EVCLIDIS ELEMENTORVM LIBRI XV.

Quibus, cùm ad omnem Mathematicæ ſcientiæ partem, tùm ad quamlibet Geometriæ tractationem, facilis comparatur aditus.

COLONIAE,
Apud Maternum Cholinum.
M. D. LXXXVII.
Cum Gratiâ & Priuilegio Cæſ. Maieſt.

AD CANDIDVM LECTOREM ST. GRACILIS præfatio.

PERMAGNI referre sẽper existimaui, lector beneuole, quantum quisq; studij & diligentiæ ad percipienda scientiarum elementa adhibeat, quibus nõ satis cognitis, aut perperã intellectis si vel digitum progredi tentes, erroris caliginem animis offundas, non veritatis lucem rebus obscuris adferas. Sed principiorum quanta sint in disciplinis momenta, haud facilè credat, qui rerum naturam ipsa specie, non viribus metiatur. Vt enim corporum quæ oriuntur & intereũt, vilissima tenuissimaq; videntur initia: ita rerum æternarum & admirabilium, quibus nobilissimæ artes continentur, elementa ad speciem sunt exilia, ad vires & facultatem quã maxima. Quis nõ videt ex fici tãtulo grano, vt ait Tullius, aut ex acino vinaceo, aut ex cæterarum frugum aut stirpium minutissimis seminibus tantos truncos ramosq; procreari? Nã Mathematicorũ initia illa quidem dictu audituq; perexigua, quantã theorematum syluã nobis pe-

pererunt? Ex quo intelligi potest, vt in ipsis seminibus, sic & in artium principijs inesse vim earũ rerum quæ ex his progignuntur. Præclarè igitur Aristoteles, vt alia permulta, μέγιστον ἴσως ἀρχὴ παντός, κ᾽ ὅσῳ κράτιστον τῇ δυνάμει, τοσούτῳ μικρότατον, οὖσα τῷ μεγέθει χαλεπόν ἐστιν ὀφθῆναι. Quocirca committendum non est, vt non bene prouisa & diligenter explorata scientiarum principia quibus propositarum quarumque rerũ veritas sit demonstranda, vel constituas, vel constituta approbes: Cauendum etiam, vt ne tantulum quidem fallaci & captiosa interpretatione turpiter deceptus, à vera principiorum ratione temerè deflectas. Nam qui initio fortè aberrauerit, is vt tandem in maximis versetur erroribus, necesse est: cum ex vno erroris capite, densiores sensim tenebræ rebus clarissimis obducantur. Quid tam varias veterum physiologorum sententias non modò cum rerum veritate pugnantes, sed vehementer etiam inter se dissentientes nobis inuexit? Equidem haud scio, fueritne vlla potior tanti dissidij causa, quàm quòd ex principijs partim falsis, partim non consentaneis ductas rationes probando adhiberent. Fit enim plerunque, vt qui non rectè de artium rerumq́; elementis sentiunt, ad præfinitas quasdam opiniones suas omnia reuocare studeant. Pythagorei, vt meminit Aristoteles, cum denarij numeri summam perfectionem cœlo tribuerent, nec plures tamen quàm nouem sphæras cernerent, decimam affingere ausi sunt terræ aduersam.

sam quam ἀντίχθονα appellarunt. Illi enim, uniuersitatis rerumq́; singularum naturam ex numeris ceu principijs æstimantes, ea protulerũt quæ φαινομένοις congruere nusquam sunt cognita Nam ridicula Democriti, Anaximenis, Melissi Anaxagoræ, Anaximandri, & reliquorum id genus physiologorum somnia ex falsis illa quidem orta naturæ principijs, sed ad Mathematicum nihil aut parum spectantia sciens prætereo Nonnullos attingam qui repetitis altius vel aliter ac decuit positis rerum initijs, cum physicis multa turbarunt, tum Mathematicos oppugnatione principiorum pessimè mulctarũt. Ex planis figuris corpora cõstituit Timæus: Geometrarum hic quidem principia cuniculis oppugnantur. Nam & superficies seu extremitates crassitudinem habebunt, & lineæ latitudinem: denique puncta non erunt indiuidua sed linearum partes. Prædicant Democritus atque Leucippus illas atomos suas, & indiuidua corpuscula. Concedit Xenocrates impartibiles quasdã magnitudines. Hic verò Geometriæ fundamẽta apertè petuntur, & funditus euertuntur: quibus dirutis nihil equidẽ aliud video restare, quàm vt amplissima Mathematicorum theatra repentè concidant. Iacebunt ergo, si dijs placet, tot præclara Geometrarum de asymmetris & alogis magnitudinibus theoremata. Quid enim causæ dicas cur indiuidua linea hanc quidem metiatur, illam verò metiri non queat? Siguidem quod minimum in vnoquaque *genere reperitur, id communis omnium mensura esse solet. Innumerabilia profectò sunt illa, quæ ex falsis eiusmodi decretis absurda consequuntur: & horum permulta quidem Mathematicus, sed longè plura colligit Physicus. Quid varia* ψευδογραφημάτων *genera commemorē, quæ ex hoc uno fonte tam longè latéque diffusa fluxisse videntur? Notissimus est Antiphontis tetragonismus, qui Geometrarum & ipse principia non parum labefecit, cùm rectæ lineæ curuam posuit æqualem. Longum esset mihi singula percensere, præsertim ad alia properanti: Hoc ergo certum fixum, & in perpetuum ratum esse oportet, quod sapiēter monet Aristoteles.* σπουδαστέον ὅπως ὁρισθῶσι καλῶς αἱ ἀρχαὶ· μεγάλην γὰρ ἔχουσι ῥοπὴν πρὸς ἑπόμενα. *Nam principijs illa congruere debent, quæ sequuntur. Quòd si tantum perspicitur in istis exilioribus Geometriæ initijs, quæ puncto, linea, superficie definiuntur, momentum, vt ne hæc quidem sine summo impendentis ruinæ periculo conuelli aut oppugnari possint, quanta quæso vis putanda est huius* στοιχειώσεως, *quam collatis tot præstātissimorum artificum inuentis, mira quadam ordinis solertia contexuit Euclides, vniuersæ Matheseos elementa complexu suo coercentem? Vt igitur omnibus rebus instructior & paratior quisque ad hoc studium libentius accedat, & singula vel minutissima exactius secum reputet atque perdiscat, operæprecium censui, in primo institutionis aditu vestibuloq́ue, præcipua quædam capita,*

quibus

quibus tota ferè Mathematicæ scientiæ ratio intelligatur, breuiter explicare: tum ea quæ sunt Geometriæ propria, diligẽter persequi: Euclidis deniq; in extruenda hac στοιχειώσει consilium sedulò ac fideliter exponere. Quæ ferè omnia ex Aristotelis potissimum ducta fontibus, nemini inuisa fore confido, qui modò ingenuum animi candorem ad legendum attulerit. Ac de Mathematicæ diuisione primùm dicamus.

Mathematicæ in primis scientiæ studiosos fuisse Pythagoreos, non modò historicorum, sed etiam philosophorum libri declarant. His ergo placuit, vt in partes quatuor vniuersum distribuatur Mathematicæ scientiæ genus, quarum duas περὶ τὸ ποσὸν, reliquas περὶ τὸ πηλίκον versari statuerunt. Nam & τὸ ποσὸν vel sine vlla cõparatione ipsum per se cognosci, vel certa quadam ratione comparatũ spectari: in illo Arithmeticam, in hoc versari Musicam: & τὸ πηλίκον partim quiescere, partim moueri quidem: illud Geometriæ propositum esse: quod verò sua sponte motu cietur, Astronomiæ. Sed ne quis falso putet, Mathematicam scientiam, quòd in vtroque quanti genere cernitur, idcirco inanem videri (si quidem non solum magnitudinis diuisio, sed etiam multitudinis accretio infinitè progredi potest) meminisse decet, τὸ πηλίκον καὶ τὸ ποσὸν, quæ subiecto Mathematicæ generi imposita sunt à Pythagoreis nomina, non cuiuscunque modi quantitatem significare, sed eam demum, quæ tum multitudine tum magnitudine

tudine sit definita, & suis circumscripta terminis. Quis enim ullam infiniti scientiam defendat? Hoc scitum est, quod non semel docet Aristoteles, infinitum ne cogitatione quidem complecti quenquam posse. Itaque ex infinita multitudinis & magnitudinis δυνάμει *finitam hæc scientia decerpit & amplectitur naturam quam tractet, & in qua versetur. Nam de vulgari Geometrarum cõsuetudine quid sentiendum sit, cum data interdum magnitudine infinita aut fabricantur aliquid, aut proprias generis subiecti affectiones exquirunt, disertè monet Aristoteles,* οὐδὲ νῦν *(de Mathematicis loquens)* δέονται τοῦ ἀπείρου, οὐδὲ χρῶνται, ἀλλὰ μόνον εἶναι ὅσην ἂν βούλωνται, πεπερασμένην. *Quamobrem disputatio ea qua infinitum refellitur, Mathematicorum decretis rationibusq́; non aduersatur, nec eorum apodixes labefacit: Etenim tali infinito opus illis nequaquam est, quod exitu nullo peragrari possit, nec talem ponunt infinitam magnitudinẽ: sed quantãcunq́; velit aliquis effingere, ea ut suppetat, infinitam præcipiunt. Quinetiã non modò immensa magnitudine opus non habent Mathematici, sed ne maxima quidem: cum instar maximæ minima quæque in partes totidem pari ratione diuidi queat. Alteram Mathematicæ diuisionẽ attulit Geminus, vir (quantum ex Proclo cõijcere licet)* μαθημάτων *laude clarissimus. Eam, quæ superiore plenior & accuratior fortè visa est, cũ doctissimè pertractarit sua in decimum Euclidis*

præfa-

præfatione P. Montaureus vir senatorius, & regiæ bibliothecæ præfectus, leuiter attingam. Nam ex duobus rerum velut summis generibus τῶν νοητῶν καὶ τῶν αἰσθητῶν. quæ res sub intelligentiam cadunt, Arithmetica & Geometria attribuit Geminus: quæ verò in sensus incurrūt, Astrologiæ, Musicæ, Supputratrici, Opticæ, Geodesiæ & Mechanicæ adiudicauit. Ad hāc certè diuisionem spectasse videtur Aristoteles, cùm Astrologiam Opticam, harmonicam φυσικωτέρας τῶν μαθημάτων nominat, vt quæ naturalibus & Mathematicis interiecta sint, ac velut ex vtrisque mixtæ disciplinæ: Siquidem genera subiecta à Phisicis mutuantur, causas verò in demonstrationibus ex superiore aliqua scientia repetunt. Id quod Aristoteles ipse apertissimè testatur, ἐνταῦθα γὰρ, φησὶ, τὸ μὲν ὅτι, τῶν αἰσθητικῶν εἰδέναι τὸ δὲ διότι, τῶν μαθηματικῶν. Sequitur, vt quid Mathematica conueniat cum Phisica & prima Philosophia: quid ipsa ab vtraque differat, paucis ostendamus. Illud quidem omnium commune est, quòd in veri contemplatione sunt positæ, ob idq; θεωρητικαὶ à Græcis dicuntur. Nam cùm διάνοια siue ratio & mens omnis sit vel πρακτικὴ, vel θεωρητικὴ, totidem scientiarum sint genera necesse est. Quòd si Physica, Mathematica, & prima Philosophia, nec in agendo, nec in efficiendo sunt occupatæ, hoc certè perspicuum est, eas omnes in cognitione contemplationéque necessariò versari. Cùm enim rerum non modò agendarum, sed etiam efficienda-

ciendarum principia in agente vel efficiente cõsistant; illarum quidem προαίρεσις. harum autem vel mens, vel ars, vel vis quedam & facultas: rerum profectò naturalium, Mathematicarum, atque diuinarum principia in rebus ipsis, non in philosophis inclusa latent. Atque hæc vna in omnes valet ratio, quæ θεωρητικὰς esse colligat. Iam verò Mathematica separatim cũ Physica congruit, quòd vtraque versatur in cognitione formarum corpori naturali inhærentiũ. Nam Mathematicus plana solida, longitudines & puncta contemplatur, quæ omnia in corpore naturali à naturali quoque philosopho tractantur. Mathematica item & prima philosophia hoc inter se propriè conueniunt, quòd cognitionem vtraque persequitur formarum, quoad immobiles, & à concretione materiæ sunt liberæ. Nam tametsi Mathematicæ formæ re vera per se nõ cohærent, cogitatione tamen à materia & motu separantur, οὐδὲ γίνεται ψεῦδος χωριζόντων, vt ait Aristoteles. De cognatione & societate breuiter diximus, iam quid intersit videamus. Vnaquæq; mathematicarũ certum quoddam rerum genus propositum habet, in quo versetur, vt Geometria quantitatem & continuationem aliorum in vnam partem, aliorũ in duas, quorundam in tres, eorúmque quatenus quanta sunt & continua, affectiones cognoscit. Prima autem philosophia, cùm sit omnium communis, vniuersum Entis genus, quæq; ei accidũt & conueniunt hoc ipso quòd est, consyderat.

Ad

Adhac, Mathematica eam modò naturã amplectitur, quæ quanquam non mouetur, separari tamen seiungiq́ue nisi mente & cogitatione à materia non potest, ob eamq́ue causam, ἐξ ἀφαιρέσεως dici consueuit. Sed Prima philosophia in ijs versatur, quæ & seiuncta, & æterna, & ab omni motu per se soluta sunt ac libera. Cæterùm Physica & Mathematica quanquam subiecto discrepare non videntur, modo tamen rationeq́ue differunt cognitionis & contemplationis, unde dissimilitudo quoque scientiarum sequitur. Etenim Mathematicæ species nihil re vera sunt aliud, quàm corporis naturalis extremitates, quas cogitatione ab omni motu & materia separatas Mathematicus contemplatur: sed easdem consectatur Physicorum ars, quatenus cum materia comprehensæ sunt, & corpora motui obnoxia circunscribunt. Ex quo fit vt quæcunque in Mathematicis incommoditates accidunt, eædem etiam in naturalibus rebus videantur accidere, non autem vicissim. Multa enim in naturalibus sequuntur incommoda, quæ nihil ad Mathematicum attinent, διὰ τὸ, inquit Aristoteles, τὰ μὲν ἐξ ἀφαιρέσεως λέγεται, τὰ μαθηματικὰ, τὰ δὲ φυσικὰ ἐκ προσθέσεως. Siquidem res cũ materia deuinctas contemplatur physicus: Mathematicus verò rẽ cognoscit circũscriptis ijs omnibus quæ sensu percipiuntur, vt grauitate leuitate duritie, mollitie, & præterea calore, frigore, alijsq́ue contrariorum paribus, quæ sub sensum subiecta sunt: tantum autem relinquit

quanti-

quantitatem & continuum. Itaq; Mathematicorum ars in ijs quæ immobilia sunt, cernitur. (τὰ γὰρ μαθηματικὰ τῶν ὄντων ἄνευ κινήσεώς ἐστιν, ἔξω τῶν περὶ τὴν ἀστρολογίαν) quæ verò in naturæ obscuritate posita est, res quidem quæ nec separari nec motu vacare possunt, contemplatur. Idquod in vtroque scientiæ genere perspicuum esse potest siue res subiectas definias, siue proprietates earum demonstres. Etenim numerus, linea figura rectum, inflexum, æquale, rotundum, vniuersa denique Mathematicus quæ tractat & profitetur, absque motu explicari docerique possunt: χωριστὰ γὰρ τῇ νοήσει κινήσεώς ἐστι: Phisicæ autem sine motione species nequaquam possunt intelligi. Quis enim hominis plantæ, ignis, ossiũ carnis naturam & proprietates sine motu, qui materiam sequitur, perspiciat? Siquidem tantisper substantia quæque naturalis constare dici solet, quoad opus & munus suum, agendo patiẽdoque tueri ac sustinere valeat: qua certè amissa δύναμις, ne nomen quidem nisi ὁμωνύμως retinet. Sed Mathematico ad explicandas circuli aut trianguli proprietates, nullum adferre potest vsum materiæ vt auri, ligni, ferri, in qua insunt, consideratio, quin eò verius eiusmodi rerum, quarum species tanquam materia vacantes efformemus animo, naturam complectemur, quòd coniunctione materiæ quasi adulterari deprauarique videntur. Quocirca Mathematicæ species eodem modo quo κοῖλον, siue concauitas, sine motu & subiecto, definitione explicare

cognos-

cognoscíq; possunt: naturales verò cùm eam vim habeant, quam, vt ita dicam, simitas cum materia comprehensæ sunt, nec absque ea separatim possunt intelligi: quibus exemplis quid inter Phisicas & Mathematicas species intersit, haud difficile est animaduertere. Illis certè non semel est vsus Aristoteles. Valeant ergo Protagoræ sophismata, Geometras hoc nomine refellentis, quod circulus normam puncto non attingat. Nã diuina Geometrarum theoremata, qui sensu æstimabit, vix quicquam reperiet quod Geometræ concedendum videatur. Quid enim ex his quæ sensum mouent, ita rectum aut rotundum dici potest, vt à Geometra ponitur? Nec verò absurdum est aut vitiosum, quòd lineas in puluere descriptas pro rectis aut rotundis assumit, quæ nec rectæ sunt nec rotundæ, ac ne latitudinis quidem expertes. Siquidem non ijs vtitur Geometra quasi inde vim habeat conclusio, sed eorum quæ discenti intelligenda relinquuntur, rudem ceu imaginem proponit. Nam qui primum instituuntur, hi ductu quodam & velut χειραγωγία sensuum opus habent, vt ad illa quæ sola intelligentia percipiuntur, aditum sibi comparare queant. Sed tamen existimandum non est rebus Mathematicis omnino negari materiam, ac non eam tantùm quæ sensum afficit. Est enim materia alia quæ sub sensum cadit, alia quæ animo & ratione intelligitur. Illam αἰσθητὴν, hanc νοητὴν vocat Aristoteles. Sensu percipitur, vt æs, vt lignum, omnísque materia quæ moueri potest

test. Animo & ratione cernitur ea quæ in rebus sensilibus inest sed nō quatenus sensu percipiuntur, quales sunt res Mathematicorum. Unde ab Aristotele scriptū legimus ἐπὶ τῶν ἐν ἀφαιρέσει ὄντων rectum se habere ut simum: μετὰ συνεχοῦς γὰρ: quasi velit ipsius recti quod Mathematicorum est, suam esse materiam, non minus quā simi quod ad Physicos pertinet. Nam licet res Mathematicæ sensili vacent materia, nō sunt tamen indiuiduæ, sed propter continuationem partitioni semper obnoxiæ, cuius ratione dici possunt suæ materia non omnino carere: quin aliud videtur τὸ εἶναι γραμμῆ, aliud quoad continuationi adiuncta intelligitur linea. Illud enim ceu forma in materia proprietatum causa est, quas sine materia percipere non licet. Hæc est societatis & dissidij Mathematicæ cum Physica & prima Philosophia ratio. Nunc autem de nominis etymo & notatione pauca quædam afferamus. Nam si quæ iudicio & ratione imposita sunt rebus nomina, ea certè non temere indita fuisse credendum est, quibus scientias appellari placuit. Sed neque otiosa semper haberi debet ista etymologiæ indagatio, cùm ad rei etiam dubiæ fidem sæpe non parum valeat recta nominis interpretatio. Sic enim Aristoteles ducto ex verborum ratione argumento, ἀυτομάτου, μετὰ βουλῆσ, αἰθέρος, aliarumq; rerum naturam ex parte confirmauit. Quoniam igitur Pythagoras Mathematicam scientiam nō modò studiosè coluit, sed etiā repetitis à capite principijs, geometricā

contem-

contemplationem in liberalis disciplinæ formā composuit, & perspectis absq; materia solius intelligentiæ adminiculo theorematibus, tractationem περὶ τῶν ἀλόγων, & κοσμικῶν σχημάτων cōstitutionem excogitauit: credibile est, Pythagoram, aut certè Pythagoreos, qui et ipsi doctoris sui studia libenter amplexi sunt, huic scientiæ id nomen dedisse, quod cum suis placitis atq; decretis congrueret, rerumq; propositarum naturam quoquo modo declararet. Ita cùm existimarent illi, omnem disciplinam, quæ μάθησις dicitur, ἀνάμνησιν esse quandam. i. recordationem et repetitionem eius scientiæ, cuius antè quam in corpus immigraret, compos fuerit anima, quemadmodum Plato quoq; in Menone, Phædone, & alijs aliquot locis videtur astruxisse: animaduerterent autem eiusmodi recordationem, quæ non posset multis ex rebus perspici, ex his potissimum scientijs demonstrari, si quis nimirum, ait Plato, ἐπὶ τὰ διαγράμματα ἄγῃ, probabile est equidem Mathematicas à Pythagoreis artes κατ' ἐξοχὴν fuisse nominatas, vt ex quibus μάθησις, id est æternarum in anima rationum recordatio διαφερόντως et pcipuè intelligi posset. Cuius etiā rei fidē nobis diuinus fecit Plato, q̃ in Menone Socratem induxit hoc argumēti genere persuadere cupientem discere nihil esse aliud quam suarum ipsius rationum animum recordari. Etenim Socrates pusionem quendam, vt Tullij verbis vtar, interrogat de geometrica dimensione quadrati: ad ea sic ille respondet vt puer, & tamen

men tam faciles interrogationes sunt, ut gradatim respondens eodem perueniat quò si Geometrica didicisset. Aliam nominis huius rationem Anatolius exposuit, ut est apud Rhodiginum, quòd cùm cæteræ disciplinæ deprehendi vel non docente aliquo possint omnes, Mathematica sub nullius cognitionem veniant, nisi preeunte aliquo, cuius solertia succidantur vepreta, vel exurantur, & superciliosa complanentur aspreta. Ita enim Cælius: quod quam vim habeat, nõ est huius loci curiosius perscrutari. Equidem M Tullius Mathematicos in magna rerum obscuritate, recondita arte multiplicíq; ac subtili versari scribit: sed quis nescit idipsum cum alijs grauioribus scientijs esse commune? Est enim, vel eodem auctore Tullio, omnis cognitio multis obstructa difficultatibus maximaq; est & in ipsis rebus obscuritas, & in iudicijs nostris infirmitas, nec ullus est, modò interius paulò Physica penetrarit, qui non facilè sit expertus, quam multi undique emergant, rerum naturalium causas inquirentibus, inexplicabiles labyrinthi. Sunt qui ex demonstrationum firmitate nominari Mathematicas opinantur: cuius etiam rationis momentum alio seorsim loco expendendum fuerit. Quocirca primam verbi notationem, quam sequutus est Proclus, nobis retinendam censeo. Hactenus de uniuerso Mathematicæ genere, quanta potui & perspicuitate & breuitate dixi. Sequitur ut de Geometria separatim atque ordine ea disseram, quæ initio sum pollicitus.

tus. Est autem Geometria, ut definit Proclus scientia, quæ versatur in cognitione magnitudinũ figurarum, & quibus hæ continentur, extremorum, item rationum & affectionum, quæ in illis cernuntur ac inhærent: ipsa quidem progrediens à puncto indiuiduo per lineas & superficies, dũ ad solida conscendat, variasq́ ipsorum differentias patefaciat. Quũmque omnis scientia demonstratiua, ut docet Aristoteles, tribus quasi momentis contineatur, genere subiecto, cuius proprietates ipsa scientia exquirit & contemplatur: caussis & principijs, ex quibus primis demonstrationes conficiuntur: & proprietatibus, quæ de genere subiecto per se enunciantur: Geometriæ quidem subiectum in lineis, triangulis, quadrangulis, circulis, planis, solidis, atque omnino figuris & magnitudinibus, earúmque extremitatibus consistit. His autem inhærent diuisiones, rationes, tactus, æqualitates, παραβολαὶ ὑπερβολαὶ ἐλλεῖψεις, atque alia generis eiusdem propè innumerabilia. Postulata verò & Axiomata ex quibus hæc inesse demonstrantur, eiusmodi ferè sunt: Quouis centro & interuallo circulum describere. Si ab æqualibus æqualia detrahas, quæ relinquuntur esse æqualia, cæteraq́ id genus permulta, quæ licet omnium sint cõmunia, ac demonstrandum tamen tum sunt accommodata, cùm ad certum quoddam genus traducuntur. Sed cùm præcipua videatur Arithmetica & Geometriæ inter Mathematicas dignatio, cur Arithmetica sit ἀκριβεστέρα &

 exactior

exactior quam Geometria paucis explicandum arbitror. Hic verò & Aristotelem sequemur ducem, qui scientiam cum scientia ita comparat, ut accuratiorem esse velit eam, quæ rei causam docet, quam quæ rem esse tantùm declarat, deinde quæ in rebus sub intelligentiam cadentibus versatur, quàm quæ in rebus sensum mouentibus cernitur. Sic enim & Arithmetica quàm Musica, & Geometria quàm Optica & Stereometria quàm Mechanica exactior esse intelligitur. Postremò quæ ex simplicioribus initijs constat, quàm quæ aliqua adiectione compositis vtitur. Atque hac quidem ratione Geometriæ præstat Arithmetica, quòd illius initium ex additione dicatur, huius sit simplicius. Est enim punctum, vt Pythagoreis placet, vnitas quæ situ obtinet: vnitas verò punctum est quod situ vacat. Ex quo percipitur, numerorum quàm magnitudinum simplicius esse elementum, numerosque magnitudinibus esse puriores, & à concretione materiæ magis disiunctos. Hæc quanquam nemini sunt dubia, habet & ipsa tamen Geometria quo se plurimum efferat, opibúsque suis ac rerum vbertate multiplici vel cum Arithmetica certet: id quod tute facilè deprehendas cùm ad infinitam magnitudinis diuisionem, quam respuit multitudo, animum conuerteris. Nunc quæ sit Arithmeticæ & Geometriæ societas, videamus. Nam theorematum quæ demonstratione illustrantur, quædam sunt vtriusque scientiæ communia, quædam verò singu-

singularum propria. Etenim quòd omnis proportio sit ῥητὸς siue rationalis, Arithmeticæ soli conuenit, nequaquam Geometriæ. in qua sunt etiam ἄῤῥητοι. seu irrationales proportiones. item, quadratorum gnomas minimo definitos esse. Arithmeticæ proprium (si quidem in Geometria nihil tale minimum esse potest) sed ad Geometriam propriè spectant situs. qui in numeris locum non habent: tactus, qui quidem à continuis admittuntur: ἄλογον. quoniam vbi diuisio infinitè procedit, ibi etiam τὸ ἄλογον esse solet. Communia porrò vtriusque sunt illa, quæ ex sectionibus eueniunt, quas Euclides libro secundo est persequutus: nisi quòd sectio per extremam & mediam rationem in numeris nusquam reperiri potest. Iam verò ex theorematibus eiusmodi communibus, alia quidem ex Geometria ad Arthmeticam traducuntur: alia contrà ex Arithmetica in Geometriam trāsferuntur: quædam verò perinde vtrique scientiæ conueniunt, vt quæ ex vniuersa arte Mathematica in vtranque harum conueniant. Nā & alterna ratio, & rationum conuersiones, compositiones, diuisiones hoc modo communia sunt vtriusque. Quæ autem sunt περὶ συμμέτρων, id est, de commensurabilibus, Arithmetica quidem primùm cognoscit & contemplatur: secundo loco Geometria Arithmeticam imitata. Quare & commensurabiles magnitudines illæ dicuntur, quæ rationem inter se habent quam numerus ad numerum, per-

inde quasi commensuratio & συμμετρία in numeris primum consistat (Ubi enim numerus, ibi & σύμμετρον cernitur: & ubi σύμμετρον, illic etiam numerus) sed quæ triangulorum sunt & quadrangulorum, à Geometra primùm considerantur: tum analogia quadam Arithmeticus eadem illa in numeris contemplatur. De Geometriæ diuisione, hoc adijciendum puto, quòd Geometriæ pars altera in planis figuris cernitur quæ solam latitudinem longitudini coniunctam habent: altera verò solidas contemplatur, quæ ad duplex illud internallum crassitudinem adsciscunt. Illam generali Geometriæ nomine veteres appellarunt: hanc propriè Stereometriam dixerunt. Ita Geometriam cum Optica, & Stereometriam cum Mechanica non rarò comparat Aristoteles. Sed illius cognitio huius inuẽtionem multis seculis antecessit, si modò Stereometriam ne Socratis quidem ætate vllam fuisse omnino verum est, quemadmodum à Platone scriptum videtur. Ad Geometriæ vtilitatem accedo, quæ quanquam suapte vi & dignitate ipsa per se nititur, nullius vsus aut actionis ministerio mancipata (vt de Mathematicis omnibus scientijs concedit in Politico Socrates) si quid ex ea tamen vtilitatis externæ quæritur, Dij boni quã latos, quàm vberes, quũ varios fructus fundit? Nec verò audiendus est vel Aristippus, vel Sophistarum alius, qui Mathematicorum artes idcirco repudiet, quòd ex fine nihil docere videantur, eiusque quod melius aut deteri-

deterius nullam habeant rationem. Ut enim nihil causæ dicas, cur sit melius triãguli, verbi gratia, tres angulos duobus esse rectis æquales: minimè tamen fuerit consentaneum; Geometriæ cognitionem ut inutilem exagitare, criminari, explodere, quasi quæ finem & bonum quò referatur, habeat nullum. Multas haud dubiè solius contemplationis beneficio citra materiæ contagionem adfert Geometria commoditates partim proprias, partim cum uniuerso genere communes. Cùm enim Geometria, ut scripsit Plato, eius quod semper est cognitionem profiteatur, ad veritatem excitabit illa quidem animum, & ad ritè philosophandum cuiusque mentem comparabit. Quinetiam ad disciplinas omnes facilius perdiscendas, attigeris necne Geometriam, quãti referre censes? Nam ubi cum materia coniũgitur, nónne præstantissimas procreat artes, Geodæsiam, Mechaniam, Opticam, quarum omnium usu, mortalium vitam summis beneficijs complectitur? Etenim bellica instrumenta, urbiumq; propugnacula, quibus munitæ urbes hostium vim propulsarent, his adiutricibus fabricata est: montium ambitus & altitudines, locorumque situs nobis indicauit: dimetiendorum & mari & terra itinerum rationem præscripsit: trutinas & stateras, quibus exacta numerorum æqualitas in ciuitate retineatur, composuit: uniuersi ordinem simulachris expressit, multáque quæ hominum fidem superarent, omnibus persuasit. Ubique extant præclara in eam rem testimonia. Illud memorabile, quòd Archimedi rex Hiero tribuit. Nam extructo vastæ molis nauigio, quod Hiero Ægyptiorum regi Ptolemæo mitteret, cùm vniuersa Syracusanorum multitudo collectis simul viribus nauem trahere non posset effecissetq́; Archimedes, vt solus Hiero illam subduceret, admiratus viri scientiã rex ἀπὸ ταύτης, ἔφη, τῆς ἡμέρας περὶ παντὸς ἀρχιμήδει λέγοντι πιστευτέον. Quid? quòd Archimedes idem, vt est apud Plutarchum, Hieroni scripsit datis viribus datum pondus moueri posse? fretusq́; demonstrationis robore, illud sæpe iactarit, si terram haberet alteram vbi pedem figeret, ad eam, nostram hanc se transmouere posse? Quid varia αὐτομάτων machinarumque genera ad vsus necessarios comparata memorem? Innumerabilia profectò sunt illa, & admiratione dignissima quibus prisci homines incredibili quodam ad philosophandum studio concitati, inopem mortalium vitam artis huius præsidio subleuarunt: tametsi memoriæ sit proditum, Platonem Eudoxo & Archytæ vitio vertisse, quõd Geometrica problemata ad sensilia & organica abducerent. Sic enim corrumpi ab illis & labefieri Geometriæ præstantiam, quæ ab intelligibilibus & incorporeis rebus ad sensiles & corporeas prolaberetur. Quapropter ridicula idem scripsit Plato Geometrarum esse vocabula, quæ quasi ad opus & actionem spectent ita sonare videntur. Quid enim est quadrare, si non opus facere? Quid addere, pro-

producere, applicare? Multa quidem sunt eiusmodi nomina, quibus necessariò & tanquam coacti Geometræ vtuntur. quippe cùm alia desint in hoc genere commodiora. Sic ergo censuit Plato, sic Aristoteles sic denique philosophi omnes, Geometriam ipsam cognitionis gratia exercendam, nec ex aliquo vsu externo sed ex rerum νοητῶν intelligentia æstimandam esse. Exposita breuius quàm res tanta dici possit, vtilitatis ratione, Geometriæ ortum, qui in hac rerum periodo ex historicorum monumentis nobis est cognitus deinceps aperiamus. Geometria apud Ægyptios inuenta, (ne ab Adamo, Setho, Noah, quos cognitione rerum multiplici valuisse constat, eam repetamus) ex terrarum dimensione, vt verbi præ se fert ratio, ortum habuisse dicitur: cùm anniuersaria Nili inundatione & incrementis limo obducti agrorum termini confunderentur. Geometriam enim, sicut & reliquas disciplinas, in vsu quàm in arte prius fuisse aiũt. Quod sanè mirum videri non debet, vt & huius & aliarum scientiarum inuentio ab vsu cœperit ac necessitate. Etenim tempus, rerum vsus, ipsa necessitas ingenium excitat, et ignauiã acuit. Deindè quicquid ortum habuit (vt tradunt Physici) ab inchoato & imperfecto processit ad perfectum. Sic artium & scientiarum principia experientiæ beneficio collecta sunt: experientia verò à memoria fluxit, quæ & ipsa à sensu primum manauit. Nam quod scribit Aristoteles, Mathematicas artes, *comparatis rebus omnibus ad vitam necessarijs, in Ægypto fuisse constitutas, quòd ibi sacerdotes omnium concessu in otio degerent: non negat ille adductos necessitate homines ad excogitandam, verbi gratia, terræ dimetiendæ rationem, quæ theorematum deinde inuestigationi causam dederit: sed hoc confirmat, præclara eiusmodi theorematum inuenta, quibus extructa Geometriæ disciplina constat, ad vsus vitæ necessarios ab illis non esse expetita. Itaque vetus ipsum Geometriæ nomen ab illa terræ partiundæ finiúmque regundorum ratione posteà recessit, & in certa quadam affectionum magnitudini per se inhærentium scientia propriè remãsit. Quemadmodum igitur in mercium & contractuum gratiam supputandi ratio, quam secuta est accurata numerorum cognitio, à Phœnicibus initium duxit: ita etiam apud Ægyptios, ex ea quam cõmemoraui causa ortũ habuit Geometria. Hãc certè, vt id obiter dicam, Thales in Græciam ex Ægypto primùm transtulit: cui non paucæ deinceps à Pythagora, Hippocrate, Chio, Platone, Archyta Tarentino, alijsq; cõpluribus, ad Euclidis tempora factæ sunt rerum magnarum accessiones. Cæterùm de Euclidis ætate id solùm addam, quod à Proclo memoriæ mandatum accepimus. Is enim commemoratis aliquot Platonis tum æqualibus tùm discipulis, subijcit, non multò ætate posteriorem illis fuisse Euclidem eum, qui Elementa conscripsit, & multa ab Eudoxo collecta, in ordinẽ luculentũ*

com-

composuit, multaq; à Theateto inchoata perfecit, queq; mollius ab aliis demonstrata fuerant, ad firmissimas & certissimas apodixes reuocauit. Vixit autem, inquit ille, sub primo Ptolemaeo. Etenim ferunt Euclidam à Ptolemaeo quõdam interrogatum numqua esset via ad Geometriam magis compendiaria, quàm sit ista στοιχείωσις respondisse, μὴ εἶναι βασιλικὴν ἀτραπὸν ἐπὶ γεωμετρίαν. Deinde subiungit, Euclidem natu quidem esse minorem Platone, maiorem verò Eratosthene & Archimede (hi enim aequales erant) cùm Archimedes Euclidis mentionem faciat. Quòd si quis egregiam Euclidis laudem quam cùm ex aliis scriptionibus accuratissimis, tùm ex hac Geometrica στοιχειώσει cõsequutus est in qua diuinus rerum ordo sapientissimis quibusque hominibus magnae semper admirationi fuit, is Proclum studiosè legat, quò rei veritatẽ illustriorem reddat grauissimi testis auctoritas. Superest igitur vt finem videamus, quò Euclidis elementa referri, & cuius causa in id studium incumbere oporteat. Et quidem si res quae tractantur, consyderes: in tota hac tractatione nihil aliud quaeri dixeris, quàm vt κοσμικὰ quae vocantur, σχήματα (fuit enim Euclides professione & instituto Platonicus) Cubus, Icosaedrũ, Octaedrum, Pyramis & Dodecaedrum certa quadã suorum & inter se laterum, & ad sphaerae diametrum ratione eidem sphaerae inscripta cõprehendantur. Huc enim pertinet Epigrammation illud vetus, quod in Geometrica Michaelis

 Pselli

Pselli συνόψει *scriptum legitur.*

Σχήματα πέντε τὰ πλάτωνος, πυθαγόρας σοφὸς εὗρε,
πυθαγόρας σοφὸς εὗρε, πλάτων δ' ἀριδήλ' ἐδίδαξεν,
Εὐκλείδης ἐπὶ τοῖσι κλέος περικαλλὲς ἔτευξεν.

Quòd si discentis institutionem spectes, illud certè fuerit propositum, vt huiusmodi elementorum cognitione informatus discentis animus, ad quamlibet non modò Geometriæ, sed & aliarũ Mathematicæ partium tractationem idoneus paratusq; accedat. Nam tametsi institutionem hanc solus sibi Geometra vendicare videtur, & tanquam in possessionem suam venerit, alios excludere posse: inde tamen per multa suo quodam modo iure decerpit Arithmeticus, pleraque Musicus, non pauca detrahit Astrologus, Opticus, Logisticus, Mechanicus, itemq; cæteri: nec vllus est denique artifex præclarus, qui in huius se possessionis societatem cupidè nõ offerat, partemq; sibi concedi postulet. Hinc στοιχείωσις *absolutum operi nomen, &* στοιχειωτὴς *dictus Euclides. Sed quid longius prouehor? Nam quod ad hanc rem attinet, tam copiosè & eruditè scripsit (vt alia complura) eo ipso, quem dixi, loco P. Montaureus, vt nihil desiderio loci reliquerit. Quæ verò ad dicendum nobis erant proposita, hactenus pro ingenij nostri tenuitate omnia mihi perfecisse videor. Nam tametsi & hæc eadem & alia pleraque multo fortè præclariora ab hominibus doctissimis,*

qui

qui tùm acumine ingenij, tùm admirabili quodam lepore dicendi semper floruerunt grauiùs, splendidiùs, vberiùs tractari posse scio: tamen experiri libuit numquid etiam nobis diuino sit concessum munere, quod rudes in hac philosophiæ partè discipulos adiuuare aut certè excitare queat. Huc accessit quòd ista recens elementorum editio, in qua nihil non parum fuisset studij, aliquid à nobis efflagitare videbatur, quod eius commendationem adaugeret. Cùm enim vir doctissimus Io Magnienus Mathematicarum artium in hac Parrhisiorum Academia professor verè regius, nostrum hunc typographum in excudendis Mathematicorum libris diligentissimum, ad hanc Elementorum editionem sæpè & multum esset adhortatus, eiusque impulsu permulta sibi iam comparasset typographus ad hanc rem necessaria, citò interuenit, malùm Ioannis Magnieni mors insperata, quæ tam graue inflixit Academiæ vulnus, cui ne post multos quidem annorum circuitus cicatrix obduci vlla posse videatur. Quamobrem amisso instituti huius operis duce, typographus, qui nec sumptus antea factos sibi perire, nec studiosos, quibus id muneris erat pollicitus, sua spe cadere vellet, ad me venit, & impensè rogauit, vt meam propositæ editioni operam & studium nauarem, quod cùm denegaret occupatio nostra, iuberet officij ratio, feci equidem rogatus, vt quæ subobscurè vel parum commode in sermonem Latinum è Græco translata

vi-

videbantur, clariore, aptiore & fideliore interpretatione nostra (quod cuiusq; pace dictum volo) lucem acciperent. Id quod in omnibus ferè libris posterioribus tute primo obtutu perspicias. Nam in sex prioribus nō tantum temporis quātum in cæteros ponere nobis licuit: decimi autem interpretatio, qua melior nulla potuit adferri, P. Montaureo solida debetur. Atq; vt ad perspicuitatem facilitatemque nihil tibi deesse queraris, adscripta sunt propositionibus singulis vel lineares figuræ, vel punctorum tanquam vnitatum notulæ, quæ Theonis apodixin illustrēt: illæ quidem magnitudinum, hæ autem numerorum indices. subscriptis etiam ciphrarum, vt vocant, characteribus, qui propositum quemuis numerum exprimant. ob eamq; causam eiusmodi vnitatum notulæ, quæ pro numeri amplitudine maius paginæ spatium occuparent, pauciores sæpius depictæ sunt aut in lineas etiam commutatæ. Nam literarum vt a, b, c characteres non modò numeris & numerorum partibus nominādis sunt accommodati, sed etiam generales esse numerorum, vt magnitudinum affectiones testantur. Adiecta sunt insuper quibusdam locis non pœnitenda Theonis scholia, siue mauis lemmata, quæ quidem longè plura accessissent, si plus otij & temporis vacui nobis fuisset relictū, quod huic studio impartiremus. Hanc igitur operam boni consule, & quæ obuia erunt impressionis vitia, candidè emenda. Vale. Lutetiæ 4. Idus April. 1557.

EVCLIDIS ELEMENTVM PRIMVM.

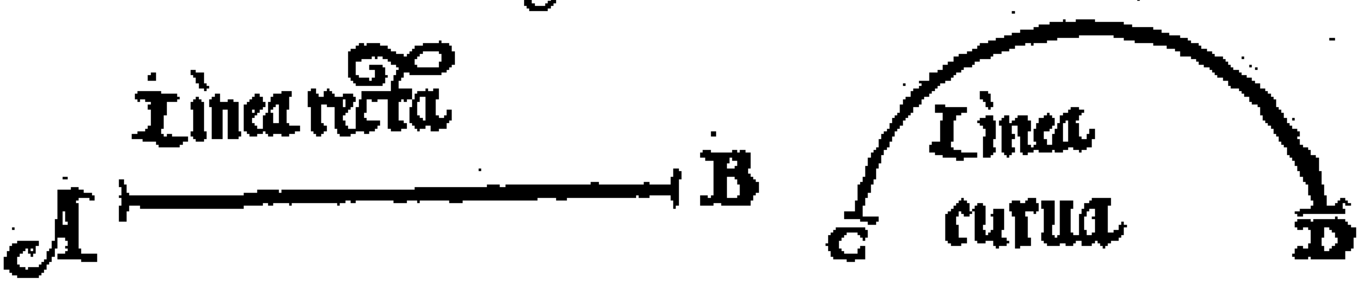
DEFINITIONES.

I

PVnctum eſt, cuius pars nulla eſt. Punctum

2

Linea verò, longitudo latitudinis expers.



3

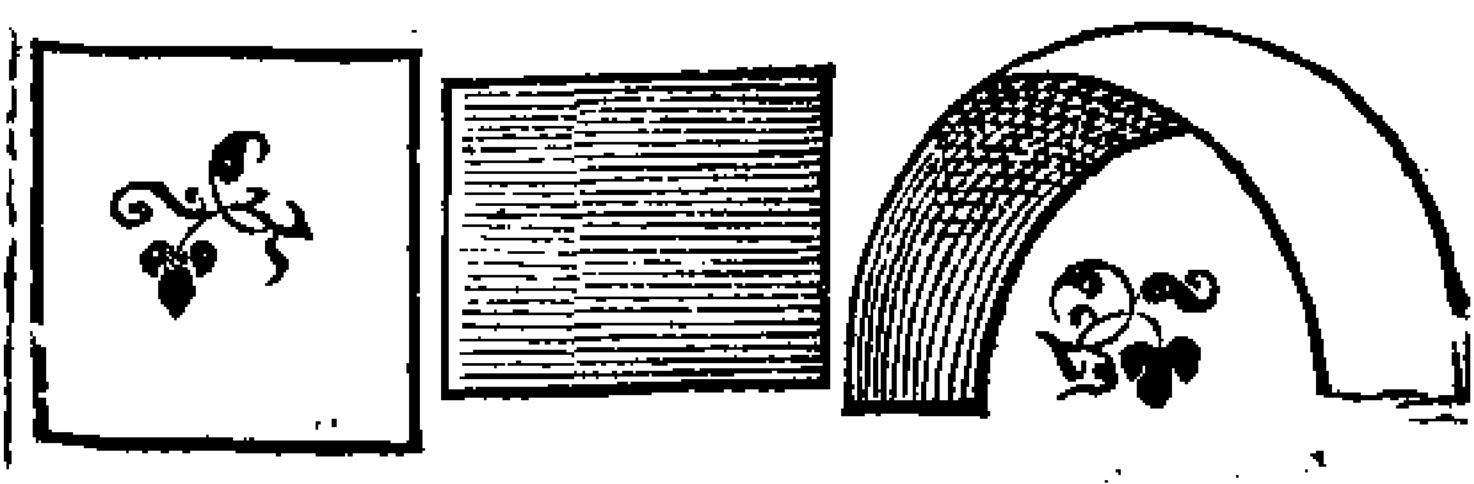
Lineæ autem termini, ſunt puncta.

4

Recta linea eſt, quæ ex æquo ſua interiacet puncta.

5

Superficies eſt, quæ longitudinem latitudinemq́ue tantùm habet.



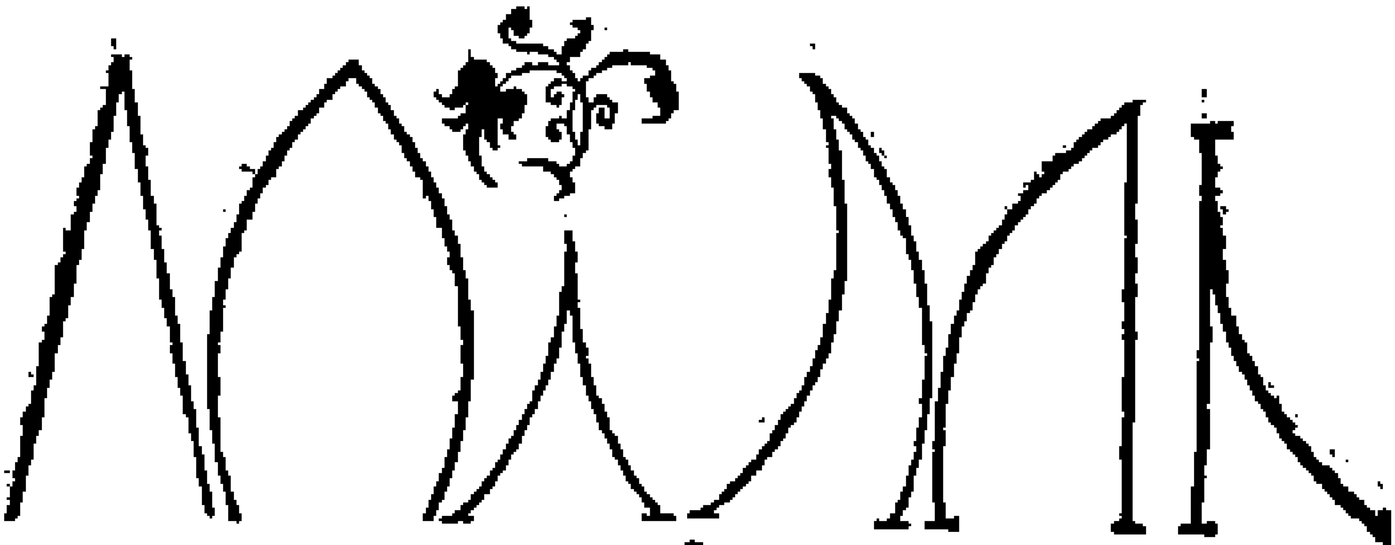
6 Super-

6

Superficiei extrema, ſunt lineæ.

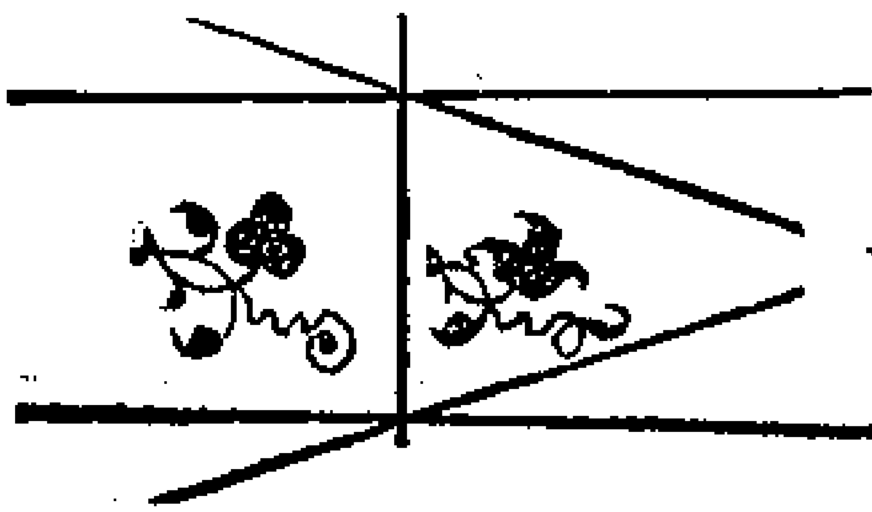
7

Plana ſuperficies eſt, quæ ex æquo ſuas interiacet lineas.



8

Planus angulus eſt duarũ linearũ in plano ſe mutuò tangentium, & non in directum iacẽtium, alterius ad alteram inclinatio.



9.

Cùm autem quæ angulum continẽt lineæ, rectæ fuerint, rectilineus ille angulus appellatur.

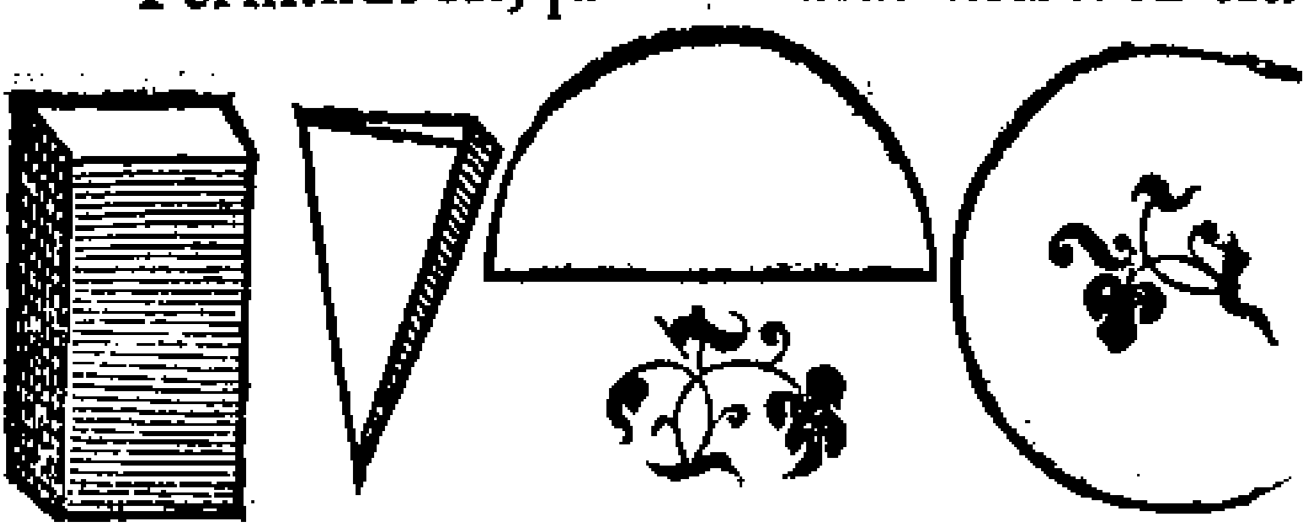
10

Cùm verò recta linea ſuper rectam conſiſtens lineã, eos qui ſunt deinceps angulos ęquales, inter ſe fecerit: rectus eſt vterq; æquaqualium angulorũ:& quæ insistit recta linea, perpẽdicularis vocatur eius, cui insistit.

11
Obtusus angulus est, qui recto maior est.

12
Acutus verò, qui minor est recto.

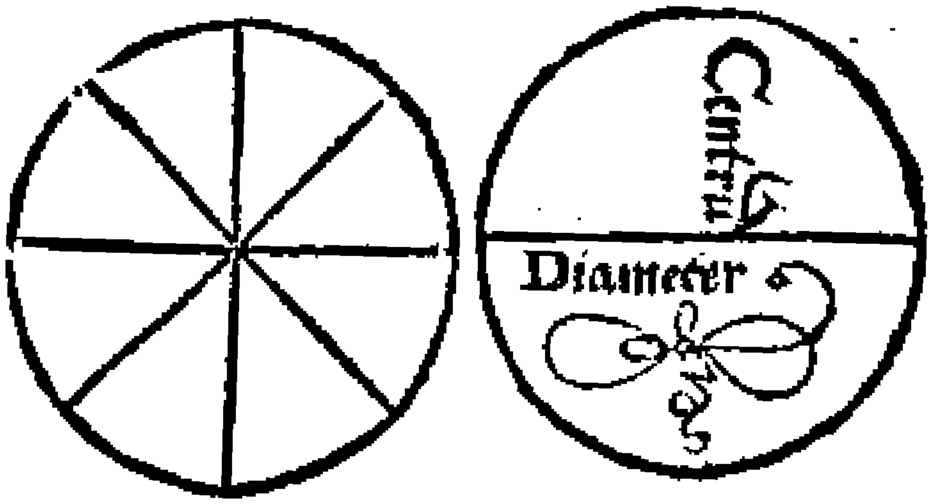
13
Terminus est, quod alicuius extremũ est.



14
Figura est, quæ sub aliquo, vel aliquibus terminis comprehenditur.

15
Circulus est figura plana sub vna linea cõprehẽsa, quę peripheria appellatur: ad quã ab vno puncto eorum, quæ intra figuram sunt

ſunt poſita, cadentes omnes rectæ lineæ inter ſe ſunt æquales.



16

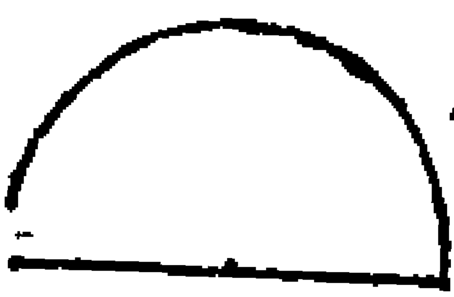
Hoc verò punctum, centrum circuli appellatur.

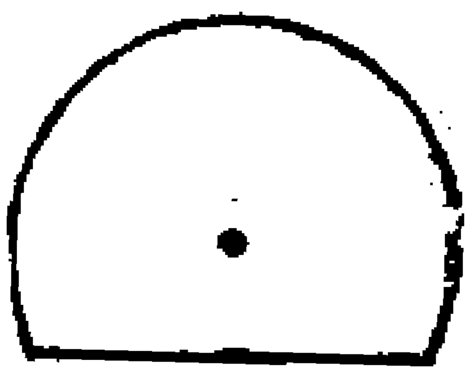
17

Diameter autem circuli, eſt recta quædam linea per cẽtrum ducta, & ex vtraque parte in circuli peripheriam terminata, quæ circulum bifariam ſecat.

18

Semicirculus eſt figura, quæ cõtinetur ſub diametro, & ſub ea linea, quæ de circuli peripheria aufertur.





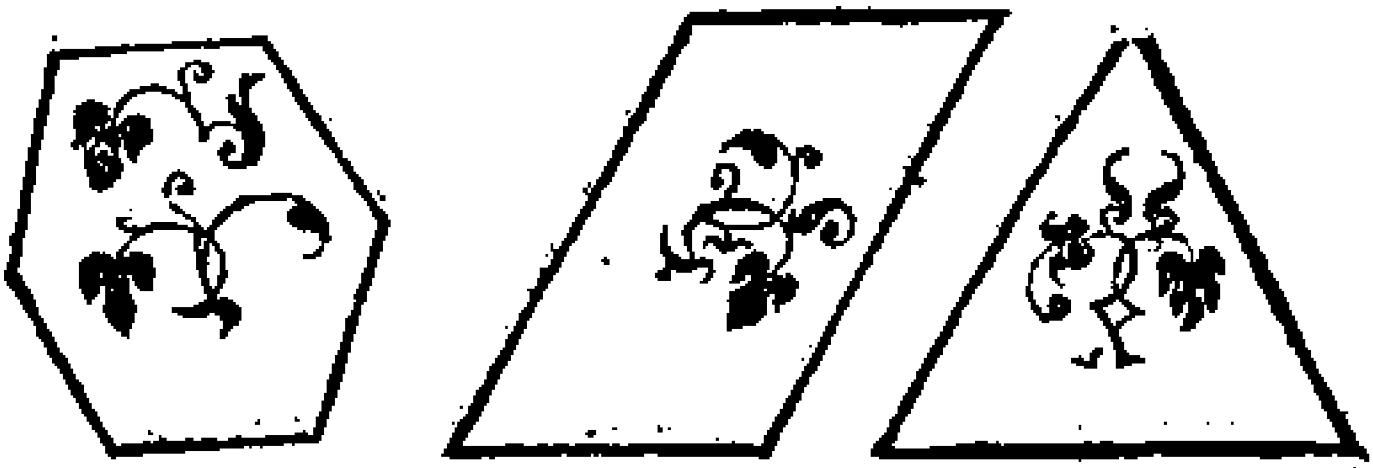
19

Segmentum circuli, eſt figura, quæ ſub recta linea & circuli peripheria continetur.

20 Recti

20

Rectilineæ figuræ, ſunt quæ ſub rectis lineis continentur.



21

Trilateræ quidem, quæ ſub tribus.

22

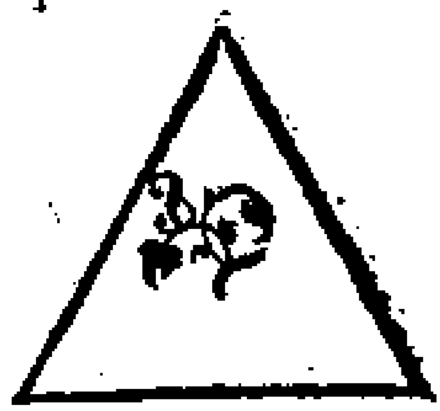
Quadrilateræ, quæ ſub quatuor.

23

Multilateræ verò, quæ ſub pluribus quàm quatuor rectis lineis comprehenduntur.

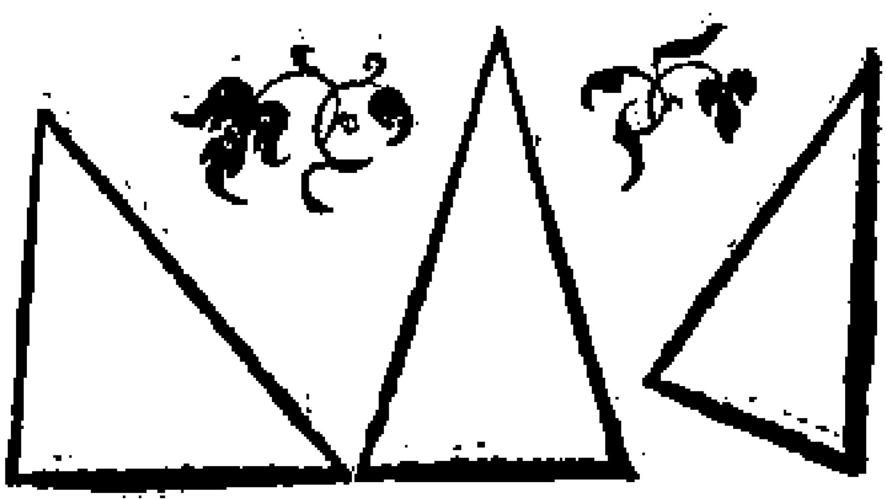
24

Trilaterarũ porrò figurarum, æquilaterum eſt triangulum, quod tria latera habet æqualia.



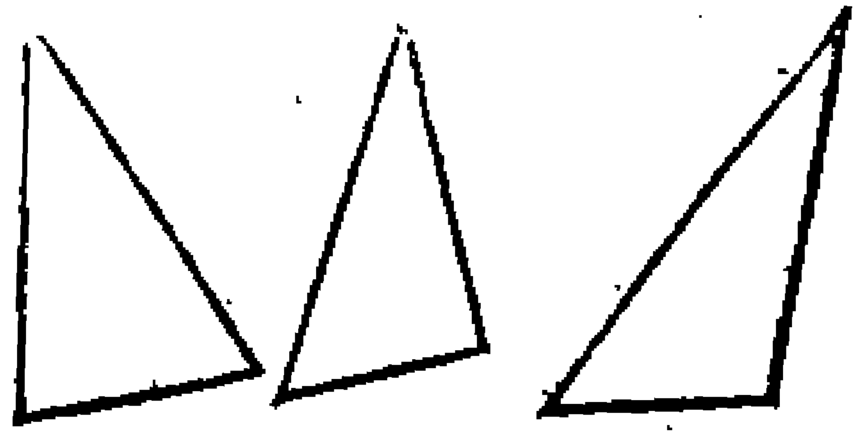
25

Iſoſceles autem eſt quod duo tantum æqualia habet latera.



26

Scalenū verò, eſt qđ tria inequalia habet latera



27

Ad hæc etiam, trilaterarum figurarum, rectangulum quidem triangulum eſt, quod rectum angulum habet.

28

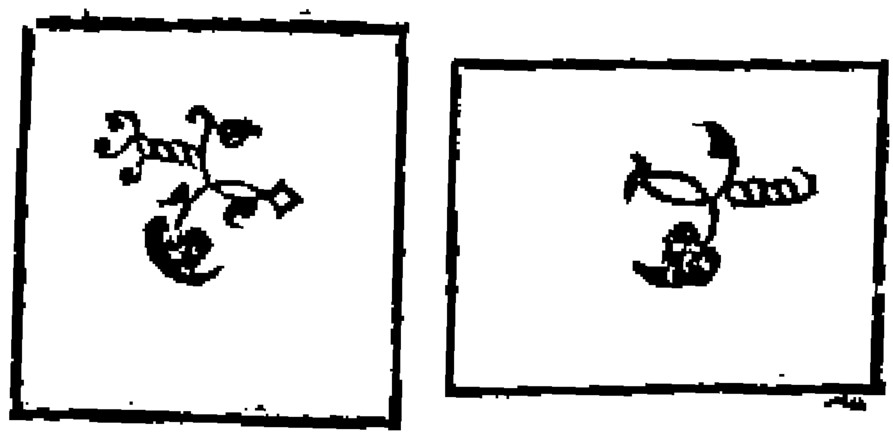
Amblygonium autem, quod obtuſum angulum habet.

29

Oxygonium verò, quod tres habet acutos angulos.

30

Quadrilaterarum autem figurarum, quadratū quidē eſt qđ & æquilaterū & rectangulum eſt.



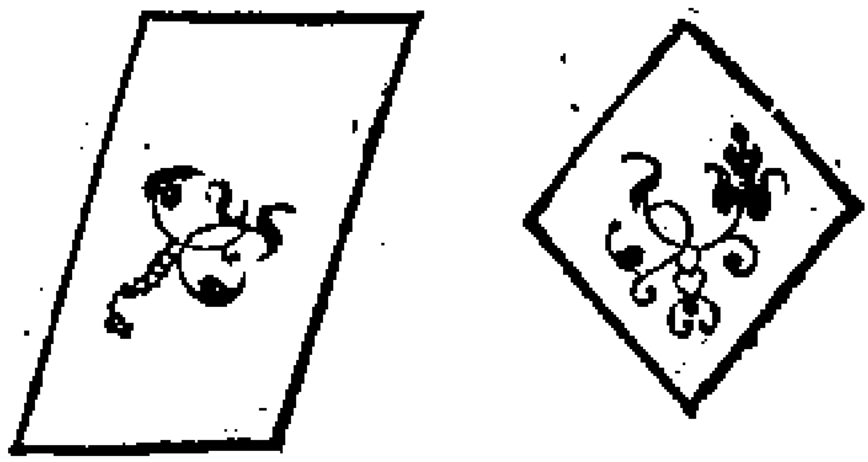
31

Altera parte longior figura eſt, quæ rectāgula quidem, at æquilatera non eſt.

32 Rhom-

32

Rhõbus aũtem, qui æquilaterum & rectangulum est.



33

Rhomboides verò, quæ aduerſa & latera & angulos habens inter ſe æqualia, neque æquilatera eſt, neque rectangula.

34

Preter has autem reliquæ quadrilateræ figuræ, trapezia appellentur.

35

Parallelæ rectæ lineæ ſunt quæ, cùm in eodem ſint plano, & ex vtraque parte in infinitum producantur, in neutram ſibi mutuò incidunt.

Poſtulata.

1

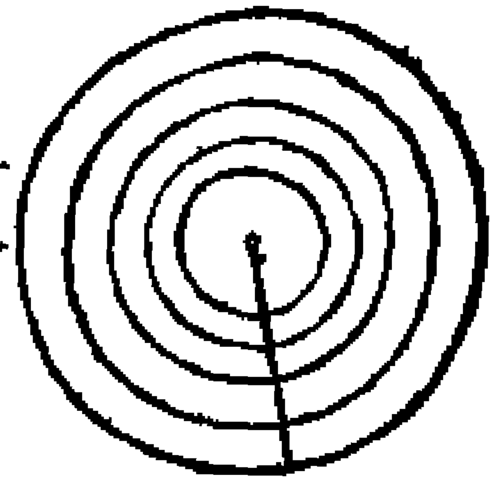
Poſtuletur, vt à quouis puncto in quoduis punctum, rectam lineã ducere cõcedatur.

2

Et rectam lineam terminatam in continuum recta producere.

3

In quouis centro & interuallo circulũ deſcribere.



Communes notiones.

1.

Quæ eidẽ æqualia, & inter ſe ſunt æqualia.

2

Et ſi æqualibus æqualia adiecta ſint, tota ſunt æqualia.

3

Et ſi ab æqualibus æqualia ablata ſint, quæ relinquuntur ſunt æqualia.

4

Et ſi inæqualibus æqualia adiecta ſint, tota ſunt inæqualia.

5

Et ſi ab inęqualibus ęqualia ablata ſint, reliqua ſunt inæqualia.

6

Quæ ~~eiuſdem duplicia~~ ſunt, inter ſe ſunt æqualia.

Et

7

Et quæ eiusdem sunt dimidia, inter se æqualia sunt.

8

Et quæ sibi mutuò congruunt, ea inter se sunt æqualia.

9

Totum est sua parte maius.

10

Item, omnes recti anguli sunt inter se æquales.

11

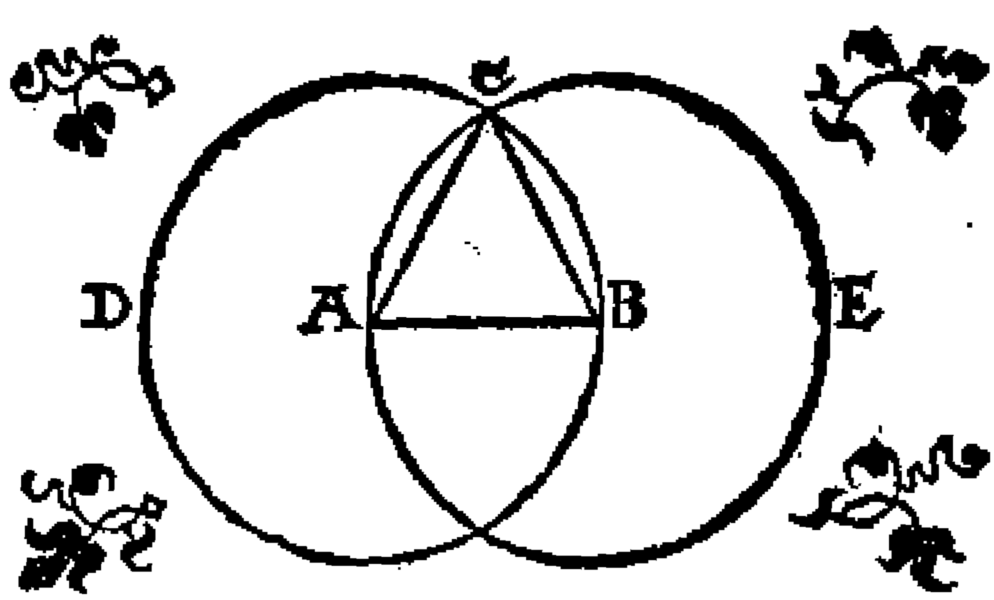
Et si in duas rectas lineas altera recta incidens, internos ad casdéque partes angulos duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ sibi mutuò incident ad eas partes, vbi sunt anguli duobus rectis minores.

12

Duæ rectæ lineæ spatium non comprehédunt.

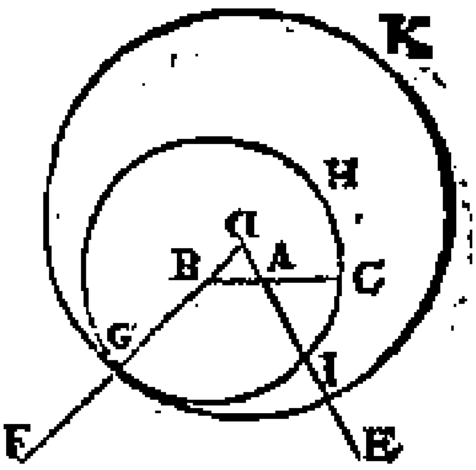
Problema 1. Propositio 1.

Super data recta linea terminata, triangulū æquilaterum constituere.



Problema 2. Propositio 2.

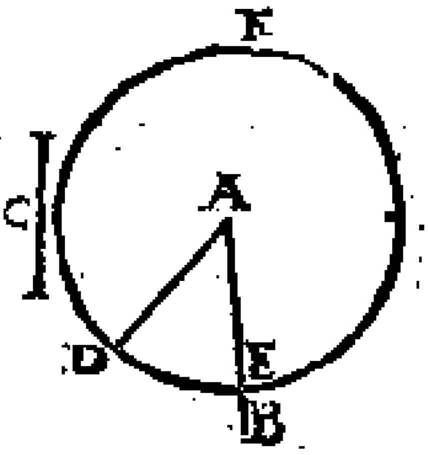
Ad datum punctum, datæ rectæ lineæ, ęqualem rectam lineam ponere.



Problema 3. Propositio 3.

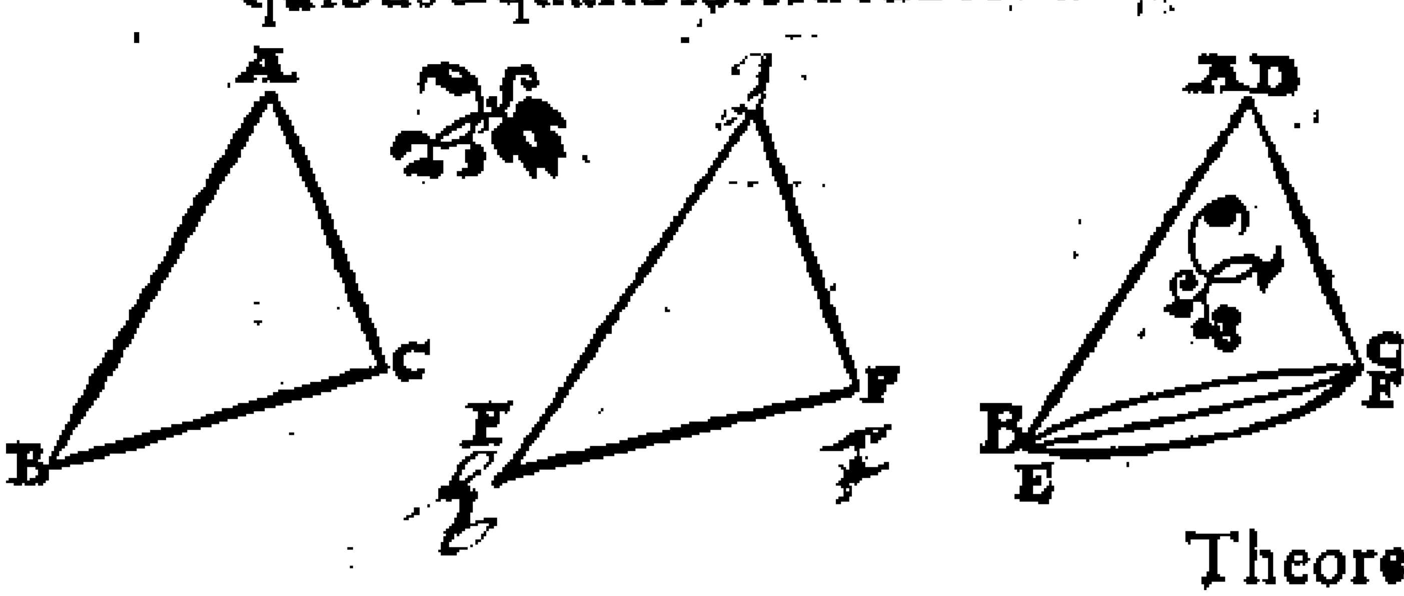
§ 30. 4. 4.

Duabus datis rectis lineis inęqualibus, de maiore æqualẽ minori rectam lineã detrahere.



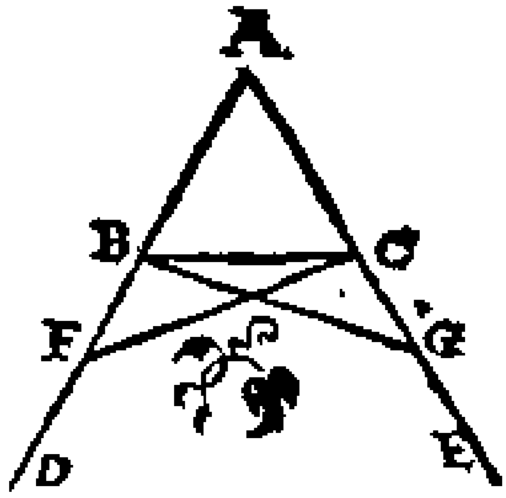
Theorema primum. Propositio 4.

Si duo triangula duo latera duobus lateribus æqualia habeant, vtrunque vtriq;, habeant verò & angulum angulo æqualẽ sub æqualibus rectis lineis contentum: & basin basi æqualem habebunt, eritq; triãgulum triãgulo æquale, ac reliqui anguli reliquis angulis æquales erũt, vterque vtrique, sub quibus æqualia latera subtenduntur.



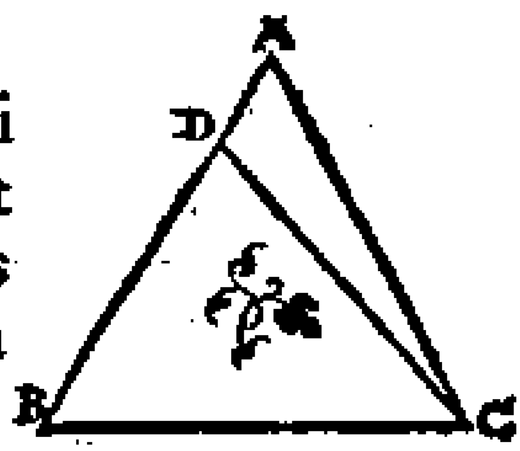
Theore-

Theorema 2. Propositio 5.



Isoscelium triangulorũ qui ad basim sunt anguli, inter se sunt æquales; & si vlterius productæ sint æquales illæ rectæ lineæ, qui sub basi sunt anguli, inter se ęquales erunt.

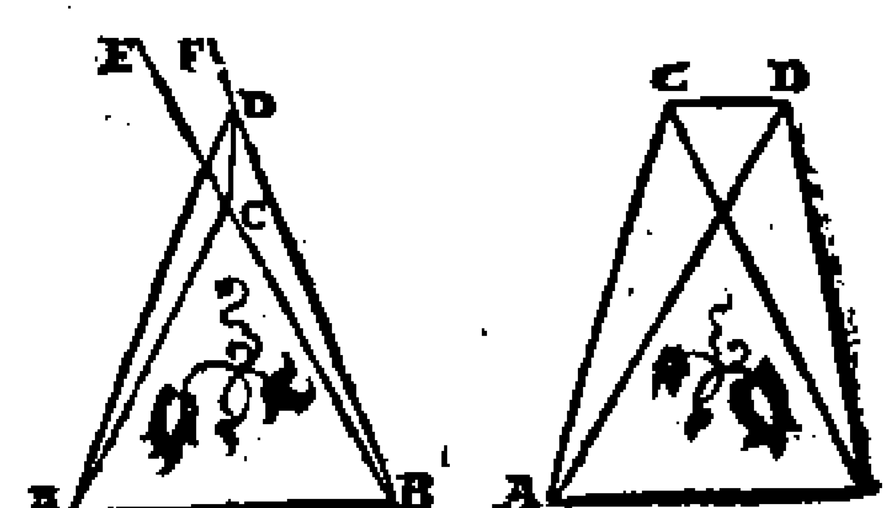
Theorema 3. Propositio 6.



