EVCLIDIS
ELEMENTORVM
LIBRI XV.

QVIBVS, CVM AD OM-
NEM MATHEMATICAE SCIEN-
TIAE PARTEM, TVM AD QVAMLI-
bet Geometriæ tractationem
facilis comparatur
aditus.

COLONIAE
Apud Gosuinum Cholinum
M. D. C.
Cum Gratia & Priuilegio.

AD CANDIDVM LECTOREM ST. GRACILIS praefatio.

PERMAGNI referre semper existimaui, Lector beneuole, quantum quisque studij & diligentiae ad percipienda scientiarum elementa adhibeat, quibus non satis cognitis, aut perperam intellectis, si vel digitum progredi tentes, erroris caliginem animis offundas, non veritatis lucem rebus obscuris adferas. Sed principiorum quanta sint in disciplinis momenta, haud facile credat, qui rerum naturam ipsa specie, non viribus metiatur. Vt enim corporum, quae oriuntur & intereunt, vilissima tenuissimaque videntur in[...] ita rerum aeternarum & admirabilium, quibus nobilissimae artes continentur, elementa ad speciem sunt exilia, ad vires & facultatem quàm maxima. Quis non videt ex fici tantulo grano, vt ait Tullius, aut ex acino vinaceo, aut ex caeterarum frugum aut stirpium minutissimis seminibus tantos truncos ramosque procreari? Nam Mathematicorum initia illa quidem dictu audituque perexigua, quantam theorematum syluam nobis pe-

perierunt? Ex quo intelligi potest, vt in ipsis seminibus, sic et in artium principijs inesse vim earum rerum, quæ ex his progignuntur. Præclarè igitur Aristoteles, vt alia permulta, μέγιστον ἴσως ἀρχὴ παντός, ὅσῳ κράτιστον τῇ δυνάμει, τοσούτῳ μικρότατον, ὂν τῷ μεγέθει χαλεπόν ἐστιν ὀφθῆναι. Quocirca committendum non est, vt non bene prouisa & dilıgēter explorata scientiarum principia, quibus propositarum quarumq́ rerum veritas sit demonstranda, vel constituas, vel constituta approbes. Cauendum etiam, vt ne tantulum quidem fallaci & captiosa interpretatione turpiter deceptus, à vera principiorū ratione temerè deflectas. Nam qui initio fortè aberrauerit, is vt tandē in maximis versetur erroribus, necesse est: cū ex vno erroris capite densiores sensim tenebræ rebus clarißimis obducātur. Quid tam varias veterum physiologorum sententias non modo cū rerum veritate pugnantes, sed vehementer etiam inter se dissentiētes nobis inuexit? Equidem haud scio, fuerit ne vlla potior tanti dißidij causa, quā quod ex principijs partim falsis, partim non cōsentaneis ductas rationes probando adhiberent. Fit enim plerunque, vt qui non rectè de artium rerumq́ elementis sentiunt, ad præfinitas quasdam opiniones suas omnia reuocari studeant. Pythagorei, vt meminit Aristoteles, cum denarij numeri summam perfectionem cœlo tribuerent, nec plures tamen quam nouem sphæras cernerent, decimam affingere ausi sunt terræ aduer-

ſam, quam ἀντίχθονα appallarunt. Illi enim vniuerſitatis rerumq́; ſingularum naturam ex numeris ſeu principijs æſtimantes, ea protulerunt quæ φαινομένοις congruere nuſquam ſunt cognita. Nam ridicula Democriti, Anaximenis, Meliſſi, Anaxagoræ, Anaximandri, & reliquorum id genus phyſiologorum ſomnia, ex falſis illa quidẽ orta naturæ principijs, ſed ad Mathematicum nihil aut parum ſpectantia, ſciens prætereo. Nonnullos attingam, qui repetitis altiùs vel aliter ac decuit poſitis rerum initijs, cum phyſicis multa turbarunt; tum Mathematicos oppugnatione principiorum peßimè mulctarunt. Ex planis figuris corpora conſtituit Timæus: Geometrarum hic quidẽ principia cuniculis oppugnantur. Nam & ſuperficies ſeu extremitates craßitudinem habebunt & lineæ latitudinem: denique puncta non erunt indiuidua, ſed linearum partes. Prædicant Democritus atque Leucippus illas atomos ſuas, & indiuidua corpuſcula. Concedit Xenocrates impartibiles quaſdam magnitudines. Hic verò Geometriæ fundamenta apertè petuntur, & funditus euertuntur: quibus dirutis nihil equidẽ aliud video reſtare, quàm vt amplißima Mathematicorum theatra repente concidant. Iacebunt ergò, ſi dijs placet, tot præclara Geometrarum de aſymmetris & alogis magnitudinibus theoremata. Quid enim cauſa dicas, cur indiuidua linea hanc quidem metiatur, illam verò metiri non queat? Siquidem quod minimũ in vnoquoq;

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genere reperitur, id communis omnium mensura esse solet. Innumerabilia profectò sunt illa, quæ ex falsis eiusmodi decretis absurda consequũtur, & horum permulta quidem Mathematicus, sed longè plura colligit Physicus. Quid varia ψευδογραφημάτων genera commemorem, quæ ex hoc vno fonte tam longè lateq; diffusa fluxisse videntur? Notißimus est Antiphontis terragonismus, qui Geometrarum & ipse principia non parum labefecit, cùm rectæ lineæ curuam posuit æqualem. Lõgũ esset mihi singula percensere præsertim ad alia properanti: Hoc ergo certum, fixum, & in perpetuum ratũ esse oportet, quod sapienter monet Aristoteles, σπουδαστέον ὅπως ὁρισθῶσι καλῶς αἱ ἀρχαί· μεγάλην γὰρ ἔχουσι ῥοπὴν πρὸς ἑπόμενα. Nam principijs illa congruere debent, quæ sequũtur. Quòd si tantum perspicitur in istis exilioribus Geometriæ initijs, quæ pũcto, linea, superficiè definiuntur, momentum, vt ne hæc quidem sine summo impendentis ruinæ periculo conuelli aut oppugnari poßint, quanta quæso vis putanda est huius στοιχειώσεως, quam collatis tot præstantißimorum artificum inuentis, mira quadam ordinis solertia contexuit Euclides, vniuersa Matheseos elementa complexu suo coercentẽ. Vt igitur omnibus rebus instructior & paratior quisq; ad hoc studium libentius accedat, & singula vel minutißima exactius secum reputet atque perdiscat, operæ precium censui, in primo institutionis aditu vestibuloq; præcipua quædem capita quibus tota

ferè

ferè Mathematicæ sciẽtiæ ratio intelligatur, breuiter explicare: tum ea quæ sunt Geometriæ propria, diligẽter persequi: Euclidis deniq; in extruenda hac στοιχειώσει cõsiliũ sedulò ac fideliter exponere. Quæ ferè omnia ex Aristotelis potißimum ducta fõtibus, nemini inuisa fore cõfido, qui modo ingenuũ animi cãdorem ad legendũ attulerit. Ac de Mathematicæ diuisione primum dicamus.

Mathematicæ in primis scientiæ studiosos fuisse Pythagoreos, non modò historicorum, sed etiam philosophorum libri declarant. His ergo placuit, vt in partes quatuor vniuersum distribuatur Mathematicæ scientiæ genus, quarũ duas περὶ τὸ ποσὸν, reliquas περὶ τὸ πηλίκον versari statuerunt. Nam & τὸ ποσὸν vel sine vlla comparatione ipsum per se cognosci, vel certa quadam ratione cõparatum spectari: in illo Arithmeticam, in hoc versari Musicam: & τὸ πηλίκον partim quiescere, partim moueri quidem: Illud Geometriæ propositum esse: quod verò sua sponte motu cietur, Astronomiæ. Sed ne quis falsò putet, Mathematicam scientiam, quòd in vtroque quanti genere cernitur, idcirco inanem videri (si quidem non solum magnitudinis diuisio sed etiam multitudinis accretio infinitè progredi potest) meminisse decet, τὸ πηλίκον καὶ τὸ ποσὸν, quæ subiecto Mathematicæ generi imposita sunt à Pythagoreis nomina, non cuiuscunque modi quantitatem significare, sed eam demum, quæ tum multitudine tum magnitudine sit defi-

pita, & suis circumscripta terminis. Quis enim vllam infiniti sciētiam defendat? Hoc scitum est, quod non semel docet Aristoteles, infinitum ne cogitatione quidem complecti quenquam posse. Itaque ex infinita multitudinis & magnitudinis δυνάμει finitā hac sciētia decerpit & amplectitur naturā, quam tractet, & in qua versetur. Nā de vulgari Geometrarum cōsuetudine quid sentiendum sit, cum data interdum magnitudine infinita haud fabricātur aliquid, aut proprias generis subiecti affectiones exquirunt, disertè monet Aristoteles, οὐδὲ νῦν (de Mathematicis) loquens δέονται τοῦ ἀπείρου, οὐδὲ χρῶνται, ἀλλὰ μόνον εἶναι ὅσην ἂν βούλωνται, πεπερασμένην. Quamobrem disputatio ea qua infinitum refellitur, Mathematicorum decretis rationibusq; non aduersatur, nec eorum apodixes labefacit. Etenim tali infinito opus illis nequaquam est, quod exitu nullo peragrari poßit, nec talem ponunt infinitam magnitudinem: sed quātamcunq; velit aliquis effingere, ea vt suppetat, infinitam pręcipiunt. Quin etiam non modò immensa magnitudine opus non habent Mathematici, sed ne maxima quidem: cum instar maximę minima quęque in partes totidem pari ratione diuidi queat. Alteram Mathematicę diuisionem attulit Geminus, vir (quantum ex Proclo conijcere licet) μαθημάτων laude clarißimus. Eā, quę superiore plenior & accuratior fortè visa est, cum doctißimè pertractarit sua in decimum Euclidis

prӕfapræfatione P. Montaureus vir senatorius, & regiæ bibliothecæ præfectus, leuiter attingam. Nam ex duobus rerum velut summis generibus τῶν νοητῶν καὶ τῶν αἰσθητῶν, quæ res sub intelligentiã cadũt, Arithmeticæ & Geometriæ attribuit Geminus: quæ verò in sensus incurrunt, Astrologiæ, Musicæ Supputatrici, Opticæ, Geodesiæ & Mechanicæ adiudicauit. Ad hanc certè diuisionem spectasse videtur Aristoteles, cum Astrologiam Opticã, harmonicã φυσικώτερας τῶν μαθημάτων nominat, vt quæ naturalibus & Mathematicis interiectæ sint, ac velut ex vtriusque mixtæ disciplinæ: Siquidem genera subiecta à Physicis mutuantur, causas verò in demonstrationibus ex superiore aliqua sciẽtia repetũt. Id quod Aristoteles ipse apertissimè testatur, ἐνταῦθα γὰρ, φησὶ, τὸ μὲν ὅτι, τῶν αἰσθητικῶν εἰδέναι τὸ διότι τῶν μαθηματικῶν. Sequitur, vt quid Mathematica conueniat cum Physica et prima Philolosophia: quid ipsa ab vtraque differat, paucis ostendamus. Illud quidem omnium commune est, quod in veri contemplatione sunt posita, ob idque θεωρητικαὶ Græcis dicuntur. Nam cùm διάνοια siue ratio & mẽs omnis sit vel πρακτικὴ vel θεωρητικὴ, totidem scientiarum sint genera necesse est. Quòd si Physica, Mathematica, & prima Philosophia, nec in agendo, nec in efficiendo sunt occupatæ, hoc certè perspicuum est, eas omnes in cognitione contemplationeque necessariò versari. Cùm enim rerum non modò agendarum, sed etiam efficiendarum *principia in agente vel efficiente consistant, illarum quidem προαίρεσις, harum autem vel mens vel ars, vel vis quædam & facultas: rerum profectò naturalium. Mathematicarum, atque diuinarum principia in rebus ipsis, non in philosophis inclusa latent. Atque hæc vna in omnes valet ratio, quæ θεωρητικὰς esse colligat. Iam verò Mathematica separatim cum Physica congruit, quòd vtraque versatur in cognitione formarum corpori naturali inhærentium. Nam Mathematicus plana, solida, longitudines & puncta contemplatur, quæ omnia in corpore naturali à naturali quoque philosopho tractantur. Mathematica itē & prima philosophia hoc inter se propriè conueniunt, quòd cognitionem vtraque persequitur formarum, quoad immobiles, & cōcretione materiæ sunt liberæ. Nam tametsi Mathematicæ formæ reuera per se non cohærent, cogitatione tamen à materia & motu separantur, οὐδὲ γίνεται ψεῦδος χωριζόντων. vt ait Aristoteles. De cognatione & societate breuiter diximus, iam quid intersit videamus. Vnaquaq; Mathematicarum certum quoddam rerum genus propositum habet, in quo versetur, vt Geometria quantitatem & continuationem aliorum in vnā partem, aliorum in duas, quorūdam in tres, eorumq; quatenus quanta sunt & continua, affectiones cognoscit. Prima autē philosophia, cùm sit omniū communis, vniuersum Entis genus quæq; ei accidunt & conueniunt hoc ipso quòd est, considerat.*

Ad

Adhæc, Mathematica eam modò naturam amplectitur, quæ quanquam non mouetur, separari tamen seiungiq́ nisi mente & cogitationn à materia non potest, ob eamq́ causam ἐξ ἀφαιρέσεως dici consueuit. Sed Prima philosophia in iis versatur, quæ & seiuncta, & æterna, & ab omni motu per se soluta sunt ac libera. Cæterùm Physica & Mathematica quanquam subiecto discrepare non videtur, modo tamen rationeq́ differũt cognitionis & contemplationis, vnde dissimilitudo quoq́ scientiarum sequitur. Etenim Mathematicæ species nihil reuera sunt aliud, quàm corporis naturalis extremitates, quas cogitatione ab omni motu & materia sepatatas Mathematicus cõtemplatur: sed easdem cõsectatur Physicorũ ars, quatenus cum materia comprehensa sunt, & corpora motui obnoxia circumscribunt. Ex quo fit, vt quæcunque in Mathematicis incommoditates accidunt, eadem etiam in naturalibus rebus videantur accidere, nõ autem vicissim. Multa enim in naturalibus sequuntur incommoda, quæ nihil ad Mathematicum attinent, διὰ τὸ, inquit Aristoteles, τὰ μὲν δ᾽ἐξ ἀφαιρέσεως, λέγεται, τά μαθηματικὰ, τὰ δὲ φυσικά ἐκ προσθέσεως. Siquidem res cum materia deuinctas cõtemplatur Physicus: Mathematicus verò rem cognoscit circumscriptis iis omnibus, quæ sensu percipiũtur, vt grauitate, leuitate, duritie, mollitie & præterea calore, frigore, aliisq́ contrariorum paribus, quæ sub sensum subiecta sunt: tãtum autem relinquit

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quantitatem & continuum, Itaque Mathematicorũ ars in ijs quæ immobilia sunt cernitur (τὰ γὰρ μαθηματικὰ τῶν ὄντων ἄνευ κινήσεώς ἐστιν ἔξω τῶν περὶ τὴν ἀστρολογίαν) quæ verò in naturæ obscuritate posita est, res quidẽ quæ nec separari nec motu vacare possunt contemplatur. Id quod in vtroque scientiæ genere perspicuum esse potest, siue res subiectas definias, siue proprietates earum demõstres. Etenim numerus, linea, figura, rectũ, inflexum, æquale rotundũ. vniuersa denique Mathematicus quæ tractat & profitetur absque motu explicari docerique possunt: χοριστὰ γὰρ τῇ νοήσει κινήσεώς ἐστι: Physica autẽ sine monitione species nequaquã possunt intelligi. Quis enim hominis, plãtæ, ignis, ossium, carnis naturam & proprietates sine motu, qui materiam sequitur, perspiciat? Siquidem tãtisper substantia quæque naturalis cõstare dici solet, quoad opus & munus suũ agendo patiendoque tueri ac sustinere valeat, quæ certè amissa δυνάμει, ne nomẽ quidem nisi ὁμωνύμως retinet. Sed Mathematico ad explicandas circuli aut trianguli proprietates, nullum adferre potest vsum, materiæ, vt auri, ligni, ferri, in qua insunt, consideratio, quin eò verius eiusmodi rerum, quarum species tanquam materia vacantes efformemus animo, naturam complectemur, quòd coniunctione materiæ quasi adulterari deprauarique videntur. Quocirca Mathematicæ species eodem modo quo κοιλὸν, siue concauitas, sine motu & subiecto, definitione explicare cog-

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nosciq́ possunt: naturales verò cùm eam vim habeant, quam vt ita dicam, simitas cum materia comprehensa sunt, nec absque ea separatim possunt intelligi: quibus exẽplis quid inter Physicas & Mathematicas species intersit, haud difficilè est animaduertere. Illis certè non semel est vsus Aristoteles. Valeant ergò Protagoræ sophismata, Geometras hoc nomine refellentis, quod circulus normam puncto non attingat. Nam diuina Geometrarum theoremata, qui sensu æstimabit, vix quicquam reperiet quod Geometra concedendum videatur. Quid enim ex his quæ sensum mouent, ita rectum aut rotundum dici potest, vt à Geometra ponitur? Nec verò absurdum est aut vitiosum, quòd lineas in puluere descriptas pro rectis aut rotundis assumit, quæ nec recta sunt nec rotũda, ac ne latitudinis quidem expertes. Siquidẽ non ijs vtitur Geometra, quasi inde vim habeat conclusio, sed eorum quæ discenti intelligenda relinquuntur, rudem seu imaginem proponit. Nã qui primum instituũtur, hi ductu quodam & velut χειραγογία sensum opus habẽt, vt ad illa quæ sola intelligentia percipiuntur, aditum sibi comparare queant. Sed tamen existimandum non est rebus Mathematicis omnino negari materiam ac non eam tantùm quæ sensum afficit. Est enim materia alia quæ sub sensum cadit, alia quæ animo & ratione intelligitur. Illam αἰσθητὴν hãc νοητὴν vocat Aristoteles. Sensu percipitur, vt æs, vt lignum, omnisq́ue materia quæ moueri potest, *rest. Animo & ratione cernitur ea quæ in rebus sensilibus inest, sed non quatenus sensu percipiuntur, quales sunt res Mathematicorum. Vnde ab Aristotele scriptum legimus ἐπὶ τῶν ἐν ἀφαιρέσει ὄντων rectum se habere vt simum: μετὰ συνεχοῦς γὰρ: quasi velit ipsius recti quod Mathematicorum est, suam esse materiam, non minus quã simi quod ad Physicos pertinet. Nam licet res Mathematicæ sensili vacent materia, non sunt tamẽ indiuiduæ, sed propter continuationem partitioni semper obnoxiæ, cuius ratione dici possunt sua materia non omnino carere: quin aliud videtur τὸ εἶναι γραμμῇ, aliud quoad continuationi adiuncta intelligitur linea. Illud enim ceu forma in materia proprietatum causa est, quæ sine materia percipere non licet. Hæc est societatis & dissidij Mathematicæ cum physica & prima philosophia ratio. Nũc autem de nominis etymo & notatione pauca quædã afferamus. Nam si quæ iudicio & ratione imposita sunt rebus nomina, ea certè non temerè indita fuisse credẽdum est, quibus sciẽtias appellari placuit, sed neq; otiosa semper haberi debet ista etymologiæ indagatio, cũ ad rei etiam dubiæ fidem sæpe non parum valeat recta nominis interpretatio. Sic enim Aristoteles ducto ex verborũ ratione argumento αὐτομάτου, μετὰ βολῆς, ἐλευθέρος, aliarumq; rerum naturam ex parte confirmauit. Quoniã igitur Pythagoras Mathematicã scientiã non modò studiosè coluit, sed etiã repetitis à capite principijs, geometricã*

contem-

templationem in liberalis disciplinae formam composuit, & perfectis absq; materia solius intelligentiae adminiculo theorematibus, tractationẽ περὶ τῶν ἀλόγων, & κοσμικῶν Σχημάτων constitutionem excogitauit: credibile est, Pythagoram, aut certè Pythagoreos, qui & ipsi doctoris sui studia libenter amplexi sunt, huic scientiae id nomẽ dedisse, quod cum suis placitis atq; decretis congrueret rerumq; propositarum naturam quoquo modo declararet. Ita cũ existimarent illi, omnem disciplinam, quae μάθησις dicitur, ἀνάμνησιν esse quandam. i. recordationem et repetitionẽ eius scientiae, cuius antè quam in corpus immigraret, compos fuerit anima; quemadmodum Plato quoque in Menone, Phaedone, & alijs aliquot locis videtur astruxisse: animaduerterent autem eiusmodi recordationem, quae nõ posset multis ex rebus perspici, ex his potißimum scientijs demõstrari, si quis nimirum, ait Plato, ἐπὶ τὰ διαγράμματα ἄγῃ, probabile est quidem Mathematicas à Pythagoreis artes κατ᾽ ἐξοχὴν fuisse nominatas, vt ex quibus μάθησις, id est aeternarũ in anima rationum recordatio, διαφερόντως & praecipuè intelligi posset. Cuius etiam rei fidem nobis diuinus fecit Plato, qui in Menone Socratem induxit hoc argumẽti genere persuadere cupientẽ, discere nihil esse aliud quam suarum ipsius rationũ animũ recordari. Etenim Socrates pusionem quendã vt Tullij verbis vtar, interrogat de geometrica dimẽsione quadrati: ad ea sic ille respõdet vt puer, et tamen

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mentam faciles interrogationes sunt, vt gradatim respōdens, eodem perueniat, quo si Geometrica didicisset. Aliam nominis huius ratione Anatolius exposuit. vt est apud Rhodiginum, quòd cū cætera disciplinæ deprehendi vel non docente aliquo poßint, omnes Mathematicæ sub nullius cognitionem veniant, nisi præeunti aliquo, cuius solertia succidantur vepreta, vel exurantur, & superciliosa complanentur aspreta. Ita enim Cælius: quod quam vim habeat, non est huius loci curiosiùs perscrutari. Equidem M. Tullius Mathematicos in magna rerū obscuritate, recondita arte, multiplicique, ac subtili versari scribit: sed quis nescit idipsum cum aliys grauioribus scientiys esse commune? Est enim, vel eodem auctore Tullio, omnis cognitio multis obstructa difficultatibus, maximaque est & in ipsis rebus obscuritas, & in iudicijs nostris infirmitas, nec vllus est, modò interius paulò Physica penetrarit, qui non facilè sit expertus, quam multi vndique emergant, rerum naturalium causas inquirentibus, inexplicabiles labyrinthi. Sunt qui ex demonstrationum firmitate nominari Mathamaticas opinantur: cuius etiam rationis momētum alio seorsim loco expendēdum fuerit. Quocirca primam verbi notationem, quam sequutus est Proclus, nobis retinendam censeo. Hactenus de vniuerso Mathematica genere, quanta potui & perspicuitate & breuitate dixi. Sequitur vt de Geometria separatim atque ordine ea differā, quæ initio sum pollicitus

*tus. Est autem Geometria, vt definit Proclus sci-
entia, quæ versatur in cognitione magnitudinum
figurarum, & quibus hæ continentur, extremo-
rum, item rationum & affectionum, quæ in illis
cernuntur ac inhærent: ipsa quidem progrediens
à puncto indiuiduo per lineas & superficies, dum
ad solida conscendat, variasq́; ipsorum differen-
tias patefaciat. Quumq́; omnis scientia demon-
stratiua, vt docet Aristoteles, tribus quasi momen-
tis cōtineatur, genere subiecto, cuius proprietates
ipsa scientia exquirit & contemplatur: causis &
principijs ex quibus primis demonstrationes cō-
ficiuntur: & proprietatibus, quæ de genere subie-
cto per se enunciantur: Geometriæ quidem subie-
ctum in lineis, triangulis, quadrangulis, circulis,
planis, solidis atque omnino figuris & magnitudi-
nibus, earumq́; extremitatibus consistit. His autē
inhærent diuisiones, rationes, tactus, æqualitates,
παραβολαὶ, ὑπερβολαὶ, ἐλλείψεις, atq; alia gerīris
eiusdem propè innumerabilia. Postulata verò &
Axiomata, ex quibus hæc inesse demonstrantur,
eiusmodi ferè sunt: Quouis centro & interuallo
circulum describere. Si ab æqualibus æqualia de-
trahas, quæ relinquuntur esse æqualia, cæteraq́;
id genus permulta, quæ licet omnium sint com-
munia, ad demonstrandum tamen tum sunt ac-
commodata, cùm ad certum quoddam genus
traducuntur. Sed cùm præcipua videatur A-
rithmeticæ & Geometriæ inter Mathematicas
dignatio, cur Arithmetica sit ἀκριβεστέρα &*

 exactior

exactior quam Geometria paucis explicandis a
bitror. Hic verò & Aristotelem sequemur a
cem, qui scientiam cum scientia ita comparat,
accuratiorem velit esse eam, quæ rei causam a
cet, quam quæ rem esse tantùm declarat, dein
quæ in rebus sub intelligentiam cadentibus ver
tur, quàm quæ in rebus sensum mouentibus ce
nitur. Sic enim & Aribtmetica quam Musica,
Geometria quàm Optica, & Stereometria quà
Mechanica exactior esse intelligitur, Postrem
quæ ex simplicioribus initijs constat, quàm qu
aliqua adiectione compositis vtitur. Atque ha
quidem ratione Geometriæ præstat Arithmeti
ca, quòd illius initium ex additione dicatur, hu
ius sit simplicius. Est enim punctum, vt Pythago
reis placet, vnitas quæ situm obtinet: vnitas ver
punctum est quam situ vacat. Ex quo percipitur
numerorum quàm magnitudinum simplicius esse
elementum, numerosque magnitudinibus esse pu
riores, & à concretione materiæ magis disiun
ctos. Hæc quamquam nemini sunt dubia, habe
& ipsa tamē Geometria quo se plurimū efferat,
opibusq; suis ac rerum vbertate multiplici vel
cum Arithmetica certet: id quod tute facilè de-
prehendas cùm ad infinitam magnitudinis diui-
sionem, quam respuit multitudo, animum con-
uerteris. Nūc quæ sit Arithmeticæ & Geometriæ
societas videamus. Nam theorematum quæ de-
monstratione illustrantur, quædam sunt vtri-
usque scientiæ communia, quædam verò singula-
rum

m propria. Etenim quòd omnis proportio sit τος siue rationalis, Arithmetica soli conuenit, quaquam Geometria, in qua sunt etiā ἄῤῥητοι u irrationales proportiones. item, quadrato- m gnωmas minimo definitos esse, Arithmetica oprium (si quidem in Geometria nihil tale mi- mum esse potest) sed ad Geometriam propriè ectant situs, qui in numeris locum non habent ctus, qui quidem à continuis admittuntur: λογικν, quoniam vbi diuisio infinitè procedit, ibi iam τὸ ἄλογον esse solet. Communia porrò v- iusque sunt illa, quæ ex sectionibus eueniunt, uas Euclides libro secundo est persequutus: nisi uod sectio per extremam & mediam rationem numeris nusquam reperiri potest. Iam verò ex eωrematibus eiusmodi communibus, alia qui- em ex Geometria ad Arithmeticam traducun- ur: alia contra ex Arithmetica in Geometriam ansferuntur: quædam verò perinde vtrique ientiæ conueniunt, vt quæ ex vniuersa arte athematica in vtramq; harum conueniant. Nam & alterna ratio, & rationum conuersio- es, compositiones, diuisiones hoc modo commu- ia sunt vtriusque. Quæ autem sunt περὶ συμ- ετρων, id est, de commensurabilibus, Arith- etica quidem primum cognoscit & contem- latur: secundo loco Geometria Arithmeti- am imitata. Quare & commensurabiles mag- itudines illæ dicuntur, quæ rationem inter habent, quam numerus ad numeram, per-

 inde

sunde quasi commensuratio & συμμετρία in nu-meris primùm consistat (Vbi enim numerus, ibi σύμμετρον cernitur: & vbi σύμμετρον, illic etia numerus) sed quæ triangulorũ sunt, & quadra-gulorum, à Geometra primũ considerantur: tu analogia quadam Arithmeticus eadem illa numeris contemplatur. De Geometriæ diuisio hoc adijciendum puto, quòd Geometriæ pars a tera in planis figuris cernitur, quæ solam latitu-nem longitudini coniunctam habent: altera ve solidas contemplatur, quæ ad duplex illud inte-uallum crassitudinem adsciscunt. Illam genera Geometriæ nomine veteres appellarunt: hanc pr priè Stereometriam dixerunt. Ita Geometria cum Optica & Stereometriam cum Mechanic non rarò comparat Aristoteles. Sed illius cogniti huius inuentionem multis seculis antecessit, si m dò Stereometriam ne Socratis quidem ætate vl lam fuisse omnino verum est, quemadmodum Platone scriptum videtur. Ad Geometriæ vtilita tem accedo, quæ quanquam suapte vi & digni tate ipsa per se nititur, nullius vsus aut action ministerio mancipata (vt de Mathematicis om nibus scientijs concedit in Politico Socrates) quid ex ea tamen vtilitatis externæ quæritu Dij boni quàm latos, quàm vberes, quàm vario fructus fundit? Nec verò audiendus est vel Ari stippus, vel Sophistarum alius, qui Mathemati corum artes idcirco repudiet, quòd ex fine nihi docere videantur, eiusque quod melius aut de

teriu

terius nullam habeant rationem. Vt enim nihil causa dicas, cur sit melius trianguli, verbi gratia, tres angulos duobus esse rectis æquales: minimè tamen fuerit consentaneum Geometriæ cognitionem vt inutilem exagitare, criminari, explodere, quasi quæ finem & bonum quò referatur, habeat nullum. Multas haud dubiè solius contemplationis beneficio citra materiæ contagionem adfert Geometria cõmoditates partim proprias, partim cum vniuerso genere communes. Cùm enim Geometria, vt scripsit Plato, eius quod semper est cognitionem profiteatur, ad veritatem excitabit illa quidem animum, & ad ritè philosophandum cuiusque mentem comparabit. Quin etiam ad disciplinas omnes facilius perdiscendas, attigeris necne Geometriam quanti referre cẽses? Nam vbi cum materia coniungitur, nónne præstantissimas procreat artes, Geodesiam, Mechaniam, Opticam, quarum omnium vsu, mortalium vitam summis beneficijs complectitur? Etenim bellica instrumẽta, vrbiumq; propugnacula, quibus munitæ vrbes hostium vim propulsarent, his adiutricibus fabricata est: montium ambitus & altitudines, locorumq; situs nobis indicauit: dimetiendorum & mari & terra itinerum rationem præscripsit: trutinas & stateras, quibus exacta numerorũ æqualitas in ciuitate retineatur, cõposuit: vniuersi ordinem simulachris expreßit, multaq; quæ hominum fidem superarent, omnibus persuasit. Vbiq; extant præclara in eam rem

 testimo-

testimonia. Illud memorabile, quòd Archime
rex Hiero tribuit. Nàm extructo vastæ molis n
uigio, quòd Hiero AEgyptiorum regi Ptolom
mitteret, cum vniuersa Syracusanorum multit
do collectis simul viribus nauem trahere non po
set effecissetq́ Archimedes, vt solus Hiero illa
subduceret, admiratus viri scientiam rex ἀπ
ταύτης, ἔφη, τῆς ἡμέρας περὶ παντος ἀρχιμή
λέγοντι πιστευτέον. *Quid? quòd Archimedes id*
vt est apud Plutarchum, Hieroni scriptis datis v
ribus datum pondus moueri posse? fretusq́ demi
strationis robore illud sæpe iactarit, si terram h
beret alteram vbi pedem figeret, ad eam nostra
hanc se transmouere posse? Quid varia, αὐτομα
των *machinarumq́ genera ad vsus necessario*
comparata memorem? Innumerabilia profect
sunt illa, & admiratione dignißima quibus pri
ci homines incredibili quodam ad philosophan
dum studio concitati, inopem mortalium vita
artis huius præsidio subleuarum: tametsi memo
ria sit proditum, Platonem Eudoxo & Archyt
vitio vertisse, quòd Geometrica problemata a
sensilia & organica abducerent. Sic enim cor
rumpi ab illis & labefieri Geometriæ præstanti
am, quæ ab intelligibilibus & incorporeis reb
ad sensiles & corporeas prolaberetur. Qua
propter ridicula idem scripsit Plato Geometra
rum esse vocabula, quæ quasi ad opus & actio
nem spectent, ita sonare videntur. Quid enim est
quadrare si non opus facere? Quid addere, pro
ducere,

lucere, applicare? Multa quidem sunt eiusmodi nomina, quibus necessario & tanquam coacti Geometræ vtuntur, quippe cum alia desint in hoc genere commodiora. Sic ergo censuit Plato, sic Aristoteles, sic denique philosophi omnes, Geometriam ipsam cognitionis gratia exercendam, nec ex aliquo vsu externo sed ex rerum νοητῶν intelligentia æstimandam esse. Exposita breuius quā res tanta dici possit, vtilitatis ratione, Geometriæ ortum, qui in hac rerum periodo ex historicorum monumentis nobis est cognitus deinceps aperiamus. Geometria apud Aegyptios inuenta, (ne ab Adamo, Setho, Noah, quos cognitione rerum multiplici valuisse constat, eam repetamus), ex terrarum dimentione, vt verbi præ se fert ratio, ortū habuisse dicitur: cùm anniuersaria Nili inundatione & incrementis limo obductiæ grorum termini confunderētur. Geometriam enim, sicut & reliquas disciplinas, in vsu quam in arte prius fuisse aiunt. Quod sane mirum videri non debet, vt & huius & aliarum scientiarum inuentio ab vsu cœperit ac necessitate. Etenim tempus, rerum vsus, ipsa necessitas ingenium excitat, & ignauiam acuit. Deinde quicquid ortum habuit (vt tradant Physici) ab inchoato & imperfecto processit ad perfectum. Sic artium & scientiarum principia experientia beneficio collecta sunt: experientia verò à memoria fluxit, quæ & ipsa à sensu primùm manauit. Nam quod scribit Aristoteles, Mathematicas artes,

compa-

comparatis rebus omnibus ad vitã necessarijs, in Aegypto fuisse constitutas, quòd ibi sacerdotes omnium concessu in otio degerent: non negat ille adductos necessitate homines ad excogitandam verbi gratia terræ dimetiendæ rationem, quæ theorematum deinde inuestigationi causam dederit: sed hoc confirmat, præclara eiusmodi theorematũ inuenta, quibus extructa Geometriæ disciplina cõstat, ad vsus vitæ necessarios ab illis non esse expetita. Itaque vetus ipsum Geometriæ nomen ab illa terræ partiundæ finiumq; regendorũ ratione postea recessit, & in certa quadam affectionum magnitudini per se inhærentium sciẽtia propria remãsit. Quemadmodum igitur in mercium & contractuum gratiam supputandi ratio, quam secuta est accurata numerorum cognitio, à Phœnicibus initium duxit: ita etiam apud AEgyptios, ex ea, quam commemoraui causa ortum habuit Geometria. Hanc certè vt id obiter dicã, Thales in Græciam ex AEgypto primum trãstulit: cui non pauca deinceps à Pythagora, Hippocrate, Chio, Platone, Archyta Tarentino, alijsq; compluribus, ad Euclidis tempora facta sunt rerum magnarum accessiones. Cæterùm de Euclidis ætate id solum addam, quod à Proclo memoriæ mandatum accepimus. Is enim commemoratis aliquot Platonis tum æqualibus tum discipulis subijcit, non multò ætate posteriorem illis fuisse Euclidem eum, qui elementa conscripsit, & multa ab Eudoxo collecta, in ordinem luculentum

com-

composuit, multaq; à Theateto inchoata perfecit, queq; mollius ab alijs demonstrata fuerāt, ad firmißimas & certißimas apodixes renocauit. Vixit autē, inquit ille, sub primo Ptolemæo. Etenim ferunt Euclidam à Ptolemæo quondam interrogatum, num qua esset via ad Geometriam magis cōpendiaria, quàm sit ista στοιχείωσις respondisse: μὴ εἶναι βασιλικὴν ἀτραπὸν ἐπὶ γεωμετρίαν. Deinde subiūgit, Euclidem natu quidem esse minorem Platone, maiorē verò Eratosthene & Archimede (hi enim æquales erāt) cùm Archimedes Euclidis mentionem faciat. Quod si quis egregiā Euclidis laudem, quam cùm ex alijs scriptionibus accuratißimis, tū ex hac Geometria στοιχείωσις cōsequutus est, in qua diuinus rerum ordo sapiētißimis quibusq; hominibus magnæ semper admirationi fuit, is Proclum studiosè legat, quò rei veritatem illustriorem reddat grauißimi testis auctoritas. Superest igitur vt finem videamus, quò Euclidis elementa referri, & cuius causa in id studium incumbere oporteat. Et quidem si res, quæ tractantur, consideres; in tota hac tractatione nihil aliud quæri dixeris, quā vt Κοσμικὰ, quæ vocātur σχήματα (fuit enim Euclides professione & instituto Platonicus) Cubus Icosaedrum, Octaedrum, Pyramis, & Dodecaedrum certa quadam suorum & inter se laterū, & ad sphæræ diametrum ratione eidem sphæræ inscripta comprehendantur. Huc enim pertinet Epigrammation illud vetus, quod in Geometrica Michaelis

Pselli συνόψει *scriptum legitur.*

Σχήματα πέντε ἃ πλάτωνος, πυθαγόρας σοφὸς εὗρε.

πυθαγόρας σοφὸς εὗρε, πλάτων δ' ἀρίδηλ' ἐδίδαξεν.

Εὐθαλείδης ἐπὶ τοῖσι κλέος γιριχαλλὲς ἔτευξεν.

