EVCLIDIS ELEMENTORVM LIBRI XV.

QVIBVS, CVM AD OMNEM MATHEMATICAE SCIENTIAE PARTEM, TVM AD QVAMLIbet Geometriæ tractationem facilis comparatur aditus.

COLONIAE

Apud Gosuinum Cholinum

M. D. C X I I.

Cum Gratia & Priuilegio.

AD CANDIDVM LECTOREM ST. GRACILIS praefatio.

PERMAGNI *referre semper existimaui, lector beneuole, quantum quisque studij & diligentiæ ad percipienda scientiarum elementa adhibeat, quibus non satis cognitis, aut perperam intellectis si vel digitũ progrediantur errores caliginem animis offundas, non veritatis lucem rebus obscuris adferas. Sed principiorum quanta sint in disciplinis momenta, haud facile credat, qui rerum naturam ipsa specie, non viribus metiatur. Vt enim corporum, quæ oriuntur & intereunt, vilissima tenuissimaq́ue videntur initia: ita rerum æternarum & admirabilium, quibus nobilissimæ artes continentur, elementa ad speciem sunt exilia, ad vires & facultatem quam maxima. Quis non videt ex fici tantulo grano, vt ait Tullius, aut ex acino vinaceo, aut ex cæterarum frugum aut stirpium minutissimis seminibus tantos truncos ramosq́ue procreari? Nam Mathematicorum initia illa quidem dictu audituq́ue perexigua, quantã theorematum syluam nobis pe-* Simile.

pererunt? Ex quo intelligi potest, vt in ipsis seminibus, sic & in artium principijs inesse vim earum rerum, quæ ex his progignuntur. Præclarè igitur Aristoteles, vt alia permulta, μέγιστον ἴσως ἀρχὴ παντὸς, καὶ ὅσῳ κράτιστον τῇ δυνάμει, τοσούτῳ μικρότατον, ὂν τῷ μεγέθει χαλεπόν ἐστιν ὀφθῆναι. Quocirca committendum non est, vt non bene prouisa & diligenter explorata scientiarum principia, quibus propositarum quarumque rerum veritas sit demonstranda, vel constituas, vel constituta approbes: Cauendum etiam, vt ne tantulum quidem fallaci & captiosa interpretatione turpiter deceptus, à vera principiorum ratione temerè deflectas. Nam qui initio fortè aberrauerit, is vt tandem in maximis versetur erroribus, necesse est: cum ex vno erroris capite densiores sensim tenebræ rebus clarißimis obducantur. Quid tam varias veterum physiologorum sententias non modò cùm rerum veritate pugnantes, sed vehementer etiam inter se dissentientes nobis inuexit? Equidem haud scio, fuerit ne vlla potior tanti dißidij causa, quam quòd ex principijs partim falsis, partim non consentaneis ductas rationes probando adhiberent. Fit enim plerunque, vt qui non rectè de artium rerumq́ue elementis sentiunt, ad præfinitas quasdam opiniones suas omnia reuocare studeant. Pythagorei, vt meminit Aristoteles, cum denarij numeri summam perfectionem cœlo tribuerent, nec plures tamen quàm nouem sphæras cernerent, decimam affingere ausi sunt terræ aduer-

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sam, [illegible] ἀντίχθονα appellarunt. Illi enim vniuersitatis rerumque singularum naturam ex numeris ceu principijs æstimantes, ea protulerunt quæ φαινομένοις congruere nusquam sunt cognita. Nam ridicula Democriti, Anaximenis, Melissi, Anaxagoræ, Anaximandri, & reliquorum id genus physiologorum somnia, ex falsis illa quidem orta naturæ principijs, sed ad Mathematicum nihil aut parum spectantia, sciens prætereo. Non nullos attingam qui repetitis altius vel aliter ac decuit positis rerum initijs, cum physicis multa turbarunt; tum Mathematicos oppugnatione principiorum pessimè mulctarunt. Ex planis figuris corpora constituit Timæus: Geometrarum hic quidem principia cuniculis oppugnantur. Nam & superficies seu extremitates crassitudinem habebunt, & lineæ latitudinem: denique puncta non erunt indiuidua, sed linearum partes. Prædicant Democritus atque Leucippus illas atomos suas, & indiuidua corpuscula. Concedit Xenocrates impartibiles quasdam magnitudines. Hic verò Geometriæ fundamenta apertè petuntur, & funditus euertuntur: quibus dirutis nihil equidem aliud video restare, quàm vt amplissima Mathematicorum theatra repentè concidant. Iacebunt ergò, si dijs placet, tot præclara Geometrarum de asymmetris & alogis magnitudinibus theoremata. Quid enim causæ dicas, cur indiuidua linea hanc quidem metiatur, illam verò meteri non queat? Siquidem quod minimum in vnoquoque

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genere reperitur, id communis omnium censura esse solet. Innumerabilia profectò sunt illa, quæ ex falsis eiusmodi decretis absurda consequuntur: & horum permulta quidem Mathematicus, sed longè plura colligit Physicus. Quid varia ψευδογραφημάτων genera commemorem, quæ ex hoc vno fonte tam longè latéque diffusa fluxisse videntur? Notißimus est Antiphontis tetragonismus, qui Geometrarum & ipse principia non parum labefecit, cùm recta lineæ curuam posuit æqualem. Longum esset mihi singula percensere, præsertim ad alia properanti: Hoc ergo certum fixum, & in perpetuum ratum esse oportet, quod sapienter monet Aristoteles, Σπουδαστέον ὅπως ὁρισθῶσι καλῶς αἱ ἀρχαὶ μεγάλην γὰρ ἔχουσι ῥοπὴν. πρὸς ἑπόμενα. Nam principijs illa congruere debent, quæ sequuntur. Quòd si tantum perspicitur in istis exilioribus Geometriæ initijs, quæ puncto, linea, superficie definiuntur, momentum, vt ne hæc quidem sine summo impendentis ruinæ periculo conuelli aut oppugnari poßint, quanta quæso vis putanda est huius στοιχειώσεως, quam collatis tot præstantißimorum artificum inuentis, mira quadam ordinis solertia contexuit Euclides, vniuersæ Matheseos elementa complexu suo coercentem? Vt igitur omnibus rebus instructior & paratior quisq; ad hoc studiũ libentius accedat, & singula vel minutißima exactius secum reputet atq; perdiscat, operæpreciũ censui, in primo institutionis aditu vestibuloq; præcipua quædam capita, quibus

quibus tota ferè Mathematicæ scientiæ ratio intelligatur, breuiter explicare: tum ea quæ sunt Geometriæ propria, diligenter persequi: Euclidis denique in extruenda hac στοιχειώσεως consilium sedulò ac fideliter exponere. Quæ ferè omnia ex Aristotelis potissimũ ducta fontibus, nemimi inuisa fore cõfido, qui modo ingenuũ animi candorẽ ad legendũ attulerit. Ac de Mathematicæ diuisione primũ dicamus.

Mathematicæ in primis scientiæ studiosos fuisse Pythagoreos, non modò historicorum, sed etiam philosophorum libri declarant. His ergo placuit, vt in partes quatuor vniuersum distribuatur Mathematicæ sciẽtiæ genus, quarum duas περὶ τὸ ποσὸν, reliquas περὶ τὸ πηλίκον versari statuerunt. Nam & τὸ ποσὸν vel sine vlla comparatione ipsum per se cognosci, vel certâ quadam ratione comparatum spectari: in illo Arithmeticam, in hoc versari Musicam: & τὸ πηλίκον partim quiescere, partim moueri quidem: illud Geometriæ propositum esse: quod verò sua sponte motu cietur, Astronomiæ. Sed ne quis falsò putet, Mathematicam scientiam, quòd in vtroque quanti genere cernitur, idcirco inanem videri (si quidem non solum magnitudinis diuisio sed etiam multitudinis accretio infinitè progredi potest) meminisse decet, τὸ πηλίκον καὶ τὸ ποσὸν, quæ subiecto Mathematicæ generi imposita sunt à Pythagoreis nomina, non cuiuscunque modi quantitatem significare, sed eam demum, quæ tum multitudine tum magnitudine sit defi-

ulta, & suis circumscripta terminis. Quis enim vllam infiniti scientiam defendat?. Hoc scitum est, quod non semel docet Aristoteles, infinitum ne cogitatione quidem complecti quenquam posse. Itaque ex infinita multitudinis & magnitudinis δυνάμει *finitam hæc scientia decerpit & amplectitur naturam, quam tractet, & in qua versetur. Nam de vulgari Geometrarum consuetudine quid sentiendum sit, cum data interdum magnitudine infinita aut fabricantur aliquid, aut proprias generis subiecti affectiones exquirunt, disertè monet Aristoteles,* ουδὲν ῦν *(de Mathematicis) loquens* δέονται τοῦ ἀπείρου, ουδὲ χρῶνται, ἀλλὰ μόνον εἶναι ὅσην ἂν βούλωνται, πεπερασμένην. *Quamobrem disputatio ea qua infinitum refellitur, Mathematicorum decretis rationibusqúe non aduersatur, nec eorum apodixes labefacit. Etenim tali infinito opus illis nequaquam est, quod exitu nullo peragrari possit, nec talem ponunt infinitam magnitudinem: sed quantamcunque velit aliquis effingere, ea vt suppetat, infinitam præcipiunt. Quinetiam non modo immensa magnitudine opus non habent Mathematici, sed ne maxima quidem: cum instar maximæ minima quæque in partes totidem pari ratione diuidi queat. Alteram Mathematicæ diuisionẽ attulit Geminus, vir (quantum ex Proclo conijcere licet)* μαθημάτων *laude clarissimus. Eam, quæ superiore plenior & accuratior fortè visa est, cũ doctissimè pertractarit sua in decimum Euclidis*

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præfatione P. Montaureus vir senatorius, & regiæ bibliothecæ præfectus, leuiter attingam. Nam ex duobus rerum velut summis generibus τῶν νοητῶν καὶ τῶν αἰσθητῶν, quæ res sub intelligentiā cadunt, Arithmeticæ & Geometriæ attribuit Geminus: quæ verò in sensus incurrunt, Astrologiæ, Musicæ Supputatrici, Opticæ, Geodesiæ & Mechanicæ adiudicauit. Ad hanc certè diuisionem spectasse videtur Aristoteles, cùm Astrologiam Opticam, harmonicam φυσικωτέρας τῶν μαθημάτων nominat, vt quæ naturalibus & Mathematicis interiectæ sint, ac velut ex vtrisque mixtæ disciplinæ: Siquidem genera subiecta à Physicis mutuantur, causas verò in demonstrationibus ex superiore aliqua scientia repetunt. Id quod Aristoteles ipse apertissimè testatur, ἐνταῦθα γὰρ, φησὶ, τὸ μὲν ὅτι, τῶν αἰσθητικῶν. εἰδέναι τὸ διότι, τῶν μαθηματικῶν. Sequitur, vt quid Mathematicæ conueniat cum Physica & prima Philosophia: quid ipsa ab vtraque differat, paucis ostendamus. Illud quidem omnium commune est, quòd in veri contemplatione sunt positæ, ob idque θεωρητικαὶ à Græcis dicuntur. Nam cùm διάνοια siue ratio & mens omnis sit vel πρακτικὴ, vel θεωρητικὴ, totidem scientiarum sint genera necesse est. Quòd si Physica, Mathematica, & prima Philosophia, nec in agendo, nec in efficiendo sunt occupatæ, hoc certè perspicuum est, eas omnes in cognitione contemplationéque necessario versari. Cùm enim rerum non modò agendarum, sed etiam effidiendarum principia in agente vel afficiente consistant, illarum quidem προαίρεσις, harum autem vel mens vel ars, vel vis quædam & facultas: rerum profectò naturalium. Mathematicarum, atque diuinarum principia in rebus ipsis, non in philosophis inclusa latent. Atque hæc vna in omnes valet ratio, qua θεωρητικὰς esse colligat. Iam verò Mathematica separatim cum Physica congruit, quòd vtraque versatur in cognitione formarum corpori naturali inhærentium. Nam Mathematicus plana, solida, longitudines & puncta contemplatur, quæ omnia in corpore naturali à naturali quoque philosopho tractantur. Mathematica item & prima philosophia hoc inter se propriè conueniunt, quòd cognitionem vtraque persequitur formarum, quoad immobiles, & à concretione materiæ sunt liberæ. Nam tametsi Mathematicæ formæ re vera per se non cohærent, cogitatione tamen à materia & motu separantur, οὐδὲ γίνεται ψεῦδος χωριζόντων. vt ait Aristoteles. De cognatione & societate breuiter diximus, iam quid intersit videamus. Vnaquæq; mathematicarum certum quoddam rerum genus propositum habet, in quo versetur, vt Geometria quantitatem & continuationem aliorum in vnam partem, aliorum in duas, quorundam in tres, eorumq; quatenus quanta sunt & continua, affectiones cognoscit. Prima autem philosophia, cùm sit omnium communis, vniuersum Entis genus, quæq; ei accidunt & conueniant hoc ipso quòd est, considerat.

Ad hæc, Mathematica eam modò naturam amplectitur, quæ quanquam non mouetur, separari tamen seiungiq́; nisi mente & cogitatione à materia non potest, ob eamq́; causam ἐξ ἀφαιρέσεως dici consueuit. Sed Prima philosophia in ijs versatur, quæ & seiuncta, & æterna, & ab omni motu per se soluta sunt ac libera. Cæterùm Physica & Mathematica quanquã subiecto discrepare non videntur, modo tamen rationeq́; differũt cognitionis & contemplationis, vnde dissimilitudo quoque scientiarum sequitur. Etenim Mathematicæ species nihil re vera sunt aliud, quàm corporis naturalis extremitates, quas cogitatione ob omni motu & materia separatas Mathematicus contemplatur: sed easdem consectatur Physicorũ ars, quatenus cum materia comprehensæ sunt, & corpora motui obnoxia circumscribunt. Ex quo fit, vt quæcunque in Mathematicis incommoditates accidunt, eadem etiam in naturalibus rebus videantur accidere, non autem vicissim. Multa enim in naturalibus sequuntur incommoda, quæ nihil ad Mathematicum attinent, διὰ τὸ, *inquit Aristoteles,* τὰ μὲν ἐξ ἀφαιρέσεως λέγεται, τὰ μαθηματικά, τὰ δὲ φυσικὰ ἐκ προσθέσεως. *Siquidem res cum materia deuinctas cõtẽplatur physicus: Mathematicus verò rẽ cognoscit circũscriptis ijs omnibus, quæ sensu percipiũtur, vt grauitate, leuitate, duritie, mollitie, & præterea calore, frigore, alijsq́; contrariorum paribus, quæ sub sensum subiecta sunt: tantum autem relinquit*

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quantitatem & continuum. Itaq; Mathematicorum ars in iis quæ immobilia sunt cernitur (τὰ γὰρ μαθηματικὰ τῶν ὄντων ἄνευ κινήσεώς ἐστιν ἔξω τῶν περὶ τὴν ἀστρολογίαν) quæ verò in naturæ obscuritate posita est, res quidem quæ nec separari nec motu vacare possunt contemplatur. Id quod in vtroque scientiæ genere perspicuum esse potest siue res subiectas definias, siue proprietates earum demonstres. Etenim numerus, linea, figura, rectum, inflexum, æquale, rotundũ, vniuersa denique Mathematicus quæ tractat & profitetur, absque motu explicari docerique possũt: χωριστὰ γὰρ τῇ νοήσει κινήσεώς ἐστι: Physica autẽ sine monitione species nequaquam possunt intelligi. Quis enim hominis, plantæ, ignis, ossium, carnis naturam & proprietates sine motu, qui materiam sequitur, perspiciat? Siquidem tantisper substantia quæque naturalis constare dici solet, quo ad opus & munus suum, agendo patiendóque tueri ac sustinere valeat: quæ certè amissa δυνάμει, ne nomen quidem nisi ὁμωνύμως retinet. Sed Mathematico ad explicandas circuli aut trianguli proprietates, nullum adferre potest vsum, materiæ, vt auri, ligni. ferri, in qua insunt, consideratio, quin eò verius eiusmodi rerum, quarum species tanquam materia vacantes efformemus animo, naturam complectemur, quòd coniunctione materiæ quasi adulterari diprauarique videntur. Quo circa Mathematica species eodem modo quo κοῖλον, siue concauitas, sine motu & subiecto, definitione explicare cog-

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nosciq; possunt: naturales verò cùm eam vim habeant, quam vt ita dicam, simitas cum materia comprehensa sunt, nec absque ea separatim possunt intelligi: quibus exemplis quid inter Physicas & Mathematicas species intersit, haud difficile est animaduertere. Illis certè non semel est vsus Aristoteles. Valeant ergò Protagoræ sophismata, Geometras hoc nomine refellentis, quod circulus normam puncto non attingat. Nam diuina Geometrarum theoremata, qui sensu æstimabit, vix quicquam reperiet quod Geometræ concedendum videatur. Quid enim ex his quæ sensum mouent, ita rectum aut rotundum dici potest, vt à Geometra ponitur? Nec verò absurdum est aut vitiosum, quòd lineas in puluere descriptas pro rectis aut rotundis assumit, quæ nec recta sunt nec rotunda, ac ne latitudinis quidem expertes. Siquidem non ijs vtitur Geometra, quasi inde vim habeat conclusio, sed eorum quæ discenti intelligenda relinquuntur, rudem ceu imaginem proponit. Nam qui primum instituuntur, hi ductu quodam & velut χειραγωγία sensuum opus habent, vt ad illa quæ sola intelligentia percipiuntur, aditum sibi comparare queant. Sed tamen existimandum non est rebus Mathematicis omnino negari materiam ac non eam tantùm quæ sensum afficit. Est enim materia alia quæ sub sensum cadit, alia quæ animo & ratione intelligitur. Illam αἰσθητὴν hanc νοητὴν vocat Aristoteles. Sensu percipitur, vt æs, vt lignum, omnisque materia quæ, moueri potest

test. Animo & ratione cernitur ea quæ in rebus sensilibus inest, sed non quatenus sensu percipiuntur, quales sunt res Mathematicorum. Vnde ab Aristotele scriptum legimus ἐπὶ τῶν ἐν ἀφαιρέσει ὄντων rectum se habere vt simum: μετὰ συνεχοῦς γάρ: quasi velit ipsius recti quod Mathematicorum est, suam esse materiam, non minus quam simi quod ad Physicos pertinet. Nam licet res Mathematicæ sensili vacent materia, non sunt tamen indiuiduæ, sed propter continuationem partitioni semper obnoxiæ, cuius ratione dici possunt sua materia non omnino carere: quin aliud videtur τὸ εἶναι γραμμῇ, aliud quoad continuationi adiuncta intelligitur linea. Illud enim ceu forma in materia proprietatum causa est, quas sine materia percipere non licet. Hæc est societatis & dissidij Mathematicæ cum Physica & prima Philosophia ratio. Nunc autem de nominis etymo & notatione pauca quædam afferamus. Nam si quæ iudicio & ratione imposita sunt rebus nomina, ea certè non temerè indita fuisse credendum est, quibus scientias appellari placuit, sed neque otiosa semper haberi debet ista etymologiæ indagatio, cùm ad rei etiam dubiæ fidem sæpe non parum valeat recta nominis interpretatio. Sic enim Aristoteles ducto ex verborum ratione argumento αὐτομάτου, μεταβολῆς, αἰθέρος, aliarumque rerum naturam ex parte confirmauit. Quoniam igitur Pythagoras Mathematicam scientiam non modò studiosè coluit, sed etiam repetitis à capite principijs, geometricam

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contemplationem in liberalis disciplinæ formam composuit, & perspectis absq; materia solius intelligentiæ adminiculo theorematibus, tractationem περὶ τῶν ἀλόγων, & κοσμικῶν σχημάτων constitutionem excogitauit: credibile est, Pythagoram, aut certè Pythagoreos, qui & ipsi doctoris sui studia libenter amplexi sunt, huic scientiæ id nomen dedisse, quod cum suis placitis atq; decretis congrueret rerumq; propositarum naturam quoquo modo declararet. Ita cùm existimarēt illi, omnem disciplinam, quæ μάθησις dicitur, ἀνάμνησιν esse quandam .i. recordationem & repetionem eius scientiæ, cuius antè quam in corpus immigraret, compos fuerit anima, quemadmodum Plato quoque in Menone, Phædone, & alijs aliquot locis videtur astruxisse: animaduerterent autem eiusmodi recordationem, quæ non posset multis ex rebus perspici, ex his potißimum scientijs demonstrari, si quis nimirum, ait Plato, ἐπὶ τὰ διαγράμματα ἄγῃ, probabile est equidem Mathematicas à Pythagoreis artes κατ' ἐξοχὴν fuisse nominatas, vt ex quibus μάθησις, id est æternarum in anima rationum recordatio διαφερόντως & principuè intelligi posset. Cuius etiam rei fidem nobis diuinus fecit Plato, qui in Menone Socratem induxit hoc argumēti genere persuadere cupientem, discere nihil esse aliud quam suarum ipsius rationum animum recordari. Etenim Socrates pusionem quendam vt Tullij verbis vtar, interrogat de geometrica dimensione quadrati: ad ea sic ille respondet vt puer, et tamen

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men tam faciles interrogationes sunt, vt gradatim respondens, eodem perueniat quò si Geometrica didicisset. Aliam nominis huius rationem Anatolius exposuit, vt est apud Rhodiginum, quòd cùm caetera disciplinae deprehendi vel non docente aliquo poßint omnes, Mathematica sub nullius cognitionem veniant, nisi praeeunte aliquo, cuius solertia succidantur vepreta, vel exurantur, & superciliosa complanentur aspreta. Ita enim Caelius: quod quam vim habeat, non est huius loci curiosius perscrutari. Equidem M. Tullius Mathematicos in magna rerum obscuritate, recondita arte multiplicíq; ac subtili versari scribit: sed quis nescit idipsum cum alijs grauioribus scientijs esse commune? Est enim, vel eodem auctore Tullio, omnis cognitio multis obstructa difficultatibus, maximáq; est & in ipsis rebus obscuritas, & in iudicijs nostris infirmitas, nec vllus est, modò interius paulò Physica penetrarit, qui non facilè sit expertus, quam multi vndique emergant, rerum naturalium causas inquirentibus, inexplicabiles labyrinthi. Sunt qui ex demonstrationum firmitate nominari Mathematicas opinantur: cuius etiam rationis momentum alio seorsim loco expendendum fuerit. Quocirca primam verbi notationem, quam sequutus est Proclus, nobis retinendam censeo. Hactenus de vniuerso Mathematicae genere, quanta potui & perspicuitate & breuitate dixi. Sequitur vt de Geometria separatim atque ordine ea disseram, quae initio sum pollici-

tus. Est autem Geometria, vt definit Proclus scientia, quæ versatur in cognitione magnitudinum figurarum, & quibus hæ continentur, extremorum, item rationum & affectionum, quæ in illis cernuntur ac inhærent: ipsa quidem progrediens à puncto indiuiduo per lineas & superficies, dum ad solida conscendat, variasq; ipsorum differentias patefaciat. Qúmque omnis scientia demonstratiua, vt docet Aristoteles, tribus quasi momentis contineatur, genere subiecto, cuius proprietates ipsa scientia exquirit & cõtemplatur: causis & principijs ex quibus primis demonstrationes conficiuntur: & proprietatibus, quæ de genere subiecto per se enunciantur: Geometriæ quidem subiectum in lineis, triangulis, quadrangulis, circulis, planis, solidis atque omnino figuris & magnitudinibus, earúmque extremitatibus consistit. His autem inhærent diuisiones, rationes, tactus, æqualitates, παραβολαὶ ὑπερβολαὶ ἐλλείψεις, atque alia generis eiusdem propè innumerabilia. Postulata verò & Axiomata ex quibus hæc inesse demonstrantur, eiusmodi ferè sunt: Quouis centro & interuallo circulum describere. Si ab æqualibus æqualia detrahas, quæ relinquuntur esse æqualia, cæteraq; id genus permulta, quæ licet omnium sint communia, ac demonstrandum tamen tum sunt accommodata, cum ad certum quoddam genus traducuntur. Sed cùm præcipua videatur Arithmeticæ & Geometriæ inter Mathematicas dignatio, cur Arithmetica sit ἀκριβεστέρα &

exactior quam Geometria paucis explicandum arbitror. Hic verò & Aristotelem sequemur ducem, qui scientiam cum scientia ita comparat, vt accuratiorem esse velit eam, quæ rei causam docet, quam quæ rem esse tantùm declarat, deinde quæ in rebus sub intelligentiam cadentibus versatur, quàm quæ in rebus sensum mouentibus cernitur. Sic enim & Arithmetica quàm Musica, & Geometria quàm Optica, & Stereometria quàm Mechanica exactior esse intelligitur. Postremò quæ ex simplicioribus initijs constat, quàm quæ aliqua adiectione compositis vtitur. Atque hac quidem ratione Geometriæ præstat Arithmetica, quòd illius initium ex additione dicatur, huius sit simplicius. Est enim punctum, vt Pythagoreis placet, vnitas quæ situm obtinet: vnitas verò punctum est quod situ vacat. Ex quo percipitur, numerorum quàm magnitudinum simplicius esse elementum, numerosque magnitudinibus esse puriores, & à concretione materiæ magis disiunctos. Hæc quanquam nemini sunt dubia, habet & ipsa tamen Geometria quo se plurimum efferat, opibúsque suis ac rerum vbertate multiplici vel cum Arithmetica certet: id quod tute facilè deprehendas cùm ad infinitam magnitudinis diuisionem, quam respuit multitudo, animum conuerteris. Nunc quæ sit Arithmetica & Geometriæ societas, videamus. Nam theorematum quæ demonstratione illustrantur, quædam sunt vtriusque scientiæ communia, quædam verò singu-

gularum propria. Etenim quòd omnis proportio sit ῥητὸς siue rationalis, Arithmeticæ soli conuenit, nequaquam Geometriæ, in qua sunt etiam ἄῤῥητοι seu irrationales proportiones. item, quadratorum gnωmas minimo definitos esse, Arithmeticæ proprium (si quidem in Geometria nihil tale minimum esse potest) sed ad Geometriam propriè spectant situs, qui in numeris locum non habent tactus, qui quidem à continuis admittuntur: ἄλογον, quoniam vbi diuisio infinitè procedit, ibi etiam τὸ ἄλογον esse solet. Communia porrò vtriusque sunt illa, quæ ex sectionibus eueniunt, quas Euclides libro secundo est persequutus: nisi quòd sectio per extremam & mediam rationem in numeris nusquam reperiri potest. Iam vero ex theωrematibus eiusmodi communibus, alia quidem ex Geometria ad Arithmeticam traducuntur: alia contrà ex Arithmetica in Geometriam transferuntur: quædam verò perinde vtrique scientiæ conueniunt, vt quæ ex vniuersa arte Mathematica in vtranque harum conueniant. Nam & alterna ratio, & rationum conuersiones, compositiones, diuisiones hoc modo communia sunt vtriusque. Quæ autem sunt περὶ συμμετρων, id est, de commensurabilibus, Arithmetica quidem primùm cognoscit & contemplatur: secundo loco Geometria Arithmeticam imitata. Quare & commensurabiles magnitudines illæ dicuntur, quæ rationem inter se habent, quam numerus ad numeram, per-

inde quasi cōmensuratio & συμμετρία in numeris primùm consistat (Vbi enim numerus ibi & σύμμετρον cernitur: & vbi σύμμετρον, illic etiam numerus) sed quæ triangulorum sunt & quadrangulorum, à Geometra primum considerantur: tùm analogia quadam Arithmeticus eadem illa in numeris contemplatur. De Geometriæ diuisione hoc adijciendum puto, quòd Geometriæ pars altera in planis figuris cernitur quæ solam latitudinem longitudini coniunctam habent: altera verò solidas contemplatur, quæ ad duplex illud interuallum craßitudinem adsciscunt. Illam generali Geometriæ nomine veteres appellarunt: hanc propriè Stereometriam dixerunt. Ita Geometriam cum Optica & Stereometriam cum Mechanica non raró comparat Aristoteles. Sed illius cognitio huius inuentionem multis seculis anteceßit, si modò Stereometriam ne Socratis quidem ætate vllam fuisse omnino verum est, quemadmodum à Platone scriptum videtur. Ad Geometriæ vtilitatem accedo, quæ quanquam suapte vi & dignitate ipsa per se nititur, nulllius vsus aut actionis ministerio mancipata (vt de Mathematicis omnibus scientijs concedit in Politico Socrates) si quid ex ea tamen vtilitatis externæ quæritur, Dij boni quam latos, quàm vberes, quum varios fructus fundit? Nec verò audiendus est vel Aristippus vel Sophistarum alius, qui Mathematicorum artes idcirco repudiet, quòd ex fine nihil docere videantur, eiusque quod melius aut deteri-

deterius nullam habeant rationem. Vt enim nihil causæ dicas, cur sit melius trianguli, verbi gratia, tres angulos duobus esse rectis æquales: minime tamen fuerit consentaneum Geometriæ cognitionem vt inutilem exagitare, criminari, explodere, quasi quæ finem & bonum quò referatur, habeat nullum. Multas haud dubiè solius contemplationis beneficio citra materiæ contagionem adfert Geometria commoditates partim proprias, partim cum vniuerso genere communes. Cùm enim Geometria, vt scripsit Plato, *eius quod semper est cognitionem profiteatur, ad veritatem excitabit illa quidem animum, & ad ritè philosophandum cuiusque mentem comparabit. Quinetiam ad disciplinas omnes facilius perdiscendas, attigeris necne Geometriam quanti referre censes? Nam vbi cùm materia coniungitur, nónne præstantißimas procreat artes. Geodesiam, Mechaniam, Opticam, quarum omnium vsu, mortalium vitam summis beneficijs complectitur? Etenim bellica instrumenta, vrbiumq́ propugnacula, quibus munitæ vrbes hostium vim propulsarent, his adiutricibus fabricata est: montium ambitus & altitudines, locorumque situs nobis indicauit: dimetiendorum & mari & terra itinerum rationem præscripsit: trutinas & stateras, quibus exacta numerorum æqualitas in ciuitate retineatur, composuit: vniuersi ordinem simulacbris expreßit, multáque quæ hominum fidem superarent, omnibus persuasit. Vbique extant præclara in eam rem*

 testimo-

teſtimonia. Illud memorabile, quòd Archimedi rex Hiero tribuit. Nam extructo vaſtæ molis nauigio, quod Hiero Ægyptiorum regi Ptolemæo mitteret, cùm vniuerſa Syracuſanorum multitudo collectis ſimul viribus nauem trahere non poſſet, effeciſſetq́; Archimedes, vt ſolus Hiero illam ſubduceret, admiratus viri ſcientiam rex ἀπὸ ταύτης, ἔφη, τῆς ἡμέρας περὶ παντὸς ἀρχιμήδει λέγοντι πιστευτέον. *Quid? quòd Archimedes idem, vt eſt apud Plutarchū, Hieroni ſcriptis datis viribus datum pondus moueri poſſe? fretusq́; demonſtrationis robore illud ſæpe iactarit, ſi terram haberet alteram vbi pedem figeret, ad eam, noſtram hanc ſe tranſmouere poſſe? Quid varia,* αὐτομάτων *machinarúmque genera ad vſus neceſſarios comparata memorem? Innumerabilia profectò ſunt illa, & admiratione digniſſima quibus priſci homines incredibili quodam ad philoſophandum ſtudio concitati, inopem mortalium vitam artis huius præſidio ſubleuarunt: tametſi memoriæ ſit proditum, Platonem Eudoxo & Archytæ vitio vertiſſe, quòd Geometrica problemata ad ſenſilia & organica abducerent. Sic enim corrumpi ab illis & labefieri Geometriæ præſtantiam, quæ ab intelligibilibus & incorporeis rebus ad ſenſiles & corporeas prolaberetur. Quapropter ridicula idem ſcripſit Plato Geometrarum eſſe vocabula, quæ quaſi ad opus & actionem ſpectent, ita ſonare videntur. Quid enim eſt quadrare ſi non opus facere? Quid addere,*

pro

producere, applicare? Multa quidem sunt eiusmodi nomina, quibus necessariò & tanquam coacti Geometræ vtuntur, quippe cùm alia desint in hoc genere commodiora. Sic ergo censuit Plato, sic Aristoteles sic denique philosophi omnes, Geometriam ipsam cognitionis gratia exercendam, nec ex aliquo vsu externo sed ex rerum νοητῶν intelligentia æstimandam esse. Exposita breuius quàm res tanta dici possit, vtilitatis ratione, Geometriæ ortum, qui in hac rerum periodo ex historicorum monumentis nobis est cognitus deinceps aperiamus. Geometria apud AEgyptios inuenta, (ne ab Adamo, Setho, Noah, quos cognitione rerum multiplici valuisse constat, eam repetamus) ex terrarum dimensione, vt verbi præ se fert ratio, ortum habuisse dicitur: cùm anniuersaria Nili inundatione & incrementis limo obducti agrorum termini confunderentur. Geometriam enim, sicut & reliquas disciplinas, in vsu quàm in arte prius fuisse aiunt. Quod sanè mirum videri non debet, vt & huius & aliarum scientiarum inuentio ab vsu cœperit ac necessitate. Etenim tempus, rerum vsus, ipsa necessitas ingenium excitat, & ignauiam acuit. Deinde quicquid ortum habuit (vt tradunt Physici) ab inchoato & imperfecto processit ad perfectum. Sic artium & scientiarum principia experientiæ beneficio collecta sunt: experientia verò à memoria fluxit, quæ & ipsa à sensu primùm manauit. Nam quod scribit Aristoteles, Mathematicas artes, *comparatis rebus omnibus ad vitam necessarijs, in AEgypto fuisse constitutas, quòd ibi sacerdotes omnium concessu in otio degerent: non negat ille adductos necessitate homines ad excogitandam, verbi gratia terræ dimetiendæ rationem, quæ theorematum deinde inuestigationi causam dederit: sed hoc confirmat, præclara eiusmodi theorematum inuenta, quibus extructa Geometriæ disciplina constat, ad vsus vitæ necessarios ab illis non esse expetita. Itaque vetus ipsum Geometriæ nomen ab illa terræ partiundæ finiúmque regendorum ratione posteà recessit, & in certa quadam affectionum magnitudini per se inhærentium scienta propriè remansit. Quemadmodum igitur in mercium & contractuum gratiam supputandi ratio, quam secuta est accurata numerorum cognitio, à Phœnicibus initium duxit: ita etiam apud AEgyptios, ex ea quam commemoraui causa ortum habuit Geometria. Hanc certè, vt id obiter dicam, Thales in Græciam ex AEgypto primum transtulit: cui non paucæ deinceps à Pythagora, Hipprocrate, Chio, Platone, Archyta Tarentino, alijsq; compluribus, ad Euclidis tempora factæ sunt rerum magnarum accessiones. Cæterùm de Euclidis ætate id solùm addam, quod à Proclo memoriæ mandatum accepimus. Is enim commemoratis aliquot Platonis tum æqualibus tùm discipulis, subijcit, non multò ætate posteriorem illis fuisse Euclidem eum, qui Elementa conscripsit, & multa ab Eudoxo collecta, in ordinem luculentum* *composuit, multaq́; à Theateto inchoata perfecit, quaq́; mollius ab alijs demonstrata fuerant, ad firmissimas & certissimas apodixes reuocauit. Vixit autem, inquit ille sub primo Ptolemaeo. Et enim ferunt Euclidam à Ptolemaeo quondam interrogatum num qua esset via ad Geometriam magis compendiaria, quàm sit ista* ϛοιχείωσις *respondisse* μὴ εἶναι βασιλικὴν ἀτραπὸν ἐπὶ γεωμετρίαν. *Diende subiungit, Euclidem natu quidem esse minorem Platone, maiorem verò Eratosthene & Archimede (hi enim quales erant) cùm Archimedes Euclidis mētionem faciat. Quòd si quis egregiam Euclidis laudem quam cùm ex alijs scriptionibus accuratissimus, tùm ex hac Geometria* στοιχειώσει *cōsequutus est in qua diuinus rerum ordo sapientissimis quibusque hominibus magnae semper admirationi fuit, is Proclum studiosè legat, quò rei veritatem illustriorem reddat grauissimi testis auctoritas. Superest igitur vt finem videamus, quò Euclidis elementa referri, & cuius causa in id studium incumbere, oporteat. Et quidem si res quae tractantur, consideres: in tota hac tractatione nihil aliud quaeri dixeris, quàm vt* Κοσμικὰ *quae vocantur* σχήματα *(fuit enim Euclides professione & instituto Platonicus) Cubus Icosaedrum, Octaedrum, Pyramis & Dodecaedrum certa quadam suorum & inter se laterum, & ad sphaerae diametrum ratione eidem sphaerae inscripta comprehendantur. Huc enim pertinet Epigrammation illud vetus, quod in Geometrica Michaelis*

Pselli συνόψει scriptum legitur.