Si trianguli duo anguli æquales inter se fuerint & sub ęqualibus angulis subtensa latera æqualia inter se erunt.

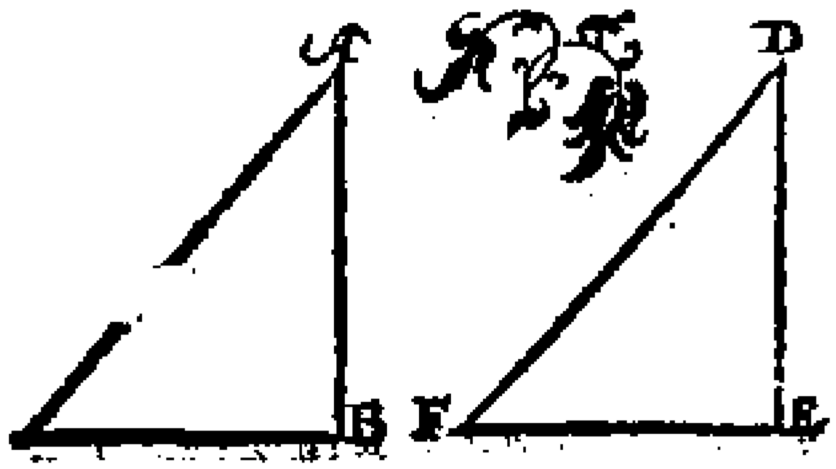
Theorema 4. Propositio 7.

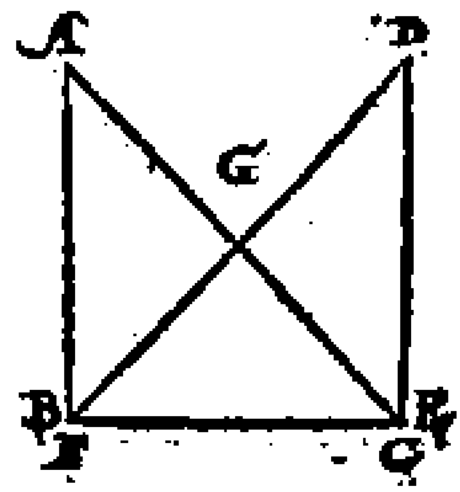
Super eadẽ recta linea, duabus eisdem rectis lineis alię duę rectę lineę ęquales, vtraque vtrique, nõ constituentur, ad aliud atque aliud punctũ, ad easdẽ partes eosdẽque terminos cum duabus initio ductis rectis lineis habentes.



Theorema 5. Propoſitio 8.

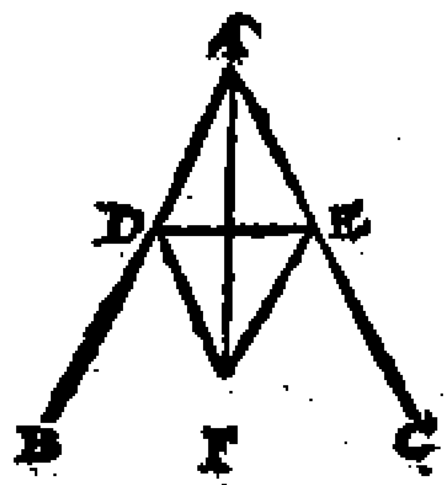
Si duo triãgula duo latera habuerint duobus lateribus, vtrunque vtriq;, æqualia: habuerint verò & baſim baſi æqualē: angulũ quoque ſub æqualibus rectis lineis cõtentum angulo æqualem habebunt.





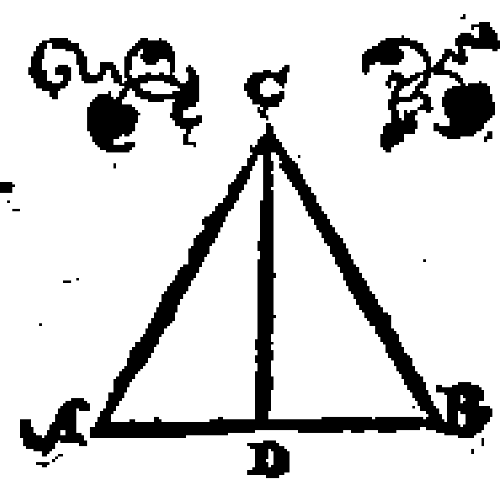
Problema 4. Propoſitio 9.

Datum angulum rectilineum bifariam ſecare.



Problema 5. Propoſitio 10.

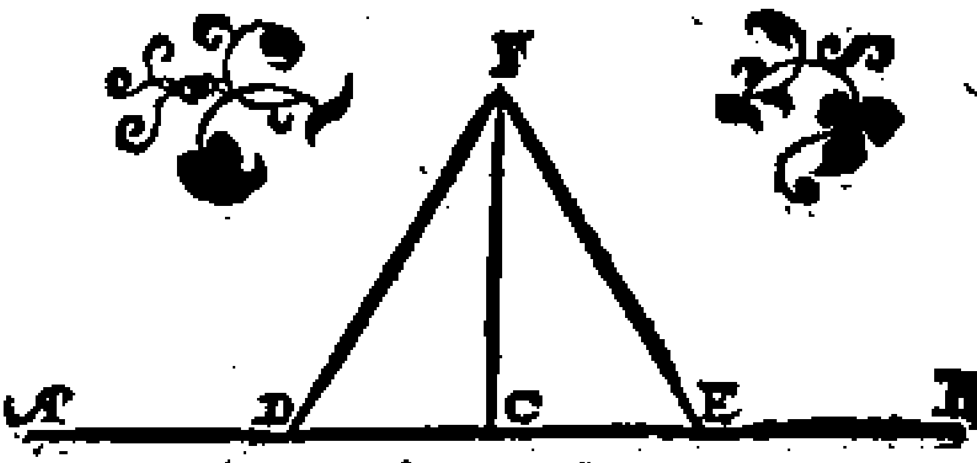
Datam rectam lineã finitam bifariam ſecare.



Proble-

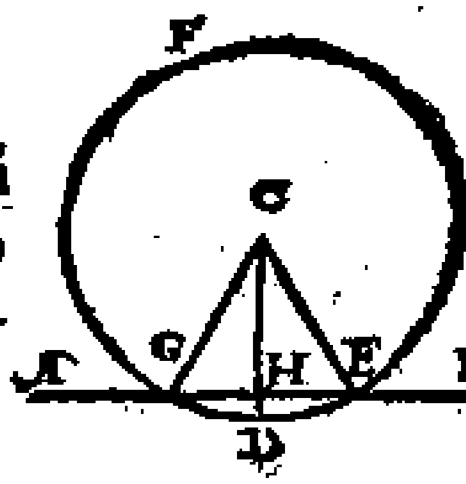
Problema 6. Propositio 11.

Data recta linea, à puncto in ea dato, re-



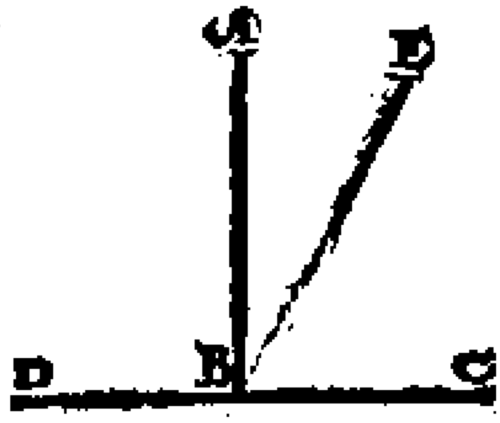
ctam lineam ad angulos rectos excitare.

Problema 7. Propositio 12.



Super datam rectā lineā infinitam, à dato puncto quod in ea non est, perpendicularem rectam deducere.

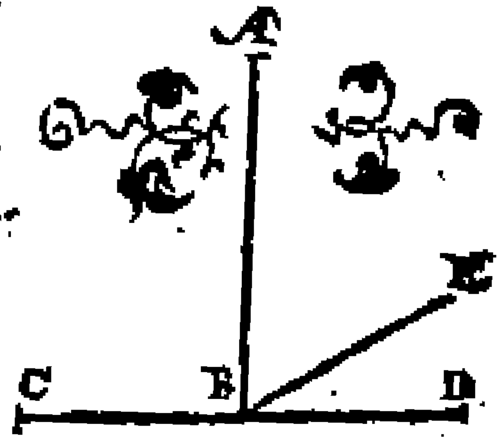
Theorema 6. Propositio 13.



Cùm recta linea super rectā consistens lineā angulos facit, aut duos rectos, aut duobus rectis æquales efficiet.

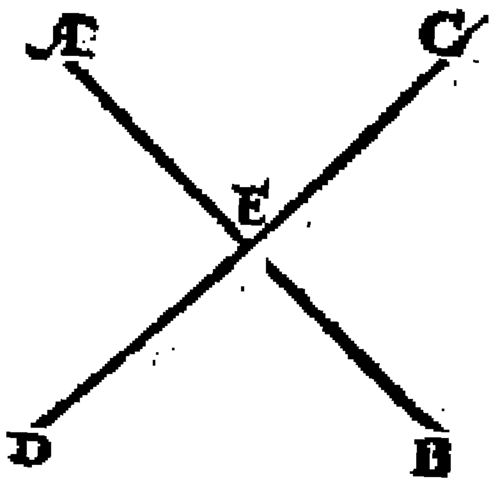
Theorema 7. Propositio 14.

Si ad aliquam rectam lineam, atque ad eius punctũ, duę rectæ lineæ nõ ad eaſdem partes ductæ, eos qui ſunt deinceps angulos duobus rectis æquales fecerint, in directum erunt inter ſe ipſæ rectæ lineæ.



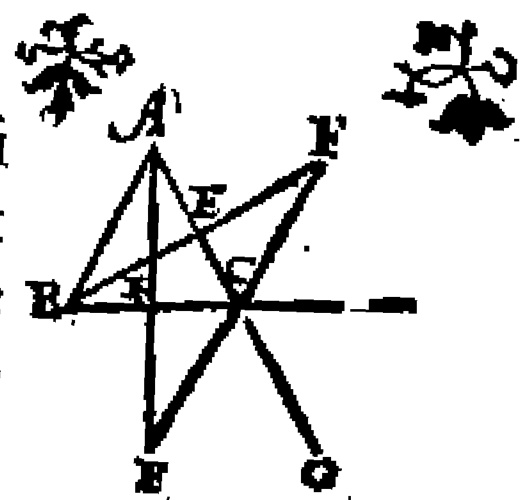
Theorema 8. Propoſitio 15.

Si duę rectę lineę ſe mutuò ſecuerint, angulos qui ad verticem ſunt, æquales inter ſe efficient.



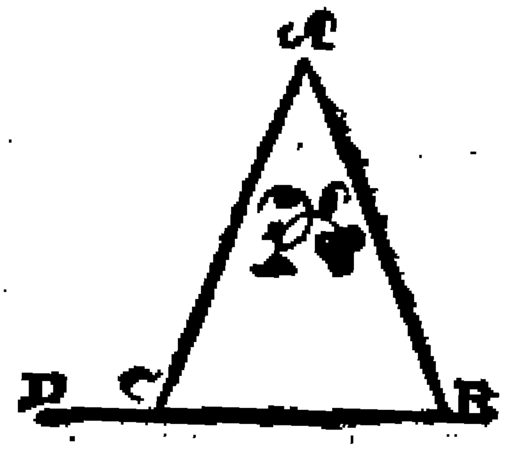
Theorema 9. Propoſitio 16.

Cuiuſcunque trianguli vno latere producto, externus angulus vtroque interno & oppoſito maior eſt.



Theorema 10. Propoſitio 17.

Cuiuſcunque trianguli duo anguli duobus rectis ſunt minores omnifariam ſumpti.



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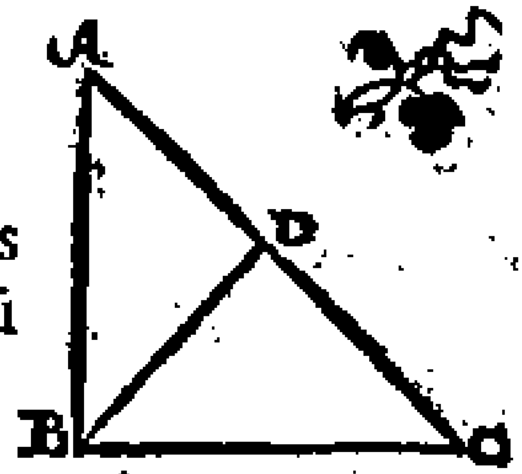
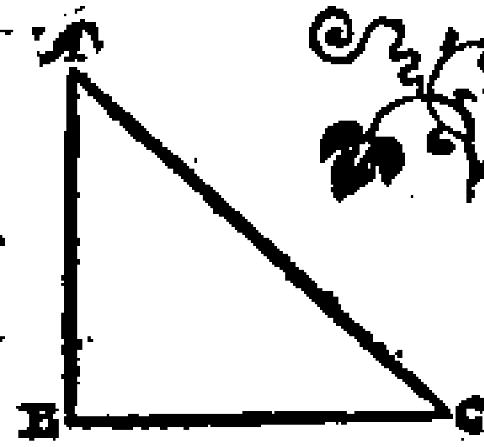
Theorema 11. Propositio 18.



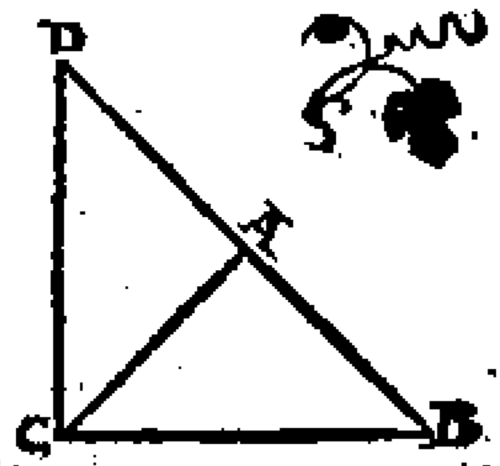
Omnis trianguli maius latus maiorem angulũ subtendit.

Theorema 12. Propositio 19.



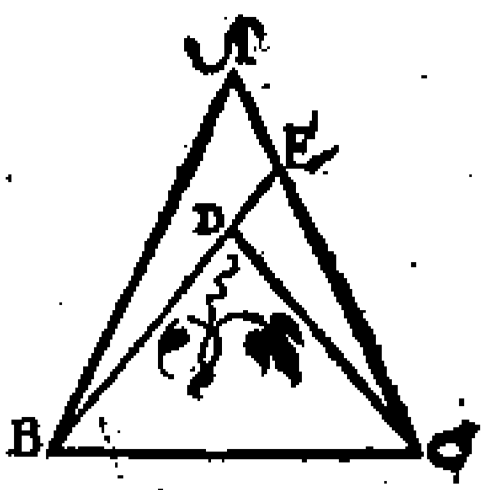
Omnis trianguli maior angulus, maiori lateri subtenditur.

Theorema 13. Propositio 20.



Omnis triãguli duo latera reliquo sunt maiora, quomodocũq; assũpta.

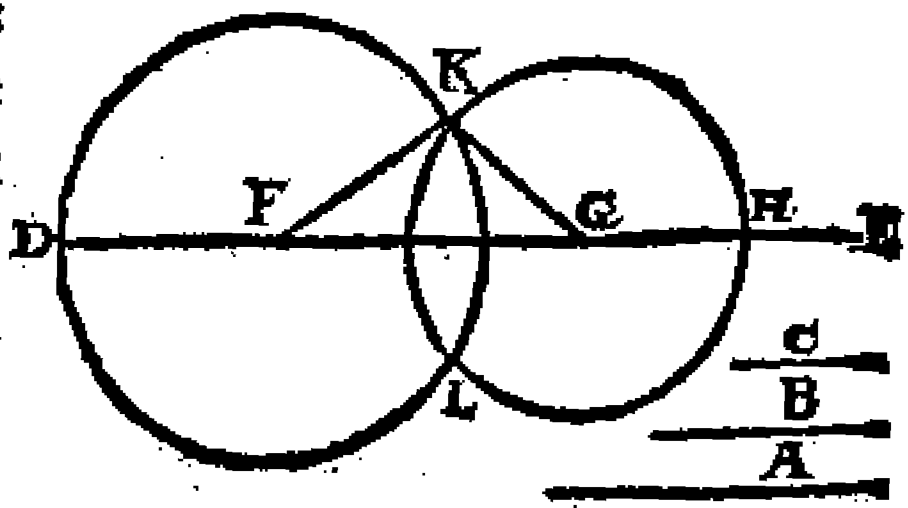
Theorema 14. Propositio 21.



Si super triãguli vno latere, ab extremitatibus duæ rectę lineę, interius cõstitutæ fuerint, hę cõstitutæ reliquis triãguli duobus lateribus minores quidem erunt, minorem verò angulum continebunt.

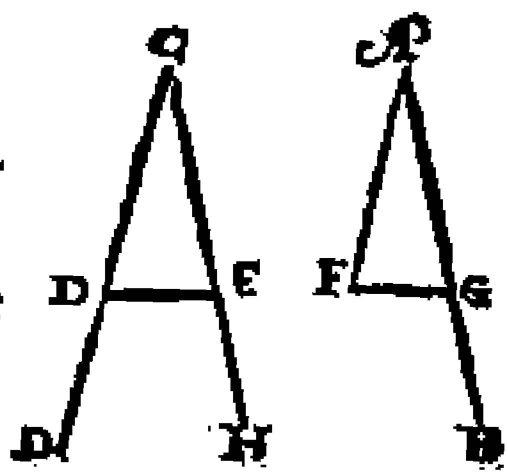
Pro-

Problema 8. Propoſitio 22.



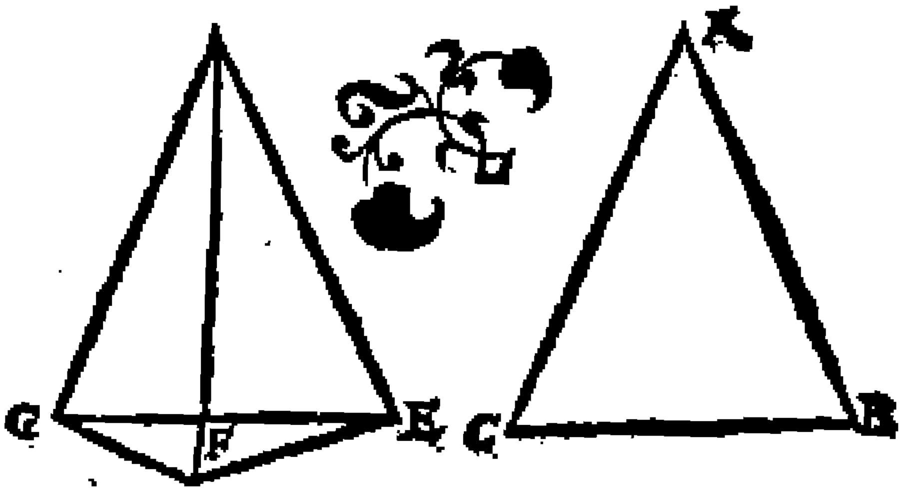
Ex tribus rectis lineis quæ sũt tribus datis rectis lineis æquales, triangulum conſtituere. Oportet autem duas reliqua eſſe maiores omnifariam ſumptas: quoniã vniuſcuiuſq; trianguli duo latera omnifariam ſumpta reliquo ſunt maiora.

Problema 9. Propoſitio 23.



Ad datam rectam lineã datumq́; in ea punctum dato angulo rectilineo æqualem angulum rectilineum conſtituere.

Theorema 15. Propoſitio 24.



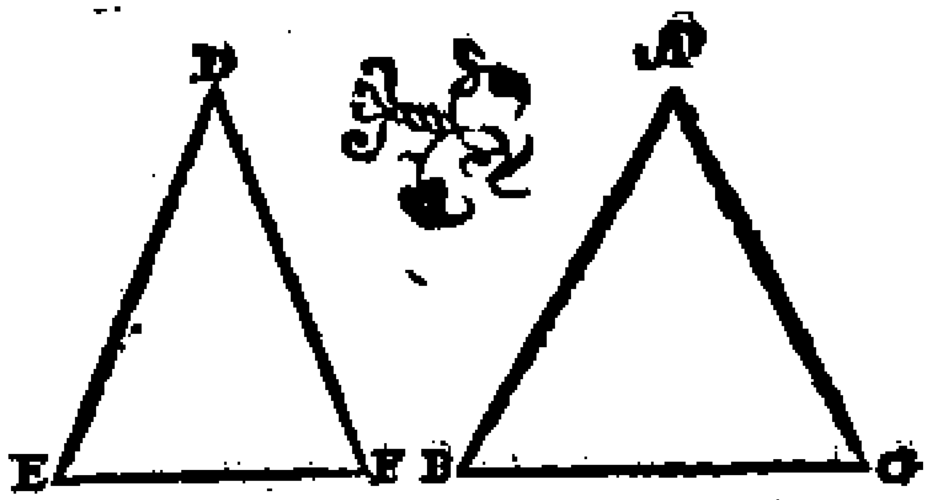
Si duo triãgula duo latera duobus lateribus æqualia habuerint, vtrũq; vtriq;, angulũ verò angu

lo

lo maiorem sub æqualibus rectis lineis cõtentum:& basin basi maiorem habebunt.

Theorema 16. Propositio 25.

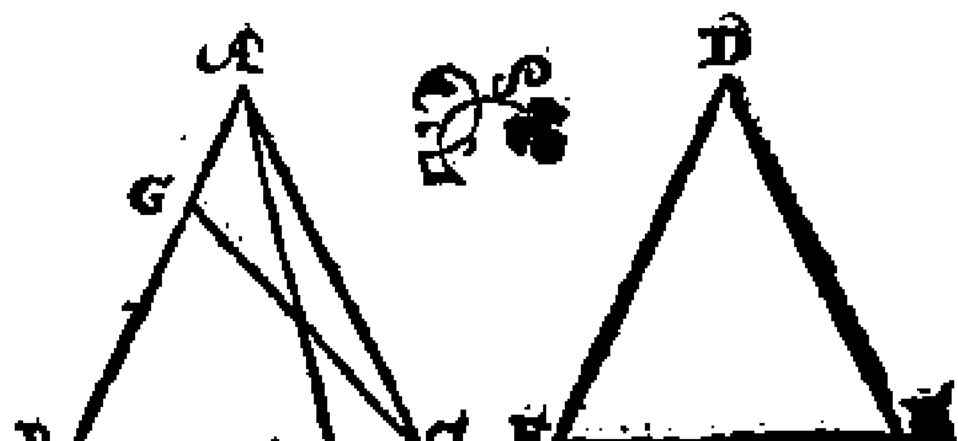
Si duo triangula duo latera duobus lateribus æqualia habuerint, vtrunque vtrique, basin verò basi maioré: & angulum sub ęqualib⁹ rectis li-



neis contentum angulo maioré habebunt.

Theorema 17. Propositio 26.

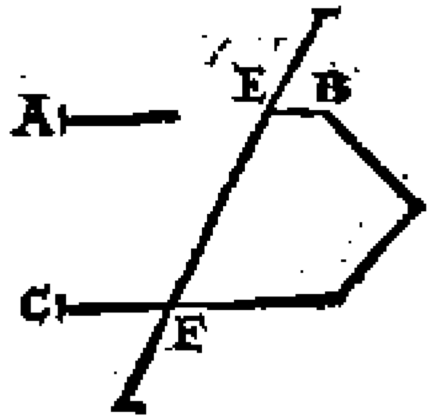
Si duo triangula duos angulos duobus angulis æquales habuerint, vtrũque vtrique, vnumq́; latus vni lateri æquale, siue quod æqualibus adiacet angulis, seu quod vni ęqualium angulorũ subtenditur:& reliqua latera reliquis lateribus ęqualia vtrũq; vtriq;,



& reliquum angulũ reliquo angulo æqualem habebunt.

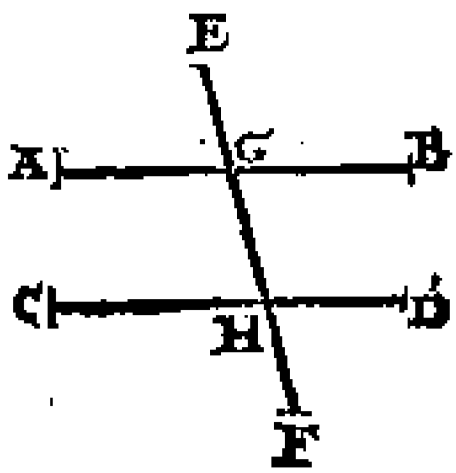
Theore-

Theorema 18. Propositio 27.



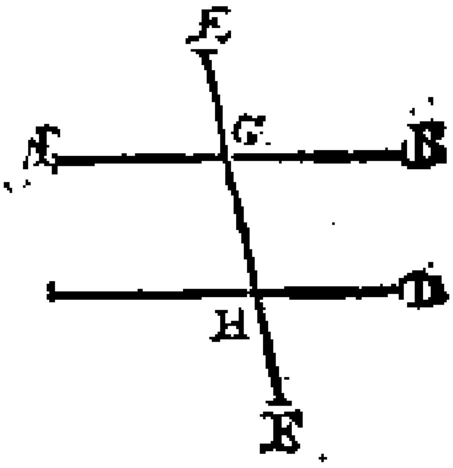
Si in duas rectas lineas recta incidēs linea alternatĩm āgulos ęquales inter se fecerit: parallelæ erunt inter se illæ rectæ lineæ.

Theorema 19. Propositio 28.



Si in duas rectas lineas recta incidens linea externum angulũ interno, & opposito, & ad easdem partes æqualē fecerit, aut internos & ad easdē partes duobus rectis æquales: parallelæ erunt inter se ipsæ rectæ lineæ.

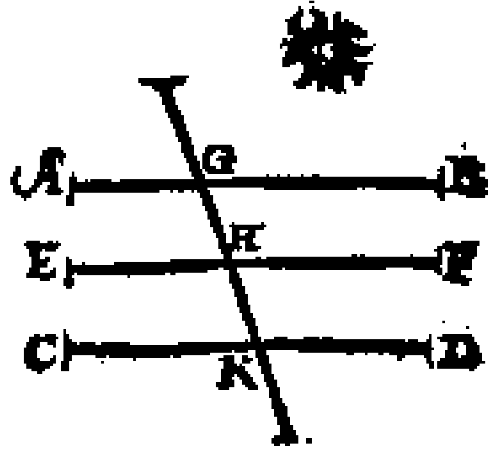
Theorema 20. Propositio 29.



In parallelas rectas lineas recta incidens linea: & alternatim angulos inter se æquales efficit & externum interno & opposito & ad easdem partes æqualem, & internos & ad easdem partes duobus rectis æquales facit.

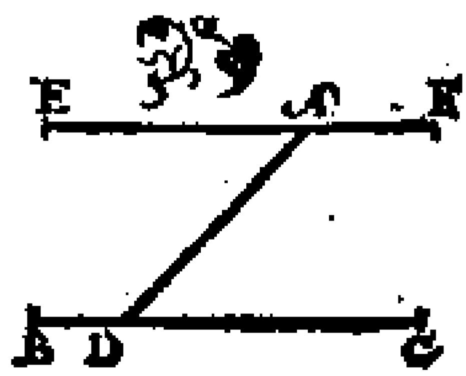
Theore-

Theorema 21. Propoſitio 30.



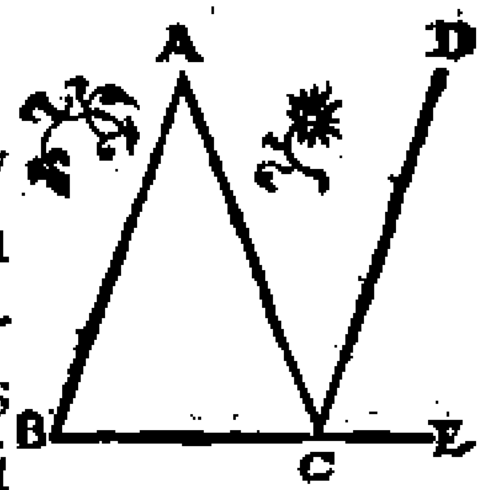
Quæ eidem rectæ lineę, parallelæ, & inter ſe ſunt parallelæ.

Problema 10. Propoſitio 31.



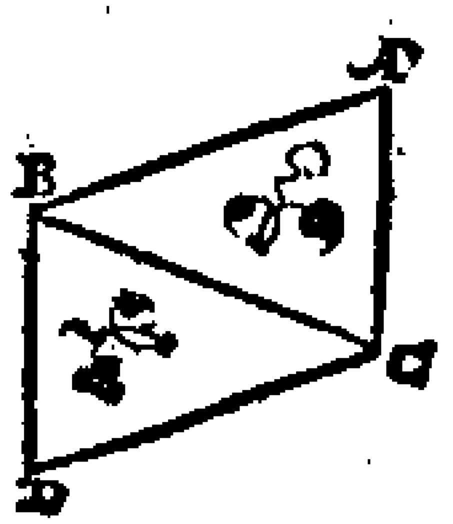
A dato puncto datæ rectæ lineæ parallelam rectam lineam ducere.

Theorema 22. Propoſitio 32.



Cuiuſcunque trianguli vno latere vlterius producto: externus angulꝰ duobus internis & oppoſitis eſt æqualis. Et trianguli tres interni anguli duobus ſunt rectis æquales.

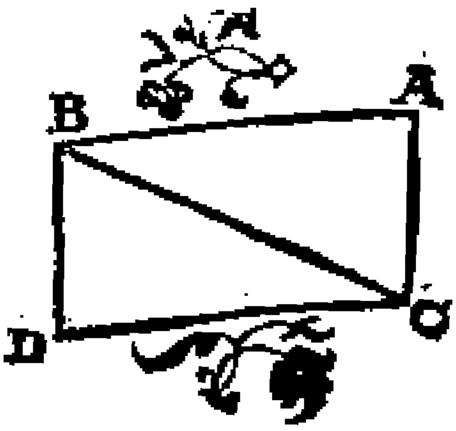
Theorema 23. Propoſitio 33.



Rectæ lineę quę ęquales & parallelas lineas ad partes eaſdem coniungunt, & ipſæ æquales & parallelæ ſunt.

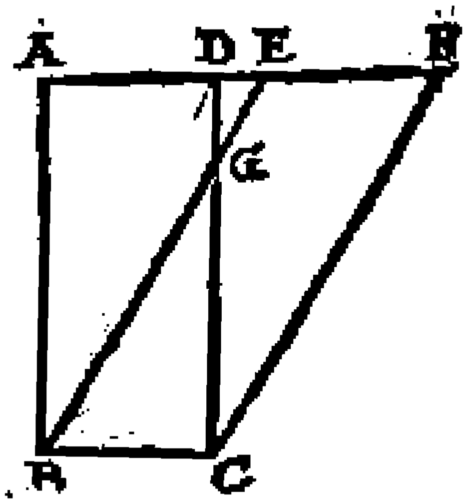
Theore-

Theorema 24. Propositio 34.



Parallelogrammorũ spatiorum æqualia sunt inter se, quæ ex aduerso & latera & anguli: atque illa bifariã secat diameter.

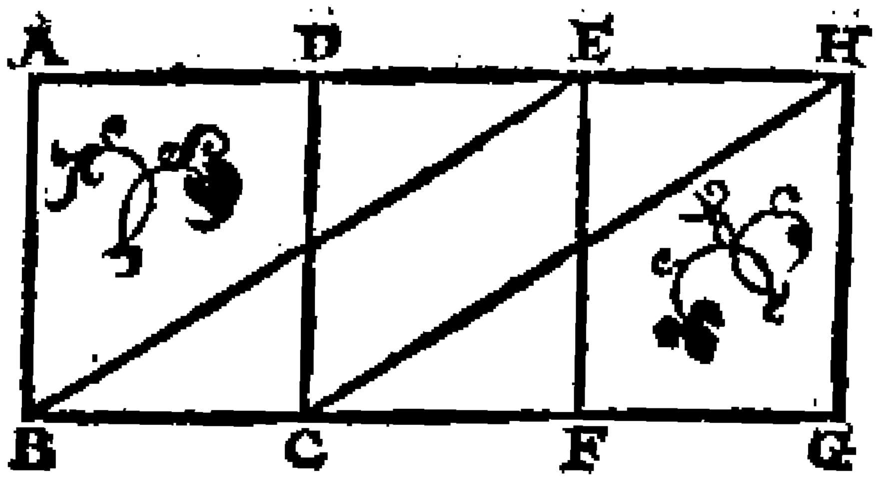
Theorema 25. Propositio 35.



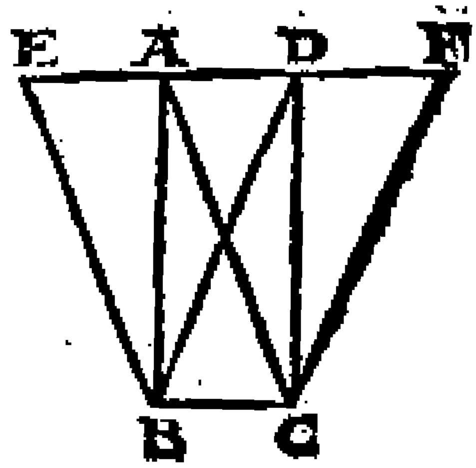
Parallelogramma super eadem basi, & in eisdem parallelis cõstituta, inter se sunt æqualia.

Theorema 26. Propositio 36.

Parallelogrãma super æqualibus basib⁹ & in eisdẽ parallelis cõstituta inter se sunt æqualia.



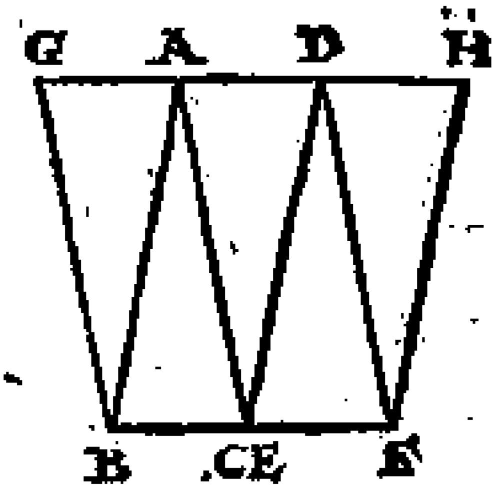
Theorema 27. Prositio 37.



Triãgula super eadẽ basi cõstituta, & in eisdẽ parallelis, inter se sunt ęqualia.

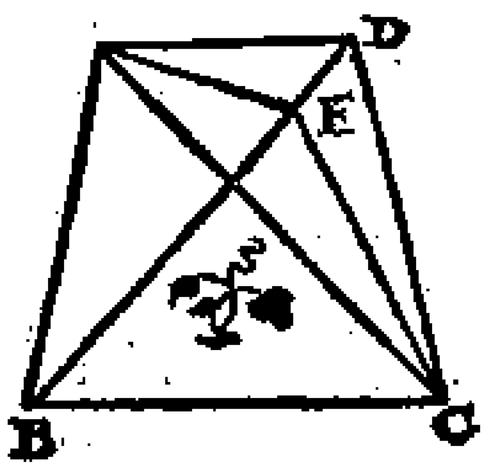
Theore-

Theorema 28. Propositio 38.



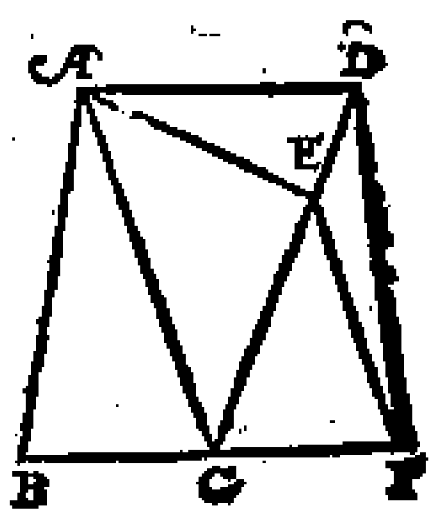
Triangula super æquali bus basibus cõstituta & in eisdem parallelis, inter se sunt æqualia.

Theorema 29. Propositio 39.



Triãgula æqualia super eadem basi, & ad easdem partes cõstituta: & in eisdem sunt parallelis.

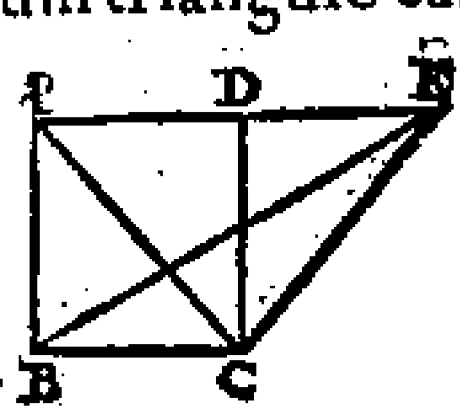
Theorema 30. Propositio 40.



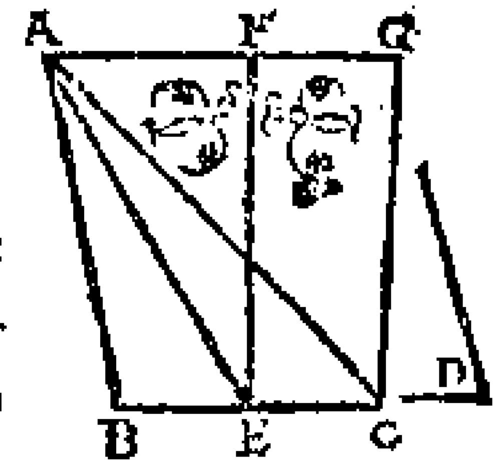
Triãgula æqualia super æqualibus basibus, & ad easdem partes cõstituta, & in eisdẽ sunt parallelis.

Theorema 31. Propositio 41.

Si parallelogrammum cum triangulo eandem basin habuerit, nõ eisdemq; fuerit parallelis, duplum erit parallelogrammum ipsius triãguli.

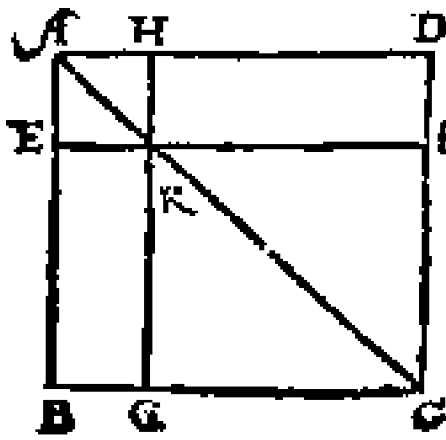


Problema 11. Propositio 42.



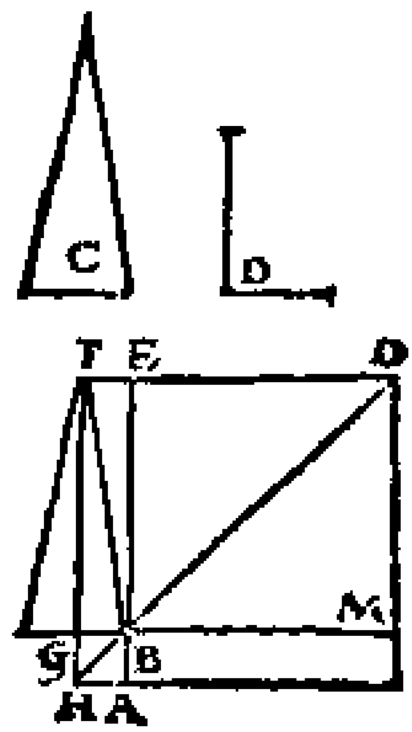
Dato triangulo æquale parallelogrammũ constituere in dato angulo rectilineo.

Theorema 32. Propositio 43.



In omni parallelogrammo, complementa eorũ quę circa diametrũ sunt parallelogrammorum, inter se sunt æqualia.

Problema 12. Propositio 44.

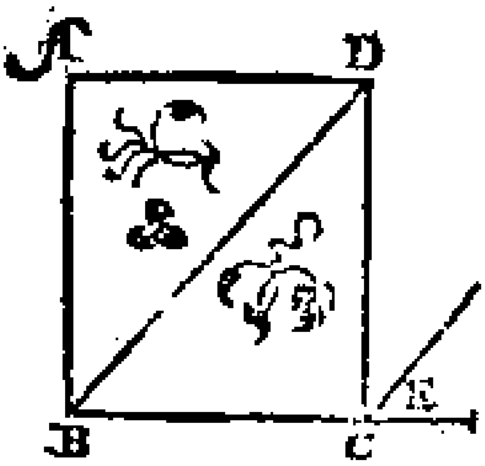


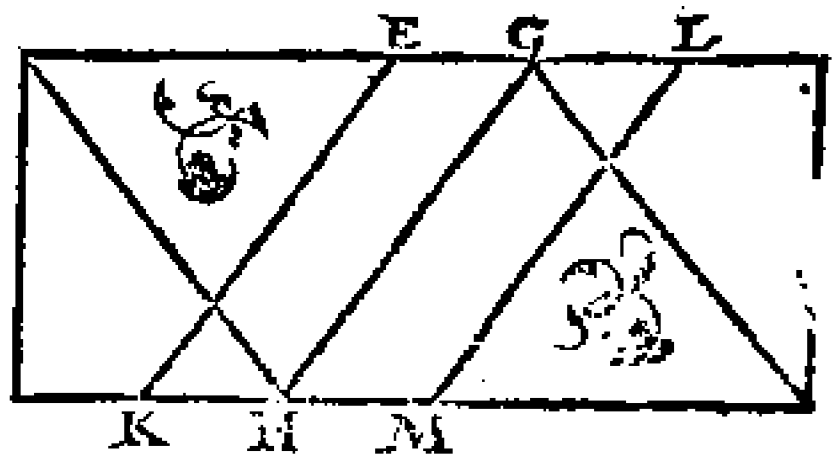
Ad datam rectam lineã, dato triangulo æquale parallelogrammum applicare in dato angulo rectilineo.

Problema 13. Propopotio 45.

Dato rectilineo æquale parallelogrammũ consti-

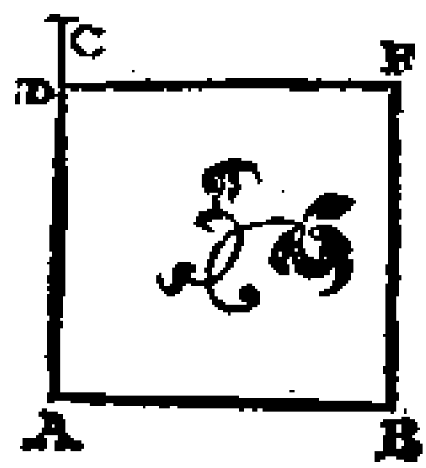
constituere in dato angulo rectilineo.





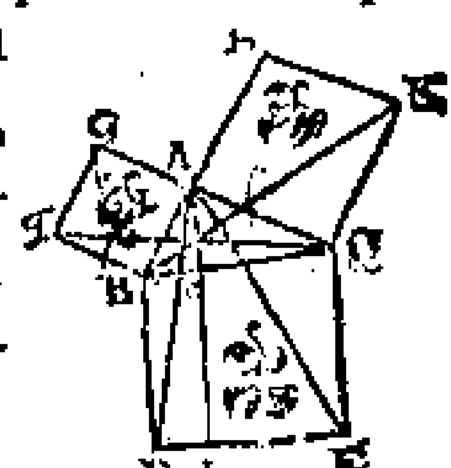
Problema 14. Propositio 46.

A data recta linea quadratum describere.



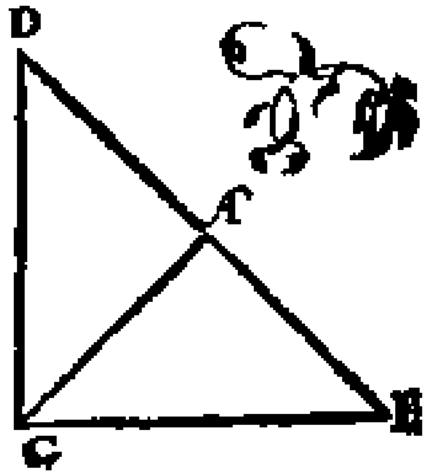
Theorema 33. Propositio 47.

In rectangulis triãgulis, quadratum quod à latere rectum angulũ subtendẽte describitur, æquale est eis, quę à lateribus rectum angulum continentibus describũtur, quadratis.



Theorema 34. Propositio 48.

Si quadratum quod ab vno laterum triãguli deſcribitur, æquale ſit eis quæ à reliquis triãguli lateribus deſcribũtur, quadratis: angulus comprehenſus ſub reliquis duobus trianguli lateribus, rectus eſt.



FINIS ELEMENTI I.

EVCLI-

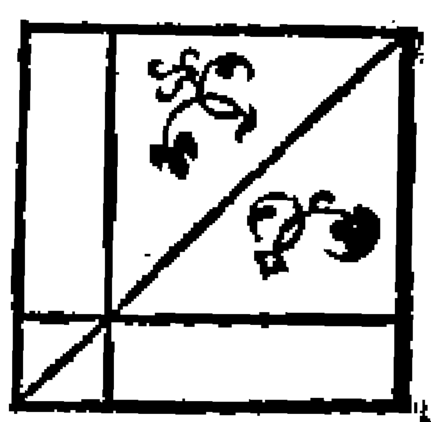
EVCLIDIS ELEMENTVM SECVNDVM.

DEFINITIONES.

1

OMne parallelogrammum rectangulum contineri dicitur sub rectis duabus lineis, quæ rectum comprehendunt angulum.

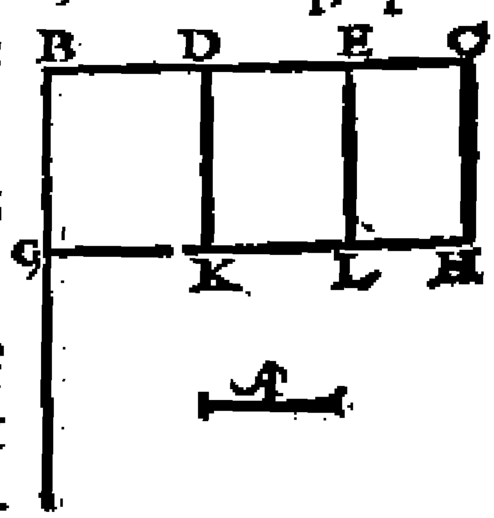
2



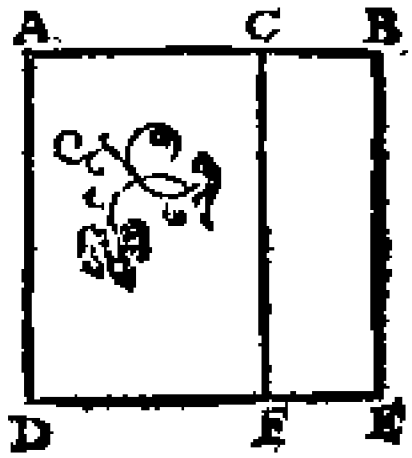
In omni parallelogrammo spatio, vnumquodlibet eorũ, quæ circa diametrum illius sunt parallelogrammorum, cũ duobus complementis, Gnomo vocetur.

Theorema 1. Propositio 1.

Si fuerint duæ rectæ lineæ, seceturq; ipsarũ altera in quotcunque segmenta: rectangulum comprehensum sub illis duab⁹ rectis lineis, æquale est eis rectangulis, quę sub insecta & quolibet segmentorum comprehenduntur.



Theorema 2. Propositio 2.

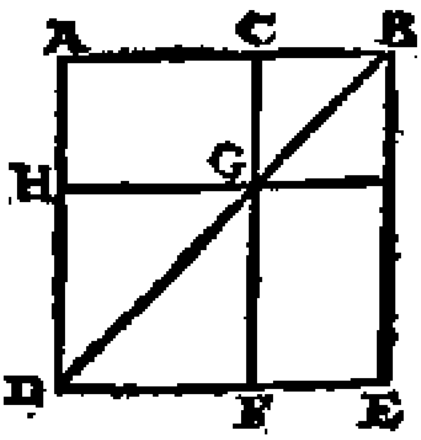


Si recta linea secta sit vtcunque, rectãgula quæ sub tota & quolibet segmentorum cõprehenduntur æqualia sunt ei, quod à tota sit, quadrato.

Theorema 3. Propositio 3.

Si recta linea secta sit vtcunque, rectangulum sub tota & vno segmẽtorum comprehensum, equale est & isti quod sub segmentis cõprehenditur rectãgulo, & illi, quod à prædicto segmento describitur, quadrato.

Theorema 4. Propositio 4.

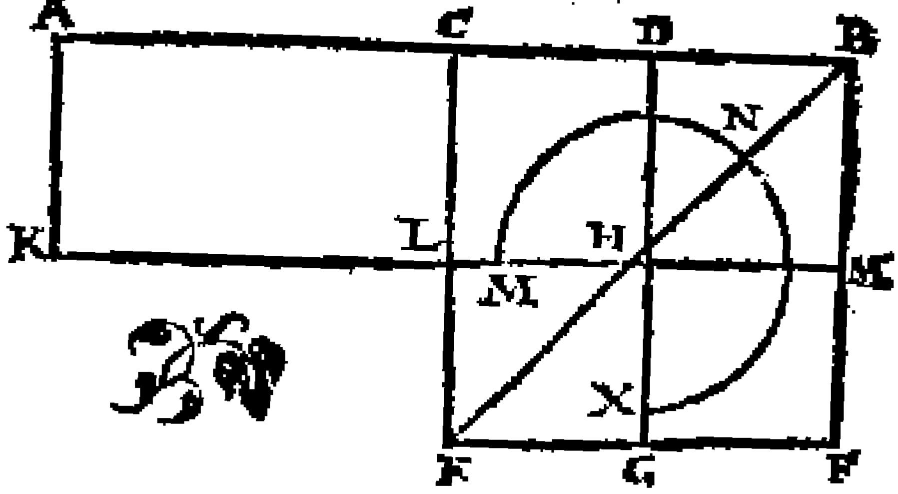


Si recta linea secta sit vtcunque: quadratũ quod à tota describitur, equale est & illis quæ à segmẽtis describũtur quadratis, & ei quod bis sub segmentis compreheditur rectangulo.

Theorema 5. Propositio 5.

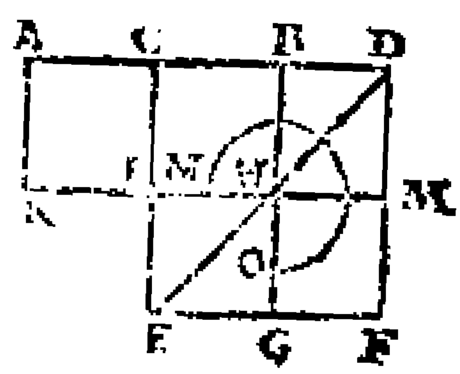
Si recta linea secetur in æqualia & non æqualia: rectangulum sub inæqualibus segmentis

mentis totius comprehẽſũ, vnà cum quadrato quod ab intermedia ſectionum, æquale eſt ei quod à dimidia deſcribitur, quadrato.



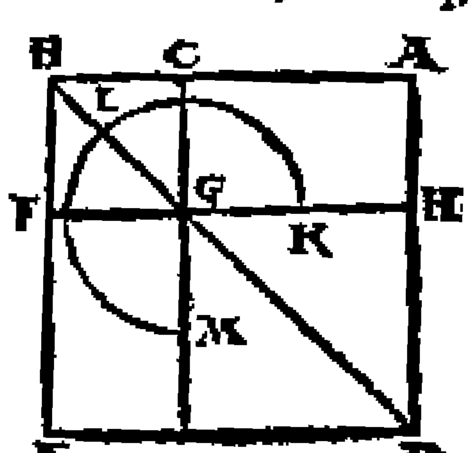
Theorema 6. Propoſitio 6.

Si recta linea bifariam ſecetur, & illi recta quædam linea in rectum adijciatur, rectãgulum comprehẽſum ſub tota cum adiecta & adiecta ſimul cum quadrato à dimidia, æquale eſt quadrato à linea, quæ tũ ex dimidia, tum ex adiecta componitur, tanquam ab vna deſcripto.



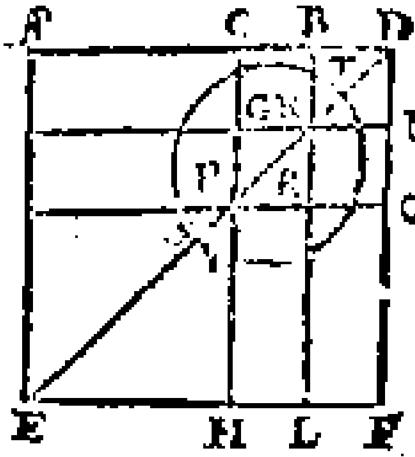
Theorema 7. Propoſitio 7.

Si recta linea ſecetur vtcunque: quod à tota, quodq́ue ab vno ſegmentorum, vtraq; ſimul quadrata, æqualia ſunt & illi quod bis ſub tota & dicto ſegmento comprehenditur, rectãgulo, & illi q̃ à reliquo ſegmento fit, quadrato.

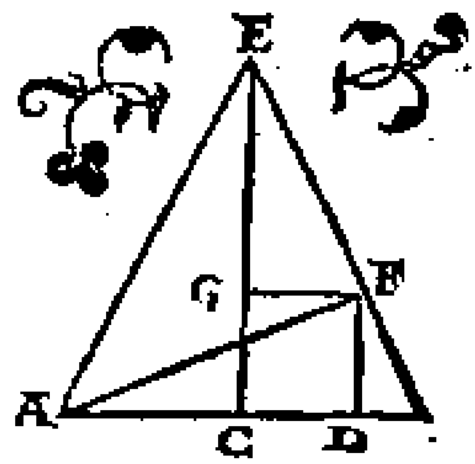


Theorema 8. Propositio 8.

Si recta linea secetur vtcunque rectangulum quater comprehẽsum sub tota & vno segmentorum, cũ eo quod à reliquo segmento fit, quadrato, æquale est ei quod à tota & dicto segmento, tanquam ab vna linea describitur, quadrato.

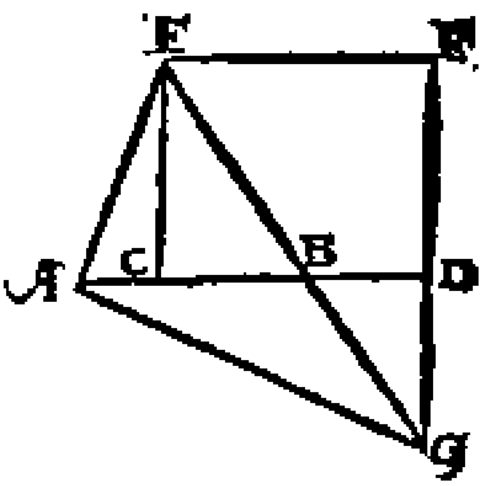


Theorema 9. Propositio 9.



Si recta linea secetur in æqualia & non æqualia: quadrata quæ ab inæqualibus totius segmẽtis fiunt, duplicia sunt & eius quod à dimidia, & eius quod ab intermedia sectionum fit, quadratorum.

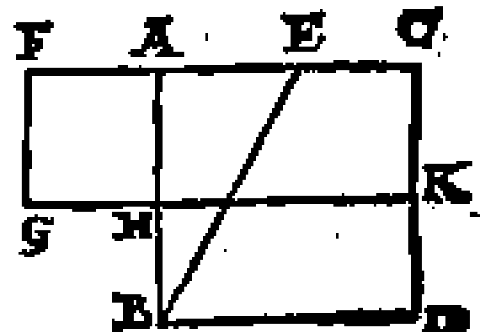
Theorema 10. Propositio 10.



Si recta linea secetur bifariam, adijciatur autem ei in rectum quæpiam recta linea: quod à tota cũ adiuncta, & quod ab adiuncta, vtraq; simul quadrata, duplicia sunt, & eius quod à dimidia, & eius quod à cõposita ex dimi-

dimidia & adiuncta, tãquam ab vna descriptum sit, quadratorum.

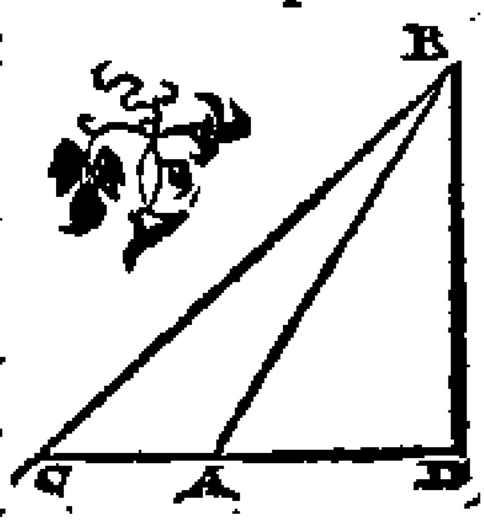
Problema I. Propositio II.



Datam rectam lineã secare, vt comprehensum sub tota & altero segmẽtorum rectangulum, æquale sit ei quod à reliquo segmento fit, quadrato.

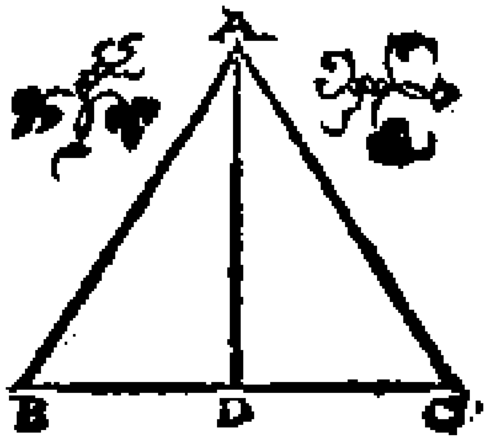
Theorema II. Propositio 12.

In amblygonijs triangulis, quadratũ quod fit à latere angulum obtusum subtendẽte, maius est quadratis, quæ fiunt à lateribus obtusum angulum comprehendentibus, pro quantitate rectanguli bis comprehẽsi & ab vno laterũ quæ sunt circa obtusum angulũ, in quod cùm protractum fuerit, cadit perpendicularis, & ab assumpta exterius linea sub perpendiculari prope angulũ obtusum.



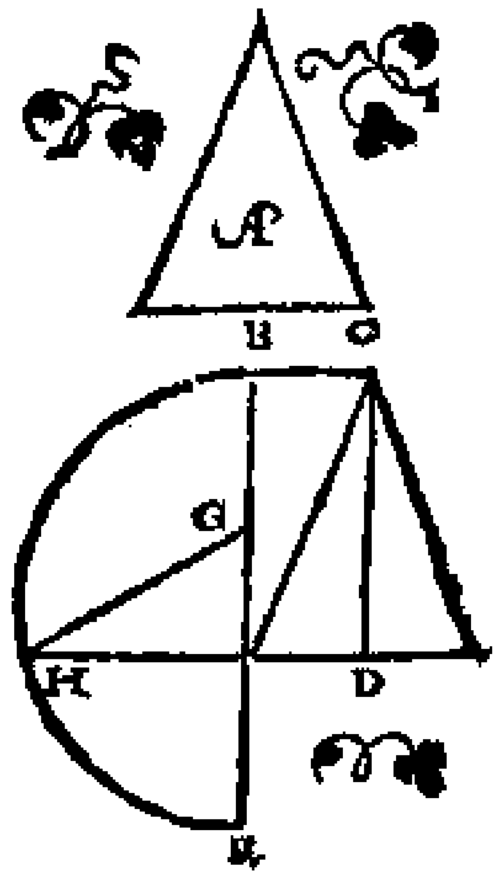
Theorema 12. Propositio 13.

In oxygonijs triangulis, quadratũ à latere angulum acutum subtendente, minus est quadratis quæ fiunt à lateribus acutũ angulum cõprehendentibus, pro quantitate rectanguli bis comprehensi, & ab vno laterum, quæ sunt circa acutum angulum, in quod perpẽdicularis cadit, & ab assumpta interius linea sub perpendiculari prope acutũ angulum.



Problema 2. Propositio 14.

Dato rectilineo æquale quadratũ constituere.



ELEMENTI II FINIS.

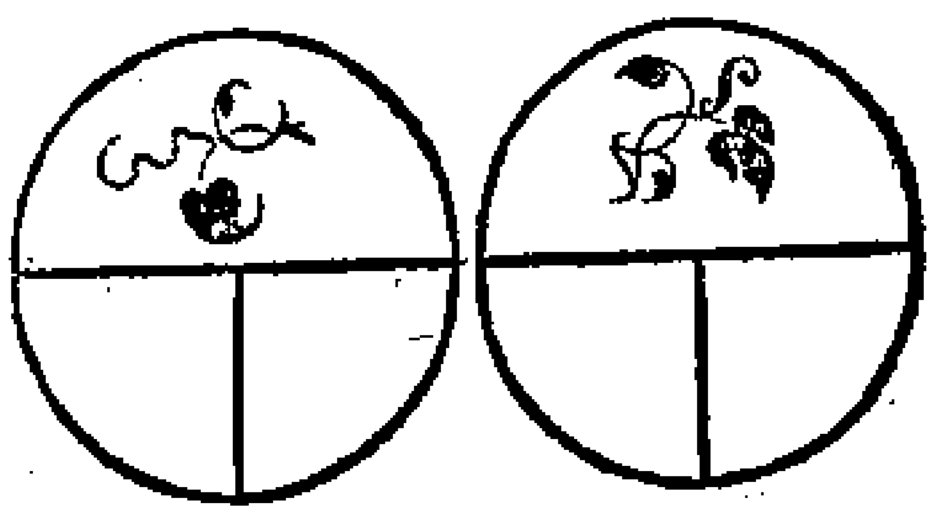
EVCLI-

EVCLIDIS ELEMENTVM TERTIVM.

DEFINITIONES.

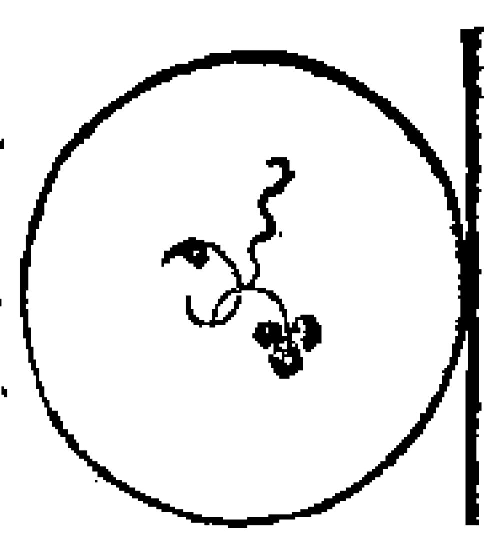
1

Aequales circuli sunt, quorũ diametri sũt ęquales vel quorũ quæ ex centris rectæ lineę sunt æquales.



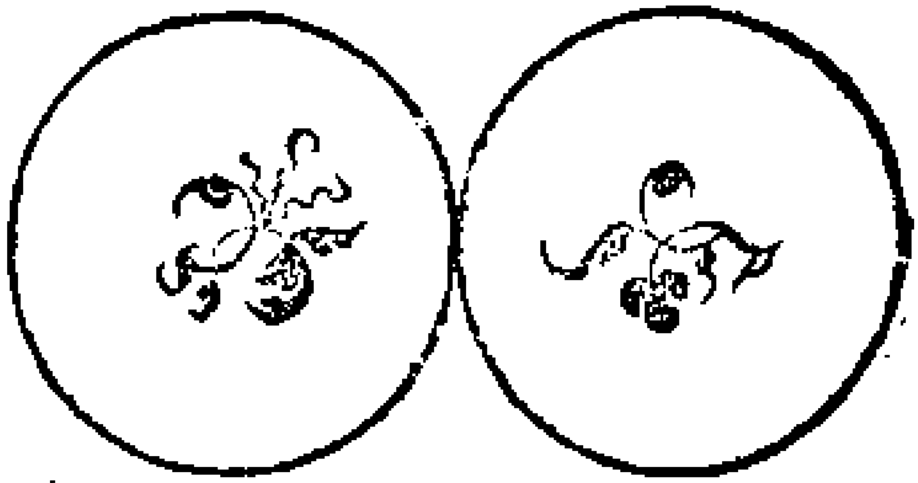
2

Recta linea circulũ tangere dicitur, quæ cùm circulum tangat, si producatur, circulum non secat.



3

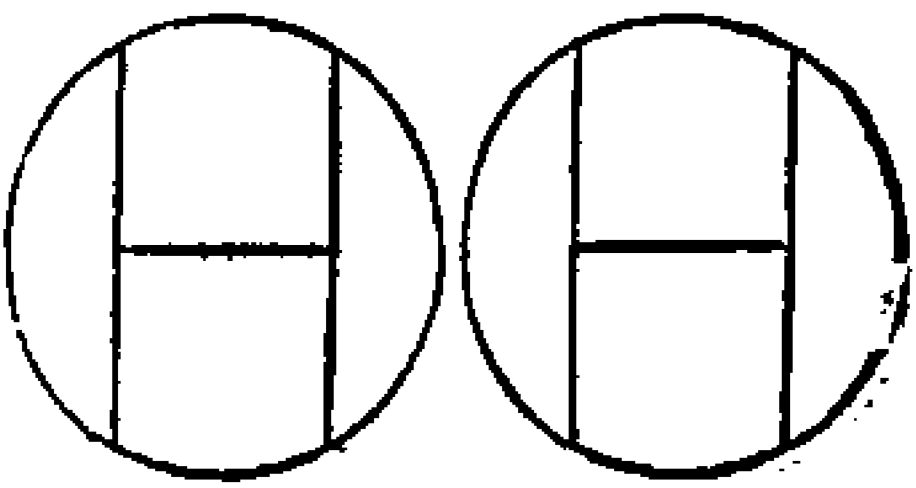
Circuli sese mutuò tãgere dicũtur: qui sese mutuo



tangentes, sese mutuò non secant.

4

In circulo æqualiter distare à centro rectæ lineæ dicuntur, cùm perpendiculares, quæ à cẽtro in ipsas ducuntur, sunt æquales. Lõgius autem abesse illa dicitur in quã maior ꝑpẽdicularis cadit.



5

Segmentũ circuli est, figura quæ sub recta linea & circuli peripheria comprehenditur.

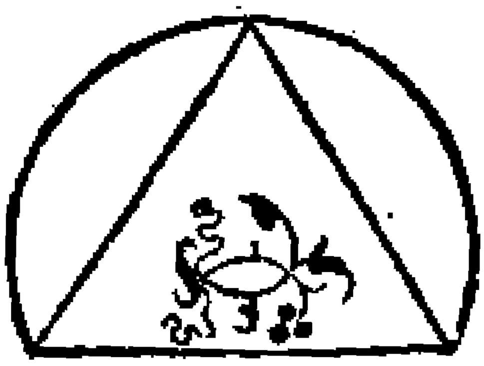
6

Segmenti autẽ angulus est, qui sub recta linea

nea & circuli peripheria comprehẽditur.

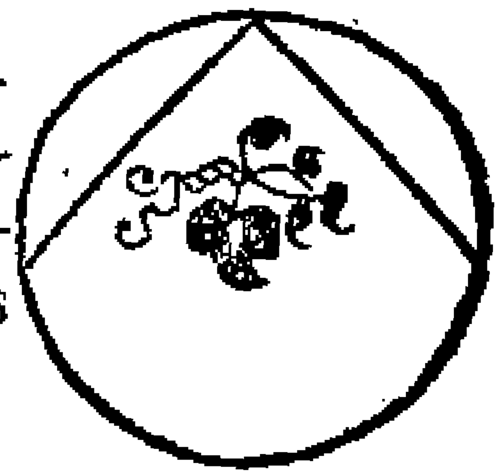
7

In segmento autem angulus est, cùm in segmenti peripheria sumptũ fuerit quodpiam pũctum, & ab illo in terminos rectæ ei⁹ lineę, quę segmenti basis est, adiunctę fuerint rectę lineę: is, inquam, angulus ab adiunctis illis lineis comprehensus.



8

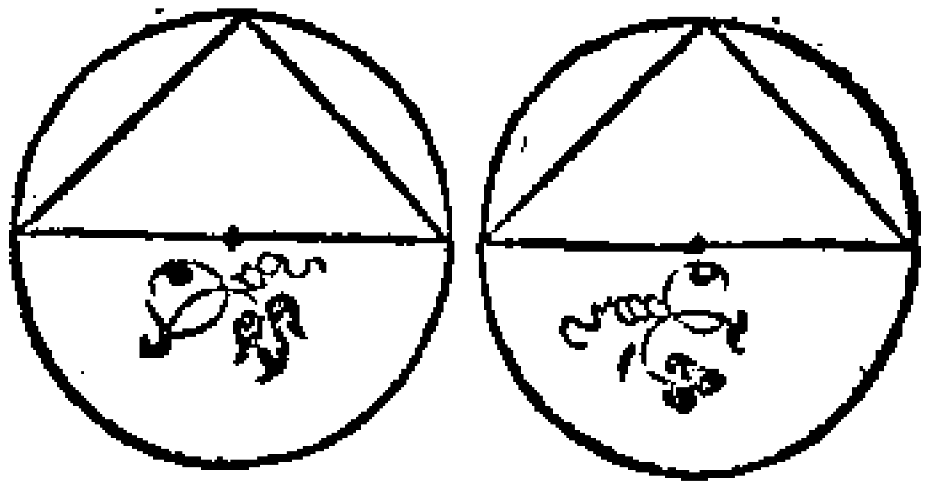
Cùm verò comprehendentes angulum rectę lineæ aliquã assumunt peripheriam, illi angulus insistere dicitur.



Sector autem circuli est cùm ad ipsius circuli cẽtrum constitutus fuerit angulus, cõprehensa nimirum figura, & à rectis lineis angulũ continentibus, & à peripheria ab illis assumpta.

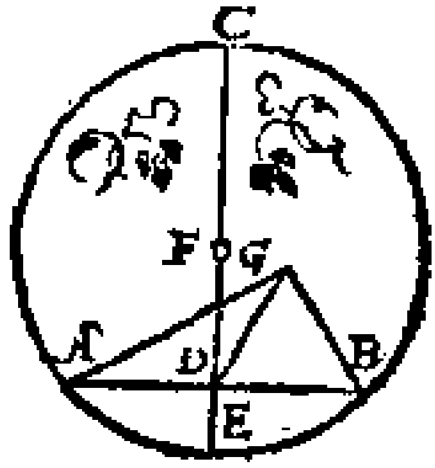
10

Similia circuli segmenta sunt, quę angulos capiunt capiũt ęquales aut in q b⁹ anguli inter se sunt ęquales



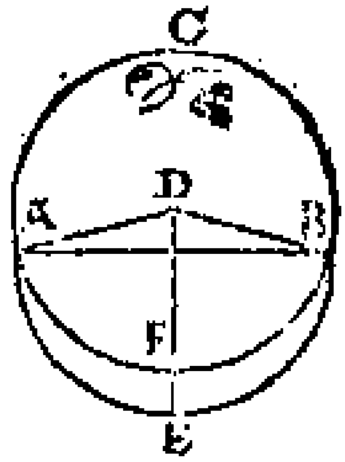
Problema 1. Propositio 1.

Dati circuli cẽtrum reperire.



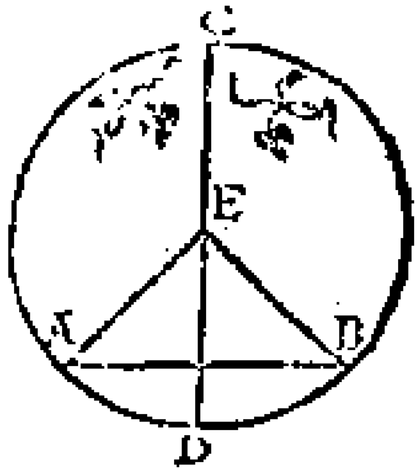
Theorema 1. Propositio 2.

Si in circuli peripheria duo quælibet pũcta accepta fuerint, recta linea quæ ad ipsa puncta adiugitur, intra circulum cadet.



Theorema 2. Propositio 3.

Si in circulo rectá quædam linea per centrum extensa quandam non per centrum extensam bifariam secet: & ad angulos rectos ipsam secabit. Et si ad angulos rectos eam secet, bifariam quoque eam secabit.



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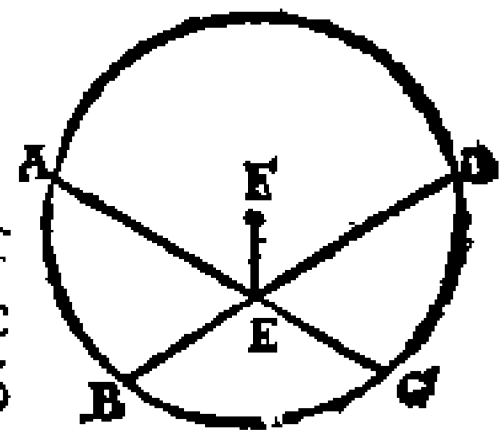
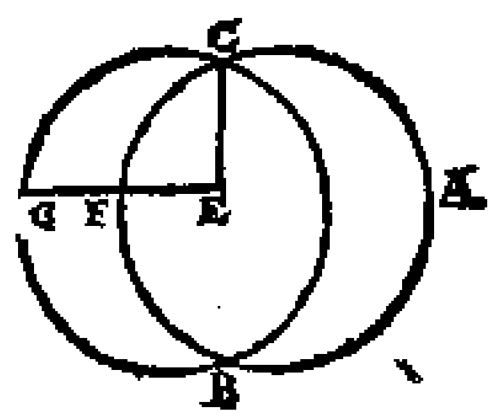
Theorema 3. Propositio 4.



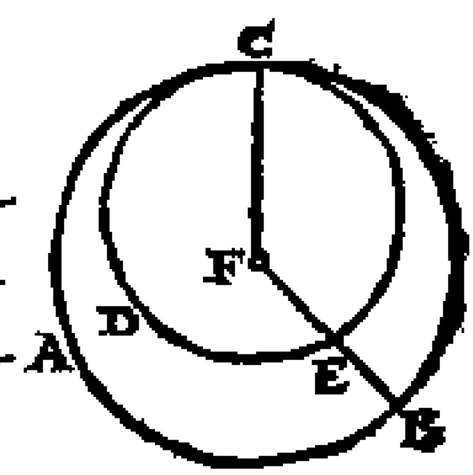
Si in circulo duę rectę lineæ sese mutuò secent non per centrum extẽsæ sese mutuò bifariam nõ secabunt.

Theorema 4. Propositio 5.



Si duo circuli sese mutuò secent, non erit illorum idem centrum.

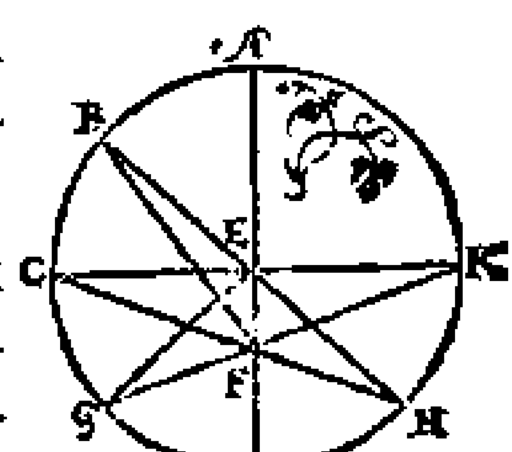
Theorema 5. Propositio 6.



Si duo circuli sese mutuò interius tangant, eorum non erit idem centrum.

Theorema 6. Propositio 7.

