4

EVCLIDIS ELEMENTORVM LIBRI XV.

A. m. b. 1454

QVIBVS, CVM AD OMNEM MATHEMATICAE SCIENTIAE PARTEM, TVM AD QVAMLIbet Geometriæ tractationem facilis comparatur aditus.

COLONIAE
Apud Gosuinum Cholinum
M. D. C.
Cum Gratia & Priuilegio.

Euclides Geometer tempore Ptolemaei, Primi Regis Aegypti, vixit ante Christum annis 291.

AD CANDIDVM LECTOREM ST. GRACILIS praefatio.

PERMAGNI referre semper existimaui, Lector beneuole, quantum quisq; studij & diligentiæ ad percipienda scientiarum elementa adhibeat, quibus non satis cognitis, aut perperam intellectis, si vel digitum progredi tentes, erroris caliginem animis offundas, non veritatis lucem rebus obscuris adferas. Sed principiorum quanta sint in disciplinis momenta, haud facilè credat, qui rerum naturam ipsa specie, non viribus metiatur. Vt enim corporum, quæ oriuntur & intereunt, vilißima tenuiß. maq; videntur initia: ita rerum æternarū & admirabilium, quibus nobilißimæ artes continentur, elementa ad speciem sunt exilia, ad vires & facultatem quàm maxima. Quis non videt ex fici tantulo grano, vt ait Tullius, aut ex acino vinaceo, aut ex cæterarum frugum aut stirpium minutißimis seminibus tantos truncos ramosq; procreari? Nam Mathematicorum initia illa quidem dictu audituque perexigua, quantam theorematum syluam nobis pe-

pererunt? Ex quo intelligi potest, vt in ipsis seminibus, sic et in artium principijs inesse vim earum rerum, quae ex his progignuntur. Praeclarè igitur Aristoteles, vt alia permulta, μέγιστον ἴσως ἀρχὴ παντός, ᾗ ὅσῳ κράτιστον τῇ δυνάμει, τοσούτῳ μικρότατον, ουτῷ μεγέθει χαλεπόν ἐστιν ὀφθῆναι. *Quocirca committendum non est, vt non benè prouisa & diligēter explorata scientiarum principia, quibus propositarum quarumq; rerum veritas sit demonstranda, vel constituas, vel constituta approbes. Cauendum etiam, vt ne tantulum quidem fallaci & captiosa interpretatione turpiter deceptus, à vera principiorū ratione temerè deflectas. Nam qui initio fortè aberrauerit, is vt tandē in maximis versetur erroribus, necesse est: cū ex vno erroris capite densiores sensim tenebrae rebus clarißimis obducātur. Quid tam varias veterum physiologorum sententias non modo cū rerum veritate pugnantes, sed vehementer etiam inter se dissentiētes nobis inuexit? Equidem haud scio, fuerit ne vlla potior tanti dißidij causa, quā quod ex principijs partim falsis, partim non cōsentaneis ductas rationes probando adhiberent. Fit enim plerunque, vt qui non rectè de artium rerumq; elementis sentiunt, ad praefinitas quasdam opiniones suas omnia reuocari studeant. Pythagorei, vt meminit Aristoteles, cum denarij numeri summam perfectionem coelo tribuerent, nec plures tamen quam nouem sphaeras cernerent, decimam affingere ausi sunt terrae aduer-*

sam

sum, quam ἀντίχθονα appellarunt. Illi enim vniuersitatis rerumq́ singularum naturam ex numeris seu principijs æstimantes, ea protulerunt quæ φαινομένοις congruere nusquam sunt cognita. Nam ridicula Democriti, Anaximenis, Melissi, Anaxagoræ, Anaximandri, & reliquorum id genus physiologorum somnia, ex falsis illa quidē orta natura principijs, sed ad Mathematicum nihil aut parum spectantia, sciens prætereo. Nonnullos attingam, qui repetitis altiùs vel aliter ac decuit positis rerum initijs, cum physicis multa turbarunt; tum Mathematicos oppugnatione principiorum pessimè mulctarunt. Ex planis figuris corpora constituit Timæus: Geometrarum hic quidē principia cuniculis oppugnantur. Nam & superficies seu extremitates crassitudinem habebunt & lineæ latitudinem: denique puncta non erunt indiuidua, sed linearum partes. Prædicant Democritus atque Leucippus illas atomos suas, & indiuidua corpuscula. Concedit Xenocrates impartibiles quasdam magnitudines. Hic verò Geometriæ fundamenta apertè petuntur, & funditus euertuntur: quibus dirutis nihil equidē aliud video restare, quàm vt amplissima Mathematicorum theatra repente concidant. Iacebunt ergò, si dijs placet, tot præclara Geometrarum de asymmetris & alogis magnitudinibus theoremata. Quid enim causæ dicas, cur indiuidua linea hanc quidem metiatur, illam verò metiri non queat? Siquidem quod minimū in vnoquoq́

 genere

genere reperitur, id communis omnium mensura esse solet. Innumerabilia profectò sunt illa, quæ ex falsis eiusmodi decretis absurda consequũtur, & horum permulta quidem Mathematicus, sed longè plura colligit Physicus. Quid varia ψευδογραφημάτων genera commemorem, quæ ex hoc vno fonte tam longè lateq; diffusa fluxisse videntur? Notißimus est Antiphontis tetragonismus, qui Geometrarum & ipse principia non parum labefecit, cùm rectæ lineæ curuam posuit æqualem. Lõgũ esset mihi singula percensere præsertim ad alia properanti. Hoc ergo certum, fixum, & in perpetuum ratũ esse oportet, quod sapienter monet Aristoteles, Σπυδαστέον ὅπως ὁρισθῶσι καλῶς αἱ ἀρχαί· μεγάλην γὰρ ἔχουσι ῥοπὴν πρὸς ἑπόμενα. Nam principijs illa congruere debent, quæ sequũtur. Quòd si tantum perspicitur in istis exilioribus Geometriæ initijs, quæ pũcto, linea, superficiè definiuntur, momentum, vt ne hæc quidem sine summo impendentis ruinæ periculo conuelli aut appugnari poßint, quanta quæso vis putanda est huius στοιχειώσεως, quam collatis tot præstantißimorum artificum inuentis, mira quadam ordinis solertia contexuit Euclides, vniuersa Matheseos elementa complexu suo coercentẽ. Vt igitur omnibus rebus instructior & paratior quisq; ad hoc studium libentius accedat, & singula vel minutißima exactius secum reputet atque perdiscat, operæ precium censui, in primo institutionis aditu, vestibuloq; præcipua quædem capita quibus tota ferè Mathematicæ sciẽtiæ ratio intelligatur, breuiter explicare: tum ea quæ sunt Geometriæ propria, diligẽter persequi: Euclidis deniq; in extruenda hac στοιχειώσει cõsiliũ sedulò ac fideliter exponere. Quæ ferè omnia ex Aristotelis potissimum ducta fõtibus, nemini inuisa fore cõfido, qui modò ingenuũ animi cãdorem ad legendũ attulerit. Ac de Mathematicæ diuisione primum dicamus.

Mathematicæ in primis scientiæ studiosos fuisse Pythagoreos, non modò historicorum, sed etiam philosophorum libri declarant. His ergo placuit, vt in partes quatuor vniuersum distribuatur Mathematicæ scientiæ genus, quarũ duas περὶ τὸ ποσὸν, reliquas περὶ τὸ πηλίκον versari statuerunt. Nam & τὸ ποσὸν vel sine vlla comparatione ipsum per se cognosci, vel certa quadam ratione cõparatum spectari: in illo Arithmeticam, in hoc versari Musicam: & τὸ πηλίκον partim quiescere, partim moueri quidem: Illud Geometriæ propositum esse: quod verò sua sponte motu cietur, Astronomiæ. Sed ne quis falsò putet, Mathematicam scientiam, quòd in vtroque quanti genere cernitur, idcirco inanem videri (si quidem non solum magnitudinis diuisio sed etiam multitudinis accretio infinitè progredi potest) meminisse decet, τὸ πηλίκον καὶ τὸ ποσὸν, quæ subiecto Mathematicæ generi imposita sunt à Pythagoreis nomina, non cuiuscunque modi quantitatem significare, sed eam demum, quæ tum multitudine tum magnitudine sit definita, & suis circumscripta terminis. Quis enim vllam infiniti sciētiam defendat? Hoc scitum est, quod non semel docet Aristoteles, infinitum ne cogitatione quidem complecti quenquam posse. Itaque ex infinita multitudinis & magnitudinis δυνάμει finitā hac sciētia decerpit & amplectitur naturā, quam tractet, & in qua versetur. Nā de vulgari Geometrarum cōsuetudine quid sentiendum sit, cum data interdum magnitudine infinita haud fabricātur aliquid, aut proprias generis subiecti affectiones exquirūnt, disertè monet Aristoteles, οὐδὲ νῦν (de Mathematicis) loquens δέονται τοῦ ἀπείρου, οὐδὲ χρῶνται, ἀλλὰ μόνον εἶναι ὅσην ἂν βούλωνται, πεπερασμένην. Quamobrem disputatio ea qua infinitum refellitur, Mathematicarum decretis rationibusq; non aduersatur, nec eorum apodixes labefacit. Etenim tali infinito opus illis nequaquam est, quod exitu nullo peragrari poßit, nec talem ponunt infinitam magnitudinem: sed quātamcunq; velit aliquis effingere, ea vt suppetat, infinitam praecipiunt. Quin etiam non modò immensa magnitudine opus non habent Mathematici, sed ne maxima quidem: cum instar maxima minima quaque in partes totidem pari ratione diuidi queat. Alteram Mathematica diuisionem attulit Geminus, vir (quantum ex Proclo coniicere licet) μαθημάτων laude clarißimus. Eā, qua superiore plenior & accuratior fortè visa est, cum doctißimè pertractarit sua in decimum Euclidis *præfatione P. Montaureus vir senatorius, & regiæ bibliothecæ præfectus, leuiter attingam. Nam ex duobus rerum velut summis generibus τῶν νοητῶν καὶ τῶν αἰσθητῶν, quæ res sub intelligentiā cadūt, Arithmeticæ & Geometriæ attribuit Geminus: quæ verò in sensus incurrunt, Astrologiæ, Musicæ Supputatrici, Opticæ, Geodesiæ & Mechanicæ adiudicauit. Ad hanc certè diuisionem spectasse videtur Aristoteles, cum Astrologiam Opticā, harmonicā φυσικωτέρας τῶν μαθημάτων nominat, vt quæ naturalibus & Mathematicis interiecta sint, ac velut ex vtriusq; mixta disciplinæ: Siquidem genera subiecta à Physicis mutuantur, caussas verò in demonstrationibus ex superiore aliqua sciētia repetūt. Id quod Aristoteles ipse apertissimè testatur, ἐνταῦθα γὰρ, φησὶ, τὸ μὲν ὅτι, τῶν αἰσθητικῶν εἰδέναι τὸ διότι τῶν μαθηματικῶν. Sequitur, vt quid Mathematicæ conueniat cum Physica et prima Philolosophia: quid ipsa ab vtraque differat, paucis ostendamus. Illud quidem omnium commune est, quod in veri contemplatione sunt posita, ob idq; θεωρητικαὶ à Græcis dicuntur. Nam cùm διάνοια siue ratio & mēs omnis sit vel πρακτική vel θεοριτική, totidem scientiarum sint genera necesse est. Quòd si Physica, Mathematica, & prima Philosophia, nec in agendo, nec in efficiendo sunt occupata, hoc certè perspicuum est, eas omnes in cognitione contemplationeq; necessariò versari. Cùm enim rerum non modò agendarum, sed etiam efficiendarum* principia in agente vel efficiente consistunt, illarum quidem προαίρεσις, harum autem vel mens vel ars, vel vis quædam & facultas: rerum profectò naturalium. Mathematicarum, atque diuinarum principia in rebus rebus ipsis, non in philosophis inclusa latent. Atque hæc vna in omnes valet ratio, quæ θεωρητικὰς esse colligat. Iam verò Mathematica separatim cum Physica congruit, quòd vtraque versatur in cognitione formarum corpori naturali inhærentium. Nam Mathematicus plana, solida, longitudines & puncta contemplatur, quæ omnia in corpore naturali à naturali quoque philosopho tractantur. Mathematica item & prima philosophia hoc inter se propriè conueniunt, quòd cognitionem vtraque persequitur formarum, quoad immobilæ, & cõcretione materiæ sunt liberæ. Nam tametsi Mathematicæ formæ reuera per se non cohærent, cogitatione tamen à materia & motu separantur, οὐδὲ γίνεται ψεῦδος χωριζόντων. vt ait Aristoteles. De cognatione & societate breuiter diximus, iam quid intersit videamus. Vnaquæq; Mathematicarum certum quoddam rerum genus propositum habet, in quo versetur, vt Geometria quantitatem & continuationem aliorum in vnã partem, aliorum in duas, quorũdam in tres, eorumq; quatenus quanta sunt & continua, affectiones cognoscit. Prima autẽ philosophia, cùm sit omniũ communis, vniuersum Entis genus quæq; ei accidunt & conueniunt hoc ipso quòd est, considerat.

Ad

Adhæc, Mathematica eam modò naturam amplectitur, quæ quanquam non mouetur, separari tamen seiungiq́ nisi mente & cogitationn à materia non potest, ob eamq́ causam ἐξ ἀφαιρέσεως dici consueuit. Sed Prima philosophia in ijs versatur, quæ & seiuncta, & æterna, & ab omni motu per se soluta sunt ac libera. Cæterùm Physica & Mathematica quanquam subiecto discrepare non vidētur, modo tamen rationeq́ differūt cognitionis & contemplationis, vnde dissimilitudo quoq́ scientiarum sequitur. Etenim Mathematicæ species nihil reuera sunt aliud, quàm corporis naturalis extremitates, quas cogitatione ab omni motu & materia separatas Mathematicus cōtemplatur: sed easdem cōsectatur Physicorū ars, quatenus cum materia comprehensa sunt, & corpora motui obnoxia circumscribunt. Ex quo fit, vt quæcunque in Mathematicis incommoditates accidunt, eædem etiam in naturalibus rebus videantur accidere, nō autem vicissim. Multa enim in naturalibus sequuntur incommoda, quæ nihil ad Mathematicum attinent, διὰ τὸ, inquit Aristoteles, τὰ μὲν ἀἐξ φαιρέσεως, λέγεται, τὰ μαθηματικὰ, τὰ δὲ φυσικὰ ἐκ προσθέσεως. Siquidem res cum materia deuinctas cōtemplatur Physicus: Mathematicus verò rem cognoscit circumscriptis ijs omnibus, quæ sensu percipiūtur, vt grauitate, leuitate, duritie, mollitie & præterea calore, frigore, alijsq́ contrariorum paribus, quæ sub sensum subiecta sunt: tātum autem relinquit *quantitatem & continuum, Itaque Mathematicorũ ars in ijs quæ immobilia sunt cernitur (τὰ γὰρ μαθηματικὰ τῶν ὄντων ἄνευ κινήσεώς ἐστιν ἔξω τῶν περὶ τὴν ἀστρολογίαν) quæ verò in naturæ obscuritate posita est, res quidẽ quæ nec separari nec motu vacare possunt contemplatur. Id quod in vtroque scientiæ genere perspicuum esse potest, siue res subiectas definias, siue proprietates earum demõstres. Etenim numerus, linea, figura, rectũ, inflexum, æquale rotundũ: vniuersa deniq; Mathematicus quæ tractat & profitetur absque motu explicari doceriq; possunt: χωριστὰ γὰρ τῇ νοήσει κινήσεώς ἐστι: Physica autẽ sine monitione species nequaquã possunt intelligi. Quis enim hominis, plãtæ, ignis, oßium, carnis naturam & proprietates sine motu, qui materiam sequitur, perspiciat? Siquidem tãtisper substantia quæque naturalis cõstare dici solet, quoad opus & munus suũ agendo patiendoq; tueri ac sustinere valeat, qua certè amissa δυνάμει, ne nomẽ quidem nisi ὁμωνύμως retinet. Sed Mathematico ad explicandas circuli aut trianguli proprietates, nullum adferre potest vsum, materiæ, vt auri, ligni, ferri, in qua insunt, consideratio, quin eò verius eiusmodi rerum, quarum species tanquam materia vacantes efformemus animo, naturam complectemur, quòd coniunctione materiæ quasi adulterari deprauariq; videntur. Quocirca Mathematicæ species eodem modo quæ κοῖλον, siue concauitas, sine motu & subiecto, definitione explicare cognos-*

nesciq; possunt: naturales verò cùm eam vim habeant, quam vt ita dicam, simitas cum materia comprehensa sunt, nec absque ea separatim possunt intelligi: quibus exēplis quid inter Physicas & Mathematicas species intersit, haud difficilè est animaduertere. Illis certè non semel est vsus Aristoteles. Valeant ergò Protagoræ sophismata, Geometras hoc nomine refellentis, quod circulus normam puncto non attingat. Nam diuina Geometrarum theoremata, qui sensu æstimabit, vix quicquam reperiet, quod Geometræ concedendum videatur. Quid enim ex his quæ sensum mouent, ita rectum aut rotundum dici potest, vt à Geometra ponitur? Nec verò absurdum est aut vitiosum, quòd lineas in puluere descriptas pro rectis aut rotundis assumit, quæ nec recta sunt nec rotūda, ac ne latitudinis quidem expertes. Siquidē non ijs vtitur Geometra, quasi inde vim habeat conclusio, sed eorum quæ discenti intelligenda relinquuntur, rudem seu imaginem proponit. Nā qui primum instituūtur, hi ductu quodam & velut χειραγογία sensum opus habēt, vt ad illa quæ sola intelligentia percipiuntur, aditum sibi comparare queant. Sed tamen existimandum non est rebus Mathematicis omnino negari materiam ac non eam tantùm quæ sensum afficit. Est enim materia alia quæ sub sensum cadit, alia quæ animo & ratione intelligitur. Illam αἰσθητὴν hāc νοτὴν vocat Aristoteles. Sensu percipitur, vt æs, & lignum, omnisque materia quæ moueri potest;

test. Animo & ratione cernitur ea quæ in rebus sensilibus inest, sed non quatenus sensu percipiuntur, quales sunt res Mathematicorum. Vnde ab Aristotele scriptum legimus ἐπὶ τῶν ἐν ἀφαιρέσει ὄντων rectum se habere vt simum: μετὰ συνεχοῦς γάρ: quasi velit ipsius recti quod Mathematicorum est, suam esse materiam, non minus quā simi quod ad Physicos pertinet. Nam licet res Mathematicæ sensili vacent materia, non sunt tamē individuæ, sed propter continuationem partitionī semper obnoxiæ, cuius ratione dici possunt suà materia non omnino carere: quin aliud videtur τὸ εἶναι γραμμῇ, aliud quoad continuationi adiuncta intelligitur linea. Illud enim ceu formà in materia proprietatum causa est, quæ sine materia percipere non licet. Hæc est societatis & dissidij Mathematicæ cum physica & prima philosophia ratio. Nūc autem de nominis etymo & notatione pauca quædā afferamus, Nam si quæ iudicio & ratione imposita sunt rebus nomina, ea certè non temerè indita fuisse credēdum est, quibus sciētias appellari placuit, sed neq; otiosa semper haberi debet ista etymologiæ indagatio, cū ad rei etiam dubiæ fidem sæpe non parum valeat recta nominis interpretatio. Sic enim Aristoteles ducto ex verborū ratione argumento ἀτόμου, μεταβολῆς, αἰθέρος, aliarumq; rerum naturam ex parte confirmauit. Quoniā igitur Pythagoras Mathematicā scientiā non modò studiosè coluit, sed etiā repetitā à capite principijs, geometricā

contem-

,templationem in liberalis disciplina formam composuit, & perfectis absque materia solius intelligentiæ adminiculo theorematibus, tractationẽ περὶ τῶν ἀλόγων, & κοσμικῶν σχημάτων constitutionem excogitauit: credibile est, Pythagoram, aut certè Pythagoreos, qui & ipsi doctoris sui studia libenter amplexi sunt, huic scientiæ id nomẽ dedisse, quod cum suis placitis atque decretis congrueret rerumque propositarum naturam quoquo modo declararet. Ita cũ existimarent illi, omnem disciplinam, quæ μάθησις dicitur, ἀνάμνησιν esse quandam. i. recordationem et repetitionẽ eius scientiæ, cuius antè quam in corpus immigraret, compos fuerit anima, quemadmodum Plato quoque in Menone, Phædone, & alijs aliquot locis videtur astruxisse: animaduerterent autem eiusmodi recordationem, quæ nõ posset multis ex rebus perspici, ex his potissimum scientijs demõstrari, si quis nimirum, ait Plato, ἐπὶ τὰ διαγράμματα ἄγῃ, probabile est quidem Mathematicas à Pythagoreis artes κατ' ἐξοχὴν fuisse nominatas, vt ex quibus μάθησις, id est æternarũ in anima rationum recordatio διαφερόντως & præcipuè intelligi posset. Cuius etiam rei fidem nobis diuinus fecit Plato, qui in Menone Socratem induxit hoc argumẽti genere persuadere cupientẽ, discere nihil esse aliud quam suarum ipsius rationũ animũ recordari. Etenim Socrates pusionem quendã vt Tullij verbis vtar, interrogat de geometrica dimẽsione quadrati: ad ea sic ille respõdet vt puer, et tamen *mentam faciles interrogationes sunt, vt gradatim respõdens, eodem perueniat, quo si Geometrica didicisset. Aliam nominis huius rationẽ Anatolius exposuit. vt est apud Rhodiginum, quòd cũ cæteræ disciplinæ deprehendi vel non docente aliquo poßint, omnes Mathematicæ sub nullius cognitionem veniant, nisi praeunti aliquo, cuius solertia succidantur vepreta, vel exurantur, & superciliosa complanentur asprета. Ita enim Cælius: quod quam vim habeat, non est huius loci curiosiùs perscrutari. Equidem M. Tullius Mathematicos in magna rerũ obscuritate, recondita arte, multiplicíq; ac subtili versari scribit: sed quis nescit idipsum cum alijs grauioribus scientijs esse commune? Est enim, vel eodem auctore Tullio, omnis cognitio multis obstructa difficultatibus, maximaq; est & in ipsis rebus obscuritas, & in iudicijs nostris infirmitas, nec vllus est, modò interius paulò Physica penetrarit, qui non facilè sit expertus, quam multi vndique emergant, rerum naturalium causas inquirentibus, inexplicabiles labyrinthi. Sunt qui ex demonstrationum firmitate nominari Mathamaticas opinantur: cuius etiam rationis momẽtum alio seorsim loco expendẽdum fuerit. Quocirca primam verbi notationem, quam sequutus est Proclus, nobis retinendam censeo. Hactenus de vniuerso Mathematicæ genere, quanta potui & perspicuitate & breuitate dixi. Sequitur vt de Geometria separatim atq; ordine ea disserã, quæ initio sum pollicitus* *rum. Est autem Geometria, vt definit Proclus scientia, quæ versatur in cognitione magnitudinum figurarum, & quibus hæ continentur, extremorum, item rationum & affectionum, quæ in illis cernuntur ac inhærent: ipsa quidem progrediens à puncto indiuiduo per lineas & superficies, dum ad solida conscendat, variasq; ipsorum differentias patefaciat. Quumq; omnis scientia demonstratiua, vt docet Aristoteles, tribus quasi momentis cõtineatur, genere subiecto, cuius proprietates ipsa scientia exquirit & contemplatur: causis & principiis ex quibus primis demonstrationes cõficiuntur: & proprietatibus, quæ de genere subiecto per se enunciantur: Geometriæ quidem subiectum in lineis, triangulis, quadrangulis, circulis, planis, solidis atque omnino figuris & magnitudinibus, earumq; extremitatibus consistit. His autẽ inhærent diuisiones, rationes, tactus, æqualitates,* παραβολαὶ, ὑπερβολαὶ, ἐλλείψεις, *atq; alia generis eiusdem propè innumerabilia. Postulata verò & Axiomata, ex quibus hæc inesse demonstrantur, eiusmodi ferè sunt: Quouis centro & interuallo circulum describere. Si ab æqualibus æqualia detrahas, quæ relinquuntur esse æqualia, cæteraq; id genus permulta, quæ licet omnium sint communia, ad demonstrandum tamen tum sunt accommodata, cùm ad certum quoddam genus traducuntur. Sed cùm præcipua videatur Arithmetica & Geometria inter Mathematicas dignatio, cur Arithmetica sit* ἀκριβεστέρα *&*

exactior quam Geometria paucis explicandis arbitror. Hic verò & Aristotelem sequemur ducem, qui scientiam cum scientia ita comparat, vt accuratiorem velit esse eam, quæ rei causam docet, quam quæ rem esse tantùm declarat, deinde quæ in rebus sub intelligentiam cadentibus versatur, quàm quæ in rebus sensum mouentibus cernitur. Sic enim & Arithmetica quam Musica, & Geometria quàm Optica, & Stereometria quàm Mechanica exactior esse intelligitur, Postremò quæ ex simplicioribus initijs constat, quàm quæ aliqua adiectione compositis vtitur. Atque hac quidem ratione Geometria præstat Arithmetica, quòd illius initium ex additione dicatur, huius sit simplicius. Est enim punctum, vt Pythagoreis placet, vnitas quæ situm obtinet: vnitas verò punctum est quam situ vacat. Ex quo percipitur, numerorum quàm magnitudinum simplicius esse elementum, numerosque magnitudinibus esse puriores, & à concretione materiæ magis disiunctos. Hæc quamquam nemini sunt dubia, habet & ipsa tamẽ Geometria quo se plurimũ efferat, opibusq; suis ac rerum vbertate multiplici vel cum Arithmetica certet: id quod tute facilè deprehendas cùm ad infinitam magnitudinis diuisionem, quam respuit multitudo, animum conuerteris. Nũc quæ sit Arithmeticæ & Geometriæ societas videamus. Nam theorematum quæ demonstratione illustrantur, quædam sunt vtriusque scientiæ communia, quædam verò singularum

rum propria. Etenim quòd omnis proportio sit ῥητὸς siue rationalis, Arithmetica soli conuenit, nequaquam Geometriæ, in qua sunt etiã ἄρρητοι seu irrationales proportiones. item, quadratorum gnomas minimo definitos esse, Arithmeticæ proprium (si quidem in Geometria nihil tale minimum esse potest) sed ad Geometriam propriè spectant situs, qui in numeris locum non habent tactus, qui quidem à continuis admittuntur: εἴλογην, quoniam vbi diuisio infinitè procedit, ibi etiam τὸ ἄλογον esse solet. Communia porrò vtriusque sunt illa, quæ ex sectionibus eueniunt, quas Euclides libro secundo est persequutus: nisi quod sectio per extremam & mediam rationem in numeris nusquam reperiri potest. Iam verò ex theorematibus eiusmodi communibus, alia quidem ex Geometria ad Arithmeticam traducuntur: alia contra ex Arithmetica in Geometriam transferuntur: quædam verò perinde vtrique scientiæ conueniunt, vt quæ ex vniuersa arte Mathematica in vtramq; harum conueniant. Nam & alterna ratio, & rationum conuersiones, compositiones, diuisiones hoc modo communia sunt vtriusque. Quæ autem sunt περὶ συμμέτρων, id est, de commensurabilibus, Arithmetica quidem primum cognoscit & contemplatur: secundo loco Geometria Arithmeticam imitata. Quare & commensurabiles magnitudines illę dicuntur, quæ rationem inter se habent, quam numerus ad numerum, per-