Σχήματα πέντε τὰ πλάτωνος, πυθαγόρας σοφὸς εὗρε.

πυθαγόρας σοφὸς εὗρε, πλάτων δ᾽ ἀρίδηλ᾽ ἐδίδαξεν.

Εὐκλείδης ἐπὶ τοῖσι κλέος περικαλλὲς ἔτευξεν.

Quòd si discentis institutionem spectes, illud certè fuerit propositum, vt huiusmodi elementorum cognitione informatus discentis animus, ad quamlibet non modò Geometriæ, sed & aliarum Mathematicæ partium tractationem idoneus paratusq́; accedat. Nam tametsi institutionem hanc solus sibi Geometra vendicare videtur, & tanquam in possessionem suam venerit, alios excludere posse: inde tamen per multa suo quodam modo iure decerpit Arithmeticus, pleraque Musicus non pauca detrahit. Astrologus, Opticus, Logisticus, Machanicus, item'q; cæteri: nec vllus est denique artifex præclarus, qui in huius se possessionis societatem cupide non offerat, partem'q́; sibi cōcedi postulet. Hinc στοιχειώσεις *absolutum operi nomen, &* στοιχειωτὴς *dictus Euclides. Sed quid longius prouehor? Nam quod ad hanc rem attinet, tam copiosè & eruditè scripsit (vt alia complura) eo ipso, quem dixi, loco P. Montaureus, vt nihil desiderio loci reliquerit. Quæ verò ad dicendum nobis erant proposita hactenus pro ingenij nostri tenuitate omnia mihi perfecisse videor. Nam tametsi & hæc eadem & alia pleraque multo fortè præclariora ab hominibus doctißimis, qui* tùm acumine ingenij, tùm admirabili quodam lepore dicendi semper floruerunt grauiùs, splendidiùs, vberiùs tractari posse scio: tamen experiri libuit numquid etiam nobis diuino sit concessum munere, quod rudes in hac philosophiæ parte discipulos adiuuare aut certè excitare queat. Huc acceßit quòd ista recens elementorum editio, in qua nihil non parum fuisset studij, aliquid à nobis efflagitare videbatur, quod eius commendationem adaugeret. Cùm enim vir doctißimus Io. Magnienus Mathematicarum artium in hac Parrhisiorum Academia professor verè regius, nostrum hunc typographum in excudendis Mathematicorum libris diligentißimum, ad hanc Elementorum editionem sæpè & multum esset adhortatus, eiusque impulsu permulta sibi iam comparasset typographus ad hanc rem necessaria, citò interuenit, malùm Ioannis Magnieni mors insperata, quæ tam graue inflixit Academiæ vulnus, cui ne post multos quidem annorum circuitus cicatrix obduci vlla posse videatur. Quamobrem amisso instituti huius operis duce, typographus, qui nec sumptus antea factos sibi perire, nec studiosos, quibus id muneris erat pollicitus, sua spe cadere vellet, ad me venit, & impensè rogauit, vt meam propositæ editioni operam & studium nauarem, quod cùm denegaret occupatio nostra, iuberet officij ratio, feci equidem rogatus, vt quæ subobscurè vel parum commode in sermonem Latinum è Græco translata vi-

videbantur, clariore, aptiore & fideliore interpretatione nostra (quod cuiusque pace dictum volo) lucem acciperent. Id quod in omnibus ferè libris posterioribus tute primo obtutu perspicias. Nam in sex prioribus non tantum temporis quantum in cæteres ponere nobis: licuit decimi autem interpretatio, qua melior nulla potuit adfert, P. Montaureo solida debetur. Atque vt ad perspicuitatem facilitatemque nihil tibi deesse queraris, adscripta sunt propositionibus singulis vel lineares figuræ, vel punctorum tanquam vnitatum notulæ, quæ Theonis apodixin illustrent illæ quidem magnitudinum, hæ autem numerorum indices, subscriptis etiam ciphrarum, vt vocant, characteribus, qui propositum quemuis numerum exprimant, ob eamque causam eiusmodi vnitatum notulæ, quæ pro numeri amplitudine maius paginæ spatium occuparent, pauciores sæpius depictæ sunt aut in lineas etiam commutatæ. Nam literarum vt a, b, c characteres non modò numeris & numerorum partibus nominandis sunt accommodati, sed etiam generales esse numerorum, vt magnitudinum affectiones testantur. Adiecta sunt insuper quibusdam locis non pœnitenda Theonis scholia, siue mauis lemmata, quæ quidem longè plura accessissent, si plus otij & temporis vacui nobis fuisset relictum, quod huic studio impartiremus. Hanc igitur operam boni consule, & quæ obuia erunt impreßionis vitia, candidè emenda. Vale. Lutetiæ 4. Idus April. 1557.

EVCLIDIS ELEMENTVM PRIMVM

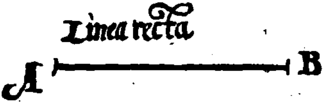
DEFINITIONES.

1

PVnctum est, cuius pars nulla est. Punctum

2

Linea verò, longitudo latitudinis expers.



3

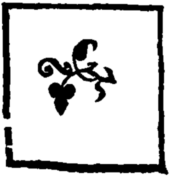
Lineæ autem termini, sunt puncta.

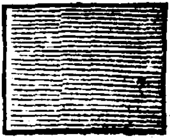
4

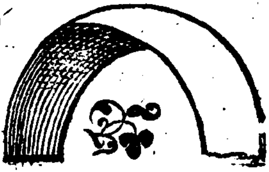
Recta linea est, quæ ex æquo sua interiacet puncta.

5

Superficies est, quæ longitudinem latitudinemque tantùm habet.





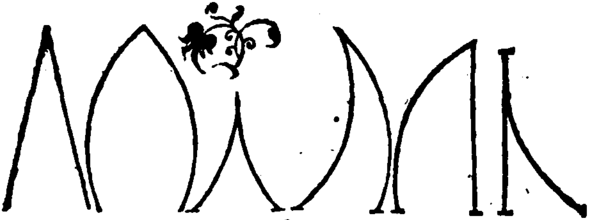


6

Superficiei extrema. ſunt lineæ.

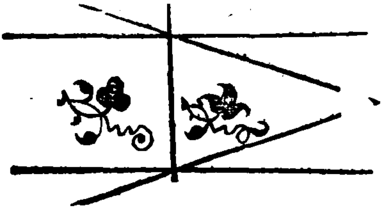
7

Plana ſuperficies eſt, quæ ex æquo ſuas interiacet lineas.



8

Planus angulus eſt duarũ linearũ in plano ſe mutuò tangentium, & non in directum



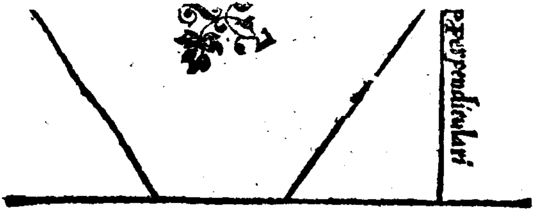
iacétium, alterius ad alterá inclinatio.

9

Cùm autem quæ angulum continét lineæ, rectæ fuerint, rectilineus ille angulus appellatur.

10

Cùm verò recta linea ſuper rectam conſiſtens lineã, eos qui ſunt deinceps angulos ęquales, inter ſe fecerit: rectus eſt vterq; æquaqualium angulorum: quæ insistit recta linea, perpẽdicularis vocatur ei⁹, cui insistit.



11

Obtusus angulus est, qui recto maior est.

12

Acutus verò, qui minor est recto.

13

Terminus est, quod alicuius extremũ est.

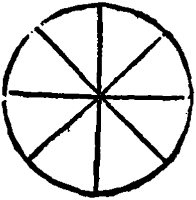
14

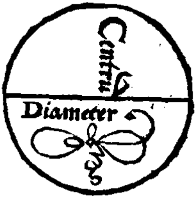
Figura est, quæ sub aliquo, vel aliquibus terminis comprehenditur.

15

Circulus est, figura plana sub vna linea cõprehẽsa, quę peripheria appllatur: ad quam ab vno puncto eorum. quæ intra figuram sunt

ſunt po ſita, cadentes omnes rectæ lineæ inter ſe ſunt æquales.





16

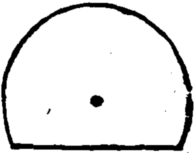
Hoc verò punctum, centrum circuli appellatur.

17

Diameter autem circuli, eſt recta quædam linea per centrum ducta, & ex vtraque parte in circuli peripheriam terminata, quæ circulum bifariam ſecat.

18

Semicirculus eſt figura, quæ continetur ſub diametro, & ſub ea linea, quæ de circuli peripheria aufertur.



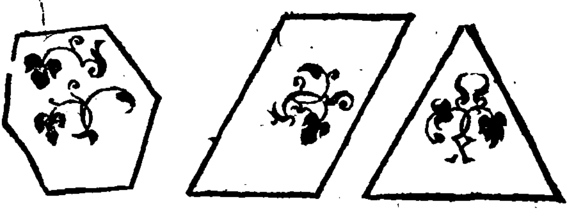
19

Segmentum circuli, eſt figura, quæ ſub recta linea & circuli peripheria continentur.

20 Recti

20.

Rectilineæ figuræ, sunt quæ sub rectis lineis continentur.



21

Trilateræ quidem, quæ sub tribus.

22

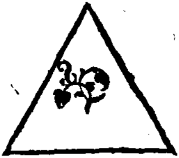
Quadrilateræ, quæ sub quatuor.

23

Multilateræ verò, quæ sub pluribus quàm quatuor rectis lineis comprehenduntur.

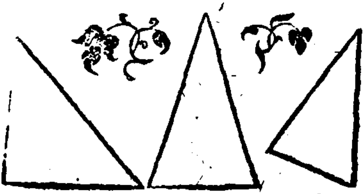
24

Trilaterarũ, porrò figurarum, æquilaterum est triangulum, quod tria latera habet æqualia.



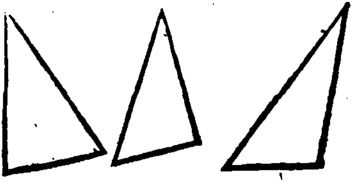
25

Isosceles autem est quod duo tantum æqualia habet latera.



26

Scalenũ verò, eſt qđ tria inęqualia habet latera.



23

Ad hæc etiam trilaterarum figurarum, rectangulum quidem triangulum eſt, quod rectum angulum habet.

28

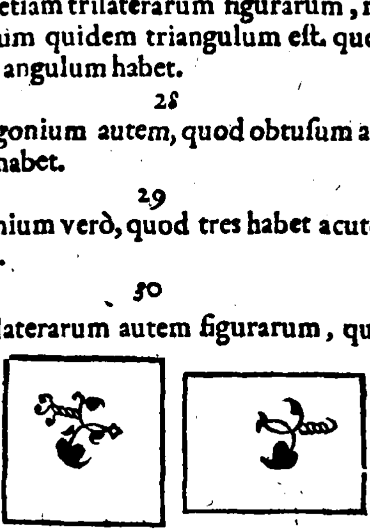
Amblygonium autem, quod obtuſum angulum habet.

29

Oxygonium verò, quod tres habet acutos angulos.

30

Quadrilaterarum autem figurarum, quadratũ quidẽ eſt qđ & æquilaterũ & rectangulum eſt.

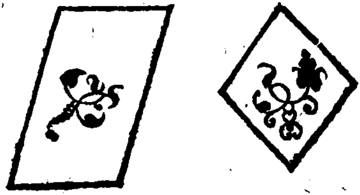


31

Altera parte longior figura eſt, quæ rectãgula quidem, at æquilatera non eſt.

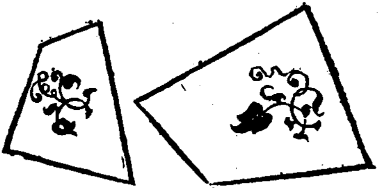
32 Rhom-

32
Rhó-bus autem, qui æquilaterum & rectangulum est.



33
Rhomboides verò, quæ aduersa & latera & angulos habens inter se æqualia, neque æquilatera est, neque rectangula.

34
Pręter has autẽ reliquæ quadrilateræ figuræ, trapezia appellentur.



35
Parallelæ rectæ lineæ sunt quæ, cùm in eodem sint plano, & ex vtraque parte in infinitum producantur, in neutram sibi mutuò incidunt.

Postulata.

1
Postuletur, vt à quouis puncto in quoduis

 pun-

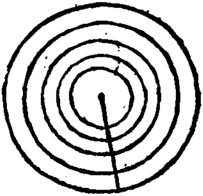
punctum, rectam lineã ducere cõcedatur.

2

Et rectam lineam terminatam in continuum recta producere.

3

In quouis centro & interuallo circulũ describere.



Communes notiones.

1.

Quæ eidẽ æqualia, & inter se sunt æqualia.

2

Et si æqualibus æqualia adiecta sint, tota sunt æqualia.

3

Et si ab æqualibus æqualia ablata sint, quæ relinquuntur sunt æqualia.

4

Et si inæqualibus æqualia adiecta sint, tota sunt inæqualia.

5

Et si ab inęqualibus æqualia ablata sint, reliqua sunt inæqualia.

6

Quæ eiusdem duplicia sunt, inter se sunt æqualia.

Et

7

Et quæ eiusdem sunt dimidia, inter se æqualia sunt.

8

Et quæ sibi mutuò congruunt, ea inter se sunt æqualia.

9

Totum est sua parte maius.

10

Item, omnes recti anguli sunt inter se æquales.

11

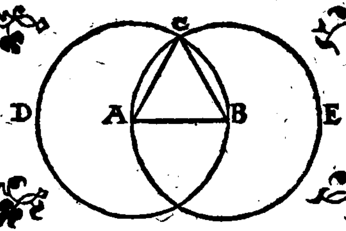
Et si in duas rectas lineas altera recta incidens, internos ad easdéque partes angulos, duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ sibi mutuò incident ad eas partes, vbi sunt anguli duobus rectis mimores.

12

Duæ rectæ lineæ spatium non comprehendunt.

Problema 1. Propositio 1.

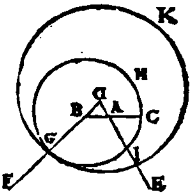
Super data recta linea terminata, triangulū



æquilaterum constituere.

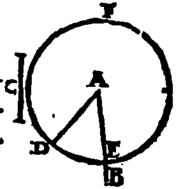
Problema 2. Propositio 2.

Ad datum punctum, datæ rectæ lineæ, ęqualem rectam lineam ponere.



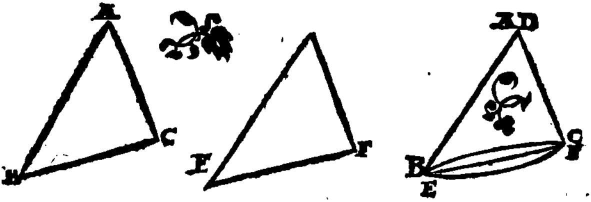
Problema 3, Propositio 3.

Duabus datis rectis lineis inęqualibus, de maiore æqualẽ minori rectam lineã detrahere.



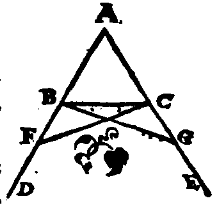
Theorema primum. Propositio 4.

Si duo triangula duo latera duobus lateribus æqualia habeant, vtrunque vtriq;, habeant verò & angulum angulo æqualẽ sub æqualibus rectis lineis contentum: & basin basi æqualem habebunt, eritq; triãgulum triãgulo æquale, ac reliqui anguli reliquis angulis æquales erũt, vterque vtrique, sub quibus æqualia latera subtenduntur,



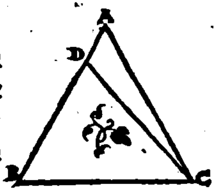
Theore-

Theorema 2. Propoſitio 5.



Iſoſcelium tringulorũ qui ab baſim ſunt anguli, inter ſe ſunt æquales; & ſi vlterius productæ ſint æquales illę rectæ lineæ, qui ſub baſi ſunt anguli, inter ſe equales erunt.

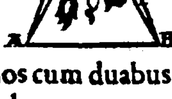
Problema 3. Propoſitio 6.

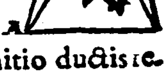


Si trianguli duo anguli æquales inter ſe fuerint & ſub ęqualibus angulis ſubtenſa latera æqualia inter ſe erunt.

Theorema 46. Propoſitio 7.

Super eadẽ recta linea, duabus eiſdem rectis lineis alię duę rectę lineæ ęquales, vtraquę vtrique nõ conſtituentur, ad aliud atque aliud punctũ, ad eaſdẽ partes eoſdẽ

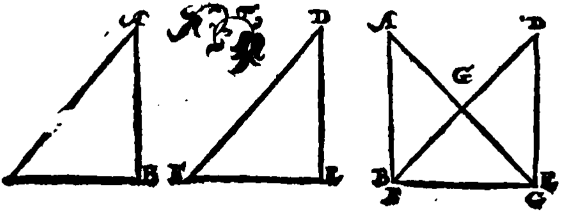




que terminos cum duabus initio ductis rectis lineis habentes.

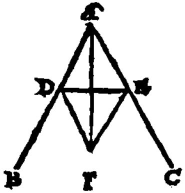
Theorema 5. Propositio 8.

Si duo triãgula duo latera habuerint duobus lateribus, vtrunque vtriq; æqualia: habuerint verò & basim basi æqualẽ: angulũ quoque sub æqualibus rectis lineis cõtentum angulo æqualem habebunt.



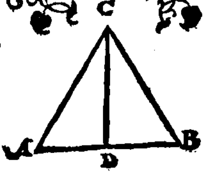
Problema 4. Porpopositio 9.

Datum angulum rectilineum bifariam secare.



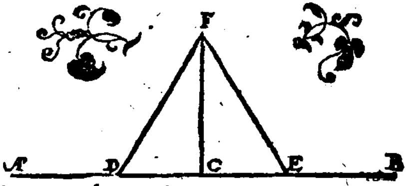
Problema 5. Propositio 10.

Datam rectam lineã finitam bifariam secare.



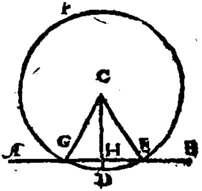
Problema 6. Propositio 11.

Data recta linea, à puncto in ea dato, rectam lineam ad angulos rectos excitare.

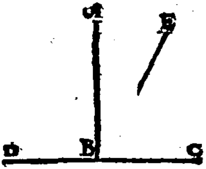


Problema 7. Propositio 12.

Super datam rectā lineā infinitam, à dato puncto quod in ea non est, perpendicularem rectam deducere.



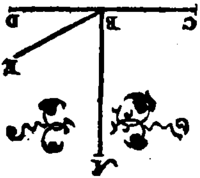
Theorema 6. Propositio 13.



Cùm recta linea super rectā consistens lineā angulos facit, aut duos rectos, aut duobus rectis æquales efficiet.

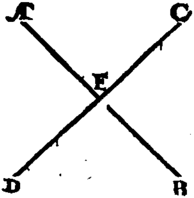
Theorema 7. Propositio 14.

Si ad aliquam rectam lineam, atque ad eius punctū, duę rectę lineæ nō ad eaſdem partes ductæ, eos qui ſunt deinceps angulos duobus rectis æquales fecerint, in directum erunt inter ſe ipſæ rectæ lineæ.



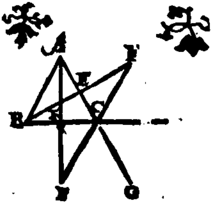
Theorema 8. Propoſitio 15.

Si duę rectę lineę ſe mutuò ſecuerint, angulos qui ad verticem ſunt, æquales inter ſe efficient.



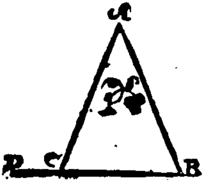
Theorema 9. Propoſitio 16.

Cuiuſcunque trianguli vno latere producto, externus angulus vtroque interno & oppoſito maior eſt.



Theorema 10. Propoſitio 17.

Cuiuſcunque trianguli duo auguli duobus rectis ſunt minores omnifariam ſumpti.



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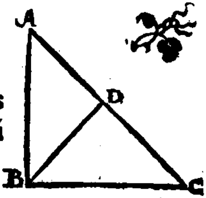
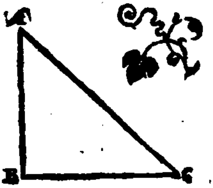
Theorema 11. Propoſitio 18.



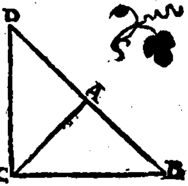
Omnis trianguli maius latus maiorem angulú ſubtendit.

Theorema 12. Propoſitio 16.



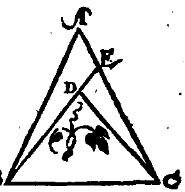
Omnis trianguli lateri angulus, maiori lateri ſubtenditur.

Theorema 13. Propoſitio 20.



Omnis triáguli duo latera reliquo ſunt maiora, quomodocumq; aſſumpta.

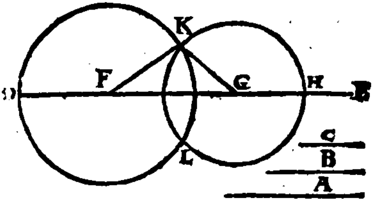
Theorema 14. Propoſitio 12.



Si ſuper triáguli vno latere, ab extremitatibus duæ rectę lineę, interius cóſtitutæ fuerint, hę cóſtitutæ reliquis triáguli duobus lateribus minores quidem erunt, minorem verò angulum continebunt.

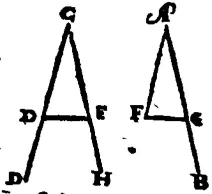
Pro-

Problema 8. Propositio 22.



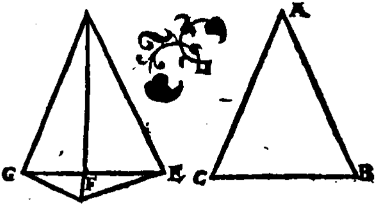
Ex tribus rectis lineis quæ sũt tribus datis rectis lineis æquales, triangulũ constituere. Oportet autem duas reliqua esse maiores omnifariam sumptas: quoniá vniuscuiusq; trianguli duo latera omnifariam sumpta reliquo sunt maiora.

Problema 9. Propositio 23.



Ad datam rectam lineã datumq́; in ea punctum dato angulo rectilineo ęqualem angulum rectilineum constituere.

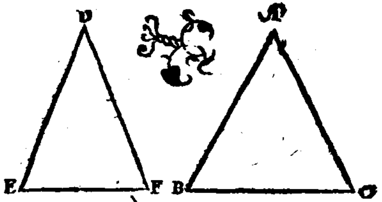
Theorema 15. Propositio 24.



Si duo triãgula duo latera duobus lateribus ęqualia habuerint, vtrũq; vtriq;, angulũ verò angu lo lo maiorem ſub æqualibus rectis lineis cō-tentum:& baſin baſi maiorem habebunt.

Theorema 16. Propoſitio 25.

Sì duo triangula duo latera duobus lateribus æqualia habuerint, vtrunque vtrique, baſin verò baſi maiorē: & angulum ſub æqualib' rectis



lineis contentum angulo maiorē habebūt.

Theorema 17. Propoſitio 26.

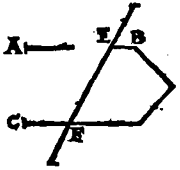
Si duo triangla duos angulos duobus angulis æquales habuerint, vtrúque vtrique, vnumq; latus vni lateri æquale, ſiue, quod æqualibus adiacet angulis, ſeu quod vni æqualium angulorū ſubtenditur: & reliqua latera reliquis lateribꝰ ęqualia vtrūq; vtriq;,

& reliquum angulū reliquo angulo æqualem habebunt.

Theore-

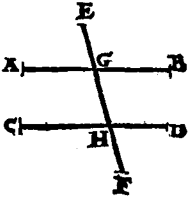
Theorema 18. Propoſitio 27.

Si in duas rectas lineas recta incidẽs linea alternatim ãgulos ęquales inter ſe fecerit: parallelę erunt inter ſe illæ rectæ lineæ.



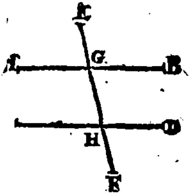
Theorema 19. Propoſitio 28.

Si in duas rectas lineas recta incidens linea externum angulũ interno, & oppoſito, & ad eaſdem partes æqualẽ fecerit, aut internos & ad eaſdẽ partes duobus rectis æquales: parellelæ erunt inter ſe ipſæ rectæ lineæ.



Theorema 20. Propoſitio 29.

In parallelas rectas lineas recta incidens linea: & alternatim angulos inter ſe æquales efficit & externum interno & oppoſito & ad eaſdem partes æqualem, & internos & ad eaſdem partes duobus rectis æquales facit.



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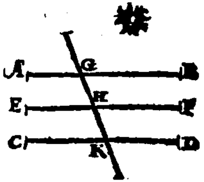
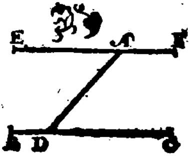
Theorema 21. Propoſitio 30.



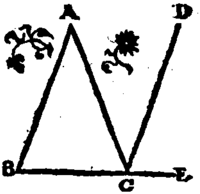
Quæ eidem rectæ lineę, *parallelę*, & inter ſe ſunt parallelæ.

Problema 10. Propoſitio 31.



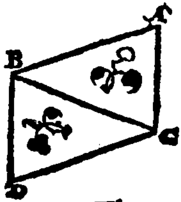
A dato puncto datæ rectæ lineæ parallelam rectam lineam ducere.

Theorema *22*. Propoſitio. *32*.



Cuiuſcunque trianguli vno latere vlterius producto: externus angulus duobus internis & oppoſitis eſt æqualis. Et trianguli tres interni anguli duobus ſunt rectis æquales.

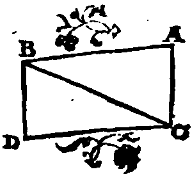
Theorema *23*. Propoſitio 33.



Rectæ lineę quę ęquales & parallelas lineas ad partes eaſdem coniungunt, & ipſæ æquales & parallelæ ſunt.

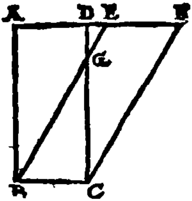
Theore-

Theorema 24. Propoſitio 34.



Parallelogrammorũ ſpatiorum æqualia ſunt inter ſe, quæ ex aduerſo & latera & anguli: atque illa bifariã ſecat diameter.

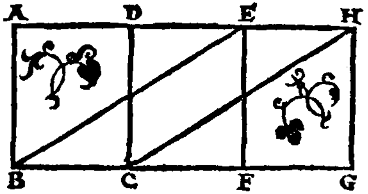
Theorema 25. Propoſitio 35.



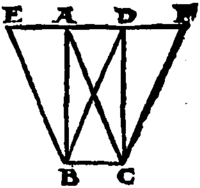
Parallelogramma ſuper eadem baſi, & in eiſdem parallelis cõſtituta, inter ſe ſunt æqualia.

Theorema 26 Propoſitio 36.

Parallelogrãma ſuper æqualibus baſibus & in eiſdẽ parallelis cõſtituta inter ſe ſunt æqualia.



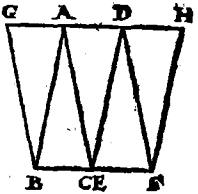
Theorema 27. Propoſitio 37.



Triãgula ſuper eadẽ baſi cõſtituta, & in eiſdẽ parallelis, inter ſe ſunt ęqualia.

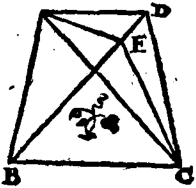
Theore-

Theorema 28. Propositio 38.



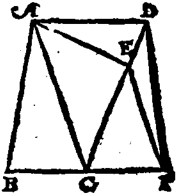
Triangula ſuper ęqualibus baſibus conſtituta et in eiſdem parallelis, inter ſe ſunt æqualia.

Theorema 29. Propoſitio 39.



Triangula æqualia ſuper eadem baſi, & ad eaſdem partes conſtituta: & in eiſdem ſunt Parallelis.

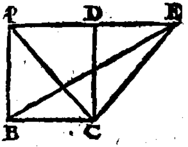
Theorema 30. Propoſitio 40.



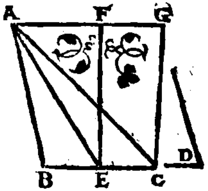
Triangula æqualia ſuper æqualibus baſibus, & ad eaſdem partes conſtituta, & in eiſdem ſunt parallelis.

Theorema 31. Propoſitio 41.