Si in diametro circuli quodpiam sumatur punctum, quod circuli cẽtrum non sit, ab eoque pũcto in circulũ quædam rectæ lineæ cadant: maxima quidem erit ea in qua centrũ, minima verò reliqua: aliarum verò propinquior illi quæ per centrũ du-

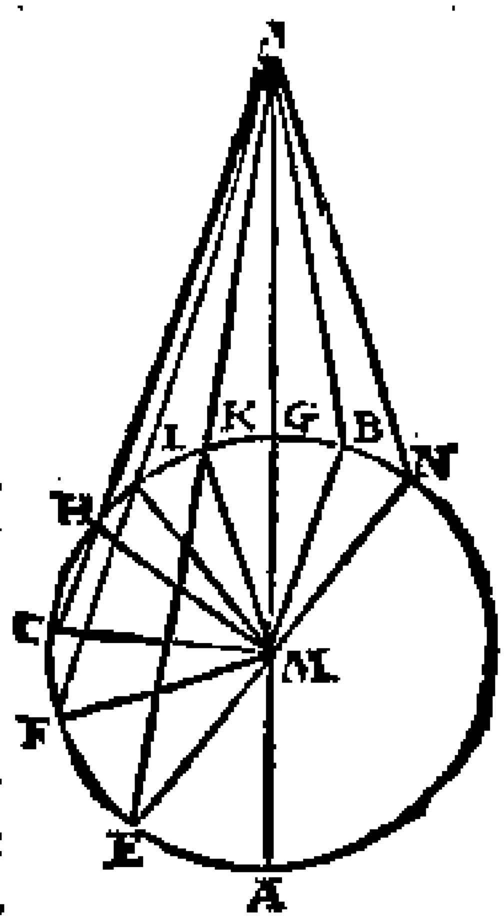


citur

citur, remotiore ſemper maior eſt. Duę autem ſolùm rectæ lineæ æquales ab eodem puncto in circulum cadũt ad vtraſque partes minimæ.

Theorema 7. Propoſitio 8.

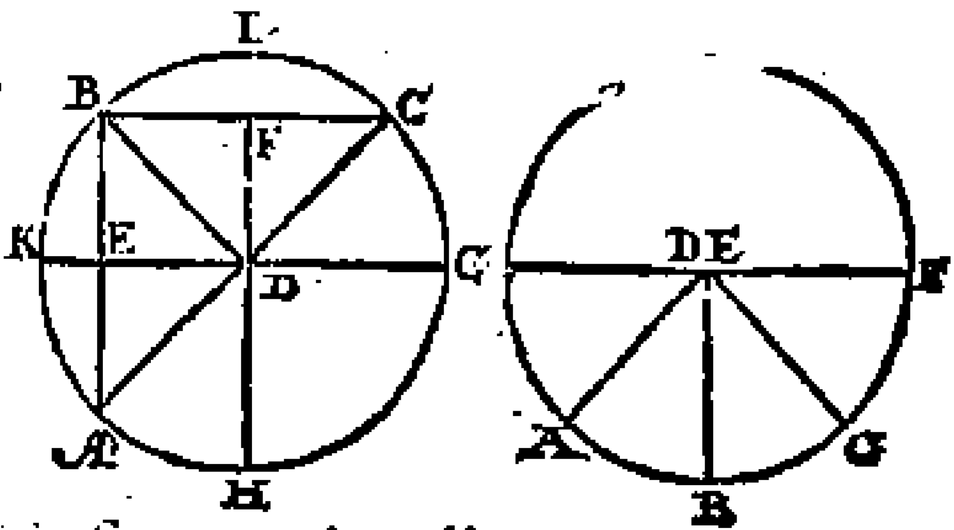
Si extra circulum ſumatur punctũ quodpiam, ab eoque puncto ad circulum deducantur rectæ quædam lineæ, quarum vna quidem per centrum protẽdatur, reliquæ verò vt libet: in cauam peripheriam cadẽtium rectarum linearum maxima quidem eſt illa, quæ per centrum ducitur: aliarum autem propinquior ei, quę per centrum tranſit, remotiore ſemper maior eſt: in cõuexam verò peripheriã cadentium rectarũ linearum minima quidẽ eſt illa, quæ inter punctum & diametrum interponitur: aliarum autem, ea quæ propinquior eſt minimæ, remotiore ſemper minor eſt. Duæ autem tantùm rectæ lineæ æquales ab eo puncto in ipſum circulum cadũt, ad vtraſque partes minimæ.



Theore-

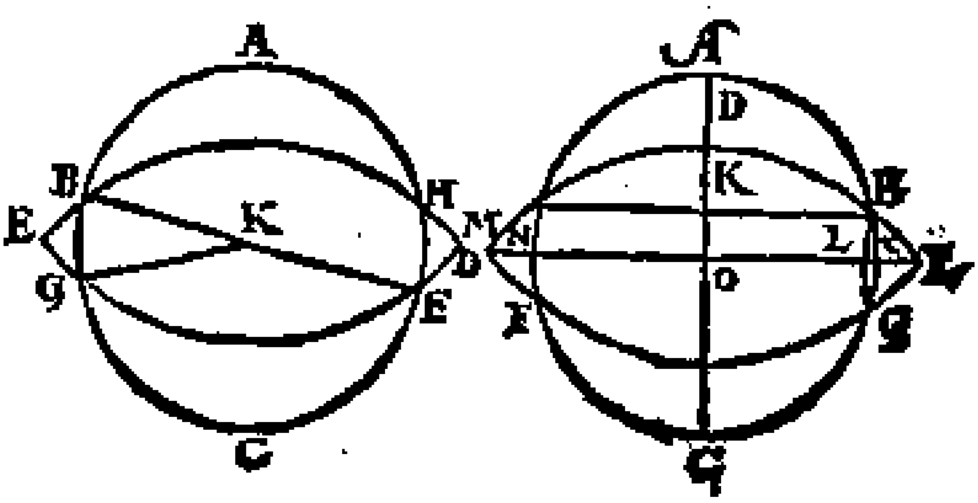
Theorema 8. Propositio 9.

Si in circulo acceptum fuerit pũctum aliquod, & ab eo puncto ad circulum cadant plures quã duę rectę lineæ, æquales, acceptũ punctũ centrum ipsius est circuli.



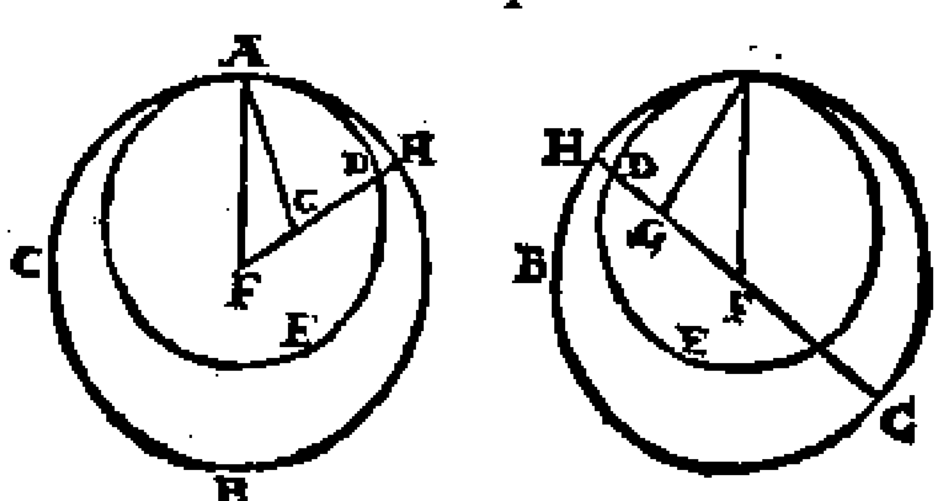
Theorema 9. Propositio 10.

Circulus circulum in pluribus quàm duob9 pũctis non secat.



Theorema 10. Propositio 11.

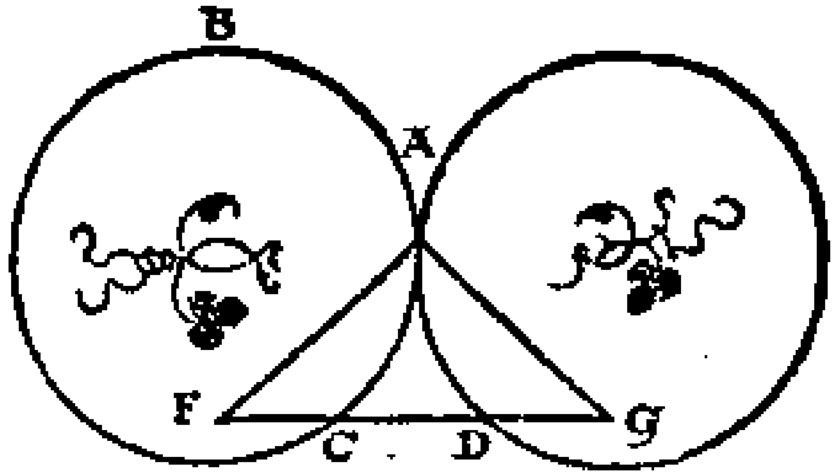
Si duo circuli sese intus cõtingãt, atque accepta



fuerint eorum centra, ad eorum cẽtra adiuncta recta linea & producta in cõtactum circulorum cadet.

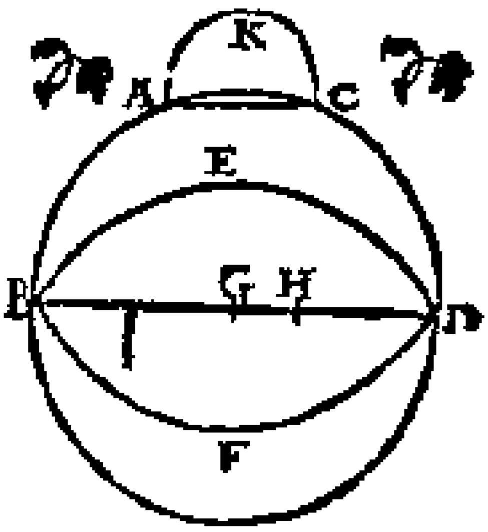
Theorema 11. Propoſitio 12.

Si duo circuli ſeſe exterius cõtingãt, linea recta q̃ ad cẽtra eorũ adiũgitur, ꝑ cõtactũ illũ trãſibit.



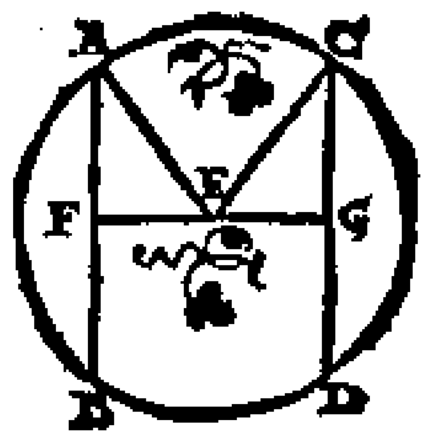
Theorema 12. Propoſitio 13.

Circulus circulum non tangit in pluribus punctis, quã vno, ſiue intus ſiue extra tangat.



Theorema 13. Propoſitio 14.

In circulo æquales rectæ lineæ æqualiter diſtant à centro. Et quæ ęqualiter diſtant à centro, æquales ſunt inter ſe.



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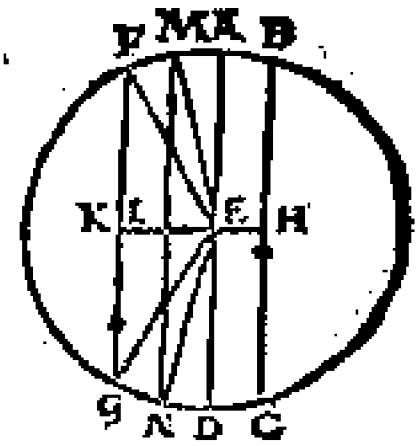
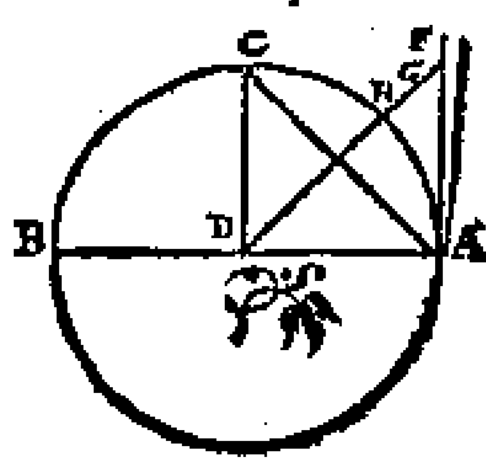
Theorema 14. Propositio 15.



In circulo maxima quidem linea est diameter: aliarum autem propinquior centro, remotiore semper maior.

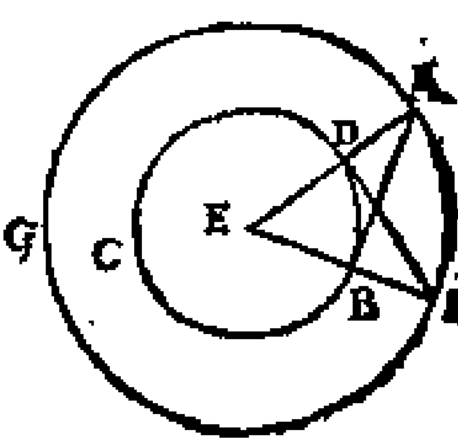
Theorema 15. Propositio 16.

Quæ ab extremitate diametri cuiusq; circuli ad angulos rectos ducitur, extra ipsũ circulum cadet, & in locum inter ipsam rectam lineam & peripheriam comprehensum, altera recta linea nõ cadet Et semicirculi quidem angulus quouis angulo acuto rectilineo maior est, reliquus autem minor,

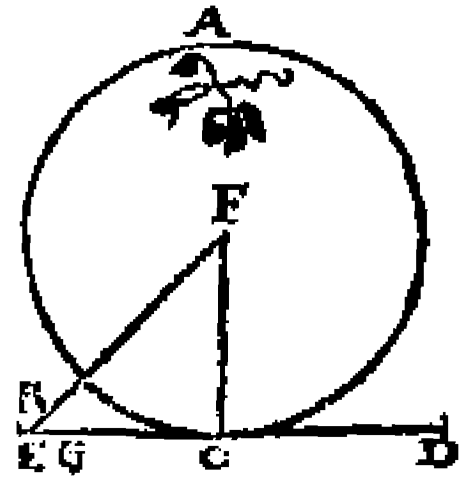


Problema 2. Propositio 17.

A dato puncto rectam lineam ducere, quæ datum tangat circulum.

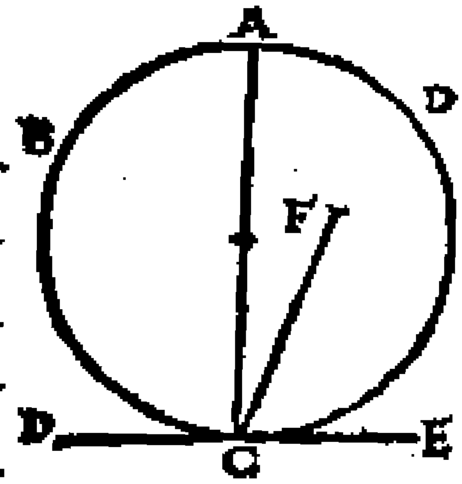


Theorema 16. Propositio 18.



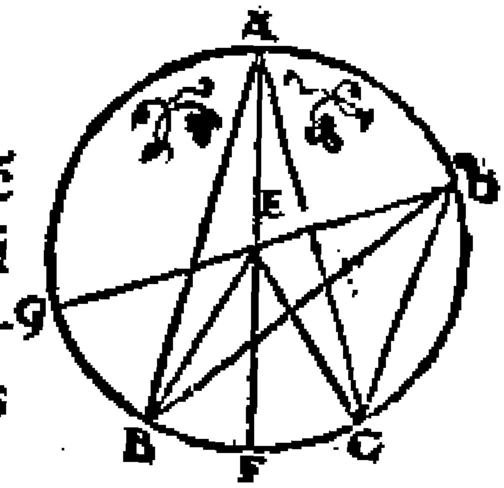
Si circulum tangat recta quæpiam linea, à centro autem ad cõtactum adiungatur recta quædam linea: quæ adiũcta fuerit ad ipsam contingentem perpendicularis erit.

Theorema 17. Propositio 19.



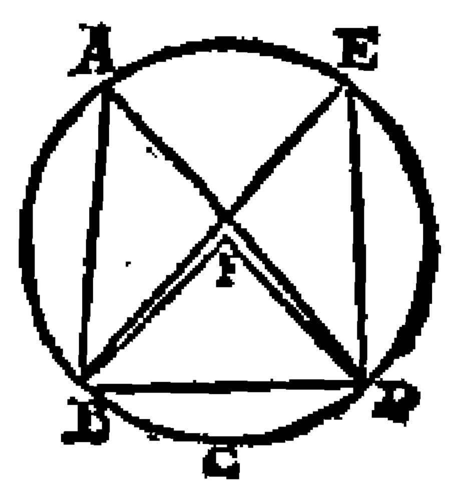
Si circulum tetigerit recta quæpiam linea, à contactu autem recta linea ad angulos rectos ipsi tãgenti excitetur, in excitata erit centrum circuli.

Theorema 18. Propositio 20.



In circulo angulus ad cẽtrum duplex est anguli ad peripheriam, cũ fuerit eadẽ peripheria basis angulorum.

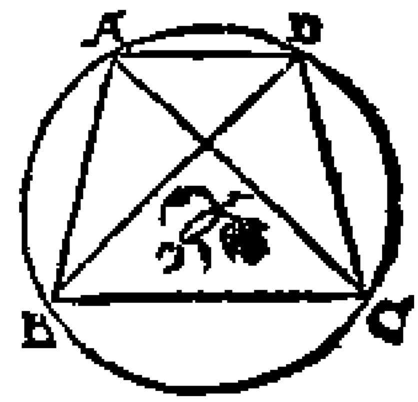
Theorema 19. Propositio 21.



In circulo, qui in eodem segmẽto sunt anguli, sunt inter se æquales.

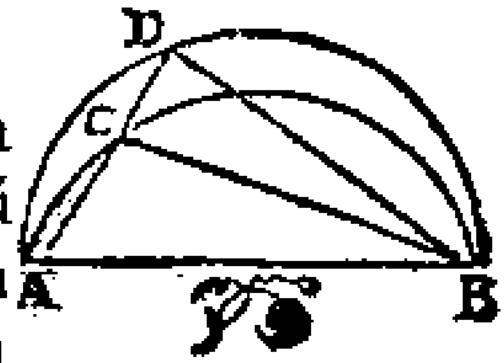
Theore-

Theorema 20. Propositio 22.



Quadrilaterorum in circulis descriptorũ anguli qui ex aduerso, duob⁹ rectis sunt æquales.

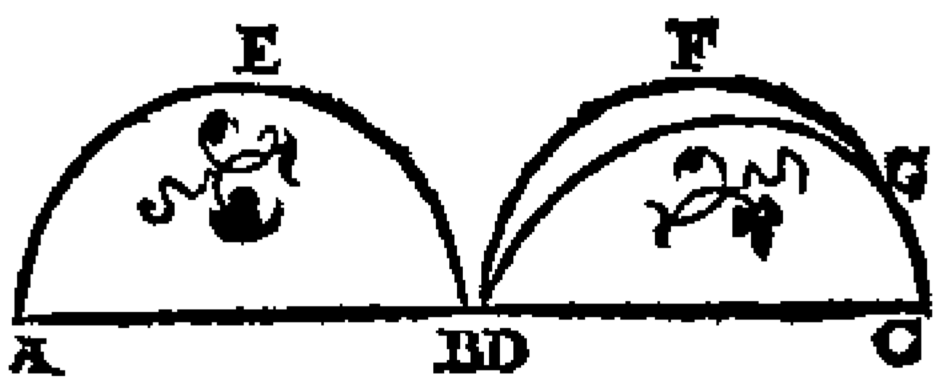
Theorema 21. Propositio 23.



Super eadem recta linea duo segmenta circulorũ similia & inæqualia non constituẽtur ad easdem partes.

Theorema 22. Propositio 24.

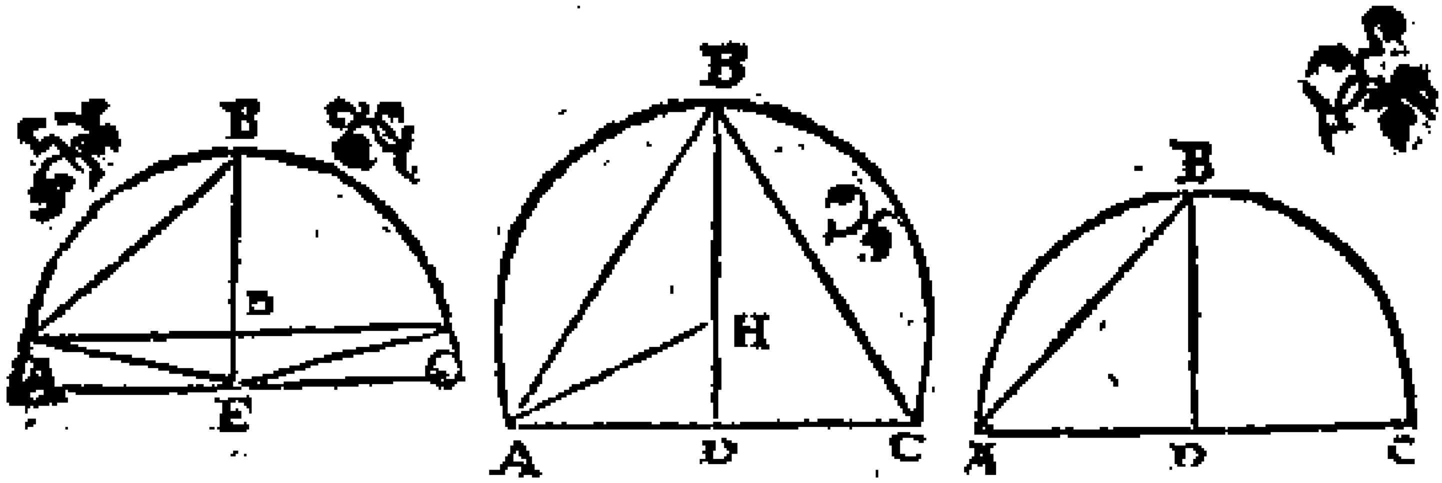
Super ę qualib⁹ rectis li neis similia circùlo rum segmenta sunt inter se æqualia.



Problema 3. Propositio 25.

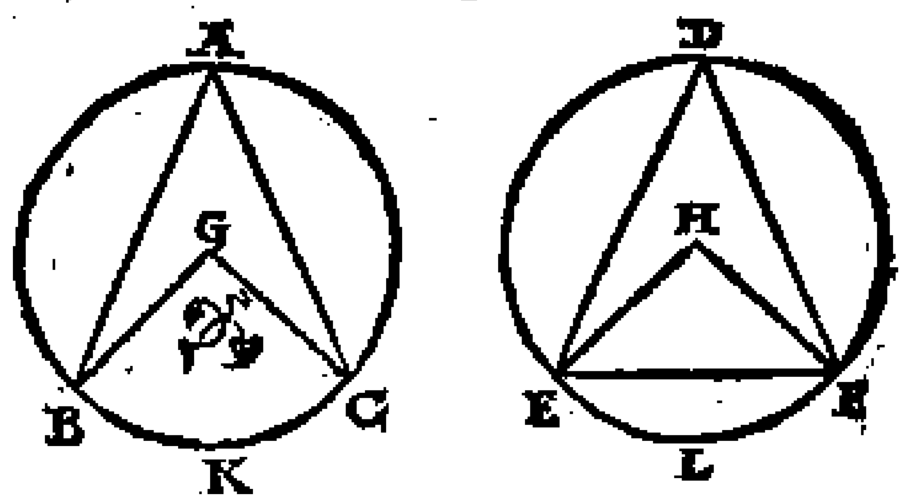
Circuli segmento dato, describere circulũ cuius

cuius eſt ſegmentum.



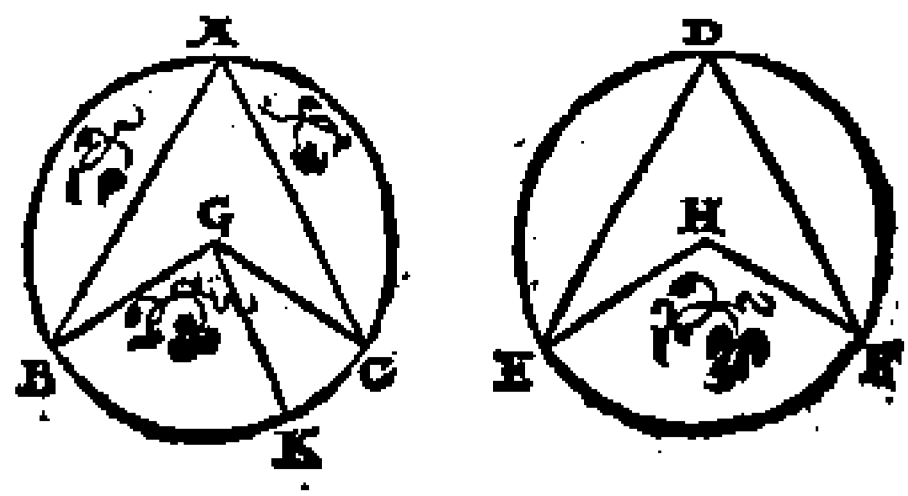
Theorema 23. Propoſitio 26.

In æqualibus circulis, æquales anguli ęqualib. peripherijs inſiſtũt ſiue ad cẽtra, ſiue ad peripherias conſtituti inſiſtant.



Theorema 24. Propoſitio 27.

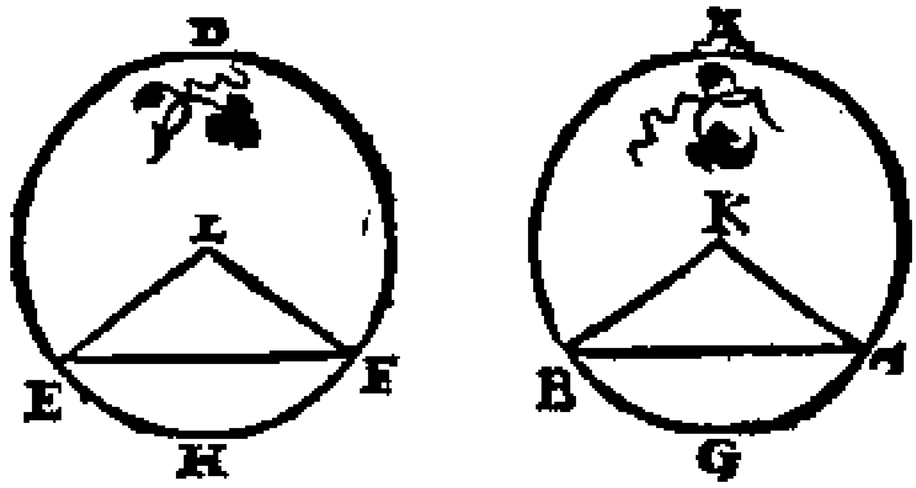
In æqualibus circulis, anguli qui æqualib⁹ peripherijs inſiſtũt ſunt inter ſe ęquales ſiue ad cẽtra, ſiue ad per.pherias cõſtituti inſiſtãt.



Theore-

Theorema 25. Propoſitio 28.

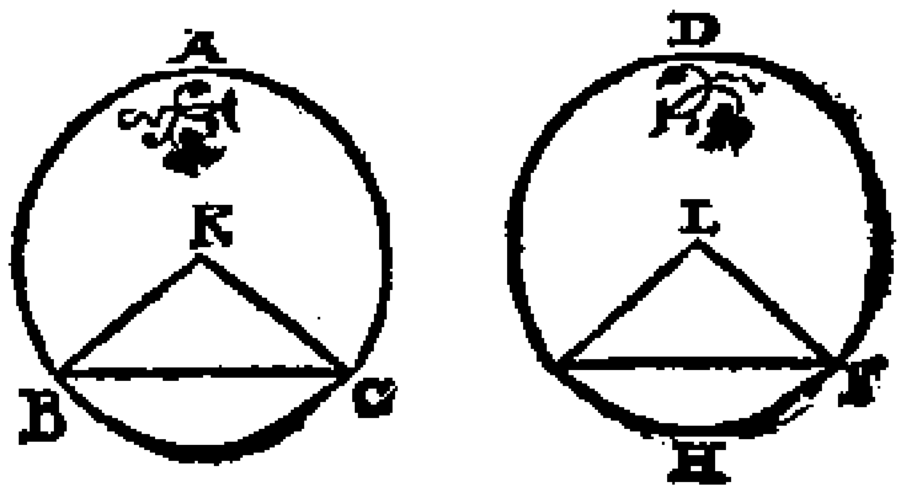
In æqualibus circulis æquales rectæ lineæ ęquales peri-pherias auferũt maiorẽ quidẽ maiori,



minorem autem minori.

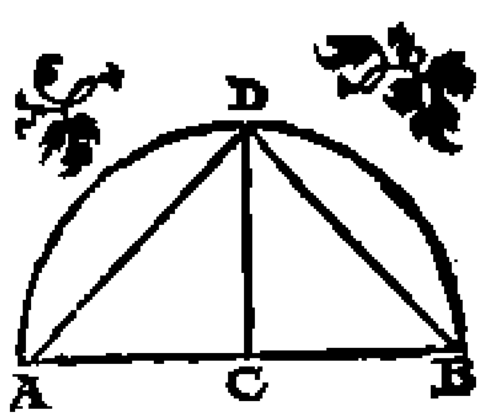
Theorema 26. Propoſitio 29.

In ęqua libꝰ cir-culis, æ-quales periphe rias æ-quales



rectæ lineæ ſubtendunt.

Problema 4. Propo-ſitio 30.



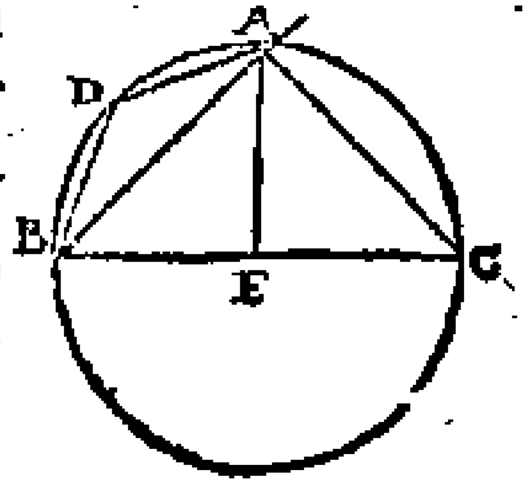
Datam peripheriam bi-fariam ſecare.

Theorema 27. Propo-ſitio 31.

In circulo angulus qui in ſemicirculo, re-

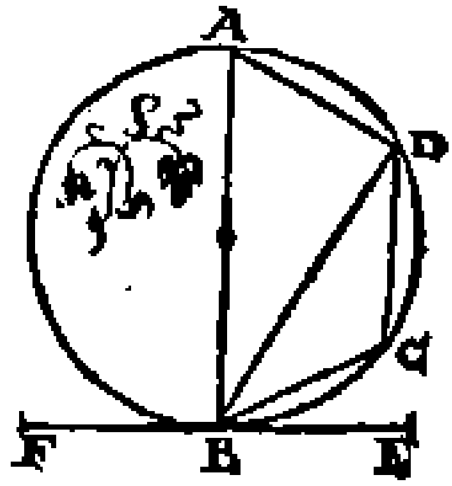
 ctus

ctus est: qui autem in maiore segmēto, minor recto: qui verò in minore segmēto, maior est recto. Et insuper angulus maioris segmēti, recto quidē maior est, minoris autē segmēti angulus, minor est recto.



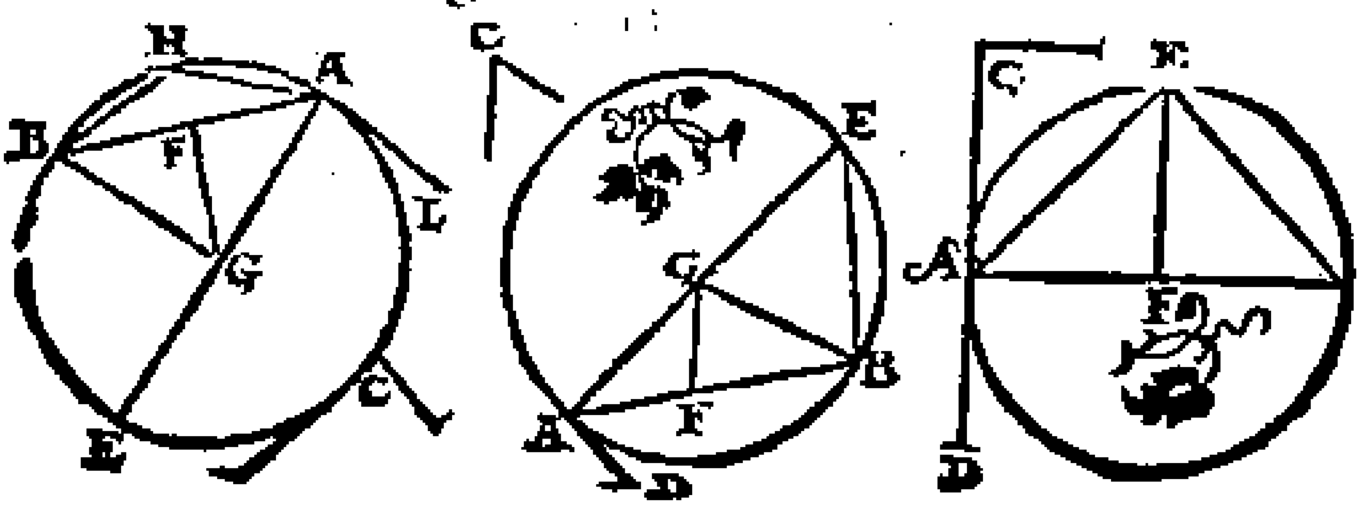
Theorema 28. Propositio 32.

Si circulum tetigerit aliqua recta linea, à contactu autē producatur quædam recta linea circulum secans: anguli quos ad contingentem facit, equales sunt ijs qui in alternis circuli segmētis consistunt, angulis.



Problema 5. Propositio 33.

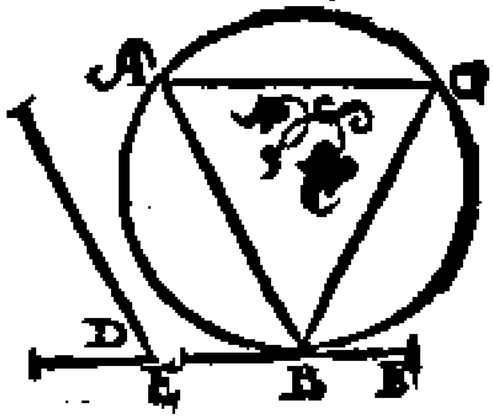
Super data recta linea describere segmentum circuli quod capiat angulum æqualē dato angulo rectilineo.



Proble-

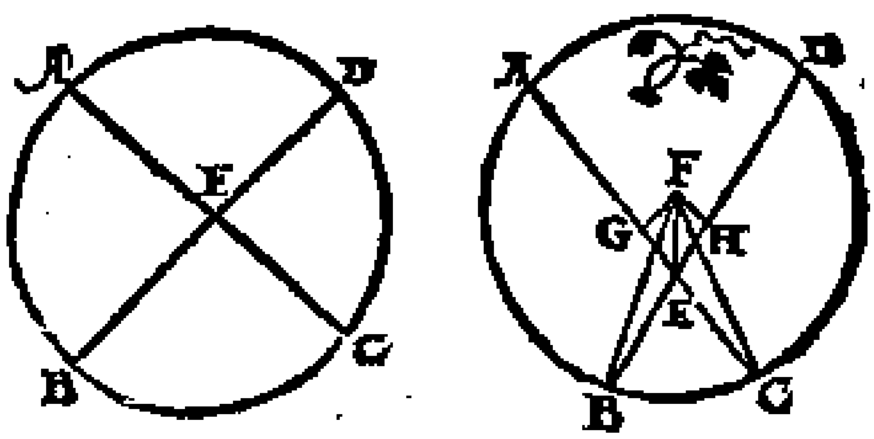
Problema 6. Propositio 34.

A dato circulo ſegmentum abſcindere capiens angulum æqualem dato angulo rectilineo.



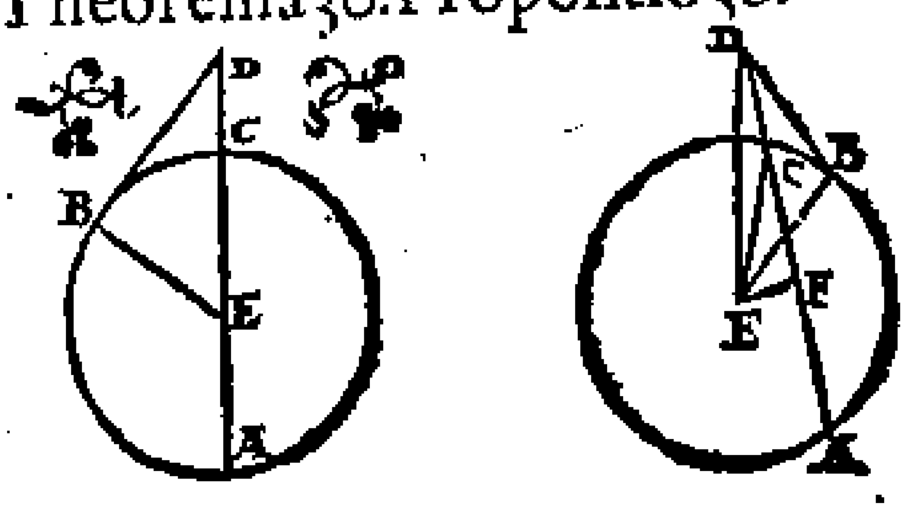
Theorema 29. Propoſitio 35.

Si in circulo duæ rectæ lineæ ſeſe mutuò ſecuerint, rectangulum cóprehenſum ſub ſegmẽtis vniꝰ, æquale eſt ei, quod ſub ſegmẽtis alterius comprehenditur, rectangulo.



Theorema 30. Propoſitio 36.

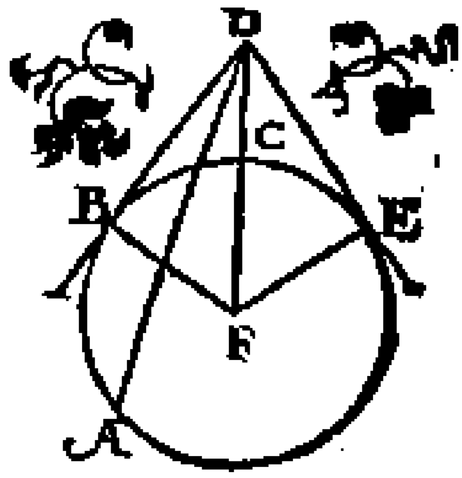
Si extra circulũ ſumatur pũctũ aliquod, ab eoq́ue in circulũ cadant duę rectę lineę, quarum altera quidem circulum ſecet, altera



verò tangat: quod ſub tota ſecante & exterius inter punctum & cōuexam peripheriam aſſumpta comprehenditur rectangulum, æquale erit ei, quod à tangente deſcribitur, quadrato.

Theorema 31. Propoſitio 37.

Si extra circulū ſumatur punctū aliquod, ab eoq́; puncto in circulum cadant duæ rectæ lineæ, quarum altera circulum ſecet, altera in eum incidat, ſit autē quod ſub tota ſecante & exterius inter punctū & cōuexam peripheriam aſſumpta, comprehenditur rectāgulum, æquale ei, quod ab incidēte deſcribitur quadrato: incidens ipſa circulum tanget.



ELEMENTI III FINIS.

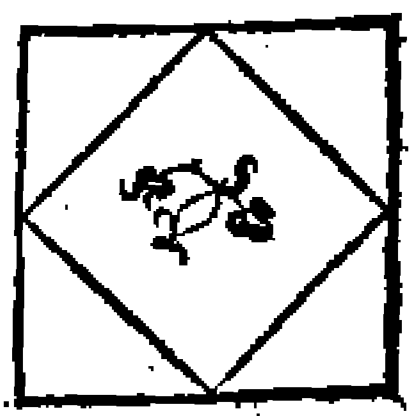
EVCLI-

EVCLIDIS ELEMENTVM QVARTVM.

DEFINITIONES.

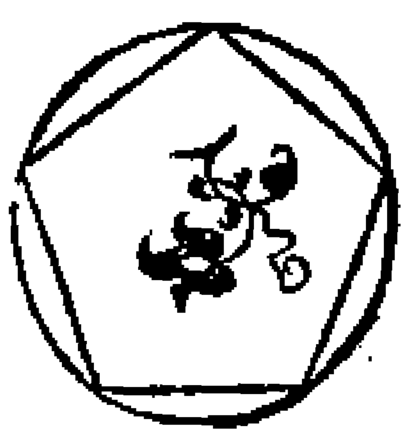
1

Figura rectilinea in figura rectilinea inscribi dicitur, cũ singuli eius figuræ quæ inscribitur, anguli singula latera eius, in qua inscribitur, tangunt.



2

Similiter & figura circum figurã describi dicitur, quum singula eius quæ circũscribitur, latera singulos eiꝰ figurę angulos tetigerint, circum quam illa describitur.



3

Figura rectilinea in circulo inscribi dicitur, quũ singuli eius figuræ q̃ inscribitur, angu-

anguli tetigerint circuli peripheriam.

4

Figura verò rectilinea circa circulũ deſcribi dicitur, quũ ſingula latera eius, quæ circumſcribitur, circuli peripheriã tangunt.

5

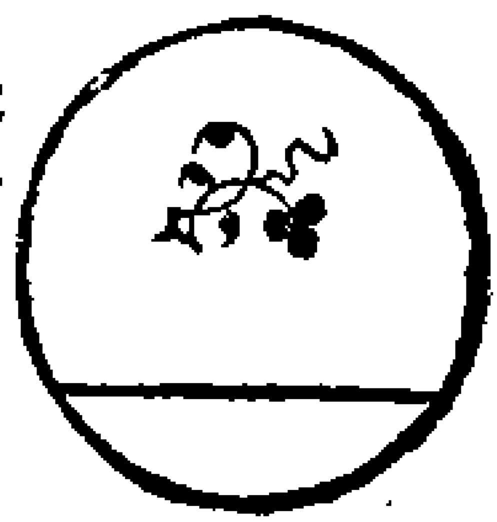
Similiter & circulus in figura rectilinea inſcribi dicitur, quum circuli peripheria ſingula latera tãgit eiꝰ figuræ, cui inſcribitur.

6

Circulus autem circum figurã deſcribi dicitur, quũ circuli peripheria ſingulos tãgit eius figuræ, quam circunſcribit, angulos.

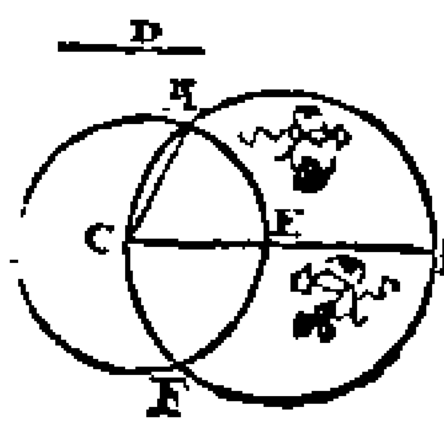
7

Recta linea in circulo accommodari ſeu coaptari dicitur, quum eius extrema in circuli peripheria fuerint.



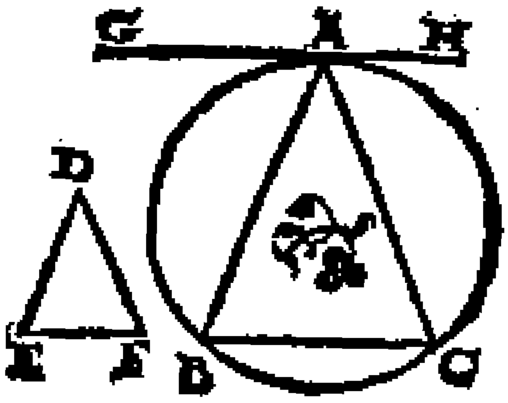
Problema 1. Propoſitio 1.

In dato circulo, rectam lineam accommodare æqualem datæ rectæ lineę quæ circuli diametro nõ ſit maior.



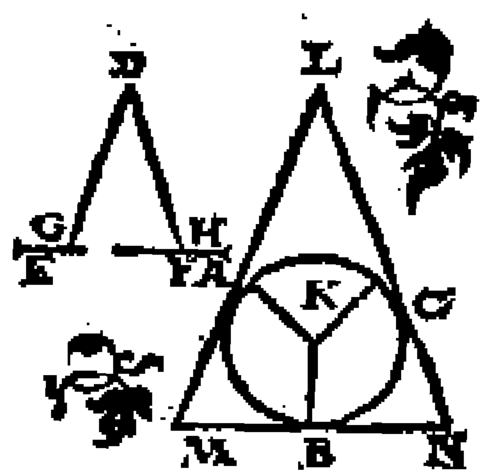
Problema 2. Propoſitio 2.

In dato circulo, triangulũ deſcribere dato triangulo æquiangulum.



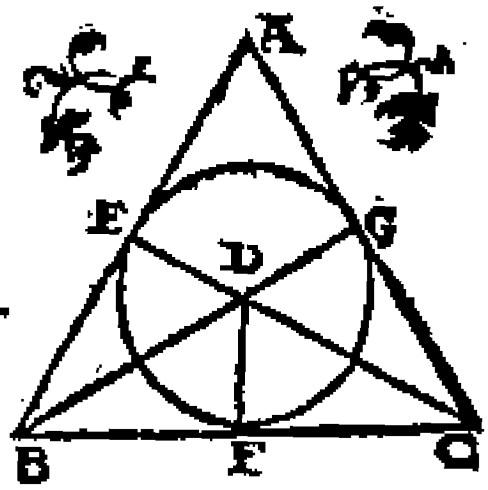
Problema 3. Propoſitio 3.

Circa datum circulũ triangulũ, deſcribere dato triangulo æquiangulũ.



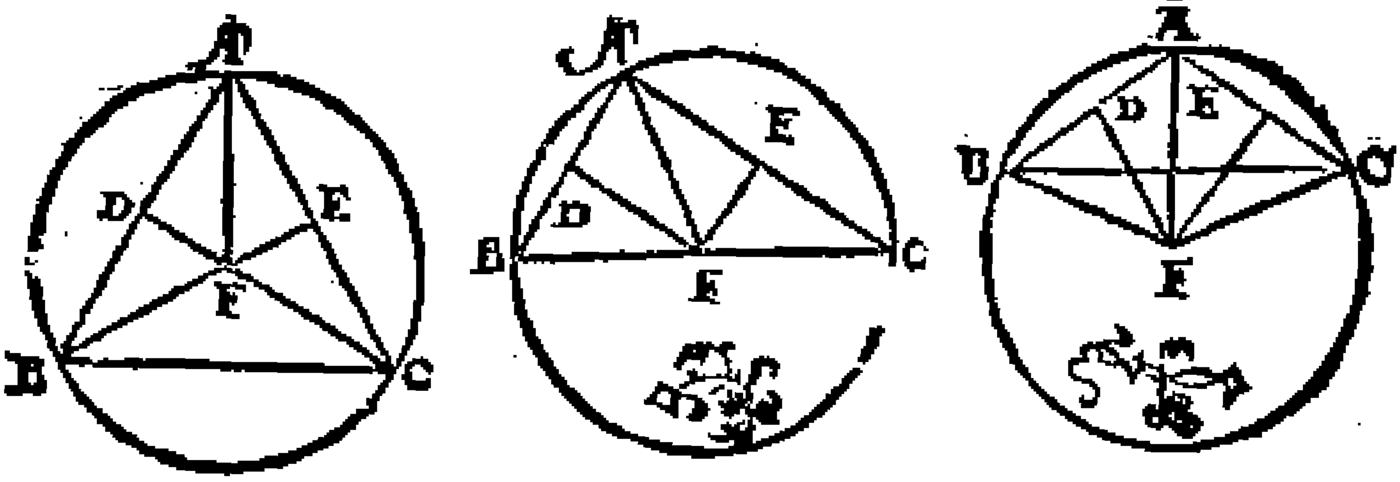
Problema 4. Propoſitio 4.

In dato triangulo circulum inſcribere.



Problema 5. Propoſitio 5.

Circa datum triangulum, circulum deſcribere.



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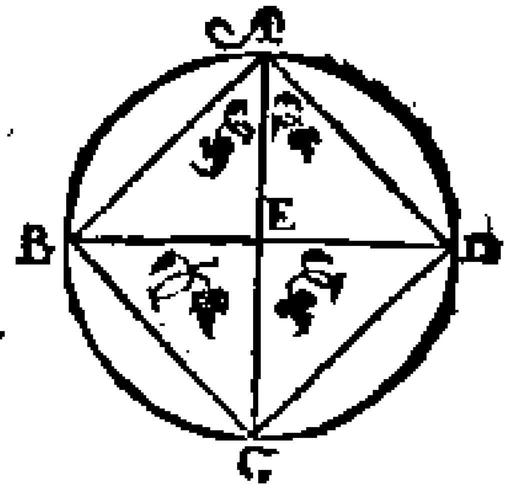
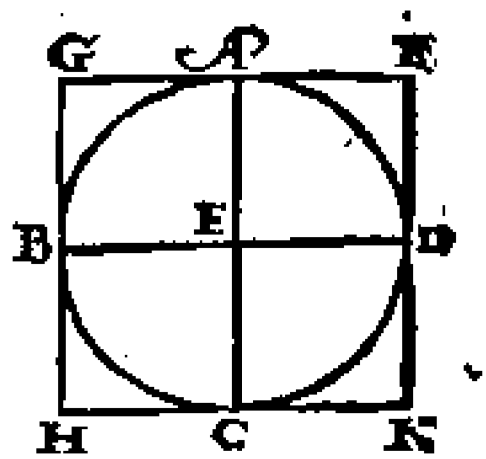
Problema 6. Propositio 6.

In dato circulo quadratum describere.



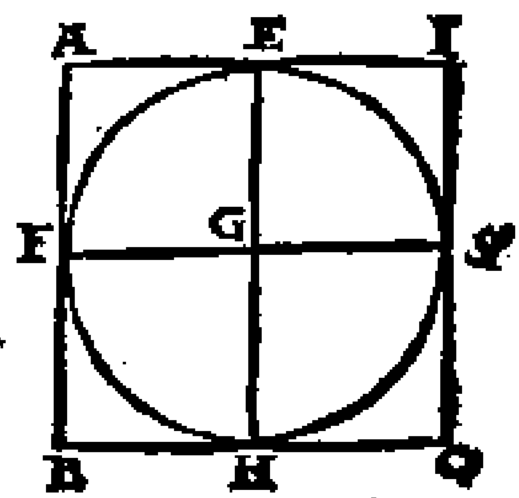
Problema 7. Propositio 7.

Circa datum circulum, quadratum describere.



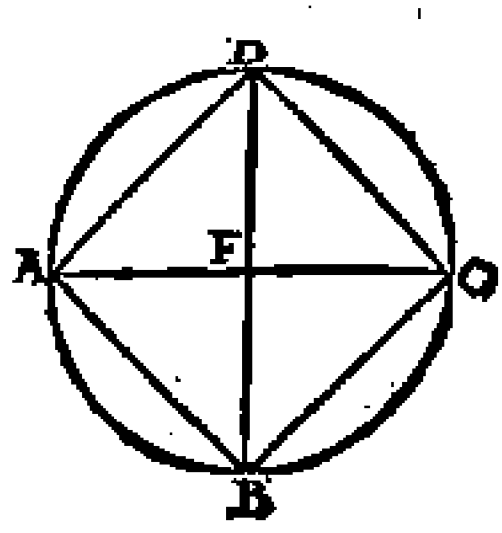
Problema 8. Propositio 8.

In dato quadrato circulum inscribere.



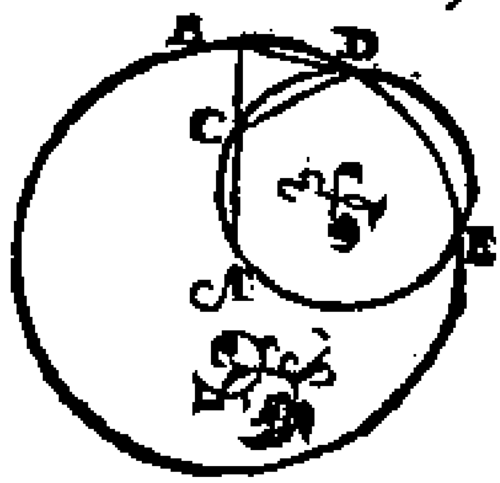
Problema 9. Propositio 9.

Circa datum quadratũ, circulum describere.



Proble-

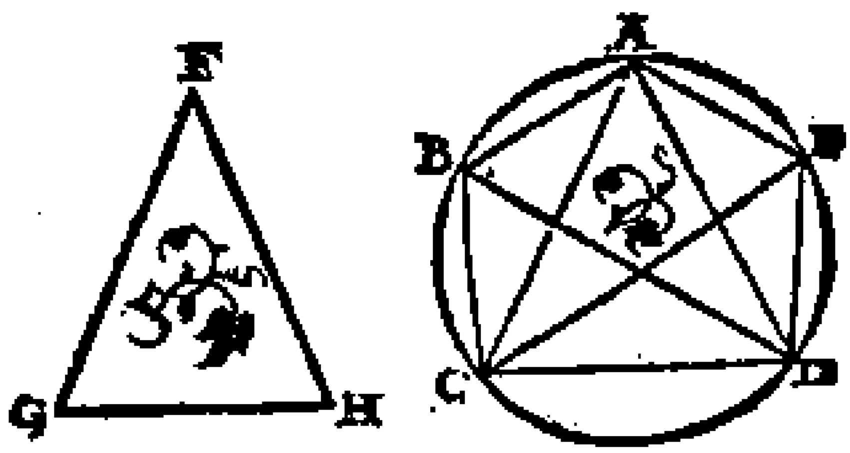
Problema 10. Propoſitio 10.



Iſoſceles triangulum cōſtituere, quod habeat vtrunque eorum, qui ad baſin ſunt, angulorum, duplum reliqui.

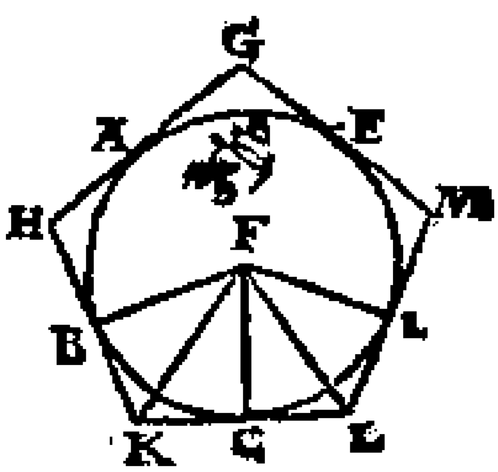
Theorema 11. Propoſitio 11.

In dato circulo, pentagonū æquilaterū & æquiāgulum inſcribere.



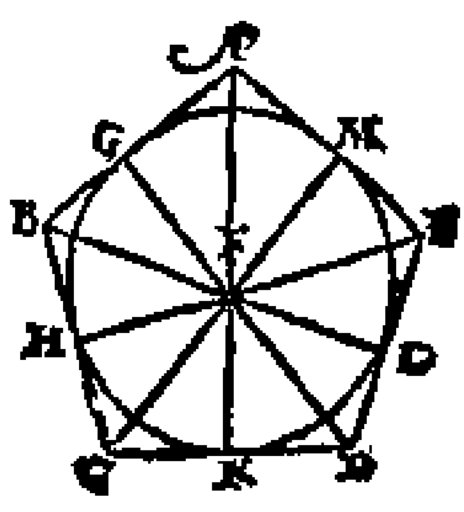
Problema 12. Propoſitio 12.

Circa datum circulum pentagonum æquilaterum & æquiangulū deſcribere.



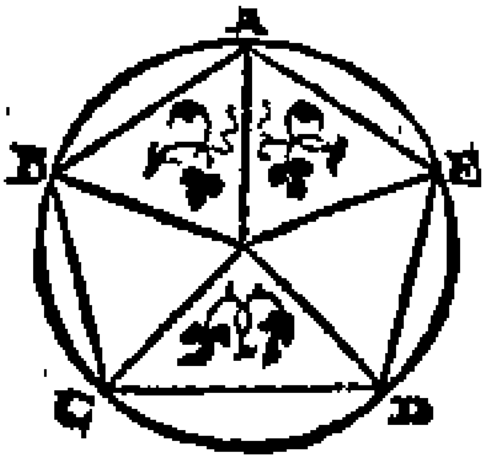
Problema 13. Propoſitio 13.

In dato pētagono æquilatero & æquiāgulo, circulum inſcribere.



Proble-

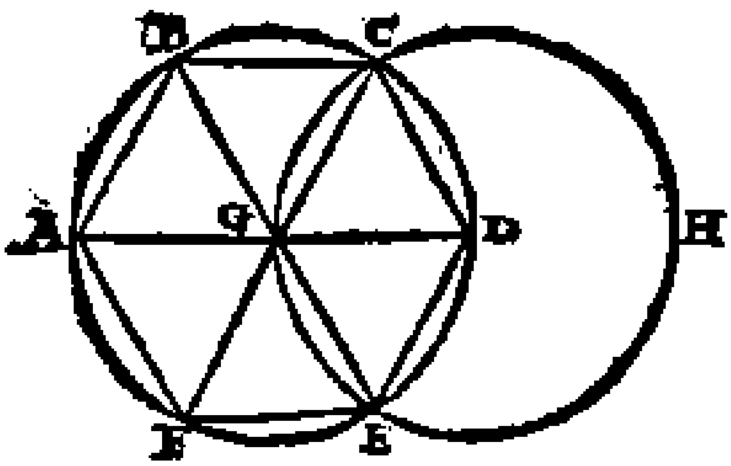
Problema 14. Propoſitio 14.

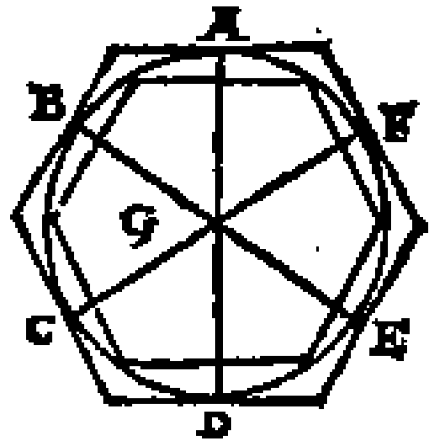


Circa datũ pentagonũ, æquilaterum & æquiangulum, circulum deſcribere.

Problema 15. Propoſitio 15.

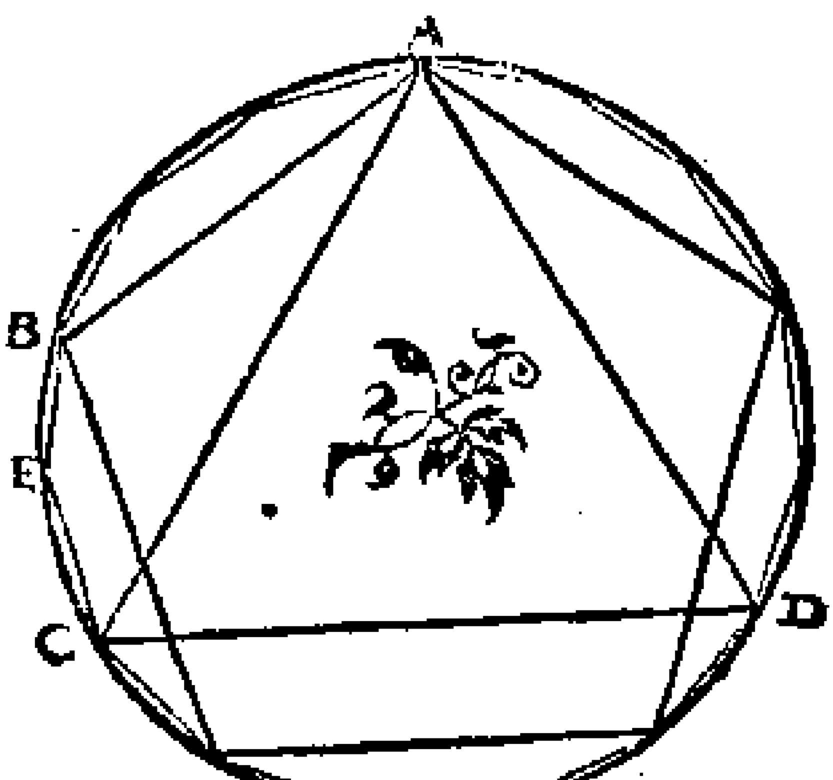
In dato circulo hexagonum & æquilaterũ & æquiangulum inſcribere.





Propoſitio 16. Theorema 16.

In dato circulo quintidecagonum & æquilaterũ & ęquiangulum deſcribere.



Elementi quarti finis.

EVCLIDIS ELEMENTVM QVINTVM.

DEFINITIONES.

1

PARS eſt magnitudo magnitudinis minor maioris, quum minor metitur maiorem.

2

Multiplex autem eſt maior minoris, cùm minor metitur maiorem.

3

Ratio, eſt duarum magnitudinum eiuſdē generis mutua quædam ſecundum quantitatem habitudo.

4

Proportio verò, eſt rationum ſimilitudo.

5

Rationē habere inter ſe magnitudinis dicuntur, quæ poſſunt multiplicatæ ſeſe mutuò ſuperare.

6

In eadem ratione magnitudines dicūtur eſſe, prima ad ſecundam, & tertia ad quartā: cùm primæ & tertiæ æquè multiplicia, à ſecundæ & quartæ æquè multiplicibus, qualiſcunq; ſit hæc multiplicatio, vtrunq; ab vtroque: vel vnà deficiunt, vel vnà ęqualia ſunt, vel vnà excedũt, ſi ea ſumãtur quæ inter ſe reſpondent.

7

Eandem autem habẽtes rationem magnitudines, proportionales vocentur.

8

Cũm verò æquè multiplicium, multiplex primę magnitudines exceſſerit multiplicẽ ſecũdæ, at multiplex tertiæ non exceſſerit multiplicem quartæ: tunc prima ad ſecundam, maiorem rationem habere dicetur, quàm tertia ad quartam.

9

Proportio autẽ in tribus terminis paucißimis conſiſtit.

10

Cùm autem tres magnitudines proportionales fuerint, prima ad tertiam, duplicatã rationẽ habere dicitur eius, quam habet ad ſecundam. At cùm quatuor magnitudines proportionales fuerint, prima ad quartã, triplicatam rationem habere dicitur eius quam habet ad ſecundam: & ſemper deinceps vno amplius, quandiu proportio extiterit.

11

Homologę, ſeu ſimiles ratione magnitudines dicuntur, antecedentes quidẽ antece-

denti-

dentibus, consequẽtes verò cõsequẽtibus.

12

Alterna ratio, est sumptio antecedẽtis cõparati ad antecedentem, & cõsequentis ad consequentem.

13

Inuersa ratio, est sumptio cõsequentis, ceu antecedentis, ad antecedentẽ velut ad cõsequentem.

14

Compositio rationis, est sumptio antecedentis cũ consequente ceu vnius, ad ipsum consequentem.

15

Diuisio rationis, est sumptio excessus quo consequentem superat antecedens ad ipsum consequentem.

16

Conuersio rationis, est sumptio antecedẽtis ad excessum, quo superat antecedẽs ipsum consequentem.

17

Ex æqualitate ratio est, si plures duab⁹ sint magnitudines, & his alię multitudine pares quæ binæ sumantur, & in eadem ratione: quũ vt in primis magnitudinibus prima ad vltimam, sic & in secundis magnitudinibus prima ad vltimã sese habuerit, vel aliter, sumptio extremorũ per subductionem mediorum.

18

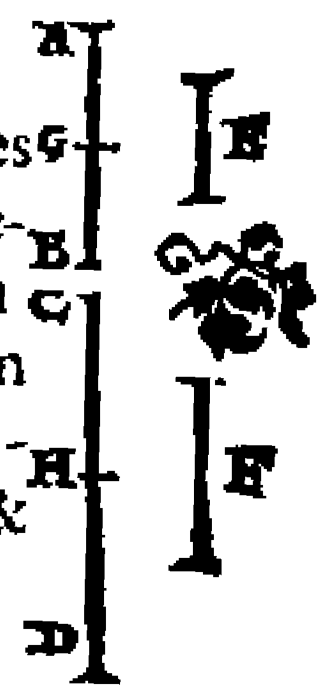
Ordinata proportio est, cùm fuerit quem admodum antecedens ad consequentem, ita antecedens ad consequentem: fuerit etiam vt consequẽs ad aliud quidpiam, ita consequens ad aliud quidpiam.

19

Perturbata autẽ proportio est, tribus positis magnitudinibus, & alijs quæ sint his multitudine pares, cùm vt in primis quidẽ magnitudinibus se habet antecedẽs ad cõsequentem, ita in secũdis magnitudinibus antecedens ad consequentem: vt autem in primis magnitudinibus consequens ad aliud quidpiam, sic in secundis magnitudinibus aliud quidpiam ad antecedentem.

Theorema 1. Propositio 1.

Si sint quotcũque magnitudines quotcunque magnitudinum æqualium numero singulę singularum æquè multiplices, quàm multiplex est vnius vna magnitudo, tam multiplices erunt, & omnes omnium.



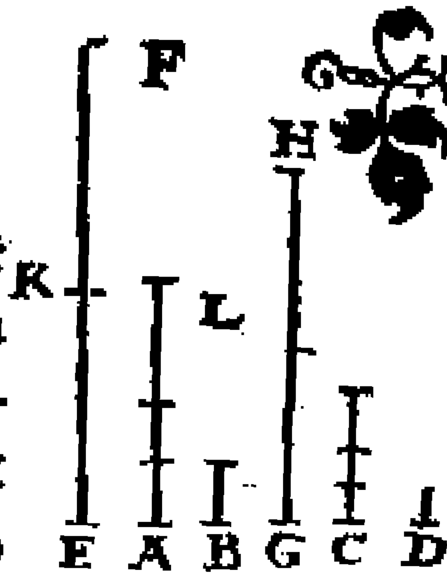
Theorema 2. Propositio 2.

Si prima secundæ æquè fuerit multiplex,

atque

atque tertia quartę, fuerit autē & quinta secūdę æquè multiplex, atque sexta quartæ: erit & cōposita prima cū quinta, secūdæ ęquè multiplex atque tertia cum sexta, quartæ.

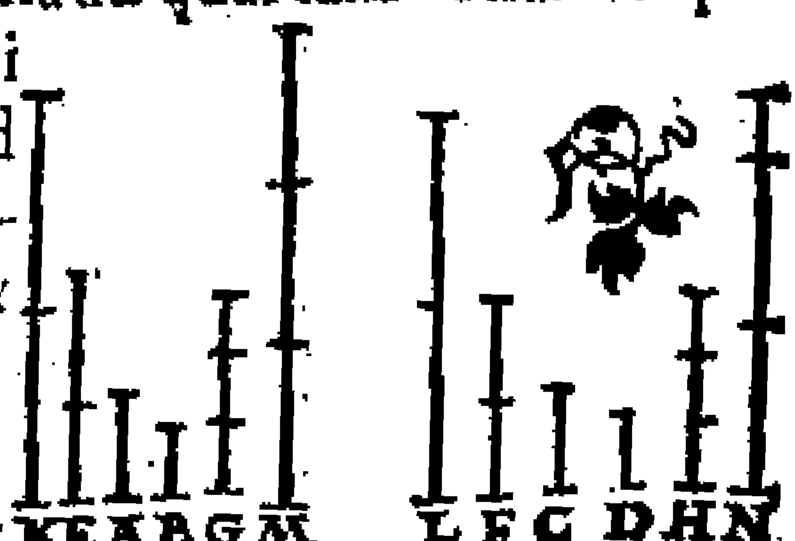
Theorema 3. Propositio 3.



Si sit prima secūdæ æquè multiplex atque tertia quartæ, sumātur autē æquè multiplices primę & tertiæ erit & ex æquo sumptarum vtraque vtriusque ęquè multiplex, altera quidem secundæ, altera autē quartæ.

Theorema 4. Propositio 4.

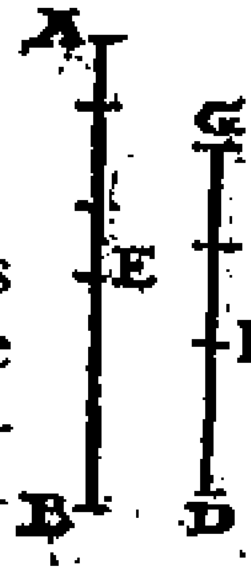
Si prima ad secundam, eandē habuerit rationem, & tertia ad quartam: etiam æque multiplices primę & tertiæ, ad æquè multiplices secundæ & quartæ iuxta quāuis multiplicationē, eandem habebunt rationem, si prout inter se



 respon-

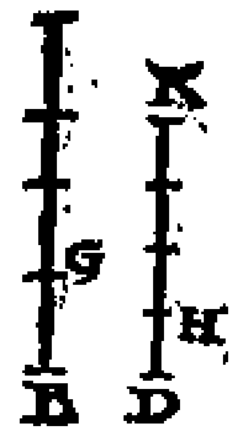
reſpondent, ita ſumptæ fuerint.

Theorema 5. Propoſitio 5.



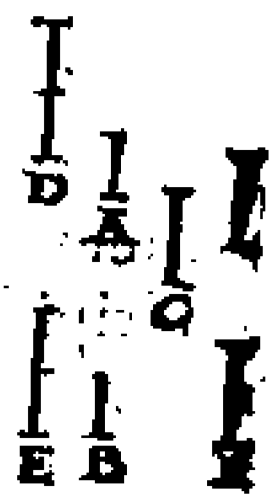
Si magnitudo magnitudinis æquè fuerit multiplex, atque ablata ablatæ: etiam reliqua reliquæ ita multiplex erit, vt tota totius.

Theorema 6. Propoſitio 6.



Si duæ magnitudines, duarum magnitudinum ſint æquè multiplices, & detractę quædam ſint earundem æquè multiplices: & reliquæ eiſdem aut ęquales ſunt, aut æquè ipſarum multiplices.

Theorema 7. Propoſitio 7.



Aequales ad eandem, eadem habent rationem: & eandem ad æquales.

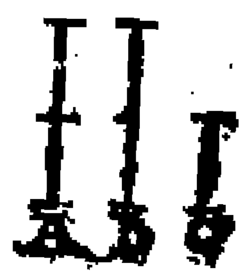
Theorema 8. Propoſitio 8.

Inæqualium magnitudinũ maior, ad eandem

dem maiorem rationẽ habet, quàm minor: & eadem ad minorem: maiorem rationem habet, quàm ad maiorem.

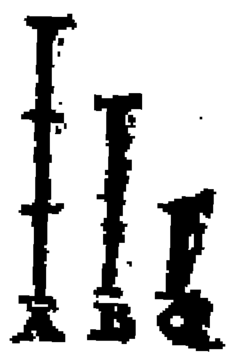
Theorema 9. Propoſitio 9.

Quæ ad eandem, eandem habẽt rationem, æquales ſũt inter ſe: & ad quas eadem, eandem habet rationem, eæ quoque ſunt inter ſe æquales.



Theorema 10. Propoſitio 10.

Ad eandem magnitudinẽ, rationem habentium, quæ maiorem rationẽ habet, illa maior eſt, ad quam autem eadem maiorem rationem habet, illa minor eſt.

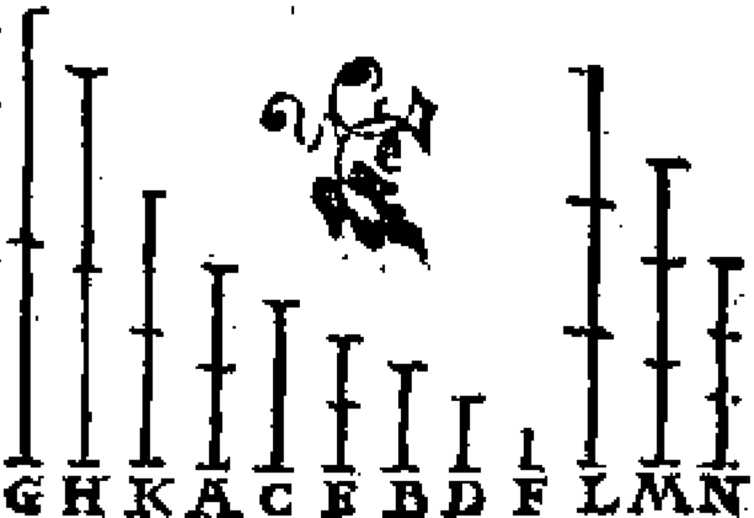


Theorema 11. Propoſitio 11.

Quæ eidem ſunt eædem rationes, & inter ſe ſunt eædem.

Theorema 12. Propoſitio 12.

Si ſint magnitudines quotcunque proportionales, quemadmodũ ſe habuerit vna antecedentium ad vnã conſequentiũ, ita ſe habebunt omnes antecedentes ad omnes conſequentes.



Theorema 13. Propoſitio 13.

Si prima ad ſecundam, eandẽ habuerit rationem, quam tertia ad quartã, tertia verò ad quartam, maiorem rationem habuerit, quàm quinta ad ſextam: prima quoque ad ſecũdam maiorem rationem habebit, quàm quinta ad ſextam.

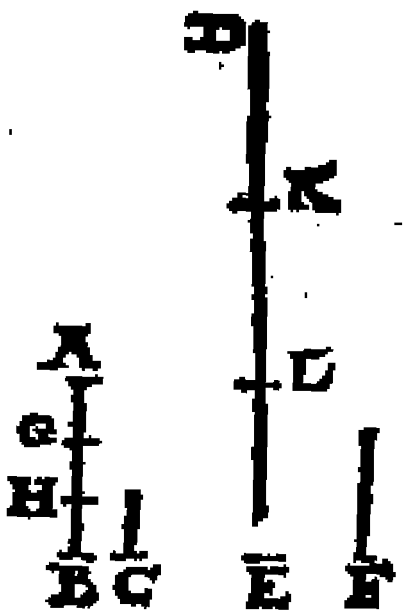
Theorema 14. Propoſitio 14.

Si prima ad ſecundam eandem habuerit rationẽ, quam tertia ad quartam, prima verò quàm tertia maior fuerit: erit & ſecunda maior quàm quarta. Quòd ſi prima fuerit æqualis tertiæ, erit

& secunda æqualis quartæ: si verò minor, & minor erit.

Theorema 15. Propositio 15.

Partes cum pariter multiplicibus in eadé sunt ratione, si prout sibi mutuo respondent, ita sumantur.

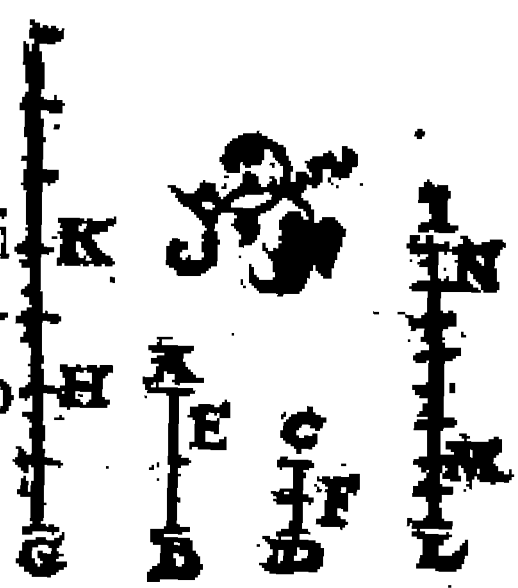


Theorema 16. Propositio 16.

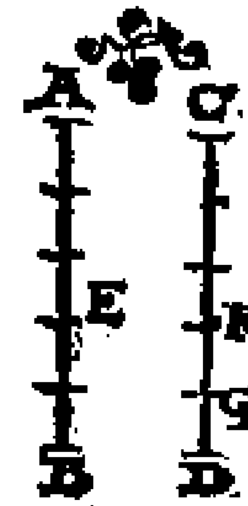
Si quatuor magnitudines proportionales fuerint, & vicissim proportionales erunt.

Theorema 17. Propositio 17.

Si compositæ magnitudines proportionales fuerint hæ quoq; diuisæ proportionales erunt.

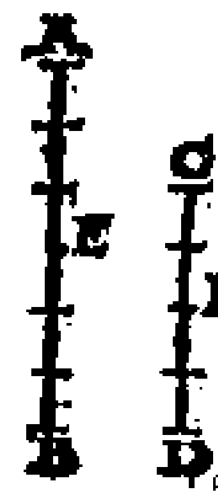


Theorema 18. Propositio 18.



Si diuisæ magnitudines sint proportionales, hæ quoque compositæ proportionales erunt.

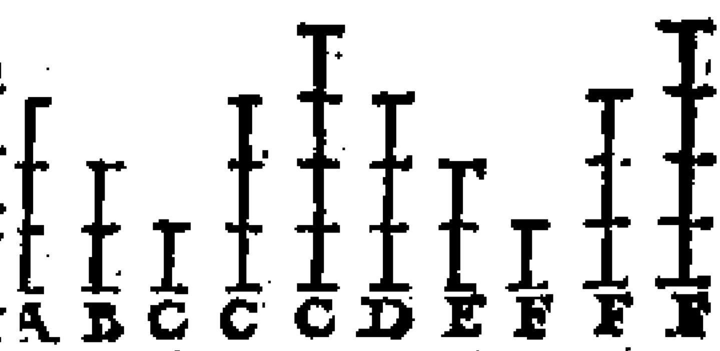
Theorema 19. Propositio 19.