Inde quasi commensuratio & συμμετρία in numeris primùm consistat (Vbi enim numerus, ibi & σύμμετρον cernitur: & vbi σύμμετρον, illic etiam numerus) sed quæ triangulorũ sunt, & quadrangulorum, à Geometra primũ considerantur: tum analogia quadam Arithmeticus eadem illa in numeris contemplatur. De Geometriæ diuisione hoc adijciendum puto, quòd Geometriæ pars altera in planis figuris cernitur, quæ solam latitudinem longitudini coniunctam habent: altera verò solidas contemplatur, quæ ad duplex illud interuallum craßitudinem adsciscunt. Illam generali Geometriæ nomine veteres appellarunt: hanc propriè Stereometriam dixerunt. Ita Geometriam cum Optica & Stereometriam cum Mechanica non rarò comparat Aristoteles. Sed illius cognitio huius inuentionem multis seculis antecesßit, si modò Stereometriam ne Socratis quidem ætate vltam fuisse omnino verum est, quemadmodum à Platone scriptum videtur. Ad Geometriæ vtilitatem accedo, quæ quanquam suapte vi & dignitate ipsa per se nititur, nullius vsus aut actionis ministerio mancipata (vt de Mathematicis omnibus scientijs concedit in Politico Socrates) si quid ex ea tamen vtilitatis externæ quæritur, Dij boni quàm latos, quàm vberes, quàm varios fructus fundit? Nec verò audiendus est vel Aristippus, vel Sophistarum alius, qui Mathematicorum artes idcirco repudiet, quòd ex fine nihil docere videantur, eiusque quod melius aut de-

terius

terius nullam habeant rationem. Vt enim nihil causæ dicas, cur sit melius trianguli, verbi gratia, tres angulos duobus esse rectis æquales: minimè tamen fuerit consentaneum Geometriæ cognitionem vt inutilem exagitare, criminari, explodere, quasi quæ finem & bonum quò referatur, habeat nullum. Multas haud dubiè solius contemplationis beneficio citra materiæ contagionem adfert Geometria cõmoditates partim proprias, partim cum vniuerso genere communes. Cùm enim Geometria, vt scripsit Plato, eius quod semper est cognitionem profiteatur, ad veritatem excitabit illa quidem animum, & ad ritè philosophandum cuiusque mentem comparabit. Quin etiam ad disciplinas omnes facilius perdiscendas, attigeris necne Geometriam quanti referre cẽses? Nam vbi cum materia coniungitur, nónne præstantissimas procreat artes, Geodesiam, Mechaniam, Opticam, quarum omnium vsu, mortalium vitam summis beneficijs complectitur? Etenim bellica instrumẽta, vrbiumq́ propugnacula, quibus munitæ vrbes hostium vim propulsarent, his adiutricibus fabricata est: montium ambitus & altitudines, locorumq́ situs nobis indicauit: dimetiendorum & mari & terra itinerum rationem præscripsit: trutinas & stateras, quibus exacta numerorũ æqualitas in ciuitate retineatur, cõposuit: vniuersi ordinem simulachris expreßit, multaq́ quæ hominum fidem superarent, omnibus persuasit. Vbiq́ extant præclara in eam rem *testimonia. Illud memorabile, quòd Archimedi rex Hiero tribuit. Nam extructo vastæ molis nauigio, quòd Hiero AEgyptiorum regi Ptolomæo mitteret, cum vniuersa Syracusanorum multitudo collectis simul viribus nauem trahere non posset effecissetq́ Archimedes, vt solus Hiero illam subduceret, admiratus viri scientiam rex ἀπὸ ταύτης, ἔφη, τῆς ἡμέρας περὶ παντος ἀρχιμήδει λέγοντι πιστευτέον. Quid? quòd Archimedes idẽ, vt est apud Plutarchum, Hieroni scriptis datis viribus datum pondus moueri posse? fretusq́ demõstrationis robore illud sæpe iactarit, si terram haberet alteram vbi pedem figeret, ad eam nostram hanc se transmouere posse? Quid varia, αὐτομάτων machinarumq́ genera ad vsus necessarios comparata memorem? Innumerabilia profectò sunt illa, & admiratione dignißima quibus prisci homines incredibili quodam ad philosophandum studio concitati, inopem mortalium vitam artis huius præsidio subleuarum: tametsi memoriæ sit proditum, Platonem Eudoxo & Archytæ vitio vertisse, quòd Geometrica problemata ad sensilia & organica abducerent. Sic enim corrumpi ab illis & labefieri Geometriæ præstantiam, quæ ab intelligibilibus & incorporeis rebus ad sensiles & corporeas prolaberetur. Quapropter ridicula idem scripsit Plato Geometrarum esse vocabula, quæ quasi ad opus & actionem spectent, ita sonare videntur. Quid enim est quadrare si non opus facere? Quid addere, producere,*

ducere, applicare? Multa quidem sunt eiusmodi nomina, quibus necessario & tanquam coacti Geometræ vtuntur, quippe cum alia desint in hoc genere commodiora. Sic ergo censuit Plato, sic Aristoteles, sic denique philosophi omnes, Geometriam ipsam cognitionis gratia exercendam, nec ex aliquo vsu externo sed ex rerum νοητῶν intelligentia æstimandam esse. Exposita breuiter quā res tanta dici possit, vtilitatis ratione, Geometriæ ortum, qui in hac rerum periodo ex historicorum monumentis nobis est cognitus deinceps aperiamus. Geometria apud Aegyptios inuenta, (ne ab Adamo, Setho, Noah, quos cognitione rerum multiplici valuisse constat, eam repetamus) ex terrarum dimensione, vt verbi præ se fert ratio, ortū habuisse dicitur: cùm anniuersaria Nili inundatione & incrementis limo obductis agrorum termini confunderētur. Geometriam enim, sicut & reliquas disciplinas, in vsu quam in arte prius fuisse aiunt. Quod sane mirum videri non debet, vt & huius & aliarum scientiarum inuentio ab vsu cœperit ac necessitate. Etenim tempus, rerum vsus, ipsa necessitas ingenium excitat, & ignauiam acuit. Deinde quicquid ortum habuit (vt tradunt Physici) ab inchoato & imperfecto processit ad perfectum. Sic artium & scientiarum principia experientiæ beneficio collecta sunt: experientia verò à memoria fluxit, quæ & ipsa à sensu primùm manauit. Nam quod scribit Aristoteles, Mathematicas artes,

compacomperatu rebus omnibus ad vitã necessarijs, in Aegypto fuisse constitutas, quòd ibi sacerdotes omnium concessu in otio degerent: non negat ille adductos necessitate homines ad excogitandam, verbi gratia terræ dimetiendæ rationem, quæ theorematum deinde inuestigationi causam dederit: sed hoc confirmat, præclara eiusmodi theorematũ inuenta, quibus extructa Geometriæ disciplina cõstat, ad vsus vitæ necessarios ab illis non esse expetita. Itaque vetus ipsum Geometriæ nomen ab illa terræ partiundæ finiumq; regendorũ ratione postea recessit, & in certa quadam affectionum magnitudini per se inhærentium sciẽtia propria remãsit. Quemadmodum igitur in mercium & contractuum gratiam supputandi ratio, quam secuta est accurata numerorum cognitio, à Phœnicibus initium duxit: ita etiam apud AEgyptios, ex ea quam commemoraui causa ortum habuit Geometria. Hanc certè, vt id obiter dicã, Thales in Græciam ex AEgypto primum trãstulit: cui non pauca deinceps à Pythagora, Hippocrate, Chio, Platone, Archyta Tarentino, alijsq; compluribus, ad Euclidis tempora factæ sunt rerum magnarum accessiones. Cæterùm de Euclidis ætate id solum addam, quod à Proclo memoriæ mandatum accepimus. Is enim commemoratis aliquot Platonis tum æqualibus tum discipulis subijcit, non multò ætate posteriorem illis fuisse Euclidem eum, qui elementa conscripsit, & multa ab Eudoxo collecta, in ordinem luculentum

com-

composuit, multaq; à Theateto inchoata perfecit, quaq; mollius ab alijs demonstrata fuerat, ad firmißimas & certißimas apodíxes reuocauit. Vixit autẽ, inquit ille, sub primo Ptolemao. Etenim ferunt Euclidam à Ptolemao quondam interrogatum, num qua esset via ad Geometriam magis cõpendiaria, quàm sit ista στοιχείωσις respondisse: μὴ εἶναι βασιλικὴν ἀτραπὸν ἐπὶ γεωμετρίαν. Deinde subiũgit, Euclidem natu quidem esse minorem Platone, maiorẽ verò Eratosthene & Archimede (hi enim aquales erãt) cùm Archimedes Euclidis mentionem faciat. Quod si quis egregiã Euclidis laudem, quam cùm ex alijs scriptionibus accuratißimis, tũ ex hac Geometria στοιχειώσει cõsequutus est, in qua diuinus rerum ordo sapiẽtißimis quibusq; hominibus magna semper admirationi fuit, is Proclum studiosè legat, quò rei veritatem illustriorem reddat grauißimi testis auctoritas. Superest igitur vt finem videamus, quò Euclidis elementa referri, & cuius causa in id studium incumbere oporteat. Et quidem si res, qua tractantur, consideres, in tota hac tractatione nihil aliud quari dixeris, quã vt Cosmicà, qua vocãtur σχήματα (fuit enim Euclides professione & instituto Platonicus) Cubus Icosaedrum, Octaedrum, Pyramis, & Dodecaedrum certa quadam suorum & inter se laterũ, & ad sphara diametrum ratione eidem sphara inscripta comprehendantur. Huc enim pertinet Epigrammation illud vetus, quod in Geometrica Michaelis

 Pselli

Pselli συνόψει *scriptum legitur.*

Σχήματα πέντε Πλάτωνος, Πυθαγόρας σοφὸς εὗρε.

Πυθαγόρας σοφὸς εὗρε, Πλάτων δ' ἀριδήλ' ἐδίδαξεν.

Εὐκλείδης ἐπὶ τοῖσι κλέος περικαλλὲς ἔτευξεν.

Quòd si discentis institutionem spectes, illud certè fuerit propositum, vt huiusmodi elementorum cognitione informatus discentis animus, ad quamlibet non modo Geometriæ, sed & aliarum Mathematicæ partium tractationem idoneus paratusq; accedat. Nam tametsi institutionem hãc solus sibi Geometra vendicare videtur, & tanquã in possessionẽ suam venerit, alios excludere posse: inde tamen per multa suo quodam modo iure decerpit Arithmeticus, plreaque Musicus non pauca detrahit. Astrologus, Opticus, Logisticus, Mechanicus, itemq; cæteri: nec vllus est deniq; artifex præclarus, qui in huius se possessionis societatẽ cupide non offerat, partemq; sibi concedi postulet. στοιχείωσις *absolutum opere nomen, &* στοιχειωτὴς *dictus Euclides. Sed quid longius prouehor? Nã quod ad hanc rem attinet, tam copiosè & eruditè scripsit (vt alia complura) eo ipso, quem dixi, loco P. Montaureus, vt nihil desiderio loci reliquerit. Quæ verò ad dicendum nobis erant proposita hactenus pro ingenij nostri tenuitate omnia mihi perfecisse videor. Nam tametsi & hæc eadem & alia pleraque multo fortè præclariora ab hominibus doctißimis, qui tum acumine ingenij,*

nij,

nij, tùm admirabili quodam lepore dicendi semper floruerunt grauiùs, splendidiùs, vberiùs tractari posse scio: tamen experiri libuit: numquid etiam nobis diuino sit concessum munere, quod rudes in hac philosophiæ parte discipulos adiuuare aut certè excitare queat. Huc accessit quòd ista recens elementorum editio, in qua nihil non parum fuisset studij, aliquid à nobis efflagitare videbatur, quod eius cõmendationem adaugeret. Cum enim vir doctißimus Io. Magnienus Mathematicarum artium in hac Parisiorum Academia professor verè regius, nostrum hunc typographum in excudendis Mathematicorum libris diligentißimum, ad hanc Elementorum editionem sæpè & multum esset adhortatus, eiusque impulsu permulta sibi iam comparasset typographus ad hanc rem necessaria, citò interuenit, malùm Ioannis Magnieni mors insperata, quæ tam grauè inflixit Academiæ vulnus, cui ne post multos quidem annorum circuitus cicatrix obduci vlla posse videatur. Quamobrem amisso instituti huius operis duce, typographus, qui nec sumptus antea factos sibi perire, nec studiosos, quibus id muneris erat pollicitus, sua spe cadere vellet, ad me venit, & impensè rogauit, vt meam propositæ editioni operam & studium nauarem, quòd cùm denegaret occupatio nostra, iuberet officij ratio, feci equidem rogatus, vt quæ subobscurè vel parum commodè in sermonem Latinum è Græco translata videbantur, *clariore, aptiore & fideliore interpretatione nostra (quod cuiusq; pace dictum volo) lucē acciperent. Id quod in omnibus ferè libris posterioribus tute primo obtutu perspicias. Nam in sex prioribus non tantum temporis quantum in cæteris ponere nobis licuit: decimi autem interpretatio, qua melior nulla potuit adferri, P. Mōtaureo solida debetur. Atque vt ad perspicuitatem facilitatemq; nihil tibi deesse queraris, adscripta sunt propositionibus singulis vel lineares figuræ, vel pūctorum tanquam vnitatum notulæ, quæ Theonis apodixin illustrent illæ quidem magnitudinum, hæ autem numerorum indices, subscriptis etiam ciphrarum, vt vocant, characteribus, qui propositum quemuis numerum exprimāt, ob eamq; causam eiusmodi vnitatum notulæ, quæ pro numeri amplitudine maius paginæ spatium occuparent, pauciores sæpius depictæ sunt, aut in lineas etiam cōmutatæ. Nam litterarum vt a, b, c, characteres non modò numeris & numerorum partibus nominandis sunt accommodati, sed etiam generales esse numerorum, vt magnitudinum affectiones testantur. Adiecta sunt insuper quibusdam locis non pœnitēda Theonis scholia, siue mauis lemmata, quæ quidem longè plura accessissent, si plus otij & temporis vacui nobis fuisset relictū, quod huic studio impartiremus. Hanc igitur operam boni consule, & quæ obuia erunt impreßionis vitia, candidè emenda. Vale. Lutetiæ 4. Idus April. 1557.*

EVCLI-

EVCLIDIS ELEMENTVM PRIMVM.

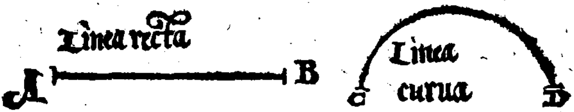
DEFINITIONES.

1.

PVnctum est, cuius pars nulla est. Punctum

2.

Linea verò, longitudo latitudinis expers.



3.

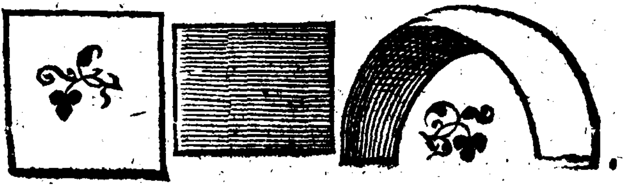
Lineæ autem termini, sunt puncta.

4.

Recta linea est, quæ ex æquo sua interiacet puncta.

5.

Superficies est, quæ longitudinem latitudinemque tantùm habet.



c.Super-

6.

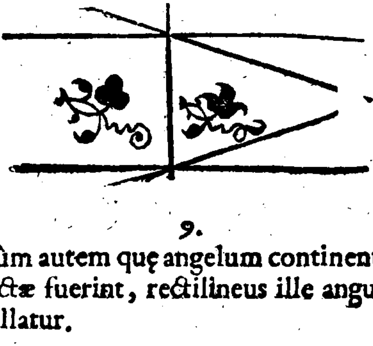
Superficiei extrema ſunt liniæ.

7.

Plana ſuperficies eſt, quæ ex æquo ſuas interiacet lineas.

8.

Planus angelus eſt duarum linearũ in plano ſe mutuò tangentium, & non indirectum



iacẽtium alterius ad alterã inclinatio.

9.

Cùm autem quę angelum continent lineę, rectæ fuerint, rectilineus ille angulus appellatur.

10.

Cùm verò recta linea ſuper rectam conſiſtens lineam, eos qui ſunt deinceps angelos æquales, inter ſe fecerit: rectus eſt vterque

æqua-

æqualium angelorum: quæ insistit recta linea, perpendicularis vocatur eius, cui insistit.

11.

Obtusus angulus est, qui recto maior est.

12.

Acutus verò, qui minus est recto.

13.

Terminus est, quod alicuius extremum est.

14.

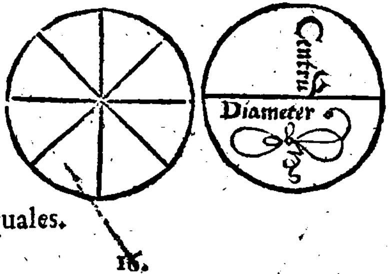
Figura est, quæ sub aliquo, vt aliquibus terminis comprehenditur.

15.

Circulus est, figura plana sub vna linea cõprehẽsa, quæ peripheria appellatur: ad quã ab vno puncto eorum, quæ intra figuram

sunt

ſunt po ſita, cadentes omnes rectę lineæ inter ſe ſunt æquales.



16.

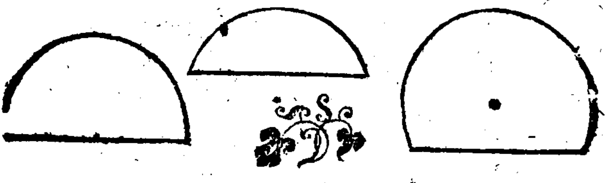
Hoc verò punctum, centrum circuli appellatur.

17.

Diameter autem circuli, eſt recta quædam linea per centrum ducta, & ex vtraque parte in circuli peripheriam terminata, quæ circulum bifariam ſecat.

18.

Semicirculus eſt figura, quę continetur ſub diametro, & ſub ea linea, quæ de circuli periferia aufertur.



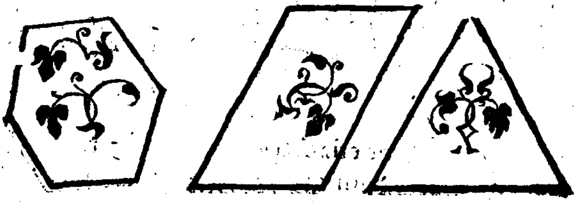
19.

Segmentum circuli, eſt figura, quæ ſub recta linea & circuli peripheria continentur.

20. Recti.

20.

Rectilineæ figuræ, sub quæ sub rectis leneis continentur.



21.

Trilateræ quidem, quæ sub tribus.

22.

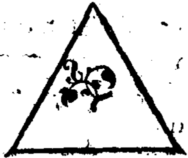
Quadrilateræ, quæ sub quatuor.

23.

Multilateræ verò, quæ sub pluribus quàm quatuor rectis lineis comprehenduntur.

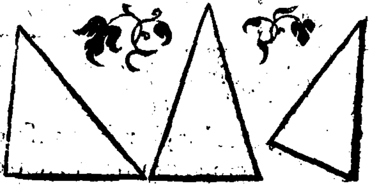
24.

Trilaterum, porrò figurarum, æquilaterum est triangulum, quod tria latera habet æqualia.



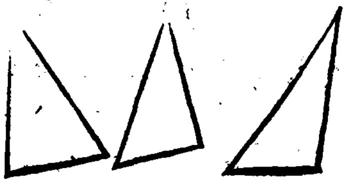
25.

Isosceles autem est quod duo tantum æqualia habet latera.



26.

Scalenum verò, est quib' tria inæqualia habet latera.



27.

Ad hanc etiam trilaterarum figurarum, rectangulum quidem triangulum est, quod rectum angulum habet.

28.

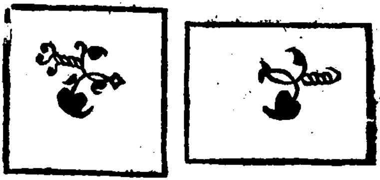
Amblygonium autem, quod obtusum angulum habet.

29.

Oxigonium verò, quod tres habet acutos angulos.

30.

Quadrilaterarum autem figurarum, quadratũ quidẽ est quid & æquilaterũ & rectangulum est.



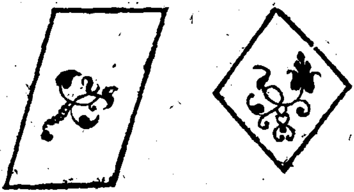
31.

Altera parte longior figura est, quæ rectangula quidem, at æquilatera non est.

32. Rhom-

32.

Rhō-
bus au
tem,
qui æ-
quila-
terum
& re-
ctangulum est.



33.

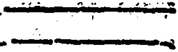
Rhomboides verò, quæ aduersa & latera & angulos habens inter se æqualia, neque æquilatera est, neque rectangula.

34.

Prçter
has au
tē re-
liquæ
qua-
drila-
teræ figuræ, trapezia appellentur.

35.

Parallele rectæ lineæ sunt quæ, cùm in eodem sint plano, & ex vtraque parte in infinitum producantur, in neutram sibi mutuò incidunt.



Postulata.

1.

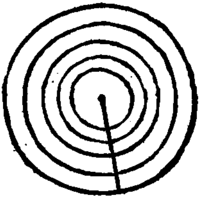
Postuletur, vt à quouis puncto in quoduis punctum, rectam lineam ducere conceda-tur.

2.

Et rectam lineam terminatam in continuum recta producere.

3.

In quouis centro & interuallo circulum describere.



Communes notiones.

1.

Quæ eidem ęqualia, & inter se sunt æqualia.

2.

Et si æqualibus æqualia adiecta sint, tota sunt æqualia.

3.

Et si ab æqualibus æqualia ablata sint, quæ relinquuntur sunt æqualia.

4.

Et si inæqualibus æqualia adiecta sint, tota sunt inæqualia.

5.

Et si ab inæqualibus æqualia ablata sint, reliqua sunt inæqualia.

6.

Quæ eiusdem duplicia sunt, inter se sunt æqualia.

Et

7.

Et quæ eiusdem sunt dimidia, inter se ęqualia sunt.

8.

Et quæ sibi mutuò congruunt, ea inter se sunt æqualia.

9.

Totum est sua parte maius.

10.

Item, omnes recti anguli sunt inter se ęquales.

11.

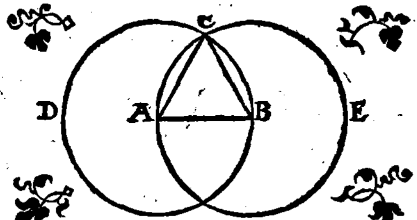
Et si duas rectas lineas altera recta incidens, inter nos & easdemque partes angelos, duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ sibi mutuò incident ad eas partes, vbi sunt anguli duobus rectis mimores.

12.

Duæ rectæ lineæ spatium non comprehendunt.

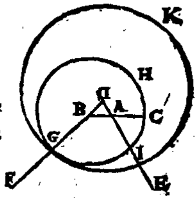
Problema 1. Propositio 1.

Super data recta linea termi nata, trian-



gulum æquilaterum constituere.

Problema 2. Propositio 2.



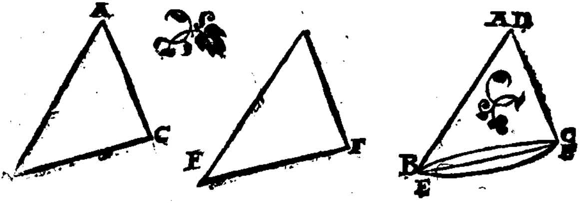
Ad datum punctum, datæ rectæ lineæ, æqualem rectam lineam ponere.

Problema 3. Propositio 3.

Duabus datis rectis lineis inæqualibus, de maiore æqualẽ minori rectam lineam detrahere.

Theorema primum Proposio 4.

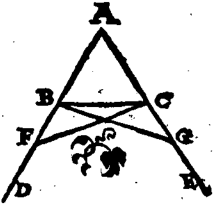
Si duo triangula duo latera duobus lateribus æqualia habeant, vtrunque vtrique, habeant verò & angulum angulo æqualẽ sub æqualibus rectis lineis contentum: & basin basi æqualem habebunt, eritque triangulũ triangulo æquale, ac reliqui anguli reliquis angulis æquales erunt, vterque vtrique, sub quibus æqualia latera subtenduntur.



Theo-

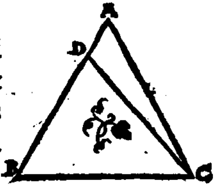
Theorema 2. Propoſitio 5.

Iſoſceleum triangulorũ qui ab baſim ſunt anguli, inter ſe ſunt æquales; & ſi vlterius productæ ſint æquales illæ rectę lineæ, qui ſub baſi ſunt anguli, inter ſe æquales erunt.



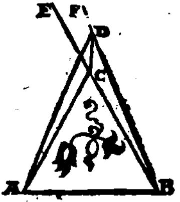
Problema 3. Propoſitio 6.

Si trianguli duo anguli æquales inter ſe fuerint & ſub æqualibus angulis ſubtenſa latera æqualia inter ſe erunt.



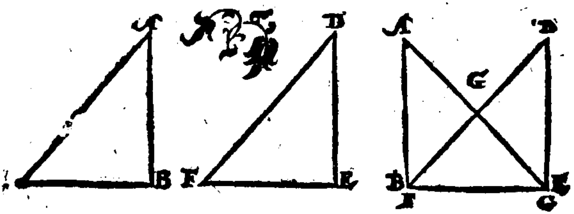
Thorema 4. Propoſitio 7.

Super eandem recta linea duabus eiuſdé rectis lineis alię duę rectę lineæ æquales, vtraque vtrique non conſtituentur, ad aliud atque aliud punctũ ad eaſdé partes eoſdéque terminos cum duabus initio ductis rectis lineis habentes.

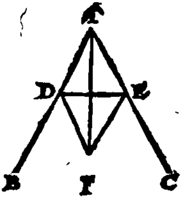


Theorema 5. Propositio 8.

Si duo triangula duo latera habuerint duobus lateribus, vtrunq; vtrique æqualia; habuerint verò & basim basi æqualem: angulum quoque sub æqualibus rectis lineis contentum angulo æqualem habebunt.

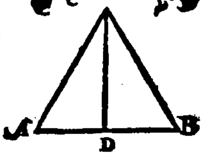


Problema 4. Propositio 9.



Datum augulum rectilineum bifariam secare.

Problema 5. Propositio 10.

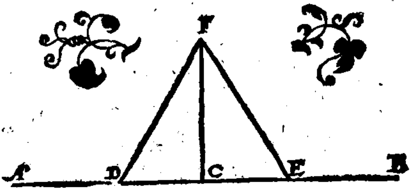


Datam rectam lineam finitam bifariam secare.

Proble-

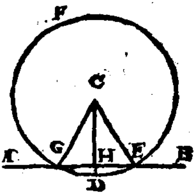
Problema 6. Propositio 11.

Data recta linea, à puncto in ea dato rectam lineam ad angulos rectos excitare.



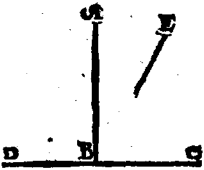
Problema 7. Propositio 12.

Super datam rectam lineam infinitam, à dato puncto quod in ea non est, perpendicularem rectam deducere.



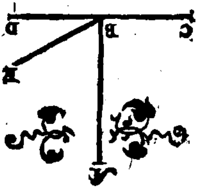
Theorema 6. Proprositio 13.

Cùm recta linea super rectam cõsistens lineam angulos facit, aut duos rectos, aut duobus rectis æquales efficiet.

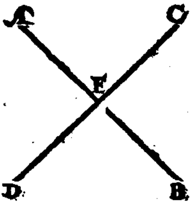


Theorema 7. Propositio 14.

Si ad aliquam rectam lineam, atque ad eius punctũ, duæ rectę lineæ nõ ad easdem partes ductæ, eos qui sunt deinceps angulos duobus rectis æquales fecerint, indirectum erunt inter se ipse rectæ lineæ.

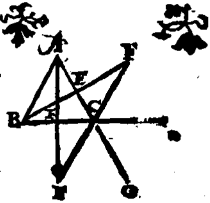


Theorema 8. Propositio 15.



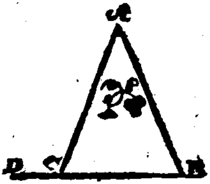
Si duæ rectę lineę se mutuò secuerint, angulos qui ad verticem sunt, æquales inter se efficient.

Theorema 9. Propositio 16.



Cuiuscunque trianguli vno latere producto, externus angulus vtroque interno & opposito maior est.

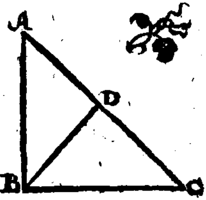
Theorema 10. Propositio 17.



Cuiuscunque trianguli duo anguli duobus rectis sunt minores omnifariam sumpti.

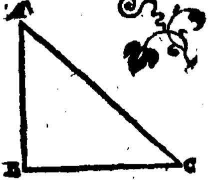
Theo-

Theorema 11. Propoſitio 18.



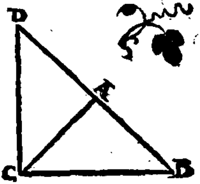
Omnis trianguli maius latus maiorem angulum ſubtendit.

Theorema 12. Propoſitio 19.



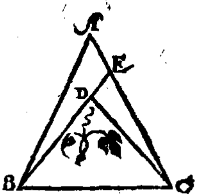
Omnis trianguli lateri angulus maiori lateri ſubtenditur.

Theorema 13. Propoſitio 20.



Omnis trianguli duo latera reliquo ſunt maiora, quomodocumque aſſumpta.

Theorema 14. Propoſitio 21.

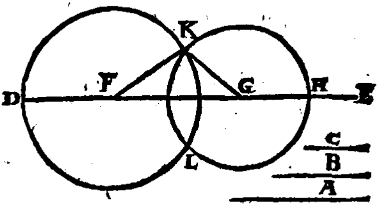


Si ſuper trianguli vno latere, ab extremitatibus duæ rectę lineæ, interius conſtitutæ fuerint, hæ conſtitutæ reliquis trianguli duobus lateribus minores quidem erunt, minorem verò angulũ continebunt.

Pro-

Problema 8. Propoſitio 22.

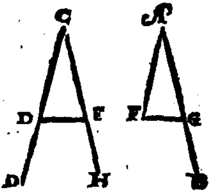
Ex tribus rectis lineis quæ ſunt trib. datis rectis lineis æquales,



triangulum conſtituere. Oportet autem duas reliqua eſſe maiores omnifariã ſumptas: quoniam vniuſcuiuſque trianguli duo latera omnifariam ſumpta reliquo ſunt maiora.

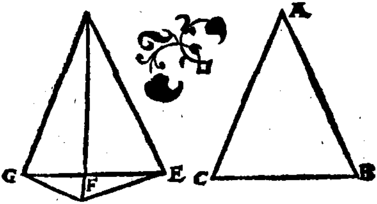
Problema 9. Propoſitio 23.

Ad datam rectam lineã datumq́; in ea punctum dato angelo recti lineo æqualem angulum recti lineum conſtituere.



Theorema 15. Propoſitio 24.

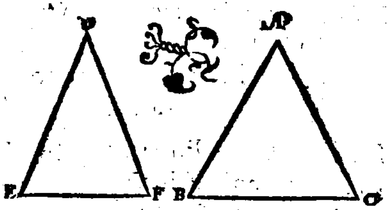
Si duo triãgula duo latera duobus lateribus æqualia habuerint, vtrumque vtrique angulũ verò angu-



angulo maiorem sub æqualibus rectis lineis contentum: & basin basi maiorẽ habebunt.

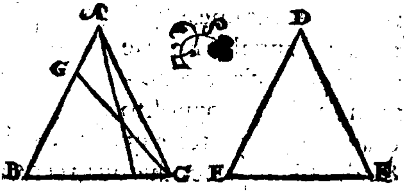
Theorema 16. Propositio 25.

Si duo triangula duo latera duobus lateribus æqualia habuerint, vtrunque vtrique, basin verò basi maiorẽ: & angulum sub æqualibus rectis lineis contentum angulo maiorem habebunt.



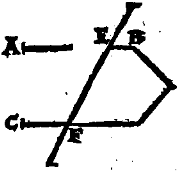
Theorema 17. Propositio 26.

Si duo triangula duos angulos duobus angulis æquales habuerint, vtrunque vtrique, vnumque latus vni lateri æquale, siue, quod æqualibus adiacet angulis, seu quod vni æqualium angulorum subtẽditur: & reliqua latera, reliquis lateribus æqualia vtrunque vtrique & reliquum angulum reliquo angulo ęqualem habebunt.



Theore-

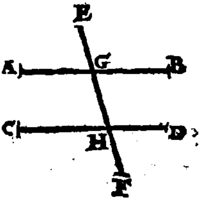
Theorema 18. Propositio 27.



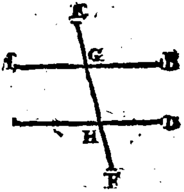
Si in duas rectas lineas recta incidens linea alternatim angulos æquales inter se fecerit: parallelæ erunt inter se illæ rectæ lineæ.

Theorema 19. Propositio 28.

Si in duas rectas lineas recta incidens linea externum angulũ interno, & opposito, & ad easdem partes æqualẽ fecerit, aut internos & ad easdẽ partes duobus rectis æquales: parallelæ erunt inter se ipsæ rectæ lineæ.



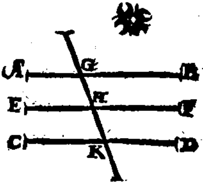
Theorema 20. Propositio 29.



In parallelas rectas lineas recta incidens linea: & alternatim angulos inter se æquales efficit & externum interno & opposito & ad easdem partes æqualem, & internos & ad easdem partes duobus rectis æquales facit.

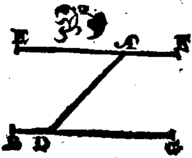
Theo-

Theorema 21. Propositio 30.



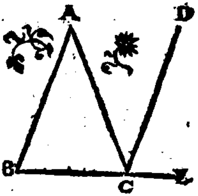
Quæ eidem rectæ lineæ, parallelæ, & inter se sunt parallelæ.

Problema 10. Propositio 31.



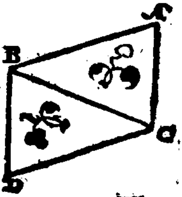
A dato puncto datæ rectæ lineæ parallelam rectam lineam ducere.

Theorema 22. Propositio. 32.



Cuiuscunq; trianguli vno latere vlterius producto: externus angelus duobus internis & oppositis est æqualis. Et trianguli tres interni anguli duobus sunt rectis æquales.

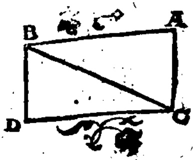
Theorema 23. Propositio 33.



Rectæ lineæ quę ęquales & parallelas lineas ad partes easdem coniungunt, & ipsæ æquales & parallelæ sunt.

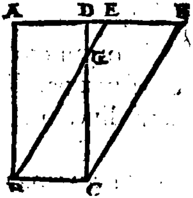
Theo-

Theorema 24. Propoſitio 34.



Parallelogrammorũ ſpatiorum æqualia ſunt inter ſe, quæ ex aduerſo & latera & anguli: atque illa bifariam ſecat diameter.

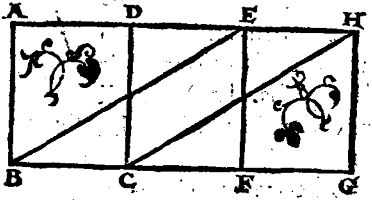
Theorema 25. Propoſitio 35.



Parallelogramma ſuper eadem baſi, & in eiſdem parallelis cõſtituta inter ſe ſunt æqualia.

Theorema 26. Propoſitio 36.

Parallelogrãma ſuper æqualibus baſibus & in eiſdẽ parallelis cõſtituta inter ſe sũt æqualia.

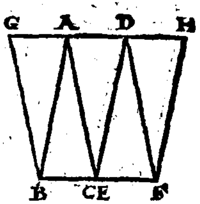


Theorema 27. Propoſitio 37.

Triangula ſuper eandem baſibus conſtituta, & in eiſdem parallelis, inter ſe ſunt æqualia.

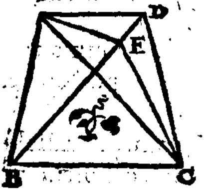
Theore-

Theorema 28. Propositio 37.



