EVCLIDIS ELEMENTORVM LIBRI XV.

QVIBVS, CVM AD OMNEM MATHEMATICAE SCIENTIAE PARTEM, TVM AD QVAMLIbet Geometriæ tractationem facilis comparatur aditus.

COLONIAE
Apud Gosuinum Cholinum
M. D. CXII.
Cum Gratia & Priuilegio.

AD CANDIDVM LECTOREM ST. GRACILIS praefatio.

PERMAGNI *referre semper existimaui, lector beneuole, quantum quisque studij & diligentiæ ad percipienda scientiarum elementa adhibeat, quibus non satis cognitis, aut perperam intellectis si vel digitũ progredientes erroris caliginem animis offundas, non veritatis lucem rebus obscuris adferas. Sed principiorum quanta sint in disciplinis momenta, haud facile credat, qui rerum naturam ipsa specie, non viribus metiatur. Vt enim corporum, quæ oriuntur & intereunt, vilißima tenuißimaq́ videntur initia: ita rerum æternarum & admirabilium, quibus nobilißimæ artes continentur, elementa ad speciem sunt exilia, ad vires & facultatem quam maxima. Quis non videt ex fici tantulo grano, vt ait Tullius, aut ex acino vinaceo, aut ex cæterarum frugum aut stirpium minutißimis seminibus tantos truncos ramosq́, procreari? Nam Mathematicorum initia illa quidem dictu audituq́, perexigua, quantã theorematum syluam nobis pe-*

pererunt? Ex quo intelligi potest, vt in ipsis seminibus, sic & in artium principijs inesse vim earum rerum, quæ ex his progignuntur. Præclarè igitur Aristoteles, vt alia permulta, μέγιστον ἴσως ἀρχὴ παντὸς, διὸ καὶ χαλεπώτατον· ὅσῳ γὰρ κράτιστον τῇ δυνάμει, τοσούτῳ μικρότατον, οὖσα τῷ μεγέθει χαλεπόν ἐστιν ὀφθῆναι. Quocirca committendum non est, vt non benè prouisa & diligenter explorata scientiarum principia, quibus propositarum quarumque rerum veritas sit demonstranda, vel constituas, vel constituta approbes: Cauendum etiam, vt ne tantulum quidem fallaci & captiosa interpretatione turpiter deceptus, à vera principiorum ratione temerè deflectas. Nam qui initio fortè aberrauerit, is vt tandem in maximis versetur erroribus, necesse est: cum ex vno erroris capite densiores sensim tenebræ rebus clarißimis obducantur. Quid tam varias veterum physiologorum sententias non modò cùm rerum veritate pugnantes, sed vehementer etiam inter se dissentientes nobis inuexit? Equidem haud scio, fuerit ne vlla potior tanti dißidij causa, quàm quòd ex principijs partim falsis, partim non consentaneis ductas rationes probando adhiberent. Fit enim plerunque, vt qui non rectè de artium rerumq́ elementis sentiunt, ad præfinitas quasdam opiniones suas omnia reuocare studeant. Pythagorei, vt meminit Aristoteles, cum denarij numeri summam perfectionem cælo tribuerent, nec plures tamen quàm nouem sphæras cernerent, decimam affingere ausi sunt terræ aduer-

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sam, quam ἀντίχθονα appellarunt. Illi enim vniuersitatis rerumq́; singularum naturam ex numeris ceu principijs æstimantes, ea protulerunt quæ φαινομένοις congruere nusquam sunt cognita. Nam ridicula Democriti, Anaximenis, Melißi Anaxagoræ, Anaximandri, & reliquorum id genus physiologorum somnia, ex falsis illa quidem orta naturæ principijs, sed ad Mathematicum nihil aut parum spectantia, sciens prætereo Nonnullos attingam qui repetitis altius vel aliter ac decuit positis rerum initijs, cum physicis multa turbarunt; tum Mathematicos oppugnatione principiorum peßimè mulctarunt. Ex planis figuris corpora constituit Timæus: Geometrarum hic quidem principia cuniculis oppugnantur. Nam & superficies seu extremitates craßitudinem habebunt, & lineæ latitudinem: denique puncta non erunt indiuidua, sed linearum partes. Prædicant Democritus atque Leucippus illas atomos suas, & indiuidua corpuscula. Concedit Xenocrates impartibiles quasdam magnitudines. Hic verò Geometriæ fundamenta apertè petuntur, & funditus euertuntur: quibus dirutis nihil equidem aliud video restare, quàm vt amplißima Mathematicorum theatra repentè concidant. Iacebunt ergò, si dijs placet, tot præclara Geometrarum de asymmetris & alogis magnitudinibus theoremata. Quid enim causa dicas, cur indiuidua linea hanc quidem metiatur, illam vero meteri non queat? Siquidem quod minimum in vnoquoque genere reperitur, id communis omnium mensura esse solet. Innumerabilia profectò sunt illa, quæ ex falsis eiusmodi decretis absurda consequuntur: & horum permulta quidem Mathematicus, sed longè plura colligit Physicus. Quid varia ψευδογραφημάτων genera commemorem, quæ ex hoc vno fonte tam longè latéque diffusa fluxisse videntur? Notißimus est Antiphontis tetragonismus, qui Geometrarum & ipse principia non parum labefecit, cùm rectæ lineæ curuam posuit æqualem. Longum esset mihi singula percensere, præsertim ad alia properanti: Hoc ergo certum fixum, & in perpetuum ratum esse oportet, quod sapienter monet Aristoteles, Σπουδαστέον ὅπως ὁρισθῶσι καλῶς αἱ ἀρχαὶ μεγάλην γὰρ ἔχουσι ῥοπὴν πρὸς ἑπόμενα. Nam principijs illa congruere debent, quæ sequuntur. Quòd si tantum perspicitur in istis exilioribus Geometriæ initijs, quæ puncto, linea, superficie definiuntur, momentum, vt ne hæc quidem sine summo impendentis ruinæ periculo conuelli aut oppugnari poßint, quanta quæso vis putanda est huius στοιχειώσεως, quam collatis tot præstantißimorum artificum inuentis, mira quadam ordinis solertia contexuit Euclides, vniuersæ Matheseos elementa complexu suo coercentem? Vt igitur omnibus rebus instructior & paratior quisque ad hoc studiũ libentius accedat, & singulaque vel minutißima exactius secum reputet atque perdiscat, operæpreciũ censui, in primo institutionis aditu vestibuloque præcipua quædam capita,

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quibus tota ferè Mathematicæ scientiæ ratio intelligatur, breuiter explicare: tum ea quæ sunt Geometriæ propria, diligenter persequi. Euclidis denique in extruenda hac στοιχειώσει consilium sedulò ac fideliter exponere. Quæ ferè omnia ex Aristotelis potißimũ ducta fontibus, nemini inuisa fore cõfido, qui modo ingenuũ animi candorẽ ad legendũ attulerit. Ac de Mathematicæ diuisione primũ dicamus.

Mathematicæ in primis scientiæ studiosos fuisse Pythagoreos, non modò historicorum, sed etiam philosophorum libri declarant. His ergo placuit, vt in partes quatuor vniuersum distribuatur Mathematicæ sciẽtiæ genus, quarum duas περὶ τὸ ποσὸν, reliquas περὶ τὸ πηλίκον versari statuerunt. Nam & τὸ ποσὸν vel sine vlla comparatione ipsum per se cognosci, vel certa quadam ratione comparatum spectari: in illo Arithmeticam, in hoc versari Musicam: & τὸ πηλίκον partim quiescere, partim moueri quidem: illud Geometriæ propositum esse: quod verò sua sponte motu cietur, Astronomiæ. Sed ne quis falsò putet, Mathematicam scientiam, quòd in vtroque quanti genere cernitur, idcirco inanem videri (si quidem non solum magnitudinis diuisio sed etiam multitudinis accretio infinitè progredi potest) meminisse decet, τὸ πηλίκον καὶ τὸ ποσὸν, quæ subiecto Mathematicæ generi imposita sunt à Pythagoreis nomina, non cuiuscunque modi quantitatem significare, sed eam demum, quæ tum multitudine tum magnitudine sit defi-

nita, & suis circumscripta terminis. Quis enim vllam infiniti scientiam defendat? Hoc scitum est, quod non semel docet Aristoteles, infinitum ne cogitatione quidem complecti unquam posse. Itaque ex infinita multitudinis & magnitudinis δυνάμει finitam hæc scientia decerpit & amplectitur naturam, quam tractet, & in qua versetur. Nam de vulgari Geometrarum consuetudine quid sentiendum sit, cum data interdum magnitudine infinita aut fabricantur aliquid, aut proprias generis subiecti affectiones exquirunt, disertè monet Aristoteles, οὐδὲ νῦν (de Mathematicis) loquens δέονται τοῦ ἀπείρου, οὐδὲ χρῶνται, ἀλλὰ μόνον εἶναι ὅσην ἂν βούλωνται, πεπερασμένην. Quamobrem disputatio ea qua infinitum refellitur, Mathematicorum decretis rationibusquê non aduersatur, nec eorum apodixen labefacit. Etenim tali infinito opus illis nequaquam est, quod exitu nullo peragrari possit, nec talem ponunt infinitam magnitudinem: sed quantamcunque velit aliquis effingere, ea vt suppetat, infinitam præcipiunt. Quinetiam non modo immensa magnitudine opus non habent Mathematici, sed ne maxima quidem: cum instar maximæ minima quæque in partes totidem pari ratione diuidi queat. Alteram Mathematicæ diuisionē attulit Geminus, vir (quantum ex Proclo conijcere licet) μαθημάτων laude clarissimus. Eam, quæ superiore plenior & accuratior fortè visa est, cū doctissimè pertractarit sua in decimum Euclidis præfa.

præfatione P. Montaureus vir senatorius, & regiæ bibliothecæ præfectus, leuiter attingam. Nam ex duobus rerum velut summis generibus τῶν νοητῶν καὶ τῶν αἰσθητῶν, quæ res sub intelligentiā cadunt, Arithmeticæ & Geomitriæ attribuit Geminus: quæ verò in sensus incurrunt, Astrologiæ, Musicæ Supputatrici, Opticæ, Geodesiæ & Mechanicæ adiudicauit. Ad hanc certè diuisionem spectasse videtur Aristoteles, cùm Astrologiam Opticam, harmonicam, φυσικωτέρας τῶν μαθημάτων nominat, vt quæ naturalibus & Mathematicis interiectæ sint, ac velut ex vtrisque mixtæ disciplinæ: Siquidem genera subiecta à Physicis mutuantur, causas verò in demonstrationibus ex superiore aliqua scientia repetunt. Id quod Aristoteles ipse apertissimè testatur, ἐνταῦθα γὰρ, φησὶ, τὸ μὲν ὅτι, τῶν αἰσθητικῶν εἰδέναι τὸ διότι, τῶν μαθηματικῶν. Sequitur, vt quid Mathematicæ conueniat cum Physica & prima Philosophia: quid ipsa ab vtraque differat, paucis ostendamus. Illud quidem omnium commune est, quòd in veri contemplatione sunt positæ, ob idque θεωρητικαὶ à Græcis dicuntur. Nam cùm διάνοια siue ratio & mens omnis sit vel πρακτικὴ, vel θεωρητικὴ, totidem scientiarum sint genera necesse est. Quòd si Physica, Mathematica, & prima Philosophia, nec in agendo, nec in efficiendo sunt occupatæ, hoc certè perspicuum est, eas omnes in cognitione contemplationéque necessario versari. Cùm enim rerum non modò agendarum, sed etiam efficiendarum principia in agente vel afficiente consistant, illarum quidem προαίρεσις, harum autem vel mens vel ars, vel vis quædam & facultas: rerum profectò naturalium. Mathematicarum, atque diuinarum principia in rebus ipsis, non in philosophis inclusa latent. Atque hæc vna in omnes valet ratio, quæ θεωρητικὰς esse colligat. Iam verò Mathematica separatim cum Physica congruit, quòd vtraque versatur in cognitione formarum corpori naturali inhærentium. Nam Mathematicus plana, solida, longitudines & puncta contemplatur, quæ omnia in corpore naturali à naturali quoque philosopho tractantur. Mathematica item & prima philosophia hoc inter se propriè conueniunt, quòd cognitionem vtraque persequitur formarum, quoad immobiles, & à concretione materiæ sunt liberæ. Nam tametsi Mathematicæ formæ re vera per se non cohærent, cogitatione tamen à materia & motu separantur, οὐδὲ γίνεται ψεῦδος χωριζόντων. vt ait Aristoteles. De cognatione & societate breuiter diximus, iam quid intersit videamus. Vnaquæq; mathematicarum certum quoddam rerum genus propositum habet, in quo versetur, vt Geometria quantitatem & continuationem aliorum in vnam partem, aliorum in duas, quorundam in tres, eorumq; quatenus quanta sunt & continua, affectiones cognoscit. Prima autem philosophia, cùm sit omnium communis, vniuersum Entis genus, quæq; ei accidunt & conueniunt hoc ipso quòd est, considerat.

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Ad hæc, Mathematica eam modò naturam amplectitur, quæ quanquam non mouetur, separari tamen seiungiq́; nisi mente & cogitatione à materia non potest, ob eamq́; causam ἐξ ἀφαιρέσεως dici consueuit. Sed Prima philosophia in ijs versatur, quæ & seiuncta, & æterna, & ab omni motu per se soluta sunt ac libera. Cæterùm Physica & Mathematica quanquã subiecto discrepare non videntur, modo tamen rationeq́; differũt cognitionis & contemplationis, vnde dißimilitudo quoque scientiarum sequitur. Etenim Mathematicæ species nihil re vera sunt aliud, quàm corporis naturalis extremitates, quas cogitatione ob omni motu & materia separatas Mathematicus contemplatur: sed easdem consectatur Physicorũ ars, quatenus cum materia comprehensæ sunt, & corpora motui obnoxia circumscribunt. Ex quo fit, vt quæcunque in Mathematicis incommoditates accidunt, eædem etiam in naturalibus rebus videantur accidere, non autem vicißim. Multa enim in naturalibus sequuntur incommoda, quæ nihil ad Mathematicum attinent, διὰ τὸ, inquit Aristoteles, τὰ μὲν ἐξ ἀφαιρέσεως λέγεται, τὰ μαθηματικά, τὰ δὲ φυσικὰ ἐκ προσθέσεως. Siquidem res cum materia deuinctas cõtẽplatur physicus: Mathematicus verò rẽ cognoscit circũscriptis ijs omnibus, quæ sensu percipiũtur, vt grauitate, leuitate, duritie, mollitie, & præterea calore, frigore, alijsq́; contrariorum paribus, quæ sub sensum subiecta sunt: tantum autem relinquit quan-

quantitatem & continuum. Itaq; Mathematicorum ars in iis quæ immobilia sunt cernitur (τὰ γὰρ μαθηματικὰ τῶν ὄντων ἄνευ κινήσεώς ἐςιν ἔξω τῶν περὶ τὴν ἀςρολογίαν) quæ verò in naturæ obscuritate posita est, res quidem quæ nec separari nec motu vacare possunt contemplatur. Id quod in vtroque scientiæ genere perspicuum esse potest siue res subiectas definias, siue proprietates earum demonstres. Etenim numerus, linea, figura, rectum, inflexum, æquale, rotundũ, vniuersa deniq; Mathematicus quæ tractat & profitetur, absque motu explicari doceriq; possũt: χωριςὰ γὰρ τῇ νοήσει κινήσεώς ἐςι: Physicæ autẽ sine motione species nequaquam possunt intelligi. Quis enim hominis, plantæ, ignis, ossium, carnis naturam & proprietates sine motu, qui materiam sequitur, perspiciat? Siquidem tantisper substantia quæque naturalis constare dici solet, quo ad opus & munus suum, agendo patiendóque tueri ac sustinere valeat: qua certè amissa δυνάμει, ne nomen quidem nisi ὁμωνύμως retinet. Sed Mathematico ad explicandas circuli aut trianguli proprietates, nullum adferre potest vsum, materiæ, vt auri, ligni, ferri, in qua insunt, consideratio, quin eò verius eiusmodi rerum, quarum species tanquam materia vacantes efformemus animo, naturam complectemur, quòd coniunctione materiæ quasi adulterari dipravarique videntur. Quo circa Mathematicæ species eodem modo quo κοιλὸν, siue concauitas, sine motu & subiecto, definitione explicare cognosnosciq́; possunt: naturales verò cùm eam vim habeant, quam vt ita dicam, simitas cum materia comprehensa sunt, nec absque ea separatim possunt intelligi: quibus exemplis quid inter Physicas & Mathematicas species intersit, haud difficile est animaduertere. Illis certè non semel est vsus Aristoteles. Valeant ergò Protagoræ sophismata, Geometras hoc nomine refellentis, quod circulus normam puncto non attingat. Nam diuina Geometrarum theoremata, qui sensu æstimabit, vix quicquam reperiet quod Geometræ concedendum videatur. Quid enim ex his quæ sensum mouent, ita rectum aut rotundum dici potest, vt à Geometra ponitur? Nec verò absurdum est aut vitiosum, quòd lineas in puluere descriptas pro rectis aut rotundis assumit, quæ nec rectæ sunt nec rotundæ, ac ne latitudinis quidem expertes. Siquidem non ijs vtitur Geometra, quasi inde vim habeat conclusio, sed eorum quæ discenti intelligenda relinquuntur, rudem ceu imaginem proponit. Nam qui primum instituuntur, hi ductu quodam & velut χειραγωγία sensuum opus habent, vt ad illa quæ sola intelligentia percipiuntur, aditum sibi comparare queant. Sed tamen existimandum non est rebus Mathematicis omnino negari materiam ac non eam tantùm quæ sensum afficit. Est enim materia alia quæ sub sensum cadit, alia quæ animo & ratione intelligitur. Illam αἰσθητὴν hanc νοητὴν vocat Aristoteles. Sensu percipitur, vt æs, vt lignum, omnisq́ue materia quæ, moueri po-

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test. Animo & ratione cernitur ea quæ in rebus sensilibus inest, sed non quatenus sensu percipiuntur, quales sunt res Mathematicorum. Vnde ab Aristotele scriptum legimus ἐπὶ τῶν ἐν ἀφαιρέσει ὄντων rectum se habere vt simum: μετὰ συνεχοῦς γὰρ: quasi velit ipsius recti quod Mathematicorum est, suam esse materiam, non minus quam simi quod ad Physicos pertinet. Nam licet res Mathematicæ sensili vacent materia, non sunt tamen indiuiduæ, sed propter continuationem partitioni semper obnoxiæ, cuius ratione dici possunt sua materia non omnino carere: quin aliud videtur τὸ εἶναι γραμμῇ, aliud quoad continuationi adiuncta intelligitur linea. Illud enim ceu forma in materia proprietatum causa est, quas sine materia percipere non licet. Hæc est societatis & dissidij Mathematicæ cum Physica & prima Philosophia ratio. Nunc autem de nominis etymo & notatione pauca quædam asseramus. Nam si quæ iudicio & ratione imposita sunt rebus nomina, ea certè non temerè indita fuisse credendum est, quibus scientias appellari placuit, sed neque otiosa semper haberi debet ista etymologiæ indagatio, cùm ad rei etiam dubiæ fidem sæpe non parum valeat recta nominis interpretatio. Sic enim Aristoteles ducto ex verborum ratione argumento αὐτομάτου, μεταβολῆς, αἰθέρος, aliarumq́; rerum naturam ex parte confirmauit. Quoniam igitur Pythagoras Mathematicam scientiam non modò studiosè coluit, sed etiam repetitis à capite principijs, geometricam

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contemplationem in liberalis disciplinæ formam composuit, & perspectis absq; materia solius intelligentiæ adminiculo theorematibus, tractationem περὶ τῶν ἀλόγων, & κοσμικῶν σχημάτων constitutionem excogitauit: credibile est, Pythagoram, aut certè Pythagoreos, qui & ipsi doctoris sui studia libenter amplexi sunt, huic scientiæ id nomen dedisse, quod cum suis placitis atq; decretis congrueret rerumq; propositarum naturam quoquo modo declararet. Ita cùm existimarēt illi, omnem disciplinam, quæ μάθησις dicitur, ἀνάμνησιν esse quandam .i. recordationem & repetitionem eius scientiæ, cuius antè quam in corpus immigraret, compos fuerit anima, quemadmodum Plato quoque in Menone, Phædone, & alijs aliquot locis videtur astruxisse: animaduerterent autem eiusmodi recordationem, quæ non posset multis ex rebus perspici, ex his potißimum scientijs demonstrari, si quis nimirum, ait Plato, ἐπὶ τὰ διαγράμματα ἄγῃ, probabile est equidem Mathematicas à Pythagoreis artes κατ' ἐξοχὴν fuisse nominatas, vt ex quibus μάθησις, id est æternarum in anima rationum recordatio διαφερόντως & principuè intelligi posset. Cuius etiam rei fidem nobis diuinus fecit Plato, qui in Menone Socratem induxit hoc argumēti genere persuadere cupientem, discere nihil esse aliud quam suarum ipsius rationum animum recordari. Etenim Socrates pusionem quendam vt Tullij verbis vtar, interrogat de geometrica dimensione quadrati: ad ea sic ille respondet vt puer, et ta-

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men tam faciles interrogationes sunt, vt gradatim respondens, eodem perueniat quò si Geometrica didicisset. Aliam nominis huius rationem Anatolius exposuit, vt est apud Rhodiginum, quòd cùm caeterae disciplinae deprehendi vel non docente aliquo possint omnes, Mathematica sub nullius cognitionem veniant, nisi praeeunte aliquo, cuius solertia succidantur vepreta, vel exurantur, & superciliosa complanentur aspreta. Ita enim Caelius: quod quam vim habeat, non est huius loci curiosius perscrutari. Equidem M. Tullius Mathematicos in magna rerum obscuritate, recondita arte multiplicíque ac subtili versari scribit: sed quis nescit idipsum cum alijs grauioribus scientijs esse commune? Est enim, vel eodem auctore Tullio, omnis cognitio multis obstructa difficultatibus, maxiмáque est & in ipsis rebus obscuritas, & in iudicijs nostris infirmitas, nec vllus est, modò interius paulò Physica penetrarit, qui non facilè sit expertus, quam multi vndique emergant, rerum naturalium causas inquirentibus, inexplicabiles labyrinthi. Sunt qui ex demonstrationum firmitate nominari Mathematicas opinantur: cuius etiam rationis momentum alio seorsim loco expendendum fuerit. Quocirca primam verbi notationem, quam sequutus est Proclus, nobis retinendam censeo. Hactenus de vniuerso Mathematicae genere, quanta potui & perspicuitate & breuitate dixi. Sequitur vt de Geometria separatim atque ordine ea disseram, quae initio sum pollici-

tus

ius. Est autem Geometria, vt definit Proclus sci entia, quæ versatur in cognitione magnitudinum figurarum, & quibus hæ continentur, extremorum, item rationum & affectionum, quæ in illis cernuntur ac inhærent: ipsa quidem progrediens à puncto indiuiduo per lineas & superficies, dum ad solida conscendat, variasq́ ipsorum differentias patefaciat. Quúmque omnis scientia demonstratiua, vt docet Aristoteles, tribus quasi momentis contineatur, genere subiecto, cuius proprietates ipsa scientia exquirit & cōtemplatur: causis & principijs ex quibus primis demonstrationes conficiuntur: & proprietatibus, quæ de genere subiecto per se enunciantur: Geometriæ quidem subiectum in lineis, triangulis, quadrangulis, circulis, planis, solidis atque omnino figuris & magnitudinibus, earúmque extremitatibus consistit. His autem inhærent diuisiones, rationes, tactus, æqualitates, παραβολαὶ ὑπερβολαὶ ἐλλείψεις, atque alia generis eiusdem propè innumerabilia. Postulata verò & Axiomata ex quibus hæc inesse demonstrantur, eiusmodi ferè sunt: Quouis centro & interuallo circulum describere. Si ab æqualibus æqualia detrahas, quæ relinquuntur esse æqualia, cæteraq́ id genus permulta, quæ licet omnium sint communia, ac demonstrandum tamen tum sunt accommodata, cum ad certum quoddam genus traducuntur. Sed cùm præcipua videatur Arithmeticæ & Geometriæ inter Mathematicas dignatio, cur Arithmetica sit ἀκριβεστέρα &

exactior quam Geometria paucis explicandum arbitror. Hic verò & Aristotelem sequemur ducem, qui scientiam cum scientia ita comparat, vt accuratiorem esse velit eam, quæ rei causam docet, quam quæ rem esse tantùm declarat, deinde quæ in rebus sub intelligentiam cadentibus versatur, quàm quæ in rebus sensum mouentibus cernitur. Sic enim & Arithmetica quàm Musica, & Geometria quàm Optica, & Stereometria quàm Mechanica exactior esse intelligitur. Postremò quæ ex simplicioribus initijs constat, quàm quæ aliqua adiectione componitur. Atque hac quidem ratione Geometriæ præstat Arithmetica, quòd illius initium ex additione dicatur, huius sit simplicius. Est enim punctum, vt Pythagoreis placet, vnitas quæ situm obtinet: vnitas verò punctum est quod situ vacat. Ex quo percipitur, numerorum quàm magnitudinum simplicius esse elementum, numerosque magnitudinibus esse puriores, & à concretione materiæ magis disiunctos. Hæc quanquam nemini sunt dubia, habet & ipsa tamen Geometria quo se plurimum efferat, opibúsque suis ac rerum vbertate multiplici vel cum Arithmetica certet: id quod tute facilè deprehendas cùm ad infinitam magnitudinis diuisionem, quam respuit multitudo, animum conuerteris. Nunc quæ sit Arithmeticæ & Geometriæ societas, videamus. Nam theorematum quæ demonstratione illustrantur, quædam sunt vtriusque scientiæ communia, quædam verò singularum propria. Etenim quòd omnis proportio sit ῥητὸς siue rationalis, Arithmeticæ soli conuenit, nequaquam Geometriæ, in qua sunt etiam ἄρρητοι seu irrationales proportiones. item, quadratorum gnomas minimo definitos esse, Arithmeticæ proprium (si quidem in Geometria nihil tale minimum esse potest) sed ad Geometriam propriè spectant situs, qui in numeris locum non habent tactus, qui quidem à continuis admittuntur: ἄλογον, quoniam vbi diuisio infinitè procedit, ibi etiam τὸ ἄλογον esse solet. Communia porrò vtriusque sunt illa, quæ ex sectionibus eueniunt, quas Euclides libro secundo est persequutus: nisi quòd sectio per extremam & mediam rationem in numeris nusquam reperiri potest. Iam vero ex theorematibus eiusmodi communibus, alia quidem ex Geometria ad Arithmeticam traducuntur: alia contrà ex Arithmetica in Geometriam transferuntur: quædam verò perinde vtrique scientiæ conueniunt, vt quæ ex vniuersa arte Mathematica in vtranque harum conueniant. Nam & alterna ratio, & rationum conuersiones, compositiones, diuisiones hoc modo communia sunt vtriusque. Quæ autem sunt περὶ συμμέτρων, id est, de commensurabilibus, Arithmetica quidem primùm cognoscit & contemplatur: secundo loco Geometria Arithmeticam imitata. Quare & commensurabiles magnitudines illæ dicuntur, quæ rationem inter se habent, quam numerus ad numerum, perinde quasi cōmensuratio & συμμετρία in numeris primùm consistat (Vbi enim numerus ibi & σύμμετρον cernitur: & vbi σύμμετρον, illic etiam numerus) sed quæ triangulorum sunt & quadrangulorum, à Geometra primùm considerantur: tùm analogia quadam Arithmeticus eadem illa in numeris contemplatur. De Geometriæ diuisione hoc adijciendum puto, quòd Geometriæ pars altera in planis figuris cernitur quæ solam latitudinem longitudini coniunctam habent: altera verò solidas contemplatur, quæ ad duplex illud interuallum crassitudinem adsciscunt. Illam generali Geometriæ nomine veteres appellarunt: hanc propriè Stereometriam dixerunt. Ita Geometriam cum Optica & Stereometriam cum Mechanica non rarò comparat Aristoteles. Sed illius cognitio huius inuentionem multis seculis antecessit, si modò Stereometriam ne Socratis quidem ætate vllam fuisse omnino verum est, quemadmodum à Platone scriptum videtur. Ad Geometriæ vtilitatem accedo, quæ quanquam suapte vi & dignitate ipsa per se nititur, nulllius vsus aut actionis ministerio mancipata (vt de Mathematicis omnibus scientijs concedit in Politico Socrates) si quid ex ea tamen vtilitatis externæ quæritur, Dij boni quàm latos, quàm vberes, quàm varios fructus fundit? Nec verò audiendus est vel Aristippus vel Sophistarum alius, qui Mathematicorum artes idcirco repudiet, quòd ex fine nihil docere videantur, eiusque quod melius aut deteri-

lterius nullam habeant rationem. Vt enim nihil
ausæ dicas, cur sit melius trianguli, verbi gratia,
res angulos duobus esse rectis æquales: minimè ta-
men fuerit consentaneum Geometriæ cognitionem
t inutilem exagitare, criminari, explodere, quasi
uæ finem & bonum quò referatur, habeat nullum.
Multas haud dubiè solius contemplationis beneficio
citra materiæ contagionem adfert Geometria com-
moditates partim proprias, partim cum vniuerso
genere communes. Cùm enim Geometria, vt scri-
psit Plato, eius quod semper est cognitionem profi-
teatur, ad veritatem excitabit illa quidem ani-
mum, & ad ritè philosophandum cuiusque men-
tem comparabit. Quinetiam ad disciplinas om-
nes facilius perdiscendas, attigeris necne Geome-
triam quanti referre censes? Nam vbi cum mate-
ria coniungitur, nónne præstantissimas procreat
artes. Geodesiam, Mechaniam, Opticam, qua-
rum omnium vsu, mortalium vitam summis be-
neficijs complectitur? Etenim bellica instrumen-
ta, vrbiumq; propugnacula, quibus munitæ vrbes
hostium vim propulsarent, his adiutricibus fa-
bricata est: montium ambitus & altitudines, loco-
rumque situs nobis indicauit: dimetiendorum
in mari & terra itinerum rationem præscripsit:
trutinas & stateras, quibus exacta numerorum
æqualitas in ciuitate retineatur, composuit: v-
niuersi ordinem simulachris expressit, multá-
que hominum fidem superarent, omnibus
persuasit. Vbique extant præclara in eam rem

testimonia. Illud memorabile, quòd Archimedi rex Hiero tribuit. Nam extructo vastæ molis nauigio, quod Hiero Ægyptiorum regi Ptolemæo mitteret, cùm vniuersa Syracusanorum multitudo collectis simul viribus nauem trahere non posset effecissetq́, Archimedes, vt solus Hiero illam subduceret, admiratus viri scientiam rex ἀπὸ ταύτης ἔφη, τῆς ἡμέρας περὶ παντος ἀρχιμήδῃ λέγοντι πιστευτέον. Quid? quòd Archimedes idem, vt est apud Plutarchū, Hieroni scriptis datis viribus datum pondus moueri posse? fretusq́, demonstrationis robore illud sæpe iactarit, si terram haberet alteram vbi pedem figeret, ad eam, nostram hanc se transmouere posse? Quid varia, αὐτομάτων machinarúmque genera ad vsus necessarios comparata memorem? Innumerabilia profectò sunt illa, & admiratione dignißima quibus prisci homines incredibili quodam ad philosophandum studio concitati, inopem mortalium vitam artis huius præsidio subleuarunt: tametsi memoriæ sit proditum, Platonem Eudoxo & Archytæ vitio vertisse, quòd Geometrica problemata ad sensilia & organica abducerent. Sic enim corrumpi ab illis & labefieri Geometriæ præstantiam, quæ ab intelligibilibus & incorporeis rebus ad sensiles & corporeas prolaberetur. Quapropter ridicula idem scripsit Plato Geometrarum esse vocabula, quæ quasi ad opus & actionem spectent, ita sonare videntur. Quid enim est quadrare si non opus facere? Quid addere, producere, applicare? Multa quidem sunt eiusmodi nomina, quibus necessariò & tanquam coacti Geometræ vtuntur, quippe cùm alia desint in hoc genere commodiora. Sic ergo censuit Plato, sic Aristoteles sic denique philosophi omnes, Geometriam ipsam cognitionis gratia exercendam, nec ex aliquo vsu externo sed ex rerum τοῦ ὄντος intelligentia æstimandam esse. Exposita breuius quàm res tanta dici possit, vtilitatis ratione, Geometriæ ortum, qui in hac rerum periodo ex historicorum monumentis nobis est cognitus deinceps aperiamus. Geometria apud AEgyptios inuenta, (ne ab Adamo, Setho, Noah, quos cognitione rerum multiplici valuisse constat, eam repetamus) ex terrarum dimensione, vt verbi præ se fert ratio, ortum habuisse dicitur: cùm anniuersaria Nili inundatione & incrementis limo obducti agrorum termini confunderentur. Geometriam enim, sicut & reliquas disciplinas, in vsu quàm in arte prius fuisse aiunt. Quod sanè mirum videri non debet, vt & huius & aliarum scientiarum inuentio ab vsu cœperit ac necessitate. Etenim tempus, rerum vsus, ipsa necessitas ingenium excitat, & ignauiam acuit. Deinde quicquid ortum habuit (vt tradunt Physici) ab inchoato & imperfecto processit ad perfectum. Sic artium & scientiarum principia experientiæ beneficio collecta sunt: experientia verò à memoria fluxit, quæ & ipsa à sensu primùm manauit. Nam quod scribit Aristoteles, Mathematicas artes, comparatis rebus omnibus ad vitam neceſſarijs, in AEgypto fuiſſe conſtitutas, quòd ibi ſacerdotes omnium conceſſu in otio degerent: non negat ille adductos neceſſitate homines ad excogitandam, verbi gratia terræ dimetiendæ rationem, quæ theorematum deinde inueſtigationi cauſam dederit: ſed hoc confirmat, præclara eiuſmodi theorematum inuenta, quibus extructa Geometriæ diſciplina conſtat, ad vſus vitæ neceſſarios ab illis non eſſe expetita. Itaque vetus ipſum Geometriæ nomen ab illa terræ partiundæ finiúmque regendorum ratione poſteà receſſit, & in certa quadam affectionum magnitudini per ſe inhærentium ſcientia propriè remanſit. Quemadmodum igitur in mercium & contractuum gratiam ſupputandi ratio, quam ſecuta eſt accurata numerorum cognitio, à Phœnicibus initium duxit: ita etiam apud AEgyptios, ex ea quam commemoraui cauſa ortum habuit Geometria. Hanc certè, vt id obiter dicam, Thales in Græciam ex AEgypto primum tranſtulit: cui non paucæ deinceps à Pythagora, Hipprocrate Chio, Platone, Archyta Tarentino, alijſq; compluribus, ad Euclidis tempora factæ ſunt rerum magnarum acceſſiones. Cæterùm de Euclidis ætate id ſolùm addam, quod à Proclo memoriæ mandatum accepimus. Is enim commemoratis aliquot Platonis tum æqualibus tùm diſcipulis, ſubijcit, non multò ætate poſteriorem illis fuiſſe Euclidem eum, qui Elementa conſcripſit, & multa ab Eudoxo collecta, in ordinem luculentum

com-

composuit, multaq́; à Theateto inchoata perfecit, quaq́; mollius ab alijs demonstrata fuerant, ad firmißimas & certißimas apodíxes reuocauit. Vixit autem, inquit ille sub primo Ptolemæo. Et enim ferunt Euclidam à Ptolemæo quendam interrogatum num qua esset via ad Geometriam magis compendiaria, quàm sit ista στοιχείωσις respondisse μὴ εἶναι βασιλικὴν ἀτραπὸν ἐπὶ γεωμετρίαν. Diende subiungit, Euclidem natu quidem esse minorem Platone, maiorem verò Eratosthene & Archimede (hi enim quales erant) cùm Archimedes Euclidis mètionem faciat. Quòd si quis egregiam Euclidis laudem quam cùm ex alijs scriptionibus accuratißimis, tùm ex hac Geometria στοιχειώσεως cōsequutus est in qua diuinus rerum ordo sapientißimis quibusque hominibus magnæ semper admirationi fuit, Proclum studiosè legat, quò rei veritatem illustriorem reddat grauißimi testis auctoritas. Superest igitur vt finem videamus, quò Euclidis elementa referri, & cuius causa in id studium incumbere, oporteat. Et quidem si res quæ tractantur, consideres: in tota hac tractatione nihil aliud quæri dixeris, quam vt σωματικὰ quæ vocantur σχήματα (fuit enim Euclides profeßione & instituto Platonicus) Cubus Icosaedrum, Octaedrum, Pyramis & Dodecaedrum certæ quadam suarum & inter se laterum, & ad sphæræ diametrum ratione eidem sphæræ inscripta comprehendantur. Huc enim pertinet Epigrammation illud vetus, quod in Geometrica Michaelis

Pselli συνόψει scriptum legitur.

Σχήματα πέντε τὰ πλάτωνος, πυθαγόρας σοφὸς
εὗρε.

πυθαγόρας σοφὸς εὗρε, πλάτων δ' ἀριδήλ' ἐδί-
δαξεν.

Εὐκλείδης ἐπὶ τοῖσι κλέος περικαλλὲς ἔτευξεν.

Quòd si discentis institutionem spectes, illud certè fuerit propositum, vt huiusmodi elementorum cognitione informatus discentis animus, ad quamlibet non modò Geometriæ, sed & aliarum Mathematicæ partium tractationem idoneus paratusq́; accedat. Nam tametsi institutionem hanc solus sibi Geometra vendicare videtur, & tanquam in possessionem suam venerit, alios excludere posse: inde tamen per multa suo quodam modo iure decerpit Arithmeticus, pleraque Musicus non pauca detrahit. Astrologus, Opticus, Logisticus, Machanicus, itemq́; cæteri: nec vllus est denique artifex præclarus, qui in huius se possessionis societatem cupide non offerat, partemq́; sibi cōcedi postulet. Hinc στοιχείωσις absolutum operi nomen, & στοιχειωτὴς dictus Euclides. Sed quid longius prouehor? Nam quod ad hanc rem attinet, tam copiosè & eruditè scripsit (vt alia complura) eo ipso, quem dixi, loco P. Montaureus, vt nihil desiderio loci reliquerit. Quæ verò ad dicendum nobis erant proposita hactenus pro ingenij nostri tenuitate omnia mihi perfecisse videor. Nam tametsi & hæc eadem & alia pleraque multo fortè præclariora ab hominibus doctißimis, qui

tum

tùm acumine ingenij, tùm admirabili quodam lepore dicendi semper floruerunt grauiùs, splendidiùs, vberiùs tractari posse scio: tamen experiri libuit numquid etiam nobis diuino sit concessum munere, quod rudes in hac philosophiæ parte discipulos adiuuare aut certè excitare queat. Huc accessit quòd ista recens elementorum editio, in qua nihil non parum fuisset studij, aliquid à nobis efflagitare videbatur, quod eius commendationem adaugeret. Cùm enim vir doctissimus Io. Magnienus Mathematicarum artium in hac Pharrhisiorum Academia professor verè regius, nostrum hunc typographum in excudendis Mathematicorum libris diligentissimum, ad hanc Elementorum editionem sæpè & multum esset adhortatus, eiusque impulsu permulta sibi iam comparasset typographus ad hanc rem necessaria, citò interuenit, malùm Ioannis Magnieni mors insperata, quæ tam graue inflixit Academiæ vulnus, cui ne post multos quidem annorum circuitus cicatrix obduci vlla posse videatur. Quamobrem amisso instituti huius operis duce, typographus, qui nec sumptus antea factos sibi perire, nec studiosos, quibus id muneris erat pollicitus, sua spe cadere vellet, ad me venit, & impensè rogauit, vt meam propositæ editioni operam & studium nauarem, quod cùm denegaret occupatio nostra, iuberet officij ratio, feci equidem rogatus, vt quæ subobscurè vel parum commodè in sermonem Latinum è Græco translata

vi-

videbantur, clariore, aptiore & fideliore interpretatione nostra (quod cuiusq; pace dictum volo) lucem acciperent. Id quod in omnibus ferè libris posterioribus tute primo obtutu perspicias. Nam in sex prioribus non tantum temporis quantum in cæteris ponere nobis licuit decimi autem interpretatio, qua melior nulla potuit adferri, P. Montaureo solida debetur. Atq; vt ad perspicuitatem facilitatemq; nihil tibi deesse queraris, adscripta sunt propositionibus singulis vel lineares figuræ, vel punctorum tanquam vnitatum notulæ, quæ Theonis apodixin illustrent illæ quidem magnitudinum, hæ autem numerorum indices, subscriptis etiam ciphrarum, vt vocant, characteribus, qui propositum quemuis numerum exprimant, ob eamq; causam eiusmodi vnitatum notulæ, quæ pro numeri amplitudine maius paginæ spatium occuparent, pauciores sæpius depictæ sunt aut in lineas etiam commutatæ. Nam literarum vt a, b, c characteres non modò numeris & numerorum partibus nominandis sunt accommodati, sed etiam generales esse numerorum, vt magnitudinum affectiones testantur. Adiecta sunt insuper quibusdam locis non pœnitenda Theonis scholia, siue mauis lemmata, quæ quidem longè plura accessissent, si plus otij & temporis vacui nobis fuisset relictum, quod huic studio impartiremus. Hanc igitur operam boni consule, & quæ obuia erunt impressionis vitia, candidè emenda. Vale. Lutetiæ 4. Idus April. 1557.

EVCLIDIS ELEMENTVM PRIMVM

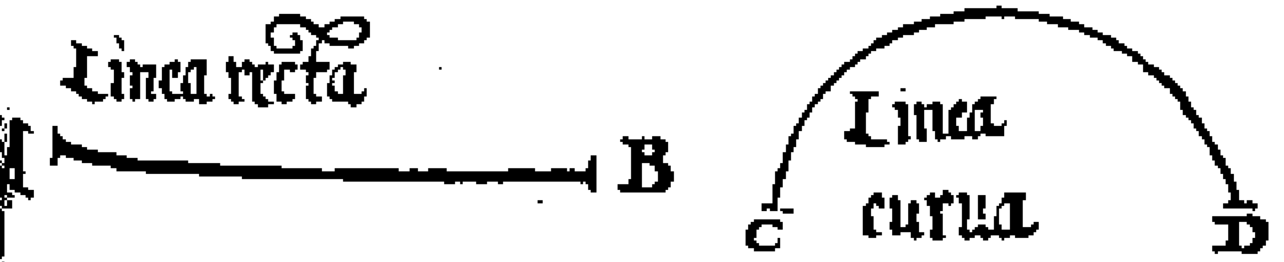
DEFINITIONES.

1

Punctum est, cuius pars nulla est. Punctum

2

Linea verò, longitudo latitudinis expers.



3

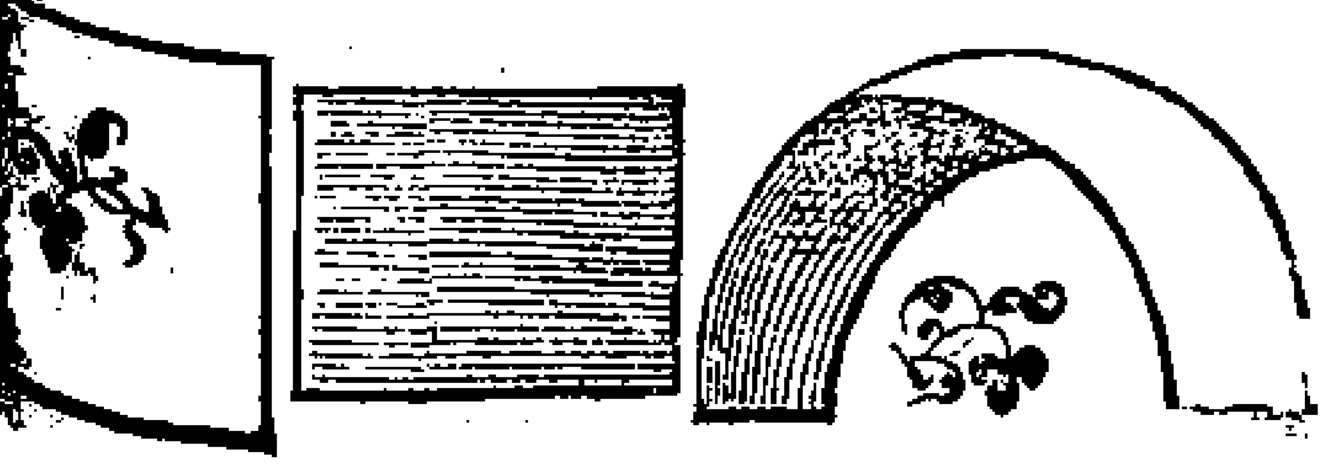
Lineæ autem termini, sunt puncta.

4

Recta linea est, quæ ex æquo sua interiacet puncta.

5

Superficies est, quæ longitudinem latitudinemque tantùm habet.



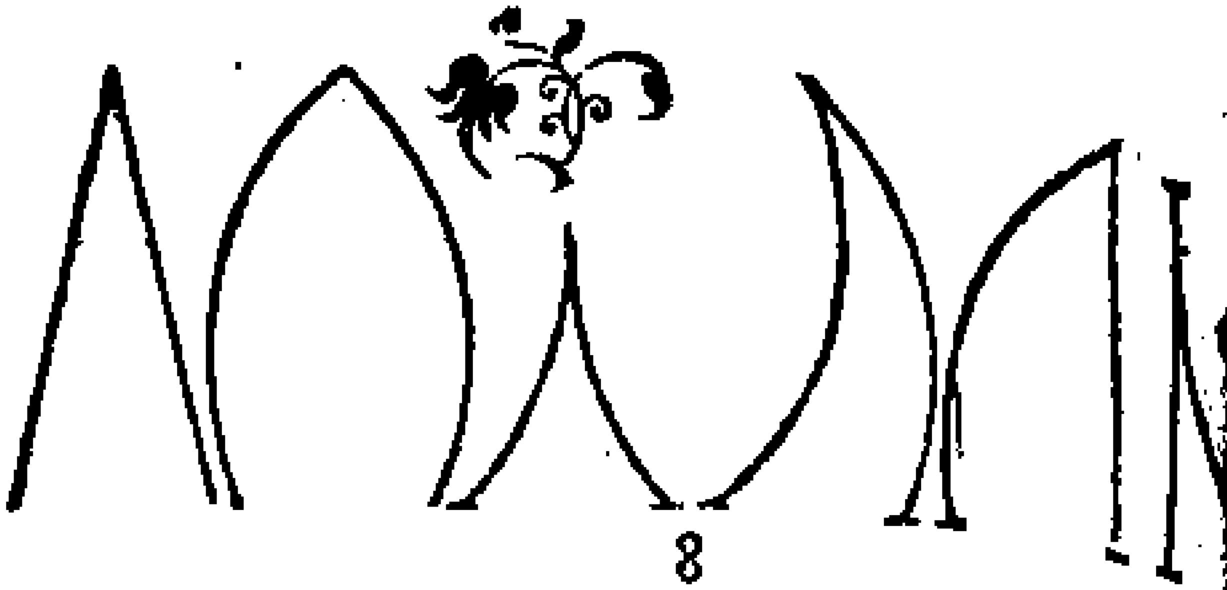
6 Super.

6

Superficiei extrema. sunt lineæ.

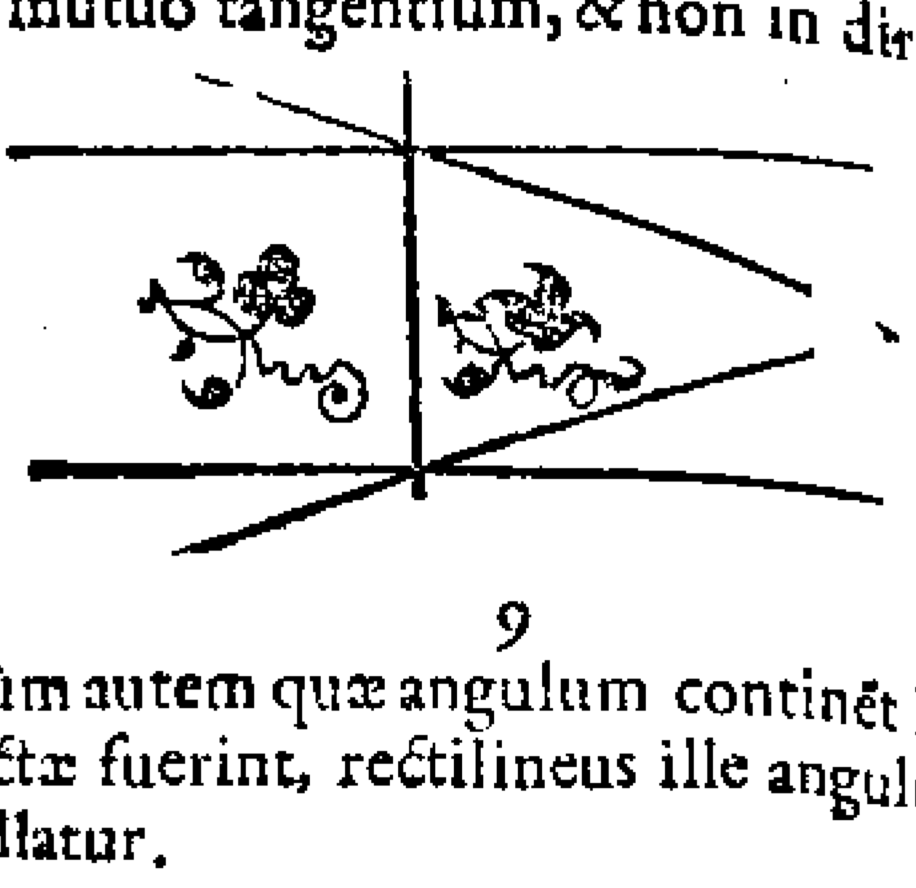
7

Plana superficies est, quæ ex æquo suas interiacet lineas.



8

Planus angulus est duarũ linearũ in plano se mutuò tangentium, & non in directum iacẽtium alterius ad alterã inclinatio.



9

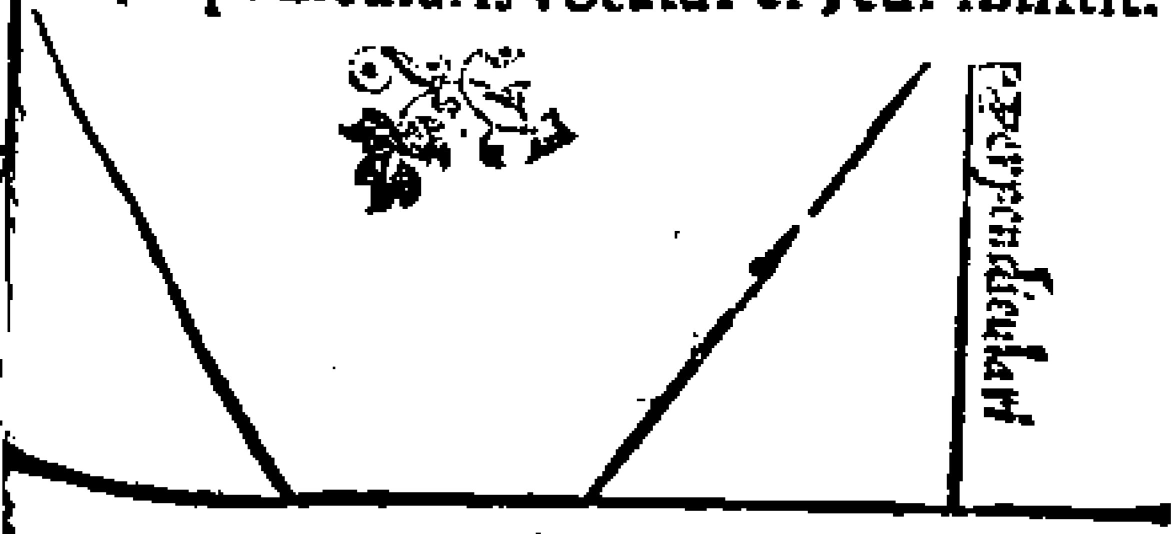
Cùm autem quæ angulum continẽt lineæ, rectæ fuerint, rectilineus ille angulus appellatur.

10

Cùm verò recta linea super rectam consistens lineã, eos qui sunt deinceps angulos æquales, inter se fecerit: rectus est vterq; æ-

qua.

qualium angulorum: quæ insistit recta linea, perpẽdicularis vocatur eiꝰ, cui insistit.



11

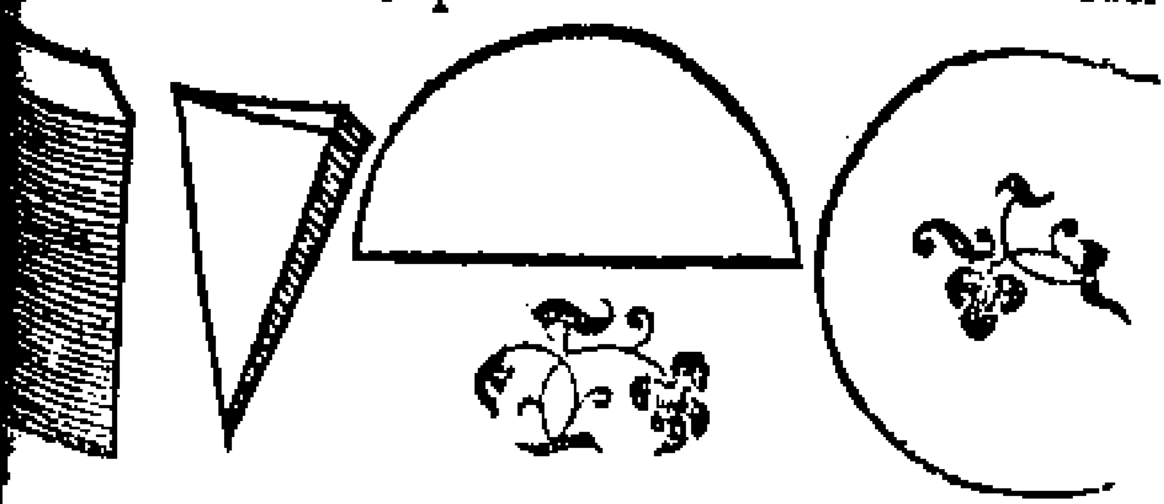
Obtusus angulus est, qui recto maior est.

12

Acutus verò, qui minor est recto.

13

Terminus est, quod alicuius extremũ est.



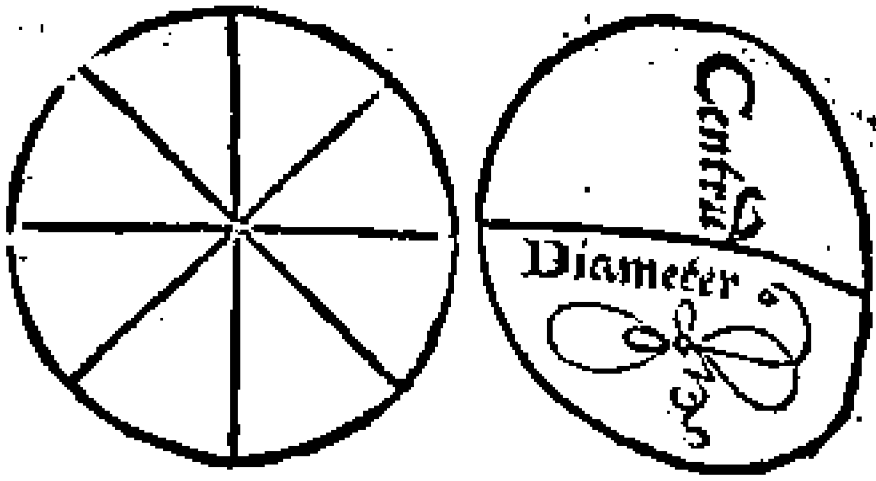
14

Figura est, quæ sub aliquo, vel aliquibus terminis comprehenditur.

15

Circulus est, figura plana sub vna linea cõprehẽsa, quę peripheria appellatur: ad quam ab vno puncto eorum, quæ intra figuram sunt

ſunt poſita, cadentes omnes rectæ lineæ inter ſe ſunt æquales.



16

Hoc verò punctum, centrum circuli appellatur.

17

Diameter autem circuli, eſt recta quædam linea per centrum ducta, & ex vtraque parte in circuli peripheriam terminata, quæ circulum bifariam ſecat.

18

Semicirculus eſt figura, quæ continetur ſub diametro, & ſub ea linea, quæ de circuli peripheria aufertur.

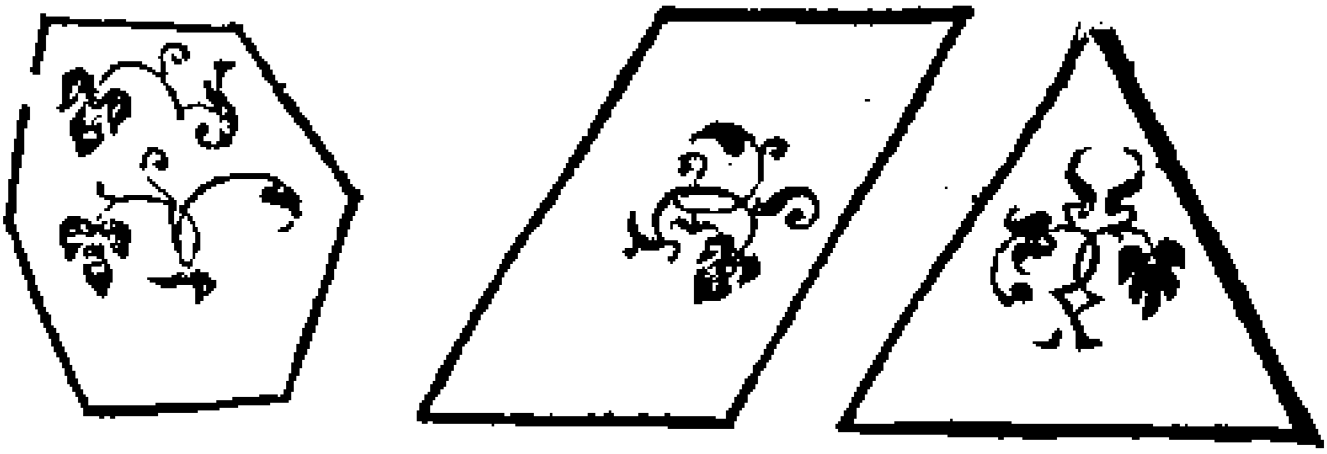
19

Segmentum circuli, eſt figura, quæ ſub recta linea & circuli peripheria continentur.

20 Recti

20.

Rectilineæ figuræ, ſunt quæ ſub rectis lineis continentur.



21

Trilateræ quidem, quæ ſub tribus.

22

Quadrilateræ, quæ ſub quatuor.

23

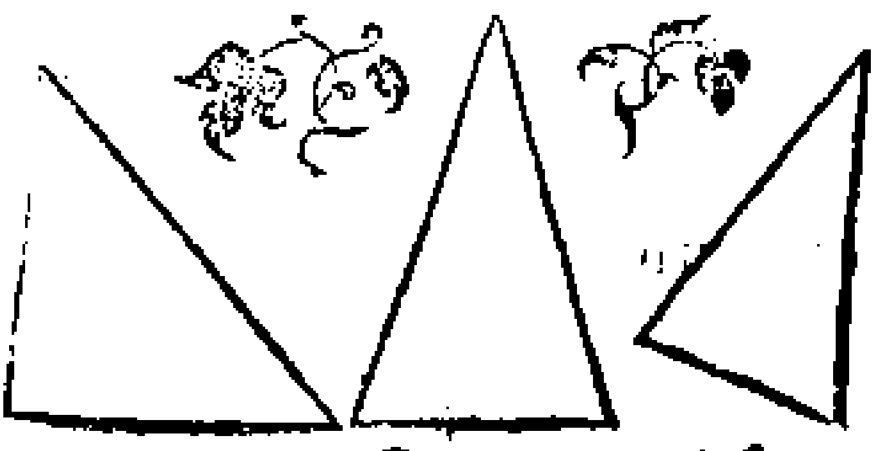
Multilateræ verò, quæ ſub pluribus quàm quatuor rectis lineis comprehenduntur.

24

Trilaterarũ, porrò figurarum, æquilaterum eſt triangulum, quod tria latera habet æqualia.

25

Iſoſceles autem eſt quod duo tantum æqualia habet latera.



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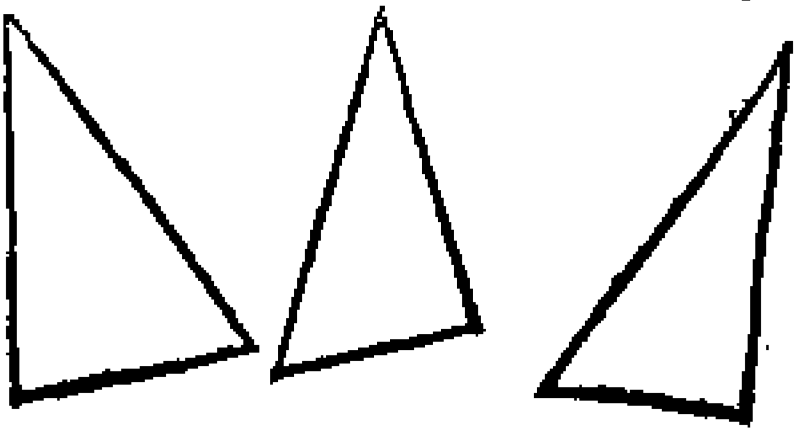
26. Sca-

26

Scalenũ verò, eſt qđ tria inęqualia habet latera.



23

Ad hæc ètiam trilaterarum figurarum, re-ctangulum quidem triangulum eſt, quod rectum angulum habet.

28

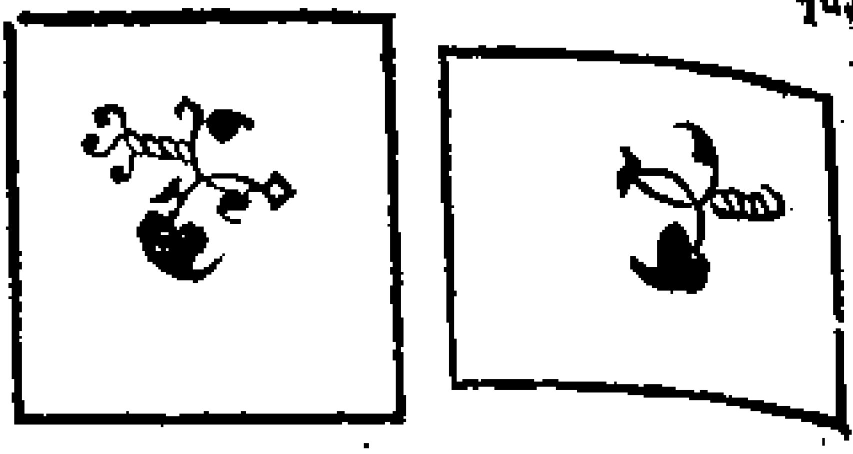
Amblygonium autem, quod obtuſum an-gulum habet.

29

Oxygonium verò, quod tres habet acutos angulos.

30

Quadrilaterarum autem figurarum, qua-dratũ quidẽ eſt qđ & æquilaterũ & re-ctangulum eſt.



31

Altera parte longior figura eſt, quæ recta-gula quidem, at æquilatera non eſt.

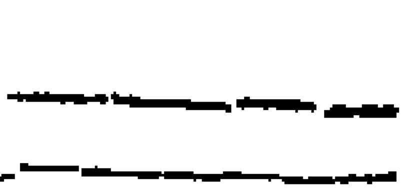
32 Rhom-

32
Rhõbus autem, qui æquilaterum & rectangulum est.

33
Rhomboides verò, quæ aduersa & latera & angulos habens inter se æqualia, neque æquilatera est, neque rectangula.

34
Prӕter has autē reliquæ quadrilateræ figuræ, trapezia appellentur.

35
Parallelæ rectæ lineæ sunt quæ, cùm in eodem sint plano, & ex vtraque parte in infinitum producantur, in neutram sibi mutuò incidunt.



Postulata.

I
Postuletur, vt à quouis puncto in quoduis

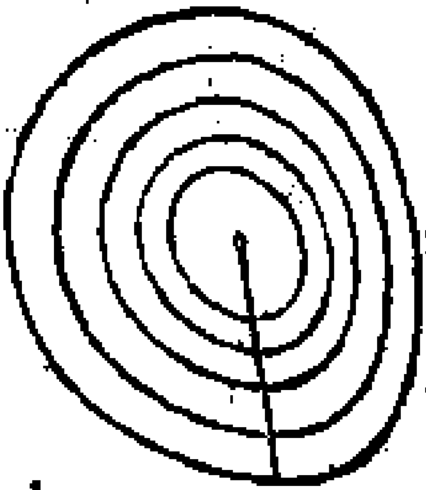
punctum, rectam lineã ducere cócedatur.

2

Et rectam lineam terminatam in continuum recta producere.

3

In quouis centro & interuallo circulũ describere.



Communes notiones.

1.

Quæ eidẽ æqualia, & inter se sunt æqualia.

2

Et si æqualibus æqualia adiecta sint, tota sunt æqualia.

3

Et si ab æqualibus æqualia ablata sint, quæ relinquuntur sunt æqualia.

4

Et si inæqualibus æqualia adiecta sint, tota sunt inæqualia.

5

Et si ab inęqualibus æqualia ablata sint, reliqua sunt inæqualia.

6

Quæ eiusdem duplicia sunt, inter se sunt æqualia.

Et

7

Et quæ eiusdem sunt dimidia, inter se æqualia sunt.

8

Et quæ sibi mutuò congruunt, ea inter se sunt æqualia.

9

Totum est sua parte maius.

10

Item, omnes recti anguli sunt inter se æquales.

11

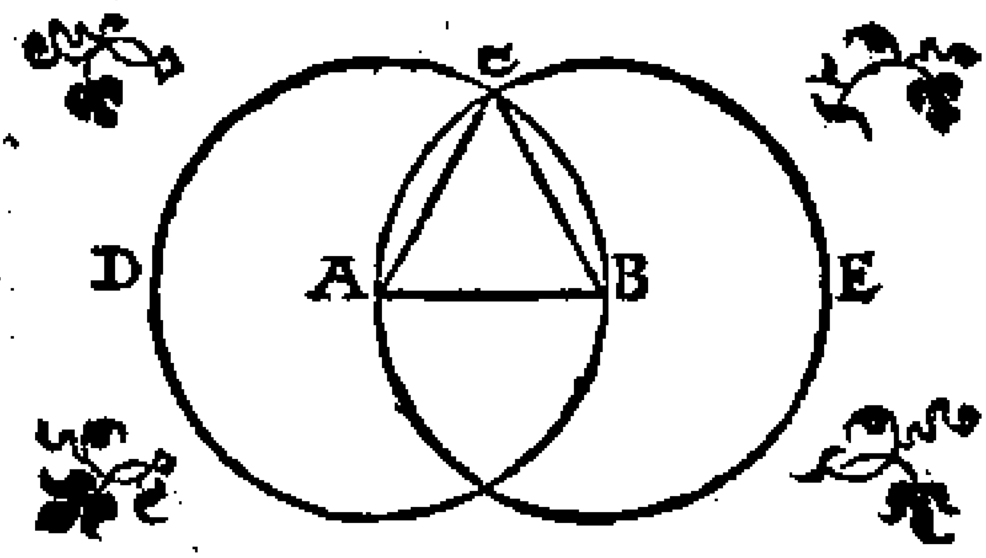
Etsi in duas rectas lineas altera recta incidens, inter nos ad easdéque partes angulos, duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ sibi mutuò incident ad eas partes, vbi sunt anguli duobus rectis mimores.

12

Duæ rectæ lineæ spatium non comprehendunt.

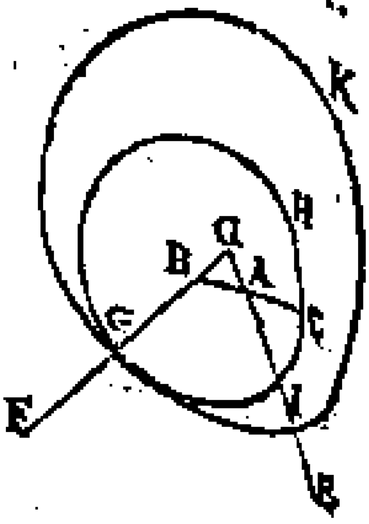
Problema 1. Propositio 1.

uper ta cta nea rmi ata, riang ulū



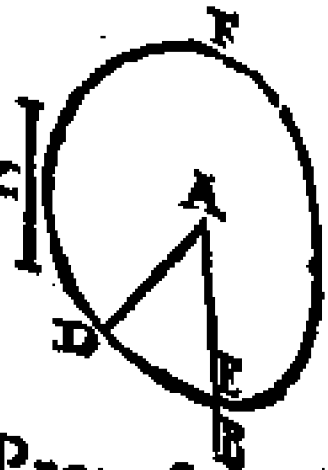
uilaterum constituere.

Problema 2. Propositio 2.



Ad datum punctum, datæ rectæ lineæ, ęqualem rectam lineam ponere.

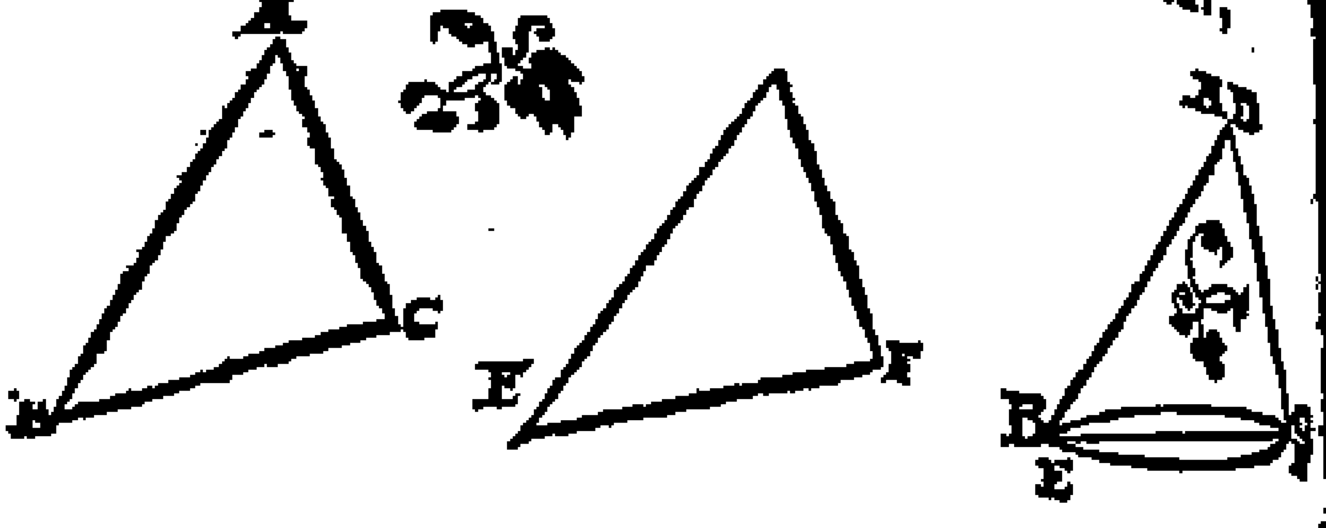
Problema 3. Propositio 3.



Duabus datis rectis lineis inęqualibus, de maiore æqualẽ minori rectam lineã detrahere.

Theorema primum. Propositio 4.

Si duo triangula duo latera duobus lateribus æqualia habeant, vtrunque vtriq; habeant verò & angulum angulo æqualẽ sub æqualibus rectis lineis contentum: & basi basi æqualem habebunt, eritq; triãgulum triãgulo æquale, ac reliqui anguli reliquis angulis æquales erũt, vterque vtrique, sub quibus æqualia latera subtenduntur.



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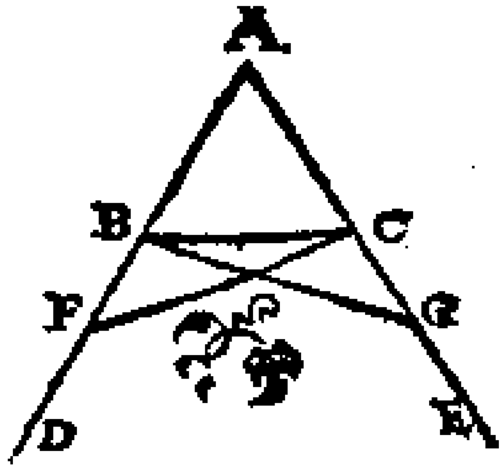
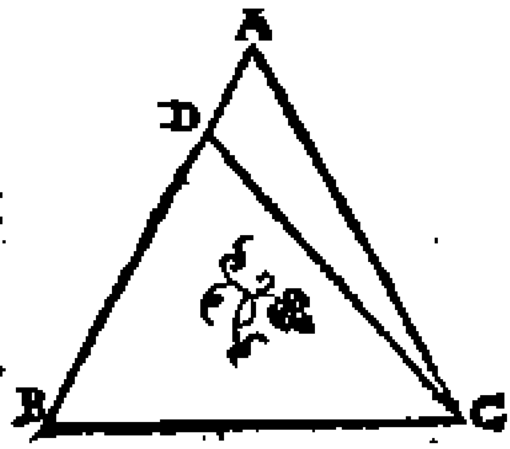
Theorema 2. Propositio 5.