Si quemadmodum totũ ad totum, ita ablatum se habuerit ad ablatum : & reliquum ad reliquum, vt totum ad totum se habebit.

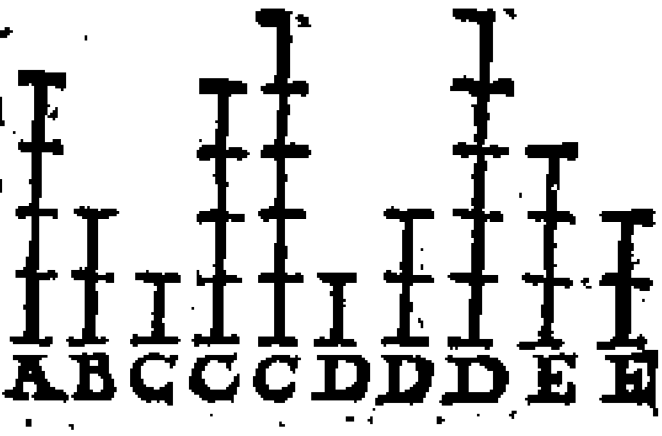
Theorema 20. Propositio 20.

Si sint tres magnitudines, & aliæ ipsis æquales numero, quæ binæ & in eadem ratione sumatur, ex æquo autẽ prima quàm tertia maior fuerit; erit & quarta, quã sexta maior. Quòd si prima tertiæ fuerit æqualis, erit & quarta æqualis sextæ : sin illa minor, hæc quoque minor erit.



Theorema 21. Propositio 21.

Si sint tres magnitudines, & alię ipsis æquales numero quæ binæ & in eadem ratione sumãtur, fueritque perturbata earũ proportio ex æquo autem prima quàm tertia maior fuerit erit & quarta quã sexta maior: quòd si prima tertiæ fuerit æqualis, erit & quarta æqualis sextæ: sin illa minor, hæc quoque minor erit.



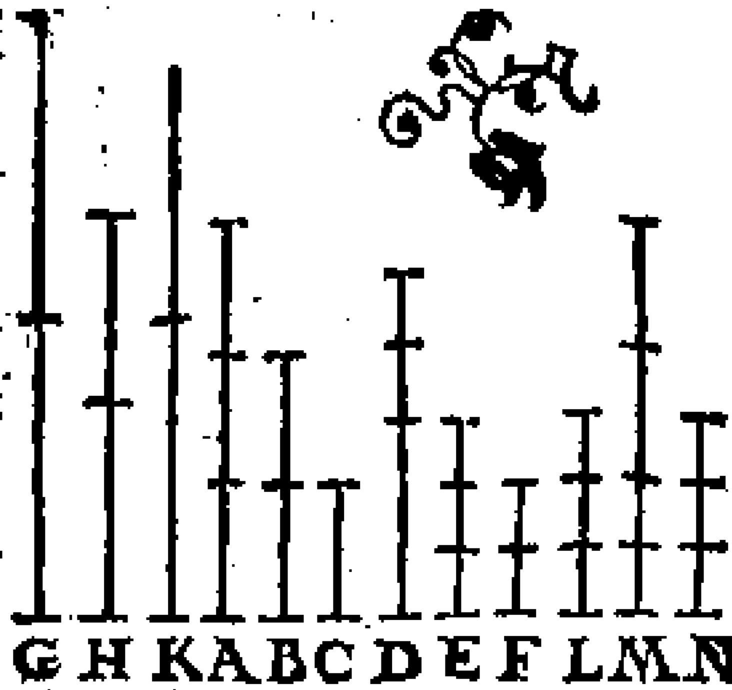
Theorema 22. Propositio 22.

Si sint quotcunq; magnitudines, & alię ipsis ęquales numero, quæ binæ in eadẽ ratione sumantur, et & ex æqualitate in eadem ratione erunt.

Theorema 23. Propositio 23.

Si sint tres magnitudines, aliæq; ipsis ęquales

les numero, q̃ binæ in eadem ratione sumã-tur, fuerit autẽ perturbata ea-rũ proportio: etiã ex æquali-te in eadem ra-tione erunt.



Theorema 24. Propo-sitio 24.

Si prima ad secundam, eandem habuerit rationẽ, quam tertia ad quartam, habuerit autem & quinta ad secundam eandẽ ra-tionem, quã sexta ad quartam: etiam cõposita prima cũ quin-ta ad secundam eandẽ habebit rationem, quã tertia cum sexta ad quartã.

Theorema 25. Propo-sitio 25.

Si quatuor magnitudines proportionales fuerint, ma-xima & minima reliquis duabus maiores erunt.

ELEMENTI V. FINIS.

EVCLIDIS ELEMENTVM SEXTVM.

DEFINITIONES.

1

Similes figurę rectilineæ sunt, quæ & angulos singulos singulis æquales habẽt, atque etiam latera, quæ circum angulos æquales, proportionalia.

2

Reciprocæ autem figuræ sunt, cùm in vtraque figura antecedentes & consequẽtes rationum termini fuerint.

3

Secundum extremam & mediam rationẽ recta linea secta esse dicitur, cùm vt tota ad maius segmentum, ita maius ad minus se habuerit.

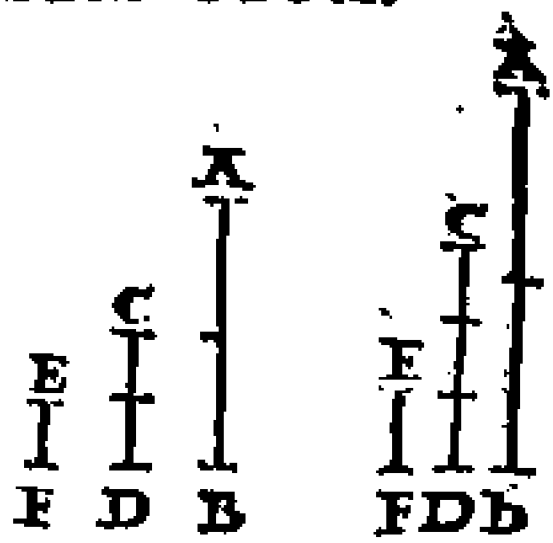
4

Altitudo cuiusque figurę, est linea perpẽdicularis à vertice ad basin deducta.

5 Ra-

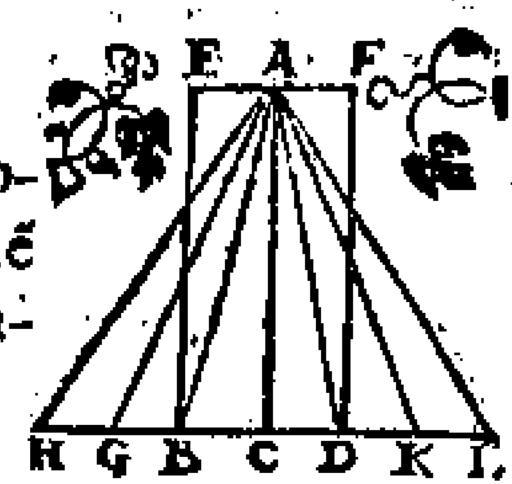
5

Ratio ex rationibus cō poni dicitur, cū rationum quantitates inter se multiplicatæ aliquā effecerint rationem.



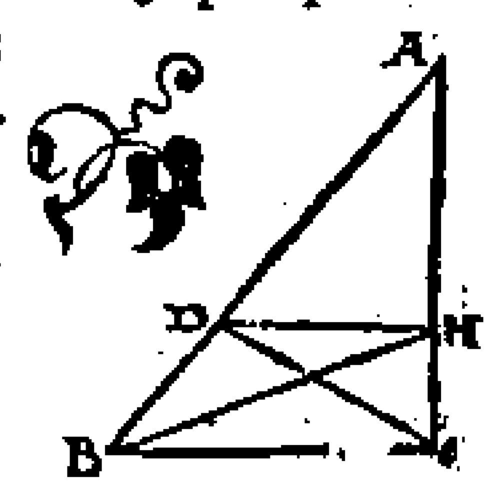
Theorema 1. Propositio 1.

Triangula & parallelogramma, quorum eadē fuerit altitudo, ita se habent inter se vt bases.



Theorema 2. Propositio 2.

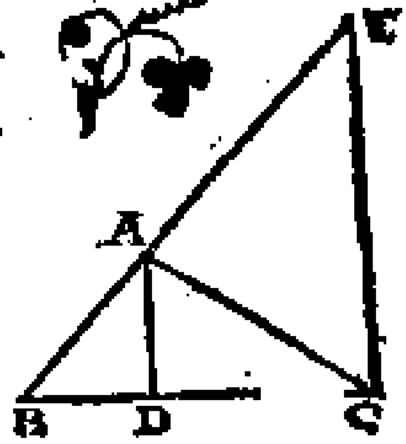
Si ad vnum trianguli latus parallela ducta fuerit recta quædam linea: hęc proportionaliter secabit, ipsius triāguli latera. Et si triāguli latera proportionaliter secta fuerint: quæ ad sectiones adiūcta fuerit recta linea, erit ad reliquum ipsius trianguli latus parallela.



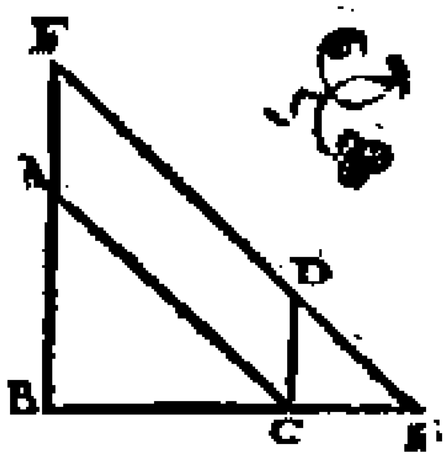
Theorema 3. Propositio 3.

Si trianguli angulus bifariā sectus sit, secās autem angulum recta linea secuerit & basim: basis segmenta eandem habebunt rationem,

tionem, quam reliqua ipsius trianguli latera. Et si basis segmenta eandem habeant rationem, quam reliqua ipsius trianguli latera, recta linea, quæ à vertice ad sectionem producitur, ea bifariã secat trianguli ipsius angulum.

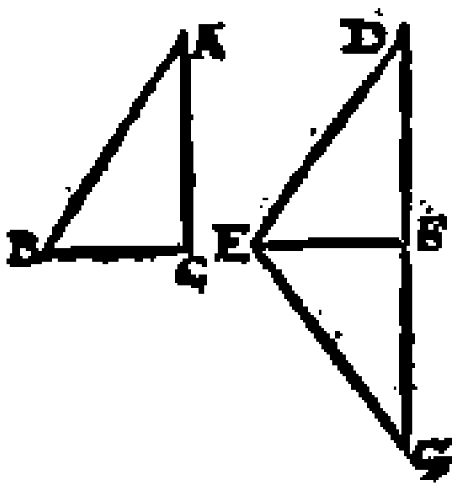


Theorema 4. Propositio 4.



Aequiangulorum triangulorum proportionalia sunt latera, quæ circum ęquales angulos, & homologa sunt latera, quæ æqualibꝰ angulis subtenduntur.

Theorema 5. Propositio 5.

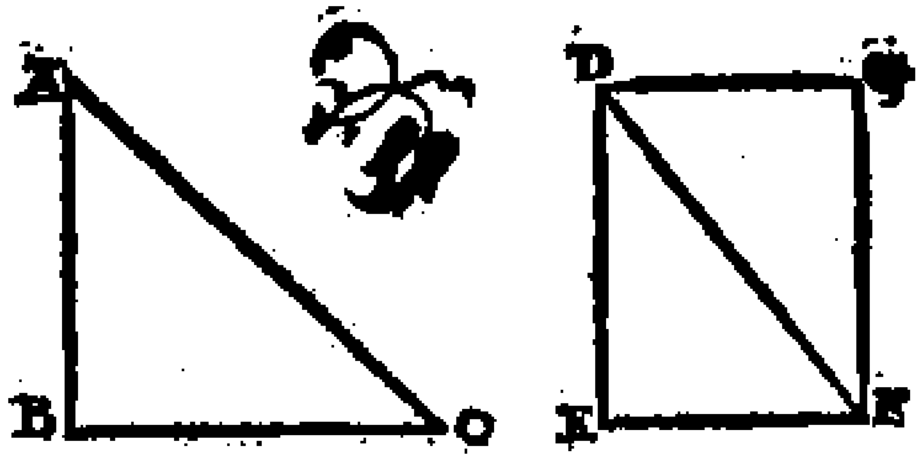


Si duo triãgula latera proportionalia habeant, ęquiangula erunt triangula, & æquales habebunt eos angulos, sub quibus homologa latera subtenduntur.

Theorema 6. Propositio 6.

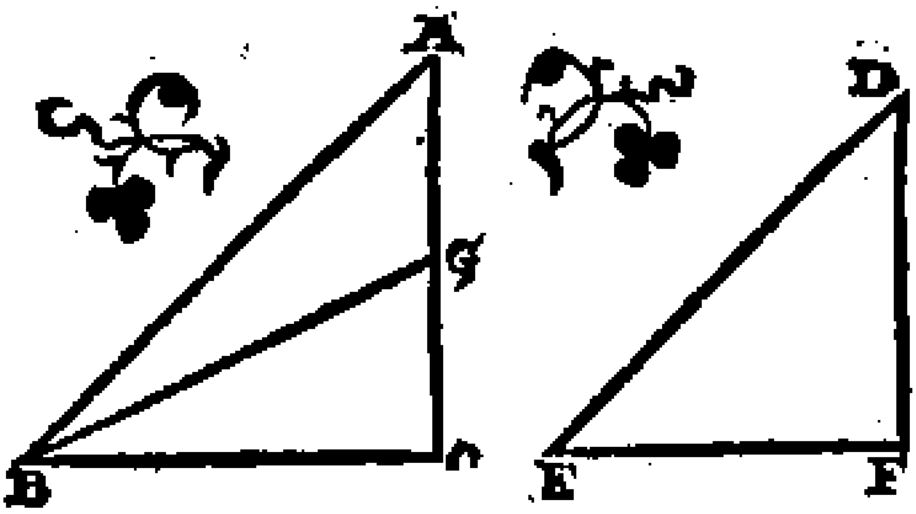
Si duo triangula vnum angulum vni angulo æqualẽ, & circũ ęquales angulos latera ꝓportionalia habuerint, ęquiangula erũt trian-

triangula,ęqualesq́; habebunt angulos ſub quibus homologa latera ſubtenduntur.



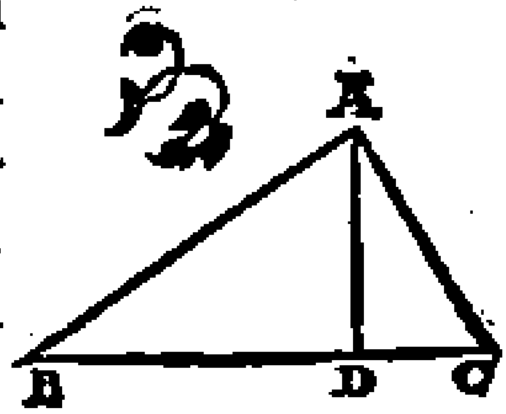
Theorema 7.Propoſitio 7.

Si duo triangula vnum angulum vni angulo æqualem,circũ autem alios angulos latera proportionalia habeant, reliquorum verò ſimul vtrunq; aut minorem aut non minoré recto: æquiangula erunt triangula,& ęquales habebunt eos angulos, circum quos ꝓportionalia ſunt latera.

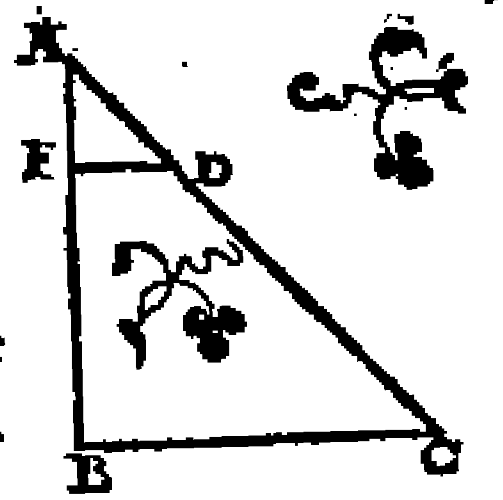


Theorema 8:Propoſitio 8.

Si in triangulo rectangulo,ab angulo recto in baſin perpendicularis ducta ſit,quę ad perpendicularem triangula,tum toti triangulo, tum ipſa inter ſe ſimilia ſunt.

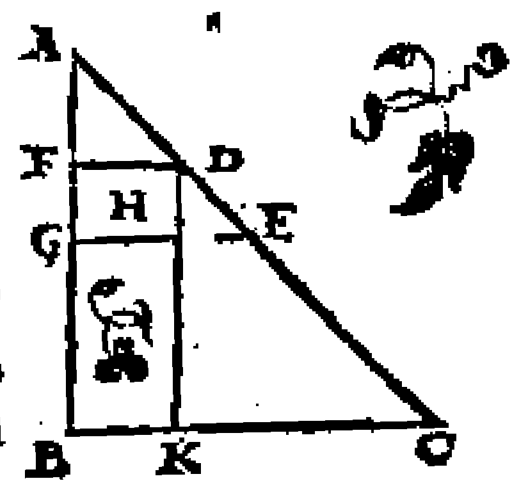


Proble-



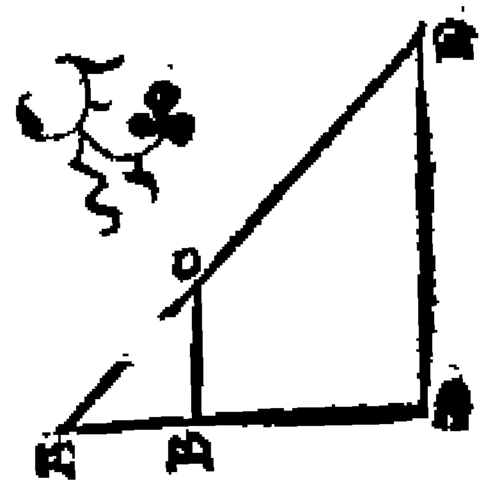
Problema 1. Propositio 9.

A data recta linea imperatam partem auferre.



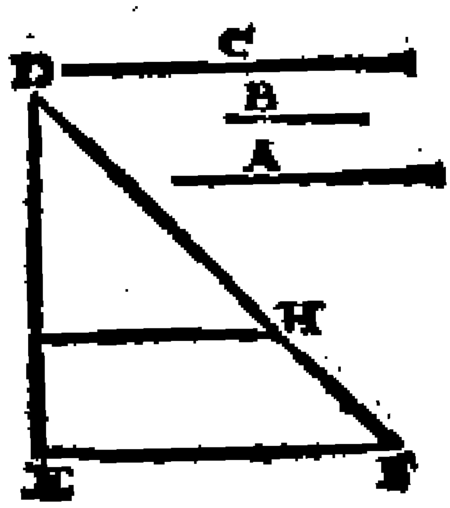
Problema 2. Propositio 10.

Datam rectam lineā in sectam similiter secare, vt data altera recta secta fuerit.



Problema 3. Propositio 11.

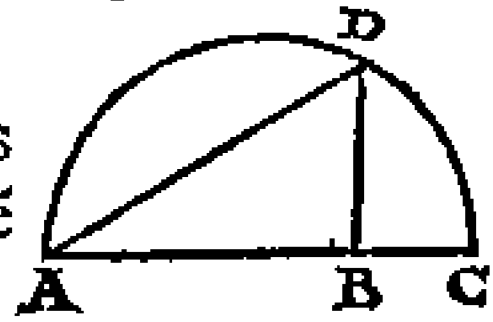
Duabus datis rectis lineis, tertiam proportionalem adinuenire.



Problema 4. Propositio 12.

Tribꝰ datis rectis lineis, quartam proportionalem adinuenire.

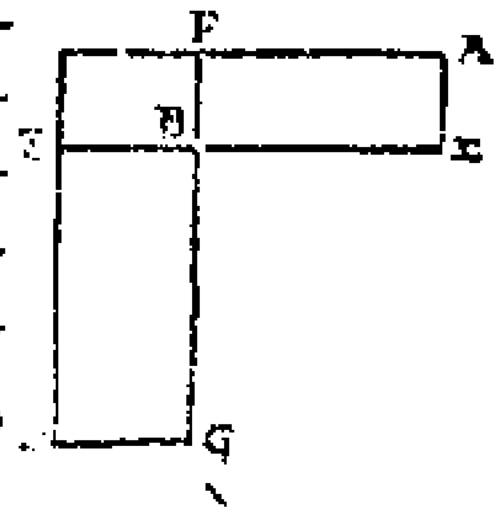
Problema 5. Propoſitio 13.



Duabꝰ datis rectis lineis mediam proportionalē adinuenire.

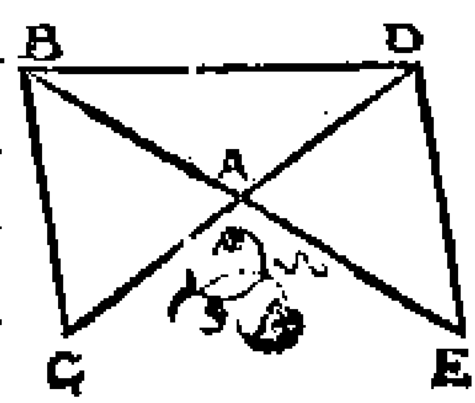
Theorema 9. Propoſitio 14.

Aequalium, & vnū vni æqualem habentiū angulum parallelogrammorum reciproca ſunt latera, quæ circum æquales angulos: & quorū parallelogrammorū vnū angulum vni angulo æqualem habentiū reciproca ſunt latera, quæ circum æquales angulos, illa ſunt æqualia.



Theorema 10. Propoſitio 15.

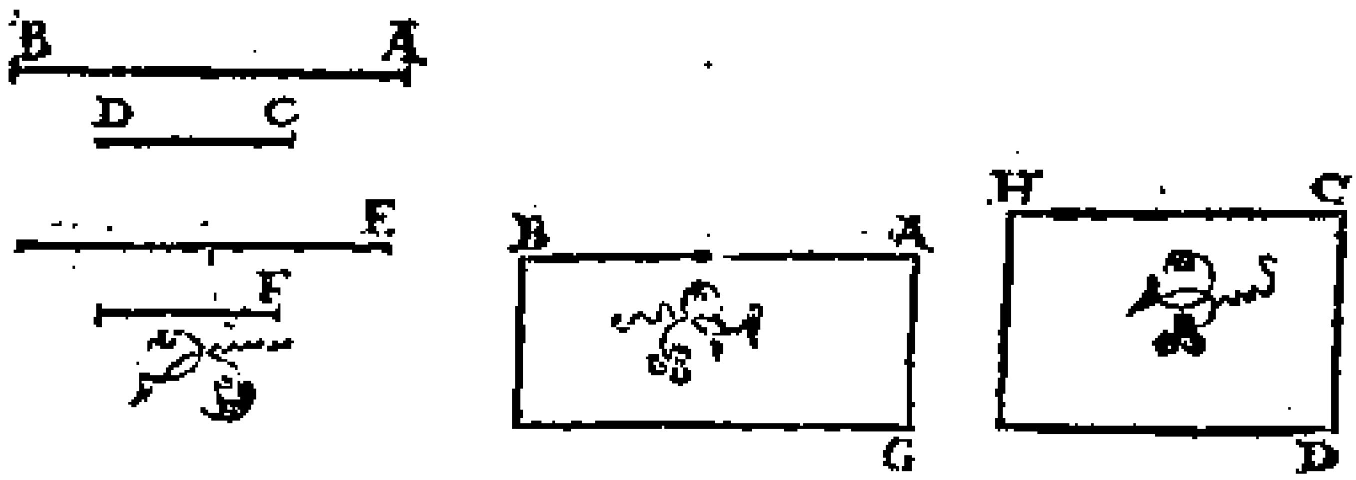
Aequalium, & vnū angulum vni æqualem habentiū triangulorū reciproca ſunt latera, quæ circum æquales angulos: & quorū triangulorū vnum angulum vni ęqualem habentium reciproca ſunt latera, q̄ circum ęquales angulos, illa ſunt æqualia.



Theo-

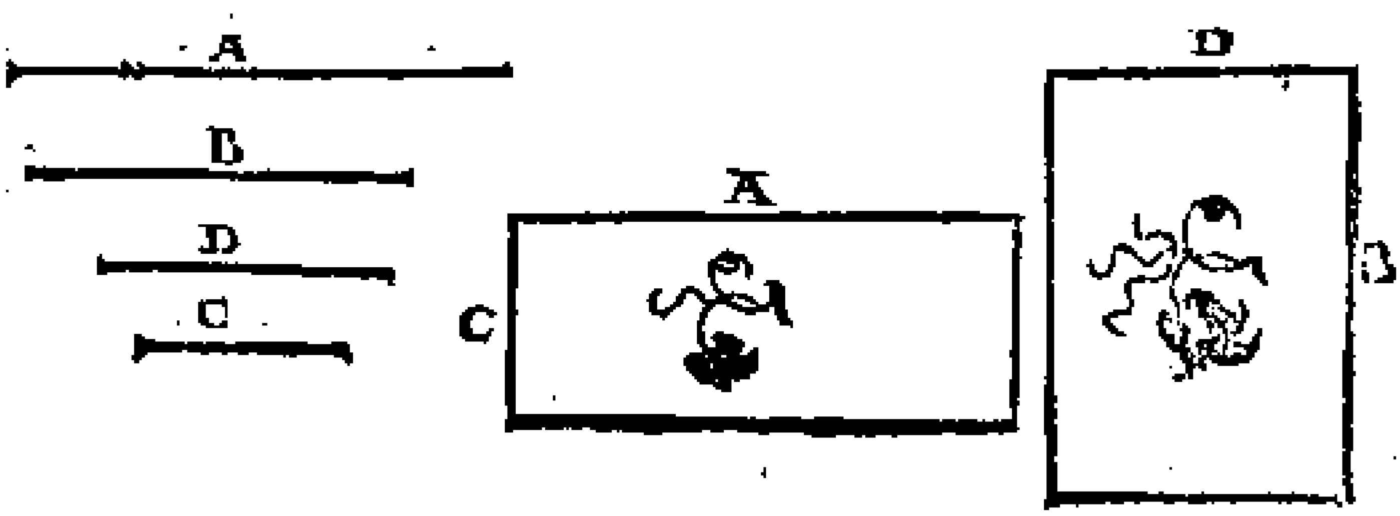
Theorema 11. Propoſitio 16.

Si quatuor rectæ lineæ proportionales fuerint, quod ſub extremis comprehenditur rectangulũ, æquale eſt ei, quod ſub medijs comprehenditur rectãgulo. Et ſi ſub extremis comprehenſum rectangulum æquale fuerit ei, quod ſub medijs cõtinetur rectãgulo, illæ quatuor rectæ lineæ proportionales erunt.



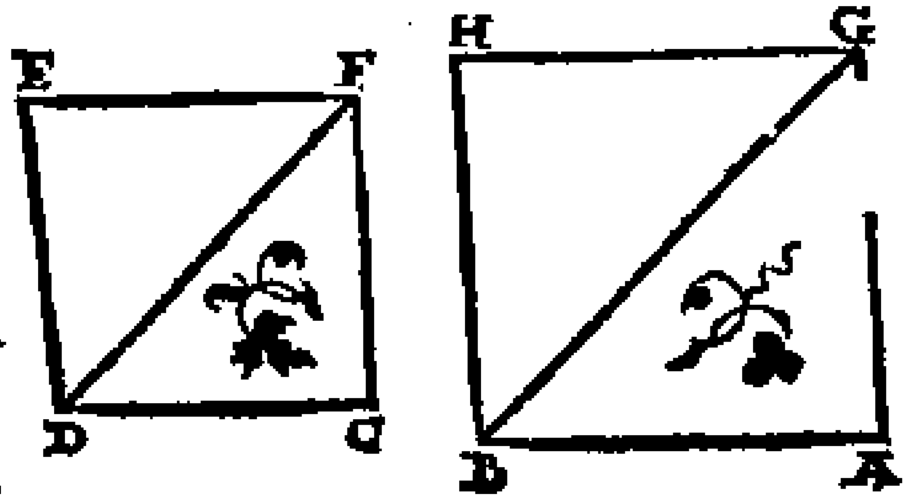
Theorema 12. Propoſitio 17.

Si tres rectæ lineæ ſint proportionales, q̃d ſub extremis comprehenditur rectangulũ ęquale eſt ei, quod à media deſcribitur quadrato: & ſi ſub extremis comprehẽſum rectangulum æquale ſit ei quod à media deſcribitur quadrato, illæ tres rectæ lineæ ꝓportionales erunt.



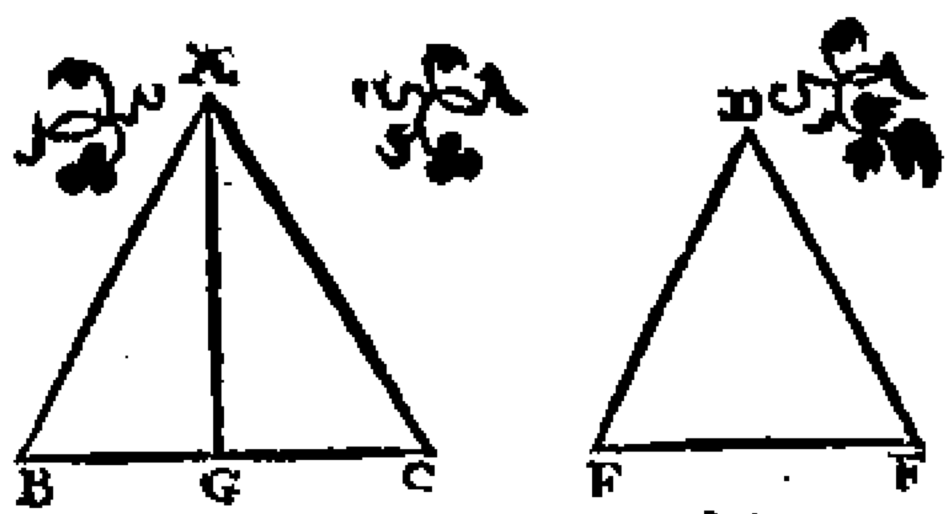
Problema 6. Propositio 18.

A data recta linea, dato recti lineo simile similiterq; positum rectilineum describere.



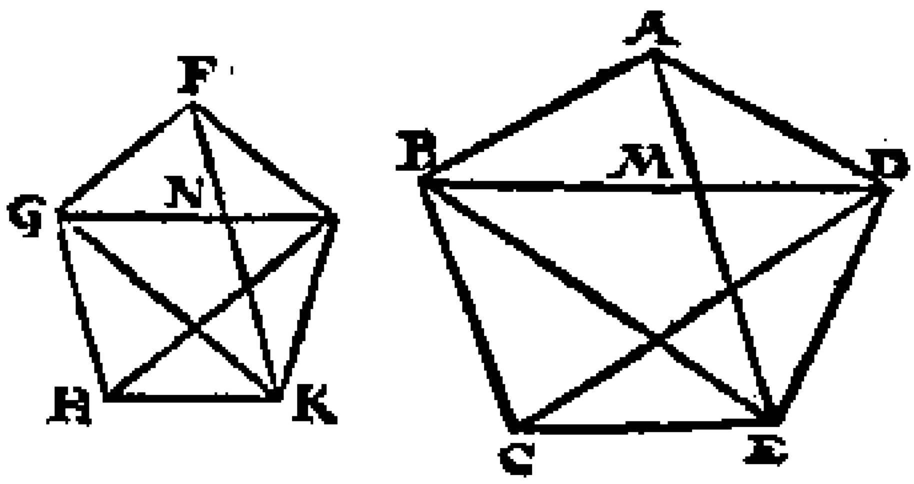
Theorema 13. Propositio 19.

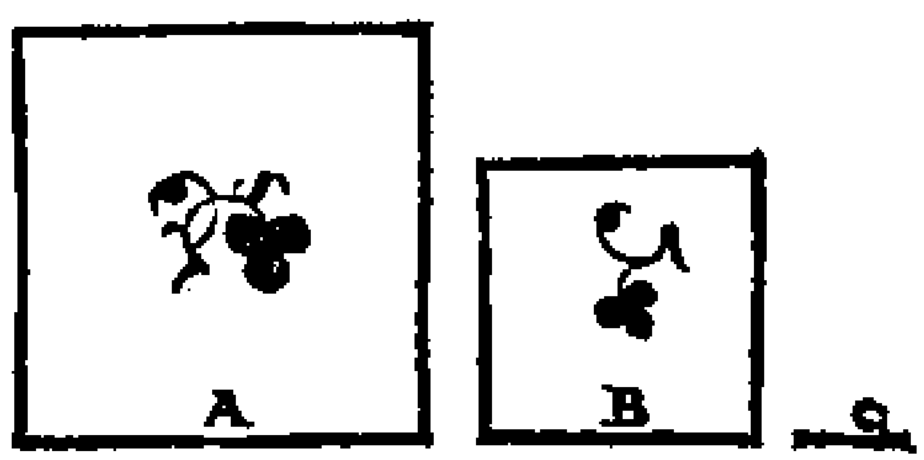
Similia triãgula inter se sũt in duplicata ratióe laterũ homologorũ.



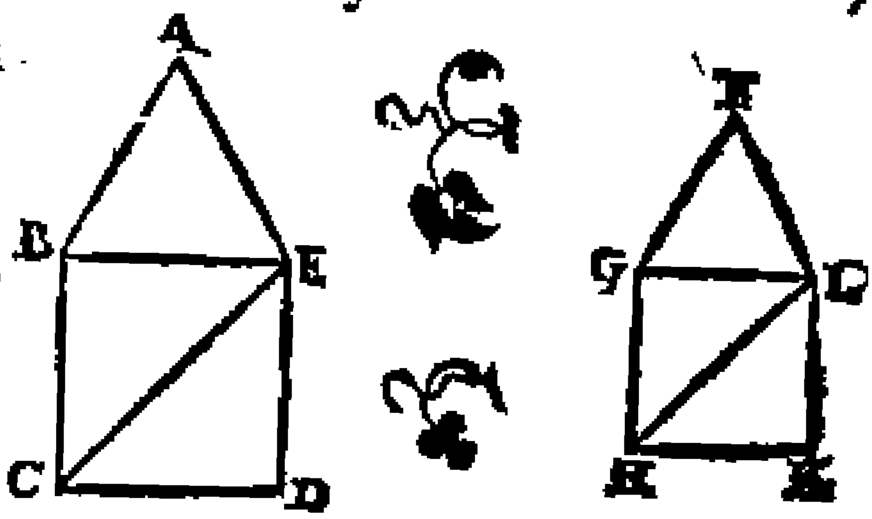
Theore. 14. Propositio 20.

Similia polygœna in similia triangula diuiduntur, & numero æqualia, & homologa totis. Et po



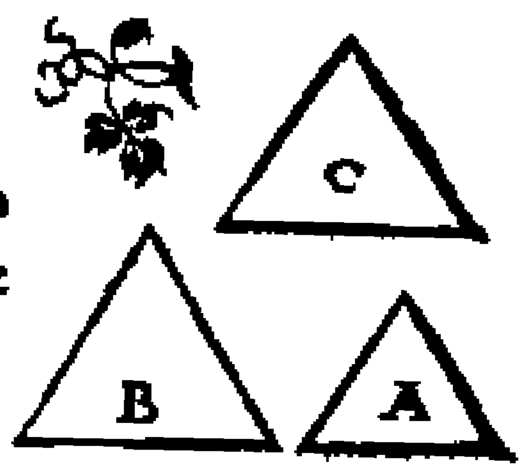


lygona dupplicatam habent eã inter ſe rationē, quã latus homologum ad homologum latus.



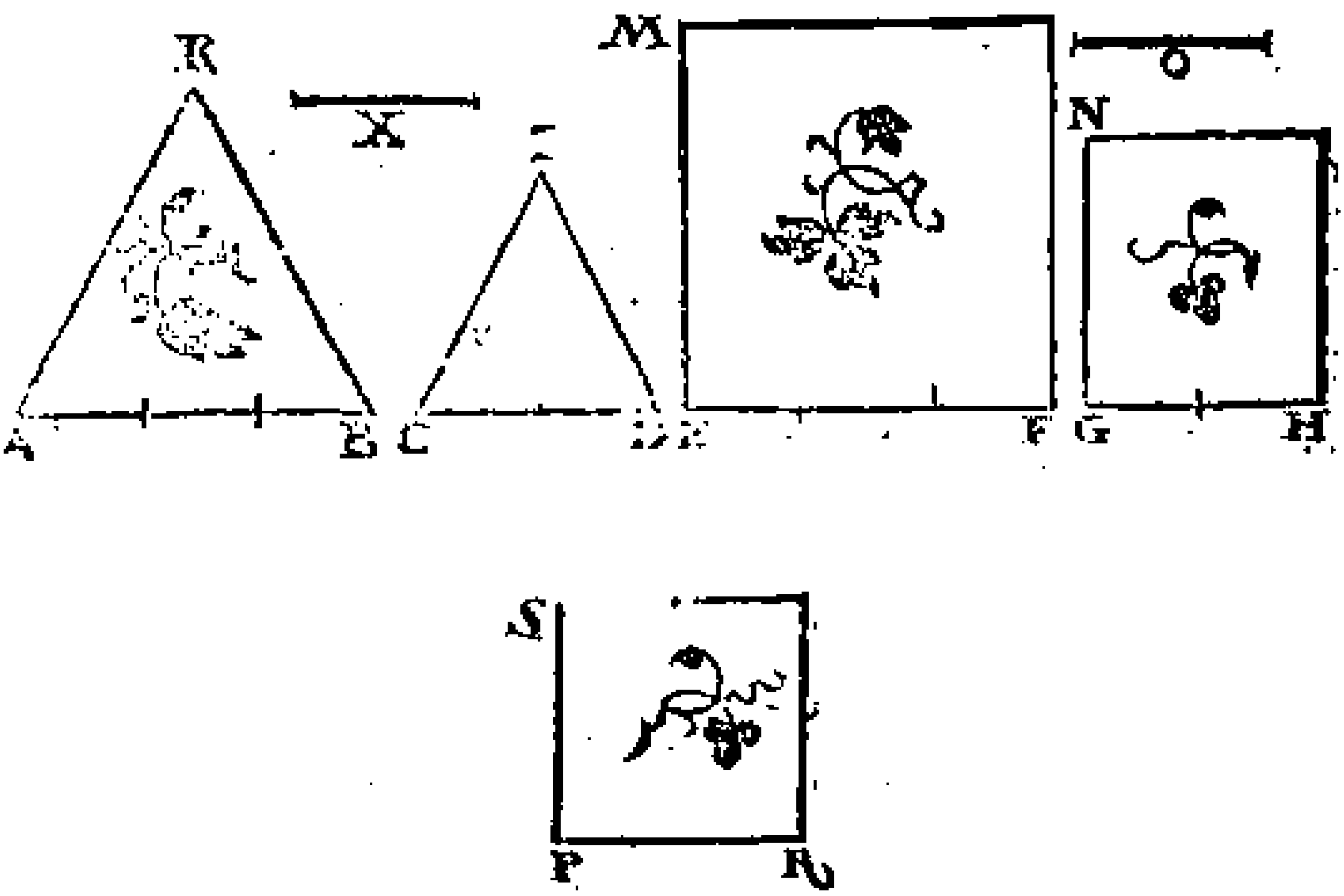
Theorema 15. Propoſitio 21.

Quæ eidem rectilineo ſunt ſimilia, & inter ſe ſunt ſimilia.

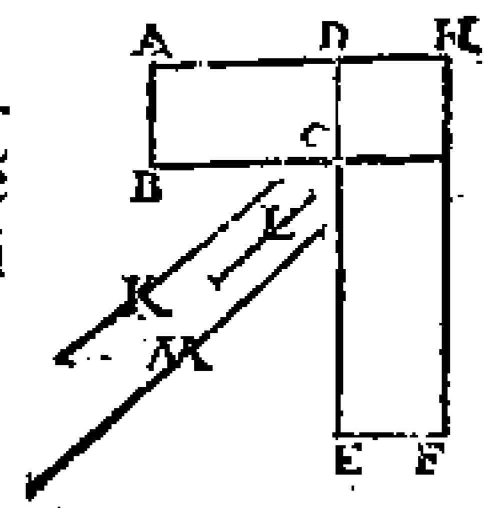


Theorema 16. Propoſitio 22.

Si quatuor rectę lineæ proportionales fuerint: & ab eis rectilinea ſimilia ſimiliterq́; deſcripta proportionalia erũt. Et ſi à rectis lineis ſimilia ſimiliterq́; deſcripta rectilinea proportionalia fuerint, ipſæ etiam rectæ lineæ proportionales erunt.

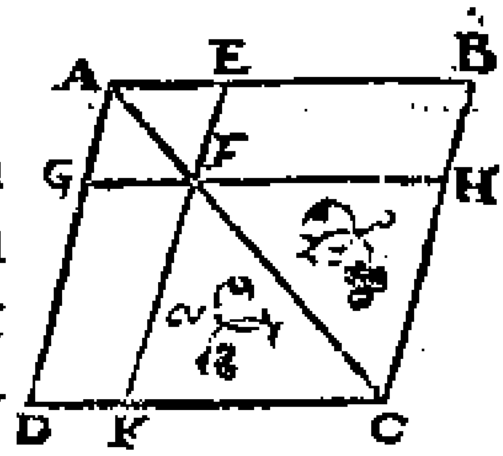


Theorema 17. Propositio 23.



Aequiangula parallelogramma inter se rationē habent eam, quę ex lateribus componitur.

Theorema 18. Propositio 24.

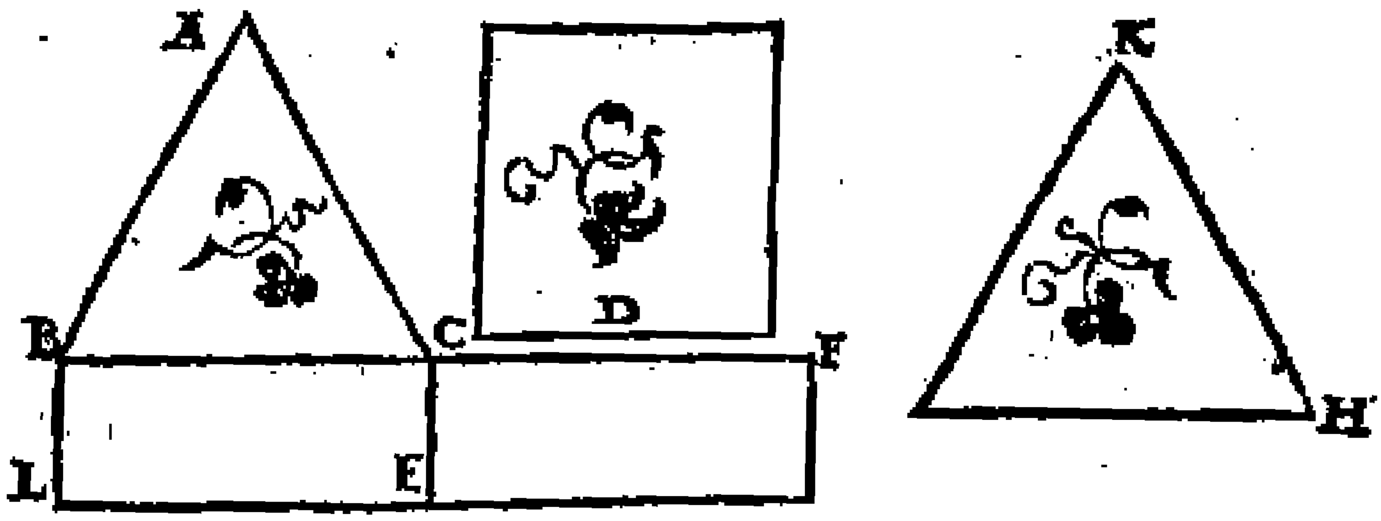


In omni parallelogrammo, quæ circa diametrū sunt parallelogrāma, & toti & inter se sunt similia.

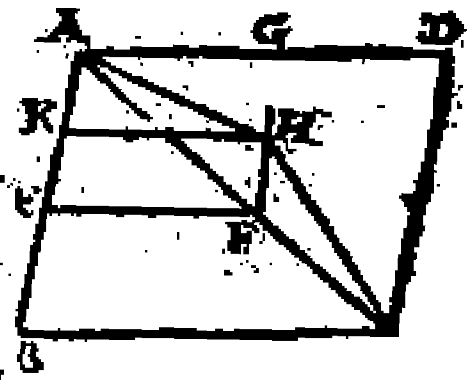
Proble-

Problema 7. Propositio 25.

Dato rectilinco simile, & alteri dato æquale idem constituere.



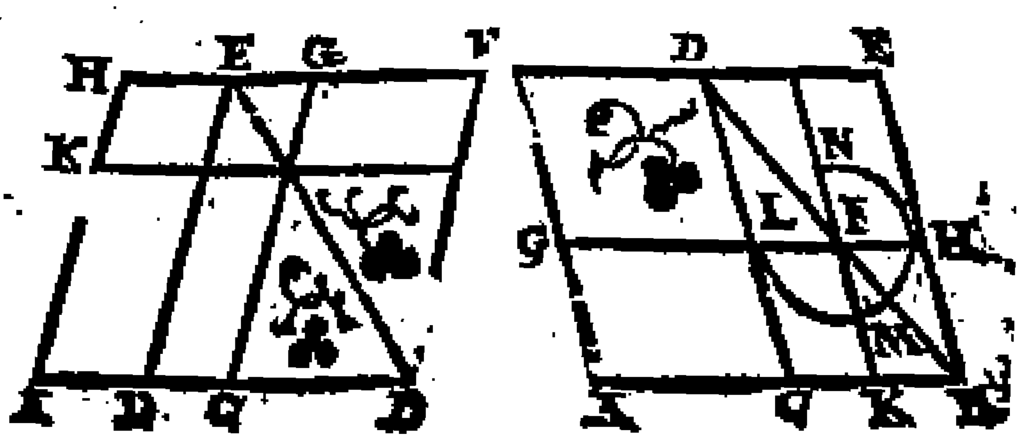
Theorema 19. Propositio 26.



Si à parallelogrãmo parallelogrammũ ablatũ sit, & simile toti & similiter positum communẽ cum eo habens angulum, hoc circum eandem cum toto diametrum consistit.

Theorema 20. Propositio 27.

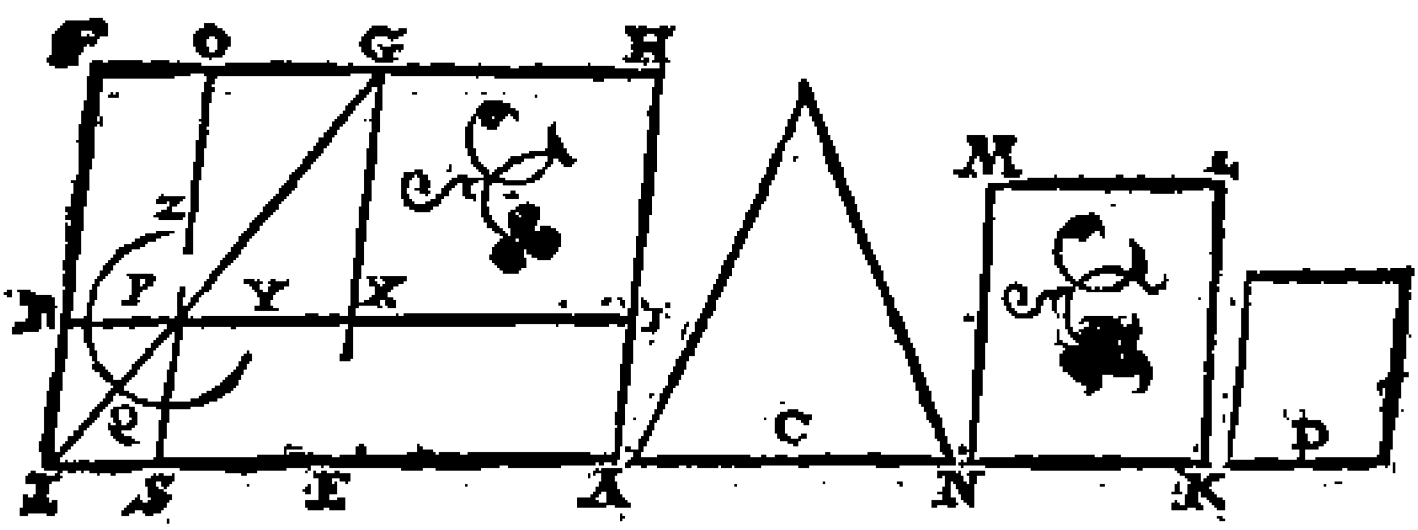
Omnium parallelogrãmorum secũdum eandem rectam lineam applicatorũ deficiẽtiumq; figuris parallelogrãmis si-



imilibus similiterque positis ei, quod à dimidia describitur, maximum, id est, quod ad dimidiam applicatur parallelogrammum simile existens defectui.

Problema 8. Propositio 28.

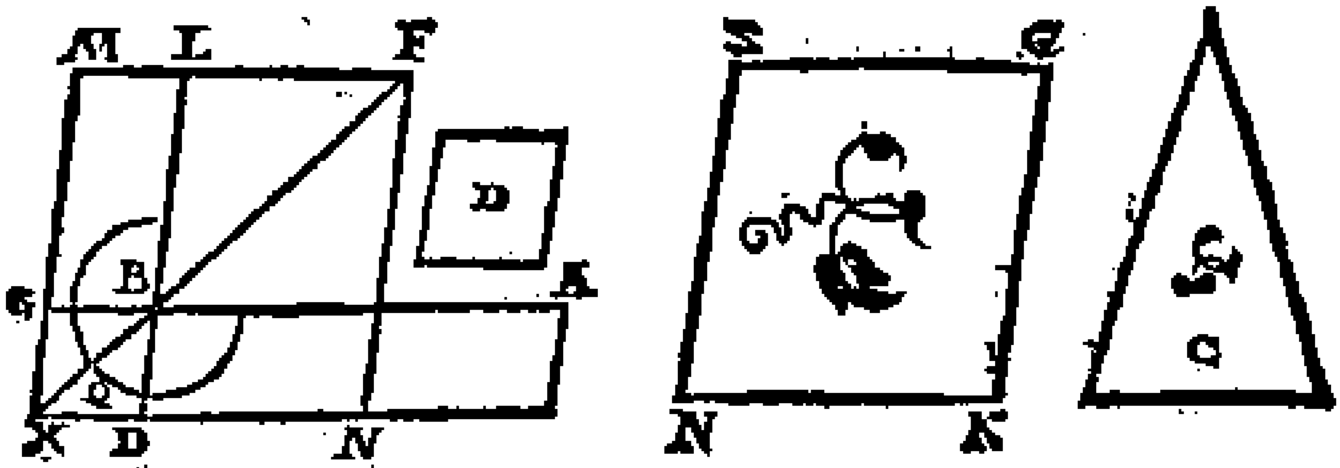
Ad datam lineam rectam, dato rectilineo æquale parallelogrammum applicare deficiens figura parallelogramma, quę similis sit alteri rectilineo dato. Oportet autẽ datum rectilineum, cui æquale applicandũm est, non maius esse eo quod ad dimidiam applicatur, cùm similes sint defectus & eius quod à dimidia describitur, & eius cui simile deesse debet.



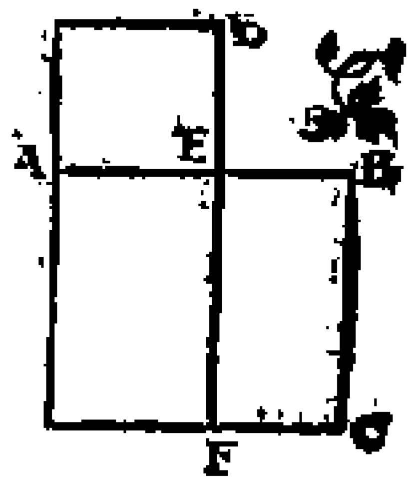
Problema 9. Propositio 29.

Ad datam rectam lineam, dato rectilineo æquale parallelogrammũ applicare, excedẽs figura parallelogrãma, quæ similis sit paral-

parallelogrammo alteri dato.



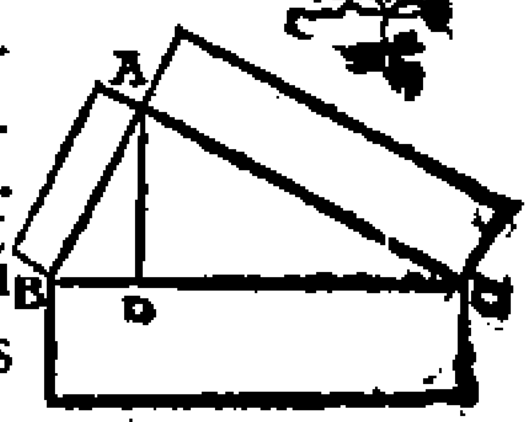
Problema 10. Propoſitio 30.



Propoſitam rectam lineam terminatã, extrema ac media ratione ſecare.

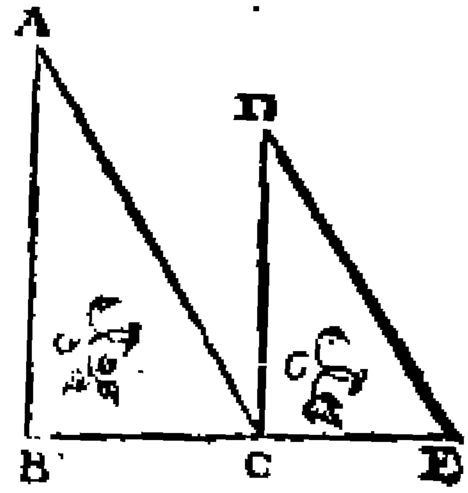
Theorema 21. Propoſitio 31.

In rectangulis triãgulis, figura quęuis à latere rectũ angulũ ſubtendẽte deſcripta ęqualis eſt figuris, quæ priori illi ſimiles & ſimiliter poſitæ à lateribus rectũ angulum cõtinentibus deſcribuntur.



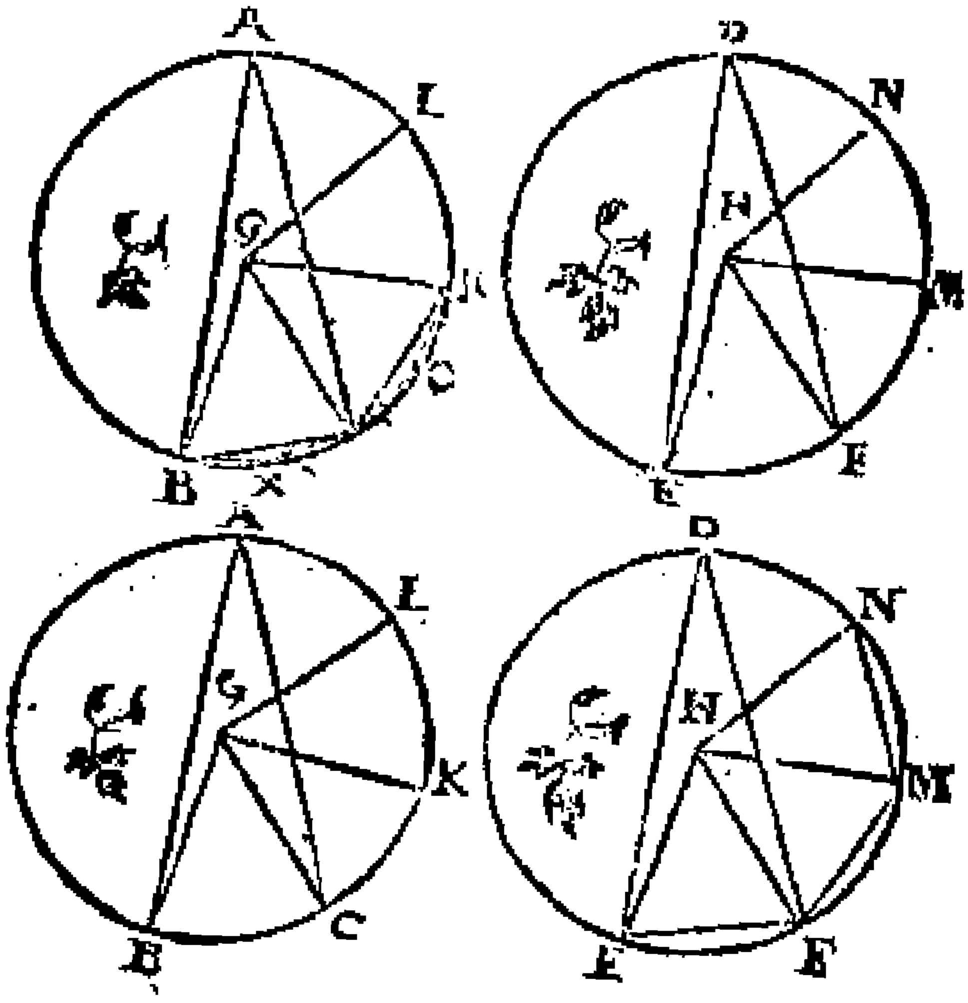
Theorema 22. Propoſitio 32.

Si duo triãgula, quę duo latera duobus lateribus proportionalia habeant, ſecũdum vnum angulũ compoſita fuerint, ita vt homologa eorum latera ſint etiam parallela, tũ reliqua illorum triangulorum latera in rectam lineam collocata reperientur.



Theorema 23. Propoſitio 33.

Inæqualibus circulis anguli eandẽ habent rationem cũ ipſis peripherijs in quibus inſiſtũt, ſiue ad cẽtra, ſiue ad peripherias cõſtituti, illis inſiſtant peripherijs In ſuper verò & ſectores quippe qui ad cẽtra cõſiſtũt.



ELEMENTI VI. FINIS.

EVCLIDIS ELEMENTVM SEPTIMVM.

DEFINITIONES.

1

Vnitas, est secũdum quam entium quodque dicitur vnum.

2

Numerus autem, ex vnitatibus composita multitudo.

3

Pars, est numerus numeri minori maioris, cùm minor metitur maiorem.

4

Partes autem, cùm non metitur.

5

Multiplex verò, maior minoris, cùm maiorem metitur minor.

6

Par numerus est, qui bifariam diuiditur.

7

Impar verò, qui bifariam non diuiditur: vel, qui vnitate differt à pari.

8

Pariter par numerus est, quem par numerus metitur per numerum parem.

9

Pariter autem impar, eſt quem par numerus metitur per numerum imparem.

10

Impariter verò impar numerus, eſt quem impar numerus metitur per numerum imparem.

11.

Primus numerus, eſt quem vnitas ſola metitur.

12

Primi inter ſe numeri ſunt, quos ſola vnitas menſura communis metitur.

13

Compoſitus numerus eſt, quem numerus quiſpiam metitur.

14

Compoſiti autẽ inter ſe numeri ſunt, quos numerus aliquis menſura communis metitur.

15

Numerus numerum multiplicare dicitur, cùm toties compoſitus fuerit is, qui multiplicatur, quot ſunt in illo multiplicante vnitates, & procreatus fuerit aliquis.

16

Cùm autẽ duo numeri mutuò ſeſe multiplicantes quempiam faciũt, qui factus erit planus appellabitur, qui verò numeri mutuò ſeſe multiplicarint, illi⁹ latera dicẽtur.

17 Cùm

17

Cùm verò tres numeri mutuò sese multiplicantes quempiã faciunt, qui procreatus erit solidus appellabitur, qui autem numeri mutuò sese multiplicarint, illius latera dicentur.

18

Quadratus numer⁹ est, qui ęqualiter æqualis: vel, qui à duobus ęqualibus numeris cõtinetur.

19

Cubus verò, qui æqualiter æqualis æqualiter: vel, qui à tribus æqualibus numeris cõtinetur.

20

Numeri proportionales sunt, cùm primus secundi, & tertius quarti æque multiplex est, vel eadem pars, vel eædem partes.

21

Similes plani & solidi numeri sunt, qui proportionalia habent latera.

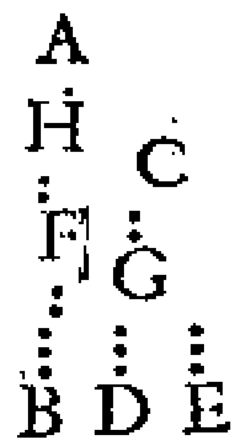
22

Perfectus numerus est, qui suis ipsius partibus est æqualis.

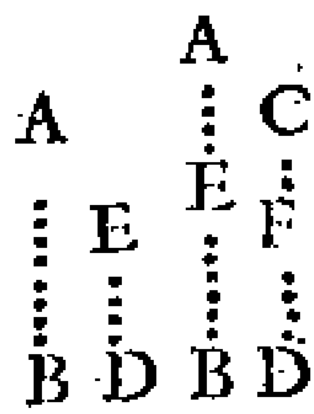
Theorema 1. Propositio 1.

Duobus numeris inæqualibus propositis,

tis, ſi detrahatur ſemper minor de maiore, alterna, quadam detractione, neque reliquus vnquam metiatur præcedentem quoad aſſumpta ſit vnitas: qui principio propoſiti ſunt, numeri primi inter ſe erunt.

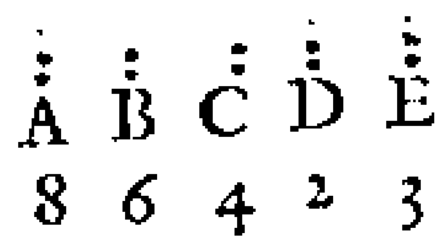


Problema 1. Propoſitio 2.

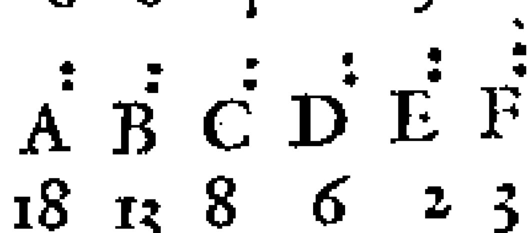


Duobus numeris datis non primis inter ſe, maximã eorum communem menſuram reperire.

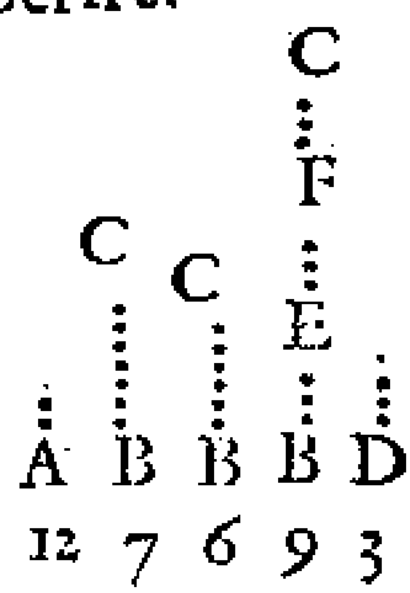
Problema 2. Propoſitio 3.



Tribus numeris datis non primis inter ſe, maximam eorum communem menſuram reperire.



Theorema 2. Propoſitio 4.

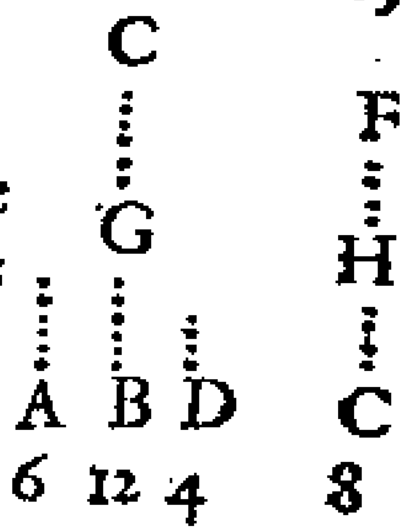


Omnis numerus, cuiuſq; numeri minor maioris aut pars eſt, aut partes.

Theore-

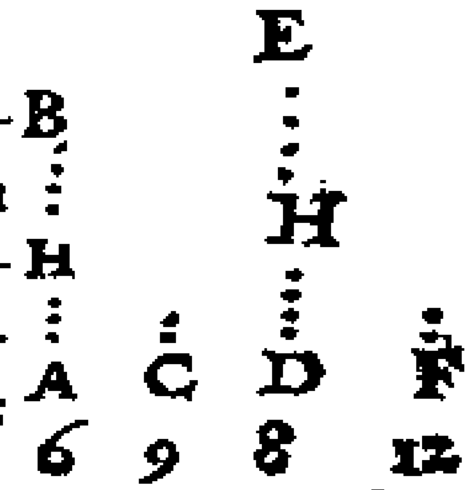
Theorema 3. Propositio 5.

Si numerus numeri pars fuerit, & alter alterius eadẽ pars & simul vterque vtriusque simul eadem pars erit, quæ vnus est vnius.



Theor. 4. Propo. 6.

Si numer⁹ sit numeri partes, & alter alterius eędem partes, & simul vterque vtriusque simul eædẽ partes erunt, quæ sunt vnus vnius.

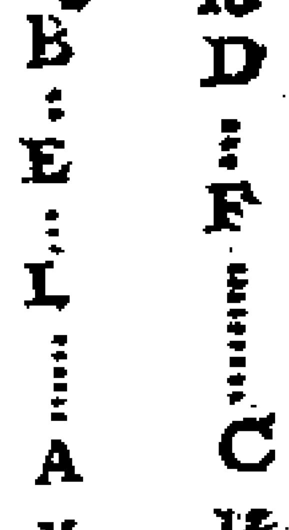


Theorema 5. Propositio 7.

Si numerus numeri eadẽ sit pars quæ detractus detracti, & reliquus reliqui eadẽ pars erit, quæ totus est totius.

Theorema 6. Propositio 8.

Si numerus numeri eædẽ sint partes quæ detractus detracti, & reliquus reliqui eędẽ partes erunt, quæ sunt totus totius.

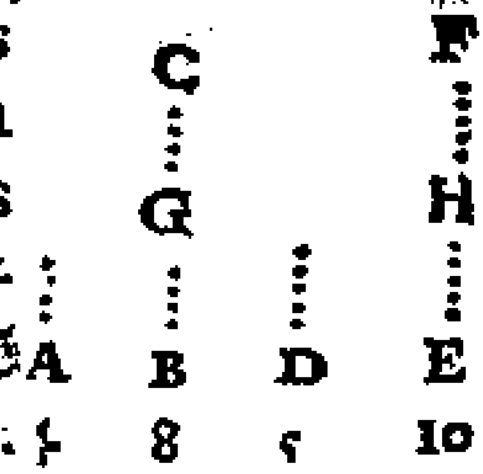


Theo-

G..M.K..N.H.

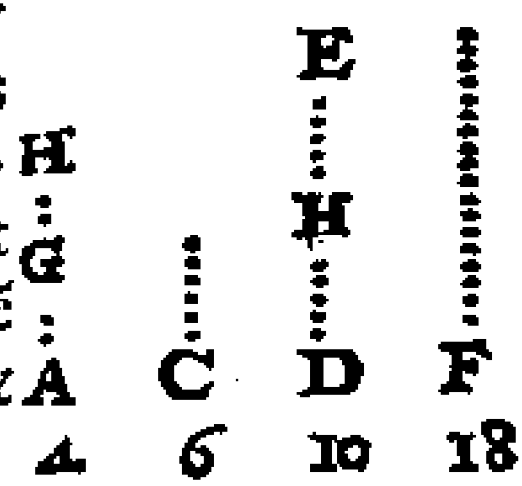
Theorema 7. Propositio 9.

Si numerus numeri pars sit, & alter alterius eadem pars, & vicissim quæ pars est vel partes primus tertij, eadē pars erit vel eędē partes, & secūdus quarti.



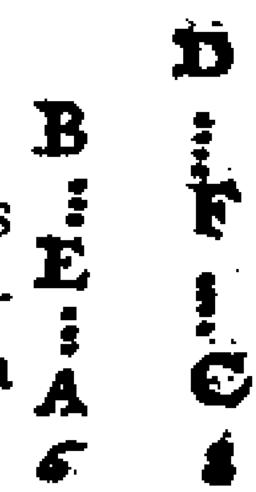
Theorema 8. Propositio 10.

Si numerus numeri partes sint, & alter alterius eædē partes, etiam vicissim quæ sunt partes aut pars primus tertij, eædē partes erunt vel pars & secundus quarti.



Theorema 9. Propositio 11.

Si quemadmodum se habet totus ad totum, ita detractus ad detractum, & reliquus ad reliquum ita habebit vt totus ad totum.



Theorema 10. Propositio 12.

Si sint quotcunque numeri proportionales, quemadmodum se habet vnus antecedentium ad vnum sequētium, ita se habe-

A	B	C	D
9	6	3	2

habebunt omnes antecedentes ad omnes consequentes.

Theorema 11. Propositio 13.

Si quatuor numeri sint proportionales, & vicissim ꝓportionales erunt.

A	B	C	D
12	4	9	3

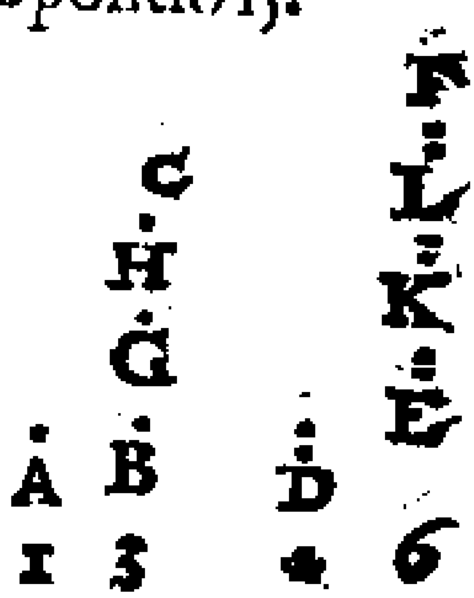
Theorema 12. Propositio 14.

Si sint quotcunque numeri, & alij illis æquales multitudine, qui bini sumantur & in eadem ratione: etiam ex æqualitate in eadem ratione erunt.

A	B	C	D	E	F
12	6	3	8	4	2

Theorema 13. Propositio 15.

Si vnitas numerum quẽpiã metiatur, aliter verò numerus alium quendã numerũ æquè metiatur, & vicissim vnitas tertium numerũ æquè metietur, atque secundis quartum.



Theorema 14. Propositio 16.

Si duo numeri mutuò sese multiplicãtes faciant aliquos qui ex illis geniti fuerint, inter se æquales erunt.

E	A	B	C	D
1	2	4	8	8

Theorema 15. Propositio 17.

Si numer⁹ duos numeros multiplicãs faciat aliquos, qui ex illis procreati erunt, eandem rationem habebunt, quam multiplicati.

I	A	B	C	D	E
1	3	4	5	12	15

Theorema 16. Propositio 18.

Si duo numeri numerum quempiam multiplicantes faciant aliquos, geniti ex illis eandem habebunt rationem, quam qui illum multiplicarunt.

A	B	C	D	E
4	5	3	12	15

Theorema 17. Propositio 19.

Si quatuor numeri sint proportionales, q̃ ex primo & quarto fit, ęqualis erit ei qui ex secundo & tertio: & si qui ex primo & quarto fit numerus æqualis sit ei qui ex secũdo & tertio, illi quatuor numeri proportionales erunt.

A	B	C	D	E	F	G
6	4	3	2	12	12	18

Theorema 18. Propositio 20.

Si tres numeri sint proportionales, qui ab extremis cõtinetur, ęqualis est ei qui à medio

dio efficitur. Et si qui ab extremis cõtinetur æqualis sit ei qui à medio describitur, illi tres nũmeri proportionales erunt.

A	B	C
9	6	4.
D		
6		

Theorema 19. Propositio 21.

Minimi numeri omniũ, qui eandem cũ eis rationem habẽt, æqualiter metiũtur numeros eandem rationem habẽtes, maior quidẽ maiorem, minor verò minorem.

D	L		
G	H		
C	E	A	B
4	3	8	6

Theorema 20. Propositio 22.

Si tres sint numeri & alij multitudine illis æquales, qui bini sumantur & in eadem ratione, sit autem perturbata eorũ proportio, etiam ex æqualitate in eadem ratione erunt.

A	B	C	D	E	F
6	4	3	12	8	6

Theorema 21. Propositio 23.

Primi inter se numeri minimi sunt omniũ eandem cum eis rationem habentium.

A	B	E	C	D
5	6	2	4	3

H 2 Theo-

Theorema 22. Propoſitio 24.

Minimi numeri omnium eandem cum eis rationem habentiũ, primi ſunt inter ſe.

A	B	C	D	E
8	6	4	3	2

Theorema 23. Propoſitio 25.

Si duo numeri ſint primi inter ſe, qui alterutrum illorum metitur numerus, is ad reliquũ primus erit.

A	B	C	D
6	7	3	4

Theorema 24. Propoſitio 26.

Si duo numeri ad quẽpiam numerũ primi ſint, ad eundẽ primus is quoque futurus eſt, qui ab illis productus fuerit.

A	B	C	D	E	F
5	3	5	5	3	2

Theorema 25. Propoſitio 27.

Si duo numeri primi ſint inter ſe, q ab vno eorũ gignitur ad reliquum primus erit.

B	A	C	D
	7	6	3

Theorema 26. Propoſitio 28.

Si duo numeri ad duos numeros ambo ad vtrunque primi ſint, & qui ex eis gignentur, primi inter ſe erunt.

A	B	E	C	D	F
3	5	15	2	4	8

Theore-

Theorema 27. Propositio 29.

Si duo numeri primi sint inter se, & multiplicãs vterq; seipsum procreet aliquẽ, qui ex ijs producti fuerint, primi inter se erũt. Quòd si numeri initio propositi multiplicãtes eos qui producti sunt, effecerint aliquos, hi quoq; inter se primi erũt, & circa extremos idẽ hoc semper eueniet.

:	:	:	:	:	:
A	C	E	B	D	F
3	6	27	4	16	63

Theorema 28. Propositio 30.

Si duo numeri primi sint inter se, etiam simul vterq; ad vtrunq; illorũ primus erit. Et si simul vterq; ad vnum aliquem eorũ primus sit, etiam qui initio positi sunt numeri, primi inter se erunt.

	C	
:	:	:
A	B	D
7	5	4

Theorema 29. Propositio 31.

Omnis primus numerus ad omnem numerum quem non metitur, primus est.

:	:	:
A	B	C
7	10	5

Theorema 30. Propositio 32.

Si duo numeri sese mutuò multiplicãtes faciant aliquem, hũc aũt ab illis productum metiatur primus quidam numerus, is alterum etiã metitur eorum qui initio positi erant.

:	:	:	:	:
A	B	C	D	E
2	6	12	3	4

Theorema 31. Propositio 33.

Omnem compositum numerum aliquis primus metietur.

A	B	C
27	9	3

Theorema 32. Propositio 34.

Omnis numerus aut primus est, aut cum aliquis primus metitur.

A	A	
3	6	3

Problema 3. Propositio 35.

Numeris datis quotcunque, reperire minimos omnium qui eandem cum illis rationem habeant.

A	B	C	D	E	F	G	H	K	I	M
6	8	12	2	3	4	6	2	3	4	3

Problema 4. Propositio 36.

B				
A	C	D	E	F
7	12	8	4	5

Duobus numeris datis reperire quem illi minimũ metiantur numerum.

A					
F	E	C	D	G	H
6	9	12	9	2	3

Theorema 33. Propositio 37.

Si duo numeri numerū quempiam metiantur, & minimus quem illi metiuntur eundem metietur.

F

A	B	E	C
2	3	6	12

Problema 5. Propositio 38.

Tribus numeris datis reperire quem minimum numerū illi metiantur.

A	B	C	D	E
3	4	6	12	8

A	B	C	D	E	F
3	6	8	12	24	16

Theorema 34. Propositio 39.

Si numerum quispiam numerus metiatur, mensus partem habebit metienti cognominem.

A	B	C	D
12	4	3	1

Theorema 35. Propositio 40.

Si numerus partem habuerit quālibet, illum metietur numerus parti cognominis.

A	B	C	D
8	4	2	1

Problema 6. Propositio 41.

Numerum reperire, qui minimus cùm sit, datas habeat partes.

A	B	C	G	H
2	3	4	12	10

ELEMENTI VII. FINIS.

EVCLIDIS
ELEMENTVM
OCTAVVM.

Theorema 1. Propoſitio 1.

Si ſint quotcũq; numeri deinceps proportionales, quorum extremi ſint inter ſe, primi, minimi ſunt omnium eandem cum eis rationem habentium.

