæ detractæ ſit medium; parallelogrammum verò bis ex eiſdem contentum, ſit rationale: Reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie rationali totam ſuperficiem mediam. A C B

Theorema 61. Propoſitio 79.

Si de linea recta detrahatur recta potentia incommenſurabilis toti lineæ; compoſitũ autem ex quadratis totius, & lineæ detractæ, ſit medium: Parallelogrammum verò bis ex ijſdem ſit etiam medium: præterea ſint quadrata ipſarum incommenſurabilia parallelogrammo bis ex ijſdem contento, Reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie media totam ſuperficiem mediam,



Theo-

Theorema 62. Propositio 80.

Residuo vnica tantùm linea recta coniungitur rationalis, potentia tantùm commensurabilis toti lineæ.

A B B D

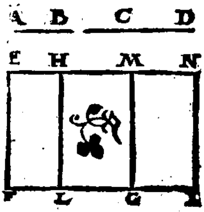
Theorema 63. Propositio 81.

Residuo medio primo vnica tantùm linea coniũgitur media, potentia tantùm commensurabilis toti, ipsa cum tota rationale continens.

A B C D

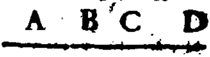
Theorema 64. Propositio 82.

Residuo medio secundo vnica tantùm coniũgitur recta linea media, potẽtia tantùm commensurabilis toti, ipsa cum tota medium continens.



Theorema 65. Propositio 83.

Lineæ minori vnica tantum recta linea con iungitur, potentia incommensurabilis toti, faciens cum tota compositum ex quadratis ipsa-

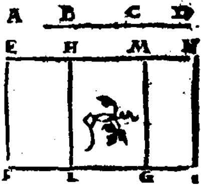


ipsarum rationale; id verò parallelogrammum, quod bis ex ipsis fit, medium.

Theorema 66. Propositio 84.

Lineæ facienti cum superficie rationali totam superficiem mediam, vnica tantùm cõiungitur linea recta, potentia incommensurabilis toti; faciens autem cum tota compositum ex quadratis ipsarum, medium; Id verò, quod fit bis ex ipsis, rationale.

Theorema 67. Propositio 85.



Lineæ cum media superficie facienti totam superficiem mediã, vnica tantum coniungitur linea, potentia toti incõmensurabilis, faciens cũ tota compositum ex quadratis ipsarum, medium, id verò, quod bis ex ipsis etiam medium: & præterea faciens compositum ex quadratis ipsarum incommensurabile ei, quod fit bis ex ipsis

DE-

DEFINITIONES:

TERTIAE, NEMPE APOTOMARUM, *seu Residuorum.*

Proposita linea rationali, & Residuo, si tota nēpe composita ex ipso Residuo, & linea illi coniuncta, seu congruente, plus possit, quā coniuncta, quadrato rectæ lineæ sibi longitudine commensurabilis;

1.

Si quidē tota lineæ propositæ rationali sit longitudine commensurabilis; Vocetur residuum primum, seu Apotome prima.

2.

Si verò coniuncta fuerit longitudine commensurabilis propositæ rationali; ipsa autem tota plus possit, quam coniuncta, quadrato lineæ, sibi longitudine commensurabilis: Vocetur Residuum secundū, seu Apotome secunda.

3.

Si verò neutra linearum fuerit longitudine commensurabilis propositæ rationali; possit autem ipsa tota plusquam coniuncta, quadrato lineæ sibi longitudine commensurabilis; Vocetur Residuum tertium, seu Apotome tertia.

Rursus si tota possit plus, quàm coniuncta seu congruens, quadrato rectę lineæ sibi longitudine incommensurabilis;

4 Et

4.

Et quidem si tota fuerit longitudine commensurabilis ipsi rationali; Vocetur Residuum quartum, seu Apotome quarta.

5.

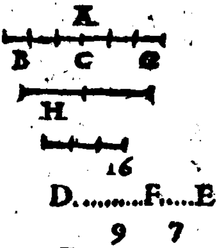
Si verò coniuncta fuerit longitudine cõmensurabilis ipsi rationali; & tota plus possit, quam coniuncta, quadrato lineæ sibi lõgitudine incommensurabilis, Vocetur Residuum quintum, seu Apotome quinta.

6.

Si denique neutra linearum fuerit commensurabilis longitudine ipsi rationali; fueritque tota potentior, quam coniuncta; quadrato lineæ sibi longitudine incommensurabilis; Vocetur Residuum sextum, seu Apotome sexta.

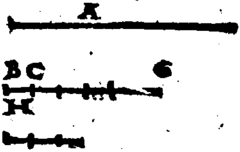
Problema 19. Propositio 86.

Reperire primum Residuum.



Problema 20. Propositio 87.

Reperire secundum Residuum.



Pro

D........F....E

27 9

E........

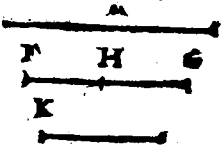
Problema 21. Propositio 88.

12

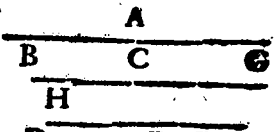
B........E....C

9 7

Reperire tertium Residuum.



Problema 22. Propositio 89.

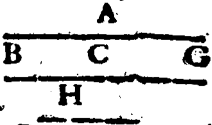


Reperire quartum Residuum.

D..........F....E

16 4

Problema 23. Propositio 90.



Reperire quintum Residuum.

D..........F.....E

25 7

Problema 24. Propositio 91.

Reperire sextum Residuum.

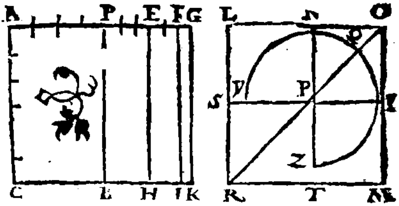
E......... 13

B.....D.....C

Theo-

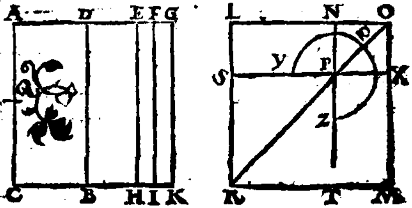
Theorema 68. Propositio 92.

Si superficies contineatur sub linea rationali, & residuo primo; recta linea, quæ illam superficiem potest, est residuum.



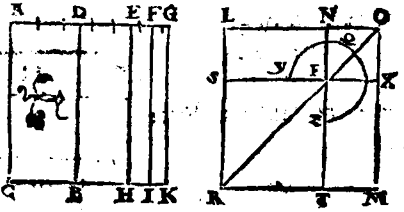
Theorema 69. Propositio 93.

Si superficies contineatur sub linea rationali, & residuo secundo; recta linea, quæ illam superficiem potest, est residuum mediale primum, seu mediæ Apotome prima.



Theorema 70. Propositio 94.

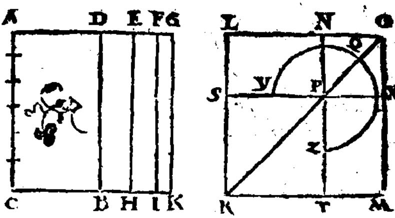
Si superficies cotineatur sub linea rationali,



tionali, & reſiduo tertio; recta linea, quæ illam ſuperficiem poteſt, eſt reſiduum mediale ſecundum, ſeu mediæ eſt Apotome ſecunda.

Theorema 71. Propoſitio 95.

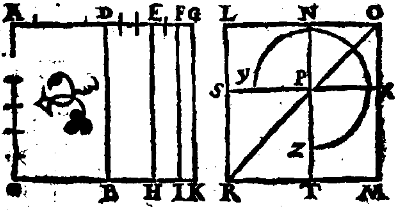
Si ſuperficies contineatur ſub linea rationali, & reſiduo quarto; recta linea, quæ



illam ſuperficiem poteſt, eſt linea minor.

Theorema 72. Propoſitio 96.

Si ſuperficies contineatur ſub linea rationali, & reſiduo quinto; recta linea, quæ illam ſuperficiem poteſt, eſt ea, quæ dicitur cum



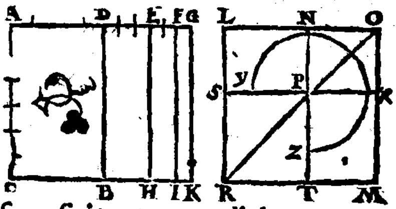
rationali ſuperficie faciẽs totam medialem

Theorema 73. Propoſitio 97.

Si ſuperficies cõtineatur ſub linea rationali, &

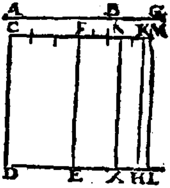
&

& Residuò sexto; recta linea, quæ illam superficiē potest, est ea, quæ dicitur faciens cum



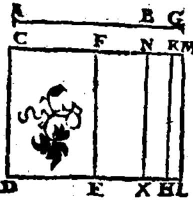
mediali superficie totam medialem.

Theorema 74. Propositio 98.



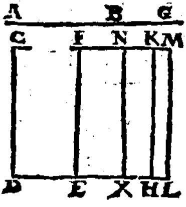
Quadratum residui ad lineam rationalem applicatum, facit alterum latus residuum primum.

Theorema 75. Propositio 99.



Quadratum residui medialis primi ad rationalem applicatum, facit alterum latus, residuum secundum.

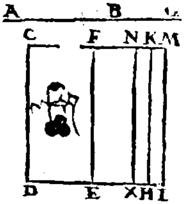
Theorema 76. Propositio 100.



Quadratum residui medialis secundi ad rationalem applicatum, facit alterum latus residuum tertium.

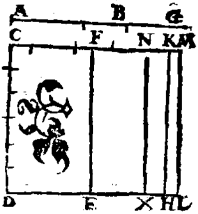
Theo-

Theorema 77. Propositio 101.



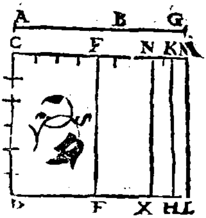
Quadratum lineę minoris ad rationalem applicatum, facit alterum latus residuum quartum.

Theorema 78. Propositio 102.



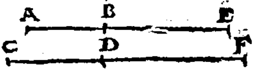
Quadratum lineæ cum rationali superficie facientis totam medialem, ad rationalem applicatum, facit alterum latus residuum quintum.

Theorema 79. Propositio 103.



Quadratum lineæ cum mediali superficie faciẽtis totam medialem, ad rationalem applicatum, facit alterum latus residuum sextum.

Theorema 80. Propositio 104.



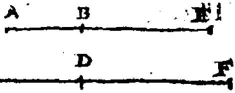
Recta linea residuo commensurabilis longitudine; est & ipsa residuum, seu in ordine eadem.

Theorema 81. Propositio 105.

Recta linea commensurabilis residuo me-

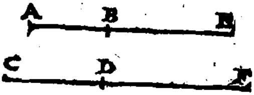
 diali,

diali, eſt & ipſa reſiduum mediale, & eiuſdem ordinis; ſeu in ordine eadem.



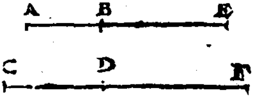
Theorema 82. Propoſitio 106.

Recta linea cõmenſurabilis lineæ minori: eſt & ipſa linea minor.



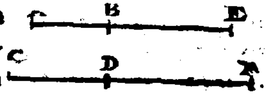
Theorema 83. Propoſitio 107.

Recta linea commenſurabilis lineæ cum rationali ſuperficie facienti totam medialem; eſt & ipſa linea cum rationali ſuperficie faciens totam medialem.