Quòd si discentis institutionem spectes, illud certè fuerit propositum, vt huiusmodi elementorum cognitione informatus discentis animus, ad quamlibet non modo Geometriæ, sed & aliarum Mathematicæ partium tractationem idoneus paratusque accedat. Nam tametsi institutionem hāc solus sibi Geometra vendicare videtur, & tanquā in possessionē suam venerit, alios excludere posse: inde tamen per multa suo quodam modo iure decerpit Arithmeticus, plreaque Musicus non pauca detrahit. Astrologus, Opticus, Logisticus, Machanicus, itemque cæteri: nec vllus est denique artifex præclarus, qui in huius se possessionis societatē cupide non offerat, partemque sibi concedi postulet. στοιχείωσις *absolutum operi nomen, &* στοιχειωτὴς *dictus Euclides. Sed quid longius prouehor? Nā quod ad hanc rem attinet, tam copiosè & eruditè scripsit (vt alia complura) eo ipso, quem dixi, loco P. Montaureus, vt nihil desiderio loci reliquerit. Quæ verò ad dicendum nobis erant proposita hactenus pro ingenij nostri tenuitate omnia mihi perfecisse videor. Nam tametsi & hæc eadem & alia pleráque multo fortè præclariora ab hominibus doctissimis, qui tum acumine ingenij,*

nij, tùm admirabili quodam lepore dicendi semper floruerunt grauiùs, splendidiùs, vberiùs tractari posse scio: tamen experiri libuit: numquid etiam nobis diuino sit concessum munere, quod rudes in hac philosophiæ parte discipulos adiuuare aut certè excitare queat. Huc accessit quòd ista recens elementorum editio, in qua nihil non parum fuisset studij, aliquid à nobis efflagitare videbatur, quod eius cõmendationem adaugeret. Cum enim vir doctissimus Io. Magnienus Mathematicarum artium in hac Pharisiorum Academia professor verè regius, nostrum hunc typographum in excudendis Mathematicorum libris diligentissimum, ad hanc Elementorum editionem sæpè & multum esset adhortatus, eiusque impulsu permulta sibi iam comparasset typographus ad hanc rem necessaria, citò interuenit, malùm Ioannis Magnieni mors insperata, quæ tam grauè inflexit Academiæ vulnus, cui ne post multos quidem annorum circuitus cicatrix obduci vlla posse videatur. Quamobrem amisso instituti huius operis duce, typographus, qui nec sumptus antea factos sibi perire, nec studiosos, quibus id muneris erat pollicitus, sua spe cadere vellet, ad me venit, & impensè rogauit, vt meam propositæ editioni operam & studium nauarem, quòd cùm denegaret occupatio nostra, iuberet officij ratio, feci equidem rogatus, vt quæ subobscurè vel parum commodè in sermonem Latinum è Græco translata videbantur, clariore, aptiore & fideliore interpretatione nostra (quod cuiusq; pace dictum volo) lucē acciperent. Id quod in omnibus ferè libris posterioribus tute primo obtutu perspicias. Nam in sex prioribus non tantum temporis quantum in cæteris ponere nobis licuit: decimi autem interpretatio, qua melior nulla potuit adferri, P. Mōtaureo solida debetur. Atque vt ad perspicuitatem facilitatemq; nihil tibi deesse queraris, adscripta sunt propositionibus singulis vel lineares figuræ, vel pūctorum tanquam vnitatum notulæ, quæ Theonis apodixin illustrent illæ quidem magnitudinum, hæ autem numerorum indices, subscriptis etiam ciphrarum, vt vocant, characteribus, qui propositum quemuis numerum exprimāt, ob eamq; causam eiusmodi vnitatum notulæ, quæ pro numeri amplitudine maius paginæ spatium occuparent, pauciores sæpius depictæ sunt, aut in lineas etiam cōmutatæ. Nam litterarum vt a, b, c, characteres non modò numeris & numerorum partibus nominandis sunt accommodati, sed etiam generales esse numerorum, vt magnitudinum affectiones testantur. Adiecta sunt insuper quibusdam locis non pœnitēda Theonis scholia, siue mauis lemmata, quæ quidem longè plura accessissent, si plus otij & temporis vacui nobis fuisset relictū, quod huic studio impartiremus. Hanc igitur operam boni consule, & quæ obuia erunt impressionis vitia, candidè emenda. Vale. Lutetiæ 4. Idus April. 1557.

EVCLI-

EVCLIDIS ELEMENTVM PRIMVM.

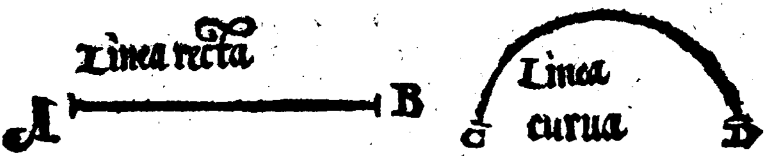
DEFINITIONES.

1.

PVnctum eſt, cuius pars nulla eſt. Punctum

2.

Linea verò, longitudo latitudinis expers.



3.

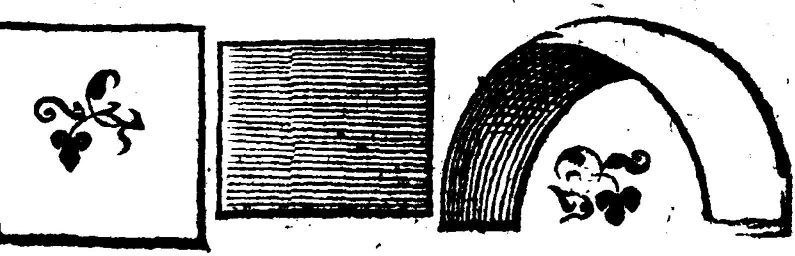
Lineæ autem termini, ſunt puncta.

4.

Recta linea eſt, quæ ex æquo ſua interiacet puncta.

5.

Superficies eſt, quæ longitudinem latitudinemque tantùm habet.

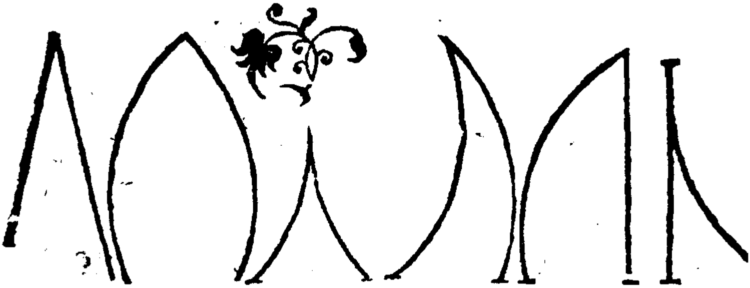


6.

Superficiei extrema sunt liniæ.

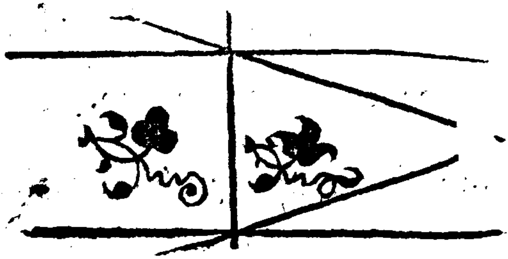
7.

Plana superficies est, quæ ex æquo suas interiacet lineas.



8.

Planus angelus est duarum linearũ in plano
nec in vna lineam coalescentiũ
se mutuò tangentium, & non indirectum iacẽtium alterius ad alterã inclinatio.



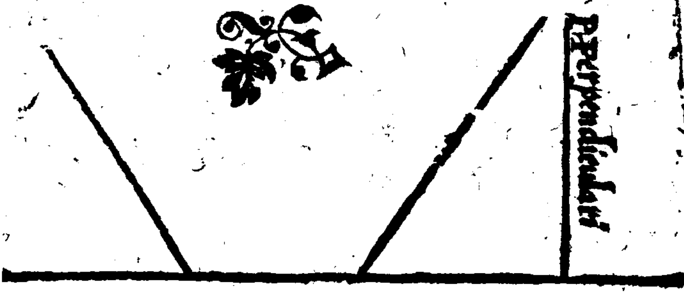
9.

Cùm autem quę angelum continent lineę, rectæ fuerint, rectilineus ille angulus appellatur.

10.

Cùm verò recta linea super rectam consistens lineam, eos qui sunt deinceps angelos æquales, inter se fecerit: rectus est vterque æqua-

æqualium angelorum: quæ inſiſtit recta linea, perpendicularis vocatur eius, cui inſiſtit.



11.

Obtuſus angulus eſt, qui recto maior eſt.

12.

Acutus verò, qui minus eſt recto.

13.

Terminus eſt, quod alicuius extremum eſt.

14.

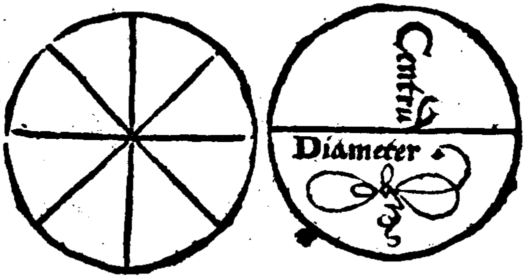
Figura eſt, quæ ſub aliquo, vt aliquibus terminis comprehenditur.

15.

Circulus eſt, figura plana ſub vna linea cõprehéſa, quæ peripheria appellatur: ad quã ab vno puncto eorum, quæ intra figuram

ſunt

ſunt poſita, cadentes omnes rectę lineæ inter ſe ſunt æquales.



16.

Hoc verò punctum, centrum circuli appellatur.

17.

Diameter autem circuli, eſt recta quædam linea per centrum ducta, & ex vtraque parte in circuli peripheriam terminata, quæ circulum bifariam ſecat.

18.

Semicirculus eſt figura, quę continetur ſub diametro, & ſub ea linea, quæ de circuli periferia auſertur.

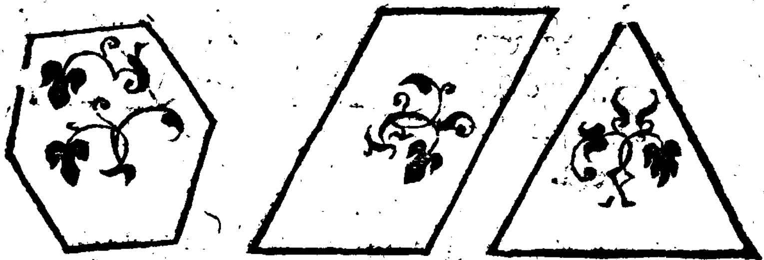
19.

Segmentum circuli, eſt figura, quæ ſub recta linea & circuli peripheria continentur.

20. Recti.

20.

Rectilineæ figuræ, ſub quæ ſub rectis leneis continentur.



21.

Trilateræ quidem, quæ ſub tribus.

22.

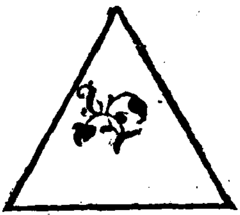
Quadrilateræ, quæ ſub quatuor.

23.

Multilateræ verò, quæ ſub pluribus quàm quatuor rectis lineis comprehenduntur.

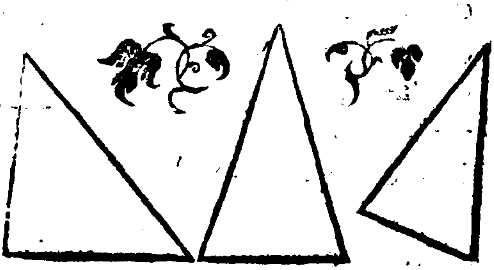
24.

Trilaterum, porrò figurarum, æquilaterum eſt triangulum, quod tria latera habet æqualia.



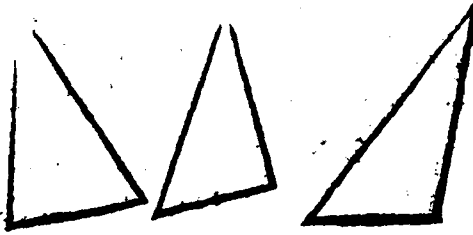
25.

Iſoſceles autem eſt quod duo tantum æqualia habet latera.



26.

Scalenum verò, est quibꝰ tria inæqualia habet latera.



27.

Ad hanc etiam trilaterarum figurarum, rectangulum quidem triangulum est, quod rectum angulum habet.

28.

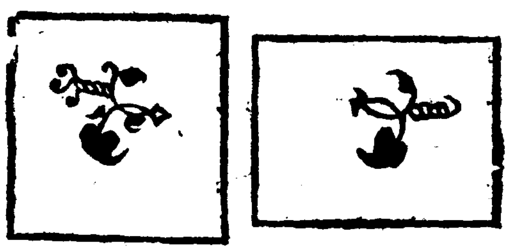
Amblygonium autem, quod obtusum angulum habet.

29.

Oxigonium verò, quod tres habet acutos angulos.

30.

Quadrilaterarum autem figurarum, quadratũ quidẽ est quiđ & æquilaterũ & rectangulum est.

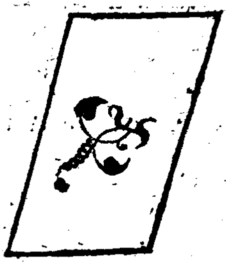


31.

Altera parte longior figura est, quæ rectangula quidem, at æquilatera non est.

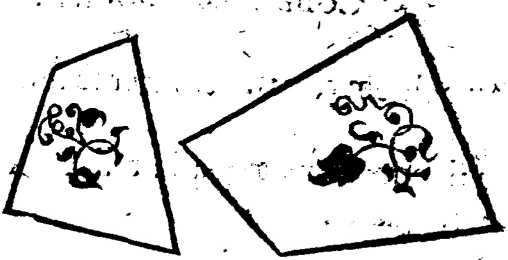
32. Rhom

32.
Rhõbus autem, qui æquilaterum & rectangulum est.



33.
Rhomboides verò, quæ aduerſa & latera & angulos habens inter ſe æqualia, neque æquilatera eſt, neque rectangula.

34.
Prçter has autẽ reliquæ quadrilateræ figuræ, trapezia appellentur.



35.
Parallele rectæ lineæ ſunt quæ, cùm in eodem ſint plano, & ex vtraque parte in infinitum producantur, in neutram ſibi mutuò incidunt.

Poſtulata.

1.
Poſtuletur, vt à quouis puncto in quoduis

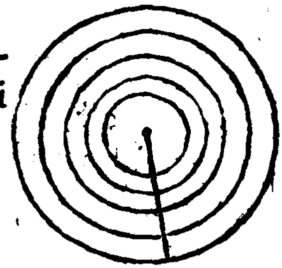
punctum, rectam lineam ducere concedatur.

2.

Et rectam lineam terminatam in continuum recta producere.

3.

In quouis centro & interuallo circulum describere.



Communes notiones.

1.

Quæ eidem ęqualia, & inter se sunt æqualia.

2.

Et si æqualibus æqualia adiecta sint, tota sunt æqualia.

3.

Et si ab æqualibus æqualia ablata sint, quæ relinquuntur sunt æqualia.

4.

Et si inæqualibus æqualia adiecta sint, tota sunt inæqualia.

5.

Et si ab inæqualibus æqualia ablata sint, reliqua sunt inæqualia.

6.

Quæ eiusdem duplicia sunt, inter se sunt æqualia.

Et

7.

Et quæ eiuſdem ſunt dimidia, inter ſe ęqualia ſunt.

8.

Et quæ ſibi mutuò congruunt, ea inter ſe ſunt æqualia.

9.

Totum eſt ſua parte maius.

10.

Item, omnes recti anguli ſunt inter ſe ęquales.

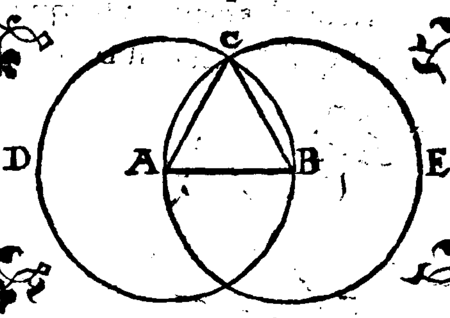
11.

Et ſi duas rectas lineas altera recta incidens, inter nos & eaſdemque partes angelos, duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ ſibi mutuò incident ad eas partes, vbi ſunt anguli duobus rectis mimores.

12.

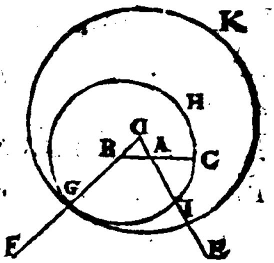
Duæ rectæ lineæ ſpatium non comprehendunt.

Problema 1. Propoſitio 1.



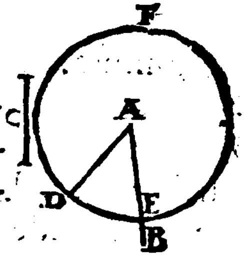
Super data recta linea termi nata, triangulum æquilaterum conſtituere.

Problema 2. Propositio 2.



Ad datum punctum, datæ rectæ lineæ, æqualem rectam lineam ponere.

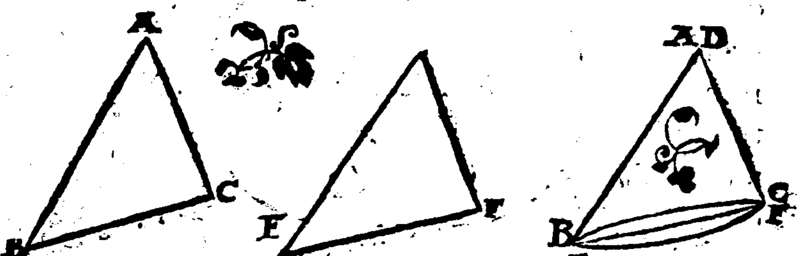
Problema 3. Propositio 3.



Duabus datis rectis lineis inæqualibus, de maiore æqualē minori rectam lineam detrahere.

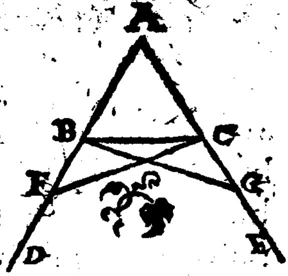
Theorema primum Proposio 4.

Si duo triangula duo latera duobus lateribus æqualia habeant, vtrunque vtrique, habeant verò & angulum angulo æqualē sub æqualibus rectis lineis contentum: & basim basi æqualem habebunt, eritque triangulū triangulo æquale, ac reliqui anguli reliquis angulis æquales erunt, vterque vtrique, sub quibus æqualia latera subtenduntur.



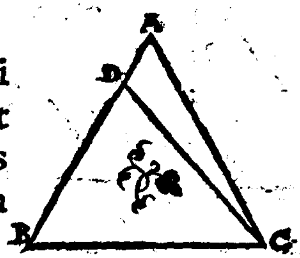
Theo-

Theorema 2. Propoſitio 5.



Iſoſceleum triangulorũ, qui ab baſim ſunt anguli, inter ſe ſunt æquales; & ſi vlterius productæ ſint æquales illæ rectę lineæ, qui ſub baſi ſunt anguli, inter ſe æquales erunt.

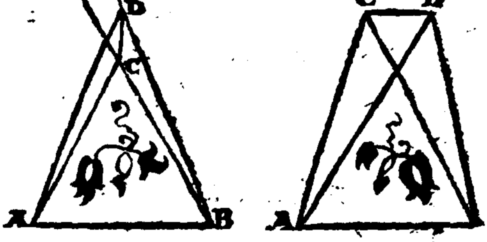
Problema 3. Propoſitio 6.



Si trianguli duo anguli æquales inter ſe fuerint & ſub æqualibus angulis ſubtenſa latera æqualia inter ſe erunt.

Thorema 4. Propoſitio 7.

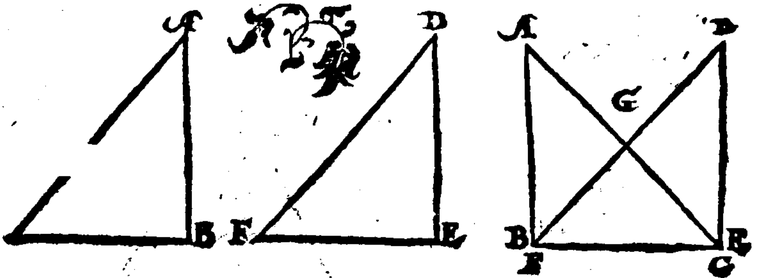
Super eandem recta linea duabus eiuſdẽ rectis lineis aliç duç rectç lineæ æquales, vtraque vtrique non conſtituentur, ad aliud atque aliud punctũ ad eaſdẽ partes eoſdẽ



que terminos cum duabus initio ductis rectis lineis habentes.

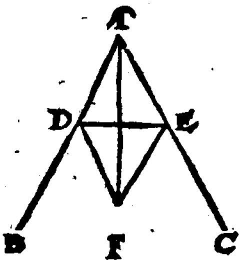
Theorema 5. Propositio 8.

Si duo triangula duo latera habuerint duobus lateribus, vtrunq; vtrique æqualia; habuerint verò & basim basi æqualem: angulum quoque sub æqualibus rectis lineis contentum angulo æqualem habebunt.



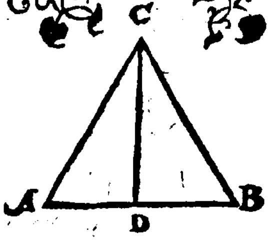
Problema 4. Propositio 9.

Datum augulum rectilineum bifariam secare.



Problema 5. Propositio 10.

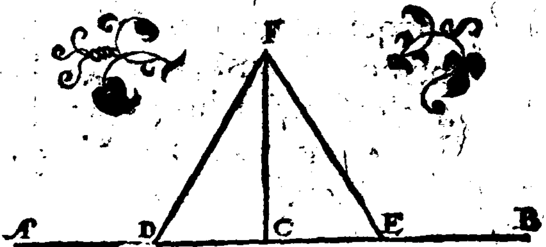
Datam rectam lineam finitam bifariam secare.



Proble-

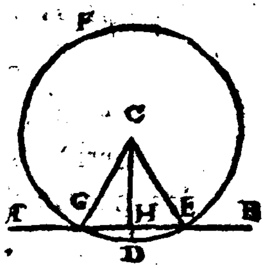
Problema 6. Propositio 11.

Data recta linea, à puncto in ea dato rectam lineam ad angulos rectos excitare.



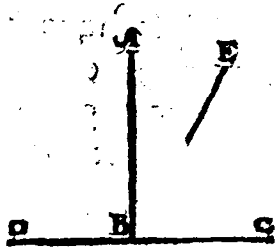
Problema 7. Propositio 12.

Super datam rectam lineam infinitam, à dato puncto quod in ea non est, perpendicularem rectam deducere.



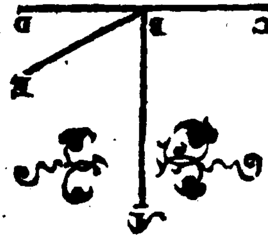
Theorema 6. Propositio 13.

Cùm recta linea super rectam cõsistens lineam angulos facit, aut duos rectos, aut duobus rectis æquales efficiet.



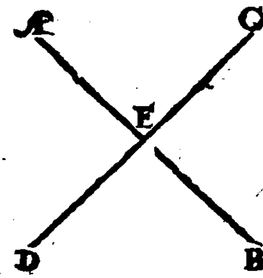
Theorema 7. Propositio 14.

Si ad aliquam rectam lineam, atque ad eius punctũ, duæ rectę lineæ nõ ad easdem partes ductæ, eos qui sunt deinceps angulos duobus rectis æquales fecerint, indirectum erunt inter se ipse rectæ lineæ.



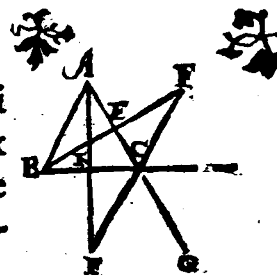
Theorema 8. Propositio 15.

Si duæ rectę lineę se mutuò secuerint, angulos qui ad verticem sunt, æquales inter se efficient.



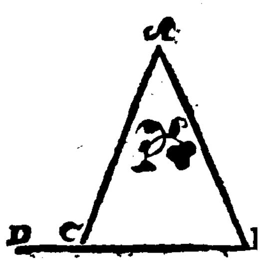
Theorema 9. Propositio 16.

Cuiuscunque trianguli vno latere producto, externus angulus vtroque interno & opposito maior est.



Theorema 10. Propositio 17.

Cuiuscunque trianguli duo anguli duobus rectis sunt minores omnifariam sumpti.



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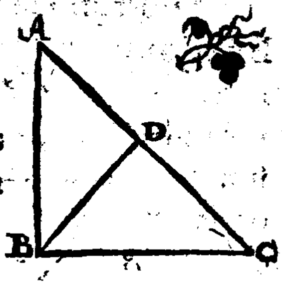
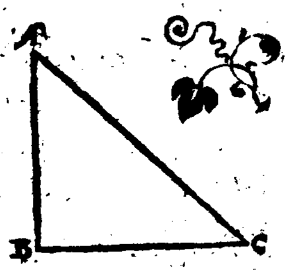
Theorema 11. Propositio 18.



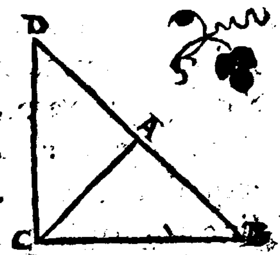
Omnis trianguli maius latus maiorem angulum subtendit.

Theorema 12. Propositio 19.



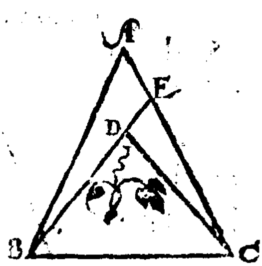
Omnis trianguli lateri angulus maiori lateri subtenditur.

Theorema 13. Propositio 20.



Omnis trianguli duo latera reliquo sunt maiora, quomodocumque assumpta.

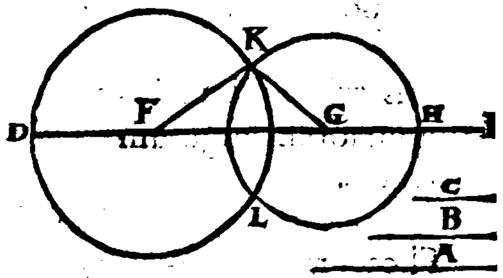
Theorema 14. Propositio 21.



Si super trianguli vno latere, ab extremitatibus duæ rectę lineæ, interius constitutæ fuerint, hæ constitutæ reliquis trianguli duobus lateribus minores quidem erunt, minorem verò angulũ continebunt.

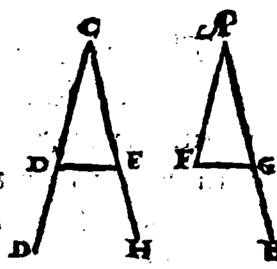
Pro-

Problema 8. Propoſitio 22.



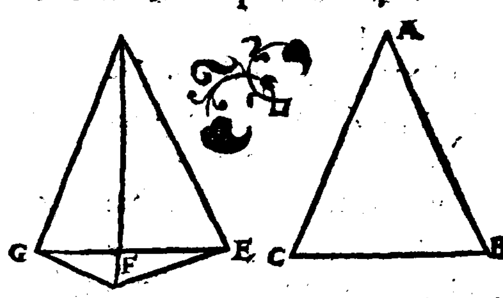
Ex tribus rectis lineis quæ ſunt trib. datis rectis lineis æquales, triangulum conſtituere. Oportet autem duas reliqua eſſe maiores omnifariã ſumptas: quoniam vniuſcuiuſque trianguli duo latera omnifariam ſumpta reliquo ſunt maiora.

Problema 9. Propoſitio 23.



Ad datam rectam lineã datumq́; in ea punctum dato angelo recti lineo æqualem angulum recti lineum conſtituere.

Theorema 15. Propoſitio 24.



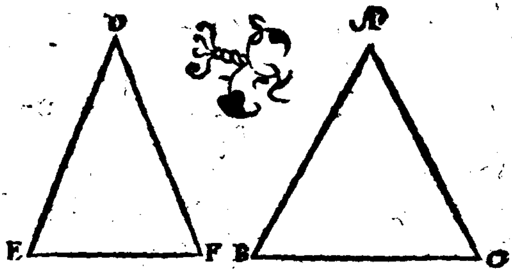
Si duo triãgula duo latera duobus lateribus æqualia habuerint, vtrumque vtrique angulũ verò

angu-

angulo maiorem sub æqualibus rectis lineis contentum: & basin basi maioré habebunt.

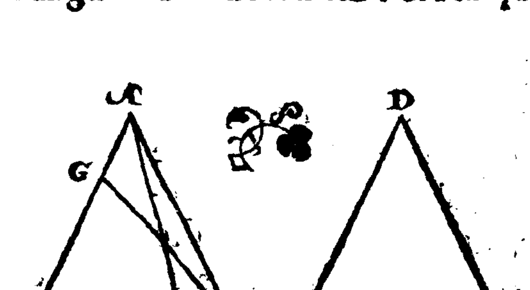
Theorema 16. Propositio 25.

Si duo triangula duo latera duobus lateribus æqualia habuerint, vtrunque vtrique, basin verò basi maioré: & angulum sub æqualibus rectis lineis contentum angulo maiorem habebunt.



Theorema 17. Propositio 26.

Si duo triangula duos angulos duobus angulis æquales habuerint, vtrunque vtrique, vnumque latus vni lateri æquale, siue, quod æqualibus adiacet angulis, seu quod vni æqualium angulorum subtéditur: & reliqua latera. reliquis laterib. æqualia vtrunq; vtriq;

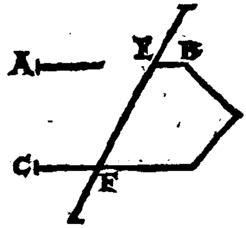


& reliquum angulum reliquo angulo ęqualem habebunt.

Theore-

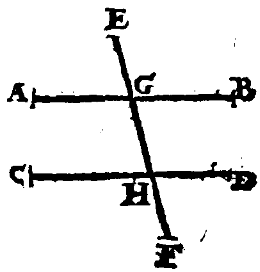
Theorema 18. Propositio 27.

Si in duas rectas lineas recta incidens linea alternatim angulos æquales inter se fecerit: parallelæ erunt inter se illæ rectæ lineæ.



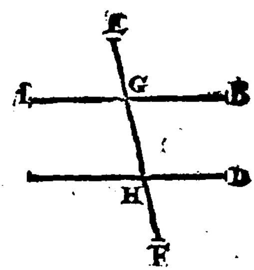
Theorema 19. Propositio 28.

Si in duas rectas lineas recta incidens linea externum angulũ interno, & opposito, & ad easdem partes æqualẽ fecerit, aut internos & ad easdẽ partes duobus rectis æquales: parallelæ erunt inter se ipsæ rectæ lineæ.



Theorema 20. Propositio 29.

In parallelas rectas lineas recta incidens linea: & alternatim angulos inter se æquales efficit & externum interno & opposito & ad easdem partes æqualem, & internos & ad easdem partes duobus rectis æquales facit.



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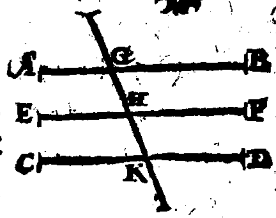
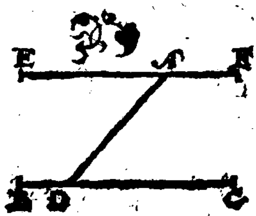
Theorema 21. Propositio 30.



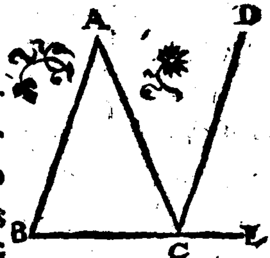
Quæ eidem rectæ lineæ, parallelæ, & inter se sunt parallelæ.

Problema 10. Propositio 31.



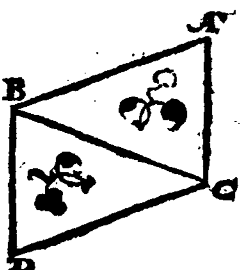
A dato puncto datæ rectæ lineæ parallelam rectam lineam ducere.

Theorema 22. Propositio, 32.



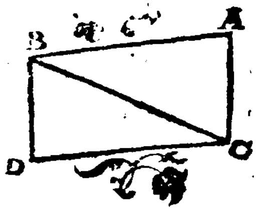
Cuiuscunq; trianguli vno latere vlterius producto: externus angelus duobus internis & oppositis est æqualis. Et trianguli tres interni anguli duobus sunt rectis æquales.

Theorema 23. Propositio 33.



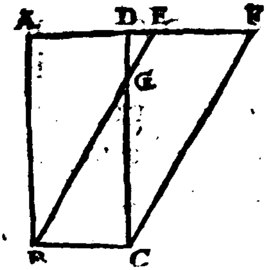
Rectæ lineæ quę ęquales & parallelas lineas ad partes easdem coniungunt, & ipsæ æquales & parallelæ sunt.

Theorema 24. Propoſitio 34.



Parallelogrammorū ſpatiorum æqualia ſunt inter ſe, quæ ex aduerſo & latera & anguli: atque illa bifariam ſecat diameter.

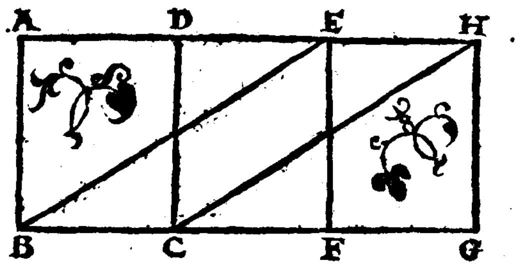
Theorema 25. Propoſitio 35.



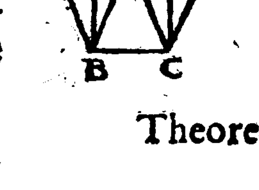
Parallelogramma ſuper eadem baſi, & in eiſdem parallelis cōſtituta inter ſe ſunt æqualia.

Theorema 26. Propoſitio 36.

Parallelogrāma ſuper æqualibus baſibus & in eiſdē parallelis cōſtituta inter ſe sūt æqualia.



Theorema 27. Propoſitio 37.

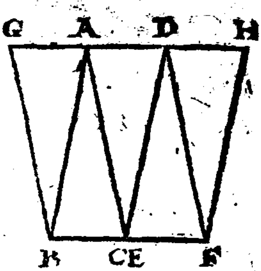


Triangula ſuper eandem baſibus conſtituta, & in eiſdem parallelis, inter ſe ſunt æqualia.

Theore-

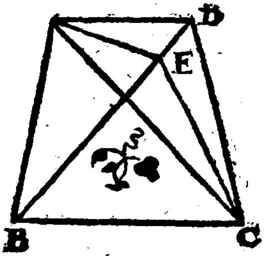
Theorema 28. Propositio 37.

Triangula super æqualibus basibus constituta & in eisdem parallelis, inter se sunt æqualia.



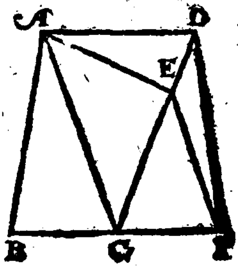
Theorema 29. Propositio 39.

Triangula æqualia super eandem basi, & ad easdem partes constituta: & in eisdem sunt parallelis.



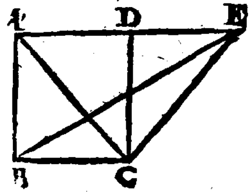
Thorema 30. Propositio 40.

Triangula æqualia super equalibus basibus, & ad easdem partes constituta, & in eisdem sunt parallelis.



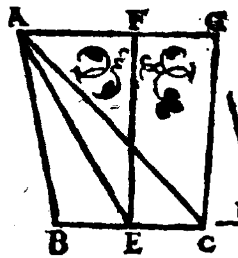
Theorema 31. Propositio 41.

Si parallelogrammum cum triangulo eandem basin habueris, non eisdemque fuerit parallelis, duplum erit parallelogrammum ipsius trianguli.



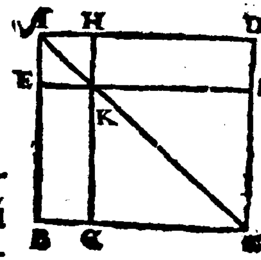
Problema 11. Propositio 42.

Dato triangulo æquale parallelogrammũ constitute in dato angulo rectilineo.



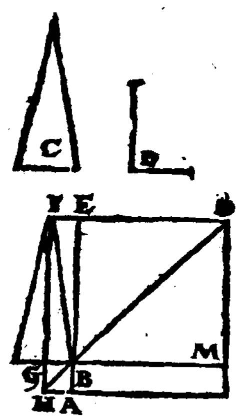
Theorema 32. Propositio 43.

In omni parallelogrammo, complementa eorũ quæ circa diametrũ sunt parallelogrammorũ, inter se sunt æqualia.



Problema 12. Propositio 44.

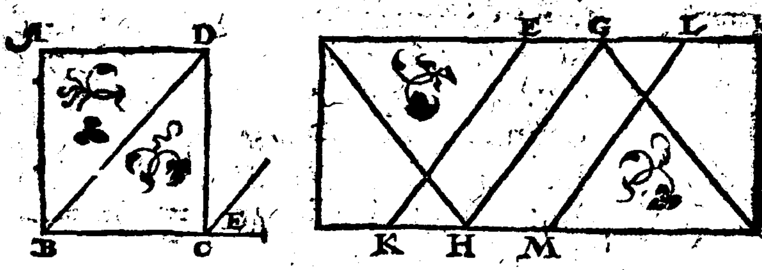
Ad datam rectam lineã dato triangulo æquale parallelogrammum applicare in dato angulo rectilineo.



Problema 13. Propositio 45.

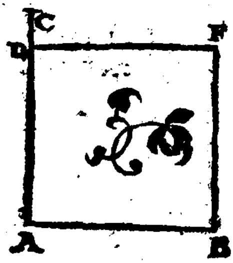
Dato rectilineo æquale parallelogramm
const

constituere in dato angulo rectilineo.



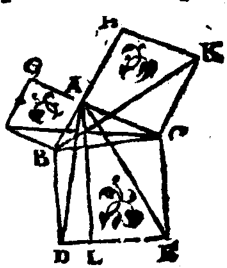
Theorema 41.Propositio 4.

A data recta linea quadratum describere.



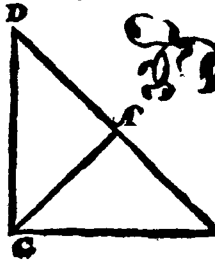
Theorema 33.Propositio 47.

In rectangulis triangulis, quadratum quod à latere rectum angulum ſubtendente deſcribitur, æquale eſt eis, quæ à lateribus rectum angulum continentibus deſcribũtur, quadratis.



Theorema 34.Propoſitio 48.

Si quadratum quod ab vno laterum trianguli describitur, æquale sit eis quæ à reliquis trianguli lateribus describuntur, quadratis: angulus comprehensus sub reliquis duobus trianguli lateribus, rectus est.



FINIS ELEMENTI I.

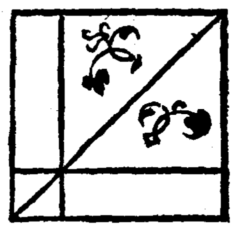
EVCLIDIS ELEMENTVM SECVNDVM.

DEFINITIONES.

1.

OMNE parallelogrammum rectangulum contineri dicitur ſub rectis duabus lineis, quæ rectum comprehendunt angulum.

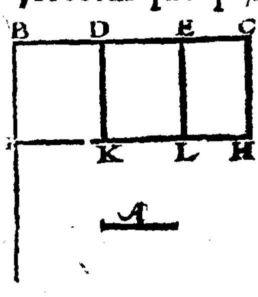
2.



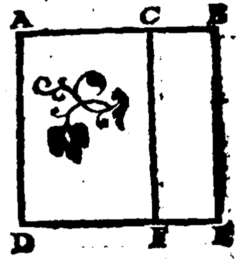
In omni parallelogrammo ſpatio, vnum quodlibet eorum, quæ circa diametrum illius ſunt parallelogrammorum, cum duobus cõplementis, Gnomo vocetur.

Theorema 1 Propoſitio 1.

Si fuerint duæ rectæ lineæ, ſeceturque ipſarum altera in quotcunq; ſegmenta: rectangulum comprehenſum ſub illis duabus rectis lineis, æquale eſt eis rectangulis, quæ ſub inſecta & quolibet ſegmẽtorum comprehenduntur.



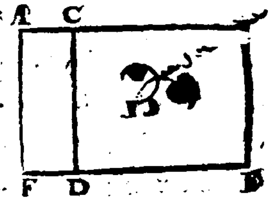
Theorema 2. Propoſitio 2.



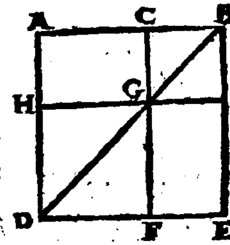
Si recta linea ſecta ſit vtcunq; rectangula quæ ſub tota & quolibet ſegmentorum comprææhendũtur ęqualia ſunt ei, quod à tota fit, quadrato.