A	B	C	D	E	F	G	H
8	12	18	27	6	8	12	18

Problema 1. Propoſitio 1.

Numeros reperire deinceps proportionales minimos, quotcunq; iuſſerit quiſpiam in data ratione.

A	B	C	D	E	F	G	H	K
3	4	9	12	16	27	36	48	64

Theorema 2. Propoſitio 3.
Conuerſa primæ.

Si ſint quotcũq; numeri deinceps proportionales minimi habentium eandem cum eis rationem, illorum extremi ſunt inter ſe primi.

A	B	C	D	E	F	G	H	K	L	M	N	O
27	16	48	64	3	4	9	12	16	27	36	48	64

Pro-

Problema 2. Propositio 4.

Rationibus datis quotcunque in minimis numeris reperire numeros deinceps minimos in datis rationibus.

A	B	C	D	E	F	H	G	K	L	N	X	M	O
3	4	2	3	4	5	6	8	12	15	4	6	10	12

Theorema 3. Propositio 5.

Plani numeri rationem inter se habent ex lateribus compositam.

A	L	B	C	D	E	F	G	H	K
18	22	32	3	6	4	8	9	12	16

Theorema 4. Propositio 6.

A	B	C	D	E	F	G	H
16	24	36	54	82	4	6	9

Si sint quotlibet numeri deinceps proportionales, primus autem secundum non metiatur, neque alius quispiam vllum metietur.

Theorema 5. Propoſitio 7.

Si ſint quotcunque numeri deinceps proportionales, primus autem extremum metiatur, is etiam ſecundum metietur.

A B C D
4 6 12 24

Theorema 6. Propoſitio 8.

Si inter duos numeros medij continua proportione incidant numeri, quot inter eos medij continua proportione incidunt numeri, tot & inter alios eandem cum illis habentes rationem medij continua proportione incident.

A C D B G H K L C M N F
4 9 27 81 1 3 9 27 2 6 18 54

Theorema 7. Propoſitio 9.

Si duo numeri ſint inter ſe primi, & inter eos medij cõtinua proportione incidãt numeri, quot inter illos medij continua proportione incidunt numeri, totidẽ & inter vtrunque eorum ac vnitatẽ deinceps medij continua proportione incident.

A M H E F N C K X G D L O B
27 27 9 36 3 36 1 12 48 4 48 16 64 64

Theo-

Theorema 8. Propoſitio 10.

Si inter duos numeros & vnitatẽ continuè proportionales incidãt numeri quot inter vtrunque ipſorum & vnitatẽ deinceps medij continua proportione incidunt numeri, totidẽ & inter illos medij continua proportione incident.

A	E	K	D	H	C	L	F	G	B
27	9	36	3	12	1	48	4	16	64

Theorema 9. Propoſitio 11.

Duorum quadratorũ numerorum vnus medius proportionalis eſt numerꝰ: & quadratus ad quadratum duplicatam habet lateris ad latus rationem.

A	C	E	D	B
9	3	12	4	16

Theorema 10. Propoſitio 12.

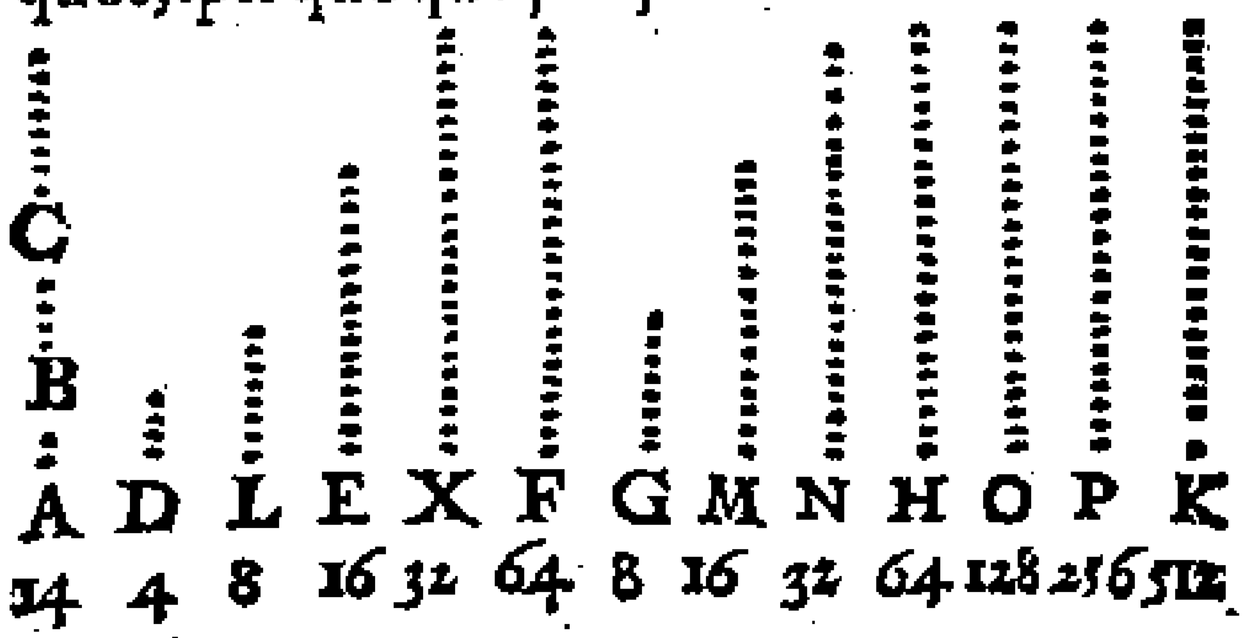
Duorum cuborum numerorũ duo medij proportionales ſunt numeri: & cubus ad cubum triplicatam habet lateris ad latus rationem.

A	H	K	B	C	D	E	F	G
27	36	48	64	3	4	9	12	16

Theo-

Theorema 11. Propositio 13.

Si ſint quotlibet numeri deinceps proportionales, & multiplicans quiſq; ſeipſum faciat aliquos, qui ab illis producti fuerint, proportionales erũt: & ſi numeri primùm poſiti, ex ſuo in procreatos ductu faciãt aliquos, ipſi quoque proportionales erunt.



Theorema 12. Propoſitio 14.

Si quadratus numerus quadratum numerũ metiatur, & latus vnius metietur latus alterius. Et ſi vnius quadrati latus metiatur latus alterius, & quadratus quadratum metietur.

A	E	B	C	D
9	12	16	3	4

Theorema 13. Propoſitio 15.

Si cubus numerus cubum numerũ metiatur, & latus vni⁹ metietur alterius latus. Et ſi latus vnius cubi latus alterius metiatur,

tum

tum cubus cubum metietur.

A	H	K	B	C	D	E	F	G
8	16	28	64	2	4	4	8	16

Theorema 14. Propositio 16.

Si quadratus numerus quadratum numerum non metiatur, neque latus vnius metietur alterius latus.
Et si latus vnius quadrati nõ metiatur latus alterius, neque quadratus quadratum metietur.

A	B	C	D
9	16	3	4

Theorema 15. Propositio 17.

Si cubus numerus cubum numerũ nõ metiatur, neque latus vnius latus alterius metietur.
Et si latus cubi alicuius latus alterius nõ metiatur, neque cubus cubũ metietur.

A	B	C	D
8	27	9	11

Theorema 16. Propositio 18.

Duorum similium planorum numerorũ vnus medius proportionalis est numerus, & planus ad planũ duplicatam habet literis homologi

A	G	B	C	D	E	F
12	18	27	2	6	3	9

logi ad latus homologum rationem.

Theorema 17. Propoſitio 19.

Inter duos ſimiles numeros ſolidos, duo medij proportionales incidunt numeri: & ſolidus ad ſimilem ſolidum triplicatam rationem habet lateris homologi ad latus homologum.

A	N	X	B	C	D	E	F	G	H	K	M	L
8	12	18	27	2	2	2	3	3	3	4	6	9

Theorema 18. Propoſitio 20.

Si inter duos numeros vn⁹ medius ꝓportionalis incidat numer⁹; ſimiles plani erunt illi numeri.

A	C	B	D	E	F	G
18	24	33	3	4	6	8

Theorema 19. Propoſitio 21.

Si inter duos numeros duo medij proportionales incidant numeri; ſimiles ſolidi ſunt illi numeri.

A	C	D	B	E	F	G	H	K	L	M
27	36	44	64	9	12	16	3	3	3	4

Theo-

Theorema 20. Propoſitio 22.

Si tres numeri deinceps ſint proportionales, primus autem ſit quadratus, & tertius quadratus erit.

A	B	D
9	15	25

Theorema 21. Propoſitio 23.

Si quatuor numeri deinceps ſint proportionales, primus autem ſit cubus, & quartus cubus erit.

A	B	C	D
8	12	18	27

Theorema 22. Propoſitio 24.

Si duo numeri rationem habeant inter ſe, quam quadratus numerus ad quadratum numerum, primus autem ſit quadratus, & ſecundus quadratus erit.

A		B	C		D
4	6	9	16	24	36

Theorema 23. Propoſitio 25.

Si numeri duo rationem inter ſe habeant, quam cubus numerus ad cubum numerũ; primus autem cubus ſit, & ſecundus cubus erit.

A	E	F	B	C			D
8	12	18	27	64	95	140	216

Theo-

Theorema 24. Propositio 26.

Similes plani numeri rationem inter se habent, quam quadratus numerus ad quadratum numerum.

A	C	B	D	E	F
18	24	32	9	12	16

Theorema 25. Propositio 27.

Similes solidi numeri rationem habēt inter se, quam cubus numerus ad cubum numerum.

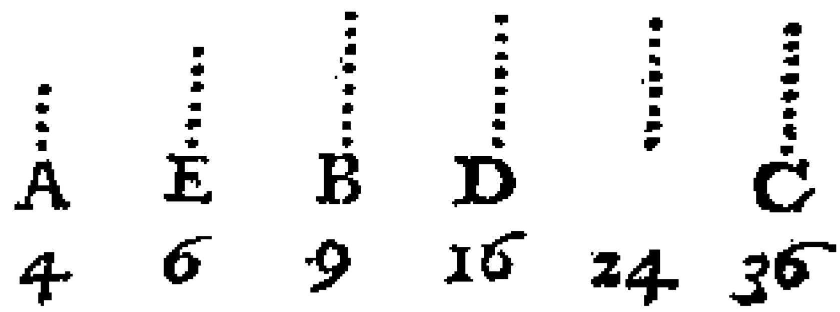
A	C	D	B	E	F	G	H
36	24	16	54	8	12	18	27

ELEMENTI VIII. FINIS.

EVCLIDIS ELEMENTVM NONVM.

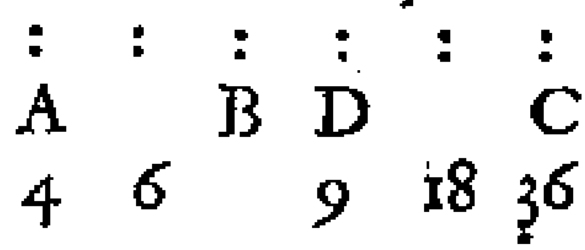
Theorema 1. Propositio 1.

Si duo similes plani numeri mutuò sese multiplicãtes quendã procreent, productus quadratus erit.



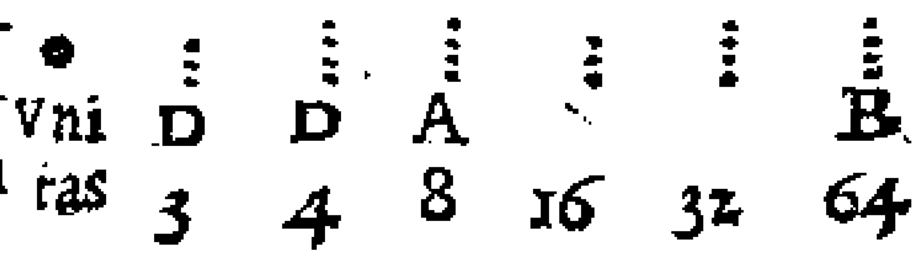
Theorema 2. Propositio 2.

Si duo numeri mutuò sese multiplicantes quadratum faciant, illi similes sunt plani.



Theorema 3. Propositio 3.

Si cubus numerus seipsum multiplicãs ꝓcreet aliquẽ, productus cubus erit.



Theorema 4. Propositio 4.

Si cubus numerus cubũ numerum multiplicans quendam procreet, procreatus cubus erit.

A	B	D	C
8	27	64	216

Theorema 5. Propositio 5.

Si cubus numerus numerum quendã multiplicans cubum procreet, & multiplicatus cubus erit.

A	B	D	D
27	64	729	1728

Theorema 6. Propositio 6.

Si numerus ſeipſum multiplicans cubum procreet, & ipſe cubus erit.

A	B	C
27	729	19683

Theorema 7. Propositio 7.

Si compoſitus numerus quẽdam numerũ multiplicans quempiam procreet, productus ſolidus erit.

A	B	C	D	E
6	8	48	2	3

Theorema 8. Propoſitio 8.

Si ab vnitate quotlibet numeri deinceps ꝓportionales ſint, tertius ab vnitate quadratus eſt, & vnũ intermittentes omnes: quartus autẽ cubus, & duobus intermiſsis omnes:

nes.ſeptimus verò cubus ſimul & æquadratus, & quinque intermiſſis omnes

Vnitas	A	B	C	D	E	F
	3	9	27	81	243	729

Theorema 9. Propoſitio 9.

Si ab vnitate ſint quotcunque numeri deinceps proportionales, ſit autem quadratus is qui vnitatem ſequitur, & reliqui oés quadrati erũt. Quòd ſi qui vnitatem ſequitur cubus ſit, & reliqui omnes cubi erunt.

quadrati.		cubi
531441	F	732969
59049	E	531441
6561	D	59049
729	C	6561
81	B	729
9	A	81
	o	
	Vnitas.	

Theorema 10. Propoſitio 10.

Si ab vnitate numeri quotcunque proportionales ſint, non ſit autẽ quadratus is qui vnitatem ſequitur, neq; alius vllus quadratus erit, dẽptis tertio ab vnitate ac omnibus

Vnitas	A	B	C	D	E	F
o	3	9	36	81	243	729

nibus vnum intermittẽtibus. Quòd ſi qui vnitatem ſequitur, cubus non ſit, neque alius vllus cubus erit, demptis quarto ab vnitate ac omnibus duos intermittentibus.

Theorema 11. Propoſitio 11.

Si ab vnitate numeri quotlibet deinceps proportionales ſint, minor maiorẽ metitur per quempiam eorum qui in proportionalibus ſunt numeris.

A	D	C	D	E
1	2	4	8	16

Theorema 12. Propoſitio 12.

Si ab vnitate quotlibet numeri ſint ꝓportionales, quot primorum numerorũ vltimum metiuntur, totidem & eum qui vnitati proximus eſt, metientur.

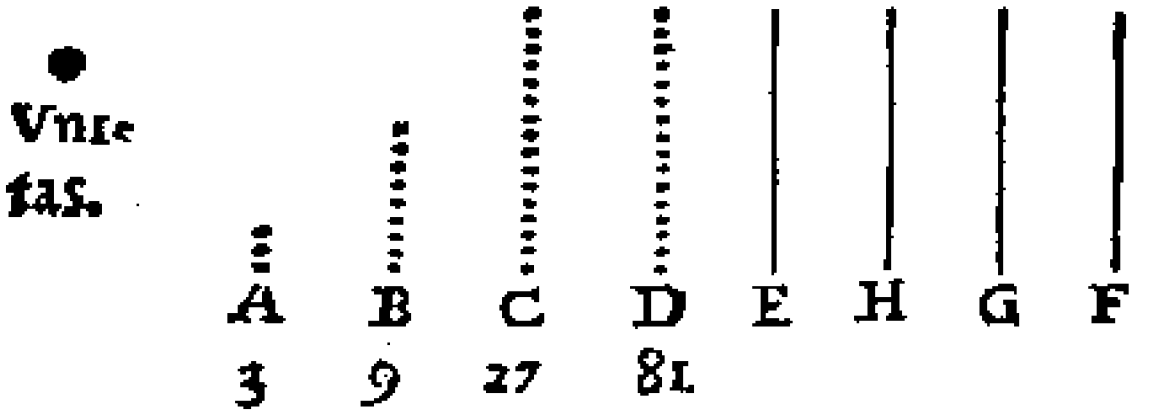
Vnitas.	A	B	C	D	E	H	G	F
	4	16	64	259	2	8	32	128

Theorema 13. Propoſitio 13.

Si ab vnitate ſint quotcunq; numeri deinceps proportionales, primus autẽ ſit qui vnitatẽ ſequitur, maximũ nullus alius me-

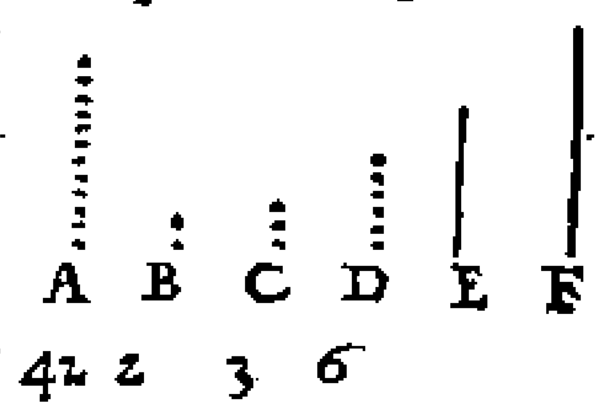
tie-

tietur, ijs exceptis qui in proportionalib9 sunt numeris.



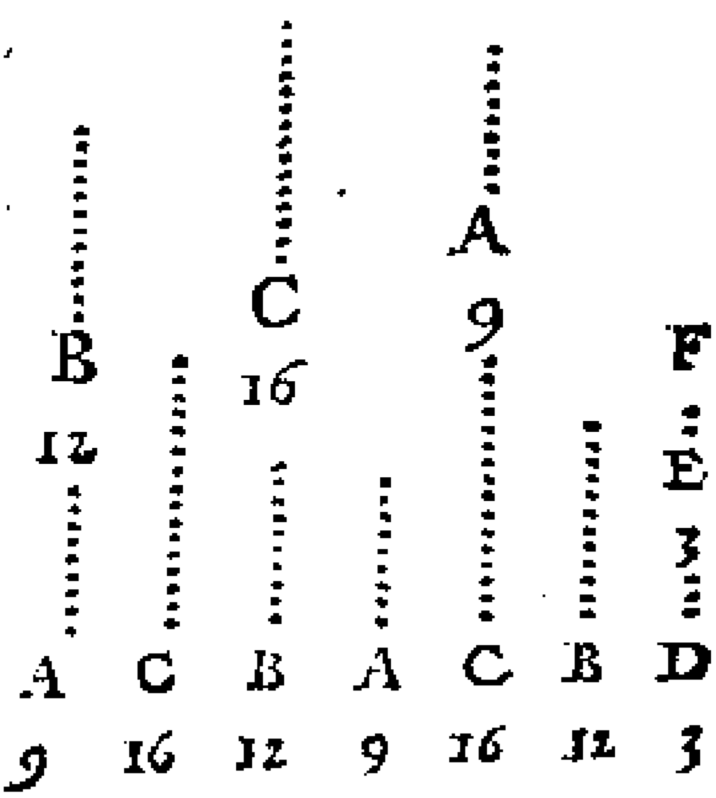
Theorema 14. Propositio 14.

Si minimum numerum primi aliquot numeri metiãtur, nullus alius numerus primus illum metietur, ijs exceptis qui primò metiuntur.



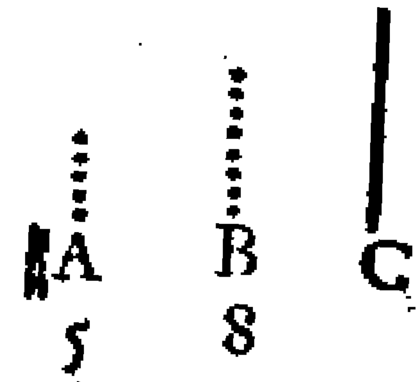
Theorema 15. Propositio 15.

Si tres numeri deinceps proportionalis sint minimi, eadẽ cum ipsis habẽtium rationẽ, duo quilibet compositi ad tertium primi erunt.



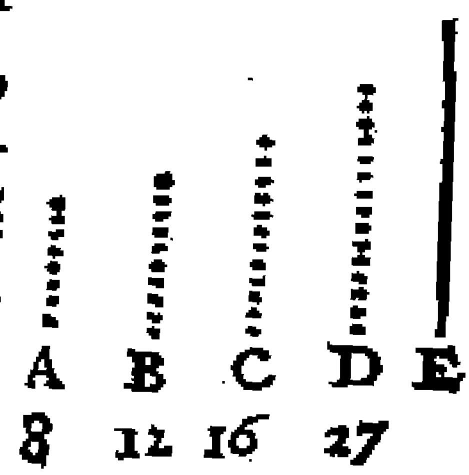
Theorema 16. Propoſitio 16.

Si duo numeri ſint inter ſe primi, non ſe habebit quẽadmodum primus ad ſecũdum, ita ſecundus ad quépiam alium.



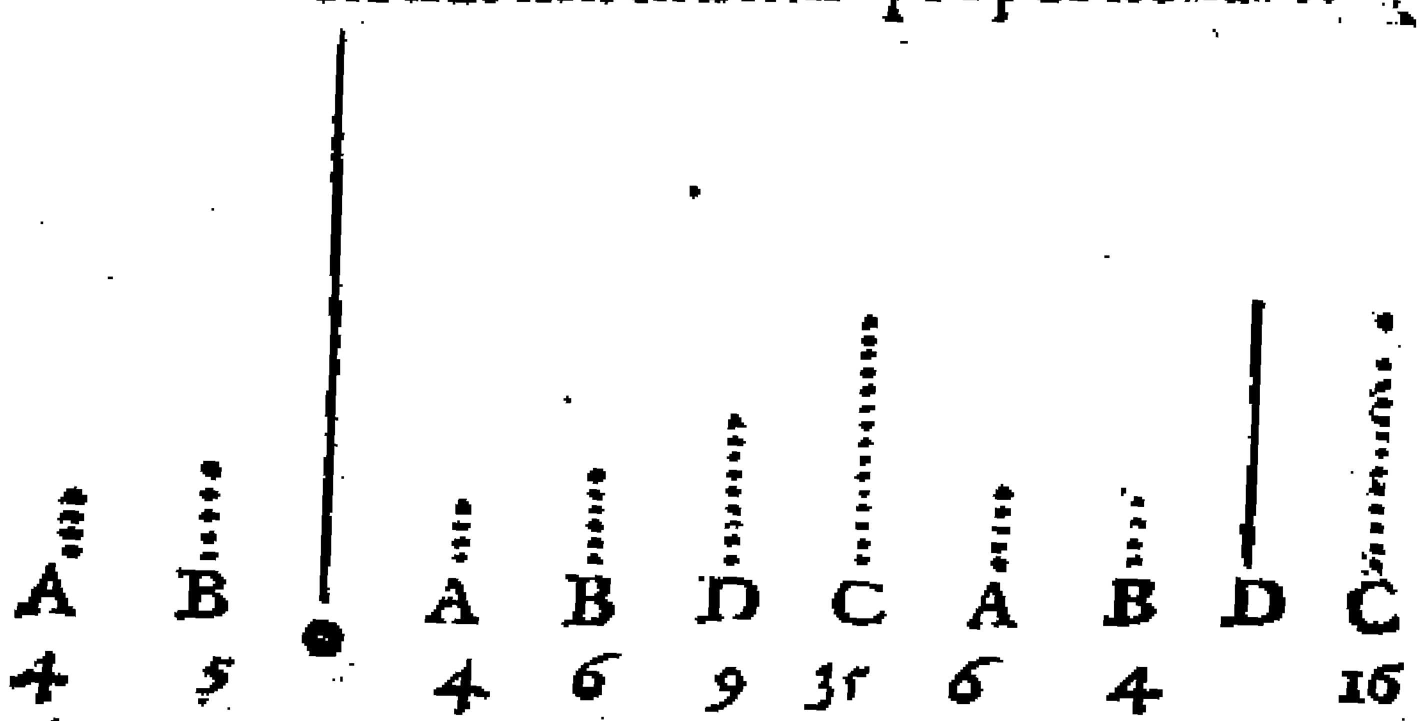
Theorema 17. Propoſitio 17.

Si ſint quotlibet numeri deinceps ꝓportionales, quorum extremi ſint inter ſe primi, non erit quẽ admodum primus ad ſecundum, ita vltimus ad quempiam alium.



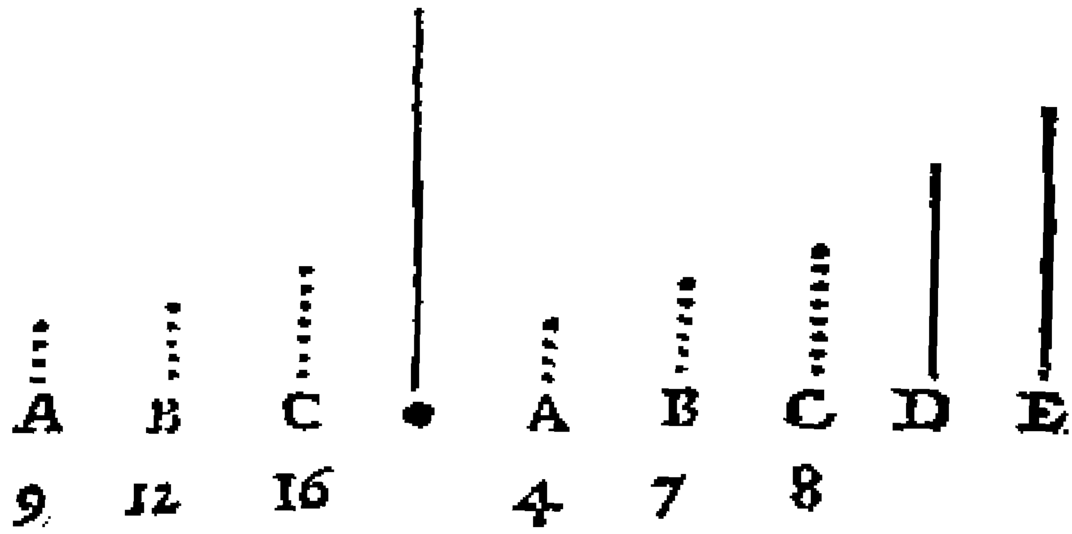
Theorema 18. Propoſitio 18.

Duobꝰ numeris datis, cõſiderare poſſitne tertius illis inueniri proportionalis.



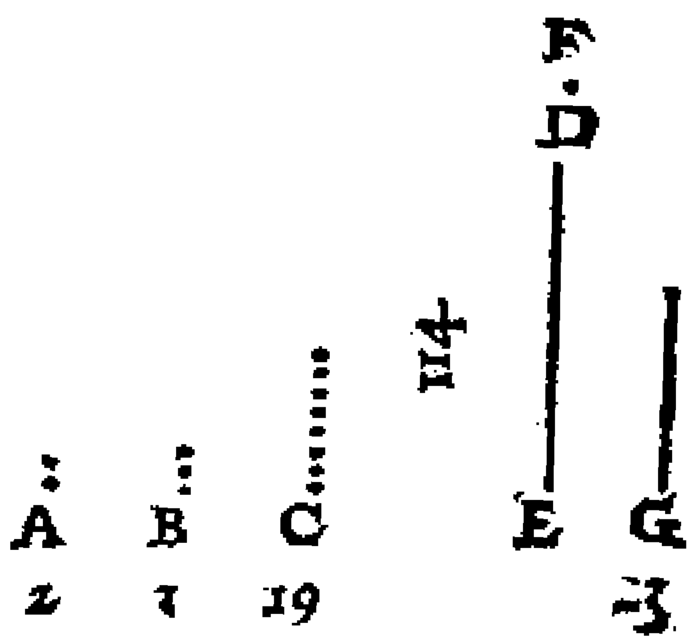
Theorema 19. Propositio 19.

Tribus numeris datis, considerare possitne quartus illis reperiri proportionalis.



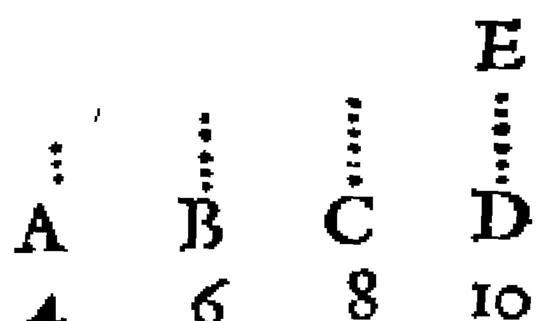
Theorema 20. Propositio 20.

Primi numeri plures sunt quacúque proposita multitudine primorum numerorum.



Theorema 21. Propositio 21.

Si pares numeri quot libet compositi sint, totus est par.



Theorema 22. Propositio 22.

Si impares numeri quotlibet compositi sint, sit autem par illorum multitudo, totus par erit.

E
A B C D
5 9 7 3

Theorema 23. Propositio 23.

Si impares numeri quotcunque cõpositi sint, sit autem impar illorũ multitudo, & totus impar erit.

A B C E
5 7 8 1

Theorema 24. Propositio 24.

B
A C
6 4

Si de pari numero par detractus sit, & reliquus par erit.

Theorema 25. Propositio 25.

B
A C D
8 1 4

Si de pari numero impar detractus sit, & reliquus impar erit.

Theorema 26. Propositio 26.

Si de impari numero impar detractus sit, & reliquus par erit.

B
A C D
4 6 1

Theo

Theorema 27. Propo-
ſitio 27.

Si ab impari numero par ablatus ſit, reliquus impar erit.

E
A D C
1 4 4

Theorema 28. Pro-
poſitio 28.

Si impar numerus parem multiplicãs, procreet quẽpiam, procreatus par erit.

A B C
3 4 12

Theorema 29. Propo-
ſitio 29.

Si impar numerus imparẽ numerum multiplicans quemdam procreet, procreatus impar erit.

A B C
3 5 15

Theorema 30. Propo-
ſitio 30.

Si impar numerus parẽ numerum metiatur, & illius dimidium metietur.

A C B
3 6 18

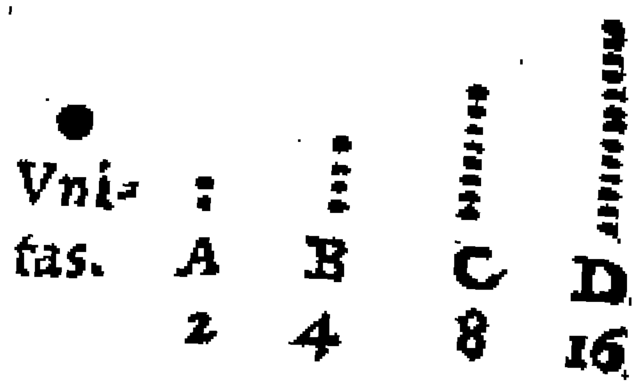
Theorema 31. Pro-
poſitio 31.

Si impar numerus ad numerum quempiã primus ſit, & ad illius duplum primus erit.

A B C D
7 8 16

Theorema 32. Propoſitio 32.

Numerorum, qui à binario dupli ſunt, vnuſquiſq; pariter par eſt tantùm.

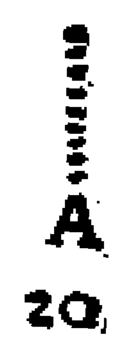


Theorema 33. Propoſitio 33.

Si numerus dimidiū impar habeat, pariter impar eſt tantùm.

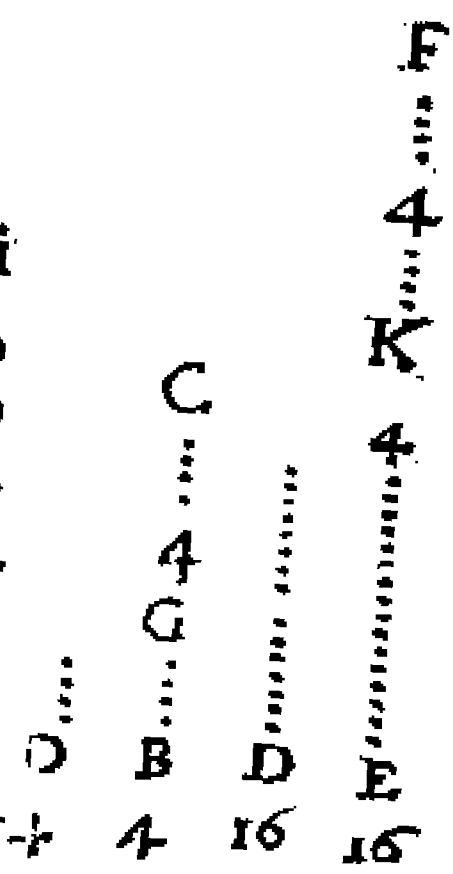
Theorema 34. Propoſitio 34.

Si par numerus nec ſit dupl⁹ à binario, nec dimidium impar habeat, pariter par eſt, & pariter impar.



Theorema 35. Propoſitio 35.

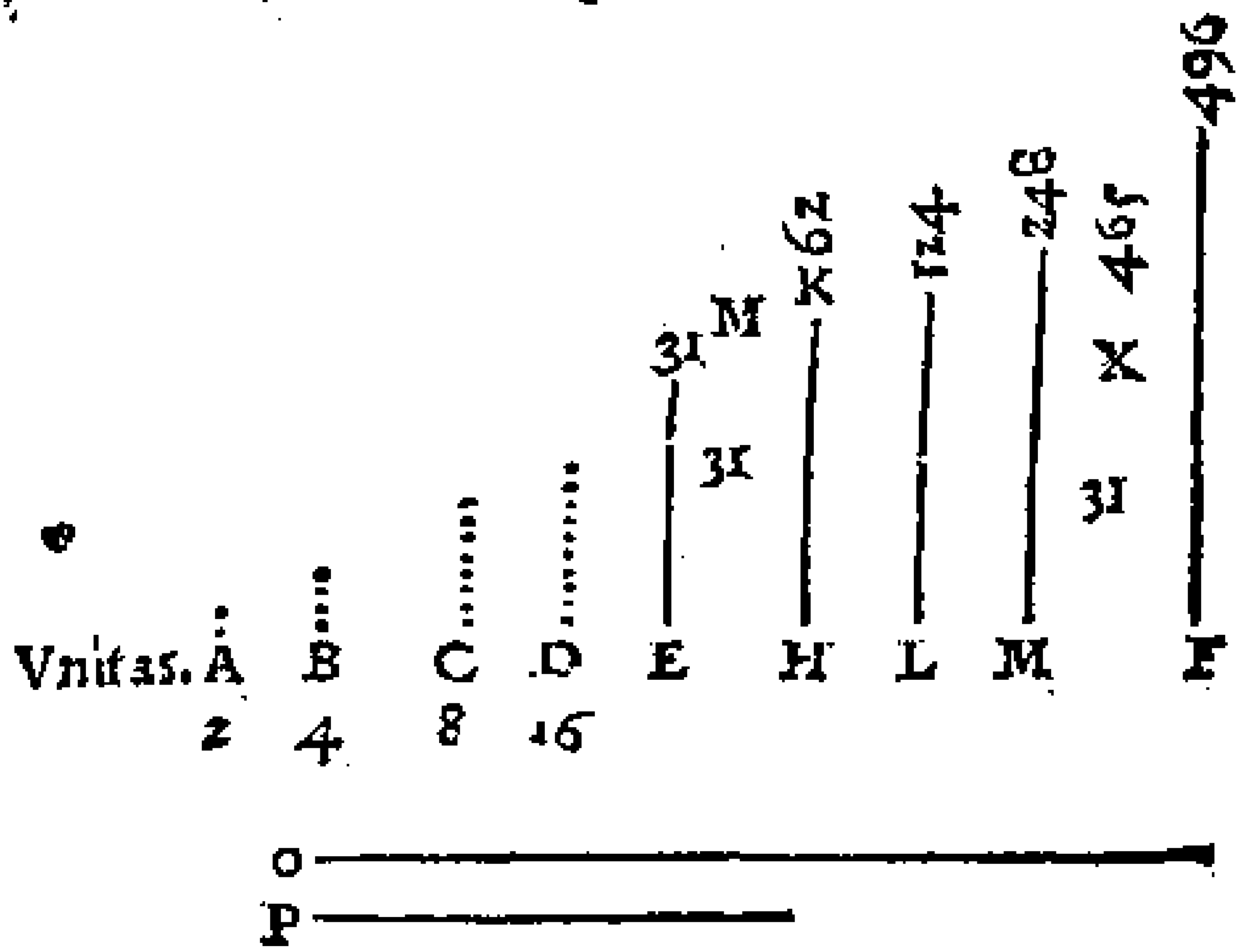
Si ſint quotlibet numeri deinceps proportionales, detrahātur autē de ſecūdo & vltimo æquales ipſi primo, erit quemadmodū ſecūdi exceſſus ad primū, ita vltimi exceſſus ad omnes qui vltimum antecedunt.



Theo-

Theorema 36. Propositio 36.

Si ab vnitate numeri quotlibet deinceps expositi sunt in duplici proportione quoad totus compositus primus factus sit, isq; totus in vltimum multiplicatus quempiá procreet, procreatus perfectus erit.



ELEMENTI IX. FINIS.

EVCLIDIS ELEMENTVM DECIMVM.

DEFINITIONES.

1.

Commenſurabiles magnitudines dicuntur illæ, quas eadem menſura metietur.

2

Incommenſurabiles verò magnitudines dicuntur hæ, quarum nullam menſuram communem contingit reperiri.

3

Lineæ rectæ potentia cõmenſurabiles sũt, quarum quadrata vna eadem ſuperficies ſiue area metitur.

4

Incommenſurabiles verò lineæ ſunt, quarum quadrata, quæ metiatur area cõmunis, reperiri nulla poteſt.

5

Hæc cùm ita ſint, oſtendi poteſt quòd quãtacunque linea recta nobis proponatur, exiſtunt etiã aliæ lineæ innumerabiles eidem commenſurabiles, aliæ item incommenſurabiles, hæ quidem longitudine & poten-

potentia: illæ verò potentia tantùm. Vocetur igitur linea recta, quantacunq; proponatur, ῥητὴ, id est rationalis.

6

Lineæ quoq; illi ῥητῇ commensurabiles siue longitudine & potentia, siue potentia tantùm, vocentur & ipsæ ῥηταί, id est rationales.

7

Quæ verò lineæ sunt incommensurabiles illi τῇ ῥητῇ, id est primo loco rationali, vocẽtur ἄλογοι, id est irrationales.

8

Et quadratum quod à linea proposita describitur, quam ῥητὴν vocari volumus, vocetur ῥητόν.

9

Et quæ sunt huic commensurabilia, vocẽtur ῥητά.

10

Quæ verò sunt illi quadrato ῥητῷ scilicet incommensurabilia, vocentur ἄλογα, id est surda.

11

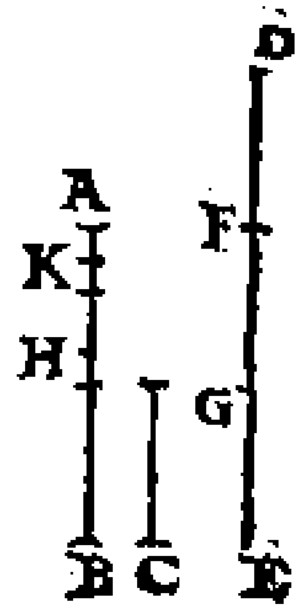
Et lineæ quæ illa incommẽsurabilia describunt, vocentur ἄλογοι Et quidem si illa incommẽsurabilia fuerint quadrata, ipsa corũ latera vocabũtur ἄλογοι lineæ. quòd si quadrata quidem non fuerint, verũ aliæ quæpiã superficies siue figuræ rectilineæ,

tunc

tunc verò lineæ illæ quæ describunt quadrata æqualia figuris rectilineis, vocentur ἄλογαι.

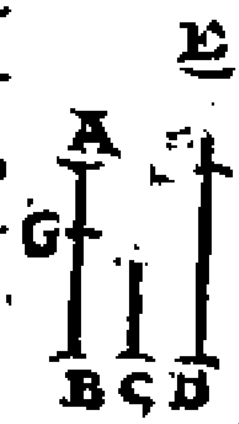
Theorema 1. Propositio 1.

Duobus magnitudinibus inæqualibus propositis, si de maiore detrahatur plus dimidio, & rursus de residuo iterũ detrahatur plus dimidio, idq́; semper fiat: relinquetur quædam magnitudo minor altera minore ex duabus propositis.



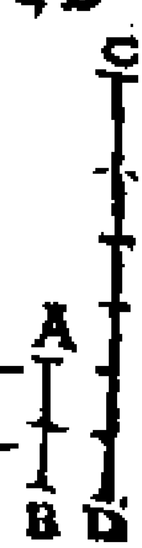
Theorema 2. Propositio 2.

Duab⁹ magnitudinibus propositis inæqualibus, si detrahatur semper minor de maiore, alterna quadam detractione neque residuum vnquam metiatur id quod ante se metiebatur, incommensurabiles sunt illæ magnitudines.



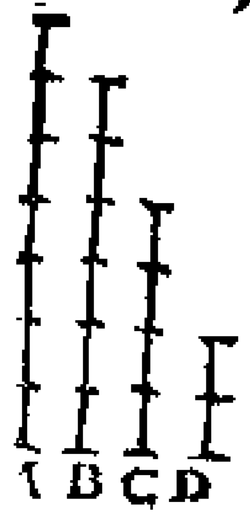
Problema 1. Propositio 3.

Duabus magnitudinibus commẽsurabilibus datis, maximam ipsarum communem mensuram reperire.



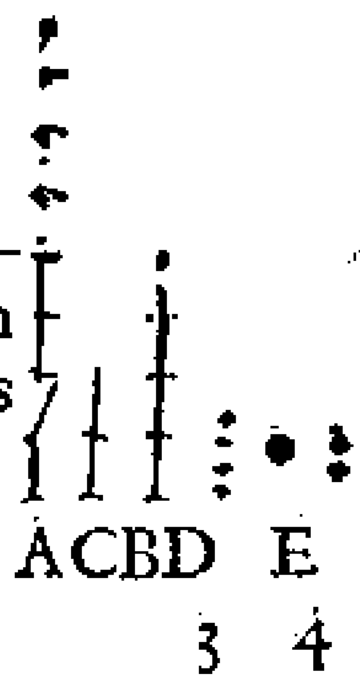
Proble-

Problema 2. Propoſitio 4.



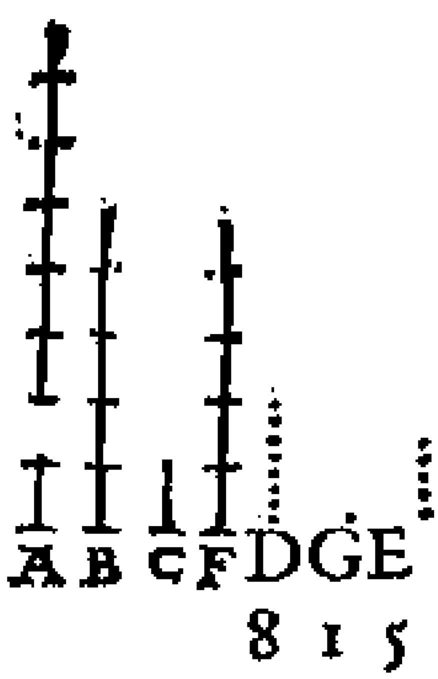
Tribus magnitudinibus cõmenſurabilibus datis, maximam ipſarũ communẽ menſuram reperire.

Theorema 3. Propoſitio 5.



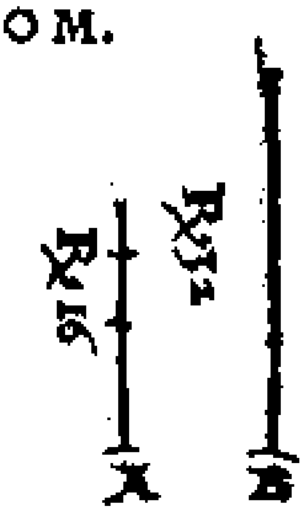
Cõmenſurabiles magnitudines inter ſe proportionẽ eam habent, quam habet numerus ad numerum.

Theorema 4. Propoſitio 6.



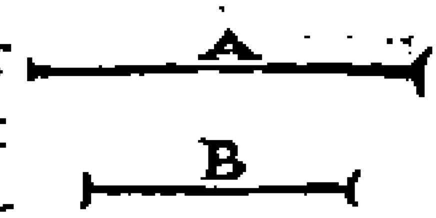
Si duæ magnitudines proportionem eam habent inter ſe quam numerus ad numerum, commenſurabiles ſunt illæ magnitudines.

Theorema 5. Propoſitio 7.



Incommenſurabiles magnitudines inter ſe proportionem non habẽt, quam numerus ad numerum.

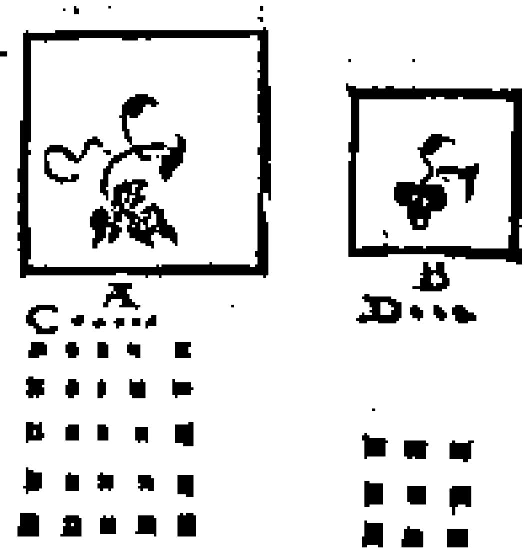
Theorema 6. Propoſitio 8.



Si duæ magnitudines inter ſe proportionem nõ habẽt quam numerus ad numerum, incommenſurabiles illæ ſunt magnitudines.

Theorema 7. Propoſitio 9.

Quadrata, quæ deſcribũtur à rectis lineis lõgitudine cõmenſurabilibus, inter ſe proportionem habent quã numerus quadratus ad alium numerum quadratũ: Et quadrata habentia proportionẽ

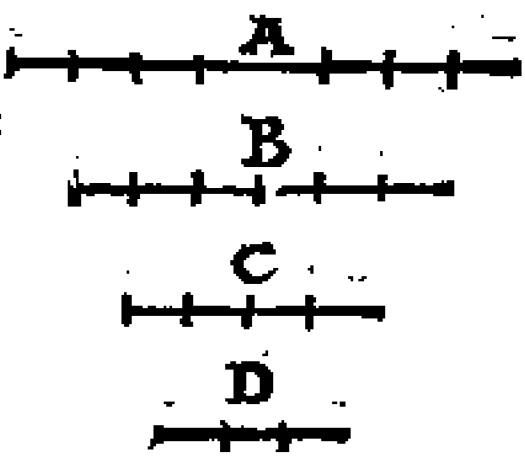


inter ſe quã quadratus numerus ad numerum quadratũ, habent quoque latera longitudine commẽſurabilia. Quadrata verò quæ

quæ deſcribuntur à lineis longitudine incõmenſurabilibus, proportioně nõ habět inter ſe, quam quadratus numerus ad numerum alium quadratum. Et quadrata nõ habentia proportionem inter ſe quam numerus quadratus ad numerum quadratũ, neque latera habebunt longitudine commenſurabilia.

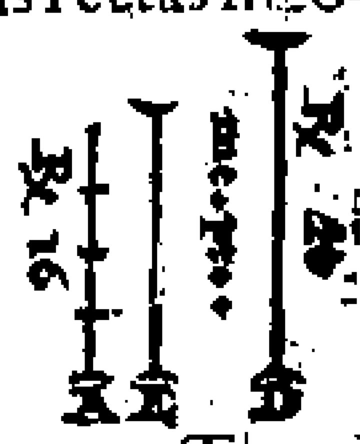
Theorema 8. Propoſitio 10.

Si quatuor magnitudines fuerint proportionales, prima verò ſecũdæ fuerit cõmenſurabilis, tertia quoque quartę commẽſurabilis erit. quòd ſi prima ſecũdæ fuerit incommẽſurabilis, tertia quoq; quartæ incommenſurabilis erit.

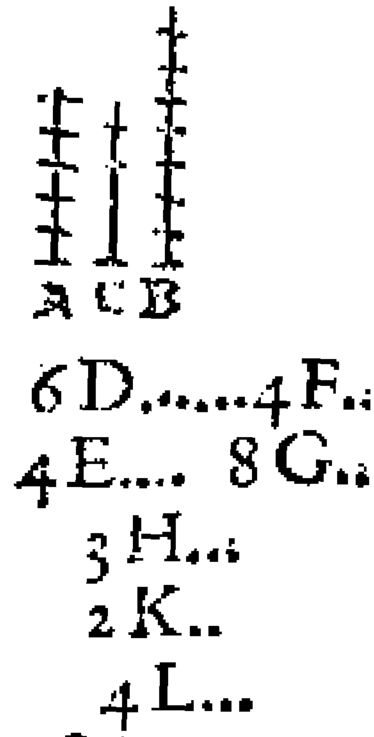


Problema 3. Propoſitio 11.

Propoſitæ lineæ rectæ (quam ῥητὴν vocari diximus) reperire duas lineas rectas incõmenſurabiles, hanc quidem longitudine tãtùm, illam verò non longitudine tantùm, ſed etiam potentia incommẽſurabilem.

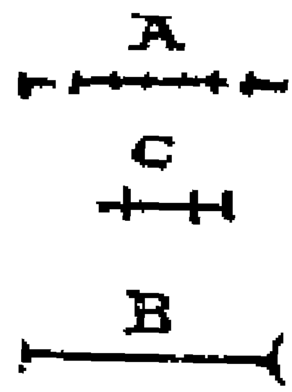


Theorema 9. Propoſitio 12.



Magnitudines quæ eidē magnitudini ſunt cō menſurabiles, inter ſe quoque ſunt commenſurabiles.

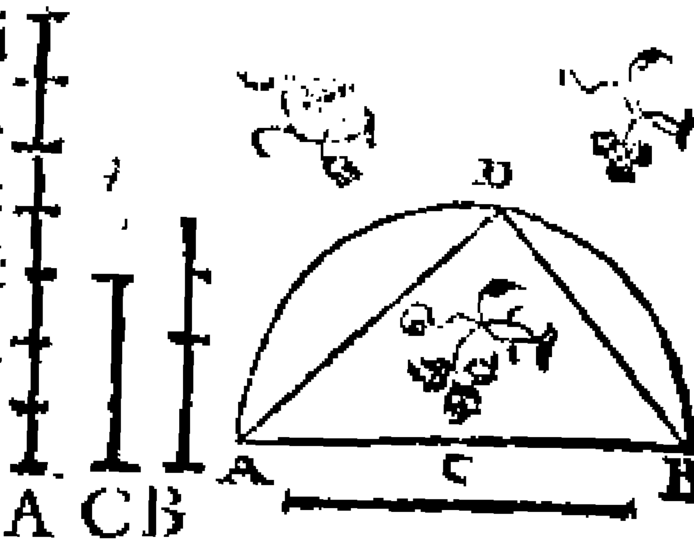
Theorema 10. Propoſitio 13.



Si ex duabus magnitudinib⁹ hæc quidem cōmenſurabilis ſit tertię magnitudini, illa verò eidem incommenſurabilis, incōmenſurabiles ſunt illæ duæ magnitudines.

Theorema 11. Propoſitio 14.

Si duarum magnitudinum commenſurabiliū altera fuerit incōmenſurabilis magnitudini alteri cuipiam tertiæ. reliqua quoque magnitudo eidē tertiæ incommē ſurabilis erit.

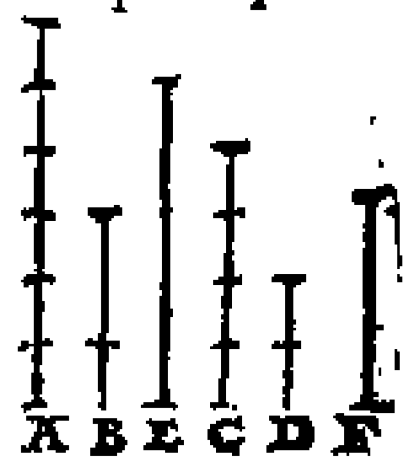


Theorema 12. Propoſitio 15.

Si quatuor rectæ proportionales fuerint,

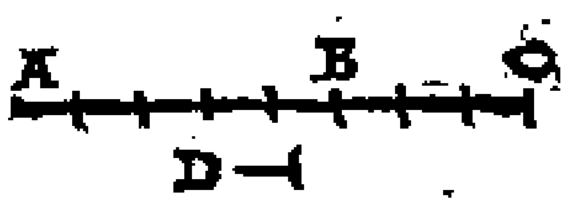
poſſit

posit autem prima plusquã secunda tãto quantum est quadratum lineæ sibi commẽsurabilis longitudine:tertia quoq; poterit plusquàm quarta tanto quantum est quadratũ lineæ sibi commensurabilis lõgitudine.Quòd si prima posit plusquã secũda quadrato lineę sibi longitudine incommẽsurabilis: tertia quoque poterit plusquã quarta quadrato lineæ sibi incõmẽsurabilis lõgitudine.



Theorema 13.Propositio 16.

Si duæ magnitudines commensurabiles componantur, tota magnitudo cõposita singulis partibus commensurabilis erit, quòd si tota magnitudo cõposita alterutri parti commensurabilis fuerit,illæ duæ quoque partes cõmẽsurabiles erunt.

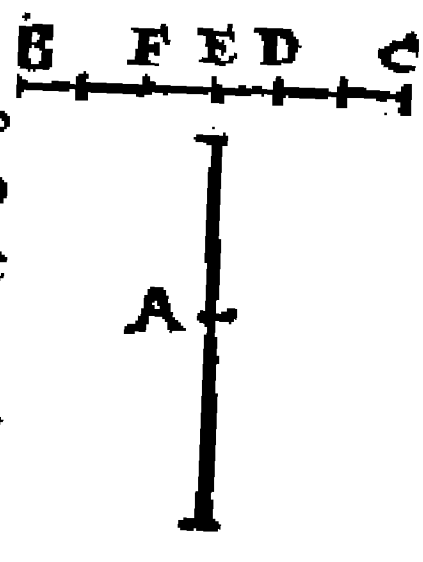


Theorema 14.Propositio 17.

Si duæ magnitudines incõmensurabiles cõponantur,ipsa quoque tota magnitudo singulis partibus componentibus incommẽsurabilis erit . Quòd si tota alteri parti incõmensurabilis fuerit,illæ quoque primę magnitudines inter se incommẽsurabiles erunt.

Theorema 15. Propositio 18.

Si fuerint duæ rectæ lineæ inæquales, & quartæ parti quadrati quod describitur à minore, æquale parallelogrammũ applicetur secundũ maiorem, ex qua maiore tãtum excurrat extra latus parallelogrãmi, quantum est alterum latus ipsius parallelogrammi: si præterea parallelogrammũ sui applicatione diuidat lineã illam in partes inter se commẽsurabiles longitudine, illa maior linea tanto plus potest quã minor, quantũ est quadratum lineæ sibi commensurabilis longitudine. Quòd si maior plus posit quàm minor, tanto quantũ est quadratum lineæ sibi cõmensurabilis lõgitudine, & præterea quartæ parti quadrati lineæ minoris ęquale parallelogrãmum applicetur secundum maiorẽ, ex qua maiore tantum excurrat extra latus parallelogrãmi, quantum est alterum latus ipsius parallelogrammi, parallelogrammum sui applicatione diuidit maiorẽ in partes inter se longitudine commẽsurabiles.

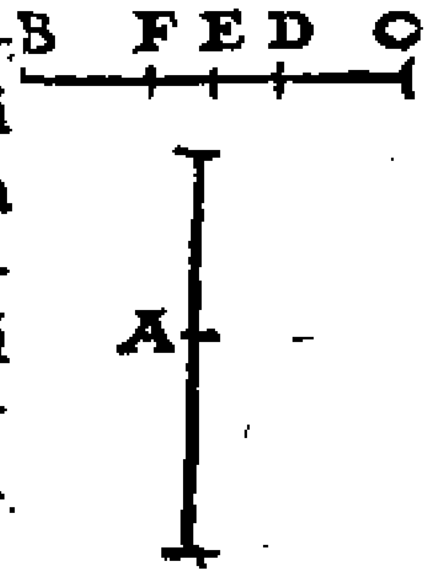


Theorema 16. Propositio 19.

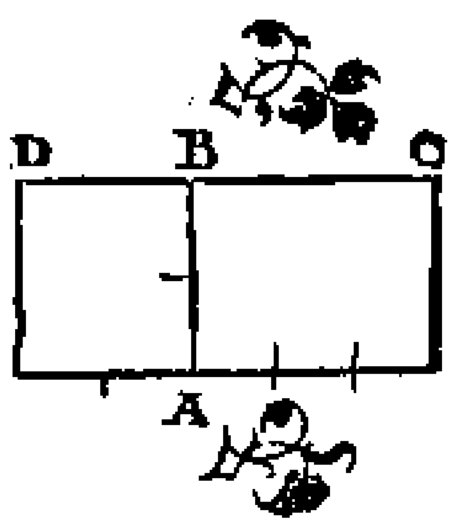
Si fuerint duæ rectæ inæquales, quartæ autem parti quadrati lineæ minoris ęquale

le

le parallelogrammorum ſecundum lineá maiorem applicetur, ex qua linea tantum excurrat extra latus parallelográmi, quátum eſt alterum latus eiuſdem parallelogrammi: ſi parallelogrammũ præterea ſui applicatione diuidat lineã in partes inter ſe longitudine incommẽſurabiles, maior illa linea tantò plus poteſt quàm minor, quantum eſt quadratum lineæ ſibi maiori incommenſurabilis longitudine. Quòd ſi maior linea tãtò plus poſsit quàm minor, quantum eſt quadratum lineæ incómenſurabilis ſibi lógitudine: & preterea quartæ parti quadrati lineę minoris ęquale parallelogrammũ applicętur ſecũdum maiorem, ex qua tantũ excurrat extra latus parallelogrã mi, quantum eſt alterum latus ipſius: parallelogrammum ſui applicatione diuidit maiorem in partes inter ſe incommenſurabiles lógitudine.



Theorema 17. Propoſitio 20.

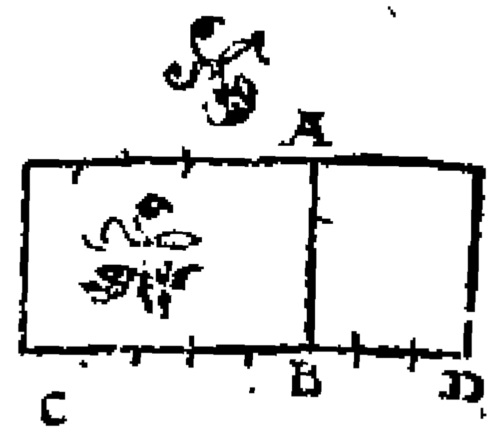


Superficies rectangula contéta ex lineis rectis rationalib⁹ lógitudine commenſurabilibus ſecundum

cundum vnum aliquem modũ ex antedictis, rationalis est.

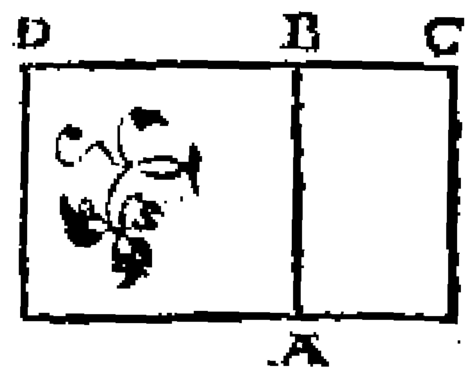
Theorema 18. Propositio 21.

Si rationale secundũ lineam rationalem applicetur, habebit alterũ latus lineã rationalem & cõmensurabilem longitudine lineæ cui rationale parallelogrammũ applicatur.



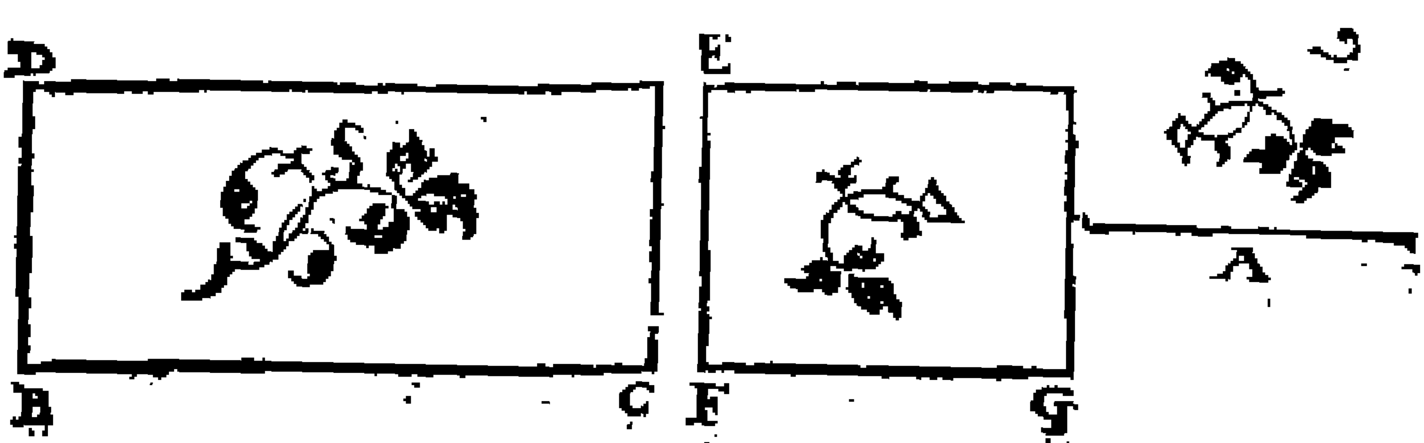
Theorema 19. Propositio 22.

Superficies rectangula cõtenta duabus lineis rectis rationalibus potentia tantum cõmẽsurabilibus, irrationalis est. Linea autem quæ illam superficiem potest, irrationalis & ipsa est: vocetur verò medialis.



Theorema 20. Propositio 23.

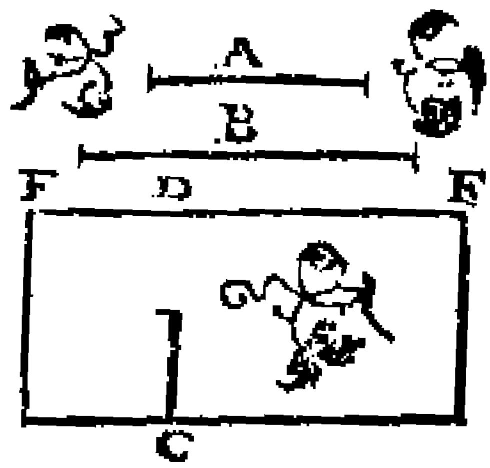
Quadrati lineæ medialis applicati secun-



dum

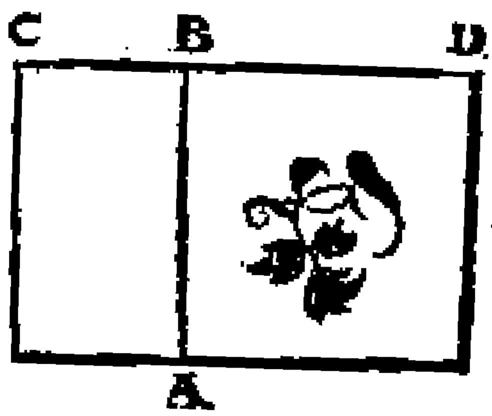
dum lineam rationalem, alterum latus est linea rationalis, & incõmensurabilis lõgitudine lineæ secundum quam applicatur.

Theorema 21. Propositio 24.



Linea recta mediali cõmẽsurabilis, est ipsa quoque medialis.

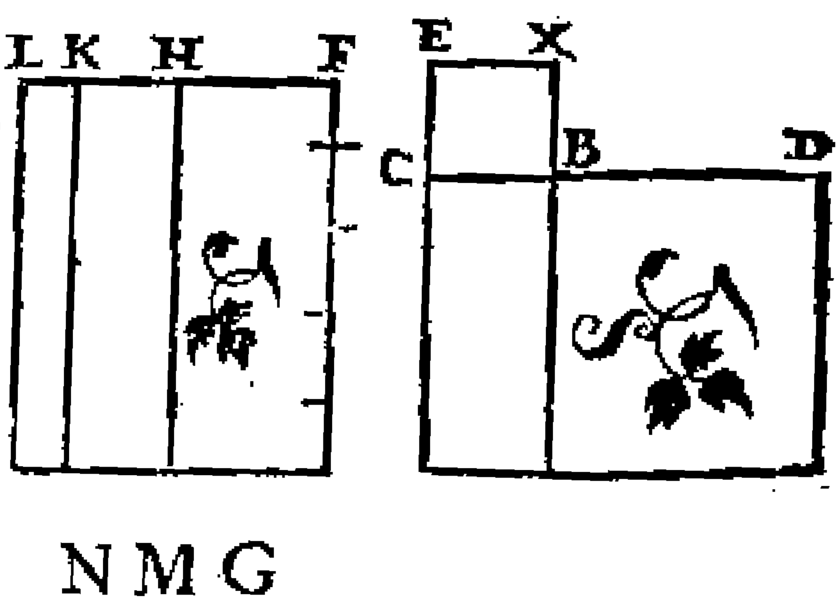
Theorema 22. Propositio 25.



Parallelogrammũ rectãgulum cõtentum ex lineis medialibus longitudine cõmensurabilibus, mediale est.

Theorema 23. Propositio 26.

Parallelogrammum rectangulum cõprehensum duabus lineis medialibus potentia tãtùm cõmensurabilib⁹, vel rationale est, vel mediale.



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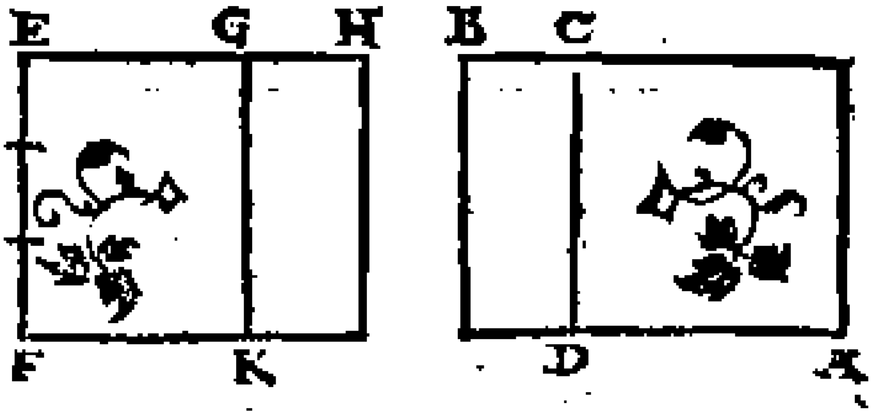
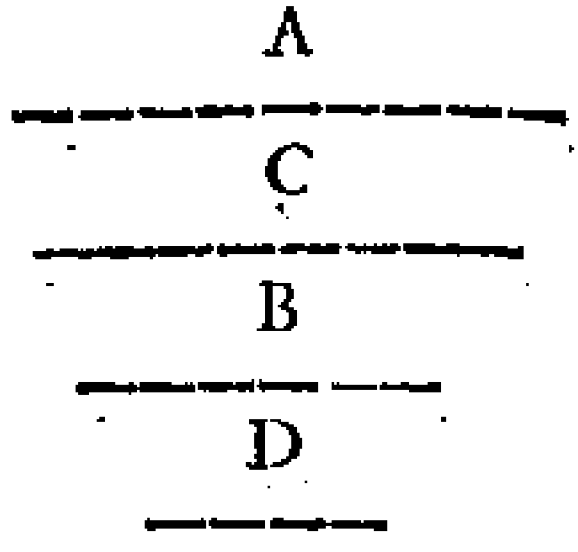
Theorema 24. Propoſitio 27.

Mediale nó eſt maius quàm mediale ſuperficie rationali.



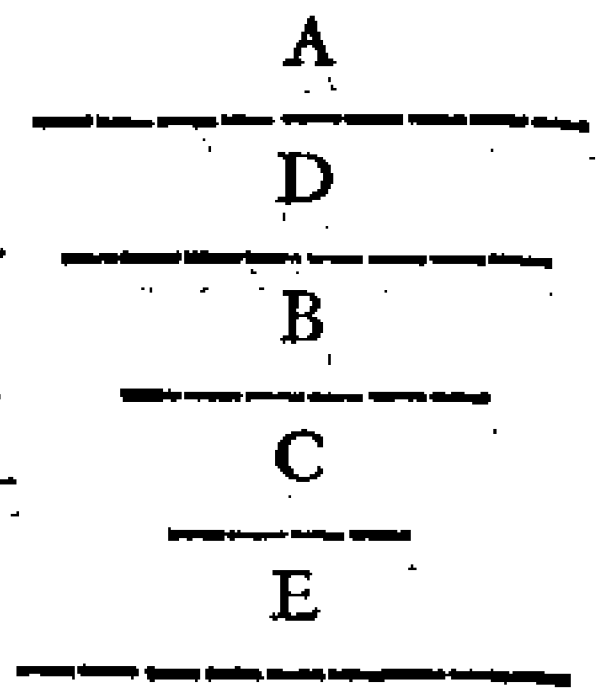
Proble. 4. Propoſitio 28.

Mediales lineas inuenire potentia tantùm commenſurabiles rationale comprehendentes.



Problema 5. Propoſitio 29.

Mediales lineas inuenire potentia tantùm commenſurabiles mediale comprehendendentes.



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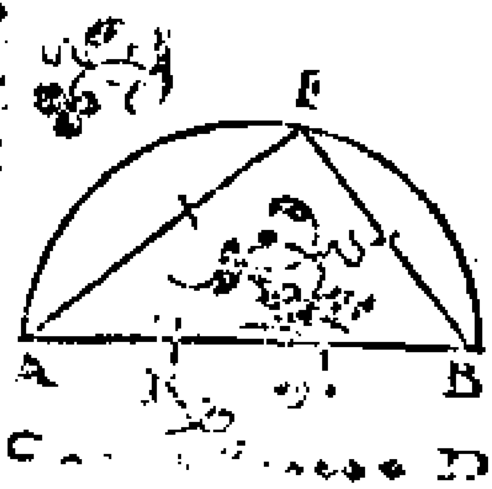
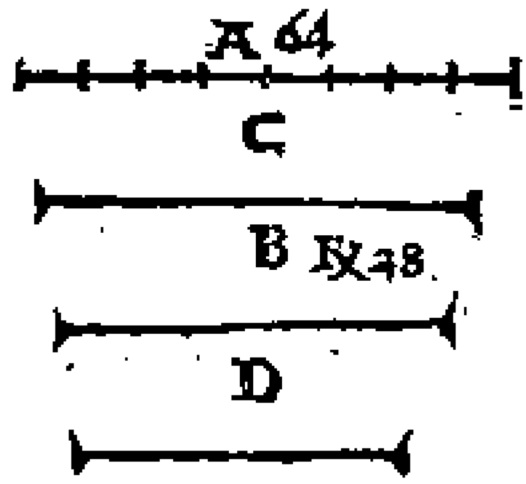
Problema 6. Propositio 30.

Reperire duas rationales potẽtia tantùm commẽsurabiles huiusmodi, vt maior ex illis possit plus quàm minor quadrato lineæ sibi commensurabilis longitudine.



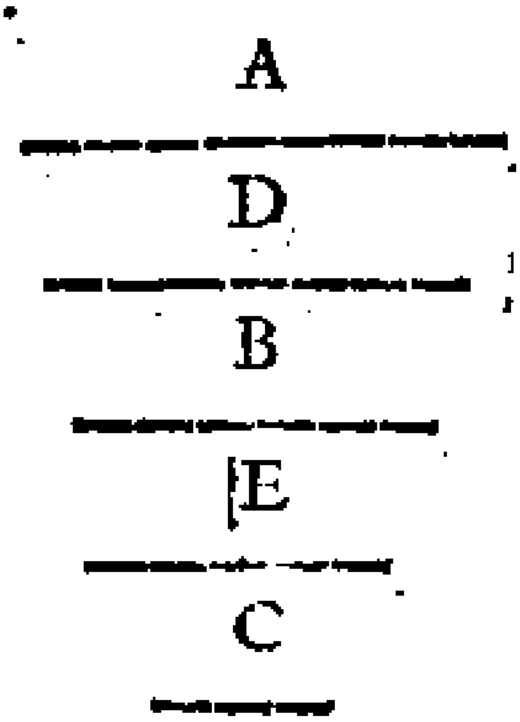
Problema 7. Propositio 31.

Reperire duas lineas mediales potentia tãtùm commensurabiles rationalem superficiẽ cõtinentes, tales inquã vt maior possit plus quàm minor quadrato lineæ sibi commẽsurabilis longitudine.



Problema 8. Propositio 32.

Reperire duas lineas mediales potentia tantùm commensurabiles medialem superficiem continentes, huiusmodi vt maior plus possit quàm minor quadrato lineæ sibi commensurabilis longitudine.



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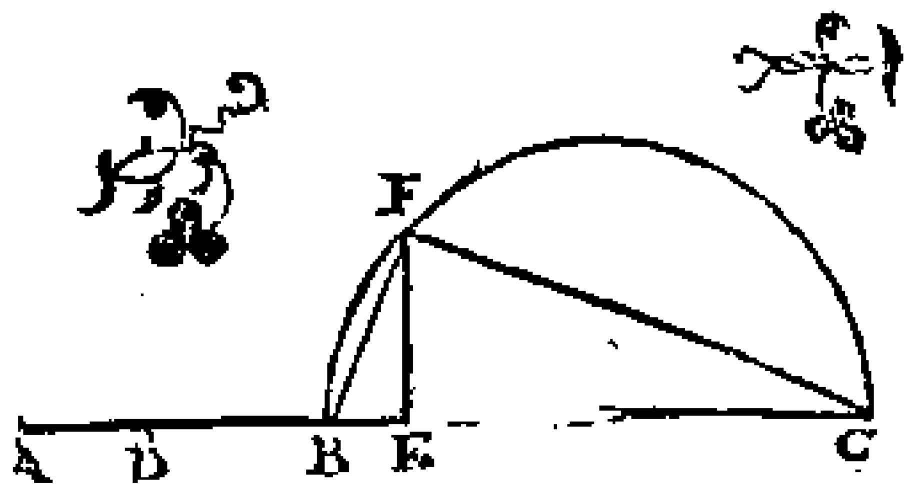
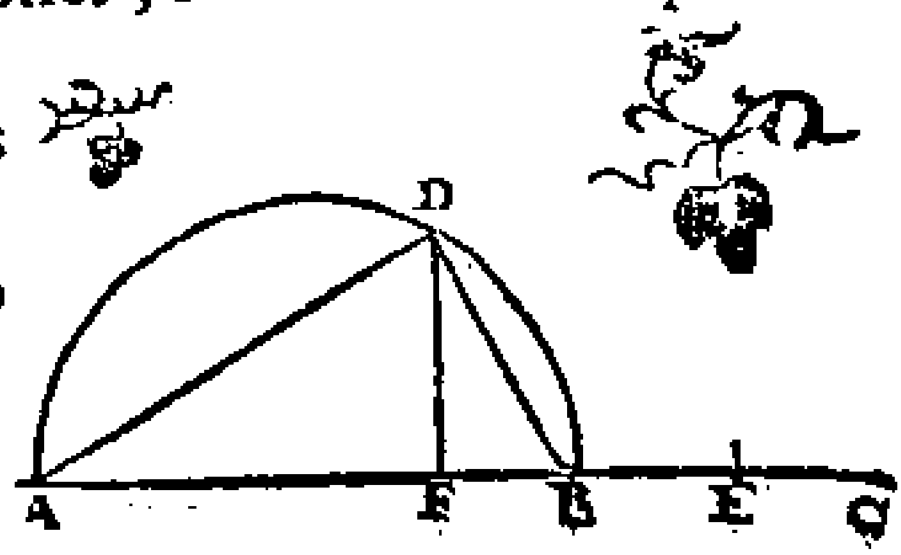
Problema 9. Propositio 33.

Reperire duas rectas potétia incommēsurabiles, quarum quadrata simul addita faciant superficiē rationalē, parallelogrāmum verò ex ipsis contentum sit mediale.



Problema 10. Propositio 34.