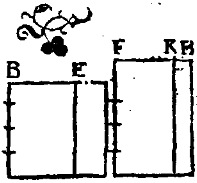
Theorema 84. Propoſitio 108.

Recta linea commenſurabilis lineæ cum mediali ſuperficie facienti totam medialem; eſt & ipſa cum mediali ſuperficie faciens totam medialem.



Theorema 85. Propoſitio 109.

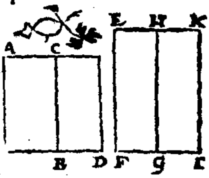
Si de ſuperficie rationali detrahatur ſuperficies medialis; recta linea, quæ reliquam ſuperficiem poteſt, eſt alterutra ex duabus irrationalibus, aut reſiduum, aut linea minor.



Theo-

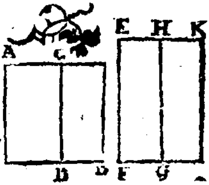
Theorema 86. Propositio 110.

Si de superficie mediali detrahatur superficies rationalis; aliæ duæ irrationales fiunt, aut residuum mediale primum, aut cum rationali superficies totam medialem.



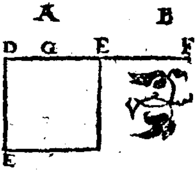
Theorema 87. Propositio 111.

Si de superficie mediali detrahatur superficies medialis, quæ sit incommēsurabilis toti; reliquæ duæ fiunt irrationales, aut residuum mediale secūdum, aut cum mediali superficie faciēs totam medialem.



Theorema 88. Propositio 112.

Linea, quæ residuum dicitur, non est eadem cum ea, quæ dicitur Binomium.



SCHOLIVM EX THEONE, ZAMberto, Campano, & P. Clauio.

Ex his demonstratis facilè intelligitur, quod recta linea, quæ residuum dicitur, & cæteræ quinque eam consequentes irrationales, neque lineæ mediali, neque sibi ipsæ inter se sunt eædem.

Nam quadratum lineæ medialis secundum rationalem applicatum, facit alterum latus, rationalem lineam, longitudine incommensurabilem ei, seu ad quam applicatur, per propos. 23. libri decimi.

Quadratum, verò residui secundum rationalem applicatum, facit alterum latus, residuum primum per 98.

Quadratum verò residui medialis primi secundum rationalem applicatum, facit alterum latus, residuum secundum per 99.

Quadratum verò residui medialis secundi, facit alterum latus, residuum tertium, per 100.

Quadratum verò lineæ minoris, facit alterum latus residuum quartum, per 101.

Quadratum verò lineæ cum rationali superficie facientis totam medialem facit alterum latus residuum quintum, per 102.

Quadratum verò lineæ cum mediali superficie facientis totam medialem, secundum rationalem applicatum, facit alterum latus residuum sextum, per 103.

Cùm

Cum igitur dicta latera, quæ sunt latitudines cuiusq; parallelogrammi vnicuiq; quadrato æqualis, & ad rationalem, applicati, differant & à primo latere, & ipsa inter se, (nam à primo differunt: quoniam sunt residua non eiusdem ordinis) constat ipsas quoq; lineas irrationales inter se differentes esse. Et quoniam demonstratum est, residuum non esse idem quod Binomium: quadrata autem residui, & quinq; linearum irrationalium illud consequentium, ad rationalem applicata, faciunt altera latera ex residuis eiusdem ordinis, cuius sunt & residua, quorum quadrata applicantur rationali, similiter & quadrata Binomij, & quinq; linearum irrationalium illud consequentium, ad rationalem applicata, faciunt altera latera ex Binomijs eiusdem ordinis, cuius sunt & Binomia, quorum quadrata applicantur rationali. Ergo lineæ irrationales, quæ consequuntur Binomium, & quæ consequuntur residuum, sunt inter se differentes. Quare dictæ lineæ omnes irrationales sunt numero, 13. subsequentes.

1. Media linea; quæ vulgò medialis appellatur: propos. 22.

2. Linea ex binis nominibus (vulgò Binomium:) cuius sex sunt species inuentæ: propos. 37.

3. Ex binis medijs prima; vulgò Bimediale primum: propos. 38.

4. Ex binis medijs secunda; vulgò Bimediale secundum: propos. 39.

5. Maior. propos. 40.

6. Linea rationale, ac medium potens: propos. 41.

7. Bina media potens: propos. 42

8. Apotome (vulgò residuum:) cuius etiam species sex sunt reperta: propos 74

9. Media Apotome prima; vulgò residuum: propos. 75

10. Media Apotome secunda, vulgò residuum mediale secundum: propos 76

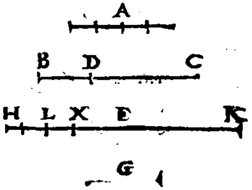
11. Minor: propos 77

12. Linea cum rationali medium totum efficiens; vulgò linea cum rationali superficie totam medialem faciens: propos. 78

13. Cum medio medium totum efficiens; vulgò linea cum mediali superficie totam medialem faciens: propos. 79

Theorema 89. Propositio 113.

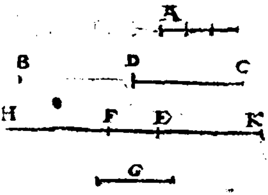
Quadratum lineæ rationalis ad Binomium applicatum, facit alterum latus residuum, cuius nomina sunt commensurabilia Binomij nominibus, & in eadem proportione præterea id, quod fit residuum, eundem ordinem retinet, quem Binomium.



Theorema 90. Propositio 114.

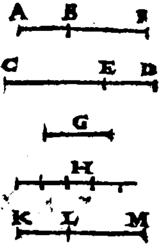
Quadratum lineæ rationalis ad residuum applicatum, facit alterum latus Binomium, cuius

cuius nomina ſunt commenſurabilia nominibus reſidui & in eadem proportione; præterea id, quod fit Binomium eſt eiuſdem ordinis, cuius & reſiduum.



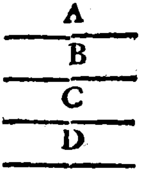
Theorema 91. Propoſitio 115.

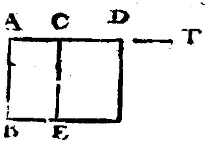
Si parallelogrammum contineatur ex reſiduo, & Binomio, cuius nomina ſunt commenſurabilia nominibus reſidui, & in eadem proportione; recta linea, quæ illam ſuperficiem poteſt, eſt rationalis.



Theorema 92. Propoſitio. 116.

Ex linea media naſcuntur lineæ irrationales inumerabiles, quarum nulla vlli antedictarum eadem ſit,

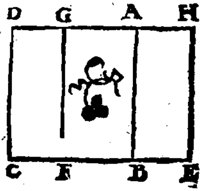




Theorema 103. Propositio 117.

E...H..E
E....

Propositum sit nobis demonstrare, in figuris quadratis diametrum esse lõgitudine incommensurabilem ipsi lateri.



FINIS ELEMENTI X.

EVCLI-

EVCLIDIS ELEMENTVM VNDECIMVM ET SOLIDORVM primum.

DEFINITIONES.

1.

Solidum, est quod longitudinem, latitudinem, & crassitudinem habet.

2.

Solidi autem extremum est superficies.

3.

Linea recta est ad planum recta, cùm ad rectas omnes lineas, à quibus illa tãgitur, quæque in proposito sunt plano, rectos angulos efficit.

4.

Planũ ad planũ rectum est, cùm rectæ lineæ, quæ cõmuni planorũ sectioni ad rectos angulos in vno planorũ ducũtur, alteri plano ad recto sunt angulos.

5.

Rectæ lineæ ad planum inclinatio est angulus acutus, ipsa insistente linea, & adiuncta altera comprehensus, cùm à sublimi recta illius lineæ termino deducta fuerit perpendicularis, atque à puncto, quod perpendicularis in ipso plano fecerit, ad propositæ illius lineæ extremum, quod in eodem est plano, altera recta linea fuerit adiuncta.

6.

Plani ad planum inclinatio, est angulus acutus rectis lineis contentus, quæ in vtroque planorum ad idem communis sectionis pûctum ductæ, rectos ipsi sectioni angulos efficiunt.

7.

Planum similiter inclinatum esse ad planum, atq; alterum ad alterum dicitur, cùm dicti inclinationum anguli inter se sunt æquales.

8.

Parallela plana sunt, quæ inter se non incidunt, nec concurrunt.

9.

Similes figuræ solidæ sunt, quæ similibus planis, multitudine æqualibus continentur.

10.

Aequales, & similes figuræ solidæ sunt, quæ similibus planis, multitudine & magnitudine æqualibus continentur.

11.

Solidus angulus est plurimum, quàm duarum linearum, quæ se mutuò contingant, nec in eadem sint superficie, ad omnes lineas inclinatio.

Aliter.

Solidus angulus est, qui pluribus, quam duobus planis angulis, in eodem plano non consistentibus, sed ad vnum punctum collectis continetur.

12. Pyra-

12.

Pyramis est figura solida, quæ planis continetur, ab vno plano ad vnum punctum collecta.

13.

Prisma figura est solida, quæ planis continetur, quorum aduersa duo sunt, & æqualia, & similia, & parallela; alia verò parallelogramma.

14.

Sphæra est figura, quæ conuerso circum quiescentem diametrum semicirculo continetur, cùm in eundem rursus locum restitutus fuerit, vnde moueri cœperat.

Aliter ex Theodosio.

Sphæra est figura solida, sub vna superficie comprehensa, ad quam ab vno puncto eorum, quæ intra figuram sunt posita, cadentes omnes rectæ lineæ, inter se sunt æquales.

15.

Axis autem sphæræ est, quiescens illa linea recta, per centrum ducta, circum quam semicirculus conuertitur.

16

Centrum verò Sphæræ est idem, quod & semicirculi.

17.

Diameter autem sphæræ est, recta quædam linea per centrum ducta, & vtrinque à sphæræ superficie terminata.

18. Conus

18.

Conus est figura, quæ sub conuerso circum quiescens alterum latus eorum, quę rectum angulum continent, orthogonio triangulo continetur, cum in eundem rursus locum illud triangulum restitutum fuerit, vnde moueri cœperat.

Atque si quiescens recta linea æqualis sit alteri, quæ circum rectum angulum conuertitur, orthogonius erit Conus: sin minor, amblygonius: si verò maior, oxygonius.

19.

Axis autem Coni est, quiescens illa recta linea, circum quam triangulum vertitur.

20.

Basis verò Coni est, circulus qui à circumducta linea recta describitur.

21.

Cylindrus est figura, quæ sub conuerso circumquiescens alterum latus eorum, quæ rectum angulum continent, parallelogrammo orthogonio compręhenditur, cum in eundem rusus locum restitutum fuerit illud parallelogrammum, vndę moueri cœperat.

22.

Axis autem Cylindri est quiescens illa recta linea, circum quam parallelogrammum vertitur.

23. Bases

23.

Bases verò cylindri sunt circuli, à duobus aduersus lateribus, quæ circum aguntur, descripti.

24.

Similes coli, & cylindri sunt, quorum & axes, & basium diametri proportionales sunt.

25.

Cubus hexandrum est figura solida, quæ sub sex quadratis æqualibus continetur.

26.

Tetraedrum est figura solida, quæ sub triangulis quatuor æqualibus, & æquilateris continetur.

27.

Octaedrum figura est solida, quæ sub octo triangulis æqualibus, & æquilateris continetur.

28.

Dedecaedrum figura est solida, quæ sub duodecim pentagonis æqualibus, æquilateris, & æquiangulis continetur.

