Omnia sunt hominum tenui pendentia filo:
Et subito casu quae valuere ruunt.
Pro naplnění moci ani draho zlatem
tvůj drženo (?) Pán vše kralovství svoje
mnoho národů naboření.

EVCLIDIS ELEMENTORVM LIBRI XV.

Quibus, cùm ad omnem Mathematicæ scientiæ partem, tùm ad quamlibet Geometriæ tractationem, facilis comparatur aditus.

Collegii Caesarei

[redacted]

Soc. Jesu

Bra

16

COLONIAE,
Apud Maternum Cholinum.
M. D. LXXXVII.
Cum Gratia & Priuilegio Cæs. Maiest.

AD CANDIDVM LECTOREM ST. GRACILIS præfatio.

PERMAGNI referre sẽper existimaui, lector beneuole, quantum quisq; studij & diligentiæ ad percipienda scientiarum elementa adhibeat, quibus nõ satis cognitis, aut perperã, intellectis, si vel digitum progredi tentes, erroris caliginem animis offundas, non veritatis lucem rebus obscuris adferas. Sed principiorum quanta sint in disciplinis momenta, haud facilè credat, qui rerum naturam ipsa specie non viribus metiatur. Vt enim corporum quæ oriuntur & intereũt vilissima tenuissimaq; videntur initia: ita rerum æternarum & admirabilium, quibus nobilissimæ artes continentur, elementa ad speciem sunt exilia, ad vires & facultatem quã maxima. Quis nõ videt ex fici tãtulo grano, vt ait Tullius, aut ex acino vinaceo, aut ex cæterarum frugum aut stirpium minutissimis seminibus tantos truncos ramosq; procreari? Nã Mathematicorũ initia illa quidem dictu audituq; perexigua, quantã theorematum syluã nobis pe-

pererunt? Ex quo intelligi potest, vt in ipsis semi nibus, sic & in artium principijs inesse vim earũ rerum quæ ex his progignuntur. Præclarè igitur Aristoteles, vt alia permulta, μέγιστον ἴσως ἀρχὴ παντὸς, καὶ ὅσῳ κράτιστον τῇ δυνάμει, τοσούτῳ μικρότατον, ὂν τῷ μεγέθει χαλεπόν ἐστιν ὀφθῆναι. Quocirca committendum non est, vt non bene prouisa & diligenter explorata scientiarum principia, quibus propositarum quarumque rerũ veritas sit demonstranda, vel constituas, vel constituta approbes: Cauendum etiam, vt ne tantulum quidem fallaci & captiosa interpretatione turpiter deceptus, à vera principiorum ratione temerè deflectas. Nam qui initio fortè aberrauerit, is vt tandem in maximis versetur erroribus, necesse est: cum ex vno erroris capite, densiores sensim tenebræ rebus clarissimis obducantur. Quid tam varias veterum physiologorum sententias non modò cum rerum veritate pugnantes, sed vehementer etiam inter se dissentientes nobis inuexit? Equidem haud scio, fueritne vlla potior tanti dissidij causa, quàm quòd ex principijs partim falsis, partim non consentaneis ductas rationes probando adhiberent. Fit enim plerunque, vt qui non rectè de artium rerumq; elementis sentiunt, ad præfinitas quasdam opiniones suas omnia reuocare studeant. Pythagorei, vt meminit Aristoteles, cum denarij numeri summam perfectinem cœlo tribuerent, nec plures tamen quàm nouem sphæras cernerent, decimam affingere ausi sunt terræ aduer-

sam

sum, quam ἀντίχθονα appellarunt. Illi enim universitatis rerumq; singularum naturam ex numeris ceu principijs æstimantes, ea protulerũt quæ φαινομένοις congruere nusquam sunt cognita Nam ridicula Democriti, Anaximenis, Melissi Anaxagoræ, Anaximandri, & reliquorum id genus physiologorum somnia ex falsis illa quidem orta naturæ principijs sed ad Mathematicum nihil aut parum spectantia. sciens præterea Nonnullos attingam qui repetitis altius vel aliter ac decuit positis rerum initijs, cum physicis multa turbarunt, tum Mathematicos oppugnatione principiorum pessimè mulctarũt. Ex planis figuris corpora cõstituit Timæus: Geometrarum hic quidem principia cuniculis oppugnantur. Nam & superficies seu extremitates crassitudinem habebunt, & lineæ latitudinem: denique puncta non erunt indiuidua, sed linearum partes. Prædicant Democritus atque Leucippus illas atomos suas, & indiuidua corpuscula. Concedit Xenocrates impartibiles quasdã magnitudines. Hic verò Geometriæ fundamẽta apertè petuntur, & funditus euertuntur: quibus dirutis nihil equidẽ aliud video restare, quàm vt amplissima Mathematicorum theatra repentè concidant. Iacebunt ergo, si dijs placet, tot præclara Geometrarum de asymmetris & alogis magnitudinibus theoremata. Quid enim causæ dicas cur indiuidua linea hanc quidem metiatur, illam verò metiri non queat? Siquidem quod minimum in vnoquoque

 genere

genere reperitur, id communis omnium mensura esse solet. Innumerabilia profectò sunt illa, quæ ex falsis eiusmodi decretis absurda consequuntur: & horum permulta quidem Mathematicus, sed longe plura colligit Physicus. Quid varia ψευδογραφημάτων genera commemorẽ, quæ ex hoc vno fonte tam longè latéque diffusa fluxisse videntur? Notissimus est Antiphontis tetragonismus, qui Geometrarum & ipse principia non parum labefecit, cùm rectæ lineæ curuam posuit æqualem. Longum esset mihi singula percensere, præsertim ad alia properanti: Hoc ergo certum fixum, & in perpetuum ratum esse oportet, quod sapiẽter monet Aristoteles, σπουδαστέον ὅπως ὁρισθῶσι καλῶς αἱ ἀρχαὶ μεγάλην γὰρ ἔχουσι ῥοπὴν πρὸς τὰ ἑπόμενα. *Nam principijs illa congruere debent, quæ sequuntur. Quòd si tantum perspicitur in istis exilioribus Geometriæ initijs quæ puncto, linea, superficie definiuntur, momentum, vt ne hæc quidem sine summo impendentis ruinæ periculo conuelli aut oppugnari possint, quanta quæso vis putanda est huius* στοιχειώσεως, *quam collatis tot præstãtissimorum artificum inuentis, mira quadam ordinis solertia contexuit Euclides, vniuersæ Matheseos elementa complexu suo coercentem? Vt igitur omnibus rebus instructior & paratior quisque ad hoc studium libentius accedat, & singula vel minutissima exactius secum reputet atque perdiscat, operæprecium censui, in primo institutionis aditu vestibuloq; præcipua quadam capita,*

quibus

quibus tota ferè Mathematicæ scientiæ ratio intelligatur, breuiter explicare: tum ea quæ sunt Geometriæ propria, diligẽter persequi: Euclidis deniq; in extruenda hac στοιχειώσει consilium sedulò ac fideliter exponere. Quæ ferè omnia ex Aristotelis potissimum ducta fontibus, nemini inuisa fore confido, qui modò ingenuum animi candorem ad legendum attulerit. Ac de Mathematicæ diuisione primùm dicamus.

Mathematicæ in primis scientiæ studiosos fuisse Pythagoreos, non modò historicorum, sed etiam philosophorum libri declarant. His ergo placuit, vt in partes quatuor vniuersum distribuatur Mathematicæ scientiæ genus, quarum duas περὶ τὸ ποσὸν, reliquas περὶ τὸ πηλίκον versari statuerunt. Nam & τὸ ποσὸν vel sine vlla cõparatione ipsum per se cognosci, vel certa quadam ratione comparatũ spectari: in illo Arithmeticam, in hoc versari Musicam: & τὸ πηλίκον partim quiescere, partim moueri quidem: illud Geometriæ propositum esse: quod verò sua sponte motu cietur, Astronomiæ. Sed ne quis falso putet, Mathematicam scientiam, quòd in vtroque quanti genere cernitur, idcirco inanem videri (si quidem non solum magnitudinis diuisio, sed etiam multitudinis accretio infinitè progredi potest) meminisse decet, τὸ πηλίκον καὶ τὸ ποσὸν, quæ subiecto Mathematicæ generi imposita sunt à Pythagoreis nomina, non cuiuscunque modi quantitatem significare, sed eam demum, quæ tum multitudine tum magnitudi-

tudine sit definita, & suis circumscripta terminis. Quis enim ullam infiniti scientiam defendat? Hoc scitum est, quod non semel docet Aristoteles, infinitum ne cogitatione quidem complecti quenquam posse. Itaque ex infinita multitudinis & magnitudinis δυνάμει finitam hac scientia decerpit & amplectitur naturam, quam tractet, & in qua versetur. Nam de vulgari Geometrarum cõsuetudine quid sentiendum sit, cum data interdum magnitudine infinita aut fabricantur aliquid, aut proprias generis subiecti affectiones exquirunt, disertè monet Aristoteles, οὐδὲ νῦν (de Mathematicis loquens) δέονται τοῦ ἀπείρου, οὐδὲ χρῶνται, ἀλλὰ μόνον εἶναι ὅσην ἂν βούλωνται, πεπερασμένην. Quamobrem disputatio ea qua infinitum refellitur, Mathematicorum decretis rationibusq; non aduersatur, nec eorum apodixes labefacit. Etenim tali infinito opus illis nequaquam est, quod exitus nullo peragrari possit, nec talem ponunt infinitam magnitudinẽ: sed quantãcunq; velit aliquis effingere, ea ut suppetat, infinitam præcipiunt. Quinetiã non modò immensa magnitudine opus non habent Mathematici, sed ne maxima quidem: cum instar maximæ minima quæque in partes totidem pari ratione diuidi queat. Alteram Mathematicæ diuisionẽ attulit Geminus, vir (quantum ex Proclo cõÿcere licet) μαθημάτων laude clarissimus. Eam, quæ superiore plenior & accuratior fortè visa est, cũ doctissimè pertractarit sua in decimum Euclidis

præfa-

præfatione P. Montaureus vir senatorius, & regiæ bibliothecæ præfectus, leniter attingam. Nam ex duobus rerum velut summis generibus τῶν νοητῶν καὶ τῶν αἰσθητῶν, quæ res sub intelligentiam cadunt, Arithmetica & Geometriæ attribuit Geminus: quæ verò in sensus incurrūt, Astrologia, Musica, Supputratici, Optica, Geodæsiæ & Mechanica adiudicauit. Ad hāc certè diuisionem spectasse videtur Aristoteles, cùm Astrologiam Opticam, harmonicam φυσικωτέρας τῶν μαθημάτων nominat, vt quæ naturalibus & Mathematicis interiecta sint, ac velut ex vtrisque mixta disciplinæ: Siquidem genera subiecta à Phisicis mutuantur, causas verò in demonstrationibus ex superiore aliqua scientia repetunt. Id quod Aristoteles ipse apertissimè testatur, ἐνταῦθα γὰρ, φησὶ, τὸ μὲν ὅτι, τῶν αἰσθητικῶν εἰδέναι τὸ δὲ διότι, τῶν μαθηματικῶν. Sequitur, vt quid Mathematica conueniat cum Phisica & prima Philosophia: quid ipsa ab vtraque differat, paucis ostendamus. Illud quidem omnium commune est, quòd in veri contemplatione sunt posita, ob idq; θεωρητικαὶ à Græcis dicuntur. Nam cùm διάνοια siue ratio & mens omnis sit vel πρακτικὴ, vel θεωρητικὴ, totidem scientiarum sint genera necesse est. Quòd si Physica, Mathematica, & prima Philosophia, nec in agendo, nec in efficiendo sunt occupatæ, hoc certè perspicuum est, eas omnes in cognitione cōtemplationéque necessariò versari. Cùm enim rerum non modò agendarum, sed etiam effi-
 cienda-

ξiendarum principia in agente vel efficiente cõ-sistant, illarum quidem προαίρεσις, harum autem vel mens, vel ars, vel vis quædam & facultas: rerum profectò naturalium. Mathematicarum, atque diuinarum principia in rebus ipsis, non in philosophis inclusa latent. Atque hæc vna in omnes valet ratio, quæ θεωρητικὰς esse colligat. Iam verò Mathematica separatim cũ Physica congruit, quòd vtraque versatur in cognitione formarum corpori naturali inhærentiũ. Nam Mathematicus plana, solida, longitudines & puncta contemplatur, quæ omnia in corpore naturali à naturali quoque philosopho tractantur. Mathematica item & prima philosophia hoc inter se propriè conueniunt, quòd cognitionem vtraque persequitur formarum, quoad immobiles, & à concretione materiæ sunt liberæ. Nam tametsi Mathematicæ formæ re vera per se nõ cohærent, cogitatione tamen à materia & motu separantur, οὐδὲ γίνεται ψεῦδος χωριζόντων, vt ait Aristoteles. De cognatione & societate breuiter diximus, iam quid intersit videamus. Vnaquæq; mathematicarũ certum quoddam rerum genus propositum habet, in quo versetur, vt Geometria quantitatem & continuationem aliorum in vnam partem, aliorũ in duas, quorundam in tres, eorúmque quatenus quanta sunt & continua, affectiones cognoscit. Prima autem philosophia, cùm sit omnium communis, vniuersum Entis genus, quæq; ei accidũt & conueniunt hoc ipso quòd est, considerat.

Ad

Adhac, Mathematica eam modò naturā amplectitur, quæ quanquam non mouetur, separari tamen seiungiq; nisi mente & cogitatione à materia non potest, ob eamq; causam, ἐξ ἀφαιρέσεως dici consueuit. Sed Prima philosophia in ijs versatur, quæ & seiuncta, & æterna, & ab omni motu per se soluta sunt ac libera. Cæterùm Physica & Mathematica quanquam subiecto discrepare non videntur, modo tamen rationeq; differunt cognitionis & contemplationis, vnde dissimilitudo quoque scientiarum sequitur. Etenim Mathematicæ species nihil re vera sunt aliud, quàm corporis naturalis extremitates, quas cogitatione ab omni motu & materia separatas Mathematicus contemplatur: sed easdem consectatur Physicorum ars, quatenus cum materia comprehensæ sunt, & corpora motui obnoxia circunscribunt. Ex quo fit vt quæcunque in Mathematicis incommoditates accidunt, eadem etiam in naturalibus rebus videantur accidere, non autem vicissim. Multa enim in naturalibus sequuntur incommoda, quæ nihil ad Mathematicum attinent, διὰ τὸ, inquit Aristoteles, τὰ μὲν ἐξ ἀφαιρέσεως λέγεται, τὰ μαθηματικὰ, τὰ δὲ φυσικὰ ἐκ προσθέσεως. Siquidem res cū materia deuinctas contemplatur physicus: Mathematicus verò rē cognoscit circūscriptis ijs omnibus quæ sensu percipiuntur, vt grauitate leuitate, duritie, mollitie, & prætereà calore, frigore, aliisque contrariorum paribus, quæ sub sensum subiecta sunt: tantum autem relinquit

quanti-

quantitatem & continuum. Itaq; Mathematicorum ars in iis quæ immobilia sunt, cernitur. (τὰ γὰρ μαθηματικὰ τῶν ὄντων ἄνευ κινήσεώς ἐστιν, ἔξω τῶν περὶ τὴν ἀστρολογίαν) quæ verò in naturæ obscuritate posita est, res quidem quæ nec separari nec motu vacare possunt, contemplatur. Idquod in vtroque scientiæ genere perspicuum esse potest siue res subiectas definias, siue proprietates earum demonstres. Etenim numerus, linea, figura rectum, inflexum, æquale, rotundum, vniuersa denique Mathematicus quæ tractat & profitetur, absque motu explicari docerique possunt: χωριστὰ γὰρ τῇ νοήσει κινήσεώς ἐστι: Phisicæ autem sine motione species nequaquam possunt intelligi. Quis enim hominis plantæ, ignis, ossium, carnis naturam & proprietates sine motu, qui materiam sequitur, perspiciat? Siquidem tantisper substantia quæque naturalis constare dici solet, quoad opus & munus suum, agendo patiendoque tueri ac sustinere valeat: qua certe amissa δυνάμει, ne nomen quidem nisi ὁμωνύμως retinet. Sed Mathematico ad explicandas circuli aut trianguli proprietates, nullum adferre potest vsum materiæ vt auri, ligni, ferri, in qua insunt, consideratio, quin eò verius eiusmodi rerum, quarum species tanquam materia vacantes efformemus animo, naturam complectemur, quod coniunctione materiæ quasi adulterari deprauarique videntur. Quocirca Mathematicæ species eodem modo quo κοιλὸν, siue concauitas, sine motu & subiecto, definitione explicare cognos-

cognoscique possunt: naturales verò cùm eam vim habeant, quam, ut ita dicam simitas cum materia comprehensæ sunt, nec absque ea separatim possunt intelligi: quibus exemplis quid inter Phisicas & Mathematicas species intersit, haud difficile est animaduertere. Illis certè non semel est usus Aristoteles. Valeant ergo Protagoræ sophismata, Geometras hoc nomine refellentis, quod circulus normam puncto non attingat. Nā diuina Geometrarum theoremata, qui sensu æstimabit, vix quicquam reperiet quod Geometræ concedendum videatur. Quid enim ex his quæ sensum mouent, ita rectum aut rotundum dici potest, ut à Geometra ponitur? Nec verò absurdum est aut vitiosum, quòd lineas in puluere descriptas pro rectis aut rotundis assumit, quæ nec rectæ sunt nec rotundæ, ac ne latitudinis quidem expertes. Siquidem non ijs utitur Geometra quasi inde vim habeat conclusio, sed eorum quæ discenti intelligenda relinquuntur, rudem ceu imaginem proponit. Nam qui primum instituuntur, hi ductu quodam & velut χειραγωγία sensuum opus habent, ut ad illa quæ sola intelligentia percipiuntur, aditum sibi comparare queant. Sed tamen existimandum non est rebus Mathematicis omnino negari materiam, ac non eam tantùm qua sensum afficit. Est enim materia alia quæ sub sensum cadit, alia quæ animo & ratione intelligitur. Illam αἰσθητὴν, hanc νοητὴν vocat Aristoteles. Sensu percipitur, ut æs, ut lignum, omnisque materia quæ moueri potest

test. Animo & ratione cernitur ea quæ in rebus sensilibus inest, sed nō quatenus sensu percipiuntur, quales sunt res Mathematicorum. Unde ab Aristotele scriptū legimus ἐπὶ τῶν ἐν ἀφαιρέσει ὄντων rectum se habere ut simum: μετὰ συνεχοῦς γὰρ: quasi velit ipsius recti quod Mathematicorum est, suam esse materiam, non minus quā simi quod ad Physicos pertinet. Nam licet res Mathematicæ sensili vacent materia, nō sunt tamen indiuiduæ, sed propter continuationem partitioni semper obnoxiæ, cuius ratione dici possunt sua materia non omnino carere: quin aliud videtur τὸ εἶναι γραμμῇ, aliud quoad continuationi adiuncta intelligitur linea. Illud enim ceu forma in materia proprietatum causa est, quas sine materia percipere non licet. Hæc est societatis & dissidij Mathematicæ cum Physica & prima Philosophia ratio. Nunc autem de nominis etymo & notatione pauca quædam afferamus. Nam si quæ iudicio & ratione imposita sunt rebus nomina, ea certè non temere indita fuisse credendum est, quibus scientias appellari placuit. Sed neque otiosa semper haberi debet ista etymologiæ indagatio, cùm ad rei etiam dubiæ fidem sæpe non parum valeat recta nominis interpretatio. Sic enim Aristoteles ducto ex verborum ratione argumento, αὐτομάτου, μετὰ βολῆς, αἰθέρος, aliarumq; rerum naturam ex parte confirmauit. Quoniam igitur Pythagoras Mathematicam scientiam nō modò studiosè coluit, sed etiā repetitis à capite principijs, geometricā

contem-

contemplationem in liberalis disciplinæ formā composuit, & perspectis absq; materia solius intelligentiæ adminiculo theorematibus, tractationem περὶ τῶν ἀλόγων, & κοσμικῶν σχημάτων cōstitutionem excogitauit: credibile est, Pythagoram, aut certè Pythagoreos, qui et ipsi doctoris sui studia libenter amplexi sunt, huic scientiæ id nomen dedisse, quod cum suis placitis atq; decretis congrueret, rerumq; propositarum naturam quoquo modo declararet. Ita cùm existimarent illi, omnem disciplinam, quæ μάθησις dicitur, ἀνάμνησιν esse quandam. i. recordationem et repetitionem eius scientiæ, cuius antè quam in corpus immigraret, compos fuerit anima, quemadmodum Plato quoq; in Menone, Phædone, & alijs aliquot locis videtur astruxisse: animaduerterent autem eiusmodi recordationem, quæ non posset multis ex rebus perspici, ex his potissimum scientijs demonstrari, si quis nimirum, ait Plato, ἐπὶ τὰ διαγράμματα ἄγῃ. probabile est equidem Mathematicas à Pythagoreis artes κατ᾽ ἐξοχὴν fuisse nominatas, vt ex quibus μάθησις, id est æternarum in anima rationum recordatio διαφερόντως et pcipuè intelligi posset. Cuius etiā rei fidē nobis diuinus fecit Plato, q in Menone Socratem induxit hoc argumēti genere persuadere cupientem discere nihil esse aliud quam suarum ipsius rationum animum recordari. Etenim Socrates pusionem quendam, vt Tullij verbis vtar, interrogat de geometrica dimensione quadrati: ad ea sic ille respondet vt puer, & tamen

men tam faciles interrogationes sunt, vt gradatim respondens eodem perueniat quò si Geometrica didicisset. Aliam nominis huius rationem Anatolius exposuit, vt est apud Rhodiginum, quòd cùm catera disciplina deprehendi vel non docente aliquo possint omnes. Mathematica sub nullius cognitionem veniant, nisi praeeunte aliquo, cuius solertia succidantur vepreta, vel exurantur, & supercil:osa complanentur aspreta. Ita enim Calius: quod quam vim habeat, nō est huius loci curiosius perscrutari. Equidem M Tullius Mathematicos in magna rerum obscuritate recondita arte multiplicíq; ac subtili versari scribit: sed quis nescit idipsum cum alijs grauioribus scientijs esse commune? Est enim, vel eodem auctore Tullio, omnis cognitio multis obstructa difficultatibus, maximaq; est & in ipsis rebus obscuritas, & in iudicijs nostris infirmitas, nec vllus est, modò interius paulò Physica penetrarit, qui non facile sit expertus, quam multi undique emergant, rerum naturalium causas inquirentibus, inexplicabiles labyrinthi. Sunt qui ex demonstrationum firmitate nominari Mathematicas opinantur: cuius etiam rationis momentum alio seorsim loco expendendum fuerit. Quocirca primam verbi notationem, quam sequutus est Proclus, nobis retinendam censeo. Hactenus de vniuerso Mathematica genere, quanta potui & perspicuitate & breuitate dixi. Sequitur vt de Geometria separatim atque ordine ea disseram, qua initio sum pollicitus.

*tus. Est autem Geometria, ut definit Proclus sci-
entia quæ versatur in cognitione magnitudinũ
figurarum, & quibus hæ continentur, extremo-
rum, item rationum & affectionum quæ in illis
cernuntur ac inhærent: ipsa quidem progrediens
à puncto indiuiduo per lineas & superficies, dũ
ad solida conscendat, variasq; ipsorum diffe-
rentias patefaciat. Quúmque omnis scientia
demonstratiua, ut docet Aristoteles tribus qua-
si momentis contineatur, genere subiecto, cuius
proprietates ipsa scientia exquirit & contem-
platur: causis & principijs ex quibus primis de-
monstrationes conficiuntur: & proprietatibus,
quæ de genere subiecto per se enunciantur: Geo-
metriæ quidem subiectum in lineis, triangulis,
quadrangulis circulis, planis, solidis atque om-
nino figuris & magnitudinibus, earúmque ex-
tremitatibus consistit. His autem inhærent diui-
siones, rationes, tactus, æqualitates,* παραβολαὶ ὑπερβολαὶ ἐλλεῖψεισ, *atque alia generis eiusdem
propè innumerabilia. Postulata verò & Axio-
mata ex quibus hæc inesse demonstrantur, eius-
modi ferè sunt: Quouis centro & interuallo cir-
culum describere. Si ab æqualibus æqualia de-
trahas, quæ relinquuntur esse æqualia, cæteraq;
id genus permulta, quæ licet omnium sint cõmu-
nia ac demonstrandum tamen tum sunt accom-
modata, cùm ad certum quoddam genus tra-
ducuntur. Sed cùm præcipua videatur Arith-
metica & Geometriæ inter Mathematicas
dignatio; cur Arithmetica sit* ἀκριβεστέρα *&*

 exactior

exactior quam Geometria paucis explicandum arbitror. Hic verò & Aristotelem sequemur ducem, qui scientiam cum scientia ita comparat, vt accuratiorem esse velit eam, quæ rei caussam docet, quam quæ rem esse tantùm declarat, deinde quæ in rebus sub intelligentiam cadentibus versatur, quàm quæ in rebus sensum mouentibus cernitur. Sic enim & Arithmetica quàm Musica, & Geometria quàm Optica & Stereometria quàm Mechanica exactior esse intelligitur. Postremò quæ ex simplicioribus initijs constat, quàm quæ aliqua adiectione compositis vtitur. Atque hac quidem ratione Geometriæ præstat Arithmetica, quòd illius initium ex additione dicatur, huius sit simplicius. Est enim punctum, vt Pythagoreis placet, vnitas quæ situm obtinet: vnitas verò punctum est quod situ vacat. Ex quo percipitur, numerorum quàm magnitudinum simplicius esse elementum, numerosque magnitudinibus esse puriores, & à concretione materiæ magis disiunctos. Hæc quanquam nemini sunt dubia, habet & ipsa tamen Geometria quo se plurimum efferat, opibúsque suis ac rerum vbertate multiplici vel cum Arithmetica certet: id quod tute facilè deprehendas cùm ad infinitam magnitudinis diuisionem, quam respuit multitudo, animum conuerteris. Nunc quæ sit Arithmeticæ & Geometriæ societas, videamus. Nam theorematum quæ demonstratione illustrantur, quædam sunt vtriusque scientiæ communia, quædam verò singu-

singularum propria. Etenim quòd omnis proportio sit ῥητὸς siue rationalis, Arithmeticæ soli conuenit, nequaquam Geometriæ, in qua sunt etiam ἄῤῥητοι, seu irrationales proportiones. item, quadratorum gnomas minimo definitos esse, Arithmeticæ proprium (si quidem in Geometria nihil tale minimum esse potest) sed ad Geometriam propriè spectant situs, qui in numeris locum non habent: tactus, qui quidem à continuis admittuntur: ἄλογον, quoniam vbi diuisio infinitè procedit, ibi etiam τὸ ἄλογον esse solet. Communia porrò vtriusque sunt illa, quæ ex sectionibus eueniunt, quas Euclides libro secundo est persequutus: nisi quòd sectio per extremam & mediam rationem in numeris nusquam reperiri potest. Iam verò ex theorematibus eiusmodi communibus, alia quidem ex Geometria ad Arthmeticam traducuntur: alia contrà ex Arithmetica in Geometriam trãsferuntur: quædam verò perinde vtrique scientiæ conueniunt, vt quæ ex vniuersa arte Mathematica in vtranque harum conueniant. Nã & alterna ratio, & rationum conuersiones, compositiones, diuisiones hoc modo communia sunt vtriusque. Quæ autem sunt περὶ συμμέτρων, id est, de commensurabilibus, Arithmetica quidem primum cognoscit & contemplatur: secundo loco Geometria Arithmeticam imitata. Quare & commensurabiles magnitudines illæ dicuntur, quæ rationem inter se habent quam numerus ad numerum, per-

inde quasi commensuratio & συμμετρία in numeris primum consistat (Ubi enim numerus, ibi & σύμμετρον cernitur: & ubi σύμμετρον, illic etiam numerus) sed quæ triangulorum sunt & quadrangulorum, à Geometra primùm considerantur: tum analogia quadam Arithmeticus eadem illa in numeris contemplatur. De Geometriæ diuisione, hoc adijciendum puto, quòd Geometriæ pars altera in planis figuris cernitur, quæ solam latitudinem longitudini coniunctam habent: altera verò solidas contemplatur, quæ ad duplex illud interuallum crassitudinem adsciscunt. Illam generali Geometriæ nomine veteres appellarunt: hanc propriè Stereometriam dixerunt. Ita Geometriam cum Optica & Stereometriam cum Mechanica non rarò comparat Aristoteles. Sed illius cognitio huius inuentionem multis seculis antecessit, si modò Stereometriam ne Socratis quidem ætate ullam fuisse omnino verum est, quemadmodum à Platone scriptum videtur. Ad Geometriæ vtilitatem accedo, quæ quanquam suapte vi & dignitate ipsa per se nititur, nullius vsus aut actionis ministerio mancipata (vt de Mathematicis omnibus scientijs concedit in Politico Socrates) si quid ex ea tamen vtilitatis externæ quæritur, Dij boni quā latos, quàm vberes, quā varios fructus fundit? Nec verò audiendus est vel Aristippus vel Sophistarum alius, qui Mathematicorum artes idcirco repudiet, quòd ex fine nihil docere videantur, eiusque quod melius aut