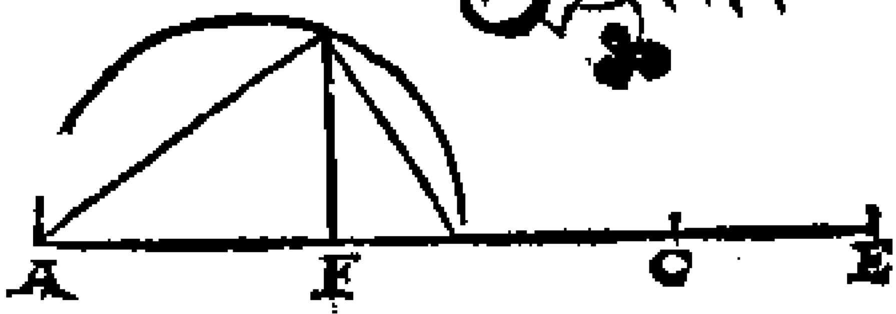
Reperire lineas duas rectas potentia incōmensurabiles, conficientes compositū ex ipsarum quadratis mediale, parallelogrammū verò ex ipsis contentum rationale.



Problema 11. Propositio 35.

Reperire duas lineas rectas potentia incōmensurabiles, cōficientes id quod ex ipsarum quadratis cōponitur mediale, simulque

que parallelogrammũ ex ipſis contẽtum, mediale, quod præterea parallelogrãmũ ſit incõmenſurabile cõpoſito ex quadratis ipſarum.



PRINCIPIVM SENARIOrum per compoſitionem.

Theorema 25. Propoſitio 36.

Si duæ rationales potẽtia tantùm commẽſurabiles cõponantur, tota linea erit irrationalis. Vocetur autem Binomium.

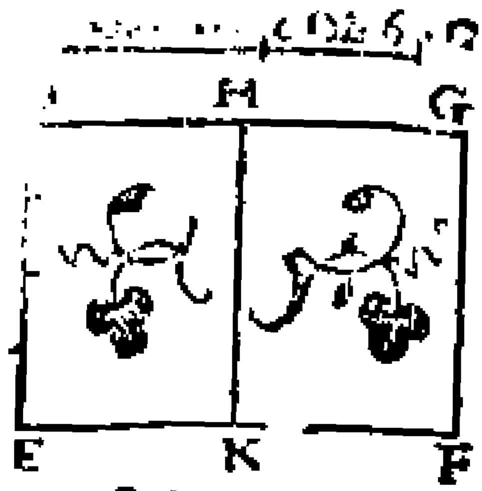
Theorema 26. Propoſitio 37.

Si duæ mediales potentia tantùm cõmenſurabiles rationale continentes cõponantur, tota linea eſt irrationalis, vocetur autem Bimediale prius.

Theo-

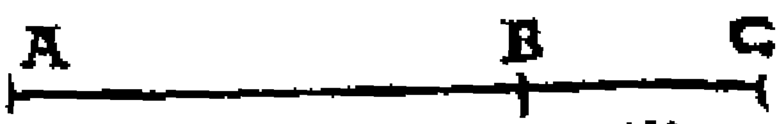
Theorema 27. Propositio 38.

Si duæ mediales potētia tantùm cōmensurabiles mediale continentes cō ponātur, tota linea est irrationalis: vocetur autē Bimediale secundum.



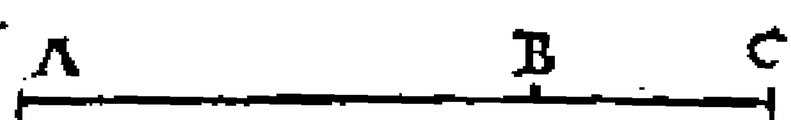
Theorema 28. Propositio 39.

Si duæ rectæ potentia incommēsurabiles cōponantur, conficientes compositum ex quadratis ipsarū rationale, parallelogrammum verò ex ipsis contentū mediale, tota linea recta est irrationalis. Vocetur autē linea maior.



Theorema 29. Propositio 40.

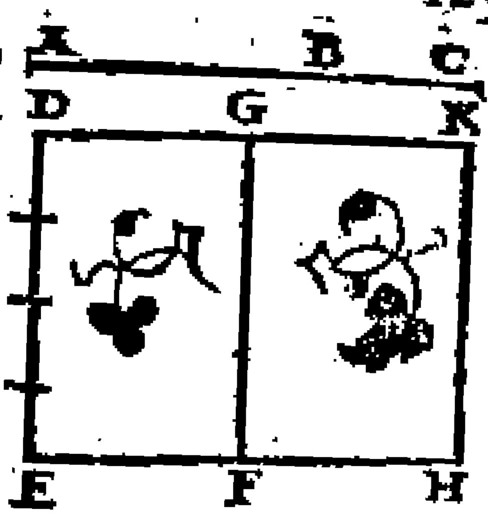
Si duæ rectæ potentia incōmensurabiles componantur, conficientes compositū ex ipsarū quadratis mediale, id verò ꝙ fit ex ipsis, rationale, tota linea est irrationalis. Vocetur autem potens rationale & mediale.



Theorema 30. Propositio 41.

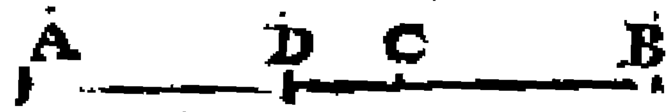
Si duæ rectę potentia incommensurabiles componantur, conficientes cōpositum ex quadratis ipsarum mediale, & quod cōtinetur

netur ex ipsis, mediale, & præterea incómensurabile cóposito ex quadratis ipsarú, tota linea est irrationalis. Vocetur autem potens duo medialia.



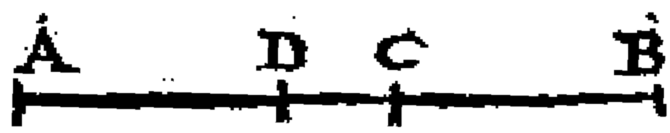
Theorema 31. Propositio 42.

Binomium in vnico tantùm púcto diuiditur in sua nomina, id est in lineas ex quibus componitur.

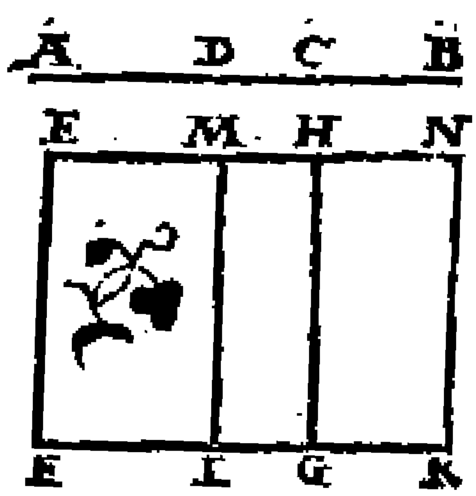


Theorema 32. Propositio 43.

Bimediale prius in vnico tantú puncto diuiditur in sua. nomina.



Problema 33. Propositio 44.



Bimediale secundum in vnico tantú puncto diuiditur in sua nomina.

Problema 34. Propositio 45.

Linea maior in vnico tátùm in púcto diuiditur in sua nomina.

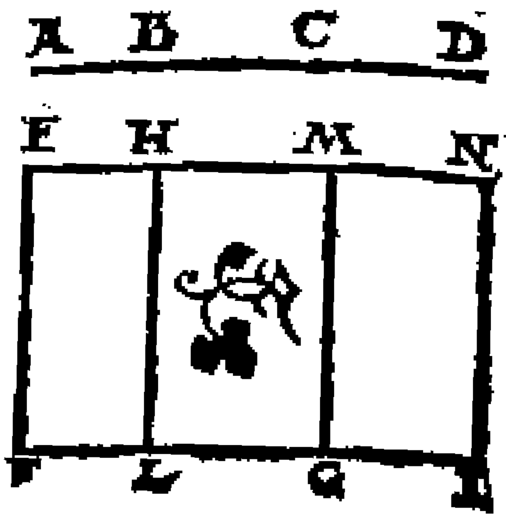
Theo-

Theorema 35. Propositio 46.

Linea potés rationale & mediale in vnico tātùm pūcto diuiditur in ſua nomina.

Theorema 36. Propositio 47.

Linea potés duo medialia in vnico tantùm pūcto diuiditur in ſua nomina.



DEFINITIONES ſecundæ.

Propoſita linea rationali, & binomio diuiſo in ſua nomina, cuius binomij maius nomen, id eſt, maior portio poſsit pluſquam minus nomen quadrato lineæ ſibi, maiori inquam nomini, commenſurabilis longitudine.

I

Si quidé maius nomen fuerit cōmenſurabile longitudine propoſitæ lineę rationali, vocetur tota linea Binomium primum.

2

Si verò minus nomen, id eſt minor portio Binomij, fuerit cōmenſurabile lōgitudine ne propoſitæ lineæ rationali, vocetur tota linea Binomium ſecundum.

3

Si verò neutrum nomen fuerit commẽſurabile longitudine propoſitæ lineæ rationali, vocetur Binomium tertium.

Rurſus ſi maius nomen poſſit pluſquã minus nomen quadrato lineæ ſibi incommenſurabilis longitudine.

4

Si quidem maius nomen eſt commenſurabile lõgitudine propoſitæ lineæ rationali, vocetur tota linea Binomium quartum.

5

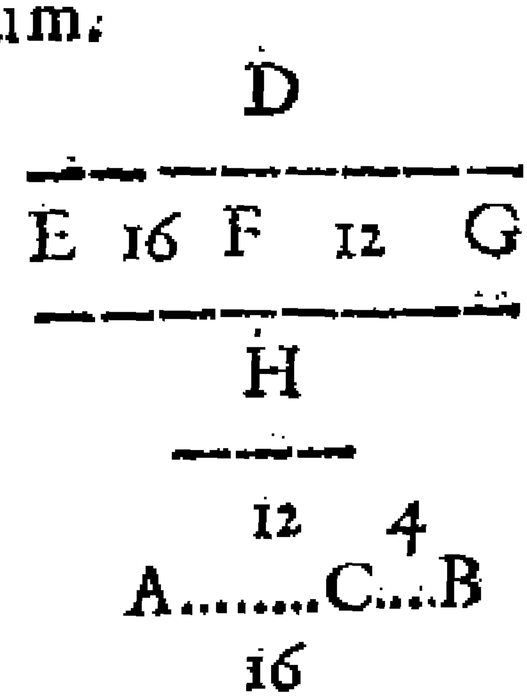
Si verò minus nomen fuerit cõmenſurabile longitudine lineæ rationali, vocetur Binomium quintum.

6

Si verò neutrum nomen fuerit longitudine commenſurabile lineæ rationali, vocetur illa Binomium ſextum.

Proble.12. Propoſitio 48.

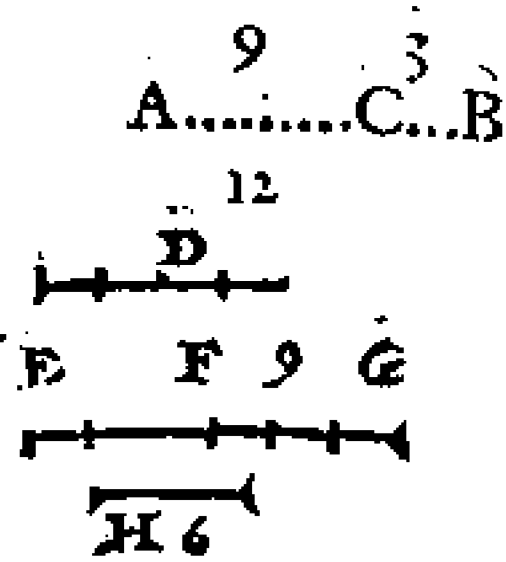
Reperire Binomium primum.



Proble-

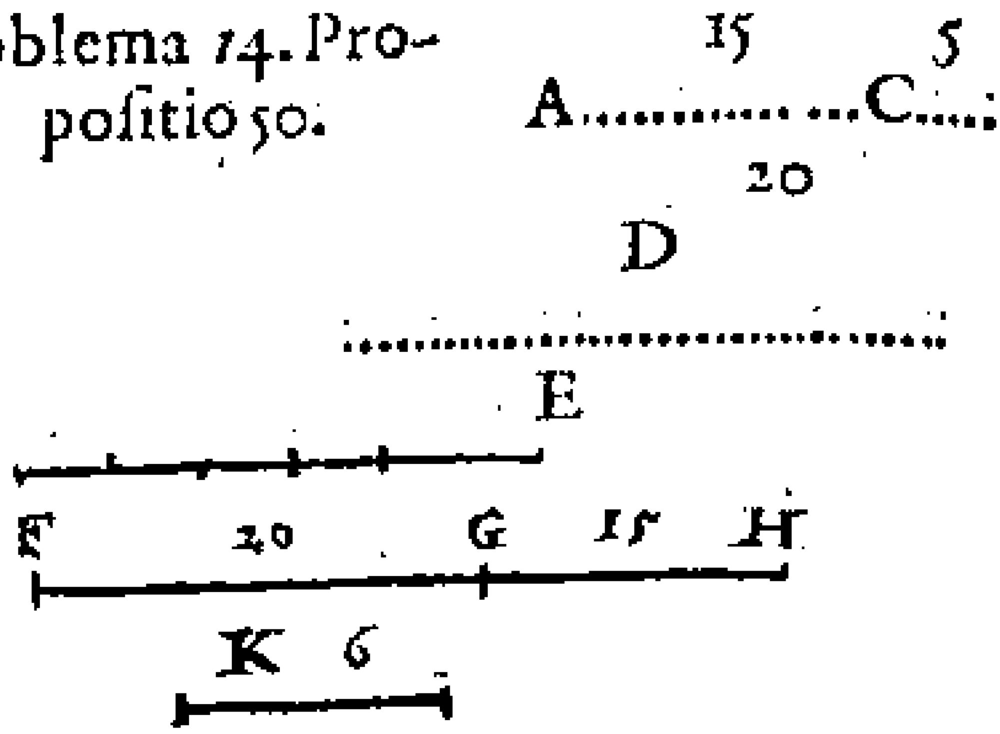
Problema 13. Propositio 49.

Reperire Binomium secundum.



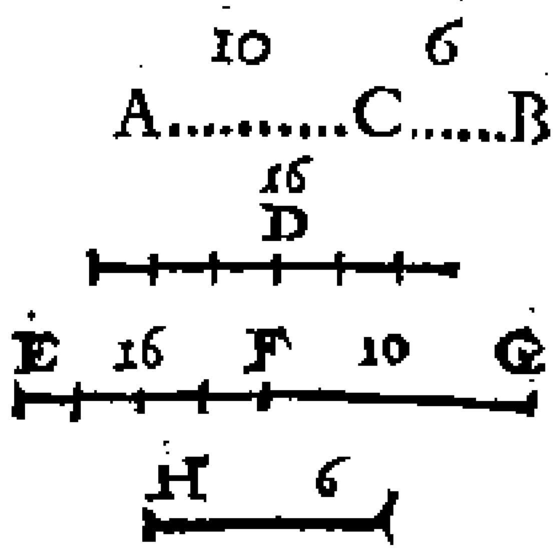
Problema 14. Propositio 50.

Reperire Binomium tertium.



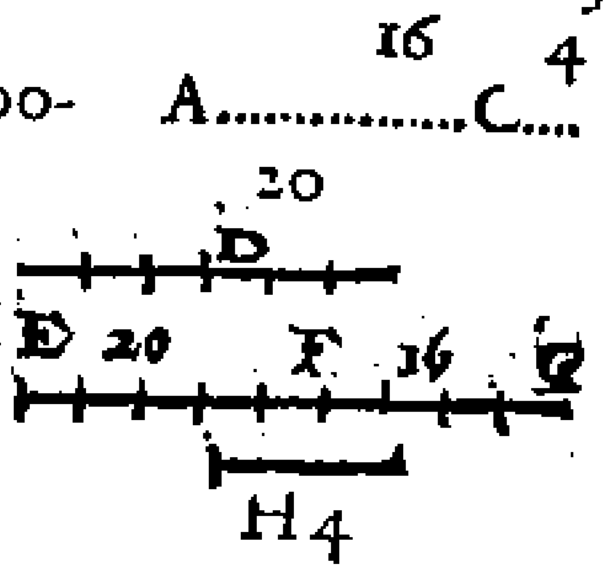
Problema 15. Propositio 51.

Reperire Binomium quartum.



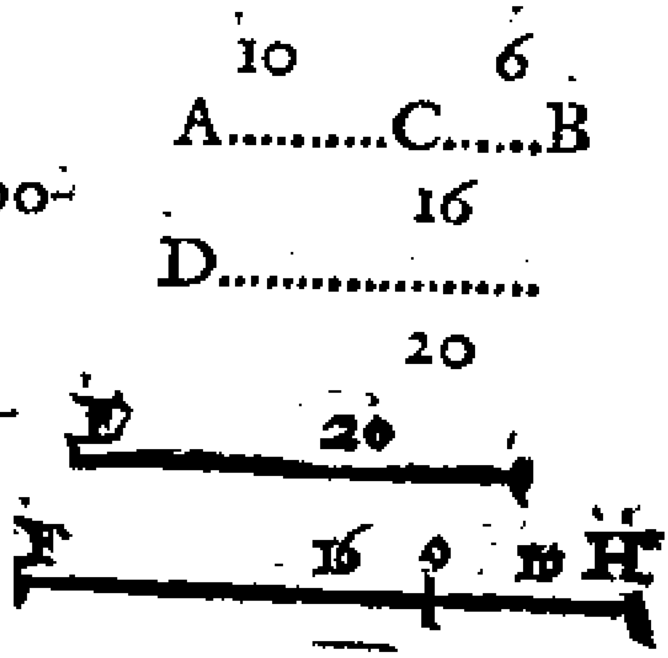
Proble-

Problema 16. Propositio 52.



Reperire Binomiũ quintum.

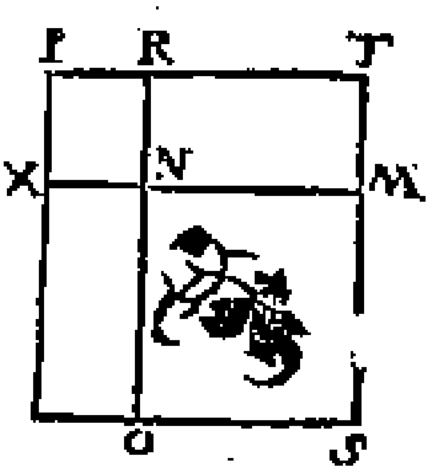
Probl. 17. Propositio 53.

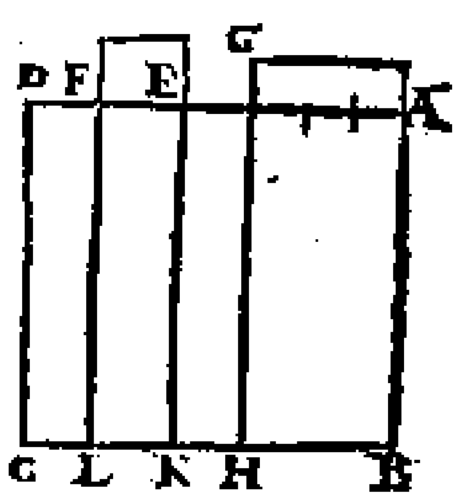


Reperire Binonimum sextum.

Theorema 37. Propositio 54.

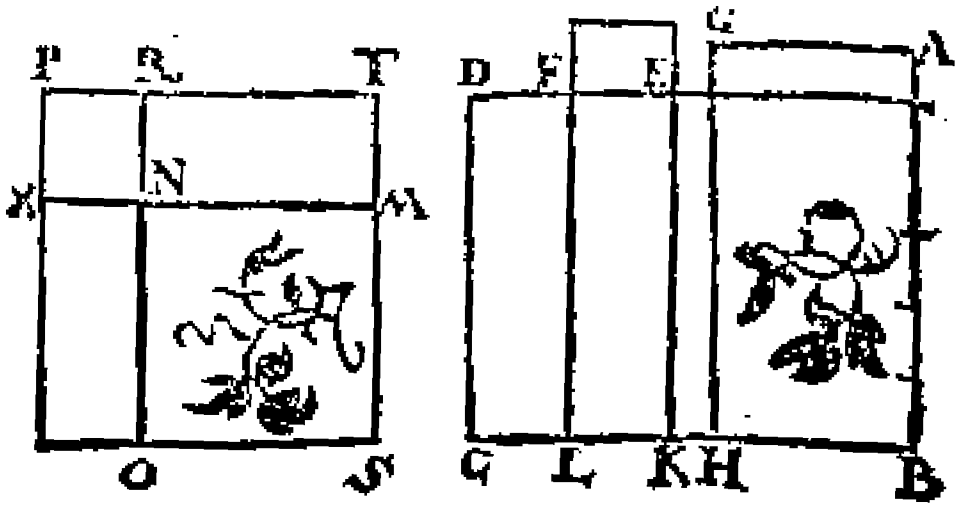
Si superficies contẽta fuerit ex rationali & Binomio primo, linea quę illã superficiem potest, est irrationalis, quæ Binomiũ vocatur.





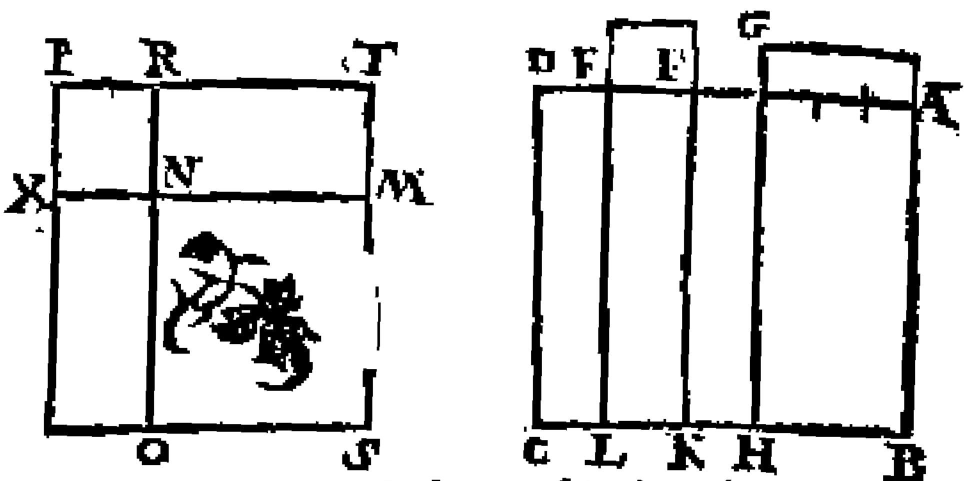
Theorema 38. Propositio 55.

Si superficies cõtenta fuerit ex linea rationali & Binomio secundo, linea potens illã superficiẽ est irrationalis, quę Bimediale primum vocatur.



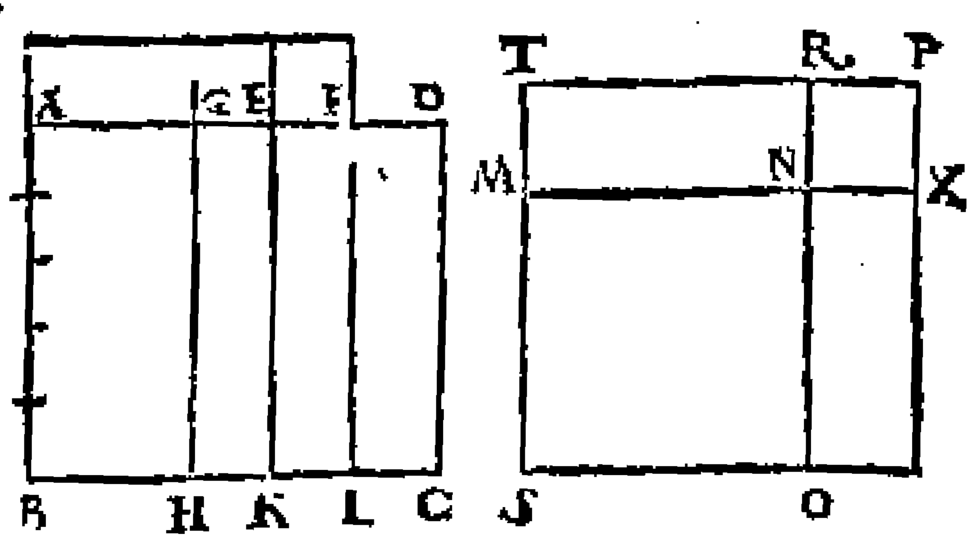
Theorema 39. Propositio 56.

Si superficies cõtineatur ex rationali & Binomio tertio, linea q̃ illã superficiem potest, est irrationalis q̃ dicitur Bimediale secundũ.



Theorema 40. Propositio 57.

Si superficies cõtineatur ex rationali & Bino-

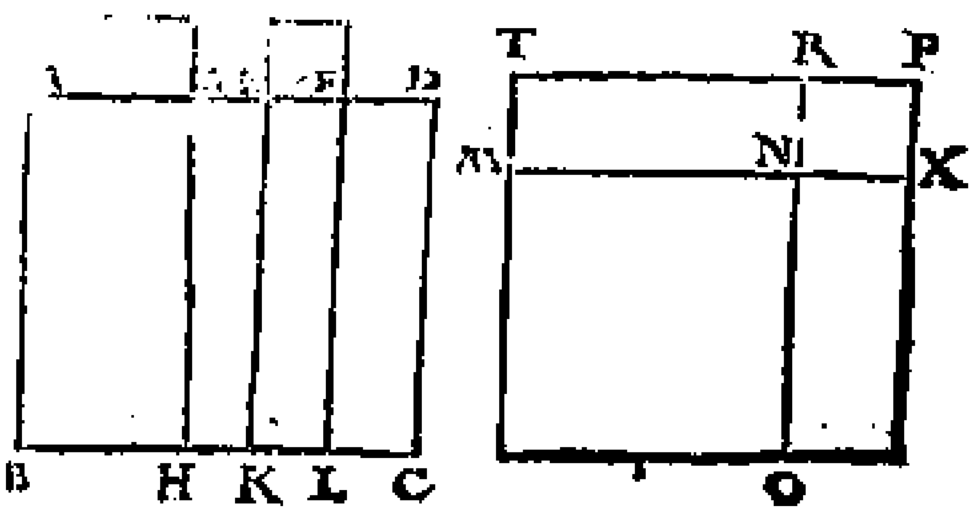


mio

mio quarto, linea potens superficiem illã, est irrationalis, quæ dicitur maior.

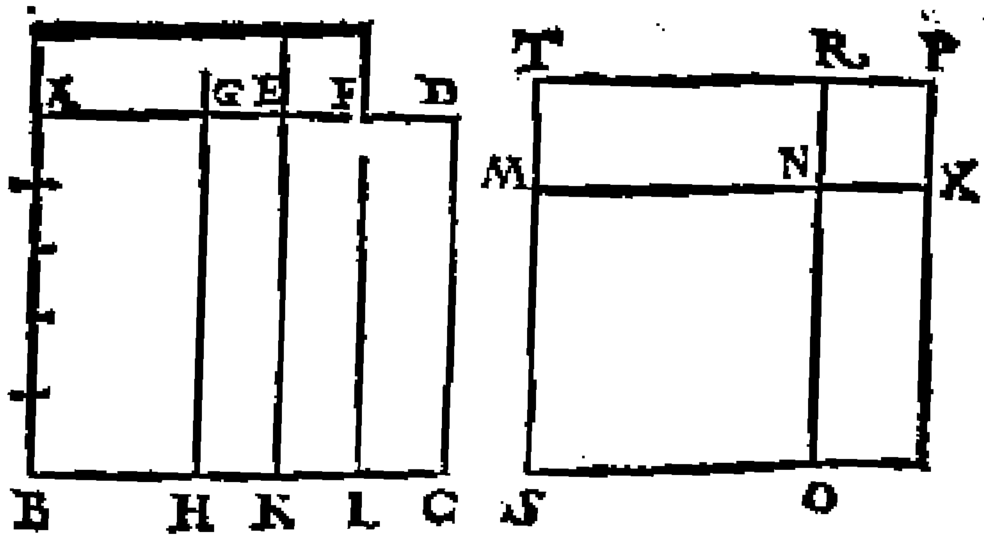
Theorema 41. Propositio 58.

Si superficies cõtineatur ex rationali & Binomio quinto, linea quæ illam superficiẽ potest est irrationalis, quę dicitur potẽs rationale & mediale.

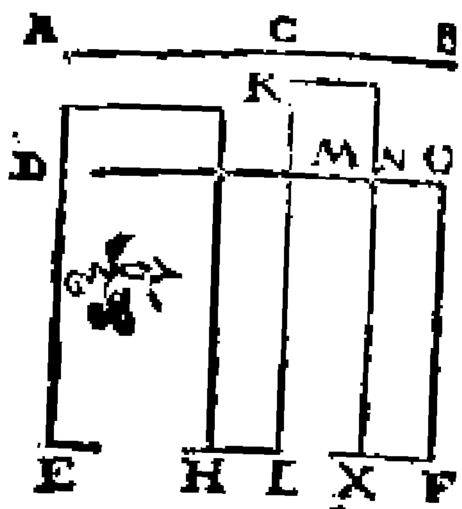


Theorema 42. Propositio 59.

Si superficies contineatur ex rationali & Binomio sexto, linea quæ illam superficiem potest est irrationalis, quæ dicitur potens duo mediala.

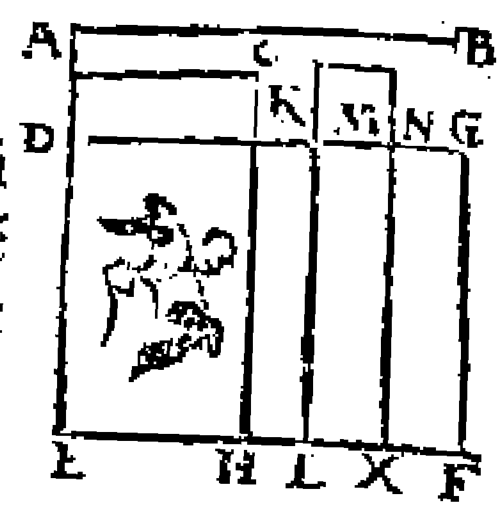


Theorema 43. Propositio 60.



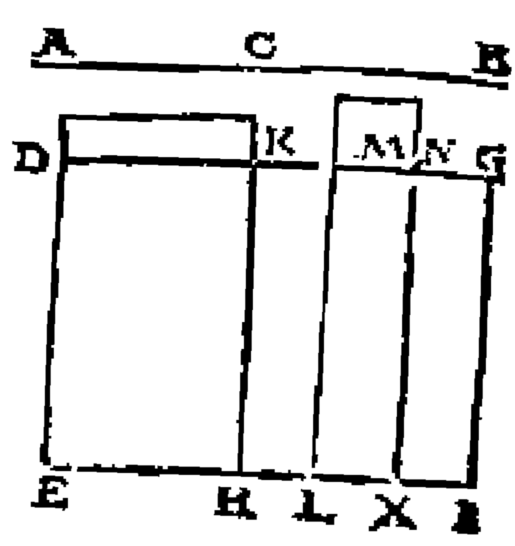
Quadratum Binomij secundum lineam rationalem applicatum, facit alterum latus Binomium primum.

Theorema 44. Propositio 61.



Quadratũ Bimedialis primi secundum rationalẽ lineam applicatum, facit alterum latus Binomiũ secundum.

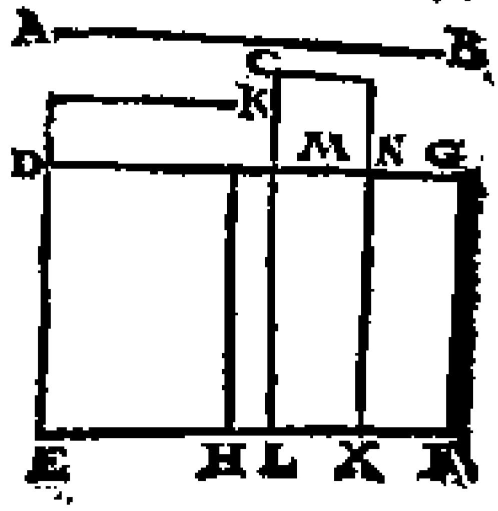
Theorema 45. Propositio 62.



Quadratũ Bimedialis secũdi secundũ rationalẽ applicatum, facit alterũ latus Binominũ tertiũ.

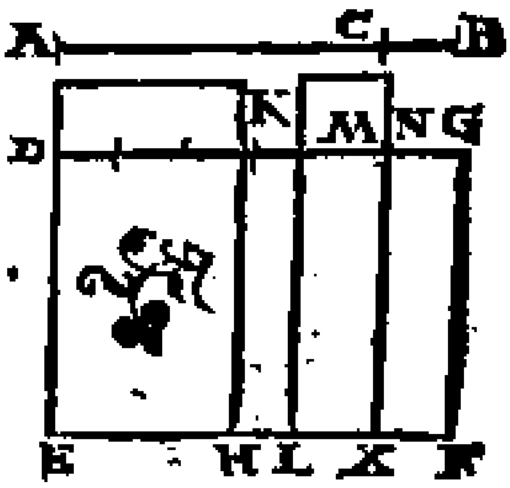
Theo-

Theorema 46. Propoſitio 63.



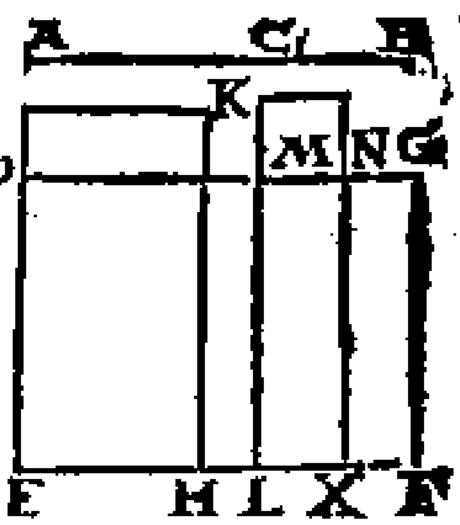
Quadratum lineæ maioris ſecundum lineam rationalem applicatum, facit alterum latus Binomium quartum.

Theorema 47. Propoſitio 64.



Quadratum lineæ potétis rationale & mediale, ſecundum rationalé applicatũ, facit alterum latus Binomium quintũ.

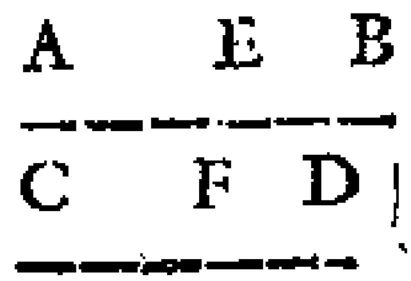
Theorema 48. Propoſitio 65.



Quadratum lineę potentis duo medialia ſecundũ rationalem applicatum, facit alterum latus Binomium ſextum.

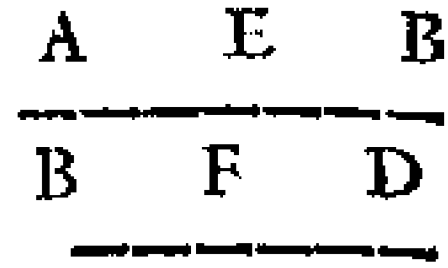
Theorema 49. Propoſitio 66.

Linea longitudine commenſurabilis Binomio eſt & ipſa Binomium eiuſdé ordinis.



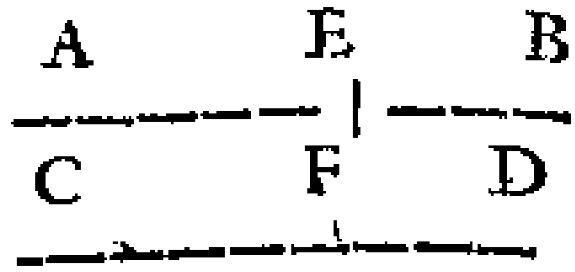
Theorema 50. Propositio 67.

Linea longitudine commensurabilis alteri bimedialium, est & ipsa bimediale etiam eiusdẽ ordinis.



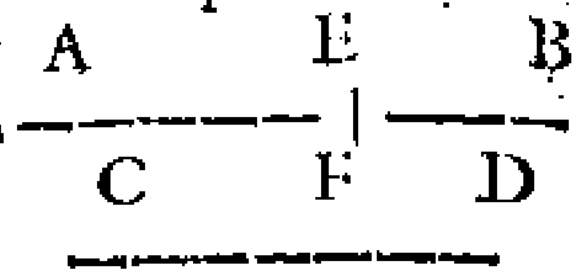
Theorema 51. Propositio 68.

Linea cõmẽsurabilis lineæ maiori, est & ipsa maior.



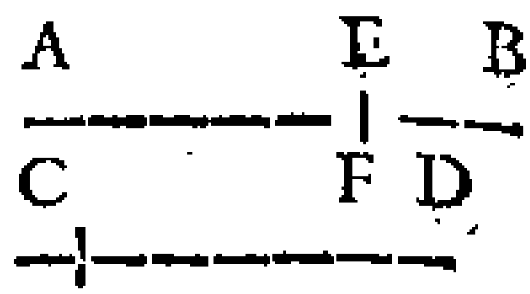
Teorema 52. Propositio 69.

Linea commẽsurabilis lineæ potenti rationale & mediale, est & ipsa linea potens rationale & mediale.



Theorema 53. Propositio 70.

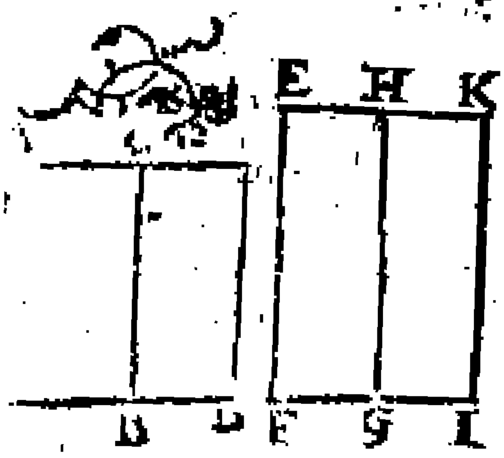
Linea commensurabilis lineæ potenti duo medialia, est & ipsa linea potens duo medialia.



Theorema 54. Propositio 71.

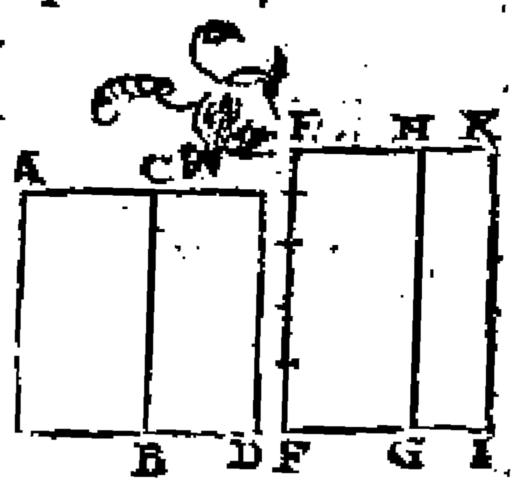
Si duæ superficies rationalis & medialis simul componantur, linea que totã superficiem

ciem compositã potest, est vna ex quatuor irrationalibus, vel ea quæ dicitur Binomium, vel bimediale primum, vel linea maior, vel linea potés rationale & mediale.



Theorema 55. Propositio 72.

Si duæ superficies mediales incõmensurabiles simul componãtur, fiunt reliquæ duæ lineę irrationales, vel bimediale secundum, vel linea potés duo medialia



SCHOLIVM.

Binomium & cæteræ consequentes lineæ irrationales, neque sunt eædem cum linea mediali, neque ipsæ inter se.

Nam quadratum lineæ medialis applicatum secundum lineam rationalem, facit alterum latus lineam rationalem, & longitudine incommensurabilem lineæ secundum quam applicatur, hoc est, lineæ rationali, per 23.

Quadratum verò Binomij secundum rationalem applicatum, facit alterum latus Binomium primum, per 60.

Quadratum verò Bimedialis primi secundum rationalem applicatum, facit alterum latus Binomiũ secundum, per 61.

Quadratum verò Bimedialis secundi secundum rationalem applicatum, facit alterum latus Binomiũ tertium, per 62.

Quadratum verò lineæ maioris secundum rationalem applicatum, facit alterum latus Binomiũ quartum, per 63.

Quadratum verò lineæ potẽtis rationale & mediale secundum rationalem applicatum, facit alterũ latus Binomium quintum, per 64.

Quadratum verò lineæ potentis duo medialia secũdum rationalem applicatum, facit alterum latus Binomium sextum, per 65.

Cùm igitur dicta latera, quæ latitudines vocantur, differant & à prima latitudine, quoniam est rationalis, cùm inter se quoque differant, eo quia sunt Binomia diuersorum ordinum: manifestum est ipsas lineas irrationales, differentes esse inter se.

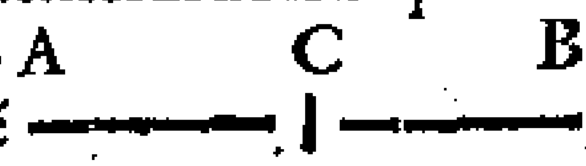
SECVN.

SECVNDVS ORDO ALterius sermonis, qui est de detractione.

Principium senariorum per detractionẽ.

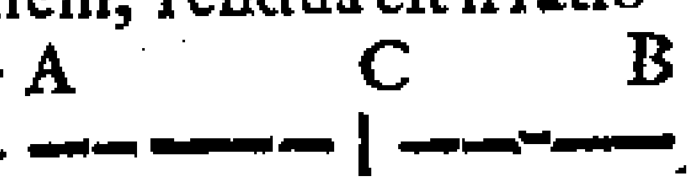
Theorema 56. Propositio 73.

Si de linea rationali detrahatur rationalis potentia tantùm commẽsurabilis ipsi toti, residua est irrationalis, vocetur autẽ Residuum.



Theorema 57. Propositio 74.

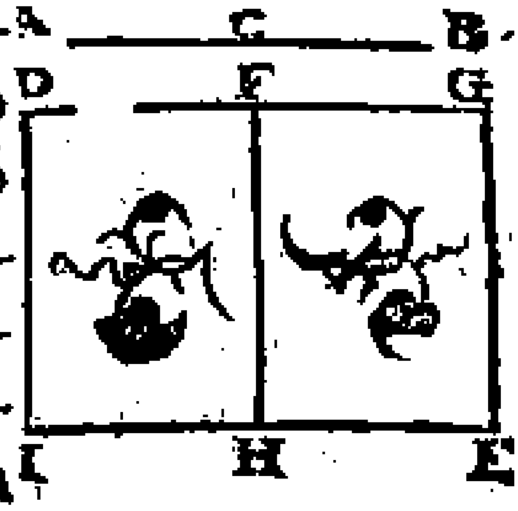
Si de linea mediali detrahatur medialis potentia tantũ cõmensurabilis toti lineæ, quæ verò detracta est cũ tota contineat superficiem rationalem, residua est irrationalis. Vocetur autem Residuũ mediale primum.



Theorema 58. Propositio 75.

Si de linea mediali detrahatur medialis

tentia tantùm commensurabilis toti, quæ verò detracta est, cùm tota cõtineat superficiẽ medialẽ, reliqua est irrationalis. Vocetur autem Residuũ mediale secundum.



Theorema 57. Propositio 76.

Si de linea recta detrahatur recta potẽtia incommensurabilis toti, compositum autem ex quadratis totius lineæ & lineæ detractæ sit rationale, parallelogrammũ verò ex ijsdem contentum sit mediale, reliqua linea erit irrationalis. A C B Vocetur autem linea minor.

Theorema 58. Propositio 77.

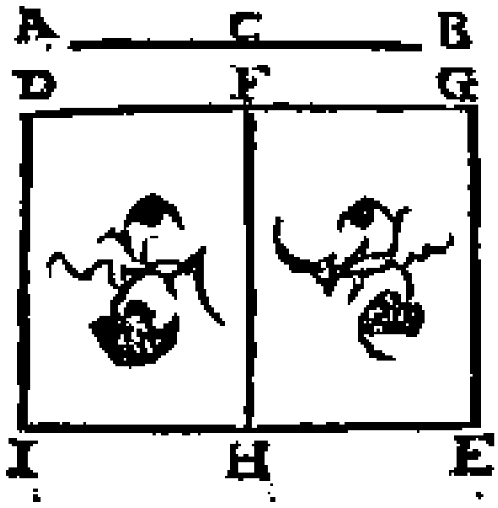
Si de linea recta detrahatur recta potentia

tia incommensurabilis toti lineæ, compositũ autem ex quadratis totius & lineæ detractæ sit mediale, parallelogrammum verò bis ex eisdem contentum sit rationale, reliqua linea est irrationalis. Vocetur autem linea faciens cum superficie rationali totam superficiem medialem.

Theorema 59. Propositio 78.

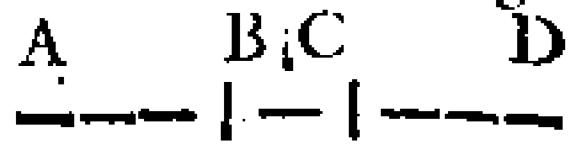
Si de linea recta detrahatur recta potentia incommensurabilis toti lineæ, compositum autem ex quadratis totius & lineæ detractæ sit mediale, parallelogrammum verò bis ex ijsdem sit etiam mediale: præterea sint quadrata ipsarum incommensurabilia parallelogrammo bis ex ijsdem cõtento, reliqua linea est irrationalis. Vocetur autem linea faciens cum superficie

ficie mediali totam ſuperficiem medialem.



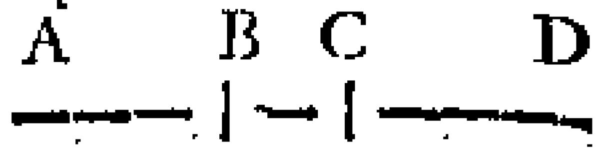
Theorema 60. Propoſitio 79.

Reſiduo vnica tantùm linea recta cõiungitur rationalis, potentia tantùm commenſurabilis toti lineæ.

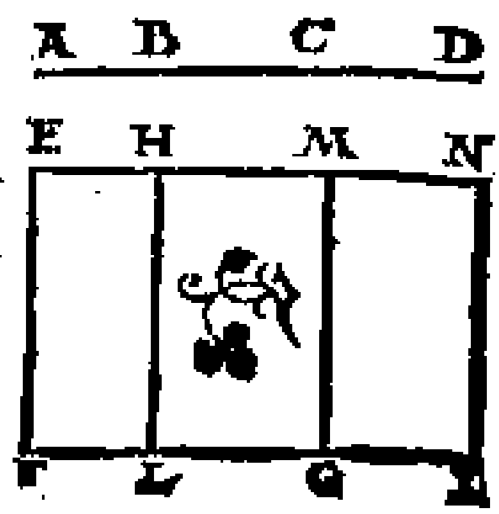


Theorema 61. Propoſitio 80.

Reſiduo mediali primo vnica tãtùm linea coniungitur medialis, potentia tantùm cõmenſurabilis toti, ipſa cum tota continẽs rationale.



Theorema 62. Propoſitio 81.



Reſiduo mediali ſecundo vnica tantùm cõiungitur medialis, potentia tantùm commẽſurabilia toti ipſa cum tota continens mediale.

Theorema 63. Propoſitio 82.

Lineæ minori vnica tantùm recta cõiungitur

tur potentia incommenſurabilis toti, faciens cum tota compoſitum ex quadratis ipſarum rationale, id verò parallelogrammum, quod bis ex ipſis fit, mediale.

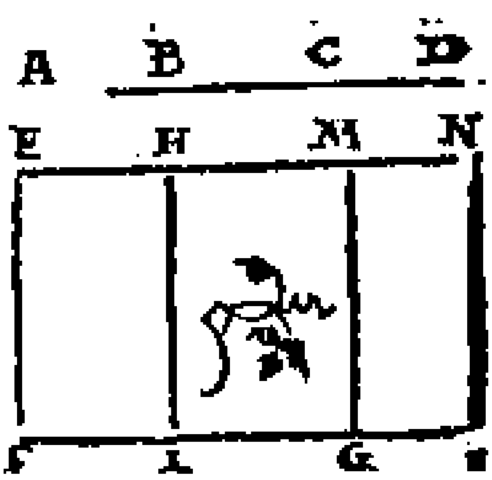
A B C D

Theorema 64. Propoſitio 83.

Lineæ facienti cum ſuperficie rationali totam ſuperficiem medialem, vnica tantùm coniungitur linea recta potentia incommenſurabilis toti, faciens autem cum tota compoſitum ex quadratis ipſarũ, mediale, id verò quod fit bis ex ipſis, rationale.

A B C D

Theorema 65. Propoſitio 84.



Lineæ cum mediali ſuperficie facienti totam ſuperficiem medialem, vnica tantùm coniũgitur linea potẽtia toti incommenſurabilis, faciens cum tota compoſitũ ex quadratis ipſarũ mediale, id verò quod fit bis ex ipſis etiam mediale, & præterea faciens compoſitum ex quadratis ipſarum incommenſurabile ei quod fit bis ex ipſis.

DE-

DEFINITIONES TERTIAE.

Propoſita linea rationali & reſiduo.

1

Si quidem tota, nempe compoſita ex ipſo reſiduo & linea illi cõiuncta, plus poteſt quàm coniuncta, quadrato lineæ ſibi cõmenſurabilis longitudine, fueritq́; tota longitudine commẽſurabilis lineæ propoſitæ rationali, reſiduũ ipſum vocetur Reſiduum primum.

2

Si verò coniuncta fuerit longitudine commenſurabilis rationali, ipſa autem tota plus poſſit quam coniuncta, quadrato lineæ ſibi longitudine commenſurabilis; reſiduum vocetur Reſiduum ſecundũ.

3

Si verò neutra linearũ fuerit longitudine commenſurabilis rationali, poſſit autem ipſa tota plusquam coniuncta, quadrato lineæ ſibi longitudine commẽſurabilis, vocetur Reſiduum tertium.

Rurſus ſi tota poſſit plus quàm coniuncta, quadrato lineæ ſibi longitudine incõmenſurabilis.

4

Et quidem ſi tota fuerit longitudine com men-

mensurabilis ipsi rationali, vocetur Residuum quartum.

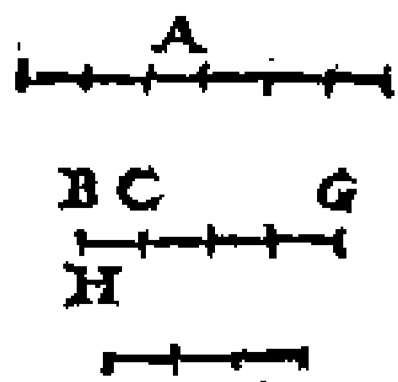
5

Si verò coniuncta fuerit lógitudine commensurabilis rationali,& tota plus possit quàm coniuncta,quadrato lineæ sibi longitudine incómensurabilis, vocetur Residuum quintum.

6

Si verò neutra linearum fuerit cómensurabilis lógitudine ipsi rationali, fueritque tota potentior quã coniuncta,quadrato lineæ sibi lógitudine incommẽsurabilis,vocetur Residuum sextum.

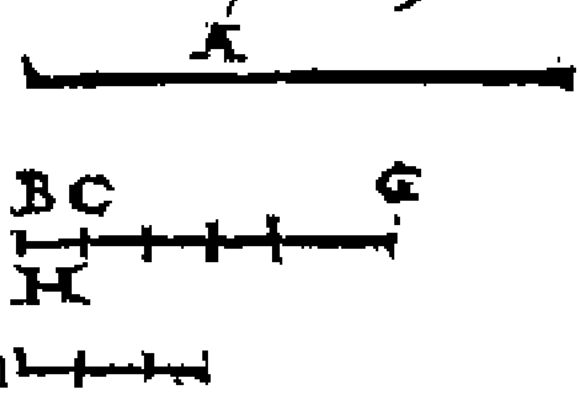
Problema 18. Propositio 85.



Reperire primum Residuum.

16
D.......F........E
7 9

Problema 19.Propositio 86.



Reperire secundum Residuum.

D.........F.........E
27 9

Proble-

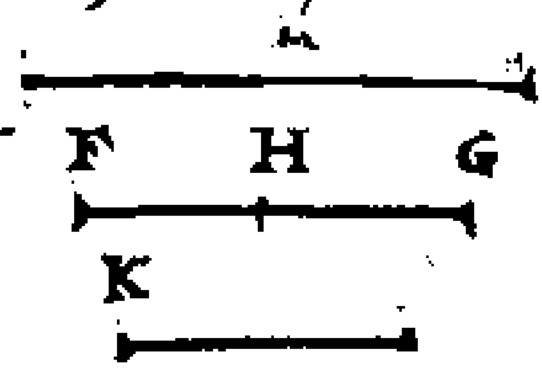
E............

12

Problema 20. Propositio 87.

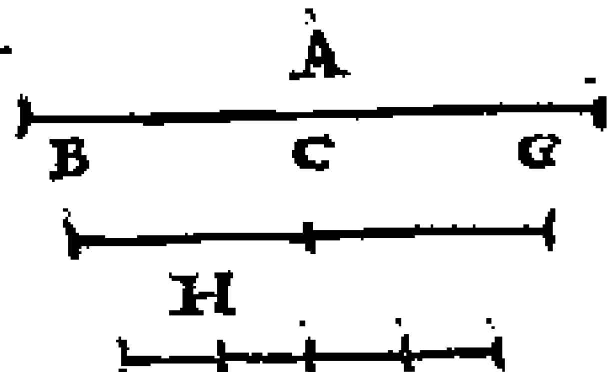
B.........D.......C

9 7



Reperire tertium Residuum.

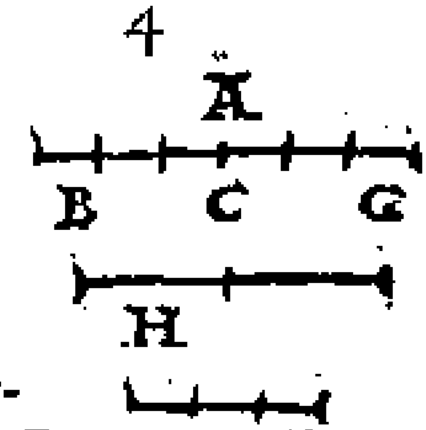
Probl. 21. Propositio 88.



Reperire quartū Residuum. D...............F....E

16 4

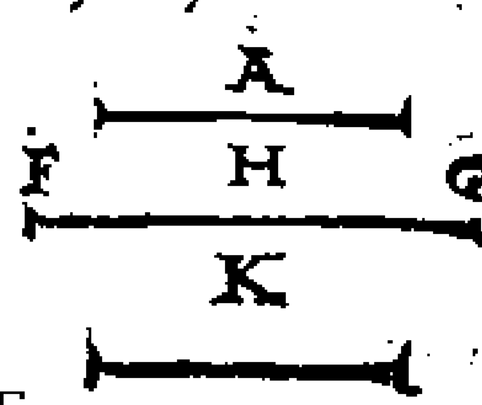
Problema 22. Propositio 89.



Reperire quintum Residuum. D.......F....E

25 7

Problema 23. Propositio 90.



Reperire sextum Residuum.

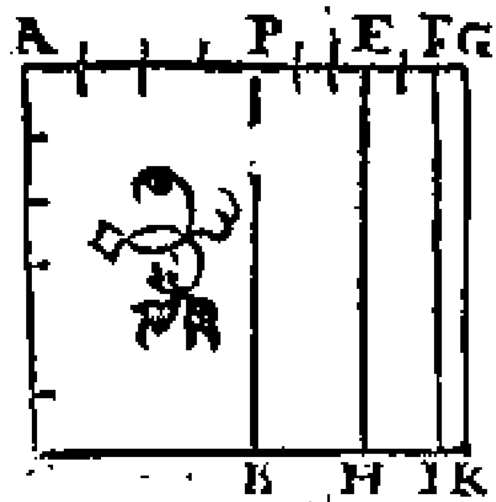
E..............

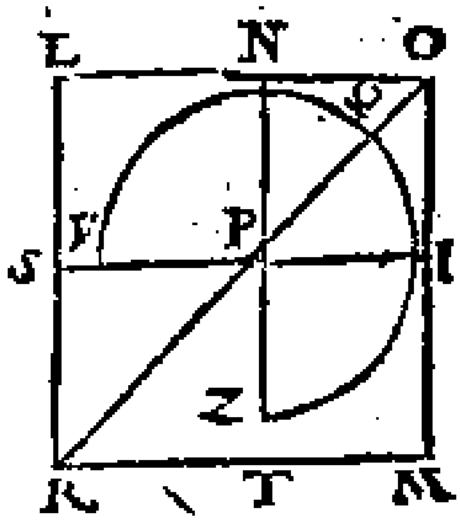
15

Theo- B........D......C

Theorema 66. Propositio 91.

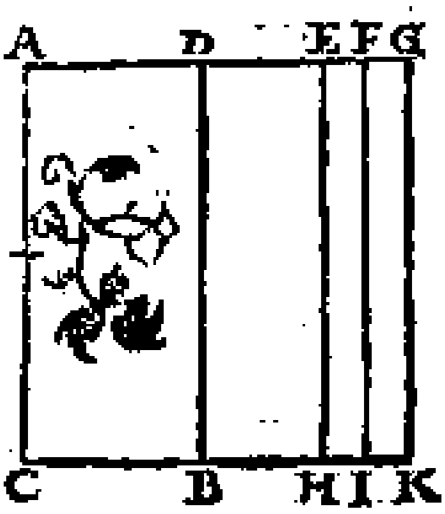
Si superficies cõtineatur ex linea rationali & residuo primo linea, quæ illã sup-ficiem potest, est residuum.

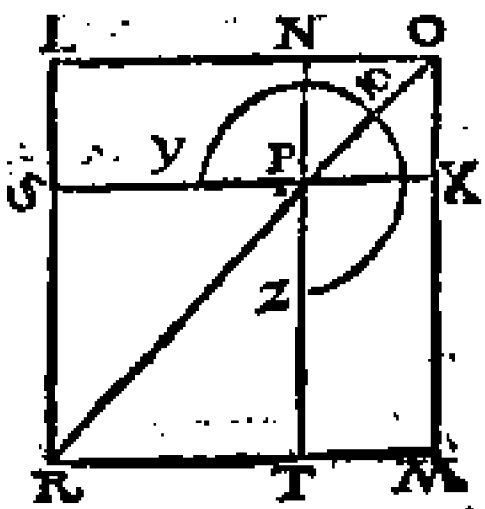




Theorema 67. Propositio 92.

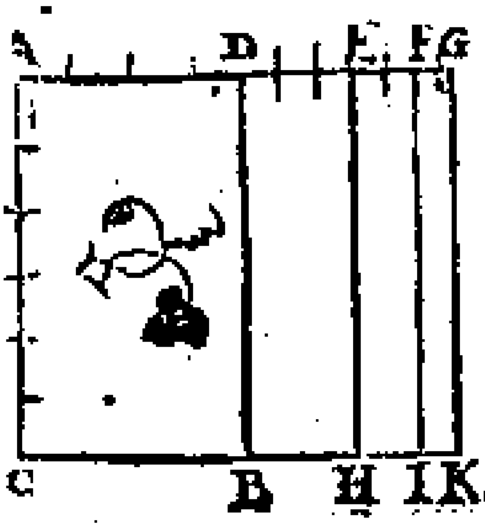
Si superficies cõtineatur ex linea rationali & residuo secũdo, linea q̃ illam supficié potest, est residuum mediale primum.

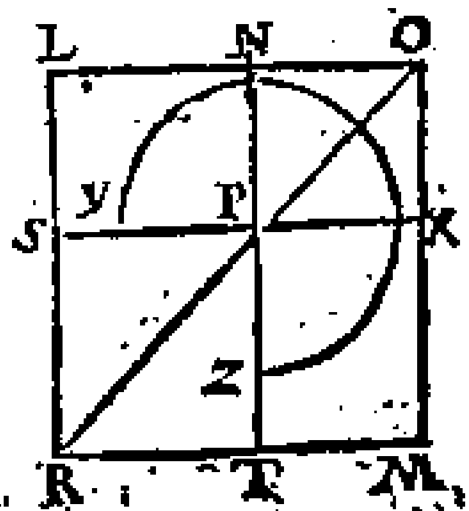




Theorema 68. Propositio 93.

Si supficies cõtineatur ex linea rationali & residuo ter-

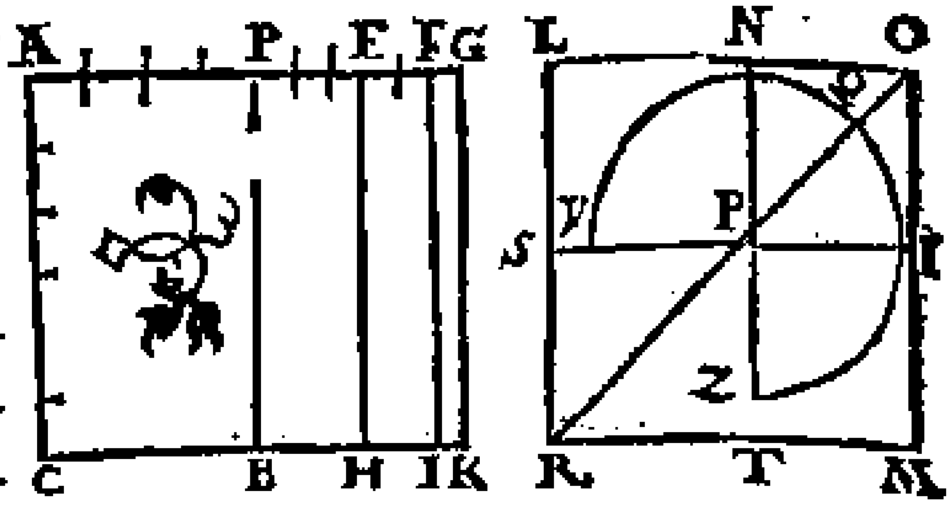




tio, linea quæ illam ſuperficiem poteſt, eſt reſiduum mediale ſecundum.

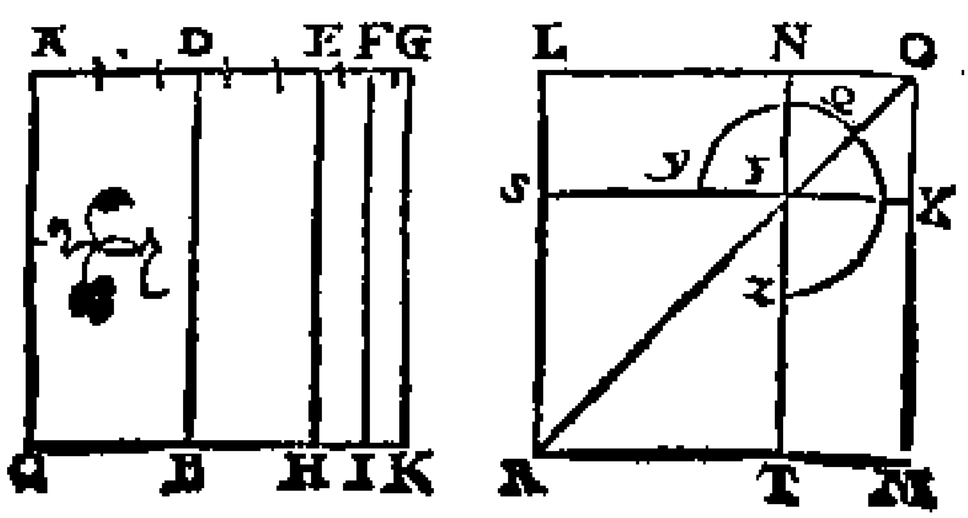
Theorema 69. Propoſitio 94.

Si ſuperficies cõtineatur ex linea rationali & reſiduo quarto linea q̃ illã ſuperficiem poteſt, eſt linea minor.



Theorema 70. Propoſitio 95.

Si ſuperficies cõtineatur ex linea rationali & reſiduo quinto, linea quæ illã ſuperficiẽ poteſt, eſt ea quæ dicitur cum rationali ſuperficie faciens totã medialem.

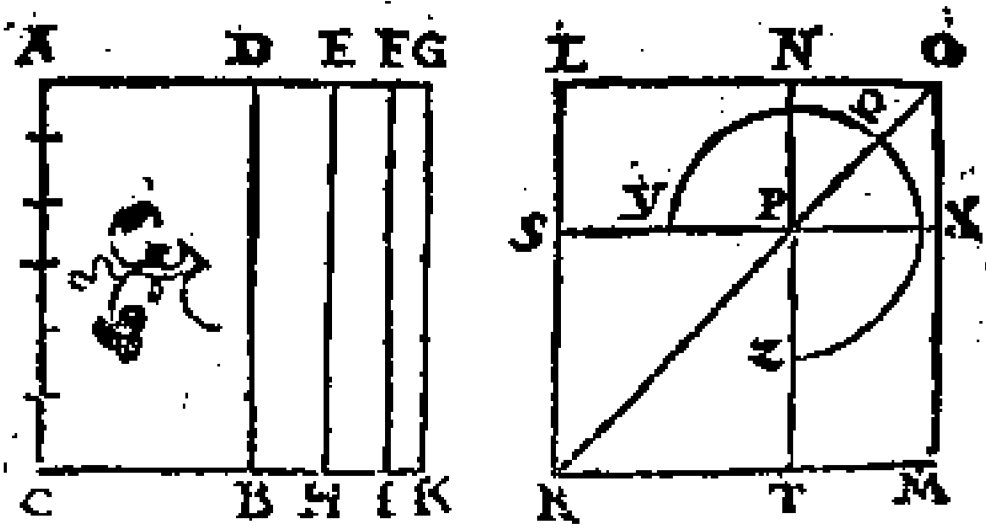


Theorema 71. Propoſitio 96.

Si ſuperficies cõtineatur ex linea rationali &

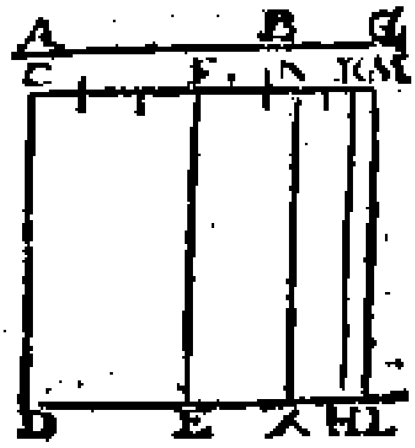
& residuo sexto, linea quæ illam superficiē

pōt, est ea quę dicitur faciēs cum mediali



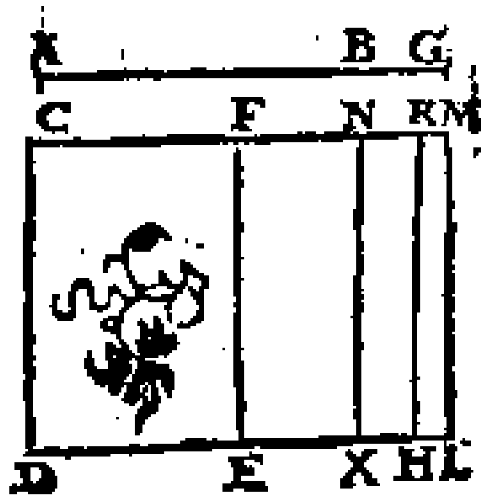
superficie totam medialem.

Theorema 72. Propositio 97.



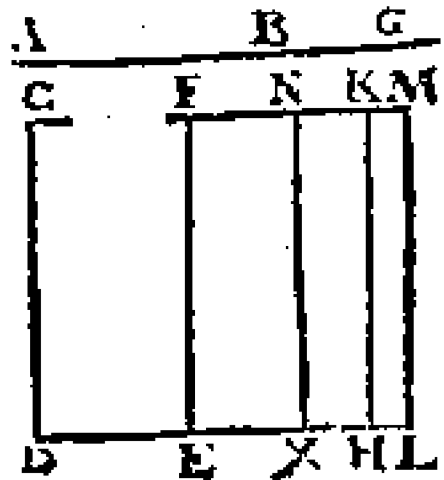
Quadratum residui secundum lineam rationalem applicatum, facit alterum lat⁹ residuum primum.

Theorema 73. Propositio 98.



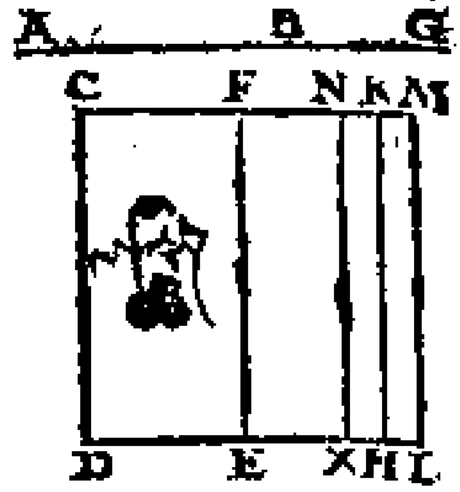
Quadratum residui medialis primi secundū rationalem applicatū, facit alterū latus residuū secundum.

Theorema 74. Propositio 99.



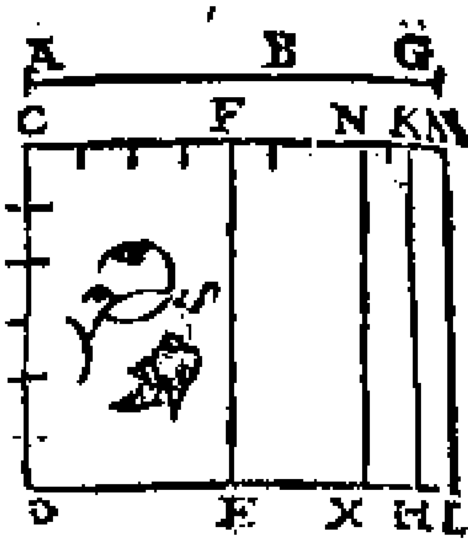
Quadratū residui medialis secūdi secundum rationalē applicatū, facit alterū latus residuū tertium.

Theorema 75. Propositio 100.



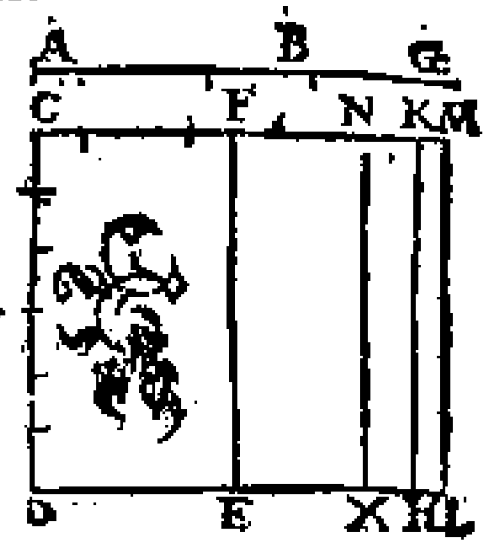
Quadratũ lineæ minoris ſecundum rationalẽ applicatum, facit alterũ latus reſiduum quartũ.

Theorema 76. Propoſitio 101.



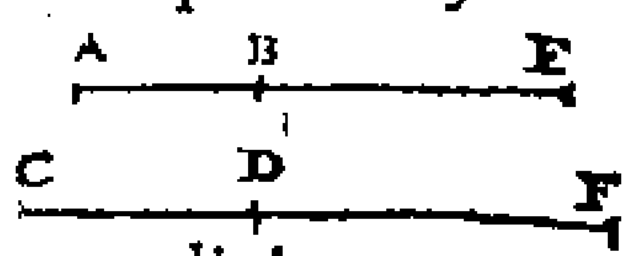
Quadratũ lineæ cum rationali ſuperficie faciẽtis totam medialem, ſecũdum rationalem applicatum, facit alterum latus reſiduum quintum.

Theorema 77. Propoſitio 102.



Quadratum lineæ cum mediali ſuperficie faciẽtis totam medialem, ſecundum rationalem applicatum, facit alterum latus reſiduum ſextum.

Theorema 78. Propoſitio 103.

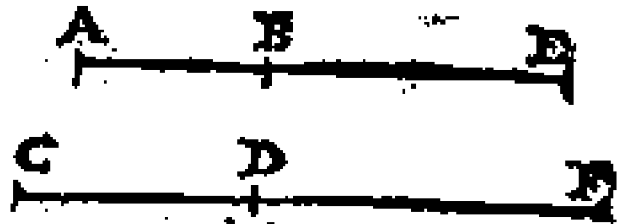


Linea reſiduo cõmẽſurabilis lõgitudine, eſt & ipſa reſiduum, & eiuſdem ordinis.

Theorema 79. Propoſitio 104.

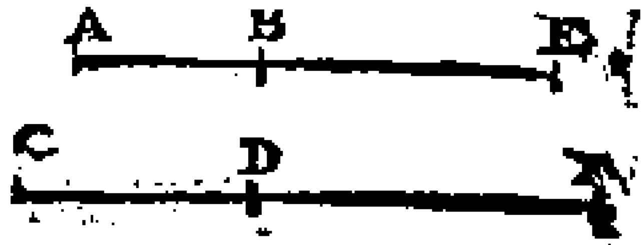
Linea cõmenſurabilis reſiduo mediali, eſt

&



& ipſa reſiduũ mediale, & eiuſdẽ ordinis.

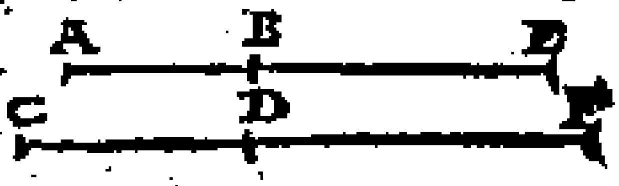
Theorema 80. Propoſitio 105.



Linea commẽſurabilis lineæ minori, eſt & ipſa linea minor.

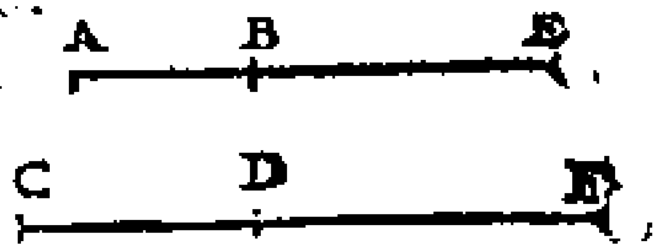
Theorema 81. Propoſitio 106.

Linea commenſurabilis lineæ cũ rationali ſuperficie facienti totã medialẽ, eſt & ipſa linea cum rationali ſuperficie faciens totam medialem.



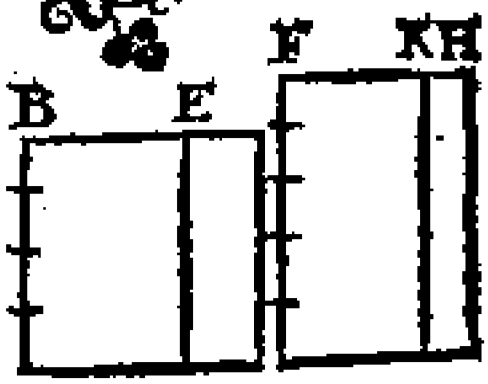
Theorema 82. Propoſitio 107.

Linea cõmẽſurabilis lineæ cũ mediali ſuperficie facienti totam medialem, eſt & ipſa cum mediali ſuperficie faciens totam medialem.



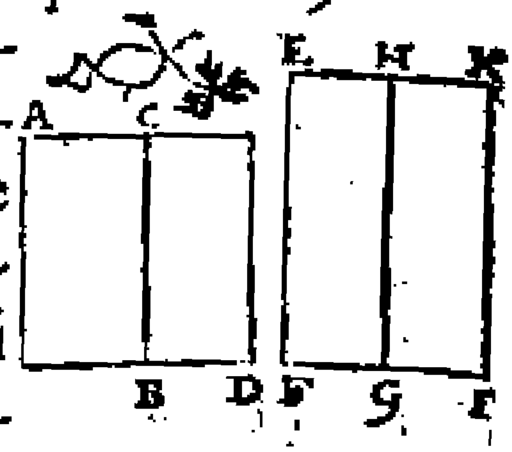
Theorema 83. Propoſitio 108.

Si de ſuperficie rationali detrahatur ſuperficies medialis, linea quæ reliquã ſuperficiem poteſt, eſt alterutra ex duabus irrationalibus, aut reſiduũ, aut linea minor.



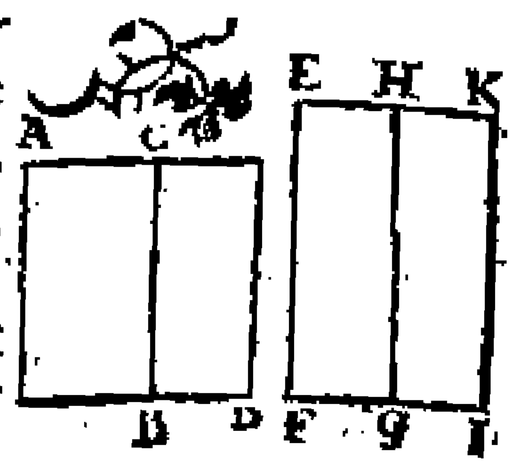
Theorema 84. Propoſitio 109.