29.

Eicosaedrum figura est solida, quæ sub triangulis viginti æqualibus, & ęquilateris cótinetur.

30.

Parallelepipedum est figura solidá, quæ sub sex figuris quadrilateris, quarum quæ ex aduerso, parallelæ sunt, continetur.

31. Soli-

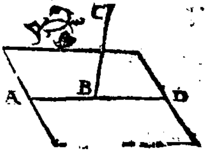
31.

Solida figura in ſolida dicitur inſcribi, quando omnes anguli figuræ inſcriptæ conſtituantur, vel in angulis, vel in lateribus, vel deniq; in planis figuræ, cui inſcribitur.

32.

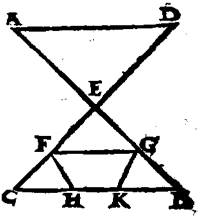
Solida figura ſolidæ figuræ viciſſim circumſcribi dicitur, quando vel anguli, vel latera, vel deniq; plana figuræ circumſcriptæ tangunt omnes angulos figuræ, circum quam deſcribitur.

Theorema 1. Propoſitio 1.



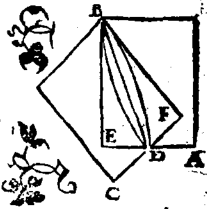
Quædā rectæ lineæ pars in ſubiecto quidem non eſt plano, quædam verò in ſublimi.

Theorema 2. Propoſitio 2.



Si duæ rectæ lineæ ſe mutuo ſecent, in vno ſunt plano: atque triangulum omne in vno eſt plano.

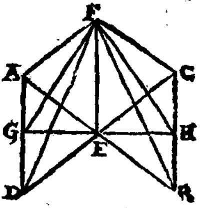
Theorema 3. Propoſitio 3.



Si duo plana ſe mutuò ſecent, communis eorum ſectio eſt recta linea.

Theo-

Theorema 4. Propoſitio 4.

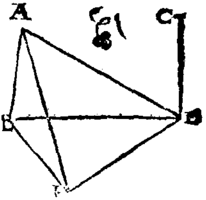


Si recta linea, rectis duabus lineis ſe mutuò ſecantibus, in communi ſectione ad rectos angulos inſiſtat: illa, ducto etiam per ipſas plano, ad angulos rectos erit.

Theorema 5. Propoſitio 5.

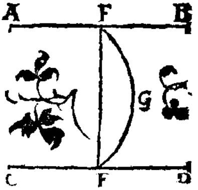
Si recta linea, rectis tribus lineis ſe mutuò tangentibus, incommuni ſectione ad rectos angulos inſiſtat: illæ tres rectæ in vno ſunt plano.

Theorema 6. Propoſitio 6.



Si duæ rectę lineę eidem plano ad rectos ſint angulos: parallelæ erunt illæ rectæ lineæ.

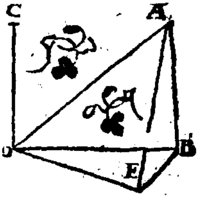
Theorema 7. Propoſitio 7.



Si duæ ſint parallelæ rectæ lineæ, in quarum vtraq; ſumpta ſint quælibet puncta: illa linea, quę ad hæc puncta adiungitur, in eodem eſt cum parallelis plano.

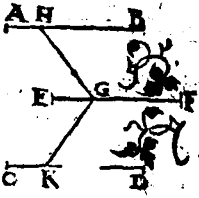
Theo-

Theorema 8. Propositio 8.



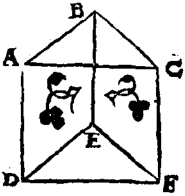
Si duæ ſint parallelæ rectæ lineæ, quarum altera ad rectos cuidam plano ſit angulos: & reliqua eidem plano ad rectos an gulos erit.

Theorema 9. Propoſitio 9.



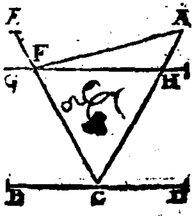
Quæ eidem rectæ lineæ ſunt parallelæ, ſed non in eodem cum illa plano: hę quoque ſunt inter ſe parallelæ.

Theorema 10. Propoſitio 10.



Si duæ rectæ lineę ſe mutuò tangentes ad duas rectas ſe mutuò tangentes ſint parrallelæ, non autē in eodem plano; illæ angulos æquales comprehendent.

Problema 1. Propoſitio 11.



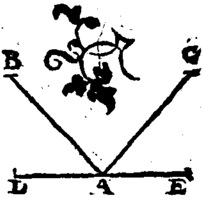
A dato puncto in ſublimi, ad ſubiectum planū perpendicularem rectā lineam ducere.

Pro-

Problema 2. Propositio. 12.

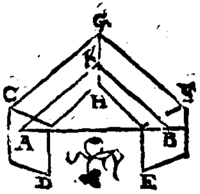
Dato plano, à puncto, quod in illo datum est, ad rectos angulos rectam lineam excitare.

Theorema 11. Propositio 13.



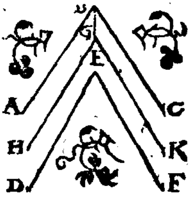
Dato plano, à púcto quod in illo datum est, duæ rectæ lineæ ad rectos angulos non excitabuntur, ad easdem partes.

Theorema 12. Propositio 14.



Ad quæ plana, eadem recta linea recta est; illa sunt parallela.

Theorema 13. Propositio 15.



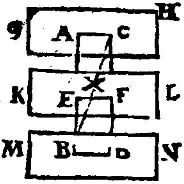
Si duæ rectæ lineæ se mutuo tangentes, ad duas rectas se mutuò tangentes sint parallelæ, non in eodem consistentes plano: parrallela sunt, quæ per illas ducuntur, plana.

Theorema 14. Propoſitio 16.

Si duo plana parallela plano quopiam ſecentur; communes illorum ſectiones ſunt parallelæ.

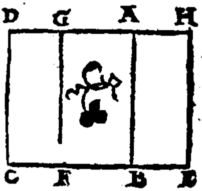
Theorema 15. Propoſitio 17.

Si duæ rectæ lineæ parallelis planis ſecentur; in eaſdem proportiones ſecabuntur.



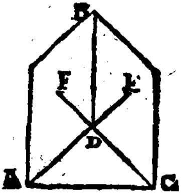
Theorema 16. Propoſitio 18.

Si recta linea plano cuipiam ad rectos ſit angulos; illa etiam omnia, quæ per ipſam plana, ad rectos eidem plano angulos erunt.



Theorema 17. Propoſitio 19.

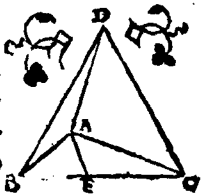
Si duo plana ſe mutuò ſecantia, plano cuidam ad rectos ſint angulos; communis etiam illorum ſectio ad rectos eidem plano angulos erit.



Theo-

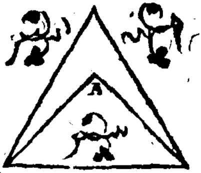
Theorema 18 Propositio 20.

Si angulus solidus sub planis tribus angulis contineatur: ex his duo quilibet, vtut assumpti, tertio sunt maiores.



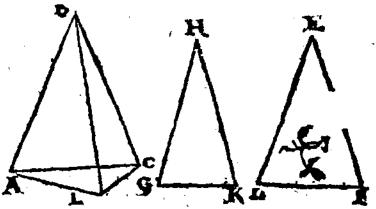
Theorema 19. Propositio 21.

Solidus omnis angulus sub minoribus quàm rectis quatuor angulis planis, continetur.



Theorema 20. Propositio 22.

Si plani tres anguli æqualibus rectis contineantur lineis, quorum duo vt libet assumpti, tertio sint maiores; triangulum constitui potest ex lineis æquales. illas rectas coniungentibus.

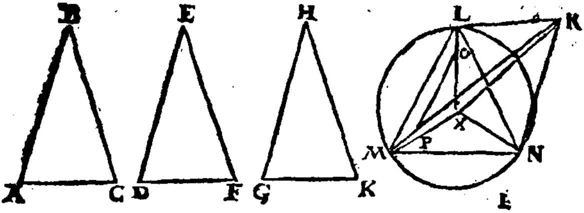


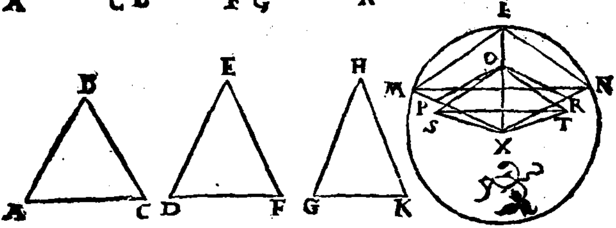
Problema 3. Propositio 23.

Ex planis tribus angulis, quorum duo, vt libet assumpti, tertio sint maiores, solidum

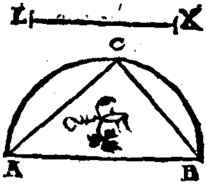
 angu-

angulum conſtituere. Oportet autem illos tres angulos rectis quatuor eſſe minores.

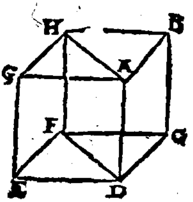




Theorema 21. Propoſitio 24.

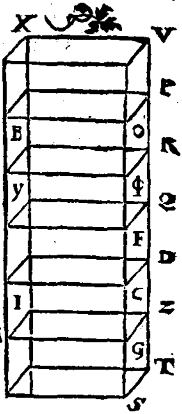


Si ſolidum ſub parallelis planis contineatur; aduerſa illius plana, ſunt parallelogramma, ſimilia, & æqualia.



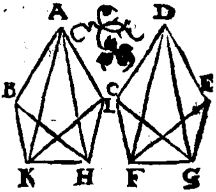
Theo-

Theorema 22. Propositio 25.



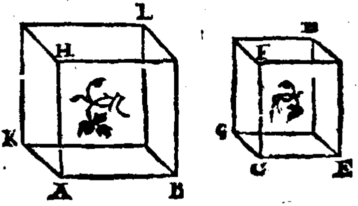
Si solidum parrallelopipedum, seu parallelis planis contentum plano secetur, aduersis planis parallelo: erit quemadmodum basis ad basim, ita solidum ad solidum.

Problema 4. propositio 26.



Ad datam rectam lineã, eiusque punctum, angulum solidum constituere, solido angulo dato æqualem.

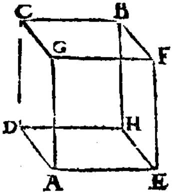
Problema 5. Propopositio 27.

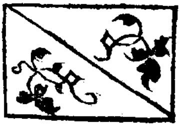


A data recta linea, dato solido parallelis planis comprehenso, simile, & similiter positum solidũ parallelis planis contentũ describere.

Theorema 23. Propositio 28.

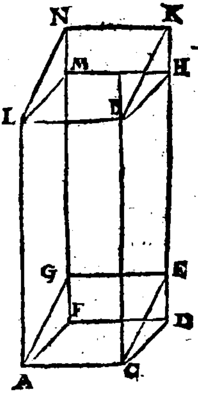
Si solidum parallelis planis comprehensum ducto per aduersorum planorum diagonios plano, sectum sit: illud solidũ ab hoc plano bifariam secabitur.





Theorema 24. Propositio 29.

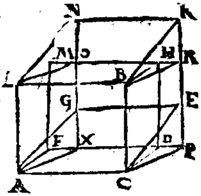
Solida parallelapipeda, seu parallelis planis cõprehensa, quæ super eandem basim, & in eadem sunt altitudine, quorum insistentes lineæ in ijsdem collocantur rectis lineis: illa sunt inter se æqualia.



