EVCLIDIS ELEMENTORVM LIBRI XV.

Quibus, cùm ad omnem Mathematicæ ſcientiæ partem, tùm ad quamlibet Geometriæ tractationem, facilis comparatur aditus.

COLONIAE,
Apud Maternum Cholinum.
M. D. LXXXVII.
Cum Gratia & Priuilegio Cæſ. Maieſt.

AD CANDIDVM LECTOREM ST. GRACILIS praefatio.

PERMAGNI *referre sẽper existimaui, lector beneuole, quantum quisq; studij & diligentiæ ad percipienda scientiarum elementa adhibeat, quibus nõ satis cognitis, aut perperã, intellectis si vel digitum progredi tentes, erroris caliginem animis offundas, non veritatis lucem rebus obscuris adferas. Sed principiorum quanta sint in disciplinis momenta, haud facilè credat, qui rerum naturam ipsa specie, non viribus metiatur. Vt enim corporum quæ oriuntur & intereũt, vilissima tenuissimaq; videntur initia: ita rerum æternarum & admirabilium, quibus nobilissimæ artes continentur, elementa ad speciem sunt exilia, ad vires & facultatem quã maxima. Quis nõ videt ex fici tãtulo grano, vt ait Tullius, aut ex acino vinaceo, aut ex cæterarum frugum aut stirpium minutissimis seminibus tantos truncos ramosq; procreari? Nã Mathematicorũ initia illa quidem dictu audituq; perexigua, quantã theorematum syluã nobis pe-*

pererunt? Ex quo intelligi potest, vt in ipsis semi nibus, sic & in artium principiis inesse vim earũ rerum, quæ ex his progignuntur. Præclarè igitur Aristoteles, vt alia permulta, μέγιστον ἴσως ἀρχὴ παντὸς, καὶ ὅσῳ κράτιστον τῇ δυνάμει, τοσούτῳ μικρότατον, οὐ μεγέθει χαλεπόν ἐστιν ὀφθῆναι. Quocirca committendum non est, vt non bene prouisa & diligenter explorata scientiarum principia, quibus propositarum quarumque rerũ veritas sit demonstranda, vel constituas, vel constituta approbes: Cauendum etiam, vt ne tantulum quidem fallaci & captiosa interpretatione turpiter deceptus, à vera principiorum ratione temerè deflectas. Nam qui initio fortè aberrauerit, is vt tandem in maximis verſetur erroribus, necesse est: cum ex vno erroris capite, densiores sensim tenebræ rebus clarissimis obducantur. Quid tam varias veterum physiologorum sententias non modò cum rerum veritate pugnantes, sed vehementer etiam inter se dissentientes nobis inuexit? Equidem haud scio, fueritne vlla potior tanti dissidij causa, quàm quòd ex principijs partim falsis, partim non consentaneis ductas rationes probando adhiberent. Fit enim plerunque, vt qui non rectè de artium rerumq́ue elementis sentiunt, ad præfinitas quasdam opiniones suas omnia reuocare studeant. Pythagorei, vt meminit Aristoteles, cum denarij numeri summam perfectinem cœlo tribuerent, nec plures tamen quàm nouem sphæras cernerent, decimam affingere ausi sunt terræ aduer-

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sam quam ἀντίχθονα appellarunt. Illi enim vniuersitatis rerumq́ singularum naturam ex numeris ceu principijs æstimantes, ea protulerũt quæ φαινομένοις congruere nusquam sunt cognita. Nam ridicula Democriti, Anaximenis, Melissi Anaxagoræ, Anaximandri, & reliquorum id genus physiologorum somnia ex falsis illa quidem orta naturæ principijs, sed ad Mathematicum nihil aut parum spectantia sciens prætereo. Nonnullos attingam qui repetitis altius vel aliter ac decuit positis rerum initijs, cum physicis multa turbarunt, tum Mathematicos oppugnatione principiorum pessimè mulctarũt. Ex planis figuris corpora cõstituit Timæus: Geometrarum hic quidem principia cuniculis oppugnantur. Nam & superficies seu extremitates crassitudinem habebunt, & lineæ latitudinem: denique puncta non erunt indiuidua sed linearum partes. Prædicant Democritus atque Leucippus illas atomos suas, & indiuidua corpuscula. Concedit Xenocrates impartibiles quasdã magnitudines. Hic verò Geometriæ fundamẽta apertè petuntur, & funditus euertuntur: quibus dirutis nihil equidẽ aliud video restare, quàm vt amplissima Mathematicorum theatra repentè concidant. Iacebunt ergo, si dijs placet, tot præclara Geometrarum de asymmetris & alogis magnitudinibus theoremata. Quid enim causæ dicas cur indiuidua linea hanc quidem metiatur, illam verò metiri non queat? Siquidem quod minimum in vnoquoque *tudine sit definita, & suis circumscripta terminis. Quis enim vllam infiniti scientiam defendat? Hoc scitum est, quod non semel docet Aristoteles, infinitum ne cogitatione quidem complecti quenquam posse. Itaque ex infinita multitudinis & magnitudinis* δυνάμει *finitam hæc scientia decerpit & amplectitur naturam quam tractet, & in qua versetur. Nam de vulgari Geometrarum cõsuetudine quid sentiendum sit, cum data interdum magnitudine infinita aut fabricantur aliquid, aut proprias generis subiecti affectiones exquirunt, diserte monet Aristoteles,* οὐδὲ νῦν *(de Mathematicis loquens)* δέονται τοῦ ἀπείρου, οὐδὲ χρῶνται, ἀλλὰ μόνον εἶναι ὅσην ἂν βούλωνται, πεπερασμένην. *Quamobrem disputatio ea qua infinitum refellitur, Mathematicorum decretis rationibusq; non aduersatur, nec eorum apodixes labefacit: Etenim tali infinito opus illis nequaquam est, quod exitu nullo peragrari possit, nec talem ponunt infinitam magnitudinẽ: sed quantãcunq; velit aliquis effingere, ea vt suppetat, infinitam præcipiunt. Quinetiã non modò immensa magnitudine opus non habent Mathematici, sed ne maxima quidem: cum instar maxima minima quæque in partes totidem pari ratione diuidi queat. Alteram Mathematica diuisionẽ attulit Geminus, vir (quantum ex Proclo cõijcere licet)* μαθημάτων *laude clarissimus. Eam, quæ superiore plenior & accuratior forte visa est, cũ doctissimè pertractarit sua in decimum Euclidis*

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præfatione P. Montaureus vir senatorius, & regiæ bibliothecæ præfectus, leuiter attingam. Nam ex duobus rerum velut summis generibus τῶν νοητῶν καὶ τῶν αἰσθητῶν, quæ res sub intelligentiam cadunt, Arithmetica & Geometria attribuit Geminus: quæ verò in sensus incurrũt, Astrologia, Musica, Supputratici, Optica, Geodesiæ & Mechanica adiudicauit. Ad hãc certè diuisionem spectasse videtur Aristoteles, cùm Astrologiam Opticam, harmonicam φυσικωτέρας τῶν μαθημάτων nominat, vt quæ naturalibus & Muthematicis interiecta sint, ac velut ex vtrisque mixta disciplinæ: Siquidem genera subiecta à Phisicis mutuantur, causas verò in demonstrationibus ex superiore aliqua scientia repetunt. Idquod Aristoteles ipse apertissimè testatur, ἐνταῦθα γὰρ, φησὶ, τὸ μὲν ὅτι, τῶν αἰσθητικῶν εἰδέναι τὸ δὲ διότι, τῶν μαθηματικῶν. Sequitur, vt quid Mathematica conueniat cum Phisica & prima Philosophia: quid ipsa ab vtraque differat, paucis ostendamus. Illud quidem omnium commune est, quòd in veri contemplatione sunt posita, ob idq; θεωρητικαὶ à Græcis dicuntur. Nam cùm διάνοια siue ratio & mens omnis sit vel πρακτική, vel θεωρητική, totidem scientiarum sint genera necesse est. Quòd si Physica, Mathematica, & prima Philosophia, nec in agendo, nec in efficiendo sunt occupatæ, hoc certè perspicuum est, eas omnes in cognitione cõtemplationéque necessariò versari. Cùm enim rerum non modò agendarum, sed etiam efficienda-

quantitatem & continuum. Itaq; Mathematicorum ars in ijs quæ immobilia sunt, cernitur. (τὰ γὰρ μαθηματικὰ τῶν ὄντων ἄνευ κινήσεώς ἐστιν, δ' ἔξω τῶν περὶ τὴν ἀστρολογίαν) quæ verò in natura obscuritate posita est, res quidem quæ nec separari nec motu vacare possunt, contemplatur. Idquod in vtroque scientiæ genere perspicuum esse potest siue res subiectas definias, siue proprietates earum demonstres. Etenim numerus, linea, figura rectum, inflexum, æquale, rotundum, vniuersa denique Mathematicus quæ tractat & profitetur, absque motu explicari docerique possunt: χωριστὰ γὰρ τῇ νοήσει κινήσεώς ἐστι: Phisica autem sine motione species nequaquam possunt intelligi. Quis enim hominis plantæ, ignis, ossium, carnis naturam & proprietates sine motu, qui materiam sequitur, perspiciat? Siquidem tantisper substantia quæque naturalis constare dici solet, quoad opus & munus suum, agendo patiendoque tueri ac sustinere valeat: quá certe amissa δυνάμει, ne nomen quidem nisi ὁμωνύμως retinet. Sed Mathematico ad explicandas circuli aut trianguli proprietates, nullum adferre potest vsum materiæ vt auri, ligni, ferri, in qua insunt, consideratio, quin eò verius eiusmodi rerum, quarum species tanquam materia vacantes efformemus animo, naturam complectemur, quod coniunctione materiæ quasi adulterari deprauarique videntur. Quocirca Mathematicæ species eodem modo quo κοῖλον, siue concauitas, sine motu & subiecto, definitione explicare

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cognoſciq; poſſunt: naturales verò cùm eam vim habeant quam, vt ita dicam ſimitas cum materia comprehenſæ ſunt, nec abſque ea ſeparatim poſſunt intelligi: quibus exemplis quid inter Phiſicas & Mathematicas ſpecies interſit, haud difficile eſt animaduertere. Illis certè non ſemel eſt vſus Ariſtoteles. Valeant ergo Protagoræ ſophiſmata, Geometras hoc nomine refellentis, quod circulus normam puncto non attingat. Nã diuina Geometrarum theoremata, qui ſenſu æſtimabit, vix quicquam reperiet quod Geometræ concedendum videatur. Quid enim ex his quæ ſenſum mouent, ita rectum aut rotundum dici poteſt, vt à Geometra ponitur? Nec verò abſurdum eſt aut vitioſum, quòd lineas in puluere deſcriptas pro rectis aut rotundis aſſumit, quæ nec rectæ ſunt nec rotundæ, ac ne latitudinis quidem expertes. Siquidem non iis vtitur Geometra quaſi inde vim habeat concluſio, ſed eorum quæ diſcenti intelligenda relinquuntur, rudem ceu imaginem proponit. Nam qui primum inſtituuntur, hi ductu quodam & velut χειραγωγία ſenſuum opus habent, vt ad illa quæ ſola intelligentia percipiuntur, aditum ſibi comparare queant. Sed tamen exiſtimandum non eſt rebus Mathematicis omnino negari materiam, ac non eam tantùm quæ ſenſum afficit. Eſt enim materia alia quæ ſub ſenſum cadit, alia quæ animo & ratione intelligitur. Illam αἰσθητὴν, hanc νοητὴν vocat Ariſtoteles. Senſu percipitur, vt æs, vt lignum, omniſque materia quæ moueri po-

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test. Animo & ratione cernitur ea quæ in rebus sensilibus inest, sed nō quatenus sensu percipiuntur, quales sunt res Mathematicorum. Unde ab Aristotele scriptū legimus ἐπὶ τῶν ἐν ἀφαιρέσει ὄντων *rectum se habere ut simum:* μετὰ συνεχοῦς γὰρ*: quasi velit ipsius recti quod Mathematicorum est, suam esse materiam, non minus quā simi quod ad Physicos pertinet. Nam licet res Mathematicæ sensili vacent materia, nō sunt tamen indiuiduæ, sed propter continuationem partitioni semper obnoxiæ, cuius ratione dici possunt sua materia non omnino carere: quin aliud videtur* τὸ εἶναι γραμμῆ, *aliud quoad continuationi adiuncta intelligitur linea. Illud enim ceu forma in materia proprietatum causa est, quas sine materia percipere non licet. Hec est societatis & dissidij Mathematicæ cum Physica & prima Philosophia ratio. Nunc autem de nominis etymo & notatione pauca quædam afferamus. Nam si quæ iudicio & ratione imposita sunt rebus nomina, ea certè non temere indita fuisse credendum est, quibus scientias appellari placuit. Sed neque otiosa semper haberi debet ista etymologiæ indagatio, cum ad rei etiam dubiæ fidem sæpe non parum valeat recta nominis interpretatio. Sic enim Aristoteles ducto ex verborum ratione argumento,* αὐτομάτου, μετὰ βουλῆς, αἰθέρος, *aliarumque rerum naturam ex parte confirmauit. Quoniam igitur Pythagoras Mathematicam scientiam nō modò studiosè coluit, sed etiā repetitis à capite principijs, geometricā contem-*

contemplationem in liberalis disciplinæ formã composuit, & perspectis absq; materia solius intelligentiæ adminiculo theorematibus, tractationem περὶ τῶν ἀλόγων, & κοσμικῶν σχημάτων cõstitutionem excogitauit: credibile est, Pythagoram, aut certè Pythagoreos, qui et ipsi doctoris sui studia libenter amplexi sunt, huic scientiæ id nomen dedisse, quod cum suis placitis atq; decretis congrueret, rerumq; propositarum naturam quoquo modo declararet. Ita cùm existimarent illi, omnem disciplinam, quæ μάθησις dicitur, ἀνάμνησιν esse quandam. i. recordationem et repetitionem eius scientiæ, cuius antè quam in corpus immigraret, compos fuerit anima, quemadmodum Plato quoq; in Menone, Phædone, & alijs aliquot locis videtur astruxisse: animaduerterent autem eiusmodi recordationem, quæ non posset multis ex rebus perspici, ex his potissimum scientijs demonstrari, si quis nimirum, ait Plato, ἐπὶ τὰ διαγράμματα ἄγῃ, probabile est equidem Mathematicas à Pythagoreis artes κατ᾽ ἐξοχὴν fuisse nominatas, vt ex quibus μάθησις, id est æternarum in anima rationum recordatio διαφερόντως et p̃cipuè intelligi posset. Cuius etiã rei fidẽ nobis diuinus fecit Plato, q̃ in Menone Socratem induxit hoc argumẽti genere persuadere cupientem discere nihil esse aliud quam suarum ipsius rationum animum recordari. Etenim Socrates pusionem quendam, vt Tullij verbis vtar, interrogat de geometrica dimensione quadrati: ad ea sic ille respondet vt puer, & tamen

men tam faciles interrogationes sunt, vt gradatim respondens eodem perueniat quò si Geometrica didicisset. Aliam nominis huius rationem Anatolius exposuit, vt est apud Rhodiginum, quòd cùm cæteræ disciplinæ deprehendi vel non docente aliquo possint omnes, Mathematicæ sub nullius cognitionem veniant, nisi præeunte aliquo, cuius solertia succidantur vepreta, vel exurantur, & superciliosa complanentur aspreta. Ita enim Cælius: quod quam vim habeat, nõ est huius loci curiosius perscrutari. Equidem M Tullius Mathematicos in magna rerum obscuritate, recondita arte multiplicíq; ac subtili versari scribit: sed quis nescit idipsum cum alijs grauioribus scientijs esse commune? Est enim, vel eodem auctore Tullio, omnis cognitio multis obstructa difficultatibus maximáq; est & in ipsis rebus obscuritas, & in iudicijs nostris infirmitas, nec vllus est, modò interius paulò Physica penetrarit, qui non facilè sit expertus, quàm multi vndique emergant, rerum naturalium causas inquirentibus, inexplicabiles labyrinthi. Sunt qui ex demonstrationum firmitate nominari Mathematicas opinantur: cuius etiam rationis momentum alio seorsim loco expendendum fuerit. Quocirca primam verbi notationem, quam sequutus est Proclus, nobis retinendam censeo. Hactenus de vniuerso Mathematicæ genere, quanta potui & perspicuitate & breuitate dixi. Sequitur vt de Geometria separatim atque ordine ea differam, quæ initio sum pollicitus.

tus. Est autem Geometria, vt definit Proclus scientia, quæ versatur in cognitione magnitudinũ figurarum, & quibus hæ continentur, extremorum, item rationum & affectionum, quæ in illis cernuntur ac inhærent: ipsa quidem progrediens à puncto indiuiduo per lineas & superficies, dũ ad solida conscendat, variasq́; ipsorum differentias patefaciat. Quúmque omnis scientia demonstratiua, vt docet Aristoteles, tribus quasi momentis contineatur, genere subiecto, cuius proprietates ipsa scientia exquirit & contemplatur: causis & principijs, ex quibus primis demonstrationes conficiuntur: & proprietatibus, quæ de genere subiecto per se enunciantur: Geometriæ quidem subiectum in lineis, triangulis, quadrangulis, circulis, planis, solidis, atque omnino figuris & magnitudinibus, earúmque extremitatibus consistit. His autem inhærent diuisiones, rationes, tactus, æqualitates, παραβολαὶ ὑπερβολαὶ ἐλλείψεισ, *atque alia generis eiusdem propè innumerabilia. Postulata verò & Axiomata ex quibus hæc inesse demonstrantur, eiusmodi ferè sunt: Quouis centro & internallo circulum describere. Si ab æqualibus æqualia detrahas, quæ relinquuntur esse æqualia, cæteraq́; id genus permulta, quæ licet omnium sint cõmunia, ac demonstrandum tamen tum sunt accommodata, cùm ad certum quoddam genus traducuntur. Sed cùm præcipua videatur Arithmetica & Geometriæ inter Mathematicas dignatio; cur Arithmetica sit* ἀκριβεστέρα *&*

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exactior quam Geometria paucis explicandum arbitror. Hic verò & Aristotelem sequemur ducem, qui scientiam cum scientia ita comparat, vt accuratiorem esse velit eam, quæ rei causam docet, quam quæ rem esse tantùm declarat, deinde quæ in rebus sub intelligentiam cadentibus versatur, quàm quæ in rebus sensum mouentibus cernitur. Sic enim & Arithmetica quàm Musica, & Geometria quàm Optica & Stereometria quàm Mechanica exactior esse intelligitur. Postremò quæ ex simplicioribus initijs constat, quàm quæ aliqua adiectione compositis vtitur. Atque hac quidem ratione Geometriæ præstat Arithmetica, quòd illius initium ex additione dicatur, huius sit simplicius. Est enim punctum, vt Pythagoreis placet, vnitas quæ situ obtinet: vnitas verò punctum est quod situ vacat. Ex quo percipitur, numerorum quàm magnitudinum simplicius esse elementum, numerosque magnitudinibus esse puriores, & à concretione materiæ magis disiunctos. Hæc quanquam nemini sunt dubia, habet & ipsa tamen Geometria quo se plurimum efferat, opibúsque suis ac rerum vbertate multiplici vel cum Arithmetica certet: id quod tute facilè deprehendas cùm ad infinitam magnitudinis diuisionem, quam respuit multitudo, animum conuerteris. Nunc quæ sit Arithmeticæ & Geometriæ societas, videamus. Nam theorematum quæ demonstratione illustrantur, quædam sunt vtriusque scientiæ communia, quædam verò

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singularum propria. Etenim quòd omnis proportio sit ῥητὸς siue rationalis, Arithmeticæ soli conuenit, nequaquam Geometriæ, in qua sunt etiam ἄῤῥητοι. seu irrationales proportiones. item, quadratorum gnωmas minimo definitos esse. Arithmeticæ proprium (si quidem in Geometria nihil tale minimum esse potest) sed ad Geometriam propriè spectant situs. qui in numeris locum non habent: tactus, qui quidem à continuis admittuntur: ἄλογον, quoniam vbi diuisio infinitè procedit, ibi etiam τὸ ἄλογον esse solet. Communia porrò vtriusque sunt illa, quæ ex sectionibus eueniunt, quas Euclides libro secundo est persequutus: nisi quòd sectio per extremam & mediam rationem in numeris nusquam reperiri potest. Iam verò ex theωrematibus eiusmodi communibus, alia quidem ex Geometria ad Arthmeticam traducuntur: alia contrà ex Arithmetica in Geometriam trāsferuntur: quædam verò perinde vtrique scientiæ conueniunt, vt quæ ex vniuersa arte Mathematica in vtranque harum conueniant. Nā & alterna ratio, & rationum conuersiones, compositiones, diuisiones hoc modo communia sunt vtriusque. Quæ autem sunt περὶ συμμέτρων, id est, de commensurabilibus, Arithmetica quidem primùm cognoscit & contemplatur: secundo loco Geometria Arithmeticam imitata. Quare & commensurabiles magnitudines illæ dicuntur, quæ rationem inter se habent quam numerus ad numerum, per-

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inde quasi commensuratio & συμμετρία in numeris primùm consistat (Vbi enim numerus, ibi & σύμμετρον cernitur: & vbi σύμμετρον, illic etiam numerus) sed quæ triangulorum sunt & quadrangulorum, à Geometra primùm considerantur: tùm analogia quadam Arithmeticus eadem illa in numeris contemplatur. De Geometriæ diuisione, hoc adijciendum puto, quòd Geometriæ pars altera in planis figuris cernitur quæ solam latitudinem longitudini coniunctam habent: altera verò solidas contemplatur, quæ ad duplex illud interuallum crassitudinem adsciscunt. Illam generali Geometriæ nomine veteres appellarunt: hanc propriè Stereometriam dixerunt. Ita Geometriam cum Optica, & Stereometriam cum Mechanica non rarò comparat Aristoteles. Sed illius cognitio huius inuẽtionem multis seculis antecessit, si modò Stereometriam ne Socratis quidem ætate vllam fuisse omnino verum est, quemadmodum à Platone scriptum videtur. Ad Geometriæ vtilitatem accedo, quæ quanquam suapte vi & dignitate ipsa per se nititur, nullius vsus aut actionis ministerio mancipata (vt de Mathematicis omnibus scientijs concedit in Politico Socrates) si quid ex ea tamen vtilitatis externæ quæritur, Dij boni quã latos, quàm vberes, quũ varios fructus fundit? Nec verò audiendus est vel Aristippus, vel Sophistarum alius, qui Mathematicorum artes idcirco repudiet, quòd ex fine nihil docere videantur, eiusque quod melius aut deteri-

deterius nullam habeant rationem Ut enim nihil causae dicas, cur sit melius, triãguli, verbi gratia, tres angulos duobus esse rectis aequales: minimè tamen fuerit consentaneum Geometriae cognitionem ut inutilem exagitare, criminari, explodere, quasi quae finem & bonum quò referatur, habeat nullum. Multas haud dubiè solius contemplationis beneficio citra materiae contagionem adfert Geometria commoditates partim proprias, partim cum uniuerso genere communes. Cùm enim Geometria, ut scripsit Plato, eius quod semper est cognitionem profiteatur, ad veritatem excitabit illa quidem animum, & ad ritè philosophandum cuiusque mentem comparabit. Quinetiam ad disciplinas omnes facilius perdiscendas, attigeris necne Geometriam, quãti referre censes? Nam ubi cum materia coniũgitur, nónne praestantissimas procreat artes, Geodaesiam, Mechaniam, Opticam, quarum omnium usu, mortalium vitam summis beneficijs complectitur? Etenim bellica instrumenta, urbiumq; propugnacula, quibus munitae urbes hostium vim propulsarent, his adiutricibus fabricata est: montium ambitus & altitudines, locorumque situs nobis indicauit: dimetiendorum & mari & terra itinerum rationem praescripsit: trutinas & stateras, quibus exacta numerorum aequalitas in ciuitate retineatur, composuit: uniuersi ordinem simulachris expressit, multáque quae hominum fidem superarent, omnibus persuasit. Ubique extant praeclara in eam rem *testimonia. Illud memorabile, quòd Archimedi rex Hiero tribuit. Nam extructo vastæ molis nauigio, quod Hiero Ægyptiorum regi Ptolemæo mitteret, cùm vniuersa Syracusanorum multitudo collectis simul viribus nauem trahere non posset effecissetq́; Archimedes, vt solus Hiero illam subduceret, admiratus viri scientiã rex ἀπὸ ταύτης, ἔφη, τῆς ἡμέρας περὶ παντὸς ἀρχιμήδει λέγοντι πιστευτέον. Quid? quòd Archimedes idem, vt est apud Plutarchum, Hieroni scripsit datis viribus datum pondus moueri posse? fretusq́; demonstrationis robore, illud sæpe iactarit, si terram haberet alteram vbi pedem figeret, ad eam, nostram hanc se transmouere posse? Quid varia, αὐτομάτων machinarumq́ue genera ad vsus necessarios comparata memorem? Innumerabilia profectò sunt illa, & admiratione dignissima quibus prisci homines incredibili quodam ad philosophandum studio concitati, inopem mortalium vitam artis huius præsidio subleuarunt: tametsi memoriæ sit proditum, Platonem Eudoxo & Archytæ vitio vertisse, quōd Geometrica problemata ad sensilia & organica abducerent. Sic enim corrumpi ab illis & labefieri Geometriæ præstantiam, quæ ab intelligibilibus & incorporeis rebus ad sensiles & corporeas prolaberetur. Quapropter ridicula idem scripsit Plato Geometrarum esse vocabula, quæ quasi ad opus & actionem spectent, ita sonare videntur. Quid enim est quadrare, si non opus facere? Quid addere,*

pro-

producere, applicare? Multa quidem sunt eiusmodi nomina, quibus necessariò & tanquam coacti Geometræ vtuntur, quippe cùm alia desint in hoc genere commodiora. Sic ergo censuit Plato, sic Aristoteles sic denique philosophi omnes, Geometriam ipsam cognitionis gratia exercendam, nec ex aliquo vsu externo sed ex rerum νοητῶν intelligentia æstimandam esse. Exposita breuius quàm res tanta dici possit, vtilitatis ratione, Geometriæ ortum, qui in hac rerum periodo ex historicorum monumentis nobis est cognitus deinceps aperiamus. Geometria apud Ægyptios inuenta, (ne ab Adamo, Setho, Noah, quos cognitione rerum multiplici valuisse constat, eam repetamus) ex terrarum dimensione, vt verbi præ se fert ratio, ortum habuisse dicitur: cùm anniuersaria Nili inundatione & incrementis limo obducti agrorum termini confunderentur. Geometriam enim, sicut & reliquas disciplinas, in vsu quàm in arte prius fuisse aiũt. Quod sanè mirum videri non debet, vt & huius & aliarum scientiarum inuentio ab vsu cœperit ac necessitate. Etenim tempus, rerum vsus, ipsa necessitas ingenium excitat, et ignauiã acuit. Deinde quicquid ortum habuit (vt tradunt Physici) ab inchoato & imperfecto processit ad perfectum. Sic artium & scientiarum principia experientiæ beneficio collecta sunt: experientia verò à memoria fluxit, quæ & ipsa à sensu primùm manauit. Nam quod scribit Aristoteles, Mathematicas artes, *comparatis rebus omnibus ad vitam necessarijs, in Ægypto fuisse constitutas, quòd ibi sacerdotes omnium concessu in otio degerent: non negat ille adductos necessitate homines ad excogitandam, verbi gratia, terræ dimetiendæ rationem, quæ theorematum deinde inuestigationi caussam dederit: sed hoc confirmat, præclara eiusmodi theorematum inuenta, quibus extructa Geometriæ disciplina constat, ad vsus vitæ necessarios ab illis non esse expetita. Itaque vetus ipsum Geometriæ nomen ab illa terræ partiundæ finiúmque regundorum ratione posteà recessit, & in certa quadam affectionum magnitudini per se inhærẽtium scientia propriè remãsit. Quemadmodum igitur in mercium & contractuum gratiam, supputandi ratio, quam secuta est accurata numerorum cognitio, à Phœnicibus initium duxit; ita etiam apud Ægyptios, ex ea quam cõmemoraui causa ortũ habuit Geometria. Hãc certè, vt id obiter dicam, Thales in Græciam ex Ægypto primùm transtulit: cui non pauca deinceps à Pythagora, Hippocrate, Chio, Platone, Archyta Tarentino, alijsq; cõpluribus, ad Euclidis tempora facta sunt rerum magnarum accessiones. Cæterùm de Euclidis ætate id solùm addam, quod à Proclo memoriæ mandatum accepimus. Is enim commemoratis aliquot Platonis tum æqualibus tùm discipulis, subijcit, non multò ætate posteriorem illis fuisse Euclidem eum, qui Elementa conscripsit, & multa ab Eudoxo collecta, in ordinẽ luculentũ* composuit, multaq; à Theateto inchoata perfecit, queq; mollius ab alijs demonstrata fuerant, ad firmissimas & certissimas apodixes reuocauit. Vixit autem, inquit ille, sub primo Ptolemæo. Etenim ferunt Euclidam à Ptolemæo quõdam interrogatum numqua esset via ad Geometriam magis compendiaria, quàm sit ista στοιχειώσις, respondisse, μὴ εἶναι βασιλικὴν ἀτραπὸν ἐπὶ γεωμετρίαν. Deinde subiungit, Euclidem natu quidem esse minorem Platone, maiorem verò Eratosthene & Archimede (hi enim æquales erant) cùm Archimedes Euclidis mentionem faciat. Quòd si quis egregiam Euclidis laudem quam cùm ex alijs scriptionibus accuratissimis, tùm ex hac Geometrica στοιχειώσει cõsequutus est in qua diuinus rerum ordo sapientissimis quibusque hominibus magnæ semper admirationi fuit, is Proclum studiosè legat, quò rei veritatẽ illustriorem reddat grauissimi testis auctoritas. Superest igitur vt finem videamus, quò Euclidis elementa referri, & cuius causa in id studium incumbere oporteat. Et quidem si res quæ tractantur, consyderes: in tota hac tractatione nihil aliud quæri dixeris, quàm vt κοσμικὰ quæ vocantur, σχήματα (fuit enim Euclides professione & instituto Platonicus) Cubus, Icosaedrũ, Octaedrum, Pyramis & Dodecaedrum certa quadã suorum & inter se laterum, & ad sphæræ diametrum ratione eidem sphæræ inscripta cõprehendantur. Huc enim pertinet Epigrammation illud vetus, quod in Geometrica Michaelis

Pselli συνόψει *scriptum legitur.*

Σχήματα πέντε τοῦ πλάτωνος, πυθαγόρας σοφὸς
εὗρε,
Πυθαγόρας σοφὸς εὗρε, πλάτων δ' ἀριδήλ' ἐδί-
δαξεν,
Εὐκλείδης ἐπὶ τοῖσι κλέος περικαλλὲς ἔτευξεν.

Quòd si discentis institutionem spectes, illud certè fuerit propositum, vt huiusmodi elementorum cognitione informatus discentis animus, ad quamlibet non modò Geometriæ, sed & aliarũ Mathematicæ partium tractationem idoneus paratusq́ accedat. Nam tametsi institutionem hanc solus sibi Geometra vendicare videtur, & tanquam in possessionem suam venerit, alios excludere posse: inde tamen per multa suo quodam modo iure decerpit Arithmeticus, pleraque Musicus, non pauca detrahit Astrologus, Opticus, Logisticus, Mechanicus, itemq́ cæteri: nec vllus est denique artifex præclarus, qui in huius se possessionis societatem cupidè nõ offerat, partemq́ sibi concedi postulet. Hinc στοιχείωσις *absolutum operi nomen, &* στοιχειωτὴς *dictus Euclides. Sed quid longius prouehor? Nam quod ad hanc rem attinet, tam copiosè & eruditè scripsit (vt alia complura) eo ipso, quem dixi, loco P. Montaureus, vt nihil desiderio loci reliquerit. Quæ verò ad dicendum nobis erant proposita, hactenus pro ingenij nostri tenuitate omnia mihi perfecisse videor. Nam tametsi & hæc eadem & alia pleraque multo fortè præclariora ab hominibus doctissimis,*

qui

qui tùm acumine ingenij, tùm admirabili quodam lepore dicendi semper floruerunt grauiùs, splendidiùs, vberiùs tractari posse scio: tamen experiri libuit numquid etiam nobis diuino sit concessum munere, quod rudes in hac philosophiæ partè discipulos adiuuare aut certè excitare queat. Huc accessit quòd ista recens elementorum editio, in qua nihil non parum fuisset studij, aliquid à nobis efflagitare videbatur, quod eius commendationem adaugeret. Cùm enim vir doctissimus Io Magnienus Mathematicarum artium in hac Parrhisiorum Academia professor verè regius, nostrum hunc typographum in excudendis Mathematicorum libris diligentissimum, ad hanc Elementorum editionem sæpè & multum esset adhortatus, eiusque impulsu permulta sibi iam comparasset typographus ad hanc rem necessaria, citò interuenit, malùm Ioannis Magnieni mors insperata, quæ tam graue inflixit Academiæ vulnus, cui ne post multos quidem annorum circuitus cicatrix obduci vlla posse videatur. Quamobrem amisso instituti huius operis duce, typographus, qui nec sumptus antea factos sibi perire, nec studiosos, quibus id muneris erat pollicitus, sua spe cadere vellet, ad me venit, & impensè rogauit, vt meam propositæ editioni operam & studium nauarem, quod cùm denegaret occupatio nostra, iuberet officij ratio, feci equidem rogatus, vt quæ subobscurè vel parum commode in sermonem Latinum è Græco translata

vi-

videbantur, clariore, aptiore & fideliore interpretatione nostra (quod cuiusq; pace dictum volo) lucem acciperent. Id quod in omnibus ferè libris posterioribus tute primo obtutu perspicias. Nam in sex prioribus nō tantum temporis quātum in cæteros ponere nobis licuit: decimi autem interpretatio, qua melior nulla potuit adferri, P. Montaureo solida debetur. Atq; ut ad perspicuitatem facilitatemque nihil tibi deesse querariss, adscripta sunt propositionibus singulis vel lineares figuræ, vel punctorum tanquam unitatum notulæ, quæ Theonis apodixin illustrēt: illæ quidem magnitudinum, hæ autem numerorum indices. subscriptis etiam ciphrarum, ut vocant, characteribus, qui propositum quemuis numerum exprimant. ob eamq; causam eiusmodi unitatum notulæ, quæ pro numeri amplitudine maius paginæ spatium occuparent, pauciores sæpius depictæ sunt aut in lineas etiam commutatæ. Nam literarum ut a, b, c characteres non modo numeris & numerorum partibus nominādis sunt accommodati, sed etiam generales esse numerorum, ut magnitudinum affectiones testantur. Adiecta sunt insuper quibusdam locis non pœnitenda Theonis scholia, siue mauis lemmata, quæ quidem longè plura accessissent, si plus otij & temporis vacui nobis fuisset relictū, quod huic studio impartiremus. Hanc igitur operam boni consule, & quæ obuia erunt impressionis vitia, candidè emenda. Vale. Lutetiæ 4. Idus April. 1557.

