EVCLIDIS ELEMENTORVM LIBRI XV.

QVIBVS, CVM AD OMNEM MATHEMATICAE SCIENTIAE PARTEM, TVM AD QVAMLIbet Geometriæ tractationem facilis comparatur aditus.

Cat. Collegii Societatis JESU Pragae

Musei Mathematici

COLONIAE
Apud Gosuinum Cholinum
M. D. CXII.
Cum Gratia & Priuilegio.

AD CANDIDVM LECTOREM ST. GRACILIS praefatio.

PERMAGNI referre semper existimaui, lector beneuole, quantum quisque studij & diligentiæ ad percipienda scientiarum elementa adhibeat, quibus non satis cognitis, aut perperam intellectis si vel digitũ progredi tentes erroris caliginem animis offundas, non veritatis lucem rebus obscuris adferas. Sed principiorum quanta sint in disciplinis momenta, haud facilè credat, qui rerum naturam ipsa specie, non viribus metiatur. Vt enim corporum, quæ oriuntur & intereunt, vilißima tenuißimaq; videntur initia: ita rerum æternarum & admirabilium, quibus nobilißimæ artes continentur, elementa ad speciem sunt exilia, ad vires & facultatem quam maxima. Quis non videt ex fici tantulo grano, vt ait Tullius, aut ex acino vinaceo, aut ex cæterarum frugum aut stirpium minutißimis seminibus tantos truncos ramosq; procreari? Nam Mathematicorum initia illa quidem dictu audituq; perexigua, quantã theorematum syluam nobis pe-

pererunt? Ex quo intelligi potest, vt in ipsis seminibus, sic & in artium principijs inesse vim earum rerum, quae ex his progignuntur. Praeclarè igitur Aristoteles, vt alia permulta, μέγιστον ἴσως ἀρχὴ παντὸς, καὶ ὅσῳ κράτιστον τῇ δυνάμει, τοσούτῳ μικρότατον, οὐδὲ μεγέθει χαλεπόν ἐστιν ὀφθῆναι. Quocirca committendum non est, vt non bene prouisa & diligenter explorata scientiarum principia, quibus propositarum quarumque rerum veritas sit demonstranda, vel constituas, vel constituta approbes: Cauendum etiam, vt ne tantulum quidem fallaci & captiosa interpretatione turpiter deceptus, à vera principiorum ratione temerè deflectas. Nam qui initio fortè aberrauerit, is vt tandem in maximis versetur erroribus, necesse est: cum ex vno erroris capite densiores sensim tenebrae rebus clarissimis obducantur. Quid tam varias veterum physiologorum sententias non modò cùm rerum veritate pugnantes, sed vehementer etiam inter se dissentientes nobis inuexit? Equidem haud scio, fuerit ne vlla potior tanti dissidij causa, quam quòd ex principijs partim falsis, partim non consentaneis ductas rationes probando adhiberent. Fit enim plerunque, vt qui non rectè de artium rerumq; elementis sentiunt, ad praefinitas quasdam opiniones suas omnia reuocare studeant. Pythagorei, vt meminit Aristoteles, cum denarij numeri summam perfectionem cælo tribuerent, nec plures tamen quàm nouem sphaeras cernerent, decimam affingere ausi sunt terrae aduer-

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sum, quam ἀντίχθονα *appellarunt. Illi enim vniuersitatis rerumq́; singularum naturam ex numeris ceu principijs aestimantes, ea protulerunt quae* φαινομένοις *congruere nusquam sunt cognita. Nam ridicula Democriti, Anaximenis, Melißi Anaxagorae, Anaximandri, & reliquorum id genus physiologorum somnia, ex falsis illa quidem orta natura principijs, sed ad Mathematicum nihil aut parum spectantia, sciens praetereo Nonnullos attingam qui repetitis altius vel aliter ac decuit positis rerum initijs, cum physicis multa turbarunt; tum Mathematicos oppugnatione principiorum peßimè mulctarunt. Ex planis figuris corpora constituit Timaeus: Geometrarum hic quidem principia cuniculis oppugnantur. Nam & superficies seu extremitates craßitudinem habebunt, & lineae latitudinem: denique puncta non erunt indiuidua, sed linearum partes. Praedicant Democritus atque Leucippus illas atomos suas, & indiuidua corpuscula. Concedit Xenocrates impartibiles quasdam magnitudines. Hic verò Geometriae fundamenta apertè petuntur, & funditus euertuntur: quibus dirutis nihil equidem aliud video restare, quàm vt amplißima Mathematicorum theatra repentè concidant. Iacebant ergò, si dijs placet, tot praeclara Geometrarum de asymmetris & alogis magnitudinibus theoremata. Quid enim causa dicas, cur indiuidua linea hanc quidem metiatur, illam verò meteri non queat? Siquidem quod minimum in vnoquoque* *genere reperitur, id communis omnium mensura esse solet. Innumerabilia profectò sunt illa, quæ ex falsis eiusmodi decretis absurda consequuntur: & horum permulta quidem Mathematicus, sed longè plura colligit Physicus. Quid varia* ψευδογραφημάτων *genera commemorem, quæ ex hoc vno fonte tam longè latéque diffusa fluxisse videntur? Notißimus est Antiphontis tetragonismus, qui Geometrarum & ipse principia non parum labefecit, cùm rectæ lineæ curuam posuit æqualem. Longum esset mihi singula percensere, præsertim ad alia properanti: Hoc ergo certum fixum, & in perpetuum ratum esse oportet, quod sapienter monet Aristoteles,* Σπουδαστέον ὅπως ὁρισθῶσι καλῶς αἱ ἀρχαὶ μεγάλην γὰρ ἔχουσι ῥοπὴν πρὸς τὰ ἑπόμενα. *Nam principijs illa congruere debent, quæ sequuntur. Quòd si tantum perspicitur in istis exilioribus Geometriæ initijs, quæ puncto, linea, superficie definiuntur, momentum, vt ne hæc quidem sine summo impendentis ruinæ periculo conuelli aut oppugnari poßint, quanta quæso vis putanda est huius* στοιχειώσεως, *quam collatis tot præstantißimorum artificum inuentis, mira quadam ordinis solertia contexuit Euclides, vniuersæ Matheseos elementa complexu suo coercentem? Vt igitur omnibus rebus instructior & paratior quisque ad hoc studium libentius accedat, & singula, vel minutißima exactius secum reputet atque perdiscat, operæpretium censui, in primo institutionis aditu vestibuloque præcipua quædam capita,*

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quibus tota ferè Mathematicæ scientiæ ratio intelligatur, breuiter explicare: tum ea quæ sunt Geometriæ propria, diligenter persequi: Euclidis denique in extruenda hac στοιχειώσει consilium sedulò ac fideliter exponere. Quæ ferè omnia ex Aristotelis potissimũ ducta fontibus, nemini inuisa fore cõfido, qui modo ingenuũ animi candorẽ ad legendũ attulerit. Ac de Mathematicæ diuisione primũ dicamus.

Mathematicæ in primis scientiæ studiosos fuisse Pythagoreos, non modò historicorum, sed etiam philosophorum libri declarant. His ergo placuit, vt in partes quatuor vniuersum distribuatur Mathematicæ sciẽtiæ genus, quarum duas περὶ τὸ ποσὸν, reliquas περὶ τὸ πηλίκον versari statuerunt. Nam & τὸ ποσὸν vel sine vlla comparatione ipsum per se cognosci, vel certa quadam ratione comparatum spectari: in illo Arithmeticam, in hoc versari Musicam: & τὸ πηλίκον partim quiescere, partim moueri quidem: illud Geometriæ propositum esse: quod verò sua sponte motu cietur, Astronomiæ. Sed ne quis falsò putet, Mathematicam scientiam, quòd in vtroque quanti genere cernitur, idcirco inanem videri (si quidem non solum magnitudinis diuisio sed etiam multitudinis accretio infinitè progredi potest) meminisse decet, τὸ πηλίκον καὶ τὸ ποσὸν, quæ subiecto Mathematicæ generi imposita sunt à Pythagoreis nomina, non cuiuscunque modi quantitatem significare, sed eam demum, quæ tum multitudine tum magnitudine sit defi-

nita, & suis circumscripta terminis. Quis enim vllam infiniti scientiam defendat? Hoc scitum est, quod non semel docet Aristoteles, infinitum ne cogitatione quidem complecti nunquam posse. Itaque ex infinita multitudinis & magnitudinis δυνάμει *finitam hæc scientia decerpit & amplectitur naturam, quam tractet, & in qua versetur. Nam de vulgari Geometrarum consuetudine quid sentiendum sit, cum data interdum magnitudine infinita aut fabricantur aliquid, aut proprias generis subiecti affectiones exquirunt, disertè monet Aristoteles,* οὐδὲ νῦν *(de Mathematicis) loquens* δέονται τοῦ ἀπείρου, οὐδὲ χρῶνται, ἀλλὰ μόνον εἶναι ὅσην ἂν βούλωνται, πεπερασμένην. *Quamobrem disputatio ea qua infinitum refellitur, Mathematicorum decretis rationibusque non aduersatur, nec eorum apodixes labefacit. Etenim tali infinito opus illis nequaquam est, quod exitu nullo peragrari possit, nec talem ponunt infinitam magnitudinem: sed quantamcunque velit aliquis effingere, ea vt suppetat, infinitam præcipiunt. Quinetiam non modò immensa magnitudine opus non habent Mathematici, sed ne maxima quidem: cum instar maximæ minima quæque in partes totidem pari ratione diuidi queat. Alteram Mathematicæ diuisionē attulit Geminus, vir (quantum ex Proclo conijcere licet)* μαθημάτων *laude clarissimus. Eam, quæ superiore plenior & accuratior fortè visa est, cū doctissimè pertractarit sua in decimum Euclidis*

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præfatione P. Montaureus vir ſenatorius, & regiæ bibliothecæ præfectus, leuiter attingam. Nam ex duobus rerum velut ſummis generibus τῶν νοητῶν καὶ τῶν αἰσθητῶν, quæ res ſub intelligentiã cadunt, Arithmeticæ & Geomitriæ attribuit Geminus: quæ verò in ſenſus incurrunt, Aſtrologiæ, Muſicæ Supputatrici, Opticæ, Geodeſiæ & Mechanicæ adiudicauit. Ad hanc certè diuiſionem ſpectaſſe videtur Ariſtoteles, cùm Aſtrologiam Opticam, harmonicam φυσικωτέρας τῶν μαθημάτων nominat, vt quæ naturalibus & Mathematicis interiectæ ſint, ac velut ex vtriſque mixtæ diſciplinæ: Siquidem genera ſubiecta à Phyſicis mutuantur, cauſas verò in demonſtrationibus ex ſuperiore aliqua ſcientia repetunt. Id quod Ariſtoteles ipſe apertiſsimè teſtatur, ἐνταῦθα γὰρ, φησὶ, τὸ μὲν ὅτι, τῶν αἰσθητικῶν εἰδέναι τὸ διότι, τῶν μαθηματικῶν. Sequitur, vt quid Mathematicæ conueniat cum Phyſica & prima Philoſophia: quid ipſa ab vtraque differat, paucis oſtendamus. Illud quidem omnium commune eſt, quòd in veri contemplatione ſunt poſitæ, ob idquè θεωρητικαὶ à Græcis dicuntur. Nam cùm διάνοια ſiue ratio & mens omnis ſit vel πρακτικὴ, vel θεωρητικὴ, totidem ſcientiarum ſint genera neceſſe eſt. Quòd ſi Phyſica, Mathematica, & prima Philoſophia, nec in agendo, nec in efficiendo ſunt occupatæ, hoc certè perſpicuum eſt, eas omnes in cognitione contemplationéque neceſſario verſari. Cùm enim rerum non modò agendarum, ſed etiam effi-

ciendarum principia in agente vel efficiente consistant, illarum quidem προαίρεσις, harum autem vel mens vel ars, vel vis quadam & facultas: rerum profectò naturalium. Mathematicarum, atque diuinarum principia in rebus ipsis, non in philosophis inclusa latent. Atque haec vna in omnes valet ratio, qua θεωρητικὰς esse colligat. Iam verò Mathematica separatim cum Physica congruit, quòd vtraque versatur in cognitione formarum corpori naturali inhaerentium. Nam Mathematicus plana, solida, longitudines & puncta contemplatur, quae omnia in corpore naturali à naturali quoque philosopho tractantur. Mathematica item & prima philosophia hoc inter se propriè conueniunt, quòd cognitionem vtraque persequitur formarum, quoad immobiles, & à con-concretione materiae sunt liberae. Nam tametsi Mathematicae formae re vera per se non cohaerent, cogitatione tamen à materia & motu separantur, οὐδὲ γίνεται ψεῦδος χωριζόντων. vt ait Aristoteles. De cognatione & societate breuiter diximus, iam quid intersit videamus. Vnaquaeque mathematicarum certum quoddam rerum genus propositum habet, in quo versetur, vt Geometria quantitatem & continuationem aliorum in vnam partem, aliorum in duas, quorundam in tres, eorumque quatenus quanta sunt & continua, affectiones cognoscit. Prima autem philosophia, cùm sit omnium communis, vniuersum Entis genus, quaeque ei accidunt & conueniunt hoc ipso quòd est, considerat.

Ad hæc, Mathematica eam modò naturam amplectitur, quæ quanquam non mouetur, separari tamen seiungiq; nisi mente & cogitatione à materia non potest, ob eamq; causam ἐξ ἀφαιρέσεως dici consueuit. Sed Prima philosophia in iis versatur, quæ & seiuncta, & æterna, & ab omni motu per se soluta sunt ac libera. Cæterùm Physica & Mathematica quanquã subiecto discrepare non videntur, modo tamen rationeq; differũt cognitionis & contemplationis, vnde dissimilitudo quoque scientiarum sequitur. Etenim Mathematicæ species nihil re vera sunt aliud, quàm corporis naturalis extremitates, quas cogitatione ob omni motu & materia separatas Mathematicus contemplatur: sed easdem consectatur Physicorũ ars, quatenus cum materia comprehensa sunt, & corpora motui obnoxia circumscribunt. Ex quo fit, vt quæcunque in Mathematicis incommoditates accidunt, eædem etiam in naturalibus rebus videantur accidere, non autem vicissim. Multa enim in naturalibus sequuntur incommoda, quæ nihil ad Mathematicum attinent, διὰ τὸ, inquit Aristoteles, τὰ μὲν ἐξ ἀφαιρέσεως λέγεται, τὰ μαθηματικὰ, τὰ δὲ φυσικὰ ἐκ προσθέσεως. Siquidem res cum materia deuinctas contẽplatur physicus: Mathematicus verò rẽ cognoscit circũscriptis iis omnibus, quæ sensũ percipiũtur, vt grauitate, leuitate, duritie, mollitie, & præterea calore, frigore, aliisq; contrariorum paribus, quæ sub sensum subiecta sunt: tantum autem relinquit
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quantitatem & continuum. Itaq; Mathematicorum ars in iis quæ immobilia sunt cernitur (τὰ γὰρ μαθηματικὰ τῶν ὄντων ἄνευ κινήσεώς ἐστιν, ἔξω τῶν περὶ τὴν ἀστρολογίαν) quæ verò in naturæ obscuritate posita est, res quidem quæ nec separari nec motu vacare possunt contemplatur. Id quod in vtroque scientiæ genere perspicuum esse potest siue res subiectas definias, siue proprietates earum demonstres. Etenim numerus, linea, figura, rectum, inflexum, æquale, rotundū, vniuersa deniq; Mathematicus quæ tractat & profitetur, absque motu explicari doceriq; possūt: χωριστὰ γὰρ τῇ νοήσει κινήσεώς ἐστι: Physica autē sine motione species nequaquam possunt intelligi. Quis enim hominis, plantæ, ignis, ossium, carnis naturam & proprietates sine motu, qui materiam sequitur, perspiciat? Siquidem tantisper substantia quæque naturalis constare dici solet, quo ad opus & munus suum, agendo patiendóque tueri ac sustinere valeat: qua certè amissa δυνάμει, ne nomen quidem nisi ὁμωνύμως retinet. Sed Mathematico ad explicandas circuli aut trianguli proprietates, nullum adferre potest vsum. materiæ, vt auri, ligni, ferri, in qua insunt, consideratio, quin eò verius eiusmodi rerum, quarum species tanquam materia vacantes efformemus animo, naturam complectemur, quòd coniunctione materiæ quasi adulterari dipravarique videntur. Quo circa Mathematica species eodem modo quo κοιλὸν, siue concauitas, sine motu & subiecto, definitione explicare cognos-

noſciq́ poſſunt: naturales verò cùm eam vim habeant, quam vt ita dicam, ſimitas cum materia comprehenſa ſunt, nec abſque ea ſeparatim poſſunt intelligi: quibus exemplis quid inter Phyſicas & Mathematicas ſpecies interſit, haud difficile eſt animaduertere. Illis certè non ſemel eſt vſus Ariſtoteles. Valeant ergò Protagoræ ſophiſmata, Geometras hoc nomine refellentis, quod circulus normam puncto non attingat. Nam diuina Geometrarum theoremata, qui ſenſu æſtimabit, vix quicquam reperiet quod Geometræ concedendum videatur. Quid enim ex his quæ ſenſum mouent, ita rectum aut rotundum dici poteſt, vt a Geometra ponitur? Nec verò abſurdum eſt aut vitioſum, quòd lineas in puluere deſcriptas pro rectis aut rotundis aſſumit, quæ nec rectæ ſunt nec rotundæ, ac ne latitudinis quidem expertes. Siquidem non ijs vtitur Geometra, quaſi inde vim habeat concluſio, ſed eorum quæ diſcenti intelligenda relinquuntur, rudem ceu imaginem proponit. Nam qui primum inſtituuntur, hi ductu quodam & velut χειραγωγία ſenſuum opus habent, vt ad illa quæ ſola intelligentia percipiuntur, aditum ſibi comparare queant. Sed tamen exiſtimandum non eſt rebus Mathematicis omnino negari materiam ac non eam tantùm quæ ſenſum afficit. Eſt enim materia alia quæ ſub ſenſum cadit, alia quæ animo & ratione intelligitur. Illam αἰσθητὴν hanc νοητὴν vocat Ariſtoteles. Senſu percipitur, vt æs, vt lignum, omniſque materia quæ moueri poteſt

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rest. *Animo & ratione cernitur ea quæ in rebus sensilibus inest, sed non quatenus sensu percipiuntur, quales sunt res Mathematicorum. Vnde ab Aristotele scriptum legimus* ἐπὶ τῶν ἐν ἀφαιρέσει ὄντων *rectum se habere vt simum:* μετὰ συνεχοῦς γάρ: *quasi velit ipsius recti quod Mathematicorum est, suam esse materiam, non minus quam simi quod ad Physicos pertinet. Nam licet res Mathematicæ sensili vacent materia, non sunt tamen indiuiduæ, sed propter continuationem partitioni semper obnoxiæ, cuius ratione dici possunt sua materia non omnino carere: quin aliud videtur* τὸ εἶναι γραμμῇ, *aliud quoad continuationi adiuncta intelligitur linea. Illud enim ceu forma in materia proprietatum causa est, quas sine materia percipere non licet. Hæc est societatis & dissidij Mathematicæ cum Physica & prima Philosophia ratio. Nunc autem de nominis etymo & notatione pauca quædam afferamus. Nam si quæ iudicio & ratione imposita sunt rebus nomina, ea certè non temerè indita fuisse credendum est, quibus scientias appellari placuit, sed neque otiosa semper haberi debet ista etymologiæ indagatio, cùm ad rei etiam dubiæ fidem sæpe non parum valeat recta nominis interpretatio. Sic enim Aristoteles ducto ex verborum ratione argumento* αὐτομάτου, μεταβολῆς, αἰθέρος, *aliarumq́; rerum naturam ex parte confirmauit. Quoniam igitur Pythagoras Mathematicam scientiam non modò studiosè coluit, sed etiam repetitis à capite principijs, geometricam*

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contemplationem in liberalis disciplinæ formam composuit, & perspectis absq; materia solius intelligentiæ adminiculo theorematibus, tractationem περὶ τῶν ἀλόγων, & κοσμικῶν σχημάτων constitutionem excogitauit: credibile est, Pythagoram, aut certè Pythagoreos, qui & ipsi doctoris sui studia libenter amplexi sunt, huic scientiæ id nomen dedisse, quod cum suis placitis atq; decretis congrueret rerumq́; propositarum naturam quoquo modo declararet. Ita cùm existimarẽt illi, omnem disciplinam, quæ μάθησις dicitur, ἀνάμνησιν esse quandam .i. recordationem & repetionem eius scientiæ, cuius antè quam in corpus immigraret, compos fuerit anima, quemadmodum Plato quoque in Menone, Phædone, & alijs aliquot locis videtur astruxisse: animaduerterent autem eiusmodi recordationem, quæ non posset multis ex rebus perspici, ex his potißimum scientijs demonstrari, si quis nimirum, ait Plato, ἐπὶ τὰ διαγράμματα ἄγῃ, probabile est equidem Mathematicas à Pythagoreis artes κατ' ἐξοχὴν fuisse nominatas, vt ex quibus μάθησις, id est æternarum in anima rationum recordatio διαφερόντως & principuè intelligi posset. Cuius etiam rei fidem nobis diuinus fecit Plato, qui in Menone Socratem induxit hoc argumẽti genere persuadere cupientem, discere nihil esse aliud quam suarum ipsius rationum animum recordari. Etenim Socrates pusionem quendam vt Tullij verbis vtar, interrogat de geometrica dimensione quadrati: ad ea sic ille respondet vt puer, et tamen *mentam faciles interrogationes sunt, vt gradatim respondens, eodem perueniat quò si Geometrica didicisset. Aliam nominis huius rationem Anatolius exposuit, vt est apud Rhodiginum, quòd cùm caetera disciplinae deprehendi vel non docente aliquo possint omnes, Mathematica sub nullius cognitionem veniant, nisi praeunte aliquo, cuius solertia succidantur vepreta, vel exurantur, & superciliosa complanentur aspreta. Ita enim Caelius: quod quam vim habeat, non est huius loci curiosius perscrutari. Equidem M. Tullius Mathematicos in magna rerum obscuritate, recondita arte multiplicią; ac subtili versari scribit: sed quis nescit idipsum cum alijs grauioribus scientijs esse commune? Est enim, vel eodem auctore Tullio, omnis cognitio multis obstructa difficultatibus, maximaq; est & in ipsis rebus obscuritas, & in iudicijs nostris infirmitas, nec vllus est, modò interim paulò Physica penetrarit, qui non facilè sit expertus, quam multi vndique emergant, rerum naturalium causas inquirentibus, inexplicabiles labyrinthi. Sunt qui ex demonstrationum firmitate nominari Mathematicas opinantur: cuius etiam rationis momentum alio seorsim loco expendendum fuerit. Quocirca primam verbi notationem, quam sequutus est Proclus, nobis retinendam censeo. Hactenus de vniuerso Mathematica genere, quanta potui & perspicuitate & breuitate dixi. Sequitur vt de Geometria separatim atque ordine ea disseram, quae initio sum pollicitus*

tus. Est *autem Geometria, vt definit Proclus scientia, quæ versatur in cognitione magnitudinum figurarum, & quibus hæ continentur, extremorum, item rationum & affectionum, quæ in illis cernuntur ac inhærent: ipsa quidem progrediens à puncto indiuiduo per lineas & superficies, dum ad solida conscendat, variasq́; ipsorum differentias patefaciat. Quúmque omnis scientia demonstratiua, vt docet Aristoteles, tribus quasi momentis contineatur, genere subiecto, cuius proprietates ipsa scientia exquirit & cõtemplatur: causis & principijs ex quibus primis demonstrationes conficiuntur: & proprietatibus, quæ de genere subiecto per se enunciantur:* Geometriæ *quidem subiectum in lineis, triangulis, quadrangulis, circulis, planis, solidis atque omnino figuris & magnitudinibus, earúmque extremitatibus consistit. His autem inhærent diuisiones, rationes, tactus, æqualitates,* παραβολαὶ ὑπερβολαὶ ἐλλείψεις, *atque alia generis eiusdem propè innumerabilia. Postulata verò & Axiomata ex quibus hæc inesse demonstrantur, eiusmodi ferè sunt: Quouis centro & interuallo circulum describere. Si ab æqualibus æqualia detrahas, quæ relinquuntur esse æqualia, cæteraq́; id genus permulta, quæ licet omnium sint communia, ac demonstrandum tamen tum sunt accommodata, cum ad certum quoddam genus traducuntur. Sed cùm præcipua videatur Arithmetica & Geometria inter Mathematicas dignatio, cur Arithmetica sit* ἀκριβεστέρα *&*

exactior quam Geometria paucis explicandum arbitror. Hic verò & Aristotelem sequemur ducem, qui scientiam cum scientia ita comparat, vt accuratiorem esse velit eam, quæ rei causam docet, quam quæ rem esse tantùm declarat, deinde quæ in rebus sub intelligentiam cadentibus versatur, quàm quæ in rebus sensum mouentibus cernitur. Sic enim & Arithmetica quàm Musica, & Geometria quàm Optica, & Stereometria quàm Mechanica exactior esse intelligitur. Postremò quæ ex simplicioribus initijs constat, quàm quæ aliqua adiectione compositis vtitur. Atque hac quidem ratione Geometriæ præstat Arithmetica, quòd illius initium ex additione dicatur, huius sit simplicius. Est enim punctum, vt Pythagoreis placet, vnitas quæ situm obtinet: vnitas verò punctum est quod situ vacat. Ex quo percipitur, numerorum quàm magnitudinum simplicius esse elementum, numerosque magnitudinibus esse puriores, & à concretione materiæ magis disiunctos. Hæc quanquam nemini sunt dubia, habet & ipsa tamen Geometria quo se plurimum efferat, opibúsque suis ac rerum vbertate multiplici vel cum Arithmetica certet: id quod tute facilè deprehendas cùm ad infinitam magnitudinis diuisionem, quam respuit multitudo, animum conuerteris. Nunc quæ sit Arithmeticæ & Geometriæ societas, videamus. Nam theorematum quæ demonstratione illustrantur, quædam sunt vtriusque scientiæ communia, quædam verò singu-

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gularum propria. Etenim quòd omnis proportio sit ῥητὸς siue rationalis, Arithmeticæ soli conuenit, nequaquam Geometriæ, in qua sunt etiam ἄῤῥητοι seu irrationales proportiones. item, quadratorum gnωmas minimo definitos esse, Arithmeticæ proprium (si quidem in Geometria nihil tale minimum esse potest) sed ad Geometriam propriè spectant situs, qui in numeris locum non habent tactus, qui quidem à continuis admittuntur: ἄλογον, quoniam vbi diuisio infinitè procedit, ibi etiam τὸ ἄλογον esse solet. Communia porrò vtriusque sunt illa, quæ ex sectionibus eueniunt, quas Euclides *libro secundo est persequutus: nisi quòd sectio per extremam & mediam rationem in numeris nusquam reperiri potest. Iam vero ex theωrematibus eiusmodi communibus, alia quidem ex Geometria ad Arithmeticam traducuntur: alia contrà ex Arithmetica in Geometriam transferuntur: quædam verò perinde vtrique scientiæ conueniunt, vt quæ ex vniuersa arte Mathematica in vtranque harum conueniant. Nam & alterna ratio, & rationum conuersiones, compositiones, diuisiones hoc modo communia sunt vtriusque. Quæ autem sunt περὶ συμμέτρων, id est, de commensurabilibus, Arithmetica quidem primùm cognoscit & contemplatur: secundo loco Geometria Arithmeticam imitata. Quare & commensurabiles magnitudines illæ dicuntur, quæ rationem inter se habent, quam numerus ad numeram, per-*

inde quasi cōmensuratio & συμμετρία in numeris primùm consistat (Vbi enim numerus ibi & σύμμετρον cernitur: & vbi σύμμετρον, illic etiam numerus) sed quæ triangulorum sunt & quadrangulorum, à Geometra primum considerantur: tùm analogia quadam Arithmeticus eadem illa in numeris contemplatur. De Geometriæ diuisione hoc adijciendum puto, quòd Geometriæ pars altera in planis figuris cernitur quæ solam latitudinem longitudini coniunctam habent: altera verò solidas contemplatur, quæ ad duplex illud interuallum craßitudinem adsciscunt. Illam generali Geometriæ nomine veteres appellarunt: hanc propriè Stereometriam dixerunt. Ita Geometriam cum Optica & Stereometriam cum Mechanica non rarò comparat Aristoteles. Sed illius cognitio huius inuentionem multis seculis anteceßit, si modò Stereometriam ne Socratis quidem ætate vllam fuisse omnino verum est, quemadmodum à Platone scriptum videtur. Ad Geometriæ vtilitatem accedo, quæ quanquam suapte vi & dignitate ipsa per se nititur, nulllius vsus aut actionis ministerio mancipata (vt de Mathematicis omnibus scientijs concedit in Politico Socrates) si quid ex ea tamen vtilitatis externæ quæritur, Dij boni quam latos, quàm vberes, quum varios fructus fundit? Nec verò audiendus est vel Aristippus vel Sophistarum alius, qui Mathematicorum artes idcirco repudiet, quòd ex fine nihil docere videantur, eiusque quod melius aut

deteri-

deterius nullam habeant rationem. Vt enim nihil causæ dicas, cur sit melius trianguli, verbi gratia, tres angulos duobus esse rectis æquales; minime tamen fuerit consentaneum Geometriæ cognitionem vt inutilem exagitare, criminari, explodere, quasi quæ finem & bonum quò referatur, habeat nullum. Multas haud dubiè solius contemplationis beneficio citra materiæ contagionem adfert Geometria commoditates partim proprias, partim cum vniuerso genere communes. Cùm enim Geometria, vt scripsit Plato, eius quod semper est cognitionem profiteatur, ad veritatem excitabit illa quidem animum, & ad ritè philosophandum cuiusque mentem comparabit. Quinetiam ad disciplinas omnes facilius perdiscendas, attigeris necne Geometriam quanti referre censes? Nam vbi cum materia coniungitur, nónne præstantißimas procreat artes. Geodesiam, Mechaniam, Opticam, quarum omnium vsu, mortalium vitam summis beneficijs complectitur? Etenim bellica instrumenta, vrbiumq; propugnacula, quibus munitæ vrbes hostium vim propulsarent, his adiutricibus fabricata est: montium ambitus & altitudines, locorumque situs nobis indicauit: dimetiendorum & mari & terra itinerum rationem præscripsit: trutinas & stateras, quibus exacta numerorum æqualitas in ciuitate retineatur, composuit: vniuersi ordinem simulachris expreßit, multáque quæ hominum fidem superarent, omnibus persuasit. Vbique extant præclara in eam rem testimonia. Illud memorabile, quòd Archimedi rex Hiero tribuit. Nam extructo vastæ molis nauigio, quod Hiero Aegyptiorum regi Ptolemæo mitteret, cùm vniuersa Syracusanorum multitudo collectis simul viribus nauem trahere non posset effecissetq́; Archimedes, vt solus Hiero illam subduceret, admiratus viri scientiam rex ἀπὸ ταύτης, ἔφη, τῆς ἡμέρας περὶ παντὸς ἀρχιμήδει λέγοντι πιστευτέον. Quid? quòd Archimedes idem, vt est apud Plutarchũ, Hieroni scriptis datis viribus datum pondus moueri posse? fretusq́; demonstrationis robore illud sæpe iactarit, si terram haberet alteram vbi pedem figeret, ad eam, nostram hanc se transmouere posse? Quid varia, αὐτομάτων machinarûmque genera ad vsus necessarios comparata memorem? Innumerabilia profectò sunt illa, & admiratione dignissima quibus prisci homines incredibili quodam ad philosophandum studio concitati, inopem mortalium vitam artis huius præsidio subleuarunt: tametsi memoriæ sit proditum, Platonem Eudoxo & Archytæ vitio vertisse, quòd Geometrica problemata ad sensilia & organica abducerent. Sic enim corrumpi ab illis & labefieri Geometriæ præstantiam, quæ ab intelligibilibus & incorporeis rebus ad sensiles & corporeas prolaberetur. Quapropter ridicula idem scripsit Plato Geometrarum esse vocabula, quæ quasi ad opus & actionem spectent, ita sonare videntur. Quid enim est quadrare si non opus facere? Quid addere, producere, applicare? Multa quidem sunt eiusmodi nomina, quibus necessariò & tanquam coacti Geometræ vtuntur, quippe cùm alia desint in hoc genere commodiora. Sic ergo censuit Plato, sic Aristoteles sic denique philosophi omnes, Geometriam ipsam cognitionis gratia exercendam: nec ex aliquo vsu externo sed ex rerum νοητῶν intelligentia æstimandam esse. Exposita breuius quàm res tanta dici poßit, vtilitatis ratione, Geometriæ ortum, qui in hac rerum periodo ex historicorum monumentis nobis est cognitus deinceps aperiamus. Geometria apud AEgyptios inuenta, (ne ab Adamo, Setho, Noah, quos cognitione rerum multiplici valuisse constat, eam repetamus) ex terrarum dimensione, vt verbi præ se fert ratio, ortum habuisse dicitur: cùm anniuersaria Nili inundatione & incremenits limo obducti agrorum termini confunderentur. Geometriam enim, sicut & reliquas disciplinas, in vsu quàm in arte prius fuisse aiunt. Quod sanè mirum videri non debet, vt & huius & aliarum scientiarum inuentio ab vsu cœperit ac neceßitate. Etenim tempus, rerum vsus, ipsa neceßitas ingenium excitat, & ignauiam acuit. Deinde quicquid ortum habuit (vt tradunt Physici) ab incoato & imperfecto proceßit ad perfectum. Sic artium & scientiarum principia experientia beneficio collecta sunt: experientia verò à memoria fluxit, quæ & ipsa à sensu primùm manauit. Nam quod scribit Aristoteles, Mathematicas artes,

 compa-

comparatis rebus omnibus ad vitam necessarijs, in AEgypto fuisse constitutas, quòd ibi sacerdotes omnium concessu in otio degerent: non negat ille adductos necessitate homines ad excogitandam, verbi gratia terræ dimetiendæ rationem, quæ theorematum deinde inuestigationi causam dederit: sed hoc confirmat, præclara eiusmodi theorematum inuenta, quibus extructa Geometriæ disciplina constat, ad vsus vitæ necessarios ab illis non esse expetita. Itaque vetus ipsum Geometriæ nomen ab illa terræ partiundæ finiúmque regendorum ratione posteà recessit, & in certa quadam affectionum magnitudini per se inhærentium scienta propriè remansit. Quemadmodum igitur in mercium & contractuum gratiam supputandi ratio, quam secuta est accurata numerorum cognitio, à Phœnicibus initium duxit: ita etiam apud AEgyptios, ex ea quam commemoraui causa ortum habuit Geometria. Hanc certè, vt id obiter dicam, Thales in Græciam ex AEgypto primum transtulit: cui non pauca deinceps à Pythagora, Hipprocrate, Chio, Platone, Archyta Tarentino, alijsq; compluribus, ad Euclidis tempora factæ sunt rerum magnarum accessiones. Cæterùm de Euclidis ætate id solùm addam, quod à Proclo memoriæ mandatum accepimus. Is enim commemoratis aliquot Platonis tum æqualibus tùm discipulis, subijcit, non multò ætate posteriorem illis fuisse Euclidem eum, qui Elementa conscripsit, & multa ab Eudoxo collecta, in ordinem luculentum

com-

composuit, multaq́; à Theateto inchoata perfecit, quaeq́; mollius ab alijs demonstrata fuerant, ad firmissimas & certissimas apodixes reuocauit. Vixit autem, inquit ille sub primo Ptolemaeo. Et enim ferunt Euclidam à Ptolemaeo quondam interrogatum num qua esset via ad Geometriam magis compendiaria, quàm sit ista στοιχείωσις *respondisse* μὴ εἶναι βασιλικὴν ἀτραπὸν ἐπὶ γεωμετρίαν. *Diende subiungit, Euclidem natu quidem esse minorem Platone, maiorem verò Eratosthene & Archimede (hi enim quales erant) cùm Archimedes Euclidis mētionem faciat. Quòd si quis egregiam Euclidis laudem quam cùm ex alijs scriptionibus accuratißimis, tùm ex hac Geometria* στοιχειώσει *cōsequutus est in qua diuinus rerum ordo sapientißimis quibusque hominibus magnae semper admirationi fuit, is Proclum studiosè legat, quò rei veritatem illustriorem reddat grauißimi testis auctoritas. Superest igitur vt finem videamus, quò Euclidis elementa referri, & cuius causa in id studium incumbere, oporteat. Et quidem si res quae tractantur, consideres: in tota hac tractatione nihil aliud quaeri dixeris, quàm vt* κοσμικὰ *quae vocantur* σχήματα *(fuit enim Euclides profeßione & instituto Platonicus) Cubus Icosaedrum, Octaedrum, Pyramis & Dodecaedrum certae quadam suorum & inter se laterum, & ad sphaerae diametrum ratione eidem sphaerae inscripta comprehendantur. Huc enim pertinet Epigrammation illud vetus, quod in Geometrica Michaelis*

B 5 Pselli

Pselli συνόψει *sriptum legitur.*

Σχήματα πέντε ἃ Πλάτωνος, Πυθαγόρας σοφὸς εὗρε.