Theo-

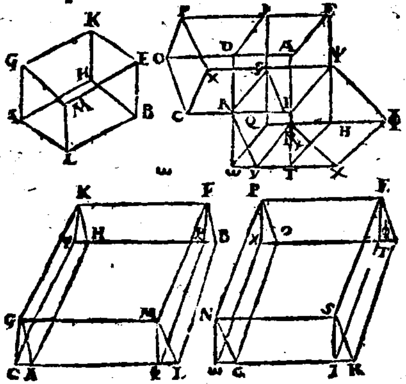
Theorema 25. Propositio 30.

Solida parallelis planis circumscripta, quæ super eandem basim, & in eadem sunt altitudine, quorum insistentes lineæ non in ijsdem reperiuntur rectis lineis; illa sunt inter se æqualia.



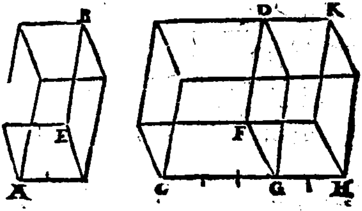
Theorema 26. Propositio 31.

Solida Parallelelis planis, circumscripta, quæ super æqualibus basibus, & in eadem sunt altitudine: æqualia sunt inter se.



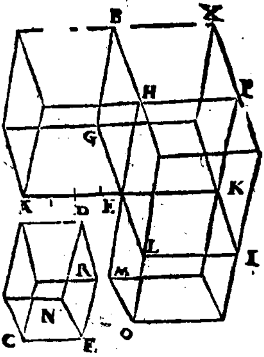
Theorema 27. Propositio 32.

Solida parallelis planis circumscripta, quæ eiusdem sunt altitudinis; eam habent inter se proportionẽ, quam bases.



Theorema 28. Propositio 33.

Similia solida parallelis planis circumscripta habent inter se proportionem homologorum laterum triplicatam.



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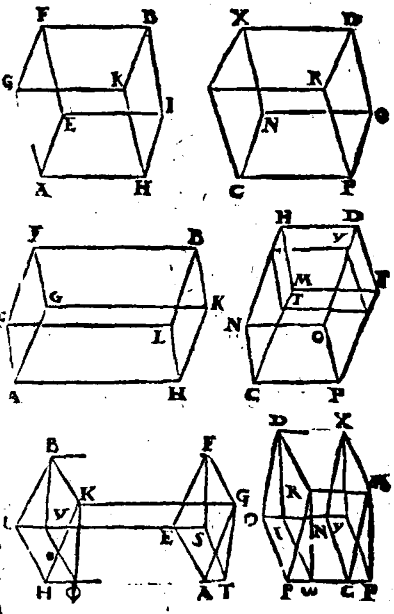
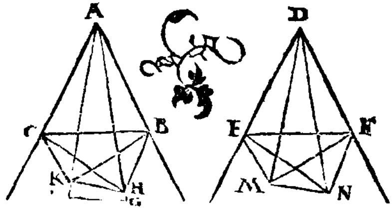
Theorema 29. Propoſitio 34.



Aequalium ſolidorũ parallelis planis contentorum baſes, cum altitudinibus reciprocantur. Et ſolida parallelis planis contenta, quorum baſes cum altitudinibus reciprocantur; illa ſunt æqualia.

Theorema 30. Propoſitio 35.

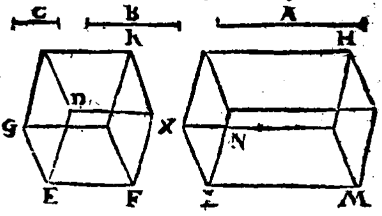
Si duo plani ſint anguli æquales, quorum vertibus sublimes rectæ lineæ insistant, quæ cum lineis primò positis angulos contineant æquales, vtrunque vtrique: in sublimibus autem lineis quælibet sumpta sint puncta, & ab his ad plana, in quibus consistunt anguli primum positi, ductæ sint perpenpendiculares; ab earum verò punctis, quæ in planis signata fuerint, ad angulos primum positos adiūctæ sint rectæ lineæ;



hæ cum sublimibus æquales angulos comprehendent.

Theorema 31. Propositio 36.

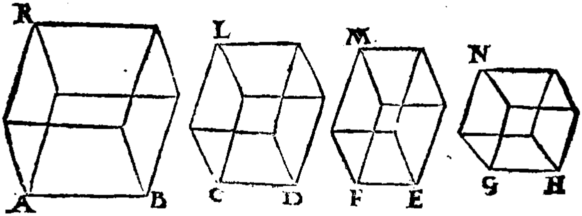
Si rectæ tres lineæ sint proportionales; quod ex his tribus fit solidum parallelis planis contentum, æquale est descripto à media linea solido parallelis planis comprehenso, quod æquilaterum quidem sit, sed antedicto æquiangulum.



Theo-

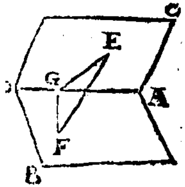
Theorema 32, Propositio 37.

Si rectæ quatuor lineæ sint proportionales, illa quoque solida parallelis planis contenta, quæ ab ipsis lineis & similia & similiter describuntur, proportionalia erunt. Et si solida parallelis planis comprehensa, quæ & similia & similiter describuntur, sint proportionalia; illæ quoque rectæ lineæ proportionales erunt.



Theorema 33. Propositio 38.

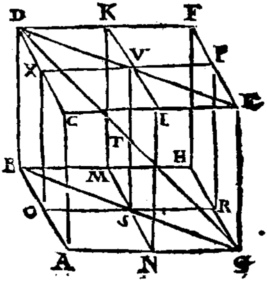
Si planum ad planum rectum sit; & à quodam puncto eorum, quæ in vno sunt planorum, perpendicularis ad alterum planum ducta sit: illa, quæ ducitur perpendicularis, in communem cadet ipsorum planorum sectionem.



Theorema 34. Propositio 39.

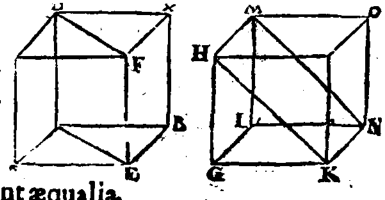
Si in solido parallelis planis circumscripto, aduer-

aduersorum planorum lateribus bifariam sectis, educta sint per sectiones plana; communis illorum planorum sectio, & solidi parallelis planis circumscripti diameter, se mutuò bifariam secabūt.



Theorema 35. Propositio 40.

Si duo sint æqualis altitudinis prismata; quorum hoc quidem basim habeat parallelogrammum; illud verò triangulum sit autem parallelogrammū triangvli duplum: illa prismata erunt æqualia.



FINIS ELEMENTI XI.

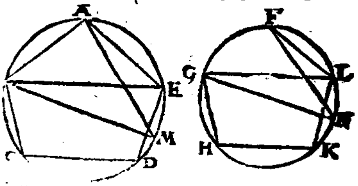
EVCLI-

EVCLIDIS ELEMENTVM DVODECIMVM

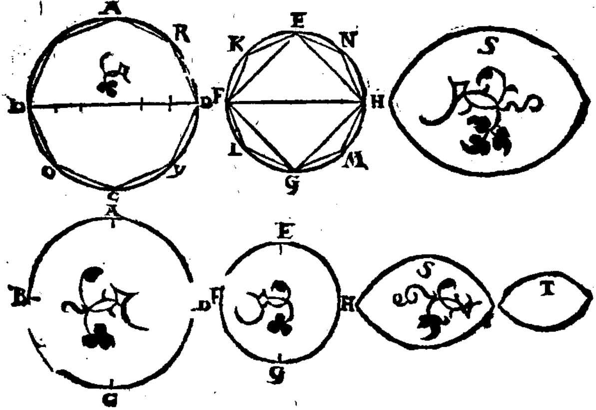
ET SOLIDORVM seucndum.

Theorema 1. Propositio 1.

Similia, quæ sunt in circulis polygona, proportionē habent inter se, quā descripta à diametris quadrata.



Theorema 2. Propositio 2.

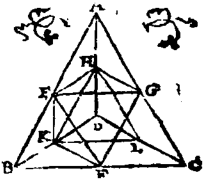


Cir-

Circuli eam interſe proportionem habent, quàm deſcripta à diametris quadrata.

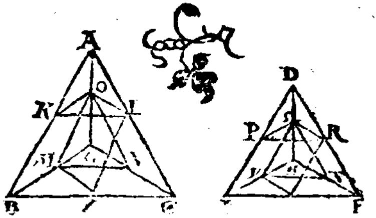
Theorema 3. Propoſitio 3.

Omnis pyramis trigonam habens baſim, in duas diuiditur pyramides non tantùm æquales & ſimiles inter ſe, ſed toti etiam pyramidi ſimiles, quarum trigonæ ſunt baſes; atque in duo priſmata æqualia, quæ duo priſmata dimidio pyramidis totius ſunt maiora.



Theorema 4. Propoſitio 4.

Si duæ eiuſdem altitudinis pyramides trigonas habeant baſes; ſit autem illarum vtraque diuiſa & in duas pyramides inter ſe æquales, totique ſimiles; & in duo priſmata æqualia; Ac eodem modo diuidatur vtraq; pyramidum, quæ ex ſuperiore diuiſione natæ ſunt; idque perpetuo fiat: quemadmodũ

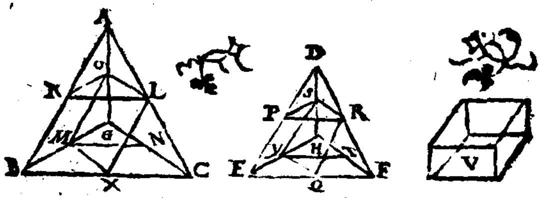


ſe habet vnius pyramidis baſis ad alterius pyramidis baſim; ita & omnia, quæ in vna pyra-

pyramide, priſmata ad omnia, quæ in altera pyramide, priſmata multitudine æqualia.

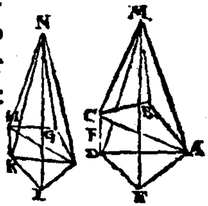
Theorema 5. Propoſitio 5.

Pyramides eiuſdem altitudinis, quarum trigonæ ſunt baſes; eam inter ſe proportionem habent, quàm ipſæ baſes.



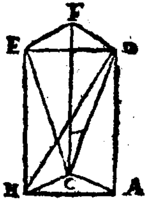
Theorema 6 Propoſitio 6.

Pyramides eiuſdem altitudinis, quarum polygonæ ſunt baſes, eam inter ſe proportionem habent quam ipſæ baſes.



Theorema 7. Propoſitio 7.

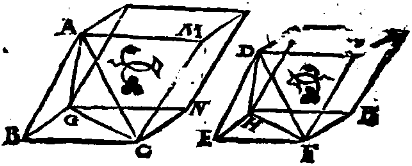
Omne priſma trigonam habens baſim, diuiditur in tres pyramides inter ſe æquales, quarum trigonæ ſunt baſes.



Theo-

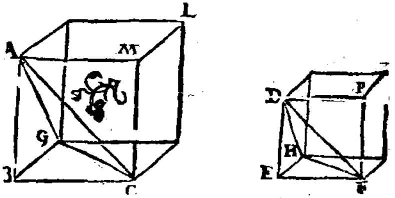
Theorema 8. Propositio 8.