EVCLIDIS ELEMENTVM PRIMVM.

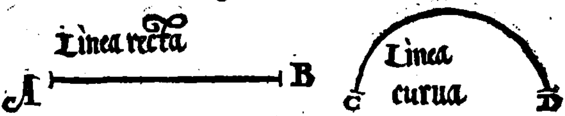
DEFINITIONES.

1

PVnctum est, cuius pars nulla est. Punctum

2

Linea verò, longitudo latitudinis expers.



3

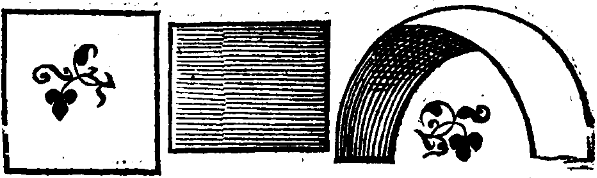
Lineæ autem termini, sunt puncta.

4

Recta linea est, quæ ex æquo sua interiacet puncta.

5

Superficies est, quæ longitudinem latitudinemque tantùm habet.



6

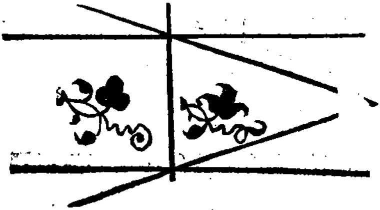
Superficiei extrema,ſunt lineæ.

7

Plana ſuperficies eſt,quæ ex æquo ſuas interiacet lineas.

8

Planus angulus eſt duarũ linearũ in plano ſe mutuò tangentium, & non in directum



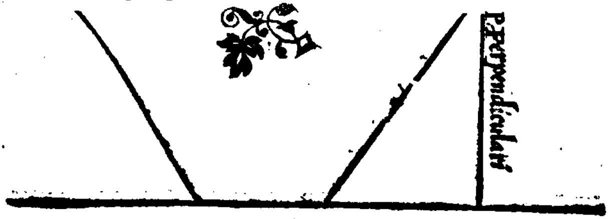
iacẽtium,alterius ad alteram inclinatio.

9.

Cùm autem quæ angulum continẽt lineæ, rectæ fuerint, rectilineus ille angulus appellatur.

10

Cùm verò recta linea ſuper rectam conſiſtens lineã, eos qui ſunt deinceps angulos ęquales,inter ſe fecerit:rectus eſt vterq; æqua-qualium angulorũ:& quæ inſiſtit recta linea,perpẽdicularis vocatur ei,cui inſiſtit



11

Obtuſus angulus eſt, qui recto maior eſt.

12

Acutus verò,qui minor eſt recto.

13

Terminus eſt,quod alicuius extremũ eſt.

14

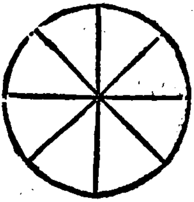
Figura eſt,quæ ſub aliquo , vel aliquibus terminis comprehenditur.

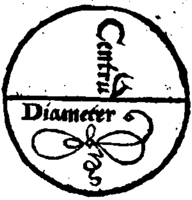
15

Circulus eſt figura plana ſub vna linea cõprehẽſa,quę peripheria appellatur:ad quã ab vno puncto eorum, quæ intra figuram

ſunt

ſunt po ſita, cadentes omnes rectæ lineæ inter ſe ſunt æquales.





16

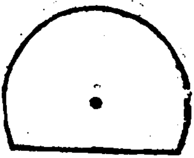
Hoc verò punctum, centrum circuli appellatur.

17

Diameter autem circuli, eſt recta quædam linea per cētrum ducta, & ex vtraque parte in circuli peripheriam terminata, quæ circulum bifariam ſecat.

18

Semicirculus eſt figura, quæ cōtinetur ſub diametro, & ſub ea linea, quæ de circuli peripheria aufertur.



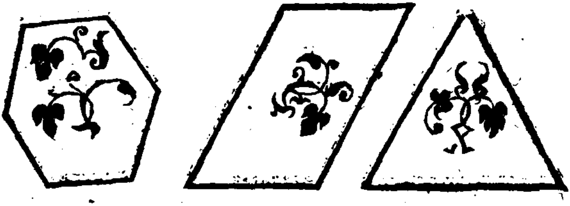
19

Segmentum circuli, eſt figura, quæ ſub recta linea & circuli peripheria continetur.

20 Recti

20

Rectilineæ figuræ, ſunt quæ ſub rectis lineis continentur.



21

Trilateræ quidem, quæ ſub tribus.

22

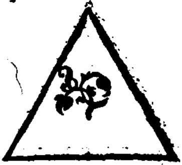
Quadrilateræ, quæ ſub quatuor.

23

Multilateræ verò, quæ ſub pluribus quàm quatuor rectis lineis comprehenduntur.

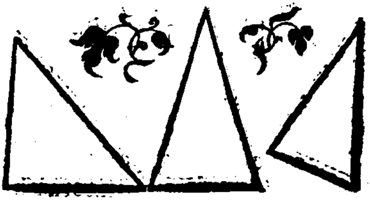
24

Trilaterarũ porrò figurarum, æquilaterum eſt triangulum, quod tria latera habet æqualia.



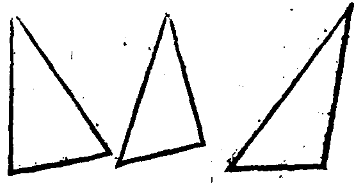
25

Iſoſceles autem eſt quod duo tantum æqualia habet latera.



26

Scalenũ verò, eſt qđ tria inęqualia habet laterà



27

Ad hæc etiam, trilaterarum figurarũ, rectangulum quidem triangulum eſt, quod rectum angulum habet.

28

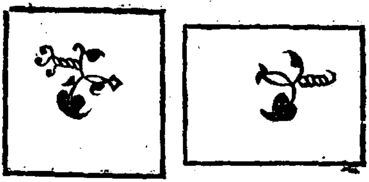
Amblygonium autem, quod obtuſum angulum habet.

29

Oxygonium verò, quod tres habet acutos angulos.

30

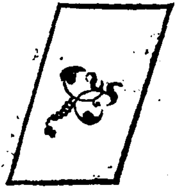
Quadrilaterarum autem figurarum, quadratũ quidẽ eſt qđ & æquilaterũ & rectangulum eſt.



31

Altera parte longior figura eſt, quæ rectãgula quidem, at æquilatera non eſt.

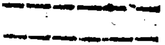
32
Rhombus autem, qui æquilaterum & rectangulum eſt.



33
Rhomboides verò, quæ aduerſa & latera & angulos habens inter ſe æqualia, neque æquilatera eſt, neque rectangula.

34
Preter has autem reliquæ quadrilateræ figuræ, trapezia appellentur.

35
Parallelæ rectæ lineæ ſunt quæ, cùm in eodem ſint plano, & ex vtraque parte in infinitum producantur, in neutram ſibi mutuò incidunt.



Poſtulata.

1
Poſtuletur, vt à quouis puncto in quoduis

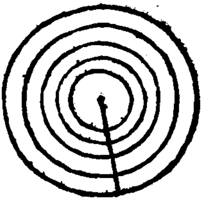
punctum,rectam lineā ducere cōcedatur.

2

Et rectam lineam terminatam in continuum recta producere.

3

In quouis centro & interuallo circulū deſcribere.



Communes notiones.

1.

Quæ eidē æqualia,& inter ſe ſunt æqualia.

2

Et ſi æqualibus æqualia adiecta ſint , tota ſunt æqualia.

3

Et ſi ab æqualibus æqualia ablata ſint,quæ relinquuntur ſunt æqualia.

4

Et ſi inæqualibus æqualia adiecta ſint, tota ſunt inæqualia.

5

Et ſi ab inęqualibus ęqualia ablata ſint, reliqua ſunt inæqualia.

6

Quæ ~~eiuſdem duplicia~~ ſunt, inter ſe ſunt æqualia.

Et

7

Et quæ eiuſdem ſunt dimidia, inter ſe æqualia ſunt.

8

Et quæ ſibi mutuò congruunt, ea inter ſe ſunt æqualia.

9

Totum eſt ſua parte maius.

10

Item, omnes recti anguli ſunt inter ſe ęquales.

11

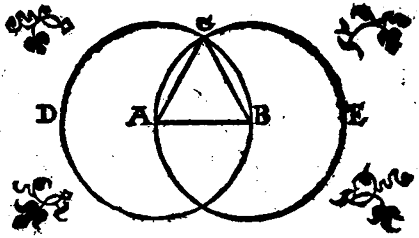
Et ſi in duas rectas lineas altera recta incidens, internos ad eaſdémque partes angulos duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ ſibi mutuò incident ad eas partes, vbi ſunt anguli duobus rectis minores.

12

Duæ rectæ lineæ ſpatium non comprehédunt.

Problema 1. Propoſitio 1. §35, A22

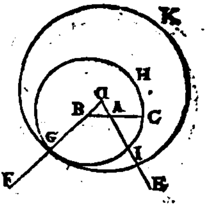
Super data recta linea terminata, triangulũ



æquilaterum conſtituere.

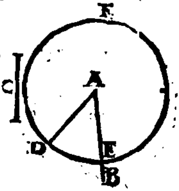
Problema 2. Propositio 2.

Ad datum punctum, datæ rectæ lineæ, ęqualem rectam lineam ponere.



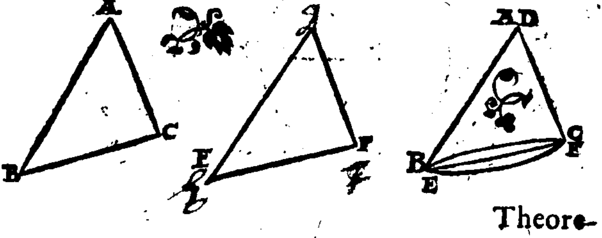
Problema 3. Propositio 3.

§ 30. 1. 4. Duabus datis rectis lineis inęqualibus, de maiore æqualē minori rectam lineā detrahere.



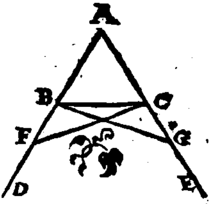
Theorema primum. Propositio 4.

Si duo triangula duo latera duobus lateribus æqualia habeant, vtrunque vtriq;, habeant verò & angulum angulo æqualē sub æqualibus rectis lineis contentum; & basin basi æqualem habebunt, eritq; triāgulum triāgulo æquale, ac reliqui anguli reliquis angulis æquales erūt, vterque vtrique, sub quibus æqualia latera subtenduntur.



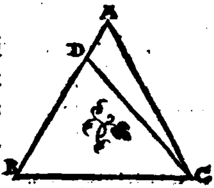
Theore-

Theorema 2. Propoſitio 5.



Iſoſcelium triangulorũ qui ad baſim ſunt anguli, inter ſe ſunt æquales; & ſi vlterius productæ ſint æquales illæ rectæ lineæ, qui ſub baſi ſunt anguli, inter ſe ęquales erunt.

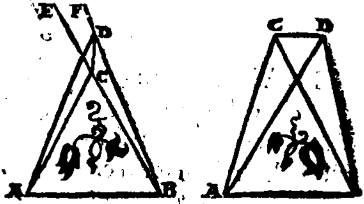
Theorema 3. Propoſitio 6.



Si trianguli duo anguli æquales inter ſe fuerint & ſub ęqualibus angulis ſubtenſa latera æqualia inter ſe erunt.

Theorema 4. Propoſitio 7.

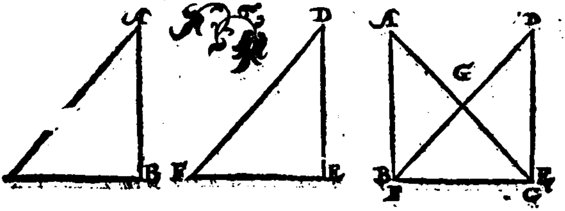
Super eadẽ recta linea, duabus eiſdem rectis lineis alię duę rectę lineę ęquales, vtraque vtrique, nõ conſtituentur, ad aliud atque aliud punctũ, ad eaſdẽ partes eoſdẽ



que terminos cum duabus initio ductis rectis lineis habentes.

Theorema 5. Propoſitio 8.

Si duo triãgula duo latera habuerint duobus lateribus, vtrunque vtriq;, æqualia: habuerint verò & baſim baſi æqualẽ: angulũ quoque ſub æqualibus rectis lineis cõtentum angulo æqualem habebunt.

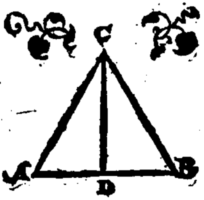


Problema 4. Propoſitio 9.

Datum angulum rectilineum bifariam ſecare.

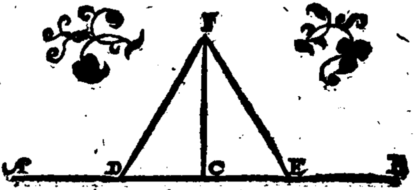
Problema 5. Propoſitio 10.

Datam rectam lineã finitam bifariam ſecare.



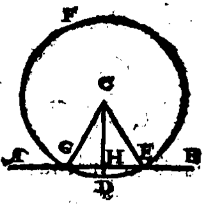
Proble-

Problema 6. Propositio 11.



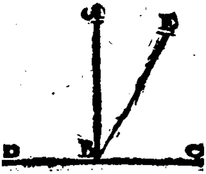
Data recta linea, à puncto in ea dato, rectam lineam ad angulos rectos excitare.

Problema 7. Propositio 12.



Super datam rectã lineã infinitam, à dato puncto quod in ea non est, perpendicularem rectam deducere.

Theorema 6. Propositio 13.



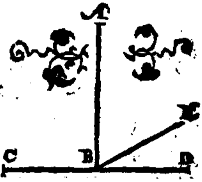
Cùm recta linea super rectã consistens lineã angulos facit, aut duos rectos, aut duobus rectis æquales efficiet.

Theorema 7. Propositio 14.

Si ad aliquam rectam lineam, atque ad eius

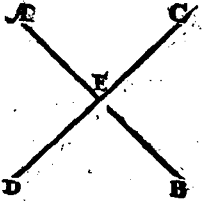
 punctum

punctũ, duę rectæ lineæ nõ ad eaſdem partes ductæ, eos qui ſunt deinceps angulos duobus rectis æquales fecerint, in directum erunt inter ſe ipſæ rectæ lineæ.



Theorema 8. Propoſitio 15.

Si duę rectę lineę ſe mutuò ſecuerint, angulos qui ad verticem ſunt, æquales inter ſe efficient.

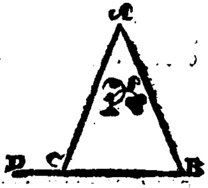


Theorema 9. Propoſitio 16.

Cuiuſcunque trianguli vno latere producto, externus angulus vtroque interno & oppoſito maior eſt.

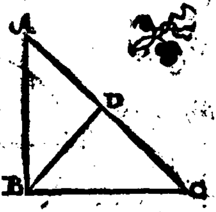
Theorema 10. Propoſitio 17.

Cuiuſcunque trianguli duo anguli duobus rectis ſunt minores omnifariam ſumpti.



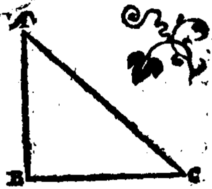
Theore-

Theorema 11. Propoſitio 18.



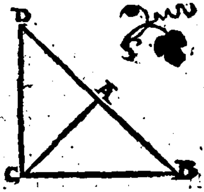
Omnis trianguli maius latus maiorem angulū ſubtendit.

Theorema 12. Propoſitio 19.



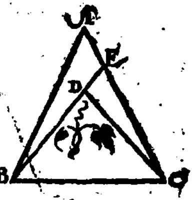
Omnis trianguli maior angulus, maiori lateri ſubtenditur.

Theorema 13. Propoſitio 20.



Omnis triãguli duo latera reliquo ſunt maiora, quomodocũq; aſsũpta.

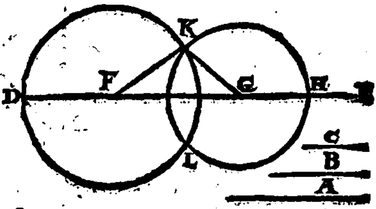
Theorema 14. Propoſitio 21.



Si ſuper triãguli vno latere, ab extremitatibus duæ rectę lineę, interius cõſtitutæ fuerint, hę cõſtitutæ reliquis triãguli duobus lateribus minores quidem erunt, minorem verò angulum continebunt.

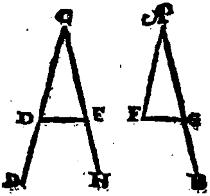
Pro-

Problema 8. Propositio 22.



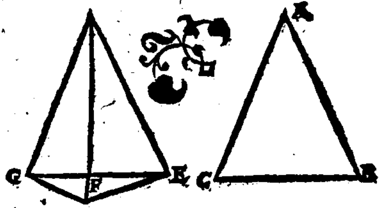
Ex tribus rectis lineis quæ sũt tribus datis rectis lineis æquales, triangulum constituere. Oportet autem duas reliqua esse maiores omnifariam sumptas: quoniã vniuscuiusq; trianguli duo latera omnifariam sumpta reliquo sunt maiora.

Problema 9. Propositio 23.



Ad datam rectam lineã datumq́; in ea punctum dato angulo rectilineo ę qualem angulum rectilineum constituere.

Theorema 15. Propositio 24.

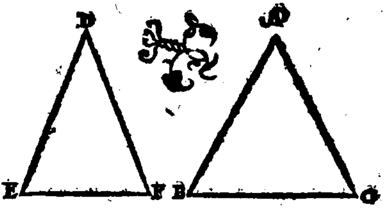


Si duo triãgula duo latera duobus lateribus ęqualia habuerint, vtrũq; vtriq;, angulũ verò angu
lo

lo maiorem sub æqualibus rectis lineis cõ-
tentum:& basin basi maiorem habebunt.

Theorema 16. Propositio 25.

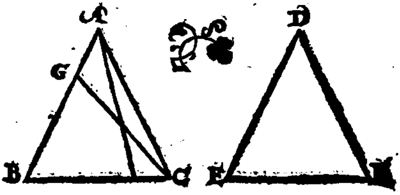
Si duo triangula duo latera duobus lateri
bus æqualia habuerint, vtrunque vtrique,
basin ve
rò basi
maiorẽ:
& angu
lum sub
ęqualibꝰ
rectis li-



neis contentum angulo maiorẽ habebunt.

Theorema 17. Propositio 26.

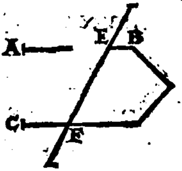
Si duo triangula duos angulos duobus an-
gulis æquales habuerint, vtrũque vtrique,
vnumq; latus vni lateri æquale, siue quod
æqualibus adiacet angulis, seu quod vni ę-
qualium angulorũ subtenditur:& reliqua
latera
reli-
quis la-
teribus
ęqualia
vtrũq;
vtriq;,



& reliquum angulũ reliquo angulo æqua-
lem habebunt.

Theore-

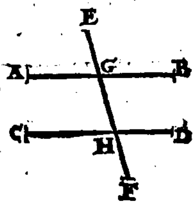
Theorema 18. Propositio 27.



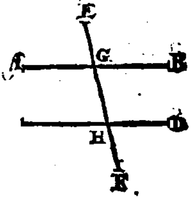
Si in duas rectas lineas recta incidẽs linea alternatim ãgulos equales inter se fecerit: parallelæ erunt inter se illæ rectæ lineæ.

Theorema 19. Propositio 28.

Si in duas rectas lineas recta incidens linea externum angulũ interno, & opposito, & ad easdem partes æqualẽ fecerit, aut internos & ad easdẽ partes duobus rectis æquales: parallelæ erunt inter se ipsæ rectæ lineæ.



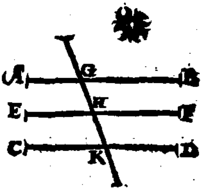
Theorema 20. Propositio 29.



In parallelas rectas lineas recta incidens linea: & alternatim angulos inter se æquales efficit & externum interno & opposito & ad easdem partes æqualem, & internos & ad easdem partes duobus rectis æquales facit.

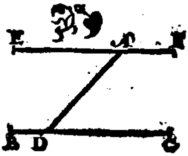
Theore-

Theorema 21. Pro-
positio 30.



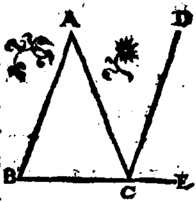
Quæ eidem rectæ lineę, parallelæ, & inter se sunt parallelæ.

Problema 10. Pro-
positio 31.



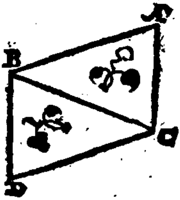
A dato puncto datæ rectæ lineæ parallelam rectam lineam ducere.

Theorema 22. Pro-
positio 32.



Cuiuscunque trianguli vno latere vlterius producto: externus angul⁹ duobus internis & oppositis est æqualis. Et trianguli tres interni anguli duobus sunt rectis æquales.

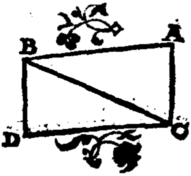
Theorema 23. Pro-
sitio 33.



Rectæ lineę quę ęquales & parallelas lineas ad partes easdem coniungunt, & ipsæ æquales & parallelæ sunt.

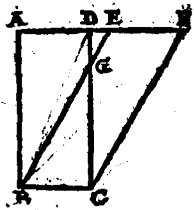
Theore-

Theorema 24. Propositio 34.



Parallelogrammorũ spatiorum æqualia sunt inter se, quæ ex aduerso & latera & anguli: atque illa bifariã secat diameter.

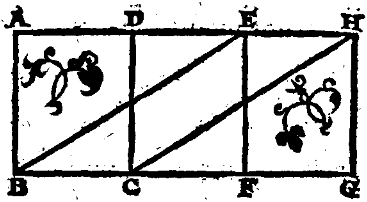
Theorema 25. Propositio 35.



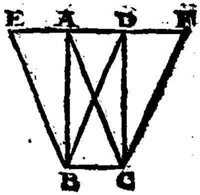
Parallelogramma super eadem basi, & in eisdem parallelis cõstituta, inter se sunt æqualia.

Theorema 26. Propositio 36.

Parallelogrãma super æqualibus basib⁹ & in eisdẽ parallelis cõstituta inter se sunt æqualia.



Theorema 27. Propositio 37.

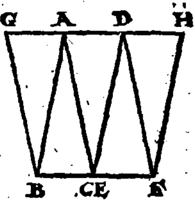


Triãgula super eadẽ basi cõstituta, & in eisdẽ parallelis, inter se sunt ęqualia.

Theore-

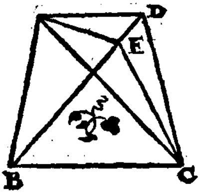
Theorema 28. Propoſitio 38.

Triangula ſuper æqualibus baſibus cōſtituta & in eiſdem parallelis, inter ſe ſunt æqualia.



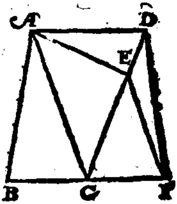
Theorema 29. Propoſitio 39.

Triãgula æqualia ſuper eadem baſi, & ad eaſdem partes cōſtituta: & in eiſdem ſunt parallelis.



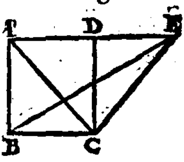
Theorema 30. Propoſitio 40.

Triãgula æqualia ſuper æqualibus baſibus, & ad eaſdem partes cōſtituta, & in eiſdē ſunt parallelis.

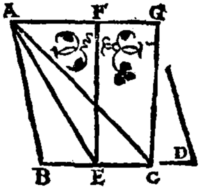


Theorema 31. Propoſitio 41.

Si parallelogrammum cum triangulo eandem baſin habuerit, nō eiſdemq́; fuerit parallelis, duplum erit parallelogrammum ipſius triãguli.

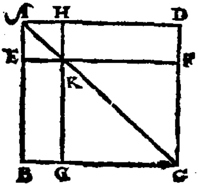


Problema 11. Propoſitio 42.



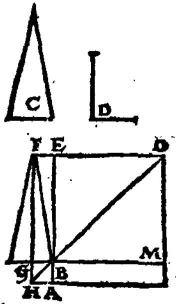
Dato triangulo æquale parallelogrammũ conſtituere in dato angulo rectilineo.

Theorema 32. Propoſitio 43.



In omni parallelogrammo, complementa eorũ quę circa diametrũ ſunt parallelogrammorum, inter ſe ſunt æqualia.

Problema 12. Propoſitio 44.

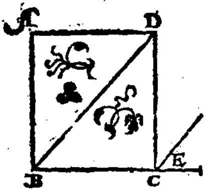


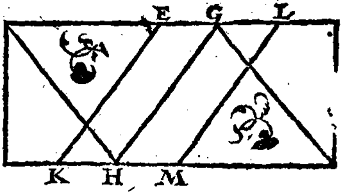
Ad datam rectam lineã, dato triangulo æquale parallelogrammum applicare in dato angulo rectilineo.

Problema 13. Propopoſitio 45.

Dato rectilineo æquale parallelogrammũ conſti-

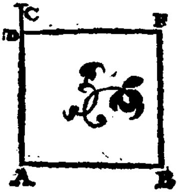
constituere in dato angulo rectilineo.





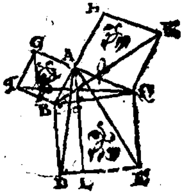
Problema 14. Propositio 46.

A data recta linea quadratum describere.



Theorema 33. Propositio 47.

In rectangulis triãgulis, quadratum quod à latere rectum angulũ subtendẽte describitur, æquale est eis, quę à lateribus rectum angulum continentibus describũtur, quadratis:

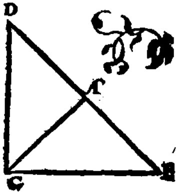


Theorema 34. Propositio 48.

Si quadratum quod ab vno laterum triã-

 guli

guli describitur, æquale sit eis quæ à reliquis triãguli lateribus describũtur, quadratis: angulus comprehensus sub reliquis duobus trianguli lateribus, rectus est.



FINIS ELEMENTI I.

EVCLI-

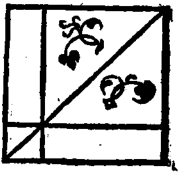
EVCLIDIS ELEMENTVM SECVNDVM.

DEFINITIONES.

I

OMne parallelogrammum rectangulum contineri dicitur sub rectis duabus lineis, quæ rectum comprehendunt angulum.

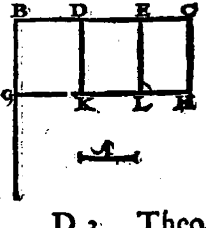
2



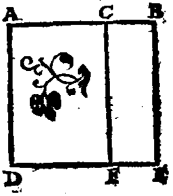
In omni parallelogrammo spatio, vnumquodlibet eorũ, quæ circa diametrum illius sunt parallelogrammorum, cũ duobus complementis, Gnomo vocetur.

Theorema 1. Propositio 1.

Si fuerint duæ rectæ lineæ, seceturq́; ipsarũ altera in quotcunque segmenta: rectangulum comprehensum sub illis duab⁹ rectis lineis, æquale est eis rectangulis, quę sub insecta & quolibet segmentorum comprehenduntur.



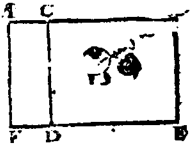
Theorema 2. Propositio 2.



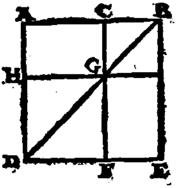
Si recta linea secta sit vtcunq;, rectãgula quæ sub tota & quolibet segmentorum cõprehenduntur æqualia sunt ei, quod à tota fit, quadrato.

Theorema 3. Propositio 3.

Si recta linea secta sit vtcunque, rectangulum sub tota & vno segmẽtorum comprehensum, equale est & illi quod sub segmentis cõprehenditur rectãgulo, & illi, quod à prædicto segmento describitur, quadrato.



Theorema 4. Propositio 4.

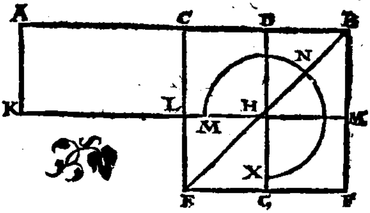


Si recta linea secta sit vtcunque: quadratũ quod à tota describitur, equale est & illis quæ à segmẽtis describũtur quadratis, & ei quod bis sub segmentis comprehẽditur rectangulo.

Theorema 5. Propositio 5.

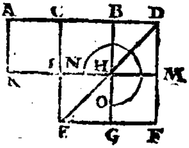
Si recta linea secetur in æqualia & non æqualia: rectangulum sub inæqualibus segmentis

mentis totius comprehẽsũ, vnà cum quadrato quod ab intermedia ſectionum, æquale eſt ei quod à dimidia deſcribitur, quadrato.



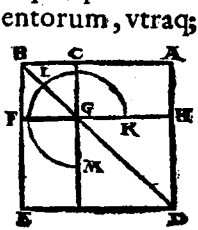
Theorema 6. Propoſitio 6.

Si recta linea bifariam ſecetur, & illi recta quædam linea in rectum adijciatur, rectãgulum compreheſum ſub tota cum adiecta & adiecta ſimul cum quadrato à dimidia, æquale eſt quadrato à linea, quæ tũ ex dimidia, tum ex adiecta componitur, tanquam ab vna deſcripto.



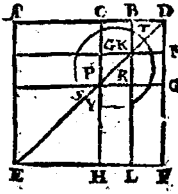
Theorema 7. Propoſitio 7.

Si recta linea ſecetur vtcunque; quod à tota, quodque ab vno ſegmentorum, vtraq; ſimul quadrata, æqualia ſunt & illi quod bis ſub tota & dicto ſegmento comprehenditur, rectãgulo, & illi ꝙ à reliquo ſegmento fit, quadrato.

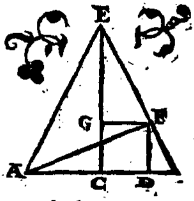


Theorema 8. Propoſitio 8.

Si recta linea ſecetur vtcunque rectangulum quater compreheſum ſub tota & vno ſegmentorum, cũ eo quod à reliquo ſegmento fit, quadrato, æquale eſt ei quod à tota & dicto ſegmento, tanquam ab vna linea deſcribitur, quadrato.



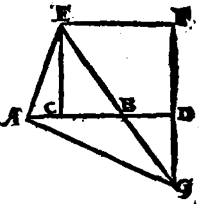
Theorema 9. Propoſitio 9.



Si recta linea ſecetur in æqualia & non æqualia: quadrata quæ ab inęqualibus totius ſegmẽtis fiunt, duplicia ſunt & eius quod à dimidia, & eius quod ab intermedia ſectionum fit, quadratorum.

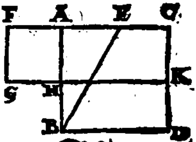
Theorema 10. Propoſitio 10.

Si recta linea ſecetur biſariam, adijciatur autem ei in rectum quępiam recta linea: quod à tota cũ adiuncta, & quod ab adiuncta, vtraq; ſimul quadrata, duplicia ſunt, & eius quod à dimidia, & eius quod à cõpoſita ex dimi-



dimidia & adiuncta, táquam ab vna descriptum sit, quadratorum.

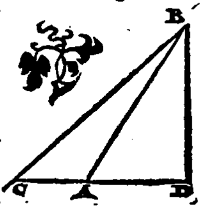
Problema 1. Propositio 11.



Datam rectam lineã secare, vt comprehensum sub tota & altero segmẽtorum rectangulum, æquale sit ei quod à reliquo segmento fit, quadrato.

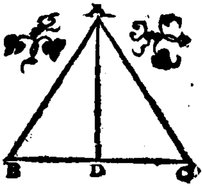
Theorema 11. Propositio 12.

In amblygonijs triangulis, quadratũ quod fit à latere angulum obtusum subtendẽte, maius est quadratis, quæ fiunt à lateribus obtusum angulum comprehendentibus, pro quantitate rectanguli bis comprehẽsi & ab vno laterũ quæ sunt circa obtusum angulũ, in quod cùm protractum fuerit, cadit perpendicularis, & ab assumpta exterius linea sub perpendiculari prope angulũ obtusum.



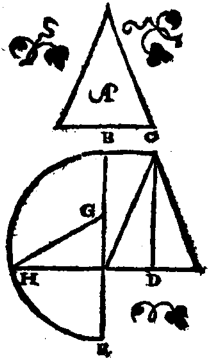
Theorema 12. Propoſitio 13.

In oxygonijs triangulis, quadratū à latere angulum acutum ſubtendente, minus eſt quadratis quæ fiunt à lateribus acutū angulum cōprehendentibus, pro quantitate rectanguli bis comprehenſi, & ab vno late rum, quæ ſunt circa acutum angulum, in quod perpēdicularis cadit, & ab aſſumpta interius linea ſub perpendiculari prope acutū angulum.



Problema 2. Propoſitio 14.

Dato rectilineo æquale quadratū conſtituere.



ELEMENTI II FINIS.

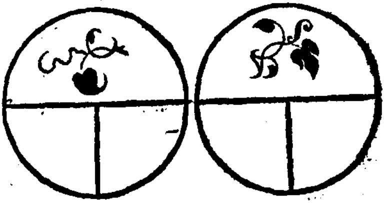
EVCLI-

EVCLIDIS ELEMENTVM TERTIVM.

DEFINITIONES.

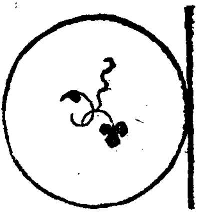
1

Aequales circuli ſunt, quorũ diametri sũt ęquales vel quorũ quæ ex centris rectæ lineę ſunt æquales.



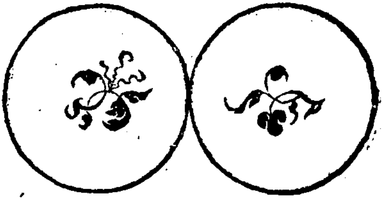
2

Recta linea circulũ tangere dicitur, quæ cùm circulum tangat, ſi producatur, circulum non ſecat.



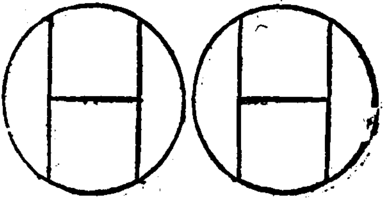
3

Circuli ſeſe mutuò tã-gere dicũtur: qui ſeſe mutuo tangentes, ſeſe mutuò non ſecant.



4.

In circulo æqualiter diſtare à centro rectæ lineæ dicuntur, cùm perpendiculares, quæ à cétro in ipſas ducuntur, ſunt æquales. Ló-gius autem abeſſe illa dicitur in quã maior ꝑ-pédicularis cadit.



