EVCLIDIS ELEMENTORVM LIBRI XV.

QVIBVS, CVM AD OMNEM MATHEMATICAE SCIENTIAE PARTEM, TVM AD QVAMLIbet Geometriæ tractationem facilis comparatur aditus.

COLONIAE

Apud Gosuinum Cholinum

M. D. C X I I.

Cum Gratia & Priuilegio.

AD CANDIDVM LECTOREM ST. GRACILIS praefatio.

PERMAGNI *referre semper existimaui, lector beneuole, quantum quisque studij & diligentiæ ad percipienda scientiarum elementa adhibeat, quibus non satis cognitis, aut perperam intellectis si vel digitũ progredi tentes erroris caliginem animis offundas, non veritatis lucem rebus obscuris adferas. Sed principiorum quanta sint in disciplinis momenta, haud facilè credat, qui rerum naturam ipsa specie, non viribus metiatur. Vt enim corporum, quæ oriuntur & intereunt, vilissima tenuissimaq; videntur initia: ita rerum æternarum & admirabilium, quibus nobilissimæ artes continentur, elementa ad speciem sunt exilia, ad vires & facultatem quam maxima. Quis non videt ex fici tantulo grano, vt ait* Tullius, *aut ex acino vinaceo, aut ex cæterarum frugum aut stirpium minutissimis seminibus tantos truncos ramosq; procreari? Nam Mathematicorum initia illa quidem dictu auditúq; perexigua, quantã theorematum syluam nobis pe-*

pererunt? Ex quo intelligi poteſt, vt in ipſis ſeminibus, ſic & in artium principijs ineſſe vim earum rerum; quæ ex his progignantur. Præclarè igitur Ariſtoteles, vt alia permulta, μέγιστον ἴσως ἀρχὴ παντὸς, καὶ ὅσῳ κράτιστον τῇ δυνάμει, τοσούτῳ μικρότατον, οὐδὲ μεγέθει χαλεπόν ἐστιν ὀφθῆναι. Quocirca committendum non eſt, vt non bene prouiſa & diligenter explorata ſcientiarum principia, quibus propoſitarum quarumque rerum veritas ſit demonſtranda, vel conſtituas, vel conſtituta approbes: Cauendum etiam, vt ne tantulum quidem fallaci & captioſa interpretatione turpiter deceptus, à vera principiorum ratione temerè deflectas. Nam qui initio fortè aberrauerit, is vt tandem in maximis verſetur erroribus, neceſſe eſt: cum ex vno erroris capite denſiores ſenſim tenebræ rebus clariſſimis obducantur. Quid tam varias veterum phyſiologorum ſententias non modò cùm rerum veritate pugnantes, ſed vehementer etiam inter ſe diſſentientes nobis inuexit? Equidem haud ſcio, fuerit ne vlla potior tanti diſſidij cauſa, quam quòd ex principijs partim falſis, partim non conſentaneis ductas rationes probando adhiberent. Fit enim plerunque, vt qui non rectè de artium rerumq́ elementis ſentiunt, ad præfinitas quaſdam opiniones ſuas omnia reuocare ſtudeant. Pythagorei, vt meminit Ariſtoteles, cum denarij numeri ſummam perfectionem cœlo tribuerent, nec plures tamen quàm nouem ſphæras cernerent, decimam affingere auſi ſunt terram aduer-

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sam, quam ἀντίχθονα appellarunt. Illi enim vniuersitatis rerumq́; singularum naturam ex numeris ceu principijs æstimantes, ea protulerunt quæ φαινομένοις congruere nusquam sunt cognita. Nam ridicula Democriti, Anaximenis, Melissi, Anaxagoræ, Anaximandri, & reliquorum id genus physiologorum somnia, ex falsis illa quidem orta naturæ principijs, sed ad Mathematicum nihil aut parum spectantia, sciens prætereo. Non nullos attingam qui repetitis altius vel aliter ac decuit positis rerum initijs, cum physicis multa turbarunt; tum Mathematicos oppugnatione principiorum pessimè mulctarunt. Ex planis figuris corpora constituit Timæus: Geometrarum hic quidem principia cuniculis oppugnantur. Nam & superficies seu extremitates crassitudinem habebunt, & lineæ latitudinem: denique puncta non erunt indiuidua, sed linearum partes. Prædicant Democritus atque Leucippus illas atomos suas, & indiuidua corpuscula. Concedit Xenocrates impartibiles quasdam magnitudines. Hic verò Geometriæ fundamenta apertè petuntur, & funditus euertuntur: quibus dirutis nihil equidem aliud video restare, quàm vt amplissima Mathematicorum theatra repentè concidant. Iacebunt ergò, si dijs placet, tot præclara Geometrarum de asymmetris & alogis magnitudinibus theoremata. Quid enim causæ dicas, cur indiuidua linea hanc quidem metiatur, illam verò metiri non queat? Siquidem quod minimum in vnoquoque *genere reperitur, id communis omnium mensura esse solet. Innumerabilia profectò sunt illa, quæ ex falsis eiusmodi decretis absurda consequuntur: & horum permulta quidem Mathematicus, sed longè plura colligit Physicus. Quid varia* ψευδογραφημάτων *genera commemorem, quæ ex hoc vno fonte tam longè latéque diffusa fluxisse videntur? Notißimus est Antiphontis tetragonismus, qui Geometrarum & ipse principia non parum labefecit, cùm rectæ lineæ curuam posuit æqualem. Longum esset mihi singula percensere, præsertim ad alia properanti: Hoc ergo certum fixum, & in perpetuum ratum esse oportet, quod sapienter monet Aristoteles,* Σπουδαστέον ὅπως ὁρισθῶσι καλῶς αἱ ἀρχαὶ μεγάλην γὰρ ἔχουσι ῥοπὴν πρὸς ἑπόμενα. *Nam principijs illa congruere debent, quæ sequuntur. Quòd si tantum perspicitur in istis exilioribus Geometriæ initijs, quæ puncto, linea, superficie definiuntur, momentum, vt ne hæc quidem sine summo impendentis ruinæ periculo conuelli aut oppugnari poßint, quanta quæso vis putanda est huius* στοιχειώσεως, *quam collatis tot præstantißimorum artificum inuentis, mira quadam ordinis solertia contexuit Euclides, vniuersæ Matheseos elementa complexu suo coercentem? Vt igitur omnibus rebus instructior & paratior quisq; ad hoc studiū libentius accedat, & singula vel minutißima exactius secum repulet atq; perdiscat, operæpreciū censui, in primo institutionis aditu vestibuloq; præcipua quædem capita,*

quibus

quibus tota ferè Mathematicæ scientiæ ratio intelligatur, breuiter explicare: tum ea quæ sunt Geometriæ propria, diligenter persequi: Euclidis deniq; in extruenda hac στοιχειώσει consilium sedulò ac fideliter exponere. Quæ ferè omnia ex Aristotelis potissimũ ducta fontibus, nemini inuisa fore cõfido, qui modo ingenuũ animi candorẽ ad legendũ attulerit. Ac de Mathematicæ diuisione primũ dicamus.

Mathematicæ in primis scientiæ studiosos fuisse Pythagoreos, non modò historicorum, sed etiam philosophorum libri declarant. His ergo placuit, vt in partes quatuor vniuersum distribuatur Mathematicæ sciẽtiæ genus, quarum duas περὶ τὸ ποσὸν, reliquas περὶ τὸ πηλίκον versari statuerunt. Nam & τὸ ποσὸν vel sine vlla comparatione ipsum per se cognosci, vel certa quadam ratione comparatum spectari: in illo Arithmeticam, in hoc versari Musicam: & τὸ πηλίκον partim quiescere, partim moueri quidem: illud Geometriæ propositum esse: quod verò sua sponte motu cietur, Astronomiæ. Sed ne quis falsò putet, Mathematicam scientiam, quòd in vtroque quanti genere cernitur, idcirco inanem videri (si quidem non solum magnitudinis diuisio sed etiam multitudinis accretio infinitè progredi potest) meminisse decet, τὸ πηλίκον καὶ τὸ ποσὸν, quæ subiecto Mathematicæ generi imposita sunt à Pythagoreis nomina, non cuiuscunque modi quantitatem significare, sed eam demum, quæ tum multitudine tum magnitudine sit defi-

nita, & suis circumscripta terminis. Quis enim vllam infiniti scientiam defendat? Hoc scitum est, quod non semel docet Aristoteles, infinitum ne cogitatione quidem complecti quenquam posse. Itaque ex infinita multitudinis & magnitudinis δυνάμει *finitam hæc scientia decerpit & amplectitur naturam, quam tractet, & in qua versetur.* Nam de vulgari Geometrarum *consuetudine quid sentiendum sit, cum data interdum magnitudine infinita aut fabricantur aliquid, aut proprias generis subiecti affectiones exquirunt, disertè monet Aristoteles,* ουδὲ νῦν *(de Mathematicis) loquens* δέονται τοῦ ἀπείρου, ουδὲ χρῶνται, ἀλλὰ μόνον εἶναι ὅσην ἂν βούλωνται, πεπερασμένην. *Quamobrem disputatio ea qua infinitum refellitur,* Mathematicorum *decretis rationibusquè non aduersatur, nec eorum apodixes labefacit.* Etenim *tali infinito opus illis nequaquam est, quod exitu nullo peragrari possit, nec talem ponunt infinitam magnitudinem: sed quantamcunque velit aliquis effingere, ea vt suppetat, infinitam præcipiunt. Quinetiam non modò immensa magnitudine opus non habent* Mathematici, *sed ne maxima quidem: cum instar maximæ minima quæque in partes totidem pari ratione diuidi queat.* Alteram Mathematicæ *diuisionē attulit* Geminus, *vir (quantum ex* Proclo *conijcere licet)* μαθημάτων *laude clarissimus.* Eam, *quæ superiore plenior & accuratior fortè visa est, cū doctissimè pertractarit sua in decimum* Euclidis *præfa-*

præfatione P. Montaureus vir senatorius, & regiæ bibliothecæ præfectus, leuiter attingam. Nam ex duobus rerum velut summis generibus τῶν νοητῶν καὶ τῶν αἰσθητῶν, quæ res sub intelligentiā cadunt, Arithmeticæ & Geomitriæ attribuit Geminus: quæ verò in sensus incurrunt, Astrologiæ, Musicæ Supputatrici, Opticæ, Geodesiæ & Mechanicæ adiudicauit. Ad hanc certè diuisionem spectasse videtur Aristoteles, cùm Astrologiam Opticam, harmonicam φυσικωτέρας τῶν μαθημάτων nominat, vt quæ naturalibus & Mathematicis interiectæ sint, ac velut ex vtrisque mixtæ disciplinæ: Siquidem genera subiecta à Physicis mutuantur, causas verò in demonstrationibus ex superiore aliqua scientia repetunt. Id quod Aristoteles ipse apertißimè testatur, ἐνταῦθα γὰρ, φησὶ, τὸ μὲν ὅτι, τῶν αἰσθητικῶν εἰδέναι τὸ διότι, τῶν μαθηματικῶν. Sequitur, vt quid Mathematica conueniat cum Physica & prima Philosophia: quid ipsa ab vtraque differat, paucis ostendamus. Illud quidem omnium commune est, quòd in veri contemplatione sunt positæ, ob idqúe θεωρητικαὶ à Græcis dicuntur. Nam cùm διάνοια siue ratio & mens omnis sit vel πρακτικὴ, vel θεωρητικὴ, totidem scientiarum sint genera necesse est. Quòd si Physica, Mathematica, & prima Philosophia, nec in agendo, nec in efficiendo sunt occupatæ, hoc certè perspicuum est, eas omnes in cognitione contemplationéque necessario versari. Cùm enim rerum non modò agendarum, sed etiam effi-

ciendarum principia in agente vel afficiente consistant, illarum quidem προαίρεσις, harum autem vel mens vel ars, vel vis quædam & facultas: rerum profectò naturalium. Mathematicarum, atque diuinarum principia in rebus ipsis, non in philosophis inclusa latent. Atque hæc vna in omnes valet ratio, quæ θεωρητικὰς esse colligat. Iam verò Mathematica separatim cum Physica congruit, quòd vtraque versatur in cognitione formarum corpori naturali inhærentium. Nam Mathematicus plana, solida, longitudines & puncta contemplatur, quæ omnia in corpore naturali à naturali quoque philosopho tractantur. Mathematica item & prima philosophia hoc inter se propriè conueniunt, quòd cognitionem vtraque persequitur formarum, quoad immobiles, & à concretione materiæ sunt liberæ. Nam tametsi Mathematicæ formæ re vera per se non cohærent, cogitatione tamen à materia & motu separantur, ὀυδὲ γίνεται ψεῦδος χωριζόντων. vt ait Aristoteles. De cognatione & societate breuiter diximus, iam quid intersit videamus. Vnaquæque mathematicarum certum quoddam rerum genus propositum habet, in quo versetur, vt Geometria quantitatem & continuationem aliorum in vnam partem, aliorum in duas, quorundam in tres, eorumque quatenus quanta sunt & continua, affectiones cognoscit. Prima autem philosophia, cùm sit omnium communis, vniuersum Entis genus, quæque ei accidunt & conueniunt hoc ipso quòd est, considerat.

Ad

Ad hæc, Mathematica eam modò naturam amplectitur, quæ quanquam non mouetur, separari tamen seiungiq; nisi mente & cogitatione à materia non potest, ob eamq; causam ἐξ ἀφαιρέσεως dici consueuit. Sed Prima philosophia in ijs versatur, quæ & seiuncta, & æterna, & ab omni motu per se soluta sunt ac libera. Cæterùm Physica & Mathematica quanquã subiecto discrepare non videntur, modo tamen rationeq; differũt cognitionis & contemplationis, vnde dißimilitudo quoque scientiarum sequitur. Etenim Mathematicæ species nihil re vera sunt aliud, quàm corporis naturalis extremitates, quas cogitatione ob omni motu & materia separatas Mathematicus contemplatur: sed easdem consectatur Physicorũ ars, quatenus cum materia comprehensæ sunt, & corpora motui obnoxia circumscribunt. Ex quo fit, vt quæcunque in Mathematicis incommoditates accidunt, eadem etiam in naturalibus rebus videantur accidere, non autem vicißim. Multa enim in naturalibus sequuntur incommoda, quæ nihil ad Mathematicum attinent, διὰ τὸ, inquit Aristoteles, τὰ μὲν ἐξ ἀφαιρέσεως λέγεται, τὰ μαθηματικὰ, τὰ δὲ φυσικὰ ἐκ προσθέσεως. Siquidem res cum materia deuinctas contẽplatur physicus: Mathematicus verò rẽ cognoscit circũscriptis ijs omnibus, quæ sensu percipiũtur, vt grauitate, leuitate, duritie, mollitie, & præterea calore, frigore, alijsq; contrariorum paribus, quæ sub sensum subiecta sunt: tantum autem relinquit

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quantitatem & continuum. Itaq; Mathematicorum ars in ijs quæ immobilia sunt cernitur (τὰ γὰρ μαθηματικὰ τῶν ὄντων ἄνευ κινήσεώς ἐστι πλὴν τῶν περὶ τὴν ἀστρολογίαν) quæ verò in naturæ obscuritate posita est, res quidem quæ nec separari nec motu vacare possunt contemplatur. Id quod in vtroque scientiæ genere perspicuum esse potest siue res subiectas definias, siue proprietates earum demonstres. Etenim numerus, linea, figura, rectum, inflexum, æquale, rotundū, vniuersa deniq; Mathematicus quæ tractat & profitetur, absque motu explicari doceriq; possūt: χωριστὰ γὰρ τῇ νοήσει κινήσεώς ἐστι: Physicæ autē sine motione species nequaquam possunt intelligi. Quis enim hominis, plantæ, ignis, ossium, carnis naturam & proprietates sine motu, qui materiam sequitur, perspiciat? Siquidem tantisper substantia quæque naturalis constare dici solet, quo ad opus & munus suum, agendo patiendóque tueri ac sustinere valeat: qua certè amissa δυνάμει, ne nomen quidem nisi ὁμωνύμως retinet. Sed Mathematico ad explicandas circuli aut trianguli proprietates, nullum adferre potest vsum, materiæ, vt auri, ligni, ferri, in qua insunt, consideratio, quin eò verius eiusmodi rerum, quarum species tanquam materia vacantes efformemus animo, naturam complectemur, quòd coniunctione materiæ quasi adulterari diprauaríque videntur. Quo circa Mathematicæ species eodem modo quo κοιλὸν, siue concauitas, sine motu & subiecto, definitione explicare cognos-

vosciq; possunt: naturales verò cùm eam vim habeant, quam vt ita dicam, simitas cum materia comprehensa sunt, nec absque ea separatìm possunt intelligi: quibus exemplis quid inter Physicas & Mathematicas species intersit, haud difficile est animaduertere. Illis certè non semel est vsus Aristoteles. Valeant ergò Protagoræ sophismata, Geometras hoc nomine refellentis, quod circulus normam puncto non attingat. Nam diuina Geometrarum theoremata, qui sensu æstimabit, vix quicquam reperiet quod Geometræ concedendum videatur. Quid enim ex his quæ sensum mouent, ita rectum aut rotundum dici potest, vt à Geometra ponitur? Nec verò absurdum est aut vitiosum, quòd lineas in puluere descriptas pro rectis aut rotundis assumit, quæ nec rectæ sunt nec rotundæ, ac ne latitudinis quidem expertes. Siquidem non ijs vtitur Geometra, quasi inde vim habeat conclusio, sed eorum quæ discenti intelligenda relinquuntur, rudem ceu imaginem proponit. Nam qui primum instituuntur, hi ductu quodam & velut χειραγογία sensuum opus habent, vt ad illa quæ sola intelligentia percipiuntur, aditum sibi comparare queant. Sed tamen existimandum non est rebus Mathematicis omnino negari materiam ac non eam tantùm quæ sensum afficit. Est enim materia alia quæ sub sensum cadit, alia quæ animo & ratione intelligitur. Illam αἰσθητὴν hanc νοητὴν vocat Aristoteles. Sensu percipitur, vt æs, vt lignum, omnisque materia quæ, moueri potest

est. Animo & ratione cernitur ea quæ in rebus sensilibus inest, sed non quatenus sensu percipiuntur, quales sunt res Mathematicorum. Vnde ab Aristotele scriptum legimus ἐπὶ τῶν ἐν ἀφαιρέσει ὄντων rectum se habere vt simum: μετὰ συνεχοῦς γάρ: quasi velit ipsius recti quod Mathematicorum est, suam esse materiam, non minus quam simi quod ad Physicos pertinet. Nam licet res Mathematicæ sensili vacent materia, non sunt tamen indiuiduæ, sed propter continuationem partitioni semper obnoxiæ, cuius ratione dici possunt sua materia non omnino carere: quin aliud videtur τὸ εἶναι γραμμῇ, aliud quoad continuationi adiuncta intelligitur linea. Illud enim ceu forma in materia proprietatum causa est, quas sine materia percipere non licet. Hæc est societatis & dissidij Mathematicæ cum Physica & prima Philosophia ratio. Nunc autem de nominis etymo & notatione pauca quædam afferamus. Nam si quæ iudicio & ratione imposita sunt rebus nomina, ea certè non temerè indita fuisse credendum est, quibus scientias appellari placuit, sed neque otiosa semper haberi debet ista etymologiæ indagatio, cùm ad rei etiam dubiæ fidem sæpe non parum valeat recta nominis interpretatio. Sic enim Aristoteles ducto ex verborum ratione argumento αὐτομάτου, μεταβολῆς, αἰθέρος, aliarumque rerum naturam ex parte confirmauit. Quoniam igitur Pythagoras Mathematicam scientiam non modò studiosè coluit, sed etiam repetitis à capite principijs, geometricam

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contemplationem in liberalis disciplinæ formam composuit, & perspectis absq; materia solius intelligentiæ adminiculo theorematibus, tractationem περὶ τῶν ἀλόγων, & κοσμικῶν σχημάτων constitutionem excogitauit: credibile est, Pythagoram, aut certè Pythagoreos, qui & ipsi doctoris sui studia libenter amplexi sunt, huic scientiæ id nomen dedisse, quod cum suis placitis atq; decretis congrueret rerumq; propositarum naturam quoquo modo declararet. Ita cùm existimarēt illi, omnem disciplinam, quæ μάθησις dicitur, ἀνάμνησιν esse quandam .i. recordationem & repetionem vim scientiæ, cuius antè quam in corpus immigraret, compos fuerit anima, quemadmodum Plato quoque in Menone, Phædone, & alijs aliquot locis videtur astruxisse: animaduerterent autem eiusmodi recordationem, quæ non posset multis ex rebus perspici, ex his potißimum scientijs demonstrari, si quis nimirum, ait Plato, ἐπὶ τὰ διαγράμματα ἄγῃ, probabile est equidem Mathematicas à Pythagoreis artes κατ' ἐξοχὴν fuisse nominatas, vt ex quibus μάθησις, id est æternarum in anima rationum recordatio διαφερόντως & principuè intelligi posset. Cuius etiam rei fidem nobis diuinus fecit Plato, qui in Menone Socratem induxit hoc argumēti genere persuadere cupientem, discere nihil esse aliud quam suarum ipsius rationum animum recordari. Etenim Socrates pusionem quendam vt Tullij verbis vtar, interrogat dè geometrica dimensione quadrati: ad ea sic ille respondet vt puer, et tamen *mentàm faciles interrogationes sunt, vt gradatim respondens, eodem perueniat quò si Geometrica didicisset. Aliam nominis huius rationem Anatolius exposuit, vt est apud Rhodiginum, quòd cùm caetera disciplinae deprehendi vel non docente aliquo possint omnes, Mathematica sub nullius cognitionem veniant, nisi praeeunte aliquo, cuius solertia succidantur vepreta, vel exurantur, & superciliosa complanentur aspreta. Ita enim Caelius: quod quam vim habeat, non est huius loci curiosius perscrutari. Equidem M. Tullius Mathematicos in magna rerum obscuritate, recondita arte multiplicià, ac subtili versari scribit: sed quis nescit idipsum cum alijs grauioribus scientijs esse commune? Est enim, vel eodem auctore Tullio, omnis cognitio multis obstructa difficultatibus, maximaq; est & in ipsis rebus obscuritas, & in iudicijs nostris infirmitas, nec vllus est, modò interius paulò Physica penetrarit, qui non facilè sit expertus, quam multi vndique emergant, rerum naturalium causas inquirentibus, inexplicabiles labyrinthi. Sunt qui ex demonstrationum firmitate nominari Mathematicas opinantur: cuius etiam rationis momentum alio seorsim loco expendendum fuerit. Quocirca primam verbi notationem, quam sequutus est Proclus, nobis retinendam censeo. Hactenus de vniuerso Mathematica genere, quanta potui & perspicuitate & breuitate dixi. Sequitur vt de Geometria separatim atque ordine ea disseram, quae initio sum pollici-*

tus. Est autem Geometria, vt definit Proclus sci entia, quæ versatur in cognitione magnitudinum figurarum, & quibus hæ continentur, extremorum, item rationum & affectionum, quæ in illis cernuntur ac inhærent: ipsa quidem progrediens à puncto indiuiduo per lineas & superficies, dum ad solida conscendat, variasq́ue ipsorum differentias patefaciat. Quúmque omnis scientia demonstratiua, vt docet Aristoteles, tribus quasi momentis contineatur, genere subiecto, cuius proprietates ipsa scientia exquirit & cõtemplatur: causis & principijs ex quibus primis demonstrationes conficiuntur: & proprietatibus, quæ de genere subiecto per se enunciantur: Geometriæ quidem subiectum in lineis, triangulis, quadrangulis, circulis, planis, solidis atque omnino figuris & magnitudinibus, earúmque extremitatibus consistit. His autem inhærent diuisiones, rationes, tactus, æqualitates, παραβολαὶ ὑπερβολαὶ ἐλλείψεις, atque alia generis eiusdem propè innumerabilia. Postulata verò & Axiomata ex quibus hæc inesse demonstrantur, eiusmodi ferè sunt: Quouis centro & interuallo circulum describere. Si ab æqualibus æqualia detrahas, quæ relinquuntur esse æqualia, cæteraq́ue id genus permulta, quæ licet omnium sint communia, ac demonstrandum tamen tum sunt accommodata, cum ad certum quoddam genus traducuntur. Sed cùm præcipua videatur Arithmetica & Geometriæ inter Mathematicas dignatio, cur Arithmetica sit ἀκριβεστέρα &

exactior quam Geometria paucis explicandum arbitror. Hic verò & Aristotelem sequemur ducem, qui scientiam cum scientia ita comparat, vt accuratiorem eſse velit eam, quæ rei causam docet, quam quæ rem esse tantùm declarat; deinde quæ in rebus sub intelligentiam cadentibus versatur, quàm quæ in rebus sensum mouentibus cernitur. Sic enim & Arithmetica quàm Musica, & Geometria quàm Optica, & Stereometria quàm Mechanica exactior esse intelligitur. Postremò quæ ex simplicioribus initijs constat, quàm quæ aliqua adiectione compositis vtitur. Atque hac quidem ratione Geometriæ præstat Arithmetica, quòd illius initium ex additione dicatur, huius sit simplicius. Est enim punctum, vt Pythagoreis placet, vnitas quæ situm obtinet: vnitas verò punctum est quod situ vacat. Ex quo percipitur, numerorum quàm magnitudinum simplicius esse elementum, numerosque magnitudinibus esse puriores, & à concretione materiæ magis disiunctos. Hæc quanquam nemini sunt dubia, habet & ipsa tamen Geometria quo se plurimum efferat; opibúsque suis ac rerum vbertate multiplici vel cum Arithmetica certet: id quod tute facilè deprehendas cùm ad infinitam magnitudinis diuisionem, quam respuit multitudo, animum converteris. Nunc quæ sit Arithmetica & Geometriæ societas, videamus. Nam theorematum quæ demonstratione illustrantur, quædam sunt vtriusque scientiæ communia, quædam verò sin-

gularum propria. Etenim quòd omnis proportio sit ῥητὸς siue rationalis, Arithmeticæ soli conuenit, nequaquam Geometriæ, in qua sunt etiam ἄῤῥητοι seu irrationales proportiones. item, quadratorum gnomas minimo definitos esse, Arithmeticæ proprium (si quidem in Geometria nihil tale minimum esse potest) sed ad Geometriam propriè spectant situs, qui in numeris locum non habent tactus, qui quidem à continuis admittuntur: ἄλογον, quoniam vbi diuisio infinitè procedit, ibi etiam τὸ ἄλογον esse solet. Communia porrò vtriusque sunt illa, quæ ex sectionibus eueniunt, quas Euclides libro secundo est persequutus: nisi quòd sectio per extremam & mediam rationem in numeris nusquam reperiri potest. Iam vero ex theorematibus eiusmodi communibus, alia quidem ex Geometria ad Arithmeticam traducuntur: alia contrà ex Arithmetica in Geometriam transferuntur: quædam verò perinde vtrique scientiæ conueniunt, vt quæ ex vniuersa arte Mathematica in vtranque harum conueniant. Nam & alterna ratio, & rationum conuersiones, compositiones, diuisiones hoc modo communia sunt vtriusque. Quæ autem sunt περὶ συμμέτρων, id est, de commensurabilibus, Arithmetica quidem primùm cognoscit & contemplatur: secundo loco Geometria Arithmeticam imitata. Quare & commensurabiles magnitudines illæ dicuntur, quæ rationem inter se habent, quam numerus ad numerum, perinde quasi cõmensuratio & συμμετρία in numeris primùm consistat (Vbi enim numerus ibi & σύμμετρον cernitur: & vbi σύμμετρον, illic etiam numerus) sed quæ triangulorum sunt & quadrangulorum, à Geometra primum considerantur: tùm analogia quadam Arithmeticus eadem illa in numeris contemplatur. De Geometriæ diuisione hoc adijciendum puto, quòd Geometriæ pars altera in planis figuris cernitur quæ solam latitudinem longitudini coniunctam habent: altera verò solidas contemplatur, quæ ad duplex illud interuallum craßitudinem adsciscunt. Illam generali Geometriæ nomine veteres appellarunt: hanc propriè Stereometriam dixerunt. Ita Geometriam cum Optica & Stereometriam cum Mechanica non rarò comparat Aristoteles. Sed illius cognitio huius inuentionem multis seculis antecesßit, si modò Stereometriam ne Socratis quidem ætate vllam fuisse omnino verum est, quemadmodum à Platone scriptum videtur. Ad Geometriæ vtilitatem accedo, quæ quanquam suapte vi & dignitate ipsa per se nititur, nulllius vsus aut actionis ministerio mancipata (vt de Mathematicis omnibus scientijs concedit in Politico Socrates) si quid ex ea tamen vtilitatis externæ quæritur, Dij boni quam latos, quàm vberes, quum varios fructus fundit? Nec verò audiendus est vel Aristippus vel Sophistarum alius, qui Mathematicorum artes idcirco repudiet, quòd ex fine nihil docere videantur, eiusque quod melius aut

deteri-

deterius nullam habeant rationem. Vt enim nihil causæ dicas, cur sit melius trianguli, verbi gratia, tres angulos duobus esse rectis æquales: minimè tamen fuerit consentaneum Geometriæ cognitionem, vt inutilem exagitare, criminari, explodere, quasi, quæ finem & bonum quò referatur, habeat nullum. Multas haud dubiè solius contemplationis beneficio citra materiæ contagionem adfert Geometria commoditates partim proprias, partim cum vniuerso genere communes. Cùm enim Geometria, vt scripsit Plato, eius quod semper est cognitionem profiteatur, ad veritatem excitabit illa quidem animum, & ad ritè philosophandum cuiusque mentem comparabit. Quinetiam ad disciplinas omnes facilius perdiscendas, attigeris necne Geometriam quanti referre censes? Nam vbi cum materia coniungitur, nónne præstantißimas procreat artes. Geodesiam, Mechaniam, Opticam, quarum omnium vsu, mortalium vitam summis beneficijs complectitur? Etenim bellica instrumenta, vrbiumq; propugnacula, quibus munitæ vrbes hostium vim propulsarent, his adiutricibus fabricata est: montium ambitus & altitudines, locorumque situs nobis indicauit: dimetiendorum & mari & terra itinerum rationem præscripsit: trutinas & stateras, quibus exacta numerorum æqualitas in ciuitate retineatur, composuit: vniuersi ordinem simulachris expreßit, multáque quæ hominum fidem superarent, omnibus persuasit. Vbique extant præclara in eam rem

 testimo-

testimonia. Illud memorabile, quòd Archimedi rex Hiero tribuit. Nam extructo vastæ molis nauigio, quod Hiero Ægyptiorum regi Ptolemæo mitteret, cùm vniuersa Syracusanorum multitudo collectis simul viribus nauem trahere non posset effecissetq́; Archimedes, vt solus Hiero illam subduceret, admiratus viri scientiam rex ἀπὸ ταύτης, ἔφη, τῆς ἡμέρας περὶ παντος ἀρχιμήδει λέγοντι πιστευτέον. Quid? quòd Archimedes idem, vt est apud Plutarchũ, Hieroni scriptis datis viribus datum pondus moueri posse? fretusq́; demonstrationis robore illud sæpe iactarit, si terram haberet alteram vbi pedem figeret, ad eam, nostram hanc se transmouere posse? Quid varia, αὐτοματων machinarúmque genera ad vsus necessarios comparata memorem? Innumerabilia profectò sunt illa, & admiratione dignißima quibus prisci homines incredibili quodam ad philosophandum studio concitati, inopem mortalium vitam artis huius præsidio subleuarunt: tametsi memoriæ sit proditum, Platonem Eudoxo & Archytæ vitio vertisse, quòd Geometrica problemata ad sensilia & organica abducerent. Sic enim corrumpi ab illis & labefieri Geometriæ præstantiam, quæ ab intelligibilibus & incorporeis rebus ad sensiles & corporeas prolaberetur. Quapropter ridicula idem scripsit Plato Geometrarum esse vocabula, quæ quasi ad opus & actionem spectent, ita sonare videntur. Quid enim est quadrare si non opus facere? Quid addere,

pro

producere, applicare? Multa quidem sunt eiusmodi nomina, quibus necessariò & tanquam coacti Geometræ vtuntur, quippe cùm alia desint in hoc genere commodiora. Sic ergo censuit Plato, sic Aristoteles sic denique philosophi omnes, Geometriam ipsam cognitionis gratia exercendam, nec ex aliquo vsu externo sed ex rerum νοητῶν intelligentia æstimandam esse. Exposita breuius quàm res tanta dici possit, vtilitatis ratione, Geometriæ ortum, qui in hac rerum periodo ex historicorum monumentis nobis est cognitus deinceps aperiamus. Geometria apud AEgyptios inuenta, (ne ab Adamo, Setho, Noah, quos cognitione rerum multiplici valuisse constat, eam repetamus) ex terrarum dimensione, vt verbi præ se fert ratio, ortum habuisse dicitur: cùm anniuersaria Nili inundatione & incrementis limo obducti, agrorum termini confunderentur. Geometriam enim, sicut & reliquas disciplinas, in vsu quàm in arte prius fuisse aiunt. Quod sanè mirum videri non debet, vt & huius & aliarum scientiarum inuentio ab vsu cœperit ac necessitate. Etenim tempus, rerum vsus, ipsa necessitas ingenium excitat, & ignauiam acuit. Deinde quicquid ortum habuit (vt tradunt Physici) ab inchoato & imperfecto processit ad perfectum. Sic artium & scientiarum principia experientiæ beneficio collecta sunt: experientia verò à memoria fluxit, quæ & ipsa à sensu primùm manauit. Nam quod scribit Aristoteles, Mathematicas artes,

 compa-

comparatis rebus omnibus ad vitam necessarijs, in AEgypto fuisse constitutas, quòd ibi sacerdotes omnium concessu in otio degerent: non negat ille adductos necessitate homines ad excogitandam, verbi gratia terræ dimetiendæ rationem, quæ theorematum deinde inuestigationi causam dederit: sed hoc confirmat, præclara eiusmodi theorematum inuenta, quibus extructa Geometriæ disciplina constat, ad vsus vitæ necessarios ab illis non esse expetita. Itaque vetus ipsum Geometriæ nomen ab illa terræ partiundæ finiúmque regendorum ratione posteà recessit, & in certa quadam affectionum magnitudini per se inhærentium scienta propriè remansit. Quemadmodum igitur in mercium & contractuum gratiam supputandi ratio, quam secuta est accurata numerorum cognitio, à Phœnicibus initium duxit: ita etiam apud AEgyptios, ex ea quam commemoraui causa ortum habuit Geometria. Hanc certè, vt id obiter dicam, Thales in Græciam ex AEgypto primum transtulit: cui non pauca deinceps à Pythagora, Hipprocrate, Chio, Platone, Archyta Tarentino, alijsq; compluribus, ad Euclidis tempora facta sunt rerum magnarum accessiones. Cæterùm de Euclidis ætate id solùm addam, quod à Proclo memoriæ mandatum accepimus. Is enim commemoratis aliquot Platonis tum æqualibus tùm discipulis, subijcit, non multò ætate posteriorem illis fuisse Euclidem eum, qui Elementa conscripsit, & multa ab Eudoxo collecta, in ordinem luculentum composuit, multaq; à Theateto inchoata perfecit, quaeq; mollius ab alijs demonstrata fuerant, ad firmissimas & certissimas apodixes reuocauit. Vixit autem, inquit ille sub primo Ptolemaeo. Et enim ferunt Euclidam à Ptolemaeo quondam interrogatum num qua esset via ad Geometriam magis compendiaria, quàm sit ista στοιχείωσις, respondisse μὴ εἶναι βασιλικὴν ἀτραπὸν ἐπὶ γεωμετρίαν.