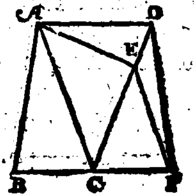
Triangula super æqualibus basibus constituta & in eisdem parallelis, inter se sunt æqualia.

Theorema 29. Propositio 39.



Triangula æqualia super eandem basi, & ad easdem partes constituta: & in eisdem sunt parallelis.

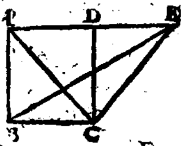
Thorema 30. Propositio 40.



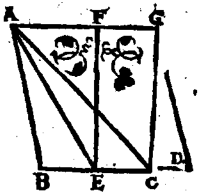
Triangula æqualia super ęqualibus basibus, & ad easdem partes constituta, & in eisdem sunt parallelis.

Theorema 31. Propositio 41.

Si parallelogrammum cum triangulo eandem basin habueris, non eisdemque fuerit parallelis, duplum erit parallelogrammum ipsius trianguli.

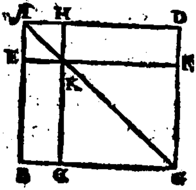


Problema 11. Propositio 42.



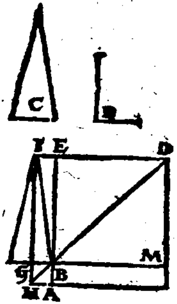
Dato triangulo æquale parallelogrammũ constitute in dato angulo rectilineo.

Theorema 32. Propositio 43.



In omni parallelogrammo, complementa eorũ quæ circa diametrũ sunt parallelogrammorũ, inter se sunt æqualia.

Problema 12. Propositio 44.

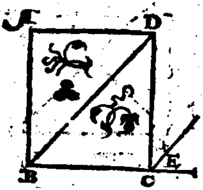


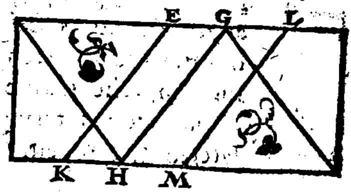
Ad datam rectam lineã dato triangulo æquale parallelogrammum applicare in dato angulo rectilineo.

Problema 13. Propositio 45.

Dato rectilineo æquale parallelogrammũ consti-

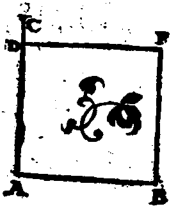
constituere in dato angulo rectilineo.





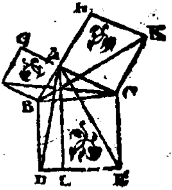
Theorema 41. Propositio 4.

A data recta linea quadratum describere.



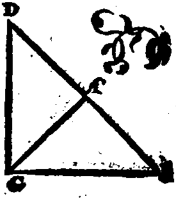
Theorema 33. Propositio 47.

In rectangulis triangulis, quadratum quod à latere rectum angulum subtendente describitur, æquale est eis, quæ à lateribus rectum angulum continentibus describũtur, quadratis.



Theorema 34. Propositio 48.

Si quadratum quod ab vno laterum trianguli deſcribitur, æquale ſit eis quæ à reliquis trianguli lateribus deſcribuntur, quadratis: angulus comprehenſus ſub reliquis duobus trianguli lateribus, rectus eſt.



FINIS ELEMENTI I.

EVCLIDIS ELEMENTVM SECVNDVM.

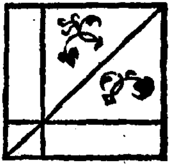
DEFINITIONES.

1.

OMNE parallelogrammum rectangulum contineri dicitur sub rectis duabus lineis, quæ rectum comprehendunt angulum.

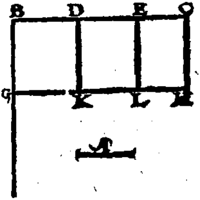
2.

In omni parallelogrammo spatio, vnum quodlibet eorum, quæ circa diametrum illius sunt parallelogrammorum, cum duobus cōplementis, Gnomo vocetur.

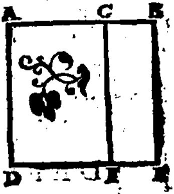


Theorema 1. Propositio 1.

Si fuerint duæ rectæ lineæ, seceturque ipsarum altera in quotcunq; segmenta: rectangulum comprehensum sub illis duabus rectis lineis, æquale est eis rectangulis, quæ sub insecta & quolibet segmētorum comprehenduntur.



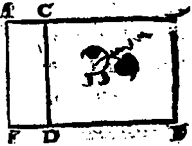
Theorema 2. Propositio 2.



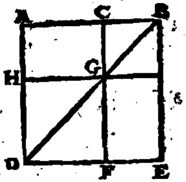
Si recta linea secta sit vtcunq; rectangula quæ sub tota & quolibet segmentorum compræhendũtur, ęqualia sunt ei, quod à tota fit, quadrato.

Theorema 3. Propopositio. 3.

Si recta linea secta sit vtcunque rectangulum sub tota & vno segmentorum compręhensum, æquale est & illi quod sub segmentis comprehenditur rectangulo & illi, quod à prædicto segmẽto describitur, quadrato.



Theorema 4. Propositio 4.

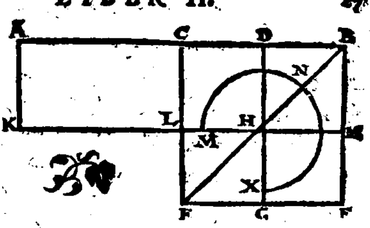


Si recta linea secta sit vtcunq; quadratum quod à tota describitur æquale est & illis quæ à segmentis describuntur quadratis, & ei quod bis sub segmentis compręhenditur rectangulo.

Theorema 5. Propositio 5.

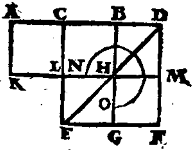
Si recta linea secetur in æqualia & non æqualia: rectangulum sub inæqualibus segmentis

mentis totius comprehensũ, vnà cum quadrato quod ab inter media sectionum, æquale est ei quod à dimidia describitur, quadrato.



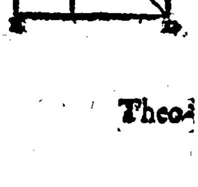
Theorema 6. Propositio 6.

Si recta linea bifariam secetur, & illi recta quædam linea in rectum adijciatur, rectangulum comprehensum sub tota cum adiecta & adiecta simul cum quadrato à dimidia, æquale est quadrato à linea quæ tũ ex dimidia, tum ex adiecta componitur, tanquam ab vna



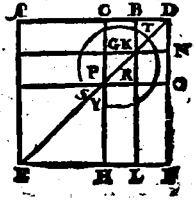
Theorema 7. Propositio 7.

Si recta linea secetur vtcunque, quod à tota, quodq; ab vno segmẽtorum, vtraq; simul quadrata, æqualia sunt & illi quod bis sub tota & dicto segmento cõprehenditur rectangulo, & illi quod à reliquo segmẽto fit, quadrato.



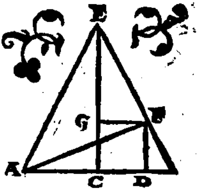
Theorema 8. Propositio 8.

Si recta linea secetur vtcunque rectangulū quater comprehensum sub tota & vno segmentorum, cum eo quod à reliquo segmēto fit, quadrato, æquale est ei quod à tota & dicto segmēto, tanquā ab vna linea describitur quadrato.



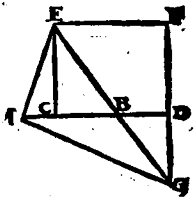
Theorema 9. Propositio 9.

Si recta linea secetur in æqualia & non æqualia: quadrata quę ab inæqualibus totius segmentis fiunt, duplicia sunt & eius quod à dimidia, & eius quod ab intermedia sectionum fit, quadratorum.



Theorema 10. Propositio 10.

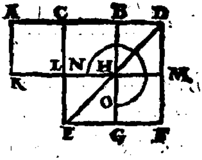
Si recta linea secetur bifariam, adijciatur autem ei in rectum quæpiam recta linea: quod à tota cū adiuncta & quod ab adiuncta, vtraq; simul quadrata, duplicia sunt, & eius quod à dimidia, & eius quod à composita ex di-



ex dimidia & adiuncta; tanquam ab vna descriptum sit quadratorum.

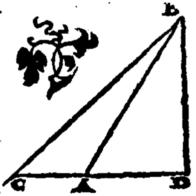
Problema 1. Propositio 11.

Datam rectam lineã secare, vt comprehensum sub tota & altero segmẽtorum rectangulum, æquale sit ei quod à reliquo segmento fit, quadrato.



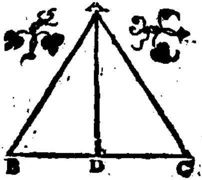
Theorema 11. Propositio 12.

In amblygonijs triãgulis, quadratum quod fit à latere angulum obtusum subtendente, maius est quadratis, quæ fiunt à lateribus obtusum angulum comprehendentibus, pro quantitate rectanguli bis comprehensi & ab vno laterũ quæ sunt circa obtusum angulũ, in quod cùm protractum fuerit, cadit perpendicularis, & ab assumpta exterius linea sub perpendiculari prope angulũ obtusum.

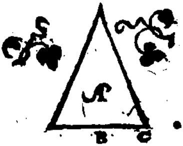


Theorema 12. Propositio 28.

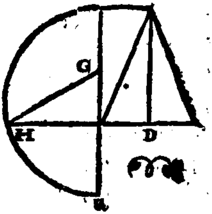
In oxygonijs triangulis quadratum à latere angulum acutum subtendente, minus est quadratis quæ fiunt à lateribus acutum angulum comprehendentibus, pro quantitate rectanguli bis comprehensi, & ab vna laterum, quæ sunt circa acutum angulum, in quod perpendicularis cadit, & ab assumpta interius linea sub perpendiculari prope acutum angulum.



Problema 2. Propositio 14.



Dato rectilineo æquale quadratum constituere.



ELEMENTI II. FINIS.

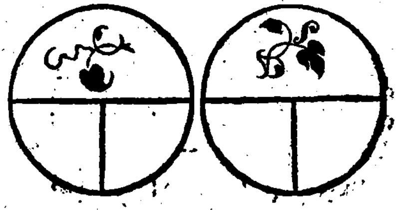
EVCLI-

EVCLIDIS ELEMENTVM TERTIVM.

DEFINITIONES.

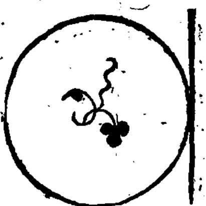
1.

Aequales circuli ſunt quorũ diametri ſunt ęquales vel quorũ quæ ex centris, rectæ lineæ sũt æquales.



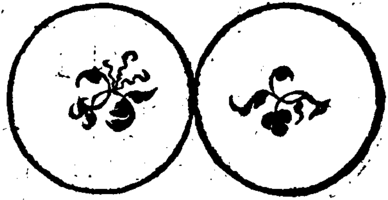
2.

Recta linea circulum tágere dicitur, quæ cùm circulum tangat, ſi producatur, circulum non ſecat.



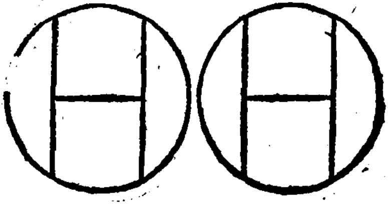
3.

Circuli sese mutuò tangere dicuntur: qui sese mutuo tangentes, sese mutuo non secant.



4.

In circulo æqualiter distare à centro rectæ lineæ dicuntur, cùm perpendiculares, quæ à centro in ipsas ducuntur, sunt æquales. Lõgius autem abesse illa dicitur in quá maior perpendicularis cadit.



5.

Segmentum circuli est, figura quæ sub recta linea & circuli peripheria comprehenditur.

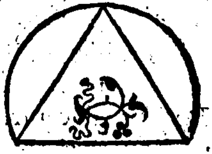
6.

Segmenti autem angelus est, qui sub recta linea

linea & circuli peripheria comprehenditur.

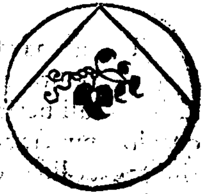
7.

In segmento autem angulus est, cum in segmenti peripheria sumptum fuerit quodpiam punctum, & ab illo in terminos rectæ eius lineæ, quæ segmenti basis est, adiūctę fuerint rectæ lineæ: is, inquam, angulus ab adiūctis illis lineis comprehensus.



8.

Cùm verò comprehendentes angulum rectæ lineæ aliquam assumitur peripheriam, illi angulus insistere dicitur.



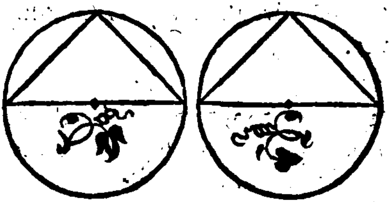
9.

Sector autem circuli est, cùm ad ipsius circuli cētrum constitutus fuerit angulus, comprahensa nimirum figura, & à rectis lineis angulum continentibus, & à peripheria ab illis assumpta.

10.

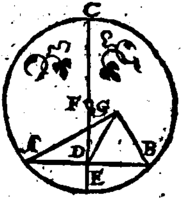
Similia circuli segmenta sunt, quæ angulos capiunt

capiunt
ęquales
aut in
quibus
anguli
inter ſe
ſunt æ-
quales.



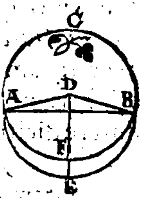
Problema 1. Propoſitio 1.

Dati circuli centrum reperire.



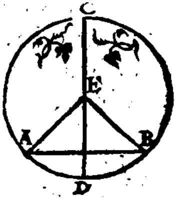
Theorema 1. Propoſitio 2.

Si in circuli peripheria duo quælibet puncta accepta fuerint, recta linea quæ ad ipſa puncta adiungitur, intra circulum cadet.



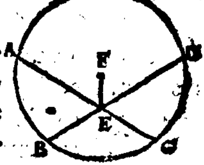
Theorema 2. Propoſitio 3.

Si in circulo recta quædam linea per centrum extenſa quandam non per centrum extenſam bifariam ſecet: & ad angulos rectos ipſam ſecabit. Et ſi ad angulos rectos eam ſecet, bifariam quoque eam ſecabit.



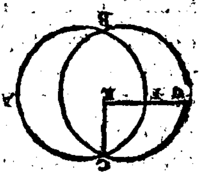
Theo-

Theorema 3. Propositio 4.



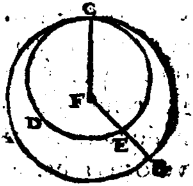
Si in circulo duæ rectę lineæ sese mutuo secent nō per centrum extensæ sese mutuò bifariam non secabunt.

Theorema 4. Propositio 5.



Si duo circuli sese mutuò secent, non erit illorum idem centrum.

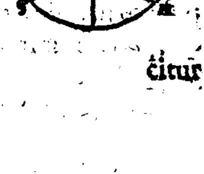
Theorema 5. Propositio 6.



Si duo circuli sese mutuò interius tangent, eorum non erit idem centrum.

Theorema 6. Propositio. 7.

Si in diametro circuli quodpiam sumatur punctum, quod circuli centrum non sit, ab eoq; puncto in circulum quædam rectæ lineæ cadant: maxima quidem erit ea in qua centrū, minima verò reliqua, aliarum verò propinquior illi quæ per centrum du-

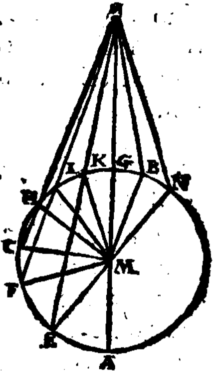


citur

citur, remotiore semper maior est. Duæ autem solùm rectæ lineæ æquales ab eodem puncto in circulum cadunt ad vtrasque partes minimæ.

Theorema 7. Propositio 8.

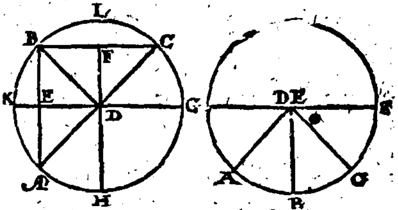
Si extra circulum sumatur punctum quodpiam, ab eoque puncto ad circulum deducantur rectæ quædam lineæ, quarum vna quidem per centrum protendatur, reliquæ verò vt libet: in cauam peripheriam cadentium rectarum linearum minima quidem est illa, quæ per centrum ducitur: aliarum autem propinquior ei, quæ per centrum transit, remotiore semper maior est: in conuexam verò peripheriã cadentium rectarum linearũ minima quidem est illa, quæ inter punctum & diametrum interponitur: aliarum autem, ea quæ propinquior est minimæ; remotiore semper minor est. Duæ autem tantùm rectæ lineæ æquales ab eo puncto in ipsum circulum cadunt, ad vtrasque partes minimæ.



Theo-

Theorema 8. Propositio 9.

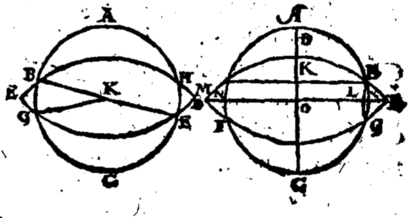
Si in circulo acceptum fuerit punctum aliquod, & ab eo puncto ad circulum cadent plures quam duæ rectæ, lineæ, æquales, acceptũ



punctum centrum ipsius erit circuli.

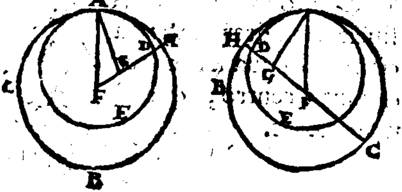
Theorema 9. Propositio 10.

Circulus circulum in pluribus quàm duob⁹ punctis non secat.



Theorema 10. Propositio 11.

Si duo circuli sese intus cõtingãnt atque accepta

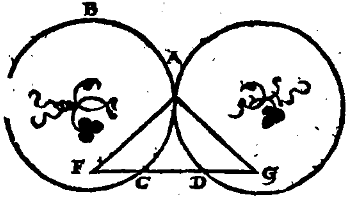


fuerint

fuerint eorum centra, ad eorum cẽntra adiuncta recta linea & producta in contactum circulorum cadet.

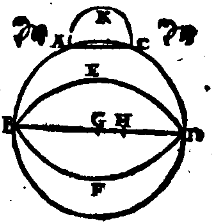
Theorema 11. Propositio 12.

Si duo circuli sese exterius cõtingant, linea recta quæ ad cẽtra eorũ ad iũgitur, per cõtactũ illũ trãsibit.



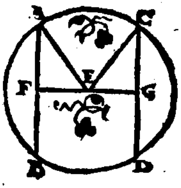
Theorema 12. Propositio 13.

Circulum circulum non tangit in pluribus punctis, quem vno, siue intus siue extra tangat.



Theorema 13. Propopositio. 14.

In circulo æquales rectæ lineæ æqualiter distant à centro. Et quæ æqualiter distant à centro, quales sunt inter se.



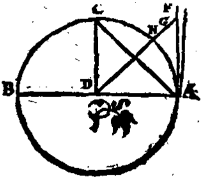
Theore-

Theorema 14. Propositio 15.

In circulo maxima, quidem linea est diameter: aliarum autem propinquior centro, remotiore semper maior.

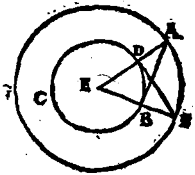
Theorema 15. Propositio 16.

Quæ ab extremitate diametri cuiusque circuli ad angulos rectos ducitur, extra ipsum circulum cadet, & in locum inter ipsam rectam lineam & peripheriam comprehensum, altera recta linea nó cadet. Et semicirculi quidem angulus quouis angulo acuto rectilineo maior est, reliquus autem minor.

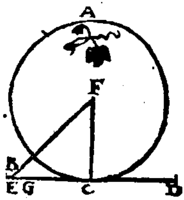


Problema 16. Propositio 17.

A dato puncto rectam lineam ducere, quæ datum tangat circulum.

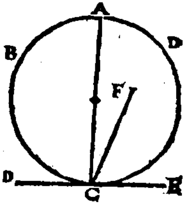


Theorema 16. Propoſitio 18.



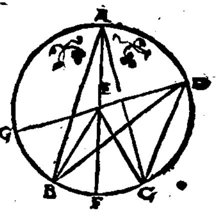
Si circulum tangat recta quæpiam linea, à centro autem ad contactum adiungatur recta quædam linea: quæ adiũcta fuerit ad ipſam contingentem perpendicularis erit.

Theorema 17. Propoſitio 19.



Si circulum tetigerit recta quæpiam linea, à contactu autem recta linea ad ãgulos rectos ipſi tamgenti excitetur, in excitata erit centrum circuli.

Theorema 18. Propoſitio 20.



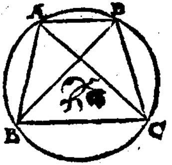
In circulo anguluſ ad cẽtrũ duplex eſt anguli ad peripheriam, cũ fuerit eadẽ peripheria baſis angulorum.

Theorema 19. Propoſitio 21.

In circulo, qui in eodem ſegmẽto ſunt anguli, ſunt inter ſe æquales.

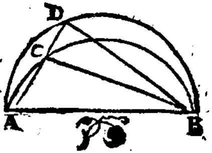
Theo-

Theorema 20. Propoſitio 22.



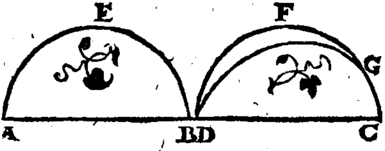
Quadrilaterorum in circulis deſcriptorum anguli qui ex aduerſo, duobus rectis ſunt æquales.

Theorema 21. Propoſitio 23.



Super eadem recta linea duo ſegmenta circulorũ ſimilia & inæqualia non conſtituentur ad eaſdem partes.

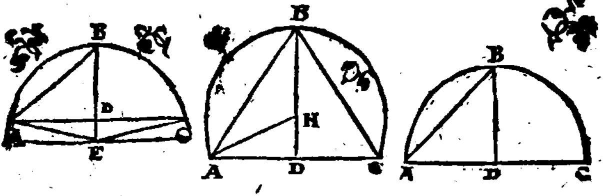
Theorema 22. Propoſitio. 24.



Super ę qualibꝰ rectis lineis ſimilia circulorum ſegmenta ſunt inter ſe æqualia.

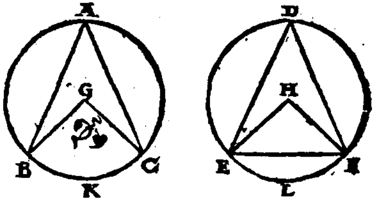
Problema 23. Propoſitio 25.

Circuli ſegmento dato, deſcribere circulũ cuius eſt ſegmentum.



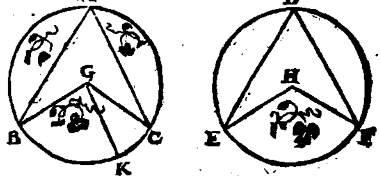
Theorema 22. Propoſitio 26.

In æqualibus circulis, æquales anguli æqualib. peripherijs inſiſtũt ſiue ad cẽtra, ſiue ad peripherias conſtituti inſiſtant.



Theorema 24. Propoſitio 27.

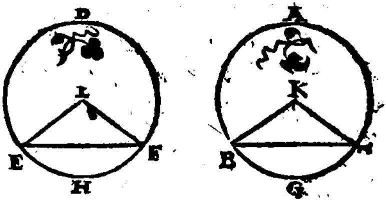
In æqualibus circulis, anguli qui æqualibus peripherijs inſiſtũt ſunt inter ſe ęquales ſiue ad centra, ſiue ad peripherias cõſtituti inſiſtãt.



Theo-

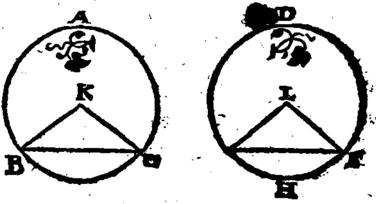
Theorema 25. Propositio. 28.

In æqualibus circulis æquales rectæ lineæ ęquales peripherias auferũt maioré quidé maiori, minorem autem minori.



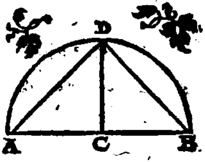
Theorema 26. Propositio 29.

In æqualib. circulis, æquales peripherias æquales rectæ lineæ subtendunt.



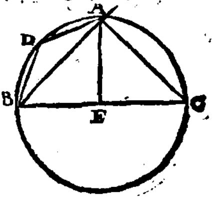
Theorema 4. Propositio 30.

Datam peripheriam bifariam secare.



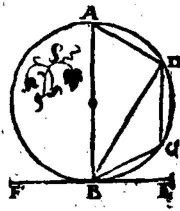
Problema 27. Propositio 31.

In circulo angulus qui in semicirculo, rectus eſt: qui autem in maiore ſegmento, minor recto: qui verò in minore ſegmēto, maior eſt recto. Et inſuper angulus maioris ſegmenti, recto quidem maior eſt, minoris autē ſegmēti angulus minor eſt recto.



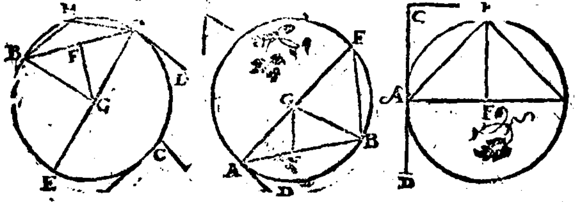
Theorema 28. Propoſitio 32.

Si circulum tetigerit aliqua recta linea, à contactu autē producatur quædam recta linea circulum ſecans: anguli quos ad contingentem facit æquales ſunt ijs qui in alternis circuli ſegmētis conſiſtunt, angulis.



Problema 5. Propoſitio 33.

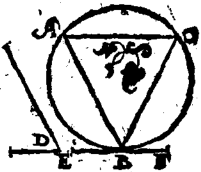
Super data recta linea deſcribere ſegmentū circuli quod capiat angulum æqualem dato angulo rectilineo.



Proble-

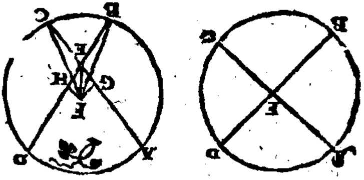
Problema 6. Propositio 34

A dato circulo ſegmentum abſcindere capiens angulum æqualem dato angulo rectilineo.



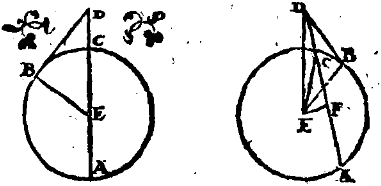
Theorema 29. Propoſitio 35.

Si in circulo duæ rectæ lineæ ſeſe mutuò ſecuerint, rectangulum comprehenſum ſub ſegmētis vnius, æquale eſt ei, quod ſub ſegmentis alterius comprehenditur, rectangulo.



Theorema 30. Propoſitio 36.

Si extra circulũ ſumatur pũctũ aliquod,



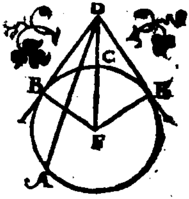
ab eoq́ue in circulũ cadant duæ rectæ lineæ, quarum altera quidem circulũ ſecet, altera

 verò

verò tangat: quod sub tota secante & exterius inter punctum & conuexam peripheriam assumpta comprehenditur rectangulum, æquale erit ei, quod à tangente describitur, quadrato

Theorema 31. Propositio 37.

Si extra circulũ sumatur punctum aliquod, ab eoque puncto in circulum cadant duæ rectæ lineæ, quarum altera circulum secet, altera in eum incidat, sit autem quod sub tota secãte & exterius inter punctum & conuexã peripheriam assumpta, comprehenditur rectangulum, æquale ei, quod quadrato, incidens ipsa circulum tanget.



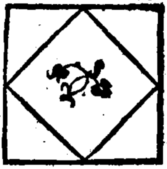
ELEMENTI III. FINIS.

EVCLIDIS ELEMENTVM QVARTVM.

DEFINITIONES.

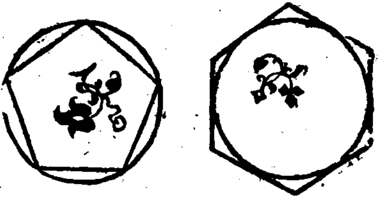
1.

Figura rectilinea in figura rectilinea inſcribi dicitur, cũ ſinguli eius figuræ quæ inſcribitur, anguli ſinguli latera eius, in qua inſcribitur, tangunt.



2.

Similiter & figura circum figuram deſcribi dicitur, quum ſingula eius quę circumſcribitur, latera ſingulos eius figurę angulos tetigerint, circum quam ille deſcribitur.



3.

Figura rectilinea in circulo inſcribi dicitur, quũ ſinguli eius figuræ quæ inſcribitur, angu-

anguli tetigerint circuli peripheriam.

4.

Figura verò rectilinea circa circulum deſcribi dicitur, quum ſingula latera eius, quæ circumſcribitur, circuli peripheriam tangunt.

5.

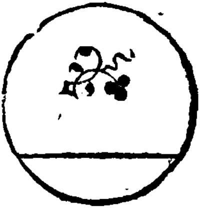
Similiter & circulus in figura rectilinea inſcribi dicitur, quũ circuli peripheria ſingula latera tangit eius figuræ, cui inſcribitur.

6.

Circulus autem circum figuram deſcribi dicitur, quum circuli peripheria ſingulos tangit eius figuræ, quam circumſcribit, angulos.

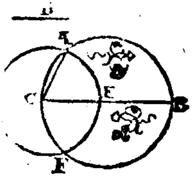
7.

Recta linea in circulo accommodari ſeu coaptari dicitur, quum eius extrema in circuli peripheria fuerint.



Problema 1. Propoſitio 1.

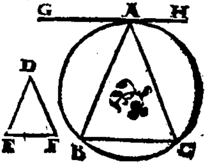
In dato circulo, rectam lineam accommodare æqualem datæ rectæ lineæ quæ circuli diamatro nõ ſit maior.



Pro-

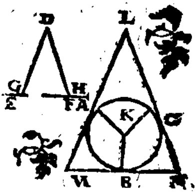
Problema 2. Propositio 3.

In dato circulo, triangulũ describere dato triangulo æquiangulum.



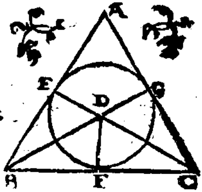
Problema 3 Propositio 3.

Circa datum circulũ triangulũ, describere dato triangulo ęquiangulum.



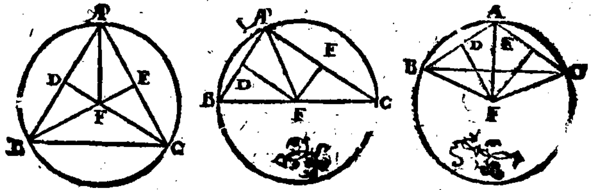
Problema 4. Propositio 4.

In dato triangulo circulum inscribere.



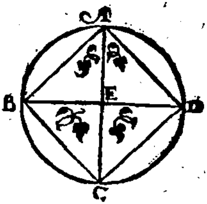
Problema 5. Propositio 5.

Circa datum triangulum, circulum describere.



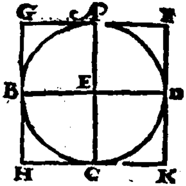
Pro-

Problema 6. Propositio 6.



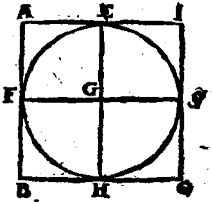
In dato circulo quadratum describere.

Problema 7. Propositio 7.



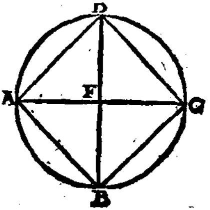
Circa datum circulum, quadratum describere.

Problema 8. Propositio 8.



In dato quadrato circulum inscribere.

Problema 9. Propositio 9.