Πυθαγόρας σοφὸς εὗρε, Πλάτων δ᾽ ἀρίδηλ᾽ ἐδίδαξεν.

Εὐκλείδης ἐπὶ τοῖσι κλέος περικαλλὲς ἔτευξεν.

Quòd si discentis institutionem spectes, illud certè fuerit propositum, vt huiusmodi elementorum cognitione informatus discentis animus, ad quamlibet non modò Geometriæ, sed & aliarum Mathematicæ partium tractationem idoneus paratusq; accedat. Nam tametsi institutionem hanc solus sibi Geometra vendicare videtur, & tanquam in possessionem suam venerit, alios excludere posse: inde tamen per multa suo quodam modo iure decerpit Arithmeticus, pleraque Musicus non pauca detrahit. Astrologus, Opticus, Logisticus, Machanicus, itemq; cæteri: nec vllus est denique artifex præclarus qui in huius se possessionis societatem cupide non offerat, partemq; sibi cõcedi postulet. Hinc στοιχείωσις *absolutum operi nomen, &* στοιχειωτὴς *dictus Euclides. Sed quid longius prouehor? Nam quod ad hanc rem attinet, tam copiosè & eruditè scripsit (vt alia complura) eo ipso, quem dixi, loco P. Montaurens, vt nihil desiderio loci reliquerit. Quæ verò ad dicendum nobis erant proposita hactenus pro ingenij nostri tenuitate omnia mihi perfecisse videor. Nam tametsi & hæc eadem & alia pleraque multo fortè præclariora ab hominibus doctissimis, qui* *tùm acumine ingenij, tùm admirabili quodam lepore dicendi semper floruerunt grauiùs, splendidiùs, vberiùs tractari posse scio: tamen experiri libuit numquid etiam nobis diuino sit concessum munere, quod rudes in hac philosophiæ parte discipulos adiuuare aut certè excitare queat. Huc accessit quòd ista recens elementorum editio, in qua nihil non parum fuisset studij, aliquid à nobis efflagitare videbatur, quod eius commendationem adaugeret. Cùm enim vir doctissimus Io. Magnienus Mathematicarum artium in hac Pharrhisiorum Academia professor verè regius, nostrum hunc typographum in excudendis Mathematicorum libris diligentissimum, ad hanc Elementorum editionem sæpè & multum esset adhortatus, eiusque impulsu permulta sibi iam comparasset typographus ad hanc rem necessaria, citò interuenit, malùm Ioannis Magnieni mors insperata, quæ tam graue inflixit Academiæ vulnus, cui ne post multos quidem annorum circuitus cicatrix obduci vlla posse videatur. Quamobrem amisso instituti huius operis duce, typographus, qui nec sumptus antea factos sibi perire, nec studiosos, quibus id muneris erat pollicitus, sua spe cadere vellet, ad me venit, & impensè rogauit, vt meam propositæ editioni operam & studium nauarem, quòd cùm denegaret occupatio nostra, iuberet officij ratio, feci equidem rogatus, vt quæ subobscurè vel parum commodè in sermonem Latinum è Græco translata*

vi-

videbantur, clariore, aptiore & fideliore interpretatione nostra (quod cuiusq; pace dictum volo) lucem acciperent. Id quod in omnibus ferè libris posterioribus tute primo obtutu perspicias. Nam in sex prioribus non tantum temporis quantum in cæteres ponere nobis licuit decimi autem interpretatio, quæ melior nulla potuit adfert, P. Montaureo solida debetur. Atq; vt ad perspicuitatem facilitatemq; nihil tibi deesse queraris, adscripta sunt propositionibus singulis vel lineares figuræ, vel punctorum tanquam vnitatum notulæ, quæ Theonis apodixin illustrent illæ quidem magnitudinum, hæ autem numerorum indices, subscriptis etiam ciphrarum, vt vocant, characteribus, qui propositum quemuis numerum exprimant, ob eamq; causam eiusmodi vnitatum notulæ, quæ pro numeri amplitudine maius paginæ spatium occuparent, pauciores sæpius depictæ sunt aut in lineas etiam commutatæ. Nam literarum vt a, b, c characteres non modò numeris & numerorum partibus nominandis sunt accommodati, sed etiam generales esse numerorum, vt magnitudinum affectiones testantur. Adiecta sunt insuper quibusdam locis non pœnitenda Theonis scholia, siue mauis lemmata, quæ quidem longè plura accessissent, si plus otij & temporis vacui nobis fuisset relictum, quod huic studio impartiremus. Hanc igitur operam boni consule, & quæ obuia erunt impreßionis vitia, candidè emenda. Vale. Lutetiæ 4. Idus April. 1557.

EVCLIDIS ELEMENTVM PRIMVM

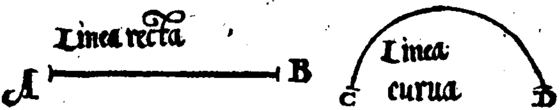
DEFINITIONES.

1

PVnctum est, cuius pars nulla est. Punctum

2

Linea verò, longitudo latitudinis expers.



3

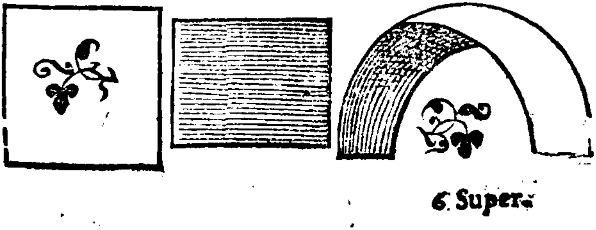
Lineæ autem termini, sunt puncta.

4

Recta linea est, quæ ex æquo sua interiacet puncta.

5

Superficies est, quæ longitudinem latitudinemque tantùm habet.

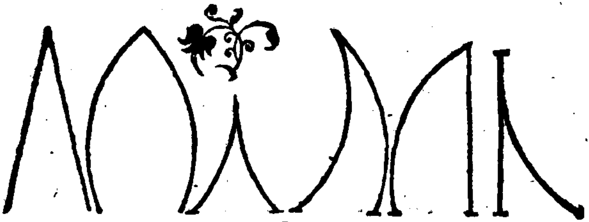


6

Superficiei extrema. ſunt lineæ.

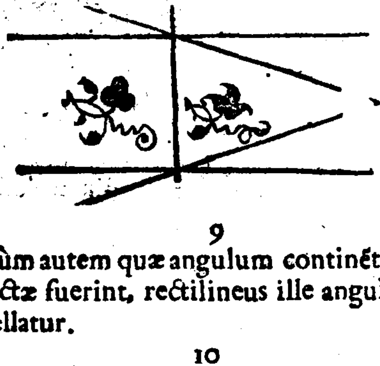
7

Plana ſuperficies eſt, quæ ex æquo ſuas interiacet lineas.



8

Planus angulus eſt duarū linearū in plano ſe mutuò tangentium, & non in directum iacétium, alterius ad alterā inclinatio.



9

Cùm autem quæ angulum continét lineæ, rectæ fuerint, rectilineus ille angulus appellatur.

10

Cùm verò recta linea ſuper rectam conſiſtens lineā, eos qui ſunt deinceps angulos ęquales, inter ſe fecerit: rectus eſt vterq; æ-

qua-

qualium angulorum: quæ insistit recta linea, perpédicularis vocatur ei', cui insistit.

11

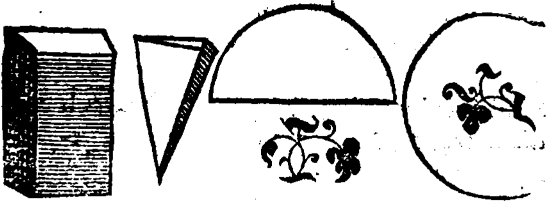
Obtusus angulus est, qui recto maior est.

12

Acutus verò, qui minor est recto.

13

Terminus est, quod alicuius extremũ est.



14

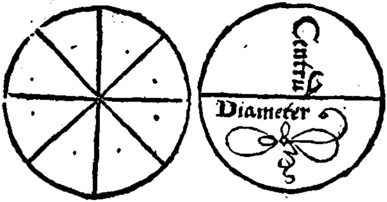
Figura est, quæ sub aliquo, vel aliquibus terminis comprehenditur.

15

Circulus est, figura plana sub vna linea cóprehésa, quę peripheria applatur: ad quam ab vno puncto eorum. quæ intra figuram

sunt

ſunt po ſita, cadentes omnes rectæ lineæ inter ſe ſunt æquales.



16

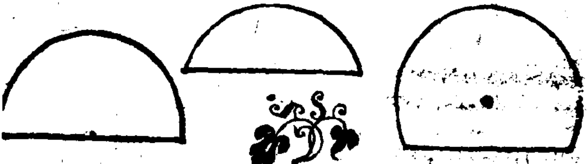
Hoc verò punctum, centrum circuli appellatur.

17

Diameter autem circuli, eſt recta quædam linea per centrum ducta, & ex vtraque parte in circuli pheripetiam terminata, quæ circulum bifariam ſecat.

18

Semicirculus eſt figura, quæ continetur ſub diametro, & ſub ea linea, quæ de circuli peripheria aufertur.



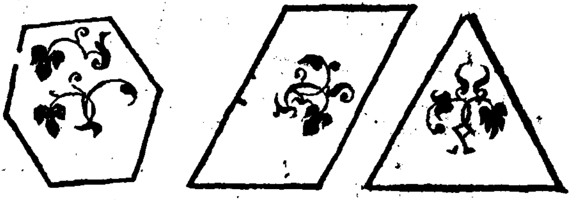
19

Segmentum circuli, eſt figura, quæ ſub recta linea & circuli peripheria continentur.

20 Recti

20.

Rectilineæ figuræ, sunt quæ sub rectis lineis continentur.



21

Trilateræ quidem, quæ sub tribus.

22

Quadrilateræ, quæ sub quatuor.

23

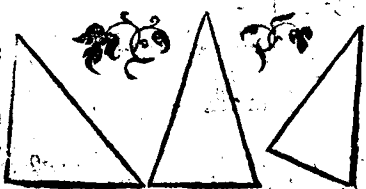
Multilateræ verò, quæ sub pluribus quàm quatuor rectis lineis comprehenduntur.

24

Trilaterarũ, porrò figurarum, æquilaterum est triangulum, quod tria latera habet æqualia.

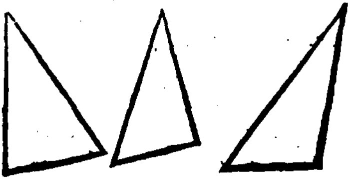
25

Isosceles autem est quod duo tantum æqualia habet latera.



26

Scalenū verò, est qđ tria inæqualia habet latera.



23

Ad hæc etiam trilaterarum figurarum, rectangulum quidem triangulum est. quod rectum angulum habet.

28

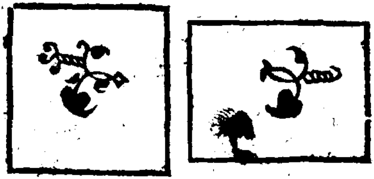
Amblygonium autem, quod obtusum angulum habet.

29

Oxygonium verò, quod tres habet acutos angulos.

30

Quadrilaterarum autem figurarum, quadratū quidē est qđ & æquilaterū & rectangulum est.



31

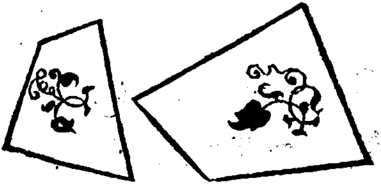
Altera parte longior figura est, quæ rectāgula quidem, at æquilatera non est.

32 Rhom-

32
Rhó-
bus au
tem,
qui æ-
qui la-
terum
& re-


ctangulum eſt.

33
Rhomboides verò, quæ aduerſa & latera & angulos habens inter ſe æqualia, neque æquilatera eſt, neque rectangula.

34
Prȩter
has au
tē re-
liquæ
qua-
drila-

teræ figuræ, trapezia appellentur.

35
Parallelæ rectæ lineæ ſunt quæ, cùm in eodem ſint plano, & ex vtraque parte in infinitum producantur, in neutram ſibi mutuò incidunt.

Poſtulata.

1
Poſtuletur, vt à quouis puncto in quoduis

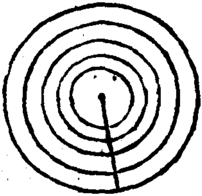
punctum, rectam lineã ducere cõcedatur.

2

Et rectam lineam terminatam in continuum recta producere.

3

In quouis centro & interuallo circulũ describere.



Communes notiones

1.

Quæ eidẽ æqualia, & inter se sunt æqualia.

2

Et si æqualibus æqualia adiecta sint, tota sunt æqualia.

3

Et si ab æqualibus æqualia ablata sint, quæ relinquuntur sunt æqualia.

4

Et si inæqualibus æqualia adiecta sint, tota sunt inæqualia.

5

Et si ab inęqualibus æqualia ablata sint, reliqua sunt inæqualia.

6

Quæ eiusdem duplicia sunt, inter se sunt æqualia.

Et

7

Et quæ eiusdem sunt dimidia, inter se æqualia sunt.

8

Et quæ sibi mutuò congruunt, ea inter se sunt æqualia.

9

Totum est sua parte maius.

10

Item, omnes recti anguli sunt inter se ęquales.

11

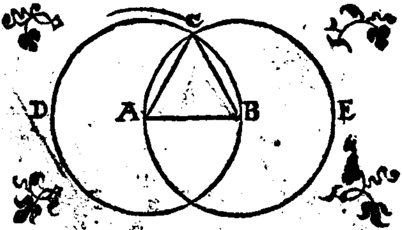
Et si in duas rectas lineas altera recta incidens, internos ad easdéque partes angulos, duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ sibi mutuò incident ad eas partes, vbi sunt anguli duobus rectis minores.

12

Duæ rectæ lineæ spatium non comprehendunt.

Problema 1. Propositio 1.

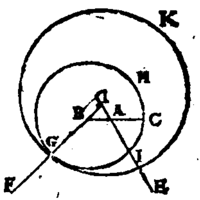
Super data recta linea terminata, triangulú



æquilaterum constituere.

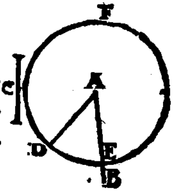
Problema 2. Propoſitio 2.

Ad datum punctum, datæ rectæ lineæ, ęqualem rectam lineam ponere.



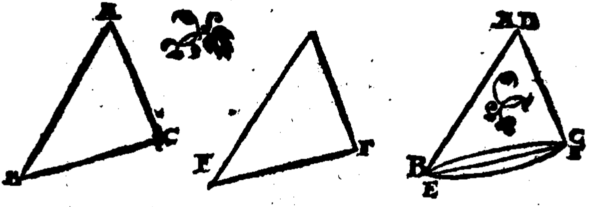
Problema 3, Propoſitio 3.

Duabus datis rectis lineis inęqualibus, de maiore æqualẽ minori rectam lineā detrahere.



Theorema primum. Propoſitio 4.

Si duo triangula duo latera duobus lateribus æqualia habeant, vtrunque vtriq́;, habeant verò & angulum angulo æqualẽ ſub æqualibus rectis lineis contentum: & baſin baſi æqualem habebunt, eritq; triāgulum triāgulo æquale, ac reliqui anguli reliquis angulis æquales erūt, vterque vtrique, ſub quibus æqualia latera ſubtenduntur,



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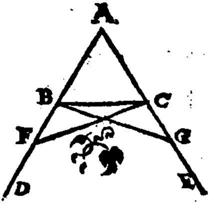
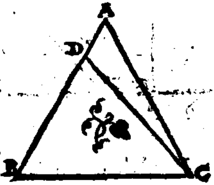
Theorema 2. Propoſitio 5.



Iſoſcelium tringulorũ qui ab baſim ſunt anguli, inter ſe ſunt æquales; & ſi vlterius productæ ſint æquales illę rectæ lineæ, qui ſub baſi ſunt anguli, inter ſe equales erunt.

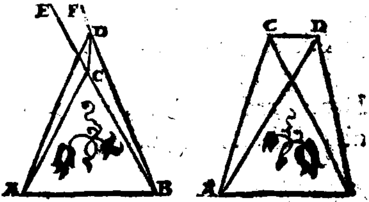
Problema 3. Propoſitio 6.



Si trianguli duo anguli æquales inter ſe fuerint & ſub ęqualibus angulis ſubtenſa latera æqualia inter ſe erunt.

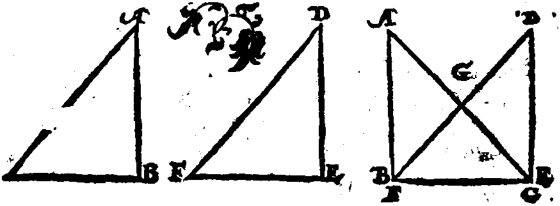
Theorema 46. Propoſitio 7.

Super eadẽ recta linea, duabus eiſdem rectis lineis aliȩ duȩ rectȩ lineæ ęquales, vtraque vtrique nõ conſtituentur, ad aliud atque aliud punctũ, ad eaſdẽ partes eoſdẽ que terminos cum duabus initio ductis rectis lineis habentes.



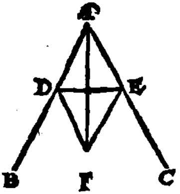
Theorema 5. Propositio 8.

Si duo triãgula duo latera habuerint duobus lateribus, vtrunque vtriq; æqualia: habuerint verò & basim basi æqualẽ: angulũ quoque sub æqualibus rectis lineis cõtentum angulo æqualem habebunt.



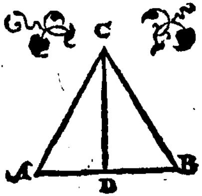
Problema 4. Porpopositio 9.

Datum angulum rectilineum bifariam secare.



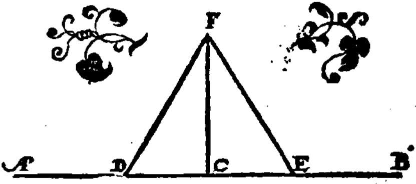
Problema 5. Propositio 10.

Datam rectam lineã finitam bifariam secare.



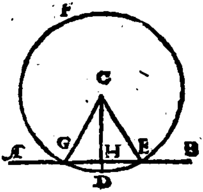
Problema 6. Propoſitio 11.

Data recta linea, à puncto in ea dato, rectam lineam ad angulos rectos excitare.



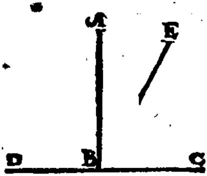
Problema 7. Propoſitio 12.

Super datam rectã lineã infinitam, à dato puncto quod in ea non eſt, perpendicularem rectam deducere.



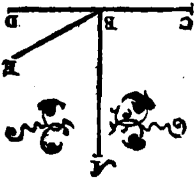
Theorema 6. Propoſitio 13.

Cùm recta linea ſuper rectã conſiſtens lineã angulos facit, aut duos rectos, aut duobus rectis æquales efficiet.



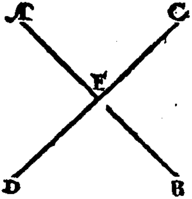
Theorema 7. Propoſitio 14.

Si ad aliquam rectam lineam, atque ad eius punctũ, duę rectęlineæ nõ ad eaſdem partes ductæ, eos qui ſunt deinceps angulos duobus rectis æquales fecerint, in directum erunt inter ſe ipſæ rectæ lineæ.



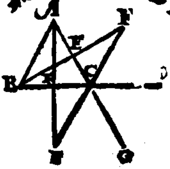
Theorema 8. Propoſitio 15.

Si duę rectę lineę ſe mutuò ſecuerint, angulos qui ad verticem ſunt, æquales inter ſe efficient.



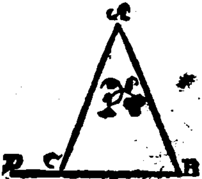
Theorema 9. Propoſitio 16.

Cuiuſcunque trianguli vno latere producto, externus angulus vtroque interno & oppoſito maior eſt.



Theorema 10. Propoſitio 17.

Cuiuſcunque trianguli duo auguli duobus rectis ſunt minores omnifariam ſumpti.



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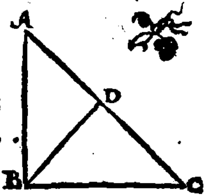
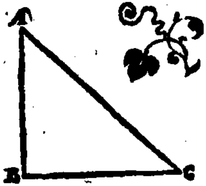
Theorema 11. Propositio 18.



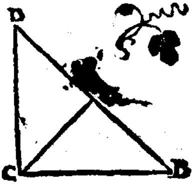
Omnis trianguli maius latus maiorem angulũ subtendit.

Theorema 12. Propositio 16.



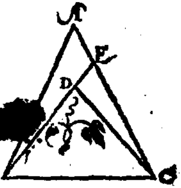
Omnis trianguli lateri angulus, maiori lateri subtenditur.

Theorema 13. Propositio 20.



Omnis triãguli duo latera reliquo sunt maiora, quomodocumq; assumpta.

Theorema 14. Propositio 12.



Si super triãguli vno latere, ab extremitatibus duæ rectę lineę, interiu cõstitutæ fuerint, hę cõstitutæ reliquis triãguli duobus lateribus minores quidem erunt, minorem verò angulum continebunt.

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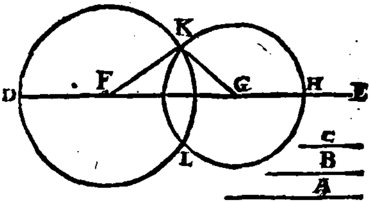
Problema 8. Propoſitio 22.

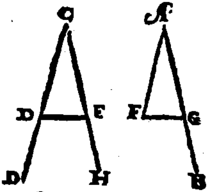
Ex tribus rectis lineis quæ ſũt tribus datis rectis lineis æquales,



triangulũ conſtituere. Oportet autem duas reliqua eſſe maiores omnifariam ſumptas: quoniã vniuſcuiuſq; trianguli duo latera omnifariam ſumpta reliquo ſunt maiora.

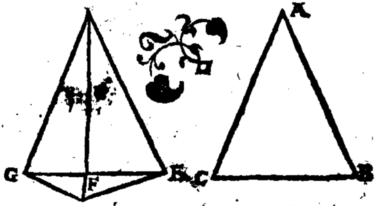
Problema 9. Propoſitio 23.

Ad datam rectam lineã datumq; in ea punctum dato angulo rectilineo ęqualem angulum rectilineum conſtituere.



Theorema 15. Propoſitio 24.

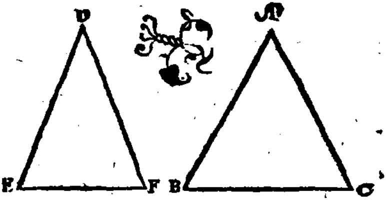
Si duo triãgula duo latera duobus lateribus ęqualia



habuerint, vtrũq; vtriq;, angulũ verò angulo lo maiorem sub æqualibus rectis lineis cō-tentum: & basin basi maiorem habebunt.

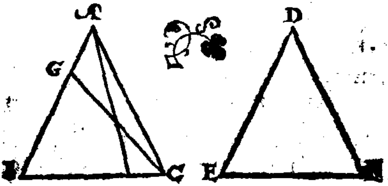
Theorema 16. Propositio 25.

Si duo triangula duo latera duobus lateribus æqualia habuerint, vtrunque vtrique, basin verò basi maiorē: & angulum sub æqualib⁹ rectis lineis contentum angulo maiorē habebūt.



Theorema 17. Propositio 26.

Si duo triangla duos angulos duobus angulis æquales habuerint, vtrūque vtrique, vnumq́; latus vni lateri æquale, siue, quod æqualibus adiacet angulis, seu quod vni æqualium angulorū subtenditur: & reliqua latera reliquis lateribus ęqualia vtrūq; vtriq; & reliquum angulū reliquo angulo æqualem habebunt.



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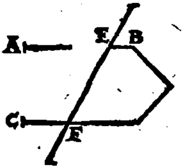
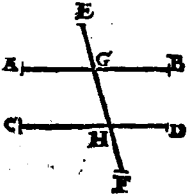
Theorema 18. Propositio 27.



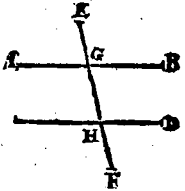
Si in duas rectas lineas recta incidẽs linea alternatim ãgulos ęquales inter se fecerit: parallelę erunt inter se illæ rectæ lineæ.

Theorema 19. Propositio 28.



Si in duas rectas lineas recta incidens linea externum angulũ interno, & opposito, & ad easdem partes æqualẽ fecerit, aut internos & ad easdẽ partes duobus rectis æquales: parellelæ erunt inter se ipsæ rectæ lineæ.

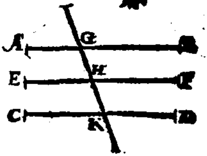
Theorema 20. Propositio 29.



In parallelas rectas lineas recta incidens linea: & alternatim angulos inter se æquales efficit & externum interno & opposito & ad easdem partes æqualem, & inter nos & ad easdem partes duobus rectis æquales facit.

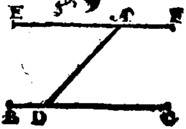
Theore-

Theorema 21. Propositio 30.



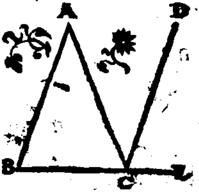
Quæ eidem rectæ lineę, parallelę, & inter se sunt parallelæ.

Problema 10. Propositio 31.



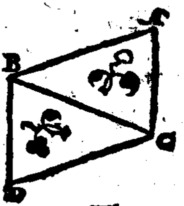
A dato puncto datæ rectæ lineæ parallelam rectam lineam ducere.

Theorema 22. Propositio. 32.



Cuiuscunque trianguli vno latere vlterius producto: externus angulus duobus internis & oppositis est æqualis. Et trianguli tres interni anguli duobus sunt rectis æquales.

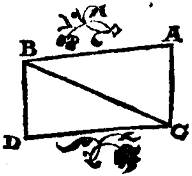
Theorema 23. Propositio 33.



Rectæ lineę quę ęquales & parallelas lineas ad partes easdem coniungunt, & ipsæ æquales & parallelæ sunt.

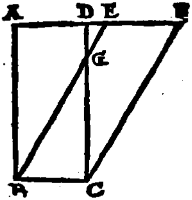
Theore-

Theorema 24. Propositio 34.



Parallelogrammorũ ſpatiorum æqualia ſunt inter ſe, quæ ex aduerſo & latera & anguli: atque illa bifariã ſecat diameter.

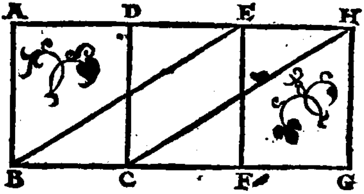
Theorema 25. Propoſitio 35.



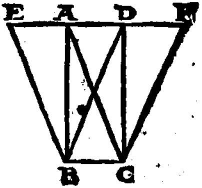
Parallelogramma ſuper eadem baſi, & in eiſdem parallelis cõſtituta, inter ſe ſunt æqualia.

Theorema 26. Propoſitio 36.

Parallelogrãma ſuper æqualibus baſibus & in eiſdẽ parallelis cõſtituta inter ſe ſunt æqualia.



Theorema 27. Propoſitio 37.

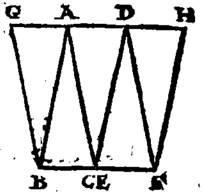


Triãgula ſuper eadẽ baſi cõſtituta, & in eiſdẽ parallelis, inter ſe ſunt ęqualia.

Theore-

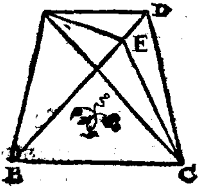
Theorema 28. Propositio 38.

Triangula super ęqualibus basibus constituta et in eisdem parallelis, inter se sunt æqualia.



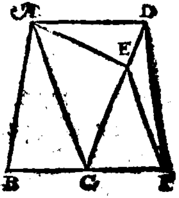
Theorema 29. Propositio 39.

Triangula æqualia super eadem basi, & ad easdem partes constituta: & in eisdem sunt Parallelis.



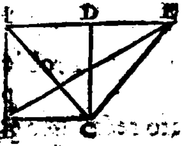
Theorema 30. Propositio 40.

Triangula æqualia super æqualibus basibus, & ad easdem partes constituta, & in eisdem sunt parallelis.