Diende subiungit, Euclidem natu quidem esse minorem Platone, maiorem verò Eratosthene & Archimede (hi enim quales erant) cùm Archimedes Euclidis mētionem faciat. Quòd si quis egregiam Euclidis laudem quam cùm ex alijs scriptionibus accuratissimis, tùm ex hac Geometria στοιχειώσει cōsequutus est in qua diuinus rerum ordo sapientissimis quibusque hominibus magna semper admirationi fuit, is Proclum studiosè legat, quo rei veritatem illustriorem reddat grauissimi testis auctoritas. Superest igitur vt finem videamus, quò Euclidis elementa referri, & cuius causa in id studium incumbere, oporteat. Et quidem si res quae tractantur, consideres: in tota hac tractatione nihil aliud quaeri dixeris, quàm vt κοσμικὰ quae vocantur σχήματα (fuit enim Euclides professione & instituto Platonicus) Cubus Icosaedrum, Octaedrum, Pyramis & Dodecaedrum certa quadam suorum & inter se laterum, & ad sphaera diametrum ratione eidem sphaera inscripta comprehendantur. Huc enim pertinet Epigrammation illud vetus, quod in Geometrica Michaelis

Pselli συνόψει scriptum legitur.

Σχήματα πέντε ἃ πλάτωνος, πυθαγόρας σοφὸς εὗρε.

πυθαγόρας σοφὸς εὗρε, πλάτων δ' ἀρίδηλ' ἐδίδαξεν.

Εὐκλείδης ἐπὶ τοῖσι κλέος περικαλλὲς ἔτευξεν.

Quòd si discentis institutionem spectes, illud certè fuerit propositum, vt huiusmodi elementorum cognitione informatus discentis animus, ad quamlibet non modò Geometriæ*, sed & aliarum Mathematicæ partium tractationem idoneus paratusq́ accedat. Nam tametsi institutionem hanc solus sibi* Geometra *vendicare videtur, & tanquam in possessionem suam venerit, alios excludere posse: inde tamen per multa suo quodam modo iure decerpit* Arithmeticus*, pleraque* Musicus *non pauca detrahit,* Astrologus, Opticus, Logisticus, Machanicus*, itemq́ cæteri: nec vllus est denique artifex præclarus, qui in huius se possessionis societatem cupide non offerat, partemq́ sibi cõcedi postulet. Hinc* στοιχείωσις *absolutum operi nomen, &* στοιχειωτὴς *dictus* Euclides*. Sed quid longius prouehor? Nam quod ad hanc rem attinet, tam copiosè & eruditè scripsit (vt alia complura) eo ipso, quem dixi, loco* P. Montaureus*, vt nihil desiderio loci reliquerit. Quæ verò ad dicendum nobis erant proposita hactenus pro ingenij nostri tenuitate omnia mihi perfecisse videor. Nam tametsi & hæc eadem & alia pleraque multo fortè præclariora ab hominibus doctißimis, qui*

tum

tùm acumine ingenij, tùm admirabili quodam lepore dicendi semper floruerunt grauiùs, splendidiùs, vberiùs tractari posse scio: tamen experiri libuit numquid etiam nobis diuino sit concessum munere, quod rudes in hac philosophiæ parte discipulos adiuuare aut certè excitare queat. Huc accessit quòd ista recens elementorum editio, in qua nihil non parum fuisset studij, aliquid à nobis efflagitare videbatur, quod eius commendationem adaugeret. Cùm enim vir doctißimus Io. Magnienus Mathematicarum artium in hac Pharrhisiorum Academia professor verè regius, nostrum hunc typographum in excudendis Mathematicorum libris diligentißimum, ad hanc Elementorum editionem sæpè & multum esset adhortatus, eiusque impulsu permulta sibi iam comparasset typographus ad hanc rem necessaria, citò interuenit, malùm Ioannis Magnieni mors insperata, quæ tam graue inflixit Academiæ vulnus, cui ne post multos quidem annorum circuitus cicatrix obduci vlla posse videatur. Quamobrem amisso instituti huius operis duce, typographus, qui nec sumptus antéa factos sibi perire, nec studiosos, quibus id muneris erat pollicitus, sua spe cadere vellet, ad me venit, & impensè rogauit, vt meam propositæ editioni operam & studium nauarem, quòd cùm denegaret occupatio nostra, iuberet officij ratio, feci equidem rogatus, vt quæ subobscurè vel parum commode in sermonem Latinum è Græco translata videbantur, clariore, aptiore & fideliore interpretatione nostra (quod cuiusq; pace dictum volo) lucem acciperent. Id quod in omnibus ferè libris posterioribus tute primo obtutu perspicias. Nam in sex prioribus non tantum temporis quantum in cæteros ponere nobis licuit: decimi autem interpretatio, qua melior nulla potuit adferri, P. Montaureo solida debetur. Atq; vt ad perspicuitatem facilitatemq; nihil tibi deesse queraris, adscripta sunt propositionibus singulis vel lineares figuræ, vel punctorum tanquam vnitatum notulæ, quæ Theonis apodixin illustrent illæ quidem magnitudinum, hæ autem numerorum indices, subscriptis etiam ciphrarum, vt vocant, characteribus, qui propositum quemuis numerum exprimant, ob eamq; causam eiusmodi vnitatum notulæ, quæ pro numeri amplitudine maius paginæ spatium occuparent, pauciores sæpius depictæ sunt aut in lineas etiam commutatæ. Nam literarum vt a, b, c characteres non modò numeris & numerorum partibus nominandis sunt accommodati, sed etiam generales esse numerorum, vt magnitudinum affectiones testantur. Adiecta sunt insuper quibusdam locis non pœnitenda Theonis scholia, siue mauis lemmata, quæ quidem longè plura accessissent, si plus otij & temporis vacui nobis fuisset relictum, quod huic studio impartiremus. Hanc igitur operam boni consule, & quæ obuia erunt impreßionis vitia, candidè emenda. Vale. Lutetiæ 4. Idus April. 1557.

EVCLIDIS ELEMENTVM PRIMVM

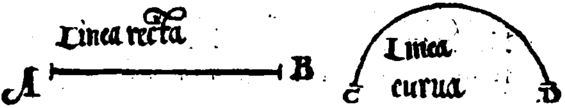
DEFINITIONES.

1

PVnctum est, cuius pars nulla est. Punctum

2

Linea verò, longitudo latitudinis expers.



3

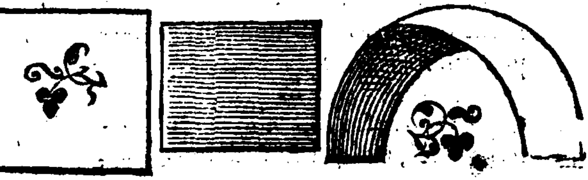
Lineæ autem termini, sunt puncta.

4

Recta linea est, quæ ex æquo sua interiacet puncta.

5

Superficies est, quæ longitudinem latitudinemque tantùm habet.



6 Super.

6

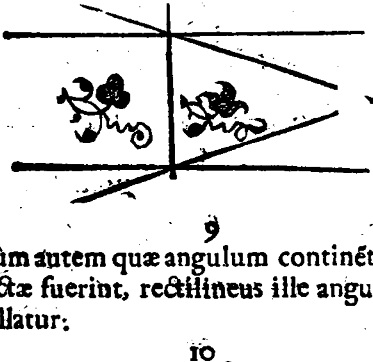
Superficiei extrema. ſunt lineæ.

7

Plana ſuperficies eſt, quæ ex æquo ſuas interiacet lineas.

8

Planus angulus eſt duarũ linearũ in plano ſe mutuò tangentium, & non in directum iacẽtium alterius ad alterã inclinatio.



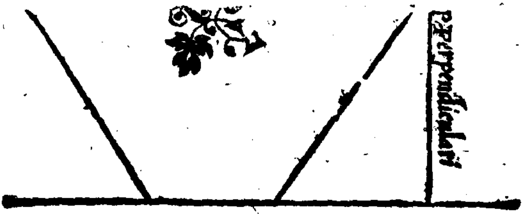
9

Cùm autem quæ angulum continẽt lineæ, rectæ fuerint, rectilineus ille angulus appellatur.

10

Cùm verò recta linea ſuper rectam conſiſtens lineã, eos qui ſunt deinceps angulos ęquales, inter ſe fecerit: rectus eſt vterq; æqua-

quaſium angulorum : quæ inſiſtit recta li-
nea, perpẽdicularis vocatur ei', cui inſiſtit.



11

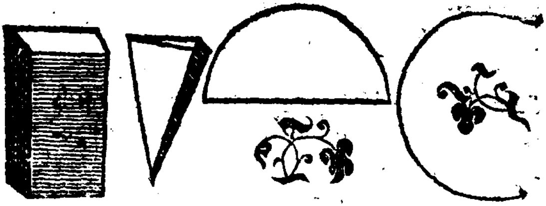
Obtuſus angulus eſt, qui recto maior eſt.

12

Acutus verò, qui minor eſt recto.

13

Terminus eſt, quod alicuius extremũ eſt.



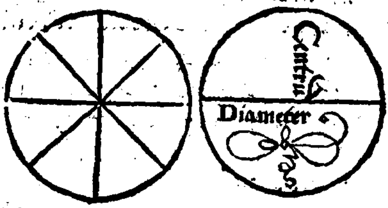
14

Figura eſt, quæ ſub aliquo, vel aliquibus terminis comprehenditur.

15

Circulus eſt, figura plana ſub vna linea cõ-
prehẽſa, quę peripheria appllatur : ad quam
ab vno puncto eorum. quæ intra figuram
ſunt

ſunt po ſita, cadentes omnes rectæ lineæ inter ſe ſunt æquales.



16

Hoc verò punctum, centrum circuli appellatur.

17

Diameter autem circuli, eſt recta quædam linea per centrum ducta, & ex vtraque parte in circuli peripheriam terminata, quæ circulum biſariam ſecat.

18

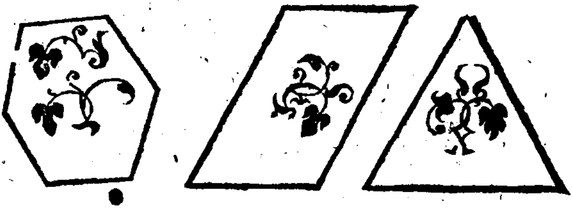
Semicirculus eſt figura, quæ continetur ſub diametro, & ſub ea linea, quæ de circuli peripheria aufertur.

19

Segmentum circuli, eſt figura, quæ ſub recta linea & circuli peripheria continentur.

20.

Rectilineæ figuræ, sunt quæ sub rectis lineis continentur.



21

Trilateræ quidem, quæ sub tribus.

22

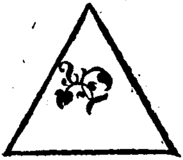
Quadrilateræ, quæ sub quatuor.

23

Multilateræ verò, quæ sub pluribus quàm quatuor rectis lineis comprehenduntur.

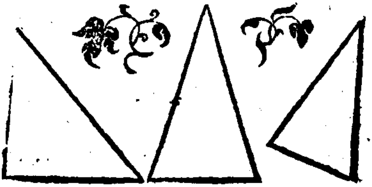
24

Trilaterarũ, porrò figurarum, æquilaterum est triangulum, quod tria latera habet æqualia.



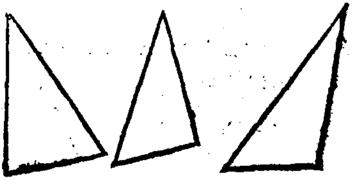
25

Isosceles autem est quod duo tantum æqualia habet latera.



26

Scalenũ verò, eſt qđ tria inęqualia habet latera.



27

Ad hæc etiam trilaterarum figurarum, rectangulum quidem triangulum eſt, quod rectum angulum habet.

28

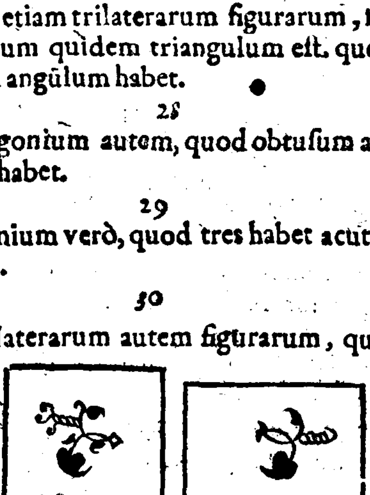
Amblygonium autem, quod obtuſum angulum habet.

29

Oxygonium verò, quod tres habet acutos angulos.

30

Quadrilaterarum autem figurarum, quadratũ quidẽ eſt qđ & æquilaterũ & rectangulum eſt.

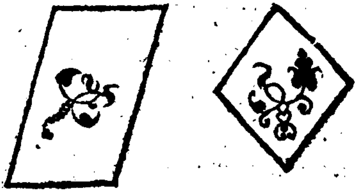


31

Altera parte longior figura eſt, quæ rectãgula quidem, at æquilatera non eſt.

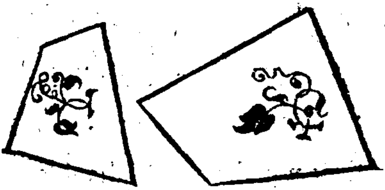
32 Rhom-

32
Rhóbus autem, qui æquilaterum & rectangulum est.



33
Rhomboides verò, quæ aduersa & latera & angulos habens inter se æqualia, neque æquilatera est, neque rectangula.

34
Præter has autẽ reliquæ quadrilateræ figuræ, trapezia appellentur.



35
Parallelæ rectæ lineæ sunt quæ, cùm in eodem sint plano, & ex vtraque parte in infinitum producantur, in neutram sibi mutuò incidunt.

Postulata.

1
Postuletur, vt à quouis puncto in quoduis

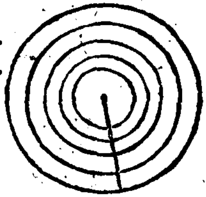
punctum, rectam lineá ducere cócedatur.

2

Et rectam lineam terminatam in continuum recta producere.

3

In quouis centro & interuallo circulũ describere.



Communes notiones.

1.

Quæ eidẽ æqualia, & inter se sunt æqualia.

2

Et si æqualibus æqualia adiecta sint, tota sunt æqualia.

3

Et si ab æqualibus æqualia ablata sint, quæ relinquuntur sunt æqualia.

4

Et si inæqualibus æqualia adiecta sint, tota sunt inæqualia.

5

Et si ab inęqualibus æqualia ablata sint, reliqua sunt inæqualia.

6

Quæ eiusdem duplicia sunt, inter se sunt æqualia.

Et

7

Et quæ eiusdem sunt dimidia, inter se æqualia sunt.

8

Et quæ sibi mutuò congruunt, ea inter se sunt æqualia.

9

Totum est sua parte maius.

10

Item, omnes recti anguli sunt inter se ęquales.

11

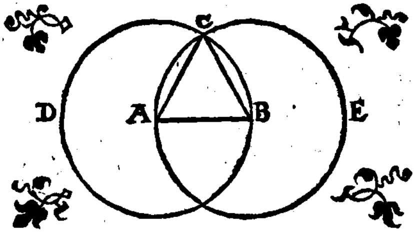
Et si in duas rectas lineas altera recta incidens, inter nos ad easdémque partes angulos, duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ sibi mutuò incident ad eas partes, vbi sunt anguli duobus rectis mimores.

12

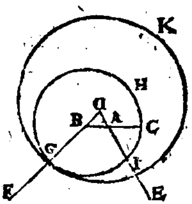
Duæ rectæ lineæ spatium non comprehendunt.

Problema 1. Propositio 1.

Super data recta linea terminata, triangulú æquilaterum constituere.

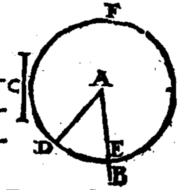


Problema 2. Propositio 2.



Ad datum punctum, datæ rectæ lineæ, ęqualem rectam lineam ponere.

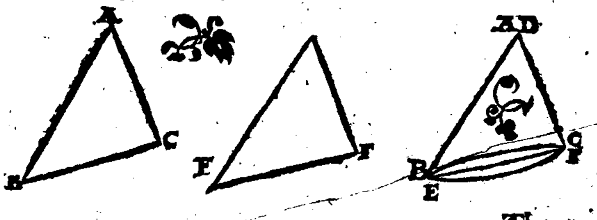
Problema 3. Propositio 3.



Duabus datis rectis lineis inęqualibus, de maiore æqualẽ minori rectam lineã detrahere.

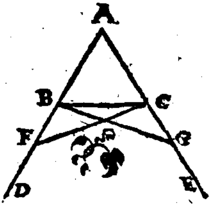
Theorema primum. Propositio 4.

Si duo triangula duo latera duobus lateribus æqualia habeant, vtrunque vtriq;, habeant verò & angulum angulo æqualẽ sub æqualibus rectis lineis contentum: & basin basi æqualem habebunt, eritq; triãgulum triãgulo æquale, ac reliqui anguli reliquis angulis æquales erũt, vterque vtrique, sub quibus æqualia latera subtenduntur,



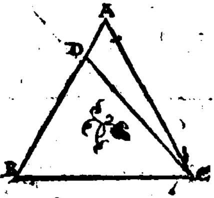
Theore-

Theorema 2. Pro-
poſitio 5.



Iſoſcelium tringulorũ qui ab baſim ſunt anguli, inter ſe ſunt æquales; & ſi vlterius productæ ſint æquales illę rectæ lineæ, qui ſub baſi ſunt anguli, inter ſe equales erunt.

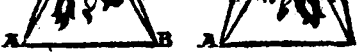
Problema 3. Pro-
poſitio 6.



Si trianguli duo anguli æquales inter ſe fuerint & ſub ęqualibus angulis ſubtenſa latera æqualia inter ſe erunt.

Theorema 46. Propoſitio 7.

Super eadẽ recta linea, duabus eiſdem rectis lineis alię duę rectę lineæ ęquales, vtraque vtrique nõ conſtituentur, ad aliud atque aliud punctũ, ad eaſdẽ partes eoſdẽ

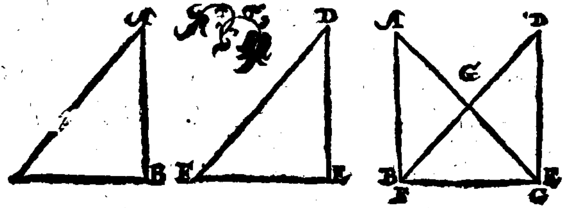


que terminos cum duabus initio ductis rectis lineis habentes.

 Theo-

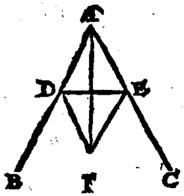
Theorema 5. Propositio 8.

Si duo triãgula duo latera habuerint duobus lateribus, vtrunque vtriq; æqualia: habuerint verò & basim basi æqualẽ: angulũ quoque sub æqualibus rectis lineis cõtentum angulo æqualem habebunt.



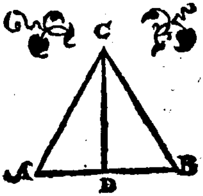
Problema 4. Porpopositio 9.

Datum angulum rectilineum bifariam secare.

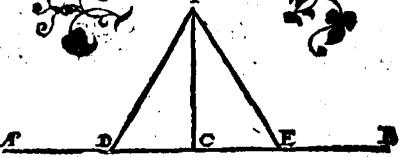


Problema 5. Propositio 10.

Datam rectam lineã finitam bifariam secare.

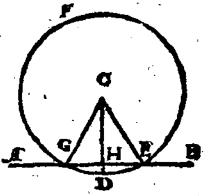


Problema 6. Propoſitio 11.



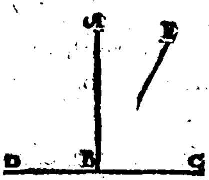
Data recta linea, à puncto in ea dato, rectam lineam ad angulos rectos excitare.

Problema 7. Propoſitio 12.



Super datam rectā lineā infinitam, à dato puncto quod in ea non eſt, perpendicularem rectam deducere.

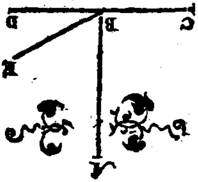
Theorema 6. Propoſitio 13.



Cùm recta linea ſuper rectā conſiſtens lineā angulos facit, aut duos rectos, aut duobus rectis æquales efficiet.

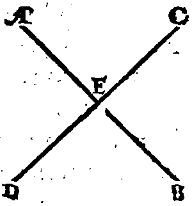
Theorema 7. Propoſitio 14.

Si ad aliquam rectam lineam, atque ad eius punctũ, duę rectę lineæ nõ ad easdem partes ductæ, eos qui sunt deinceps angulos duobus rectis æquales fecerint, in directum erunt inter se ipsæ rectæ lineæ.



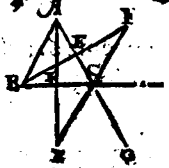
Theorema 8. Propositio 15.

Si duę rectę lineę se mutuò secuerint, angulos qui ad verticem sunt, æquales inter se efficient.



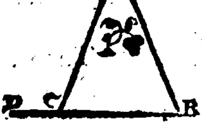
Theorema 9. Propositio 16.

Cuiuscunque trianguli vno latere producto, externus angulus vtroque interno & opposito maior est.



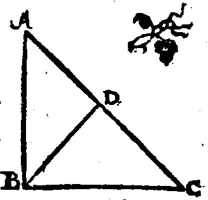
Theorema 10. Propositio 17.

Cuiuscunque trianguli duo auguli duobus rectis sunt minores omnifariam sumpti.



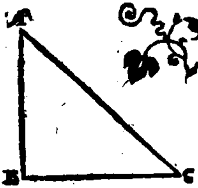
Theore-

Theorema 11. Propositio 18.



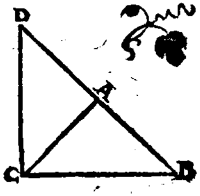
Omnis trianguli maius latus maiorem angulũ subtendit.

Theorema 12. Propositio 16.



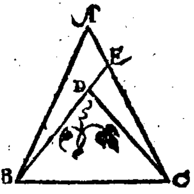
Omnis trianguli lateri angulus, maiori lateri subtenditur.

Theorema 13. Propositio 20.



Omnis triãguli duo latera reliquo sunt maiora, quomodocumq; assumpta.

Theorema 14. Propositio 12.

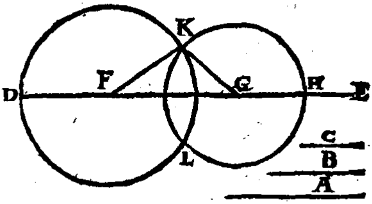


Si super triãguli vno latere, ab extremitatibus duæ rectę lineę, interius cõstitutæ fuerint, hę cõstitutæ reliquis triãguli duobus lateribus minores quidem erunt, minorem verò angulum continebunt.

Pro-

Problema 8. Propoſitio 22.

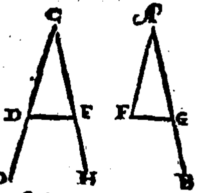
Ex tribus rectis lineis quæ sũt tribus datis rectis lineis æquales,



triangulũ conſtituere. Oportet autem duas reliqua eſſe maiores omnifariam ſumptas: quoniá vniuſcuiuſq; trianguli duo latera omnifariam ſumpta reliquo ſunt maiora.

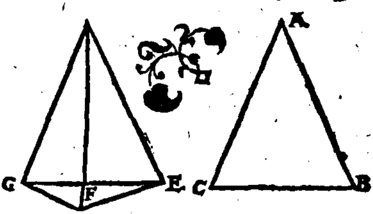
Problema 9. Propoſitio 23.

Ad datam rectam lineã datumq; in ea punctum dato angulo rectilineo ęqualem angulum rectilineum conſtituere.



Theorema 15. Propoſitio 24.

Si duo triãgula duo latera duobus lateribus ęqualia

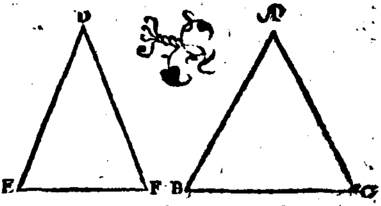


habuerint; vtrũq; vtriq;, angulũ verò angu lo

lo maiorem ſub æqualibus rectis lineis cō-
tentum: & baſin baſi maiorem habebunt.

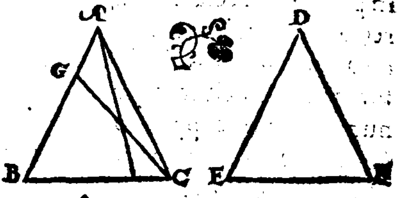
Theorema 16. Propoſitio 25.

Si duo triangula duo latera duobus lateribus æqualia habuerint, vtrunque vtrique, baſin verò baſi maiorē: & angulum ſub æqualib' rectis lineis contentum angulo maiorē habebūt.



Theorema 17. Propoſitio 26.

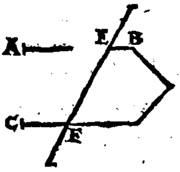
Si duo triangla duos angulos duobus angulis æquales habuerint, vtrūque vtrique, vnumq; latus vni lateri æquale, ſiue, quod æqualibus adiacet angulis, ſeu quod vni æqualium angulorū ſubtenditur: & reliqua latera reliquis lateribꝰ ęqualia vtrūq; vtriq;, & reliquum angulū reliquo angulo æqualem habebunt.



Theore-

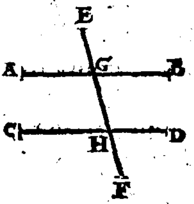
Theorema 18. Propositio 27.

Si in duas rectas lineas recta incidẽs linea alternatim ãgulos ęquales inter se fecerit: parallelę erunt inter se illæ rectæ lineæ.



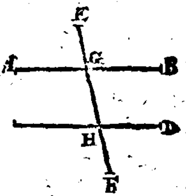
Theorema 19. Propositio 28.

Si in duas rectas lineas recta incidens linea externum angulũ interno, & opposito, & ad easdem partes æqualẽ fecerit, aut internos & ad easdẽ partes duobus rectis æquales: parellelæ erunt inter se ipsæ rectæ lineæ.



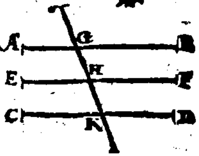
Theorema 20. Propositio 29.

In parallelas rectas lineas recta incidens linea: & alternatim angulos inter se æquales efficit & externum interno & opposito & ad easdem partes æqualem, & inter nos & ad easdem partes duobus rectis æquales facit.



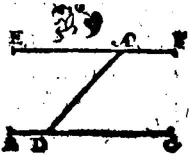
Theore-

Theorema 21. Propoſitio 30.



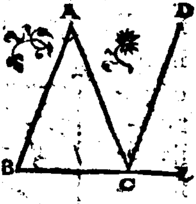
Quæ eidem rectæ lineę, parallelę, & inter ſe ſunt parallelæ.

Problema 10. Propoſitio 31.



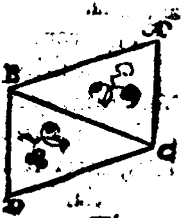
A dato puncto datæ rectæ lineæ parallelam rectam lineam ducere.

Theorema 22. Propoſitio 32.



Cuiuſcunque trianguli vno latere vlterius producto: externus angulus duobus internis & oppoſitis eſt æqualis. Et trianguli tres interni anguli duobus ſunt rectis æquales.

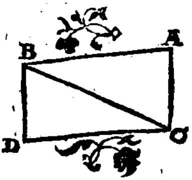
Theorema 23. Propoſitio 33.



Rectæ lineę quę ęquales & parallelas lineas ad partes eaſdem coniungunt, & ipſæ æquales & parallelæ ſunt.

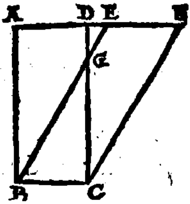
Theore-

Theorema 24. Propoſitio 34.



Parallelogrammorũ ſpatiorum æqualia ſunt inter ſe, quæ ex aduerſo & latera & anguli: atque illa bifariã ſecat diameter.

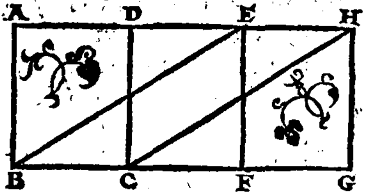
Theorema 25. Propoſitio 35.



Parallelogramma ſuper eadem baſi, & in eiſdem parallelis cõſtituta, inter ſe ſunt æqualia.

Theorema 26. Propoſitio 36.

Parallelogrãma ſuper æqualibus baſibus & in eiſdẽ parallelis cõſtituta inter ſe ſunt æqualia.

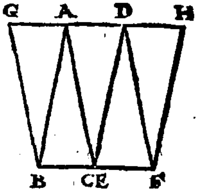


Theorema 27. Propoſitio 37.

Triãgula ſuper eadẽ baſi cõſtituta, & in eiſdẽ parallelis, inter ſe ſunt ęqualia.

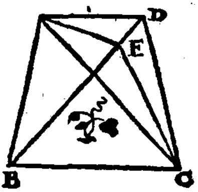
Theore-

Theorema 28. Propositio 38.