5

Segmentũ circuli eſt, figura quæ ſub recta linea & circuli peripheria comprehenditur.

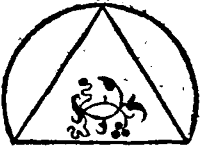
6

Segmenti auté angulus eſt, qui ſub recta linea

nea & circuli peripheria comprehẽditur.

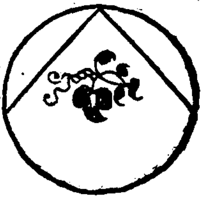
7

In ſegmento autem angulus eſt, cùm in ſegmenti peripheria ſumptũ fuerit quodpiam pũctum, & ab illo in terminos rectæ eiꝰ lineę, quę ſegmenti baſis eſt, adiunctę fuerint rectę lineę: is, inquam, angulus ab adiunctis illis lineis comprehenſus.



8

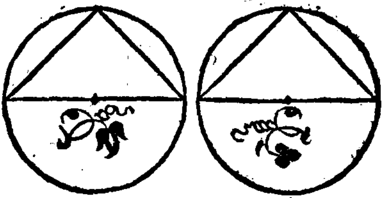
Cùm verò comprehendentes angulum rectę lineæ aliquã aſſumunt peripheriam, illi angulus inſiſtere dicitur.



Sector autem circuli eſt cùm ad ipſius circuli cẽtrum conſtitutus fuerit angulus, cõprehenſa nimirum figura, & à rectis lineis angulũ continentibus, & à peripheria ab illis aſſumpta.

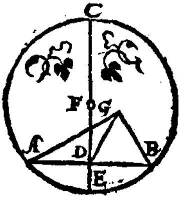
10

Similia circuli ſegmenta ſunt, quę angulos capiunt capiũt ęquales aut in q̃ b⁹ anguli inter se sunt ęquales



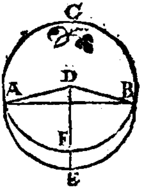
Problema 1. Propositio 1.

Dati circuli cẽtrum reperire.



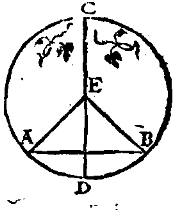
Theorema 1. Propositio 2.

Si in circuli peripheria duo quælibet pũcta accepta fuerint, recta linea quæ ad ipsa puncta adiũgitur, intra circulum cadet.



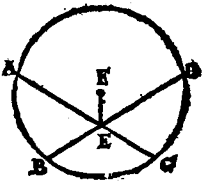
Theorema 2. Propositio 3.

Si in circulo rectá quædam linea per centrum extensa quandam non per centrum extensam bifariam secet: & ad angulos rectos ipsam secabit. Et si ad angulos rectos eam secet, bifariam quoque eam secabit.



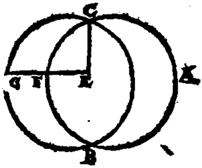
Theore-

Theorema 3. Propoſitio 4.



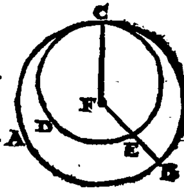
Si in circulo duę rectę lineæ ſeſe mutuò ſecent non per centrum extẽſæ ſeſe mutuò biſariam nõ ſecabunt.

Theorema 4. Propoſitio 5.



Si duo circuli ſeſe mutuò ſecent, non erit illorum idem centrum.

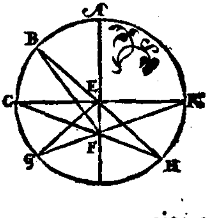
Theorema 5. Propoſitio 6.



Si duo circuli ſeſe mutuò interius tangant, eorum non erit idem cẽtrum.

Theorema 6. Propoſitio 7.

Si in diametro circuli quodpiam ſumatur punctum, quod circuli cẽtrum non ſit, ab eoque pũcto in circulũ quædam rectæ lineæ cadant: maxima quidem erit ea in qua centrũ, minima verò reliqua: aliarum verò propinquior illi quæ per centrũ du-

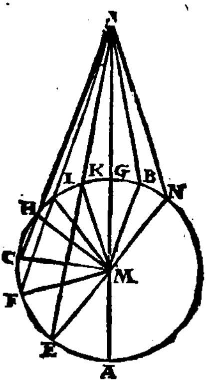


citur

citur, remotiore ſemper maior eſt. Duę autem ſolùm rectæ lineæ æquales ab eodem puncto in circulum cadũt ad vtraſque partes minimæ.

Theorema 7. Propoſitio 8.

Si extra circulum ſumatur punctũ quodpiam, ab eoque puncto ad circulum deducantur rectæ quædam lineæ, quarum vna quidem per centrum protẽdatur, reliquæ verò vt libet: in cauam peripheriam cadẽtium rectarum linearum maxima quidem eſt illa, quæ per centrum ducitur: aliarum autem propinquior ei, quę per centrum tranſit, remotiore ſemper maior eſt: in cõuexam verò peripheriã caden tium rectarũ linearum minima quidẽ eſt illa, quæ inter punctum & diametrum interponitur: aliarum autem, ea quæ propinquior eſt minimæ, remotiore ſemper minor eſt. Duæ

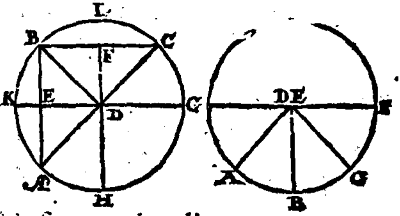


autem tantùm rectæ lineæ æquales ab eo puncto in ipſum circulum cadũt, ad vtraſque partes minimæ.

Theore-

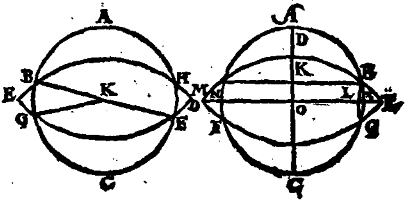
Theorema 8. Propositio 9.

Si in circulo acceptum fuerit pũctum aliquod, & ab eo puncto ad circulum cadant plures quã duę rectę lineæ, æquales, acceptũ punctũ centrum ipsius est circuli.



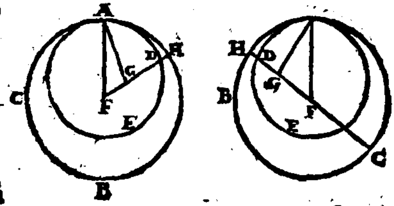
Theorema 9. Propositio 10.

Circulus circulum in pluribus quàm duob⁹ pũctis non secat.



Theorema 10. Propositio 11.

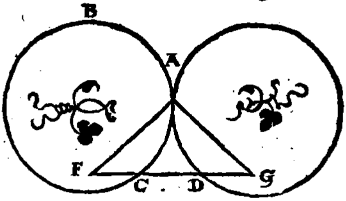
Si duo circuli sese intus cõtingãt, atque accepta



ſuerint eorum centra, ad eorum cẽtra adiuncta recta linea & producta in cõtactum circulorum cadet.

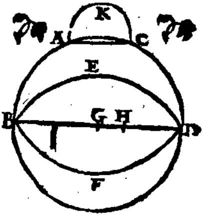
Theorema 11. Propoſitio 12.

Si duo circuli ſeſe exterius cõtingãt, linea recta q̃ ad cẽtra eorũ adiũgitur, ꝑ cõtactũ illũ trãſibit.



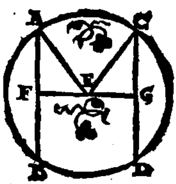
Theorema 12. Propoſitio 13.

Circulus circulum non tangit in pluribus punctis, quã vno, ſiue intus ſiue extra tangat.



Theorema 13. Propoſitio 14.

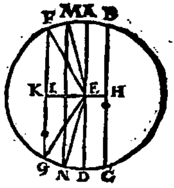
In circulo æquales rectæ lineæ æqualiter diſtant à centro. Et quæ ęqualiter diſtant à centro, æquales ſunt inter ſe.



Theore-

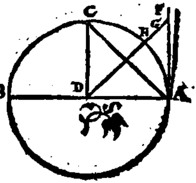
Theorema 14. Propo-
ſitio 15.

In circulo maxima quidem linea eſt diameter: aliarum autem propinquior centro, remotiore ſemper maior.



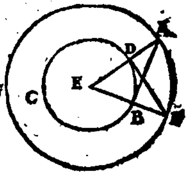
Theorema 15. Propoſitio 16.

Quæ ab extremitate diametri cuiuſq; circuli ad angulos rectos ducitur, extra ipsũ circulum cadet, & in locum inter ipſam re ctam lineam & peripheriam comprehenſum, al tera recta linea nõ cadet Et ſemicirculi quidem angulus quouis angulo acuto rectilineo maior eſt, reliquus autem minor,

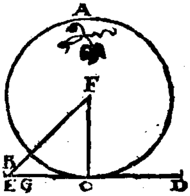


Problema 2. Propo-
ſitio 17.

A dato puncto rectam lineam ducere, quæ datum tangat circulum.

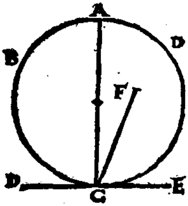


Theorema 16. Propoſitio 18.



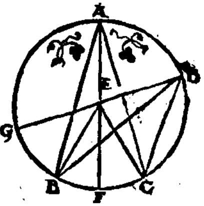
Si circulum tangat recta quæpiam linea, à centro autem ad cōtactum adiungatur recta quædam linea: quæ adiūcta fuerit ad ipſam contingentem perpendicularis erit.

Theorema 17. Propoſitio 19.



Si circulum tetigerit recta quæpiam linea, à contactu autem recta linea ad angulos rectos ipſi tāgenti excitetur, in excitata erit centrum circuli.

Theorema 18. Propoſitio 20.



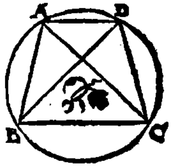
In circulo angulus ad cē trum duplex eſt anguli ad peripheriam, cū fuerit eadē peripheria baſis angulorum.

Theorema 19. Propoſitio 21.

In circulo, qui in eodem ſegmēto ſunt anguli, ſunt inter ſe æquales.

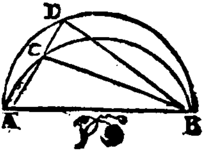
Theore-

Theorema 20. Propoſitio 22.



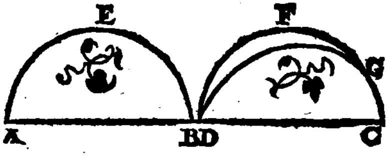
Quadrilaterorum in circulis deſcriptorũ anguli qui ex aduerſo, duobꝰ rectis ſunt æquales.

Theorema 21. Propoſitio 23.



Super eadem recta linea duo ſegmenta circulorũ ſimilia & inæqualia non conſtituẽtur ad eaſdem partes.

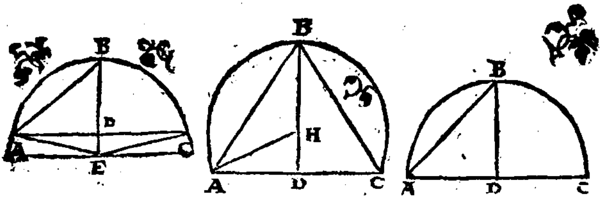
Theorema 22. Propoſitio 24.



Super ę qualibꝰ rectis lineis ſimilia circulorum ſegmenta ſunt inter ſe æqualia.

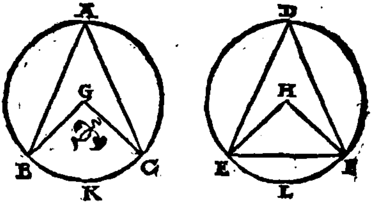
Problema 3. Propoſitio 25.

Circuli ſegmento dato, deſcribere circulũ cuius est segmentum.



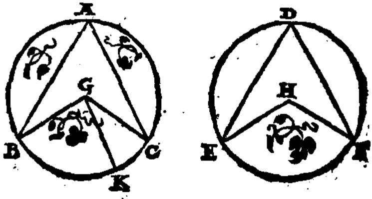
Theorema 23. Propositio 26.

In æqualibus circulis, æquales anguli æqua lib. pe riphe rijs in sistũt siue ad cẽ- tra, si- ue ad peripherias constituti insistant.



Theorema 24. Propositio 27.

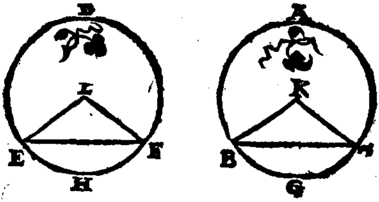
In æqualibus circulis, anguli qui æqualib⁹ peri- pherijs insistũt sunt in- ter se æ- quales siue ad cẽtra, siue ad peripherias cõstituti insistãt.



Theore-

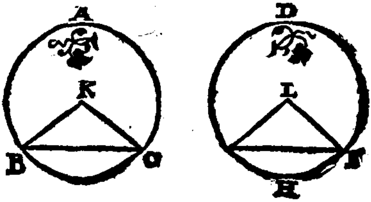
Theorema 25. Propositio 28.

In æqualibus circulis æquales rectæ lineæ equales peripherias auferũt maiorẽ quidẽ maiori, minorem autem minori.



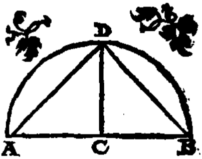
Theorema 26. Propositio 29.

In equalib⁹ circulis, æquales peripherias æquales rectæ lineæ subtendunt.



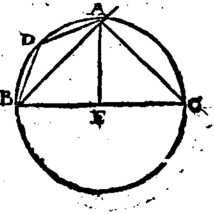
Problema 4. Propositio 30.

Datam peripheriam bifariam secare.



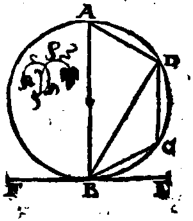
Theorema 27. Propositio 31.

In circulo angulus qui in semicirculo, rectus eſt:qui autem in maiore ſegméto, minor recto : qui verò in minore ſegméto, maior eſt recto. Et inſuper angulus maioris ſegméti, recto quidé maior eſt, minoris auté ſegméti angulus, minor eſt recto.



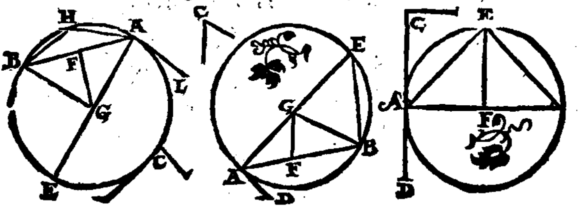
Theorema 28. Propoſitio 32.

Si circulum tetigerit aliqua recta linea, à contactu auté producatur quædam recta linea circulum ſecans: anguli quos ad contingentem facit, ęquales ſunt ijs qui in alternis circuli ſegmẽtis conſiſtunt, angulis.



Problema 5. Propoſitio 33.

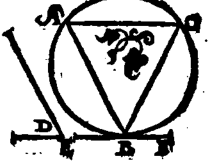
Super data recta linea deſcribere ſegmentum circuli quod capiat angulum æqualé dato angulo rectilineo.



Proble-

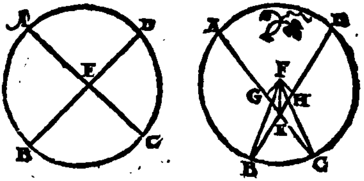
Problema 6. Propoſitio 34.

A dato circulo ſegmentum abſcindere capiens angulum æqualem dato angulo rectilineo.



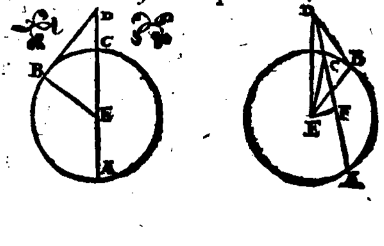
Theorema 29. Propoſitio 35.

Si in circulo duæ rectæ lineæ ſeſe mutuò ſecuerint, rectangulum cõprehenſum ſub ſegmẽtis vniꝰ, æquale eſt ei, quod ſub ſegmẽtis alterius comprehenditur, rectangulo.



Theorema 30. Propoſitio 36.

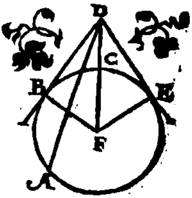
Si extra circulũ ſumatur pũctũ aliquod, ab eoque in circulũ cadant duę rectę lineę, quarum altera quidem circulum ſecet, altera



verò tangat: quod ſub tota ſecante & exterius inter punctum & cõuexam peripheriam aſſumpta comprehenditur rectangulum, æquale erit ei, quod à tangente deſcribitur, quadrato.

Theorema 31. Propoſitio 37.

Si extra circulũ ſumatur punctũ aliquod, ab eoq́; puncto in circulum cadant duæ rectæ lineæ, quarum altera circulum ſecet, altera in eum incidat, ſit autẽ quód ſub tota ſecante & exterius inter punctũ & cõuexam peripheriam aſſumpta, comprehenditur rectãgulum, æquale ei, quod ab incidẽte deſcribitur quadrato: incidens ipſa circulum tanget.



ELEMENTI III FINIS.

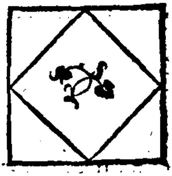
EVCLI-

EVCLIDIS ELEMENTVM QVARTVM.

DEFINITIONES.

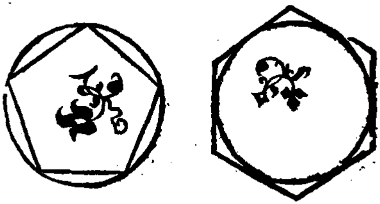
1

Figura rectilinea in figura rectilinea inſcribi dicitur, cũ ſinguli eius figuræ quæ inſcribitur, anguli ſingula latera eius, in qua inſcribitur, tangunt.



2

Similiter & figura cireum figurã deſcribi dicitur, quum ſingula eius quæ circũſcribitur, latera ſingulos eius figuræ angulos tetigerint, circum quam illa deſcribitur.



3

Figura rectilinea in circulo inſcribi dicitur, quũ ſinguli eius figuræ q̃ inſcribitur, angu-

anguli tetigerint circuli peripheriam.

4

Figura verò rectilinea circa circulú describi dicitur, quū singula latera eius, quæ circum scribitur, circuli peripheriã tangunt.

5

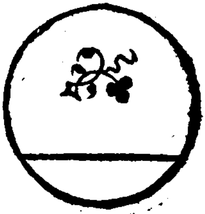
Similiter & circulus in figura rectilinea inscribi dicitur, quum circuli peripheria singula latera tãgit ei⁹ figuræ, cui inscribitur.

6

Circulus autem circum figurã describi dicitur, quū circuli peripheria singulos tãgit eius figuræ, quam circunscribit, angulos.

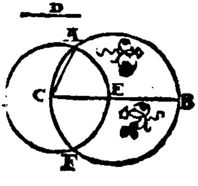
7

Recta linea in circulo accommodari seu coaptari dicitur, quum eius extrema in circuli peripheria fuerint.



Problema 1. Propositio 1.

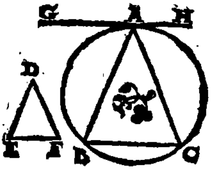
In dato circulo, rectam lineam accommodare æqualem datæ rectæ lineę quæ circuli diametro nó sit maior.



Proble-

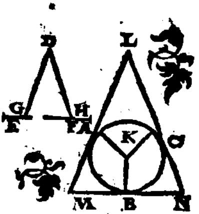
Problema 2. Propoſitio 2.

In dato circulo, triangulũ deſcribere dato triangulo æquiangulum.



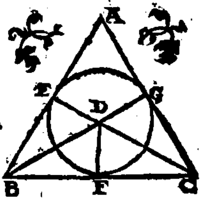
Problema 3. Propoſitio 3.

Circa datum circulũ triangulũ, deſcribere dato triangulo æquiangulũ.



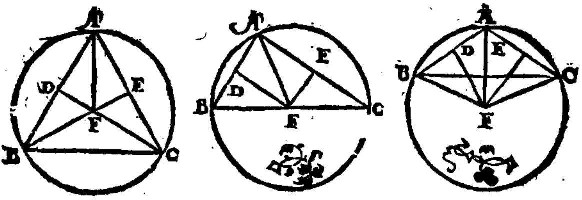
Problema 4. Propoſitio 4.

In dato triangulo circulum inſcribere.



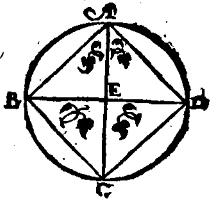
Problema 5. Propoſitio 5.

Circa datum triangulum, circulum deſcribere.



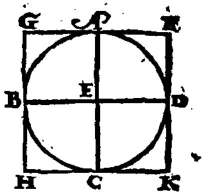
Pro-

Problema 6. Propositio 6.



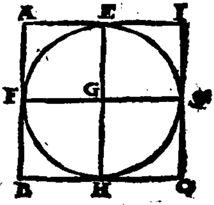
In dato circulo quadratum describere.

Problema 7. Propositio 7.



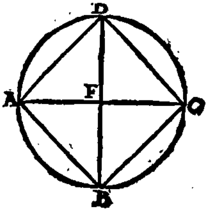
Circa datum circulum, quadratum describere.

Problema 8. Propositio 8.



In dato quadrato circulum inscribere.

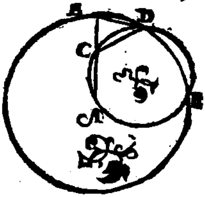
Problema 9. Propositio 9.



Circa datum quadratū, circulum describere.

Proble-

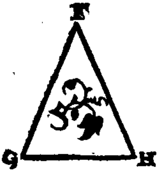
Problema 10. Propoſitio 10.



Iſoſceles triangulum cõſtituere, quod habeat vtrunque eorum, qui ad baſin ſunt, angulorum, duplum reliqui.

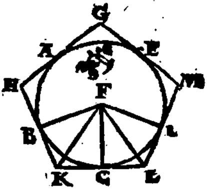
Theorema 11. Propoſitio 11.

In dato circulo, pentagonũ æquilaterũ & æquiãgulum inſcribere.



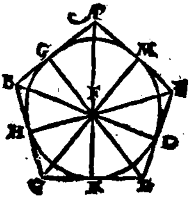
Problema 12. Propoſitio 12.

Circa datum circulum pentagonum æquilaterum & æquiangulũ deſcribere.



Problema 13. Propoſitio 13.

In dato pẽtagono æquilatero & æquiãgulo, circulum inſcribere.



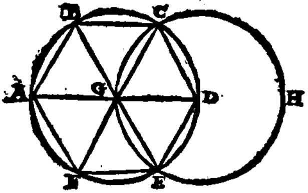
Proble-

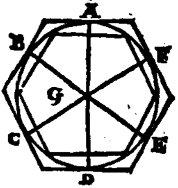
Problema 14. Propositio 14.

Circa datũ pentagonũ, æquilaterum & æquiangulum, circulum deſcribere.

Problema 15. Propoſitio 15.

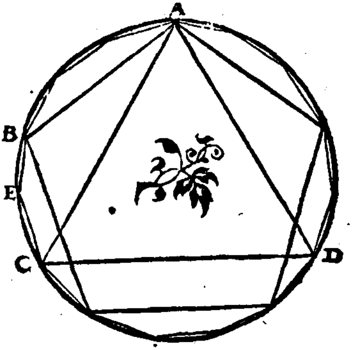
In dato circulo hexagonum & æquilaterũ & æquiangulum inſcribere.





Propoſitio 16. Theorema 16.

In dato circulo quintidecagonum & æquilaterũ & ęquiangulum deſcribere.



Elementi quarti finis.

EVCLIDIS ELEMENTVM QVINTVM.

DEFINITIONES.

1

PARS est magnitudo magnitudinis minor maioris, quum minor metitur maiorem.

2

Multiplex autem est maior minoris, cùm minor metitur maiorem.

3

Ratio, est duarum magnitudinum eiusdẽ generis mutua quædam secundum quantitatem habitudo.

4

Proportio verò, est rationum similitudo.

5

Rationẽ habere inter se magnitudinis dicuntur, quæ possunt multiplicatæ sese mutuò superare.

6

In eadem ratione magnitudines dicũtur esse, prima ad secundam, & tertia ad quartã: cùm primæ & tertiæ æquè multiplicia, à secundæ & quartæ æquè multiplicibus, qualiſcunq; ſit hæc multiplicatio, vtrunq; ab vtroque: vel vnà deficiunt, vel vnà ęqualia ſunt, vel vnà excedũt, ſi ea ſumãtur quæ inter ſe reſpondent.

7

Eandem autem habẽtes rationem magnitudines, proportionales vocentur.

8

Cũm verò æquè multiplicium, multiplex primę magnitudines exceſſerit multiplicẽ ſecũdæ, at multiplex tertiæ non exceſſerit multiplicem quartæ: tunc prima ad ſecundam, maiorem rationem habere dicetur, quàm tertia ad quartam.

9

Proportio autẽ in tribus terminis paucißimis conſiſtit.

10

Cùm autem tres magnitudines proportionales fuerint, prima ad tertiam, duplicatã rationẽ habere dicitur eius, quam habet ad ſecundam. At cùm quatuor magnitudines proportionales fuerint, prima ad quartã, triplicatam rationem habere dicitur eius quam habet ad ſecundam: & ſemper deinceps vno amplius, quandiu proportio extiterit.

11

Homologę, ſeu ſimiles ratione magnitudines dicuntur, antecedentes quidẽ antecedentidentibus, consequẽtes verò cõsequẽtibus.

12

Alterna ratio, est sumptio antecedẽtis cõparati ad antecedentem, & cõsequentis ad consequentem.

13

Inuersa ratio, est sumptio cõsequentis, ceu antecedentis, ad antecedentẽ velut ad cõsequentem.

14

Compositio rationis, est sumptio antecedentis cũ consequente ceu vnius, ad ipsum consequentem.

15

Diuisio rationis, est sumptio excessus quò consequentem superat antecedens ad ipsum consequentem.

16

Conuersio rationis, est sumptio antecedẽtis ad excessum, quo superat antecedẽs ipsum consequentem.

17

Ex æqualitate ratio est, si plures duab⁹ sint magnitudines, & his alię multitudine pares quæ binæ sumantur, & in eadem ratione: quũ vt in primis magnitudinibus prima ad vltimam, sic & in secundis magnitudinibus prima ad vltimã sese habuerit, vel aliter, sumptio extremorũ per subductionem mediorum.

 18 Or-

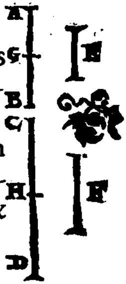
18

Ordinata proportio est, cùm fuerit quem admodum antecedens ad consequentem, ita antecedens ad consequentem : fuerit etiam vt consequēs ad aliud quidpiam, ita consequens ad aliud quidpiam.

19

Perturbata autē proportio est, tribus positis magnitudinibus, & alijs quæ sint his multitudine pares, cùm vt in primis quidē magnitudinibus se habet antecedēs ad cōsequentem, ita in secūdis magnitudinibus antecedens ad consequentem: vt autem in primis magnitudinibus consequens ad aliud quidpiam, sic in secundis magnitudinibus aliud quidpiam ad antecedentem.

Theorema 1. Propositio 1.

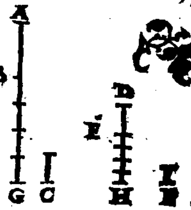


Si sint quotcūque magnitudines quotcunque magnitudinum æqualium numero singulę singularum æquè multiplices, quàm multiplex est vnius vna magnitudo, tam multiplices erunt, & omnes omnium.

Theorema 2. Propositio 2.

Si prima secundæ æquè fuerit multiplex, atque

atque tertia quartę, fuerit autē & quinta secūdę æquè multiplex, atque sexta quartæ: erit & cōposita prima cū quinta, secūdæ ęquè multiplex atque tertia cum sexta, quartæ.



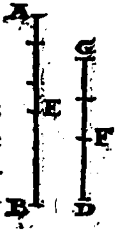
Theorema 3. Propositio 3.

Si sit prima secūdæ æquè multiplex atque tertia quartæ, sumātur autē æquè multiplices primę & tertiæ erit & ex æquo sumptarum vtraque vtriusque ęquè multiplex, altera quidem secundæ, altera autē quartæ.

Theorema 4. Propositio 4.

Si prima ad secundam, eandē habuerit rationem, & tertia ad quartam: etiam æque multiplices primę & tertiæ, ad æquè multiplices secundæ & quartæ iuxta quāuis multiplicationē, eandem habebunt rationem, si prout inter se

respondent, ita sumptæ fuerint.



Theorema 5. Propositio 5.

Si magnitudo magnitudinis æquè fuerit multiplex, atque ablata ablatæ: etiam reliqua reliquæ ita multiplex erit, vt tota totius.

Theorema 6. Propositio 6.

Si duæ magnitudines, duarum magnitudinum sint æquè multiplices, & detractę quædam sint earundem æquè multiplices: & reliquæ eisdem aut ęquales sunt, aut æquè ipsarum multiplices.

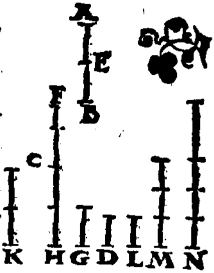
Theorema 7. Propositio 7.

Aequales ad eandem, eadem habent rationem: & eandem ad æquales.

Theorema 8. Propositio 8.

Inæqualium magnitudinũ maior, ad eandem

dem maiorem rationé habet, quàm minor: & eadem ad minorem: maiorem rationem habet, quàm ad maiorem.



Theorema 9. Propositio 9.

Quæ ad eandem, eandem habét rationem, æquales sũt inter se: & ad quas eadem, eandem habet rationem, eæ quoque sunt inter se æquales.

Theorema 10. Propositio 10.

Ad eandem magnitudiné, rationem habentium, quæ maiorem rationé habet, illa maior est, ad quam autem eadem maiorem rationem habet, illa minor est.

Theorema 11. Propositio 11.

Quæ eidem sunt eædem rationes, & inter se sunt eædem,

Theorema 12. Propositio 12.

Si sint magnitudines quotcunque proportionales, quemadmodū se habuerit vna antecedentium ad vnā

consequentiū, ita se habebunt omnes antecedentes ad omnes consequentes.

Theorema 13. Propositio 13.

Si prima ad secundam, eandē habuerit rationem, quam tertia ad quartā, tertia verò ad quartam, maiorem rationem habuerit, quàm quinta ad sextam, prima quoque ad secūdam maiorem rationem habebit, quàm quinta ad sextam.

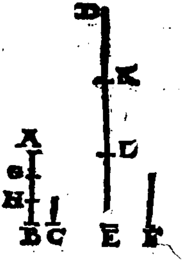
Theorema 14. Propositio 14.

Si prima ad secundam eandem habuerit rationē, quam tertia ad quartam, prima verò quàm tertia maior fuerit: erit & secunda maior quàm quarta. Quòd si prima fuerit æqualis tertiæ, erit

&

& ſecunda æqualis quartæ: ſi verò minor, & minor erit.

Theorema 15. Propoſitio 15.



Partes cum pariter multiplicibus in eadẽ ſunt ratione, ſi prout ſibi mutuo reſpondent, ita ſumantur.

Theorema 16. Propoſitio 16.

Si quatuor magnitudines proportionales fuerint, & viciſſim proportionales erunt.

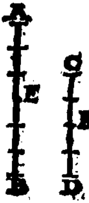
Theorema 17. Propoſitio 17.

Si compoſitæ magnitudines proportionales fuerint hæ quoq; diuiſæ proportionales erunt.

Theorema 18. Propositio 18.

Si diuisæ magnitudines sint proportionales, hæ quoque compositæ proportionales erunt.

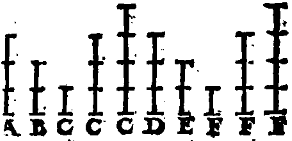
Theorema 19. Propositio 19.



Si quemadmodum totũ ad totum, ita ablatum se habuerit ad ablatum: & reliquum ad reliquum, vt totum ad totum se habebit.

Theorema 20. Propositio 20.

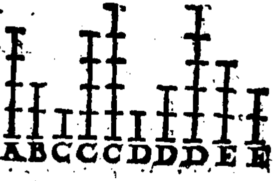
Si sint tres magnitudines, & aliæ ipsis æquales numero, quæ binæ & in eadem ratione sumatur, ex æquo autẽ prima quàm tertia maior fuerit; erit & quarta, quã sexta maior. Quòd si prima tertiæ fuerit æqualis, erit & quarta æqualis sextæ: sin illa minor, hæc quoque minor erit.



Theore-

Theorema 21. Propositio 21.

Si sint tres magnitudines, & alię ipsis æquales numero quæ binæ & in eadem ratione sumãtur, fueritque perturbata earũ proportio ex æquo autem prima quàm tertia maior fuerit erit & quarta quã sexta maior: quòd si prima tertiæ fuerit æqualis, erit & quarta æqualis sextæ: sin illa minor, hæc quoque minor erit.



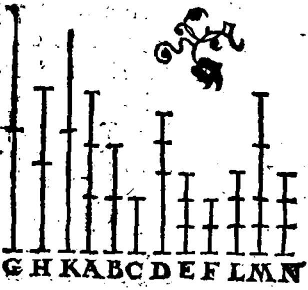
Theorema 22. Propositio 22.

Si sint quotcunq; magnitudines, & alię ipsis ęquales numero, quæ binæ in eadẽ ratione sumantur, et & ex æqualitate in eadem ratione erunt.

Theorema 23. Propositio 23.

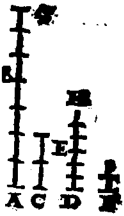
Si sint tres magnitudines, aliæq; ipsis ęquales

les numero, q̃ binæ in eadem ratione sumãtur, fuerit autẽ perturbata earũ proportio: etiã ex æqualite in eadem ratione erunt.



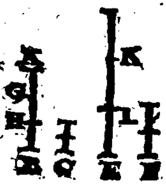
Theorema 24. Propositio 24.

Si prima ad secundam, eandem habuerit rationẽ, quam tertia ad quartam, habuerit autem & quinta ad secundam eandẽ rationem, quã sexta ad quartam: etiam cõposita prima cũ quinta ad secundam eandẽ habebit rationem, quã tertia cum sexta ad quartã.



Theorema 25. Propositio 25.

Si quatuor magnitudines proportionales fuerint, maxima & minima reliquis duabus maiores erunt.



ELEMENTI V. FINIS.

EVCLIDIS ELEMENTVM SEXTVM.

DEFINITIONES.

1

Similes figuræ rectilineæ sunt, quæ & angulos singulos singulis æquales habẽt, atque etiam latera, quæ circum angulos æquales, proportionalia.

2

Reciprocæ autem figuræ sunt, cùm in vtraque figura antecedentes & consequẽtes rationum termini fuerint.

3

Secundum extremam & mediam rationẽ recta linea secta esse dicitur, cùm vt tota ad maius segmentum, ita maius ad minus se habuerit.