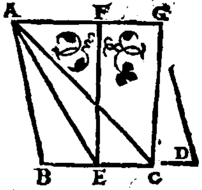


Theorema 31. Propositio 41.

Si parallelogrammum cum triangulo eandem basin habueris, non eisdemque fuerit parallelis, duplum erit parallelogrammum ipsius trianguli.

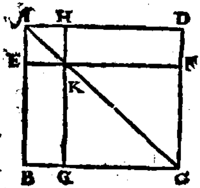


Problema 11. Propositio 42.



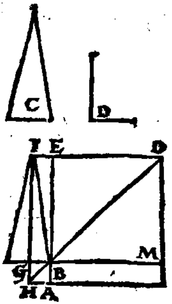
Dato triangulo æquale parallelogrammũ constiture in dato angulo rectilineo.

Theorema 32. Propositio 43.



In omni parallelogrammo, complementa eorũ quę circa diametrũ sunt parallelogrammorũ, inter se sunt æqualia.

Problema 12. Propositio 44.

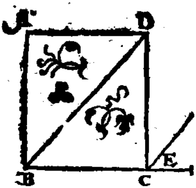


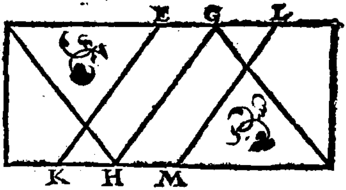
Ad datam rectam lineã dato triangolo æquale parallelogrammum applicare in dato angulo rectilineo.

Problema 13. Propopotio 45.

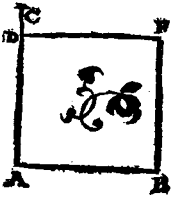
Dato rectilineo ęquale parallelongrammũ consti-

constituere in dato angulo rectilineo.





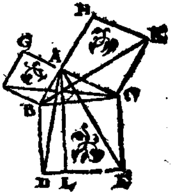
Theorema 41. Propositio 4.



A data recta linea quadratum describere.

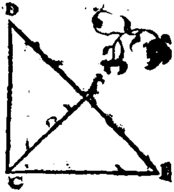
Theorema 33. Propositio 47.

In rectangulis triangulis, quadratum quod à latere rectum angulum subtendente describitur, æquale est eis, quæ à lateribus rectum angulum continentibus describũtur, quadratis.



Theorema 34. Propositio 48.

Si quadratum quod ab vno laterum trianguli describitur, æquale sit eis quæ à reliquis trianguli lateribus describuntur, quadratis: angulus comprehensus sub reliquis duobus trianguli lateribus, rectus est.



FINIS ELEMENTI I.

EVCLIDIS ELEMENTVM SECVNDVM.

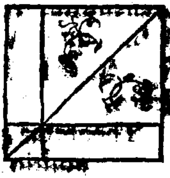
DEFINITIONES

1.

OMne parallelogrammum rectangulum contineri dicitur sub rectis duabus lineis, quę rectum comprehendunt angulum.

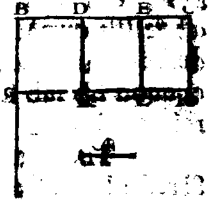
2.

In omni parallelogrammo spatio, vnumquodlibet eorum, quæ circa *diametrum* illius sunt parallelogrammorum, cum duobus cōplementis, Gnomo vocetur.



Theorema 1. Propositio 1.

Si fuerint duæ rectæ lineæ, seceturque ipsarum altera in quotcunq; segmenta: rectangulum comprehensum sub illis duabus rectis lineis, æquale est eis rectangulis, quæ sub insecta & quolibet segmētorum comprehenduntur.



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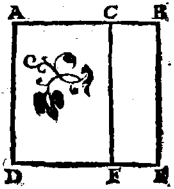
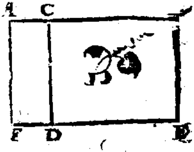


Theorema 2. Propoſitio 2.

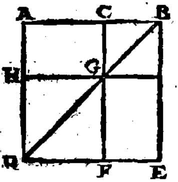
Si recta linea ſecta ſit vtcunq;, rectāgula quæ ſub tota & quolibet ſegmentorum cōprehenduntur æqualia ſunt ei, quod à tota fit, quadrato.

Theorema 3. Propoſitio 3.

Si recta linea ſecta ſit vtcunque, rectangulum ſub tota & vno ſegmētorum comprehenſum, æquale eſt & illi quod ſub ſegmentis cōprehenditur rectangulo & illi, quod à prædicto ſegmēto deſcribitur, quadrato.



Theorema 4. Propoſitio 4.



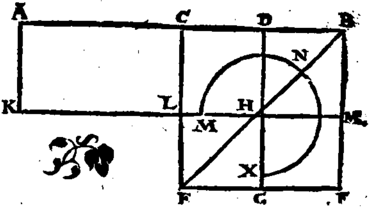
Si recta linea ſecta ſit vtcunque; quadratū quod à tota deſcribitur, æquale eſt & illis quæ à ſegmētis deſcribūtur quadratis, & ei quod bis ſub ſegmentis comprehēditur rectangulo.

Theorema 5. Propoſitio 5.

Si recta lenea ſecetur in æqualia & non æqualia: rectangulum ſub inæqualibus ſeg-

mentis

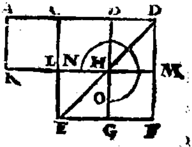
mentis totius comprehẽsũ, vnà cum quadrato quod ab inter me-



dia sectionum, æquale est ei quod à dimidia describitur, quadrato.

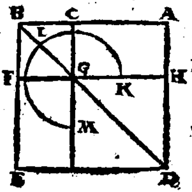
Theorema 6 Propositio 6.

Si recta linea bifariam secetur, & illi recta quædam linea in rectum adijciatur, rectãgulum comprehensum sub tota cum adiecta & adiecta simul cum quadrato à dimidia, æquale est quadrato à linea quæ tũ ex dimidia, tum ex adiecta componitur, tanquam ab vna descripto.



Theorema 7. Propositio 7.

Si recta linea secetur vtcunque, quod à tota, quodq́ue ab vno segmentorum, vtraq; simul quadrata, æqualia sunt & illi quod bis sub tota & dicto segemento comprehenditur, rectangulo, & illi ꝗ à reliquo segmento fit, quadrato.



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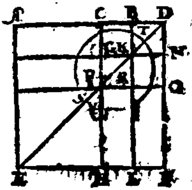
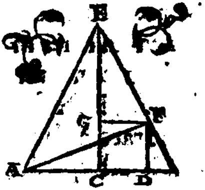
Theorema 8. Propositio 8.

Si recta linea secetur vtcunq; rectangulum quater comprehensum sub tota & vno segmentorum, cum eo quod à reliquo segmẽto fit, quadrato, ęquale est ei quod à tota & dicto segmento, tanquã ab vna linea describitur quadrato.



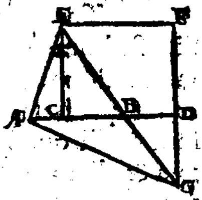
Theorema 9. Propositio 9.

Si recta linea secetur in æqualia & non æqualia; quadrata quæ ab inequalibus totius segmẽtis fiunt, duplicia sunt & eius quod à dimidia, & eius quod ab intermedia sectionum fit, quadratorum.



Theorema 10. Propositio 10.

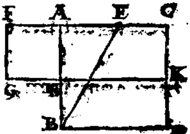
Si recta linea secetur bifariam, adijciatur autem ei in rectum quępiam recta linea: quod à tota cũ adiuncta, & quod ab adiuncta, vtraq; simul quadrata, duplicia sunt, & eius quod à dimidia, & eius quod à composita ex dimidia & adiuncta,



iuncta, tanquã ab vna descriptum sit quadratorum.

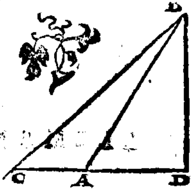
Problema I. Propositio II.

Datam rectam lineã secare, vt comprehensum sub tota & altero segmẽtorum rectangulum, æquale sit ei quod à reliquo segmento fit, quadrato.



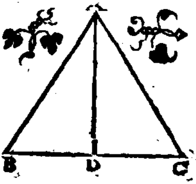
Theorema II. Propositio 12.

In amblygonijs triangulis, quadratũ quod fit à latere angulum obtusum subtendẽte, maius est quadratis, quæ fiunt à lateribus obtusum angulum comprehendentibus pro quantitate rectanguli bis comprehen & ab vno laterũ quę sunt circa obtusum angulũ, in quod cum protractum fuerit, cadit perpendicularis, & ab assumpta exterius linea sub perpendiculari prope angulũ obtusum.



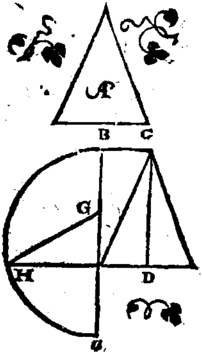
Theorema 12. Propoſitio 13

In oxygonijs triangulis quadratũ à latere angulum acutum ſubtendente, minus eſt quadratis quæ fiunt à lateribus acutũ angulum cõprehendentibus, pro quantitate rectanguli bis comprehenſi, & ab vno laterum, quæ ſunt circa acutum angulum, in quod perpẽdicularis cadit, & ab aſſumpta interius linea ſub perpendiculari prope acutũ angulum.



Problema 2. Propoſitio 14.

Dato rectilineo æquale quadratũ conſtituere.



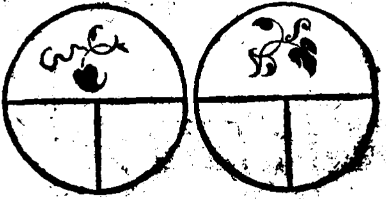
ELEMENTI II. FINIS.

EVCLIDIS ELEMENTVM TERTIVM.

DEFINITIONES.

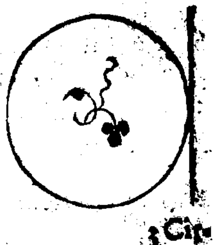
1.

Aequales circuli sunt quorum diametri sũt ęquales, vel quorũ quæ ex centris, rectæ lineę sunt æquales.



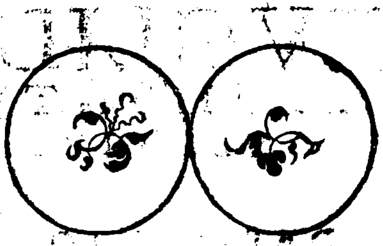
2.

Recta linea circulũ tangere dicitur, quæ cùm circulun tangat, si producatur, circulum non secat.



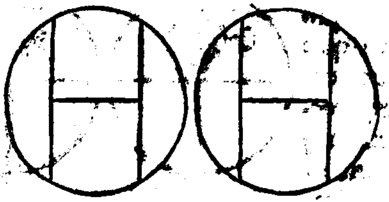
3

Circuli ſeſe mutuò tangere dicuntur: qui ſeſe mutuo tangentes, ſeſe mutuò non ſecant.



4

In circulo æqualiter diſtare à centro rectæ lineæ dicuntur, cùm perpendiculares quæ à cẽtro in ipſas ducuntur, ſunt æquales. Lõgius autem abeſſe illa dicitur in quã maior perpendicularis cadit.



5

Segmentum circuli eſt, figura quæ ſub recta linea & circuli peripheria comprehenditur.

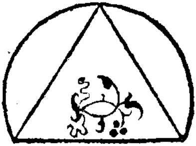
6

Segmenti autem angulus eſt, qui ſub recta linea

linea & circuli peripheria comprehēditur

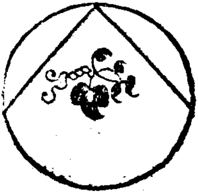
7

In segmento autem angulus est, cum in segmenti peripheria sumptum fuerit quodpiam punctum, & ab illo in terminos rectæ eius lineæ, quæ segmenti basis est, adiūctę fuerint rectæ lineæ: is, inquam, angulus ab adiunctis illis lineis comprehēsus.



8

Cùm verò comprehendentes angulum rectæ lineæ aliquam assumitur peripheriam, illi angulus insistere dicitur.



9

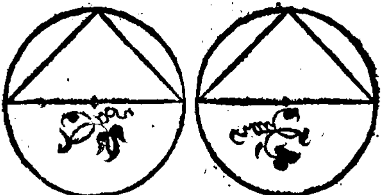
Sector autem circuli est, cùm ad ipsius circuli cētrum constitutus fuerit angulus, comprehensa nimirum figura, & à rectis lineis angulum continentibus, & à peripheria ab illis assumpta.

10

Similia circuli segmenta sunt, quæ angulos

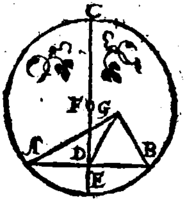
capiunt

capiunt ęquales aut in q b'anguli inter se sunt ęquales



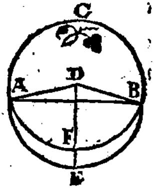
Problema 1. Propositio 1.

Dati circuli centrum reperire.



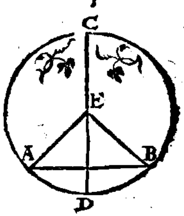
Theorema 1. Propositio 2.

Si in circuli peripheria duo quælibet puncta accepta fuerint, recta linea quæ ad ipsa puncta adiungitur, intra circulum cadet.



Theorema 2. Propositio 3.

Si in circulo recta quædam lenea per centrum extensa quandam non per centrum extensam bifariam secet: & ad angulos rectos ipsam secabit. Et si ad angulos rectos eam secet, bifariam quoque eam secabit,



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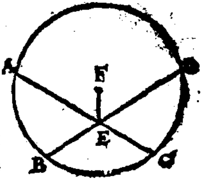
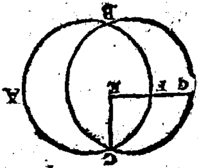
Theorema 3. Propositio 4.



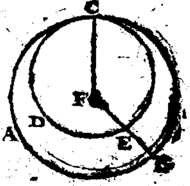
Si in circulo duę rectę lineæ sese mutuo secent non per centrum extēsæ sese mutuò bifariam non secabunt.

Theorema 4. Propositio 5.



Si duo circuli sese mutuò secent, non erit illorum idem centrum.

Theorema 5. Propositio 6.



Si duo circuli sese mutuò interius tangant, eorum non erit idem centrum.

Theorema 6. Propositio 7.

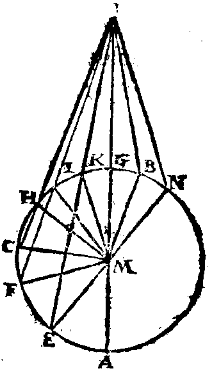
Si in diametro circuli quodpiam sumatur punctum, quod circuli centrum non sit, ab eoque puncto in circulū quædam rectæ lineæ cadant: maxima quidem erit ea in qua centrū, minima verò reliqua: aliarum verò propinquior illi quę per centrum du-

citur

citur, remotiore semper maior est. Duæ autem solùm rectę lineę æquales ab eodem puncto in circulum cadunt ad vtrasq; partes minimæ.

Theorema 7. Propositio 8.

Si extra circulum sumatur punctum quodpiam, ab eoque puncto ad circulum deducantur rectæ quædam lineæ, quarum vna quidem per centrum protendatur, reliquę verò vt libet: in cauam peripheriam cadentium rectarum linearum minima quidem est illa, quæ per centrum ducitur: aliarum autem propinquior ei, quæ per centrum transit, remotiore semper maior est: in conuexam verò peripheriã cadentium rectarũ linearum minima quidem est illa, quæ inter punctum & diametrum interponitur: aliarum autem, ea quę propinquior est minimæ, remotiore semper minor est. Duæ

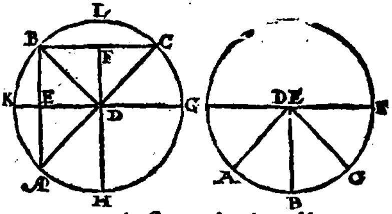


autem tantùm rectæ lineæ æquales ab eo puncto in ipsum circulum cadunt, ad vtrasque partes minimæ.

Theo-

Theorema 8. Propositio 9.

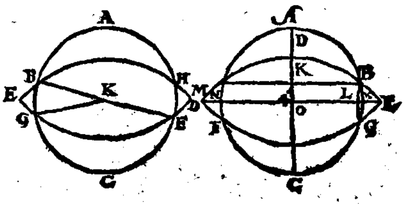
Si in circulo acceptum fuerit pūctum aliquod, & ab eo puncto ad circulum cadant plures quam duę rectæ lineæ, æquales, acceptū



punctum centrum ipsius erit circuli.

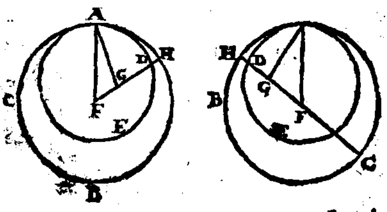
Theorema 9. Propositio 10.

Circū lus circulum in pluribus quàm duob' punctis non secat.



Theorema 10. Propositio 11.

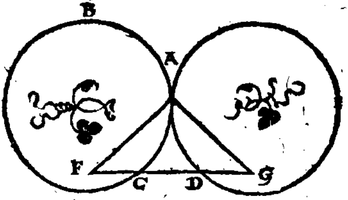
Si duo circuli sese intus cōtingāt, atque accepta



fuerint eorum centra, ad eorum cétra adiuncta recta linea & producta ni cõtactum circulorum cadet.

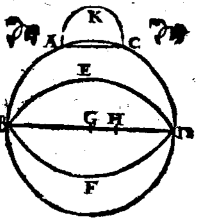
Theorema 11. Propoſitio 12.

Si duo circuli ſeſe exterius contingãt, linea recta q̃ ad cétra eorũ ad iũgitur, ꝑ contactũ illũ trãſibit.



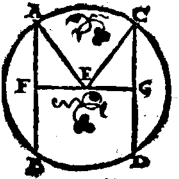
Theorema 12. Propoſitio 13.

Circulum circulum non tangit in pluribus punctis, quam vno, ſiue intus ſiue extra tangat.



Theorema 13. Propoſitio 23.

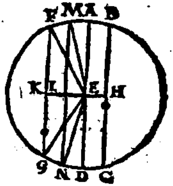
In circulo æquales rectæ lineæ æqualiter diſtant à centro. Et quæ ęqualiter diſtant à centro, quales ſunt inter ſe.



Theore-

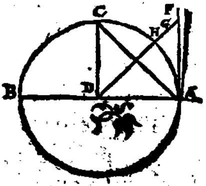
Theorema 14. Propositio 15.

In circulo maxima, quidem linea eſt diameter: aliarum autem propinquior centro, remotiore ſemper maior.



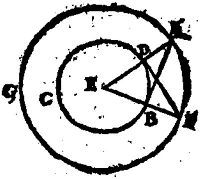
Theorema 51. Propoſitio 16.

Quæ ab extremitate diametri cuiuſq; circuli ad angulos rectos ducitur, extra ipsũ circulum cadet, & in locum inter ipſam rectam lineam & peripheriam comprehenſum, altera recta linea nõ cadet Et ſemicirculi quidem angulus quouis augulo acuto rectilineo maior eſt, reliquus autem minor,

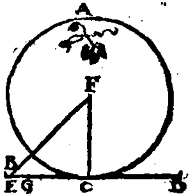


Problema 2. Propoſitio 17.

A dato puncto rectam lineam ducere, quæ datum tangat circulum.

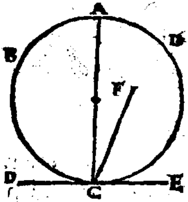


Theorema 16. Propositio 18.



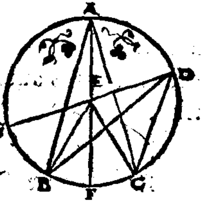
Si circulum tangat recta quæpiam linea, à centro autem ad cōtactum adiungatur recta quædam linea: quę adiūcta fuerit ad ipsam contingentem perpendicularis erit.

Theorema 17. Propositio 19.



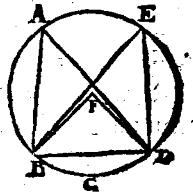
Si circulum tetigerit recta quæpiam linea, à contactu autem recta linea ad angulos rectos ipsi tāgenti excitetur, in excitata erit centrum circuli.

Theorema 18. Propositio 20.



In circulo angulus ad cētrum duplex est anguli ad peripheriam, cū fuerit eadē peripheria basis angulorum.

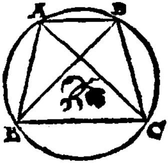
Theorema 19. Propositio 21.



In circulo, qui in eodem segmēto sunt anguli, sunt inter se æquales.

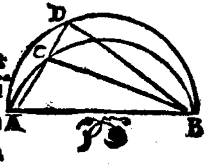
Theore-

Theorema 20. Propositio 22.



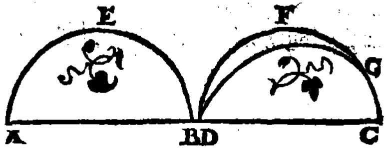
Quadrilaterorum in circulis descriptorũ anguli qui ex aduerso, duob' rectis sunt æquales.

Theorema 21. Propositio 23.



Super eadem recta linea duo segmenta circulorũ similia & inæqualia non constituẽtur ad easdem partes.

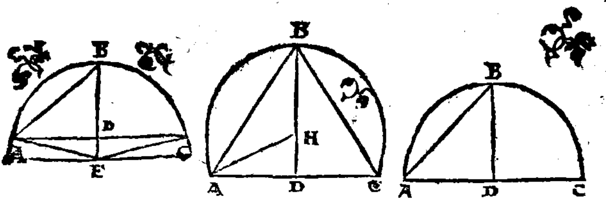
Theorema 22. Propositio 24.



Super ę qualib' rectis li neis similia circulorum segmenta sunt inter se æqualia.

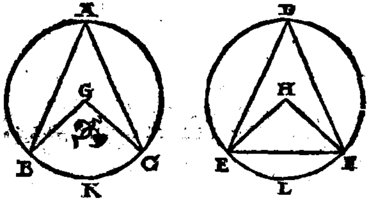
Problema 3. Propositio 25.

Circuli segmento dato, describere circulũ cuius eſt ſegmentum.



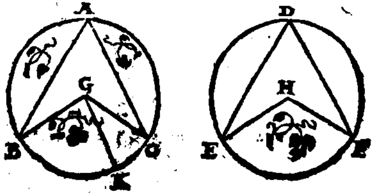
Theorema 23.Propoſitio 26.

In æqualibus circulis, æquales anguli ęqualib.periphe rijs inſiſtũt ſiue ad cẽtra, ſiue ad peripherias conſtituti inſiſtant.



Theorema 24 Propoſitin 27.

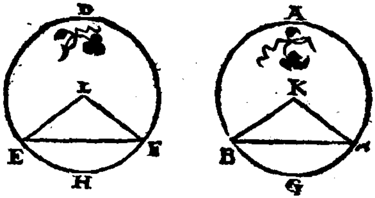
In æqualibus circulis, anguli qui ęqualibus peripherijs inſiſtũt ſunt inter ſe ęquales ſiue ad cẽtra, ſiue ad peripherias cõſtituti inſiſtãt.



Theore-

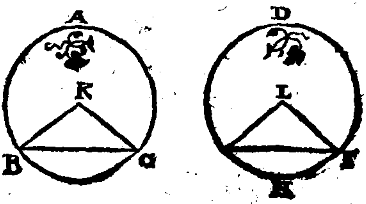
Theorema 25. Propositio 28.

In æqualibus circulis æquales rectæ lineæ ęquales peripherias auferũt maiorẽ quidẽ maiori, minorem autem minori.



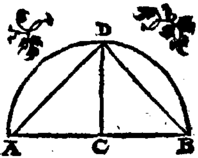
Theorema 26. Propositio 29.

In ęqualib⁹ circulis, æquales periphe rias æquales rectæ lineæ subtendunt.



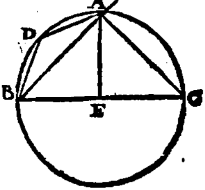
Problema 4. Propositio 30.

Datam peripheriam bifariam secare.



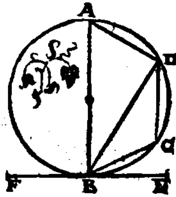
Theorema 27. Propositio 13.

In circulo angulus qui in semicirculo, rectus est: qui autem in maoire segmẽto, minor recto: qui verò in minore segmẽto, maior est recto. Et insuper angulus maioris segmenti, recto quidem maior est, minoris autẽ segmẽti angulus, minor est recto.



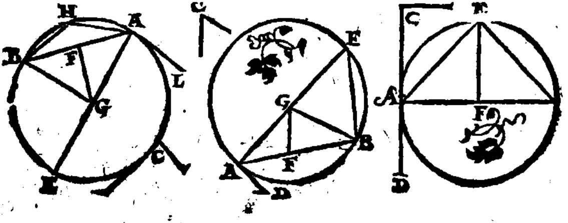
Theorema 28. Propositio 32.

Si circulum tetigerit aliqua recta linea, à contactu autẽ producatur quædam recta linea circulum secans: anguli quos ad contingentem facit ęquales sunt ijs qui in alternis circuli segmẽtis consistunt, angulis.



Problema 5. Propositio 33.

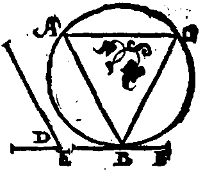
Super data recta linea describere segmentum circuli quod capiat angulum æqualẽ dato angulo rectilineo.



Proble-

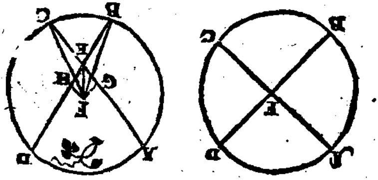
Problema 6. Propositio. 34.

A dato circulo segmentum abscindere capiens angulum æqualem dato angulo rectilineo.



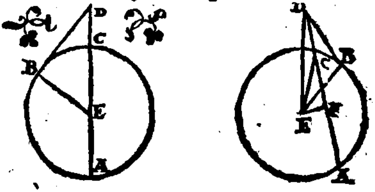
Theorema 29. Propositio 35.

*S*i in circulo duæ rectæ lineæ sese mutuò secuerint, rectangulum comprehésum sub segmétis vni᷒, æquale est ei, quod sub segmentis alterius comprehenditur, rectangulo.



Theorema 30. Propositio 36.

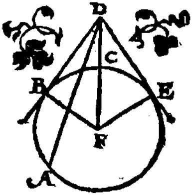
Si extra circulũ sumatur pũctũ aliquod,



ab eoq́ue in circulũ cadant duę rectę lineæ, quarũ altera quidem circulum secet, altera verò tangat: quod ſub tota ſecante & exterius inter punctum & conuexam peripheriam aſſumpta comprehenditur rectangulum, æquale erit ei, quod à tangente deſcribitur, quadrato.

Theorema 31. Propoſitio 37.

Si extra circulũ ſumatur punctun aliquod, ab eoq; puncto in circulum cadant duæ rectæ lineæ, quarum altera circulum ſecet, altera in eum incidat, ſit autẽ quod ſub tota ſecante & exterius inter punctum & conuexã peripheriam aſſumpta, comprehenditur rectangulum, æquale ei, quod quadrato: incidens ipſa circulum tanget.



ELEMENTI III. FINIS.

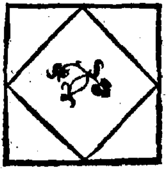
EVCLI.

EVCLIDIS ELEMENTVM QVARTVM.

DEFINITIONES.

1.

Figura rectilinea in figura rectilinea inſcribi dicitur, cũ ſinguli eius figuræ quæ inſcribitur, anguli ſingula latera eius, in qua inſcribitur, tangunt.



2

Similiter & figura circum figurã deſcribi dicitur, quum ſingula eius quæ circũſcribitur, latera ſingulos ei⁹ figuræ angulos tetigerint, circum quam illa deſcribitur.

3

Figura rectilinea in circulo inſcribi dicitur, quũ ſinguli eius figuræ quę inſcribitur, angu-

anguli tetigerint circuli peripheriam.

4

Figura verò rectilinea circa circulũ deſcribi dicitur, quũ ſingula latera eius, quæ circum ſcribitur, circuli peripheriá tangunt.

5

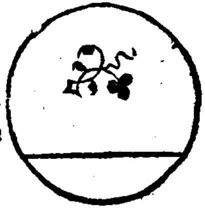
Similiter & circulus in figura rectilinea inſcribi dicitur, quum circuli peripheria ſingula latera tãgit eius figuræ, cui inſcribitur.

6

Circulus autem circum figurã deſcribi dicitur, quũ circuli peripheria ſingulos tãgit eius figuræ, quam circunſcribit, angulos.

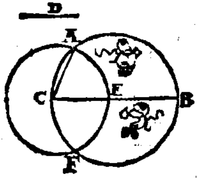
7

Recta linea in circulo accommodari ſeu coaptari dicitur, quum eius extrema in circuli peripheria fuerint.



Problema 1. Propoſitio 1.

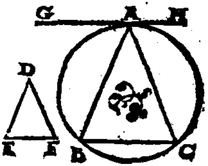
In dato circulo, rẽctam lineam accommodare æqualem datæ rectæ lineæ quæ circuli diametro nó ſit maior.



Proble-

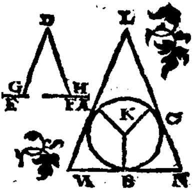
Problema 2. Propositio 3.

In dato circulo, triangulũ describere dato triangulo æquiangulum.



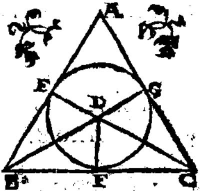
Problema 3. Propositio 3.

Circa datum circulũ triangulũ, describere dato triangulo æquiangulũ.



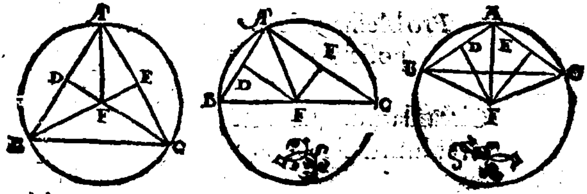
Problema 4. Propositio 4.

In dato triangulo circulum inscribere.



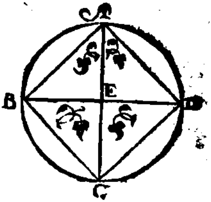
Problema 5. Propositio 5.

Circa datum triangulum, circulum describere.



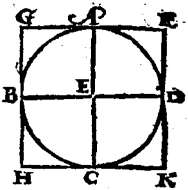
Problema 6. Propositio 6.

In dato circulo quadratum describere.



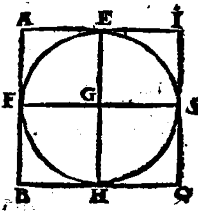
Problema 7. Propositio 7.

Circa datum circulum, quadratum describere.



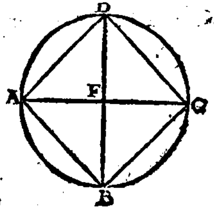
Problema 8. Propositio. 8.

In dato quadrato circulum inscribere.



Problema 9. Propositio 9.

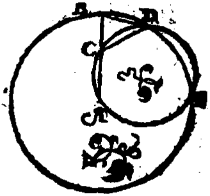
Circa datum quadratū, circulum describere.



Proble-

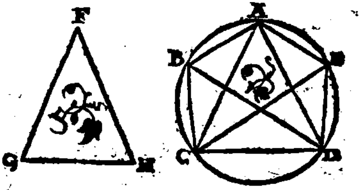
Problema 10. Propositio. 10

Ifosceles triágulum constituere, quod habeat vtrunque eorum, qui ad duplum reliqui.



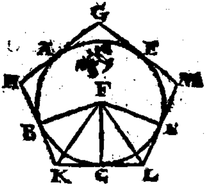
Theorema 11. Propositio 11.

In dato circulo, pentagonũ æquilaterũ & æquiãgulum inscribere.



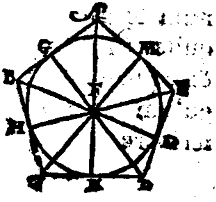
Problema 12. Propositio 12.

Circa datum circulum, pentagonum æquilaterum æquiangulum describere.



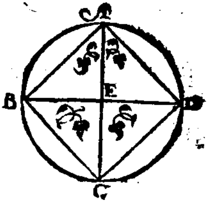
Problema 13. Propositio 13.

In dato pẽtagono æquilatero & æquiãgulo circulum inscribere.



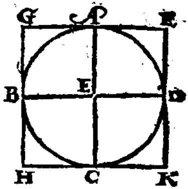
Problema 6. Propositio 6.

In dato circulo quadratum describere.



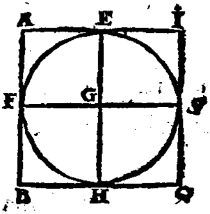
Problema 7. Propositio 7.

Circa datum circulum, quadratum describere.