Si parallelogrammum cum triangulo eandem baſin habuerit, non eiſdemque fuerit parallelis, duplum erit parallelogrammum ipſius trianguli.

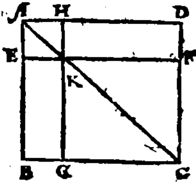


Problema 11. Propositio 42.



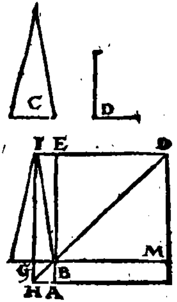
Dato triangulo æquale parallelogrammū constituere in dato angulo rectilineo.

Theorema 32. Propositio 43.



In omni parallelogrammo, complementa eorū quę circa diametrū sunt parallelogrammorū, inter se sunt æqualia.

Problema 12. Propositio 44.

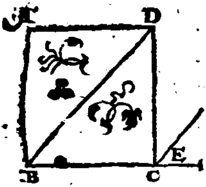


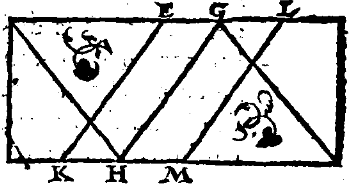
Ad datam rectam lineā dato triangolo æquale parallelogrammum applicare in dato angulo rectilineo.

Problema 13. Propopotio 45.

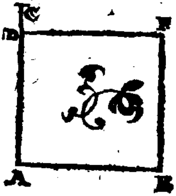
Dato rectilineo ęquale parallelongrammū consti-

constituere in dato angulo rectilineo.





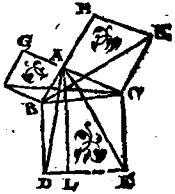
Theorema 41. Propositio 4.



A data recta linea quadratum describere.

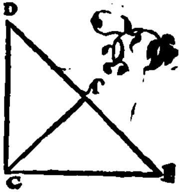
Theorema 33. Propositio 47.

In rectangulis triangulis, quadratum quod à latere rectum angulum subtendente describitur, æquale est eis, quæ à lateribus rectum angulum continentibus describũtur, quadratis.



Theorema 34. Propositio 48.

Si quadratum quod ab vno laterum trianguli deſcribitur, æquale ſit eis quæ à reliquis trianguli lateribus deſcribuntur, quadratis: angulus comprehenſus ſub reliquis duobus trianguli lateribus, rectus eſt.



FINIS ELEMENTI I.

EVCLIDIS ELEMENTVM SECVNDVM.

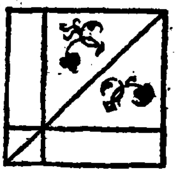
DEFINITIONES.

1.

OMne parallelogrammum rectangulum contineri dicitur sub rectis duabus lineis, quę rectum comprehendunt angulum.

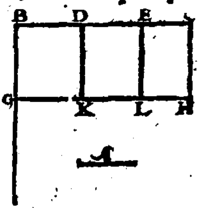
2.

In omni parallelogrammo *spatio*, vnumquodlibet eorum, quæ circa diametrum illius sunt parallelogrammorum, cum duobus cõplementis, Gnomo vocetur.

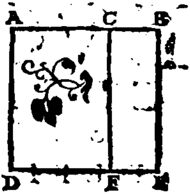


Theorema 1. Propositio *1.*

Si fuerint duæ rectæ lineæ, seceturque ipsarum altera in quotcunq; segmenta: rectangulum comprehensum sub illis duabus rectis lineis, æquale est eis rectangulis, quæ sub insecta & quolibet segmẽtorum comprehenduntur.



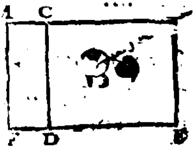
Theorema 2. Propositio 2.



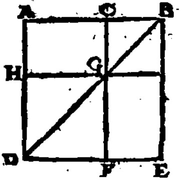
Si recta linea secta sit vtcunq;, rectágula quæ sub tota & quolibet segmentorum cóprehenduntur æqualia sunt ei, quod à tota fit, quadrato.

Theorema 3. Propositio 3.

Si recta linea secta sit vtcunque, rectangulum sub tota & vno segmẽtorum comprehensum, ęquale est & illi quod sub segmentis cóprehenditur rectangulo & illi, quod à prædicto segmẽto describitur, quadrato.



Theorema 4. Propositio 4.

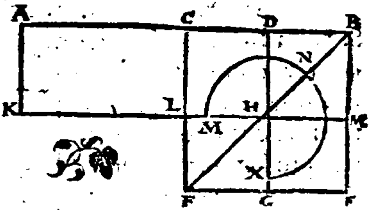


Si recta linea secta sit vtcunque: quadratũ quod à tota describitur, ęquale est & illis quæ à segmẽtis describũtur quadratis, & ei quod bis sub segmentis compreh ẽditur rectangulo.

Theorema 5. Propositio 5.

Si recta lenea secetur in æqualia & non æqualia: rectangulum sub inæqualibus segmentis

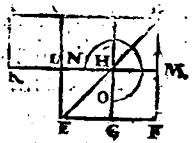
mentis to- tius com- prehéſũ, vnà cum quadrato quod ab inter me-



dia ſectionum, æquale eſt ei quod à dimi- día deſcribitur, quadrato.

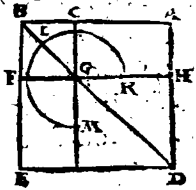
Theorema 6 Propoſitio 6.

Si recta linea bifariam ſecetur, & illi recta quædam linea in rectum adijciatur, rectã- gulum comprehenſum ſub tota cum adie- cta & adiecta ſimul cum quadrato à dimidia, æ- quale eſt quadrato à li- nea quæ tũ ex dimidia, tum ex adiecta compo- nitur, tanquam ab vna deſcripto.



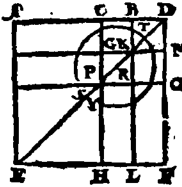
Theorema 7. Propoſitio 7.

Si recta linea ſecetur vt cunque, quod à to- ta, quodque ab vno ſegmentorum, vtraq; ſimul quadrata, æqualia ſunt & illi quod bis ſub tota & dicto ſegemento comprehenditur, rectan- gulo, & illi ꝙ à reliquo ſegmento fit, quadrato.

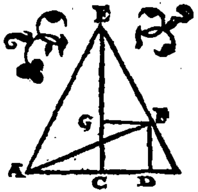


Theorema 8. Propositio 8.

Si recta linea secetur vtcunq; rectangulum quater comprehensum sub tota & vno segmentorum, cum eo quod à reliquo segmẽto fit, quadrato, ęquale est ei quod à tota & dicto segmento, tanquã ab vna linea describitur quadrato.

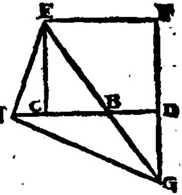


Theorema 9. Propositio 9.



Si recta linea secetur in æqualia & non æqualia: quadrata quæ ab inęqualibus totius segmẽtis fiunt, duplicia sunt & eius quod à dimidia, & eius quod ab intermedia sectionum fit, quadratorum.

Theorema 10. Propositio 10.

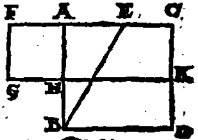


Si recta linea secetur bifariam, adijciatur autem ei in rectum quępiam recta linea; quod à tota cũ adiuncta, & quod ab adiuncta, vtraq; simul quadrata, duplicia sunt, & eius quod à dimidia, & eius quod à composita ex dimidia & adiuncta.

iuncta, tanquã ab vna descriptum sit quadratorum.

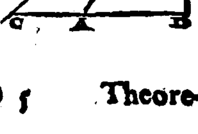
Problema 1. Propositio 11.

Datam rectam lineã secare, vt comprehensum sub tota & altero segmẽtorum rectangulum, æquale sit ei quod à reliquo segmento fit, quadrato.



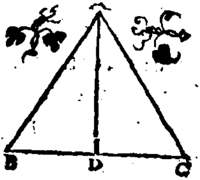
Theorema 11. Propositio 12.

In amblygonijs triangulis, quadratũ quod fit à latere angulum obtusum subtendẽte, maius est quadratis, quæ fiunt à lateribus obtusum angulum comprehendentibus, pro quantitate rectanguli bis comprehensi & ab vno laterũ quę sunt circa obtusum angulũ, in quod cùm protractum fuerit, cadit perpendicularis, & ab assumpta exterius linea sub perpendiculari prope angulũ obtusum.



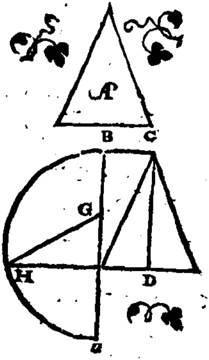
Theorema 12. Propositio 28.

In oxygonijs triangulis quadratũ à latere angulum acutum subtendente, minus est quadratis quæ fiunt à lateribus acutũ angulum cõprehendentibus, pro quantitate rectanguli bis comprehensi, & ab vno laterum, quæ sunt circa acutum angulum, in quod perpẽdicularis cadit, & ab assumpta interius linea sub perpendiculari prope acutũ angulum.



Problema 2. Propositio 14.

Dato rectilineo æquale quadratũ constituere.



ELEMENTI II. FINIS.

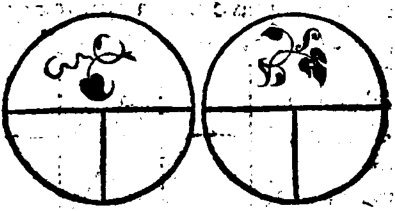
EVCLI.

EVCLIDIS ELEMENTVM TERTIVM.

DEFINITIONES.

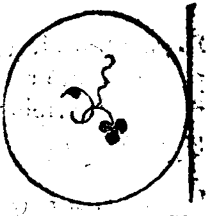
1.

Aequales circuli sunt quorum diametri sūt ęquales vel quorū quæ ex centris, rectæ lineę sunt æquales.



2.

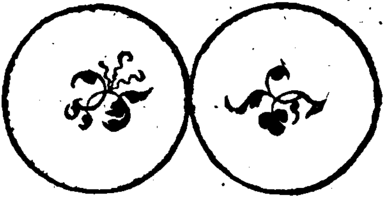
Recta linea circulū tangere dicitur, quæ cùm circulum tangat, si producatur, circulum non secat.



3 Cir-

3

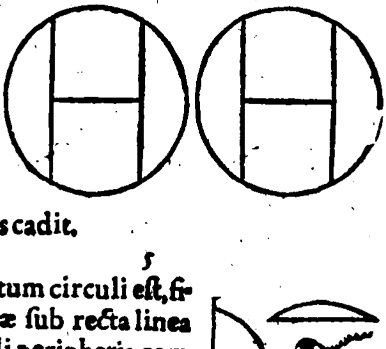
Circuli ſeſe mutuò tangere dicuntur: qui ſeſe mutuo



tangentes, ſeſe mutuo non ſecant.

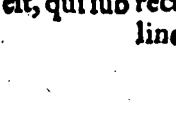
4

In circulo æqualiter diſtare à centro rectæ lineæ dicuntur, cùm perpendiculares, quæ à cétro in ipſas ducuntur, ſunt æquales. Lógius autem abeſſe illa dicitur in quã maior perpendicularis cadit.



5

Segmentum circuli eſt, figura quæ ſub recta linea & circuli peripheria comprehenditur.



6

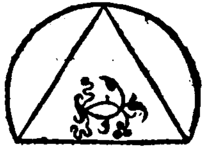
Segmenti autem angulus eſt, qui ſub recta

linea

linea & circuli peripheria comprehẽditur

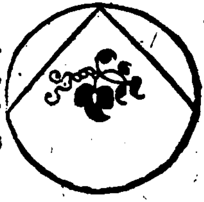
7

In segmento autem angulus est, cum in segmenti peripheria sumptum fuerit quodpiam punctum, & ab illo in terminos rectæ eius lineæ, quæ segmenti basis est, adiũctę fuerint rectæ lineæ: is, inquam, angulus ab adiunctis illis lineis comprehẽsus.



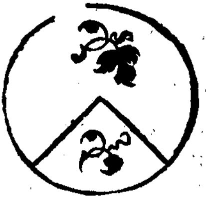
8

Cùm verò comprehendentes angulum rectæ lineæ aliquam assumitur peripheriam, illi angulus insistere dicitur.



9

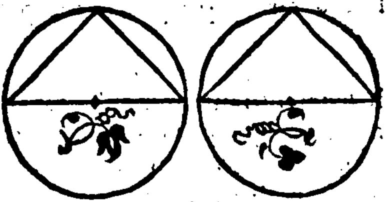
Sector autem circuli est, cùm ad ipsius circuli cẽtrum constitutus fuerit angulus, comprehensa nimirum figura, & à rectis lineis angulum continentibus, & à peripheria ab illis assumpta.



10

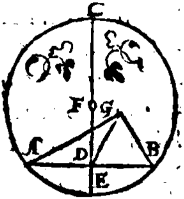
Similia circuli segmenta sunt, quæ angulos capiunt

capiunt ęquales aut in q b'anguli inter se sunt ęquales



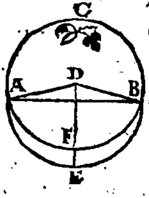
Problema 1. Propositio 1.

Dati circuli centrum reperire.



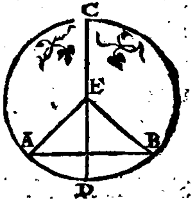
Theorema 1. Propositio 2.

Si in circuli peripheria duo quælibet puncta accepta fuerint, recta linea quæ ad ipsa puncta adiungitur, intra circulum cadet.



Theorema 2. Propositio 3.

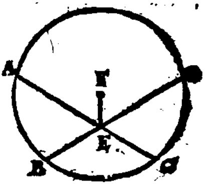
Si in circulo recta quædam lenea per centrum extensa quandam non per centrum extensam bifariam secet: & ad angulos rectos ipsam secabit. Et si ad angulos rectos eam secet, bifariam quoque eam secabit.



Theore-

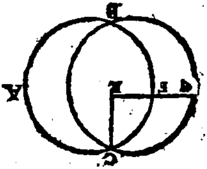
Theorema 3. Propoſitio 4.

Si in circulo duę rectę lineæ ſeſe mutuo ſecent non per centrum extēſæ ſeſę mutuò bifariam non ſecabunt.



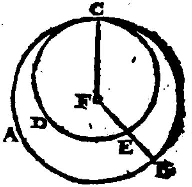
Theorema 4. Propoſitio 5.

Si duo circuli ſeſe mutuò ſecent, non erit illorum idem centrum.



Theorema 5. Propoſitio 6.

Si duo circuli ſeſe mutuò interius tangant, eorum non erit idem centrum.



Theorema 6. Propoſitio 7.

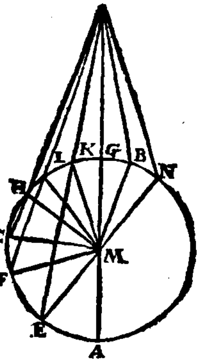
Si in diametro circuli quodpiam ſumatur punctum, quod circuli centrum non ſit, ab eoque puncto in circulũ quædam rectæ lineæ cadant: maxima quidem erit ea in qua centrũ, minima verò reliqua: aliarum verò propinquior illi quę per centrum du-

citur

citur, remotiore ſemper maior eſt. Duæ autem ſolùm rectę lineę æquales ab eodem puncto in circulum cadunt ad vtraſq; partes minimæ.

Theorema 7. Propoſitio 8.

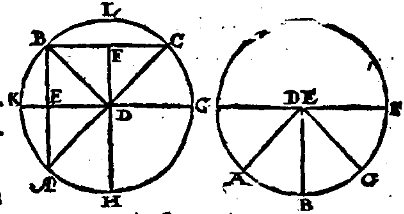
Si extra circulum ſumatur punctum quodpiam, ab eoque puncto ad circulum deducantur rectæ quædam lineæ, quarum vna quidem per centrum protendatur, reliquę verò vt libet: in cauam peripheriam cadentium rectarum linearum minima quidem eſt illa, quæ per centrum ducitur: aliarum autem propinquior ei, quæ per centrum tranſit, remotiore ſemper maior eſt: in conuexam verò peripheriã cadentium rectarũ linearum minima quidem eſt illa, quæ inter punctum & diametrum interponitur: aliarum autem, ea quę propinquior eſt minimæ, remotiore ſemper minor eſt. Duæ autem tantùm rectæ lineæ æquales ab eo puncto in ipſum circulum cadunt, ad vtraſque partes minimæ.



Theo-

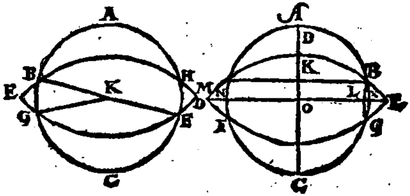
Theorema 8. Propositio 9.

Si in circulo acceptum fuerit pũctum aliquod, & ab eo puncto ad circulum cadant *plures* *quam* duę rectæ lineæ, æquales, acceptũ punctum centrum ipsius erit circuli.



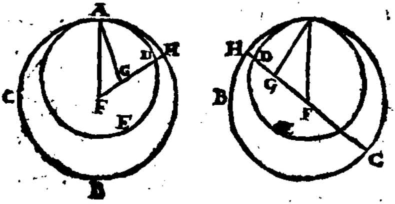
Theorema 9. Propositio 10.

Circũlus cir*culum* in pluribus quàm duob' punctis non secat.



Theorema 10. Propositio 11.

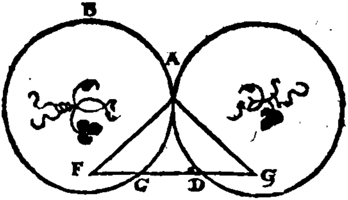
Si duo circuli sese intus cõtingãt, atque accepta



fuerint eorum centra, ad eorum cẽtra adiuncta recta linea & producta ni cõtactum circulorum cadet.

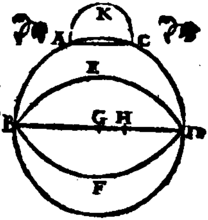
Theorema 11. Propositio 12.

Si duo circuli sese exterius contingãt, linea recta q̃ ad cẽtra eorũ adiũgitur, ꝑ contactũ illũ trãsibit.



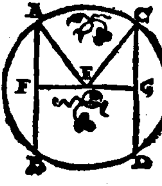
Theorema 12. Propositio 13.

Circulum circulum non tangit in pluribus punctis, quam vno, siue intus siue extra tangat.



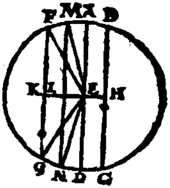
Theorema 13. Propositio 23.

In circulo æquales rectæ lineæ æqualiter distant à centro. Et quæ ęqualiter distant à centro, æquales sunt inter se.



Theore-

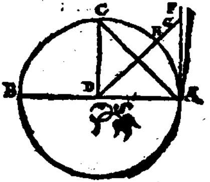
Theorema 14. Propositio 15.



In circulo maxima, quidem linea est diameter: aliarum autem propinquior centro, remotiore semper maior.

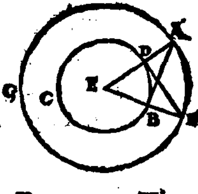
Theorema 11. Propositio 16.

Quæ ab extremitate diametri cuiusq; circuli ad angulos rectos ducitur, extra ipsũ circulum cadet, & in locum inter ipsam rectam lineam & peripheriam comprehensum, altera recta linea nõ cadet Et semicirculi quidem angulus quouis augulo acuto rectilineo maior est, reliquus autem minor,

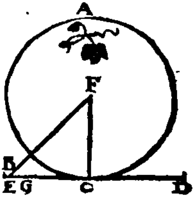


Problema 2. Propositio 17.

A dato puncto rectam lineam ducere, quæ datum tangat circulum.

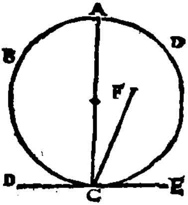


Theorema 16. Propoſitio 18.



Si circulum tangat recta quæpiam linea, à centro autem ad cõtactum adiungatur recta quædam linea: quę adiũcta fuerit ad ipſam contingentem perpendicularis erit.

Theorema 17. Propoſitio 19.



Si circulum tetigerit recta quæpiam linea, à contactu autem recta linea ad angulos rectos ipſi tã genti excitetur, in excitata erit centrum circuli.

Theorema 18. Propoſitio 20.

In circulo angulus ad cẽ trum duplex eſt anguli ad peripheriam, cũ fue rit eadẽ peripheria baſis angulorum.

Theorema 19. Propoſitio 21.

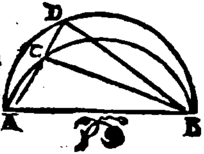
In circulo, qui in eodem ſegmẽto ſunt anguli, ſunt inter ſe æquales.

Theore-

Theorema 20. Propositio 22.

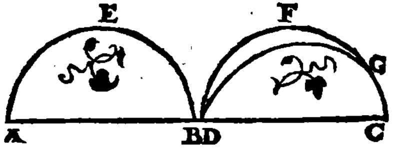
Quadrilaterorum in circulis descriptorũ anguli qui ex aduerso, duob' rectis sunt æquales.

Theorema 21. Propositio 23.



Super eadem recta linea duo segmenta circulorũ similia & inæqualia non constituẽtur ad easdem partes.

Theorema 22. Propositio 24.



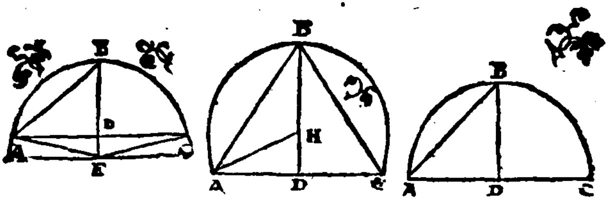
Super ęqualib' rectis lineis similia circulorum segmenta sunt inter se æqualia.

Problema 23. Propositio 25.

Circuli segmento dato, describere circulũ

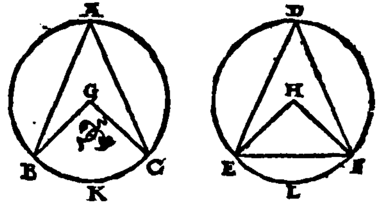
 cuius

cuius est segmentum.



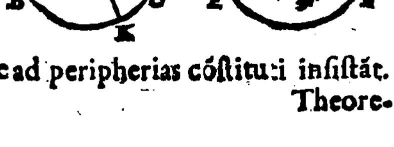
Theorema 22. Propositio 26.

In æqualibus circulis, æquales anguli ęqualib. peripherijs insistũt siue ad cẽtra, siue ad peripherias constituti insistant.



Theorema 24 Propositin 27.

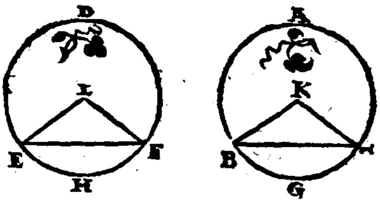
In æqualibus circulis, anguli qui ęqualibus peripherijs insistũt sunt inter se ęquales siue ad cẽtra, siue ad peripherias cõstituti insistãt.



Theore-

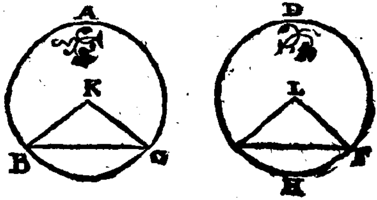
Theorema 25. Propositio 28.

In æqualibus circulis æquales rectæ lineæ ęquales peripherias auferũt maiorẽ quidẽ maiori, minorem autem minori.



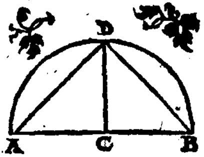
Theorema 26. Propositio 29.

In ęqualib' circulis, æquales peripherias æquales rectæ lineæ subtendunt.



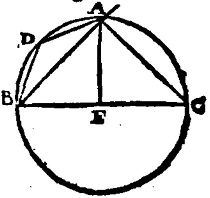
Problema 4. Propositio 30.

Datam peripheriam bifariam secare.



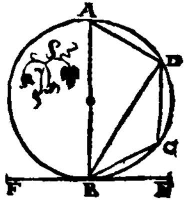
Theorema 27. Propositio 13.

In circulo angulus qui in semicirculo, rectus est: quī autem in maoire ſegmēto, minor recto: qui verò in minore ſegmēto, maior eſt recto. Et inſuper angulus maioris ſegmenti, recto quidem maior eſt, minoris autē ſegmēti angulus, minor eſt recto.



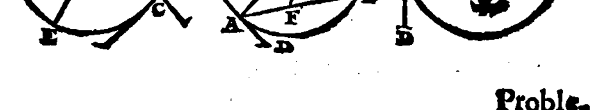
Theorema 28. Propoſitio 32.

Si circulum tetigerit aliqua recta linea, à contactu autē producatur quædam recta linea circulum ſecans: anguli quos ad contingentem facit ęquales ſunt ijs qui in alternis circuli ſegmētis conſiſtunt, angulis.



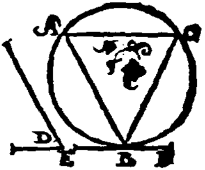
Problema 5. Propoſitio 33.

Super data recta linea deſcribere ſegmentum circuli quod capiat angulum æqualē dato angulo rectilineo.



Proble-

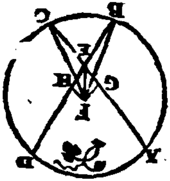
Problema 6. Propositio. 34.

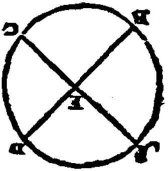


A dato circulo ſegmentum abſcindere capiens angulum æqualem dato angulo rectilineo.

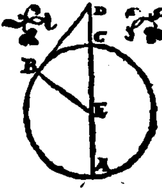
Theorema 29. Propoſitio 35.

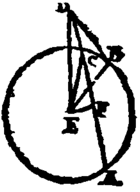
Si in circulo duæ rectæ lineæ ſeſe mutuò ſecuerint, rectangulum comprehēſum ſub ſegmē-tis vni°, æquale eſt ei, quod ſub ſegmentis alterius comprehenditur, rectangulo.





Theorema 30. Propoſitio 36.

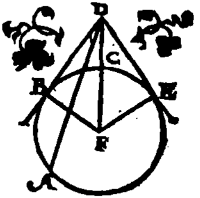




Si extra circulū ſumatur pūctū aliquod, ab eoque in circulū cadant duę rectę lineæ, quarū altera quidem circulum ſecet, altera verò tangat: quod sub tota secante & exterius inter punctum & conuexam peripheriam assumpta comprehenditur rectangulum, æquale erit ei, quod à tangente describitur, quadrato.

Theorema 31. Propositio 37.

Si extra circulũ sumatur punctun aliquod, ab eoq; puncto in circulum cadant duæ rectæ lineæ, quarum altera circulum secet, altera in eum incidat, sit autẽ quod sub tota secante & exterius inter punctum & conuexã peripheriam assumpta, comprehenditur rectangulum, æquale ei, quod quadrato: incidens ipsa circulum tanget.



ELEMENTI II. FINIS.

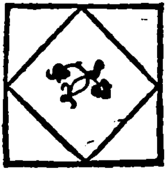
EVCLI.

EVCLIDIS ELEMENTVM QVARTVM.

DEFINITIONES.

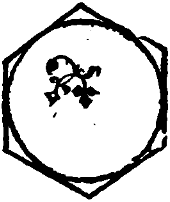
1.

Figura rectilinea in figura rectilinea inscribi dicitur, cũ singuli eius figuræ quæ inscribitur, anguli singula latera eius, in qua inscribitur, tangunt.



2

Similiter & figura circum figurá' describi dicitur, quum singula eius quæ circũscribitur, latera singulos eiˀ figurę angulos tetigerint, circum quam illa describitur.



3

Figura rectilinea in circulo inscribi dicitur, quú singuli eius figuræ quę inscribitur, anguli tetigerint circuli peripheriam.

4

Figura verò rectilinea circa circulũ descri- bi dicitur, quũ singula latera eius, quæ cir- cum scribitur, circuli peripheriã tangunt.

5

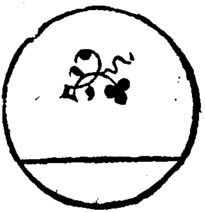
Similiter & circulus in figura rectilinea in scribi dicitur, quum circuli peripheria sin- gula latera tãgit eius figurę, cui inscribitur.

6

Circulus autem circum figurã describi di- citur, quũ circuli peripheria singulos tãgit eius figuræ, quam circunscribit, angulos.

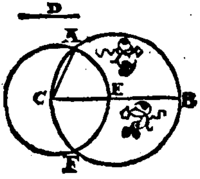
7

Recta linea in circulo ac commodari seu coapta- ri dicitur, quum eius ex- trema in circuli periphe ria fuerint.



Problema I. Pro- positio I.

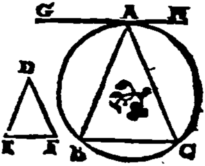
In dato circulo, rectam lineam accommodare ę- qualem datæ rectæ lineæ quæ circuli diametro nõ sit maior.



Proble-

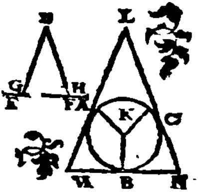
Problema 2. Propositio 3.

In dato circulo, triangulũ describere dato triangulo æquiangulum.



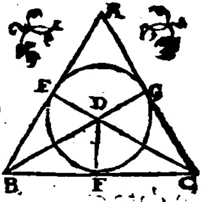
Problema 3. Propositio 3.

Circa datum circulũ triangulũ, describere dato triangulo æquiangulũ.



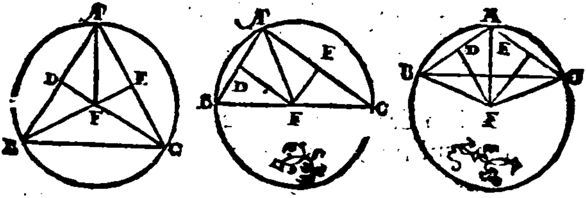
Problema 4. Propopositio 4.

In dato triangulo circulum inscribere.

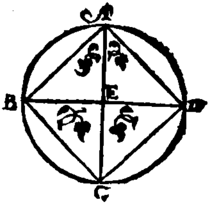


Problema 5. Propositio 5.

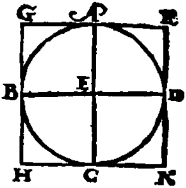
Circa datum triangulum, circulum describere.



Problema 6. Propositio 6.

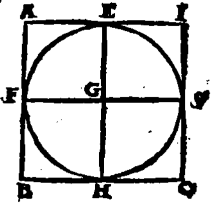


In dato circulo quadratum deſcribere.



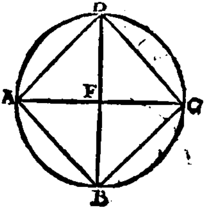
Problema 7. Propoſitio 7.

Circa datum circulum, quadratum deſcribere.



Problema 8. Propoſitio. 8.

In dato quadrato circulum inſcribere.

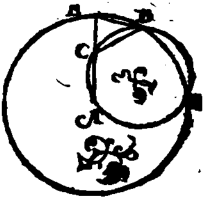


Problema 9. Propoſitio 9.

Circa datum quadratũ, circulum deſcribere.

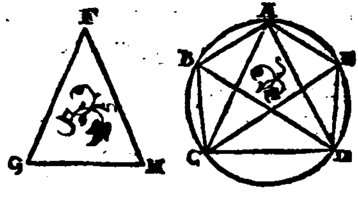
Proble.

**Problema 10. Propo-
positio. 10**



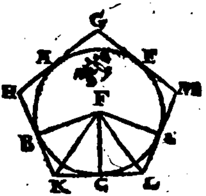
Isosceles triãgulum constituere, quod habeat vtrunque eorum, qui ad duplum reliqui.

Theorema 11. Propositio 11.



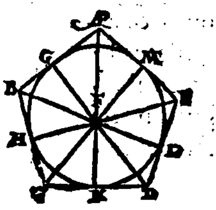
In dato circulo, pentagonũ æquilaterũ & æquiãgulum inscribere.

Problema 12. Propositio 12.



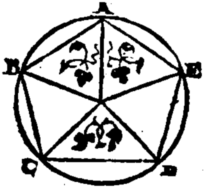
Circa datum circulum, pentagonum æquilaterum æquiangulum describere.

Problema 13. Propositio 13.



In dato pẽtagono æquilatero & æquiãgulo circulum inscribere.

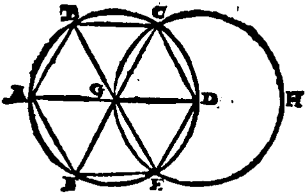
Problema 14. Propositio 14.