Similes pyramides, quæ trigonas habent bases in triplicata sunt homologorum laterum proportione.



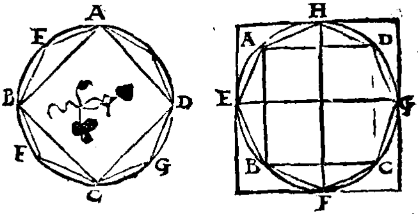
Theorema 9. Propositio 9.

Aequalium pyramidum, & trigonas bases habentium, reciprocantur bases cum altitudinibus. Et quarum pyramidum trigonas bases habẽtium reciprocãtur bases cum altitudinibus; illæ sunt æquales.



Theorema 10. Propositio 10.

Omnis conꝰ tertia pars est cylindri

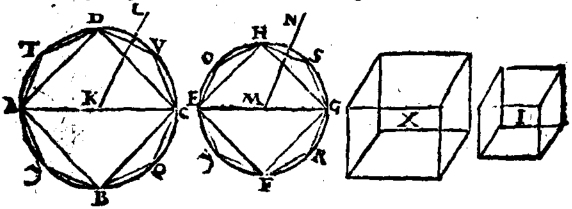


eandem

eandem cum ipſo cono baſim habentis, & altitudinem æqualem.

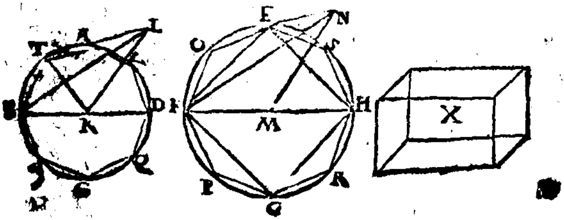
Theorema 11. Propoſitio 11.

Coni, & cylindri eiuſdem altitudinis, eam inter ſe proportionem habent, quam baſes.



Theorema 12. Propoſitio 12.

Similes coni, & cylindri, triplicatam habẽt, inter ſe proportionem diametrorum, quæ ſunt in baſibus.



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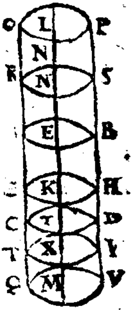
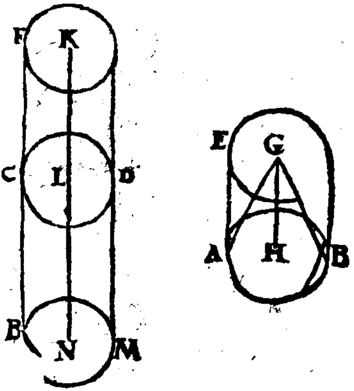
Theorema 13. Propositio 13.



Si cylindrus plano sectus sit aduersis planis parallelo: Erit quẽadmodum cylindrus ad cylindrum, ita axis ad axem.

Theorema 14. Propositio 14.

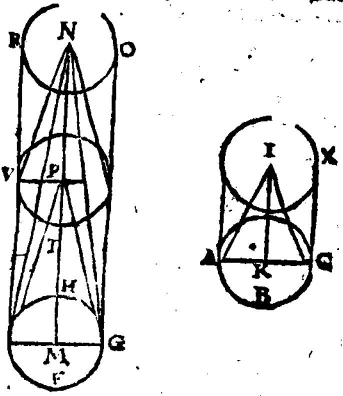
Coni, & cylindri, qui in æqualibus sunt basibus; eam habent inter se proportionẽ, quam altitudines.



Theo-

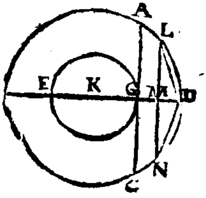
Theorema 15. Propositio 15.

Aequalium conorum, & cylindrorum bases cum altitudinibus reciprocãtur. Et quorum conorum & cylindrorum rum bases cum altitudinibus reciprocãtur; illi sunt æquales.



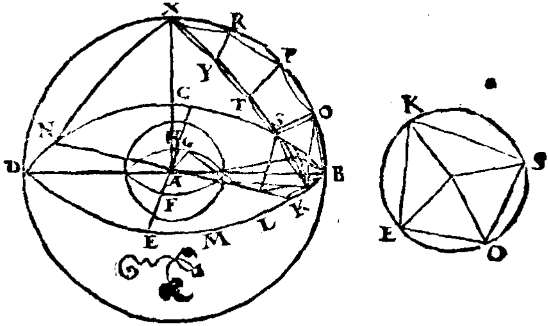
Problema 1. Propositio 16.

Duobus circulis circa idem centrum existentibus, in maiore circulo polygonum æqualium, pariumque laterum inscribere, quod minorem circulum non tãgat.



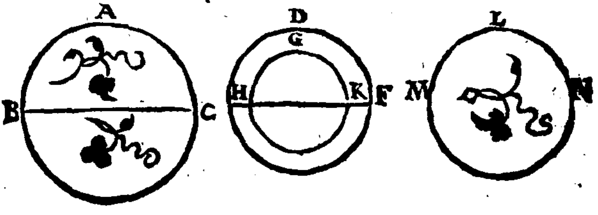
Problema 2. Propositio 17.

Duabus sphæris circa idem centrum exsistentibus, in maiori sphæra solidum polyedrum inscribere, quod minoris sphæræ superficiem non tangat.



Theorema 16. Propositio 18.

Sphæræ inter se proportionem habent suarum diametrorum triplicatam.

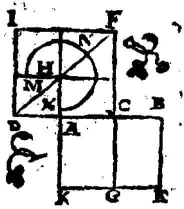


FINIS ELEMENTI XII.

EVCLIDIS ELEMENTVM DECIMVM TERTIVM, ET SOLIDORVM tertium.

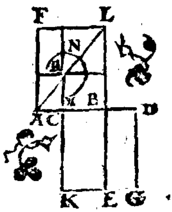
Theorema 1. Propositio 1.

Si recta linea per extremam & mediam proportionem secta sit; maius segmentū, quod totius lineæ dimidium assumpserit, quintuplum potest eius, quòd à totius dimidia describitur, quadrati.

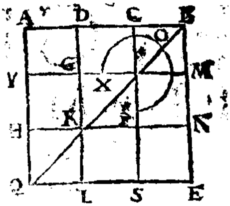


Theorema 2. Propositio 2.

Si recta linea sui ipsius segmenti quintuplum possit, & dupla segmenti huius linea per extremam & mediam proportionem secetur, maius segmentum reliqua pars est liniæ primùm positæ.

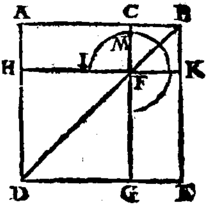


Theorema 3. Propoſitio 3.



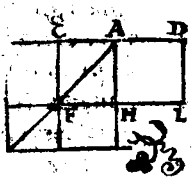
Si recta linea per extremam & mediam proportionem ſecta ſit; minus ſegmentum, quod maioris ſegmenti dimidium aſſumpſerit, quintuplum poteſt eius, quòd à maioris ſegmenti dimidio deſcribitur quadrati.

Theorema 4. Propoſitio 4.



Si recta linea per extremam & mediam proportionem ſecta ſit: quòd à tota, quodque à minore ſegmento, ſimul vtraque quadrata, tripla ſunt eius, quod à maiore ſegmento deſcribitur, quadrati.

Theorema 5. Propoſitio 5.



Si ad rectam lineam, quæ per extremam & mediam proportionē ſecatur, adiuncta ſit altera ſegmento maiori æqualis: tota hæc linea recta per extremam & mediam proportio-

nem

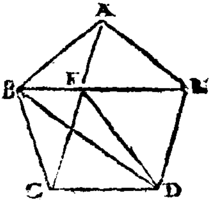
nem secta; estque maius segmentum recta linea primùm posita.

Theorema 6. Propositio 6.

Si recta linea ῥητὴ, siue rationalis, per extremam & mediam proportionem facta sit; vtrunque segmentorum ἄλογος, siue irrationalis, est linea, quæ dicitur Residuum, seu Apotome.

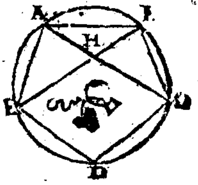
Theorema 7. Propositio 7.

Si pentagoni æquilateri tres sint æquales anguli, siue qui deinceps, siue qui non deinceps sequuntur: illud pentagonum erit æquiangulum.



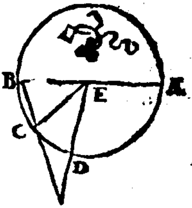
Theorema 8. Propositio 8.

Si pentagoni æquilateri, & æquianguli duos qui deinceps sequuntur, angulos rectæ subtendãt lineæ; illę per extremam & mediam proportionem se mutuò secant; earumque maiora segmenta, ipsius pentagoni lateri sunt æqualia.



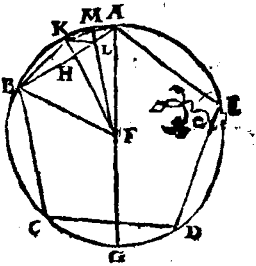
Theorema 9. Propoſitio 9.

Si latus hexagoni, & latus decagoni eidem circulo inſcriptorum compoſita ſint; tota recta linea per extremam & mediam proportionem ſecta eſt; eiuſque ſegmentum maius, eſt hexagoni latus.



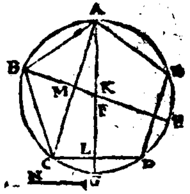
Theorema 10. Propoſitio 10.

Si in circulo pentagonum æquilaterũ inſcriptum ſit; pentagoni latus poteſt & latus hexagoni, & latus decagoni, eidem circulo inſcriptorum.



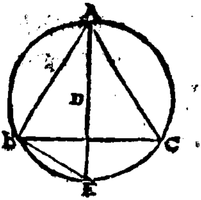
Theorema 11. Propoſitio 11.

Si in circulo ῥητὸν, ſeu rationalem diametrum habente, inſcriptum ſit pentagonum æquilaterum; pentagoni latus irrationalis eſt linea, quæ vocatur minor.



Theo-

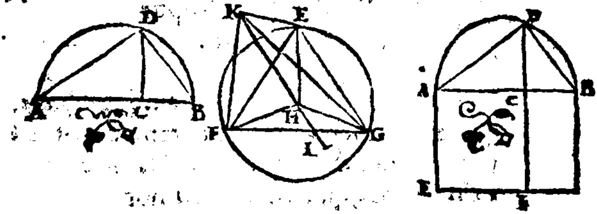
Theorema 12. Propoſitio 12.



Si in circulo inſcriptum ſit triangulum æquilaterum; huius trianguli latus potentia triplum eſt eius lineæ, quæ ex circuli centro ducitur.

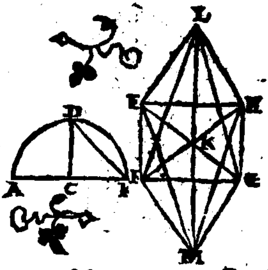
Problema 1. Propoſitio 13.

Pyramidem conſtituere; & data ſphæra cõplecti: atque docere quod illius ſphæræ diameter potentia ſeſquialtera ſit lateris ipſius pyramidis.



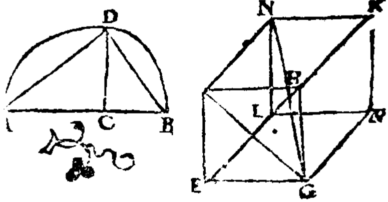
Theorema 2. Propoſitio. 14.