Theorema 3. Propopoſitio. 3.

Si recta linea ſecta ſit vtcunque rectangulum ſub tota & vno ſegmentorum comprehenſum, æquale eſt & illi quod ſub ſegmentis comprehenditur rectangulo & illi, quod à prædicto ſegmẽto deſcribitur, quadrato.



Theorema 4. Propoſitio 4.

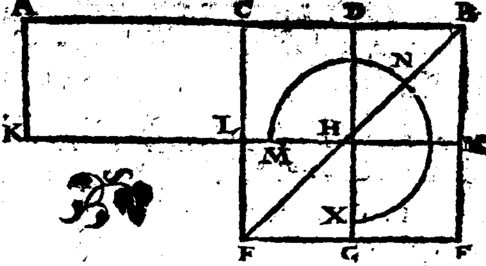


Si recta linea ſecta ſit vtcunq; : quadratum quod à tota deſcribitur æquale eſt & illis quæ à ſegmentis deſcribuntur quadratis, & ei quod bis ſub ſegmentis comprehenditur rectangulo.

Theorema 5. Propoſitio 5.

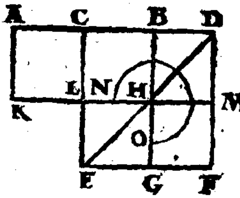
Si recta linea ſecetur in æqualia & non æqualia: rectangulum ſub inæqualibus ſegmentis

mentis to-tius comprehensú, vnà cum quadrato quod ab inter media ſectionum, æquale eſt ei quod à dimidia deſcribitur, quadrato.



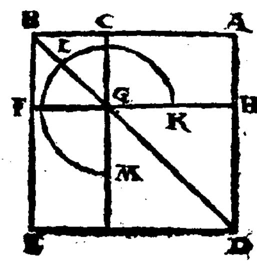
Theorema 6. Propoſitio 6.

Si recta linea bifariam ſecetur, & illi recta quædam linea in rectum adijciatur, rectangulum comprehenſum ſub tota cum adiecta & adiecta ſimul cum quadrato à dimidia, æquale eſt quadrato à linea quæ tú ex dimidia, tum ex adiecta componitur, tanquam ab vna



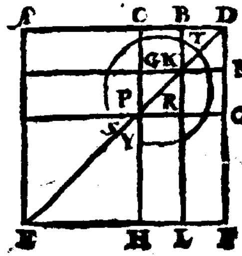
Theorema 7. Propoſitio 7.

Si recta linea ſecetur vtcunque, quod à tota, quodq; ab vno ſegmẽtorum, vtraq; ſimul quadrata, æqualia ſunt & illi quod bis ſub tota & dicto ſegmento cóprehenditur rectangulo, & illi quod à reliquo ſegmẽto fit, quadrato.



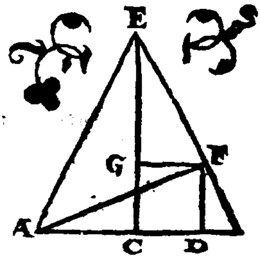
Theorema 8. Propoſitio 8.

Si recta linea ſecetur vtcunque rectangulũ quater comprehenſum ſub tota & vno ſegmentorum, cum eo quod à reliquo ſegméto fit, quadrato, ęquale eſt ei quod à tota & dicto ſegméto, tanquã ab vna linea deſcribitur quadrato.



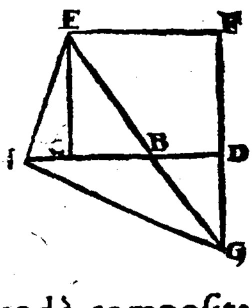
Theorema 9. Propoſitio 9.

Si recta linea ſecetur in æqualia & non æqualia: quadrata quę ab inæqualibus totius ſegmentis fiunt, duplicia ſunt & eius quod à dimidia, & eius quod ab intermedia ſectionum fit, quadratorum.



Theorema 10. Propoſitio 10.

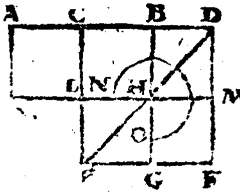
Si recta linea ſecetur bifariam, adijciatur autem ei in rectum quæpiam recta linea: quod à tota cũ adiuncta & quod ab adiuncta, vtraq; ſimul quadrata, duplicia ſunt, & eius quod à dimidia, & eius quod à compoſita ex di-



ex dimidia & adiuncta; tanquam ab vna descriptum sit quadratorum.

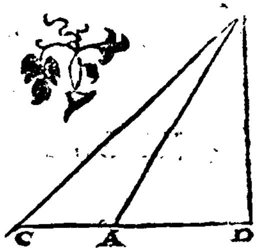
Problema 1. Propositio 11.

Datam rectam lineã secare, vt comprehensum sub tota & altero segmẽtorum rectangulum, æquale sit ei quod à reliquo segmento fit, quadrato.



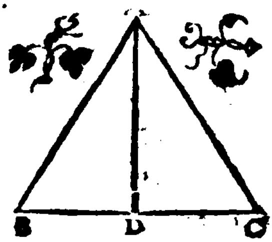
Theorema 11. Propositio 12.

In amblygonijs triãgulis, quadratum quod fit à latere angulum obtusum subtendente, maius est quadratis, quæ fiunt à lateribus obtusum angulum comprehendentibus, pro quantitate rectanguli bis comprehensi & ab vno laterũ quæ sunt circa obtusum angulũ, in quod cùm protractum fuerit, cadit perpendicularis, & ab assumpta exterius linea sub perpendiculari prope angulũ obtusum.



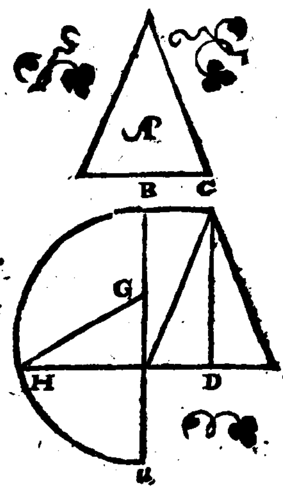
Theorema 12. Propositio 28.

In oxygonijs triangulis quadratum à latere angulum acutum subtendente, minus est quadratis, quæ fiunt à lateribus acutum angulum comprehendentibus, pro quantitate rectanguli bis comprehensi, & ab vna laterum, quæ sunt circa acutum angulum, in quod perpendicularis cadit, & ab assumpta interius linea sub perpendiculari prope acutum angulum.



Problema 2. Propositio 14.

Dato rectilineo æquale quadratum constituere.



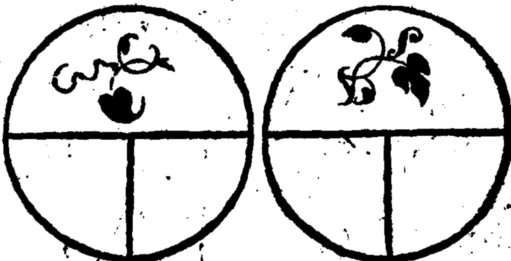
ELEMENTI II. FINIS.

EVCLIDIS ELEMENTVM TERTIVM.

DEFINITIONES.

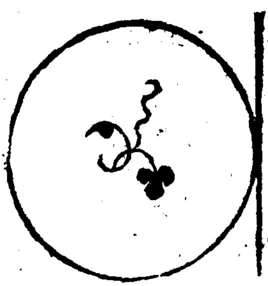
1.

Aequales circuli ſunt quorũ diametri ſunt ęquales vel quorũ quæ ex centris, rectæ lineæ sũt æquales.



2.

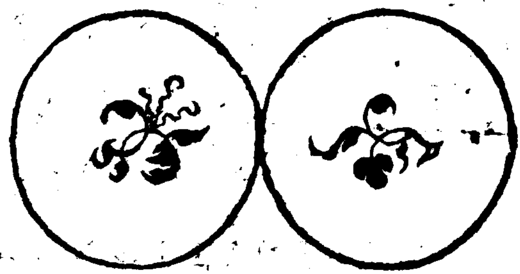
Recta linea circulum tãgere dicitur, quæ cùm circulum tangat, ſi producatur, circulum non ſecat.



3. Cir-

3.

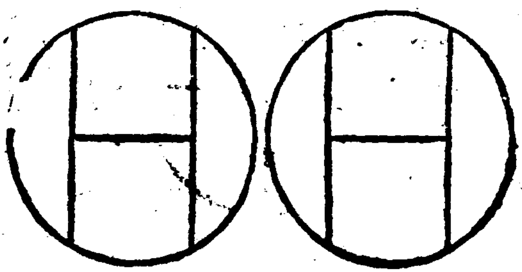
Circuli ſeſe mutuò tangere dicuntur: qui ſeſe mutuo



tangentes, ſeſe mutuo non ſecant.

4.

In circulo æqualiter diſtare à centro rectæ lineæ dicuntur, cùm perpendiculares, quæ à centro in ipſas ducuntur, ſunt æquales. Lógius autem abeſſe illa dicitur in quã maior perpendicularis cadit.



5.

Segmentum circuli eſt, figura quæ ſub recta linea & circuli peripheria comprehenditur.

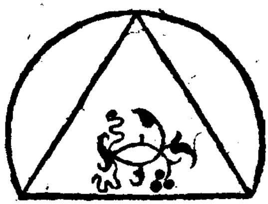
6.

Segmenti autem angelus eſt, qui ſub recta linea

linea & circuli peripheria comprehenditur.

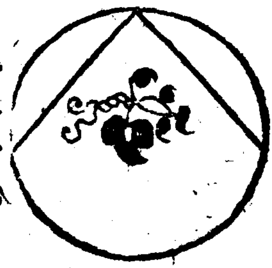
7.

In segmento autem angulus est, cum in segmenti peripheria sumptum fuerit quodpiam punctum, & ab illo in terminos rectæ eius lineæ, quæ segmenti basis est, adiũctę fuerint rectæ lineæ: is, inquam, angulus ab adiũctis illis lineis comprehensus.



8.

Cùm verò comprehendentes angulum rectæ lineæ aliquam assumitur peripheriam, illi angulus insistere dicitur.



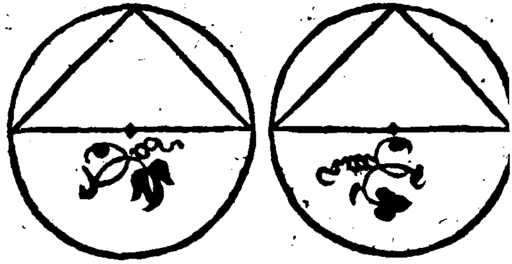
9.

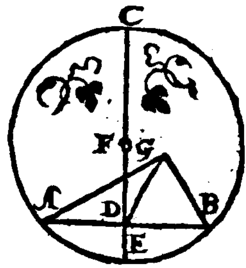
Sector autem circuli est, cùm ad ipsius circuli cẽtrum constitutus fuerit angulus, comprænhensa nimirum figura, & à rectis lineis angulum continentibus, & à peripheria ab illis assumpta.

10.

Similia circuli segmenta sunt, quæ angulos capiunt

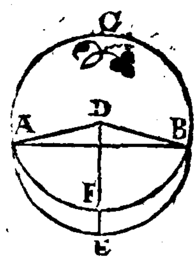
capiunt ęquales aut in quibus anguli inter ſe ſunt æquales.





Problema 1. Propoſitio 1.

Dati circuli centrum reperire.

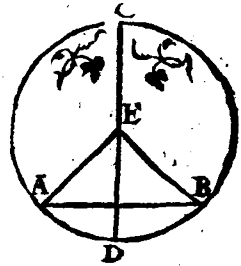


Theorema 1. Propoſitio 2.

Si in circuli peripheria duo quælibet puncta accepta fuerint, recta linea quæ ad ipſa puncta adiungitur, intra circulum cadet.

Theorema 2. Propoſitio 3.

Si in circulo recta quædam linea per centrum extenſa quandam non per centrum extenſam bifariam ſecet: & ad angulos rectos ipſam ſecabit. Et ſi ad angulos rectos eam ſecet, bifariam quoque eam ſecabit.



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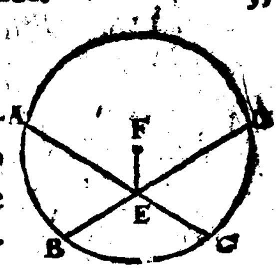
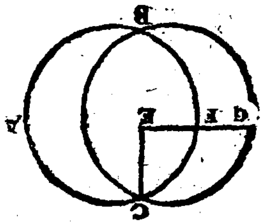
Theorema 3. Propositio 4.



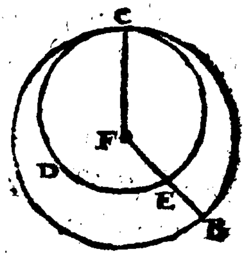
Si in circulo duæ rectę lineæ ſeſe mutuo ſecent nõ per centrum extenſæ ſeſe mutuò bifariam non ſecabunt.

Theorema 4. Propoſitio 5.



Si duo circuli ſeſe mutuò ſecent, non erit illorum idem centrum.

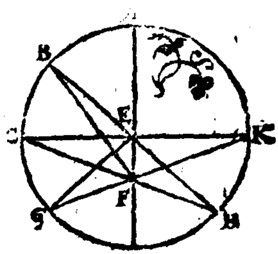
Theorema 5. Propoſitio 6.



Si duo circuli ſeſe mutuò interius tangent, eorum non erit idem centrum.

Theorema 6. Propoſitio 7.

Si in diametro circuli quodpiam ſumatur punctum, quod circuli centrum non ſit, ab eoq́; puncto in circulum quædam rectæ lineæ cadant: maxima quidem erit ea in qua centrũ, minima verò reliqua, aliarum verò propinquior illi quæ per centrum du-

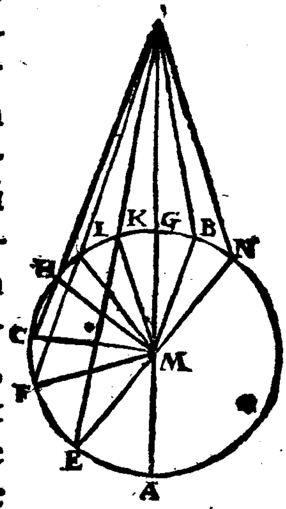


citur

citur, remotiore semper maior est. Duæ autem solùm rectæ lineæ æquales ab eodem puncto in circulum cadunt ad vtrasque partes minimæ.

Theorema 7. Propositio 8.

Si extra circulum sumatur punctum quodpiam, ab eoque puncto ad circulum deducantur rectæ quædam lineæ, quarum vna quidem per centrum protendatur, reliquæ verò vt libet: in cauam peripheriam cadentium rectarum linearum minima quidem est illa, quæ per centrum ducitur: aliarum autem propinquior ei, quæ per centrum transit, remotiore semper maior est: in conuexam verò peripheriã cadentium rectarum linearũ minima quidem est illa, quæ inter punctum & diametrum interponitur: aliarum autem, ea quæ propinquior est minimæ, remotiore semper minor est. Duæ autem tantùm rectæ lineæ æquales ab eo puncto in ipsum circulum cadunt, ad vtrasque partes minimæ.



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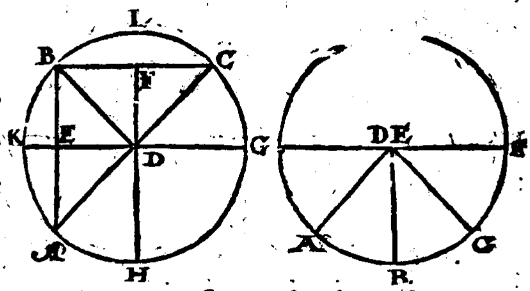
Theorema 8. Propositio 9.

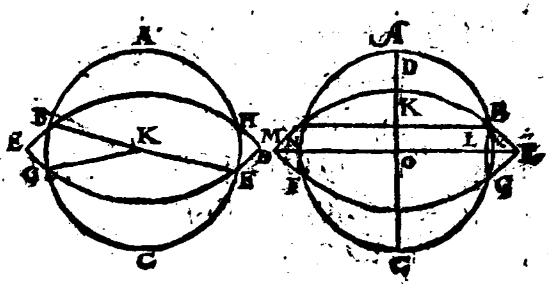
Si in circulo acceptum fuerit punctum aliquod, & ab eo puncto ad circulum cadent plures quam duæ rectæ, lineæ, æquales, acceptú



punctum centrum ipsius erit circuli.

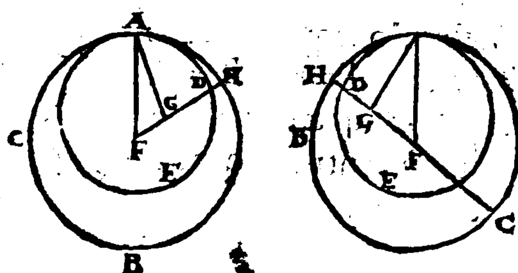
Theorema 9. Propositio 10.

Circulus circulum in pluribus quàm duobus punctis non secat.



Theorema 10. Propositio 11.

Si duo circuli sese intus cótingãt atque accepta

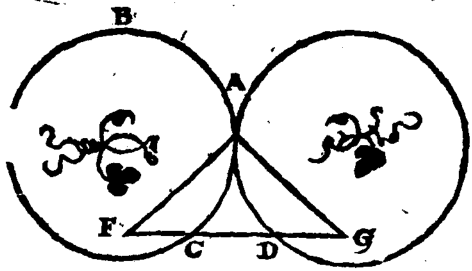


E fuerint

fuerint eorum centra, ad eorum centra ad
iuncta recta linea & producta in contactu
circulorum cadet.

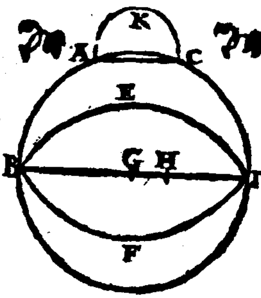
Theorema 11. Propositio 12.

Si duo circuli sese exterius cõtingant, line
recta q̃ ad cẽtra eorũ adiũgitur, per cõtactũ illũ trãsibit.



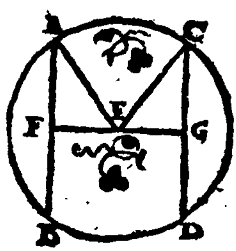
Theorema 12. Propositio 13.

Circulum circulum non tangit in pluribus punctis, quem vno, siue intus siue extra tangat.



Theorema 13. Propopositio. 14.

In circulo æquales rectæ lineæ æqualiter distant à centro. Et quæ æqualiter distant à centro, æquales sunt inter se.



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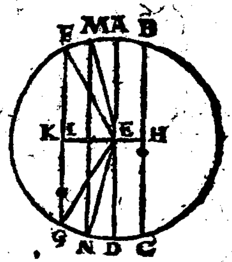
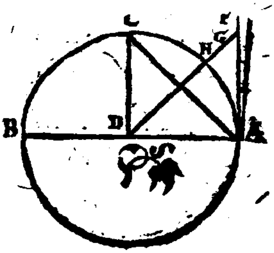
Theorema 14. Propositio 15.



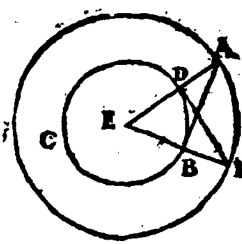
In circulo maxima, quidem linea est diameter: aliarum autem propinquior centro, remotiore semper maior.

Theorema 15. Propositio 16.

Quæ ab extremitate diametri cuiusque circuli ad angulos rectos ducitur, extra ipsum circulum cadet, & in locum inter ipsam rectam lineam & peripheriam comprehensum, altera recta linea nõ cadet. Et semicirculi quidem angulus quouis angulo acuto rectilineo maior est, reliquus autem minor.



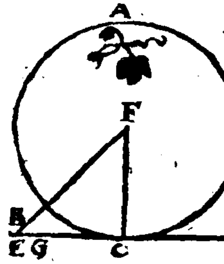
Problema 16. Propositio 17.



A dato puncto rectam lineam ducere, quæ datum tangat circulum.

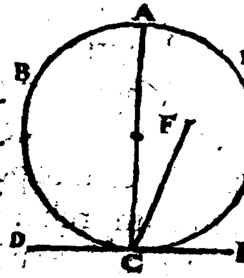
 Theo-

Theorema 16. Propositio 18.



Si circulum tangat recta quæpiam linea, à centro autem ad contactum adiungatur recta quædam linea: quæ adiũcta fuerit ad ipsam contingentem perpendicula erit.

Theorema 17. Propositio 19.



Si circulum tetigerit recta quæpiam linea, à contactu autem recta linea ad ãgulos rectos ipsi tangenti excitetur, in excitata erit centrum circuli.

Theorema 18. Propositio 20.

In circulo angulus ad cẽtrum duplex est anguli ad peripheriam, cũ fuerit eadẽ peripheria basis angulorum.

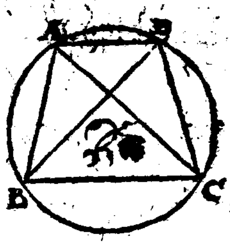
Theorema 19. Propositio 21.

In circulo, qui in eodem segmẽto sunt anguli, sunt inter se æquales.

Theo-

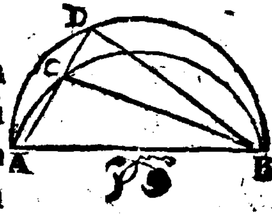
Theorema 20. Pro-
positio 22.

Quadrilaterorum in cir-
ulis descriptorum angu
i qui ex aduerso, duobus
ectis sunt æquales.



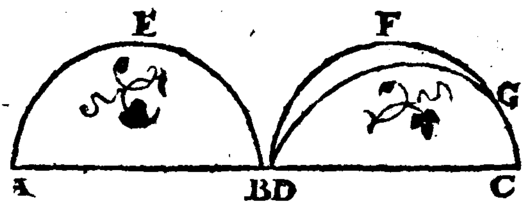
Theorema 21. Pro-
positio 23.

iuper eadem recta linea
luo segmenta circulorũ
imilia & inæqualia non
onstituentur ad easdem
partes.



Theorema 22. Propositio. 24.

Super æ
qualib'
rectis li
neis si-
milia
circulo
rum se-
gmenta sunt inter se æqualia.

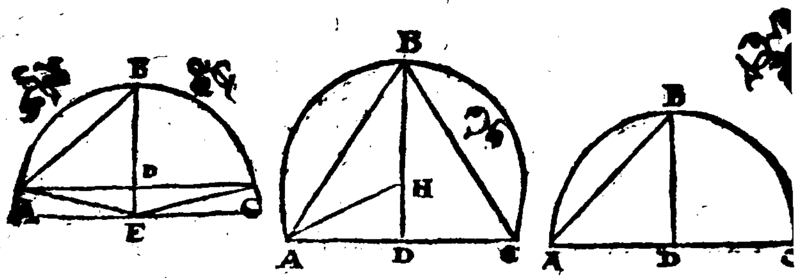


Problema 23. Propo-
sitio 25.

Circuli segmento dato, describere circu'ũ

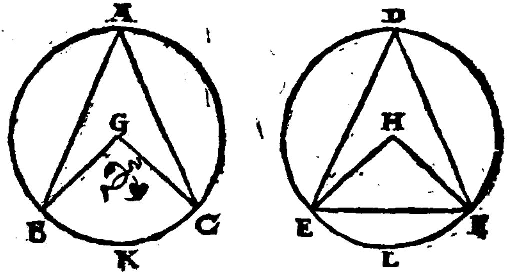
 cuius

cuius est segmentum.



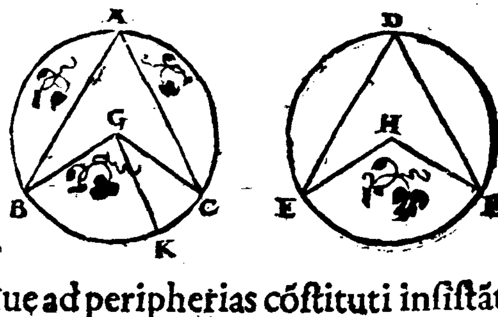
Theorema 22. Propositio 26.

In æqualibus circulis, æquales anguli æqua lib.pe riphe rijs in sistũt siue ad cẽtra, siue ad peripherias constituti insistant.



Theorema 24. Propositio 27.

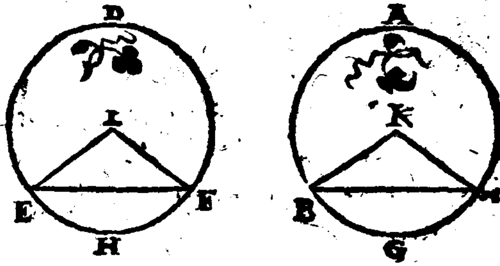
In æqualibus circulis, anguli qui æqualibus peripherijs insistũt sunt inter se ęquales siue ad centra, siue ad peripherias cõstituti insistãt.



Theo-

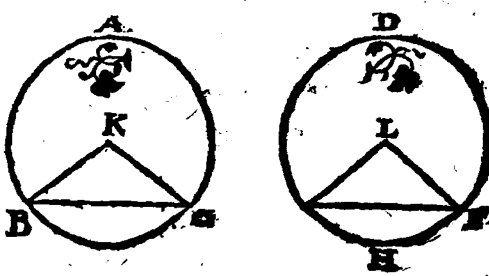
Theorema 25. Propositio. 28.

In æqualibus circulis æquales rectæ lineæ æquales peripherias auferũt maiorẽ quidẽ maiori, minorem autem minori.



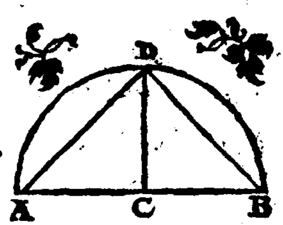
Theorema 26. Propositio 29.

In æqualib. circulis, æquales peripherias æquales rectæ lineæ subtendunt.



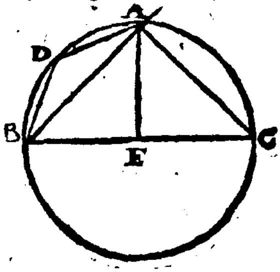
Theorema 4. Propositio 30.

Datam peripheriam bifariam secare.



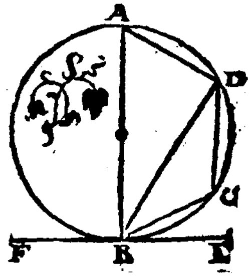
Problema 27. Propositio 31.

In circulo angulus qui in semicirculo, rectus est: qui autem in maiore segmento, minor recto: qui verò in minore segméto, maior est recto. Et insuper angulus maioris segmenti, recto quidem maior est, minoris auté segméti angulus minor est recto.



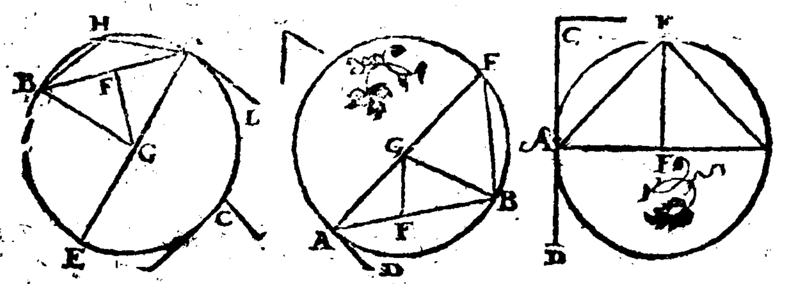
Theorema 28. Propositio 32.

Si circulum tetigerit aliqua recta linea, à contactu auté producatur quædam recta linea circulum secans: anguli quos ad contingentem facit æquales sunt ijs qui in alternis circuli segmétis consistunt, angulis.



Problema 5. Propositio 33.

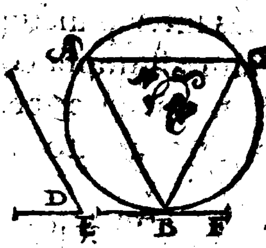
Super data recta linea describere segmentú circuli quod capiat angulum æqualem dato angulo rectilineo.



Proble

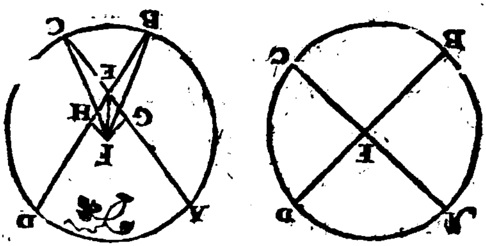
Problema 6. Propoſitio 34

A dato circulo ſegmentum abſcindere capiens angulum æqualem dato angulo rectilineo.



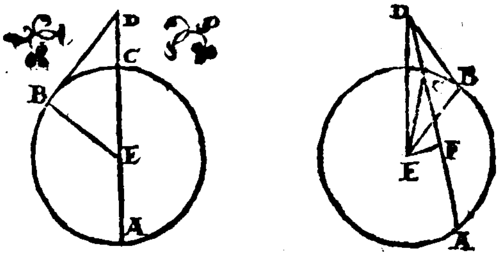
Theorema 29. Propoſitio 35.

Si in circulo duæ rectæ lineæ ſeſe mutuò ſecuerint, rectangulum comprehenſum ſub ſegmẽtis vnius, æquale eſt ei, quod ſub ſegmentis alterius comprehenditur, rectangulo.



Theorema 30. Propoſitio 36.

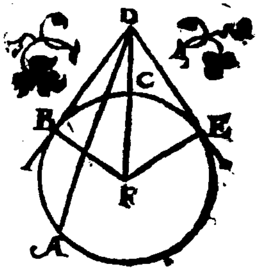
Si extra circulũ ſumatur pũctũ aliquod, ab eoque in circulũ cadant duæ rectæ lineæ, quarum altera quidem circulũ ſecet, altera



verò tangat: quod sub tota secante & exterius inter punctum & conuexam peripheriam assumpta comprehenditur rectangulum, æquale erit ei, quod à tangente describitur, quadrato

Theorema 31. Propositio 37.

Si extra circulũ sumatur punctum aliquod, ab eoque puncto in circulum cadant duæ rectæ lineæ, quarum altera circulum secet, altera in eum incidat, sit autem quod sub tota secãte & exterius inter punctum & conuexã peripheriam assumpta, comprehenditur rectangulum, æquale ei, quod quadrato, incidens ipsa circulum tanget.



ELEMENTI III. FINIS.

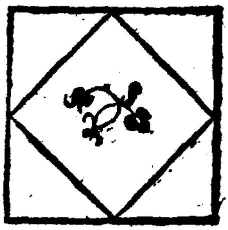
EVCLI

EVCLIDIS ELEMENTVM QVARTVM.

DEFINITIONES.

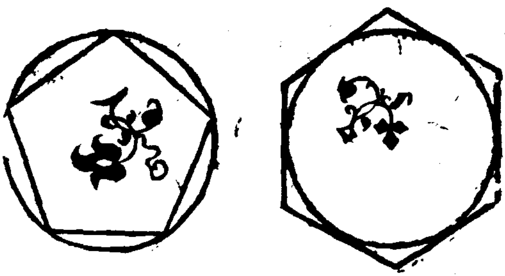
1.

Figura rectilinea in figura rectilinea inſcribi dicitur, cũ ſinguli eius figuræ quæ inſcribitur, anguli ſinguli latera eius, in qua inſcribitur, tangunt.



2.

Similiter & figura circum figuram deſcribi dicitur, quum ſingula eius quę circumſcribitur, latera ſingulos eius figurę angulos tetigerint, circum quam ille deſcribitur.



3.

Figura rectilinea in circulo inſcribi dicitur, quũ ſinguli eius figuræ quæ inſcribitur, angu-

anguli tetigerint circuli peripheriam.

4.

Figura verò rectilinea circa circulum describi dicitur, quum singula latera eius, quæ circum scribitur, circuli peripheriam tangunt.

5.

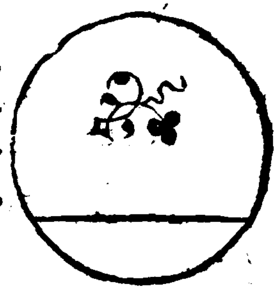
Similiter & circulus in figura rectilinea inscribi dicitur, quũ circuli peripheria singula latera tangit eius figuræ, cui inscribitur.

6.

Circulus autem circum figuram describi dicitur, quum circuli peripheria singulos tangit eius figuræ, quam circumscribit, angulos.

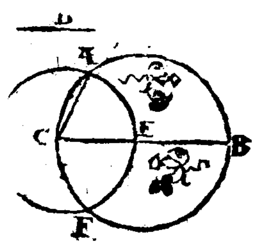
7.

Recta linea in circulo accommodari seu coaptari dicitur, quum eius extrema in circuli peripheria fuerint.



Problema 1. Propositio. 1.

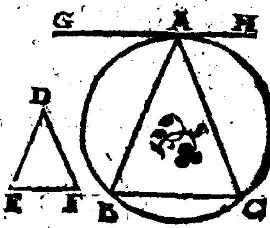
In dato circulo, rectam lineam accommodare æqualem datæ rectæ lineæ quæ circuli diamatro nõ sit maior.



Pro-

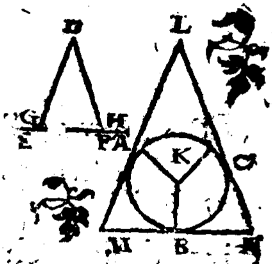
Problema 2. Pro-
poſitio 3.

In dato circulo, triangulũ deſcribere dato triangulo æquiangulum.



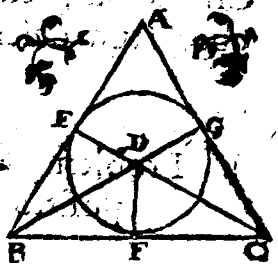
Problema 3 Pro
poſitio 3.

Circa datum circulũ triangulũ, deſcribere dato triangulo ęquiangulum.



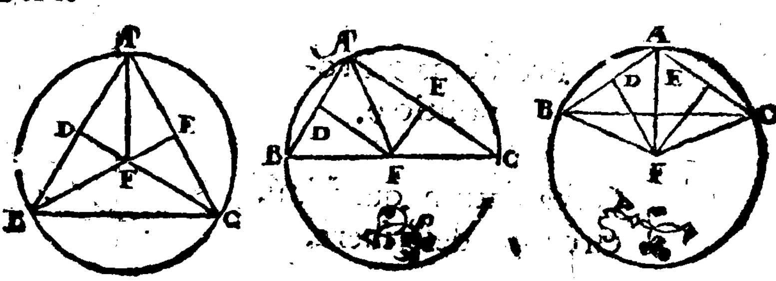
Problema 4. Pro-
poſitio 4.

In dato triangulo circulum inſcribere.



Problema 5. Propoſitio 5.

Circa datum triangulum, circulum deſcribere.



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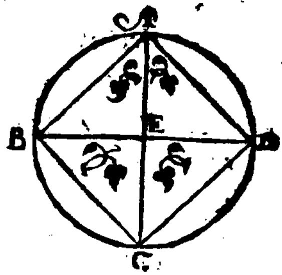
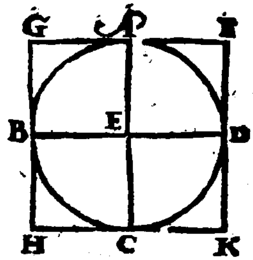
Problema 6. Propositio 6.

In dato circulo quadratum describere.



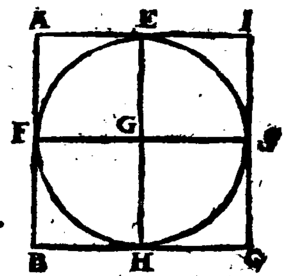
Problema 7. Propositio 7.

Circa datum circulum, quadratum describere.



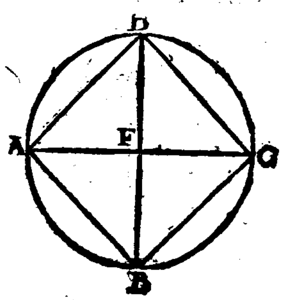
Problema 8. Propositio 8.

In dato quadrato circulum inscribere.



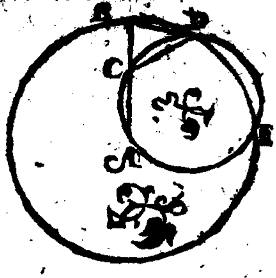
Problema 9. Propositio 9.

Circa datum quadratũ, circulum describere.



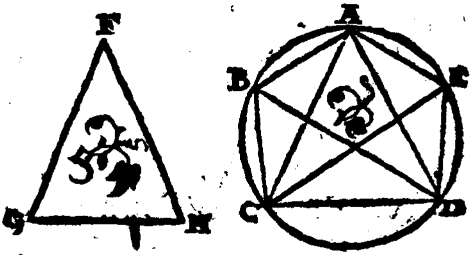
Problema 10. Propositio 10.

Isosceles triangulum cõstituere, quod habeat vtrunque eorum, qui ad basim sunt angulorum, duplum reliqui.



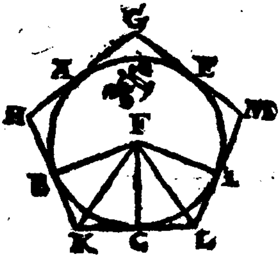
Theorema 11. Propositio 11.

In dato circulo, pentagonũ æquilaterũ & æquiãgulum inscribere.



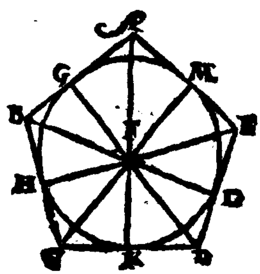
Problema 12. Propositio 12.

Circa datum circulum, pentagonum æquilaterum æquiangulum describere.



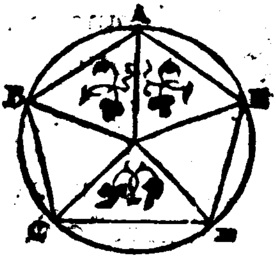
Problema 13. Propositio 13.

In dato pentagono equilatero & æquiãgulo cirlum inscribere.



Prbole.

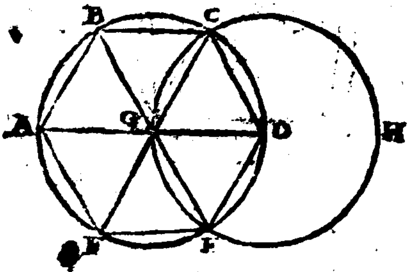
Problema 14. Propositio 14.

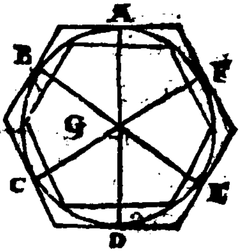


Circa datum pentagonú, æquilaterum & æquiangulum, circulum describere.

Problema 15. Propositio 15.

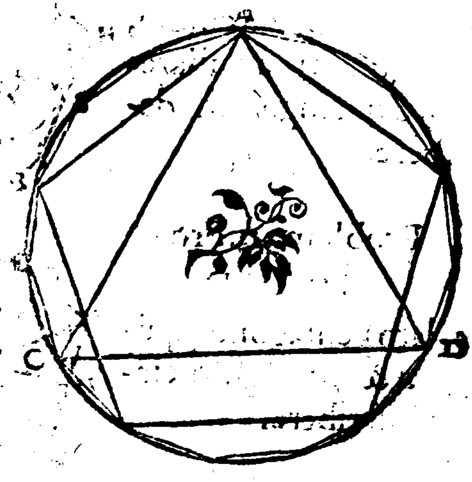
In dato circulo hexagonum & æquilaterú & æquiangulum inscribere.