deteri-

deterius nullam habeant rationem Ut enim nihil causæ dicas, cur sit melius triāguli, verbi gratia, tres angulos duobus esse rectis æquales: minimè tamen fuerit consentaneum, Geometriæ cognitionem ut inutilem exagitare, criminari, explodere, quasi quæ finem & bonum quò referatur, habeat nullum. Multas haud dubiè solius contemplationis beneficio citra materiæ contagionem adfert Geometria commoditates partim proprias, partim cum uniuerso genere communes. Cùm enim Geometria, ut scripsit Plato, eius quod semper est cognitionem profiteatur, ad veritatem excitabit illa quidem animum, & ad ritè philosophandum cuiusque mentem comparabit. Quinetiam ad disciplinas omnes facilius perdiscendas, attigeris necne Geometriam quāti referre censes? Nam ubi cum materia coniūgitur, nónne præstantissimas procreat artes, Geodæsiam, Mechaniam, Opticam, quarum omnium usu, mortalium vitam summis beneficijs complectitur? Etenim bellica instrumenta, urbiumq; propugnacula, quibus munitæ urbes hostium vim propulsarent, his adiutricibus fabricata est: montium ambitus & altitudines, locorumque situs nobis indicauit: dimetiendorum & mari & terra itinerum rationem præscripsit: trutinas & stateras, quibus exacta numerorum æqualitas in ciuitate retineatur, composuit: uniuersi ordinem simulachris expressit, multáque quæ hominum fidem superarent, omnibus persuasit. Vbique extant præclara in eam rem *testimonia. Illud memorabile, quod Archimedi rex Hiero tribuit. Nam extructo vastæ molis nauigio, quod Hiero Ægyptiorum regi Ptolemæo mitteret, cùm vniuersa Syracusanorum multitudo collectis simul viribus nauem trahere non posset effecissetq; Archimedes, vt solus Hiero illam subduceret, admiratus viri scientiã rex* ἀπὸ ταύτης, ἔφη, τῆς ἡμέρας περὶ παντὸς ἀρχιμήδῃ λέγοντι πιστευτέον. *Quid? quod Archimedes idem, vt est apud Plutarchum, Hieroni scripsit datis viribus datum pondus moueri posse: fretusq; demonstrationis robore, illud sæpe iactarit, si terram haberet alteram vbi pedem figeret, ad eam, nostram hanc se transmouere posse? Quid varia* αὐτομάτων *machinarum que genera ad vsus necessarios comparata memorem? Innumerabilia profectò sunt illa, & admiratione dignissima, quibus prisci homines incredibili quodam ad philosophandum studio concitati, inopem mortalium vitam artis huius præsidio subleuarunt: tametsi memoriæ sit proditum, Platonem Eudoxo & Archytæ vitio vertisse, quòd Geometrica problemata ad sensilia & organica abducerent. Sic enim corrumpi ab illis & labefieri Geometriæ præstantiam, quæ ab intelligibilibus & incorporeis rebus ad sensiles & corporeas proluberetur. Quapropter ridicula idem scripsit Plato Geometrarum esse vocabula, quæ quasi ad opus & actionem spectent, ita sonare videntur. Quid enim est quadrare si non opus facere? Quid addere,*

pro-

producere, applicare? Multa quidem sunt eiusmodi nomina, quibus necessariò & tanquam coacti Geometræ vtuntur, quippe cùm alia desint in hoc genere commodiora. Sic ergo censuit Plato, sic Aristoteles, sic denique philosophi omnes, Geometriam ipsam cognitionis gratia exercendam, nec ex aliquo vsu externo sed ex rerum νοητῶν intelligentia æstimandam esse. Exposita breuius quàm res tanta dici possit, vtilitatis ratione, Geometriæ ortum, qui in hac rerum periodo ex historicorum monumentis nobis est cognitus deinceps aperiamus. Geometria apud Ægyptios inuenta, (ne ab Adamo, Setho, Noah, quos cognitione rerum multiplici valuisse constat eam repetamus) ex terrarum dimensione, vt verbi præ se fert ratio, ortum habuisse dicitur: cùm anniuersaria Nili inundatione & incrementis limo obducti agrorum termini confunderentur. Geometriam enim, sicut & reliquas disciplinas, in vsu quàm in arte prius fuisse aiūt. Quod sanè mirum videri non debet, vt & huius & aliarum scientiarum inuentio ab vsu cœperit ac necessitate. Etenim tempus, rerum vsus, ipsa necessitas ingenium excitat, et ignauiā acuit. Deindè quicquid ortum habuit (vt tradunt Physici) ab inchoato & imperfecto processit ad perfectum. Sic artium & scientiarum principia experientiæ beneficio collecta sunt: experientia verò à memoria fluxit, quæ & ipsa à sensu primùm manauit. Nam quod scribit Aristoteles, Mathematicas artes,

 compa-

comparatis rebus omnibus ad vitam necessariis, in Ægypto fuisse constitutas, quòd ibi sacerdotes omnium concessu in otio degerent: non negat ille adductos necessitate homines ad excogitandam, verbi gratia terræ dimetiendæ rationem, quæ theorematum deinde inuestigationi causam dederit: sed hoc confirmat, præclara eiusmodi theorematum inuenta, quibus extructa Geometriæ disciplina constat, ad vsus vitæ necessarios ab illis non esse expetita. Itaque vetus ipsum Geometriæ nomen ab illa terræ partiundæ finiúmque regundorum ratione posteà recessit, & in certa quadam affectionum magnitudini per se inhærentium scientia propriè remāsit. Quemadmodum igitur in mercium & contractuum gratiam supputandi ratio, quam secuta est accurata numerorum cognitio, à Phœnicibus initium duxit: ita etiam apud Ægyptios, ex ea quam cōmemoraui causa ortū habuit Geometria. Hāc certè, vt id obiter dicam, Thales in Græciam ex Ægypto primùm transtulit: cui non paucæ deinceps à Pythagora, Hippocrate, Chio, Platone, Archyta Tarentino, aliisq; cōpluribus, ad Euclidis tempora factæ sunt rerum magnarum accessiones. Cæterùm de Euclidis ætate id solùm addam, quod à Proclo memoriæ mandatum accepimus. Is enim commemoratis aliquot Platonis tum æqualibus tùm discipulis, subiicit, non multò ætate posteriorem illis fuisse Euclidem eum, qui Elementa conscripsit, & multa ab Eudoxo collecta, in ordinē luculentū

com-

composuit, multaq; à Theæteto inchoata perfecit. quæq; mollius ab alijs demonstrata fuerant, ad firmissimas & certissimas apodixes reuocauit. Vixit autem, inquit ille, sub primo Ptolemæo. Etenim ferunt Euclidam à Ptolemæo quodam interrogatum numqua esset via ad Geometriam magis compendiaria, quàm sit ista στοιχειώσις respondisse, μὴ εἶναι βασιλικὴν ἀτραπὸν ἐπὶ γεωμετρίαν. Deinde subiungit, Euclidem natu quidem esse minorem Platone, maiorem verò Eratosthené & Archimede (hi enim æquales erant) cùm Archimedes Euclidis mentionem faciat. Quòd si quis egregiam Euclidis laudem quam cùm ex alijs scriptionibus accuratissimis, tùm ex hac Geometrica στοιχειώσει cõsequutus est in qua diuinus rerum ordo sapientissimis quibusque hominibus magnæ semper admirationi fuit, is Proclum studiosè legat, quò rei veritatẽ illustriorem reddat grauissimi testis auctoritas. Superest igitur vt finem videamus, quò Euclidis elementa referri, & cuius causa in id studium incumbere oporteat. Et quidem si res quæ tractantur, consyderes: in tota hac tractatione nihil aliud quæri dixeris, quàm vt κοσμικὰ quæ vocantur. σχήματα (fuit enim Euclides professione & instituto Platonicus) Cubus, Icosaedrũ, Octaedrum, Pyramis & Dodecaedrum certa quadã suorum & inter se laterum, & ad sphæræ diametrum ratione eidem sphæræ inscripta cõprehendantur. Huc enim pertinet Epigrammation illud vetus, quod in Geometrica Michaelis

Pselli συνόψει *scriptum legitur.*

Σχήματα πέντε τοῦ πλάτωνος, πυθαγόρας σοφὸς
εὗρε,
πυθαγόρας σοφὸς εὗρε, πλάτων δ' ἀρίδηλ' ἐδί-
δαξεν,
Εὐκλείδης ἐπὶ τοῖσι κλέος περικαλλὲς ἔτευξεν.

Quòd si discentis institutionem spectes, illud certè fuerit propositum, vt huiusmodi elementorum cognitione informatus discentis animus, ad quamlibet non modò Geometriæ, sed & aliarũ Mathematicæ partium tractationem idoneus paratusq; accedat. Nam tametsi institutionem hanc solus sibi Geometra vendicare videtur, & tanquam in possessionem suam venerit, alios excludere posse: inde tamen per multa suo quodam modo iure decerpit Arithmeticus, pleraque Musicus, non pauca detrahit Astrologus, Opticus, Logisticus, Mechanicus, itemq; cæteri; nec vllus est denique artifex præclarus, qui in huius se possessionis societatem cupidè nõ offerat, partemq; sibi concedi postulet. Hinc στοιχείωσις *absolutum operi nomen, &* στοιχειωτὴς *dictus Euclides. Sed quid longius prouehor? Nam quod ad hanc rem attinet, tam copiosè & eruditè scripsit (vt alia complura) eo ipso, quem dixi, loco P. Montaureus, vt nihil desiderio loci reliquerit. Quæ verò ad dicendum nobis erant proposita, hactenus pro ingenij nostri tenuitate omnia mihi perfecisse videor. Nam tametsi & hæc eadem & alia pleraque multo fortè præclariora ab hominibus doctissimis,*

qui

qui tum acumine ingenij, tum admirabili quodam lepore dicendi semper floruerunt grauius, splendidius, uberius tractari posse scio: tamen experiri libuit numquid etiam nobis diuino sit concessum munere, quod rudes in hac philosophiæ parte discipulos adiuuare aut certè excitare queat. Huc accessit quòd ista recens elementorum editio, in qua nihil non parum fuisset studij, aliquid à nobis efflagitare videbatur, quod eius commendationem adaugeret. Cum enim vir doctissimus Io Magnienus Mathematicarum artium in hac Parrhisiorum Academia professor verè regius, nostrum hunc typographum in excudendis Mathematicorum libris diligentissimum, ad hanc Elementorum editionem sæpè & multum esset adhortatus, eiusque impulsu permulta sibi iam comparasset typographus ad hanc rem necessaria, citò interuenit, malum Ioannis Magnieni mors insperata, quæ tam graue inflixit Academiæ vulnus, cui ne post multos quidem annorum circuitus cicatrix obduci vlla posse videatur. Quamobrem amisso instituti huius operis duce, typographus, qui nec sumptus antea factos sibi perire, nec studiosos, quibus id muneris erat pollicitus, sua spe cadere vellet, ad me venit, & impensè rogauit, vt meam propositæ editioni operam & studium nauarem, quod cum denegaret occupatio nostra, iuberet officij ratio, feci equidem rogatus, vt quæ subobscurè vel parum commode in sermonem Latinum è Græco translata

vi-

videbantur, clariore, aptiore & fideliore interpretatione nostra (quod cuiusq; pace dictum volo) lucem acciperent. Id quod in omnibus ferè libris posterioribus tute primo obtutu perspicias. Nam in sex prioribus nō tantum temporis quātum in cæteres ponere nobis: licuit decimi autem interpretatio, qua melior nulla potuit adferri, P. Montaureo solida debetur. Atq; vt ad perspicuitatem facilitatemque nihil tibi deesse queraris, adscripta sunt propositionibus singulis vel lineares figuræ, vel punctorum tanquam vnitatum notulæ, quæ Theonis apodixin illustrēt: illæ quidem magnitudinum, hæ autem numerorum indices. subscriptis etiam ciphrarum, vt vocant, characteribus, qui propositum quemuis numerum exprimant. ob eamq; causam eiusmodi vnitatum notulæ, quæ pro numeri amplitudine maius paginæ spatium occuparent, pauciores sæpius depictæ sunt aut in lineas etiam commutatæ. Nam literarum vt a, b, c characteres non modò numeris & numerorum partibus nominādis sunt accommodati, sed etiam generales esse numerorum, vt magnitudinum affectiones testantur. Adiecta sunt insuper quibusdam locis non pœnitenda Theonis scholia, siue mauis lemmata, quæ quidem longè plura accessissent, si plus otij & temporis vacui nobis fuisset relictū, quod huic studio impartiremus. Hanc igitur operam boni consule, & quæ obuia erunt impressionis vitia, candidè emenda. Vale. Lutetiæ 4. Idus April. 1557.

EVCLIDIS ELEMENTVM PRIMVM.

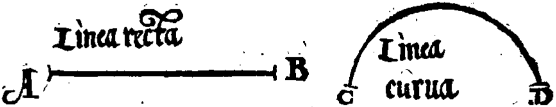
DEFINITIONES.

1

PVnctum eſt, cuius pars nulla eſt. Punctum

2

Linea verò, longitudo latitudinis expers.



3

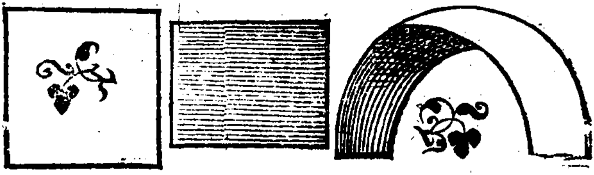
Lineæ autem termini, ſunt puncta.

4

Recta linea eſt, quæ ex æquo ſua interiacet puncta.

5

Superficies eſt, quæ longitudinem latitudinemq́ue tantùm habet.



6 Super=

6

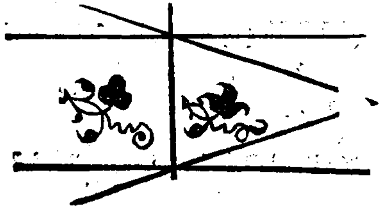
Superficiei extrema,ſunt lineæ.

7

Plana ſuperficies eſt,quæ ex æquo ſuas interiacet lineas.

8

Planus angulus eſt duarũ linearũ in plano ſe mutuò tangentium, & non in directum



iacétium,alterius ad alteram inclinatio.

9.

Cùm autem quæ angulum continẽt lineæ, rectæ fuerint, rectilineus ille angulus appellatur.

10

Cùm verò recta linea ſuper rectam conſiſtens lineã, eos qui ſunt deinceps angulos ęquales,inter ſe fecerit:rectus eſt vterq; æquâqualium angulorũ:& quæ insistit recta linea, perpēdicularis vocatur ei⁹, cui insistit.

11

Obtusus angulus est, qui recto maior est.

12

Acutus verò, qui minor est recto.

13

Terminus est, quod alicuius extremũ est.

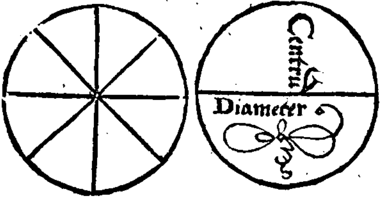
14

Figura est, quæ sub aliquo, vel aliquibus terminis comprehenditur.

15

Circulus est figura plana sub vna linea cō-prehēsa, quę peripheria appellatur: ad quā ab vno puncto eorum, quæ intra figuram sunt

sunt posita, cadentes omnes rectæ lineæ inter se sunt æquales.



16

Hoc verò punctum, centrum circuli appellatur.

17

Diameter autem circuli, est recta quædam linea per cētrum ducta, & ex vtraque parte in circuli peripheriam terminata, quæ circulum bifariam secat.

18

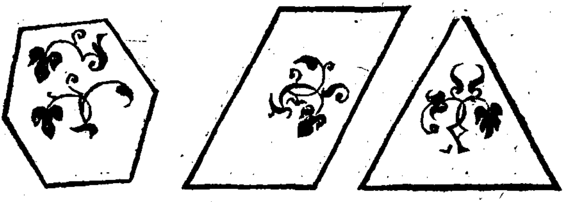
Semicirculus est figura, quæ cōtinetur sub diametro, & sub ea linea, quæ de circuli peripheria aufertur.

19

Segmentum circuli, est figura, quæ sub recta linea & circuli peripheria continetur.

20

Rectilineæ figuræ, ſunt quæ ſub rectis lineis continentur.



21

Trilateræ quidem, quæ ſub tribus.

22

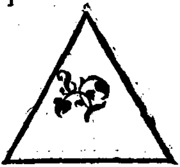
Quadrilateræ, quæ ſub quatuor.

23

Multilateræ verò, quæ ſub pluribus quàm quatuor rectis lineis comprehenduntur.

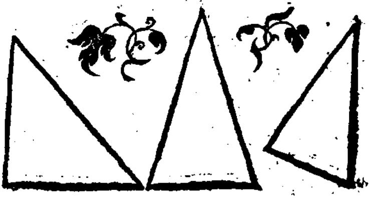
24

Trilaterarũ porrò figurarum, æquilaterum eſt triangulum, quod tria latera habet æqualia.



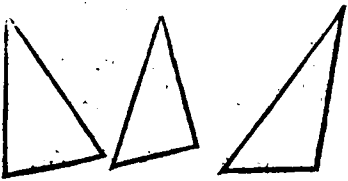
25

Iſoſceles autem eſt quod duo tantum æqualia habet latera.



26

Scalenũ verò, eſt qđ tria in ęqualia habet latera



27

Ad hæc etiam, trilaterarum figurarum, rectangulum quidem triangulum eſt, quod rectum angulum habet.

28

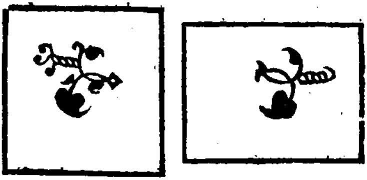
Amblygonium autem, quod obtuſum angulum habet.

29

Oxygonium verò, quod tres habet acutos angulos.

30

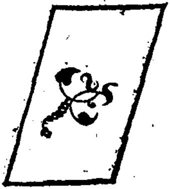
Quadrilaterarum autem figurarum, quadratũ quidẽ eſt qđ & æquilaterũ & rectangulum eſt.



31

Altera parte longior figura eſt, quæ rectágula quidem, æquilatera non eſt.

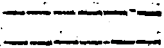
32
Rhóbus autem, qui æquilaterum & rectangulum est.



33
Rhomboides verò, quæ aduersa & latera & angulos habens inter se æqualia, neque æquilatera est, neque rectangula.

34
Preter has autem reliquæ quadrilateræ figuræ, trapezia appellentur.

35
Parallelæ rectæ lineæ sunt quæ, cùm in eodem sint plano, & ex vtraque parte in infinitum producantur, in neutram sibi mutuò incidunt.



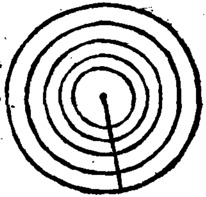
Postulata.

1
Postuletur, vt à quouis puncto in quoduis

punctum, rectam lineã ducere cócedatur.

2

Et rectam lineam terminatam in continuum recta producere.



3

In quouis centro & interuallo circulũ describere.

Communes notiones.

1.

Quæ eidẽ æqualia, & inter se sunt æqualia.

2

Et si æqualibus æqualia adiecta sint, tota sunt æqualia.

3

Et si ab æqualibus æqualia ablata sint, quæ relinquuntur sunt æqualia.

4

Et si inæqualibus æqualia adiecta sint, tota sunt inæqualia.

5

Et si ab inęqualibus ęqualia ablata sint, reliqua sunt inæqualia.

6

Quæ eiusdem duplicia sunt, inter se sunt æqualia.

Et

7

Et quæ eiuſdem ſunt dimidia, inter ſe æqualia ſunt.

8

Et quæ ſibi mutuò congruunt, ea inter ſe ſunt æqualia.

9

Totum eſt ſua parte maius.

10

Item, omnes recti anguli ſunt inter ſe ęquales.

11

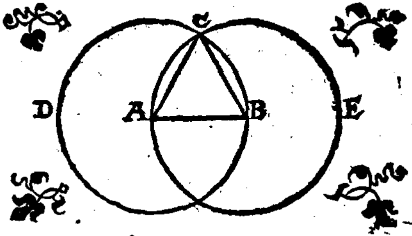
Et ſi in duas rectas lineas altera recta incidens, internos ad eaſdẽque partes angulos duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ ſibi mutuò incident ad eas partes, vbi ſunt anguli duobus rectis minores.

12

Duæ rectæ lineæ ſpatium non comprehédunt.

Problema 1. Propoſitio 1.

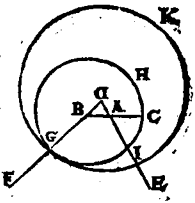
Super data recta linea terminata, triangulũ



æquilaterum conſtituere.

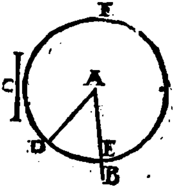
Problęma 2. Propoſitio 2.

Ad datum punctum, datæ rectæ lineæ, ęqualem rectam lineam ponere.



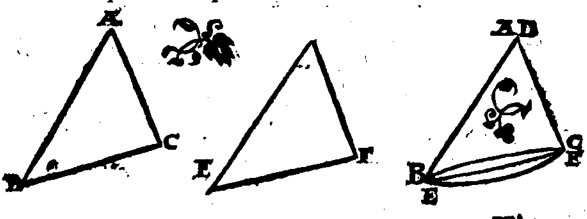
Problèma 3. Propoſitio 3.

Duabus datis rectis lineis inęqualibus, de maiore æqualẽ minori rectam lineã detrahere.



Theorema primum. Propoſitio 4.

Si duo triangula duo latera duobus lateribus æqualia habeant, vtrunque vtriq;, habeant verò & angulum angulo æqualẽ ſub æqualibus rectis lineis contentum: & baſin baſi æqualem habebunt, eritq; triãgulum triãgulo æquale, ac reliqui anguli reliquis angulis æquales erũt, vterque vtrique, ſub quibus æqualia latera ſubtenduntur.



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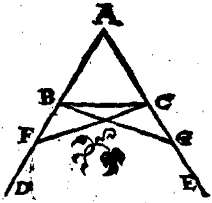
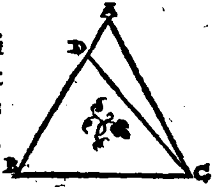
Theorema 2. Propoſitio 5.



Iſoſcelium triangulorũ qui ad baſim ſunt anguli, inter ſe ſunt æquales; & ſi vlterius productæ ſint æquales illæ rectæ lineæ, qui ſub baſi ſunt anguli, inter ſe ęquales erunt.

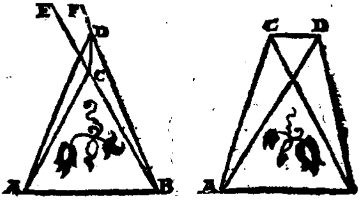
Theorema 3. Propoſitio 6.



Si trianguli duo anguli æquales inter ſe fuerint & ſub ęqualibus angulis ſubtenſa latera æqualia inter ſe erunt.

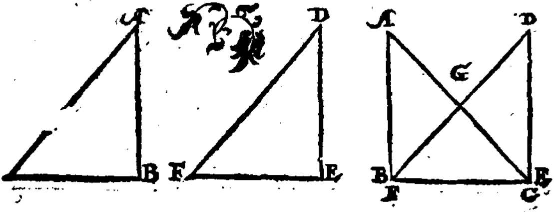
Theorema 4. Propoſitio 7.

Super eadẽ recta linea, duabus eiſdem rectis lineis alię duę rectę lineę ęquales, vtraque vtrique, nõ conſtituentur, ad aliud atque aliud punctũ, ad eaſdẽ partes eoſdẽ que terminos cum duabus initio ductis rectis lineis habentes.

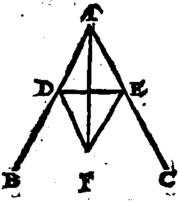


Theorema 5. Propoſitio 8.

Si duo triãgula duo latera habuerint duobus lateribus,vtrunque vtriq;,æqualia:habuerint verò & baſim baſi æqualẽ: angulũ quoque ſub æqualibus rectis lineis cõtentum angulo æqualem habebunt.



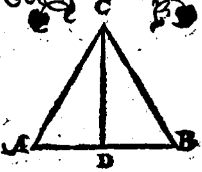
Problema 4. Propoſitio 9.



Datum angulum rectilineum bifariam ſecare.

Problema 5. Propoſitio 10.

Datam rectam lineã finitam bifariam ſecare.



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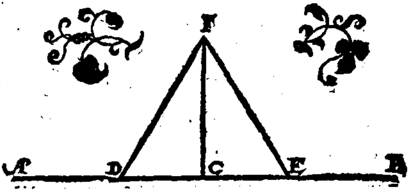
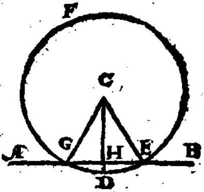
Problema 6. Propositio 11.



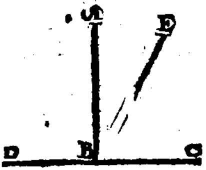
Data recta linea, à puncto in ea dato, rectam lineam ad angulos rectos excitare.

Problema 7. Propositio 12.



Super datam rectã lineã infinitam, à dato puncto quod in ea non est, perpendicularem rectam deducere.

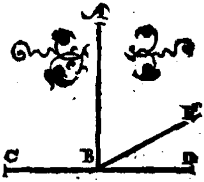
Theorema 6. Propositio 13.



Cùm recta linea super rectã consistens lineã angulos facit, aut duos rectos, aut duobus rectis æquales efficiet.

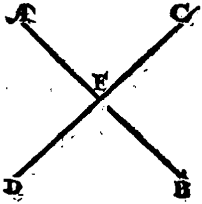
Theorema 7. Propositio 14.

Si ad aliquam rectam lineam, atque ad eius punctú, duę rectæ lineæ nó ad easdem partes ductæ, eos qui sunt deinceps angulos duobus rectis æquales fecerint, in directum erunt inter se ipsæ rectæ lineæ.



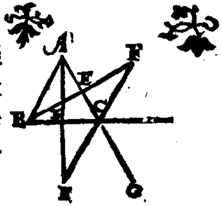
Theorema 8. Propositio 15.

Si duę rectę lineę se mutuò secuerint, angulos qui ad verticem sunt, æquales inter se efficient.



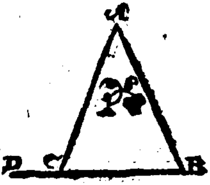
Theorema 9. Propositio 16.

Cuiuscunque trianguli vno latere producto, externus angulus vtroque interno & opposito maior est.



Theorema 10. Propositio 17.

Cuiuscunque trianguli duo anguli duobus rectis sunt minores omnifariam sumpti.



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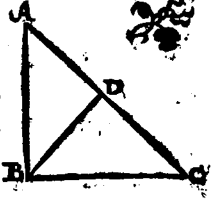
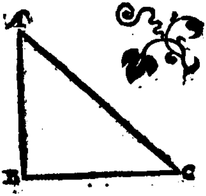
Theorema 11. Propoſitio 18.



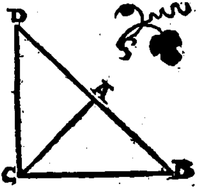
Omnis trianguli maius latus maiorem angulũ ſubtendit.

Theorema 12. Propoſitio 19.



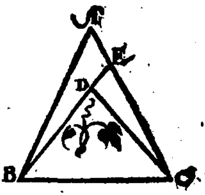
Omnis trianguli maior angulus, maiori lateri ſubtenditur.

Theorema 13. Propoſitio 20.



Omnis triãguli duo latera reliquo ſunt maiora, quomodocũq; aſsũpta.

Theorema 14. Propoſitio 21.

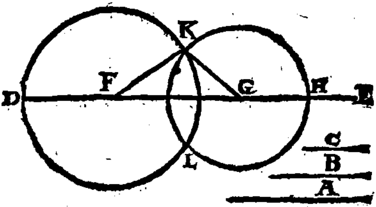


Si ſuper triãguli vno latere, ab extremitatibus duæ rectę lineę, interius cõſtitutæ fuerint, hę cõſtitutæ reliquis triãguli duobus lateribus minores quidem erunt, minorem verò angulum continebunt.

Pro-

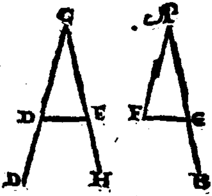
Problema 8. Propoſitio 22.