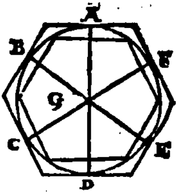


Circa datũ pentagonũ, æquilaterum & æquiangulum, circulum describere.

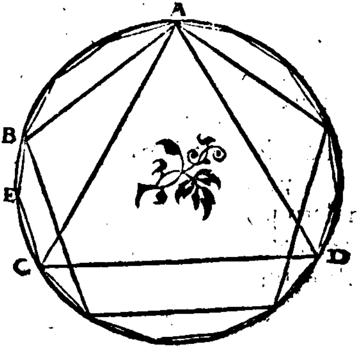
Problema 15. Propositio 15.

In dato circulo hexagonum & æquilaterũ & æquiangulum inscribere.





propositio 16. Theorema 16.



In dato circulo quintidecagonum & æquilaterũ & æquiangulum describere.

Elementi quarti finis.

EVCLIDIS ELEMENTVM QVINTVM.

DEFINITIONES.

1

PARS est magnitudo magnitudinis minor maioris, quum minor metitur maiorem.

2

Multiplex autem est maior minoris, cùm minor metitur maiorem.

3

Ratio, est duarum magnitudinum eiusdē generis mutua quædam secundum quantitalem habitudo.

4

Proprotio verò, est rationum similitudo.

5

Rationē habere inter se magnitudinis dicuntur, quæ possunt multiplicatæ sese mutu superar e

6.

In eadem ratione magnitudines dicūtur esse, prima ad secundam, & tertia ad quartá, cum primæ & tertiæ æquè multiplicia, à secundæ & quartæ æquè multiplicibus,

F qualis-

qualiſcunq; ſit hæc multiplicatio, vtrumq; ab vtroque: vel vnà deficiunt, vel vnà ęqualia ſunt, vel vnà excedũt, ſi ea ſumãtur quæ inter ſe reſpondent.

7

Eandam autem habẽtes rationem magnitudines, proportionales vocentur.

8

Cum verò æquè multiplicium, multiplex primę magitudines exceſſerit? multiplicẽ ſecundæ, at multiplex tertiæ non exceſſerit multiplicem quartæ: tunc prima ad ſecundam, maiorem rationem habere dicetur, quàm tertia ad quartam.

9

Proportio autẽ in tribus terminis pauciſſimis conſiſtit.

10

Cùm autem tres magnititudines proportionales, fuerint, prima ad tertiam, duplicatã rationẽ habere dicitur eius quam habet ad ſecundam. At cùm quatuor magnitudines proportionales, fuerint, prima ad quartam, triplicatam rationem habere dicitur eius quam habet ad ſecundam: & ſemper deinceps vno amplius, quãdiu proportio extiterit.

11

Homologę, ſeu ſimiles ratioue magnitudines dicuntur, antecedentes quidẽ antece-

denti-

dentibus, consequétés verò cósequẽtibus.

12

Alterna ratio, est sumptio antecedentis cóparati ad antecedentem, & cõsequentis ad consequentem.

13

Inuersa ratio, est sumptio cõsequentis, ceu antecedentis, ad antecedenté velut ad consequentem.

14.

Compositio rationis, est sumptio antecedentis cũ consequente ceu vnius ad ipsum consequentem.

15

Diuisio rationis, est sumptio excessus quo consequentem superat antecedens ad ipsum consequentem.

16

Conuersio rationis, est sumptio antecedentis ad excessum, quo superat antecedens ipsum cõsequentem.

17

Ex ęqualitate ratio est, si plures duabus sint magnitudines, & his alię multitudine pares quæ binæ sumantur, & in eadem ratione: quum vt in primis magnitudinibus prima ad vltimam sic & in secundis magnitudinibus prima ad vltimã sese habuerit, vel aliter, sumptio extremorũ per subductionem mediorum.

18

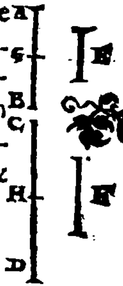
Ordinata proportio est, cùm fuerit quemadmodum antecedens ad consequentem ita antecedens ad consequentem: fuerit etiam vt consequens ad aliud quidpiam. ita consequens ad aliud quidpiam.

19

Perturbata autẽ proportio est, tribus positis magnitudinibus, & alij quæ sint his multitudine pares, cùm vt in primis quidẽ magnitudinibus se habet antecedẽs ad cõsequentem, ita in secũdis magnitudinibus antecedens ad consequentem: vt autem in primis magnitudinibus consequens ad aliud quidpiam ad antecedentem.

Theorema 1. Propositio 1.

Si sint quotcũque magnitudines quotcunque magnitudinum æqualium numero singulę singularum æquè multiplices, quàm multiplex est vnius vna magnitudo, tam multiplices erunt, & omnes omnium.

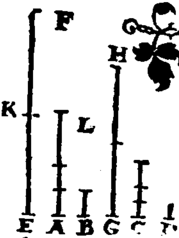


Theorema 2. Propositio 2.

Si prima secundæ æquè fuerit multiplex, atque

atque tertia quartę, fuerit autē & quinta ſecūdę æquè multiplex, atque ſexta quartæ: erit & cōpoſita prima cū quinta, ſecūdæ ęquè multiplex atque tertia cum ſexta, quartæ.

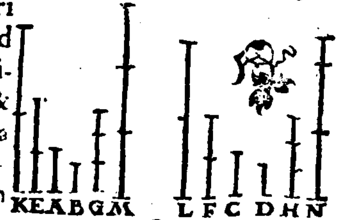
Theorema 3. Propoſitio 3.



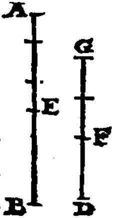
Si ſi prima ſecundæ æquè multiplex atque tertia quartæ, ſumātur autē æquè multiplices primę & tertiæ erit & ex æquo ſumptarum vtraque vtriuſque ęquè multiplex, altera quidem ſecundæ, altera autē

Theorema 4. Propoſitio 4.

Si prima ad ſecundam, eandē habuerit rationem, & tertia ad quartam: etiam æquè multiplices primę & tertiæ, ad æquè multiplices ſecundæ & quartæ iuxta quāuis multiplicationē, eandem habebunt rationem, ſi prout inter ſe



respondent, ita sumptæ fuerint.



Theorema 5. Propositio 5.

Si magnitudo magnitudinis æquè fuerit multiplex, atque ablata ablatæ: etiam reliqua reliquæ ita multiplex erit, vt tota totius.

Theorema 6. Propositio 6.

Si duæ magnitudines, duarum magnitudinum sint æquè multiplices, & detractę quædam sint earundem æquè multiplices: & reliquæ eisdem aut ęquales sunt, aut æquè ipsarum multiplices.

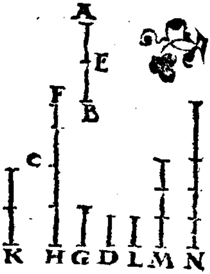
Theorema 7. Propositio 7.

Aequales ad eandem, eadem habent rationem: & eandem ad æquales.

Theorema 8. Propositio 8.

In æqualium magnitudinũ maior, ad eandem

dem maiorem ratione habet, quàm minor: & eadem ad minorem: maiorem rationem habet, quàm ad maiorem.



Theorema 9. Propositio 9.

Quæ ad eandem, eandem habẽt rationem, æquales sunt inter se: & ad quas eadem, eandem habet rationem, eæ quoque sunt inter se æquales.

Thorema 10. Propositio 10.

Ad eandem magnitudiné ratio nem habentium, quę maiorem rationé habet, illa maior est ad quam auté eadem maiorem rationem habet, illa minor est.

Theorema 11. Propositio 11.

Quæ eidem sunt eædem ratioues, & inter se sunt eędem.

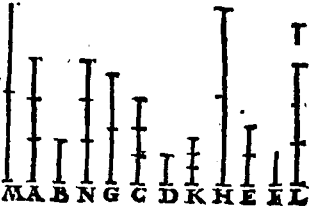
Theorema 12. Propoſitio 12.

Si ſint magnitudines quotcunque proportionales, quem admodũ ſe habuerit vna antecedentium ad vnã

conſequentiũi, ta ſe habebunt omnes antecedenses ad omnes coſequentes.

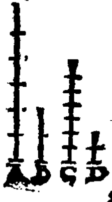
Theorema 13. Propoſitio 13.

Si prima ad ſecundum, eandẽ habuerit rationem, quam tertia ad quartã, tertia verò ad quartam maiorem rationem habuerit, quàm quinta ad ſextam. prima quoque ad ſecũdam maiorem rationem habebit, quàm quinta ad ſextam.

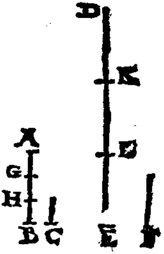


Theorema 14. Propoſitio 14.

Si prima ad ſecundam eandem habuerit rationẽ, quam tertia ad quartam, prima verò quàm tertia maior fuerit: erit & ſecunda maior quàm quarta. Quòd ſi prima fuerit æqualis tertiæ, erit



& secunda æqualis quartæ: si verò minor, & minor erit.

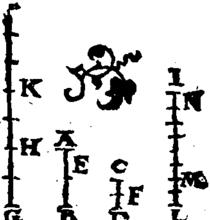


Theorema 15. Propositio 15.

Partes cum pariter multiplicibus in eadẽ sunt ratione, si prout sibi mutuo respondent, ita sumantur.

Theorema 16. Propositio 16.

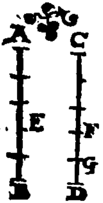
Si quatuor magnitudines proportionales fuerint, & vicissim proportionales erunt.



Theorema 17. Propositio 17.

Si compositæ magnitudines porportionales fuerint hæ quoq; diuisæ proportionales erunt.

Theorema 18. Propositio 18.



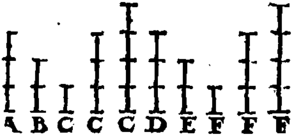
Si diuisæ magnitudines sint proportionales, hæ quoque compositæ proportionales orunt.

Theorema 19. Propositio 19.

Si quemadmodum totũ ad totum, ita ablatum se habuerit ad ablatum: & reliquum ad reliquum, vt totum ad totum se habebit.

Theorema 20. Propositio 20.

Si sint tres magnitudines, & aliæ ipsis æquales numero, quæ binæ & in eadem ratione sumatur, ex æquo autē prima quàm tertia maior fuerit: erit & quarta, quã sexta maior. Quòd si

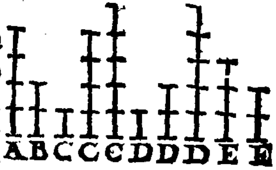


prima tertiæ fuerit æqualis, erit & quarto æqualis sextæ: sin illa minor, hæc quoque minor erit.

Theore-

Theorema 21. Propositio 21.

Si sint tres magnitudines, & aliæ ipsis æquales numero quæ binæ & in eadem ratione sumãtur, fuerit que perturbata earũ proportio ex æquo autem prima quàm tertia maior fuerit erit & quarta quã sexta maior: quòd si prima tertiæ fuerit æqualis, erit & quarta æqualis sextæ: sin illa minor, hæc quoque minor erit.



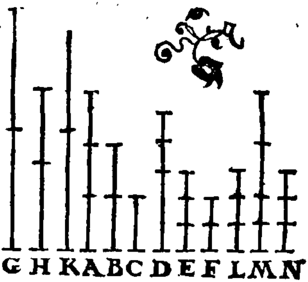
Theorema 22. Propositio 22.

Si sint quotcunq; magnitudines, & aliæ ipsis æquales numero, quæ binæ in eadẽ ratione sumantur, et ex æqualitate in eadem ratione erunt.

Theorema 23. Propositio 32.

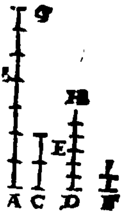
Si sint tres magnitudines, aliæq; ipsis æquales

les numero, q̄ binę in eadem ratione sumātur, fuerit autē perturbata earū proportio: etiā ex æqualite in eadem ratione erunt.



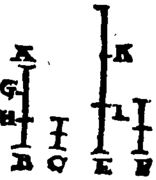
Theorema 24. Propositio 24.

Si prima ad secundam, eandem habuerit rationē, quam tertia ad quartam, habuerit autem & quinta ad secundam, eandē rationem, quā sexta ad quartam: etiam cōposita prima cū quinta ad secundam eandē habebit rationē quam tertia cum sexta ad quartam.



Theorema 25. Propositio 25.

Si quatuor magnitudines proportionales fuerint, maxima & minima reliqui duabus maiores erunt



ELEMENTI V. FINIS.

EVCLIDIS ELEMENTVM SEXTVM.

DEFINITIONES.

1

SImiles figuræ rectilineæ ſunt, quæ & angulos ſingulos ſingulis æquales habẽt, atque etiam latera, quę circum angulos æquales, proportionalia,

2

Reciprocæ autem figuræ ſunt, cùm in vtraque figura antecedentes & conſequentes rationum termini fuerint.

3

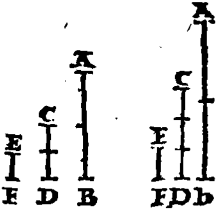
Secundum extremam & mediam rationẽ recta linea ſecta eſſe dicitur, cùm vt tota ad maius ſegmentum, ita maius ad minus ſe habuerit.

4

Altitudo cuiuſque figurę, eſt linea perpendicularis à vertice ad baſin deducta

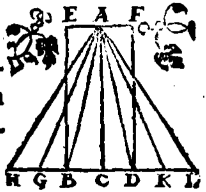
5.

Ratio ex rationibus cõponi dicitur, cũ rationum quantitates inter se multiplicatæ aliqua effecerint rationem.



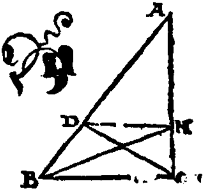
Theorema 1. Propositio. 1

Triangula & parallelogramma, quorum eadem fuerit altitudo, ita se habent inter se vt bases.



Theorema 2. Propositio 2.

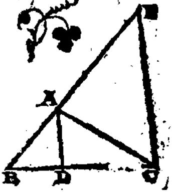
Si ad vnum trianguli latus parallela ducta fuerit recta quædam linea: hęc proportionaliter secabit, ipsius triãguli latera. Et si trianguli latera proportionaliter secta fuerint: quæ ad sectiones adiũcta fuerit recta linea, erit ad reliquum ipsius trianguli latus parallela.



Theorema 3. Propositio 3.

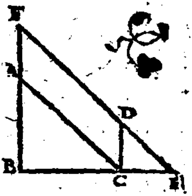
Si trianguli angulus bifariam sectus sit, secans autẽ augulum recta linea secuerit & basim: basis segmenta eandem habebunt rationem,

tionem, quam reliqua ipsius trianguli latera. Et si basis segmenta eandem habeant rationem quam reliqua ipsius trianguli latera, recta linea, quæ a vertice ad sectionem producitur, ea bifariā secat trianguli ipsius angulum.



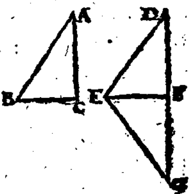
Theorema 4. Propositio 4

Aequiangulorum triangulorum proportianolia sunt latera, quę circum ęquales angulos, & homologa sunt latera, quę ęqualibus angulis subtenduntur.



Theorema 5. Propositio 5.

Si duo triāgula latera proportionalia habeant, ęquiangula erunt triangula, & æquales habebunt eos angulos, sub quib⁹ homologa latera subtenduntur.

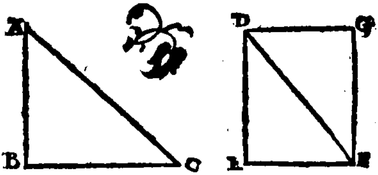


Theorema 6. Propositio 6.

Si duo triangula vnum angulum vni angulo æqualē, & circum ęquales angulos latera ꝓportionalia habuerint, ęquiangula erunt

trian

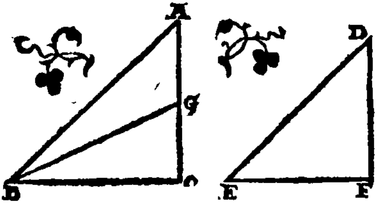
triangu-
la, ęqua-
lesq; ha-
bebunt
angulos
ſub qui-
bus ho-



mologa latera ſubtenduntur.

Theorema 7. Propoſitio 7.

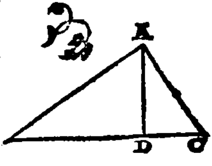
Si duo triangula vnum angulum vni angulo æqualem, circũ autem alios angulos latera proportionalia habent, reliquorum vero ſimul vtrunq; aut minórem aut non minorẽ



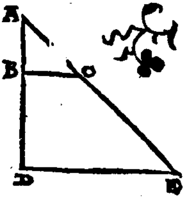
recto: æquiangula erunt triangula, & æquales habebunt eos angulos circum quos proportionalia ſunt latera.

Theorema 8. Propoſitio 8.

Si in triangulo rectangulo, ab angulo recto in baſin perpendicularis ducta ſit quę ad perpendicularem triangulo, tum toti triangulo, tum ipſa inter ſe ſimilia ſunt.

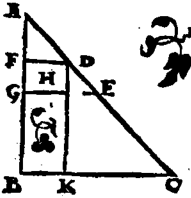


Problema 1. Propositio 9.



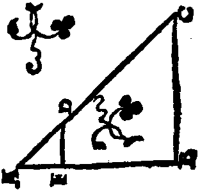
A data recta linea imperatam partem auferre.

Problema 2. Propositio 10.



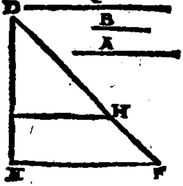
Datam rectá lineam in sectam similiter secare, vt data altera recta secta fuerit.

Problema 3. Propositio 11.



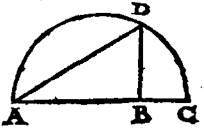
Duabus datis rectis lineis, tertiam propotionalem ad inuenire.

Problema 4. Propositio 12.



Trib' datis rectis lineis quartam proportionalem adiuenire.

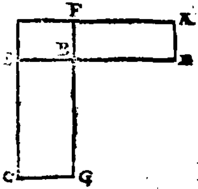
Problema 5. Propo-
sitio 13.



Duab' datis rectis lineis mediam proportionalē adinuenire.

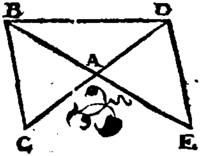
Theorema 9 Propositio 14.

Aequalium, & vnū vni æqualem habentiū angulum parallelogrammorum reciproca sunt latera, quæ circum æquales angulos. & quorū parallelogrammorū vnum angulum vni angulo æqualem habentiū reciproca sunt latera, quæ circum æquales angulos, illa sunt æqualia.



Theorema 10. Propositio 15.

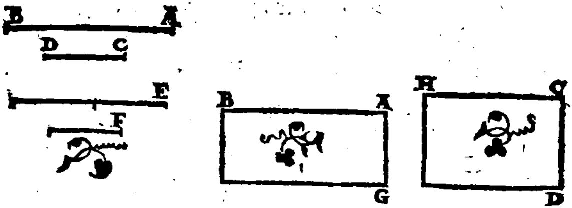
Aequalium, & vnū angulum vni æqualem habentiū triangulorū reciproca sunt latera, quæ circum æquales angulos. & quorū triangulorū vnum angulum vni æqualem habentium reciproca sunt latera, q̄ circum æquales angulos, illa sunt æqualia.



Theo-

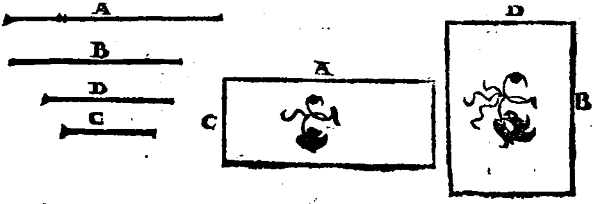
Theorema 11. Propoſitio 16.

Si quatuor rectæ lineæ proportionales fuerint, quod ſub extremis comprehenditur rectangulũ, æquale eſt ei, quod ſub medijs comprehenditur rectágulo. Et ſi ſub extremis comprehenſum rectangulum æquale fuerit ei, quod ſub medijs continetur rectã gulo, illæ quatuor rectæ lineæ proportionales erunt.



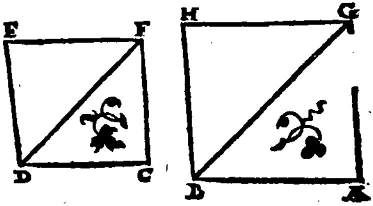
Theorema 12. Propoſitio 17.

Si tres rectæ lineæ ſint proportionales, qđ ſub extremis cõprehenditur rectangulũ ęquale eſt ei, quod à media deſcribitur quadrato: & ſi ſub extremis comprehẽſum rectangulum æquale ſit ei quod à media deſcribitur quadrato, illæ tres rectę lineę proportionales erunt.



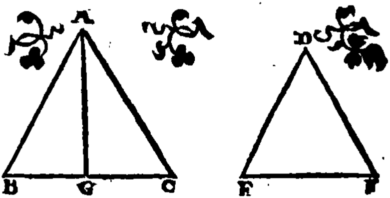
Problema 6. Propoſitio 18.

A data recta linea, dato rectilineo ſimile ſimiliterq; poſitum rectilineum deſcribere.



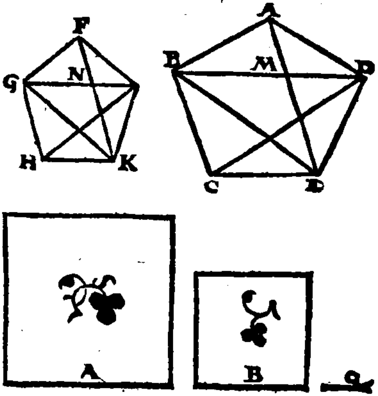
Theorema 13. Propoſitio 19.

Similia triãgula inter ſe ſunt in duplicata ratione laterũ homologorũ.

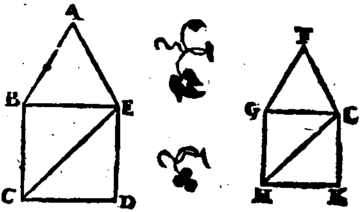


Theore. 14. Propoſitio 20.

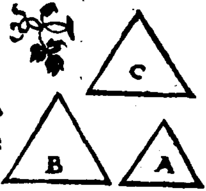
Similia polygona in ſimilia triãgula diuidũtur, & numero ęqualia, & homologa totis. Et po



lygona duplicatam habent eā inter se rationē, quā latus homologū ad homologum latus.



Theorema 15. Propositio 21.

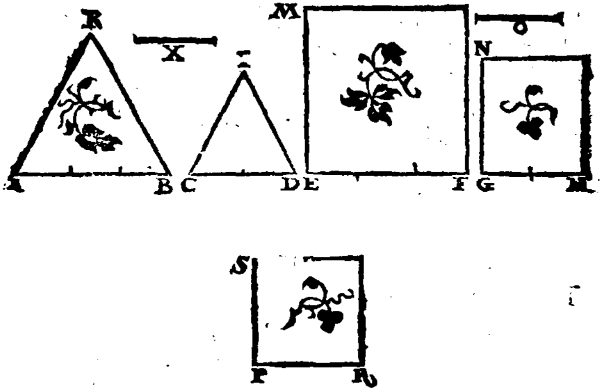


Quæ eidem rectilineo sunt similia, & inter se sunt similia.

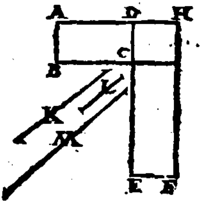
Theorema 16. Propositio 22.

Si quatuor rectę lineæ proportionales fuerint: & ab eis rectilinea similia similiterq; descripta proportionalia erūt. Et si à rectis lineis similia similiterq; descripta rectilinea proportionalia fuerint, ipsæ etiam re-

&æ lineæ proportionales erunt.

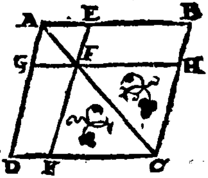


Theorema 17. Propoſitio 23.



Aequiangula parallelogramma inter ſe rationẽ habent eum, quę ex lateribus componitur.

Theorema 18. Propoſitio 24.

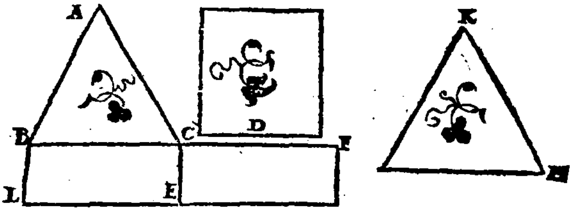


In omni parallelogrammo, quæ circa diametrũ ſunt parallelogrãma, & toti & inter ſe ſunt ſimilia.

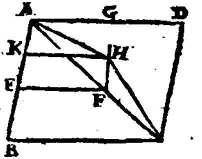
Proble-

Problema 7. Propoſitio 25.

Dato rectilineo ſimile, & alteri dato ęquale idem conſtituere.



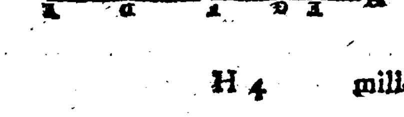
Theorema 19. Propoſitio 26.



Si à parallelogrãmo parallelogrammũ ablatũ ſit, & ſimile toti & ſimiliter poſitũ communẽ cum eo habens angulum, hoc circum eandem cum toto diametrum conſiſtit.

Theorema 20. Propoſitio 27.

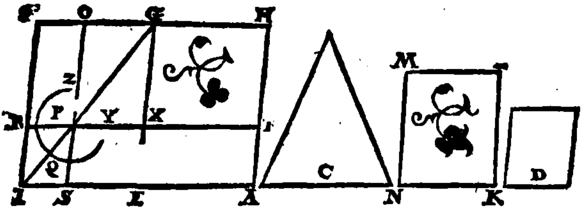
Omnium parallelogrãmorum ſecũdum eandem rectam lineam applicatorũ deficiẽtiumq; figuris parallelográmis ſi



milibus ſimiliterque poſitis ei, quod à dimidia deſcribitur, maximum, id eſt, quod ad dimidiam applicatur parallelogrammum ſimile exiſtens defectui.

Problema 8. propoſitio 28.

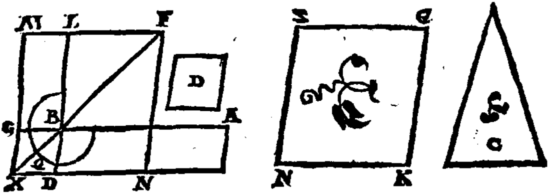
Ad datam lineam rectam, dato rectilineo æquale parallelogrammum applicare deficiens figura parallelogramma, quę ſimilis ſit alteri rectilineo dato. Oportet autẽ datum rectilineum, cui æquale applicandum eſt, non maius eſſe eo quod ad dimidiam applicatur, cum ſimiles ſint defectus & eius quod à dimidia deſcribitur, & eius cui ſimile deeſſe debet.



Problema 9. Propoſitio. 29.

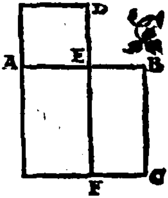
Ad datam rectam lineam, dato rectilineo æquale parallelogrammũ applicare, excedẽs figura parallelográma, quæ ſimilis ſit paral-

parallelogrammo alteri dato.



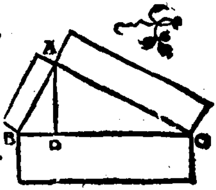
Prblema 10. Propo-poſitio 30.

Propoſitam rectam lineam terminatã, extrema ac media ratione ſecare.



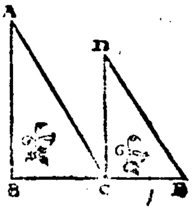
Theorema 21. Propoſitio .31

In rectangulis triãgulis, figura quęuis à latere rectũ angulũ ſubtendẽte deſcripta ęqualis eſt figuris, quæ priori illi ſimiles & ſimiliter poſitę à lateribus rectũ angulum continentibus deſcribuntur.



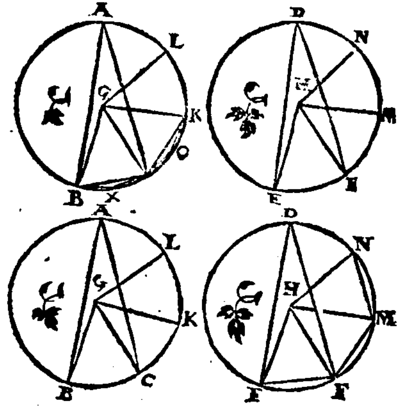
Theorema 22. Propo-ſitio 32.

Si duo triãgula, quę duo latera duobus lateribus proportionalia habeant, ſecũdum vnum angulũ compoſita fuerint, ita vt homologa eorum latera ſint etiam parallela, tum reliqua illorum ttiangulorum latera in rectam lineam collocata reperientur.



Theorema 23. Propoſitio 33.

In æqualibus circulis anguli eandẽ habent rationem, cũ ipſis peripherijs in quibus inſiſtũt, ſiue ad cẽtra ſiue ad peripherias conſtituti, illis inſiſtant peripherijs. Inſuper verò & ſectores quippe qui ad cẽtra cõſiſtũt.



ELEMENTI VI. FINIS.

EVCLIDIS ELEMENTVM SEPTIMVM.

DEFINITIONES.

1

Vnitas, est secũdum quam entium quodque dicitur vnum.

2

Numerus autem, ex vnitatibus composita multitudo.

3

Pars, est numerus numeri minori maioris, cùm minor metitur maiorem.

4

Partes autem, cùm non metitur.

5

Multiplex verò, maior minoris, cùm maiorem metitur minor.

6

Par numerus est, qui bifariã non diuiditur.

7

Impar verò, qui bifariam non diuiditur, vel, qui vnitate differt à pari.

8

Pariter par numerus est, quem par numerus metitur per numerum parem.

9 Pari.

9.

Pariter autem impar, est quem par numerus metitur per numerum imparem.

10.

Impariter verò impar numerus, est quem impar numerus metitur per numerum imparem.

11.

Primus numerus, est quem vnitas sola metitur.

12.

Primi inter se numeri sunt, quos sola vnitas mensura communis metitur.

13.

Compositus numerus est, quem numerus quispiam metitur.

14.

Compositi autẽ inter se numeri sunt, quos numerus aliquis mensura communis metitur.

15.

Numerus numerum multiplicare dicitur, cùm toties compositus fuerit is, qui multiplicatur, quot sunt in illo multiplicante vnitates, & procreatus fuerit aliquis.

16.

Cùm autẽ duo numeri mutuò sese multiplicantes quempiam faciũt, qui factus erit planus appellabitur, qui verò numeri mutuò sese multiplicarint, illi' latera dicẽtur.

17.Cùm

17

Cùm verò tres numeri mutuò sese multiplicantes quempiã faciunt, qui procreatus erit solidus apellabitur, qua autem numeri mutuò sese multiplicarint, illius, latera dicentur.

18.

Quadratus numerus est, qui ęqualiter ęqualis: vel, qui à duobus ęqualibus numeris cõtinetur.

19.

Cubus verò, qui à tribus æqualibus æqualiter: vel, quia tribus æqualibus numeris continetur.

10

Numeri proportionales sunt, cum primus secundi, & tertitius quarti æquè multiplex est, vt eadem pars, vel eædem partes.

21

Similes plani & solidi numeri sunt, qui proportionalia habent latera.

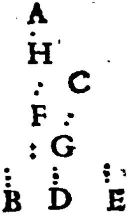
22

Perfectus numerus est, qui suis ipsius partibus est æqualis.

Theorema 1. Propositio 1.

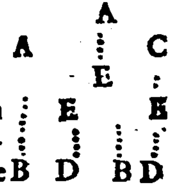
Duobus numeris in æqualibus propositis,

tis, si detrahatur semper minor. de maiore, alterna, quadam detractione; neque reliquus vnquam metiatur præcedentem quo ad assumpta sit vnitas: qui principio propositi suntnumeri primi inter se erunt.

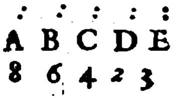


Porblema 1. Propositio 2.

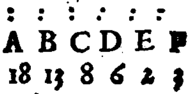
Duobus numeris datis non primis inter se, maximã eorũ cõmunẽ mensurã reperire



Broblema 2. Propositio. 3

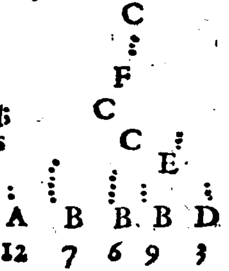


Tribus numeris datis non primis inter se, maximam eorum communem mensuram reperire.



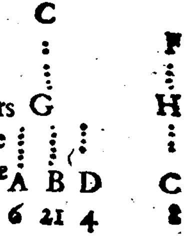
Theorema 18. Propositio 8.