Isoscelium tringulorũ qui ab basim sunt anguli, inter se sunt æquales; & si vlterius productæ sint æquales illę rectæ lineæ, qui sub basi sunt anguli, inter se equales erunt.

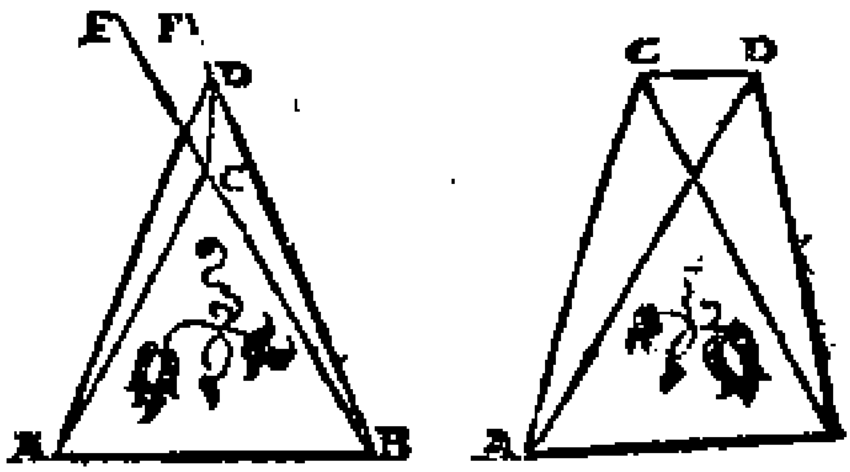
Problema 3. Propositio 6.



Si trianguli duo anguli æquales inter se fuerint & sub ęqualibus angulis subtensa latera æqualia inter se erunt.

Theorema 46. Propositio 7.

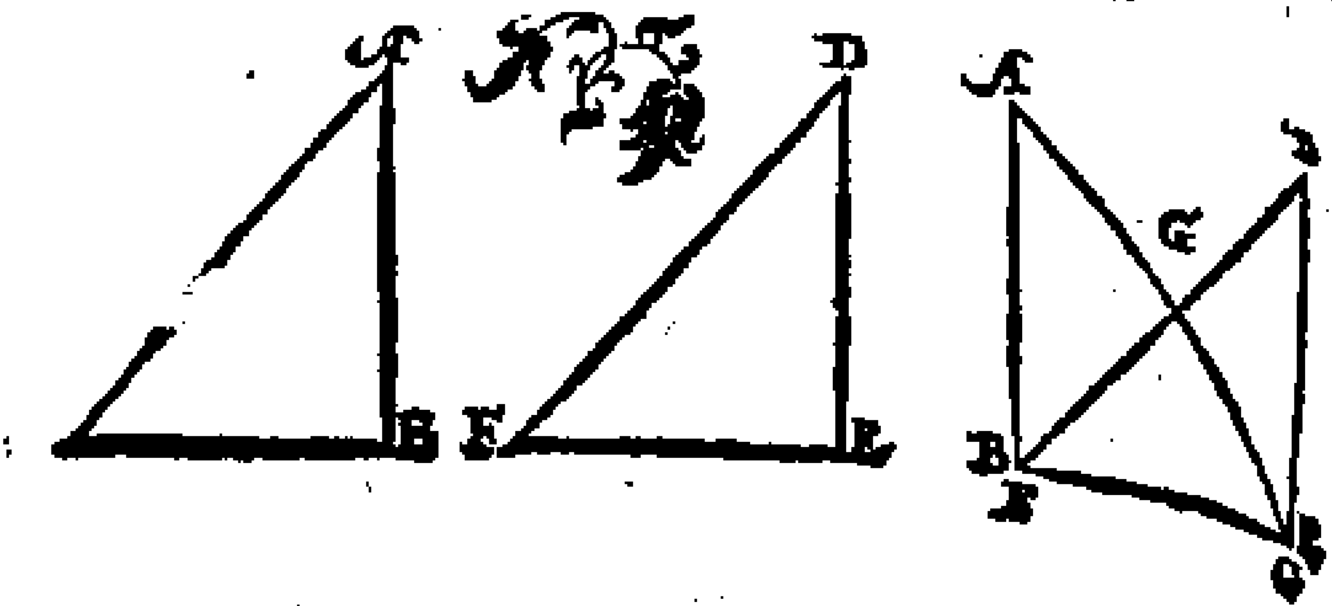
Super eadẽ recta linea, duabus eisdem rectis lineis aliȩ duȩ rectȩ lineæ ęquales, vtraque vtrique nõ constituentur, ad aliud atque aliud punctũ, ad easdẽ partes eosdẽ



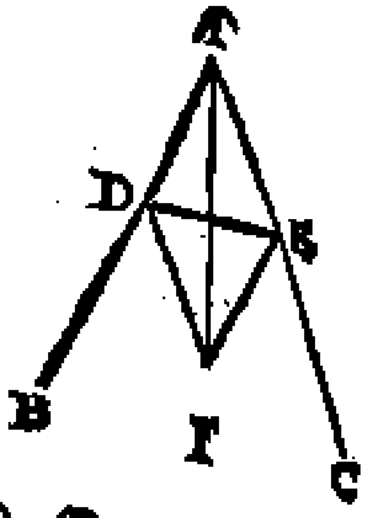
quę terminos cum duabus initio ductis rectis lineis habentes.

Theorema 5. Propo-
ſitio 8.

Si duo triágula duo latera habuerint duobus lateribus, vtrunque vtriq; æqualia: habuerint verò & baſim baſi æqualē: angulū quoque ſub æqualibus rectis lineis cõtētum angulo æqualem habebunt.



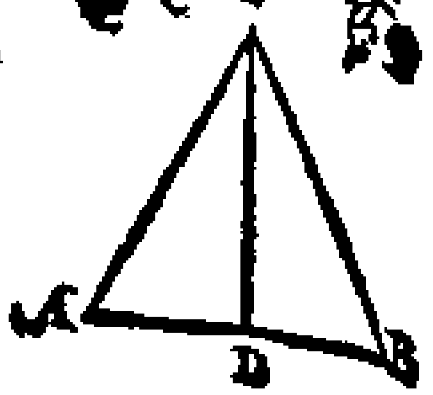
Problema 4. Popo-
poſitio 9.



Datum angulum rectilineum bifariam ſecare.

Problema 5. Pro-
poſitio 10.

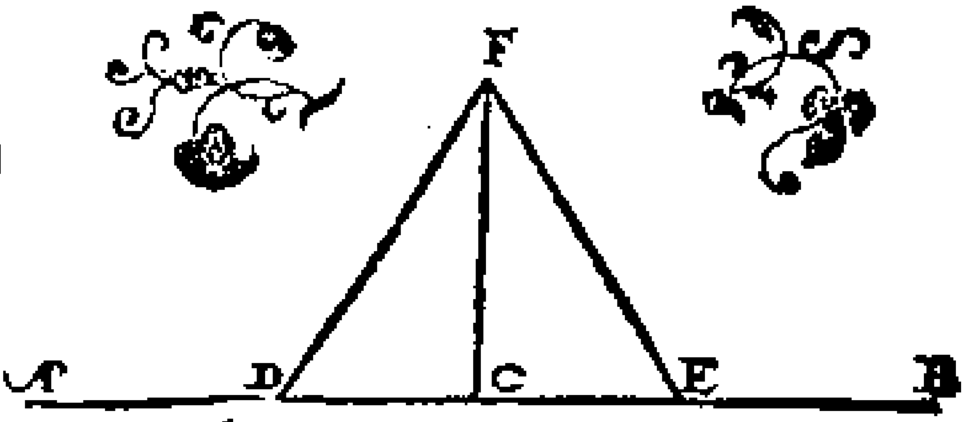
Datam rectam lineā finitam bifariam ſecare.



Proble.

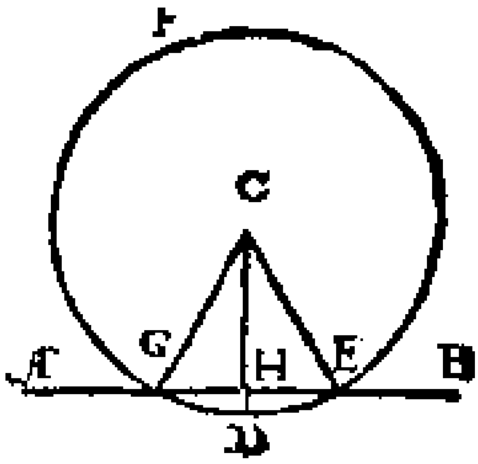
Problema 6. Propositio 11.

Data recta linea, à puncto in ea dato, re-



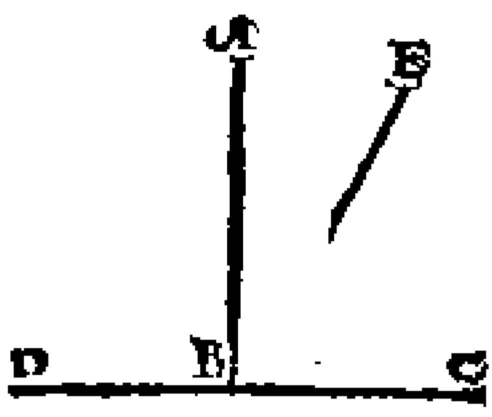
ctam lineam ad angulos rectos excitare.

Problema 7. Propositio 12.



Super datam rectã lineã infinitam, à dato puncto quod in ea non est, perpendicularem rectam deducere.

Theorema 6. Propositio 13.



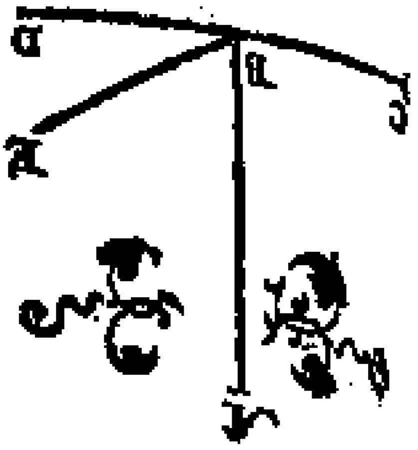
Cùm recta linea super rectã consistens lineã angulos facit, aut duos rectos, aut duobus rectis æquales efficiet.

Theorema 7. Propositio 14.

Si ad aliquam rectam lineam, atque ad eius

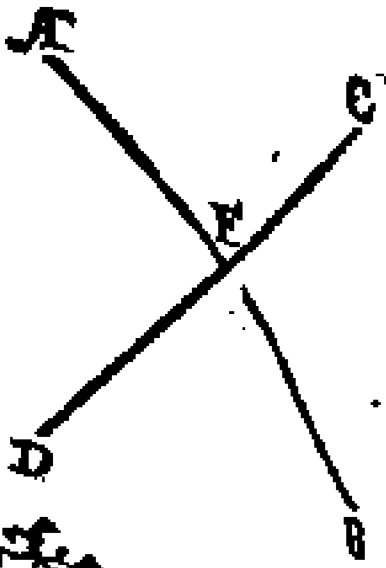
 punctum

punctú, duę rectęlineæ nó ad eaſdem partes ductæ, eos qui ſunt deinceps angulos duobus rectis æquales fecerint, in directum erunt inter ſe ipſæ rectæ lineæ.



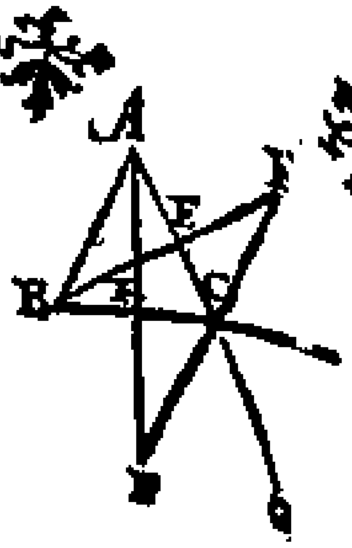
Theorema 8. Propoſitio 15.

Si duę rectę lineę ſe mutuò ſecuerint, angulos qui ad verticem ſunt, æquales inter ſe efficient.



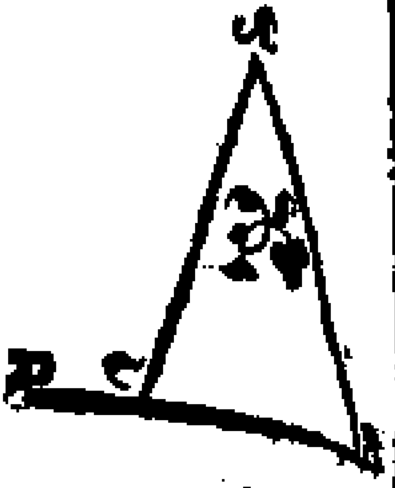
Theorema 9. Propoſitio 16.

Cuiuſcunque trianguli vno latere producto, externus angulus vtroque interno & oppoſito maior eſt.



Theorema 10. Propoſitio 17.

Cuiuſcunque trianguli duo auguli duobus rectis ſunt minores omnifariam ſumpti.



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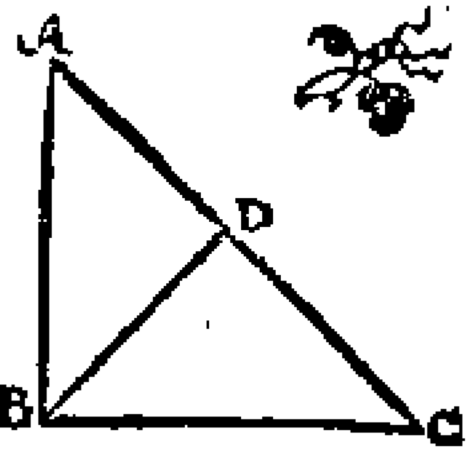
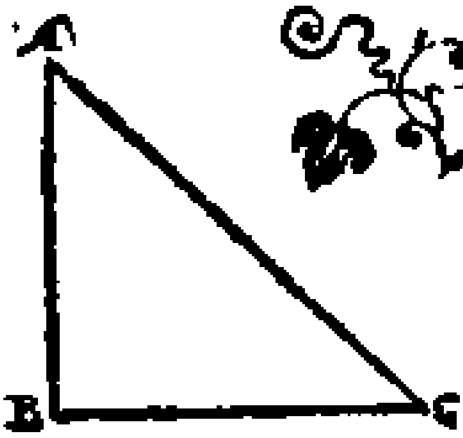
Theorema 11. Propoſitio 18.



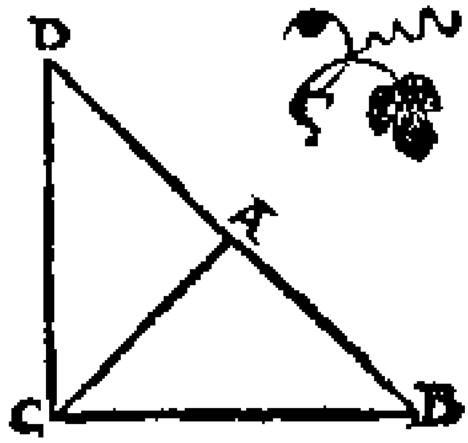
Omnis trianguli maius latus maiorem angulũ ſubtendit.

Theorema 12. Propoſitio 16.



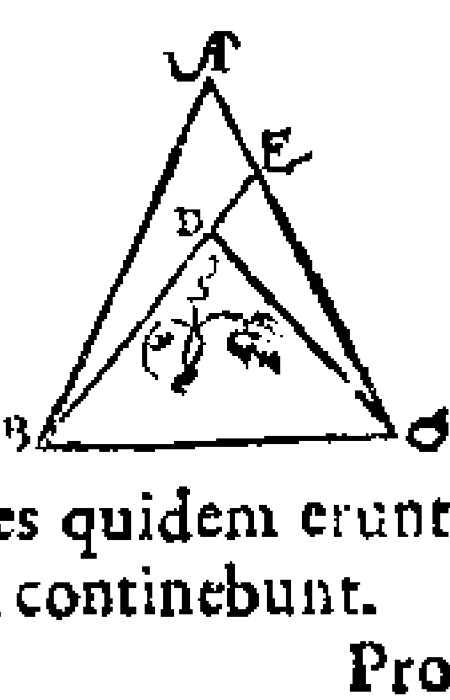
Omnis trianguli lateri angulus, maiori lateri ſubtenditur.

Theorema 13. Propoſitio 20.



Omnis triãguli duo latera reliquo ſunt maiora, quomodocumq; aſſumpta.

Theorema 14. Propoſitio 12.

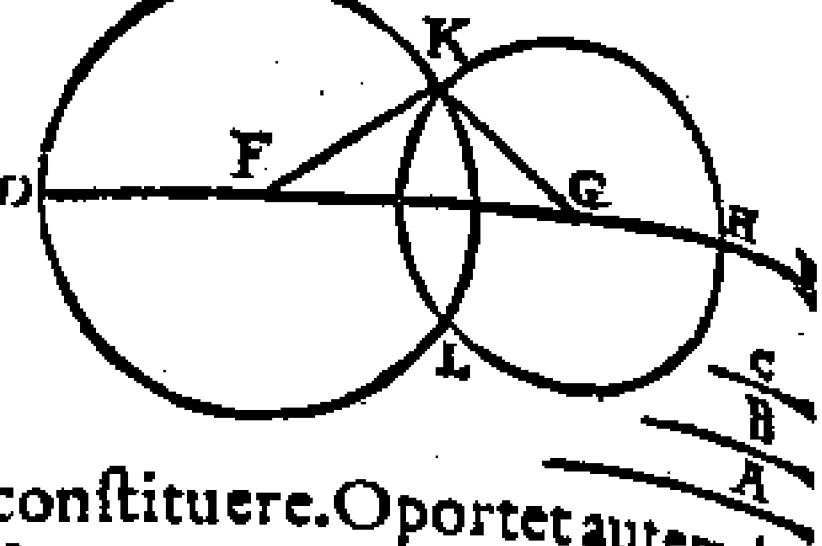


Si ſuper triãguli vno latere, ab extremitatibus duæ rectę lineę, interius cõſtitutæ fuerint, hę cõſtitutæ reliquis triãguli duobus lateribus minores quidem erunt, minorem verò angulum continebunt.

Pro-

Problema 8. Propositio 22.

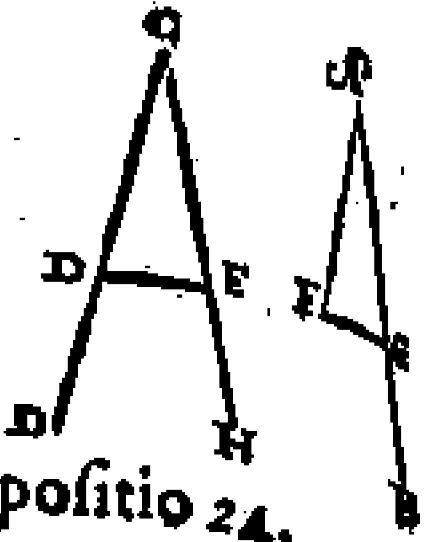
Ex tribus rectis lineis quæ sũt tribus datis rectis lineis æquales,



triangulũ constituere. Oportet autem duas reliqua esse maiores omnifariam sumptas: quoniá vniuscuiusq; trianguli duo latera omnifariam sumpta reliquo sunt maiora.

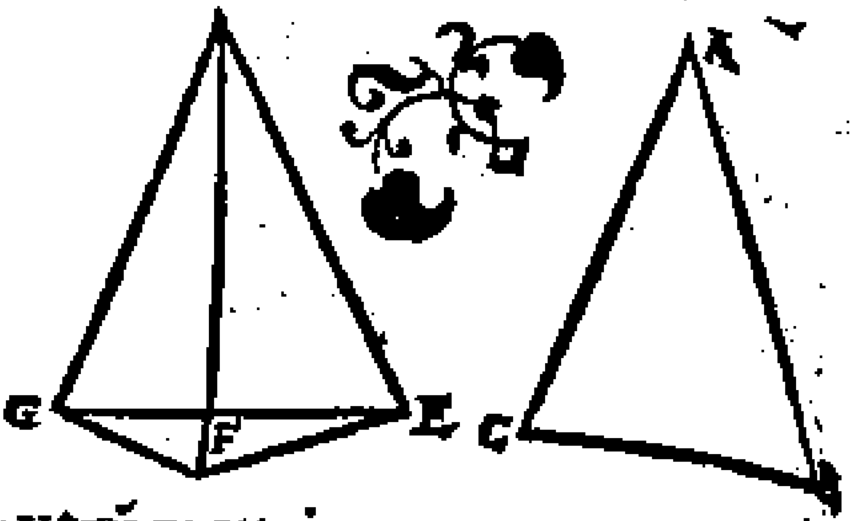
Problema 9. Propositio 23.

Ad datam rectam lineã datumq; in ea punctum dato angulo rectilineo æqualem angulum rectilineum constituere.



Theorema 15. Propositio 24.

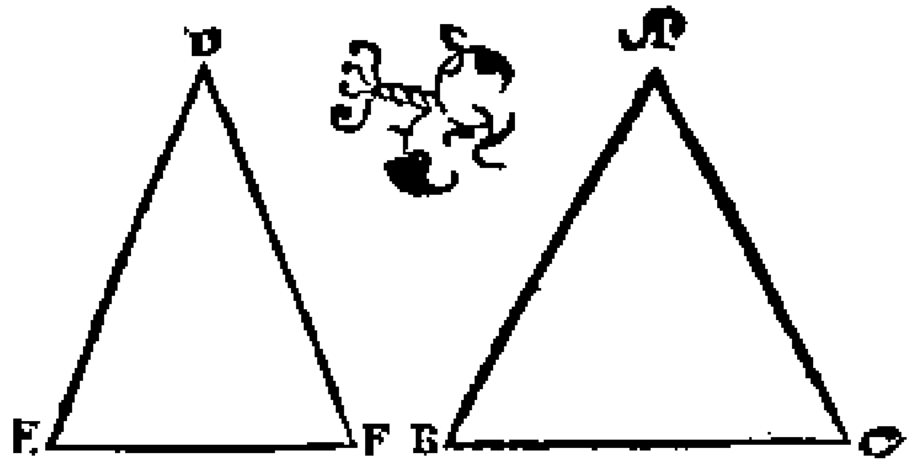
Si duo triãgula duo latera duobus lateribus æqualia



habuerint, vtrũq; vtriq;, angulũ verò angulo lo maiorem ſub æqualibus rectis lineis cõtentum: & baſin baſi maiorem habebunt.

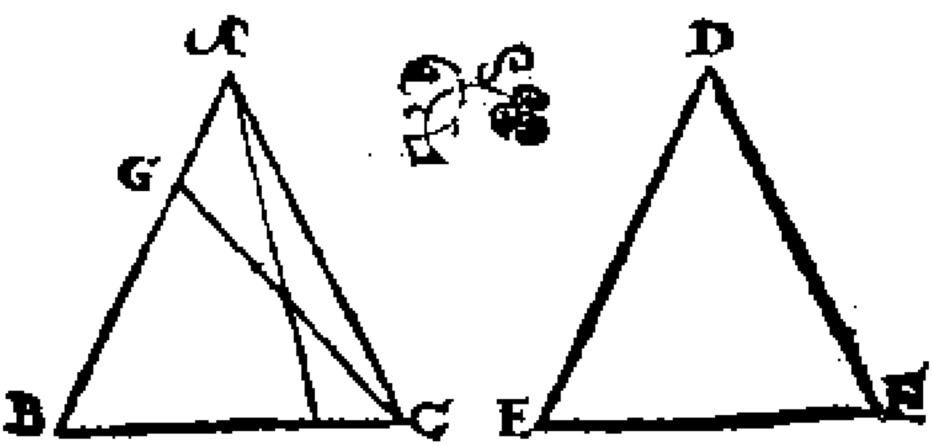
Theorema 16. Propoſitio 25.

Si duo triangula duo latera duobus lateribus æqualia habuerint, vtrunque vtrique, baſin verò baſi maiorē: & angulum ſub æqualib⁹ rectis lineis contentum angulo maiorē habebũt.



Theorema 17. Propoſitio 26.

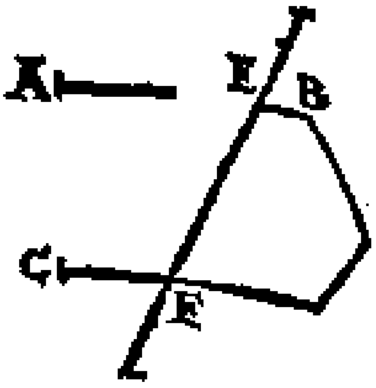
Si duo triangla duos angulos duobus angulis æquales habuerint, vtrũque vtrique, vnumq́; latus vni lateri æquale, ſiue, quod æqualibus adiacet angulis, ſeu quod vni æqualium angulorũ ſubtenditur: & reliqua latera reliquis lateribɂ ęqualia vtrũq; vtriq;, & reliquum angulũ reliquo angulo æqualem habebunt.



Theore-

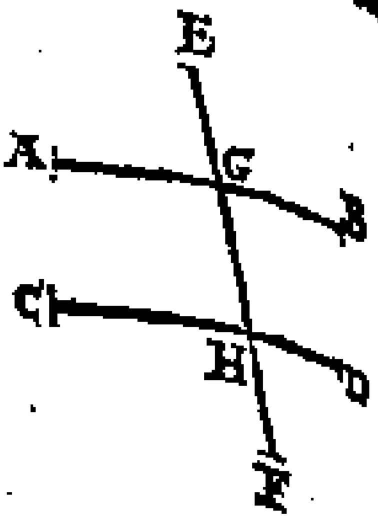
Theorema 18. Propositio 27.

Si in duas rectas lineas recta incidés linea alternatim águlos ęquales inter se fecerit: parallelę erunt inter se illæ rectæ lineæ.



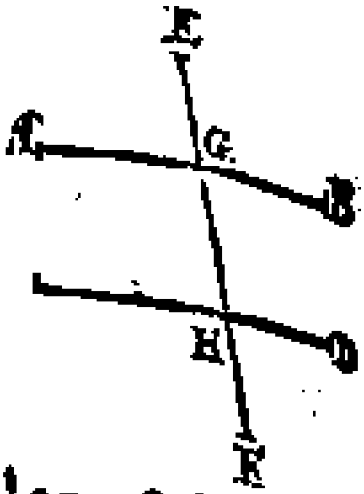
Theorema 19. Propositio 28.

Sí in duas rectas lineas recta incidens linea externum angulú interno, & opposito, & ad easdem partes æqualé fecerit, aut internos & ad easdé partes duobus rectis æquales: parellelæ erunt inter se ipsæ rectæ lineæ.



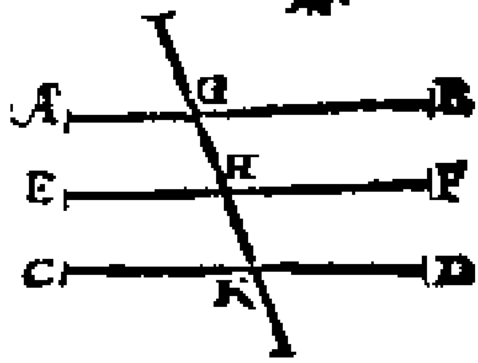
Theorema 20. Propositio 29.

In parallelas rectas lineas recta incidens linea: & alternatim angulos inter se æquales efficit & externum interno & opposito & ad easdem partes æqualem, & inter nos & ad easdem partes duobus rectis æquales facit.



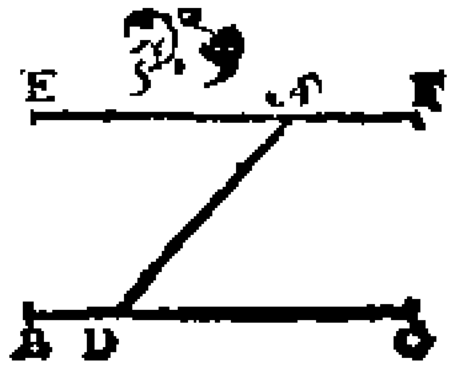
Theore.

Theorema 21. Propositio 30.



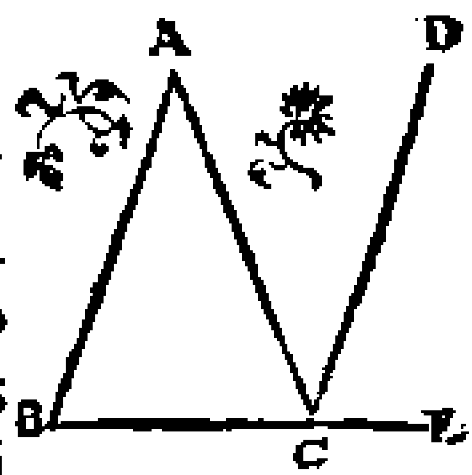
Quæ eidem rectæ lineę, parallelę, & inter se sunt parallelæ.

Problema 10. Propositio 31.



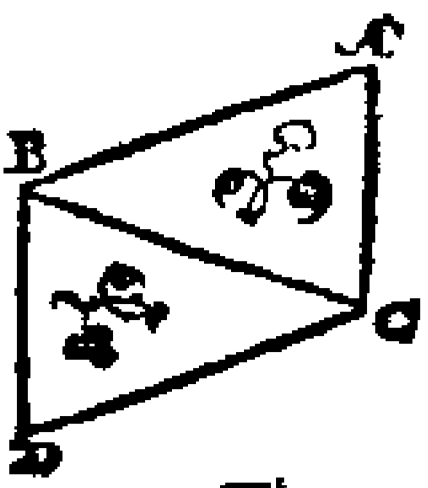
A dato puncto datæ rectæ lineæ parallelam rectam lineam ducere.

Theorema 22. Propositio. 32.



Cuiuscunque trianguli vno latere vlterius producto: externus angulus duobus internis & oppositis est æqualis. Et trianguli tres interni anguli duobus sunt rectis æquales.

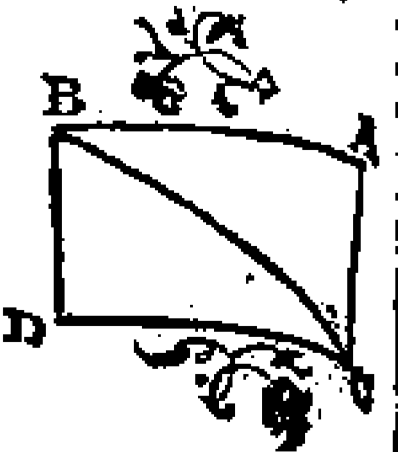
Theorema 23. Propositio 33.



Rectæ lineę quę equales & parallelas lineas ad partes easdem coniungunt, & ipsæ æquales & parallelæ sunt.

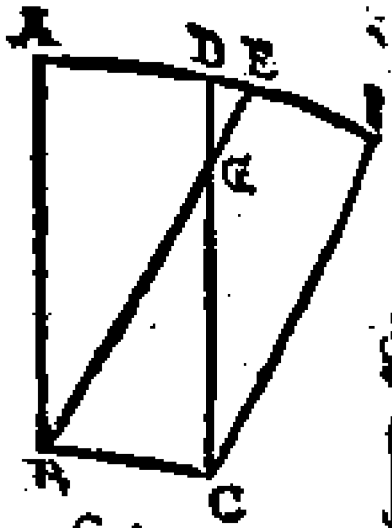
Theore-

Theorema 24. Propositio 34.



Parallelogrammorũ spatiorum æqualia sunt inter se, quæ ex aduerso & latera & anguli: atque illa bifariã secat diameter.

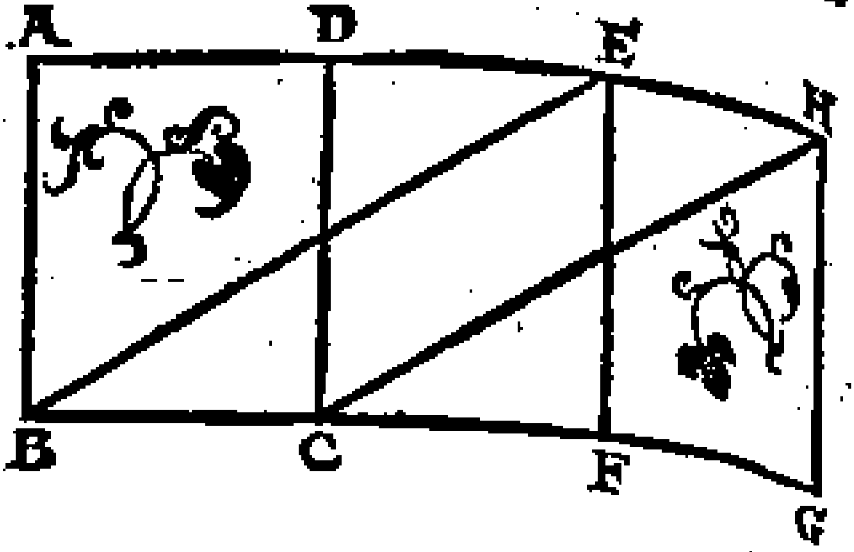
Theorema 25. Propositio 35.



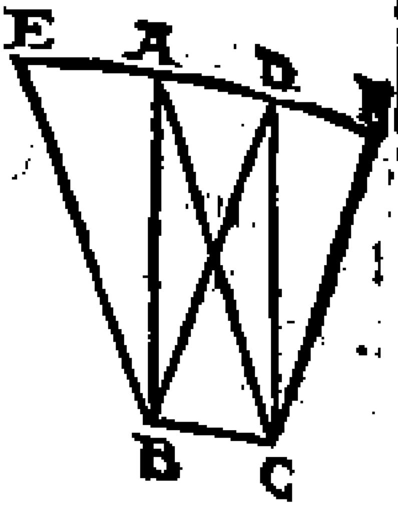
Parallelogramma super eadem basi, & in eisdem parallelis cõstituta, inter se sunt æqualia.

Theorema 26. Propositio 36.

Parallelográma super æqualibus basibus & in eisdé parallelis cõstituta inter se sunt æqualia.



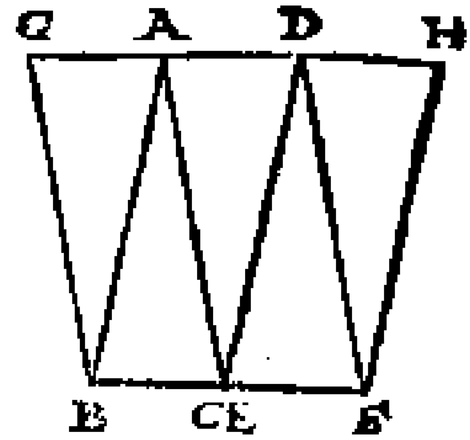
Theorema 27. Propositio 37.



Triãgula super eadé basi cõstituta, & in eisdé parallelis, inter se sunt ęqualia.

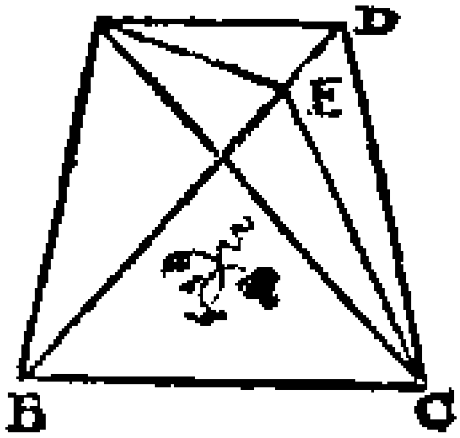
Theore.

Theorema 28. Propositio 38.



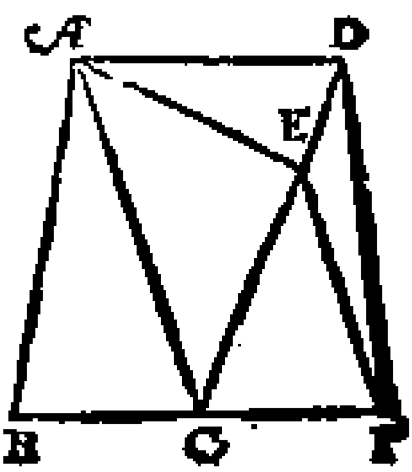
Triangula ſuper ęqualibus baſibus conſtituta et in eiſdem parallelis, inter ſe ſunt æqualia.

Theorema 29. Propoſitio 39.



Triangula æqualia ſuper eadem baſi, & ad eaſdem partes conſtituta: & in eiſdem ſunt Parallelis.

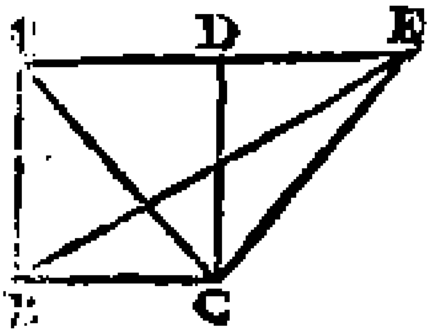
Theorema 30. Propoſitio 40.



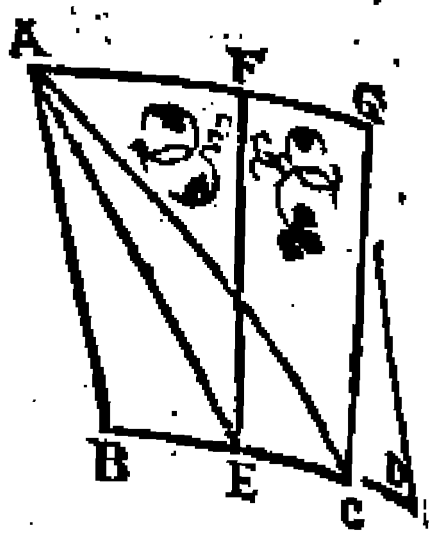
Triangula æqualia ſuper æqualibus baſibus, & ad eaſdem partes conſtituta, & in eiſdem ſunt parallelis.

Theorema 31. Propoſitio 41.

Parallelogrammum cum triangulo eandem baſin habueris, non eidemque fuerit parallelis, duplum erit parallelogrammum ipſius trianguli.

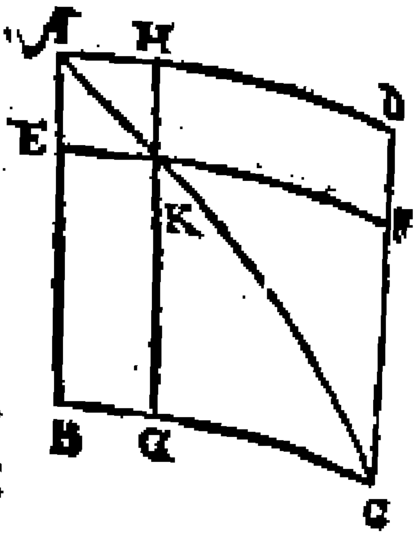


Problema 11. Propositio 42.



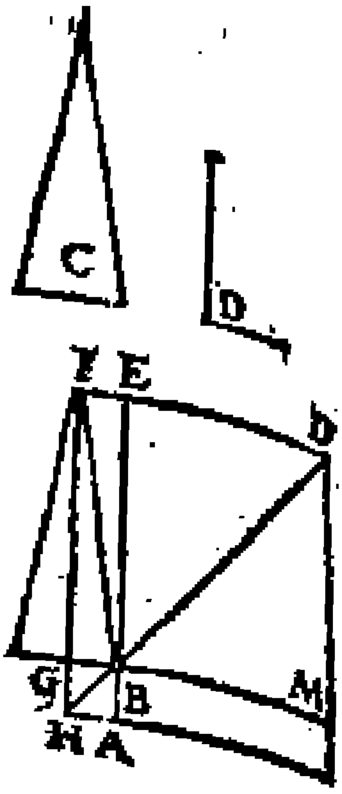
Dato triangulo æquale parallelogrammũ constiture in dato angulo rectilineo.

Theorema 32. Propositio 43.



In omni parallelogrammo, complementa eorũ quę circa diametrũ sunt parallelogrammorũ, inter se sunt æqualia.

Problema 12. Propositio 44.

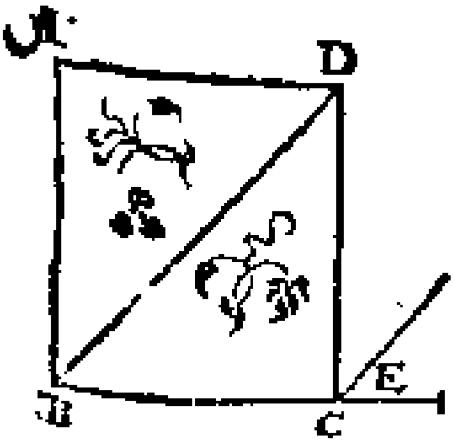


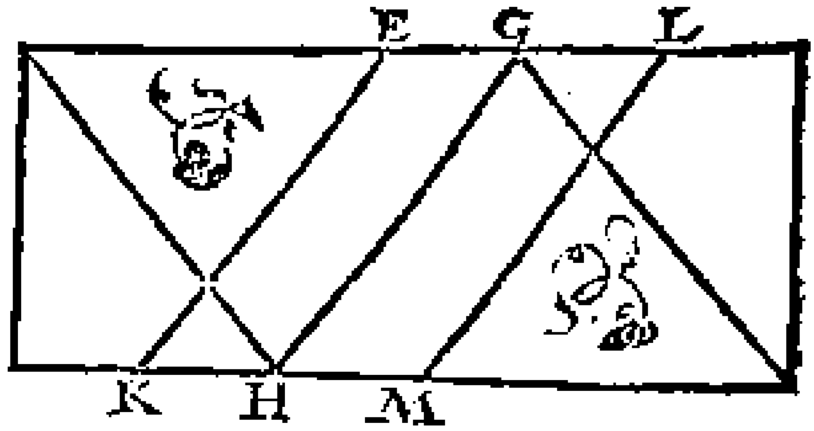
Ad datam rectam lineã dato triangolo æquale parallelogrammum applicare in dato angulo rectilineo.

Problema 13. Propopotio 45.

Dato rectilineo ęquale parallelongrammũ consti-

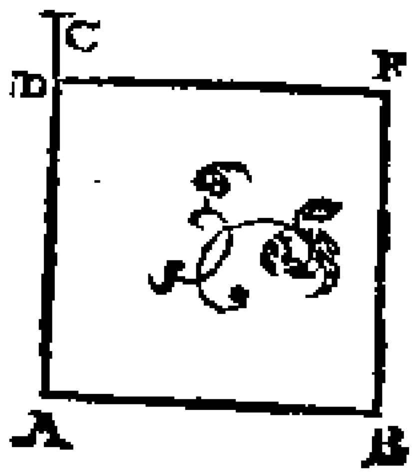
constituere in dato angulo rectilineo.





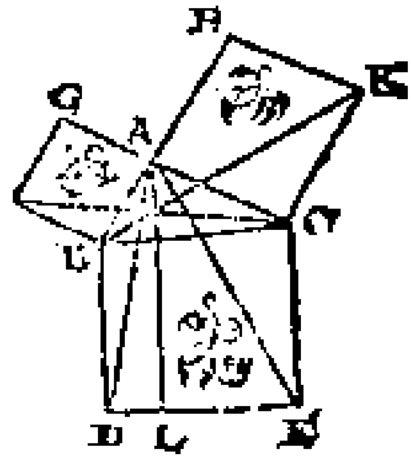
Theorema 41. Propositio 4.

A data recta linea quadratum describere.



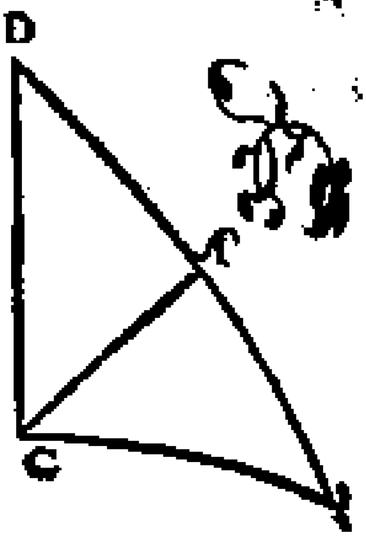
Theorema 33. Propositio 47.

In rectangulis triangulis, quadratum quod à latere rectum angulum subtendente describitur, æquale est eis, quæ à lateribus rectum angulum continentibus describũtur, quadratis.



Theorema 34. Propositio 48.

Si quadratum quod ab vno laterum trianguli describitur, æquale sit eis quæ à reliquis trianguli lateribus describuntur, quadratis: angulus comprehensus sub reliquis duobus trianguli lateribus, rectus est.



FINIS ELEMENTI I.

EVCLIDIS ELEMENTVM SECVNDVM.

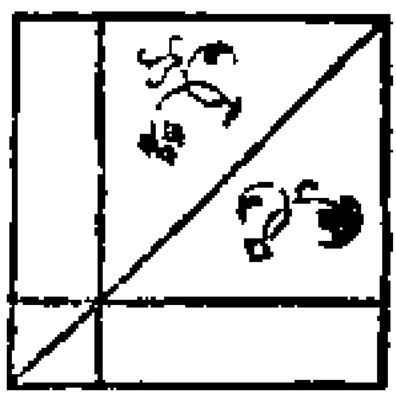
DEFINITIONES.

1.

OMne parallelogrammum rectangulum contineri dicitur sub rectis duabus lineis, quę rectum comprehendunt angulum.

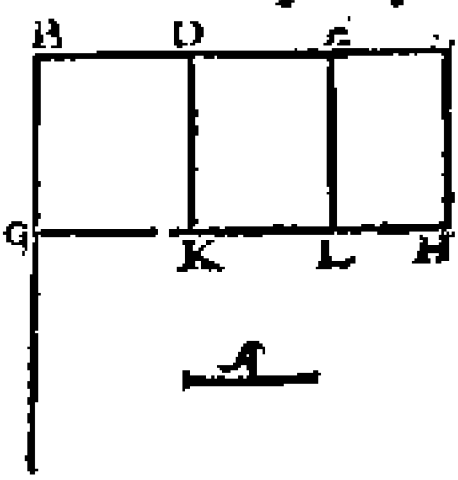
2.

In omni parallelogrammo spatio, vnumquodlibet eorum, quæ circa diametrum illius sunt parallelogrammorum, cum duobus cõplementis, Gnomo vocetur.



Theorema 1. Propositio 1.

Si fuerint duæ rectæ lineæ, seceturque ipsarum altera in quotcunq; segmenta: rectangulum comprehensum sub illis duabus rectis lineis, æquale est eis rectangulis, quæ sub insecta & quolibet segmẽtorum comprehenduntur.

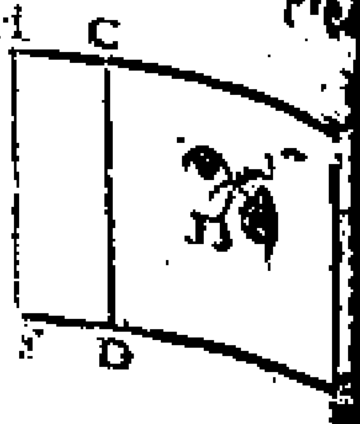


Theorema 2. Propoſitio 2.

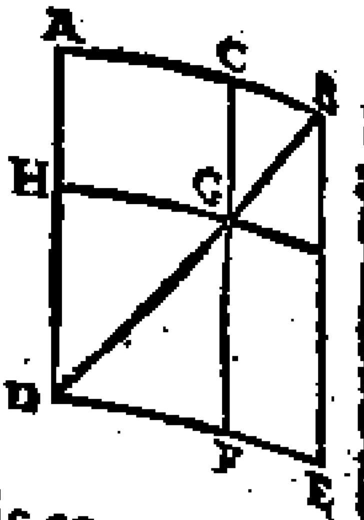
Si recta linea ſecta ſit vtcunq;, rectágula quæ ſub tota & quolibet ſegmentorum cóprehenduntur æqualia ſunt ei, quod à tota fit, quadrato.

Theorema 3. Propoſitio 3.

Si recta linea ſecta ſit vtcunque, rectangulum ſub tota & vno ſegmẽtorum comprehenſum, ęquale eſt & illi quod ſub ſegmentis cóprehenditur rectangulo & illi, quod à prædicto ſegmẽto deſcribitur, quadrato.



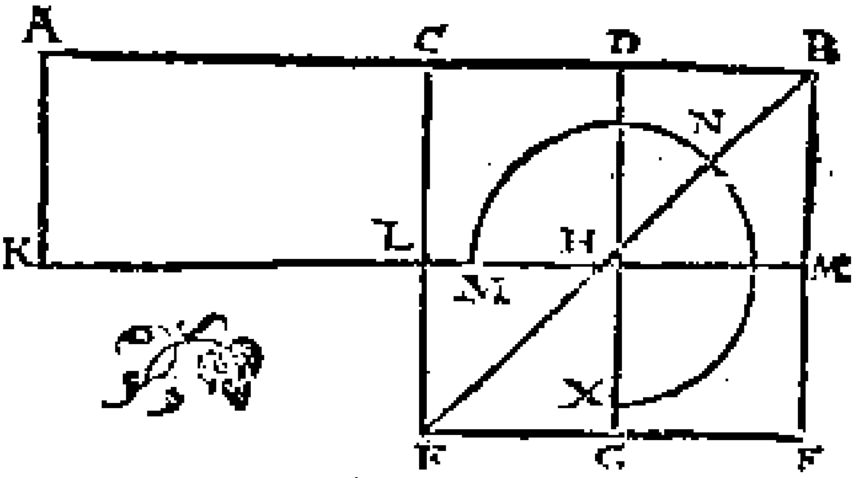
Theorema 4. Propoſitio 4.



Si recta linea ſecta ſit vtcunque: quadratũ quod à tota deſcribitur, ęquale eſt & illis quæ à ſegmẽtis deſcribũtur quadratis, & ei quod bis ſub ſegmentis comprehéditur rectangulo.

Theorema 5. Propoſitio 5.

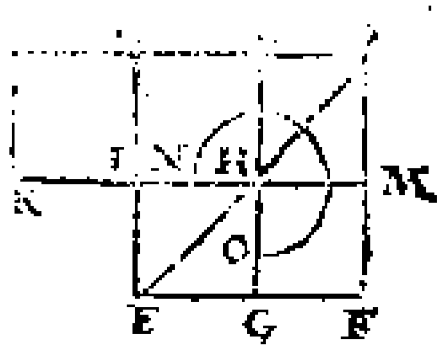
Si recta lenea ſecetur in æqualia & non æqualia: rectangulum ſub inæqualibus ſegmentis



mentis to tius comprehēſū, vnà cum quadrato quod ab inter media ſectionum, æquale eſt ei quod à dimidia deſcribitur, quadrato.

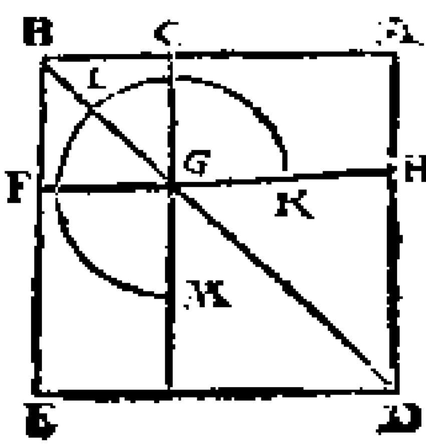
Theorema 6. Propoſitio 6.

Si recta linea bifariam ſecetur, & illi recta quædam linea in rectum adijciatur, rectágulum comprehenſum ſub tota cum adiecta & adiecta ſimul cum quadrato à dimidia, æquale eſt quadrato à linea quæ tū ex dimidia, tum ex adiecta componitur, tanquam ab vna deſcripto.



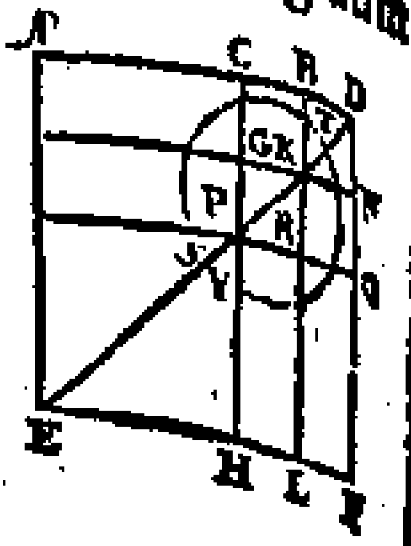
Theorema 7. Propoſitio 7.

Si recta linea ſecetur vt cunque, quod à tota, quodque ab vno ſegmentorum, vtraq; ſimul quadrata, æqualia ſunt & illi quod bis ſub tota & dicto ſegemento comprehenditur, rectangulo, & illi ꝙ à reliquo ſegmento fit, quadrato.

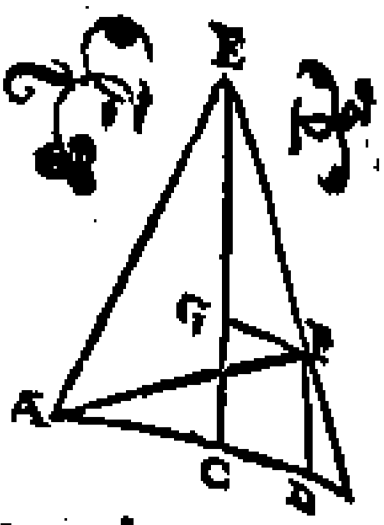


Theorema 8. Propositio 8.

Si recta linea secetur vtcunq; rectangulum quater comprehensum sub tota & vno segmentorum, cum eo quod à reliquo segméto fit, quadrato, ęquale est ei quod à tota & dicto segmento, tanquá ab vna linea describitur quadrato.



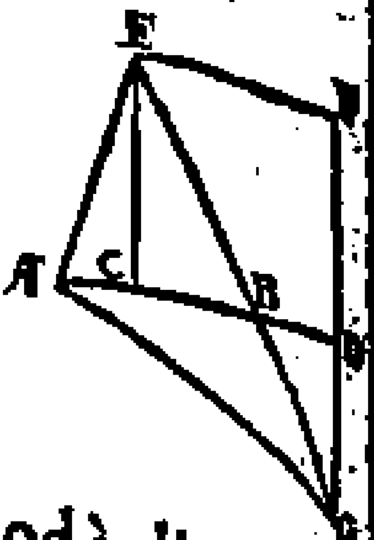
Theorema 9. Propositio 9.



Si recta linea secetur in æqualia & non æqualia: quadrata quæ ab inęqualibus totius segmétis fiunt, duplicia sunt & eius quod à dimidia, & eius quod ab intermedia sectionum fit, quadratorum.

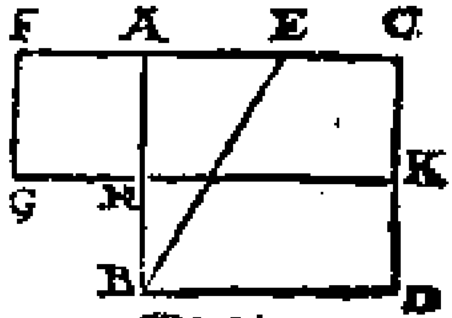
Theorema 10. Propositio 10.

Si recta linea secetur bifariam, adijciatur autem ei in rectum quępiam recta linea: quod à tota cũ adiuncta, & quod ab adiuncta, vtraq; simul quadrata, duplicia sunt, & eius quod à dimidia, & eius quod à composita ex dimidia & adiuncta.



iuncta, tanquã ab vna descriptum sit quadratorum.

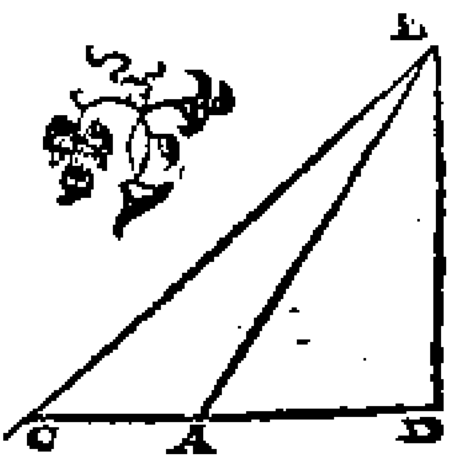
Problema 1. Propositio 11.



Datam rectam lineã secare, vt comprehensum sub tota & altero segmẽtorum rectangulum, æquale sit ei quod à reliquo segmento fit, quadrato.

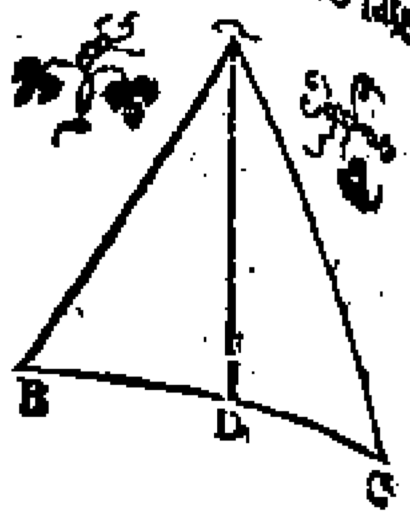
Theorema 11. Propositio 12.

In amblygonijs triangulis, quadratũ quod fit à latere angulum obtusum subtendẽte, maius est quadratis, quæ fiunt à lateribus obtusum angulum comprehendentibus, pro quantitate rectanguli bis comprehensi & ab vno laterũ quę sunt circa obtusum angulũ, in quod cum protractum fuerit, cadit perpendicularis, & ab assumpta exterius linea sub perpendiculari prope angulũ obtusum.



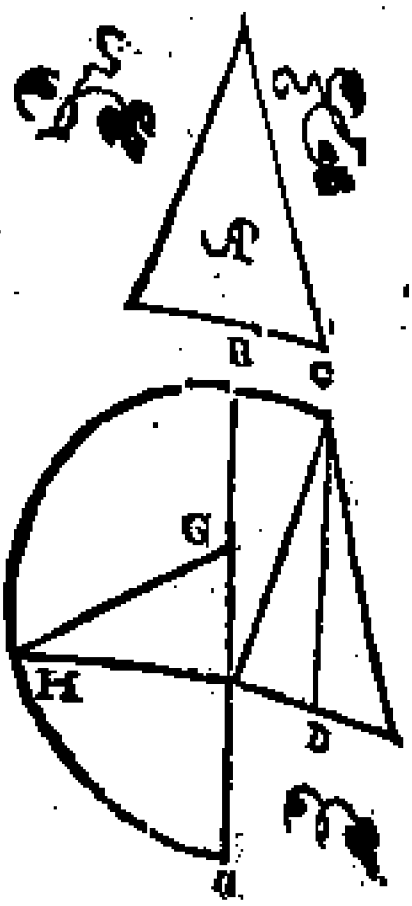
Theorema 12. Propoſitio 28.

In oxygonijs triangulis quadratũ à latere angulum acutum ſubtendente, minus eſſe quadratis quæ fiunt à lateribus acutũ angulum cóprehendentibus, pro quantitate rectanguli bis comprehenſi, & ab vno laterum, quæ ſunt circa acutum angulum, in quod perpẽdicularis cadit, & ab aſſumpta interius linea ſub perpendiculari prope acutũ angulum.



Problema 2. Propoſitio 14.

Dato rectilineo æquale quadratũ conſtituere.



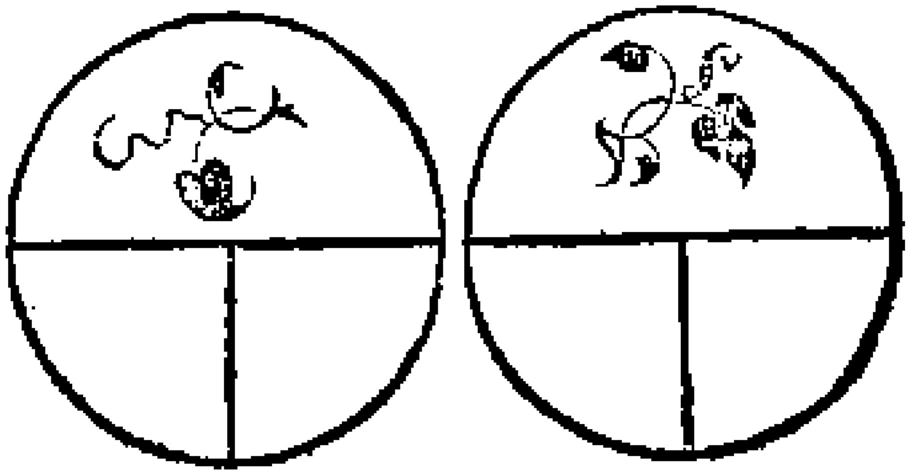
ELEMENTI II. FINIS.

EVCLIDIS ELEMENTVM TERTIVM.

DEFINITIONES.

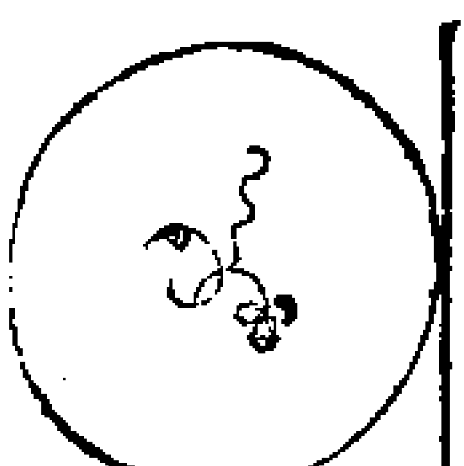
1.

Aequales circuli sunt quorum diametri sũt ęquales vel quo rũ quæ ex centris, rectæ lineę sunt æquales.



2.

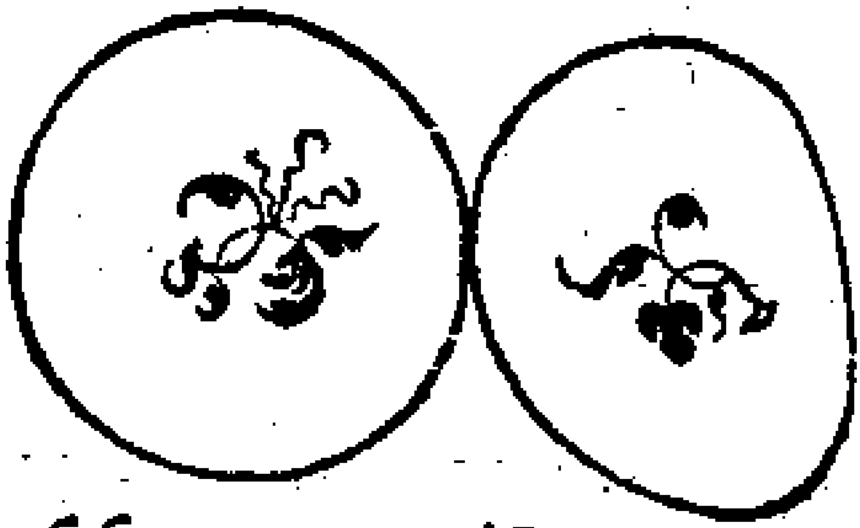
Recta linea circulũ tangere dicitur, quæ cùm circulun tangat, si producatur, circulum non secat.



3 Cir.

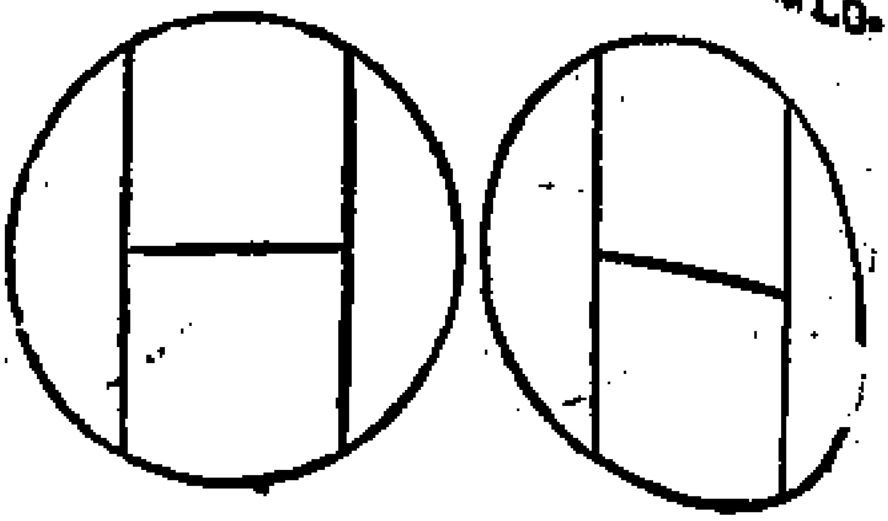
3

Circuli ſeſe mutuò tangere dicuntur: qui ſeſe mutuo tangentes, ſeſe mutuo non ſecant.



4

In circulo æqualiter diſtare à centro rectæ lineæ dicuntur, cùm perpendiculares, quæ à cẽtro in ipſas ducuntur, ſunt æquales. Lõgius autem abeſſe illa dicitur in quã maior perpendicularis cadit.



5

Segmentum circuli eſt, figura quæ ſub recta linea & circuli peripheria comprehenditur.

6

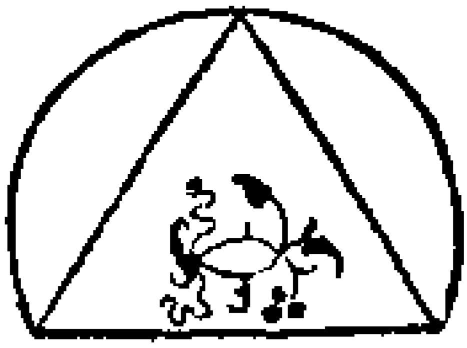
Segmenti autem angulus eſt, qui ſub recta

linea

linea & circuli peripheria comprehẽditur

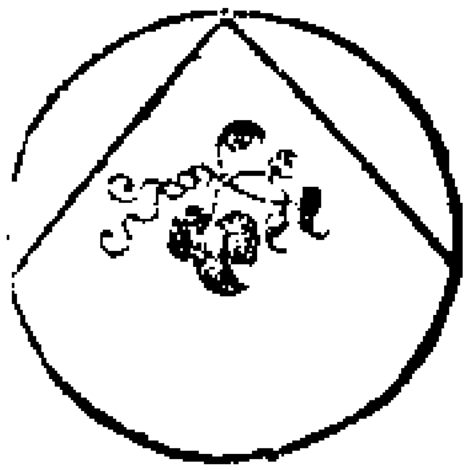
7

In ſegmento autem angulus eſt, cum in ſegmenti peripheria ſumptum fuerit quodpiam punctum, & ab illo in terminos rectæ eius lineæ, quæ ſegmenti baſis eſt, adiũctę fuerint rectæ lineæ: is, inquam, angulus ab adiunctis illis lineis comprehẽſus.



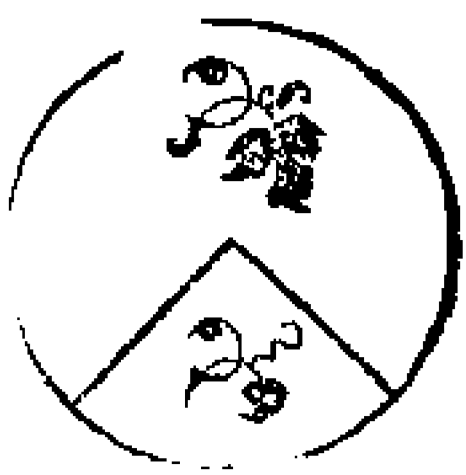
8

Cùm verò comprehendentes angulum rectæ lineæ aliquam aſſumitur peripheriam, illi angulus inſiſtere dicitur.



9

Sector autem circuli eſt, cùm ad ipſius circuli cẽtrum conſtitutus fuerit angulus, comprehenſa nimirum figura, & à rectis lineis angulum continentibus, & à peripheria ab illis aſſumpta.

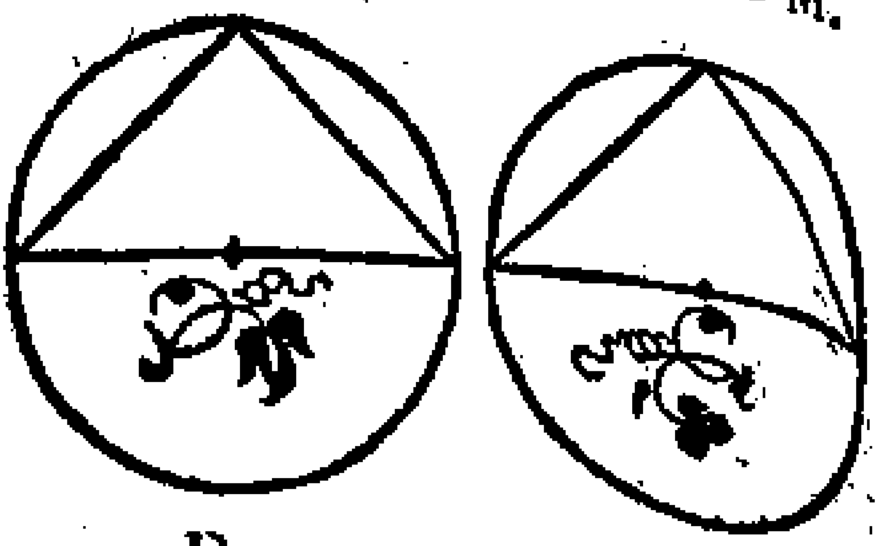


10

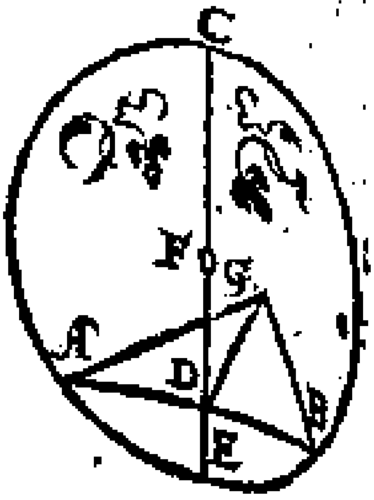
Similia circuli ſegmenta ſunt, quæ angulos

capiunt

capiunt ęquales aut in q b'anguli inter se sunt ęquales

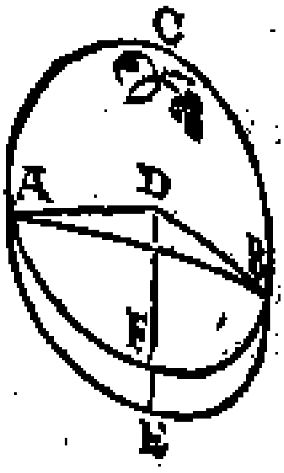


Problema 1. Propositio 1.



Dati circuli centrum reperire.

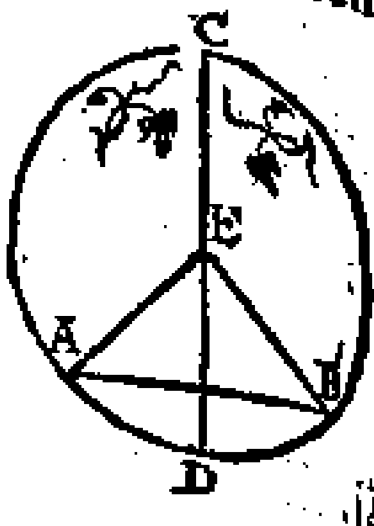
Theorema 1. Propositio 2.



Si in circuli peripheria duo quælibet puncta accepta fuerint, recta linea quæ ad ipsa puncta adiungitur, intra circulum cadet.

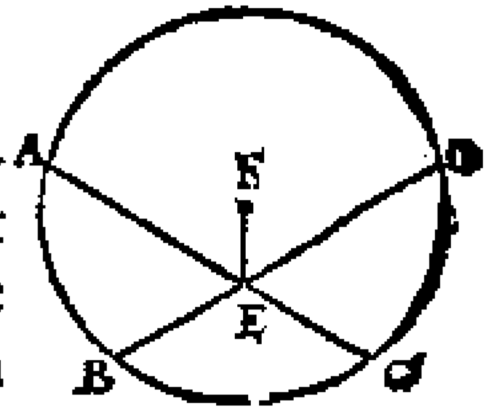
Theorema 2. Propositio 3.

Si in circulo recta quædam lenea per centrum extensa quandam non per centrum extensam bifariam secet: & ad angulos rectos ipsam secabit. Et si ad angulos rectos eam secet, bifariam quoque eam secabit.



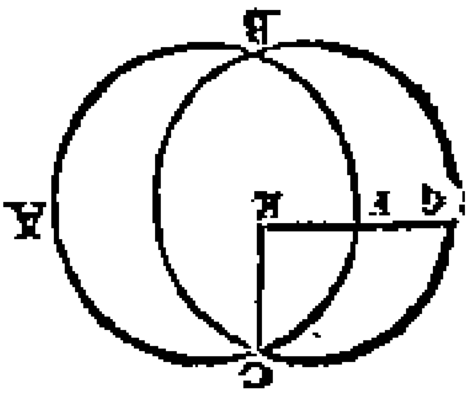
Theore.

Theorema 3. Propositio 4.



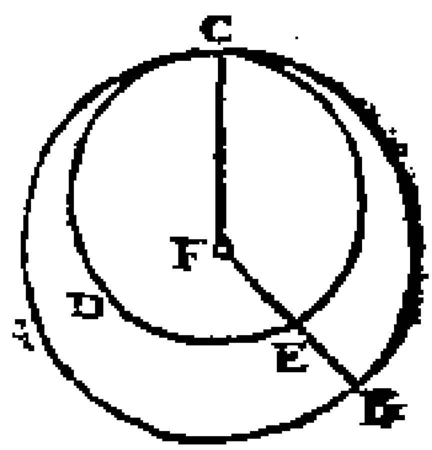
Si in circulo duę rectę lineæ sese mutuo secent non per centrum extēsæ sese mutuò bifariam non secabunt.

Theorema 4. Propositio 5.



