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EVCLIDIS ELEMENTORVM LIBRI XV.

QVIBVS, CVM AD OMNEM MATHEMATICAE SCIENTIAE PARTEM, TVM AD QVAMLIbet Geometriæ tractationem facilis comparatur aditus.

COLONIAE
Apud Gosuinum Cholinum
M. D. CXII.
Cum Gratia & Priuilegio.

AD CANDIDVM LECTOREM ST. GRACILIS praefatio.

PERMAGNI referre semper existimaui, lector beneuole, quantum quisque studij & diligentiæ ad percipienda scientiarum elementa adhibeat, quibus non satis cognitis, aut perperam intellectis, si vel digitū progredi tentes erroris caliginem animis offundas, non veritatis lucem rebus obscuris adferas. Sed principiorum quanta sint in disciplinis momenta, haud facilè credat, qui rerum naturam ipsa specie, non viribus metiatur. Vt enim corporum, quæ oriuntur & intereunt, vilißima tenuißimaq́; videntur initia: ita rerum æternarum & admirabilium, quibus nobilißimæ artes continentur, elementa ad speciem sunt exilia, ad vires & facultatem quam maxima. Quis non videt ex fici tantulo grano, vt ait Tullius, aut ex acino vinaceo, aut ex cæterarum frugum aut stirpium minutißimis seminibus tantos truncos ramosq́; procreari? Nam Mathematicorum initia illa quidem dictu audituq́; perexigua, quantā theorematum syluam nobis pe-

pererunt? Ex quo intelligi potest, vt in ipsis seminibus, sic & in artium principijs inesse vim earum rerum, quæ ex his progignuntur. Præclarè igitur Aristoteles, vt alia permulta, μέγιστον ἴσως ἀρχὴ παντὸς, καὶ ὅσῳ κράτιστον τῇ δυνάμει, τοσούτῳ μικρότατον, οὖσα μεγέθει χαλεπόν ἐστιν ὀφθῆναι. Quocirca committendum non est, vt non bene prouisa & diligenter explorata scientiarum principia, quibus propositarum quarumque rerum veritas sit demonstranda, vel constituas, vel constituta approbes: Cauendum etiam, vt ne tantulum quidem fallaci & captiosa interpretatione turpiter deceptus, à vera principiorum ratione temerè deflectas. Nam qui initio fortè aberrauerit, is vt tandem in maximis versetur erroribus, necesse est: cum ex vno errorum capite densiores sensim tenebræ rebus clarissimis obducantur. Quid tam varias veterum physiologorum sententias non modò cùm rerum veritate pugnantes, sed vehementer etiam inter se dissentientes nobis inuexit? Equidem haud scio, fuerit ne vlla potior tanti dissidij causa, quam quòd ex principijs partim falsis, partim non consentaneis ductas rationes probando adhiberent. Fit enim plerunque, vt qui non rectè de artium rerumq́ue elementis sentiunt, ad præfinitas quasdam opiniones suas omnia reuocare studeant. Pythagorei, vt meminit Aristoteles, cum denarij numeri summam perfectionem cælo tribuerent, nec plures tamen quàm nouem sphæras cernerent, decimam affingere ausi sunt terræ aduer-

sani, quam ἀντίχθονα *appellarunt. Illi enim uniuersitatis rerumque singularum naturam ex numeris ceu principijs æstimantes, ea protulerunt quæ* φαινομένοις *congruere nusquam sunt cognita. Nam ridicula Democriti, Anaximenis, Melissi Anaxagoræ, Anaximandri, & reliquorum id genus physiologorum somnia, ex falsis illa quidem orta naturæ principijs, sed ad Mathematicum nihil aut parum spectantia, sciens prætereo Nonnullos attingam qui repetitis altius vel aliter ac decuit positis rerum initijs, cum physicis multa turbarunt; tum Mathematicos oppugnatione principiorum pessimè mulctarunt. Ex planis figuris corpora constituit Timæus: Geometrarum hic quidem principia cuniculis oppugnantur. Nam & superficies seu extremitates craßitudinem habebunt, & lineæ latitudinem: denique puncta non erunt indiuidua, sed linearum partes. Prædicant Democritus atque Leucippus illas atomos suas, & indiuidua corpuscula. Concedit Xenocrates impartibiles quasdam magnitudines. Hic verò Geometriæ fundamenta apertè petuntur, & funditus euertuntur: quibus dirutis nihil equidem aliud video restare, quàm vt amplißima Mathematicorum theatra repentè concidant. Iacebunt ergò, si dijs placet, tot præclara Geometrarum de asymmetris & alogis magnitudinibus theoremata. Quid enim causa dicas, cur indiuidua linea hanc quidem metiatur, illam verò meteri non queat? Siquidem quod minimum in vnoquoque* *genere reperitur, id communis omnium mensura esse solet. Innumerabilia profectò sunt illa, quæ ex falsis eiusmodi decretis absurda consequuntur: & horum permulta quidem Mathematicus, sed longè plura colligit Physicus. Quid varia* ψευδογραφημάτων *genera commemorem, quæ ex hoc vno fonte tam longè latéque diffusa fluxisse videntur? Notißimus est Antiphontis tetragonismus, qui Geometrarum & ipse principia non parum labefecit, cùm rectæ lineæ curuam posuit æqualem. Longum esset mihi singula percensere, præsertim ad alia properanti: Hoc ergo certum fixum, & in perpetuum ratum esse oportet, quod sapienter monet Aristoteles,* σπουδαστέον ὅπως ὁρισθῶσι καλῶς αἱ ἀρχαὶ μεγάλην γὰρ ἔχουσι ῥοπὴν πρὸς ἑπόμενα. *Nam principijs illa congruere debent, quæ sequuntur. Quòd si tantum perspicitur in istis exilioribus Geometriæ initijs, quæ puncto, linea, superficie definiuntur, momentum, vt ne hæc quidem sine summo impendentis ruinæ periculo conuelli aut oppugnari poßint, quanta quæso vis putanda est huius* στοιχειώσεως, *quam collatis tot præstantißimorum artificum inuentis, mira quadam ordinis solertia contexuit Euclides, vniuersa Matheseos elementa complexu suo coercentem? Vt igitur omnibus rebus instructior & paratior quisq; ad hoc studiũ libentius accedat, & singula vel minutißima exactius secum reputet atq; perdiscat, operæpreciũ censui, in primo institutionis aditu vestibuloq; præcipua quædem capita,*

quibus

quibus tota ferè Mathematicæ scientiæ ratio intelligatur, breuiter explicare: tum ea quæ sunt Geometriæ propria, diligenter persequi: Euclidis denique in extruenda hac στοιχειώσεως consilium sedulò ac fideliter exponere. Quæ ferè omnia ex Aristotelis potissimũ ducta fontibus, nemini inuisa fore cõfido, qui modo ingenui animi candorẽ ad legendũ attulerit. Ac de Mathematicæ diuisione primũ dicamus.

Mathematicæ in primis scientiæ studiosos fuisse Pythagoreos, non modò historicorum, sed etiam philosophorum libri declarant. His ergo placuit, vt in partes quatuor vniuersum distribuatur Mathematicæ sciẽtiæ genus, quarum duas περὶ τὸ ποσὸν, reliquas περὶ τὸ πηλίκον versari statuerunt. Nam & τὸ ποσὸν vel sine vlla comparatione ipsum per se cognosci, vel certa quadam ratione comparatum spectari: in illo Arithmeticam, in hoc versari Musicam: & τὸ πηλίκον partim quiescere, partim moueri quidem: illud Geometriæ propositum esse: quod verò sua sponte motu cietur, Astronomiæ. Sed ne quis falsò putet, Mathematicam scientiam, quòd in vtroque quanti genere cernitur, idcirco inanem videri (si quidem non solum magnitudinis diuisio sed etiam multitudinis accretio infinitè progredi potest) meminisse decet, τὸ πηλίκον καὶ τὸ ποσὸν, quæ subiecto Mathematicæ generi imposita sunt à Pythagoreis nomina, non cuiuscunque modi quantitatem significare, sed eam demum, quæ tum multitudine tum magnitudine sit defi-

dita, & suis circumscripta terminis. Quis' enim vllam infiniti scientiam defendat? Hoc scitum est, quod non semel docet Aristoteles, infinitum ne cogitatione quidem complecti nunquam posse. Itaque ex infinita multitudinis & magnitudinis δυνάμει *finitam hac scientia decerpit & amplectitur naturam, quam tractet, & in qua versetur. Nam de vulgari Geometrarum consuetudine quid sentiendum sit, cum data interdum magnitudine infinita aut fabricentur aliquid, aut proprias generis subiecti affectiones exquirunt, disertè monet Aristoteles,* ὀυδὲ νῦν *(de Mathematicis) loquens* δέονται τοῦ ἀπείρου, ὀυδὲ χρῶνται, ἀλλὰ μόνον εἶναι ὅσην ἂν βούλωνται, πεπερασμένην. *Quamobrem disputatio ea qua infinitum refellitur, Mathematicorum decretis rationibusque non aduersatur, nec eorum apodixes labefacit. Etenim tali infinito opus illis nequaquam est, quod exitu nullo peragrari possit, nec talem ponunt infinitam magnitudinem: sed quantamcunque velit aliquis effingere, ea vt suppetat, infinitam praecipiunt. Quinetiam non modò immensa magnitudine opus non habent Mathematici, sed ne maxima quidem: cum instar maxima minima quaque in partes totidem pari ratione diuidi queat. Alteram Mathematicae diuisionẽ attulit Geminus, vir (quantum ex Proclo coniicere licet)* μαθημάτων *laude clarißimus. Eam, quae superiore plenior & accuratior fortè visa est, cũ doctißimè pertractarit sua in decimum Euclidis*

praefapræfatione P. Montaureus vir senatorius, & regiæ bibliothecæ præfectus, leuiter attingam. Nam ex duobus rerum velut summis generibus τῶν νοητῶν καὶ τῶν αἰσθητῶν, quæ res sub intelligentiā cadunt, Arithmetica & Geometria attribuit Geminus: quæ verò in sensus incurrunt, Astrologiæ, Musicæ Supputatrici, Optica, Geodesia & Mechanica adiudicauit. Ad hanc certè diuisionem spectasse videtur Aristoteles, cum Astrologiam Opticam, harmoniam, φυσικωτέρας τῶν μαθημάτων nominat, vt quæ naturalibus & Mathematicis interiectæ sint, ac velut ex vtrisque mixtæ disciplinæ: Siquidem genera subiecta à Physicis mutuantur, causas verò in demonstrationibus ex superiore aliqua scientia repetunt. Id quod Aristoteles ipse apertissimè testatur, ἐνταῦθα γὰρ, φησὶ, τὸ μὲν ὅτι, τῶν αἰσθητικῶν εἰδέναι τὸ διότι, τῶν μαθηματικῶν. Sequitur, vt quid Mathematica conueniat cum Physica & prima Philosophia: quid ipsa ab vtraque differat, paucis ostendamus. Illud quidem omnium commune est, quòd in veri contemplatione sunt posita, ob idqúe θεωρητικαὶ à Græcis dicuntur. Nam cùm διάνοια siue ratio & mens omnis sit vel πρακτικὴ, vel θεωρητικὴ, totidem scientiarum sint genera necesse est. Quòd si Physica, Mathematica, & prima Philosophia, nec in agendo, nec in efficiendo sunt occupatæ, hoc certè perspicuum est, eas omnes in cognitione contemplationéque necessario versari. Cùm enim rerum non modò agendarum, sed etiam effi-

ciendarum principia in agente vel efficiente consistant, illarum quidem προαίρεσις, harum autem vel mens vel ars, vel vis quædam & facultas: rerum profectò naturalium. Mathematicarum, atque diuinarum principia in rebus ipsis, non in philosophis inclusa latent. Atque hæc vna in omnes valet ratio, quæ θεωρητικὰς esse colligat. Iam verò Mathematica separatim cum Physica congruit, quòd vtraque versatur in cognitione formarum corpori naturali inhærentium. Nam Mathematicus plana, solida, longitudines & puncta contemplatur, quæ omnia in corpore naturali à naturali quoque philosopho tractantur. Mathematica item & prima philosophia hoc inter se propriè conueniunt, quòd cognitionem vtraque persequitur formarum, quoad immobiles, & à concretione materiæ sunt liberæ. Nam tametsi Mathematicæ formæ re vera per se non cohærent, cogitatione tamen à materia & motu separantur, οὐδὲ γίνεται ψεῦδος χωριζόντων. vt ait Aristoteles. De cognatione & societate breuiter diximus, iam quid intersit videamus. Vnaquæq; mathematicarum certum quoddam rerum genus propositum habet, in quo versetur, vt Geometria quantitatem & continuationem aliorum in vnam partem, aliorum in duas, quorundam in tres, eorumq; quatenus quanta sunt & continua, affectiones cognoscit. Prima autem philosophia, cùm sit omnium communis, vniuersum Entis genus, quæq; ei accidunt & conueniunt hoc ipso quòd est, considera-

Ad hæc, Mathematica eam modò naturam amplectitur, quæquanquam non mouetur, separari tamen seiungiq; nisi mente & cogitatione à materia non potest, ob eamq; causam ἐξ ἀφαιρέσεως dici consueuit. Sed Prima philosophia in ijs versatur, quæ & seiuncta, & æterna, & ab omni motu per se soluta sunt ac libera. Cæterùm Physica & Mathematica quanquā subiecto discrepare non videntur, modo tamen rationeq; differūt cognitionis & contemplationis, vnde dissimilitudo quoque scientiarum sequitur. Etenim Mathematicæ species nihil re vera sunt aliud, quàm corporis naturalis extremitates, quas cogitatione ob omni motu & materia separatas Mathematicus contemplatur: sed easdem consectatur Physicorū ars, quatenus cum materia comprehensa sunt, & corpora motui obnoxia circumscribunt. Ex quo fit, vt quæcunque in Mathematicis incommoditates accidunt, eædem etiam in naturalibus rebus videantur accidere, non autem vicissim. Multa enim in naturalibus sequuntur incommoda, quæ nihil ad Mathematicum attinent, διὰ τὸ, inquit Aristoteles, τὰ μὲν ἐξ ἀφαιρέσεως λέγεται, τὰ μαθηματικὰ, τὰ δὲ φυσικὰ ἐκ προσθέσεως. Siquidem res cum materia deuinctas contēplatur physicus: Mathematicus verò rē cognoscit circū scriptis ijs omnibus, quæ sensu percipiūtur, vt grauitate, leuitate, duritie, mollitie, & præterea calore, frigore, alijsq; contrariorum paribus, quæ sub sensum subiecta sunt: tantum autem relinquit

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quantitatum & continuum. Itaq; Mathemati-corum ars in ijs quæ immobilia sunt cernitur (τὰ γὰρ μαθηματικὰ τῶν ὄντων ἄνευ κινήσεώς ἐστιν πλὴν τῶν περὶ τὴν ἀστρολογίαν) quæ verò in naturæ obscuritate posita est, res quidem quæ nec separari nec motu vacare possunt contemplatur. Id quod in vtroque scientiæ genere perspicuum esse potest siue res subiectas definias, siue proprietates earum demonstres. Etenim numerus, linea, figura, rectum, inflexum, æquale, rotundū, vniuersa deniq; Mathematicus quæ tractat & profitetur, absque motu explicari doceriq; poßūt: χωριστὰ γὰρ τῇ νοήσει κινήσεώς ἐστι: Physica autē sine motione species nequaquam possunt intelligi. Quis enim hominis, plantæ, ignis, oßium, carnis naturam & proprietates sine motu, qui materiam sequitur, perspiciat? Siquidem tantisper substantia quæque naturalis constare dici solet, quo ad opus & munus suum, agendo patiendóque tueri ac sustinere valeat: quâ certè amissa δυνάμει, ne nomen quidem nisi ὁμωνύμως retinet. Sed Mathematico ad explicandas circuli aut trianguli proprietates, nullum adferre potest vsum, materiæ, vt auri, ligni, ferri, in qua insunt, consideratio, quin eò verius eiusmodi rerum, quarum species tanquam materia vacantes efformemus animo, naturam complectemur, quòd coniunctione materiæ quasi adulterari deprauaríque videntur. Quo circa Mathematica species eodem modo quo κοῖλον, siue concauitas, sine motu & subiecto, definitione explicare cog-

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nosciq; possunt: naturales verò cùm eam vim habeant, quam vt ita dicam, similitas cum materia comprehensa sunt, nec absque ea separatim possunt intelligi: quibus exemplis quid inter Physicas & Mathematicas species intersit, haud difficile est animaduertere. Illis certè non semel est vsus Aristoteles. Valeant ergò Protagoræ sophismata, Geometras hoc nomine refellentis, quod circulus normam puncto non attingat. Nam diuina Geometrarum theoremata, qui sensu æstimabit, vix quicquam reperiet quod Geometræ concedendum videatur. Quid enim ex his quæ sensum mouent, ita rectum aut rotundum dici potest, vt à Geometra ponitur? Nec verò absurdum est aut vitiosum, quòd lineas in puluere descriptas pro rectis aut rotundis assumit, quæ nec rectæ sunt nec rotundæ, ac ne latitudinis quidem expertes. Siquidem non ijs vtitur Geometra, quasi inde vim habeat conclusio, sed eorum quæ discenti intelligenda relinquuntur, rudem ceu imaginem proponit. Nam qui primum instituuntur, hi ductu quodam & velut χειραγωγία sensuum opus habent, vt ad illa quæ sola intelligentia percipiuntur, aditum sibi comparare queant. Sed tamen existimandum non est rebus Mathematicis omnino negari materiam ac non eam tantùm quæ sensum afficit. Est enim materia alia quæ sub sensum cadit, alia quæ animo & ratione intelligitur. Illam αἰσθητὴν hanc νοητὴν vocat Aristoteles. Sensu percipitur, vt æs, vt lignum, omnisque materia quæ moueri potest *rest. Animo & ratione cernitur ea qua in rebus sensilibus inest, sed non quatenus sensu percipiuntur, quales sunt res Mathematicorum. Vnde ab Aristotele scriptum legimus ἐπὶ τῶν ἐν ἀφαιρέσει ὄντων rectum se habere vt simum: μετὰ συνεχοῦς γάρ: quasi velit ipsius recti quod Mathematicorum est, suam esse materiam, non minus quam simi quod ad Physicos pertinet. Nam licet res Mathematica sensili vacent materia, non sunt tamen indiuidua, sed propter continuationem partitioni semper obnoxia, cuius ratione dici possunt sua materia non omnino carere: quin aliud videtur τὸ εἶναι γραμμῇ, aliud quoad continuationi adiuncta intelligitur linea. Illud enim ceu forma in materia proprietatum causa est, quas sine materia percipere non licet. Hac est societatis & dissidij Mathematica cum Physica & prima Philosophia ratio. Nunc autem de nominis etymo & notatione pauca quadam afferamus. Nam si qua iudicio & ratione imposita sunt rebus nomina, ea certè non temerè indita fuisse credendum est, quibus scientias appellari placuit, sed neque otiosa semper haberi debet ista etymologia indagatio, cùm ad rei etiam dubia fidem sape non parum valeat recta nominis interpretatio. Sic enim Aristoteles ducto ex verborum ratione argumento αὐτομάτου, μετὰ βουλῆς, αἰθέρος, aliarumq; rerum naturam ex parte confirmauit. Quoniam igitur Pythagoras Mathematicam scientiam non modò studiosè coluit, sed etiam repetitis à capite principijs geometricam*

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contemplationem in liberalis disciplinæ formam composuit, & perspectis absque materia solius intelligentiæ adminiculo theorematibus, tractationem περὶ τῶν ἀλόγων, & κοσμικῶν σχημάτων constitutionem excogitauit: credibile est, Pythagoram, aut certè Pythagoreos, qui & ipsi doctoris sui studia libenter amplexi sunt, huic scientiæ id nomen dedisse, quod cum suis placitis atque decretis congrueret rerumque propositarum naturam quoquo modo declararet. Ita cùm existimarent illi, omnem disciplinam, quæ μάθησις dicitur, ἀνάμνησιν esse quandam .i. recordationem & repetitionem eius scientiæ, cuius antè quam in corpus immigraret, compos fuerit anima, quemadmodum Plato quoque in Menone, Phædone, & alijs aliquot locis videtur astruxisse: animaduerterent autem eiusmodi recordationem, quæ non posset multis ex rebus perspici, ex his potissimum scientijs demonstrari, si quis nimirum, ait Plato, ἐπὶ τὰ διαγράμματα ἄγῃ, probabile est equidem Mathematicas à Pythagoreis artes κατ᾽ ἐξοχὴν fuisse nominatas, vt ex quibus μάθησις, id est æternarum in anima rationum recordatio διαφερόντως & principuè intelligi posset. Cuius etiam rei fidem nobis diuinus fecit Plato, qui in Menone Socratem induxit hoc argumenti genere persuadere cupientem, discere nihil esse aliud quam suarum ipsius rationum animum recordari. Etenim Socrates pusionem quendam vt Tullij verbis vtar, interrogat de geometrica dimensione quadrati: ad ea sic ille respondet vt puer, et tamen

men tam faciles interrogationes sunt, vt gradatim respondens, eodem perueniat quò si Geometrica didicisset. Aliam nominis huius rationem Anatolius exposuit, vt est apud Rhodiginum, quòd cùm caetera disciplinae deprehendi vel non docente aliquo possint omnes, Mathematica sub nullius cognitionem veniant, nisi praeeunte aliquo, cuius solertia succidantur vepreta, vel exurantur, & superciliosa complanentur aspreta. Ita enim Caelius: quod quam vim habeat, non est huius loci curiosius perscrutari. Equidem M. Tullius Mathematicos in magna rerum obscuritate, recondita arte multiplicíq; ac subtili versari scribit: sed quis nescit idipsum cum alijs grauioribus scientijs esse commune? Est enim, vel eodem auctore Tullio, omnis cognitio multis obstructa difficultatibus, maximaq; est & in ipsis rebus obscuritas, & in iudicijs nostris infirmitas, nec vllus est, modò interim paulò Physica penetrarit, qui non facilè sit expertus, quam multi vndique emergant, rerum naturalium causas inquirentibus, inexplicabiles labyrinthi. Sunt qui ex demonstrationum firmitate nominari Mathematicas opinantur: cuius etiam rationis momentum alio seorsim locò expendendum fuerit. Quocirca primam verbi notationem, quam sequutus est Proclus, nobis retinendam censeo. Hactenus de vniuerso Mathematicae genere, quanta potui & perspicuitate & breuitate dixi. Sequitur vt de Geometria separatim atque ordine ea disseram, quae initio sum pollicitus *us. Est autem Geometria, vt definit Proclus scientia, quæ versatur in cognitione magnitudinum figurarum, & quibus hæ continentur, extremorum, item rationum & affectionum, quæ in illis cernuntur ac inhærent: ipsa quidem progrediens à puncto indiuiduo per lineas & superficies, dum ad solida conscendat, variasq; ipsorum differentias patefaciat. Qúmque omnis scientia demonstratiua, vt docet Aristoteles, tribus quasi momentis contineatur, genere subiecto, cuius proprietates ipsa scientia exquirit & cõtemplatur: causis & principijs ex quibus primis demonstrationes conficiuntur: & proprietatibus, quæ de genere subiecto per se enunciantur: Geometriæ quidem subiectum in lineis, triangulis, quadrangulis, circulis, planis, solidis atque omnino figuris & magnitudinibus, earúmque extremitatibus consistit. His autem inhærent diuisiones, rationes, tactus, æqualitates, παραβολαὶ ὑπερβολαὶ ἐλλείψεις, atque alia generis eiusdem propè innumerabilia. Postulata verò & Axiomata ex quibus hæc inesse demonstrantur, eiusmodi ferè sunt: Quouis centro & interuallo circulum describere. Si ab æqualibus æqualia detrahas, quæ relinquuntur esse æqualia, cæteraq; id genus permulta, quæ licet omnium sint communia, ac demonstrandum tamen tum sunt accommodata, cum ad certum quoddam genus traducuntur. Sed cùm præcipua videatur Arithmetica & Geometriæ inter Mathematicas dignatio, cur Arithmetica sit ἀκριβεστέρα &*

exactior quam Geometria paucis explicandum arbitror. Hic verò & Aristotelem sequemur ducem, qui scientiam cum scientia ita comparat, vt accuratiorem eße velit eam, quæ rei causam docet, quam quæ rem esse tantùm declarat, deinde quæ in rebus sub intelligentiam cadentibus versatur, quàm quæ in rebus sensum mouentibus cernitur. Sic enim & Arithmetica quàm Musica, & Geometria quàm Optica, & Stereometria quàm Mechanica exactior esse intelligitur. Postremò quæ ex simplicioribus initijs constat, quàm quæ aliqua adiectione compositis vititur. Atque hac quidem ratione Geometriæ præstat Arithmetica, quòd illius initium ex additione dicatur, huius sit simplicius. Est enim punctum, vt Pythagoreis placet, vnitas quæ situm obtinet: vnitas verò punctum est quod situ vacat. Ex quo percipitur, numerorum quàm magnitudinum simplicius esse elementum, numerosque magnitudinibus esse puriores, & à concretione materiæ magis disiunctos. Hæc quanquam nemini sunt dubia, habet & ipsa tamen Geometria quo se plurimum efferat, opibúsque suis ac rerum vbertate multiplici vel cum Arithmetica certet: id quod tute facilè deprehendas cùm ad infinitam magnitudinis diuisionem, quam respuit multitudo, animum conuerteris. Nunc quæ sit Arithmeticæ & Geometriæ societas, videamus. Nam theorematum quæ demonstratione illustrantur, quædam sunt vtriusque scientiæ communia, quædam verò singugularum propria. Etenim quòd omnis proportio sit ῥητὸς siue rationalis, Arithmeticæ soli conuenit, nequaquam Geometriæ, in qua sunt etiam ἄῤῥητοι seu irrationales proportiones. item, quadratorum gnωmas minimo definitos esse, Arithmeticæ proprium (si quidem in Geometria nihil tale minimum esse potest) sed ad Geometriam propriè spectant situs, qui in numeris locum non habent tactus, qui quidem à continuis admittuntur: ἄλογον, quoniam vbi diuisio infinitè procedit, ibi etiam τὸ ἄλογον esse solet. Communia porrò vtriusque sunt illa, quæ ex sectionibus eueniunt, quas Euclides libro secundo est persequutus: nisi quòd sectio per extremam & mediam rationem in numeris nusquam reperiri potest. Iam vero ex theωrematibus eiusmodi communibus, alia quidem ex Geometria ad Arithmeticam traducuntur: alia contrà ex Arithmetica in Geometriam transferuntur: quædam verò perinde vtrique scientiæ conueniunt, vt quæ ex vniuersa arte Mathematica in vtranque harum conueniant. Nam & alterna ratio, & rationum conuersiones, compositiones, diuisiones hoc modo communia sunt vtriusque. Quæ autem sunt περὶ συμμέτρων, id est, de commensurabilibus, Arithmetica quidem primùm cognoscit & contemplatur: secundo loco Geometria Arithmeticam imitata. Quare & commensurabiles magnitudines illæ dicuntur, quæ rationem inter se habent, quam numerus ad numerum, perinde

inde quasi cõmensuratio & συμμετρία in numeris primùm consistat (Vbi enim numerus ibi & σύμμετρον cernitur: & vbi σύμμετρον, illic etiam numerus) sed quæ triangulorum sunt & quadrangulorum, à Geometra primum considerantur: tùm analogia quadam Arithmeticus eadem illa in numeris contemplatur. De Geometriæ diuisione hoc adijciendum puto, quòd Geometriæ pars altera in planis figuris cernitur quæ solam latitudinem longitudini coniunctam habent: altera verò solidas contemplatur, quæ ad duplex illud interuallum crassitudinem adsciscunt. Illam generali Geometriæ nomine veteres appellarunt: hanc propriè Stereometriam dixerunt. Ita Geometriam cum Optica & Stereometriam cum Mechanica non rarò comparat Aristoteles. Sed illius cognitio huius inuentionem multis seculis antecessit, si modò Stereometriam ne Socratis quidem ætate vllam fuisse omnino verum est, quemadmodum à Platone scriptum videtur. Ad Geometriæ vtilitatem accedo, quæ quanquam suapte vi & dignitate ipsa per se nititur, nullius vsus aut actionis ministerio mancipata (vt de Mathematicis omnibus scientijs concedit in Politico Socrates) si quid ex ea tamen vtilitatis externæ quæritur, Dij boni quam latos, quàm vberes, quum varios fructus fundit? Nec verò audiendus est vel Aristippus vel Sophistarum alius, qui Mathematicorum artes idcirco repudiet, quòd ex fine nihil docere videantur, eiusque quod melius aut deteri-

leterius nullam habeant rationem. Vt enim nihil causæ dicas, cur sit melius trianguli, verbi gratia, tres angulos duobus esse rectis æquales: minime tamen fuerit consentaneum Geometriæ cognitionem vt inutilem exagitare, criminari, explodere, quasi quæ finem & bonum quò referatur, habeat nullum. Multas haud dubiè solius contemplationis beneficio citra materiæ contagionem adfert Geometria commoditates partim proprias, partim cum vniuerso genere communes. Cùm enim Geometria, vt scripsit Plato, eius quod semper est cognitionem profiteatur, ad veritatem excitabit illa quidem animum, & ad ritè philosophandum cuiusque mentem comparabit. Quinetiam ad disciplinas omnes facilius perdiscendas, attigeris necne Geometriam quanti referre censes? Nam vbi cum materia coniungitur, nónne præstantißimas procreat artes. Geodesiam, Mechaniam, Opticam, quarum omnium vsu, mortalium vitam summis beneficijs complectitur? Etenim bellica instrumenta, vrbiumq; propugnacula, quibus munitæ vrbes hostium vim propulsarent, his adiutricibus fabricata est: montium ambitus & altitudines, locorumque situs nobis indicauit: dimetiendorum & mari & terra itinerum rationem præscripsit: trutinas & stateras, quibus exacta numerorum æqualitas in ciuitate retineatur, composuit: vniuersi ordinem simulacbris expreßit, multáque quæ hominum fidem superarent, omnibus persuasit. Vbique extant præclara in eam rem

 testimo-

testimonia. Illud memorabile, quòd Archimedi rex Hiero tribuit. Nam extructo. vasta molis nauigio, quod Hiero AEgyptiorum regi Ptolemao mitteret, cùm vniuersa Syracusanorum multitudo collectis simul viribus nauem trahere non posset effecisseq́, Archimedes, vt solus Hiero illam subduceret, admiratus viri scientiam rex ἀπὸ ταύτης, ἔφη, τῆς ἡμέρας περὶ παντος ἀρχιμήδει λέγοντι πιστευτέον. Quid? quòd Archimedes idem, vt est apud Plutarchũ, Hieroni scriptis datis viribus datum pondus moueri posse? fretusq́ demonstrationis robore illud sæpe iactarit, si terram haberet alteram vbi pedem figeret, ad eam, nostram hanc se transmouere posse? Quid varia, αὐτομάτων machinarúmque genera ad vsus necessarios comparata memorem? Innumerabilia profecto sunt illa, & admiratione dignißima quibus prisci homines incredibili quodam ad philosophandum studio concitati, inopem mortalium vitam artis huius præsidio subleuarunt: tametsi memoriæ sit proditum, Platonem Eudoxo & Archyta vitio vertisse, quòd Geometrica problemata ad sensilia & organica abducerent. Sic enim corrumpi ab illis & labefieri Geometriæ præstantiam, quæ ab intelligibilibus & incorporeis rebus ad sensiles & corporeas prolaberetur. Quapropter ridicula idem scripsit Plato Geometrarum esse vocabula, quæ quasi ad opus & actionem spectent, ita sonare videntur. Quid enim est quadrare si non opus facere? Quid addere, *producere, applicare? Multa quidem sunt eiusmodi nomina, quibus necessariò & tanquam coacti Geometræ vtuntur, quippe cùm alia desint in hoc genere commodiora. Sic ergo censuit Plato, sic Aristoteles sic denique philosophi omnes, Geometriam ipsam cognitionis gratia exercendam, nec ex aliquo vsu externo sed ex rerum νοητῶν intelligentia æstimandam esse. Exposita breuius quàm res tanta dici possit, vtilitatis ratione, Geometriæ ortum, qui in hac rerum periodo ex historicorum monumentis nobis est cognitus deinceps aperiamus. Geometria apud AEgyptios inuenta, (ne ab Adamo, Setho, Noah, quos cognitione rerum multiplici valuisse constat, eam repetamus) ex terrarum dimensione, vt verbi præ se fert ratio, ortum habuisse dicitur: cùm anniuersaria Nili inundatione & incremenits limo obducti agrorum termini confunderentur. Geometriam enim, sicut & reliquas disciplinas, in vsu quàm in arte prius fuisse aiunt. Quod sanè mirum videri non debet, vt & huius & aliarum scientiarum inuentio ab vsu cœperit ac necessitate. Etenim tempus, rerum vsus, ipsa necessitas ingenium excitat, & ignauiam acuit. Deinde quicquid ortum habuit (vt tradunt Physici) ab incohato & imperfecto processit ad perfectum. Sic artium & scientiarum principia experientiæ beneficio collecta sunt: experientia verò à memoria fluxit, quæ & ipsa à sensu primùm manauit. Nam quod scribit Aristoteles, Mathematicas artes,* comparatis rebus omnibus ad vitam necessarijs, in AEgypto fuisse constitutas, quòd ibi sacerdotes omnium concessu in otio degerent: non negat, ille adductos necessitate homines ad excogitandam, verbi gratia terræ dimetiendæ rationem, quæ theorematum deinde inuestigationi causam dederit: sed hoc confirmat, præclara eiusmodi theorematum inuenta, quibus extructa Geometriæ disciplina constat, ad vsus vitæ necessarios ab illis non esse expetita. Itaque vetus ipsum Geometriæ nomen ab illa terræ partiundæ finiúmque regendorum ratione posteà recessit, & in certa quadam affectionum magnitudini per se inhærentium scientia propriè remansit. Quemadmodum igitur in mercium & contractuum gratiam supputandi ratio, quam secuta est accurata numerorum cognitio, à Phœnicibus initium duxit: ita etiam apud AEgyptios, ex ea quam commemoraui causa ortum habuit Geometria. Hanc certè, vt id obiter dicam, Thales in Græciam ex AEgypto primum transtulit: cui non pauca deinceps à Pythagora, Hipprocrate, Chio, Platone, Archyta Tarentino, alijsq́ complaribus, ad Euclidis tempora facta sunt rerum magnarum accessiones. Cæterùm de Euclidis ætate id solùm addam, quod à Proclo memoriæ mandatum accepimus. Is enim commemoratis aliquot Platonis tum æqualibus tùm discipulis, subijcit, non multò ætate posteriorem illis fuisse Euclidem eum, qui Elementa conscripsit, & multa ab Eudoxo collecta, in ordinem luculentum

com-

composuit, multaq; à Theateto inchoata perfecit, quæq; mollius ab alijs demonstrata fuerant, ad firmissimas & certissimas apodixes reuocauit. Vixit autem, inquit ille sub primo Ptolemæo. Et enim ferunt Euclidam à Ptolemæo quondam interrogatum num qua esset via ad Geometriam magis compendiaria, quàm sit ista στοιχείωσις respondisse μὴ εἶναι βασιλικὴν ἀτραπὸν ἐπὶ γεωμετρίαν. Diende subiungit, Euclidem natu quidem esse minorem Platone, maiorem verò Eratosthene & Archimede (hi enim quales erant) cùm Archimedes Euclidis mētionem faciat. Quòd si quis egregiam Euclidis laudem quam cùm ex alijs scriptionibus accuratissimis, tùm ex hac Geometria στοιχειώσει cōsequutus est in qua diuinus rerum ordo sapientissimis quibusque hominibus magnæ semper admirationi fuit, is Proclum studiosè legat, quò rei veritatem illustriorem reddat grauissimi testis auctoritas. Superest igitur vt finem videamus, quò Euclidis elementa referri, & cuius causa in id studium incumbere, oporteat. Et quidem si res quæ tractantur, consideres: in tota hac tractatione nihil aliud quæri dixeris, quàm vt κοσμικὰ quæ vocantur σχήματα (fuit enim Euclides professione & instituto Platonicus) Cubus Icosaedrum, Octaedrum, Pyramis & Dodecaedrum certa quadam suorum & inter se laterum, & ad sphæræ diametrum ratione eidem sphæræ inscripta comprehendantur. Huc enim pertinet Epigrammation illud vetus, quod in Geometrica Michaelis

Pselli συνόψει sriptum legitur.