Ex tribus rectis lineis quæ sũt tribus datis rectis lineis æquales, triangulum conſtituere. Oportet autem duas reliqua eſſe maiores omnifariam ſumptas: quoniã vniuſcuiuſq; trianguli duo latera omnifariam ſumpta reliquo ſunt maiora.



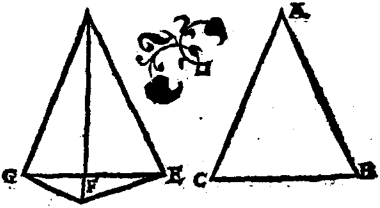
Problema 9. Propoſitio 23.

Ad datam rectam lineã datumq́; in ea punctum dato angulo rectilineo ę qualem angulum rectilineum conſtituere.



Theorema 15. Propoſitio 24.

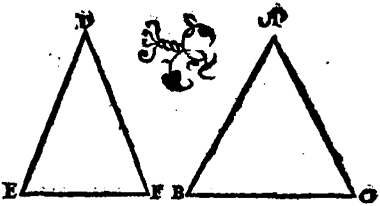
Si duo triãgula duo latera duobus lateribus ę qualia habuerint, vtrũq; vtriq;, angulũ verò angulo



lo maiorem ſub æqualibus rectis lineis cõtentum:& baſin baſi maiorem habebunt.

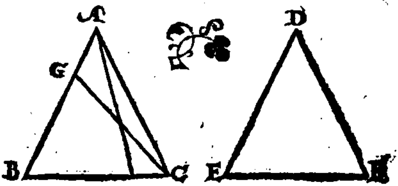
Theorema 16. Propoſitio 25.

Si duo triangula duo latera duobus lateribus æqualia habuerint, vtrunque vtrique, baſin verò baſi maiorẽ: & angulum ſub ęqualib⁹ rectis lineis contentum angulo maiorẽ habebunt.



Theorema 17. Propoſitio 26.

Si duo triangula duos angulos duobus angulis æquales habuerint, vtrũque vtrique, vnumq; latus vni lateri æquale, ſiue quod æqualibus adiacet angulis, ſeu quod vni ęqualium angulorũ ſubtenditur:& reliqua latera reliquis lateribus ęqualia vtrũq; vtriq;, & reliquum angulũ reliquo angulo æqualem habebunt.



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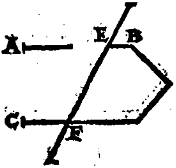
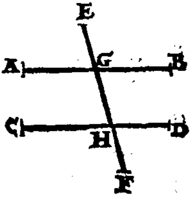
Theorema 18. Propositio 27.



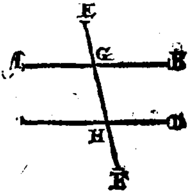
Si in duas rectas lineas recta incidẽs linea alternatim ãgulos ęquales inter se fecerit: parallelæ erunt inter se illæ rectæ lineæ.

Theorema 19. Propositio 28.



Si in duas rectas lineas recta incidens linea externum angulũ interno, & opposito, & ad easdem partes æqualẽ fecerit, aut internos & ad easdẽ partes duobus rectis æquales: parallelæ erunt inter se ipsæ rectæ lineæ.

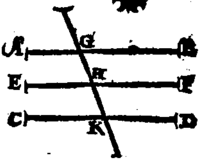
Theorema 20. Propositio 29.



In parallelas rectas lineas recta incidens linea: & alternatim angulos inter se æquales efficit & externum interno & opposito & ad easdem partes æqualem, & internos & ad easdem partes duobus rectis æquales facit.

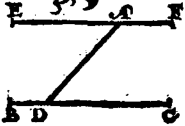
Theore-

Theorema 21. Propoſitio 30.



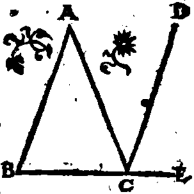
Quæ eidem rectæ lineę, parallelæ, & inter ſe ſunt parallelæ.

Problema 10. Propoſitio 31.



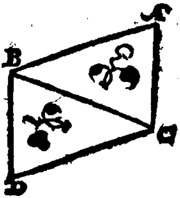
A dato puncto datæ rectæ lineæ parallelam rectam lineam ducere.

Theorema 22. Propoſitio 32.



Cuiuſcunque trianguli vno latere vlterius producto: externus angulꝰ duobus internis & oppoſitis eſt æqualis. Et trianguli tres interni anguli duobus ſunt rectis æquales.

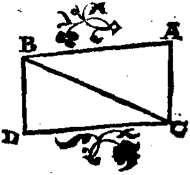
Theorema 23. Proſitio 33.



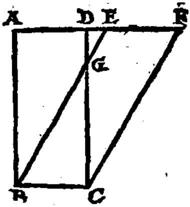
Rectæ lineę quę ęquales & parallelas lineas ad partes eaſdem coniungunt, & ipſæ æquales & parallelæ ſunt.

Theore-

Theorema 24. Propoſitio 34.



Parallelogrammorũ ſpatiorum æqualia ſunt inter ſe, quæ ex aduerſo & latera & anguli: atque e illa bifariã ſecat diameter.

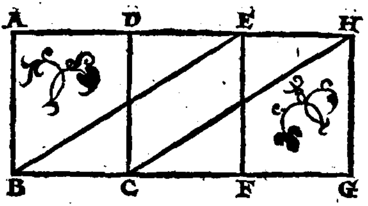


Theorema 25. Propoſitio 35.

Parallelogramma ſuper eadem baſi, & in eiſdem parallelis cõſtituta, inter ſe ſunt æqualia.

Theorema 26. Propoſitio 36.

Parallelogrãma ſuper æqualibus baſib⁹ & in eiſdẽ parallelis cõſtituta inter ſe ſunt æqualia.



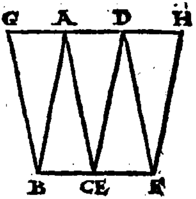
Theorema 27. Propoſitio 37.

Triãgula ſuper eadẽ baſi cõſtituta, & in eiſdẽ parallelis, inter ſe ſunt equalia.

Theore-

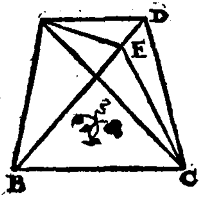
Theorema 28. Propositio 38.

Triangula ſuper æqualibus baſibus cõſtituta & in eiſdem parallelis, inter ſe ſunt æqualia.



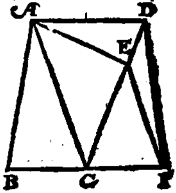
Theorema 29. Propoſitio 39.

Triãgula æqualia ſuper eadem baſi, & ad eaſdem partes cõſtituta: & in eiſdem ſunt parallelis.



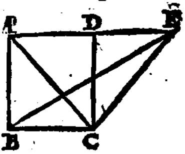
Theorema 30. Propoſitio 40.

Triãgula æqualia ſuper æqualibus baſibus, & ad eaſdem partes cõſtituta, & in eiſdẽ ſunt parallelis.

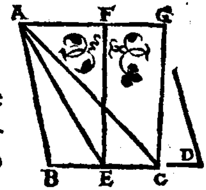


Theorema 31. Propoſitio 41.

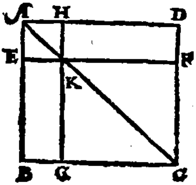
Si parallelogrammum cum triangulo eandem baſin habuerit, non eiſdemq; fuerit parallelis, duplum erit parallelogrammum ipſius triãguli.



Problema 11. Propoſitio 42.



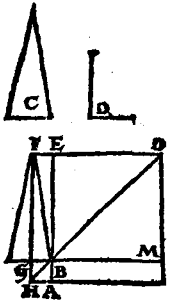
Dato triangulo æquale parallelogrammũ conſtituere in dato angulo rectilineo.



Theorema 32. Propoſitio 43.

In omni parallelogrammo, complementa corũ quę circa diametrũ ſunt parallelogrammorum, inter ſe ſunt æqualia.

Problema 12. Propoſitio 44.

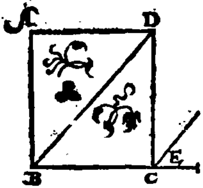


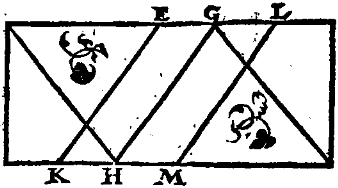
Ad datam rectam lineã, dato triangulo æquale parallelogrammum applicare in dato angulo rectilineo.

Problema 13. Propopotio 45.

Dato rectilineo æquale parallelogrammũ conſti-

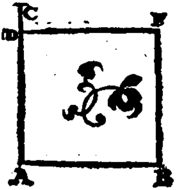
conſtituere in dato angulo rectilineo.





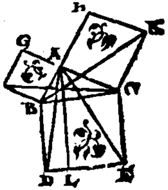
Problema 14. Propoſitio 46.

A data recta linea quadratum deſcribere.



Theorema 33. Propoſitio 47.

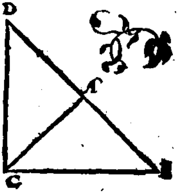
In rectangulis triãgulis, quadratum quod à latere rectum angulũ ſubtendẽte deſcribitur, æquale eſt eis, quę à lateribus rectum angulum continentibus deſcribũtur, quadratis.



Theorema 34. Propoſitio 48.

Si quadratum quod ab vno lateru

guli deſcribitur, æquale ſit eis quæ à reliquis triãguli lateribus deſcribũtur, quadratis: angulus comprehenſus ſub reliquis duobus trianguli lateribus, rectus eſt.



FINIS ELEMENTI I.

EVCLI-

EVCLIDIS ELEMENTVM SECVNDVM.

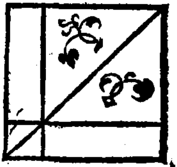
DEFINITIONES.

I

OMne parallelogrammum rectangulum contineri dicitur sub rectis duabus lineis, quæ rectum comprehendunt angulum.

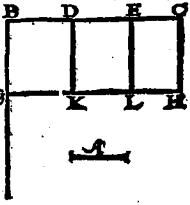
2

In omni parallelogrammo spatio, vnumquodlibet eorũ, quæ circa diametrum illius sunt parallelogrammorum, cũ duobus complementis, Gnomo vocetur.

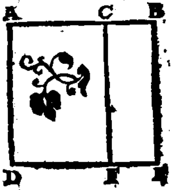


Theorema I. Propositio I.

Si fuerint duæ rectæ lineæ, seceturq; ipsarũ altera in quotcunque segmenta: rectangulum comprehensum sub illis duab9 rectis lineis, æquale est eis rectangulis, quę sub insecta & quolibet segmentorum comprehenduntur.



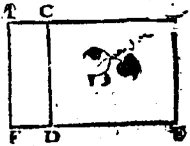
Theorema 2. Propoſitio 2.



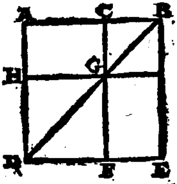
Si recta linea ſecta ſit vtcunq;, rectãgula quæ ſub tota & quolibet ſegmentorum cõprehenduntur æqualia ſunt ei, quod à tota fit, quadrato.

Theorema 3. Propoſitio 3.

Si recta linea ſecta ſit vtcunque, rectangulum ſub tota & vno ſegmẽtorum comprehenſum, ęquale eſt & illi quod ſub ſegmentis cõprehenditur rectãgulo, & illi, quod à prædicto ſegmento deſcribitur, quadrato.



Theorema 4. Propoſitio 4.

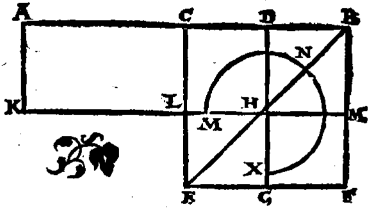


Si recta linea ſecta ſit vtcunque: quadratũ quod à tota deſcribitur, ęquale eſt & illis quæ à ſegmẽtis deſcribũtur quadratis, & ei quod bis ſub ſegmentis comprehẽditur rectangulo.

Theorema 5. Propoſitio 5.

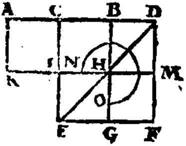
Si recta linea ſecetur in æqualia & non æqualia: rectangulum ſub inæqualibus ſegmentis

mentis totius comprehẽſũ, vnà cum quadrato quod ab intermedia ſectionum, æquale eſt ei quod à dimidia deſcribitur, quadrato.



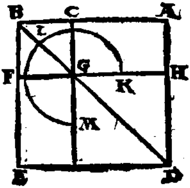
Theorema 6. Propoſitio 6.

Si recta linea bifariam ſecetur, & illi recta quædam linea in rectum adijciatur, rectãgulum comprehẽſum ſub tota cum adiecta & adiecta ſimul cum quadrato à dimidia, æquale eſt quadrato à linea, quæ tũ ex dimidia, tum ex adiecta componitur, tanquam ab vna deſcripto.



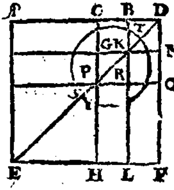
Theorema 7. Propoſitio 7.

Si recta linea ſecetur vtcunque: quod à tota, quodq́ue ab vno ſegmentorum, vtraq; ſimul quadrata, æqualia ſunt & illi quod bis ſub tota & dicto ſegmento comprehenditur, rectãgulo, & illi ꝙ à reliquo ſegmento fit, quadrato.



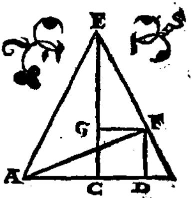
Theorema 8. Propositio 8.

Si recta linea secetur vtcunque rectangulum quater comprehésum sub tota & vno segmentorum, cũ eo quod à reliquo segmento fit, quadrato, æquale est ei quod à tota & dicto segmento, tanquam ab vna linea describitur, quadrato.



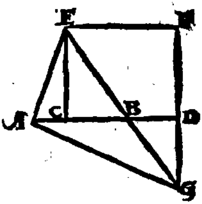
Theorema 9. Propositio 9.

Si recta linea secetur in æqualia & non æqualia: quadrata quæ ab inequalibus totius segmẽtis fiunt, duplicia sunt & eius quod à dimidia, & eius quod ab intermedia sectionum fit, quadratorum.



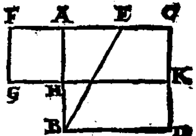
Theorema 10. Propositio 10.

Si recta linea secetur bifariam, adijciatur autem ei in rectum quępiam recta linea: quod à tota cũ adiuncta, & quod ab adiuncta, vtraq; simul quadrata, duplicia sunt, & eius quod à dimidia, & eius quod à cõposita ex dimi-



dimidia & adiuncta, tãquam ab vna descriptum sit, quadratorum.

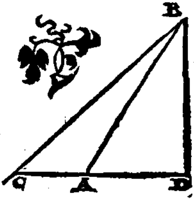
Problema I. Propositio II.



Datam rectam lineã secare, vt comprehensum sub tota & altero segmẽtorum rectangulum, æquale sit ei quod à reliquo segmento fit, quadrato.

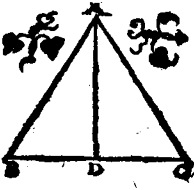
Theorema II. Propositio 12.

In amblygonijs triangulis, quadratũ quod fit à latere angulum obtusum subtendẽte, maius est quadratis, quæ fiunt à lateribus obtusum angulum comprehendentibus, pro quantitate rectanguli bis comprehẽsi & ab vno laterũ quæ sunt circa obtusum angulũ, in quod cùm protractum fuerit, cadit perpendicularis, & ab assumpta exterius linea sub perpendiculari prope angulũ obtusum.

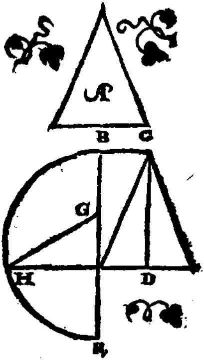


Theorema 12. Propoſitio 13.

In oxygonijs triangulis, quadratũ à latere angulum acutum ſubtendente, minus eſt quadratis quæ fiunt à lateribus acutũ angulum cõprehendentibus, pro quantitate rectanguli bis comprehenſi, & ab vno late rum, quæ ſunt circa acutum angulum, in quod perpẽdicularis cadit, & ab aſſumpta interius linea ſub perpendiculari prope acutũ angulum.



Problema 2. Propoſitio 14.



Dato rectilineo æquale quadratũ conſtituere.

ELEMENTI II FINIS.

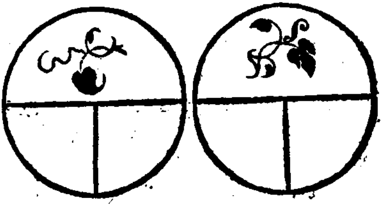
EVCLI-

EVCLIDIS ELEMENTVM TERTIVM.

DEFINITIONES.

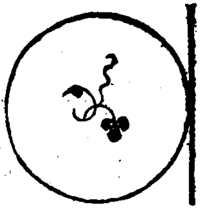
1

Aequales circuli ſunt, quorũ diametri sũt ęquales vel quorũ quæ ex centris rectæ lineę ſunt æquales.



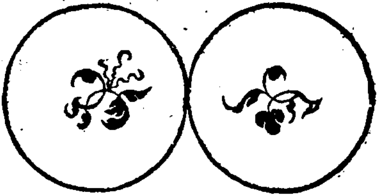
2

Recta linea circulũ tangere dicitur, quæ cùm circulum tangat, ſi producatur, circulum non ſecat.



3

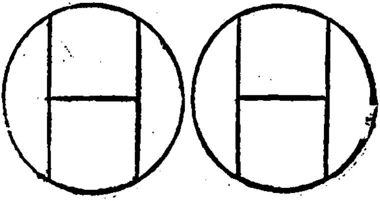
Circuli ſeſe mutuò tãgere dicũtur: qui ſeſe mutuò



tangentes, ſeſe mutuò non ſecant.

4

In circulo æqualiter diſtare à centro rectæ lineæ dicuntur, cùm perpendiculares, quæ à cẽtro in ipſas ducuntur, ſunt æquales. Lõgius autem abeſſe illa dicitur in quã maior ꝑ pẽdicularis cadit.



5

Segmentũ circuli eſt, figura quæ ſub recta linea & circuli peripheria comprehenditur.

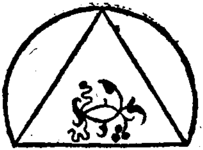
6

Segmenti autẽ angulus eſt, qui ſub recta linea

nea & circuli peripheria compreheditur.

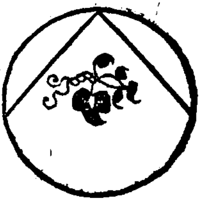
7

In ſegmento autem angulus eſt, cùm in ſegmenti peripheria ſumptũ fuerit quodpiam pũctum, & ab illo in terminos rectæ ei9 lineę, quę ſegmenti baſis eſt, adiunctę fuerint rectę lineę: is, inquam, angulus ab adiunctis illis lineis comprehenſus.

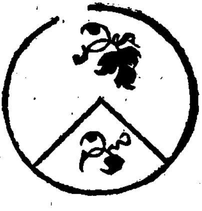


8

Cùm verò comprehendentes angulum rectę lineæ aliquã aſſumunt peripheriam, illi angulus inſiſtere dicitur.

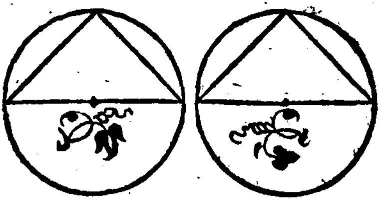


Sector autem circuli eſt cùm ad ipſius circuli cẽtrum conſtitutus fuerit angulus, cõprehenſa nimirum figura, & à rectis lineis angulũ continentibus, & à peripheria ab illis aſſumpta.



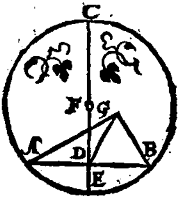
10

Similia circuli ſegmenta ſunt, quę angulos capiunt capiũt ęquales aut in q. b⁹ anguli inter ſe ſunt ęquales



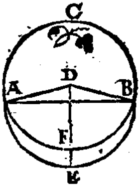
Problema 1. Propoſitio 1.

Dati circuli cẽtrum reperire.



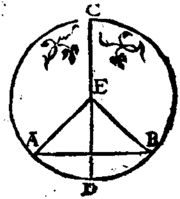
Theorema 1. Propoſitio 2.

Si in circuli peripheria duo quælibet pũcta accepta fuerint, recta linea quæ ad ipſa puncta adiũgitur, intra circulum cadet.



Theorema 2. Propoſitio 3.

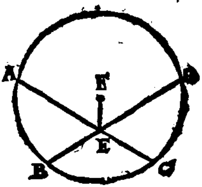
Si in circulo recta quædam linea per centrum extenſa quandam non per centrum extenſam bifariam ſecet: & ad angulos rectos ipſam ſecabit. Et ſi ad angulos rectos eam ſecet, bifariam quoque eam ſecabit.



Theore-

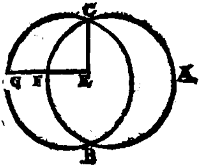
Theorema 3. Propo-
ſitio 4.

Sì in circulo duę rectę li-neæ ſeſe mutuò ſecent non per centrum extẽſæ ſeſe mutuò biſariam nõ ſecabunt.



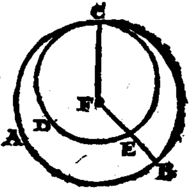
Theorema 4. Pro-
poſitio 5.

Sì duo circuli ſeſe mutuò ſecent, non erit illorum idem centrum.



Theorema 5. Pro-
poſitio 6.

Si duo circuli ſeſe mutuò interiùs tangant, eorum non erit idem centrum.



Theorema 6. Propoſitio 7.

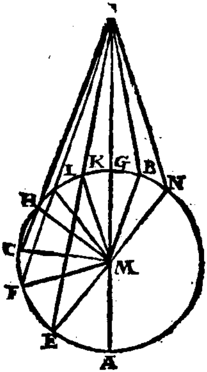
Si in diametro circuli quodpiam ſumatur punctum, quod circuli cẽtrum non ſit, ab eoque pũcto in circulũ quædam rectæ lineæ cadant: maxima quidem erit ea in qua centrũ, minima verò reliqua: aliarum verò propinquior illi quæ per centrũ du-

citur

citur, remotiore semper maior est. Duę autem solùm rectæ lineæ æquales ab eodem puncto in circulum cadũt ad vtrasque partes minimæ.

Theorema 7. Propositio 8.

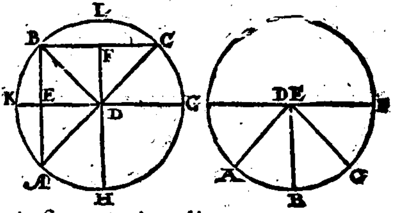
Si extra circulum sumatur punctũ quodpiam, ab eoque puncto ad circulum deducantur rectæ quædam lineæ, quarum vna quidem per centrum procedatur, reliquæ verò vt libet: in cauam peripheriam cadẽtium rectarum linearum maxima quidem est illa, quæ per centrum ducitur: aliarum autem propinquior ei, quę per centrum transit, remotiore semper maior est: in cõuexam verò peripheriã cadentium rectarũ linearum minima quidẽ est illa, quæ inter punctum & diametrum interponitur: aliarum autem, ea quæ propinquior est minimæ, remotiore semper minor est. Duæ autem tantùm rectæ lineæ æquales ab eo puncto in ipsum circulum cadũt, ad vtrasque partes minimæ.



Theore-

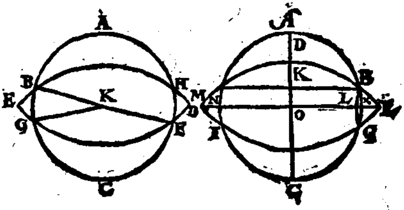
Theorema 8. Propositio 9.

Si in circulo acceptum fuerit pũctum aliquod, & ab eo puncto ad circulum cadant plures quã duę rectę lineæ, æquales, acceptũ punctũ centrum ipsius est circuli.



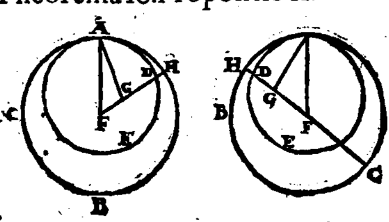
Theorema 9. Propositio 10.

Circulus circulum in pluribus quàm duob⁹ pũctis non secat.



Theorema 10. Propositio 11.

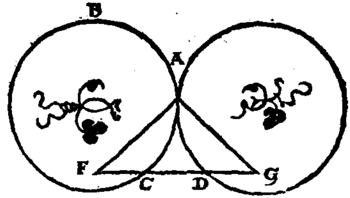
Si duo circuli sese intus cõtingãt, atque accepta



fuerint eorum centra, ad eorum cẽtra adiuncta recta linea & producta in cõtactum circulorum cadet.

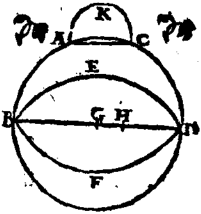
Theorema 11. Propositio 12.

Si duo circuli sese exterius cõtingãt, linea recta q̃ ad cẽtra eorũ adiũgitur, ꝑ cõtactũ illũ trãsibit.



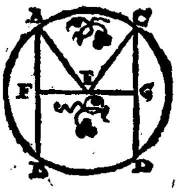
Theorema 12. Propositio 13.

Circulus circulum non tangit in pluribus punctis, quã vno, siue intus siue extra tangat.



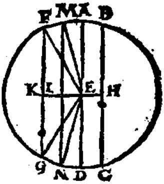
Theorema 13. Propositio 14.

In circulo æquales rectæ lineæ æqualiter distant à centro. Et quæ ęqualiter distant à centro, æquales sunt inter se.



Theore-

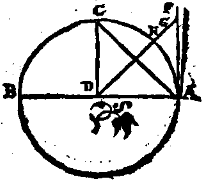
Theorema 14. Propositio 15.



In circulo maxima quidem linea eſt diameter: aliarum autem propinquior centro, remotiore ſemper maior.

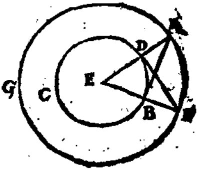
Theorema 15. Propoſitio 16.

Quæ ab extremitate diametri cuiuſq; circuli ad angulos rectos ducitur, extra ipſũ circulum cadet, & in locum inter ipſam rectam lineam & peripheriam comprehenſum, altera recta linea nõ cadet Et ſemicirculi quidem angulus quouis angulo acuto rectilineo maior eſt, reliquus autem minor;

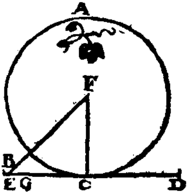


Problema 2. Propoſitio 17.

A dato puncto rectam lineam ducere, quæ datum tangat circulum.

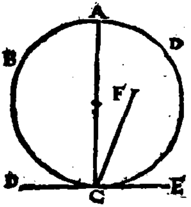


Theorema 16. Propositio 18.



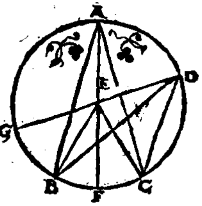
Si circulum tangat recta quæpiam linea, à centro autem ad cõtactum adiungatur recta quædam linea: quæ adiũcta fuerit ad ipsam contingentem perpendicularis erit.

Theorema 17. Propositio 19.



Si circulum tetigerit recta quæpiam linea, à contactu autem recta linea ad angulos rectos ipsi tãgenti excitetur, in excitata erit centrum circuli.

Theorema 18. Propositio 20.



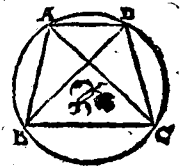
In circulo angulus ad cẽtrum duplex est anguli ad peripheriam, cũ fuerit eadẽ peripheria basis angulorum.

Theorema 19. Propositio 21.

In circulo, qui in eodem segmẽto sunt anguli, sunt inter se æquales.

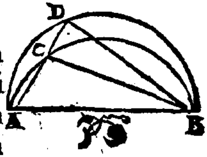
Theore-

Theorema 20. Propositio 22.



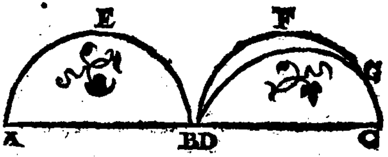
Quadrilaterorum in circulis descriptorũ anguli qui ex aduerso, duob⁹ rectis sunt æquales.

Theorema 21. Propositio 23.



Super eadem recta linea duo segmenta circulorũ similia & inæqualia non constituẽtur ad easdem partes.

Theorema 22. Propositio 24.



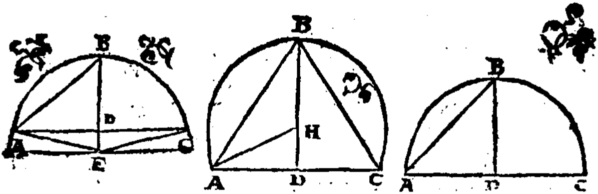
Super ęqualib⁹ rectis lineis similia circulorum segmenta sunt inter se æqualia.