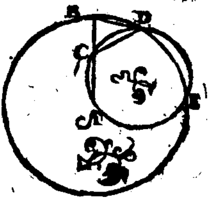


Circa datum quadratũ, circulum describere.

Pro-

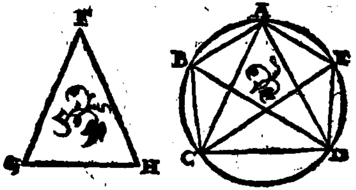
Problema 10. Propositio 10.

Isosceles triangulum cōstituere, quod habeat vtrunque eorum, qui ad duplum reliqui.



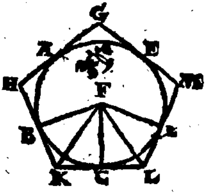
Theorema 11. Propositio 11.

In dato circulo, pentagonū æquilaterū & æquiágulum inscribere.



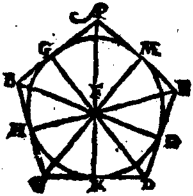
Problema 12. Propositio 12.

Circa datum circulum, pentagonum æquilaterum æquiangulum describere.



Problema 13. Propositio 13.

In dato pentagono ęquilatero & æquiágulo cirlum inscribere.

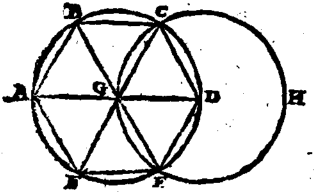


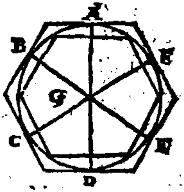
Problema 14. Propositio 14.

Circa datum pentagonũ, æquilaterum & æquiangulum, circulum describere.

Problema 15. Propositio 15.

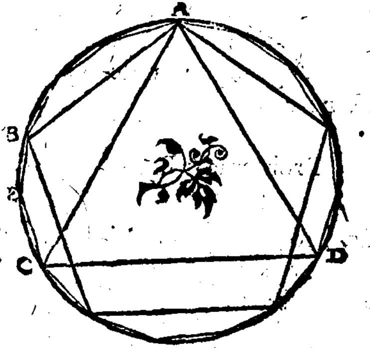
In dato circulo hexagonum & æquilaterũ & æquiangulum inscribere.





Propositio 16. Theorema 16.

In dato circulo quintidecagonum & æquilaterũ & æquiangulum describere.



Elementi quarti finis.

EVCLIDIS ELEMENTVM QVINTVM.

DEFINITIONES.

1.

PARS est magnitudo magnitudinis minor maioris, quum minor metitur maiorem.

2.

Multiplex autem est maior minoris, cum minor metitur maiorem.

3.

Ratio est duarum magnitudinum eiusdem generis mutua quædam secundum quantitatem habitudo.

4.

Proportio verò est rationum similitudo.

5.

Rationem habere inter se magnitudinis dicuntur, quæ possunt multiplicatæ sese mutu superare.

6.

In eadem ratione magnitudinis dicuntur esse, prima ad secundam, & tertia ad quartam, cum primæ & tertiæ æquè multiplicia, à secundæ & quartæ æquè multiplicibus,

 qualis-

qualiſcunque ſit hæc multiplicatio, vtrunq; ab vtroque: vel vna deficiunt, vel vna æqualia ſunt, vel vna excedũt, ſi ea ſumantur quæ inter ſe reſpondent.

7.

Eandem autem habentes rationem magnitudines, proportionales vocentur.

8.

Cum verò æquè multiplicium, multiplex primæ magnitudines exceſſerit multiplicẽ ſecundæ, at multiplex tertiæ non exceſſerit multiplicem quartæ: tunc prima ad ſecundam, maiorem rationem habere dicetur, quàm tertia ad quartam.

9.

Proportio autem in tribus terminis pauciſſimis conſiſtit.

10.

Cùm autem tres magnitudines proportionales, fuerint, prima ad tertiam, duplicatam rationẽ habere dicitur eius quam habet ad ſecundam. At cùm quatuor magnitudines proportionales fuerint, prima ad quartam, triplicatam rationem habere dicitur eius quam habet ad ſecundam: & ſemper deinceps vno amplius, quamdiu proportio extiterit.

11.

Homologæ, ſeu ſimiles ratione magnitudines dicuntur, antecedentes quidem antecedentidentibus, consequentes verò consequentib.

12.

Alterna ratio, est sumptio antecedentis cōparati ad antecedentem, & consequentis ad consequentem.

13.

Inuersa ratio, est sumptio consequentis, ceu antecedentis, ad antecedentem velut ad cōsequentem.

14.

Compositio rationis, est sumptio antecedētis cū consequente ceu vnius ad ipsum consequentem.

15.

Diuisio rationis, est sumptio excessus quo cōsequētem superat antecedens ad ipsum consequentem.

16.

Conuersio rationis, est sumptio antecedentis ad excessum, quo superat antecedens ipsum consequentem.

17.

Ex æqualitate ratio est, si plures duabus sint magnitudines, & hìs aliæ multitudine pares quæ binæ sumantur, & in eadem ratione: quùm vt in primis magnitudinibus prima ad vltimam sic & in secundis magnitudinibus prima ad vltimam sese habuerit, vel aliter, sumptio extremorum per subductionē mediorum.

18.

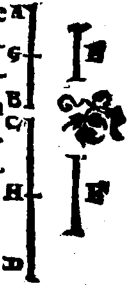
Ordinata proportio est, cùm fuerit quemadmodum antecedens ad consequentem ita antecedens ad consequentem: fuerit etiam vt consequens ad aliud quidpiam, ita consequens ad aliud quidpiam.

19.

Perturbata autem proportio est, tribus positis magnitudinibus, & alij quæ sint his multitudine pares, cùm vt in primis quidẽ magnitudinibus se habet antecedens ad cõsequentem, ita in secundis magnitudinibus antecedens ad consequentem: vt autem in primis magnitudinibus consequẽs ad aliud quidpiam ad antecedentem.

Theorema 1. Propositio 1.

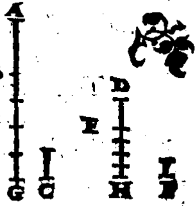
Si sint quotcunque magnitudine quotcunque magnitudinum æqualium numero singulæ singularum æquè multiplices, quàm multiplex est vnius vna magnitudo, tam multiplices erunt, & omnes omnium.



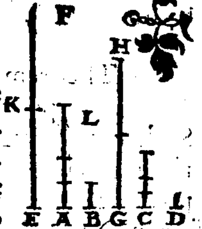
Theorema 2. Propositio 2.

Si prima secundæ æquè fuerit multiplex, atque

atque tertia quartæ, fuerit autẽ & quinta secũdæ æquè multiplex, atque sexta quartæ: erit & cõposita prima cũ quinta, secundę æquè multiplex atque tertia cum sexta, quartæ.



Theorema 3. Propositio 3.

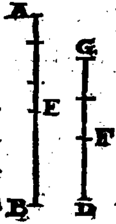


Si prima secundæ æquè multiplex atque tertia quartæ sumantur autem æquè multiplices primæ & tertiæ erit & ex æquo sumptarum vtraque vtriusque æquè multiplex, altera quidem secundæ, altera autem.

Theorema 4. Propositio 4.

Si prima ad secundam, eandem habuerit rationem, & tertia ad quartam: etiam æquè multiplices primæ & tertię ad æquè multiplices secundæ & quartæ iuxta quãuis multiplicationẽ, eandem habebunt rationem, si prout inter se respon-

respondent, ita sumptæ fuerint.



Theorema 5. Propositio 5.

Si magnitudo magnitudinis æquè fuerit multiplex, atque ablata ablatæ: etiam reliqua reliquæ ita multiplex erit, vt tota totius.

Theorema 6. Propositio 6.

Si duæ magnitudines, duarum magnitudinum sint æquè multiplices, & detractæ quædam sint earundem æquè multiplices: & reliquæ eisdem aut æquales sunt, aut æquè ipsarum multiplices.

Theorema 7. Propositio 7.

AEquales ad eandem, eadem habent rationem: & eandem ad æquales.

Theorema 8. Propositio 8.

In æqualium magnitudinum maior, ad eandem

dem maiorem ratione habet, quàm minor: & eadem ad minorem: maiorem rationem habet, quàm ad maiorem.

Theorema 9. Propositio 9.

Quæ ad eandem, eandem habent rationem, æquales sunt inter se: & ad quas eadem, eandem habet rationem, eæ quoque sunt inter se æquales.

Theorema 10. Propositio 10.

Ad eandem magnitudiné ratio nem habentium, quæ maiorem rationé habet, illa maior est ad quam auté eadem maiorem rationem habet, illa minor est.

Theorema 11. Propositio 11.

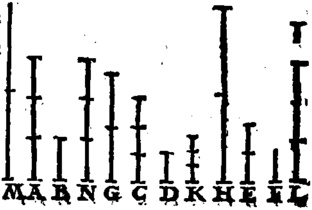
Quæ eidem sunt eædem rationes, & inter se sunt eædem.

Theorema 12. Propositio 12.

Si ſint magnitudines quotcunque proportionales, quemadmodũ ſe habuerit vna antecedentium ad vnã cõſequentium, ita ſe habebunt omnes antecedentes ad omnes conſequentes.

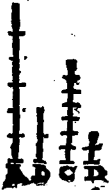
Theorema 13. Propoſitio 13.

Si prima ad ſecundum, eandem habuerit rationem, quam tertia ad quartam, tertia verò ad quartam maiorem rationem habuerit, quàm quinta ad ſextam, prima quoq; ad ſecundam maiorem rationem habebit, quàm quinta ad ſextam.



Theorema 14. Propoſitio 14.

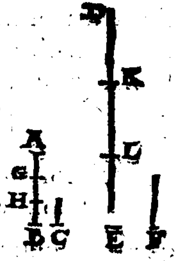
Si prima ad ſecundam eandem habuerit rationem, quam tertia ad quartam, prima verò quàm tertia maior fuerit: erit & ſecunda maior quam quarta. Quòd ſi prima fuerit æqualis tertiæ, erit



& secunda æqualis quartæ: si verò minor, & minor erit.

Theorema 15. Propopositio. 15.

Partes cum pariter multiplicibus in eadem sunt ratione, si prout sibi mutuo respondent, ita sumantur.

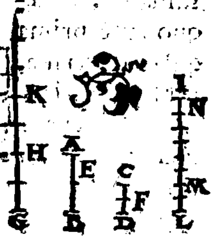


Theorema 16. Propositio 16.

Si quatuor magnitudines proportionales fuerint, vt vicissim proportionales erunt.

Theorema 17. Propositio 17.

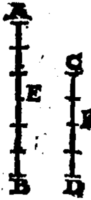
Si compositæ magnitudines proportionales fuerint hæ quoq; diuisæ proportionales erunt.



Theorema 18. Propositio 18.

Si diuisæ magnitudines sint proportionales, hæ quoque compositæ proportionales erunt.

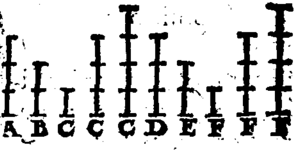
Theorema 19. Propositio 19.



Si quemadmodum totú ad totum, ita ablatum se habuerit ad ablatum: & reliquum ad reliquum, vt totum ad totum se habebit.

Theorema 20. Propositio 20.

Si sint tres magnitudines, & aliæ ipsis æquales numero, quæ binæ & in eadem ratione sumatur, ex æquo auté prima quàm tertia maior fuerit: erit & quarta, quã sexta maior. Quòd si

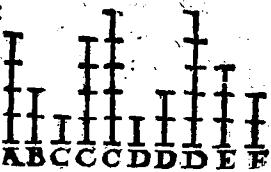


prima tertiæ fuerit æqualis, erit & quarto æqualis sextæ: sin illa minor, hæc quoque minor erit.

Theo-

Theorema 21. Propositio 21.

Si sint tres magnitudines & aliæ ipsis æquales numero quæ binæ & in eadem ratione sumantur, fuerit que perturbata earũ proportio ex æquo autem prima quam tertia maior fuerit



erit & quarta quam sexta maior: quòd prima tertiæ fuerit æqualis, erit & quarta æqualis sextæ: sin illa minor, hæc quoque minor erit.

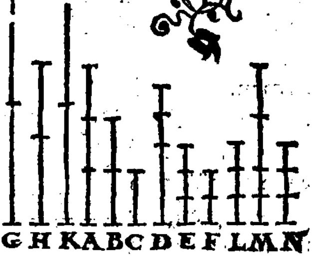
Theorema 22. Propositio 22.

Si sint quotcunq; magnitudines, & aliæ ipsis equales numero, quæ binæ in eadẽ ratione sumantur, & ex æqualitate in eadem ratione erunt.

Theorema 23. Propositio 23.

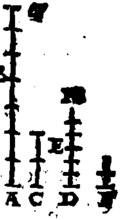
Si sint tres magnitudiés, aliæq; ipsis æquales nume-

numero, quam binæ in eadem ratione ſumantur, fuerit autē perturbata earū proportio: etiā ex ęqualitate in eadem ratione erunt.



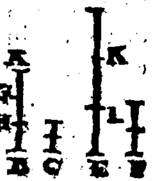
Theorema 24. Propoſitio 24.

Si prima ad ſecundam, eandem habuerit rationem, quam tertia ad quartam, habuerit autem & quinta ad ſecundam, eandem rationem quam ſexta ad quartam: etiam compoſita prima cū quinta ad ſecundam eandem habebit rationem, quam tertia cum ſexta ad quartam.



Theorema 25. Propoſitio 25.

Si quatuor magnitudines proportionales fuerint, maxima & minima reliqui duabus maiores erunt.



ELEMENTI V. FINIS.

EVCLIDIS ELEMEVTVM SEXTVM.

DEFINITIONES.

1.

SImiles figuræ rectilineæ ſunt, quę & angulos ſingulos ſingulis æquales habent, atque etiam latera, quæ circum angulos æquales, proportionalia.

2.

Reciprocæ autem figuræ ſunt, cum in vtraque figura antecedentes & conſequentes rationum termini fuerint.

3.

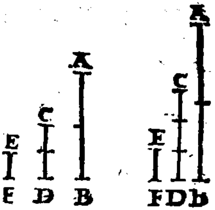
Secundum extremam & mediam rationem recta linea ſecta eſſe dicitur, cum vt tota ad maius ſegmentum, ita maius ad minus ſe habuerit.

4.

Altitudo cuiuſque figuræ, eſt linea perpendicularis à vertice ad baſin deducta.

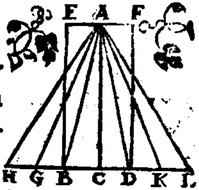
5.

Ratio ex rationibus cō poni dicitur, cum ratio num quantitatis inter ſe multiplicatæ aliqua effecerint rationem.



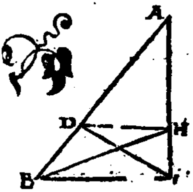
Theorema 1. Propoſitio 1.

Triangula & parallelogramma, quorum eadem fuerit altitudo, ita ſe habent inter ſe vt baſes.



Theorema 2. Propoſitio 2.

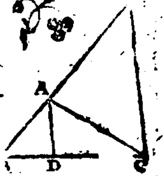
Si ad vnum trianguli latus parallela ducta fuerit recta quædam linea: hæc proportionaliter ſecabit, ipſius trianguli latera. Etſi trianguli latera proportionaliter ſecta fuerint: quæ ad ſectiones adiūcta fuerit recta linea, erit ad reliquum ipſius trianguli latus parallela.



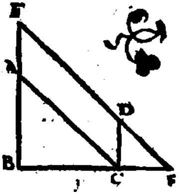
Theorema 3. Propoſitio 3.

Si trianguli angulus bifariam ſectus ſit, ſecans autem angulum recta linea ſecuerit & baſim: baſis ſegmenta eandem habebunt rationem

tionem, quam reliqua ipsius trianguli latera. Et si basis segmenta eandem habeant rationem quam reliqua ipsius trianguli latera, recta linea, quæ à vertice ad sectionem producitur, ea bifariá secat trianguli ipsius angulum.



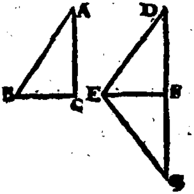
Theorema 4. Propositio 4.



Aequiangulorum triangulorum proportianolia sunt latera, quæ circum æquales angulos, & homologa sunt latera, quæ æqualibus angulis subtenduntur.

Theorema 5. Propositio 5.

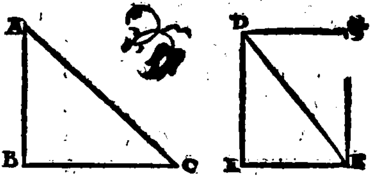
Si duo triágula latera proportionalia habeant, æquiangula erunt triangula, & æquales habebunt eos angulos, sub quib⁹ homologa latera subtenduntur.



Theorema 6. Propositio 6.

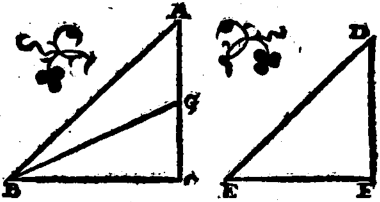
Si duo triangula vnum angulum vni angulo æqualé, & circum æquales angulos latera proportionalia habuerint, æquiangula erút trian-

triangula, ęqualesq; habebunt angulos ſub quibus homologa latera ſubtenduntur.



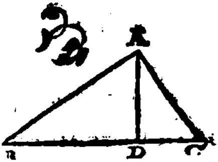
Theorema 7. Propoſitio 7.

Si duo triangula vnum angulum vni angulo æqualem, circum autem alios angulos latera proportionalia habent, reliquorum vero ſimul vtrunq; aut minorem aut non minoré recto: æquiangula erunt triangula, & æquales habebunt eos angulos circum quos proportionalia ſunt latera.



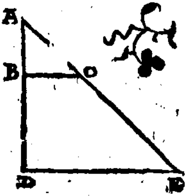
Theorema 8. Propoſitio 8.

Si in triangulo rectangulo, ab angulo recto in baſin perpendicularis ducta ſit quæ ad perpendicularem triangulo, tum toti triangulo, tum ipſa inter ſe ſimilia ſunt.



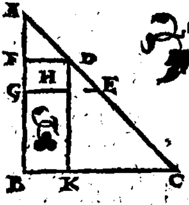
Pro-

Problema 1. Propositio 9.



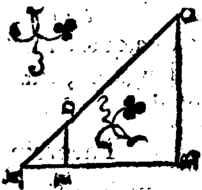
A data recta linea imperatam partem auferre.

Problema 2. Propositio 10.



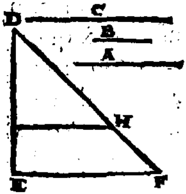
Datam rectam lineam in sectam similiter secare, vt data altera recta secta fuerit.

Problema 3. Propositio 11.



Duabus datis rectis lineis, tertiam proportionalem ad inuenire.

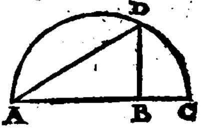
Problema 4. Propositio 12.



Tribus datis rectis lineis quartam proportionalẽ ad inuenire.

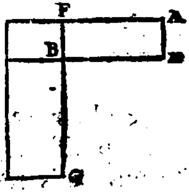
Problema 5. Propoſitio 13.

Duabꝰ datis rectis lineis mediam proportionalẽ adinuenire.



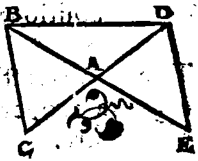
Theorema 9. Propoſitio 14.

AEqualium, & vnũ vni æqualem habentiũ angulum parallelogrammorum reciproca ſunt latera, quæ circum æquales angulos, & quorum parallelogrã morum vnum angulũ vni angulo æqualem habentium reciproca ſunt latera, quæ circum æquales angulos, illa ſunt æqualia.



Theorema 10. Propoſitio 15.

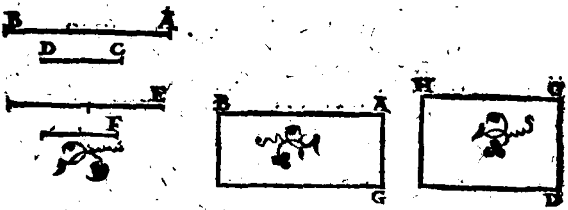
Aequalium & vnum angulum vni ęqualem habentium triangulorũ reciproca ſunt latera, quæ circum æquales angulos: & quorum triangulorum vnum angulum vni æqualem habentium reciproca ſunt latera, quæ circum æquales angulos, illa ſunt æqualia.



Theo-

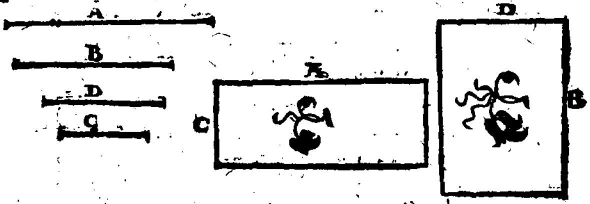
Theorema 11. Propositio 16.

Si quatuor rectæ lineæ proportiolales fuerint, quod sub extremis comprehenditur rectangulum, ęquale est ei, quod sub medijs comprehenditur rectangulo. Et si sub extremis comprehensum rectangulum æquale fuerit ei, quod sub medijs continetur rectágulo illæ quatuor rectæ lineæ proportionales erunt.

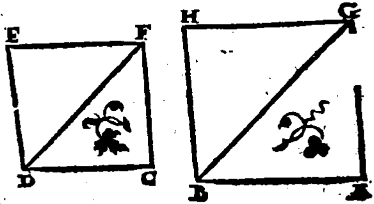


Theorema 12. Propositio 17.

Si tres rectæ lineę sint proportionales, quod sub extremis compreh éditur rectangulum æquale est ei, quod à media describitur quadrato: & si sub extremis comprehensum rectangulum æquale sit ei, quod à media describitur quadrato, illę tres rectæ lineę proportionales erunt.

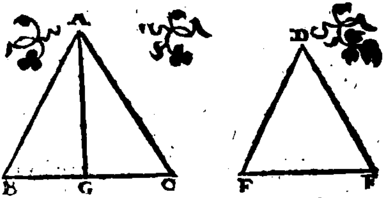


Problema 6. Propositio 18.



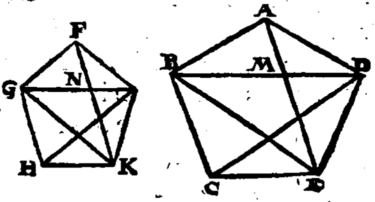
A data recta linea, dato recti lineo simile similiterq; positum rectilineum describere.

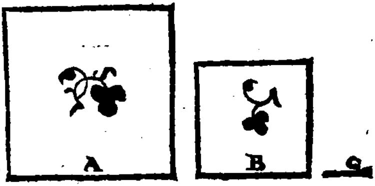
Theorema 13. Propositio 19.



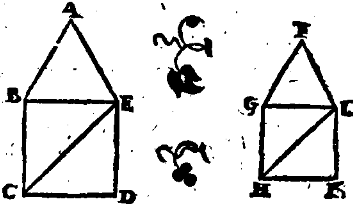
Similia triãgula inter se sunt in duplicata ratione laterũ homologorũ.

Theore. 14: Propositio 20



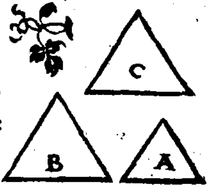


Similia polygona in similia triangula diuidũtur, & numero æqualia, et homologa totis. Et polygona duplicatã habẽt eam inter ſe rationẽ, quã latus homologũ ad homologum latus.



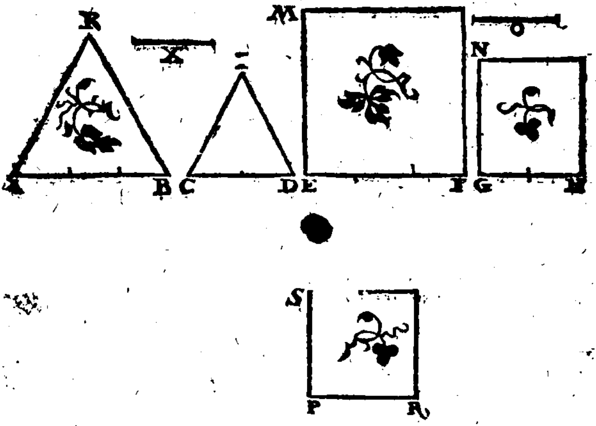
Theorema 15. Propoſitio 21.

Quæ eidem rectilineo ſunt ſimilia, & inter ſe ſunt ſimilia.

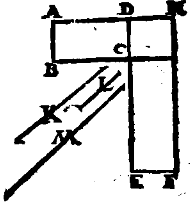


Theorema 16. Propoſitio 22.

Si quatuor rectæ lineæ proportionales fuerint: & ab eis rectilinea ſimili ſimiliterq́ue deſcripta proportionalia erũt. Et ſi à rectis lineis ſimilia ſimiliterq́ue deſcripta rectilinea proportionalia fuerint, ipſæ etiam rectæ lineæ proportionales erunt.

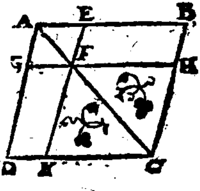


Problema 17. Propositio 23.



Aequiangula parallelogramma inter se rationẽ habent eum, quæ ex lateribus componitur.

Theorema 17. Propositio 24.

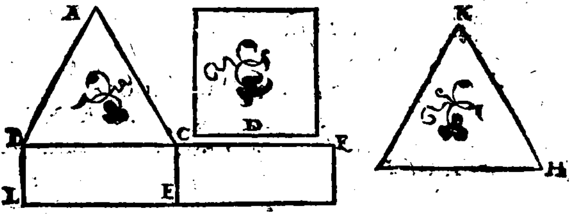


In omni parallelogrammo, quæ circa diametrũ sunt parallelogrãma, & toti & inter se sunt similia.

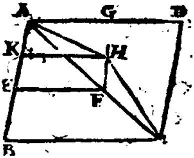
Proble-

Theorema 7. Propoſitio 25.

Dato recto lineo ſimile, & alteri dato ęquale idem conſtituere.



Theorema 19. Propoſitio 26.



Si à parallelogrãmo parallelogrammũ ablatum ſit, & ſimile toti & ſimiliter poſitum communé cum eo habens angulum, hoc circum eandem cum tota diametrum conſiſtit.

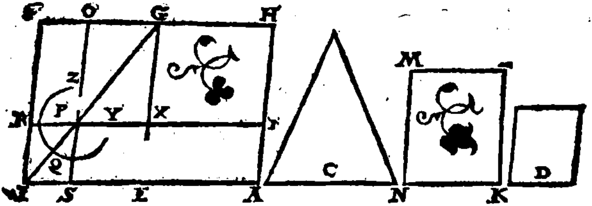
Theorema 20. Propoſitio 27.

Omnium parallelogrammorum ſecundum eandem rectam lineam applicatorum deficiétiumq; figuris parallelogrãmis ſi-

milibus ſimiliterque poſitis ei, quod à dimidia deſcribitur, maximum, id eſt, quod ad dimidiam applicatur parallelogrammũ ſimile exiſtens defectui.

Problema 8. Propoſitio 28.

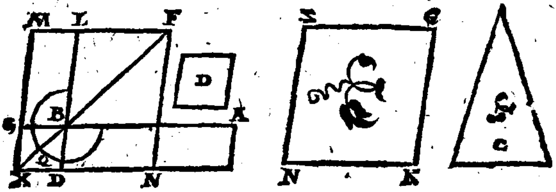
Ad datam lineam rectam, dato rectilineo æquale parallelogrammum applicare deficiens figura parallelogramma, quæ ſimilis ſit alteri rectilineo dato. Oportet autem datum rectilineum, cui æquale applicandum eſt, non maius eſſe eo quod ad dimidiam applicatur, cum ſimiles ſint defectus & eius quod à dimidia deſcribitur, & eius cui ſimile deeſſe debet.



Problema 9. Propo-
ſitio 29.

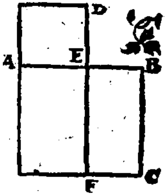
Ad datam rectam lineam, dato rectilineo æquale parallelogrammum applicare, excedens figura parallelogramma, quæ ſimilis ſit

sit parallelogrammo alteri dato.



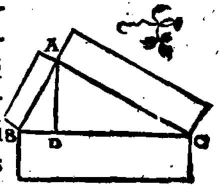
Problema 10. Propositio 30.

Propositam rectam lineam terminatam, extrema ac media ratione secare.



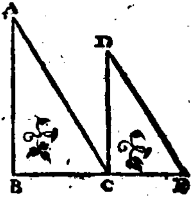
Theorema 21. Propositio 31.

In rectangulis triangulis, figura quæuis à latere rectũ angulum subtendēte descripta ęqualis est figuris, quæ priori illi similes & similiter positæ à lateribus rectũ angulum continentibus describuntur.



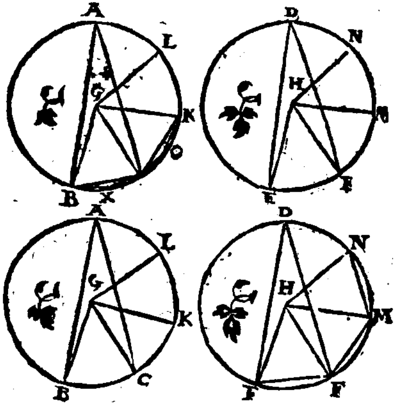
Theorema 22. Propositio 32.

Si duo triangula, quæ duo latera duobus lateribus proportionalia habeant, secundum vnum angulum composita fuerint, ita vt homologa eorum latera sint etiam parallela, tum reliqua illorum triangulorum latera in rectam lineam collocata reperirentur.



Theorema 23. Propositio 33.

In æqualibus circulis anguli eandem habent rationẽ, cum ipsis peripherijs in quibus insistunt, siue ad centra siue ad peripherias cõstituti, illis insistant peripherijs. Insuper verò & sectores quippe qui ad cẽtra cõsistũt.



ELEMENTI VI. FINIS.

EVCLIDIS ELEMENTVM SEPTIMVM.

DEFINITIONES.

1.

VNitas, est secundũ quam entium quodque dicitur.

2.

Numerus autem, ex vnitatibus composita multitudo.

3.

Pars, est numerus numeri minor maioris, cùm minor metitur maiorem.

4.

Partes autem, cùm non metitur.

5.

Multiplex verò, maior minoris, cùm maiorem metitur minor.

6.

Par numerus est, qui bifariam nõ diuiditur.

7.

Impar verò, qui bifariam nõ diuiditur: vel, qui vnitate differt à pari.

8.

Pariter par numerus est, quem par numerus metitur per numerum parem.

9. Pari-

9.

Pariter autem impar, eſt quem par numerus metitur per numerum imparem.

10.

Impariter verò impar numerus, eſt quem impar numerus metitur per numerum imparem.

11.

Primus numerus, eſt quem vnitas ſola metitur.

12.

Primi inter ſe numeri ſunt, quos ſola vnitas menſura communis metitur.

13.

Compoſitus numerus eſt, quem numerus quiſpiam metitur.

14.

Compoſiti autẽ inter ſe numeri ſunt, quos numerus aliquis menſura communis metitur.

15.

Numerus numerum multiplicare dicitur, cùm toties compoſitus fuerit is, qui multiplicatur, quot ſunt in illo multiplicante vnitates, & procreatus fuerit aliquis.

16.

Cùm autem duo numeri mutuò ſeſe multiplicantes quempiã faciunt, qui factus erit planus appellabitur, qui verò numeri mutuò ſeſe multiplicarint, illius latera dicentur

17.

Cùm verò tres numeri mutuò sese multiplicantes quempiam faciũt, qui procreatus erit solidus appellabitur, qua autem numeri mutuò sese multiplicarint, illius, latera dicentur.

18.

Quadratus numerus est, qui ęqualiter æqualis: vel, qui à duabus æqualibus numeris cótinetur.

19.