Σχήματα πέντε τὰ πλάτωνος, πυθαγόρας σοφὸς εὗρε.

πυθαγόρας σοφὸς εὗρε, πλάτων δ' ἀριδήλ' ἐδίδαξεν.

Εὐκλείδης ἐπὶ τοῖσι κλέος περικαλλὲς ἔτευξεν.

Quòd si discentis institutionem spectes, illud certè fuerit propositum, vt huiusmodi elementorum cognitione informatus discentis animus, ad quamlibet non modò Geometriæ, sed & aliarum Mathematicæ partium tractationem idoneus paratusq; accedat. Nam tametsi institutionem hanc solus sibi Geometra vendicare videtur, & tanquam in possessionem suam venerit, alios excludere posse: inde tamen per multa suo quodam modo iure decerpit Arithmeticus, pleraque Musicus non pauca detrahit. Astrologus, Opticus, Logisticus, Machanicus, itemq; cæteri: nec vllus est denique artifex præclarus, qui in huius se possessionis societatem cupidè non offerat, partemq; sibi cōcedi postulet. Hinc στοιχείωσις absolutum operi nomen, & στοιχειωτὴς dictus Euclides. Sed quid longius prouehor? Nam quod ad hanc rem attinet, tam copiosè & eruditè scripsit (vt alia complura) eo ipso, quem dixi, loco P. Montaureus, vt nihil desiderio loci reliquerit. Quæ verò ad dicendum nobis erant proposita hactenus pro ingenij nostri tenuitate omnia mihi perfecisse videor. Nam tametsi & hæc eadem & alia pleraque multo fortè præclariora ab hominibus doctissimis, qui

tam

tùm acumine ingenij, tùm admirabili quodam lepore dicendi semper floruerunt grauiùs, splendidiùs, vberiùs tractari posse scio: tamen experiri libuit numquid etiam nobis diuino sit concessum munere, quod rudes in hac philosophiæ parte discipulos adiuuare aut certè excitare queat. Huc accessit quòd ista recens elementorum editio, in qua nihil non parum fuisset studij, aliquid à nobis efflagitare videbatur, quod eius commendationem adaugeret. Cùm enim vir doctissimus Io. Magnienus Mathematicarum artium in hac Pharrhisiorum Academia professor verè regius, nostrum hunc typographum in excudendis Mathematicorum libris diligentissimum, ad hanc Elementorum editionem sæpè & multum esset adhortatus, eiusque impulsu permulta sibi iam comparasset typographus ad hanc rem necessaria, citò interuenit, malùm Ioannis Magnieni mors insperata, quæ tam graue inflixit Academiæ vulnus, cui ne post multos quidem annorum circuitus cicatrix obduci vlla posse videatur. Quamobrem amisso instituti huius operis duce, typographus, qui nec sumptus antea factos sibi perire, nec studiosos, quibus id muneris erat pollicitus, sua spe cadere vellet, ad me venit, & impensè rogauit, vt meam propositæ editioni operam & studium nauarem, quòd cùm denegaret occupatio nostra, iuberet officij ratio, feci equidem rogatus, vt quæ subobscurè vel parum commode in sermonem Latinum è Græco translata

vi-

videbantur, clariore, aptiore & fideliore interpretatione nostra (quod cuiusq; pace dictum volo) lucem acciperent. Id quod in omnibus ferè libris posterioribus tute primo obtutu perspicias. Nam in sex prioribus non tantum temporis quantum in cæteres ponere nobis licuit: decimi autem interpretatio, qua melior nulla potuit adfert, P. Montaureo solida debetur. Atq; vt ad perspicuitatem facilitatemq; nihil tibi deesse queraris, adscripta sunt propositionibus singulis vel lineares figuræ, vel punctorum tanquam vnitatum notulæ, quæ Theonis apodixin illustrent illæ quidem magnitudinum, hæ autem numerorum indices, subscriptis etiam ciphrarum, vt vocant, characteribus, qui propositum quemuis numerum exprimant, ob eamq; causam eiusmodi vnitatum notulæ, quæ pro numeri amplitudine maius paginæ spatium occuparent, pauciores sæpius depictæ sunt aut in lineas etiam commutatæ. Nam literarum vt a, b, c characteres non modò numeris & numerorum partibus nominandis sunt accommodati, sed etiam generalas esse numerorum, vt magnitudinum affectiones testantur. Adiecta sunt insuper quibusdam locis non pœnitenda Theonis scholia, siue mauis lemmata, quæ quidem longè plura accessissent, si plus otij & temporis vacui nobis fuisset relictum, quod huic studio impartiremus. Hanc igitur operam boni consule, & quæ obuia erunt impreßionis vitia, candidè emenda. Vale. Lutetiæ 4. Idus April. 1557.

EVCLIDIS ELEMENTVM PRIMVM

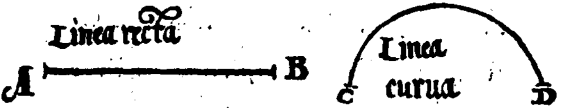
DEFINITIONES.

1

Punctum est, cuius pars nulla est. Punctum

2

Linea verò, longitudo latitudinis expers.



3

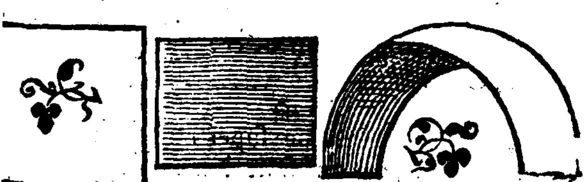
Lineæ autem termini, sunt puncta.

4

Recta linea est, quæ ex æquo sua interiacet puncta.

5

Superficies est, quæ longitudinem latitudinemque tantùm habet.



6 Super.

6

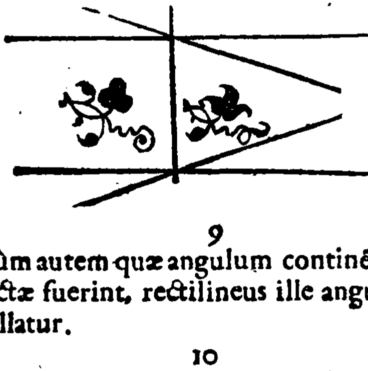
Superficiei extrema. ſunt lineæ.

7

Plana ſuperficies eſt, quæ ex æquo ſuas interiacet lineas.

8

Planus angulus eſt duarũ linearũ in plano ſe mutuò tangentium, & non in directum



iacẽtium alterius ad alterã inclinatio.

9

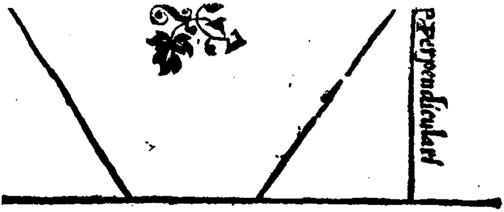
Cùm autem quæ angulum continẽt lineæ, rectæ fuerint, rectilineus ille angulus appellatur.

10

Cùm verò recta linea ſuper rectam conſiſtens lineã, eos qui ſunt deinceps angulos ęquales, inter ſe fecerit: rectus eſt vterq; æ-

qua-

qualium angulorum : quæ inſiſtit recta linea, perpẽdicularis vocatur ei', cui inſiſtit.



11

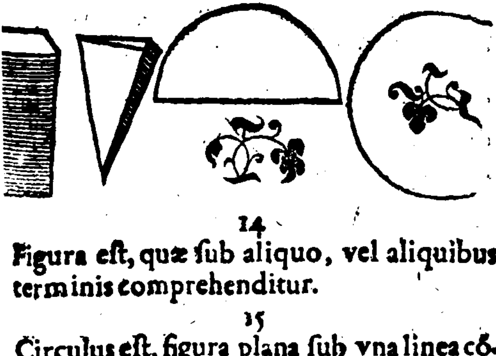
Obtuſus angulus eſt, qui recto maior eſt.

12

Acutus verò, qui minor eſt recto.

13

Terminus eſt, quod alicuius extremũ eſt.



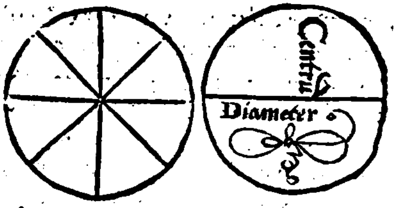
14

Figura eſt, quæ ſub aliquo, vel aliquibus terminis comprehenditur.

15

Circulus eſt, figura plana ſub vna linea cõprehẽſa, quę peripheria appliatur: ad quam ab vno puncto eorum. quæ intra figuram ſunt

ſunt poſita, cadentes omnes rectæ lineæ inter ſe ſunt æquales.



16

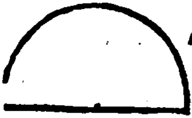
Hoc verò punctum, centrum circuli appellatur.

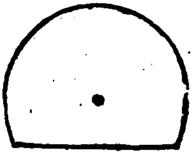
17

Diameter autem circuli, eſt recta quædam linea per centrum ducta, & ex vtraque parte in circuli peripheriam terminata, quæ circulum bifariam ſecat.

18

Semicirculus eſt figura, quæ continetur ſub diametro, & ſub ea linea, quæ de circuli peripheria aufertur.





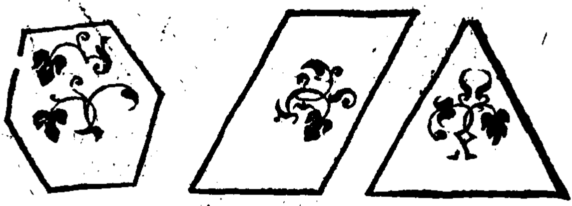
19

Segmentum circuli, eſt figura, quæ ſub recta linea & circuli peripheria continentur.

20 Recti

20.

Rectilineæ figuræ, ſunt quæ ſub rectis lineis continentur.



21

Trilateræ quidem, quæ ſub tribus.

22

Quadrilateræ, quæ ſub quatuor.

23

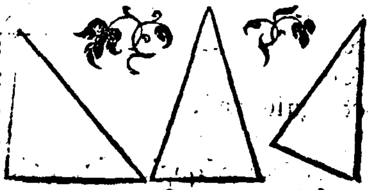
Multilateræ verò, quæ ſub pluribus quàm quatuor rectis lineis comprehenduntur.

24

Trilaterarũ, porrò figurarum, æquilaterum eſt triangulum, quod tria latera habet æqualia.

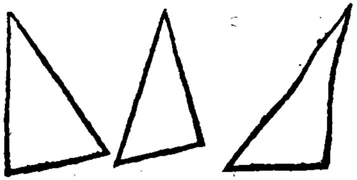
25

Iſoſceles autem eſt quod duo tantum æqualia habet latera.



26

Scalenũ verò, eſt qđ tria inæqualia habet latera.



23

Ad hæc etiam trilaterarum figurarum, rectangulum quidem triangulum eſt, quod rectum angulum habet.

28

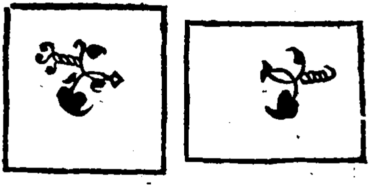
Amblygonium autem, quod obtuſum angulum habet.

29

Oxygonium verò, quod tres habet acutos angulos.

30

Quadrilaterarum autem figurarum, quadratũ quidẽ eſt qđ & æquilaterũ & rectangulum eſt.

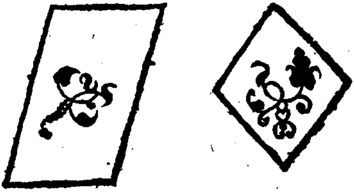


31

Altera parte longior figura eſt, quæ rectãgula quidem, at æquilatera non eſt.

32 Rhom-

32
Rhōbus autem, qui æqui laterum & rectangulum eſt.



33
Rhomboides verò, quæ aduerſa & latera & angulos habens inter ſe æqualia, neque æquilatera eſt, neque rectangula.

34
Preter has autē reliquæ quadrilateræ figuræ, trapezia appellentur.

35
Parallelæ rectæ lineæ ſunt quæ, cũm in eodem ſint plano, & ex vtraque parte in infinitum producantur, in neutram ſibi mutuò incidunt.

Poſtulata.

I
Poſtuletur, vt à quouis puncto in quoduis

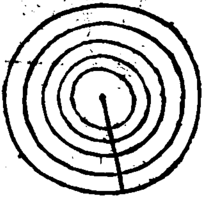
punctum, rectam lineã ducere cõcedatur.

2

Et rectam lineam terminatam in continuum recta producere.

3

In quouis centro & interuallo circulũ describere.



Communes notiones.

1.

Quæ eidẽ æqualia, & inter se sunt æqualia.

2

Et si æqualibus æqualia adiecta sint, tota sunt æqualia.

3

Et si ab æqualibus æqualia ablata sint, quæ relinquuntur sunt æqualia.

4

Et si inæqualibus æqualia adiecta sint, tota sunt inæqualia.

5

Et si ab inæqualibus æqualia ablata sint, reliqua sunt inæqualia.

6

Quæ eiusdem duplicia sunt, inter se sunt æqualia.

Et

7

Et quæ eiuſdem ſunt dimidia, inter ſe æqualia ſunt.

8

Et quæ ſibi mutuò congruunt, ea inter ſe ſuntæqualia.

9

Totum eſt ſua parte maius.

10

Item, omnes recti anguli ſunt inter ſe ęquales.

11

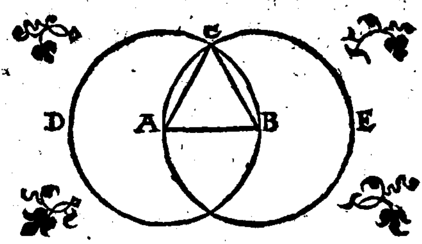
Et ſi in duas rectas lineas altera recta incidens, internos ad eaſdémque partes angulos, duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ ſibi mutuò incident ad eas partes, vbi ſunt anguli duobus rectis minores.

12

Duæ rectæ lineæ ſpatium non comprehendunt.

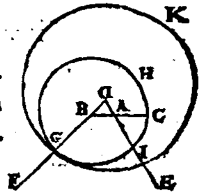
Problema 1. Propoſitio 1.

Super data recta linea termi nata, triangulũ



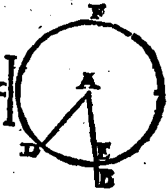
æquilaterum conſtituere.

Problema 2. Propositio 2.



Ad datum punctum, datæ rectæ lineæ, ęqualem rectam lineam ponere.

Problema 3. Propositio 3.



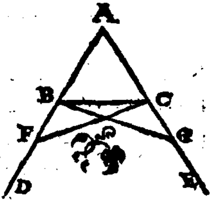
Duabus datis rectis lineis inęqualibus, de maiore æqualē minori rectam lineā detrahere.

Theorema primum. Propositio 4.

Si duo triangula duo latera duobus lateribus æqualia habeant, vtrunque vtriq́;, habeant verò & angulum angulo æqualē sub æqualibus rectis lineis contentum: & basin basi æqualem habebunt, eritq́; triāgulum triāgulo æquale, ac reliqui anguli reliquis angulis æquales erūt, vterque vtrique, sub quibus æqualia latera subtenduntur,

Theore-

Theorema 2. Propositio 5.



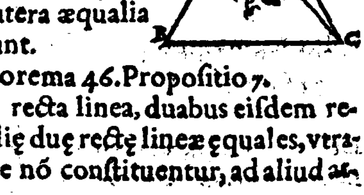
Isoscelium tringulorũ qui ab basim sunt anguli, inter se sunt æquales; & si vlterius productæ sint æquales illę rectæ lineæ, qui sub basi sunt anguli, inter se equales erunt.

Problema 3. Propositio 6.

Si trianguli duo anguli æquales inter se fuerint & sub ęqualibus angulis subtensa latera æqualia inter se erunt.

Theorema 46. Propositio 7.

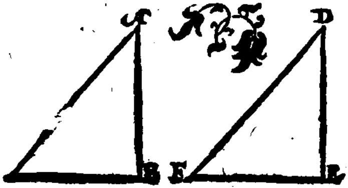
Super eadẽ recta linea, duabus eisdem rectis lineis aliç duç rectę lineæ ęquales, vtraque vtrique nõ constituentur, ad aliud atque aliud punctũ, ad easdẽ partes eosdẽ

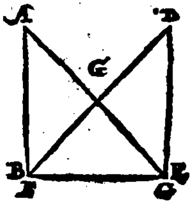


que terminos cum duabus initio ductis rectis lineis habentes.

Theorema 5. Propositio 8.

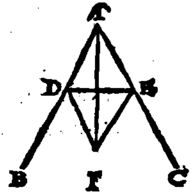
Si duo triãgula duo latera habuerint duobus lateribus, vtrunque vtriq; æqualia: habuerint verò & basim basi æqualẽ: angulũ quoque sub æqualibus rectis lineis cõtentum angulo æqualem habebunt.





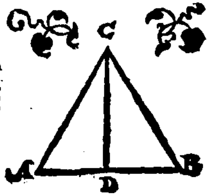
Problema 4. Propositio 9.

Datum angulum rectilineum bifariam secare.



Problema 5. Propositio 10.

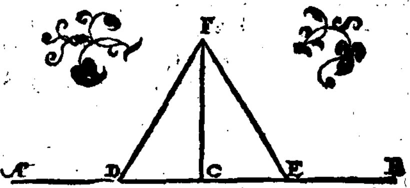
Datam rectam lineã finitam bifariam secare.



Proble-

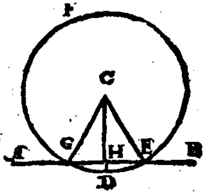
Problema 6. Propositio 11.

Data recta linea, à puncto in ea dato, re-



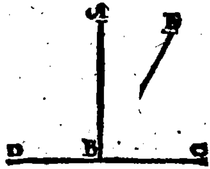
ctam lineam ad angulos rectos excitare.

Problema 7. Propositio 12.



Super datam rectã lineã infinitam, à dato puncto quod in ea non est, perpendicularem rectam deducere.

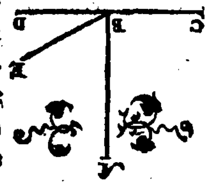
Theorema 6. Propositio 13.



Cùm recta linea super rectã consistens lineã angulos facit, aut duos rectos, aut duobus rectis æquales efficiet.

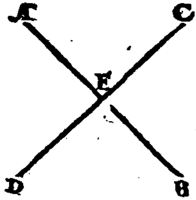
Theorema 7. Propositio 14.

Si ad aliquam rectam lineam, atque ad eius punctũ, duę rectęlineæ nó ad eaſdem partes ductæ, eos qui ſunt deinceps angulos duobus rectis æquales fecerint, in directum erunt inter ſe ipſæ rectæ lineæ.



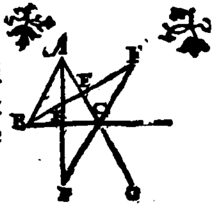
Theorema 8. Propoſitio 15.

Si duę rectę lineę ſe mutuò ſecuerint, angulos qui ad verticem ſunt, æquales inter ſe efficient.



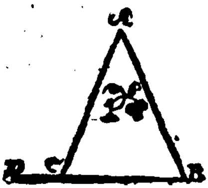
Theorema 9. Propoſitio 16.

Cuiuſcunque trianguli vno latere producto, externus angulus vtroque interno & oppoſito maior eſt.



Theorema 10. Propoſitio 17.

Cuiuſcunque trianguli duo auguli duobus rectis ſunt minores omnifariam ſumpti.



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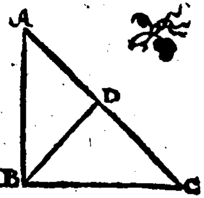
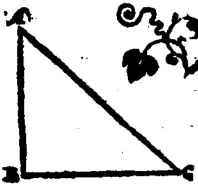
Theorema 11. Propositio 18.



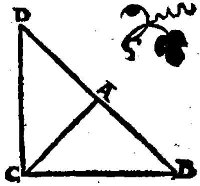
Omnis trianguli maius latus maiorem angulũ subtendit.

Theorema 12. Propositio 16.



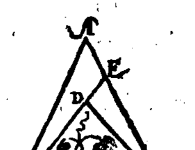
Omnis trianguli lateri angulus, maiori lateri subtenditur.

Theorema 13. Propositio 20.



Omnis triãguli duo latera reliquo sunt maiora, quomodocũmq; assumpta.

Theorema 14. Propositio 12.

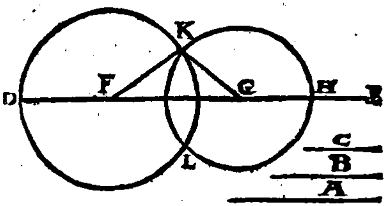


Si super triãguli vno latere, ab extremitatibus duæ rectę lineę, interius cõstitutæ fuerint, hę cõstitutæ reliquis triãguli duobus lateribus minores quidem erunt, minorem verò angulum continebunt.

Pro-

Problema 8. Propoſitio 22.

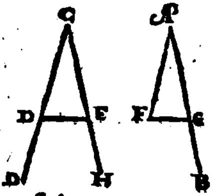
Ex tribus rectis line is quæ sũt tribus datis rectis ſineis æquales,



triangulũ conſtituere. Oportet autem duas reliqua eſſe maiores omnifariám ſumptas: quoniá vniuſcuiuſq; trianguli duo latera omnifariam ſumpta reliquo ſunt maiora.

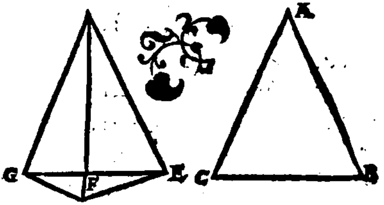
Problema 9. Propoſitio 23.

Ad datam rectam lineã datumq; in ea punctum dato angulo rectilineo ęqualem angulum rectilineum conſtituere.



Theorema 15. Propoſitio 24.

Si duo triãgula duo latera duobus lateribus ęqualia

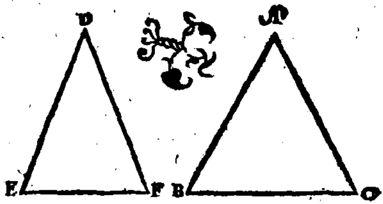


habuerint, vtrũq; vtriq;, angulũ verò angu lo

lo maiorem sub æqualibus rectis lineis cō-
tentum: & basin basi maiorem habebunt.

Theorema 16. Propositio 25.

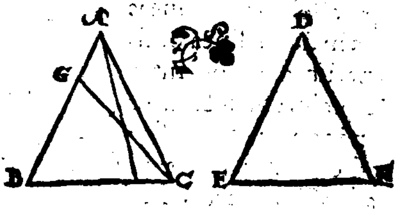
Si duo triangula duo latera duobus lateri-
bus æqualia habuerint, vtrunque vtrique,
basin ve-
rò basi
maiorē:
& angu-
lum sub
æquali-
b' rectis



lineis contentum angulo maiorē habebūt.

Theorema 17. Propositio 26.

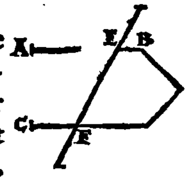
Si duo triangla duos angulos duobus an-
gulis æquales habuerint, vtrūque vtrique,
vnumq; latus vni lateri æquale, siue, quod
æqualibus adiacet angulis, seu quod vni æ-
qualium angulorū subtenditur: & reliqua
latera
reli-
quis la
terib'
ęqua-
lia
vtrūq;
vtriq;,



& reliquum angulū reliquo angulo æqua-
lem habebunt.

Theore-

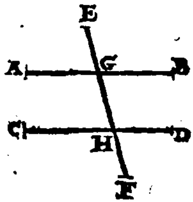
Theorema 18. Propoſitio 27.



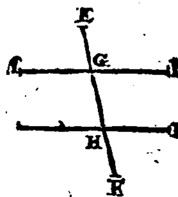
Si in duas rectas lineas recta incidēs linea alternatim āgulos ęquales inter ſe fecerit: parallelę erunt inter ſe illæ rectæ lineæ.

Theorema 19. Propoſitio 28.

Si in duas rectas lineas recta incidens linea externum angulū interno, & oppoſito, & ad eaſdem partes æqualē fecerit, aut internos & ad eaſdē partes duobus rectis æquales: parellelæ erunt inter ſe ipſæ rectæ lineæ.



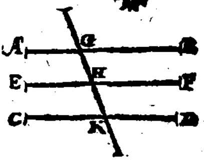
Theorema 20. Propoſitio 29.



In parallelas rectas lineas recta incidens linea: & alternatim angulos inter ſe æquales efficit & externum interno & oppoſito & ad eaſdem partes æqualem, & inter nos & ad eaſdem partes duobus rectis æquales facit.

Theore-

Theorema 21. Propositio 30.

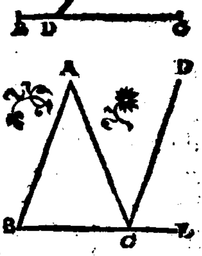


Quæ eidem rectæ lineę, parallelę, & inter se sunt parallelæ.

Problema 10. Propositio 31.

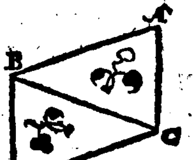
A dato puncto datæ rectæ lineæ parallelam rectam lineam ducere.

Theoremà 22. Propositio. 32.



Cuiuscunque trianguli vno latere vlterius producto: externus angulus duobus internis & oppositis est æqualis. Et trianguli tres interni anguli duobus sunt rectis æquales.

Theorema 23. Propositio 33.

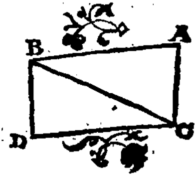


Rectæ lineę quę ęquales & parallelas lineas ad partes easdem coniungunt, & ipsæ æquales & parallelæ sunt.

Theore-

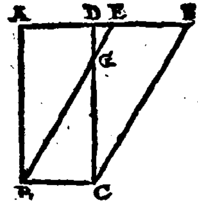
Theorema 24. Propoſitio 34.

Parallelogrammorū ſpatiorum æqualia ſunt inter ſe, quæ ex aduerſo & latera & anguli: atque illa bifariā ſecat diameter.



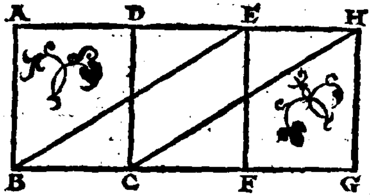
Theorema 25. Propoſitio 35.

Parallelogramma ſuper eadem baſi, & in eiſdem parallelis cōſtituta, inter ſe ſunt æqualia.



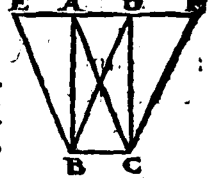
Theorema 26. Propoſitio 36.

Parallelogrāma ſuper æqualibus baſibus & in eiſdē parallelis cōſtituta inter ſe ſunt æqualia.



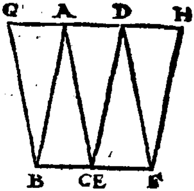
Theorema 27. Propoſitio 37.

Triāgula ſuper eadē baſi cōſtituta, & in eiſdē parallelis, inter ſe ſunt ęqualia.



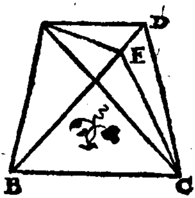
Theore-

Theorema 28. Propositio 38.



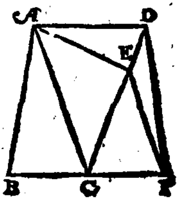
Triangula super equalibus basibus constituta et in eisdem parallelis, inter se sunt æqualia.

Theorema 29. Propositio 39.



Triangula æqualia super eadem basi, & ad easdem partes constituta: & in eisdem sunt Parallelis.

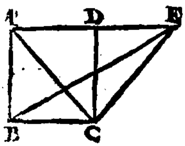
Theorema 30. Propositio 40.



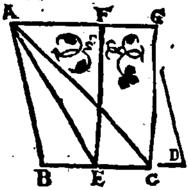
Triangula æqualia super æqualibus basibus, & ad easdem partes constituta, & in eisdem sunt parallelis.

Theorema 31. Propositio 41.

Si parallelogrammum cum triangulo eandem basin habueris, non eisdemque fuerit parallelis, duplum erit parallelogrammum ipsius trianguli.

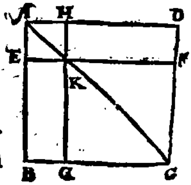


Problema 11. Propositio 42.