Si duo circuli sese mutuò secent, non erit illorum idem centrum.

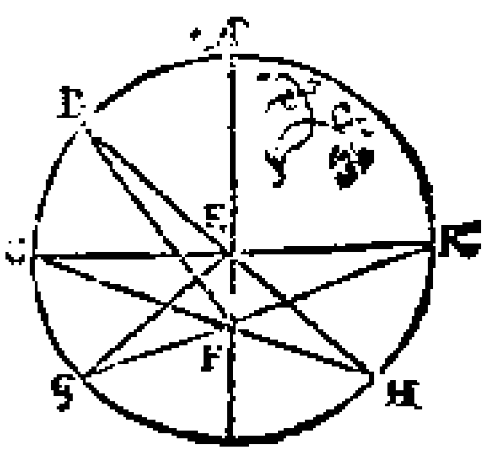
Theorema 5. Propositio 6.



Si duo circuli sese mutuò interius tangant, eorum non erit idem centrum.

Theorema 6. Propositio 7.

Si in diametro circuli quodpiam sumatur punctum, quod circuli centrum non sit, ab eoque puncto in circulū quædam rectæ lineæ cadant: maxima quidem erit ea in qua centrū, minima verò reliqua: aliarum verò propinquior illi quę per centrum du-

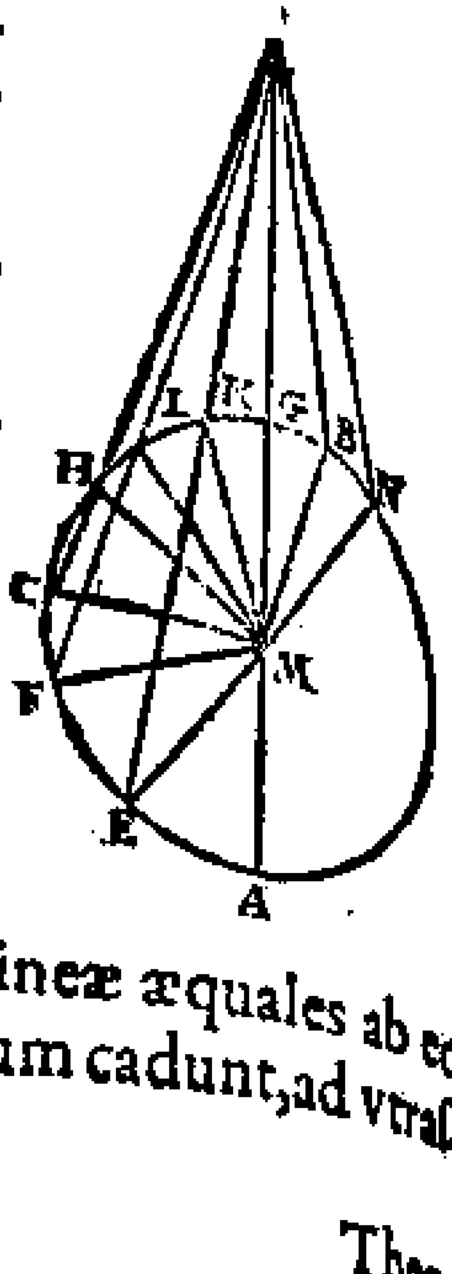


citur

citur, remotiore ſemper maior eſt. Duæ autem ſolùm rectę lineę æquales ab eodem puncto in circulum cadunt ad vtraſq; partes minimæ.

Theorema 7. Propoſitio 8.

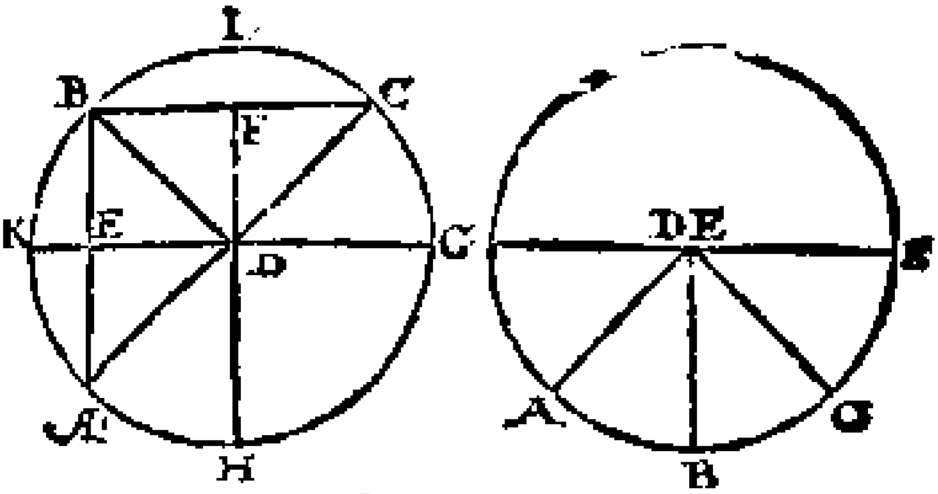
Si extra circulum ſumatur punctum quodpiam, ab eoque puncto ad circulum deducantur rectæ quædam lineæ, quarum vna quidem per centrum protendatur, reliquę verò vt libet: in cauam peripheriam cadentium rectarum linearum minima quidem eſt illa, quæ per centrum ducitur: aliarum autem propinquior ei, quæ per centrum tranſit, remotiore ſemper maior eſt: in conuexam verò peripheriá cadentium rectarũ linearum minima quidem eſt illa, quæ inter punctum & diametrum interponitur: aliarum autem, ea quę propinquior eſt minimæ, remotiore ſemper minor eſt. Duæ autem tantùm rectæ lineæ æquales ab eo puncto in ipſum circulum cadunt, ad vtraſque partes minimæ.



Theo-

Theorema 8. Propositio 9.

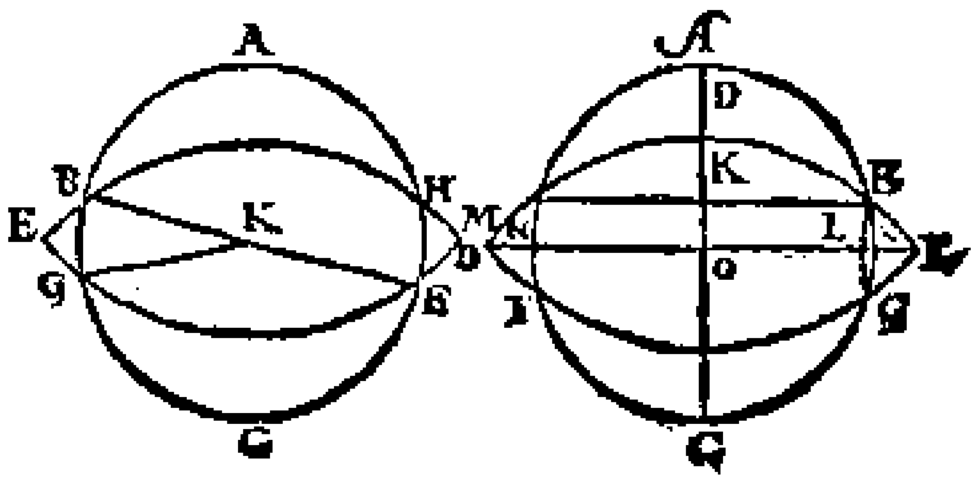
Si in circulo acceptum fuerit pūctum aliquod, & ab eo puncto ad circulum cadant plures quam duę rectæ lineæ, æquales, acceptū



punctum centrum ipsius erit circuli.

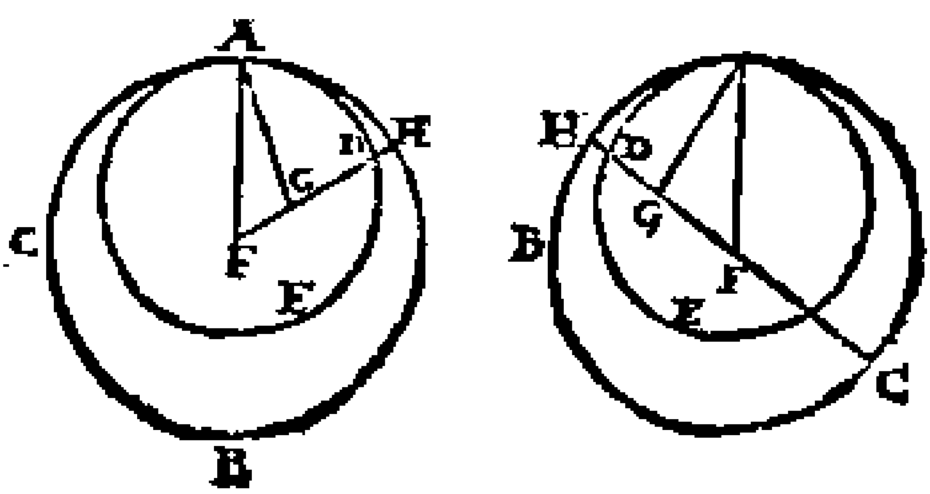
Theorema 9. Propositio 10.

Circū lus circulum in pluribus quàm duobus punctis non secat.



Theorema 10. Propositio 11.

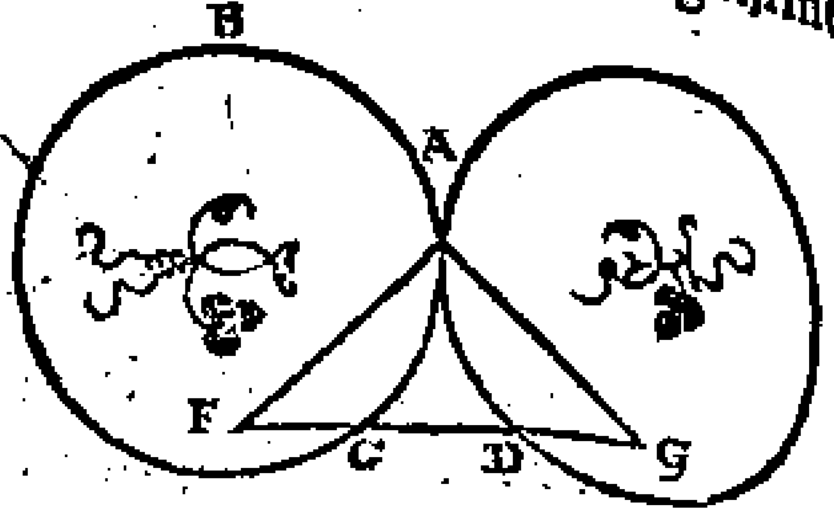
Si duo circuli sese intus cōtingāt, atque accepta



fuerint eorum centra, ad eorum cētra adiuncta recta linea & producta ni cōtactum circulorum cadet.

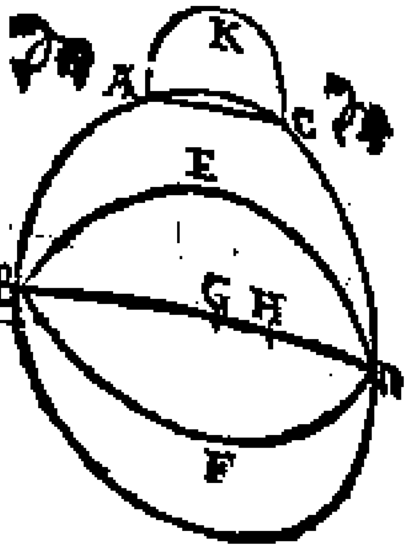
Theorema 11. Propositio 12.

Si duo circuli sese exterius contingāt, linea recta q̃ ad cētra eorū adiūgitur, p contactū illū trāsibit.



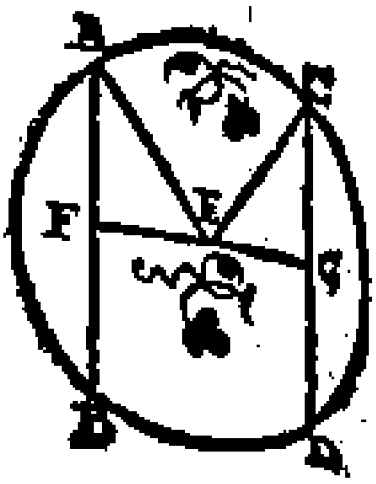
Theorema 12. Propositio 13.

Circulum circulum non tangit in pluribus punctis, quam vno, siue intus siue extra tangat.



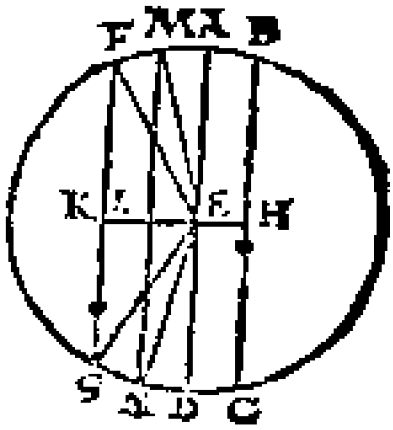
Theorema 13. Propositio 23.

In circulo æquales rectæ lineæ æqualiter distant à centro. Et quæ ęqualiter distant à centro, quales sunt inter se.



Theor.

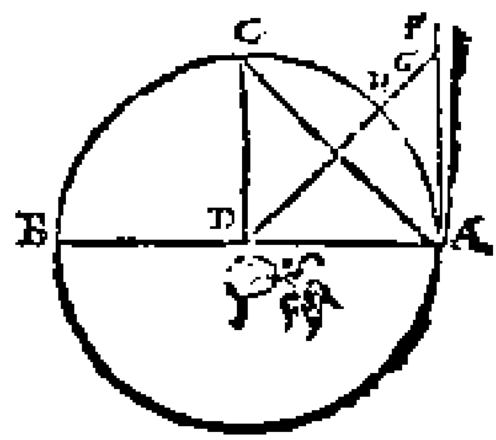
Theorema 14. Propositio 15.



In circulo maxima, quidem linea est diameter: aliarum autem propinquior centro, remotiore semper maior.

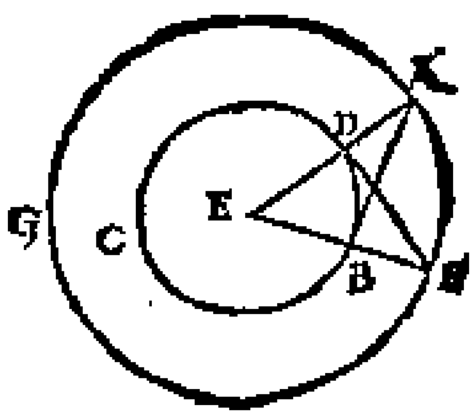
Theorema 51. Propositio 16.

Quæ ab extremitate diametri cuiusq; circuli ad angulos rectos ducitur, extra ipsú circulum cadet, & in locum inter ipsam rectam lineam & peripheriam comprehensum, altera recta linea nó cadet Et semicirculi quidem angulus quouis augulo acuto rectilineo maior est, reliquus autem minor,

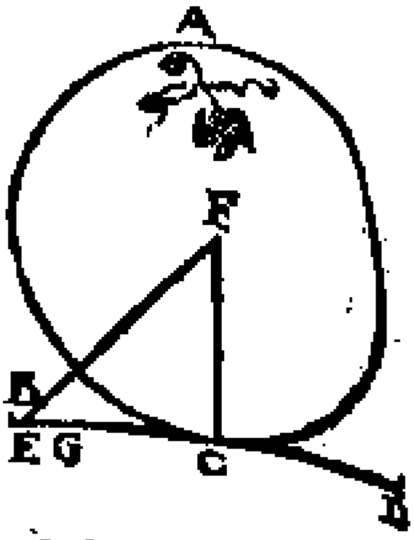


Problema 2. Propositio 17.

A dato puncto rectam lineam ducere, quæ datum tangat circulum.

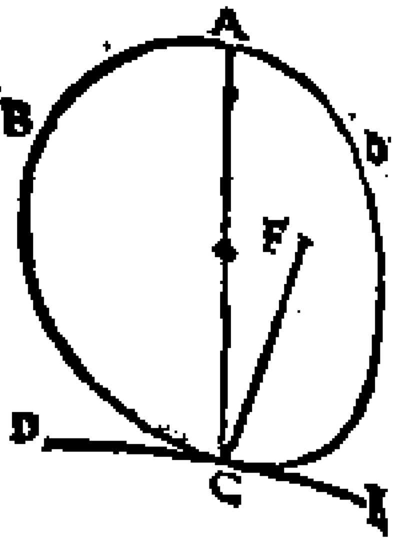


Theorema 16. Propositio 18.



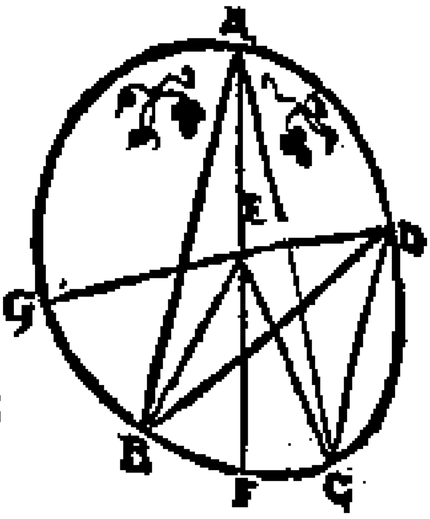
Si circulum tangat recta quæpiam linea, à centro autem ad cótactum adiungatur recta quædam linea: quę adiũcta fuerit ad ipsam contingentem perpendicularis erit.

Theorema 17. Propositio 19.



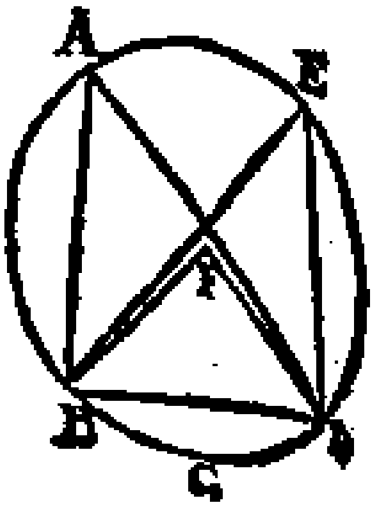
Si circulum tetigerit recta quæpiam linea, à contactu autem recta linea ad angulos rectos ipsi tãgenti excitetur, in excitata erit centrum circuli.

Theorema 18. Propositio 20.



In circulo angulus ad cẽtrum duplex est anguli ad peripheriam, cũ fuerit eadẽ peripheria basis angulorum.

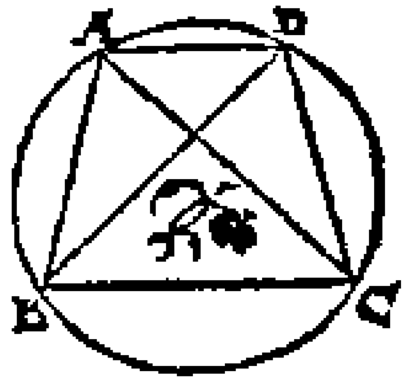
Theorema 19. Propositio 21.



In circulo, qui in eodem segméto sunt anguli, sunt inter se æquales.

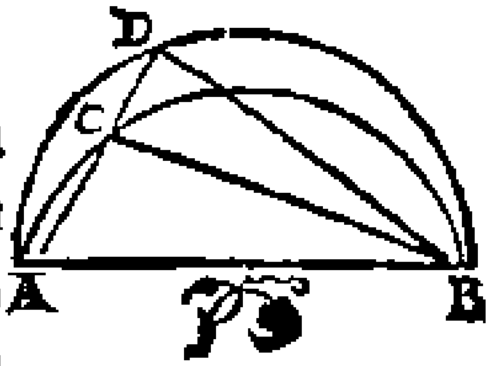
Theore-

Theorema 20. Propoſitio 22.



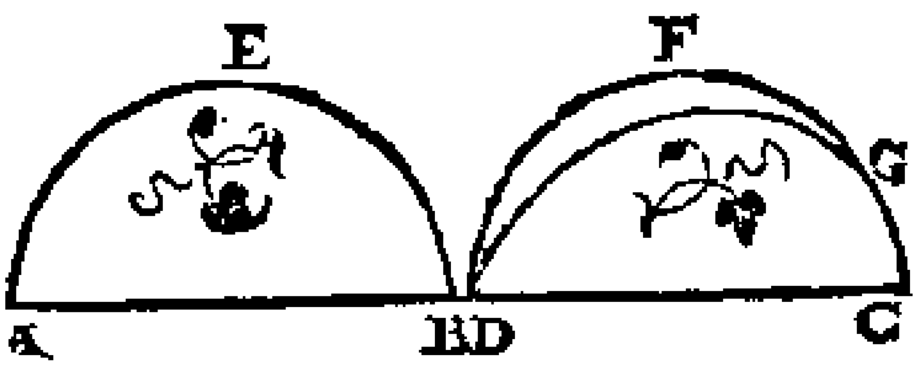
Quadrilaterorum in circulis deſcriptorũ anguli qui ex aduerſo, duobꝰ rectis ſunt æquales.

Theorema 21. Propoſitio 23.



Super eadem recta linea duo ſegmenta circulorũ ſimilia & inæqualia non conſtituẽtur ad eaſdem partes.

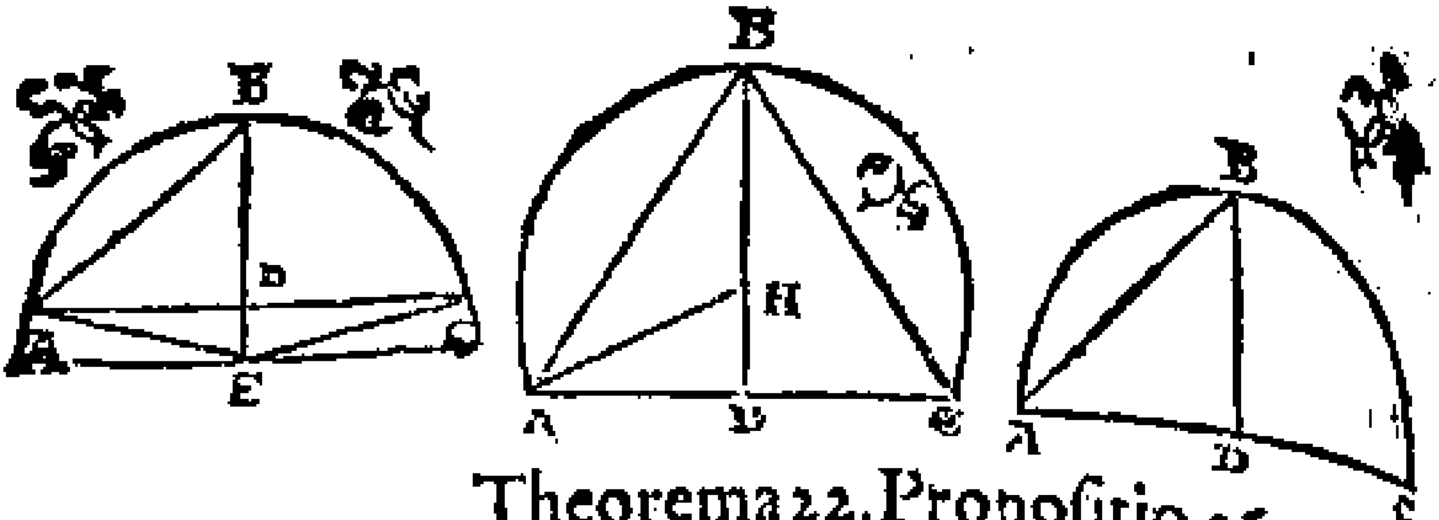
Theorema 22. Propoſitio 24.



Super ęqualibꝰ rectis lineis ſimilia circulorum ſegmenta ſunt inter ſe æqualia.

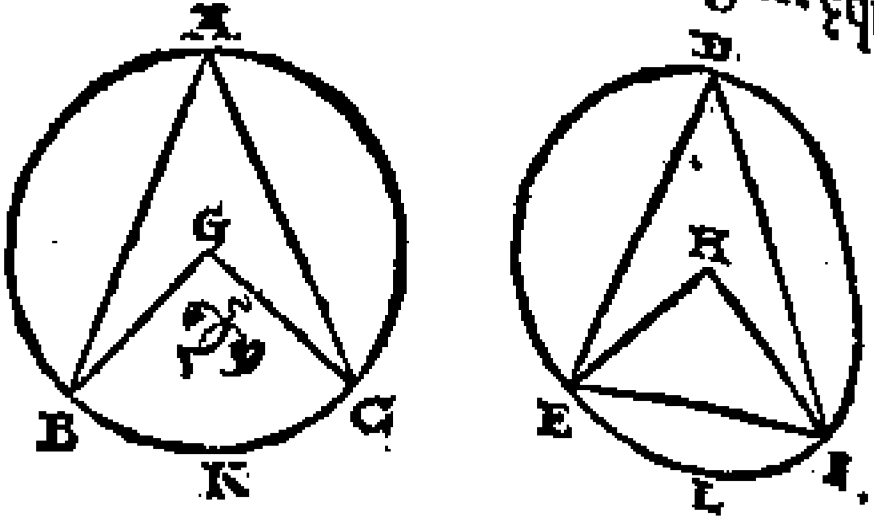
Problema 23. Propoſitio 25.

Circuli ſegmento dato, deſcribere circulũ cuius est ſegmentum.



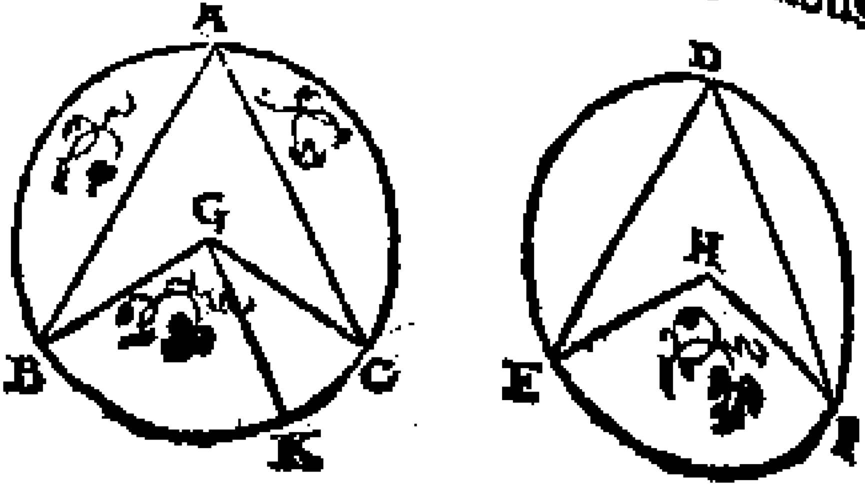
Theorema 22. Propoſitio 26.

In æqualibus circulis, æquales anguli ęqualib. peripherijs inſiſtűt ſiue ad cétra, ſiue ad peripherias conſtituti inſiſtant.



Theorema 24 Propoſitin 27.

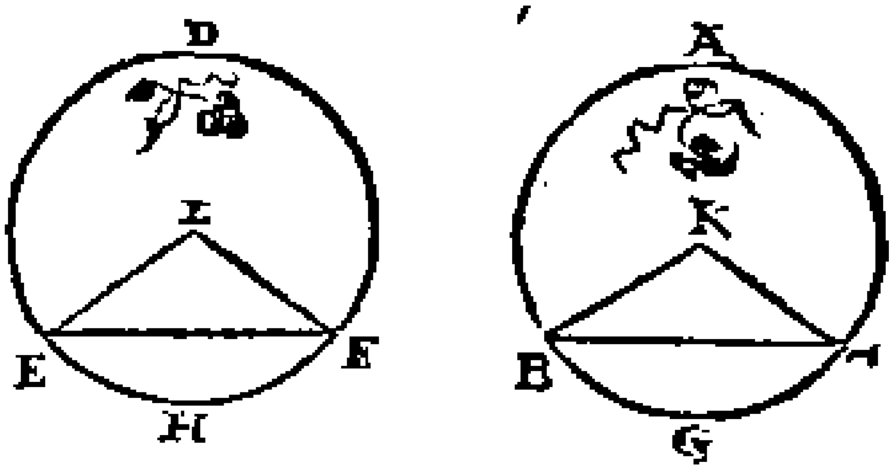
In æqualibus circulis, anguli qui ęqualibus peripherijs inſiſtűt ſunt inter ſe ęquales ſiue ad cétra, ſiue ad peripherias cōſtituti inſiſtāt.



Theore.

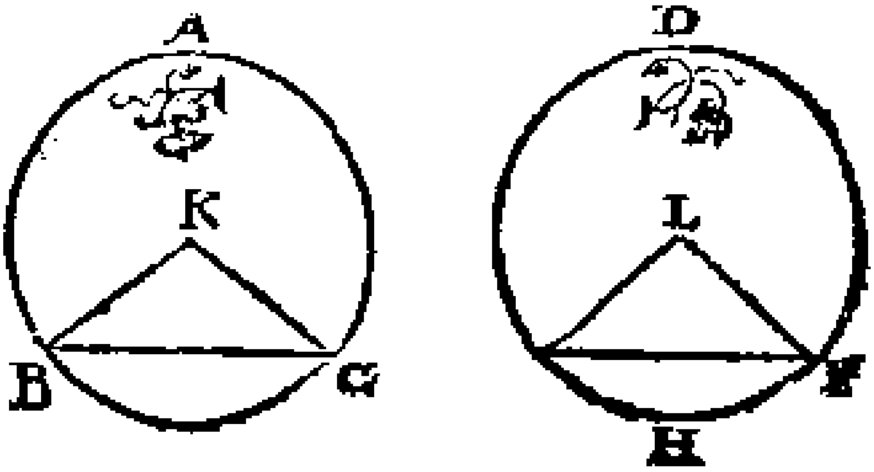
Theorema 25 Propositio 28.

In æqualibus circulis æquales rectæ lineæ ęquales peripherias auferũt maiorẽ quidẽ maiori, minorem autem minori.



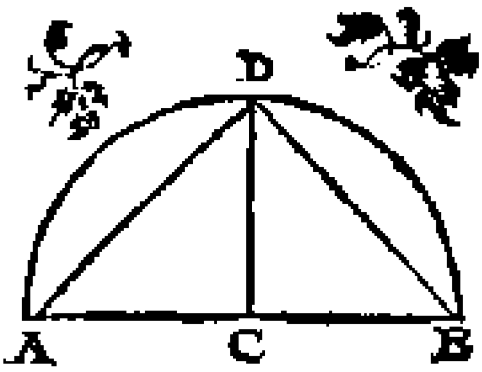
Theorema 26. Propositio 29.

In ęqualibꝰ circulis, æquales peripherias æquales rectæ lineæ subtendunt.



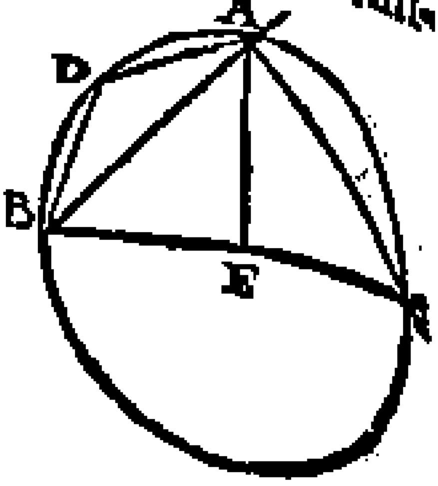
Problema 4. Propositio 30.

Datam peripheriam bifariam secare.



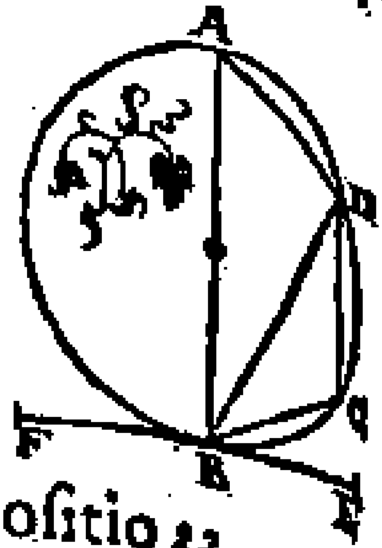
Theorema 27. Propositio 13.

In circulo angulus qui in semicirculo, re ctus est: qui autem in maoire segmẽto, minor recto: qui verò in minore segmẽto, maior est recto. Et insuper angulus maioris segmenti, recto quidem maior est, minoris autẽ segmẽti angulus, minor est recto.



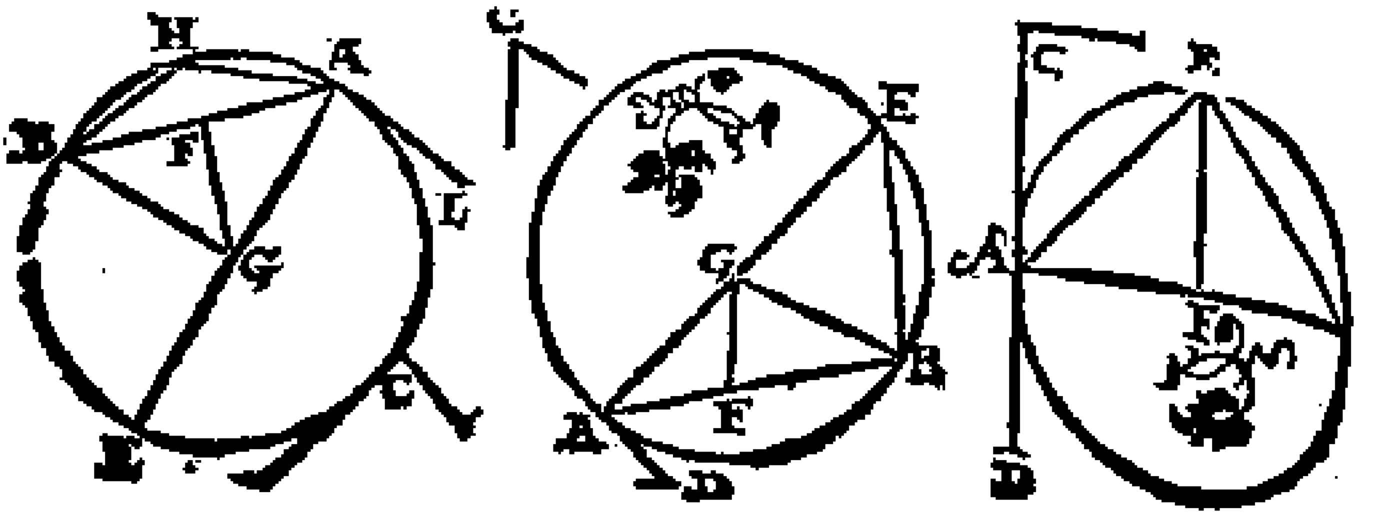
Theorema 28. Propositio 32.

Si circulum tetigerit aliqua recta linea, à contactu autẽ producatur quædam recta linea circulum secans: anguli quos ad contingentem facit ęquales sunt ijs qui in alternis circuli segmẽtis consistunt, angulis.



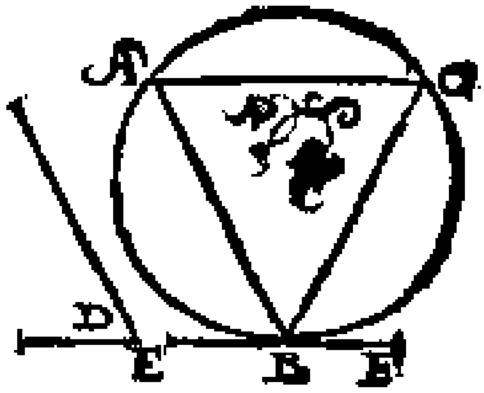
Problema 5. Propositio 33.

Super data recta linea describere segmentum circuli quod capiat angulum æqualẽ dato angulo rectilineo.



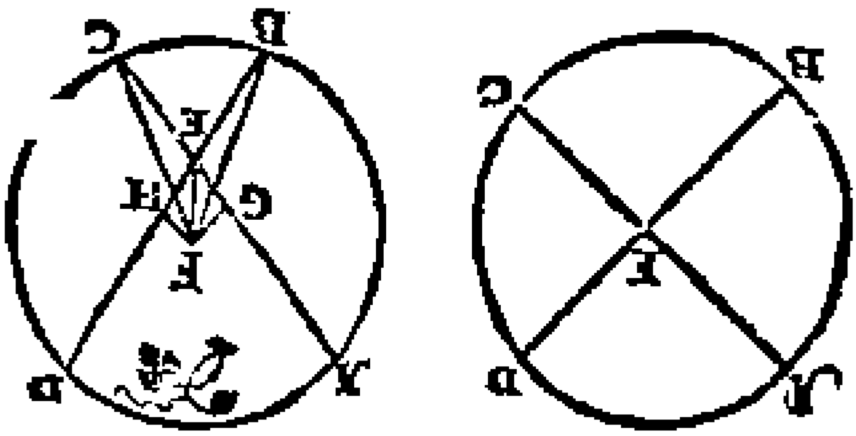
Problema 6. Propoſitio. 34.

A dato circulo ſegmentum abſcindere capiens angulum æqualem dato angulo rectilineo.



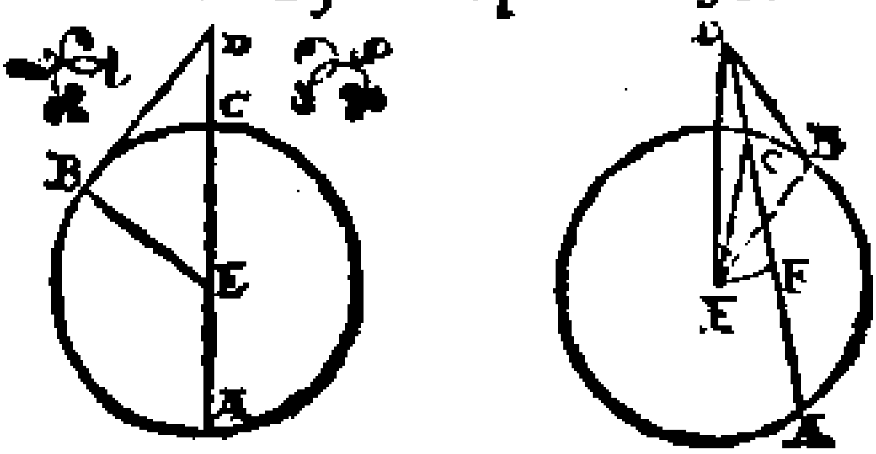
Theorema 29. Propoſitio 35.

Si in circulo duæ rectæ lineæ ſeſe mutuò ſecuerint, rectangulum comprehéſum ſub ſegmétis vniꝰ, æquale eſt ei, quod ſub ſegmentis alterius comprehenditur, rectangulo.



Theorema 30. Propoſitio 36.

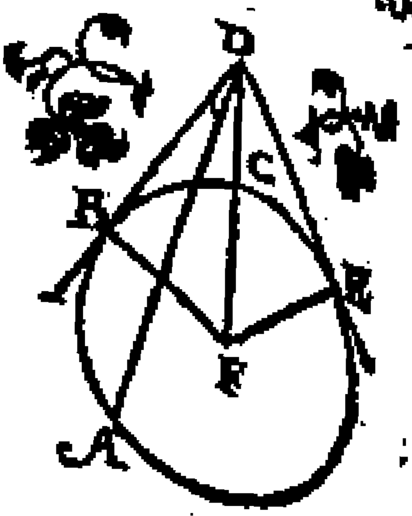
Si extra circulú ſumatur pūctū aliquod, ab eoq́ue in circulú cadant duę rectę lineæ, quarú altera quidem circulum ſecet, altera



verò tangat: quod sub tota secante & exterius inter punctum & conuexam peripheriam assumpta comprehenditur rectangulum, æquale erit ei, quod à tangente describitur, quadrato,

Theorema 31. Propositio 37.

Si extra circulũ sumatur punctun aliquod, ab eoq; puncto in circulum cadant duæ rectæ lineæ, quarum altera circulum secet, altera in eum incidat, sit autẽ quod sub tota secante & exterius inter punctum & conuexã peripheriam assumpta, comprehenditur rectangulum, æquale ei, quod quadrato: incidens ipsa circulum tanget.



ELEMENTI II. FINIS.

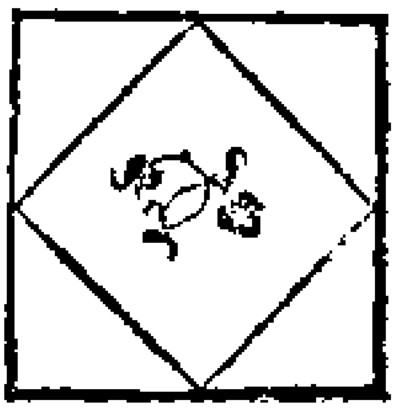
EVCLI

EVCLIDIS ELEMENTVM QVARTVM.

DEFINITIONES.

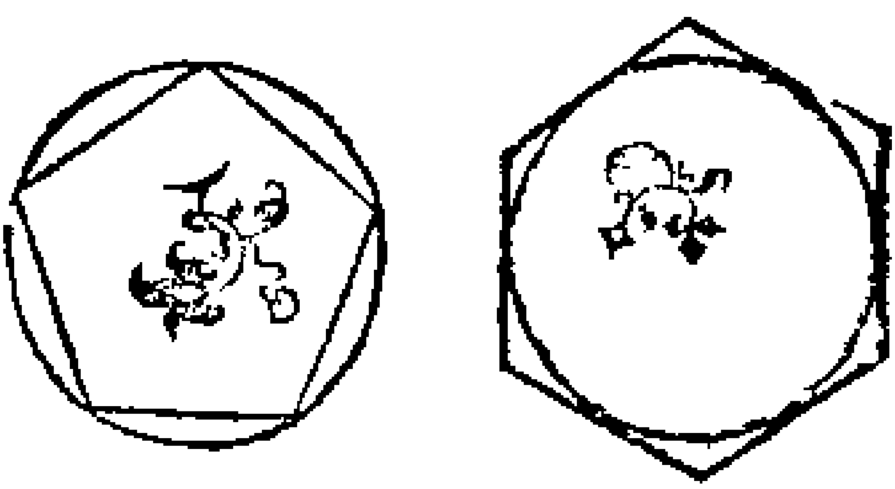
1.

Figura rectilinea in figura rectilinea inscribi dicitur, cũ singuli eius figuræ quæ inscribitur, anguli singula latera eius, in qua inscribitur, tangunt.



2

Similiter & figura circum figurã describi dicitur, quum singula eius quæ circũscribitur, latera singulos eius figuræ angulos tetigerint, circum quam illa describitur.



3

Figura rectilinea in circulo inscribi dicitur, quũ singuli eius figuræ quę inscribitur, angu-

anguli tetigerint circuli peripheriam.

4

Figura verò rectilinea circa circulũ describi dicitur, quũ singula latera eius, quæ circum scribitur, circuli peripheriã tangunt.

5

Similiter & circulus in figura rectilinea inscribi dicitur, quum circuli peripheria singula latera tãgit eius figuræ, cui inscribitur.

6

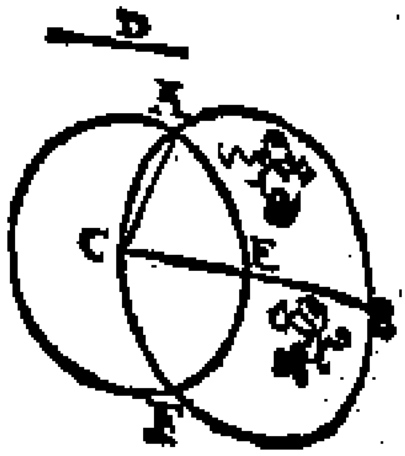
Circulus autem circum figurã describi dicitur, quũ circuli peripheria singulos tãgit eius figuræ, quam circunscribit, angulos.

7

Recta linea in circulo accommodari seu coaptari dicitur, quum eius extrema in circuli peripheria fuerint.

Problema I. Propositio I.

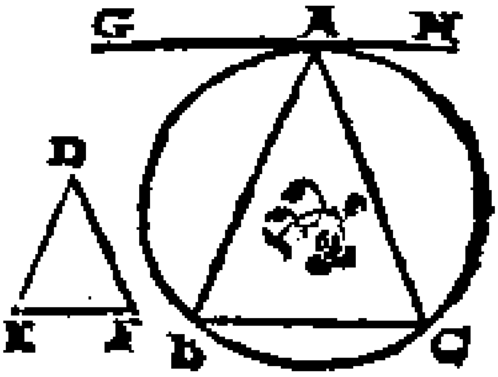
In dato circulo, rectam lineam accommodare equalem datæ rectæ lineæ quæ circuli diametro nõ sit maior.



Proble-

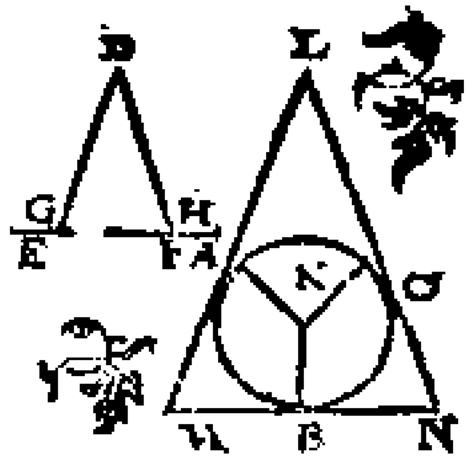
Problema 2. Propositio 3.

In dato circulo, triangulũ describere dato triangulo æquiangulum.



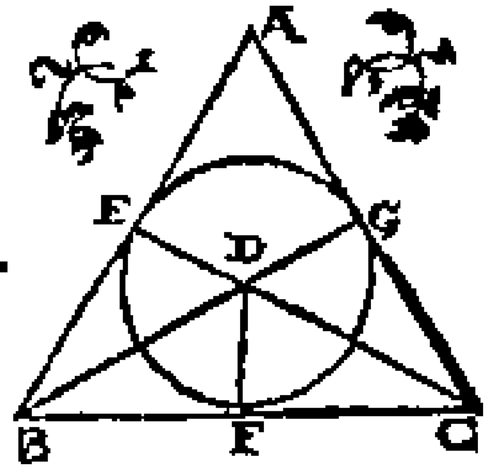
Problema 3. Propositio 3.

Circa datum circulũ triangulũ, describere dato triangulo æquiangulũ.



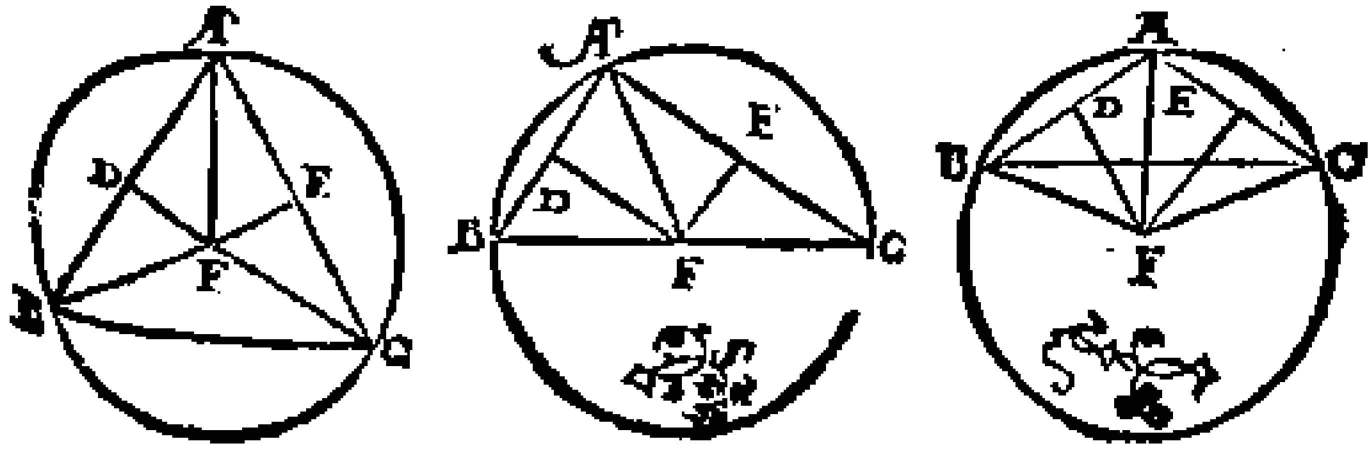
Problema 4. Propositio 4.

In dato triangulo circulum inscribere.

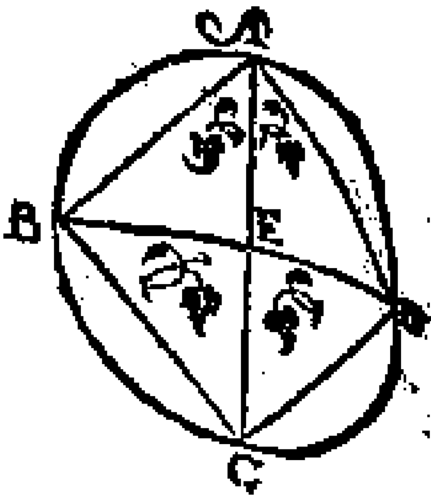


Problema 5. Propositio 5.

Circa datum triangulum, circulum describere.

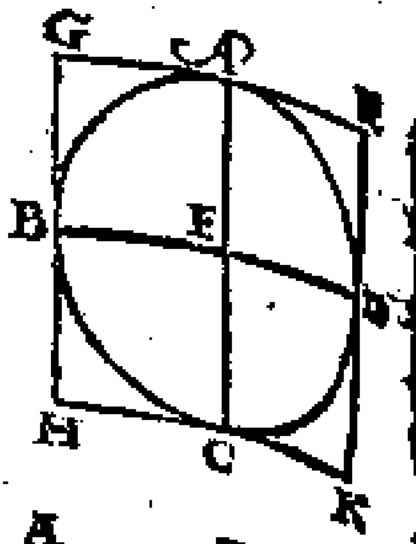


Problema 6. Propositio 6.



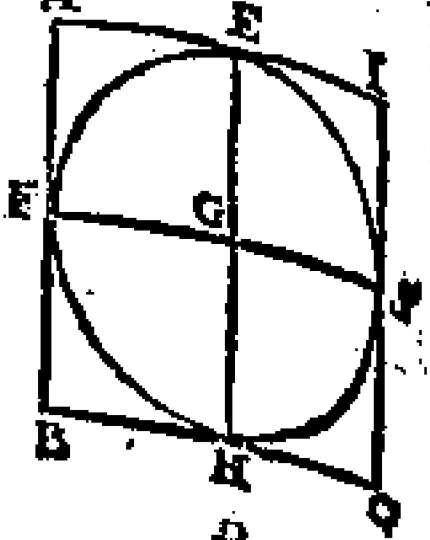
In dato circulo quadratum describere.

Problema 7. Propositio 7.



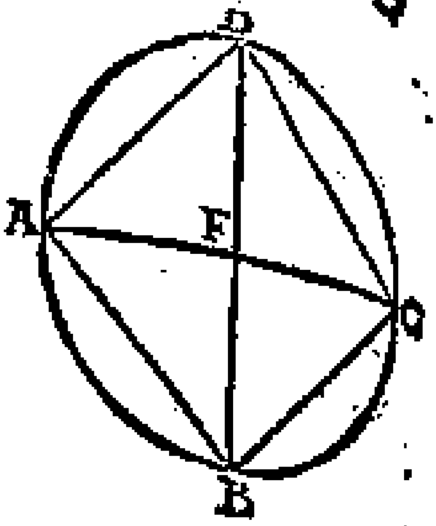
Circa datum circulum, quadratum describere.

Problema 8. Propositio. 8.



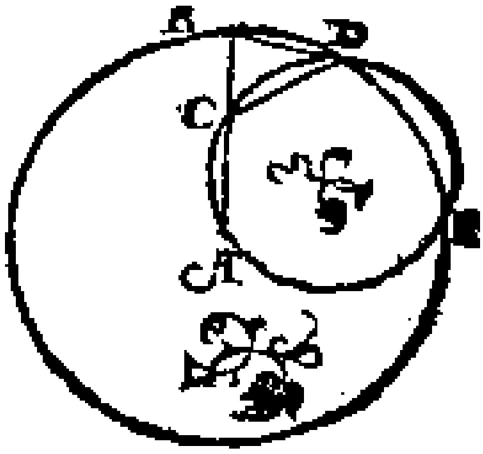
In dato quadrato circulum inscribere.

Problema 9. Propositio 9.



Circa datum quadratū, circulum describere.

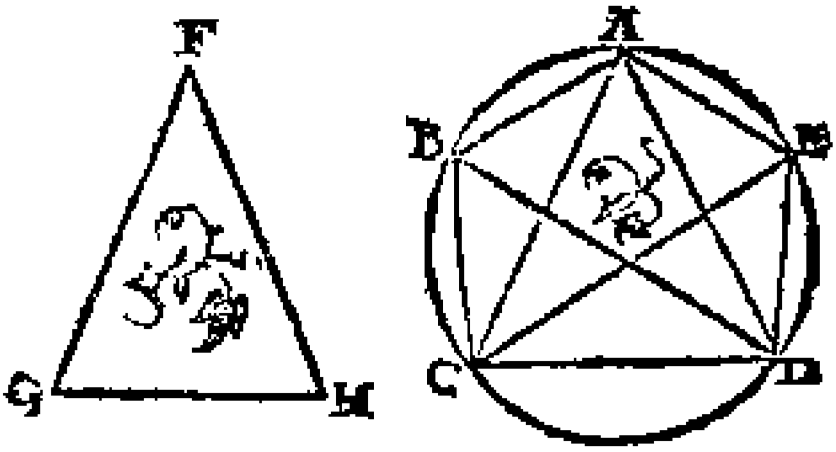
Problema 10. Propopoſitio. 10



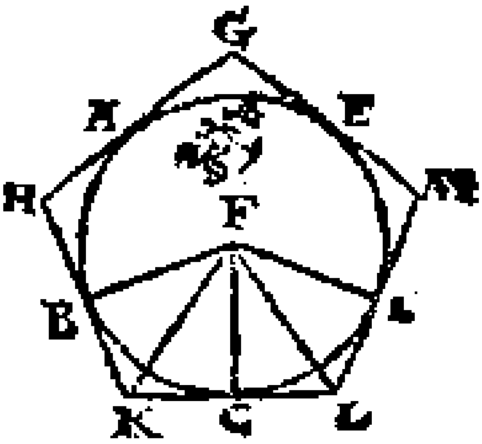
ſoſceles triãgulum con-
ſtituere, quod habeat v-
trunque eorum, qui ad
duplum reliqui.

Theorema 11. Propoſitio 11.

In dato
circulo,
pentago-
nũ æqui-
laterũ &
æquiãgu
lum in-
ſcribere.

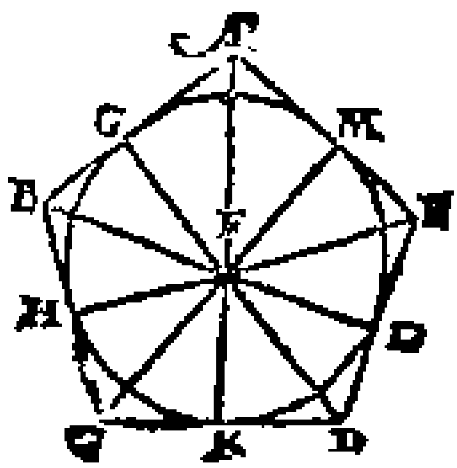


Problema 12. Propoſitio 12.



Circa datum circulum, pentagonum æquilaterum æquiangulum deſcribere.

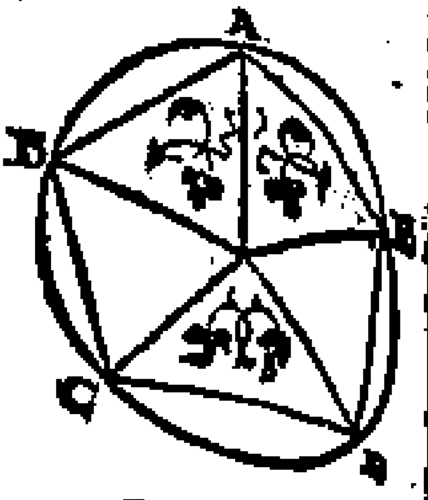
Problema 13. Propoſitio 13.



In dato pẽtagono æquilatero & æquiãgulo circulum inſcribere.

Proble.

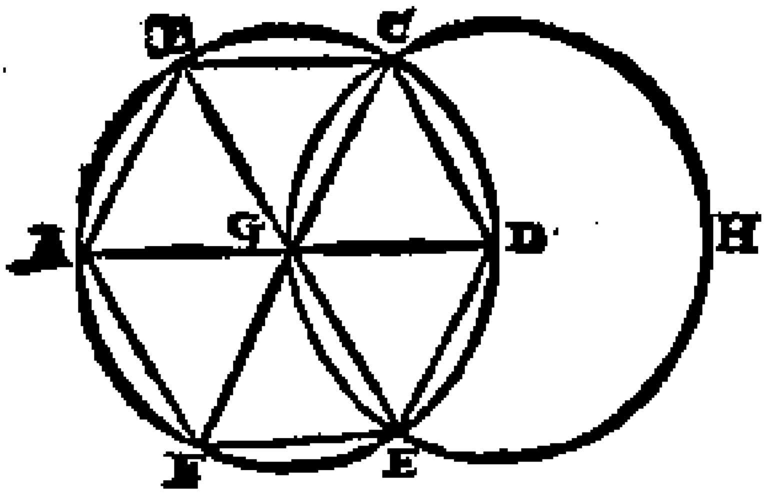
Problema 14. Propositio 14.

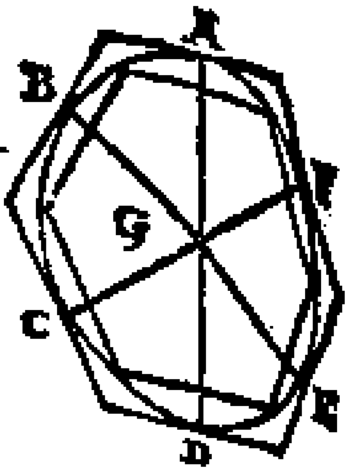


Circa datú pentagonú, æquilaterum & æquiangulum, circulum describere.

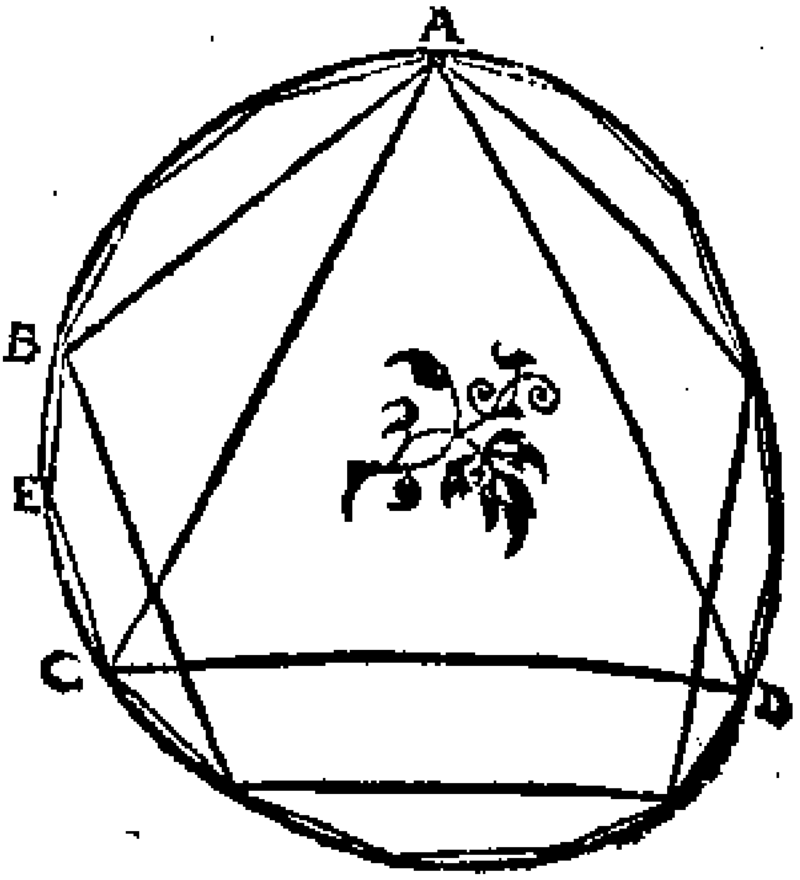
Problema 15. Propositio 15.

In dato circulo hexagonum & æquilaterum & æquiangulum inscribere.





propositio 16. Theorema 16.



In dato circulo quintidecagonum & æquilaterú & æquiangulum describere.

Elementi quarti finis.

EVCLIDIS ELEMENTVM QVINTVM.

DEFINITIONES.

1

PARS est magnitudo magnitudinis minor maioris, quum minor metitur maiorem.

2

Multiplex autem est maior minoris, cùm minor metitur maiorem.

3

Ratio, est duarum magnitudinum eiusdē generis mutua quædam secundum quantitatem habitudo.

4

Proprotio verò, est rationum similitudo.

5

Rationē habere inter se magnitudinis dicuntur, quæ possunt multiplicatæ sese mutuo superar e

6.

In eadem ratione magnitudines dicūtur esse, prima ad secundam, & tertia ad quartā, cum primæ & tertiæ æquè multiplicia, à secundæ & quartæ æquè multiplicibus, qualiſcunq; ſit hæc multiplicatio, vtrunq; ab vtroque: vel vnà deficiunt, vel vnà ęqualia ſunt, vel vnà excedũt, ſi ea ſumãtur quæ inter ſe reſpondent.

7

Eandam autem habẽtes rationem magnitudines, proportionales vocentur.

8

Cum verò æquè multiplicium, multiplex primę magitudines exceſſerit? multiplicẽ ſecundæ, at multiplex tertiæ non exceſſerit multiplicem quartæ: tunc prima ad ſecundam, maiorem rationem habere dicetur, quàm tertia ad quartam.

9

Proportio autẽ in tribus terminis pauciſſimis conſiſtit.

10

Cùm autem tres magnititudines proportionales, fuerint, prima ad tertiam, duplicatã rationẽ habere dicitur eius quam habet ad ſecundam. At cùm quatuor magnitudines proportionales, fuerint, prima ad quartam, triplicatam rationem habere dicitur eius quam habet ad ſecundam: & ſemper deinceps vno amplius, quãdiu proportio extiterit.

11

Homologę, ſeu ſimiles ratioue magnitudines dicuntur, antecedentes quidẽ antecedentidentibus, consequétés verò cósequétibus.

12

Alterna ratio, est sumptio antecedentis cõparati ad antecedentem, & cõsequentis ad consequentem.

13

Inuersa ratio, est sumptio cõsequentis, ceu antecedentis, ad antecedenté velut ad consequentem.

14.

Compositio rationis, est sumptio antecedentis cũ consequente ceu vnius ad ipsum consequentem.

15

Diuisio rationis, est sumptio excessus quo consequentem superat antecedens ad ipsum consequentem.

16

Conuersio rationis, est sumptio antecedentis ad excessum, quo superat antecedens ipsum cõsequentem.

17

Ex ęqualitate ratio est, si plures duabus sint magnitudines, & his alię multitudine pares quæ binæ sumantur, & in eadem ratione: quum vt in primis magnitudinibus prima ad vltimam sic & in secundis magnitudinibus prima ad vltimã sese habuerit, vel aliter, sumptio extremorũ per subductionem mediorum.

18

Ordinata proportio est, cùm fuerit quemadmodum antecedens ad consequentem ita antecedens ad consequentem: fuerit etiam vt consequens ad aliud quidpiam, ita consequens ad aliud quidpiam.

19

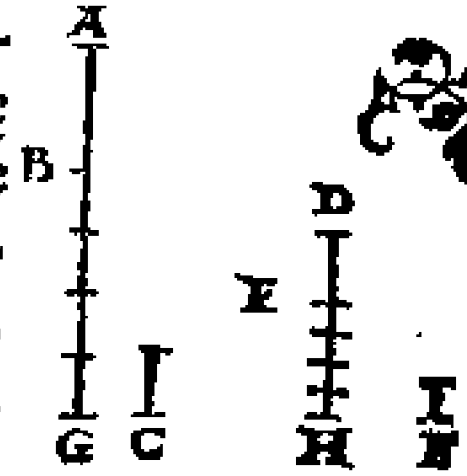
Perturbata autẽ proportio est, tribus positis magnitudinibus, & alij quæ sint his multitudine pares, cùm vt in primis quidẽ magnitudinibus se habet antecedẽs ad cõsequentem, ita in secũdis magnitudinibus antecedens ad consequentem: vt autem in primis magnitudinibus consequens ad aliud quidpiam ad antecedentem.

Theorema 1. Propositio 1.

Si sint quotcũque magnitudines quotcunque magnitudinum æqualium numero singulę singularum æquè multiplices, quàm multiplex est vnius vna magnitudo, tam multiplices erunt, & omnes omnium.

Theorema 2. Propositio 2.

Si prima secundæ æquè fuerit multiplex, atque



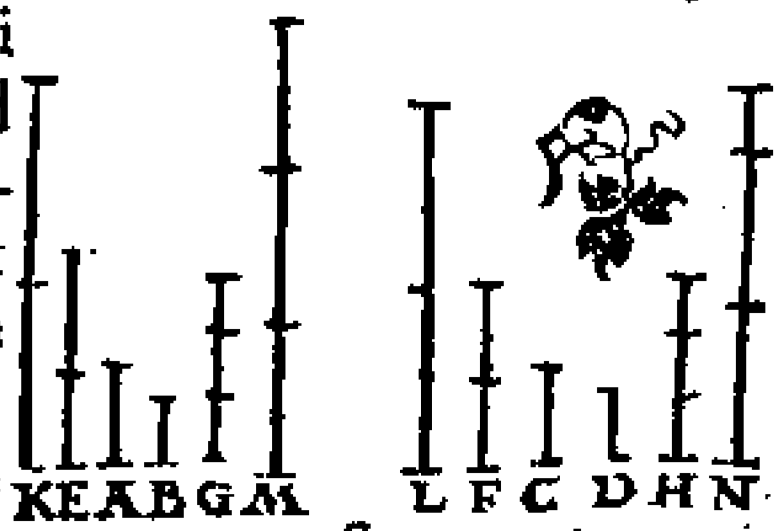
atque tertia quartę, fuerit autē & quinta ſecūdę ęquè multiplex, atque ſexta quartæ: erit & cōpoſita prima cū quinta, ſecūdæ ęquè multiplex atque tertia cum ſexta, quartæ.

Theorema 3. Propoſitio 3.

Si ſi prima ſecundæ æquè multiplex atque tertia quartæ, ſumātur autē æquè multiplices primę & tertiæ erit & ex æquo ſumptarum vtraque vtriuſque ęquè multiplex, altera quidem ſecundæ, altera autē

Theorema 4. Propoſitio 4.

Si prima ad ſecundam, eandē habuerit rationem, & tertia ad quartam: etiam æquè multiplices primę & tertiæ, ad ęquè multiplices ſecundæ & quartæ iuxta quāuis multiplicationē, eandem habebunt rationem, ſi prout inter ſe



respondent, ita sumptæ fuerint.

Theorema 5. Propositio 5.

Si magnitudo magnitudinis æquè fuerit multiplex, atque ablata ablatæ: etiam reliqua reliquæ ita multiplex erit, vt tota totius.

Theorema 6. Propositio 6.

Si duæ magnitudines, duarum magnitudinum sint æquè multiplices, & detractę quædam sint earundem æquè multiplices: & reliquæ eisdem aut ęquales sunt, aut æquè ipsarum multiplices.

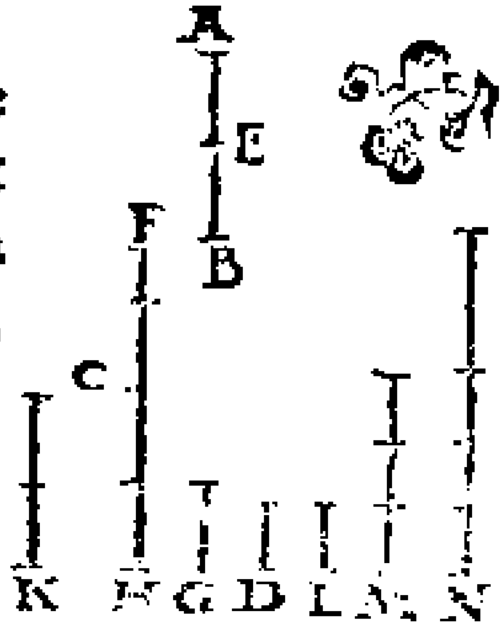
Theorema 7. Propositio 7.

Aequales ad eandem, eadem habent rationem: & eandem ad æquales.

Theorema 8. Propositio 8.

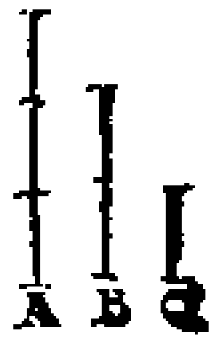
In æqualium magnitudinũ maior, ad eandem

dem maiorem ratione habet, quàm minor: & eadem ad minorem: maiorem rationem habet, quàm ad maiorem.



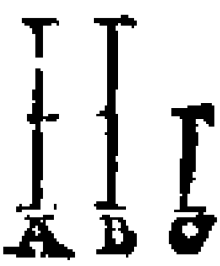
Theorema 9. Propositio 9.

Quæ ad eandem, eandem habẽt rationem, æquales sunt inter se: & ad quas eadem, eandem habet rationem, eæ quoque sunt inter se æquales.



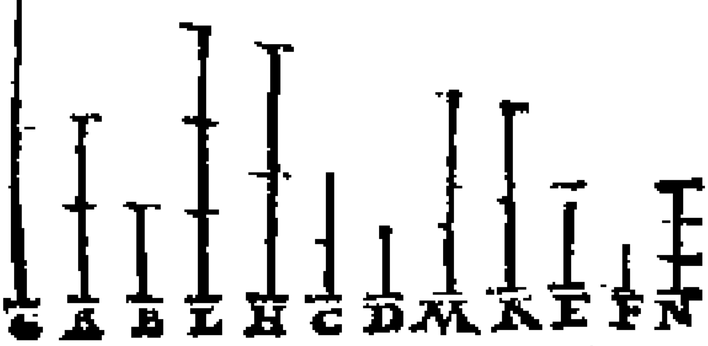
Thorema 10. Propositio 10.

Ad eandem magnitudiné rationem habentium, quę maiorem rationé habet, illa maior est ad quam auté eadem maiorem rationem habet, illa minor est.



Theorema 11. Propositio 11.

Quæ eidem sunt eædem rationes, & inter se sunt eędem.

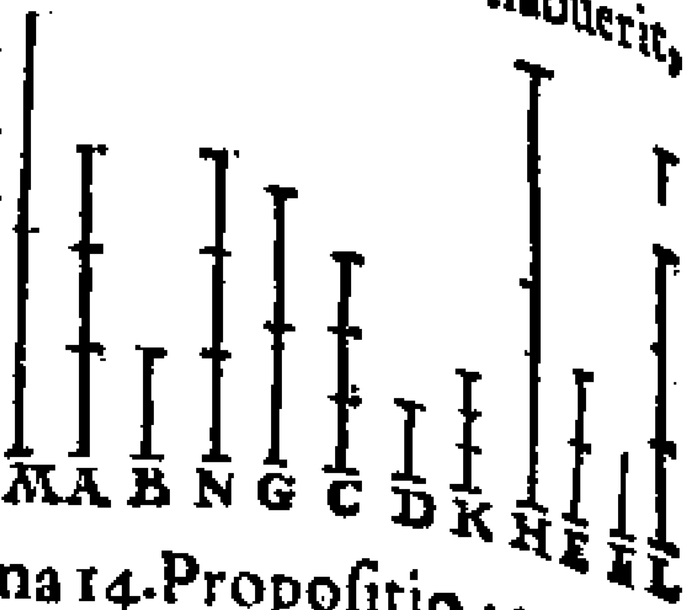


Theorema 12. Propositio 12.

Si sint magnitudines quotcunque proportionales, quem admodũ se habuerit vna antecedentium ad vnã consequentiũ, ta se habebunt omnes antecedenses ad omnes cõsequentes.

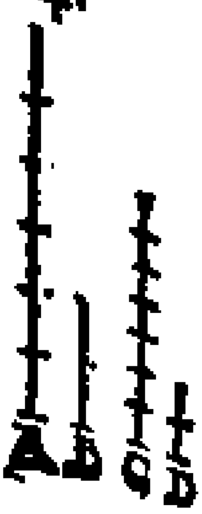
Theorema 13. Propositio 13.

Si prima ad secundum, eandẽ habuerit rationem, quam tertia ad quartã, tertia verò ad quartam maiorem rationem habuerit, quàm quinta ad sextam. prima quoque ad secũdam maiorem rationem habebit, quàm quinta ad sextam.



Theorema 14. Propositio 14.

Si prima ad secundam eandem habuerit rationẽ, quam tertia ad quartam, prima verò quàm tertia maior fuerit: erit & secunda maior quàm quarta. Quòd si prima fuerit æqualis tertiæ, erit

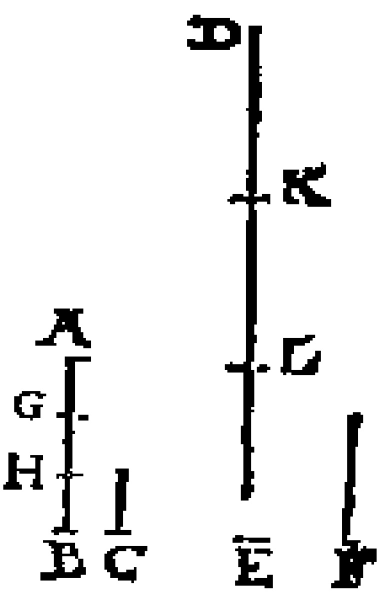


&

& secunda æqualis quartæ: si verò minor, & minor erit.