Problema 3. Propositio 25.

Circuli segmento dato, describere circulũ

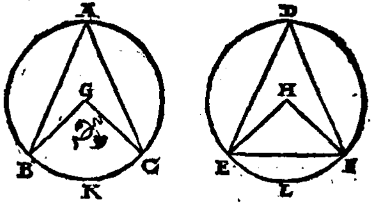
 cuius

cuius eſt ſegmentum.



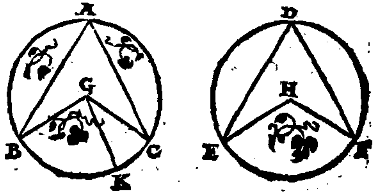
Theorema 23. Propoſitio 26.

In æqualibus circulis, æquales anguli ęqualib. peripherijs inſiſtũt ſiue ad cẽtra, ſiue ad peripherias conſtituti inſiſtant.



Theorema 24. Propoſitio 27.

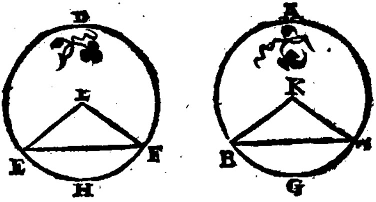
In æqualibus circulis, anguli qui æqualib9 peripherijs inſiſtũt ſunt inter ſe ęquales ſiue ad cẽtra, ſiue ad peripherias cõſtituti inſiſtãt.



Theore-

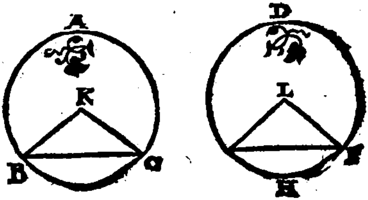
Theorema 25. Propositio 28.

In æqualibus circulis æquales rectæ lineæ ęquales peripherias auferũt maiorẽ quidẽ maiori, minorem autem minori.



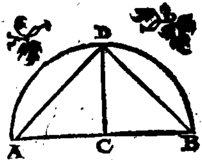
Theorema 26. Propositio 29.

In ęqualib9 circulis, æquales peripherias æquales rectæ lineæ subtendunt.



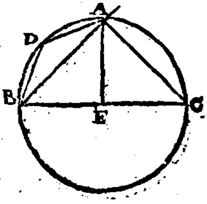
Problema 4. Propositio 30.

Datam peripheriam bifariam secare.



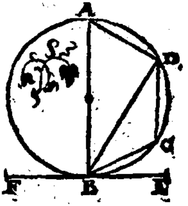
Theorema 27. Propositio 31.

In circulo angulus qui in semicirculo, rectus eſt:qui autem in maiore ſegmēto,minor recto : qui verò in minore ſegmēto, maior eſt recto.Et inſuper angulus maioris ſegmēti, recto quidē maior eſt, minoris autē ſegmēti angulus,minor eſt recto.



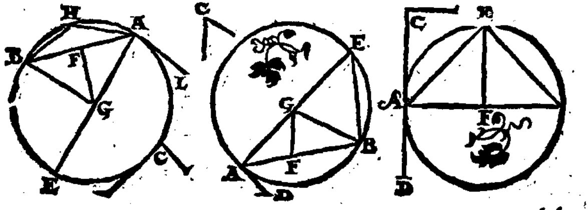
Theorema 28.Propoſitio 32.

Si circulum tetigerit aliqua recta linea, à contactu autē producatur quædam recta linea circulum ſecans: anguli quos ad contingentem facit,ęquales ſunt ijs qui in alternis circuli ſegmētis conſiſtunt, angulis.



Problema 5. Propoſitio 33.

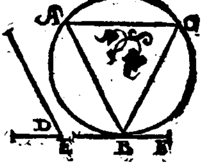
Super data recta linea deſcribere ſegmentum circuli quod capiat angulum æqualē dato angulo rectilineo.



Proble-

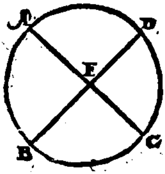
Problema 6. Propositio 34.

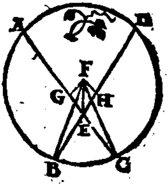
A dato circulo segmentum abscindere capiens angulum æqualem dato angulo rectilineo.



Theorema 29. Propositio 35.

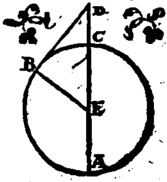
Si in circulo duæ rectæ lineæ sese mutuò secuerint, rectangulum cõprehensum sub segmẽtis vnius, æquale est ei, quod sub segmẽtis alterius comprehenditur, rectangulo.

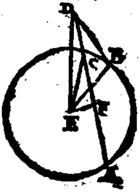




Theorema 30. Propositio 36.

Si extra circulũ sumatur pũctũ aliquod, ab eoque in circulũ cadant duę rectę lineę, quarum altera quidem circulum secet, altera

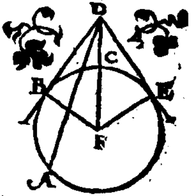




verò tangat: quod ſub tota ſecante & exterius inter punctum & cõuexam peripheriam aſſumpta comprehenditur rectangulum, æquale erit ei, quod à tangente deſcribitur, quadrato.

Theorema 31. Propoſitio 37.

Si extra circulũ ſumatur punctũ aliquod, ab eoq; puncto in circulum cadant duæ rectæ lineæ, quarum altera circulum ſecet, altera in eum incidat, ſit autẽ quod ſub tota ſecante & exterius inter punctũ & cõuexam peripheriam aſſumpta, comprehenditur rectãgulum, æquale ei, quod ab incidẽte deſcribitur quadrato: incidens ipſa circulum tanget.



ELEMENTI III FINIS.

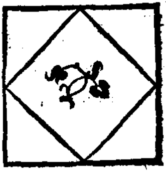
EVCLI-

EVCLIDIS ELEMENTVM QVARTVM.

DEFINITIONES.

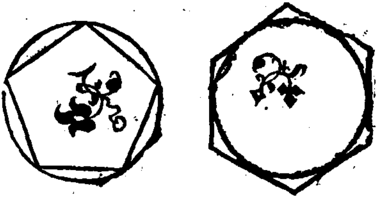
1

Figura rectilinea in figura rectilinea inscribi dicitur, cũ singuli eius figuræ quæ inscribitur, anguli singula latera eius, in qua inscribitur, tangunt.



2

Similiter & figura circum figurã describi dicitur, quum singula eius quæ circũscribitur, latera singulos eius figuræ angulos tetigerint, circum quam illa describitur.



3

Figura rectilinea in circulo inscribi dicitur, quũ singuli eius figuræ q̃ inscribitur, angu-

anguli tetigerint circuli peripheriam.

4

Figura verò rectilinea circa circulũ describi dicitur, quũ singula latera eius, quæ circum scribitur, circuli peripheriã tangunt.

5

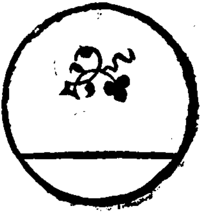
Similiter & circulus in figura rectilinea inscribi dicitur, quum circuli peripheria singula latera tãgit eiꝰ figuræ, cui inscribitur.

6

Circulus autem circum figurã describi dicitur, quũ circuli peripheria singulos tãgit eius figuræ, quam circunscribit, angulos.

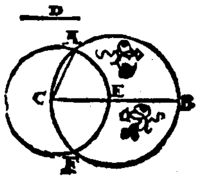
7

Recta linea in circulo accommodari seu coaptari dicitur, quum eius extrema in circuli peripheria fuerint.

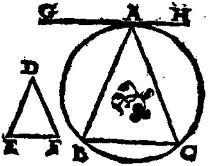


Problema 1. Propositio 1.

In dato circulo, rectam lineam accommodare ç-qualem datæ rectæ lineę quæ circuli diametro nõ sit maior.

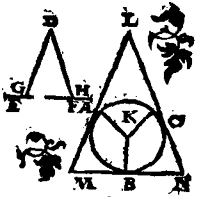


Problema 2. Propoſitio 2.



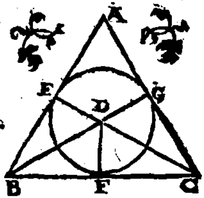
In dato circulo, triangulũ deſcribere dato triangulo æquiangulum.

Problema 3. Propoſitio 3.



Circa datum circulũ triangulũ, deſcribere dato triangulo æquiangulũ.

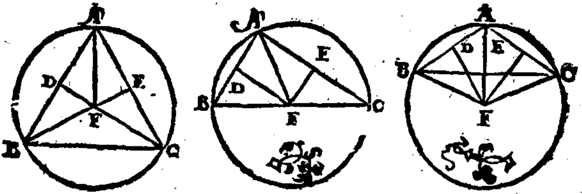
Problema 4. Propoſitio 4.



In dato triangulo circulum inſcribere.

Problema 5. Propoſitio 5.

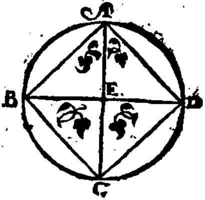
Circa datum triangulum, circulum deſcribere.



Pro-

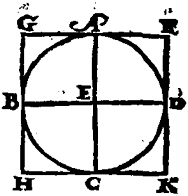
Problema 6. Propo-
ſitio 6.

In dato circulo quadratum deſcribere.



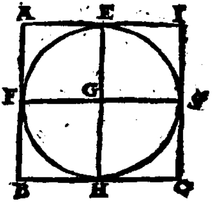
Problema 7. Propo-
ſitio 7.

Circa datum circulum, quadratum deſcribere.



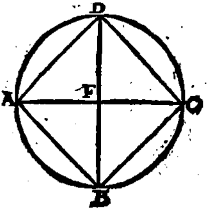
Problema 8. Propo-
ſitio 8.

In dato quadrato circulum inſcribere.



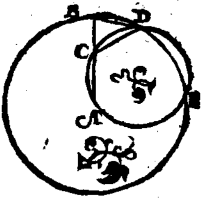
Problema 9. Propo-
ſitio 9.

Circa datum quadratū, circulum deſcribere.



Proble-

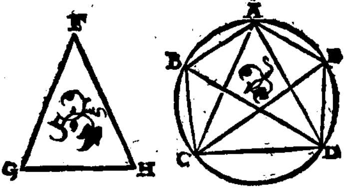
Problema 10. Propositio 10.



Isosceles triangulum cóstituere, quod habeat vtrunque eorum, qui ad basin sunt, angulorum, duplum reliqui.

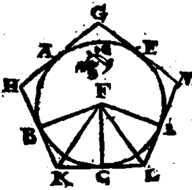
Theorema 11. Propositio 11.

In dato circulo, pentagonũ æquilaterũ & æquiágulum inscribere.



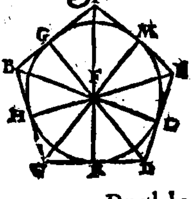
Problema 12. Propositio 12.

Circa datum circulum, pentagonum æquilaterum & æquiangulũ describere.



Problema 13. Propositio 13.

In dato pétagono æquilatero & æquiágulo, circulum inscribere.



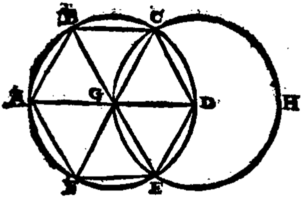
Proble-

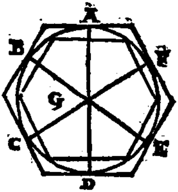
Problema 14. Propoſitio 14.

Circa datũ pentagonũ, æquilaterum & æquiangulum, circulum deſcribere.

Problema 15. Propoſitio 15.

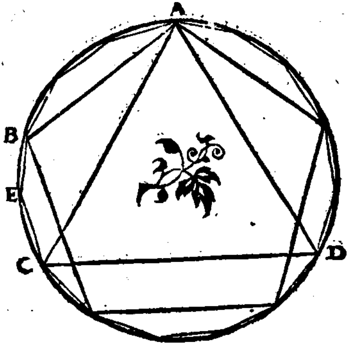
In dato circulo hexagonum & æquilaterũ & æquiangulum inſcribere.





Propoſitio 16. Theorema 16.

In dato circulo quintidecagonum & æquilaterũ & ęquiangulum deſcribere.



Elementi quarti finis.

EVCLIDIS ELEMENTVM QVINTVM.

DEFINITIONES.

1

PARS est magnitudo magnitudinis minor maioris, quum minor metitur maiorem.

2

Multiplex autem est maior minoris, cùm minor metitur maiorem.

3

Ratio, est duarum magnitudinum eiusdẽ generis mutua quædam secundum quantitatem habitudo.

4

Proportio verò, est rationum similitudo.

5

Rationẽ habere inter se magnitudinis dicuntur, quæ possunt multiplicatæ sese mutuò superare.

6

In eadem ratione magnitudines dicũtur esse, prima ad secundam, & tertia ad quartã: cùm primæ & tertiæ æquè multiplicia, à secundæ & quartæ æquè multiplicibus,

 qualis-

qualiſcunq; ſit hæc multiplicatio, vtrunq; ab vtroque: vel vnà deficiunt, vel vnà ęqualia ſunt, vel vnà excedũt, ſi ea ſumãtur quæ inter ſe reſpondent.

7

Eandem autem habẽtes rationem magnitudines, proportionales vocentur.

8

Cũm verò æquè multiplicium, multiplex primę magnitudines exceſſerit multiplicẽ ſecũdæ, at multiplex tertiæ non exceſſerit multiplicem quartæ: tunc prima ad ſecundam, maiorem rationem habere dicetur, quàm tertia ad quartam.

9

Proportio autẽ in tribus terminis paucisſimis conſiſtit.

10

Cùm autem tres magnitudines proportionales fuerint, prima ad tertiam, duplicatã rationẽ habere dicitur eius, quam habet ad ſecundam. At cùm quatuor magnitudines proportionales fuerint, prima ad quartã, triplicatam rationem habere dicitur eius quam habet ad ſecundam: & ſemper deinceps vno amplius, quandiu proportio extiterit.

11

Homologę, ſeu ſimiles ratione magnitudines dicuntur, antecedentes quidẽ antecedentidentibus,consequētes verò cōsequētibus.

12

Alterna ratio, est sumptio antecedētis cōparati ad antecedentem,& cōsequentis ad consequentem.

13

Inuersa ratio,est sumptio cōsequentis,ceu antecedentis,ad antecedenté velut ad cōsequentem.

14

Compositio rationis,est sumptio antecedentis cū consequente ceu vnius,ad ipsum consequentem.

15

Diuisio rationis,est sumptio excessus quo consequentem superat antecedens ad ipsum consequentem.

16

Conuersio rationis,est sumptio antecedētis ad excessum,quo superat antecedés ipsum consequentem.

17

Ex æqualitate ratio est,si plures duab⁹ sint magnitudines, & his alię multitudine pares quæ binæ sumantur, & in eadem ratione: quū vt in primis magnitudinibus prima ad vltimam,sic & in secundis magnitudinibus prima ad vltimā sese habuerit,vel aliter,sumptio extremorū per subductionem mediorum.

18

Ordinata proportio est, cùm fuerit quem admodum antecedens ad consequentem, ita antecedens ad consequentem : fuerit etiam vt consequẽs ad aliud quidpiam, ità consequens ad aliud quidpiam.

19

Perturbata autẽ proportio est, tribus positis magnitudinibus, & alijs quæ sint his multitudine pares, cùm vt in primis quidẽ magnitudinibus se habet antecedẽs ad cõsequentem, ita in secũdis magnitudinibus antecedens ad consequentem: vt autem in primis magnitudinibus consequens ad aliud quidpiam, sic in secundis magnitudinibus aliud quidpiam ad antecedentem.

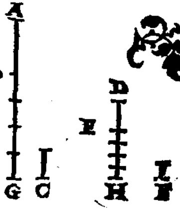
Theorema 1. Propositio 1.

Si sint quotcũque magnitudines quotcunque magnitudinum æqualium numero singulę singularum æquè multiplices, quàm multiplex est vnius vna magnitudo, tam multiplices erunt, & omnes omnium.

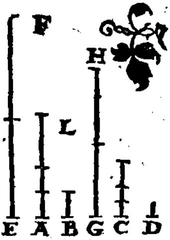
Theorema 2. Propositio 2.

Si prima secundæ æquè fuerit multiplex, atque

atque tertia quartę, fuc-
rit autē & quinta secūdę
æquè multiplex, atque
sexta quartæ: erit & cō-
posita prima cū quinta,
secūdæ ęquè multiplex
atque tertia cum sexta,
quartæ.



Theorema 3. Propositio 3.



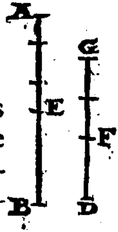
Si sit prima secūdæ æquè multiplex atque tertia quartæ, sumātur autē æquè multiplices primę & tertiæ erit & ex æquo sumptarum vtraque vtriusque ęquè multiplex, altera quidem secundæ, altera autē quartæ.

Theorema 4. Propositio 4.

Si prima ad secundam, eandē habuerit rationem, & tertia ad quartam: etiam æque multiplicespri mę & tertiæ, ad æquè multiplices secundæ & quartæ iuxta quāuis multi plicationē, ea-

dem habebunt rationem, si prout inter se respon-

reſpondent, ita ſumptæ fuerint.



Theorema 5. Propo-
ſitio 5.

Si magnitudo magnitudinis æquè fuerit multiplex, atque ablata ablatæ: etiam reliqua reliquæ ita multiplex erit, vt tota totius.

Theorema 6. Propo-
ſitio 6.

Si duæ magnitudines, duarum magnitudinum ſint æquè multiplices, & detractę quædam ſint earundem æquè multiplices: & reliquæ eiſdem aut ęquales ſunt, aut æquè ipſarum multiplices.

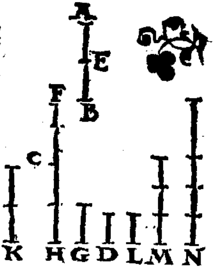
Theorema 7. Propo-
ſitio 7.

Aequales ad eandem, eadem habent rationem: & eandem ad æquales.

Theorema 8. Propo-
ſitio 8.

Inæqualium magnitudinũ maior, ad eandem

dem maiorem rationē habet, quàm minor: & eadem ad minorem: maiorem rationem habet, quàm ad maiorem.



Theorema 9. Propoſitio 9.

Quæ ad eandem, eandem habēt rationem, æquales sūt inter ſe: & ad quas eadem, eandem habet rationem, eæ quoque ſunt inter ſe æquales.

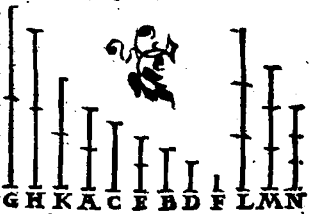
Theorema 10. Propoſitio 10.

Ad eandem magnitudinē, rationem habentium, quæ maiorem rationē habet, illa maior eſt, ad quam autem eadem maiorem rationem habet, illa minor eſt.

Theorema 11. Propoſitio 11.

Quæ eidem ſunt eædem rationes, & inter ſe ſunt eædem.

Theorema 12. Propoſitio 12.



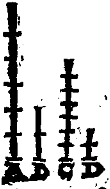
Si ſint magnitudines quotcunque proportionales, quemadmodũ ſe habuerit vna antecedentium ad vnã conſequentiũ, ita ſe habebunt omnes antecedentes ad omnes conſequentes.

Theorema 13. Propoſitio 13.

Si prima ad ſecundam, eandẽ habuerit rationem; quam tertia ad quartã, tertia verò ad quartam, maiorem rationem habuerit, quàm quinta ad ſextam. prima quoque ad ſecũdam maiorem rationem habebit, quàm quinta ad ſextam.

Theorema 14. Propoſitio 14.

Si prima ad ſecundam eandem habuerit rationẽ, quam tertia ad quartàm, prima verò quàm tertia maior fuerit: erit & ſecunda maior quàm quarta. Quòd ſi prima fuerit æqualis tertiæ, erit

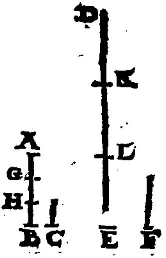


&

& ſecunda æqualis quartæ: ſi verò minor, & minor erit.

Theorema 15. Propoſitio 15.

Partes cum pariter multiplicibus in eadē ſunt ratione, ſi prout ſibi mutuo reſpondent, ita ſumantur.

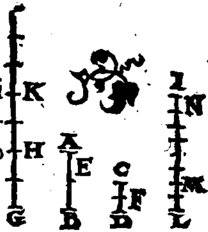


Theorema 16. Propoſitio 16.

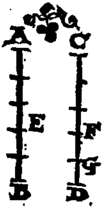
Si quatuor magnitudines proportionales fuerint, & viciſſim proportionales erunt.

Theorema 17. Propoſitio 17.

Si compoſitæ magnitudines proportionales fuerint hæ quoq; diuiſæ proportionales erunt.



Theorema 18. Propositio 18.



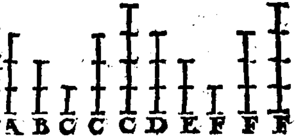
Si diuisæ magnitudines sint proportionales, hæ quoque compositæ proportionales erunt.

Theorema 19. Propositio 19.

Si quemadmodum totũ ad totum, ita ablatum se habuerit ad ablatum: & reliquum ad reliquum, vt totum ad totum se habebit.

Theorema 20. Propositio 20.

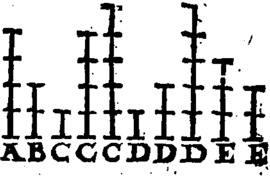
Si sint tres magnitudines, & aliæ ipsis æquales numero, quæ binæ & in eadem ratione sumatur, ex æquo autẽ prima quàm tertia maior fuerit: erit & quarta, quã sexta maior. Quòd si prima tertiæ fuerit æqualis, erit & quarta æqualis sextæ: sin illa minor, hæc quoque minor erit.



Theore-

Theorema 21. Propositio 21.

Si ſint tres magnitudines,& aliȩ ipſis æquales numero quæ binæ & in eadem ratione ſumãtur, fueritq́ue perturbata earũ proportio ex æquo autem prima quàm tertia maior fuerit erit & quarta quã ſexta maior: quòd ſi prima tertiæ fuerit æqualis, erit & quarta æqualis ſextæ: ſin illa minor, hæc quoque minor erit.



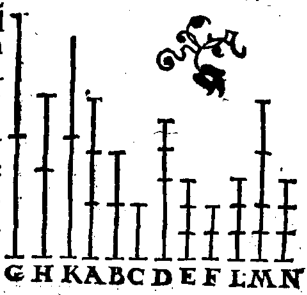
Theorema 22. Propoſitio 22.

Si ſint quotcũnq; magnitudines,& aliȩ ipſis ȩquales numero, quæ binæ in eadẽ ratione ſumantur, et & ex æqualitate in eadem ratione erunt.

Theorema 23. Propoſitio 23.

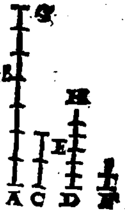
Si ſint tres magnitudines, aliæq́; ipſis ȩquales

les numero, q̄ binæ in eadem ratione sumā-tur, fuerit autē perturbata earū proportio: etiā ex æqualite in eadem ratione erunt.



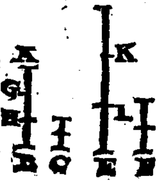
Theorema 24. Propositio 24.

Si prima ad secundam, eandem habuerit rationē, quam tertia ad quartam, habuerit autem & quinta ad secundam eandē rationem, quā sexta ad quartam: etiam cōposita prima cū quinta ad secundam eandē habebit rationem, quā tertia cum sexta ad quartā.



Theorema 25. Propositio 25.

Si quatuor magnitudines proportionales fuerint, maxima & minima reliquis duabus maiores erunt.



ELEMENTI V. FINIS.

EVCLIDIS ELEMENTVM SEXTVM.

DEFINITIONES.

1

Similes figurę rectilineæ sunt, quæ & angulos singulos singulis æquales habēt, atque etiam latera, quæ circum angulos æquales, proportionalia.

2

Reciprocæ autem figuræ sunt, cùm in vtraque figura antecedentes & consequētes rationum termini fuerint.

3

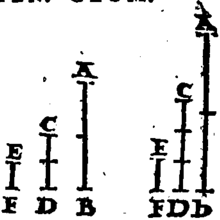
Secundum extremam & mediam rationē recta linea secta esse dicitur, cùm vt tota ad maius segmentum, ita maius ad minus se habuerit.

4

Altitudo cuiusque figurę, est linea perpēdicularis à vertice ad basin deducta.

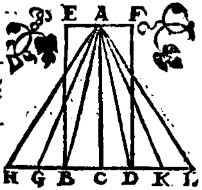
5

Ratio ex rationibus cõponi dicitur, cũ rationum quantitates inter ſe multiplicatæ aliquã effecerint rationem.



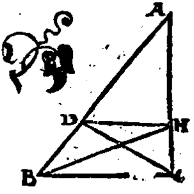
Theorema 1. Propoſitio 1.

Triangula & parallelogramma, quorum eadẽ fuerit altitudo, ita ſe habent inter ſe vt baſes.



Theorema 2. Propoſitio 2.

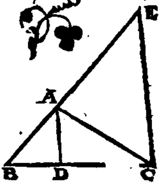
Si ad vnum trianguli latus parallela ducta fuerit recta quædam linea: hęc proportionaliter ſecabit, ipſius triãguli latera. Et ſi triãguli latera proportionaliter ſecta fuerint: quæ ad ſectiones adiũcta fuerit recta linea, erit ad reliquum ipſius trianguli latus parallela.



Theorema 3. Propoſitio 3.

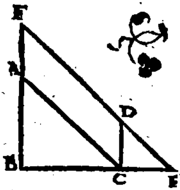
Si trianguli angulus bifariã ſectus ſit, ſecãs autem angulum recta linea ſecuerit & baſim: baſis ſegmenta eandem habebunt rationem,

tionem,quam reliqua ipsius trianguli latera. Et si basis segmenta eandem habeant rationem quam reliqua ipsius trianguli latera, recta linea, quæ à vertice ad sectionem producitur, ea bifariã secat trianguli ipsius angulum.



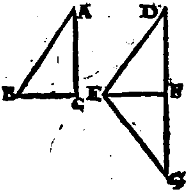
Theorema 4. Propositio 4.

Aequiangulorum triangulorum proportionalia sunt latera,quæ circum ęquales angulos,& homologa sunt latera, quæ æqualib⁹ angulis subtenduntur.



Theorema 5.Propositio 5.

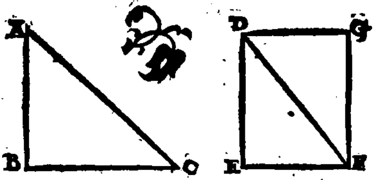
Si duo triãgula latera proportionalia habeant,ęquiangula erunt triangula, & æquales habebunt eos angulos,sub quibus homologa latera subtenduntur.



Theorema 6.Propositio 6.

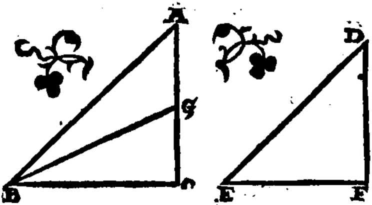
Si duo triangula vnum angulum vni angulo æqualẽ, & circũ ęquales angulos latęra ꝓportionalia habuerint, ęquiangula erũt trian-

triangula, ęqualesq́; habebunt angulos sub quibus homologa latera subtenduntur.



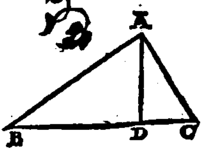
Theorema 7. Propositio 7.

Si duo triangula vnum angulum vni angulo æqualem, circũ autem alios angulos latera proportionalia habeant, reliquorum verò simul vtrunq; aut minorem aut non minorẽ recto: æquiangula erunt triangula, & ęquales habebunt eos angulos, circum quos ꝓportionalia sunt latera.



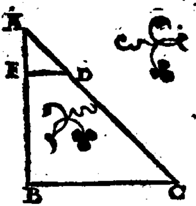
Theorema 8. Propositio 8.

Si in triangulo rectangulo, ab angulo recto in basin perpendicularis ducta sit, quę ad perpendicularem triangula, tum toti triangulo, tum ipsa inter se similia sunt.



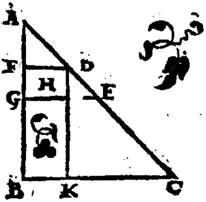
Proble-

Problema 1. Propositio 9.



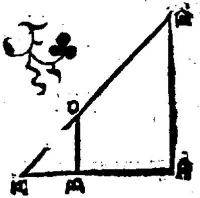
A data recta linea imperatam partem auferre.

Problema 2. Propositio 10.