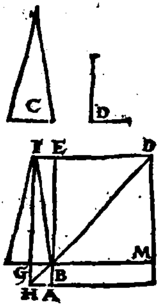
Dato triangulo æquale parallelogrammũ constiture in dato angulo rectilineo.

Theorema 32. Propositio 43.



In omni parallelogrammo, complementa eorũ quę circa diametrũ sunt parallelogrammorũ, inter se sunt æqualia.

Problema 12. Propositio 44.

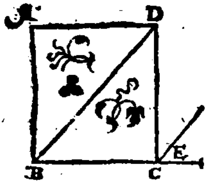


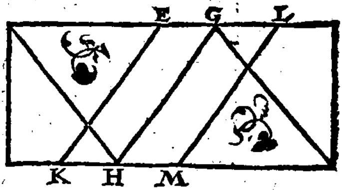
Ad datam rectam lineã dato triangolo æquale parallelogrammum applicare in dato angulo rectilineo.

Problema 13. Propopotio 45.

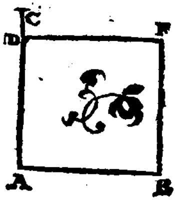
Dato rectilineo ęquale parallelongrammũ consti-

conſtituere in dato angulo rectilineo.





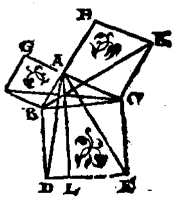
Theorema 41. Propoſitio 4.



A data recta linea quadratum deſcribere.

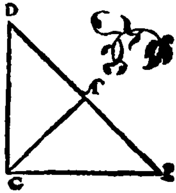
Theorema 33. Propoſitio 47.

In rectangulis triangulis, quadratum quod à latere rectum angulum ſubtendente deſcribitur, æquale eſt eis, quæ à lateribus rectum angulum continentibus deſcribũtur, quadratis.



Theorema 34. Propoſitio 48.

Si quadratum quod ab vno laterum trianguli deſcribitur, æquale ſit eis quæ à reliquis trianguli lateribus deſcribuntur, quadratis: angulus comprehenſus ſub reliquis duobus trianguli lateribus, rectus eſt.



FINIS ELEMENTI I.

EVCLIDIS ELEMENTVM SECVNDVM.

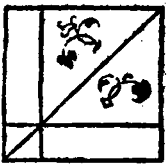
DEFINITIONES.

1.

OMne parallelogrammum rectangulum contineri dicitur sub rectis duabus lineis, quę rectum comprehendunt angulum.

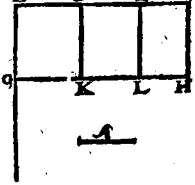
2.

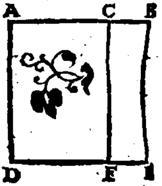
In omni parallelogrammo spatio, vnumquodlibet eorum, quæ circa diametrum illius sunt parallelogrammorum, cum duobus cõplementis, Gnomo vocetur.



Theorema 1. Propositio *1.*

Si fuerint duæ rectæ lineæ, seceturque ipsarum altera in quotcunq; segmenta: rectangulum comprehensum sub illis duabus rectis lineis, æquale est eis rectangulis, quæ sub insecta & quolibet segmẽtorum comprehenduntur.

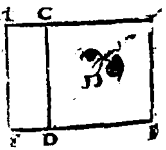




Theorema 2. Propoſitio 2.

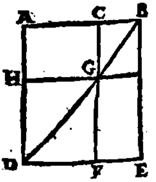
Si recta linea ſecta ſit vtcunq;, rectãgula quæ ſub tota & quolibet ſegmentorum cõprehenduntur æqualia ſunt ei, quod à tota fit, quadrato.

Theorema 3. Propoſitio 3.



Si recta linea ſecta ſit vtcunque, rectangulum ſub tota & vno ſegmẽtorum comprehenſum, ęquale eſt & illi quod ſub ſegmentis cõprehenditur rectangulo & illi, quod à prædicto ſegmẽto deſcribitur, quadrato.

Theorema 4. Propoſitio 4.



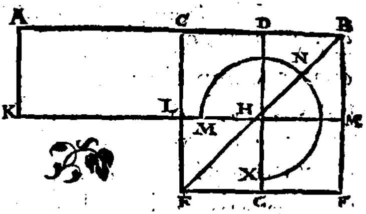
Si recta linea ſecta ſit vtcunque: quadratũ quod à tota deſcribitur, ęquale eſt & illis quæ à ſegmẽtis deſcribũtur quadratis, & ei quod bis ſub ſegmentis comprehẽditur rectangulo.

Theorema 5. Propoſitio 5.

Si recta lenea ſecetur in æqualia & non æqualia: rectangulum ſub inæqualibus ſeg-

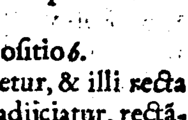
mentis

mentis totius comprehésũ, vnà cum quadrato quod ab inter media sectionum, æquale est ei quod à dimidia describitur, quadrato.



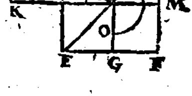
Theorema 6. Propositio 6.

Si recta linea bifariam secetur, & illi recta quædam linea in rectum adijciatur, rectãgulum comprehensum sub tota cum adiecta & adiecta simul cum quadrato à dimidia, æquale est quadrato à linea quæ tũ ex dimidia, tum ex adiecta componitur, tanquam ab vna descripto.



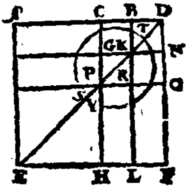
Theorema 7. Propositio 7.

Si recta linea secetur vt cunque, quod à tota, quodq́ue ab vno segmentorum, vtraq; simul quadrata, æqualia sunt & illi quod bis sub tota & dicto segemento comprehenditur, rectangulo, & illi ꝙ à reliquo segmento fit, quadrato.



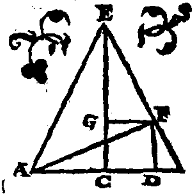
Theorema 8. Propoſitio 8.

Si recta linea ſecetur vtcunq; rectangulum quater comprehenſum ſub tota & vno ſegmentorum, cum eo quod à reliquo ſegméto fit, quadrato, ęquale eſt ei quod à tota & dicto ſegmento, tanquá ab vna linea deſcribitur quadrato.



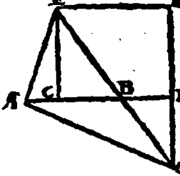
Theorema 9. Propoſitio 9.

Si recta linea ſecetur in æqualia & non æqualia: quadrata quæ ab inęqualibus totius ſegmẽtis fiunt, duplicia ſunt & eius quod à dimidia, & eius quod ab intermedia ſectionum fit, quadratorum.



Theorema 10. Propoſitio 10.

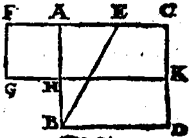
Si recta linea ſecetur bi fariam, adijciatur autem ei in rectum quępiam recta linea: quod à tota cũ adiuncta, & quod ab adiuncta, vtraq; ſimul quadrata, duplicia ſunt, & eius quod à dimidia, & eius quod à compoſita ex dimidia & adiuncta.



iuncta, tanquã ab vna descriptum sit quadratorum.

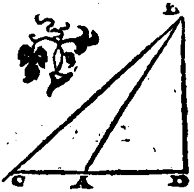
Problema 1. Propositio 11.

Datam rectam lineã secare, vt comprehensum sub tota & altero segmẽtorum rectangulum, æquale sit ei quod à reliquo segmento fit, quadrato.



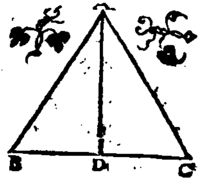
Theorema 11. Propositio 12.

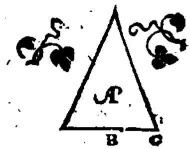
In amblygonijs triangulis, quadratũ quod fit à latere angulum obtusum subtendẽte, maius est quadratis, quæ fiunt à lateribus obtusum angulum comprehendentibus, pro quantitate rectanguli bis comprehensi & ab vno laterũ quę sunt circa obtusum angulũ, in quod cum protractum fuerit, cadit perpendicularis, & ab assumpta exterius linea sub perpendiculari prope angulũ obtusum.



Theorema 12. Propositio 28.

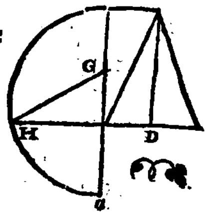
In oxygonijs triangulis quadratũ à latere angulum acutum ſubtendente, minus eſt quadratis quæ fiunt à lateribus acutũ angulum cõprehendentibus, pro quantitate rectanguli bis comprehenſi, & ab vno late rum, quæ ſunt circa acu tum angulum, in quod perpẽdicularis cadit, & ab aſſumpta interius linea ſub perpendiculari prope acutũ angulum.





Problema 2. Propoſitio 14.

Dato rectilineo æquale quadratũ conſtituere.



ELEMENTI II. FINIS.

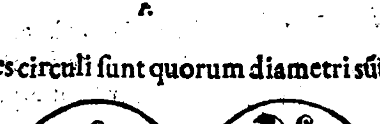
EVCLI.

EVCLIDIS ELEMENTVM TERTIVM.

DEFINITIONES.

1.

Aequales circuli sunt quorum diametri sūt ęquales vel quorū quæ ex centris, rectæ lineę sunt æquales.



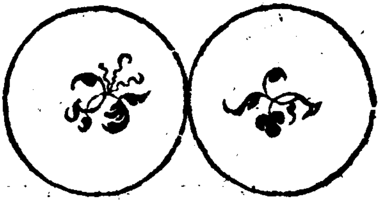
2.

Recta linea circulū tangere dicitur, quæ cùm circulun tangat, si producatur, circulum non secat.

3 Cir-

3

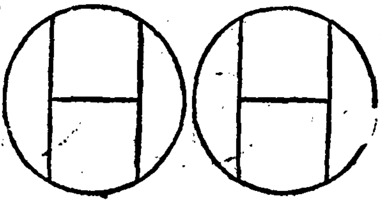
Circuli ſeſe mutuò tangere dicuntur: qui ſeſe mutuo



tangentes, ſeſe mutuo non ſecant.

4

In circulo æqualiter diſtare à centro rectæ lineæ dicuntur, cùm perpendiculares, quæ à cẽtro in ipſas ducuntur, ſunt æquales. Lõgius autem abeſſe illa dicitur in quã maior perpendicularis cadit.



5

Segmentum circuli eſt, figura quæ ſub recta linea & circuli peripheria comprehenditur.

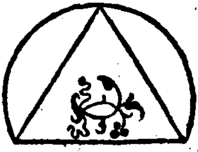
6

Segmenti autem angulus eſt, qui ſub recta linea

linea & circuli peripheria comprehẽditur

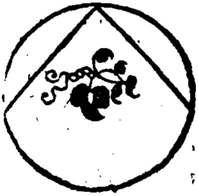
7

In ſegmento autem angulus eſt, cum in ſegmenti peripheria ſumptum fuerit quodpiam punctum, & ab illo in terminos rectæ eius lineæ, quæ ſegmenti baſis eſt, adiũctę fuerint rectæ lineæ: is, inquam, angulus ab adiunctis illis lineis comprehẽſus.



8

Cùm verò comprehendentes angulum rectæ lineæ aliquam aſſumitur peripheriam, illi angulus inſiſtere dicitur.



9

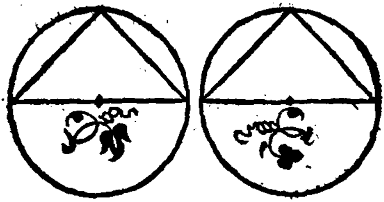
Sector autem circuli eſt, cùm ad ipſius circuli cẽtrum conſtitutus fuerit angulus, comprehenſa nimirum figura, & à rectis lineis angulum continentibus, & à peripheria ab illis aſſumpta.

10

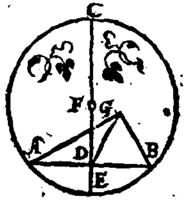
Similia circuli ſegmenta ſunt, quæ angulos

capiunt

capiunt ęquales aut in q. b'anguli inter ſe ſunt ęquales

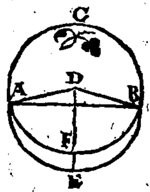


Problema 1. Propoſitio 1.



Dati circuli centrum reperire.

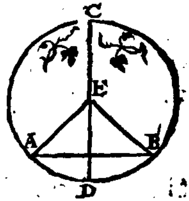
Theorema 1. Propoſitio 2.



Si in circuli peripheria duo quælibet puncta accepta fuerint, recta linea quæ ad ipſa puncta adiungitur, intra circulum cadet.

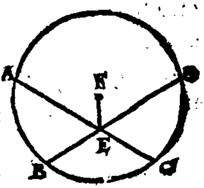
Theorema 2. Propoſitio 3.

Si in circulo recta quædam lenea per centrum extenſa quandam non per centrum extenſam bifariam ſecet: & ad angulos rectos ipſam ſecabit. Et ſi ad angulos rectos eam ſecet, bifariam quoque eam ſecabit.



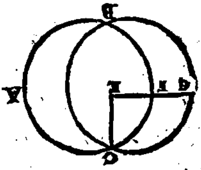
Theore-

Theorema 3. Propositio 4.



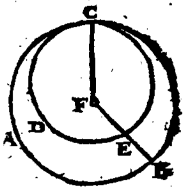
Si in circulo duę rectę lineæ sese mutuo secent non per centrum extẽsæ sese mutuò bifariam non secabunt.

Theorema 4. Propositio 5.



Si duo circuli sese mutuò secent, non erit illorum idem centrum.

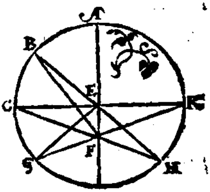
Theorema 5. Propositio 6.



Si duo circuli sese mutuò interius tangant, eorum non erit idem centrum.

Theorema 6. Propositio 7.

Si in diametro circuli quodpiam sumatur punctum, quod circuli centrum non sit, ab eoque puncto in circulũ quædam rectæ lineæ cadant: maxima quidem erit ea in qua centrũ, minima verò reliqua: aliarum verò propinquior illi quę per centrum du-

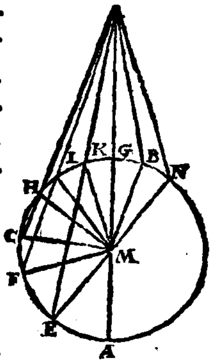


citur

citur, remotiore ſemper maior eſt. Duæ autem ſolùm rectę lineę æquales ab eodem puncto in circulum cadunt ad vtraſq; partes minimæ.

Theorema 7. Propoſitio 8.

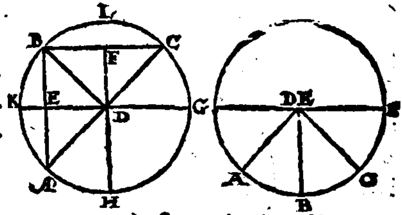
Si extra circulum ſumatur punctum quodpiam, ab eoque puncto ad circulum deducantur rectæ quædam lineæ, quarum vna quidem per centrum protendatur, reliquę verò vt libet: in cauam peripheriam cadentium rectarum linearum minima quidem eſt illa, quæ per centrum ducitur: aliarum autem propinquior ei, quæ per centrum tranſit, remotiore ſemper maior eſt: in conuexam verò peripheriã cadentium rectarũ linearum minima quidem eſt illa, quæ inter punctum & diametrum interponitur: aliarum autem, ea quę propinquior eſt minimæ, remotiore ſemper minor eſt. Duæ autem tantùm rectæ lineæ æquales ab eo puncto in ipſum circulum cadunt, ad vtraſque partes minimæ.



Theo-

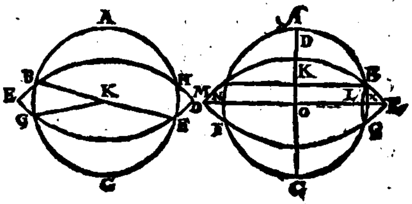
Theorema 8. Propositio 9.

Si in circulo acceptum fuerit pũctum aliquod, & ab eo puncto ad circulum cadant plures quam duę rectæ lineæ, æquales, acceptũ punctum centrum ipsius erit circuli.



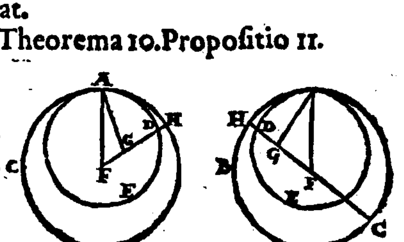
Theorema 9. Propositio 10.

Circũlus circulum in pluribus quàm duob' punctis non secat.



Theorema 10. Propositio 11.

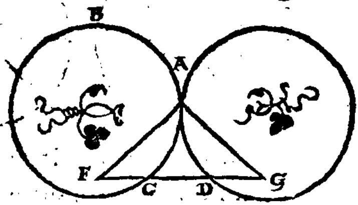
Si duo circuli sese intus cõtingãt, atque accepta



fuerint eorum centra, ad eorum cētra adiuncta recta linea & producta ni cōtactum circulorum cadet.

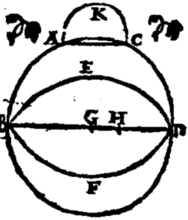
Theorema 11. Propoſitio 12.

Si duo circuli ſeſe exterius contingāt, linea recta q̃ ad cētra eorū adiūgitur, p contactū iſtū trāſibit.



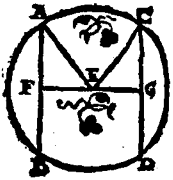
Theorema 12. Propoſitio 13.

Circulum circulum non tangit in pluribus punctis, quam vno, ſiue intus ſiue extra tangat.



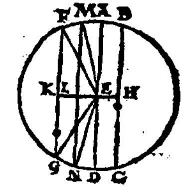
Theorema 13. Propoſitio 23.

In circulo æquales rectæ lineæ æqualiter diſtant à centro. Et quæ ęqualiter diſtant à centro, quales ſunt inter ſe.



Theore-

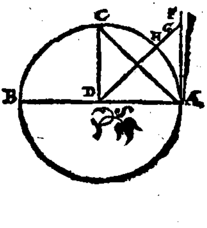
Theorema 14. Propositio 15.



In circulo maxima, quidem linea est diameter: aliarum autem propinquior centro, remotiore semper maior.

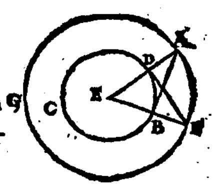
Theorema 51. Propositio 16.

Quæ ab extremitate diametri cuiusq; circuli ad angulos rectos ducitur, extra ipsũ circulum cadet, & in locum inter ipsam rectam lineam & peripheriam comprehensum, altera recta linea nõ cadet Et semicirculi quidem angulus quouis augulo acuto rectilineo maior est, reliquus autem minor,

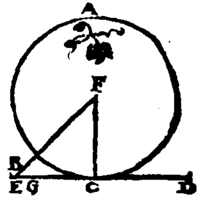


Problema 2. Propositio 17.

A dato puncto rectam lineam ducere, quæ datum tangat circulum.

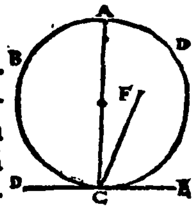


Theorema 16. Propositio 18.



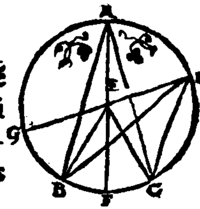
Si circulum tangat recta quæpiam linea, à centro autem ad cótactum adiungatur recta quædam linea: quę adiūcta fuerit ad ipsam contingentem perpendicularis erit.

Theorema 17. Propositio 19.



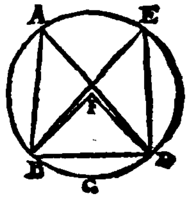
Si circulum tetigerit recta quæpiam linea, à contactu autem recta linea ad angulos rectos ipsi tágenti excitetur, in excitata erit centrum circuli.

Theorema 18. Propositio 20.



In circulo angulus ad cétrum duplex est anguli ad peripheriam, cū fuerit eadé peripheria basis angulorum.

Theorema 19. Propositio 21.



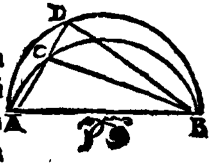
In circulo, qui in eodem segméto sunt anguli, sunt inter se æquales.

Theore-

Theorema 20. Propositio 22.

Quadrilaterorum in circulis descriptorũ anguli qui ex aduerso, duob⁹ rectis sunt æquales.

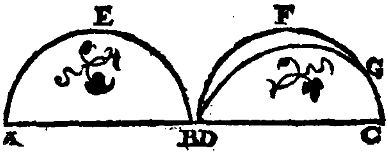
Theorema 21. Propositio 23.



Super eadem recta linea duo segmenta circulorũ similia & inæqualia non constituẽtur ad easdem partes.

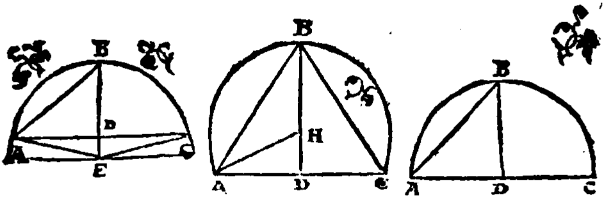
Theorema 22. Propositio 24.

Super ęqualib⁹ rectis lineis similia circulorum segmenta sunt inter se æqualia.



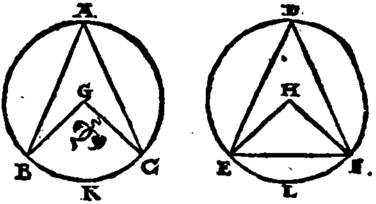
Problema 23. Propositio 25.

Circuli segmento dato, describere circulũ cuius est segmentum.



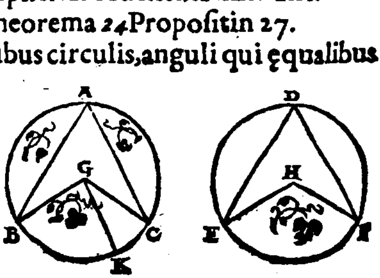
Theorema 22. Propositio 26.

In æqualibus circulis, æquales anguli ęqualib. peripherijs insistūt siue ad cētra, siue ad peripherias constituti insistant.



Theorema 24 Propositin 27.

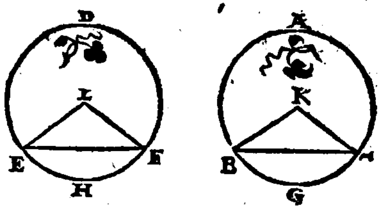
In æqualibus circulis, anguli qui ęqualibus peripherijs insistūt sunt inter se ęquales siue ad cētra, siue ad peripherias cōstituti insistāt



Theore-

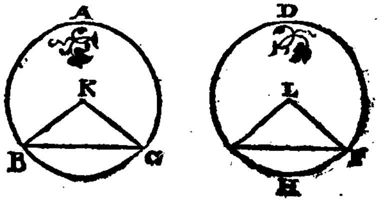
Theorema 25. Propositio 28.

In æqualibus circulis æquales rectæ lineæ equales peri- pherias. auferũt maiorẽ quidẽ maiori, minorem autem minori.



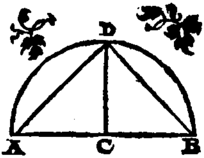
Theorema 26. Propositio 29.

In equa lib⁹ cir- culis, æ- quales periphé rias æ- quales rectæ lineæ subtendunt.



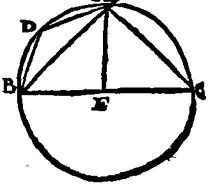
Problema 4. Propositio 30.

Datam peripheriam bifariam secare.



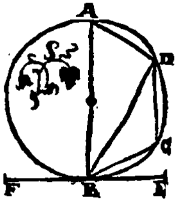
Theorema 27. Propositio 13.

In circulo angulus qui in semicirculo, re ctus est: qui autem in maoire segmẽto, minor recto: qui verò in minore segmẽto, maior est recto. Et insuper angulus maioris segmenti, recto quidem maior est, minoris autẽ segmẽti angulus, minor est recto.



Theorema 28. Propositio 32.

Si circulum tetigerit aliqua recta linea, à contactu autẽ producatur quædam recta linea circulum secans: anguli quos ad contingentem facit ęquales sunt ijs qui in alternis circuli segmẽtis consistunt, angulis.

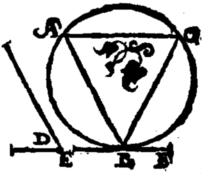


Problema 5. Propositio 33.

Super data recta linea describere segmentum circuli quod capiat angulum æqualẽ dato angulo rectilineo.

Proble-

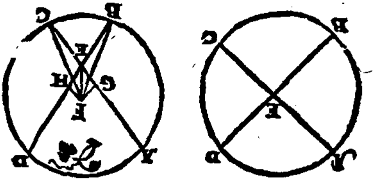
Problema 6. Propoſitio. 34.



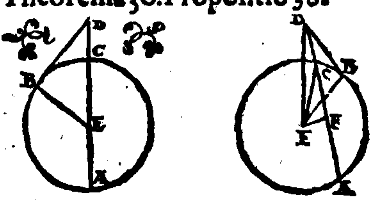
A dato circulo ſegmentum abſcindere capiens angulum æqualem dato angulo rectilineo.

Theorema 29. Propoſitio 35.

Si in circulo duæ rectæ lineæ ſeſe mutuò ſecuerint, rectangulum comprehēſum ſub ſegmētis vniꝰ, æquale eſt ei, quod ſub ſegmentis alterius comprehenditur, rectangulo.



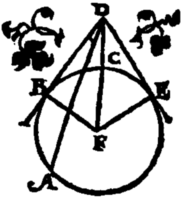
Theorema 30. Propoſitio 36.



Si extra circulũ ſumatur pũctũ aliquod, ab eoq́ue in circulũ cadant duę rectę lineæ, quarũ altera quidem circulum ſecet, altera verò tangat: quod sub tota secante & exterius inter punctum & conuexam peripheriam assumpta comprehenditur rectangulum, æquale erit ei, quod à tangente describitur, quadrato.

Theorema 31. Propositio 37.

Si extra circulũ sumatur punctun aliquod, ab eoq; puncto in circulum cadant duæ rectæ lineæ, quarum altera circulum secet, altera in eum incidat, sit autẽ quod sub tota secante & exterius inter punctum & conuexã peripheriam assumpta, comprehenditur rectangulum, æquale ei, quod quadrato: incidens ipsa circulum tanget.



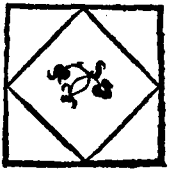
ELEMENTI II. FINIS.

EVCLIDIS ELEMENTVM QVARTVM.

DEFINITIONES.

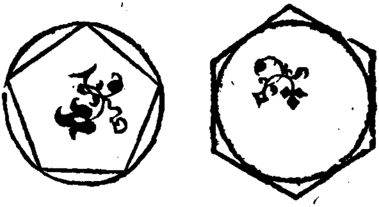
1.

FIgura rectilinea in figura rectilinea in- ſcribi dicitur, cũ ſinguli ius figura quæ inſcri- ɪitur, anguli ſingula la- era eius, in qua inſcri- ɪitur, tangunt.



2

imiliter & figura circum figurã deſcribi licitur, quum ſingula eius quæ circũſcri- ɪitur, la era ſin ;ulos e- ' figu- ę angu os teti- ;erint,



:ircum quam illa deſcribitur.

3

Figura rectilinea in circulo inſcribi dici- ur, quũ ſinguli eius figuræ quę inſcribitur, angu-

anguli tetigerint circuli peripheriam.

4

Figura verò rectilinea circa circulũ descri-bi dicitur, quũ singula latera eius, quæ circum scribitur, circuli peripheriã tangunt.

5

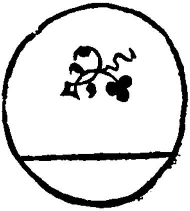
Similiter & circulus in figura rectilinea inscribi dicitur, quum circuli peripheria singula latera tãgit eius figurę, cui inscribitur.

6

Circulus autem circum figurã describi dicitur, quũ circuli peripheria singulos tãgit eius figuræ, quam circunscribit, angulos.

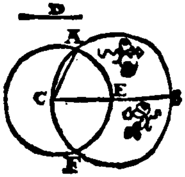
7

Recta linea in circulo accommodari seu coaptari dicitur, quum eius extrema in circuli peripheria fuerint.



Problema 1. Propositio 1.

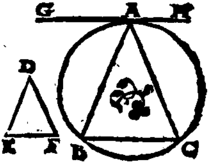
In dato circulo, rẽctam lineam accommodare ęqualem datæ rectæ lineæ quæ circuli diametro nõ sit maior.



Proble-

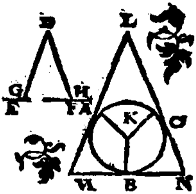
Problema 2. Propoſitio 3.

In dato circulo, triangulũ deſcribere dato triangulo æquiangulum.



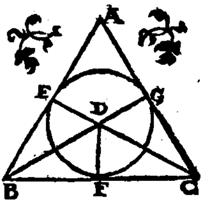
Problema 3. Propoſitio 3.

Circa datum circulũ triangulũ, deſcribere dato triangulo æquiangulũ.



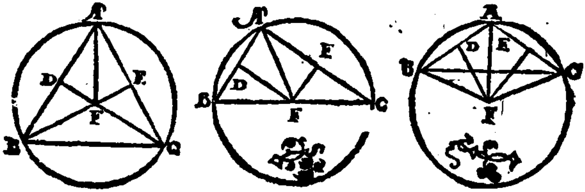
Problema 4. Propoſitio 4.

In dato triangulo circulum inſcribere.



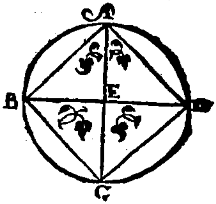
Problema 5. Propoſitio 5.

Circa datum triangulum, circulum deſcribere.



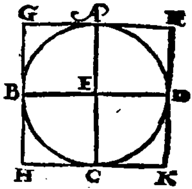
Pro

Problema 6. Propositio 6.



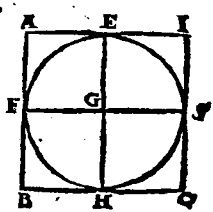
In dato circulo quadratum describere.

Problema 7. Propositio 7.



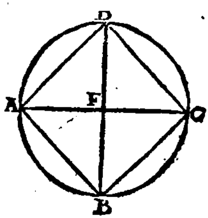
Circa datum circulum, quadratum describere.

Problema 8. Propositio 8.



In dato quadrato circulum inscribere.

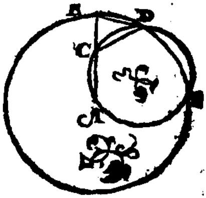
Problema 9. Propositio 9.



Circa datum quadratũ, circulum describere.

Proble-

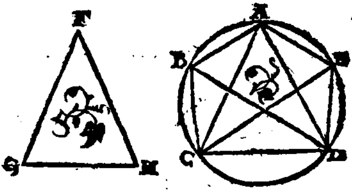
Problema 10. Propo-
positio. 10



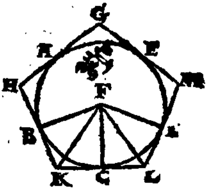
Isosceles triãgulum con-
stituere, quod habeat v-
trunque eorum, qui ad
duplum reliqui.

Theorema 11. Propositio 11.

In dato
circulo,
pentago-
nũ æqui-
laterũ &
æquiãgu
lum in-
scribere.

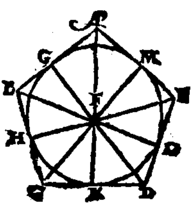


Problema 12. Pro-
positio 12.



Circa datum circulum,
pentagonum æquilate-
rum æquiangulum de-
scribere.

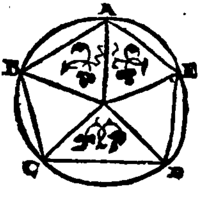
Problema 13. Pro-
positio 13.



In dato pẽtagono æqui-
latero & æquiãgulo cir-
culum inscribere.

Proble-

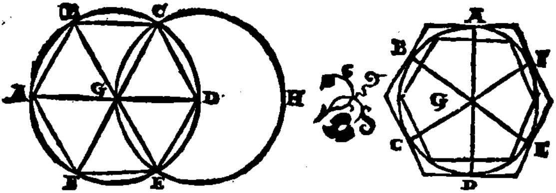
Problema 14. Propositio 14.