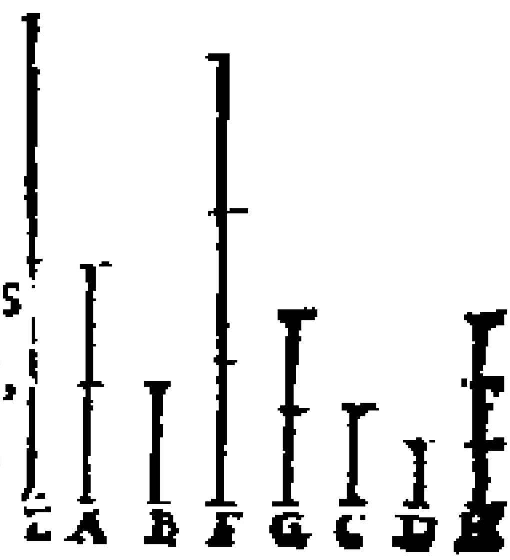
Theorema 15. Propositio 15.

Partes cum pariter multiplicibus in eadẽ sunt ratione, si prout sibi mutuo respondent, ita sumantur.



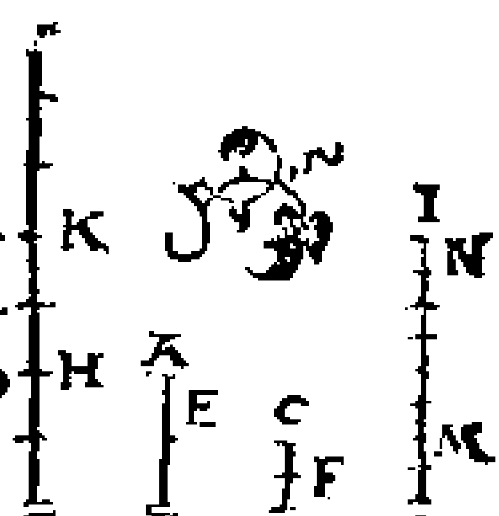
Theorema 16. Propositio 16.

Si quatuor magnitudines proportionales fuerint, & vicissim proportionales erunt.

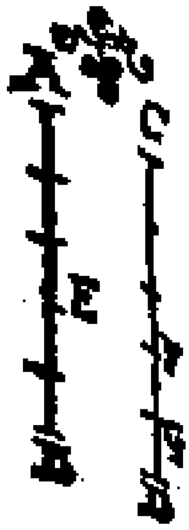


Theorema 17. Propositio 17.

Si compositæ magnitudines porportionales fuerint hæ quoq; diuisæ proportionales erunt.

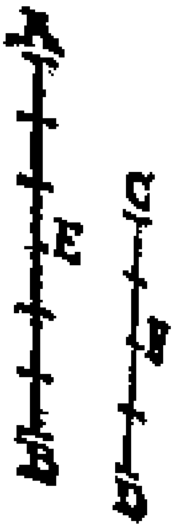


Theorema 18. Propositio 18.



Si diuisæ magnitudines sint proportionales, hæ quoque compositæ proportionales erunt.

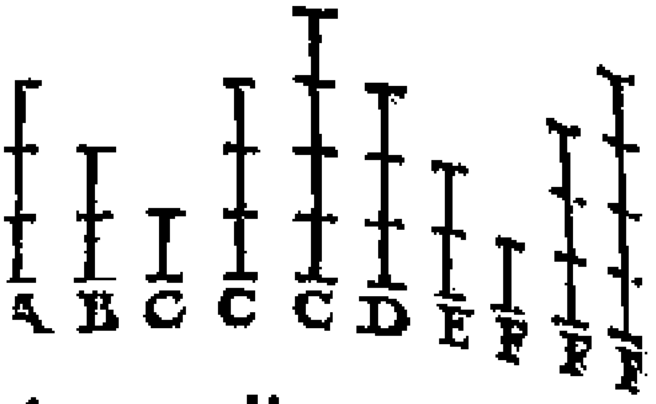
Theorema 19. Propositio 19.



Si quemadmodum totũ ad totum, ita ablatum se habuerit ad ablatum: & reliquum ad reliquum, vt totum ad totum se habebit.

Theorema 20. Propositio 20.

Si sint tres magnitudines, & aliæ ipsis æquales numero, quæ binæ & in eadem ratione sumatur, ex æquo auté prima quàm tertia maior fuerit: erit & quarta, quã sexta maior. Quòd si

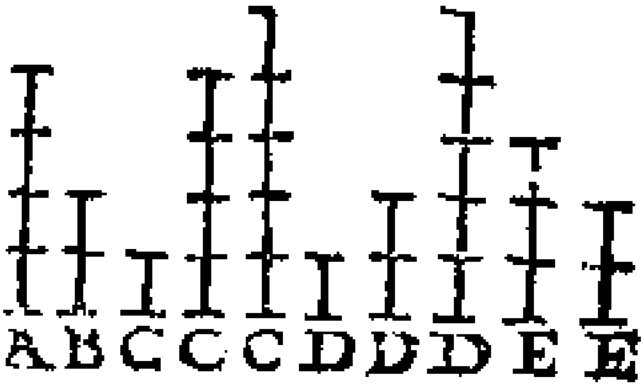


prima tertiæ fuerit æqualis, erit & quarta æqualis sextæ: sin illa minor, hæc quoque minor erit.

Theore-

Theorema 21. Propoſitio 21.

Si ſint tres magnitudines, & alię ipſis æquales numero quæ binæ & in eadem ratione ſumãtur, fuerit que perturbata earũ proportio ex æquo autem prima quàm tertia maior fuerit erit & quarta quã ſexta maior: quòd ſi prima tertiæ fuerit æqualis, erit & quarta æqualis ſextæ: ſin illa minor, hæc quoque minor erit.



Theorema 22. Propoſitio 22.

Si ſint quotcunq; magnitudines, & alię ipſis ęquales numero, quæ binæ in eadẽ ratione ſumantur, et ex æqualitate in eadem ratione erunt.

Theorema 23. Propoſitio 32.

Si ſint tres magnitudines, aliæq; ipſis ęquales

les

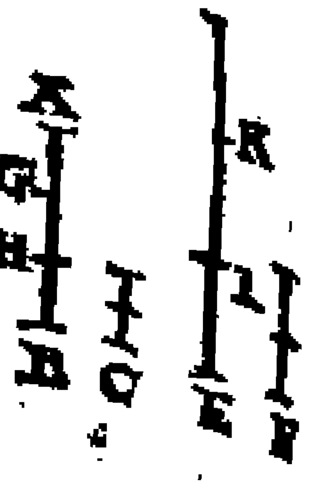
les numero, q̃ binę in eadem ratione ſumã-tur, fuerit auté perturbata ea-rũ proportio: etiã ex æquali-te in eadem ra tione erunt.

Theorema 24. Propoſitio 24.

Si prima ad ſecundam, eandem habuerit rationẽ, quam tertia ad quartam, habuerit autem & quinta ad ſecundam, eandẽ rationem, quã ſexta ad quartam: etiam cõpoſita prima cũ quinta ad ſecundam eandẽ habebit rationẽ quam tertia cum ſexta ad quartam.

Theorema 25. Propoſitio 25.

Si quatuor magnitudines proportionales fuerint, maxima & minima reliquis duabus maiores erunt



ELEMENTI V. FINIS.

EVCLIDIS ELEMENTVM SEXTVM.

DEFINITIONES.

1

SImiles figuræ rectilineæ sunt, quæ & angulos singulos singulis æquales habēt, atque etiam latera, quę circum angulos æquales, proportionalia,

2

Reciprocæ autem figuræ sunt, cum in vtraque figura antecedentes & consequentes rationum termini fuerint.

3

Secundum extremam & mediam rationē recta linea secta esse dicitur, cùm vt tota ad maius segmentum, ita maius ad minus se habuerit.

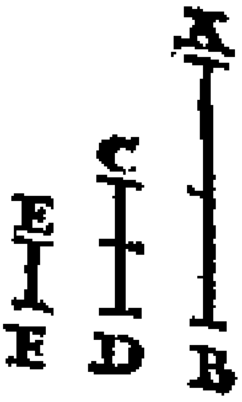
4

Altitudo cuiusque figurę, est linea perpendicularis à vertice ad basin deducta

5 Ra-

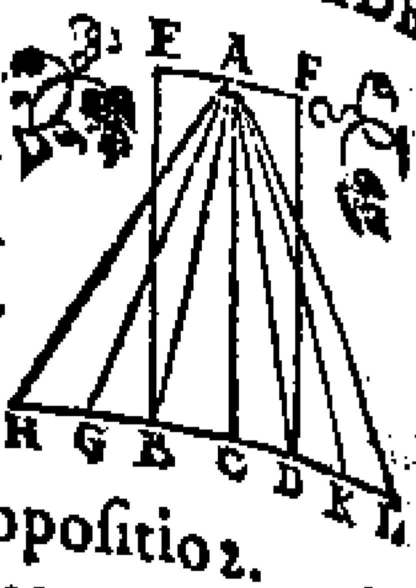
5.

Ratio ex rationibus cō poni dicitur, cū rationum quantitates inter In multiplicatæ aliqua effecerint rationem.



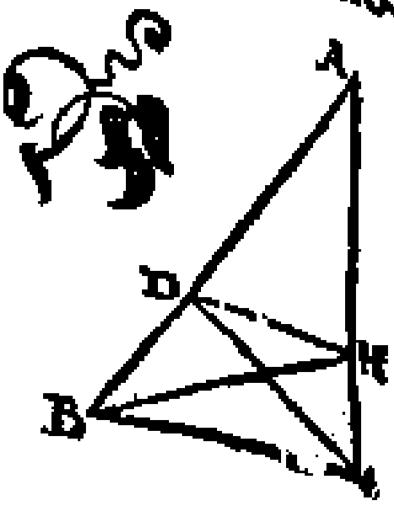
Theorema 1. Propositio. 1

Triangula & parallelogramma, quorum eadem fuerit altitudo, ita ſe habent inter ſe vt baſes.



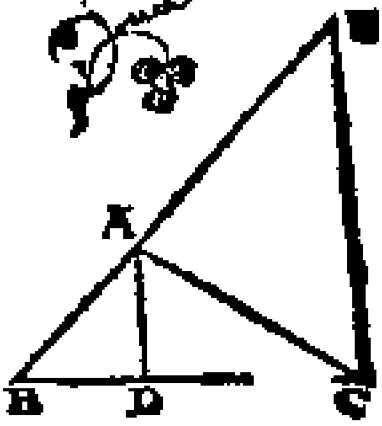
Theorema 2. Propoſitio 2.

Si ad vnum trianguli latus parallela ducta fuerit recta quædam linea: hęc proportionaliter ſecabit, ipſius triāguli latera. Et ſi trianguli latera proportionaliter ſecta fuerint: quæ ad ſectiones adiūcta fuerit recta linea, erit ad reliquum ipſius trianguli latus parallela.



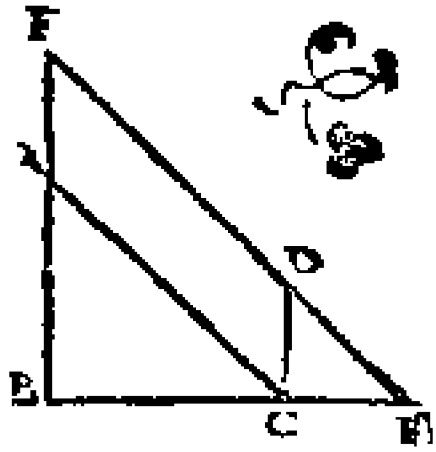
Theorema 3. Propoſitio 3.

Si trianguli angulus bifariam ſectus ſit, ſecans autē augulum recta linea ſecuerit & baſim: baſis ſegmenta eandem habebunt rationem,



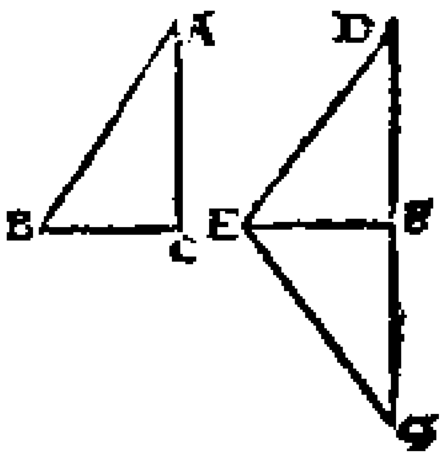
tionem, quam reliqua ipsius trianguli latera, Et si basis segmenta eandem habeant rationem quam reliqua ipsius trianguli latera, recta linea, quæ à vertice ad sectionem producitur, ea bifariã secat trianguli ipsius angulum.

Theorema 4. Propositio 4



Aequiangulorum triangulorum proportianolia sunt latera, quę circum ęquales angulos, & homologa sunt latera, quę ęqualibus angulis subtenduntur.

Theorema 5. Propositio 5.



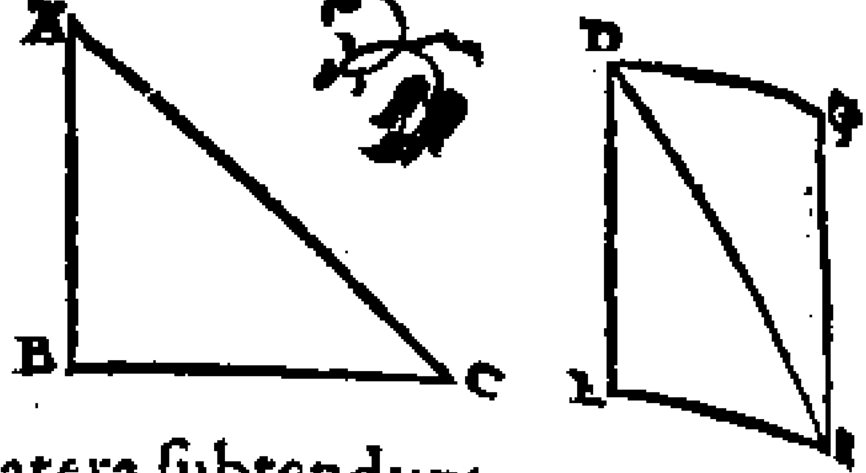
Si duo triãgula latera proportionalia habeant, ęquiangula erunt triangula, & æquales habebunt eos angulos, sub quibꝰ homologa latera subtenduntur.

Theorema 6. Propositio 6.

Si duo triangula vnum angulũ vni angulo æqualẽ, & circum ęquales angulos latera ꝓportionalia habuerint, ęquiangula erunt

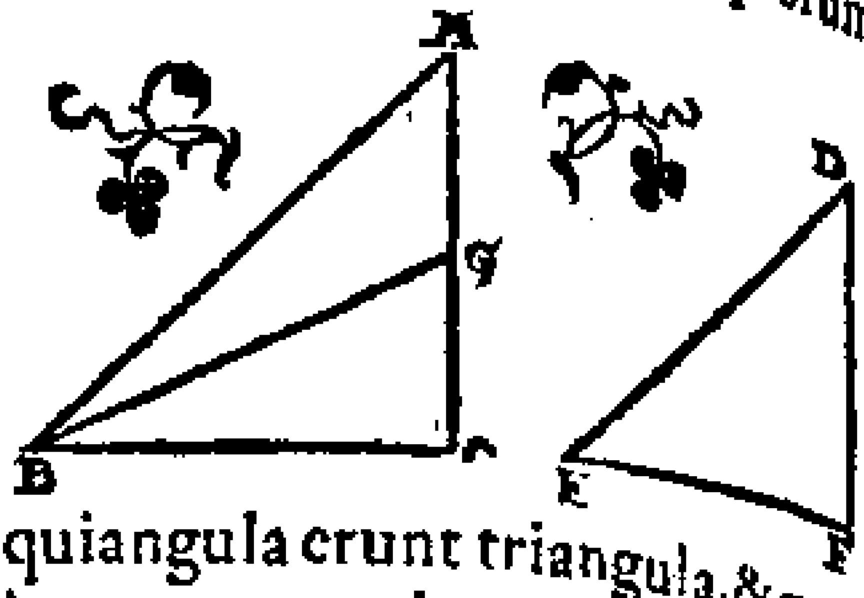
trian

triangula, ęqualesq́; habebunt angulos ſub quibus homologa latera ſubtenduntur.



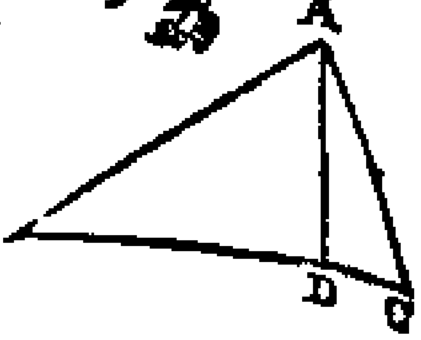
Theorema 7. Propoſitio 7.

Si duo triangula vnum angulum vni angulo æqualem, circũ autem alios angulos latera proportionalia habent, reliquorum vero ſimul vtrunq; aut minorem aut non minorẽ recto: æquiangula erunt triangula, & æquales habebunt eos angulos circum quos proportionalia ſunt latera.



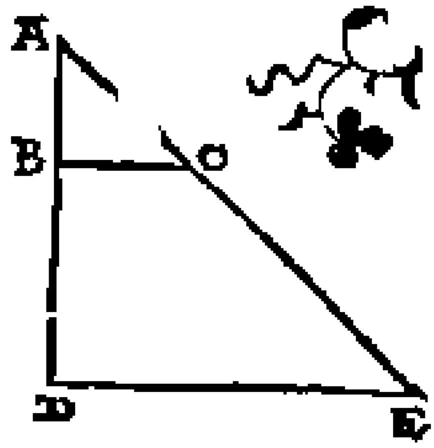
Theorema 8. Propoſitio 8.

Si in triangulo rectangulo, ab angulo recto in baſin perpendicularis ducta ſit que ad perpendicularem triangulo, tum toti triangulo, tum ipſa inter ſe ſimilia ſunt.



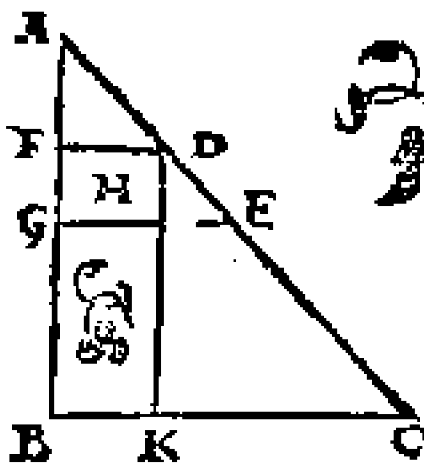
Problema 1. Propositio 9.

A data recta linea imperatam partem auferre.



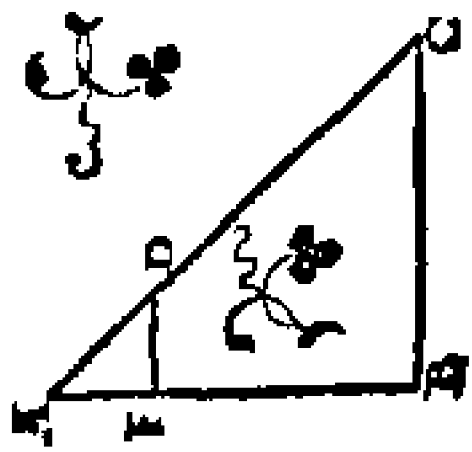
Problema 2. Propositio 10.

Datam rectã lineam in sectam similiter secare, vt data altera recta secta fuerit.



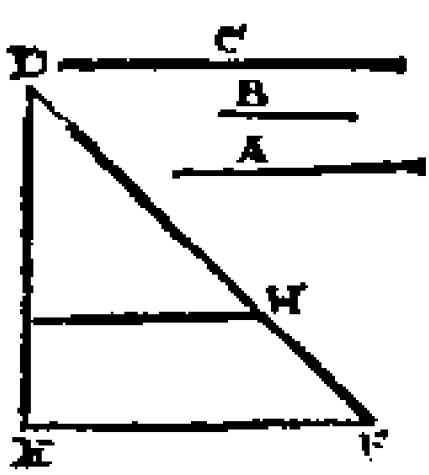
Problema 3. Propositio 11.

Duabus datis rectis lineis, tertiam proportionalem ad inuenire.



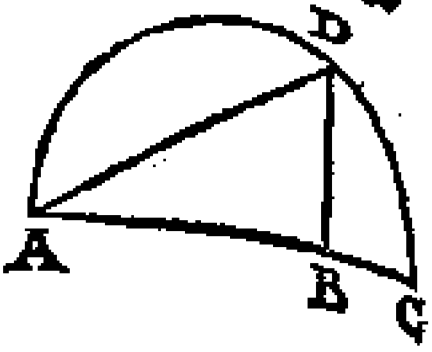
Problema 4. Propositio 12.

Tribꝰ datis rectis lineis quartam proportionalem adinuenire.



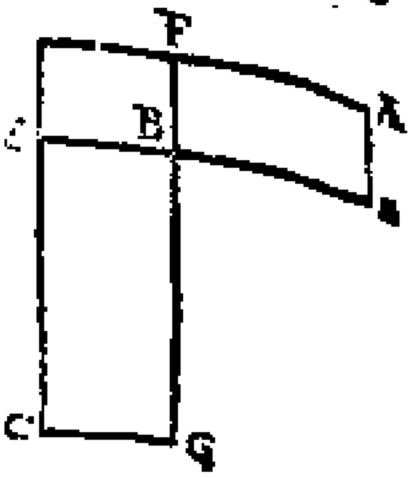
Problema 5. Propositio 13.

Duab' datis rectis lineis mediam proportionalé adinuenire.



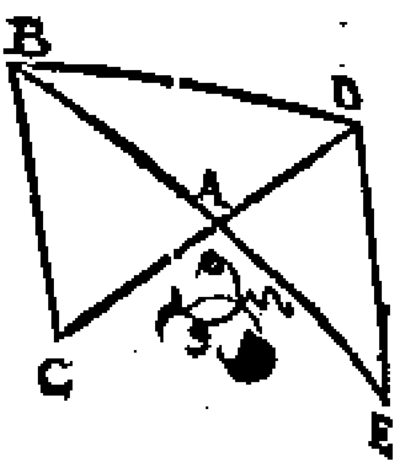
Theorema 9. Propositio 14.

Aequalium, & vnú vni æqualem habentiú angulum parallelogrammorum reciproca sunt latera, quæ circum æquales angulos. & quorú parallelogrammorú vnum angulum vni angulo æqualem habentiú reciproca sunt latera, quæ circum æquales angulos, illa sunt æqualia.



Theorema 10. Propositio 15.

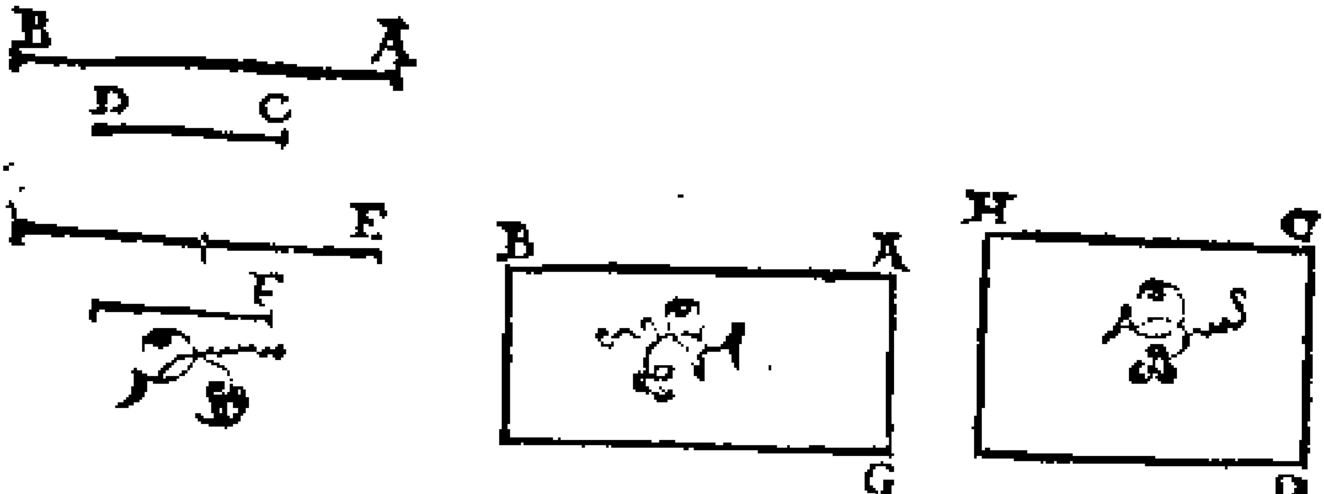
Aequalium, & vnú angulum vni æqualem habentiú triangulorú reciproca sunt latera, quæ circum æquales angulos: & quorú triangulorú vnum angulum vni æqualem habentium reciproca sunt latera, q́ circum æquales angulos, illa sunt æqualia.



Theo.

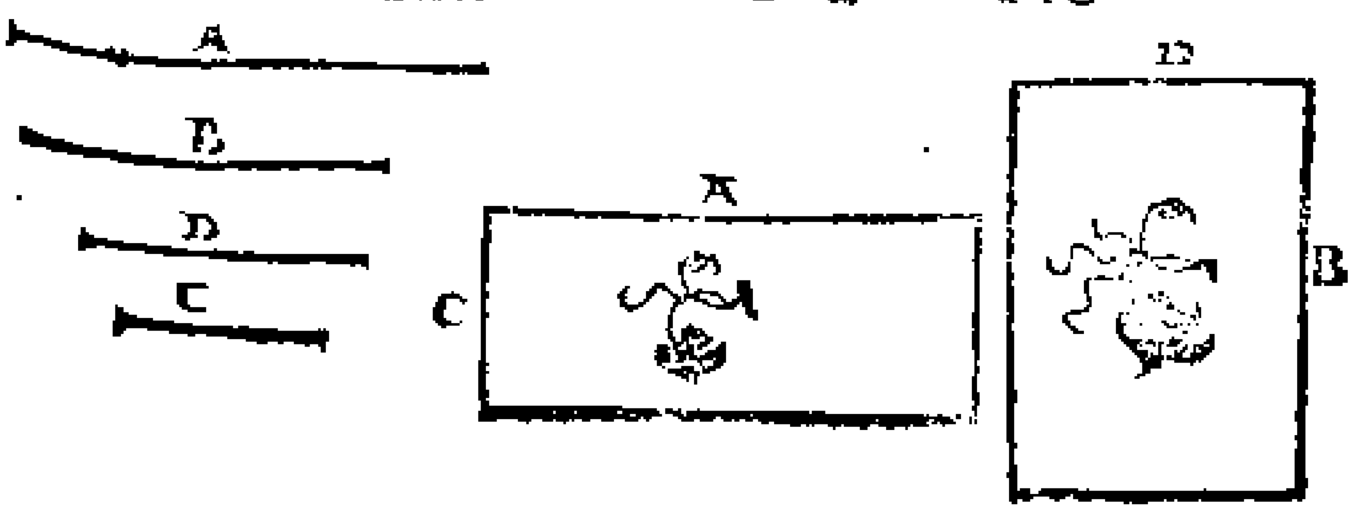
Theorema 11. Propositio 16.

Si quatuor rectæ lineæ proportionales fuerint, quod sub extremis comprehenditur rectangulũ, æquale est ei, quod sub medijs comprehenditur rectãgulo. Et si sub extremis comprehensum rectangulum æquale fuerit ei, quod sub medijs continetur rectã gulo, illæ quatuor rectæ lineæ proportionales erunt.



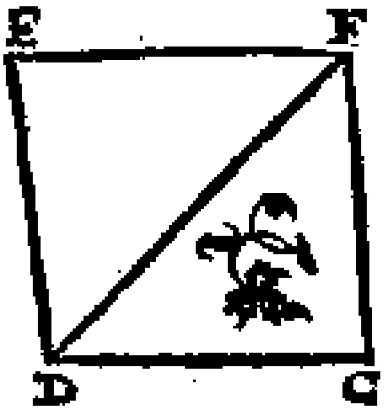
Theorema 12. Propositio 17.

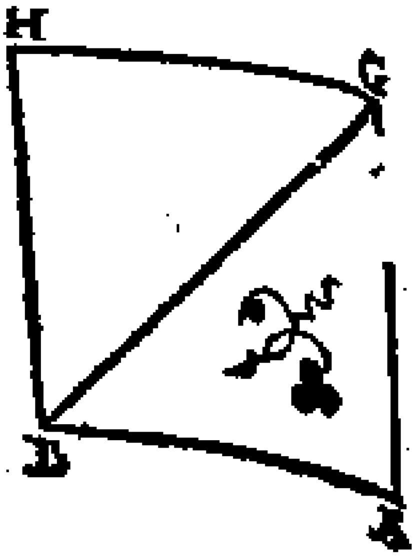
Si tres rectæ lineæ sint proportionales, qđ sub extremis cõprehenditur rectangulũ ęquale est ei, quod à media describitur quadrato: & si sub extremis comprehẽsum rectangulum æquale sit ei quod à media describitur quadrato, illæ tres rectę lineę proportionales erunt.



Problema 6. Propositio 18.

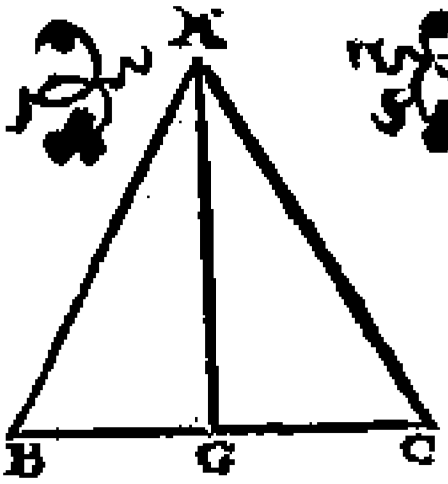
A data recta linea, dato rectilineo simile similiterq; positum rectilineum describere.

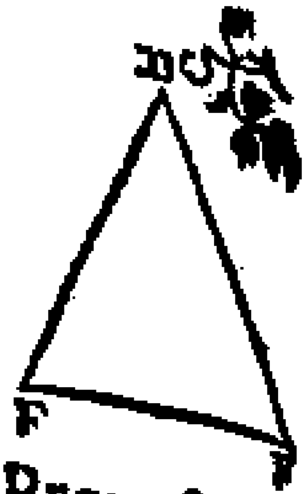




Theorema 13. Propositio 19.

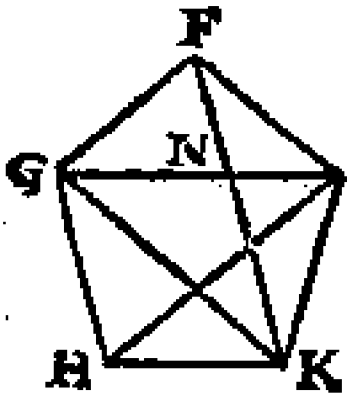
Similia triãgula inter se sunt in duplicata ratione laterũ homologorũ.

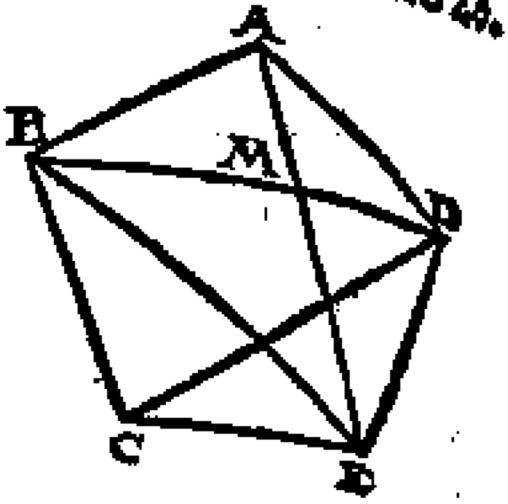


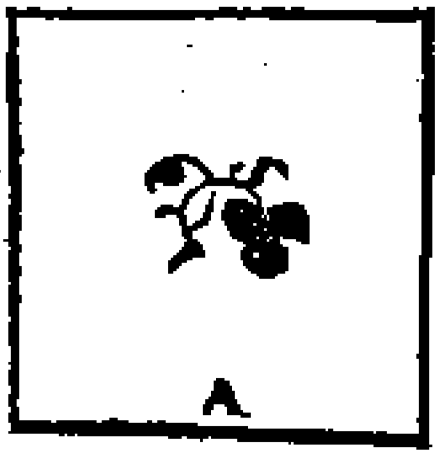


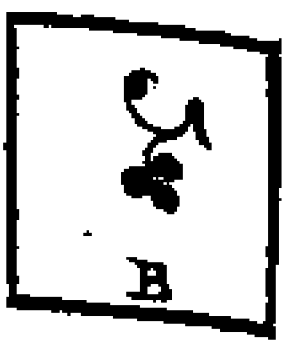
Theore. 14. Propositio 20.

Similia polygona in similia triãgula diuidũtur, & numero equalia, & homologa totis, Et po

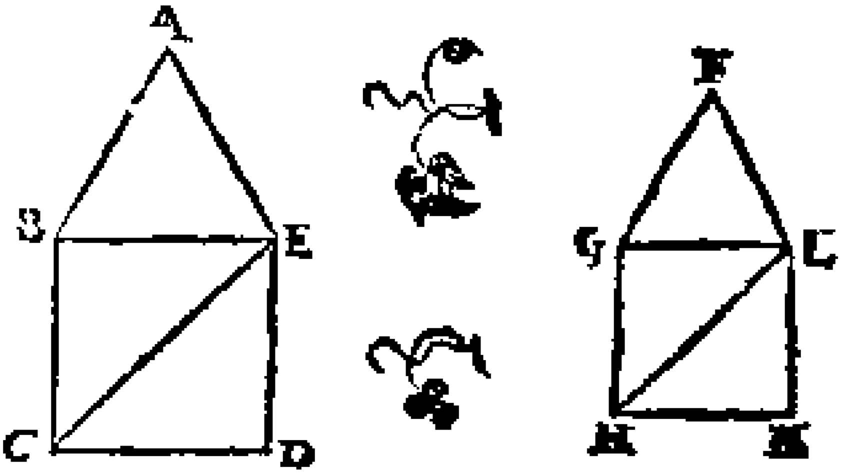




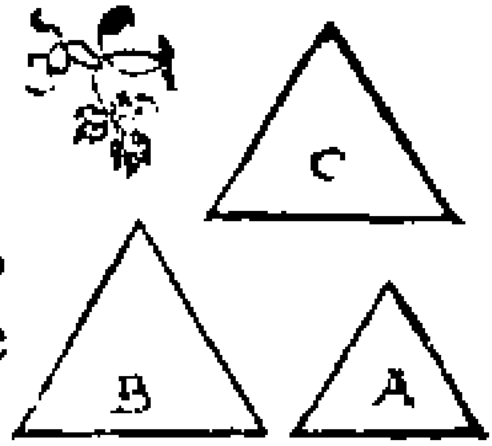




lygona duplicatam habent eã inter se rationẽ, quã latus homologũ ad homologum latus.



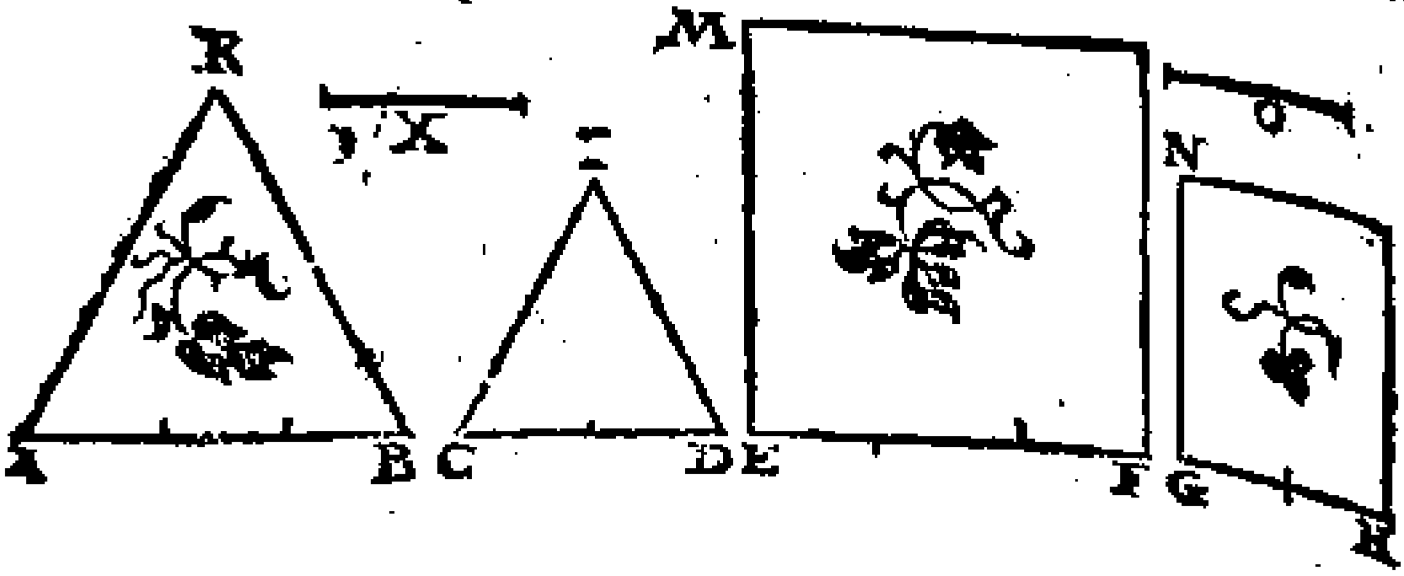
Theorema 15. Propositio 21.



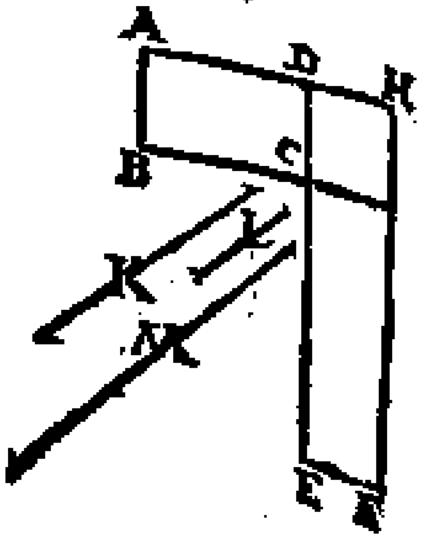
Quæ eidem rectilineo sunt similia, & inter se sunt similia.

Theorema 16. Propositio 22.

Si quatuor rectę lineæ proportionales fuerint: & ab eis rectilinea similia similiterq́; descripta proportionalia erũt. Et si à rectis lineis similia similiterq́; descripta rectilinea proportionalia fuerint, ipsæ etiam rectæ lineæ proportionales erunt.

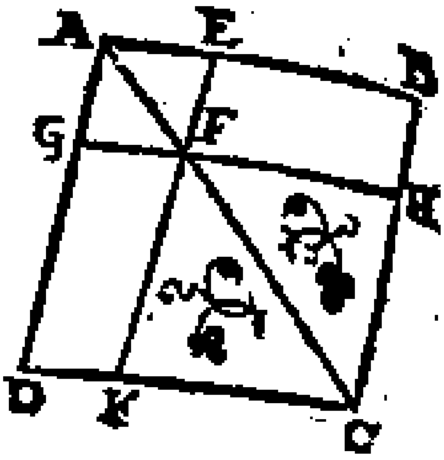


Theorema 17. Propositio 23.



Aequiangula parallelogramma inter se rationé habent eum, quę ex lateribus componitur.

Theorema 18. Propositio 24.

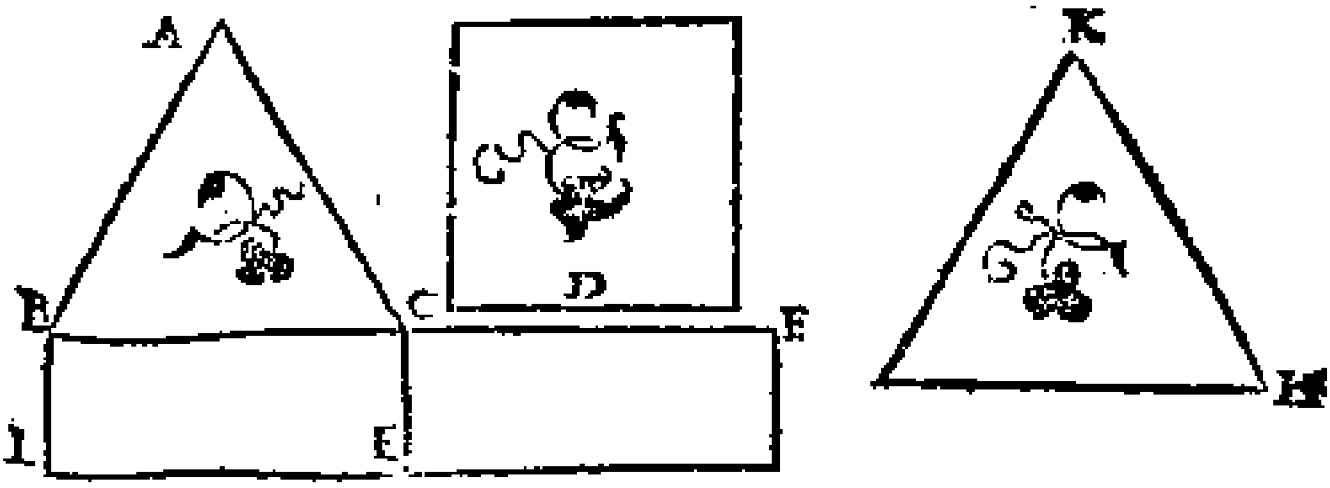


In omni parallelogrammo, quæ circa diametrú sunt parallelográma, & toti & inter se sunt similia.

Proble.

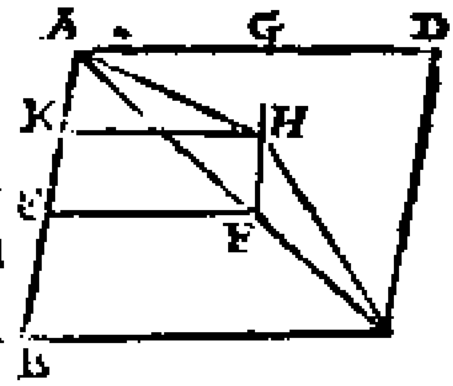
Problema 7. Propositio 25.

Dato rectilineo simile, & alteri dato æquale idem constituere.



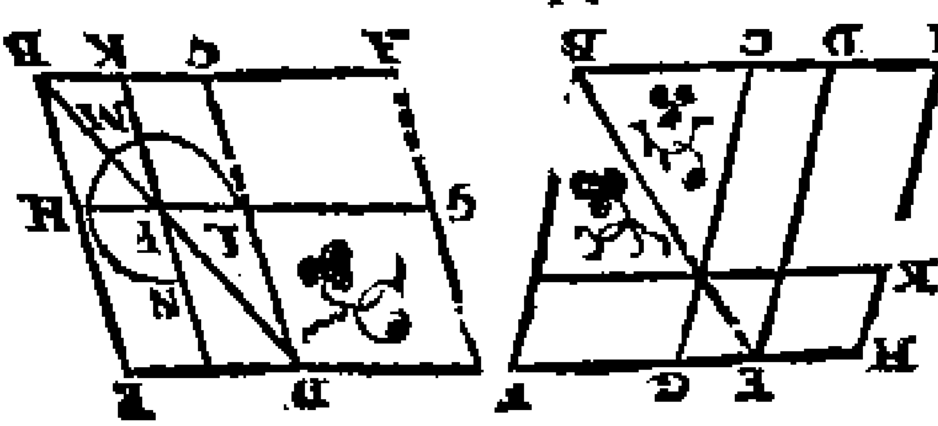
Theorema 19. Propositio 26.

Si à parallelogrãmo parallelogrammũ ablatũ sit, & simile toti & similiter positũ communẽ cum eo habens angulum, hoc circum eandem cum toto diametrum consistit.



Theorema 20. Propositio 27.

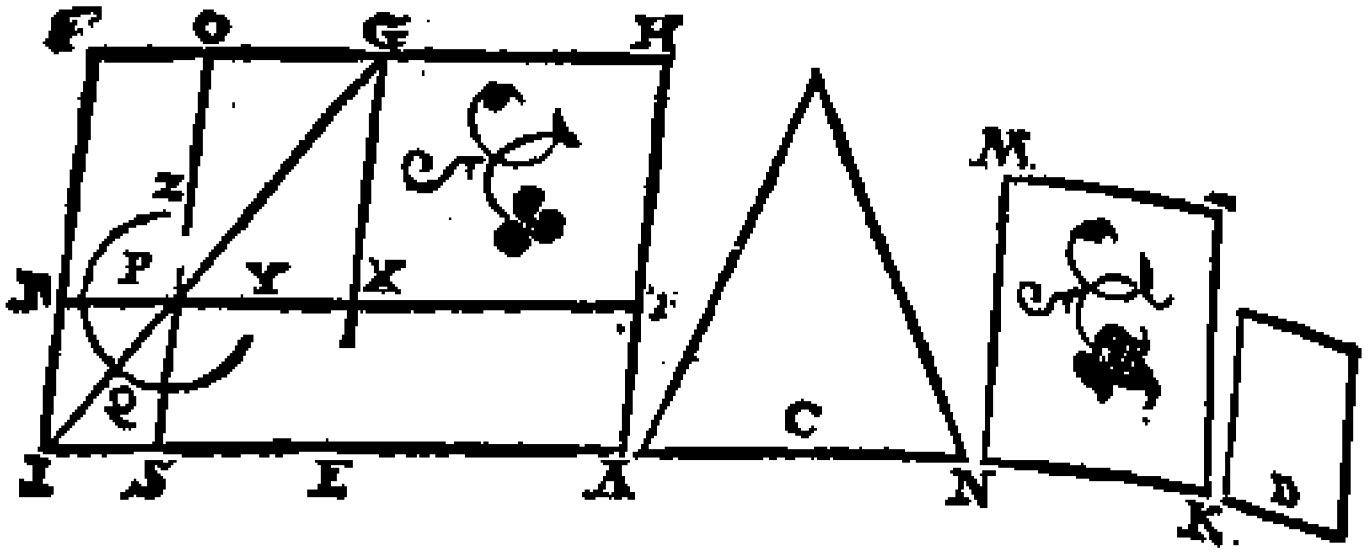
Omnium parallelogrãmorum secũdum eandem rectam lineam applicatorũ deficiẽtiumq; figuris parallelogrãmis si



milibus ſimiliterque poſitis ei, quod à di-
midia deſcribitur, maximum, id eſt, quod
ad dimidiam applicatur parallelogram-
mum ſimile exiſtens defectui.

Problema 8. propoſitio 28.

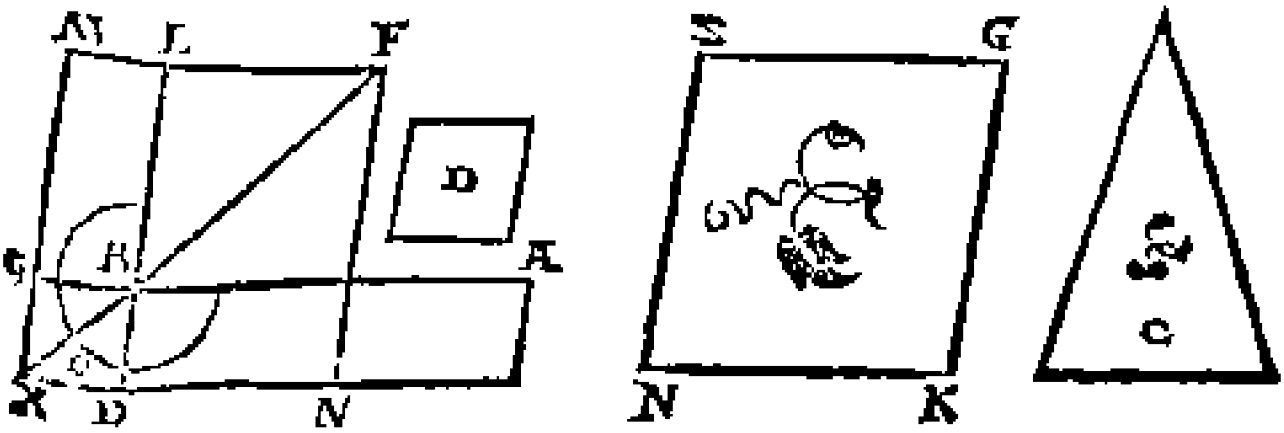
Ad datam lineam rectam, dato rectilineo
æquale parallelogrammum applicare de-
ficiens figura parallelogramma, quę ſimi-
lis ſit alteri rectilineo dato. Oportet autẽ
datum rectilineum, cui æquale applican-
dum eſt, non maius eſſe eo quod ad dimi-
diam applicatur, cum ſimiles ſint defectus
& eius quod à dimidia deſcribitur, & eius
cui ſimile deeſſe debet.



Problema 9. Propo-
ſitio. 29.

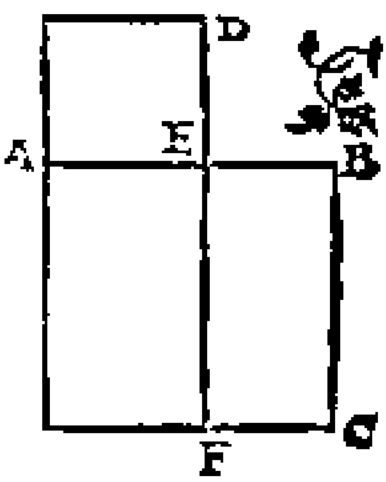
Ad datam rectam lineam, dato rectilineo
æquale parallelogrammũ applicare, exce-
dẽs figura parallelográma, quæ ſimilis ſit
paral-

parallelogrammo alteri dato.



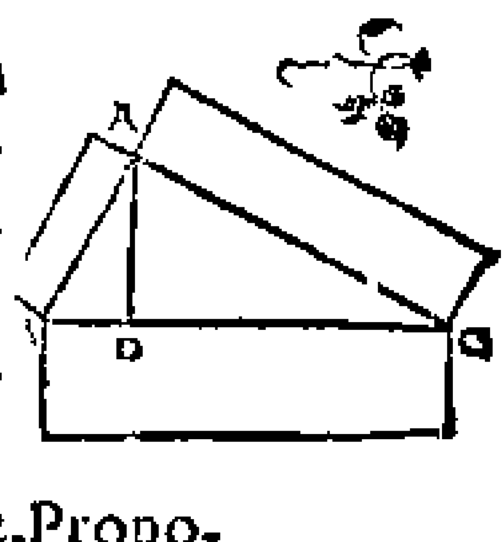
Prblema 10. Propositio 30.

Propositam rectam lineam terminatã, extrema ac media ratione secare.



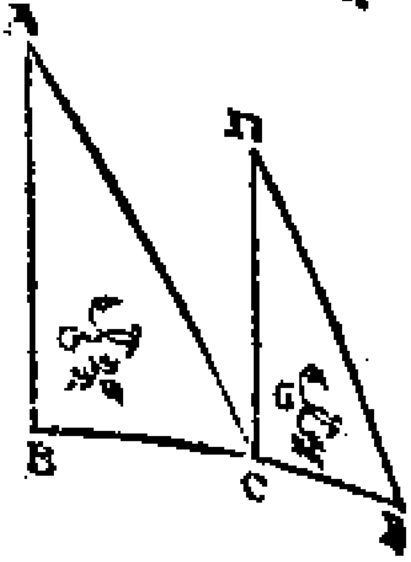
Theorema 21. Propositio 31

In rectangulis triãgulis, figura quęuis à latere rectũ angulũ subtendẽte descripta ęqualis est figuris, quæ priori illi similes & similiter positę à lateribus rectũ angulum continentibus describuntur.



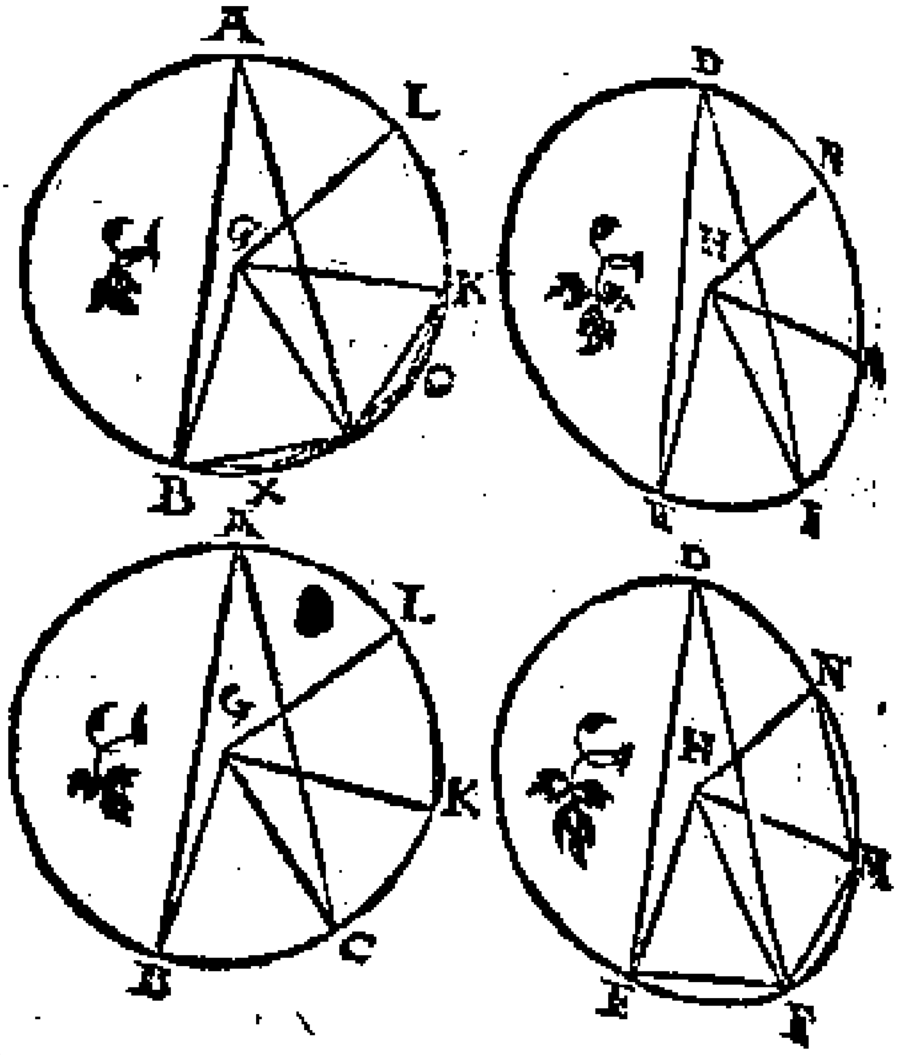
Theorema 22. Propositio 32.

Si duo triãgula, quę duo latera duobus lateribus proportionalia habeant, secũdum vnum angulū composita fuerint, ita vt homologa eorum latera sint etiam parallela, tum reliqua illorum ttiangulorum latera in rectam lineam collocata reperientur.



Theorema 23. Propositio 33.

In æqualibus circulis anguli eandē habent rationem. cū ipsis peripherijs in quibus insistūt, siue ad cētra siue ad peripherias constituti, illis insistant peripherijs In super verò & sectores qppe qui ad cētra cōsistūt.



ELEMENTI VI. FINIS.

EVCLIDIS ELEMENTVM SEPTIMVM.

DEFINITIONES.

1

Vnitas, eſt ſecũdum quam entium quodque dicitur vnum.

2

Numerus autem, ex vnitatibus compoſita multitudo.

3

Pars, eſt numerus numeri minori maioris, cùm minor metitur maiorem.

4

Partes autem, cùm non metitur.

5

Multiplex verò, maior minoris, cùm maiorem metitur minor.

6

Par numerus eſt, qui bifariã non diuiditur.

7

Impar verò, qui bifariam non diuiditur, vel, qui vnitate differt à pari.

8

Pariter par numerus eſt, quem par numerus metitur per numerum parem.

9.

Pariter autem impar, est quem par numerus metitur per numerum imparem.

10.

Impariter verò impar numerus, est quem impar numerus metitur per numerum imparem.

11.

Primus numerus, est quem vnitas sola metitur.

12.

Primi inter se numeri sunt, quos sola vnitas mensura communis metitur.

13.

Compositus numerus est, quem numerus quispiam metitur.

14.

Compositi autẽ inter se numeri sunt, quos numerus aliquis mensura communis metitur.

15.

Numerus numerum multiplicare dicitur, cùm toties compositus fuerit is, qui multiplicatur, quot sunt in illo multiplicante vnitates, & procreatus fuerit aliquis.

16.

Cùm autẽ duo numeri mutuò sese multiplicantes quempiam faciũt, qui factus erit planus appellabitur, qui verò numeri mutuò sese multiplicarint, illi⁹ latera dicẽtur.

17. Cùm

17

Cùm verò tres numeri mutuò sese multiplicantes quempiā faciunt, qui procreatus erit solidus apellabitur, qua autem numeri mutuò sese multiplicarint, illius, latera dicentur.

18.

Quadratus numerus est, qui ęqualiter ęqualis: vel, qui à duobus ęqualibus numeris cótinetur.

19.

Cubus verò, qui à tribus æqualibus æqualiter: vel, quia tribus æqualibus numeris continetur.

10

Numeri proportionales sunt, cum primus secundi, & tertitius quarti æquè multiplex est, vt eadem pars, vel eædem partes.

21

Similes plani & solidi numeri sunt, qui proportionalia habent latera.

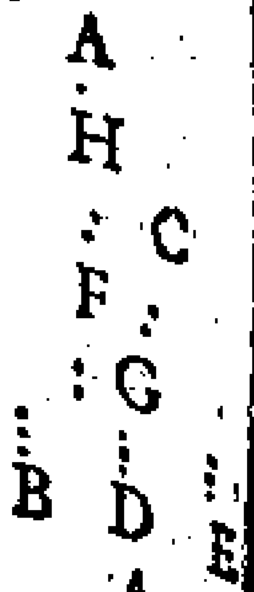
22

Perfectus numerus est, qui suis ipsius partibus est æqualis.

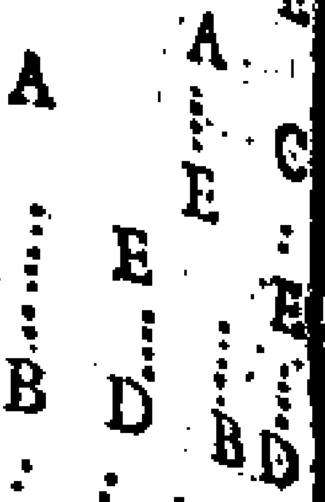
Theorema 1. Propositio 1.

Duobus numeris in æqualibus propositis,

tis, si detrahatur semper minor de maiore, alterna, quadam detractione; neque reliquus vnquam metiatur præcedentem quo ad assumpta sit vnitas: qui principio propositi sunt numeri primi inter se erunt.

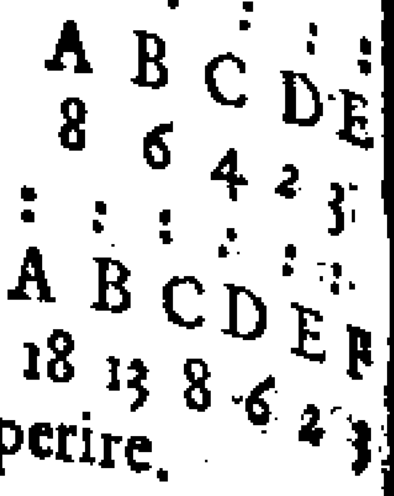


Porblema 1. Propositio 2.



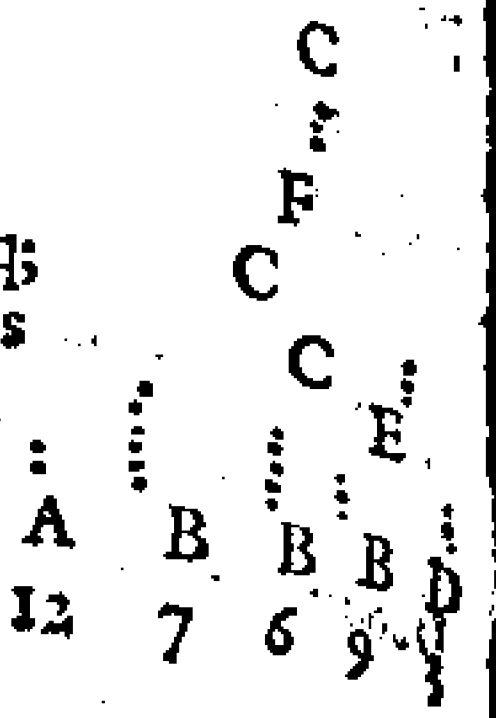
Duobus numeris datis non primis inter se, maximã eorũ cõmunẽ mensurã reperire

Broblema 2. Propositio. 3



Tribus numeris datis non primis inter se, maximam eorum communem mensuram reperire.

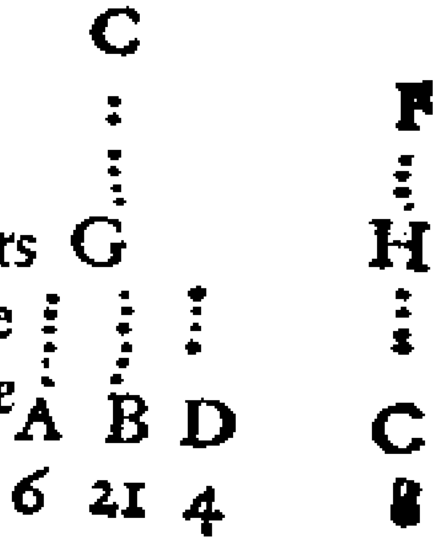
Theorema 18. Propositio 8.



Omnis numerus, cuiusq; numeri minor maioris aut par est, aut partes.

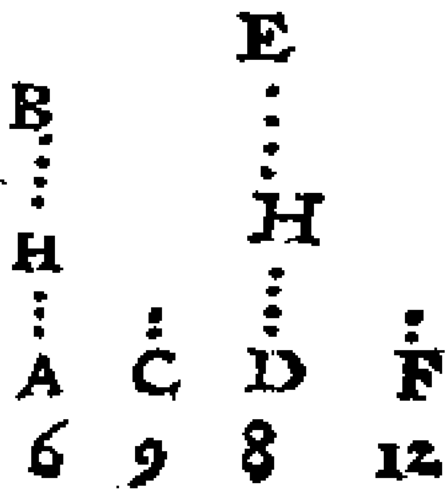
Theore.

Theorema 3. Propositio 5.



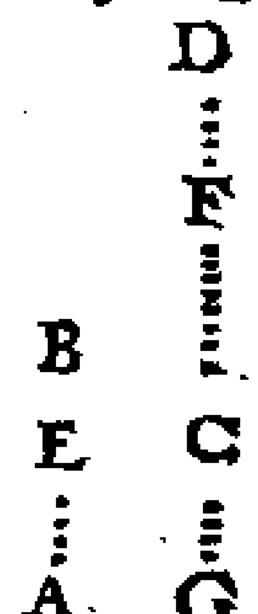
Si numeris numeri par fuerit, & alter alterius eadẽ pars & simul vterque vtriusque simul eadem pars erit, quæ vnus est vnius.

Theor. 4. Propo. 6.



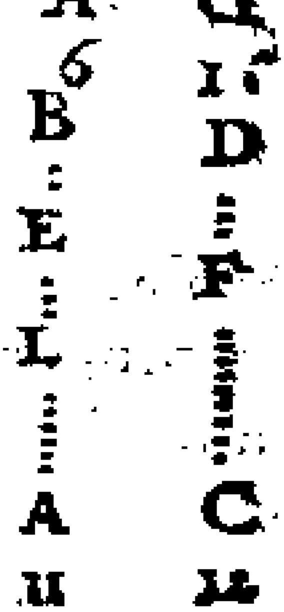
Si numer⁹ sit numeri partes, & alter alterius eędem partes, & simul vterque vtriusque simul eædẽ partes erunt, quæ sunt vnus vnius.

Theorema 5. Propositio 7.



Si numerus numeri eadẽ si pars quæ detractus detracti, & reliquus reliqui eadẽ pars erit, quæ totus est totius.

Theorema 6. Propositio. 8.

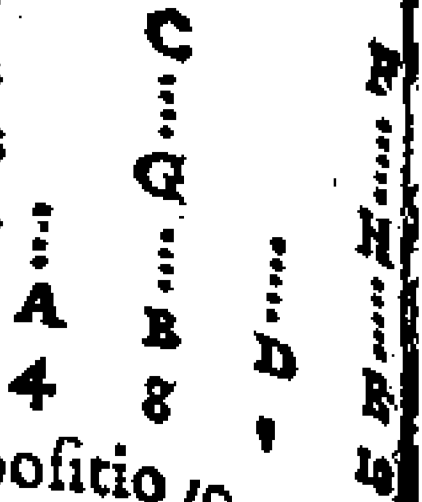


Si numerus numeri eædẽ sint partes quæ detractus detracti, & reliquus reliqui eędẽ partes erũt, quę sũt totus totius. Theo...

G..M.K..N,H.

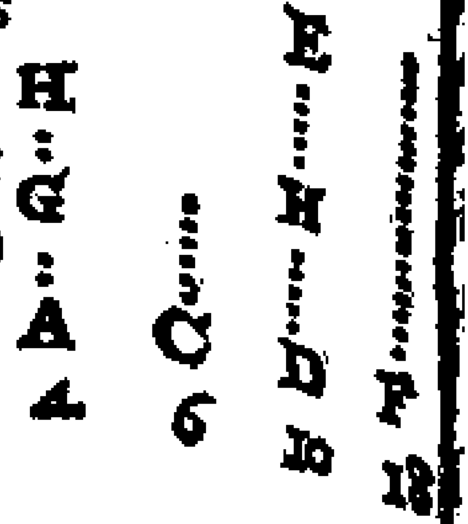
Theorema 7. Propositio 9.

Si numerus numeris pars sit, & alter alterius eadem pars, & viciſſim quæ pars eſt vel partes primus tertij, eadem pars erit eędem partes, & ſecūdus quarti.



Theorema 8. Propoſitio 10.

Si numerus numeri partes ſint, & aliter alterius eadē partes, etiam viciſſim quæ ſunt partes aut pars primus tertij, eędem partes erūt vel pars & ſecundus quarti.

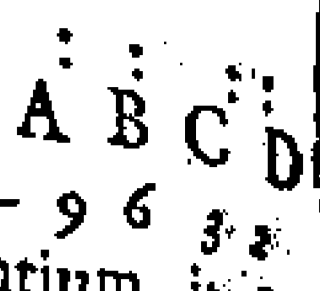


Theorema 9. Propoſitio 11.

Si quemadmodum ſe habet totus ad totum, ita detractus ad detractum, & reliquus ad reliquum ita habebit vt totus ad totum

Theorema 10. Propoſitio 12.

Si ſint quotcunque numeri proportionales; quemadmodum ſe habet vnus antecedentium ad vnum ſequentium, ita ſe habe-



habebunt omnes antecedentes ad omnes consequentes.

Theorema 11. Propositio 13.

Si quatuor numeri sint proportionales, & vicissim proportionales erunt.

A	B	C	D
12	4	9	3

Theorema 12 Propositio 14

Si sint quotcunque numeri, & alij illis æquales multitudine, qui bini sumantur & in eadem ratione: etiam ex æqualitate in eadem ratione erunt

A	B	C	D	E	F
12	6	3	8	4	2

Theorema 13. Propositio 15.

Si vnitas numerum quempiã metiatur, aliter verò numerus alium quendã numerũ æquè metiatur, & vicissim vnitas tertium numerũ æquę metietur, atque secundus quartum.

F
L
C K
H E
G
A B D
1 3 6

Theorema 14. Propositio 16

Si duo numeri mutuò sese multiplicãtes faciant aliquos qui ex illis geniti fuerint, inter se æquales erunt.

E	A	B	C	D
1	2	4	8	8

Theorema 15. Propositio 17.

Si numer⁹ duos numeros multiplicãs faciat aliquos, qui ex illis procreati erunt, eadem rationem habebunt, quam multiplicati.

I	A	B	C	D	E
1	3	4	5	12	15

Theorema 16. Propositio 18.

Si duo numeri numerum quempiam multiplicantes faciant aliquos, geniti ex illis eandem habebunt rationem, quam qui illum multiplicarunt.

A	B	C	D	E
4	5	3	12	15

Theorema 17. Propositio 19.

Si quatuor numeri sint proportionales, qui ex primo & quarto fit, ęqualis erit ei qui ex secundo & tertio: & si qui ex primo & quarto fit numerus æqualis sit ei qui ex secũdo & tertio, illi quatuor numeri sint proportionales erunt.

A	B	C	D	E	F	G
6	4	3	2	12	12	18

Theorema 18. Propositio 20.

Si tres numeri sint proportionales, qui ab extremis cótinetur, ęqualis est ei qui à medio

io efficitur. Et ſi qui ab ex-
emis cõtinetur æqualis ſit
qui à medio deſcribitur, il
tres numeri proportiona-
es erunt.

A B C
9 6 4
D
6

Theorema 19. Propoſitio. 21.

Minimi numeri omniũ, ui eandem cũ eis ratio- em habẽt, æqualiter me- tur numeros eandem ionem habẽtes, maior uidem maiorem, minor erò minorem

D L
G H
C E A B
4 3 8 6

Theorema 20. Propoſitio 22.

res ſint numeri & alij multitudine illis uales, qui bini ſumantur & in eadem ra- ne, ſit autem perturbata eorũ propor- etiam ex æ- litate in ea- m ratione e- nt.

A B C D E F
6 4 3 12 8 6

Theorema 21. Propoſitio 23.

mi inter ſe numeri minimi ſunt omniũ dem cum eis ra- em habentium.

A B E C D
5 6 2 4 5
H 2

Theo-

Theorema 22 Propositio 24.

Minimi uumeri omnium eandem cum eis rationem habentiũ, primus sunt inter erit

A B C D E
8 6 4 3 2

Theorema 23. Propositio 2ſ.

Si duo nu meri ſint prlmi inter ſe, qui alte utrum illorum metitur numerus, is ad' reliquũ primus erit.

A B C D
6 7 3 4

Theorema 24. Propotio 2ſ.

Si duo oumtri ad qué piam numerum primi ſint, an eundem primus is quoque futurus eſt, qui ab illis productus fuerit.

B A C D E
5 5 5 3

Theorema 25. Propoſitio 27.

Si duo numerì pirmi ſint inter ſe. q ab vno corũ gignitur ad reliquum primus erit.

B A C D
7 6 3

Theorema 26. Propoſitio 28.

Si duo numeri ad duos numeros ambo ad vtrunque primi ſint, & qui ex eis gignentur, primi inter ſe èrunt.

A B E C D F
3 5 15 2 4 8

Theore

Theorema 27. Propositio 29.

duo numeri primi sint inter se, & multiplicãs vterq; seipsum procreet aliquem, qui ex ijs producti fuerint, primi inter se erũt. Quòd si numeri initio propositi multiplicantes eos qui producti sunt, effecerint aliquos, hi quoq; inter se primi erunt; & circa extremos idem hoc semper eueniet.

A	C	E	B	D	F
3	6	27	4	16	63

Theorema 28. Propositio. 30.

duo numeri primi sint inter se, etiam simul vterq; ad vtrunq; illorum primus erit. Et si simul vterq; ad vnum aliquem eorum primus sit, etiam qui initio positi sunt numeri, primi inter sunt erunt.

A	B	C	D
7	5		4

Theorema 29. Propositio 31.

Omnis primus numerus ad omnem numerum quem non metitur, primus est.

A	B	C
7	10	5

Theorema 30. Proositpio 32.

duo numeri sese mutuo multiplicãtes faciant aliquem, hũc aũt ab illis productum metiatur primus quidam numerus, is alterum etiam metitur eorum qui initio positi erant.

A	B	C	D	E
3	6	12	3	4

Theorema 31. Propositio 33.

Omnem compositum numerum aliquis primus metietur.

A B C
27 9 3

Theorema 32. Propositio 34.

Omnis numerus aut primus est, aut eum aliquis primus metitur.

A A
3 6

Problema 3. Propositio 35.

Numeris datis quotcunque, reperire minimos omnium qui eandem cum illis rationem habeant.

A B C D E F G H K L M
6 8 12 2 3 4 6 2 3 4 3

Problema 4. Propositio 36.

Duobus numeris datis reperire quẽ illi minimum metiantur numerum.

B
A C D E F
7 12 8 4 5

A
F E C D G H
6 9 12 9 2 3

Theorema 33. Propositio 37

Si duo numeri numerũ quempiam metiantur, & minimus quem illi metiuntur eundem metietur.

F
A B E C
2 3 6 12

Problema 5. Propositio 38.

Tribus numeris datis reperire quem minimum numerũ illi metiantur.

A B C D E
3 4 6 12 8

A B C D E F
3 6 8 12 24 16

Theorema 34. Propositio 39.