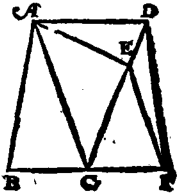
Triangula super ęqualibus basibus constituta et in eisdem parallelis, inter se sunt æqualia.

Theorema 29. Propositio 39.



Triangula æqualia super eadem basi, & ad easdem partes constituta: & in eisdem sunt Parallelis.

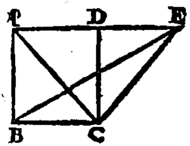
Theorema 30. Propositio 40.



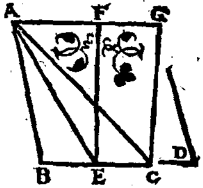
Triangula æqualia super æqualibus basibus, & ad easdem partes constituta, & in eisdem sunt parallelis.

Theorema 31. Propositio 41.

Si parallelogrammum cum triangulo eandem basin habueris, non eisdemque fuerit parallelis, duplum erit parallelogrammum ipsius trianguli.

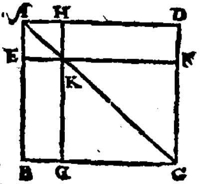


Problema 11. Propositio 42.



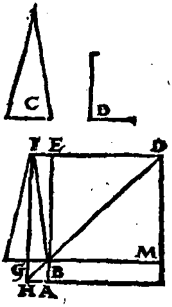
Dato triangulo æquale parallelogrammũ constiture in dato angulo rectilineo.

Theorema 32. Propositio 43.



In omni parallelogrammo, complementa eorũ quę circa diametrũ sunt parallelogrammorũ, inter se sunt æqualia.

Problema 12. Propositio 44.

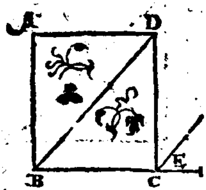


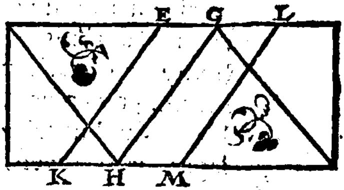
Ad datam rectam lineã dato triangolo æquale parallelogrammum applicare in dato angulo rectilineo.

Problema 13. Propopotio 45.

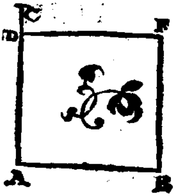
Dato rectilineo ęquale parallelongrammũ consti-

conſtituere in dato angulo rectilineo.





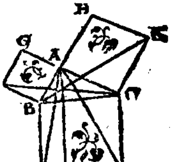
Theorema 41. Propoſitio 4.



A data recta linea quadratum deſcribere.

Theorema 33. Propoſitio 47.

In rectangulis triangulis, quadratum quod à latere rectum angulum ſubtendente deſcribitur, æquale eſt eis, quæ à lateribus rectum angulum continentibus deſcribũtur, quadratis.

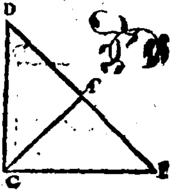


Theorema 34. Propoſitio 48.

Si quadratum quod ab vno laterum trian-

 guli

guli deſcribitur, æquale ſit eis quæ à reliquis trianguli lateribus deſcribuntur, quadratis: angulus comprehenſus ſub reliquis duobus trianguli lateribus, rectus eſt.



FINIS ELEMENTI I.

EVCLIDIS ELEMENTVM SECVNDVM.

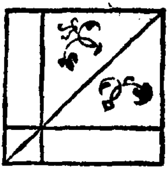
DEFINITIONES.

1.

OMne parallelogrammum rectangulum contineri dicitur sub rectis duabus lineis, quę rectum comprehendunt angulum.

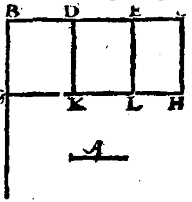
2.

In omni parallelogrammo ſpatio, vnumquodlibet eorum, quæ circa diametrum illius ſunt parallelogrammorum, cum duobus cōplementis, Gnomo vocetur.

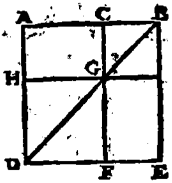


Theorema 1. Propoſitio 1.

Si fuerint duæ rectæ lineæ, ſeceturque ipſarum altera in quotcunq; ſegmenta: rectangulum comprehenſum ſub illis duabus rectis lineis, æquale eſt eis rectangulis, quæ ſub inſecta & quolibet ſegmētorum comprehenduntur.



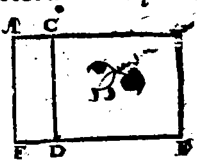
Theorema 2. Propositio 2.



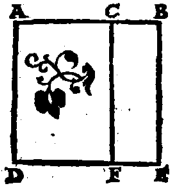
Si recta linea secta sit vt cunq;, rectãgula quæ sub tota & quolibet segmentorum cõprehenduntur æqualia sunt ei, quod à tota fit, quadrato.

Theorema 3. Propositio 3.

Si recta linea secta sit vtcunque, rectangulum sub tota & vno segmẽtorum comprehensum, ęquale est & illi quod sub segmentis cõprehenditur rectangulo & illi, quod à prædicto segmẽto describitur, quadrato.



Theorema 4. Propositio 4.

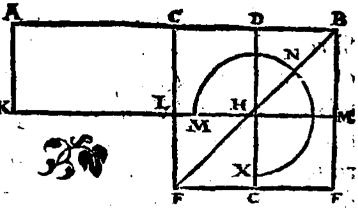


Si recta linea secta sit vtcunque: quadratũ quod à tota describitur, ęquale est & illis quæ à segmẽtis describũtur quadratis, & ei quod bis sub segmentis comprehẽditur rectangulo.

Theorema 5. Propositio 5.

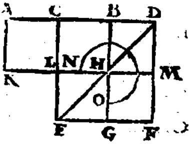
Si recta lenea secetur in æqualia & non æqualia: rectan gulum sub inæqualibus segmentis

mentis totius compreheńsú, vnà cum quadrato quod ab inter media sectionum, æquale eſt ei quod à dimidia deſcribitur, quadrato.



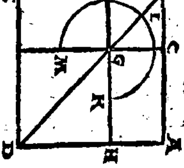
Theorema 6. Propoſitio 6.

Si recta linea bifariam ſecetur, & illi recta quædam linea in rectum adijciatur, rectágulum comprehenſum ſub tota cum adiecta & adiecta ſimul cum quadrato à dimidia, æquale eſt quadrato à linea quæ tú ex dimidia, nitur, tanquam ab vna deſcripto.



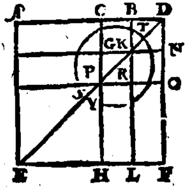
Theorema 7. Propoſitio 7.

Si recta linea ſecetur vt cunque, quod à tota, quoque ab vno ſegmentorum, vtraq; ſimul quadrata, æqualia ſunt & illi quod bis ſub tota & dicto ſegemento comprehenditur, rectangulo, & illi q̃ à reliquo ſegmento fit, quadrato.



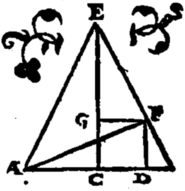
Theorema 8. Propositio 8.

Si recta linea secetur vtcunq; rectangulum quater comprehensum sub tota & vno segmentorum, cum eo quod à reliquo segmẽto fit, quadrato, ęquale est ei quod à tota & dicto segmento, tanquã ab vna linea describitur quadrato.



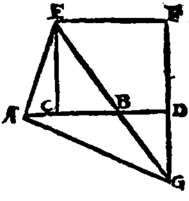
Theorema 9. Propositio 9.

Si recta linea secetur in æqualia & non æqualia: quadrata quæ ab inęqualibus totius segmẽtis fiunt, duplicia sunt & eius quod à dimidia, & eius quod ab intermedia sectionum fit, quadratorum.



Theorema 10. Propositio 10.

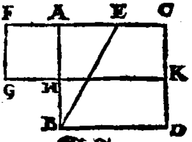
Si recta linea secetur bifariam, adijciatur autem ei in rectum quępiam recta linea: quod à tota cũ adiuncta, & quod ab adiuncta, vtraq; simul quadrata, duplicia sunt, & eius quod à dimidia, & eius quod à composita ex dimidia & adiuncta,



iuncta, tanquã ab vna descriptum sit quadratorum.

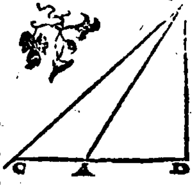
Problema 1. Propositio 11.

Datam rectam lineã secare, vt comprehensum sub tota & altero segmẽtorum rectangulum, æquale sit ei quod à reliquo segmento fit, quadrato.



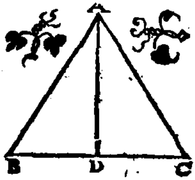
Theorema 11. Propositio 12.

In amblygonijs triangulis, quadratũ quod fit à latere angulum obtusum subtendẽte, maius est quadratis, quæ fiunt à lateribus obtusum angulum comprehendentibus, pro quantitate rectanguli bis comprehensi & ab vno laterũ quę sunt circa obtusum angulũ, in quod cùm protractum fuerit, cadit perpendicularis, & ab assumpta exterius linea sub perpendiculari prope angulũ obtusum.



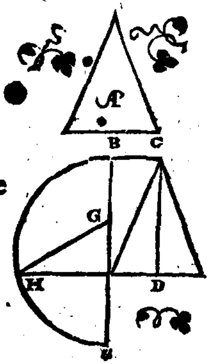
Theorema 12. Propositio 28.

In oxygonijs triangulis quadratū à latere angulum acutum ſubtendente, minus eſt quadratis quæ fiunt à lateribus acutū angulum cōprehendentibus, pro quantitate rectanguli bis comprehenſi, & ab vno late rum, quæ ſunt circa acutum angulum, in quod perpēdicularis cadit, & ab aſſumpta interius linea ſub perpendiculari prope acutū angulum.



Problema 2. Propoſitio 14.

Dato rectilineo æquale quadratū conſtituere.



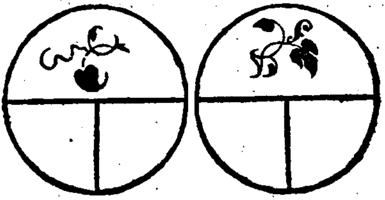
ELEMENTI II. FINIS.

EVCLIDIS ELEMENTVM TERTIVM.

DEFINITIONES.

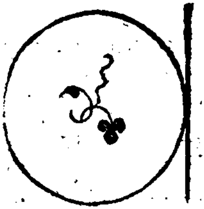
1.

Aequales circuli sunt quorum diametri sũt
ęquales vel quorũ quæ ex centris, rectæ lineę sunt æquales.



2.

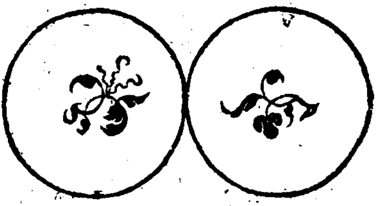
Recta linea circulũ tangere dicitur, quæ cùm circulun tangat, si producatur, circulum non secat.



3 Cir-

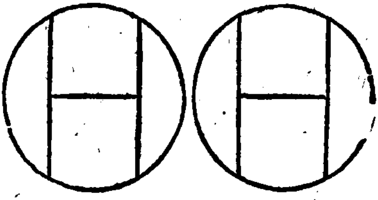
3

Circuli ſeſe mutuò tangere dicuntur: qui ſeſe mutuo tangentes, ſeſe mutuo non ſecant.



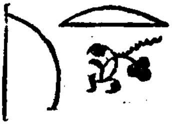
4

In circulo æqualiter diſtare à centro rectæ lineæ dicuntur, cùm perpendiculares, quæ à cẽtro in ipſas ducuntur, ſunt æquales. Lõgius autem abeſſe illa dicitur in quã maior perpendicularis cadit.



5

Segmentum circuli eſt, figura quæ ſub recta linea & circuli peripheria comprehenditur.



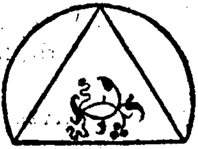
6

Segmenti autem angulus eſt, qui ſub recta linea

linea & circuli peripheria comprehēditur

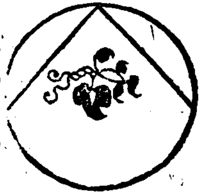
7

In segmento autem angulus est, cum in segmenti peripheria sumptum fuerit quodpiam punctum, & ab illo in terminos rectæ eius lineæ, quæ segmenti basis est, adiūctę fuerint rectæ lineæ: is, inquam, angulus ab adiunctis illis lineis comprehēsus.



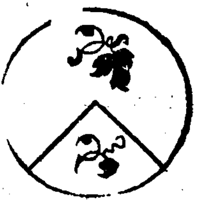
8

Cùm verò comprehendentes angulum rectæ lineæ aliquam assumitur peripheriam, illi angulus insistere dicitur.



9

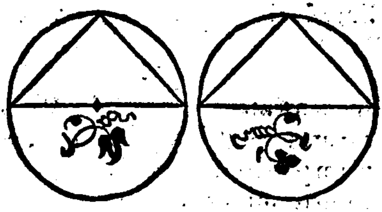
Sector autem circuli est, cùm ad ipsius circuli cētrum constitutus fuerit angulus, comprehensa nimirum figura, & à rectis lineis angulum continentibus, & à peripheria ab illis assumpta.



10

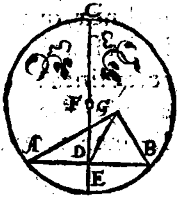
Similia circuli segmenta sunt, quæ angulos capiunt

capiunt
ęquales
aut in q
b'angu-
li inter
se sunt
ęquales



Problema 1. Propositio 1.

Dati circuli centrum reperire.

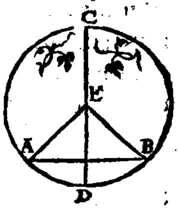


Theorema 1. Propositio 2.

Si in circuli peripheria duo quælibet puncta accepta fuerint, recta linea quæ ad ipsa puncta adiungitur, intra circulum cadet.

Theorema 2. Propositio 3.

Si in circulo recta quædam lenea per centrum extensa quandam non per centrum extensam bifariam secet: & ad angulos rectos ipsam secabit. Et si ad angulos rectos eam secet, bifariam quoque eam secabit.



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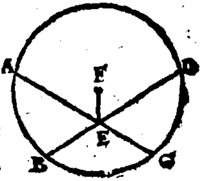
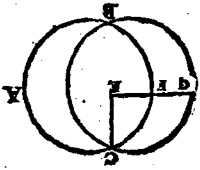
Theorema 3. Propositio 4.



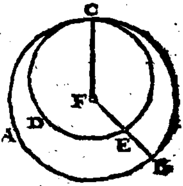
Si in circulo duę rectę lineæ sese mutuo secent non per centrum extẽsæ sese mutuò bifariam non secabunt.

Theorema 4. Propositio 5.



Si duo circuli sese mutuò secent, non erit illorum idem centrum.

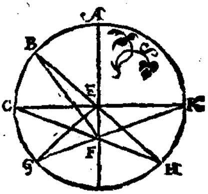
Theorema 5. Propositio 6.



Si duo circuli sese mutuò interius tangant, eorum non erit idem centrum.

Theorema 6. Propositio 7.

Si in diametro circuli quodpiam sumatur punctum, quod circuli centrum non sit, ab eoq́ue puncto in circulũ quædam rectæ lineæ cadant: maxima quidem erit ea in qua centrũ, minima verò reliqua: aliarum verò propinquior illi quę per centrum du-

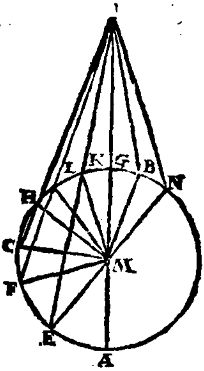


citur

citur, remotiore ſemper maior eſt. Duæ autem ſolùm rectę lineę æquales ab eodem puncto in circulum cadunt ad vtraſq; partes minimæ.

Theorema 7. Propoſitio 8.

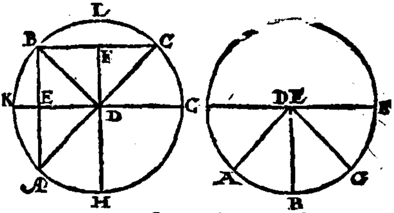
Si extra circulum ſumatur punctum quodpiam, ab eoque puncto ad circulum deducantur rectæ quædam lineæ, quarum vna quidem per centrum protendatur, reliquę verò vt libet: in cauam peripheriam cadentium rectarum linearum minima quidem eſt illa, quæ per centrum ducitur: aliarum autem propinquior ei, quæ per centrum tranſit, remotiore ſemper maior eſt: in conuexam verò peripheriã cadentium rectarũ linearum minima quidem eſt illa, quæ inter punctum & diametrum interponitur: aliarum autem, ea quę propinquior eſt minimæ, remotiore ſemper minor eſt. Duæ autem tantùm rectæ lineæ æquales ab eo puncto in ipſum circulum cadunt, ad vtraſque partes minimæ.



Theo-

Theorema 8. Propositio 9.

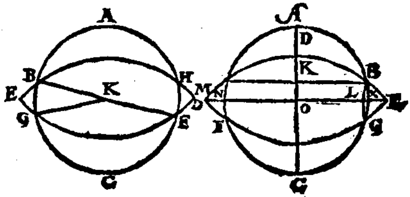
Si in circulo acceptum fuerit pūctum aliquod, & ab eo puncto ad circulum cadant plures quam duę rectæ lineæ, æquales, acceptū



punctum centrum ipsius erit circuli.

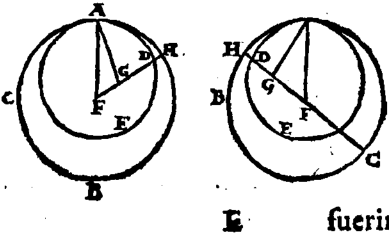
Theorema 9. Propositio 10.

Circū lus circulum in pluribus quàm duob' punctis non secat.



Theorema 10. Propositio 11.

Si duo circuli sese intus cōtingāt, atque accepta

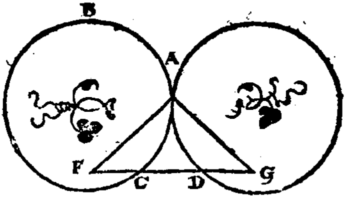


fuerint

fuerint eorum centra, ad eorum cẽtra adiuncta recta linea & producta ni cõtactum circulorum cadet.

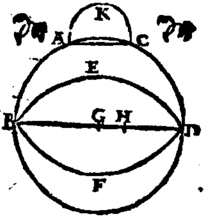
Theorema 11. Propoſitio 12.

Si duo circuli ſeſe exterius contingãt, linea recta q̃ ad cẽtra eorũ ad iũgitur, ꝑ contactũ illũ trãſibit.



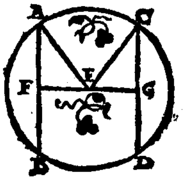
Theorema 12. Propoſitio 13.

Circulum circulum non tangit in pluribus punctis, quam vno, ſiue intus ſiue extra tangat.



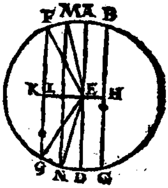
Theorema 13, Propoſitio 23.

In circulo æquales rectæ lineæ æqualiter diſtant à centro. Et quæ ęqualiter diſtant à centro, quales ſunt inter ſe.



Theore-

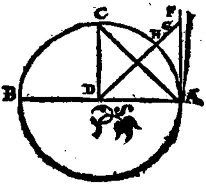
Theorema 14. Propositio 15.



In circulo maxima, quidem linea est diameter: aliarum autem propinquior centro, remotiore semper maior.

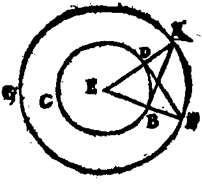
Theorema 51. Propositio 16.

Quæ ab extremitate diametri cuiusq; circuli ad angulos rectos ducitur, extra ipsũ circulum cadet, & in locum inter ipsam rectam lineam & peripheriam comprehensum, altera recta linea nõ cadet Et semicirculi quidem angulus quouis angulo acuto rectilineo maior est, reliquus autem minor,

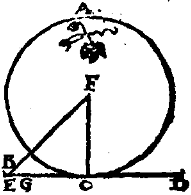


Problema 2. Propositio 17.

A dato puncto rectam lineam ducere, quæ datum tangat circulum.

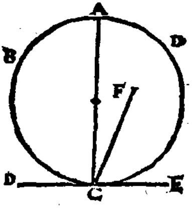


Theorema 16. Propositio 18.



Si circulum tangat recta quæpiam linea, à centro autem ad cõtactum adiungatur recta quædam linea: quę adiũcta fuerit ad ipsam contingentem perpendicularis erit.

Theorema 17. Propositio 19.

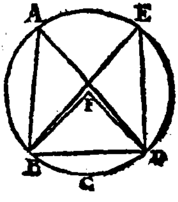


Si circulum tetigerit recta quæpiam linea, à contactu autem recta linea ad angulos rectos ipsi tãgenti excitetur, in excitata erit centrum circuli.

Theorema 18. Propositio 20.

In circulo angulus ad cẽtrum duplex est anguli ad peripheriam, cũ fuerit eadẽ peripheria basis angulorum.

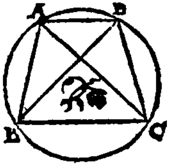
Theorema 19. Propositio 21.



In circulo, qui in eodem segmẽto sunt anguli, sunt inter se æquales.

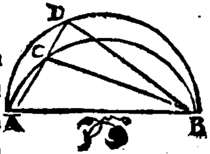
Theore-

Theorema 20. Propoſitio 22.



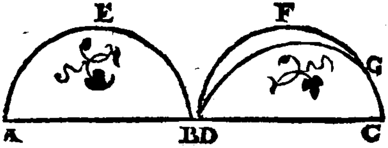
Quadrilaterorum in circulis deſcriptorũ anguli qui ex aduerſo, duobꝰ rectis ſunt æquales.

Theorema 21. Propoſitio 23.



Super eadem recta linea duo ſegmenta circulorũ ſimilia & inæqualia non conſtituẽtur ad eaſdem partes.

Theorema 22. Propoſitio 24.



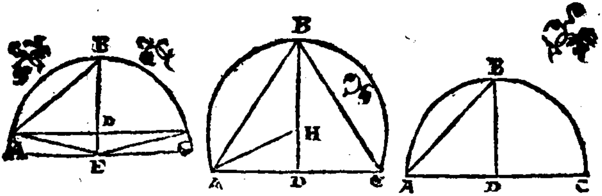
Super ę qualibꝰ rectis lineis ſimilia circulorum ſegmenta ſunt inter ſe æqualia.

Problema 23. Propoſitio 25.

Circuli ſegmento dato, deſcribere circulũ

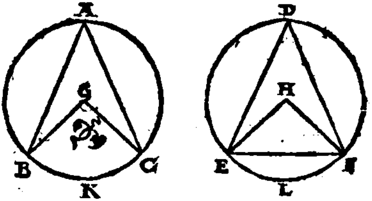
 cuius

cuius est segmentum.



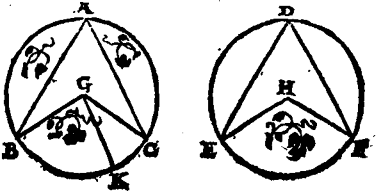
Theorema 22. Propositio 26.

In æqualibus circulis, æquales anguli ęqualib. periphe rijs insistũt siue ad cẽtra, siue ad peripherias constituti insistant.



Theorema 24 Propositin 27.

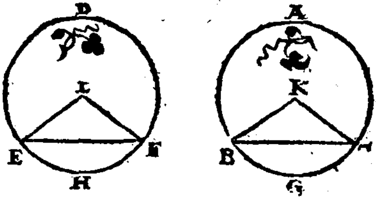
In æqualibus circulis, anguli qui ęqualibus peripherijs insistũt sunt inter se ęquales siue ad cẽtra, siue ad peripherias cõstitu:i insistãt.



Theore-

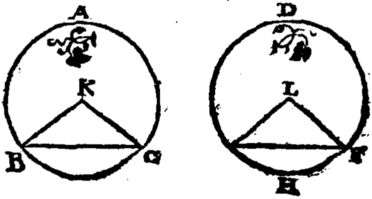
Theorema 25. Propositio 28.

In æqualibus circulis æquales rectæ lineæ ęquales peripherias auferũt maiorẽ quidẽ maiori, minorem autem minori.



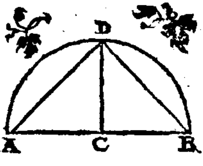
Theorema 26. Propositio 29.

In ęqualib⁹ circulis, æquales peripherias æquales rectæ lineæ subtendunt.



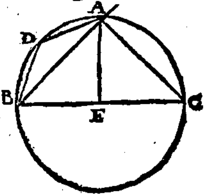
Problema 4. Propositio 30.

Datam peripheriam bifariam secare.



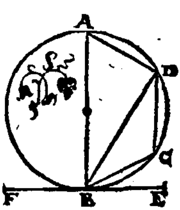
Theorema 27. Propositio 13.

In circulo angulus qui in semicirculo, rectus eſt: qui autem in maoire ſegmẽto, minor recto: qui verò in minore ſegmẽto, maior eſt recto. Et inſuper angulus maioris ſegmenti, recto quidem maior eſt, minoris autẽ ſegmẽti angulus, minor eſt recto.



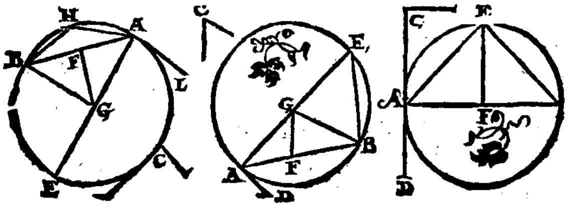
Theorema 28. Propoſitio 32.

Si circulum tetigerit aliqua recta linea, à contactu autẽ producatur quædam recta linea circulum ſecans: anguli quos ad contingentem facit ęquales ſunt ijs qui in alternis circuli ſegmẽtis conſiſtunt, angulis.



Problema 5. Propoſitio 33.

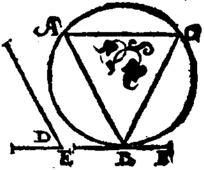
Super data recta linea deſcribere ſegmentum circuli quod capiat angulum æqualẽ dato angulo rectilineo.



Proble-

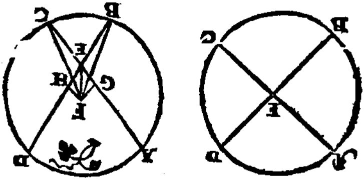
Problema 6. Propoſitio 34.

A dato circulo ſegmentum abſcindere capiens angulum æqualem dato angulo rectilineo.



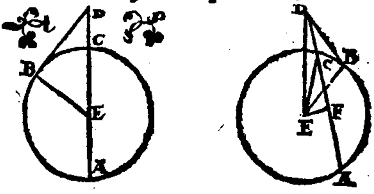
Theorema 29. Propoſitio 35.

Si in circulo duæ rectæ lineæ ſeſe mutuò ſecuerint, rectangulum comprehēſum ſub ſegmētis vniꝰ, æquale eſt ei, quod ſub ſegmentis alterius comprehenditur, rectangulo.



Theorema 30. Propoſitio 36.

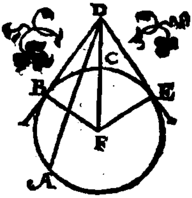
Si extra circulũ ſumatur pũctũ aliquod, ab eoq̃ue in circulũ cadant duę rectę lineæ, quarũ altera quidem circulum ſecet, altera



verò tangat: quod ſub tota ſecante & exterius inter punctum & conuexam peripheriam aſſumpta comprehenditur rectangulum, æquale erit ei, quod à tangente deſcribitur, quadrato,

Theorema 31. Propoſitio 37.

Si extra circulũ ſumatur punctun aliquod, ab eoq; puncto in circulum cadant duæ rectæ lineæ, quarum altera circulum ſecet, altera in eum incidat, ſit autẽ quod ſub tota ſecante & exterius inter punctum & conuexã peripheriam aſſumpta, comprehenditur rectangulum, æquale ei, quod quadrato: incidens ipſa circulum tanget.



ELEMENTI II. FINIS.

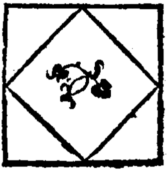
EVCLI.

EVCLIDIS ELEMENTVM QVARTVM.

DEFINITIONES.

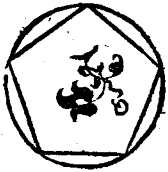
1.

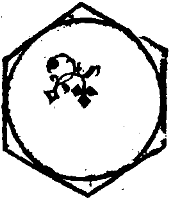
Figura rectilinea in figura rectilinea inſcribi dicitur, cũ ſinguli eius figura quæ inſcribitur, anguli ſingula latera eius, in qua inſcribitur, tangunt.



2

Similiter & figura circum figurã deſcribi dicitur, quum ſingula eius quæ circũſcribitur, latera ſingulos eiꝰ figurę angulos tetigerint, circum quam illa deſcribitur.





3

Figura rectilinea in circulo inſcribi dicitur, quũ ſinguli eius figuræ quę inſcribitur, angu-

anguli tetigerint circuli peripheriam.

4

Figura verò rectilinea circa circulũ deſcribi dicitur, quũ ſingula latera eius, quæ circum ſcribitur, circuli peripheriã tangunt.

5

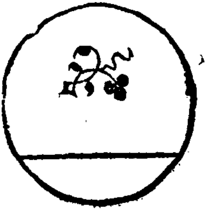
Similiter & circulus in figura rectilinea inſcribi dicitur, quum circuli peripheria ſingula latera tãgit eius figuræ, cui inſcribitur.

6

Circulus autem circum figurã deſcribi dicitur, quũ circuli peripheria ſingulos tãgit eius figuræ, quam circunſcribit, angulos.

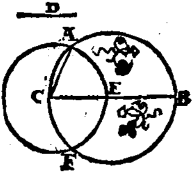
7

Recta linea in circulo accommodari ſeu coaptari dicitur, quum eius extrema in circuli peripheria fuerint.



Problema 1. Propoſitio 1.

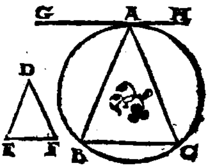
In dato circulo, rectam lineam accommodare ęqualem datæ rectæ lineæ quæ circuli diametro nõ ſit maior.



Proble-

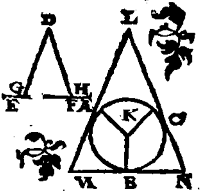
Problema 2. Propoſitio 3.

In dato circulo, triangulũ deſcribere dato triangulo æquiangulum.



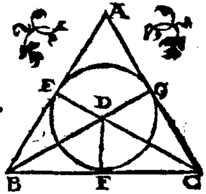
Problema 3. Propoſitio 3.

Circa datum circulũ triangulũ, deſcribere dato triangulo æquiangulũ.



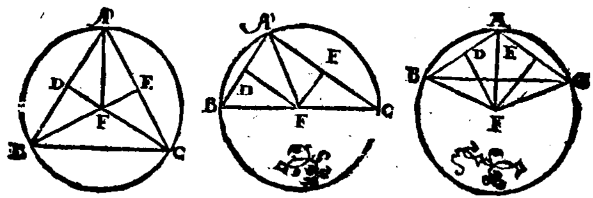
Problema 4. Propopoſitio 4.

In dato triangulo circulum inſcribere.

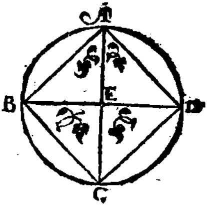


Problema 5. Propoſitio 5.

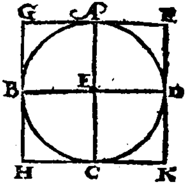
Circa datum triangulum, circulum deſcribere.



Problema 6. Propositio 6.

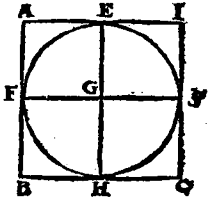


In dato circulo quadratum deſcribere.



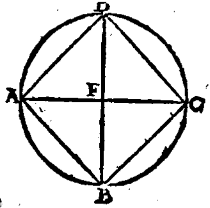
Problema 7. Propoſitio 7.

Circa datum circulum, quadratum deſcribere.



Problema 8. Propoſitio. 8.