Circa datũ pentagonũ, æquilaterum & æquiangulum, circulum describere.

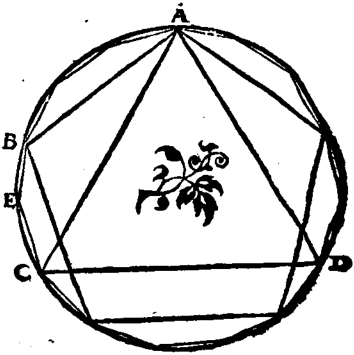
Problema 15. Propositio 15.

In dato circulo hexagonum & æquilaterũ & æquiangulum inscribere.



propositio 16. Theorema 16.

In dato circulo quintidecagonum & æquilaterũ & æquiangulum describere.



Elementi quarti finis.

EVCLIDIS ELEMENTVM QVINTVM.

DEFINITIONES.

1

PARS est magnitudo magnitudinis minor maioris, quum minor metitur maiorem.

2

Multiplex autem est maior minoris, cùm minor metitur maiorem.

3

Ratio, est duarum magnitudinum eiusdẽ generis mutua quædam secundum quantitalem habitudo.

4

Proprotio verò, est rationum similitudo.

5

Rationẽ habere inter se magnitudinis dicuntur, quæ possunt multiplicatæ sese mutu superar e

6.

In eadem ratione magnitudines dicũtur esse, prima ad secundam, & tertia ad quartã, cum primæ & tertiæ æquè multiplicia, à secundæ & quartæ æquè multiplicibus, qualiſcunq; ſit hæc multiplicatio, vtrumq; ab vtroque: vel vnà deficiunt, vel vnà ęqualia ſunt, vel vnà excedũt, ſi ea ſumãtur quæ inter ſe reſpondent.

7

Eandam autem habẽtes rationem magnitudines, proportionales vocentur.

8

Cum verò æquè multiplicium, multiplex primę magitudines exceſſerit? multiplicẽ ſecundæ, at multiplex tertiæ non exceſſerit multiplicem quartæ: tunc prima ad ſecundam, maiorem rationem habere dicetur, quàm tertia ad quartam.

9

Proportio autẽ in tribus terminis pauciſſimis conſiſtit.

10

Cùm autem tres magnititudines proportionales, fuerint, prima ad tertiam, duplicatã rationẽ habere dicitur eius quam habet ad ſecundam. At cùm quatuor magnitudines proportionales, fuerint, prima ad quartam, triplicatam rationem habere dicitur eius quam habet ad ſecundam: & ſemper deinceps vno amplius, quãdiu proportio extiterit.

11

Homo logę, ſeu ſimiles ratioue magnitudines dicuntur, antecedentes quidẽ antecedentidentibus, consequẽtẽs verò cõsequẽtibus.

12

Alterna ratio, est sumptio antecedentis cõparati ad antecedentem, & cõsequentis ad consequentem.

13

Inuersa ratio, est sumptio cõsequentis, ceu antecedentis, ad antecedentẽ velut ad consequentem.

14

Compositio rationis, est sumptio antecedentis cũ consequente ceu vnius ad ipsum consequentem.

15

Diuisio rationis, est sumptio excessus quo consequentem superat antecedens ad ipsum consequentem.

16

Conuersio rationis, est sumptio antecedentis ad excessum, quo superat antecedens ipsum cosequentem.

17

Ex ęqualitate ratio est, si plures duabus sint magnitudines, & his alię multitudine pares quæ binæ sumantur, & in eadem ratione: quum vt in primis magnitudinibus prima ad vltimam sic & in secundis magnitudinibus prima ad vltimã sese habuerit, vel aliter, sumptio extremorũ per subductionem mediorum.

 18 Or-

18

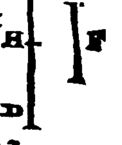
Ordinata proportio eſt, cùm fuerit quemadmodum antecedens ad conſequentem ita antecedens ad conſequentem : fuerit etiam vt conſequens ad aliud quidpiam. ita conſequens ad aliud quidpiam.

19

Perturbata autẽ proportio eſt, tribus poſitis magnitudinibus, & alij quæ ſint his multitudine pares, cùm vt in primis quidẽ magnitudinibus ſe habet antecedẽs ad cõſequentem, ita in ſecũdis magnitudinibus antecedens ad conſequentem : vt autem in primis magnitudinibus conſequens ad aliud quidpiam ad antecedentem.

Theorema 1. Propoſitio 1.

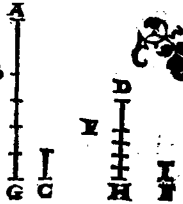
Si ſint quotcũque magnitudines quotcunque magnitudinum æqualium numero ſingulę ſingularum æquè multiplices, quàm multiplex eſt vnius vna magnitudo, tam multiplices erunt, & omnes omnium.



Theorema 2. Propoſitio 2.

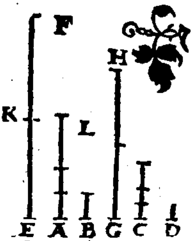
Si prima ſecundæ æquè fuerit multiplex,

atque



atque tertia quartę, fuerit autẽ & quinta ſecũdę æquè multiplex, atque ſexta quartæ: erit & cõpoſita prima cũ quinta, ſecũdæ ęquè multiplex atque tertia cum ſexta, quartæ.

Theorema 3. Propoſitio 3.

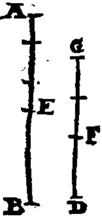


Si ſi prima ſecundæ æquè multiplex atque tertia quartæ, ſumãtur autẽ æquè multiplices primę & tertiæ erit & ex æquo ſumptarum vtraque vtriuſque ęquè multiplex, altera quidem ſecundæ, altera autẽ

Theorema 4. Propoſitio. 4.

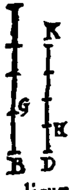
Si prima ad ſecundam, eandẽ habuerit rationem, & tert;a ad quartam: etiam æquè multiplicespri mę & tertiæ, ad æquè multiplices ſecundæ & quartæ iuxta quãuis multiplicationẽ, ean

dem habebunt rationem, ſi prout inter ſe respondent, ita sumptæ fuerint.



Theorema 5. Propositio 5.

Si magnitudo magnitudinis æquè fuerit multiplex, atque ablata ablatæ: etiam reliqua reliquæ ita multiplex erit, vt tota totius.



Theorema 6. Propositio 6.

Si duæ magnitudines, duarum magnitudinum sint æquè multiplices, & detractę quædam sint earundem æquè multiplices: & reliquæ eisdem aut ęquales sunt, aut æquè ipsarum multiplices.

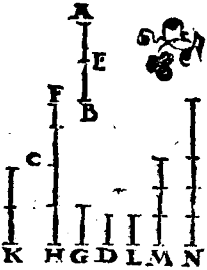
Theorema 7. Propositio 7.

Aequales ad eandem, eadem habent rationem: & eandem ad æquales.

Theorema 8. Propositio 8.

In æqualium magnitudinũ maior, ad eandem

dem maiorem ratione habet, quàm minor: & eadem ad minorem: maiorem rationem habet, quàm ad maiorem.



Theorema 9. Propositio 9.

Quæ ad eandem, eandem habét rationem, æquales sunt inter se: & ad quas eadem, eandem habet rationem, eæ quoque sunt inter se æquales.

Thorema 10. Propositio 10.

Ad eandem magnitudiné rationem habentium, quę maiorem rationé habet, illa maior est ad quam auté eadem maiorem rationem habet, illa minor est.

Theorema 11. Propositio 11.

Quæ eidem sunt eædem ratioues, & inter se sunt eędem.

Theorema 12. Propoſitio 12.

Si ſint magnitudines quotcunque proportionales, quem admodũ ſe habuerit vna antecedentium ad vnã

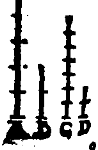
conſequentiũi, ta ſe habebunt omnes antecedenses ad omnes coſequentes.

Theorema 13. Propoſitio 13.

Si prima ad ſecundum, eandẽ habuerit rationem, quam tertia ad quartã, tertia verò ad quartam maiorem rationem habuerit, quàm quinta ad ſextam. prima quoque ad ſecũdam maiorem rationem habebit, quàm quinta ad ſextam.

Theorema 14. Propoſitio 14.

Si prima ad ſecundam eandem habuerit rationẽ, quam tertia ad quartam, prima verò quàm tertia maior fuerit: erit & ſecunda maior quàm quarta. Quòd ſi prima fuerit æqualis tertiæ, erit

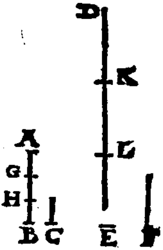


&

& secunda æqualis quartæ: si verò minor, & minor erit.

Theorema 15. Propositio 15.

Partes cum pariter multiplicibus in eadé sunt ratione, si prout sibi mutuo respondent, ita sumantur.

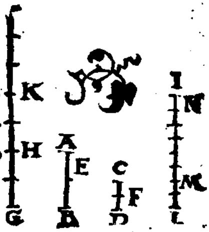


Theorema 16. Propositio 16.

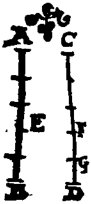
Si quatuor magnitudines proportionales fuerint, & vicissim proportionales erunt.

Theorema 17. Propositio 17.

Si compositæ magnitudines porportionales fuerint hæ quoq; diuisæ proportionales erunt.



Theorema 18. Propositio 18.



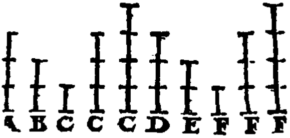
Si diuisæ magnitudines sint proportionales, hæ quoque compositæ proportionales erunt.

Theorema 19. Propositio 19.

Si quemadmodum totũ ad totum, ita ablatum se habuerit ad ablatum: & reliquum ad reliquum, vt totum ad totum se habebit.

Theorema 20. Propositio 20.

Si sint tres magnitudines, & aliæ ipsis æquales numero, quæ binæ & in eadem ratione sumatur, ex æquo autẽ prima quàm tertia maior fuerit: erit & quarta, quã sexta maior. Quòd si

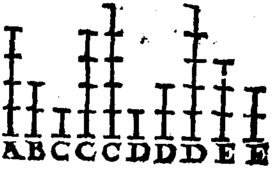


prima tertiæ fuerit æqualis, erit & quarto æqualis sextæ: sin illa minor, hæc quoque minor erit.

Theore-

Theorema 21. Propositio 21.

Si sint tres magnitudines, & aliæ ipsis æquales numero quæ binæ & in eadem ratione sumātur, fuerit que perturbata earū proportio ex æquo autem prima quàm tertia maior fuerit erit & quarta quā sexta maior: quòd si prima tertiæ fuerit æqualis, erit & quarta æqualis sextæ: si illa minor, hæc quoque minor erit.



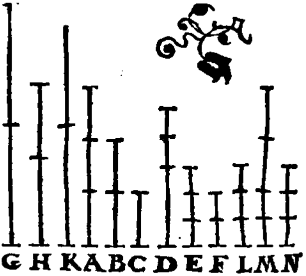
Theorema 22. Propositio 22.

Si sint quotcunq; magnitudines, & aliç ipsis çquales numero, quæ binæ in eadē ratione sumantur, et ex æqualitate in eadem ratione erunt.

Theorema 23. Propositio 32.

Si sint tres magnitudines, aliæq; ipsis çquales

les numero, q̃ binę in eadem ratione sumã-tur, fuerit autẽ perturbata ea-rũ proportio: etiã ex æqualite in eadem ratione erunt.

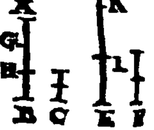


Theorema 24. Propositio 24.

Si prima ad secundam, eandem habuerit rationẽ, quam tertia ad quartam, habuerit autem & quinta ad secundam, eandẽ rationem, quã sexta ad quartam: etiam cõposita prima cũ quinta ad secundam eandẽ habebit rationẽ quam tertia cum sexta ad quartam.

Theorema 25. Propositio 25.

Si quatuor magnitudines proportionales fuerint, maxima & minima reliquis duabus maiores erunt



ELEMENTI V. FINIS.

EVCLIDIS ELEMENTVM SEXTVM.

DEFINITIONES.

1

Similes figuræ rectilineæ sunt, quæ & angulos singulos singulis æquales habēt, atque etiam latera, quę circum angulos æquales, proportionalia,

2

Reciprocæ autem figuræ sunt, cum in vtraque figura antecedentes & consequentes rationum termini fuerint.

3

Secundum extremam & mediam rationē recta linea secta esse dicitur, cùm vt tota ad maius segmentum, ita maius ad minus se habuerit.

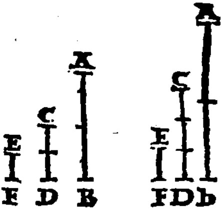
4

Altitudo cuiusque figurę, est linea perpendicularis à vertice ad basin deducta

5 Ra-

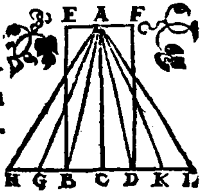
5.

Ratio ex rationibus cō poni dicitur, cū rationum quantitates inter ſe multiplicatæ aliqua effecerint rationem.



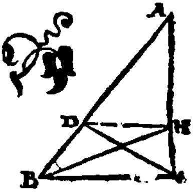
Theorema 1. Propoſitio. 1

Triangula & parallelogramma, quorum eadem fuerit altitudo, ita ſe habent inter ſe vt baſes.



Theorema 2. Propoſitio 2.

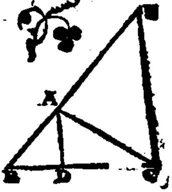
Si ad vnum trianguli latus parallela ducta fuerit recta quædam linea: hęc proportionaliter ſecabit, ipſius triāguli latera. Et ſi trianguli latera proportionaliter ſecta fuerint: quæ ad ſectiones adiūcta fuerit recta linea, erit ad reliquum ipſius trianguli latus parallela.



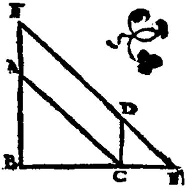
Theorema 3. Propoſitio 3.

Si trianguli angulus bifariam ſectus ſit, ſecans autē augulum recta linea ſecuerit & baſim: baſis ſegmenta eandem habebunt rationem,

tionem, quam reliqua ipsius trianguli latera, Et si basis segmenta eandem habeant rationem quam reliqua ipsius trianguli latera, recta linea, quæ à vertice ad sectionem producitur, ea bifariá secat trianguli ipsius angulum.

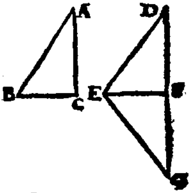


Theorema 4. Propositio 4



Aequiangulorum triangulorum proportianolia sunt latera, quę circum ęquales angulos, & homologa sunt latera, quę ęqualibus angulis subtenduntur.

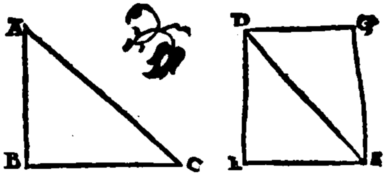
Theorema 5. Propositio 5.



Si duo triágula latera proportionalia habeant, ęquiangula erunt triangula, & æquales habebunt eos angulos, sub quib⁹ homologa latera subtenduntur.

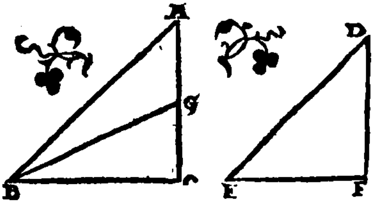
Theorema 6. Propositio 6.

Si duo triangula vnum angulùm vni angulo æqualé, & circum ęquales angulos latera pportionalia habuerint, ęquiangula erunt triang triangula, ęqualesq́; habebunt angulos ſub quibus homologa latera ſubtenduntur.



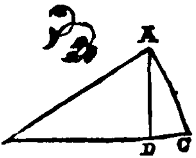
Theorema 7. Propoſitio 7.

Si duo triangula vnum angulum vni angulo æqualem, circũ autem alios angulos latera proportionalia habent, reliquorum vero ſimul vtrunq; aut minórem aut non minoré recto: æquiangula erunt triangula, & æquales habebunt eos angulos circum quos proportionalia ſunt latera.

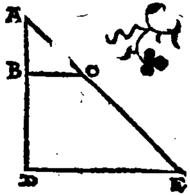


Theorema 8. Propoſitio 8.

Si in triangulo rectangulo, ab angulo recto in baſin perpendicularis ducta ſit quę ad perpendicularem triangulo, tum toti triangulo, tum ipſa inter ſe ſimilia ſunt.

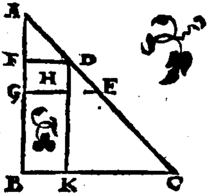


Problema 1. Propositio 9.



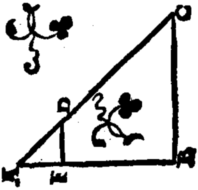
A data recta linea imperatam partem auferre.

Problema 2. Propositio 10.



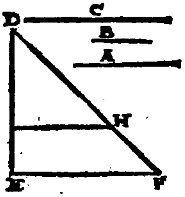
Datam rectã lineam in sectam similiter secare, vt data altera recta secta fuerit.

Problema 3. Propositio 11.



Duabus datis rectis lineis, tertiam propotionalem ad inuenire.

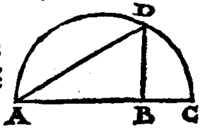
Problema 4. Propositio 12.



Trib⁹ datis rectis lineis quartam proportionalem adiuenire.

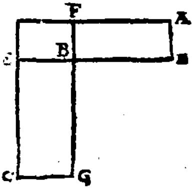
Problema 5. Propositio 13.

Duab' datis rectis lineis mediam proportionalẽ adinuenire.



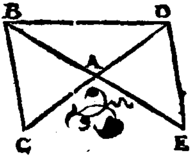
Theorema 9. Propositio 14.

Aequalium, & vnũ vni æqualem habentiũ angulum parallelogrammorum reciproca sunt latera, quæ circum æquales angulos. & quorũ parallelogrammorũ vnum angulum vni angulo æqualem habentiũ reciproca sunt latera, quæ circum æquales angulos, illa sunt æqualia.



Theorema 10. Propositio 15.

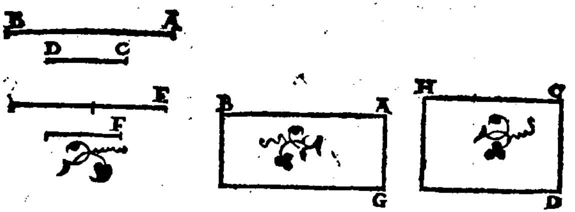
Aequalium, & vnũ angulum vni æqualem habentiũ triangulorũ reciproca sunt latera, quæ circum æquales angulos. & quorũ triangulorũ vnum angulum vni æqualem habentium reciproca sunt latera, q̃ circum æquales angulos, illa sunt æqualia.



Theo-

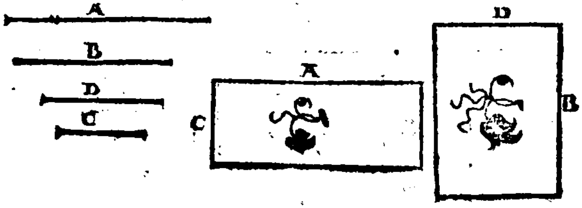
Theorema 11. Propositio 16.

Si quatuor rectæ lineæ proportionales fue rint, quod sub extremis comprehenditur rectangulũ, æquale est ei, quod sub medijs comprehenditur rectãgulo. Et si sub extremis comprehensum rectangulum æquale fuerit ei, quod sub medijs continetur rectã gulo, illæ quatuor rectæ lineæ proportionales erunt.



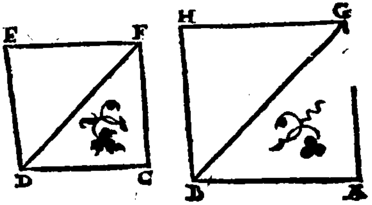
Theorema 12. Propositio 17.

Si tres rectæ lineæ sint proportionales, qđ sub extremis cõprehenditur rectangulũ ę quale est ei, quod à media describitur quadrato: & si sub extremis comprehẽsum rectangulum æquale sit ei quod à media describitur quadrato, illæ tres rectę lineę proportionales erunt.



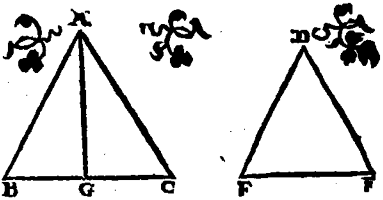
Problema 6. Propoſitio 18.

A data recta linea, dato rectilineo ſimile ſimiliterq; poſitum rectilineum deſcribere.



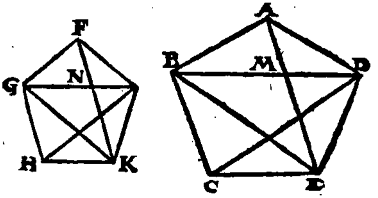
Theorema 13. Propoſitio 19.

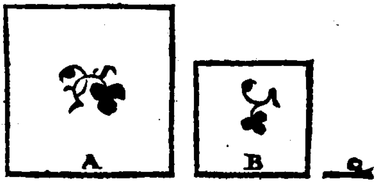
Similia triãgula inter ſe ſunt in duplicata ratione laterũ homologorũ.



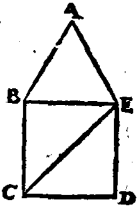
Theore. 14. Propoſitio 20.

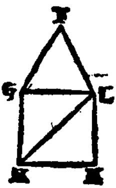
Similia polygona in ſimilia triãgula diuidũtur, & numero ęqualia, & homologa totis, Et po





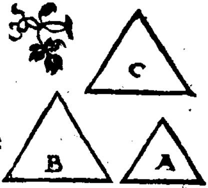
lygona duplicatam habent eã inter se rationẽ, quã latus homologũ ad homologum latus.





Theorema 15. Propositio 21.

Quæ eidem rectilineo sunt similia, & inter se sunt similia.

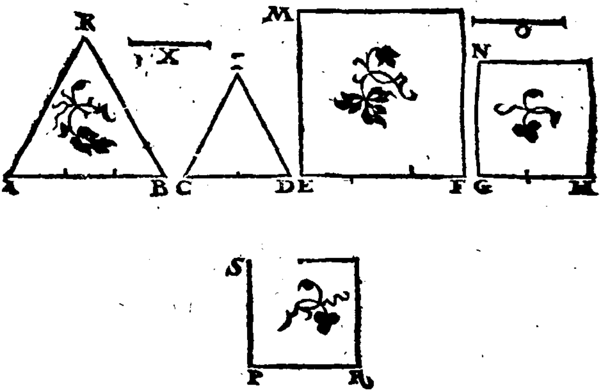


Theorema 16. Propositio 22.

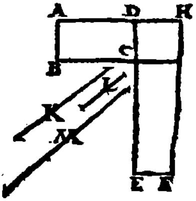
Si quatuor rectę lineæ proportionales fuerint; & ab eis rectilinea similia similiterq; descripta proportionalia erũt. Et si à rectis lineis similia similiterq; descripta rectilinea proportionalia fuerint, ipsæ etiam re-

 ctæ

&æ lineæ proportionales erunt.

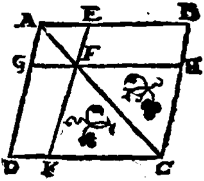


Theorema 17. Propositio 23.



Aequiangula parallelogramma inter se rationẽ habent eum, quę ex lateribus componitur.

Theorema 18. Propositio 24.

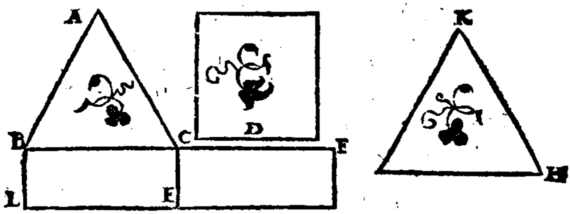


In omni parallelogrammo, quæ circa diametrũ sunt parallelogrãma, & toti & inter se sunt similia.

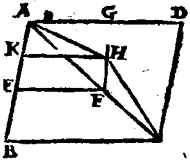
Proble-

Problema 7. Propositio 25.

Dato rectilineo simile, & alteri dato ęquale idem constituere.



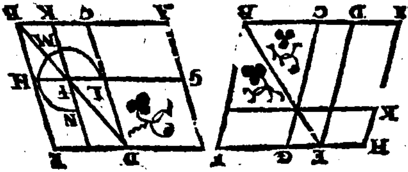
Theorema 19. Propositio 26.



Si à parallelogrãmo parallelogrammũ ablatũ sit, & simile toti & similiter positũ communẽ cum eo habens angulum, hoc circum eandem cum toto diametrum consistit.

Theorema 20. Propositio 27.

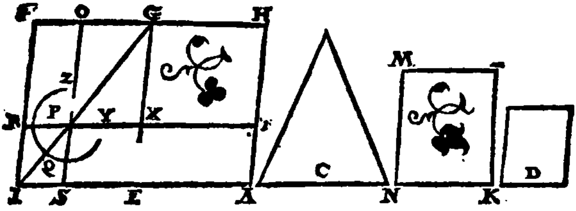
Omnium parallelogrãmorum secũdum eandem rectam lineam applicatorũ deficiẽtiumq; figuris parallelogrãmis si-



milibus ſimiliterque poſitis ei, quod à dimidia deſcribitur, maximum, id eſt, quod ad dimidiam applicatur parallelogrammum ſimile exiſtens defectui.

Problema 8. propoſitio 28.

Ad datam lineam rectam, dato rectilineo æquale parallelogrammum applicare deficiens figura parallelogramma, quę ſimilis ſit alteri rectilineo dato. Oportet autẽ datum rectilineum, cui æquale applicandum eſt, non maius eſſe eo quod ad dimidiam applicatur, cum ſimiles ſint defectus & eius quod à dimidia deſcribitur, & eius cui ſimile deeſſe debet.

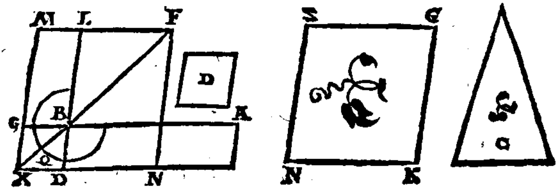


Problema 9. Propoſitio. 29.

Ad datam rectam lineam, dato rectilineo æquale parallelogrammũ applicare, excedẽs figura parallelogrãma, quæ ſimilis ſit

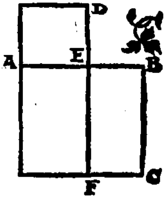
paral-

parallelogrammo alteri dato.



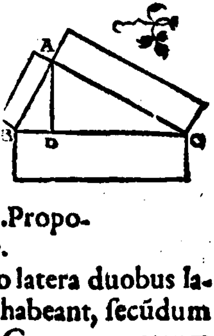
Prblema 10. Propo-
positio 30.

Propositam rectam lineam terminatã, extrema ac media ratione secare.



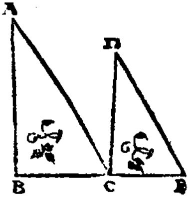
Theorema 21. Propositio 31

In rectangulis triãgulis, figura quęuis à latere rectũ angulũ subtendẽte descripta ęqualis est figuris, quæ priori illi similes & similiter positę à lateribus rectũ angulum continentibus describuntur.



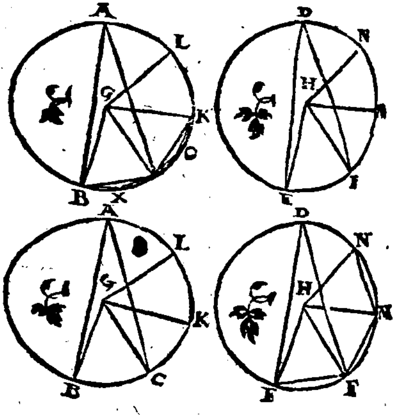
Theorema 22. Propo-
sitio 32.

Si duo triãgula, quę duo latera duobus lateribus proportionalia habeant, secũdum vnum angulū composita fuerint, ita vt homologa eorum latera sint etiam parallela, tum reliqua illorum ttiangulorum latera in rectam lineam collocata reperientur.



Theorema 23. Propositio 33.

In æqualibus circulis anguli eandē habent rationem, cū ipsis peripherijs in quibus insistūt, siue ad cētra siue ad peripherias constituti, illis insistant peripherijs In super verò & sectores quippe qui ad cētra cōsistūt.



ELEMENTI VI. FINIS.

EVCLIDIS ELEMENTVM SEPTIMVM.

DEFINITIONES.

1

Vnitas, est secũdum quam entium quodque dicitur vnum.

2

Numerus autem, ex vnitatibus composita multitudo.

3

Pars, est numerus numeri minor maioris, cùm minor metitur maiorem.

4

Partes autem, cùm non metitur.

5

Multiplex verò, maior minoris, cùm maiorem metitur minor.

6

Par numerus est, qui bifariã non diuiditur.

7

Impar verò, qui bifariam non diuiditur, vel, qui vnitate differt à pari.

8

Pariter par numerus est, quem par numerus metitur per numerum parem.

9 Pari-

9.

Pariter autem impar, eſt quem par numerus metitur per numerum imparem.

10.

Impariter verò impar numerus, eſt quem impar numerus metitur per numerum imparem.

11.

Primus numerus, eſt quem vnitas ſola metitur.

12.

Primi inter ſe numeri ſunt, quos ſola vnitas menſura communis metitur.

13.

Compoſitus numerus eſt, quem numerus quiſpiam metitur.

14.

Compoſiti autē inter ſe numeri ſunt, quos numerus aliquis menſura communis metitur.

15.

Numerus numerum multiplicare dicitur, cùm toties compoſitus fuerit is, qui multiplicatur, quot ſunt in illo multiplicante vnitates, & procreatus fuerit aliquis.

16.

Cùm autē duo numeri mutuò ſeſe multiplicantes quem piam faciūt, qui factus erit planus appellabitur, qui verò numeri mutuò ſeſe multiplicarint, illi' latera dicētur.

17. Cùm

17

Cùm verò tres numeri mutuò sese multiplicantes quempiã faciunt, qui procreatus erit solidus apellabitur, qua autem numeri mutuò sese multiplicarint, illius, latera dicentur.

18.

Quadratus numerus est, qui ęqualiter ęqualis: vel, qui à duobus ęqualibus numeris cótinetur.

19.

Cubus verò, qui à tribus æqualibus æqualiter: vel, quia tribus æqualibus numeris continetur.

10

Numeri proportionales sunt, cum primus secundi, & tertitius quarti æquè multiplex est, vt eadem pars, vel eædem partes.

21

Similes plani & solidi numeri sunt, qui proportionalia habent latera.

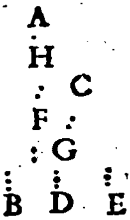
22

Perfectus numerus est, qui suis ipsius partibus est æqualis.

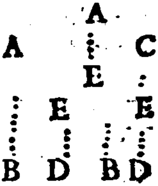
Theorema 1. Propositio 1.

Duobus numeris in æqualibus propositis,

tis,si detrahatur semper minor. de maiore,alterna,quadam detractione; neque reliquus vnquam metiatur præcedentem quo ad assumpta sit vnitas: qui principio propositi suntnumeri primi inter se erunt.

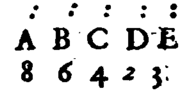


Porblema 1. Propositio 2.

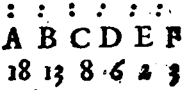


Duobus numeris datis non primis inter se, maximã eorũ cõmunẽ mensurã reperire

Broblema 2. Propositio. 3

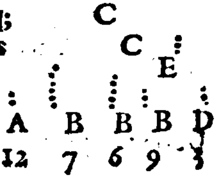


Tribus numeris datis non primis inter se, maximam eorum communem mensuram reperire.



Theorema 18. Propositio 8.

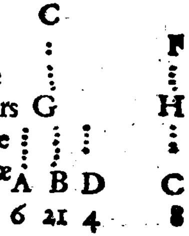
Omnis numerus,cuiusq; numeri minor maioris aut par est,aut partes.



Theore-

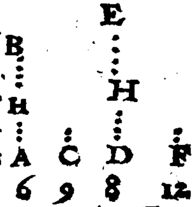
Theorema 3. Propositio 5.

Si numeris numeri par fuerit, & alter alterius eadẽ pars & simul vterque vtriusque simul eadem pars erit, quæ vnus est vnius.