Propositio 16. Theorema 16.

In dato circulo quintidecagonum & æquilaterú & æquiangulum describere.



Elementi quarti finis.

EVCLIDIS ELEMENTVM QVINTVM.

DEFINITIONES.

1.

PARS est magnitudo magnitudinis minor maioris, quum minor metitur maiorem.

2.

Multiplex autem est maior minoris, cùm minor metitur maiorem.

3.

Ratio est duarum magnitudinum eiusdem generis mutua quædam secundum quantitatem habitudo.

4.

Proportio verò est rationum similitudo.

5.

Rationem habere inter se magnitudinis dicuntur, quæ possunt multiplicatæ sese mutu superare.

6.

In eadem ratione magnitudinis dicuntur esse, prima ad secundam, & tertia ad quartam, cum primæ & tertiæ æquè multiplicia, à secundæ & quartæ æquè multiplicibus, qualis-

qualiſcunque ſit hæc multiplicatio, vtrum ab vtroque: vel vna deficiunt, vel vna æqualia ſunt, vel vna excedũt, ſi ea ſumantur quæ inter ſe reſpondent.

7.

Eandem autem habentes rationem magnitudines, proportionales vocentur.

8.

Cum verò æquè multiplicium, multiplex primæ magnitudines exceſſerit multiplicem ſecundæ, at multiplex tertiæ non exceſſerit multiplicem quartæ: tunc prima ad ſecundam, maiorem rationem habere dicetur quàm tertia ad quartam.

9.

Proportio autem in tribus terminis pauciſſimis conſiſtit.

10.

Cùm autem tres magnitudines proportionales, fuerint, prima ad tertiam, duplicatam rationẽ habere dicitur eius quam habet ad ſecundam. At cùm quatuor magnitudines proportionales fuerint, prima ad quartam triplicatam rationem habere dicitur eius quam habet ad ſecundam: & ſemper deinceps vno amplius, quamdiu proportio extiterit.

11.

Homologæ, ſeu ſimiles ratione magnitudines dicuntur, antecedentes quidem antecedentidentibus, consequentes verò consequentib.

12.

Alterna ratio, est sumptio antecedentis cõparati ad antecedentem, & consequentis ad consequentem.

13.

Inuersa ratio, est sumptio consequentis, ceu antecedentis, ad antecedentem velut ad cõsequentem.

14.

Compositio rationis, est sumptio antecedẽtis cũ consequente ceu vnius ad ipsum consequentem.

15.

Diuisio rationis, est sumptio excessus quo consequẽtem superat antecedens ad ipsum consequentem.

16.

Conuersio rationis, est sumptio antecedentis ad excessum, quo superat antecedens ipsum consequentem.

17.

Ex æqualitate ratio est, si plures duabus sint magnitudines, & his aliæ multitudine pares quæ binæ sumantur, & in eadem ratione, quum vt in primis magnitudinibus prima ad vltimam sic & in secundis magnitudinibus prima ad vltimam sese habuerit, vel aliter, sumptio extremorum per subductionẽ mediorum.

18.

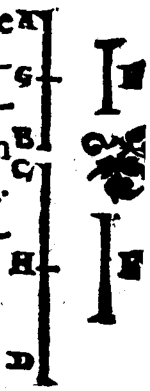
Ordinata proportio eſt, cùm fuerit quem admodum antecedens ad conſequentem i antecedens ad conſequentem: fuerit etiar vt conſequens ad aliud quidpiam, ita conſ quens ad aliud quidpiam.

19.

Perturbata autem proportio eſt, tribus po ſitis magnitudinibus, & alij quæ ſint h multitudine pares, cùm vt in primis quic magnitudinibus ſe habet antecedens ad cō ſequentem, ita in ſecundis magnitudinibu antecedens ad conſequentem: vt autem i primis magnitudinibus conſequēs ad aliu quidpiam ad antecedentem.

Theorema 1. Propo- ſitio 1.

Si ſint quotcunque magnitudines quotcunque magnitudinum æqualium numero ſingulæ ſingularum æquè multiplices, quàm multiplex eſt vnius vna magnitudo, tam multiplices erunt, & omnes omnium.

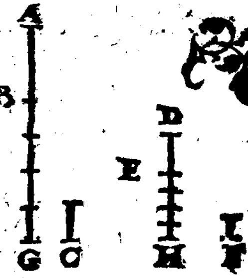


Theorema 2. Propoſitio 2.

Si prima ſecundæ æquè fuerit multiplex

atqu

atque tertia quartæ, fuerit autẽ & quinta secũdæ æquè multiplex, atque sexta quartæ: erit & cõposita prima cũ quinta, secundę æquè multiplex atque tertia cum sexta, quartæ.

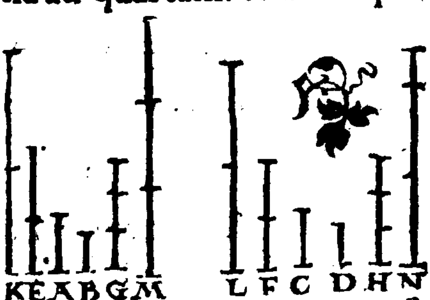


Theorema 3. Propositio 3.

Si prima secundæ æquè multiplex atque tertia quartæ sumantur autem æquè multiplices primæ & tertiæ erit & ex æquo sumptarum vtraque vtriusque æquè multiplex; altera quidem secundæ, altera autem.

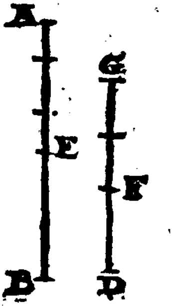
Theorema 4. Propositio 4.

Si prima ad secundam, eandem habuerit rationem, & tertia ad quartam: etiam æquè multiplices primæ & tertię, ad ęquè multiplices secundæ & quartæ iuxta quãuis multiplicationẽ, eandem habebunt rationem, si prout inter se



 respon-

respondent, ita sumptæ fuerint.



Theorema 5. Propositio 5.

Si magnitudo magnitudinis æquè fuerit multiplex, atque ablata ablatæ: etiam reliqua reliquæ ita multiplex erit, vt tota totius.

Theorema 6. Propositio 6.

Si duæ magnitudines, duarum magnitudinum sint æquè multiplices, & detractæ quædam sint earundem æquè multiplices: & reliquæ eisdem aut æquales sunt, aut æquè ipsarum multiplices.

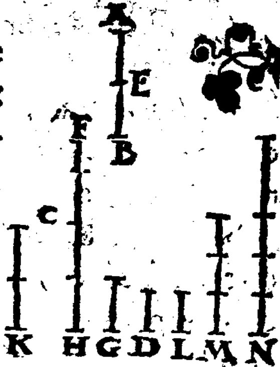
Theorema 7. Propositio 7.

AEquales ad eandem, eadem habent rationem: & eandem ad æquales.

Theorema 8. Propositio 8.

In equalium magnitudinum maior, ad eandem

dem maiorem ratione habet, quàm minor: & eadem ad minorem: maiorem rationem habet, quàm ad maiorem.



Theorema 9. Propositio 9.

Quæ ad eandem, eandem habent rationem, æquales sunt inter se: & ad quas eadem, eandem habet rationem, eæ quoque sunt inter se æquales.

Theorema 10. Propositio 10.

Ad eandem magnitudiné rationem habentium, quæ maiorem rationé habet, illa maior est ad quamauté eadem maiorem rationem habet, illa minor est.

Theorema 11. Propositio 11.

Quæ eidem sunt eædem rationes, & inter se sunt eædem.

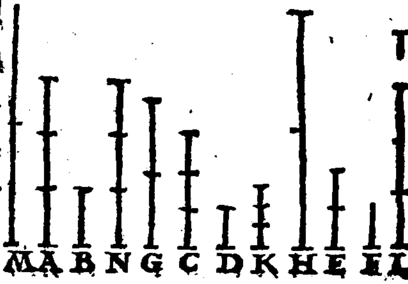
Theorema 12. Propoſitio 12.

Si ſint magnitudines quotcunque proportionales, quemadmodũ ſe habuerit vna antecedentium ad vnã

cõſequentium, ita ſe habebunt omnes antecedentes ad omnes conſequentes.

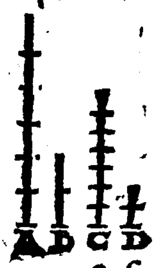
Theorema 13. Propoſitio 13.

Si prima ad ſecundum, eandem habuerit rationem, quam tertia ad quartam, tertia verò ad quartam maiorem rationem habuerit, qũàm quinta ad ſextam, prima quoq; ad ſecundam maiorem rationem habebit, quàm quinta ad ſextam.



Theorema 14. Propoſitio 14.

Si prima ad ſecundam eandem habuerit rationem, quam tertia ad quartam, prima verò quàm tertia maior fuerit: erit & ſecunda maior quam quarta. Quòd ſi prima fuerit æqualis tertiæ, erit

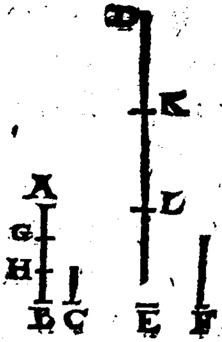


&ſe-

& secunda æqualis quartæ: si verò minor, & minor erit.

Theorema 15. Propositio. 15.

Partes cum pariter multiplicibus in eadem sunt ratione, si prout sibi mutuo respondent, ita sumantur.

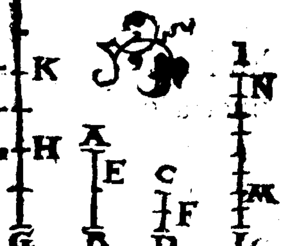


Theorema 16. Propositio 16.

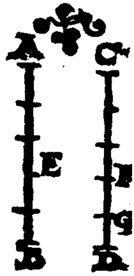
Si quatuor magnitudines proportionales fuerint, vt vicissim proportionales erunt.

Theorema 17. Propositio 17.

Si compositę magnitudines proportionales fuerint hæ quoq; diuisæ proportionales erunt.

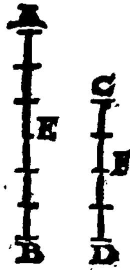


Theorema 18. Propoſitio 18.



Si diuiſæ magnitudines ſint proportionales, hæ quoque compoſitæ proportionales erunt.

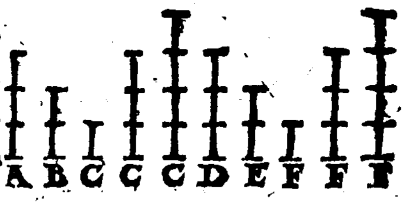
Theorema 19. Propoſitio 19.



Si quemadmodum totú ad totum, ita ablatum ſe habuerit ad ablatum: & reliquum ad reliquum, vt totum ad totum ſe habebit.

Theorema 20. Propoſitio 20.

Si ſint tres magnitudines, & aliæ ipſis æquales numero, quæ binæ & in eadem ratione ſumatur, ex æquo autẽ prima quàm tertia maior fuerit: erit & quarta, quá ſexta maior. Quòd ſi prima tertiæ fuerit æqualis, erit & quarto æqualis ſextæ: ſin illa minor, hæc quoque minor erit.



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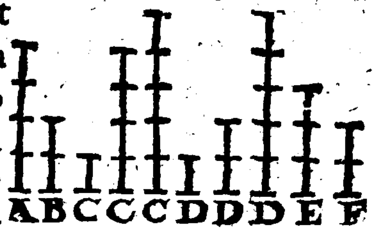
Theorema 21. Propositio 21.

Si sint tres magnitudines & aliæ ipsis æquales numero quæ binæ & in eadem ratione sumantur, fuerit que perturbata earũ proportio ex æquo autem prima quam tertia maior fuerit



erit & quarta quam sexta maior: quòd si prima tertiæ fuerit æqualis, erit & quarta æqualis sextæ: sin illa minor, hæc quoque minor erit.

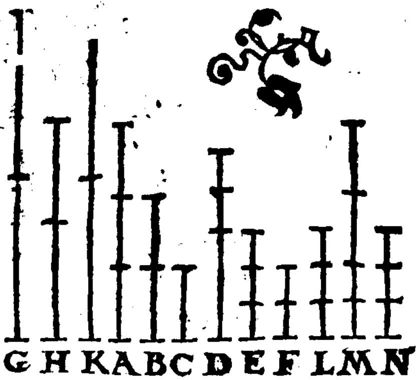
Theorema 22. Propositio 22.

Si sint quotcunq; magnitudines, & alię ipsis ęquales numero, quę binæ in eadẽ ratione sumantur, & ex æqualitate in eadem ratione erunt.

Theorema 23. Propositio 23.

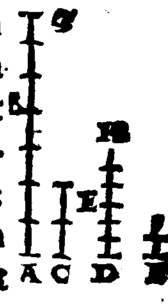
Si sint tres magnitudiẽs, aliæq; ipsis æquales nume-

numero, quam binæ in eadem ratione ſumantur, fuerit autẽ perturbata earũ proportio: etiã ex ęqualite in eadem ratione erunt.



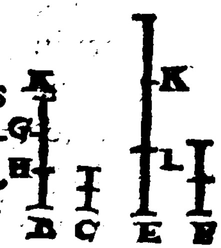
Theorema 24. Propoſitio 24.

Si prima ad ſecundam, eandem habuerit rationem, quam tertia ad quartam, habuerit autem & quinta ad ſecundam, eandem rationem quam ſexta ad quartam: etiam compoſita prima cũ quinta ad ſecundam eandem habebit rationem, quam tertia cum ſexta ad quartam.



Theorema 25. Propoſitio 25.

Si quatuor magnitudines proportionales fuerint, maxima & minima reliqui duabus maiores erunt.



ELEMENTI V. FINIS.

EVCLIDIS ELEMEVTVM SEXTVM.

DEFINITIONES.

1.

SImiles figuræ rectilineæ sunt, quę & angulos singulos singulis æquales habent, atque etiam latera, quæ circum angulos æquales, proportionalia.

2.

Reciprocæ autem figuræ sunt, cum in vtraque figura antecedentes & consequentes rationum termini fuerint.

3.

Secundum extremam & mediam rationem recta linea secta esse dicitur, cum vt tota ad maius segmentum, ita maius ad minus se habuerit.

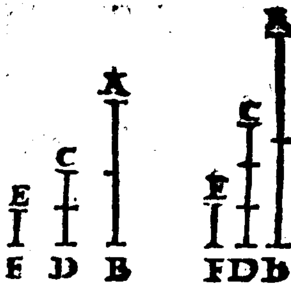
4.

Altitudo cuiusque figuræ, est linea perpendicularis à vertice ad basin deducta.

5. Ra-

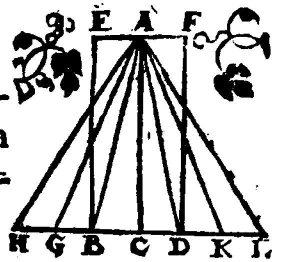
5.

Ratio ex rationibus cōponi dicitur, cum ratio num quantitatis inter ſe multiplicatæ aliqua effecerint rationem.



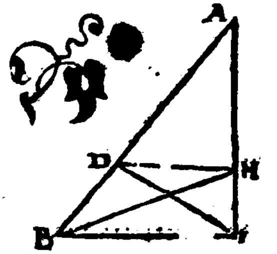
Theorema 1. Propoſitio 1.

Triangula & parallelogramma, quorum eadem fuerit altitudo, ita ſe habent inter ſe vt baſes.



Theorema 2. Propoſitio 2.

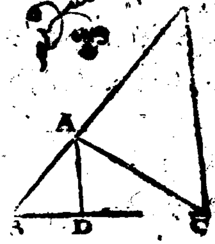
Si ad vnum trianguli latus parallela ducta fuerit recta quædam linea: hæc proportionaliter ſecabit, ipſius trianguli latera. Etſi trianguli latera proportionaliter ſecta fuerint: quæ ad ſectiones adiūcta fuerit recta linea, erit ad reliquum ipſius trianguli latus parallela.



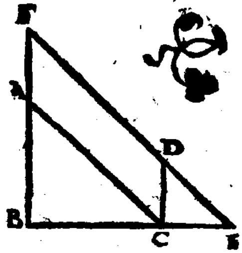
Theorema 3. Propoſitio 3.

Si trianguli angulus biſariam ſectus ſit, ſecans autem angulum recta linea ſecuerit & baſim: baſis ſegmenta eandem habebunt rationem

tionem, quam reliqua ipsius trianguli latera. Et si basis segmenta eandem habeant rationem quam reliqua ipsius trianguli latera, recta linea, quæ à vertice ad sectionem producitur, ea bifariá secat trianguli ipsius angulum.

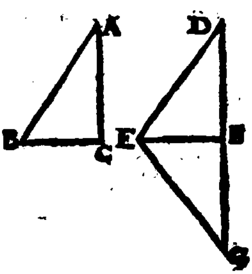


Theorema 4. Propositio 4.



Aequiangulorum triangulorum proportianolia sunt latera, quæ circum æquales angulos, & homologa sunt latera, quæ æqualibus angulis subtenduntur.

Theorema 5. Propositio 5.

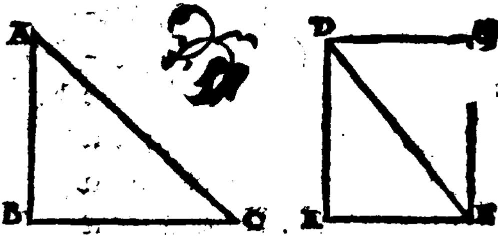


Si duo triágula latera proportionalia habeant, æquiangula erunt triangula, & æquales habebunt eos angulos, sub quib⁹ homologa latera subtenduntur.

Theorema 6. Propositio 6.

Si duo triangula vnum angulum vni angulo æqualẽ, & circum æquales angulos latera proportionalia habuerint, æquiangula erũt trian-

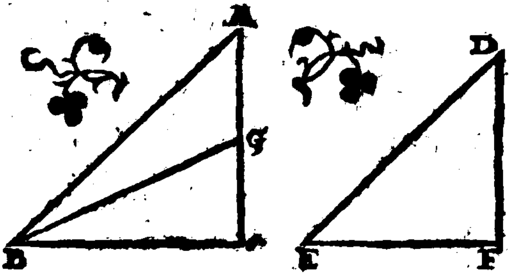
triangula, ęqualesq́; habebunt angulos ſub quibus ho-



mologa latera ſubtenduntur.

Theorema 7. Propoſitio 7.

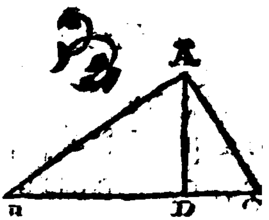
Si duo triangula vnum angulum vni angulo æqualem, circum autem alios angulos latera proportionalia habent, reliquorum vero ſimul vtrunq; aut minorem aut non minoré



recto: æquiangula erunt triangula, & æquales habebunt eos angulos circum quos proportionalia ſunt latera.

Theorema 8. Propoſitio 8.

Si in triangulo rectangulo, ab angulo recto in baſin perpendicularis ducta ſit quæ ad perpendicularem triangulo, tum toti triangulo, tum ipſa inter ſe ſimilia ſunt.



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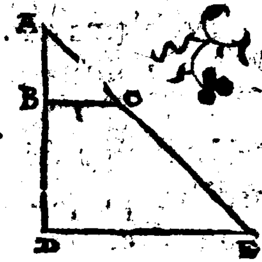
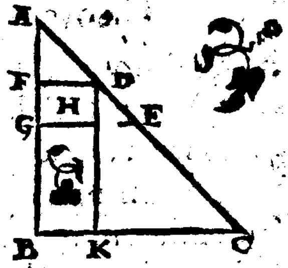
Problema 1. Propositio 9.



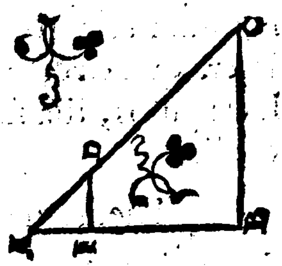
A data recta linea imperatam partem auferre.

Problema 2. Propositio 10.



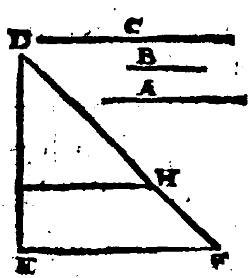
Datam rectam lineam insectam similiter secare, vt data altera recta secta fuerit.

Problema 3. Propositio 11.



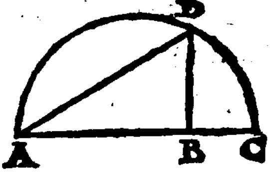
Duabus datis rectis lineis, tertiam proportionalem ad inuenire.

Problema 4. Propositio 12.



Tribus datis rectis lineis quartam proportionalé ad inuenire.

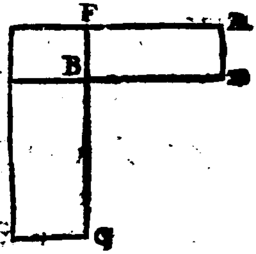
Problema 5. Propositio 13.



Duab' datis rectis lineis mediam proportionalē adinuenire.

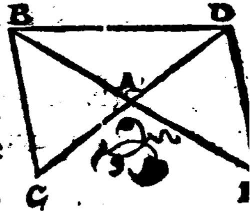
Theorema 9. Propositio 14.

AEqualium, & vnū vni æqualem habentiū angulum parallelogrammorum reciproca sunt latera, quæ circum æquales angulos: & quorum parallelogrāmorum vnum angulū vni angulo æqualem habentium reciproca sunt latera, quæ circum æquales angulos, illa sunt æqualia.



Theorema 10. Propositio 15.

Aequalium & vnum angulum vni ęqualem habentium triangulorū reciproca sunt latera, quæ circum æquales angulos: & quorum triangulorum vnum angulum vni æqualem habentium reciproca sunt latera, quæ circum æquales angulos, illa sunt æqualia.



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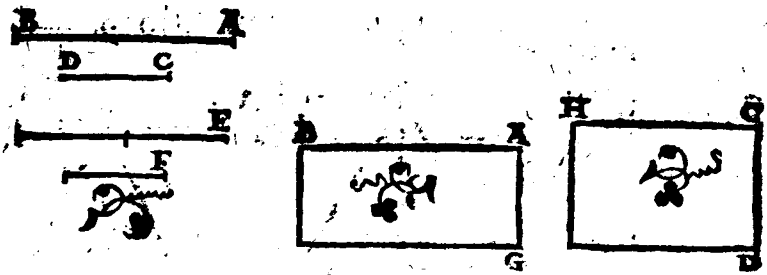
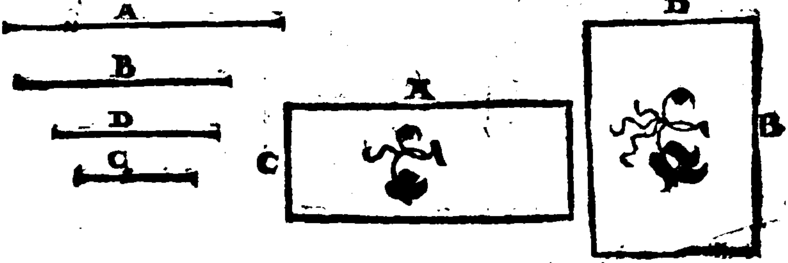
Theorema 11. Propositio 16.

Si quatuor rectæ lineæ proportiolales fuerint, quod sub extremis comprehenditur rectangulum, ęquale ęst ei, quod sub medijs comprehenditur rectangulo. Et si sub extremis comprehensum rectangulum æquale fuerit ei, quod sub medijs continetur rectāgulo illæ quatuor rectæ lineæ proportionales erunt.



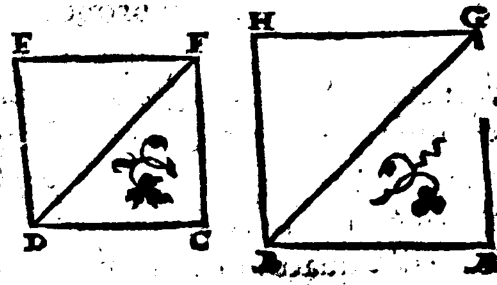
Theorema 12. Propositio 17.

Si tres rectæ lineę sint proportionales, quod sub extremis comprehēditur rectangulum æquale est ei, quod à media describitur quadrato: & si sub extremis comprehensum rectangulum æquale sit ei, quod à media describitur quadrato, illę tres rectæ lineę proportionales erunt.



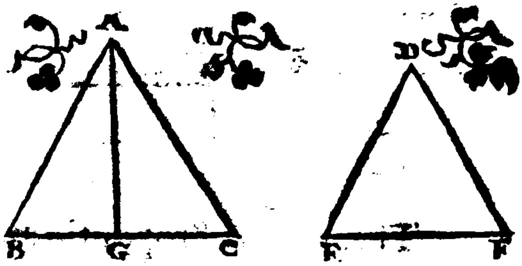
Problema 6. Propoſitio 18.

A data recta linea, dato rectilineo ſimile ſimiliterq; poſitum rectilineum deſcribere.



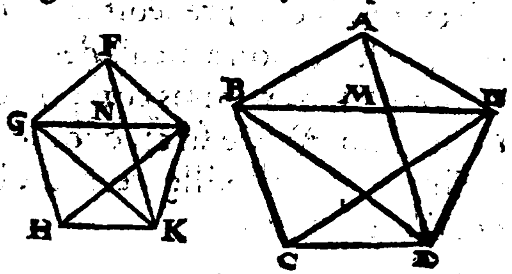
Theorema 13. Propoſitio 19.

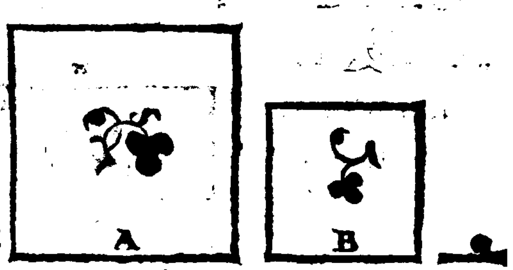
Similia triãgula inter ſe ſunt in duplicata ratione laterũ homologorũ.



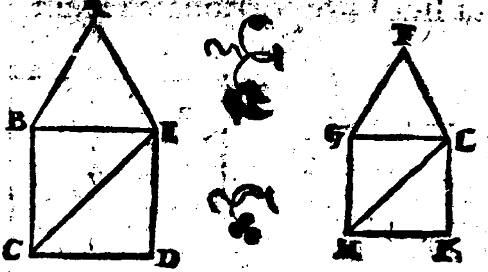
Theore. 14: Propoſitio 20.

Similia polygona in ſimilia triangula diuidũtur, & numero æqualia, et homologa totis. Et



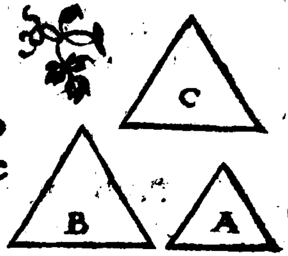


polygona duplicatá habét eám inter ſe rationé, quá latus homologú ad homologum latus.



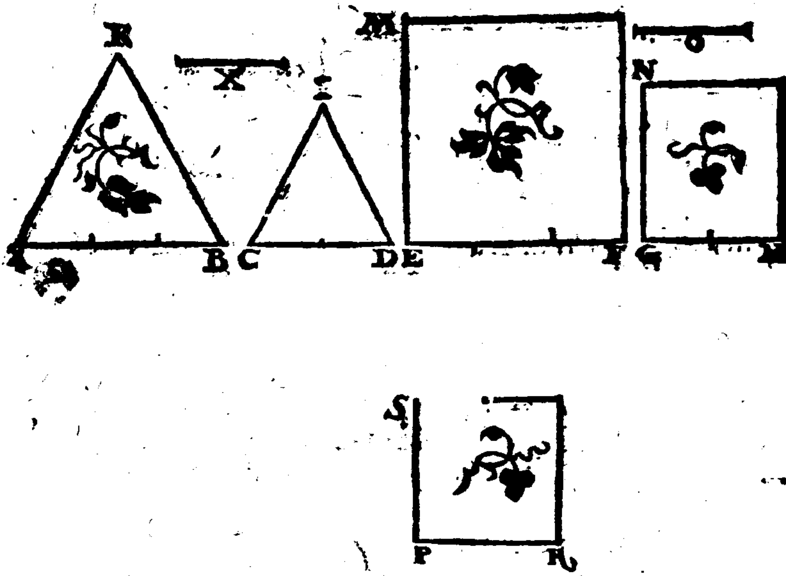
Theorema 15. Propoſitio 21.

Quæ eidem rectilineo ſunt ſimilia, & inter ſe ſunt ſimilia.

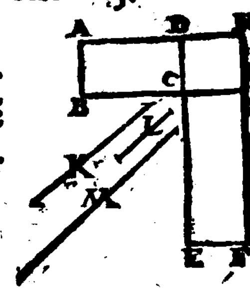


Theorema 16. Propoſitio 22.

Si quatuor rectæ lineæ proportionales fuerint: & ab eis rectilinea ſimili ſimiliterque deſcripta proportionalia erũt. Et ſi à rectis lineis ſimilia ſimiliterque deſcripta rectilinea proportionalia fuerint, ipſæ etiam rectæ lineæ proportionales erunt.

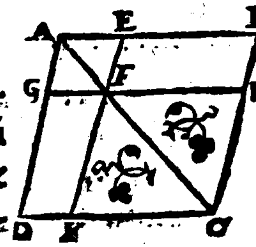


Problema 17. Propositio 23.



AEquiangula parallelogramma inter se rationẽ habent eum, quæ ex lateribus componitur.

Theorema 17. Propositio 24.



In omni parallelogrammo, quæ circa diametrũ sunt parallelogrãma, & toti & inter se sunt similia.

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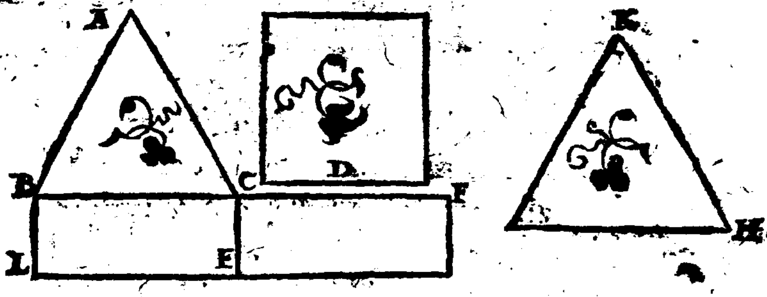
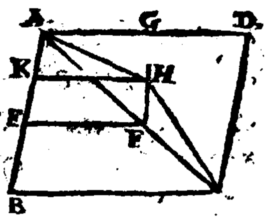
Theorema 7. Propoſitio 25.

Dato recto lineo ſimile, & alteri dato æquale idem conſtituere.



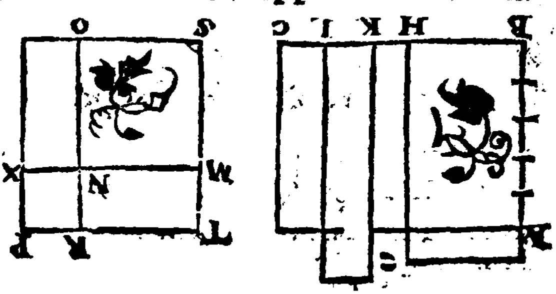
Theorema 19. Propoſitio 26.



Si à parallelogrãmo parallelogrammũ ablatum ſit, & ſimile toti & ſimiliter poſitum communẽ cum eo habens angulum, hoc circum eandem cum tota diametrum conſiſtit.

Theorema 20. Propoſitio 27.

Omnium parallelogrammorum ſecundum eandem rectam lineam applicatorum deficiẽtiumq; figuris parallelogrã mis ſi-

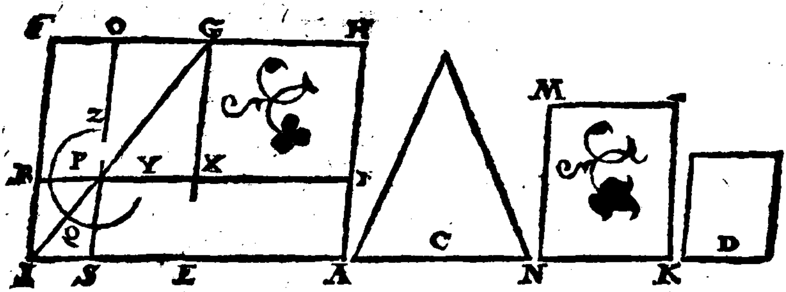


 milia

milibus similiterque positis ei, quod à dimidia describitur, maximum, id est, quod ad dimidiam applicatur parallelogrammũ simile existens defectui.

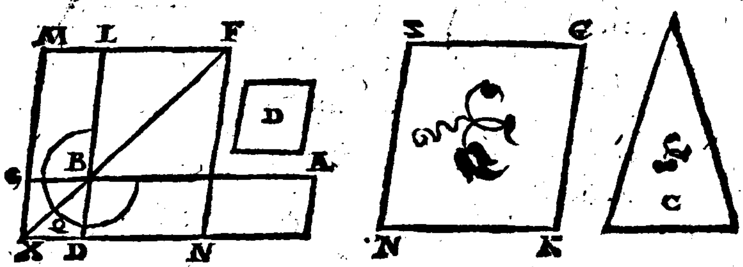
Problema 8. Propositio 28.

Ad datam lineam rectam, dato rectilineo æquale parallelogrammum applicare deficiens figura parallelogramma, quæ similis sit alteri rectilineo dato. Oportet autem datum rectilineum, cui æquale applicandum est, non maius esse eo quod ad dimidiam applicatur, cum similes sint defectus & eius quod à dimidia describitur, & eius cui simile deesse debet.

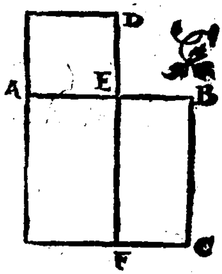


Problema 9. Propositio 29.

Ad datam rectam lineam, dato rectilineo æquale parallelogrammum applicare, excedens figura parallelogramma, quæ similis sit fit parallelogrammo alteri dato.



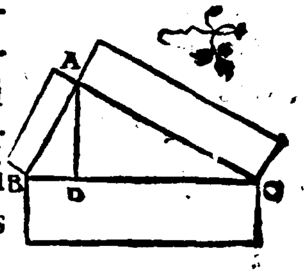
Problema 10. Propositio 30.



Propositam rectam lineam terminatam, extrema ac media ratione secare.

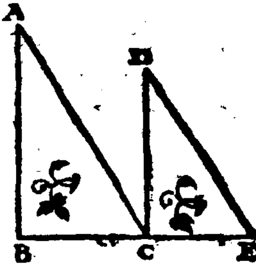
Theorema 21. Propositio 31.

In rectangulis triangulis, figura quæuis à latere rectũ angulum subtendéte descripta ęqualis est figuris, quæ priori illi similes & similiter positæ à lateribus rectũ angulum continentibus describuntur.



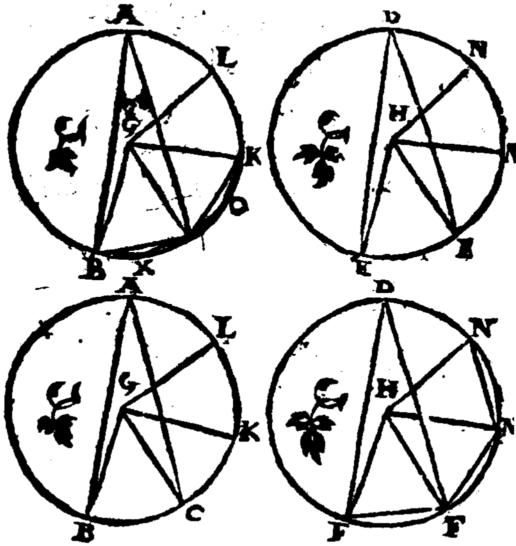
Theorema 22. Propositio 32.

Si duo triangula, quæ duo latera duobus lateribus proportionalia habeant, secundum vnum angulum composita fuerint, ita vt homologa eorum latera sint etiam parallela, tum reliqua illorum triangulorum latera in rectam lineam collocata reperirentur.



Theorema 23. Propositio 33.

In æqualibus circulis anguli eandem haben rationé, cum ipsis peripherijs in quibus insistunt, siue ad centra siue ad peripherias cóstituti, illis insistant peripherijs. In super verò & sectores quippe qui ad cétra cósistút.



ELEMENTI VI. FINIS.

EVCLIDIS ELEMENTVM SEPTIMVM.

DEFINITIONES.

1.

VNitas, est secundũ quam entium quodque dicitur.

2.

Numerus autem, ex vnitatibus composita multitudo.

3.

Pars, est numerus numeri minori maioris, cùm minor metitur maiorem.

4.

Partes autem, cùm non metitur.

5.

Multiplex verò, maior minoris, cùm maiorem metitur minor.

6.

Par numerus est, qui bifariam nõ diuiditur.

7.

Impar verò, qui bifariam nõ diuiditur: vel qui vnitate differt à pari.

8.

Pariter par numerus est, quem par numerus metitur per numerum parem.

9.Pari-

9.

Pariter autem impar, est quem par numerus metitur per numerum imparem.

10.

Impariter verò impar numerus, est quem impar numerus metitur per numerum imparem.

11.

Primus numerus, est quem vnitas sola metitur.

12.

Primi inter se numeri sunt, quos sola vnitas mensura communis metitur.

13.

Compositus numerus est, quem numerus quispiam metitur.

14.

Compositi autē inter se numeri sunt, quos numerus aliquis mensura communis metitur.

15.

Numerus numerum multiplicare dicitur, cum toties compositus fuerit is, qui multiplicatur, quot sunt in illo multiplicante vnitates, & procreatus fuerit aliquis.

16.

Cùm autem duo numeri mutuò sese multiplicantes quempiā faciunt, qui factus erit planus appellabitur, qui verò numeri mutuò sese multiplicarint, illius latera dicentur

15. Cùm

17.

Cùm verò tres numeri mutuò sese multiplicantes quempiam faciũt, qui procreatus erit solidus appellabitur, qua autem numeri mutuò sese multiplicarint, illius, latera dicentur.

18.

Quadratus numerus est, qui ęqualiter æqualis: vel, qui à duabus æqualibus numeris cõtinetur.

19.

Cubus verò, qui à tribus æqualibus æqualiter: vel, quia tribus æqualibus numeris continetur.

20.

Numeri proportionales sunt, cum primus secundi, & tertius quarti æquè multiplex est, vt eadem pars, vel eædem partes.

21.

Similes plani & solidi numeri sunt, qui proportionalia habent latera.

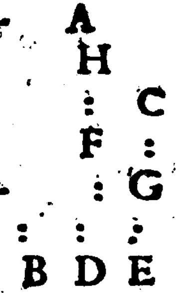
22.

Perfectus numerus est, qui suis ipsius partibus est æqualis.

Theorema 1. Propositio 1.