In dato quadrato circulum inſcribere.

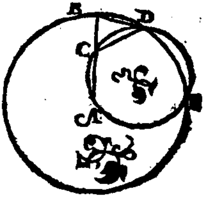


Problema 9. Propoſitio 9.

Circa datum quadratũ, circulum deſcribere.

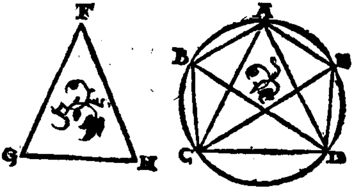
Proble-

Problema 10. Propositio. 10



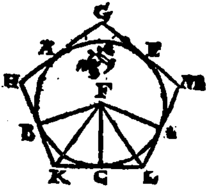
Isosceles triãgulum constituere, quod habeat vtrunque eorum, qui ad basim sunt angulorum, duplum reliqui.

Theorema 11. Propositio 11.



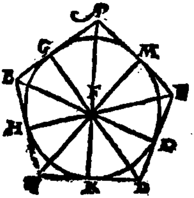
In dato circulo, pentagonũ æquilaterũ & æquiãgulum inscribere.

Problema 12. Propositio 12.



Circa datum circulum, pentagonum æquilaterum æquiangulum describere.

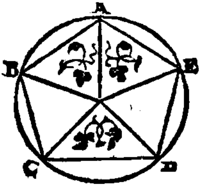
Problema 13. Propositio 13.



In dato pẽtagono æquilatero & æquiãgulo circulum inscribere.

Proble

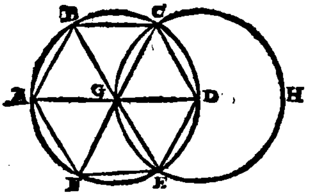
Problema 14. Propoſitio 14.

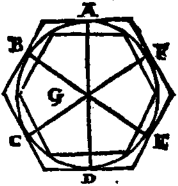


Circa datũ pentagonũ, æquilaterum & æquiangulum, circulum deſcribere.

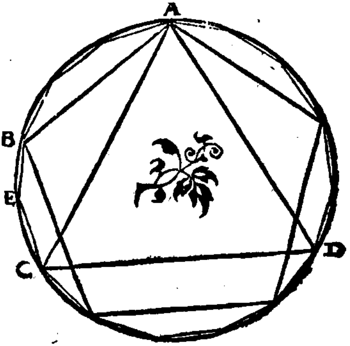
Problema 15. Propoſitio 15.

In dato circulo hexagonum & æquilaterũ & æquiangulum inſcribere.





propoſitio 16. Theorema 16.



In dato circulo quintidecagonum & æquilaterũ & æquiangulum deſcribere.

Elementi quarti finis.

EVCLIDIS ELEMENTVM QVINTVM.

DEFINITIONES.

1

PARS est magnitudo magnitudinis minor maioris, quum minor metitur maiorem.

2

Multiplex autem est maior minoris, cùm minor metitur maiorem.

3

Ratio, est duarum magnitudinum eiusdẽ generis mutua quædam secundum quantitatem habitudo.

4

Proprotio verò, est rationum similitudo.

5

Rationẽ habere inter se magnitudinis dicuntur, quæ possunt multiplicatæ sese mutu superare

6.

In eadem ratione magnitudines dicũtur esse, prima ad secundam, & tertia ad quartã, cum primæ & tertiæ æquè multiplicia, à secundæ & quartæ æquè multiplicibus, qualiſcunq; ſit hæc multiplicatio, vtrumq; ab vtroque: vel vnà deficiunt, vel vnà ęqualia ſunt, vel vnà excedũt, ſi ea ſumãtur quæ inter ſe reſpondent.

7

Eandam autem habẽtes rationem magnitudines, proportionales vocentur.

8

Cum verò æquè multiplicium, multiplex primę magitudines exceſſerit? multiplicẽ ſecundæ, at multiplex tertiæ non exceſſerit multiplicem quartæ: tunc prima ad ſecundam, maiorem rationem habere dicetur, quàm tertia ad quartam.

9

Proportio autẽ in tribus terminis pauciſſimis conſiſtit.

10

Cùm autem tres magnititudines proportionales, fuerint, prima ad tertiam, duplicatã rationẽ habere dicitur eius quam habet ad ſecundam. At cùm quatuor magnitudines proportionales, fuerint, prima ad quartam, triplicatam rationem habere dicitur eius quam habet ad ſecundam: & ſemper deinceps vno amplius, quãdiu proportio extiterit.

11

Homo logę, ſeu ſimiles ratioue magnitudines dicuntur, antecedentes quidẽ antecedentidentibus, conſequẽtẽs verò cõſequẽtibus.

12

Alterna ratio, eſt ſumptio antecedentis cõparati ad antecedentem, & cõſequentis ad conſequentem.

13

Inuerſa ratio, eſt ſumptio cõſequentis, ceu antecedentis, ad antecedentẽ velut ad conſequentem.

14.

Compoſitio rationis, eſt ſumptio antecedentis cũ conſequente ceu vnius ad ipſum conſequentem.

15

Diuiſio rationis, eſt ſumptio exceſſus quo conſequentem ſuperat antecedens ad ipſum conſequentem.

16

Conuerſio rationis, eſt ſumptio antecedentis ad exceſſum, quo ſuperat antecedens ipſum coſequentem.

17

Ex ęqualitate ratio eſt, ſi plures duabus ſint magnitudines, & his alię multitudine pares quæ binæ ſumantur, & in eadem ratione: quum vt in primis magnitudinibus prima ad vltimam ſic & in ſecundis magnitudinibus prima ad vltimã ſeſe habuerit, vel aliter, ſumptio extremorũ per ſubductionem mediorum.

18

Ordinata proportio eſt, cùm fuerit quemadmodum antecedens ad conſequentem ita antecedens ad conſequentem : fuerit etiam vt conſequens ad aliud quidpiam. ita conſequens ad aliud quidpiam.

19

Perturbata autē proportio eſt, tribus poſitis magnitudinibus, & alij quæ ſint his multitudine pares, cùm vt in primis quidē magnitudinibus ſe habet antecedēs ad cōſequentem, ita in ſecūdis magnitudinibus antecedens ad conſequentem : vt autem in primis magnitudinibus conſequens ad aliud quidpiam ad antecedentem.

Theorema 1. Propoſitio 1.

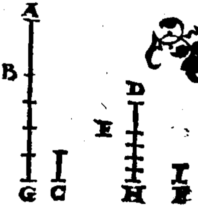
Si ſint quotcūque magnitudines quotcunque magnitudinum æqualium numero ſingulę ſingularum æquè multiplices, quàm multiplex eſt vnius vna magnitudo, tam multiplices erunt, & omnes omnium.

Theorema 2. Propoſitio 2.

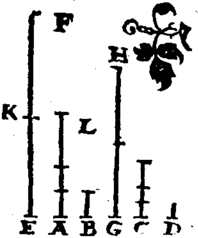
Si prima ſecundæ æquè fuerit multiplex,

atque

atque tertia quartę, fuerit autẽ & quinta secũdę æquè multiplex, atque sexta quartæ: erit & cõposita prima cũ quinta, secũdæ ęquè multiplex atque tertia cum sexta, quartæ.



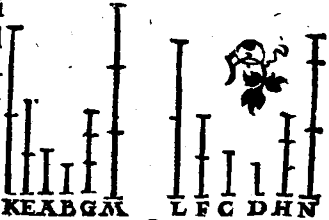
Theorema 3. Propositio 3.



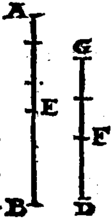
Si si prima secundæ æquè multiplex atque tertia quartæ, sumãtur autẽ æquè multiplices primę & tertiæ erit & ex æquo sumptarum vtraque vtriusque ęquè multiplex, altera quidem secundæ, altera autẽ

Theorema 4. Propositio 4.

Si prima ad secundam, eandẽ habuerit rationem, & tertia ad quartam: etiam æquè multiplices primę & tertiæ, ad æquè multiplices secundæ & quartæ iuxta quãuis multiplicationẽ, eandem habebunt rationem, si prout inter se



reſpondent, ita ſumptæ fuerint.

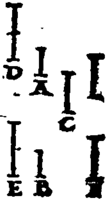


Theorema 5. Propoſitio 5.

Si magnitudo magnitudinis æquè fuerit multiplex, atque ablata ablatæ: etiam reliqua reliquæ ita multiplex erit, vt tota totius.

Theorema 6. Propoſitio 6.

Si duæ magnitudines, duarum magnitudinum ſint æquè multiplices, & detractę quædam ſint earundem æquè multiplices: & reliquæ eiſdem aut ęquales ſunt, aut æquè ipſarum multiplices.



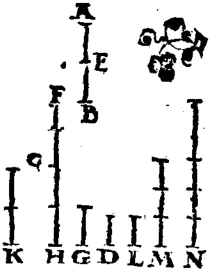
Theorema 7. Propoſitio 7.

Aequales ad eandem, eadem habent rationem: & eandem ad æquales.

Theorema 8. Propoſitio 8.

Inæqualium magnitudinũ maior, ad eandem

dem maiorem ratione habet, quàm minor: & eadem ad minorem: maiorem rationem habet, quàm ad maiorem.



Theorema 9. Propoſitio 9.

Quæ ad eandem, eandem habẽt rationem, æquales ſunt inter ſe: & ad quas eadem, eandem habet rationem, eæ quoque ſunt inter ſe æquales.

Thorema 10. Propoſitio 10.

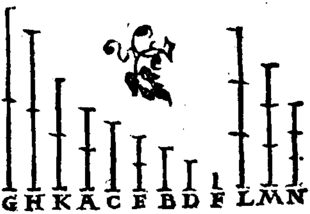
Ad eandem magnitudinẽ rationem habentium, quę maiorem rationẽ habet, illa maior eſt ad quam autẽ eadem maiorem rationem habet, illa minor eſt.

Theorema 11. Propoſitio 11.

Quæ eidem ſunt eædem ratioues, & inter ſe ſunt eędem.

Theorema 12. Propoſitio 12.

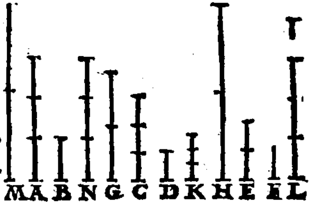
Si ſint magnitudines quotcunque proportionales, quem admodũ ſe habuerit vna antecedentium ad vnã



conſequentiũi, ta ſe habebunt omnes antecedenſes ad omnes coſequentes.

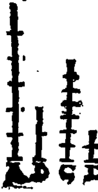
Theorema 13. Propoſitio 13.

Si prima ad ſecundum, eandẽ habuerit rationem, quam tertia ad quartã, tertia verò ad quartam maiorem rationem habuerit, quàm quinta ad ſextam. prima quoque ad ſecũdam maiorem rationem habebit, quàm quinta ad ſextam.



Theorema 14. Propoſitio 14.

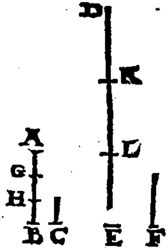
Si prima ad ſecundam eandem habuerit rationẽ, quam tertia ad quartam, prima verò quàm tertia maior fuerit: erit & ſecunda maior quàm quarta. Quòd ſi prima fuerit æqualis tertiæ, erit



&

& secunda æqualis quartæ: si verò minor, & minor erit.

Theorema 15. Propositio 15.



Partes cum pariter multiplicibus in eadẽ sunt ratione, si prout sibi mutuo respondent, ita sumantur.

Theorema 16. Propositio 16.

Si quatuor magnitudines proportionales fuerint, & vicissim proportionales erunt.

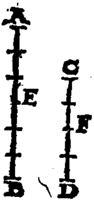
Theorema 17. Propositio 17.

Si compositæ magnitudines porportionales fuerint hæ quoq; diuisæ proportionales erunt.

Theorema 18. Propoſitio 18.

Si diuiſæ magnitudines ſint proportionales, hæ quoque compoſitæ proportionales orunt.

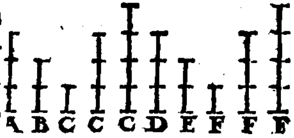
Theorema 19. Propoſitio 19.



Si quemadmodum totũ ad totum, ita ablatum ſe habuerit ad ablatum: & reliquum ad reliquum, vt totum ad totum ſe habebit.

Theorema 20. Propoſitio 20.

Si ſint tres magnitudines, & aliæ ipſis æquales numero, quæ binæ & in eadem ratione ſumatur, ex æquo autẽ prima quàm tertia maior fuerit: erit & quarta, quã ſexta maior. Quòd ſi prima tertiæ fuerit æqualis, erit & quarto æqualis ſextæ: ſin illa minor, hæc quoque minor erit.



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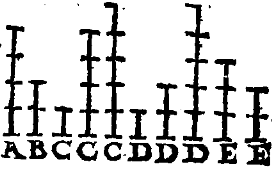
Theorema 21. Propoſitio 21.

Si ſint tres magnitudines, & alię ipſis æquales numero quæ binæ & in eadem ratione ſumátur, fuerít que perturbata earũ proportio ex æquo autem prima quàm tertia maior fuerit erit & quarta



quã ſexta maior: quòd ſi prima tertiæ fuerit æqualis, erit & quarta æqualis ſextæ: ſin illa minor, hæc quoque minor erit.

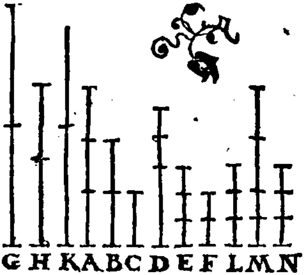
Theorema 22. Propoſitio 22.

Si ſint quotcunq; magnitudines, & alię ipſis ęquales numero, quæ binæ in eadẽ ratione ſumantur, et ex æqualitate in eadem ratione erunt.

Theorema 23. Propoſitio 23.

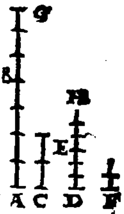
Si ſint tres magnitudines, aliæq; ipſis ęquales

les numero, q̃ binę in eadem ratione sumã-tur, fuerit auté perturbata ea-rũ proportio: etiá ex æquali-te in eadem ra tione erunt.



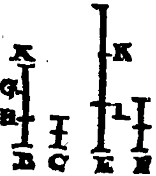
Theorema 24. Propositio 24.

Si prima ad ſecundam, eandem habuerit rationẽ, quam tertia ad quartam, habuerit autem & quinta ad ſecundam, eandẽ rationem, quã ſexta ad quartam: etiam cõpoſita prima cũ quinta ad ſecundam eandẽ habebit rationẽ, quam tertia cum ſexta ad quartam.



Theorema 25. Propositio 25.

Si quatuor magnitudines proportionales fuerint, maxima & minima reliquis duabus maiores erunt



ELEMENTI V. FINIS.

EVCLIDIS ELEMENTVM SEXTVM.

DEFINITIONES.

1

Similes figuræ rectilineæ sunt, quæ & angulos singulos singulis æquales habēt, atque etiam latera, quę circum angulos æquales, proportionalia,

2

Reciprocæ autem figuræ sunt, cum in vtraque figura antecedentes & consequentes rationum termini fuerint.

3

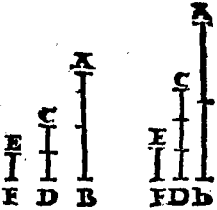
Secundum extremam & mediam rationē recta linea secta esse dicitur, cùm vt tota ad maius segmentum, ita maius ad minus se habuerit.

4

Altitudo cuiusque figurę, est linea perpendicularis à vertice ad basin deducta

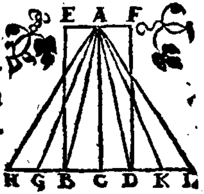
5.

Ratio ex rationibus cõponi dicitur, cũ rationum quantitates inter ſe multiplicatæ aliqua effecerint rationem.



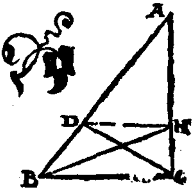
Theorema 1. Propoſitio, 1

Triangula & parallelogramma, quorum eadem fuerit altitudo, ita ſe habent inter ſe vt baſes.



Theorema 2. Propoſitio 2.

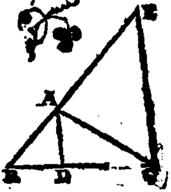
Si ad vnum trianguli latus parallela ducta fuerit recta quædam linea: hęc proportionaliter ſecabit, ipſius triãguli latera. Et ſi trianguli latera proportionaliter ſecta fuerint: quæ ad ſectiones adiũcta fuerit recta linea, erit ad reliquum ipſius trianguli latus parallela.



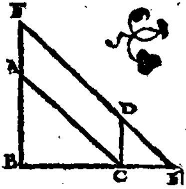
Theorema 3. Propoſitio 3.

Si trianguli angulus biſariam ſectus, ſit, ſecans autẽ augulum recta linea ſecuerit & baſim: baſis ſegmenta eandem habebunt rationem,

tionem, quam reliqua ipsius trianguli latera, Et si basis segmenta eandem habeant rationem quam reliqua ipsius trianguli latera, recta linea, quæ à vertice ad sectionem producitur, ea bifariã secat trianguli ipsius angulum.

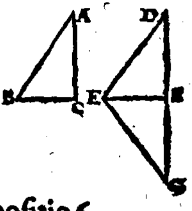


Theorema 4. Propositio 4



Aequiangulorum triangulorum proportianolia sunt latera, quę circum ęquales angulos, & homologa sunt latera, quę ęqualibus angulis subtenduntur.

Theorema 5. Propositio 5.



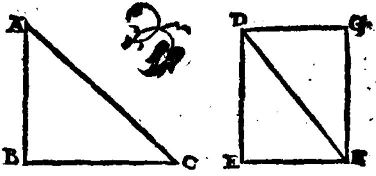
Si duo triãgula latera proportionalia habeant, ęquiangula erunt triangula, & æquales habebunt eos angulos, sub quib' homologa latera subtenduntur.

Theorema 6. Propositio 6.

Si duo triangula vnum angulũm vni angulo æqualẽ, & circum ęquales angulos latera pportionalia habuerint, ęquiangula erunt

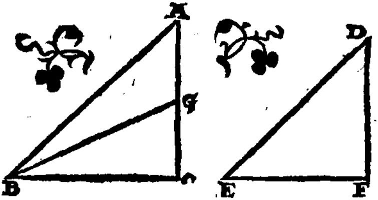
trian

triangula, ęqualesq́; habebunt angulos ſub quibus homologa latera ſubtenduntur.



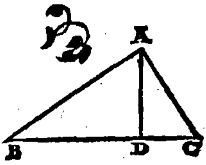
Theorema 7. Propoſitio 7.

Si duo triangula vnum angulum vni angulo æqualem, circũ autem alios angulos latera proportionalia habent, reliquorum vero ſimul vtrunq; aut minórem aut non minoré recto: æquiangula erunt triangula, & æquales habebunt eos angulos circum quos proportionalia ſunt latera.

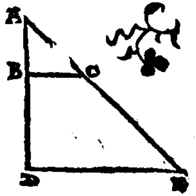


Theorema 8. Propoſitio 8.

Si in triangulo rectangulo, ab angulo recto in baſin perpendicularis ducta ſit quę ad perpendicularem triangulo, tum toti triangulo, tum ipſa inter ſe ſimilia ſunt.

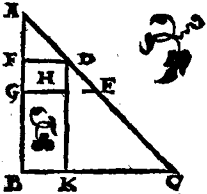


Problema 1. Propositio 9.



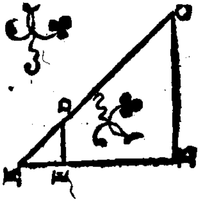
A data recta linea imperatam partem auferre.

Problema 2. Propositio 10.



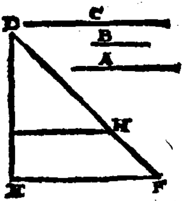
Datam rectã lineam in sectam similiter secare, vt data altera recta secta fuerit.

Problema 3. Propositio 11.



Duabus datis rectis lineis, tertiam propotionalem ad inuenire.

Problema 4. Propositio 12.



Trib' datis rectis lineis quartam proportionalem adiuenire.

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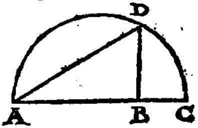
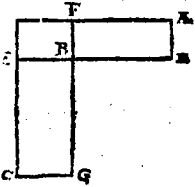
Problema 5. Propositio 13.



Duab' datis rectis lineis mediam proportionalē adinuenire.

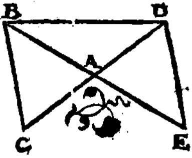
Theorema 9 Propositio 14.

Aequalium, & vnū vni æqualem habentiū angulum parallelogrammorum reciproca sunt latera, quæ circum æquales angulos. & quorū parallelogrammorū vnum angulum vni angulo æqualem habentiū reciproca sunt latera, quæ circum æquales angulos, illa sunt æqualia.



Theorema 10. Propositio 15.

Aequalium, & vnū angulum vni æqualem habentiū triangulorū reciproca sunt latera, quæ circum æquales angulos. & quorū triangulorū vnum angulum vni æqualem habentium reciproca sunt latera, q̄ circum æquales angulos, illa sunt æqualia.



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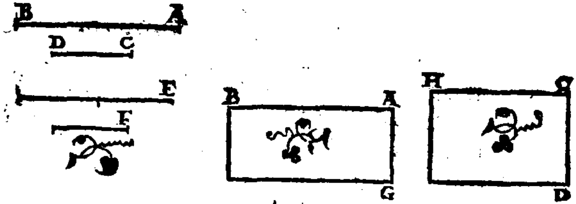
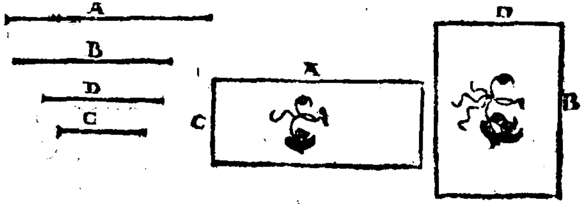
Theorema 11. Propoſitio 16.

Si quatuor rectæ lineæ proportionales fuerint, quod ſub extremis comprehenditur rectangulũ, æquale eſt ei, quod ſub medijs comprehenditur rectãgulo. Et ſi ſub extremis comprehenſum rectangulum æquale fuerit ei, quod ſub medijs continetur rectãgulo, illæ quatuor rectæ lineæ proportionales erunt.



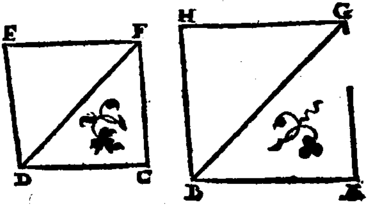
Theorema 12. Propoſitio 17.

Si tres rectæ lineæ ſint proportionales, qd ſub extremis cõprehenditur rectangulũ ęquale eſt ei, quod à media deſcribitur quadrato: & ſi ſub extremis comprehẽſum rectangulum æquale ſit ei quod à media deſcribitur quadrato, illæ tres rectę lineę proportionales erunt.



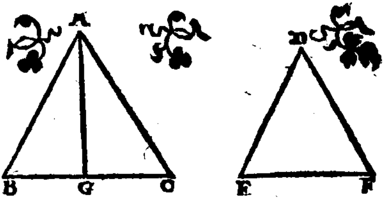
Problema 6. Propoſitio 18.

A data recta linea, dato rectilineo ſimile ſimiliterq́; poſitum rectilineum deſcribere.



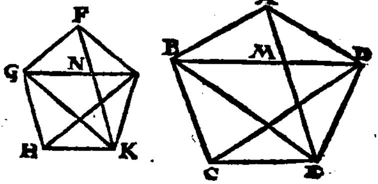
Theorema 13. Propoſitio 19.

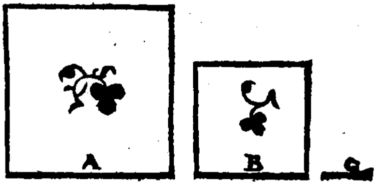
Similia triãgula inter ſe ſunt in duplicata ratione laterũ homologorũ.



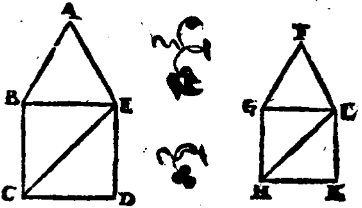
Theore. 14. Propoſitio 20.

Similia polygona in ſimilia triãgula diuidũtur, & numero ęqualia, & homologa totis, Et po



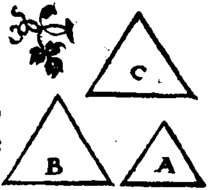


lygona duplicatam habent eã inter se rationẽ, quã latus homologũ ad homologum latus.



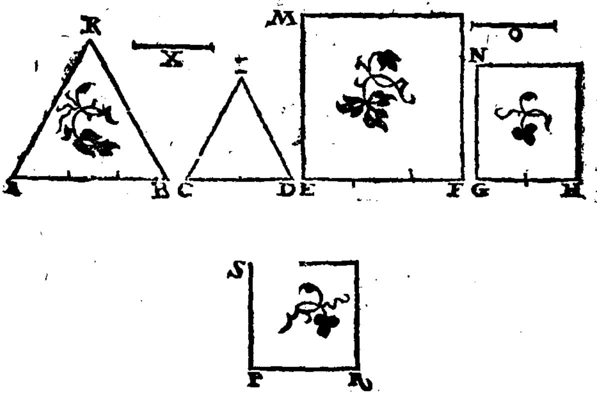
Theorema 15. Propositio 21.

Quæ eidem rectilineo sunt similia, & inter se sunt similia.



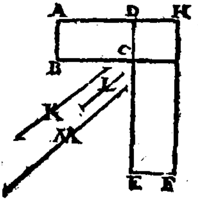
Theorema 16. Propositio 22.

Si quatuor rectę lineæ proportionales fuerint: & ab eis rectilinea similia similiterq; descripta proportionalia erũt. Et si à rectis lineis similia similiterq; descripta rectilinea proportionalia fuerint, ipsæ etiam rectæ lineæ proportionales erunt.



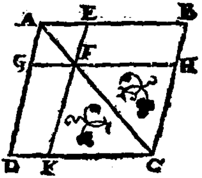
Theorema 17. Propositio 23.

Aequiangula parallelogramma inter se rationē habent eum, quę ex lateribus componitur.



Theorema 18. Propositio 24.

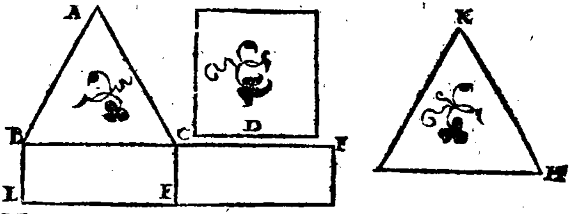
In omni parallelogrammo, quæ circa diametrū sunt parallelogrāma, & toti & inter se sunt similia.



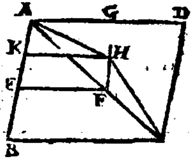
Proble-

Problema 7. Propositio 25.

Dato rectilineo simile, & alteri dato ęquale idem constituere.



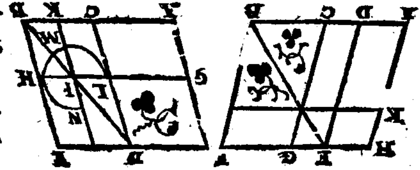
Theorema 19. Propositio 26.



Si à parallelogrãmo parallelogrammũ ablatũ sit, & simile toti & similiter positũ communẽ cum eo habens angulum, hoc circum eandem cum toto diametrum consistit.

Theorema 20. Propositio 27.

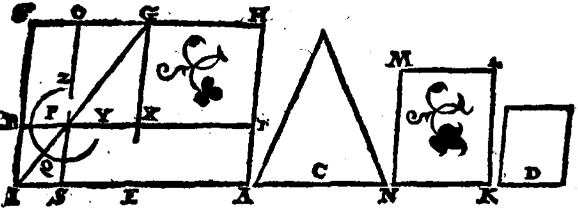
Omnium parallelogrãmorum secũdum eandem rectam lineam applicatorũ deficiẽtiumq; figuris parallelogrãmis si



milibus ſimiliterque poſitis ei, quod à dimidia deſcribitur, maximum, id eſt, quod ad dimidiam applicatur parallelogrammum ſimile exiſtens defectui.

Problema 8. propoſitio 28.

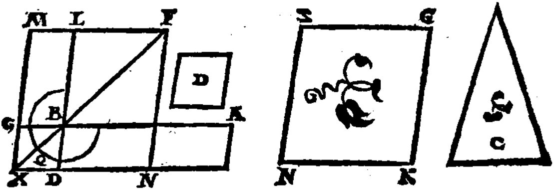
Ad datam lineam rectam, dato rectilineo æquale parallelogrammum applicare deficiens figura parallelogramma, quę ſimilis ſit alteri rectilineo dato. Oportet autẽ datum rectilineum, cui æquale applicandum eſt, nõ maius eſſe eo quod ad dimidiam applicatur, cum ſimiles ſint defectus & eius quod à dimidia deſcribitur, & eius cui ſimile deeſſe debet.



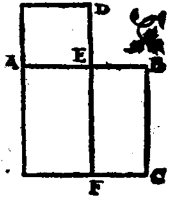
Problema 9. Propoſitio. 29.

Ad datam rectam lineam, dato rectilineo æquale parallelogrammũ applicare, excedẽs figura parallelogrãma, quæ ſimilis ſit paral-

parallelogrammo alteri dato.



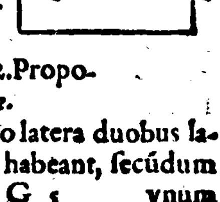
Prblema 10. Propo- positio 30.



Propositam rectam lineam terminatā, extrema ac media ratione secare.

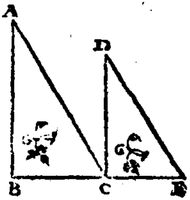
Theorema 21. Propositio .31

In rectangulis triāgulis, figura quęuis à latere rectū angulū subtendēte descripta ęqualis est figuris, quæ priori illi similes & similiter positę à lateribus rectū angulum continentibus describuntur.



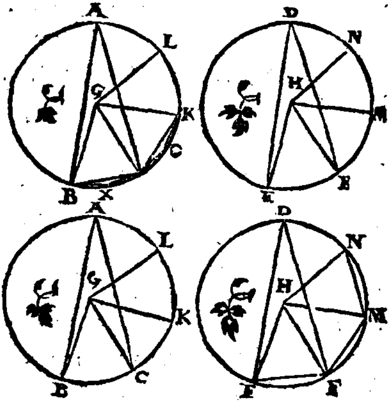
Theorema 22. Propo- sitio 32.

Si duo triāgula, quę duo latera duobus lateribus proportionalia habeant, secūdum vnum angulũ composita fuerint, ita vt homologa eorum latera ſint etiam parallela, tum reliqua illorum ttiangulorum latera in rectam lineam collocata reperientur.



Theorema 23. Propoſitio 33.

In æqualibus circulis anguli eandẽ habent rationem, cũ ipſis peripherijs in quibus inſiſtũt, ſiue ad cẽtra ſiue ad peripherias conſtituti, illis inſiſtant periphẽrijs In ſuper verò & ſectores quippe qui ad cẽtra cõſiſtũt.



ELEMENTI VI. FINIS.

EVCLIDIS ELEMENTVM SEPTIMVM.

DEFINITIONES.

1

Vnitas, eſt ſecũdum quam entium quodque dicitur vnum.

2

Numerus autem, ex vnitatibus compoſita multitudo.

3

Pars, eſt numerus numeri minori maioris, cùm minor metitur maiorem.

4

Partes autem, cùm non metitur.

5

Multiplex verò, maior minoris, cùm maiorem metitur minor.

6

Par numerus eſt, qui bifariá non diuiditur.

7

Impar verò, qui bifariam non diuiditur: vel, qui vnitate differt à pari.

8

Pariter par numerus eſt, quem par numerus metitur per numerum parem.

9 Pari.

9.

Pariter autem impar, est quem par numerus metitur per numerum imparem.

10.

Impariter verò impar numerus, est quem impar numerus metitur per numerum imparem.

11.

Primus numerus, est quem vnitas sola metitur.

12.

Primi inter se numeri sunt, quos sola vnitas mensura communis metitur.

13.

Compositus numerus est, quem numerus quispiam metitur.