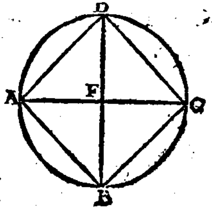
Problema 8. Propositio 8.

In dato quadrato circulum inscribere.



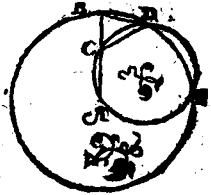
Problema 9. Propositio 9.

Circa datum quadratū, circulum describere.



Proble-

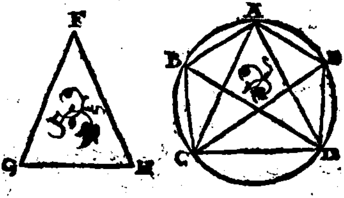
Problema 10. Propositio. 10



Isosceles triãgulum constituere, quod habeat vtrunque eorum, qui ad duplum reliqui.

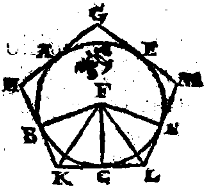
Theorema 11. Propositio 11.

In dato circulo, pentagonũ æquilaterũ & æquiãgulum inscribere.



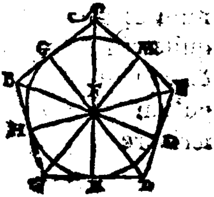
Problema 12. Propositio 12.

Circa datum circulum, pentagonum æquilaterum æquiangulum describere.



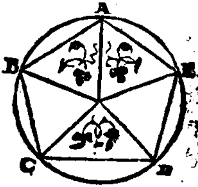
Problema 13. Propositio 13.

In dato pẽtagono æquilatero & æquiãgulo circulum inscribere.



Proble-

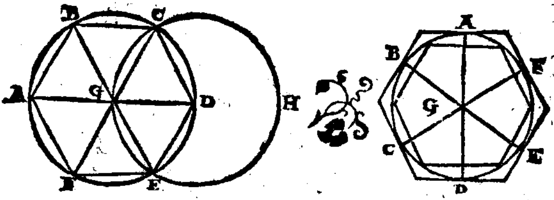
Problema 14. Propositio 14.



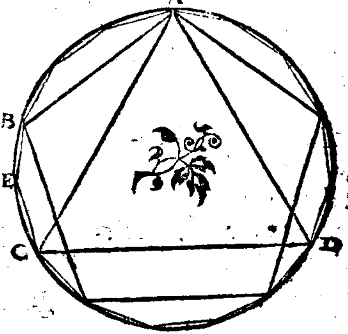
Circa datū pentagonū, æquilaterum & æquiangulum, circulum deſcribere.

Problema 15. Propoſitio 15.

In dato circulo hexagonum & æquilaterū & æquiangulum inſcribere.



propoſitio 16. Theorema 16.



In dato circulo quintidecagonum & æquilaterū & æquiangulum deſcribere.

Elementi quarti finis.

EVCLIDIS ELEMENTVM QVINTVM.

DEFINITIONES.

1

PARS est magnitudo magnitudinis minor maioris, quum minor metitur maiorem.

2

Multiplex autem est maior minoris, cùm minor metitur maiorem.

3

Ratio, est duarum magnitudinum eiusdẽ generis mutua quædam secundum quantitatem habitudo.

4

Proprotio verò, est rationum similitudo.

5

Rationẽ habere inter se magnitudinis dicuntur, quæ possunt multiplicatæ sese mutu superar e

6.

In eadem ratione magnitudines dicũtur esse, prima ad secundam, & tertia ad quartã, cum primæ & tertiæ æquè multiplicia, à secundæ & quartæ æquè multiplicibus,

 qualis-

qualiſcunq; ſit hæc multiplicatio, vtrumq; ab vtroque: vel vnà deficiunt, vel vnà ęqualia ſunt, vel vnà excedũt, ſi eа ſumãtur quæ inter ſe reſpondent.

7

Eandam autem habẽtes rationem magnitudines, proportionales vocentur.

8

Cum verò æquè multiplicium, multiplex primę magitudines exceſſerit? multiplicẽ ſecundæ, at multiplex tertiæ non exceſſerit multiplicem quartæ: tunc prima ad ſecundam, maiorem rationem habere dicetur, quàm tertia ad quartam.

9

Proportio autẽ in tribus terminis pauciſſimis conſiſtit.

10

Cùm autem tres magnititudines proportionales, fuerint, prima ad tertiam, duplicatã rationẽ habere dicitur eius quam habet ad ſecundam. At cùm quatuor magnitudines proportionales, fuerint, prima ad quartam, triplicatam rationem habere dicitur eius quam habet ad ſecundam: & ſemper deinceps vno amplius, quãdiu proportio extiterit.

11

Homo logę, ſeu ſimiles ratioue magnitudines dicuntur, antecedentes quidẽ antecedentidentibus, consequētēs verò cōsequētibus.

12

Alterna ratio, est sumptio antecedentis cōparati ad antecedentem, & cōsequentis ad consequentem.

13

Inuersa ratio, est sumptio cōsequentis ceu antecedentis, ad antecedentē velut ad consequentem.

14.

Compositio rationis, est sumptio antecedentis cū consequente ceu vnius ad ipsum consequentem.

15

Diuisio rationis, est sumptio excessus quo consequentem superat antecedens ad ipsum consequentem.

16

Conuersio rationis, est sumptio antecedentis ad excessum, quo superat antecedens ipsum cosequentem.

17

Ex ęqualitate ratio est, si plures duabus sint magnitudines, & his alię multitudine pares quæ binæ sumantur, & in eadem ratione: quum vt in primis magnitudinibus prima ad vltimam sic & in secundis magnitudinibus prima ad vltimā sese habuerit, vel aliter, sumptio extremorū per subductionem mediorum.

18

Ordinata proportio eſt, cùm fuerit quemadmodum antecedens ad conſequentem antecedens ad conſequentem : fuerit vt conſequens ad aliud quidpiam. ita uens ad aliud quidpiam.

19

ata autẽ proportio eſt, tribus po agnitudinibus, & alij quæ ſint his titudine pares, cùm vt in primis quidẽ magnitudinibus ſe habet antecedẽs ad cõſequentem, ita in ſecũdis magnitudinibus antecedens ad conſequentem : vt autem in primis magnitudinibus conſequens ad aliud quidpiam ad antecedentem.

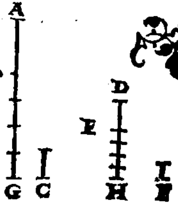
Theorema 1. Propoſitio 1.

Si ſint quotcũque magnitudines quotcunque magnitudinum æqualium numero ſingulę ſingularum æquè multiplices, quàm multiplex eſt vnius vna magnitudo, tam multiplices erunt, & omnes omnium.

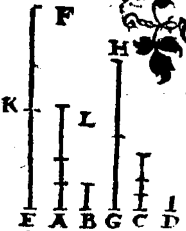
Theorema 2. Propoſitio 2.

Si prima ſecundæ æquè fuerit multiplex, atque

atque tertia quartę, fuerit autē & quinta ſecūdę æquè multiplex, atque ſexta quartæ: erit & cōpoſita prima cū quinta, ſecūdæ ęquè multiplex atque tertia cum ſexta, quartæ.



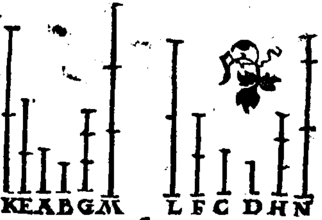
Theorema 3. Propoſitio 3.



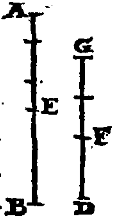
Si ſi prima ſecundæ æquè multiplex atque tertia quartæ, ſumātur autē æquè multiplices primę & tertiæ erit & ex æquo ſumptarum vtraque vtriuſque ęquè multiplex, altera quidem ſecundæ, altera autē

Theorema 4. Propoſitio 4.

Si prima ad ſecundam, eandē habuerit rationem, & tertia ad quartam: etiam æquè multiplices primę & tertiæ, ad æquè multiplices ſecūndæ & quartæ iuxta quāuis multiplicationē, eandem habebunt rationem, ſi prout inter ſe



respondent, ita sumptæ fuerint.



Theorema 5. Propositio 5.

Si magnitudo magnitudinis æquè fuerit multiplex, atque ablata ablatæ: etiam reliqua reliquæ ita multiplex erit, vt tota totius.

Theorema 6. Propositio 6.

Si duæ magnitudines, duarum magnitudinum sint æquè multiplices, & detractę quædam sint earundem æquè multiplices: & reliquæ eisdem aut ęquales sunt, aut æquè ipsarum multiplices.

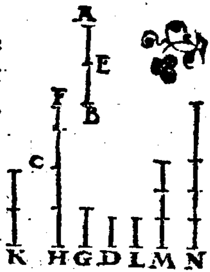
Theorema 7. Propositio 7.

Aequales ad eandem, eadem habent rationem: & eandem ad æquales.

Theorema 8. Propositio 8.

In æqualium magnitudinũ maior, ad eandem

dem maiorem ratione habet, quàm minor: & eadem ad minorem: maiorem rationem habet, quàm ad maiorem.



Theorema 9. Propositio 9.

Quæ ad eandem, eandem habét rationem, æquales sunt inter se: & ad quas eadem, eandem habet rationem, eæ quoque sunt inter se æquales.

Thorema 10. Propositio 10.

Ad eandem magnitudiné rationem habentium, quę maiorem rationé habet, illa maior est ad quam auté eadem maiorem rationem habet, illa minor est.

Theorema 11. Propositio 11.

Quæ eidem sunt eædem rationes, & inter se sunt eędem.

Theorema 12. Propoſitio 12.

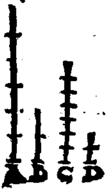
Si ſint magnitudines quotcunque proportionales, quem admodũ ſe habuerit vna antecedentium ad vnã conſequentiũ, ta ſe habebunt omnes antecedenses ad omnes coſequentes.

Theorema 13. Propoſitio 13.

Si prima ad ſecundum, eandẽ habuerit rationem, quam tertia ad quartã, tertia verò ad quartam maiorem rationem habuerit, quàm quinta ad ſextam, prima quoque ad ſecũdam maiorem rationem habebit, quàm quinta ad ſextam.

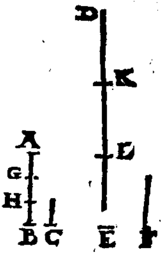
Theorema 14. Propoſitio 14.

Si prima ad ſecundam eandem habuerit rationẽ, quam tertia ad quartam, prima verò quàm tertia maior fuerit: erit & ſecunda maior quàm quarta. Quòd ſi prima fuerit æqualis tertiæ, erit



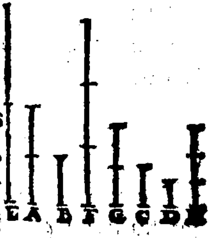
& secunda æqualis quartæ: si verò minor, & minor erit.

Theorema 15. Propositio 15.



Partes cum pariter multiplicibus in eadē sunt ratione, si prout sibi mutuò respondent, ita sumantur.

Theorema 16. Propositio 16.



Si quatuor magnitudines proportionales fuerint, & vicissim proportionales erunt.

Theorema 17. Propositio 17.

Si compositæ magnitudines porportionales fuerint hæ quoq; diuisæ proportionales erunt.

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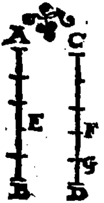
Theorema 18. Propositio 18.



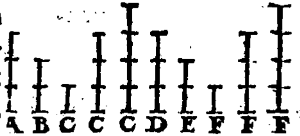
Si diuisæ magnitudines sint proportionales, hæ quoque compositæ proportionales erunt.

Theorema 19. Propositio 19.

Si quemadmodum totũ ad totum, ita ablatum se habuerit ad ablatum: & reliquum ad reliquum, vt totum ad totum se habebit.

Theorema 20. Propositio 20.

Si sint tres magnitudines, & aliæ ipsis æquales numero, quæ binæ & in eadem ratione sumatur, ex æquo autẽ prima quàm tertia maior fuerit: erit & quarta, quã sexta maior. Quòd si

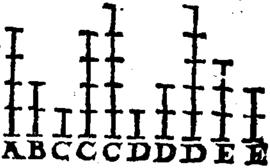


prima tertiæ fuerit æqualis, erit & quarto æqualis sextæ: sin illa minor, hæc quoque minor erit.

Theore-

Theorema 21. Propositio 21.

Si sint tres magnitudines, & alię ipsis æquales numero quæ binæ & in eadem ratione sumãtur, fuerit que perturbata earũ proportio ex æquo autem prima quàm tertia maior fuerit erit & quarta



quã sexta maior: quòd si prima tertiæ fuerit æqualis, erit & quarta æqualis sextæ: sin illa minor, hæc quoque minor erit.

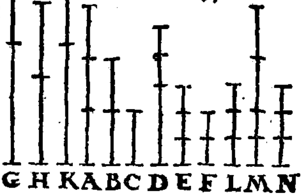
Theorema 22. Propositio 22.

Si sint quotcunq; magnitudines, & alię ipsis ęquales numero, quæ binæ in eadẽ ratione sumantur, et ex æqualitate in eadem ratione erunt.

Theorema 23. Propositio 23.

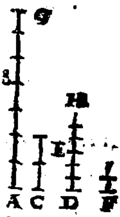
Si sint tres magnitudines, aliæq; ipsis ęquales

les numero, q̃ binę in eadem ratione ſumã-tur, fuerit autẽ perturbata earũ proportio: etiã ex æqualite in eadem ratione erunt.



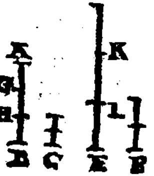
Theorema 24. Propoſitio 24.

Si prima ad ſecundam, eandem habuerit rationẽ, quam tertia ad quartam, habuerit autem & quinta ad ſecundam, eandẽ rationem, quã ſexta ad quartam: etiam cõpoſita prima cũ quinta ad ſecundam eandẽ habebit rationẽ quam tertia cum ſexta ad quartam.



Theorema 25. Propoſitio 25.

Si quatuor magnitudines proportionales fuerint, maxima & minima reliquis duabus maiores erunt



ELEMENTI V. FINIS.

EVCLIDIS ELEMENTVM SEXTVM.

DEFINITIONES.

I

Similes figuræ rectilineæ ſunt, quæ & angulos ſingulos ſingulis æquales habẽt, atque etiam latera, quę circum angulos æquales, proportionalia,

2

Reciprocæ autem figuræ ſunt, cum in vtraque figura antecedentes & conſequentes rationum termini fuerint.

3

Secundum extremam & mediam rationẽ recta linea ſecta eſſe dicitur, cum vt tota ad maius ſegmentum, ita maius ad minus ſe habuerit.

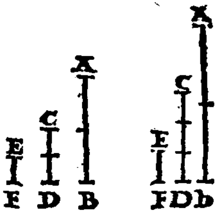
4

Altitudo cuiuſque figurę, eſt linea perpendicularis à vertice ad baſin deducta

5 Ra-

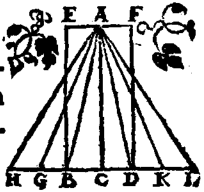
5.

Ratio ex rationibus cõponi dicitur, cũ ratio-num quantitates inter ſe multiplicatæ aliqua effecerint rationem.



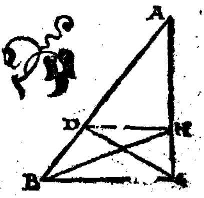
Theorema 1. Propoſitio. 1

Triangula & parallelogramma, quorum eadem fuerit altitudo, ita ſe habent inter ſe vt baſes.



Theorema 2. Propoſitio 2.

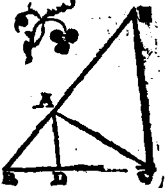
Si ad vnum trianguli latus parallela ducta fuerit recta quædam linea: hęc proportionaliter ſecabit, ipſius triãguli latera. Et ſi trianguli latera proportionaliter ſecta fuerint: quæ ad ſectiones adiũcta fuerit recta linea, erit ad reliquum ipſius trianguli latus parallela.



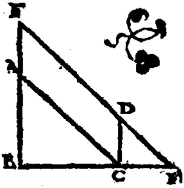
Theorema 3. Propoſitio 3.

Si trianguli angulus bifariam ſectus ſit, ſecans autẽ augulum recta linea ſecuerit & baſim: baſis ſegmenta eandem habebunt rationem,

tionem, quam reliqua ipsius trianguli latera, Et si basis segmenta eandem habeant rationem quam reliqua ipsius trianguli latera, recta linea, quæ à vertice ad sectionem producitur, ea bifariã secat trianguli ipsius angulum.

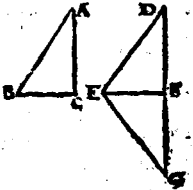


Theorema 4. Propositio 4



Aequiangulorum triangulorum proportianolia sunt latera, quę circum ęquales angulos, & homologa sunt latera, quę ęqualibus angulis subtenduntur.

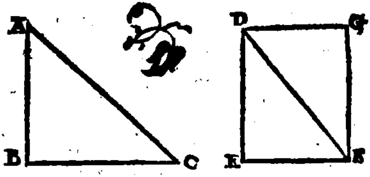
Theorema 5. Propositio 5.



Si duo triãgula latera proportionalia habeant, ęquiangula erunt triangula, & æquales habebunt eos angulos, sub quib⁹ homologa latera subtenduntur.

Theorema 6. Propositio 6.

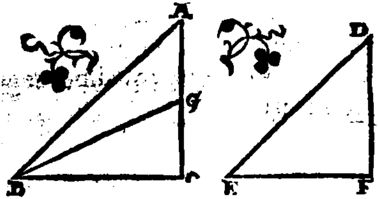
Si duo triangula vnum angulùm vni angulo æqualẽ, & circum ęquales angulos latera proportionalia habuerint, ęquiangula erunt trian triangu-la, ęqua-lesq; ha-bębunt angulos ſub quibus ho-



mologa latera ſubtenduntur.

Theorema 7. Propoſitio 7.

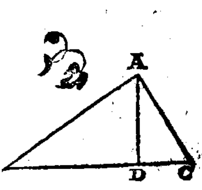
Si duo triangula vnum angulum vni angulo æqualem, circũ autem alios angulos latera proportionalia habent, reliquorum vero ſimul vtrunq; aut minórem aut non minoré



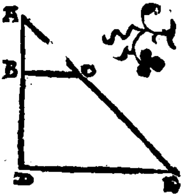
recto: æquiangula erunt triangula, & æquales habebunt eos angulos circum quos proportionalia ſunt latera.

Theorema 8. Propoſitio 8.

Si in triangulo rectangulo, ab angulo recto in baſin perpendicularis ducta ſit quę ad perpendicularem triangulo, tum toti triangulo, tum ipſa inter ſe ſimilia ſunt.

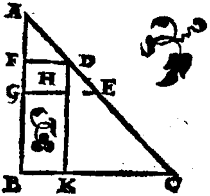


Problema 1. Propositio 9.



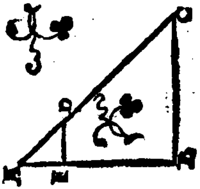
A data recta linea imperatam partem auferre.

Problema 2. Propositio 10.



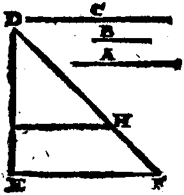
Datam rectã lineam in sectam similiter secare, vt data altera recta secta fuerit.

Problema 3. Propositio 11.



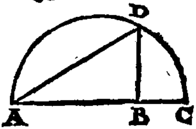
Duabus datis rectis lineis, tertiam propotionalem ad inuenire.

Problema 4. Propositio 12.



Trib⁹ datis rectis lineis quartam proportionalem adiuenire.

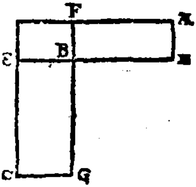
Problema 5. Propo-
sitio 13.



Duab' datis rectis lineis mediam proportionalē adinuenire.

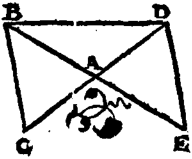
Theorema 9. Propositio 14.

Aequalium, & vnū vni æqualem habentiū angulum parallelogrammorum reciproca sunt latera, quæ circum æquales angulos: & quorū parallelogrammorū vnum angulum vni angulo æqualem habentiū reciproca sunt latera, quæ circum æquales angulos, illa sunt æqualia.



Theorema 10. Propositio 15.

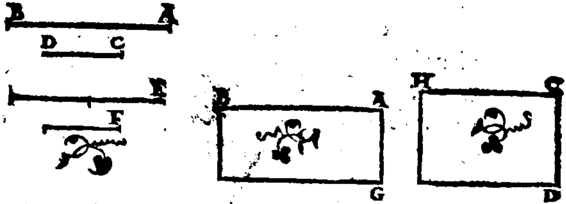
Aequalium, & vnū angulum vni æqualem habentiū triangulorū reciproca sunt latera, quæ circum æquales angulos: & quorū triangulorū vnum angulum vni æqualem habentium reciproca sunt latera, q̄ circum æquales angulos, illa sunt æqualia.



Theo-

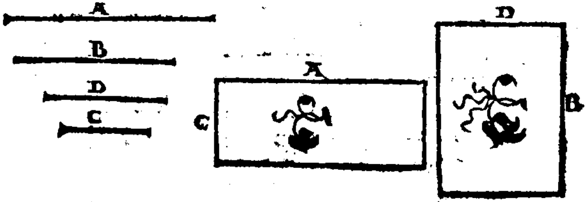
Theorema 11. Propoſitio 16.

Si quatuor rectæ lineæ proportionales fuerint, quod ſub extremis comprehenditur rectangulũ, æquale eſt ei, quod ſub medijs comprehenditur rectãgulo. Et ſi ſub extremis comprehenſum rectangulum æquale fuerit ei, quod ſub medijs continetur rectãgulo, illæ quatuor rectæ lineæ proportionales erunt.



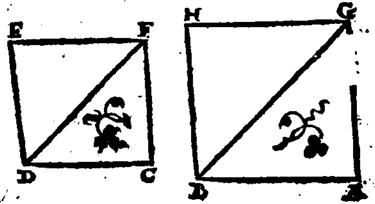
Theorema 12. Propoſitio 17.

Si tres rectæ lineæ ſint proportionales, q̃d ſub extremis cõprehenditur rectangulũ ęquale eſt ei, quod à media deſcribitur quadrato; & ſi ſub extremis comprehẽſum rectangulum æquale ſit ei quod à media deſcribitur quadrato, illæ tres rectę lineę proportionales erunt.



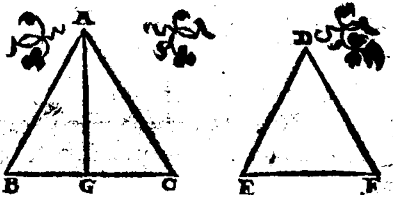
Problema 6. Propoſitio 18.

A data recta linea, dato recti lineo ſimile ſimiliterq; poſitum rectilineum deſcribere.



Theorema 13. Propoſitio 19.

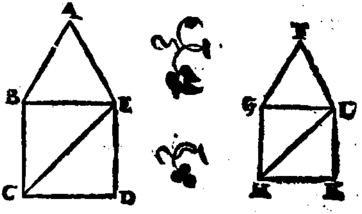
Similia triãgula inter ſe ſunt in duplicata ratione laterũ homologorũ.



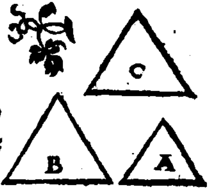
Theore. 14. Propoſitio 20.

Similia polygona in ſimilia triãgula diuidũtur, & numero ęqualia, & homologa totis. Et po

lygona duplicatam habent eã inter ſe rationẽ, quã latus homologũ ad homologum latus.



Theorema 15. Propoſitio 21.

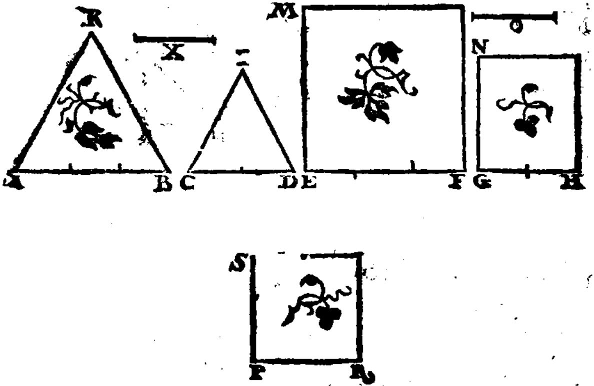


Quæ eidem rectilineo ſunt ſimilia, & inter ſe ſunt ſimilia.

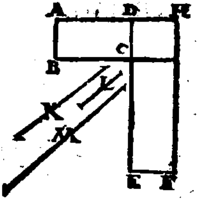
Theorema 16. Propoſitio 22.

Si quatuor rectę lineæ proportionales fuerint: & ab eis rectilinea ſimilia ſimiliterq́; deſcripta proportionalia erũt. Et ſi à rectis lineis ſimilia ſimiliterq́; deſcripta rectilinea proportionalia fuerint, ipſæ etiam recta

&æ lineæ proportionales erunt.

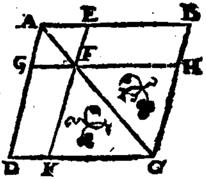


Theorema 17. Propositio 23.



Aequiangula parallelogramma inter se rationẽ habent eum, quę ex lateribus componitur.

Theorema 18. Propositio 24.

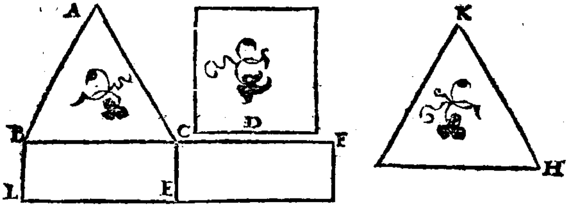


In omni parallelogrammo, quæ circa diametrũ sunt parallelogrãma, & toti & inter se sunt similia.

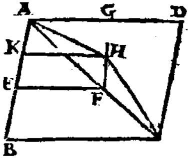
Proble-

Problema 7. Propoſitio 25.

Dato rectilineo ſimile, & alteri dato æquale idem conſtituere.



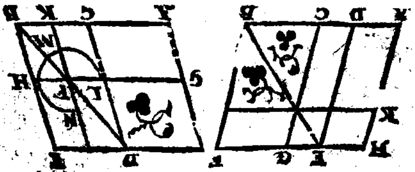
Theorema 19. Propoſitio 26.



Si à parallelogrãmo parallelogrammũ ablatũ ſit, & ſimile toti & ſimiliter poſitũ communẽ cum eo habens angulum, hoc circum eandem cum toto diametrum conſiſtit.

Theorema 20. Propoſitio 27.

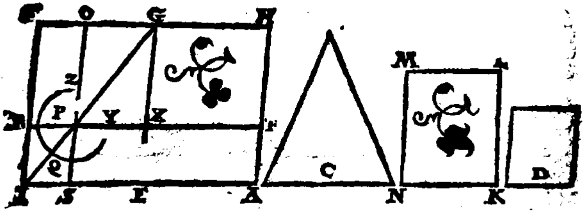
Omnium parallelogrãmorum ſecũdum eandem rectam lineam applicatorũ deficiẽtiumq́, figuris parallelogrãmis ſi



milibus similiterque positis ei, quod à dimidia describitur, maximum, id est. quod ad dimidiam applicatur parallelogrammum simile existens defectui.

Problema 8. propositio 28.

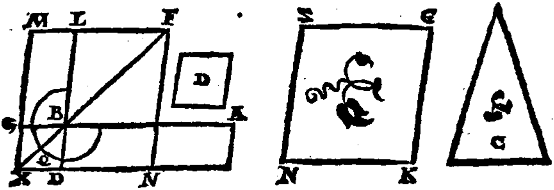
Ad datam lineam rectam, dato rectilineo æquale parallelogrammum applicare deficiens figura parallelogramma, quę similis sit alteri rectilineo dato. Oportet autē datum rectilineum, cui æquale applicandum est, non maius esse eo quod ad dimidiam applicatur, cum similes sint defectus & eius quod à dimidia describitur, & eius cui simile deesse debet.



Problema 9. Propositio. 29.

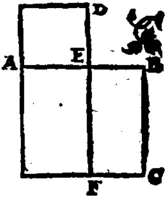
Ad datam rectam lineam, dato rectilineo æquale parallelogrammũ applicare, excedēs figura parallelogrāma, quæ similis sit paral-

parallelogrammo alteri dato.



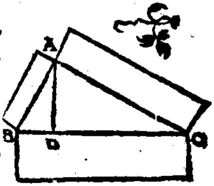
Prblema 10. Propo-
positio 30.

Propositam rectam lineam terminatã, extrema ac media ratione secare.



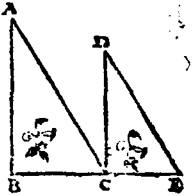
Theorema 21. Propositio .31

In rectangulis triãgulis, figura quęuis à latere rectũ angulũ subtendẽte descripta ęqualis est figuris, quæ priori illi similes & similiter positę à lateribus rectũ angulum continentibus describuntur.



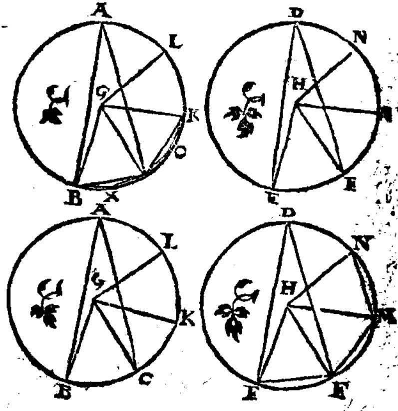
Theorema 22. Propo-
sitio 32.

Si duo triãgula, quę duo latera duobus lateribus proportionalia habeant, secũdum vnum angulũ composita fuerint, ita vt homologa eorum latera sint etiam parallela, tum reliqua illorum ttiangulorum latera in rectam lineam collocata reperientur.



Theorema 23. Propositio 33.

In æqualibus circulis anguli eandẽ habent rationem, cũ ipsis peripherijs in quibus insistũt, siue ad cẽtra siue ad peripherias constituti, illis insistant peripherijs In super verò & sectores quippe qui ad cẽtra cõsistũt.



ELEMENTI VI. FINIS.

EVCLIDIS ELEMENTVM SEPTIMVM.

DEFINITIONES.

1

Vnitas, eſt ſecũdum quam entium quodque dicitur vnum.

2

Numerus autem, ex vnitatibus compoſita multitudo.

3

Pars, eſt numerus numeri minori maioris, cùm minor metitur maiorem.

4

Partes autem, cùm non metitur.

5

Multiplex verò, maior minoris, cùm maiorem metitur minor.

6

Par numerus eſt, qui bifariã non diuiditur.

7

Impar verò, qui bifariam non diuiditur: vel, qui vnitate differt à pari.

8

Pariter par numerus eſt, quem par numerus metitur per numerum parem.

9 Pari.

9.

Pariter autem impar, est quem par numerus metitur per numerum imparem.

10.

Impariter verò impar numerus, est quem impar numerus metitur per numerum imparem.

11.

Primus numerus, est quem vnitas sola metitur.

12.

Primi inter se numeri sunt, quos sola vnitas mensura communis metitur.

13.

Compositus numerus est, quem numerus quispiam metitur.

14.

Compositi autē inter se numeri sunt, quos numerus aliquis mensura communis metitur.

15.

Numerus numerum multiplicare dicitur, cùm toties compositus fuerit is, qui multiplicatur, quot sunt in illo multiplicante vnitates, & procreatus fuerit aliquis.

16.

Cùm autē duo numeri mutuò sese multiplicantes quempiam faciūt, qui factus erit planus appellabitur, qui verò numeri mutuò sese multiplicarint, illi' latera dicētur.

17. Cùm

17

Cùm verò tres numeri mutuò sese multiplicantes quempiã faciunt, qui procreatus erit solidus apellabitur, qua autem numeri mutuò sese multiplicarint, illius, latera dicentur.

18.

Quadratus numerus est, qui ęqualiter ęqualis: vel, qui à duobus ęqualibus numeris cótinetur.

19.

Cubus verò, qui à tribus æqualibus æqualiter: vel, quia tribus æqualibus numeris continetur.

10

Numeri proportionales sunt, cum primus secundi, & tertitius quarti æquè multiplex est, vt eadem pars, vel eædem partes.

21

Similes plani & solidi numeri sunt, qui proportionalia habent latera.

22

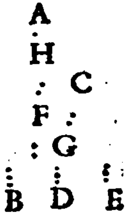
Perfectus numerus est, qui suis ipsius partibus est æqualis.

Theorema 1. Propositio 1.