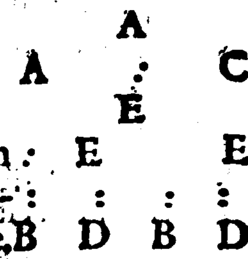
Duobus numeris in æqualibus propositis,

tis, si detrahatur semper minor de maiore, alterna, quadam detractione; neque reliquus vnquam metiatur præcedentem quo ad assumpta sit vnitas: qui principio propositi sunt numeri primi inter se erunt.

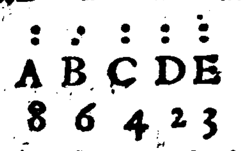


Problema 1. Propositio 2.

Duobus numeris datis non primis inter se, maximã eorũ cõmunẽ mẽsurã reperire.



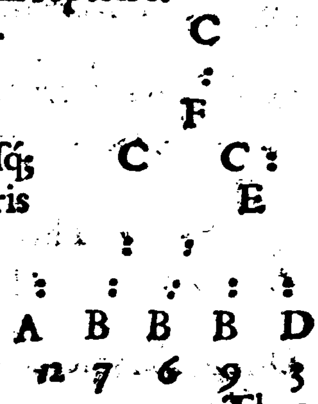
Problema 2. Propositio 3.



Tribus numeris datis non primis inter se, maximam eorum communem mensuram reperire.

Theorema 18. Propositio 8.

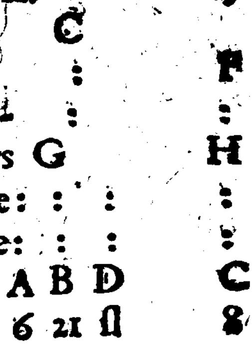
Omnis numerus cuiusq; numeri minor maioris aut par est, aut partes.



Theo-

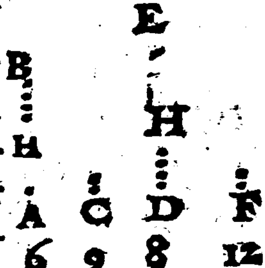
Theorema 3. Propositio 5.

Si numerus numeri par fuerit, & alter alterius eadẽ pars & simul vterque vtriusque simul eadem pars erit, quæ vnus est vnius.



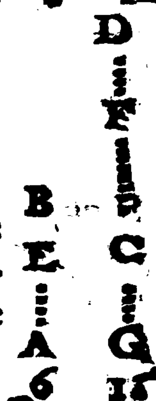
Theor. 4. Propo. 6.

Si numerus sit numeri partes, & alter alterius eædem partes, & simul vterque vtriusque simul eædem partes erunt, quæ sunt vnus vnius.



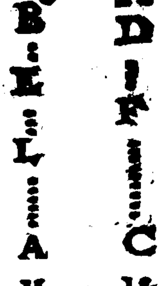
Theorema 5. Propositio 7.

Si numerus numeri eadẽ si pars quæ detractus detracti, & reliquus reliqui eadẽ pars erit, quæ totus est totius.



Theorema 6. Propositio. 8.

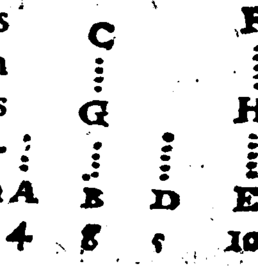
Si numerus numeri eædẽ sint partes quæ detractus detracti & reliquus reliqui eædẽ partes erũt, quæ sunt totus totius.



G.M.K..N.H.

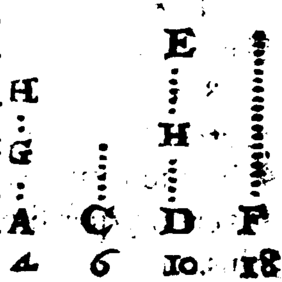
Theorema 7. Propositio 9.

Si numerus numeris pars sit, & alter alterius eadem pars, & vicissim quæ pars est vel partes primus tertij, eadem pars erit eædem partes, & secundus quarti.



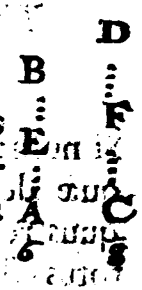
Theorema 8. Propositio 10.

Si numerus numeri partes sint, & aliter alterius eadẽ partes, etiam vicissim quæ sunt partes aut pars primus tertij, eædem partes erũt vel pars & secundus quarti.



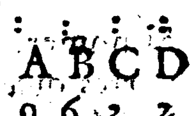
Theorema 9. Propositio 11.

Si quemadmodum se habet totus ad totum, ita detractus ad detractum, & reliquus ad reliquum ita habebit vt totus ad totum.



Theorema 10. Propositio 12.

Si sint quotcunque numeri proportionales; quemadmodum se habet vnus antecedentium ad vnum sequentium, ita se habe-



habebunt omnes antecedentes ad omnes consequentes.

Theorema 11. Propositio 13.

Si quatuor numeri sint proportionales, & vicissim proportionales erũt.

A	B	C	D
12	4	9	3

Theorema 12. Propositio 14.

Si sint quotcunque numeri, & alij illis æquales multitudine, qui bini sumantur & in eadem ratione, etiam ex æqualitate in eadem ratione erunt.

A	B	C	D	E	F
12	6	3	8	4	2

Theorema 13. Propositio 15.

Si vnitas numerum quãpiã metiatur, aliter verò numerus alium quendam numerũ æquè metiatur, & vicissim vnitas tertium numerũ æquè metietur, atque secundus quartum.

C H G A B D L K E F

1 3 6

Theorema 14. Propositio 16.

Si duo numeri mutuò sese multiplicãtes faciant aliquos qui ex illis geniti fuerint, inter se æquales erunt.

E	A	B	C	D
1	2	4	8	8

Theorema 15. Propoſitio 17.

Si numerus duos numeros multiplicans faciat aliquos, qui ex illis procreati erunt, eadem rationem habebunt quam multiplicati.

I	A	B	C	D	E
1	3	4	5	12	15

Theorema 16. Propoſitio 18.

Si duo numeri numerum quempiam multiplicantes faciant aliquos, geniti ex illis eandem habebunt rationem, quam qui illum multiplicarunt.

A	B	C	D	E
4	5	3	12	15

Theorema 17. Propoſitio 19.

Si quatuor numeri ſint proportionales, qui ex primo & quarto fit, æqualis erit ei qui ex ſecundo & tertio: & ſi qui ex primo & quarto ſit numerus æqualis ſit ei qui ex ſecundo & tertio, illi quatuor numeri ſint proportionales erunt.

A	B	C	D	E	F	G
6	4	3	2	12	12	18

Thorema 18. Propoſitio 20.

Si tres numeri ſint proportionales, qui ab extremis cõtinetur, æqualis eſt ei qui à me-

dio

dio efficitur. Et si qui ab extremis cõtinetur æqualis sit ei qui à medio describitur, illi tres numeri proportionales erunt.

A	B	C
9	6	4
D		
6		

Theorema [illegible] Propositio 21.

Minimi numeri omniũ, qui eandem cum eis rationem habẽt, æqualiter metiuntur numeros eandem rationem habentes, maior quidem maiorem, minor verò minorem.

D	L		
G	H		
C	E	A	B
4	3	8	6

Theorema 20. Propositio 22.

Si tres sint numeri & alij multitudine illis æquales, qui bini sumantur & in eadem ratione, sit autem perturbata eorum proportio, etiam ex æqualitate in eadem ratione erunt

A	B	C	D	E	F
6	4	3	12	8	6

Theorema 21. Propositio 23.

Primi inter se numeri minimi sunt omniũ eandem cum eis rationem habentium.

A	B	E	C	D
5	6	2	4	3

Theorema 22. Propositio 24.

Minimi numeri omnium eandem cum eis rationem habẽtium, primus sunt inter se.

A B C D E
7 6 4 3 2

Theorema 23. Propositio 25.

Si duo numeri sint primi inter se, qui alter vtrum eorum metitur numerus, is ad reliquũ primus erit.

A B C D
6 7 3 4

Theorema 24. Propositio 26.

Si duo numeri ad quépiam numerum primi sint, an eundem primus is quoque futurus est, qui ab illis productus fuerit.

5
B

A C D E F
5 5 5 3 2

Theorema 25. Propositio 27.

Si duo numeri primi sint inter se, quæ ab vno eorum gignitur ad reliquũ primus erit.

B

A C D
7 6 3

Theorema 26. Propositio 28.

Si duo numeri ad duos numeros ambo ad vtrunque primi sint, & qui ex eis gignentur, primi inter se erunt.

A B E C D F
3 5 15 2 4 8

Theore-

Theorema 27. Propositio 29.

Si duo numeri primi sint inter se, & multiplicans vterque seipsum procreet aliquem, qui ex ijs producti fuerint, primi inter se erűt. Quòd si numeri initio propositi multiplicantes eos qui producti sunt, effecerint aliquos, hi quoque inter se primi erunt, & circa extremos idem hoc semper eueniet.

A	C	E	B	D	F
3	6	27	4	16	63

Theorema 28. Propositio 30.

Si duo numeri primi sunt inter se, etiam simul vterq; ad vtrunq; illorum primus erit. Et si simul vterq; ad vnum aliquem eorum primus sit etiam qui initio positi sunt numeri, primi inter sunt erunt.

C

A	B	D
7	5	4

Theorema 29. Propositio 31.

Omnis primus numerus ad omnem numerum quem non metitur, primus est.

A	B	C
7	10	5

Theorema 30. Propositio 32.

Si duo numeri sese mutuo multiplicātes faciant aliquem, hunc autē ab illis productū metiatur primis quidam numerus, is alterum etiam metitur eorum qui initio positi erant.

A	B	C	D	E
3	6	12	3	4

Theorema 31. Propositio 33.

Omnem compositum numerum aliquis primus metietur.

A B C
27 9 3

Theorema 32. Propositio 34.

Omnis numerus aut primus est, aut eum aliquis primus metitur.

A A
3 6 8

Problema 3. Propositio 35.

Numerus datis quotcunque, reperire minimos omnium qui eandem cum illis rationem habeant.

A B C D E F G H K I M
6 8 12 2 3 4 6 2 3 4 3

Problema 4. Propositio 36.

B A C D E F
7 12 8 4 5

Duobus numeris datis reperire quē illi minimum metiantur numerum.

A F E C D G H
6 9 12 9 2 3

Theorema 33. Propositio 37.

Si duo numeri numerum quempiam metiantur, & minimus quem illi metiuntur eundum metietur.

F

A B E C

2 3 6 12

Problema 5. Propositio 38.

Tribus numeris datis reperire quem minimum numerũ illi metiantur.

A B C D E

3 4 6 12 8

A B C D E F

3 6 8 12 24 16

Theorema 34. Propositio 39.

Si numerum quispiam numerus metiatur, mensus partem habebit metienti cognominem.

A B C D

12 4 3 1

Theorema 35. Propositio 40.

Si numerus partem habuerit quamlibet, illum metietur numeru[s] parti cognominis.

A B C D

8 4 2 1

Problema 6. Propositio 41.

Numerum reperire, qui minimus cùm sit, datas habeat partes.

A B C G H

2 3 4 12 10

Elementi septimi finis.

EVCLIDIS ELEMENTVM OCTAVVM.

Theorema 1 Propositio 1.

Si sunt quotcunq; numeri deinceps proportionales, quorum extremi sint inter se, primi, minimi sunt omnium eandem cum eis rationem habentium.

A	B	C	D	E	F	G	H
8	12	18	27	6	8	12	18

Problema 1. Propositio 2.

Numerus reperire deinceps proportionales minimos quotcunque iusserit quispiam in data ratione.

A	B	C	D	E	F	G	H	K
3	4	9	12	16	27	36	48	64

Theorema 2. Propositio 3.

Conuersa primæ.

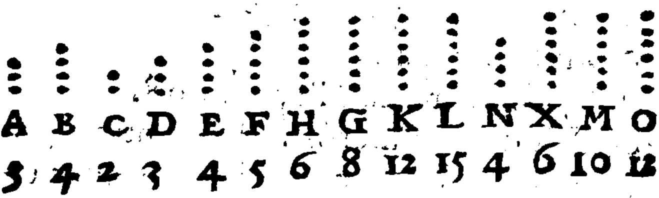
Si sint quotcunq; numeri deinceps proportionales minimi habentium eandem cum eis rationem, illorum extremi sunt inter se primi.

A	B	C	D	E	F	G	H	K	L	M	N	O
27	16	48	64	3	4	9	12	16	27	36	48	64

Proble-

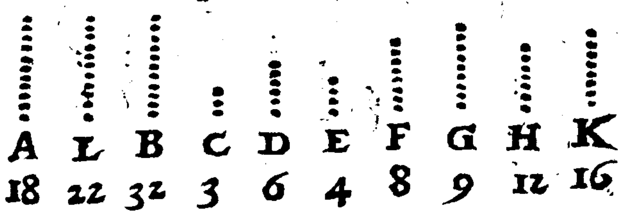
Problema 3. Propositio 4.

Rationibus datis quotcunque in minimis numeris reperire numeros deinceps minimos in datis rationibus.



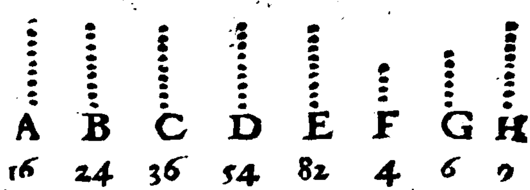
Theorema 3. Propositio 5.

Plani numeri rationem inter se habent ex lateribus compositam.



Theorema 4. Propositio 6.

Si sint quotlibet numeri deinceps.



proportionales, primus autem secundum non metiatur, neque alius quispiam vllum metietur.

Theorema 5. Propositio 7.

Si sint quotcunque numeri deinceps proportionales, primus autem extremum metiatur, is etiam secundum metietur.

A	B	C	D
4	6	12	24

Theorema 6. Propositio 8.

Si inter duos numeros medij continui proportione indicant numeri, quot inter eos medij continua proportione incidunt numeri, tot & inter alios eandem cum illis habentes rationem medij continua proportione incident.

A	C	D	B	G	H	K	L	C	M	N	F
4	9	27	81	1	3	9	27	2	6	18	54

Theorema 7. Propositio 9.

Si duo numeri sunt inter se primi, & inte eos medij cótinua proportione incidát nu meri, quot inter alios medij continua pro portione incidunt numeri, totidem et inte vtrunque eorum ac vnitatem deinceps me dij continua proportione incident.

A	M	H	E	F	N	C	K	X	G	D	L	O	1
27	27	9	36	3	36	1	12	48	4	48	16	64	6

Theo

Theorema 8. Propositio 10.

Si inter duos numeros & vnitatem cõtinuè proportionales incidant numeri quot inter vtrumque ipsorum & vnitatẽ deinceps medij continua proportione incidunt numeri, totidẽ & inter illos medij continua proportione incident.

A	E	K	D	H	C	L	F	G	B
27	9	36	3	12	1	48	4	16	64

Theorema 9. Propositio 11.

Duorum quadratorum numerorum vnus medius proportionalis eſt numerus: & quadratus ad quadratum duplicatam habet lateris ad latus rationem.

A	C	E	D	B
9	3	12	4	16

Theorema 10. Propoſitio 12.

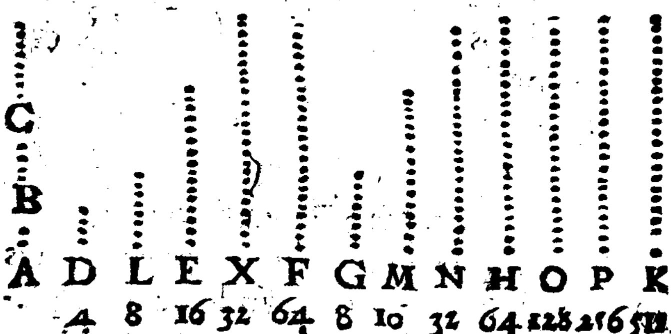
Duorum cuborum numerorum duo medij proportionales ſunt numerorũ, duo medij cubum triplicatam habet lateris ad latus rationem.

A	H	K	B	C	D	E	F	G
27	36	48	64	3	4	9	12	16

Theo-

Theorema 11. Propositio 13.

Si sint quotlibet numeri deinceps proportionales & multiplicans quisque seipsum faciat aliquos, qui ab illis producti fuerint, proportionales erunt, & si numeri primùm positi ex suo in procreatos ductu faciant aliquo, ipsi quoque proportionales erunt.



Theorema 12. Propositio 14.

Si quadratus numerus quadratum numerū metiatur, & latus vnius metietur latus alterius. Et si vnius quadrati latus metiatur latus alterius, & quadratus quadratum metietur.

:	:	:	:	:
A	E	B	C	D
9	12	16	8	4

Theorema 13. Propositio 15.

Si cubus numerus cubum numerum metiatur, & latus vnius metietur alterius latus. Et si latus vnius cubi latus alterius metiatur,

tum

tum cubus cubum metietur.

A	H	K	B	C	D	E	F	G
8	16	28	64	2	4	4	8	16

Theorema 14. Propositio 16.

Si quadratus numerus quadratum numerum non metiatur, neque latus vnius metietur alterius latus. Etsi latus vnius quadrati nó metiatur latus alterius, neq; quadratus quadratum metietur.

A	B	C	D
9	16	3	4

Theorema 15. Propositio 17.

Si cubus numerus cubum numerũ non metiatur, neque latus vnius latus alterius metietur. Et si latus cubi alicuius latus alterius non metiatur, neque cubus cubum metietur.

A	B	C	D
8	27	9	11

Theorema 16 Propositio 18.

Duorum similium planorum numerorum vnus medius proportionalis est numerus, & planus ad planum duplicatam habet literis homologi-

A	G	B	C	D	E	F
12	18	27	2	6	5	9

logi ad latus homologum rationem.

Theorema 17. Propositio 19.

Inter duos similes numeros solidos, du[o] medij proportionales incidunt numeri: & solidus ad similem solidum triplicatam rationem habet lateris homologi ad latu[s] homologum.

A N X B C D E F G H K M L
8 12 18 27 2 2 2 3 3 3 4 6 9

Theorema 18. Propositio 20

Si inter duos numeros vnus medius proportionalis incidat numerus, similes plani erunt illi numeri.

A C B D E F G
18 24 33 3 4 6 8

Theorema 19. Propositio 21.

Si inter duos numeros duo medij proportionales incidãt numeri, similes solidi sun[t] illi numeri.

A C D B E F G H K L M
27 36 44 64 9 12 16 3 3 3 4

Theo

Theorema 20. propositio 22.

Si tres numeri deinceps ſint proportionales, primus autem ſit quadratus, & tertius quadratus erit.

A B D
9 15 25

Theorema 21. propoſitio 23.

Si quatuor numeri deinceps ſint proportionales, primus autem ſit cubus, & quartus cubus erit.

A B C D
8 12 18 27

Theorema 22. propoſitio 24.

Si duo numeri rationem habeant inter ſe, quam quadratus numerus ad quadratum numerum, primus autem ſit quadratus erit, & ſecundus quadratus erit.

A B C D
4 6 9 16 24 36

Theorema 23. propoſitio 25.

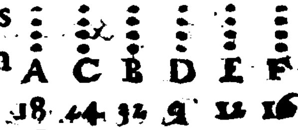
Si numeri duo rationem inter ſe habeant, quam cubus numerus ad cubum numerum, primus autem cubus ſit, & ſecundus cubus erit.

A E F B C D
8 12 18 2. 64 95 140 ..

Theo-

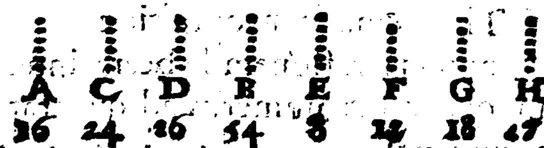
Theorema 24. Propositio 26.

Similes plani numeri rationem inter se hàbent, quam quadratus numerus ad quadratum nemurum.



Theorema 25. Proposisio 27.

Similes solidi numeri rationem habent inter se, quam cubus numerus ad cubum numerum.



ELEMENTI VIII. FINIS.

EV.

EVCLIDIS ELEMENTVM NONVM.

Theorema 1. Propositio 1.

Si duo similes plani numeri mutuò sese multiplantes quēdam procreent, productus quadratus.

A	E	B	D		C
4	6	9	16	24	36

Theorema 2. Propositio 2.

Si duo numeri mutuò sese multiplicantes quadratum faciant, illi similes sunt plani.

A		B	D		C
4	6	9	18		36

Theorema 3. Propositio 3.

Si cubus numerus seipsum multiplicans procreet aliquē, productus cubus erit.

Vnitas	D	D	A			B
	3	4	8	16	32	64

Theorema 4. Propositio 4.

Si cubus numerus cubũ numerum multiplicans quendam procreet, procreatus cubus erit.

A	B	D	C
8	27	64	216

Theorema 5. Propositio 5.

Si cubus numerus numerum quendam multiplicans cubum procreet, & multiplicatus cubus erit.

A	B	C	D		
27	64	729	17	18	

Theorema 6. Propositio 6.

Si numerus ſeipſum multiplicans cubum procreet, & ipſe cubus erit.

A	B	C
27	729	19683

Theorema 7. Propoſitio 7.

Si compoſitus numerus quendam numerũ multiplicans quempiam procreet, productus ſolidus erit.

A	B	C	D	E
6	8	48	2	3

Theorema 8. Propoſitio 8.

Si ab vnitate quotlibet numeri deinceps proportionales ſint, tertius ab vnitate quadratus eſt, & vnum intermittentes omnes: quartus autem cubus, & duobus intermiſsis omnes

nes: septimus verò cubus simul & quadratus, & quinque intermissis omnes

Vnitas	A	B	C	D	E	F
	3	9	27	81	243	729

Theorema 9. Propositio 9.

Si ab vnitate sint quotcunque numeri: deinceps proportionales, sit autem quadratus is, qui vnitatẽ sequitur, & reliqui omnes quadrati erũt. Quòd si qui vnitatem sequitur cubus sit, & reliqui omnes cubi erunt.

quadrati		cubi
531441	F	732969
59040	E	531441
6561	D	59049
723	C	6561
81	B	729
9	A	81
	o	
	vnitas.	

Theorema 10. Propositio 10.

Si ab vnitate numeri quotcunque proportionales sint, non sit autem quadratus is qui vnitatem sequitur, neq; alius vllus quadratus erit, demptis tertio ab vnitate ac omnibus

o	A	B	C	D	E	F
Vnitas.	3	9	36	81	243	729

nibus vnum intermittentibus. Quòd si qui vnitatem sequitur, cubus non sit, neque alius vllus cubus erit, demptis quarto ab vnitate ac omnibus duos intermittentibus.

Theorema 11. Propositio 11.

Si sib vnitate numeri quotlibet deinceps proportionales sint, minor maiorem metitur per quempiam eorum qui in proportionalibus sunt numeris.

A	B	C	D	E
1	2	4	8	16

Theorema 12. Propositio 12.

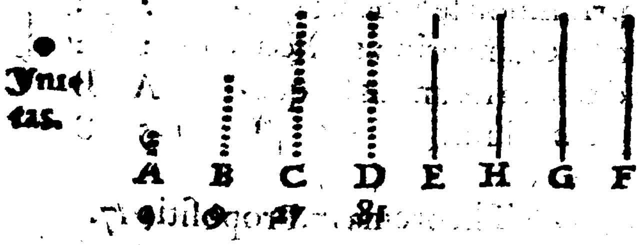
Si ab vnitate quotlibet numeri sint proportionales, quot primorum numerorum vltimũ metiuntur, totidem & eum qui vnitati proximus est, metientur.

Vnitas.	A	B	C	D	E	H	G	F
	4	16	64	259	2	8	32	128

Theorema 13. Propositio 13.

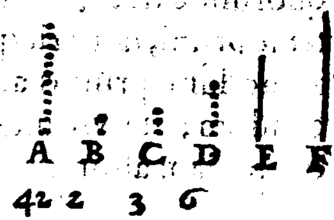
Si ab vnitate sint quotcunque numeri deinceps proportionales, primus autem sit qui vnitaté sequitur, maximũ nullus alius metietur,

tietur, ijs exceptis qui in proportionalibus ſint numeris.



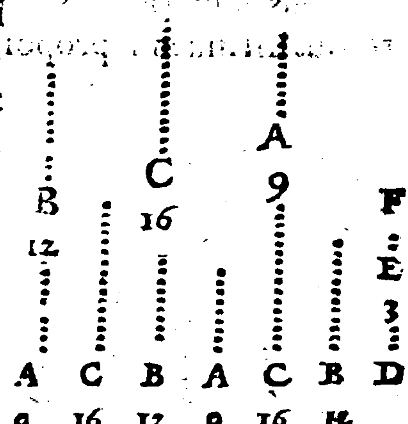
Theorema 14. propoſitio 14.

Si minimum numerum primi aliquot numeri metiãtur, nullus alius numerus primus illum metietur, ijs exceptis qui primo metiuntur.



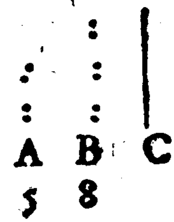
Theorema 15. propoſitio 15.

Si tres numeri deinceps proportionales ſint minimi, eadem cum ipſis habẽtium rationem, duo quilibet compoſiti ad tertium primi erunt.



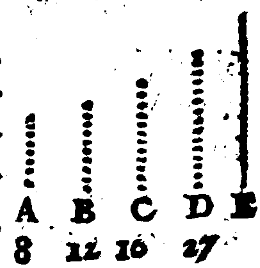
Theorema 16. Propoſitio 16.

Si duo numeri ſint inter ſe primi, non ſe habebit quẽadmodum primus ad ſecũdum, ita ſecundus ad quẽpiam alium.



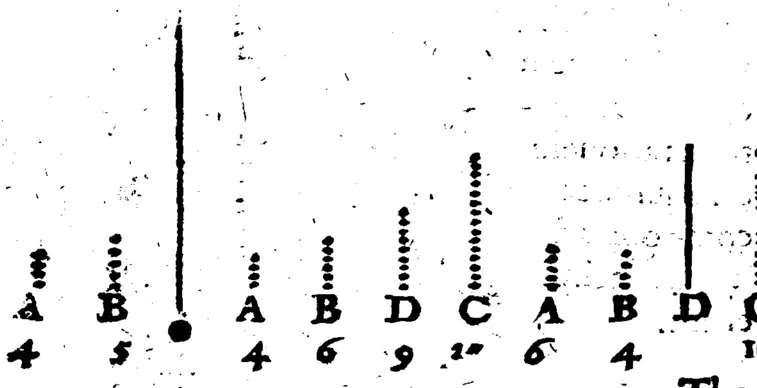
Theorema 17. Propoſitio 17.

Si ſint quotlibet numeri deinceps proportionales, quorum extremi ſint inter ſe primi, non erit quẽadmodum primus ad ſecundum, ita vltimus ad quempiam alium.



Theorema 18. Propoſitio 18.

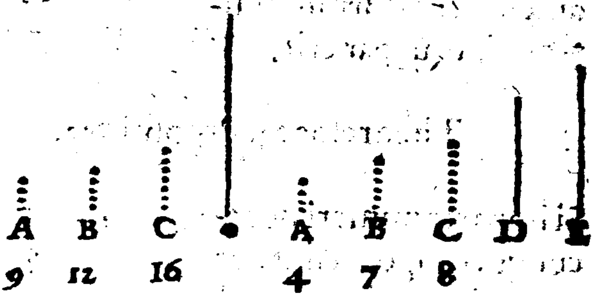
Duobus numeris datis, cõſiderare noſcitne tertius illi inueniri proportionalis.



Theo

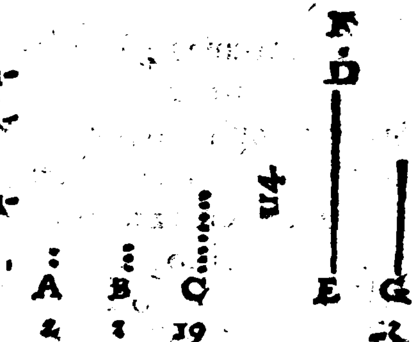
Theorema 19. Propositio 19.

Tribus numeris datis, considerare possitne quartus illis reperiri proportionalis.



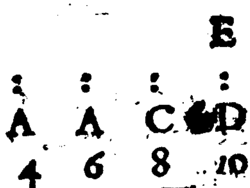
Theorema 20. Propositio 20.

Primi numeri plures sunt quacúque proposita multitudine primorum numerorum.



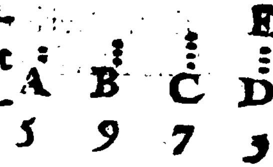
Theorema 21. Propositio 21.

Si pares numeri quotlibet compositi sint, totus est par.



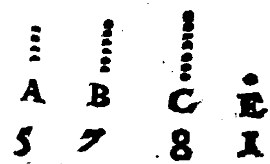
Theorema 22. Propositio 22.

Si impares numeri quotlibet compositi sint, sit autem par illorum multitudo, totus par erit.

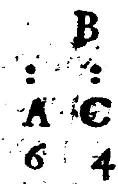


Theorema 23. Propsitio 23.

Si impares numeri quotcunq; compositi sint, sit autem impar illorũ multitudo, & totus impar erit.

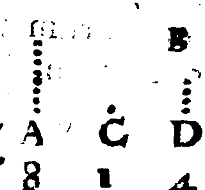


Theorema 24. Propositio 24.



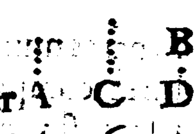
Si de pari numero par detractus sit, & reliquus par erit.

Theorema 25. Propositio 25.



Si de pari numero impar detractus sit, & reliquus impar erit.

Theorema 26. Propositio 26.



Si de impari numero impar detractus sit, & reliquus par erit.

Theore-

Theorema 27. Propositio 27.

Si ab impari numero par ablatus sit, reliquus impar erit.

B

A D C

1 4 4

Theorema 28. Propositio 28.

Si impar numerus parem multiplicãs, procreet quempiam, procreatus par erit.

A B C

3 4 21

Theorema 29. Propositio 29.

Si impar numerus imparẽ numerum multiplicans quendam procreet, procreatus impar erit.

A B C

3 5 15

Theorema 30. Propositio 30.

Si impar numerus parem numerum metiatur, & illius dimidium metietur.

A C B

3 6 18

Theorema 31. Propositio 31.

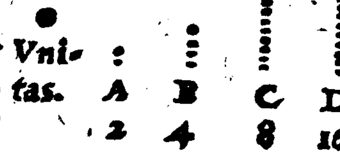
Si impar numerus ad numerum quempiã primus sit & ad alius duplum primus erit.

A B C D

7 8 16

Theorema 32. Propositio 32.

Numerorum qui à binario dupli sunt, vnusquisq; pariter par est tantum.



Theorema 33. Propositio 33.

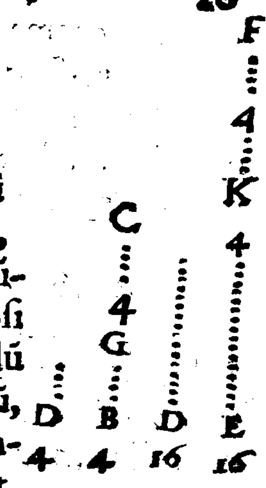
Si numerus dimidiũ impar habeat, pariter impar est tantùm.

Theorema 34. Propositio 34.

Si par numerus nec sit dupli à binario, nec dimidium impar habeat, pariter par eum, & pariter impar.

Theorema 35. Propositio 35.

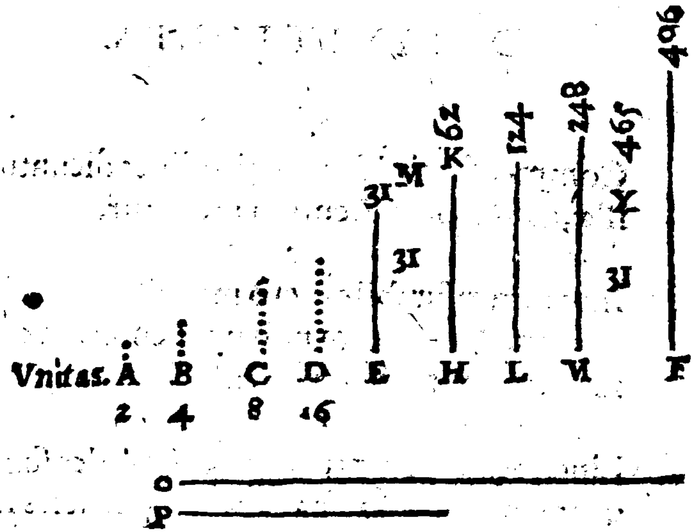
Si sint, quotlibet numeri deinceps proportionales, detrahantur autem de secũdo & vltimo æquales ipsi primo, erit quemadmodũ secundi excessus ad primũ, ita vltimi excessus ad omnes qui vltimũ antecedunt.



Theo-

Theorema 16. propositio 16.

Si ab vnitate numeri quotlibet deinceps expositi sunt in duplici proportione quoad totus compositus primus factus sit, isque totus in vltimum multiplicatus quempiam procreet, procreatus perfectus erit.



ELEMENTI IX. FINIS.

EV.

EVCLIDIS ELEMENTVM DECIMVM.

DEFINITIONES.

1.

Commensurabiles magnitudines dicuntur illæ, quas eadem mensura metietur.

2.

Incommensurabiles verò magnitudines dicuntur, hæ quarum nullam mensuram cõmunem contingit reperiri.

3.

Lineæ rectæ potentiâ cõmensurabiles sunt, quarum quadrata vna eadem superficies siue area metitur.

4.

Incommensurabiles verò lineæ sunt, quarũ quadrata, quæ metiatur area communis, reperiri nulla potest.

5.

Hęc cùm ita sint, ostendi potest quòd quantacunque linea recta nobis proponatur, existũt etiam aliæ lineæ innumerabiles eidem commensurabiles, aliæ item incommensurabiles, hæ quidem longitudine & potentia:

ia, illa verò potentia tantùm. Vocetur igitur linea recta, quantacunq; proponatur, id est rationalis.

6.

Linea quoq; illi ῥητῇ commensurabiles siue longitudine & potentia, siue potentia tantùm, vocentur & ipsæ ῥηταί, id est rationales.

7.

Quæ verò lineæ sunt incommensurabiles illi τῇ ῥητῇ, id est primo loco rationali, vocẽtur ἄλογοι, id est, irrationales.

8.

Et quadratum quod linea proposita describitur, quam ῥητὴν vocari volumus, vocentur ῥητόν.

9.

Et quæ sunt huic commensurabilia, vocentur ῥητά.

10.

Quę verò sunt illi quadrato ῥητῷ scilicet incommensurabilia, vocentur ἄλογα, id est, surda.

11.

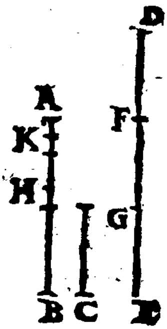
Et lineæ quæ illa, incommensurabilia describunt, vocentur ἄλογοι. Et quidam se illa incommensurabilia fuerint quadrata, ipsa eorum latera vocabũtur ἄλογοι lineæ, quòd si quadrata quidem non fuerint, verum aliæ quæpiam superficies siue figuræ rectilineæ,

tunc

tunc vero lineæ illæ quæ describunt quadrata æqualia figuris rectilineis, vocentur ἄλογοι.

Theorema 1. Propositio 1.

Duabus magnitudinibus inæqualibus propositis, si de maiore detrahatur plus dimidio, & rursus de residuo iterum detrahatur plus dimidio, idque semper fiat: relinquetur quædam magnitudo minor altera minore ex duabus propositis.



Theorema 2. Propositio 2.

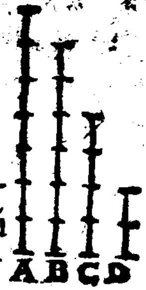
Duabus magnitudinibus propositis inęqualibus, si detrahatur semper minor de maiore alterna quadam detractione neque residuum vnquam metiatur id quod ante se metiebatur, incommensurabiles sunt illæ magnitudines.

Problema 1. Propositio 3.

Duabus magnitudinibus commensurabilibus datis, maximam ipsarum communem mensuram reperire.

Proble-

Problema 2. Propositio 4.

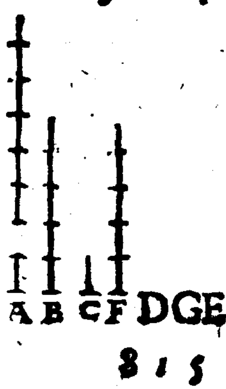


Tribus magnitudinibus commensurabilibus datis, maximam ipsarũ communem mensuram reperire.

Theorema 3 Propositio 5.

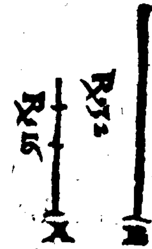
Commensurabiles magnitudines inter se proportionẽ eam habent, quam habet numerus ad numerum.

Theorema 4. Propositio 6.



Si duæ magnitudines proportionem eam habent inter se quam numerus ad numerum, commensurabiles sunt illæ magnitudines.

Theore-



Theorema 5. Propositio 7.

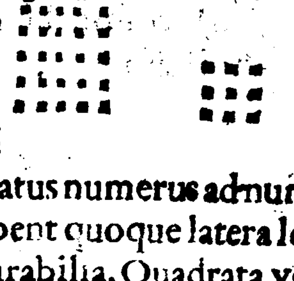
Incommensurabiles magnitudines inter se proportionem non habent, quam numerus ad numerum.

Theorema 6. Propositio 8.

Si duæ magnitudines inter se proportionem nō habēt quam numerus ad numerum, incommensurabiles illæ sunt magnitudines.

Theorema 7. Propositio 9.

Quadrata, quæ describuntur à rectis lineis longitudine cōmensurabilibus, inter se proportionem habēt quam numerus quadratus ad alium numerum quadratum. Et quadrata habentia proportionē



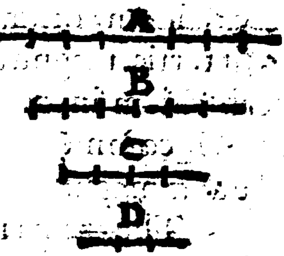
inter se quam quadratus numerus ad numerum quadratum, habent quoque latera longitudine commensurabilia. Quadrata verò

quæ

quæ deſcribuntur à lineis longitudine incõmenſurabilibus proportionẽ non habent inter ſe; quam quadratus numerus ad numerum alium quadratum. Et quadrata non habentia proportionem inter ſe quam numerus quadratus ad numerum quadratum; neque latera habebunt longitudine commenſurabilia.

Theorema 8. Propoſitio 10.

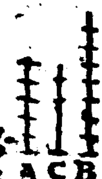
Si quatuor magnitudines fuerint proportionales, prima verò ſecundæ fuerit cõmenſurabilis, tertia quoque quartæ cõmmẽſurabilis erit, quòd ſi prima ſecũdæ fuerit incommenſurabilis, tertia quoque quartæ incommenſurabilis erit.



Problema 3. Propoſitio 11.

Propoſitæ lineæ rectæ (quam ῥητὴν vocari diximus) reperire duas lineas rectas incommenſurabiles, hanc quidem longitudine tantùm, illam verò non longitudine tantùm; ſed etiam potentia incommenſurabilem.

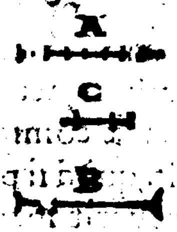
Theorema 6. Propositio 12.