Si de ſuperficie mediali detrahatur ſuperficies rationalis, aliæ duæ irrationales fiũt, aut residuum mediale primũ aut cũ rationali ſuperficiem faciens totam medialem.



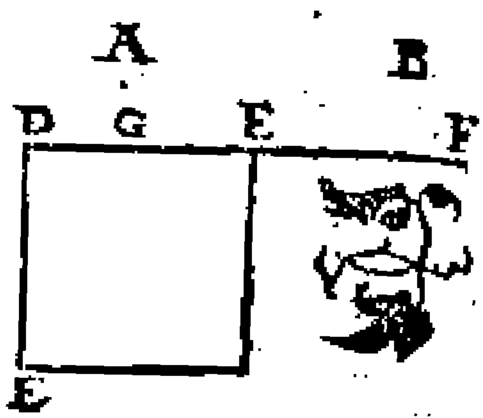
Theorema 85. Propoſitio 110.

Si de ſuperficie mediali detrahatur ſuperficies medialis q̃ ſit incõmẽſurabilis toti, reliquæ duæ fiunt irrationales, aut reſiduum mediale ſecundum, aut cũ mediali ſuperficie faciens totam medialem.



Theorema 86. Propoſitio 111.

Linea quæ Reſiduum dicitur, non eſt eadem cum ea quæ dicitur, Binomium.



SCHO-

SCHOLIVM.

Linea quæ Residuum dicitur, & cæteræ quinque eã consequentes irrationales, neque lineæ mediali neque sibi ipsæ inter se sunt eædem. Nam quadratum lineæ medialis secundum rationalem applicatum, facit alterum latus, rationalem lineam longitudine incommensurabilem ei, secundum quam applicatur, per 23.

Quadratum verò residui secundum rationalem applicatum, facit alterum latus residuum primum per 97.

Quadratum verò residui medialis primi secundum rationalem applicatum, facit alterum latus residuum secundum, per 98.

Quadratum verò residui medialis secundi, facit alterum latus residuum tertium, per 99.

Quadratum verò lineæ minoris, facit alterum latus residuum quartum, per 100.

Quadratum verò lineæ cum rationali superficie facientis totam medialem, facit alterum latus residuum quintum, per 101.

Quadratum verò lineæ cum mediali superficie facientis totam medialem, secundum rationalem applicatum, facit alterum latus residuum sextum, per 102.

Cùm igitur dicta latera, quæ sunt latitudines cuiusque parallelogrammi vnicuique quadrato æqualis & secundum rationalem applicati, differant & à primo latere, & ipsa inter se (nam à primo differunt: quoniam sunt resi-

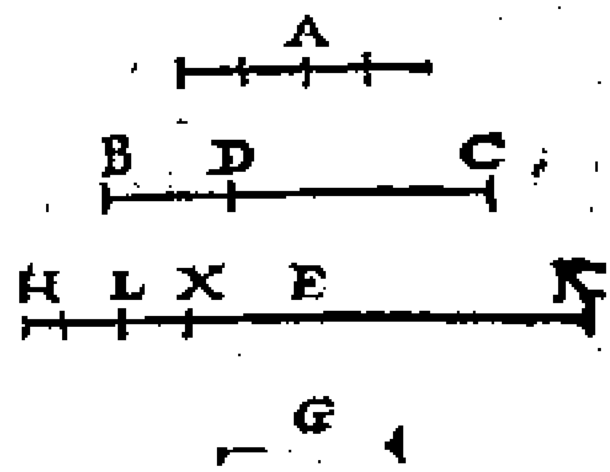
dua non eiusdem ordinis) constat ipsas quoque lineas irrationales inter se differentes esse. Et quoniam demonstratum est, Residuum non esse idem quod Binomium, quadrata autem residui & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex residuis eiusdem ordinis cuius sunt & residua, quorum quadrata applicantur rationali similiter & quadrata Binomij & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex Binomijs eiusdem ordinis, cuius sunt & Binomia, quorum quadrata applicantur rationali. Ergo lineæ irrationales quæ cõsequuntur Binomium, & quæ consequuntur residuum, sunt inter se differentes. Quare dictæ lineæ omnes irrationales sunt numero 13.

1 Medialis.
2 Binomium.
3 Bimediale primum.
4 Bimediale secundum.
5 Maior.
6 Potens rationale & mediale.
7 Potẽs duo medialia.
8 Residuum
9 Residuum mediale primum.
10 Residuum mediale secundum.
11 Minor.
12 Faciens cum rationali superficie totam medialem.
13 Faciens cum mediali superficie totam medialem.

Theo-

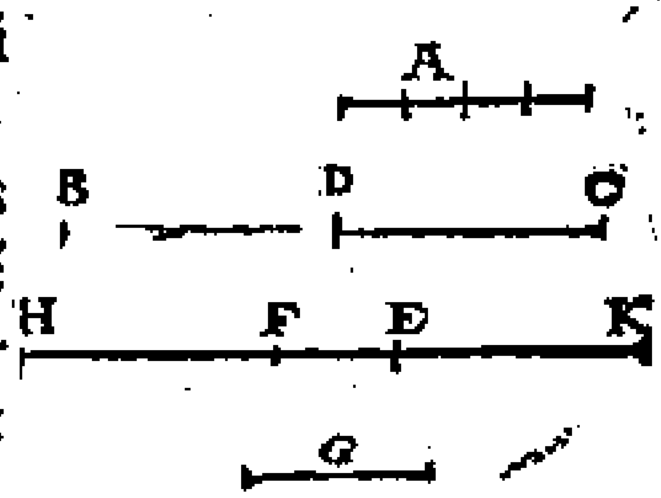
Theorema 87. Propoſitio 112.

Quadratū lineæ rationalis ſecundum Binomiū applicatū, facit alterū latus reſiduū, cuius nomina ſunt cõmenſurabilia Binomij nominibus, & in eadẽ proportione præterea id quod fit Reſiduum, eundem ordinem retinet quem Binomium.



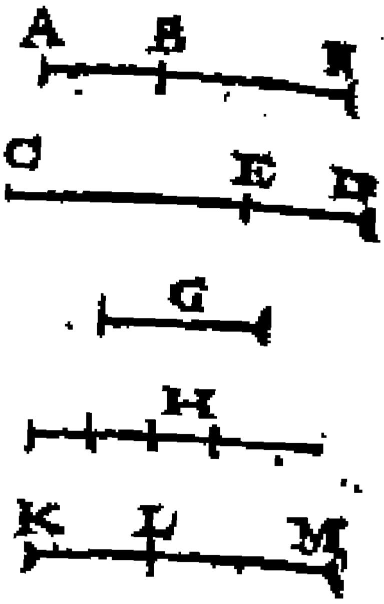
Theorema 88. Propoſitio 113.

Quadratū lineæ rationalis ſecūdum reſiduum applicatū, facit alterum latus Binomium, cuius nomina ſunt cõmenſurabilia nominibus reſidui & in eadẽ proportione: pręterea id quod fit Binomiū, eſt eiuſdem ordinis, cuius & Reſiduum.



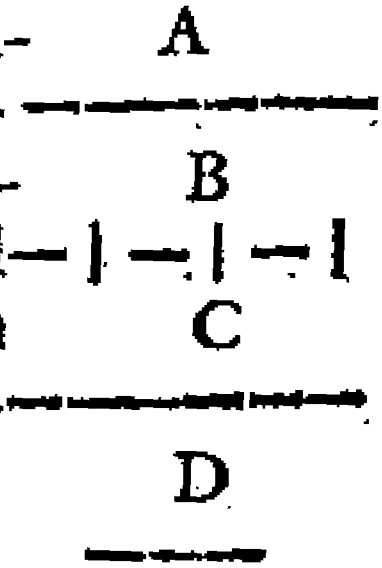
Theorema 89. Propoſitio 114.

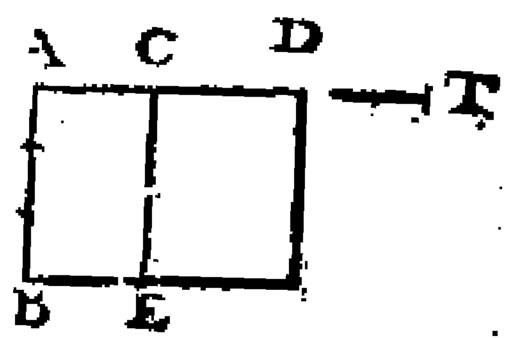
Si parallelogrammū contineatur ex reſiduo & Binomio, cuius nomina sunt cómensurabilia nominibus residui & in eadẽ proportione, linea quæ illam superficiem potest, est rationalis.



Theorema 90. Propositio 115.

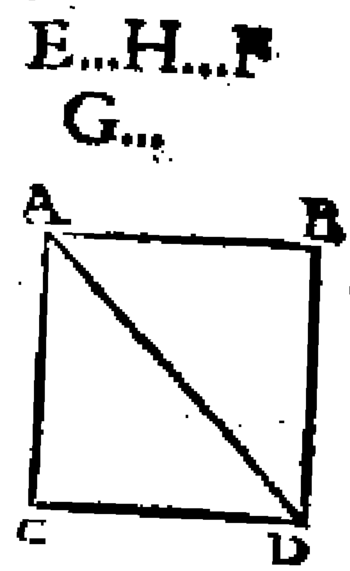
Ex linea mediali nascũtur lineę irrationales innumerabiles, quarum nulla vlli ante dictarum eadem sit.





Propositio 116.

Propositũ nobis esto demõstrare in figuris quadratis diametrũ esse longitudine incómensurabilem ipsi lateri.



ELEMENTI X. FINIS.

EVCLIDIS ELEMENTVM VNDECIMVM, ET SOLIDORVM

primum.

DEFINITIONES.

1

Solidum, est quod longitudinem, latitunem, & crassitudinem habet.

2

Solidi autem extremum est superficies.

3

Linea recta est ad planum recta, cùm ad rectas oēs lineas, à quibus illa tangitur, quæque in proposito sunt plano, rectos angulos efficit.

4

Planum ad planum rectum est, cùm rectæ lineæ, quæ cōmuni planorum sectioni ad rectos angulos in vno planorum ducūtur, alteri plano ad rectos sunt angulos.

5

Rectæ lineę ad planū inclinatio, acutus est angulus, ipsa insistente linea & adiūcta altera comprehēsus, cùm à sublimi rectę illius linee termino deducta fuerit ꝑpēdicularis,

laris, atq; à puncto quod perpendicularis in ipso plano fecerit, ad propositę illius lineæ extremum, quod in eodē est plano, altera recta linea fuerit adiuncta.

6

Plani ad planū inclinatio, acutus est angulus rectis lineis contentus, quæ in vtroque planorum ad idem cōmunis sectionis pūctū ductæ, rectos ipsi sectioni angulos efficiunt.

7

Planum similiter inclinatum esse ad planum, atq; alterum ad alterum dicitur, cū dicti inclinationum anguli inter se sunt ęquales.

8

Parallela plana, sunt quæ eodem non incidunt, nec concurrunt.

9

Similes figuræ solidæ, sunt quæ similibus planis, multitudine æqualibus cōtinētur.

10

Aequales & similes figurę solidę sunt, quę similibus planis, multitudine & magnitudine æqualibus continentur.

11

Solidus angulus, est plurium quàm duarū linearum, quæ se mutuò contingant, nec in eadem sint superficie, ad omnes lineas inclinatio.

Aliter.

Aliter.

Solidus angulus, eſt qui pluribus quã duobus planis angulis in eodem non cõſiſtentibus plano, ſed ad vnũ punctũ collectis, continetur.

12

Pyramis, eſt figura ſolida quæ planis cõtinetur, ab vno plano ad vnum punctũ collecta.

13

Priſma, figura eſt ſolida quæ planis cõtinetur, quorum aduerſa duo ſunt & æqualia & ſimilia & parallela, alia verò parallelogramma.

14

Sphera eſt figura, quæ cõuerſo circum quieſcentem diametrum ſemicirculo cõtinetur, cùm in eundem rurſus locũ reſtitutus fuerit, vnde moueri cœperat.

15

Axis autem ſphæræ, eſt quieſcẽs illa linea circum quam ſemicirculus conuertitur.

16

Centrum verò Sphæræ eſt idẽ, quod & ſemicirculi.

17

Diameter autem Sphæræ, eſt recta quædã linea per centrum ducta, & vtrinq; à ſphæræ ſuperficie terminata.

Cœnus

18

Cωnus est figura, quæ conuerso circũquiescens alterum latus eorum quæ rectũ angulum continent, orthogωnio triangulo continetur, cùm in eundem rursus locum illud triangulum restitutum fuerit, vnde moueri cœperat. Atq; si quiescens recta linea æqualis sit alteri, quę circum rectũ angulum conuertitur, rectangulus erit Cωnus: sin minor, amblygωnius: si verò maior, oxygonius.

19

Axis autem Cωni, est quiescens illa linea, circum quam triangulum vertitur.

20

Basis verò Cωni, circulus est, qui à circũducta linea recta describitur.

21

Cylindrus figura est, quæ conuerso circũ quiescens alterum latus eorum quæ rectũ angulum continẽt, parallelogrammo orthogωnio comprehenditur, cùm in eundẽ rursus locum restitutum fuerit illud parallelogrammũ, vnde moueri cœperat.

22

Axis autem Cylindri, est quiescens illa recta linea, circũ quam parallelogrammum vertitur.

23

Bases verò cylindri, sunt circuli à duobus ad-

aduersus lateribus quæ circumaguntur, descripti.

24

Similes coni & cylindri, sunt quorũ & axes & basiũ diametri proportionales sunt.

25

Cubus est figura solida, quæ sex quadratis æqualibus continetur.

26

Tetraedũm est figura, quæ triãgulis quatuor æqualibus & æquilateris continetur.

27

Octaedrũ figura est solida, quæ octo triangulis æqualibus & æquilateris cõtinetur.

28

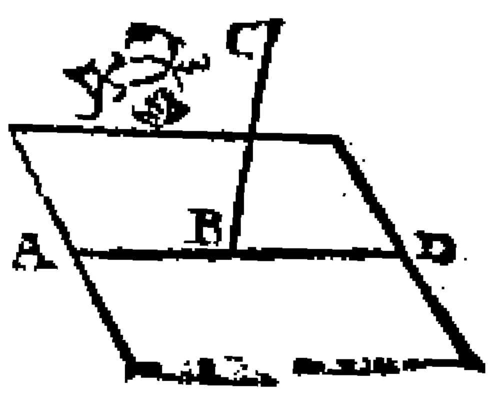
Dedecaedrum figura est solida, quæ duodecim pentagonis æqualibus, æquilateris, & æquiangulis continetur.

29

Eicosaedrum figura est solida, quę triãgulis viginti æqualibus & æquilateris continetur.

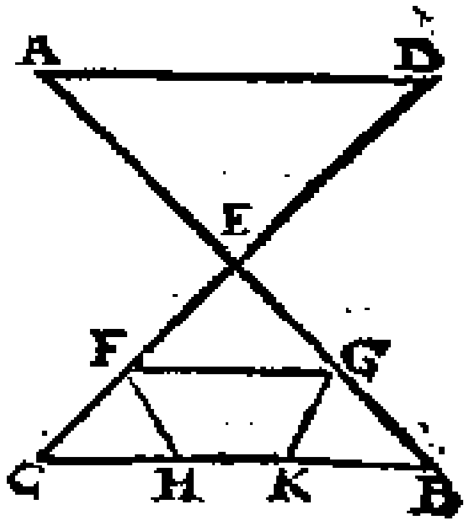
Theorema 1. Propositio 1.

Quædã rectæ lineę pars in subiecto quidem nõ est plano, quædã verò in sublimi.



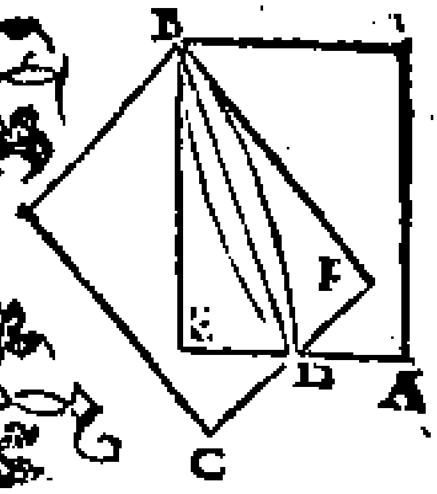
Theo-

Theorema 2. Propositio 2.



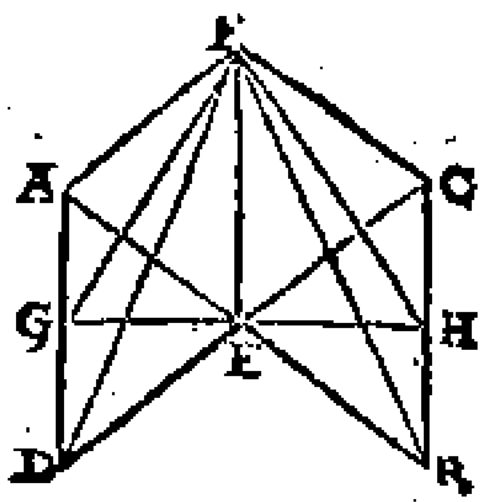
Si duę rectę lineę se mutuò secẽt, in vno sũt plano: atque triangulum omne in vnó est plano.

Theorema 3. Propositio 3.



Si duo plana se mutuò secent, communis eorũ sectio est recta linea.

Theorema 4. Propositio 4.



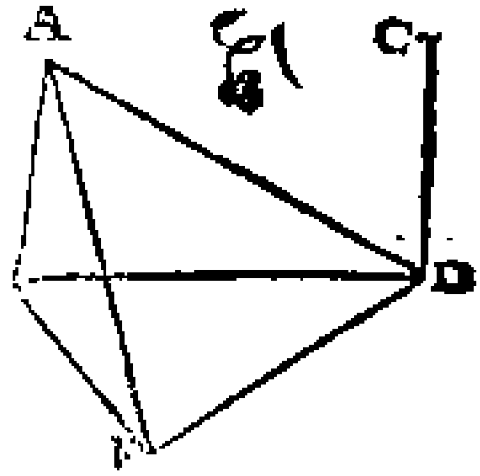
Si recta linea rectis dua bus lineis se mutuò secantibus, in cõmuni sectione ad rectos angulos insistat illá ducto etiam per ipsas plano ad angulos rectos erit.

Theorema 5. Propositio 5.

Si recta linea rectis tribus lineis se mutuò tangentibus, in communi sectione ad rectos angulos insistat, illę tres rectę in vno sunt plano.

Theo-

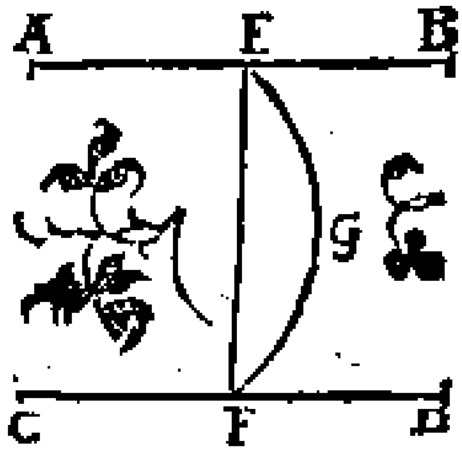
Theorema 6. Propositio 6.



Si duæ rectæ lineæ eidẽ plano ad rectos sint angulos, parallelæ erũt illæ rectæ lineæ.

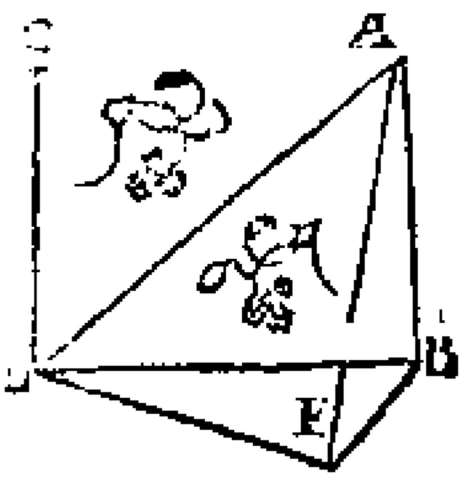
Theorema 7. Propositio 7.

Si duæ sint parallelæ rectæ lineæ, in quarũ vtraque sumpta sint quę libet pũcta, illa linea quę ad hæc puncta adiungitur, in eodem est cũ parallelis plano.



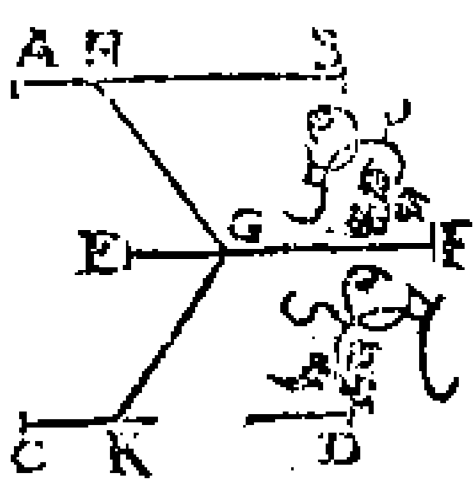
Theorema 8. Propositio 8.

Si duæ sint parallelæ rectæ lineæ, quarum altera ad rectos cuidam plano sit angulos, & reliqua eidẽ plano ad rectos angulos erit.

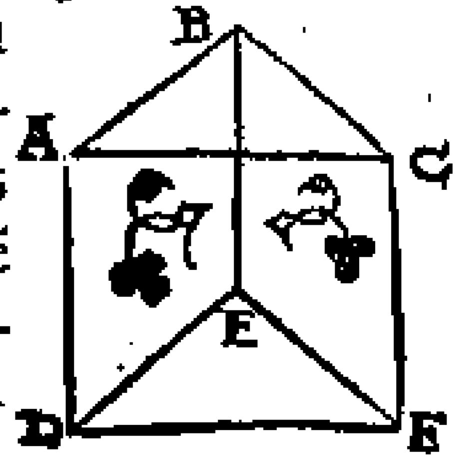


Theorema 9. Propositio 9.

Quæ eidem rectæ lineæ sunt parallelæ, sed nõ in eodem cũ illa plano, hæ quoque sunt inter se parallelæ.

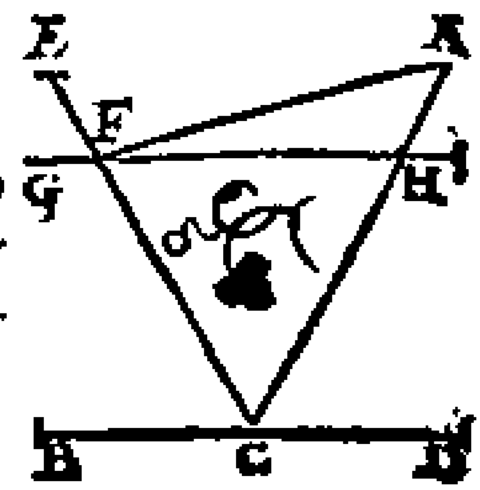


Theorema 10. Propositio 10.



Si duę rectę lineæ se mutuò tangẽtes ad duas rectas se mutuò tangentes sint parallelæ, non autẽ in eodem plano, illę angulos æquales comprehendent.

Proble. 1. Propositio 11.

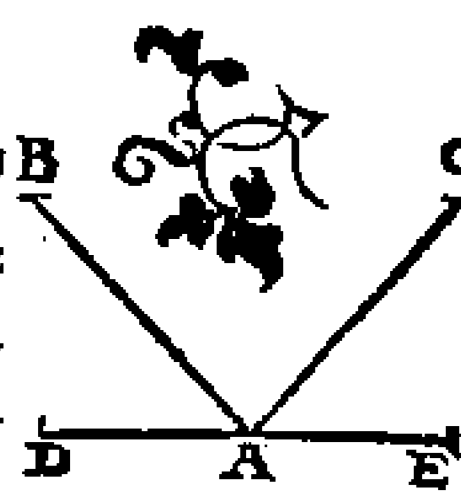


A dato sublimi puncto, in subiectum planũ perpendicularem rectã lineam ducere.

Problema 2. Propositio 12.

Dato plano, à puncto quod in illo datum est, ad rectos angulos rectam lineam excitare.

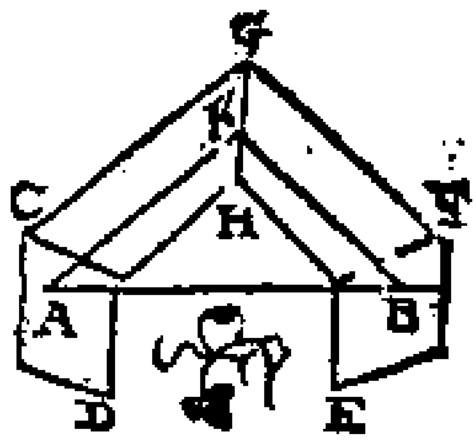
Theorema 11. Propositio 13.



Dato plano, à pũcto qđ in illo datum est, duæ rectæ lineæ ad rectos angulos non excitabuntur ad easdem partes.

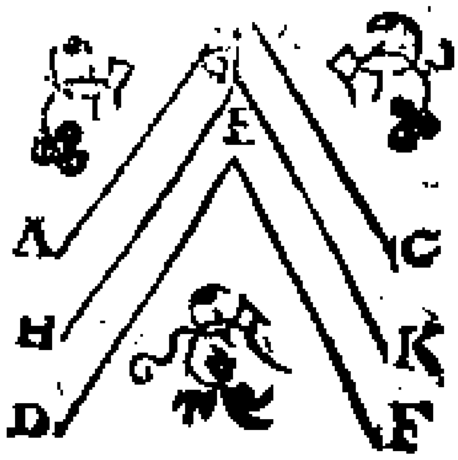
Theo-

Theorema 12. Propo-
ſitio 14.



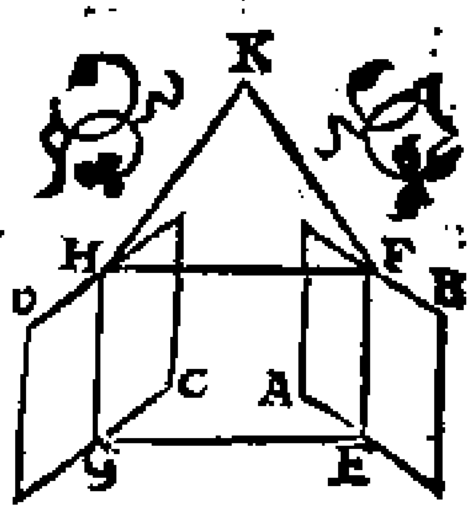
Ad quæ plana, eadem recta linea recta eſt, illa ſút parallela.

Theorema 13. Propoſitio 15.



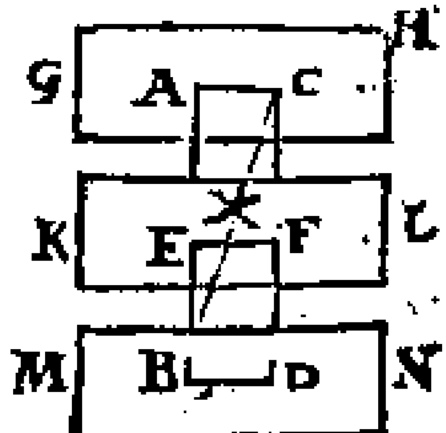
Si duæ rectæ lineæ ſe mutuò tangentes ad duas rectas ſe mutuò tangentes ſint parallelæ, non in eodem conſiſtentes plano, parallela ſunt quæ per illas ducuntur plana.

Theorema 14. Propo-
ſitio 16.



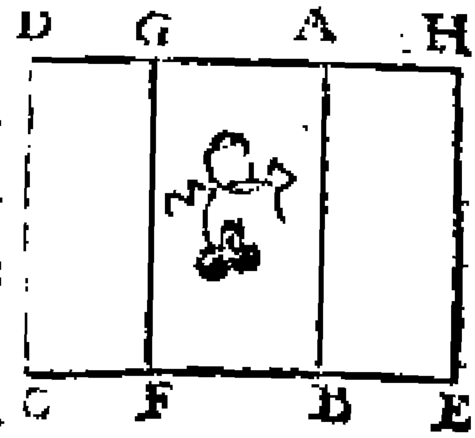
Si duo plana parallela plano quopiam ſecẽtur, communes illorum ſectiones ſunt parallelæ.

Theorema 15. Propo-
ſitio 17.



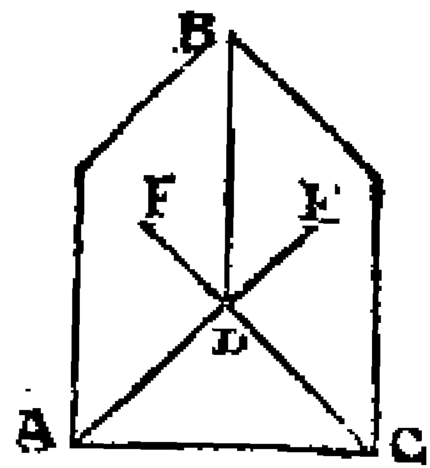
Si duæ rectæ lineę parallelis planis ſecẽtur, in eaſdem rationes ſecabũtur.

Theorema 16. Propositio 18.



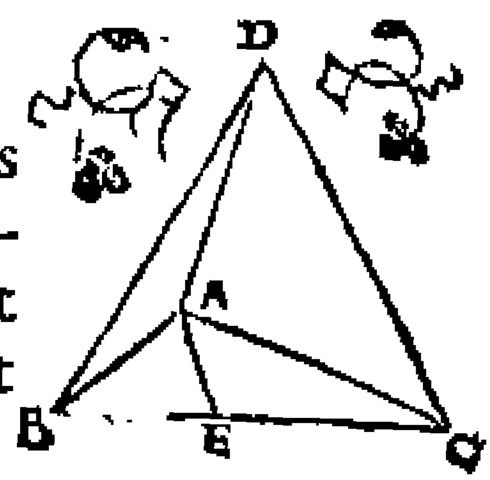
Si recta linea plano cuipiam ad rectos sit angulos, illa etiam omnia quę per ipsam plana, ad rectos eidem plano angulos erunt.

Theorema 17. Propositio 19.



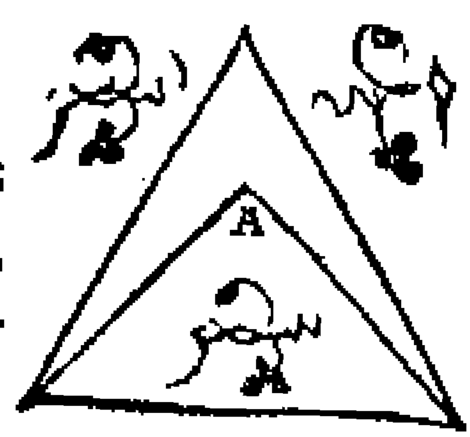
Si duo plana se mutuò secantia plano cuidã ad rectos sint angulos, cõmunis etiam illorum sectio ad rectos eidẽ plano angulos erit.

Theorema 18. Propositio 20.



Si angulus solidus planis tribus angulis contineatur, ex his duo quilibet vtut assumpti tertio sunt maiores.

Theorema 19. Propositio 21.

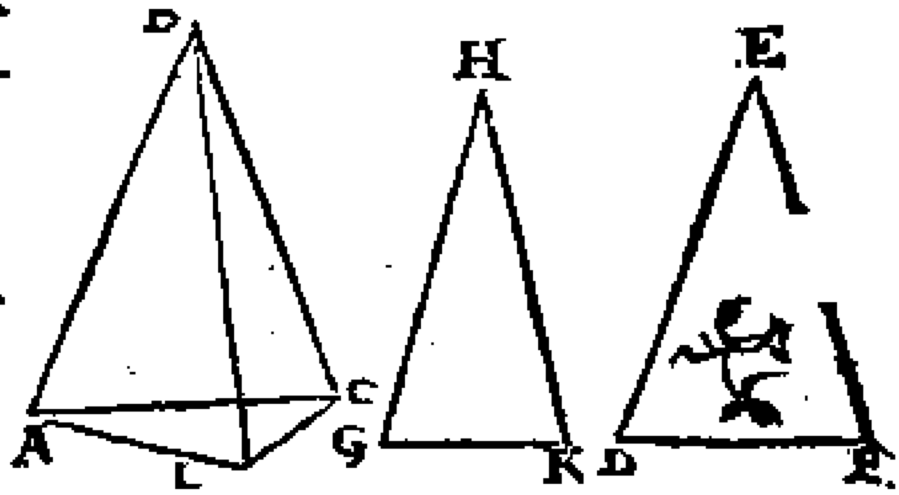


Solidus omnis angulus minoribus continetur, quàm rectis quatuor angulis planis.

Theo-

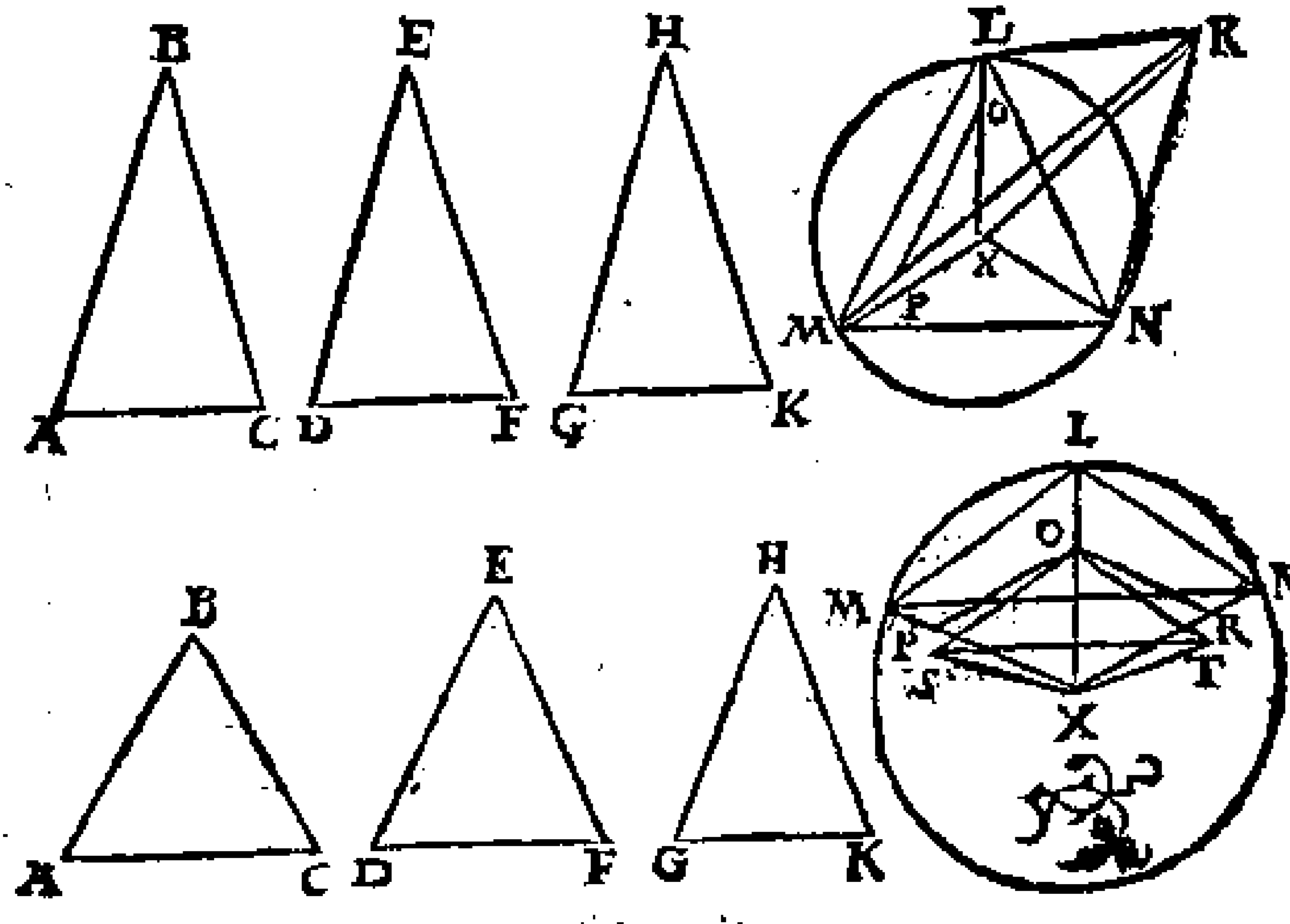
Theorema 20. Propoſitio 22.

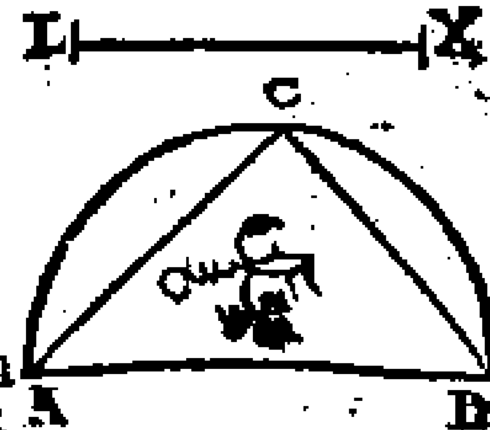
Si plani tres anguli ęqualibus rectis cõtineantur lineis, quorũ duo vt libet aſſumpti, tertio ſint maiores, triangulum conſtitui poteſt ex lineis æquales, illas rectas coniungẽtibus.



Problema 3. Propoſitio 23.

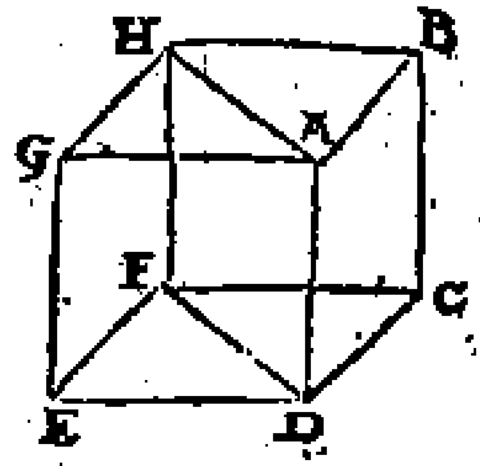
Ex planis tribus angulis, quorũ duo vt libet aſſumpti tertio ſint maiores, ſolidũ angulum conſtituere. Decet autem illos tres angulos rectis quatuor eſſe minores.

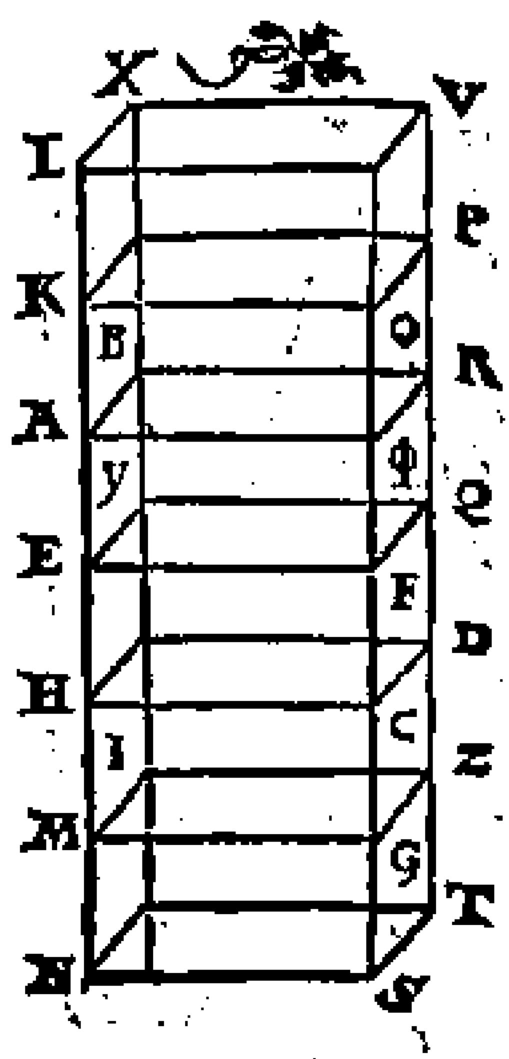




Theorema 21. Propositio 24.

Si solidum parallelis planis contineatur, aduersa illius plana & æqualia sunt & parallelogramma.





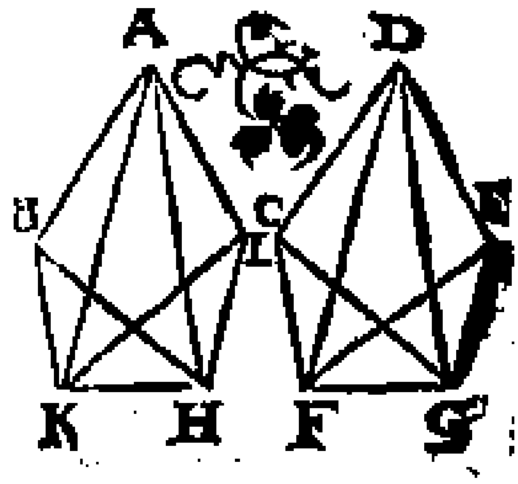
Theorema 22. Propositio 25.

Si solidum parallelis planis contentū plano secetur aduersis planis parallelo; erit quéadmodum basis ad basim, ita solidum ad solidum.

Proble-

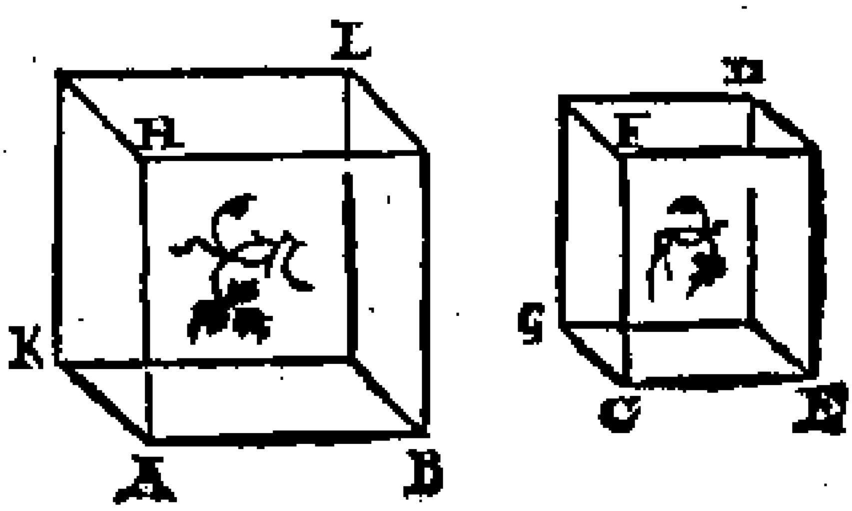
Problema 4. Propoſitio 26.

Ad datam rectam lineá eiuſque punctũ, angulum ſolidum conſtituere ſolido angulo dato æqualem.



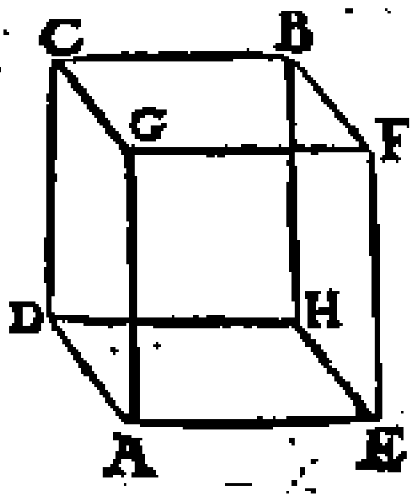
Problema 5. Propoſitio 27.

A data recta, dato ſolido parallelis planis comprehenſo ſimile & ſimiliter poſitum ſolidũ parallelis planis contẽtum deſcribere.

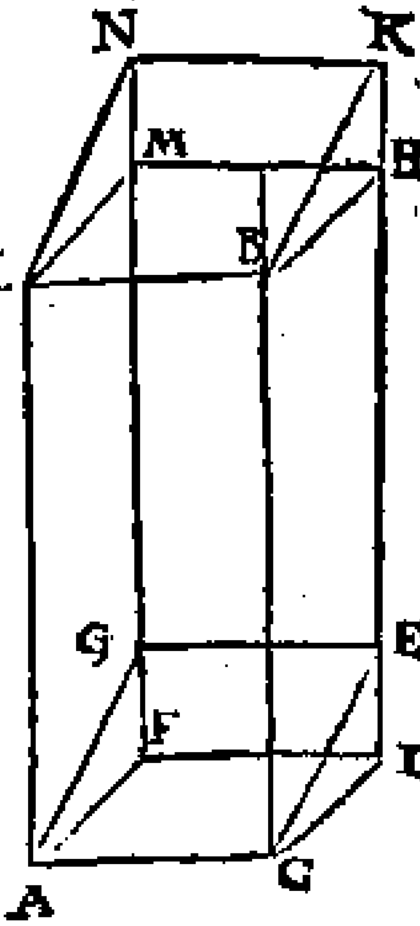


Theorema 23. Propoſitio 28.

Si ſolidũ parallelis planis comprehẽſum, ducto per aduerſorum planorum diagonios plano ſectũ ſit, illud ſolidũ ab hoc plano bifariàm ſecabitur.



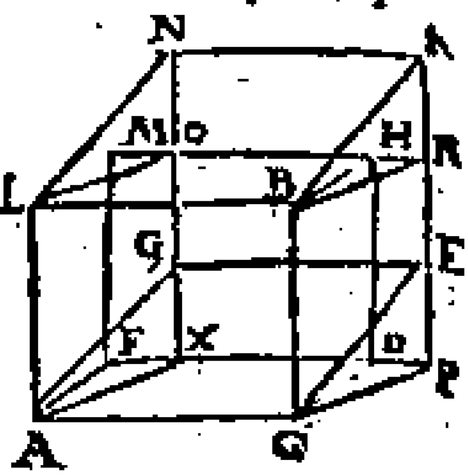
Theorema 34. Propositio 29.



Solida parallelis planis compreh̃ẽsa, quę super eandem basim, & in eadẽ sunt altitudine, quorum insistẽtes lineæ in ijsdem collocantur rectis lineis, illa sũt inter se æqualia.

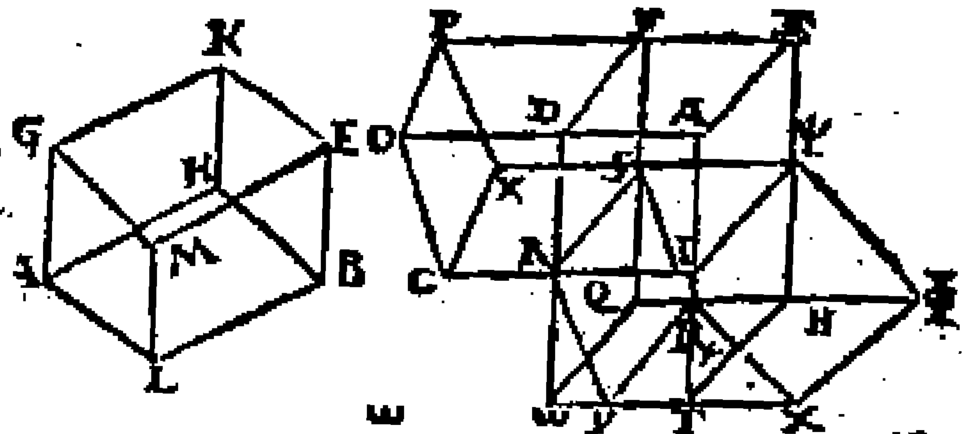
Theorema 25. Propositio 30.

Solida parallelis planis circunscripta, quæ super eandẽ basim & in eadẽ sũt altitudine, quorum insistentes lineæ nõ in ijsdem reperiuntur rectis lineis, illa sunt inter se æqualia.

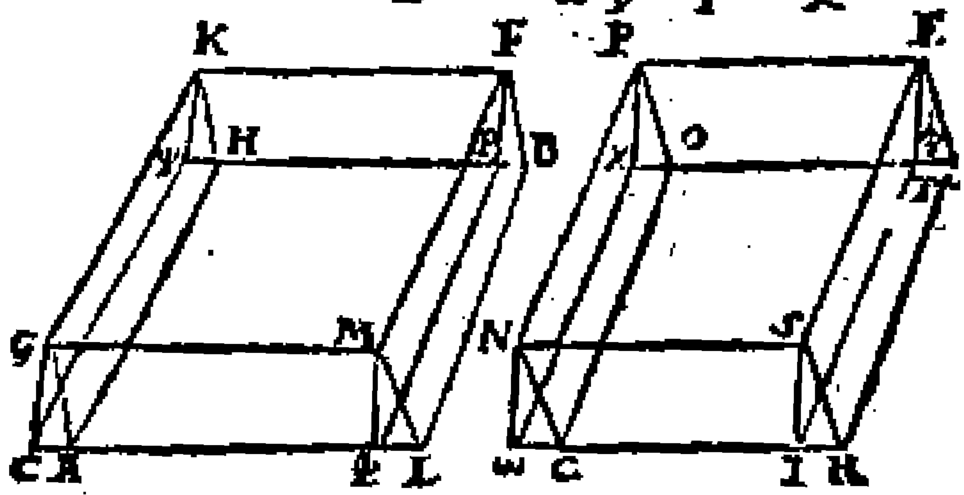


Theorema 26. Propositio 31.

Solida parallelis planis circunscripta, quę in

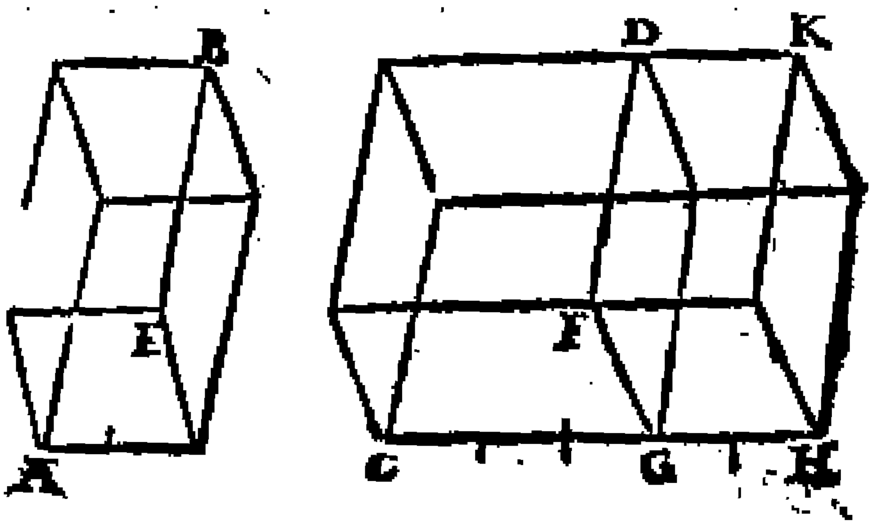


in eadē ſunt altitudine, æqualia ſunt inter ſe.



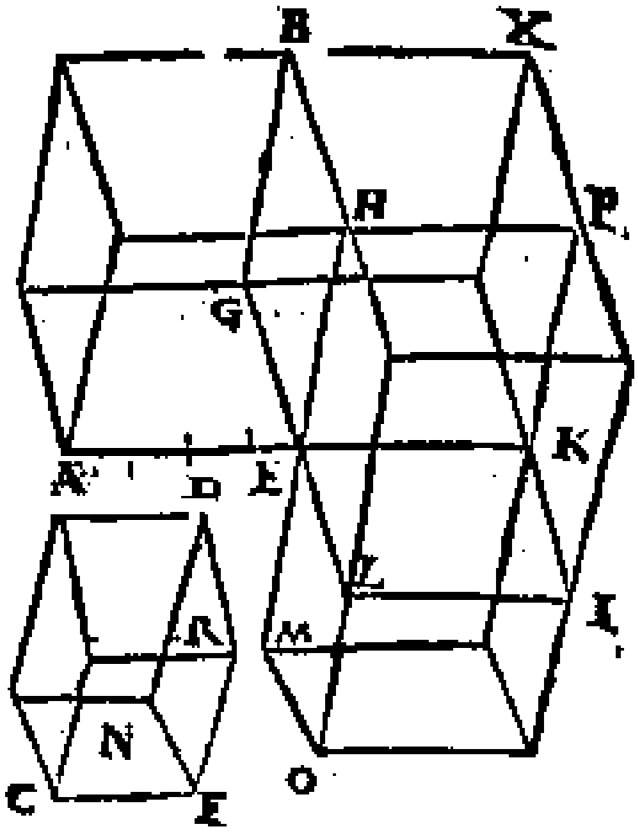
Theorema 27. Propoſitio 32.

Solida parallelis planis circunſcripta quæ eiuſdem ſunt altitudinis, eam habēt inter ſe rationē, quam baſes.



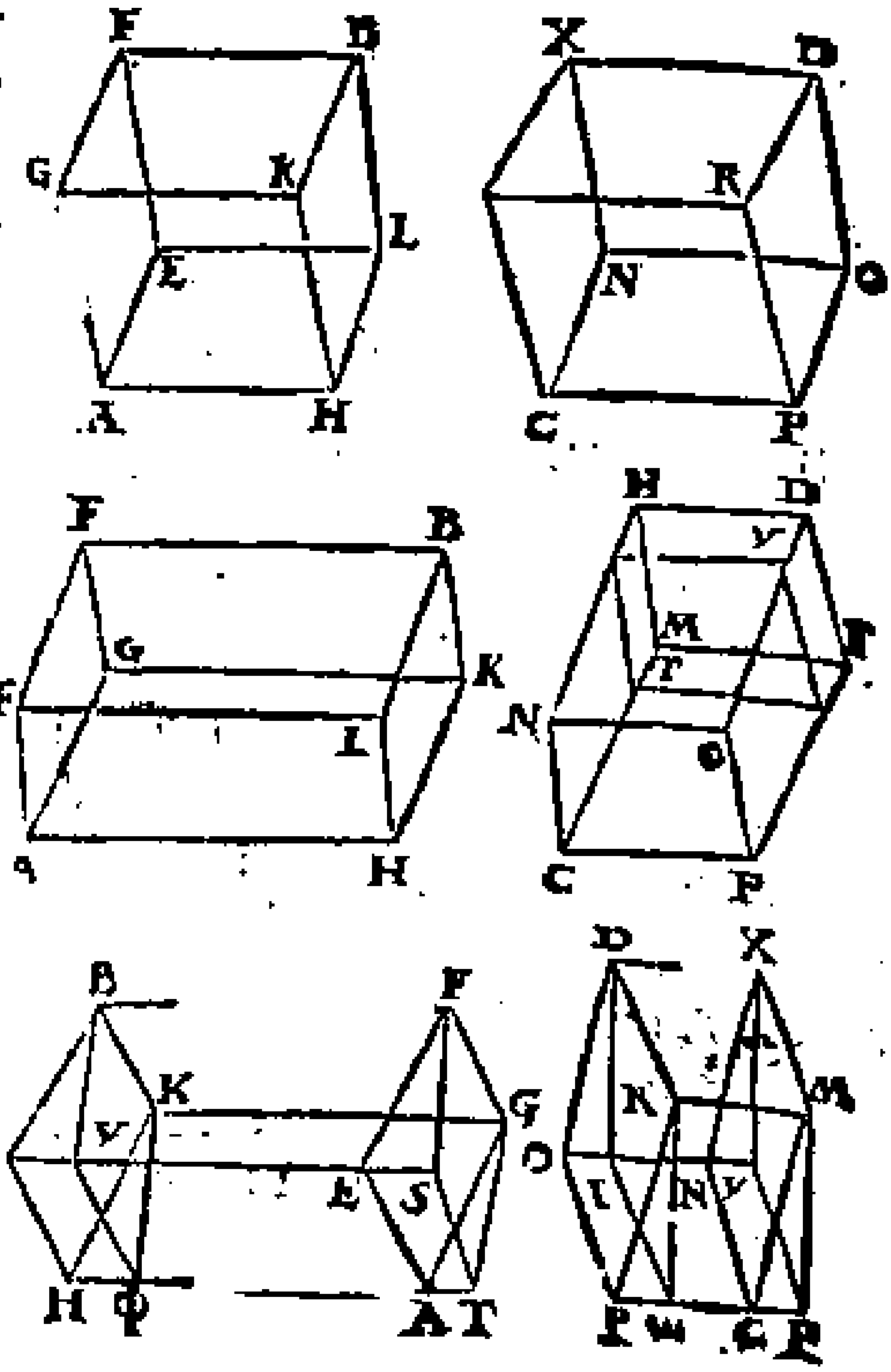
Theor.28.Propositio 33.

Similia solida parallelis planis circunscripta habēt inter se rationē homologorum laterum triplicatam.



Theor.29.Propositio 34.

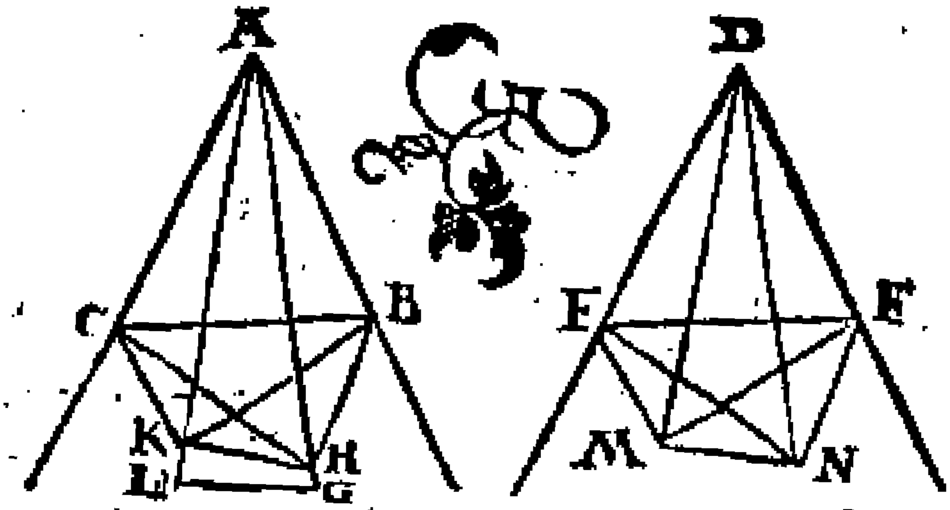
Aequalium solidorū parallelis planis cōtentorum bases cū altitudinibus reciprocātur. Et solida parallelis planis cōtenta, quorū bases cum altitudi-



nibus reciprocantur, illa ſunt æqualia.

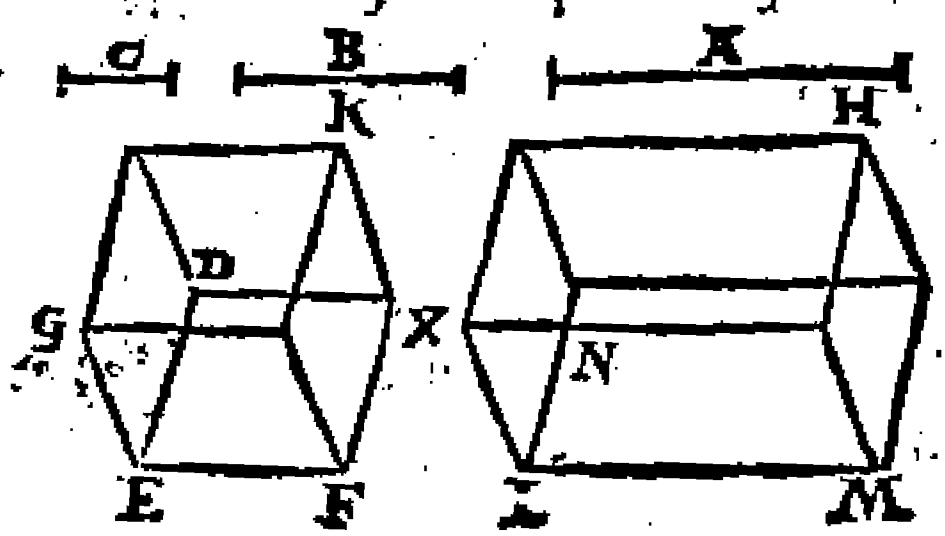
Theorema 30. Propoſitio 35.

Si duo plani ſint anguli æquales, quorũ verticibus ſublimes rectæ lineæ inſiſtant, quæ cum lineis primò poſitis angulos cõtineant æquales, vtrunq; vtriq;, in ſublimibus autem lineis quælibet ſumpta ſint pũcta, & ab his ad plana in quibus conſiſtunt anguli primũ poſiti, ductæ ſint perpendiculares, ab earũ verò punctis, quæ in planis ſignata fuerint, ad angulos primũ poſitos adiũctæ ſint, rectæ lineę, hæ cũ ſublimibus æquales angulos comprehendent.



Theorema 31. Propoſitio 36.

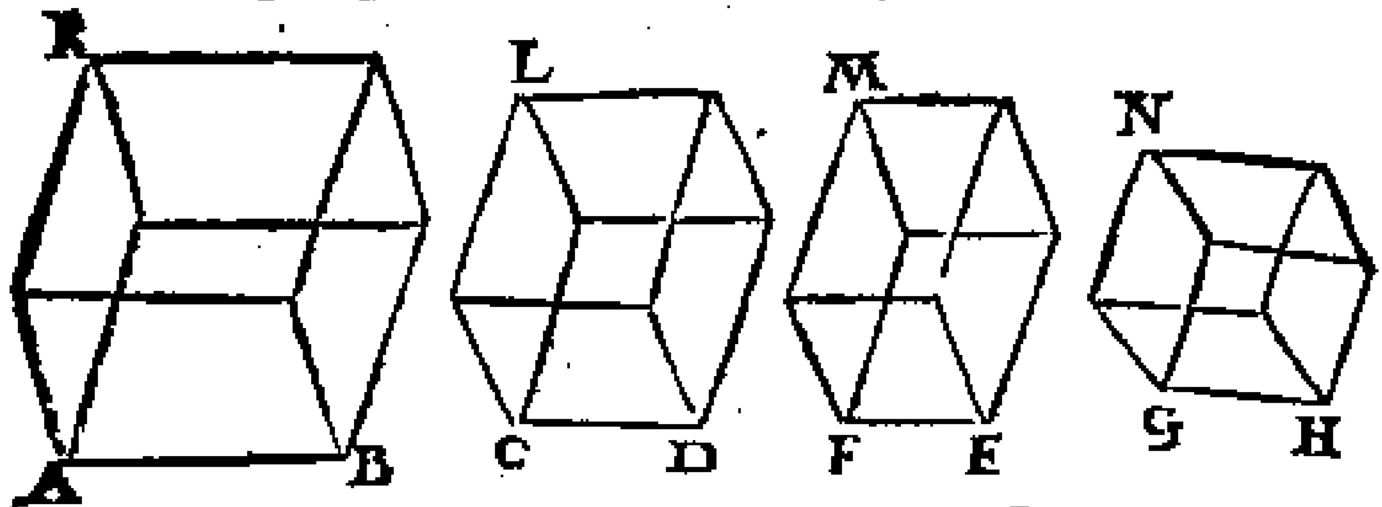
Si rectæ tres lineæ ſint proporionales, quod



ex his tribus fit solidum parallelis planis contentum, æquale est descripto à media linea solido parallelis planis comprehēso, quod æquilaterum quidem sit, sed antedicto æquiangulum.

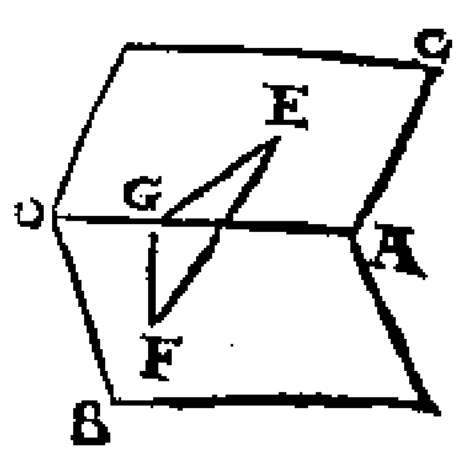
Theorema 32. Propositio 37.

Si rectæ quatuor lineæ sint proportionales, illa quoque solida parallelis planis cōtenta, quæ ab ipsis lineis & similia & similiter describuntur, proportionalia erūt. Et si solida parallelis planis cōprehensa, quæ & similia & similiter describuntur; sint proportionalia, illæ quoque rectæ lineæ proportionales erunt.



Theorema 33. Propositio 38.

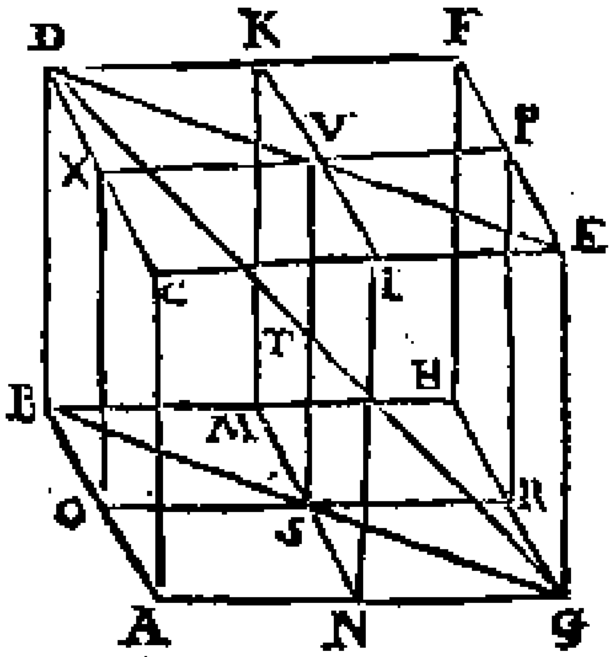
Si planum ad planū rēctum sit, & à quodā puncto eorū quæ in vno sunt planorū perpendicularis ad alterum ducta sit, illa quæ ducitur perpēdicularis, in cōmunē cadet planorū sectionē.



Theo-

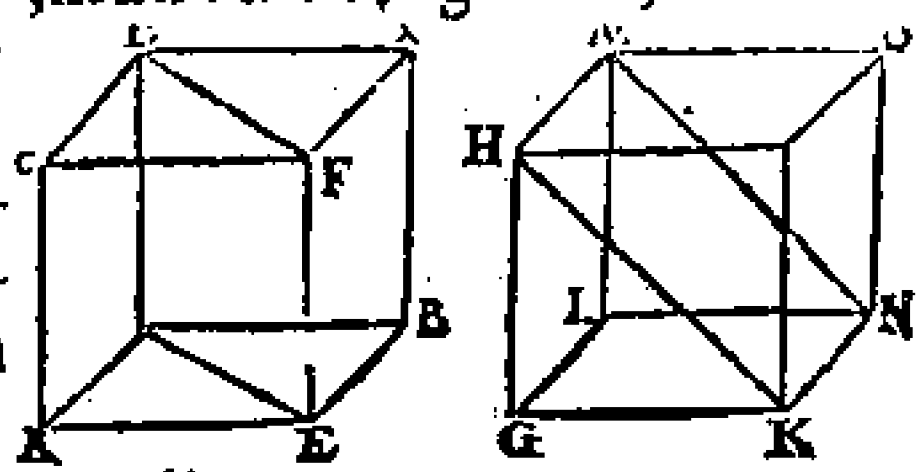
Theorema 34. Propositio 39.

Si in ſolido parallelis planis circũſcripto, aduerſorum planorum lateribus biſariã ſectis, educta ſint per ſectiones plana, cõmunis illa planorũ ſectio & ſolidi parallelis plani circũſcripti diameter, ſe mutuò biſariam ſecant.



Theorema 35. Propoſitio 40.

Si duo ſint æqualis altitudinis priſmata, quorũ hoc quidẽ baſim habeat parallelogrammũ, illud verò triãgulum, ſit autem parallelogrammũ triãguli duplũ, illa priſmata erunt æqualia.

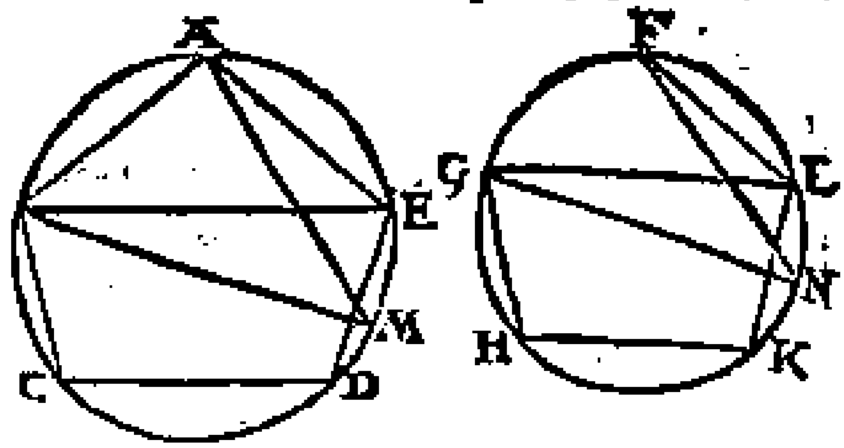


ELEMENTI XI. FINIS.

EVCLIDIS ELEMENTVM DVODECIMVM. ET SOLIDORVM ſecundum.

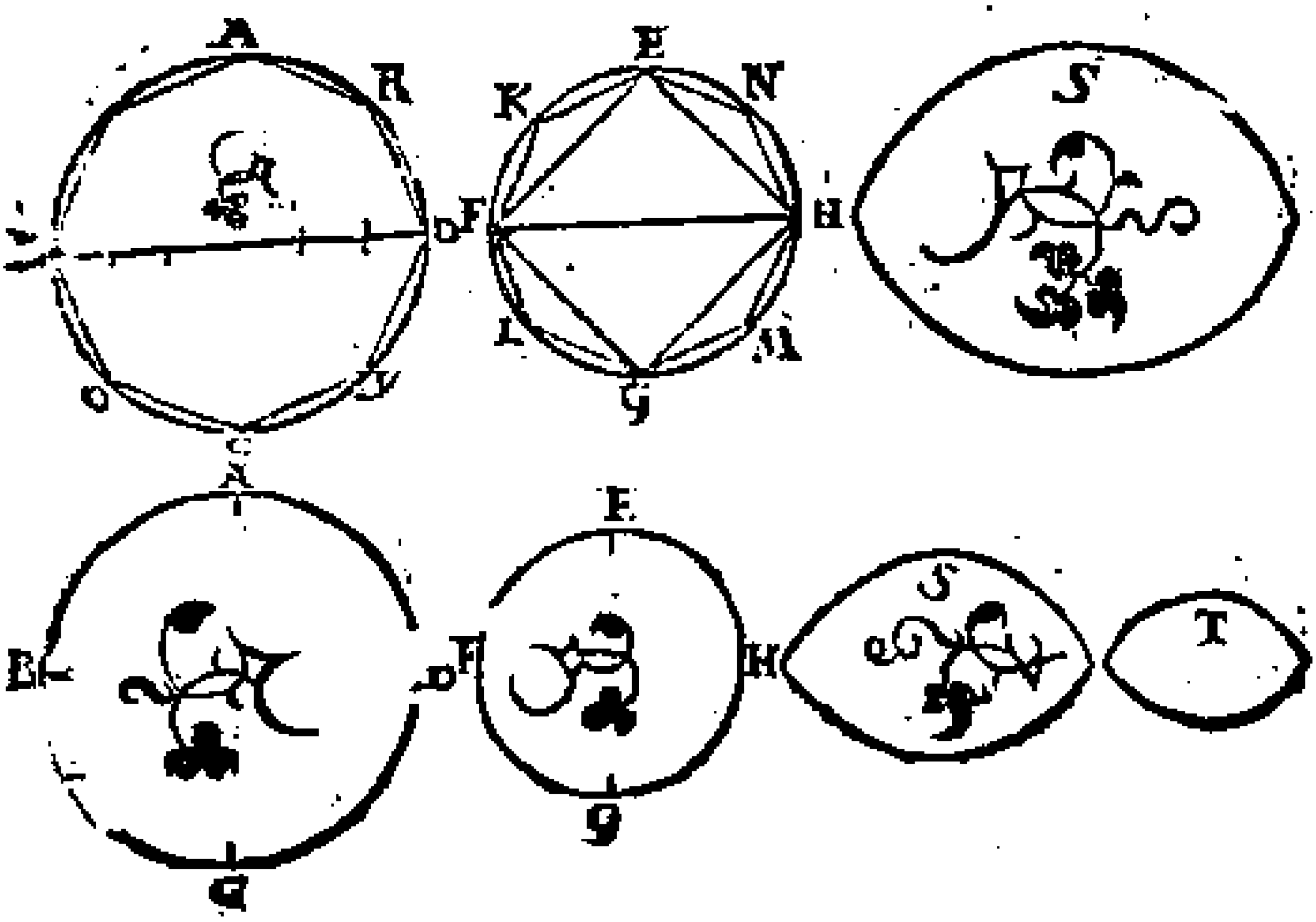
Theorema 1. Propoſitio 1.

Similia, quæ ſunt in circulis polygona, rationē habēt inter ſe, quam deſcripta à diametris quadrata.



Theorema 2. Propoſitio 2.

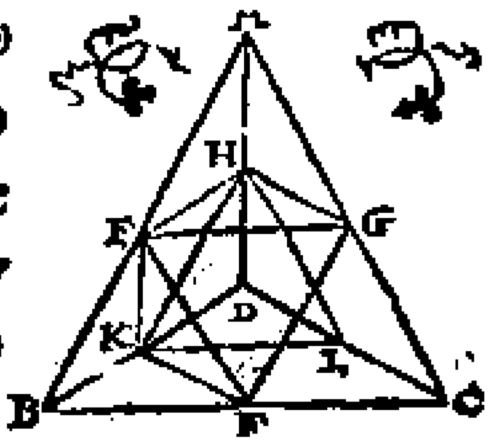
Circuli eam inter ſe rationē habent, quam



descripta à diametris quadrata.

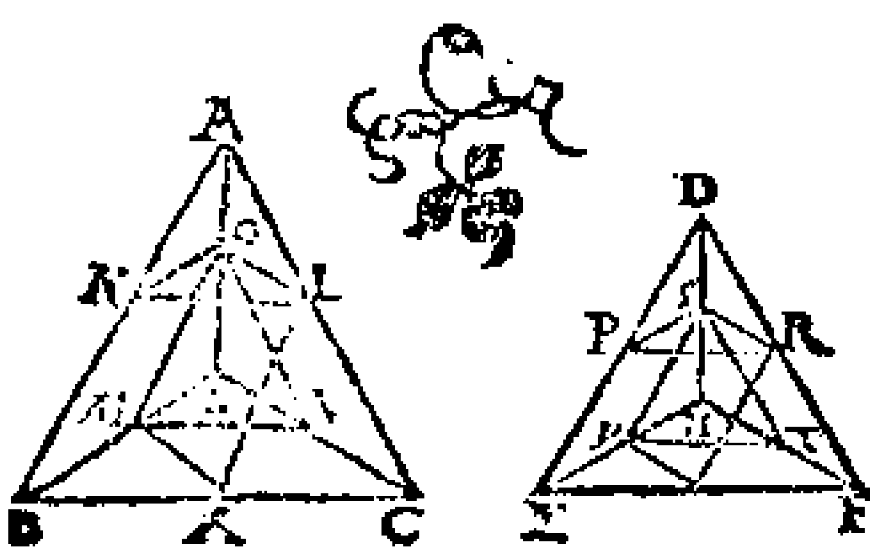
Theorema 3. Propositio 3.

Omnis pyramis trigωnā habens basim, in duas diuiditur pyramidas non tantū æquales & similes inter se, sed toti etiā pyramidi similes, quarum trigωnæ sunt bases, atq; in duo prismata æqualia, quæ duo prismata dimidio pyramidis totius sunt maiora.



Theorema 4. Propositio 4.