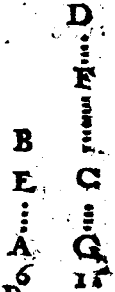
Theor. 4. Propo. 6.

Si numer' sit numeri partes, & alter alterius eędem partes, & simul vterque vtriusque simul eædẽ partes erunt, quæ sunt vnus vnius.



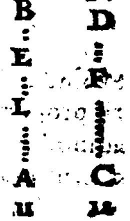
Theorema 5. Propositio 7.

Si numerus numeri eadẽ si pars quæ detractus detracti, & reliquus reliqui eadẽ pars erit, quæ totus est totius.



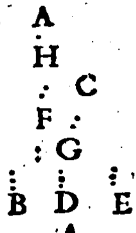
Theorema 6. Propositio. 8.

Si numerus numeri eædẽ sint partes quæ detractus detracti & reliquus reliqui eędẽ partes erũt, quę sũt totus totius. Theo-

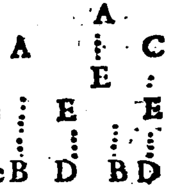


G..M.K..N,H.

tis, si detrahatur semper minor. de maiore, alterna, quadam detractione; neque reliquus vnquam metiatur præcedentem quo ad assumpta sit vnitas: qui principio propositi suntnumeri primi inter se erunt.

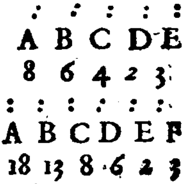


Porblema 1. Propositio 2.



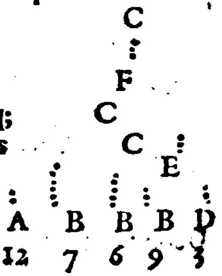
Duobus numeris datis non primis inter se, maximã eorũ cõmunẽ mensurã reperire

Broblema 2. Propositio. 3



Tribus numeris datis non primis inter se, maximam eorum communem mensuram reperire.

Theorema 18. Propositio 8.

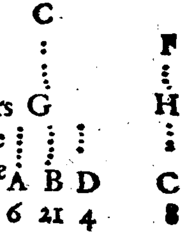


Omnis numerus, cuiusq; numeri minor maioris aut par est, aut partes.

Theore-

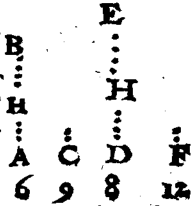
Theorema 3. Propo-
sitio 5.

Si numeris numeri par fuerit, & alter alterius eadẽ pars & simul vterque vtriusque simul eadem pars erit, quæ vnus est vnius.



Theor. 4. Propo. 6.

Si numer' sit numeri partes, & alter alterius eędem partes, & simul vterque vtriusque simul eædẽ partes erunt, quæ sunt vnus vnius.



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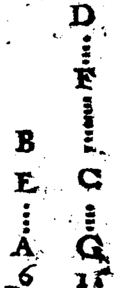
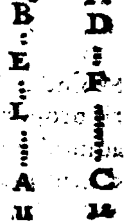


Theorema 6. Propositio. 8.

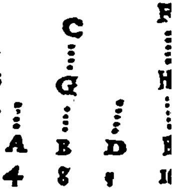
Si numerus numeri eædẽ sint partes quæ detractus detracti, & reliquus reliqui eędẽ partes erũt, quę sũt totus totius. Theo-



G..M.K..N,H.

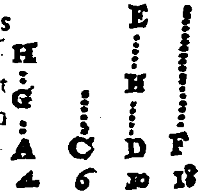
Theorema 7. Propositio 9.

Si numerus numeris pars sit, & alter alterius eadem pars, & viciſſim quæ pars eſt vel partes primus tertij, eadem pars erit eędem partes, & ſecũdus quarti.



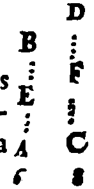
Theorema 8. Propoſitio 10.

Si numerus numeri partes ſint, & aliter alterius eadẽ partes, etiam viciſſim quæ ſunt partes aut pars primus tertij, eędem partes erũt vel pars & ſecundus quarti.



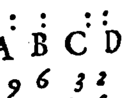
Theorema 9. Propoſitio 11.

Si quemadmodum ſe habet totus ad totum, ita detractus ad detractum, & reliquus ad reliquum ita habebit vt totus ad totum



Theorema 10. Propoſitio 12.

Si ſint quotcunque numeri proportionales; quemadmodum ſe habet vnus antecedentium ad vnum ſequentium, ita ſe habe-



habebunt omnes antecedentes ad omnes consequentes.

Theorema 11. Propositio 13.

Si quatuor numeri sint proportionales, & viciſsim ꝓportionales erunt.

:	:	:	:
A	B	C	D
12	4	9	3

Theorema 12 Propositio 14

Si ſint quotcunque numeri, & alij illis æquales multitudine, qui bini ſumantur & in eadem ratione: etiam ex æqualitate in eadem ratione erunt

:	:	:	:	:	:
A	B	C	D	E	F
12	6	3	8	4	2

Theorema 13. Propoſitio 15.

Si vnitas numerum quépiã metiatur, aliter verò numerus alium quendã numerũ æquè metiatur, & viciſſim vnitas tertium numerũ æquę metietur, atque ſecundis quartum.

F
L
C K
H E
G
A B D
1 3 2 6

Theorema 14. Propoſitio 16

Si duo numeri mutuò ſeſe multiplicãtes faciant aliquos qui ex illis geniti fuerint, inter ſe æquales erunt.

.	:	:	:	:
E	A	B	C	D
1	2	4	8	8

Theorema 15. Propoſitio 17.

Si numer⁹ duos numeros multiplicás faciat aliquos, qui ex illis procreati erunt, eadem rationem habebunt, quam multiplicati.

:	:	:	:	:	:
I	A	B	C	D	E
1	3	4	5	12	15

Theorema 16. Propoſitio 18.

Si duo numeri numerum quempiam multiplicantes faciant aliquos, geniti ex illis eandem habebunt rationem, quam qui illum multiplicarunt.

:	:	:	:	:
A	B	C	D	E
4	5	3	12	15

Theorema 17. Propoſitio 19.

Si quatuor numeri ſint proportionales, ꝙ ex primo & quarto fit, ęqualis erit ei qui ex ſecundo & tertio: & ſi qui ex primo & quarto ſit numerus æqualis ſit ei qui ex ſecũdo & tertio, illi quatuor numeri ſint propor tionales erunt.

:	:	:	:	:	:	:
A	B	C	D	E	F	G
6	4	3	2	12	12	18

Theorema 18. Propoſitio 20.

Si tres numeri ſint proportionales, qui ab extremis cõtinetur, ęqualis eſt ei qui à medio

dio efficitur. Et si qui ab extremis cõtinetur æqualis sit ei qui à medio describitur, illi tres numeri proportionales erunt.

A	B	C
9	6	4
D		
6		

Theorema 19. Propositio. 21.

Minimi numeri omniũ, qui eandem cũ eis rationem habẽt, æqualiter metiũtur numeros eandem rationem habẽtes, maior quidem maiorem, minor verò minorem

D	L		
G	H		
C	E	A	B
4	3	8	6

Theorema 20. Propositio 22.

Si tres sint numeri & alij multitudine illis æquales, qui bini sumantur & in eadem ratione, sit autem perturbata eorũ proportio, etiam ex æqualitate in eadem ratione erunt.

A	B	C	D	E	F
6	4	3	12	8	6

Theorema 21. Propositio 23.

Primi inter se numeri minimi sunt omniũ eandem cum eis rationem habentium.

A	B	E	C	D
5	6	2	4	3

Theorema 22 Propositio 24.

Minimi uumeri omnium eandem cum eis rationem habentiũ, primus sunt inter erit

A	B	C	D	E
8	6	4	3	2

Theorema 23. Propositio 25.

Si duo numeri sint primi inter se, qui alter utrum illorum metitur numerus, is ad reliquũ primus erit.

A	B	C	D
6	7	3	4

Theorema 24. Propotio 25.

Si duo oumtri ad qué piam numerum primi sint, an eundem primus is quoque futurus est, qui ab illis productus fuerit.

B	A	C	D	E	F
	5	5	5	3	2

Theorema 25. Propositio 27.

Si duo numeri pirmi sint inter se. q ab vno eorũ gignitur ad reliquum primus erit.

B	A	C	D
	7	6	3

Theorema 26. Propositio 28.

Si duo numeri ad duos numeros ambo ad vtrunque primi sint, & qui ex eis gignentur, primi inter se ẽrunt.

A	B	E	C	D	F
3	5	15	2	4	8

Theore

Theorema 27. Propositio 29.

duo numeri primi ſint inter ſe, & multi-
icās vterq; ſeipſum procreet aliquem, qui
: ijs producti fuerint, primi inter ſe erūt.
uòd ſi numeri initio propoſiti multipli
ntes eos qui producti ſunt, effecerint ali-
uos, hi quoq; inter ſe primi erunt, & circa
ctremos idem hoc
mper eueniet.

A	C	E	B	D	F
3	6	27	4	16	63

Theorema 28. Propoſitio. 30.

duo numeri primi ſint inter ſe, etiam ſi.
ul vterq; ad vtrunq; illorum primus erit.
ſi ſimul vterq; ad vnum aliquem eorum
imus ſit, etiam qui ini-
poſiti ſunt numeri,
imi inter ſunt erunt.

C

A	B	D
7	5	4

Theorema 29. Propoſitio 31.

mnis primus numerus ad om
m numerum quem non me-
ur, primus eſt.

A	B	C
7	10	5

Theoremā 30. Proofitpio 32.

duo numeri ſeſe mutuo multiplicātes fa-
ınt aliquem, hūc aūt ab illis productum
etiatur primus qui-
m numerus, is alte-
m etiam metitur eo-
m qui initio poſiti erant.

A	B	C	D	E
3	6	12	3	4

Theorema 31. Propoſitio 33.

Omnem compoſitum numerum aliquis primus metietur.

A B C
27 9 3

Theorema 32. Propoſitio 34.

Omnis numerus aut primus eſt, aut eum aliquis primus metitur.

A A
3 6

Problema 3. Propoſitio 35.

Numeris datis quotcunque, reperire minimos omnium qui eandem cum illis rationem habeant.

A B C D E F G H K I M
6 8 12 2 3 4 6 2 3 4 3

Problema 4. Propoſitio 36.

B
A C D E F
7 12 8 4 5

Duobus numeris datis reperire quẽ illi minimum metiantur numerum.

A
F E C D G H
6 9 12 9 3 3

Theo-

Theorema 33. Propoſitio 37

Si duo numeri numerũ quempiam metiantur, & minimus quem illi metiuntur eundem metietur.

F

A B E C

2 3 6 12

Problema 5. Propoſitio 38.

Tribus numeris datis reperire quem minimum numerũ illi metiantur.

A B C D E

3 4 6 12 8

A B C D E F

3 6 8 12 24 16

Theorema 34. Propſitio 39.

Si numerum quiſpiam numerus metiatur, menſus partem habebit metienti cognominem.

A B C D

12 4 3 1

Theorema 35. Propoſitio 40.

Si numerus partem habuerit quãlibet, illum metietur numerus parti cognominis.

A B C D

8 4 2 1

Problema 6. Propoſitio 41.

Numerum reperire, qui minimus cùm ſit, datas habeat partes.

A B C G H

2 3 4 12 10

ELEMENTI VII. FINIS.

EVCLIDIS ELEMENTVM OCTAVVM.

Theorema 1. Propositio 1.

Si sunt quotcunq; numeri deinceps proportionales, quorum extremi sint inter se, primini, minimi sunt omnium eandem cum eis rationem habentium.

A	B	C	D	E	F	G	H
8	12	18	27	6	8	12	18

Problema 1. Propositio 2.

Numeros reperire deinceps proportionales minimos, quotcunq; iusserit quispiam in data ratione.

A	B	C	D	E	F	G	H	K
3	4	9	12	16	27	36	49	64

Theorema 2. Propositio 3.

Conuersa primæ.

Si sint quotcunq; numeri deinceps proportionales minimi habentium eandem cum eis rationem, illorum extremi sunt inter se primi.

A	B	C	D	E	F	G	H	K	L	M	N	O
27	16	48	64	3	4	9	12	16	27	36	48	64

Problema 2. Propositio 4.

Rationibus datis quotcunque in minimis numeris reperire numeros deinceps minimos in datis rationibus.

A	B	C	D	E	F	H	G	K	L	N	X	M	O
3	4	2	3	4	5	6	8	12	15	4	6	10	12

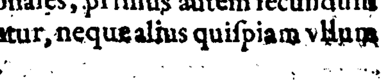
Theorema 3. Propositio 5.

Plani numeri rationem inter se habent ex lateribus compositam.

A	L	B	C	D	E	F	G	H	K
18	22	32	3	6	4	8	9	12	16

Theorema 4. Propositio 4.

Si sint quotlibet numeri deinceps



A	B	C	D	E	F	G	H
16	24	36	54	81	4	6	9

proportionales, primus autem secundum non metiatur, neque alius quispiam vllum metietur.

Theorema 5. Propositio 7.

Si sint quotcunque numeri deinceps proportionales, primus autem extremum metiatur, is etiam secundum metietur.

A	B	C	D
4	6	12	24

Theorema 6. Propositio 8.

Si inter duos numeros medij continua proportione incidant numeri, quot inter eos medij continua proportione incidunt numeri, tot & inter alios eandem cum illis habentes rationem medij continua proportione incident.

A	C	D	B	G	H	K	L	C	M	N	F
4	9	27	81	1	3	9	27	2	6	18	54

Theorema 7. Propositio 9.

Si duo numeri sint inter se primi, & inter eos medij cōtinua proportione incidāt numeri, quot inter illos medij continua proportione incidunt numeri, totidē & inter vtrunque eorum ac vnitatē deinceps medij continua proportione incident.

A	M	H	E	F	N	C	K	X	G	D	L	O	B
27	27	9	36	3	36	1	12	48	4	48	16	64	64

Theorema 8. Propoſitio 10.

Si inter duos numeros & vnitatē continuè proportionales incidāt numeri quot inter vtrumque ipſorum & vnitatē deinceps medij continua proportione incidunt numeri, totidē & inter illos medij continua proportione incident.

A	E	K	D	H	C	L	F	G	B
27	9	36	3	12	1	48	4	16	64

Theorema 9. Propoſitio 11.

Duorum quadratorū numerorum vnus medius proportionalis eſt numerus: & quadratus ad quadratum duplicatam habet lateris ad latus rationem.

A	C	E	D	B
9	3	12	4	16

Theorema 10. Propoſitio 12.

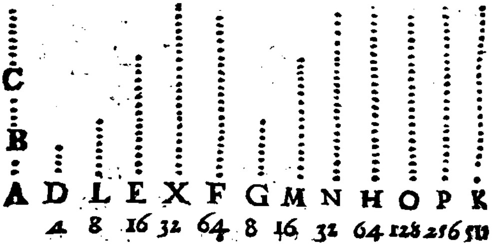
Duorum cuborum numerorū duo medij proportionales ſunt numerorū duo medij cubum triplicatam habet lateris ad latus rationem.

A	H	K	B	C	D	E	F	G
27	36	48	64	3	4	9	12	16

Theo-

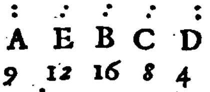
Thorema 10. Propositio 13.

Si sint quotlibet numeri deinceps proportionales, & multiplicans quisq; seipsum faciat aliquos, qui ab illis producti fuerint, proportionales erũt, & si numeri primùm positi, ex suo in proceatos ductu faciãt aliquo, ipsi quoque proportionales erunt



Theorema 12. Propositio 14.

Si quadratus numerus quadratum numerũ metiatur, & latus vnius metietur latus alterius. Et si vnius quadrati latus metiatur latus alterius, & quadratus quadratum metietur.



Theorema 13. Propositio 15.

Si cubus numerus cubum numerũ metiatur, & latus vni⁹ metietur alterius latus. Et si latus vnius cubi latus alterius metiatur,

tum

tum cubus cubum metietur.

A	H	K	B	C	D	E	F	G
8	16	28	64	2	4	4	8	16

Theorema 14. Propoſitio 9.

Si quadratus numerus quadratum numerum non metiatur, neque latus vnius metietur alterius latus.

Etſi latus vni⁹ quadrati non metiatur latus alterius, neque quadratus quadratum metietur.

A	B	C	D
9	16	3	4

Theorema 15. Propoſitio 17.

Si cubus numerus cubum numerũ non metiatur, neque latus vnius latus alterius metietur.

Et ſi latus cubi alicuius latus alterius non metiatur, neque cubus cubum metietur.

A	B	C	D
8	27	9	11

Theorema 16. Propoſitio 18.

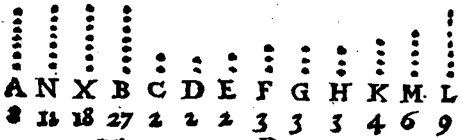
Duorum ſimilium planorum numerorum vnus medius proportionalis eſt numerus, & planus ad planum duplicatam habet literis homologi-

A	G	B	C	D	E	F
12	18	27	2	6	3	9

logi ad latus homologum rationem.

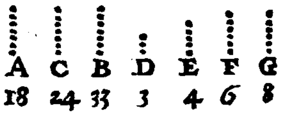
Theorema 17. Propositio 19.

Inter duos similes numeros solidos, duo medij proportionales incidunt numeri: & solidus ad similem solidum triplicatam rationem habet lateris homologi ad latus homologum.



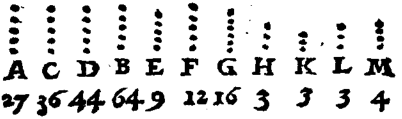
Theorema 18. Propositio 20.

Si inter duos numeros vnus medius proportionalis incidat numerus, similes plani erunt illi numeri.



Theorema 19 Propositio 21.

Si inter duos numeros duo medij proportionales incidant numeri, similes solidi sunt illi numeri.



Theo-

Theorema 20. Propositio 22.

Si tres numeri deinceps ſint proportionales, primus autem ſit quadratus, & tertius quadratus erit.

A	B	D
9	15	25

Theorema 21. Propoſitio 23.

Si quatuor numeri deinceps ſint proportionales, primus autem ſit cubus, & quartus cubus erit.

A	B	C	D
8	12	18	27

Theorema 22. Propoſitio 25.

Si duo numeri rationem habeant inter ſe, quam quadratus numerus ad quadratum numerum, primus autem ſit quadratus erit, & ſecunndus quadratus erit.

A		B	C		D
4	6	9	16	24	36

Theorema 23. Propoſitio 25.

Si numeri duo rationem inter ſe habeant, quam cubus numerus ad cubum numerú, primus autem cubus ſit, & ſecundus cubus erit.

A	E	F	B	C			D
8	12	18	27	64	95	140	216

The

Theorema 24. Propositio 26.

Similes plani numeri rationem inter se habent, quam quadratus numerus ad quadratum numerum.

A	C	B	D	E	F
18	24	32	9	12	16

Theorema 25. Propositio 27.

Similes solidi numeri rationem habent inter se, quam cubus numerus ad cubum numerum.

A	C	D	B	E	F	G	H
16	24	36	54	8	12	18	27

ELEMENTI VIII. FINIS.

EVCLI.

EVCLIDIS ELEMENTVM NONVM.

Theorema 1. Propositio 1.

Si duo similes plani numeri mutuò sese multiplicā- tes quendā procreent, productus quadratus

A	E	B	D		C
4	6	9	16	24	36

Theorema 2. Propositio 2.

Si duo numeri mutuò sese multiplicantes quadratum faciant, illi similes sunt plani.

A		B	D		C
4	6		9	18	36

Theorema 3. Propositio 3.

Si cubus numerus seipsum multiplicās procreet aliqué, productus cubus erit.

Vnitas	D	D	A			B
	3	4	8	16	32	64

Theorema 4. Propositio 4.

Si cubus numerus cubũ numerum multiplicans quendam procreet, procreatus cubus erit.

A	B	D	C
8	27	64	216

Theorema 5. Propositio 5.

Si cubus numerus numerum quendã multiplicans cubum procreet, & multiplicatus cubus erit.

A	B	D	D
27	64	729	1728

Theorema 6. Propositio 6.

Si numerus seipsum multiplicans cubum procreet, & ipse cubus erit.

A	B	C
27	729	19683

Theorema 7. Propositio 7.

Si compositus numerus quendam numerũ multiplicans quempiam procreet, productus solidus erit.

A	B	C	D	E
6	8	48	2	3

Theorema 8. Propositio 8.

Si ab vnitate quotlibet numeri deinceps ꝓportionales sint, tertius ab vnitate quadratus est, & vnũ intermittentes omnes: quartus autẽ cubus, & duobus intermissis omnes

nes: ſeptimus verò cubus ſimul & æquadratus, & quinque intermiſſis omnes.

Vnitas	A	B	C	D	E	F
	3	9	27	81	243	729

Theorema 9. Propoſitio 9.

Si ab vnitate ſint quotcunqne numeri deinceps proportionales, ſit autem quadratus is qui vnitatem ſequitur, & reliqui oẽs quadrati erũt. Quòd ſi qui vnitatem ſequitur cubus ſit, & reliqui omnes cubi erunt.

quadrati		cubi
531441	F	732969
59049	E	531441
6561	D	59049
729	C	6561
81	B	729
9	A	81
	o	
	vnitas.	

Theorema 10. Propoſitio 10.

Si ab vnitate numeri qnotcunque proportionales ſint, non ſit autẽ quadratus is qui vnitatem ſequitur, neq; alius vllus quadratus erit, demptis tertio ab vnitate ac omnibus

o Vnitas.	A	B	C	D	E	F
	3	9	36	81	243	729

nibus vnum intermittentibus. Quòd si qui vnitatem ſequitur, cubus non ſit, neque alius vllus cubus erit, demptis quarto ab vnitate ac omnibus duos intermittentibus.

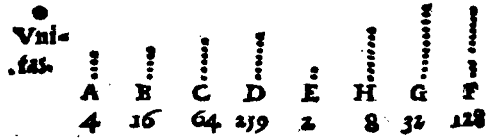
Theorema 11. Propoſitio 11.

Si ab vnitate numeri quotlibet deinceps proportionales ſint, minor maiorem metitur per quempiam eorum qui in proportionalibus ſunt numeris.

. : : : :
A D C D E
1 2 4 8 16

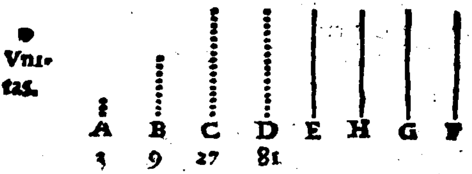
Theorema 12. Propoſitio 12.

Si ab vnitate quolibet numeri ſint proportionales, quot primorum numerorũ vltimum metiuntur, totidem & eum qui vnitati proximus eſt, metientur.



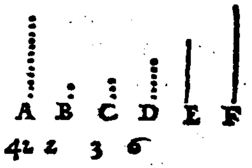
Theorema 13. Propoſitio 13.

Si ab vnitate ſint quotcunq; numeri deinceps proportionales, primus autẽ ſit qui vnitatẽ ſequitur, maximũ nullus alius metietietur, ijs exceptis qui in proportionalibus ſunt numeris.



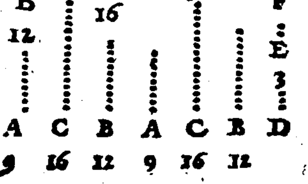
Theorema 14. Propoſitio 14.

Si minimum numerum primi aliquot numeri metiãtur, nullus alius numerus primus illum metietur, ijs exceptis qui primò metiuntur.



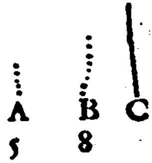
Theorema 15. Propoſitio 15.

Si tres numeri deinceps proportionales ſint minimi, eadem cum ipſis habẽtium rationem, duo quilibet compoſiti ad tertium primi erunt.



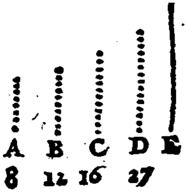
Theorema 16. propoſitio 16.

Si duo numeri ſint inter ſe primi, non ſe habebit quē-admodum primus ad ſecū dum, ita ſecundus ad quē-piam alium.



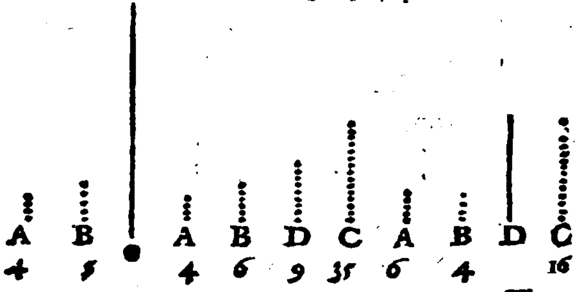
Theorema 17. Propoſitio 18.

Si ſint quotlibet numeri deinceps ꝓportionales, quorum extremi ſint inter ſe primi, non erit. quē admodum primus ad ſecundum, ita vltimus ad quempiam alium.



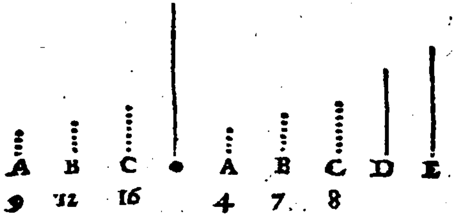
Theorema 18. Propoſitio 18.

Duobus numeris datis, cõſiderare noſſitne tertius illi inueniri proportionalis.



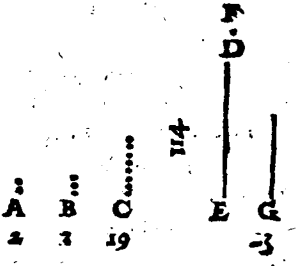
Theorema 19. Propositio 19.

Tribus numeris datis, conſiderare poſſit-ne quartus *illis* reperiri proportionalis.



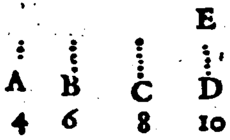
Theorema 20. Propo- ſitio 20.

Primi numeri plures ſunt qua cũque propoſi- ta multitudine primorum nu- merorum.



Theorema 12. Propoſitio 12.

Si pares numeri quot libet compoſiti ſint, totus eſt par.



Theorema 22. Propositio 22.

Si impares numeri quotlibet compositi sint, sit autem par illorum multitudo, totus par erit.

E
A B C D
5 9 7 5

Theorema 23. Propositio 23.

Si impares numeri quotcunque cõpositi sint, sit autem impar illorũ multitudo, & totus impar erit.

A B C E
5 7 8 1

B
A C
6 4

Theorema 24. Propositio 24.

Si de pari numero par detractus sit, & reliquus par erit.

Theorema 25. Propositio 25.

Si de pari numerò impar detractus sit, & reliquus impar erit.

B
A C D
8 1 4

Theorema 26. Propositio 26.

Si de impari numero impar detractus sit, & reliquus par erit.

B
A C D
4 6 1

Theo-

Theorema 27. Propositio 27.

Si ab impari numero par ablatus sit, reliquus impar erit.

A D C
1 4 4

Theorema 28. Propositio 28.

Si impar numerus parem multiplicans, procreet quempiam, procreatus par erit.

A B C
3 4 12

Theorema 29. Propositio. 29.

Si impar numerus imparẽ numerum multiplicans quemdam procreet, procreatus impar erit.

A B C
3 5 15

Theorema 30. Propositio 30.

Si impar numerus parẽ numerum metiatur, & illius dimidium metietur.

A C B
3 6 18

Theorema 31. Propositio 31.

Si impar numerus ad numerum quẽpiam primus sit, & ad illius duplum primus erit

A B C D
7 8 16

Theorema 32. Propositio 32.

Numerorum, qui à binario dupli sunt, vnusquisq; pariter par est tantùm.

Vnitas. A 2 B 4 C 8 D 16

Theorema 33. Propositio 33.

Si numerus dimidiũ impar habeat, pariter impar est tantùm.

A 20

Theorema 34. Propositio 34.

Si par numerus nec sit dupl' à binario, nec dimidium impar habeat, pariter par eũ, & pariter impar.

A 20

Theorema 35. Propositio 35.

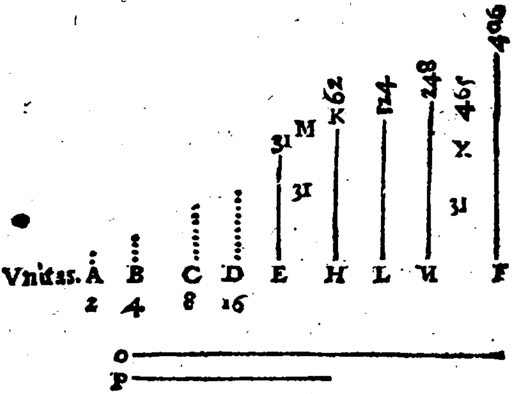
Si sint, quotlibet numeri deinceps proportionales, detrahãtur aũt de secundo & vltimo æquales ipsi primo, erit quemadmodum secũdi excessus ad primũ, ita vltimi excessus ad omnes qui vltimum antecedunt.

D 4 B 4 C 4 G D 16 F 4 K 4 E 16

Thee

Theorema 16. Propositio 16.

Si ab vnitate numeri quotlibet deinceps expositi sunt in duplici proportione quoad totus compositus primus factus sit, isq; totus in vltimum multiplicatus quempiã procreet, procreatus perfectus erit.



ELEMENTI IX. FINIS

EVCLI.

EVCLIDIS ELEMENTVM DECIMVM.

DEFINITIONES.

1

Commenſurabiles magnitudines dicuntur illæ, quas eadem menſura metietur.

2

Incommenſurabiles verò magnitudines dicuntur, hæ quarum nullam menſuram communem contingit reperiri.

3

Lineæ rectæ potentia cõmenſurabiles ſunt, quarum quadrata vna eadem ſuperficies ſiue area metitur.

4

Incommenſurabiles verò lineæ ſunt, quarum quadrata, quæ metiatur area communis, reperiri nulla poteſt.

5

Hęc cùm ita ſint, oſtendi poteſt quòd quantacunque linea recta nobis proponatur, exiſtunt etiam aliæ lineæ innumerabiles eidem commenſurabiles, aliæ item incommenſurabiles, hæ quidem longitudine & poten-

potentia: illæ verò potentia tantùm. Vocetur igitur linea recta. quantacunq; proponatur, ῥητὴ, id est rationali.

6

Lineæ quoq; illi ῥητῇ commensurabiles siue longitudine & potentia, siue potentia tantùm, vocentur & ipsæ ῥηταί, id est rationales.

7

Quæ verò lineæ sunt incommensurabiles illi τῇ ῥητῇ, id est primo loco rationali, vocẽtur ἄλογοι, id est irrationales.

8

Et quadratum quod à linea proposita describitur, quam ῥητὴν vocari volumus, vocetur ῥητόν.

9

Et quæ sunt huic commensurabilia, vocẽtur ῥητά.

10

Quæ verò sunt illi quadrato ῥητῷ scilicet incommensurabilia, vocentur ἄλογα, id est surda.

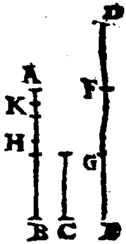
11

Et lineę quę illa incommensurabilia describunt, vocentur ἄλογοι. Et quidem si illa incommensurabilia fuerint quadrata, ipsa eorũ latera vocabuntur ἄλογοι lineæ. quòd si quadrata quidem non fuerint, verũ aliæ quępiã superficies siue figuræ rectilineæ, tunc

tunc verò lineæ illæ quæ deſcribunt quadrata æqualia figuris rectilineis, vocentur ἄλογοι.

Theorema 1. Propoſitio 1.

Duabus magnitudinibus inæqualibus propoſitis, ſi de maiore detrahatur plus dimidio, & rurſus de reſiduo iterũ detrahatur plus dimidio, idq́ſ ſemper fiat: relinquetur quædam magnitudo minor altera minore ex duabus propoſitis.



Theorema 2 Propoſitio. 2.

Duabus magnitudinibus propoſitis inæqualibus, ſi detrahatur ſemper minor de maiore, alterna quadam detractione neque reſiduum vnquam metiatur id quod ante ſe metiebatur, incommenſurabiles ſunt illæ magnitudines.

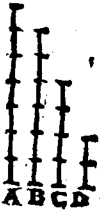
Problema 1. Propoſitio 3.

Duabus magnitudinibus commẽſurabilibus datis, maximam ipſarum communem menſuram reperire.

Proble-

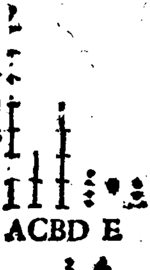
Problema 2. Propositio 4.