Magnitudines quæ eidem magnitudini sunt commésurabiles, inter se quoque sunt commensurabiles.

6D.. 4F..
4E.... 8G..
3H...
2K..
4L...

Theorema 10. Propositio 13.



Si ex duabus magnitudinibus hæc quidem commensurabilis sit tertiæ magnitudini, illa verò eidem incommensurabilis, incommésurabiles sunt illæ duæ magnitudines.

Theorema 11. Propositio 14.

Si duarum magnitudinum commensurabilium altera fuerit incommensurabilis magnitudine alteri cuipiam tertię, reliqua quoq; magnitudo eidem tertię incommensurabilis erit.

Theorema 12. Propositio 15.

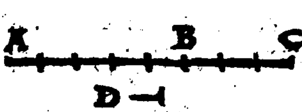
Si quatuor rectæ proportionales fuerint,

posit

posit autem prima plusquam secunda tāto quantum est quadratum lineæ sibi commē-surabilis longitudine: tertia quoq; potuerit plusquam quarta tanto quantum est quadratum lineæ sibi commensurabilis longitudini. Quòd si prima posit plusquam secunda quadrato lineę sibi longitudine incommensurabilis: tertia quoque poterit plusquam quarta quadrato lineæ sibi incō-mensurabilis lōgitudine.

Theorema 13. Propositio 16.

Si duæ magnitudines commensurabiles componantur, tota magnitudo composita singulis partibus commensurabilis erit, quòd si tota magnitudo composita alterutri parti commensurabilis fuerit, illæ duæ quoque partes cōmensurabiles erunt.

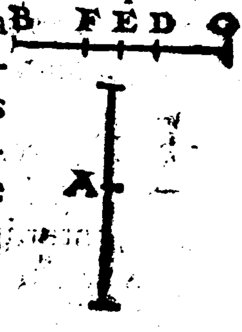


Theorema 14. Propositio 17.

Si duæ magnitudines incommensurabiles componantur, ipsa quoque tota magnitudo singulis partibus componentibus incommensurabilis erit. Quòd si tota alteri parti incommensurabilis fuerit, illæ quoque primæ magnitudines inter se incommensurabiles erunt.

Theorema 15. Propositio 18.

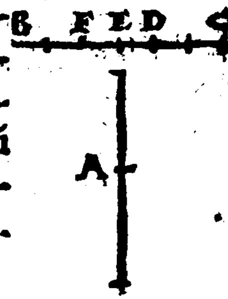
Si fuerint duæ rectæ lineæ inæquales, & quartæ parti quadrati quod describitur à minore æquale parallelogrammum applicetur secundum maiorem, ex qua maiore tantum excurrat extra latus parallelogrammi, quantum est alterum latus ipsius parallelogrammi: si præterea parallelogrammũ sui applicatione diuidat lineam illã in partes inter se commensurabiles longitudine, illa maior linea tanto plus potest quàm minor, quantum est quadratum lineæ sibi cõmensurabilis longitudine. Quòd si maior plus posit quam minor, tanto quantum est quadratum lineæ sibi commensurabilis lõgitudine, & præterea quartæ parti quadrati lineę minoris ęquale parallelogrammum applicetur secundum maiorem, ex qua maiore tantum excurrat extra latus parallelogrammi, quã-tum est alterum latus ipsius parallelogrammi parallelogrammum sui applicatione diuidit maiorẽ in partes inter se longitudine commensurabiles.



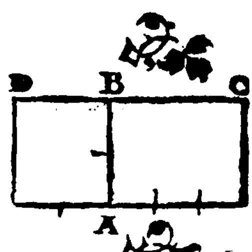
Theorema 16. Propositio 19.

Si fuerint duæ rectæ inæquales, quartæ autem parti quadrati lineæ minoris æquale paral-

parallelogrammorum ſecundum lineam maiorem applicetur, ex qua linea tantum excurrat extra latus parallelogrammi, quantum eſt alterum latus eiuſdem parallelogrammi; ſi parallelogrammum preterea ſui applicatione diuidat lineam in partes inter ſe longitudine incommenſurabiles, maior illa linea tantò plus poteſt quàm minor quantum eſt quadratum lineæ ſibi minori commenſurabiles longitudine. Quod ſi maior linea tantò plus poſsit quàm minor, quantum eſt quadratum lineæ incommenſurabilis ſibi longitudine: & præterea quartæ parti quadrati lineæ minoris æquale parallelogrammum applicetur ſecũdum maiorem ex qua tantum excurrat extra latus parallelogrãmi, quantùm eſt alterum latus ipſius: parallelogrammũ ſui applicatione diuidit maiorem in partes inter ſe incõmenſurabiles longitudine.



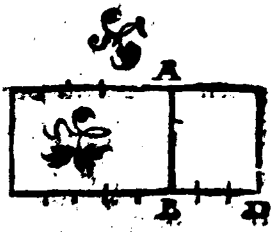
Theorema 17. Propoſitio 20.



Superficies rectanguli contenta ex lineis rectas rationalibus longitudine commenſurabilibus ſecundum vnum aliquem modum ex antedictis, rationalis est.

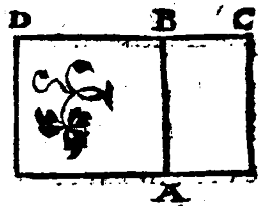
Theorema 18. Propositio 21.

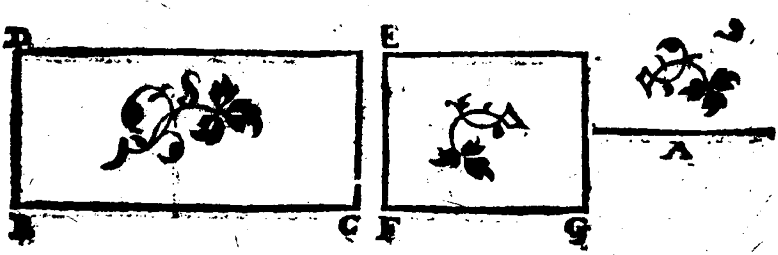
Si rationale secũdum lineam rationalem applicetur, habebit alterum latus lineam rationalem & cõmensurabilem longitudine lineæ cui rationale parallelogrammũ applicatur.



Theorema 39. Propositio 56.

Superficies rectangula contenta duabus lineis rectis rationalibus potentia tantum cõmensurabilibus, irrationalis est. Linea autem quæ illam superficiem potest, irrationalis & ipsa est: vocetur verò medialis dum

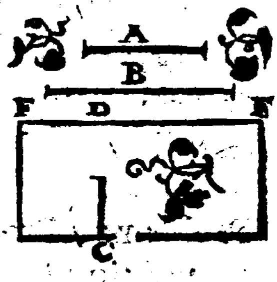




line-

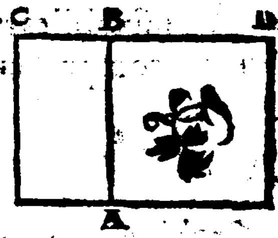
lineam rationalem, alterum latus est linea rationalis, & incommensurabilis longitudine lineæ secundum quam applicatur.

Theorema 21. Propositio 24.



Linea recta mediali commẽsurabilis, est ipsa quoque medialis.

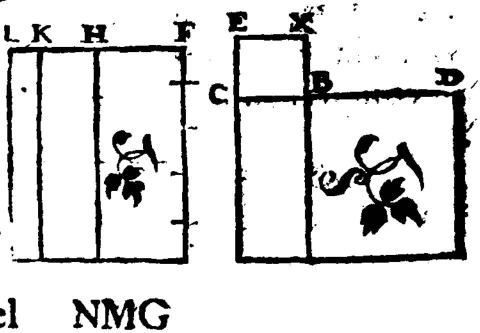
Theorema 22. Propositio 25.



Parallelogrammum rectangulum contentũ ex lineis medialibus longitudine commẽsurabilibus, mediale est.

Theorema 23. Propositio 26.

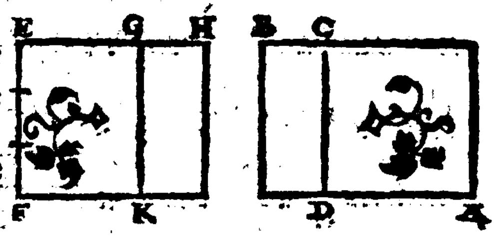
Parallelogrammum rectangulum comprehensum duabus lineis medialibus potentia tantũ cõmensurabilibus, vel rationale est, vel mediale.



 Theo-

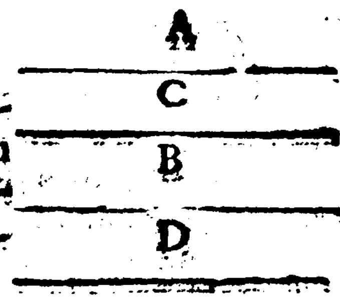
Theorema 24. Propoſitio 27.

Mediale nō eſt maius quàm mediale ſuperficie rationali.



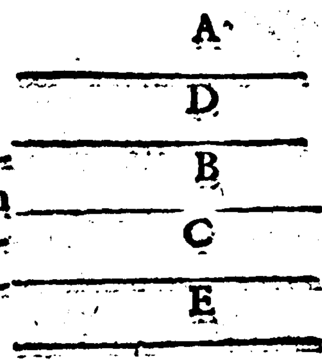
Problema 4. Propoſitio 28.

Mediales lineas inuenire potentia tantùm commenſurabiles rationale comprehendentes.



Problema 5. Propoſitio 29.

Mediales lineas inuenire potentia tantùm commenſurabiles mediale comprehendentes.

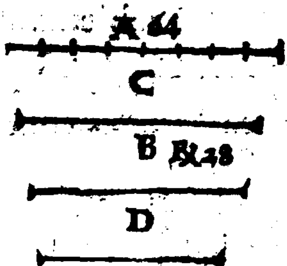


Problema 6. Propositio 30.

Reperire duas rationales potentia tantum commẽsurabiles huiusmodi, vt maior ex illis posſit plus quàm minor quadrato lineæ ſibi commenſurabilis longitudine.

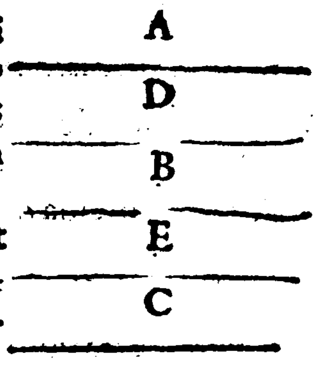
Problema 7. Propoſitio 31.

Reperire duas lineas medialis potentia tantùm commenſurabiles rationalem ſuperficiẽ continẽtes, tales inquã vt maior poſsit plus quàm minor quadrato lineæ ſibi commenſurabilis longitudine.



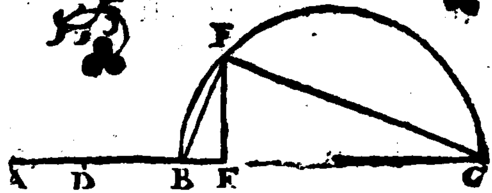
Problema 8. Propoſitio 32.

Reperire duas lineas mediales potentia tantùm commenſurabiles medialem ſuperficiem continentes, huiuſmodi vt maior plus poſsit quàm minor quadrato lineæ ſibi commenſurabilis longitudine.



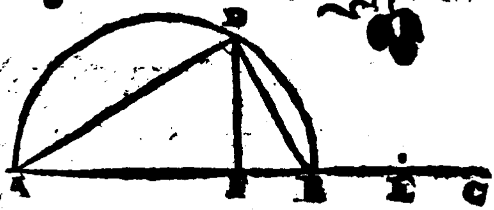
Problema 9. Propositio 33.

Reperire duas rectas potentia incommensurabiles, quarum quadrata simul addita faciant superficié rationalé, parallelográmú verò ex ipsis contentum sit mediale.



Problema 10. Propositio 34.

Reperire lineas duas rectas potentia incommensurabiles conficientes compositum ex ipsarum quadratis mediale, parallelogrammú verò ex ipsis contentum rationale.



Problema 11. Propositio 85,

Reperire duas lineas rectas potentia inconmensurabiles, conficientes id quod ex ipsarum quadratis componitur meale, simul-

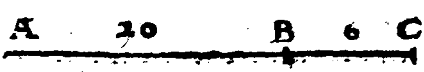
quę

que parallelogrammum ex ipsis contẽtum, mediale, quod præterea parallelogrammũ sit incõmensurabile composito ex quadratis ipsarum.

PRINCIPIVM SENARIOrum per compositionem.

Theorema 25. Propositio 36.

Si duæ rationales potentia tantùm commẽsurabilis. Vocetur autem Binomium.



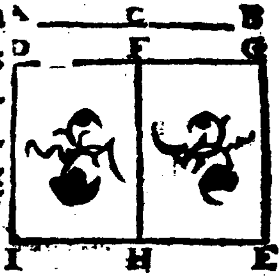
Theorema 26. Propositio 37.

Si duę mediales potentia tantùm commensurabiles rationale continentes componantur, tota linea est irrationalis, vocetur autem Bimediale prius.

Theo-

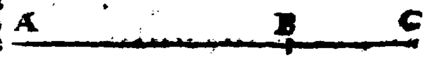
Theorema 27. Propositio 38.

Si duæ mediales potẽtia tantùm commẽſurabiles mediale continentes cõponãtur, tota linea eſt irrationalis: vocatur autẽ Bimediale ſecundum.



Theorema 28. Propoſitio 39.

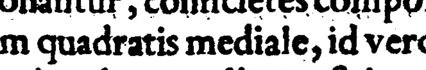
Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex quadratis ipſarum rationale parallelogrammum verò ex ipſis contentum mediale, tota linea recta



eſt irrationalis. Vocetur autem linea maior.

Theorema 29. Propoſitio 40.

Si duæ rectæ potentia incommenſurabiles componantur, conficiẽtes compoſitum ex ipſarum quadratis mediale, id vero ꝙ fit ex ipſis, rationale, tota linea eſt irrationales. Vocetur au-

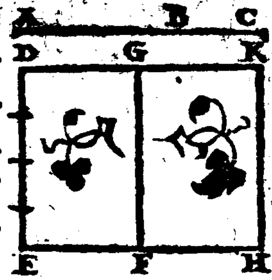


tem potens rationale & mediale.

Theorema 30. Propoſitio 41.

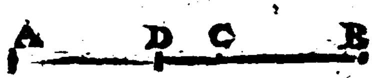
Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex quadratis ipſarum mediale, & quod continetur

netur ex ipsis, mediale, & præterea incommẽsurabile composito ex quadratis ipsarum tota linea est irrationalis. Vocetur autem potens duo medialia.



Theorema 31. Propositio 42.

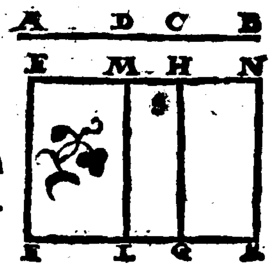
Binomium in vnico tantùm puncto diuiditur in sua nomina, id est in lineas ex quibus componitur.



Theorema 32. Propositio 43.

Bimediale prius in vnico tantum puncto diuiditur in sua nomina.

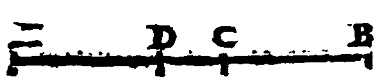
Problema 33. Propositio 44.



Bimediale secundum in vnico tantũ puncto diuiditur in sua nomina.

Problema 34. Propositio 45.

Linea maior in vnico tantùm in pũcto diuiditur in sua nomina.



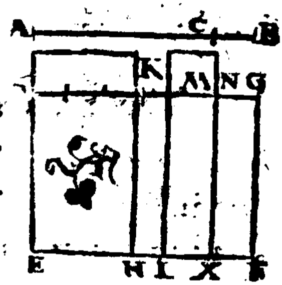
Theo-

Theorema 35. Propositio 46.

Linea potens rationale & mediale in vnico tantùm puncto diuiditur in sua nomina.

Theotema 36. Propositio 47.

Linea potens duo medialia in vnico tantum puncto diuiditur in sua nomina.



DEFINITIONES secundæ.

Proposita linea rationali, & binomio diuiso in sua nomine, cuius binomij maius nomen, id est, maior portio possit plusquam minus nomen quadrato lineæ sibi, maiori inquam nomini, commensurabilis longitudine.

1.

Si quidem maius nomen fuerit commensurabile longitudine propositę lineæ rationali, vocetur tota linea Binomium primum.

2.

Si verò minus nomen, id est minor portio binomij, fuerit commensurabile longitudine

dine

dine propositæ lineæ rationali, vocetur tota linea Binomium secundum.

3.

Si verò neutrum nomen fuerit commensurabile longitudine propositæ lineæ rationali, vocetur Binomium tertium.

Rursus si maius nomé possit plusquam minus nomen quadrato lineæ sibi incommensurabiles longitudine.

4.

Si quidem maius nomen est commensurabile longitudine propositæ lineæ rationali, vocetur tota linea Binomium quartum.

5.

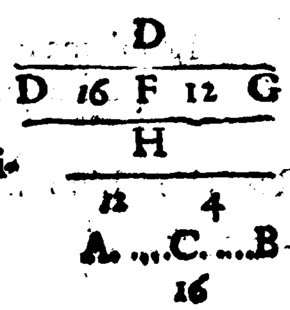
Si verò minus nomen fuerit commensuràbile longitudine lineæ raaionali, vocetur Binomium quintum.

6.

Si verò neutrum nomen fuerit longitudine commensurabile lineæ rationali, vocetur illa Binomium sextum.

Problema 12. Propositio 48.

Reperire Binomium primum.



Pro-

Problema 13. Propositio 49.

9 3
A.....C...B
12

D

D F 9 G

H 6

Reperire Binonium secundum.

Problema 14. Propositio 50.

15 5
A...M....C..B
20

D

....

E

Reperire Binomium tertiū.

F 20 G 15 H

K 6

Problema 15. Propositio 51.

10 6
A.....C....B
16

Reperire Binomium quartum.

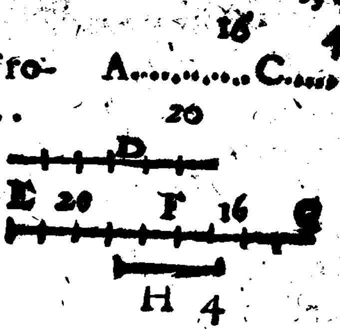
E 16 F 10 G

H 6

Proble-

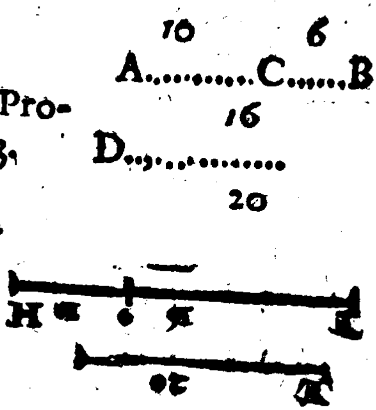
Problema 16. Propositio 52.

Reperire Binomium quintum.



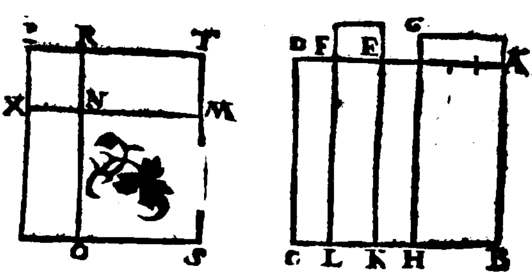
Problema 17. Propositio 53.

Reperire Binonium sextum.



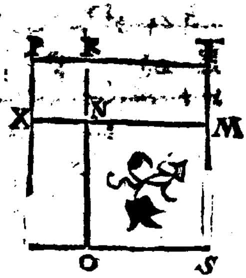
Theorema 37. Propositio 54.

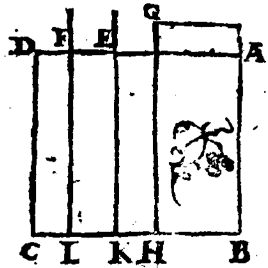
Si superficies contenta fuerit ex rationali & Binomio primo, linea quæ illã superficiẽ potest, est irrationalis, quæ Binomium vocatur.



Theorema 38. Propositio 55

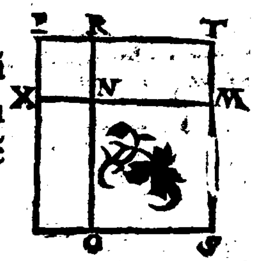
Si superficies contenta fuerit ex linea rationali & Binomio secundo, linea potens illam superficié est irrationalis quæ Bimediale primum vocatur.

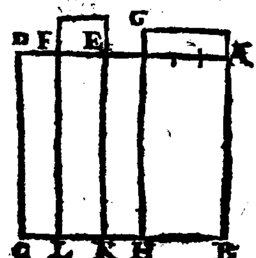




Theorema 39. Propositio 56.

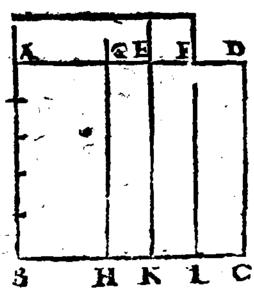
Si superficies contineatur ex rationali & Binomio tertio, linea quã illam superficié potest, est irrationalis quam dicitur Bimediale secũdum.

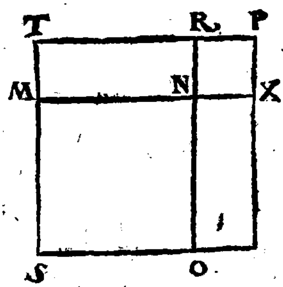




Theorema 40. Propositio 57.

Si superficies contineatur ex rationali



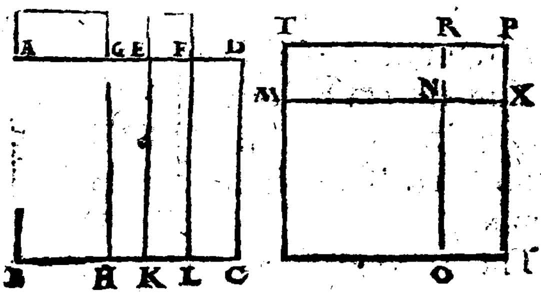


nali

nali & Binomio quarto, linea potens superficiem illam, est irrationalis, quæ dicitur maior.

Theorema 41. Propositio 58.

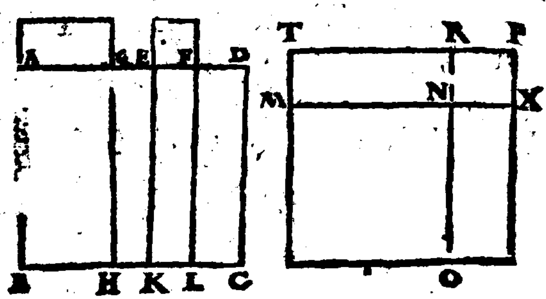
Si superficies contineatur ex rationali & Binomio quinto, linea quæ illam superficiem



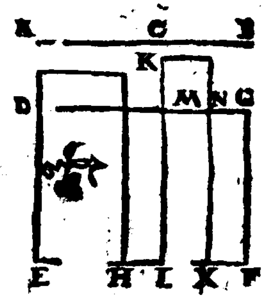
potest, est irrationalis, quæ dicitur potens rationale & mediale.

Theorema 42. Propositio 59.

Si superficies continеatur ex rationali & Binomio sexto, linea quæ illam superficiem potest est irrationalis, quæ dicitur potens duo medialia.

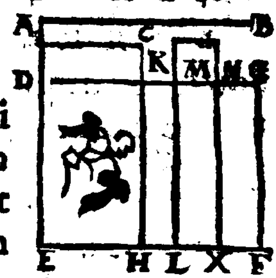


Theorema 43. Pro-
poſitio 60.



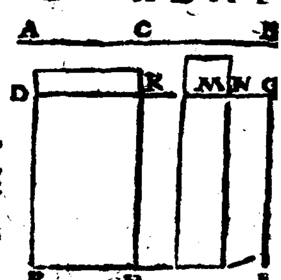
Quadratum Binomij ſecundum lineam rationalem applicatum, facit alterum latus Binomium primum.

Theorema 44. Pro-
poſitio 61.



Quadratũ Bimedialis primi ſecundum rationalem lineam applicatum, facit alterum latus Binomium ſecundum.

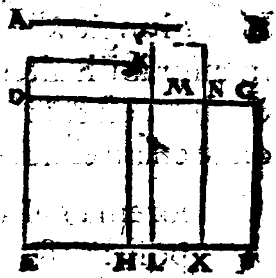
Theorema 45. Pro-
poſitio 62.



Quadratũ Bimedialis ſecundi ſecundũ rationalẽ applicatum, facit alterũ latus Binomium tertiũ.

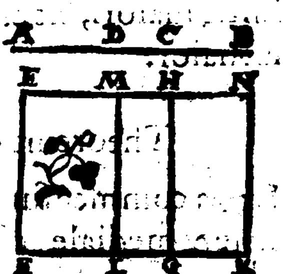
Theo-

Theorema 46. Propositio 63.



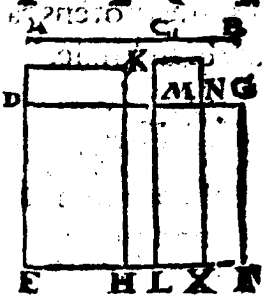
Quadratum lineæ maioris secundum lineam rationalem applicatum, facit alterum latus Binomium quartum.

Theorema 47. Propositio 64.



Quadratum lineæ potẽtis rationale & mediale secundum rationalẽ applicatum, facit alterũ latus Binomium quintũ.

Theorema 48. Propositio 65.

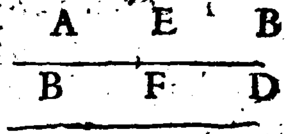


Quadratum lineæ potentis duo medialia secundũ rationalem applicatum, facit alterum latus Binomium sextum.

Theorema 49. Propositio 66.

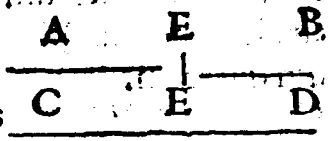
Linea longitudine commensurabilis Binomio est & ipsa Binomium eiusdem ordinis.

Theorema 50 Propositio 67.



Linea longitudine commensurabilis alteri bimedialum, est & ipsa bimediale etiam eiusdem ordinis.

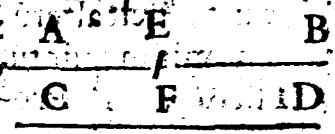
Theorema 51. Propositio 68.



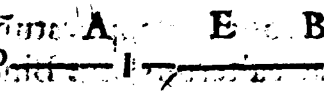
Linea cōmensurabilis lineæ maiori, est & ipsa maior.

Theorema 52 Propositio 69.

Linea commensurabilis liniæ potenti rationale & mediale, est & ipsa linea potens rationale & mediale.



Theorema 53 Propositio 70.

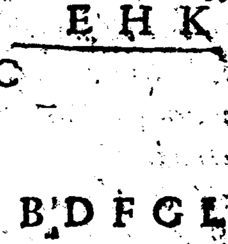


Linea commensurabilis, lineæ potenti duo medialia, est & ipsa linea potens duo medialia.

Theorema 54. Propositio 71.

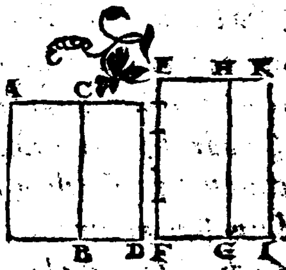
Si duæ superficies rationalis & medialis simul componantur, linea quæ totā superficiem

ciem compositam potest, est vna ex quatuor irrationalibus, vna ex quę dicitur Binomium, vel bimediale primum, vel linea maior, vel linea potens rationale & mediale.



Theorema 55. Propositio 72.

Si duæ superficies mediales incommensurabiles simul componantur fiũt reliquæ duæ lineæ irrationales, vel bimediale secundum, vel linea potens duo medialia.



SCHOLIVM.

Binomium & cætera consequentes lineæ irrationales, neque sunt eædem cum linea mediali, neq; ipsa inter se.

Nam quadratum lineæ medialis applicatum secundum lineam rationalem facit alterum latus lineam rationalem, & longitudine incommensurabilem lineæ secundum quam applicatur, hoc est lineæ rationali, per 23.

Quadratum verò Binomij secundum rationalem applicatum, facit alterum latus Binomium primũ per 60.

Quadratum verò Bimedialis primi secundum rationalem applicatum, facit alterum latus Binomium secundum, per 61.

Quadratum verò Bimedialis secundi secundum rationalem applicatum, facit alterum latus Binomium tertium, per 62.

Quadratum verò lineæ maioris secundum rationalem applicatum facit alterum latus Binomium quartum, per 63.

Quadratum verò lineæ potentis rationale & mediale secundum rationalẽ applicatum facit alterum latus Binomium quintum, per 64.

Quadratum verò lineæ potentis duo medialia secundum rationalem applicatum, facit alterum latus Binomium sextum, per 65.

Cùm igitur dicta latera, quæ latitudines vocantur, differant & à prima latitudine, quoniam est rationalis, cùm inter se quoque differant, eo quia sunt Binomia diuersorum ordinum: manifestum est ipsas lineas irrationales, differentes esse inter se.

SECVN.

SECVNDVS ORDO ALterius sermonis, qui est de detractione.

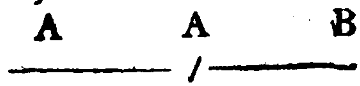
Principium senariorum per detractionem.

Theorema 57. Propositio 74.

Si de linea rationali detrahatur rationalis potentia tantùm commensurabilis ipsi toti residua est irrationalis, vocetur autem Residuum.

Theorema 56 Propopositio 37.

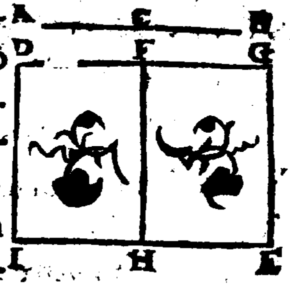
Si de linea mediali detrahatur medialis potentia tantum commensurabilis toti lineæ, quę verò detracta est cum tota contineat superficiem rationalem, residua est irrationalis. Vocetur autem Residuum mediale primum.



Theorema 58. Propositio 75.

Si de linea mediali detrahatur medialis

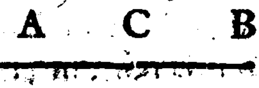
 poten-



tentia tantùm commensurabilis toti, quæ verò detracta est, cùm tota cótineat superficié medialem, reliqua est irrationalis. Vocetur autem Residuum mediale secundum.

Theorema 57. Propositio 76.

Si de linea recta detrahatur recta potentia incommensurabilis toti, compositum autem ex quadratis totius lineæ & lineæ detracta sit rationale, parallelogrammum verò ex eisdem contentum sit mediale, reliqua linea erit irrationalis. Vocetur autem linea minor.

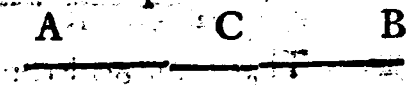


Theorema 58 Propositio 77.

Si de linea recta detrahatur recta poten-

tia

tia incommensurabilis toti lineæ compositum autem ex quadratis totius & lineæ detractæ sit mediale, parallelogrammum verò bis ex eisdem contentum sit rationale, reliqua linea est irrationalis. Vocetur autem linea faciens cum superficie rationali totam superficiem medialem.

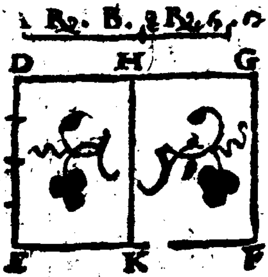


Theorema 59. Propositio 78.

Si de linea recta detrahatur recta potentia incommensurabilis toti lineæ, compositum autem ex quadratis totius & lineæ detractæ sit mediale, parallelogrammum verò bis ex iisdem sit etiam mediale: præterea sint quadrata ipsarum incommensurabilia parallelogrammo bis ex iisdem contento, reliqua linea est irrationalis.

Vocetur autem linea faciens cum superficie

ficie mediali totam superficiem medialem.



Theorema 60. Propositio 79.

Residuo vnica tantùm linea recta cóiungitur rationalis potentia tantùm cōmensurabilis toti lineæ rationalem.

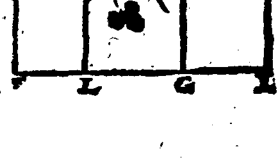
A B B D

Theorema 61. Propositio 80.

Residuo mediali primo vnica tantùm linea coniungitur medialis potentia tantùm commensurabilis toti, ipsa cum tota continés.

A BC D

Theorema 62. Propositio 81.

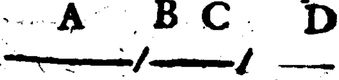


Residuo mediali secundo vnica tantum coniungitur medialis, potentia tantùm commēsurabilia toti ipsa cum tota continens mediale.

Theorema 63. Propositio 82.

Lineę minori vnica tantùm recta coniungitur

tur potentia incommensurabilis toti, faciens cum tota compositum ex quadratis ipsarum rationale, id



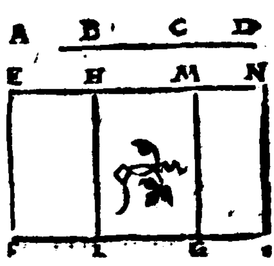
verò parallelogrammum, quod bis ex ipsis fit, mediale.

Theorema 64. Propositio 83.

Lineæ facienti cum superficie rationali totam superficiem medialem, vnica tantùm coniungitur linea recta potentia incommensurabilis toti, faciens autem cum tota compositum ex quadratis ipsarũ, mediale, id verò quod fit bis ex ipsis, rationale.

Theorema 65. Propositio 84.

Lineæ cum mediali superficie facienti totam superficiem medialem, vnica tantum coniũgitur linea potẽtia toti incommẽsurabilis, faciens cum tota compositũ ex quadratis ipsarũ mediale, id verò quod fit bis ex ipsis etiam mediale, & præterea faciens compositum ex quadratis ipsarum incommensurabile ei quod fit bis ex ipsis.



DE-

DEFINITIONES. TERTIAE.

Proposita linea rationali & residuo.

1.

Si quidem tota, nempe composita ex ipso residuo & linea illi cōiuncta, plus potest quàm coniuncta, quadrato lineæ sibi cōmensurabilis longitudine, fueritq; tota longitudine commésurabilis lineæ propositæ rationali, residuum ipsum vocetur Residuum primum.

2.

Si verò coniuncta fuerit longitudine commensurabilis rationali, ipsa autem tota plus possit quam coniuncta, quadrato lineæ, sibi longitudine commensurabilis, residuum vocetur Residuum secundum.

3.

Si verò neutra linearum fuerit longitudine commensurabilis rationali possit autem ipsa tota plusquam coniuncta, quadrato lineæ sibi longitudine commensurabilis, vocetur Residuum tertium.

Rursus si tota possit plus quàm coniuncta, quadrato lineę sibi longitudine incommensueabilis.

4.

Et quidem si tota fuerit longitudine commenmēſurabilis ipſi rationali, vocetur Reſiduū quartum.

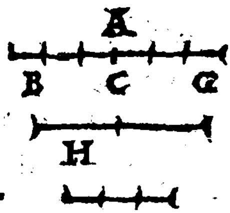
5.

Si verò coniuncta fuerit longitudine commenſurabilis rationali, & tota plus poſſit quàm coniuncta, quadrato lineæ ſibi longitudine incommēſurabilis, vocetur Reſiduum quintum.

6.

Si verò neutra linearum fuerit commenſurabilis longitudine ipſi rationali, fuerítque tota potentior quàm coniūcta, quadrato lineæ ſibi longitudine incommenſurabilis, vocetur Reſiduum ſextum.

Problema 18. Propoſitio 85.



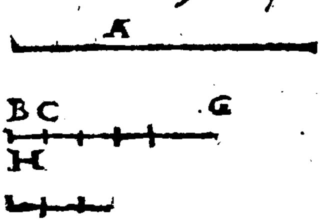
Reperire primum Reſiduum.

16

D.........F.....E

9 7

Problema 19. Propoſitio 86.



Reperire ſecundum Reſiduum.

D........F.. .E

27. 9

Proble-

E..........

Problema 20. Propositio 87.

12

B.........E....C

9 7

Reperire tertium Residuum.

F H G

K

Probl. 21. Propositio 88.

A

B C G

H

Reperire quartum Residuum

D...............F....E

16. 4

Problema 22. Propositio 89.

K

B C G

Reperire quintum Residuum.

H

D.........F.....E

25 7

Problema 23 Propositio 90.

F H G

Reperire sextum Residuum.

K

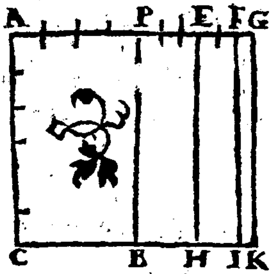
E.......... 13

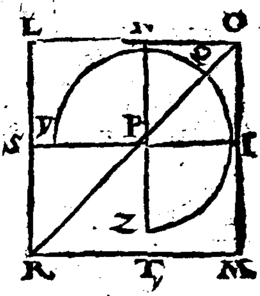
B......D.. ..C

Theo-

Theorema 66. Propositio 91.

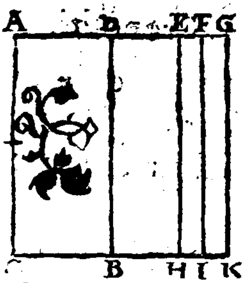
Si superficies contineatur ex linea rationali & residuo primo llnea, quę illam superficiem potest, est residuum.

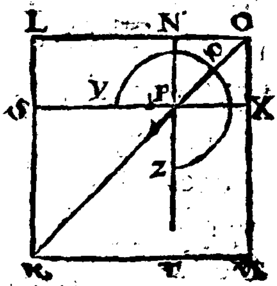




Theorema 67. Propositio 92

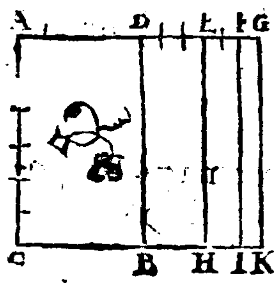
Si superficies contineatur ex linea rationali & residuo secundo linea, qá illam superficié potest, est residuum mediale primum.

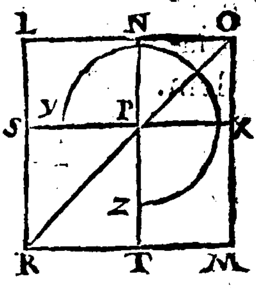




Theorema 68. Propositio 93.

Si superficies cótineatur ex linea rationali & residuo ter-



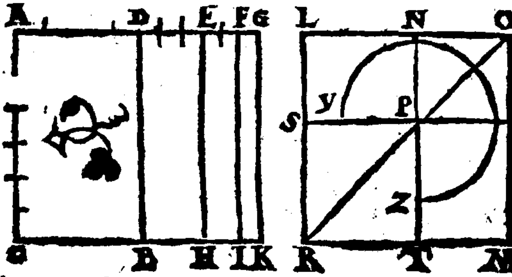


tio, linea quæ illam superficiem potest, e
residuum mediale secundum.

Theorema 69. Propositio 94.