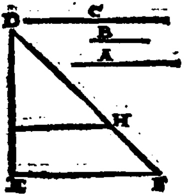
Datam rectam lineā insectam similiter secare, vt data altera recta secta fuerit.

Problema 3. Propositio 11.



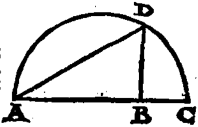
Duabus datis rectis lineis, tertiam proportionalem adinuenire.

Problema 4. Propositio 12.



Tribꝰ datis rectis lineis, quartam proportionalem adinuenire.

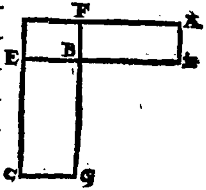
Problema 5. Propoſitio 13.



Duabꝰ datis rectis lineis mediam proportionalē adinuenire.

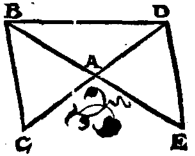
Theorema 9. Propoſitio 14.

Aequalium,& vnū vni æqualem habentiū angulum parallelogrammorum reciproca ſunt latera, quæ circum æquales angulos:& quorū parallelogrammorū vnū angulum vni angulo æqualem habentiū reciproca ſunt latera, quæ circum æquales angulos, illa ſunt æqualia.



Theorema 10. Propoſitio 15.

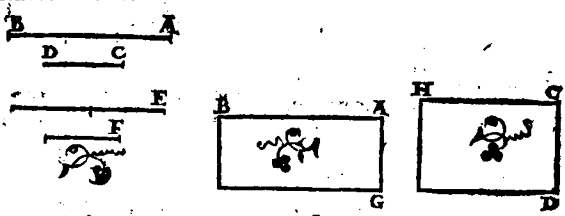
Aequalium,& vnū angulum vni æqualem habentiū triangulorū reciproca ſunt latera, quæ circum æquales angulos: & quorū triangulorū vnum angulum vni ęqualem habentium reciproca ſunt latera, q̄ circum ęquales angulos, illa ſunt æqualia.



Theo-

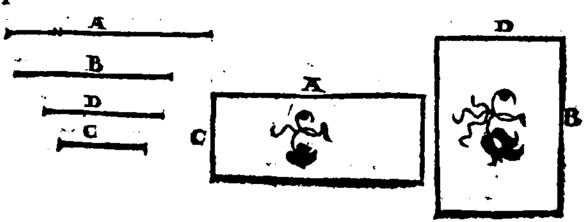
Theorema 11. Propoſitio 16.

Si quatuor rectæ lineæ proportionales fuerint, quod ſub extremis comprehenditur rectangulũ, æquale eſt ei, quod ſub medijs comprehenditur rectãgulo. Et ſi ſub extremis comprehenſum rectangulum æquale fuerit ei, quod ſub medijs cõtinetur rectãgulo, illæ quatuor rectæ lineæ proportionales erunt.



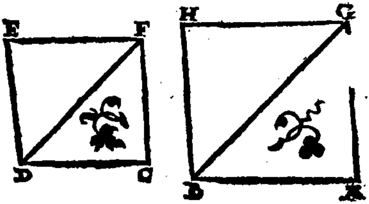
Theorema 12. Propoſitio 17.

Si tres rectæ lineæ ſint proportionales, qđ ſub extremis comprehenditur rectangulũ æquale eſt ei, quod à media deſcribitur quadrato: & ſi ſub extremis comprehẽſum rectangulum æquale ſit ei quod à media deſcribitur quadrato, illæ tres rectæ lineæ ꝑportionales erunt.



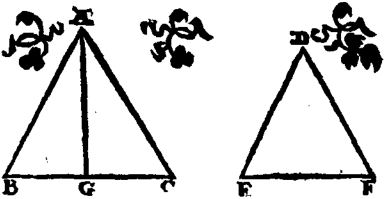
Problema 6. Propoſitio 18.

A data recta linea, dato rectilineo ſimile ſimiliterq́; poſitum rectilineum deſcribere.



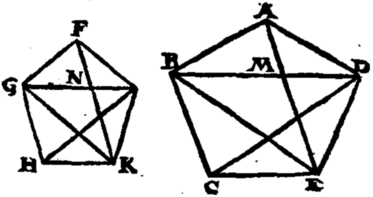
Theorema 13. Propoſitio 19.

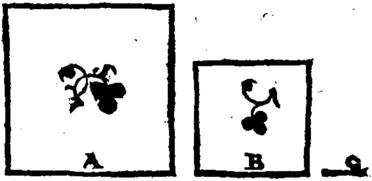
Similia triãgula inter ſe ſũt in duplicata ratiõe laterũ homologorũ.



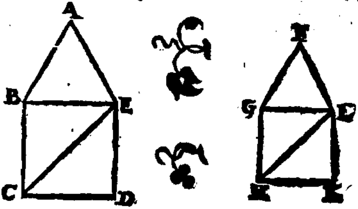
Theore. 14. Propoſitio 20.

Similia polygona in ſimilia triangula diuiduntur, & numero æqualia, & homologa totis. Et po



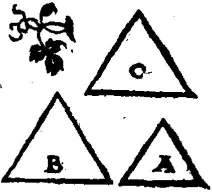


lygona du plicatam habent cā inter ſe ra- tionē, quā latus ho- mologum ad homologum latus.



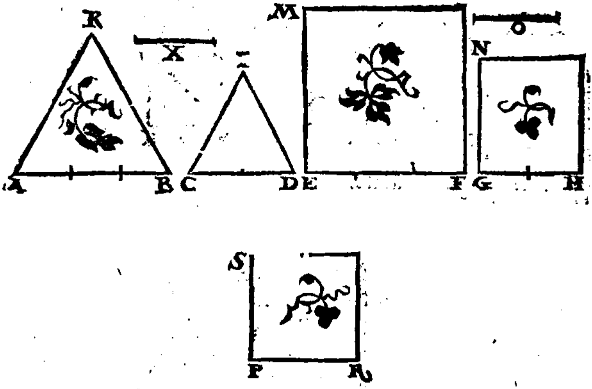
Theorema 15. Pro- poſitio 21.

Quæ eidem rectilineo ſunt ſimilia, & inter ſe ſunt ſimilia.



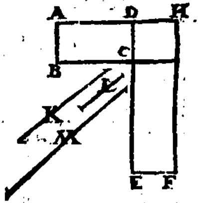
Theorema 16. Pro- poſitio 22.

Si quatuor rectę lineæ proportionales fue rint: & ab eis rectilinea ſimilia ſimiliterq́; deſcripta proportionalia erũt. Et ſi à rectis lineis ſimilia ſimiliterq́; deſcripta rectili- nea proportionalia fuerint, ipſæ etiam rectæ lineæ proportionales erunt.



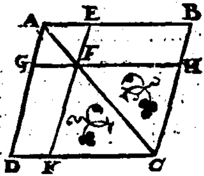
Theorema 17. Propositio 23.

[Aequiangula parallelogramma inter se rationẽ habent eam, quę ex lateribus componitur.



Theorema 18. Propositio 24.

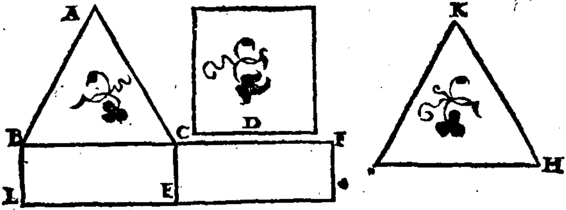
In omni parallelogrammo, quæ circa diametrũ sunt parallelogrãma, & toti & inter se sunt similia.



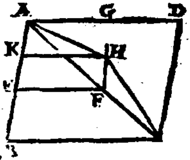
Proble-

Problema 7. Propositio 25.

Dato rectilineo simile, & alteri dato ęquale idem constituere.



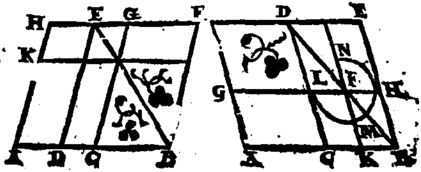
Theorema 19. Propositio 26.



Si à parallelogrãmo parallelogrammũ ablatũ sit, & simile toti & similiter positum communẽ cum eo habens angulum, hoc circum eandem cum toto diametrum consistit.

Theorema 20. Propositio 27.

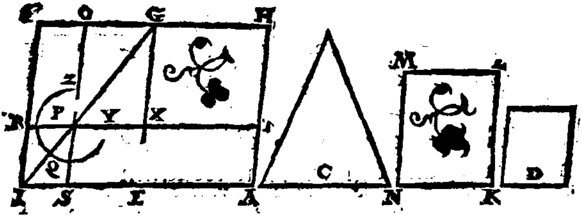
Omnium parallelogrãmorum secũdum eandem rectam lineam applicatorũ deficiẽtiumq; figuris parallelogrãmis si



milibus similiterque positis ei, quod à dimidia describitur, maximum, id est, quod ad dimidiam applicatur parallelogrammum simile existens defectui.

Problema 8. Propositio 28.

Ad datam lineam rectam, dato rectilineo, æquale parallelogrammum applicare deficiens figura parallelogramma, quę similis sit alteri rectilineo dato. Oportet autē datum rectilineum, cui æquale applicandum est, non maius esse eo quod ad dimidiam applicatur, cùm similes sint defectus & eius quod à dimidia describitur, & eius cui simile deesse debet.

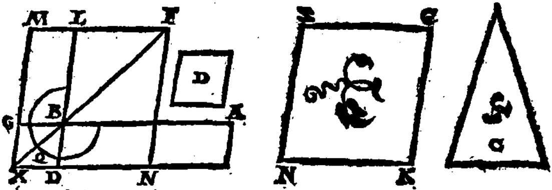


Problema 9. Propositio 29.

Ad datam rectam lineam, dato rectilineo æquale parallelogrammū applicare, excedēs figura parallelogrāma, quæ similis sit

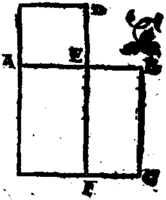
paral-

parallelogrammo alteri dato.



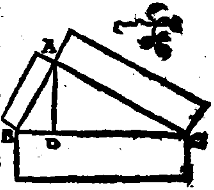
Problema 10. Propoſitio 30.

Propoſitam rectam lineam terminatã, extrema ac media ratione ſecare.



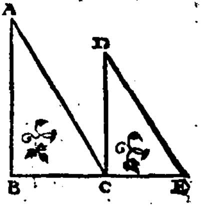
Theorema 21. Propoſitio 31.

In rectangulis triãgulis, figura quęuis à latere rectũ angulũ ſubtendẽte deſcripta ęqualis eſt figuris, quæ priori illi ſimiles & ſimiliter poſitæ à lateribus rectũ angulum cõtinentibus deſcribuntur.



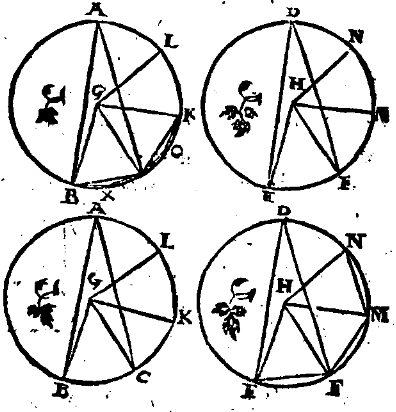
Theorema 22. Propoſitio 32.

Si duo triãgula, quę duo latera duobus lateribus proportionalia habeant, ſecũdum vnum angulũ composita fuerint, ita vt homologa eorum latera ſint etiam parallela, tũ reliqua illorum triangulorum latera in rectam lineam collocata reperientur.



Theorema 23. Propoſitio 33.

Inæqualibus circulis anguli eandẽ habent rationem cũ ipſis peripherijs in quibus inſiſtũt, ſiue ad cẽtra, ſiue ad peripherias cõſtituti, illis inſiſtant peripherijs. Inſuper verò & ſectores quippe qui ad cẽtra cõſiſtũt.



ELEMENTI VI. FINIS.

EVCLIDIS ELEMENTVM SEPTIMVM.

DEFINITIONES.

1.

Vnitas, est secūdum quam entium quodque dicitur vnum.

2.

Numerus autem, ex vnitatibus composita multitudo.

3.

Pars, est numerus numeri minori maioris, cùm minor metitur maiorem.

4

Partes autem, cùm non metitur.

5

Multiplex verò, maior minoris, cùm maiorem metitur minor.

6

Par numerus est, qui bifariam diuiditur.

7

Impar verò, qui bifariam non diuiditur: vel, qui vnitate differt à pari.

8

Pariter par numerus est, quem par numerus metitur per numerum parem.

9 Pari

9

Pariter autem impar, eſt quem par numerus metitur per numerum imparem.

10

Impariter verò impar numerus, eſt quem impar numerus metitur per numerum imparem.

11.

Primus numerus, eſt quem vnitas ſola metitur.

12

Primi inter ſe numeri ſunt, quos ſola vnitas menſura communis metitur.

13

Compoſitus numerus eſt, quem numerus quiſpiam metitur.

14

Compoſiti autẽ inter ſe numeri ſunt, quos numerus aliquis menſura communis metitur.

15

Numerus numerum multiplicare dicitur, cùm toties compoſitus fuerit is, qui multiplicatur, quot ſunt in illo multiplicante vnitates, & procreatus fuerit aliquis.

16

Cùm autẽ duo numeri mutuò ſeſe multiplicantes quempiam faciũt, qui factus erit planus appellabitur, qui verò numeri mutuò ſeſe multiplicarint, illi⁹ latera dicẽtur.

17 Cùm

17

Cùm verò tres numeri mutuò ſeſe multiplicantes quempiã faciunt, qui procreatus erit ſolidus appellabitur, qui autem numeri mutuò ſeſe multiplicarint, illius latera dicentur.

18

Quadratus numerꝰ eſt, qui ęqualiter æqualis: vel, qui à duobus ęqualibus numeris cõtinetur.

19

Cubus verò, qui æqualiter æqualis æqualiter: vel, qui à tribus æqualibus numeris cõtinetur.

19

Numeri proportionales ſunt, cùm primus ſecundi, & tertius quarti æque multiplex eſt, vel eadem pars, vel eædem partes.

21

Similes plani & ſolidi numeri ſunt, qui proportionalia habent latera.

22

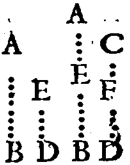
Perfectus numerus eſt, qui ſuis ipſius partibus eſt æqualis.

Theorema 1. Propoſitio 1.

Duobus numeris inæqualibus propoſitis,

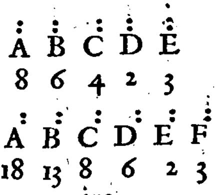
tis, si detrahatur semper minor de maiore, alterna, quadam detractione, neque reliquus vnquam metiatur præcedentem quoad assumpta sit vnitas: qui principio propositi sunt, numeri primi inter se erunt.

Problema 1. Propositio 2.



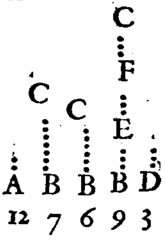
Duobus numeris datis non primis inter se, maximã eorum communem mensuram reperire.

Problema 2. Propositio 3.



Tribus numeris datis non primis inter se, maximam eorum communem mensuram reperire.

Theorema 2. Propositio 4.

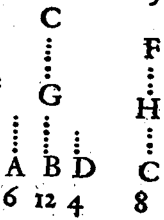


Omnis numerus, cuiusq; numeri minor maioris aut pars est, aut partes.

Theore-

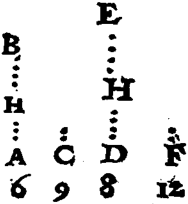
Theorema 3. Propositio 5.

Si numerus numeri pars fuerit, & alter alterius eadẽ pars & simul vterque vtriusque simul eadem pars erit, quæ vnus est vnius.



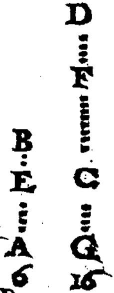
Theor. 4. Propo. 6.

Si numer⁹ sit numeri partes, & alter alterius eędem partes, & simul vterque vtriusque simul eædẽ partes erunt, quæ sunt vnus vnius.



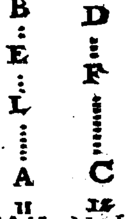
Theorema 5. Propositio 7.

Si numerus numeri eadẽ sit pars quæ detractus detracti, & reliquus reliqui eadẽ pars erit, quæ totus est totius.



Theorema 6. Propositio 8.

Si numerus numeri eædẽ sint partes quæ detractus detracti, & reliquus reliqui eędẽ partes erunt, quæ sunt totus totius.

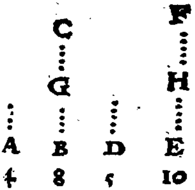


Theo-

G..M.K...N.H.

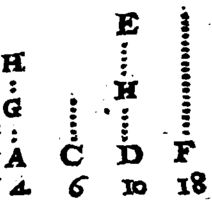
Theorema 7. Propositio 9.

Si numerus numeri pars ſit, & alter alterius eadem pars, & viciſsim quæ pars eſt vel partes primus tertij, eadẽ pars erit vel eędẽ partes, & ſecũdus quarti.



Theorema 8. Propoſitio 10.

Si numerus numeri partes ſint, & alter alterius eædẽ partes, etiam viciſſim quæ ſunt partes aut pars primus tertij, eædẽ partes erunt vel pars & ſecundus quarti.



Theorema 9. Propoſitio 11.

Si quemadmodum ſe habet totus ad totum, ita detractus ad detractum, & reliquus ad reliquum ita habebit vt totus ad totum.

Theorema 10. Propoſitio 12.

Si ſint quotcunque numeri proportionales, quemadmodum ſe habet vnus antecedentium ad vnum ſequẽtium, ita ſe habe-

A	B	C	D
9	6	3	2

habebunt omnes antecedentes ad omnes consequentes.

Theorema 11. Propositio 13.

Si quatuor numeri sint proportionales, & vicissim proportionales erunt.

:	:	:	:
A	B	C	D
12	4	9	3

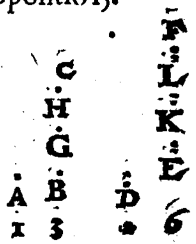
Theorema 12. Propositio 14.

Si sint quotcunque numeri, & alij illis æquales multitudine, qui bini sumantur & in eadem ratione: etiam ex æqualitate in eadem ratione erunt.

:	:	:	:	:	:
A	B	C	D	E	F
12	6	3	8	4	2

Theorema 13. Propositio 15.

Si vnitas numerum quempiã metiatur, aliter verò numerus alium quendã numerũ æquè metiatur, & vicissim vnitas tertium numerũ æquè metietur, atque secundis quartum.



Theorema 14. Propositio 16.

Si duo numeri mutuò sese multiplicãtes faciant aliquos qui ex illis geniti fuerint; inter se æquales erunt.

	:	:	:	:
E	A	B	C	D
1	2	4	8	8

Theorema 15. Propositio 17.

Si numer⁹ duos numeros multiplicãs faciat aliquos, qui ex illis procreati erunt, eandem rationem habebunt, quam multiplicati.

I	A	B	C	D	E
1	3	4	5	12	15

Theorema 16. Propositio 18.

Si duo numeri numerum quempiam multiplicantes faciant aliquos, geniti ex illis eandem habebunt rationem, quam qui illum multiplicarunt.

A	B	C	D	E
4	5	3	12	15

Theorema 17. Propositio 19.

Si quatuor numeri sint proportionales, q̃ ex primo & quarto fit, ęqualis erit ei qui ex secundo & tertio: & si qui ex primo & quarto fit numerus æqualis sit ei qui ex secũdo & tertio, illi quatuor numeri propor tionales erunt.

A	B	C	D	E	F	G
6	4	3	2	12	12	18

Theorema 18. Propositio 20.

Si tres numeri sint proportionales, qui ab extremis cõtinetur, ęqualis est ei qui à medio

dio efficitur. Et si qui ab extremis cōtinetur æqualis sit ei qui à medio describitur, illi tres numeri proportionales erunt.

:	:	:
A	B	C
9	6	4
:		
D		
6		

Theorema 19. Propositio 21.

Minimi numeri omniū, qui eandem cū eis rationem habēt, æqualiter metiūtur numeros eandem rationem habētes, maior quidē maiorem, minor verò minorem.

D	L		
:	:		
G	H		
:	:	:	:
C	E	A	B
4	3	8	6

Theorema 20. Propositio 22.

Si tres sint numeri & alij multitudine illis æquales, qui bini sumantur & in eadem ratione, sit autem perturbata eorū proportio, etiam ex æqualitate in eadem ratione erunt.

:	:	:	:	:	:
A	B	C	D	E	F
6	4	3	12	8	6

Theorema 21. Propositio 23.

Primi inter se numeri minimi sunt omniū eandem cum eis rationem habentium.

:	:	:	:	:
A	B	E	C	D
5	6	2	4	3

Theorema 22. Propositio 24.

Minimi numeri omnium eandem cum eis rationem habentiũ, primi sunt inter se.

A	B	C	D	E
8	6	4	3	2

Theorema 23. Propositio 25.

Si duo numeri sint primi inter se, qui alterutrum illorum metitur numerus, is ad reliquũ primus erit.

A	B	C	D
6	7	3	4

Theorema 24. Propositio 26.

Si duo numeri ad quẽpiam numerũ primi sint, ad eundẽ primus is quoque futurus est, qui ab illis productus fuerit.

B	A	C	D	E	F
3	5	5	5	3	2

Theorema 25. Propositio 27.

Si duo numeri primi sint inter se, q ab vno eorũ gignitur ad reliquum primus erit.

B	A	C	D
	7	6	3

Theorema 26. Propositio 28.

Si duo numeri ad duos numeros ambo ad vtrunque primi sint, & qui ex eis gignentur, primi inter se erunt.

A	B	E	C	D	F
3	5	15	2	4	8

Theore-

Theorema 27. Propoſitio 29.

Si duo numeri primi ſint inter ſe,& multiplicãs vterq; ſeipſum procreet aliquẽ, qui ex ijs producti fuerint,primi inter ſe erũt. Quòd ſi numeri initio propoſiti multiplicãtes eos qui producti ſunt, effecerint aliquos,hi quoq; inter ſe primi erũt, & circa extremos idẽ hoc ſemper eueniet.

A	C	E	B	D	F
3	6	27	4	16	63

Theorema 28. Propoſitio 30.

Si duo numeri primi ſint inter ſe,etiam ſimul vterq; ad vtrunq; illorũ primus erit. Et ſi ſimul vterq; ad vnum aliquem eorũ primus ſit,etiam qui initio poſiti ſunt numeri, primi inter ſe erunt.

C

A	B	D
7	5	4

Theorema 29. Propoſitio 31.

Omnis primus numerus ad omnem numerum quem non metitur,primus eſt.

A	B	C
7	10	5

Theorema 30. Propoſitio 32.

Si duo numeri ſeſe mutuò multiplicãtes faciant aliquem, hũc aũt ab illis productum metiatur primus quidam numerus, is alterum etiã metitur eorum qui initio poſiti erant.

A	B	C	D	E
2	6	12	3	4

Theorema 31. Propositio 33.

Omnem compositum numerum aliquis primus metietur.

A	B	C
27	9	3

Theorema 32. Propositio 34.

Omnis numerus aut primus est, aut eum aliquis primus metitur.

A	A	
3	6	3

Problema 3. Propositio 35.

Numeris datis quotcunque, reperire minimos omnium qui eandem cum illis rationem habeant.

A	B	C	D	E	F	G	H	K	I	M
6	8	12	2	3	4	6	2	3	4	3

Problema 4. Propositio 36.

B A	C	D	E	F
7	12	8	4	5

Duobus numeris datis reperire quem illi minimũ metiantur numerum.

A F	E	C	D	G	H
6	9	12	9	2	3

Theo-

Theorema 33. Propoſitio 37.

Si duo numeri numerũ quempiam metiantur, & minimus quem illi metiuntur eundem metietur.

	B	E	C
2	3	6	12

F

Problema 5. Propoſitio 38.

Tribus numeris datis reperire quem minimum numerũ illi metiantur.

A	B	C	D	E
3	4	6	12	8

A	B	C	D	E	F
3	6	8	12	24	16

Theorema 34. Propoſitio 39.

Si numerum quiſpiam numerus metiatur, menſus partem habebit metienti cognominem.

A	B	C	D
12	4	3	1

Theorema 35. Propoſitio 40.

Si numerus partem habuerit quálibet, illum metietur numerus parti cognominis.

A	B	C	D
8	4	2	1

Problema 6. Propoſitio 41.

Numerum reperire, qui minimus cùm ſit, datas habeat partes.

A	B	C	G	H
2	3	4	12	10

ELEMENTI VII. FINIS.

EVCLIDIS ELEMENTVM OCTAVVM.

Theorema 1. Propositio 1.

Si sint quotcũq; numeri deinceps proportionales, quorum extremi sint inter se, primi, minimi sunt omnium eandem cum eis rationem habentium.

A	B	C	D	E	F	G	H
8	12	18	27	6	8	12	18

Problema 1. Propositio 1.

Numeros reperire deinceps proportionales minimos, quotcunq; iusserit quispiam in data ratione.

A	B	C	D	E	F	G	H	K
3	4	9	12	16	27	36	49	64

Theorema 2. Propositio 3.

Conuersa primæ.

Si sint quotcũq; numeri deinceps proportionales minimi habentium eandem cum eis rationem, illorum extremi sunt inter se primi.

A	B	C	D	E	F	G	H	K	L	M	N	O
27	16	48	64	3	4	9	12	16	27	36	48	64

Pro-

Problema 2. Propoſitio 4.

Rationibus datis quotcunque in minimis numeris reperire numeros deinceps minimos in datis rationibus.

A	B	C	D	E	F	H	G	K	L	N	X	M	O
3	4	2	3	4	5	6	8	12	15	4	6	10	12

Theorema 3. Propoſitio 5.

Plani numeri rationem inter ſe habent ex lateribus compoſitam.

A	L	B	C	D	E	F	G	H	K
18	22	32	3	6	4	8	9	12	16

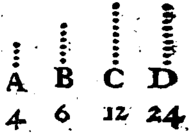
Theorema 4. Propoſitio 6.

Si ſint quotlibet numeri deinceps proportionales, primus autem ſecundum non metiatur, neque alius quiſpiam vllum metietur.

A	B	C	D	E	F	G	H
16	24	36	54	82	4	6	9

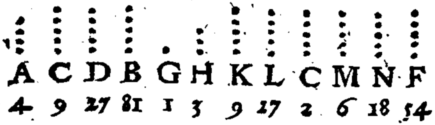
Theorema 5. Propositio 7.

Si ſint quotcunque numeri deinceps proportionales, primus autem extremum metiatur, is etiam ſecundum metietur.



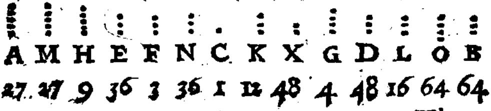
Theorema 6. Propoſitio 8.

Si inter duos numeros medij continua proportione incidant numeri, quot inter eos medij continua proportione incidunt numeri, tot & inter alios eandem cum illis habentes rationem medij continua proportione incident.



Theorema 7. Propoſitio 9.

Si duo numeri ſint inter ſe primi, & inter eos medij cõtinua proportione incidãt numeri, quot inter illos medij continua proportione incidunt numeri, totidẽ & inter vtrunque eorum ac vnitatẽ deinceps medij continua proportione incident.



Theo-

Theorema 8. Propoſitio 10.

Si inter duos numeros & vnitatē continuè proportionales incidāt numeri quot inter vtrunque ipſorum & vnitatē deinceps medij continua proportione incidunt numeri, totidē & inter illos medij continua proportione incident.

A	E	K	D	H	C	L	F	G	B
27	9	36	3	12	1	48	4	16	64

Theorema 9. Propoſitio 11.

Duorum quadratorū numerorum vnus medius proportionalis eſt numer⁹: & quadratus ad quadratum duplicatam habet lateris ad latus rationem.

A	C	E	D	B
9	3	12	4	16

Theorema 10. Propoſitio 12.

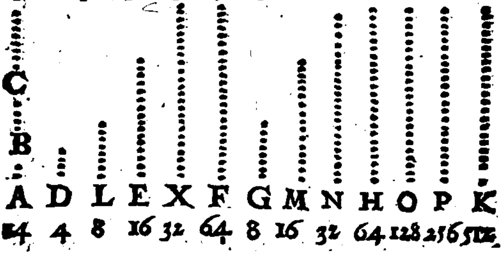
Duorum cuborum numerorū duo medij proportionales ſunt numeri: & cubus ad cubum triplicatam habet lateris ad latus rationem.

A	H	K	B	C	D	E	F	G
27	36	48	64	3	4	9	12	16

Theo-

Theorema 11. Propositio 13.

Si sint quotlibet numeri deinceps proportionales, & multiplicans quisq; seipsum faciat aliquos, qui ab illis producti fuerint, proportionales erũt: & si numeri primùm positi, ex suo in procreatos ductu faciãt aliquos, ipsi quoque proportionales erunt.