4

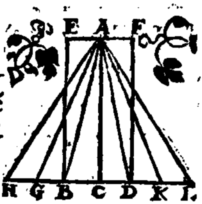
Altitudo cuiusque figuræ, est linea perpẽdicularis à vertice ad basin deducta.

5

Ratio ex rationibus cõponi dicitur, cũ rationum quantitates inter se multiplicatæ aliquã effecerint rationem.

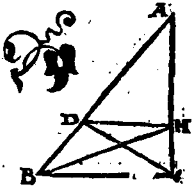
Theorema 1. Propositio 1.

Triangula & parallelogramma, quorum eadẽ fuerit altitudo, ita se habent inter se vt bases.



Theorema 2. Propositio 2.

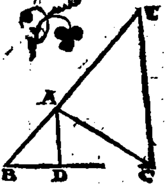
Si ad vnum trianguli latus parallela ducta fuerit recta quædam linea: hęc proportionaliter secabit, ipsius triãguli latera. Et si triãguli latera proportionaliter secta fuerint: quæ ad sectiones adiũcta fuerit recta linea, erit ad reliquum ipsius trianguli latus parallela.



Theorema 3. Propositio 3.

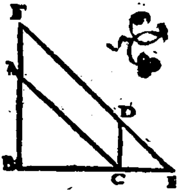
Si trianguli angulus bifariã sectus sit, secãs autem angulum recta linea secuerit & basim: basis segmenta eandem habebunt rationem,

tionem, quam reliqua ipsius trianguli latera. Et si basis segmenta eandem habeant rationem, quam reliqua ipsius trianguli latera, recta linea, quæ à vertice ad sectionem producitur, ea bifariã secat trianguli ipsius angulum.



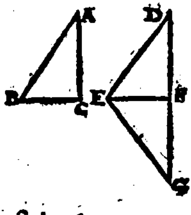
Theorema 4. Propositio 4.

Aequiangulorum triangulorum proportionalia sunt latera, quæ circum ęquales angulos, & homologa sunt latera, quæ æqualibꝰ angulis subtenduntur.



Theorema 5. Propositio 5.

Si duo triãgula latera proportionalia habeant, ęquiangula erunt triangula, & æquales habebunt eos angulos, sub quibus homologa latera subtenduntur.



Theorema 6. Propositio 6.

Si duo triangula vnum angulum vni angulo æqualẽ, & circũ ęquales angulos latera ꝓportionalia habuerint, ęquiangula erũt trian-

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angulos
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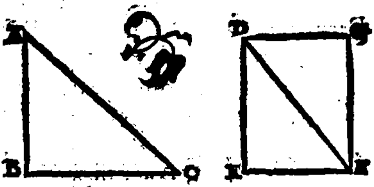
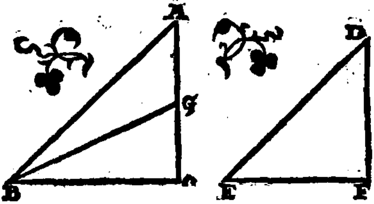
mologa latera ſubtenduntur.



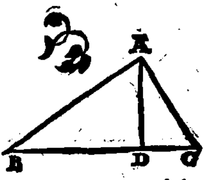
Theorema 7. Propoſitio 7.

Si duo triangula vnum angulum vni angulo æqualem, circũ autem alios angulos latera proportionalia habeant, reliquorum verò ſimul vtrunq; aut minorem aut non minorẽ recto: æquiangula erunt triangula, & ęquales habebunt eos angulos, circum quos ꝓportionalia ſunt latera.



Theorema 8. Propoſitio 8.

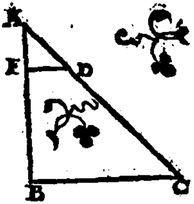
Si in triangulo rectangulo, ab angulo recto in baſin perpendicularis ducta ſit, quę ad perpendicularem triangula, tum toti triangulo, tum ipſa inter ſe ſimilia ſunt.



Proble-

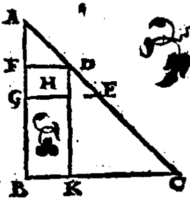
Problema 1. Propoſitio 9.

A data recta linea imperatam partem auferre.



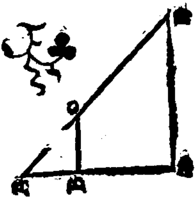
Problema 2. Propoſitio 10.

Datam rectam lineā inſectam ſimiliter ſecare, vt data altera recta ſecta fuerit.



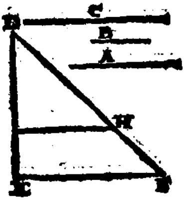
Problema 3. Propoſitio 11.

Duabus datis rectis lineis, tertiam proportionalem adinuenire.

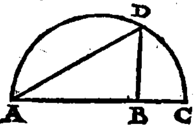


Problema 4. Propoſitio 12.

Tribꝰ datis rectis lineis, quartam proportionalem adinuenire.



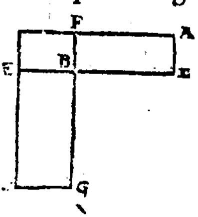
Problema 5. Propoſitio 13.



Duabꝰ datis rectis lineis mediam proportionalẽ adinuenire.

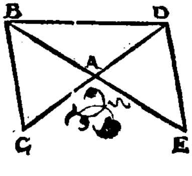
Theorema 9. Propoſitio 14.

Aequalium, & vnũ vni æqualem habentiũ angulum parallelogrammorum reciproca ſunt latera, quæ circum æquales angulos: & quorũ parallelogrammorũ vnũ angulum vni angulo æqualem habentiũ reciproca ſunt latera, quæ circum æquales angulos, illa ſunt æqualia.



Theorema 10. Propoſitio 15.

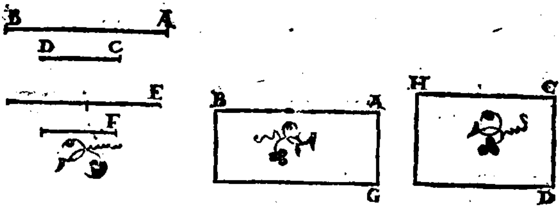
Aequalium, & vnũ angulum vni æqualem habentiũ triangulorũ reciproca ſunt latera, quæ circum æquales angulos: & quorũ triangulorũ vnum angulum vni ęqualem habentium reciproca ſunt latera, q̃ circum ęquales angulos, illa ſunt æqualia.



Theo-

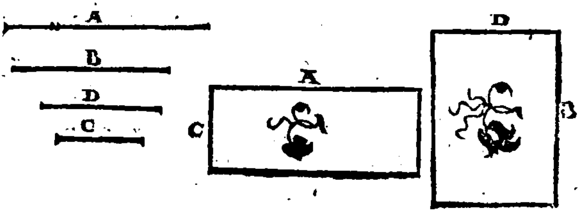
Theorema 11. Propoſitio 16.

Si quatuor rectæ lineæ proportionales fuerint, quod ſub extremis comprehenditur rectangulũ, æquale eſt ei, quod ſub medijs comprehenditur rectãgulo. Et ſi ſub extremis comprehenſum rectangulum æquale fuerit ei, quod ſub medijs cõtinetur rectãgulo, illæ quatuor rectæ lineæ proportionales erunt.



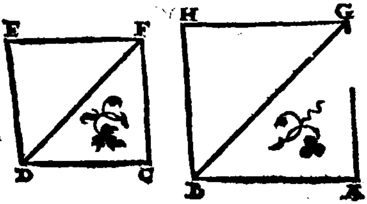
Theorema 12. Propoſitio 17.

Si tres rectæ lineæ ſint proportionales, qđ ſub extremis comprehenditur rectangulũ ęquale eſt ei, quod à media deſcribitur quadrato: & ſi ſub extremis comprehẽſum rectangulum æquale ſit ei quod à media deſcribitur quadrato, illæ tres rectæ lineæ ꝓportionales erunt. G 2 Pro-



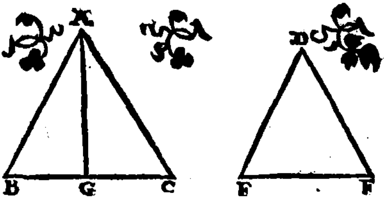
Problema 6. Propoſitio 18.

A data recta linea, dato recti lineo ſimile ſimiliterq; poſitum rectilineum deſcribere.



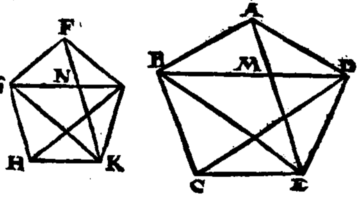
Theorema 13. Propoſitio 19.

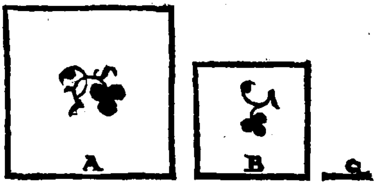
Similia triãgula inter ſe ſũt in duplicata ratiõe laterũ homologorũ.



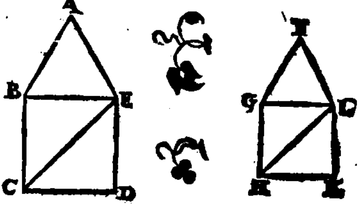
Theore. 14. Propoſitio 20.

Similia polygωna in ſimilia triangula diuiduntur, & numero æqualia, & homologa totis. Et po



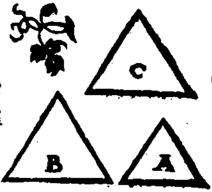


lygonadu plicatam habent eā inter se rationē, quā latus homologum ad homologum latus.



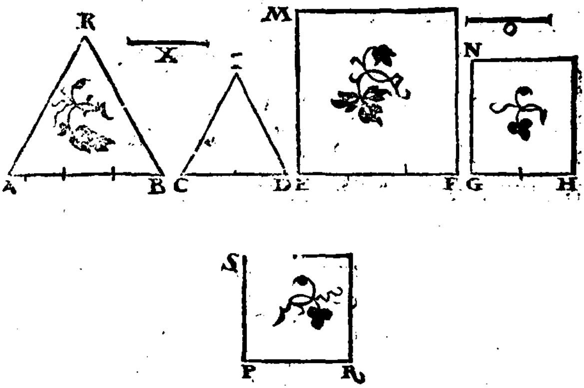
Theorema 15. Propositio 21.

Quæ eidem rectilineo sunt similia, & inter se sunt similia.

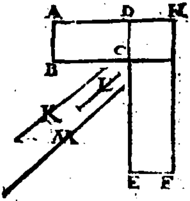


Theorema 16. Propositio 22.

Si quatuor rectę lineæ proportionales fuerint: & ab eis rectilinea similia similiterq; descripta proportionalia erūt. Et si à rectis lineis similia similiterq; descripta rectilinea proportionalia fuerint, ipsæ etiam rectæ lineæ proportionales erunt.

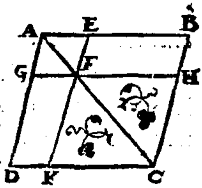


Theorema 17. Propositio 23.



Aequiangula parallelogramma inter se rationẽ habent eam, quę ex lateribus componitur.

Theorema 18. Propositio 24.

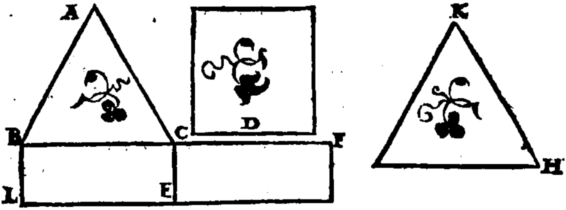


In omni parallelogrammo, quæ circa diametrũ sunt parallelogrãma, & toti & inter se sunt similia.

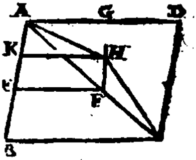
Proble-

Problema 7. Propoſitio 25.

Dato rectilineo ſimile, & alteri dato æquale idem conſtituere.



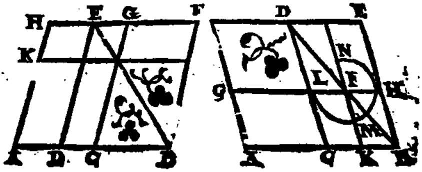
Theorema 19. Propoſitio 26.



Si à parallelogrāmo parallelogrammū ablatū ſit, & ſimile toti & ſimiliter poſitum communē cum eo habens angulum, hoc circum eandem cum toto diametrum conſiſtit.

Theorema 20. Propoſitio 27.

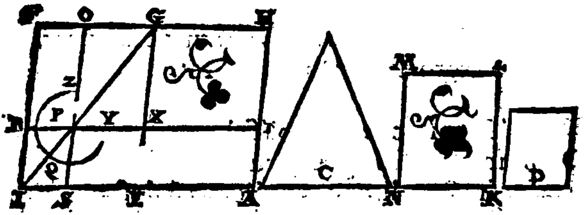
Omnium parallelogrāmorum ſecūdum eandem rectam lineam applicatorū deficiētiumq́; figuris parallelogrāmis ſi



milibus similiterque positis ei, quod à dimidia describitur, maximum, id est, quod ad dimidiam applicatur parallelogrammum simile existens defectui.

Problema 8. Propositio 28.

Ad datam lineam rectam, dato rectilineo æquale parallelogrammum applicare deficiens figura parallelogramma, quę similis sit alteri rectilineo dato. Oportet autẽ datum rectilineum, cui æquale applicandũm est, non maius esse eo quod ad dimidiam applicatur, cùm similes sint defectus & eius quod à dimidia describitur, & eius cui simile deesse debet.

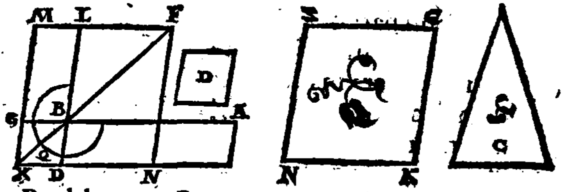


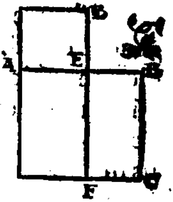
Problema 9. Propositio 29.

Ad datam rectam lineam, dato rectilineo æquale parallelogrammũ applicare, excedẽs figura parallelogramma, quæ similis sit

paral-

parallelogrammo alteri dato.



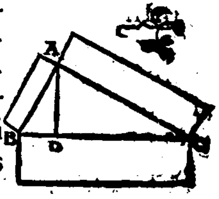


Problema 10. Propoſitio 30.

Propoſitam rectam lineam terminatã, extrema ac media ratione ſecare.

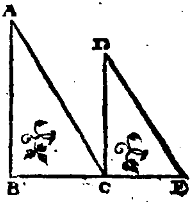
Theorema 21. Propoſitio 31.

In rectangulis triãgulis, figura quęuis à latere rectũ angulũ ſubtendẽte deſcripta ęqualis eſt figuris, quæ priori illi ſimiles & ſimiliter poſitæ à lateribus rectũ angulum cõtinentibus deſcribuntur.



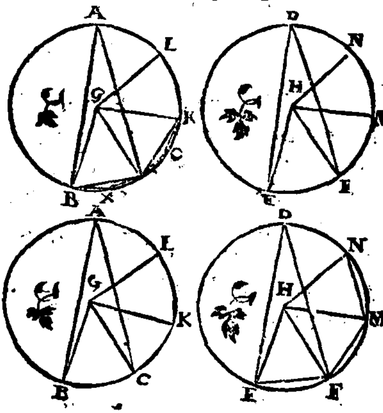
Theorema 22. Propoſitio 32.

Si duo triãgula, quę duo latera duobus lateribus proportionalia habeant, ſecũdum vnum angulũ compoſita fuerint, ita vt homologa eorum latera ſint etiam parallela, tũ reliqua illorum triangulorum latera in rectam lineam collocata reperientur.



Theorema 23. Propoſitio 33.

Inæqualibus circulis anguli eandẽ habent rationem cũ ipſis peripherijs in quibus inſiſtũt, ſiue ad cẽtra, ſiue ad peripherias cõſtituti, illis inſiſtant peripherijs. Inſuper verò & ſectores, quippe qui ad cẽtra cõſiſtũt.



ELEMENTI VI. FINIS.

EVCLIDIS ELEMENTVM SEPTIMVM.

DEFINITIONES.

1

Vnitas, est secũdum quam entium quodque dicitur vnum.

2

Numerus autem, ex vnitatibus composita multitudo.

3

Pars, est numerus numeri minori maioris, cùm minor metitur maiorem.

4

Partes autem, cùm non metitur.

5

Multiplex verò, maior minoris, cùm maiorem metitur minor.

6

Par numerus est, qui bifariam diuiditur.

7

Impar verò, qui bifariam non diuiditur: vel, qui vnitate differt à pari.

8

Pariter par numerus est, quem par numerus metitur per numerum parem.

9

Pariter autem impar, eſt quem par numerus metitur per numerum imparem.

10

Impariter verò impar numerus, eſt quem impar numerus metitur per numerum imparem.

11.

Primus numerus, eſt quem vnitas ſola metitur.

12

Primi inter ſe numeri ſunt, quos ſola vnitas menſura communis metitur.

13

Compoſitus numerus eſt, quem numerus quiſpiam metitur.

14

Compoſiti autẽ inter ſe numeri ſunt, quos numerus aliquis menſura communis metitur.

15

Numerus numerum multiplicare dicitur, cùm toties compoſitus fuerit is, qui multiplicatur, quot ſunt in illo multiplicante vnitates, & procreatus fuerit aliquis.

16

Cùm autẽ duo numeri mutuò ſeſe multiplicantes quempiam faciũt, qui factus erit planus appellabitur, qui verò numeri mutuò ſeſe multiplicarint, illi⁹ latera dicẽtur.

17 Cùm

17

Cùm verò tres numeri mutuò ſeſe multi-plicantes quempiã faciunt, qui procreatus erit ſolidus appellabitur, qui autem nume ri mutuò ſeſe multiplicarint, illius latera dicentur.

18

Quadratus numer⁹ eſt, qui ęqualiter æqua lis: vel, qui à duobus ęqualibus numeris cõ tinetur.

19

Cubus verò, qui æqualiter æqualis æquali-ter: vel, qui à tribus æqualibus numeris cõ-tinetur.

20

Numeri proportionales ſunt, cùm primus ſecundi, & tertius quarti æque multiplex eſt, vel eadem pars, vel eædem partes.

21

Similes plani & ſolidi numeri ſunt, qui pro portionalia habent latera.

22

Perfectus numerus eſt, qui ſuis ipſius par-tibus eſt æqualis.

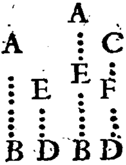
Theorema 1. Propo-ſitio 1.

Duobus numeris inæqualibus propoſi-tis,

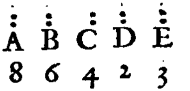
tis, si detrahatur semper minor de maiore, alterna, quadam detractione, neque reliquus vnquam metiatur præcedentem quoad assumpta sit vnitas: qui principio propositi sunt, numeri primi inter se erunt.

Problema 1. Propositio 2.

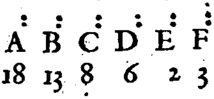
Duobus numeris datis non primis inter se, maximã eorum communem mensuram reperire.



Problema 2. Propositio 3.

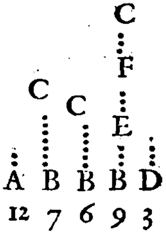


Tribus numeris datis non primis inter se, maximam eorum communem mensuram reperire.



Theorema 2. Propositio 4.

Omnis numerus, cuiusq; numeri minor maioris aut pars est, aut partes.



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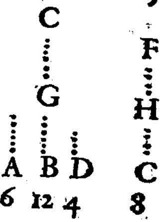
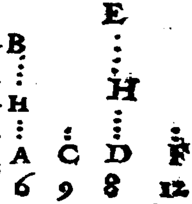


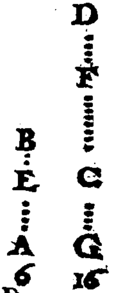
Theorema 3. Propositio 5.

Si numerus numeri pars fuerit,& alter alterius eadē pars & simul vterque vtriusque simul eadem pars erit, quæ vnus est vnius.



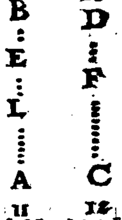
Theor.4.Propo.6.

Si numer⁹ sit numeri partes,& alter alterius eędem partes,& simul vterque vtriusque simul eadē partes erunt, quæ sunt vnus vnius.



Theorema 5. Propositio 7.

Si numerus numeri eadē sit pars quæ detractus detracti, & reliquus reliqui eadē pars erit, quæ totus est totius.



Theorema 6. Propositio 8.

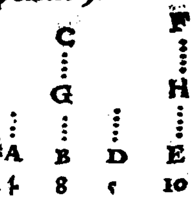
Si numerus numeri eædē sint partes quæ detractus detracti, & reliquus reliqui eędē partes erunt, quæ sunt totus totius.

Theo-

G..M.K...N.H.

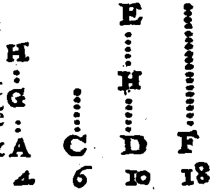
Theorema 7. Propositio 9.

Si numerus numeri pars sit, & alter alterius eadem pars, & vicissim quæ pars est vel partes primus tertij, eadé pars erit vel eędé partes, & secũdus quarti.



Theorema 8. Propositio 10.

Si numerus numeri partes sint, & alter alterius ęædé partes, etiam vicissim quæ sunt partes aut pars primus tertij, eædé partes erunt vel pars & secundus quarti.

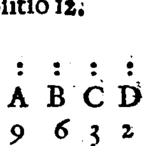


Theorema 9. Propositio 11.

Si quemadmodum se habet totus ad totum, ita detractus ad detractum, & reliquus ad reliquum ita habebit vt totus ad totum.

Theorema 10. Propositio 12.

Si sint quotcunque numeri proportionales, quemadmodum se habet vnus antecedentium ad vnum sequẽtium, ita se habe-



habebunt omnes antecedentes ad omnes consequentes.

Theorema 11. Propositio 13.

Si quatuor numeri sint proportionales, & vicissim ꝓportionales erunt.

A	B	C	D
12	4	9	3

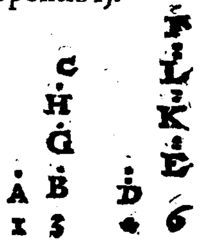
Theorema 12. Propositio 14.

Si sint quotcunque numeri, & alij illis æquales multitudine, qui bini sumantur & in eadem ratione: etiam ex æqualitate in eadem ratione erunt.

A	B	C	D	E	F
12	6	3	8	4	2

Theorema 13. Propositio 15.

Si vnitas numerum quẽpiã metiatur, aliter verò numerus alium quendã numerũ æquè metiatur, & vicissim vnitas tertium numerũ æquè metietur, atque secundus quartum.



Theorema 14. Propositio 16.

Si duo numeri mutuò sese multiplicãtes faciant aliquos qui ex illis geniti fuerint, inter se æquales erunt.

E	A	B	C	D
1	2	4	8	8

Theorema 15. Propositio 17.

Si numerꝰ duos numeros multiplicãs faciat aliquos, qui ex illis procreati erunt, eandem rationem habebunt, quam multiplicati.

I	A	B	C	D	E
1	3	4	5	12	15

Theorema 16. Propositio 18.

Si duo numeri numerum quempiam multiplicantes faciant aliquos, geniti ex illis eandem habebunt rationem, quam qui illum multiplicarunt.

A	B	C	D	E
4	5	3	12	15

Theorema 17. Propositio 19.

Si quatuor numeri ſint proportionales, q̃ ex primo & quarto fit, ęqualis erit ei qui ex ſecundo & tertio: & ſi qui ex primo & quarto fit numerus æqualis ſit ei qui ex ſecũdo & tertio, illi quatuor numeri proportionales erunt.

A	B	C	D	E	F	G
6	4	3	2	12	12	18

Theorema 18. Propositio 20.

Si tres numeri ſint proportionales, qui ab extremis cõtinetur, ęqualis eſt ei qui à medio

dio efficitur. Et si qui ab extremis cõtinetur æqualis sit ei qui à medio describitur, illi tres nũmeri proportionales erunt.

A	B	C
9	6	4.
D		
6		

Theorema 19. Propositio 21.

Minimi numeri omniũ, qui eandem cũ eis rationem habẽt, æqualiter metiũtur numeros eandem rationem habẽtes, maior quidẽ maiorem, minor verò minorem.

D	L		
G	H		
C	E	A	B
4	3	8	6

Theorema 20. Propositio 22.

Si tres sint numeri & alij multitudine illis æquales, qui bini sumantur & in eadem ratione, sit autem perturbata eorũ proportio, etiam ex æqualitate in eadem ratione erũnt.

A	B	C	D	E	F
6	4	3	12	8	6

Theorema 21. Propositio 23.

Primi inter se numeri minimi sunt omniũ eandem cum eis rationem habentium.

A	B	E	C	D
5	6	2	4	3

Theorema 22. Propositio 24.

Minimi numeri omnium eandem cum eis rationem habentiũ, primi sunt inter se.

A	B	C	D	E
8	6	4	3	2

Theorema 23. Propositio 25.

Si duo numeri sint primi inter se, qui alterutrum illorum metitur numerus, is ad reliquũ primus erit.

A	B	C	D
6	7	3	4

Theorema 24. Propositio 26.

Si duo numeri ad quẽpiam numerũ primi sint, ad eundẽ primus is quoque futurus est, qui ab illis productus fuerit.

B	A	C	D	E	F
3	5	5	5	3	2

Theorema 25. Propositio 27.

Si duo numeri primi sint inter se, q abvno eorũ gignitur ad reliquum primus erit.

B	A	C	D
	7	6	3

Theorema 26. Propositio 28.

Si duo numeri ad duos numeros ambo ad vtrunque primi sint, & qui ex eis gignentur, primi inter se erunt.

A	B	E	C	D	F
3	5	15	2	4	8

Theore-

Theorema 27. Propositio 29.

Si duo numeri primi sint inter se, & multiplicãs vterq; seipsum procreet aliquẽ, qui ex ijs producti fuerint, primi inter se erũt. Quòd si numeri initio propositi multiplicãtes eos qui producti sunt, effecerint aliquos, hi quoq; inter se primi erũt, & circa extremos idẽ hoc semper eueniet.

:	:	:	:	:	:
A	C	E	B	D	F
3	6	27	4	16	63

Theorema 28. Propositio 30.

Si duo numeri primi sint inter se, etiam simul vterq; ad vtrunq; illorũ primus erit. Et si simul vterq; ad vnum aliquem eorũ primus sit, etiam qui initio positi sunt numeri, primi inter se erunt.

	C	
:	:	:
A	B	D
7	5	4

Theorema 29. Propositio 31.

Omnis primus numerus ad omnem numerum quem non metitur, primus est.

:	:	:
A	B	C
7	10	5

Theorema 30. Propositio 32.

8i duo numeri sese mutuò multiplicãtes faciant aliquem, hũc aũt ab illis productum metiatur primus quidam numerus, is alterum etiã metitur eorum qui initio positi erant.

:	:	:	:	:
A	B	C	D	E
2	6	12	3	4

Theorema 31. Propositio 33.

Omnem compositum numerum aliquis primus metietur.

A	B	C
27	9	3

Theorema 32. Propositio 34.

Omnis numerus aut primus est, aut eum aliquis primus metitur.

A	A	
3	6	3

Problema 3. Propositio 35.

Numeris datis quotcunque, reperire minimos omnium qui eandem cum illis rationem habeant.

A	B	C	D	E	F	G	H	K	I	M
6	8	12	2	3	4	6	2	3	4	3

Problema 4. Propositio 36.

B				
A	C	D	E	F
7	12	8	4	5

Duobus numeris datis reperire quem illi minimũ metiantur numerum.

A					
F	E	C	D	G	H
6	9	12	9	2	3

Theo-

Theorema 33. Propoſitio 37.

Si duo numeri numerũ quempiam metiantur, & minimus quem illi metiuntur eundem metietur.

A	B	E	C	F
2	3	6	12	

Problema 5. Propoſitio 38.

Tribus numeris datis reperire quem minimum numerũ illi metiantur.

A	B	C	D	E	
3	4	6	12	8	
A	B	C	D	E	F
3	6	8	12	24	16

Theorema 34. Propoſitio 39.

Si numerum quiſpiam numerus metiatur, menſus partem habebit metienti cognominem.

A	B	C	D
12	4	3	1

Theorema 35. Propoſitio 40.

Si numerus partem habuerit quãlibet, illum metietur numerus parti cognominis.

A	B	C	D
8	4	2	1

Problema 6. Propoſitio 41.

Numerum reperire, qui minimus cùm ſit, datas habeat partes.

A	B	C	G	H
2	3	4	12	19

ELEMENTI VII. FINIS.

EVCLIDIS ELEMENTVM OCTAVVM.

Theorema 1. Propositio 1.

Si sint quotcũq; numeri deinceps proportionales, quorum extremi sint inter se, primi, minimi sunt omnium eandem cum eis rationem habentium.

A	B	C	D	E	F	G	H
8	12	18	27	6	8	12	18

Problema 1. Propositio 1.

Numeros reperire deinceps proportionales minimos, quotcunq; iusserit quispiam in data ratione.

A	B	C	D	E	F	G	H	K
3	4	9	12	16	27	36	49	64

Theorema 2. Propositio 3.

Conuersa primæ.

Si sint quotcũq; numeri deinceps proportionales minimi habentium eandem cum eis rationem, illorum extremi sunt inter se primi.

A	B	C	D	E	F	G	H	K	L	M	N	O
27	16	48	64	3	4	9	12	16	27	36	48	64

Pro-

Problema 2. Propositio 4.

Rationibus datis quotcunque in minimis numeris reperire numeros deinceps minimos in datis rationibus.

A	B	C	D	E	F	H	G	K	L	N	X	M	O
3	4	2	3	4	5	6	8	12	15	4	6	10	12

Theorema 3. Propositio 5.

Plani numeri rationem inter se habent ex lateribus compositam.

A	L	B	C	D	E	F	G	H	K
18	22	32	3	6	4	8	9	12	16

Theorema 4. Propositio 6.

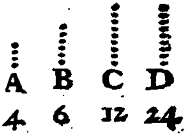
A	B	C	D	E	F	G	H
16	24	36	54	82	4	6	9

Si sint quotlibet numeri deinceps proportionales, primus autem secundum non metiatur, neque alius quispiam vllum metietur.

H 5 Theo-

Theorema 5. Propoſitio 7.

Si ſint quotcunque numeri deinceps proportionales, primus autem extremum metiatur, is etiam ſecundum metietur.

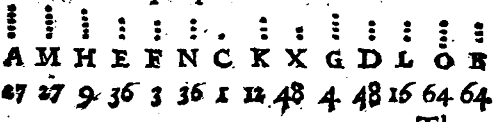


Theorema 6. Propoſitio 8.

Si inter duos numeros medij continua ꝓportione incidant numeri, quot inter eos medij continua proportione incidunt numeri, tot & inter alios eandem cum illis habentes rationem medij continua proportione incident.

Theorema 7. Propoſitio 9.

Si duo numeri ſint inter ſe primi, & inter eos medij cõtinua proportione incidãt numeri, quot inter illos medij continua proportione incidunt numeri, totidẽ & inter vtrunque eorum ac vnitatẽ deinceps medij continua proportione incident.



Theo-

Theorema 8. Propositio 10.

Si inter duos numeros & vnitatẽ continuè proportionales incidãt numeri quot inter vtrunque ipsorum & vnitatẽ deinceps medij continua proportione incidunt numeri, totidẽ & inter illos medij continua proportione incident.

A	E	K	D	H	C	L	F	G	B
27	9	36	3	12	1	48	4	16	64

Theorema 9. Propositio 11.

Duorum quadratorũ numerorum vnus medius proportionalis est numer⁹: & quadratus ad quadratum duplicatam habet lateris ad latus rationem.

A	C	E	D	B
9	3	12	4	16

Theorema 10. Propositio 12.

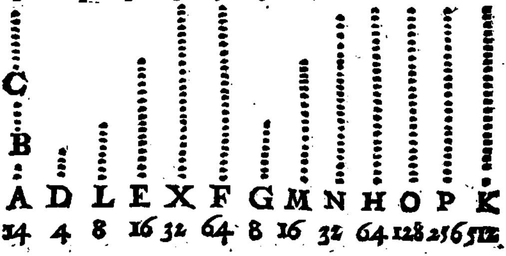
Duorum cuborum numerorũ duo medij proportionales sunt numeri: & cubus ad cubum triplicatam habet lateris ad latus rationem.

A	H	K	B	C	D	E	F	G
27	36	48	64	3	4	9	12	16

Theo-

Theorema 11. Propositio 13.

Si sint quotlibet numeri deinceps proportionales, & multiplicans quisq; seipsum faciat aliquos, qui ab illis producti fuerint, proportionales erũt: & si numeri primùm positi, ex suo in procreatos ductu faciãt aliquos, ipsi quoque proportionales erunt.



Theorema 12. Propositio 14.

Si quadratus numerus quadratum numerũ metiatur, & latus vnius metietur latus alterius. Et si vnius quadrati latus metiatur latus alterius, & quadratus quadratum metietur.

A	E	B	C	D
9	12	16	3	4

Theorema 13. Propositio 15.

Si cubus numerus cubum numerũ metiatur, & latus vniꝰ metietur alterius latus. Et si latus vnius cubi latus alterius metiatur, tum

tum cubus cubum metietur.

A	H	K	B	C	D	E	F	G
8	16	28	64	2	4	4	8	16

Theorema 14. Propositio 16.

Si quadratus numerus quadratum numerum non metiatur, neque latus vnius metietur alterius latus.

Et si latus vni⁹ quadrati nõ metiatur latus alterius, neque quadratus quadratum metietur.

A	B	C	D
9	16	3	4

Theorema 15. Propositio 17.

Si cubus numerus cubum numerũ nõ metiatur, neque latus vni⁹ latus alterius metietur.

Et si latus cubi alicuius latus alterius nõ metiatur, neque cubus cubũ metietur.

A	B	C	D
8	27	9	11

Theorema 16. Propositio 18.

Duorum similium planorum numerorũ vnus medius proportionalis est numerus, & planus ad planũ duplicatam habet literis homologi

A	G	B	C	D	E	F
12	18	27	2	6	3	9

logi ad latus homologum rationem.

Theorema 17. Propositio 19.

Inter duos similes numeros solidos, duo medij proportionales incidunt numeri: & solidus ad similem solidum triplicatam rationem habet lateris homologi ad latus homologum.

A	N	X	B	C	D	E	F	G	H	K	M	L
8	12	18	27	2	2	2	3	3	3	4	6	9

Theorema 18. Propositio 20.

Si inter duos numeros vn⁹ medius proportionalis incidat numer⁹; similes plani erunt illi numeri.

A	C	B	D	E	F	G
18	24	32	3	4	6	8

Theorema 19. Propositio 21.