Omnis numerus, cuiusq; numeri minor maioris aut par est, aut partes.



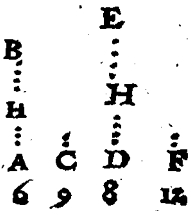
Theore-

Theorema 3. Propositio 5.



Si numeris numeri par fuerit, & alter alterius eadẽ pars & simul vterque vtriusque simul eadem pars erit, quæ vnus est vnius.

Theor. 4. Propo. 6.

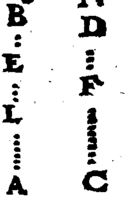


Si numer⁹ sit numeri partes, & alter alterius eędem partes, & simul vterque vtriusque simul eædẽ partes erunt, quæ sunt vnus vnius.

Theorema 5. Propositio 7.

Si numerus numeri eadẽ si pars quæ detractus detracti, & reliquus reliqui eadẽ pars erit, quæ totus est totius.

Theorema 6. Propositio. 8.

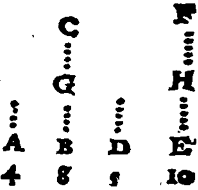


Si numerus numeri eædẽ sint partes quæ detractus detracti & reliquus reliqui eędẽ partes erũt, quę sũt totus totius. Theo-

G..M.K. .N.H.

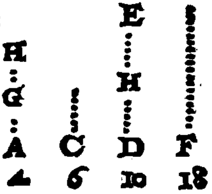
Theorema 7. Propositio 9.

Si numerus numeris pars sit, & alter alterius eadem pars, & viciſſim quæ pars eſt vel partes primus tertij, eadem pars erit eędem partes, & ſecũdus quarti.



Theorema 8. Propoſitio 10.

Si numerus numeri partes ſint, & aliter alterius eadẽ partes, etiam viciſſim quæ ſunt partes aut pars primus tertij, eędem partes erũt vel pars & ſecundus quarti.



Theorema 9. Propoſitio 11.

Si quemadmodum ſe habet totus ad totum, ita detractus ad detractum, & reliquus ad reliquum ita habebit vt totus ad totum

Theorema 10. Propoſitio 12.

Si ſint quotcunque numeri proportionales; quemadmodum ſe habet vnus antecedentium ad vnum ſequentium, ita ſe habe-

A B C D
9 6 3 2

habebunt omnes antecedentes ad omnes consequentes.

Theorema 11. Propositio 13.

Si quatuor numeri sint proportionales, & vicissim proportionales erunt.

:	:	:	:
A	B	C	D
12	4	9	3

Theorema 12. Propositio 14.

Si sint quotcunque numeri, & alij illis æquales multitudine, qui bini sumantur & in eadem ratione: etiam ex æqualitate in eadem ratione erunt

:	:	:	:	:	:
A	B	C	D	E	F
12	6	3	8	4	2

Theorema 13, Propositio 15.

Si vnitas numerum quempiã metiatur, aliter verò numerus alium quendã numerũ æquè metiatur, & vicissim vnitas tertium numerũ æquę metietur, atque secundis quartum.

F L K E C H G A B D

1 3 2 6

Theorema 14. Propositio 16

Si duo numeri mutuò sese multiplicãtes faciant aliquos qui ex illis geniti fuerint, inter se æquales erunt.

.	:	:	:	:
E	A	B	C	D
1	2	4	8	8

Theorema 15. Propoſitio 17.

Si numerꝰ duos numeros multiplicãs faciat aliquos, qui ex illis procreati erunt, eadem rationem habebunt, quam multiplicati.

:	:	:	:	:	:
I	A	B	C	D	E
1	3	4	5	12	15

Theorema 16. Propoſitio 18.

Si duo numeri numerum quempiam multiplicantes faciant aliquos, geniti ex illis eandem habebunt rationem, quam qui illum multiplicarunt.

:	:	:	:	:
A	B	C	D	E
4	5	3	12	15

Theorema 17. Propoſitio 19.

Si quatuor numeri ſint proportionales, ꝙ ex primo & quarto fit, ęqualis erit ei qui ex ſecundo & tertio: & ſi qui ex primo & quarto ſit numerus æqualis ſit ei qui ex ſecũdo & tertio, illi quatuor numeri ſint proportionales erunt.

:	:	:	:	:	:	:
A	B	C	D	E	F	G
6	4	3	2	12	12	18

Theorema 18. Propoſitio 20.

Si tres numeri ſint proportionales, qui ab extremis cõtinetur, ęqualis eſt ei qui à medio

dio efficitur. Et ſi qui ab extremis cõtinetur æqualis ſit ei qui à medio deſcribitur, illi tres numeri proportionales erunt.

A	B	C	D
9	6	4	6

Theorema 19. Propoſitio. 21.

Minimi numeri omniũ, qui eandem cũ eis rationem habẽt, æqualiter metiũtur numeros eandem rationem habẽtes, maior quidem maiorem, minor verò minorem

D	L		
G	H		
C	E	A	B
4	3	8	6

Theorema 20. Propoſitio 22.

Si tres ſint numeri & alij multitudine illis æquales, qui bini ſumantur & in eadem ratione, ſit autem perturbata eorũ proportio, etiam ex æqualitate in eadem ratione erunt.

A	B	C	D	E	F
6	4	3	12	8	6

Theorema 21. Propoſitio 23.

Primi inter ſe numeri minimi ſunt omniũ eandem cum eis rationem habentium.

A	B	E	C	D
5	6	2	4	3

Theorema 22. Propositio 24.

Minimi numeri omnium eandem cum eis rationem habentiũ, primus sunt inter se.

A B C D E
8 6 4 3 2

Theorema 23. Propositio 25.

Si duo numeri sint primi inter se, qui alterutrum illorum metitur numerus, is ad reliquũ primus erit.

A B C D
6 7 3 4

Theorema 24. Propotio 25.

Si duo numeri ad quẽpiam numerum primi sint, an eundem primus is quoque futurus est, qui ab illis productus fuerit.

B A C D E F
5 5 5 3 2

Theorema 25. Propositio 27.

Si duo numeri pirmi sint inter se, q ab vno eorũ gignitur ad reliquum primus erit.

B
A C D
7 6 3

Theorema 26. Propositio 28.

Si duo numeri ad duos numeros ambo ad vtrunque primi sint, & qui ex eis gignentur, primi inter se erunt.

A B E C D F
3 5 15 2 4 8

Theore-

Theorema 27. Propositio 29.

Si duo numeri primi sint inter se, & multiplicãs vterq; seipsum procreet aliquem, qui ex ijs producti fuerint, primi inter se erũt. Quòd si numeri initio propositi multiplicantes eos qui producti sunt, effecerint aliquos, hi quoq; inter se primi erunt, & circa extremos idem hoc semper eueniet.

A	C	E	B	D	F
3	6	27	4	16	63

Theorema 28. Propositio. 30.

Si duo numeri primi sint inter se, etiam simul vterq; ad vtrunq; illorum primus erit. Et si simul vterq; ad vnum aliquem eorum primus sit, etiam qui initio positi sunt numeri, primi inter sunt erunt.

C

A	B	D
7	5	4

Theorema 29. Propositio 31.

Omnis primus numerus ad omnem numerum quem non metitur, primus est.

A	B	C
7	10	5

Theorema 30. Propositio 32.

Si duo numeri sese mutuo multiplicãtes faciant aliquem, hũc aũt ab illis productum metiatur primus quidam numerus, is alterum etiam metitur eorum qui initio positi erant.

A	B	C	D	E
3	6	12	3	4

Theorema 31. Propositio 33.

Omnem compositum numerum aliquis primus metietur.

A B C
27 9 3

Theorema 32. Propositio 34.

Omnis numerus aut primus est, aut eum aliquis primus metitur.

A A
3 6 3

Problema 3. Propositio 35.

Numeris datis quotcunque, reperire minimos omnium qui eandem cum illis rationem habeant.

A B C D E F G H K I M
6 8 12 2 3 4 6 2 3 4 3

Problema 4. Propositio 36.

B
A C D E F
7 12 8 4 5

Duobus numeris datis reperire quẽ illi minimum metiantur numerum.

A
F E C D G H
6 9 12 9 2 5

Theo-

Theorema 33. Propositio 37.

Si duo numeri numerũ quempiam metiantur, & minimus quem illi metiuntur eundem metietur.

A	B	E	F G
2	3	6	12

Problema 5. Propositio 38.

Tribus numeris datis reperire quem minimum numerũ illi metiantur.

A	B	C	D	E
3	4	6	12	8

A	B	C	D	E	F
3	6	8	12	24	16

Theorema 34. Propositio 39.

Si numerum quispiam numerus metiatur, mensus partem habebit metienti cognominem.

A	B	C	D
12	4	3	1

Theorema 35. Propositio 40.

Si numerus partem habuerit quãlibet, illum metietur numerus parti cognominis.

A	B	C	D
8	4	2	1

Problema 6. Propositio 41.

Numerum reperire, qui minimus cùm sit, datas habeat partes.

A	B	C	G	H
2	3	4	12	10

ELEMENTI VII. FINIS.

EVCLIDIS ELEMENTVM OCTAVVM.

Theorema 1. Propositio 1.

Si sunt quotcũq; numeri deinceps proportionales, quorum extremi sint inter se, primi, minimi sunt omnium eandem cum eis rationem habentium.

A	B	C	D	E	F	G	H
8	12	18	27	6	8	12	18

Problema I. Propositio I.

Numeros reperire deinceps proportionales minimos, quotcunq; iusserit quispiam in data ratione.

A	B	C	D	E	F	G	H	K
3	4	9	12	16	27	36	49	64

Theorema 2. Propositio 3.

Conuersa primæ.

Si sint quotcũq; numeri deinceps proportionales minimi habentium eandem cum eis rationem, illorum extremi sunt inter se primi.

A	B	C	D	E	F	G	H	K	L	M	N	O
27	16	48	64	3	4	9	12	16	27	36	48	64

Problema 2. Propoſitio 4.

Rationibus datis quotcunque in minimis numeris reperire numeros deinceps minimos in datis rationibus.

A	B	C	D	E	F	H	G	K	L	N	X	M	O
5	4	2	3	4	5	6	8	12	15	4	6	10	12

Theorema 3. Propoſitio 5.

Plani numeri rationem inter ſe habent ex lateribus compoſitam.

A	L	B	C	D	E	F	G	H	K
18	22.	32	3	6	4	8	9	12	16

Theorema 4. Propoſitio 4.

Si ſint quotlibet numeri deinceps proportionales, primus autem ſecundum non metiatur, neque alius quiſpiam vllum metietur.

A	B	C	D	E	F	G	H
16	24	36	54	82	4	6	9

Theorema 5. Propositio 7.

Si ſint quotcunque numeri deinceps proportionales, primus autem extremum metiatur, is etiam ſecundum metietur.

A	B	C	D
4	6	12	24

Theorema 6. Propoſitio 8.

Si inter duos numeros medij continua proportione incidant numeri, quot inter eos medij continua proportione incidunt numeri, tot & inter alios eandem cum illis habentes rationem medij continua proportione incident.

A	C	D	B	G	H	K	L	C	M	N	F
4	9	27	81	1	3	9	27	2	6	18	54

Theorema 7. Propoſitio 9.

Si duo numeri ſint inter ſe primi, & inter eos medij cõtinua proportione incidãt numeri, quot inter illos medij continua proportione incidunt numeri, totidẽ & inter vtrunque eorum ac vnitatẽ deinceps medij continua proportione incident.

A	M	H	E	F	N	C	K	X	G	D	L	O	B
27	27	9	36	3	36	1	12	48	4	48	16	64	64

Theo-

Theorema 8. Propoſitio 10.

Si inter duos numeros & vnitaté continuè proportionales incidát numeri quot inter vtrumque ipſorum & vnitaté deinceps medij continua proportione incidunt numeri, totidé & inter illos medij continua proportione incident.

A	E	K	D	H	C	L	F	G	B
27	9	36	3	12	1	48	4	16	64

Theorema 9. Propoſitio 11.

Duorum quadratorũ numerorum vnus medius proportionalis eſt numerus: & quadratus ad quadratum duplicatam habet lateris ad latus rationem.

A	C	E	D	B
9	3	12	4	16

Theorema 10. Propoſitio 12.

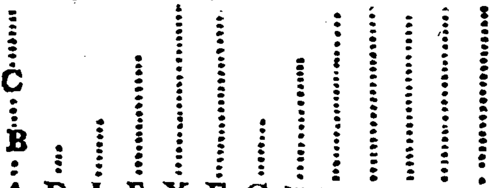
Duorum cuborum numerorũ duo medij proportionales ſunt numerorũ duo medij cubum triplicatam habet lateris ad latus rationem.

A	H	K	B	C	D	E	F	G
27	36	48	64	3	4	9	12	16

Theo.

Thorema 10. Propoſitio 13.

Si ſint quotlibet numeri deinceps proportionales, & multiplicans quiſq; ſeipſum faciat aliquos, qui ab illis producti fuerint, proportionales erũt, & ſi numeri primùm poſiti ex ſuo in proceatos ductu faciãt aliquo, ipſi quoque proportionales erunt



C B

A D L E X F G M N H O P K

4 8 16 32 64 8 16 32 64 128 256 512

Theorema 12. Propoſitio 14.

Si quadratus numerus quadratum numerũ metiatur, & latus vnius metietur latus alterius. Et ſi vnius quadrati latus metiatur latus alterius, & quadratus quadratum metietur.

A E B C D

9 12 16 8 4

Theorema 13. Propoſitio 15.

Si cubus numerus cubum numerũ metiatur, & latus vni' metietur alterius latus. Et ſi latus vnius cubi latus alterius metiatur,

tum

tum cubus cubum metietur.

A	H	K	B	C	D	E	F	G
8	16	28	64	2	4	4	8	16

Theorema 14. Propoſitio 9.

Si quadratus numerus quadratum numerum non metiatur, neque latus vnius metietur alterius latus.

Etſi latus vni⁹ quadrati non metiatur latus alterius, neque quadratus quadratum metietur.

A	B	C	D
9	16	3	4

Theorema 15. Propoſitio 17.

Si cubus numerus cubum numerũ non metiatur, neque latus vnius latus alterius metietur. Et ſi latus cubi alicuius latus alterius non metiatur, neque cubus cubum metietur.

A	B	C	D
8	27	9	11

Theorema 16. Propoſitio 18.

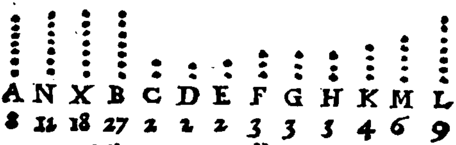
Duorum ſimilium planorum numerorum vnus medius proportionalis eſt numerus, & planus ad planum duplicatam habet literis homologi-

A	G	B	C	D	E	F
12	18	27	2	6	3	9

logi ad latus homologum rationem.

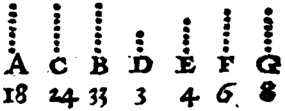
Theorema 17. Propositio 19.

Inter duos similes numeros solidos, duo medij proportionales incidunt numeri: & solidus ad similem solidum triplicatam rationem habet lateris homologi ad latus homologum.



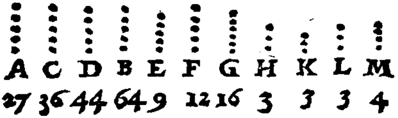
Theorema 18. Propositio 20.

Si inter duos numeros vnus medius proportionalis incidat numer9, similes plani erunt illi numeri.



Theorema 19 Propositio 21.

Si inter duos numeros duo medij proportionales incidant numeri, similes solidi sunt illi numeri.



Theo-

Theorema 20. Propositio 22.

Si tres numeri deinceps sint proportionales, primus autem sit quadratus, & tertius quadratus erit.

A	B	D
9	15	25

Theorema 21. Propositio 23.

Si quatuor numeri deinceps sint proportionales, primus autem sit cubus, & quartus cubus erit.

A	B	C	D
8	12	18	27

Theorema 22. Propositio 25.

Si duo numeri rationem habeant inter se, quam quadratus numerus ad quadratum numerum, primus autem sit quadratus erit, & secunndus quadratus erit.

A		B	C		D
4	6	9	16	24	36

Theorema 23. Propositio 25.

Si numeri duo rationem inter se habeant, quam cubus numerus ad cubum numerũ, primus autem cubus sit, & secundus cubus erit.

A	E	F	B	C			D
8	12	18	27	64	95	140	216

The

Theorema 24. Propositio 26.

Similes plani numeri rationem inter se habent, quam quadratus numerus ad quadratum numerum.

A	C	B	D	E	F
18	24	32	9	12	16

Theorema 25. Propositio 27.

Similes solidi numeri rationem habent inter se, quam cubus numerus ad cubum numerum.

A	C	D	B	E	F	G	H
16	24	36	54	8	12	18	27

ELEMENTI VIII. FINIS.

EVCLI.

EVCLIDIS ELEMENTVM NONVM.

Theorema 1. Propositio 1.

Si duo similes plani numeri mutuò sese multiplicãtes quendã procreent, productus quadratus erit.

A	E	B	D		C
4	6	9	16	24	36

Theorema 2. Propositio 2.

Si duo numeri mutuò sese multiplicantes quadratum faciant, illi similes sunt plani.

A		B	D		C
4	6	9		18	36

Theorema 3. Propositio 3.

Si cubus numerus seipsum multiplicãs procreet aliquẽ, productus cubus erit.

Vnitas	D	D	A			B
	3	4	8	16	32	64

Theorema 4. Propositio 4.

Si cubus numerus cubũ numerum multiplicans quendam procreet, procreatus cubus erit.

A	B	D	C
8	27	64	216

Theorema 5. Propositio 5.

Si cubus numerus numerum quendã multiplicans cubum procreet, & multiplicatus cubus erit.

A	B	D	D
27	64	729	1728

Theorema 6. Propositio 6.

Si numerus seipsum multiplicans cubum procreet, & ipse cubus erit.

A	B	C
27	729	19683

Theorema 7. Propositio 7.

Si compositus numerus quendam numerũ multiplicans quempiam procreet, productus solidus erit.

A	B	C	D	E
6	8	48	2	3

Theorema 8. Propositio 8.

Si ab vnitate quotlibet numeri deinceps ꝓportionales sint, tertius ab vnitate quadratus est, & vnũ intermittentes omnes: quartus autẽ cubus, & duobus intermissis omnes

nes: septimus verò cubus simul & æquadratus, & quinque intermissis omnes.

Vnitas	A	B	C	D	E	F
	3	9	27	81	243	729

Theorema 9. Propositio 9.

Si ab vnitate sint quotcunque numeri deinceps proportionales, sit autem quadratus is qui vnitatem sequitur, & reliqui oẽs quadrati erũt. Quòd si qui vnitatem sequitur cubus sit, & reliqui omnes cubi erunt.

quadrati		cubi
531441	F	732969
59049	E	531441
6561	D	59049
729	C	6561
81	B	729
9	A	81
	o	
	vnitas.	

Theorema 10. Propositio 10.

Si ab vnitate numeri quotcunque proportionales sint, non sit autẽ quadratus is qui vnitatem sequitur, neq; alius vllus quadratus erit, demptis tertio ab vnitate ac omnibus

o Vnitas.	A	B	C	D	E	F
	3	9	36	81	243	729

nibus vnum intermittentibus. Quòd si qui vnitatem sequitur, cubus non sit, neque alius vllus cubus erit, demptis quarto ab vnitate ac omnibus duos intermittentibus.

Theorema 11. Propositio 11.

Si ab vnitate numeri quotlibet deinceps proportionales sint, minor maiorem metitur per quempiam eorum qui in proportionalibus sunt numeris.

.	:	:	:	:
A	D	C	D	E
1	2	4	8	16

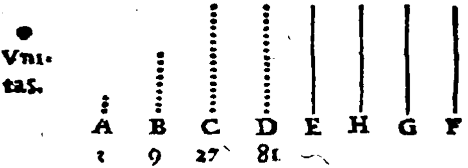
Theorema 12. Propositio 12.

Si ab vnitate quolibet numeri sint proportionales, quot primorum numerorũ vltimum metiuntur, totidem & eum qui vnitati proximus est, metientur.

Vnitas.	A	B	C	D	E	H	G	F
	4	16	64	259	2	8	32	128

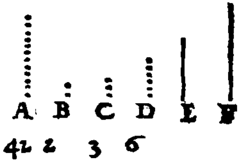
Theorema 13. Propositio 13.

Si ab vnitate sint quotcunq; numeri deinceps proportionales, primus autẽ sit qui vnitatẽ sequitur, maximũ nullus alius metietietur, ijs exceptis qui in proportionalibus sunt numeris.



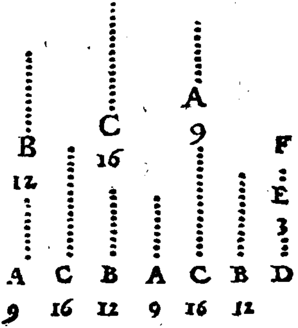
Theorema 14. Propositio 14.

Si minimum numerum primi aliquot numeri metiãtur, nullus alius numerus primus illum metietur, ijs exceptis qui primò metiuntur.



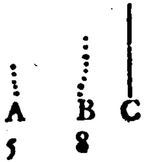
Theorema 15. Propositio 15.

Si tres numeri deinceps proportionales sint minimi, eadem cum ipsis habẽtium rationem, duo quilibet compositi ad tertium primi erunt.



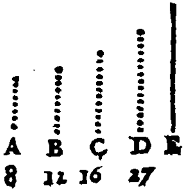
Theorema 16. propoſitio 16.

Si duo numeri ſint inter ſe primi, non ſe habebit quẽadmodum primus ad ſecũdum, ita ſecundus ad quẽpiam alium.



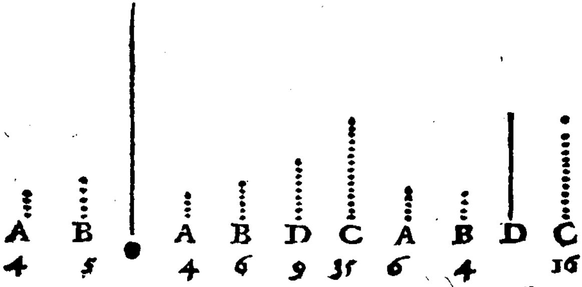
Theorema 17. Propoſitio 18.

Si ſint quotlibet numeri deinceps ꝓportionales, quorum extremi ſint inter ſe primi, non erit. quẽadmodum primus ad ſecundum, ita vltimus ad quempiam alium.



Theorema 18. Propoſitio 18.

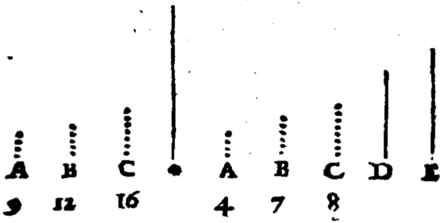
Duobus numeris datis, cõſiderare noſſitne tertius illi inueniri proportionalis.



Theo-

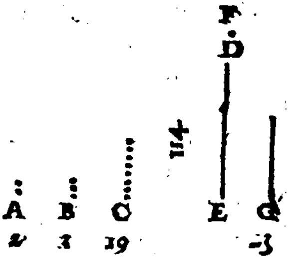
Theorema 19. Propositio 19.

Tribus numeris datis, considerare possitne quartus *illis* reperiri proportionalis.



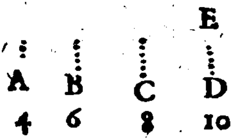
Theorema 20. Propositio 20.

Primi numeri plures sunt quacunque proposita multitudine primorum numerorum.



Theorema 12. Propositio 12.

Si pares numeri quotlibet compositi sint; totus est par.



Theorema 22. Propositio 22.

Si impares numeri quotlibet compositi sint, sit autem par illorum multitudo, totus par erit.

E

A B C D

5 9 7 3

Theorema 23. Propositio 23.

Si impares numeri quotcunque cópositi sint, sit autem impar illorũ multitudo, & totus impar erit.

A B C E

5 7 8 1

Theorema 24. Propositio 24.

Si de pari numero par detractus sit, & reliquus par erit.

B

A C

6 4

Theorema 25. Propositio 25.

Si de pari numerò impar detractus sit, & reliquus impar erit.

B

A C D

8 1 4

Theorema 26. Propositio 26.

Si de impari numero impar detractus sit, & reliquus par erit.

B

A C D

4 6 1

Theorema 27. Propositio 27.

Si ab impari numero par ablatus sit, reliquus impar erit.

A D C
1 4 4

Theorema 28. Propositio 28.

Si impar numerus parem multiplicãs, procreet quempiam, procreatus par erit.

A B C
3 4 28

Theorema 29. Propositio. 29.

Si impar numerus imparẽ numerum multiplicans quemdam procreet, procreatus impar erit.

A B C
3 5 15

Theorema 30. Propositio 30.

Si impar numerus parẽ numerum metiatur, & illius dimidium metietur.

A C B
3 6 18

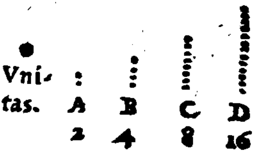
Theorema 31. Propositio 31.

Si impar numerus ad numerum quẽpiam primus sit, & ad illius duplum primus erit

A B C D
7 8 16

Theorema 32. Propositio 32.

Numerorum, qui à binario dupli sunt, vnusquisq; pariter par est tantùm.



Theorema 33. Propositio 33.

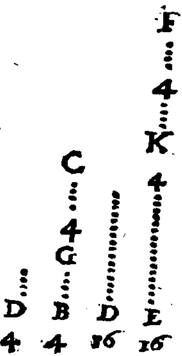
Si numerus dimidiú impar habeat, pariter impar est tantùm.

Theorema 34. Propositio 34.

Si par numerus nec sit dupl' à binario, nec dimidium impar habeat, pariter par eũ, & pariter impar.

Theorema 35. Propositio 35.

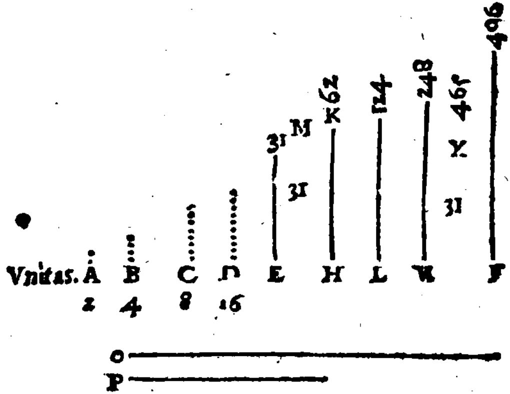
Si sint, quotlibet numeri deinceps proportionales, detrahãtur aũt de secundo & vltimo æquales ipsi primo, erit quemadmodum secũdi excessus ad primũ, ita vltimi excessus ad omnes qui vltimum antecedunt.



Theo-

Theorema 16. Propositio 16.

Si ab vnitate numeri quotlibet deinceps expositi sunt in duplici proportione quoad totus compositus primus factus sit, isq; totus in vltimum multiplicatus quempiã procreet, procreatus perfectus erit.



ELEMENTI IX. FINIS

EVCLI.

EVCLIDIS ELEMENTVM DECIMVM.

DEFINITIONES.

1

Commensurabiles magnitudines dicuntur illæ, quas eadem mensura metietur.

2

Incommensurabiles verò magnitudines dicuntur, hæ quarum nullam mensuram communem contingit reperiri.

3

Lineæ rectæ potentia cōmensurabiles sunt, quarum quadrata vna ead m superficies siue area metitur.

4

Incommensurabiles verò lineæ sunt, quarum quadrata, quæ metiatur area communis, reperiri nulla potest.

5

Hęc cùm ita sint, ostendi potest quòd quantacunque linea recta nobis proponatur, existunt etiam aliæ lineæ innumerabiles eidem commensurabiles, aliæ item incommensurabiles, hæ quidem longitudine & poten-

potentia: illæ verò potentia tantùm. Vocetur igitur linea recta, quantacunq; proponatur, ῥητή, id est rationalis.

6

Lineæ quoq; illi ῥητῇ commensurabiles siue longitudine & potentia, siue potentia tantùm, vocentur & ipsæ ῥηταί, id est rationales.

7

Quæ verò lineæ sunt incommensurabiles illi τῇ ῥητῇ, id est primo loco rationali, vocẽtur ἄλογοι, id est irrationales.

8

Et quadratum quod à linea proposita describitur, quam ῥητὴν vocari volumus, vocetur ῥητόν.

9

Et quæ sunt huic commensurabilia, vocẽtur ῥητά.

10

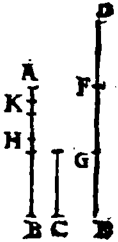
Quæ verò sunt illi quadrato ῥητῷ scilicet incommensurabilia, vocentur ἄλογα, id est surda.

11

Et lineę quę illa incommensurabilia describunt, vocentur ἄλογοι. Et quidem si illa incommensurabilia fuerint quadrata, ipsa eorũ latera vocabuntur ἄλογοι lineæ, quòd si quadrata quidem non fuerint, verũ alias quæpiã superficies siue figuræ rectilineæ, tunc

tunc verò lineæ illæ quæ deſcribunt quadrata æqualia figuris rectilineis, vocentur ἄλογοι.

Theorema 1. Propoſitio 1.



Duabus magnitudinibus inæqualibus propoſitis, ſi de maiore detrahatur plus dimidio, & rurſus de reſiduo iterũ detrahatur plus dimidio, idq́; ſemper fiat: relinquetur quædam magnitudo minor altera minore ex duabus propoſitis.

Theorema 2 Propoſitio. 2.

Duabus magnitudinibus propoſitis inęqualibus, ſi detrahatur ſemper minor de maiore, alterna quadam detractione neque reſiduum vnquam metiatur id quod ante ſe metiebatur, incommenſurabiles ſunt illæ magnitudines.

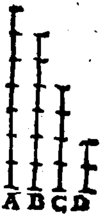
Problema 1. Propoſitio 3.

Duabus magnitudinibus commẽſurabilibus datis, maximam ipſarum communem menſuram reperire.

Proble-

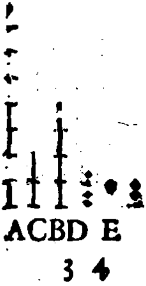
Problema 2. Propositio 4.

Tribus magnitudinibus cõmensurabilibus datis, maximam ipsarũ communẽ mensuram reperire.



Theorema 3. Propositio 5.

Cõmensurabiles magnitudines inter se proportionẽ eam habent, quam habet numerus ad numerum.

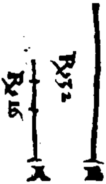


Theorema 4. Propositio 6.

Si duæ magnitudines proportionem eam habent inter se quam numerus ad numerum, commensurabiles sunt illæ magnitudines.

Theo-

Theorema 5. Propoſitio 7.



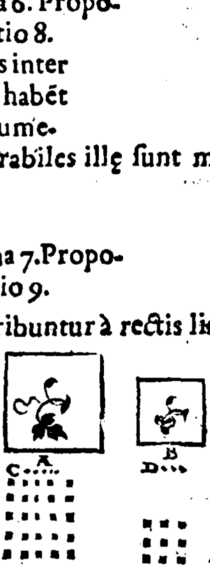
Incommenſurabiles magnitudines inter ſe proportionem non habẽt, quam numerus ad numerum.

Theorema 6. Propoſitio 8.

Si duæ magnitudines inter ſe proportionem nõ habẽt quam numerus ad numerum, incommenſurabiles illę ſunt magnitudines.

Theorema 7. Propoſitio 9.

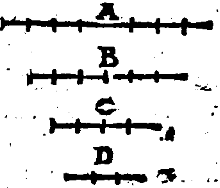
Quadrata, quæ deſcribuntur à rectis lineis lõgitudine cõmenſurabilibus, inter ſe proportionem habent quã numerus quadratus ad alium numerum quadratũ. Et quadrata habentia proportionẽ



inter ſe quã quadratus numerus ad numerum quadratũ, habent quoque latera longitudine commẽſurabilia. Quadrata verò quæ deſcribuntur à lineis longitudine in cōmenſurabilibus, proportionē non habent inter ſe, quam quadratus numerus ad numerum alium quadratum. Et quadrata non habentia proportionem inter ſe quam numerus quadratus ad numerum quadratum, neque latera habebunt longitudine commnnſurabilia.

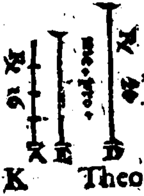
Theorema 8. Propoſitio 10.

Si quatuor magnitudines fuerint proportionales, prima verò ſecundæ fuerit commēſurabilis, tertia quoque quartę commenſurabilis erit. quòd ſi prima ſecūdæ fuerit incommenſurabilis, tertia quoq; quartæ incommenſurabilis erit.