Theorema 12. Propositio 14.

Si quadratus numerus quadratum numerũ metiatur, & latus vnius metietur latus alterius. Et si vnius quadrati latus metiatur latus alterius, & quadratus quadratum metietur.

A	E	B	C	D
9	12	16	3	4

Theorema 13. Propositio 15.

Si cubus numerus cubum numerũ metiatur, & latus vnius metietur alterius latus. Et si latus vnius cubi latus alterius metiatur,

tum

tum cubus cubum metietur.

A H K B C D E F G

8 16 28 64 2 4 4 8 16

Theorema 14. Propositio 16.

Si quadratus numerus quadratum numerum non metiatur, neque latus vnius metietur alterius latus.

Et si latus vni9 quadrati nó metiatur latus alterius, neque quadratus quadratum metietur.

A B C D

9 16 3 4

Theorema 15. Propositio 17.

Si cubus numerus cubum numerũ nó mẽtiatur, neque latus vni9 latus alterius metietur.

Et si latus cubi alicuius latus alterius nó metiatur, neque cubus cubũ metietur.

A B C D

8 27 9 11

Theorema 16. Propositio 18.

Duorum similium planorum numerorũ vnus medius proportionalis est numerus, & planus ad planũ duplicatam habet literis homo-

A G B C D E F

12 18 27 2 6 3 9

logi

logi ad latus homologum rationem.

Theorema 17. Propositio 19.

Inter duos similes numeros solidos, duo medij proportionales incidunt numeri: & solidus ad similem solidum triplicatam rationem habet lateris homologi ad latus homologum.

A	N	X	B	C	D	E	F	G	H	K	M	L
8	12	18	27	2	2	2	3	3	3	4	6	9

Theorema 18. Propositio 20.

Si inter duos numeros vn^9 medius ꝓportionalis incidat numer^9, similes plani erunt illi numeri.

A	C	B	D	E	F	G
18	24	33	3	4	6	8

Theorema 19. Propositio 21.

Si inter duos numeros duo medij proportionales incidant numeri, similes solidi sunt illi numeri.

A	C	D	B	E	F	G	H	K	L	M
27	36	44	64	9	12	16	3	3	3	4

Theo-

Theorema 20. Propositio 22.

Si tres numeri deinceps sint proportionales, primus autem sit quadratus, & tertius quadratus erit.

A	B	D
9	15	25

Theorema 21. Propositio 23.

Si quatuor numeri deinceps sint proportionales, primus autem sit cubus, & quartus cubus erit.

A	B	C	D
8	12	18	27

Theorema 22. Propositio 24.

Si duo numeri rationem habeant inter se, quam quadratus numerus ad quadratum numerum, primus autem sit quadratus, & secundus quadratus erit.

A		B	C		D
4	6	9	16	24	36

Theorema 23. Propositio 25.

Si numeri duo rationem inter se habeant, quam cubus numerus ad cubum numerũ, primus autem cubus sit, & secundus cubus erit.

A	E	F	B	C			D
8	12	18	27	64	95	140	216

Theo-

Theorema 24. Propositio 26.

Similes plani numeri rationem inter se habent, quam quadratus numerus ad quadratum numerum.

A	C	B	D	E	F
18	24	32	9	12	16

Theorema 25. Propositio 27.

Similes solidi numeri rationem habẽt inter se, quam cubus numerus ad cubum numerum.

A	C	D	B	E	F	G	H
16	24	36	54	8	12	18	27

ELEMENTI VIII. FINIS.

EVCLI-

EVCLIDIS ELEMENTVM NONVM.

Theorema 1. Propositio 1.

Si duo similes plani numeri mutuò sese multiplicãtes quendã procreent, productus quadratus erit.

A	E	B	D		C
4	6	9	16	24	36

Theorema 2. Propositio 2.

Si duo numeri mutuò sese multiplicantes quadratum faciant, illi similes sunt plani.

A		B	D		C
4	6		9	18	36

Theorema 3. Propositio 3.

Si cubus numerus seipsum multiplicãs pcreet aliquẽ, productus cubus erit.

Vnitas	D	D	A			B
	3	4	8	16	32	64

Theorema 4. Propositio 4.

Si cubus numerus cubũ numerum multiplicans quendam procreet, procreatus cubus erit.

A	B	D	C
8	27	64	216

Theorema 5. Propositio 5.

Si cubus numerus numerum quendã multiplicans cubum procreet, & multiplicatus cubus erit.

A	B	D	D
27	64	729	17 28

Theorema 6. Propositio 6.

Si numerus seipsum multiplicans cubum procreet, & ipse cubus erit.

A	B	C
27	729	19683

Theorema 7. Propositio 7.

Si compositus numerus quẽdam numerũ multiplicans quempiam procreet, productus solidus erit.

A	B	C	D	E
6	8	48	2	3

Theorema 8. Propositio 8.

Si ab vnitate quotlibet numeri deinceps ꝓportionales sint, tertius ab vnitate quadratus est, & vnũ intermittentes omnes: quartus autẽ cubus, & duobus intermissis omnes:

nes. ſeptimus verò cubus ſimul & æquadratus, & quinque intermiſſis omnes

Vnitas	A	B	C	D	E	F
	3	9	27	81	243	729

Theorema 9. Propoſitio 9.

Si ab vnitate ſint quotcunque numeri deinceps proportionales, ſit autem quadratus is qui vnitatem ſequitur, & reliqui oés quadrati erũt. Quòd ſi qui vnitatem ſequitur cubus ſit, & reliqui omnes cubi erunt.

quadrati.		cubi
531441	F	732969
59049	E	531441
6561	D	59049
729	C	6561
81	B	729
9	A	81
	o	
	Vnitas.	

Theorema 10. Propoſitio 10.

Si ab vnitate numeri quotcunque proportionales ſint, non ſit autẽ quadratus is qui vnitatem ſequitur, neq; alius vllus quadratus erit, dẽptis tertio ab vnitate ac omnibus

Vnitas.	A	B	C	D	E	F
o	3	9	36	81	243	729

nibus vnum intermittẽtibus. Quòd ſi qui vnitatem ſequitur, cubus non ſit, neque alius vllus cubus erit, demptis quarto ab vnitate ac omnibus duos intermittentibus.

Theorema 11. Propoſitio 11.

❡ Si ab vnitate numeri quotlibet deinceps proportionales ſint, minor maiorē metitur per quempiam eorum qui in proportionalibus ſunt numeris.

A	D	C	D	E
1	2	4	8	16

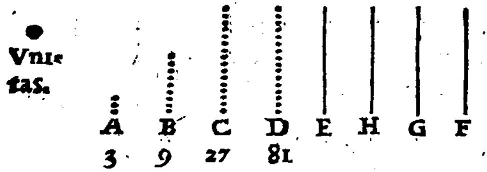
Theorema 12. Propoſitio 12.

Si ab vnitate quotlibet numeri ſint ꝓportionales, quot primorum numerorū vltimum metiuntur, totidem & eum qui vnitati proximus eſt, metientur.

Vnitas.	A	B	C	D	E	H	G	F
	4	16	64	259	2	8	32	128

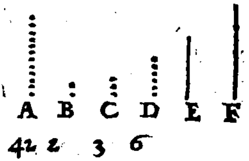
Theorema 13. Propoſitio 13.

Si ab vnitate ſint quotcunq; numeri deinceps proportionales, primus autē ſit qui vnitatē ſequitur, maximū nullus alius metietietur, ijs exceptis qui in proportionalibꝰ ſunt numeris.



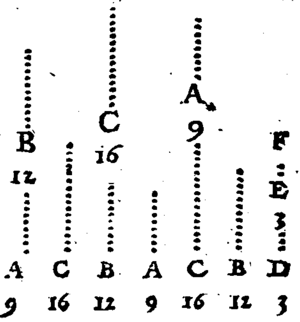
Theorema 14. Propóſitio 14.

Si minimum numerum primi aliquot numeri metiātur, nullus alius numerus primus illum metietur, ijs exceptis qui primò metiuntur.



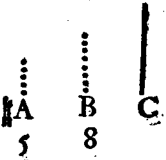
Theorema 15. Propoſitio 15.

Si tres numeri deinceps proportionalisſint minimi, eadē cum ipſis habē tium rationē, duo quilibet compoſiti ad tertium primi erunt.



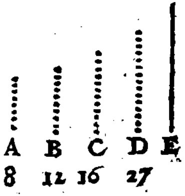
Theorema 16. Propoſitio 16.

Si duo numeri ſint inter ſe primi, non ſe habebit quẽ-admodum primus ad ſecũdum, ita ſecundus ad quẽpiam alium.



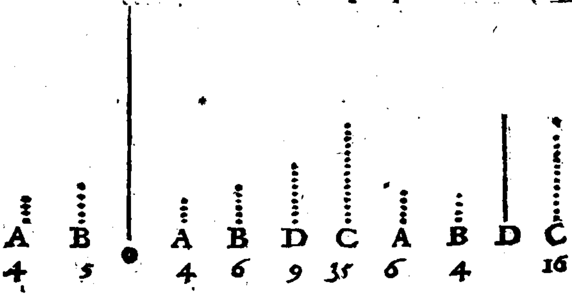
Theorema 17. Propoſitio 17.

Si ſint quotlibet numeri deinceps ꝓportionales, quorum extremi ſint inter ſe primi, non erit quẽ admodum primus ad ſecundum, ita vltimus ad quempiam alium.



Theorema 18. Propoſitio 18.

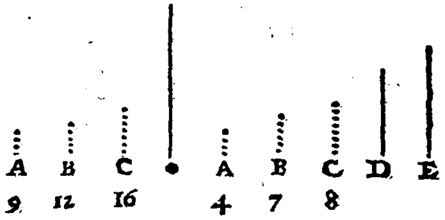
Duobꝰ numeris datis, cõſiderare poſsitne tertius illis inueniri proportionalis.



Theore-

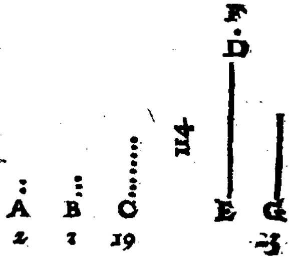
Theorema 19. Propositio 19.

Tribus numeris datis; considerare possitne quartus illis reperiri proportionalis.



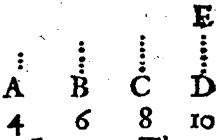
Theorema 20. Propositio 20.

Primi numeri plures sunt quã cũque proposita multitudine primorum numerorum.



Theorema 21. Propositio 21.

Si pares numeri quotlibet compositi sint, totus est par.



Theorema 22. Propositio 22.

Si impares numeri quotlibet compositi sint, sit autem par illorum multitudo, totus par erit.

A B C D E
5 9 7 5

Theorema 23. Propositio 23.

Si impares numeri quotcunque cōpositi sint, sit autem impar illoru multitudo, & totus impar erit.

A B C E
5 7 8 1

Theorema 24. Propositio 24.

Si de pari numero par detractus sit, & reliquus par erit.

A B C
6 4

Theorema 25. Propositio 25.

Si de pari numero impar detractus sit, & reliquus impar erit.

A B C D
8 1 4

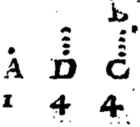
Theorema 26. Propositio 26.

Si de impari numero impar detractus sit, & reliquus par erit.

A B C D
4 6 1

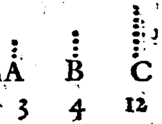
Theo

Theorema 27. Propositio 27.



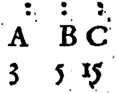
Si ab impari numero par ablatus sit, reliquus impar erit.

Theorema 28. Propositio 28.



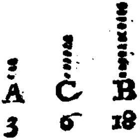
Si impar numerus parem multiplicās, procreet quēpiam, procreatus par erit.

Theorema 29. Propositio 29.



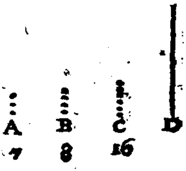
Si impar numerus imparē numerum multiplicans quemdam procreet, procreatus impar erit.

Theorema 30. Propositio 30.



Si impar numerus parē numerum metiatur, & illius dimidium metietur.

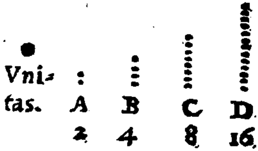
Theorema 31. Propositio 31.



Si impar numerus ad numerum quempiā primus sit, & ad illius duplum primus erit.

Theorema 32. Propoſitio 32.

Numerorum, qui à binario dupli ſunt, vnuſquiſq; pariter par eſt tantùm.



Theorema 33. Propoſitio 33.

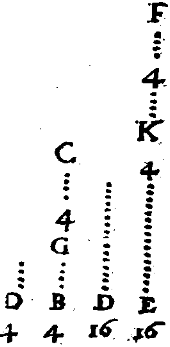
Si numerus dimidiũ impar habeat, pariter impar eſt tantùm.

Theorema 34. Propoſitio 34.

Si par numerus nec ſit duplꝰ à binario, nec dimidium impar habeat, pariter par eſt, & pariter impar.

Theorema 35. Propoſitio 35.

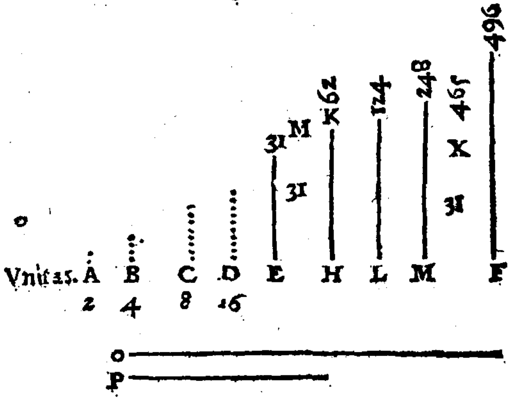
Si ſint quotlibet numeri deinceps proportionales, detrahãtur autẽ de ſecũdo & vltimo æquales ipſi primo, erit quemadmodũ ſecũdi exceſſus ad primũ, ita vltimi exceſſus ad omnes qui vltimum antecedunt.



Theo-

Theorema 36. Propositio 36.

Si ab vnitate numeri quotlibet deinceps expositi sunt in duplici proportione quoad totus compositus primus factus sit, isq́; totus in vltimum multiplicatus quempiá procreet, procreatus perfectus erit.



ELEMENTI IX. FINIS.

EVCLIDIS ELEMENTVM DECIMVM.

DEFINITIONES.

1.

Commenſurabiles magnitudines dicuntur illæ, quas eadem menſura metietur.

2

Incommenſurabiles verò magnitudines dicuntur hæ, quarum nullam menſuram communem contingit reperiri.

3

Lineæ rectæ potentia cõmenſurabiles sũt, quarum quadrata vna eadem ſuperficies ſiue area metitur.

4

Incommenſurabiles verò lineæ ſunt, quarum quadrata, quæ metiatur area cõmunis, reperiri nulla poteſt.

5

Hæc cùm ita ſint, oſtendi poteſt quòd quãtacunque linea recta nobis proponatur, exiſtunt etiã aliæ lineæ innumerabiles eidem commenſurabiles, aliæ item incommenſurabiles, hæ quidem longitudine & poten-

potentia: illæ verò potentia tantùm. Vocetur igitur linea recta, quantacunq; proponatur, ῥητὴ, id est rationalis.

6

Lineæ quoq; illi ῥητῇ commensurabiles siue longitudine & potentia, siue potentia tantùm, vocentur & ipsæ ῥηταί, id est rationales.

7

Quæ verò lineæ sunt incommensurabiles illi τῇ ῥητῇ, id est primo loco rationali, vocētur ἄλογοι, id est irrationales.

8

Et quadratum quod à linea proposita describitur, quam ῥητὴν vocari volumus, vocetur ῥητὸν.

9

Et quæ sunt huic commensurabilia, vocētur ῥητά.

10

Quæ verò sunt illi quadrato ῥητῷ scilicet incommensurabilia, vocentur ἄλογα, id est surda.

11

Et lineæ quæ illa incommēsurabilia describunt, vocentur ἄλογοι Et quidem si illa incommēsurabilia fuerint quadrata, ipsa corū latera vocabūtur ἄλογοι lineæ. quòd si quadrata quidem non fuerint, verū aliæ quæpiā superficies siue figuræ rectilineæ, tunc

tunc verò lineæ illæ quæ deſcribunt quadrata æqualia figuris rectilineis, vocentur ἄλογοι.

Theorema 1. Propoſitio 1.

Duabus magnitudinibus inæqualibus propoſitis, ſi de maiore detrahatur plus dimidio, & rurſus de reſiduo iterũ detrahatur plus dimidio, idq́; ſemper ſiat: relinquetur quædam magnitudo minor altera minore ex duabus propoſitis.

Theorema 2. Propoſitio 2.

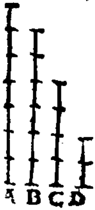
Duab⁹ magnitudinibus propoſitis inæqualibus, ſi detrahatur ſemper minor de maiore, alterna quadam detractione neque reſiduum vnquam metiatur id quod ante ſe metiebatur, incommenſurabiles ſunt illæ magnitudines.

Problema 1. Propoſitio 3.

Duabus magnitudinibus commẽſurabilibus datis, maximam ipſarum communem menſuram reperire.

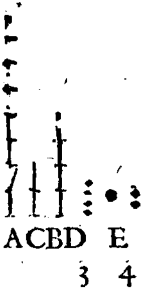
Proble-

Problema 2. Propoſitio 4.



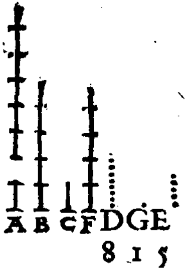
Tribus magnitudinibus cõmenſurabilibus datis, maximam ipſarũ communẽ menſuram reperire.

Theorema 3. Propoſitio 5.



Cõmenſurabiles magnitudines inter ſe proportionẽ eam habent, quam habet numerus ad numerum.

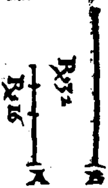
Theorema 4. Propoſitio 6.



Si duæ magnitudines proportionem eam habent inter ſe quam numerus ad numerum, commenſurabiles ſunt illæ magnitudines.

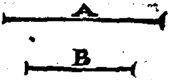
Theo-

Theorema 5. Propoſitio 7.



Incommenſurabiles magnitudines inter ſe proportionem non habēt, quam numerus ad numerum.

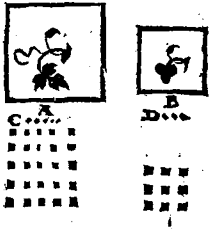
Theorema 6. Propoſitio 8.



Si duæ magnitudines inter ſe proportionem nō habēt quam numerus ad numerum, incommenſurabiles illæ ſunt magnitudines.

Theorema 7. Propoſitio 9.

Quadrata, quæ deſcribūtur à rectis lineis lōgitudine cōmenſurabilibus, inter ſe proportionem habent quā numerus quadratus ad alium numerum quadratū. Et quadrata habentia proportionē inter ſe quā quadratus numerus ad numerum quadratū, habent quoque latera longitudine commenſurabilia. Quadrata verò

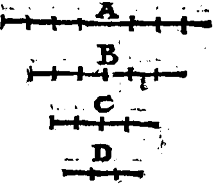


quæ

quæ deſcribuntur à lineis longitudine incómenſurabilibus, proportioné nó habẽt inter ſe, quam quadratus numerus ad numerum alium quadratum. Et quadrata nó habentia proportionem inter ſe quam numerus quadratus ad numerum quadratũ, neque latera habebunt longitudine commenſurabilia.

Theorema 8. Propoſitio 10.

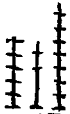
Si quatuor magnitudines fuerint proportionales, prima verò ſecũdæ fuerit cõmenſurabilis, tertia quoque quartę comméſurabilis erit: quòd ſi prima ſecũdæ fuerit incomméſurabilis, tertia quoq; quartæ incommenſurabilis erit.



Problema 3. Propoſitio 11.

Propoſitæ lineæ rectæ (quam ῥητὴν vocari diximus) reperire duas lineas rectas incómenſurabiles, hanc quidem longitudine tãtùm, illam verò non longitudine tantùm, ſed etiam potentia incomméſurabilem.

Theorema 9. Propoſitio 12.

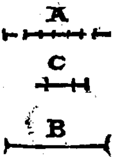


A C B

Magnitudines quæ eidẽ magnitudini ſunt cõ menſurabiles, inter ſe quoque ſunt commenſurabiles.

6 D...... 4 F..
4 E.... 8 G..
3 H...
2 K..
4 L...

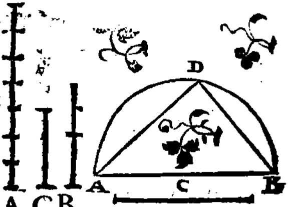
Theorema 10. Propoſitio 13.



Si ex duabus magnitudinib⁹ hæc quidem cõmenſurabilis ſit tertię magnitudini, illa verò eidem incommenſurabilis, incõmenſurabiles ſunt illæ duæ magnitudines.

Theorema 11. Propoſitio 14.

Si duarum magnitudinum commenſurabiliũ altera fuerit incõmenſurabilis magnitudini alteri cuipiam tertiæ. reliqua quoque magnitudo eidẽ tertiæ incommẽſurabilis erit.



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Theorema 12. Propoſitio 15.

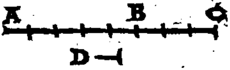
Si quatuor rectæ proportionales fuerint,

poſſit

posſit autem prima pluſquã ſecunda tãtò quantum eſt quadratum lineæ ſibi commẽſurabilis longitudine: tertia quoq; poterit pluſquàm quarta tanto quantum eſt quadratũ lineæ ſibi commenſurabilis lõgitudine. Quòd ſi prima poſſit pluſquã ſecũda quadrato lineę ſibi longitudine incommẽſurabilis: tertia quoque poterit pluſquã quarta quadrato lineæ ſibi incõmẽſurabilis lõgitudine.

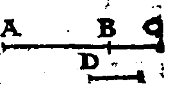
Theorema 13. Propoſitio 16.

Si duæ magnitudines commenſurabiles componantur, tota magnitudo cõpoſita ſingulis partibus commenſurabilis erit, quòd ſi tota magnitudo cõpoſita alterutri parti commenſurabilis fuerit, illæ duæ quoque partes cõmẽſurabiles erunt.



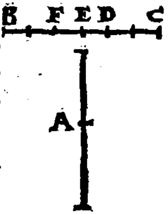
Theorema 14. Propoſitio 17.

Si duæ magnitudines incõmenſurabiles cõponantur, ipſa quoque tota magnitudo ſingulis partibus componentibus incommẽſurabilis erit. Quòd ſi tota alteri parti incõmenſurabilis fuerit, illæ quoque primę magnitudines inter ſe incõmmẽſurabiles erunt.



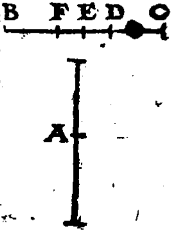
Theorema 15. Propositio 18.

Si fuerint duæ rectæ lineæ inæquales, & quartæ parti quadrati quod describitur à minore, æquale parallelogrammũ applicetur secundũ maiorem, ex qua maiore tãtum excurrat extra latus parallelogrãmi, quantum est alterum latus ipsius parallelogrammi: si præterea parallelogrammũ sui applicatione diuidat lineã illam in partes inter se commẽsurabiles longitudine, illa maior linea tanto plus potest quã minor, quantũ est quadratum lineæ sibi commensurabilis longitudine. Quòd si maior plus posit quàm minor, tanto quantũ est quadratum lineæ sibi cõmensurabilis lõgitudine, &ettereà quartæ parti quadrati lineæ minoris ęquale parallelogrãmum applicetur secundum maioré, ex qua maiore tantum excurrat extra latus parallelogrâmi, quantum est alterum latus ipsius parallelogrammi, parallelogrammum sui applicatione diuidit maioré in partes inter se longitudine commẽsurabiles.

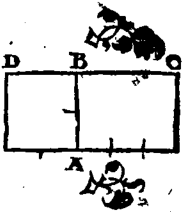


Theorema 16. Propositio 19.

Si fuerint duæ rectæ inæquales, quartæ autem parti quadrati lineæ minoris ęquale le parallelogrammorum ſecundum lineã maiorem applicetur, ex qua linea tantum excurrat extra latus parallelogrãmi, quãtum eſt alterum latus eiuſdem parallelogrammi: ſi parallelogrammũ præterea ſui applicatione diuidat lineã in partes inter ſe longitudine incommẽſurabiles, maior illa linea tantò plus poteſt quàm minor, quantum eſt quadratum lineæ ſibi maiori incommenſurabilis longitudine. Quòd ſi maior linea tãtò plus poſsit quàm minor, quantum eſt quadratum lineæ incõmenſurabilis ſibi lõgitudine: & preterea quartæ parti quadrati lineę minoris ęquale parallelogrammũ applicetur ſecũdum maiorem, ex qua tantũ excurrat extra latus parallelogrãmi, quantum eſt alterum latus ipſius: parallelogrammum ſui applicatione diuidit maiorem in partes inter ſe incommenſurabiles lõgitudine.



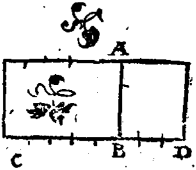
Theorema 17. Propoſitio 20.



Superficies rectangula contẽta ex lineis rectis rationalib⁹ lõgitudine commenſurabilibus ſecundum vnum aliquem modũ ex antedictis, rationalis eſt.

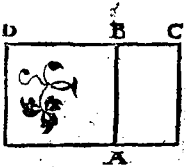
Theorema 18. Propoſitio 21.

Si rationale ſecundũ lineam rationalem applicetur, habebit alterũ latus lineã rationalem & cõmenſurabilem longitudine lineæ cui rationale parallelogrammũ applicatur.



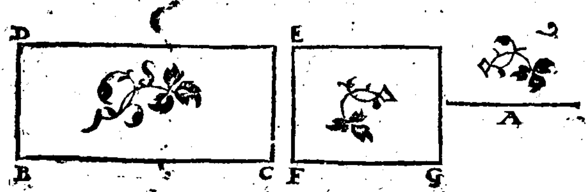
Theorema 19. Propoſitio 22.

Superficies rectangula cõtenta duabus lineis rectis rationalibus potentia tantum cõmẽſurabilibus, irrationalis eſt. Linea autem quæ illam ſuperficiem poteſt, irrationalis & ipſa eſt: vocetur verò medialis.



Theorema 20. Propoſitio 23.

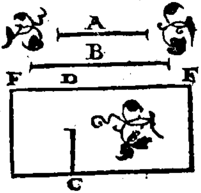
Quadrati lineæ medialis applicati ſecun-



dum

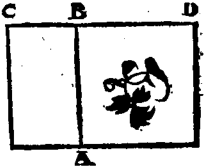
dum lineam rationalem, alterum latus est linea rationalis, & incõmensurabilis lõgitudine lineæ secundum quam applicatur.

Theorema 21. Propositio 24.



Linea recta mediali cõmẽsurabilis, est ipsa quoque medialis.

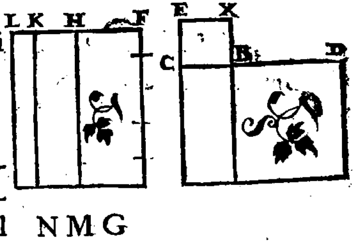
Theorema 22. Propositio 25.



Parallelogrammũ rectãgulum cõtentum ex lineis medialibus longitudine cõmensurabilibus, mediale est.

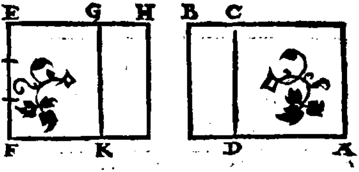
Theorema 23. Propositio 26.

Parallelogrammum rectangulum cõprehensum duabus lineis medialibus potentia tãtùm cõmensurabilib9, vel rationale est, vel mediale.



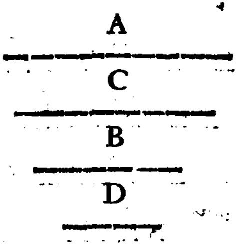
Theorema 24. Propositio 27.

Mediale nõ est maius quàm mediale superficie rationali.



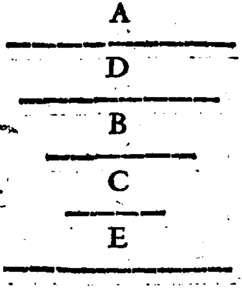
Proble. 4. Propositio 28.

Mediales lineas inuenire potentia tantùm commensurabiles rationale comprehendentes.



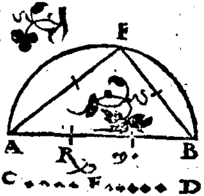
Problema 5. Propositio 29.

Mediales lineas inuenire potentia tantùm commensurabiles mediale comprehendendentes.