Si inter duos numeros duo medij proportionales incidant numeri; similes solidi sunt illi numeri,

A	C	D	B	E	F	G	H	K	L	M
27	36	48	64	9	12	16	3	3	3	4

Theo-

Theorema 20. Propoſitio 22.

Si tres numeri deinceps ſint proportionales, primus autem ſit quadratus, & tertius quadratus erit.

A	B	D
9	15	25

Theorema 21. Propoſitio 23.

Si quatuor numeri deinceps ſint proportionales, primus autem ſit cubus, & quartus cubus erit.

A	B	C	D
8	12	18	27

Theorema 22. Propoſitio 24.

Si duo numeri rationem habeant inter ſe, quam quadratus numerus ad quadratum numerum, primus autem ſit quadratus, & ſecundus quadratus erit.

A		B	C		D
4	6	9	16	24	36

Theorema 23. Propoſitio 25.

Si numeri duo rationem inter ſe habeant, quam cubus numerus ad cubum numerũ, primus autem cubus ſit, & ſecundus cubus erit.

A	E	F	B	C			D
8	12	18	27	64	95	140	216

Theo-

Theorema 24. Propositio 26.

Similes plani numeri rationem inter se habent, quam quadratus numerus ad quadratum numerum.

A	C	B	D	E	F
18	24	32	9	12	16

Theorema 25. Propositio 27.

Similes solidi numeri rationem habét inter se, quam cubus numerus ad cubum numerum.

A	C	D	B	E	F	G	H
16	24	36	54	8	12	18	27

ELEMENTI VIII. FINIS.

EVCLIDIS ELEMENTVM NONVM.

Theorema 1. Propositio 1.

Si duo similes plani numeri mutuò sese multiplicãtes quendã procreent, productus quadratus erit.

A	E	B	D		C
4	6	9	16	24	36

Theorema 2. Propositio 2.

Si duo numeri mutuò sese multiplicantes quadratum faciant, illi similes sunt plani.

A		B	D		C
4	6	9		18	36

Theorema 3. Propositio 3.

Si cubus numerus seipsum multiplicãs pcreet aliquẽ, productus cũbus erit.

Vni	D	D	A			B
tas	3	4	8	16	32	64

Theorema 4. Propositio 4.

Si cubus numerus cubũ numerum multiplicans quendam procreet, procreatus cubus erit.

A	B	D	C
8	27	64	216

Theorema 5. Propositio 5.

Si cubus numerus numerum quendã multiplicans cubum procreet, & multiplicatus cubus erit.

A	B	D	D
27	64	729	17 28

Theorema 6. Propositio 6.

Si numerus seipsum multiplicans cubum procreet, & ipse cubus erit.

A	B	C
27	729	19683

Theorema 7. Propositio 7.

Si compositus numerus quẽdam numerũ multiplicans quempiam procreet, productus solidus erit.

A	B	C	D	E
6	8	48	2	3

Theorema 8. Propositio 8.

Si ab vnitate quotlibet numeri deinceps ꝓportionales sint, tertius ab vnitate quadratus est, & vnũ intermittentes omnes: quartus autẽ cubus, & duobus intermissis omnes:

nes. septimus verò cubus simul & æquadratus, & quinque intermissis omnes

Vnitas	A	B	C	D	E	F
	3	9	27	81	243	729

Theorema 9. Propositio 9.

Si ab vnitate sint quotcunque numeri deinceps proportionales, sit autem quadratus is qui vnitatem sequitur, & reliqui oēs quadrati erūt. Quòd si qui vnitatem sequitur cubus sit, & reliqui omnes cubi erunt.

quadrati.		cubi
531441	F	732969
59049	E	531441
6561	D	59049
729	C	6561
81	B	729
9	A	81
	o	
	Vnitas.	

Theorema 10. Propositio 10.

Si ab vnitate numeri quotcunque proportionales sint, non sit autē quadratus is qui vnitatem sequitur, neq; alius vllus quadratus erit, dēptis tertio ab vnitate ac omnibus

o	A	B	C	D	E	F
Vnitas.	3	9	36	81	243	729

nibus vnum intermittẽtibus. Quòd si qui vnitatem sequitur, cubus non sit, neque alius vllus cubus erit, demptis quarto ab vnitate ac omnibus duos intermittentibus.

Theorema 11. Propositio 11.

Si ab vnitate numeri quotlibet deinceps proportionales sint, minor maiorẽ metitur per quempiam eorum qui in proportionalibus sunt numeris.

.	:	:	:	:
A	D	C	D	E
1	2	4	8	16

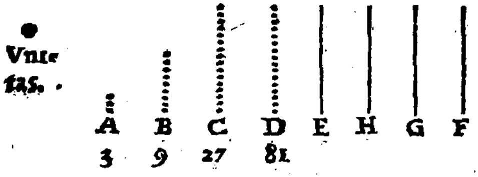
Theorema 12. Propositio 12.

Si ab vnitate quotlibet numeri sint proportionales, quot primorum numerorũ vltimum metiuntur, totidem & eum qui vnitati proximus est, metientur.

Vnitas.	A	B	C	D	E	H	G	F
	4	16	64	259	2	8	32	128

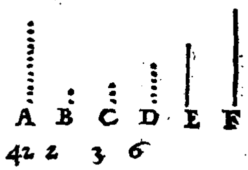
Theorema 13. Propositio 13.

Si ab vnitate sint quotcunq; numeri deinceps proportionales, primus autẽ sit qui vnitatẽ sequitur, maximũ nullus alius metietietur, ijs exceptis qui in proportionalib⁹ ſunt numeris.



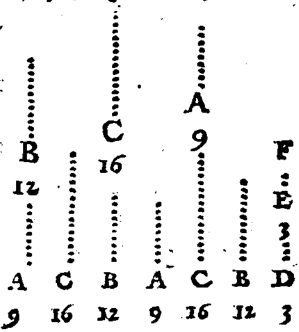
Theorema 14. Propoſitio 14.

Si minimum numerum primi aliquot numeri metiãtur, nullus alius numerus primus illum metietur, ijs exceptis qui primò metiuntur.



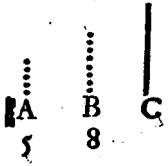
Theorema 15. Propoſitio 15.

Si tres numeri deinceps proportionales ſint minimi, eadẽ cum ipſis habẽtium rationẽ, duo quilibet compoſiti ad tertium primi erunt.



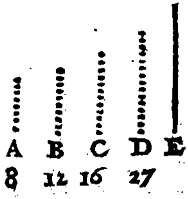
Theorema 16. Propoſitio 16.

Si duo numeri ſint inter ſe primi, non ſe habebit quẽadmodum primus ad ſecũdum, ita ſecundus ad quẽpiam alium.



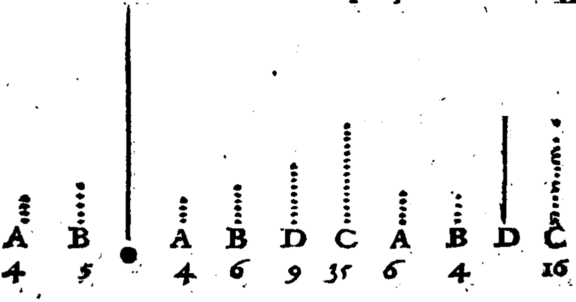
Theorema 17. Propoſitio 17.

Si ſint quotlibet numeri deinceps ꝓportionales, quorum extremi ſint inter ſe primi, non erit quẽ admodum primus ad ſecundum, ita vltimus ad quempiam alium.



Theorema 18. Propoſitio 18.

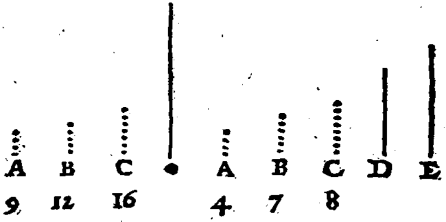
Duobꝰ numeris datis, cõſiderare poſſitne tertius illis inueniri proportionalis.



Theore-

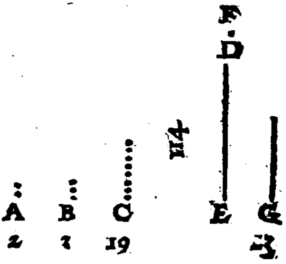
Theorema 19. Propositio 19.

Tribus numeris datis, considerare possitne quartus illis reperiri proportionalis.



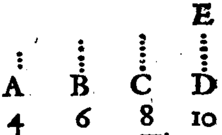
Theorema 20. Propositio 20.

Primi numeri plures sunt quacúque proposita multitudine primorum numerorum.



Theorema 21. Propositio 21.

Si pares numeri quotlibet compositi sint, totus est par.



Theorema 22. Propositio 22.

Si impares numeri quotlibet compositi sint, sit autem par illorum multitudo, totus par erit.

A B C D E

5 9 7 5

Theorema 23. Propositio 23.

Si impares numeri quotcunque cõpositi sint, sit autem impar illorũ multitudo, & totus impar erit.

A B C E I

5 7 8 1

Theorema 24. Propositio 24.

Si de pari numero par detractus sit, & reliquus par erit.

A B C

6 4

Theorema 25. Propositio 25.

Si de pari numero impar detractus sit, & reliquus impar erit.

A B C D

8 1 4

Theorema 26. Propositio 26.

Si de impari numero impar detractus sit, & reliquus par erit.

A B C D

4 6 1

Theo

Theorema 27. Propositio 27.

Si ab impari numero par ablatus sit, reliquus impar erit.

A D C
1 4 4

Theorema 28. Propositio 28.

Si impar numerus parem multiplicãs, procreet quépiam, procreatus par erit.

A B C
3 4 12

Theorema 29. Propositio 29.

Si impar numerus imparé numerum multiplicans quemdam procreet, procreatus impar erit.

A B C
3 5 15

Theorema 30. Propositio 30.

Si impar numerus paré numerum metiatur, & illius dimidium metietur.

A C B
3 6 18

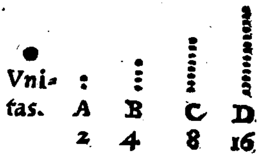
Theorema 31. Propositio 31.

Si impar numerus ad numerum quempiã primus sit, & ad illius duplum primus erit.

A B C D
7 8 16

Theorema 32. Propositio 32.

Numerorum, qui à binario dupli sunt, vnusquisq; pariter par est tantùm.



Theorema 33. Propositio 33.

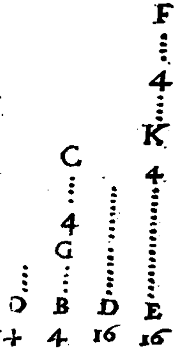
Si numerus dimidiũ impar habeat, pariter impar est tantùm.

Theorema 34. Propositio 34.

Si par numerus nec sit dupl⁹ à binario, nec dimidium impar habeat, pariter par est, & pariter impar.

Theorema 35. Propositio 35.

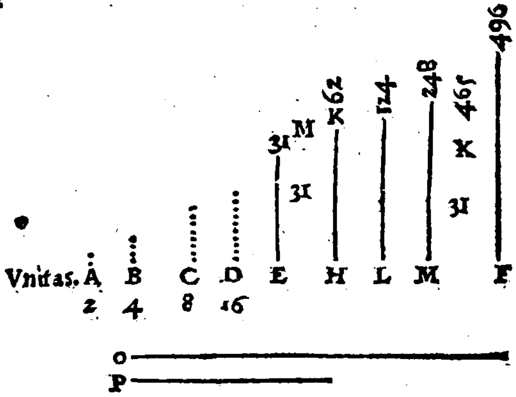
Si sint quotlibet numeri deinceps proportionales, detrahãtur autẽ de secũdo & vltimo æquales ipsi primo, erit quemadmodũ secũdi excessus ad primũ, ita vltimi excessus ad omnes qui vltimum antecedunt.



Theo-

Theorema 36. Propositio 36.

Si ab vnitate numeri quotlibet deinceps expositi sunt in duplici proportione quoad totus compositus primus factus sit, isq; totus in vltimum multiplicatus quempiã procreet, procreatus perfectus erit.



ELEMENTI IX. FINIS.

EVCLI-

EVCLIDIS ELEMENTVM DECIMVM.

DEFINITIONES.

1.

Commensurabiles magnitudines dicuntur illæ, quas eadem mensura metietur.

2

Incommensurabiles verò magnitudines dicuntur hæ, quarum nullam mensuram communem contingit reperiri.

3

Lineæ rectæ potentia cõmensurabiles sũt, quarum quadrata vna eadem superficies siue area metitur.

4

Incommensurabiles verò lineæ sunt, quarum quadrata, quæ metiatur area cõmunis, reperiri nulla potest.

5

Hæc cùm ita sint, ostendi potest quòd quãtacunque linea recta nobis proponatur, existunt etiã aliæ lineæ innumerabiles eidem commensurabiles, aliæ item incommensurabiles, hæ quidem longitudine & poten-

potentia:illæ verò potentia tantùm. Vocetur igitur linea recta, quantacunq; proponatur, ῥητὴ, id est rationalis.

6

Lineæ quoq; illi ῥητῇ commensurabiles siue longitudine & potentia, siue potentia tantùm, vocentur & ipsæ ῥηταί, id est rationales.

7

Quæ verò lineæ sunt incommensurabiles illi τῇ ῥητῇ, id est primo loco rationali, vocētur ἄλογοι, id est irrationales.

8

Et quadratum quod à linea proposita describitur, quam ῥητὴν vocari volumus, vocetur ῥητόν.

9

Et quæ sunt huic commensurabilia, vocētur ῥητά.

10

Quæ verò sunt illi quadrato ῥητῷ scilicet incommensurabilia, vocentur ἄλογα, id est surda.

11

Et lineæ quæ illa incommēsurabilia describunt, vocentur ἄλογοι Et quidem si illa incommēsurabilia fuerint quadrata, ipsa corū latera vocabūtur ἄλογοι lineæ. quòd si quadrata quidem non fuerint, verū aliæ quæpiā superficies siue figuræ rectilineæ, tunc

tunc verò lineæ illæ quæ deſcribunt quadrata æqualia figuris rectilineis, vocentur ἄλογοι.

Theorema 1. Propoſitio 1.

Duabus magnitudinibus inæqualibus propoſitis, ſi de maiore detrahatur plus dimidio, & rurſus de reſiduo iterũ detrahatur plus dimidio, idq́; ſemper ſiat: relinquetur quædam magnitudo minor altera minore ex duabus propoſitis.

Theorema 2. Propoſitio 2.

Duab⁹ magnitudinibus propoſitis inæqualibus, ſi detrahatur ſemper minor de maiore, alterna quadam detractione neque reſiduum vnquam metiatur id quod ante ſe metiebatur, incommenſurabiles ſunt illæ magnitudines.

Problema 1. Propoſitio 3.

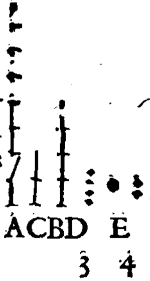
Duabus magnitudinibus cõmẽſurabilibus datis, maximam ipſarum communem menſuram reperire.

Proble-

Problema 2. Propositio 4.

Tribus magnitudinibus cõmensurabilibus datis, maximam ipsarũ communẽ mensuram reperire.

Theorema 3. Propositio 5.



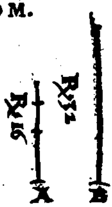
Cõmensurabiles magnitudines inter se proportionẽ eam habent, quam habet numerus ad numerum.

Theorema 4. Propositio 6.

Si duæ magnitudines proportionem eam habent inter se quam numerus ad numerum, commensurabiles sunt illæ magnitudines.

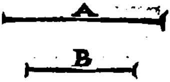
Theo-

Theorema 5. Propositio 7.



Incommenſurabiles magnitudines inter ſe proportionem non habēt, quam numerus ad numerum.

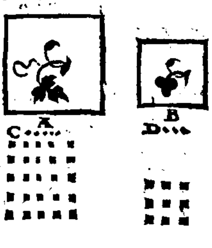
Theorema 6. Propoſitio 8.



Si duæ magnitudines inter ſe proportionem nō habēt quam numerus ad numerum, incommenſurabiles illæ ſunt magnitudines.

Theorema 7. Propoſitio 9.

Quadrata, quæ deſcribūtur à rectis lineis lōgitudine cōmenſurabilibus, inter ſe proportionem habent quā numerus quadratus ad alium numerum quadratū: Et quadrata habentia proportionē

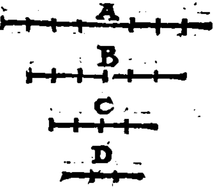


inter ſe quā quadratus numerus ad numerum quadratū, habent quoque latera longitudine commēſurabilia. Quadrata verò quæ

quæ deſcribuntur à lineis longitudine incómenſurabilibus, proportioné nó habét inter ſe, quam quadratus numerus ad numerum alium quadratum. Et quadrata nó habentia proportionem inter ſe quam numerus quadratus ad numerum quadratú, neque latera habebunt longitudine commenſurabilia.

Theorema 8. Propoſitio 10.

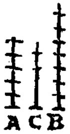
Si quatuor magnitudines fuerint proportionales, prima verò ſecúdæ ſuerit cómenſurabilis, tertia quoque quartę comméſurabilis erit. quòd ſi prima ſecúdæ fuerit incomméſurabilis, tertia quoq; quartæ incommenſurabilis erit.



Problema 3. Propoſitio 11.

Propoſitæ lineæ rectæ (quam ῥητὴν vocari diximus) reperire duas lineas rectas incómenſurabiles, hanc quidem longitudine tãtùm, illam verò non longitudine tantùm, ſed etiam potentia incommẽſurabilem.

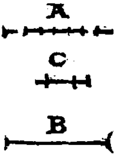
Theorema 9. Propositio 12.



Magnitudines quæ eidē magnitudini sunt cō mensurabiles, inter se quoque sunt commensurabiles.

6 D...... 4 F..
4 E.... 8 G..
3 H...
2 K..
4 L...

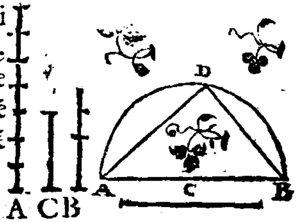
Theorema 10. Propositio 13.



Si ex duabus magnitudinibꝰ hæc quidem cōmensurabilis sit tertię magnitudini, illa verò eidem incommensurabilis, incōmensurabiles sunt illæ duæ magnitudines.

Theorema 11. Propositio 14.

Si duarum magnitudinum commensurabiliū altera fuerit incōmensurabilis magnitudini alteri cuipiam tertiæ, reliqua quoque magnitudo eidē tertiæ incommēsurabilis erit.



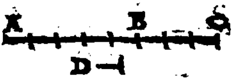
Theorema 12. Propositio 15.

Si quatuor rectæ proportionales fuerint, possit

posſit autem prima pluſquã ſecunda tãto quantum eſt quadratum lineæ ſibi commẽſurabilis longitudine:tertia quoq; poterit pluſquàm quarta tanto quantum eſt quadratũ lineæ ſibi commenſurabilis lõgitudine. Quòd ſi prima poſſit pluſquã ſecũda quadrato lineę ſibi longitudine incommẽſurabilis: tertia quoque poterit pluſquã quarta quadrato lineæ ſibi incõmẽſurabilis lõgitudine.

Theorema 13. Propoſitio 16.

Si duæ magnitudines commenſurabiles componantur, tota magnitudo cõpoſita ſingulis partibus commenſurabilis erit, quòd ſi tota magnitudo cõpoſita alterutri parti commenſurabilis fuerit, illæ duæ quoque partes cõmẽſurabiles erunt.

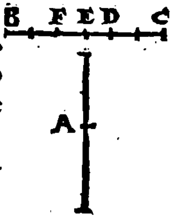


Theorema 14. Propoſitio 17.

Si duæ magnitudines incõmenſurabiles cõponantur, ipſa quoque tota magnitudo ſingulis partibus componentibus incommẽſurabilis erit. Quòd ſi tota alteri parti incõmenſurabilis fuerit, illæ quoque primę magnitudines inter ſe incommẽſurabiles erunt.

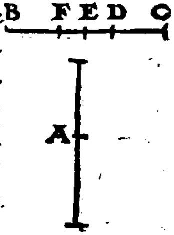
Theorema 15. Propositio 18.

Si fuerint duæ rectæ lineæ inæquales, & quartæ parti quadrati quod describitur à minore, æquale parallelogrammũ applicetur secundũ maiorem, ex qua maiore tãtum excurrat extra latus parallelogrãmi, quantum est alterum latus ipsius parallelogrammi: si præterea parallelogrammũ sui applicatione diuidat lineã illam in partes inter se commẽsurabiles longitudine, illa maior linea tanto plus potest quã minor, quantũ est quadratum lineæ sibi commensurabilis longitudine. Quòd si maior plus possit quàm minor, tanto quantũ est quadratum lineæ sibi cõmensurabilis lõgitudine, & præterea quartæ parti quadrati lineæ minoris ęquale parallelogrãmum applicetur secundum maiorẽ, ex qua maiore tantum excurrat extra latus parallelogrãmi, quantum est alterum latus ipsius parallelogrammi, parallelogrammum sui applicatione diuidit maiorẽ in partes inter se longitudine commẽsurabiles.

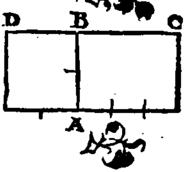


Theorema 16. Propositio 19.

Si fuerint duæ rectæ inæquales, quartæ autem parti quadrati lineæ minoris ęquale le parallelogrammorum ſecundum lineā maiorem applicetur, ex qua linea tantum excurrat extra latus parallelogrāmi, quātum eſt alterum latus eiuſdem parallelogrammi: ſi parallelogrammū præterea ſui applicatione diuidat lineā in partes inter ſe longitudine incommēſurabiles, maior illa linea tantò plus poteſt quàm minor, quantum eſt quadratum lineæ ſibi maiori incommenſurabilis longitudine. Quòd ſi maior linea tātò plus poſsit quàm minor, quantum eſt quadratum lineæ incómenſurabilis ſibi lógitudine: & preterea quartæ parti quadrati lineę minoris ęquale parallelogrammū applicetur ſecūdum maiorem, ex qua tantū excurrat extra latus parallelogrāmi, quantum eſt alterum latus ipſius: parallelogrammum ſui applicatione diuidit maiorem in partes inter ſe incommenſurabiles lógitudine.



Theorema 17. Propoſitio 20.

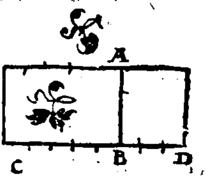


Superficies rectangula contéta ex lineis rectis rationalib⁹ lógitudine commenſurabilibus ſe-

cundum vnum aliquem modũ ex antedi-
ctis, rationalis eſt.

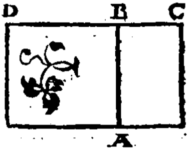
Theorema 18. Propoſitio 21.

Si rationale ſecundũ lineam rationalem applicetur, habebit alterũ latus lineã rationalem & cõmenſurabilem longitudine lineæ cui rationale parallelogrammũ applicatur.



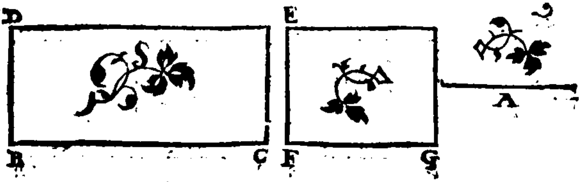
Theorema 19. Propoſitio 22.

Superficies rectangula cõtenta duabus lineis rectis rationalibus potentia tantum cõmẽſurabilibus, irrationalis eſt. Linea autem quæ illam ſuperficiem poteſt, irrationalis & ipſa eſt: vocetur verò medialis.



Theorema 20. Propoſitio 23.

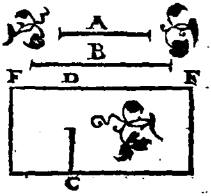
Quadrati lineæ medialis applicati ſecun-



dum

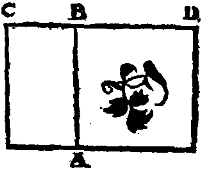
dum lineam rationalem, alterum latus est linea rationalis, & incómensurabilis lógitudine lineæ secundum quam applicatur.

Theorema 21. Propositio 24.



Linea recta mediali cómésurabilis, est ipsa quoque medialis.

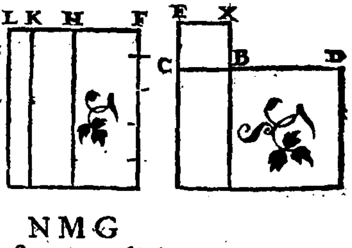
Theorema 22. Propositio 25.



Parallelogrammū rectā gulum cōtentum ex lineis medialibus longitudine cōmensurabilibus, mediale est.

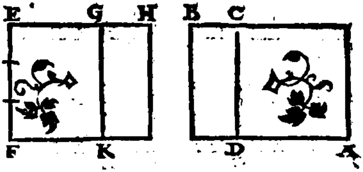
Theorema 23. Propositio 26.

Parallelogrammum rectangulum cópre-hensum duabus lineis medialibus potentia tātùm cómensurabilibus, vel rationale est, vel mediale.



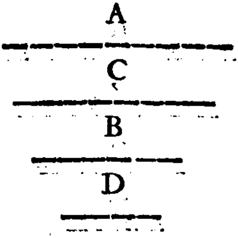
Theorema 24. Propoſitio 27.

Mediale nõ eſt maius quàm mediale ſuperficie rationali.



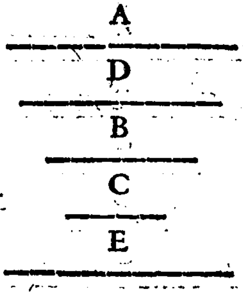
Proble. 4. Propoſitio 28.

Mediales lineas inuenire potentia tantùm commenſurabiles rationale comprehendentes.



Problema 5. Propoſitio 29.

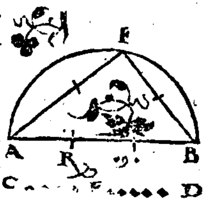
Mediales lineas inuenire potentia tantùm commenſurabiles mediale comprehendendentes.



Pro-

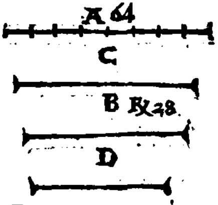
Problema 6. Propoſitio 30.

Reperire duas rationales potẽtia tantùm commẽſurabiles huiuſmodi, vt maior ex illis poſſit plus quàm minor quadrato lineæ ſibi commenſurabilis longitudine.



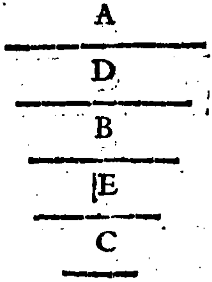
Problema 7. Propoſitio 31.

Reperire duas lineas mediales potentia tã-tùm commenſurabiles rationalem ſuperficiẽ cõtinentes, tales inquã vt maior poſſit plus quàm minor quadrato lineæ ſibi commẽſurabilis longitudine.



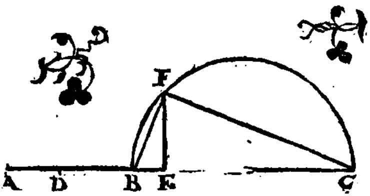
Problema 8. Propoſitio 32.

Reperire duas lineas mediales potentia tantùm commenſurabiles medialem ſuperficiem continentes, huiuſmodi vt maior plus poſſit quàm minor quadrato lineæ ſibi commenſurabilis longitudine.



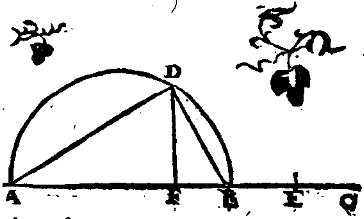
Problema 9. Propositio 33.

Reperire duas rectas potētia incommēsurabiles, quarum quadrata simul addita faciant superficiē rationalē, parallelogrāmum verò ex ipsis contentum sit mediale.



Problema 10. Propositio 34.

Reperire lineas duas rectas potentia incōmensurabiles, conficientes compositū ex ipsarum quadratis mediale, parallelogrammū verò ex ipsis contentum rationale.



Problema 11. Propositio 35.

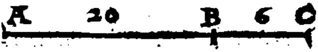
Reperire duas lineas rectas potentia incōmensurabiles, cōficientes id quod ex ipsarum quadratis cōponitur mediale, simulque

que parallelogrammũ ex ipſis contẽtum, mediale, quod præterea parallelogrãmũ ſit incõmenſurabile cõpoſito ex quadratis ipſarum.

PRINCIPIVM SENARIOrum per compoſitionem.

Theorema 25. Propoſitio 36.

Si duæ rationales potẽtia tantùm commẽſurabiles cõponantur, tota linea erit irrationalis. Vocetur autem Binomium.



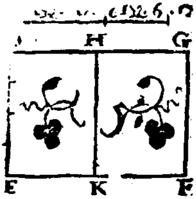
Theorema 26. Propoſitio 37.

Si duæ mediales potentia tantùm cõmenſurabiles rationale continentes cõponantur, tota linea eſt irrationalis, vocetur autem Bimediale prius.

Theo-

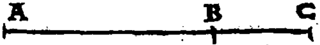
Theorema 27. Propoſitio 38.

Si duæ mediales potẽtia tantùm cõmenſurabiles mediale continentes cõ ponãtur, tota linea eſt irrationalis: vocetur autẽ Bimediale ſecundum.



Theorema 28. Propoſitio 39.

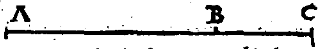
Si duæ rectæ potentia incommẽſurabiles cõponantur, conficientes compoſitum ex quadratis ipſarũ rationale, parallelogrammum verò ex ipſis contentũ mediale, tota linea recta



eſt irrationalis. Vocetur autẽ linea maior.

Theorema 29. Propoſitio 40.

Si duæ rectæ potentia incõmenſurabiles componantur, conficientes compoſitũ ex ipſarũ quadratis mediale, id verò quod fit ex ipſis, rationale, tota linea eſt irrationalis. Vocetur au

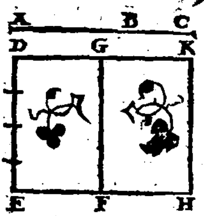


tem potens rationale & mediale.

Theorema 30. Propoſitio 41.

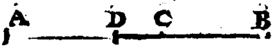
Si duæ rectę potentia incommenſurabiles componantur, conficientes cõpoſitum ex quadratis ipſarum mediale, & quod cõtinetur

netur ex ipsis, mediale, & præterea incõmensurabile cõposito ex quadratis ipsarũ, tota linea est irrationalis. Vocetur autem potens duo medialia.



Theorema 31. Propositio 42.

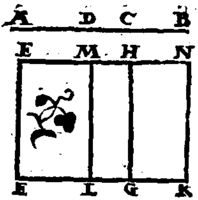
Binomium in vnico tantùm pũcto diuiditur in sua nomina, id est in lineas ex quibus componitur.



Theorema 32. Propositio 43.

Bimediale prius in vnico tantũ puncto diuiditur in sua nomina.

Problema 33. Propositio 44.



Bimediale secundum in vnico tantũ puncto diuiditur in sua nomina.

Problema 34. Propositio 45.

Linea maior in vnico tãtùm in pũcto diuiditur in sua nomina.

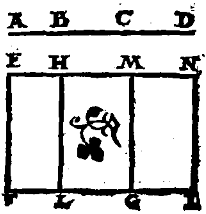
Theo-

Theorema 35. Propositio 46.

Linea potẽs rationale & mediale in vnico tãtùm pũcto diuiditur in ſua nomina.

Theorema 36. Propoſitio 47.

Linea potẽs duo medialia in vnico tantùm pũcto diuiditur in ſua nomina.



DEFINITIONES ſecundæ.

Propoſita linea rationali, & binomio diuiſo in ſua nomina, cuius binomij maius nomen, id eſt, maior portio poſsit pluſquam minus nomen quadrato lineæ ſibi, maiori inquam nomini, commenſurabilis longitudine.

1

Si quidẽ maius nomen fuerit cõmenſurabile longitudine propoſitæ lineę rationali, vocetur tota linea Binomium primum.

2

Si verò minus nomen, id eſt minor portio Binomij, fuerit cõmenſurabile lõgitudine ne propositæ lineæ rationali, vocetur tota linea Binomium secundum.

3

Si verò neutrum nomen fuerit commẽsurabile longitudine propositæ lineæ rationali, vocetur Binomium tertium.

Rursus si maius nomen possit plusquã minus nomen quadrato lineæ sibi incommensurabilis longitudine.

4

Si quidem maius nomen est commensurabile lõgitudine propositæ lineæ rationali, vocetur tota linea Binomium quartum.

5

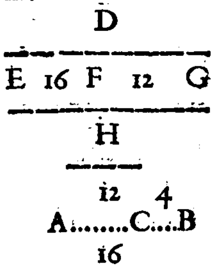
Si verò minus nomen fuerit cõmensurabile longitudine lineæ rationali, vocetur Binomium quintum.

6

Si verò neutrum nomen fuerit longitudine commensurabile lineæ rationali, vocetur illa Binomium sextum.

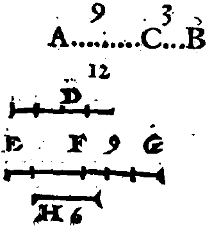
Proble. 12. Propositio 48.

Reperire Binomium primum.



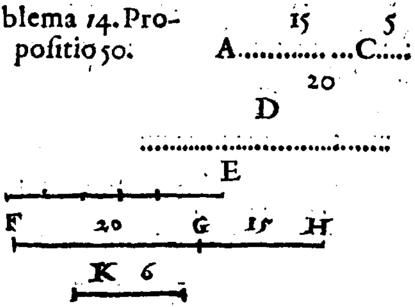
Proble-

Problema 13. Propositio 49.



Reperire Binomium secundum.

Problema 14. Propositio 50.



Reperire Binomium tertiū:

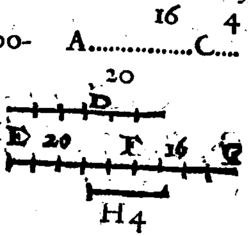
Problema 15. Propositio 51.

Reperire Binomium quartum.

Proble-

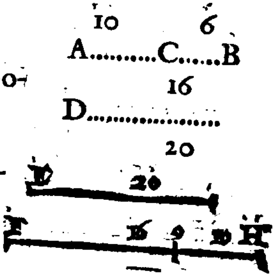
Problema 16. Propositio 52.

Reperire Binomiũ quintum.



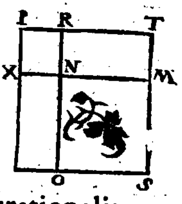
Probl. 17. Propopositio 53.

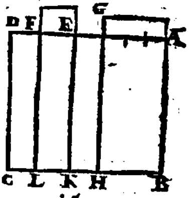
Reperire Binonimum sextum.



Theorema 37. Propositio 54.