Problema 3. Propoſitio 11.

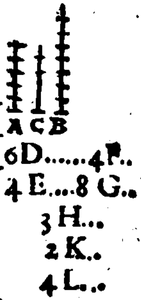
Propoſitæ lineæ rectæ (quam ῥητὴν vocari diximus) reperire duas lineas rectas incommenſurabiles, hanc quidem longitudiue tantùm, illam verò non longitudine tantùm, ſed etiam potentia incommēſurabilem.



Theo-

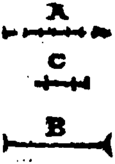
Theorema 6. Propositio 12.

Magnitudines quæ eidem magnitudini ſunt commenſurabiles, inter ſe quoque ſunt commenſurabiles.



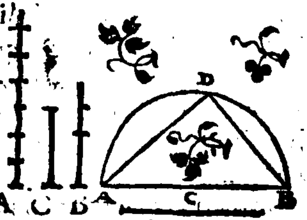
Theorema 10. Propoſitio 13.

Si ex duabus magnitudinibus hæc quidem commenſurabilis ſit tertię magnitudini, illa verò eidem incommenſurabilis, incommēſurabiles ſunt illæ duæ magnitudines.



Theorema 11. Propoſitio 14.

Si duarum magnitudinum commenſurabilium altera fuerit incommenſurabilis magnitudini alteri cuipiam tertię, reliqua quoq; magnitudo eidem tertiæ incommenſurabilis erit.



Theorema 12. Propoſitio 15.

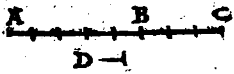
Si quatuor rectæ proportionales fuerint,

poſſit

possit autem prima plusquã secunda tanto quantum est quadratum lineę sibi commensurabilis longitudine: tertia quoq; ponteris plusquam quarta tanto quantum est quadratum lineæ sibi commensurabilis longitudine. Quòd si prima possit plusquam secunda quadrato lineę sibi longitudine incommensurabilis: tertia quoque poterit plusquam quarta quadrato lineę sibi incõmẽsurablilis longitudine.

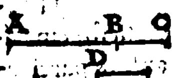
Theorema 13. Propositio 16.

Si duæ magnitudines commensurabiles componantur, tota magnitudo composita singulis partibus commensurabilis erit, quòd si tota magnitudo composita alteruтri parti commensurabilis fuerit, illæ duæ quoque partes cõmensurabiles erunt.



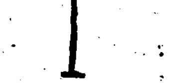
Theorema 14. Propositio 17.

Si duæ magnitudines incommensurabiles componantur, ipsa quoque tota magnitudo singulis partibus componentibus incommensurabilis erit. Quòd si tota alteri parti incommensurabilis fuerit, illæ quoque primæ magnitudines inter se incommensurabiles erunt.



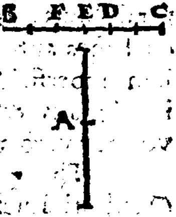
Theorema 15. Propositio 18.

Si fuerint duæ rectæ lineæ inæquales, & quartæ parti quadrati quod describitur à minore æquale parallelongrammú applicecetur secundú maiorem, ex qua maiore tátum excurrat extra latus parallelogrammi, quantum est alterum latus ipsius parallelogrammi: si præterea parallelogrammum sui applicatione diuidat lineá illam in partes inter se commensurabiles longitudine, illa maior linea tanto plus potest quam minor, quantú est quadratum lineæ sibi commensurabilis longitudine. Quòd si maior quadratú lineæ sibi commensurabilis lonplus possit quàm minor, tanto quantú est gitudine, & prætereà quartæ parti quadrati lineę minoris ęquale parallelogrammum applicetur secundum maiorem, ex qua maiore tantum excurrat extra latus pallelogrammi, quantum est alterum latus ipsius parallelongrammi parallelogammum sui applicatione diuidit maioré in partes inter se longitudine commensurabiles.

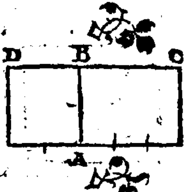


Theorema 16. Propositio 19.

Si fuerint duæ rectæ inæquales, quartæ autem parti quadrati lineæ minoris æquase parallelogrammorum secundum lineam maiorem applicetur, ex qua linea tantum excurrat extra latus parallelogrammi, quantum est alterum latus eiusdem parallelogrammi: si parallelogrammum praeterea sui applicatione diuidat lineam in partes inter se longitudine incommensurabiles, maior illa linea tantò plus potest, quàm minor quantum est quadratum lineae sibi maiori incommensurabiles longitudine. Quod si maior linea tantò plus possit quàm minor, quantum est quadratum lineae incommensurabilis sibi longitudine: & praeterea quartae parti quadrati lineae minoris aequale parallelogrammum applicetur secundum maiorem ex qua tantum excurrat extra latus parallelogrāmi, quantū est alterum latus ipsius: parallelogrammum sui applicatione diuidit maiorem in partes inter se incommensurabiles longitudine.



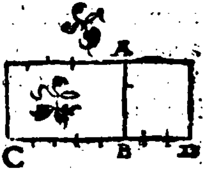
Theorema 17. Propositio 20.



Superficies rectangula contenta ex lineis rectis rationalibus longitudine commensurabilibus secundum

cundum vnum aliquem modum ex antedi-
ctis, rationalis est.

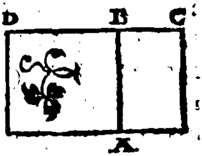
Theorema 18. Propositio 21.



Si ratinonale secundú lineam rationalem applicetur, habebit alterum latus lineam rationalem & cómensurabilem longitudine lineæ cui rationale parallelogrammum applicatur.

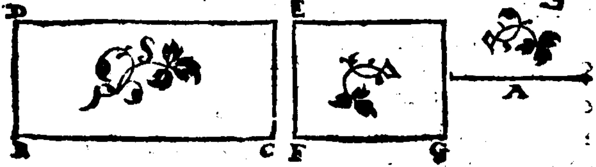
Theorema 39. Propositio 56.

Surperficies rerctangula contenta duabus lineis rectis rationalibus potentia tantum commésurabilibus, irrationalis est. Linea autem quæ illam superficiem potest, irrationalis & ipsa est: vocetur verò medialis



Theorema 20. Propositio 23.

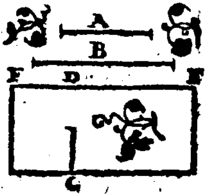
Quadrati lineæ medialis applicati secun-



dum

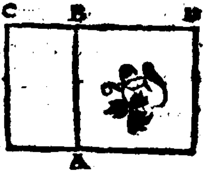
dum lineam rationalem, alterum latus est linea rationalis, & incommensurabilis longitudine lineæ secundum quam applicatur.

Theorema 21. Propositio 24



Linea recta mediali commẽsurabilis, est ipsa quoque medialis.

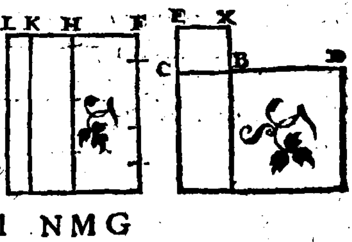
Theorema 22. Propositio 25



Parallelongrammum rectangulum contẽtum ex lineis medialibus longitudine commensurabilibus, mediale est.

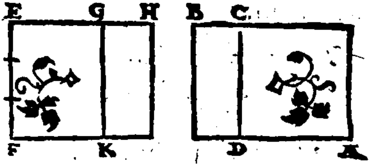
Theorema 23. Propositio 26.

Parallelogrammum rectangulum comprehensum duabus lineis medialibus potentia tãtùm cõmensurabilibus, vel rationale est, vel mediale.



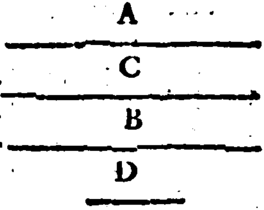
Theorema 24. Propositio 27.

Mediale nó est maius quàm mediale superficie rationali.



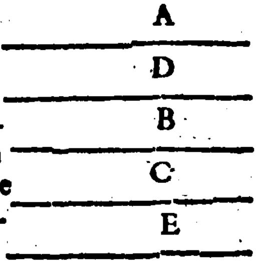
Problema. 4. Propositio 28.

Mediales lines inuenire potentia tantùm commensurabiles rationale comprehendentes.



Problema 5. Propositio 29.

Mediales lineas inuenire potentia tantùm commensurabiles mediale comprehendendentes



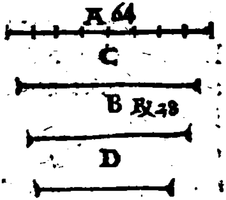
Pro-

Problema 6. Propositio 30.

Reperire duas rationales potentia tantum commẽsurabiles huiusmodi, vt maior ex illis possit plus quàm minor quadrato lineæ sibi commensurabilis longitudine.

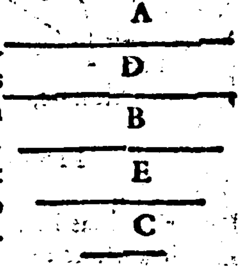
Problema 7. Propositio 31.

Reperire duas lineas medialis potentia tantùm commensurabiles rationalem superficiẽ continẽtes, tales in quã vt maior possit plus quàm minor quadrato lineæ sibi commẽsurabilis longitudine.



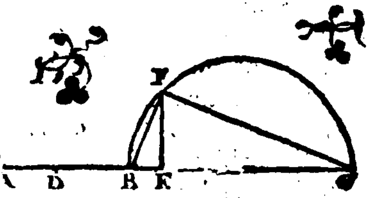
Problema 8. Propositio 32.

Reperire duas lineas mediales potentia tantùm commensurabiles medialem superficiem continentes, huiusmodi vt maior plus possit quàm minor quadrato lineæ sibi commensurabilis longitudine.



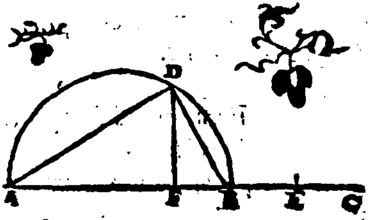
Problema 9. Propositio 33.

Réperire duas rectas poutentia incommēsurabiles, quarum quadrata simul addita faciant superficiē rationalē, parallelogrammum verò ex ipsis conetntum sit mediale.



Problema 10. Propositio 34.

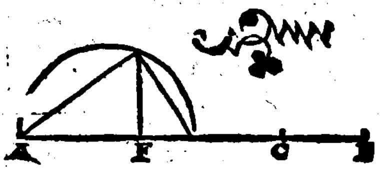
Reperire lineas duas rectas potentia incommensurabiles conficentes compositnm ex ipsarum quadratis mediale, parallelogrammū verò ex ipsis contentum rationale,



Problema 11. Propositio 85.

Reperire duas lineas rectas potentia incōmensurabiles, conficientes id quod ex ipsarum quadratis componitur meale, simulque

que parallelogrammú ex ipsis contentum, mediale, quod præterea parallelogrammú sit incómensurabile compoto ex quadratis ipsarum.



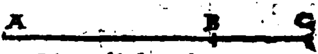
PRINCIPIVM SENARIOrum per compositionem.

Thorema 25. Propositio 36.

Si duæ rationales potentia tantùm commensurabilis. Vocetur autem Binommum.

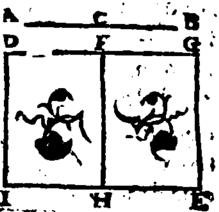
Theorema 26, Propositio 37.

Si duæ mediales potentia tantòm commensurabiles rationale continentes componantur, tota linea est irrationalis, vocetur autem Bimediale prius.



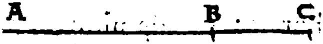
Theorema 27. Propositio 38.

Si duæ mediales Potétia tantùm comméſurabiles mediale continétes componãtur, tota linea eſt irrationalis: vocetur auté Bimediale ſecundum.



Theorema 28. Propoſitio 36.

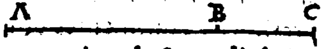
Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex quadratis ipſarum rationale, parallelogrammum verò ex ipſis contentum mediale, tota linea recta



eſt irrationalis. Vocetur autẽ linea maior.

Theorema 39. Propoſitio 40.

Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex ipſarum quadratis mediale, id vero ꝙ fit ex ipſis, rationale, tota linea eſt irrationales. Vocetur au-

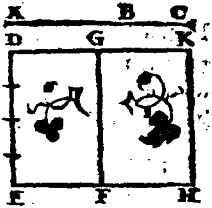


tem potens rationale & mediale.

Theorema 30. Propoſitio 41.

Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex quadratis ipſarum mediale, & quod conti-

netur



netur ex ipsis, mediale, & præterea incõmensurabile composito ex quadratis ipsarum tota linea est irrationalis. Vocetur autem potens duo medialia.

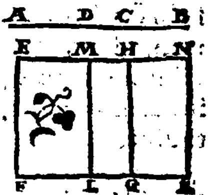
Theorema 31. Propositio 42.

Binominum in vnico tantùm pũcto diuiditur in sua nomina, id est in lineas ex quibus componitur.

Theorema 32. Propositio 43.

Bimediale prius in vnico tantum puncto diuiditur in sua nomina.

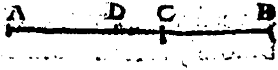
Problema 33. Propositio 44



Bimediale secundum in vnico tantũ puncto diuiditur in sua nomina.

Problema 34. Propositio 45.

Linea maior in vnico tantùm in pũcto diuiditur in sua nomina.



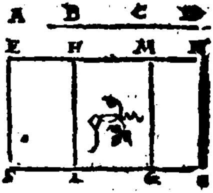
Theo-

Theorema 35. Propositio 46.

Linea potens rationale & mediale in vnico tantùm puncto diuiditur in sua nomina.

Theoroma 36. Propositio 47

Linea ootèns duo medialia in vnico tantùm puncto diuiditur in sua nomina.



DEFINITIONES. secundæ.

Proposita linea rationali, & binomio diuiso in sua nomine, cuius binomij maius n omen, id est, maior portio possit plusquam minus nomen quadrato lineæ sibi, maiori inquam nomini, commensurabilis longitudine.

1.

Si quidē maius nomen fuerit commensurabile longitudine propositæ lineæ rationali, vocetur toto linea Binomum primium.

2

Si veró minus nomen, id est minor portio binomij, fuerit commensurabile longitudi ne ne propositæ lineæ rationali, vocetur tota linea Binomium secundum.

3

Si verò neutrum nomen fuerit commensurabile longitudine propositæ lineæ rationali, vocetur Binomium tertium.

Rursus si maius nomen possit plusquam minus nomen quadrato lineæ sibi incommensurabilis longitudine.

4

Si quidem maius nomen est commensurabile longitudine propositæ lineæ rationali, vocetur tota linea Binomium quartum.

5

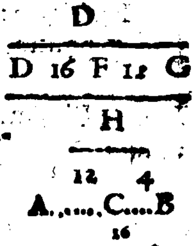
Si verò minus nomen fuerit commensurabile longitudine lineæ rationali, vocetur Binomium quintum.

6

Si verò neutrum nomen fuerit longitudine commensurabile lineæ rationali, vocetur illa Binomium sextum.

Proble. 12. Propositio 48

Reperire Binomium primum.



Pro-

Problema 13. Propositio 49

9 3
A........C...B
12

D

Reperire Binomium secudum.

E F 9 G

H 6

Problema 14. Propositio 50.

15 5
A..............C....
20

D

E

Reperire Binomium tertiũ.

F 20 G 15 H

K 6

Problema 15. Propositio 51.

10 6
A........C......B
16
D

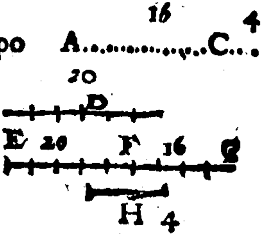
Repere Binomiũ quartum.

E 16 F 10 G

H 6

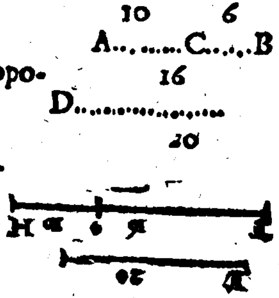
Proble-

Problema 16. Propositio 52



Reperire Binomium quintum.

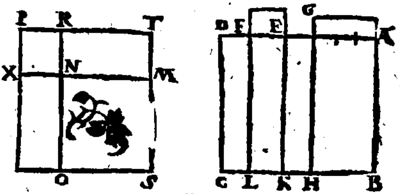
Probl. 17. Propositio 53.



Reperire Binonimum sextum.

Theorema 37. Propositio 54.

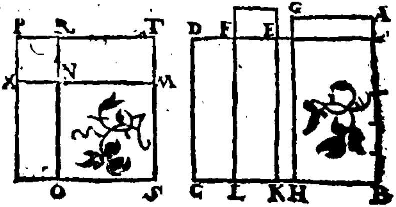
Si superficies contéta fuerit ex rationali & Binomio primo, linea quę illã superficiem potest, est irrationalis, quæ Binomiũ vocatur.



 Theore-

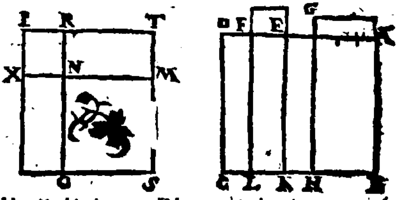
Theorema 38. Propositio 55.

Si superficies cõtenta fuerit ex linea rationali & Binomio secundo, linea potens illam superficiẽ est irrationalis quę Bimediale primum vocatur.

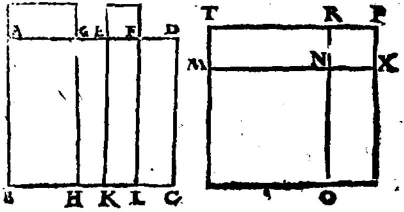


Theorema 39. Propositio 56.

Si superficies cótineatur ex rationali & Binomio tertio, líñea q̃ illam superficiem potest, est irrationalis q̃ dicitur Bimediale secundú.



Theorema 40. Propositio 57.

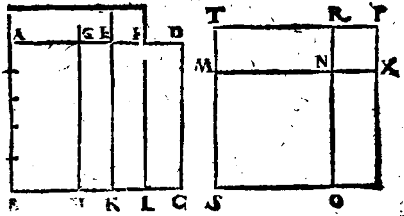


Si superficies contineatur ex rationali & Boin mio

mio quarto. linea potens superficiem illam, est irrationalis, quæ dicitur maior.

Theorema 41. Propositio 58.

Si suptrficies contineatur ex rationali & Binomio quinto, linea quæ illam superficiem potest est irrationalis, quę dicitur potés rationale & mediale.

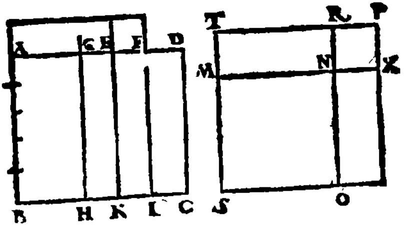


Therema 42. Propositio 59

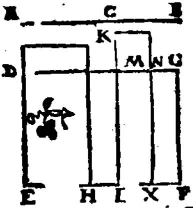
Si superficies contineatur ex rationali & Binomio sexto, linea quæ illam superficiem potest est irrationalis, quæ dicitur

 potens

potens tuo mediala.

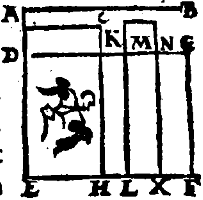


Theorema 43. Propositio 60.



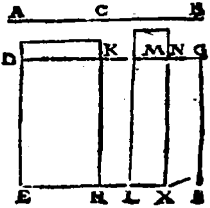
Quadratum Binomij secundum lineam rationalem applicatum, facit alterum latus Binomium primum.

Theorema 44. Propositio 61.



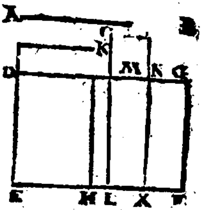
Quadratũ Bimedialis primi secundum rationalem lineam applicatum, facit alterum latus Binomium secundum.

Theorema 45. Propositio 62.



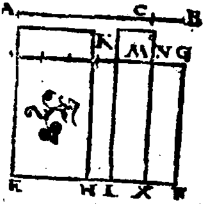
Quadratũ Bimedialis secũdi secundũ rationalem applicatum, facit alterũ latus Binominũ tertium.

Theorema 46. ropopoſitio 63



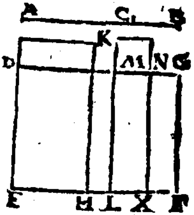
Quadratum lineæ maioris ſecundum lineam rationalem applicatum, facit alterum latus Binomium quartum.

Theorema 47. Propoſitio 64.



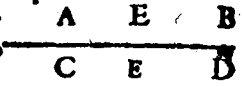
Quadratum lineæ potẽtis rationale & mediale ſecundum rationalẽ applicatũ, facit alterum latusBinomium quintum.

Theorema 48. Propoſitio 65



Quadratum lineę potentis duo medialia ſecundũ rationalem applicatum, facit alterum latus Binomium ſextum.

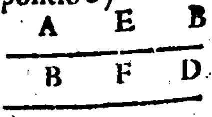
Theorema 49. Propoſitio 66.



Linea lõgitudine commenſurabilis Binomio eſt & ipſa Binomium eiuſdem ordinis.

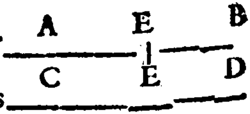
Theorema 50. Propositio 67

Linea longitudine commensurabilis alteri bimedialum, est & ipsa bimediale etiam eiusdē ordinis.



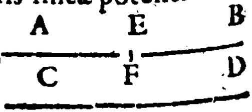
Theorema 51. Propositio 68

Linea cōmensurabilis lineæ maiori, est & ipsa maior.



Theorema 52. Propositio 69.

Linea commensurabilis lineæ potenti rationale & mediale, est & ipsa linea potens rationale & mediale.



Theorema 53. Propositio 70.

Linea commensurabilis lineæ potenti duo medialia, est & ipsa linea potens duo medialia.

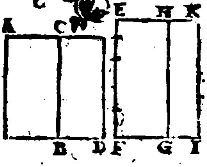
Theorema 54 Propositio 71

Si duæ superficies rationalis & medialis simul componantur, linea quę totā superficiem

ciem compositã potest, est vna ex quatuor irrationalibus, vna ex quæ dicitur Binomium, vel bimediale primum. vel linea maior, vel linea potẽs rationale & mediale.

Theorema 55. Propositio 72.

Si duæ superficies mediales incommensurabiles simul componantur fiunt reliquæ duæ lineę irrationales, vel bimediale secundum, vel linea potẽs duo medialia



SCHOLIVM.

Binomium & cæteræ consequentes lineæ irrationales, neque sunt eadem cum lineâ mediali, neque ipsæ inter se.

Nam quadratum lineæ medialis applicatum secundum lineam rationalem facit alterum latus lineam rationalem, & longitudine incommensurabilem lineæ secundum quam applicatur, hoc est, lineæ rationali, per 23.

Quadratum verò Binomij secundum rationalem applicatum, facit alterum latus Binomium primum, per 60.

Quadratum verò Bimedialis primi secundum rationalem applicatum facit alterum latus Binomium secundum per 61

Quadratum verò Bimedialis secundi secundum rationalem applicatum; facit alterum latus Binomiũ tertium per 62.

Quadratum verò lineæ maioris secundum rationalem applicatum facit alterum latus Binomiũ quartum, per 63

Quadratum verò lineæ potẽtis rationale & mediale secundum rationalem applicatum, facit alterũ latus Binomium quintum, per 64.

Quadratum vero lineæ potentis duo medialia secũdum rationalem applicatum, facit alterum latus Binomium sextum, per 65.

Cùm igitur dicta latera, quæ latitudines vocantur, differant & à prima latitudine, quoniam est rationalis, cùm inter se quoque differant, eo quia sunt Binomia diuersorum ordinum: manifestum est ipsas lineas irrationales, differentes esse inter se.

SECVN-

SECVNDVS ORDO ALterius sermonis, qui est de detractione.

Principium senariorum per detractionem.

Theorema 57. Propositio 74.

Si de linea rationali detrahatur rationalis potentia tantùm commensurabilis ipsi toti, residua est irrationalis, vocetur autem Residuum.

A A A

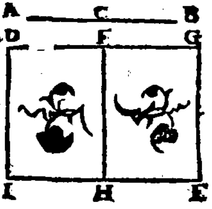
Theorema 56. Propositio 57.

Si de linea mediali detrahatur medialis potentia tantũ commensurabilis toti lineæ, quæ verò detracta est cũ tota contineat superficiem rationalem, residua est irrationalis. Vocetur autem Residuum mediale primum.

A A B

Theorema 58 Propositio. 75.

Si de linea mediali detrahatur medialis



tentia tantùm commensurabilis toti, quæ verò detracta est, cùm tota cō tineat superficiē medialē, reliqua est irrationalis. Vocetur autem Residuū mediale secundum

Theorema 57. Propositio 76

Si de linea recta detrahatur recta potētia incommensurabilis toti, compositum autem ex quadratis totius lineæ & lineæ detractæ sit rationale, parallelogrammum verò ex ijsdem contentum sit mediale, reliqua linea erit irrationalis. Vocetur autem linea minor.

A C B

Theorema 58. Propositio 77.

Si de linea recta detrahatur recta potentia

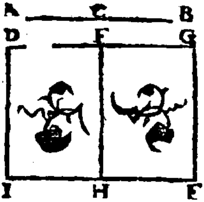
tia incommenſurabilis toti lineæ compoſitum autem ex quadratis totius & lineæ detractæ ſit mediale, parallelogrammum verò bis ex eiſdem contentum ſit rationale, reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie rationali totam ſuperficiem medialem.

Theorema 59. Propoſitio 78.

Si de linea recta detrahatur recta potentia incommenſurabilis toti lineæ, compoſitum autem ex quadratis totius & lineæ detractæ ſit mediale, parallelogrammum verò bis ex ijſdem ſit etiam mediale: præterea ſint quardata ipſarum incommenſurabilia parrallelogrammo bis ex ijſdem contento, reliqua linea eſt irrationalis.

Vocetur autem linea faciens cum ſuperficie

ficie mediali totam ſuper ficiem medialem.



Theorema 60. Propoſitio 79.

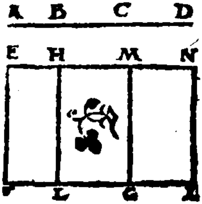
Reſiduo vnica tantùm linea recta cõiungitur rationalis, potentia tantùm cõmenſurabilis toti lineæ rationalem.

Theorema 61 Propoſitio 80.

Reſiduo mediali primo vnica tãtùm linea coniungitur medialis, potentia tantùm cõmenſurabilis toti, ipſa cum tota continens

Theorema 62. Propoſitio 81.

Reſiduo mediali ſecundo vnica tantùm coniungitur mediails, potentia tantùm commẽſurabilia toti ipſa cum tota continens mediale.



Theorema 63. Propoſitio 82.

Lineæ minori vnica tantùm recta cõiungitur

tur potentia incommensurabilis toti, faciens cum tota compositum ex quadratis ipsarum rationale, id verò parallelogrammum, quod bis ex ipsis fit, mediale.

A B C D

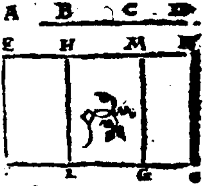
Theorema 64. Propositio 83.

Lineæ facienti cum superficie rationali totam superficiem medialem, vnica tantùm coniungitur linea recta potentia incommensurabilis toti, faciens autem cum tota compositum ex quadratis ipsarũ, mediale, id verò quod fit bis ex ipsis, rationale.

A B C D

Theorema 65. Propositio 84.

Lineæ cum mediali superficie facienti totam superficiem medialem, vnica tantum coniũgitur linea potẽtia toti incommensurabilis, faciens cum tota compositũ ex quadratis ipsarũ mediale, id verò quod fit bis ex ipsis etiam mediale, & prætereà faciens compositum ex quadratis ipsarum incommensurabile ei quod fit bis ex ipsis.



DEFINITIONES TERTIAE

Propoſita linea rationali & reſiduo.

1

Si quidem tota, nempe compoſita ex ipſo reſiduo & linea illi cõiuncta, plus poteſt quàm coniuncta, quadrato lineæ ſibi cõmenſurabilis longitudine, fueritq; tota longitudine commẽſurabilis lineæ propoſitæ rationali, reſiduũ ipſum vocetur Reſiduum primum.

2

Si verò coniuncta fuerit longitudine commenſurabilis rationali, ipſa autem tota plus poſſit quam coniuncta, quadrato lineæ ſibi longitudine commenſurabilis, reſiduum vocetur Reſiduum ſecundũ.

3

Si verò neutra linearũ fuerit longitudine commenſurabilis rationali poſſit autem ipſa tota pluſquam coniuncta, quadrato lineæ ſibi longitudine commẽſurabilis, vocetur Reſiduum tertium.

Rurſus ſi tota poſſit plus quàm coniuncta, quadrato lineæ ſibi longitudine incommenſurabilis

4

Et quidem ſi tota fuerit longitudine commenmenſurabilis ipſi rationali, vocetur Reſiquum quartum.

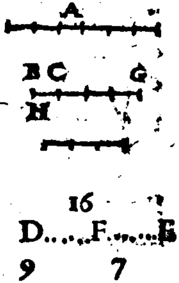
5

Si verò coniuncta fuerit lõgitudine commenſurabilis rationali, & tota plus poſſit quàm coniuncta, quadrato lineæ ſibi longitudine incõmenſurabilis, vocetur Reſiduum quintum.

6

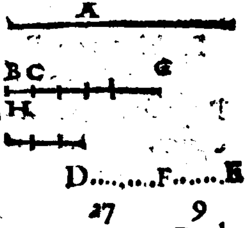
Si verò neutra linearum fuerit cõmenſurabilis longitudine ipſi rationali, fueritque tota potentior quã coniuncta, quadrato lineæ ſibi longitudine incommenſurabilis, vocetur Reſiduum ſextum.

Problema 18. Propoſitio 85.



Reperire primum Reſiduum.

Problema 19. Propoſitio 86.



Reperire ſecundum Reſiduum.

Prob-

E.........

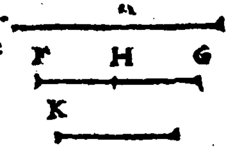
12

B........E......C

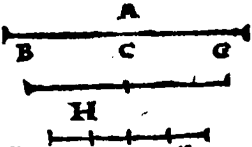
9 7

Problema 02. Propositio 87.

Reperire tertium Residuum.



Probl. 21. Propositio 88.

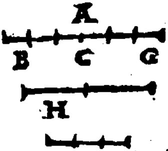


Reperire quartum Residuum.

D... ....F....E

16 4

Problema 22. Propositio 89



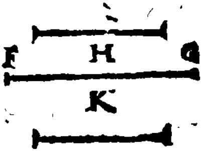
Reperire quintum Residuum

D......F....E

25 7

Problema 23. Propositio 90.

Reperire sextum Residuum.

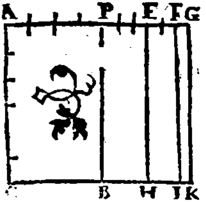


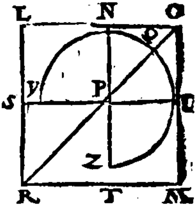
E........... 13

Theo- B.......D......C

Theorema 66. Propositio 91.

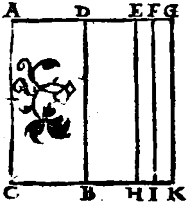
Si superficies contineatur ex linea rationali & residuo primo linea, quæ illam superficiem potest, est residuum.

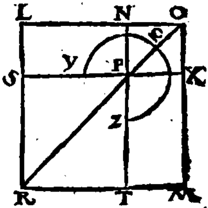




Theorema 67. Propositio 92.

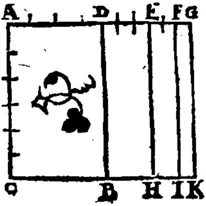
Si superficies cõtineatur ex linea rationali & residuo secundo, linea q̃ illam sup̃ficiẽ potest, est residuum mediale primum.

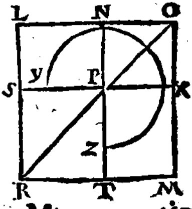




Theorema 68. Propositio 93.

Si superficies cõtineatur ex linea rationali & residuo ter-

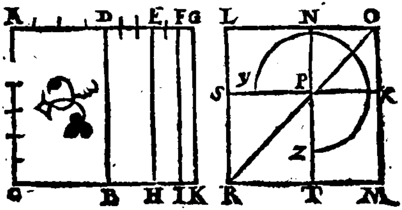




tio, linea quæ illam ſuperficiem poteſt, eſt reſiduum mediale ſecundum.