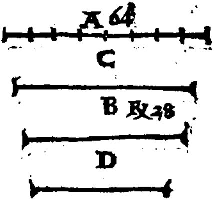
Problema 6. Propositio 30.

Reperire duas rationales potétia tantùm commésurabiles huiusmodi, vt maior ex illis possit plus quàm minor quadrato lineæ sibi commensurabilis longitudine.



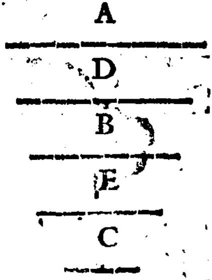
Problema 7. Propositio 31.

Reperire duas lineas mediales potentia tátùm commensurabiles rationalem superficié cótinentes, tales inquã vt maior possit plus quàm minor quadrato lineæ sibi commésurabilis longitudine.



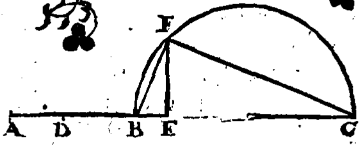
Problema 8. Propositio 32.

Reperire duas lineas mediales potentia tantùm commensurabiles medialem superficiem continentes, huiusmodi vt maior plus possit quàm minor quadrato lineæ sibi commensurabilis longitudine.



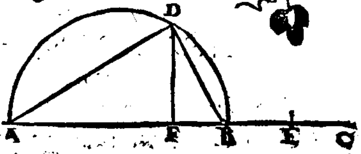
Problema 9. Propoſitio 33.

Reperire duas rectas potētia incommēſurabiles, quarum quadrata ſimul addita faciant ſuperficiē rationalē, parallelogrāmum verò ex ipſis contentum ſit mediale.



Problema 10. Propoſitio 34.

Reperire lineas duas rectas potentia incōmenſurabiles, cōficientes compoſitū ex ipſarum quadratis mediale, parallelogrammū verò ex ipſis contentum rationale.



Problema 11. Propoſitio 35.

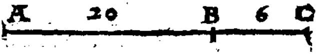
Reperire duas lineas rectas potentia incōmenſurabiles, cōficientes id quod ex ipſarum quadratis cōponitur mediale, ſimulque

que parallelogrammũ ex ipsis cōtētum, mediale, quod præterea parallelogrãmũ sit incōmensurabile cōposito ex quadratis ipsarum.

PRINCIPIVM SENARIOrum per compositionem.

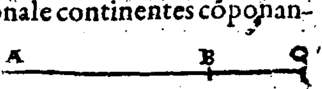
Theorema 25. Propositio 36.

Si duæ rationales potētia tantùm commēsurabiles cōponantur, tota linea erit irrationalis. Vocetur autem Binomium.



Theorema 26. Propositio 37.

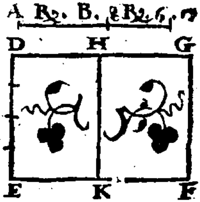
Si duæ mediales potentia tantùm cōmensurabiles rationale continentes cōponantur, tota linea est irrationalis, vocetur autem Bimediale prius.



Theo-

Theorema 27. Propositio 38.

Si duæ mediales potétia tantùm cómensurabiles mediale continentes cõ ponãtur, tota linea est irrationalis: vocetur auté Bimediale secundum.



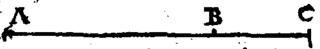
Theorema 28. Propositio 39.

Si duæ rectæ potentia incommésurabiles cõponantur, conficientes compositum ex quadratis ipsarũ rationale, parallelogrammum verò ex ipsis contentũ mediale, tota linea recta

est irrationalis. Vocetur auté linea maior.

Theorema 29. Propositio 40.

Si duæ rectæ potentia incómensurabiles componantur, conficientes compositũ ex ipsarũ quadratis mediale, id verò ꝙ fit ex ipsis, rationale, tota linea est irrationalis. Vocetur autem

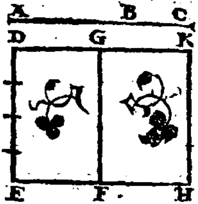


potens rationale & mediale.

Theorema 30. Propositio 41.

Si duæ rectę potentia incommensurabiles componantur, conficientes cõpositum ex quadratis ipsarum mediale, & quod cótinetur

netur ex ipſis, mediale, & præterea incõmenſurabile cõpoſito ex quadratis ipſarũ, tota linea eſt irrationalis. Vocetur autem potens duo medialia.



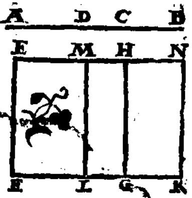
Theorema 31. Propoſitio 42.

Binomium in vnico tantùm pũcto diuiditur in ſua nomina, id eſt in lineas ex quibus componitur.

Theorema 32. Propoſitio 43.

Bimediale prius in vnico tantũ puncto diuiditur in ſua. nomina.

Problema 33. Propoſitio 44.



Bimediale ſecundum in vnico tantũ puncto diuiditur in ſua nomina.

Problema 34. Propoſitio 45.

Linea maior in vnico tãtùm in pũcto diuiditur in ſua nomina.

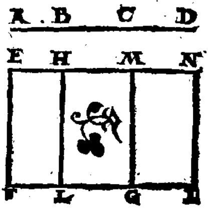
Theo-

Theorema 35. Propositio 46.

Linea potés rationale & mediale in vnico tãtùm pũcto diuiditur in ſua nomina.

Theorema 36. Propoſitio 47.

Linea potés duo medialia in vnico tantùm pũcto diuiditur in ſua nomina.



DEFINITIONES ſecundæ.

Propoſita linea rationali,& binomio diuiſo in ſua nomina, cuius binomij maius nomen, id eſt, maior portio poſsit pluſquam minus nomen quadrato lineæ ſibi, maiori inquam nomini, commenſurabilis longitudine.

I

Si quidé maius nomen fuerit cõmenſurabile longitudine propoſitæ lineę rationali, vocetur tota linea Binomium primum.

2

Si verò minus nomen, id eſt minor portio Binomij, fuerit cõmenſurabile lõgitudine ne propositæ lineæ rationali, vocetur tota linea Binomium secundum.

3

Si verò neutrum nomen fuerit commẽsurabile longitudine propositæ lineæ rationali, vocetur Binomium tertium.

Rursus si maius nomen possit plusquã minus nomen quadrato lineæ sibi incommensurabilis longitudine.

4

Si quidem maius nomen est commensurabile lõgitudine propositæ lineæ rationali, vocetur tota linea Binomium quartum.

5

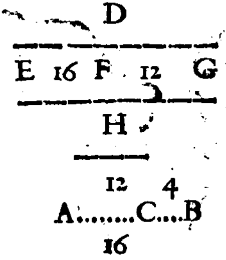
Si verò minus nomen fuerit cõmensurabile longitudine lineæ rationali, vocetur Binomium quintum.

6

Si verò neutrum nomen fuerit longitudine commensurabile lineæ rationali, vocetur illa Binomium sextum.

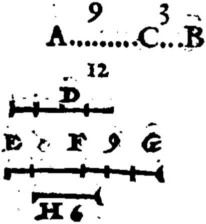
Proble.12. Propositio 48.

Reperire Binomium primum.



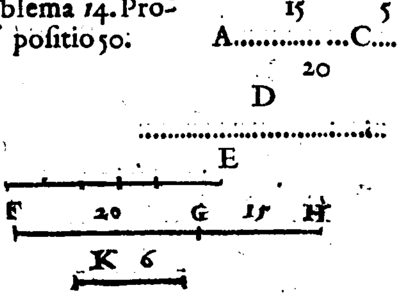
Proble-

Problema 13. Propositio 49.



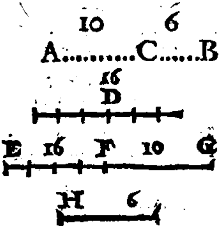
Reperire Binomium secundum.

Problema 14. Propositio 50.



Reperire Binomium tertiũ.

Problema 15. Propositio 51.

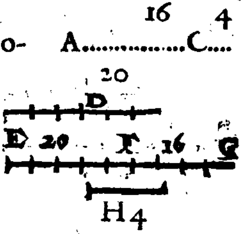


Reperire Binomium quartum.

Proble-

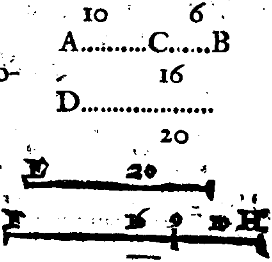
Problema 16. Propositio 52.

Reperire Binomium quintum.



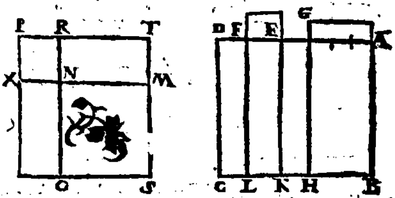
Probl. 17. Propositio 53.

Reperire Binonimum sextum.



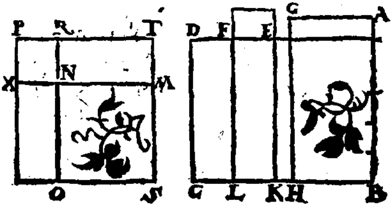
Theorema 37. Propositio 54.

Si superficies contéta fuerit ex rationali & Binomio primo, linea quę illã superficiem potest, est irrationalis, quæ Binomiũ vocatur.



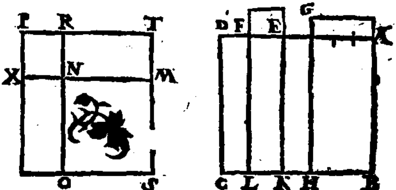
Theorema 38. Propositio 55.

Si superficies cótenta fuerit ex linea rationali & Binomio secundo, linea potens illá superficié est irrationalis, quę Bimediale primum vocatur.



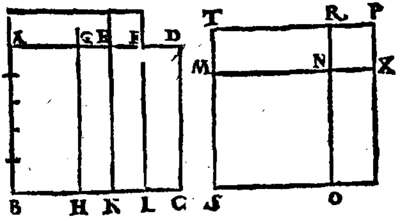
Theorema 39. Propositio 56.

Si superficies cótineatur ex rationali & Binomio tertio, linea q̃ illá superficiem potest, est irrationalis q̃ dicitur Bimediale secundú.



Theorema 40. Propositio 57.

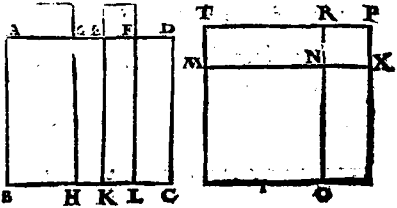
Si superficies cótineatur ex rationali & Binomio



mio quarto, linea potens ſuperficiem illã, eſt irrationalis, quæ dicitur maior.

Theorema 41. Propoſitio 58.

Si ſuperficies cõtineatur ex rationali & Binomio quinto, linea quæ illam ſuperficiẽ poteſt eſt irrationalis, quę dicitur potẽs rationale & mediale.

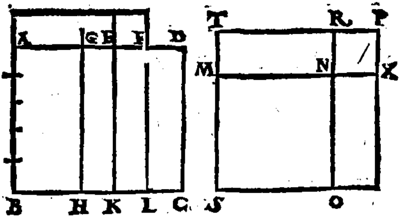


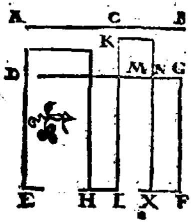
Theorema 42. Propoſitio 59.

Si ſuperficies contineatur ex rationali & Binomio ſexto, linea quæ illam ſuperficiem poteſt eſt irrationalis, quæ dicitur

L 2 potens

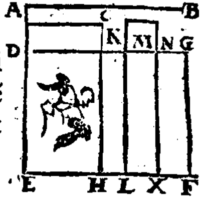
potens duo mediala.





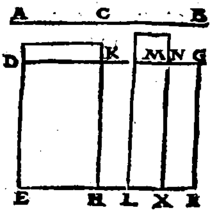
Theorema 43. Propositio 60.

Quadratum Binomij secundum lineam rationalem applicatum, facit alterum latus Binomium primum.



Theorema 44. Propositio 61.

Quadratũ Bimedialis primi secundum rationalẽ lineam applicatum, facit alterum latus Binomiũ secundum.

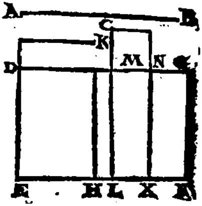


Theorema 45. Propositio 62.

Quadratũ Bimedialis secũdi secundũ rationalẽ applicatum, facit alterũ latus Binominũ tertiũ.

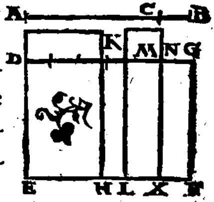
Theo-

Theorema 46. Propositio 63.



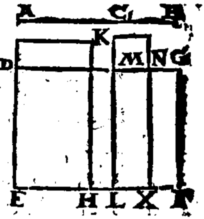
Quadratum lineæ maioris secundum lineam rationalem applicatum, facit alterum latus Binomium quartum.

Theorema 47. Propositio 64.



Quadratum lineæ potẽtis rationale & mediale secundum rationalẽ applicatũ, facit alterum latus Binomium quintũ.

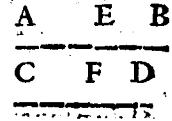
Theorema 48. Propositio 65.



Quadratum lineę potentis duo medialia secundũ rationalem applicatum, facit alterum latus Binomium sextum.

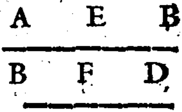
Theorema 49. Propositio 66.

Linea longitudine commensurabilis Binomio est & ipsa Binomium eiusdẽ ordinis.



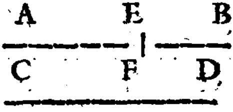
Theorema 50. Propoſitio 67.

Linea longitudine commenſurabilis alteri bimedialium, eſt & ipſa bimediale etiam eiuſdē ordinis.



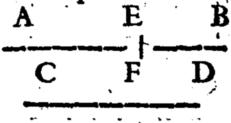
Theorema 51. Propoſitio 68.

Linea cōmēſurabilis lineæ maiori, eſt & ipſa maior.



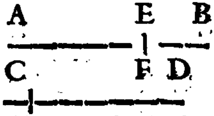
Teorema 52. Propoſitio 69.

Linea commēſurabilis lineæ potenti rationale & mediale, eſt & ipſa linea potens rationale & mediale.



Theorema 53. Propoſitio 70.

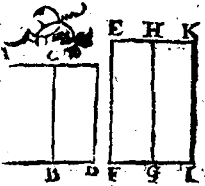
Linea commenſurabilis lineæ potenti duo medialia, eſt & ipſa linea potens duo medialia.



Theorema 54. Propoſitio 71.

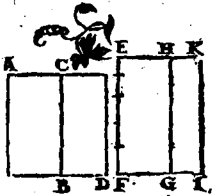
Si duæ ſuperficies rationalis & medialis ſimul componantur, linea quę totā ſuperficiem

ciem compositã potest, est vna ex quatuor irrationalibus, vel ea quæ dicitur Binomium, vel bimediale primum, vel linea maior, vel linea potẽs rationale & mediale.



Theorema 55. Propositio 72.

Si duæ superficies mediales incõmensurabiles simul componãtur, fiunt reliquæ duæ lineę irrationales, vel bimediale secundum, vel linea potẽs duo medialia



SCHOLIVM.

Binomium & cætera consequentes lineæ irrationales, neque sunt eædem cum linea mediali, neque ipsa inter se.

Nam quadratum lineæ medialis applicatum secundum lineam rationalem, facit alterum latus lineam rationalem, & longitudine incommensurabilem lineæ secundum quam applicatur, hoc est, lineæ rationali, per 23.

Quadratum verò Binomij secundum rationalem applicatum, facit alterum latus Binomium primum, per 60.

Quadratum verò Bimedialis primi secundùm rationalem applicatum, facit alterum latus Binomiū secundum, per 61.

Quadratum verò Bimedialis secundi secundum rationalem applicatum, facit alterum latus Binomiū tertium, per 62.

Quadratum verò lineæ maioris secundum rationalem applicatum, facit alterum latus Binomiū quartum, per 63.

Quadratum verò lineæ potētis rationale & mediale secundum rationalem applicatum, facit alterū latus Binomium quintum, per 64.

Quadratum verò lineæ potentis duo medialia secūdum rationalem applicatum, facit alterum latus Binomium sextum, per 65.

Cùm igitur dicta latera, quæ latitudines vocantur, differant & à prima latitudine, quoniam est rationalis, cùm inter se quoque differant, eo quia sunt Binomia diuersorum ordinum: manifestum est ipsas lineas irrationales, differentes esse inter se.

SECVN-

SECVNDVS ORDO ALterius sermonis, qui est de detractione.

Principium senariorum per detractionē.

Theorema 56. Propositio 73.

Si de linea rationali detrahatur rationalis potentia tantùm commēsurabilis ipsi toti, residua est irrationalis, vocetur autē Residuum.

A C B

Theorema 57. Propositio 74.

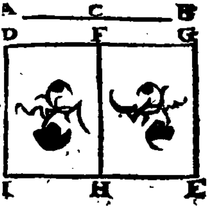
Si de linea mediali detrahatur medialis potentia tantū cōmensurabilis toti lineæ, quæ verò detracta est cū tota contineat superficiem rationalem, residua est irrationalis. Vocetur autem Residuū mediale primum.

A C B

Theorema 58. Propositio 75.

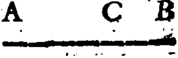
Si de linea mediali detrahatur medialis

tentia tantùm commensurabilis toti, quæ verò detracta est, cùm tota cõtineat superficiẽ medialẽ, reliqua est irrationalis. Vocetur autem Residuũ mediale secundum



Theorema 57. Propositio 76.

Si de linea recta detrahatur recta potẽtia incommensurabilis toti, compositum autem ex quadratis totius lineæ & lineæ detractæ sit rationale, parallelogrammũ verò ex ijsdem contentum sit mediale, reliqua linea erit irrationalis. Vocetur autem linea minor.



Theorema 58. Propositio 77.

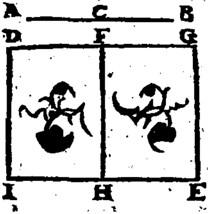
Si de linea recta detrahatur recta potentia

tia incommenſurabilis toti lineæ, compoſitũ autem ex quadratis totius & lineæ detractæ ſit mediale, parallelogrammum verò bis ex eiſdem contentum ſit rationale, reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie rationali totam ſuperficiem medialem.

Theorema 59. Propoſitio 78.

Si de linea recta detrahatur recta potentia incommenſurabilis toti lineæ, compoſitum autem ex quadratis totius & lineæ detractæ ſit mediale, parallelogrammum verò bis ex ijſdem ſit etiam mediale: præterea ſint quadrata ipſarum incommenſurabilia parallelogrammo bis ex ijſdem cõtento, reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie

ficie mediali totam ſuperficiem medialem.

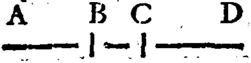


Theorema 60. Propoſitio 79.

Reſiduo vnica tantùm linea recta cóiungitur rationalis, potentia tantùm commenſurabilis toti lineæ.

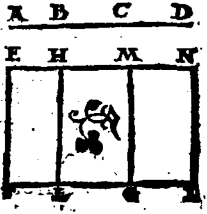
Theorema 61. Propoſitio 80.

Reſiduo mediali primo vnica tãtùm linea coniungitur medialis, potentia tantùm cómenſurabilis toti, ipſa cum tota continés rationale.



Theorema 62. Propoſitio 81.

Reſiduo mediali ſecundo vnica tantùm cóiungitur medialis, potentia tantùm commenſurabilia toti ipſa cum tota continens mediale.



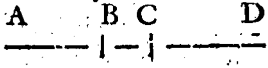
Theorema 63. Propoſitio 82.

Lineæ minori vnica tantùm recta cóiungitur

tur potentia incommenſurabilis toti, faciens cum tota compoſitum ex quadratis ipſarum rationale, id verò parallelogrammum, quod bis ex ipſis fit, mediale.

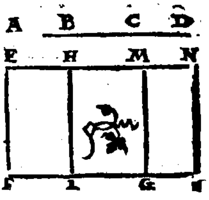
Theorema 64. Propoſitio 83.

Lineæ facienti cum ſuperficie rationali totam ſuperficiem medialem, vnica tantùm coniungitur linea recta potentia incommenſurabilis toti, faciens autem cum tota compoſitum ex quadratis ipſarũ, mediale, id verò quod fit bis ex ipſis, rationale.



Theorema 65. Propoſitio 84.

Lineæ cum mediali ſuperficie facienti totam ſuperficiem medialem, vnica tantùm coniũgitur linea potẽtia toti incommenſurabilis, faciens cum tota compoſitũ ex quadratis ipſarũ mediale, id verò quod fit bis ex ipſis etiam mediale, & prætereà faciens compoſitum ex quadratis ipſarum incommenſurabile ei quod fit bis ex ipſis.



DEFINITIONES TERTIAE.

Propoſita linea rationali & reſiduo.

1

Si quidem tota, nempe compoſita ex ipſo reſiduo & linea illi cōiuncta, plus poteſt quàm coniuncta, quadrato lineæ ſibi cōmenſurabilis longitudine, fueritq́; tota longitudine commēſurabilis lineæ propoſitæ rationali, reſiduū ipſum vocetur Reſiduum primum.

2

Si verò coniuncta fuerit longitudine com menſurabilis rationali, ipſa autem tota plus poſſit quam coniuncta, quadrato lineæ ſibi longitudine commenſurabilis, reſiduum vocetur Reſiduum ſecundū.

3

Si verò neutra lineārū fuerit longitudine commenſurabilis rationali, poſſit autem ipſa tota pluſquam coniuncta, quadrato lineæ ſibi longitudine commēſurabilis, vocetur Reſiduum tertium.

Rurſus ſi tota poſſit plus quàm coniuncta, quadrato lineæ ſibi longitudine incōmenſurabilis.

4

Et quidem ſi tota fuerit longitudine com men-

mensurabilis ipsi rationali, vocetur Residuum quartum.

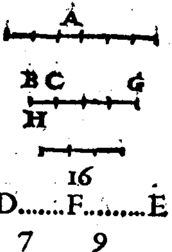
5

Si verò coniuncta fuerit lõgitudine commensurabilis rationali,& tota plus possit quàm coniuncta,quadrato lineæ sibi longitudine incõmensurabilis, vocetur Residuum quintum.

6

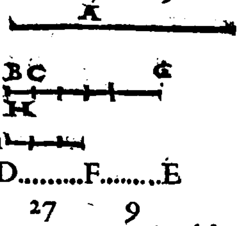
Si verò neutra linearum fuerit cõmensurabilis lõgitudine ipsi rationali, fueritque tota potentior quã coniuncta,quadrato lineæ sibi lõgitudine incommẽsurabilis,vocetur Residuum sextum.

Problema 18. Propositio 85.



Reperire primum Residuum.

Problema 19. Propositio 86.



Reperire secundum Residuum.

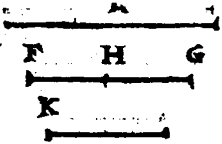
Proble-

E...........

Problema 20. Propositio 87. 12

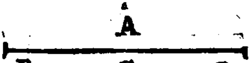
B.........D.......C

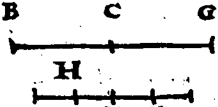
9 7



Reperire tertium Residuum.

Probl. 21. Propositio 88.

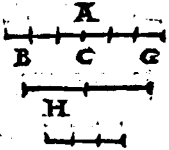




Reperire quartũ Residuum. D...............F....E

16 4

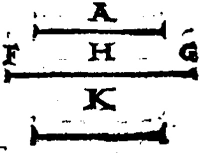
Problema 22. Propositio 89.



Reperire quintum Residuum. D.......F....E

25 7

Problema 23. Propositio 90.



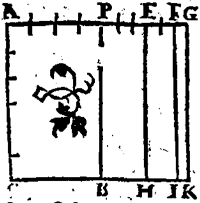
Reperire sextum Residuum. E...............

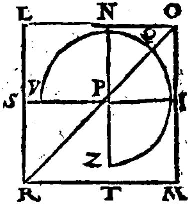
15

Theo-B........D......C

Theorema 66. Propositio 91.

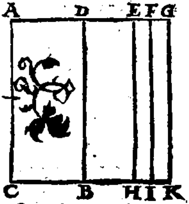
Si superficies cõtineatur ex linea rationali & residuo primo linea, quæ illã supficiem potest, est residuum.

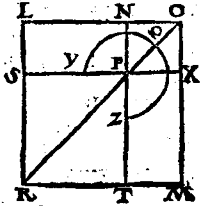




Theorema 67. Propositio 92.

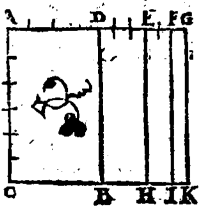
Si superficies cõtineatur ex linea rationali & residuo secũdo, linea q̃ illam supficiẽ potest, est residuum mediale primum.

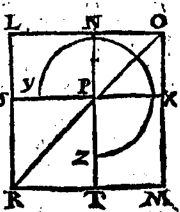




Theorema 68. Propositio 93.

Si supficies cõtineatur ex linea rationali & residuo ter-

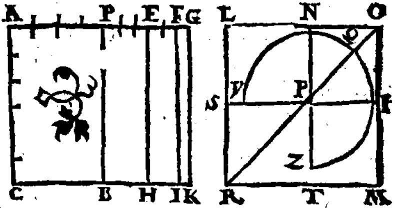




tio, linea quæ illam ſuperficiem poteſt, eſt reſiduum mediale ſecundum.

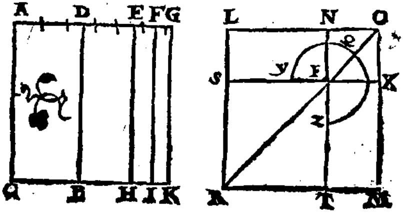
Theorema 69. Propoſitio 94.

Si ſuperficies cõtineatur ex linea rationali & reſiduo quarto linea q̃ illã ſuperficiem poteſt, eſt linea minor.



Theorema 70. Propoſitio 95.

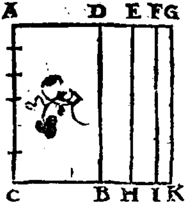
Si ſuperficies cõtineatur ex linea rationali & reſiduo quinto, linea quæ illã ſuperficiẽ poteſt, eſt ea quæ dicitur cum rationali ſuperficie faciens totã medialem.

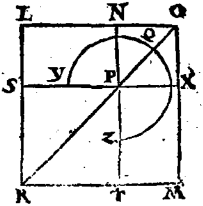


Theorema 71. Propoſitio 96.

Si ſuperficies cõtineatur ex linea rationali &

& residuo sexto, linea quæ illam superficiē pót, est ea quę dicitur faciés cum mediali

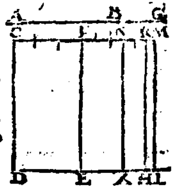




superficie totam medialem.

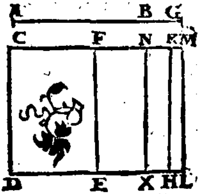
Theorema 72. Propositio 97.

Quadratum residui secundum lineam rationalem applicatum, facit alterum lat• residuum primum.



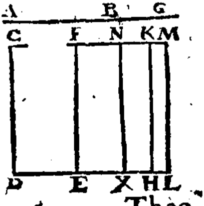
Theorema 73. Propositio 98.

Quadratum residui medialis primi secundũ rationalem applicatũ, facit alterũ latus residuũ secundum.

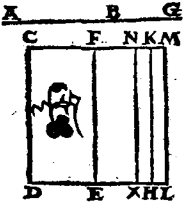


Theorema 74. Propositio 99.

Quadratũ residui medialis secũdi secundum rationalẽ applicatũ, facit alterũ latus residuũ tertium.

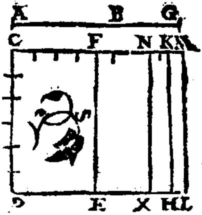


Theorema 75. Propositio 100.



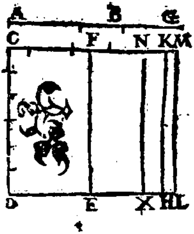
Quadratũ lineæ minoris ſecundum rationalẽ applicatum, facit alterũ latus reſiduum quartũ.

Theorema 76. Propoſitio 101.



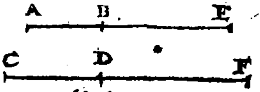
Quadratũ lineæ cum rationali ſuperficie faciẽtis totam medialem, ſecũdum rationalem applicatum, facit alterum latus reſiduum quintum.

Theorema 77. Propoſitio 102.



Quadratum lineæ cum mediali ſuperficie faciẽtis totam medialem, ſecundum rationalem applicatum, facit alterum latus reſiduum ſextum.

Theorema 78. Propoſitio 103.



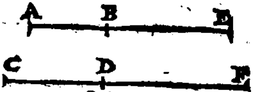
Linea reſiduo cõmẽſurabilis lõgitudine, eſt & ipſa reſiduum, & eiuſdem ordinis.