Tribus magnitudinibus cómensurabilibus datis, maximam ipsarú communé mensuram reperire.



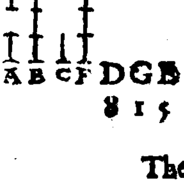
Theorema 3. Propositio 5.

Cómensurabiles magnitudines inter se proportioné eam habent, quam habet numerus ad numerum.



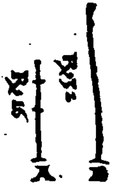
Theorema 4. Propositio 6.

Si duæ magnitudines proportionem eam habent inter se quam numerus ad numerum, commensurabiles sunt illæ magnitudines.



Theo-

Theorema 5. Propoſitio 7.



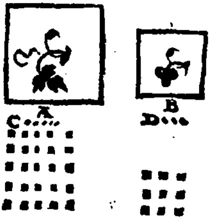
Incommenſurabiles magnitudines inter ſe proportionem non habēt, quam numerus ad numerum.

Theorema 6. Propoſitio 8.

Si duæ magnitudines inter ſe proportionem nō habēt quam numerus ad numerum, incommenſurabiles illę ſunt magnitudines.

Theorema 7. Propoſitio 9.

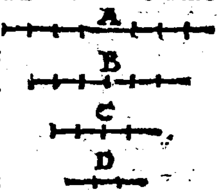
Quadrata, quæ deſcribuntur à rectis lineis lōgitudine cōmenſurabilibus, inter ſe proportionem habent quā numerus quadratus ad alium numerum quadratū. Et quadrata habentia proportionē



inter ſe quā quadratus numerus ad numerum quadratū, habent quoque latera longitudine commēſurabilia. Quadrata verò quæ deſcribuntur à lineis longitudine in cómenſurabilibus, proportioné non habent inter ſe, quam quadratus numerus ad numerum alium quadratum. Et quadrata non habentia proportionem inter ſe quam numerus quadratus ad numerum quadratum, neque latera habebunt longitudine commnnſurabilia.

Theorema 8. Propoſitio 10.

Si quatuor magnitudines fuerint proportionales, prima verò ſecundæ fuerit commẽſurabilis, tertia quoque quartę commenſurabilis erit. quòd ſi prima ſecũdæ fuerit incommenſurabilis, tertia quoq; quartæ incommenſurabilis erit.

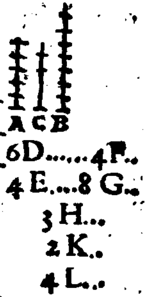


Problema 3. Propoſitio 11.

Propoſitæ lineæ rectæ (quam ῥητὴν vocari diximus) reperire duas lineas rectas incommenſurabiles, hanc quidem longitudiue tantùm, illam verò non longitudine tantùm, ſed etiam potentia incommẽſurabilem.

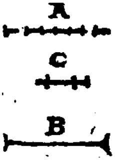
Theo-

Theorema 6. Propositio 12.



Magnitudines quæ eidem magnitudini sunt commensurabiles, inter se quoque sunt commensurabiles.

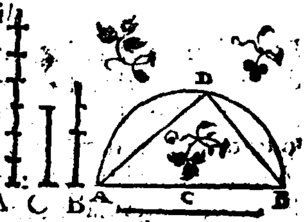
Theorema 10. Propositio 13.



Si ex duabus magnitudinibus hæc quidem commensurabilis sit tertię magnitudini, illa verò eidem incommensurabilis, incommẽsurabiles sunt illæ duæ magnitudines.

Theorema 11. Propositio 14.

Si duarum magnitudinum commensurabilium altera fuerit incommensurabilis magnitudini alteri cuipiam tertię, reliqua quoq; magnitudo eidem tertiæ incommensurabilis erit.



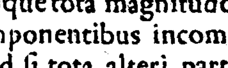
Theorema 12. Propositio 15.

Si quatuór rectæ proportionales fuerint, possit

possit autem prima plusquá secunda tanto quantum est quadratum lineę sibi commensurabilis longitudine: tertia quoq; ponterit plusquam quarta tanto quantum est quadratum lineæ sibi commensurabilis longitudine. Quòd si prima possit plusquam secunda quadrato lineę sibi longitudine incommensurabilis: tertia quoque poterit plusquam quarta quadrato lineę sibi incómésurablilis longitudine.

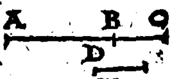
Theorema 13. Propositio 16.

Si duæ magnitudines commensurabiles componantur, tota magnitudo composita singulis partibus commensurabilis erit, quòd si tota magnitudo composita alterutri parti commensurabilis fuerit, illæ duæ quoque partes cómensurabiles erunt.



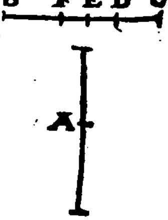
Theorema 14. Propositio 17.

Si duæ magnitudines incommensurabiles componantur, ipsa quoque tota magnitudo singulis partibus componentibus incommensurabilis erit. Quòd si tota alteri parti incommensurabilis fuerit, illæ quoque primę magnitudines inter se incommensurabiles erunt.



Theorema 15. Propositio 18.

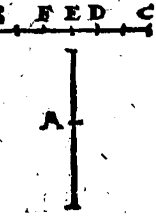
Si fuerint duæ rectæ lineæ inæquales, & quartæ parti quadrati quod describitur à minore æquale parallelongrammũ applicecetur secundũ maiorem, ex qua maiore tátum excurrat extra latus parallelogrammi, quantum est alterum latus ipsius parallelogrammi: si præterea parallelogrammum sui applicatione diuidat lineã illam in partes inter se commensurabiles longitudine, illa maior linea tanto plus potest quàm minor, quantũ est quadratum lineæ sibi commensurabilis longitudine. Quòd si maior quadratũ lineæ sibi commensurabilis lon-
plus possit quàm minor, tanto quantũ est
gitudine, & prætereà quartæ parti quadrati lineę minoris ęquale parallelogrammum applicetur secundum maiorem, ex qua maiore tantum excurrat extra



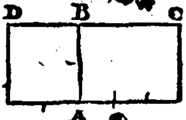
latus pallelogrammi, quantum est alterum latus ipsius parallelongrammi parallelogammum sui applicatione diuidit maiorẽ in partes inter se longitudine commensurabiles.

Theorema 16. Propositio 19.

Si fuerint duæ rectæ inæquales, quartæ autem parti quadrati lineæ minoris æquale le parallelogrammorum ſecundum lineam maiorem applicetur, ex qua linea tantum excurrat extra latus parallelogrammi, quãtum eſt alterum latus eiuſdem parallelogrammi: ſi parallelogrammum præterea ſui applicatione diuidat lineam in partes inter ſe longitudine incommenſurabiles, maior illa linea tantò plus poteſt quàm minor quantum eſt quadratum lineæ ſibi maiori incommenſurabiles longitudine. Quod ſi maior linea tantò plus poſſit quàm minor, quantum eſt quadratum lineæ incommenſurabilis ſibi longitudine: & preterea quartæ parti quadrati lineæ minoris æquale parallelogrammum applicetur ſecundum maiorem ex qua tantum excurrat extra latus parallelogrãmi, quantum eſt alterum latus ipſius: parallelogrammum ſui applicatione diuidit maiorem in partes inter ſe incommenſurabiles longitudine.



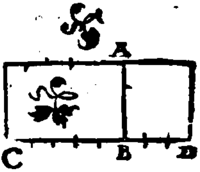
Theorema 17. Propoſitio 20.



Superficies rectangula contenta ex lineis rectis rationalibus longitudine commenſurabilibus ſecundum vnum aliquem modum ex antedictis, rationalis est.

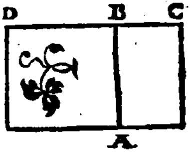
Theorema 18. Propositio 21.

Si ratinonale secundũ lineam rationalem applicetur, habebit alterum latus lineam rationalem & cõmensurabilem longitudine lineæ cui rationale parallelogrammum applicatur.



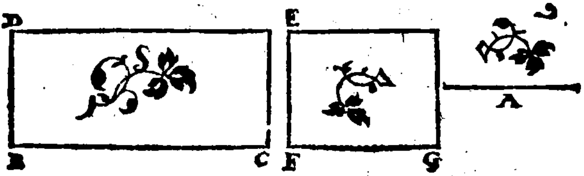
Theorema 39. Propositio 56.

Surperficies rerctangula contenta duabus lineis rectis rationalibus potentia tantum commẽsurabilibus, irrationalis est. Linea autem quæ illam superficiem potest, irrationalis & ipsa est: vocetur verò medialis



Theorema 20. Propositio 23.

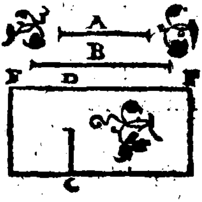
Quadrati lineæ medialis applicati secun-



dum

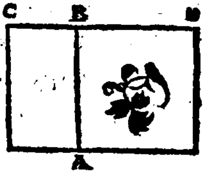
dum lineam rationalem, alterum latus est linea rationalis, & incommensurabilis longitudine lineæ secundum quam applicatur.

Theorema 21. Propositio 24



Linea recta mediali commẽsurabilis, est ipsa quoque medialis.

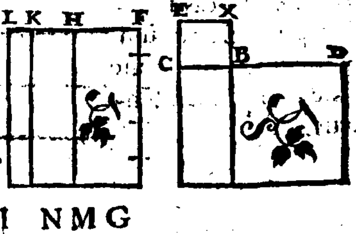
Theorema 22. Propositio 25



Parallelongrammum rectangulum contẽtum ex lineis medialibus longitudine commensurabilibus, mediale est.

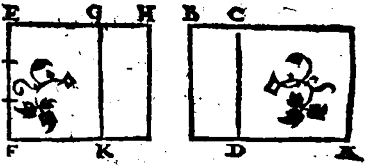
Theorema 23. Propositio 26.

Parallelogrammum rectangulum comprehensum duabus lineis medialibus potentia tãtùm cõmensurabilibus, vel rationale est, vel mediale.



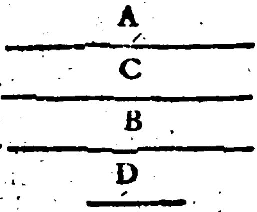
Theorema 24. Propositio 27.

Mediale nó est maius quàm mediale superficie rationali.



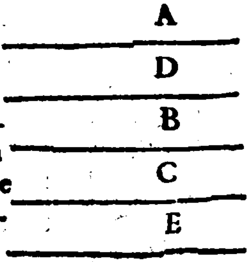
Problema. 4. Propositio 28.

Mediales lines inuenire potentia tantùm commensurabiles rationale comprehendentes.



Problema 5. Propositio 29.

Mediales lineas inuenire potentia tantùm commensurabiles mediale comprehendendentes



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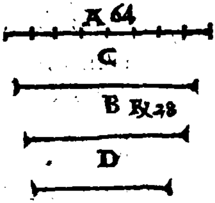
Problema 6. Propositio 30.

Reperire duas rationales potentia tantum commẽsurabiles huiusmodi, vt maior ex illis possit plus quàm minor quadrato lineæ sibi commensurabilis longitudine.

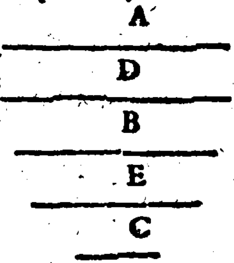
Problema 7. Propositio 31.

Reperire duas lineas medialis potentia tantùm commensurabiles rationalem superficiẽ continẽtes, tales inquã vt maior possit plus quàm minor quadrato lineæ sibi commẽsurabilis longitudine.



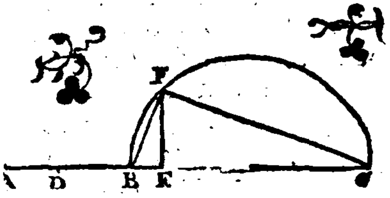
Problema 8. Propositio 32.

Reperire duas lineas mediales potentia tantùm commensurabiles medialem superficiem continentes, huiusmodi vt maior plus possit quàm minor quadrato lineæ sibi commensurabilis longitudine.



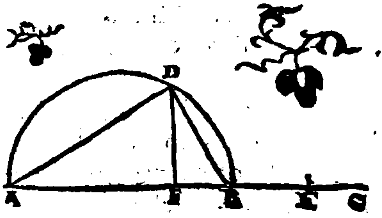
Problema 9. Propositio 33.

Réperire duas rectas poutentia incommẽsurabiles, quarum quadrata simul addita faciant superficiẽ rationalẽ, parallelogrammum verò ex ipsis conetntum sit mediale.



Problema 10. Propositio 34.

Reperire lineas duas rectas potentia incommensurabiles cónficentes compositum ex ipsarum quadratis mediale, parallelogrammũ verò ex ipsis contentum rationale,



Problema 11. Propositio 85.

Reperire duas lineas rectas potentia incõmensurabiles, conficientes id quod ex ipsarum quadratis componitur meale, simulque

que parallelogrammũ ex ipsis contentum, mediale, quod præterea parallelogrammũ sit incómensurabile composito ex quadratis ipsarum.

PRINCIPIVM SENARIOrum per compositionem.

Thorema 25. Propositio 36.

Si duæ rationales potentia tantùm commensurabilis. Vocetur autem Binommum.

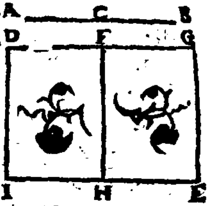
Theorema 26. Propositio 37.

Si duæ mediales potentia tantòm commensurabiles rationale continentes componantur, tota linea est irrationalis, vocetur autem Bimediale prius.

The

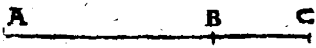
Theorema 27. Propoſitio 38.

Si duæ mediales Potētia tantùm commēſurabiles mediale cōtinētes componātur, tota linea eſt irrationalis: vocetur autē Bimediale ſecundum.



Theorema 28. Propoſitio 36.

Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex quadratis ipſarum rationale, parallelogrammum verò ex ipſis contentum mediale, tota linea recta eſt irrationalis. Vocetur autē linea maior.

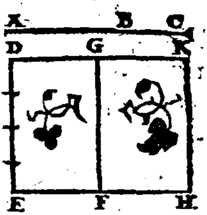


Theorema 39. Propoſitio 40.

Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex ipſarum quadratis mediale, id vero ꝙ fit ex ipſis, rationale, tota linea eſt irrationalis. Vocetur autem potens rationale & mediale.

Theorema 30. Propoſitio 41.

Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex quadratis ipſarum mediale, & quod continetur aetur ex ipsis, mediale, & præterea incõmensurabile composito ex quadratis ipsarum tota linea est irrationalis. Vocetur autem potens duo medialia.



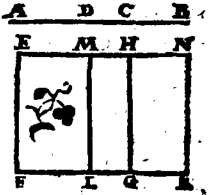
Theorema 31. Propositio 42.

Binominum in vnico tantùm pũcto diuiditur in sua nomina, id est in lineas ex quibus componitur.

Theorema 32. Propositio 43.

Bimediale prius in vnico tantum puncto diuiditur in sua nomina.

Problema 33. Propositio 44



Bimediale secundum in vnico tantũ puncto diuiditur in sua nomina.

Problema 34. Propositio 45.

Linea maior in vnico tantùm in pũcto diuiditur in sua nomina.

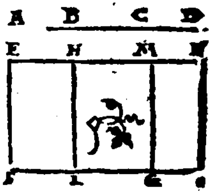
Theo-

Theorema 35. Propositio 46.

Linea potens rationale & mediale in vnico tantùm puncto diuiditur in sua nomina.

Theoroma 36. Propositio 47

Linea ootèns duo medialia in vnico tantùm puncto diuiditur in sua nomina.



DEFINITIONES. secundæ.

Proposita linea rationali, & binomio diuiso in sua nomine, cuius binomij maius nomen, id est, maior portio possit plusquam minus nomen quadrato lineæ sibi, maiori inquam nomini, commensurabilis longitudine.

1.

Si quidẽ maius nomen fuerit commensurabile longitudine propositæ lineæ rationali, vocetur toto linea Binomum primium.

2

Si veró minus nomen, id est minor portio binomij, fuerit commensurabile longitudine ne propositæ lineæ rationali, vocetur tota linea Binomium secundum.

3

Si verò neutrum nomen fuerit commensurabile longitudine propositæ lineæ rationali, vocetur Binomium tertium.

Rursus si maius nomen possit plusquam minus nomen quadrato lineæ sibi incommensurabilis longitudine.

4

Si quidem maius nomen est commensurabile longitudine propositæ lineæ rationali, vocetur tota linea Binomium quartum.

5

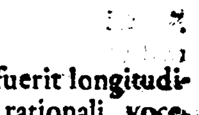
Si verò minus nomen fuerit commensurabile longitudine lineæ rationali, vocetur Binomium quintum.

6

Si verò neutrum nomen fuerit longitudine commensurabile lineæ rationali, vocetur illa Binomium sextum.

Proble. 12. Propositio 48

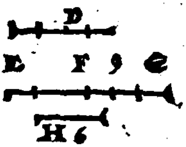
Reperire Binomium primum.



Problema 13. Propositio 49

9 3
A........C..B
12

Reperire Binomium secundum.

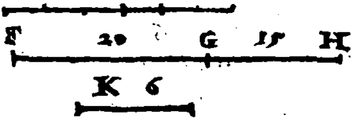


Problema 14. Propositio 50.

15 5
A..............C....
20

D
....
E

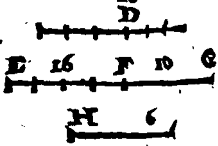
Reperire Binomium tertiũ.



Problema 15. Propositio 51.

10 6
A........C......B
16
D

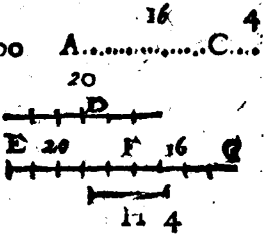
Repere Binomiũ quartum.



Proble.

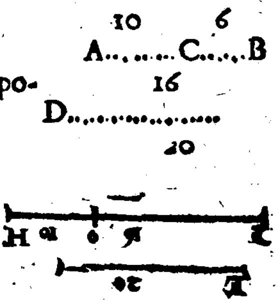
Problema 16. Propositio 52

Reperire Binomiun quintum.



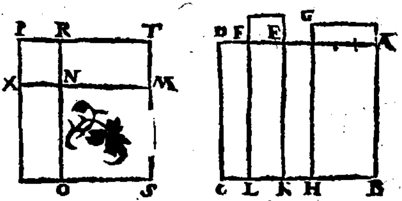
Probl. 17. Propositio 53.

Reperire Binonimum sextum.



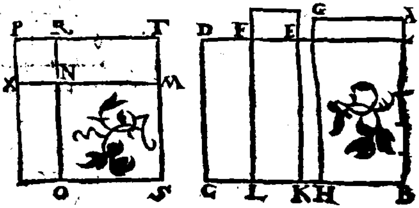
Theorema 37. Propositio 54.

Si superficies contẽta fuerit ex rationali & Binomio primo, linea quę illã superficiem potest, est irrationalis, quæ Binomiũ vocatur.



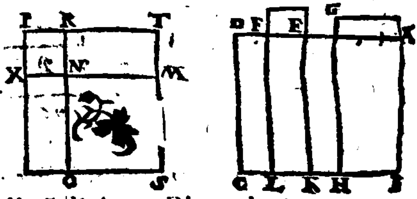
Theorema 38. Propositio 55.

Si superficies cõtenta fuerit ex linea rationali & Binomio secundo, linea potens illam superficiẽ est irrationalis quę Bimediale primum vocatur.



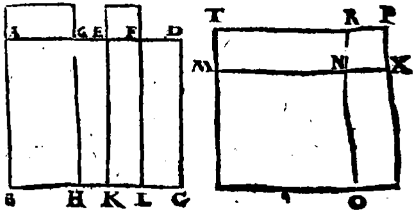
Theorema 39. Propositio 56.

Si superficies cõtineatur ex rationali & Binomio tertio, linea q̃ illam superficiem potest, est irrationalis q̃ dicitur Bimediale secundũ.



Theorema 40. Propositio 57.

Si superficies contineatur ex rationali & Boin

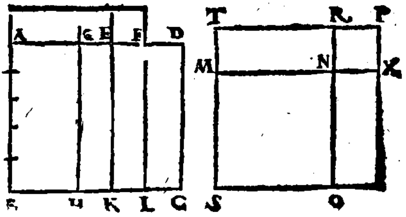


mio

mio quarto. linea potens ſuperfi.iem illam, eſt irrationalis, quæ dicitur maior.

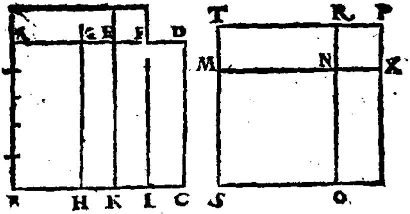
Theorema 41. Propoſitio 58.

Si ſuperficies contineatur ex rationali & Binomio quinto, linea quæ illam ſuperficiem poteſt eſt irrationalis, quę dicitur potés rationale & mediale.

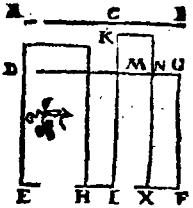


Therema 42. Propoſitio 59

Si ſuperficies contineatur ex rationali & Binomio ſexto, linea quæ illam ſuperficiem poteſt eſt irrationalis, quæ dicitur potens tuo mediala.

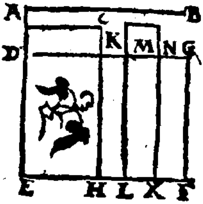


Theorema 43. Propositio 60.



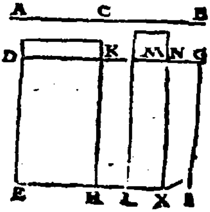
Quadratum Binomij secundum lineam rationalem applicatum, facit alterum latus Binomium primum.

Theorema 44. Propositio 61.



Quadratũ Bimedialis primi secundum rationalem lineam applicatum, facit alterum latus Binomium secundum.

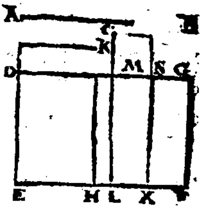
Theorema 45. Propositio 62.



Quadratũ Bimedialis secũdi secundũ rationalem applicatum, facit alterũ latus Binomimũ tertium.

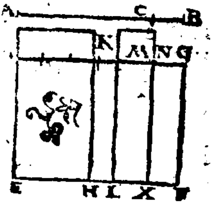
Theorema 46. ropoſitio 63

Quadratum lineæ maioris ſecundum lineam rationalem applicatum, facit alterum latus Binomium quartum.



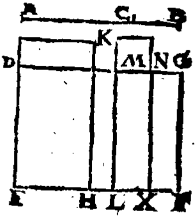
Theorema 47. Propoſitio 64.

Quadratum lineæ potẽtis rationale & mediale ſecundum rationalẽ applicatũ, facit alterum latus Binomium quintum.



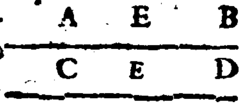
Theorema 48. Propoſitio 65

Quadratum lineę potentis duo medialia ſecundũ rationalem applicatum, facit alterum latus Binomium ſextum.



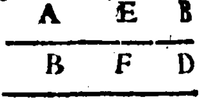
Theorema 49. Propoſitio 66.

Linea lõgitudine commenſurabilis Binomio eſt & ipſa Binomium eiuſdem ordinis.



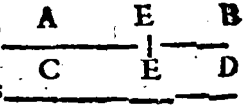
Theorema 50. Propositio 67

Linea longitudine commensurabilis alteri bimedialum, est & ipsa bimediale etiam eiusdẽ ordinis.



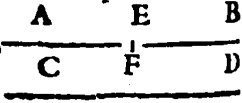
Theorema 51. Propositio 68

Linea cõmensurabilis lineæ maiori, est & ipsa maior.



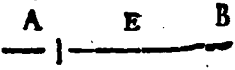
Theorema 52. Propositio 69.

Linea commensurabilis lineæ potenti rationale & mediale, est & ipsa linea potens rationale & mediale.



Theorema 53. Propositio 70.

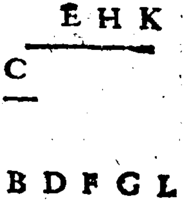
Linea commensurabilis lineæ potenti duo medialia, est & ipsa linea potens duo medialia.



Theorema 54 Propositio 71

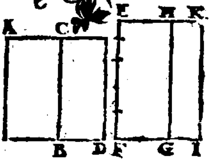
Si duæ superficies rationalis & medialis simul componantur, linea quę totã superficiem

ciem compoſitã poteſt, eſt vna ex quatuor irrationalibus, vna ex quæ dicitur Binomium, vel bimediale primum. vel linea maior, vel linea potẽs rationale & mediale.



Theorema 55. Propoſitio 72.

Si duæ ſuperficies mediales incommenſurabiles ſimul componantur fiunt reliquæ duæ lineę irrationales, vel bimediale ſecundum, vel linea potẽs duo medialia



SCHOLIVM.

Binomium & cæteræ conſequentes lineæ irrationales, neque ſunt eadem cum linea mediali, neque ipſæ inter ſe.

Nam quadratum lineæ medialis applicatum ſecundum lineam rationalem facit alterum latus lineam rationalem, & longitudine incommenſurabilem lineæ ſecundum quam applicatur, hoc eſt, lineæ rationali, per 23.

Quadratum verò Binomij ſecundum rationalem applicatum, facit alterum latus Binomium primum, per 60.

Quadratum verò Bimedialis primi secundum rationalem applicatum facit alterum latus Binomium secundum. per 61

Quadratum verò Bimedialis secundi secundum rationalem applicatum; facit alterum latus Binomiũ tertium per 62.

Quadratum verò lineæ maioris secundum rationalem applicatum facit alterum latus Binomiũ quartum, per 63

Quadratum verò lineæ potẽtis rationale & mediale secundum rationalem applicatum facit alterũ latus Binomium quintum, per 64.

Quadratum vero lineæ potentis duo medialia secũdum rationalem applicatum, facit alterum latus Binomium sextum per 65.

Cùm igitur dicta latera, quæ latitudines vocantur, differant & à prima latitudine, quoniam est rationalis, cùm inter se quoque differant, eo quia sunt Binomia diuersorum ordinum: manifestum est ipsas lineas irrationales, differentes esse inter se.

SECVN.

SECVNDVS ORDO ALterius ſermonis, qui eſt de detractione.

Principium ſenariorum per detractionem.

Theorema 57. Propoſitio 74.

Si de linea rationali detrahatur rationalis potentia tantùm commenſurabilis ipſi toti, reſidua eſt irrationalis, vocetur autem Reſiduum.

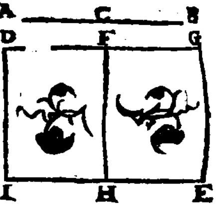
Theorema 56. Propoſitio 57.

Si de linea mediali detrahatur medialis potentia tantũ commenſurabilis toti lineæ, quæ verò detracta eſt cũ tota contineat ſuperficiem rationalem, reſidua eſt irrationalis. Vocetur autem Reſiduum mediale primum.

Theorema 58 Propoſitio. 75.

Si de linea mediali detrahatur medialis

tentia tantùm commensurabilis toti, quæ verò detracta est, cùm tota cõtineat superficiẽ medialẽ, reliqua est irrationalis. Vocetur autem Residuũ mediale secundum.



Theorema 57. Propositio 76

Si de linea recta detrahatur recta potẽtia incommensurabilis toti, compositum autem ex quadratis totius lineæ & lineæ detractæ sit rationale, parallelogrammum verò ex ijsdem contentum sit mediale, reliqua linea erit irrationalis. Vocetur autem linea minor.

Theorema 58. Propositio 77.

Si de linea recta detrahatur recta potentia

tia incommensurabilis toti lineæ compositum autem ex quadratis totius & lineæ detractæ sit mediale, parallelogrammum verò bis ex eisdem contentum sit rationale, reliqua linea est irrationalis. Vocetur autem linea faciens cum superficie rationali totam superficiem medialem.

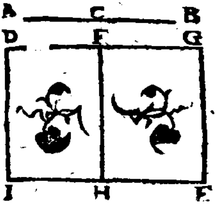
A C B

Theorema 59. Propositio 78.

Si de linea recta detrahatur recta potentia incommensurabilis toti lineæ, compositum autem ex quadratis totius & lineæ detractæ sit mediale, parallelogrammum verò bis ex ijsdem sit etiam mediale: præterea sint quardata ipsarum incommensurabilia parrallelogrammo bis ex ijsdem contento, reliqua linea est irrationalis.

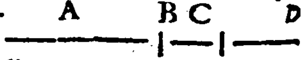
Vocetur autem linea faciens cum superficie

ficie mediali totam superficiem medialem.



Theorema 60. Propositio 79.

Residuo vnica tantùm linea recta cóiungitur rationalis potentia tantùm cómensurabilis toti lineæ rationalem.

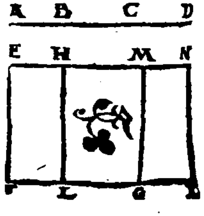


Theorema 61. Propositio 80.

Residuo mediali primò vnica tãtùm linea coniungitur medialis, potentia tantùm cómensurabilis toti, ipsa cum tota continens

Theorema 62. Propositio 81.

Residuo mediali secundo vnica tantùm coniungitur mediails, potentia tantùm commẽsurabilia toti ipsa cum tota continens mediale.

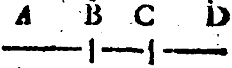


Theorema 63. Propositio 82.

Lineæ minori vnica tantùm recta cóiungitur

tur

tur potentia incommensurabilis toti, faciens cum tota compositum ex quadratis ipsarum rationale, id verò parallelogrammum, quòd bis ex ipsis sit, mediale.

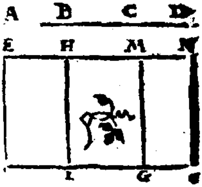


Theorema 64. Propositio 83.

Lineæ facienti cum superficie rationali totam superficiem medialem, vnica tantùm coniungitur linea recta potentia incommensurabilis toti, faciens autem cum tota compositum ex quadratis ipsarũ, mediale, id verò quod fit bis ex ipsis, rationale.

Theorema 65. Propositio 84.

Lineæ cum mediali superficie facienti totam superficiem medialem, vnica tantum coniũgitur linea potẽtia toti incommensurabilis, faciens cum tota compositũ ex quadratis ipsarũ mediale, id verò quod fit bis ex ipsis etiam mediale, & præterea faciens compositum ex quadratis ipsarum incommensurabile ei quod fit bis ex ipsis,



DE.

DEFINITIONES. TERTIAE

Propoſita linea rationali & reſiduo.

1

Si quidem tota, nempè compoſita ex ipſo reſiduo & linea illi cõiuncta, plus poteſt quàm coniuncta, quadrato lineæ ſibi cõmenſurabilis longitudine, fueritq; tota longitudine commẽſurabilis lineæ propoſitæ rationali, reſiduũ ipſum vocetur Reſiduum primum.

2

Si verò coniuncta fuerit longitudine commenſurabilis rationali, ipſa autem tota plus poſſit quam coniuncta, quadrato lineæ ſibi longitudine commenſurabilis, reſiduum vocetur Reſiduum ſecundũ.

3

Si verò neutra linearũ fuerit longitudine commenſurabilis rationali, poſſit autem ipſa tota plusquam coniuncta, quadrato lineæ ſibi longitudine commẽſurabilis, vocetur Reſiduum tertium.

Rurſus ſi tota poſſit plus quàm coniuncta quadrato lineæ ſibi longitudine incommenſurabilis

4

Et quidem ſi tota fuerit longitudine com- men-

mensurabilis ipsi rationali, vocetur Residuum quartum.

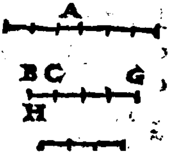
5

Si verò coniuncta fuerit lógitudine commensurabilis rationali, & tota plus possit quàm coniuncta, quadrato lineæ sibi longitudine incõmensurabilis, vocetur Residuum quintum.

6

Si verò neutra linearum fuerit cómensurabilis longitudine ipsi rationali, fueritque tota potentior quã coniuncta, quadrato lineæ sibi longitudine incommensurabilis, vocetur Residuum sextum.

Problema 18. Propositio 85.



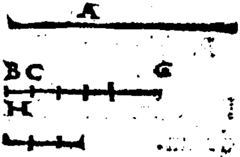
Reperire primum Residuum.