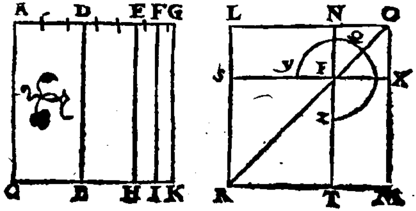
Theorema 69. Propoſitio 94.

Si ſuperficies contineatur ex linea rationali & reſiduo quarto linea q̃ illã ſuperficiem poteſt, eſt linea minor.



Theorema 70. Propoſitio 95.

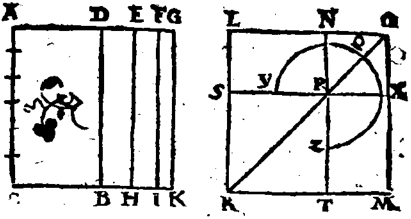
Si ſuperficies contineatur ex linea rationali & reſiduo quinto, linea quæ illã ſuperficiẽ poteſt, eſt ea quæ dicitur cum rationali ſuperficie faciens totam medialem.



Theorema 71. Propo-poſitio 96.

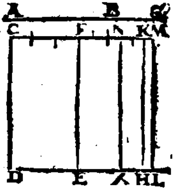
Si ſuperficies contineatur ex linea rationali &

& residuo sexto, linea quæ illam superficiē pōt, est ea quę dicitur faciēs cum mediali superficie totam medialem.



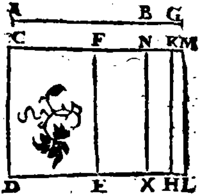
Theorema 72. Propositio 97.

Quadratum residui secundum lineam rationalem applicatum, facit alterum latus residuum primum.



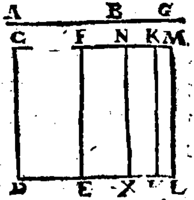
Theorema 73. Propositio 98.

Quadratum residui medialis primi secundū rationalem applicatū, facit alterum latus residuū secundum.



Theorema 74. Propositio 99.

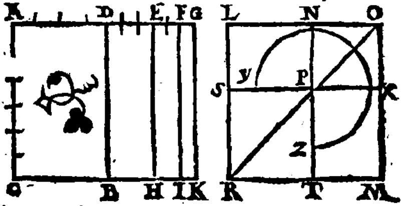
Quadratū, residui medialis secundi secundum rationalē applicatū, facit alterū latus residuum tertium.



tio,linea quæ illam ſuperficiem poteſt, eſt reſiduum mediale ſecundum.

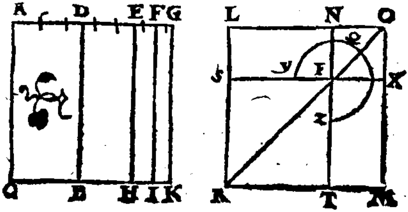
Theorema 69. Propoſitio 94.

Si ſuperficies contineatur ex linea rationali & reſiduo quarto linea q̃ illã ſuperficiem poteſt, eſt linea minor.



Theorema 70. Propoſitio 95.

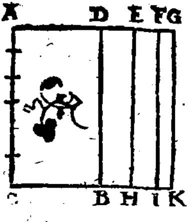
Si ſuperficies contineatur ex linea rationali & reſiduo quinto,linea quæ illã ſuperficiẽ poteſt,eſt ea quæ dicitur cum rationali ſuperficie faciens totam medialem.

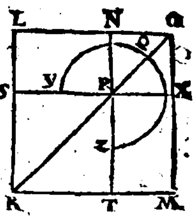


Theorema 71. Propopoſitio 96.

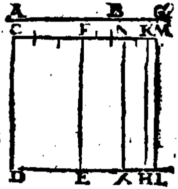
Si ſuperficies contineatur ex linea rationali &

& residuo sexto, linea quæ illam superficiē pōt, est ea quę dicitur faciēs cum mediali superficie totam medialem.



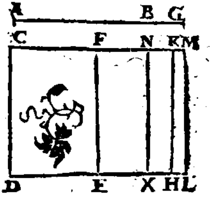


Theorema 72. Propositio 97.



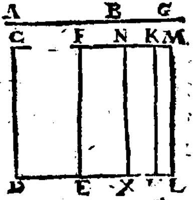
Quadratum residui secundum lineam rationalem applicatum, facit alterum latus redsiuum primum.

Theorema 73. Propositio 98.



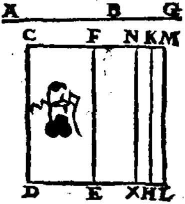
Quadratum residui medialis primi secundū rationalem applicatū, facit alterum latus residuū secundum.

Theorema 74. Propositio 99.



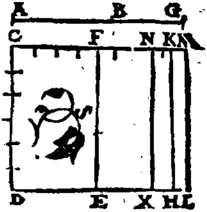
Quadratū, residui medialis secundi secundum rationalē applicatū, facit alterū latus residuum tertium.

Theorema 75. Propoſitio 100.



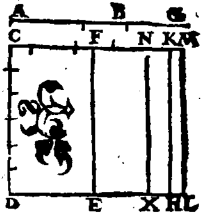
Quadratũ lineæ minóris ſecundùm rationalẽ applicatum, facit alterũ latus reſiduum quartũ.

Theorema 76. Propoſitio 101.



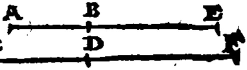
Quadratũ lineæ cum rationali ſuperficie faciẽtis totam medialem, ſecũdum rationalem applicatum, facit alterum latus reſiduum quintum.

Theorema 77. Propoſitio 102.



Quadratum lineæ cum mediali ſuperficie faciẽtis totam medialem, ſecundum rationalem applicatum, facit alterum latus reſiduum ſextum.

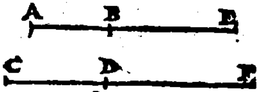
Theorema 78. Propoſitio 103.



Linea reſiduo commenſurabilis longitudine, eſt & ipſa reſiduum, & eiuſdem ordinis

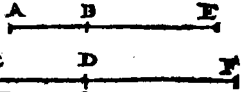
Theorema 79. Propoſitio 104.

Linea cõmenſurabilis reſiduo mediali, eſt & ipſa reſiduũ mediale, & eiuſdẽ ordinis.



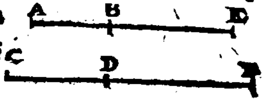
Theorema 80. Propoſitio 105.

Linea cõmẽſurabilis lineę minori, eſt et ipſa linea minor



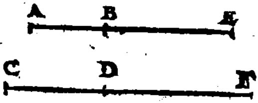
Theorema 81. Propoſitio 106.

Linea commenſurabilis lineæ cũ rationali ſuperficie facienti totã medialẽ, eſt & ipſa linea cum rationali ſuperficie faciens totam medialem.



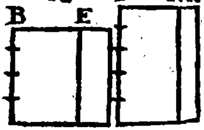
Theorema 82. Propoſitio 107.

Linea cõmẽſurabilis lineæ cũ mediali ſuperficie, facienti totam medialem, eſt & ipſa cum mediali ſuperficie faciens totam medialem.



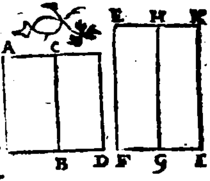
Theorema 83. Propoſitio 108.

Si de ſuderficie rationali detrahatur ſuperficies medialis, linea quæ reliquã ſuperficiem poteſt, eſt alterutra ex duabus irrationalibus, aut reſiduum, aut liuea minor.



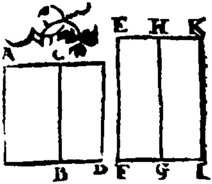
Theorema 84. Propositio 109

Si de superficie mediali detrahatur superficies rationalis, aliæ duæ irrationales, fiũt, aut residuum mediale primũ aut cũ rationali superficiem faciens totam medialem.



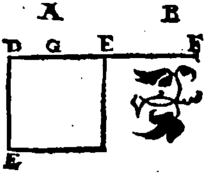
Theorema 85. Propositio 110.

Si de superficie mediali detrahatur superficies medialis q̃ sit incommẽsurabilis toti, reliquę duæ fiunt irrationales, aut residuum mediale secundum, aut cũ mediali superficie faciens totam medialem.



Theorema 86. Propositio 111.

Linea quæ Residuum dicitur, nõ est eadem cum ea quæ dicitur Binomium.



SCHO-

SCHOLIVM.

Linea quæ Residuum dicitur, & cætera quinque eã consequentes irrationales, neque lineæ mediali neque sibi ipsa inter se sunt eadem. Nam quadratum lineæ medialis secundum rationalem applicatum facit alterum latus, rationalem lineam longitudine incommensurabilem ei, secundum quam applicatur, per 23.

Quadratum, verò residui secundum rationalem applicatũ, facit alterũ latus residuũ primũ, per 97.

Quadratum verò residui medialis primi secundum rationalem applicatum, facit alterum latus residuum secundum, per 98.

Quadratum verò residui medialis secundi, facit alterum latus residuum tertium, per 99.

Quadratum verò lineæ minoris, facit alterũ latus residuum quartum, per 100.

Quadratum verò lineæ cum rationali superficie facientis totam medialem, facit alterum latus residuum quintum, per 101.

Quadratum verò lineæ cum mediali superficie facientis totam medialem, secundum rationalem applicatum, facit alterum latus residuum sextum, per 102.

Cùm igitur dicta latera, quæ sunt latitudines cuiusque parallelogrammi vnicuique quadrato æqualis & secundum rationalem, applicati, differant & à primo latere, & ipsa inter se (nam à primo differunt: quoniam sunt resi-

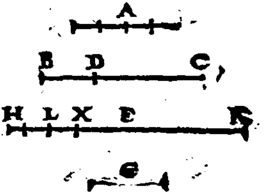
dua non eiusdem ordinis) constat ipsas quoque, lineas irrationales inter se differentes esse. Et *quoniam demonstratum est, Residuum non esse idem quod Binomium, quadrata autem residui & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex residuis eiusdem ordinis cuius sunt & residua, quorum quadrata applicantur rationali: similiter & quadrata Binomij, & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex Binomijs eiusdem ordinis, cuius sunt & Binomia, quorum quadrata applicatur rationali. Ergo lineæ irrationales quæ cõsequuntur Binomium, & quæ consequuntur residuũ, sunt inter se differẽtes. Quare dictæ lineæ omnes irrationales sunt numero 13.*

1 Medialis.
2 *Binomium.*
3 *Bimediale primum.*
4 *Bimediale secundum.*
5 Maior.
6 Potens *rationale & mediale.*
7 Potẽs *duo medialia.*
8 Residuum.
9 Residuum *mediale.*
primum.
10 *Residuum mediale secundum.*
11 Minor.
12 Faciens *cum rationali superficie totam medialem*
13 Faciens *cum mediali superficie totam medialem.*

Theo-

Theorema 87. Propositio 112.

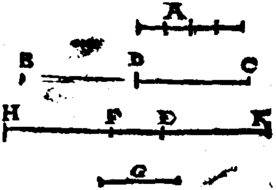
Quadratũ lineæ rationalis ſecundum Binomiũ applicatũ, facit alterum latus reſiduũ, cuius nomina ſunt cõmenſurabilia Binomij nominibus, & in eadẽ proportione



præterea id quod fit Reſiduum, eundem ordinem retinet quem Binomium.

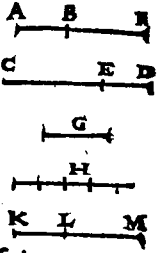
Theorema 88. Propoſitio 113.

Quadratũ lineæ rationalis ſecundum reſiduum applicatũ, facit alterum latus Binomium, cuius nomina ſunt commenſurabilia nominibus reſidui & in eadẽ proportione: preterea id quod fit Binomiũ, eſt eiuſdem ordinis, cuius & Reſiduum.



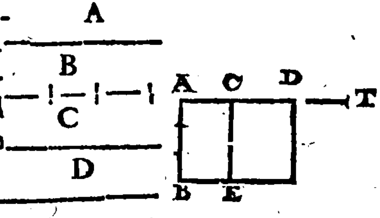
Theorema 89 Propoſitio 114.

Si parallelogrammum contineatur ex reſiduo & Binomio, cuius nomina sunt commensurabilia nominibus residui & in eadē proportiōne, linea quæ illiam superficiem potest, est rationalis,



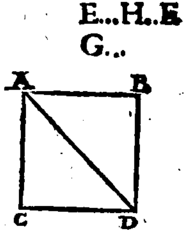
Theorema 90. Propositio 115.

Ex linea mediali nascūtur lineę irrationales innumerabiles, quarum nulla vlli ante dictarum eadem sit.



Theorema 100. Propositio 116.

Propositū nobis esto demonstrare in figuris quadratis diametrū esse longitudine incommensurabilem ipsi lateri.



ELEMENTI X. FINIS.

EVCLIDIS ELEMENTVM VNDECIMVM, ET SOLIDORVM.

prim m.

DEFINITIONES.

1

Solidum, est quod longitudinem, latitu-nem, & crassitudinem habet.

2

Solidi autem extremum est superficies.

3.

Linea recta est ad planum recta, cùm ad rectas oẽs lineas, à quibus illa tangitur, quæque in propositio sunt plano, rectos angulos efficit.

4

Planum ad planum rectum est, cum rectæ lineæ, quæ cõmuni planorum sectioni ad rectos angulos in vno planorum ducũtur, alteri plano ad rectos sunt angulos.

5

Rectæ lineę ad planũ inclinatio acutus est angulus, ipsa insistente linea & adiũcta altera cõprehẽsus, cùm à sublimi rectæ illius us lineę termino deducta fuerit ꝑpendicularis,

laris,atq; à puncto quod perpendicularis in ipso plano fecerit,ad propositę illius lineæ extremum, quod in eodem est plano, altera recta linea fuerit adiuncta.

6

Plani ad planũ inclinatio, acutus est angulus rectis lineis contentus, quæ in vtroque planorum ad idem communis sectionis pũctũ ductæ, rectos ipsi sectioni angulos efficiunt.

7

Planum similiter inclinatum esse ad planum,atq; alterum ad alterum dicitur,cum dicti inclinationum anguli inter se sunt ęquales.

8

Parallela plana,sunt quæ eodem non incidunt,nec concurrunt.

9

Similes figuræ solidæ, sunt quæ æqualibus planis,multitudine æqualibus continetur.

10

Aequales & similes figuræ solidæ sunt, quę similibus planis, multitudine & magnitudine æqualibus continentur.

11

Solidus angulus,est plurium quàm duarũ linearum, quæ se mutuò contingant, nec in eadem sint superficie, ad omnes lineas inclinatio. Aliter.

Aliter.

Solidus angulus, eſt qui pluribus quã duobus planis angulis, in eodem non cõſiſtentibus plano, ſe ad vnum punctum collectis continetur.

12

Pyramis, eſt figura ſolida quæ planis continetur, ab vno plano ad vnum punctũ collecta.

13

Priſma, figura eſt ſolida quę planis continetur, quorum aduerſa duo ſunt & æqualia & ſimilia & parallela, alia verò parallelogramma.

14

Sphęra eſt figura, quæ cõuerſo circum quieſcentem diametrum ſemicirculo continetur, cùm in eundem rurſus locũ reſtitutus fuerit, vnde moueri cœperat.

15

Axis autem ſphæræ, eſt quieſcẽs illa linea circum quam ſemicirculus conuertitur.

16

Centrum verò Sphęrę eſt idem, quod & ſemicirculi.

17

Diameter autem Sphæræ, eſt recta quædã linea per centrum ducta, & vtrinq; à ſphęra ſuperficie terminata.

Conus

18

Cωnus est figura, quæ conuerso circũ quiescens alterum latus eorum quæ rectum angulum continent, orthogωnio triangulo continetur, cùm in eundem rursus locum illud triangulum restitutum fuerit, vnde moueri cœperat. Atq; si quiescens recta linea æqualis fit alteri, quę circum rectũ angulum conuertitur, rectangulus erit Cωnus: sin minor, amblygωnius: si verò maior, oxygonius.

19

Axis autem Cωni, est quiescens illa linea circum quam triangulum vertitur.

20

Basis verò Cωni, circulus est, qui à circũducta linea recta describitur.

21

Cylindrus figura est, quæ conuerso circum quiescens alterum latus eorum quæ rectum angulum continent, parallelogrammo orthogωnio comprehenditur, cum in eũdem rursus locum restitutum fuerit illud parallelogrammũ, vnde moueri cœperat.

22

Axis autem Cylindri est quiescens illa recta linea, circum quam parallelogrammum vertitur.

23

Bases ver cylindri, sunt circuli à duobus ad-

aduerſus lateribus quæ circum aguntur, deſcripti.

24

Similes coni & cylindri, ſunt quorũ & axes & baſium diametri proportionales ſunt.

25

Cubus eſt figura ſolida, quæ ſex quadratis æqualibus continetur.

26

Tetraedum eſt figura, quæ triangulis quatuor æqualibus & æquilateris continetur.

27

Octaedrũ figura eſt ſolida, quæ octo triangulis æquilibus & æquilateris continetur.

28

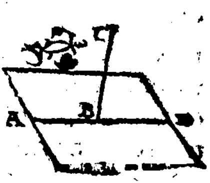
Dedecaedrum figura eſt ſolida, quæ duodelim pentagonis æqualibus, æquilateris & æquiangulis continetur.

29

Eicoſaedrum figura eſt ſolida, quę triangulis viginti æqualibus & æquilateris continetur.

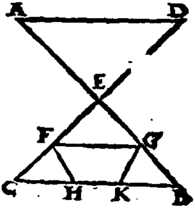
Theorema I. propoſitio I.

Quædã rectę lineę pars in ſubiecto quidem non eſt plano, quædam verò in ſublimi.



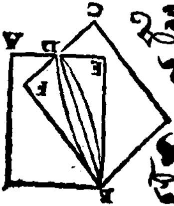
Theo

Tehorema 2. Propoſitio 2.



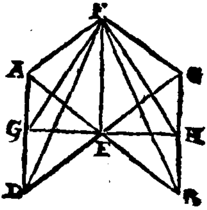
Si duę rectę lineę ſe mutuò ſecẽt, in vno sũt plano: atque triangulum omne in vno eſt plano.

Theorema 3. Propoſitio 3.



Si duo plana ſe mutuò ſecent, communis eorũ ſectio eſt recta linea.

Theorema 4 Propoſitio 4.



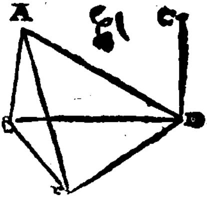
Si recta linea rectis duabus lineis ſe mutuò ſecantibus, in cõmuni ſectione ad rectos angulos inſiſtat illa ducto etiam per ipſas plano ad angulos rectos erit.

Theorema 5. Propoſitio 5.

Si recta linea rectis tribus lineis ſe mutuò tangentibus, in communi ſectione ad rectos angulos inſiſtat, illæ tres rectę in vno ſunt plano.

Theo.

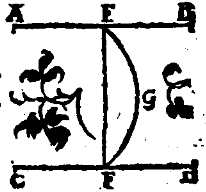
Theorema 6. Propositio 6.



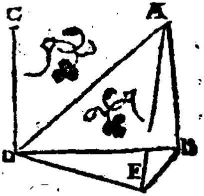
Si duæ rectæ lineæ eidẽ plano ad rectos sint angulos, parallelæ erunt illæ rectæ lineæ

Theorema 7. Propositio 7.

Si duæ sint parallelæ rectæ lineæ, in quarũ vtraque sumpta sint quę libet pũcta, illa linea quę ad hæc puncta adiungitur, in eodem est cũ parallelis plano.

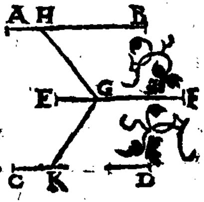


Theorema 8. Propositio 8.



Si duæ sint parallelæ rectæ lineæ, quarum altera ad rectos cuidam plano sit angulos, & reliqua eidẽ plano ad rectos angulos erit

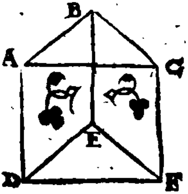
Theorema 9. Prositio 9.



Quæ eidem rectæ lineæ sunt parallelæ, sed nõ in eodem cũ illa plano, hæ quoque sunt inter se parallelæ.

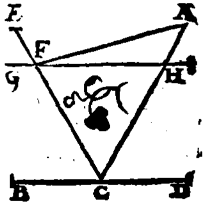
Theorema 10. Propoſitio 10.

Si duę rectę lineæ ſe mutuò tangẽtes ad duas rectas ſe mutuò tangentes ſint parallelę, non autem in eodem plano, illę angulos æquales comprehendent.



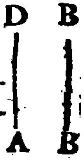
Proble. 1. Propoſitio 11.

A dato ſublimi puncto, in ſubiectum planũ perpendicularem rectam lineam ducere.



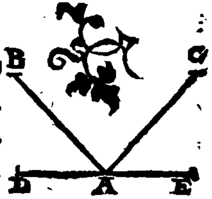
Problema 2. Propoſitio 12.

Dato plano, à pũcto quod in illo datum eſt, ad rectos angulos rectam lineam excitare.



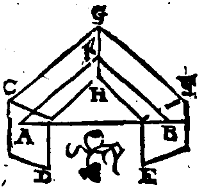
Theorema 11. Propoſitio 13.

Dato plano, à puncto qd in illo datum eſt, duæ rectæ lineæ ad rectos angulos non excitabuntur ad eaſdem partes.



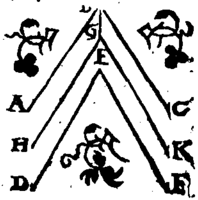
Theo-

Theorema 12. Propo-
ſitio 14.



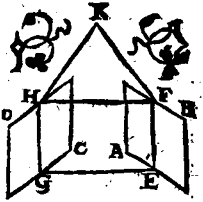
Ad quæ plana, eadem re-
cta linea recta eſt, illa sũt
parallela.

Theorema 13. Propoſitio 15.



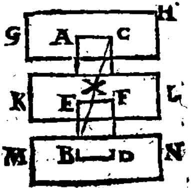
Si duæ rectæ lineæ ſe mu-
tuò tangentes ad duas re-
ctas ſe mutuò tangentes
ſint parallelæ, non in eo-
dem conſiſtentes plano,
parallela ſunt quæ per il-
las ducuntur plana.

Theorema 14. Propo-
ſitio 16.



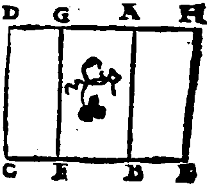
Si duo plana parallela
plano quopiã ſecentur,
communes illorum ſe-
ctiones ſunt parallelæ.

Theorema 15. Propo-
ſitio 17.



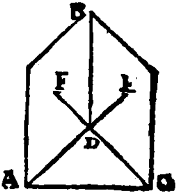
Si duæ rectæ lineę paral-
lelis planis ſecẽtur, in eaſ-
dem rationes ſecabũtur.

Theorema 16. Propositio 18.



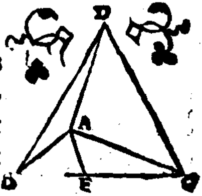
Si recta linea plano cuipiam ad rectos sit angulos, illa etiam omnia quę per ipsam plana, ad rectos eidem plano angulos erunt.

Theorema 17. Propositio 19



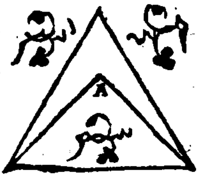
Si duo plana se mutuò secantia plano cuidã ad rectos sint angulos, communis etiam illorum sectio ad rectos eidem plano angulos erit.

Theorema 18. Propositio 20.



Si angulus solidus planis tribus angulis contineatur, ex his duo quilibet vtut assumpti tertio sunt maiores

Theorema 19. Propositio 21.

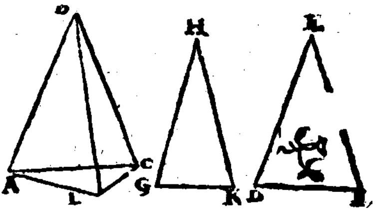


Solidus omnis angulus minoribus continetur, quàm rectis quatuor angulis planis.

Theo-

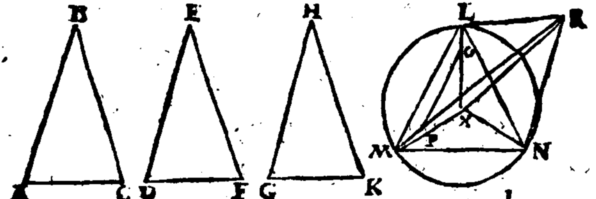
Theorema 20 Proopositio 22.

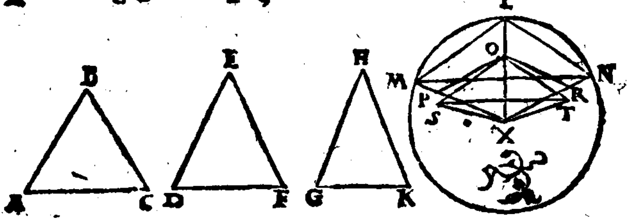
Si plani tres anguli ęqualibus rectis contineãtur lineis, quorũ duo vt libet assumpti, tertio sint maiores, triangulum constitui potest ex lineis æquales, illas rectas coniungentibus.



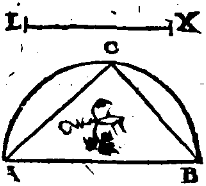
Problema 5. Propositio 23.

Ex planis tribus angulis, quorũ duo vt libet assumpti tertio sint maiores, solidũ angulum constituere. Decet autem illos tres angulos rectis quatuor esse minores.

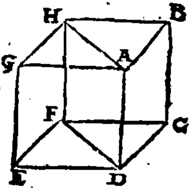




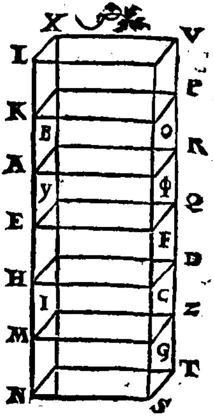
Theorema 21. Propo-
sitio 24.



Si solidum parallelis planis contineatur, aduersa illius plana & æqualia sunt & paralle logramma.



Theorema 22. Propo-
sitio 25.

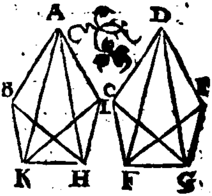


Si solidum parallelis planis contentũ plano secetur aduersis planis parallelo, erit quemadmodum basis ad basim, ita solidum ad solidum.

Proble-

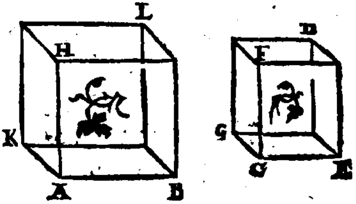
Problema 4. Propoſitio 26.

Ad datam rectam lineã eiuſque punctum, angulum ſolidum conſtituere: ſolido angulo dato æqualem.



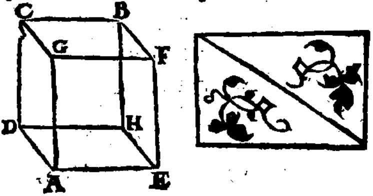
Problema 5. Propoſitio 27.

A data recta, dato ſolido parallelis planis comprehenſio ſimile & ſimiliter poſitum ſolidũ paralle lis planis cõtẽtum deſcribere.

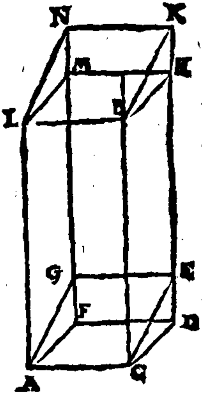


Theorema 23. Propoſitio 28.

Si ſolidũ parallelis planis comprehenſum ductor per aduerſorum planorum diagonios plano ſectũ ſit, illud ſolidum ab hoc plano biſariam ſecabitur.



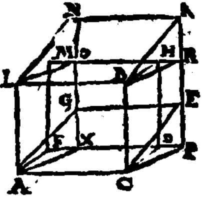
Theorema 34. Propositio 29.



Solida parallelis planis comprehēsa, quę super eandem basim. & in eadē sunt altitudine, quorum insistētes lineæ in ijsdem collocantur rectis lineis, illa sūt inter se æqualia.

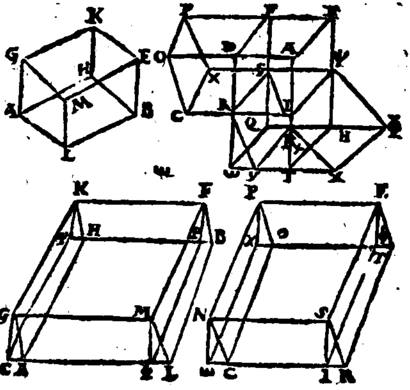
Theorema 25. Propositio 30.

Solida parallelis planis circumscripta, quæ super eandē basim & in eadē sūt altitudine, quorum insistentes lineæ nō in ijsdem reperiuntur rectis lineis, illa sunt inter se æqualia.



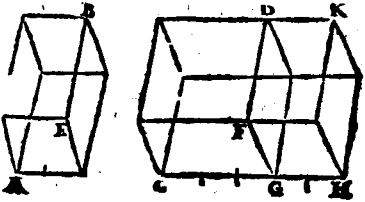
Theorema 26. Propositio 31.

Solida parallelis planis circunscripta, quæ in in eadem sũt altitudine, æqualia sũt inter se.



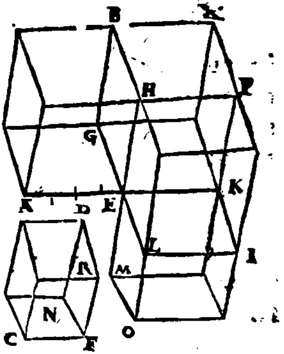
Theorema 27. Propositio 32.

Solida parallelis planis circunſcripta quæ eiuſdem ſunt altitudinis, eam habẽt inter ſe rationẽ, quam baſes.



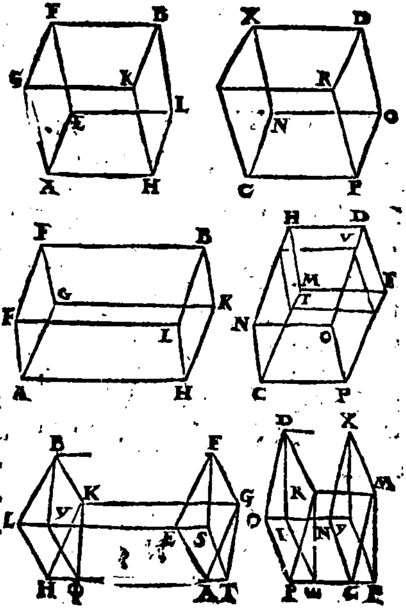
Theor 28. Propositio 33.

Similia solida parallelis, planis circũscripte habẽt inter se rationẽ homologorũ laterum triplicatam



Theor. 29. Propositio 24

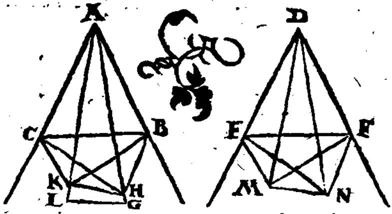
Aequalium solidorũ parallelis planis cõtẽtorum bases cũ altitudinibus reciprocantur. Et solida parallelis planis contenta, quorũ



baſes cum altitudinibus reciprocantur, illa ſunt æqualia.

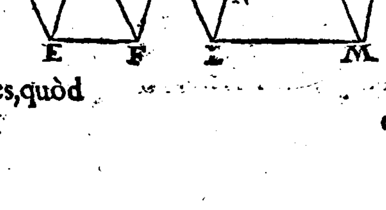
Theorema 30. Propoſitio 35.

Si duo plani ſint anguli æquales, quorum verticibus ſublimes rectæ lineæ inſiſtant, quæ cum lineis primò poſitis angulos contineant æquales, vtrunq; vtriq;, in ſublimibus autem lieneis quęlibet ſumpta ſint puncta, & ab his ad plana in quibus conſiſtunt anguli primum poſiti, ductæ ſint perpendiculares, ab earum verò punctis, quæ in planis ſignata fuerint, ad angulos primum poſitos ad iũctæ ſint rectæ lineę, hæ cũ ſublimibus æquales angulos comprehendent.



Theorema 31. Propoſitio 36.

Si rectæ tres lineæ ſint proportionales, quòd

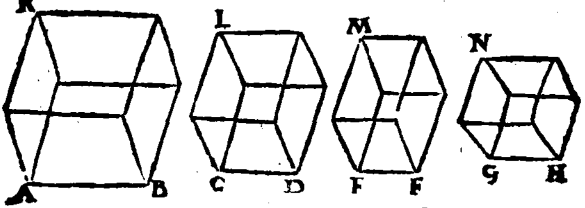


ex

ex his tribus fit solidum parallelis planis contentum, æqnale est descripto à media linea solido parallelis planis comprehẽso, quod æquilaterum quidem sit, sed antedicto æquiangulum.