14.

Compositi autẽ inter se numeri sunt, quos numerus aliquis mensura communis metitur.

15.

Numerus numerum multiplicare dicitur, cùm toties compositus fuerit is, qui multiplicatur, quot sunt in illo multiplicante vnitates, & procreatus fuerit aliquis.

16.

Cùm autẽ duo numeri mutuò sese multiplicantes quempiam faciũt, qui factus erit planus appellabitur, qui verò numeri mutuò sese multiplicarint, illi' latera dicẽtur.

17. Cùm

17

Cùm verò tres numeri mutuò sese multiplicantes quempiã faciunt, qui procreatus erit solidus apellabitur, qua autem nume ri mutuò sese multiplicarint, illius, latera dicentur.

18.

Quadratus numerus est, qui ęqualiter ęqualis: vel, qui à duobus ęqualibus numeris cõtinetur.

19.

Cubus verò, qui à tribus æqualibus æqualiter: vel, quia tribus æqualibus numeris continetur.

10

Numeri proportionales sunt, cum primus secundi, & tertitius quarti æquè multiplex est, vt eadem pars, vel eædem partes.

21

Similes plani & solidi numeri sunt, qui pro portionalia habent latera.

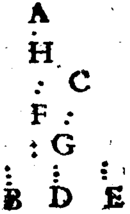
22

Perfectus numerus est, qui suis ipsius partibus est æqualis.

Theorema 1. Propositio 1.

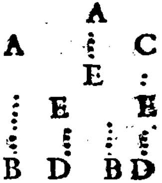
Duobus numeris in æqualibus propositis,

tis, si detrahatur semper minor. de maiore, alterna, quadam detractione; neque reliquus vnquam metiatur præcedentem quo ad assumpta sit vnitas: qui principio propositi sunt numeri primi inter se erunt.

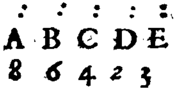


Porblema 1. Propositio 2.

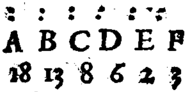
Duobus numeris datis non primis inter se, maximã eorũ cõmunẽ mensurã reperire



Broblema 2. Propositio. 3

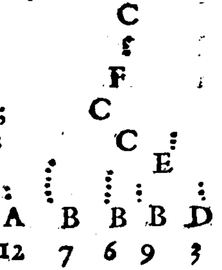


Tribus numeris datis non primis inter se, maximam eorum communem mensuram reperire.



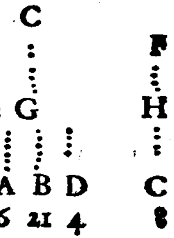
Theorema 18. Propositio 8.

Omnis numerus, cuiusq; numeri minor maioris aut par est, aut partes.



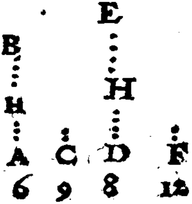
Theore-

Theorema 3 Propositio 5.



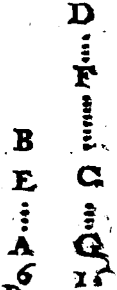
Si numeris numeri par fuerit, & alter alterius eadẽ pars & simul vterque vtriusque simul eadem pars erit, quæ vnus est vnius.

Theor. 4. Propo. 6.



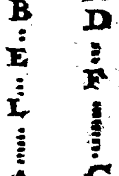
Si numer⁹ sit numeri partes, & alter alterius eędem partes, & simul vterque vtriusque simul eædẽ partes erunt, quæ sunt vnus vnius.

Theorema 5. Propositio 7.



Si numerus numeri eadẽ si pars quæ detractus detracti, & reliquus reliqui eadẽ pars erit, quæ totus est totius.

Theorema 6. Propositio. 8.

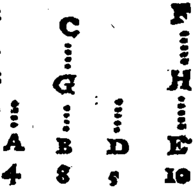


Si numerus numeri eædẽ sint partes quæ detractus detracti & reliquus reliqui eędẽ partes erũt, quę sũt totus totius. Theo-

G..M.K. .N.H.

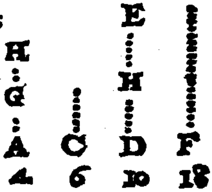
Theorema 7. Propoſitio 9.

Si numerus numeris pars ſit, & alter alterius eadem pars, & viciſſim quæ pars eſt vel partes primus tertij, eadem pars erit eędem partes, & ſecūdus quarti.



Theorema 8. Propoſitio 10.

Si numerus numeri partes ſint, & aliter alterius eadē partes, etiam viciſſim quæ ſunt partes aut pars primus tertij, eędem partes erūt vel pars & ſecundus quarti.

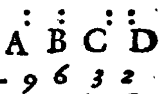


Theorema 9. Propoſitio 11.

Si quemadmodum ſe habet totus ad totum, ita detractus ad detractum, & reliquus ad reliquum ita habebit vt totus ad totum

Theorema 10. Propoſitio 12.

Si ſint quotcunque numeri proportionales; quemadmodum ſe habet vnus antecedentium ad vnum ſequentium, ita ſe habe-



habe-

habebunt omnes antecedentes ad omnes consequentes.

Theorema 11. Propositio 13.

Si quatuor numeri sint	:	:	:	:
proportionales, & viciſ-	A	B	C	D
ſim pportionales erunt.	12	4	9	3

Theorema 12 Propoſitio 14

Si ſint quotcunque	:	:	:	:	:	:
numeri, & alij illis	A	B	C	D	E	F
æquales multitudi-	12	6	3	8	4	2

ne, qui bini ſumantur & in eadem ratione: etiam ex æqualitate in eadem ratione erunt

Theorema 13. Propoſitio 15.

Si vnitas numerum quépiã metiatur, aliter verò numerus alium quendã numerũ æquè metiatur, & viciſſim vnitas tertium numerũ æquę metietur, atque ſecundis quartum.

	C			F
	H			L
	G			K
				E
A	B		D	
1	3		2	6

Theorema 14. Pro-
poſitio 16

Si duo numeri mu-	.	:	:	:	:
tuò ſeſe multiplicã	E	A	B	C	D
tes faciant aliquos	1	2	4	8	8

qui ex illis geniti fuerint, inter ſe æquales erunt.

Theorema 15. Propoſitio 17.

Si numer' duos numeros multiplicãs faciat aliquos, qui ex illis procreati erunt, eadem rationem habebunt, quam multiplicati.

:	:	:	:	:	:
I	A	B	C	D	E
1	3	4	5	12	15

Theorema 16. Propoſitio 18.

Si duo numeri numerum quempiam multiplicantes faciant aliquos, geniti ex illis eandem habebunt rationem, quam qui illum multiplicarunt.

:	:	:	:	:
A	B	C	D	E
4	5	3	12	15

Theorema 17. Propoſitio 19.

Si quatuor numeri ſint proportionales, ꝗ ex primo & quarto fit, ęqualis erit ei qui ex ſecundo & tertio: & ſi qui ex primo & quarto ſit numerus æqualis ſit ei qui ex ſecũdo & tertio, illi quatuor numeri ſint proportionales erunt.

:	:	:	:	:	:	:
A	B	C	D	E	F	G
6	4	3	2	12	12	18

Theorema 18. Propoſitio 20.

Si tres numeri ſint proportionales, qui ab extremis cõtinetur, ęqualis eſt ei qui à me-

dio

dio efficitur. Et si qui ab extremis cõtinetur æqualis sit ei qui à medio describitur, illi tres numeri proportionales erunt.

A	B	C
9	6	4
D		
6		

Theorema 19. Propositio. 21.

Minimi numeri omniũ, qui eandem cũ eis rationem habẽt, æqualiter metiũtur numeros eandem rationem habẽtes, maior quidem maiorem, minor verò minorem

D	L		
G	H		
C	E	A	B
4	3	8	6

Theorema 20. Propositio 22.

Si tres sint numeri & alij multitudine illis æquales, qui bini sumantur & in eadem ratione, sit autem perturbata eorũ proportio, etiam ex æqualitate in eadem ratione erunt.

A	B	C	D	E	F
6	4	3	12	8	6

Theorema 21. Propositio 23.

Primi inter se numeri minimi sunt omniũ eandem cum eis rationem habentium.

A	B	E	C	D
5	6	2	4	8

Theorema 22. Propositio 24.

Minimi numeri omnium eandem cum eis rationem habentiũ, primus sunt inter se.

A B C D E
8 6 4 3 2

Theorema 23. Propositio 25.

Si duo numeri sint primi inter se, qui alterutrum illorum metitur numerus, is ad reliquũ primus erit.

A B C D
6 7 3 4

Theorema 24. Propotio 25.

Si duo numeri ad quépiam numerum primi sint, an eundem primus is quoque futurus est, qui ab illis productus fuerit.

B
3

A C D E F
5 5 5 3 2

Theorema 25. Propositio 27.

Si duo numeri pirmi sint inter se, q ab vno eorũ gignitur ad reliquum primus erit.

B

A C D
7 6 3

Theorema 26. Propositio 28.

Si duo numeri ad duos numeros ambo ad vtrunque primi sint, & qui ex eis gignentur, primi inter se erunt.

A B E C D F
3 5 15 2 4 8

Theore-

Theorema 27. Propositio 29.

Si duo numeri primi sint inter se, & multiplicãs vterq; seipsum procreet aliquem, qui ex ijs producti fuerint, primi inter se erũt. Quòd si numeri initio propositi multiplicantes eos qui producti sunt, effecerint aliquos, hi quoq; inter se primi erunt, & circa extremos idem hoc semper eueniet.

A	C	E	B	D	F
3	6	27	4	16	63

Theorema 28. Propositio. 30.

Si duo numeri primi sint inter se, etiam simul vterq; ad vtrunq; illorum primus erit. Et si simul vterq; ad vnum aliquem eorum primus sit, etiam qui initio positi sunt numeri, primi inter sunt erunt.

A	B	D	C
7	5	4	

Theorema 29. Propositio 31.

Omnis primus numerus ad omnem numerum quem non metitur, primus est.

A	B	C
7	10	5

Theorema 30. Propositio 32.

Si duo numeri sese mutuo multiplicãtes faciant aliquem, hũc aũt ab illis productum metiatur primus quidam numerus, is alterum etiam metitur eorum qui initio positi erant.

A	B	C	D	E
3	6	12	3	4

Theorema 31. Propositio 33.

Omnem compositum numerum aliquis primus metietur.

A B C
27 9 3

Theorema 32. Propositio 34.

Omnis numerus aut primus est, aut eum aliquis primus metitur.

A A
3 6 8

Problema 3. Propositio 35.

Numeris datis quotcunque, reperire minimos omnium qui eandem cum illis rationem habeant.

A B C D E F G H K I M
6 8 12 2 3 4 6 2 3 4 3

Problema 4. Propositio 36.

B
A C D E F
7 12 8 4 5

Duobus numeris datis reperire quem illi minimum metiantur numerum.

A
F E C D G H
6 9 12 9 2 3

Theo-

Theorema 33. Propositio 37

Si duo numeri numerũ quempiam metiantur, & minimus quem illi metiuntur eundem metietur.

A B E G F

2 3 6 12

Problema 5. Propositio 38.

Tribus numeris datis reperire quem minimum numerũ illi metiantur.

A B C D E

3 4 6 12 8

A B C D E F

3 6 8 12 24 16

Theorema 34. Propositio 39.

Si numerum quispiam numerus metiatur, mensus partem habebit metienti cognominem.

A B C D

12 4 3 1

Theorema 35. Propositio 40.

Si numerus partem habuerit quãlibet, illum metietur numerus parti cognominis.

A B C D

8 4 2 1

Problema 6. Propositio 41.

Numerum reperire, qui minimus cùm sit, datas habeat partes.

A B C G H

2 3 4 12 10

ELEMENTI VII. FINIS.

EVCLIDIS ELEMENTVM OCTAVVM.

Theorema 1. Propositio 1.

Si sunt quotcũq; numeri deinceps proportionales, quorum extremi sint inter se, primi, minimi sunt omnium eandem cum eis rationem habentium.

A	B	C	D	E	F	G	H
8	12	18	27	6	8	12	18

Problema 1. Propositio 2.

Numeros reperire deinceps proportionales minimos, quotcunq; iusserit quispiam in data ratione.

A	B	C	D	E	F	G	H	K
3	4	9	12	16	27	36	49	64

Theorema 2. Propositio 3.

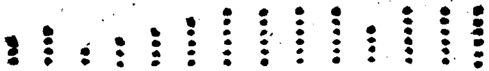
Conuersa primæ.

Si sint quotcũq; numeri deinceps proportionales minimi habentium eandem cum eis rationem, illorum extremi sunt inter se primi.

A	B	C	D	E	F	G	H	K	L	M	N	O
27	36	48	64	3	4	9	12	16	27	36	48	64

Problema 2. Propositio 4.

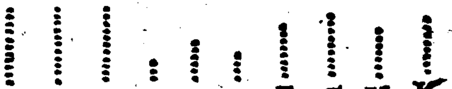
Rationibus datis quotcunque in minimis numeris reperire numeros deinceps minimos in datis rationibus.



A	B	C	D	E	F	H	G	K	L	N	X	M	O
3	4	2	3	4	5	6	8	12	15	4	6	10	12

Theorema 3. Propositio 5.

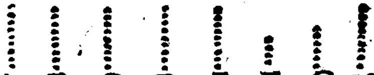
Plani numeri rationem inter se habent ex lateribus compositam.



A	L	B	C	D	E	F	G	H	K
18	22	32	3	6	4	8	9	12	16

Theorema 4. Propositio 4.

Si sint quotlibet numeri deinceps



A	B	C	D	E	F	G	H
16	24	36	54	81	4	6	9

proportionales, primus autem secundum non metiatur, neque alius quispiam ullum metietur.

Theorema 5. Propositio 7.

Si sint quotcunque numeri deinceps proportionales, primus autem extremum metiatur, is etiam secundum metietur.

A	B	C	D
4	6	12	24

Theorema 6. Propositio 8.

Si inter duos numeros medij continua ꝓportione incidant numeri, quot inter eos medij continua proportione incidunt numeri, tot & inter alios eandem cum illis habentes rationem medij continua proportione incident.

A	C	D	B	G	H	K	L	C	M	N	F
4	9	27	81	1	3	9	27	2	6	18	54

Theorema 7. Propositio 9.

Si duo numeri sint inter se primi, & inter eos medij cótinua proportione incidát numeri, quot inter illos medij continua proportione incidunt numeri, totidé & inter vtrunque eorum ac vnitaté deinceps medij continua proportione incident.

A	M	H	E	F	N	C	K	X	G	D	L	O	B
27	27	9	36	1	36	1	12	48	4	48	16	64	64

Theo-

Theorema 8. Propositio 10.

Si inter duos numeros & vnitatẽ continuè proportionales incidãt numeri quot inter vtrumque ipsorum & vnitatẽ deinceps medij continua proportione incidunt numeri, totidẽ & inter illos medij continua proportione incident.

A	E	K	D	H	C	L	F	G	B
27	9	36	3	12	1	48	4	16	64

Theorema 9. Propositio 11.

Duorum quadratorũ numerorum vnus medius proportionalis est numerus: & quadratus ad quadratum duplicatam habet lateris ad latus rationem.

A	C	E	D	B
9	3	12	4	16

Theorema 10. Propositio 12.

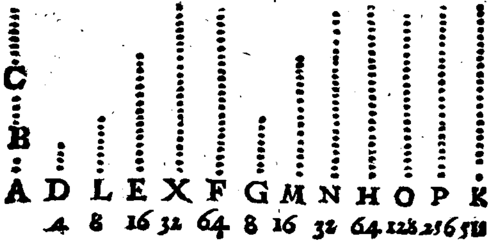
Duorum cuborum numerorũ duo medij proportionales sunt numerorũ duo medij cubum triplicatam habet lateris ad latus rationem.

A	H	K	B	C	D	E	F	G
27	36	48	64	3	4	9	12	16

Theo-

Thorema 10. Propositio 13.

Si sint quotlibet numeri deinceps proportionales, & multiplicans quisq; seipsum faciat aliquos, qui ab illis producti fuerint, proportionales erũt, & si numeri primùm positi, ex suo in proceatos ductu faciãt aliquo, ipsi quoque proportionales erunt



Theorema 12. Propositio 14.

Si quadratus numerus quadratum numerũ metiatur, & latus vnius metietur latus alterius. Et si vnius quadrati latus metiatur latus alterius, & quadratus quadratum metietur.

A E B C D
9 12 16 8 4

Theorema 13. Propositio 15.

Si cubus numerus cubum numerũ metiatur, & latus vni' metietur alterius latus. Et si latus vnius cubi latus alterius metiatur,

tum

tum cubus cubum metietur.

A	H	K	B	C	D	E	F	G
8	16	28	64	2	4	4	8	16

Theorema 14. Propositio 9.

Si quadratus numerus quadratum numerum non metiatur, neque latus vnius metietur alterius latus.

Etsi latus vni' quadrati non metiatur latus alterius, neque quadratus quadratum metietur.

A	B	C	D
9	16	3	4

Theorema 15. Propositio 17.

Si cubus numerus cubum numerũ non metiatur, neque latus vnius latus alterius metietur. Et si latus cubi alicuius latus alterius non metiatur, neque cubus cubum metietur.

A	B	C	D
8	27	9	12

Theorema 16. Propositio 18.

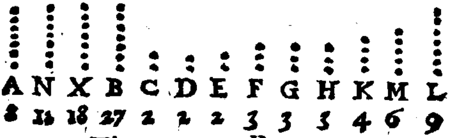
Duorum similium planorum numerorum vnus medius proportionalis est numerus, & planus ad planum duplicatam habet literis homologi-

A	G	B	C	D	E	F
12	18	27	2	6	3	9

logi ad latus homologum rationem.

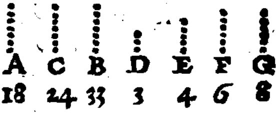
Theorema 17. Propositio 19.

Inter duos similes numeros solidos, duo medij proportionales incidunt numeri: & solidus ad similem solidum triplicatam rationem habet lateris homologi ad latus homologum.



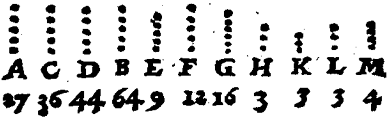
Theorema 18. Propositio 20.

Si inter duos numeros vnus medius ꝓportionalis incidat numer⁹, similes plani erunt illi numeri.



Theorema 19 Propositio 21.

Si inter duos numeros duo medij proportionales incidant numeri, similes solidi sunt illi numeri.



Theorema 20. Propositio 22.

Si tres numeri deinceps sint proportionales, primus autem sit quadratus, & tertius quadratus erit.

A	B	D
9	15	25

Theorema 21. Propositio 23.

Si quatuor numeri deinceps sint proportionales, primus autem sit cubus, & quartus cubus erit.

A	B	C	D
8	12	18	27

Theorema 22. Propositio 25.

Si duo numeri rationem habeant inter se, quam quadratus numerus ad quadratum numerum, primus autem sit quadratus erit, & secunndus quadratus erit.

A		B	C		D
4	6	9	16	24	36

Theorema 23. Propositio 25.

Si numeri duo rationem inter se habeant, quam cubus numerus ad cubum numerũ, primus autem cubus sit, & secundus cubus erit.

A	E	F	B	C			D
8	12	18	27	64	95	140	216

The

Theorema 24. Propositio 26.

Similes plani numeri rationem inter se habent, quam quadratus numerus ad quadratum numerum.

A	C	B	D	E	F
18	24	32	9	12	16

Theorema 25. Propositio 27.

Similes solidi numeri rationem habent inter se, quam cubus numerus ad cubum numerum.

A	C	D	B	E	F	G	H
16	24	36	54	8	12	18	27

ELEMENTI VIII. FINIS.

EVCLI.

EVCLIDIS ELEMENTVM NONVM.

Theorema 1. Propositio 1.

Si duo similes plani numeri mutuò sese multiplicã-tes quendã procreent, productus quadratus

A	E	B	D		C
4	6	9	16	24	36

Theorema 2. Propositio 2.

Si duo numeri mutuò sese multiplicantes quadratum faciant, illi similes sunt plani.

A		B	D		C
4	6		9	18	36

Theorema 3. Propositio 3.

Si cubus numerus seipsum multiplicãs procreet aliquẽ, productus cubus erit.

Vnitas	D	D	A			B
	3	4	8	16	32	64

Theorema 4. Propositio 4.

Si cubus numerus cubũ numerum multiplicans quendam procreet, procreatus cubus erit.

A	B	D	C
8	27	64	216

Theorema 5. Propositio 5.

Si cubus numerus numerum quendã multiplicans cubum procreet, & multiplicatus cubus erit.

A	B	D	D
27	64	729	1728

Theorema 6. Propositio 6.

Si numerus seipsum multiplicans cubum procreet, & ipse cubus erit.

A	B	C
27	729	19683

Theorema 7. Propositio 7.

Si compositus numerus quendam numerũ multiplicans quempiam procreet, productus solidus erit.

A	B	C	D	E
6	8	48	2	3

Theorema 8. Propositio 8.

Si ab vnitate quotlibet numeri deinceps proportionales sint, tertius ab vnitate quadratus est, & vnũ intermittentes omnes: quartus autẽ cubus, & duobus intermissis omnes

nes: septimus verò cubus simul & æquadratus, & quinque intermissis omnes.

Vnitas	A	B	C	D	E	F
	3	9	27	81	243	729

Theorema 9. Propositio 9.

Si ab vnitate sint quotcunqne numeri deinceps proportionales, sit autem quadratus is qui vnitatem sequitur, & reliqui oés quadrati erũt. Quòd si qui vnitatem sequitur cubus sit, & reliqui omnes cubi erunt.

quadrati		cubi
531441	F	732969
59049	E	531441
6561	D	59049
729	C	6561
81	B	729
9	A	81
	o	
	vnitas.	

Theorema 10. Propositio 10.

Si ab vnitate numeri qnotcunque proportionales sint, non sit auté quadratus is qui vnitatem sequitur, neq; alius vllus quadratus erit, demptis tertio ab vnitate ac om

o Vnitas.	A	B	C	D	E	F
	3	9	36	81	243	729

nibus vnum intermittentibus. Quòd si qui vnitatem sequitur, cubus non sit, neque alius vllus cubus erit, demptis quarto ab vnitate ac omnibus duos intermittentibus.

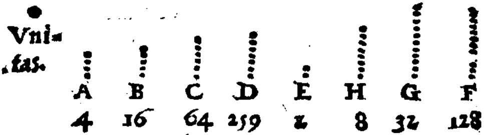
Theorema 11. Propositio 11.

Si ab vnitate numeri quotlibet deinceps proportionales sint, minor maiorem metitur per quempiam eorum qui in proportionalibus sunt numeris.

. : : : :
A D C D E
1 2 4 8 16

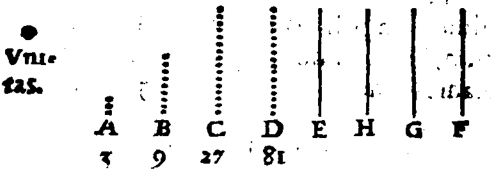
Theorema 12. Propositio 12.

Si ab vnitate quolibet numeri sint proportionales, quot primorum numerorũ vltimum metiuntur, totidem & eum qui vnitati proximus est, metientur.



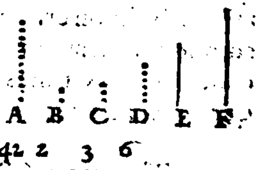
Theorema 13. Propositio 13.

Si ab vnitate sint quotcunq; numeri deinceps proportionales, primus autẽ sit qui vnitatẽ sequitur, maximũ nullus alius metietietur, ijs exceptis qui in proportionalibus ſunt numeris.



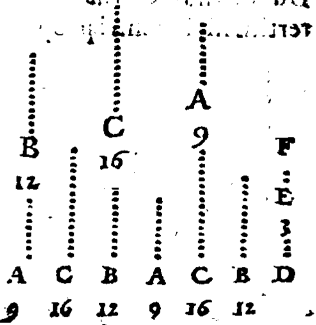
Theorema 14. Propoſitio 14.

Si minimum numerum primi aliquot numeri metiātur, nullus alius numerus primus illum metietur, ijs exceptis qui primò metiuntur.



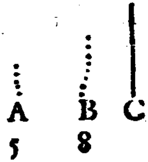
Theorema 15. Propoſitio 15.

Si tres numeri deinceps proportionales ſint minimi, eadem cum ipſis habētium rationem, duo quilibet compoſiti ad tertium primi erunt.



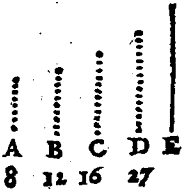
Theorema 16. propositio 16.

Si duo numeri ſint inter ſe primi, non ſe habebit quẽadmodum primus ad ſecũdum, ita ſecundus ad quẽpiam alium.



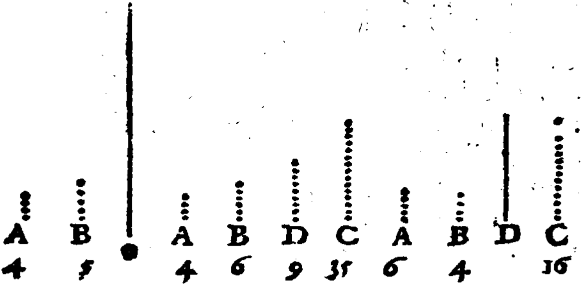
Theorema 17. Propoſitio 18.

Si ſint quotlibet numeri deinceps ꝓportionales, quorum extremi ſint inter ſe primi, non erit quẽadmodum primus ad ſecundum, ita vltimus ad quempiam alium.



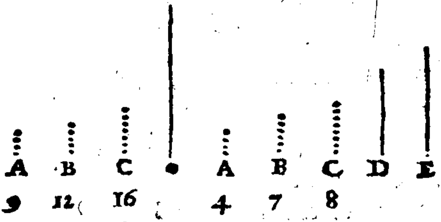
Theorema 18. Propoſitio 18.

Duobus numeris datis, cóſiderare noſſitne tertius illi inueniri proportionalis.



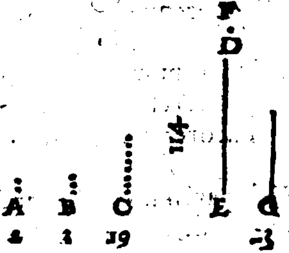
Theorema 19. Propositio 19.

Tribus numeris datis, considerare possitne quartus *illis* reperiri proportionalis.



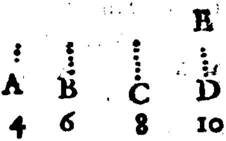
Theorema 20. Propositio 20.

Primi numeri plures sunt quacũque proposita multitudine primorum numerorum.



Theorema 12. Propositio 12.

Si pares numeri quotlibet compositi sint, totus est par.



Theorema 22. Propositio 22.

Si impares numeri quotlibet compositi sint, sit autem par illorum multitudo, totus par erit.

			E
A	B	C	D
5	9	7	3

Theorema 23. Propositio 23.

Si impares numeri quotcunque cõpositi sint, sit autem impar illorũ multitudo, & totus impar erit.

A	B	C	E
5	7	8	1

Theorema 24. Propositio 24.

Si de pari numero par detractus sit, & reliquus par erit.

	B
A	C
6	4

Theorema 25. Propositio 25.

Si de pari numerò impar detractus sit, & reliquus impar erit.

		B
A	C	D
8	1	4

Theorema 26. Propositio 26.

Si de impari numero impar detractus sit, & reliquus par erit.

		B
A	C	D
4	6	1

Theo-

Theorema 27. Propositio 27.

Si ab impari numero par ablatus sit, reliquus impar erit.

A D C
1 4 4

Theorema 28. Propositio 28.

Si impar numerus parem multiplicās, procreet quempiam, procreatus par erit.

A B C
3 4 12

Theorema 29. Propositio 29.

Si impar numerus imparē numerum multiplicans quempiam procreet, procreatus impar erit.

A B C
3 5 15

Theorema 30. Propositio 30.

Si impar numerus parē numerum metiatur, & illius dimidium metietur.

A C B
3 6 18

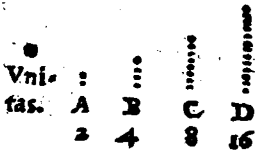
Theorema 31. Propositio 31.

Si impar numerus ad numerum quēpiam primus sit, & ad illius duplum primus erit

A B C D
7 8 16

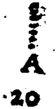
Theorema 32. Propositio 32.

Numerorum, qui à binario dupli sunt, vnusquisq; pariter par est tantùm.



Theorema 33. Propositio 33.

Si numerus dimidiũ impar habeat, pariter impar est tantùm.

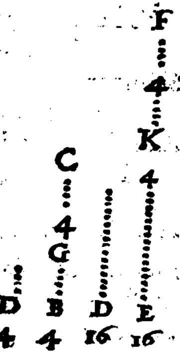


Theorema 34. Propositio 34.

Si par numerus nec sit dupl' à binario, nec dimidium impar habeat, pariter par eũ, & pariter impar.

Theorema 35. Propositio 35.

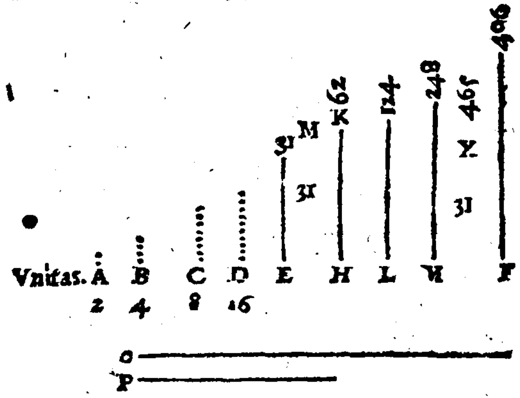
Si sint quotlibet numeri deinceps proportionales, detrahãtur aũt de secundo & vltimo æquales ipsi primo, erit quemadmodum secũdi excessus ad primũ, ita vltimi, excessus ad omnes qui vltimum antecedunt.



Theo-

Theorema 16. Propositio 16.

Si ab vnitate numeri quotlibet deinceps expositi sunt in duplici proportione quoad totus compositus primus factus sit, isq; totus in vltimum multiplicatus quempiã procreet, procreatus perfectus erit.



ELEMENTI IX. FINIS

EVCLI.

EVCLIDIS ELEMENTVM DECIMVM.

DEFINITIONES.

I

Commensurabiles magnitudines dicuntur illæ, quas eadem mensura metietur.

2

Incommensurabiles verò magnitudines dicuntur, hæ quarum nullam mensuram communem contingit reperiri.

3

Lineæ rectæ potentia cõmensurabiles sunt, quarum quadrata vna eadem superficies siue area metitur.

4

Incommensurabiles verò lineæ sunt, quarum quadrata, quæ metiatur area communis, reperiri nulla potest.

5

Hęc cùm ita sint, ostendi potest quòd quantacunque linea recta nobis proponatur, existunt etiam aliæ lineæ innumerabiles eidem commensurabiles, aliæ item incommensurabiles, hæ quidem longitudine & poten-

potentia: illæ verò potentia tantùm. Vocetur igitur linea recta, quantacunq; proponatur, ῥητὴ, id eſt rationalis.

6

Lineæ quoq; illi ῥητῇ commenſurabiles ſiue longitudine & potentia, ſiue potentia tantùm, vocentur & ipſæ ῥηταί, id eſt rationales.

7

Quæ verò lineæ ſunt incommenſurabiles illi τῇ ῥητῇ, id eſt primo loco rationali, vocẽtur ἄλογοι, id eſt irrationales.

8

Et quadratum quod à linea propoſita deſcribitur, quam ῥητὴν vocari volumus, vocetur ῥητὸν.

9

Et quæ ſunt huic commenſurabilia, vocẽtur ῥητά.

10

Quæ verò ſunt illi quadrato ῥητῷ ſcilicet incommenſurabilia, vocentur ἄλογα, id eſt ſurda.

11

Et lineę quę illa incommenſurabilia deſcribunt, vocentur ἄλογοι. Et quidem ſi illa incommenſurabilia fuerint quadrata, ipſa eorũ latera vocabuntur ἄλογοι lineæ, quòd ſi quadrata quidem non fuerint, verũ aliæ quæpiã ſuperficies ſiue figuræ rectilineæ,

tunc

tunc verò lineæ illæ quæ deſcribunt quadrata æqualia figuris rectilineis, vocentur ἄλογοι.