Si superficies contẽta fuerit ex rationali & Binomio primo, linea quę illã superficiem potest, est irrationalis, quæ Binomiũ vocatur





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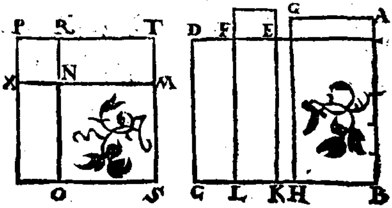
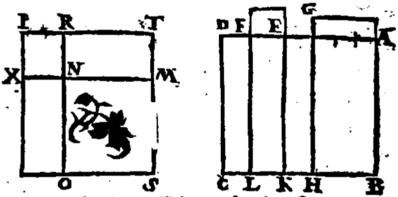
Theorema 38. Propositio 55.

Si superficies cōtenta fuerit ex linea ratio nali & Binomio secundo, linea potens illā superficiē est irrationalis, quę Bimediale primum vocatur.



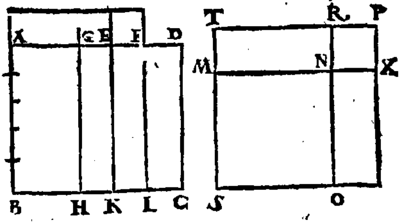
Theorema 39. Propositio 56.

Si superficies cōtineatur ex rationali & Binomio tertio, linea q̄ illā superficiem potest, est irrationalis q̄ dicitur Bimediale secundū.



Theorema 40. Propositio 57.

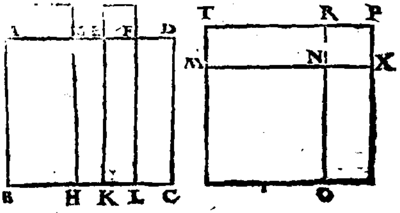
Si superficies cōtineatur ex rationali & Bino- mio



mio quarto, linea potens superficiem illā, est irrationalis, quæ dicitur maior.

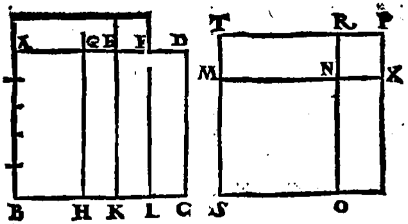
Theorema 41. Propositio 58.

Si superficies cōtineatur ex rationali & Binomio quinto, linea quæ illam superficiē potest est irrationalis, quę dicitur potēs rationale & mediale.



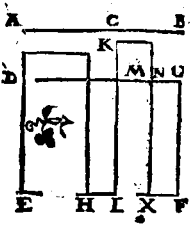
Theorema 42. Propositio 59.

Si superficies contineatur ex rationali & Binomio sexto, linea quæ illam superficiem potest est irrationalis, quæ dicitur potens duo mediala.



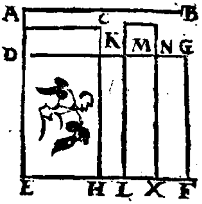
Theorema 43. Propoſitio 60.

Quadratum Binomij ſecundum lineam rationalem applicatum, facit alterum latus Binomium primum.



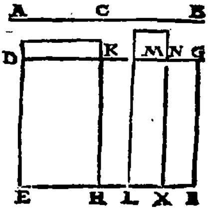
Theorema 44. Propoſitio 61.

Quadratũ Bimedialis primi ſecundum rationalẽ lineam applicatum, facit alterum latus Binomiũ ſecundum.



Theorema 45. Propoſitio 62.

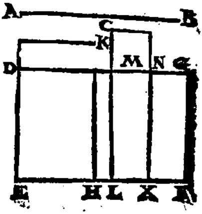
Quadratũ Bimedialis ſecũdi ſecundũ rationalẽ applicatum, facit alterũ latus Binominũ tertiũ.



Theo-

Theorema 46. Propoſitio 63.

Quadratum lineæ maioris ſecundum lineam rationalem applicatum, facit alterum latus Binomium quartum.

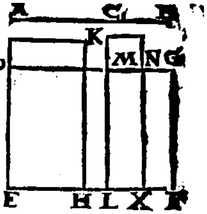


Theorema 47. Propoſitio 64.

Quadratum lineæ potẽtis rationale & mediale, ſecũndum rationalẽ applicatũ, facit alterum latus Binomium quintũ.

Theorema 48. Propoſitio 65.

Quadratum lineę potentis duo medialia ſecundũ rationalem applicatum, facit alterum latus Binomium ſextum.

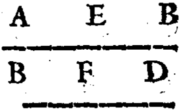


Theorema 49. Propoſitio 66.

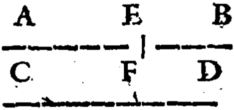
Linea longitudine commenſurabilis Binomio eſt & ipſa Binomium eiuſdẽ ordinis.

Theorema 50. Propositio 67.

Linea longitudine commensurabilis alteri bimedialium, est & ipsa bimediale etiam eiusdē ordinis.



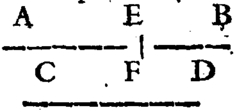
Theorema 51. Propositio 68.



Linea cōmēsurabilis lineæ maiori, est & ipsa maior.

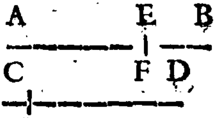
Teorema 52. Propositio 69.

Linea commēsurabilis lineæ potenti rationale & mediale, est & ipsa linea potens rationalc & mediale.



Theorema 53. Propositio 70.

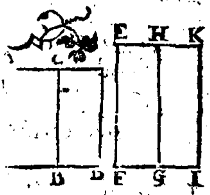
Linea commensurabilis lineæ potenti duo medialia, est & ipsa linea potens duo medialia.



Theorema 54. Propositio 71.

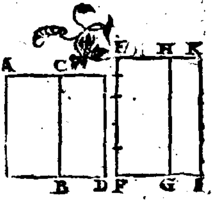
Si duæ superficies rationalis & medialis simul componantur, linea quę totā superficiem

ciem compositã potest, est vna ex quatuor irrationalibus, vel ea quæ dicitur Binomium, vel bimediale primum, vel linea maior, vel linea potẽs rationale & mediale.



Theorema 55. Propositio 72.

Si duæ superficies mediales incõmensurabiles simul componãtur, fiunt reliquæ duæ lineę irrationales, vel bimediale secundum, vel linea potẽs duo medialia



SCHOLIVM.

Binomium & cæteræ consequentes lineæ irrationales, neque sunt eædem cum linea mediali, neque ipsæ inter se.

Nam quadratum lineæ medialis applicatum secundum lineam rationalem, facit alterum latus lineam rationalem, & longitudine incommensurabilem lineæ secundum quam applicatur, hoc est, lineæ rationali, per 23.

Quadratum verò Binomij secundum rationalem applicatum, facit alterum latus Binomium primum, per 60.

Quadratum verò Bimedialis primi ſecundum rationalem applicatum, facit alterum latus Binomiū ſecundum, per 61.

Quadratum verò Bimedialis ſecundi ſecundum rationalem applicatum, facit alterum latus Binomiū tertium, per 62.

Quadratum verò lineæ maioris ſecundum rationalem applicatum, facit alterum latus Binomiū quartum, per 63.

Quadratum verò lineæ potētis rationale & mediale ſecundum rationalem applicatum, facit alterū latus Binomium quintum, per 64.

Quadratum verò lineæ potentis duo medialia ſecūdum rationalem applicatum, facit alterum latus Binomium ſextum, per 65.

Cùm igitur dicta latera, quæ latitudines vocantur, differant & à prima latitudine, quoniam eſt rationalis, cùm inter ſe quoque differant, eo quia ſunt Binomia diuerſorum ordinum: manifeſtum eſt ipſas lineas irrationales, differentes eſſe inter ſe.

SECVN-

SECVNDVS ORDO ALterius ſermonis, qui eſt de detractione.

Principium ſenariorum per detractionẽ.

Theorema 56. Propoſitio 73.

Si de linea rationali detrahatur rationalis potentia tantùm commẽſurabilis ipſi toti, reſidua eſt irrationalis, vocetur autẽ Reſiduum.

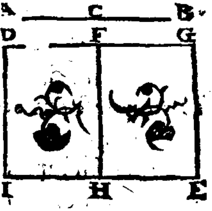
Theorema 57. Propoſitio 74.

Si de linea mediali detrahatur medialis potentia tantũ cõmenſurabilis toti lineæ, quæ verò detracta eſt cũ tota contineat ſuperficiem rationalem, reſidua eſt irrationalis. Vocetur autem Reſiduũ mediale primum.

Theorema 58. Propoſitio 75.

Si de linea mediali detrahatur medialis

tentia tantùm commensurabilis toti, quæ verò detracta est, cùm tota cõtineat superficiẽ medialẽ, reliqua est irrationalis. Vocetur autem Residuũ mediale secundum.



Theorema 57. Propositio 76.

Si de linea recta detrahatur recta potẽtia incommensurabilis toti, compositum autem ex quadratis totius lineæ & lineæ detractæ sit rationale, parallelogrammũ verò ex ijsdem contentum sit mediale, reliqua linea erit irrationalis. A C B Vocetur autem linea minor.

Theorema 58. Propositio 77.

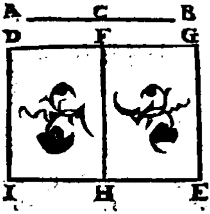
Si de linea recta detrahatur recta potentia

tia incommenſurabilis toti lineæ, compoſitũ autem ex quadratis totius & lineæ detractæ ſit medíale, parallelogrammum verò bis ex eiſdem contentum ſit rationale, reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie rationali totam ſuperficiem medialem.

Theorema 59. Propoſitio 78.

Si de linea recta detrahatur recta potentia incommenſurabilis toti lineæ, compoſitum autem ex quadratis totius & lineæ detractæ ſit mediale, parallelogrammum verò bis ex ijſdem ſit etiam mediale: præterea ſint quadrata ipſarum incommenſurabilia parallelogrammo bis ex ijſdem cõtento, reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie

ficie mediali totam ſuperficiem medialem.

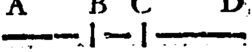


Theorema 60. Propoſitio 79.

Reſiduo vnica tantùm linea recta cõiungitur rationalis, potentia tantùm commenſurabilis toti lineæ.

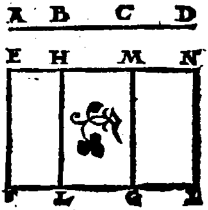
Theorema 61. Propoſitio 80.

Reſiduo mediali primo vnica tãtùm linea coniungitur medialis, potentia tantùm cõmenſurabilis toti, ipſa cum tota continẽs rationale.



Theorema 62. Propoſitio 81.

Reſiduo mediali ſecundo vnica tantùm cõiungitur medialis, potentia tantùm commẽſurabilia toti ipſa cum tota continens mediale.



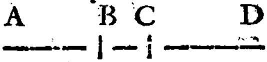
Theorema 63. Propoſitio 82.

Lineæ minori vnica tantùm recta cõiungitur

tur potentia incommenſurabilis toti, faciens cum tota compoſitum ex quadratis ipſarum rationale, id verò parallelogrammum, quod bis ex ipſis fit, mediale.

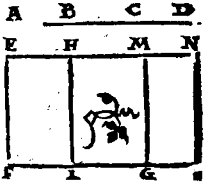
Theorema 64. Propoſitio 83.

Lineæ facienti cum ſuperficie rationali totam ſuperficiem medialem, vnica tantùm coniungitur linea recta potentia incommenſurabilis toti, faciens autem cum tota compoſitum ex quadratis ipſarũ, mediale, id verò quod fit bis ex ipſis, rationale.



Theorema 65. Propoſitio 84.

Lineæ cum mediali ſuperficie facienti totam ſuperficiem medialem, vnica tantùm coniũgitur linea potẽtia toti incommenſurabilis, faciens cum tota compoſitũ ex quadratis ipſarũ mediale, id verò quod fit bis ex ipſis etiam mediale, & prætereà faciens compoſitum ex quadratis ipſarum incommenſurabile ei quod fit bis ex ipſis.



DE-

DEFINITIONES TERTIAE.

Propoſita linea rationali & reſiduo.

1

Si quidem tota, nempe compoſita ex ipſo reſiduo & linea illi cõiuncta, plus poteſt quàm coniuncta, quadrato lineæ ſibi cõmenſurabilis longitudine, fueritq; tota longitudine commẽſurabilis lineæ propoſitæ rationali, reſiduũ ipſum vocetur Reſiduum primum.

2

Si verò coniuncta fuerit longitudine commenſurabilis rationali, ipſa autem tota plus poſſit quam coniuncta, quadrato lineæ ſibi longitudine commenſurabilis; reſiduum vocetur Reſiduum ſecundũ.

3

Si verò neutra linearũ fuerit longitudine commenſurabilis rationali, poſſit autem ipſa tota plusquam coniuncta, quadrato lineæ ſibi longitudine commẽſurabilis; vocetur Reſiduum tertium.

Rurſus ſi tota poſſit plus quàm coniuncta, quadrato lineæ ſibi longitudine incõmenſurabilis.

4

Et quidem ſi tota fuerit longitudine commenmensurabilis ipsi rationali, vocetur Residuum quartum.

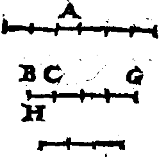
5

Si verò coniuncta fuerit lógitudine commensurabilis rationali,& tota plus possit quàm coniuncta,quadrato lineæ sibi longitudine incómensurabilis, vocetur Residuum quintum.

6

Si verò neutra linearum fuerit cõmensurabilis lõgitudine ipsi rationali, fueritque tota potentior quã coniuncta,quadrato lineæ sibi lõgitudine incommẽsurabilis,vocetur Residuum sextum.

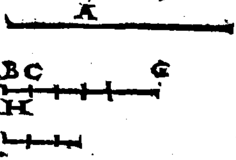
Problema 18. Propositio 85.



Reperire primum Residuum.

16
D.......F.........E
7 9

Problema 19. Propositio 86.



Reperire secundum Residuum.

D.........F.........E
27 9

Proble-

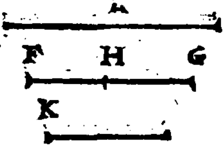
E...........

Problema 20. Propositio 87.

12

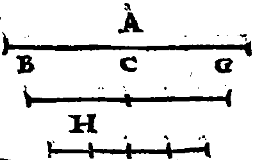
B.........D.......C

9 7



Reperire tertium Residuum.

Probl. 21. Propositio 88.



Reperire quartũ Residuum.

D.................F....E

16 4

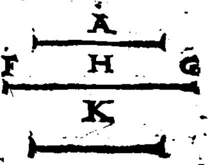
Problema 22. Propositio 89.

Reperire quintum Residuum.

D.......F....E

25 7

Problema 23. Propopositio 90.



Reperire sextum Residuum.

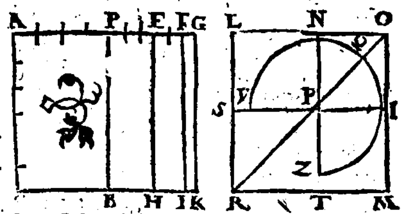
E...............

15

Theo-B........D......C

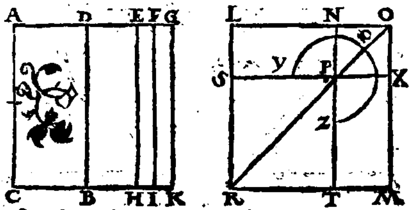
Theorema 66. Propoſitio 91.

Si ſuperficies cõtineatur ex linea rationali & reſiduo primo linea, quæ illã ſupficie m poteſt, eſt reſiduum.



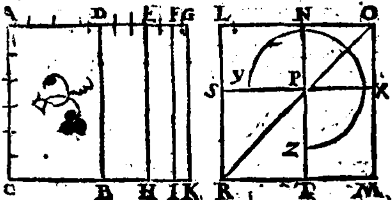
Theorema 67. Propoſitio 92.

Si ſuperficies cõtineatur ex linea rationali & reſi duo ſecũdo, linea q̃ illam ſupficiẽ poteſt, eſt reſiduum mediale primum.



Theorema 68. Propoſitio 93.

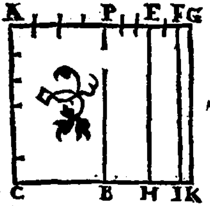
Si ſupficies cõtineatur ex linea rationali & reſiduo ter-

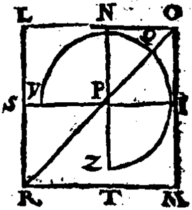


tio, linea quæ illam superficiem potest, est residuum mediale secundum.

Theorema 69. Propositio 94.

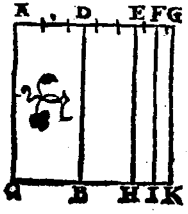
Si superficies cõtineatur ex linea rationali & resi-duo quarto linea q̃ illã su-perfici-em po-test, est linea minor.

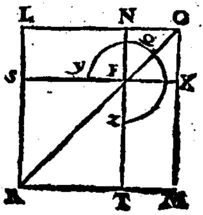




Theorema 70. Propositio 95.

Si superficies cõtineatur ex linea rationali & residuo quinto, linea quæ illã superficiẽ potest, est ea quæ dicitur cum rationali super-fi-cie fa-ciens totã me-dia-lem.



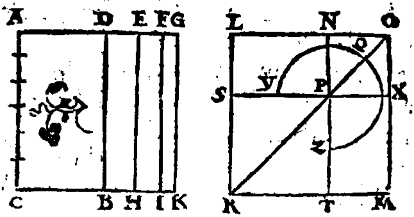


Theorema 71. Propo-sitio 96.

Si superficies cõtineatur ex linea rationali

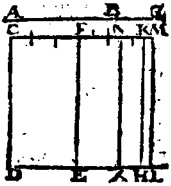
&

& reſiduo ſexto, linea quæ illam ſuperficiē pōt, eſt ea quę dicitur faciēs cum mediali



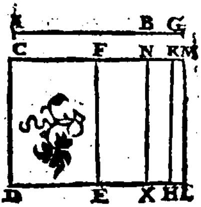
ſuperficie totam medialem.

Theorema 72. Propoſitio 97.



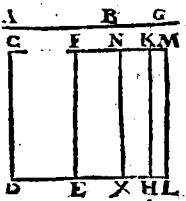
Quadratum reſidui ſecundum lineam rationalem applicatum, facit alterum lat' reſiduum primum.

Theorema 73. Propoſitio 98.



Quadratum reſidui medialis primi ſecundū rationalem applicatū, facit alterū latus reſiduū ſecundum.

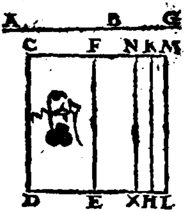
Theorema 74. Propoſitio 99.



Quadratū reſidui medialis ſecūdi ſecundum rationalē applicatū, facit alterū latus reſiduū tertium.

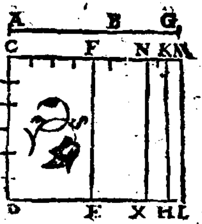
 Theo-

Theorema 75. Propoſitio 100.



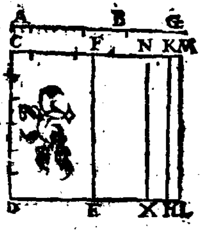
Quadratū lineæ minoris ſecundum rationalē applicatum, facit alterū latus reſiduum quartū.

Theorema 76. Propoſitio 101.



Quadratū lineæ cum rationali ſuperficie faciētis totam medialem, ſecūdum rationalem applicatum, facit alterum latus reſiduum quintum.

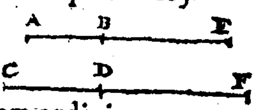
Theorema 77. Propoſitio 102.



Quadratum lineæ cum mediali ſuperficie faciētis totam medialem, ſecundum rationalem applicatum, facit alterum latus reſiduum ſextum.

Theorema 78. Propoſitio 103.

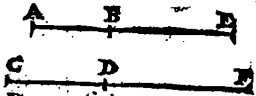
Linea reſiduo cōmēſurabilis lōgitudine, eſt & ipſa reſiduum, & eiuſdem ordinis.



Theorema 79. Propoſitio 104.

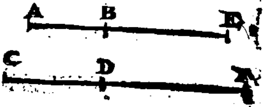
Linea cōmenſurabilis reſiduo mediali, eſt &

& ipſa reſiduũ mediale, & eiuſdẽ ordinis.



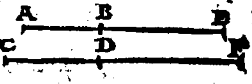
Theorema 80. Propoſitio 105.

Linea commẽſurabilis lineæ minori, eſt & ipſa linea minor.



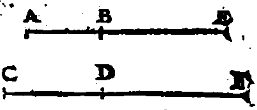
Theorema 81. Propoſitio 106.

Linea commenſurabilis lineæ cũ rationali ſuperficie facienti totã medialẽ, eſt & ipſa linea cum rationali ſuperficie faciens totam medialem.



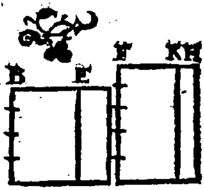
Theorema 82. Propoſitio 107.

Linea cõmmẽſurabilis lineæ cũ mediali ſuperficie facienti totam medialem, eſt & ipſa cum mediali ſuperficie faciens totam medialem.



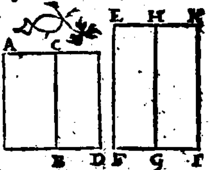
Theorema 83. Propoſitio 108.

Si de ſuperficie rationali detrahatur ſuperficies medialis, linea quæ reliquã ſuperficiem poteſt, eſt alterutra ex duabus irrationalibus, aut reſiduũ, aut linea minor.



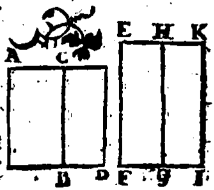
Theorema 84. Propoſitio 109.

Si de ſuperficie mediali detrahatur ſuperficies rationalis, aliæ duæ irrationales fiũt, aut reſiduum mediale primũ aut cũ rationali ſuperficiem faciens totam medialem.



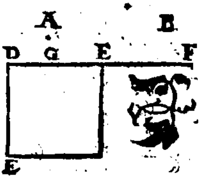
Theorema 85. Propoſitio 110.

Si de ſuperficie mediali detrahatur ſuperficies medialis q̃ ſit incommẽſurabilis toti, reliquæ duæ fiunt irrationales, aut reſiduum mediale ſecundum, aut cũ mediali ſuperficie faciens totam medialem.



Theorema 86. Propoſitio 111.

Linea quæ Reſiduum dicitur, non eſt eadem cum ea quæ dicitur Binomium.



SCHO-

SCHOLIVM.

Linea quæ Residuum dicitur, & cætera quinque eã consequentes irrationales, neque lineæ mediali neque sibi ipsæ inter se sunt eadem. Nam quadratum lineæ medialis secundum rationalem applicatum, facit alterum latus, rationalem lineam longitudine incommensurabilem ei, secundum quam applicatur, per 23.

Quadratum verò residui secundum rationalem applicatum, facit alterum latus residuum primum per 97.

Quadratum verò residui medialis primi secundum rationalem applicatum, facit alterum latus residuum secundum, per 98.

Quadratum verò residui medialis secundi, facit alterum latus residuum tertium, per 99.

Quadratum verò lineæ minoris, facit alterum latus residuum quartum, per 100.

Quadratum verò lineæ cum rationali superficie facientis totam medialem, facit alterum latus residuum quintum, per 101.

Quadratum verò lineæ cum mediali superficie facientis totam medialem, secundum rationalem applicatum, facit alterum latus residuum sextum, per 102.

Cùm igitur dicta latera, quæ sunt latitudines cuiusque parallelogrammi vnicuique quadrato æqualis & secundum rationalem applicati, differant & à primo latere, & ipsa inter se (nam à primo differunt: quoniam sunt resi-

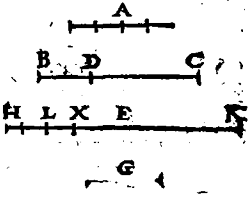
dua non eiusdem ordinis) constat ipsas quoque lineas irrationales inter se differentes esse. Et quoniam demonstratum est, Residuum non esse idem quod Binomium, quadrata autem residui & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex residuis eiusdem ordinis cuius sunt & residua, quorum quadrata applicantur rationali similiter & quadrata Binomij & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex Binomijs eiusdem ordinis, cuius sunt & Binomia, quorum quadrata applicantur rationali. Ergo lineæ irrationales quæ cõsequuntur Binomium, & quæ consequuntur residuum, sunt inter se differentes. Quare dictæ lineæ omnes irrationales sunt numero 13.

1 Medialis.
2 Binomium.
3 Bimediale primum.
4 Bimediale secundum.
5 Maior.
6 Potens rationale & mediale.
7 Potẽs duo medialia.
8 Residuum
9 Residuum mediale primum.
10 Residuum mediale secundum.
11 Minor.
12 Faciens cum rationali superficie totam medialem.
13 Faciens cum mediali superficie totam medialem.

Theo-

Theorema 87. Propositio 112.

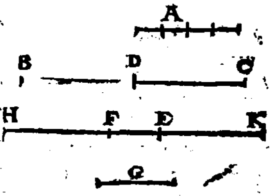
Quadratũ lineæ rationalis secundum Binomiũ applicatũ, facit alterũ latus residuũ, cuius nomina sunt cõmensurabilia Binomij nominibus, & in eadẽ proportione



præterea id quod fit Residuum, eundem ordinem retinet quem Binomium.

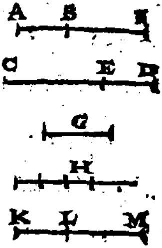
Theorema 88. Propositio 113.

Quadratũ lineæ rationalis secũdum residuum applicatũ, facit alterum latus Binomium, cuius nomina sunt cõmensurabilia nominibus residui, & in eadẽ proportione: preterea id quod fit Binomiũ, est eiusdem ordinis, cuius & Residuum.



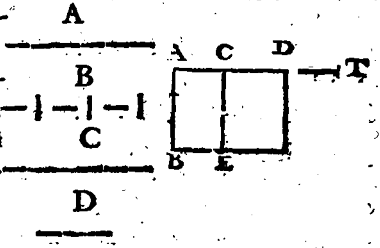
Theorema 89. Propositio 114.

Si parallelogrammũ contineatur ex residuo & Binomio, cuius nomina ſunt cómenſurabilia nominibus reſidui & in eadé proportione, linea quæ illam ſuperficiem poteſt, eſt rationalis.



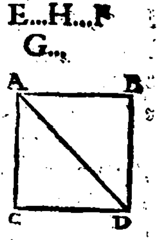
Theorema 90. Propoſitio 115.

Ex linea mediali naſcũtur lineę irrationales innumerabiles, quarum nulla vlli ante dictarum eadem ſit.



Propoſitio 116.

Propoſitũ nobis eſto demóſtrare in figuris quadratis diametrũ eſſe longitudine incómenſurabilem ipſi lateri.



ELEMENTI X. FINIS.

EVCLIDIS ELEMENTVM VNDECIMVM, ET SOLIDORVM *primum.*

DEFINITIONES.

1

Solidum, est quod longitudinem, latitu-nem, & crassitudinem habet.

2

Solidi autem extremum est superficies.

3

Linea recta est ad planum recta, cùm ad rectas oẽs lineas, à quibus illa tangitur, quæque in proposito sunt plano, rectos angulos efficit.

4

Planum ad planum rectum est, cùm rectæ lineæ, quæ cõmuni planorum sectioni ad rectos angulos in vno planorum ducũtur, alteri plano ad rectos sunt angulos.

5

Rectæ lineę ad planũ inclinatio, acutus est angulus, ipsa insistente linea & adiũcta altera comprehẽsus, cùm à sublimi rectę illius lineę termino deducta fuerit ꝑpẽdicularis,

laris, atq; à puncto quod perpendicularis in ipso plano fecerit, ad propositę illius lineæ extremum, quod in eodẽ est plano, altera recta linea fuerit adiuncta.

6

Plani ad planũ inclinatio, acutus est angulus rectis lineis contentus, quæ in vtroque planorum ad idem cõmunis sectionis pũctũ ductæ, rectos ipsi sectioni angulos efficiunt.

7

Planum similiter inclinatum esse ad planum, atq; alterum ad alterum dicitur, cũ dicti inclinationum anguli inter se sunt ęquales.

8

Parallela plana, sunt quæ eodem non incidunt, nec concurrunt.

9

Similes figuræ solidæ, sunt quæ similibus planis, multitudinẽ æqualibus cõtinẽtur.

10

Aequales & similes figurę solidę sunt, quę similibus planis, multitudine & magnitudine æqualibus continentur.

11

Solidus angulus, est plurium quàm duarũ linearum, quæ se mutuò contingant, nec in eadem sint superficie, ad omnes lineas inclinatio.

Aliter.

Aliter.

Solidus angulus, est qui pluribus quã duobus planis angulis in eodem non cõsistentibus plano, sed ad vnũ punctũ collectis, continetur.

12

Pyramis, est figura solida quæ planis cõtinetur, ab vno plano ad vnum punctũ collecta.

13

Prisma, figura est solida quæ planis cõtinetur, quorum aduersa duo sunt & æqualia & similia & parallela, alia verò parallelogramma.

14

Sphęra est figura, quæ cõuerso circum quiescentem diametrum semicirculo cõtinetur, cùm in eundem rursus locũ restitutus fuerit, vnde moueri cœperat.

15

Axis autem sphæræ, est quiescẽs illa linea circum quam semicirculus conuertitur.

16

Centrum verò Sphæræ est idẽ, quod & semicirculi.

17

Diameter autem Sphæræ, est recta quædã linea per centrum ducta, & vtrinq; à sphęræ superficie terminata.

Conus

18

Cωnus est figura, quæ conuersó circũquiescens alterum latus eorum quæ rectũ angulum continent, orthogωnio triangulo continetur, cùm in eundem rursus locum illud triangulum restitutum fuerit, vnde moueri cœperat. Atq; si quiescens recta linea æqualis sit alteri, quę circum rectũ angulum conuertitur, rectangulus erit Cωnus: sin minor, amblygωnius: si verò maior, oxygonius.

19

Axis autem Cωni, est quiescens illa linea, circum quam triangulum vertitur.

20

Basis verò Cωni, circulus est, qui à circũducta linea recta describitur.

21

Cylindrus figura est, quæ conuersó circũ quiescens alterum latus eorum quæ rectũ angulum continet, parallelogrammo orthogωnio comprehenditur, cùm in eundẽ rursus locum restitutum fuerit illud parallelogrammũ, vnde moueri cœperat.

22

Axis autem Cylindri, est quiescens illa recta linea, circũ quam parallelogrammum vertitur.

23

Bases verò cylindri, sunt circuli à duobus ad-

aduerſus lateribus quæ circumaguntur, deſcripti.

24

Similes coni & cylindri, ſunt quorũ & axes & baſiũ diametri proportionales ſunt.

25

Cubus eſt figura ſolida, quæ ſex quadratis æqualibus continetur.

26

Tetraedũm eſt figura, quæ triãgulis quatuor æqualibus & æquilateris continetur:

27

Octaedrũ figura eſt ſolida, quæ octo triangulis æqualibus & æquilateris cõtinetur.

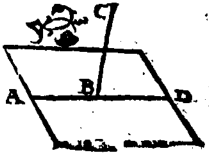
28

Dedecaedrum figura eſt ſolida, quæ duodecim pentagonis æqualibus, æquilateris, & æquiangulis continetur.

29

Eicoſaedrum figura eſt ſolida, quę triãgulis viginti æqualibus & æquilateris continetur:

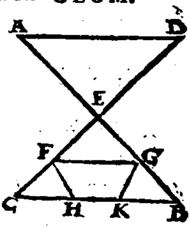
Theorema 1. Propoſitio 1.



Quædã rectæ lineę pars in ſubiecto quidem nõ eſt plano, quædã verò in ſublimi.

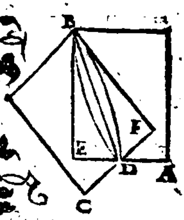
Theo-

Theorema 2. Propoſitio 2.



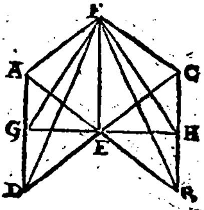
Si duę rectę lineę ſe mutuò ſecẽt, in vno sũt plano: atque triangulum omne in vnó eſt plano.

Theorema 3. Propoſitio 3.



Si duo plana ſe mutuò ſecent, communis eorũ ſectio eſt recta linea.

Theorema 4. Propoſitio 4.



Si recta linea rectis duabus lineis ſe mutuò ſecantibus, in cõmuni ſectione ad rectos angulos inſiſtat illá ducto etiam per ipſas plano ad ángulos rectos erit.

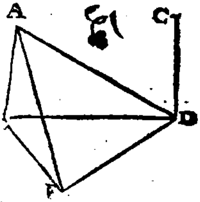
Theorema 5. Propoſitio 5.

Si recta linea rectis tribus lineis ſe mutuò tangentibus, in communi ſectione ad rectos angulos inſiſtat, illæ tres rectę in vno ſunt plano.

Theo-

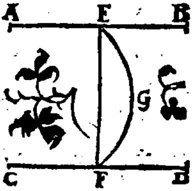
Theorema 6. Propoſitio 6.

Si duæ rectæ lineæ eidẽ plano ad rectos ſint angulos, parallelæ erũt illæ rectæ lineæ.



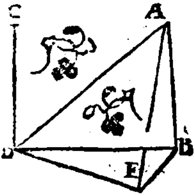
Theorema 7. Propoſitio 7.

Si duæ ſint parallelæ rectæ lineæ, in quarũ vtraque ſumpta ſint quę libet pũcta, illa linea quę ad hæc puncta adiungitur, in eodem eſt cũ parallelis plano.



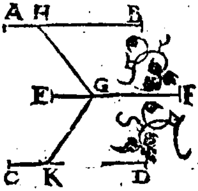
Theorema 8. Propoſitio 8.

Si duæ ſint parallelæ rectæ lineæ, quarum altera ad rectos cuidam plano ſit angulos, & reliqua eidẽ plano ad rectos angulos erit.

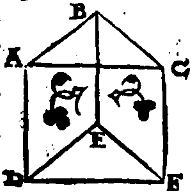


Theorema 9. Propoſitio 9.

Quæ eidem rectæ lineæ ſun parallelæ, ſed nõ in eodem cũ illa plano, hæ quoque ſunt inter ſe parallelæ.

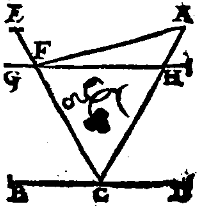


Theorema 10. Propoſitio 10.



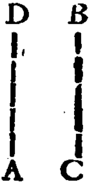
Si duę rectę lineæ ſe mutuò tangẽtes ad duas rectas ſe mutuò tangentes ſint parallelæ, non autẽ in eodem plano, illę angulos æquales comprehendent.

Proble. 1. Propoſitio 11.



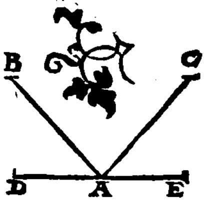
A dato ſublimi puncto, in ſubiectum planũ perpendicularem rectã lineam ducere.

Problema 2. Propoſitio 12.