Si numerum quiſpiam numerus metiatur, menſus partem habebit metienti cognominem.

A B C D
12 4 3 1

Theorema 35. Propoſitio 40.

Si numerus partem habuerit quãlibet, illum metietur numerus parti cognominis.

A B C D
8 4 2 1

Problema 6. Propoſitio 41.

Numerum reperire, qui minimus cùm ſit, datas habeat partes.

A B C G H
2 3 4 12 10

ELEMENTI VII. FINIS.

EVCLIDIS ELEMENTVM OCTAVVM.

Theorema 1. Propositio 1.

Si sunt quotcunq; numeri deinceps proportionales, quorum extremi sint inter se, primi, minimi sunt omnium eandem cum eis rationem habentium.

A	B	C	D	E	F	G	H
8	12	18	27	6	8	12	18

Problema 1. Propositio 2.

Numeros reperire deinceps proportionales minimos, quotcunq; iusserit quispiam in data ratione.

A	B	C	D	E	F	G	H	K
3	4	9	12	16	27	36	49	64

Theorema 2. Propositio 3.
Conuersa primæ.

Si sint quotcunq; numeri deinceps proportionales minimi habentium eandem cum eis rationem, illorum extremi sunt inter se primi.

A	B	C	D	E	F	G	H	K	L	M	N	O
27	16	48	64	3	4	9	12	16	27	36	48	64

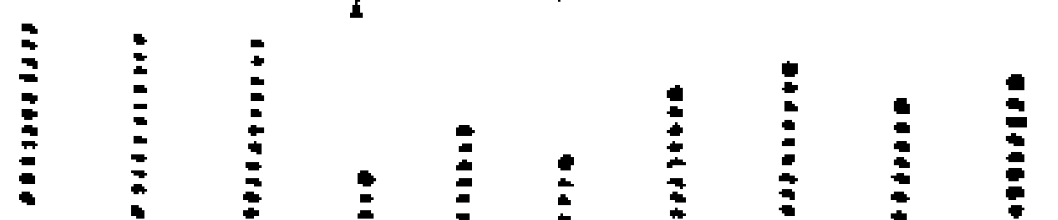
Problema 2. Propositio 4.

Rationibus datis quotcunque in minimis numeris reperire numeros deinceps minimos in datis rationibus.

A	B	C	D	E	F	H	G	K	L	N	X	M	O
3	4	2	3	4	5	6	8	12	15	4	6	10	12

Theorema 3. Propositio 5.

Plani numeri rationem inter se habent ex lateribus compositam.



A	L	B	C	D	E	F	G	H	K
18	22	32	3	6	4	8	9	12	16

Theorema 4. Propositio 4.

Si sint quotlibet numeri deinceps proportionales, primus autem secundum non metiatur, neque alius quispiam vllum metietur.

A	B	C	D	E	F	G	H
16	24	36	54	82	4	6	9

Theorema 5. Propositio 7.

Si sint quotcunque numeri deinceps proportionales, primus autem extremum metiatur, is etiam secundum metietur.

A	B	C	D
4	6	12	24

Theorema 6. Propositio 8.

Si inter duos numeros medij continua proportione incidant numeri, quot inter eos medij continua proportione incidunt numeri, tot & inter alios eandem cum illis habentes rationem medij continua proportione incident.

A	C	D	B	G	H	K	L	C	M	N	F
4	9	27	81	1	3	9	27	2	6	18	54

Theorema 7. Propositio 9.

Si duo numeri sint inter se primi, & inter eos medij cõtinua proportione incidãt numeri, quot inter illos medij continua proportione incidunt numeri, totidẽ & inter vtrunque eorum ac vnitatẽ deinceps medij continua proportione incident.

A	M	H	E	F	N	C	K	X	G	D	L	O	B
27	27	9	36	3	36	1	12	48	4	48	16	64	64

Theo-

Theorema 8. Propositio 10.

Si inter duos numeros & vnitatẽ continuè proportionales incidãt numeri quot inter vtrumque ipsorum & vnitatẽ deinceps medij continua proportione incidunt numeri, totidẽ & inter illos medij continua proportione incident.

A	E	K	D	H	C	L	F	G	B
27	9	36	3	12	1	48	4	16	64

Theorema 9. Propositio 11.

Duorum quadratorũ numerorum vnus medius proportionalis est numerus: & quadratus ad quadratum duplicatam habet lateris ad latus rationem.

A	C	E	D	B
9	3	12	4	16

Theorema 10. Propositio 12.

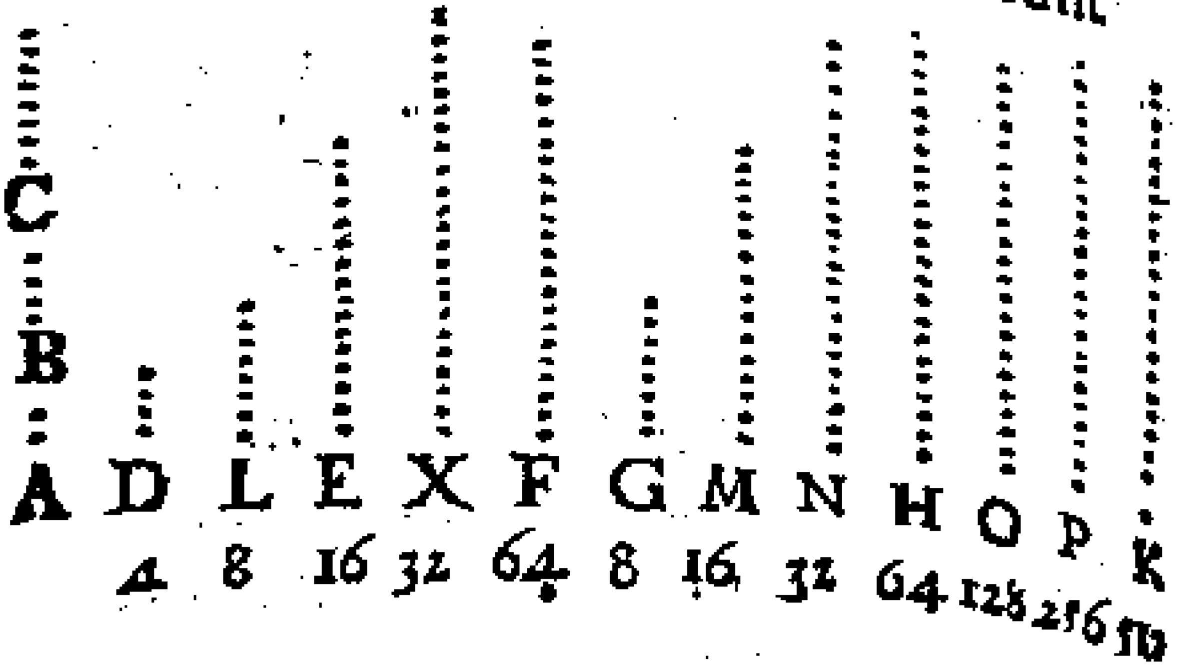
Duorum cuborum numerorũ duo medij proportionales sunt numerorũ duo medij cubum triplicatam habet lateris ad latus rationem.

A	H	K	B	C	D	E	F	G
27	36	48	64	3	4	9	12	16

Theo.

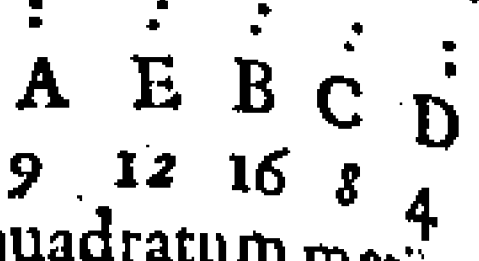
Thorema 10. Propositio 13.

Si sint quotlibet numeri deinceps proportionales, & multiplicans quisq; seipsum faciat aliquos, qui ab illis producti fuerint, proportionales erũt, & si numeri primùm positi, ex suo in procreatos ductu faciãt aliquo, ipsi quoque proportionales erunt.



Theorema 12. Propositio 14.

Si quadratus numerus quadratum numerũ metiatur, & latus vnius metietur latus alterius. Et si vnius quadrati latus metiatur latus alterius, & quadratus quadratum metietur.



Theorema 13. Propositio 15.

Si cubus numerus cubum numerũ metiatur, & latus vni⁹ metietur alterius latus. Et si latus vnius cubi latus alterius metiatur,

tum

tum cubus cubum metietur.

A H K B C D E F G
8 16 28 64 2 4 4 8 16

Theorema 14. Propositio 9.

Si quadratus numerus quadratum numerum non metiatur, neque latus vnius metietur alterius latus.

Et si latus vni⁹ quadrati non metiatur latus alterius, neque quadratus quadratum metietur.

A B C D
9 16 3 4

Theorema 15. Propositio 17.

Si cubus numerus cubum numerũ non metiatur, neque latus vnius latus alterius metietur.

Et si latus cubi alicuius latus alterius non metiatur, neque cubus cubum metietur.

A B C D
8 27 9 11

Theorema 16. Propositio 18.

Duorum similium planorum numerorum vnus medius proportionalis est numerus, & planus ad planum duplicatam habet literis homologi-

A G B C D E F
12 18 27 2 6 5 9

logi ad latus homologum rationem.

Theorema 17. Propositio 19.

Inter duos similes numeros solidos, duo medij proportionales incidunt numeri: & solidus ad similem solidum triplicatam rationem habet lateris homologi ad latus homologum.

A	N	X	B	C	D	E	F	G	H	K	M	L
8	12	18	27	2	2	2	3	3	3	4	6	9

Theorema 18. Propositio 20.

Si inter duos numeros vnus medius proportionalis incidat numerus, similes plani erunt illi numeri.

A	C	B	D	E	F	G
18	24	33	3	4	6	8

Theorema 19 Propositio 21.

Si inter duos numeros duo medij proportionales incidant numeri, similes solidi sunt illi numeri.

A	C	D	B	E	F	G	H	K	L	M
27	36	44	64	9	12	16	3	3	3	4

Theo-

Theorema 20. Propositio 22.

Si tres numeri deinceps sint proportionales, primus autem sit quadratus, & tertius quadratus erit.

A	B	D
9	15	25

Theorema 21. Propositio 23.

Si quatuor numeri deinceps sint proportionales, primus autem sit cubus, & quartus cubus erit.

A	B	C	D
8	12	18	27

Theorema 22. Propositio 25.

Si duo numeri rationem habeant inter se, quam quadratus numerus ad quadratum numerum, primus autem sit quadratus erit, & secunndus quadratus erit.

A		B	C		D
4	6	9	16	24	36

Theorema 23. Propositio 25.

Si numeri duo rationem inter se habeant, quam cubus numerus ad cubum numerú, primus autem cubus sit, & secundus cubus erit.

A	E	F	B	C			D
8	12	18	27	64	95	140	216

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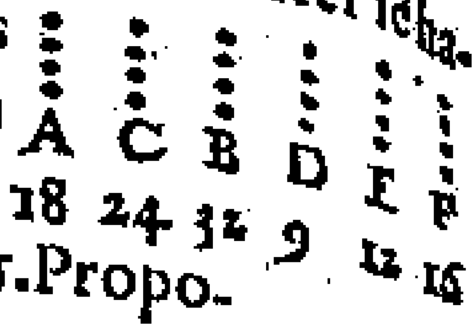
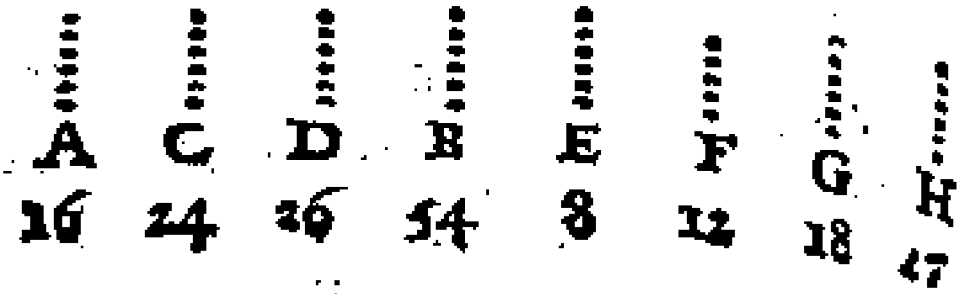
Theorema 24. Propositio 26.

Similes plani numeri rationem inter se habent, quam quadratus numerus ad quadratum numerum.



Theorema 25. Propositio 27.

Similes solidi numeri rationem habent inter se, quam cubus numerus ad cubum numerum.



ELEMENTI VIII. FINIS.

EVCLI

EVCLIDIS ELEMENTVM NONVM.

Theorema 1. Propositio 1.

Si duo similes plani numeri mutuò sese multiplicãtes quendã procreent, productus quadratus

A	E	B	D		C
4	6	9	16	24	36

Theorema 2. Propositio 2.

Si duo numeri mutuò sese multiplicantes quadratum faciant, illi similes sunt plani.

A		B	D		C
4	6		9	18	36

Theorema 3. Propositio 3.

Si cubus numerus seipsum multiplicãs procreet aliqué, productus cubus erit.

Vnitas	D	D	A			B
	3	4	8	16	32	64

Theorema 4. Propositio 4.

Si cubus numerus cubũ numerum multiplicans quendam procreet, procreatus cubus erit.

A	B	D	C
8	27	64	216

Theorema 5. Propositio 5.

Si cubus numerus numerum quendã multiplicans cubum procreet, & multiplicatus cubus erit.

A	B	D	D
27	64	729	1728

Theorema 6. Propositio 6.

Si numerus ſeipſum multiplicans cubum procreet, & ipſe cubus erit.

A	B	C
27	729	19683

Theorema 7. Propoſitio 7.

Si compoſitus numerus quendam numerũ multiplicans quempiam procreet, productus ſolidus erit.

A	B	C	D	E
6	8	48	2	3

Theorema 8. Propoſitio 8.

Si ab vnitate quotlibet numeri deinceps proportionales ſint, tertius ab vnitate quadratus eſt, & vnũ intermittentes omnes: quartus autẽ cubus, & duobus intermiſſis omnes

nes: septimus verò cubus simul & æquadratus, & quinque intermissis omnes.

Vnitas	A	B	C	D	E	F
	3	9	27	81	243	729

Theorema 9. Propositio 9.

Si ab vnitate sint quotcunque numeri deinceps proportionales, sit autem quadratus is qui vnitatem sequitur, & reliqui oẽs quadrati erũt. Quòd si qui vnitatem sequitur cubus sit, & reliqui omnes cubi erunt.

quadrati		cubi
531441	F	732969
59049	E	531441
6561	D	59049
729	C	6561
81	B	729
9	A	81
	o	
	vnitas.	

Theorema 10. Propositio 10.

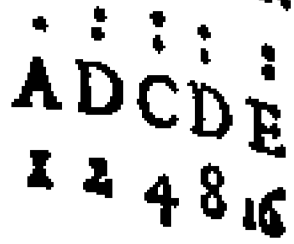
Si ab vnitate numeri quotcunque proportionales sint, non sit autẽ quadratus is qui vnitatem sequitur, neq; alius vllus quadratus erit, demptis tertio ab vnitate ac om-

o	A	B	C	D	E	F
Vnitas.	3	9	36	81	243	729

nibus vnum intermittentibus. Quòd si qui vnitatem ſequitur, cubus non ſit, neque aliud ullus cubus erit, demptis quarto ab vnitate ac omnibus duos intermittentibus.

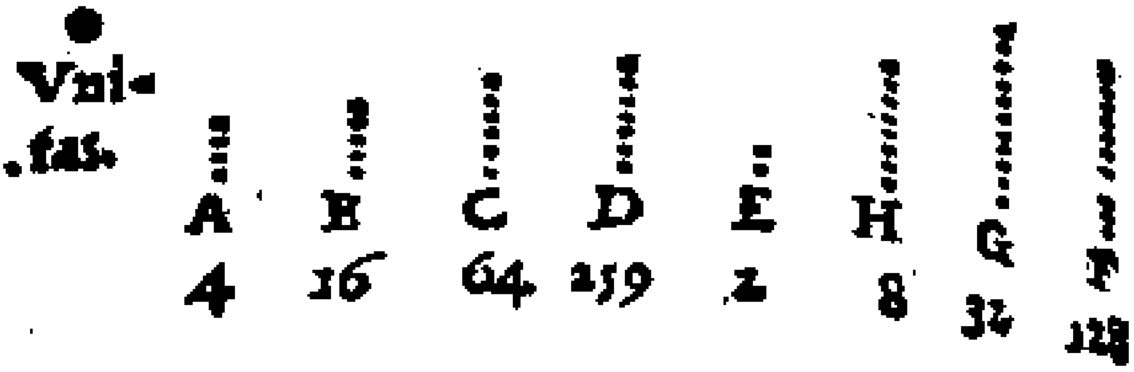
Theorema 11. Propoſitio 11.

Si ab vnitate numeri quotlibet deinceps proportionales ſint, minor maiorem metitur per quempiam eorum qui in proportionalibus ſunt numeris.



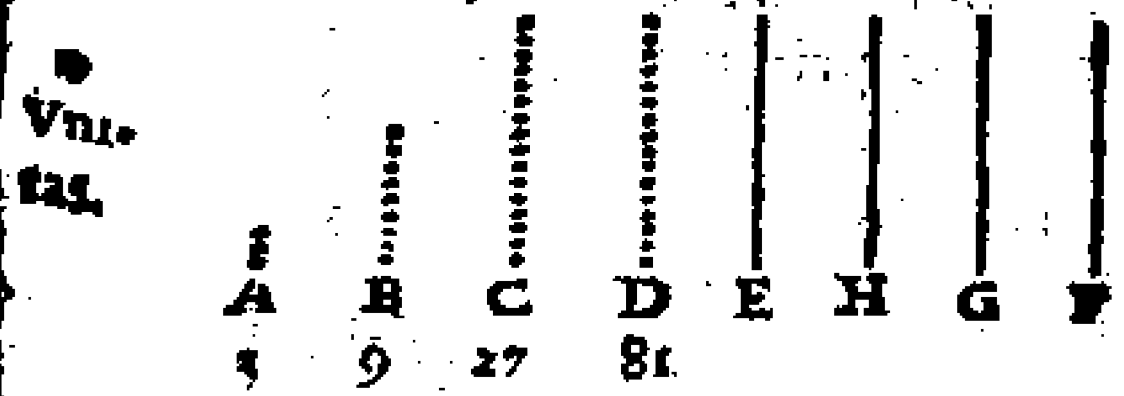
Theorema 12. Propoſitio 12.

Si ab vnitate quolibet numeri ſint proportionales, quot primorum numerorũ vltimum metiuntur, totidem & eum qui vnitati proximus eſt, metientur.



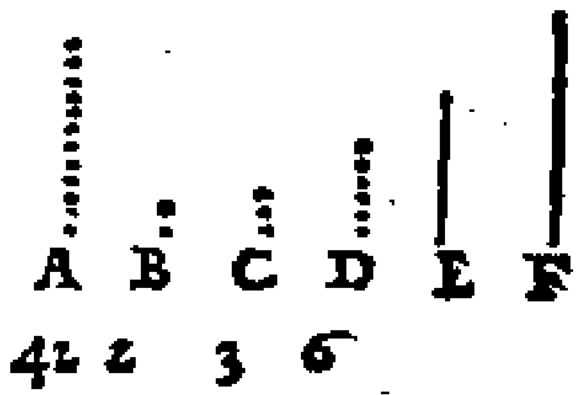
Theorema 13. Propoſitio 13.

Si ab vnitate ſint quotcunq; numeri deinceps proportionales, primus autẽ ſit qui vnitatẽ ſequitur, maximũ nullus alius metietietur, ijs exceptis qui in proportionalibus sunt numeris.



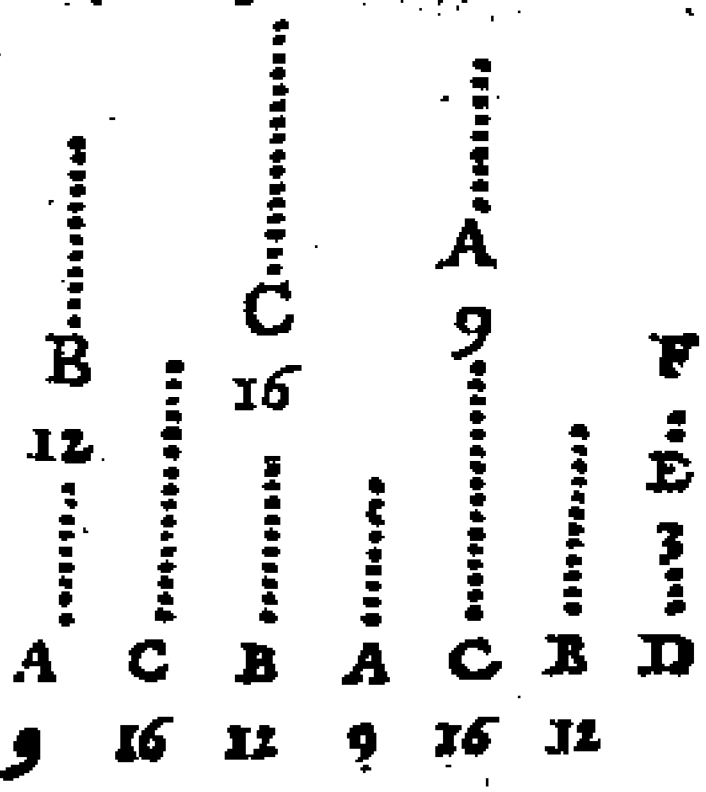
Theorema 14. Propositio 14.

Si minimum numerum primi aliquot numeri metiãtur, nullus alius numerus primus illum metietur, ijs exceptis qui primò metiuntur.



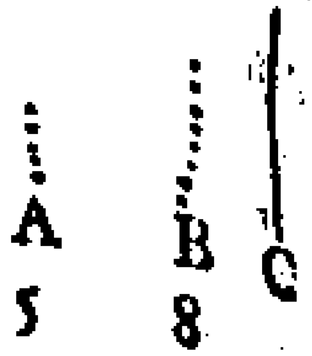
Theorema 15. Propositio 15.

Si tres numeri deinceps proportionales sint minimi, eadem cum ipsis habẽtium rationem, duo quilibet compositi ad tertium primi erunt.



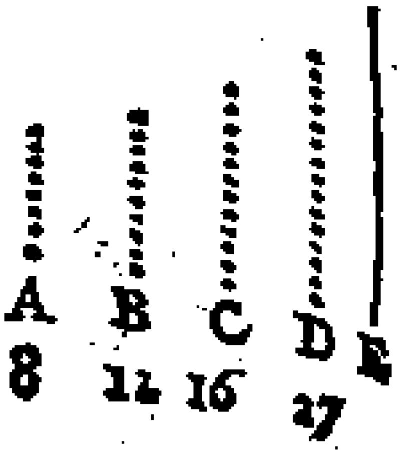
Theorema 16. propositio 16.

Si duo numeri sint inter se primi, non se habebit quẽadmodum primus ad secũdum, ita secundus ad quẽpiam alium.



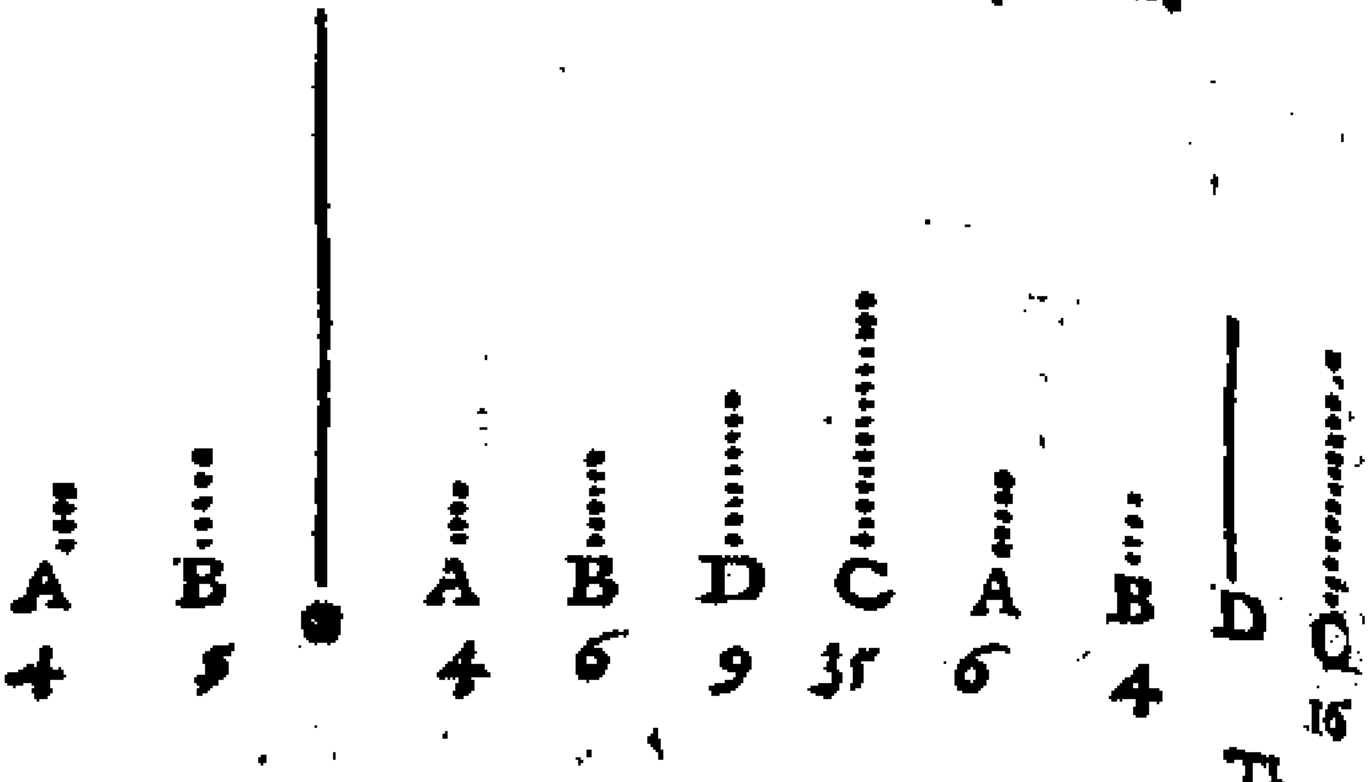
Theorema 17. Propositio 18.

Si sint quotlibet numeri deinceps ꝓportionales, quorum extremi sint inter se primi, non erit quẽadmodum primus ad secundum, ita vltimus ad quempiam alium.



Theorema 18. Propositio 18.

Duobus numeris datis, cõsiderare possit ne tertius illi inueniri proportionalis.



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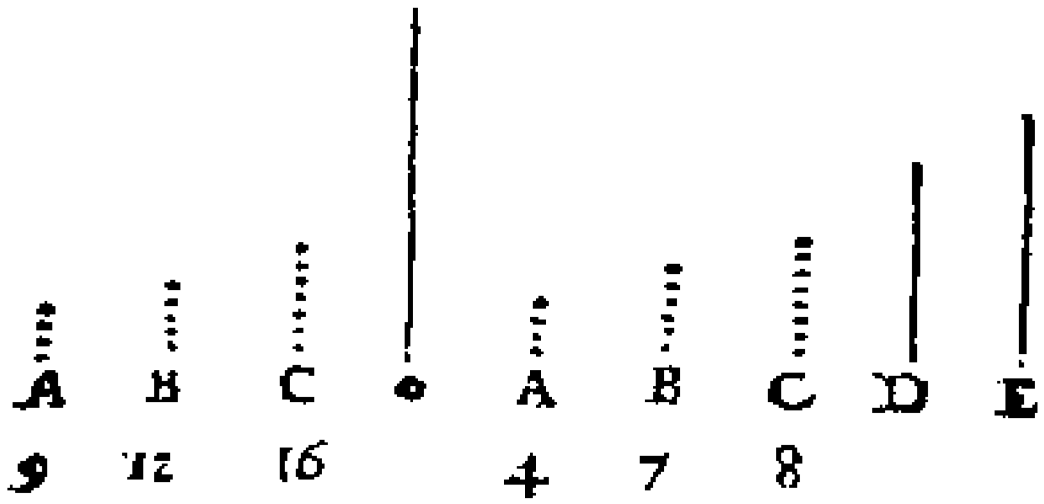
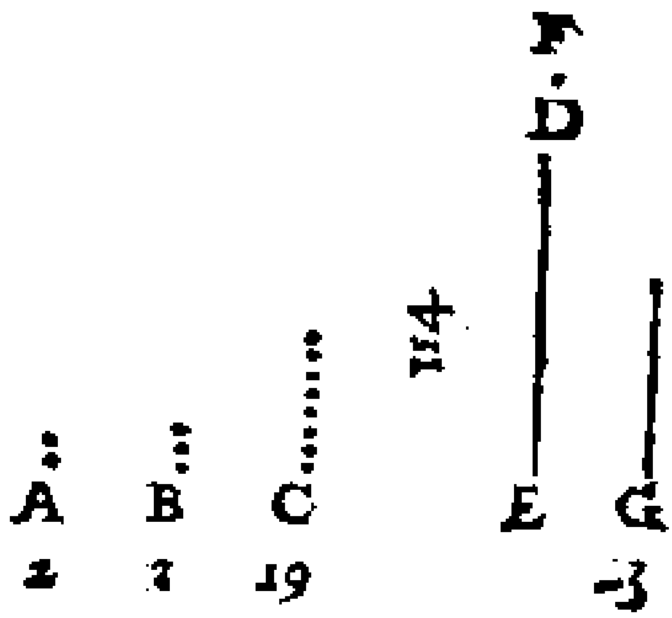
Theorema 19. Propositio 19.

Tribus numeris datis, considerare possibile quartus illis reperiri proportionalis.



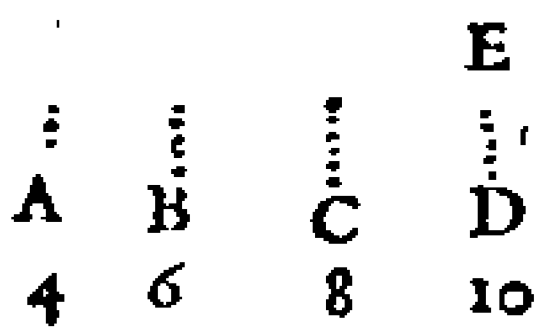
Theorema 20. Propositio 20.

Primi numeri plures sunt quacũque proposita multitudine primorum numerorum.



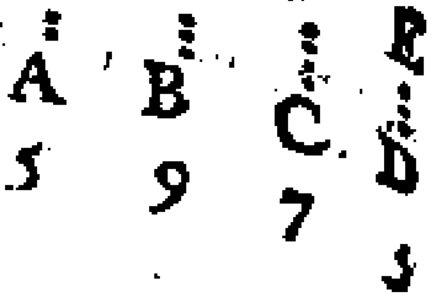
Theorema 12. Propositio 12.

Si pares numeri quotlibet compositi sint, totus est par.



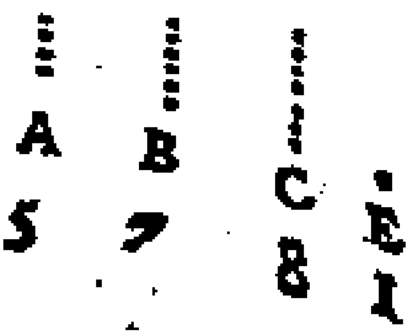
Theorema 22. Propositio 22.

Si impares numeri quotlibet compositi sint, sit autem par illorum multitudo, totus par erit.



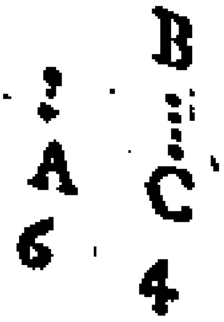
Theorema 23. Propositio 23.

Si impares numeri quotcunque cōpositi sint, sit autem impar illorū multitudo, & totus impar erit.



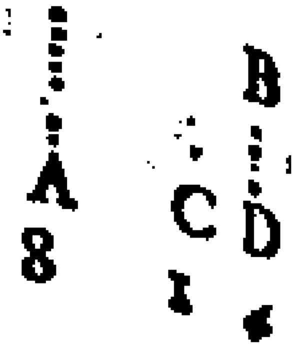
Theorema 24. Propositio 24.

Si de pari numero par detractus sit, & reliquus par erit.



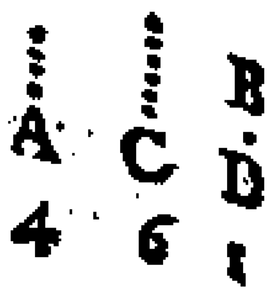
Theorema 25. Propositio 25.

Si de pari numerò impar detractus sit, & reliquus impar erit.



Theorema 26. Propositio 26.

Si de impari numero impar detractus sit, & reliquus par erit.



Theorema 27. Propositio 27.

Si ab impari numero par ablatus ſit, reliquus impar erit.

A D C
1 4 4

Theorema 28. Propoſitio 28.

Si impar numerus parem multiplicās, procreet quempiam, procreatus par erit.

A B C
3 4 12

Theorema 29. Propoſitio. 29.

Si impar numerus imparē numerum multiplicans quemdam procreet, procreatus impar erit.

A B C
3 5 15

Theorema 30. Propoſitio 30.

Si impar numerus parē numerum metiatur, & illius dimidium metietur.

A C B
3 6 18

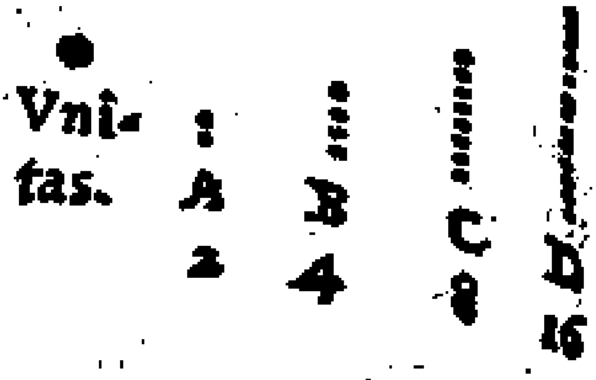
Theorema 31. Propoſitio 31.

Si impar numerus ad numerum quēpiam primus ſit, & ad illius duplum primus erit.

A B C D
7 8 16

Theorema 32. Propositio 32.

Numerorum, qui à binario dupli sunt, vnusquisq; pariter par est tantùm.



Theorema 33. Propositio 33.

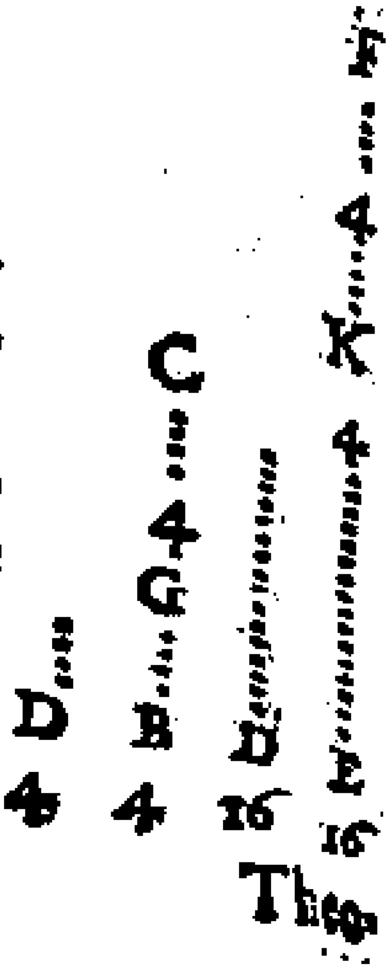
Si numerus dimidiũ impar habeat, pariter impar est tantùm.

Theorema 34. Propositio 34.

Si par numerus nec sit dupl' à binario, nec dimidium impar habeat, pariter par eũ, & pariter impar.

Theorema 35. Propositio 35.

Si sint, quotlibet numeri deinceps proportionales, detrahãtur aũt de secundo & vltimo æquales ipsi primo, erit quemadmodum secũdi excessus ad primũ, ita vltimi excessus ad omnes qui vltimum antecedunt.



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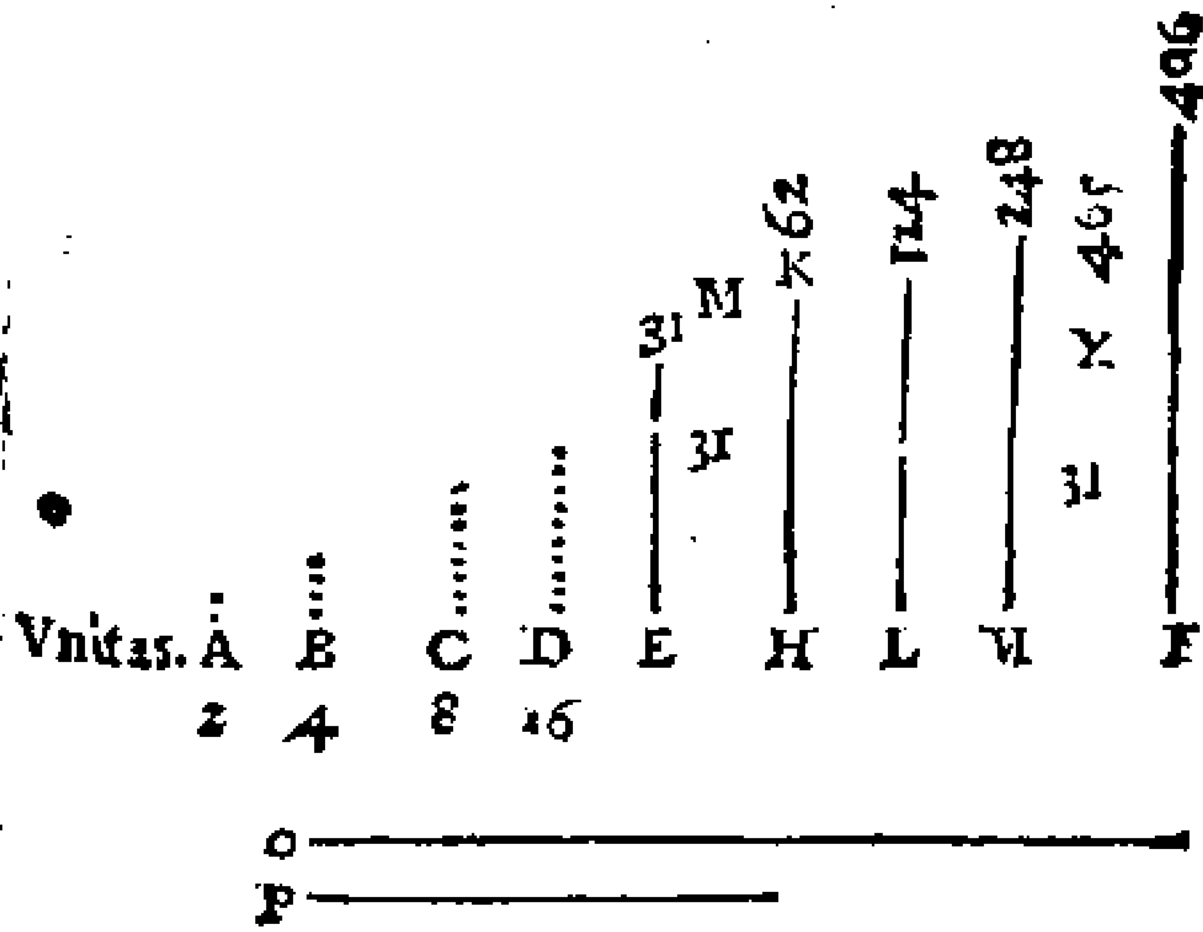
Theorema 16. Propositio 16.

Si ab vnitate numeri quotlibet deinceps expositi sunt in duplici proportione quoad totus compositus primus factus sit, isque totus in vltimum multiplicatus quempiam procreet, procreatus perfectus erit.



ELEMENTI IX. FINIS

EVCLI.

EVCLIDIS ELEMENTVM DECIMVM.

DEFINITIONES.

1

Commenſurabiles magnitudines dicuntur illæ, quas eadem menſura metietur.

2

Incommenſurabiles verò magnitudines dicuntur, hæ quarum nullam menſuram communem contingit reperiri.

3

Lineæ rectæ potentia cõmenſurabiles ſunt, quarum quadrata vna eadem ſuperficies ſiue area metitur.

4

Incommenſurabiles verò lineæ ſunt, quarum quadrata, quæ metiatur area communis, reperiri nulla poteſt.

5

Hęc cùm ita ſint, oſtendi poteſt quòd quantacunque linea recta nobis proponatur, exiſtunt etiam aliæ lineæ innumerabiles eidem commenſurabiles, aliæ item incommenſurabiles, hæ quidem longitudine & poten-

potentia: illæ verò potentia tantùm. Vocetur igitur linea recta, quantacunq; proponatur, ῥητὴ, id est rationali.

6

Lineæ quoq; illi ῥητῇ commensurabiles siue longitudine & potentia, siue potentia tantùm, vocentur & ipsæ ῥηταί, id est rationales.

7

Quæ verò lineæ sunt incommensurabiles illi τῇ ῥητῇ, id est primo loco rationali, vocētur ἄλογοι, id est irrationales.

8

Et quadratum quod à linea proposita describitur, quam ῥητὴν vocari volumus, vocetur ῥητόν.

9

Et quæ sunt huic commensurabilia, vocētur ῥητά.

10

Quæ verò sunt illi quadrato ῥητῷ scilicet incommensurabilia, vocentur ἄλογα, id est surda.

11

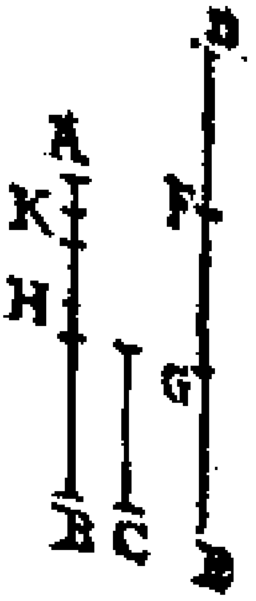
Et lineę quę illa incommensurabilia describunt, vocentur ἄλογοι. Et quidem si illa incommensurabilia fuerint quadrata, ipsa eorū latera vocabuntur ἄλογοι lineæ. quòd si quadrata quidem non fuerint, verū aliæ quæpiā superficies siue figuræ rectilineæ,

tunc

tunc verò lineæ illæ quæ deſcribunt quadrata æqualia figuris rectilineis, vocentur ἄλογοι.

Theorema 1. Propoſitio 1.

Duabus magnitudinibus inæqualibus propoſitis, ſi de maiore detrahatur plus dimidio, & rurſus de reſiduo iterũ detrahatur plus dimidio, idq́; ſemper fiat: relinquetur quædam magnitudo minor altera minore ex duabus propoſitis.



Theorema 2 Propoſitio. 2.

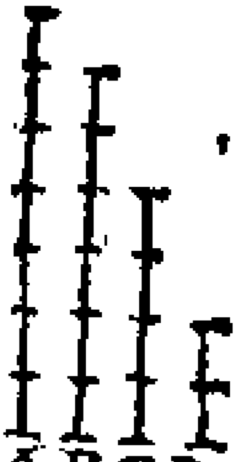
Duabus magnitudinibus propoſitis inęqualibus, ſi detrahatur ſemper minor de maiore, alterna quadam detractione neque reſiduum vnquam metiatur id quod ante ſe metiebatur, incommenſurabiles ſunt illæ magnitudines.

Problema 1. Propoſitio 3.

Duabus magnitudinibus commēſurabilibus datis, maximam ipſarum communem menſuram reperire.

Proble-

Problema 2. Propositio 4.

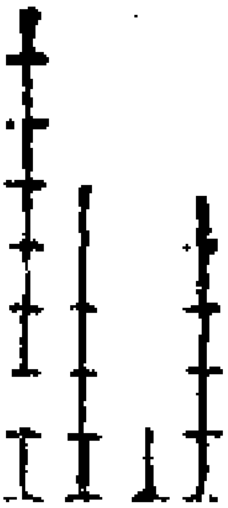


Tribus magnitudinibus cómensurabilibus datis, maximam ipsarũ communé mensuram reperire.

Theorema 3. Propositio 5.

Cõmensurabiles magnitudines inter se proportioné eam habent, quam habet numerus ad numerum.

Theorema 4. Propositio 6.



Si duæ magnitudines proportionem eam habent inter se quam numerus ad numerum, commensurabiles sunt illæ magnitudines.

Theo.

Theorema 5. Propositio 7.

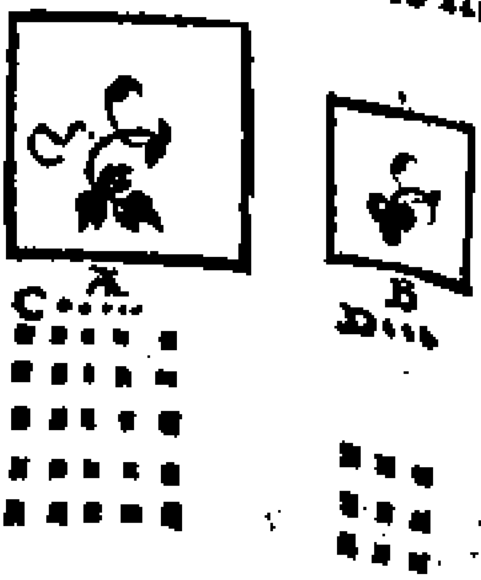
Incommensurabiles magnitudines inter se proportionem non habẽt, quam numerus ad numerum.

Theorema 6. Propositio 8.

Si duæ magnitudines inter se proportionem nõ habẽt quam numerus ad numerum, incommensurabiles illę sunt magnitudines.

Theorema 7. Propositio 9.

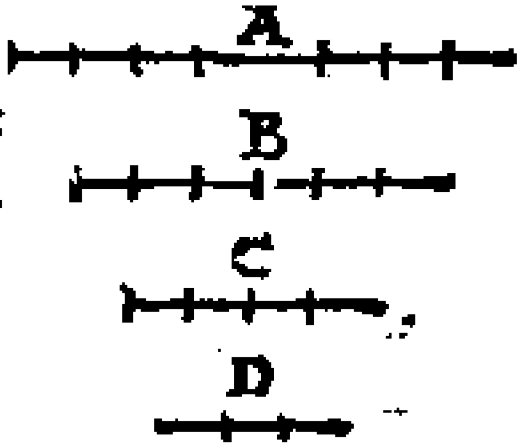
Quadrata, quæ describuntur à rectis lineis lõgitudine cõmensurabilibus, inter se proportionem habent quã numerus quadratus ad alium numerum quadratũ. Et quadrata habentia proportionẽ



inter se quã quadratus numerus ad numerum quadratũ, habent quoque latera longitudine commẽsurabilia. Quadrata verò quæ deſcribuntur à lineis longitudine incōmenſurabilibus, proportioné non habent inter ſe, quam quadratus numerus ad numerum alium quadratum. Et quadrata non habentia proportionem inter ſe quam numerus quadratus ad numerum quadratum, neque latera habebunt longitudine commnnſurabilia.

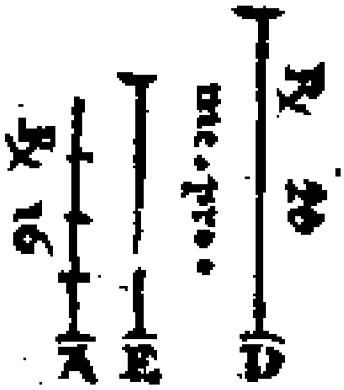
Theorema 8. Propoſitio 10.

Si quatuor magnitudines fuerint proportionales, prima verò ſecundæ fuerit commēſurabilis, tertia quoque quartę commenſurabilis erit. quòd ſi prima ſecūdæ fuerit incommenſurabilis, tertia quoq; quartæ incommenſurabilis erit.



Problema 3. Propoſitio 11.

Propoſitæ lineæ rectæ (quam ῥητὴν vocari diximus) reperire duas lineas rectas incommenſurabiles, hanc quidem longitudiue tantùm, illam verò non longitudiné tantùm, ſed etiam potentia incommēſurabilem.



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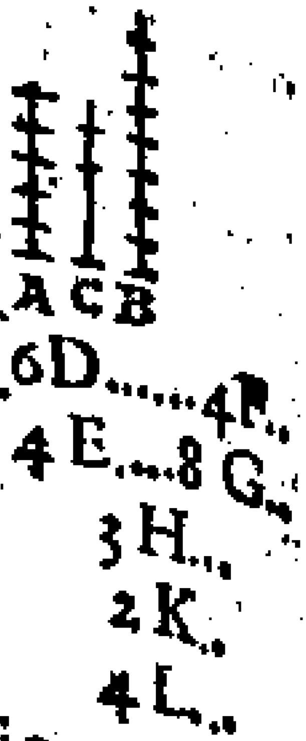
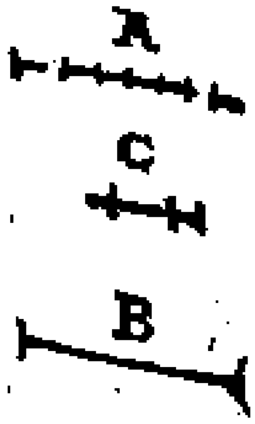
Theorema 6. Propositio 12.

Magnitudines quæ eidem magnitudini sunt commensurabiles, inter se quoque sunt commensurabiles.



Theorema 10. Propositio 13.

Si ex duabus magnitudinibus hæc quidem commensurabilis sit tertię magnitudini, illa verò eidem incommensurabilis, incommēsurabiles sunt illæ duæ magnitudines.



Theorema 11. Propositio 14.

Si duarum magnitudinum commensurabilium altera fuerit incommensurabilis magnitudini alteri cuipiam tertię, reliqua quoq; magnitudo eidem tertiæ incommensurabilis erit.

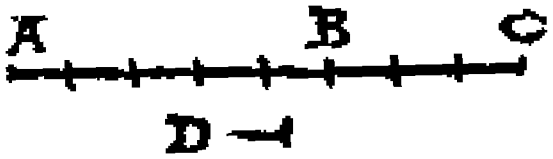
Theorema 12. Propositio 15.

Si quatuor rectæ proportionales fuerint, possit

possit autem prima plusquā secunda tanto quantum est quadratum lineę sibi commensurabilis longitudine: tertia quoq; ponterit plusquam quarta tanto quantum est quadratum lineæ sibi commensurabilis longitudine. Quòd si prima possit plusquam secunda quadrato lineę sibi longitudine incommensurabilis: tertia quoque poterit plusquam quarta quadrato lineę sibi incōmēsurablilis longitudine.

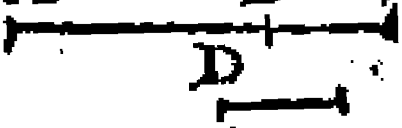
Theorema 13. Propositio 16.

Si duæ magnitudines commensurabiles componantur, tota magnitudo composita singulis partibus commensurabilis erit, quòd si tota magnitudo composita alterutri parti commensurabilis fuerit, illæ duæ quoque partes cōmensurabiles erunt.



Theorema 14. Propositio 17.

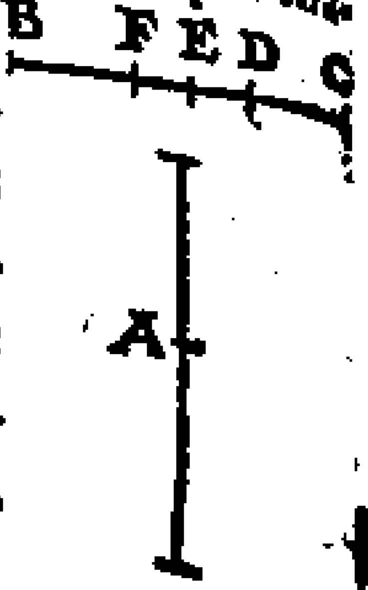
Si duæ magnitudines incommensurabiles componantur, ipsa quoque tota magnitudo singulis partibus componentibus incommensurabilis erit. Quòd si tota alteri parti incommensurabilis fuerit, illæ quoque primę magnitudines inter se incommensurabiles erunt.



 Theo-

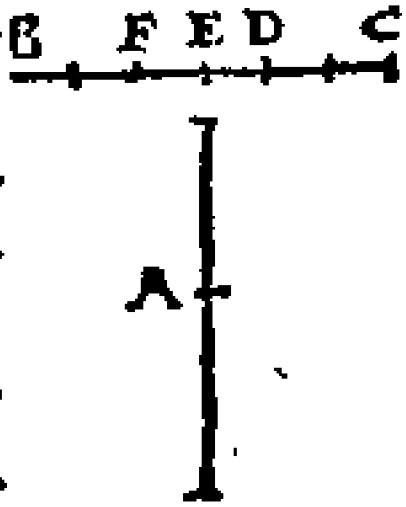
Theorema 15. Propositio 18.

Si fuerint duæ rectæ lineæ inæquales, & quartæ parti quadrati quod describitur à minore æquale parallelongrammũ applicecetur secundũ maiorem, ex qua maiore tãtum excurrat extra latus parallelogrammi, quantum est alterum latus ipsius parallelogrammi: si præterea parallelogrammum sui applicatione diuidat lineã illam in partes inter se commensurabiles longitudine, illa maior linea tanto plus potest quam minor, quantũ est quadratum lineæ sibi commensurabilis longitudine. Quòd si maior quadratũ lineæ sibi commensurabilis longitudine plus possit quàm minor, tanto quantũ est gitudine, & prætereà quartæ parti quadrati lineę minoris ęquale parallelogrammum applicetur secundum maiorem, ex qua maiore tantum excurrat extra latus pallelogrammi, quantum est alterum latus ipsius parallelongrammi parallelogammum sui applicatione diuidit maioré in partes inter se longitudine commensurabiles.

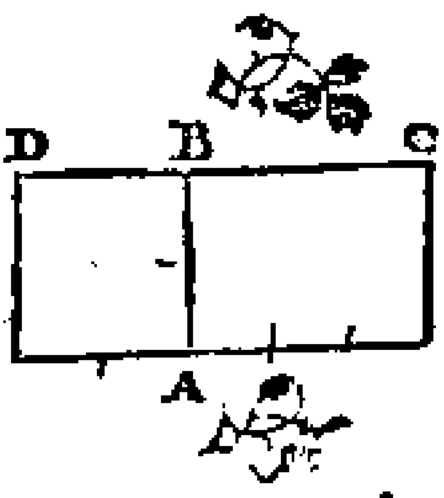


Theorema 16. Propositio 19.

Si fuerint duæ rectæ inæquales, quartæ autem parti quadrati lineæ minoris æquale le parallelogrammorum ſecundum lineam maiorem applicetur, ex qua lineæ tantum excurrat extra latus parallelogrammi, quã-tum eſt alterum latus eiuſdem parallelo-grammi: ſi parallelogrammum præterea ſui applicatione diuidat lineam in partes inter ſe longitudine incommenſurabiles, maior illa linea tantò plus poteſt quàm minor quantum eſt quadratum lineæ ſibi maiori incommenſurabiles longitudine. Quod ſi maior linea tantò plus poſſit quàm minor, quantum eſt quadratum lineæ incommen-ſurabilis ſibi longitudine: & præterea quar-tæ parti quadrati lineæ minoris æquale pa-rallelogrammum applicetur ſecundum ma-iorem ex qua tantum excur-rat extra latus parallelográ-mi, quantum eſt alterum la-tus ipſius: parallelogram-mum ſui applicatione diui-dit maiorem in partes inter ſe incommenſurabiles lon-gitudine.



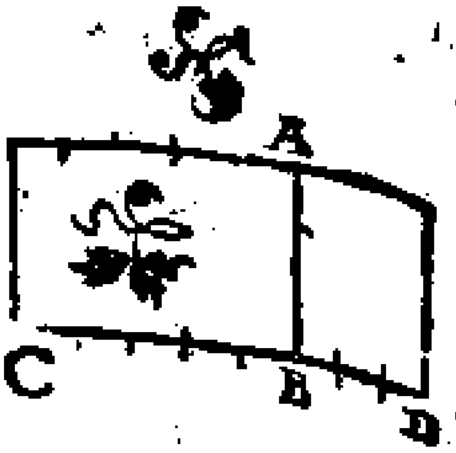
Theorema 17. Propo-ſitio 20.



Superficies rectangula contenta ex lineis rectis rationalibus longitudine commenſurabilibus ſecundum vnum aliquem modum ex antedictis, rationalis est.

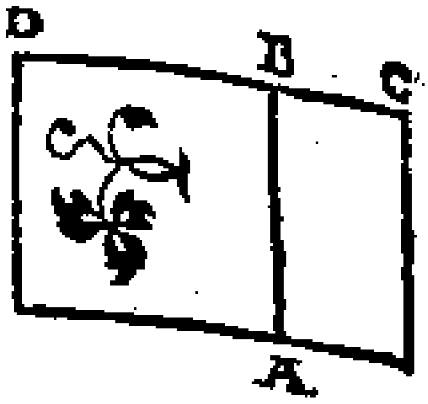
Theorema 18. Propositio 21.

Si ratinonale secundũ lineam rationalem applicetur, habebit alterum latus lineam rationalem & cōmensurabilem longitudine lineæ cui rationale parallelogrammum applicatur.



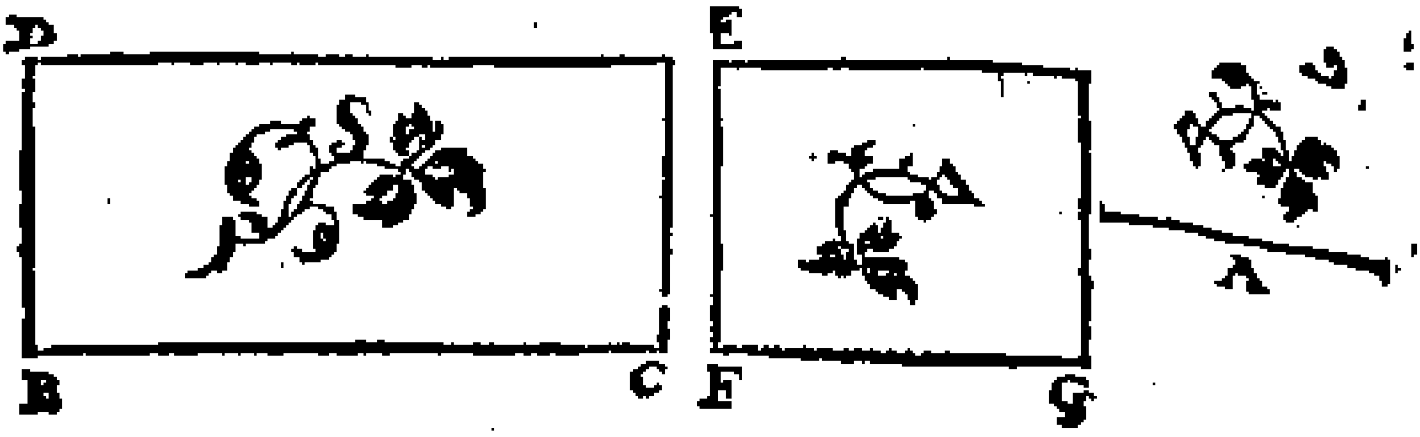
Theorema 39. Propositio 56.

Surperficies rerctangula contenta duabus lineis rectis rationalibus potentia tantum commēsurabilibus, irrationalis est. Linea autem quæ illam superficiem potest, irrationalis & ipsa est: vocetur verò medialis



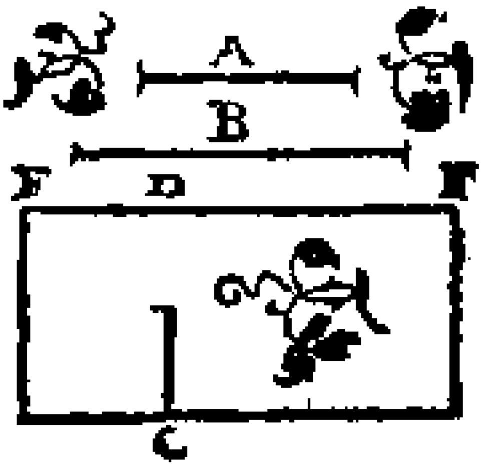
Theorema 20. Propositio 23.

Quadrati lineæ medialis applicati secun-



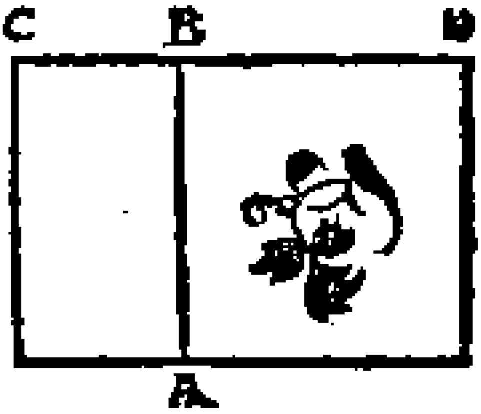
dum lineam rationalem. alterum latus est linea rationalis, & incommensurabilis longitudine lineæ secundum quam applicatur.

Theorema 21. Propositio 24



Linea recta mediali commẽsurabilis, est ipsa quoque medialis.

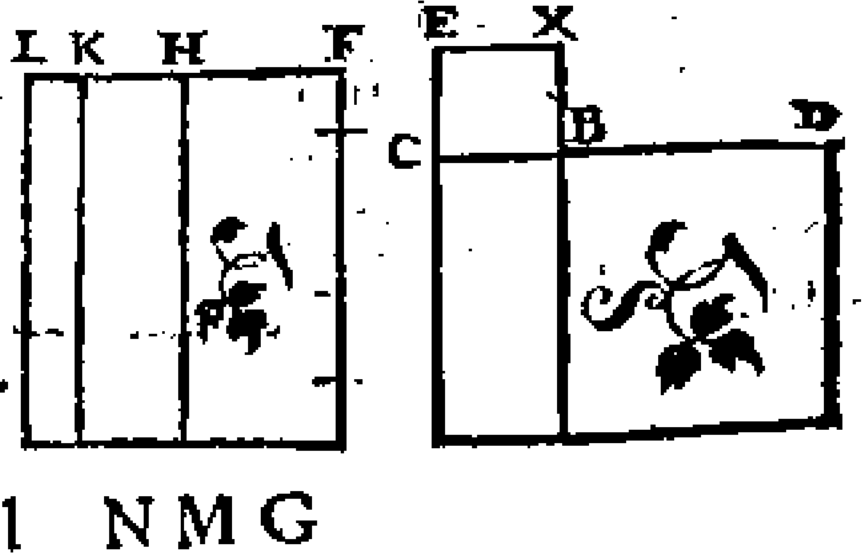
Theorema 22. Propositio 25



Parallelongrammum rectangulum contẽtum ex lineis medialibus longitudine commensurabilibus, mediale est.

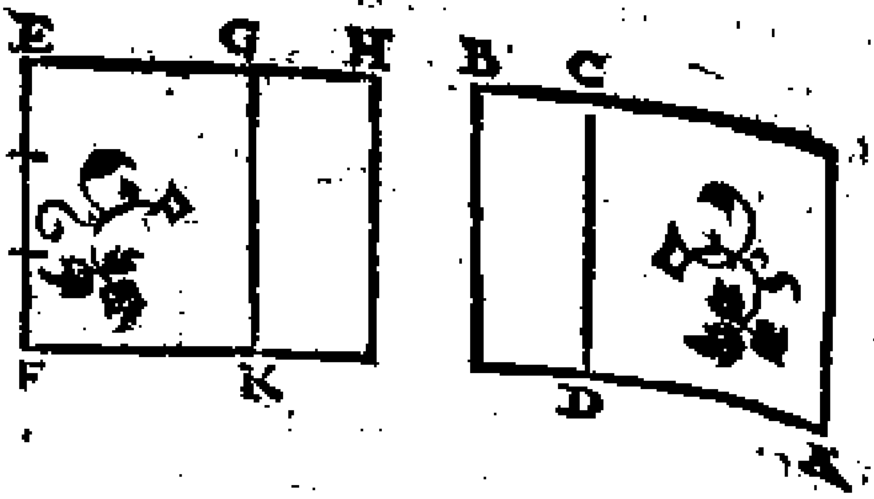
Theorema 23. Propositio 26.

Parallelogrammum rectangulum comprehensum duabus lineis medialibus potentia tãtùm cõmensurabilibus, vel rationale est, vel mediale.



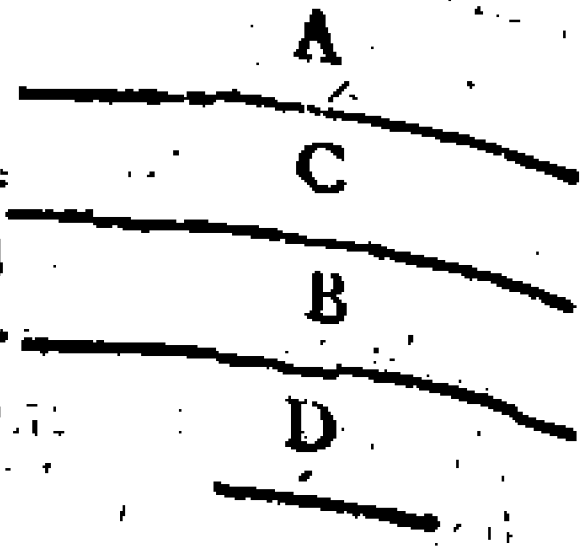
Theorema 24. Propositio 27.

Mediale nō est maius quàm mediale superficie rationali.



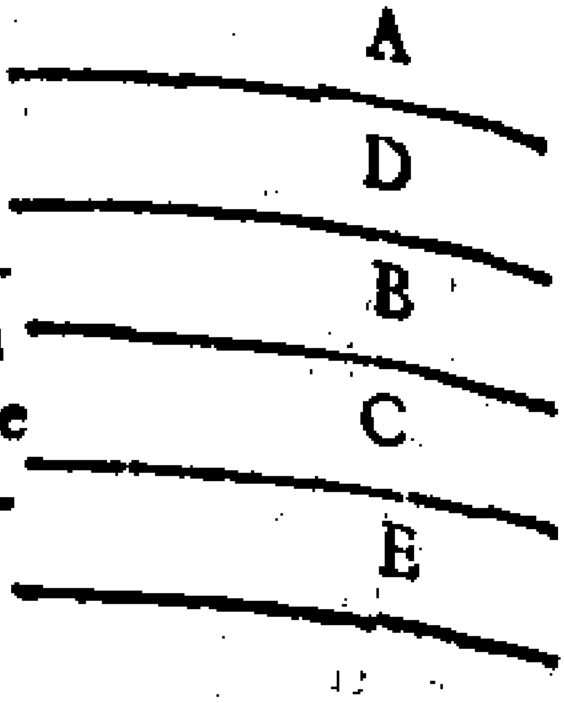
Problema. 4. Propositio 28.

Mediales lines inuenire potentia tantùm commensúrabiles rationale comprehendentes.



Problema 5. Propositio 29.

Mediales lineas inuenire potentia tantùm commensurabiles mediale comprehendendentes



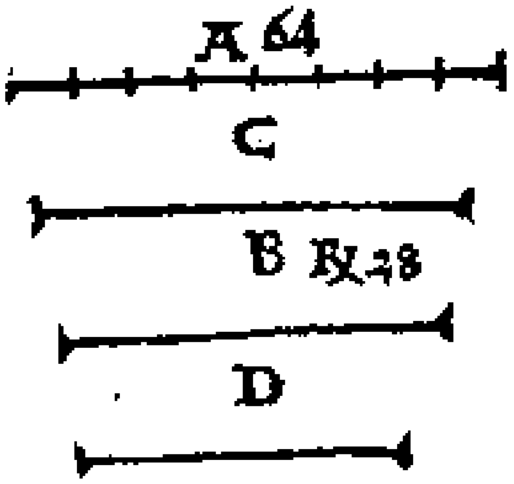
Pro-

Problema 6. Propositio 30.

Reperire duas rationales potentia tantum commẽsurabiles huiusmodi, vt maior ex illis possit plus quàm minor quadrato lineæ sibi commensurabilis longitudine.

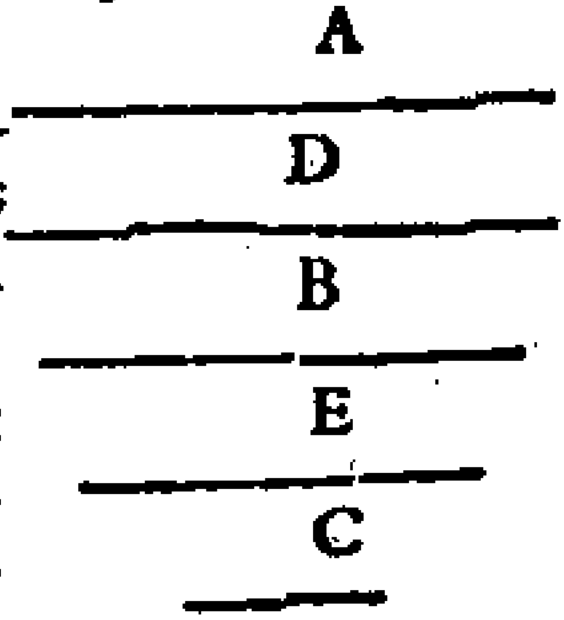
Problema 7. Propositio 31.

Reperire duas lineas medialis potentia tantùm commensurabiles rationalem superficiẽ continẽtes, tales inquã vt maior possit plus quàm minor quadrato lineæ sibi commẽsurabilis longitudine.



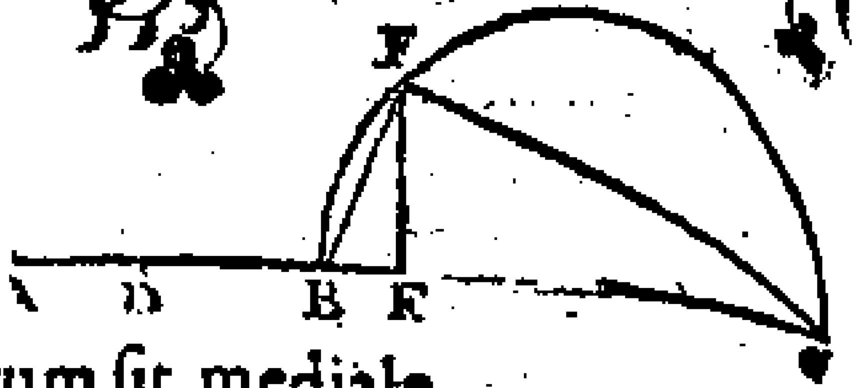
Problema 8. Propositio 32.

Reperire duas lineas mediales potentia tantùm commensurabiles medialem superficiem continentes, huiusmodi vt maior plus possit quàm minor quadrato lineæ sibi commensurabilis longitudine.



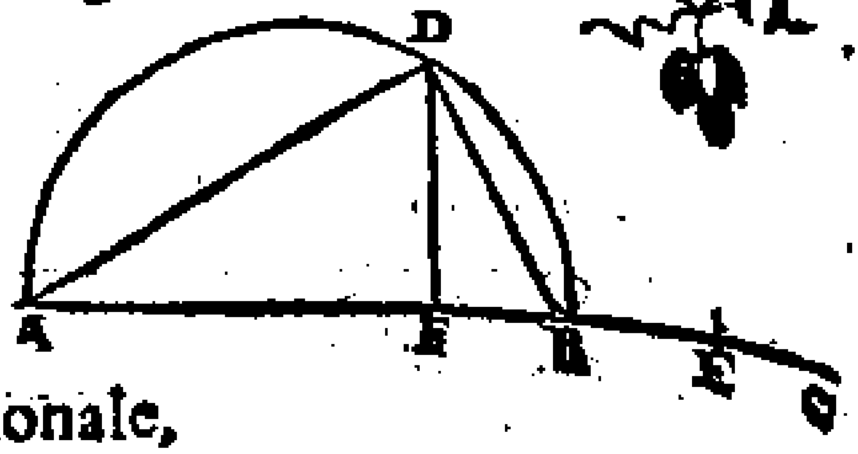
Problema 9. Propositio 33.

Réperire duas rectas poutentia incommē rabiles, quarum quadrata simul addita faciant superficiē rationalé, parallelogrammum vetò ex ipsis conetntum sit mediale.



Problema 10. Propositio 34.

Reperire lineas duas rectas potentia incommensurabiles cónficentes compositnm ex ipsarum quadratis mediale, parallelogrammū verò ex ipsis contentum rationale,



Problema 11. Propositio 85.

Reperire duas lineas rectas potentia incō mensurabiles, conficientes id quod ex ipsarum quadratis componitur meale, simul. que

que parallelogrammũ ex ipsis contentum, mediale, quod præterea parallelogrammũ sit incōmensurabile composito ex quadratis ipsarum.

PRINCIPIVM SENARIOrum per compositionem.

Thorema 25. Propositio 36.

Si duæ rationales potentia tantùm commensurabilis. Vocetur autem Binommum.

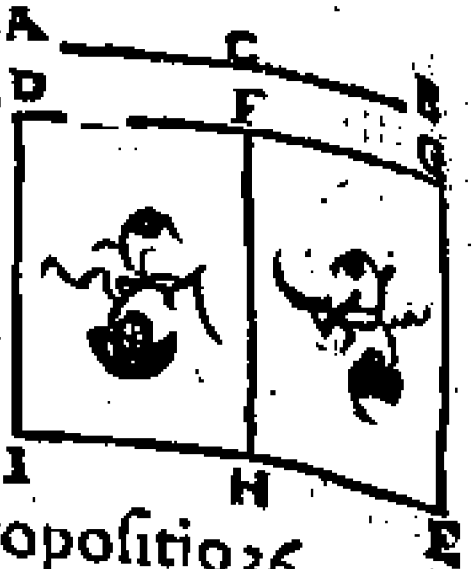
Theorema 26. Propositio 37.