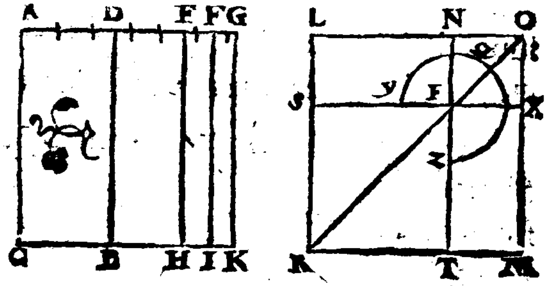
Si superficies contineatur ex linea rational

& residuo quarto linea quã illã superficiem potest, est linea minor.



Theorema 70. Propositio 95.

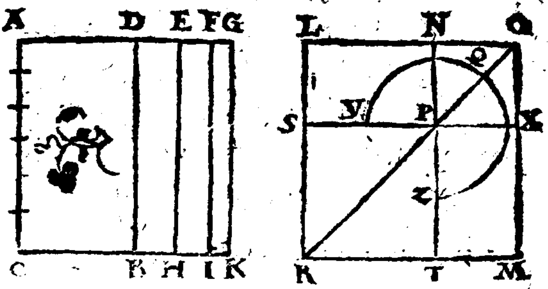
Si superficies contineatur ex linea rational
& residuo quinto, linea quę illam superfici
potest, est ea quæ dicitur cum rationali su
perficie faciens totam medialem.



Theorema 71. Propositio 96.

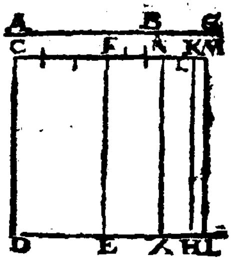
Si superficies contineatur ex linea rational
&

& residuo sexto, linea quæ illam superficié
pót,
est ea
quędi
citur
faciés
cum
me-
diali
superficié totam medialem.



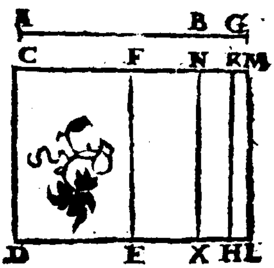
Theorema 72. Propositio 97.

Quadratum residui secundum lineam rationalem applicatum, facit alterum latus residuum primum.



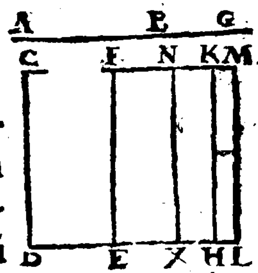
Theorema 73. Propositio 98.

Quadratum residui medialis primi secúdum rationalem applicatũ, facit alterum latus residuũ secundum.



Theorema 74. Propositio 99.

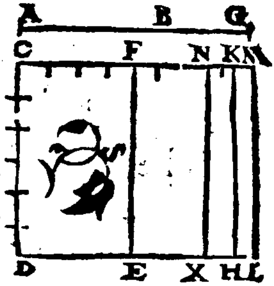
Quadratum, residui medialis secundi secundum rationalem applicatũ, facit alterum latus residuũ tertium.



Theorema 75. Propoſitio 100.

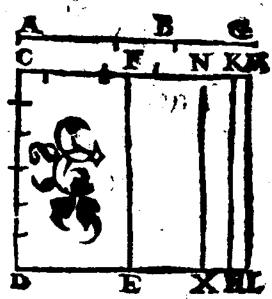
Quadratum lineę minoris ſecundum rationalẽ applicatum, facit alterũ latus reſiduum quartũ.

Theorema 76. Propoſitio 101.



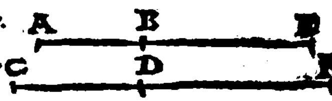
Quadratum lineæ cũ rationali ſuperficie facientis totam medialem, ſecundum rationalem applicatum, facit alterum latus reſiduum quintum.

Theorema 77. Propoſitio 102.



Quadratum lineæ cum mediali ſuperficie faciẽtis totam medialem, ſecundum rationalem applicatum, facit alterum latus reſiduum ſextum.

Theorema 78. Propoſitio 103.

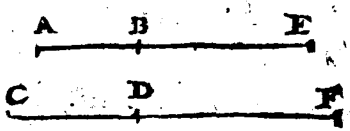


Linea reſiduo commenſurabilis longitudine, eſt & ipſa reſiduum, & eiuſdem ordinis.

Theorema 79. Propoſitio 104.

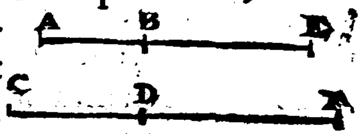
Li-

Linea commēſurabilis reſiduó mediali, eſt & ipſa reſiduū mediale, & eiuſdem ordinis.



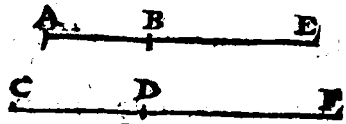
Theorema 80. Propoſitio 105.

Linea cōmēſurabilis lineæ minori, eſt & ipſa linea minor.



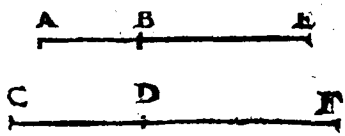
Theorema 81. Propoſitio 106.

Linea commenſurabilis lineæ cum rationali ſuperficie facienti totá medialē, eſt & ipſa linea cum rationali ſuperficie faciens totam medialem.



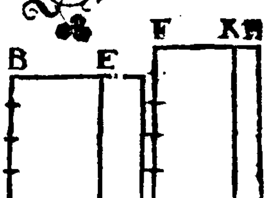
Theorema 82. Propoſitio 107.

Linea commenſurabilis lineæ cum mediali ſuperficie facienti totam medialem, eſt & ipſa cum mediali ſuperficie faciens totam medialem.



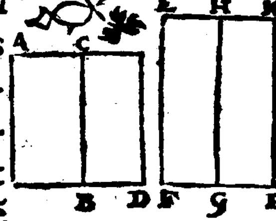
Theorema 83. Propoſitio 108.

Si de ſuperficie rationali detrahatur ſuperficies medialis, linea quæ reliquam ſuperficiem poteſt, eſt alterutra ex duabus irrationalibus, aut reſiduum, aut linea minor.



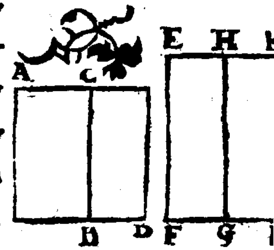
Theorema 84. Propositio 109.

Si de superficie mediali detrahatur superficies rationalis, aliæ duę irrationales, fiunt, aut residuũ mediale primũ aut cum rationali superficiẽ faciens totam medialem.



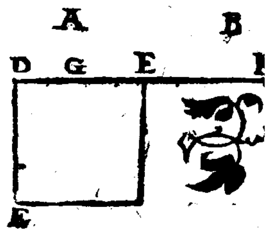
Theorema 85. Propositio 110.

Si de superficie mediali detrahatur superficies medialis quã sit incommẽsurabilis toti, reliquæ duæ fiunt irrationales, aut residuum mediale secũdum, aut cum mediali superficie faciẽs totam medialem.



Theorema 86. Propositio 111

Linea quæ Residuum dicitur, non est eadem cum ea quæ dicitur Binomium.



SCHO

SCHOLIVM.

Linea quæ Residuum dicitur, & cætera quinque eam consequentes irrationales, neq; lineæ mediali neq; sibi ipsa inter se sunt eadem. Nam quadratum lineæ medialis secundum rationalem applicatum facit alterum latus, rationalē lineam longitudine incommēsurabilem ei, secundum quam applicatur, per 23.

Quadratū, verò residui secundū rationalē applicatū, facit alterum latus residuū primū, per 97.

Quadratum verò residui medialis primi secundū rationalem applicatum, facit alterum latus residuum secundum per 98.

Quadratum verò residui medialis secundi, facit alterum latus residuum tertium, per 99.

Quadratum verò lineæ minoris, facit alterum latus residuum quartum, per 100.

Quadratum verò lineæ cum rationali superficie facientis totam medialem, facit alterum latus residuum quintum, per 101.

Quadratum verò lineæ cum mediali superficie facientis totam medialem, secundum rationalē applicatum, facit alterum latus residuum sextum, per 102.

Cùm igitur dicta latera, quæ sunt latitudines cuiusque parallelogrammi vnicuique quadrato æqualis & secundum rationalem, applicati differant & à primo latere, & ipsa inter se (nam à primo differunt: quoniam sunt resi-

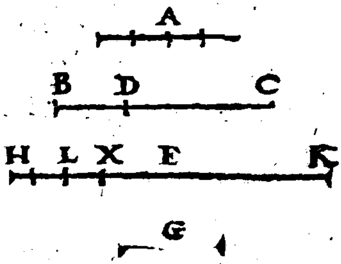
dua non eiusdem ordinis) constat ipsas quoque lineas irrationales inter se differentes esse. Et quoniam demonstratum est, Residuum nō esse idem quod Binomiū, quadrata autem residui & quinque linearū irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex residuis eiusdem ordinis cuius sunt & residua, quorum quadrata applicantur rationali similiter & quadrata Binomij & quinque linearum irrationalium illud consequentium, secūdum rationalem applicata, faciunt altera latera ex Binomijs eiusdem ordinis, cuius sunt & Binomia, quorum quadrata applicantur rationali. Ergo lineæ irrationales quæ consequuntur Binomium, & quæ consequuntur residuum, sunt inter se differentes. Quare dictæ lineæ omnes irrationales sunt numero, 13.

1. Medialis.
2. Binomium.
3. Bimediale primum.
4. Bimediale secundum.
5. Maior.
6. Potens rationale & mediale.
7. Potens duo medialia.
8. Residuum.
9. Residuum mediale primum.
10. Residuum mediale secundum.
11. Minor.
12. Faciens cum rationali superficie totā medialem.
13. Faciēs cum mediali superficie totam medialem.

Theo-

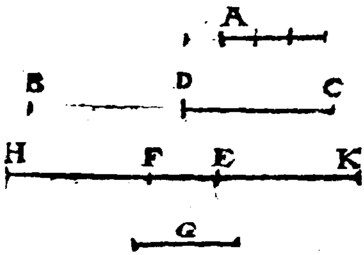
Theorema 87. Propositio 112.

Quadratum lineæ rationalis ſecundum Binomiũ applicatũ, facit alterum latus reſiduũ, cuius nomina ſunt commẽſurabilia Binomij nominibus, & in eadem proportione præterea id quod fit Reſidium, eundem ordinem retinet quem Binomium.



Theorema 88. Propoſitio 113.

Quadratum lineæ rationalis ſecundum reſiduum applicatum, facit alterum latus Binomium cuius nomina ſunt commenſurabilia nominibus reſidui & in eadem proportione: præterea id quod fit Binomium eſt eiuſdẽ ordinis, cuius & Reſiduum.

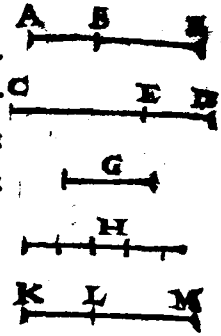


Theorema 89. Propopoſitio 114.

Si parallelogrammum contineatur ex reſi-

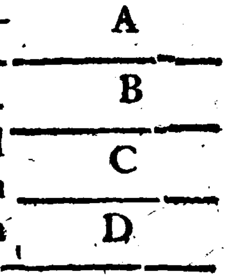
 duo

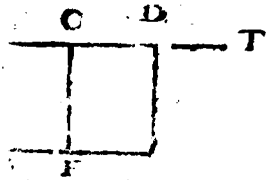
duo & Binomio, cuius nomina sunt commẽsurabilia nominibus residui & in eadem proportione, linea quæ illam superficiem potest, est rationalis.



Theorema 90. Propositio 115.

Ex linea mediali nascuntur lineæ irrationales innumerabiles, quarum nulla vlli ante dicta rum eadem sit.

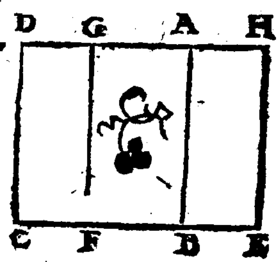




Theorema 100. Propositio 116.

Propositũ nobis esto demonstrare in figuris quadratis diametrũ esse longitudine incommensurabilem ipsi lateri.

E...H..E
E...



ELEMENTI X. FINIS.

EV-

EVCLIDIS ELEMENTVM VNDECIMVM, ET SOLIDORVM primum.

DEFINITIONES.

1.

Solidum, est quod longitudinem, latitudinem, & craſsitudinem habet.

2.

Solidi autem extremum eſt ſuperficies.

3.

Linea recta eſt ad planum recta cùm ad rectas omnes lineas, à quibus illa tãgitur, quæque in propoſito ſunt plano, rectos angulos efficit.

4.

Planum ad planum rectum eſt, cum rectæ lineæ, quæ communi planorum ſectioni ad rectos angulos in vno planorum ducuntur, alteri plano ad recto ſunt angulos.

5.

Rectæ lineę ad planum inclinatio acutus eſt angulus, ipſa inſiſtente linea & adiuncta altera cõprehenſus, cùm à ſublimi rectæ illius lineæ termino deducta fuerit perpendicularis,

laris, atque à puncto quod perpendicularis in ipso plano fecerit, ad propositæ illius lineæ extremum, quod in eodem est plano, altera recta linea fuerit adiuncta.

6.

Plani ad planum inclinatio, acutus est angulus rectis lineis contentus, quæ in vtroque planorum ad idem communis sectionis pũctum ductæ, rectos ipsi sectioni angulos efficiunt.

7.

Planum similiter inclinatum esse ad planũ, atque alterum ad alterum dicitur, cum dicti inclinationum anguli inter se sunt æquales.

8.

Parallela plana, sunt quæ eodem non incidunt, nec concurrunt.

9.

Similes figuræ solidæ, sunt quæ æqualibus planis, multitudine æqualibus continetur.

10.

Aequales & similes figuræ solidæ sunt, quæ similibus planis, multitudine & magnitudine æqualibus continentur.

11.

Solidus angulus, est plurium quàm duarum linearum, quæ se mutuo contingant, nec in eadem sint superficie, ad omnes lineas inclinatio.

Aliter

Aliter.

Solidus angulus, eſt qui pluribus quã duobus planis angulis, in eodem non conſiſtentibus plano, ſed ad vnum punctum collectis continetur.

12.

Pyramis eſt figura ſolida quæ planis continetur, ab vno plano ad vnum punctum collecta.

13.

Priſma figura eſt ſolida quæ planis continetur, quorum aduerſa duo ſunt & æqualia & ſimilia & parallela, alia verò parallelogramma.

14.

Sphæra eſt figura, quæ cõuerſo circum quieſcentem diametrum ſemicirculo continetur, cùm in eundem rurſus locum reſtitutus fuerit, vnde moueri cœperat.

15.

Axis autem ſphæræ, eſt quieſcens illa linea circum quam ſemicirculus conuertitur.

16.

Centrum verò Sphærę eſt idem, quod & ſemicirculi.

17.

Diameter autem Sphæræ, eſt recta quædam linea per centrum ducta, & vtrinque à ſphęræ ſuperficie terminata.

Conus

18.

Conus est figura, quæ conuerso circum quiescens alterum latus eorum quę rectum angulum continent, orthogonio triangulo cōtinetur, cùm in eodem rursus locum illud triangulum restitutum fuerit, vnde moueri cœperat. Atque si quiescens recta linea ęqualis sit alteri, quæ circum rectum angulum cōuertitur, rectangulus erit Conus: sin minor, amblygonius: si verò maior, oxygonius.

19.

Axis autem Coni, est quiescens illa linea, circum quam triangulum vertitur.

20

Basis verò Coni, circulus est, qui à circūducta linea recta describitur.

21.

Cylindrus figura est, quæ conuerso circumquiescens alterum latus eorum quę rectum angulum continent, parallelogrammo orthogonio comprehenditur, cum in eundem rursus locum restitutum fuerit illud parallelogrammum, vnde moueri cœperat.

22.

Axis autem Cylindri est quiescens illa recta linea, circum quam parallelogrammū vertitur.

23.

Bases verò cylindri, sunt circuli à duobus

ad-

aduersus lateribus quæ circum aguntur, descripti

24.

Similes coli & cylindri, sunt quorum & axes & basium diametri proportionales sunt.

25.

Cubus est figura solida, quæ sex quadratis æqualibus continetur.

26.

Tetraedum est figura, quæ triangulis quatuor æqualibus & æquilateris continetur.

27.

Octaedrum figura est solida, quæ octo triãgulis æqualibus & æquilateris continetur.

28.

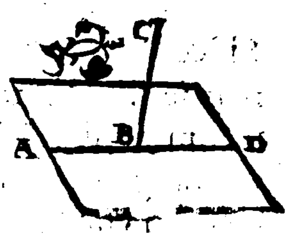
Dedecaedrum figura est solida, quæ duodecim pentagonis æqualibus, æquilateris, & æquiangulis continetur.

29.

Eicosaedrum figura est solida, quę triangulis viginti æqualibus & æquilateris continetur.

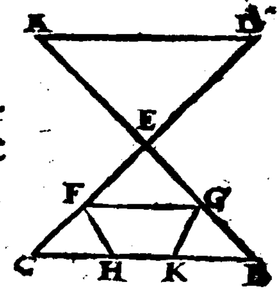
Theorema 1. Propositio 1.

Quædã rectæ lineæ pars in subiecto quidem non est plano, quædam verò in sublimi.



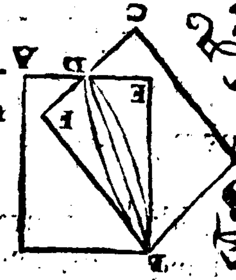
Theo-

Theorema 2. Propoſitio 2.



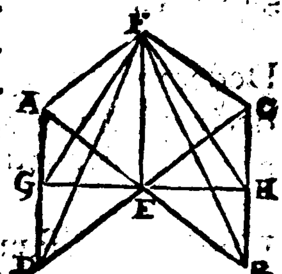
Si duæ rectę lineæ ſe mutuo ſecent, in vno ſunt plano: atque triangulum omne in vno eſt plano.

Theorema 3. Propoſitio 3.



Si duo plana ſe mutuò ſecent, communis eorum ſectio eſt recta linea.

Theorema 4. Propoſitio 4.



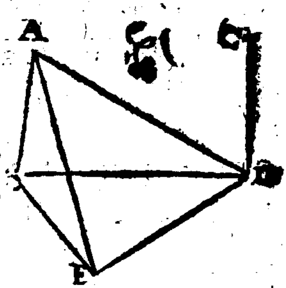
Si recta linea rectis duabus lineis ſe mutuò ſecantibus, in cõmuni ſectione ad rectos angulos inſiſtat illa ducto etiam per ipſas plano ad angulos rectos erit.

Theorema 5. Propoſitio 5.

Si recta linea rectis tribus lineis ſe mutuò tangentibus, in communi ſectione ad rectos angulos inſiſtat, illę tres rectæ in vno ſunt plano.

Theo-

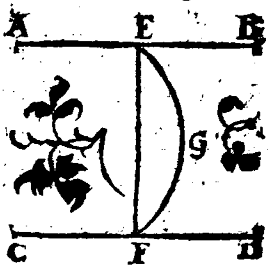
Theorema 6. Propositio 6.



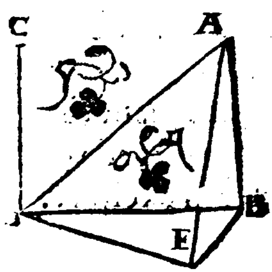
Si duæ rectę lineę eidem plano ad rectos ſint angulos, parallelæ erunt illæ rectæ lineæ.

Theorema 7. Propoſitio 7.

Si duæ ſint parallelæ rectæ lineæ, in quartũ vtraque ſumpta ſint quæ libet pũcta, illa lineæ quæ ad hæc puncta adiungitur, in eodem eſt cum parallelis plano.

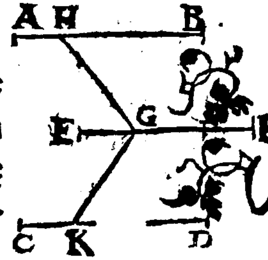


Theorema 8. Propoſitio 8.



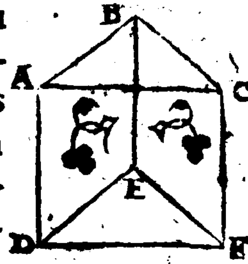
Si duæ ſint parallelæ rectæ lineæ, quarum altera ad rectos cuidam plano ſit angulos, & reliqua eidẽ plano ad rectos angulos erit.

Theorema 9. Propoſitio 9.



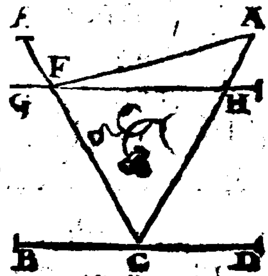
Quæ eidem rectæ lineæ ſunt parallelæ, ſed non in eodem cum illa plano, hę quoque ſunt inter ſe parallelæ.

Theorema 10. Propositio 10.



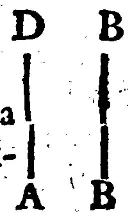
Si duæ rectæ lineæ se mutuò tangentes ad duas rectas se mutuò tangentes sint pallalelę, non autem in eodem plano, illæ angulos æquales comprehendent.

Problema 1. Propositio 11.



A dato sublimi puncto, in subiectum planũ perpendicularem rectam lineam ducere.

Problema 2. Propositio 12.



Dato plano, à pũcto quod in illo datum est, ad rectos angulos rectã lineam excitare.

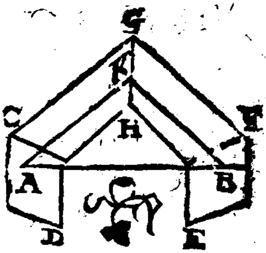
Theorema 11. Propositio 13.

dato plano, à pũcto quod in illo datum est, duæ rectæ lineæ ad rectos angulos non excitabuntur ad easdem partes.

Theo-

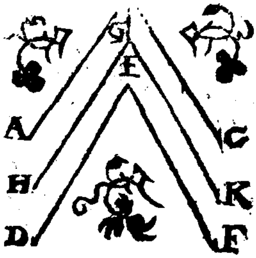
Theorema 12. Propoſitio 14.

Ad quæ plana, eadem recta linea recta eſt, illa ſunt parallela.



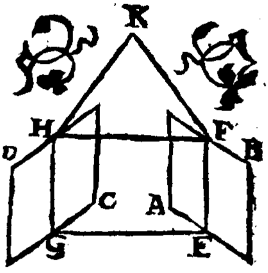
Theorema 13. Propoſitio 15.

Si duæ rectæ lineæ ſe mutuo tangentes ad duas rectas ſe mutuò tangentes ſint parallelæ, non in eodem conſiſtentes plano, parallela ſunt quæ per illas ducuntur plana.



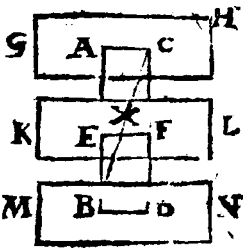
Theorema 14. Propoſitio 16.

Si duo plana parallela plano quopiam ſecentur, communes illorum ſectiones ſunt parallelæ.

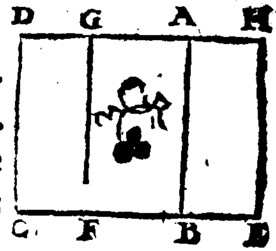


Theorema 15. Propoſitio 17.

Si duæ rectæ lineæ parallelis planis ſecētur, in eaſdēm rationes ſecabūtur.

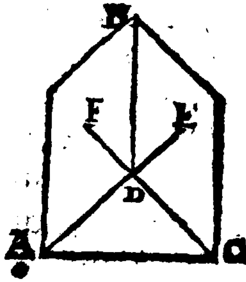


Theorema 16. Propositio 18.



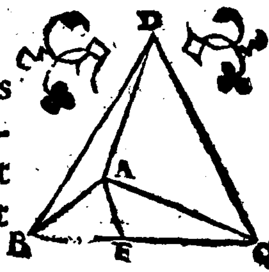
Si recta linea plano cuipiam ad rectos ſit angulos, illa etiam omnia quæ per ipſam plana, ad rectos eidem plano angulos erunt.

Theorema 17. Propoſitio 19.



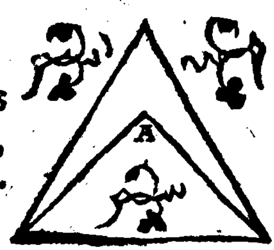
Si duo plana ſe mutuò ſecantia plano cuidã ad rectos ſint angulos, communis etiam illorum ſectio ad rectos eidem plano angulos erit.

Theorema 18. Propoſitio 20.



Si angulos ſolidus planis tribus angulis contineatur, ex his duo quilibet vtut aſſumpti tertio ſunt maiores.

Theorema 19. Propoſiti 21.

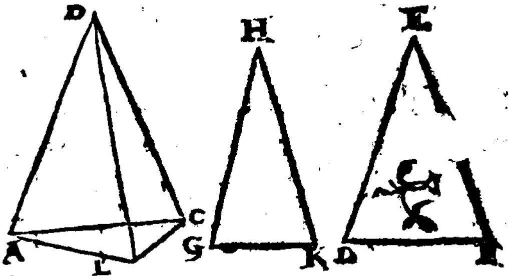


Solidus omnis angulus minoribus continetur, quàm rectis quatuor angulis planis.

Theo-

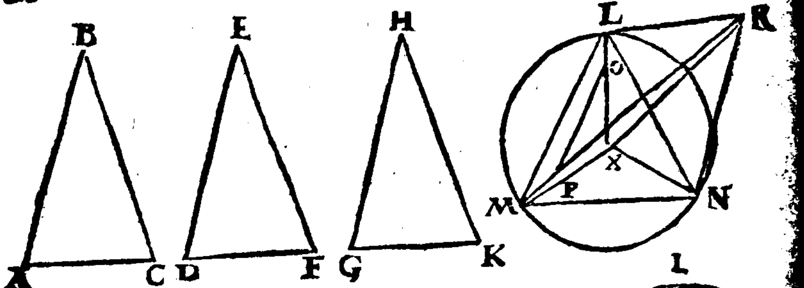
Theorema 20. Propoſitio 22.

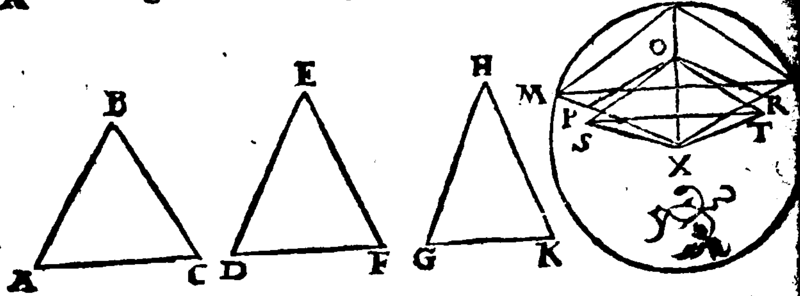
Si plani tres anguli æqualibus rectis contineantur lineis, quorũ duo vt libet aſſumpti, tertio ſint maiores, triangulum conſtitui poteſt ex lineis æquales, illas rectas coniungentibus.



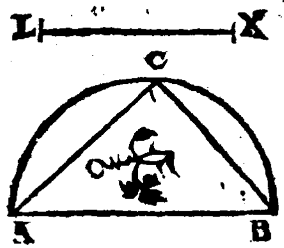
Problema 3. Propoſitio 23.

Ex planis tribus angulis, quorum duo vt libet aſſumpti tertio ſint maiores, ſolidũ angulum conſtituere. Decet autem illos tres angulos rectis quatuor eſſe minores.

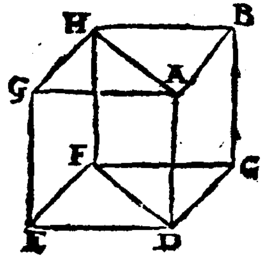




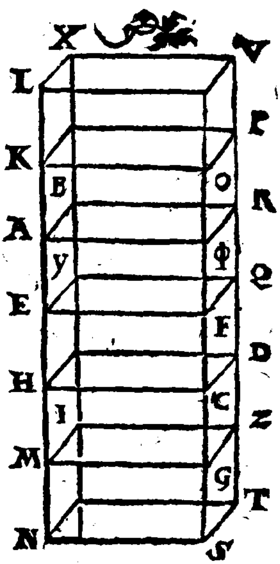
Theorema 21. Propositio 24.



Si solidum parallelis planis contineatur, aduersa illius plana & æqualia sunt & parallelogramma.



Theorema 22. Propositio 25.

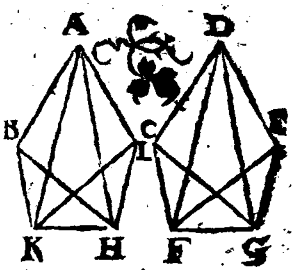


Si solidum parallelis planis contétum plano secetur aduersis planis parallelo, erit quemadmodum basis ad basim, ita solidum ad solidum.

Proble-

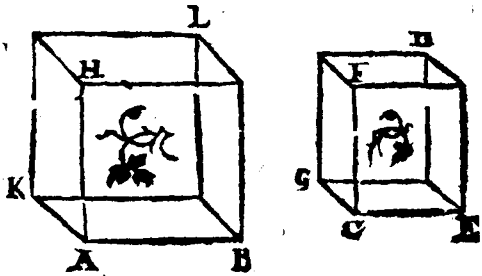
Problema 4. Propoſitio 26.

Ad datam rectam lineã eiuſque punctum, angulum ſolidum conſtituere ſolido angulo dato æqualem.



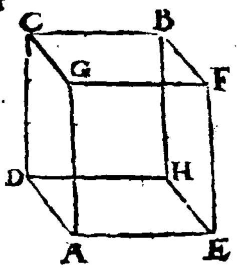
Problema 5. Propoſitio 27.

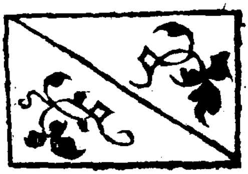
A data recta, dato ſolido parallelis planis comprehenſio ſimile & ſimiliter poſitúm ſolidũ parallelis planis contentum deſcribere.



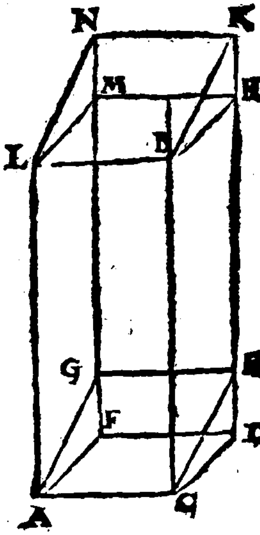
Theorema 23. Propoſitio 28.

Si ſolidum parallelis planis comprehẽſum ductor per aduerſorum planorum diagonios plano ſectũ ſit, illud ſolidum ab hoc plano bifariam ſecabitur.



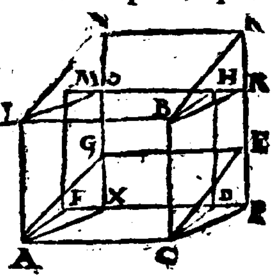


Theorema 24. Propoſitio 29.



Solida parallelis planis comprehẽſa, quæ ſuper eandem baſim & in eadẽ ſunt altitudine quorum inſiſtẽtes lineæ in iiſdem collocantur rectis lineis, illa ſunt inter ſe æqualia.

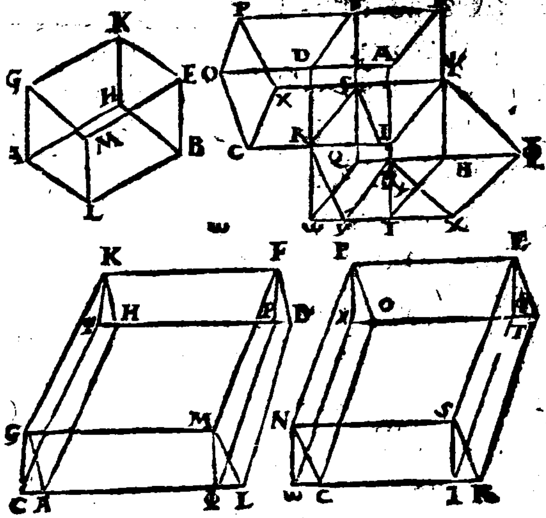
Theorema 25. Propoſitio 30.



Solida parallelis planis circumſcripta, quæ ſuper eandem baſim & in eadẽ ſunt altitudine, quorum inſiſtentes lineæ non in iiſdem reperiuntur rectis lineis, illa ſunt inter ſe æqualia.

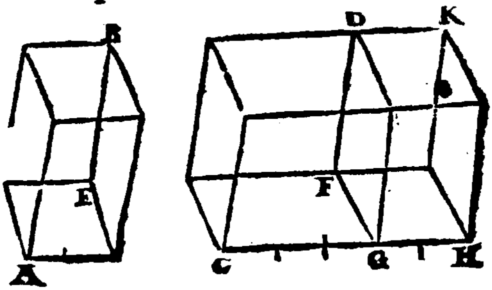
Theorema 26. Propoſitio 31.

Solida parallelis planis circumſcripta, quæ in in eadem sunt altitudine, æqualia sút inter se.



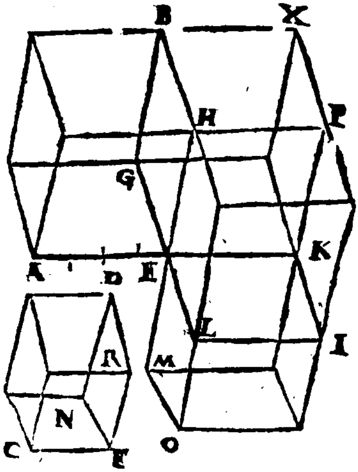
Theorema 27. Propositio 32.

Solida parallelis planis circumscripta quæ eiusdem sunt altitudinis, eam habét inter se rationem, quá bases.

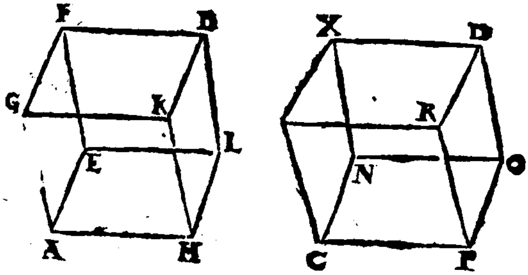


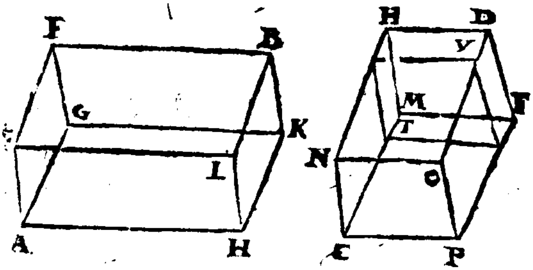
Teeore. 28. Propositio 33.

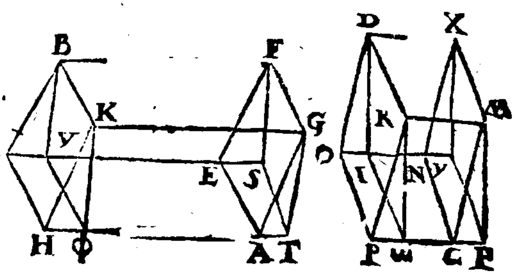
Similia solida parallelis, planis circũscripte habẽt inter se rationẽ homologorũ laterum triplicatam.



Theore. 29. Propositio 24.



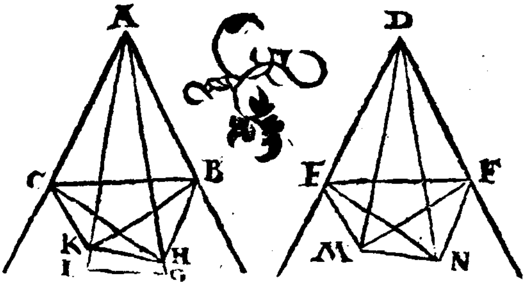




Aequalium solidorũ parallelis planis cõtẽtorum bases cũ altitudinibus reciprocantur. Et solida parallelis planis contẽta quorũ bases cum altitudinibus reciprocantur, illa ſunt æqualia.

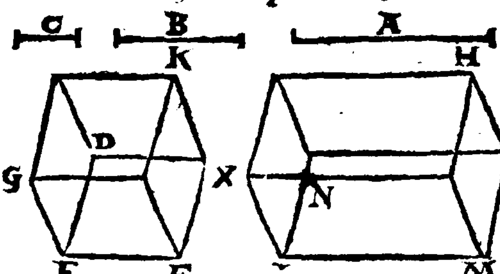
Theorema 30. Propoſitio 35.

Si duo plani ſint anguli æquales, quorum verticibus ſublimes rectæ lineæ inſiſtant, quæ cum lineis primò poſitis angulos contineant equales, vtrunq; vtrique in ſublimibus autem lineis quælibet ſumpta ſint puncta, & ab his ad plana in quibus conſiſtunt anguli primum poſiti, ductæ ſint perpendiculares, ab earum verò punctis, quæ in planis ſignata fuerint, ad angulos primum poſitos ad iūctæ ſint rectæ lineę, hæ cū ſublimibus æquales angulos comprehendent.



Theorema 31. Propoſitio 36.

Si rectæ tres lineæ ſint proportionales, quòd

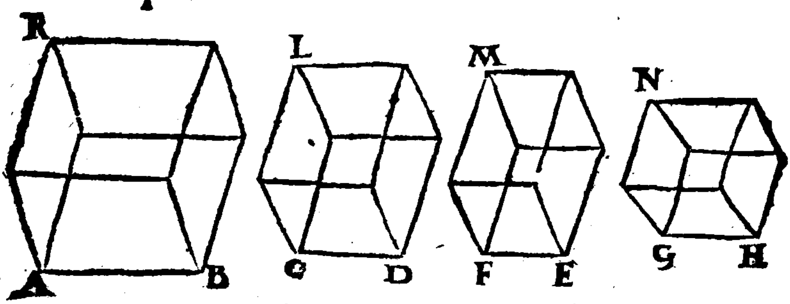


ex

ex his tribus fit solidum parallelis planis cõtentum, æquale est descripto à media linea solido parallelis planis comprehenso, quod æquilaterum quidem sit, sed antedicto æquiangulum.

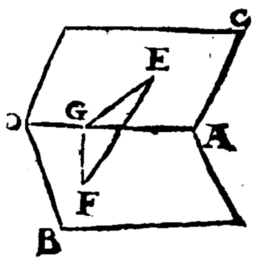
Theorema 32. Propositio 37.

Si rectæ quatuor lineæ sint proportionales, illa quoque solida parallelis planis contenta, quæ ab ipsis lineis & similia & similiter discribuntur, proportionalia erunt. Et si solida parallelis planis comprehensa, quæ & similia & similiter describuntur, sint proportionalia, illæ quoque rectæ lineæ proportionales erunt.



Theorema 33. Propositio 38.

Si planum ad planum rectum sit, & à quodam puncto eorũ quæ in vno sunt planorum perpẽdicularis ad alterum ducta sit, illa quæ ducitur perpẽdicularis, in cõmunem cadet planorum sectionem.