Dato plano, à puncto quod in illo datum eſt, ad rectos angulos rectam lineam excitare.

Theorema 11. Propoſitio 13.

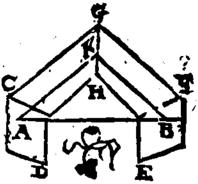


Dato plano, à pũcto qđ in illo datum eſt, duæ rectæ lineæ ad rectos angulos non excitabuntur ad eaſdem partes.

Theo-

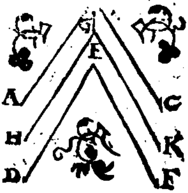
Theorema 12. Propositio 14.

Ad quæ plana, eadem recta linea recta est, illa sũt parallela.



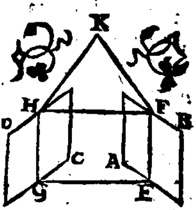
Theorema 13. Propositio 15.

Si duæ rectæ lineæ se mutuò tangentes ad duas rectas se mutuò tangentes sint parallelæ, non in eodem consistentes plano, parallela sunt quæ per illas ducuntur plana.



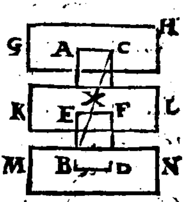
Theorema 14. Propositio 16.

Si duo plana parallela plano quopiam secẽtur, communes illorum sectiones sunt parallelæ.

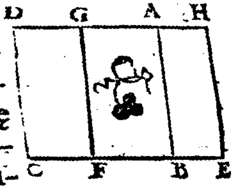


Theorema 15. Propositio 17.

Si duæ rectæ lineę parallelis planis secẽtur, in easdem rationes secabũtur.

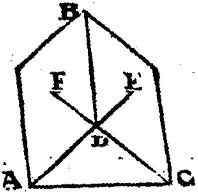


Theorema 16. Propositio 18.



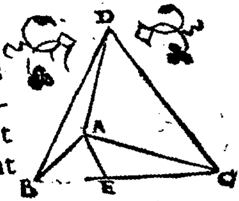
Si recta linea plano cuipiam ad rectos sit angulos, illa etiam omnia quę per ipsam plana, ad rectos eidem plano angulos erunt.

Theorema 17. Propositio 19.



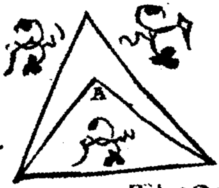
Si duo plana se mutuò secantia plano cuidã ad rectos sint angulos, cõmunis etiam illorum sectio ad rectos eidẽ plano angulos erit.

Theorema 18. Propositio 20.



Si angulus solidus planis tribus angulis contineatur, ex his duo quilibet vtut assumpti tertio sunt maiores.

Theorema 19. Propositio 21.

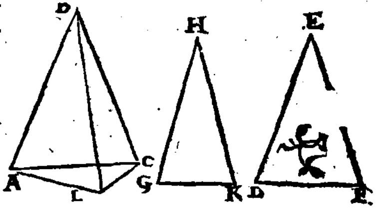


Solidus omnis angulus minoribus continetur, quàm rectis quatuor angulis planis.

Theo-

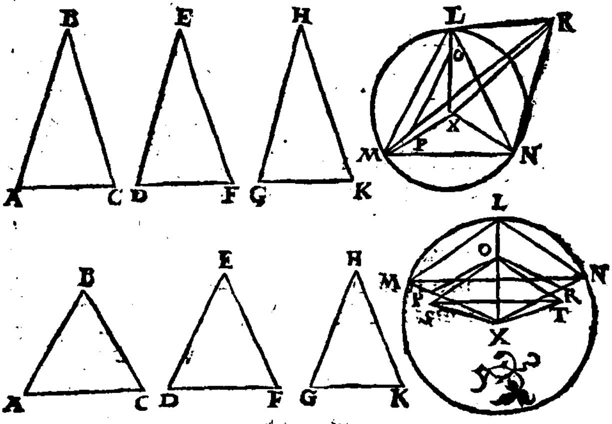
Theorema 20. Propoſitio 22.

Si plani tres anguli æqualibus rectis cõtineantur lineis, quorũ duo vt libet aſſumpti, tertio ſint maiores, triangulum conſtitui poteſt ex lineis æquales, illas rectas coniungẽtibus.

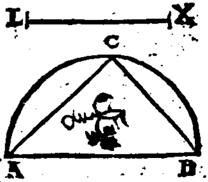


Problema 3. Propoſitio 23.

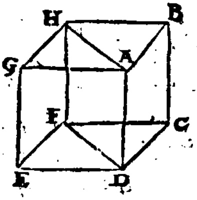
Ex planis tribus angulis, quorũ duo vt libet aſſumpti tertio ſint maiores, ſolidũ angulum conſtituere. Decet autem illos tres angulos rectis quatuor eſſe minores.



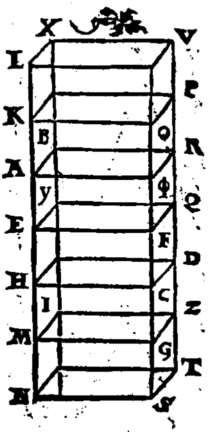
Theorema 21. Propositio 24.



Si solidum parallelis planis contineatur, aduersa illius plana & æqualia sunt & parallelogramma.



Theorema 22. Propositio 25.

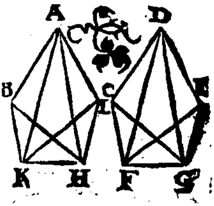


Si solidum parallelis planis contentũ plano secetur aduersis planis parallelo, erit queadmodum basis ad basim, ita solidum ad solidum.

Proble-

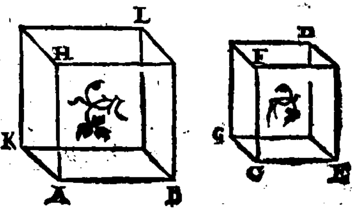
Problema 4. Propoſitio 26.

Ad datam rectam lineā eiuſque punctū, angulum ſolidum conſtituere ſolido angulo dato æqualem.



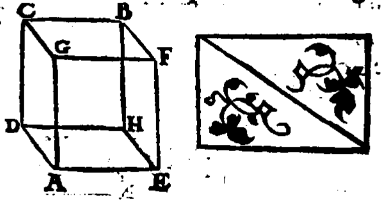
Problema 5. Propoſitio 27.

A data recta, dato ſolido parallelis planis comprehenſo ſimile & ſimiliter poſitum ſolidū parallelis planis contētum deſcribere.

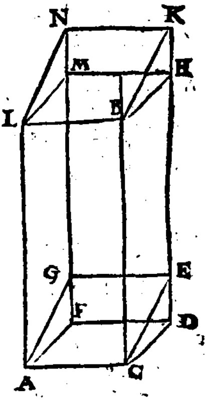


Theorema 23. Propoſitio 28.

Si ſolidū parallelis planis comprehēſum, ducto per aduerſorum planorum diagoniospla no ſectū ſit, illud ſolidū ab hoc plano bifariàm ſecabitur.



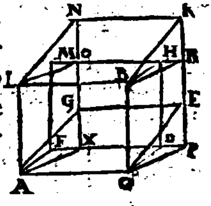
Theorema 34. Propositio 29.



Solida parallelis planis cõprehẽsa, quę super eandem basim, & in eadẽ sunt altitudine, quorum insistẽtes lineæ in ijsdem collocantur rectis lineis, illa sũt inter se æqualia.

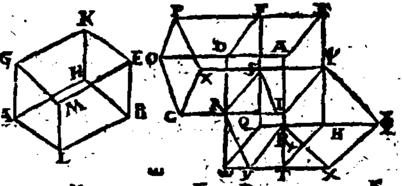
Theorema 25. Propositio 30.

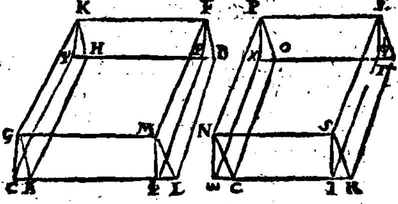
Solida parallelis planis circunscripta, quæ super eandẽ basim & in eadẽ sũt altitudine, quorum insistentes lineæ nõ in ijsdem reperiuntur rectis lineis, illa sunt inter se æqualia.



Theorema 26. Propositio 31.

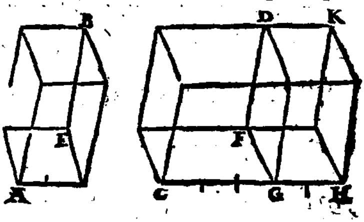
Solida parallelis planis circunscripta, quę in in eadé sunt altitudine, æqualia sunt inter se.





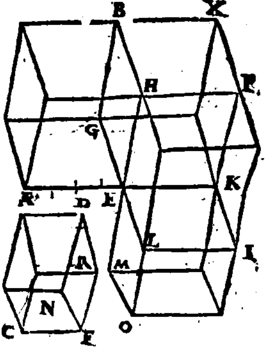
Theorema 27. Propositio 32.

Solida parallelis planis circunscripta quæ eiusdem sunt altitudinis, eam habét inter se rationé, quam bases.



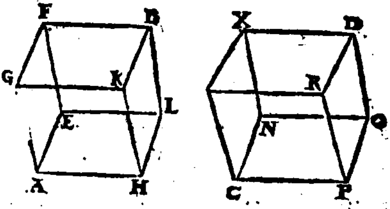
Theor.28. Propositio 33.

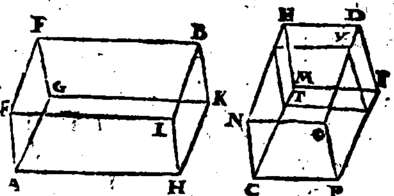
Similia solida parallelis planis circunscripta habẽt inter se rationẽ homologorum laterum triplicatam.

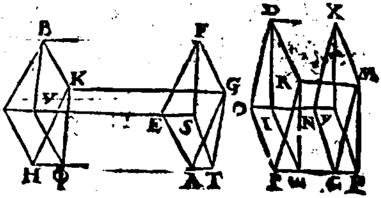


Theor.29. Propositio 34.

Aequalium solidorũ parallelis planis cótẽtorum bases cũ altitudinibus reciprocãtur. Et solida parallelis planis cótẽta, quorũ bases cum altitudi-



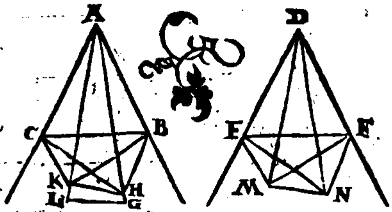




nibus reciprocantur, illa sunt æqualia.

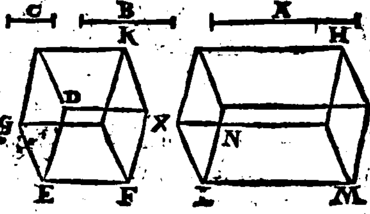
Theorema 30. Propositio 35.

Si duo plani sint anguli æquales, quorũ verticibus sublimes rectæ lineæ insistant, quæ cum lineis primò positis angulos cõtineant æquales, vtrunq; vtriq;, in sublimibus autem lineis quælibet sumpta sint pũcta, & ab his ad plana in quibus consistunt anguli primũ positi, ductæ sint perpendiculares, ab earũ verò punctis, quæ in planis signata fuerint, ad angulos primũ positos adiũctæ sint rectæ lineę, hæ cũ sublimibus æquales angulos comprehendent.



Theorema 31. Propositio 36.

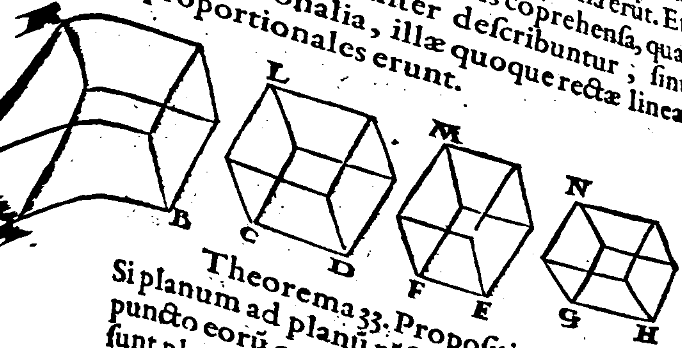
Si rectæ tres lineæ sint proporionales, quod



ex his tribus fit solidum parallelis planis contentum, æquale est descripto à media linea solido parallelis planis comprehẽso, quod æquilaterum quidem sit, sed antedicto æquiangulum.

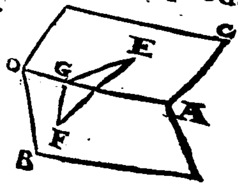
Theorema 32. Propositio 37.

Si rectæ quatuor lineæ sint proportionales, illa quoque solida parallelis planis cõtenta, quæ ab ipsis lineis & similia & similiter describuntur, proportionalia erũt. Et si solida parallelis planis cõprehensa, quæ & similia & similiter describuntur, sint proportionalia, illæ quoque rectæ lineæ proportionales erunt.



Theorema 33. Propositio 38.

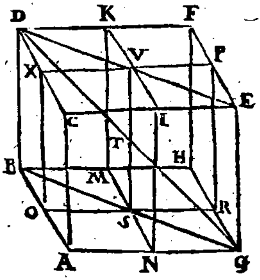
Si planum ad planũ rẽctum sit, & à quodã puncto eorũ quæ in vno sunt planorũ perpendicularis ad alterum ducta sit, illa quæ ducitur perpẽdicularis, in cõmunẽ cadet planorũ sectionẽ.



Theo-

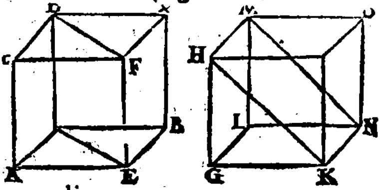
Theorema 34. Propositio 39.

Si in solido parallelis planis circũscripto, aduersorum planorum lateribus bifariã sectis, educta sint per sectiones plana, cõmunis illa planorũ sectio & solidi parallelis plani circũscripti diameter, se mutuò bifariam secant.



Theorema 35. Propositio 40.

Si duo sint æqualis altitudinis prismata, quorũ hoc quidẽ basim habeat parallelogrammũ, illud verò triãgulum, sit autem parallelogrammũ triãguli duplũ, illa prismata erunt æqualia.



ELEMENTI XI. FINIS.

ex his tribus fit solidum parallelis planis contentum, æquale est descripto à media linea solido parallelis planis compreheſo, quod æquilaterum quidem sit, sed antedicto æquiangulum.

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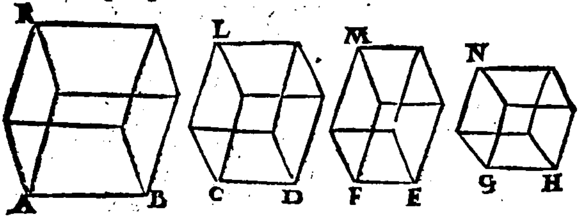

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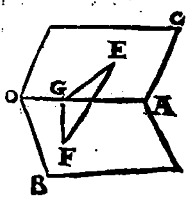
Si planum ad planũ rectum sit, & à quodã puncto eorũ quæ in vno sunt planorũ perpendicularis ad alterum ducta sit, illa quæ ducitur perpẽdicularis, in cõmunẽ cadet planorũ sectionẽ.



Theo-

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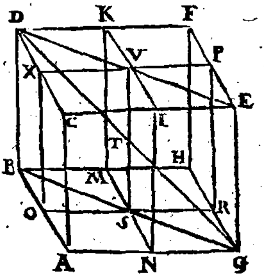
Si in solido parallelis planis circũscripto, aduersorum planorum lateribus bifariã sectis, educta sint per sectiones plana, cõmunis illa planorũ sectio & solidi parallelis plani circũscripti diameter, se mutuò bifariam secant.



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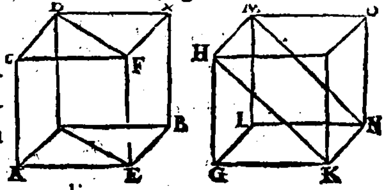

ELEMENTI XI. FINIS.

ex his tribus fit ſolidum parallelis planis contentum, æquale eſt deſcripto à media linea ſolido parallelis planis comprehẽſo, quod æquilaterum quidem ſit, ſed antedicto æquiangulum.

Theorema 32. Propoſitio 37.

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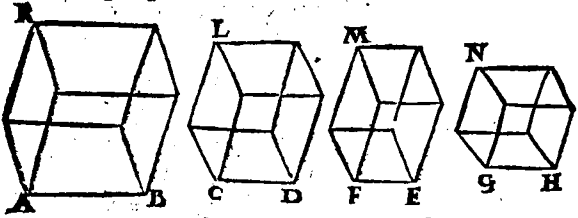

Theorema 33. Propoſitio 38.

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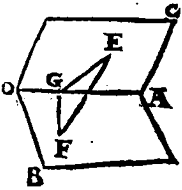

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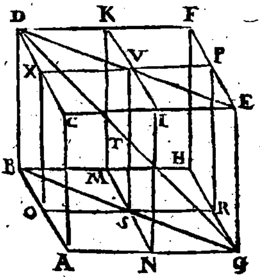

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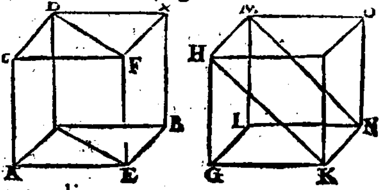

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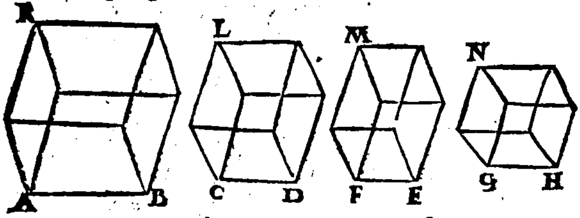

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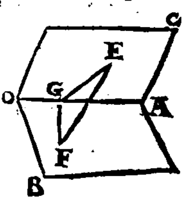

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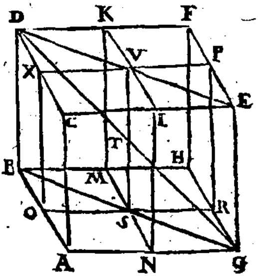
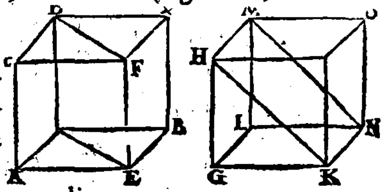
Theorema 34. Propoſitio 39.

Si in ſolido parallelis planis circũſcripto, aduerſorum planorum lateribus bifariã ſectis, educta ſint per ſectiones plana, cõmunis illa planorũ ſectio & ſolidi parallelis plani circũſcripti diameter, ſe mutuò bifariam ſecant.



Theorema 35. Propoſitio 40.

Si duo ſint æqualis altitudinis priſmata, quorũ hoc quidẽ baſim habeat parallelogrammũ, illud verò triãgulum, ſit autem parallelogrammũ triãguli duplũ, illa priſmata erunt æqualia.



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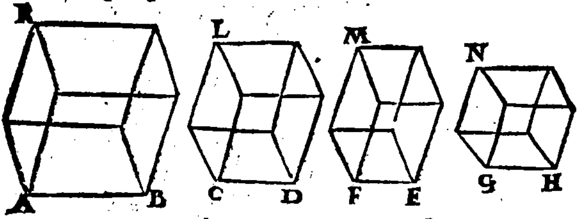

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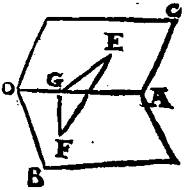

Theo-

Theorema 34. Propoſitio 39.

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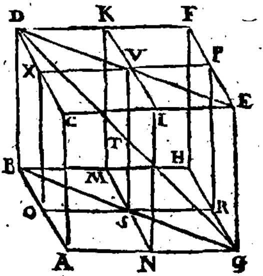
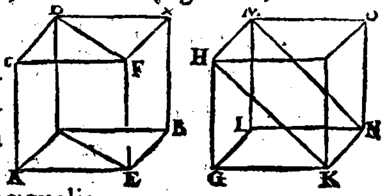


Theorema 35. Propoſitio 40.

Si duo ſint æqualis altitudinis priſmata, quorū hoc quidē baſim habeat parallelogrammū, illud verò triāgulum, ſit autem parallelogrammū triāguli duplū, illa priſmata erunt æqualia.

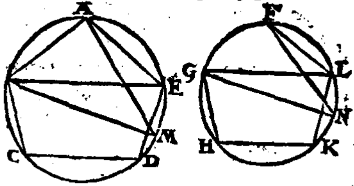


ELEMENTI XI. FINIS.

EVCLIDIS ELEMENTVM DVODECIMVM. ET SOLIDORVM ſecundum.

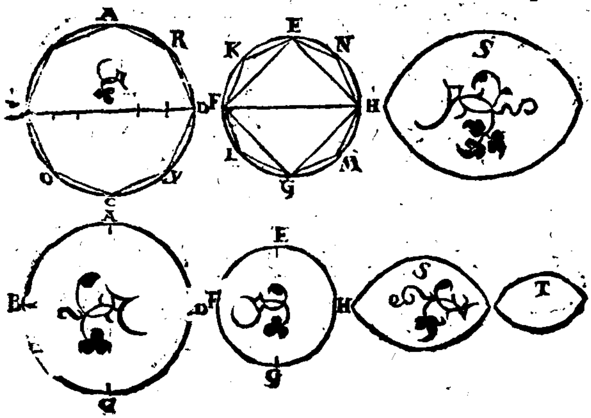
Theorema 1. Propoſitio 1.

Similia, quæ ſunt in circulis polygona, rationẽ habẽt inter ſe, quam deſcripta à diametris quadrata.



Theorema 2. Propoſitio 2.

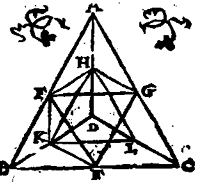
Circuli eam inter ſe rationẽ habent, quam



deſcripta à diametris quadrata.

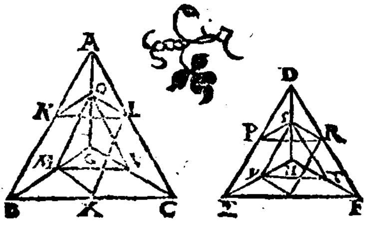
Theorema 3. Propoſitio 3.

Omnis pyramis trigωnã habens baſim, in duas diuiditur pyramidas non tantũ æquales & ſimiles inter ſe, ſed toti etiã pyramidi ſimiles, quarum trigωnæ ſunt baſes, atq; in duo priſmata æqualia, quæ duo priſmata dimidio pyramidis totius ſunt maiora.



Theorema 4. Propoſitio 4.

Si duæ eiuſdem altitudinis pyramides trigωnas habeãt baſes, ſit aũt illarum vtraque diuiſa & in duas pyramidas inter ſe ęquales totiq; ſimiles, & in duo priſmata æqualia, ac eodẽ modo diuidatur vtraq; pyramidũ quæ ex ſuperiore diuiſione natæ ſunt, idque perpetuò fiat: quemadmodũ ſe habet vnius pyramidis baſis ad alterius pyramidis baſim, ita & omnia quæ in vna pyra-

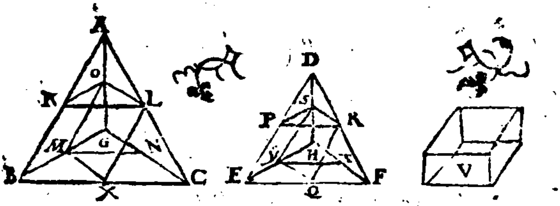


mide

mide priſmata, ad omnia quæ in altera pyramide, priſmata multitudine æqualia.

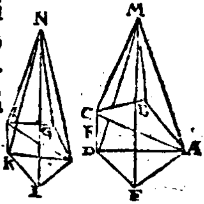
Theorema 5. Propoſitio 5.

Pyramides eiuſdem altitudinis, quarũ trigωnæ ſunt baſes, eam inter ſe rationem habent, quàm ipſæ baſes.



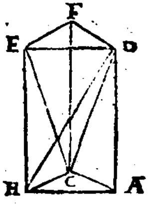
Theorema 6. Propoſitio 6.

Pyramides eiuſdem altitudinis, quarum polygωnæ ſunt baſes, eam inter ſe rationem habent, quã ipſæ baſes.



Theorema 7. Propoſitio 7.

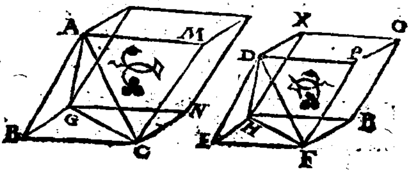
Omne priſma trigωnã habẽs baſim, diuiditur in tres pyramides inter ſe æquales, quarum trigωnæ ſunt baſes.



Theo-

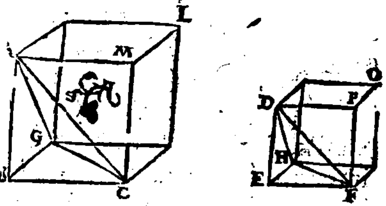
Theorema 8. Propositio 8.

Similes pyramides qui trigωnas habent ba ses, in tripli- cata sũt hõ- molo- gorũ laterũ ratiõe.



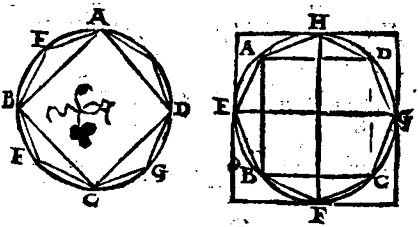
Theorema 9. Propositio 9.

Aequaliũ pyramidum & trigωnas bases hà bentium reciprocantur bases cum altitudi nibus. Et quarũ pyramidum trigωnas ba- ses haben tium reci procãtur bases cũ altitudi- nibus, il- læ sunt æquales.



Theorema 10. Propositio 10.

Om- nis cω nº ter- tia pars est cy- lindri eandẽ

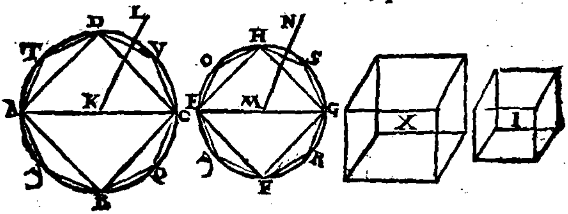


O cum

cum ipſo cono baſim habentis,& altitudinem æqualem.

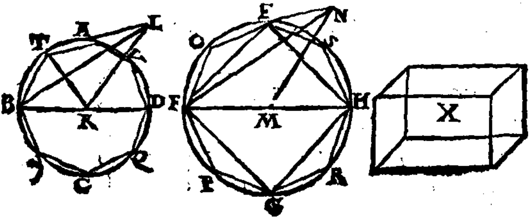
Theorema 11. Propoſitio 11.

Coni & cylindri eiuſdem altitudinis, eam inter ſe rationem habent,quam baſes.



Theorema 12. Propoſitio 12.

Similes coni & cylindri, triplicatam habẽt inter ſe rationem diametrorum, quæ ſunt in baſibus.



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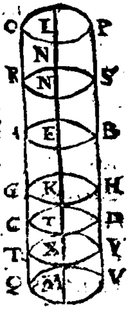
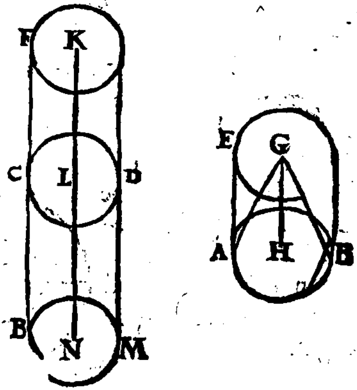
Theorema 13. Propositio 13.



Si cylindrus plano sectus sit ad uersis planis parallelo, erit quẽ admodum cylindrus ad cylindrum, ita axis ad axem.

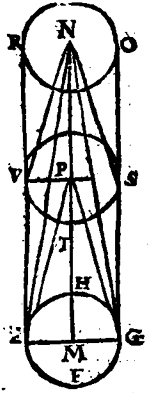
Theorema 14. Propositio 14.

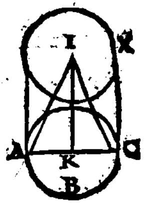
Coni & cylindri qui in æqualibus sunt basibus, eam habent inter se rationem, quam altitudines



Theorema 15. Propositio 15.

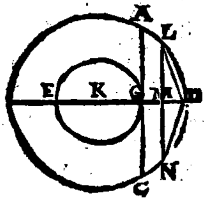
Aequalium conorum & cylindrorũ bases cũ altitudinib⁹ reciprocãtur. Et quorum conorũ & cylindrorũ bases cum altitudinib⁹ reciprocãtur, illi sunt æquales.





Problema 1. Propositio 16.

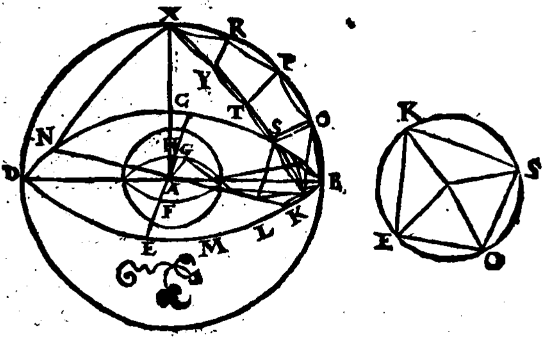
Duobus circulis circũ idem centrum consistentibus, in maiore circulo polygonum æqualium pariumq́ue laterum inscribere, quod minorem circulum nõ tangat.



Pro-

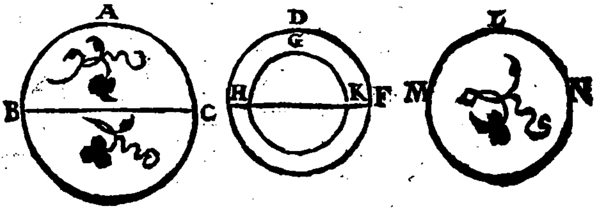
Problema 2. Propositio 17.

Duabus sphæris circum idem centrũ consistentibus, in maiore sphera solidũ polyedrum inscribere, quod minoris sphæræ superficiem non tangat.



Theorema 16. Propositio 18.

Sphæræ inter se rationem habent suarum diametrorum triplicatam.

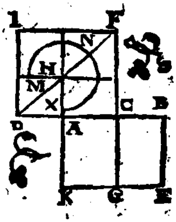


ELEMENTI XII. FINIS.

EVCLIDIS ELEMENTVM DEDECIMVMTERTIVM, ET SOLIDORVM TERTIVM.

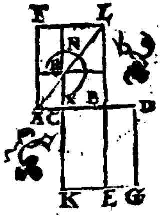
Theorema 1. Propositio 1.

Si recta linea per extremā & mediā rationē secta sit, maius segmentum quod totius lineæ dimidium assumpserit, quintuplum potest eius quadrati, quod à totius dimidia describitur.



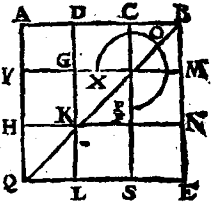
Theorema 2. Propositio 2.

Si recta linea sui ipsius segmenti quintuplū possit, & dupla segmenti huius linea per extremā & mediam rationē secetur maius segmentū reliqua pars est lineæ primum positæ.



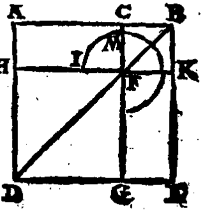
Theo-

Theorema 3. Propoſitio 3.



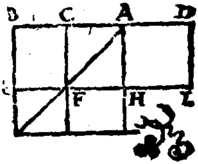
Si recta linea per extremã & mediam rationẽ ſecta ſit, minus ſegmẽtum quod maioris ſegmenti dimidium aſſumpſerit, quintuplum poteſt eius, quòd à maioris ſegmẽti dimidio deſcribitur, quadrati.

Theorema 4. Propoſitio 4.



Si recta linea per extremam & mediam rationẽ ſecta ſit, quod à tota, quodque à minore ſegmẽto ſimul vtraq; quadrata, tripla ſunt eius, quod à maiore ſegmẽto deſcribitur, quadrati.

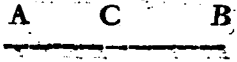
Theorema 5. Propoſitio 5.



Si ad rectam lineam, quæ per extremam & mediam rationem ſecetur, adiuncta ſit altera ſegmento maiori ęqualis, tota hæc linea recta per extremam & mediam rationẽ ſecta est, estque maius segmentum linea primùm posita.

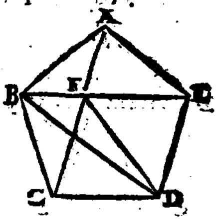
Theorema 6. Propositio 6.

Si recta linea ῥητὴ siue rationalis, per extremam & mediam rationem secta sit, vtrunque segmentorum ἄλογος siue irrationalis est linea, quæ dicitur Residuum.



Theorema 7. Propositio 7.

Si pentagωni æquilateri tres sint æquales anguli, siue q deinceps, siue qui non deinceps sequuntur, illud pētagωnum erit ęquiangulum.

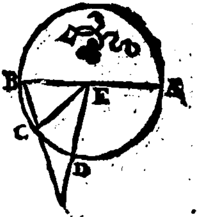


Theorema 8. Propositio 8.

Si pētagωni æquilateri & æquianguli duos qui deinceps sequuntur angulos rectę subtendāt lineæ, illæ per extremā & mediam rationem se mutuò secant, earumq; maiora segmenta, ipsius pentagωni lateri sunt æqualia.

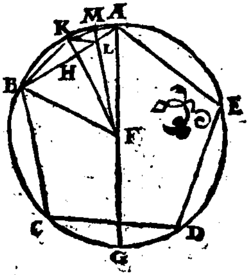
Theo-

Theorema 9. Propoſitio 9.



Si latus hexagωni & latus decagωni eidem circulo inſcriptorum compoſita ſint, tota recta linea per extremam & mediam rationem ſecta eſt, eiuſque ſegmẽtum maius, eſt hexagωni latus.

Theorema 10. Propoſitio 10.

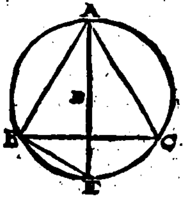


Si circulo pẽtagωnum æquilaterũ inſcriptum ſit, pẽtagωni lat⁹ poteſt & latus hexagωni & latus decagωni eidem circulo inſcriptorum.

Theorema 11. Propoſitio 11.

Si in circulo ῥητὴν habente diametrum, inſcriptũ ſit pentagωnum æquilaterũ, pentagωni latus irrationalis eſt linea, quæ vocatur Minor.

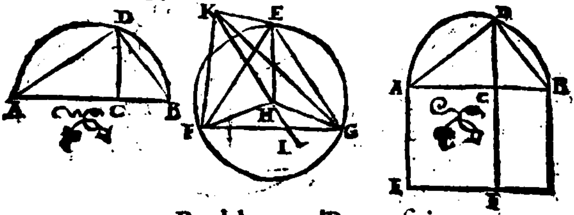
Theorema 12. Propoſitio 12.