Si duæ mediales potentia tantòm commensurabiles rationale continentes componantur, tota linea est irrationalis, vocetur autem Bimediale prius.

The-

Theorema 27. Propositio 38.

Si duæ mediales Potētia tantū commēsurabiles mediale continētes componātur, tota linea est irrationalis: vocetur autē Bimediale secundum.



Theorema 28. Propositio 36.

Si duæ rectæ potentia incommensurabiles componantur, conficientes compositum ex quadratis ipsarum rationale, parallelogrammum verò ex ipsis contentum mediale, tota linea recta est irrationalrs. Vocetur autē linea maior.

Theorema 29. Propositio 40.

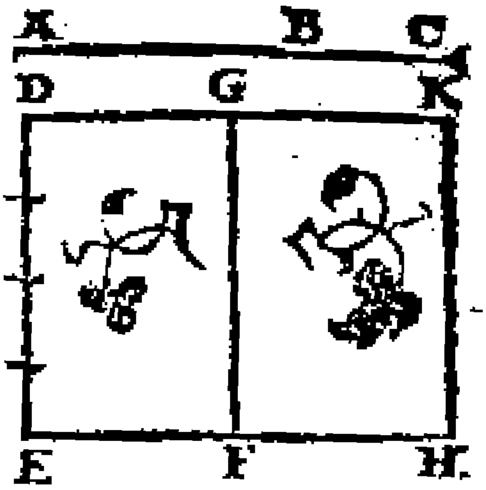
Si duæ rectæ potentia incommensurabiles componantur, conficientes compositum ex ipsarum quadratis mediale, id vero quod fit ex ipsis, rationale, tota linea est irrationales. Vocetur autem potens rationale & mediale.

Theorema 30. Propositio 41.

Si duæ rectæ potentia incommensurabiles componantur, conficientes compositum ex quadratis ipsarum mediale, & quod conti-

netur

netur ex ipsis, mediale, & præterea incōmensurabile composito ex quadratis ipsarum tota linea est irrationalis. Vocetur autem potens duo medialia.



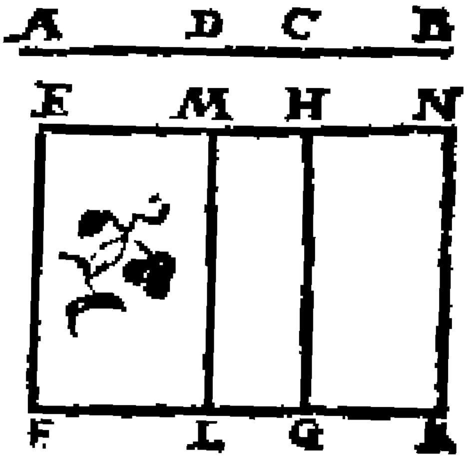
Theorema 31. Propositio 42.

Binominum in vnico tantùm pũcto diuiditur in sua nomina, id est in lineas ex quibus componitur.

Theorema 32. Propositio 43.

Bimediale prius in vnico tantum puncto diuiditur in sua nomina.

Problema 33. Propositio 44



Bimediale secundum in vnico tantũ puncto diuiditur in sua nomina.

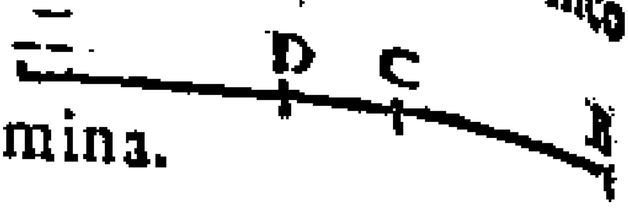
Problema 34. Propositio 45.

Linea maior in vnico tantùm in pũcto diuiditur in sua nomina.

Theo-

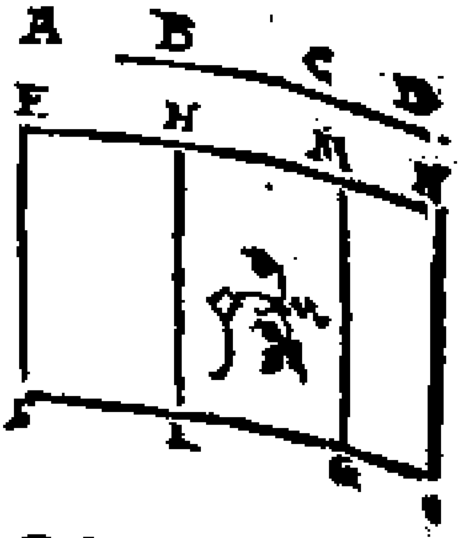
Theorema 35. Propositio 46.

Linea potens rationale & mediale in vnico tantùm puncto diuiditur in sua nomina.



Theoroma 36. Propositio 47

Linea ootèns duo media lia in vnico tantùm puncto diuiditur in sua nomina.



DEFINITIONES secundæ.

Proposita linea rationali, & binomio diuiso in sua nomine, cuius binomij maius nomen, id est, maior portio possit plusquam minus nomen quadrato lineæ sibi, maiori inquam nomini, commensurabilis longitudine.

I.

Si quidé maius nomen fuerit commensurabile longitudine propositæ lineæ rationali, vocetur toto linea Binomum primium.

2

Si veró minus nomen, id est minor portio binomij, fuerit commensurabile longitudine ne propositæ lineæ rationali, vocetur tota linea Binomium secundum.

3

Si verò neutrum nomen fuerit commensurabile longitudine propositæ lineæ rationali, vocetur Binomium tertium.

Rursus si maius nomen possit plusquam minus nomen quadrato lineæ sibi incommensurabilis longitudine.

4

Si quidem maius nomen est commensurabile longitudine propositæ lineæ rationali, vocetur tota linea Binomium quartum.

5

Si verò minus nomen fuerit commensurabile longitudine lineæ rationali, vocetur Binomium quintum.

6

Si verò neutrum nomen fuerit longitudine commensurabile lineæ rationali, vocetur illa Binomium sextum.

Proble. 12. Propositio 48

Reperire Binomium primum.

D

D 16 F 12 G

H

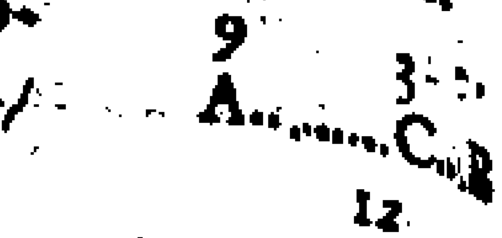
12 4

A.....C....B

16

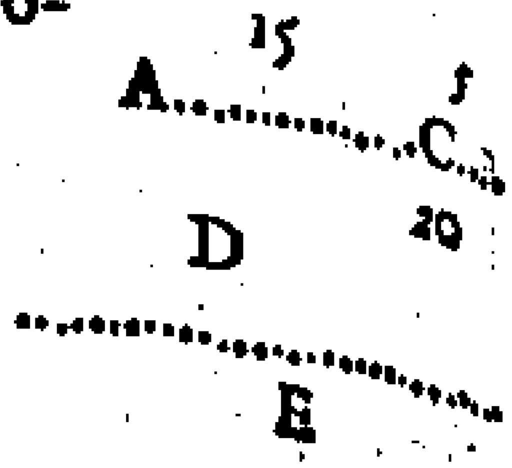
Pro-

Problema 13. Propositio 49.

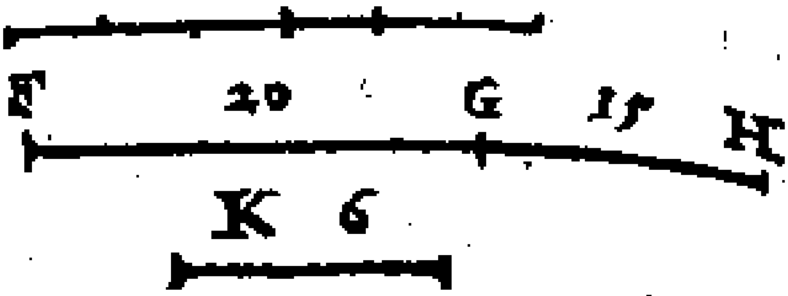


Reperire Binomium secundum.

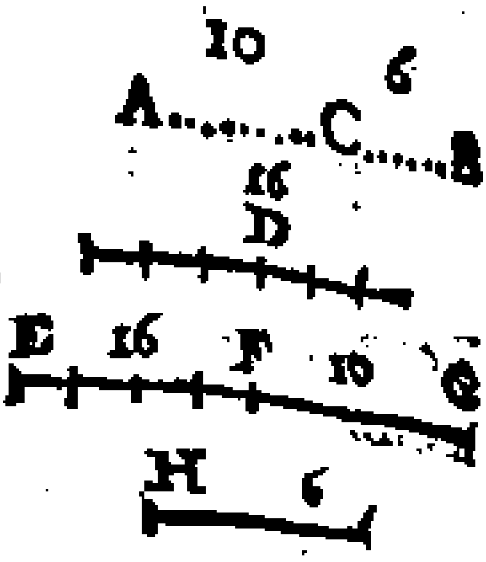
Problema 14. Propositio 50.



Reperire Binomium tertiũ.



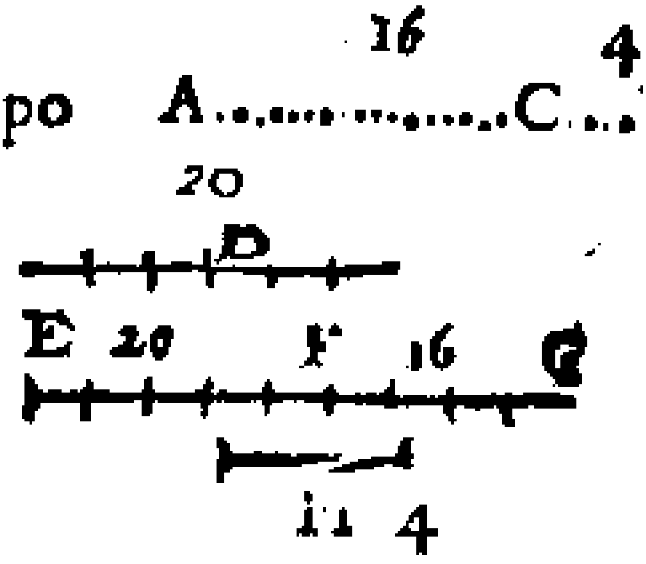
Problema 15. Propositio 51.



Repere Binomiũ quartum.

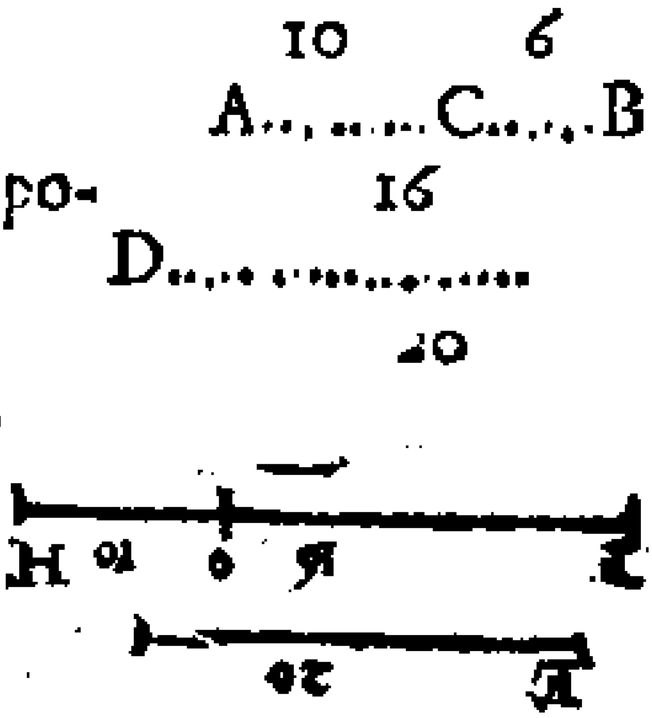
Proble.

Problema 16. Propositio 52



Reperire Binomium quintum.

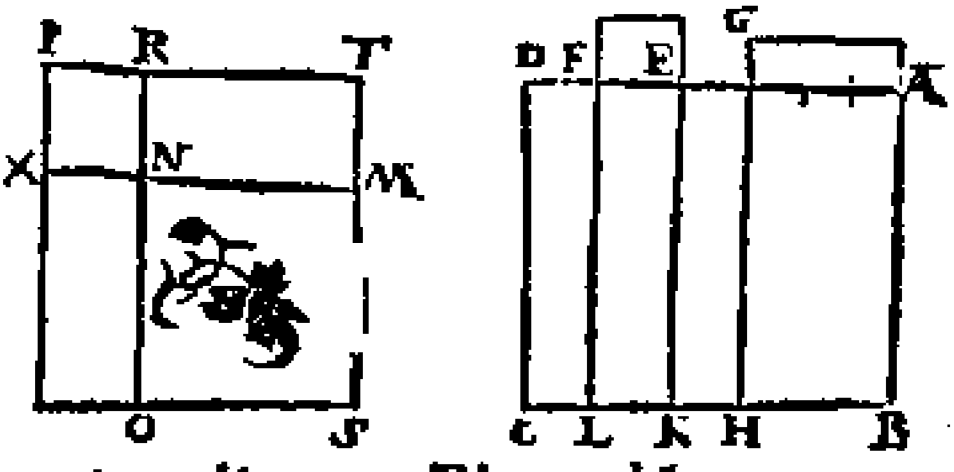
Probl. 17. Propositio 53.



Reperire Binonimum sextum.

Theorema 37. Propositio 54.

Si superficies contéta fuerit ex rationali & Bino

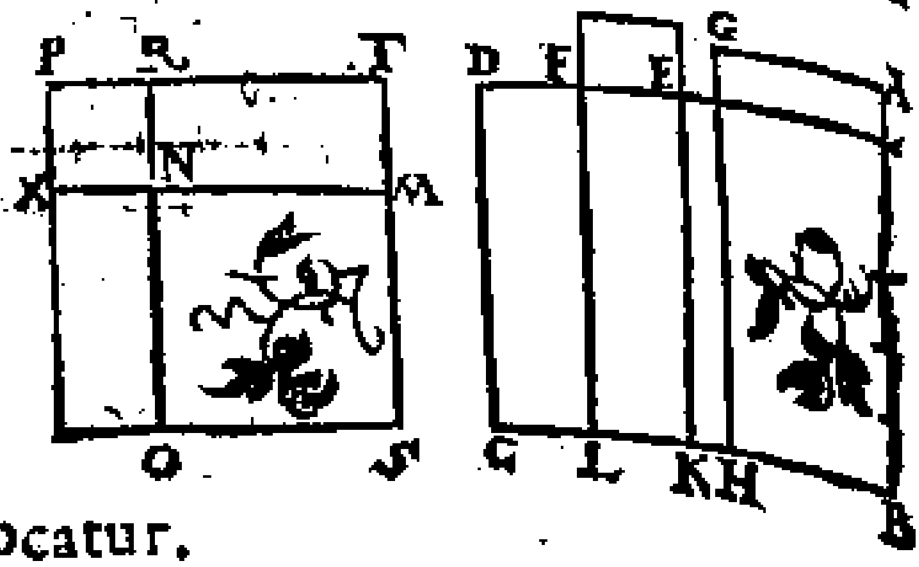


mio primo, linea quę illa superficiem potest, est irrationalis, quæ Binomiũ vocatur.

 Theore-

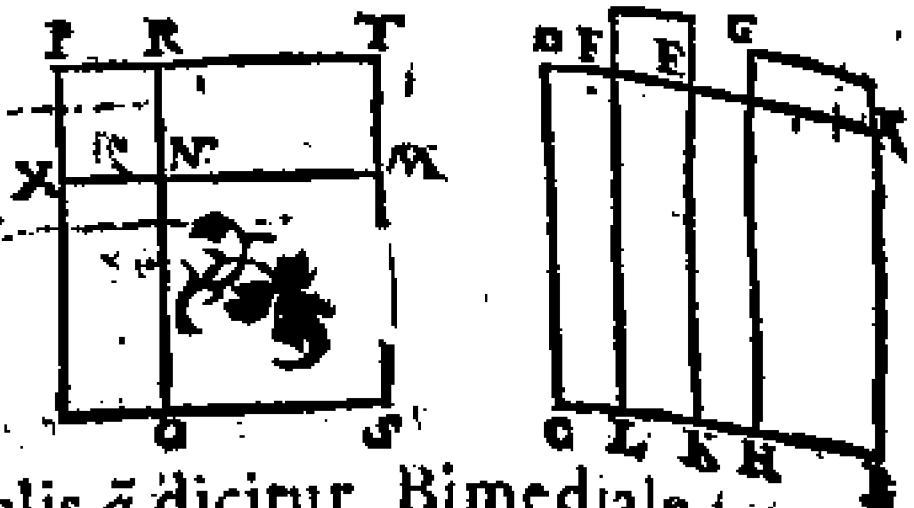
Theorema 38. Propositio 55.

Si superficies cõtenta fuerit ex linea rationali & Binomio secundo, linea potens illam superficiẽ est irrationalis quę Bimediale primum vocatur.



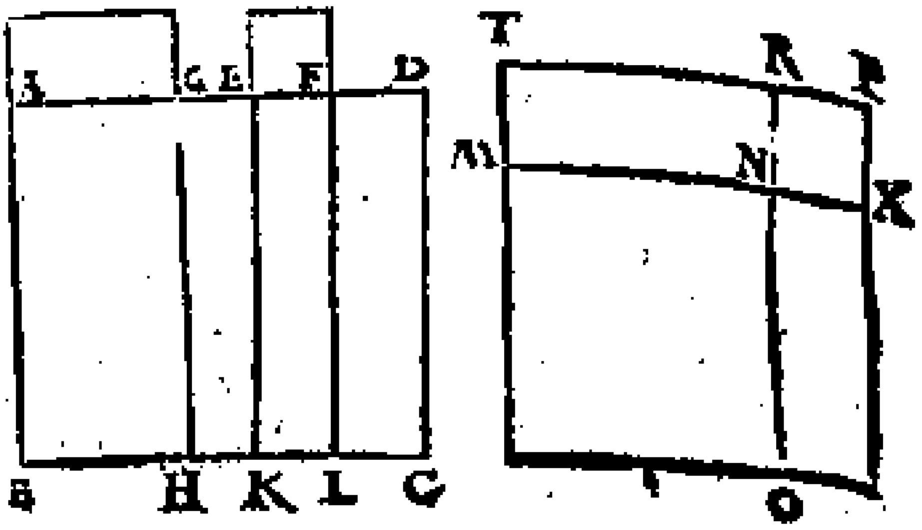
Theorema 39. Propositio 56.

Si superficies cõtineatur ex rationali & Binomio tertio, linea quę illam superficiem potest, est irrationalis quę dicitur Bimediale secundũ.



Theorema 40. Propositio 57.

Si superficies contineatur ex rationali & Boin

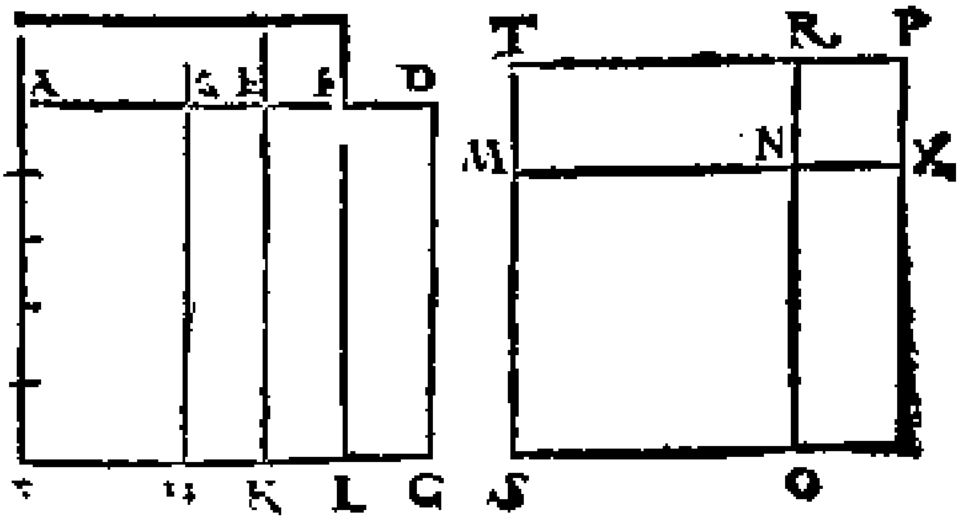


mio

mio quarto. linea potens superfi.iem illam, est irrationalis, quæ dicitur maior.

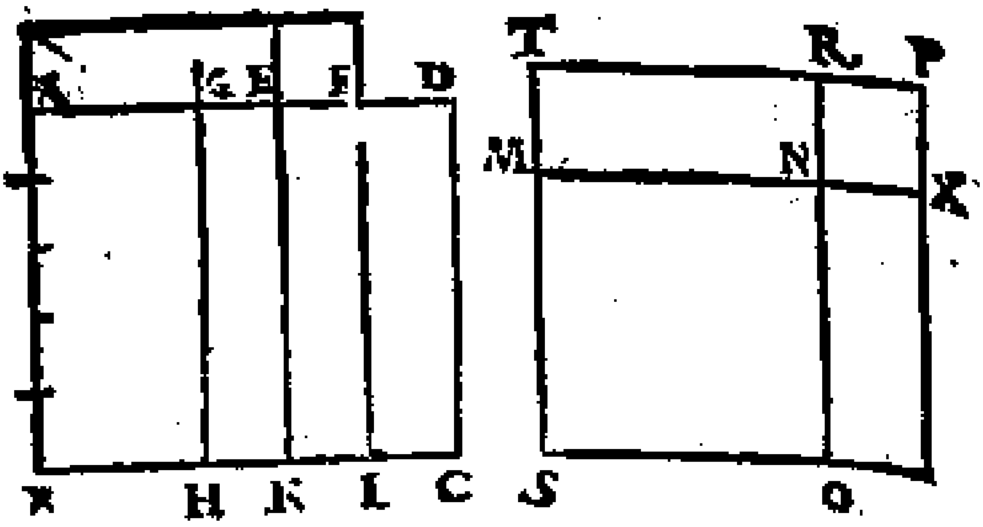
Theorema 41. Propositio 58.

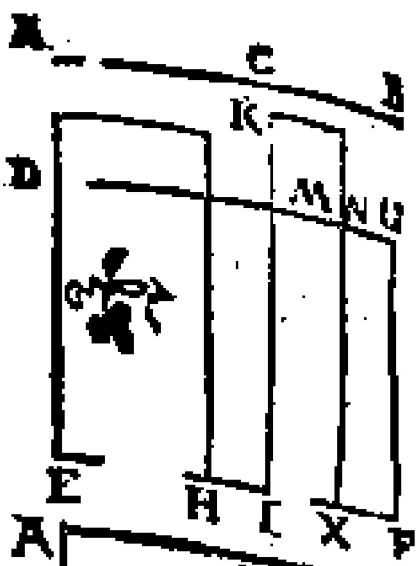
Si superficies contineatur ex rationali & Binomio quinto, linea quæ illam superficiem potest est irrationalis, quę dicitur potés rationale & mediale.



Therema 42. Propositio 59

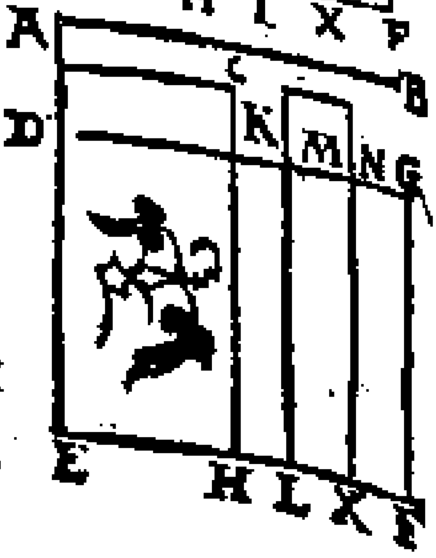
Si superficies contineatur ex rationali & Binomio sexto, linea quæ illam superficiem potest est irrationalis, quæ dicitur poténs tuo mediala.





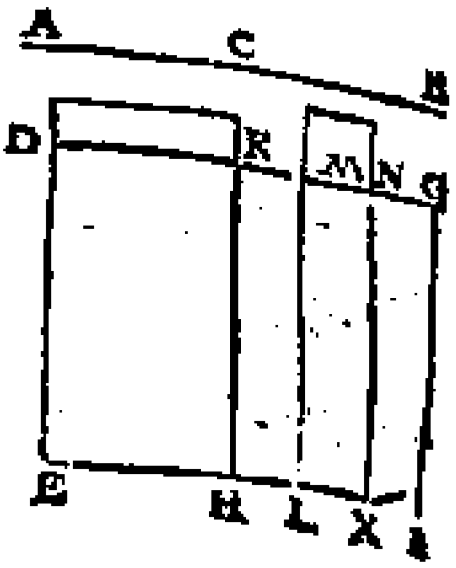
Theorema 43. Propositio 60.

Quadratum Binomij secundum lineam rationalem applicatum, facit alterum latus Binomium primum.



Theorema 44. Propositio 61.

Quadratū Bimedialis primi secundum rationalem lineam applicatum, facit alterum latus Binomium secundum.

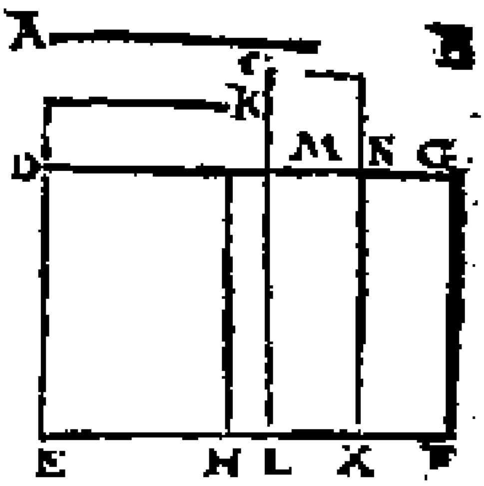


Theorema 45. Propositio 62.

Quadratū Bimedialis secūdi secundū rationalem applicatum; facit alterū latus Binomimū tertium.

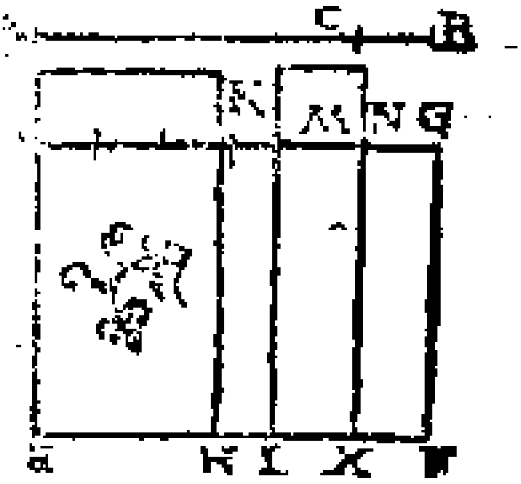
Theorema 46. [P]ropo-
positio 63

Quadratum lineæ maioris secundum lineam rationalem applicatum, facit alterum latus Binomium quartum.



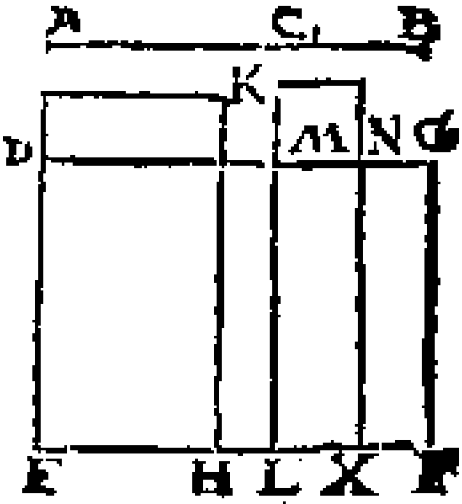
Theorema 47. Propositio 64.

Quadratum lineæ potẽtis rationale & mediale secundum rationalẽ applicatũ, facit alterum latus Binomium quintum.



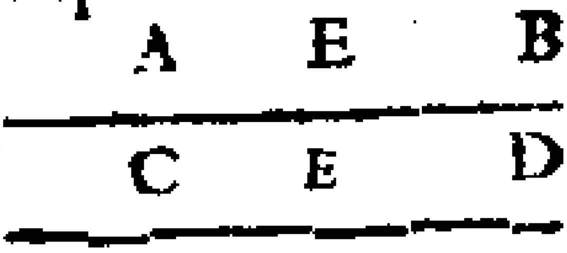
Theorema 48. Propositio 65

Quadratum lineę potentis duo medialia secundũ rationalem applicatum, facit alterum latus Binomium sextum.



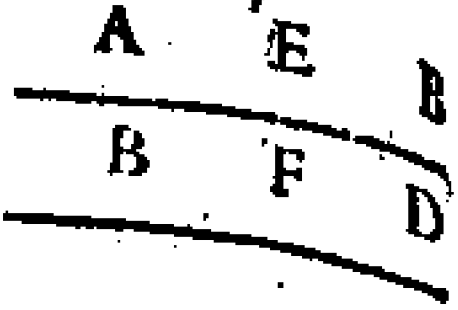
Theorema 49. Propositio 66.

Linea lõgitudine commensurabilis Binomio est & ipsa Binomium eiusdem ordinis.



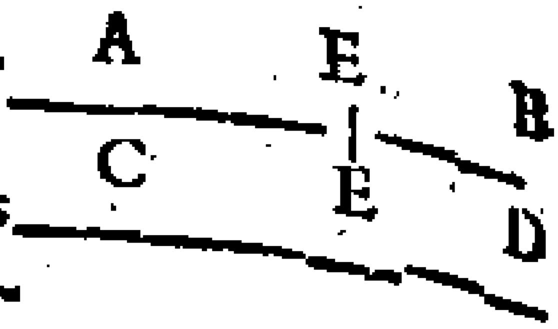
Theorema 50. Propositio 67

Linea longitudine commensurabilis alteri bimedialum, est & ipsa bimediale etiam eiusdẽ ordinis.



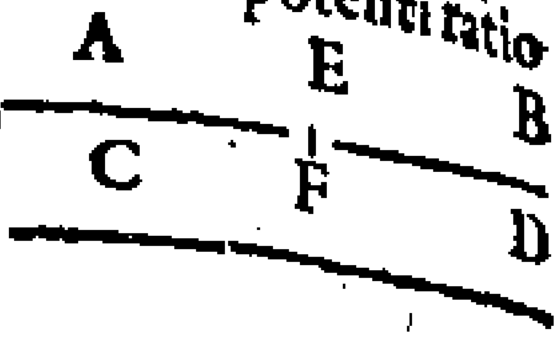
Theorema 51. Propositio 68

Linea cõmensurabilis lineæ maiori, est & ipsa maior.



Theorema 52. Propositio 69.

Linea commensurabilis lineæ potenti rationale & mediale, est & ipsa linea potens rationale & mediale.

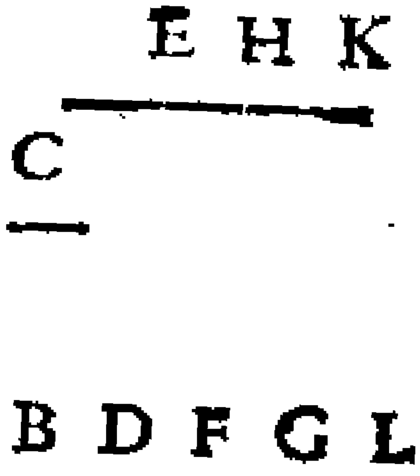


Theorema 53. Propositio 70.

Linea commensurabilis lineæ potenti duo medialia, est & ipsa linea potens duo medialia.

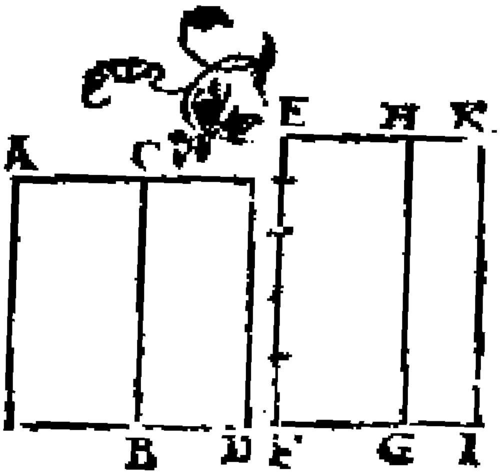
Theorema 54 Propositio 71

Si duæ superficies rationalis & medialis simul componantur, linea quę totã superficiem



ciem compositã potest, est vna ex quatuor irrationalibus, vna ex quæ dicitur Binomium, vel bimediale primum. vel linea maior, vel linea potēs rationale & mediale.

Theorema 55. Propositio 72.



Si duæ superficies mediales incommensurabiles simul componantur siunt reliquæ duæ lineę irrationales, vel bimediale secundum, vel linea potēs duo medialia

SCHOLIVM.

Binomium & cæteræ consequentes lineæ irrationales, neque sunt eadem cum linea mediali, neque ipsæ inter se.

Nam quadratum lineæ medialis applicatum secundum lineam rationalem facit alterum latus lineam rationalem, & longitudine incommensurabilem lineæ secundum quam applicatur, hoc est, lineæ rationali, per 23.

Quadratum verò Binomij secundum rationalem applicatum, facit alterum latus Binomium primum, per 60.

Quadratum verò Bimedialis primi secundum rationalem applicatum, facit alterum latus Binomium secundum, per 61

Quadratum verò Bimedialis secundi secundum rationalem applicatum; facit alterum latus Binomiũ tertium per 62.

Quadratum verò lineæ maioris secundum rationalem applicatum facit alterum latus Binomiũ quartum, per 63

Quadratum verò lineæ potẽtis rationale & mediale secundum rationalem applicatum, facit alterũ latus Binomium quintum, per 64.

Quadratum vero lineæ potentis duo medialia secũdum rationalem applicatum, facit alterum latus Binomium sextum per 65.

Cùm igitur dicta latera, quæ latitudines vocantur, differant & à prima latitudine, quoniam est rationalis, cùm inter se quoque differant, eo quia sunt Binomia diuersorum ordinum: manifestum est ipsas lineas irrationales, differentes esse inter se.

SECVN.

SECVNDVS ORDO ALterius sermonis, qui est de detractione.

Principium senariorum per detractionem.

Theorema 57. Propositio 74.

Si de linea rationali detrahatur rationalis potentia tantùm commensurabilis ipsi toti, residua est irrationalis, vocetur autem Residuum.

A A A

Theorema 56. Propositio 57.

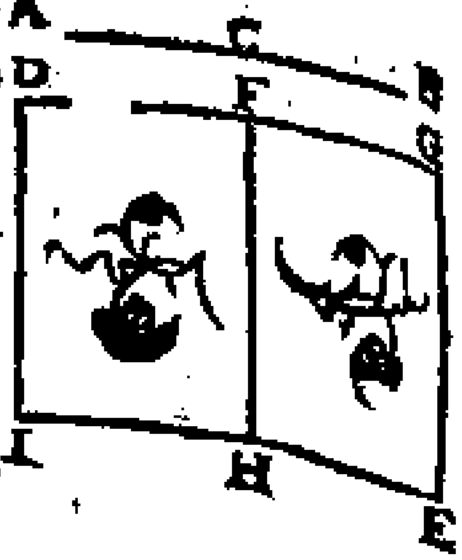
Si de linea mediali detrahatur medialis potentia tantũ commensurabilis toti lineæ, quæ verò detracta est cũ tota contineat superficiem rationalem, residua est irrationalis. Vocetur autem Residuum mediale primum.

A A B

Theorema 58 Propositio. 75.

Si de linea mediali detrahatur medialis

tentia tantùm commensurabilis toti, quæ verò detracta est, cùm tota cõtineat superficiẽ medialẽ, reliqua est irrationalis. Vocetur autem Residuũ mediale secundum.



Theorema 57. Propositio 76

Si de linea recta detrahatur recta potẽtia incommensurabilis toti, compositum autem ex quadratis totius lineæ & lineæ detractæ sit rationale, parallelogrammum verò ex ijsdem contentum sit mediale, reliqua linea erit irrationalis. Vocetur autem linea minor.

Theorema 58. Propositio 77.

Si de linea recta detrahatur recta potentia

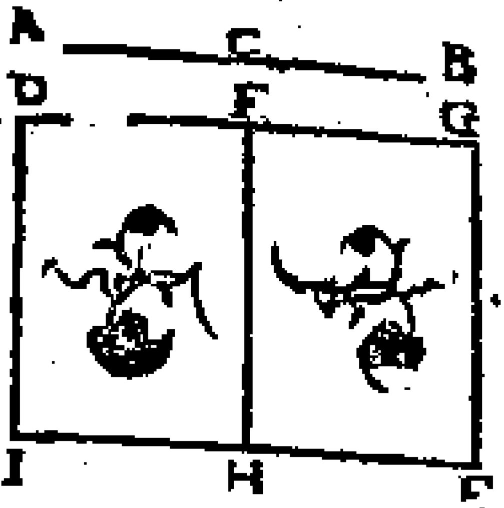
tia incommenſurabilis toti lineæ compoſitum autem ex quadratis totius & lineæ detractæ ſit mediale, paralielogrammum verò bis ex eiſdem contentum ſit rationale, reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie rationali totam ſuperficiem medialem.

Theorema 59. Propoſitio 78.

Si de linea recta detrahatur recta potentia incommenſurabilis toti lineæ, compoſitum autem ex quadratis totius & lineæ detractæ ſit mediale, parallelogrammum verò bis ex ijſdem ſit etiam mediale: præterea ſint quardata ipſarum incommenſurabilia parrallelogrammo bis ex ijſdem contento, reliqua linea eſt irrationalis.

Vocetur autem linea faciens cum ſuper-ficie

ficie mediali totam superficiem medialem.

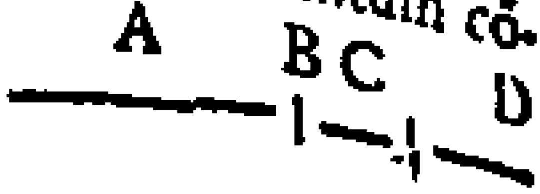


Theorema 60. Propositio 79.

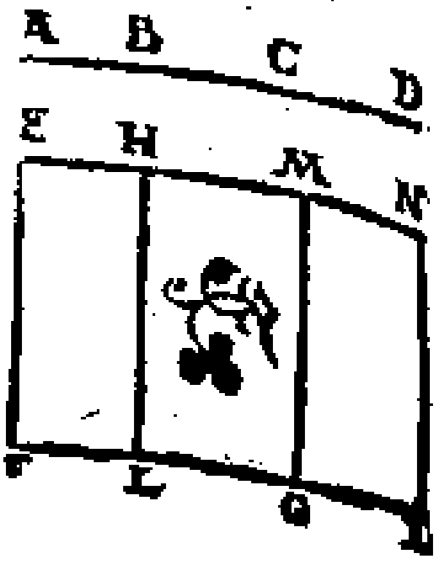
Residuo vnica tantùm linea recta cõiungitur rationalis potentia tantùm cõmensurabilis toti lineæ rationalem.

Theorema 61. Propositio 80.

Residuo mediali primò vnica tãtùm linea coniungitur medialis, potentia tantùm cõmensurabilis toti, ipsa cum tota continens



Theorema 62. Propositio 81.



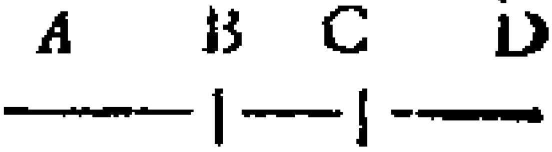
Residuo mediali secundo vnica tantùm coniungitur mediails, potentia tantùm commẽsurabilia toti ipsa cum tota continens mediale.

Theorema 63. Propositio 82.

Lineæ minori vnica tantùm recta cõiungitur

tur

tur potentia incommensurabilis toti, faciens cum tota compositum ex quadratis ipsarum rationale, id verò parallelogrammum, quod bis ex ipsis fit, mediale.

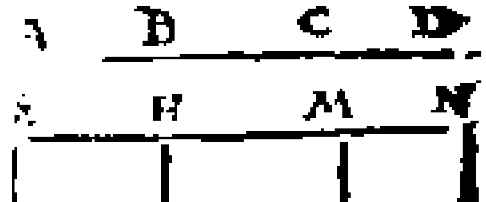


Theorema 64. Propositio 83.

Lineæ facienti cum superficie rationali totam superficiem medialem, vnica tantùm coniungitur linea recta potentia incommensurabilis toti, faciens autem cum tota compositum ex quadratis ipsarũ, mediale, id verò quod fit bis ex ipsis, rationale.

Theorema 65. Propositio 84.

Lineæ cum mediali superficie facienti totam superficiem medialem, vnica tantum coniũgitur linea potẽtia toti incommensurabilis, faciens cum tota compositũ ex quadratis ipsarũ mediale, id vero quod fit bis ex ipsis etiam mediale, & prætereà faciens compositum ex quadratis ipsarum incommensurabile ei quod fit bis ex ipsis,



DE.

DEFINITIONES TERTIAE

Proposita linea rationali & residuo.

1

Si quidem tota, nempe composita ex ipso residuo & linea illi cõiuncta, plus potest quàm coniuncta, quadrato lineæ sibi cõ. longitudine commẽsurabilis, fueritq; tota longitudine commẽsurabilis lineæ propositæ rationali, residuũ ipsum vocetur Residuum primum.

2

Si verò coniuncta fuerit longitudine commensurabilis rationali, ipsa autem tota plus possit quam coniuncta, quadrato lineæ sibi longitudine commensurabilis, residuum vocetur Residuum secundũ.

3

Si verò neutra linearũ fuerit longitudine commensurabilis rationali possit autem ipsa tota plusquam coniuncta, quadrato lineæ sibi longitudine commẽsurabilis, vocetur Residuum tertium.

Rursus si tota possit plus quàm coniuncta quadrato lineæ sibi longitudine incommensurabilis

4

Et quidem si tota fuerit longitudine commen.

menſurabilis ipſi rationali, vocetur Reſiquum quartum.

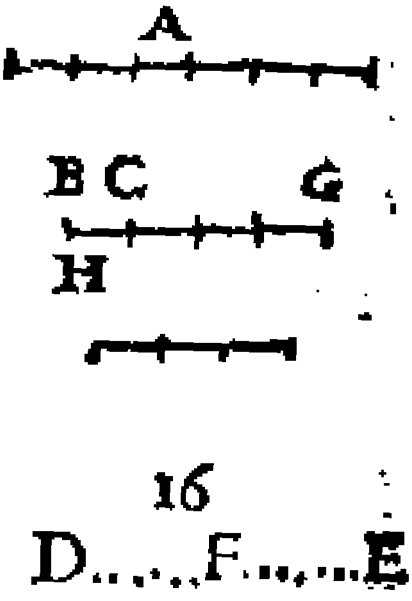
5

Si verò coniuncta fuerit lõgitudine commenſurabilis rationali, & tota plus poſſit quàm coniuncta, quadrato lineæ ſibi longitudine incõmenſurabilis, vocetur Reſiduum quintum.

6

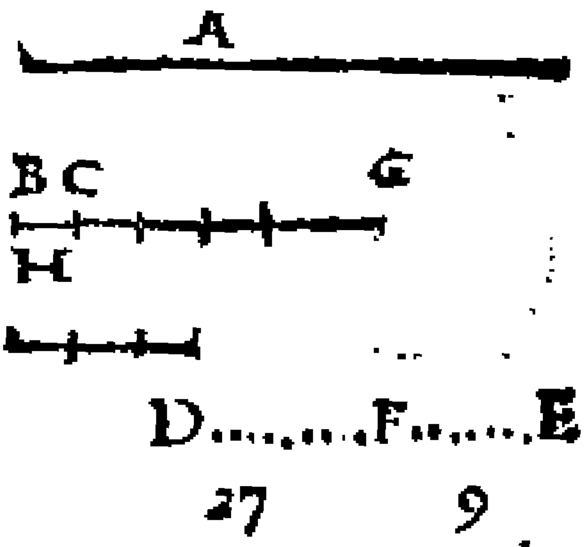
Si verò neutra linearum fuerit cõmenſurabilis longitudine ipſi rationali, fueritque tota potentior quã coniuncta, quadrato lineæ ſibi longitudine incommenſurabilis, vocetur Reſiduum ſextum.

Problema 18. Propoſitio 85.



Reperire primum Reſiduum.

Problema 19. Propoſitio 86.

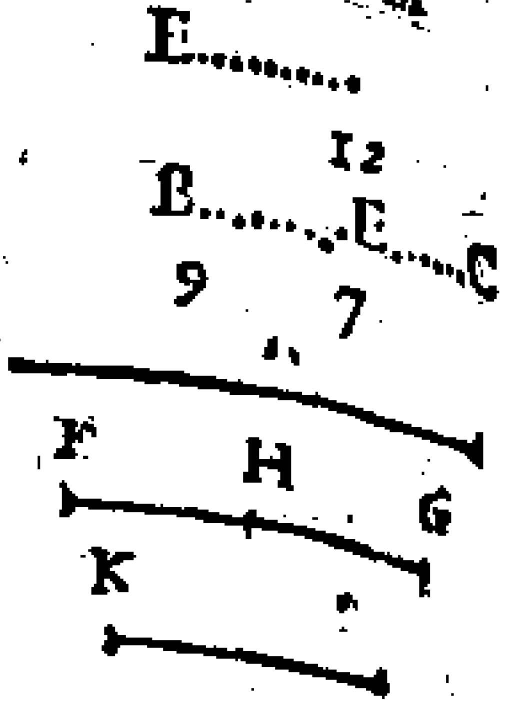


Reperire ſecundum Reſiduum.

Prob-

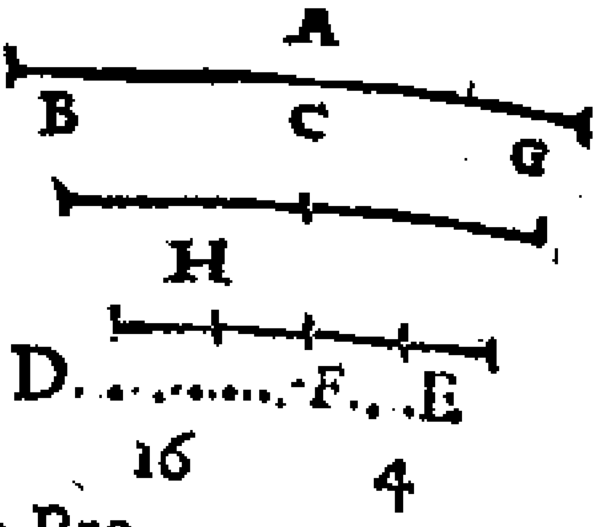
Problema 02. Propositio 87.

Reperire tertium Residuum.



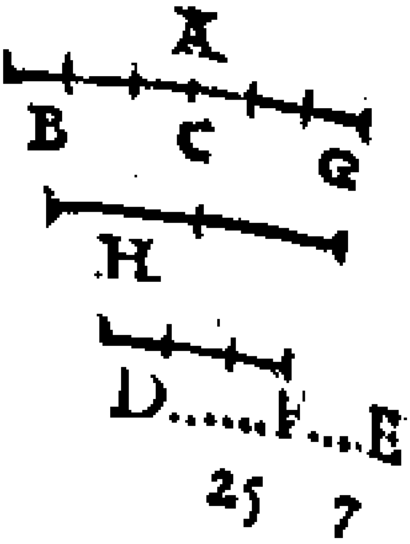
Probl. 21. Propositio 88.

Reperire quartum Residuum.



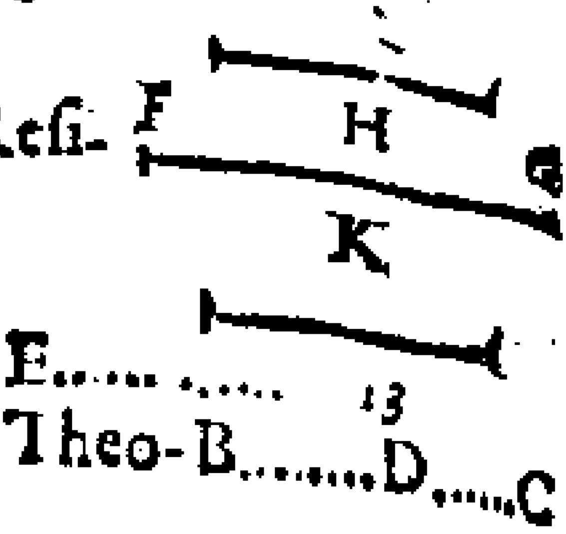
Problema 22. Propositio 89

Reperire quintum Residuum



Problema 23. Propositio 90.

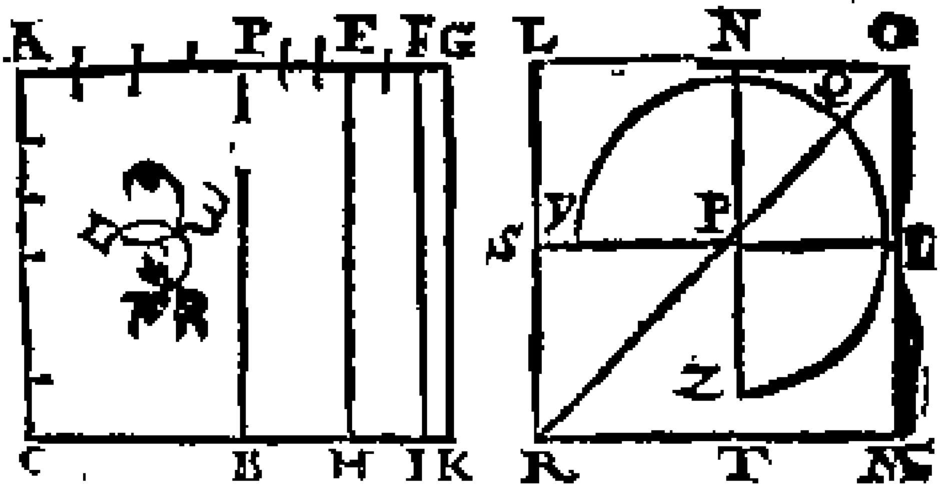
Reperire sextum Residuum.



Theo-

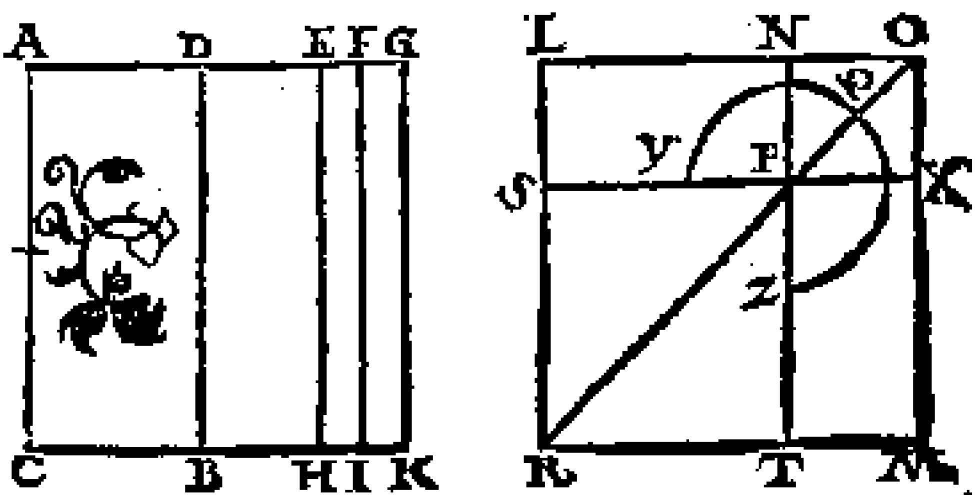
Theorema 66. Propoſitio 91.

Si ſuperficies contineatur ex linea rationali & reſiduo primo linea, quæ illam ſuperficiem poteſt, eſt reſiduum.



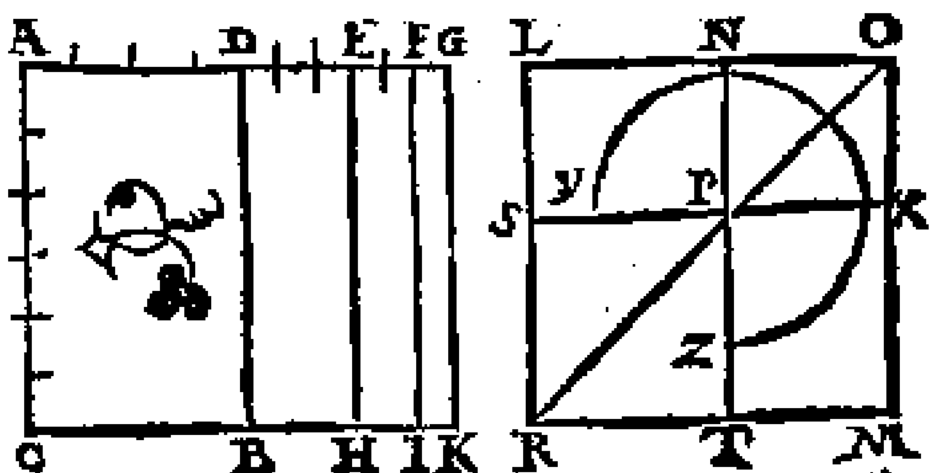
Theorema 67. Propoſitio 92.

Si ſuperficies cõtineatur ex linea rationali & reſiduo ſecundo, linea q̃ illam ſupſiciẽ poteſt, eſt reſiduum mediale primum.



Theorema 68. Propoſitio 93.

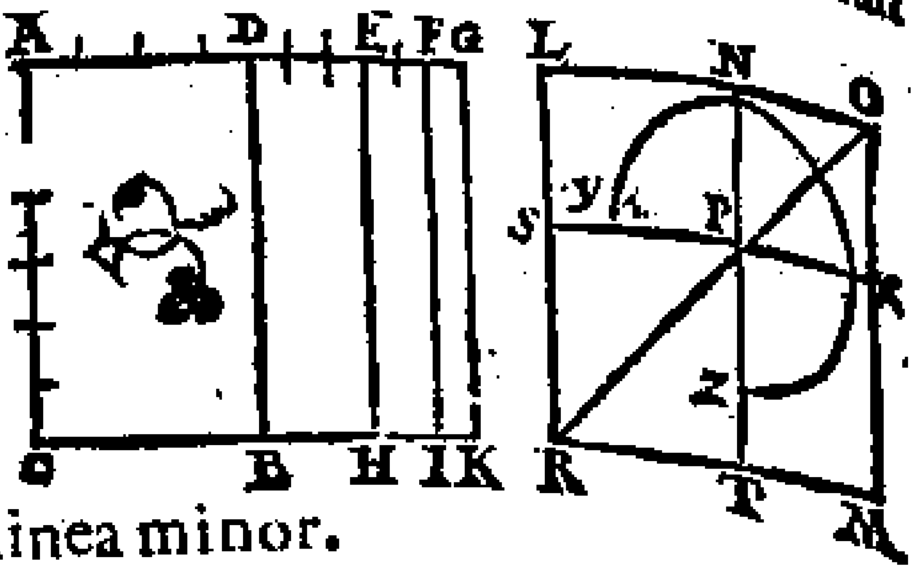
Si ſuperficies cõtineatur ex linea rationali & reſiduo ter-



tio, linea quæ illam superficiem potest, est residuum mediale secundum.

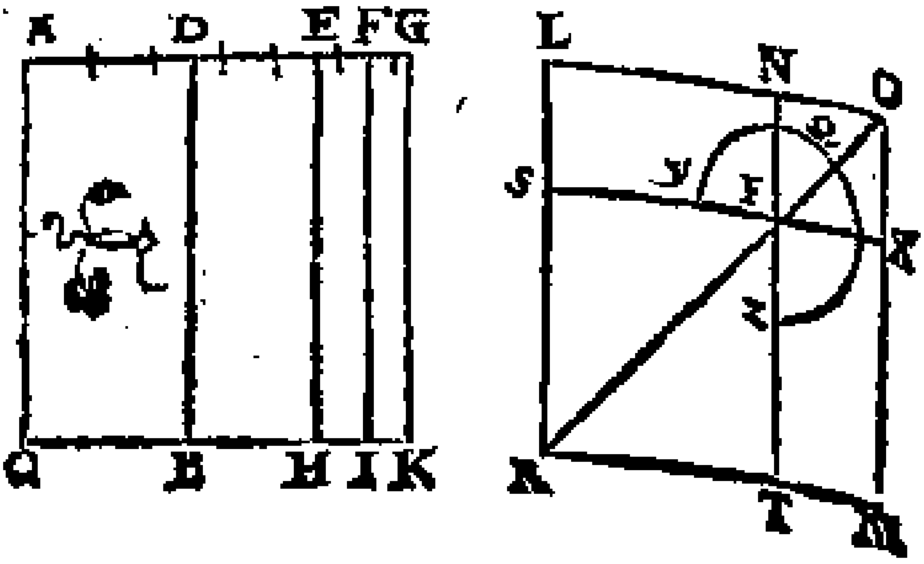
Theorema 69. Propositio 94.

Si superficies contineatur ex linea rationali & residuo quarto linea q̃ illá superficiem potest, est linea minor.



Theorema 70. Propositio 95.

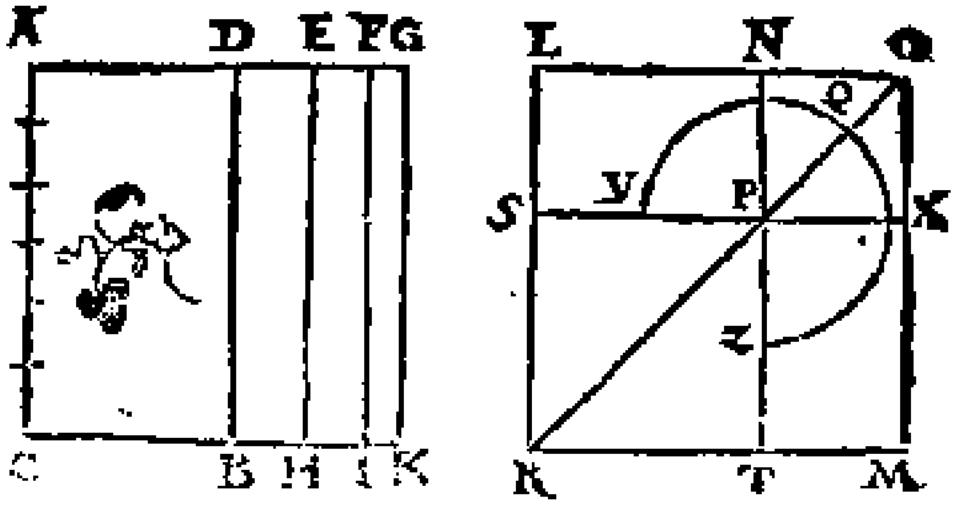
Si superficies contineatur ex linea rationali & residuo quinto, linea quæ illá superficiē potest, est ea quæ dicitur cum rationali superficie faciens totam medialem.



Theorema 71. Propo-positio 96.

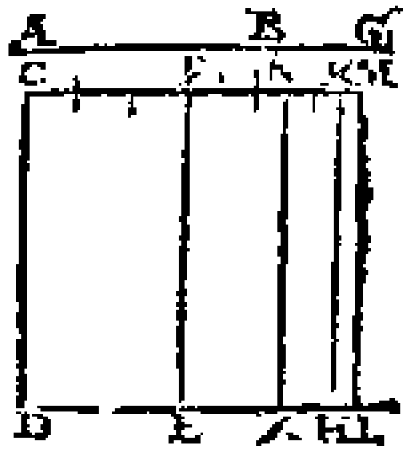
Si superficies contineatur ex linea rationali &

& residuo sexto, linea quæ illam superficiē pōt, est ea quę dicitur faciēs cum mediali superficie totam medialem.



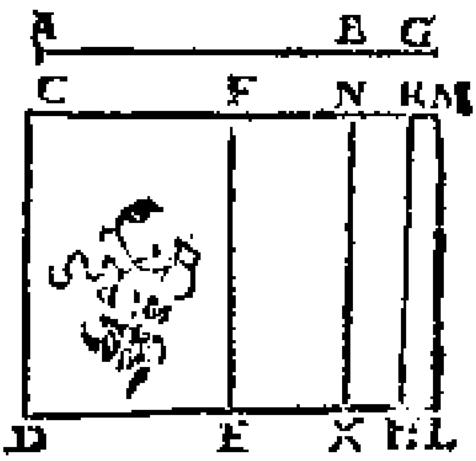
Theorema 72. Propositio 97.

Quadratum residui secundum lineam rationalem applicatum, facit alterum latus redsiuum primum.



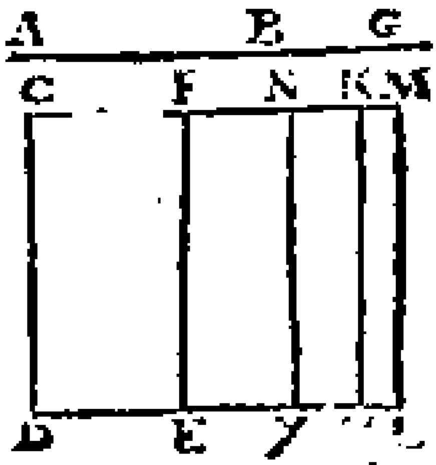
Theorema 73. Propositio 98.

Quadratum residui medialis primi secundũ rationalem applicatũ, facit alterum latus residuũ secundum.

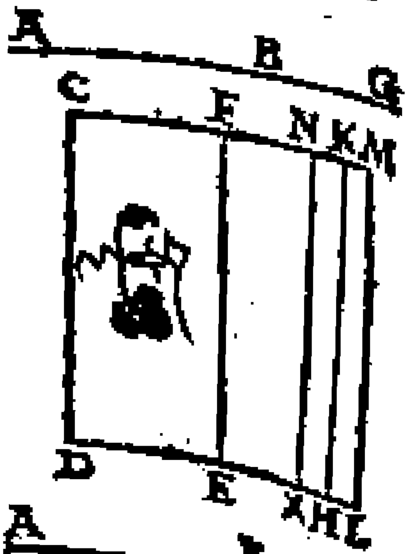


Theorema 74. Propositio 99.

Quadratũ, residui medialis secundi secundum rationalē applicatũ, facit alterũ latus residuum tertium.

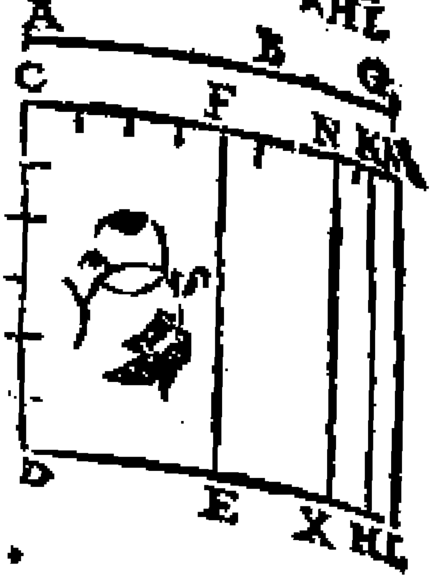


Theorema 75. Propositio 100.



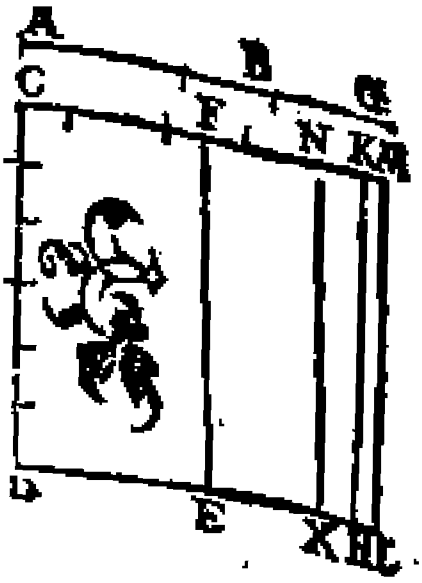
Quadratũ lineæ minoris ſecundùm rationalẽ applicatum, facit alterũ latus reſiduum quartũ.

Theorema 76. Propoſitio 101.



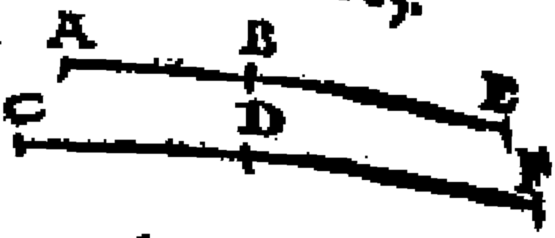
Quadratũ lineæ cum rationali ſuperficie faciẽtis totam medialem, ſecũdum rationalem applicatum, facit alterum latus reſiduum quintum.

Theorema 77. Propoſitio 102.



Quadratum lineæ cum mediali ſuperficie faciẽtis totam medialem, ſecundum rationalem applicatum, facit alterum latus reſiduum ſextum.

Theorema 78. Propoſitio 103.

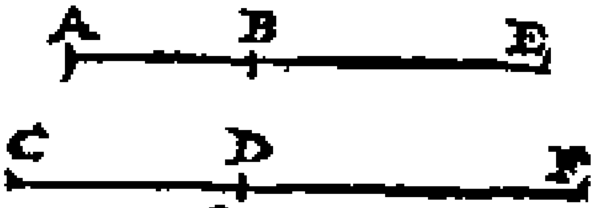


Linea reſiduo commenſurabilis longitudine, eſt & ipſa reſiduum, & eiuſdem ordinis

Theorema 79. Propoſitio 104.

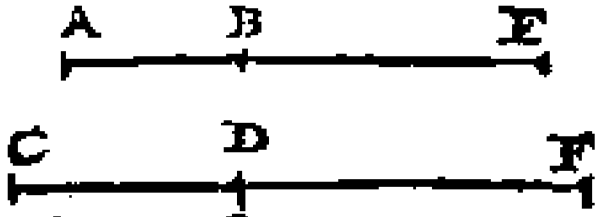
Li.

Linea cõmenſurabilis reſiduo mediali, eſt & ipſa reſiduũ mediale, & eiuſdẽ ordinis.



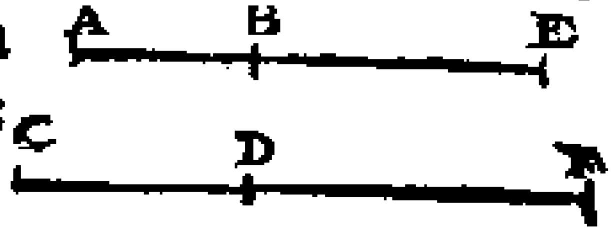
Theorema 80. Propoſitio 105.

Linea cõmẽſurabilis lineę minori, eſt et ipſa linea minor



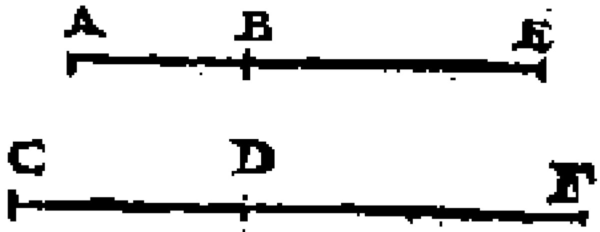
Theorema 81. Propoſitio 106.

Linea commenſurabilis lineæ cũ rationali ſuperficie facienti totã medialẽ, eſt & ipſa linea cum rationali ſuperficie faciens totam medialem.



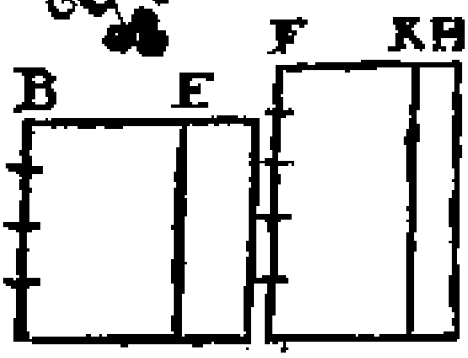
Theorema 82. Propoſitio 107.

Linea commẽſurabilis lineæ cũ mediali ſuperficie facienti totam medialem, eſt & ipſa cum mediali ſuperficie faciens totam medialem.



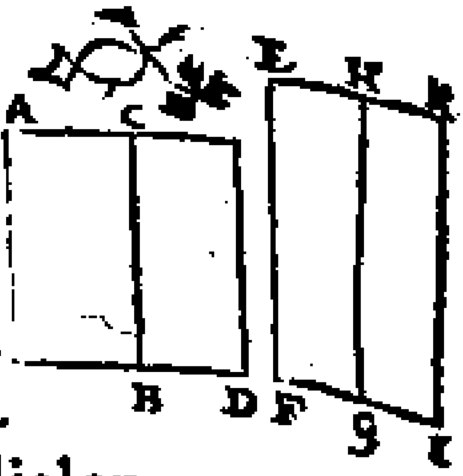
Theorema 83. Propoſitio 108.

Si de ſuderficie rationali detrahatur ſuperficies medialis, linea quæ reliquã ſuperficiem poteſt, eſt alterutra ex duabus irrationalibus, aut reſiduum, aut liuea minor.



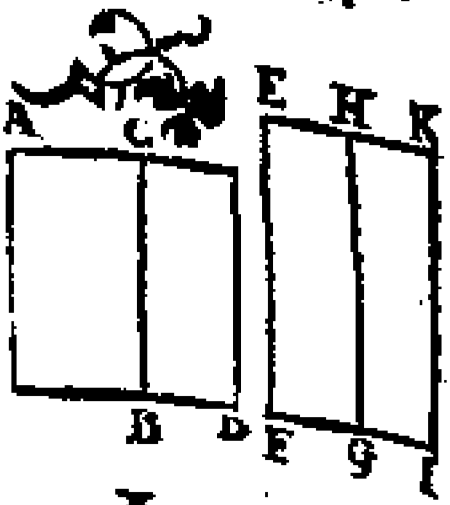
Theorema 84. Propoſitio 109

Si de ſuperficie mediali detrahatur ſuperficies rationalis, aliæ duæ irrationales, fiũt, aut reſiduum mediale primũ aut cũ rationali ſuperficiem faciens totam medialem.

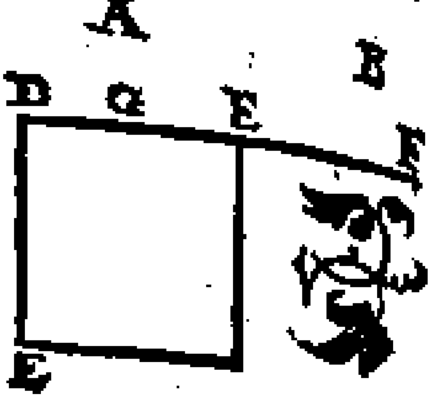


Theorema 85. Propoſitio 110.

Si de ſuperficie mediali detrahatur ſuperficies medialis q̃ ſit incommẽſurabilis toti, reliquę duæ fiunt irrationales, aut reſiduum mediale ſecundum, aut cũ mediali ſuperficie faciens totam medialem.



Theorema 86. Propoſitio 111.



Linea quæ Reſiduum dicitur, nõ eſt eadem cum ea quæ dicitur Binomium.

SCHO.

SCHOLIVM.

Linea quæ Residuum dicitur, & cæteræ quinque eã consequentes irrationales, neque lineæ mediali neque sibi ipsæ inter se sunt eædem. Nam quadratum lineæ medialis secundum rationalem applicatum facit alterum latus, rationalem lineam longitudine incommensurabilem ei, secundum quam applicatur. per 23.

Quadratum verò residui secundum rationalem applicatũ, facit alterũ latus residuũ primũ, per 97.

Quadratum verò residui medialis primi secundum rationalem applicatum facit alterum latus residuum secundum, per 98.

Quadratum verò residui medialis secundi, facit alterum latus residuum tertium, per 99.

Quadratum verò lineæ minoris, facit alterũ latus residuum quartum, per 100.

Quadratum verò lineæ cum rationali superficie facientis totam medialem, facit alterum latus residuum quintum, per 101.

Quadratum verò lineæ cum mediali superficie facientis totam medialem, secundum rationalem applicatum, facit alterum latus residuum sextum, per 102.

Cùm igitur dicta latera, quæ sunt latitudines cuiusque parallelogrammi vnicuique quadrato æqualis & secundum rationalem, applicati, differant & à primo latere, & ipsa inter se (nam à primo differunt: quoniam sunt resi-

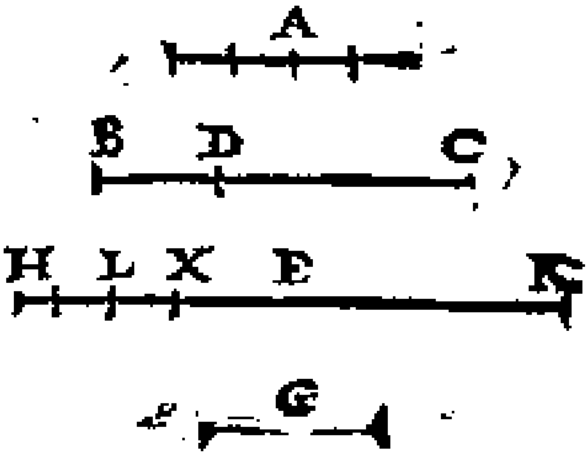
dua non eiusdem ordinis) constat ipsas quoque lineas irrationales inter se defferentes esse. quoniam demonstratum est, Residuum non esse idem quod Binomium, quadrata autem residui & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex residuis eiusdem ordinis cuius sunt & residua, quorum quadrata applicantur rationali similiter & quadrata Binomij & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex Binomijs eiusdem ordinis, cuius sunt & Binomia, quorum quadrata applicātur rationali. Ergo lineæ irrationales quæ cōsequuntur Binomium, & quæ consequuntur residuū, sunt inter se differētes. Quare dictæ lineæ omnes irrationales sunt numero 13.