Theorema 1. Propoſitio 1.

Duabus magnitudinibus inæqualibus propoſitis, ſi de maiore detrahatur plus dimidio, & rurſus de reſiduo iterũ detrahatur plus dimidio, idq́; ſemper fiat: relinquetur quædam magnitudo minor altera minore ex duabus propoſitis.

Theorema 2 Propoſitio. 2.

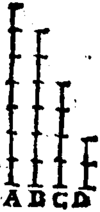
Duabus magnitudinibus propoſitis inęqualibus, ſi detrahatur ſemper minor de maiore, alterna quadam detractione neque reſiduum vnquam metiatur id quod ante ſe metiebatur, incommenſurabiles ſunt illæ magnitudines.

Problema 1. Propoſitio 3.

Duabus magnitudinibus commẽſurabilibus datis, maximam ipſarum communem menſuram reperire.

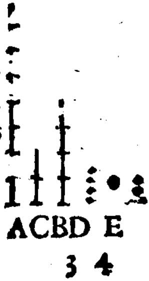
Proble-

Problema 2. Propoſitio 4.



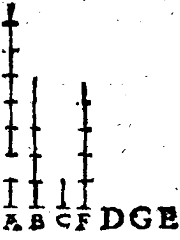
Tribus magnitudinibus cómenſurabilibus datis, maximam ipſarú communé menſuram reperire.

Theorema 3. Propoſitio 5.



Cómenſurabiles magnitudines inter ſe proportioné eam habent, quam habet numerus ad numerum.

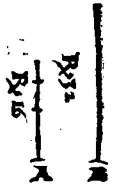
Theorema 4. Propoſitio 6.



Si duæ magnitudines proportionem eam habent inter ſe quam numerus ad numerum, commenſurabiles ſunt illæ magnitudines.

Theo-

Theorema 5. Propositio 7.



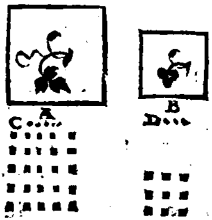
Incommensurabiles magnitudines inter se proportionem non habẽt, quam numerus ad numerum.

Theorema 6. Propositio 8.

Si duæ magnitudines inter se proportionem nõ habẽt quam numerus ad numerum, incommensurabiles illę sunt magnitudines.

Theorema 7. Propositio 9.

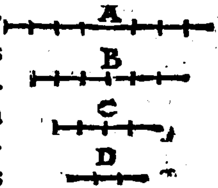
Quadrata, quæ describuntur à rectis lineis lógitudine cómensurabilibus, inter se proportionem habent quã numerus quadratus ad alium numerum quadratũ. Et quadrata habentia proportionẽ inter se quã quadratus numerus ad numerum quadratũ, habent quoque latera longitudine commẽsurabilia. Quadrata verò



quæ deſcribuntur à lineis longitudine in cómenſurabilibus, proportioné non habent inter ſe, quam quadratus numerus ad numerum alium quadratum. Et quadrata non habentia proportionem inter ſe quam numerus quadratus ad numerum quadratum, neque latera habebunt longitudine commnnſurabilia.

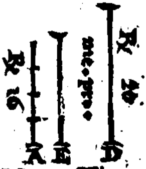
Theorema 8. Propoſitio 10.

Si quatuor magnitudines fuerint proportionales, prima verò ſecundæ fuerit commẽſurabilis, tertia quoque quartę commenſurabilis erit. quòd ſi prima ſecũdæ fuerit incommenſurabilis, tertia quoq; quartæ incommenſurabilis erit.



Problema 3. Propoſitio 11.

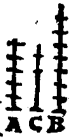
Propoſitæ lineæ rectæ (quam ῥητὴν vocari diximus) reperire duas lineas rectas incommenſurabiles, hanc quidem longitudiue tantùm, illam verò non longitudine tantùm, ſed etiam potentia incommẽſurabilem.



K

Theo-

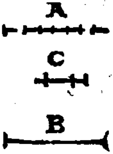
Theorema 6. Propositio 12.



Magnitudines quæ eidem magnitudini sunt commensurabiles, inter se quoque sunt commensurabiles.

6 D...... 4 F..
4 E.... 8 G..
3 H...
2 K..
4 L...

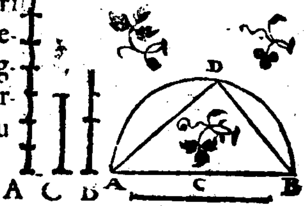
Theorema 10. Propositio 13.



Si ex duabus magnitudinibus hæc quidem commensurabilis sit tertię magnitudini, illa verò eidem incommensurabilis, incommensurabiles sunt illæ duæ magnitudines.

Theorema 11. Propositio 14.

Si duarum magnitudinum commensurabilium altera fuerit incommensurabilis magnitudini alteri cuipiam tertię, reliqua quoq; magnitudo eidem tertiæ incommensurabilis erit.



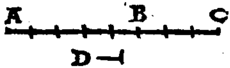
Theorema 12. Propositio 15.

Si quatuor rectæ proportionales fuerint, possit

possit autem prima plusquã secunda tanto quantum est quadratum lineę sibi commensurabilis longitudine: tertia quoq; ponterit plusquam quarta tanto quantum est quadratum lineæ sibi commensurabilis longitudine, Quòd si prima possit plusquam secunda quadrato lineę sibi longitudine incommensurabilis: tertia quoque poterit plusquam quarta quadrato lineę sibi incõmẽsurablilis longitudine.

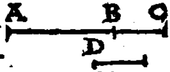
Theorema 13. Propositio 16.

Si duæ magnitudines commensurabiles componantur, tota magnitudo composita singulis partibus commensurabilis erit, quòd si tota magnitudo composita alterutri parti commensurabilis fuerit, illæ duæ quoque partes cõmensurabiles erunt.



Theorema 14. Propositio 17.

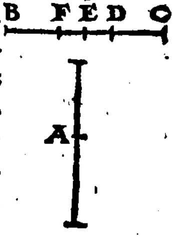
Si duæ magnitudines incommensurabiles componantur, ipsa quoque tota magnitudo singulis partibus componentibus incommensurabilis erit. Quòd si tota alteri parti incommensurabilis fuerit, illæ quoque primę magnitudines inter se incommensurabiles erunt.



Theo-

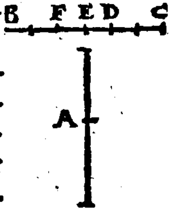
Theorema 15. Propositio 18.

Si fuerint duæ rectæ lineæ inæquales, & quartæ parti quadrati quod describitur à minore æquale parallelongrammū applicecetur secundū maiorem, ex qua maiore tātum excurrat extra latus parallelogrammi, quantum est alterum latus ipsius parallelogrammi: si præterea parallelogrammum sui applicatione diuidat lineā illam in partes inter se commensurabiles longitudine, illa maior linea tanto plus potest quam minor, quantū est quadratum lineæ sibi commensurabilis longitudine. Quòd si maior quadratū lineæ sibi commensurabilis lonplus possit quàm minor, tanto quantū est gitudine, & præterea quartæ parti quadrati lineę minoris ęquale parallelogrammum applicetur secundum maiorem, ex qua maiore tantum excurrat extra latus pallelogrammi, quantum est alterum latus ipsius parallelongrammi parallelogammum sui applicatione diuidit maiorē in partęs inter se longitudine commensurabiles.

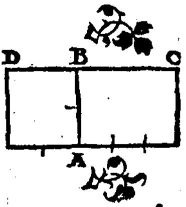


Theorema 16. Propositio 19.

Si fuerint duæ rectæ inæquales, quartæ autem parti quadrati lineæ minoris æquale le parallelogrammorum ſecundum lineam maiorem applicetur, ex qua linea tantum excurrat extra latus parallelogrammi, quantum eſt alterum latus eiuſdem parallelogrammi: ſi parallelogrammum præterea ſui applicatione diuidat lineam in partes inter ſe longitudine incommenſurabiles, maior illa linea tantò plus poteſt quàm minor quantum eſt quadratum lineæ ſibi maiori incommenſurabiles longitudine. Quod ſi maior linea tantò plus poſſit quàm minor, quantum eſt quadratum lineæ incommenſurabilis ſibi longitudine: & preterea quartæ parti quadrati lineæ minoris æquale parallelogrammum applicetur ſecundum maiorem ex qua tantum excurrat extra latus parallelogrammi, quantum eſt alterum latus ipſius: parallelogrammum ſui applicatione diuidit maiorem in partes inter ſe incommenſurabiles longitudine.



Theorema 17. Propoſitio 20.

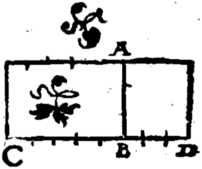


Superficies rectangula contenta ex lineis rectis rationalibus longitudine commenſurabilibus ſecundum

cundum vnum aliquem modum ex antedi-ctis, rationalis eſt.

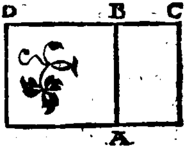
Theorema 18 Propoſitio 21.

Si ratinonale ſecundũ lineam rationalem applicetur, habebit alterum latus lineam rationalem & cõmenſurabilem longitudine lineæ cui rationale parallelogrammum applicatur.



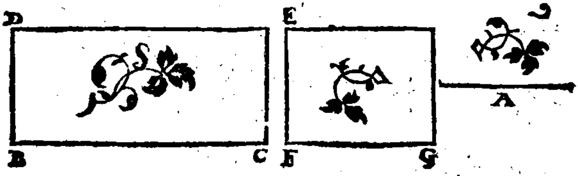
Theorema 39. Propoſitio 56.

Surperficies rerctangula contenta duabus lineis rectis rationalibus potentia tantũ commẽſurabilibus, irrationalis eſt. Linea autem quæ illam ſuperficiem poteſt, irrationalis & ipſa eſt: vocetur verò medialis



Theorema 20. Propoſitio 23.

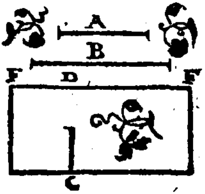
Quadrati lineæ medialis applicati ſecun-



dum

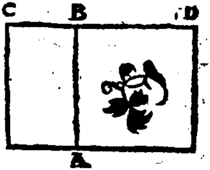
dum lineam rationalem, alterum latus est linea rationalis, & incommensurabilis longitudine lineæ secundum quam applicatur.

Theorema 21. Propositio 24



Linea recta mediali commēsurabilis, est ipsa quoque medialis.

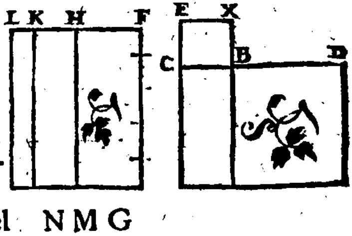
Theorema 22. Propositio 25



Parallelongrammum rectangulum contētum ex lineis medialibus longitudine commensurabilibus, mediale est.

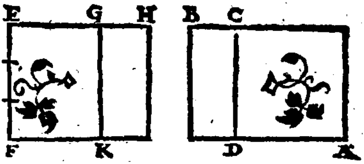
Theorema 23. Propositio 26.

Parallelogrammum rectangulum compreheusum duabus lineis medialibus potentia tātùm cōmensurabilibus, vel rationale est, vel mediale.



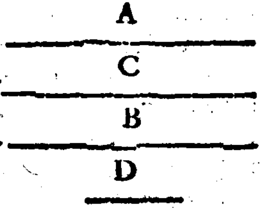
Theorema 24. Propositio 27.

Mediale nó est maius quàm mediale superficie rationali.



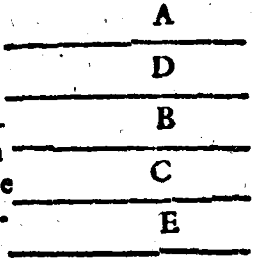
Problema.4. Propositio 28.

Mediales lines inuenire potentia tantùm commensurabiles rationale comprehendentes.



Problema 5. Propositio 29.

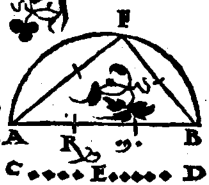
Mediales lineas inuenire potentia tantùm commensurabiles mediale comprehendendentes



Pro-

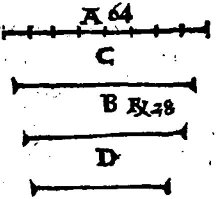
Problema 6. Propositio 30.

Reperire duas rationales potentia tantum commẽsurabiles huiusmodi, vt maior ex illis possit plus quàm minor quadrato lineæ sibi commensurabilis longitudine.



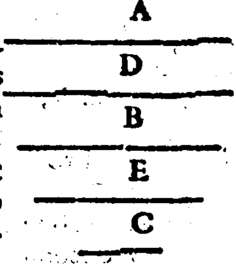
Problema 7. Propositio 31.

Reperire duas lineas medialis potentia tantùm commensurabiles rationalem superficiẽ continẽtes, tales in quã vt maior possit plus quàm minor quadrato lineæ sibi commẽsurabilis longitudine.



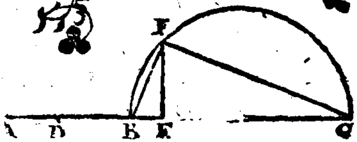
Problema 8. Propositio 32.

Reperire duas lineas mediales potentia tantùm commensurabiles medialem superficiem continentes, huiusmodi vt maior plus possit quàm minor quadrato lineæ sibi commensurabilis longitudine.



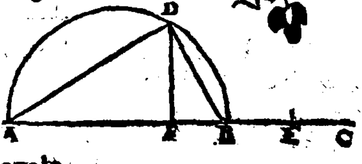
Problema 9. Propositio 33.

Réperire duas rectas poutentia incommésurabiles, quarum quadrata simul addita faciant superficié rationalé, parallelogrammum verò ex ipsis conetntum sit mediale.



Problema 10. Propositio 34.

Reperire lineas duas rectas potentia incommensurabiles conficentes compositnm ex ipsarum quadratis mediale, parallelogrammũ verò ex ipsis contentum rationale,



Problema 11. Propositio 85.

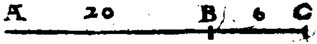
Reperire duas lineas rectas potentia incómensurabiles, conficientes id quod ex ipsaram quadratis componitur meale, simulque

que parallelogrammũ ex ipsis contentum, mediale, quod præterea parallelogrammũ sit incōmensurabile composito ex quadratis ipsarum.

PRINCIPIVM SENARIOrum per compositionem.

Thorema 25. Propositio 36.

Si duæ rationales potentia tantùm commensurabilis. Vocetur autem Binommum.



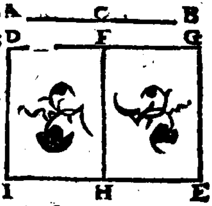
Theorema 26. Propositio 37.

Si duæ mediales potentia tantòm commensurabiles rationale continentes componantur, tota linea est irrationalis, vocetur autem Bimediale prius.

The-

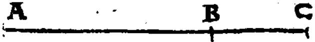
Theorema 27. Propoſitio 38.

Si duæ mediales Potētia tantùm commēſurabiles mediale continētes componātur, tota linea eſt irrationalis: vocetur autē Bimediale ſecundum.



Theorema 28. Propoſitio 36.

Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex quadratis ipſarum rationale parallelogrammum verò ex ipſis contentum mediale, tota linea recta



eſt irrationalrs. Vocetur autē linea maior.

Theorema 39. Propoſitio 40.

Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex ipſarum quadratis mediale, id vero ꝙ fit ex ipſis, rationale, tota linea eſt irrationales. Vocetur au-

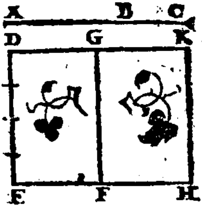
tem potens rationale & mediale.

Theorema 30. Propoſitio 41.

Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex quadratis ipſarum mediale, & quod conti-

netur

n·tur ex ipſis, mediale, & prӕterea incõmenſurabile compoſito ex quadratis ipſarum tota linea eſt irrationalis. Vocetur autem potens duo medialia.



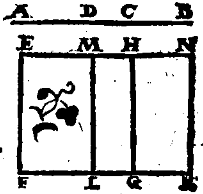
Theorema 31. Propoſitio 42.

Binominum in vnico tantùm pũcto diuiditur in ſua nomina, id eſt in lineas ex quibus componitur.

Theorema 32. Propoſitio 43.

Bimediale prius in vnico tantum puncto diuiditur in ſua nomina.

Problema 33. Propoſitio 44



Bimedíale ſecundum in vnico tantũ puncto diuiditur in ſua nomina.

Problema 34. Propoſitio 45.

Linea maior in vnico tantùm in pũcto diuiditur in ſua nomina.

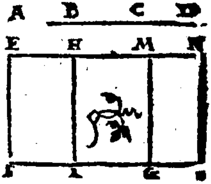
Theo-

Theorema 35. Propoſitio 46.

Linea potens rationale & mediale in vnico tantùm puncto diuiditur in ſua nomina.

Theorema 36. Propoſitio 47

Linea ootèns duo media lia in vnico tantùm puncto diuiditur in ſua nomina.



DEFINITIONES.
ſecundæ.

Propoſita linea rationali, & binomio diuiſo in ſua nomine, cuius binomij maius nomen, id eſt, maior portio poſſit pluſquam minus nomen quadrato lineæ ſibi, maiori inquam nomini, commenſurabilis longitudine.

I.

Si quidẽ maius nomen fuerit commenſurabile longitudine propoſitæ lineæ rationali, vocetur toto linea Binomum primium.

2

Si veró minus nomen, id eſt minor portio binomij, fuerit commenſurabile longitudine ne propositæ lineæ rationali, vocetur tota linea Binomium secundum.

3

Si verò neutrum nomen fuerit commensurabile longitudine propositæ lineæ rationali, vocetur Binomium tertium.

Rursus si maius nomen possit plusquam minus nomen quadrato lineæ sibi incommensurabilis longitudine.

4

Si quidem maius nomen est commensurabile longitudine propositæ lineæ rationali, vocetur tota linea Binomium quartum.

5

Si verò minus nomen fuerit commensurabile longitudine lineæ rationali, vocetur Binomium quintum.

6

Si verò neutrum nomen fuerit longitudine commensurabile lineæ rationali, vocetur illa Binomium sextum.

D

D 16 F 12 G

H

12 4
A.....C....B
16

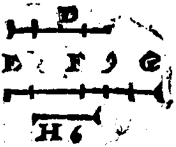
Proble.12. Propositio 48

Reperire Binomium primum.

Pro-

Problema 13. Propositio 49

9 3
A.........C...B
12

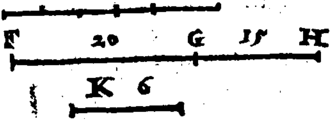


Reperire Binomium secudum.

Problema 14. Propositio 50.

15 5
A...............C....
20

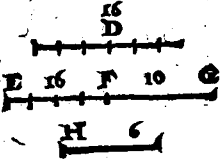
D
....
E



Reperire Binomium tertiũ.

Problema 15. Propositio 51.

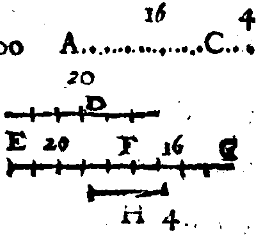
10 6
A.........C......B



Repere Binomiũ quartum.

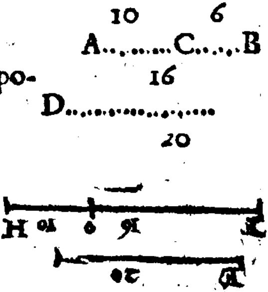
Proble-

Problema 16. Propositio 52



Reperire Binomium quintum.

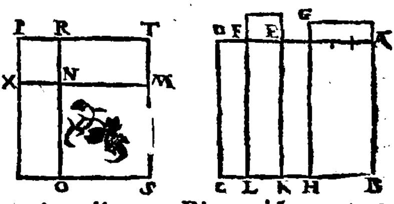
Probl. 17. Propositio 53.



Reperire Binonimum sextum.

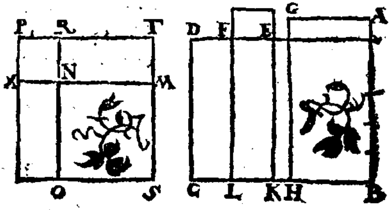
Theorema 37. Propositio 54.

Si superficies contēta fuerit ex rationali & Binomio primo, linea quę illā superficiem potest, est irrationalis, quæ Binomiū vocatur.



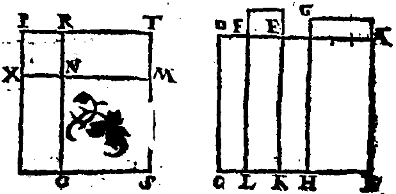
Theorema 38. Propositio 55.

Si superficies cōtenta fuerit ex linea rationali & Binomio secundo, linea potens illam superficiē est irrationalis quę Bimediale primum vocatur.



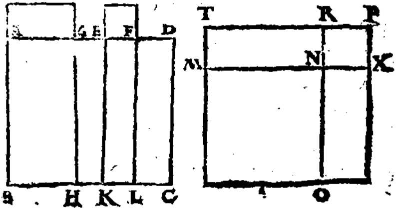
Theorema 39. Propositio 56.

Si superficies cōtineatur ex rationali & Binomio tertio, linea q̄ illam superficiem potest, est irrationalis q̄ dicitur Bimediale secundū.



Theorema 40. Propositio 57.

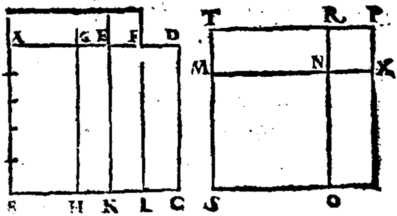
Si superficies contineatur ex rationali & Boin mio



mio quarto. linea potens ſuperficiem illam, eſt irrationalis, quæ dicitur maior.

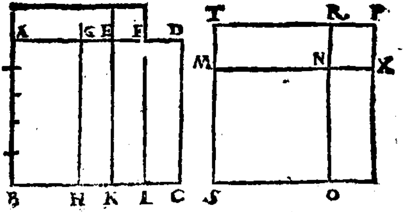
Theorema 41. Propoſitio 58.

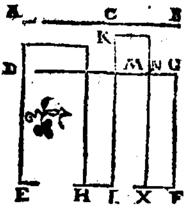
Si ſuperficies contineatur ex rationali & Binomio quinto, linea quæ illam ſuperficiem poteſt eſt irrationalis, quę dicitur potẽs rationale & mediale.



Therema 42. Propoſitio 59

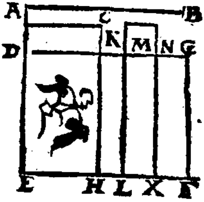
Si ſuperficies contineatur ex rationali & Binomio ſexto, linea quæ illam ſuperficiem poteſt eſt irrationalis, quæ dicitur potens tuo mediala.





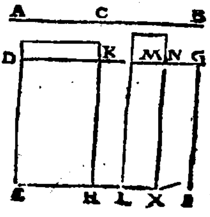
Theorema 43. Propositio 60.

Quadratum Binomij secundum lineam rationalem applicatum, facit alterum latus Binomium primum.



Theorema 44. Propositio 61.

Quadratũ Bimedialis primi secundum rationalem lineam applicatum, facit altérum latus Binomium secundum.

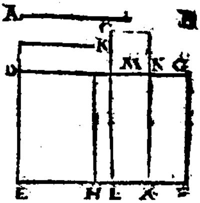


Theorema 45. Propositio 62.

Quadratũ Bimedialis secũdi secundũ rationalem applicatum, facit alterũ latus Binominũ tertium.

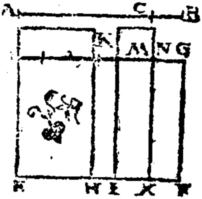
Theorema 46. ropo-
positio 63

Quadratum lineæ maioris secundum lineam rationalem applicatum, facit alterum latus Binomium quartum.



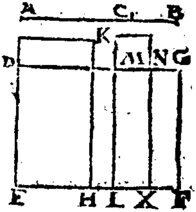
Theorema 47. Propositio 64.

Quadratum lineæ potẽtis rationale & mediale secundum rationalẽ applicatũ, facit alterum latusBinomium quintum.



Theorema 48. Propositio 65

Quadratum lineę potentis duo medialia secundũ rationalem applicatum, facit alterum latus Binomium sextum.

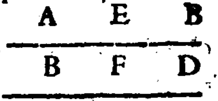


Theorema 49. Propositio 66.

Linea lõgitudine commensurabilis Binomio est & ipsa Binomium eiusdem ordinis.

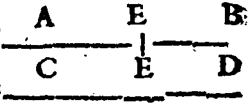
Theorema 50. Propoſitio 67

Linea longitudine commenſurabilis alteri bimedialum, eſt & ipſa bimediale etiam eiuſdẽ ordinis.



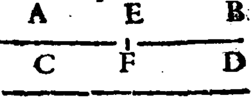
Theorema 51. Propoſitio 68

Linea cõmenſurabilis lineæ maiori, eſt & ipſa maior.



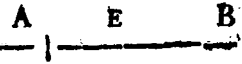
Theorema 52. Propoſitio 69.

Linea commenſurabilis lineæ potenti rationale & mediale, eſt & ipſa linea potens rationale & mediale.



Theorema 53. Propoſitio 70.

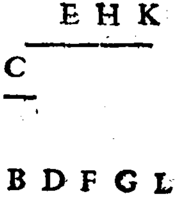
Linea commenſurabilis lineæ potenti duo medialia, eſt & ipſa linea potens duo medialia.



Theorema 54 Propoſitio 71

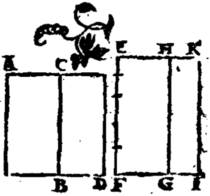
Si duæ ſuperficies rationalis & medialis ſimul componantur, linea quę totã ſuperficiem

ciem compositá potest, est vna ex quatuor irrationalibus, vna ex quæ dicitur Binomium, vel bimediale primum. vel linea maior, vel linea potés rationale & mediale.



Theorema 55. Propositio 72.

Si duæ superficies mediales incommensurabiles simul componantur fiunt reliquæ duæ lineę irrationales, vel bimediale secundum, vel linea potés duo medialia



SCHOLIVM.

Binomium & cæteræ consequentes lineæ irrationales, neque sunt eadem cum linea mediali; neque ipsæ inter se.

Nam quadratum lineæ medialis applicatum secundum lineam rationalem facit alterum latus lineam rationalem, & longitudine incommensurabilem lineæ secundum quam applicatur, hoc est, lineæ rationali, per 23.

Quadratum verò Binomij secundum rationalem applicatum, facit alterum latus Binomium primum, per 60.

 Quadra-

Quadratum verò Bimedialis primi secundum rationalem applicatum, facit alterum latus Binomium secundum. per 61

Quadratum verò Bimedialis secundi secundum rationalem applicatum; facit alterum latus Binomiũ tertium per 62.

Quadratum verò lineæ maioris secundum rationalem applicatum facit alterum latus Binomiũ quartum, per 63

Quadratum verò lineæ potẽtis rationale & mediale secundum rationalem applicatum, facit alterũ latus Binomium quintum, per 64.

Quadratum vero lineæ potentis duo medialia secũdum rationalem applicatum, facit alterum latus Binomium sextum, per 65.

Cùm igitur dicta latera, quæ latitudines vocantur, differant & à prima latitudine, quoniam est rationalis, cùm inter se quoque differant, eo quia sunt Binomia diuersorum ordinum: manifestum est ipsas lineas irrationales, differentes esse inter se.

SECVN.

SECVNDVS ORDO ALterius sermonis, qui est de detractione.

Principium senariorum per detractionem.

Theorema 57. Propositio 74.

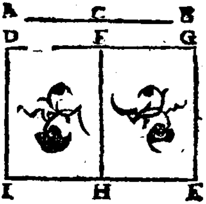
Si de linea rationali detrahatur rationalis potentia tantùm commensurabilis ipsi toti, residua est irrationalis, vocetur autem Residuum.

Theorema 56. Propositio 37.

Si de linea mediali detrahatur medialis potentia tantũ commensurabilis toti lineæ, quæ verò detracta est cũ tota contineat superficiem rationalem, residua est irrationalis. Vocetur autem Residuum mediale primum.

Theorema 58 Propositio. 75.

Si de linea mediali detrahatur medialis



tentia tantùm commensurabilis toti, quæ verò detracta est, cùm tota cōtineat superficiē medialē, reliqua est irrationalis. Vocetur autem Residuū mediale secundum.

Theorema 57. Propositio 76

Si de linea recta detrahatur recta potētia incommensurabilis toti, compositum autem ex quadratis totius lineæ & lineæ detractæ sit rationale, parallelogrammum verò ex ijsdem contentum sit mediale, reliqua linea erit irrationalis. Vocetur autem linea minor.

Theorema 58. Propositio 77.

Si de linea recta detrahatur recta potentia

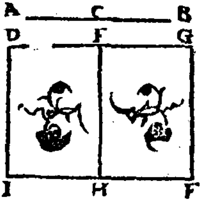
tia incommensurabilis toti lineæ compositum autem ex quadratis totius & lineæ detractæ sit mediale, parallelogrammum verò bis ex eisdem contentum sit rationale, reliqua linea est irrationalis. Vocetur autem linea faciens cum superficie rationali totam superficiem medialem.

Theorema 59. Propositio 78.

Si de linea recta detrahatur recta potentia incommensurabilis toti lineæ, compositum autem ex quadratis totius & lineæ detractæ sit mediale, parallelogrammum verò bis ex ijsdem sit etiam mediale: præterea sint quardata ipsarum incommensurabilia parrallelogrammo bis ex ijsdem contento, reliqua linea est irrationalis.

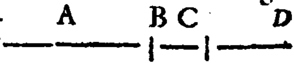
Vocetur autem linea faciens cum superficie

ficie mediali totam ſuperficiem mediałem.



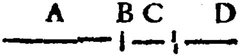
Theorema 60 Propoſitio 79.

Reſiduo vnica tantùm linea recta cóiungitur rationalis potentia tantùm cómenſurabilis toti lineæ rationalem.



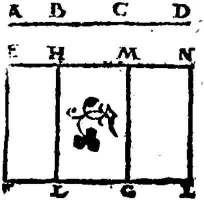
Theorema 61. Propoſitio 80.

Reſiduo mediali primo vnica tãtùm linea coniungitur medialis, potentia tantùm cómenſurabilis toti, ipſa cum tota continens



Theorema 62. Propoſitio 81.

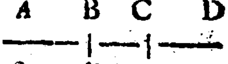
Reſiduo mediali ſecundo vnica tantùm coniungitur mediails, potentia tantùm commẽſurabilia toti ipſa cum tota continens mediale.



Theorema 63. Propoſitio 82.

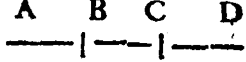
Lineæ minori vnica tantùm recta cóiungitur

tur potentia incommensurabilis toti, faciens cum tota compositum ex quadratis ipsarum rationale, id verò parallelogrammum, quod bis ex ipsis fit, mediale.



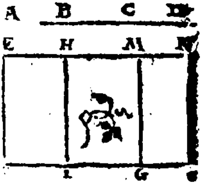
Theorema 64. Propositio 83.

Lineæ facienti cum superficie rationali totam superficiem medialem, vnica tantùm coniungitur linea recta potentia incommensurabilis toti, faciens autem cum tota compositum ex quadratis ipsarũ, mediale, id verò quod fit bis ex ipsis, rationale.



Theorema 65. Propositio 84.

Lineæ cum mediali superficie facienti totam superficiem medialem, vnica tantum coniũgitur linea potẽtia toti incommensurabilis, faciens cum tota compositũ ex quadratis ipsarũ mediale, id verò quod fit bis ex ipsis etiam mediale, & præterea faciens compositum ex quadratis ipsarum incommensurabile ei quod fit bis ex ipsis,



DE.

DEFINITIONES. TERTIAE

Proposita linea rationali & residuo.

1

Si quidem tota, nempe composita ex ipso residuo & linea illi cõiuncta, plus potest quàm coniuncta, quadrato lineæ sibi cõmensurabilis longitudine, fueritq́; tota longitudine commẽsurabilis lineæ propositæ rationali, residuũ ipsum vocetur Residuum primum.

2

Si verò coniuncta fuerit longitudine commensurabilis rationali, ipsa autem tota plus possit quam coniuncta, quadrato lineæ sibi longitudine commensurabilis, residuum vocetur Residuum secundũ.