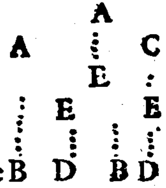
Duobus numeris inæqualibus proposi-

tis,

tis, si detrahatur semper minor. de maiore, alterna, quadam detractione; neque reliquus vnquam metiatur præcedentem quo ad assumpta sit vnitas: qui principio propositi sunt numeri primi inter se erunt.

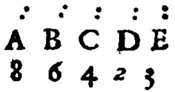


Porblema 1. Propositio 2.

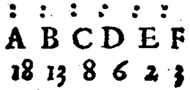


Duobus numeris datis non primis inter se, maximã eorũ cõmunẽ mensurã reperire.

Broblema 2. Propositio. 3

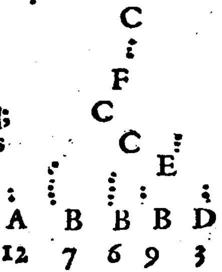


Tribus numeris datis non primis inter se, maximam eorum communem mensuram reperire.



Theorema 18. Propositio 8.

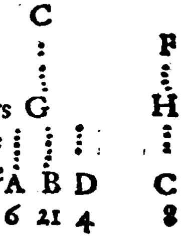
Omnis numerus, cuiusq; numeri minor maioris aut par est, aut partes.



Theore-

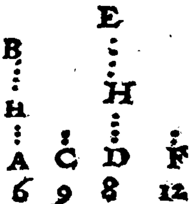
Theorema 3. Propositio 5.

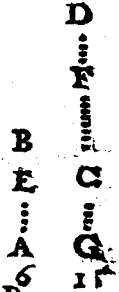
Si numeris numeri par fuerit, & alter alterius eadẽ pars & simul vterque vtriusque simul eadem pars erit, quæ vnus est vnius.



Theor. 4. Propo. 6.

Si numer⁹ sit numeri partes, & alter alterius eędem partes, & simul vterque vtriusque simul eædẽ partes erunt, quæ sunt vnus vnius.

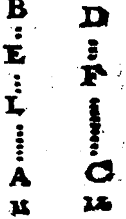




Theorema 5. Propositio 7.

Si numerus numeri eadẽ si pars quæ detractus detracti, & reliquus reliqui eadẽ pars erit, quæ totus est totius.

Theorema 6. Propositio. 8.

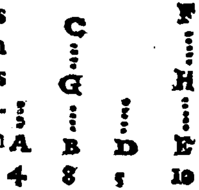


Si numerus numeri eædẽ sint partes quæ detractus detracti & reliquus reliqui eędẽ partes erũt, quę sũt totus totius. Theo-

G..M.K..N.H.

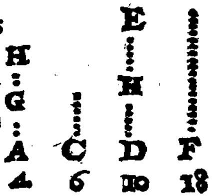
Theorema 7. Propositio 9.

Si numerus numeris pars sit, & alter alterius eadem pars, & vicissim quæ pars est vel partes primus tertij, eadem pars erit eędem partes, & secũdus quarti.



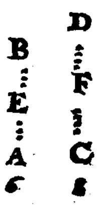
Theorema 8. Propositio 10.

Si numerus numeri partes sint, & aliter alterius eadẽ partes, etiam vicissim quæ sunt partes aut pars primus tertij, eędem partes erũt vel pars & secundus quarti.



Theorema 9. Propositio 11.

Si quemadmodum se habet totus ad totum, ita detractus ad detractum, & reliquus ad reliquum ita habebit vt totus ad totum



Theorema 10. Propositio 12.

Si sint quotcunque numeri proportionales; quemadmodum se habet vnus antecedentium ad vnum sequentium, ita se habe-

A	B	C	D
9	6	3	2

habebunt omnes antecedentes ad omnes conſequentes.

Theorema 11. Propoſitio 13.

Si quatuor numeri ſint proportionales, & viciſſim ꝓportionales erunt.

A	B	C	D
12	4	9	3

Theorema 12 Propoſitio 14

Si ſint quotcunque numeri, & alij illis æquales multitudine, qui bini ſumantur & in eadem ratione: etiam ex æqualitate in eadem ratione erunt

A	B	C	D	E	F
12	6	3	8	4	2

Theorema 13. Propoſitio 15.

Si vnitas numerum quẽpiã metiatur, aliter verò numerus alium quendã numerũ æquè metiatur, & viciſſim vnitas tertium numerũ æquę metietur, atque ſecundus quartum.

			F
	C		L
	H		K
	G		E
A	B	D	
1	3	4	6

Theorema 14. Propoſitio 16

Si duo numeri mutuò ſeſe multiplicãtes faciant aliquos qui ex illis geniti fuerint, inter ſe æquales erunt.

E	A	B	C	D
1	2	4	8	8

Theorema 15. Propositio 17.

Si numer' duos numeros multiplicãs faciat aliquos, qui ex illis procreati erunt, eadem rationem habebunt, quam multiplicati.

I	A	B	C	D	E
1	3	4	5	12	15

Theorema 16. Propositio 18.

Si duo numeri numerum quempiam multiplicantes faciant aliquos, geniti ex illis eandem habebunt rationem, quam qui illum multiplicarunt.

A	B	C	D	E
4	5	3	12	15

Theorema 17. Propositio 19.

Si quatuor numeri sint proportionales, qui ex primo & quarto fit, ęqualis erit ei qui ex secundo & tertio: & si qui ex primo & quarto sit numerus æqualis sit ei qui ex secũdo & tertio, illi quatuor numeri sint proportionales erunt.

A	B	C	D	E	F	G
6	4	3	2	12	12	18

Theorema 18. Propositio 20.

Si tres numeri sint proportionales, qui ab extremis cõtinetur, ęqualis est ei qui à medio

dio efficitur. Et si qui ab extremis cótinetur æqualis sit ei qui à medio describitur, illi tres numeri proportionales erunt.

A	B	C	D
9	6	4	6

Theorema 19. Propositio. 21.

Minimi numeri omniũ, qui eandem cũ eis rationem habẽt, æqualiter metiũtur numeros eandem rationem habétes, maior quidem maiorem, minor verò minorem

D	L		
G	H		
C	E	A	B
4	3	8	6

Theorema 20. Propositio 22.

Si tres sint numeri & alij multitudine illis æquales, qui bini sumantur & in eadem ratione, sit autem perturbata eorũ proportio, etiam ex æqualitate in eadem ratione erunt.

A	B	C	D	E	F
6	4	3	12	8	6

Theorema 21. Propositio 23.

Primi inter se numeri minimi sunt omniũ eandem cum eis rationem habentium.

A	B	E	C	D
5	6	2	4	3

Theorema 22 Propositio 24.

Minimi numeri omnium eandem cum eis rationem habentiũ, primus sunt inter se.

A	B	C	D	E
8	6	4	3	2

Theorema 23. Propositio 25.

Si duo numeri sint primi inter se, qui alterutrum illorum metitur numerus, is ad reliquũ primus erit.

A	B	C	D
6	7	3	4

Theorema 24. Propotio 25.

Si duo numeri ad quẽpiam numerum primi sint, an eundem primus is quoque futurus est. qui ab illis productus fuerit.

B	A	C	D	E	F
5	5	5	5	3	2

Theorema 25. Propositio 27.

Si duo numeri pirmi sint inter se, q̃ ab vno eorũ gignitur ad reliquum primus erit.

B	A	C	D
	7	6	3

Theorema 26. Propositio 28.

Si duo numeri ad duos numeros ambo ad vtrunque primi sint, & qui ex eis gignentur, primi inter se erunt.

A	B	E	C	D	F
3	5	15	2	4	8

Theore-

Theorema 27. Propositio 29.

Si duo numeri primi sint inter se, & multiplicãs vterq; seipsum procreet aliquem, qui ex ijs producti fuerint, primi inter se erũt. Quòd si numeri initio propositi multiplicantes eos qui producti sunt, effecerint aliquos, hi quoq; inter se primi erunt, & circa extremos idem hoc semper eueniet.

A	C	E	B	D	F
3	6	27	4	16	63

Theorema 28. Propositio. 30.

Si duo numeri primi sint inter se, etiam simul vterq; ad vtrunq; illorum primus erit. Et si simul vterq; ad vnum aliquem eorum primus sit, etiam qui initio positi sunt numeri, primi inter sunt erunt.

C

A	B	D
7	5	4

Theorema 29. Propositio 31.

Omnis primus numerus ad omnem numerum quem non metitur, primus est.

A	B	C
7	10	5

Theorema 30. Propositio 32.

Si duo numeri sese mutuo multiplicãtes faciant aliquem, hũc aũt ab illis productum metiatur primus quidam numerus, is alterum etiam metitur eorum qui initio positi erant.

A	B	C	D	E
3	6	12	3	4

Theorema 31. Propositio 33.

Omnem compositum numerum aliquis primus metietur.

A B C
27 9 3

Theorema 32. Propositio 34.

Omnis numerus aut primus est, aut eum aliquis primus metitur.

A A
3 6 1

Problema 3. Propositio 35.

Numeris datis quotcunque, reperire minimos omnium qui eandem cum illis rationem habeant.

A B C D E F G H K I M
6 8 12 2 3 4 6 2 3 4 5

Problema 4. Propositio 36.

B
A C D E F
7 12 8 4 5

Duobus numeris datis reperire quẽ illi minimum metiantur numerum.

A
F E C D G H
6 9 12 9 2 5

Theo-

Theorema 33. Propoſitio 37

Si duo numeri numerũ quempiam metiantur, & minimus quem illi metiuntur eundem metietur.

A B E C

2 3 6 12

Problema 5. Propoſitio 38.

Tribus numeris datis reperire quem minimum numerũ illi metiantur.

A B C D E

3 4 6 12 8

A B C D E F

3 6 8 12 24 16

Theorema 34. Propſitio 39.

Si numerum quiſpiam numerus metiatur, menſus partem habebit metienti cognominem.

A B C D

12 4 3 1

Theorema 35. Propoſitio 40.

Si numerus partem habuerit quãlibet, illum metietur numerus parti cognominis.

A B C D

8 4 2 1

Problema 6. Propoſitio 41.

Numerum reperire, qui minimus cùm ſit, datas habeat partes.

A B C G H

2 3 4 12 10

ELEMENTI VII. FINIS.

EVCLIDIS ELEMENTVM OCTAVVM.

Theorema 1. Propositio 1.

Si sunt quotcũq; numeri deinceps proportionales, quorum extremi sint inter se, primi, minimi sunt omnium eandem cum eis rationem habentium.

A	B	C	D	E	F	G	H
8	12	18	27	6	8	12	18

Problema 1. Propositio 2.

Numeros reperire deinceps proportionales minimos, quotcunq; iusserit quispiam in data ratione.

A	B	C	D	E	F	G	H	K
3	4	9	12	16	27	36	49	64

Theorema 2. Propositio 3.

Conuersa primæ.

Si sint quotcũq; numeri deinceps proportionales minimi habentium eandem cum eis rationem, illorum extremi sunt inter se primi.

A	B	C	D	E	F	G	H	K	L	M	N	O
27	16	48	64	3	4	9	12	16	27	36	48	64

Problema 2. Propositio 4.

Rationibus datis quotcunque in minimis numeris reperire numeros deinceps minimos in datis rationibus.

A	B	C	D	E	F	H	G	K	L	N	X	M	O
3	4	2	3	4	5	6	8	12	15	4	6	10	12

Theorema 3. Propositio 5.

Plani numeri rationem inter se habent ex lateribus compositam.

A	L	B	C	D	E	F	G	H	K
18	22	32	3	6	4	8	9	12	16

Theorema 4. Propositio 4.

Si sint quotlibet numeri deinceps

A	B	C	D	E	F	G	H
16	24	36	54	82	4	6	9

proportionales, primus autem secundum non metiatur, neque alius quispiam vllum metietur.

Theorema 5. Propositio 7.

Si sint quotcunque numeri deinceps proportionales, primus autem extremum metiatur, is etiam secundum metietur.

A	B	C	D
4	6	12	24

Theorema 6. Propositio 8.

Si inter duos numeros medij continua proportione incidant numeri, quot inter eos medij continua proportione incidunt numeri, tot & inter alios eandem cum illis habentes rationem medij continua proportione incident.

A	C	D	B	G	H	K	L	C	M	N	F
4	9	27	81	1	3	9	27	2	6	18	54

Theorema 7. Propositio 9.

Si duo numeri sint inter se primi, & inter eos medij cōtinua proportione incidāt numeri, quot inter illos medij continua proportione incidunt numeri, totidē & inter vtrunque eorum ac vnitatē deinceps medij continua proportione incident.

A	M	H	E	F	N	C	K	X	G	D	L	O	B
27	27	9	36	3	36	1	12	48	4	48	16	64	64

Theo-

Theorema 8. Propositio 10.

Si inter duos numeros & vnitatẽ continuè proportionales incidãt numeri quot inter vtrumque ipsorum & vnitatẽ deinceps medij continua proportione incidunt numeri, totidẽ & inter illos medij continua proportione incident.

A	E	K	D	H	C	L	F	G	B
27	9	36	3	12	1	48	4	16	64

Theorema 9. Propositio 11.

Duorum quadratorũ numerorum vnus medius proportionalis est numerus: & quadratus ad quadratum duplicatam habet lateris ad latus rationem.

A	C	E	D	B
9	3	12	4	16

Theorema 10. Propositio 12.

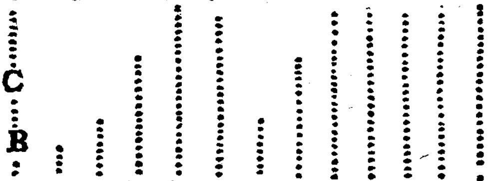
Duorum cuborum numerorũ duo medij proportionales sunt numerorũ duo medij cubum triplicatam habet lateris ad latus rationem.

A	H	K	B	C	D	E	F	G
27	36	48	64	3	4	9	12	16

Theo-

Thorema 10. Propositio 13.

Si sint quotlibet numeri deinceps proportionales, & multiplicans quisq; seipsum faciat aliquos, qui ab illis producti fuerint, proportionales erũt, & si numeri primùm positi, ex suo in proceatos ductu faciát aliquo, ipsi quoque proportionales erunt



A D L E X F G M N H O P K

4 8 16 32 64 8 16 32 64 128 256 512

Theorema 12. Propositio 14.

Si quadratus numerus quadratum numerũ metiatur, & latus vnius metietur latus alterius. Et si vnius quadrati latus metiatur latus alterius, & quadratus quadratum metietur.

A E B C D

9 12 16 8 4

Theorema 13. Propositio 15.

Si cubus numerus cubum numerũ metiatur, & latus vni⁹ metietur alterius latus. Et si latus vnius cubi latus alterius metiatur, tum

tum cubus cubum metietur.

A	H	K	B	C	D	E	F	G
8	16	28	64	2	4	4	8	16

Theorema 14. Propositio 9.

Si quadratus numerus quadratum numerum non metiatur, neque latus vnius metietur alterius latus.

Etsi latus vni' quadrati non metiatur latus alterius, neque quadratus quadratum metietur.

A	B	C	D
9	16	3	4

Theorema 15. Propositio 17.

Si cubus numerus cubum numerũ non metiatur, neque latus vnius latus alterius metietur. Et si latus cubi alicuius latus alterius non metiatur, neque cubus cubum metietur.

A	B	C	D
8	27	9	11

Theorema 16. Propositio 18.

Duorum similium planorum numerorum vnus medius proportionalis est numerus, & planus ad planum duplicatam habet literis homologi-

A	G	B	C	D	E	F
12	18	27	2	6	3	9

logi ad latus homologum rationem.

Theorema 17. Propositio 19.

Inter duos similes numeros solidos, duo medij proportionales incidunt numeri: & solidus ad similem solidum triplicatam rationem habet lateris homologi ad latus homologum.

A	N	X	B	C	D	E	F	G	H	K	M	L
8	12	18	27	2	2	2	3	3	3	4	6	9

Theorema 18. Propositio 20.

Si inter duos numeros vnus medius proportionalis incidat numerus, similes plani erunt illi numeri.

A	C	B	D	E	F	G
18	24	33	3	4	6	8

Theorema 19 Propositio 21.

Si inter duos numeros duo medij proportionales incidant numeri, similes solidi sunt illi numeri.

A	C	D	B	E	F	G	H	K	L	M
27	36	44	64	9	12	16	3	3	3	4

Theo-

Theorema 20. Propositio 22.

Si tres numeri deinceps ſint proportionales, primus autem ſit quadratus, & tertius quadratus erit.

A	B	D
9	15	25

Theorema 21. Propoſitio 23.

Si quatuor numeri deinceps ſint proportionales, primus autem ſit cubus, & quartus cubus erit.

A	B	C	D
8	12	18	27

Theorema 22. Propoſitio 25.

Si duo numeri rationem habeant inter ſe, quam quadratus numerus ad quadratum numerum, primus autem ſit quadratus erit, & ſecunndus quadratus erit.

A		B	C		D
4	6	9	16	24	36

Theorema 23. Propoſitio 25.

Si numeri duo rationem inter ſe habeant, quam cubus numerus ad cubum numerú, primus autem cubus ſit, & ſecundus cubus erit.

A	E	F	B	C			D
8	12	18	27	64	95	140	216

The

Theorema 24. Propositio 26.

Similes plani numeri rationem inter se habent, quam quadratus numerus ad quadratum numerum.

A	C	B	D	E	F
18	24	32	9	12	16

Theorema 25. Propositio 27.

Similes solidi numeri rationem habent inter se, quam cubus numerus ad cubum numerum.

A	C	D	B	E	F	G	H
16	24	36	54	8	12	18	27

ELEMENTI VIII. FINIS.

EVCLI

EVCLIDIS ELEMENTVM NONVM.

Theorema 1. Propositio 1.

Si duo similes plani numeri mutuò sese multiplicãtes quendã procreent, productus quadratus

A	E	B	D		C
4	6	9	16	24	36

Theorema 2. Propositio 2.

Si duo numeri mutuò sese multiplicantes quadratum faciant, illi similes sunt plani.

A		B	D		C
4	6	9		18	36

Theorema 3. Propositio 3.

Si cubus numerus seipsum multiplicãs procreet aliquẽ, productus cubus erit.

Vnitas	D	D	A			B
	3	4	8	16	32	64

Theorema 4. Propositio 4.

Si cubus numerus cubũ numerum multiplicans quendam procreet, procreatus cubus erit.

A	B	D	C
8	27	64	216

Theorema 5. Propositio 5.

Si cubus numerus numerum quendã multiplicans cubum procreet, & multiplicatus cubus erit.

A	B	D	D
27	64	729	1728

Theorema 6. Propositio 6.

Si numerus seipsum multiplicans cubum procreet, & ipse cubus erit.

A	B	C
27	729	19683

Theorema 7. Propositio 7.

Si compositus numerus quendam numerũ multiplicans quempiam procreet, productus solidus erit.

A	B	C	D	E
6	8	48	2	3

Theorema 8. Propositio 8.

Si ab vnitate quotlibet numeri deinceps ꝓportionales sint, tertius ab vnitate quadratus est, & vnũ intermittentes omnes: quartus autẽ cubus, & duobus intermissis omnes

nes: septimus verò cubus simul & æquadratus, & quinque intermissis omnes.

Vnitas	A	B	C	D	E	F
	3	9	27	81	243	729

Theorema 9. Propositio 9.

Si ab vnitate sint quotcunque numeri deinceps proportionales, sit autem quadratus is qui vnitatem sequitur, & reliqui oẽs quadrati erũt. Quòd si qui vnitatem sequitur cubus sit, & reliqui omnes cubi erunt.

quadrati		cubi
531441	F	732969
59049	E	531441
6561	D	59049
729	C	6561
81	B	729
9	A	81
	o	
	vnitas.	

Theorema 10. Propositio 10.

Si ab vnitate numeri quotcunque proportionales sint, non sit autẽ quadratus is qui vnitatem sequitur, neq; alius vllus quadratus erit, demptis tertio ab vnitate ac omnibus

o	A	B	C	D	E	F
Vnitas.	3	9	36	81	243	729

nibus vnum intermittentibus. Quòd si qui vnitatem sequitur, cubus non sit, neque alius vllus cubus erit, demptis quarto ab vnitate ac omnibus duos intermittentibus.

Theorema 11. Propositio 11.

Si ab vnitate numeri quotlibet deinceps proportionales sint, minor maiorem metitur per quempiam eorum qui in proportionalibus sunt numeris.

A	D	C	D	E
1	2	4	8	16

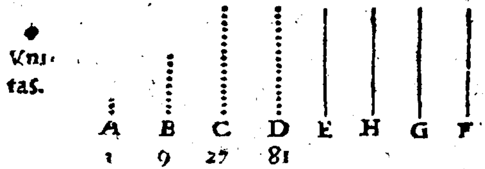
Theorema 12. Propositio 12.

Si ab vnitate quolibet numeri sint proportionales, quot primorum numerorũ vltimum metiuntur, totidem & eum qui vnitati proximus est, metientur.

Vnitas.	A	B	C	D	E	H	G	F
	4	16	64	259	2	8	32	128

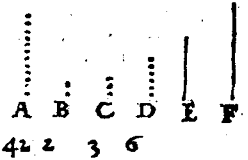
Theorema 13. Propositio 13.

Si ab vnitate sint quotcunq; numeri deinceps proportionales, primus autẽ sit qui vnitatẽ sequitur, maximũ nullus alius metietietur, ijs exceptis qui in proportionalibus ſunt numeris.



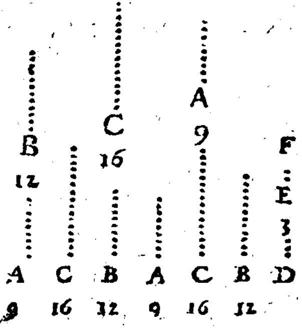
Theorema 14. Propoſitio 14.

Si minimum numerum primi aliquot numeri metiātur, nullus alius numerus primus illum metietur, ijs exceptis qui primò metiuntur.



Theorema 15. Propoſitio 15.

Si tres numeri deinceps proportionales ſint minimi, eadem cum ipſis habētium rationem, duo quilibet compoſiti ad tertium primi erunt.



nibus vnum intermittentibus. Quòd si qui vnitatem sequitur, cubus non sit, neque alius vllus cubus erit, demptis quarto ab vnitate ac omnibus duos intermittentibus.

Theorema 11. Propositio 11.

Si ab vnitate numeri quotlibet deinceps proportionales sint, minor maiorem metitur per quempiam eorum qui in proportionalibus sunt numeris.

A D C D E
1 2 4 8 16

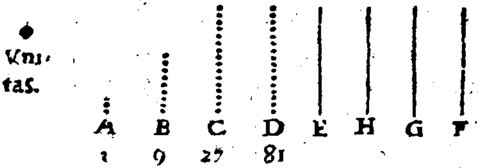
Theorema 12. Propositio 12.

Si ab vnitate quolibet numeri sint proportionales, quot primorum numerorũ vltimum metiuntur, totidem & eum qui vnitati proximus est, metientur.

Vnitas. A B C D E H G F
4 16 64 259 2 8 32 128

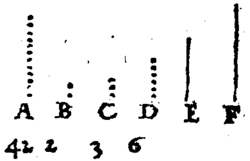
Theorema 13. Propositio 13.

Si ab vnitate sint quotcunq; numeri deinceps proportionales, primus autẽ sit qui vnitatẽ sequitur, maximũ nullus alius metietietur, ijs exceptis qui in proportionalibus sunt numeris.



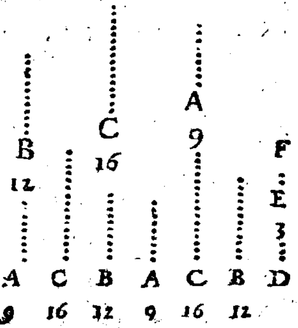
Theorema 14. Propositio 14.

Si minimum numerum primi aliquot numeri metiatur, nullus alius numerus primus illum metietur, ijs exceptis qui primò metiuntur.



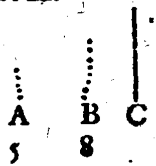
Theorema 15. Propositio 15.

Si tres numeri deinceps proportionales sint minimi, eadem cum ipsis habentium rationem, duo quilibet compositi ad tertium primi erunt.



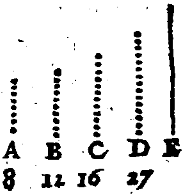
Theorema 16. propoſitio 16.

Si duo numeri ſint inter ſe primi, non ſe habebit quēadmodum primus ad ſecūdum, ita ſecundus ad quēpiam alium.



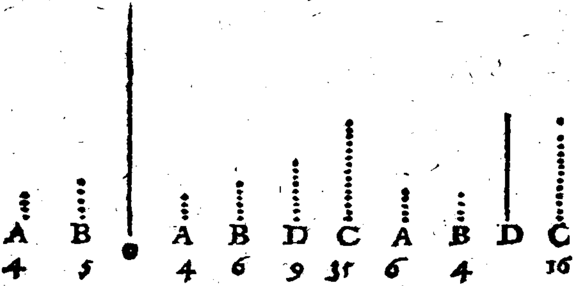
Theorema 17. Propoſitio 18.

Si ſint quotlibet numeri deinceps ꝓportionales, quorum extremi ſint inter ſe primi, non erit quēadmodum primus ad ſecundum, ita vltimus ad quempiam alium.



Theorema 18. Propoſitio 18.

Duobus numeris datis, cōſiderare noſſitne tertius illi inueniri proportionalis.



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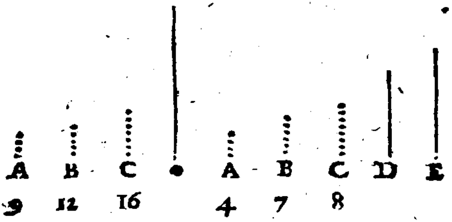
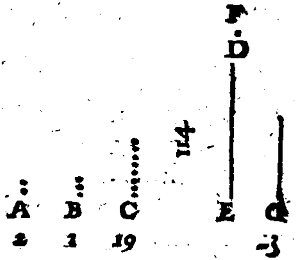
Theorema 19. Propositio 19.

Tribus numeris datis, considerare possitne quartus illis reperiri proportionalis.



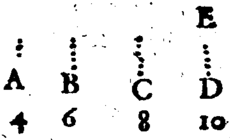
Theorema 20. Propositio 20.

Primi numeri plures sunt quacũque proposita multitudine primorum numerorum.



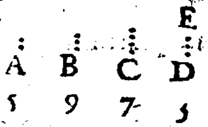
Theorema 12. Propositio 12.

Si pares numeri quotlibet compositi sint, totus est par.



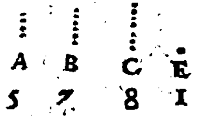
Theorema 22. Propositio 22.

Si impares numeri quotlibet compositi sint, sit autem par illorum multitudo, totus par erit.



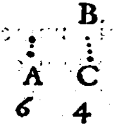
Theorema 23. Propositio 23.

Si impares numeri quotcunque cōpositi sint, sit autem impar illorū multitudo, & totus impar erit.



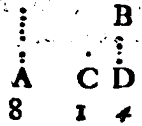
Theorema 24. Propositio 24.

Si de pari numero par detractus sit, & reliquus par erit.



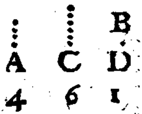
Theorema 25. Propositio 25.

Si de pari numerò impar detractus sit, & reliquus impar erit.



Theorema 26. Propositio 26.

Si de impari numero impar detractus sit, & reliquus par erit.



Theo-

Theorema 27. Propositio 27.

Si ab impari numero par ablatus sit, reliquus impar erit.

A D C
1 4 4

Theorema 28. Propositio 28.

Si impar numerus parem multiplicās, procreet quempiam, procreatus par erit.

A B C
3 4 12

Theorema 29. Propositio 29.

Si impar numerus imparē numerum multiplicans quemdam procreet, procreatus impar erit.

A B C
3 5 15

Theorema 30. Propositio 30.

Si impar numerus parē numerum metiatur, & illius dimidium metietur.

A C B
3 6 18

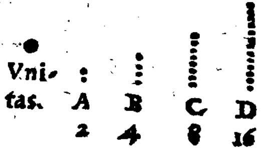
Theorema 31. Propositio 31.

Si impar numerus ad numerum quēpiam primus sit, & ad illius duplum primus erit

A B C D
7 8 16

Theorema 32. Propositio 32.

Numerorum, qui à binario dupli sunt, vnusquisq; pariter par est tantùm.



Theorema 33. Propositio 33.

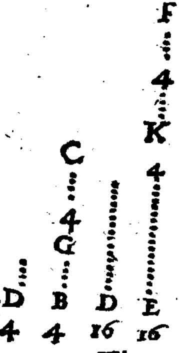
Si numerus dimidiũ impar habeat, pariter impar est tantùm.

Theorema 34. Propositio 34.

Si par numerus nec sit dupl' à binario, nec dimidium impar habeat, pariter par eũ, & pariter impar.

Theorema 35. Propositio 35.

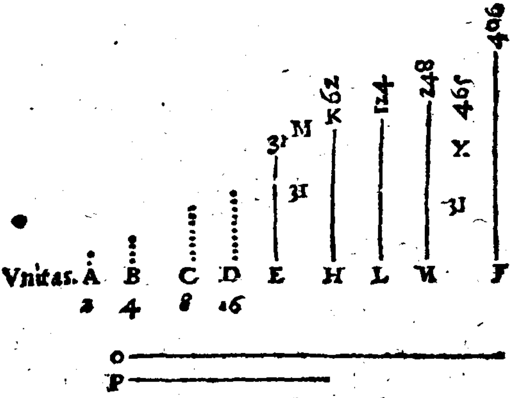
Si sint, quotlibet numeri deinceps proportionales, detrahãtur aũt de secundo & vltimo æquales ipsi primo, erit quemadmodum secũdi excessus ad primũ, ita vltimi excessus ad omnes qui vltimum antecedunt.



Theo-

Theorema 16. Propositio 16.

Si ab vnitate numeri quotlibet deinceps expositi sunt in duplici proportione quoad totus compositus primus factus sit, isq; totus in vltimum multiplicatus quempiá procreet, procreatus perfectus erit.



ELEMENTI IX. FINIS

EVCLI.

EVCLIDIS ELEMENTVM DECIMVM.

DEFINITIONES.

1

Commensurabiles magnitudines dicuntur illæ, quas eadem mensura metietur.

2

Incommensurabiles verò magnitudines dicuntur, hæ quarum nullam mensuram communem contingit reperiri.

3

Lineæ rectæ potentia cõmensurabiles sunt, quarum quadrata vna eadem superficies siue area metitur.

4

Incommensurabiles verò lineæ sunt, quarum quadrata, quæ metiatur area communis, reperiri nulla potest.

5

Hęc cùm ita sint, ostendi potest quòd quantacunque linea recta nobis proponatur, existunt etiam aliæ lineæ innumerabiles eidem commensurabiles, aliæ item incommensurabiles, hæ quidem longitudine & poten-

potentia: illæ verò potentia tantùm. Vocetur igitur linea recta, quantacunq; proponatur, ῥητὴ, id est rationali.

6

Lineæ quoq; illi ῥητῇ commensurabiles siue longitudine & potentia, siue potentia tantùm, vocentur & ipsæ ῥηταί, id est rationales.

7

Quæ verò lineæ sunt incommensurabiles illi τῇ ῥητῇ, id est primo loco rationali, vocétur ἄλογοι, id est irrationales.

8

Et quadratum quod à linea proposita describitur, quam ῥητὴν vocari volumus, vocetur ῥητόν.

9

Et quæ sunt huic commensurabilia, vocétur ῥητά.

10

Quæ verò sunt illi quadrato ῥητῷ scilicet incommensurabilia, vocentur ἄλογα, id est surda.

11

Et lineę quę illa incommensurabilia describunt, vocentur ἄλογοι. Et quidem si illa incommensurabilia fuerint quadrata, ipsa eorũ latera vocabuntur ἄλογοι lineæ. quòd si quadrata quidem non fuerint, verũ aliæ quæpiã superficies siue figuræ rectilineæ, tunc

tunc verò lineæ illæ quæ deſcribunt quadrata æqualia figuris rectilineis, vocentur ἄλογοι.

Theorema 1. Propoſitio 1.

Duabus magnitudinibus inæqualibus propoſitis, ſi de maiore detrahatur plus dimidio, & rurſus de reſiduo iterū detrahatur plus dimidio, idq́; ſemper fiat: relinquetur quædam magnitudo minor altera minore ex duabus propoſitis.

Theorema 2 Propoſitio. 2.

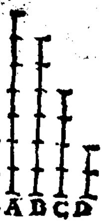
Duabus magnitudinibus propoſitis inęqualibus, ſi detrahatur ſemper minor de maiore, alterna quadam detractione neque reſiduum vnquam metiatur id quod ante ſe metiebatur, incommenſurabiles ſunt illæ magnitudines.

Problema 1. Propoſitio 3.