1 Medialis.
2 Binomium.
3 Bimediale primum.
4 Bimediale secundum.
5 Maior.
6 Potens rationale & mediale.
7 Potēs duo medialia.
8 Residuum.
9 Residuum mediale primum.
10 Residuum mediale secundum.
11 Minor.
12 Faciens cum rationali superficie totam medialem
13 Faciens cum mediali superficie totam medialem.

Theo-

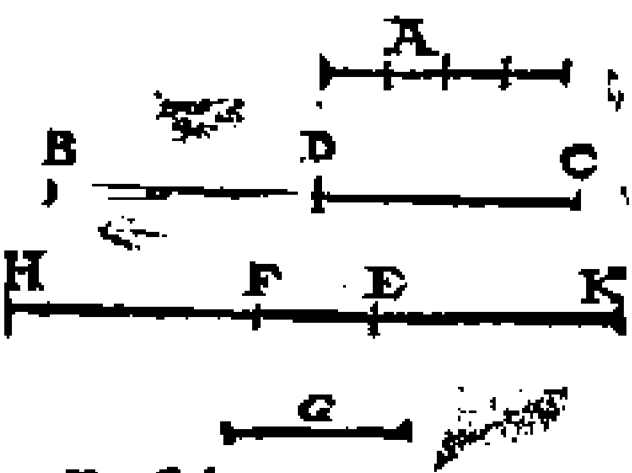
Theorema 87. Propositio 112.

Quadratũ lineæ rationalis secundum Binomiũ applicatũ, facit alterum latus residuũ, cuius nomina sunt cõmensurabilia Binomij nominibus, & in eadẽ proportione præterea id quod fit Residuum, eundem ordinem retinet quem Binomium.



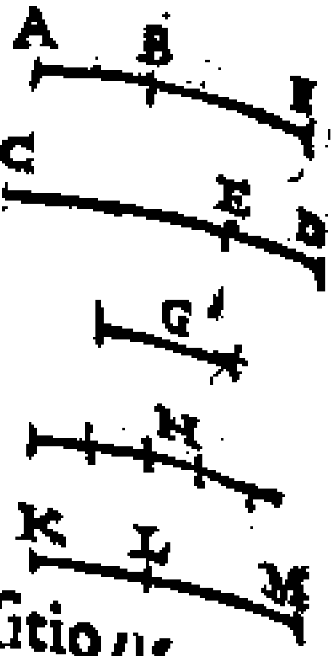
Theorema 88. Propositio 113.

Quadratũ lineæ rationalis secundum residuum applicatũ, facit alterum latus Binomium, cuius nomina sunt commensurabilia nominibus residui & in eadẽ proportione: præterea id quod fit Binomiũ, est eiusdem ordinis, cuius & Residuum.



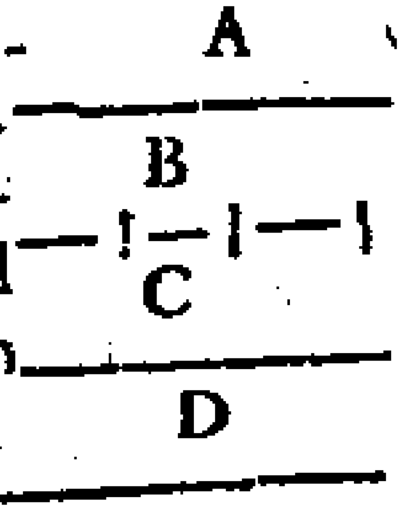
Theorema 89 Propositio 114.

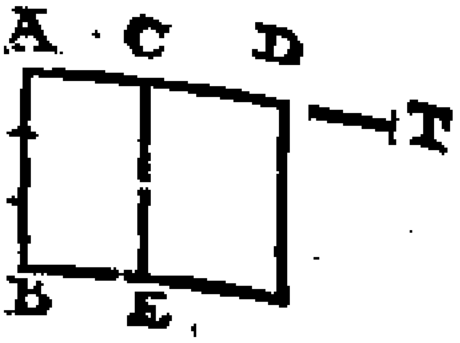
Si parallelogrammum contineatur ex residuo & Binomio, cuius nomina sunt commensurabilia nominibus residui & in eadẽ proportione, linea quæ illam superficiem potest, est rationalis.



Theorema 90. Propositio 115.

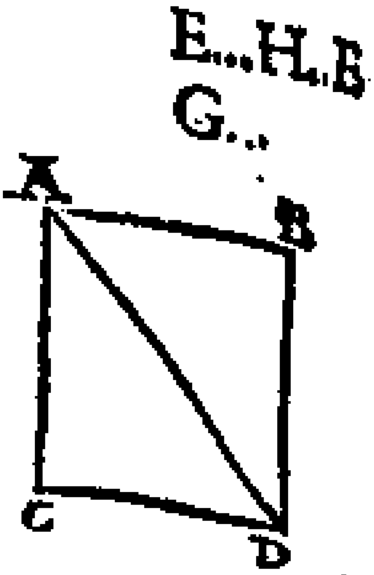
Ex linea mediali nascũtur lineę irrationales innumerabiles, quarum nulla vlli ante dictarum eadem sit.





Theorema 100. Propositio 116.

Propositũ nobis esto demonstrare in figuris quadratis diametrũ esse longitudine incommensurabilem ipsi lateri.



ELEMENTI X. FINIS.

EVCLIDIS ELEMENTVM VNDECIMVM, ET SOLIDORVM primum.

DEFINITIONES.

1

Solidum, est quod longitudinem, latitunem, & crassitudinem habet.

2

Solidi autem extremum est superficies.

3.

Linea recta est ad planum recta cùm ad rectas oẽs lineas, à quibus illa tangitur, quæque in proposito sunt plano, rectos angulos efficit.

4

Planum ad planum rectum est, cum rectæ lineæ, quæ cõmuni planorum sectioni ad rectos angulos in vno planorum ducũtur, alteri plano ad rectos sunt angulos.

5

Rectæ lineę ad planũ inclinatio acutus est angulus, ipsa insistente linea & adiũcta altera cõprehẽsus, cùm à sublimi rectæ illius lineę termino deducta fuerit ꝑpẽndicularis,

laris, atq; à puncto quod perpendicularis in ipso plano fecerit, ad propositę illius lineæ extremum, quod in eodem est plano, altera recta linea fuerit adiuncta.

6

Plani ad planũ inclinatio, acutus est angulus rectis lineis contentus, quæ in vtroque planorum ad idem communis sectionis pũctũ ductæ, rectos ipsi sectioni angulos efficiunt.

7

Planum similiter inclinatum esse ad planum, atq; alterum ad alterum dicitur, cum dicti inclinationum anguli inter se sunt ęquales.

8

Parallela plana, sunt quæ eodem non incidunt, nec concurrunt.

9

Similes figuræ solidæ, sunt quæ æqualibus planis, multitudine æqualibus continetur.

10

Aequales & similes figuræ solidæ sunt, quę similibus planis, multitudine & magnitudine æqualibus continentur.

11

Solidus angulus, est plurium quàm duarũ linearum, quæ se mutuò contingant, nec in eadem sint superficie, ad omnes lineas inclinatio.

Aliter.

Aliter.

Solidus angulus, est qui pluribus quã duobus planis angulis, in eodem non cõsistentibus plano, se ad vnum punctum collectis continetur.

12

Pyramis, est figura solida quæ planis continetur, ab vno plano ad vnum punctũ collecta.

13

Prisma, figura est solida quę planis continetur, quorum aduersa duo sunt & æqualia & similia & parallela, alia verò parallelogramma.

14

Sphęra est figura, quæ cõuerso circum quiescentem diametrum semicirculo continetur, cùm in eundem rursus locũ restitutus fuerit, vnde moueri cœperat.

15

Axis autem sphæræ, est quiescẽs illa linea circum quam semicirculus conuertitur.

16

Centrum verò Sphęrę est idem, quod & semicirculi.

17

Diameter autem Sphæræ, est recta quædã linea per centrum ducta, & vtrinq; à sphęræ superficie terminata.

Conus

18

Cωnus est figura, quæ conuerso circũ quiescens alterum latus eorum quæ rectum angulum continent, orthogωnio triangulo continetur, cùm in eundem rursus locum illud triangulum restitutum fuerit, vnde moueri cœperat. Atq; si quiescens recta linea æqualis sit alteri, quę circum rectũ angulum conuertitur, rectangulus erit Cωnus: sin minor, amblygωnius: si verò maior, oxygonius.

19

Axis autem Cωni, est quiescens illa linea, circum quam triangulum vertitur.

20

Basis verò Cωni, circulus est, qui à circũducta linea recta describitur.

21

Cylindrus figura est, quæ conuerso circum quiescens alterum latus eorum quæ rectum angulum continent, parallelogrammo orthogωnio comprehenditur, cum in eũdem rursus locum restitutum fuerit illud parallelogrammũ, vnde moueri cœperat.

22

Axis autem Cylindri est quiescens illa recta linea, circum quam parallelogrammum vertitur.

23

Bases verò cylindri, sunt circuli à duobus

ad.

aduerſus lateribus quæ circum aguntur, deſcripti.

24

Similes coni & cylindri, ſunt quorũ & axes & baſium diametri proportionales ſunt.

25

Cubus eſt figura ſolida, quæ ſex quadratis æqualibus continetur.

16

Tetraedum eſt figura, quæ triangulis quatuor æqualibus & æquilateris continetur.

17

Octaedrũ figura eſt ſolida, quæ octo triangulis æquilibus & æquilateris continetur.

28

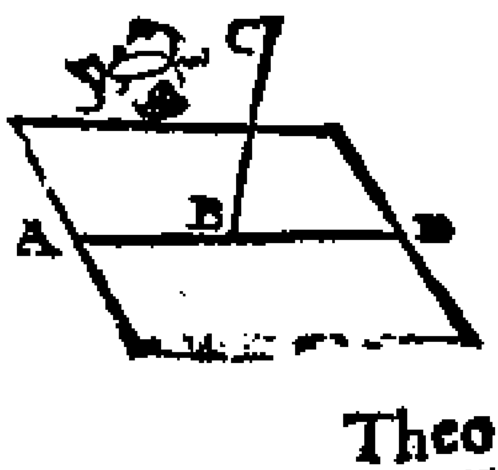
Dedecaedrum figura eſt ſolida, quæ duodelim pentagonis æqualibus, æquilateris, & æquiangulis continetur.

29

Eicoſaedrum figura eſt ſolida, quę triangulis viginti æqualibus & æquilateris continetur.

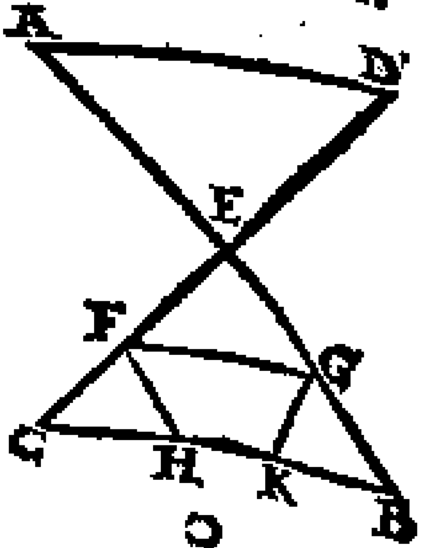
Theorema 1. propoſitio 1.

Quædã rectę lineę pars in ſubiecto quidem non eſt plano, quædam verò in ſublimi.



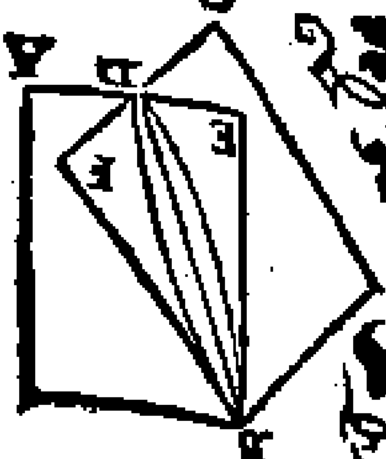
Theo

Tehorema 2. Propositio 2.



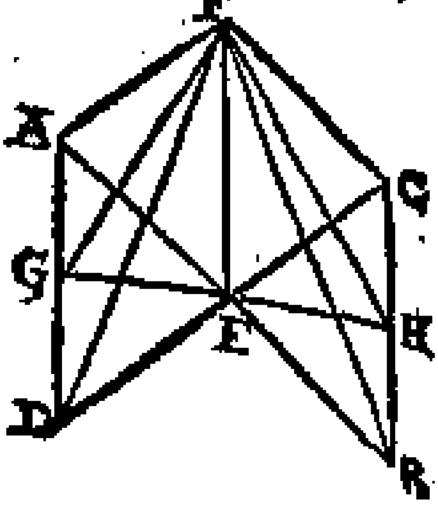
Si duę rectę lineę se mutuò secēt, in vno sūt plano: atque triangulum omne in vno est plano.

Theorema 3. Propositio 3.



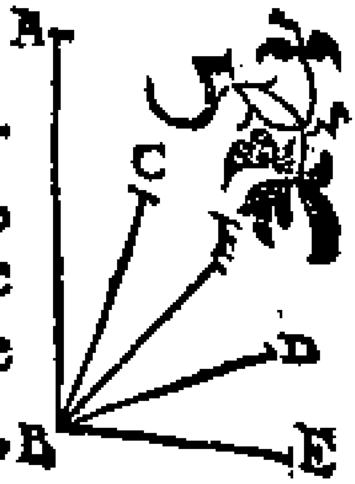
Si duo plana se mutuò secent, communis eorū sectio est recta linea.

Theorema 4. Propositio 4.



Si recta linea rectis duabus lineis se mutuò secantibus, in cōmuni sectione ad rectos angulos insistat illa ducto etiam per ipsas plano ad angulos rectos erit.

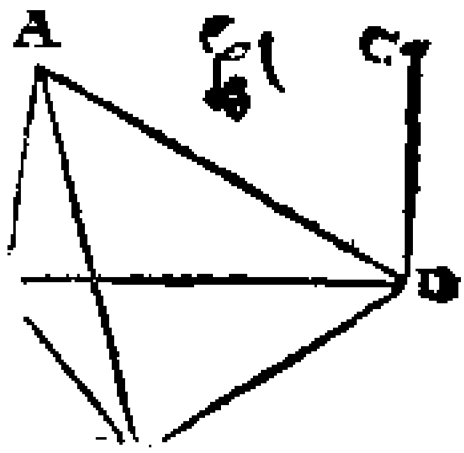
Theorema 5. Propositio 5.



Si recta linea rectis tribus lineis se mutuò tangentibus, incommuni sectione ad rectos angulos insistat, illæ tres rectę in vno sunt plano.

Theo.

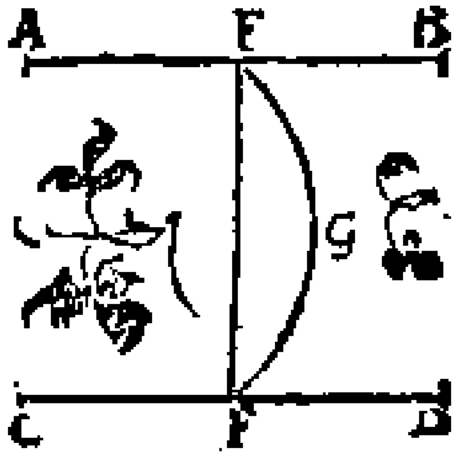
Theorema 6. Propositio 6.



Si duæ rectæ lineæ eidē plano ad rectos sint angulos, parallelæ erunt illæ rectæ lineæ

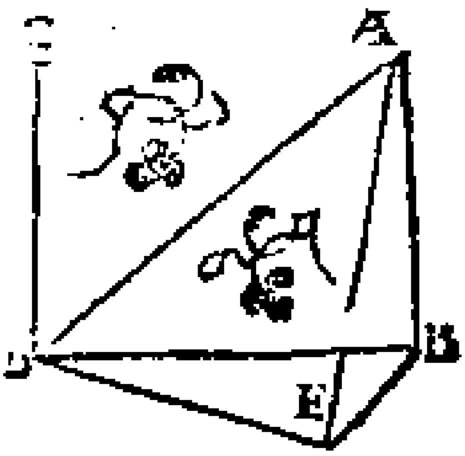
Theoréma 7. Propositio 7.

Si duæ sint parallelæ rectæ lineæ, in quarū vtraque sumpta sint quę libet pūcta, illa linea quę ad hæc puncta adiungitur, in eodem est cū parallelis plano.



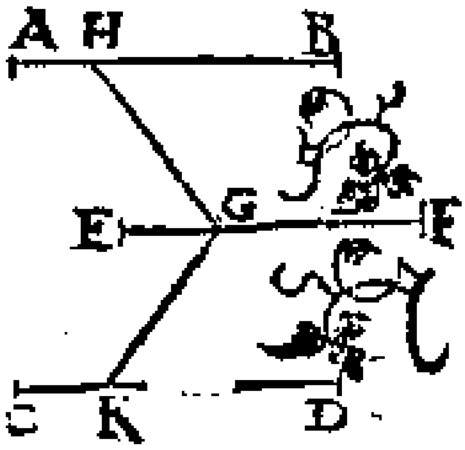
Theorema 8. Propositio 8.

Si duæ sint parallelæ rectæ lineæ, quarum altera ad rectos cuidam plano sit angulos, & reliqua eidē plano ad rectos angulos erit

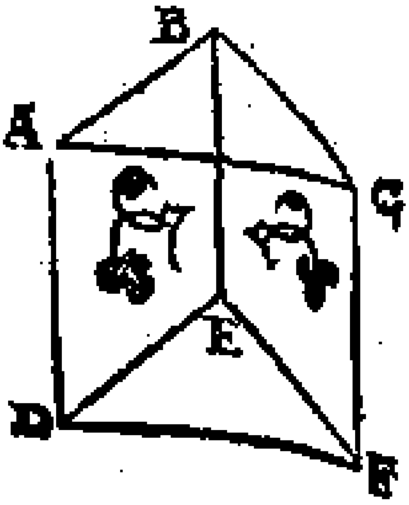


Theorema 9. Propositio 9.

Quæ eidem rectæ lineæ sunt parallelæ, sed nō in eodem cū illa plano, hæ quoque sunt inter se parallelæ.

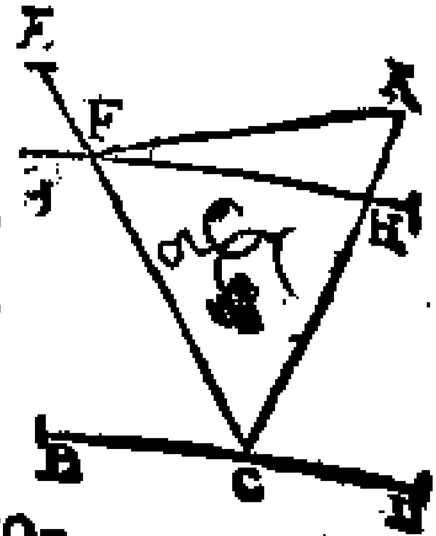


Theorema 10. Propositio 10.



Si duę rectę lineæ se mutuò tangétes ad duas rectas se mutuò tangentes sint parallelę, non autem in eodem plano, illę angulos æquales comprehendent.

Proble. 1. Propositio 11.

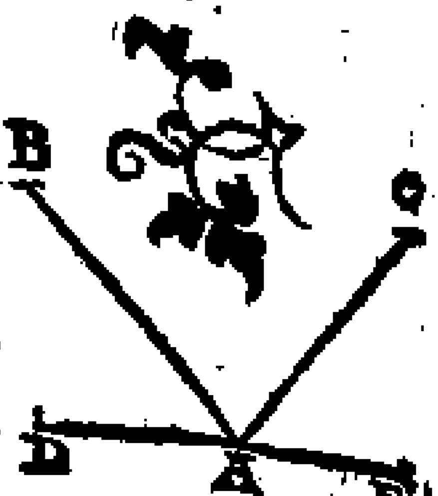


A dato sublimi puncto, in subiectum planũ perpendicularem rectam lineam ducere.

Problema 2. Propositio 12.

Dato plano, à pũcto quod in illo datum est, ad rectos angulos rectam lineam excitare.

Theorema 11. Propositio 13.

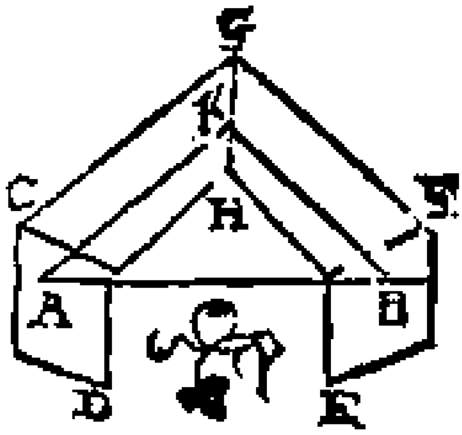


Dato plano, à puncto qđ in illo datum est, duæ rectæ lineæ ad rectos angulos non excitabuntur ad easdem partes.

Theo.

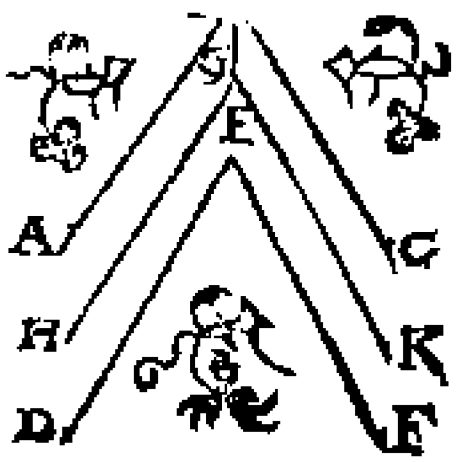
Theorema 12. Propoſitio 14.

Ad quæ plana, eadem recta linea recta eſt, illa sũt parallela.



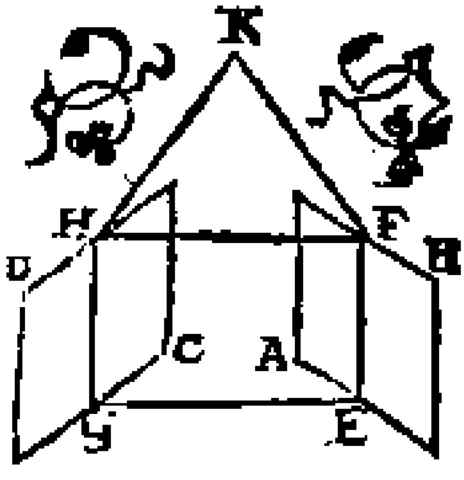
Theorema 13. Propoſitio 15.

Si duæ rectæ lineæ ſe mutuò tangentes ad duas rectas ſe mutuò tangentes ſint parallelæ, non in eodem conſiſtentes plano, parallela ſunt quæ per illas ducuntur plana.



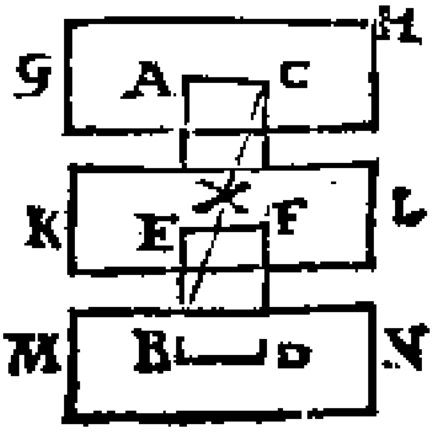
Theorema 14. Propoſitio 16.

Si duo plana parallela plano quopiã ſecentur, communes illorum ſectiones ſunt parallelæ.

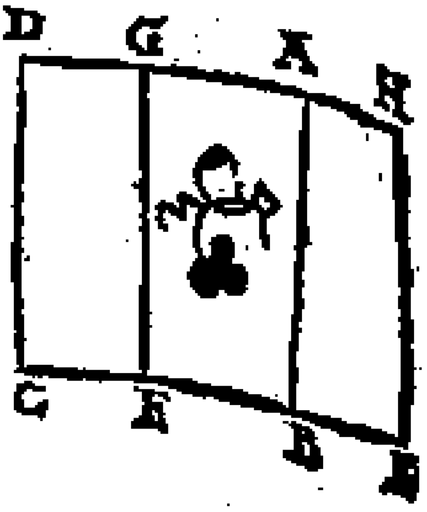


Theorema 15. Propoſitio 17.

Si duæ rectæ lineę parallelis planis ſecẽtur, in eaſdem rationes ſecabũtur.

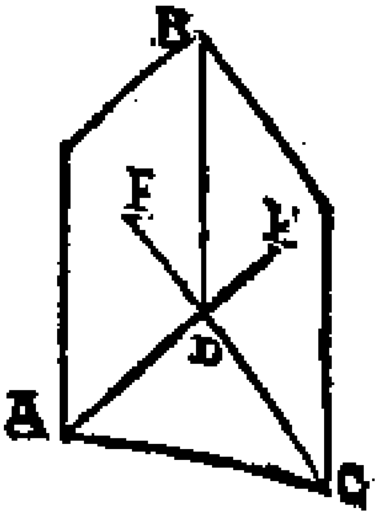


Theorema 16. Propo-
sitio 18.



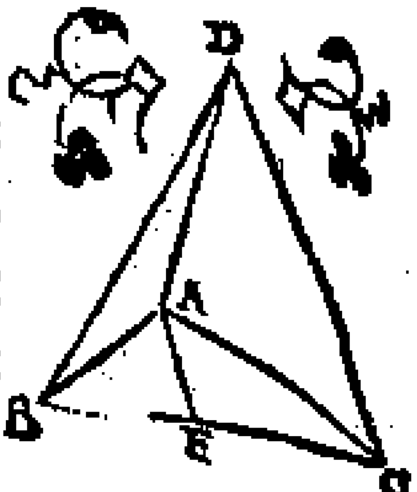
Si recta linea plano cuipiam ad rectos sit angulos, illa etiam omnia quę per ipsam plana, ad rectos eidem plano angulos erunt.

Theorema 17. Propo-
sitio 19



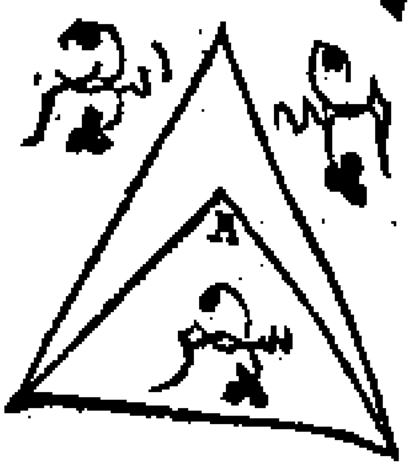
Si duo plana se mutuò secantia plano cuidá ad rectos sint angulos, communis etiam illorum sectio ad rectos eidem plano angulos erit.

Theorema 18. Propo-
sitio 20.



Si angulus solidus planis tribus angulis contineatur, ex his duo quilibet vtut assumpti tertio sunt maiores

Theorema 19. Propo-
sitio 21.

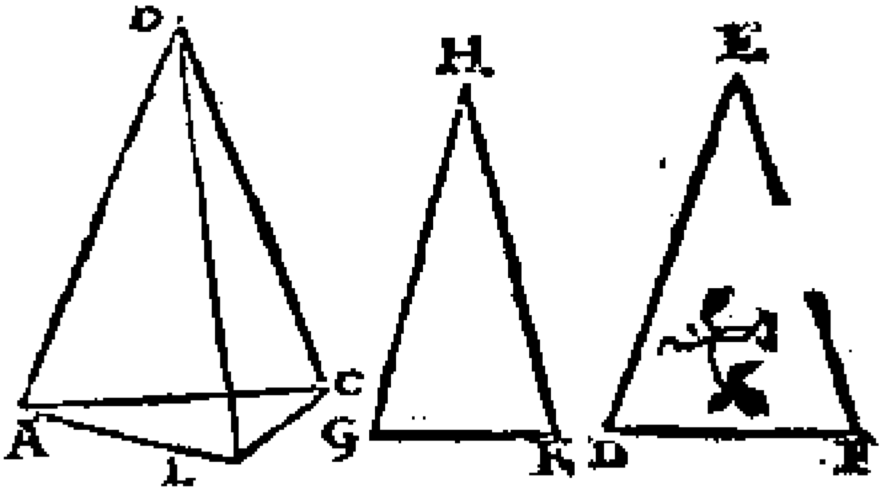


Solidus omnis angulus minoribus continetur, quàm rectis quatuor angulis planis.

Theo-

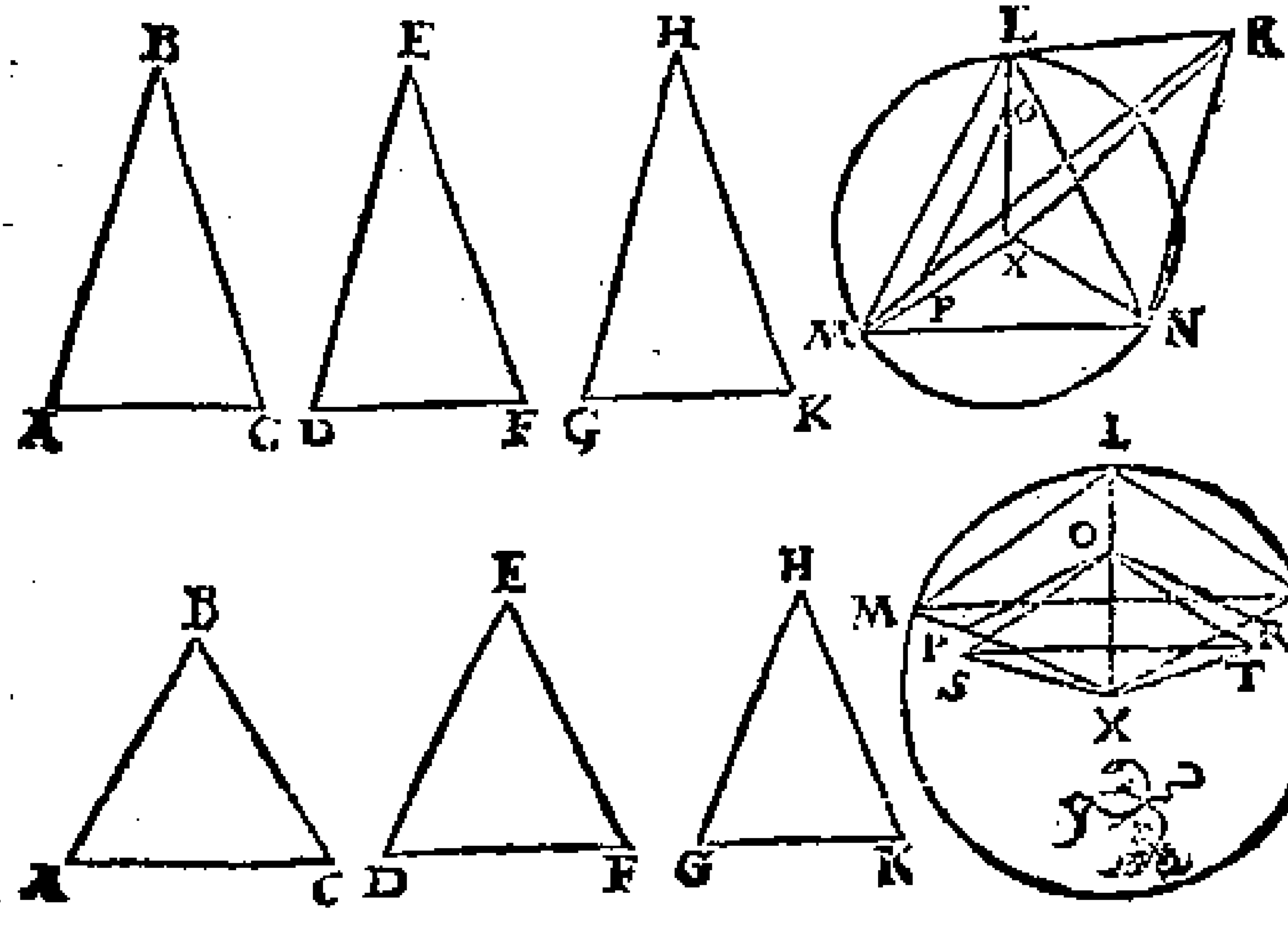
Theorema 20. Proopoſitio 22.

Si plani tres anguli ęqualibus rectis contineãtur lineis, quorũ duo vt libet aſſumpti, tertio ſint maiores, triangulum conſtitui poteſt ex lineis æquales, illas rectas coniungentibus.

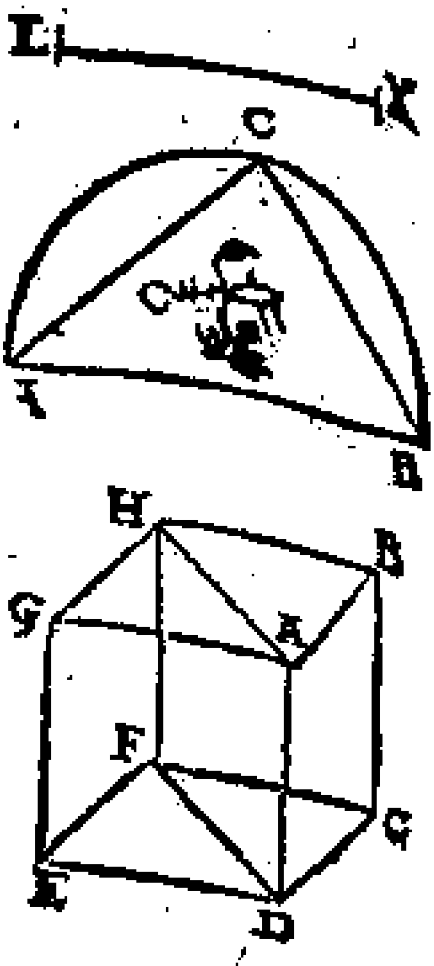


Problema 5, Propoſitio 23.

Ex planis tribus angulis, quorũ duo vt libet aſſumpti tertio ſint maiores, ſolidũ angulum conſtituere. Decet autem illos tres angulos rectis quatuor eſſe minores.

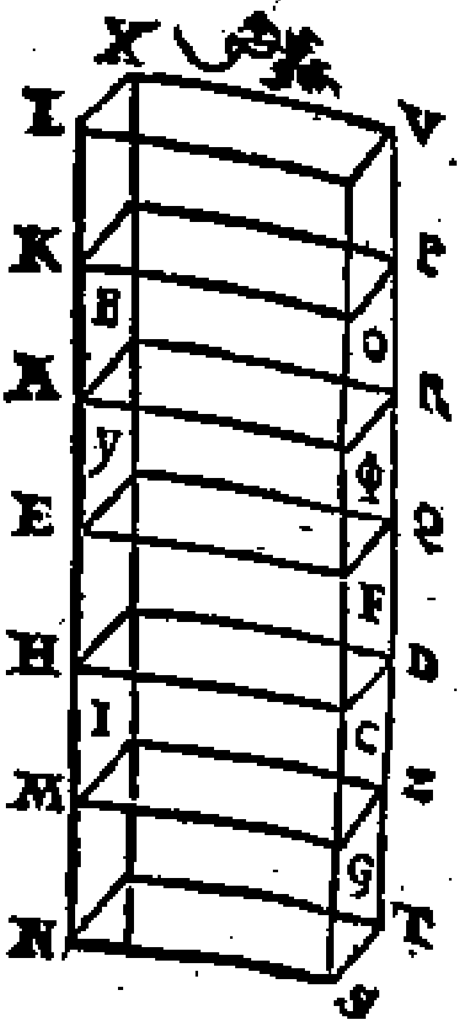


Theorema 21. Propositio 24.



Si solidum parallelis planis contineatur, aduersa illius plana & æqualia sunt & paralle logramma.

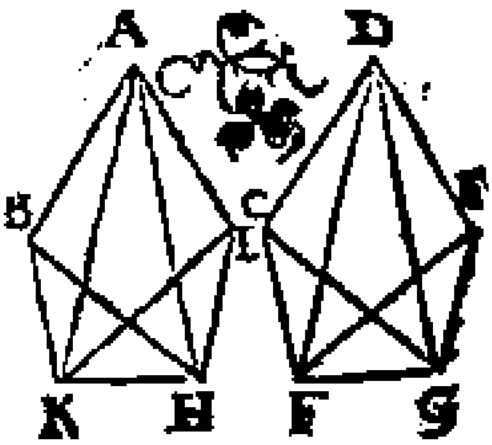
Theorema 22. Propositio 25.



Si solidum parallelis planis contentú plano secetur aduersis planis parallelo, erit quemadmodum basis ad basim, ita solidum ad solidum.

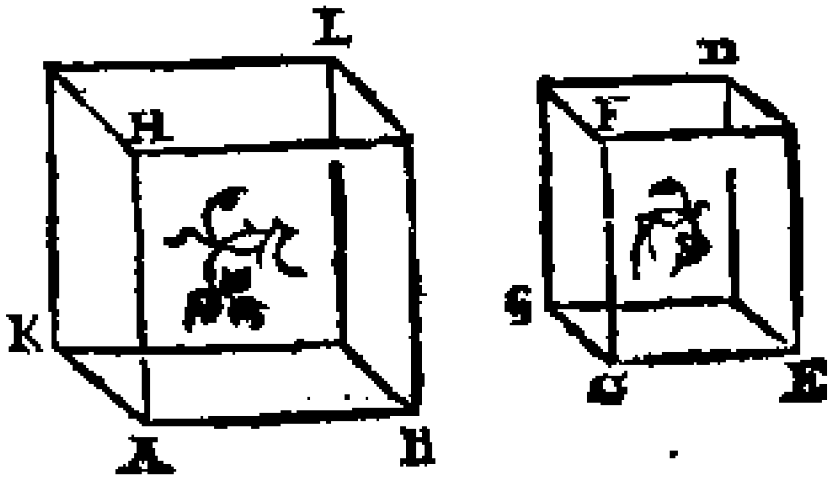
Problema 4. Propoſitio 26.

Ad datam rectam lineã eiuſque punctum, angulum ſolidum conſtituere ſolido angulo dato æqualem.



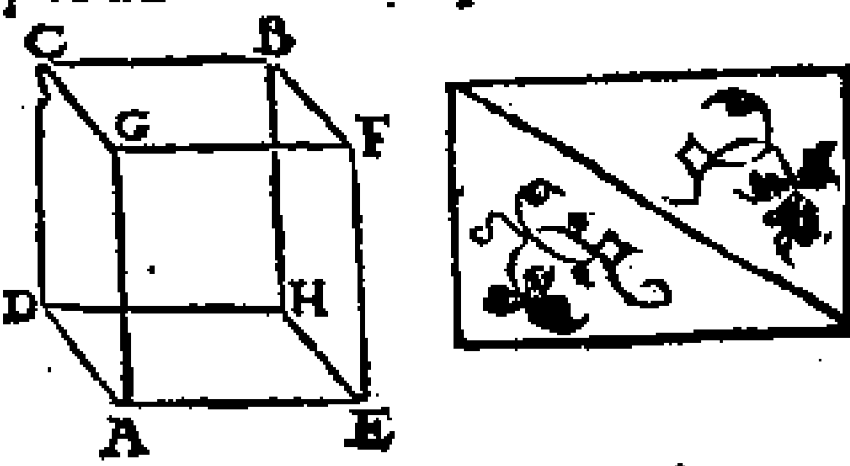
Problema 5. Propoſitio 27.

A data recta, dato ſolido parallelis planis comprehenſio ſimile & ſimiliter poſitum ſolidū parallelis planis contētum deſcribere.

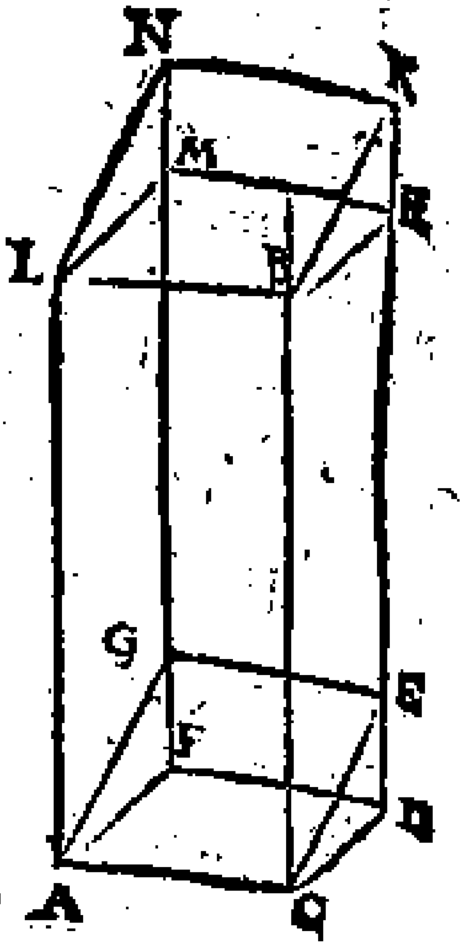


Theorema 23. Propoſitio 28.

Si ſolidū parallelis planis comprehenſum ductor per aduerſorum planorum diagonios plano ſectū ſit, illud ſolidum ab hoc plano bifariam ſecabitur.



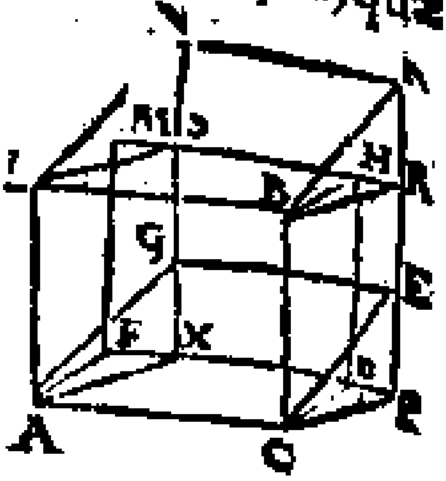
Theorema 34. Propositio 29.



Solida parallelis planis compreheſa, quę ſuper eandem baſim & in eadē ſunt altitudine, quorum inſiſtētes lineæ in ijſdem collocantur rectis lineis, illa ſūt inter ſe æqualia.

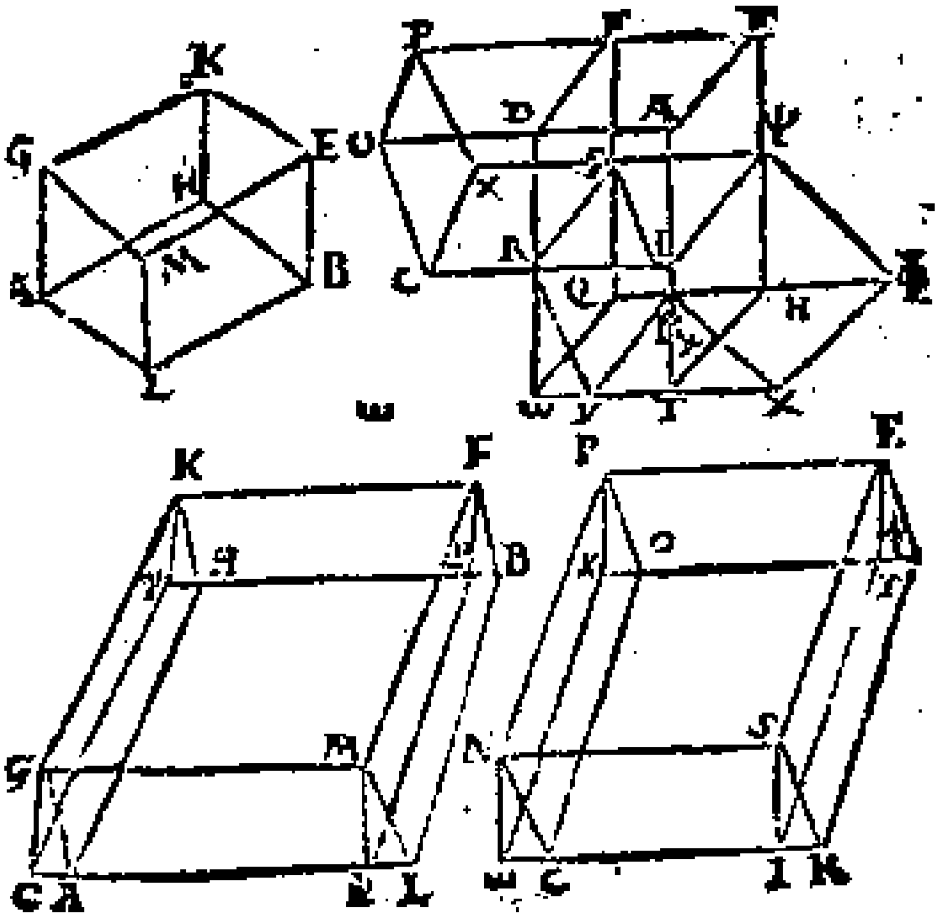
Theorema 25. Propositio 30.

Solida parallelis planis circumſcripta, quæ ſuper eandē baſim & in eadē ſūt altitudine, quorum inſiſtentes lineæ nō in ijſdem reperiuntur rectis lineis, illa ſunt inter ſe æqualia.



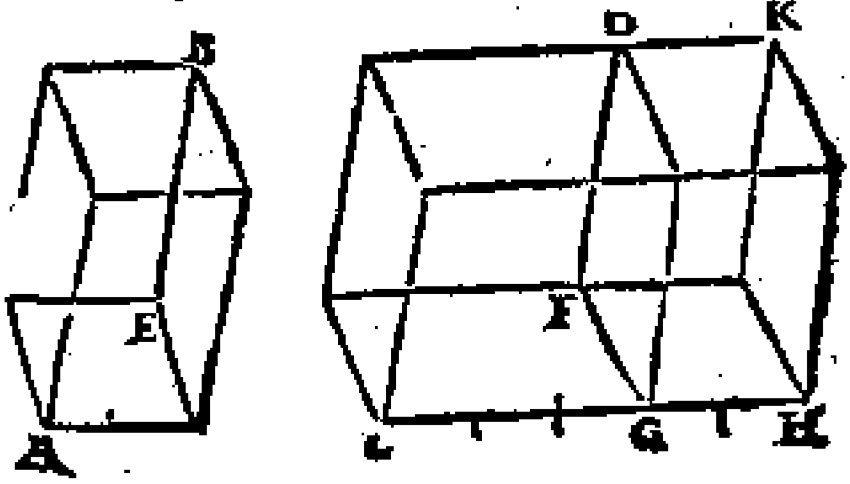
Theorema 26. Propositio 31.

Solida parallelis planis circunſcripta, quæ in in eadem sūt altitudine, æqualia sūt inter se.



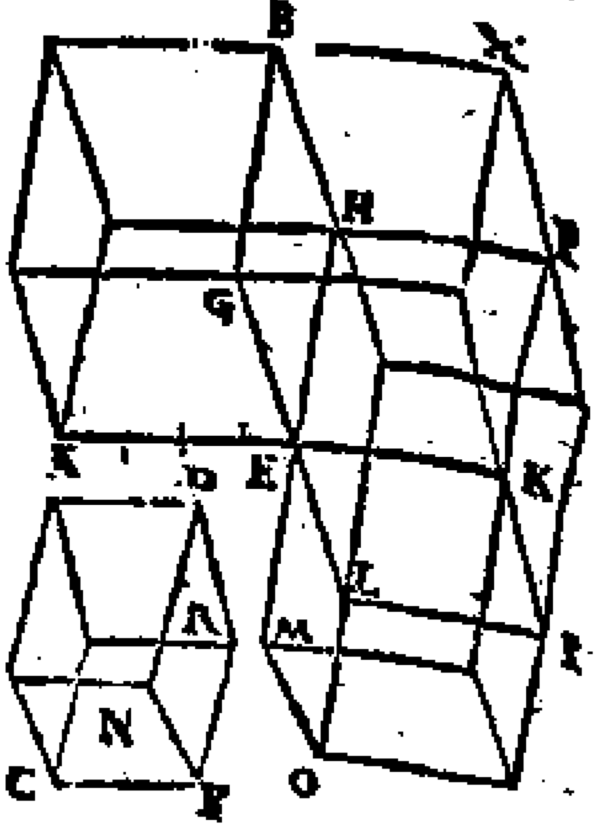
Theorema 27. Propositio 32.

Solida parallelis planis circunscripta quæ eiusdem sunt altitudinis, eam habēt inter se rationē, quam bases.

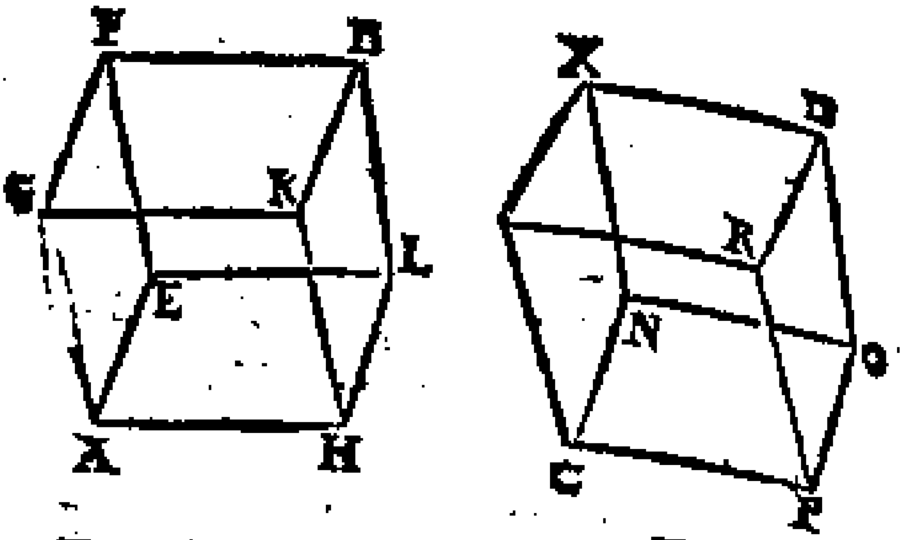


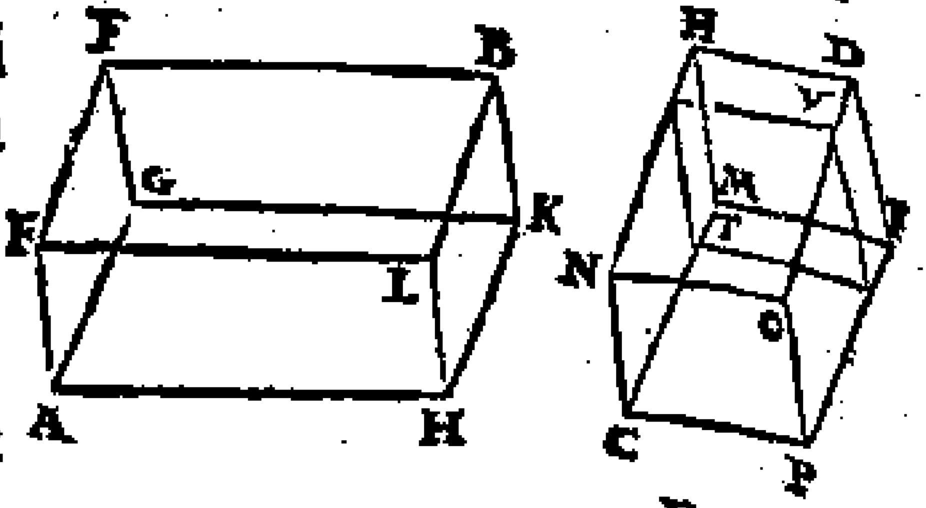
Theor 28. Propositio 23.

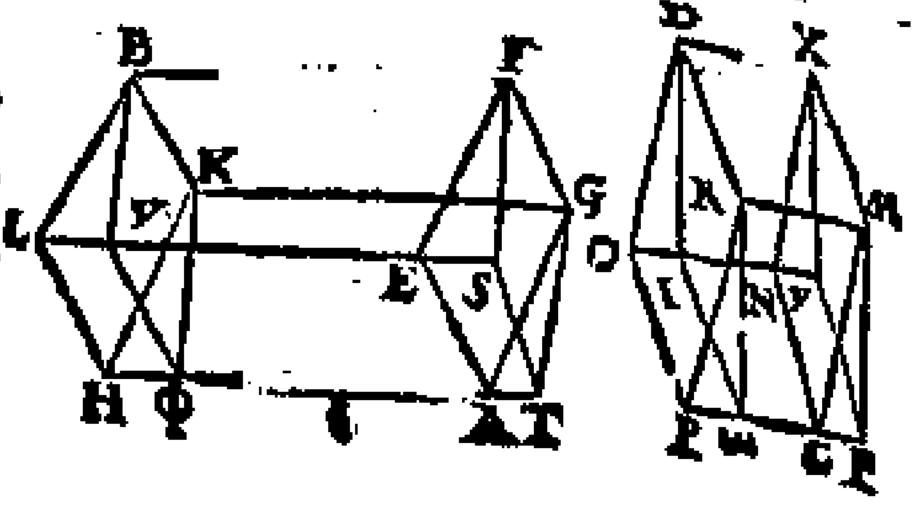
Similia solida parallelis, planis circũscripte habẽt inter se rationẽ homologorũ laterum triplicatam



Theor. 29. Propositio 24



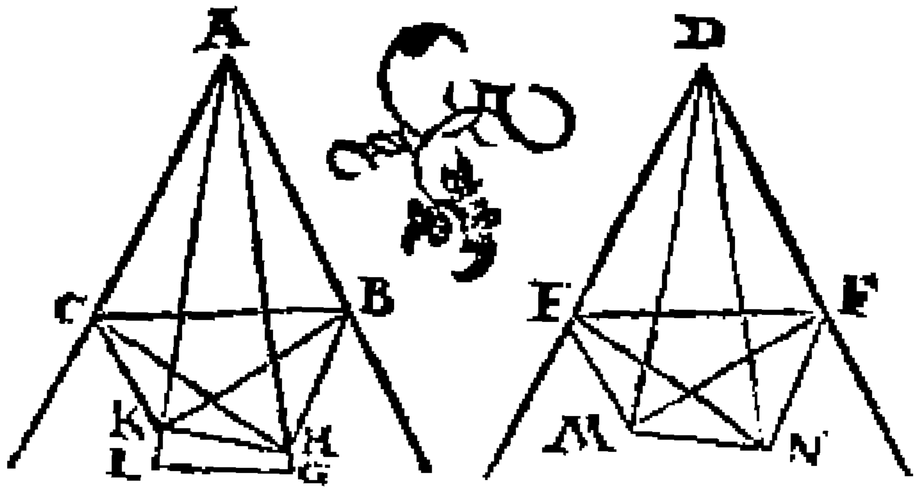




Aequalium solidorũ parallelis planis cõtẽtorum bases cũ altitudinibus reciproçantur. Et solida parallelis planis contenta, quorũ bases cum altitudinibus reciprocantur, illa sunt æqualia.

Theorema 30. Propositio 35.

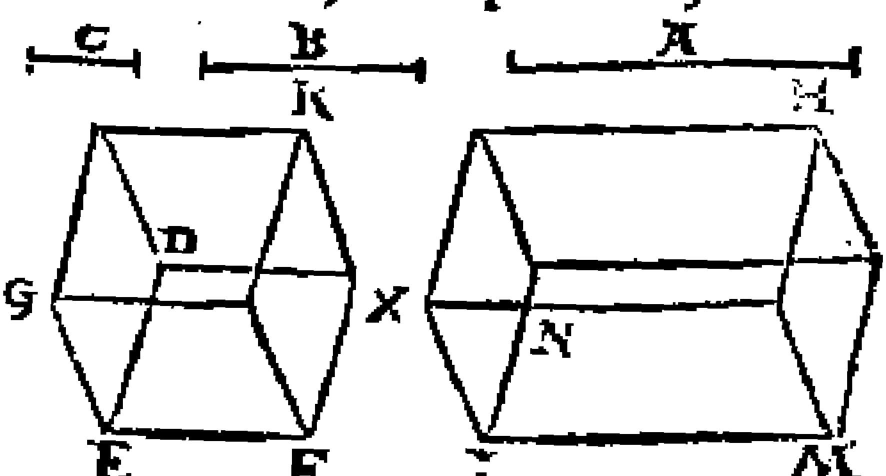
Si duo plani sint anguli æquales, quorum verticibus sublimes rectæ lineæ insistant, quæ cum lineis primò positis angulos contineant æquales, vtrunq; vtriq;, in sublimibus autem lieneis quęlibet sumpta sint puncta, & ab his ad plana in quibus consistunt anguli primum positi, ductæ sint perpendiculares, ab earum verò punctis, quæ in planis signata fuerint, ad angulos primum positos ad iũctæ sint rectæ lineę, hæ cũ subli-



mibus æquales angulos comprehendent.

Theorema 31. Propositio 36.

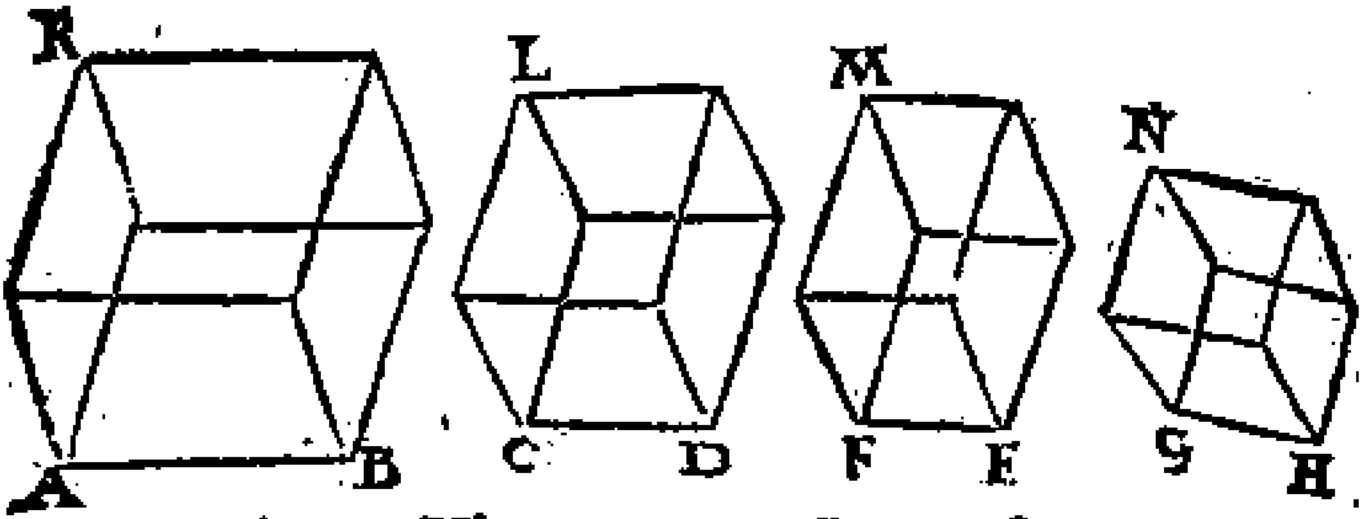
Si rectæ tres lineæ sint proportionales, quòd



ex his tribus fit ſolidum parallelis planis contentum, æquale eſt deſcripto à media linea ſolido parallelis planis compreheſo, quod æquilaterum quidem ſit, ſed antedicto æquiangulum.

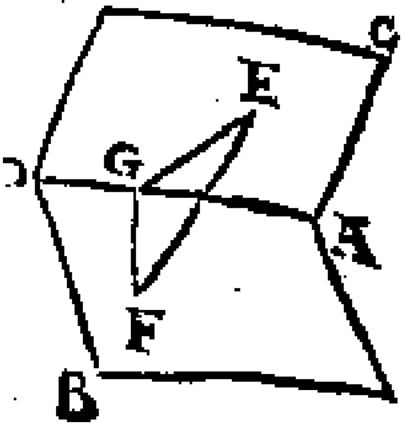
Theorema 32. Propoſitio 37.

Si rectæ quatuor lineæ ſint proportionales, illa quoque ſolida parallelis planis contenta, quæ ab ipſis lineis & ſimilia & ſimiliter diſcribuntur, proportionalia erunt. Et ſi ſolida parallelis planis comprehenſa, quę & ſimilia & ſimiliter deſcribuntur, ſint proportionalia, illæ quoque rectæ lineæ proportionales erunt.



Theorema 33. Propoſitio 38.

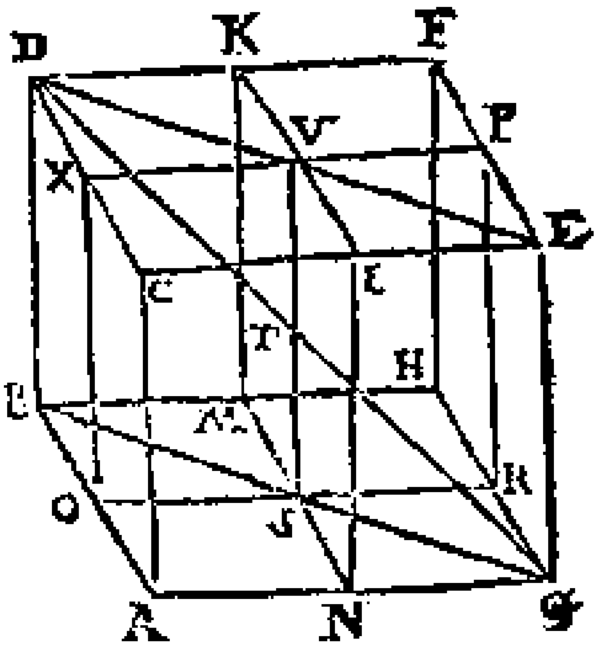
Si planum ad planum rectũ ſit, & à quodã puncto eorũ quæ in vno ſunt planorũ perpendicularis ad alterum ducta ſit, illa quæ ducitur perpẽdicularis, in communẽ cadet planorũ ſectionẽ.



Theo.

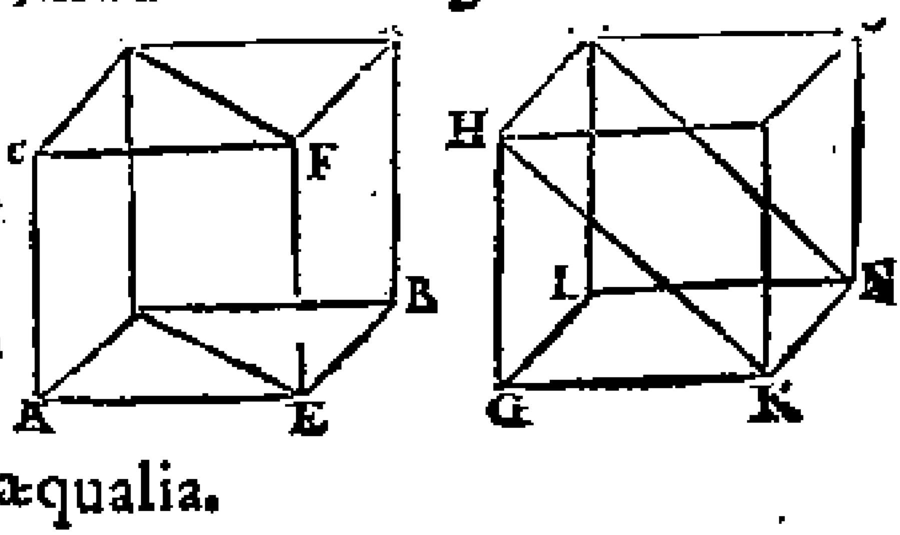
Theorema 34. Propositio 39.

Si in solido parallelis planis circunscripto, aduersorum planorum lateribus bifariam sectis, educta sint per sectiones plana, communis illa planorũ sectio & solidi parallelis plani circumscripti diameter, se mutuò bifariã secant.



Theorema 35. Propositio 40.

Si duo sint æqualis altitudinis prismata, quorũ hoc quidem basim habeat parallelogrammũ, illud verò triangulum, sit autem parallelogrammũ triãguli duplũ, illa prismata erunt æqualia.

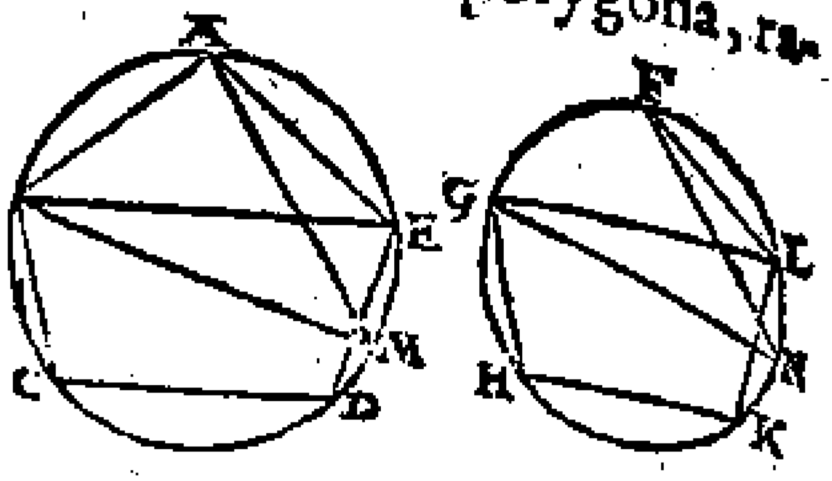


ELEMENTI XI. FINIS.

EVCLIDIS ELEMENTVM DVODECIMVM, ET SOLIDORVM secundum.

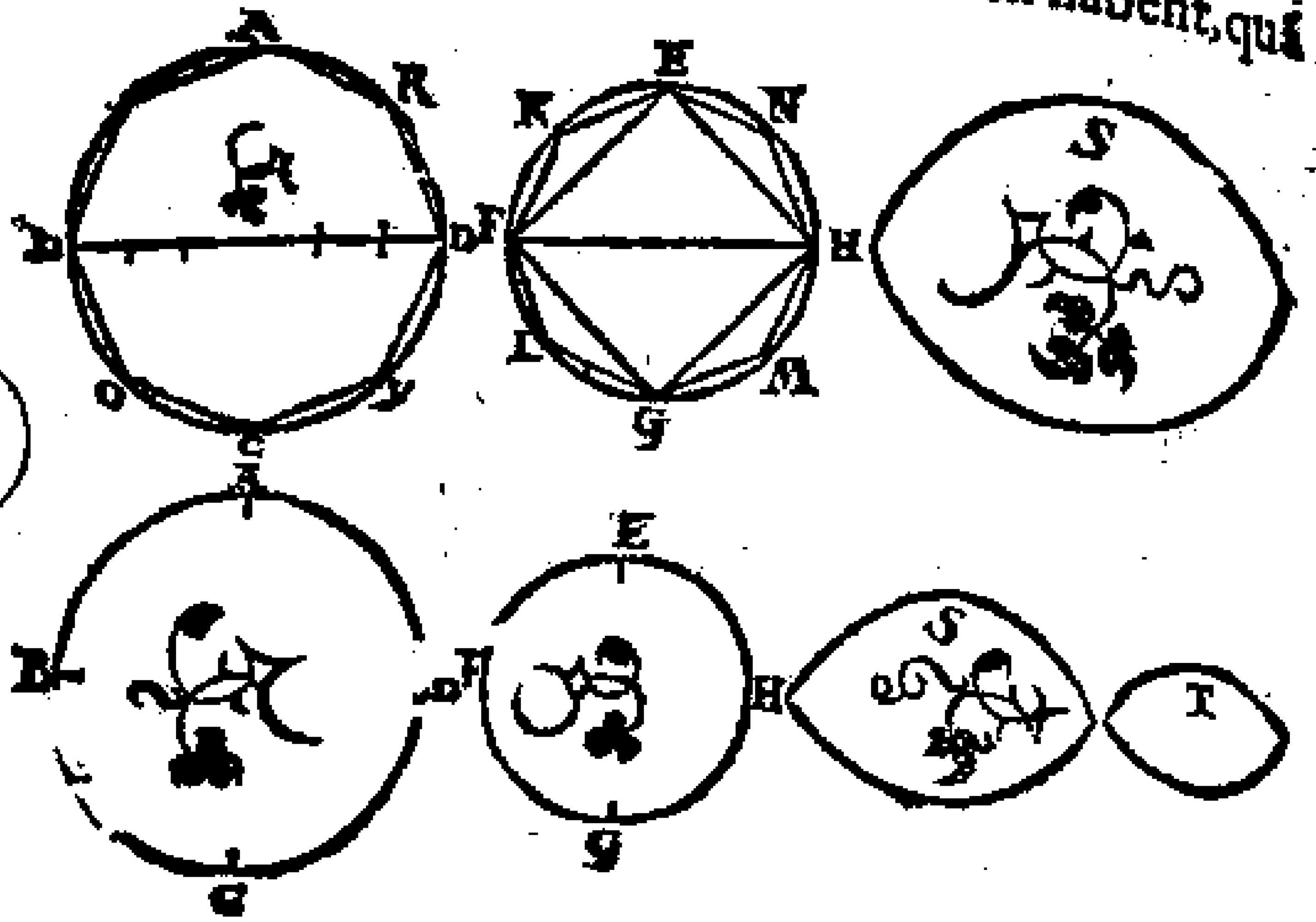
Theorema 1. Propositio 1.

Similia quæ sunt in circulis polygona, rationé habent inter se, quam descripta à diametris quadrata.



Theorema 2. Propositio 2.

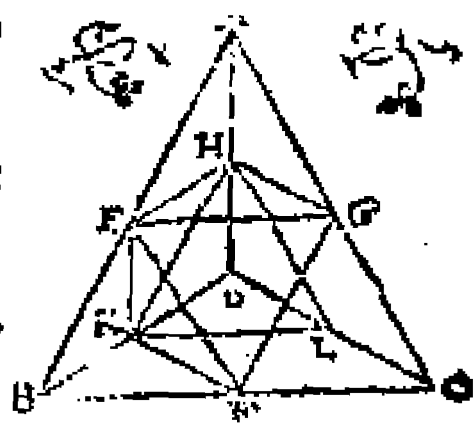
Circuli eam inter se rationem habent, quã



descripta à diametris quadrata.

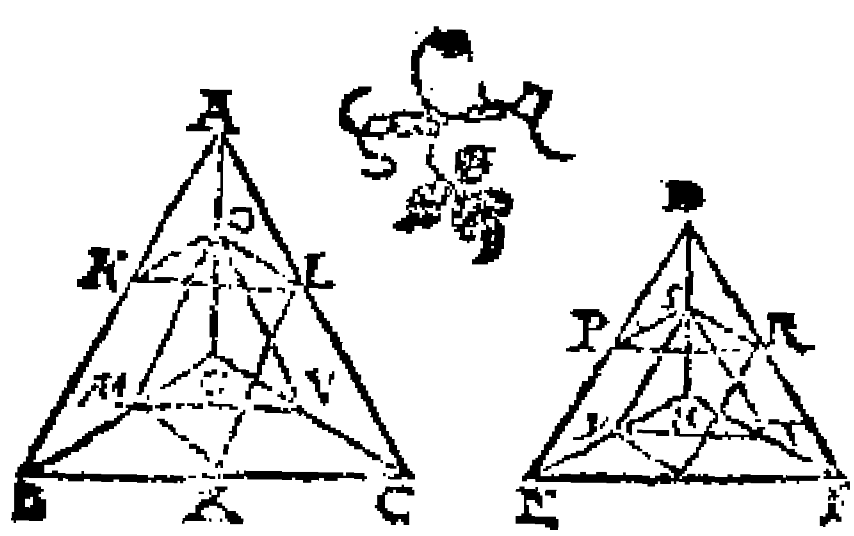
Tueorema 3. Propositio 3.

Omnis pyramis trigωnam habens basim, in duas diuiditur pyramidas non tantũ æquales & similes inter se, sed toti etiam pyramidi similes, quarum trigωnæ sunt bases, atq; in duo prismata equalia, quæ duo prismata dimidio pyramidis totius sũt maiora.



Theorema 4. Propositio 4.