3

Si verò neutra linearũ fuerit longitudine commensurabilis rationali, possit autem ipsa tota plusquam coniuncta, quadrato lineæ sibi longitudine commẽsurabilis, vocetur Residuum tertium.

Rursus si tota possit plus quàm coniuncta, quadrato lineæ sibi longitudine incommensurabilis

4

Et quidem si tota fuerit longitudine commenmensurabilis ipsi rationali, vocetur Residuum quartum.

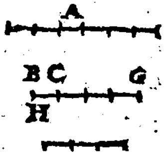
5

Si verò coniuncta fuerit lõgitudine commensurabilis rationali, & tota plus possit quàm coniuncta, quadrato lineæ sibi longitudine incõmensurabilis, vocetur Residuum quintum.

6

Si verò neutra linearum fuerit cõmensurabilis longitudine ipsi rationali, fueritque tota potentior quã coniuncta, quadrato lineæ sibi longitudine incommensurabilis, vocetur Residuum sextum.

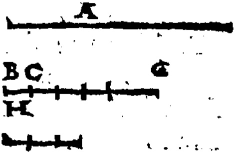
Problema 18. Propositio 85.



Reperire primum Residuum.

16
D.....F.......E
9 7

Problema 19. Propositio 86.



Reperire secundum Residuum.

D........F......E.
27 9

Prob-

E.........

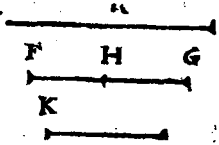
Problema 02. Propositio 87.

12

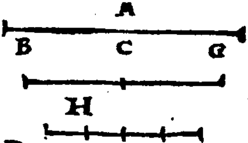
B..........E......C

9 7

Reperire tertium Residuum.



Probl. 21. Propositio 88.

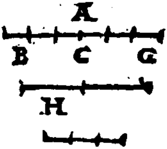


Reperire quartum Residuum.

D..............F...E

16 4

Problema 22. Propositio 89



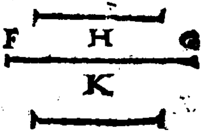
Reperire quintum Residuum

D......F....E

25 7

Problema 23. Propositio 90.

Reperire sextum Residuum.

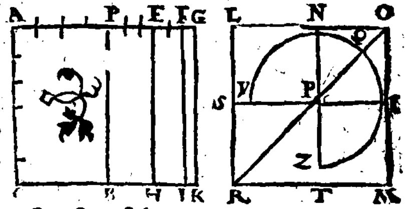


E............ 13

Theo- B........D.......C

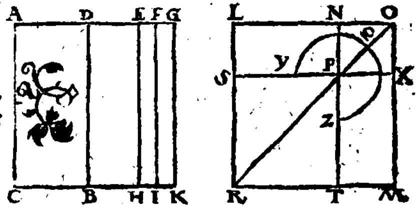
Theórema 66. Propoſitio 91.

Si ſuperficies contineatur ex linea rationali & reſiduo primo linea, quæ illam ſuperficiem poteſt, eſt reſiduum.



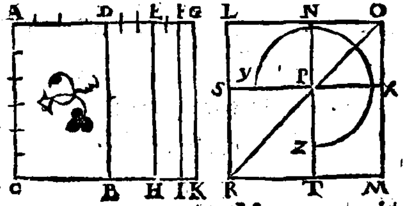
Theorema 67. Propoſitio 92.

Si ſuperficies cõtineatur ex linea rationali & reſiduo ſecundo, linea q̃ illam ſupſiciẽ poteſt, eſt reſiduum mediale primum.



Theorema 68. Propoſitio 93.

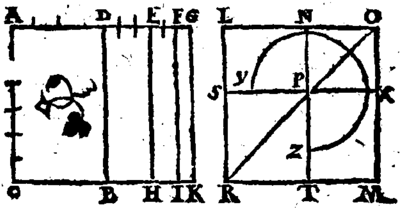
Si ſuperficies cõtineatur ex linea rationali & reſiduo ter-



tio,linea quæ illam ſuperficiem poteſt, eſt reſiduum mediale ſecundum.

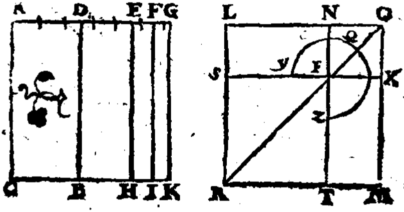
Theorema 69. Propoſitio 94.

Si ſuperficies contineatur ex linea rationali & reſiduo quarto linea q̃ illá ſuperficiem poteſt, eſt linea minor.



Theorema 70. Propoſitio 95.

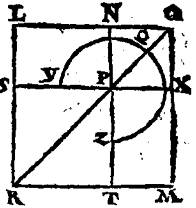
Si ſuperficies contineatur ex linea rationali & reſiduo quinto,linea quæ illá ſuperficié poteſt,eſt ea quæ dicitur cum rationali ſuperficie faciens totam medialem.



Theorema 71. Propopoſitio 96.

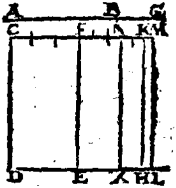
Si ſuperficies contineatur ex linea rationali &

&residuo sexto, linea quæ illam superficiẽ pót, est ea quę dicitur faciẽs cum mediali



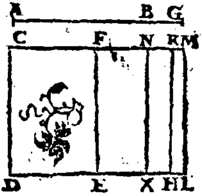
superficie totam medialem.

Theorema 72. Propositio 97.



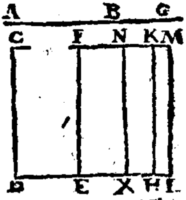
Quadratum residui secundum lineam rationalem applicatum, facit alterum latus redsiuum primum.

Theorema 73. Propositio 98.



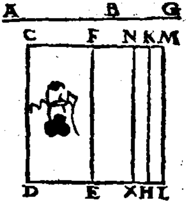
Quadratum residui medialis pɪimi secundũ rationalem applicatũ, facit alterum latus residuũ secundum.

Theorema 74. Propositio 99.



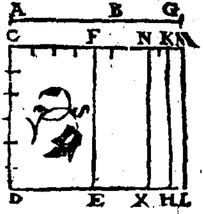
Quadratũ, residui medialis secundi secundum rationalẽ applicatũ, facit alterũ latus residuum tertium.

Theorema 75. Propositio 100.



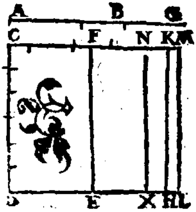
Quadratũ lineæ minoris secundùm rationalẽ applicatum, facit alterũ latus residuum quartũ.

Theorema 76. Propositio 101.



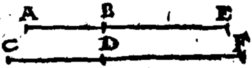
Quadratũ lineæ cum rationali superficie faciẽtis totam medialem, secũdum rationalem applicatum, facit alterum latus residuum quintum.

Theorema 77. Propositio 102.



Quadratum lineæ cum mediali superficie faciẽtis totam medialem, secundum rationalem applicatum, facit alterum latus residuum sextum.

Theorema 78. Propositio 103.

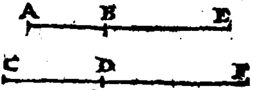


Linea residuo commensurabilis longitudine, est & ipsa residuum, & eiusdem ordinis

Theorema 79. Propositio 104.

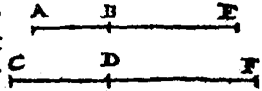
Li-

Linea cõmenſurabilis reſiduo mediali, eſt & ipſa reſiduũ mediale, & eiuſdẽ ordinis.



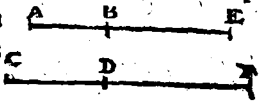
Theorema 80. Propoſitio 105.

Linea cõmẽſurabilis lineę minori, eſt et ipſa linea minor



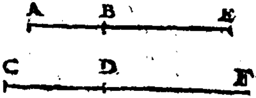
Theorema 81. Propoſitio 106.

Linea commenſurabilis lineæ cũ rationali ſuperficie facienti totã medialẽ, eſt & ipſa linea cum rationali ſuperficie faciens totam medialem.



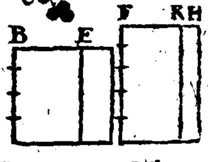
Theorema 82. Propoſitio 107.

Linea commẽſurabilis lineæ cũ mediali ſuperficie facienti totam medialem, eſt & ipſa cum mediali ſuperficie faciens totam medialem.



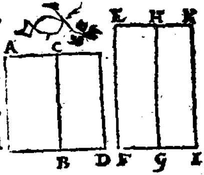
Theorema 83. Propoſitio 108.

Si de ſuderficie rationali detrahatur ſuperficies medialis, linea quæ reliquã ſuperficiem poteſt, eſt alterutra ex duabus irrationalibus, aut reſiduum, aut liuea minor.



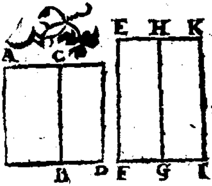
Theorema 84. Propositio 109.

Si de superficie mediali detrahatur superficies rationalis, aliæ duæ irrationales, fiũt, aut residuum mediale primũ aut cũ rationali superficiem faciens totam medialem.



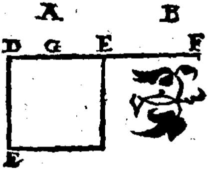
Theorema 85. Propositio 110.

Si de superficie mediali detrahatur superficies medialis q̃ sit incommẽsurabilis toti, reliquę duæ fiunt irrationales, aut residuum mediale secundum, aut cũ mediali superficie faciens totam medialem.



Theorema 86. Propositio 111.

Linea quæ Residuum dicitur, nó est eadem cum ea quæ dicitur Binomium.



SCHO-

SCHOLIVM.

Linea quæ Residuum dicitur, & cæteræ quinque eã consequentes irrationales, neque linea mediali neque sibi ipsa inter se sunt eadem. Nam quadratum lineæ medialis secundum rationalem applicatum facit alterum latus, rationalem lineam longitudine incommensurabilem ei, secundum quam applicatur, per 23.

Quadratum, verò residui secundum rationalem applicatũ, facit alterũ latus residuũ primũ, per 97.

Quadratum verò residui medialis primi secundum rationalem applicatum facit alterum latus residuum secundum, per 98.

Quadratum verò residui medialis secundi, facit alterum latus residuum tertium, per 99.

Quadratum verò lineæ minoris, facit alterũ latus residuum quartum, per 100.

Quadratum verò lineæ cum rationali superficie facientis totam medialem, facit alterum latus residuum quintum, per 101.

Quadratum verò lineæ cum mediali superficie facientis totam medialem secundum rationalem applicatum facit alterum latus residuum sextum, per 102.

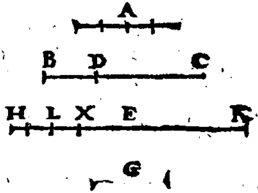
Cùm igitur dicta latera, quæ sunt latitudines cuiusque parallelogrammi vnicuique quadrato æqualis & secundum rationalem, applicati, differant & à primo latere, & ipsa inter se (nam à primo differunt: quoniam sunt residua non eiusdem ordinis) constat ipsas quoque, lineas irrationales inter se defferentes esse. Et quoniam demonstratum est, Residuum non esse idem quod Binomium, quadrata autem residui & quinque linearum irrationalium illud consequentium, secundum rationalem applicata faciunt altera latera ex residuis eiusdem ordinis cuius sunt & residua, quorum quadrata applicantur rationali similiter & quadrata Binomij & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex Binomijs eiusdem ordinis, cuius sunt & Binomia, quorum quadrata applicatur rationali. Ergo lineæ irrationales quæ cōsequuntur Binomium, & quæ consequuntur residuū, sunt inter se differētes. Quare dictæ lineæ omnes irrationales sunt numero 13.

1 Medialis.
2 Binomium.
3 Bimediale primum.
4 Bimediale secundum.
5 Maior.
6 Potens rationale & mediale.
7 Potēs duo medialia.
8 Residuum.
9 Residuum mediale primum.
10 Residuum mediale secundum.
11 Minor.
12 Faciens cum rationali superficie totam medialem.
13 Faciens cum mediali superficie totam medialem.

Theo-

Theorema 87. Propositio 112.

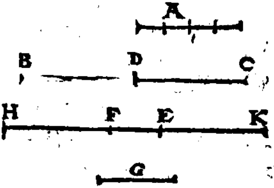
Quadratũ lineæ rationalis ſecundum Binomiũ applicatũ, facit alterum latus reſiduũ, cuius nomina ſunt cómenſurabilia Binomij nominibus, & in eadé proportione



præterea id quod fit Reſiduum, eundem ordinem retinet quem Binomium.

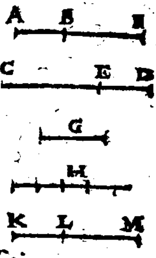
Theorema 88. Propoſitio 113.

Quadratũ lineæ rationalis ſecundum reſiduum applicatũ, facit alterum latus Binomium, cuius nomina ſunt commenſurabilia nominibus reſidui & in eadé proportione: prę terea id quod fit Binomiũ, eſt eiuſdem ordinis, cuius & Reſiduum.



Theorema 89 Propoſitio 114.

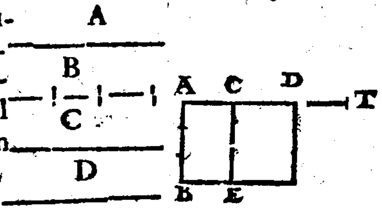
Si parallelogrammum contineatur ex reſi-



duo'& Binomio, cuius nomina ſunt commenſurabilia nominibus reſidui & in eadẽ proportione, linea quæ iliam ſuperficiem poteſt, eſt rationalis,

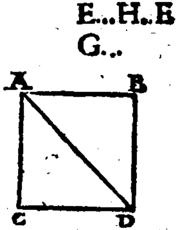
Theorema 90. Propoſitio 115.

Ex linea mediali naſcũtur lineę irrationales innumerabiles, quarum nulla vlli ante dictarum eadem ſit.



Theorema 100. Propoſitio 116.

Propoſitũ nobis eſto demonſtrare in figuris quadratis diametrũ eſſe longitudine incommenſurabilem ipſi lateri.



ELEMENTI X. FINIS.

EVCLIDIS ELEMENTVM VNDECIMVM, ET SOLIDORVM.

prim m.

DEFINITIONES.

1

Solidum, eſt quod longitudinem, latitunem, & craſſitudinem habet.

2

Solidi autem extremum eſt ſuperficies.

3.

Linea recta eſt ad planum recta, cùm ad rectas oés lineas, à quibus illa tangitur, quæque in propoſitio ſunt plano, rectos angulos efficit.

4

Planum ad planum rectum eſt, cum rectæ lineæ, quæ cõmuni planorum ſectioni ad rectos angulos in vno planorum ducũtur, alteri plano ad recte: ſunt angulos.

5

Rectæ lineę ad planũ inclinatio, acutus eſt angulus, ipſa inſiſtente linea & adiũcta altera cõprehẽſus, cùm à ſublimi rectæ illius us lineę termino deducta fuerit ꝑpẽdicularis,

laris,atq; à puncto quod perpendicularis in ipso plano fecerit,ad proposite illius lineæ extremum, quod in eodem est plano, altera recta linea fuerit adiuncta.

6

Plani ad planũ inclinatio, acutus est angulus rectis lineis contentus, quæ in vtroque planorum ad idem communis sectionis pũctũ ductæ, rectos ipsi sectioni angulos efficiunt.

7

Planum similiter inclinatum esse ad planum,atq; alterum ad alterum dicitur,cum dicti inclinationum anguli inter se sunt æquales.

8

Parallela plana,sunt quæ eodem non incidunt,nec concurrunt.

9

Similes figuræ solidæ, sunt quæ æqualibus planis,multitudine æqualibus continetur.

10

Aequales & similes figuræ solidæ sunt, quę similibus planis, multitudine & magnitudine æqualibus continentur.

11

Solidus angulus,est plurium quàm duarũ linearum, quæ se mutuò contingant, nec in eadem sint superficie, ad omnes lineas inclinatio. Aliter.

Aliter.

Solidus angulus, est qui pluribus quã duobus planis angulis, in eodem non cõsistentibus plano, se ad vnum punctum collectis continetur.

12

Pyramis, est figura solida quæ planis continetur, ab vno plano ad vnum punctũ collecta.

13

Prisma figura est solida quę planis continetur, quorum aduersa duo sunt & æqualia & similia & parallela, alia verò parallelogramma.

14

Sphęra est figura, quæ cõuerso circum quiescentem diametrum semicirculo continetur, cùm in eundem rursus locũ restitutus fuerit, vnde moueri cœperat.

15

Axis autem sphæræ, est quiescẽs illa linea circum quam semicirculus conuertitur.

16

Centrum verò Sphęrę est idem, quod & semicirculi.

17

Diameter autem Sphæræ, est recta quædã linea per centrum ducta, & vtrinq; à sphęræ superficie terminata.

Conus

18

Cωnus est figura, quæ conuerso circũ quiescens alterum latus eorum quæ rectum angulum continent, orthogωnio triangulo continetur, cùm in eundem rursus locum illud triangulum restitutum fuerit, vnde moueri cœperat. Atq; si quiescens recta linea æqualis sit alteri, quę circum rectũ angulum conuertitur, rectangulus erit Cωnus: sin minor, amblygωnius: si verò maior, oxygonius.

19

Axis autem Cωni, est quiescens illa linea. circum quam triangulum vertitur.

20

Basis verò Cωni, circulus est, qui à circũducta linea recta describitur.

21

Cylindrus figura est, quæ conuerso circum quiescens alterum latus eorum quæ rectum angulum continent, parallelogrammo orthogωnio comprehenditur, cum in eũdem rursus locum restitutum fuerit illud parallelogrammũ, vnde moueri cœperat.

22

Axis autem Cylindri est quiescens illa recta linea, circum quam parallelogrammum vertitur.

23

Bases ver cylindri, sunt circuli à duobus ad-

aduersus lateribus quæ circum aguntur, descripti.

24

Similes coni & cylindri, sunt quorũ & axes & basium diametri proportionales sunt.

25

Cubus est figura solida, quæ sex quadratis æqualibus continetur.

16

Tetraedum est figura, quæ triangulis quatuor æqualibus & æquilateris continetur.

17

Octaedrũ figura est solida, quæ octo triangulis æquilibus & æquilateris continetur.

28

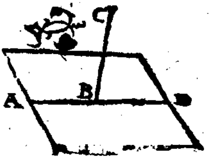
Dedecaedrum figura est solida, quæ duodelim pentagonis æqualibus, æquilateris. & æquiangulis continetur.

29

Eicosaedrum figura est solida, quę triangulis viginti æqualibus & æquilateris continetur.

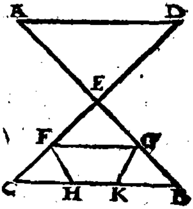
Theorema 1. propositio 1.

Quædã rectę lineę pars in subiecto quidem non est plano, quædam verò in sublimi.



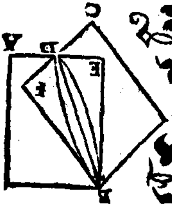
Theo

Tehorema 2. Propositio 2.



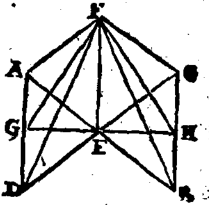
Si duę rectę lineę se mutuò secēt, in vno sūt plano: atque triangulum omne in vno est plano.

Theorema 3. Propositio 3.



Si duo plana se mutuò secent, communis eorū sectio est recta linea.

Theorema 4 Propositio 4.



Si recta linea rectis duabus lineis se mutuò secantibus, in cōmuni sectione ad rectos angulos insistat illa ducto etiam per ipsas plano ad angulos rectos erit.

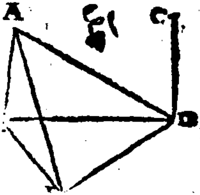
Theorema 5. Propositio 5.

Si recta linea rectis tribus lineis se mutuò tangentibus, in communi sectione ad rectos angulos insistat, illæ tres rectę in vno sunt plano.

Theo-

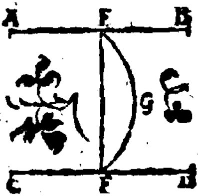
Theorema 6. Propoſitio 6.

Si duæ rectæ lineæ eidẽ plano ad rectos ſint angulos, parallelæ erunt illæ rectæ lineæ



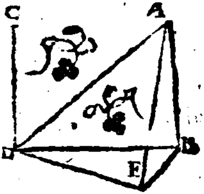
Theorema 7. Propoſitio 7.

Si duæ ſint parallelæ rectæ lineæ, in quarũ vtraque ſumpta ſint quę libet pũcta, illa linea quę ad hæc puncta adiungitur, in eodem eſt cũ parallelis plano.



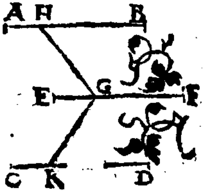
Theorema 8. Propoſitio 8.

Si duæ ſint parallelæ rectæ lineæ, quarum altera ad rectos cuidam plano ſit angulos, & reliqua eidẽ plano ad rectos angulos erit



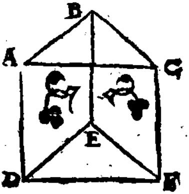
Theorema 9. Propoſitio 9.

Quæ eidem rectæ lineæ ſunt parallelæ, ſed nõ in eodem cũ illa plano, hæ quoque ſunt inter ſe parallelæ.



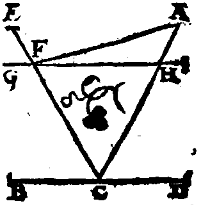
Theorema 10. Propoſitio 10.

Si duę rectę lineæ ſe mutuò tangẽtes ad duas rectas ſe mutuò tangentes ſint parallelę, non autem in eodem plano, illę angulos æquales comprehendent.



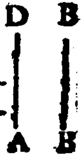
Proble. 1. Propoſitio 11.

A dato ſublimi puncto, in ſubiectum planũ perpendicularem rectam lineam ducere.



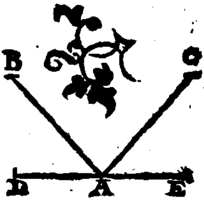
Problema 2. Propoſitio 12.

Dato plano, à pũcto quod in illo datum eſt, ad rectos angulos rectam lineam excitare.



Theorema 11. Propoſitio 13.

Dato plano, à puncto qd' in illo datum eſt, duæ rectæ lineæ ad rectos angulos non excitabuntur ad eaſdem partes.



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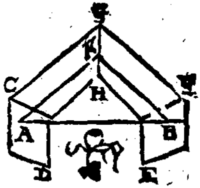
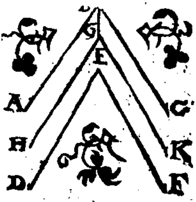
Theorema 12. Propositio 14.

Ad quæ plana, eadem recta linea recta est, illa sũt parallela.



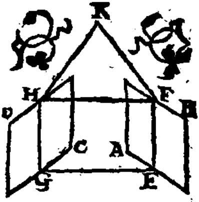
Theorema 13. Propositio 15.

Si duæ rectæ lineæ se mutuò tangentes ad duas rectas se mutuò tangentes sint parallelæ, non in eodem consistentes plano, parallela sunt quæ per illas ducuntur plana.



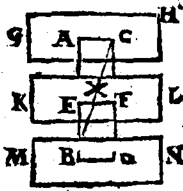
Theorema 14. Propositio 16.

Si duo plana parallela plano quopiã secentur, communes illorum sectiones sunt parallelæ.

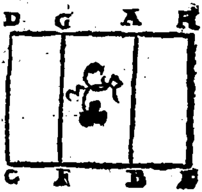


Theorema 15. Propositio 17.

Si duæ rectæ lineę parallelis planis secẽtur, in easdem rationes secabũtur.

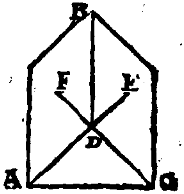


Theorema 16. Propositio 18.



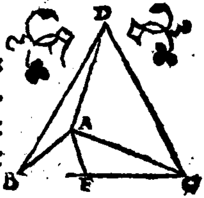
Si recta linea plano cuipiam ad rectos sit angulos, illa etiam omnia quę per ipsam plana, ad rectos eidem plano angulos erunt.

Theorema 17. Propositio 19



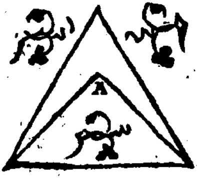
Si duo plana se mutuò secantia plano cuidã ad rectos sint angulos, communis etiam illorum sectio ad rectos eidem plano angulos erit.

Theorema 18. Propositio 20.



Si angulus solidus planis tribus angulis contineatur, ex his duo quilibet vtut assumpti tertio sunt maiores

Theorema 19. Propositio 21.

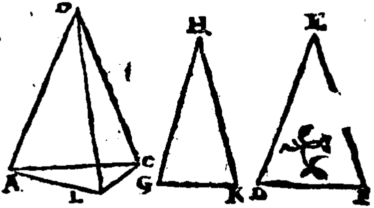


Solidus omnis angulus minoribus continetur, quàm rectis quatuor angulis planis.

Theo-

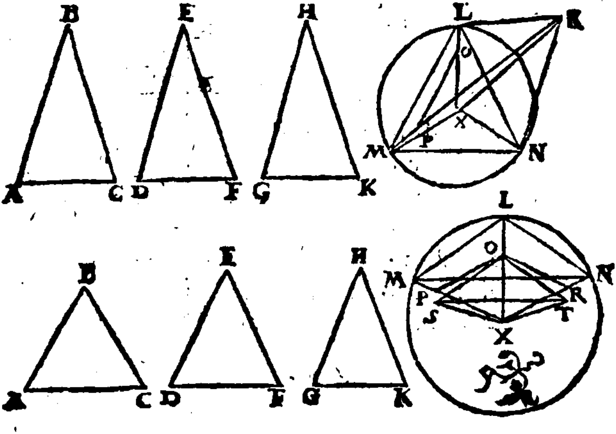
Theorema 20. Proopositio 22.

Si plani tres anguli ęqualibus rectis contineãtur lineis, quorũ duo vt libet assumpti, tertio sint maiores, triangulum constitui potest ex lineis æquales, illas rectas coniungentibus.

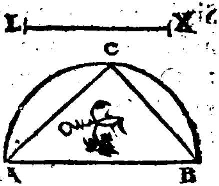


Problema 3. Propositio 23.

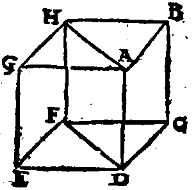
Ex planis tribus angulis, quorũ duo vt libet assumpti tertio sint maiores, solidũ angulum constituere. Decet autem illos tres angulos rectis quatuor esse minores.

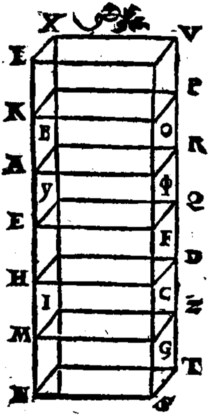


Theorema 21. Propositio 24.



Si solidum parallelis planis contineatur, aduersa illius plana & æqualia sunt & parallelogramma.





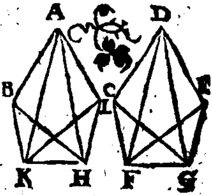
Theorema 22. Propositio 25.

Si solidum parallelis planis contentũ plano secetur aduersis planis parallelo, erit quemadmodum basis ad basim, ita solidum ad solidum.

Proble-

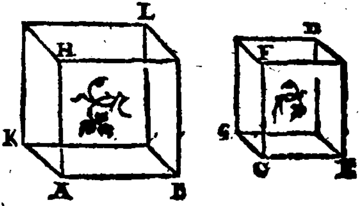
Problema 4. Propositio 26.

Ad datam rectam lineã eiusque punctum, angulum solidum constituere solido angulo dato æqualem.



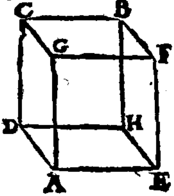
Problema 5. Propositio 27.

A data recta, dato solido parallelis planis comprehensio simile & similiter positum solidũ parallelis planis contẽtum describere.

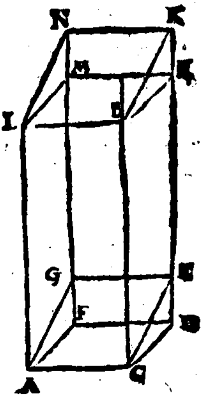


Theorema 23. Propositio 28.

Si solidũ parallelis planis comprehensum ductor per aduersorum planorum diagonios plano sectũ sit, illud solidum ab hoc plano bifariam secabitur.



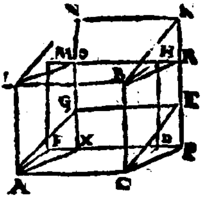
Theorema 34. Propositio 29.



Solida parallelis planis comprehēſa, quę ſuper eandem baſim. & in eadē ſunt altitudine, quorum inſiſtētes lineæ in ijſdem collocantur rectis lineis, illa ſūt inter ſe æqualia.

Theorema 25. Propositio 30.

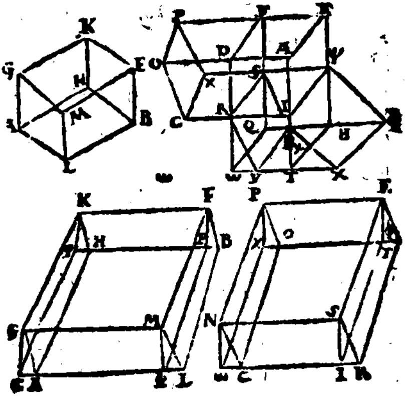
Solida parallelis planis circumſcripta, quæ ſuper eandē baſim & in eadē ſūt altitudine, quorum inſiſtentes lineæ nō in ijſdem reperiuntur rectis lineis, illa ſunt inter ſe æqualia.



Theorema 26. Propositio 31.

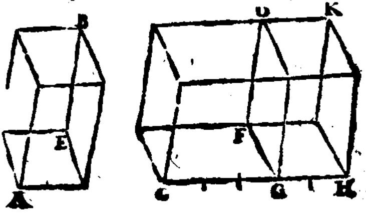
Solida parallelis planis circunſcripta, quæ in

In eadem sũt altitudine, æqualia sũt inter se.

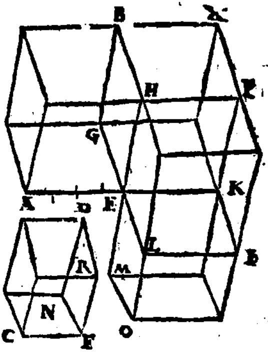


Theorema 27. Propositio 32.

Solida parallelis planis circunscripta quæ eiusdem sunt altitudinis, eam habẽt inter se rationẽ, quam bases.

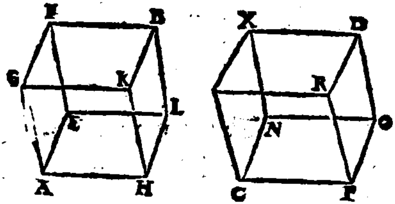


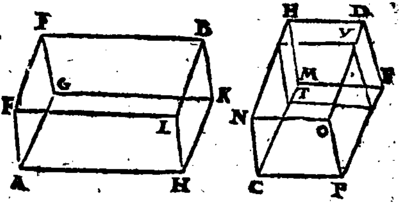
Theor 28. Propositio 23.

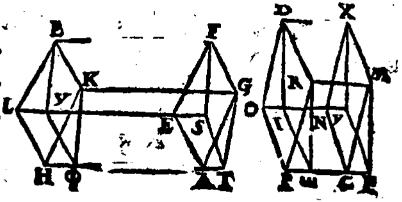


Similia solida parallelis, planis circũscripte habẽt inter se rationẽ homologorũ laterum triplicatam

Theor. 29. Propositio 24



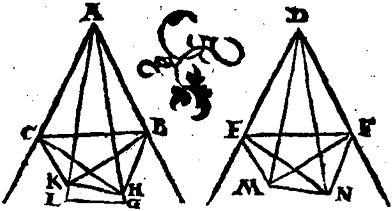




Aequalium solidorũ parallelis planis cõtentorum bases cũ altitudinibus reciprocantur. Et solida parallelis planis contenta, quorũ bases cum altitudinibus reciprocantur, illa sunt æqualia.