Cubus verò, qui à tribus æqualibus æqualiter: vel, quia tribus æqualibus numeris continetur.

20.

Numeri proportionales sunt, cum primus secundi, & tertius quarti æquè multiplex est, vt eadem pars, vel eædem partes.

21.

Similes plani & solidi numeri sunt, qui proportionalia habent latera.

22.

Perfectus numerus est, qui suis ipsius partibus est æqualis.

Theorema 1. Propositio 1.

Duobus numeris inæqualibus propositis,

tis, si detrahatur semper minor, de maiore, alterna, quadam detractione; neque reliquus vnquam metiatur præcedentem quo ad assumpta sit vnitas: qui principio propositi sunt numeri primi inter se erunt.

A
H
C
F
G
B D E

A

Problema 1. Propositio 2.

A C
E
E E
B D B D

Duobus numeris datis non primis inter se, maximã eorũ cõmunẽ mẽsurã reperire.

Problema 2. Propositio 3.

A B C D E
8 6 4 2 3

Tribus numeris datis non primis inter se, maximam eorum communem mensuram reperire.

A B C D E F
18 12 8 6 2 3

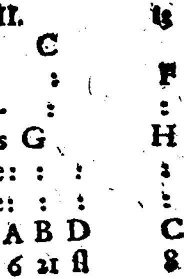
Theorema 18. Propositio 8.

C
F
C C
E

Omnis numerus cuiusq; numeri minor maioris aut par est, aut partes.

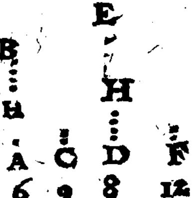
A B B B D
12 7 6 9 3

Theo-



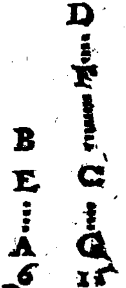
Theorema 3. Propositio 5.

Si numerus numeri par fuerit, & alter alterius eadē pars & simul vterque vtriusque: simul eadem pars erit, quæ vnus est vnius.



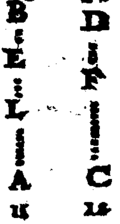
Theor. 4. Propo. 6.

Si numerus sit numeri partes, & alter alterius eædem partes, & simul vterque vtriusque simul eædem partes erunt, quæ sunt vnus vnius.



Theorema 5. Propositio 7.

Si numerus numeri eadē si pars quæ detractus detracti, & reliquus reliqui eadē pars erit, quæ totus est totius.



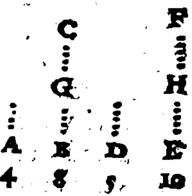
Theorema 6. Propositio. 8.

Si numerus numeri eædē sint partes quæ detractus detracti & reliquus reliqui eædē partes erūt, quæ sunt totus totius.

G.M.K..N.H.

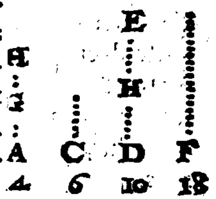
Theorema 7. Propositio 9.

Si numerus numeris pars sit, & alter alterius eadem pars, & vicissim quæ pars est vel partes primus tertij, eadem pars erit eædem partes, & secundus quarti.



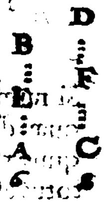
Theorema 8. Propositio 10.

Si numerus numeri partes sint, & aliter alterius eadẽ partes, etiam vicissim quæ sunt partes aut pars primus tertij, eędem partes erũt vel pars & secundus quarti.



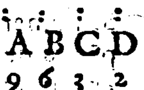
Theorema 9. Propositio 11.

Si quemadmodum se habet totus ad totum, ita detractus ad detractum; & reliquus ad reliquum ita habebit vt totus ad totum.



Theorema 10. Propositio 12.

Si sint quotcunque numeri proportionales; quemadmodum se habet vnus antecedentium ad vnum sequentium, ita se habe-



habebunt omnes antecedentes ad omnes consequentes.

Theorema 11. Propositio 13.

Si quatuor numeri sint proportionales, & vicissim proportionales erũt.

:	:	:	:
A	B	C	D
12	4	9	3

Theorema 12. Propositio 14.

Si sint quotcunque numeri, & alij illis equales multitudine, qui bini sumantur & in eadem ratione, etiam ex æqualitate in eadem ratione erunt.

:	:	:	:	:	:
A	B	C	D	E	F
12	6	3	8	4	2

Theorema 13. Propositio 15.

Si vnitas numerum quãpiã metiatur, aliter verò numerus alium quendam numerũ æquè metiatur, & vicissim vnitas tertium numerũ æquè metietur, atque secundus quartum.

F
L
K
E
C
H
G
A B D
1 3 6

Theorema 14. Propositio 16.

Si duo numeri mutuò sese multiplicãtes faciant aliquos qui ex illis geniti fuerint, inter se æquales erunt.

:	:	:	:	:
E	A	B	C	D
1	2	4	8	8

Theorema 15. Propositio 17.

Si numerus duos numeros multiplicans faciat aliquos, qui ex illis procreati erunt, eadem rationem habebunt quam multiplicati.

I	A	B	C	D	E
1	3	4	5	12	15

Theorema 16. Propositio 18.

Si duo numeri numerum quempiam multiplicantes faciant aliquos, geniti ex illis eandem habebunt rationem, quam qui illum multiplicarunt.

A	B	C	D	E
4	5	3	12	15

Theorema 17. Propositio 19.

Si quatuor numeri sint proportionales, qui ex primo & quarto fit, æqualis erit ei qui ex secundo & tertio: & si qui ex primo & quarto fit numerus æqualis sit ei qui ex secundo & tertio, illi quatuor numeri sint proportionales erunt.

A	B	C	D	E	F	G
6	4	3	2	12	12	18

Thorema 18. Propositio 20.

Si tres numeri sint proportionales, qui ab extremis cõtinetur, æqualis est ei qui à medio

dio efficitur. Et si qui ab extremis cōtinetur æqualis sit ei qui à medio describitur, illi tres numeri proportionales erunt.

A B C
9 6 4
D
6

Theorema 19. Propositio 21.

Minimi numeri omniū, qui eandem cum eis rationem habēt, equaliter metiuntur numeros eandem rationem habentes, maior quidem maiorem, minor verò minorem.

D L
G H
C E A B
4 3 8 6

Theorema 20. Propositio 22.

Si tres sint numeri & alij multitudine illis æquales, qui bini sumantur & in eadem ratione, sit autem perturbata eorum proportio, etiam ex æqualitate in eadem ratione erunt.

A	B	C	D	E	F
6	4	3	12	8	6

Theorema 21. Propositio 23.

Primi inter se numeri minimi sunt omniū eandem cum eis rationem habentium.

A	B	E	C	D
5	6	2	4	3

Theorema 22. Propositio 24.

Minimi numeri omnium eandem cum eis rationem habẽtium, primus sunt inter se.

A	B	C	D	E
7	6	4	3	2

Theorema 23. Propositio 25.

Si duo numeri sint primi inter se, qui alter vtrum eorum metitur numerus, is ad reliquũ primus erit.

A	B	C	D
6	7	3	4

Theorema 24. Propositio 26.

Si duo numeri ad quẽpiam numerum primi sint, an eundem primus is quoque futurus est, qui ab illis productus fuerit.

B	A	C	D	E	F
3	5	5	5	3	2

Theorema 25. Propositio 27.

Si duo numeri primi sint inter se, quæ ab vno eorum gignitur ad reliquũ primus erit.

B	A	C	D
	7	6	3

Theorema 26. Propositio 28.

Si duo numeri ad duos numeros ambo ad vtrunque primi sint, & qui ex eis gignentur, primi inter se erunt.

A	B	E	C	D	F
3	5	15	2	4	8

Theore-

Theorema 27. Propositio 29.

Si duo numeri primi sint inter se, & multiplicans vterque seipsum procreet aliquem, qui ex ijs producti fuerint, primi inter se erũt. Quòd si numeri initio propositi multiplicantes eos qui producti sunt, effecerint aliquos; hi quoque inter se primi erunt, & circa extremos idem hoc semper eueniet.

A	C	E	B	D	F
3	6	27	4	16	63

Theorema 28. Propositio 30.

Si duo numeri primi sunt inter se, etiam simul vterq; ad vtrunq; illorum primus erit. Et si simul vterq; ad vnnm aliquem eorum primus sit etiam qui initio positi sunt numeri, primi inter sunt erunt.

C

A	B	D
7	5	4

Theorema 29. Propositio 31.

Omnis primus numerus ad omnem numerum quem non metitur, primus est.

A	B	C
7	10	5

Theorema 30. Propositio 32.

Si duo numeri sese mutuo multiplicãtes faciant aliquem, hunc autẽ ab illis productũ metiatur primis quidam numerus, is alterum etiam metitur eorum qui initio positi erant.

A	B	C	D	E
3	6	12	3	4

Theorema 31. Propositio 33.

Omnem compositum numerum aliquis primus metietur.

A	B	C
27	9	3

Theorema 32. Propositio 34.

Omnis numerus aut primus est, aut eum aliquis primus metitur.

A	A	
3	6	8

Problema 3. Propositio 35.

Numerus datis quotcunque, reperire minimos omnium qui eandem cum illis rationem habeant.

A	B	C	D	E	F	G	H	K	I	M
6	8	12	2	3	4	6	2	3	4	3

Problema 4. Propositio 36.

B				
A	C	D	E	F
7	12	8	4	5

Duobus numeris datis reperire quẽ illi minimum metiantur numerum.

A					
F	E	C	D	G	H
6	9	12	9	2	3

Theo-

Theorema 33. Propositio 37.

Si duo numeri numerum quempiam metiantur, & minimus quem illi metiuntur eundum metietur.

A B E C F
2 3 6 12

Problema 5. Propositio 38.

Tribus numeris datis reperire quem minimum numerũ illi metiantur.

A B C D E
3 4 6 12 8

A B C D E F
3 6 8 12 24 16

Theorema 34. Propositio 39.

Si numerum quispiam numerus metiatur, mensus partem habebit metienti cognominem.

A B C D
12 4 3 1

Theorema 35. Propositio 40.

Si numerus partem habuerit quamlibet, illum metietur numerus parti cognominis.

A B C D
8 4 2 1

Problema 6. Propositio 41.

Numerum reperire, qui minimus cùm sit, datas habeat partes.

A B C G H
2 3 4 12 10

Elementi septimi finis.

EVCLIDIS ELEMENTVM OCTAVVM.

Theorema 1. Propositio 1.

Si sunt quotcunq; numeri deinceps proportionales, quorum extremi sint inter se, primi, minimi sunt omnium eandem cum eis rationem habentium.

A	B	C	D	E	F	G	H
8	12	18	27	6	8	12	18

Problema 1. Propositio 2.

Numerus reperire deinceps proportionales minimos quotcunque iusserit quispiam in data ratione.

A	B	C	D	E	F	G	H	K
1	4	7	12	16	27	36	49	64

Theorema 2. Propositio 3.

Conuersa primæ.

Si sint quotcunq; numeri deinceps proportionales minimi habentium eandem cum eis rationem, illorum extremi sunt inter se primi.

A	B	C	D	E	F	G	H	K	L	M	N	O
27	16	48	64	3	4	9	12	16	27	36	48	64

Proble-

Problema 3. Propositio 4.

Rationibus datis quotcunque in minimis numeris reperire numeros deinceps minimos in datis rationibus.

A	B	C	D	E	F	H	G	K	L	N	X	M	O
3	4	2	3	4	5	6	8	12	15	4	6	10	12

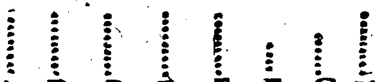
Theorema 3. Propositio 5.

Plani numeri rationem inter se habent ex lateribus compositam.

A	L	B	C	D	E	F	G	H	K
18	22	32	3	6	4	8	9	12	16

Theorema 4. Propositio 6.

Si sint quotlibet numeri deinceps.



A	B	C	D	E	F	G	H
16	24	36	54	82	4	6	9

proportionales, primus autem secundum non metiatur, neque alius quispiam vllum metietur.

Theorema 5. Propositio 7.

Si ſint quotcunque numeri deinceps proportionales, primus autem extremum metiatur, is etiam ſecundum metietur.

A B C D
4 6 12 24

Theorema 6. Propoſitio 8.

Si inter duos numeros medij continui proportione indicant numeri, quot inter eos medij continua proportione incidunt numeri, tot & inter alios eandem cum illis habentes rationem medij continua proportione incident.

A C D B G H K L C M N F
4 9 27 81 1 3 9 27 2 6 18 54

Theorema 7. Propoſitio 9.

Si duo numeri ſunt inter ſe primi, & inter eos medij cōtinua proportione incidāt numeri, quot inter alios medij continua proportione incidunt numeri, totidem et inter vtrunque eorum ac vnitatem deinceps medij continua proportione incident.

A M H E F N C K X G D L O B
27 27 9 36 3 36 1 12 48 4 48 16 64 64

Theo-

Theorema 8. Propositio 10.

Si inter duos numeros & vnitatem cõtinuè proportionales incidant numeri quot inter vtrumque ipsorum & vnitatẽ deinceps medij continua proportione incidunt numeri, totidẽ & inter illos medij continua proportione incident.

A	E	K	D	H	C	L	F	G	B
27	9	36	3	12	1	48	4	16	64

Theorema 9. Propositio 11.

Duorum quadratorum numerorum vnus medius proportionalis est numerus: & quadratus ad quadratum duplicatam habet lateris ad latus rationem.

A	C	E	D	B
9	3	12	4	16

Theorema 10. Propositio 12.

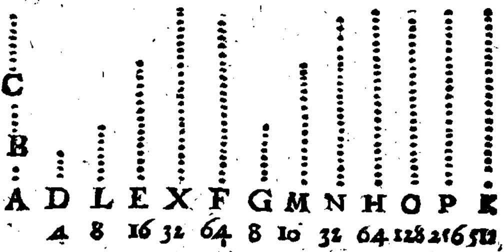
Duorum cuborum numerorum duo medij proportionales sunt numerorũ, duo medij cubum triplicatam habet lateris ad latus rationem.

A	H	K	B	C	D	E	F	G
27	36	48	64	3	4	9	12	16

Theo-

Theorema 11. Propositio 13.

Si sint quotlibet numeri deinceps proportionales & multiplicans quisque seipsum faciat aliquos, qui ab illis producti fuerint, proportionales erunt, & si numeri primùm positi ex suo in procreatos ductu faciant aliquo, ipsi quoque proportionales erunt.



Theorema 12. Propositio 14.

Si quadratus numerus quadratum numerũ metiatur, & latus vnius metietur latus alterius. Et si vnius quadrati latus metiatur latus alterius, & quadratus quadratum metietur.

A	E	B	C	D
9	12	16	8	4

Theorema 13. Propositio 15.

Si cubus numerus cubum numerum metiatur, & latus vnius metietur alterius latus. Et si latus vnius cubi latus alterius metiatur,

tum

rum cubus cubum metietur.

A	H	K	B	C	D	E	F	G
8	16	28	64	2	4	4	8	16

Theorema 14. Propositio 16.

Si quadratus numerus quadratum numerum non metiatur, neque latus vnius metietur alterius latus. Et si latus vnius quadrati nō metiatur latus alterius, neq; quadratus quadratum metietur.

A	B	C	D
9	16	3	4

Theorema 15. Propositio 17.

Si cubus numerus cubum numerū non metiatur, neque latus vnius latus alterius metietur. Et si latus cubi alicuius latus alterius non metiatur, neque cubus cubum metietur.

A	B	C	D
8	27	9	11

Theorema 16. Propositio 18.

Duorum similium planorum numerorum vnus medius proportionalis est numerus, & planus, ad planum duplicatam habet literis homologi-

A	G	B	C	D	E	F
12	18	27	2	6	3	9

logi ad latus homologum rationem.

Theorema 17. Propositio 19.

Inter duos similes numeros solidos, duo medij proportionales incidunt numeri: & solidus ad similem solidum triplicatam rationem habet lateris homologi ad latus homologum.

A	N	X	B	C	D	E	F	G	H	K	M	L
8	12	18	27	2	2	2	3	3	3	4	6	9

Theorema 18. Propositio 20

Si inter duos numeros vnus medius proportionalis incidat numerus, similes plani erunt illi numeri.

A	C	B	D	E	F	G
18	24	32	3	4	6	8

Theorema 19. Propositio 21.

Si inter duos numeros duo medij proportionales incidăt numeri, similes solidi sunt illi numeri.

A	C	D	B	E	F	G	H	K	L	M
27	36	44	64	9	12	16	3	3	3	4

Theo-

Theorema 20. propositio 22.

Si tres numeri deinceps sint proportionales, primus autem sit quadratus, & tertius quadratus erit.

A B D
9 15 25

Theorema 21. propositio 23.

Si quatuor numeri deinceps sint proportionales, primus autem sit cubus, & quartus cubus erit.

A B C D
8 12 18 27

Theorema 22. propositio 24.

Si duo numeri rationem habeant inter se, quam quadratus numerus ad quadratum numerum, primus autem sit quadratus erit, & secundus quadratus erit.

A B C D
4 6 9 16 34 36

Theorema 23. propositio 25.

Si numeri duo rationem inter se habeant, quam cubus numerus ad cubum numerum, primus autem cubus sit, & secundus cubus erit.

A E F B C D
8 12 18 27 64 95 140 216

Theo-

Theorema 24. Propositio 26.

Similes plani numeri rationem inter se habent, quam quadratus numerus ad quadratum nemurum.

A	C	B	D	E	F
18	24	32	9	12	16

Theorema 25. Propositio 27.

Similes solidi numeri rationem habent inter se, quam cubus numerus ad cubum numerum.

A	C	D	B	E	F	G	H
16	24	36	54	8	12	18	27

ELEMENTI VIII. FINIS.

EV-

EVCLIDIS ELEMENTVM NONVM.

Theorema 1. Propositio 1.

Si duo similes plani numeri mutuò sese multiplantes quédam procreent, productus quadratus.

A	E	B	D		C
4	6	9	16	24	36

Theorema 2. Propositio 2.

Si duo numeri mutuò sese multiplicantes quadratum faciant, illi similes sunt plani.

A		B	D		C
4	6		9	18	36

Theorema 3. Propositio 3.

Si cubus numerus seipsum multiplicans procreet aliqué, productus cubus erit.

D	D	A			B
3	4	8	16	32	64

Theorema 4. Propoſitio 4.

Si cubus numerus cubũ numerum multiplicans quendam procreet, procreatus cubus erit.

A	B	D	G
8	27	64	216

Theorema 5. Propoſitio 5.

Si cubus numerus numerum quendam multiplicans cubum procreet, & multiplicatus cubus erit.

A	B	C	D	
27	64	729	17	18

Theorema 6. Propoſitio 6.

Si numerus ſeipſum multiplicans cubum procreet, & ipſe cubus erit.

A	B	C
27	729	19683

Theorema 7. Propoſitio 7.

Si compoſitus numerus quendam numerũ multiplicans quempiam procreet, productus ſolidus erit.

A	B	C	D	E
6	8	48	2	8

Theorema 8. Propoſitio 8.

Si ab vnitate quotlibet numeri deinceps proportionales ſint, tertius ab vnitate quadratus eſt, & vnum intermittentes omnes: quartus autem cubus, & duobus intermiſsis omnes

nes: ſeptimus verò cubus ſimul & æquadratus, & quinque intermiſſis omnes

	Vnitas	A	B	C	D	E	F
		3	9	27	81	243	729

Theorema 9. Propoſitio 9.

Si ab vnitate ſint quotcunque numeri: deinceps proportionales, ſit autem quadratus is, qui vnitatẽ ſequitur, & reliqui omnes quadrati erũt. Quòd ſi qui vnitatem ſequitur cubus ſit, & reliqui omnes cubi erunt.

quadrati		cubi
531441	F	732969
59040	E	531441
6561	D	59049
723	C	6561
81	B	729
9	A	81
	o	
	vnitas.	

Theorema 10. Propoſitio 10.

Si ab vnitate numeri quotcunque proportionales ſint, non ſit autem quadratus is qui vnitatem ſequitur, neq; alius vllus quadratus erit, demptis tertio ab vnitate ac om-

o	A	B	C	D	E	F
Vnitas.	3	9	36	81	243	729

 nibus

nibus vnum intermittentibus. Quòd ſi qui vnitatem ſequitur, cubus non ſit, neque alius vllus cubus erit, demptis quarto ab vnitate ac omnibus duos intermittentibus.

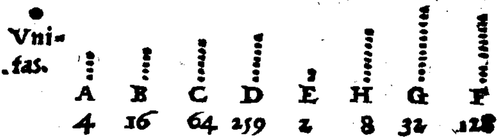
Theorema 11. Propoſitio 11.

Si ſib vnitate numeri quotlibet deinceps proportionales ſint, minor maiorem metitur per quempiam eorum qui in proportionalibus ſunt numeris.

:	:	:	:	:
A	B	C	D	E
1	2	4	8	16

Theorema 12. Propoſitio 12.

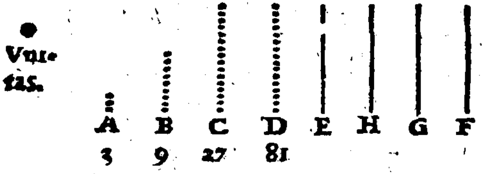
Si ab vnitate quotlibet numeri ſint proportionales, quot primorum numerorum vltimũ metiuntur, totidem & eum qui vnitati proximus eſt, metientur.



Theorema 13. Propoſitio 13.

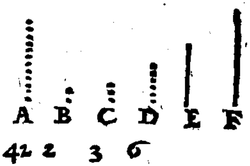
Si ab vnitate ſint quotcunque numeri deinceps proportionales, primus autem ſit qui vnitatẽ ſequitur, maximũ nullus alius metietur,

tietur, ijs exceptis qui in proportionalibus ſunt numeris.



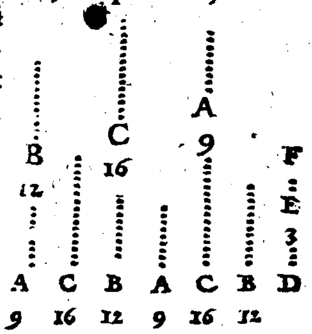
Theorema 14. propoſitio 14.

Si minimum numerum primi aliquot numeri metiātur, nullus alius numerus primus illum metietur, ijs exceptis qui primo metiuntur.



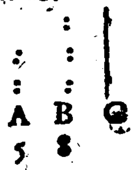
Theorema 15. propoſitio 15.

Si tres numeri deinceps proportionales ſint minimi, eadem cum ipſis habētium rationem, duo quilibet compoſiti ad tertium primi erunt.



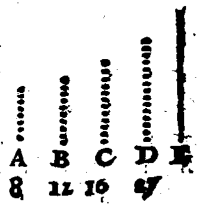
Theorema 16. Propoſitio 16.

Si duo numeri ſint inter ſe primi, non ſe habebit quẽadmodum primus ad ſecũdum, ita ſecundus ad quẽpiam alium.



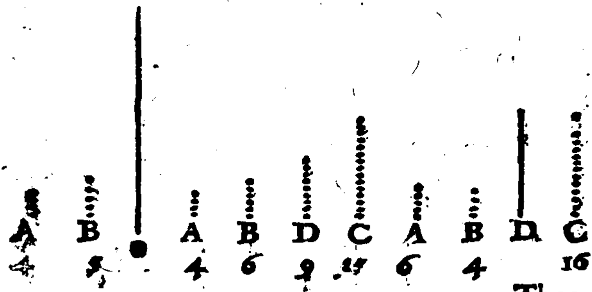
Theorema 17. Propoſitio 17.

Si ſint quotlibet numeri deinceps proportionales, quorum extremi ſint inter ſe primi, non erit quẽadmodum primus ad ſecundum, ita vltimus ad quempiam alium.



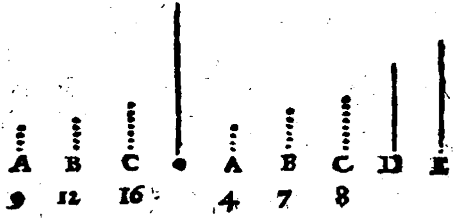
Theorema 18. Propoſitio 18.

Duobus numeris datis, cõſiderare noſcitur tertius illi inueniri proportionalis.



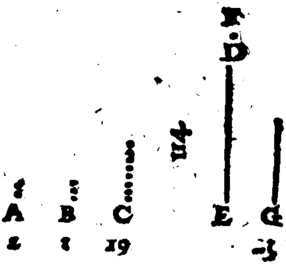
Theorema 19. Propositio 19.

Tribus numeris datis, considerare posit ne quartus illis reperiri proportionalis.



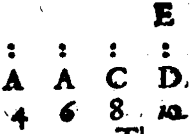
Theorema 20. Propositio 20.

Primi numeri plures sunt quacũque proposita multitudine primorum numerorum.



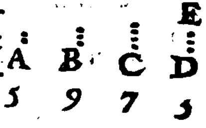
Theorema 21. Propositio 21.

Si pares numeri quotlibet compositi sint, totus est par.



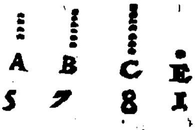
Theorema 22. Propoſitio 22.

Si impares numeri quotlibet compoſiti ſint, ſit autem par illorum multitudo, totus par erit.



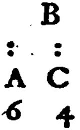
Theorema 23. Propſitio 23.

Si impares numeri quotcunq; compoſiti ſint, ſit autem impar illorũ multitudo, & totus impar erit.



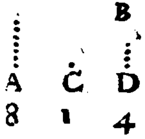
Theorema 24. Propoſitio 24.

Si de pari numero par detractus ſit, & reliquus par erit.



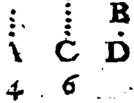
Theorema 25. Propoſitio 25.

Si de pari numerò impar detractus ſit, & reliquus impar erit.



Theorema 26. Propoſitio 26.

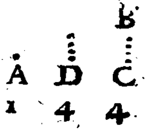
Si de impari numero impar detractus ſit, & reliquus par erit,



Theore-

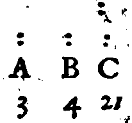
Theorema 27. Propositio 27.

Si ab impari numero par ablatus sit, reliquus impar erit.



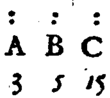
Theorema 28. Propositio 28.

Si impar numerus parem multiplicãs, procreet quempiam, procreatus par erit.



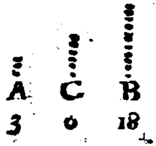
Theorema 29. Propositio 29.

Si impar numerus imparẽ numerum multiplicans quendam procreet, procreatus impar erit.



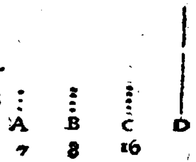
Theorema 30. Propositio 30.

Si impar numerus parem numerum metiatur, & illius dimidium metietur.

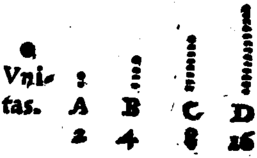


Theorema 31. Propositio 31.

Si impar numerus ad numerum quempiã primus sit & ad alius duplum primus erit.



Theorema 32. Propoſitio 32.



Numerorum qui à binario dupli ſunt, vnuſquiſq; pariter par eſt tantum.

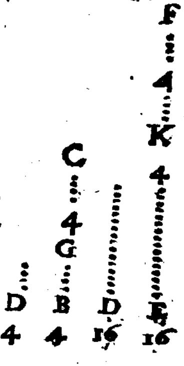
Theorema 33. Propoſitio 33.

Si numerus dimidiũ impar habeat, pariter impar eſt tantùm.

Theorema 34. Propoſitio 34.

Si par numerus nec ſit dupli à binario, nec dimidium impar habeat, pariter par eum, & pariter impar.

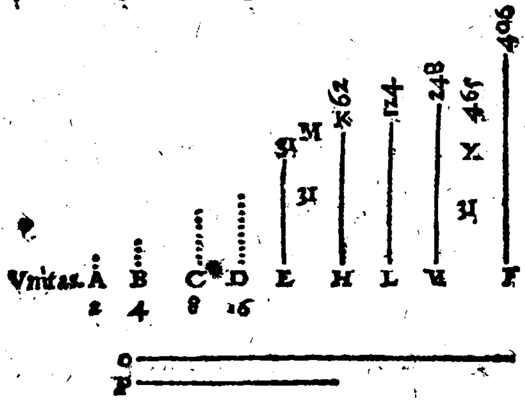
Theorema 35. Propoſitio 35.



Si ſint, quotlibet numeri deinceps proportionales, detrahantur autem de ſecũdo & vltimo æquales ipſi primo, erit quemadmodũ ſecundi exceſſus ad primũ, ita vltimi exceſſus ad omnes qui vltimũ antecedunt.

Theorema 16. Propositio 16.

Si ab vnitate numeri quotlibet deinceps expositi sunt in duplici proportione quoad totus compositus primus factus sit, isque totus in vltimum multiplicatus quempiam procreet, procreatus perfectus erit.



ELEMENTI IX. FINIS.

EVCLIDIS ELEMENTVM DECIMVM.

DEFINITIONES.

1.

Commensurabiles magnitudines dicuntur, illæ, quas eadem mensura metietur.

2.

Incommensurabiles verò magnitudines dicuntur, hæ quarum nullam mensuram cómunem contingit reperiri.

3.

Lineæ rectæ potentia cómensurabiles sunt, quarum quadrata vna eadem superficies siue area metitur.

4.

Incommensurabiles verò lineæ sunt, quarũ quadrata, quæ metiatur area communis, reperiri nulla potest.

5.

Hęc cùm ita sint, ostendi potest quòd quantacunque linea recta nobis proponatur, existũt etiam aliæ lineæ innumerabiles eidem commensurabiles, aliæ item incommensurabiles, hæ quidem longitudine & potentia:

tia, illa verò potentia tantùm. Vocetur igitur linea recta, quantacunq; proponatur, id est rationalis.

6.

Linea quoq; illi ῥητῇ commensurabiles siue longitudine & potentia, siue potentia tantùm, vocentur & ipsæ ῥηταί, id est rationales.

7.

Quæ verò lineæ sunt incommensurabiles illi τῇ ῥητῇ, id est primo loco rationali, vocétur ἄλογοι, id est, irrationales.

8.

Et quadratum quod linea proposita describitur, quam ῥητήν vocari volumus, vocentur ῥητόν.

9.

Et quæ sunt huic commensurabilia, vocentur ῥητά.

10.

Quę verò sunt illi quadrato ῥυτῷ scilicet incommensurabilia, vocentur αλογα, id est, surda.

11.

Et lineæ quæ illa, incommensurabilia describunt, vocentur ἄλογοι. Et quidam se illa incommensurabilia fuerint quadrata, ipsa eorum latera vocabũtur ἄλογοι lineæ, quòd si quadrata quidem non fuerint, verum aliæ quæpiam superficies siue figuræ rectilineæ, tunc

tunc vero lineæ illæ quæ describunt quadrata æqualia figuris rectilineis, vocentur ἄλογοι.

Theorema 1. Propositio 1.

Duabus magnitudinibus inæqualibus propositis, si de maiore detrahatur plus dimidio, & rursus de residuo iterum detrahatur plus dimidio, idque semper fiat: relinquetur quædam magnitudo minor altera minore ex duabus propositis.

Theorema 2. Propositio 2.

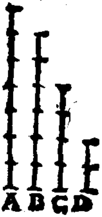
Duabus magnitudinibus propositis inęqualibus, si detrahatur semper minor de maiore alterna quadam detractione neque residuum vnquam metiatur id quod ante se metiebatur, incommensurabiles sunt illæ magnitudines.

Problema 1. Propositio 3.

Duabus magnitudinibus commensurabilibus datis, maximam ipsarum communem mensuram reperire.

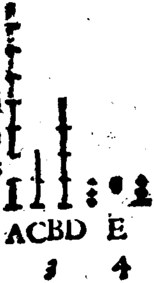
Proble-

Problema 2. Propositio 4.



Tribus magnitudinibus commensurabilibus datis, maximam ipsarũ communem mensuram reperire.