Duabus magnitudinibus commēſurabilibus datis, maximam ipſarum communem menſuram reperire.

Proble-

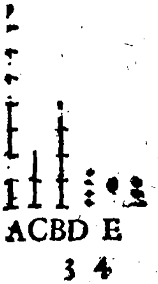
Problema 2. Propositio 4.



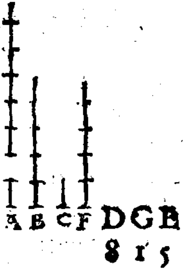
Tribus magnitudinibus cómensurabilibus datis, maximam ipsarũ communẽ mensuram reperire.

Theorema 3. Propositio 5.

Cómensurabiles magnitudines inter se proportioné eam habent, quam habet numerus ad numerum.



Theorema 4. Propositio 6.



Si duæ magnitudines proportionem eam habent inter se quam numerus ad numerum, commensurabiles sunt illæ magnitudines.

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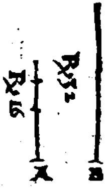
Theorema 5. Propositio 7.



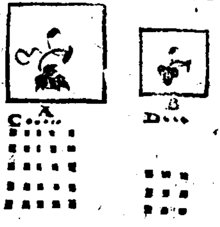
Incommensurabiles magnitudines inter se proportionem non habēt, quam numerus ad numerum.

Theorema 6. Propositio 8.

Si duæ magnitudines inter se proportionem nō habēt quam numerus ad numerum, incommensurabiles illę sunt magnitudines.

Theorema 7. Propositio 9.

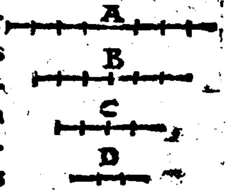
Quadrata, quæ describuntur à rectis lineis lōgitudine cōmensurabilibus, inter se proportionem habent quā numerus quadratus ad alium numerum quadratū. Et quadrata habentia proportionē



inter se quā quadratus numerus ad numerum quadratū, habent quoque latera longitudine commēsurabilia. Quadrata verò quæ deſcribuntur à lineis longitudine in cõmenſurabilibus, proportioné non habent inter ſe, quam quadratus numerus ad numerum alium quadratum. Et quadrata non habentia proportionem inter ſe quam numerus quadratus ad numerum quadratum, neque latera habebunt longitudine commnnſurabilia.

Theorema 8. Propoſitio 10.

Si quatuor magnitudines fuerint proportionales, prima verò ſecundæ fuerit commẽſurabilis, tertia quoque quartę commenſurabilis erit. quòd ſi prima ſecũdæ fuerit incommenſurabilis, tertia quoq; quartæ incommenſurabilis erit.



Problema 3. Propoſitio 11.

Propoſitæ lineæ rectæ (quam ῥητὴν vocari diximus) reperire duas lineas rectas incommenſurabiles, hanc quidem longitudiue tantùm, illam verò non longitudine tantùm, ſed etiam potentia incommẽſurabilem.

Theo-

Theorema 6. Propositio 12.

Magnitudines quæ eidem magnitudini sunt commensurabiles, inter se quoque sunt commensurabiles.

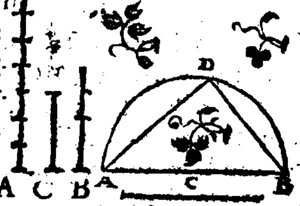
6 D...... 4 F.
4 E.... 8 G.
3 H...
2 K..
4 L...

Theorema 10. Propositio 13.

Si ex duabus magnitudinibus hæc quidem commensurabilis sit tertię magnitudini, illa verò eidem incommensurabilis, incommēsurabiles sunt illæ duę magnitudines.

Theorema 11. Propositio 14.

Si duarum magnitudinum commensurabilium altera fuerit incommensurabilis magnitudini alteri cuipiam tertię, reliqua quoq; magnitudo eidem tertiæ incommensurabilis erit.



Theorema 12. Propositio 15.

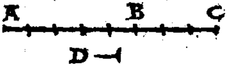
Si quatuor rectæ proportionales fuerint,

possit

possit autem prima plusquá secunda tanto quantum est quadratum lineę sibi commensurabilis longitudine: tertia quoq; poterit plusquam quarta tanto quantum est quadratum lineæ sibi commensurabilis longitudine. Quòd si prima possit plusquam secunda quadrato lineę sibi longitudine incommensurabilis: tertia quoque poterit plusquam quarta quadrato lineę sibi incómẽsurablilis longitudine.

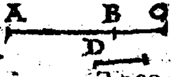
Theorema 13. Propositio 16.

Si duæ magnitudines commensurabiles componantur, tota magnitudo composita singulis partibus commensurabilis erit, quòd si tota magnitudo composita alterutri parti commensurabilis fuerit, illæ duæ quoque partes cómensurabiles erunt.



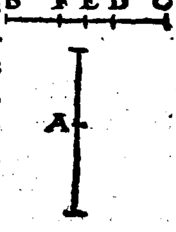
Theorema 14. Propositio 17.

Si duæ magnitudines incommensurabiles componantur, ipsa quoque tota magnitudo singulis partibus componentibus incommensurabilis erit. Quòd si tota alteri parti incommensurabilis fuerit, illæ quoque primę magnitudines inter se incommensurabiles erunt.



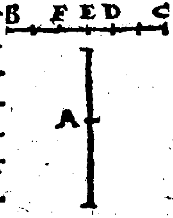
Theorema 15. Propositio 18.

Si fuerint duæ rectæ lineæ inæquales, & quartæ parti quadrati quod describitur à minore æquale parallelongrammú applicecetur secundú maiorem, ex qua maiore tátum excurrat extra latus parallelogrammí, quantum est alterum latus ipsius parallelogrammi: si præterea parallelogrammum sui applicatione diuidat lineá illam in partes inter se commensurabiles longitudine, illa maior linea tanto plus potest quam minor, quantú est quadratum lineæ sibi commensurabilis longitudine. Quòd si maior quadratú lineæ sibi commensurabilis lonplus possit quàm minor, tanto quantú est gitudine, & præterea quartæ parti quadrati lineę minoris ęquale parallelogrammum applicetur secundum maiorem, ex qua maiore tantum excurrat extra latus pallelogrammi, quantum est alterum latus ipsius parallelongrammi parallelògammum sui applicatione diuidit maioré in partes inter se longitudine commensurabiles.

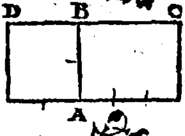


Theorema 16. Propositio 19.

Si fuerint duæ rectæ inæquales, quartæ autem parti quadrati lineæ minoris æquale le parallelogrammorum secundum lineam maiorem applicetur, ex qua linea tantum excurrat extra latus pnrallelogrammi, quàtum est alterum latus eiusdem parallelogrammi: si parallelogrammum præterea sui applicatione diuidat lineam in partes inter se longitudine incommensurabiles, maior illa linea tantò plus potest quàm minor quantum est quadratum lineæ sibi maiori incommensurabiles longitudine. Quod si maior linea tantò plus possit quàm minor, quantum est quadratum lineæ incommensurabilis sibi longitudine: & preterea quartæ parti quadrati lineæ minoris æquale parallelogrammum applicetur secundum maiorem ex qua tantum excurrat extra latus parallelográmi, quantum est alterum latus ipsius: parallelogrammum sui applicatione diuidit maiorem in partes inter se incommensurabiles longitudine.



Theorema 17. Propositio 20.

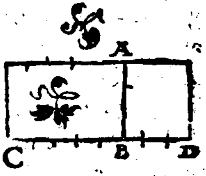


Superficies rectangula contenta ex lineis rectis rationalibus longitudine commensurabilibus secundum

cundum vnum aliquem modum ex antedictis, rationalis eſt.

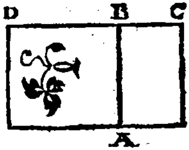
Theorema 18. Propoſitio 21.

Si ratinonale ſecundũ lineam rationalem applicetur, habebit alterum latus lineam rationalem & cõmenſurabilem longitudine lineæ cui rationale parallelogrammum applicatur.



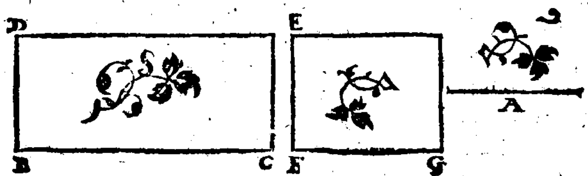
Theorema 39. Propoſitio 56.

Surperficies rerctangula contenta duabus lineis rectis rationalibus potentia tantum commẽſurabilibus, irrationalis eſt. Linea autem quæ illam ſuperficiem poteſt, irrationalis & ipſa eſt: vocetur verò medialis.



Theorema 20. Propoſitio 23.

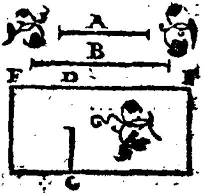
Quadrati lineæ medialis applicati ſecun-



dum

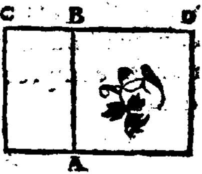
dum lineam rationalem, alterum latus est linea rationalis, & incommensurabilis longitudine lineæ secundum quam applicatur.

Theorema 21. Propositio 24



Linea recta mediali commẽsurabilis, est ipsa quoque medialis.

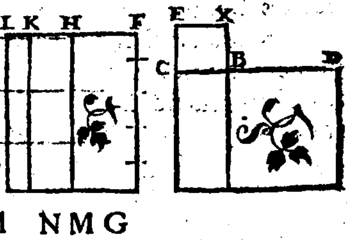
Theorema 22. Propositio 25



Parallelongrammum rectangulum contẽtum ex lineis medialibus longitudine commensurabilibus, mediale est.

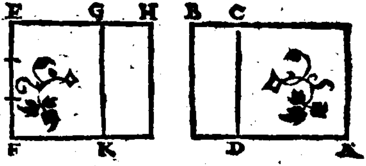
Theorema 23. Propositio 26.

Parallelogrammum rectangulum comprehensum duabus lineis medialibus potentia tãtùm cõmensurabilibus, vel rationale est, vel mediale.



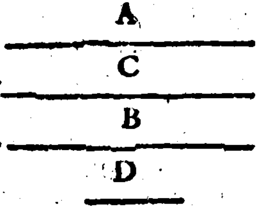
Theorema 24. Propositio 27.

Mediale nó est maius quàm mediale superficie rationali.



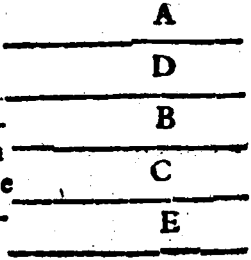
Problema. 4. Propositio 28.

Mediales lines inuenire potentia tantùm commensurabiles rationale comprehendentes.



Problema 5. Propositio 29.

Mediales lineas inuenire potentia tantùm commensurabiles me diale comprehendendentes



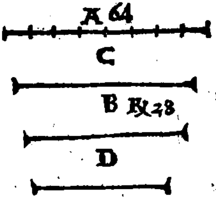
Pro-

Problema 6. Propoſitio 30.

Reperire duas rationales potentia tantum commẽ ſurabiles huiuſmodi, vt maior ex illis poſſit plus quàm minor quadrato lineæ ſibi commenſurabilis longitudine.

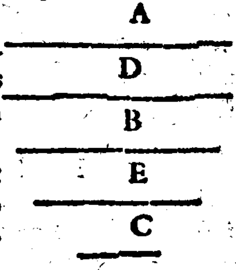
Problema 7. Propoſitio 31.

Reperire duas lineas medialis potentia tantùm commenſurabiles rationalem ſuperficiẽ continẽtes, tales inquã vt maior poſſit plus quàm minor quadrato lineæ ſibi commẽſurabilis longitudine.



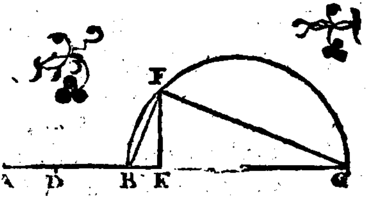
Problema 8. Propoſitio 32.

Reperire duas lineas mediales potentia tantùm commenſurabiles medialem ſuperficiem continentes, huiuſmodi vt maior plus poſſit quàm minor quadrato lineæ ſibi commenſurabilis longitudine.



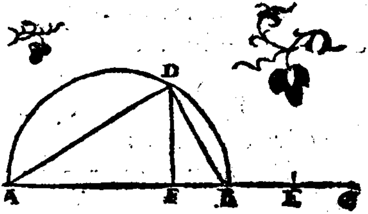
Problema 9. Propositio 33.

Reperire duas rectas poutentia incommensurabiles, quarum quadrata simul addita faciant superficiẽ rationalẽ, parallelogrammum verò ex ipsis conetntum sit mediale.



Problema 10. Propositio 34.

Reperire lineas duas rectas potentia incommensurabiles conficentes compositum ex ipsarum quadratis mediale, parallelogrammũ verò ex ipsis contentum rationale,



Problema 11. Propositio 85.

Reperire duas lineas rectas potentia incõmensurabiles, conficientes id quod ex ipsarum quadratis componitur meãle, simulque

que parallelogrammū ex ipsis contentum, mediale, quod præterea parallelogrammū sit incómensurabile composito ex quadratis ipsarum.

PRINCIPIVM SENARIORUM PER COMPOSITIONEM.

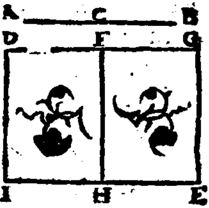
Thorema 25. Propositio 36.

Si duæ rationales potentia tantùm commensurabilis. Vocetur autem Binommum.

Theorema 26. Propositio 37.

Si duæ mediales potentia tantòm commensurabiles rationale continentes componantur, tota linea est irrationalis, vocetur autem Bimediale prius.

Theorema 27. Propositio 38.



Si duæ mediales Potétia tantùm comméſurabiles mediale continétes componãtur, tota linea eſt irrationalis: vocetur auté Bimediale ſecundum.

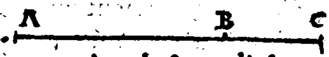
Theorema 28. Propoſitio 36.

Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex quadratis ipſarum rationale parallelogrammum verò ex ipſis contentum mediale, tota linea recta

eſt irrationalrs. Vocetur auté linea maior.

Theorema 39. Propoſitio 40.

Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex ipſarum quadratis mediale, id vero ꝙ fit ex ipſis, rationale, tota linea eſt irrationales. Vocetur au-

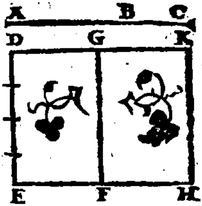


tem potens rationale & mediale.

Theorema 30. Propoſitio 41.

Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex quadratis ipſarum mediale, & quod continetur

netur ex ipsis, mediale, & præterea incõmensurabile composito ex quadratis ipsarum tota linea est irrationalis. Vocetur autem potens duo medialia.



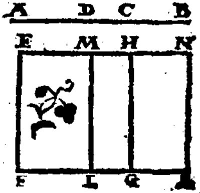
Theorema 31. Propositio 42.

Binominum in vnico tantùm pũcto diuiditur in sua nomina, id est in lineas ex quibus componitur.

Theorema 32. Propositio 43.

Bimediale prius in vnico tantum puncto diuiditur in sua nomina.

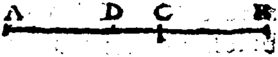
Problema 33. Propositio 44



Bimediale secundum in vnico tantũ puncto diuiditur in sua nomina.

Problema 34. Propositio 45.

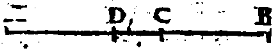
Linea maior in vnico tantùm in pũcto diuiditur in sua nomina.



Theo-

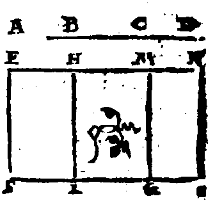
Theorema 35. Propositio 46.

Linea potens rationale & mediale in vnico tantùm puncto diuiditur in sua nomina.



Theoroma 36 Propositio 47

Linea ootèns duo medialia in vnico tantùm puncto diuiditur in sua nomina.



DEFINITIONES.
secundæ.

Proposita linea rationali, & binomio diuiso in sua nomine, cuius binomij maius nomen, id est, maior portio possit plusquam minus nomen quadrato lineæ sibi, maiori inquam nomini, commensurabilis longitudine.

I.

Si quidé maius nomen fuerit commensurabile longitudine propositæ lineæ rationali, vocetur toto linea Binomum primium.

2

Si veró minus nomen, id est minor portio binomij, fuerit commensurabile longitudi ne ne propositæ lineæ rationali, vocetur tota linea Binomium secundum.

3

Si verò neutrum nomen fuerit commensurabile longitudine propositæ lineæ rationali, vocetur Binomium tertium.

Rursus si maius nomen possit plusquam minus nomen quadrato lineæ sibi incommensurabilis longitudine.

4

Si quidem maius nomen est commensurabile longitudine propositæ lineæ rationali, vocetur tota linea Binomium quartum.

5

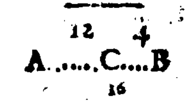
Si verò minus nomen fuerit commensurabile longitudine lineæ rationali, vocetur Binomium quintum.

6

Si verò neutrum nomen fuerit longitudine commensurabile lineæ rationali, vocetur illa Binomium sextum.

Proble.12. Propositio 48

Reperire Binomium primum.



Pro-

Problema 13. Propositio 49

9 3
A........C...B
12

D

Reperire Binomium secundum.

E F 9 G

H 6

Problema 14. Propositio 50.

15 5
A...............C....
20

D

....

E

Reperire Binomium tertiũ.

F 20 G 15 H

K 6

Problema 15. Propositio 51.

10 6
A........C......B
16
D

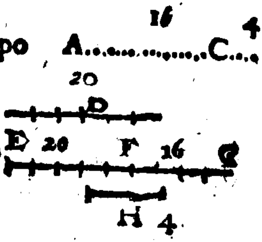
Repere Binomiũ quartum.

E 16 F 10 G

H 6

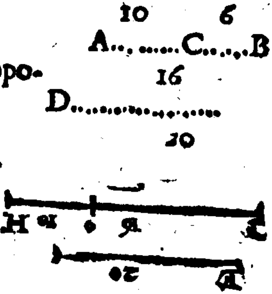
Proble-

Problema 16. Propositio 52



Reperire Binomium quintum.

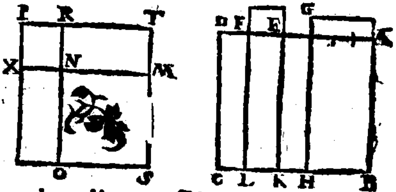
Probl. 17. Propositio 53.



Reperire Binonimum sextum.

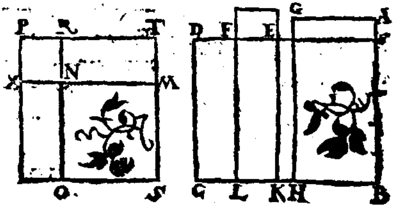
Theorema 37. Propositio 54.

Si superficies contéta fuerit ex rationali & Binomio primo, linea quę illã superficiem potest, est irrationalis, quæ Binomiũ vocatur.



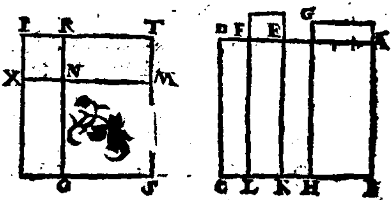
Theorema 38. Propositio 55.

Si superficies cōtenta fuerit ex linea ratio nali & Binomio secundo, linea potens illam superficiē est irrationalis quę Bimediale primum vocatur.



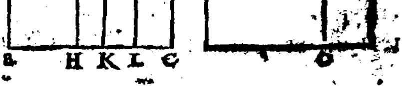
Theorema 39. Propositio 56.

Si superficies cōtineatur ex rationali & Binomio tertio, lineã q̃ illam superficiem potest, est irrationalis q̃ dicitur Bimediale secundū.



Theorema 40. Propositio 57.

Si superficies contineatur ex rationali & Boin

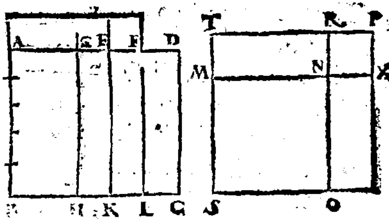


mio

mio quarto. linea potens superficiem illam, est irrationalis, quæ dicitur maior.

Theorema 41. Propositio 58.

Si superficies contineatur ex rationali & Binomio quinto, linea quæ illam superficiem potest est irrationalis, quę dicitur potés rationale & mediale.



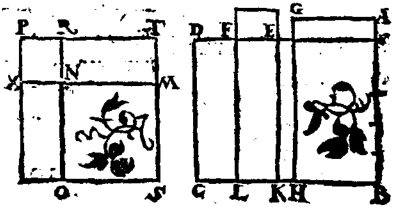
Therema 42. Propositio 59

Si superficies contineatur ex rationali & Binomio sexto, linea quæ illam superficiem potest est irrationalis, quæ dicitur

L 2 potens

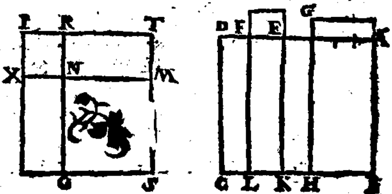
Theorema 38. Propositio 55.

Si superficies cõtenta fuerit ex linea rationali & Binomio secundo, linea potens illam superficiẽ est irrationalis quę Bimediale primum vocatur.



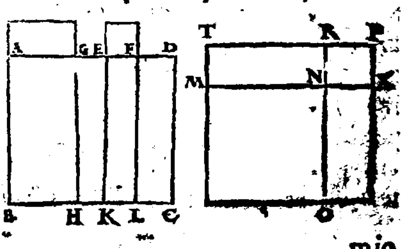
Theorema 39. Propositio 56.

Si superficies cõtineatur ex rationali & Binomio tertio, linea q̃ illam superficiem potest, est irrationalis q̃ dicitur Bimediale secundũ.



Theorema 40. Propositio 57.

Si superficies contineatur ex rationali & Boin

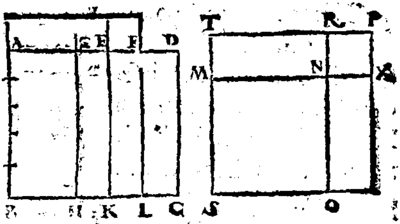


mio

mio quarto. linea potens ſuperficiem illam, eſt irrationalis, quæ dicitur maior.

Theorema 41. Propoſitio 58.

Si ſuperficies contineatur ex rationali & Binomio quinto, linea quæ illam ſuperficiem poteſt eſt irrationalis, quę dicitur potēs rationale & mediale.

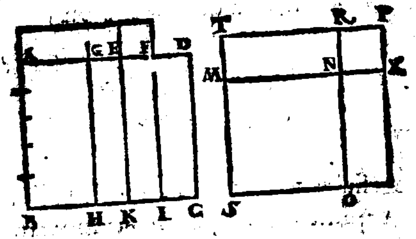


Therema 42. Propoſitio 59

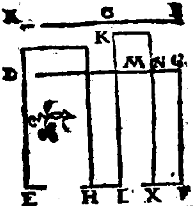
Si ſuperficies contineatur ex rationali & Binomio ſexto, linea quæ illam ſuperficiem poteſt eſt irrationalis, quæ dicitur

 potens

potens tuo mediala.

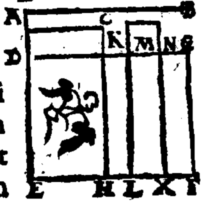


Theorema 43. Propositio 60.



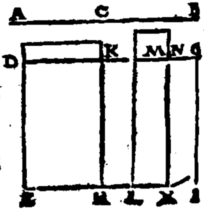
Quadratum Binomij secundum lineam rationalem applicatum, facit alterum latus Binomium primum.

Theorema 44. Propositio 61.



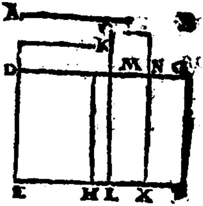
Quadratũ Bimedialis primi secundum rationalem lineam applicatum, facit alterum latus Binomium secundum.

Theorema 45. Propositio 62.



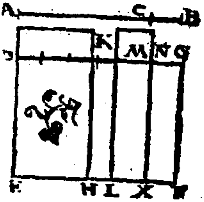
Quadratũ Bimedialis secũdi secundũ rationalem applicatum, facit alterũ latus Binominũ tertium.

Theorema 46. ropopositio 63



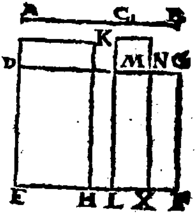
Quadratum lineæ maioris ſecundum lineam rationalem applicatum, facit alterum latus Binomium quartum.

Theorema 47. Propoſitio 64.



Quadratum lineæ potẽtis rationale & mediale ſecundum rationalẽ applicatũ, facit alterum latus Binomium quintum.

Theorema 48. Propoſitio 65



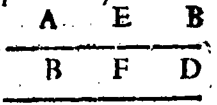
Quadratum lineę potentis duo medialia ſecundũ rationalem applicatum, facit alterum latus Binomium ſextum.

Theorema 49. Propoſitio 66.

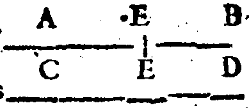
Linea lõgitudine commenſurabilis Binomio eſt & ipſa Binomium eiuſdem ordinis.

Theorema 50. Propositio 67

Linea longitudine commensurabilis alteri bimedialum, est & ipsa bimediale etiam eiusdẽ ordinis.



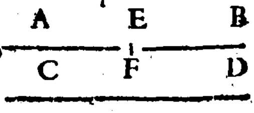
Theorema 51. Propositio 68



Linea cõmensurabilis lineæ maiori, est & ipsa maior.

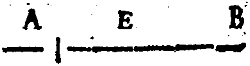
Theorema 52. Propositio 69.

Linea commensurabilis lineæ potenti rationale & mediale, est & ipsa linea potens rationale & mediale.



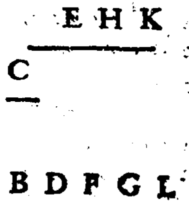
Theorema 53. Propositio 70.

Linea commensurabilis, lineæ potenti duo medialia, est & ipsa linea potens duo medialia.



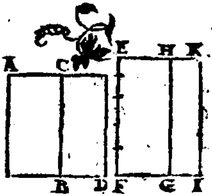
Theorema 54 Propositio 71

Si duæ superficies rationalis & medialis simul componantur, linea quę totã superficiem ètiem compositā potest, est vna ex quatuor irrationalibus, vna ex quæ dicitur Binomium, vel bimediale primum. vel linea maior, vel linea potēs rationale & mediale.



Theorema 55. Propositio 72.

Si duæ superficies mediales incommensurabiles simul componantur fiunt reliquæ duæ lineę irrationales, vel bimediale secundum, vel linea potēs duo medialia



SCHOLIVM.

Binomium & cæteræ consequentes lineæ irrationales, neque sunt eædem cum linea mediali, neque ipsæ inter se.

Nam quadratum lineæ medialis applicatum secundum lineam rationalem facit alterum latus lineam rationalem, & longitudine incommensurabilem lineæ secundum quam applicatur, hoc est, lineæ rationali, per 23.

Quadratum verò Binomij secundum rationalem applicatum, facit alterum latus Binomium primum, per 60.

Quadratum verò Bimedialis primi secundum rationalem applicatum, facit alterum latus Binomium secundum, per 61

Quadratum verò Bimedialis secundi secundum rationalem applicatum; facit alterum latus Binomiũ tertium per 62.

Quadratum verò lineæ maioris secundum rationalem applicatum. facit alterum latus Binomiũ quartum, per 63

Quadratum verò lineæ potẽtis rationale & mediale secundum rationalem applicatum, facit alterũ latus Binomium quintum, per 64.

Quadratum vero lineæ potentis duo medialia secũdum rationalem applicatum, facit alterum latus Binomium sextum, per 65.

Cùm igitur dicta latera, quæ latitudines vocantur, differant & à prima latitudine, quoniam est rationalis, cùm inter se quoque differant, eo quia sunt Binomia diuersorum ordinum: manifestum est ipsas lineas irrationales, differentes esse inter se.

SECVN.

SECVNDVS ORDO ALterius ſermonis, qui eſt de detractione.

Principium ſenariorum per detractionem.

Theorema 57. Propoſitio 74.

Si de linea rationali detrahatur rationalis potentia tantùm commenſurabilis ipſi toti, reſidua eſt irrationalis, vocetur autem Reſiduum.

A A A

Theorema 56. Propoſitio 37.

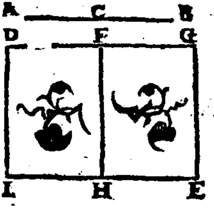
Si de linea mediali detrahatur medialis potentia tantũ commenſurabilis toti lineæ, quæ verò detracta eſt cũ tota contineat ſuperficiem rationalem, reſidua eſt irrationalis. Vocetur autem Reſiduum mediale primum.

A A B

Theorema 58 Propoſitio. 75.

Si de linea mediali detrahatur medialis

tentia tantùm commensurabilis toti, quæ verò detracta est, cùm tota cõtineat superficiẽ mediale, reliqua est irrationalis. Vocetur autem Residuũ mediale secundum.



Theorema 57. Propositio 76

Si de linea recta detrahatur recta potẽtia incommensurabilis toti, compositum autem ex quadratis totius lineæ & lineæ detractæ sit rationale, parallelogrammum verò ex ijsdem contentum sit mediale, reliqua linea erit irrationalis. Vocetur autem linea minor.

Theorema 58. Propositio 77.

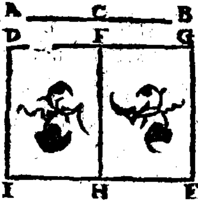
Si de linea recta detrahatur recta potentia

tia incommensurabilis toti lineæ compositum autem ex quadratis totius & lineæ detractæ sit mediale, parallelogrammum verò bis ex eisdem contentum sit rationale, reliqua linea est irrationalis. Vocetur autem linea faciens cum superficie rationali totam superficiem medialem.

Theorema 59. Propositio 78.

Si de linea recta detrahatur recta potentia incommensurabilis toti lineæ, compositum autem ex quadratis totius & lineæ detractæ sit mediale, parallelogrammum verò bis ex ijsdem sit etiam mediale: præterea sint quardata ipsarum incommensurabilia parrallelogrammo bis ex ijsdem contento, reliqua linea est irrationalis.

Vocetur autem linea faciens cum superficie mediali totam super ficiem medialem.



Theorema 60. Propositio 79.

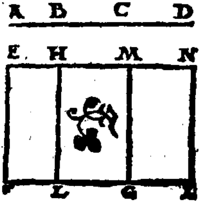
Residuo vnica tantùm linea recta cóiungitur rationalis, potentia tantùm cõmensurabilis toti lineæ rationalem.

Theorema 61. Propositio 80.

Residuo mediali primo vnica tãtùm lineæ coniungitur medialis, potentia tantùm cõmensurabilis toti, ipsa cum tota continens

Theorema 62. Propositio 81.

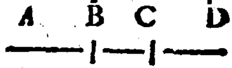
Residuo mediali secundo vnica tantùm coniungitur mediails, potentia tantùm commẽsurabilia toti ipsa cum tota continens mediale.



Theorema 63. Propositio 82.

Lineæ minori vnica tantùm recta cóiungitur

tur potentia incommensurabilis toti, faciens cum tota compositum ex quadratis ipsarum rationale, id verò parallelogrammum, quod bis ex ipsis fit, mediale.

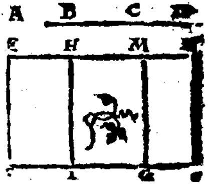


Theorema 64. Propositio 83.

Lineæ facienti cum superficie rationali totam superficiem medialem, vnica tantùm coniungitur linea recta potentia incommensurabilis toti, faciens autem cum tota compositum ex quadratis ipsarũ, mediale, id verò quod fit bis ex ipsis, rationale.

Theorema 65. Propositio 84.

Lineæ cum mediali superficie facienti totam superficiem medialem, vnica tantum coniũgitur linea potẽtia toti incommensurabilis, faciens cum tota compositũ ex quadratis ipsarũ mediale, id verò quod fit bis ex ipsis etiam mediale, & prætereà faciens compositum ex quadratis ipsarum incommensurabile ei quod fit bis ex ipsis,



DE-

DEFINITIONES TERTIAE

Proposita linea rationali & residuo.

1

Si quidem tota, nempe composita ex ipso residuo & linea illi cóiuncta, plus potest quàm coniuncta, quadrato lineæ sibi cómensurabilis longitudine, fueritq; tota longitudine commẽsurabilis lineæ propositæ rationali, residuũ ipsum vocetur Residuum primum.