16

D.....F.....E

9 7

Problema 19. Propositio 86.



Reperire secundum Residuum.

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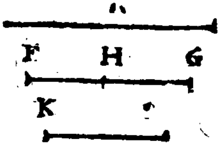
12

B........E.....C

9 7

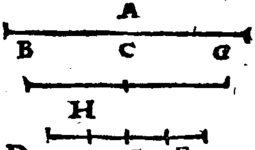
Problema 02. Propositio 87.

Reperire tertium Residuum.



Probl. 21 Propositio 88.

Reperire quartum Residuum.



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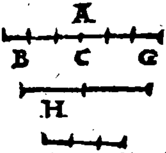
Problema 22. Propositio 89

Reperire quintum Residuum

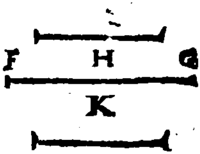


D......F....E

25 7

Problema 23. Propositio 90.

Reperire sextum Residuum.

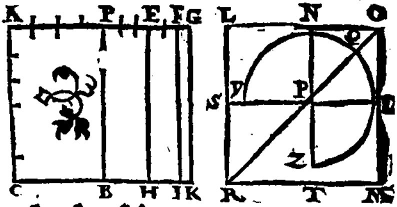


E........... 13

Theo- B........D......C

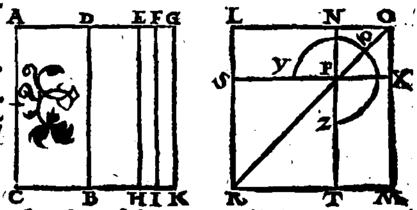
Theorema 66. Propositio 91.

Si superficies contineatur ex linea rationali & residuo primo linea, quæ illam superficiem potest, est residuum.



Theorema 67. Propositio 92.

Si superficies cõtineatur ex linea rationali & residuo secundo, linea q̃ illam supficiẽ potest, est residuum mediale primum.



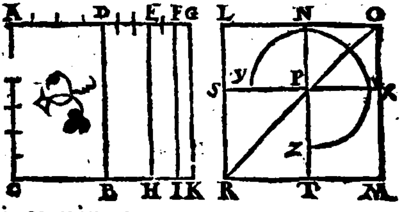
Theorema 68. Propositio 93.

Si superficies cõtineatur ex linea rationali & residuo ter-

tio, linea quæ illam ſuperficiem poteſt, eſt reſiduum mediale ſecundum.

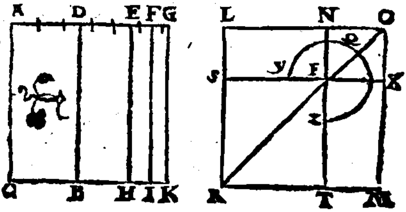
Theorema 69. Propoſitio 94.

Si ſuperficies contineatur ex linea rationali & reſiduo quarto linea q̃ illã ſuperficiem poteſt, eſt linea minor.



Theorema 70. Propoſitio 95.

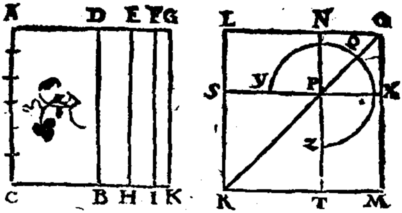
Si ſuperficies contineatur ex linea rationali & reſiduo quinto, linea quæ illã ſuperficiẽ poteſt, eſt ea quæ dicitur cum rationali ſuperficie faciens totam medialem.



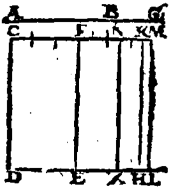
Theorema 71. Propopoſitio 96.

Si ſuperficies contineatur ex linea rationali &

& residuo sexto, *linea* quæ illam superficiē pōt, est ea quę dicitur faciēs cum mediali superficie totam medialem.

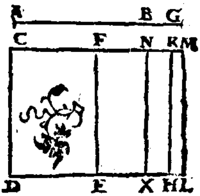


Theorema 72. Propositio 97.



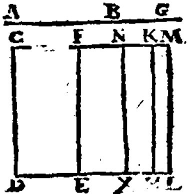
Quadratum residui secundum lineam rationalem applicatum, facit alterum latus redsiuum primum.

Theorema 73. Propositio 98.



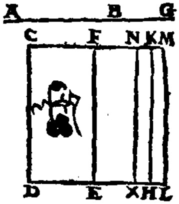
Quadratum residui medialis pıimi secundū rationalem applicatū, facit alterum latus residuū secundum.

Theorema 74. Propositio 99.



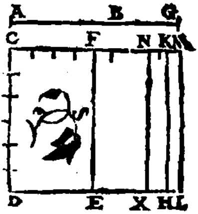
Quadratū, residui medialis secundi secundum rationalē applicatū, facit alterū latus residuum tertium.

Theorema 75. Propositio 100.



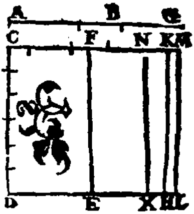
Quadratũ lineæ minoris secundùm rationalẽ applicatum, facit alterũ latus residuum quartũ.

Theorema 76. Propositio 101.



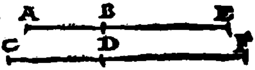
Quadratũ lineæ cum rationali superficie faciẽtis totam medialem, secũdum rationalem applicatum, facit alterum latus residuum quintum.

Theorema 77. Propositio 102.



Quadratum lineæ cum mediali superficie faciẽtis totam medialem, secundum rationalem applicatum, facit alterum latus residuum sextum.

Theorema 78. Propositio 103.

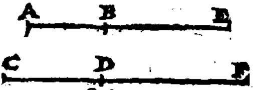


Linea residuo commensurabilis longitudine, est & ipsa residuum, & eiusdem ordinis

Theorema 79. Propositio 104.

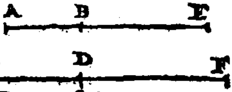
Li.

Linea cõmenſurabilis reſiduo mediali, eſt & ipſa reſiduũ mediale, & eiuſdẽ ordinis.



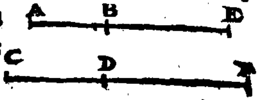
Theorema 80. Propoſitio 105.

Linea cõmẽſurabilis lineę minori, eſt et ipſa linea minor



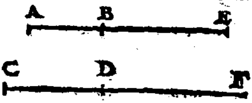
Theorema 81. Propoſitio 106.

Linea commenſurabilis lineæ cũ rationali ſuperficie facienti totã medialẽ, eſt & ipſa linea cum rationali ſuperficie faciens totam medialem.



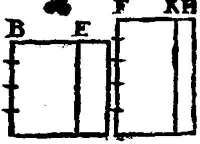
Theorema 82. Propoſitio 107.

Linea commẽſurabilis lineæ cũ mediali ſuperficie facienti totam medialem, eſt & ipſa cum mediali ſuperficie faciens totam medialem.



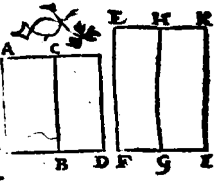
Theorema 83. Propoſitio 108.

Si de ſuderficie rationali detrahatur ſuperficies medialis, linea quæ reliquã ſuperficiem poteſt, eſt alterutra ex duabus irrationalibus, aut reſiduum, aut linea minor.



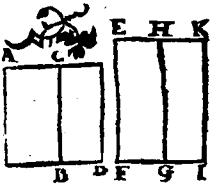
Theorema 84. Propositio 109

Si de ſuperficie mediali detrahatur ſuperficies rationalis, aliæ duæ irrationales, fiũt, aut reſiduum mediale primũ aut cũ rationali ſuperficiem faciens totam medialem.



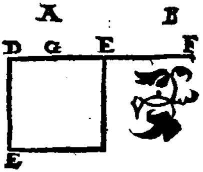
Theorema 85. Propositio 110.

Si de ſuperficie mediali detrahatur ſuperficies medialis q̃ ſit incommẽſurabilis toti, reliquę duæ fiunt irrationales, aut reſiduum mediale ſecundum, aut cũ mediali ſuperficie faciens totam medialem.



Theorema 86. Propositio 111.

Linea quæ Reſiduum dicitur, nõ eſt eadem cum ea quæ dicitur Binomium.



SCHO.

SCHOLIVM.

Linea quæ Residuum dicitur, & cætera quinque eã consequentes irrationales, neque lineæ mediali neque sibi ipsa inter se sunt eadem. Nam quadratum lineæ medialis secundum rationalem applicatum facit alterum latus, rationalem lineam longitudine incommensurabilem ei, secundum quam applicatur. per 23.

Quadratum, verò residui secundum rationalem applicatũ, facit alterũ latus residuũ primũ, per 97.

Quadratum verò residui medialis primi secundum rationalem applicatum, facit alterum latus residuum secundum, per 98.

Quadratum verò residui medialis secundi, facit alterum latus residuum tertium, per 99.

Quadratum verò lineæ minoris, facit alterũ latus residuum quartum, per 100.

Quadratum verò lineæ cum rationali superficie facientis totam medialem, facit alterum latus residuum quintum, per 101.

Quadratum verò lineæ cum mediali superficie facientis totam medialem, secundum rationalem applicatum, facit alterum latus residuum sextum, per 102.

Cùm igitur dicta latera, quæ sunt latitudines cuiusque parallelogrammi vnicuique quadrato æqualis & secundum rationalem, applicati, differant & à primo latere, & ipsa inter se (nam à primo differunt: quoniam sunt resi-

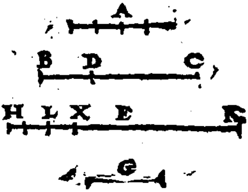
duæ non eiusdem ordinis) constat ipsas quoque lineas irrationales inter se differentes esse. Et quoniam demonstratum est, Residuum non esse idem quod Binomium, quadrata autem residui & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex residuis eiusdem ordinis cuius sunt & residua, quorum quadrata applicantur rationali similiter & quadrata Binomij & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex Binomijs eiusdem ordinis, cuius sunt & Binomia, quorum quadrata applicãtur rationali. Ergo lineæ irrationales quæ cõsequuntur Binomium, & quæ consequuntur residuũ, sunt inter se differẽtes. Quare dictæ lineæ omnes irrationales sunt numero 13.

1 Medialis.
2 Binomium.
3 Bimediale primum.
4 Bimediale secundum.
5 Maior.
6 Potens rationale & mediale.
7 Potẽs duo medialia.
8 Residuum.
9 Residuum mediale primum.
10 Residuum mediale secundum.
11 Minor.
12 Faciens cum rationali superficie totam medialem
13 Faciens cum mediali superficie totam medialem.

Theo-

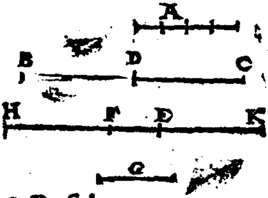
Theorema 87. Propoſitio 112.

Quadratũ lineæ rationalis ſecundum Binomiũ applicatũ, facit alterum latus reſiduũ, cuius nomina ſunt cõmenſurabilia Binomij nominibus, & in eadẽ proportione præterea id quod fit Reſiduum, eundem ordinem retinet quem Binomium.



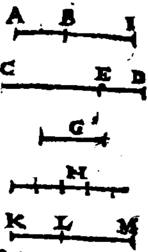
Theorema 88. Propoſitio 113.

Quadratũ lineæ rationalis ſecundum reſiduum applicatũ, facit alterum latus Binomium, cuius nomina ſunt commenſurabilia nominibus reſidui & in eadẽ proportione: prę-terea id quod fit Binomiũ, eſt eiuſdem ordinis, cuius & Reſiduum.



Theorema 89 Propoſitio 114.

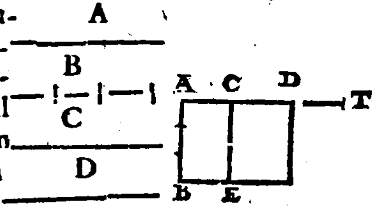
Si parallelogrammum contineatur ex reſi-



duo & Binomio, cuius nomina sunt commensurabilia nominibus residui & in eadẽ proportione, linea quæ iliam superficiem potest, est rationalis.

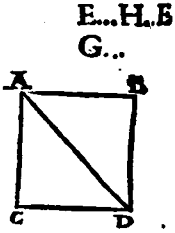
Theorema 90. Propositio 115.

Ex linea mediali nascũtur lineę irrationales innumerabiles, quarum nulla vlli ante dictarum eadem sit.



Theorema 100. Propositio 116.

Propositũ nobis esto demonstrare in figuris quadratis diametrũ esse longitudine incommensurabilem ipsi lateri.



ELEMENTI X. FINIS.

EVCLIDIS ELEMENTVM VNDECIMVM, ET SOLIDORVM.

primum.

DEFINITIONES.

1

Solidum, eſt quod longitudinem, latitunem, & craſſitudinem habet.

2

Solidi autem extremum eſt ſuperficies.

3.

Linea recta eſt ad planum recta cùm ad rectas oẽs lineas, à quibus illa tangitur, quæque in propoſitio ſunt plano, rectos angulos efficit.

4

Planum ad planum rectum eſt, cum rectæ lineæ, quæ cõmuni planorum ſectioni ad rectos angulos in vno planorum ducũtur, alteri plano ad rectos ſunt angulos.

5

Rectæ lineę ad planũ inclinatio acutus eſt angulus, ipſa inſiſtente linea & adiũcta altera cõprehẽſus, cùm à ſublimi rectæ illius lineę termino deducta fuerit ꝑpẽndicularis,

laris,atq; à puncto quod perpendicularis in ipso plano fecerit,ad propositę illius lineæ extremum, quod in eodem est plano, altera recta linea fuerit adiuncta.

6

Plani ad planũ inclinatio, acutus est angulus rectis lineis contentus, quæ in vtroque planorum ad idem communis sectionis pũctũ ductæ, rectos ipsi sectioni angulos efficiunt.

7

Planum similiter inclinatum esse ad planum,atq; alterum ad alterum dicitur,cum dicti inclinationum anguli inter se sunt ęquales.

8

Parallela plana,sunt quæ eodem non incidunt,nec concurrunt.

9

Similes figuræ solidæ, sunt quæ æqualibus planis,multitudine æqualibus continetur.

10

Aequales & similes figuræ solidæ sunt, quę similibus planis, multitudine & magnitudine æqualibus continentur.

11

Solidus angulus,est plurium quàm duarũ linearum, quæ se mutuò contingant, nec in eadem sint superficie, ad omnes lineas inclinatio. Aliter.

Aliter.

Solidus angulus, est qui pluribus quã duobus planis angulis, in eodem non cõsistentibus plano, se ad vnum punctum collectis continetur.

12

Pyramis, est figura solida quæ planis continetur, ab vno plano ad vnum punctũ collecta.

13

Prisma, figura est solida quę planis continetur, quorum aduersa duo sunt & æqualia & similia & parallela, alia verò parallelogramma.

14

Sphęra est figura, quæ cõuerso circum quiescentem diametrum semicirculo continetur, cùm in eundem rursus locũ restitutus fuerit, vnde moueri cœperat.

15

Axis autem sphæræ, est quiescẽs illa linea circum quam semicirculus conuertitur.

16

Centrum verò Sphęrę est idem, quod & semicirculi.

17

Diameter autem Sphæræ, est recta quædã linea per centrum ducta, & vtrinq; à sphęræ superficie terminata.

Conus

18

Cωnus est figura, quæ conuerso circũ quiescens alterum latus eorum quæ rectum angulum continent, orthogωnio triangulo continetur, cùm in eundem rursus locum illud triangulum restitutum fuerit, vnde moueri cœperat. Atq; si quiescens recta linea æqualis sit alteri, quę circum rectũ angulum conuertitur, rectangulus erit Cωnus: sin minor, amblygωnius: si verò maior, oxygonius.

19

Axis autem Cωni, est quiescens illa linea, circum quam triangulum vertitur.

20

Basis verò Cωni, circulus est, qui à circũducta linea recta describitur.

21

Cylindrus figura est, quæ conuerso circum quiescens alterum latus eorum quæ rectum angulum continent, parallelogrammo orthogωnio comprehenditur, cum in eũdem rursus locum restitutum fuerit illud parallelogrammũ, vnde moueri cœperat.

22

Axis autem Cylindri est quiescens illa recta linea, circum quam parallelogrammum vertitur.

23

Bases verò cylindri, sunt circuli à duobus aduerſus lateribus quæ circum aguntur, deſcripti.

24

Similes coni & cylindri, ſunt quorũ & axes & baſium diametri proportionales ſunt.

25

Cubus eſt figura ſolida, quæ ſex quadratis æqualibus continetur.

26

Tetraedum eſt figura, quæ triangulis quatuor æqualibus & æquilateris continetur.

27

Octaedrũ figura eſt ſolida, quæ octo triangulis æquilibus & æquilateris continetur.

28

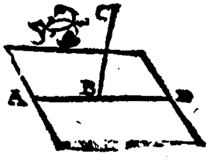
Dedecaedrum figura eſt ſolida, quæ duodelim pentagonis æqualibus, æquilateris. & æquiangulis continetur.

29

Eicoſaedrum figura eſt ſolida, quę triangulis viginti æqualibus & æquilateris continetur.

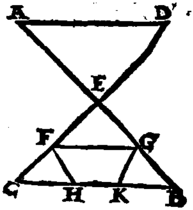
Theorema 1. propoſitio 1.

Quædã rectę lineę pars in ſubiecto quidem non eſt plano, quædam verò in ſublimi.



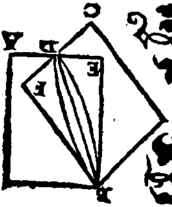
Theo

Tehorema 2. Propoſitio 2.



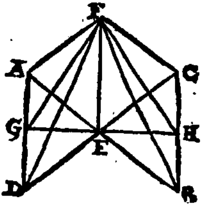
Si duę rectę lineę ſe mutuò ſecẽt, in vno sũt plano: atque triangulum omne in vno eſt plano.

Theorema 3. Propoſitio 3.



Si duo plana ſe mutuò ſecent, communis eorũ ſectio eſt recta linea.

Theorema 4. Propoſitio 4.



Si recta linea rectis duabus lineis ſe mutuò ſecantibus, in cõmuni ſectione ad rectos angulos inſiſtat illa ducto etiam per ipſas plano ad angulos rectos erit.

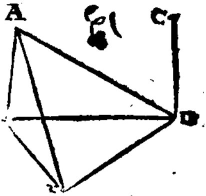
Theorema 5. Propoſitio 5.

Si recta linea rectis tribus lineis ſe mutuò tangentibus, incommuni ſectione ad rectos angulos inſiſtat, illæ tres rectę in vno ſunt plano.

Theo-

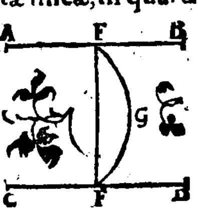
Theorema 6. Propositio 6.

Si duæ rectæ lineæ eidē plano ad rectos sint angulos, parallelæ erunt illæ rectæ lineæ



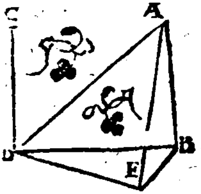
Theoréma 7. Propositio 7.

Si duæ sint parallelæ rectæ lineæ, in quarũ vtraque sumpta sint quę libet pūcta, illa linea quę ad hæc puncta adiungitur, in eodem est cũ parallelis plano.



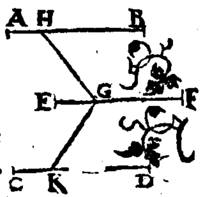
Theorema 8. Propositio 8.

Si duæ sint parallelæ rectæ lineæ, quarum altera ad rectos cuidam plano sit angulos, & reliqua eidē plano ad rectos angulos erit



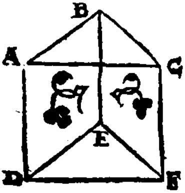
Theorema 9. Propositio 9.

Quæ eidem rectæ lineæ sunt parallelæ, sed nō in eodem cũ illa plano, hæ quoque sunt inter se parallelæ.



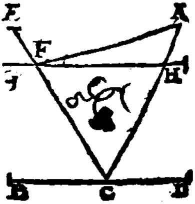
Theorema 10. Propoſitio 10.

Si duę rectę lineæ ſe mutuò tangétes ad duas rectas ſe mutuò tangentes ſint parallelę, non autem in eodem plano, illę angulos æquales comprehendent.



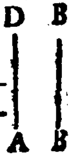
Proble. 1. Propoſitio 11.

A dato ſublimi puncto, in ſubiectum planũ perpendicularem rectam lineam ducere.



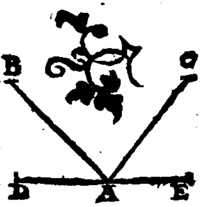
Problema 2. Propoſitio 12.

Dato plano, à púcto quod in illo datum eſt, ad rectos angulos rectam lineam excitare.



Theorema 11. Propoſitio 13.

Dato plano, à puncto qđ in illo datum eſt, duæ rectæ lineæ ad rectos angulos non excitabuntur ad eaſdem partes.



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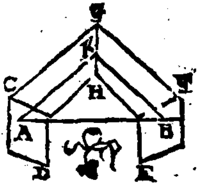
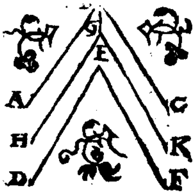
Theorema 12. Propoſitio 14.



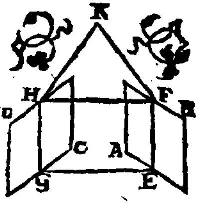
Ad quæ plana, eadem recta linea recta eſt, illa sũt parallela.

Theorema 13. Propoſitio 15.



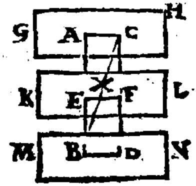
Si duæ rectæ lineæ ſe mutuò tangentes ad duas rectas ſe mutuò tangentes ſint parallelæ, non in eodem conſiſtentes plano, parallela ſunt quæ per illas ducuntur plana.

Theorema 14. Propoſitio 16.



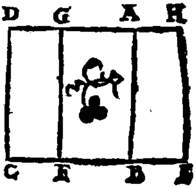
Si duo plana parallela plano quopiã ſecentur, communes illorum ſectiones ſunt parallelæ.

Theorema 15. Propoſitio 17.



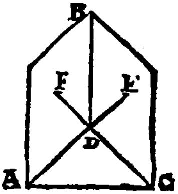
Si duæ rectæ lineę parallelis planis ſecẽtur, in eaſdem rationes ſecabũtur.

Theorema 16. Propositio 18.



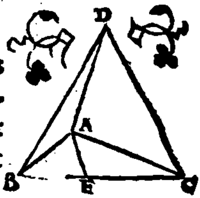
Si recta linea plano cuipiam ad rectos sit angulos, illa etiam omnia quę per ipsam plana, ad rectos eidem plano angulos erunt.

Theorema 17. Propositio 19



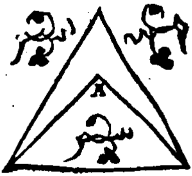
Si duo plana se mutuò secantia plano cuidã ad rectos sint angulos, communis etiam illorum sectio ad rectos eidem plano angulos erit.

Theorema 18. Propositio 20.



Si angulus solidus planis tribus angulis contineatur, ex his duo quilibet vtut assumpti tertio sunt maiores

Theorema 19. Propositio 21.

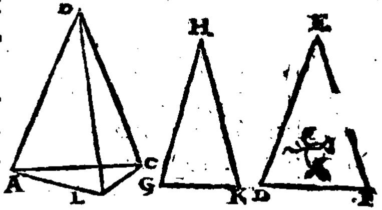


Solidus omnis angulus minoribus continetur, quàm rectis quatuor angulis planis.

Theo-

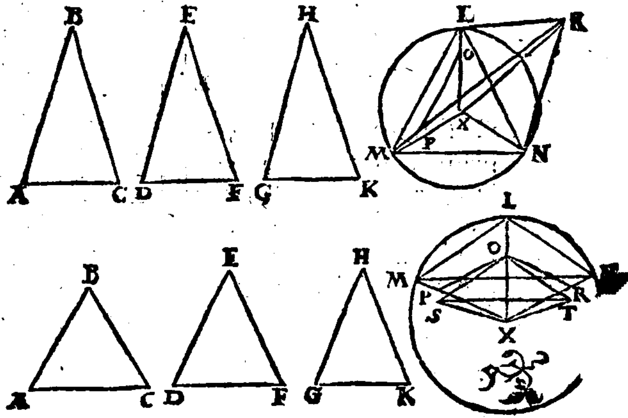
Theorema 20. Proopositio 22.

Si plani tres anguli ęqualibus rectis contineãtur lineis, quorũ duo vt libet assumpti, tertio sint maiores, triangulum constitui potest ex lineis æquales, illas rectas coniungentibus.

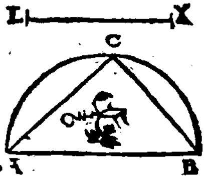


Problema 3. Propositio 23.

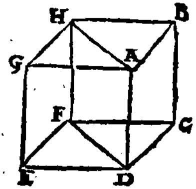
Ex planis tribus angulis, quorũ duo vt libet assumpti tertio sint maiores, solidũ angulum constituere. Decet autem illos tres angulos rectis quatuor esse minores.

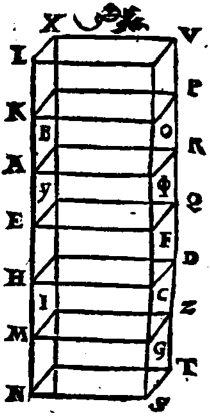


Theorema 21. Propositio 24.



Si solidum parallelis planis contineatur, aduersa illius plana & æqualia sunt & parallelogramma.





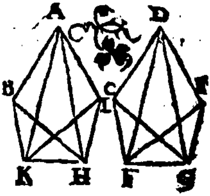
Theorema 22. Propositio 25.

Si solidum parallelis planis contentũ plano secetur aduersis planis parallelo, erit quemadmodum basis ad basim, ita solidum ad solidum.

Proble-

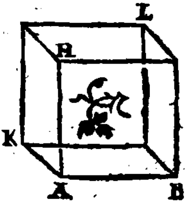
Problema 4. Propositio 26.

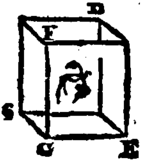
Ad datam rectam lineã eiusq́ue punctum, angulum solidum constituere solido angulo dato æqualem.



Problema 5. Propositio 27.

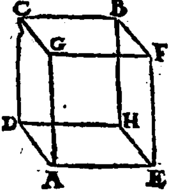
A data recta, dato solido parallelis planis comprehensio simile & similiter positum solidũ parallelis planis contẽtum describere.



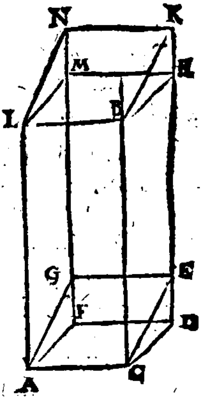


Theorema 23. Propositio 28.

Si solidũ parallelis planis comprehensum ductor per aduersorum planorum diagonios plano sectũ sit, illud solidum ab hoc plano bifariam secabitur.

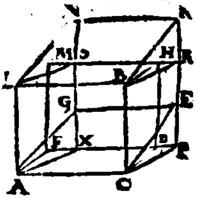


Theorema 34. Propoſitio 29.



Solida parallelis planis comprehēſa, quę ſuper eandem baſim & in eadē ſunt altitudine, quorum inſiſtētes lineæ in ijſdem collocantur rectis lineis, illa ſūt inter ſe æqualia.

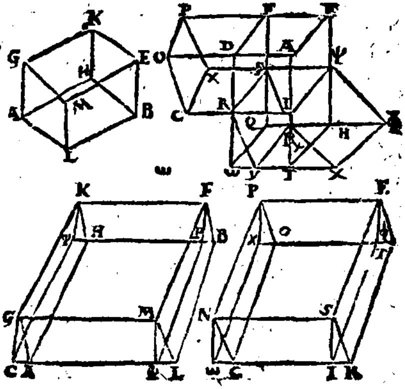
Theorema 25. Propoſitio 30.



Solida parallelis planis circumſcripta, quæ ſuper eandē baſim & in eadē ſūt altitudine, quorum inſiſtentes lineæ nō in ijſdem reperiuntur rectis lineis, illa ſunt inter ſe æqualia.

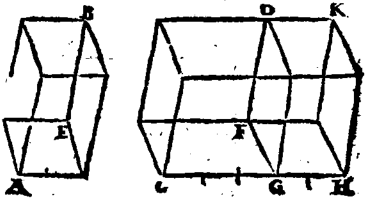
Theorema 26. Propoſitio 31.

Solida parallelis planis circunſcripta, quæ in in eadem sūt altitudine, æqualia sūt inter ſe.



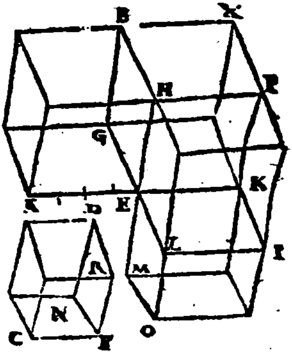
Theorema 27. Propoſitio 32.

Solida parallelis planis circunſcripta quæ eiuſdem ſunt altitudinis, eam habēt inter ſe rationē, quam baſes.



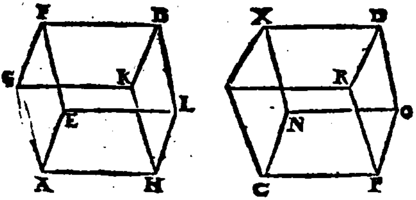
Theor 28. Propositio 23.

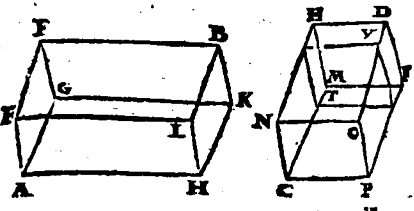
Similia solida parallelis, planis circũscripte habẽt inter se rationẽ homologorũ laterum triplicatam

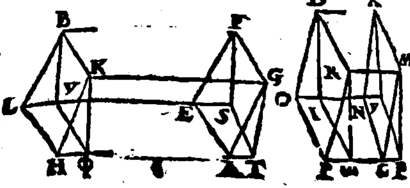


Theor. 29. Propositio 24

Aequalium solidorũ parallelis planis cõtẽtorum bases cũ altitudinibus reciprocantur. Et solida parallelis planis contenta,



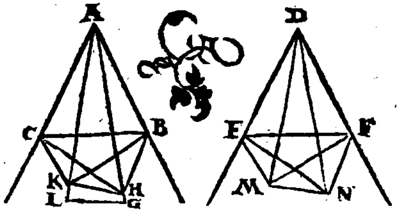




baſes cum altitudinibus reciprocantur, illa ſunt æqualia.

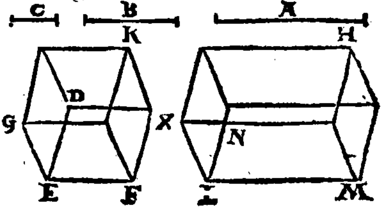
Theorema 30. Propoſitio 35.

Si duo plani ſint anguli æquales, quorum verticibus ſublimes rectæ lineæ inſiſtant, quæ cum lineis primò poſitis angulos contineant æquales, vtrunq; vtriq;, in ſublimibus autem lieneis quęlibet ſumpta ſint puncta, & ab his ad plana in quibus conſiſtunt anguli primum poſiti, ductæ ſint perpendiculares, ab earum verò punctis, quæ in planis ſignata fuerint, ad angulos primum poſitos adiūctæ ſint rectæ lineę, hæ cū ſublimibus æquales angulos comprehendent.



Theorema 31. Propoſitio 36.

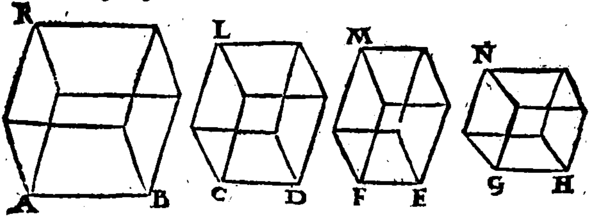
Si rectæ tres lineæ ſint proportionales, quòd



ex his tribus fit solidum parallelis planis contentum, æqnale est descripto à media linea solido parallelis planis comprehēso, quod æquilaterum quidem sit, sed antedicto æquiangulum.