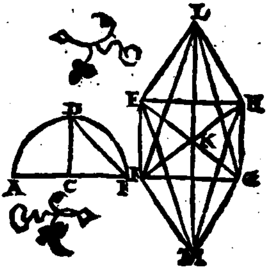
Si in circulo inſcriptum ſit triangulum æquilaterum, huius trianguli latus potentia triplum eſt eius lineæ, quæ ex circuli centro ducitur.

Problema 1. Propoſitio 13.

Pyramidem conſtituere, & data ſphærę cõplecti, atque docere illius ſphæræ diametrum potentia ſeſquialteram eſſe lateris ipſius pyramidis.



Problema 2. Propoſitio 14.

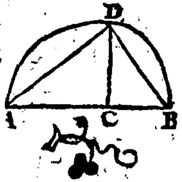


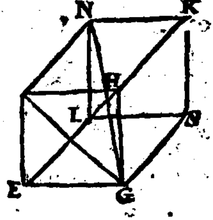
Octaedrum conſtituere, eaq; ſphęra qua pyramidẽ complecti, atque ꝓbare illius ſphæræ diametrũ potentia duplã eſſe lateris ipſi' octaedri.

Pro-

Problema 3. Propositio 15.

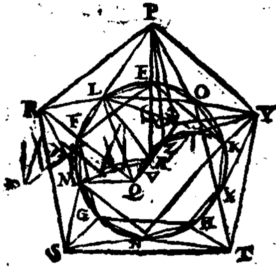
Cubum constituere, eaque sphæra qua & superiores figuras complecti, atque docere illius sphæræ diametrum potẽtia triplam esse lateris ipsius cubi.



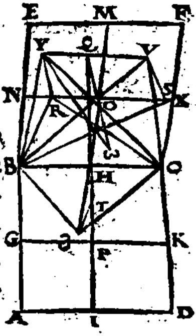


Problema 4. Propositio 16.

Icosaedrum constituere, eademque sphera qua & antedictas figuras complecti, atque probare, Icosaedri latus irrationalem esse lineam, quæ vocatur Minor.

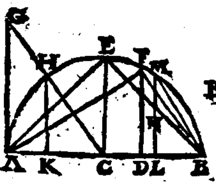


Problema 5. Propositio 17.



Dodecaedrũ cõstituere, eademq́ue sphæra qua & antedictas figuras cõplecti, atque probare dodecaedri latus irrationalem esse lineam, quæ vocatur Residuum.

Theorema 6. Propositio 18.



quinq; figurũ latera proponere, & inter se comparare.

SCHOLIVM.

Aio verò, præter dictas quinque figuras non posse aliam constitui figuram solidam, quæ planis & æquilateris & æquiangulis contineatur, inter se æqualibus. Non enim ex duobus triãgulis, sed neq; ex alijs duabus figuris solidus cõstituetur angulus Sed

Sed ex tribus triangulis, constat Pyramidis angulus.

Ex quatuor autem, Octaëdri.

Ex quinque verò, Icosaëdri.

Nam ex triangulis sex & æquilateris & æquiangulis ad idem punctum coeuntibus, non fiet angulus solidus. Cùm enim trianguli æquilateri angulus, recti vnius bessem contineat, erunt eiusmodi sex anguli rectis quatuor æquales Quod fieri non potest. Nam solidus omnis angulus minoribus quã rectis quatuor angulis continetur, per 21.11.

Ob easdem sanè causas, neque ex pluribus quã planis sex eiusmodi angulis solidus constat.

Sed ex tribus quadratis, Cubi angulus continetur.

Ex quinque, nullus potest. Rursus enim recti quatuor erunt.

Ex tribus autem pentagonis æquilateris & æquiangulis, Dodecaëdri angulus continetur.

Sed ex quatuor, nullus potest. Cùm enim pentagoni æquilateri angulus rectus sit, et quinta recti pars erunt quatuor anguli rectis quatuor maiores. Quod fieri nequit. Nec sanè ex alijs polygonis figuris solidus angulus continebitur, quòd hinc quoque absurdum sequatur. Quamobrem perspicuum est, præter dictas quinque figuras aliam figuram solidam non posse constitui, quæ ex planis æquilateris & æquiangulis contineatur.

ELEMENTI XIII. FINIS.

EVCLIDIS ELEMENTVM DECIMVMQVARTVM, VT quidam arbitrantur, vt alij verò, Hypsiclis Alexandrini, de quinque corporibus.

LIBER PRIMVS.

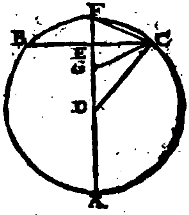
Basilides Tyrius, Protarche, Alexandriam profectus, patrique nostro ob disciplinæ societatem commendatus, longissimo peregrinationis tempore cum eo versatus est. Cúmque differerent aliquando de scripta ab Apollonio cōparatione Dodecaëdri & Icosaëdri eidem sphæra inscriptorum, quam hæc inter se habeant rationem, censuerunt ea non rectè tradidisse Apollonium: quæ à se emendata, vt de patre audire erat, literis prodiderunt. Ego autem postea incidi in alterum librũ ab Apollonio editum, qui demonstrationem accuratè complecteretur de re proposita, ex eiusq; problematis indagatione magnam equidem cepi voluptatem. Illud certè ab omnibus perspici potest, quod scripsit Apollonius, cùm sit iu omnium manibus. Quod autē diligenti, quantum conijcere licet, studio nos postea

scrip-

scripsisse videmur, id monimentis consignatum tibi nuncupandum duximus, vt qui feliciter cùm in omnibus disciplinis tum vel maximè in Geometria versatus, scitè ac prudẽter iudices ea quæ dicturi sumus ob eam verò, quæ tibi cum patre fuit, vitæ consuetudinem, quaq; nos complecteris, beneuolentiã, tractationem ipsam libenter audias. Sed iam tempus est, vt proœmio modum facientes, hanc syntaxim aggrediamur.

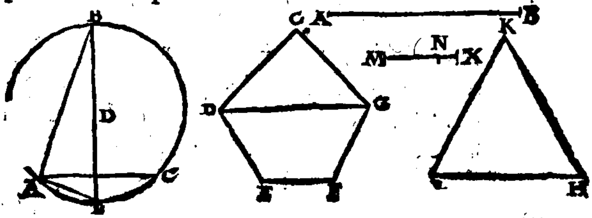
Theorema 1. Propositio 1.

Perpendicularis linea, quæ ex circuli cuiuspiam centro in latus pẽtagωni ipsi circulo inscripti ducitur, dimidia est vtriusq; simul lineæ, & eius quæ ex centro, & lateris decagωni in eodẽ circulo inscripti.



Theorema 2. Propositio 2.

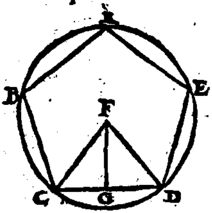
Idem circulus cõprehendit & dωdecaedri pentagωnum & icosaedri triangulũ, eidem sphæræ inscriptorum.

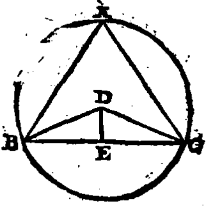


Theo

Theorema 3. Propositio 3.

Si pentagωno & æquilatero & æquiangulo circunſcriptus ſit circulus, ex cuius centro in vnum pentagωni latus ducta ſit perpendicularis: quod vno laterum & perpẽdiculari trigeſies continetur, illud



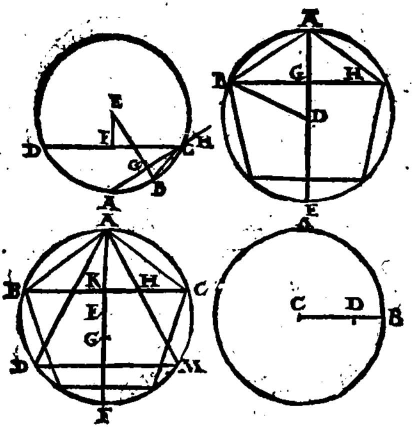


æquale eſt dωdecaedri ſuperficiei.

Theorema 4. Propoſitio 4.

Hoc perſpicuum cùm ſit, probandum eſt; quemadmodum ſe habet dωdecaedri ſuperficies ad icoſaedri ſuperficiem, ita ſe habere cubi latus ad icoſaedri latus.

Cubi



Cubi latus.

E————————

Dodecaedri.

F————————

Icoſaedri.

G————————

SCHOLIVM.

Nunc autem probandum eſt, quemadmodum ſe habet cubi latus ad icoſaëdri latus, ita ſe habere ſolidum dodecaëdri ad icoſaëdri ſolidum. Cùm enim æquales circuli comprehendant & dodecaëdri pẽtagonum & Icoſaëdri triangulum, eidem ſphæra *ſcriptorum: in ſphæris autem æquales circuli æquali interuallo diſtent à centro (ſiquidem perpendiculares à ſphæræ centro ad circulorum plana ductæ & æquales ſunt, & ad circulorum centra cadunt) idcirco lineæ, hoc eſt perpendiculares, quæ à ſphæra cẽtro ducuntur ad centrum circuli comprehendẽtis & triangulum Icoſaëdri, & pentagonũm dodecaëdri, ſunt æquales. Sũt igitur æqualis altitudinis Pyramides, quæ baſes habent ipſa dodecaëdri pentagona, et quæ Icoſaëdri triangula. At æqualis altitudinis pyramides rationem inter ſe habent eam quam baſes, ex 5. & 6. 11. Quemadmodum igitur pentagonũ ad triangulum, ita pyramis, cuius baſis quidem eſt dodecaëdri pentagonum, vertex autem ſphæræ centrum ad pyramida, cuius baſis quidem eſt Icoſaëdri triangulum, vertex autem, ſphæræ centrum. Quamobrem vt ſe habent duodecim pentagona ad viginti triangula, ita duodecim pyramides, quorum pẽtagona ſint baſes, ad viginti pyramidas, quæ trigonas habeant baſes. At pentagona duodecim ſunt dodecaëdri ſuperficies, viginti autem triangula, Icoſaëdri. Eſt igitur vt dodecaëdri ſuperficies ad Icoſaëdri ſuperficiem, ita duodecim pyramides, quæ pentagonas habeant baſes, ad viginti pyramidas, quarum trigonæ ſunt baſes. Sunt autem duodecim quidem pyramides, quæ pentagonas habeant baſes, ſolidũ dodecaëdri: viginti autem pyramides, quæ trigonas habeant baſes, Icoſaëdri ſolidum. Quare ex 11. 5. vt dodecaëdri ſuperficies ad Icoſaëdri ſuperficiem, ita ſolidum dodecaëdri ad Icoſaëdri ſolidum. Vt autem*

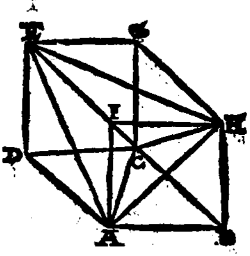
tem dodecaëdri superficies ad Icosaëdri superficiem, ita probatum est cubi latus ad Icosaëdri latus. Quemadmodum igitur cubi latus ad Icosaedri latus, ita se habet solidum dodecaëdri ad Icosaëdri solidum.

EVCLIDIS ELEMENTVM DECIMVMQVINTVM, ET Solidorum quintum, vt nonnulli putant, vt autem alij, Hypsiclis Alexandrini, de quinque corporibus,

LIBER II.

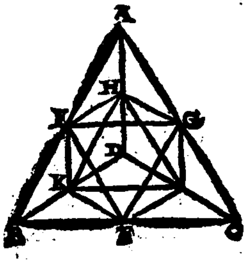
Problema 1. Propositio 1.

In dato cubo pyramida inscribere.



Problema 2. Propositio 2.

In data pyramide octaedrum inscribere.



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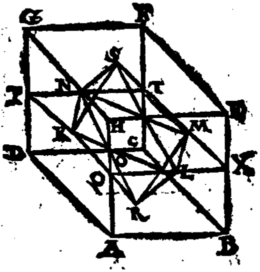
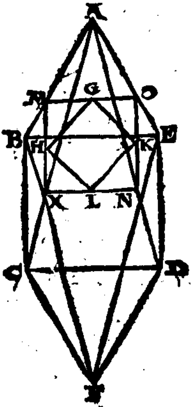
Problema 3. Propositio 3.

In dato cubo octaedrum inscribere.



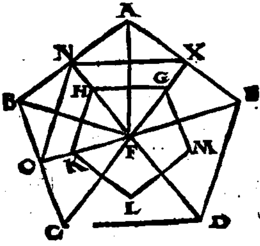
Problema 4. Propositio 4.

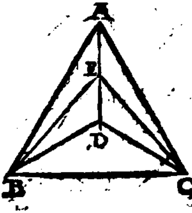
In dato octaedro cubũ inscribere.

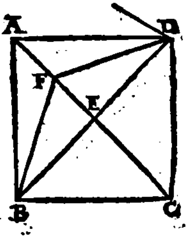


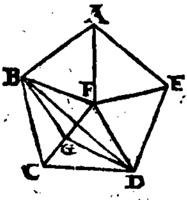
Problema 5. Propositio 5.

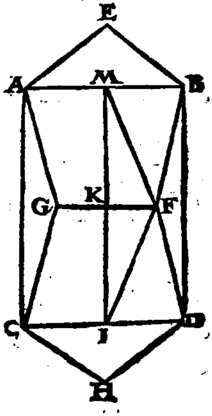
In dato Icosaedro dodecaedrum inscribere.

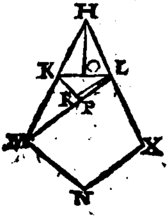












SCHO-

SCHOLIVM.

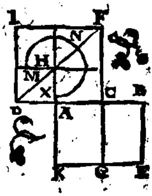
Meminisse decet, si quis nos roget, quot Icosaëdrū habeat latera, ita respondendum esse: Patet Icosaëdrum viginti contineri triangulis, quodlibet verò triangulum rectis tribus constare lineis. Quare multiplicanda sunt nobis viginti triangula in trianguli vnius latera, fiuntq; sexaginta, quorum dimidum est triginta. Ad eundem modum & in dodecaëdro. Cū enim rursus duodecim pentagona dodecaëdrum cōprehendant, itemq; pentagonum quoduis rectis quinque constet lineis, quinq; duodecies multiplicamus, fiunt sexaginta, quorum rursus dimidium est triginta. Sed cur dimidiū capimus? Quoniam vnumquodq; latus siue sit trianguli, siue pentagoni, siue quadrati, vt in cubo, iteratò sumitur. Similiter autem eadem via & in cubo & in pyramide & in octaëdro latera inuenies. Quòd si item velis singularum quoq; figurarum angelos reperire, facta eadem multiplicatione numerum procreatum partire in numerum planorū, quæ vnum solidum angulum includunt: vt quoniā triangula quinq; vnum Icosaëdri angulum continēt, partire 60. in quinque, nascūtur duodecim anguli Icosaëdri. In dodecaëdro autem tria pentagona angulum comprehendunt. partire ergo 60. in tria, & habebis dodecaëdri angulos viginti. Atq; simili ratione in reliquis figuris angulos reperies.

Finis Elementorum Euclidis.

DECIMVMTERTIVM, ET SOLIDORVM TERTIVM.

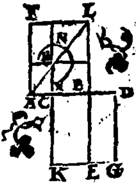
Theorema 1. Propositio 1.

Si recta linea per extremã & mediã rationẽ secta sit, maius segmentum quod totius lineæ dimidium assumpserit, quintuplum potest eius quadrati, quod à totius dimidia describitur.



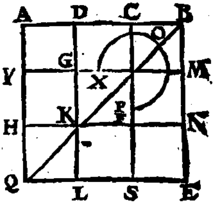
Theorema 2. Propositio 2.

Si recta linea sui ipsius segmenti quintuplũ possit, & dupla segmenti huius linea per extremã & mediam rationẽ secetur maius segmentũ reliqua pars est lineæ primum positæ.



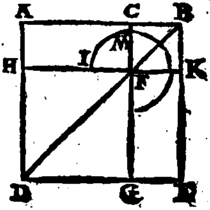
Theo-

Theorema 3. Propoſitio 3.



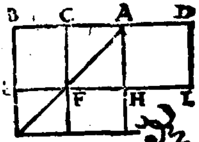
Si recta linea per extremã & mediam rationẽ ſecta ſit, minus ſegmẽtum quod maioris ſegmenti dimidium aſſumpſerit, quintuplum poteſt eius, quòd à maioris ſegmẽti dimidio deſcribitur, quadrati.

Theorema 4. Propoſitio 4.



Si recta linea per extremam & mediam rationẽ ſecta ſit, quod à tota, quodque à minore ſegmẽto ſimul vtraq; quadrata, tripla ſunt eius, quod à maiore ſegmẽto deſcribitur, quadrati.

Theorema 5. Propoſitio 5.



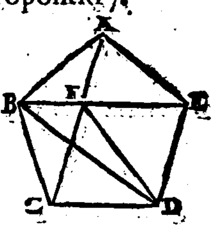
Si ad rectam lineam, quæ per extremam & mediam rationem ſecetur, adiuncta ſit altera ſegmento maiori ęqualis, tota hæc linea recta per extremam & mediam rationẽ ſecta est, estque maius segmentum linea primùm posita.

Theorema 6. Propositio 6.

Si recta linea ῥητὴ siue rationalis, per extremam & mediam rationem secta sit, vtrunque segmentorum ἄλογος siue irrationalis est linea, quæ dicitur Residuum.

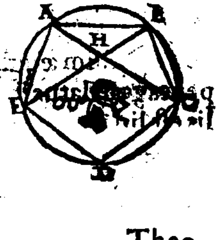
Theorema 7. Propositio 7.

Si pentagωni æquilateri tres sint æquales anguli, siue q. deinceps, siue qui non deinceps sequuntur, illud pétagωnum erit æquiangulum.



Theorema 8. Propositio 8.

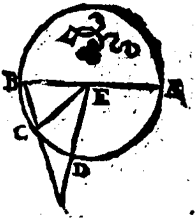
Si pétagωni æquilateri & æquianguli duos qui deinceps sequuntur angulos rectę subtendãt lineæ, illæ per extremã & mediam rationem se mutuò secant, earumq; maiora segmenta, ipsius pentagωni lateri sunt æqualia.



Theo-

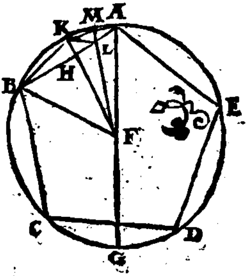
Theorema 9. Propositio 9.

Si latus hexagωni, & latus decagωni eidem circulo inſcriptorum compoſita ſint, tota recta linea per extremam & mediam rationem ſecta eſt, eiuſque ſegmẽtum maius, eſt hexagωni latus.



Theorema 10. Propoſitio 10.

Si circulo pẽtagωnum æquilaterũ inſcriptum ſit, pẽtagωni lat⁹ poteſt & latus hexagωni & latus decagωni eidem circulo inſcriptorum.

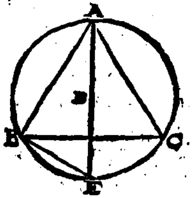


Theorema 11. Propoſitio 11.

Si in circulo ῥητὴν habente diametrum, inſcriptũ ſit pentagωnum æquilaterũ, pentagωni latus irrationalis eſt linea, quæ vocatur Minor.

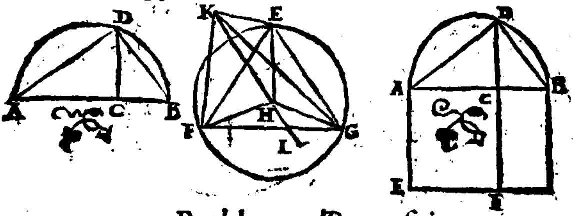
Theorema 12. Propoſitio 12.

Si in circulo inſcriptum ſit triangulum æquilaterum, huius trianguli latus potentia triplum eſt eius lineæ, quæ ex circuli centro ducitur.



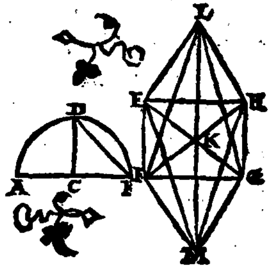
Problema 1. Propoſitio 13.

Pyramidem conſtituere, & data ſphærę cõplecti, atque docere illius ſphæræ diametrum potentia ſeſquialteram eſſe lateris ipſius pyramidis.



Problema 2. Propoſitio 14.

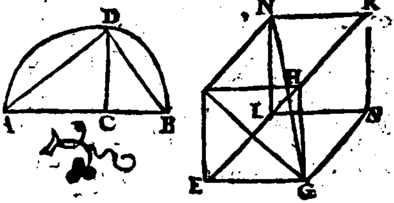
Octaedrum conſtituere, eáq; ſphęra qua pyramidẽ complecti, atque ꝓbare illius ſphæræ diametrũ potentia duplã eſſe lateris ipſiꝰ octaedri.



Pro-

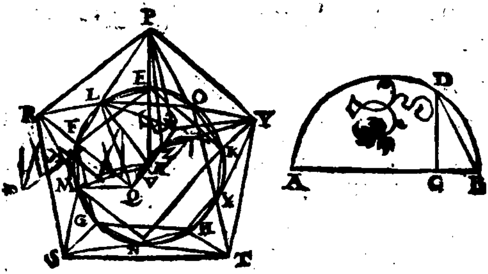
Problema 3. Propositio 15.

Cubum constituere, eaque sphæra qua & superiores figuras complecti, atque docere illius sphæræ diametrum potētia triplam esse lateris ipsius cubi.



Problema 4. Propositio 16.

Icosaedrum constituere, eademque sphęra qua & antedictas figuras complecti, atque probare, Icosaedri latus irrationalem esse lineam, quæ vocatur Minor.



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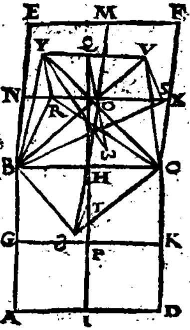
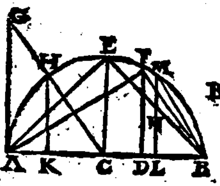


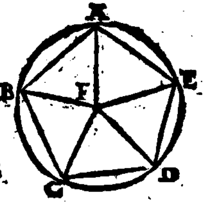
Problema 5. Propositio 17.

Dodecaedrũ cõstituere, eademque sphæra quá & antedictas figuras cõplecti, atque probare dodecaedri latus irrationalem esse lineam, quæ vocatur Residuum.

Theorema 6. Propositio 18.

quinq; figurarũ latera proponere, & inter se comparare.





SCHOLIVM.

Aio verò, præter dictas quinque figuras non posse aliam constitui figuram solidam, quæ planis & æquilateris & æquiangulis contineatur, inter se æqualibus. Non enim ex duobus triãgulis, sed neq; ex alijs duabus figuris solidus cõstituetur angulus

Sed

Sed ex tribus triangulis, constat Pyramidis angulus.

Ex quatuor autem, Octaëdri.

Ex quinque verò, Icosaëdri.

Nam ex triangulis sex & æquilateris & æquiangulis ad idem punctum coeuntibus, non fiet angulus solidus. Cùm enim trianguli æquilateri angulus, recti vnius bessem contineat, erunt eiusmodi sex anguli rectis quatuor æquales Quod fieri non potest. Nam solidus omnis angulus minoribus quã rectis quatuor angulis continetur, per 21.11.

Ob easdem sanè causas, neque ex pluribus quã planis sex eiusmodi angulis solidus constat.

Sed ex tribus quadratis, Cubi angulus continetur.

Ex quinque, nullus potest. Rursus enim recti quatuor erunt.

Ex tribus autem pentagonis æquilateris & æquiangulis, Dodecaëdri angulus continetur.

Sed ex quatuor, nullus potest. Cùm enim pentagoni æquilateri angulus rectus sit, et quinta recti pars erunt quatuor anguli rectis quatuor maiores. Quod fieri nequit. Nec sanè ex alijs polygonis figuris solidus angulus continebitur, quòd hinc quoque absurdum sequatur. Quamobrem perspicuum est, præter dictas quinque figuras aliam figuram solidam non posse constitui, quæ ex planis æquilateris & æquiangulis contineatur.

ELEMENTI XIII. FINIS.

EVCLIDIS ELEMENTVM DECIMVMQVARTVM, VT quidam arbitrantur, vt alij verò, Hypsiclis Alexandrini, de quinque corporibus.

LIBER PRIMVS.

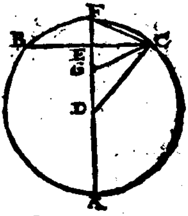
BAsilides Tyrius, Protarche, Alexandriam profectus, patrique nostro ob disciplinæ societatem commendatus, longißimo peregrinationis tempore cum eo versatus est. Cúmque differerent aliquando de scripta ab Apollonio cõparatione Dodecaëdri & Icosaëdri eidem sphæra inscriptorum, quam hæc inter se habeant rationem, censuerunt ea non rectè tradidisse Apollonium: quæ à se emendata, vt de patre audire erat, literis prodiderunt. Ego autem postea incidi in alterum librũ ab Apollonio editum, qui demonstrationem accuratè complecteretur de re proposita, ex eiusq; problematis indagatione magnam equidem cepi voluptatem. Illud certè ab omnibus perspici potest, quod scripsit Apollonius, cùm sit in omnium manibus. Quod autẽ diligenti, quantum conijcere licet, studio nos postea

scrip-

ſcripſiſſe videmur, id monimentis conſignatum tibi nuncupandum duximus, vt qui feliciter cùm in omnibus diſciplinis tum vel maximè in Geometria verſatus, ſcitè ac prudẽter iudices ea quæ dicturi ſumus ob eam verò, quæ tibi cum patre fuit, vitæ conſuetudinem, quaq; nos complecteris, beneuolentiã, tractationem ipſam libenter audias. Sed iam tempus eſt, vt præmio modum facientes, hanc ſyntaxim aggrediamur.

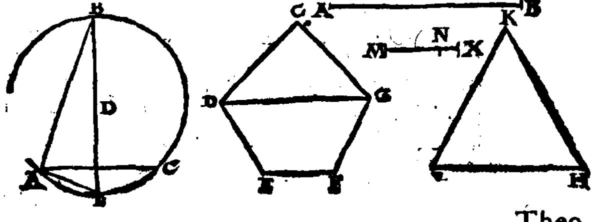
Theorema 1. Propoſitio 1.

Perpendicularis linea, quæ ex circuli cuiuſpiam centro in latus pẽtagωni ipſi circulo inſcripti ducitur, dimidia eſt vtriuſq; ſimul lineæ, & eius quæ ex centro, & lateris decagωni in eodẽ circulo inſcripti.



Theorema 2. Propoſitio 2.

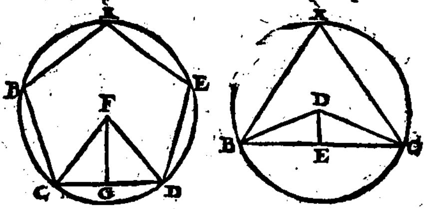
Idem circulus cõprehendit & dωdecaedri pentagωnum & icoſaedri triangulũ, eidem ſphæræ inſcriptorum.



Theo

Theorema 3. Propoſitio 3.

Si pentagωno & æquilatero & æquiangulo circunſcriptus ſit circulus, ex cuius centro in vnum pentagωni latus ducta ſit perpendicularis: quod vno laterum & perpẽdicu lari trigeſies continetur, illud

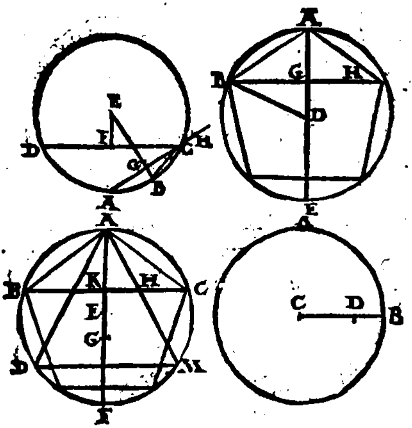


æquale eſt dωdecaedri ſuperficiei.

Theorema 4. Propoſitio 4.

Hoc perſpicuum cùm ſit, probandum eſt, quemadmodum ſe habet dωdecaedri ſuperficies ad icoſaedri ſuperficiem, ita ſe habere cubi latus ad icoſaedri latus.

Cubi



Cubi latus.

E————————

Dodecaedri.

F————————

Icoſaedri.

G————————

SCHOLIVM.

Nunc autem probandum eſt, quemadmodum ſe habet cubi latus ad icoſaëdri latus, ita ſe habere ſolidum dodecaëdri ad icoſaëdri ſolidum. Cùm enim æquales circuli comprehendant & dodecaëdri pẽtagonum & Icoſaëdri triangulum, eidem ſphæra in-

scriptorum: in sphæris autem æquales circuli æquali interuallo distent à centro (siquidem perpendiculares à sphæræ centro ad circulorum plana ductæ & æquales sunt, & ad circulorum centra cadunt) idcirco lineæ, hoc est perpendiculares, quæ à sphæræ cẽtro ducuntur ad centrum circuli comprehendẽtis & triangulum Icosaëdri, & pentagonum dodecaëdri, sunt æquales. Sũt igitur æqualis altitudinis Pyramides, quæ bases habent ipsa dodecaëdri pentagona, et quæ Icosaëdri triangula. At æqualis altitudinis pyramides rationem inter se habent eam quam bases, ex 5. & 6. 11. Quemadmodum igitur pentagonũ ad triangulum, ita pyramis, cuius basis quidem est dodecaëdri pentagonum, vertex autem sphæræ centrum ad pyramida, cuius basis quidem est Icosaëdri triangulum, vertex autem, sphæræ centrum. Quamobrem vt se habent duodecim pentagona ad viginti triangula, ita duodecim pyramides, quorum pẽtagona sint bases, ad viginti pyramidas, quæ trigonas habeant bases. At pentagona duodecim sunt dodecaëdri superficies, viginti autem triangula, Icosaëdri. Est igitur vt dodecaëdri superficies ad Icosaëdri superficiem, ita duodecim pyramides, quæ pentagonas habeant bases, ad viginti pyramidas, quarum trigonæ sunt bases. Sunt autem duodecim quidem pyramides, quæ pentagonas habeant bases, solidũ dodecaëdri: viginti autem pyramides, quæ trigonas habeant bases, Icosaëdri solidum. Quare ex 11. 5. vt dodecaëdri superficies ad Icosaëdri superficiem, ita solidum dodecaëdri ad Icosaëdri solidum. Vt autem

tem

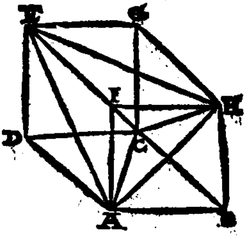
tem dodecaëdri superficies ad Icosaëdri superficiem, ita probatum est cubi latus ad Icosaëdri latus. Quemadmodum igitur cubi latus ad Icosaedri latus, ita se habet solidum dodecaëdri ad Icosaëdri solidum.

EVCLIDIS ELEMENTVM DECIMVMQVINTVM, ET Solidorum quintum, vt nonnulli putant, vt autem alij, Hypsiclis Alexandrini, de quinque corporibus,

LIBER II.

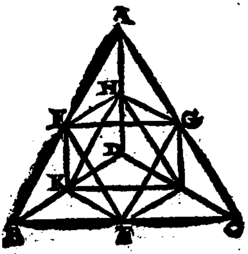
Problema 1. Propositio 1.

In dato cubo pyramida inscribere.



Problema 2. Propositio 2.

In data pyramide octaedrum inscribere.



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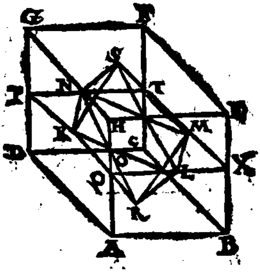
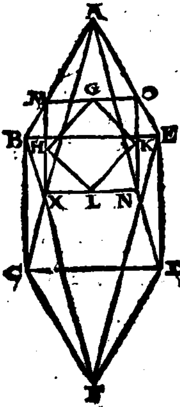
Problema 3. Propositio 3.

In dato cubo octaedrum inscribere.



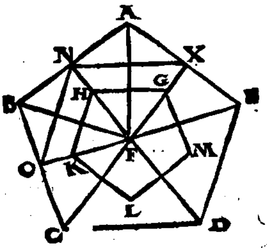
Problema 4. Propositio 4.

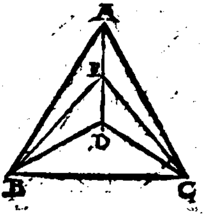
In dato octaedro cubũ inscribere.

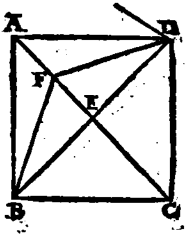


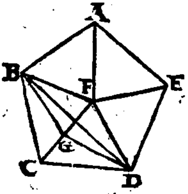
Problema 5. Propositio 5.

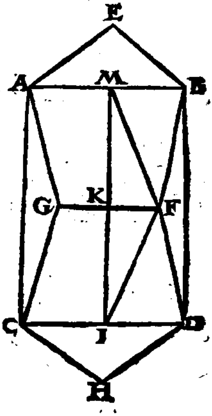
In dato Icosaedro dodecaedrum inscribere.

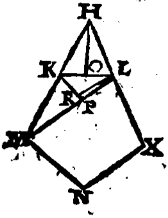












SCHO-

SCHOLIVM.

Meminisse decet, si quis nos roget, quot Icosaëdrũ habeat latera, ita respondendum esse: Patet Icosaëdrum viginti contineri triangulis, quodlibet verò triangulum rectis tribus constare lineis. Quare multiplicanda sunt nobis viginti triangula in trianguli vnius latera, fiuntq; sextaginta, quorum dimidum est triginta. Ad eundem modum & in dodecaëdro. Cũ enim rursus duodecim pentagona dodecaëdrum cõprehendant, itemq; pentagonum quoduis rectis quinque constet lineis, quinq; duodecies multiplicamus, fiunt sexaginta, quorum rursus dimidium est triginta. Sed cur dimidiũ capimus? Quoniam vnumquodq; latus siue sit trianguli, siue pentagoni, siue quadrati, vt in cubo, iteratò sumitur. Similiter autem eadem via & in cubo & in pyramide & in octaëdro latera inuenies. Quòd si item velis singularum quoq; figurarum angelos reperire, facta eadem multiplicatione numerum procreatum partire in numerum planorũ, quæ vnum solidum angulum includunt: vt quoniã triangula quinq; vnum Icosaëdri angulum continẽt, partire 60. in quinque, nascũtur duodecim anguli Icosaëdri. In dodecaëdro autem tria pentagona angulum comprehendunt. partire ergo 60. in tria, & habebis dodecaëdri angulos viginti. Atq; simimili ratione in reliquis figuris angulos reperies.

Finis Elementorum Euclidis.