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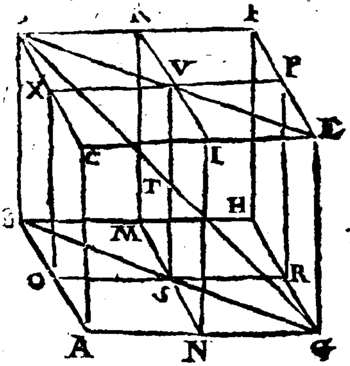
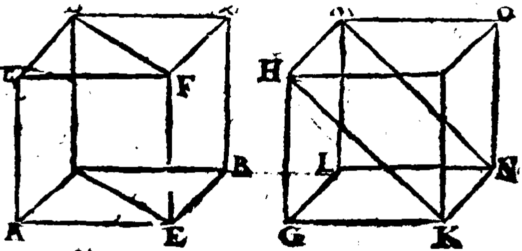
Theorema 34. Propositio 39.

Si in solido parallelis planis circumſcripto, aduerſorum planorum lateribus bifariam ſectis, educta ſint per ſectiones plana, communis illa planorũ ſectio & ſolidi parallelis plani circumſcripti diameter, ſe mutuò bifariã ſecant.



Theorema 35. Propoſitio 40.

Si duo ſint æqualis altitudinis priſmata, quorum hoc quidem baſim habeat parallelogrammum, illud verò triãgulum, ſit autẽ parallelogrammũ triãguli duplũ, illa priſmata erunt æqualia.



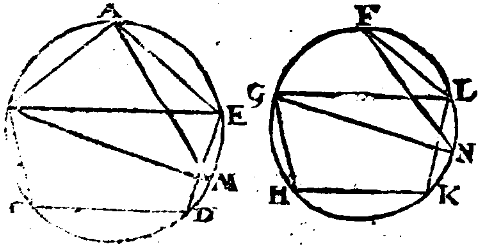
ELEMENTI XI. FINIS.

EVCLIDIS ELEMENTVM DVODECIMVM.

ET SOLIDORVM secundum.

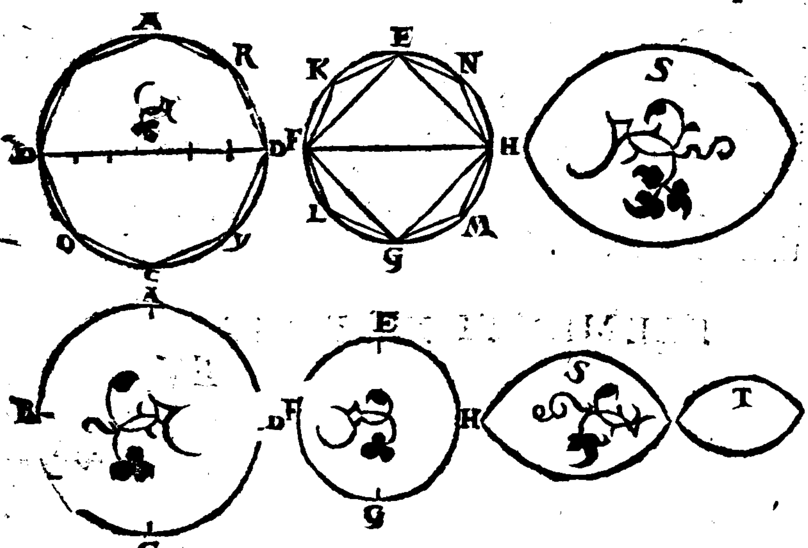
Theorema 1. Propositio 1.

Similia quæ ſunt in circulis polygona, rationem habent inter ſe, quam deſcripta à diametris quadrata.



Theoroma 2. Propoſitio 2.

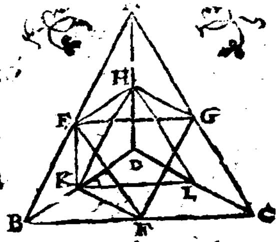
Circuli eam inter ſe rationem habent, quàm



descripta à diametris quadrata.

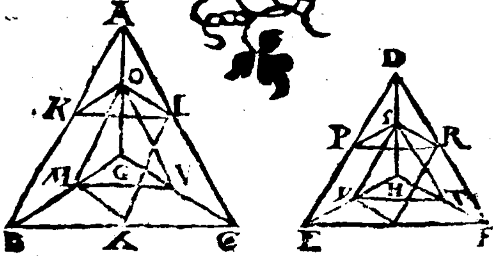
Theorema 3 Propositio 3.

Omnis pyramis trigonam habens basim, in duas diuiditur pyramidas nõ tantùm æquales & similes inter se, sed toti etiam pyramidi similes, quarum trigonæ sunt bases, atq; in duo prismata æqualia, quæ duo prismata dimidio pyramidis totius sunt maiora.



Tneorema 4. Propositio 4.

Si duæ eiusdem altitudinis pyramides trigonas habeant bases, sit autem illarũ vtraq; diuisa & in duas pyramides inter se æquales totique similes, & in duo prisma æqualia, ac eodem modo diuidatur vtraq; pyramidum quæ ex superiore diuisione natæ sunt, idque perpetuo fiat: quẽadmodum se habet vnius piramidis basis ad alterius pyramidis basim, ita & omnia quæ in vna pyra-

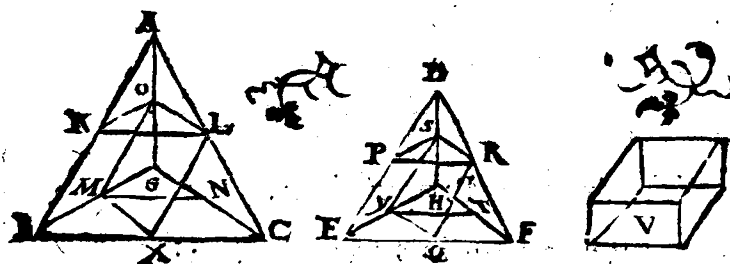


mide

mide prismata ad omnia quæ in altera pyramide, prismata multitudine æqualia.

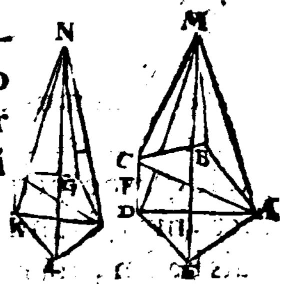
Theorema 5. Propositio 5.

Pyramides eiusdem altitudinis, quarum trigonæ sunt bases, eam inter se rationem habent, quàm ipsæ bases.



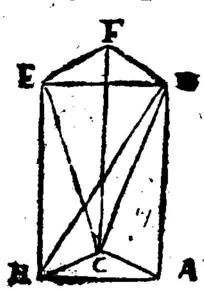
Theorema 6. Propositio 6.

Pyramides eiusdem altitudinis, quarum polygonæ sunt bases, eam inter se rationem habent, quã ipsæ bases.



Theorema 7. Propositio 7.

Omne prisma trigonã habens basim, diuiditur in tres pyramides inter se æquales, quartum trigonæ sunt bases.



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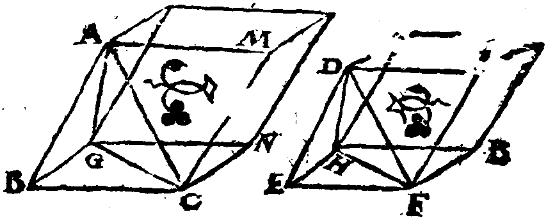
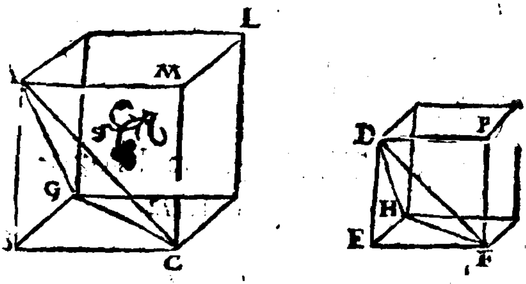
Theorema 8. Propositio 8.

Similes pyramides qui trigonas habent bases, in triplicata sũt homologorũ laterũ ratióe



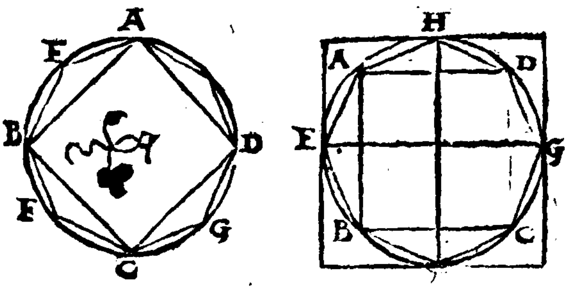
Theorema 9. Propositio 9.

Aequaliũ pyramidum & trigonas bases habentium reciprocantur bases cum altitudinibus. Et quarum pyramidum trigonas bases habétiũ reciprocãtur bases cũ altitudinibus, illæ sunt æquales.



Theorema 10. Propositio 10.

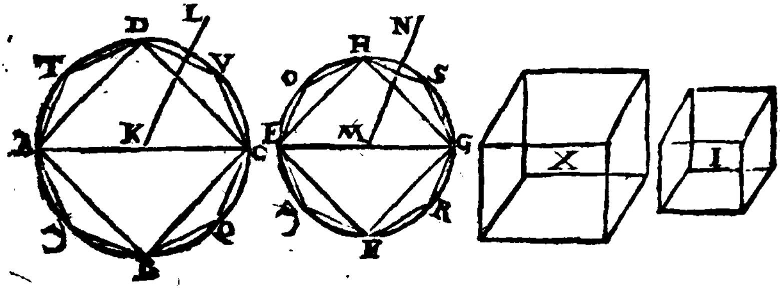
Omnis conꝰ tertia pars est cylindri



eandem cum ipso cono basim habentis, & altitudinem æqualem.

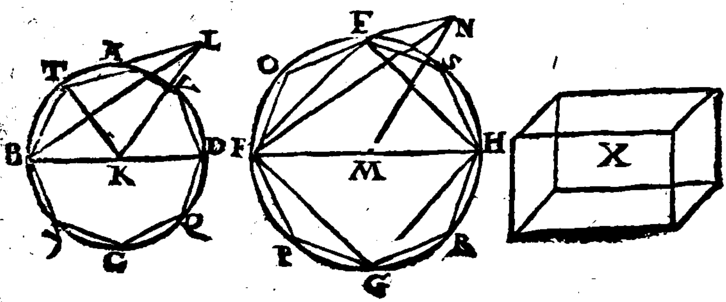
Theorema 11. Propositio 11.

Coni & cylindri eiusdem altitudinis, eam inter se rationem habent, quam bases.



Theorema 12. Propositio 12.

Similes coni & cylindri, triplicatam habent inter se rationem diametrorum, quæ sunt in basibns.



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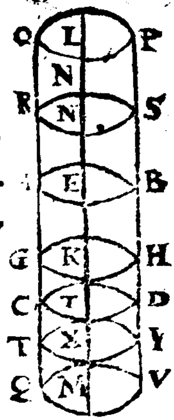
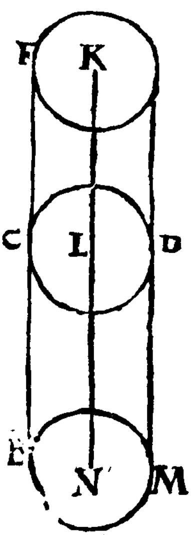
Theorema 13 Propositio 13.

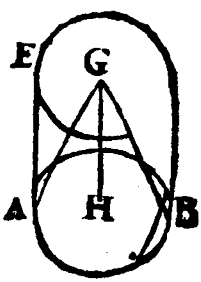


Si cylindrus plano sectus sit aduersis planis parallelo, erit quémadmodum cylindrus ad cylindrum, ita axis ad axem.

Theorema 14. Propositio 14.

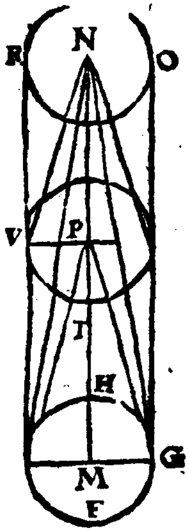
Coni & cylindri qui in æqualibus sunt basibus, eam habent inter se rationem, quam altitudines

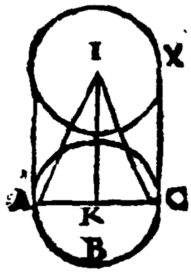




Theorema 15. Propositio 15.

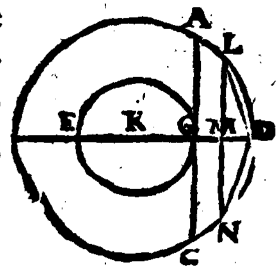
Aequalium conorum & cylindrorum bases cũ altitudinibus reciprocantur. Et quorum conorũ & cylindrorum bases cum altitudinibus reciprocantur, illi sunt ęquales.





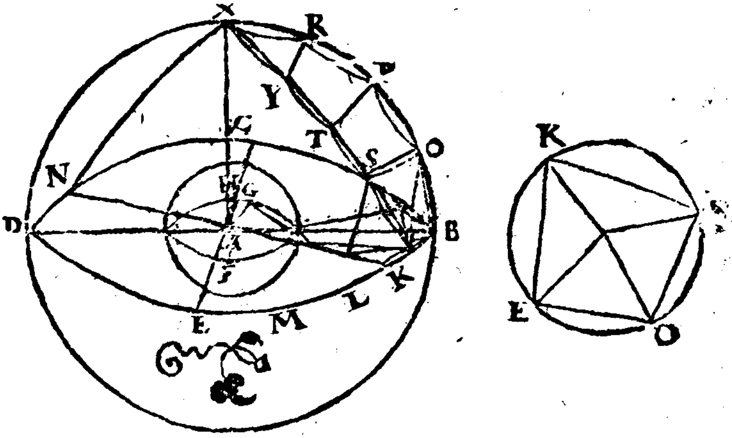
Theorema 1. Propositio 16.

Duobus circulis circum idem centrum cõsistentibus, in maiore circulô polygonum æqualium pariumque laterum inscribere, quod minorem circulum non tangat.



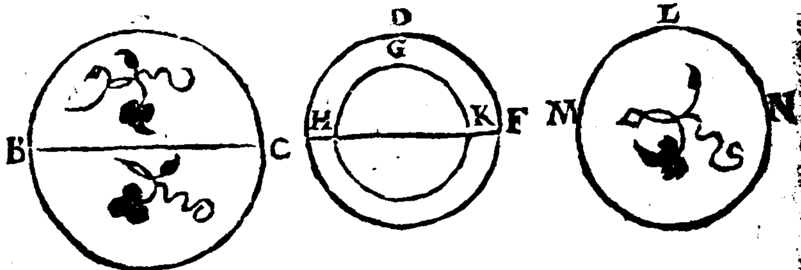
Problema 2. Propoſitio 17.

Duabus ſphęris circum idem centrum conſiſtentibus, in maiorem ſphęra ſolidum polyedrum inſcribere, quod minoris ſpæræ ſuperficiem non tangat.



Theorema 16. Propoſitio 18.

Sphæræ inter ſe rationem habent ſuarum diametrorum triplicatam.

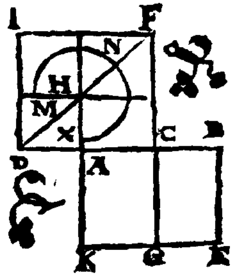


Elementi duodecimi finis.

EVCLIDIS ELEMENTVM DECIMVM TERTIVM, ET SOLIDORVM tertium.

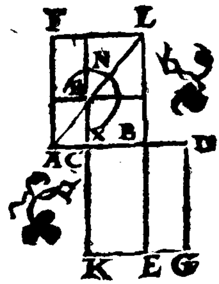
Theorema 1. Propositio 1.

Si recta linea per extremam & mediam rationẽ secta sit, maius segmentũ quod totius lineæ dimidium assumpserit, quintuplum potest eius quadrati, quod à totius dimidia describitur.



Theorema 2. Propositio 2.

Si recta linea sui ipsius segmenti quintuplum possit, & dupla segmenti huius linea per extremam & mediam rationẽ secetur maius segmẽtum reliqua pars est liniæ primum positæ.



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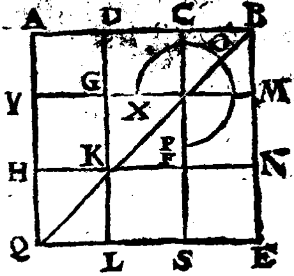
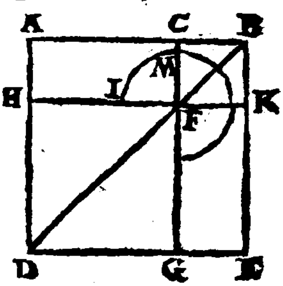
Theorema 3. Propositio 3.



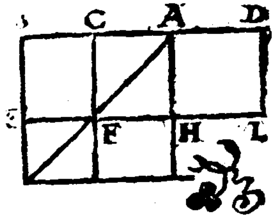
Si rectæ linea per extremam & mediam rationem secta sit minus segmẽtum quod maioris segmẽti dimidium assumpserit, quintuplum potest eius, quòd à maioris segmenti dimidio describitur, quadrati.

Theorema 4. Propositio 4.



Si recta linea per extremam & mediam rationẽ secta sit, quòd à tota, quodque à minore segmento simul vtraq; quadrata, tripla sunt eius, quod à maiore segmẽto describitur, quadrati.

Theorema 5. Propositio 5.



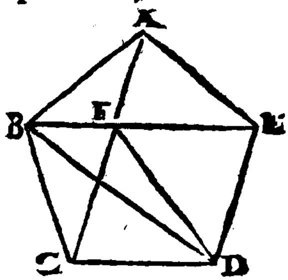
Si ad rectam lineam, quæ per extremam & mediam rationem secetur, adiuncta sit altera segmento maiori æqualis, tota hæc linea recta per extremam & mediam rationẽ secta est, estque maius segmentum linea primùm posita.

Theorema 6. Propositio 6.

Si recta linea ῥητὴ siue rationalis, per extremam & mediam rationem facta sit, vtrunque segmentorum ἄλογος siue irrationalis est linea, quæ dicitur Residuum.

Theorema 7. Propositio 7.

Si pentagoni æquilateri tres sint æquales anguli, siue quã deinceps siue qui non deinceps sequuntur, illud pentagonum erit æquiangulum.



Theorema 8. Propositio 8.

Si pentagoni æquilateri & æquianguli duos qui deinceps sequuntur angulos rectæ subtendãt lineæ, illę per extremam & mediam rationem se mutuò secant, earumque maiora segmenta, ipsius pentagoni lateri sunt æqualia.

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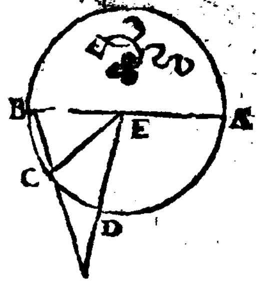
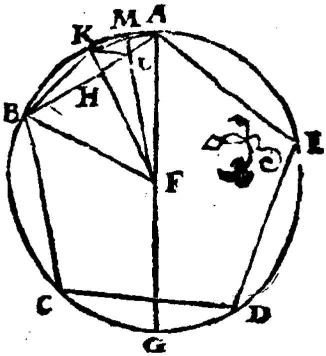
Theorema 9. Propositio 9.



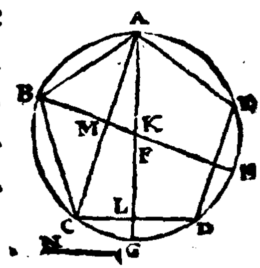
Si latus hexagoni & latus decagoni eidem circulo inſcriptorum compoſita ſint, tota recta linea per extremam & mediam rationem ſecta eſt, eiuſque ſegmẽtum maius, eſt hexagoni latus.

Theorema 10. Propoſitio 10.



Si circulo pẽtagonum ęquilaterum inſcriptum ſit, pentagoni latus poteſt & latus hexagoni & latus decagoni eidem circulo inſcriptorum.

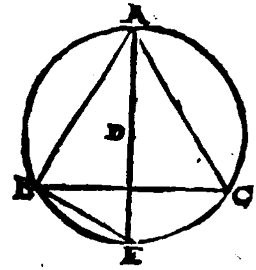
Theorema 11. Propoſitio 11.



Si in circulo ῥητὴν habente diametrum, inſcriptum ſit pentagonum æquilaterum, pentagoni latus irrationalis eſt linea quæ vocatur Minor.

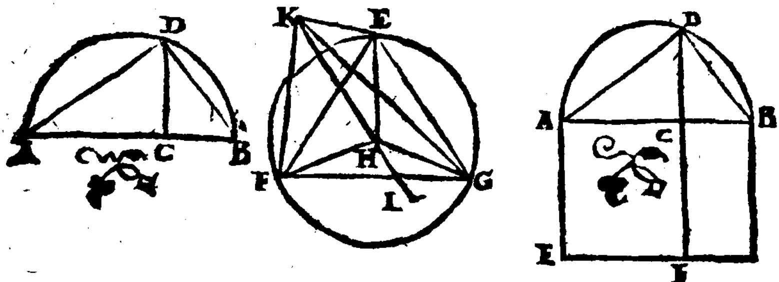
Theorema 12. Propoſitio 12.

Si in circulo inſcriptum ſit triangulum æquilaterum, huius trianguli latus potentia triplum eſt eius lineę, quæ ex circuli centro ducitur.



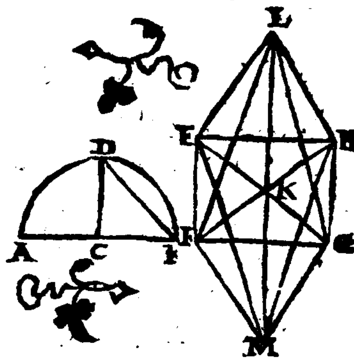
Problema 1. Propoſitio 13.

Pyramidem conſtituere, & data ſphæra cõplecti, atque docere illius ſphæræ diametrũ potentia ſeſquialteram eſſe lateris ipſius pyramidis.



Theorema 2. Propoſitio 14.

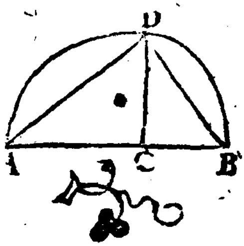
Octaedrum conſtituere, eaq; ſphæra qua pyramidẽ complecti, atque ꝓbare illius ſphæræ diametrũ potentia duplã eſſe lateris ipſius octaedri.

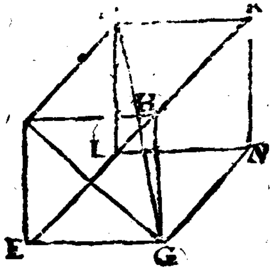


Proble.

Problema 3. Propositio 15.

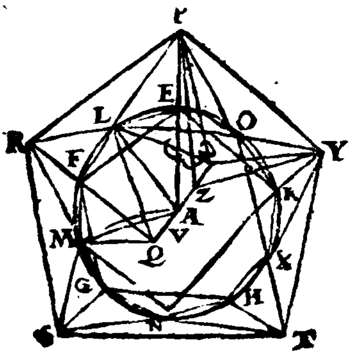
Cubum constituere, eaque sphæra qua & superiores figuras complecti, atque docere illius sphæræ diametrum potētia triplam esse lateris ipsius cubi.

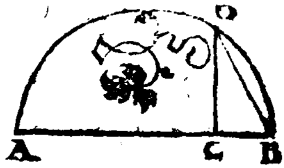


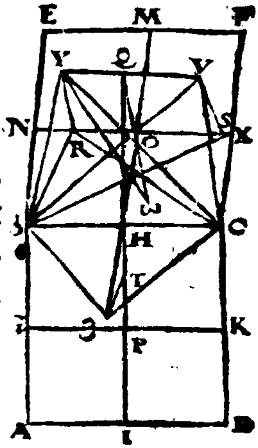


Problema 4. Propositio 16.

Icosaedrum constituere, eademque sphæra qua & antedictas figuras complecti, atque probare, Icosaedri latus irrationalem esse lineam, quæ vocatur Minor.



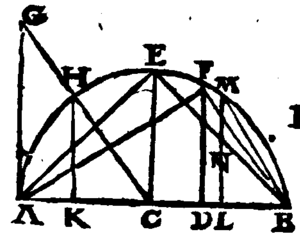


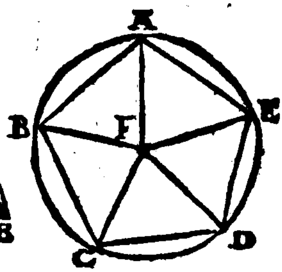


Problema 5. Propositio 17.

Dodecaedrũ constituere, eademque sphæra qua & antedictas figuras complecti, atque probare dodecaedri latus irrationalẽ esse lineam, quæ vocatur Residuum.

Theorema 6. Propositio 18.





quinq; figurarum latera proponere, & inter se comparare.

SCOLIVM.

Aio verò, præterea dictas quinque figuras nõ posse aliam constitui figuram solidam, quæ planis & æquilateris & æquiangulis cõtineatur, inter se æqualibus. Non enim ex duobus triangulis, sed neque ex aliis duabus figuris solidus cõstituetur angulus.

Sed

Sed ex tribus triãgulis, cõstat Pyramidis angulus
Ex quatuor autem, Octaedri
Ex quinque verò, Icosaedri.
Nam ex triangulis, sex & æquilateris & æquiangulis ad idem punctum coeuntibus, non fiet angulus, recti vnius bessem contineat, erunt eiusmodi sex anguli rectis quatuor æquales. Quod fieri non potest. Nam solidus omnis angulus minoribus quam rectis quatuor angulis continetur, per 21.11.
Ob easdem sanè causas, neque ex pluribus quam planis sex eiusmodi angulis solidus constat.
Sed ex tribus quadratis, Cubi angulus cõtinetur.
Ex quinque, nullus potest. Rursus enim recti quatuor erunt.
Ex tribus autem pentangonis æquilateris & æquiangulis Dodecaedri angulus continetur.
Sed ex quatuor, nullus potest. Cùm enim pentagoni æquilateri angulus rectus sit, & quinta recti pars erũt quatuor anguli rectis quatuor maiores. Quod fieri nequit. Nec sanè ex alijs polygonis figuris solidus angulus cõtinebitur, quod hinc quoque absurdum sequatur. Quamobrem perspicuum est, præter dictas quinque figuras aliã figuram solidam non posse constitui, quæ ex planis æquilateris & æquiangulis contineatur.

ELEMEMTI XIII. FINIS.

EVCLIDIS ELEMENTVM DECIMVMQVARTVM, VT quidam arbitrantur, vt alij verò, Hypsiclis Alexandrini, de quinque corporibus.

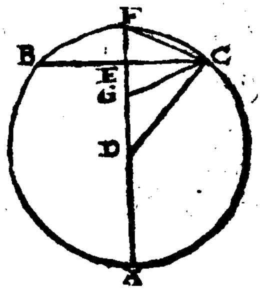
LIBER PRIMVS.

Basilides Tyrius, Protarche, Alexandriã profectus, patriq; nostro ob disciplinæ societatem commendatus, longissimo peregrinationis tempore cum eo versatus est. Cumq; differerẽt aliquãdo descripta ab Apollonio comparatione Dodecaedri & Icosaedri eidem sphæræ inscriptorum, quam hæc inter se habeant rationem, censuerunt ea non rectè tradidisse Apollonium: quæ à se non emendata, vt de patre audire erat literis prodiderunt. Ego autem postea incidi in alterum librum ab Apollonio editum, qui demonstrationem accuratè complecteretur de re proposita, ex eiusq; problematis indagatione magnam equidem cœpi voluptatem. Illud certè ab omnibus perspici potest quod scripsit Apollonius, cùm sit in omnium manibus. Quod autẽ diligenti, quantũ conijcere licet, studio nos postea scrip-

scripsisse videmur, id monimentis consignatum tibi nuncupandum duximus, vt qui feliciter cùm in omnibus disciplinis tum vel maximè in Geometria versatus, scitè ac prudẽter iudices ea quæ dicturi sumus ob eam verò, quæ tibi cum patre fuit, vitæ consuetudinem, quaq; nos complecteris, beneuolentiam, tractationem ipsam libenter audias. Sed iam tempus est, vt præmio modum facientes, hanc syntaxim aggrediamur.

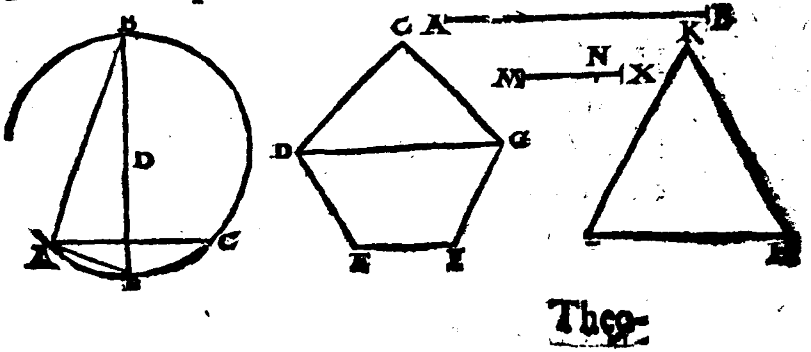
Theorema 1. Propositio 1.

Perpendicularis linea, quæ ex circuli cuiuspiam centro in latus pentagoni ipsi circulo inscripti ducitur, dimidia est vtriusque simul lineæ, & eius, quæ ex centro & lateris decagoni in eodẽ circirculo inscripti.



Theorema 2. Propositio 2.

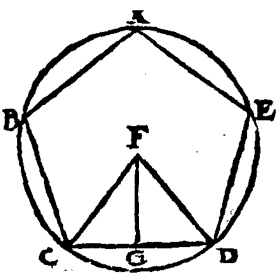
Idem circulus comprehẽdit & dodecaedri penragonum & icosaedri triãgulum, eidem sphæræ inscriptorum.

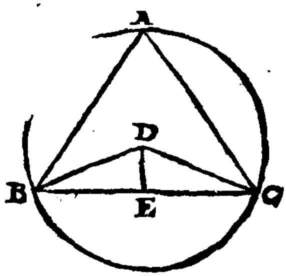


Theo-

Theorema 3. Propoſitio 3.

Si pentagono & æquilatero & æquiangulo circumſcriptus ſit circulus ex cuius centro in vnum pentagoni latus dicta ſit perpendicularis: quod vno laterum & perpendiculari trigeſies cõtinetur, illud



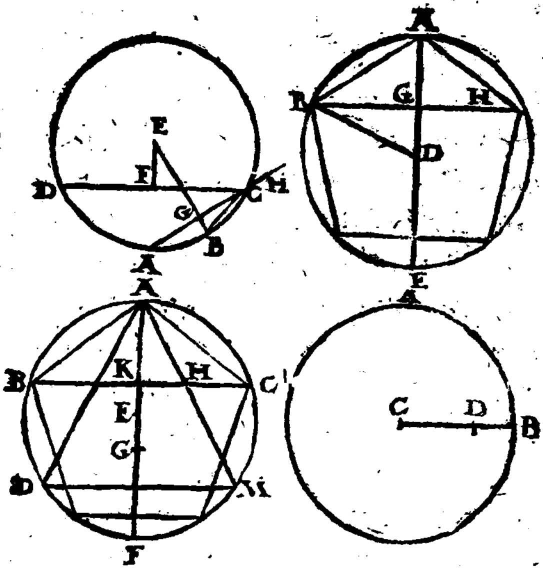


æquale eſt dodecaedri ſuperficiei.

Theorema 4. Propoſitio 4.

Hoc perſpicuum cùm ſit probandum eſt, quemadmodum ſe habet dodecaedri ſuperficies ad icoſaedri ſuperficiem ita ſe habere cubi latus ad icoſaedri latus.

Cubi



Cubilatus.

E————————

Dodecaedri.

F————————

Icosaedri.

G————————

SCHOLIVM.

Nunc autem probandum est, quemadmodum se habet cubi latus ad icosaedri latus ita se habere solidum dodecaedri ad icosaedri solidum. Cùm enim æquales circuli comprehendat & dodecaedri pentagonum & Icosaedri triangulum, eidem

P *sphæra*

sphæræ inscriptorum: in sphæris autem æquales circuli æquali interuallo distent à centro (siquidē perpendiculares à sphæræ cētro ad circulorum plana ducta & æquales sunt, & ad circulorum centra cadunt) idcirco lineæ, hoc est perpendiculares quæ à sphæræ centro ducuntur ad centrum circuli comprehendētis & triangulum Icosaedri, pētagonum dodecaedri sunt æquales. Sunt igitur æqualis altitudinis Pyramides, quæ bases habent ipsa dodecaedri pentagona, & quæ Icosaedri triangula. At æqualis altitudinis pyramides rationē inter se habent eam quam bases, ex 5. & 6. 11. Quemadmodum igitur pentagonum ad triangulum, ita pyramis, cuius basis quidem est dodecaedri pentagonum, vertex autem sphæræ centrum ad pyramida, cuius basis quidem est Icosaedri triāgulum, vertex autem sphæræ centrū. Quamobrem vt se habent duodecim pentagona ad viginti triangula, ita duodecim pyramides, quorū pentagona sint bases, ad viginti pyramidas, quæ trigonas habeant bases. At pentagona duodecim sunt dodecaedri superficies, viginti autem triangula, Icosaedri. Est igitur vt dodecaedri superficies ad Icosaedri superficiem, ita duodecim pyramides quæ pentagonas habeant bases, ad viginti pyramidas, quarum trigonæ sunt bases. Sunt autē duodecim quidem pyramides, quæ pētagonas habeant bases, solidum dodecaedri: viginti autem pyramides, quæ trigonas habeant bases, Icosaedri solidū. Quare ex 11. 5. vt dodecaedri superficies ad

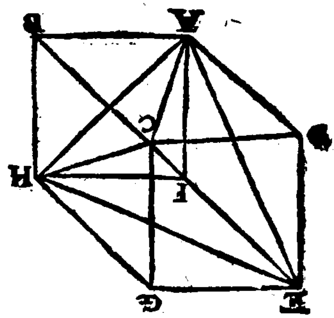
Icosae-

Icosaedri superficiem, ita solidum dodecaedri ad Icosaedri solidum. Vt autem dodecaedri superficiem, ita probatum est cubi latus ad Icosaedri latus. Quemadmodum igitur cubi latus ad Icosaedri latus, ita se habet solidum dodecaedri ad Icosaedri solidum.

ELEMENTI XIIII. FINIS.

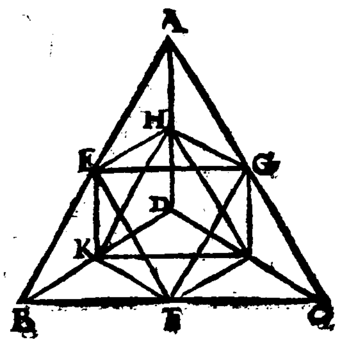
EVCLIDIS ELEMENTVM DECIMVM QVINTVM, ET Solidorum quintum, vt nonnulli putant, vt autem alij Hypsiclis Alexandrini, de quinque corporibus.

LIBER II.



Problema 1. Propositio 1.

In dato cubo pyramida inscribere.



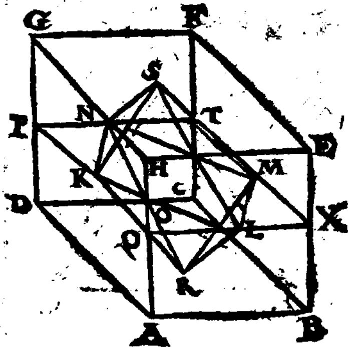
Problema 2. Propositio 2.

In data pyramide octaedrum inscribere.

Pro-

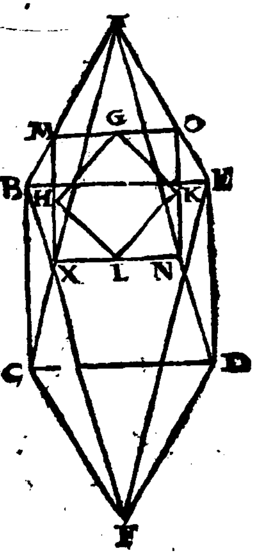
Problema 3. Propositio 3.

In dato cubo octaedrum inscribere.



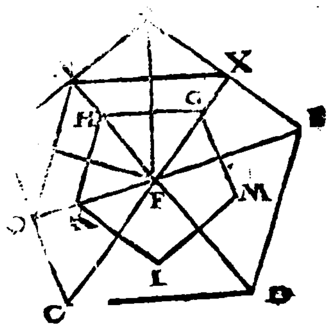
Problema 4. Propositio 4.

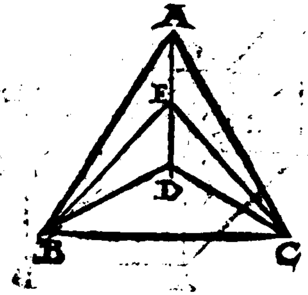
In dato octaedro cubũ inscribere.

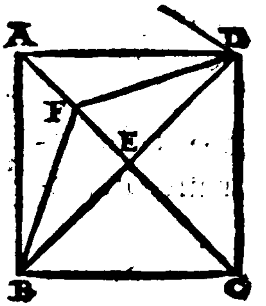


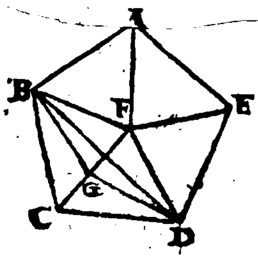
Problema 5. Propositio 5.

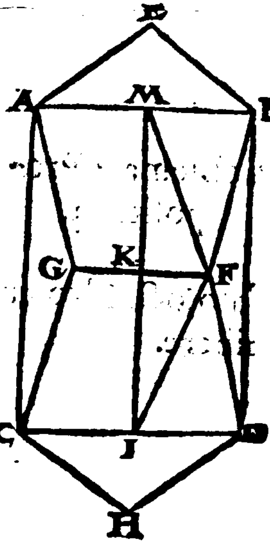
In dato Icosaedro dodecaedrum inscribere.

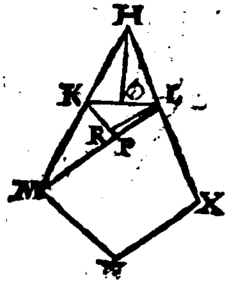












SCHO.

SCHOLIVM.

Meminisse decet, si quis nos roget, quot Icosaedrum habeat latera, ita respondendum esse: Patet Icosaedrum viginti contineri triangulis, quodlibet verò triangulum rectis tribus constare lineis. Quare multiplicanda sunt nobis viginti triangula in trianguli vnius latera, fiuntq́ sexaginta, quorum dimidium est triginta. Ad eũdem modum & in dodecaedro. Cum enim rursus duodecim pentagona dodecaedrum comprehēdant itemq́ pentagonum quoduis rectis quinque cõstet lineis, quinque duodecies multiplicamus, fiunt sexaginta, quorum rursus dimidium est triginta. Sed cur dimidium capimus? Quoniã vnumquodq́ latus siue sit triãguli, siue pentagoni, siue quadrati vt in cubo, iterato sumitur. Similiter autem eadem via & in cubo & in pyramide & in octaedro latera inuenies. Quòd si item velis singularum quoq́ figurarum angulos reperire, facta eadem multiplicatione numerum procreatum partire in numerũ planorum, quæ vnum solidum angulum includũt: vt quoniam triangula quinque vnum Icosaedri angulum continet, partire 60. in quinque, nascuntur duodecim anguli Icosaedri. In dodecaedro autem tria pentagona angulum comprehendunt, partire ergo 60. in tria, & habebis dodec. … ri angulos viginti. Atq́ simili ratione & reliquis figuris angulos reperies.

Finis Elementorum Euclidis.