Octaedrum conſtituere, eaq; ſphęra, qua pyramidem, complecti; atq; probarequod illius ſphærę diameter potentia dupla ſit lateris ipſius octaedri.



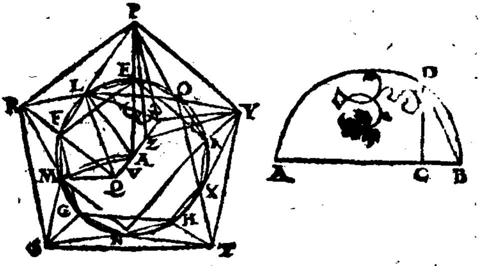
Problema 3. Propositio 15.

Cubum constituere; eaque sphæra, qua & superiores figuras complecti; atque docere quod illius sphæræ diameter potentia tripla sit lateris ipsius cubi.

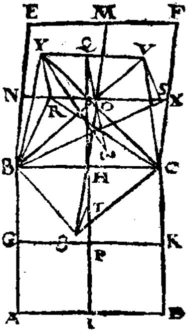


Problema 4. Propositio 16.

Icosaedrum constituere; eademque sphæra, qua & antedictas figuras, complecti; atque probare, quod illius Icosaedri latus irrationalis sit linea, quæ vocatur Minor.

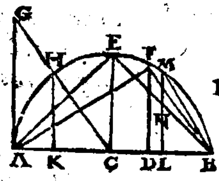


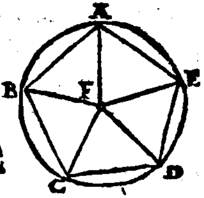
Problema 5. Propoſitio 17.



Dodecaedrum conſtituere; eademq́ue ſphæra, qua & antedictas figuras, complecti; atq; probare quod illius dodecaedri latus irrationalis ſit linea, quæ vocatur Reſiduum, ſeu Apotome.

Problema 6. Propoſitio 18.





Qūinque figurarū latera ꝓponere, & inter ſe comparare.

SCHOLIVM.

Interpretes hoc in loco demonſtrant, præter dictas quinque figuras, non poſſe aliam conſtitui figuram ſolidam, quæ ex planis & æquilateris & æquiangulis contineatur, inter ſe æqualibus.

Non enim ex duobus triangulis, neque ex aliis duabus figuris, solidus constituitur angulus; cùm saltem tres anguli plani requirantur ad solidi anguli constitutionem.

Ex tribus autem triangulis æquilateris, constat pyramidis angulus.

Ex quatuor angulus, Octaedri.

Ex quinque angulus, Icosaedri.

Nam ex triangulis, sex & æquilateris & æquiangulis ad idem punctum coeuntibus, non fiet angulus solidus: cum enim trianguli æquilateri angulus, recti vnius bessem (hoc est duas tertias partes:) cõtineat, erunt eiusmodi sex anguli rectis quatuor æquales. Quod fieri non potest. Nam solidus omnis angulus minoribus, quam rectis quatuor angulis, continetur; per propos. 21 lib. 11.

Multò ergo minus ex pluribus, quam sex planis eiusmodi angulis, solidus angulus constabit.

Sed ex tribus quadratis, Cubi angulus continetur:

Ex quatuor autem quadratis, nullus angulus solidus constitui potest. Rursus enim recti quatuor erunt. Multò ergo minus ex pluribus, quàm quatuor eiusmodi angulis solidus angulus constabit.

Ex tribus autem pentagonis æquilateris, & æquiangulis, Dodecaedri angulus continetur.

Sed ex quatuor huiusmodi angulis, nullus solidus angulus confici potest. Cùm enim pentagoni æquilateri angulus rectus sit, & quinta recti pars erunt quatuor anguli rectis quatuor maiores. Quod fieri

fieri nequit. Nec sanè ex alijs polygonis figuris solidus angulus continebitur, quod hinc quoque absurdum sequatur.

Quamobrem perspicuum est, præter dictas quinque figuras aliam figuram solidam non posse constitui, quæ ex planis æquilateris, & æquiangulis inter se æqualibus, contineatur.

Vid. Theon p.244. Et P.Clauius p.277.

FINIS ELEMENTI XIII.

EVCLIDIS ELEMENTVM DECIMVMQVARTVM ET SOLIDORVM QVARTVM, VT QVIdam arbitrantur; Vt alij verò, Hypsiclis Alexandrini, de quinque corporibus.

LIBER PRIMVS.

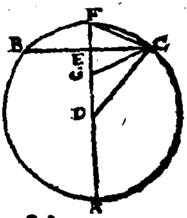
Prooemium Hypsiclis Alexandrini ad protarchum.

Basilides Tyrius, Protarche, Alexandriam profectus, patriq́ue nostro ob disciplinæ societatem commendatus, longißimo peregrinationis tempore cum eo versatus est. Cū́q́ue differerent aliquando de scripta ab Apollonio comparatione Dodecaedri, & Icosaedri eidem sphæra inscriptorum; quam hæc inter se habe[illegible]tionem, censuerunt ea non rectè tradidisse Apollonium; quæ à se emendata, vt de patre audire erat, literis prodiderunt. Ego autem postea incidi in alterum librum ab Apollonio editum, qui demonstrationem accuratè complecteretur de re proposita, ex eiusq́ue problematis indagatione magnam equidem cœpi voluptatem. Illud certè ab omnibus perspici potest, quod scripsit Apollonius, cum sit in omnium manibus. Quod autem diligenti, quantum conijcere licet, studio nos postea scripsisse videmur, id mo-

id monumentis consignatum tibi dedicandum duximus, vt qui feliciter cum in omnibus disciplinis, tum vel maximè in Geometria versatus, scitè ac prudenter iudices ea, quæ dicturi sumus: Ob eam verò, quæ tibi cum patre fuit, vitæ consuetudinem, quaq; nos complecteris, beneuolentiam, tractationem ipsam libenter audias. Sed iam tempus est, vt proœmium finem facientes, hanc syntaxim aggrediamur.

Theorema 1. Propositio 1.

Perpendicularis linea, quæ ex circuli cuiuspiam centro in latus pentagoni ipsi circulo inscripti ducitur; dimidia est vtriusque simul lineæ, & eius, quæ ex centro, & lateris decagoni in eodem circulo inscripti.



Theorema 2. Propositio 2.

Si binæ rectæ lineæ extrema, & media proportione secentur; ipsæ similiter secabuntur, in easdem scilicet proportiones.

Theorema 3. Propositio 3.

Si in circulo pentagonum æquilaterum inscribatur; quod ex latere pentagoni; & quod ex ea, quæ binis lateribus pentagonis subtenditur, recta linea; vtraque simul quadrata, quintupla sunt eius, quod ex se midiametro describitur, quadrati.

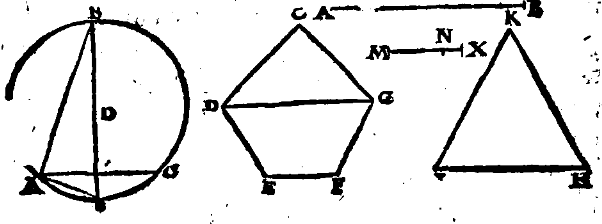
Theo-

Theorema 4. Propositio 4.

Si latus hexagoni alicuius circuli secetur extrema, & media proportione; maius illius segmentum erit latus decagoni eiusdem circuli.

Theorema 5. Propositio 5.

Idem circulus comprehendit, & dodecaedri pentagonum, & icosaedri triangulum, eidem sphæræ inscriptorum.



Theorema 6. Propositio 6.

Si pentagono, & æquilatero, & æquiangulo circumscribatur circulus; ex cuius centro ad vnum pentagoni latus ducatur linea perpendicularis: Erit, quod sub dicto latere, & perpendiculari continetur, rectangulum trigesies sumptum, dodecaedri superficiei æquale.

Theorema 7. Propositio 7.

Si ex centro circuli triangulum icosaedri cir-

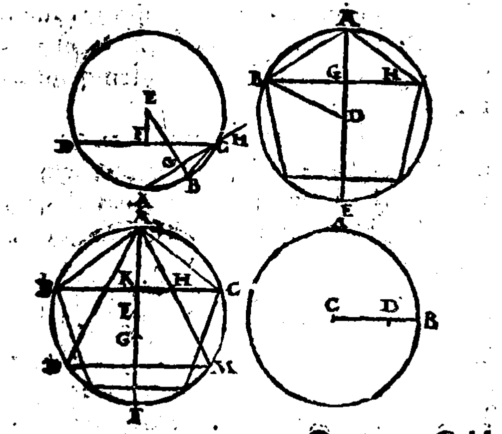
circumscribentis, linea perpendicularis ducatur ad vnum latus trianguli: Erit, quod sub dicto latere, & perpendiculari comprehenditur, rectangulum trigesies sumptum, Icosaedri superficiei æquale.

Theorema 8. Propositio 8.

Rectangulum contentum sub tribus quartis partibus diametri alicuius circuli; & sub quinque sextis partibus lineæ subtendentis angulum pentagoni æquilateri in eodem circulo descripti; æquale est dicto pētāgono.

Theorema 9. Propositio 9.

Superficies Dodecaedri ad superficiem Icosaedri, in eadem sphæra descripti, eandem proportionem habet, quàm latus cubi ad latus Icosaedri.



Cubi latus.

E

Dodecaedri, latus.

F

Icosaedri, latus.

G

Theorema 10. Propositio 10.

Si recta linea secetur extrema, & media proportione; Erit, vt recta potens id, quod à tota; & id, quod à maiori segmento, ad rectam potentem id, quod à tota, & id, quod à minori segmento; Ita latus cubi ad latus Icosaedri, in eadem sphæra cum cubo inscripti.

Theorema 11. Propositio 11.

Dodecaedrum, ad Icosaedrum in eadem cum ipso sphæra inscriptum, est, vt cubi latus, ad Icosaedri latus, in vna eademq; sphæra.

Theorema 12. Propositio 12.

Latus trianguli æquilateri potentia sesquitertium est lineæ perpendicularis ab vno angulo ad latus oppositum, seu basim, deductæ.

Theorema 13. Propositio 13.

Si sphæra (duo solida corpora, Tetraedrum & Octaedrum circumscribentis) diameter fuerit Rationalis: Erit tam superficies Tetraedri, quàm Octaedri in ea sphæra media.

Theo-

Theorema 14. Propositio 14.

Si Tetraedrum, atque octaedrum in eadem sphæra inscribantur: Erit basis Tetraedri sesquitertia baseos Octaedri; Superficies autem Octaedri sesquialtera superficiei Tetraedri.

Theorema 15. Propositio 15.

Recta linea ex angulo quouis tetraedri in sphæra inscripti, per centrum sphæræ ducta; cadit in baseos oppositæ; estq; perpendicularis ad dictam basin.

Theorema 16. Propositio 16.

Octaedrum in sphæra inscriptum, diuiditur in duas pyramides æquales, & similes, æqualium altitudinum, basis vero vtriusque pyramidis est quadratum subduplum quadrati quadrati diametri sphæræ.

Theorema 17. Propositio 17.

Tetraedrum sphæræ impositum ad Octaedrum in eadem sphæra descriptum, se habet, vt rectangulum sub linea potente viginti septem sexagesimas quartas partes quadrati lateris tetraedri; & sub linea continente octo nonas partes eiusdem lateris, comprehensum, ad quadratum diametri sphæræ.

Theorema 18. Propositio 18.

Linea perpendicularis ex quolibet angulo trianguli æquilateri ad basin oppositam demissa; tripla est eius perpendicularis, quæ ex centro trianguli ad eandem basin deducitur.