Theorema 3 Propositio 5.



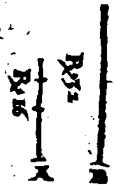
Commensurabiles magnitudines inter se proportionẽ eam habent, quam habet numerus ad numerum.

Theorema 4. Propositio 6.

Si duæ magnitudines proportionem eam habent inter se quam numerus ad numerum, commensurabiles sunt illæ magnitudines.

Theore-

Theorema 5. Propositio 7.



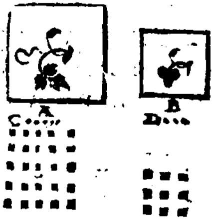
Incommensurabiles magnitudines inter se proportionem non habent, quam numerus ad numerum.

Theorema 6. Propositio 8.

Si duæ magnitudines inter se proportionem nó habét quam numerus ad numerum, incommensurabiles illæ sunt magnitudines.

Theorema 7. Propositio 9.

Quadrata, quæ describuntur à rectis lineis longitudine cómensurabilibus, inter se proportionem habét quam numerus quadratus ad alium numerum quadratú. Et quadrata habentia proportioné inter se quam quadratus numerus ad numerum quadratum, habent quoque latera longitudine commensurabilia. Quadrata verò

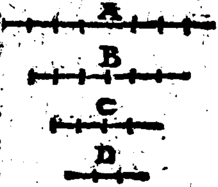


quæ

quæ describuntur à lineis longitudine incōmensurabilibus proportionē non habent inter se, quam quadratus numerus ad numerum alium quadratum. Et quadrata non habentia proportionem inter se quam numerus quadratus ad numerum quadratum, neque latera habebunt longitudine commensurabilia.

Theorema 8. Propositio 10.

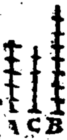
Si quatuor magnitudines fuerint proportionales, prima verò secundæ fuerit cōmensurabilis, tertia quoque quartæ commēsurabilis erit, quòd si prima secūdæ fuerit incommensurabilis, tertia quoque quartæ incommensurabilis erit.



Problema 3. Propositio 11.

Propositæ lineæ rectæ (quam ῥητὴν vocari diximus) reperire duas lineas rectas incommensurabiles, hanc quidem longitudine tantùm, illam verò non longitudine tantùm, sed etiam potentia incommensurabilem.

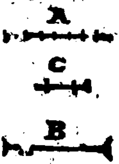
Theorema 6. Propositio 12.



Magnitudines quæ eidem magnitudini sunt commẽsurabiles, inter se quoque sunt commensurabiles.

6D.......4F..
4E.....8G..
3H...
2K..
4L...

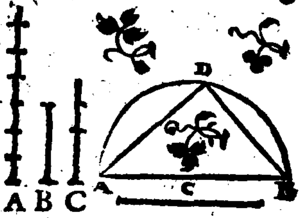
Theorema 10. Propositio 13.



Si ex duabus magnitudinibus hæc quidem commensurabilis sit tertiæ magnitudini, illa verò eidem incommensurabilis, incommẽsurabiles sunt illæ duæ magnitudines.

Theorema 11. Propositio 14.

Si duarum magnitudinum commensurabilium altera fuerit incommensurabilis magnitudine alteri cupiam tertię, reliqua quoq; magnitudo eidem tertię incommensurabilis erit.



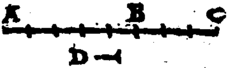
Theorema 12. Propositio 15.

Si quatuor rectæ proportionales fuerint, possit

posſit autem prima pluſquam ſecunda tãtò quantum eſt quadratum lineæ ſibi commẽſurabilis longitudine: tertia quoq; potuerit pluſquam quarta tanto quantum eſt quadratum lineæ ſibi commenſurabilis longitudini. Quòd ſi prima poſsit pluſquam ſecunda quadrato lineę ſibi longitudine incommenſurabilis: tertia quoque poterit pluſquam quartã quadrato lineæ ſibi incõmenſurabilis lõgitudine.

Theorema 13. Propoſitio 16.

Si duæ magnitudines commenſurabiles componantur, tota magnitudo compoſita ſingulis partibus commenſurabilis erit, quòd ſi tota magnitudo compoſita alterutri parti commenſurabilis fuerit, illæ duæ quoque partes cõmenſurabiles erunt.



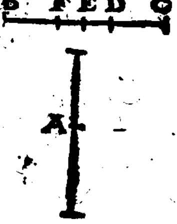
Theorema 14. Propoſitio 17.

Si duæ magnitudines incommenſurabiles componantur, ipſa quoque tota magnitudo ſingulis partibus componentibus incommenſurabilis erit. Quòd ſi tota alteri parti incommenſurabilis fuerit, illæ quoque primæ magnitudines inter ſe incommenſurabiles erunt.

K 2 Theo-

Theorema 15. Propositio 18.

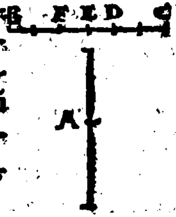
Si fuerint duæ rectæ lineæ inæquales, & quartæ parti quadrati quod describitur à minore æquale parallelogrammum applicetur secundum maiorem, ex qua maiore tantum excurrat extra latus parallelogrammi, quantum est alterum latus ipsius parallelogrammi: si præterea parallelogrammũ sui applicatione diuidat lineam illã in partes inter se commensurabiles longitudine, illa maior linea tanto plus potest quàm minor, quantum est quadratum lineæ sibi cõmensurabilis longitudine. Quòd si maior plus posit quam minor, tanto quantum est quadratum lineæ sibi commensurabilis lõgitudine, & præterea quartæ parti quadrati lineę minoris ęquale parallelogrammum applicetur secundum maiorem, ex qua maiore tantum excurrat extra latus parallelogrammi, quã-tum est alterum latus ipsius parallelogrammi parallelogrammum sui applicatione diuidit maiorẽ in partes inter se longitudine commensurabiles.



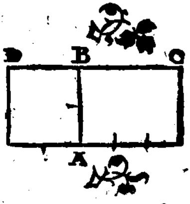
Theorema 16. Propositio 19.

Si fuerint duæ rectæ inæquales, quartæ autem parti quadrati lineæ minoris æquale paral-

parallelogrammorum secundum lineam maiorem applicetur, ex qua linea tantum excurrat extra latus parallelogrammi, quãtum est alterum latus eiusdem parallelogrammi: si parallelogrammum preterea sui applicatione diuidat lineam in partes inter se longitudine incommensurabiles, maior illa linea tantò plus potest quàm minor quantum est quadratum lineæ sibi minori commensurabiles longitudine. Quod si maior linea tantò plus posit quàm minor, quantum est quadratum lineæ incommensurabilis sibi longitudine & præterea quartæ parti quadrati lineæ minoris æquale parallelogrammum applicetur secũdum maiorem ex qua tantum excurrat extra latus parallelogrãmi, quantum est alterum latus ipsius: parallelogrammũ sui applicatione diuidit maiorem in partes inter se incõmensurabiles longitudine.



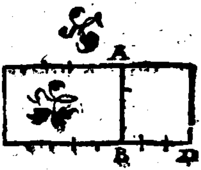
Theorema 17. Propositio 20.



Superficies rectanguli contenta ex lineis rectas rationalibus longitudine commensurabilibus secundum vnum aliquem modum ex antedictis, rationalis est.

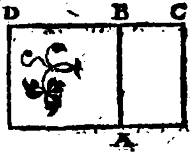
Theorema 18. Propositio 21.

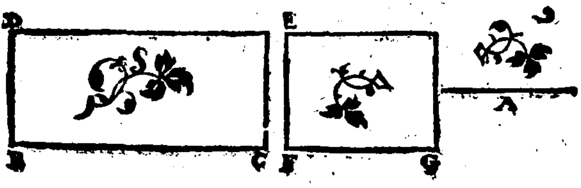
Si rationale secúdum lineam rationalem applicetur, habebit alterum latus lineam rationalem & cómensurabilem longitudine lineæ cui rationale parallelogrammú applicatur.



Theorema 39. Propositio 56.

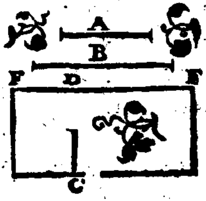
Superficies rectangula contenta duabus lineis rectis rationalibus potentia tantum cómensurabilibus, irrationalis est. Linea autem quæ illam superficiem potest, irrationalis & ipsa est: vocetur verò medialis dum





line-

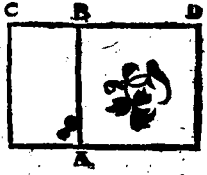
lineam rationalem, alterum latus est linea rationalis, & incommensurabilis longitudine lineæ secundum quam applicatur.



Theorema 21. Propositio 24.

Linea recta mediali commensurabilis, est ipsa quoque medialis.

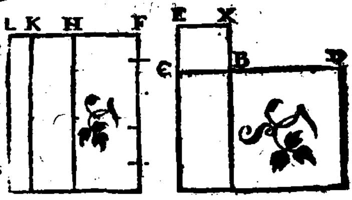
Theorema 22. Propositio 25.



Parallelogrammum rectangulum contentū ex lineis medialibus longitudine commēsurabilibus, mediale est.

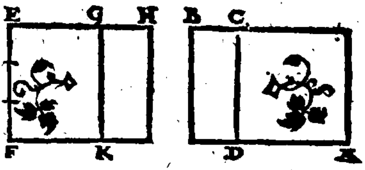
Theorema 33. Propositio 26.

Parallelogrammum rectangulum comprehensum duabus lineis medialibus potentia tantū cōmensurabilibus, vel rationale est, vel mediale.



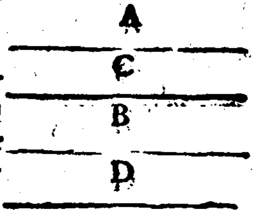
Theorema 24. Propositio 27.

Mediale nõ est maius quàm mediale superficie rationali.



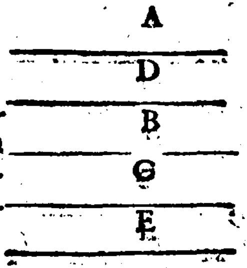
Problema 4. Propositio 28.

Mediales lineas inuenire potentia tantùm commensurabiles rationale comprehendentes.



Problema 5. Propositio 29.

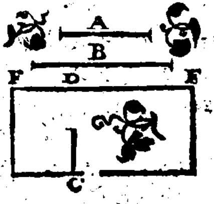
Mediales lineas inuenire potentia tantùm commensurabiles mediale comprehendentes.



Pro-

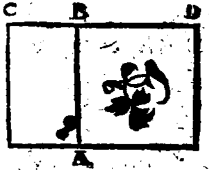
lineam rationalem, alterum latus est linea rationalis, & incommensurabilis longitudine lineæ secundum quam applicatur.

Theorema 2, Propositio 24.



Linea recta mediali commẽsurabilis, est ipsa quoque medialis.

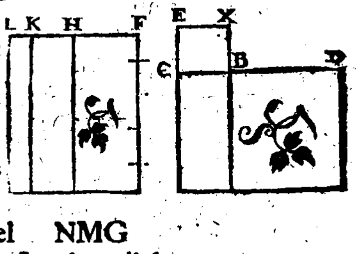
Theorema 22. Propositio 25.



Parallelogrammum rectangulum contentũ ex lineis medialibus longitudine commẽsurabilibus, mediale est.

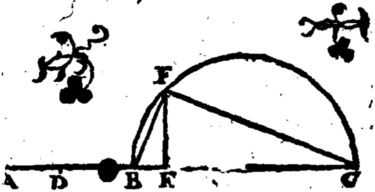
Theorema 33. Propositio 26.

Parallelogrammum rectangulum comprehensum duabus lineis medialibus potentia tantũ cõmensurabilibus, vel rationale est, vel mediale.



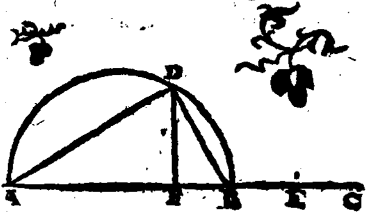
Problema 9. Propoſitio 33.

Reperire duas rectas potentia incommenſurabiles, quarum quadrata ſimul addita faciant ſuperficiē rationalē, parallelogrāmū verò ex ipſis contentum ſit mediale.



Problema 10. Propoſitio 34.

Reperire lineas duas rectas potentia incommenſurabiles conficientes compoſitum ex ipſarum quadratis mediale, parallelogrammū verò ex ipſis contentum rationale.



Problema 11. Propoſitio 35.

Reperire duas lineas rectas potentia inconmenſurabiles, conficientes id quod ex ipſarum quadratis componitur meale, ſimulque

que parallelogrammum ex ipsis contẽtum, mediale, quod præterea parallelogrammũ sit incõmensurabile composito ex quadratis ipsarum.

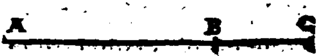
PRINCIPIVM SENARIOrum per compositionem.

Theorema 25. Propositio 36.

Si duæ rationales potentia tantùm commẽsurabilis. Vocetur autem Binomium.

Theorema 26. Propositio 37.

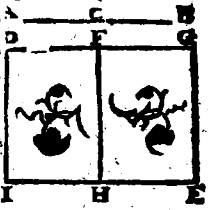
Si duę mediales potentia tantùm commensurabiles rationale continentes componantur, tota linea est irrationalis, vocetur autem Bimediale prius.



Theo-

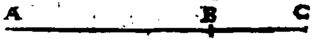
Theorema 27. Propositio 38.

Si duæ mediales potẽtia tantùm commẽsurabiles mediale continentes cõponãtur, tota linea eſt irrationalis: vocetur autẽ Bimediale ſecundum.



Theorema 28. Propoſitio 39.

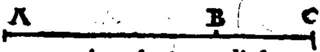
Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex quadratis ipſarum rationale parallelogrammum verò ex ipſis contentum mediale, tota linea recta



eſt irrationalis. Vocetur autem linea maior.

Theorema 29. Propoſitio 40.

Si duæ rectæ potentia incommenſurabiles componantur, conficiẽtes compoſitum ex ipſarum quadratis mediale, id vero ꝙ fit ex ipſis, rationale, tota linea eſt irrationales. Vocetur autem



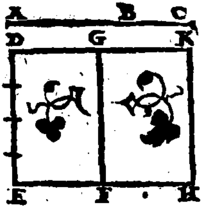
potens rationale & mediale.

Theorema 30. Propoſitio 41.

Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex quadratis ipſarum mediale, & quod conti-

netur

netur ex ipsis, mediale, & præterea incommēsurabile composito ex quadratis ipsarum tota linea est irrationalis. Vocetur autem potens duo medialia.



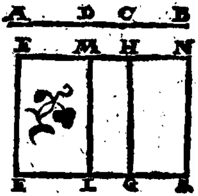
Theorema 31. Propositio 42.

Binomium in vnico tantùm puncto diuiditur in sua nomina, id est in lineas ex quibus componitur.

Theorema 32. Propositio 43.

Bimediale prius in vnico tantum puncto diuiditur in sua nomina.

Problema 33. Propositio 44.



Bimediale secundum in vnico tantū puncto diuiditur in sua nomina.

Problema 34. Propositio 45.

Linea maior in vnico tantùm in pūcto diuiditur in sua nomina.

Theo-

Theorema 35. Propositio 46.

Linea potens rationale & mediale in vnico tantùm puncto diuiditur in sua nomina.

Theorema 36. Propositio 47.

Linea potens duo medialia in vnico tantum puncto diuiditur in sua nomina.

DEFINITIONES secundæ.

Proposita linea rationali, & binomio diuiso in sua nomine; cuius binomij maius nomen, id est; maior portio possit plusquàm minus nomen quadrato lineæ sibi, maiori inquam nomini, commensurabilis longitudine.

1.

Si quidem maius nomen fuerit commensurabile longitudine propositę liniæ rationali, vocetur tota linea Binomum primum.

2.

Si verò minus nomen, id est minor portio binomij, fuerit commensurabile longitudine

dine propositæ lineæ rationali, vocetur tota linea Binomium secundum.

3.

Si verò neutrum nomen fuerit commensurabile longitudine propositæ lineæ rationali, vocetur Binomium tertium.

Rursus si maius nomē possit plusquam minus nomen quadrato lineæ sibi incommensurabiles longitudine.

4.

Si quidem maius nomen est commensurabile longitudine propositæ lineæ rationali, vocetur tota linea Binomium quartum.

5.

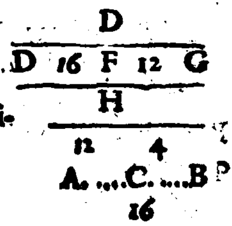
Si verò minus nomen fuerit commensurabile longitudine lineæ raaionali, vocetur Binomium quintum.

6.

Si verò neutrum nomen fuerit longitudine commensurabile lineæ rationali, vocetur illa Binomium sextum.

Problema 12. Propositio 48.

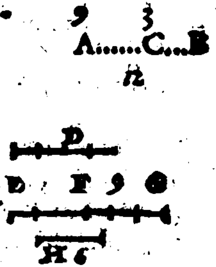
Repetire Binomium primum.



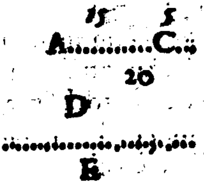
Pro-

Problema 13. Propositio 49.

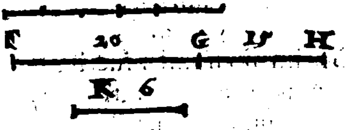
Repetire Binonium secundum.



Problema 14. Propositio 50.

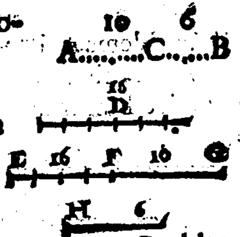


Reperire Binomium tertiũ.



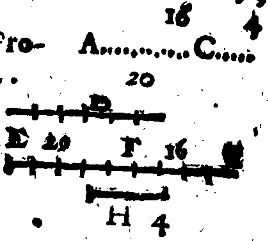
Problema 15. Propositio 51.

Repetire Binomium quartum.



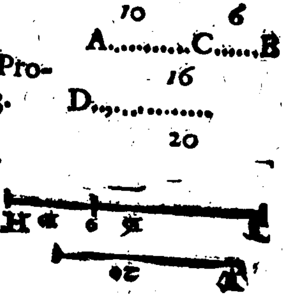
Proble-

Problema 16. Propositio 52.



Reperire Binomium quintum.

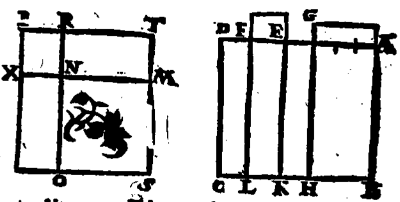
Problema 17. Propositio 53.



Reperire Binonitum sextum.

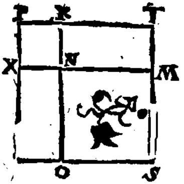
Theorema 37. Propositio 54.

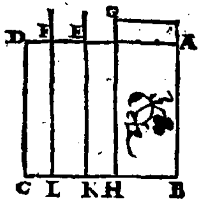
Si superficies contenta fuerit ex rationali & Binomio primo, linea quæ illã superficiẽ potest, est irrationalis, quæ Binomium vocatur.



Theorema 38. Propositio 55

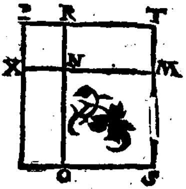
Si superficies contenta fuerit ex linea ratio-nali & Binomio secundo, linea potens illam superficié est irrationalis quæ Bimediale primum vocatur.

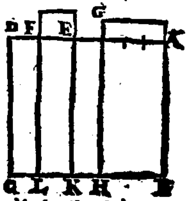




Theorema 39. Propositio 56.

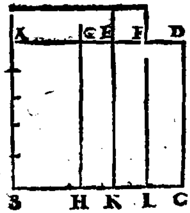
Si superficies contineatur ex rationali & Binomio tertio, linea qã illam superficié potest, est irrationalis quam dicitur Bimediale secúdum.

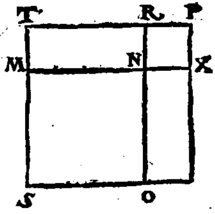




Theorema 40. Propositio 57.

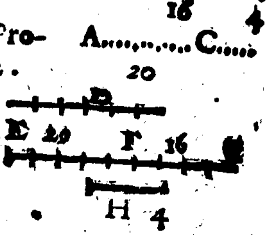
Si superficies contineatur ex ratio-





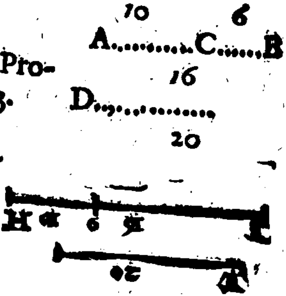
nali

Problema 16. Propositio 52.



Reperire Binomium quintum.

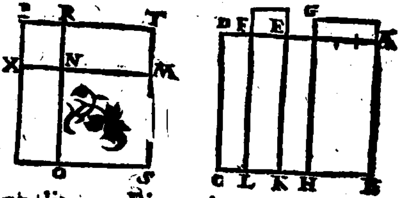
Problema 17. Propositio 53.



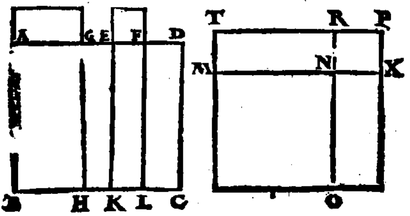
Reperire Binomium sextum.

Theorema 37. Propositio 54.

Si superficies contenta fuerit ex rationali & Binomio primo, linea quæ illã superficiē potest, est irrationalis, quæ Binomium vocatur.

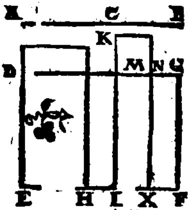


potens tuo mediala.



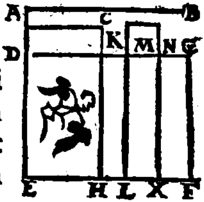
Theorema 43. Propositio 60.

Quadratum Binomij secundum lineam rationalem applicatum, facit alterum latus Binomium primum.



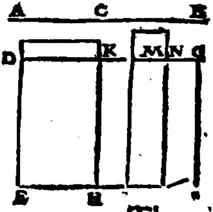
Theorema 44. Propositio 61.

Quadratũ Bimedialis primi secundum rationalem lineam applicatum, facit alterum latus Binomium secundum.



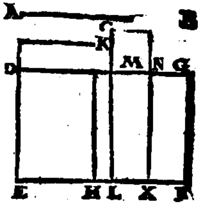
Theorema 45. Propositio 62.

Quadratũ Bimedialis secundi secundũ rationalẽ applicatum, facit alterũ latus Binomium tertiũ.



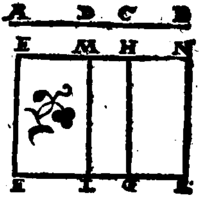
Theo-

Theorema 46. Propoſitio 63.



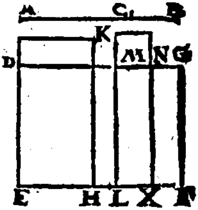
Quadratum lineæ maioris ſecundum lineam rationalem applicatum, facit alterum latus Binomium quartum.

Theorema 47. Propoſitio 64.



Quadratum lineæ potẽtis rationale & mediale ſecundum rationalẽ applicatum, facit alterũ latus Binomium quintũ.

Theorema 48. Propoſitio 65.

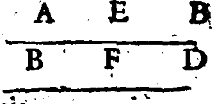


Quadratum lineæ potentis duo medialia ſecundũ rationalem applicatum, facit alterum latus Binomium ſextum.

Theorema 49. Propoſitio 66.

Linea longitudine commenſurabilis Binomio eſt & ipſa Binomium eiuſdem ordinis.

Theorema 50. Propositio 67.

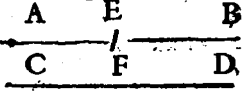


Linea longitudine commensurabilis alteri bimedialum, est & ipsa bimediale etiam eiusdem ordinis.

Theorema 51. Propositio 68.

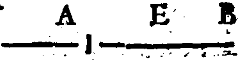
Linea cõmensurabilis lineæ maiori, est & ipsa maior.

Theorema 52 Propositio 69.



Linea commensurabilis liniæ potenti rationale & mediale, est & ipsa linea potens rationale & mediale.

Theorema 53. Propositio 70.



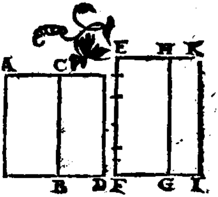
Linea commensurabilis lineæ potenti duo medialia, est & ipsa linea potens duo medialia.

Theorema 54. Propositio 71.

Si duæ superficies rationalis & medialis simul componantur, linea quæ totã superficiem

ciem compositam potest, est vna ex quatuor irrationalibus, vna ex quę dicitur Binomium, vel bimediale primum, vel linea maior, vel linea potens rationale & mediale.

Theorema 55. Propositio 72.



Si duæ superficies mediales incommensurabiles simul componantur fiūt reliquæ duæ lineæ irrationales, vel bimediale secundum, vel linea potens duo medialia.

SCHOLIVM.

Binomium & cætera consequentes lineæ irrationales, neque sunt eadem cum linea mediali, neq; ipsa inter se.

Nam quadratum lineæ medialis applicatum secundum lineam rationalem facit alterum latus lineam rationalem, & longitudine incommensurabilem lineæ secundum quam applicatur, hoc est lineæ rationali, per 23.

Quadratum verò Binomij secundum rationalem applicatum, facit alterum latus Binomium primū per 60.

Quadratum verò Bimedialis primi secundum rationalem applicatum, facit alterum latus Binomium secundum, per 61.

Quadratum verò Bimedialis secundi secundum rationalem applicatum, facit alterum latus Binomium tertium, per 62.

Quadratum verò lineæ maioris secundum rationalem applicatum facit alterum latus Binomium quartum, per 63.

Quadratum verò lineæ potentis rationale & mediale secundum rationalē applicatum facit alterum latus Binomium quintum, per 64.

Quadratum verò lineæ potentis duo medialia secundum rationalem applicatum, facit alterum latus Binomium sextum, per 65.

Cùm igitur dicta latera, quæ latitudines vocantur, differant & à prima latitudine, quoniam est rationalis, cùm inter se quoque differant, eo quia sunt Binomia diuersorum ordinum: manifestum est ipsas lineas irrationales, differentes esse inter se.

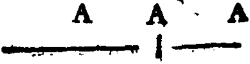
SECVN.

SECVNDVS ORDO ALterius sermonis, qui est de detractione.

Principium senariorum per detractionem.

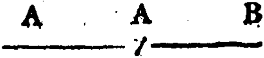
Theorema 57. Propositio 74.

Si de linea rationali detrahatur rationalis potentia tantùm commensurabilis ipsi toti, residua est irrationalis, vocetur autem Residuum.



Theorema 56. Propopositio 37.

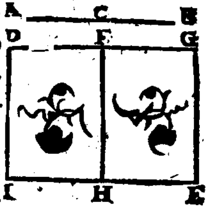
Si de linea mediali detrahatur medialis potentia tantum commensurabilis toti lineæ, quę verò detracta est cum tota contineat superficiem rationalem, residua est irrationalis. Vocetur autem Residuum mediale primum.



Theorema 58. Propositio 75.

Si de linea mediali detrahatur medialis

tentia tantùm commensurabilis toti, quæ verò detracta est, cùm tota cõtineat superficié medialem, reliqua est irrationalis. Vocetur autem Residuum mediale secundum.



Theorema 57. Propositio 76.

Si de linea recta detrahatur recta potentia incommensurabilis toti, compositum autem ex quadratis totius lineæ & lineæ detracta sit rationale, parallelogrammum verò ex eisdem contentum sit mediale, reliqua linea erit irrationalis. Vocetur autem linea minor.

Theorema 58 Propositio 77.

Si de linea recta detrahatur recta potentia

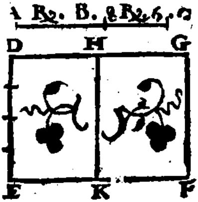
tia incommenſurabilis toti lineæ compoſitum autem ex quadratis totius & lineæ detractæ ſit mediale, parallelogrammum verò bis ex eiſdem contentum ſit rationale, reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie rationali totam ſuperficiem medialem.

Theorema 59. Propoſitio 78.

Si de linea recta detrahatur recta potentia incommenſurabilis toti lineæ, compoſitum autem ex quadratis totius & lineæ detractæ ſit mediale, parallelogrammum verò bis ex ijſdem ſit etiam mediale: præterea ſint quadrata ipſarum incommenſurabilia parallelogrammo bis ex ijſdem contento, reliqua linea eſt irrationalis.

Vocetur autem linea faciens cum ſuperficie

ficie mediali totam superficiem medialem.

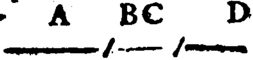


Theorema 60. Propositio 79.

Residuo vnica tantùm linea recta cõiungitur rationalis potentia tantùm cõmensurabilis toti lineæ rationalem.

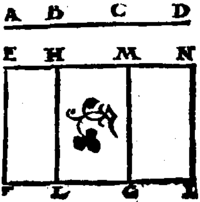
Theorema 61. Propositio 80.

Residuo mediali primo vnica tantùm linea coniungitur medialis potentia tantùm commensurabilis toti, ipsa cum tota continẽs.



Theorema 62. Propositio 81.

Residuo mediali secundo vnica tantum coniungitur medialis, potentia tantùm commẽsurabilis toti ipsa cum tota continens mediale.



Theorema 63. Propositio 82.

Lineę minori vnica tantùm recta coniungitur

tur potentia incommensurabilis toti, faciens cum tota compositum ex quadratis ipsarum rationale, id verò parallelogrammum, quod bis ex ipsis sit, mediale.

A B C D

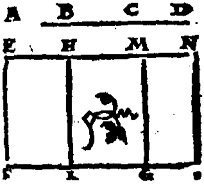
Theorema 64. Propositio 83.

Lineæ facienti cum superficie rationali totam superficiem medialem, vnica tantùm coniungitur linea recta potentia incommensurabilis toti, faciens autem cum tota compositum ex quadratis ipsarú, mediale, id verò quod fit bis ex ipsis, rationale.

A B C D

Theorema 65. Propositio 84.

Lineæ cum mediali superficie facienti totam superficiem medialem, vnica tantum conıúgitur linea potétia toti incomměsurabilis, faciens cum tota compositú ex quadratis ipsarú mediale, id verò quod fit bis ex ipsis etiam mediale, & præterea faciens compositum ex quadratis ipsarum incommensurabile ei quod fit bis ex ipsis.



DE-

DEFINITIONES TERTIAE.

Propoſita linea rationali & reſiduo.

1.

Si quidem tota, nempe compoſita ex ipſo reſiduo & linea illi cõiuncta, plus poteſt quàm coniuncta, quadrato lineæ ſibi cõmenſurabilis longitudine, fueritq́; tota longitudine commẽſurabilis lineæ propoſitæ rationali, reſiduum ipſum vocetur Reſiduum primum.

2.

Si verò coniuncta fuerit longitudine commenſurabilis rationali, ipſa autem tota plus poſsit quam coniuncta, quadrato lineæ, ſibi longitudine commenſurabilis, reſiduum vocetur Reſiduum ſecundum.

3.

Si verò neutra linearum fuerit longitudine commenſurabilis rationali poſsit autem ipſa tota pluſquam coniuncta, quadrato lineæ ſibi longitudine commenſurabilis, vocetur Reſiduum tertium.

Rurſus ſi tota poſsit plus quàm coniuncta, quadrato lineę ſibi longitudine incommenſueabilis.

4.

Et quidem ſi tota fuerit longitudine commenmēſurabilis ipſi rationali, vocetur Reſiduū quartum.

5.

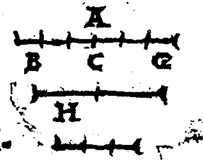
Si verò coniuncta fuerit longitudine commenſurabilis rationali, & tota plus poſſit quàm coniuncta, quadrato lineæ ſibi longitudine incommēſurabilis, vocetur Reſiduum quintum.