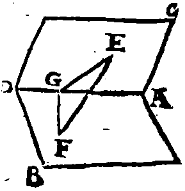
Theorema 32. Propositio 37.

Si rectæ quatuor lineæ sint proportionales, illa quoque solida parallelis planis contenta, quæ ab ipsis lineis & similia & similiter discribuntur, proportionalia erunt. Et si solida parallelis planis comprehensa, quę & similia & similiter describuntur, sint proportionalia, illæ quoque rectæ lineæ proportionales erunt.



Theorema 33. Propositio 38.

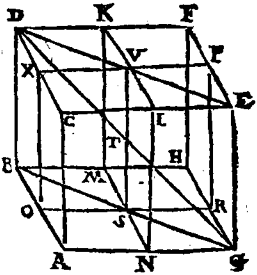
Si planum ad planum rectū sit, & à quodā puncto eorū quæ in vno sunt planorū perpendicularis ad alterum ducta sit, illa quæ ducitur perpēdicularis, in communē cadet planorū sectionē.



Theo-

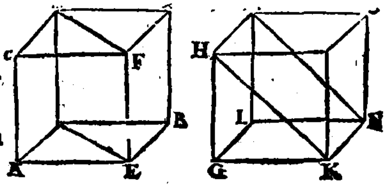
Theorema 34. Propositio 39.

Si in ſolido parallelis planis circunſcripto, aduerſorum planorum lateribus bifariam ſectis, educta ſint per ſectiones plana, communis illa planorũ ſectio & ſolidi parallelis plani circumſcripti diameter, ſe mutuò bifariã ſecant.



Theorema 35. Propoſitio 40.

Si duo ſint æqualis altitudinis priſmata, quorũ hoc quidem baſim habeat parallelogrammũ, illud verò triangulum, ſit autem parallelogrammũ triãguli duplũ, illa priſmata erunt æqualia.



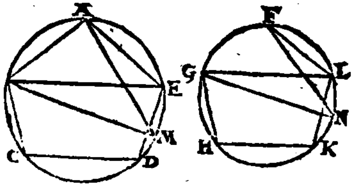
ELEMENTI XI. FINIS.

EVCLIDIS ELEMENTVM DVODECIMVM.

ET SOLIDORVM. secundum.

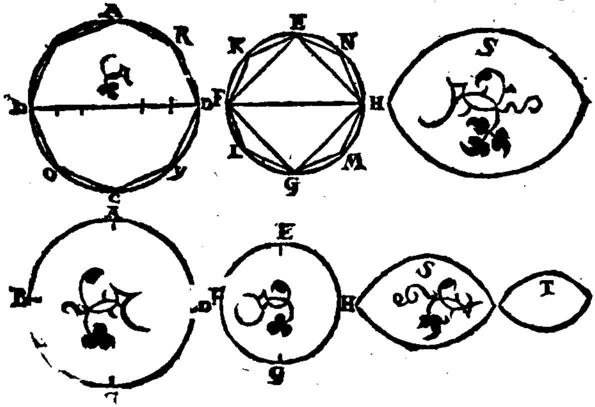
Theorema 1. Propositio 1.

Similia quæ sunt in circulis polygona, rationé habent inter se, quam descripta à diametris quadrata.



Theorema 2. Propositio 2.

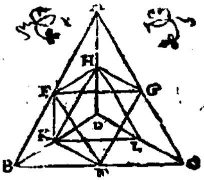
Circuli eam inter se rationem habent, quã



descripta à diametris quadrata.

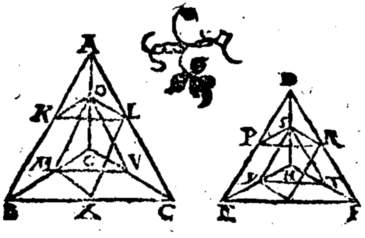
Tueorema 3. Propositio 3.

Omnis pyramis trigωnam habens basim, in duas diuiditur pyramidas non tantũ æquales & similes inter se, sed toti etiam pyramidi similes, quarum trigωnæ sunt bases, atq; in duo prismata ęqualia, quæ duo prismata dimidio pyramidis totius sũt maiora.



Theorema 4. Propositio 4.

Si duæ eiusdem altitudinis pyramides trigωnas habeãt bases, sit aũt illarum vtraque diuisa & in duas pyramidas inter se ęquales totiq; similes, & in duo prisma æqualia, ac eodem modo diuidatur vtraq; pyramidum quæ ex superiore diuisione natæ sunt, idque perpetuò fiat: quemadmodú se habet vnius pyramidis basis ad alterius pyrãidis basim, ita & omnia quę in vna pyra-

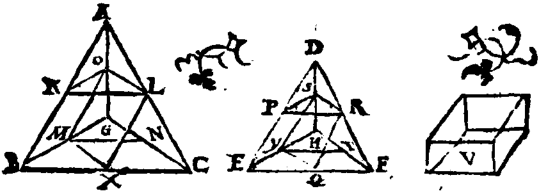


mide

mide priſmata, ad omnia quæ in altera pyramide, priſmata multitudine æqualia.

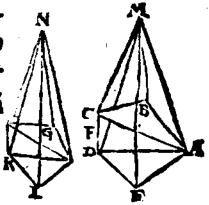
Theorema 5. Propoſitio 5.

Pyramides eiuſdem altitudinis, quarũ trigωnæ ſunt baſes, eam inter ſe rationem habent, quàm ipſæ baſes.



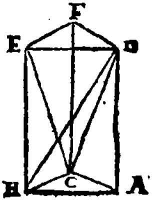
Theorema 6. Propoſitio 6.

Pyramides eiuſdẽ altitudinis, quarum polygωnæ ſunt baſes, eam inter ſe rationem habent, quã ipſæ baſes.



Theorema 7. Propoſitio 7.

Omne priſma trigωnã habẽs baſim, diuiditur in tres pyramides inter ſe æquales, quarum trigωnæ ſunt baſes



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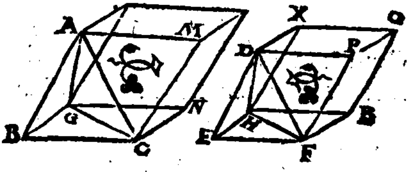
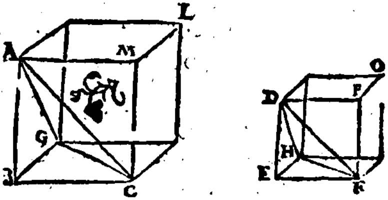
Theorema 8. Propoſitio 8.

Similes pyramides qui trigωnas habent baſes, in triplicata ſũt homologorũ laterũ ratiõe



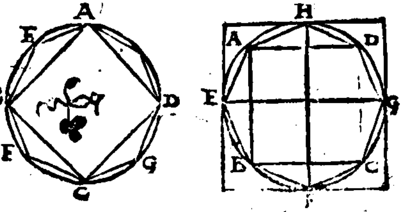
Theorema 9. Propoſitio 9.

Aequaliũ pyramidum & trigωnas baſes habentium reciprocantur baſes cum altitudinibus. Et quarũ pyramidum trigωnas baſes habentium reciprocãtur baſes cũ altitudinibus, illæ ſunt æquales.



Thorema 10. Propoſitio 10.

Omnis cωn⁹ tertia pars eſt cylindri eandẽ

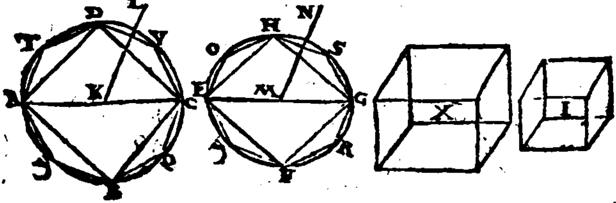


cum

cum ipſo cono baſim habentis, & altitudinem æqualem.

Theorema 11. Propoſitio 11.

Coni & cylindri eiuſdem altitudinis, eam inter ſe rationem habent, quam baſes.

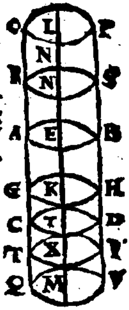


Theorema 12. Propoſitio 12.

Similes coni & cylindri, triplicatã habent inter ſe rationem diametrorum, quæ ſunt in baſibus.

Theo-

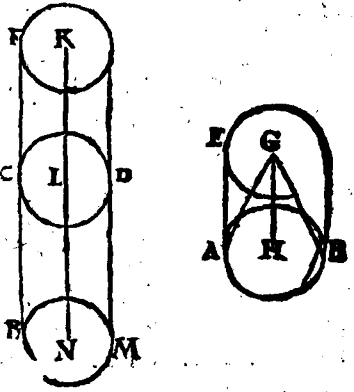
Theorema 13. Propositio 13.



Si cylindrus plano sectus sit aduersis planis parallelo, erit quéadmodum cylindrus ad cylindrum, ita axis ad axem.

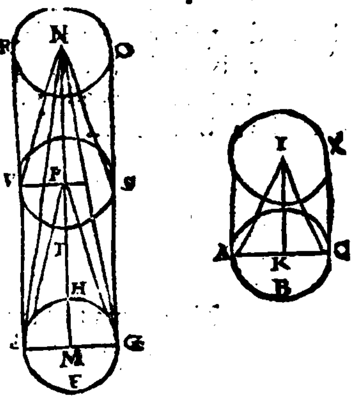
Theorema 14. Propositio 14.

Coni & cylindri qui in æqualibus sunt basibus, eam habent inter se rationem, quam altitudines



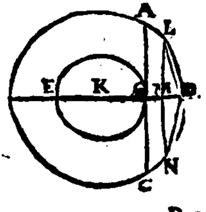
Theorema 15. Propositio 15.

Aequalium conorum & cylindrorũ bases cũ alutitudinib' reciprocátur. Et quorum conorũ & cylindrorum bases cum altitudinibus reciprocantur, illi sunt æquales.



Theoroma 1. Propositio 16.

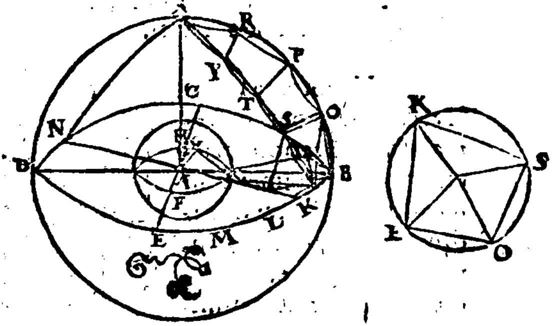
Duobus circulis circũ idem centrum consistentibus, in maiore circulo polygonum æqualium pariumque laterum inscribere, quod minorem circulũ non tangat.



Pro-

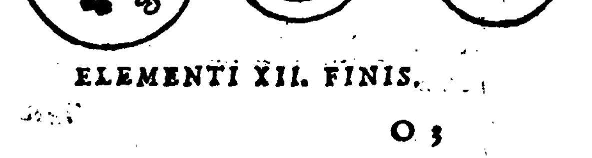
Problema 2. Propositio 17.

Duabus sphæris circum idem centrum consistentibus, in maiorem sphæra solidum polyedrum inscribere, quod minoris sphæræ superficiem non tangat.



Theorema 16. Propositio 18.

Sphæræ inter se rationem habent suarum diametrorum triplicatam.

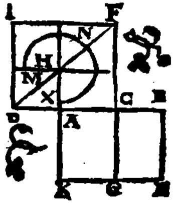


ELEMENTI XII. FINIS.

EVCLIDIS ELEMENTVM DECIMVMTERTIVM, ET SOLIDORVM TERTIVM.

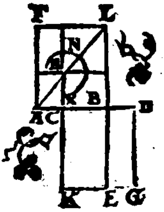
Theorema 1. Propositio 1.

Si recta linea per extremã & mediã rationem secta sit, maius segmentum quod totius lineæ dimidium assumpserit, quintuplum potest eius quadrati, quod à totius dimidia describitur.



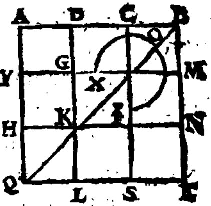
Theorema 2. Propositio 2.

Si recta linea sui ipsius segmenti quintuplum possit, & dupla segmenti huius linea per extremã & mediam rationẽ secetur maius segmentũ reliqua pars est lineæ primum positæ.



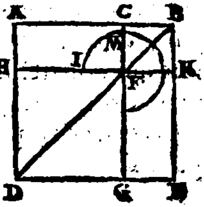
Theo-

Theorema 3. Propoſitio 3.



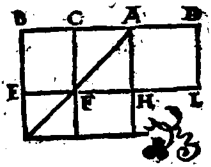
Si recta linea per extremã & mediã rationẽ ſecta ſit, minus ſegmẽtũ quod maioris ſegmenti dimidium aſſumpſerit, quintuplum poteſt eius, quod à maioris ſegmẽti dimidio deſcribitur, quadrati.

Theorema 4. Propoſitio 4.



Si recta linea per extremam & mediam rationẽ ſecta ſit, quod à tota, quodque à minore ſegmẽto ſimul vtraq; quadrata, tripla ſunt eius, quod à maiore ſegmẽto deſcribitur, quadrati.

Theorema 5. Propoſitio 5.



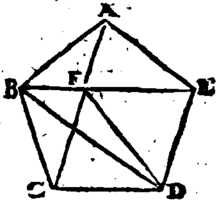
Si ad rectam lineam, quæ per extremam & mediam rationem ſecetur, adiuncta ſit altera ſegmento maiori equalis, tota hæc linea recta per extremam & mediam rationẽ ſecta est, estque maius segmentum linea primùm posita.

Theorema 6. Propositio 6.

Si recta linea ῥητὴ siue rationalis, per extremam & mediam rationem secta sit, vtrunque segmentorum ἄλογος siue irrationalis est linea, quæ dicitur Residuum.

Theorema 7. Propositio 7.

Si pentagoni æquilateri tres sint æquales anguli, siue quã deinceps siue qui non deinceps sequuntur, illud pentagonum erit æquiangulum.



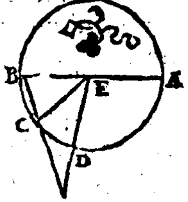
Theorema 8. Propositio 8.

Si pentagoni æquilateri & æquianguli duos qui deinceps sequuntur angulos rectæ subtendãt lineæ, illæ per extremã & mediam rationem se mutuò secant, earumq; maiora segmenta, ipsius pentagoni lateri sunt æqualia.

Theo-

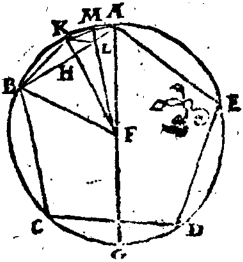
Theorema 9. Propositio 9.

Si latus hexagωni & latus decagωni eidem circulo inscriptorum composita sint, tota recta linea per extremam & mediam rationem secta est, eiusque segmētum maius, est hexagωni latus.



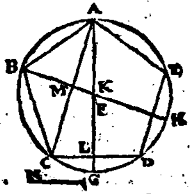
Theorema 10. Propositio 10.

Si circulo pētagωnum æquilaterum inscriptum sit, pentagωini lat° potest & latus hexagωni & latus decagωni eidem circulo inscriptorum.



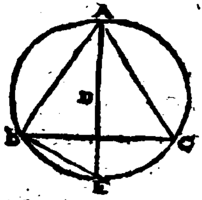
Therema 11. Propositio 11.

Si in circulo ῥητὴν habente diametrum, inscriptum sit pentagωnum æquilaterum, pentagωni latus irrationalis est linea quæ vocatur Minor.



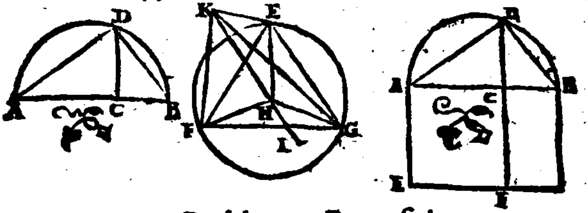
Theorema 12. Propositio 12.

Si in circulo inscriptum sit triangulum æquilaterum, huius trianguli latus potentia triplum est eius lineæ, quæ ex circuli centro ducitur.



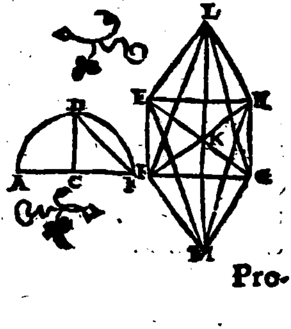
Problema 1. Propositio 13.

Pyramidem constituere, & data sphærę cõplecti, atque docere illius sphæræ diametrum potentia sesquialteram esse lateris ipsius pyramidis.



Problema 2. Propositio 14.

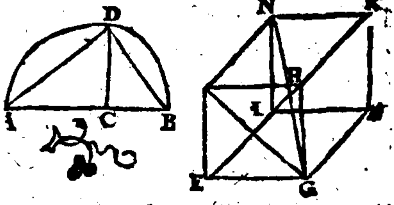
Octaedrum constituere, eaq́; sphęra qua pyramidẽ complecti, atque ꝓbare illius sphæræ diametrũ potentia duplã esse lateris ipsi' octaedri.



Pro-

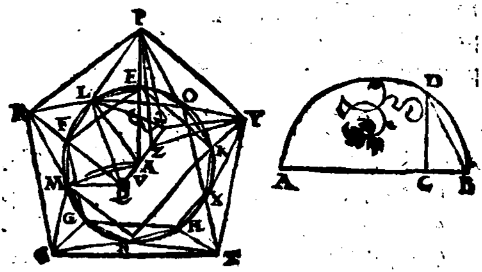
Problema 3. Propoſitio 15.

Cubum conſtituere, eaque ſphæra qua & ſuperiores figuras complecti, atque docere illius ſphæræ diametrum potētia triplā eſſe lateris ipſius cubi.



Problema 4. Propoſitio 16.

Icoſaedrum conſtituere, eademque ſphæra qua & antedictas figuras complecti, atque probare, Icoſaedri latus irrationalem eſſe lineam, quæ vocatur Minor.



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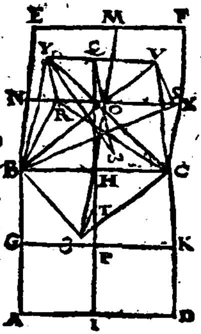
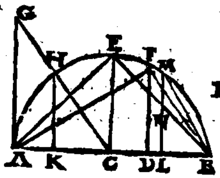
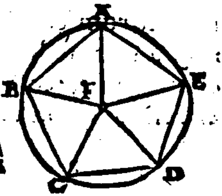


Problema 5. Propositio 17.

Dodecaedrũ constituere, eademque sphæra qua & & antedictas fuguras cóplecti, atque probare dodecaedri latus irrationalem esse lineam, quæ vocatur Residuum.

Theorema 6. Propositio 18.





quinq; figurarum latera proponere, & inter se comparare.

SCHOLIVM.

Aio verò, præter dictas quinque figuras non posse aliam constitui figuram solidã, quæ planis & æquilateris & æquiangulis cõtineatur, inter se æqualibus. Non enim ex duobus triãgulis, sed neq; ex alijs duab' figuris solidus cõstituetur angulus

Sed

Sed ex tribus triãgulis, constat Pyramidis angulus

Ex quatuor autem, Octaëdri

Ex quinque verò, Icosaëdri.

Nam ex triangulis, sex & æquilateris & æquiangulis ad idem punctum coeuntibus, non fiet angulus solidus. Cùm enim trianguli æquilateri angulus, recti vnius bessem contineat, erunt eiusmodi sex anguli rectis quatuor æquales. Quod fieri nõ potest. Nam solidus omnis angulus minor bus quã rectis quatuor angulis continetur, per 21.11.

Ob easdem sanè causas, neque ex pluribus quã planis sex eiusndoi angulis solidus constat.

Sed ex tribus quadratis, Cubi angulus continetur.

Ex quinque, nullus potest. Rursus enim recti quatuor erunt.

Ex tribus autẽ pentangonis æquilateris & æquiangulis, Dodecaëdri angulus continetur.

Sed ex quatuor, nullus potest. Cùm enim pentagoni æquilateri angulus rectus sit, et quinta recti pars erunt quatuor anguli rectis quatuor maiores. Quod fieri nequit. Nec sanè ex alijs polygonis figuris solidus angulus continebitur, quod hinc quoque absurdum sequatur. Quamobrem perspicuum est, præter dictas quinque figuras aliã figuram solidam non posse constitui, quæ ex planis æquilateris & æquiangulis contineatur.

ELEMENTI XIII. FINIS

EVCLIDIS ELEMENTVM DECIMVM QVARTVM, VT qui dam arbitrantur, vt alij verò, Hypsiclis Alexandrini, de quinque corporibus.

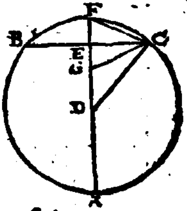
LIBER PRIMVS.

Basilides Tyrius, Protarche, Alexandriam profectus, patrique nostro ob disciplinæ societatem commendatus, longißimo peregrinationis tempore cum eo versatus est. Cùmque disserent aliquando de scripta ab Apollonio comparatione Dodecaëdri & Icosaëdri eidem sphæræ inscriptorum, quam hæc inter se habeant rationem, censuerunt ea non rectè tradidisse Apollonium: quæ à se emendata, vt de patre audire erat, literis prodiderunt. Ego autem postea incidi in alterum librum ab Apollonio editum, qui demonstrationem accuratè complecteretur de re proposita, ex eiusq; problematis indagatione magnam equidem cepi voluptatem. Illud certè ab omnibus perspici potest, quod scripsit Apollonius, cùm sit in omnium manibus. Quod autē diligenti, quantum conijcere licet, studio nos postea scrip-

ſcripſiſſe videmur, id monimentis conſignatum tibi nuncupandum duximus, vt qui feliciter cùm in omnibus diſciplinis tum vel maximè in Geometria verſatus, ſcitè ac prudẽter iudices ea quæ dicturi ſumus ob eam verò, quæ tibi cum patre fuit, vitæ conſuetudinem, quaq́; nos complecteris, beneuolentiã, tractationẽ ipſam libenter audias. Sed iam tempus eſt, vt proœmio modum facientes, hanc ſyntaxim aggrediamur.

Theorema 1. Proopoſitio 1.

Perpendicularis linea, quæ ex circuli cuiuſpiam centro in latus pentagoni ipſi circulo inſcripti ducitur, dimidia eſt vtriuſq; ſimul lineæ, & eius, quæ ex centro & lateris decagoni in eodẽ circulo inſcripti.



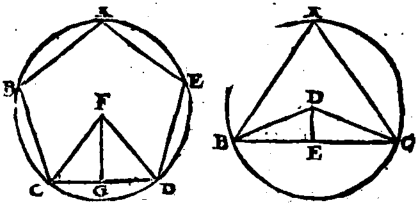
Theorema 2. Propoſitio 2.

Idem circulus cõprehendit & dodecaedri pentagonum & icoſaedri triangulum, eidẽ ſphæræ inſcriptorum.

Theo-

Theorama 3. Propositio 3.

Si pentagono & æquilatero & æquiangulo circumſcriptus ſit circulus ex cuius centro in vnum pentagoni latus dicta ſit perpendicularis: quod vno laterum & perpẽdiculari trigeſies cõtinetur, illud

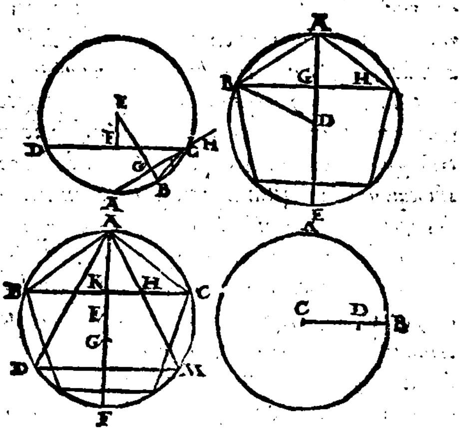


æquale eſt dodecaedri ſuperficiei.

Theorema 4. Propoſitio 4.

Hoc perſpicuum cùm ſit probandum eſt, quemadmodum ſe habet dodecaedri ſuperficies ad icoſaedri ſuperficiem ita ſe habere cubi latus ad icoſaedri latus.

Cubi



Cubilatus.

E————————

Dodecaedri.

F————————

Icosaedri.

G————————

SCHOLIVM

Nunc autem probandum est, quemadmodum se habet cubi latus ad icosaëdri latus, ita se habere solidum dodecaëdri ad icosaëdri solidum: Cùm enim æquales circuli comprehendat & dodecaëdri pentagonum & Icosaëdri triangulum, eidē sphæra in-

scriptorum: in sphæris autem æquales circuli æquali interuallo distent à centro (siquidem perpendiculares à sphæræ centro ad circulorum plana ductæ & æquales sunt, & ad circulorum centra cadunt) idcirco lineæ, hoc est perpendiculares, quæ à sphæræ cētro ducuntur ad centrum circuli comprehendētis et triangulum Icosaëdri, & pentagonum dodecaëdri sūt æquales. Sūt igitur æqualis altitudinis Pyramides, quæ bases habent ipsa dodecaëdri pentagona, et quæ Icosaëdri triangula. At æqualis altitudinis pyramides rationem inter se habent eam quam bases, ex 5. & 6. 11. Quemadmodum igitur pentagonū ad triangulum, ita pyramis, cuius basis quidem est dodecaëdri pentagonum, vertex autem sphæræ centrum ad pyramida, cuius basis quidem est Icosaëdri triangulum, vertex autem sphæræ centrum. Quamobrem vt se habent duodecim pentagona ad viginti triāgula, ita duodecim pyramides, quorum pētagonæ sint bases, ad viginti pyramidas, quæ trigonas habeant bases At pentagona duodecim sunt dodecaëdri superficies, viginti autem triangula, Icosaëdri. Est igitur vt dodecaëdri superficies ad Icosaëdri superficiem, ita duodecim pyramides, quæ pentagonas habeant bases, ad viginti pyramidas, quarum trigona sunt bases. Sunt autem duodecim quidem pyramides, quæ pentagonas habeant bases, solidū dodecaëdri: viginti autem pyramides, quæ trigonas habeant bases, Icosaëdri solidum. Quare ex 11. 5. vt dodecaëdri superficies ad Icosaëdri superficiem, ita soli-

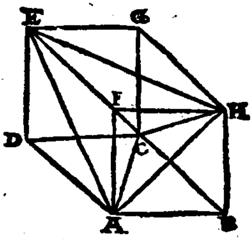
solidum dodecaëdri ad Icosaëdri solidum. Vt autem dodecaëdri superficies ad Icosaëdri superficiem, ita probatum est cubi latus ad Icosaëdri latus. Quemadmodum igitur cubi latus ad Icosaëdri latus, ita se habet solidum dodecaëdri ad Icosaëdri solidum.

EVCLIDIS ELEMENTVM DECIMVM QVINTVM, ET Solidorum quintum, vt nonnulli putant, vt autem alij, Hypsiclis Alexandrini, de quinque corporibus.

LIBER II.

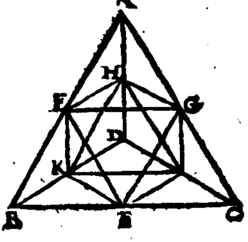
Problema 1. Propositio 1.

In dato cubo pyramida inscribere.



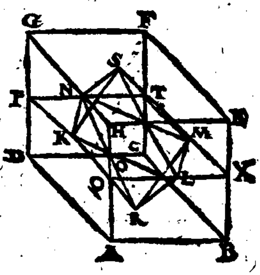
Problema 2. Propositio 2.

In data pyramide octaedrum inscribere.



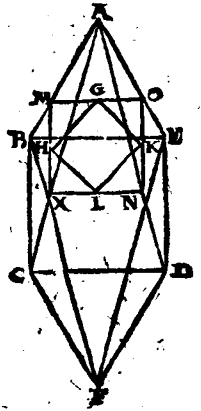
Pro-

Problema 3. Propositio 3.

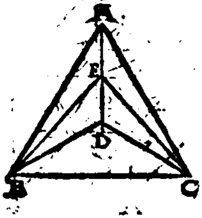


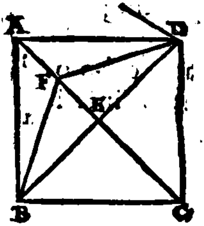
In dato cubo octaedrum inscribere.

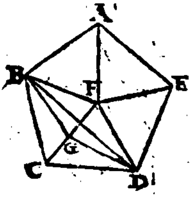
Problema 4. Propositio 4.

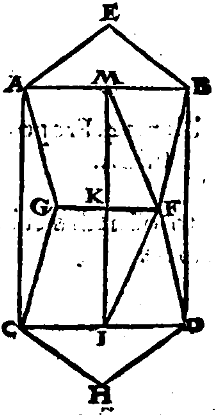


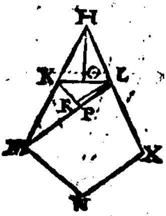
In dato octaedro cubū inscribere.









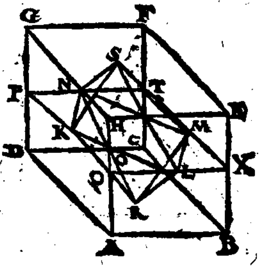


SCHO.

SCHOLIVM.

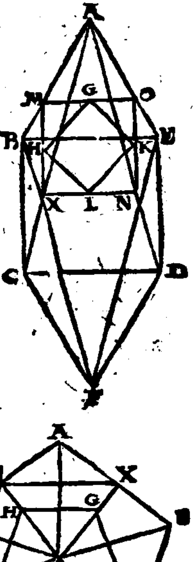
Meminisse decet, si quis nos roget, quot Icosaedrũ habeat latera, ita respondendum esse: Patet Icosaedrum viginti contineri triãgulis, quodlibet verò triangulum rectis tribus constare lineis. Quare multiplicanda sunt nobis viginti triangula in trianguli vnius latera, fiũtq́; sexaginta, quorum dimidium est triginta. Ad eundem modum & in dodecaedro. Cũ enim rursus duodecim pentagona dodecaedrum cõprehendant, itemq́; pentagonum quoduis rectis quinque constet lineis, quinq́; duodecies multiplicamus, fiunt sexaginta, quorum rursus dimidium est triginta. Sed cur dimidiũ capimus? Quoniã vnumquodq́; latus siue sit trianguli, siue pentagoni, siue quadrati vt in cubo, iteratò sumitur. Similiter autem eadem via & in cubo & in pyramide & in octaedro latera inuenies. Quòd si itẽ velis singularum quoq́; figurarum angulos reperire, facta eadem multiplicatione

Problema 3. Propositio 3.



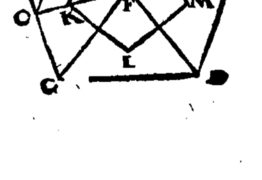
In dato cubo octaedrum inscribere.

Problema 4. Propositio 4.



In dato octaedro cubũ inscribere.

Problema 5. Propositio 5.



In dato Icosaedro dodecaedrum inscribere.

SCHOLIVM.

Meminisse decet, si quis nos roget, quot Icosaëdrũ habeat latera, ita respondendum esse: Patet Icosaedrum viginti contineri triãgulis, quodlibet verò triangulum rectis tribus constare lineis. Quare multiplicanda sunt nobis viginti triangula in trianguli vnius latera, fiuntq; sexaginta, quorum dimidium est triginta. Ad eundem modum & in dodecaedro. Cũ enim rursus duodecim pentagona dodecaedrum cõprehendant, itemq; pentagonum quoduis rectis quinque constet lineis, quinq; duodecies multiplicamus, fiunt sexaginta, quorum rursus dimidium est triginta. Sed cur dimidiũ capimus? Quoniã vnumquodq; latus siue sit trianguli, siue pentagoni, siue quadrati vt in cubo, iteratò sumitur. Similiter autem eadem via & in cubo & in pyramide & in octaedro latera inuenies. Quòd si itẽ velis singularum quoq; figurarum angulos reperire, facta eadem multiplicatione numerum procreatum partire in numerum planorũ, quæ vnum solidum angulum includũt: vt quoniã triãgula quinq; vnum Icosaëdri angulum continet, partire 60. in quinque, nascũtur duodecim anguli Icosaedri. In dodecaedro autẽ tria pẽtagona angulum comprehendunt, partire ergo 60. in tria, & habebis dodecaedri angulos viginti. Atq; simili ratione in reliquis figuris angulos reperies.

Finis Elementorum Euclidis.