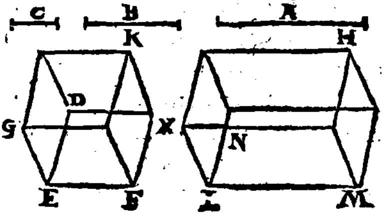
Theorema 30. Propositio 35.

Si duo plani sint anguli æquales, quorum verticibus sublimes rectæ lineæ insistant, quæ cum lineis primò positis angulos contineant æquales, vtrunq; vtriq;, in sublimibus autem lieneis quęlibet sumpta sint puncta, & ab his ad plana in quibus consistunt anguli primum positi, ductæ sint perpendiculares, ab earum verò punctis, quæ in planis signata fuerint, ad angulos primum positos ad iūctæ sint rectæ lineę, hæ cū sublimibus æquales angulos comprehendent.



Theorema 31. Propositio 36.

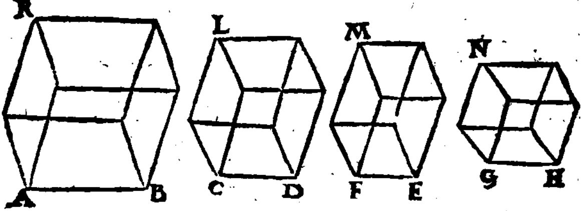
Si rectæ tres lineæ sint proportionales, quòd



ex his tribus fit solidum parallelis planis contentum, æquale est descripto à media linea solido parallelis planis compreheſo, quod æquilaterum quidem sit, sed antedicto æquiangulum.

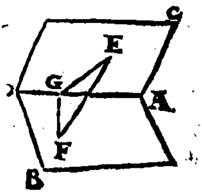
Theorema 32. Propositio 37.

Si rectæ quatuor lineæ sint proportionales, illa quoque solida parallelis planis contenta, quæ ab ipsis lineis & similia & similiter discribuntur, proportionalia erunt. Et si solida parallelis planis comprehensa, quę & similia & similiter describuntur, sint proportionalia, illæ quoque rectæ lineæ proportionales erunt.



Theorema 33. Propositio 38.

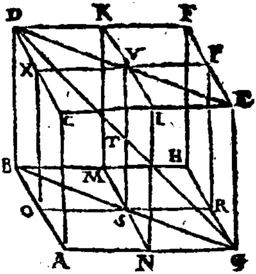
Si planum ad planum rectũ sit, & à quodã puncto corũ quæ in vno sunt planorũ perpendicularis ad alterum ducta sit, illa quæ ducitur perpẽdicularis, in communẽ cadet planorũ sectionẽ.



Theo.

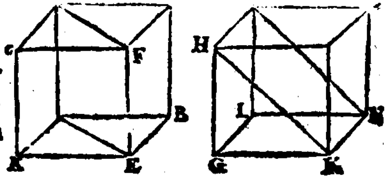
Theorema 34. Propoſitio 39.

Si in ſolido parallelis planis circunſcripto, aduerſorum planorum lateribus bifariam ſectis, educta ſint per ſectiones plana, communis illa planorũ ſectio & ſolidi parallelis plani circumſcripti diameter, ſe mutuò bifariã ſecant.



Theorema 35. Propoſitio 40.

Si duo ſint æqualis altitudinis priſmata, quorũ hoc quidem baſim habeat parallelogrammũ, illud verò triangulum, ſit autem parallelogrammũ triãguli duplũ, illa priſmata erunt æqualia.



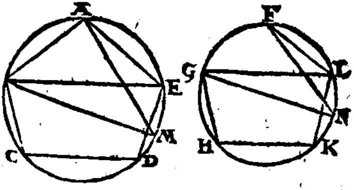
ELEMENTI XI. FINIS.

EVCLIDIS ELEMENTVM DVODECIMVM.

ET SOLIDORVM. ſecundum.

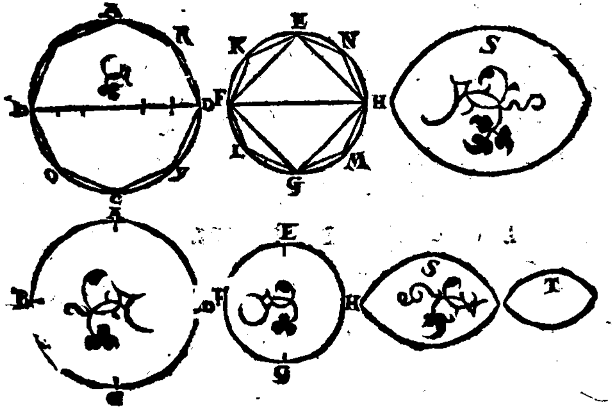
Theorema 1. Propoſitio 1.

Similia quæ ſunt in circulis polygona, rationẽ habent inter ſe, quam deſcripta à diametris quadrata.



Theorema 2. Propoſitio 2.

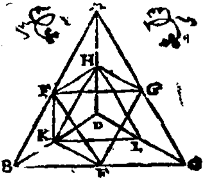
Circuli eam inter ſe rationem habent, quā



descripta à diametris quadrata.

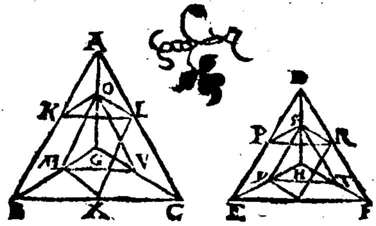
Tueorema 3. Propositio 3.

Omnis pyramis trigωnam habens basim, in duas diuiditur pyramidas non tantũ æquales & similes inter se, sed toti etiam pyramidi similes, quarum trigωnæ sunt bases, atq; in duo prismata ęqualia, quæ duo prismata dimidio pyramidis totius sũt maiora.



Theorema 4. Propositio 4.

Si duæ eiusdem altitudinis pyramides trigωnas habeãt bases, sit aũt illarum vtraque diuisa & in duas pyramidas inter se ęquales totiq; similes, & in duo prisma æqualia, ac eodem modo diuidatur vtraq; pyramidum quæ ex superiore diuisione natæ sunt, idq́ue perpetuò fiat: quemadmodũ se habet vnius pyramidis basis ad alterius pyrãidis basim, ita & omnia quę in vna pyra-

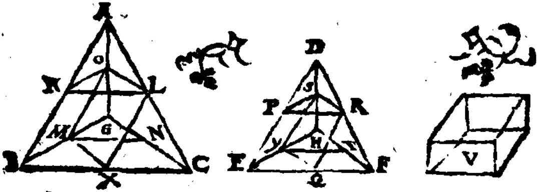


mido

mide prismata, ad omnia quæ in altera pyramide, prismata multitudine æqualia.

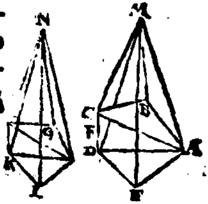
Theorema 5. Propositio 5.

Pyramides eiusdem altitudinis, quarũ trigωnæ sunt bases, eam inter se rationem habent, quàm ipsæ bases.



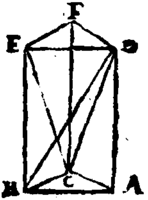
Theorema 6. Propositio 6.

Pyramides eiusdẽ altitudinis, quarum polygωnæ sunt bases, eam inter se rationem habent, quã ipsæ bases.



Theorema 7. Propositio 7.

Omne prisma trigωnũ habẽs basim, diuiditur in tres pyramides inter se æquales, quarum trigωnæ sunt bases



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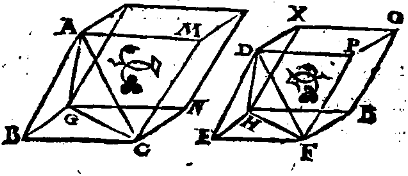
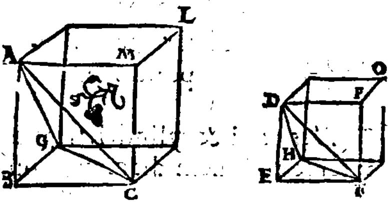
Theorema 8. Propositio 8.

Similes pyramides qui trigωnas habent bases, in triplicata sũt homologorũ laterũ ratiõe



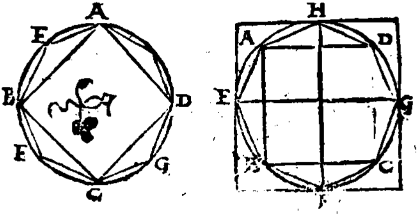
Theorema 9. Propositio 9.

Aequaliũ pyramidum & trigωnas bases habentium reciprocantur bases cum altitudinibus. Et quarũ pyramidum trigωnas bases habentium reciprocãtur bases cũ altitudinibus, illæ sunt æquales.



Thorema 10. Propositio 10.

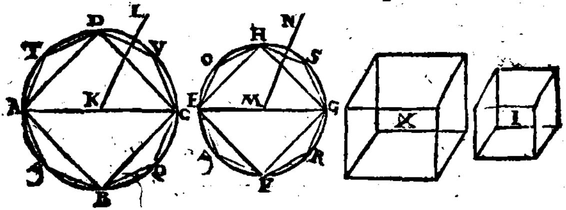
Omnis cωnus tertia pars est cylindri eandẽ



cum ipso cono basim habentis, & altitudinem æqualem.

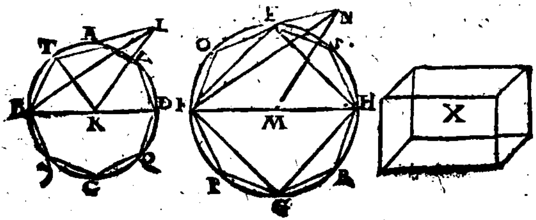
Theorema 11. Propositio 11.

Coni & cylindri eiusdem altitudinis, eam inter se rationem habent, quam bases.



Theorema 12. Propositio 12.

Similes coni & cylindri, triplicatã habent inter se rationem diametrorum, quæ sunt in basibus.



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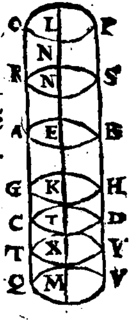
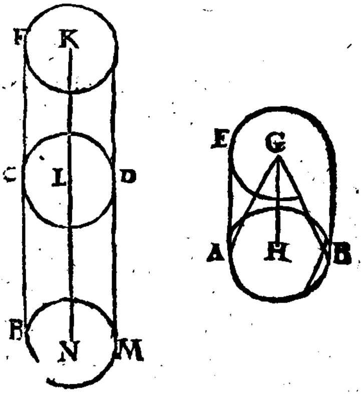
Theorema 13. Propositio 13.



Si cylindrus plano sectus sit aduersis planis parallelo, erit quemadmodum cylindrus ad cylindrum, ita axis ad axem.

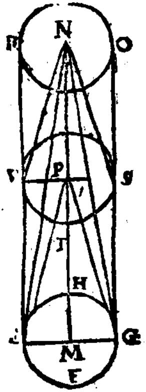
Theorema 14. Propositio 14.

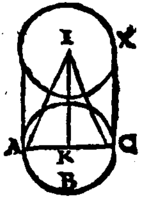
Coni & cylindri qui in æqualibus sunt basibus, eam habent inter se rationem, quam altitudines



Theorema 15. Propositio 15.

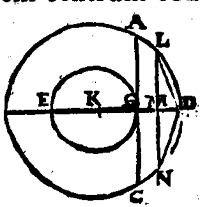
Aequalium conorum & cylindrorũ bases cũ alutitudinib' reciprocãtur. Et quorum conorũ & cylindrorum bases cum altitudinibus reciprocantur, illi sunt æquales.





Theoroma 1. Propositio 16.

Duobus circulis circũ idem centrum consistentibus, in maiore circulo polygonum æqualium pariumque laterum inscriberẽ, quod minorem circulũ non tangat.



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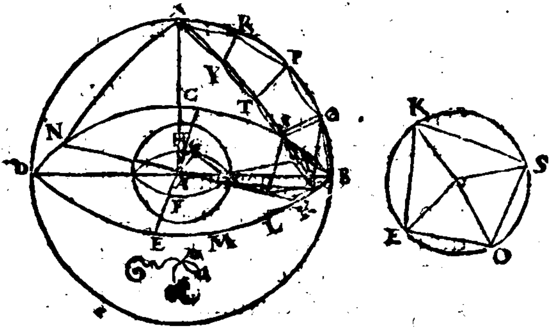
Problema 2. Propositio 17.

Duabus sphæris circum idem centrum consistentibus, in maiorem sphera solidum polyedrum inscribere, quod minoris sphæræ superficiem non tangat.



Theorema 16. Propositio 18.

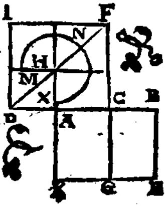
Sphæræ inter se rationem habent suarum diametrorum triplicatam.

ELEMENTI XII. FINIS.

EVCLIDIS ELEMENTVM DECIMVMTERTIVM, ET SOLIDORVM TERTIVM.

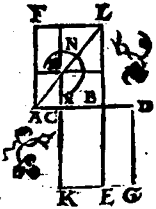
Theorema 1. Propositio 1.

Si recta linea per extremã & mediã rationem secta sit, maius segmentum quod totius lineæ dimidium assumpserit, quintuplum potest eius quadrati, quod à totius dimidia describitur.



Theorema 2. Propositio 2.

Si recta linea sui ipsius segmenti quintuplum possit, & dupla segmenti huius linea per extremã & mediam rationẽ secetur maius segmentũ reliqua pars est lineæ primum positæ.



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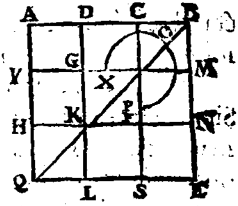
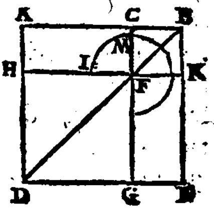
Theorema 3. Propositio 3.



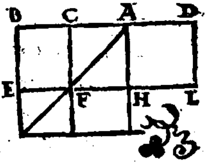
Si recta linea per extremã & mediã rationẽ secta sit, minus segmẽtũ quod maioris segmenti dimidium assumpserit, quintuplum potest eius, quòd à maioris segmẽti dimidio describitur, quadrati.

Theorema 4, Propositio 4.



Si recta linea per extremam & mediam rationẽ secta sit, quod à tota, quodque à minore segmẽto simul vtraq; quadrata, tripla sunt eius, quod à maiore segmẽto describitur, quadrati.

Theorema 5, Propositio 5.



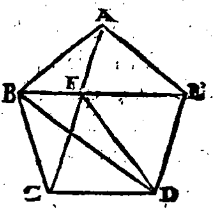
Si ad rectam lineam, quæ per extremam & mediam rationem secetur, adiuncta sit altera segmento maiori ęqualis, tota hæc linea recta per extremam & mediam rationẽ seɛta eſt, eſtque maius ſegmentum linea primùm poſita.

Theorema 6. Propoſitio 6.

Si recta linea ῥητὴ ſiue rationalis, per extremam & mediam rationem ſecta ſit, vtrunque ſegmentorum ἄλογος ſiue irrationalis eſt linea, quæ dicitur Reſiduum.

Theorema 7. Propoſitio 7.

Si pentagωni æquilateri tres ſint æquales anguli, ſiue quã deinceps ſiue qui non deinceps ſequuntur, illud pentagωnum erit æquiangulum.



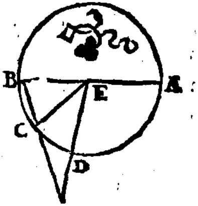
Theorema 8. Propoſitio 8.

Si pentagωni æquilateri & æquianguli duos qui deinceps ſequuntur angulos rectæ ſubtendãt lineæ, illæ per extremã & mediam rationem ſe mutuò ſecant, earumq; maiora ſegmenta, ipſius pentagωni lateri ſunt æqualia

Theo-

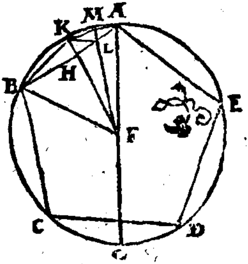
Theorema 9. Propositio 9.

Si latus hexagωni & latus decagωni eidem circulo inſcriptorum compoſita ſint, tota recta linea per extremam & mediam rationem ſecta eſt, eiuſque ſegmẽtum maius, eſt hexagωni latus.



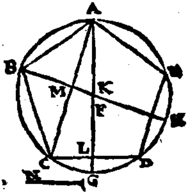
Theorema 10. Propositio 10.

Si circulo pẽtagωnum ęquilaterum inſcriptum ſit, pentagωini lat' poteſt & latus hexagωni & latus decagωni eidem circulo inſcriptorum.



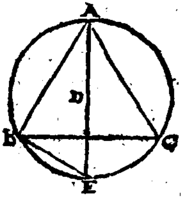
Therema 11. Propositio 11.

Si in circulo ῥητὴν habente diametrum, inſcriptum ſit pentagωnum æquilaterum, pentagωni latus irrationalis eſt linea quæ vocatur Minor.



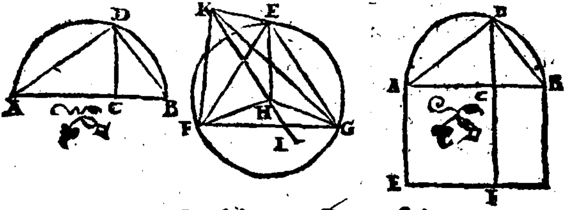
Theorema 12. Propoſitio 12.

Si in circulo inſcriptum ſit triangulum æquilaterum, huius trianguli latus potentia triplum eſt eius lineæ, quæ ex circuli centro ducitur.



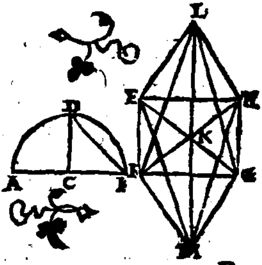
Problema 1. Propoſitio 13.

Pyramidem conſtituere, & data ſphærę cõplecti, atque docere illius ſphæræ diametrum potentia ſeſquialteram eſſe lateris ipſius pyramidis.



Problema 2. Propoſitio 14.

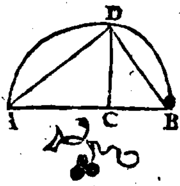
Octaedrum conſtituere, eaq́; ſphęra qua pyramidẽ complecti, atque ꝓbare illius ſphæræ diametrũ potentia duplã eſſe lateris ipſi⁹ octae dri.

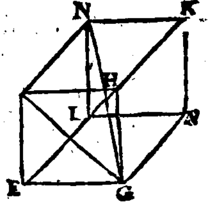


Pro-

Problema 3. Propoſitio 15.

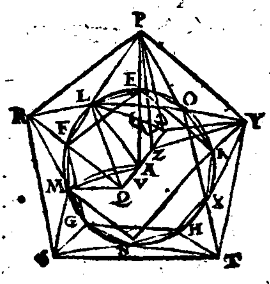
Cubum conſtituere, eaque ſphæra qua & ſuperiores figuras complecti, atque docere illius ſphæræ diametrum potẽtia triplã eſſe lateris ipſius cubi.

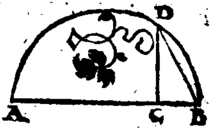




Problema 4. Propoſitio 16.

Icoſaedrum conſtituere, eademque ſphæra qua & antedictas figuras complecti, atque probare, Icoſaedri latus irrationalem eſſe lineam, quæ vocatur Minor.





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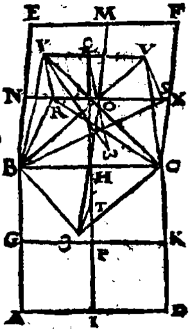
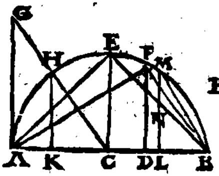
Problema 5. Propositio 17.



Dodecaedrũ constituere, eademque sphæra qua & & antedictas fuguras cõplecti, atque probare dodecaedri latus irrationalem esse lineam, quæ vocatur Residuum.

Theorema 6. Propositio 18.

quinq; figurarum latera pro-



ponere, & inter se comparare.

SCHOLIVM.

Aio verò, præter dictas quinque figuras non posse aliam constitui figuram solidã, quæ planis & æquilateris & æquiangulis cõtineatur, inter se æqualibus. Non enim ex duobus triãgulis, sed neq; ex alijs duab' figuris solidus cõstituetur angulus

Sed

Sed ex tribus triãgulis, constat Pyramidis angulus
Ex quatuor autem, Octaëdri
Ex quinque verò, Icosaëdri.
Nam ex triangulis, sex & æquilateris & æquiangulis ad idem punctum coeuntibus, non fiet angulus solidus. Cùm enim trianguli æquilateri angulus, recti vnius bessem contineat, erunt eiusmodi sex anguli rectis quatuor æquales. Quod fieri nõ potest. Nam solidus omnis angulus minoribus quã rectis quatuor angulis continetur, per 21.11.
Ob easdem sanè causas, neque ex pluribus quã planis sex eiusmdoi angulis solidus constat.
Sed ex tribus quadratis, Cubi angulus continetur.
Ex quinque, nullus potest. Rursus enim recti quatuor erunt.
Ex tribus autẽ pentangonis æquilateris & æquiangulis, Dodecaëdri angulus continetur.
Sed ex quatuor, nullus potest. Cùm enim pentagoni æquilateri angulus rectus sit, et quinta recti pars erunt quatuor anguli rectis quatuor maiores. Quod fieri nequit. Nec sanè ex alijs polygonis figuris solidus angulus continebitur, quod hinc quoque absurdum sequatur. Quamobrem perspicuum est, præter dictas quinque figuras aliã figuram solidam non posse constitui, quæ ex planis æquilateris & æquiangulis contineatur.

ELEMENTI XIII. FINIS

EVCLIDIS ELEMENTVM DECIMVM QVARTVM, VT qui dam arbitrantur, vt alij verò, Hypſiclis Alexandrini, de quinque corporibus.

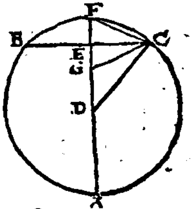
LIBER PRIMVS.

BAſilides Tyrius, Protarche, Alexandriam profectus, patrique noſtro ob diſciplinæ ſocietatem commendatus, longiſsimo peregrinationis tempore cum eo verſatus eſt. Cùmque diſſererent aliquando de ſcripta ab Apollonio comparatione Dodecaëdri & Icoſaëdri eidem ſphæræ inſcriptorum, quam hæc inter ſe habeant rationem, cenſuerunt ea non rectè tradidiſſe Apollonium: quæ à ſe emendata, vt de patre audire erat, literis prodiderunt. Ego autem poſtea incidi in alterum librum ab Apollonio editum, qui demonſtrationem accuratè complecteretur de re propoſita, ex eiuſq́; problematis indagatione magnam equidem cepi voluptatem. Illud certè ab omnibus perſpici poteſt, quod ſcripſit Apollonius, cùm ſit in omnium manibus. Quod autẽ diligenti, quantum conijcere licet, ſtudio nos poſtea ſcrip-

scripsisse videmur, id monimentis consignatum tibi nuncupandum duximus, vt qui feliciter cùm in omnibus disciplinis tum vel maximè in Geometria versatus, scitè ac prudẽter iudices ea quæ dicturi sumus ob eam verò quæ tibi cum patre fuit, vitæ consuetudinem, quaq́; nos complecteris, beneuolentiã, tractationẽ ipsam libenter audias. Sed iam tempus est, vt præmio modum facientes, hanc syntaxim aggrediamur.

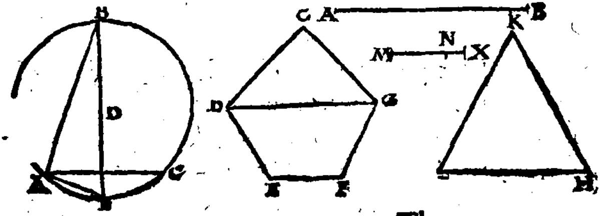
Theorema 1. Proopositio 1.

Perpendicularis linea, quæ ex circuli cuiuspiam centro in latus pentagωni ipsi circulo inscripti ducitur, dimidia est vtriusq; simul lineæ, & eius, quæ ex centro & lateris decagωni in eodẽ circulo inscripti.



Theorema 2. Propositio 2.

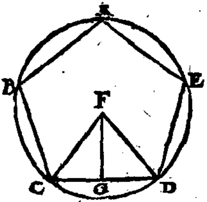
Idem circulus cõprehendit & dωdecaedri pentagωnum & icosaedri triangulum, eidẽ sphæræ inscriptorum.

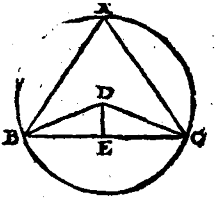


Theo-

Theorama 3. Propositio 3.

Si pentagωno & æquilatero & æquiangulo circumscriptus sit circulus ex cuius, centro in vnum pentagωni latus dicta sit perpendicularis: quod vno laterum & perpẽdiculari trigesies cõtinetur, illud



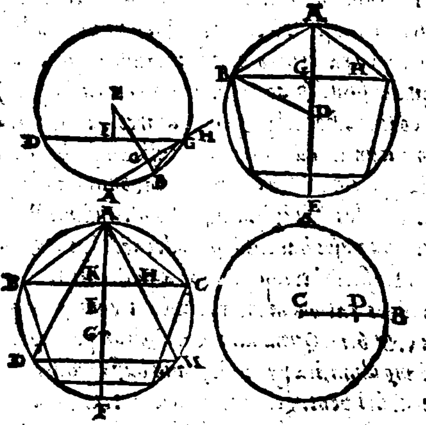


æquale est dωdecaedri superficiei.

Theorema 4. Propositio 4.

Hoc perspicuum cùm sit probandum est, quemadmodum se habet dωdecaedri superficies ad icosaedri superficiem ita se habere cubi latus ad icosaedri latus.

Cubi



Cubilatus.

E

Dodecaedri.

F

Icosaedri.

G

SCHOLIVM

Nunc autem probandum est, quemadmodum se habet cubi latus ad icosaëdri latus, ita se habere solidum dodecaëdri ad icosaëdri solidum: Cùm enim æquales circuli comprehendat & dodecaëdri pentagonum & Icosaëdri triangulum, eidē sphæræ in-

scriptorum: in sphæris autem æquales circuli æqualĩ interuallo distent à centro (siquidem perpendiculares à sphæræ centro ad circulorum plana ductæ & æquales sunt, & ad circulorum centra cadunt) idcirco lineæ, hoc est perpendiculares, quæ à sphæræ cẽtro ducuntur ad centrum circuli comprehendẽtis et triangulum Icosaëdri, & pentagonum dodecaëdri sũt æquales. Sũt igitur æqualis altitudinis Pyramides, quæ bases habent ipsa dodecaëdri pentagona, et quæ Icosaëdri triangula. At æqualis altitudinis pyramides rationem inter se habent eam quam bases, ex 5. & 6. 11. Quemadmodum igitur pentagonũ ad triangulum, ita pyramis, cuius basis quidem est dodecaëdri pentagonum, vertex autem sphæræ centrum ad pyramida, cuius basis quidem est Icosaëdri triangulum, vertex autem sphæræ centrum. Quamobrem vt se habent duodecim pentagona ad viginti triãgula, ita duodecim pyramides, quorum pẽtagona sint bases, ad viginti pyramidas, quæ trigonas habeant bases. At pentagona duodecim sunt dodecaëdri superficies, viginti autem triangula, Icosaëdri. Est igitur vt dodecaëdri superficies ad Icosaëdri superficiem, ita duodecim pyramides quæ pentagonas habeant bases, ad viginti pyramidas, quarum trigona sunt bases. Sunt autem duodecim quidem pyramides, quæ pentagonas habeant bases, solidũ dodecaëdri: viginti autem pyramides, quæ trigonas habeant bases, Icosaedri solidum. Quare ex 11. 5. vt dodecaëdri superficies ad Icosaëdri superficiem, ita

soli-

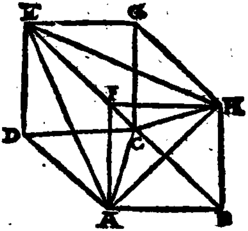
solidum dodecaëdri ad Icosaëdri solidum. Vt que-sem dodecaëdri superficies ad Icosaëdri superficiem, ita probatum est cubi latus ad Icosaëdri latus. Quemadmodum igitur cubi latus ad Icosaëdri latus, ita se habet solidum dodecaëdri ad Icosaëdri solidum.

EVCLIDIS ELEMENTVM DECIMVM QVINTVM, ET Solidorum quintum, vt nonnulli putant, vt autem alij, Hypsiclis Alexandrini, de quinque corporibus.

LIBER II.

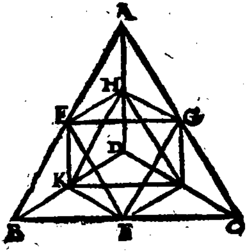
Problema 1. Propositio 1.

In dato cubo pyramida inscribere.



Problema 2. Propositio 2.

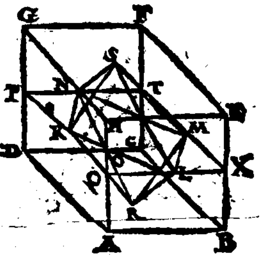
In data pyramide octaedrum inscribere.



Pro-

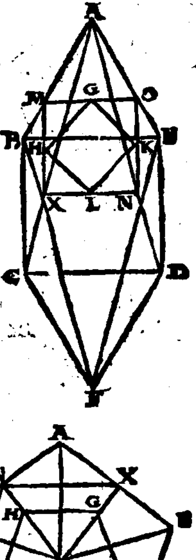
Problema 3. Propositio 3.

In dato cubo octaedrum inscribere.



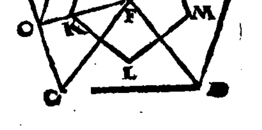
Problema 4. Propositio 4.

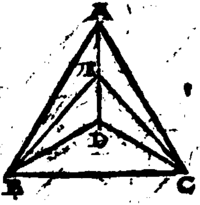
In dato octaedro cubũ inscribere

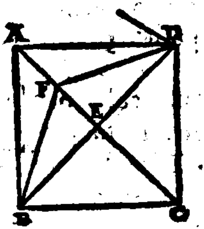


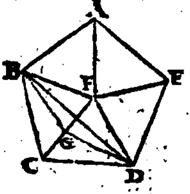
Problema 5. Propositio 5.

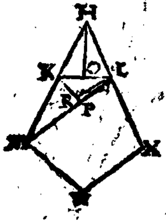
In dato Icosaedro dudecaedrum inscribere.

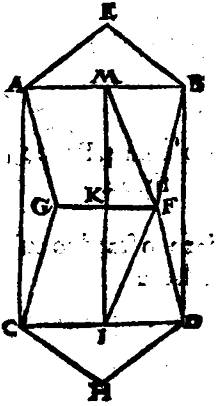












SCHO.

SCHOLIVM.

Meminiſſe decet, ſi quis nos roget, quot Icoſaëdrū habeat latera, ita reſpondendum eſſe: Patet Icoſaedrum viginti contineri triãgulis, quodlibet verò triangulum rectis tribus conſtare lineis. Quare multiplicanda ſunt nobis viginti triangula in trianguli vnius latera, ſiūtq́; ſexaginta, quorum dimidium eſt triginta. Ad eundem modum & in dodecaedro. Cū enim rurſus duodecim pentagona dodecaedrum cōprehendant, itemq́; pentagonum quoduis rectis quinque conſtet lineis, quinq́; duodecies multiplicamus, fiunt ſexaginta, quorum rurſus dimidium eſt triginta. Sed cur dimidiū capimus? Quoniã vnumquodq́; latus ſiue ſit trianguli ſiue pentagoni, ſiue quadrati vt in cubo, iterato ſumitur. Similiter autem eadem via & in cubo & in pyramide & in octaedro latera inuenies. Quòd ſi itē velis ſingularum quoq́; figurarum angulos reperire, facta eadem multiplicatione numerum procreatum partire in numerum planorū, quæ vnum ſolidum angulum includūt: vt quoniã triãgula quinq́; vnum Icoſaëdri angulum continet, partire 60. in quinque, naſcūtur duodecim anguli Icoſaedri. In dodecaedro autē tria pētagona angulum comprehendunt, partire ergo 60. in tria, & habebis dodecaedri angulos viginti. Atq́; ſimili ratione in reliquis figuris angulos reperies.

Finis Elementorum Euclidis.