Si duæ eiusdem altitudinis pyramides trigωnas habeāt bases, sit aūt illarum vtraque diuisa & in duas pyramidas inter se equales totiq; similes, & in duo prismata æqualia, ac eodē modo diuidatur vtraq; pyramidū quæ ex superiore diuisione natæ sunt, idque perpetuò fiat: quemadmodū se habet vnius pyramidis basis ad alterius pyramidis basim, ita & omnia quæ in vna pyra-

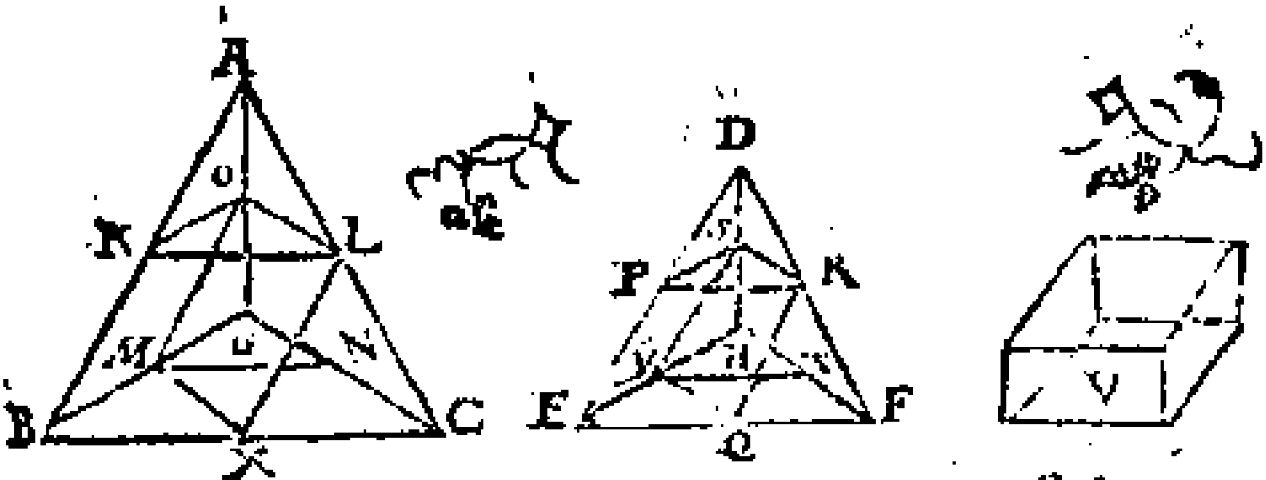


mide

mide priſmata, ad omnia quæ in altera pyramide, priſmata multitudine æqualia.

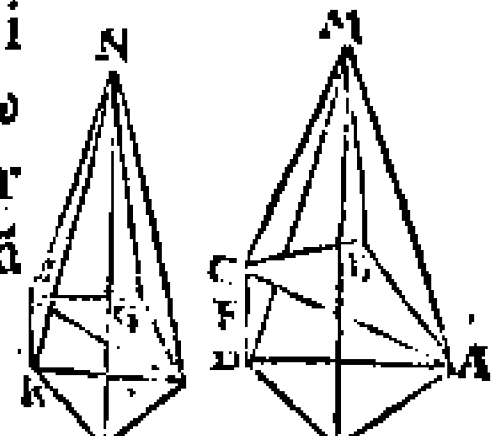
Theorema 5. Propoſitio 5.

Pyramides eiuſdem altitudinis, quarũ trigωnæ ſunt baſes, eam inter ſe rationem habent, quàm ipſæ baſes.



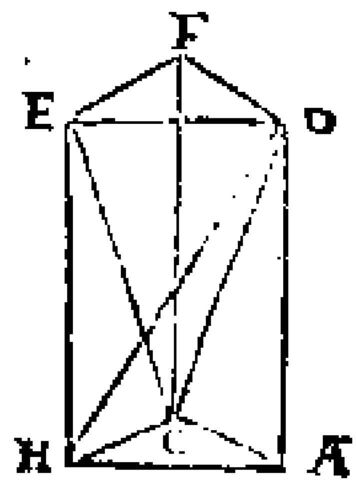
Theorema 6. Propoſitio 6.

Pyramides eiuſdem altitudinis, quarum polygωnæ ſunt baſes, eam inter ſe rationem habent, quã ipſæ baſes.



Theorema 7. Propoſitio 7.

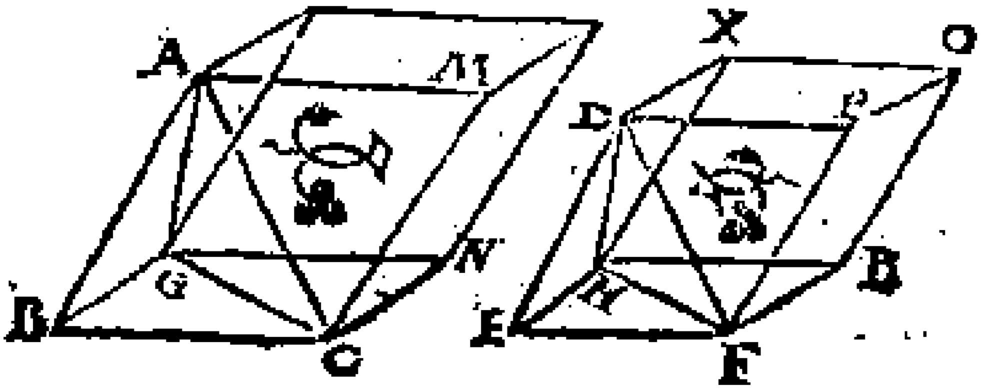
Omne priſma trigωnã habẽs baſim, diuiditur in tres pyramides inter ſe æquales, quarum trigωnæ ſunt baſes.



Theo-

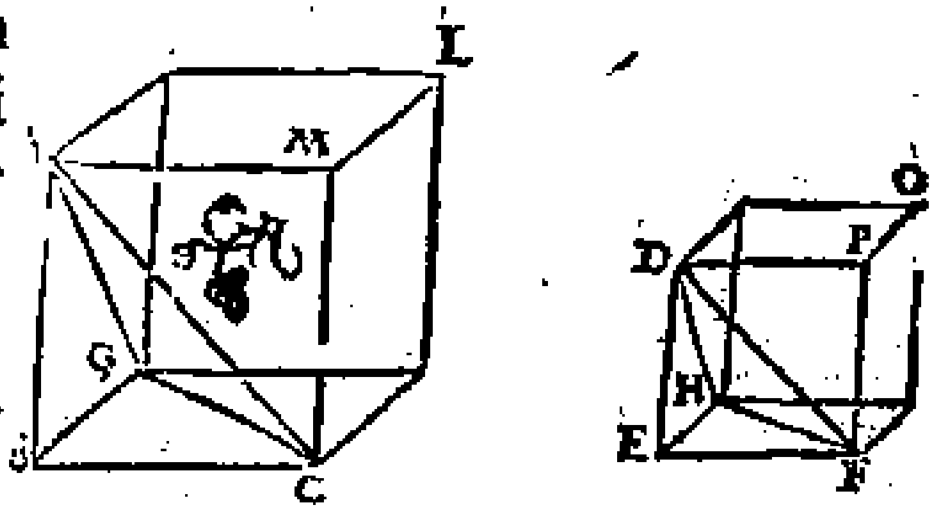
Theorema 8. Propositio 8.

Similes pyramides qui trigωnas habent bases, in triplicata sũt homologorũ laterũ ratiõe.



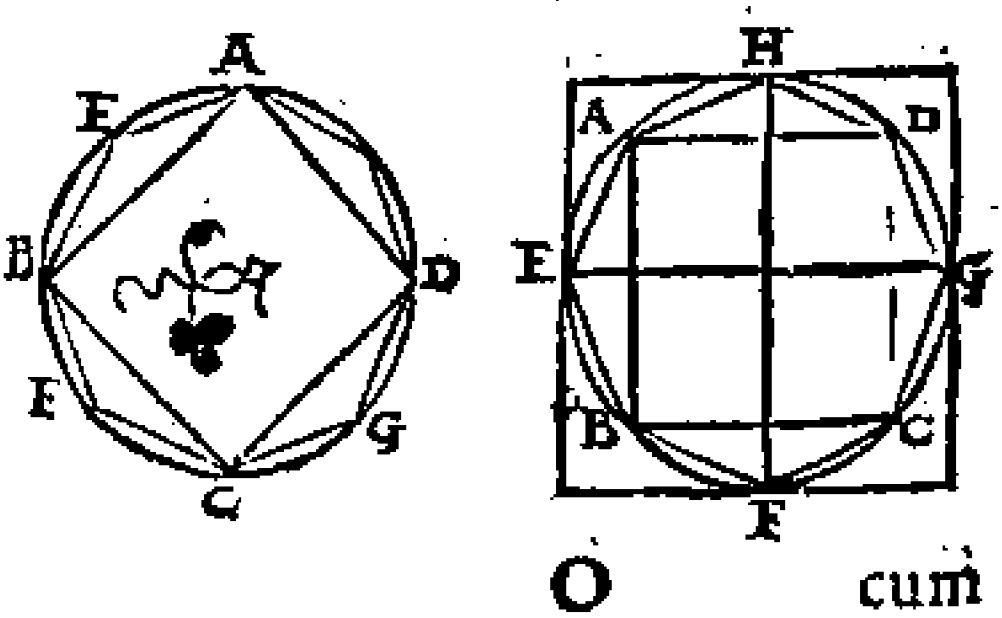
Theorema 9. Propositio 9.

Aequaliũ pyramidum & trigωnas bases hàbentium reciprocantur bases cum altitudinibus. Et quarũ pyramidum trigωnas bases habentium reciprocãtur bases cũ altitudinibus, illæ sunt æquales.



Theorema 10. Propositio 10.

Omnis cωn⁹ tertia pars est cylindri eandẽ

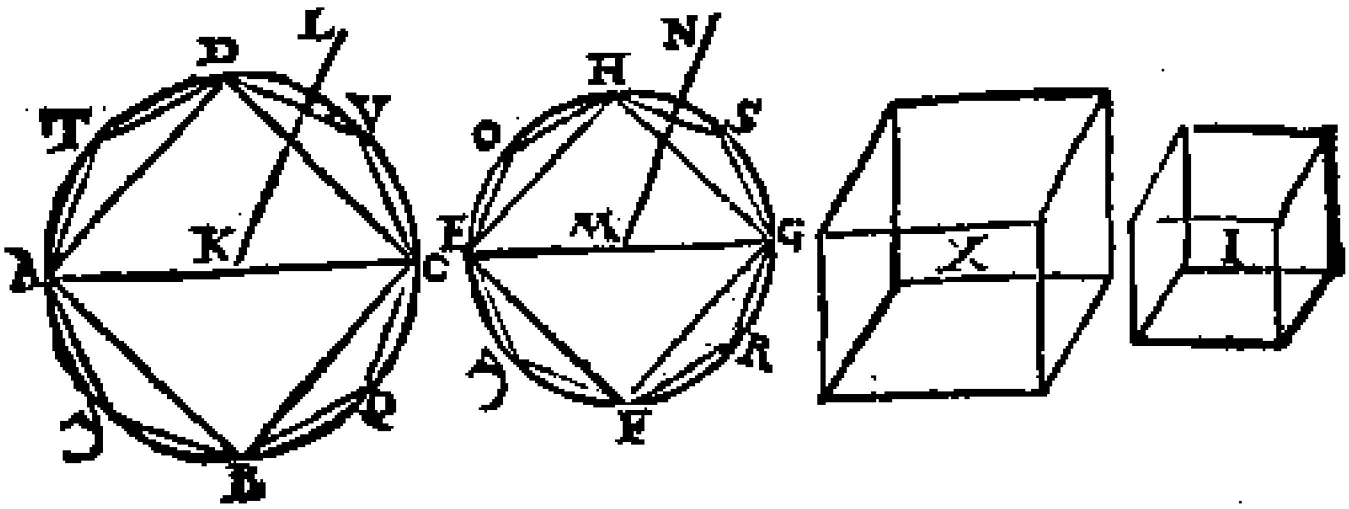


cum

cum ipso cono basim habentis, & altitudinem æqualem.

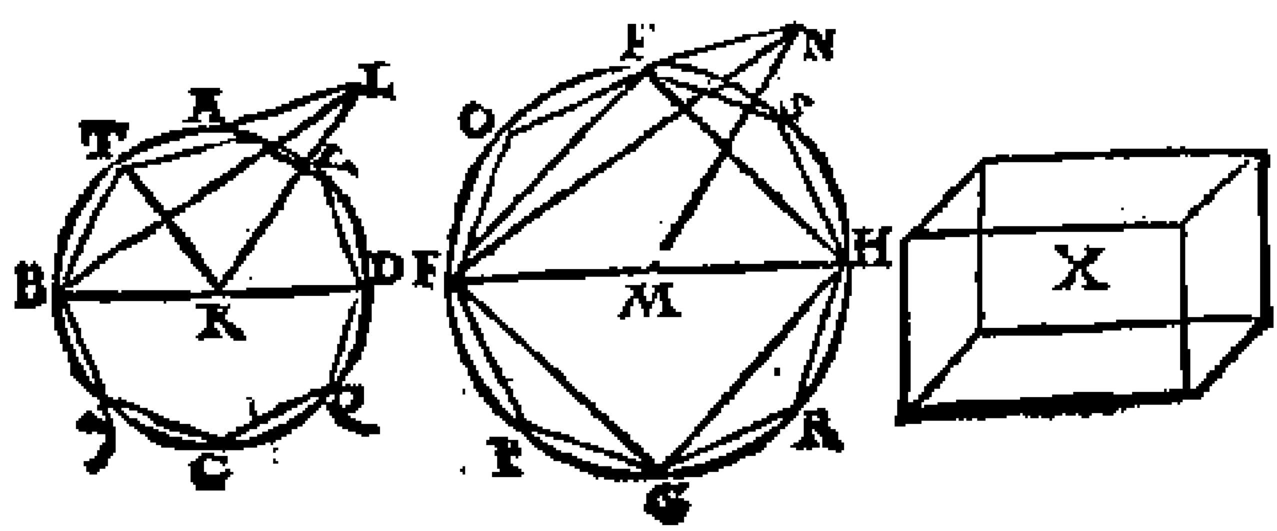
Theorema 11. Propositio 11.

Coni & cylindri eiusdem altitudinis, eam inter se rationem habent, quam bases.



Theorema 12. Propositio 12.

Similes coni & cylindri, triplicatam habēt inter se rationem diametrorum, quæ sunt in basibus.



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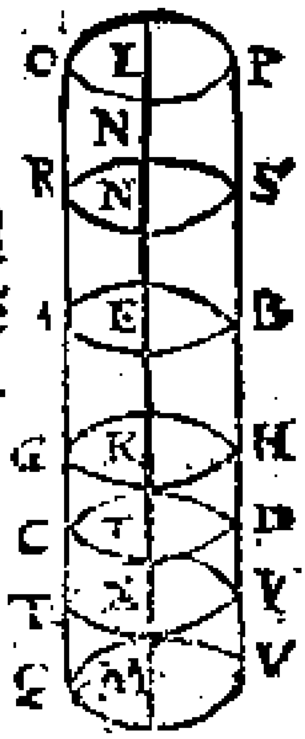
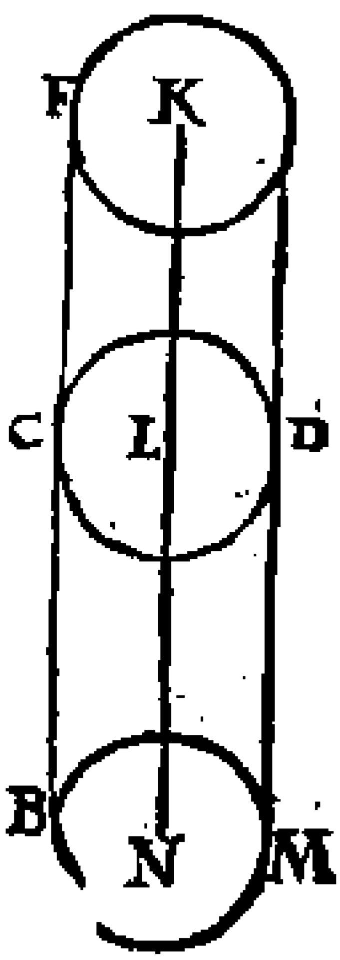
Theorema 13. Propositio 13.

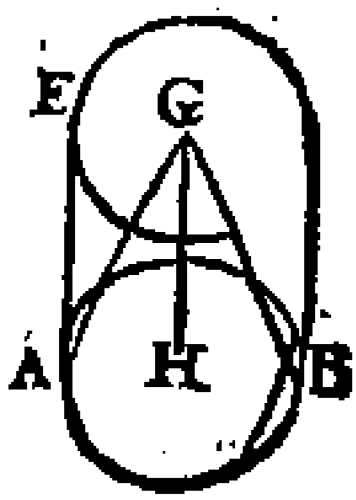


Si cylindrus plano sectus sit aduersis planis parallelo, erit quemadmodum cylindrus ad cylindrum, ita axis ad axem.

Theorema 14. Propositio 14.

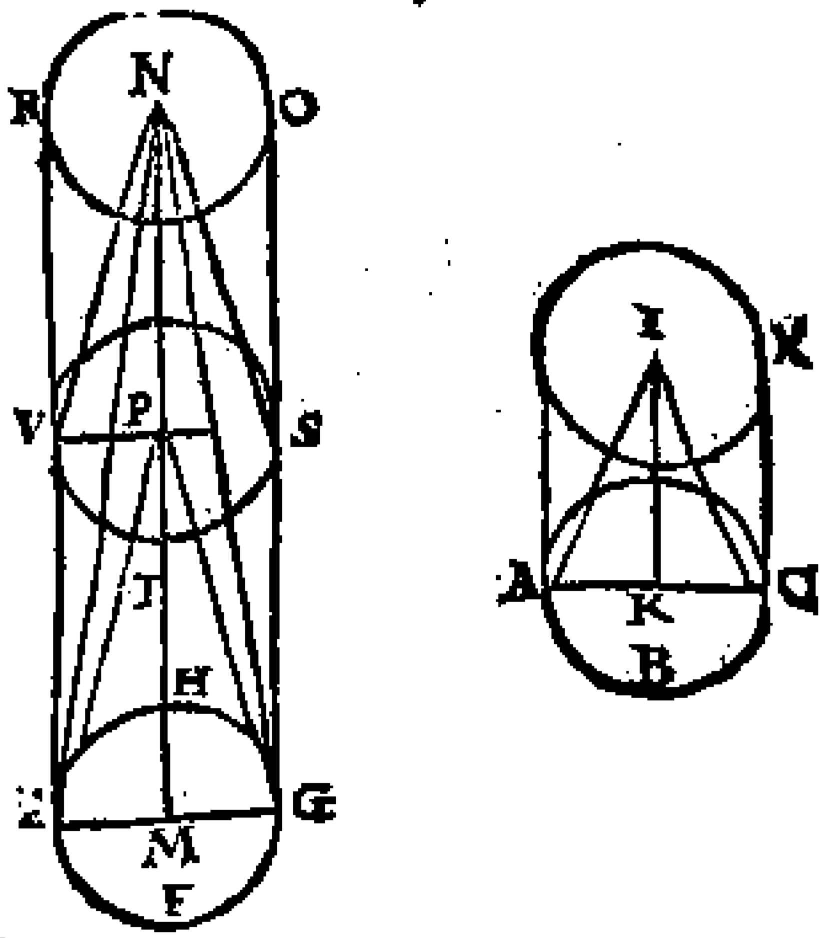
Coni & cylindri qui in æqualibus sunt basibus, eam habent inter se rationem, quam altitudines





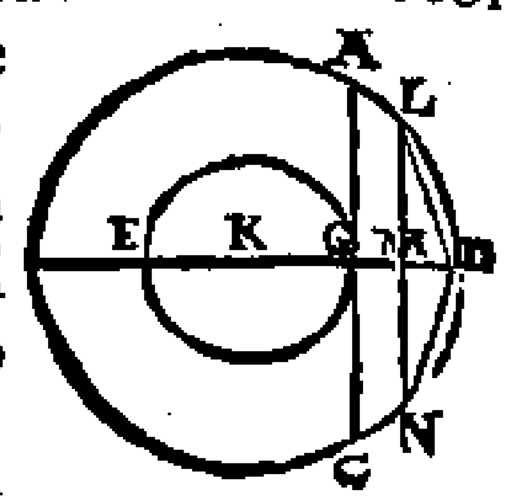
Theorema 15. Propoſitio 15.

Aequalium conorum & cylindrorũ baſes cũ altitudinibꝰ reciprocãtur. Et quorum conorũ & cylindrorũ baſes cum altitudinibꝰ reciprocãtur, illi ſunt æquales.



Problema 1. Propoſitio 16.

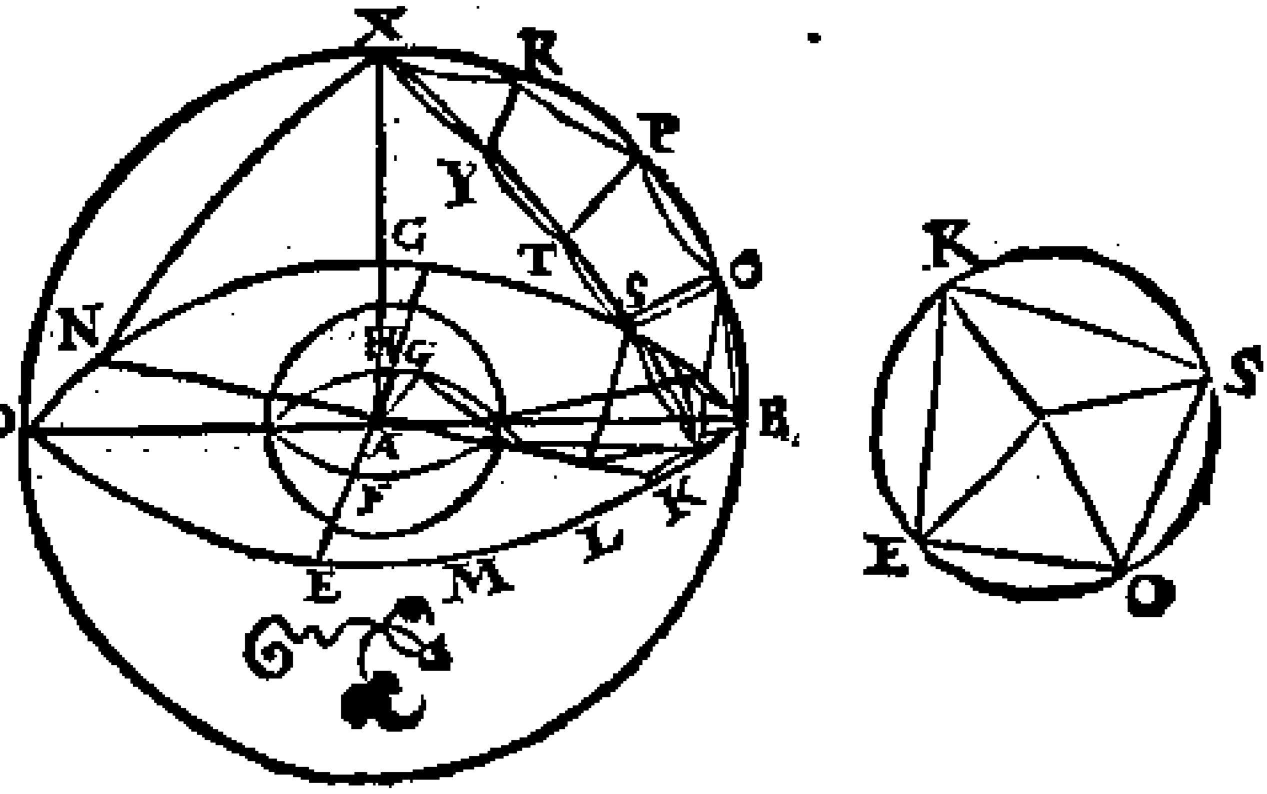
Duobus circulis circũ idem centrum conſiſtentibus, in maiore circulo polygonum æqualium pariumq́ue laterum inſcribere, quod minorem circulum nõ tangat.



Pro-

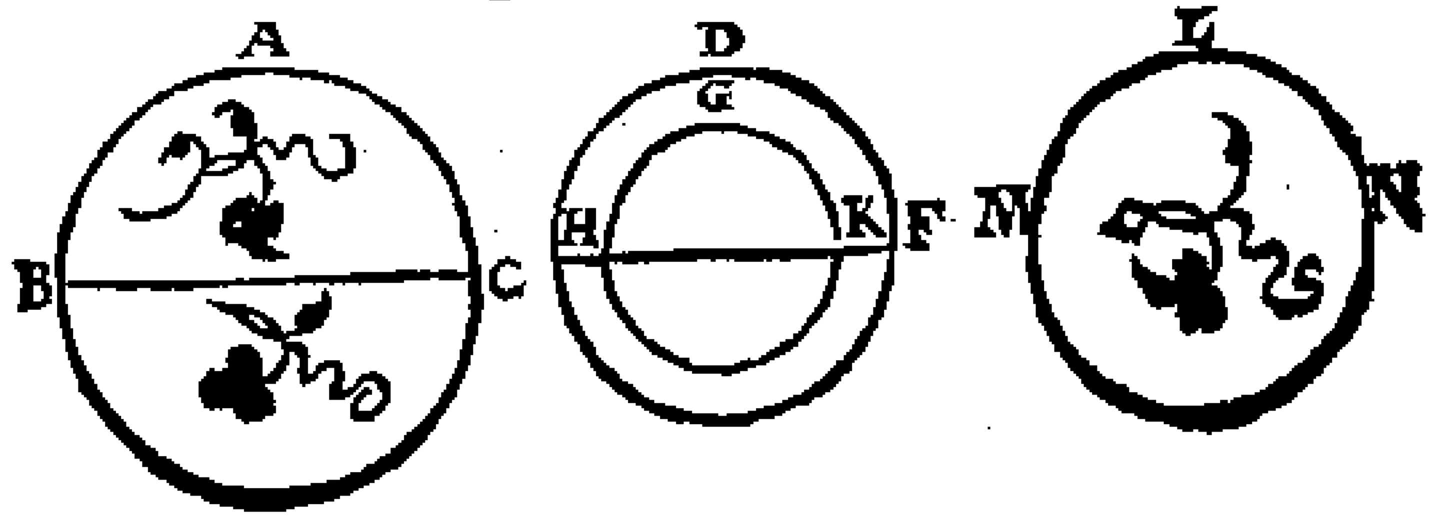
Problema 2. Propoſitio 17.

Duabus ſphæris circum idem centrũ conſiſtentibus, in mãiore ſphęra ſolidũ polyedrum inſcribere, quod minoris ſphæræ ſuperficiem non tangat.



Theorema 16. Propoſitio 18.

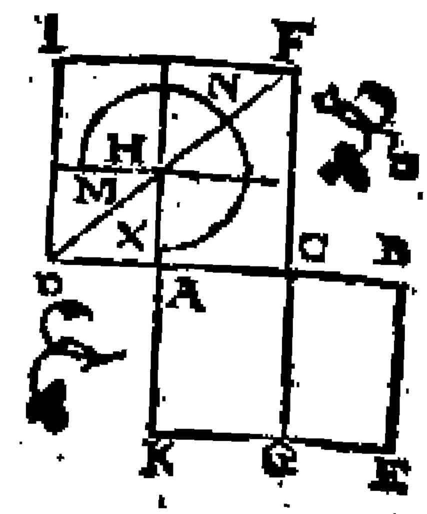
Sphæræ inter ſe rationem habent ſuarum diametrorum triplicatam.



ELEMENTI XII. FINIS.

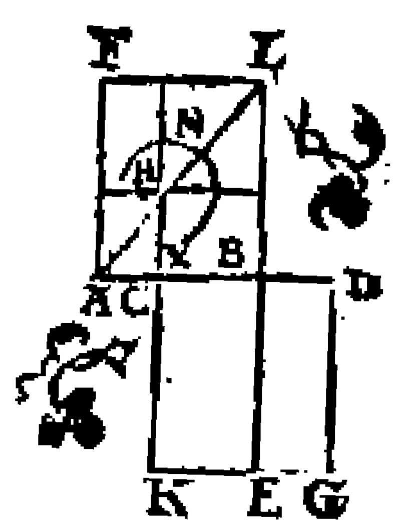
EVCLIDIS ELEMENTVM DEDECIMVMTERTIVM, ET SOLIDORVM TERTIVM.

Theorema 1. Propositio 1.



Si recta linea per extremã & mediã rationẽ secta sit, maius segmentum quod totius lineæ dimidium assumpserit, quintuplum potest eius quadrati, quod à totius dimidia describitur.

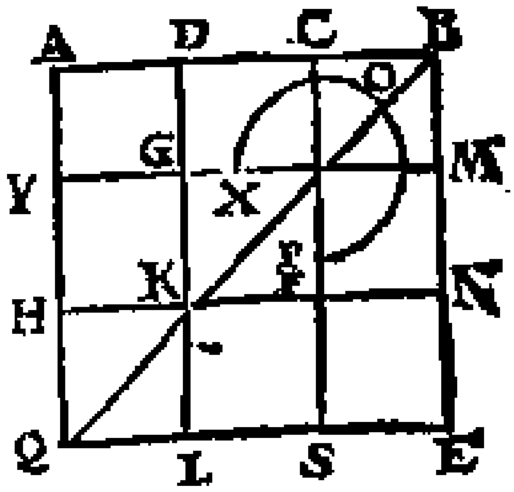
Theorema 2. Propositio 2.



Si recta linea sui ipsius segmenti quintuplũ possit, & dupla segmenti huius linea per extremã & mediam rationẽ secetur maius segmentũ reliqua pars est lineæ primùm positæ.

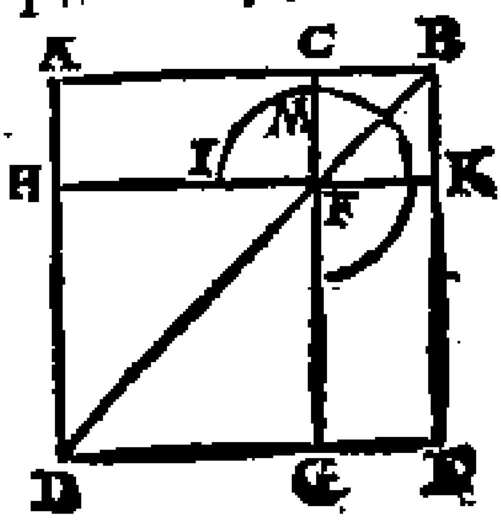
Theo-

Theorema 3. Propositio 3.



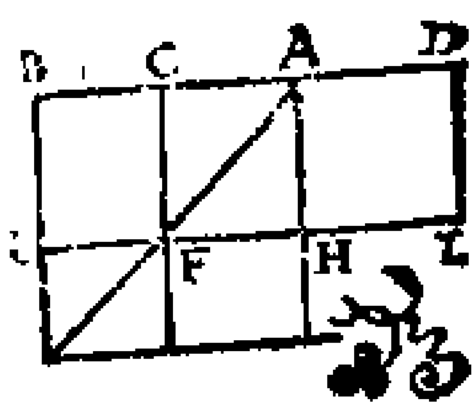
Si recta linea per extremā & mediam rationē secta sit, minus segmētum quod maioris segmenti dimidium assumpserit, quintuplum potest eius, quòd à maioris segmēti dimidio describitur, quadrati.

Theorema 4. Propositio 4.



Si recta linea per extremam & mediam rationē secta sit, quod à tota, quodque à minore segmēto simul vtraq; quadrata, tripla sunt eius, quod à maiore segmēto describitur, quadrati.

Theorema 5. Propositio 5.



Si ad rectam lineam, quæ per extremam & mediam rationem secetur, adiuncta sit altera segmento maiori ęqualis, tota hæc linea recta per extremam & mediam rationē secta

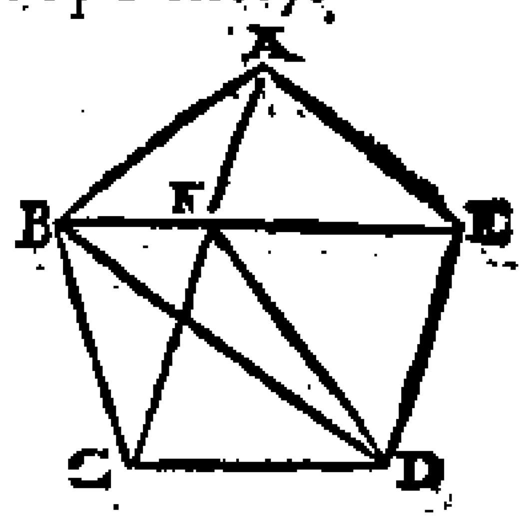
cta est, estque maius segmentum linea primùm posita.

Theorema 6. Propositio 6.

Si recta linea ῥητὴ siue rationalis, per extremam & mediam rationem secta sit, vtrunque segmentorum ἄλογος siue irrationalis est linea, quæ dicitur Residuum.

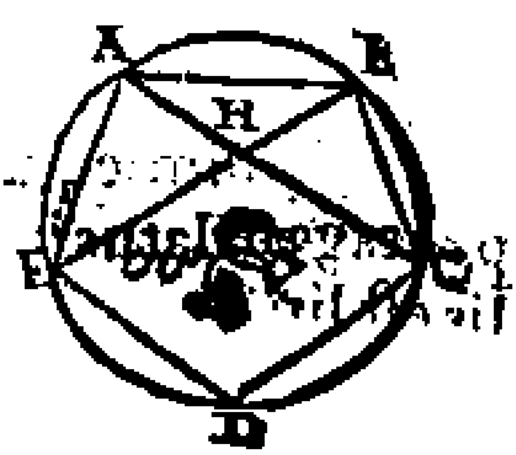
Theorema 7. Propositio 7.

Si pentagωni æquilateri tres sint æquales anguli, siue q deinceps, siue qui non deinceps sequuntur, illud pẽtagωnum erit ęquiangulum.



Theorema 8. Propositio 8.

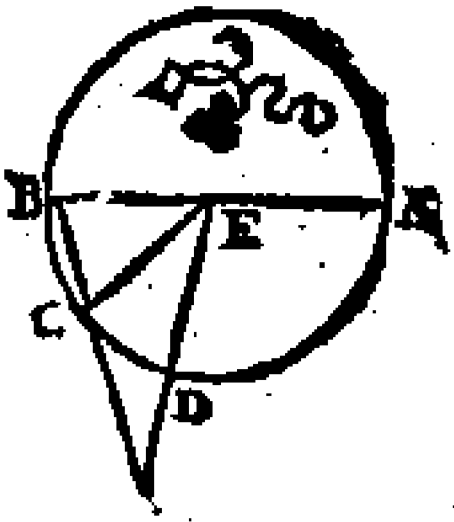
Si pẽtagωni æquilateri & æquianguli duos qui deinceps sequuntur angulos rectę subtendãt lineæ, illæ per extremã & mediam rationem se mutuò secant, earumq́; maiora segmenta, ipsius pentagωni lateri sunt æqualia.



Theo-

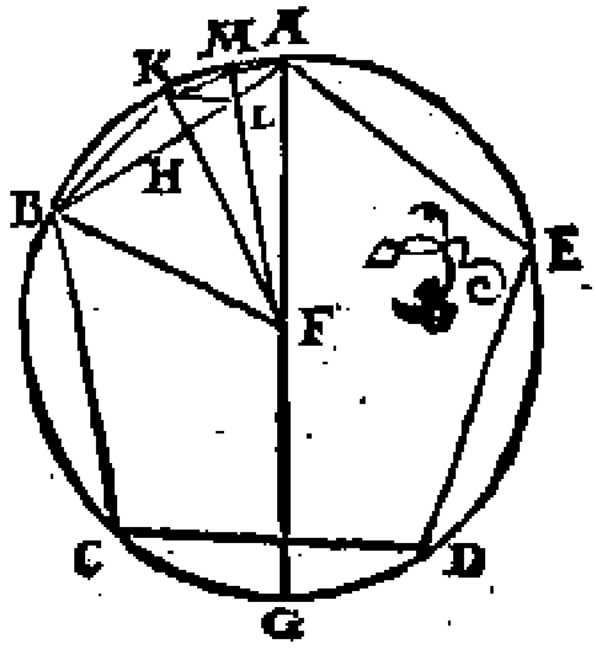
Theorema 9. Propositio 9.

Si latus hexagωni, & latus decagωni eidem circulo inscriptorum composita sint, tota recta linea per extremam & mediam rationem secta est, eiusque segmẽtum maius, est hexagωni latus.



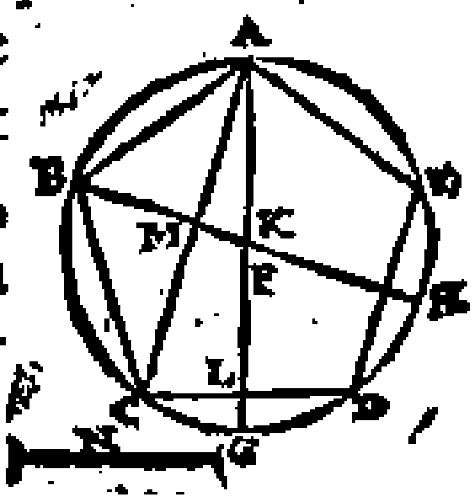
Theorema 10. Propositio 10.

Si circulo pẽtagωnum æquilaterũ inscriptum sit, pẽtagωni lat⁹ potest & latus hexagωni & latus decagωni eidem circulo inscriptorum.



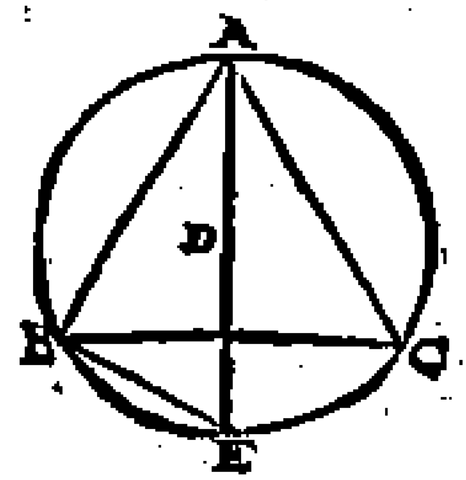
Theorema 11. Propositio 11.

Si in circulo ῥητὴν habente diametrum, inscriptũ sit pentagωnum æquilaterũ, pentagωni latus irrationalis est linea, quæ vocatur Minor.



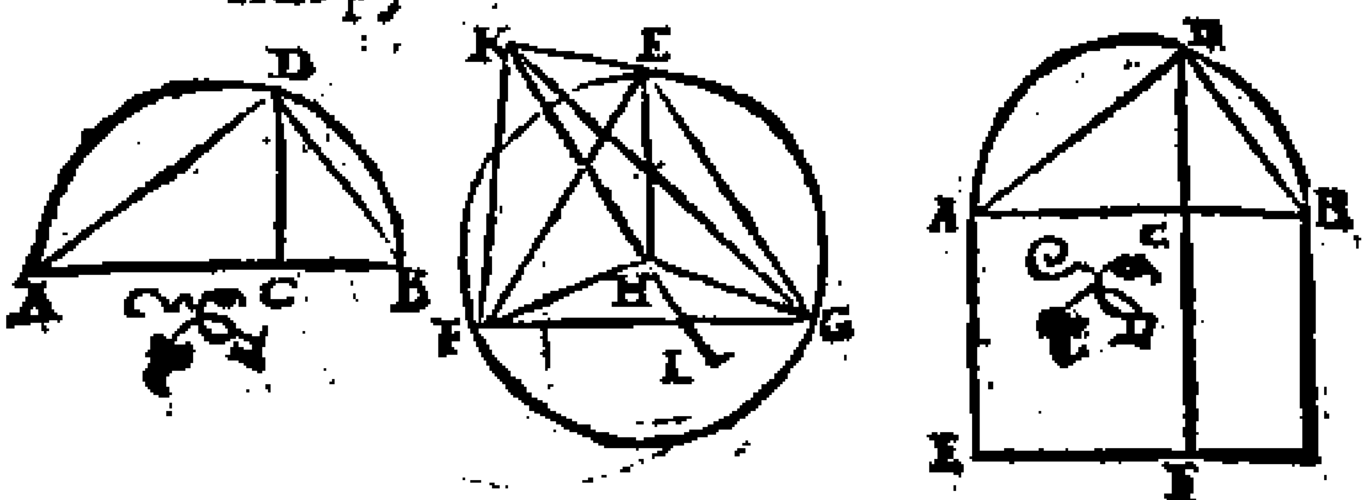
Theorema 12. Propositio 12.

Si in circulo inscriptum sit triangulum æquilaterum, huius trianguli latus potentia triplum est eius lineæ, quæ ex circuli centro ducitur.



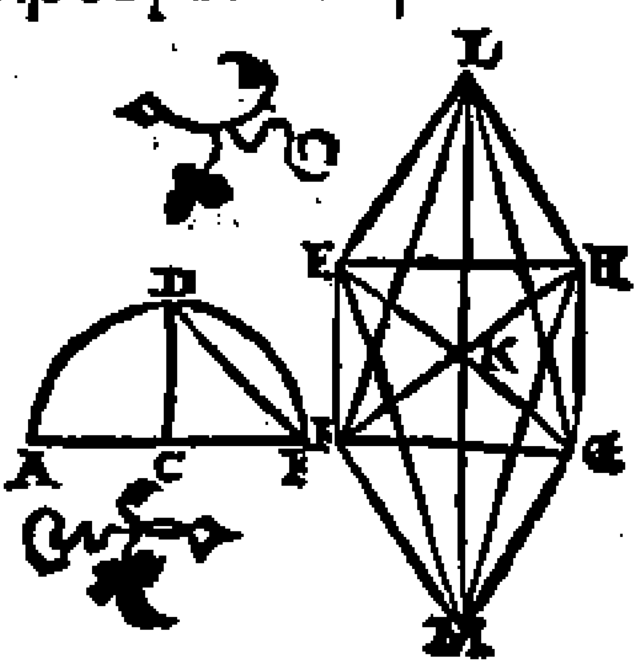
Problema 1. Propositio 13.

Pyramidem constituere, & data sphærę cōplecti, atque docere illius sphæræ diametrum potentia sesquialteram esse lateris ipsius pyramidis.



Problema 2. Propositio 14.

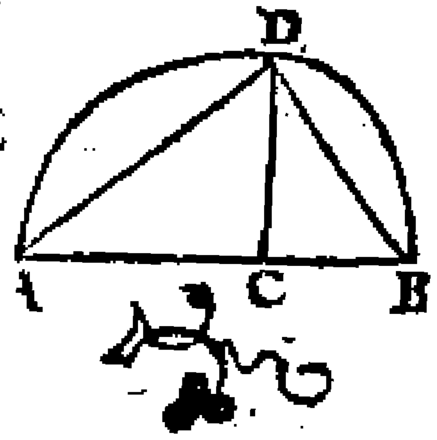
Octaedrum constituere, eaq; sphęra qua pyramidē complecti, atque pbare illius sphæræ diametrū potentia duplā esse lateris ipsius octaedri.

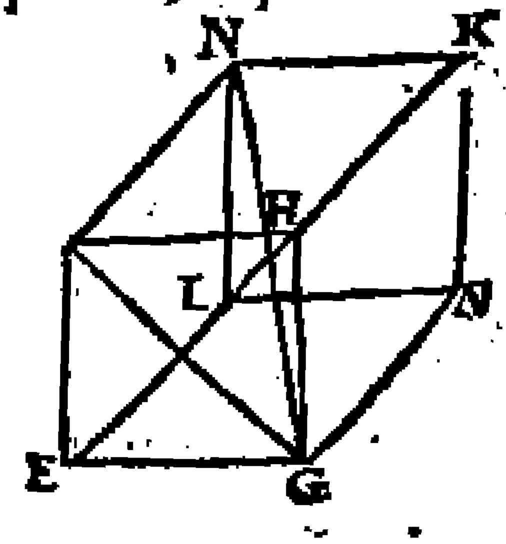


Pro-

Problema 3. Propositio 15.

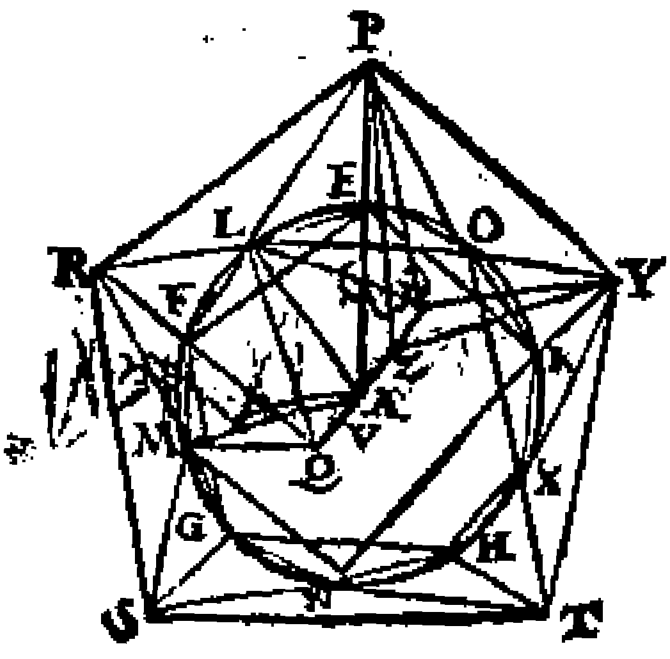
Cubum constituere, eaque sphæra qua & superiores figuras complecti, atque docere illius sphæræ diametrum potẽtia triplam esse lateris ipsius cubi.

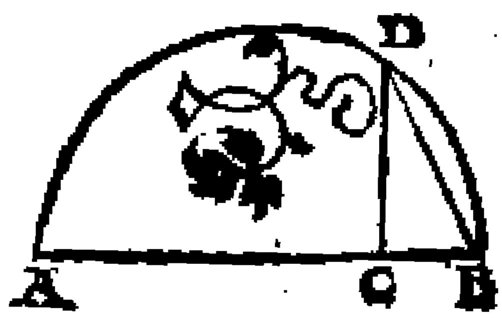


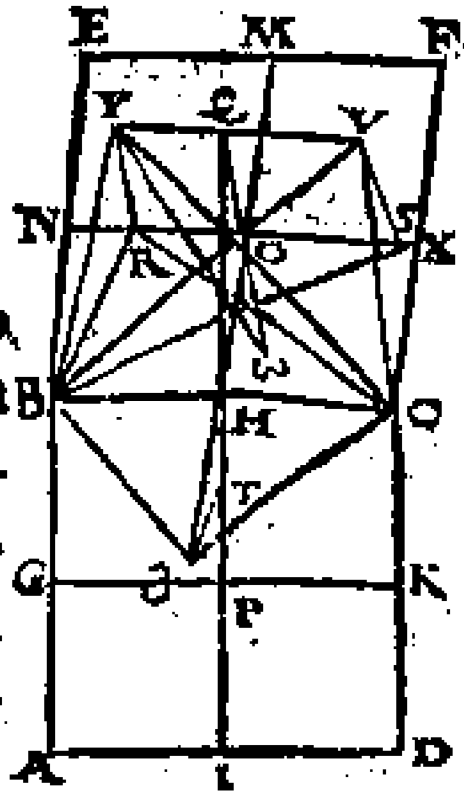


Problema 4. Propositio 16.

Icosaedrum constituere, eademque sphæra qua & antedictas figuras complecti, atque probare, Icosaedri latus irrationalem esse lineam, quæ vocatur Minor.





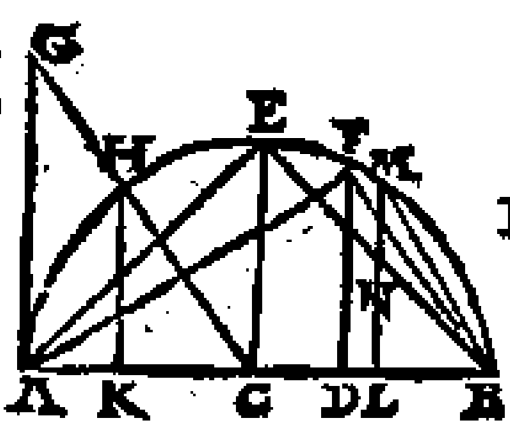


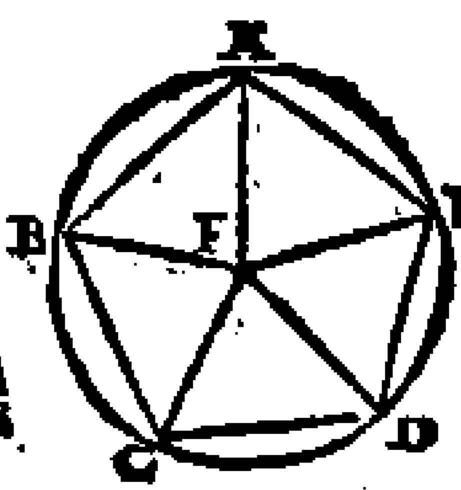
Problema 5. Propositio 17.

Dodecaedrū cōstituere, eademque sphæra qua & antedictas figuras cōplecti, atque probare dodecaedri latus irrationalem esse lineam, quæ vocatur Residuum.

Theorema 6. Propositio 18.

quinque figurarū latera proponere, & inter se comparare.





SCHOLIVM.

Aio verò, præter dictas quinque figuras non posse aliam constitui figuram solidam, quæ planis & æquilateris & æquiangulis contineatur, inter se æqualibus. Non enim ex duobus triāgulis, sed neq; ex alijs duabus figuris solidus cōstituetur angulus

Sed

Sed ex tribus triangulis, constat Pyramidis angulus.
Ex quatuor autem, Octaëdri.
Ex quinque verò, Icosaëdri.
Nam ex triangulis sex & æquilateris & æquiangulis ad idem punctum coeuntibus, non fiet angulus solidus. Cùm enim trianguli æquilateri angulus, recti vnius bessem contineat, erunt eiusmodi sex anguli rectis quatuor æquales Quod fieri non potest. Nam solidus omnis angulus minoribus quã rectis quatuor angulis continetur, per 21.11.
Ob easdem sanè causas, neque ex pluribus quã planis sex eiusmodi angulis solidus constat.
Sed ex tribus quadratis, Cubi angulus continetur.
Ex quinque, nullus potest. Rursus enim recti quatuor erunt.
Ex tribus autem pentagonis æquilateris & æquiangulis, Dodecaëdri angulus continetur.
Sed ex quatuor, nullus potest. Cùm enim pentagoni æquilateri angulus rectus sit, et quinta recti pars erunt quatuor anguli rectis quatuor maiores. Quod fieri nequit. Nec sanè ex alijs polygonis figuris solidus angulus continebitur, quòd hinc quoque absurdum sequatur. Quamobrem perspicuum est, præter dictas quinque figuras aliam figuram solidam non posse constitui, quæ ex planis æquilateris & æquiangulis contineatur.

ELEMENTI XIII. FINIS.

EVCLIDIS ELEMENTVM DECIMVMQVARTVM, VT quidam arbitrantur, vt alij verò, Hypsiclis Alexandrini, de quinque corporibus.

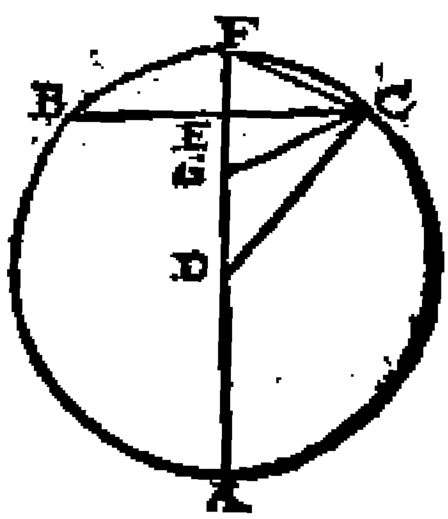
LIBER PRIMVS.

BAsilides Tyrius, Protarche, Alexandriam profectus, patrique nostro ob disciplinæ societatem commendatus, longissimo peregrinationis tempore cum eo versatus est. Cúmque disserérent aliquando de scripta ab Apollonio cõparatione Dodecaëdri & Icosaëdri eidem sphæra inscriptorum, quam hæc inter se habeant rationem, censuerunt ea non rectè tradidisse Apollonium: quæ à se emendata, vt de patre audire erat, literis prodiderunt. Ego autem postea incidi in alterum librũ ab Apollonio editum, qui demonstrationem accuratè complecteretur de re proposita, ex eiusq; problematis indagatione magnam equidem cepi voluptatem. Illud certè ab omnibus perspici potest, quod scripsit Apollonius, cùm sit in omnium manibus. Quod autẽ diligenti, quantum conijcere licet, studio nos postea scrip-

ſcripſiſſe videmur, id monimentis conſignatum tibi nuncupandum duximus, vt qui feliciter cùm in omnibus diſciplinis tum vel maximè in Geometria verſatus, ſcitè ac prudẽter iudices ea quæ dicturi ſumus ob eam verò, quæ tibi cum patre fuit, vitæ conſuetudinem, quaq; nos complecteris, beneuolentiã, tractationem ipſam libenter audias. Sed iam tempus eſt, vt proœmio modum facientes, hanc ſyntaxim aggrediamur.

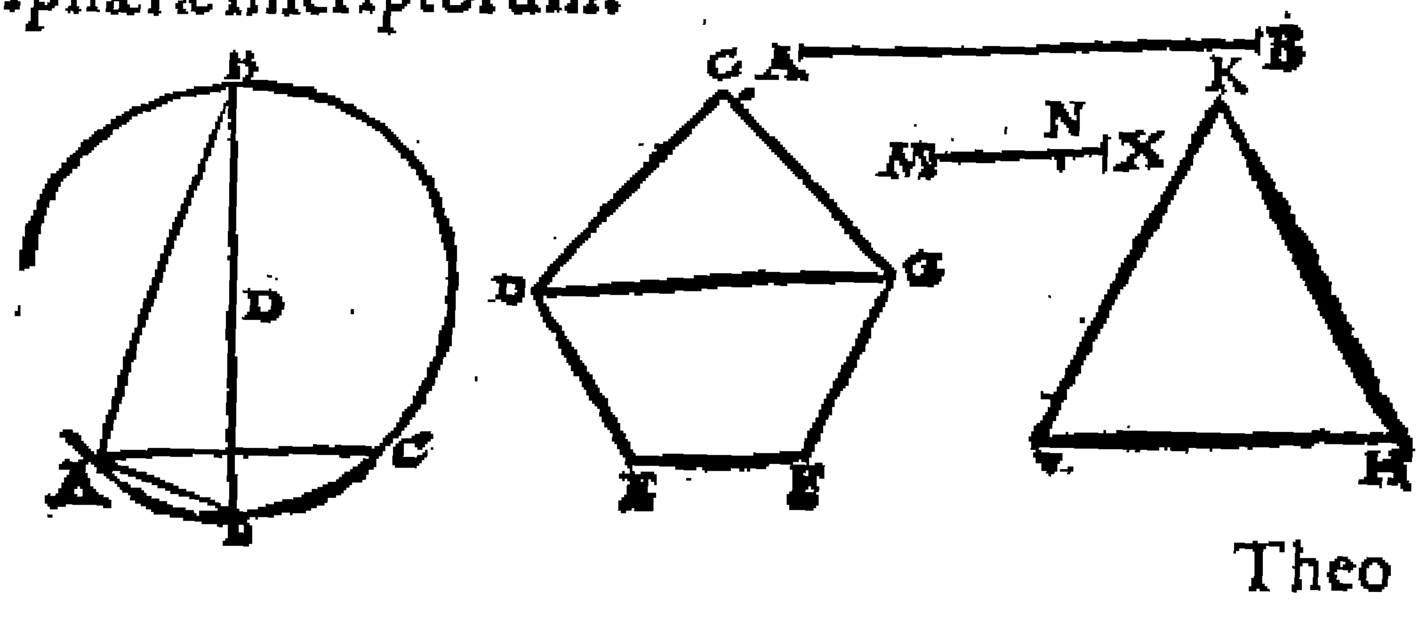
Theorema 1. Propoſitio 1.

Perpendicularis linea, quæ ex circuli cuiuſpiam centro in latus pẽtagωni ipſi circulo inſcripti ducitur, dimidia eſt vtriuſq; ſimul lineæ, & eius quæ ex centro, & lateris decagωni in eodẽ circulo inſcripti.



Theorema 2. Propoſitio 2.

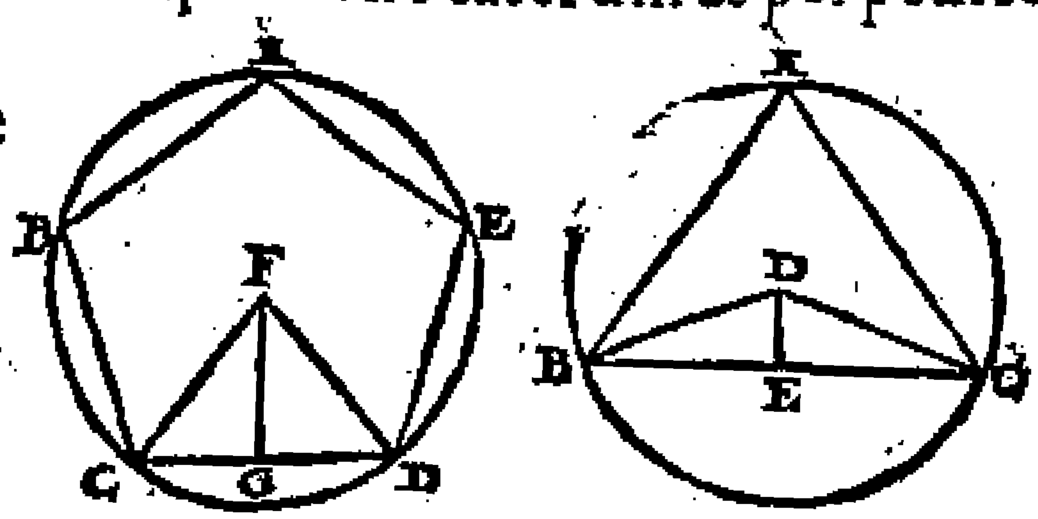
Idem circulus cõprehendit & dωdecaedri pentagωnum & icoſaedri triangulũ, eidem ſphæræ inſcriptorum.



Theo

Theorema 3. Pro-
poſitio 3.

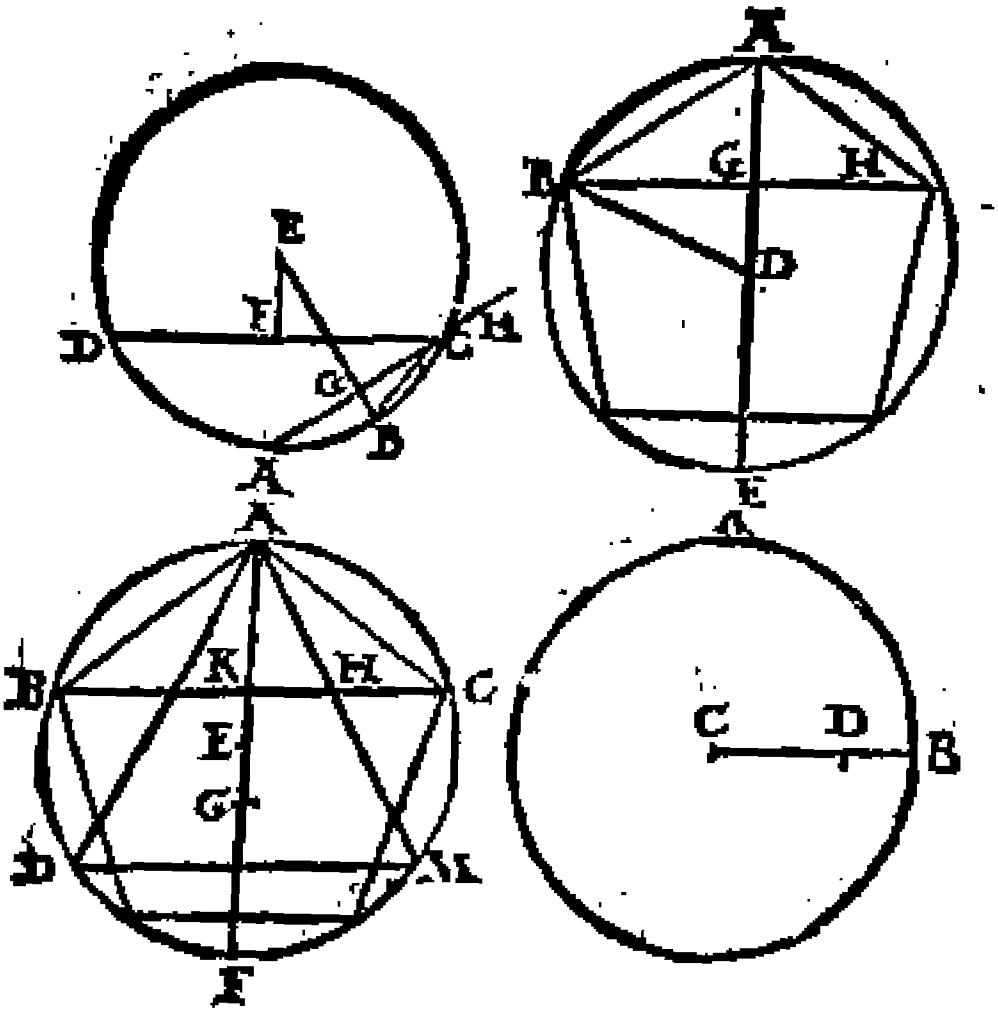
Si pentagωno & æquilatero & æquiangulo circunſcriptus ſit circulus, ex cuius centro in vnum pentagωni latus ducta ſit perpendicularis: quod vno laterum & perpēdicu lari- trige ſies con- tine- tur, illud æquale eſt dωdecaedri ſuperficiei.



Theorema 4. Pro-
poſitio 4.

Hoc perſpicuum cùm ſit, probandum eſt, quemadmodum ſe habet dωdecaedri ſuperficies ad icoſaedri ſuperficiem, ita ſe habere cubi latus ad icoſaedri latus.

Cubi



Cubi latus.

E————————

Dodecaedri.

F————————

Icosaedri.

G————————

SCHOLIVM.

Nunc autem probandum est, quemadmodum se habet cubi latus ad icosaëdri latus, ita se habere solidum dodecaëdri ad icosaëdri solidum. Cùm enim æquales circuli comprehendant & dodecaëdri pẽtagonum & Icosaëdri triangulum, eidem sphæra *scriptorum:in sphæris autem æquales circuli æquali interuallo distent à centro (siquidem perpendiculares à sphæræ centro ad circulorum plana ductæ & æquales sunt, & ad circulorum centra cadunt) idcirco lineæ, hoc est perpendiculares, quæ à sphæræ cẽtro ducuntur ad centrum circuli comprehendẽtis & triangulum Icosaëdri, & pentagonum dodecaëdri, sunt æquales. Sũt igitur æqualis altitudinis Pyramides, quæ bases habent ipsa dodecaëdri pentagona, et quæ Icosaëdri triangula. At æqualis altitudinis pyramides rationem inter se habent eam quam bases, ex 5. & 6. 11. Quemadmodum igitur pentagonũ ad triangulum, ita pyramis, cuius basis quidem est dodecaëdri pentagonum, vertex autem sphæræ centrum ad pyramida, cuius basis quidem est Icosaëdri triangulum, vertex autem sphæræ centrum. Quamobrem vt se habent duodecim pentagona ad viginti triangula, ita duodecim pyramides, quorum pẽtagonæ sint bases, ad viginti pyramidas, quæ trigonas habeant bases. At pentagona duodecim sunt dodecaëdri superficies, viginti autem triangula, Icosaëdri. Est igitur vt dodecaëdri superficies ad Icosaëdri superficiem, ita duodecim pyramides, quæ pentagonas habeant bases, ad viginti pyramidas, quarum trigonæ sunt bases. Sunt autem duodecim quidem pyramides, quæ pentagonas habeant bases, solidũ dodecaëdri: viginti autem pyramides, quæ trigonas habeant bases, Icosaëdri solidum. Quare ex 11. 5. vt dodecaëdri superficies ad Icosaëdri superficiem, ita solidum dodecaëdri ad Icosaëdri solidum. Vt autem*

tem

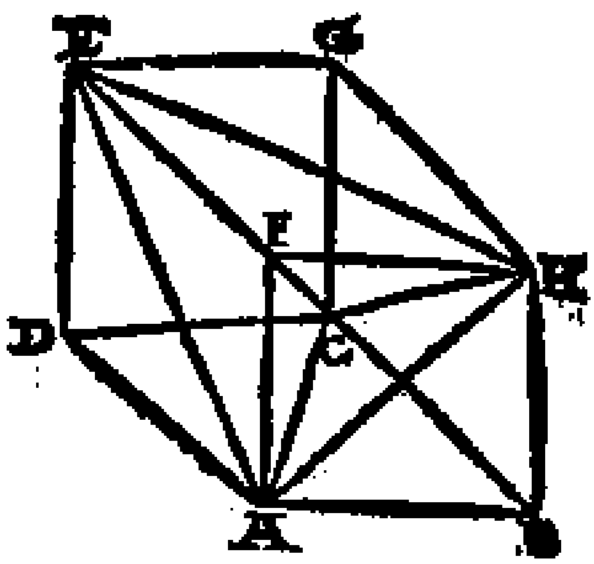
tem dodecaëdri superficies ad Icosaëdri superficiem, ita probatum est cubi latus ad Icosaëdri latus. Quemadmodum igitur cubi latus ad Icosaedri latus, ita se habet solidum dodecaëdri ad Icosaëdri solidum.

EVCLIDIS ELEMENTVM DECIMVMQVINTVM, ET Solidorum quintum, vt nonnulli putant, vt autem alij, Hypsiclis Alexandrini, de quinque corporibus,

LIBER II.

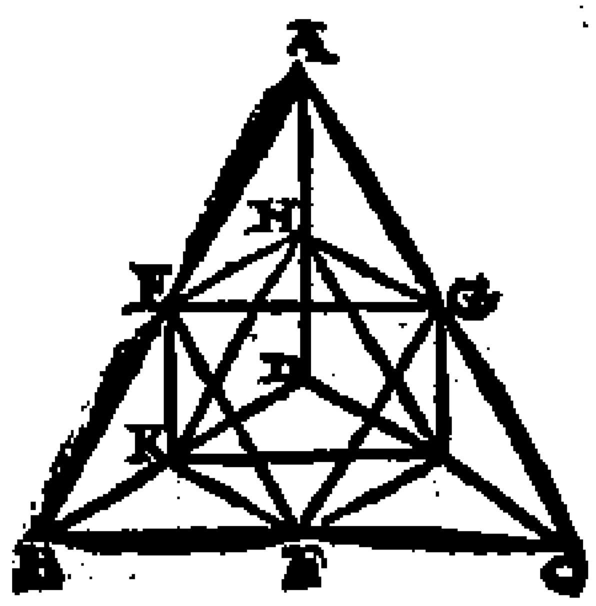
Problema 1. Propositio 1.

In dato cubo pyramida inscribere.



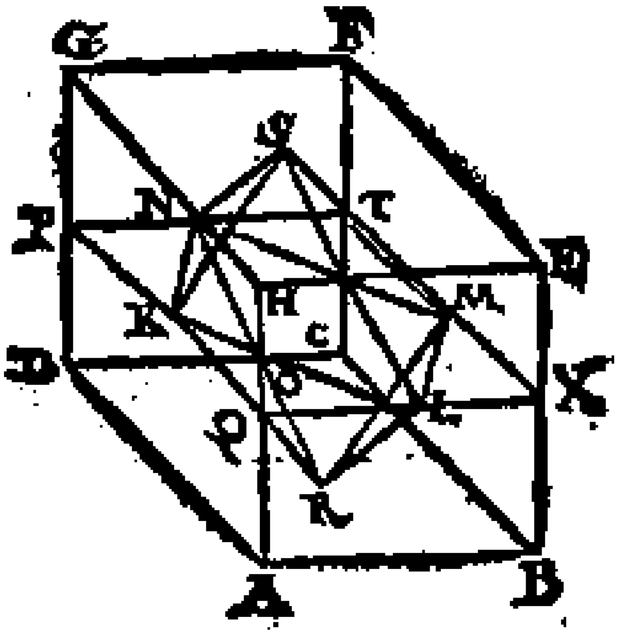
Problema 2. Propositio 2.

In data pyramide octaedrum inscribere.



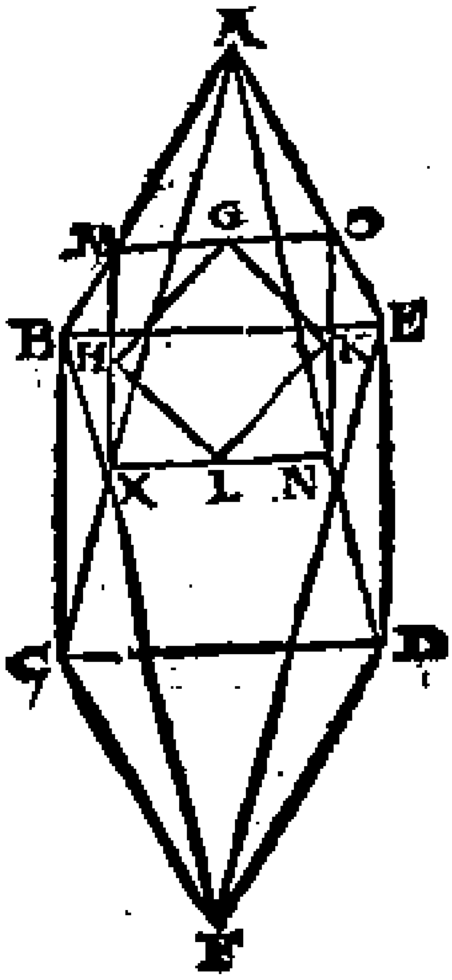
Problema 3. Propositio 3.

In dato cubo octaedrum inscribere.



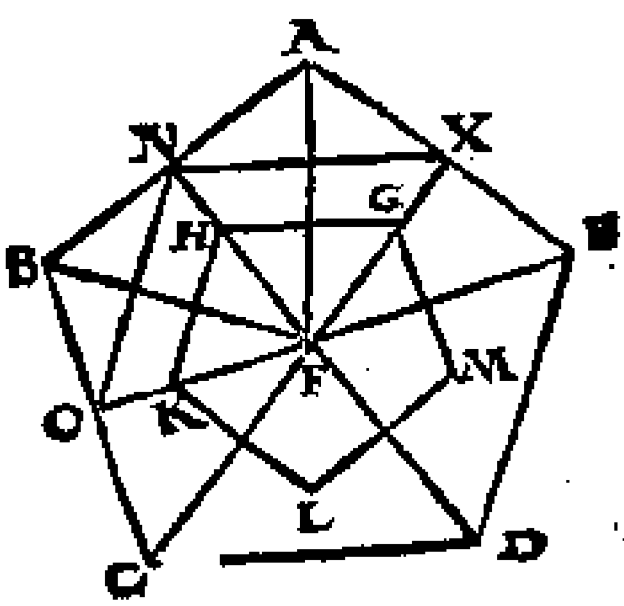
Problema 4. Propositio 4.

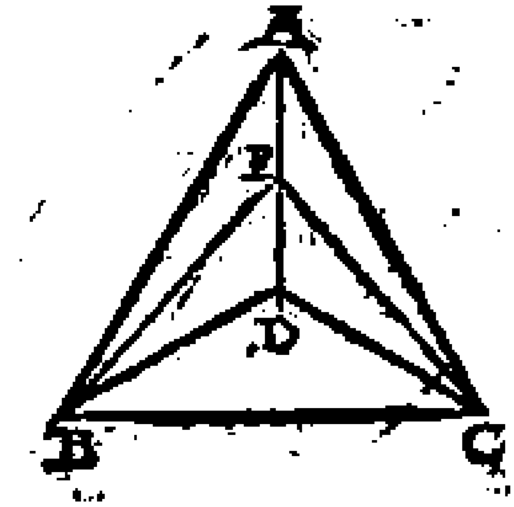
In dato octaedro cubũ inscribere.

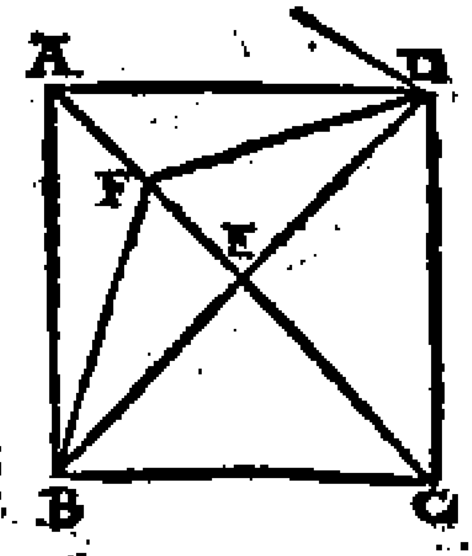


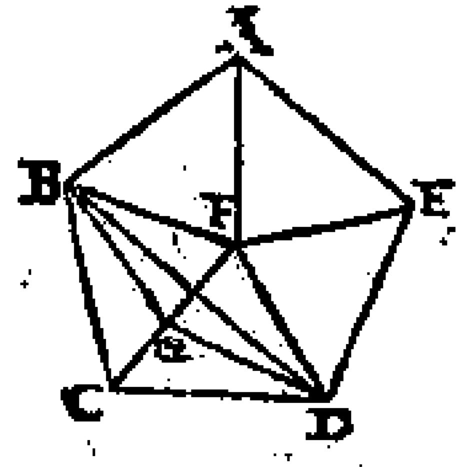
Problema 5. Propositio 5.

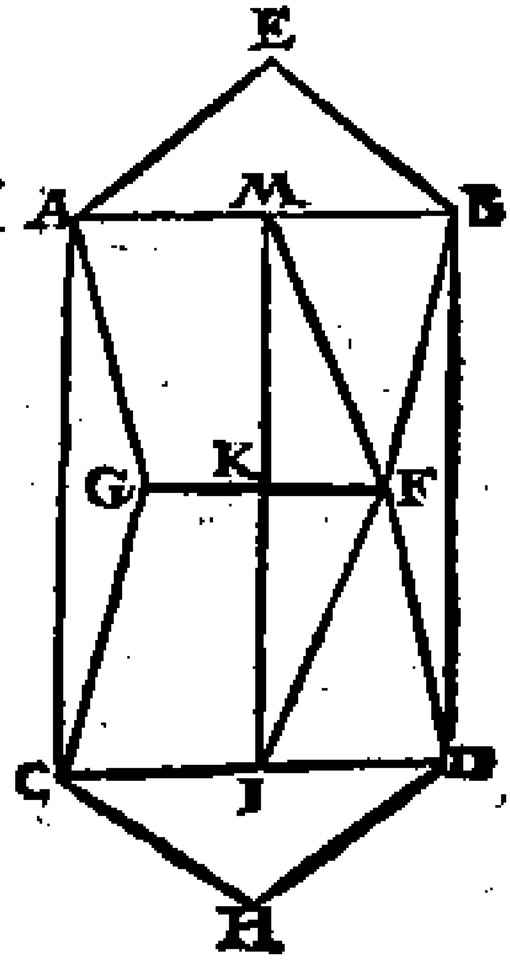
In dato Icosaedro dodecaedrum inscribere.

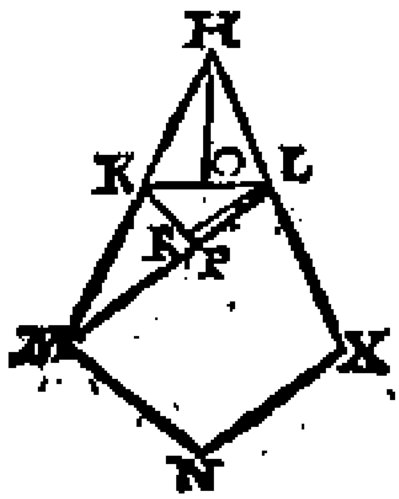












SCHO:

SCHOLIVM.

Meminisse decet, si quis nos roget, quot Icosaëdrũ habeat latera, ita respondendum esse: Patet Icosaëdrum viginti contineri triangulis, quodlibet verò triangulum rectis tribus constare lineis. Quare multiplicanda sunt nobis viginti triangula in trianguli vnius latera, fiuntq; sexaginta, quorum dimidium est triginta. Ad eundem modum & in dodecaëdro. Cũ enim rursus duodecim pentagona dodecaëdrum cõprehendant, itemq; pentagonum quoduis rectis quinque constet lineis, quinq; duodecies multiplicamus, fiunt sexaginta, quorum rursus dimidium est triginta. Sed cur dimidiũ capimus? Quoniam vnumquodq; latus siue sit trianguli, siue pentagoni, siue quadrati, vt in cubo, iteratò sumitur. Similiter autem eadem via & in cubo & in pyramide & in octaëdro latera inuenies. Quòd si item velis singularum quoq; figurarum angelos reperire, facta eadem multiplicatione numerum procreatum partire in numerum planorũ, quæ vnum solidum angulum includunt: vt quoniã triangula quinq; vnum Icosaëdri angulum continẽt, partire 60. in quinque, nascẽtur duodecim anguli Icosaëdri. In dodecaëdro autem tria pentagona angulum comprehendunt. partire ergo 60. in tria, & habebis dodecaëdri angulos viginti. Atq; simili ratione in reliquis figuris angulos reperies.

Finis Elementorum Euclidis.