Si duæ eiusdem altitudinis pyramides trigωnas habeãt bases, sit aũt illarum vtraque diuisa & in duas pyramidas inter se equales totiq; similes, & in duo prisma æqualia, ac eodem modo diuidatur vtraq; pyramidum quæ ex superiore diuisione natæ sunt, idque perpetuò fiat: quemadmodũ se habet vnius pyramidis basis ad alterius pyrãidis basim, ita & omnia que in vna pyra-

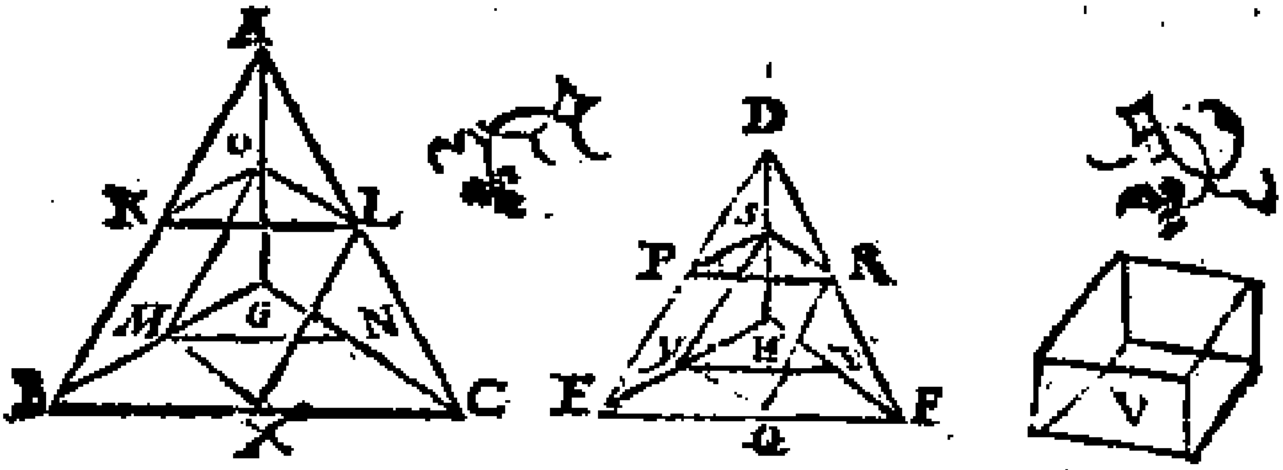


mide

mide prismata, ad omnia quæ in altera pyramide, prismata multitudine æqualia.

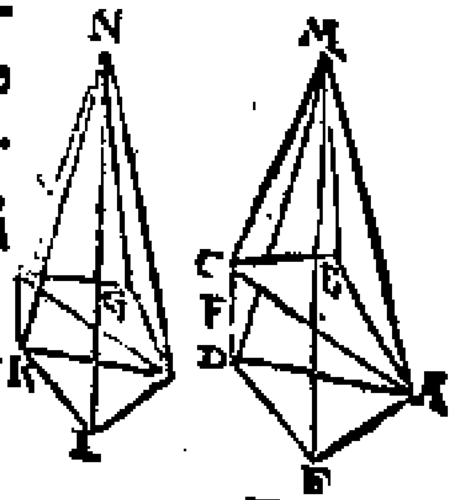
Theorema 5. Propositio 5.

Pyramides eiusdem altitudinis, quarũ trigœnæ sunt bases, eam inter se rationem habent, quàm ipsæ bases.



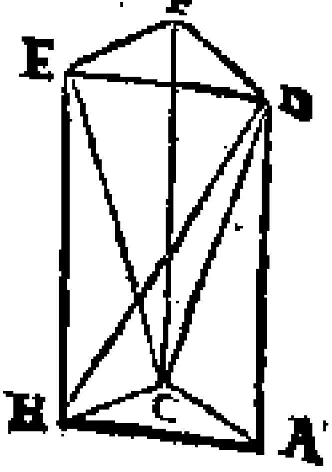
Theorema 6. Propositio 6.

Pyramides eiusdẽ altitudinis, quarum polygœnæ sunt bases, eam inter se rationem habent, quã ipsæ bases.



Theorema 7. Propositio 7.

Omne prisma trigœnã habẽs basim, diuiditur in tres pyramides inter se æquales, quarum trigœnæ sunt bases



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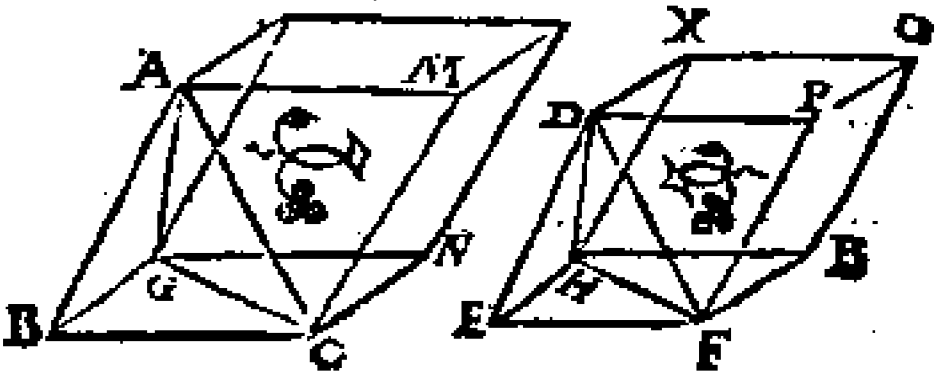
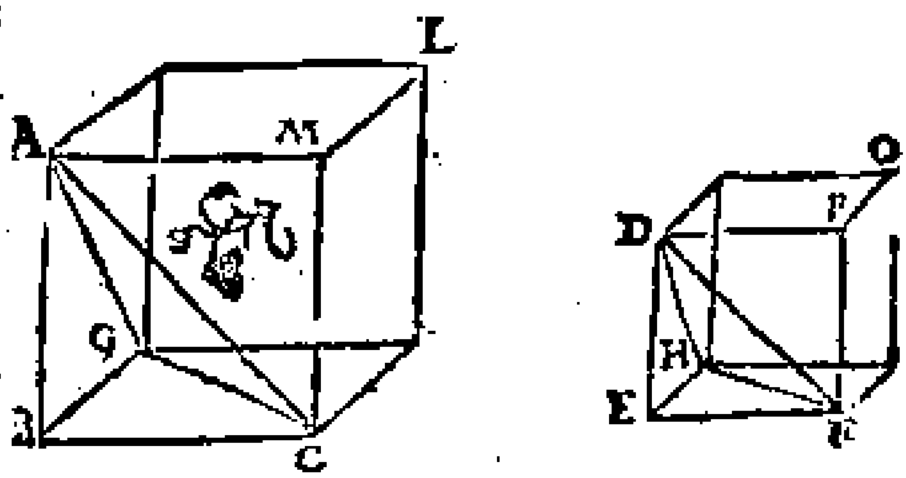
Theorema 8. Propositio 8.

Similes pyramides qui trigωnas habent bases, in triplicata sũt homologorũ laterũ ratiõe



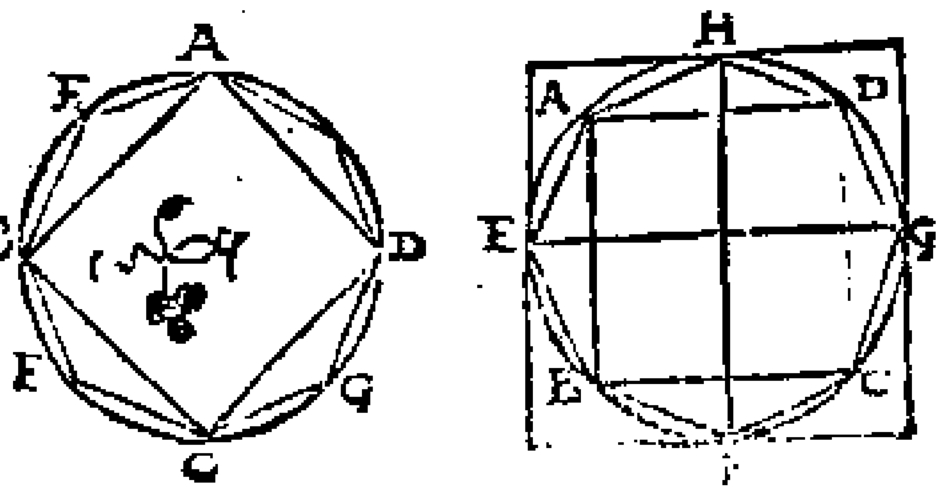
Theorema 9. Propositio 9.

Aequaliũ pyramidum & trigωnas bases habentium reciprocantur bases cum altitudinibus. Et quarũ pyramidum trigωnas bases habentium reciprocãtur bases cũ altitudinibus, illæ sunt æquales.



Thorema 10. Propositio 10.

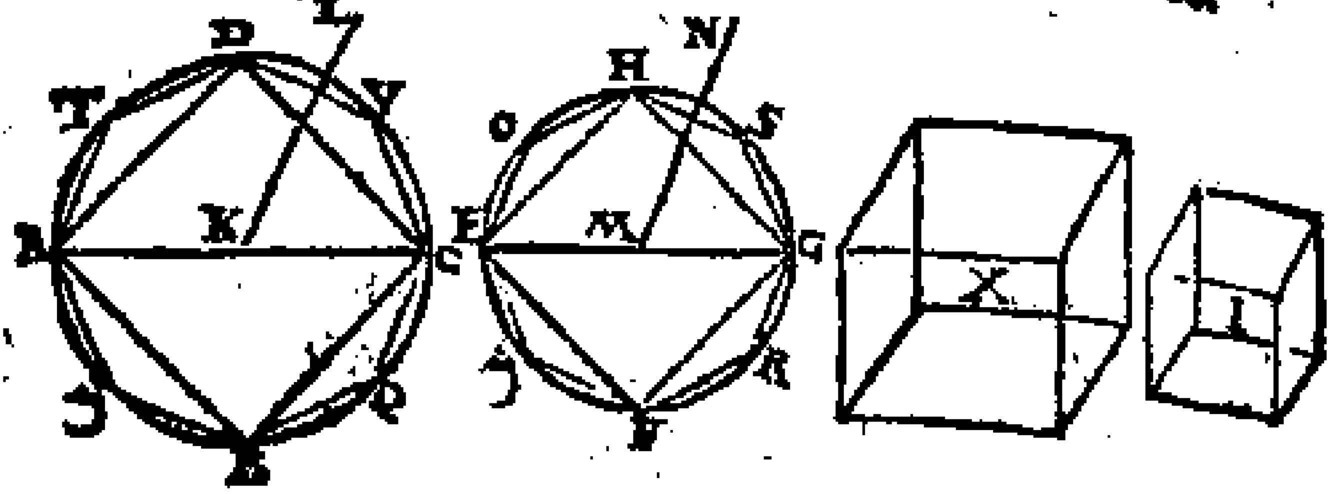
Omnis cωn° tertia pars est cylindri eãdẽ



cum ipso cono basimq; habentis, & altitudinem æqualem.

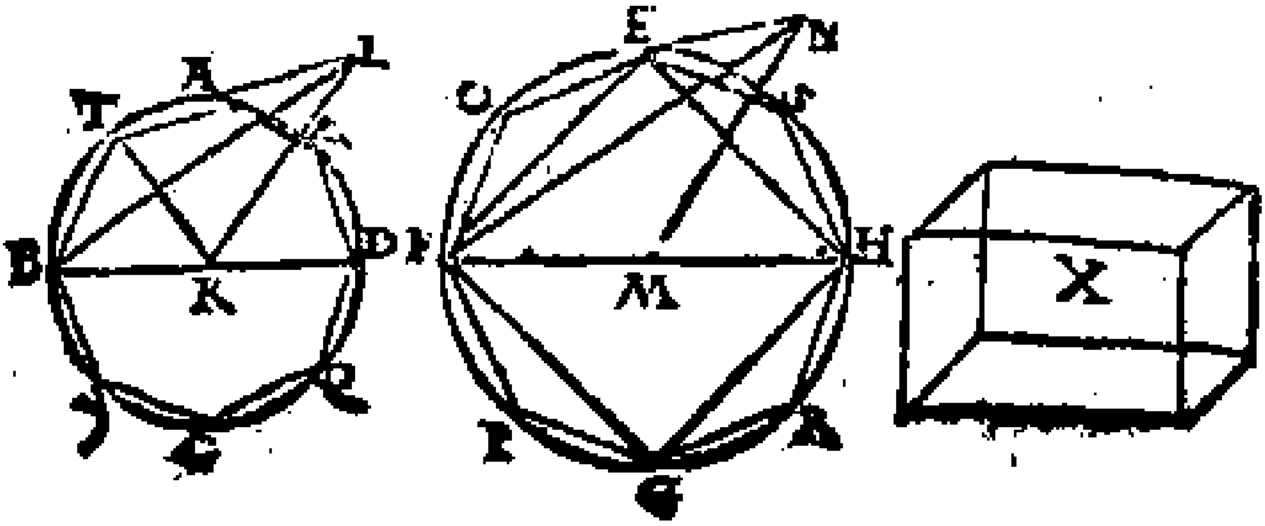
Theorema 11. Propositio 11.

Coni & cylindri eiusdem altitudinis, eam inter se rationem habent, quam bases.



Theorema 12. Propositio 12.

Similes coni & cylindri, triplicatã habent inter se rationem diametrorum, quæ sunt in basibus.



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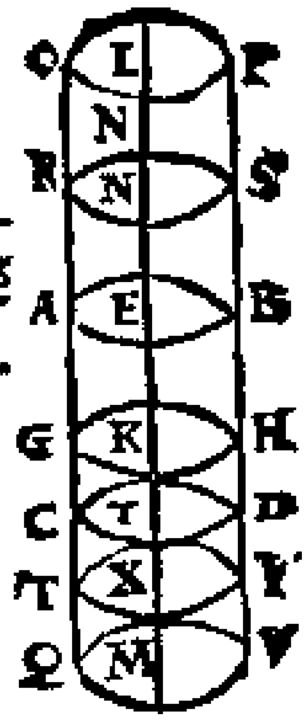
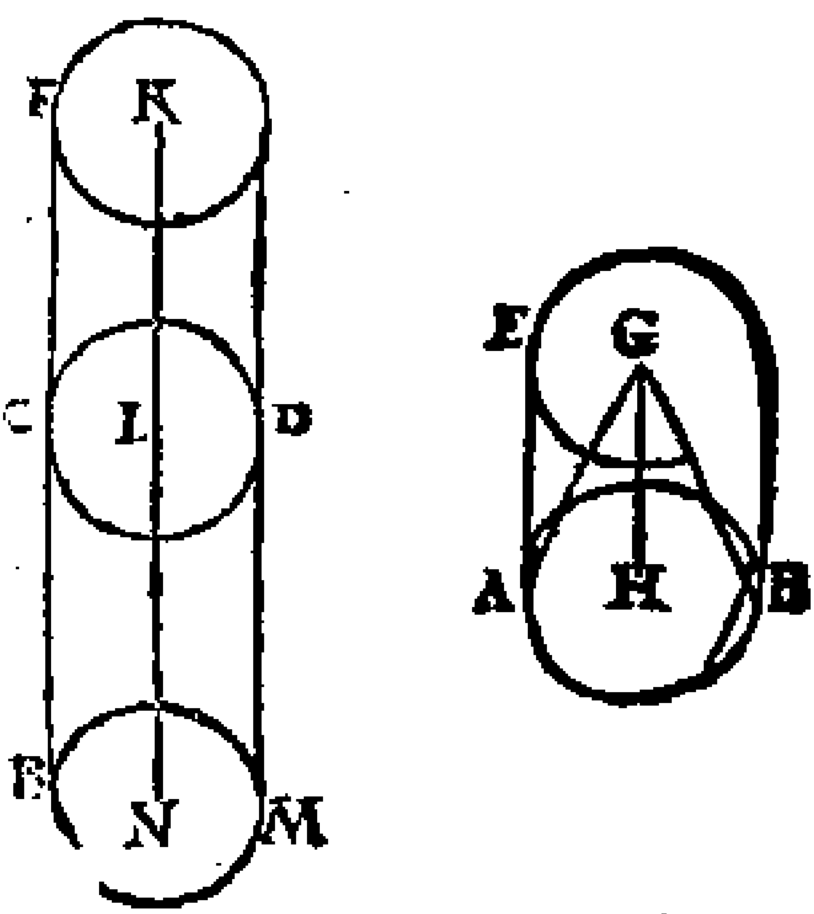
Theorema 13. Propositio 13.



Si cylindrus plano sectus sit aduersis planis parallelo, erit quẽadmodum cylindrus ad cylindrum, ita axis ad axem.

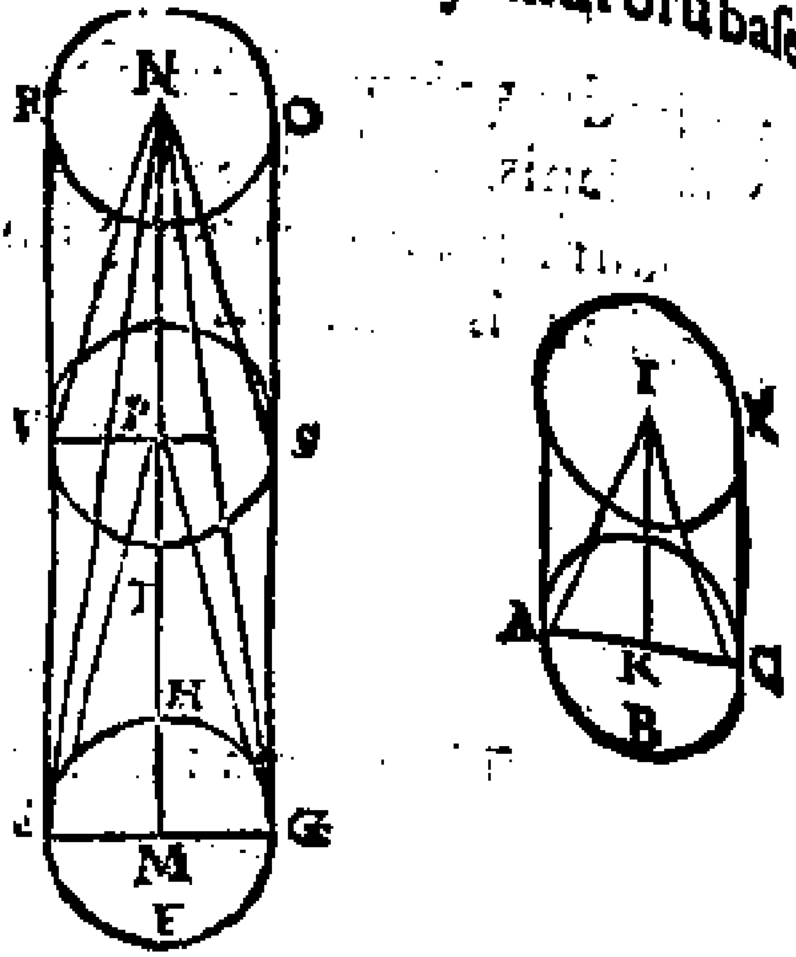
Theorema 14. Propositio 14.

Coni & cylindri qui in æqualibus sunt basibus, eam habent inter se rationem, quam altitudines



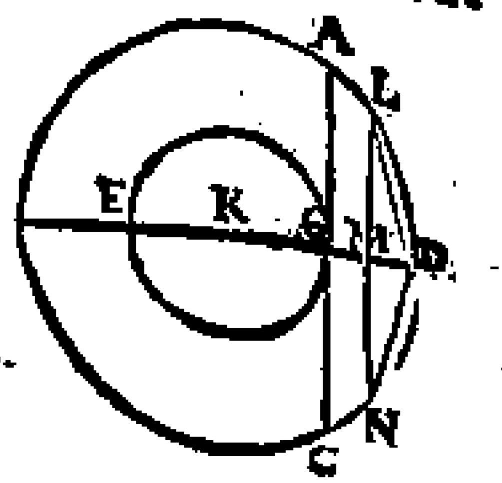
Theorema 15. Propositio 15.

Aequalium conorum & cylindrorũ bases cũ alutitudinib⁹ reciprocátur. Et quorum conorũ & cylindrorum bases cum altitudinibus reciprocantur, illi sunt æquales.



Theoroma 1. Propositio 16.

Duobus circulis circũ idem centrum consistentibus, in maiore circulo polygonum æqualium pariumque laterum inscribere, quod minorem circulũ non tangat.



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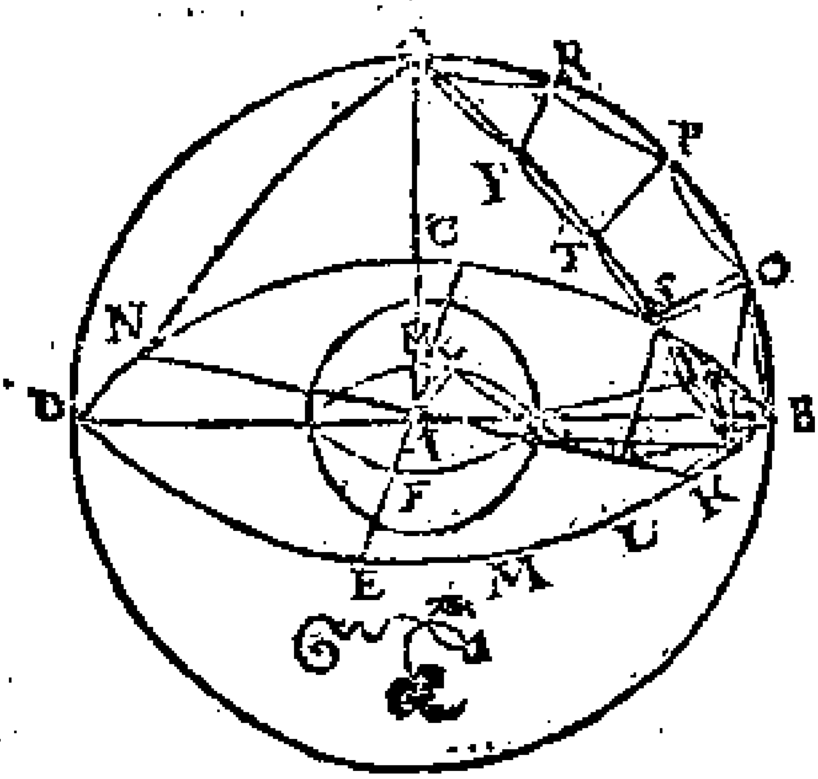
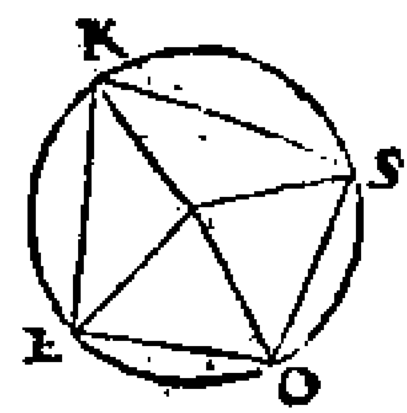
Problema 2. Propositio 17.

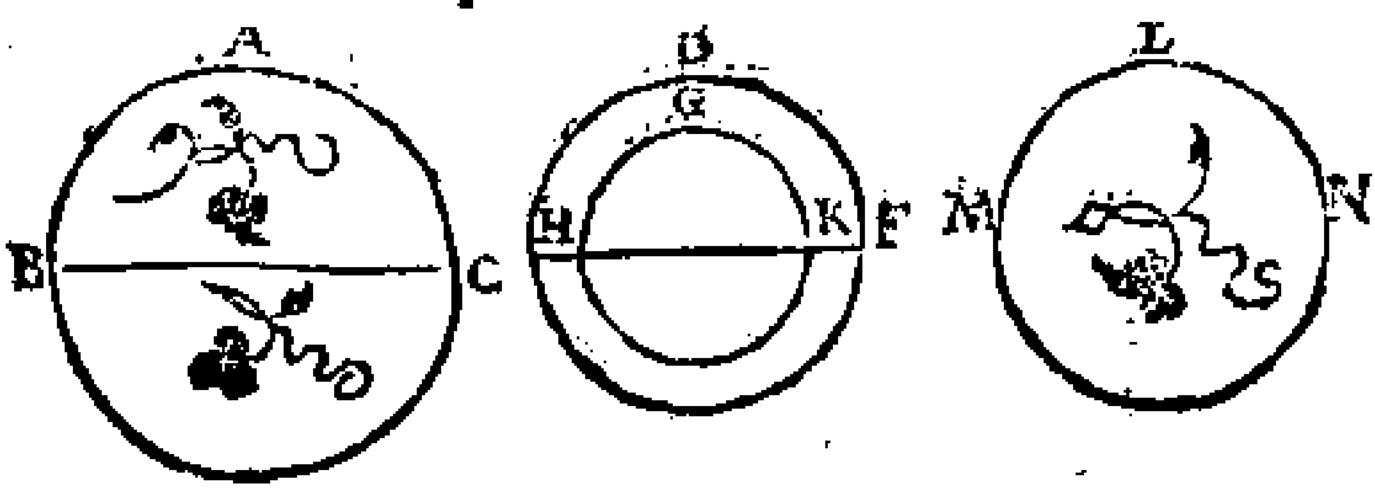
Duabus sphæris circum idem centrum cõsistentibus, in maiorem sphęra solidũ polyedrum inscribere, quod minoris sphæræ superficiem non tangat.





Theorema 16. Propositio 18.

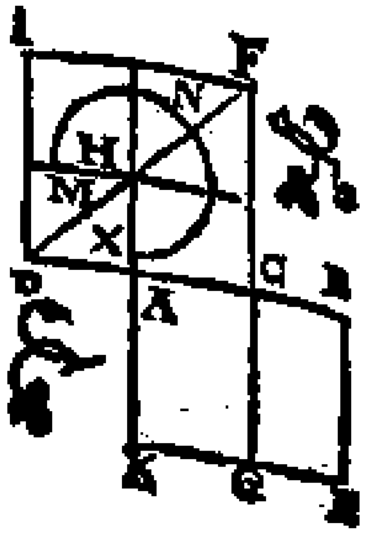
Sphæræ inter se rationem habent suarum diametrorum triplicatam.



ELEMENTI XII. FINIS.

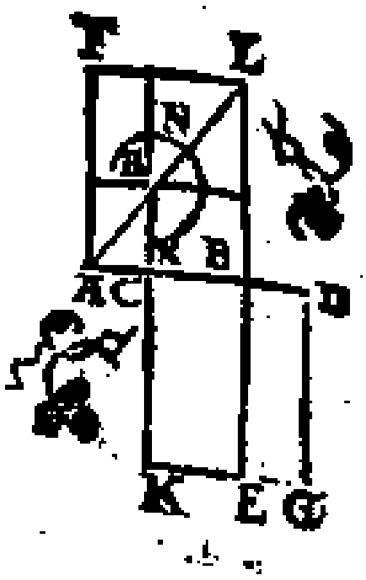
EVCLIDIS ELEMENTVM DECIMVMTERTIVM, ET SOLIDORVM TERTIVM.

Theorema 1. Propoſitio 1.



Si recta linea per extremā & mediā rationem ſecta ſit, maius ſegmentum quod totius lineæ dimidium aſſumpſerit, quintuplum poteſt eius quadrati, quod à totius dimidia deſcribitur.

Theorema 2. Propoſitio 2.



Si recta linea ſui ipſius ſegmenti quintuplum poſſit, & dupla ſegmenti huius linea per extremā & mediam rationē ſecetur maius ſegmentū reliqua pars eſt lineæ primum poſitæ.

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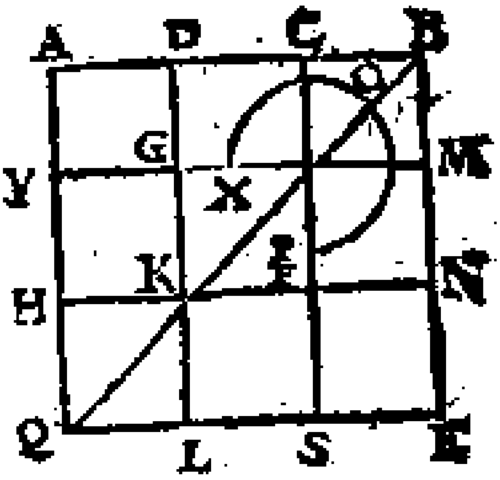
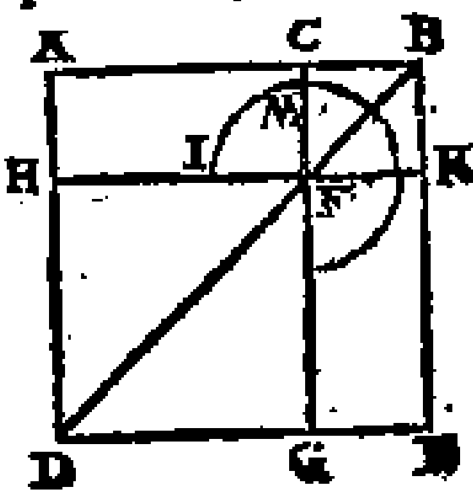
Theorema 3. Propositio 3.



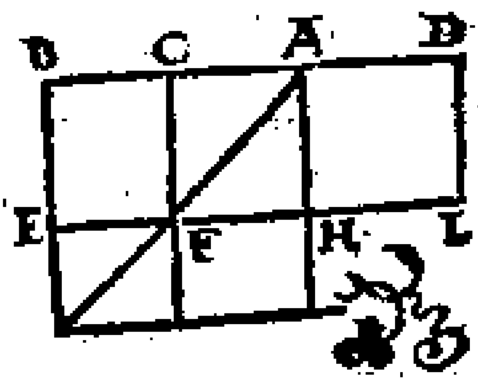
Si recta linea per extremã & mediã rationẽ secta sit, minus segmẽtũ quod maioris segmenti dimidium assumpserit, quintuplum potest eius, quod à maioris segmẽti dimidio describitur, quadrati.

Theorema 4. Propositio 4.



Si recta linea per extremam & mediam rationẽ secta sit, quod à tota, quodque à minore segmẽto simul vtraq; quadrata, tripla sunt eius, quod à maiore segmẽto describitur, quadrati.

Theorema 5. Propositio 5.

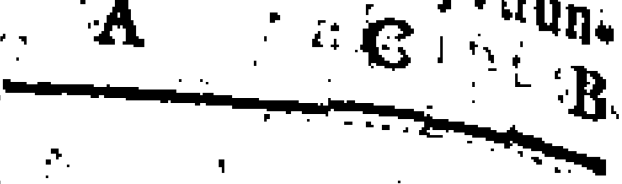


Si ad rectam lineam, quæ per extremam & mediam rationem secetur, adiuncta sit altera segmento maiori equalis, tota hæc linea recta per extremam & mediam rationẽ secta

ecta est, estque maius segmentum lineæ primùm posita.

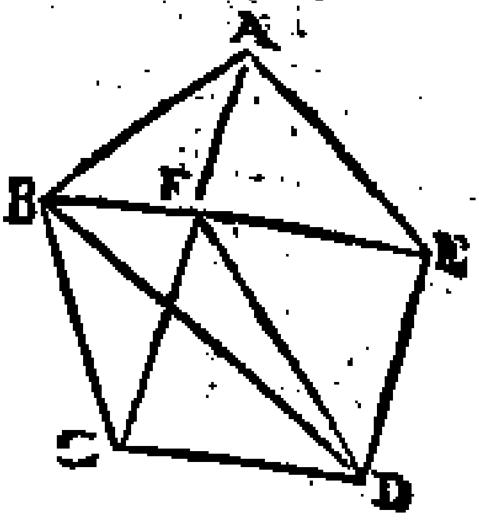
Theorema 6. Propositio 6.

Si recta linea ῥητὴ siue rationalis, per extremam & mediam rationem secta sit, vtrunque segmentorum ἄλογος siue irrationalis est linea, quæ dicitur Residuum.



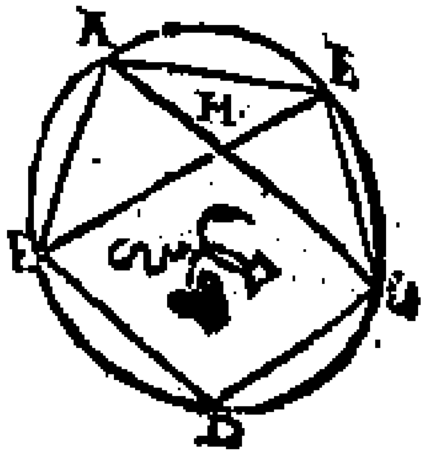
Theorema 7. Propositio 7.

Si pentagωni æquilateri tres sint æquales anguli, siue quã deinceps siue qui non deinceps sequuntur, illud pentagωnum erit ęquiangulum.



Theorema 8. Propositio 8.

Si pentagωni æquilateri & æquianguli duos qui deinceps sequuntur angulos rectæ subtendãt lineæ, illæ per extremã & mediam rationem se mutuò secant, earumq; maiora segmenta, ipsius pentagωni lateri sunt æqualia.



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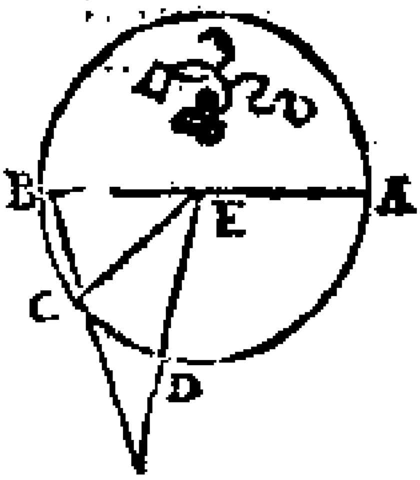
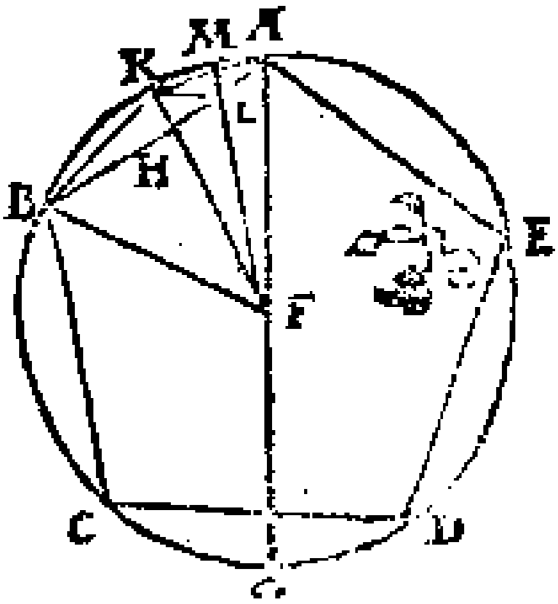
Theorema 9 Propoſitio 9.

Si latus hexagωni & latus decagωni eidem circulo inſcriptorum compoſita ſint, tota recta linea per extremam & mediam rationem ſecta eſt, eiuſque ſegmẽtum maius, eſt hexagωni latus.



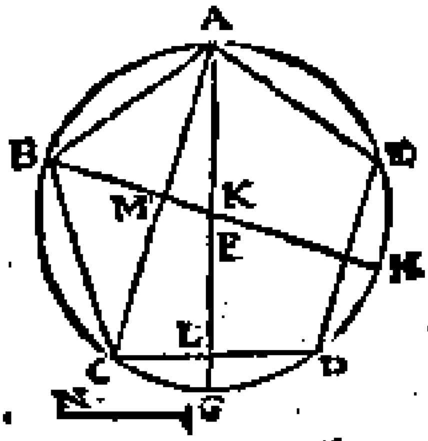
Theorema 10. Propoſitio 10.

Si circulo pẽtagωnum æquilaterum inſcriptum ſit, pentagωini lat' poteſt & latus hexagωni & latus decagωni eidem circulo inſcriptorum.



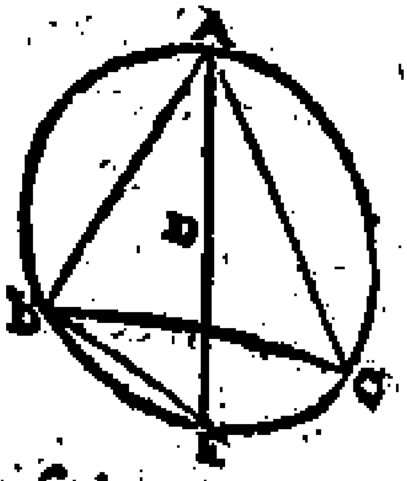
Therema 11. Propoſitio 11.

Si in circulo ῥητὴν habente diametrum, inſcriptum ſit pentagωnum æquilaterum, pentagωni latus irrationalis eſt linea quæ vocatur Minor.



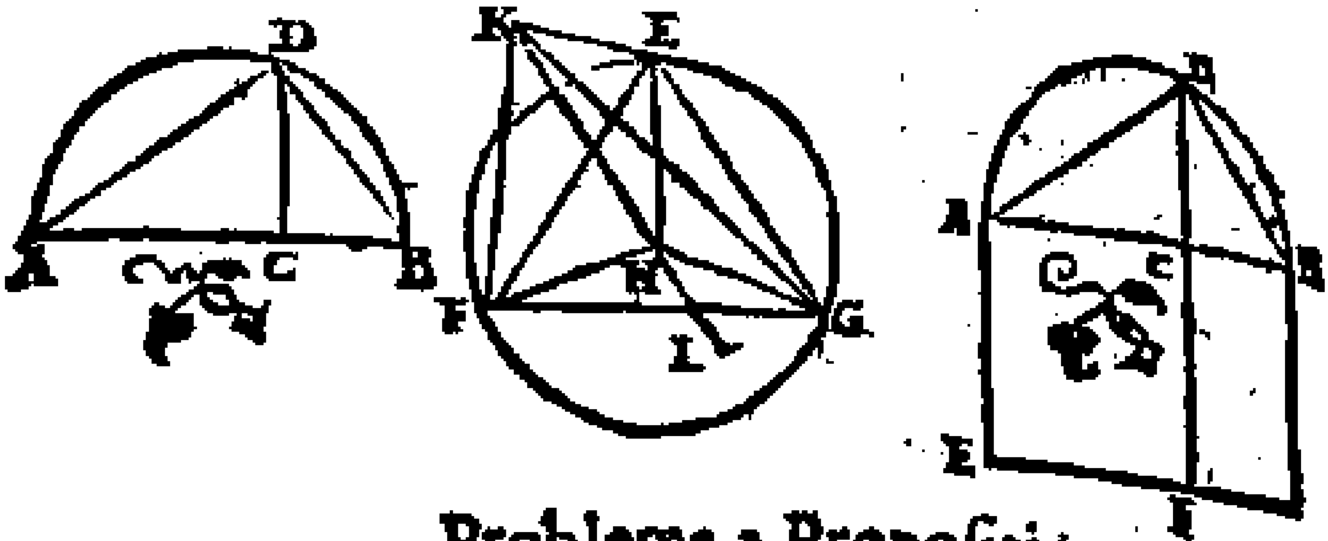
Theorema 12. Propoſitio 12.

Si in circulo inſcriptum ſit triangulum æquilaterum, huius trianguli latus potentia triplum eſt eius lineæ, quæ ex circuli centro ducitur.



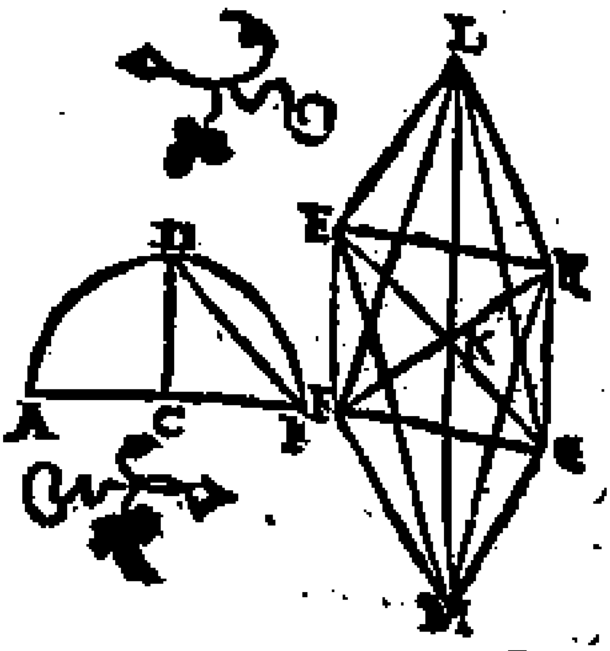
Problema 1. Propoſitio 13.

Pyramidem conſtituere, & data ſphærę cõplecti, atque docere illius ſphæræ diametrum potentia ſeſquialteram eſſe lateris ipſius pyramidis.



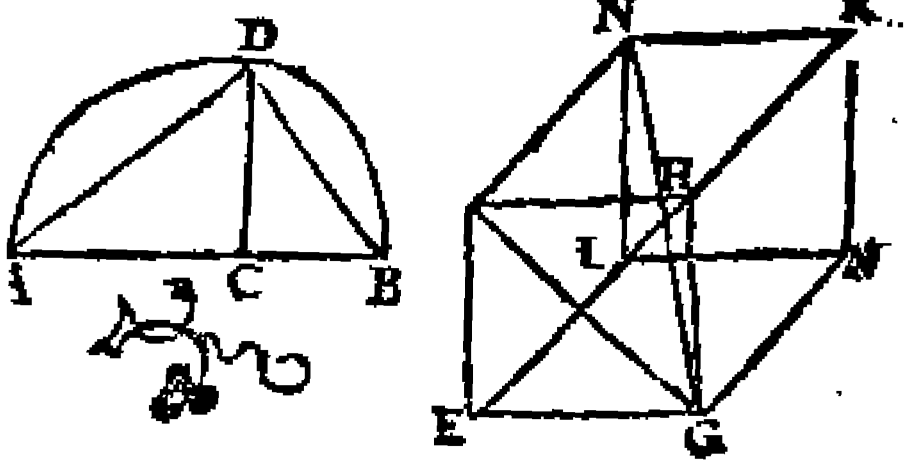
Problema 2. Propoſitio 14.

Octaedrum conſtituere, eaq; ſphęra qua pyramidẽ complecti, atque ꝓbare illius ſphæræ diametrũ potentia duplã eſſe lateris ipſiꝰ octaedri.



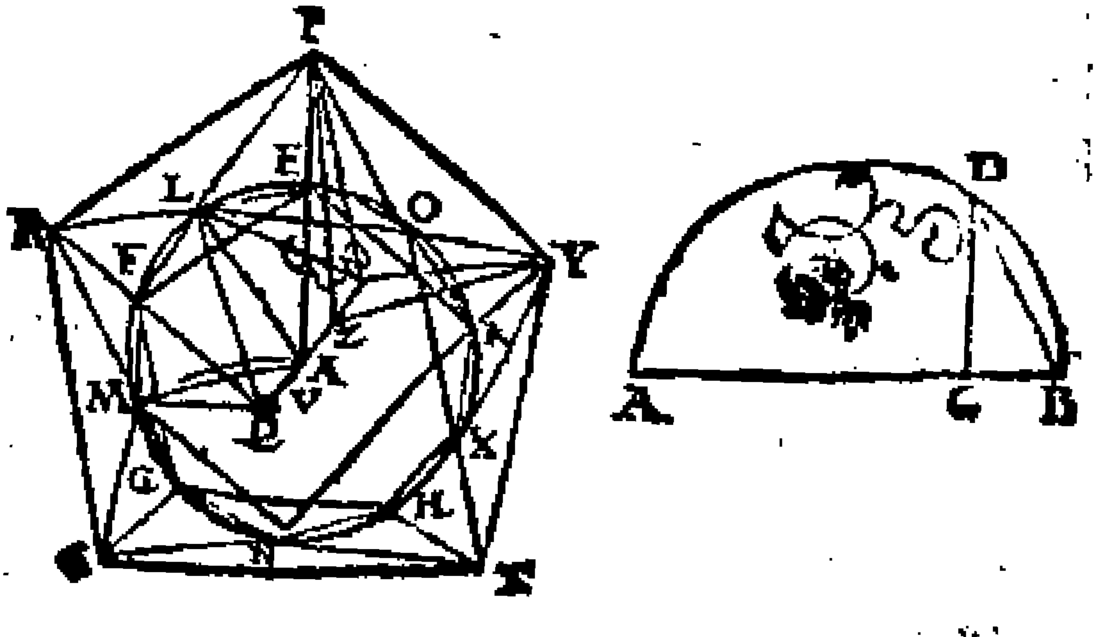
Problema 3. Propoſitio 15.

Cubum conſtituere, eaq́ue ſphæra qua & ſuperiores figuras complecti, atque docere illius ſphæræ diametrum potẽtia triplá eſſe lateris ipſius cubi.



Problema 4. Propoſitio 16.

Icoſaedrum conſtituere, eademq́ue ſphæra qua & antedictas figuras complecti, atque probare, Icoſaedri latus irrationalem eſſe lineam, quæ vocatur Minor.



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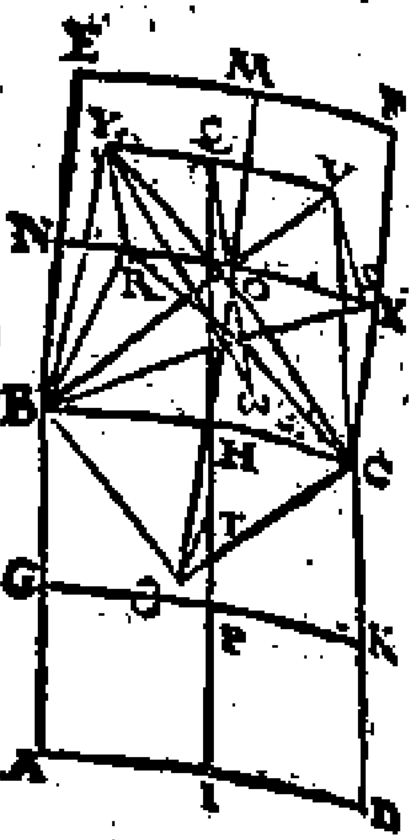
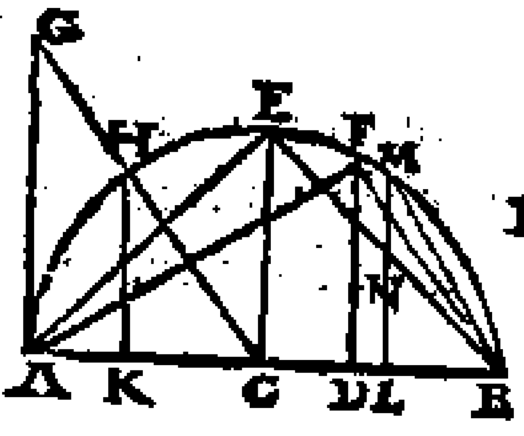
Problema 5. Propositio 17.

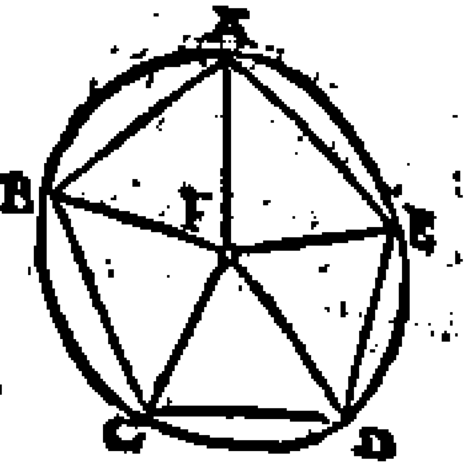


Dodecaedrū constituere, eademq́ue sphæra qua & & antedictas figuras cō-plecti, atque probare dodecaedri latus irrationalem esse lineam, quæ vocatur Residuum.

Theorema 6. Propositio 18.

quinq; figurarum latera proponere, & inter se comparare.





SCHOLIVM.

Aio verò, præter dictas quinque figuras non posse aliam constitui figuram solidā, quæ planis & æquilateris & æquiangulis cōtineatur, inter se æqualibus. Non enim ex duobus triāgulis, sed neq; ex aliis duab' figuris solidus cōstituetur angulus

Sed

Sed ex tribus triãgulis, constat Pyramidis angulus

Ex quatuor autem, Octaëdri

Ex quinque verò, Icosaëdri.

Nam ex triangulis, sex & æquilateris & æquiangulis ad idem punctum coeuntibus, non fiet angulus solidus. Cùm enim trianguli æquilateri angulus, recti vnius bessem contineat, erunt eiusmodi sex anguli rectis quatuor æquales. Quod fieri nõ potest. Nam solidus omnis angulus minoribus quã rectis quatuor angulis continetur, per 21.11.

Ob easdem sanè causas, neque ex pluribus quã planis sex eiusmodi angulis solidus constat.

Sed ex tribus quadratis, Cubi angulus continetur.

Ex quinque, nullus potest. Rursus enim recti quatuor erunt.

Ex tribus autẽ pentangonis æquilateris & æquiangulis, Dodecaëdri angulus continetur.

Sed ex quatuor, nullus potest. Cùm enim pentagoni æquilateri angulus rectus sit, et quinta recti pars erunt quatuor anguli rectis quatuor maiores. Quod fieri nequit. Nec sanè ex alijs polygonis figuris solidus angulus continebitur, quod hinc quoque absurdum sequatur. Quamobrem perspicuum est, præter dictas quinque figuras aliã figuram solidam non posse constitui, quæ ex planis æquilateris & æquiangulis contineatur.

ELEMENTI XIII. FINIS

EVCLIDIS ELEMENTVM DECIMVM QVARTVM, VT qui dam arbitrantur, vt alij verò, Hypsiclis Alexandrini, de quinque corporibus.

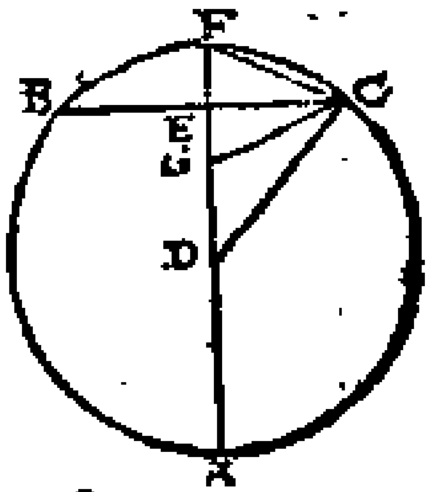
LIBER PRIMVS.

BAsilides Tyrius, Protarche, Alexandriam profectus, patrique nostro ob disciplinæ societatem commendatus, longißimo peregrinationis tempore cum eo versatus est. Cùmque disserevent aliquando de scripta ab Apollonio comparatione Dodecaëdri & Icosaëdri eidem sphæra inscriptorum, quam hæc inter se habeant rationem, censuerunt ea non rectè tradidisse Apollonium: quæ à se emendata, vt de patre audire erat, literis prodiderunt. Ego autem postea incidi in alterum librum ab Apollonio editum, qui demonstrationem accuratè complecteretur de re proposita, ex eiusq; problematis indagatione magnam equidem cepi voluptatem. Illud certè ab omnibus perspici potest, quod scripsit Apollonius, cùm sit in omnium manibus. Quod autẽ diligenti, quantum coniicere licet, studio nos postea scrip-

scripsisse videmur, id monimentis consignatum tibi nuncupandum duximus, vt qui feliciter cùm in omnibus disciplinis tum vel maximè in Geometria versatus, scitè ac prudēter iudices ea quæ dicturi sumus ob eam verò, quæ tibi cum patre fuit, vitæ consuetudinem, quaq́ue nos complecteris, beneuolentiā, tractationē ipsam libenter audias. Sed iam tempus est, vt præmio modum facientes, hanc syntaxim aggrediamur.

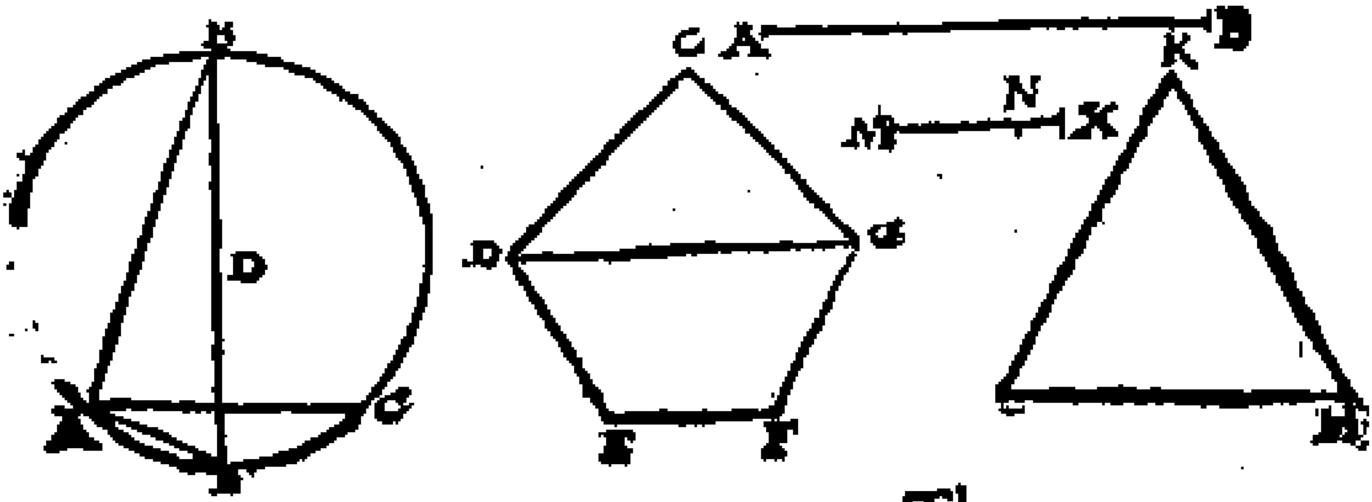
Theorema 1. Proopositio 1.

Perpendicularis linea, quæ ex circuli cuiuspiam centro in latus pentagωni ipsi circulo inscripti ducitur, dimidia est vtriusq; simul lineæ, & eius, quæ ex centro & lateris decagωni in eodē circulo inscripti.



Theorema 2. Propositio 2.

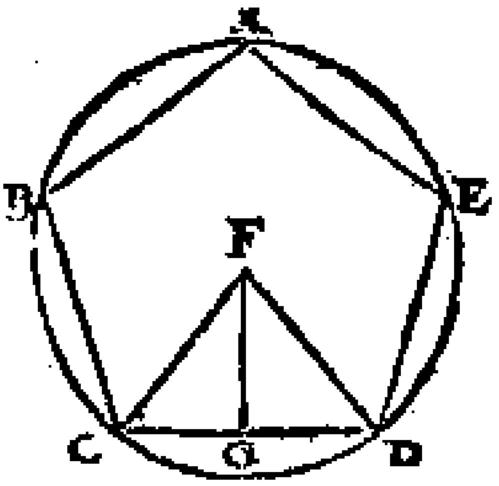
Idem circulus cóprehendit & dωdecaedri pentagωnum & icosaedri triangulum, eidē sphæræ inscriptorum.

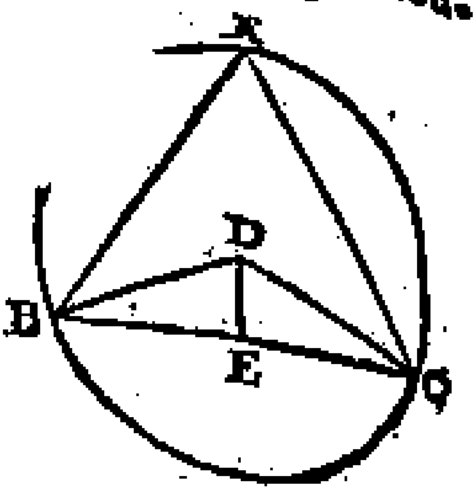


Theo-

Theorama 3. Propoſitio 3.

Si pentagωno & æquilatero & æquiangulo circumſcriptus ſit circulus ex cuius, centro in vnum pentagωni latus dicta ſit perpendicularis: quod vno laterum & perpẽdiculari trigeſies cõtinetur, illud



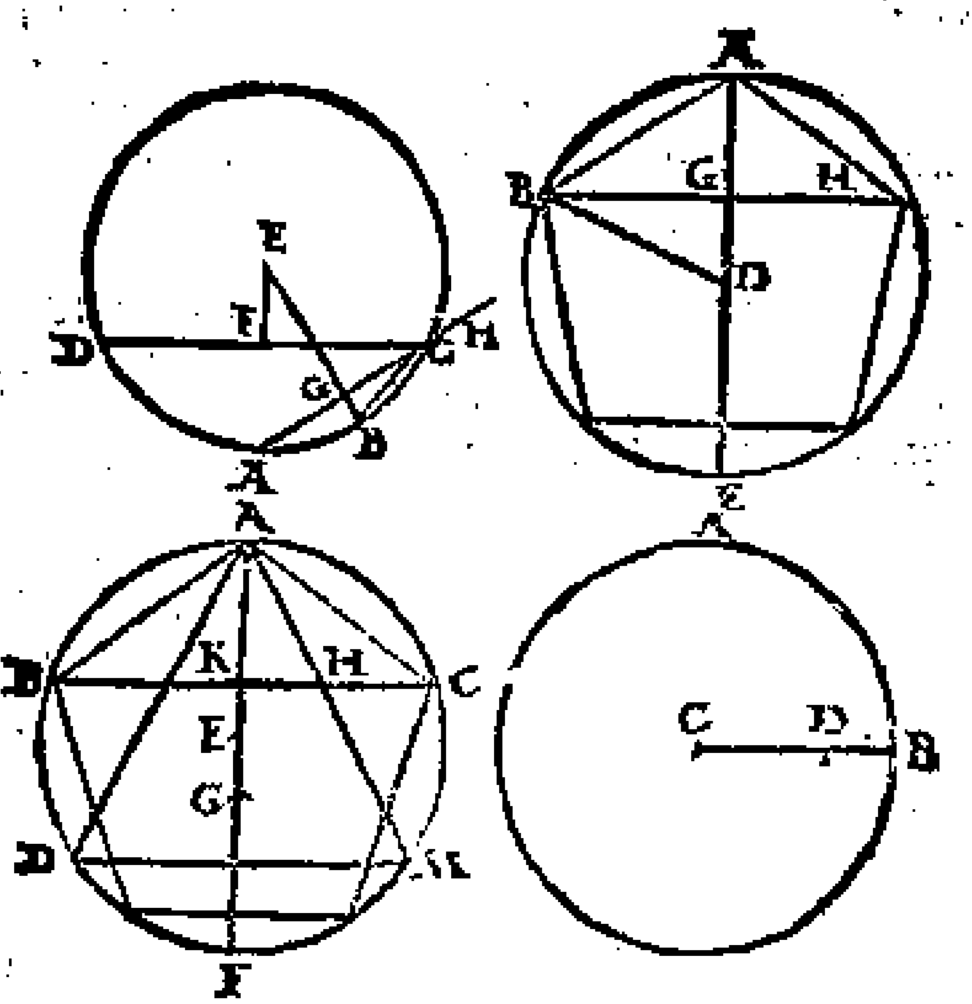


æquale eſt dωdecaedri ſuperficiei.

Theorema 4. Propoſitio 4.

Hoc perſpicuum cùm ſit probandum eſt, quemadmodum ſe habet dωdecaedri ſuperficies ad icoſaedri ſuperficiem ita ſe habere cubi latus ad icoſaedri latus.

Cubi



Cubilatus.

E————————

Dodecaedri.

F————————

Icosaedri.

G————————

SCHOLIVM

Nunc autem probandum est, quemadmodum se habet cubi latus ad icosaëdri latus, ita se habere solidum dodecaëdri ad icosaëdri solidum: Cùm enim æquales circuli comprehendat & dodecaëdri pentagonum & Icosaëdri triangulum, eidē sphæræ in-

scriptorum: in sphaeris autem aequales circuli aequali interuallo distent à centro (siquidem perpendiculares à sphaerae centro ad circulorum plana ductae & aequales sunt, & ad circulorum centra cadunt) idcirco lineae, hoc est perpendiculares, quae à sphaerae cẽtro ducuntur ad centrum circuli comprehendẽtis et triangulum Icosaëdri, & pentagonum dodecaëdri sũt aequales. Sũt igitur aequalis altitudinis Pyramides, quae bases habent ipsa dodecaëdri pentagona, et quae Icosaëdri triangula. At aequalis altitudinis pyramides rationem inter se habent eam quam bases, ex 5. & 6. 11. Quemadmodum igitur pentagonũ ad triangulum, ita pyramis, cuius basis quidem est dodecaëdri pentagonum, vertex autem sphaerae centrum ad pyramida, cuius basis quidem est Icosaëdri triangulum, vertex autem sphaerae centrum. Quamobrem vt se habent duodecim pentagona ad viginti triãgula, ita duodecim pyramides, quorum pẽtagonae sint bases, ad viginti pyramidas, quae trigonas habeant bases. At pentagona duodecim sunt dodecaëdri superficies, viginti autem triangula, Icosaëdri. Est igitur vt dodecaëdri superficies ad Icosaëdri superficiem, ita duodecim pyramides, quae pentagonas habeant bases, ad viginti pyramidas, quarum trigonae sunt bases. Sunt autem duodecim quidem pyramides, quae pentagonas habeant bases, solidũ dodecaëdri: viginti autem pyramides, quae trigonas habeant bases, Icosaëdri solidum. Quare ex 11. 5. vt dodecaëdri superficies ad Icosaëdri superficiem, ita

soli-

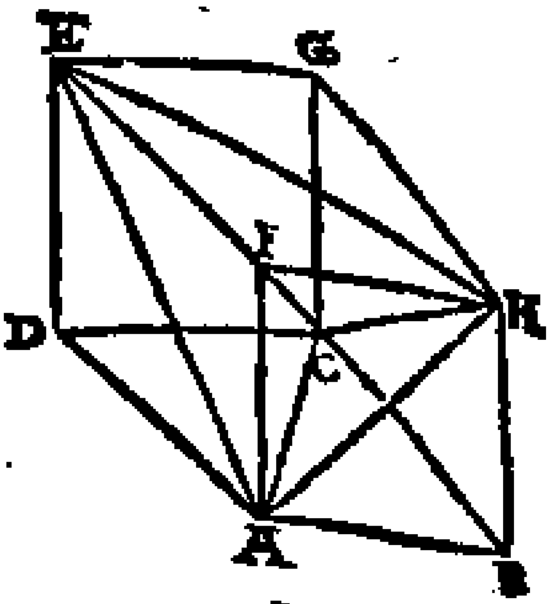
solidum dodecaëdri ad Icosaëdri solidum. Vt autem dodecaëdri superficies ad Icosaëdri superficiem, ita probatum est cubi latus ad Icosaëdri latus. Quemadmodum igitur cubi latus ad Icosaëdri latus, ita se habet solidum dodecaëdri ad Icosaëdri solidum.

EVCLIDIS ELEMENTVM DECIMVM QVINTVM, ET Solidorum quintum, vt nonnulli putant, vt autem alij, Hypsiclis Alexandrini, de quinque corporibus.

LIBER II.

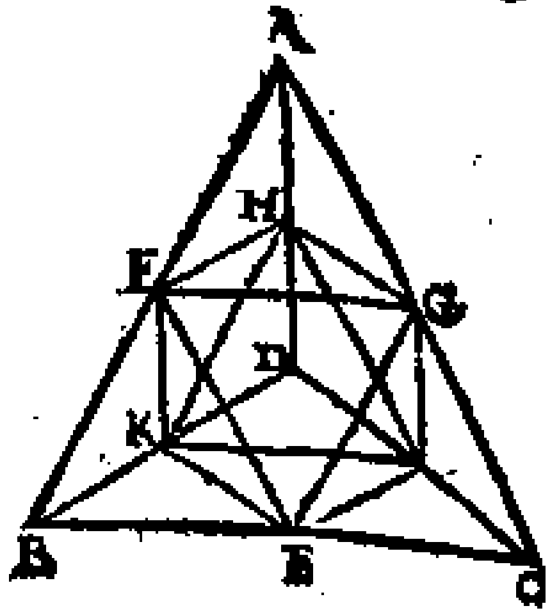
Problema 1. Propositio 1.

In dato cubo pyramida inscribere.



Problema 2. Propositio 2.

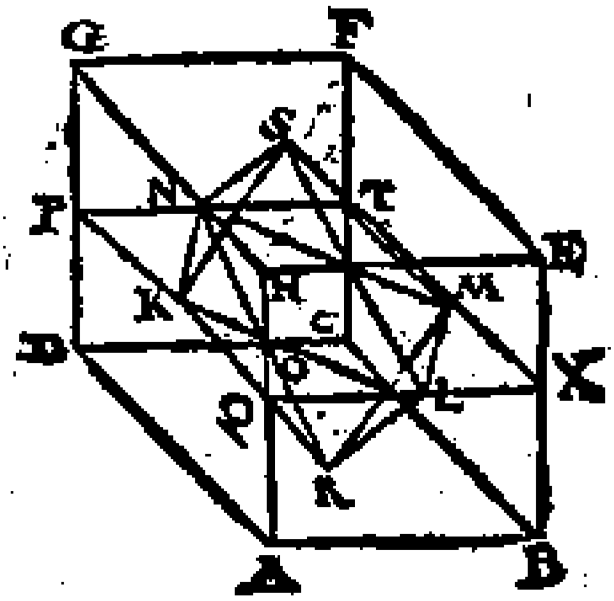
In data pyramide octaedrum inscribere.

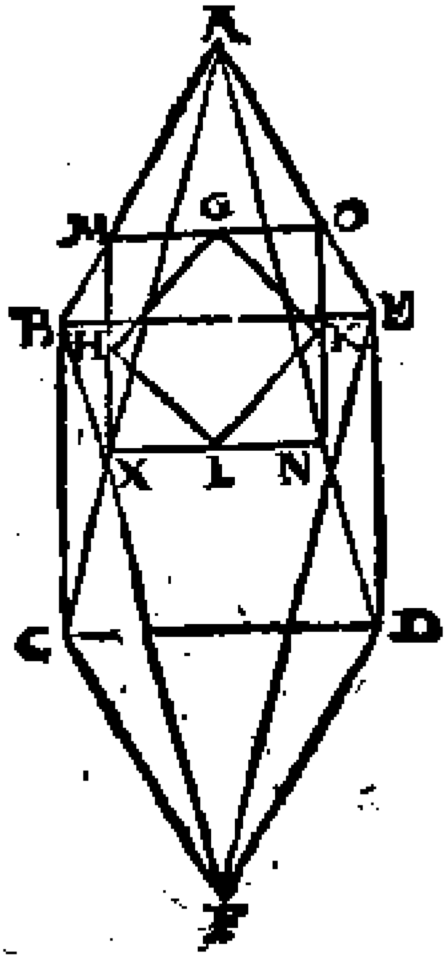


Pro-

Problema 3. Propositio 3.

In dato cubo octaedrum inscribere.



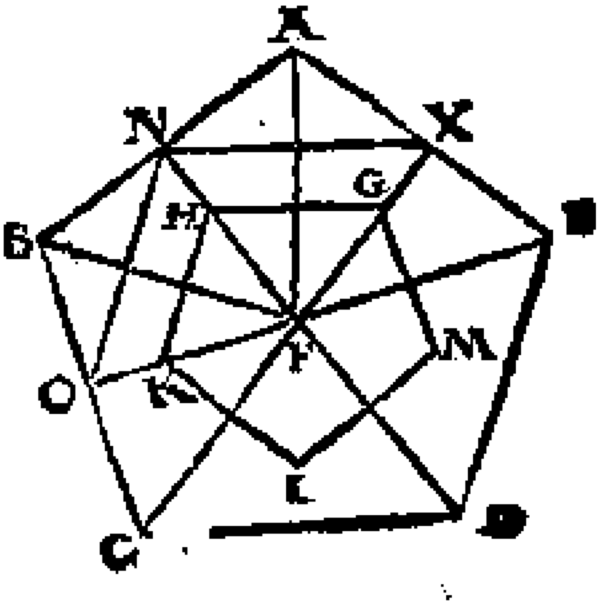


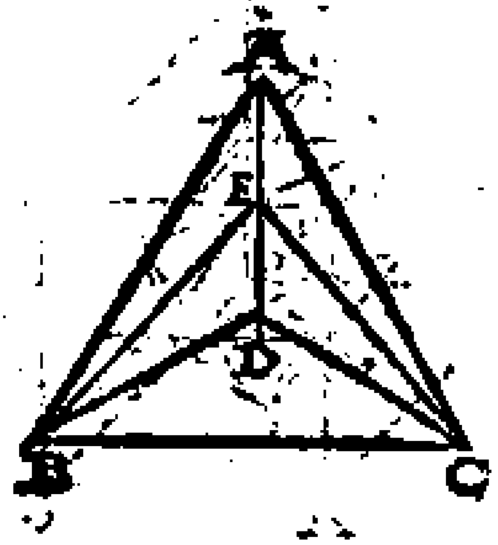
Problema 4. Propositio 4.

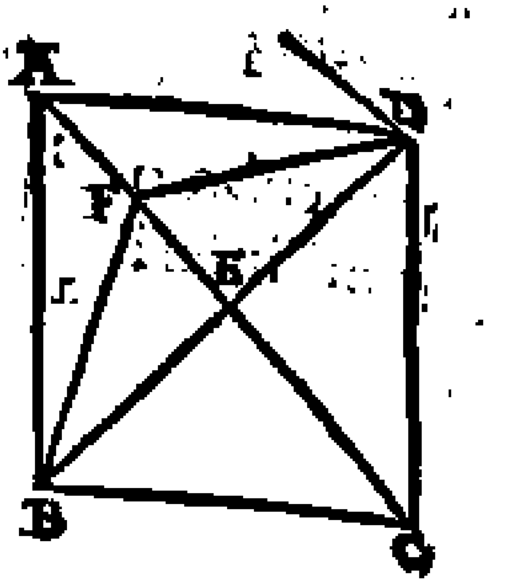
In dato octaedro cubũ inscribere.

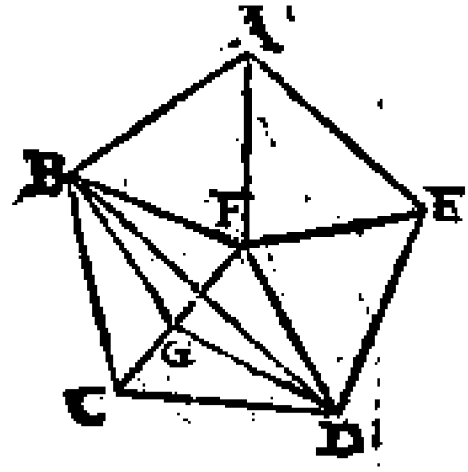
Problema 5. Propositio 5.

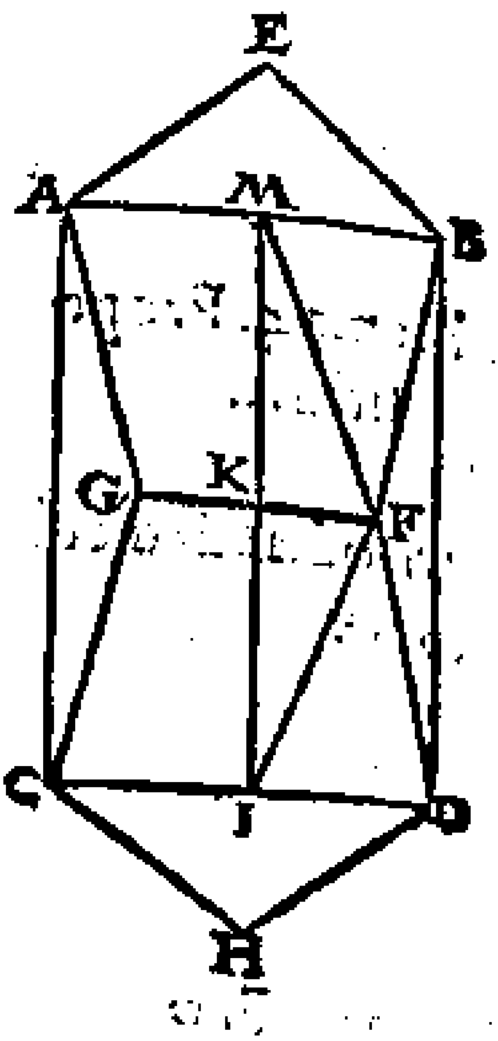
In dato Icosaedro dodecaedrum inscribere.

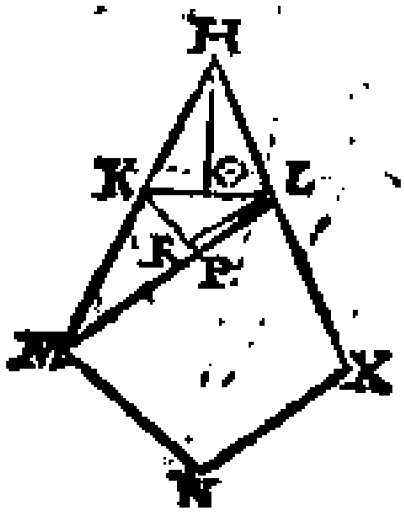












SCHO.

SCHOLIVM.

Meminisse decet, si quis nos roget, quot Icosaedrū habeat latera, ita respondendum esse: Patet Icosaedrum viginti contineri triãgulis, quodlibet verò triangulum rectis tribus constare lineis. Quare multiplicanda sunt nobis viginti triangula in trianguli vnius latera, fiuntq́ sexaginta, quorum dimidium est triginta. Ad eundem modum & in dodecaedro. Cū enim rursus duodecim pentagona dodecaedrum cōprehendant, itemq́ pentagonum quoduis rectis quinque constet lineis, quinq́ duodecies multiplicamus, fiunt sexaginta, quorum rursus dimidium est triginta. Sed cur dimidiū capimus? Quoniã vnumquodq́ latus siue sit trianguli, siue pentagoni, siue quadrati vt in cubo, iteratò sumitur. Similiter autem eadem via & in cubo & in pyramide & in octaedro latera inuenies. Quòd si itē velis singularum quoq́ figurarum angulos reperire, facta eadem multiplicatione numerum procreatum partire in numerum planorū, quæ vnum solidum angulum includūt: vt quoniã triãgula quinq́ vnum Icosaëdri angulum continet, partire 60. in quinque, nascūtur duodecim anguli Icosaedri. In dodecaedro autē tria pētagona angulum comprehendunt, partire ergo 60. in tria, & habebis dodecaedri angulos viginti. Atq́ simimili ratione in reliquis figuris angulos reperies.

Finis Elementorum Euclidis.