6.

Si verò neutra linearum fuerit commenſurabilis longitudine ipſi rationali, fueritque tota potentior quam coniūcta, quadrato lineæ ſibi longitudine incommenſurabilis, vocetur Reſiduum ſextum.

Problema 18. Propoſitio 85.

Reperire primum Reſiduum.



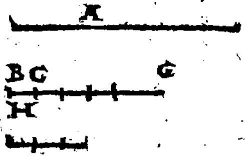
16

D.........F.....E

9 7

Problema 19. Propoſitio 86.

Reperire ſecundum Reſiduum.



D.........F.....E

27. 9

Proble-

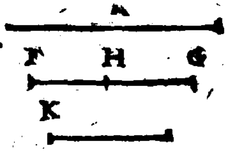
Problema 20. Propositio 87.

E.........

12

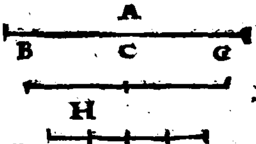
B.......E....C

9 7



Reperire tertium Residuum.

Probl. 21. Propositio 88.

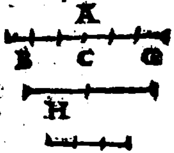


Reperire quartum Residuum

D..............F...E

4

Problema 22. Propositio 89.

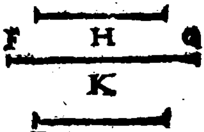


Reperire quintum Residuum.

D..........F....E

25 7

Problema 23 Propositio 90.



Reperire sextum Residuum.

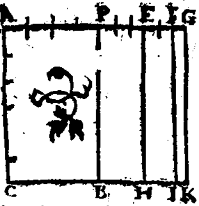
E......... 13

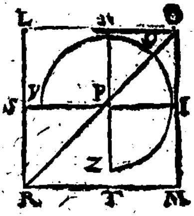
B......D.....C

Theo-

Theorema 66. Propositio 91.

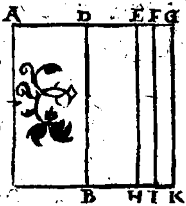
Si superficies contineatur ex linea rationali & resi- duo primo linea, quę il- lam su- perfici- em potest, est residuum.

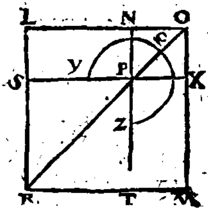




Theorema 67. Propositio 92.

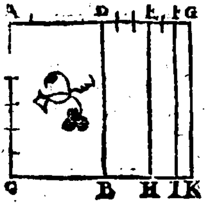
Si superficies contineatur ex linea rationali & resi- duo secun- do li- nea, quã illam superfi ciẽ potest, est residuum mediale primum.

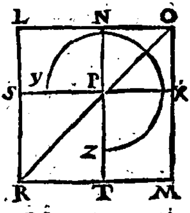




Theorema 68. Propositio 93.

Si super ficies cõ tinea- tur ex linea ra tionali & resi- duo ter-

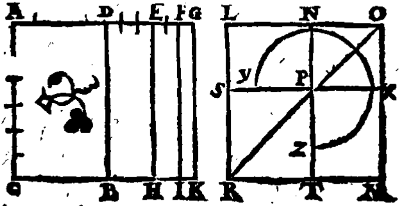




tio, linea quæ illam superficiem potest, est residuum mediale secundum.

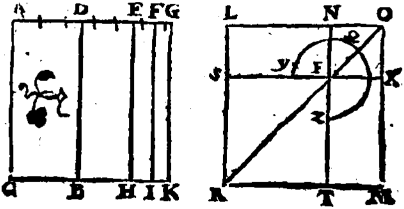
Theorema 69. Propositio 94.

Si superficies contineatur ex linea rationali & residuo quarto linea quã illã superficiem potest, est linea minor.



Theorema 70 Propositio 95.

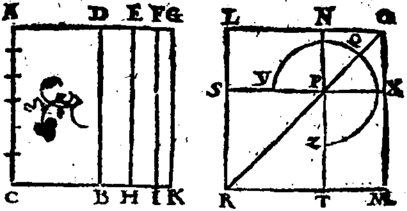
Si superficies contineatur ex linea rationali & residuo quinto, linea quę illam superficiẽ potest, est ea quæ dicitur cum rationali superficie faciens totam medialem.



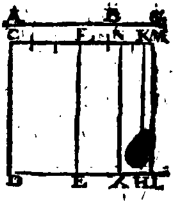
Theorema 71. Propositio 96.

Si superficies contineatur ex linea rationali &

& residuo sexto, linea quæ illam superficiē pōt, est ea quę dicitur faciēs cum mediali superficie totam medialem.

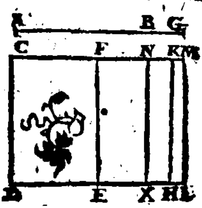


Theorema 72. Propositio 97.



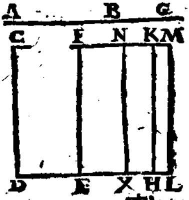
Quadratum residui secundum lineam rationalem applicatum, facit alterum latus residuum primum.

Theorema 73. Propositio 98.



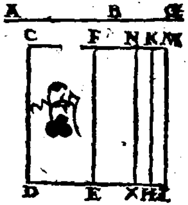
Quadratum residui medialis primi secūdum rationalem applicatū, facit alterum latus residuū secundum.

Theorema 74. Propositio 99.



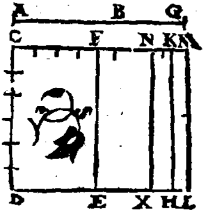
Quadratum; residui medialis secundi secundum rationalem applicatū, facit alterum latus residuū tertium.

Theorema 75. Propositio 100.



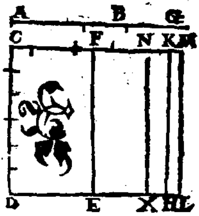
Quadratum lineę minoris secundum rationalē applicatum, facit alterū latus residuum quartū.

Theorema 76. Propositio 101.



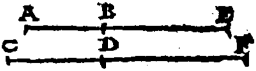
Quadratum lineæ cū rationali superficie facientis totam medialem, secundum rationalem applicatum, facit alterum latus residuum quintum.

Theorema 77. Propositio 102.



Quadratum lineæ cum mediali superficie faciētis totam medialem, secundum rationalem applicatum, facit alterum latus residuum sextum.

Theorema 78. Propositio 103.

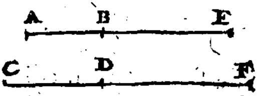


Linea residuo commensurabilis longitudine, est & ipsa residuum, & eiusdem ordinis.

Theorema 79. Propositio 104.

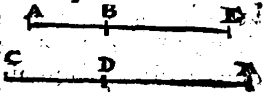
Li-

Linea comméſurabilis reſiduo mediali, eſt & ipſa reſiduũ mediale, & eiuſdem ordinis.



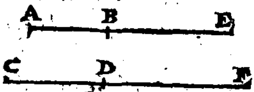
Theorema 80. Propoſitio 105.

Linea cõméſurabilis lineæ minori, eſt & ipſa linea minor.



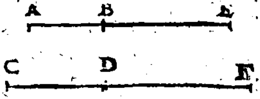
Theorema 81. Propoſitio 106.

Linea commenſurabilis lineæ cum rationali ſuperficie facienti totã medialẽ, eſt & ipſa linea cum rationali ſuperficie faciens totam medialem.



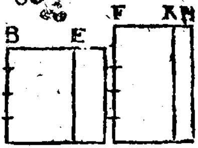
Theorema 82. Propoſitio 107.

Linea commenſurabilis lineæ cum mediali ſuperficie facienti totam medialem, eſt & ipſa cum mediali ſuperficie faciens totam medialem.



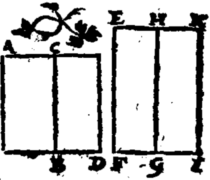
Theorema 83. Propoſitio 108.

Si de ſuperficie rationali detrahatur ſuperficies medialis, linea quæ reliquam ſuperficiem poteſt, eſt alterutra ex duabus irrationalibus, aut reſiduum, aut linea minor.



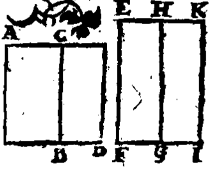
Theorema 84. Propoſitio 109.

Si de ſuperficie mediali detrahatur ſuperficies rationalis, aliæ duę irrationales, fiunt, aut reſiduũ mediale primũ aut cum rationali ſuperficiẽ faciens totam medialem.



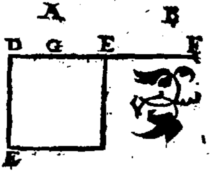
Theorema 85. Propoſitio 110.

Si de ſuperficie mediali detrahatur ſuperficies medialis quã ſit incommẽſurabilis toti, reliquæ duæ fiunt irrationales, aut reſiduum mediale ſecũdum, aut cum mediali ſuperficie faciẽs totam medialem.



Theorema 86. Propoſitio 111.

Linea quæ Reſiduum dicitur, non eſt eadem cum ea quæ dicitur Binomium.



SCHO-

SCHOLIVM.

neaqua Residuum dicitur, & cætera quinque eam consequentes irrationales, neq; lineæ mediali neq; sibi ipsa inter se sunt eadem. Nam quadratum lineæ medialis secundum rationalem applicatum facit alterum latus, rationalē lineam longitudine incommēsurabilem ei, secundum quam applicatur, per 23.

uadratū, verò residui secundū rationalē applicatū, facit alterum latus residuū primū, per 97.

Quadratum verò residui medialis primi secundū rationalem applicatum, facit alterum latus residuum secundum per 98.

Quadratum verò residui medialis secundi, facit alterum latus residuum tertium, per 99.

Quadratum verò lineæ minoris, facit alterum latus residuum quartum, per 100.

Quadratum verò lineæ cum rationali superficie facientis totam medialem, facit alterum latus residuum quintum, per 101.

Quadratum verò lineæ cum mediali superficie facientis totam medialem, secundum rationalē applicatum, facit alterum latus residuum sextum, per 102.

Cùm igitur dicta latera, quæ sunt latitudines cuiusque parallelogrammi vnicuīque quadrato æqualis & secundum rationalem, applicati differant & à primo latere, & ipsa inter se. (nam à primo differunt: quoniam sunt resi-

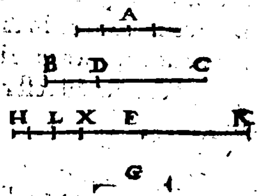
duæ non eiusdem ordinis) constat ipsas quoque, lineas irrationales inter se differentes esse. Et quoniam demonstratum est, Residuum nõ esse idem quod Binomiũ, quadrata autem residui & quinque linearũ irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex residuis eiusdem ordinis cuius sunt & residua, quorum quadrata applicantur rationali similiter & quadrata Binomij & quinque linearum irrationalium illud consequentium, secũdum rationalem applicata, faciunt altera latera ex Binomijs eiusdem ordinis cuius sunt & Binomia, quorum quadrata applicantur rationali. Ergo lineæ irrationales quæ consequuntur Binomium, & quæ consequuntur residuum, sunt inter se differentes. Quare dictæ lineæ omnes irrationales, sunt numero, 13.

1. Medialis.
2. Binomium.
3. Bimediale primum.
4. Bimediale secundum.
5. Maior.
6. Potens rationale & mediale.
7. Potens duo medialia.
8. Residuum.
9. Residuum mediale primum.
10. Residuum mediale secundum.
11. Minor.
12. Faciens cum rationali superficie totã medialem.
13. Faciẽs cum mediali superficie totam medialem.

Theo-

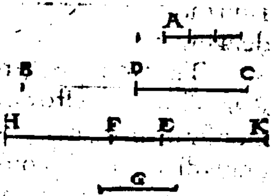
Theorema 87. Propositio 112.

Quadratum lineæ rationalis secundum Binomiũ applicatũ, facit alterum latus residuũ, cuius nomina sunt commẽsurabilia Binomij nominibus, & in eadem proportione: præterea id quod fit Residium, eundem ordinem retinet quem Binomium.



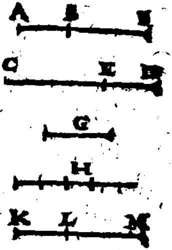
Theorema 88. Propositio 113.

Quadratum lineæ rationalis secundum residuum applicatum, facit alterum latus Binomium cuius nomina sunt commensurabilia nominibus residui & in eadem proportione: præterea id quod fit Binomium est eiusdẽ ordinis, cuius & Residuum.



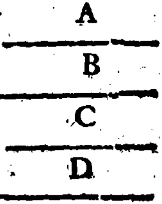
Theorema 89. Propositio 114.

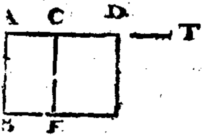
Si parallelogrammum contineatur ex residuo & Binomio, cuius nomina sunt comméſurabilia nominibus reſidui & in eadem proportione, linea quæ illam ſuperficiem poteſt, eſt rationalis.



Theorema 90. Propoſitio 115.

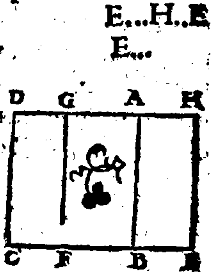
Ex linea mediali naſcuntur lineæ irrationales innumerabiles, quarum nulla vlli ante dictarum eadem ſit.





Theorema 100. Propoſitio 116.

Propoſitũ nobis eſto demonſtrare in figuris quadratis diametrũ eſſe longitudine incommenſurabilem ipſi lateri.



ELEMENTI X. FINIS.

EVCLIDIS ELEMENTVM VNDECIMVM, ET SOLIDORVM primum.

DEFINITIONES.

1.

Solidum, est quod longitudinem, latitudinem, & crassitudinem habet.

2.

Solidi autem extremum est superficies.

3.

Linea recta est ad planum recta cùm ad rectas omnes lineas, à quibus illa tãgitur, quæque in proposito sunt plano, rectos angulos efficit.

4.

Planum ad planum rectum est, cum rectæ lineæ, quæ communi planorum sectioni ad rectos angulos in vno planorum ducuntur, alteri plano ad recto sunt angulos.

5.

Rectæ lineę ad planum inclinatio acutus est angulus, ipsa insistente linea & adiuncta altera cõprehensus, cùm à sublimi rectæ illius lineæ termino deducta fuerit perpendicularis,

laris, atque à puncto quod perpendicularis in ipso plano fecerit, ad propositæ illius lineæ extremum, quod in eodem est plano, altera recta linea fuerit adiuncta.

6.

Plani ad planum inclinatio, acutus est angulus rectis lineis contentus, quæ in vtroque planorum ad idem communis sectionis pũctum ductæ, rectos ipsi sectioni angulos efficiunt.

7.

Planum similiter inclinatum esse ad planũ, atque alterum ad alterum dicitur, cum dicti inclinationum anguli inter se sunt æquales.

8.

Parallela plana, sunt quæ eodem non incidunt, nec concurrunt.

9.

Similes figuræ solidæ, sunt quæ æqualibus planis, multitudine æqualibus continetur.

10.

Aequales & similes figuræ solidæ sunt, quæ similibus planis, multitudine & magnitudine æqualibus continentur.

11.

Solidus angulus, est plurium quàm duarum linearum, quæ se mutuo contingant, nec in eadem sint superficie, ad omnes lineas inclinatio. Aliter

Aliter.

Solidus angulus, eſt qui pluribus quã duobus planis angulis, in eodem non conſiſtentibus plano, ſed ad vnum punctum collectis continetur.

12.

Pyramis eſt figura ſolida quæ planis continetur, ab vno plano ad vnum punctum collecta.

13.

Priſma figura eſt ſolida quæ planis continetur, quorum aduerſa duo ſunt & æqualia & ſimilia & parallela, alia verò parallelogramma.

14.

Sphæra eſt figura, quæ cõuerſo circum quieſcentem diametrum ſemicirculo continetur, cùm in eundem rurſus locum reſtitutus fuerit, vnde moueri cœperat.

15.

Axis autem ſphæræ, eſt quieſcens illa linea circum quam ſemicirculus conuertitur.

16.

Centrum verò Sphærę eſt idem, quod & ſemicirculi.

17.

Diameter autem Sphæræ, eſt recta quædam linea per centrum ducta, & vtrinque à ſphęræ ſuperficie terminata.

Conus

18.

Conus est figura, quæ conuerso circum quiescens alterum latus eorum quę rectum angulum continent, orthogonio triangulo cõtinetur, cùm in eodem rursus locum illud triangulum restitutum fuerit, vnde moueri cœperat. Atque si quiescens recta linea ęqualis sit alteri, quæ circum rectum angulum cõuertitur, rectangulus erit Conus: sin minor, amblygonius: si verò maior, oxygonius.

19.

Axis autem Coni, est quiescens illa linea, circum quam triangulum vertitur.

20

Basis verò Coni, circulus est, qui à circũducta linea recta describitur.

21.

Cylindrus figura est, quæ conuerso circumquiescens alterum latus eorum quę rectum angulum continent, parallelogrammo orthogonio comprehenditur, cum in eundem rursus locum restitutum fuerit illud parallelogrammum, vnde moueri cœperat.

22.

Axis autem Cylindri est quiescens illa recta linea, circum quam parallelogrammũ vertitur.

23.

Bases verò cylindri, sunt circuli à duobus

ad-

aduersus lateribus quæ circum aguntur, descripti

24.

Similes coli & cylindri, sunt quorum & axes & basium diametri proportionales sunt.

25.

Cubus est figura solida, quæ sex quadratis æqualibus continetur.

26.

Tetraedum est figura, quæ triangulis quatuor æqualibus & æquilateris continetur.

27.

Octaedrum figura est solida, quæ octo triãgulis æqualibus & æquilateris continetur.

28.

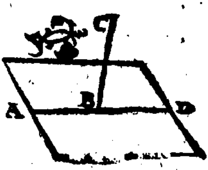
Dedecaedrum figura est solida, quæ duodecim pentagonis æqualibus, æquilateris, & æquiangulis continetur.

29.

Eicosaedrum figura est solida, quę triangulis viginti æqualibus & æquilateris continetur.

Theorema 1. Propositio 1.

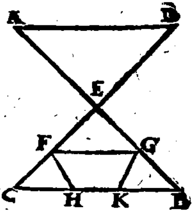
Quædã rectæ lineæ pars in subiecto quidem non est plano, quædam verò in sublimi.



Theo-

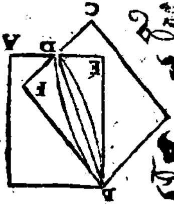
Theorema 2. Propoſitio 2.

Si duæ rectę lineæ ſe mutuo ſecent, in vno ſunt plano: atque triangulum omne in vno eſt plano.



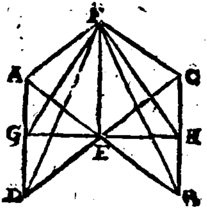
Theorema 3. Propoſitio 3.

Si duo plana ſe mutuò ſecent, communis eorum ſectio eſt recta linea.



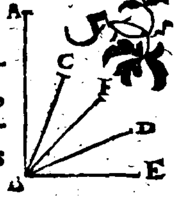
Theorema 4. Propoſitio 4.

Si recta linea rectis duabus lineis ſe mutuò ſecantibus, in cõmuni ſectione ad rectos angulos inſiſtat illa ducto etiam per ipſas plano ad angulos rectos erit.



Theorema 5. Propoſitio 5.

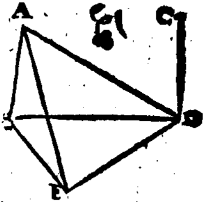
Si recta linea rectis tribus lineis ſe mutuò tangentibus, incommuni ſectione ad rectos angulos inſiſtat, illę tres rectæ in vno ſunt plano.



Theo-

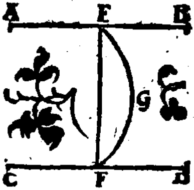
Theorema 6. Propositio 6.

Si duæ rectę lineę eidem plano ad rectos sint angulos, parallelæ erunt illæ rectæ lineæ.



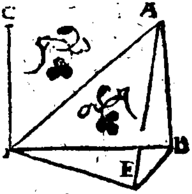
Theorema 7. Propositio 7.

Si duæ sint parallelæ rectæ lineæ, in quarũ vtraque sumpta sint quælibet pũcta, illa lineæ quæ ad hæc puncta adiungitur, in eodem est cum parallelis plano.



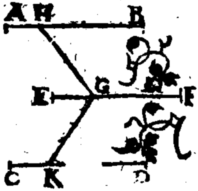
Theorema 8. Propositio 8.

Si duæ sint parallelæ rectæ lineæ, quarum altera ad rectos cuidam plano sit angulos, & reliqua eidẽ plano ad rectos angulos erit.



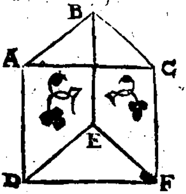
Theorema 9. Propositio 9.

Quæ eidem rectæ lineæ sunt parallelæ, sed non in eodem cum illa plano, hę quoque sunt inter se parallelæ.



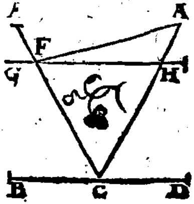
Theorema 10. Propositio 10.

Si duæ rectæ lineæ se mutuò tangentes ad duas rectas se mutuò tangentes sint pallalelę, non autem in eodem plano, illæ angulos æquales comprehendent.



Problema 1. Propositio 11.

A dato sublimi puncto, in subiectum planũ perpendicularem rectam lineam ducere.

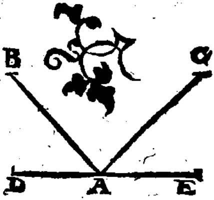


Problema 2. Propositio 12.

Dato plano, à pũcto quod in illo datum est, ad rectos angulos rectã lineam excitare.

Theorema 11. Propositio 13.

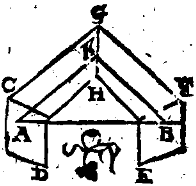
dato plano, à pũcto quod in illo datum est, duæ rectæ lineæ ad rectos angulos non excitabuntur ad easdem partes.



Theo-

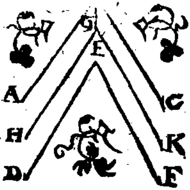
Theorema 12. Propositio 14.

Ad quæ plana, eadem recta linea recta est, illa sunt parallela.



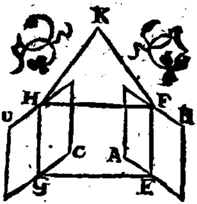
Theorema 13. Propositio 15.

Si duæ rectæ lineæ se mutuo tangentes ad duas rectas se mutuò tangentes sint parallelæ; non in eodem consistentes plano, parallela sunt quæ per illas ducuntur plana.



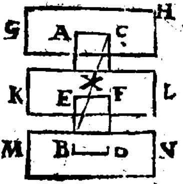
Theorema 14. Propositio 16.

Si duo plana parallela plano quopiam secentur, communes illorum sectiones sunt parallelæ.

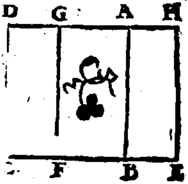


Theorema 15. Propositio 17.

Si duæ rectæ lineæ parallelis planis secētur, in easdēm rationes secabūtur.

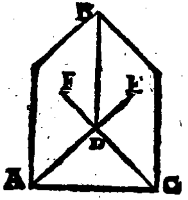


Theorema 16. Propoſitio 18.



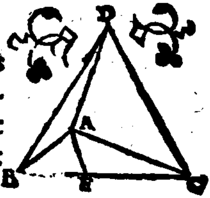
Si recta linea plano cuipiam ad rectos ſit angulos, illa etiam omnia quæ per ipſam plana, ad rectos eidem plano angulos erunt.

Theorema 17. Propoſitio 19.



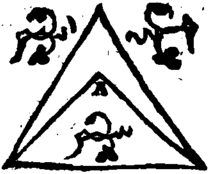
Si duo plana ſe mutuò ſecantia plano cuidã ad rectos ſint angulos, commu nis etiam illorum ſectio ad rectos eidem plano an gulos erit.

Theorema 18. Propoſitio 20.



Si angulos ſolidus planis tribus angulis contineatur, ex his duo quilibet vtut aſſumpti tertio ſunt maiores.

Theorema 19. Propoſiti 21.

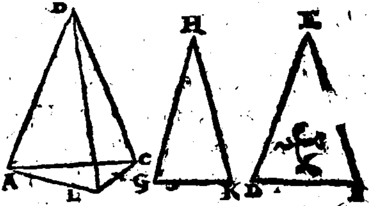


Solidus omnis angulus minoribus continetur, quàm rectis quatuor angulis planis.

Theo-

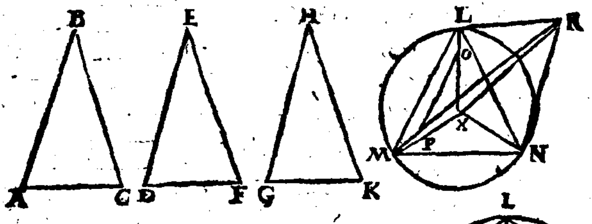
Theorema 20. Propoſitio 22.

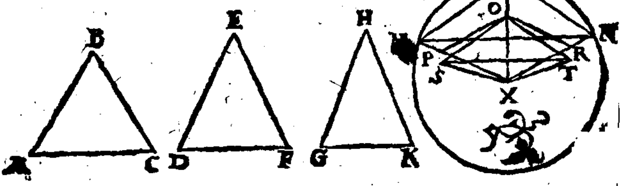
Si planì tres anguli æqualibus rectis contineantur lineis, quorũ duo vt libet aſſumpti, tertio ſint maiores, triangulum conſtitui poteſt ex lineis æquales, illas rectas coniungentibus.



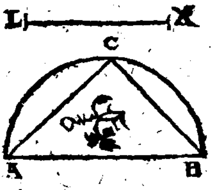
Problema 3. Propoſitio 23.

Ex planis tribus angulis, quorum duo vt libet aſſumpti tertio ſint maiores, ſolidũ angulum conſtituere. Decet autem illos tres angulos rectis quatuor eſſe minores.

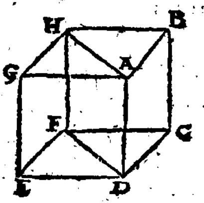




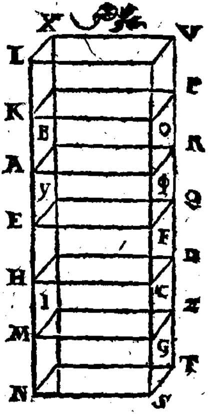
Theorema 21. Propoſitio 24.



Si ſolidum parallelis planis contineatur, aduerſa illius plana & æqualia ſunt & parallelogramma.



Theorema 22. Propoſitio 25.

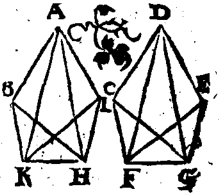


Si ſolidùm parallelis planis contẽtum plano ſecetur aduerſis planis parallelo, erit quemadmodum baſis ad baſim, ita ſolidum ad ſolidum.

Proble-

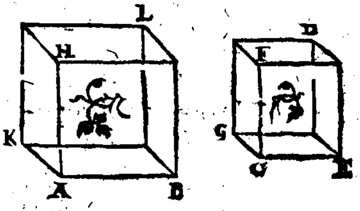
Problema 4. Propositio 26.

Ad datam rectam lineã eiuſque punctum, angulum ſolidum conſtituere ſolido angulo dato æqualem.



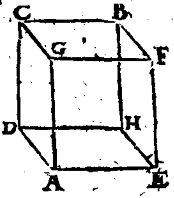
Problema 5. Propoſitio 27.

A data recta, dato ſolido parallelis planis comprehenſio ſimile & ſimiliter poſitum ſolidũ parallelis planis contentum deſcribere.

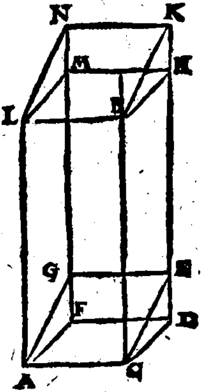


Theorema 23. Propoſitio 28.

Si ſolidum parallelis planis comprehẽſum, ductor per aduerſorum planorum diagonios plano ſectũ ſit, illud ſolidum ab hoc plano bifariam ſecabitur.



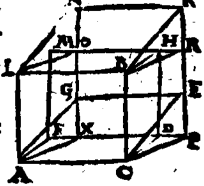
Theorema 24. Propositio 29.



Solida parallelis planis comprehēſa, quæ ſuper eandem baſim & in eadē ſunt altitudine quorum inſiſtētes lineæ in ijſdem collocantur rectis lineis, illa ſunt inter ſe æqualia.

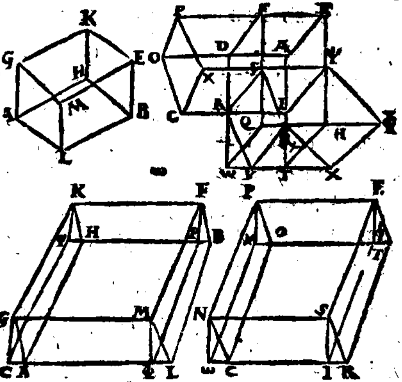
Theorema 25. Propoſitio 30.

Solida parallelis planis circumſcripta, quæ ſuper eandem baſim & in eadē ſunt altitudine, quorum inſiſtentes lineæ non in ijſdem reperiuntur rectis lineis, illa ſunt inter ſe æqualia.



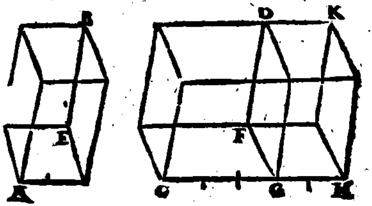
Theorema 26. Propoſitio 31.

Solida parallelis planis circumſcripta, quæ in in eadem sunt altitudine, æqualia sũt inter ſe.



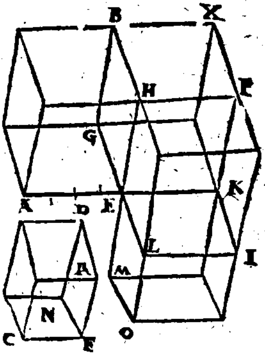
Theorema 27. Propoſitio 32.

Solida parallelis planis circumſcripta quæ eiuſdem ſunt altitudinis, eam habẽt inter ſe rationem, quã baſes.



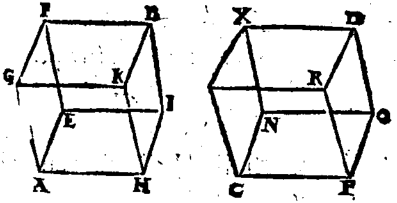
Teeore. 28. Propositio 33.

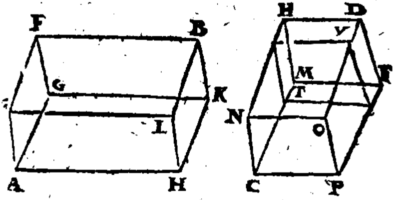
Similia solida parallelis, planis circũscripte habẽt inter se rationé homologorũ laterum triplicatam.

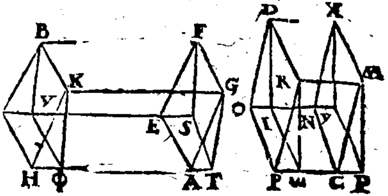


Theore. 29. Propositio 24.

Aequalium solidorũ parallelis planis cõtẽtorum bases cũ altitudinibus reciprocantur. Et solida parallelis planis contẽta quorũ bases



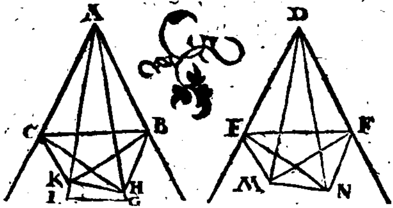




cum altitudinibus reciprocantur, illa sunt æqualia.