2

Si verò coniuncta fuerit longitudine commensurabilis rationali, ipsa autem tota plus possit quam coniuncta, quadrato lineæ, sibi longitudine commensurabilis, residuum vocetur Residuum secundũ.

3

Si verò neutra linearũ fuerit longitudine commensurabilis rationali, possit autem ipsa tota plusquam coniuncta, quadrato lineæ sibi longitudine commẽsurabilis, vocetur Residuum tertium.

Rursus si tota possit plus quàm coniuncta quadrato lineæ sibi longitudine incommensurabilis

4

Et quidem si tota fuerit longitudine commenmensurabilis ipsi, rationali, vocetur Resiquum quartum.

5

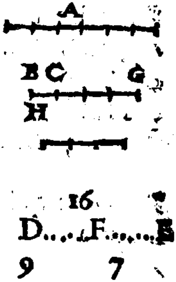
Si verò coniuncta fuerit lõgitudine commensurabilis rationali, & tota plus possit quàm coniuncta, quadrato lineæ sibi longitudine incõmensurabilis, vocetur Residuum quintum.

6

Si verò neutra linearum fuerit cõmensurabilis longitudine ipsi rationali, fueritque tota potentior quã coniuncta, quadrato lineæ sibi longitudine incommensurabilis, vocetur Residuum sextum.

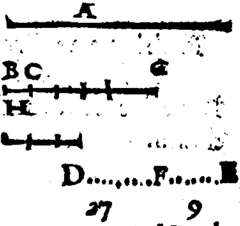
Problema 18. Propositio 85.

Reperire primum Residuum.



Problema 19. Propositio 86.

Reperire secundum Residuum.



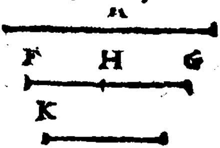
E.........

Problema 20. Propositio 87.

12

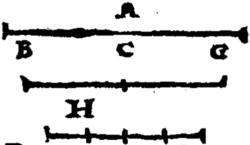
B.........E......C

9 7



Reperire tertium Residuum.

Probl. 21. Propositio 88.

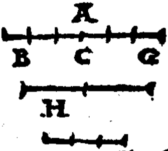


Reperire quartum Residuum.

D...........F....E

16 4

Problema 22. Propositio 89



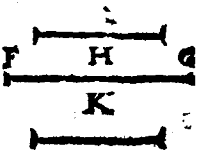
Reperire quintum Residuum

D......F....E

25 7

Problema 23. Propositio 90.

Reperire sextum Residuum.

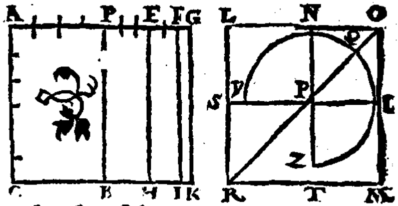


E............ 13

Theo-B........D......C

Theorema 66. Propoſitio 91.

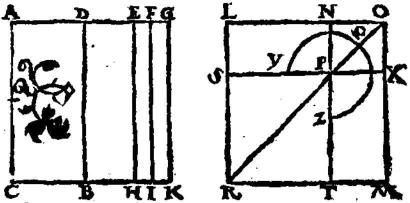
Si ſuperficies contineatur ex linea rationali & reſi-



duo primo linea, quæ illam ſuperficiem poteſt, eſt reſiduum.

Theorema 67. Propoſitio 92.

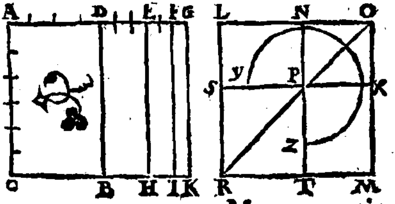
Si ſuperficies cótineatur ex linea rationali & reſi-



duo ſecundo, linea q̃ illam ſupſiciẽ poteſt, eſt reſiduum mediale primum.

Theorema 68. Propoſitio 93.

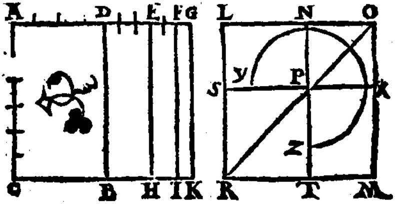
Si ſuperficies cótineatur ex linea rationali & reſiduo ter-



tio,linea quæ illam ſuperficiem poteſt, eſt reſiduum mediale ſecundum.

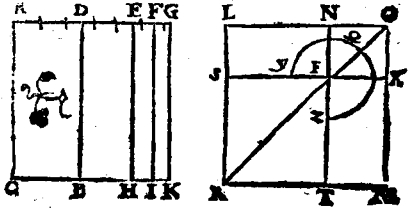
Theorema 69. Propoſitio 94.

Si ſuperficies contineatur ex linea rationali & reſiduo quarto linea q̃ illá ſuperficiem poteſt, eſt linea minor.



Theorema 70. Propoſitio 95.

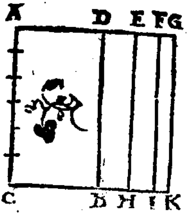
Si ſuperficies contineatur ex linea rationali & reſiduo quinto,linea qùæ illá ſuperficiẽs poteſt,eſt ea quæ dicitur cum rationali ſuperficie faciens totam medialem.

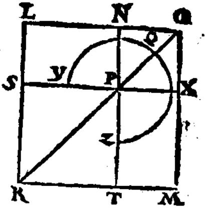


Theorema 71. Propo-
poſitio 96.

Si ſuperficies contineatur ex linea rational &

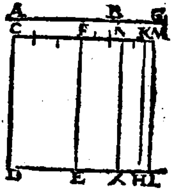
& residuo sexto, linea quæ illam superficiē pōt, est ea quę dicitur faciēs cum mediali





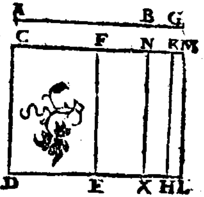
superficie totam medialem.

Theorema 72. Propositio 97.



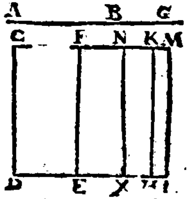
Quadratum residui secundum lineam rationalem applicatum, facit alterum latus redsiuum primum.

Theorema 73. Propositio 98.



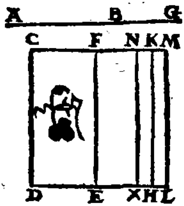
Quadratum residui medialis pι imi secundū rationalem applicatū, facit alterum latus residuū secundum.

Theorema 74. Propositio 99.



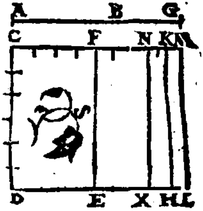
Quadratū, residui medialis secundi secundum rationalē applicatū, facit alterū latus residuum tertium.

Theorema 75. Propositio 100.



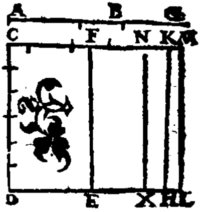
Quadratũ lineæ minoris secundùm rationalẽ applicatum, facit alterũ latus residuum quartũ.

Theorema 76. Propositio 101.



Quadratũ lineæ cum rationali superficie faciẽtis totam medialem, secũdum rationalem applicatum, facit alterum latus residuum quintum.

Theorema 77. Propositio 102.



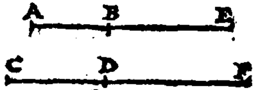
Quadratum lineæ cum mediali superficie faciẽtis totam medialem, secundum rationalem applicatum, facit alterum latus residuum sextum.

Theorema 78. Propositio 103.

Linea residuo commensurabilis longitudine, est & ipsa residuum, & eiusdem ordinis

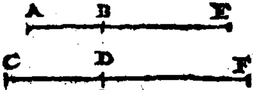
Theorema 79. Propositio 104.

Linea cõmenſurabilis reſiduo mediali, eſt & ipſa reſiduũ mediale, & eiuſdẽ ordinis.



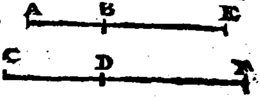
Theorema 80. Propoſitio 105.

Linea cõmẽſurabilis lineę minori, eſt et ipſa linea minor



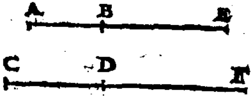
Theorema 81. Propoſitio 106.

Linea commenſurabilis lineæ cũ rationali ſuperficie facienti totã medialẽ, eſt & ipſa linea cum rationali ſuperficie faciens totam medialem.



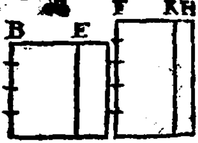
Theorema 82. Propoſitio 107.

Linea commẽſurabilis lineæ cũ mediali ſuperficie facienti totam medialem, eſt & ipſa cum mediali ſuperficie faciens totam medialem.

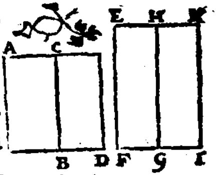


Theorema 83. Propoſitio 108.

Si de ſuderficie rationali detrahatur ſuperficies medialis, linea quæ reliquã ſuperficiem poteſt, eſt alterutra ex duabus irrationalibus, aut reſiduum, aut liuea minor.



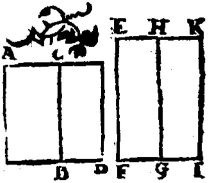
Theorema 84. Propositio 109



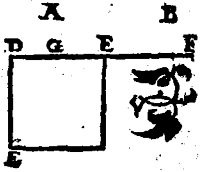
Si de ſuperficie mediali detrahatur ſuperficies rationalis, aliæ duæ irrationales, fiũt, aut reſiduum mediale primũ aut cũ rationali ſuperficiem faciens totam medialem.

Theorema 85. Propoſitio 110.

Si de ſuperficie mediali detrahatur ſuperficies medialis q̃ ſit incommẽſurabilis toti, reliquę duæ fiunt irrationales, aut reſiduum mediale ſecundum, aut cũ mediali ſuperficie faciens totam medialem.



Theorema 86. Propoſitio 111.



Linea quæ Reſiduum dicitur, nõ eſt eadem cum ea quæ dicitur Binomium.

SCHO.

SCHOLIVM.

Lineæ quæ Residuum dicitur, & cæteræ quinque eã consequentes irrationales, neque lineæ mediali neque sibi ipsæ inter se sunt eadem. Nam quadratum lineæ medialis secundum rationalem applicatum, facit alterum latus, rationalem lineam longitudine incommensurabilem ei, secundum quam applicatur, per 23.

Quadratum verò residui secundum rationalem applicatũ, facit alterũ latus residuũ primũ, per 97.

Quadratum verò residui medialis primi secundum rationalem applicatum, facit alterum latus residuum secundum, per 98.

Quadratum verò residui medialis secundi, facit alterum latus residuum tertium, per 99.

Quadratum verò lineæ minoris, facit alterũ latus residuum quartum, per 100.

Quadratum verò lineæ cum rationali superficie facientis totam medialem, facit alterum latus residuum quintum, per 101.

Quadratum verò lineæ cum mediali superficie facientis totam medialem, secundum rationalem applicatum, facit alterum latus residuum sextum, per 102.

Cùm igitur dicta latera, quæ sunt latitudines cuiusque parallelogrammi vnicuique quadrato æqualis & secundum rationalem, applicati, differant & à primo latere, & ipsa inter se (nam à primo differunt: quoniam sunt resi-

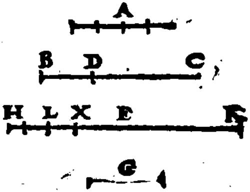
dua non eiusdem ordinis) constat ipsas quoque, lineas irrationales inter se desferentes esse. Et quoniam demonstratum est, Residuum non esse idem quod Binomium, quadrata autem residui & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex residuis eiusdem ordinis cuius sunt & residua, quorum quadrata applicantur rationali, similiter & quadrata Binomij & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex Binomijs eiusdem ordinis, cuius sunt & Binomia, quorum quadrata appli cãtur rationali. Ergo lineæ irrationales quæ cõsequuntur Binomium, & quæ consequuntur residuũ, sunt inter se differẽtes. Quare dictæ lineæ omnes irrationales sunt numero 13.

1 Medialis.
2 Binomium.
3 Bimediale primum.
4 Bimediale secundum.
5 Maior.
6 Potens rationale & mediale.
7 Potẽs duo medialia.
8 Residuum.
9 Residuum mediale primum.
10 Residuum mediale secundum.
11 Minor.
12 Faciens cum rationali superficie totam medialem
13 Faciens cum mediali superficie totam medialem.

Theo-

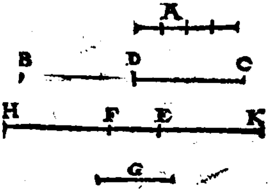
Theorema 87. Propositio 112.

Quadratũ lineæ rationalis secundum Binomiũ applicatũ, facit alterum latus residuũ, cuius nomina sunt cõmensurabilia Binomij nominibus, & in eadẽ proportione præterea id quod fit Residuum, eundem ordinem retinet quem Binomium.



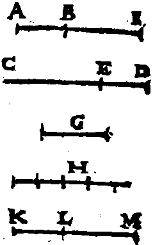
Theorema 88. Propositio 113.

Quadratũ lineæ rationalis secundum residuum applicatũ, facit alterum latus Binomium, cuius nomina sunt commensurabilia nominibus residui & in eadẽ proportione: prę terea id quod fit Binomiũ, est eiusdem ordinis, cuius & Residuum.



Theorema 89 Propositio 114.

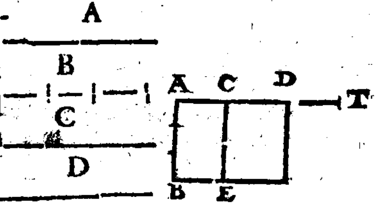
Si parallelogrammum contineatur ex resi-



duo & Binomio, cuius nomina sunt commensurabilia nominibus residui & in eadẽ proportione, linea quæ iliam superficiem potest, est rationalis.

Theorema 90. Propositio 115.

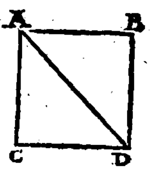
Ex linea mediali nascũtur lineę irrationales innumerabiles, quarum nulla ulli antedictarum eadem sit.



Theorema 100. Propositio 116.

Propositũ nobis esto demonstrare in figuris quadratis diametrũ esse longitudine incommensurabilem ipsi lateri.

E...H..B
G...



EVCLIDIS ELEMENTVM VNDECIMVM, ET SOLIDORVM.

prim m.

DEFINITIONES.

1

Solidum, est quod longitudinem, latitunem, & crassitudinem habet.

2

Solidi autem extremum est superficies.

3.

Linea recta est ad planum recta, cùm ad rectas oẽs lineas, à quibus illa tangitur, quæque in propositio sunt plano, rectos angulos efficit.

4

Planum ad planum rectum est, cum rectæ lineæ, quæ cõmuni planorum sectioni ad rectos angulos in vno planorum ducũtur, alteri plano ad rectos sunt angulos.

5

Rectæ lineę ad planũ inclinatio, acutus est angulus, ipsa insistente linea & adiũcta altera cõprehẽsus, cùm à sublimi rectæ illius lineę termino deducta fuerit ꝑpẽdicularis,

laris, atq; à puncto quod perpendicularis in ipso plano fecerit, ad propositę illius lineæ extremum, quod in eodem est plano, altera recta linea fuerit adiuncta.

6

Plani ad planũ inclinatio, acutus est angulus rectis lineis contentus, quæ in vtroque planorum ad idem communis sectionis pũctũ ductæ, rectos ipsi sectioni angulos efficiunt.

7

Planum similiter inclinatum esse ad planum, atq; alterum ad alterum dicitur, cum dicti inclinationum anguli inter se sunt ęquales.

8

Parallela plana, sunt quæ eodem non incidunt, nec concurrunt.

9

Similes figuræ solidæ, sunt quæ æqualibus planis, multitudine æqualibus continetur.

10

Aequales & similes figuræ solidæ sunt, quę similibus planis, multitudine & magnitudine æqualibus continentur.

11

Solidus angulus, est plurium quàm duarũ linearum, quæ se mutuò contingant, nec in eadem sint superficie, ad omnes lineas inclinatio. Aliter.

Aliter.

Solidus angulus, eſt qui pluribus quã duobus planis angulis, in eodem non cõſiſtentibus plano, ſe ad vnum punctum collectis continetur.

12

Pyramis, eſt figura ſolida quæ planis continetur, ab vno plano ad vnum punctũ collecta.

13

Priſma, figura eſt ſolida quę planis continetur, quorum aduerſa duo ſunt & æqualia & ſimilia & parallela, alia verò parallelogramma.

14

Sphęra eſt figura, quæ cõuerſo circum quieſcentem diametrum ſemicirculo continetur, cùm in eundem rurſus locũ reſtitutus fuerit, vnde moueri cœperat.

15

Axis autem ſphæræ, eſt quieſcẽs illa linea circum quam ſemicirculus conuertitur.

16

Centrum verò Sphęrę eſt idem, quod & ſemicirculi.

17

Diameter autem Sphæræ, eſt recta quædã linea per centrum ducta, & vtrinq; à ſphęræ ſuperficie terminata.

Conus

18

Cωnus est figura, quæ conuerſo circũ quieſcens alterum latus eorum quæ rectum angulum continent, orthogωnio triangulo continetur, cùm in eundem rurſus locum illud triangulum reſtitutum fuerit, vnde moueri cœperat. Atq; ſi quieſcens recta linea æqualis ſit alteri, quę circum rectũ angulum conuertitur, rectangulus erit Cωnus: ſin minor, amblygωnius: ſi verò maior, oxygonius.

19

Axis autem Cωni, eſt quieſcens illa linea, circum quam triangulum vertitur.

20

Baſis verò Cωni, circulus eſt, qui à circũducta linea recta deſcribitur.

21

Cylindrus figura eſt, quæ conuerſo circum quieſcens alterum latus eorum quæ rectum angulum continent, parallelogrammo orthogωnio comprehenditur, cum in eũdem rurſus locum reſtitutum fuerit illud parallelogrammũ, vnde moueri cœperat.

22

Axis autem Cylindri eſt quieſcens illa recta linea, circum quam parallelogrammum vertitur.

23

Baſes verò cylindri, ſunt circuli à duobus

ad-

adaersus lateribus quæ circum aguntur, descripti.

24

Similes coni & cylindri, sunt quorũ & axes & basium diametri proportionales sunt.

25

Cubus est figura solida, quæ sex quadratis æqualibus continetur.

26

Tetraedum ex figura, quæ triangulis quatuor æqualibus & æquilateris continetur.

27

Octaedrũ figura est solida, quæ octo triangulis æquilibus & æquilateris continetur.

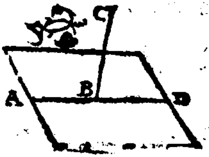
28

Dedecaedrum figura est solida, quæ duodelim pentagonis æqualibus, æquilateris, & æquiangulis continetur.

29

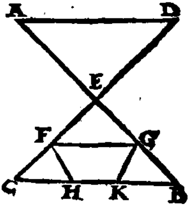
Eicosaedrum figura est solida, quę triangulis viginti æqualibus & æquilateris continetur.

Theorema 1. propositio 1.



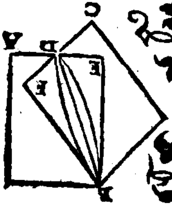
Quædã rectę linee pars in subiecto quidem non est plano, quædam verò in sublimi.

Tehorema 2.Propo-ſitio 2.



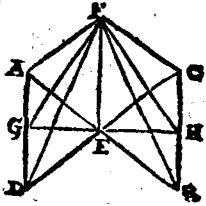
Si duę rectę lineę ſe mutuò ſecét, in vno sũt plano ∴ atque triangulum omne in vno eſt plano.

Theorema 3.Propo-ſitio 3.



Si duo plana ſe mutuò ſecent, communis eorũ ſectio eſt recta linea.

Theorema 4 Propo-ſitio 4.



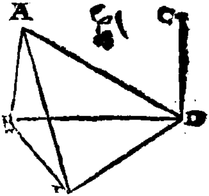
Sj recta linea rectis duabus lineis ſe mutuò ſecantibus, in cõmuni ſectione ad rectos angulos inſiſtat illa ducto etiam per ipſas plano ad angulos rectos erit.

Theorema 5.Pro-poſitio 5.

Si recta linea rectis tribus lineis ſe mutuò tangentibus, incommuni ſectione ad rectos angulos inſiſtat, illæ tres rectę in vno ſunt plano.

Theo-

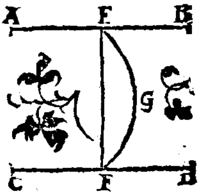
Theorema 6. Propositio 6.



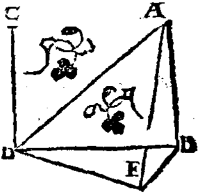
Si duæ rectæ lineæ eidē plano ad rectos sint angulos, parallelæ erunt illæ rectæ lineæ

Theorema 7. Propositio 7.

Si duæ sint parallelæ rectæ lineæ, in quarū vtraque sumpta sint quę libet pūcta, illa linea quę ad hæc puncta adiungitur, in eodem est cū parallelis plano.

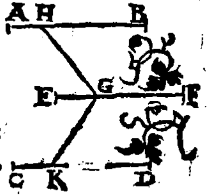


Theorema 8. Propositio 8.



Si duæ sint parallelæ rectæ lineæ, quarum altera ad rectos cuidam plano sit angulos, & reliqua eidē plano ad rectos angulos erit

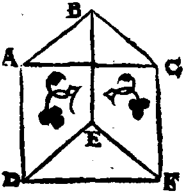
Theorema 9. Prositio 9.



Quæ eidem rectæ lineæ sunt parallelæ, sed nō in eodem cū illa plano, hæ quoque sunt inter se parallelæ.

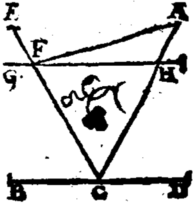
Theorema 10. Propositio 10.

Si duę rectę lineæ se mutuò tangétes ad duas rectas se mutuò tangentes sint parallelę, non autem in eodem plano, illę angulos æquales comprehendent.



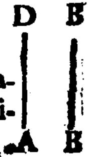
Proble. 1. Propositio 11.

A dato sublimi pũncto, in subiectum planũ perpendicularem rectam lineam ducere.



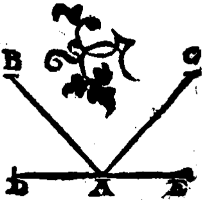
Problema 2. Propositio 12.

Dato plano, à pũcto quod in illo datum est, ad rectos angulos rectam lineam excitare.



Theorema 11. Propositio 13.

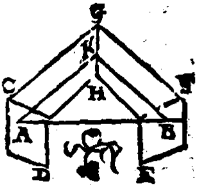
Dato plano, à puncto qđ in illo datum est, duæ rectæ lineæ ad rectos angulos non excitabuntur ad easdem partes.



Theo-

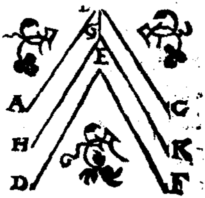
Theorema 12. Propoſitio 14.

Ad quæ plana, eadem recta linea recta eſt, illa sút parallela.



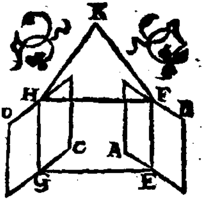
Theorema 13. Propoſitio 15.

Si duæ rectæ lineæ ſe mutuò tangentes ad duas rectas ſe mutuò tangentes ſint parallelæ, non in eodem conſiſtentes plano, parallela ſunt quæ per illas ducuntur plana.



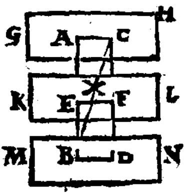
Theorema 14. Propoſitio 16.

Si duo plana parallela plano quopiã ſecentur, communes illorum ſectiones ſunt parallelæ.

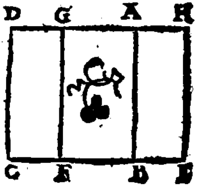


Theorema 15. Propoſitio 17.

Si duæ rectæ lineę parallelis planis ſecẽtur, in eaſdem rationes ſecabũtur.

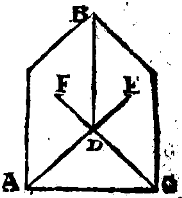


Theorema 16. Propoſitio 18.



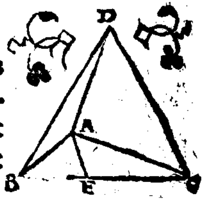
Si recta linea plano cuipiam ad rectos ſit angulos, illa etiam omnia quę per ipſam plana, ad rectos eidem plano angulos erunt.

Theorema 17. Propoſitio 19



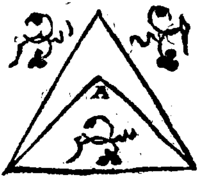
Si duo plana ſe mutuò ſecantia plano cuidā ad rectos ſint angulos, communis etiam illorum ſectio ad rectos eidem plano angulos erit.

Theorema 18. Propoſitio 20.



Si angulus ſolidus planis tribus angulis contineatur, ex his duo quilibet vtut aſſumpti tertio ſunt maiores.

Theorema 19. Propoſitio 21.

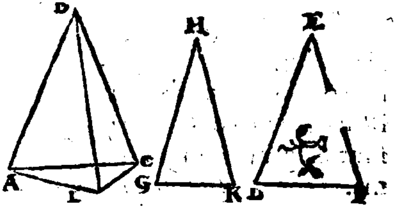


Solidus omnis angulus minoribus continetur, quàm rectis quatuor angulis planis.

Theo-

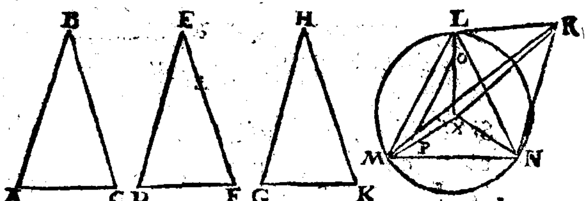
Theorema 20 Proopoſitio 22.

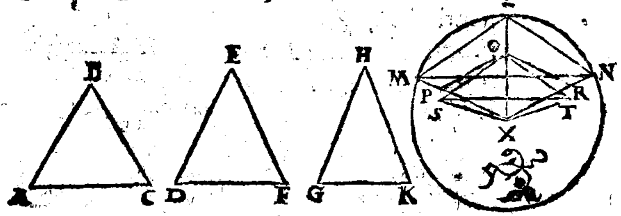
Si plani tres anguli ęqualibus rectis contineãtur lineis, quorũ duo vt libet aſſumpti, tertio ſint maiores, triangulum conſtitui poteſt ex lineis æquales, illas rectas coniungentibus.



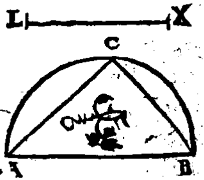
Problema 3. Propoſitio 23.

Ex planis tribus angulis, quorũ duo vt libet aſſumpti tertio ſint maiores, ſolidũ angulum conſtituere. Decet autem illos tres angulos rectis quatuor eſſe minores.

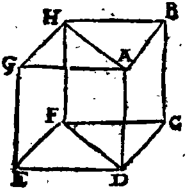




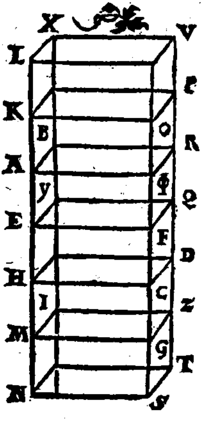
Theorema 21. Propositio 24.



Si solidum parallelis planis contineatur, aduersa illius plana & æqualia sunt & paralle logramma.



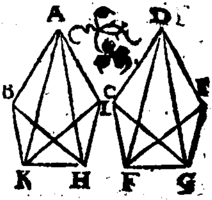
Theorema 22. Propositio 25.



Si solidum parallelis planis contentũ plano secetur aduersis planis parallelo, erit quemadmodum basis ad basim, ita solidum ad solidum.

Proble-

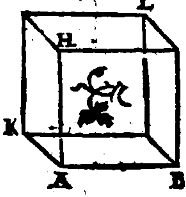
Problema 4. Propositio 26.

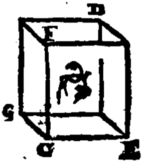


Ad datam rectam lineã eiusque punctum, angulum solidum constituere solido angulo dato æqualem.

Problema 5. Propositio 27.

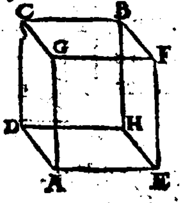
A data recta, dato solido parallelis planis comprehensio simile & similiter positum solidũ parallelis planis contẽtum describere.





Theorema 23. Propositio 28.

Si solidũ parallelis planis comprehensum ductor per aduersorum planorum diagonios plano sectũ sit, illud solidum ab hoc plano bifariam secabitur.



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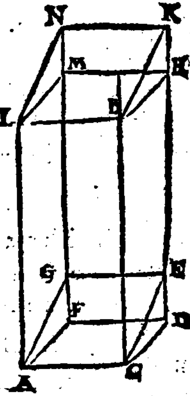
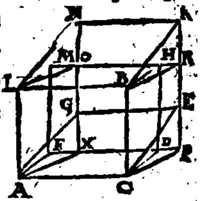
Theorema 24. Propositio 29.



Solida parallelis planis compreheſa, quę ſuper eandem baſim, & in eadē ſunt altitudine, quorum inſiſtētes lineæ in ijſdem collocantur rectis lineis, illa ſūt inter ſe æqualia.

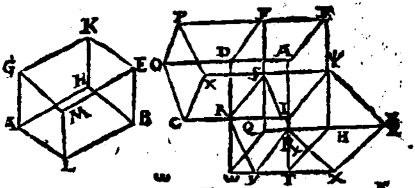
Theorema 25. Propositio 30.

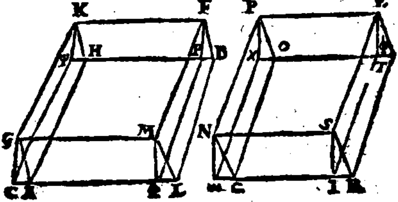
Solida parallelis planis circumſcripta, quæ ſuper eandē baſim & in eadē ſūt altitudine, quorum inſiſtentes lineæ nō in ijſdem reperiuntur rectis lineis, illa ſunt inter ſe æqualia.



Theorema 26. Propositio 31.

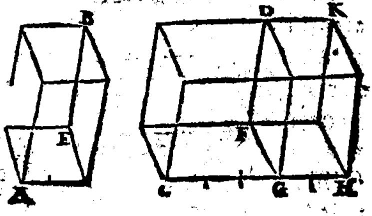
Solida parallelis planis circunſcripta, quæ in in eadem sũt altitudine, æqualia sũt inter se.





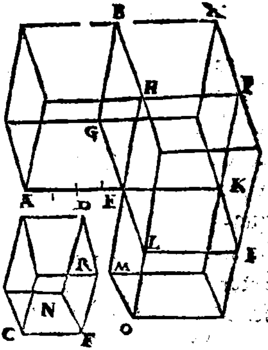
Theorema 27. Propositio 32.

Solida parallelis planis circumscripta quæ eiusdem sunt altitudinis, eam habẽt inter se rationẽ, quam bases.



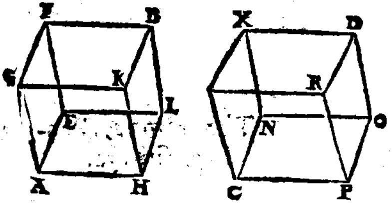
Theor 28. Propositio 33.

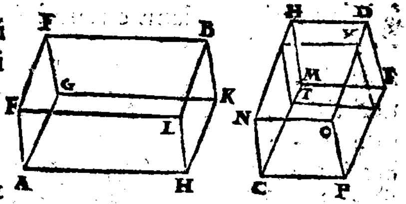
Similia solida parallelis, planis circũscripte habét inter se rationé homologorũ laterum triplicatam

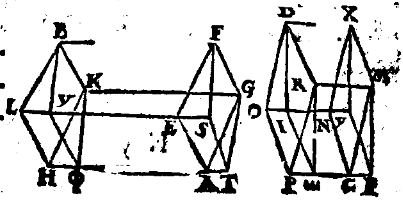


Theor. 29. Propositio 24

Aequalium solidorũ parallelis planis cótétorum bases cũ altitudinibus reciprocantur. Et solida parallelis planis contenta; quorũ



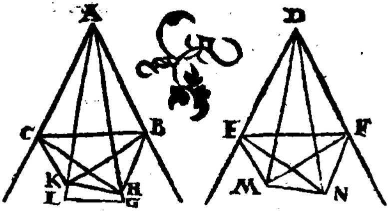




bases cum altitudinibus reciprocantur, illa sunt æqualia.

Theorema 30. Propositio 35.