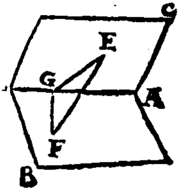
Theorema 32. Propositio 37.

Si rectæ quatuor lineæ sint proportionales, illa quoque solida parallelis planis contenta, quæ ab ipsis lineis & similia & similiter discribuntur, proportionalia erunt. Et si solida parallelis planis comprehensa, quę & similia & similiter describuntur, sint proportionalia, illæ quoque rectæ lineæ proportionales erunt.



Theorema 33. Propositio 38.

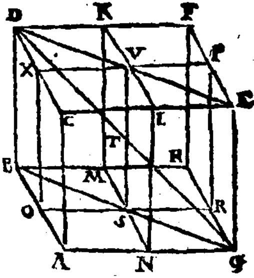
Si planum ad planum rectũ sit, & à quodã puncto eorũ quæ in vno sunt planorũ perpendicularis ad alterum ducta sit, illa quæ ducitur perpẽdicularis, in communẽ cadet planorũ sectionẽ.



Theo-

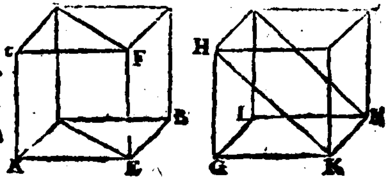
Theorema 34. Propositio 39.

Si in solido parallelis planis circunscripto, aduersorum planorum lateribus bifariam sectis, educta sint per sectiones plana, communis illa planorũ sectio & solidi paralle- lis plani circumscripti diameter, se mutuò bifariã secant.



Theorema 35. Propositio 40.

Si duo sint æqualis altitudinis prismata, quorũ hoc quidem basim habeat parallelogrammũ, illud verò triangulum, sit autem parallelogrammũ triãguli duplũ, illa prismata erunt æqualia.



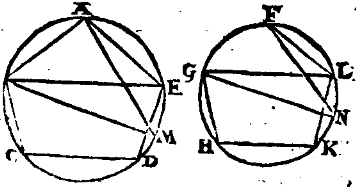
ELEMENTI XI. FINIS.

EVCLIDIS ELEMENTVM DVODECIMVM.

ET SOLIDORVM. ſecundum.

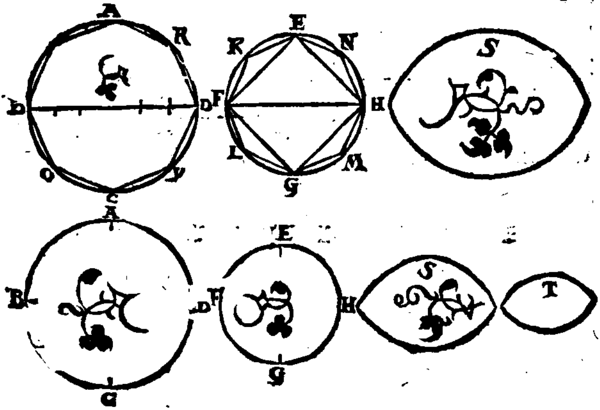
Theorema 1. Propoſitio 1.

Similia quæ ſunt in circulis polygona, rationẽ habent inter ſe, quam deſcripta à diametris quadrata.



Theorema 2. Propoſitio 2.

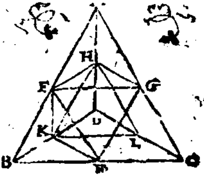
Circuli eam inter ſe rationem habent, quã



descripta à diametris quadrata.

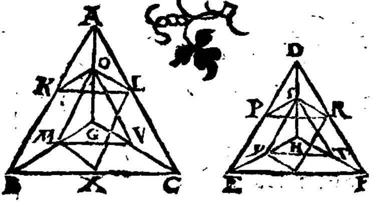
Tueorema 3. Propositio 3.

Omnis pyramis trigωnam habens basim, in duas diuiditur pyramidas non tantũ æquales & similes inter se, sed toti etiam pyramidi similes, quarum trigωnæ sunt bases, atq; in duo prismata ęqualia, quæ duo prismata dimidio pyramidis totius sũt maiora.



Theorema 4. Propositio 4.

Si duæ eiusdem altitudinis pyramides trigωnas habeãt bases, sit aũt illarum vtraque diuisa & in duas pyramidas inter se ęquales totiq; similes, & in duo prisma æqualia, ac eodem modo diuidatur vtraq; pyramidum quæ ex superiore diuisione natæ sunt, idque perpetuò fiat: quemadmodũ se habet vnius pyramidis basis ad alterius pyrãidis basim, ita & omnia quę in vna pyra-

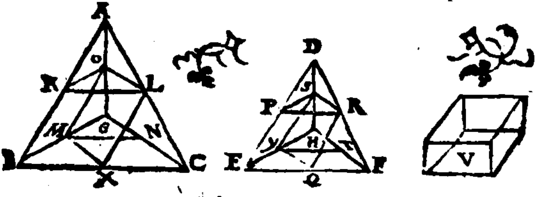


mide

mide priſmata, ad omnia quæ in altera pyramide, priſmata multitudine æqualia.

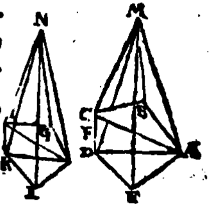
Theorema 5. Propoſitio 5.

Pyramides eiuſdem altitudinis, quarũ trigωnæ ſunt baſes, eam inter ſe rationem habent, quàm ipſæ baſes.



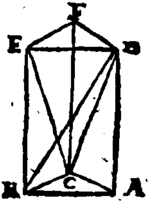
Theorema 6. Propoſitio 6.

Pyramides eiuſdẽ altitudinis, quarum polygωnæ ſunt baſes, eam inter ſe rationem habent, quã ipſæ baſes.



Theorema 7. Propoſitio 7.

Omne priſma trigωnã habẽs baſim, diuiditur in tres pyramides inter ſe æquales, quarum trigωnæ ſunt baſes



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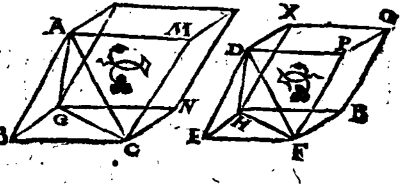
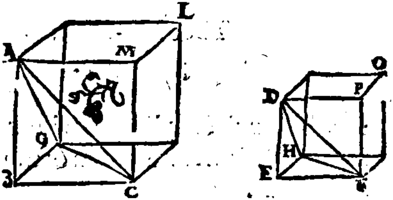
Theorema 8. Propositio 8.

Similes pyramides qui trigωnas habent bases, in triplicata sũt homologorũ laterũ ratiõe



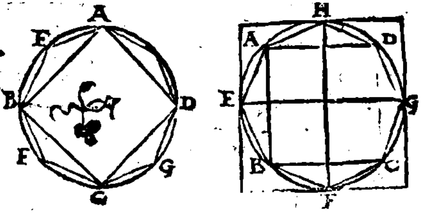
Theorema 9. Propositio 9.

Aequaliũ pyramidum & trigωnas bases habentium reciprocantur bases cum altitudinibus. Et quarũ pyramidum trigωnas bases habentium reciprocãtur bases cũ altitudinibus, illæ sunt æquales.



Thorema 10. Propositio 10.

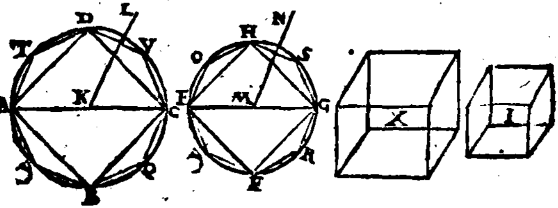
Omnis cωnus tertia pars est cylindri eandẽ



cum ipso cono basim habentis, & altitudinem æqualem.

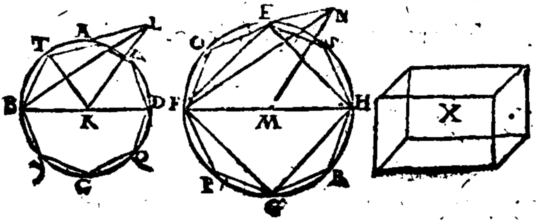
Theorema 11. Propositio 11.

Coni & cylindri eiusdem altitudinis, eam inter se rationem habent, quam bases.



Theorema 12. Propositio 12.

Similes coni & cylindri, triplicatã habent inter se rationem diametrorum, quæ sunt in basibus.



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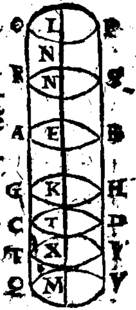
Theorema 13. Propositio 13.



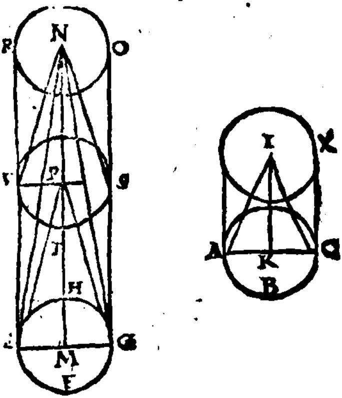
Si cylindrus plano sectus sit aduersis planis parallelo, erit quemadmodum cylindrus ad cylindrum, ita axis ad axem.

Theorema 14. Propositio 14.

Coni & cylindri qui in æqualibus sunt basibus, eam habent inter se rationem, quam altitudines

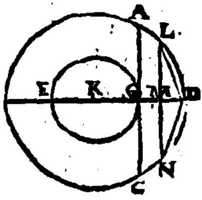
Theorema 15. Propositio 15.

Aequalium conorum & cylindrorũ bases cũ altitudinib' reciprocãtur. Et quorum conorũ & cylindrorum bases cum altitudinibus reciprocantur, illi sunt æquales.



Theoroma 1. Propositio 16.

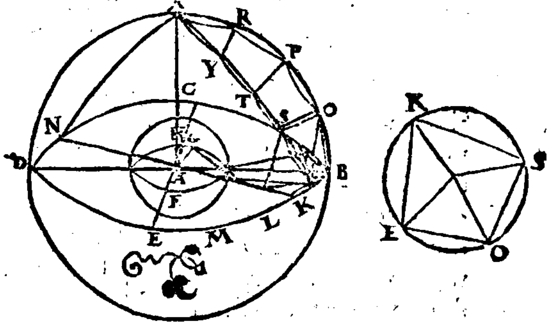
Duobus circulis circũ idem centrum consistentibus, in maiore circulo polygonum æqualium pariumque laterum inscribere, quod minorem circulũ non tangat.



Pro-

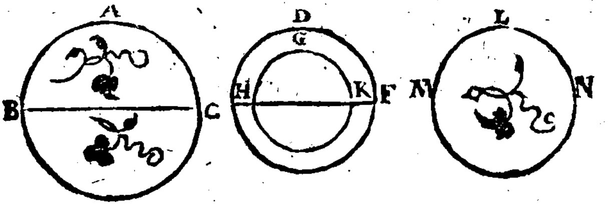
Problema 2. Propoſitio 17.

Duabus ſphæris circum idem centrum conſiſtentibus, in maiorem ſphęra ſolidum polyedrum inſcribere, quod minoris ſphæræ ſuperficiem non tangat.



Theorema 16. Propoſitio 18.

Sphæræ inter ſe rationem habent ſuarum diametrorum triplicatam.

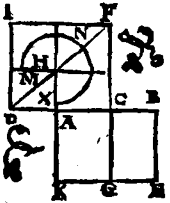


ELEMENTI XII. FINIS.

EVCLIDIS ELEMENTVM DECIMVMTERTIVM, ET SOLIDORVM TERTIVM.

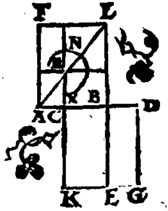
Theorema 1. Propositio 1.

Si recta linea per extremá & mediá rationem secta sit, maius segmentum quod totius lineæ dimidium assumpserit, quintuplum potest eius quadrati, quod à totius dimidia describitur.



Theorema 2. Propositio 2.

Si recta linea sui ipsius segmenti quintuplum possit, & dupla segmenti huius linea per extremá & mediam rationé secetur maius segmentũ reliqua pars est lineæ primum positæ.



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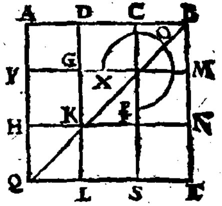
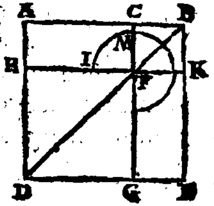
Theorema 3. Propoſitio 3.



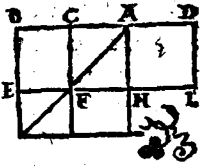
Si recta linea per extremā & mediā rationē ſecta ſit, minus ſegmētū quod maioris ſegmenti dimidium aſſumpſerit, quintuplum poteſt eius, quòd à maioris ſegmēti dimidio deſcribitur, quadrati.

Theorema 4. Propoſitio 4.



Si recta linea per extremam & mediam rationē ſecta ſit, quod à tota, quodque à minore ſegmēto ſimul vtraq; quadrata, tripla ſunt eius, quod à maiore ſegmēto deſcribitur, quadrati.

Theorema 5. Propoſitio 5.



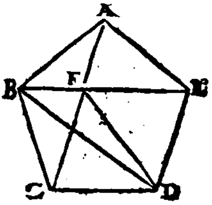
Si ad rectam lineam, quæ per extremam & mediam rationem ſecetur, adiuncta ſit altera ſegmento maiori equalis, tota hæc linea recta per extremam & mediam rationē ſecta est, estque maius segmentum linea primùm posita.

Theorema 6. Propositio 6.

Si recta linea ῥητὴ siue rationalis, per extremam & mediam rationem secta sit, vtrunque segmentorum ἄλογος siue irrationalis est linea, quæ dicitur Residuum.

Theorema 7. Propositio 7.

Si pentagωni æquilateri tres sint æquales anguli, siue quã deinceps siue qui non deinceps sequuntur, illud pentagωnum erit æquiangulum.



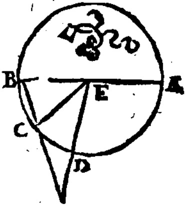
Theorema 8. Propositio 8.

Si pentagωni æquilateri & æquianguli duos qui deinceps sequuntur angulos rectæ subtendãt lineæ, illæ per extremã & mediam rationem se mutuò secant, earumq; maiora segmenta, ipsius pentagωni lateri sunt æqualia

Theo-

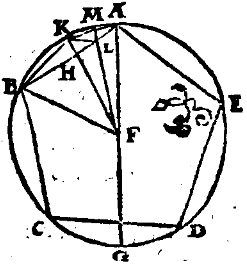
Theorema 9. Propoſitio 9.

Si latus hexagωni & latus decagωni eidem circulo inſcriptorum compoſita ſint, tota recta linea per extremam & mediam rationem ſecta eſt, eiuſque ſegmētum maius, eſt hexagωni latus.



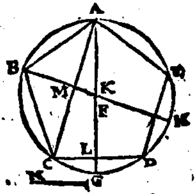
Theorema 10. Propoſitio 10.

Si circulo pētagωnum æquilaterum inſcriptum ſit, pentagωini lat⁹ poteſt & latus hexagωni & latus decagωni eidem circulo inſcriptorum.



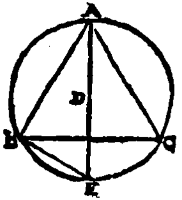
Therema 11. Propoſitio 11.

Si in circulo ῥητὴν habente diametrum, inſcriptum ſit pentagωnum æquilaterum, pentagωni latus irrationalis eſt linea quæ vocatur Minor.



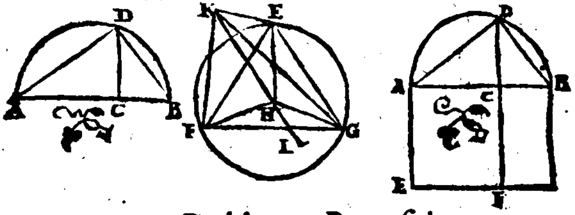
Theorema 12. Propoſitio 12.

Si in circulo inſcriptum ſit triangulum æquilaterum, huius trianguli latus potentia triplum eſt eius lineæ, quæ ex circuli centro ducitur.



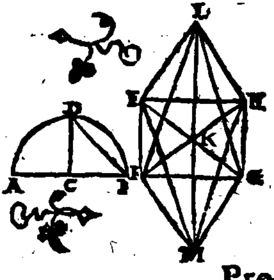
Problema 1. Propoſitio 13.

Pyramidem conſtituere. & data ſphærę cō-plecti, atque docere illius ſphæræ diametrum potentia ſeſquialteram eſſe lateris ipſius pyramidis.



Problema 2. Propoſitio 14.

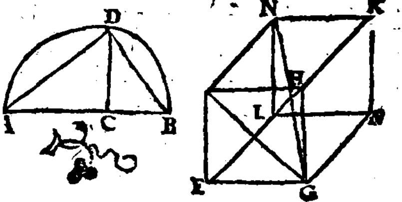
Octaedrum conſtituere, eaq́; ſphę ra qua pyramidē complecti, atque ꝓbare illius ſphæræ diametrū potentia duplā eſſe lateris ipſi' octaedri.



Pro-

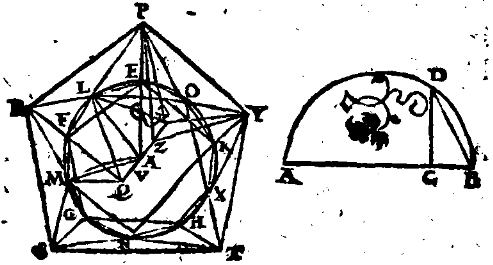
Problema 3. Propositio 15.

Cubum constituere, eaque sphæra qua & superiores figuras complecti, atque docere illius sphæræ diametrum potẽtia triplã esse lateris ipsius cubi.

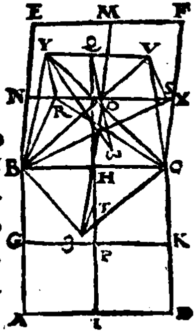


Problema 4. Propositio 16.

Icosaedrum constituere, eademque sphæra qua & antedictas figuras complecti, atque probare, Icosaedri latus irrationalem esse lineam, quæ vocatur Minor.

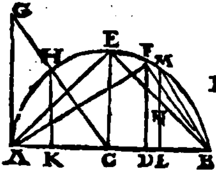


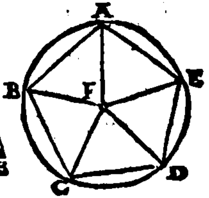
Problema 5. Propositio 17.



Dodecaedrũ constituere, eademque sphæra qua & & antedictas fuguras cõplecti, atque probare dodecaedri latus irrationalem esse lineam, quæ vocatur Residuum.

Theorema 6. Propositio 18.





quinq; figurarum latera proponere, & inter se comparare.

SCHOLIVM.

Aio verò, præter dictas quinque figuras non posse aliam constitui figuram solidã, quæ planis & æquilateris & æquiangulis cõtineatur, inter se æqualibus. Non enim ex duobus triãgulis, sed neq; ex aliis duab' figuris solidus cõstituetur angulus

Sed

Sed ex tribus triãgulis, constat Pyramidis angulus

Ex quatuor autem, Octaëdri.

Ex quinque verò, Icosaëdri.

Nam ex triangulis, sex & æquilateris & æquiangulis ad idem punctum coeuntibus, non fiet angulus solidus. Cùm enim trianguli æquilateri angulus, recti vnius bessem contineat, erunt eiusmodi sex anguli rectis quatuor æquales. Quod fieri nõ potest. Nam solidus omnis angulus minoribus quã rectis quatuor angulis continetur, per 21.11.

Ob easdem sanè causas, neque ex pluribus quã planis sex eiusmdoi angulis solidus constat.

Sed ex tribus quadratis, Cubi angulus continetur.

Ex quinque, nullus potest. Rursus enim recti quatuor erunt.

Ex tribus autẽ pentangonis æquilateris & æquiangulis, Dodecaëdri angulus continetur.

Sed ex quatuor, nullus potest. Cùm enim pentagoni æquilateri angulus rectus sit, et quinta recti pars erunt quatuor anguli rectis quatuor maiores. Quod fieri nequit. Nec sanè ex alijs polygonis figuris solidus angulus continebitur, quòd hinc quoque absurdum sequatur. Quamobrem perspicuum est; præter dictas quinque figuras aliã figuram solidam non posse constitui, quæ ex planis æquilateris & æquiangulis contineatur.

ELEMENTI XIII. FINIS

EVCLIDIS ELEMENTVM DECIMVM QVARTVM, VT qui dam arbitrantur, vt alij verò, Hypsiclis Alexandrini, de quinque corporibus.

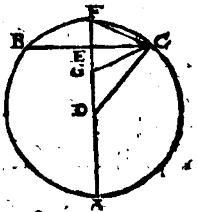
LIBER PRIMVS.

Basilides Tyrius, Protarche, Alexandriam profectus, patrique nostro ob disciplinæ societatem commendatus, longißimo peregrinationis tempore cum eo versatus est. Cùmque differerent aliquando de scripta ab Apollonio comparatione Dodecaëdri & Icosaëdri eidem sphæræ inscriptorum, quam hæc inter se habeant rationem, censuerunt ea non rectè tradidisse Apollonium: quæ à se emendata, vt de patre audire erat, literis prodiderunt. Ego autem postea incidi in alterum librum ab Apollonio editum, qui demonstrationem accuratè complecteretur de re proposita, ex eiusq́; problematis indagatione magnam equidem cepi voluptatem. Illud certè ab omnibus perspici potest, quod scripsit Apollonius, cùm sit in omnium manibus. Quod autē diligenti, quantum conijcere licet, studio nos postea scrip-

scripsisse videmur, id monimentis consignatum tibi nuncupandum duximus, vt qui feliciter cùm in omnibus disciplinis tum vel maximè in Geometria versatus, scitè ac prudẽter iudices ea quæ dicturi sumus ob eam verò, quæ tibi cum patre fuit, vitæ consuetudinem, quaq; nos complecteris, beneuolentiã, tractationẽ ipsam libenter audias. Sed iam tempus est, vt proœmio modum facientes, hanc syntaxim aggrediamur.

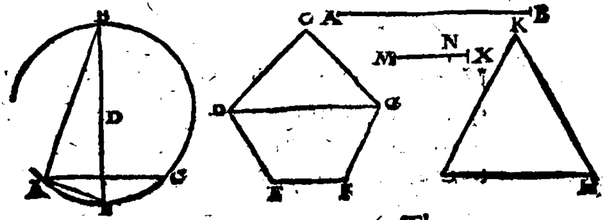
Theorema 1. Proopositio 1.

Perpendicularis linea, quæ ex circuli cuiuspiam centro in latus pentagoni ipsi circulo inscripti ducitur, dimidia est vtriusq; simul lineæ, & eius, quæ ex centro & lateris decagoni in eodẽ circulo inscripti.



Theorema 2. Propositio 2.

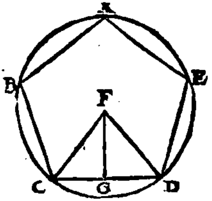
Idem circulus cõprehendit & dodecaedri pentagonum & icosaedri triãngulum, eidẽ sphæræ inscriptorum.

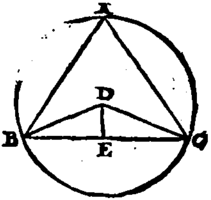


Theo-

Theorama 3. Propoſitio 3.

Si pentagωno & æquilatero & æquiangulo circumſcriptus ſit circulus ex cuius, centro in vnum pentagωni latus dicta ſit perpendicularis: quod vno laterum & perpēdiculari trigeſies cōtinetur, illud æquale eſt dωdecaedri ſuperficiei.

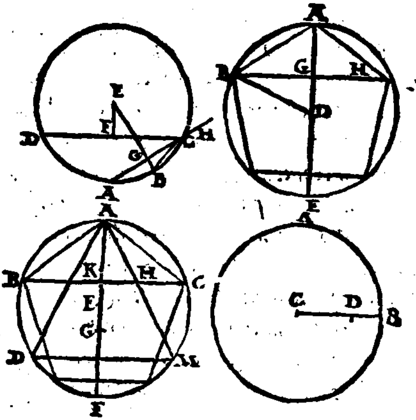




Theorema 4. Propoſitio 4.

Hoc perſpicuum cùm ſit probandum eſt, quemadmodum ſe habet dωdecaedri ſuperficies ad icoſaedri ſuperficiem ita ſe habere cubi latus ad icoſaedri latus.

Cubi



Cubilatus.

E————————

Dodecaedri.

F————————

Icosaedri.

G————————

SCHOLIVM

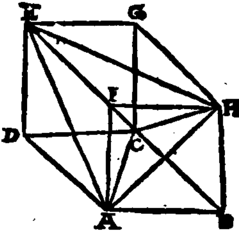
Nunc autem probandum est, quemadmodum se habet cubi latus ad icosaëdri latus, ita se habere solidum dodecaëdri ad icosaëdri solidum: Cùm enim æquales circuli comprehendat & dodecaëdri pentagonum & Icosaëdri triangulum, eidē sphæra in-

scriptorum: in sphæris autem æquales circuli æquali interuallo distent à centro (siquidem perpendiculares à sphæræ centro ad circulorum plana ductæ & æquales sunt, & ad circulorum centra cadunt) idcirco lineæ, hoc est perpendiculares, quæ à sphæræ cētro ducuntur ad centrum circuli comprehendētis et triangulum Icosaëdri, & pentagonum dodecaëdri sūt æquales. Sūt igitur æqualis altitudinis Pyramides, quæ bases habent ipsa dodecaëdri pentagona, et quæ Icosaëdri triangula. At æqualis altitudinis pyramides rationem inter se habent eam quam bases, ex 5. & 6. 11. Quemadmodum igitur pentagonū ad triangulum, ita pyramis, cuius basis quidem est dodecaëdri pentagonum, vertex autem sphæræ centrum ad pyramida, cuius basis quidem est Icosaëdri triangulum, vertex autem sphæræ centrum. Quamobrem vt se habent duodecim pentagona ad viginti triāgula, ita duodecim pyramides, quorum pētagona sint bases, ad viginti pyramidas, quæ trigonas habeant bases. At pentagona duodecim sunt dodecaëdri superficies, viginti autem triangula, Icosaëdri. Est igitur vt dodecaëdri superficies ad Icosaëdri superficiem, ita duodecim pyramides quæ pentagonas habeant bases, ad viginti pyramidas, quarum trigonæ sunt bases. Sunt autem duodecim quidem pyramides, quæ pentagonas habeant bases, solidū dodecaëdri: viginti autem pyramides, quæ trigonas habeant bases, Icosaëdri solidum. Quare ex 11. 5. vt dodecaëdri superficies ad Icosaëdri superficiem, ita soli-

solidum dodecaëdri ad Icosaëdri solidum. Vt autem dodecaëdri superficies ad Icosaëdri superficiem, ita probatum est cubi latus ad Icosaëdri latus. Quemadmodum igitur cubi latus ad Icosaëdri latus, ita se habet solidum dodecaëdri ad Icosaëdri solidum.

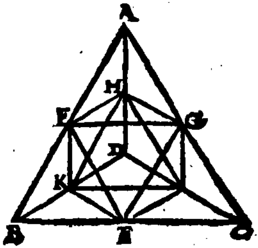
EVCLIDIS ELEMENTVM DECIMVM QVINTVM, ET Solidorum quintum, vt nonnulli putant, vt autem alij, Hypsiclis Alexandrini, de quinque corporibus.

LIBER II.



Problema 1. Propositio 1.

In dato cubo pyramida inscribere.



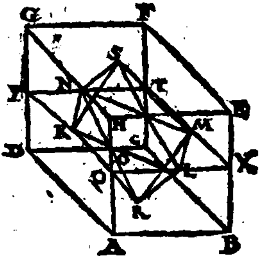
Problema 2. Propositio 2.

In data pyramide octaedrum inscribere.

Pro-

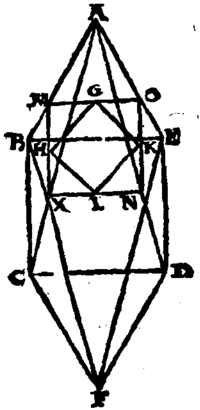
Problema 3. Propositio 3.

In dato cubo octaedrum inscribere.

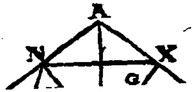


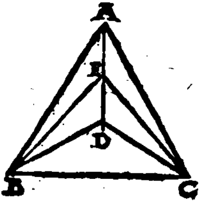
Probléma 4. Propositio 4.

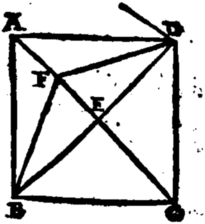
In dato octaedro cubű inscribere

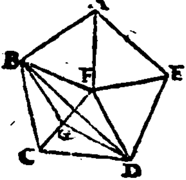


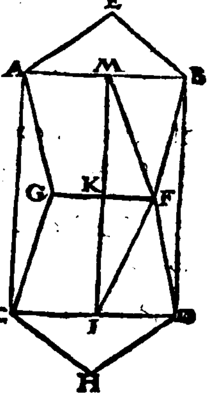
Problema 5. Propositiő 5.

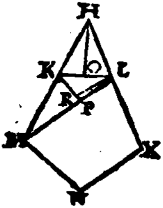












SCHO.

SCHOLIVM.

decet, si quis nos roget, quot Icosaëdrū bal ..., ita respondendum esse: Patet Icosaedrum viginti contineri triāgulis, quodlibet verò triangulum rectis tribus constare lineis. Quare multiplicanda sunt nobis viginti triangula in trianguli vnius latera, fiūtq; sexaginta, quorum dimidium est triginta. Ad eundem modum & in dodecaedro. Cū enim rursus duodecim pentagona dodecaedrum cōprehendant, itemq; pentagonum quoduis rectis quinque constet lineis, quinq; duodecies multiplicamus, ... t sexaginta, quorum rursus dimidium est triginta. Sed cur dimidiū capimus? Quoniā vnumquodq; latus siue sit trianguli, siue pentagoni, siue ... drati vt in cubo, iteratò sumitur. Similiter autem eadem via & in cubo & in pyramide & in octaedro latera in uenies. ... od si itē velis singularum quoq; figurarum angulos reperire, fa ... a eadem multiplicatione numerum procreatum partire in numerum planorū, quæ vnum solidum angulum includūt: vt quoniā triāgula quinq; vnum Icosaëdri angulum continet, partire 60. in quinque, nascūtur duodecim anguli Icosaedri. In dodecaedro autē tria pētagona angulum comprehendunt, partire ergo 60. in tria, & habebis dodecaedri an ... ti. Atq; simi-

Hyg: lib: 4

Priores astrologi cum omnes res ad XII re...
rent, ut menses et horas, et lati in dinem signi...
Itaq et signa per q res omnes significantur 12
esse voluerunt.

Macrobius annos appellat periodos singulorum
planetarum

Atlas rex Mauritaniae primus
fabricasse de sphaeram similem
celesti machinae insequitam.
Stellis: in finibus ph: lib
2 cap. 8 et 7 cap.
35

Signis sim.
motus pp
in mobi:
lis

de signorum successione sm ordinem
vel de ortu signorum inter Y et ...
8

de motu planetarum in signis sm ...
... B ... in 8 ...

Congressus siccarum stellarum in signis sicc[is] causa est siccitatis aeris, ut ♄ et ♂ in ♈ et ♌. Huius exemplum est 1540 anno, quo eclipsis ☉ in ♈ cum ☌ ♄ et ♂ in ≏ maximos aestus excitavit. Deinde etiam anno 1507 duae eclipses ☽, quarum altera accidit in ♌, altera autem in ♒, item ☌ ♃ ♄ et ☉ in ♌ causa fuit aestuum maximorum. Congressus autem stellarum humidarum in signis humidis, humiditatis causa est, ut anno 1524, ☌ ♄ et ♃ et ♀ in ♋ vel ♓ excitant humiditates

Sex hypothesibus primi cap. probat in specie ea, quae principio generaliter de Sphaera proponit

Sphaera

Hypotheses aliae de

- Caelo
  - Quod sit figurae sphaericae
  - Quod moveatur circulariter
  - Hic addi potest
  - Quod motus caeli sit uniformis et regularis
- Terra
  - De forma eius
  - de forma aquae — Quae efficiunt unum globum
  - Quod sit centrum
  - Quod sit immobilis.