Theorema 19. Propositio 19.

Si Octaedrum sphæræ inscribatur; erit semidiameter sphæræ potentia tripla eius perpendicularis, quæ ex centro sphæræ ad basin quamcunque Octaedri ducitur.

Theorema 20. Propositio 20.

Duplum quadrati ex diametro cuiuslibet sphæræ descripti, æquale est superficiei cubi in illa sphæra collocati: perpendicularis autem à centro sphæræ in aliquam basin cubi demissa, æqualis est dimidio lateris cubi.

Theorema 21. Propositio 21.

Idem circulus comprehendit, & cubi quadratum, & Octaedri triangulum, eiusdem sphæræ.

Theorema 22. Propositio 22.

Si Octaedrum, atque Tetraedrum eidem sphæræ inscribantur; Erit Octaedrum ad triplum Tetraedri, vt latus Octaedri ad latus Tetraedri.

Theorema 23. Propositio 23.

Si recta linea proposita potuerit totam aliquam lineam sectam extrema, & media ratione, & maius eius segmentum; item totam aliam similiter sectam, & minus eius segmentum: Erit maius segmentum prioris lineæ latus Icosaedri; minus autem segmentum posterioris lineæ latus Dodecaedri, eius sphæræ cuius recta linea proposita diameter existit.

Theo-

Theorema 24. Propositio 24.

Si latus Octaedri potuerit maius, & minus segmentum rectæ lineæ extrema, ac media ratione sectæ: poterit latus Icosaedri in eadem sphæra descripti, duplum minoris segmenti.

Theorema 25. Propositio 25.

Si recta linea diuisa extrema ac media ratione cum minore segmento angulum rectũ constituat, cui recta subtendatur: Erit recta linea, quæ potentia sit subdupla ipsius rectæ subtensæ, latus Octaedri eius sphæræ, in qua dictum minus segmentum latus existit dodecaedri.

Theorema 26. Propositio 26.

Si latus Tetraedri possit maius, & minus segmentum lineæ rectæ extrema ac media ratione sectæ: latus Icosaedri eidem sphæræ inscripti potentia sesquialterum est minoris segmenti.

Theorema 27. Propositio 27.

Cubus ad Octaedrum in eadem cum ipso sphæra descriptum, est, vt superficies cubi ad Octaedri superficiem: Item vt latus cubi ad semidiametrum sphæræ.

Theorema 28. Propositio 28.

Si sint quatuor lineæ rectæ continuæ proportionalis, nec non & aliæ quatuor, ita vt sit eadem antecedens omnium: Erit proportio tertiæ ad tertiam proportionis secundæ ad secundam duplicata;& proportio quartę ad quartam eiusdem proportionis secundæ ad secundam triplicata.

Theorema 29. Propositio 29.

Quadratum lateris trianguli æquilateri ad ipsum triangulum habet proportionem duplicatam proportionis lateris trianguli ad lineam medio loco proportionalem inter perpendicularem ab vno angulo ad latus oppositum ductam, & dimidium ipsius lateris.

Theorema 30. Propositio 30.

Si cubus, & Tetraedrum in eadem sphæra describantur; Erit quadratum cubi ad triangulum Tetraedri, vt latus Tetraedri ad lineam perpendicularem, quę ex vno angulo trianguli Tetraedri ad latus oppositum protrahatur.

Theorema 31. Propositio 31.

Latus Tetraedri potentia sesquialterum est axis, seu altitudinis ipsius; Axis verò, siue altitudo Tetraedri potentia sesquialtera est lateris cubi in eadem sphæra descripti.

Theorema 32. Propositio 32.

Cubus triplus est Tetraedri eidem sphæræ inscripti.

FINIS ELEMENTI XIV.

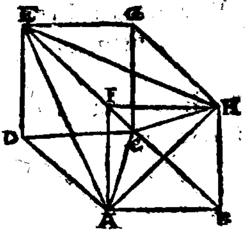
EVCLI-

EVCLIDIS ELEMENTVM DECIMVMQVINTVM ET SOLIDORVM QVINTVM, VT NONnulli putant; Vt autem alij, Hypsiclis Alexandrini, de quinque corporibus.

LIBER II.

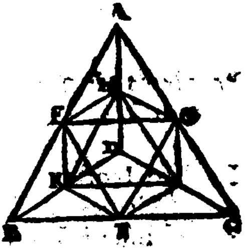
Problema 1. Propositio 1.

In dato cubo pyramidem (Tetraedrum inscribere.



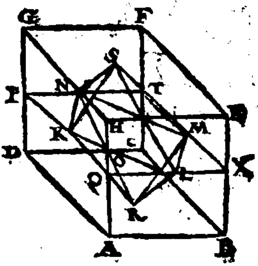
Problema 2. Propositio 2.

In data pyramide Octaedrum inscribere.



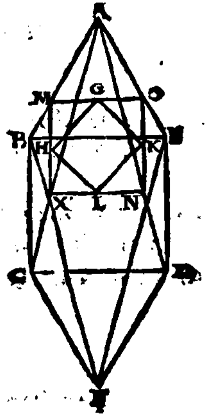
Problema 3. Propositio 3.

In dato cubo (hexaedro) Octaedrũ inscribere.



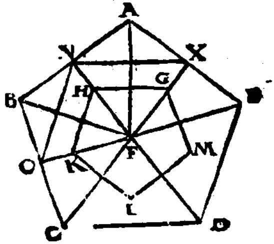
Problema 4. Propositio 4.

In dato octaedro cubũ inscribere.

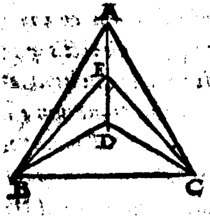


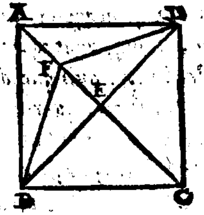
Problema 5. Propositio 5.

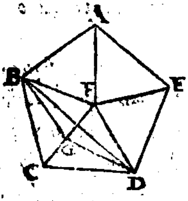
In dato Icosaedro dodecaedrum inscribere.

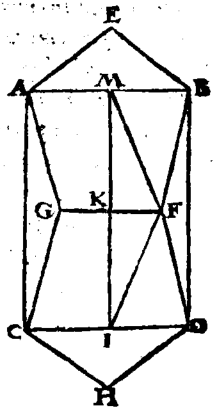


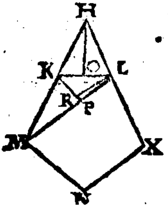
SCHOLIVM EX ZAMBERTO lib. 15. propos. 5. p. 260.











MEminisse decet, si quis nos roget, quot Icosaedrum habeat latera, ita respondendum esse: Patet Icosaedrum viginti contineri triangulis quodlibet verò triangulum rectis tribus *constare lineis. Quare multiplicanda sunt nobis viginti triangula in trianguli vnius latera fiuntq́ sexaginta, quorum dimidium est triginta. Ad eundem modum & in dodecaedro. Cum enim rursus duodecim pentagona dodecaedrum comprehendant, itemq́ pentagonum quoduis rectis quinque constet lineis; quinque duodecies multiplicamus, fiunt sexaginta, quorum rursus dimidium est triginta. Sed cur dimidium capimus? Quoniam vnumquodque latus siue sit trianguli, siue pentagoni, siue quadrati vt in cubo, iterato sumitur. Similiter autem eadem via & in cubo, & in tetraedro & in octaedro latera inuenies.*

Latera figurarum inuenienda.

Quòd si item velis singularum quoque figurarum angulos reperire, facta eadem multiplicatione numerum procreatum partire in numerum planorum, quæ vnum solidum angulum includunt: vt quoniam triangula quinque vnum Icosaedri angulum continet, partire 60. in quinque nascuntur duodecim anguli Icosaedri. In dodecaedro autem tria pentagona angulum comprehendunt, partire ergo 60. in tria, & habebis dodecaedri angulos viginti. Atque simili ratione in reliquis figuris angulos reperies, &c.

Anguli figurarum inueniendi.

Problema 6. Propositio 6.

In dato octaedro pyramidem, seu tetraedrum describere.

pro-

Problema 7. Propositio 7.
In dato Dodecaedro Icosaedrum describere.

Problema 8. Propositio 8.
In dato Dodecaedro Cubum describere.

Problema 9. Propositio 9.
In dato Dodecaedro Octaedrum describere.

Problema 10. Propositio 10.
In dato Dodecaedro pyramidem describere.

Problema 11. Propositio 11.
In dato Icosaedro Cubum describere.

Problema 12. Propositio 12.
In dato Icosaedro pyramidem describere.

Problema 13. Propositio 13.
In dato cubo Dodecaedrum describere.

Problema 14. Propositio 14.
In dato cubo Icosaedrum describere.

Problema 15. Propositio 15.
In dato Icosaedro Octaedrum describere.

Problema 16. Propositio 16.
In dato Octaedro Icosaedrum describere.

pro-

Problema 17. propositio 17.

In dato Octaedro Dodecaedrum describere.

Problema 18. Propositio 18.

In data pyramide Cubum describere: hoc est, in proposito Tetraedro hexaedrum delineare.

Problema 19. Propositio 19.

In data pyramide Icosaedrum describere.

Problema 20. Propositio 20.

In data pyramide Dodecaedrum describere.

Problema 21. Propositio 21.

In dato solido reg lari sphęram describere; hoc est, Interprete Campano; In fabricato quouis quinq; corporum regularium, sphęram fabricare.

FINIS ELEMENTI XV.

EVCLI-

EVCLIDIS ELEMENTVM DECIMVM SEXTVM, ET SOLIDORVM sextum.

Quo varia solidorum regularium sibi mutuò inscriptorum, & laterum eorundem comparationes explicantur, à Francisco Flussate Candalla, & P. Christophoro Clauio adiectam, & de quinque corporibus,

LIBER TERTIVS.

Theorema 1. Propositio 1.

SI in Dodecaedro Cubus describatur, & in hoc Cubo aliud Dodecaedrum: Erit proportio Dodecaedri exterioris ad Dodecaedrum interius proportionis eius, quam habet maius segmentum ad minus rectæ lineę diuisæ extrema, ac media ratione triplicata.

Theorema 2. Propositio 2.

Linea perpendicularis ex quouis angulo pētagoni æquilateri, & æquianguli in latus oppositum demissa; secatur à linea recta illum angulum subtendente, extrema ac media ratione.

Theo-

Theorema 3. Propositio 3.

Si ab angulis trianguli pyramidis ducantur tres lineæ rectæ, opposita latera secantes extrema ac media ratione; ita vt prope quemuis angulum sit maius segmentum vnius lateris, & minus alterius: Hæ tres sectionibus suis in medio producent basin Icosaedri in dicta pyramide descripti, inscriptam quidem alij triangulo, cuius anguli latera trianguli pyramidis secant extrema ac media ratione; & latera ipsa bifariam secantur ab angulis basis Icosaedri.

Theorema 4. Propositio 4.

Minus segmentum lateris pyramidis extrema ac media ratione secti, duplum est potẽtia lateris Icosaedri in ea pyramide descripti.

Theorema 5. Propositio 5.

Latus Cubi potentia dimidium est lateris pyramidis in eo descriptæ: Latus vero pyramidis duplum est longitudine lateris Octaedri sibi inscripti: latus denique Cubi duplum est potentia lateris sibi inscripti Octaedri.

Theorema 6. Propositio 6.

Latus Dodecaedri maius segmentum est rectæ lineæ quæ potentia est dimidia lateris pyramidis sibi inscriptæ.