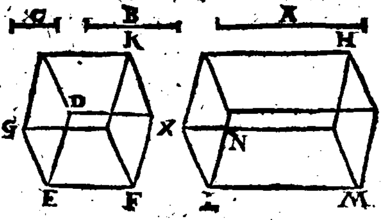
Theorema 30. Propositio 35.

Si duo plani sint anguli æquales, quorum verticibus sublimes rectæ lineæ insistant, quæ cum lineis primò positis angulos contineant equales, vtrunq; vtrique in sublimibus autem lineis quælibet sumpta sint puncta, & ab his ad plana in quibus consistunt anguli primum positi, ductæ sint perpendiculares, ab earum verò punctis, quæ in planis signata fuerint, ad angulos primum positos adiunctæ sint rectæ lineę, hæ cũ sublimibus æquales angulos comprehendent.



Theorema 31. Propositio 36.

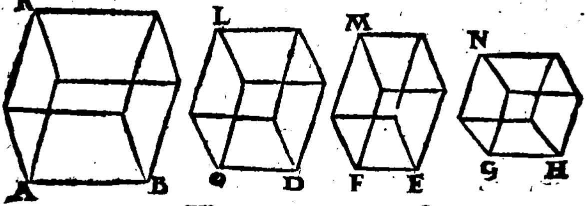
Si rectæ tres lineæ sint proportionales, quòd



ex his tribus fit solidum parallelis planis cõtentum, æquale est descripto à media linea solido parallelis planis comprehenso, quod æquilaterum quidem sit, sed antedicto æquiangulum.

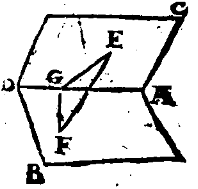
Theorema 32. Propositio 37.

Si rectæ quatuor lineæ sint proportionales, illa quoque solida parallelis planis contenta, quæ ab ipsis lineis & similia & similiter discribuntur, proportionalia erunt. Et si solida parallelis planis comprehensa, quæ & similia & similiter describuntur, sint proportionalia, illæ quoque rectæ lineæ proportionales erunt.



Theorema 33. Propositio 38.

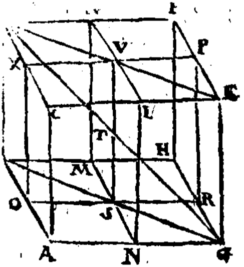
Si planum ad planum rectum sit, & à quodã puncto eorũ quæ in vno sunt planorum perpẽdicularis ad alterum ducta sit, illa quæ ducitur perpẽdicularis, in cõmunem cadet planorum sectionem.



Theo-

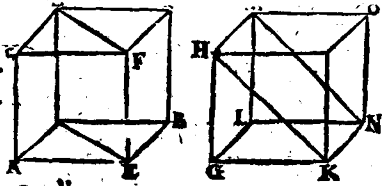
Theorema 34. Propoſitio 39.

Si in ſolido parallelis planis circumſcripto, aduerſorum planorum lateribus bifariam ſectis, educta ſint per ſectiones plana, communis illa planorũ ſectio & ſolidi parallelis plani circumſcripti diameter, ſe mutuò bifariã ſecant.



Theorema 35. Propoſitio 40.

Si duo ſint æqualis altitudinis priſmata, quorum hoc quidem baſim habeat parallelogrammum, illud verò triãgulum, ſit autẽ parallelogrammũ triãguli duplũ, illa priſmata erunt æqualia.



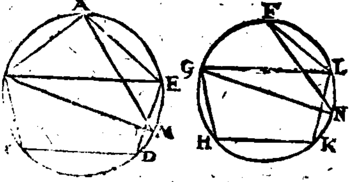
ELEMENTI XI. FINIS.

EVCLIDIS ELEMENTVM DVODECIMVM.

ET SOLIDORVM ſecundum.

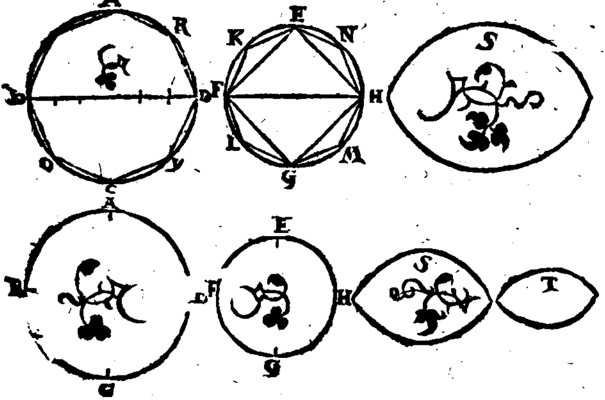
Theorema 1. Propoſitio 1.

Similia quæ ſunt in circulis polygona, rationem habent inter ſe, quam deſcripta à diametris quadrata.



Theoroma 2. Propoſitio 2.

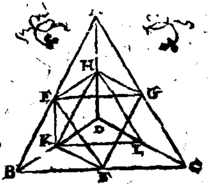
Circuli eam inter ſe rationem habent, quàm



descripta à diametris quadrata.

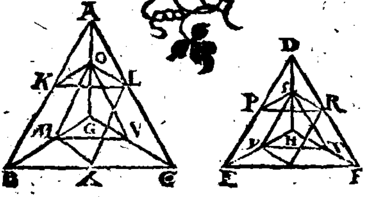
Theorema 3. Propositio 3.

Omnis pyramis trigonam habens basim, in duas diuiditur pyramidas nō tantùm æquales & similes inter se, sed toti etiam pyramidi similes, quarum trigonæ sunt bases, atq; in duo prismata æqualia, quæ duo prismata dimidio pyramidis totius sunt maiora.



Tneorema 4. Propositio 4.

Si duæ eiusdem altitudinis pyramides trigonas habeant bases, sit autem illarũ vtraq; diuisa & in duas pyramides inter se æquales totique similes, & in duo prisma æqualia, ac eodem modo diuidatur vtraq; pyramidum quæ ex superiore diuisione natæ sunt, idque perpetuo fiat: quẽadmodum se habet vnius piramidis basis ad alterius pyra midis basim, ita & omnia quæ in vna pyra-

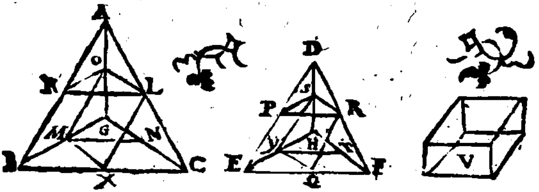


mide

mide prismata ad omnia quæ in altera pyramide, prismata multitudine æqualia.

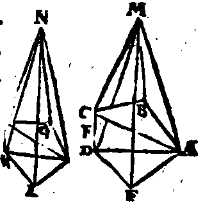
Theorema 5. Propositio 5.

Pyramides eiusdem altitudinis, quarum trigonæ sunt bases, eam inter se rationem habent, quàm ipsæ bases.



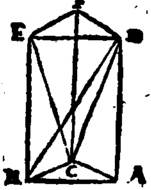
Theorema 6. Propositio 6.

Pyramides eiusdem altitudinis, quarum polygonæ sunt bases, eam inter se rationem habent, quã ipsæ bases.



Theorema 7. Propositio 7.

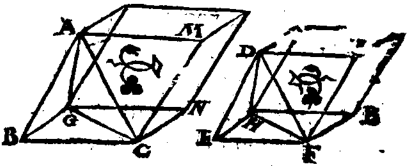
Omne prisma trigonã habens basim, diuiditur in tres pyramides inter se æquales, quartum trigonæ sunt bases.



Theo-

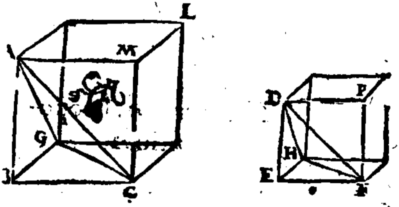
Theorema 8. Propoſitio 8.

Similes pyramides qui trigonas habent baſes, in triplicata sũt homologorũ laterũ ratiõe



Theorema 9. Propoſitio 9.

Aequaliũ pyramidum & trigonas baſes habentium reciprocantur baſes cum altitudinibus. Et quarum pyramidum trigonas baſes habẽtiũ reciprocãtur baſes cũ altitudinibus, illæ ſunt æquales.



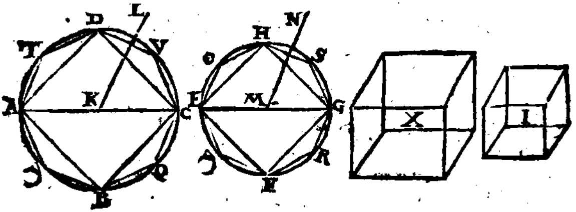
Theorema 10. Propoſitio 10.

Omnis conꝰ tertia pars eſt cylindri

eandem cum ipso cono basim habentis, & altitudinem æqualem.

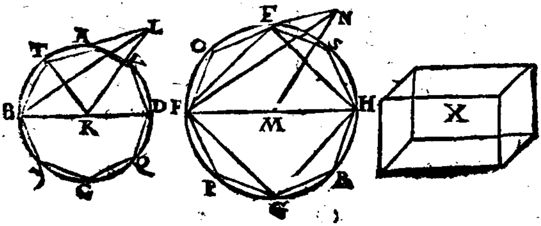
Theorema 11. Propositio 11.

Coni & cylindri eiusdem altitudinis, eam inter se rationem habent, quam bases.



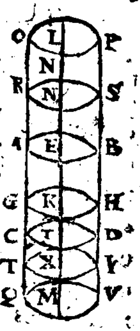
Theorema 12. Propositio 12.

Similes coni & cylindri, triplicatam habent inter se rationem diametrorum, quæ sunt in basibus.



Theo-

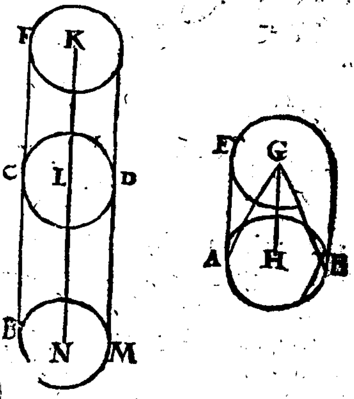
Theorema 13. Propositio 13.



Si cylindrus plano sectus sit aduersis planis parallelo, erit quemadmodum cylindrus ad cylindrum, ita axis ad axem.

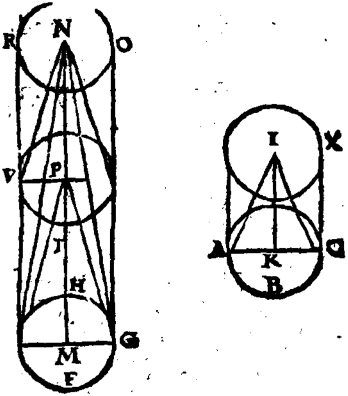
Theorema 14. Propositio 14.

Coni & cylindri qui in æqualibus sunt basibus, eam habent inter se rationem, quam altitudines



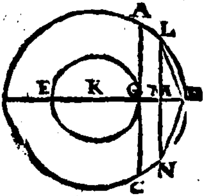
Theorema 15. Propositio 15.

Aequalium conorum & cylindrorum bases cũ altitudinibus reciprocantur. Et quorum conorũ & cylindrorum bases cum altitudinibus reciprocantur, illi sunt æquales.



Theorema 1. Propositio 16.

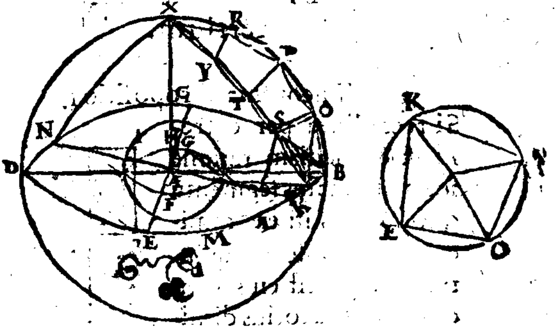
Duobus circulis circũm idem centrum cõsistentibus, in maiore circulo polygonum æqualium pariumq́ue laterum inscribere, quod minorem circulum non tangat.



Pro-

Problema 2. Propoſitio 17.

Duabus ſphæris circum idem centrum conſiſtentibus, in maiorem ſphæram ſolidum polyedrum inſcribere, quod minoris ſpæræ ſuperficiem non tangat.



Theorema 16. Propoſitio 18.

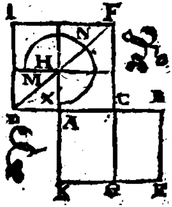
Sphæræ inter ſe rationem habent ſuarum diametrorum triplicatam.

Elementi duodecimi finis.

EVCLIDIS ELEMENTVM DECIMVM TERTIVM, ET SOLIDORVM tertium.

Theorema 1. Propositio 1.

Si recta linea per extremam & mediam rationẽ secta sit, maius segmentũ quod totius lineæ dimidium assumpserit, quintuplum potest eius quadrati, quod à totius dimidia describitur.

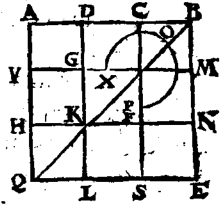


Theorema 2. Propositio 2.

Si recta linea sui ipsius segmenti quintuplum possit, & dupla segmenti huius linea per extremam & mediam rationẽ secetur maius segmẽtum reliqua pars est liniæ primum positæ.

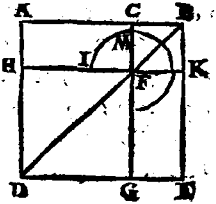
Theo-

Theorema 3. Propositio 3.



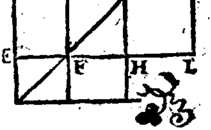
Si rectæ linea per extremam & mediam rationem secta sit minus segmētum quod maioris segmēti dimidium assumpserit, quintuplum potest eius, quòd à maioris segmenti dimidio describitur, quadrati.

Theorema 4. Propositio 4.



Si recta linea per extremam & mediam rationē secta sit, quòd à tota, quodque à minore segmento simul vtraq; quadrata, tripla sunt eius, quod à maiore segmēto describitur, quadrati.

Theorema 5. Propositio 5.



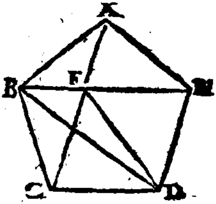
Si ad rectam lineam, quæ per extremam & mediam rationem secetur, adiuncta sit altera segmento maiori æqualis, tota hæc linea recta per extremam & mediam rationē secta est, estque maius segmentum linea primùm posita.

Theorema 6. Propositio 6.

Si recta linea ῥητὴ siue rationalis, per extremam & mediam rationém facta sit, vtrunque segmentorum ἄλογος siue irrationalis est linea, quæ dicitur Residuum.

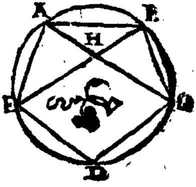
Theorema 7. Propositio 7.

Si pentagoni æquilateri tres sint æquales anguli, siue quã deinceps siue qui non deinceps sequuntur, illud pentagonum erit æquiangulum.



Theorema 8. Propositio 8.

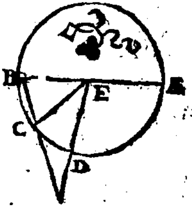
Si pentagoni æquilateri & æquianguli duos qui deinceps sequuntur angulos rectæ subtendãt lineæ, illę per extremam & mediam rationem se mutuò secant, earumque maiora segmenta, ipsius pentagoni lateri sunt æqualia.



Theo-

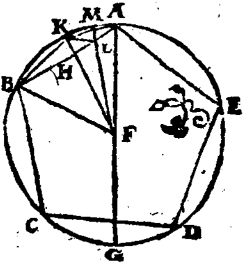
Theorema 9. Propositio 9.

Si latus hexagoni & latus decagoni eidem circulo inſcriptorum compoſita ſint, tota recta linea per extremam & mediam rationem ſecta eſt, eiuſque ſegmẽtum maius, eſt hexagoni latus.



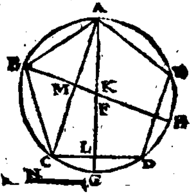
Theorema 10. Propoſitio 10.

Si circulo pẽtagonum ęquilaterum inſcriptum ſit, pentagoni latus poteſt & latus hexagoni & latus decagoni eidem circulo inſcriptorum.



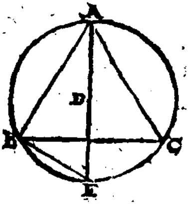
Theorema 11. Propoſitio 11.

Si in circulo ῥητὴν habente diametrum, inſcriptum ſit pentagonum æquilaterum, pentagoni latus irrationalis eſt linea quæ vocatur Minor.



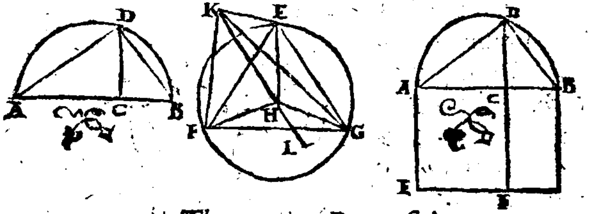
Theorema 12. Propoſitio 12.

Si in circulo inſcriptum ſit triangulum æquilaterum, huius trianguli latus potentia triplum eſt eius lineę, quæ ex circuli centro ducitur.



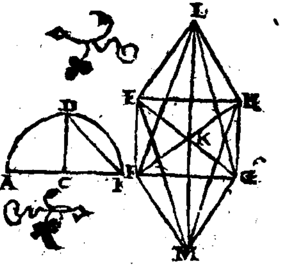
Problema 1. Propoſitio 13.

Pyramidem conſtituere, & data ſphæra cõplecti, atque docere illius ſphæræ diametrũ potentia ſeſquialteram eſſe lateris ipſius pyramidis.



Theorema 2. Propoſitio 14.

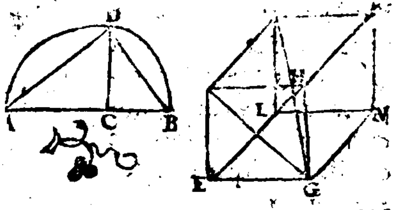
Octaedrum conſtituere, eaq́; ſphæra qua pyramidẽ complecti, atque probare illius ſphæræ diametrũ potentia duplã eſſe lateris ipſius octaedri.



Proble-

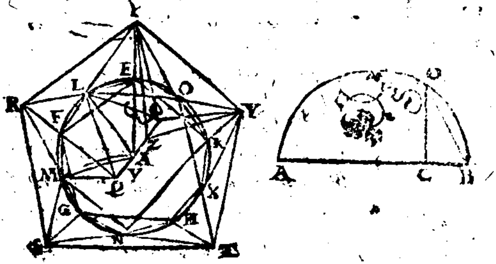
Problema 3. Propositio 15.

Cubum constituere, eaque sphæra qua & superiores figuras complecti, atque docere illius sphæræ diametrum potētia triplam esse lateris ipsius cubi.

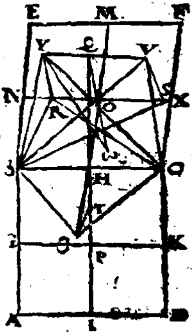


Problema 4. Propositio 16.

Icosaedrum constituere, eademque sphæra qua & antedictas figuras complecti, atque probare, Icosaedri latus irrationalem esse lineam, quæ vocatur Minor.



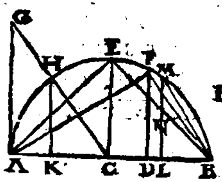
Problema 5. Propoſitio 17.

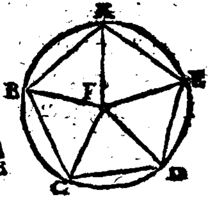


Dodecaedrũ conſtituere, eademq́ue ſphæra qua & antedictas figuras complecti, atque probare dodecaedri latus irrationalẽ eſſe lineam, quæ vocatur Reſiduum.

Theorema 6 Propoſitio 18.

quinq; figurarum latera





proponere,& inter ſe comparare.

SCOLIVM.

Aio verò, præterea dictas quinque figuras nõ poſſe aliam conſtitui figuram ſolidam, quæ planis & æquilateris & æquiangulis cõtineatur, inter ſe æqualibus. Non enim ex duobus triangulis, ſed neque ex alijs duabus figuris ſolidus cõſtituetur angulus.

Sed

Sed ex tribus triãgulis cõstat Pyramidis angulus

Ex quatuor autem, Octaedri

Ex quinque verò, Icosaedri.

Nam ex triangulis, sex & æquilateris & æquiangulis ad idem punctum coeuntibus, non fiet angulus, recti vnius bessem contineat, erunt eiusmodi sex anguli rectis quatuor æquales. Quod fieri non potest. Nam solidus omnis angulus minoribus quam rectis quatuor angulis continetur, per 21.11.

Ob easdem sanè causas, neque ex pluribus quam planis sex eiusmodi angulis solidus constat.

Sed ex tribus quadratis, Cubi angulus cõtinetur.

Ex quinque, nullus potest. Rursus enim recti quatuor erunt.

Ex tribus autem pentangonis æquilateris & æquiangulis Dodecaedri angulus continetur.

Sed ex quatuor nullus potest. Cùm enim pentagoni æquilateri angulus rectus sit, & quinta recti pars erũt quatuor anguli rectis quatuor maiores. Quod fieri nequit. Nec sanè ex alijs polygonis figuris solidus angulus cõtinebitur, quod hinc quoque absurdum sequatur. Quamobrem perspicuum est, præter dictas quinque figuras aliã figuram solidam non posse constitui, quæ ex planis æquilateris & æquiangulis contineatur.

ELEMEMTI XIII. FINIS.

EVCLIDIS ELEMENTVM DECIMVMQVARTVM, VT quidam arbitrantur, vt alij verò, Hypsiclis Alexandrini, de quinque corporibus.

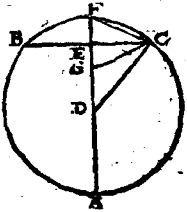
LIBER PRIMVS.

Basilides Tyrius, Protarche, Alexandriã profectus, patriq; nostro ob disciplinã societatem commendatus, longissimo peregrinationis tempore cum eo versatus est. Cumq; differerẽt aliquãdo descripta ab Apollonio comparatione Dodecaedri & Icosaedri eidem sphæra inscriptorum, quam hæc inter se habeant rationem, censuerunt ea non rectè tradidisse Apollonium: quæ a se non emendata, vt de patre audire erat literis prodiderunt. Ego autem postea incidi in alterum librum ab Apollonio editum, qui demonstrationem accuratè complecteretur de re proposita, ex eiusq; problematis indagatione magnam equidem cœpi voluptatem. Illud certè ab omnibus perspici potest quod scripsit Apollonius, cùm sit in omnium manibus. Quod autẽ diligenti, quantũ conijcere licet, studio nos postea scrip-

scripsisse videmur, id monimentis consignatum tibi nuncupandum duximus, vt qui feliciter cùm in omnibus disciplinis tum vel maximè in Geometria versatus, scitè ac prudēter iudices ea quæ dicturi sumus ob eam verò, quæ tibi cum patre fuit, vitæ consuetudinem, quaq; nos complecteris, beneuolentiam, tractationem ipsam libenter audias. Sed iam tempus est, vt proœmio modum facientes, hanc syntaxim aggrediamur.

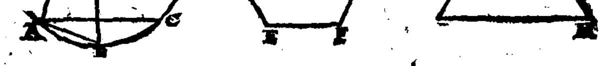
Theorema 1. Propositio 1.

Perpendicularis linea, quæ ex circuli cuiuspiam centro in latus pentagoni ipsi circulo inscripti ducitur, dimidia est vtriusque simul lineæ, & eius, quæ ex centro & lateris decagoni in eodē circirculo inscripti.



Theorema 2. Propositio 2.

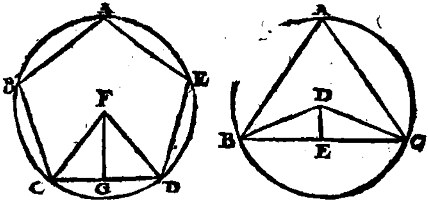
Idem circulus comprehēdit & dodecaedri penragonum & icosaedri triāgulum, eidem sphæræ inscriptorum.



Theo-

Theorema 3. Propoſitio 3.

Si pentagono & æquilatero & æquiangulo circumſcriptus ſit circulus ex cuius centro in vnum pentagoni latus dicta ſit perpendicularis: quod vno laterum & perpendiculari trigeſies cõtinetur, illud

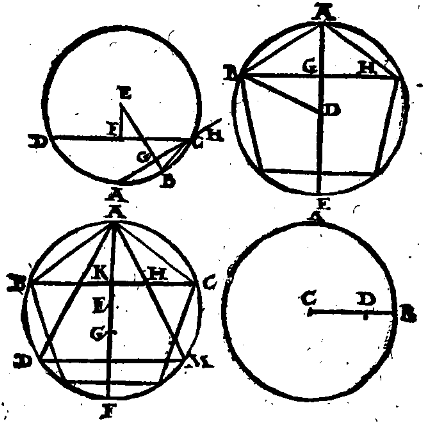


æquale eſt dodecaedri ſuperficiei.

Theorema 4. Propoſitio 4.

Hoc perſpicuum cùm ſit probandum eſt, quemadmodum ſe habet dodecaedri ſuperficies ad icoſaedri ſuperficiem ita ſe habere cubi latus ad icoſaedri latus.

Cubi



Cubilatus.

E

Dodecaedri.

F

Icosaedri.

G

SCHOLIVM.

Nunc autem probandum est, quemadmodum se habet cubi latus ad icosaedri latus ita se habere solidum dodecaedri ad icosaedri solidum. Cùm enim aequales circuli comprehendat & dodecaedri pentagonum & Icosaedri triangulum, eidem

P *sphaera*

sphæra inscriptorum: in sphæris autem æquales circuli æquali interuallo distent à centro (siquidē perpendiculares à sphæra cētro ad circulorum plana ducta & æquales sunt, & ad circulorum centra cadunt) idcirco lineæ, hoc est perpendiculares quæ à sphæra centro ducuntur ad centrum circuli comprehendētis & triangulum Icosaedri, pētagonum dodecaedri sunt æquales. Sunt igitur æqualis altitudinis Pyramides, quæ bases habent ipsa dodecaedri pentagona, & quæ Icosaedri triangula. At æqualis altitudinis pyramides rationē inter se habent eam quam bases, ex 5. & 6. 11. Quemadmodum igitur pentagonum ad triangulum, ita pyramis, cuius basis quidem est dodecaedri pentagonum, vertex autem sphæra centrum ad pyramida, cuius basis quidem est Icosaedri triāgulum, vertex autem sphæra centrū. Quamobrem vt se habent duodecim pentagona ad viginti triangula, ita duodecim pyramides, quorū pentagona sint bases, ad viginti pyramidas, quæ trigonas habeant bases. At pentagona duodecim sunt dodecaedri superficies, viginti autem triangula, Icosaedri. Est igitur vt dodecaedri superficies ad Icosaedri superficiem, ita duodecim pyramides quæ pentagonas habeant bases, ad viginti pyramidas, quarum trigonæ sunt bases. Sunt autē duodecim quidem pyramides, quæ pētagonas habeant bases, solidum dodecaedri: viginti autem pyramides, quæ trigonas habeant bases, Icosaedri solidū. Quare ex 11. 5. vt dodecaedri superficies ad

Icosae-

Icosaedri superficiem, ita solidum dodecaedri ad Icosaedri solidum. Vt autem dodecaedri superficiem, ita probatum est cubi latus ad Icosaedri latus. Quemadmodum igitur cubi latus ad Icosaedri latus, ita se habet solidum dodecaedri ad Icosaedri solidum.

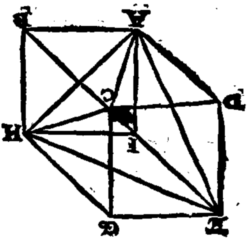
ELEMENTI XIIII. FINIS.

EVCLIDIS ELEMENTVM DECIMVM QVINTVM, ET Solidorum quintum, vt nonnulli putant, vt autem alij Hypsiclis Alexandrini, de quinque corporibus.

LIBER II.

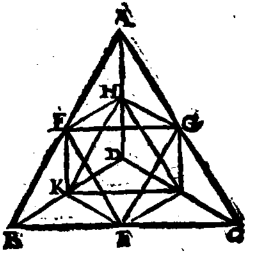
Problema 1. Propositio 1.

In dato cubo pyramida inscribere.



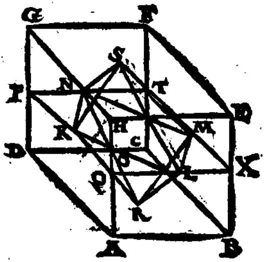
Problema 2. Propositio 2.

In data pyramide octaedrum inscribere.



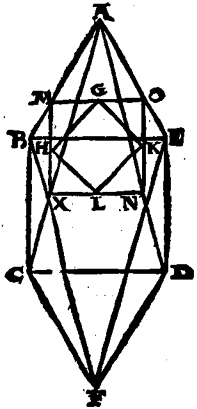
Problema 3. Propoſitio 3.

In dato cubo octaedrum inſcribere.



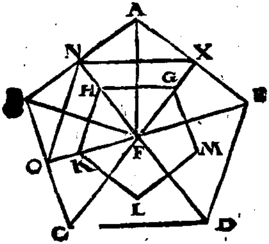
Problema 4. Propoſitio 4.

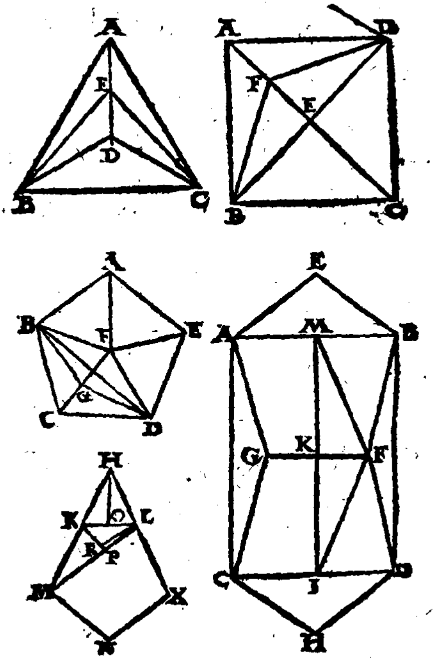
In dato octaedro cubũ inſcribere.



Problema 5. Propoſitio 5.

In dato Icoſaedro dodecaedrum inſcribere.





SCHO.

SCHOLIVM.

Meminisse decet, si quis nos roget, quot Icosaedrum habeat latera, ita respondendum esse: Patet Icosaedrum viginti contineri triangulis, quodlibet verò triangulum rectis tribus constare lineis. Quare multiplicanda sunt nobis viginti triangula in trianguli vnius latera, fiuntq́ sexaginta, quorum dimidium est triginta. Ad eũdem modum & in dodecaedro. Cum enim rursus duodecim pentagona dodecaedrum comprehẽdant itemq́ pentagonum quoduis rectis quinque cõstet lineis, quinque duodecies multiplicamus, fiunt sexaginta, quorum rursus dimidium est triginta. Sed cur dimidium capimus? Quoniã vnumquodq́ latus siue sit triãguli, siue pentagoni, siue quadrati vt in cubo, iterato sumitur. Similiter autem eadem via & in cubo & in pyramide & in octaedro latera inuenies. Quòd si item velis singularum quoq́ figurarum angulos reperire, facta eadem multiplicatione numerum procreatum partire in numerũ planorum, quæ vnum solidum angulum includũt: vt quoniam triangula quinque vnum Icosaedri angulum continet, partire 60. in quinque, nascuntur duodecim anguli Icosaedri. In dodecaedro autem tria pentagona angulum comprehendunt, partire ergo 60. in tria, & habebis dodecaedri angulos viginti. Atq́ simili ratione in reliquis figuris angulos reperies.

Finis Elementorum Euclidis.