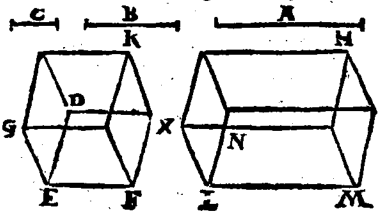
Si duo plani sint anguli æquales, quorum verticibus sublimes rectæ lineæ insistant, quæ cum lineis primò positis angulos contineant æquales, vtrunq; vtriq;, in sublimibus autem lieneis quęlibet sumpta sint puncta, & ab his ad plana in quibus consistunt anguli primum positi, ductæ sint perpendiculares, ab earum verò punctis, quæ in planis signata fuerint, ad angulos primum positos adiunctæ sint rectæ lineę, hæ cũ subli-



mibus æquales angulos comprehendent.

Theorema 31. Propositio 36.

Si rectæ tres lineæ sint proportionales, quòd

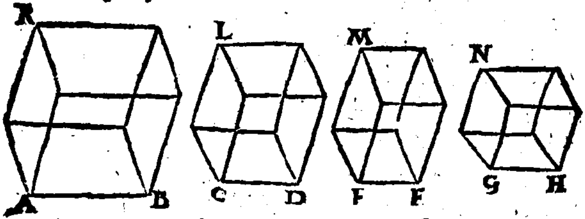


ex

ex his tribus fit solidum parallelis planis contentum, æquale est descripto à media linea solido parallelis planis comprehẽso, quod æquilaterum quidem sit, sed ante dicto æquiangulum.

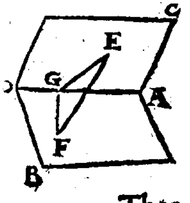
Theorema 32. Propositio 37.

Si rectæ quatuor lineæ sint proportionales, illa quoque solida parallelis planis contenta, quæ ab ipsis lineis & similia & similiter discribuntur, proportionalia erunt. Et si solida parallelis planis comprehensa, quę & similia & similiter describuntur, sint proportionalia, illæ quoque rectæ lineæ proportionales erunt.



Theorema 33. Propositio 38.

Si planum ad planum rectũ sit, & à quodã puncto eorũ quæ in vno sunt planorũ perpendicularis ad alterum ducta sit, illa quæ ducitur perpẽdicularis, in communẽ cadet planorũ sectionẽ.



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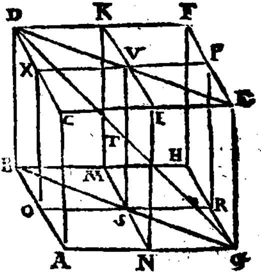
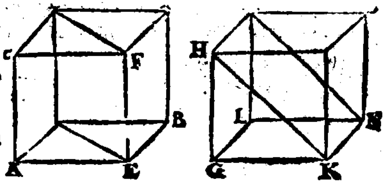
Theorema 34. Propositio 39.

Si in solido parallelis planis circunscripto, aduersorum planorum lateribus bifariam sectis, educta sint per sectiones plana, communis illa planorũ sectio & solidi parallelis plani circumscripti diameter, se mutuò bifariã secant.



Theorema 35. Propositio 40.

Si duo sint æqualis altitudinis prismata, quorũ hoc quidem basim habeat parallelogrammũ, illud verò triangulum, sit autem parallelogrammũ triãguli duplũ, illa prismata erunt æqualia.



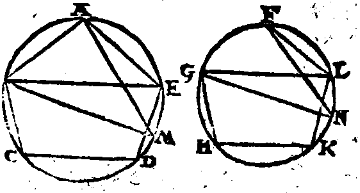
ELEMENTI XI. FINIS.

EVCLIDIS ELEMENTVM DVODECIMVM.

ET SOLIDORVM. secundum.

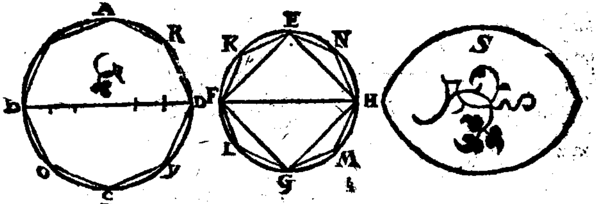
Theorema 1. Propositio 1.

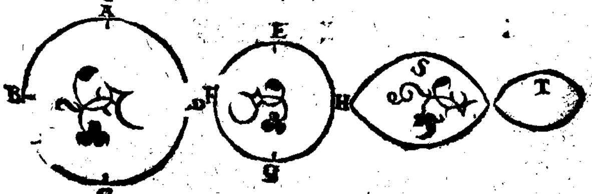
Similia quæ sunt in circulis polygona, rationē habent inter se, quam descripta à diametris quadrata.



Theorema 2. Propositio 2.

Circuli eam inter se rationem habent, quā

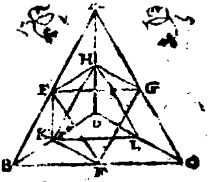




descripta à diametris quadrata.

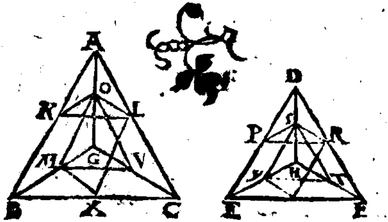
Tueorema 3. Propositio 3.

Omnis pyramis trigωnam habens basim, in duas diuiditur pyramidas non tantũ æquales & similes inter se, sed toti etiam pyramidi similes, quarum trigωnæ sunt bases, atq; in duo prismata ęqualia, quæ duo prismata dimidio pyramidis totius sũt maiora.



Theorema 4. Propositio 4.

Si duæ eiusdem altitudinis pyramides trigωnas habeãt bases, sit aũt illarum vtraque diuisa & in duas pyramidas inter se ęquales totiq; similes, & in duo prisma æqualia, ac eodem modo diuidatur vtraq; pyramidum quæ ex superiore diuisione natæ sunt, idque perpetuò fiat: quemadmodũ se habet vnius pyramidis basis ad alterius pyrãidis basim, ita & omnia quę in vna pyrá-

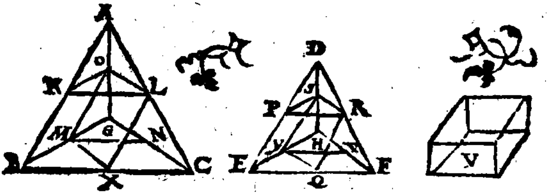


mide

mide prismata, ad omnia quæ in altera pyramide, prismata multitudine æqualia.

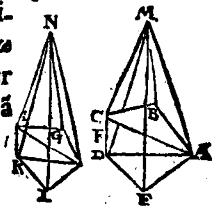
Theorema 5. Propositio 5.

Pyramides eiusdem altitudinis, quarũ trigωnæ sunt bases, eam inter se rationem habent, quàm ipsæ bases.



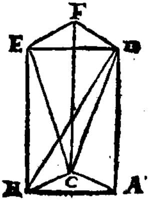
Theorema 6. Propositio 6.

Pyramides eiusdẽ altitudinis, quarum polygωnæ sunt bases, eam inter se rationem habent, quã ipsæ bases.



Theorema 7. Propositio 7.

Omne prisma trigωnã habẽs basim, diuiditur in tres pyramides inter se æquales, quarum trigωnæ sunt bases



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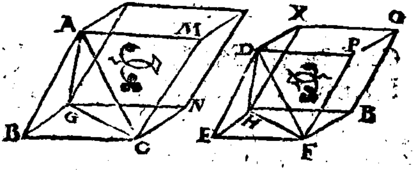
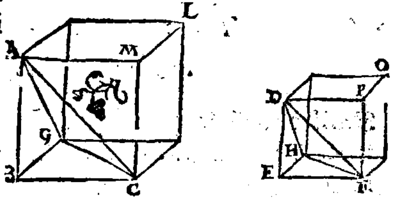
Theorema 8. Propoſitio 8.

Similes pyramides qui trigωnas habent baſes, in triplicata ſũt homologorũ laterũ ratiõe



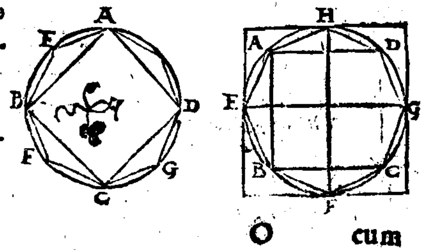
Theorema 9. Propoſitio 9.

Aequaliũ pyramidum & trigωnas baſes habentium reciprocantur baſes cum altitudinibus. Et quarũ pyramidum trigωnas baſes habentium reciprocãtur baſes cũ altitudinibus, illæ ſunt æquales.



Thorema 10. Propoſitio 10.

Omnis cωn⁹ tertia pars eſt cylindri eandẽ

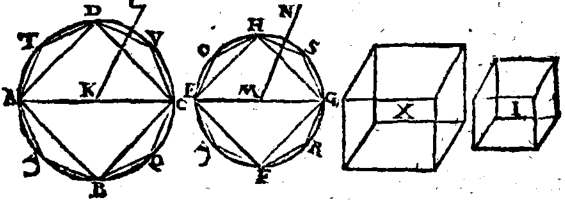


cum

cum ipſo cono baſim habentis, & altitudinem æqualem.

Theorema 11. Propoſitio 11.

Coni & cylindri eiuſdem altitudinis, eam inter ſe rationem habent, quam baſes.



Theorema 12. Propoſitio 12.

Similes coni & cylindri, triplicatã habent inter ſe rationem diametrorum, quæ ſunt in baſibus.

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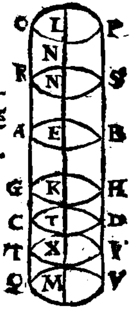
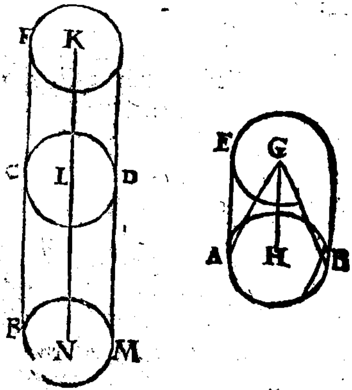
Theorema 13. Propositio 13.



Si cylindrus plano sectus sit aduersis planis parallelo, erit quẽadmodum cylindrus ad cylindrum, ita axis ad axem.

Theorema 14. Propositio 14.

Coni & cylindri qui in æqualibus sunt basibus, eam habent inter se rationem, quam altitudines



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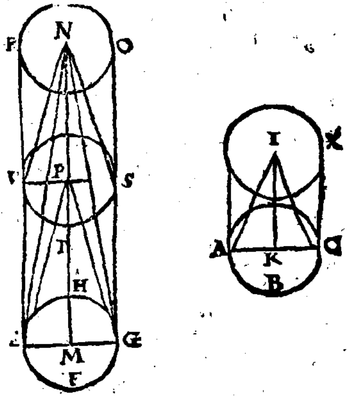
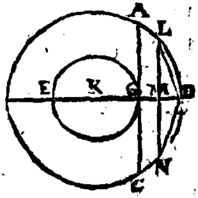
Theorema 15. Propoſitio 15.

Aequalium conorum & cylindrorũ baſes cũ alutitudinib9 reciprocãtur. Et quorum conorũ & cylindrorum baſes cum altitudinibus reciprocantur, illi ſunt æquales.



Theoroma 1. Propoſitio 16.

Duobus circulis circũ idem centrum conſiſtentibus, in maiore circulo polygonum æqualium pariumque laterum inſcribere, quod minorem circulũ non tangat.



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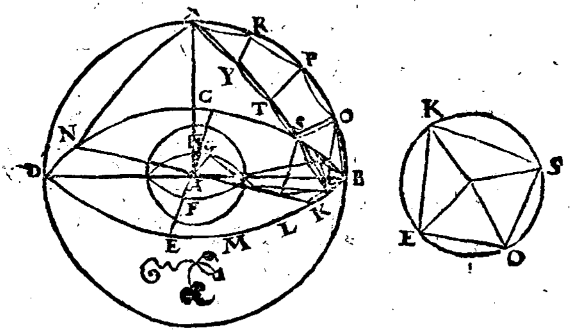
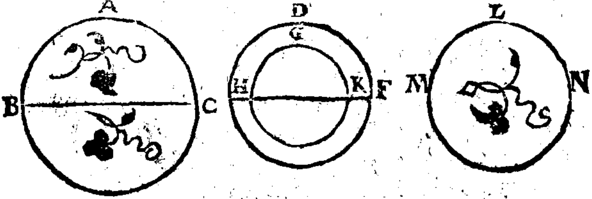
Problema 2. Propoſitio 17.

Duabus ſphæris circum idem centrum conſiſtentibus, in maiorem ſphęra ſolidum polyedrum inſcribere, quod minoris ſphæræ ſuperficiem non tangat.



Theorema 16. Propoſitio 18.

Sphæræ inter ſe rationem habent ſuarum diametrorum triplicatam.

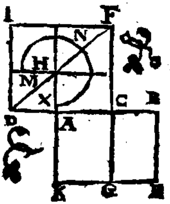


ELEMENTI XII. FINIS.

EVCLIDIS ELEMENTVM DECIMVMTERTIVM, ET SOLIDORVM TERTIVM.

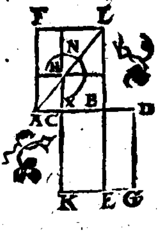
Theorema 1. Propositio 1.

Si recta linea per extremã & mediã rationem secta sit, maius segmentum quod totius lineæ dimidium assumpserit, quintuplum potest eius quadrati, quod à totius dimidia describitur.



Theorema 2. Propositio 2.

Si recta linea sui ipsius segmenti quintuplum possit, & dupla segmenti huius linea per extremã & mediam rationẽ secetur maius segmentũ reliqua pars est lineæ primum positæ.



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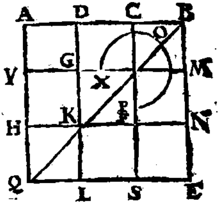
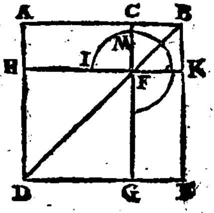
Theorema 3. Propositio 3.



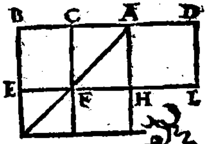
Si recta linea per extremá & mediá rationé secta sit, minus segmétú quod maioris segmenti dimidium assumpserit, quintuplum potest eius, quòd à maioris segméti dimidio describitur, quadrati.

Theorema 4. Propositio 4.



Si recta linea per extremam & mediam rationé secta sit, quod à tota, quodque à minore segméto simul vtraq; quadrata, tripla sunt eius, quod à maiore segméto describitur, quadrati.

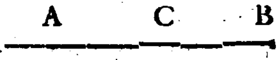
Theorema 5. Propositio 5.



Si ad rectam lineam, quæ per extremam & mediam rationem secetur, adiuncta sit altera segmento maiori equalis, tota hæc linea recta per extremam & mediam ration cta est, estque maius segmentum linea primùm posita.

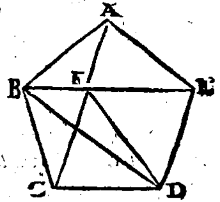
Theorema 6. Propositio 6.

Si recta linea ῥητὴ siue rationalis, per extremam & mediam rationem secta sit, vtrunque segmentorum ἄλογος siue irrationalis est linea, quæ dicitur Residuum.



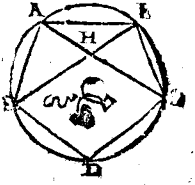
Theorema 7. Propositio 7.

Si pentagωni æquilateri tres sint æquales anguli, siue quā deinceps siue qui non deinceps sequuntur, illud pentagωnum erit æquiangulum.



Theorema 8. Propositio 8.

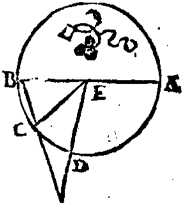
Si pentagωni æquilateri & æquianguli duos qui deinceps sequuntur angulos rectæ subtendāt lineæ, illæ per extremā & mediam rationem se mutuò secant, earumq; maiora segmenta, ipsius pentagωni lateri sunt æ-



Theo-

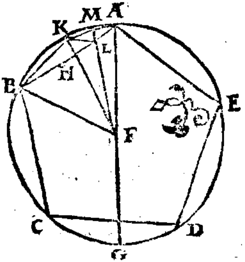
Theorema 9 Propoſitio 9.

Si latus hexagωni & latus decagωni eidem circulo inſcriptorum compoſita ſint, tota recta linea per extremam & mediam rationem ſecta eſt, eiuſque ſegmẽtum maius, eſt hexagωni latus.



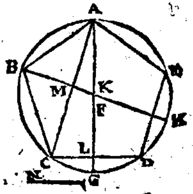
Theorema 10. Propoſitio 10.

Si circulo pẽtagωnum ęquilaterum inſcriptum ſit, pentagωini latꝰ poteſt & latus hexagωni & latus decagωni eidem circulo inſcriptorum.



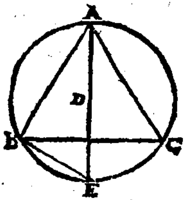
Therema 11. Propoſitio 11.

Si in circulo ῥητὴν habente diametrum, inſcriptum ſit pentagωnum æquilaterum, pentagωni latus irrationalis eſt linea quæ vocatur Minor.



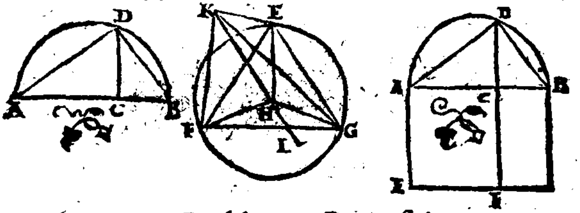
Theorema 12. Propoſitio 12.

Si in circulo inſcriptum ſit triangulum æquilaterum, huius trianguli latus potentia triplum eſt eius lineæ, quæ ex circuli centro ducitur.



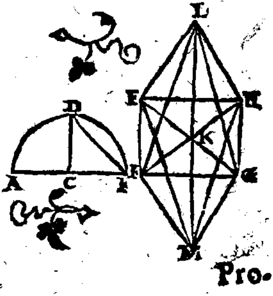
Problema 1. Propoſitio 13.

Pyramidem conſtituere, & data ſphærę cōplecti, atque docere illius ſphæræ diametrum potentia ſeſquialteram eſſe lateris ipſius pyramidis.



Problema 2. Propoſitio 14.

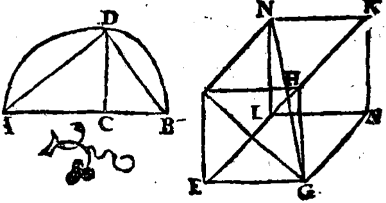
Octaedrum conſtituere, eaq; ſphęra qua pyramidē complecti, atque ꝓbare illius ſphæræ diametrū potentia duplā eſſe lateris ipſi' octaedri.



Pro-

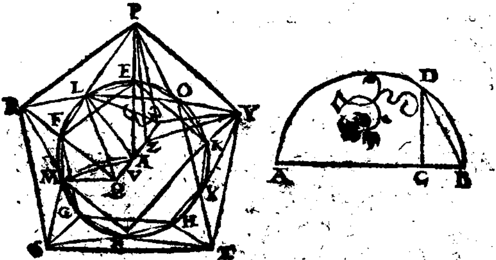
Problema 3. Propositio 15.

Cubum constituere, eaque sphæra qua & superiores figuras complecti, atque docere illius sphæræ diametrum potẽtia triplã esse lateris ipsius cubi.



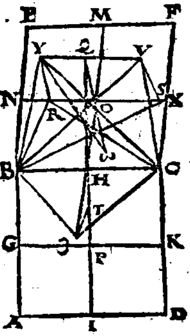
Problema 4. Propositio 16.

Icosaedrum constituere, eademque sphæra qua & antedictas figuras complecti, atque probare, Icosaedri latus irrationalem esse lineam, quæ vocatur Minor.



Pro-

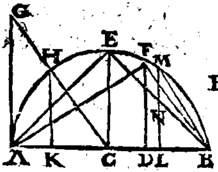
Problema 5. Propositio 17.

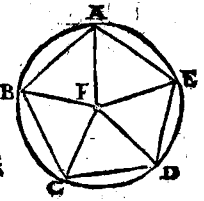


Dodecaedrũ constituere, eademque sphæra qua & & antedictas fuguras cõplecti, atque probare dodecaedri latus irrationalem esse lineam, quæ vocatur Residuum.

Theorema 6. Propositio 18.

quinq; figurarum latera pro-





ponere, & inter se comparare.

SCHOLIVM.

Aio verò, præter dictas quinque figuras non posse aliam constitui figuram solidã, quæ planis & æquilateris & æquiangulis cõtineatur, inter se æqualibus. Non enim ex duobus triãgulis, sed neq; ex alijs duab⁹ figuris solidus cõstituetur angulus

Sed

Sed ex tribus triãgulis, constat Pyramidis angulus
Ex quatuor autem, Octaëdri
Ex quinque verò, Icosaëdri.

Nam ex triangulis, sex & æquilateris & æquiangulis ad idem punctum coeuntibus, non fiet angulus solidus. Cùm enim trianguli æquilateri angulus, recti vnius bessem contineat, erunt eiusmodi sex anguli rectis quatuor æquales. Quod fieri nõ potest. Nam solidus omnis angulus minoribus quã rectis quatuor angulis continetur, per 21.11.

Ob easdem sanè causas, neque ex pluribus quã planis sex eiusmdoi angulis solidus constat.

Sed ex tribus quadratis, Cubi angulus continetur.

Ex quinque, nullus potest. Rursus enim recti quatuor erunt.

Ex tribus autẽ pentangonis æquilateris & æquiangulis, Dodecaëdri angulus continetur.

Sed ex quatuor, nullus potest. Cùm enim pentagoni æquilateri angulus rectus sit, et quinta recti pars erunt quatuor anguli rectis quatuor maiores. Quod fieri nequit. Nec sanè ex alijs polygonis figuris solidus angulus continebitur, quod hinc quoque absurdum sequatur. Quamobrem perspicuum est, præter dictas quinque figuras aliã figuram solidam non posse constitui, quæ ex planis æquilateris & æquiangulis contineatur.

ELEMENTI XIII. FINIS

EVCLIDIS ELEMENTVM DECIMVM QVARTVM, VT qui dam arbitrantur, vt alij verò, Hypsiclis Alexandrini, de quinque corporibus.

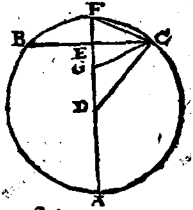
LIBER PRIMVS.

BAsilides Tyrius, Protarche, Alexandriam profectus, patrique nostro ob disciplinæ societatem commendatus, longißimo peregrinationis tempore cum eo versatus est. Cùmque disserent aliquando de scripta ab Apollonio comparatione Dodecaëdri & Icosaëdri eidem sphæræ inscriptorum, quam hæc inter se habeant rationem, censuerunt ea non rectè tradidisse Apollonium: quæ à se emendata, vt de patre audire erat, literis prodiderunt. Ego autem postea incidi in alterum librum ab Apollonio editum, qui demonstrationem accuratè complecteretur de re proposita, ex eiusq́; problematis indagatione magnam equidem cepi voluptatem. Illud certè ab omnibus perspici potest, quod scripsit Apollonius, cùm sit in omnium manibus. Quod autem diligenti, quantum conijcere licet, studio nos postea scrip-

scripsisse videmur, id monimentis consignatum tibi nuncupandum duximus, vt qui feliciter cùm in omnibus disciplinis tum vel maximè in Geometria versatus, scitè ac prudēter iudices ea quæ dicturi sumus ob eam verò, quæ tibi cum patre fuit, vitæ consuetudinem, quaq́ nos complecteris, beneuolentiā, tractationē ipsam libenter audias. Sed iam tempus est, vt præmio modum facientes, hanc syntaxim aggrediamur.

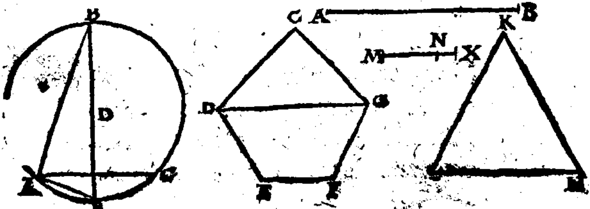
Theorema 1. Proopositio 1.

Perpendicularis linea, quæ ex circuli cuiuspiam centro in latus pentagωni ipsi circulo inscripti ducitur, dimidia est vtriusq; simul lineæ, & eius, quæ ex centro & lateris decagωni in eodē circulo inscripti.



Theorema 2. Propositio 2.

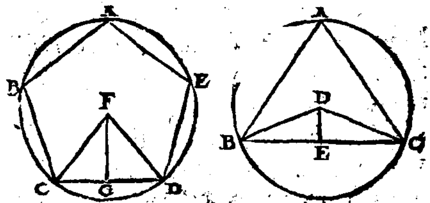
Idem circulus cōprehendit & dωdecaedri pentagωnum & icosaedri triangulum, eidē sphæræ inscriptorum.



Theo-

Theorama 3 Propositio 3.

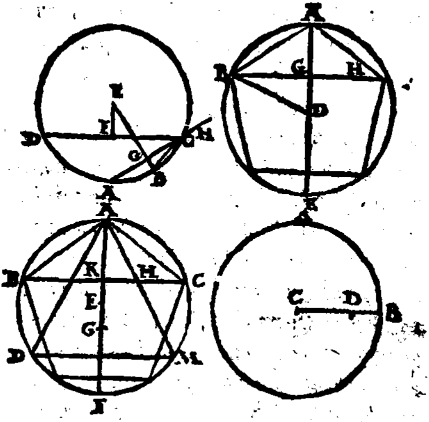
Si pentagωno & æquilatero & æquiangulo circumſcriptus ſit circulus ex cuius centro in vnum pentagωni latus dicta ſit perpendicularis: quod vno laterum & perpédiculari trigeſies có tinetur, illud æquale eſt dωdecaedri ſuperficiei.



Theorema 4. Propoſitio 4.

Hoc perſpicuum cùm ſit probandum eſt, quemadmodum ſe habet dωdecaedri ſuperficies ad icoſaedri ſuperficiem ita ſe habere cubi latus ad icoſaedri latus.

Cubi



Cubilatus.

E————

Dodecaedri.

F————

Icosaedri.

G————

SCHOLIVM

Nunc autem probandum est, quemadmodum se habet cubi latus ad icosaëdri latus, ita se habere solidum dodecaëdri ad icosaëdri solidum: Cùm enim æquales circuli comprehendat & dodecaëdri pentagonum & icosaëdri triangulum, eidẽ sphæræ in-

scriptorum in sphæris autem æquales circuli æquali interuallo distent à centro (siquidem perpendiculares à sphæræ centro ad circulorum plana ductæ & æquales sunt, & ad circulorum centra cadunt) idcirco lineæ, hoc est perpendiculares, quæ à sphæræ cētro ducuntur ad centrum circuli comprehendētis et triangulum Icosaëdri, & pentagonum dodecaëdri sūt æquales. Sūt igitur æqualis altitudinis Pyramides, quæ bases habent ipsa dodecaëdri pentagona, et quæ Icosaëdri triangula. At æqualis altitudinis pyramides rationem inter se habent eam quam bases, ex 5. & 6 11. Quemadmodum igitur pentagonū ad triangulum, ita pyramis, cuius basis quidem est dodecaëdri pentagonum, vertex autem sphæræ centrum ad pyramida, cuius basis quidem est Icosaëdri triangulum, vertex autem sphæræ centrum. Quamobrem vt se habent duodecim pentagona ad viginti triāgula, ita duodecim pyramides, quorum petagona sint bases, ad viginti pyramidas, quæ trigonas habeant bases At pentagona duodecim sunt dodecaëdri superficies, viginti autem triangula, Icosaëdri. Est igitur vt dodecaëdri superficies ad Icosaëdri superficiem, ita duodecim pyramides quæ pentagonas habeant bases, ad viginti pyramidas, quarum trigonæ sunt bases. Sunt autem duodecim quidem pyramides, quæ pentagonas habeant bases, solidū dodecaëdri: viginti autem pyramides, quæ trigonas habeant bases, Icosaëdri solidum. Quare ex 11. 5. vt dodecaëdri superficies ad Icosaëdri superficiem, ita

soli-

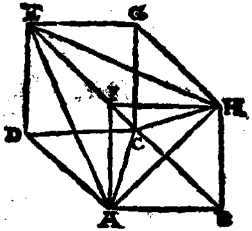
ſolidum dodecaëdri ad Icoſaëdri ſolidum. Vt autem dodecaëdri ſuperficies ad Icoſaëdri ſuperficiem, ita probatum eſt cubi latus ad Icoſaëdri latus. Quemadmodum igitur cubi latus ad Icoſaëdri latus, ita ſe habet ſolidum dodecaëdri ad Icoſaëdri ſolidum.

EVCLIDIS ELEMENTVM DECIMVM QVINTVM, ET Solidorum quintum, vt nonnulli putant, vt autem alij, Hypsiclis Alexandrini, de quinque corporibus.

LIBER II.

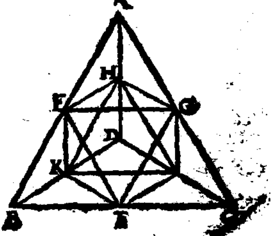
Problema 1. Propositio 1.

In dato cubo pyramida inscribere.



Problema 2. Propositio 2.

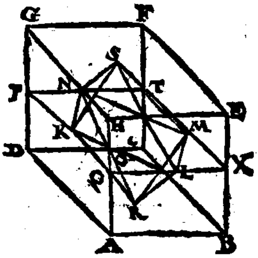
In data pyramide octaedrum inscribere.



Pro-

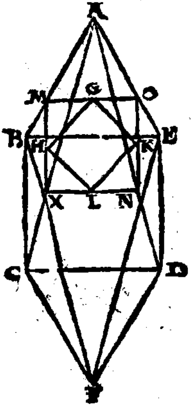
Problema 3. Propositio 3.

In dato cubo octaedrum inscribere.



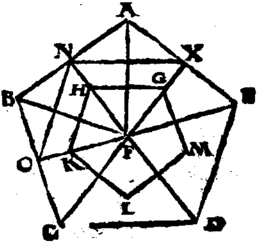
Problẽma 4. Propositio 4.

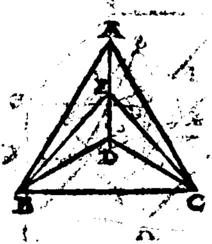
In dato octaedro cubũ inscribere

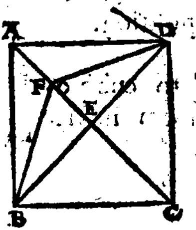


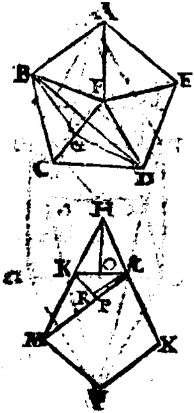
Problema 5. Propositio 5.

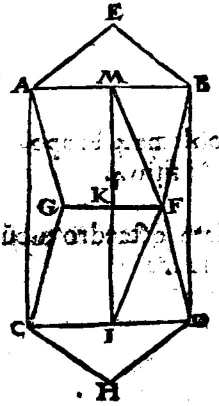
In dato Icosaedro dodecaedrum inscribere.











SCHO.

SCHOLIVM.

Meminisse decet, si quis nos roget, quot Icosaëdrũ habeat latera, ita respondendum esse: Patet Icosaedrum viginti contineri triãgulis, quodlibet verò triangulum rectis tribus constare lineis. Quare multiplicanda sunt nobis viginti triangula in trianguli vnius latera, fiuntq́; sexaginta, quorum dimidium est triginta. Ad eundem modum & in dodecaedro. Cũ enim rursus duodecim pentagona dodecaedrum cõprehendant itemq́; pentagonum quoduis rectis quinque constet lineis, quinq; duodecies multiplicamus, fiunt sexaginta, quorum rursus dimidium est triginta. Sed cur dimidiũ capimus? Quoniã vnumquodq; latus siue sit trianguli, siue pentagoni, siue quadrati vt in cubo, iterato sumitur. Similiter autem eadem via & in cubo & in pyramide & in octaedro latera inuenies. Quòd si itẽ velis singularum quoq; figurarum angulos reperire, facta eadem multiplicatione numerum procreatum partire in numerum planorũ, quæ vnum solidum angulum includũt: vt quoniã triãgula quinq; vnum Icosaëdri angulum continet, partire 60. in quinque, nascũtur duodecim anguli Icosaedri. In dodecaedro autẽ tria pẽtagona angulum comprehendunt, partire ergo 60. in tria, & habebis dodecaedri angulos viginti. Atq; simili ratione in reliquis figuris angulos reperies.

Finis Elementorum Euclidis.