Theorema 7. Propositio 7.

Si in Cubo describatur & Icosaedrum, & Dode-

Dodecaedrũ:Latus Icosaedri mediũ propor tionale erit inter latus Cubi,&Dodecaedri.

Theorema 8. Propositio 8.

Latus pyramidis potentia Octodecuplum est lateris Cubi in ea descripti.

Theorema 9. Propositio 9.

Latus pyramidis potentia Octaedecuplum est rectæ lineę extrema ac media ratione sectæ,cuius maius segmentum est latus Dodecaedri in pyramide descripti.

Theorema 10. Propositio 10.

Si in Octaedro Icosaedrum describatur: Erit latus Icosaedri potentia duplum minoris segmenti lateris Octaedri extrema ac media ratione diuisi.

Theorema 11. Propositio 11.

Latus Octaedri potentia quadruplum est sesquialterum lateris Cubi in ipso descripti.

Theorema 12. Propositio 12.

Latus Icosaedri maius segmentum est eius rectæ lineæ extrema ac media ratione sectę, quæ potentia dupla est lateris Octaedri in eo Icosaedro descripti.

Theorema 13. Propositio 13.

Latus Cubi ad latus Dodecaedri in ipso descripti proportionem habet duplicatam eius,quam habet maius segmentum ad minus rectę lineę diuisę extrema ac media ratione. Latus vero Dodecaedri ad latus cubi in ipso descri-

descripti, proportionem habet, quam minus segmentum ad maius, eiusdem rectæ lineæ.

Theorema 14. Propositio 14.
Latus octaedri sesquialterum est lateris pyramidis sibi inscriptæ.

Theorema 15. Propositio 15.
Si ex quadrato diametri Icosaedri auferatur triplum quadrati lateris cubi in eo descripti; relinquitur quadratum sesquitertium quadrati lateris Icosaedri.

Theorema 16. Propositio 16.
Latus Dodecaedri minus segmentum est rectæ lineæ extrema ac media ratione diuisæ, quę duplum potest lateris octaedri in eo descripti.

Theorema 17. Propositio 17.
Diameter Icosaedri potest, & sui ipsius lateris sesquitertium, & lateris pyramidis in eo descriptæ sesquialterum.

Theorema 18. Propositio 18.
Latus Dodecaedri ad Icosaedri sibi inscripti latus, se habet, vt minus segmentum lineæ perpendicularis ab vno angulo pentagoni ad latus oppositum ductæ, atque extrema ac media ratione diuisæ, ad partem eiusdem lineæ inter centrum pentagoni, & latus eiusdem positæ.

Problema 19. Propositio 19.
Si dimidium lateris Icosaedri extrema ac

media

media ratione sectum fuerit, minusque eius segmentum à toto latere Icosaedri sublatum; à reliqua quoque recta linea pars rursum tertia detracta: Relinquetur latus Dodecaedri in Icosaedro descripti.

Theorema 20 Propositio. 20.

Cubus sibi inscriptæ pyramidis triplus est.

Theorema 21. Propositio 21.

Pyramis sibi inscripti Octaedri dupla est.

Theorema 22. Propositio 22.

Cubus sibi inscripti Octaedri sextuplus est.

Theorema 23. Propositio 23.

Octaedrum sibi inscripti Cubi quadruplum sesquialterum est.

Theorema 24. Propositio 24.

Octaedrum sibi inscriptæ pyramidis tredecuplum sesquialterum est.

Theorema 25. Propositio 25.

Pyramis sibi inscripti Cubi noncupla est.

Theorema 26. Propositio 26.

Octaedrum ad Icosaedrum sibi inscriptum proportionem habet, quam duæ bases octaedri ad quinque bases Icosaedri.

Theorema 27. Propositio 27.

Icosaedrum ad Dodecaedrum sibi inscriptum, proportionem habet compositam ex proportione lateris Icosaedri ad latus cubi in eadem cum Icosaedro sphæra descripti; & ex proportione triplicata eius quam habet diameter Icosaedri ad rectam lineam contra basium Icosaedri oppositarum coniungentem.

Theorema 28. Propositio 28.

Dodecaedrum excedit Cubum, sibi inscriptum parallelepipedo, cuius quidem basis à quadrato Cubi deficit rectangulo contento sub latere Cubi, tertiaque parte, minoris segmenti eiusdem lateris Cubi; At verò altitudo ab altitudine, siue latere Cubi deficit, minore segmento eius lineæ, quæ dimidiati lateris Cubi segmentum minus existit.

Theorema 29. Propositio 29.

Dodecaedrum ad Icosaedrum sibi inscriptum, proportionem habet compositam ex proportione triplicata eius, quam habet diameter Dodecaedri ad rectam lineam contra basium Dodecaedri oppositarum copulantem; & ex proportione lateris Bubi ad lateris Icosaedri in eadem sphæra cum Cubo descripti.

Theorema 30. Propositio 30.

Dodecaedrum pyramidis, in qua describitur, duas nonas partes continet, minus duobus parallelepipedis; quorum vnius longitudo lateri Cubi in eadem pyramide descripti, æqualis est; latitudo verò tertiæ parti miti minoris segmenti lateris eiusdem Cubi; altitudo denique à latere eiusdem Cubi deficit minore segmento eius lineæ, quæ dimidiati lateris Cubi eiusdem minus segmẽtum existit: Alterius autem & longitudo,& latitudo lateri Cubi prædicti, est æqualis; altitudo vero minus segmentum eius lineæ quæ dimidiati lateris Cubi eiusdem segmentum minus existit;ita vt amborum altitudines simul altitudini,siue lateri eiusdem Cubi sint æquales.

Theorema 31. Propositio 31.

Octaedrum excedit sibi inscriptum Icosaedrum parallelepipedo, cuius basis est quadratum lateris Icosaedri; altitudo vero maius segmentum semidiametri Octaedri extrema ac media ratione sectæ.

DE QVINQVE CORPORVM regularium descriptione in data sphæra, ex Pappo Alexandrino.

Lemma I.

Datis duobus circulis in sphæra parallelis, dataque in vno eorum linea recta;ducare in altero diametrum huius rectæ datæ parallelam.

Lemma II.

Si in planis parallelis descripti sint duo circuli, in quibus duæ rectæ parallelæ abscindant arcus similes: Erunt duæ rectæ coniungentes extrema vnius rectæ cum centro parallelæ duabus rectis, quæ extrema alterius rectæ cum centro coniungant.

Lemma III.

Si in sphæra sint duo circuli paralleli, & æquales, in quibus ductæ sint duæ rectæ parallelæ, & æquales, ad easdem partes centrorum: Rectæ harum parallelarum extrema puncta ad easdem partes coniungentes, æquales quoque, & parallelæ sunt, & ad plana circulorum perpendiculares.

Lemma IV.

Si in sphæra sint duo circuli paralleli, & æquales, in quibus ductæ sint duæ rectæ parallelæ, & æquales, non ad easdem partes cẽtrorum: Rectæ lineæ harum parallelarum extrema puncta non ad easdem partes coniungentes, in centro sphæræ sese intersecant; ac proinde diametri sphæræ erunt, & inter se æquales: Rectæ verò lineæ earundem parallelarum extrema puncta ad easdem partes connectentes, & æquales, & parallelæ inter se ter ſe ſunt, & cum parallelis rectos angulos conſtituunt.

Lemma V.

Si in ſphæra ſint duæ rectæ parallelæ; rectæ earum puncta extrema ad eaſdem partes coniungentes, æquales inter ſe erunt: Et ſi parallelæ ſint æquales, coniungentes non ſolùm æquales, ſed & parallelæ erunt, rectoſque cum ipſis angulos conficient.

Lemma VI.

In data ſphæra duos circulos æquales, ac parallelos deſcribere; ita vt diameter ſphæræ ſit vtriuſque diametri potentia ſeſquialtera.

Problema 1. Propoſitio 1.

In data ſphæra pyramidem trigonam deſcribere.

Problema 2. Propoſitio 2.

In data ſphæra Octaedrum deſcribere.

Problema 3. Propoſitio 3.

In data ſphæra Cubum deſcribere.

Problema 4. propoſitio 4.

In data ſphæra Icoſaedrum deſcribere.

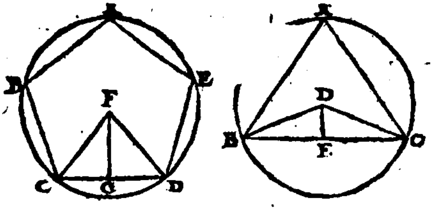
Problema 5 Propoſitio 5.

In data ſphæra Dodecaedrum deſcribere.

SCHOLIVM EX P. CLAVIO.

Ex his, quæ hoc in loco, & lib. 13. & lib. 14. demonstrata sunt, facile etiam ostenditur, si omnia quinque corpora regularia in vna eademque sphæra describantur, maximum omnium esse Dodecaëdrum: Deinde Icosaëdrum maius reliquis tribus: Tertio Cubum maiorem reliquis duobus: Octaëdrum denique Tetraëdro esse maius. Ex quo euidenter constabit, Euclidem recto ordine quinque hæc corpora construxisse, cum post Tetraëdrum statim Octaëdrum non autem Cubum constituerit. Ita enim à minoribus ad maiora progressus est.

FINIS ELEMENTI XVI. ET Vltimi.



Pagina 208. Theoremate 6. desunt hæ duæ Figuræ.

ERRATA SIC CORRIGATO.

Pagina 1. versu 5 legatur:
Punctum est, cuius pars nulla est. Punctum
p.3.v.10.vel pro vt.q. 5. v. 6. comprehenduntur pro comprehenditur. p. 8. v.6. perallelis pro perallelis. Ibid.dicuntur pro dicentur p.9.v. 5. postponantur. 4.B.C.D.v.penult·post si addatur ab.p.13.19.ad pro ab p. 17.v.14. Proposit: 18. p. 18.v.19. maiorem pro minorem.p 34.v.12.ipsas pro ipsis.p.40.v.16.circu lus pro circulum.p.41.v.20.prob.2. pro prob. 12. p. 50.v.12.figuram profiguræ.p.68. v. 2. diuidendo dimidando.Ibid.v.18. priorum pro priorem.p.71.post proport.add: ex.Ibid. v.15. excedens pro excadens. p.76.v.15.æqualia pro æquales p 77.v.1.post Et addat. si. Ibid.v.18.Problema proTheorema p.85.v.6. vnũquodque.p.88.vnius numeri p.90.v.2.& ille p. 93.v.13.& reliquus ad p. 101. v. 4. dele definitiones p. 116. v.Problema 2.p.121.v.5. primæ nempe magnitudinum p.126 v.penu. Problema 3.p.138.dele est p.146.v.10.dele lineam p.156.v.20. Reperire primũ residuum adde seu apotomen p.164.v.11.dele seu p. 169.v.20.ad rectos.p. 170.v.23.pluriũ.p.173. v. 3. aduersis Ibid.9.Cubus seu hexaedrum. p. 181. v.4.sub parallelis.p.186 v.1.verticibus.p.189:v.5.secundum. p.197.v 11 dele,rum.Ibid.v.vlt.circulũ non tangat. p.199 v.1.pro secta,lege secatur.Ibid. v.5. secta sit,p. 201.v.Problema 2.p.204.v. 9. sex autem trianguli & æquilateris.p. 297.v. 8. vt procemio finem. pag. 210. vers.26. Si spæræ duo.pa. 211. vers. 10. cadit in centum baseos ibidem vers. 17.dele quadrati.

Cætere beneuolus lector per se facile corriget.

FINIS.