Theorema 79. Propoſitio 104.

Linea cõmenſurabilis reſiduo mediali, eſt

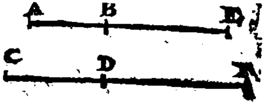
&

& ipſa reſiduũ mediale, & eiuſdẽ ordinis.



Theorema 80. Propoſitio 105.

Linea cõmẽſurabilis lineæ minori, eſt & ipſa linea minor.

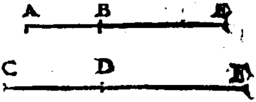


Theorema 81. Propoſitio 106.

Linea commenſurabilis lineæ cũ rationali ſuperficie facienti totã medialẽ, eſt & ipſa linea cum rationali ſuperficie faciens totam medialem.

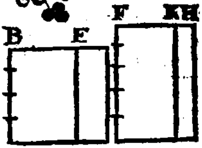
Theorema 82. Propoſitio 107.

Linea cõmẽſurabilis lineæ cũ mediali ſuperficie facienti totam medialem, eſt & ipſa cum mediali ſuperficie faciens totam medialem.

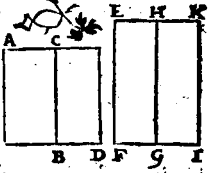


Theorema 83. Propoſitio 108.

Si de ſuperficie rationali detrahatur ſuperficies medialis, linea quæ reliquã ſuperficiem poteſt, eſt alterutra ex duabus irrationalibus, aut reſiduũ, aut linea minor.

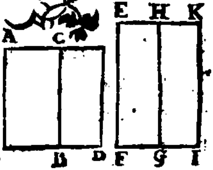


Theorema 84. Propositio 109.



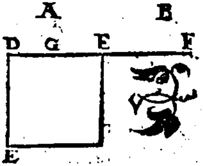
Si de superficie mediali detrahatur superficies rationalis, aliæ duæ irrationales fiũt, aut residuum mediale primũ aut cũ rationali superficiem faciens totam medialem.

Theorema 85. Propositio 110.



Si de superficie mediali detrahatur superficies medialis q̃ sit incommẽsurabilis toti, reliquæ duæ fiunt irrationales, aut residuum mediale secundum, aut cũ mediali superficie faciẽs totam medialem.

Theorema 86. Propositio 111.



Linea quæ Residuum dicitur, non est eadem cum ea quæ dicitur Binomium.

SCHO-

SCHOLIVM.

Linea quæ Residuum dicitur, & cæteræ quinque eã consequentes irrationales, neque lineæ mediali neque sibi ipsæ inter se sunt eædem. Nam quadratum lineæ medialis secundum rationalem applicatum, facit alterum latus, rationalem lineam longitudine incommensurabilem ei, secundum quam applicatur, per 23.

Quadratum verò residui secundum rationalem applicatum, facit alterum latus residuum primum per 97.

Quadratum verò residui medialis primi secundum rationalem applicatum, facit alterum latus residuum secundum, per 98.

Quadratum verò residui medialis secundi, facit alterum latus residuum tertium, per 99.

Quadratum verò lineæ minoris, facit alterum latus residuum quartum, per 100.

Quadratum verò lineæ cum rationali superficie facientis totam medialem, facit alterum latus residuum quintum, per 101.

Quadratum verò lineæ cum mediali superficie facientis totam medialem, secundum rationalem applicatum, facit alterum latus residuum sextum, per 102.

Cùm igitur dicta latera, quæ sunt latitudines cuiusque parallelogrammi vnicuique quadrato æqualis & secundum rationalem applicati, differant & à primo latere, & ipsa inter se (nam à primo differunt: quoniam sunt resi-

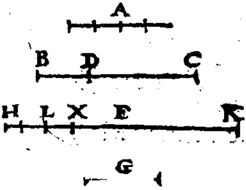
dua non eiusdem ordinis) constat ipsas quoque lineas irrationales inter se differentes esse. Et quoniam demonstratum est, Residuum non esse idem quod Binomium, quadrata autem residui & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex residuis eiusdem ordinis cuius sunt & residua, quorum quadrata applicantur rationali similiter & quadrata Binomij & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex Binomijs eiusdem ordinis, cuius sunt & Binomia, quorum quadrata applicantur rationali. Ergo linea irrationales qua cõsequuntur Binomium, & qua consequuntur residuum, sunt inter se differentes. Quare dicta linea omnes irrationales sunt numero 13.

1 Medialis.
2 Binomium.
3 Bimediale primum.
4 Bimediale secundum.
5 Maior.
6 Potens rationale & mediale.
7 Potẽs duo medialia.
8 Residuum
9 Residuum mediale primum.
10 Residuum mediale secundum.
11 Minor.
12 Faciens cum rationali superficie totam medialem.
13 Faciens cum mediali superficie totam medialem.

Theo-

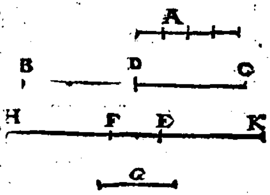
Theorema 87. Propositio 112.

Quadratũ lineæ rationalis secundum Binomiũ applicatũ, facit alterũ latus residuũ, cuius nomina sunt cõmensurabilia Binomij nominibus, & in eadẽ proportione præterea id quod fit Residuum, eundem ordinem retinet quem Binomium.



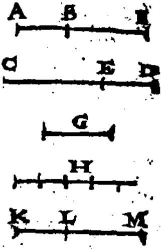
Theorema 88. Propositio 113.

Quadratũ lineæ rationalis secũdum residuum applicatũ, facit alterum latus Binomium, cuius nomina sunt cõmensurabilia nominibus residui & in eadẽ proportione: preterea id quod fit Binomiũ, est eiusdem ordinis, cuius & Residuum.



Theorema 89. Propositio 114.

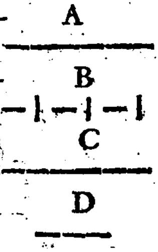
Si parallelogrammũ contineatur ex resi-

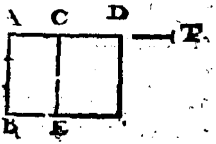


duo & Binomio, cuius nomina ſunt cōmenſurabilia nominibus reſidui & in eadē proportione, linea quæ illam ſuperficiem poteſt, eſt rationalis.

Theorema 90. Propoſitio 115.

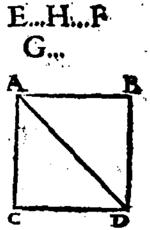
Ex linea mediali naſcūtur lineę irrationales innumerabiles, quarum nulla vlli ante dictarum eadem ſit.





Propoſitio 116.

Propoſitū nobis eſto demōſtrare in figuris quadratis diametrū eſſe longitudine incōmenſurabilem ipſi lateri.



ELEMENTI X. FINIS.

EVCLIDIS ELEMENTVM VNDECIMVM, ET SOLIDORVM *primum.*

DEFINITIONES.

1

Solidum, est quod longitudinem, latitunem,& crassitudinem habet.

2

Solidi autem extremum est superficies.

3

Linea recta est ad planum recta, cùm ad rectas oẽs lineas, à quibus illa tangitur, quæque in proposito sunt plano, rectos angulos efficit.

4

Planum ad planum rectum est, cùm rectæ lineæ, quæ cõmuni planorum sectioni ad rectos angulos in vno planorum ducũtur, alteri plano ad rectos sunt angulos.

5

Rectæ lineę ad planũ inclinatio, acutus est angulus, ipsa insistente linea & adiũcta altera comprehẽsus, cùm à sublimi rectę illius lineę termino deducta fuerit ꝑpẽdicularis,

laris, atq; à puncto quod perpendicularis in ipso plano fecerit, ad propositę illius lineæ extremum, quod in eodẽ est plano, altera recta linea fuerit adiuncta.

6

Plani ad planũ inclinatio, acutus est angulus rectis lineis contentus, quæ in vtroque planorum ad idem cõmunis sectionis pũctũ ductæ, rectos ipsi sectioni angulos efficiunt.

7

Planum similiter inclinatum esse ad planum, atq; alterum ad alterum dicitur, cũ dicti inclinationum anguli inter se sunt ęquales.

8

Parallela plana, sunt quæ eodem non incidunt, nec concurrunt.

9

Similes figuræ solidæ, sunt quæ similibus planis, multitudine æqualibus cõtinẽtur.

10

Aequales & similes figurę solidę sunt, quę similibus planis, multitudine & magnitudine æqualibus continentur.

11

Solidus angulus, est plurium quàm duarũ linearum, quæ se mutuò contingant, nec in eadem sint superficie, ad omnes lineas inclinatio.

Aliter.

Aliter.

Solidus angulus, est qui pluribus quā duobus planis angulis in eodem non cōsisten tibus plano, sed ad vnū punctū collectis, continetur.

12

Pyramis, est figura solida quæ planis cōtinetur, ab vno plano ad vnum punctū collecta.

13

Prisma, figura est solida quæ planis cōtinetur; quorum aduersa duo sunt & æqualia & similia & parallela, alia verò parallelogramma.

14

Sphęra est figura, quæ cōuerso circum quiescentem diametrum semicirculo cōtinetur, cùm in eundem rursus locū restitutus fuerit, vnde moueri cœperat.

15

Axis autem sphæræ, est quiescēs illa linea circum quam semicirculus conuertitur.

16

Centrum verò Sphæræ est idē, quod & semicirculi.

17

Diameter autem Sphæræ, est recta quædā linea per centrum ducta, & vtrinq; à sphęræ superficie terminata.

Cœnus

18

Conus est figura, quæ conuerso circũquiescens alterum latus eorum quæ rectũ angulum continent, orthogonio triangulo continetur, cùm in eundem rursus locum illud triangulum restitutum fuerit, vndè moueri cœperat. Atq; si quiescens recta linea æqualis sit alteri, quę circum rectũ angulum conuertitur, rectangulus erit Conus: sin minor, amblygonius: si verò maior, oxygonius.

19

Axis autem Coni, est quiescens illa linea, circum quam triangulum vertitur.

20

Basis verò Coni, circulus est, qui à circũducta linea recta describitur.

21

Cylindrus figura est, quæ conuerso circũ quiescens alterum latus eorum quæ rectũ angulum continẽt, parallelogrammo orthogonio comprehenditur, cùm in eundẽ rursus locum restitutum fuerit illud parallelogrammũ, vnde moueri cœperat.

22

Axis autem Cylindri, est quiescens illa recta linea, circũ quam parallelogrammum vertitur.

23

Bases verò cylindri, sunt circuli à duobus

ad-

aduersus lateribus quæ circumaguntur, descripti.

24

Similes coni & cylindri, sunt quorũ & axes & basiũ diametri proportionales sunt.

25

Cubus est figura solida, quæ sex quadratis æqualibus continetur.

26

Tetraedum est figura, quæ triãgulis quatuor æqualibus & æquilateris continetur.

27

Octaedrũ figura est solida, quæ octo triangulis æqualibus & æquilateris cõtinetur.

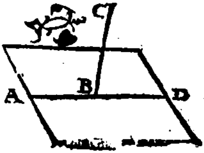
28

Dedecaedrum figura est solida, quæ duodecim pentagonis æqualibus, æquilateris, & æquiangulis continetur.

29

Eicosaedrum figura est solida, quę triãgulis viginti æqualibus & æquilateris continetur.

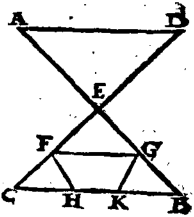
Theorema 1. Propositio 1.



Quædã rectæ lineę pars in subiecto quidem nõ est plano, quædã verò in sublimi.

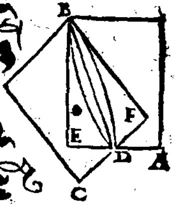
Theo-

Theorema 2. Propoſitio 2.



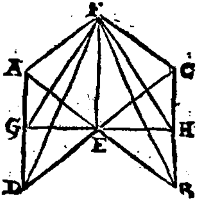
Si duę rectę lineę ſe mutuò ſecẽt, in vno ſũt plano : atque triangulum omne in vno eſt plano.

Theorema 3. Propoſitio 3.



Si duo plana ſe mutuò ſecent, communis eorũ ſectio eſt recta linea.

Theorema 4. Propoſitio 4.



Si recta linea rectis duabus lineis ſe mutuò ſecantibus, in cõmuni ſectione ad rectos angulos inſiſtat illa ducto etiam per ipſas plano ad angulos rectos erit.

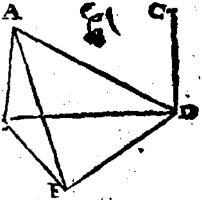
Theorema 5. Propoſitio 5.

Si recta linea rectis tribus lineis ſe mutuò tangentibus, in communi ſectione ad rectos angulos inſiſtat, illæ tres rectę in vno ſunt plano.

Theo-

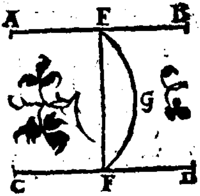
Theorema 6. Propoſitio 6.

Si duæ rectæ lineæ eidē plano ad rectos ſint angulos, parallelæ erūt illæ rectæ lineæ.



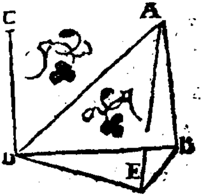
Theorema 7. Propoſitio 7.

Si duæ ſint parallelæ rectæ lineæ, in quarū vtraque ſumpta ſint quę libet pūcta, illa linea quę ad hæc puncta adiungitur, in eodem eſt cū parallelis plano.



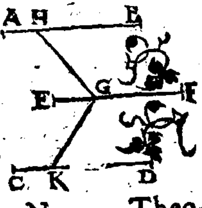
Theorema 8. Propoſitio 8.

Si duæ ſint parallelæ rectæ lineæ, quarum altera ad rectos cuidam plano ſit angulos, & reliqua eidē plano ad rectos angulos erit.

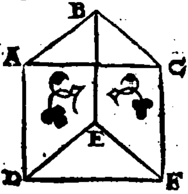


Theorema 9. Propoſitio 9.

Quæ eidem rectæ lineæ ſunt parallelæ, ſed nō in eodem cū illa plano, hæ quoque ſunt inter ſe parallelæ.

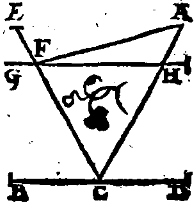


Theorema 10. Propositio 10.



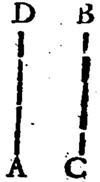
Si duę rectę lineæ se mutuò tangétes ad duas rectas se mutuò tangentes sint parallelæ, non auté in eodem plano, illę angulos æquales comprehendent.

Proble. 1. Propositio 11.



A dato sublimi puncto, in subiectum planũ perpendicularem rectã lineam ducere.

Problema 2. Propositio 12.



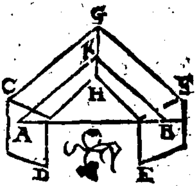
Dato plano, à puncto quod in illo datum est, ad rectos angulos rectam lineam excitare.

Theorema 11. Propositio 13.

Dato plano, à pũcto qd' in illo datum est, duæ rectæ lineæ ad rectos angulos non excitabuntur ad easdem partes.

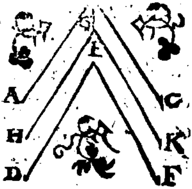
Theo-

Theorema 12. Propoſitio 14.



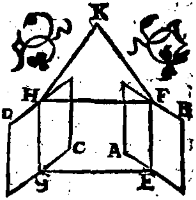
Ad quæ plana, eadem recta linea recta eſt, illa sũt parallela.

Theorema 13. Propoſitio 15.



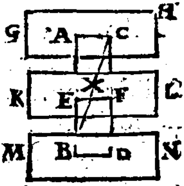
Si duæ rectæ lineæ ſe mutuò tangentes ad duas rectas ſe mutuò tangentes ſint parallelæ, non in eodem conſiſtentes plano, parallela ſunt quæ per illas ducuntur plana.

Theorema 14. Propoſitio 16.



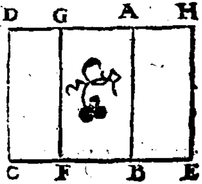
Si duo plana parallela plano quopiam ſecētur, communes illorum ſectiones ſunt parallelæ.

Theorema 15. Propoſitio 17.



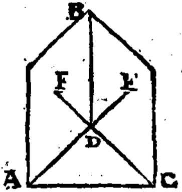
Si duæ rectæ lineę parallelis planis ſecētur, in eaſdem rationes ſecabũtur.

Theorema 16. Propositio 18.



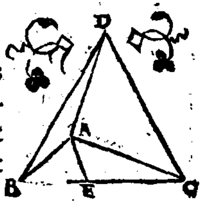
Si recta linea plano cuipiam ad rectos sit angulos, illa etiam omnia quę per ipsam plana, ad rectos eidem plano angulos erunt.

Theorema 17. Propositio 19.



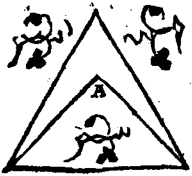
Si duo plana se mutuò secantia plano cuidã ad rectos sint angulos, cõmunis etiam illorum sectio ad rectos eidẽ plano angulos erit.

Theorema 18. Propositio 20.



Si angulus solidus planis tribus angulis contineatur, ex his duo quilibet vtut assumpti tertio sunt maiores.

Theorema 19. Propositio 21.

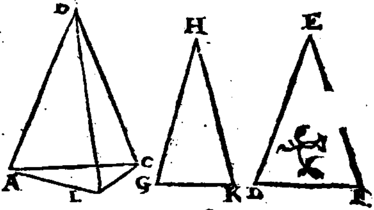


Solidus omnis angulus minoribus continetur, quàm rectis quatuor angulis planis.

Theo-

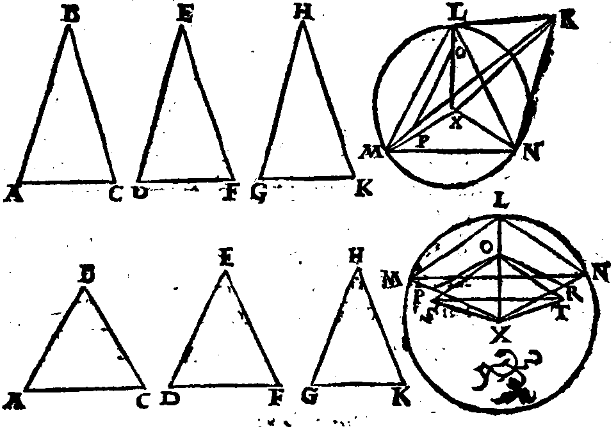
Theorema 20. Propositio 22.

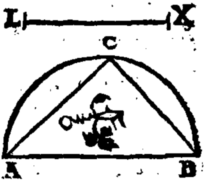
Si plani tres anguli ęqualibus rectis cõtineantur lineis, quorũ duo vt libet assumpti, tertio sint maiores, triangulum constitui potest ex lineis æquales, illas rectas coniungẽtibus.



Problema 3. Propositio 23.

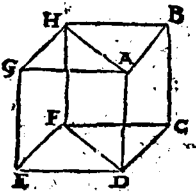
Ex planis tribus angulis, quorũ duo vt libet assumpti tertio sint maiores, solidũ angulum constituere. Decet autem illos tres angulos rectis quatuor esse minores.

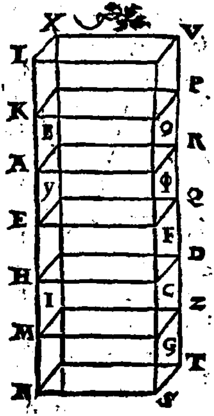




Theorema 21. Propositio 24.

Si solidum parallelis planis contineatur, aduersa illius plana & æqualia sunt & parallelogramma.





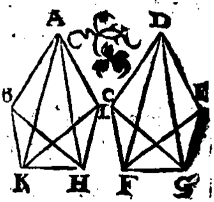
Theorema 22. Propositio 25.

Si solidum parallelis planis contentũ plano secetur aduersis planis parallelo, erit quéadmodum basis ad basim, ita solidum ad solidum.

Proble-

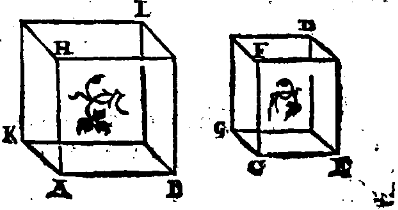
Problema 4. Propositio 26.

Ad datam rectam lineā eiusque punctū, angulum solidum constituere solido angulo dato æqualem.



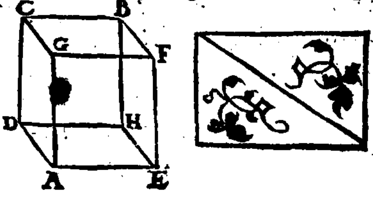
Problema 5. Propositio 27.

A data recta, dato solido parallelis planis comprehenso simile & similiter positum solidū parallelis planis contētum describere.

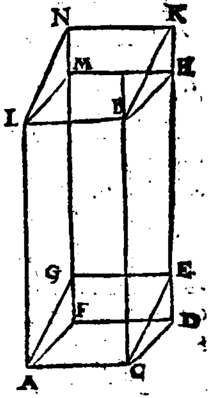


Theorema 23. Propositio 28.

Si solidū parallelis planis comprehēsum, ducto per aduersorum planorum diagoniosplano sectū sit, illud solidū ab hoc plano bifariàm secabitur.



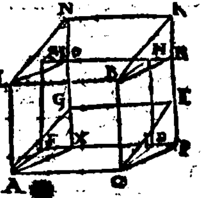
Theorema 24. Propositio 29.



Solida parallelis planis compreheſa, quę ſuper eandem baſim, & in eadē ſunt altitudine, quorum inſiſtētes lineæ in ijſdem collocantur rectis lineis, illa sūt inter ſe æqualia.

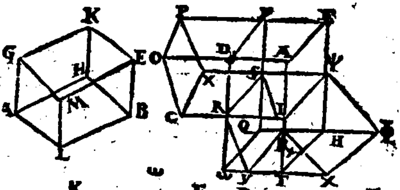
Theorema 25. Propositio 30.

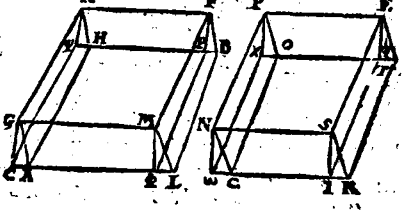
Solida parallelis planis circunſcripta, quæ ſuper eandē baſim & in eadē ſūt altitudine, quorum inſiſtentes lineæ nō in ijſdem reperiuntur rectis lineis, illa ſunt inter ſe æqualia.



Theorema 26. Propositio 31.

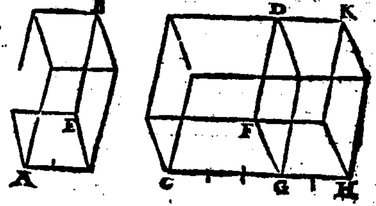
Solida parallelis planis circunſcripta, quę in in eadē sunt altitudine, æqualia sunt inter se.





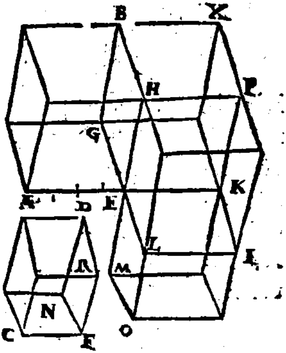
Theorema 27. Propositio 32.

Solida parallelis planis circunſcripta quæ eiuſdem ſunt altitudinis, eam habēt inter ſe rationē, quam baſes.



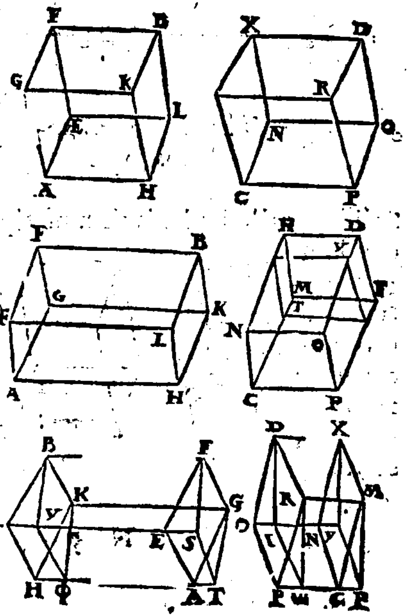
Theor.28.Propositio 33.

Similia solida parallelis planis circunscripta habẽt inter se rationẽ homologorum laterum triplicatam.



Theor.29. Propositio 34.

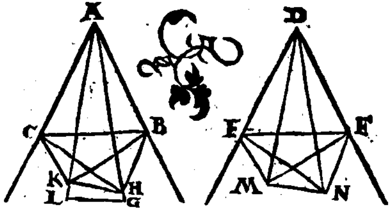
Aequalium solidorũ parallelis planis cõtẽtorum bases cũ altitudinibus reciprocãtur. Et solida parallelis planis cõtẽta, quorũ bases cum altitudi-



nibus reciprocantur, illa sunt æqualia.

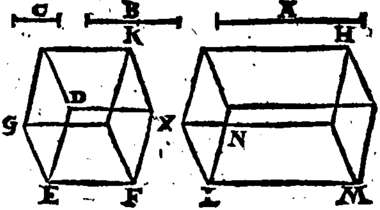
Theorema 30. Propositio 35.

Si duo plani sint anguli æquales, quorũ verticibus sublimes rectæ lineæ insistant, quæ cum lineis primò positis angulos cõtineant æquales, vtrunq; vtriq;, in sublimibus autem lineis quælibet sumpta sint pũcta, & ab his ad plana in quibus consistunt anguli primũ positi, ductæ sint perpendiculares, ab earũ verò punctis, quæ in planis signata fuerint, ad angulos primũ positos adiũctæ sint rectæ lineę, hæ cũ sublimibus æquales angulos comprehendent.



Theorema 31. Propositio 36.

Si rectæ tres lineæ sint proporionales, quod

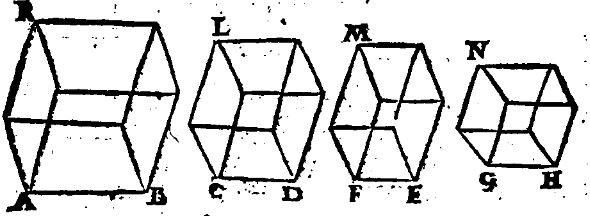


ex

ex his tribus fit ſolidum parallelis planis contentum, æquale eſt deſcripto à media linea ſolido parallelis planis comprehéſo, quod æquilaterum quidem ſit, ſed antedicto æquiangulum.

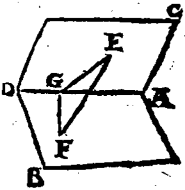
Theorema 32. Propoſitio 37.

Si rectæ quatuor lineæ ſint proportionales, illa quoque ſolida parallelis planis cõtenta, quæ ab ipſis lineis & ſimilia & ſimiliter deſcribuntur, proportionalia erũt. Et ſi ſolida parallelis planis cõprehenſa, quæ & ſimilia & ſimiliter deſcribuntur, ſint proportionalia, illæ quoque rectæ lineæ proportionales erunt.



Theorema 33. Propoſitio 38.

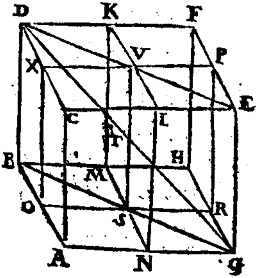
Si planum ad planũ rectum ſit, & à quodã puncto eorũ quæ in vno ſunt planorũ perpendicularis ad alterum ducta ſit, illa quæ ducitur perpẽdicularis, in cõmunẽ cadet planorũ ſectionẽ.



Theo-

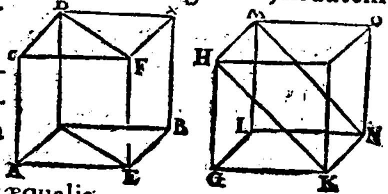
Theorema 34. Propositio 39.

Si in solido parallelis planis circũscripto, aduersorum planorum lateribus bifariã sectis, educta sint per sectiones plana, cõmunis illa planorũ sectio & solidi parallelis plani circũscripti diameter, se mutuò bifariam secant.



Theorema 35. Propositio 40.

Si duo sint æqualis altitudinis prismata, quorũ hoc quidẽ basim habeat parallelogrammũ, illud verò triãgulum, sit autem parallelogrammũ triãguli duplũ, illa prismata erunt æqualia.

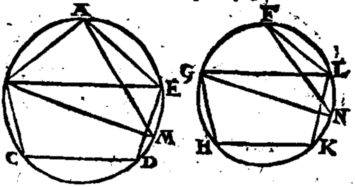


ELEMENTI XI. FINIS.

EVCLIDIS ELEMENTVM DVODECIMVM. ET SOLIDORVM ſecundum.

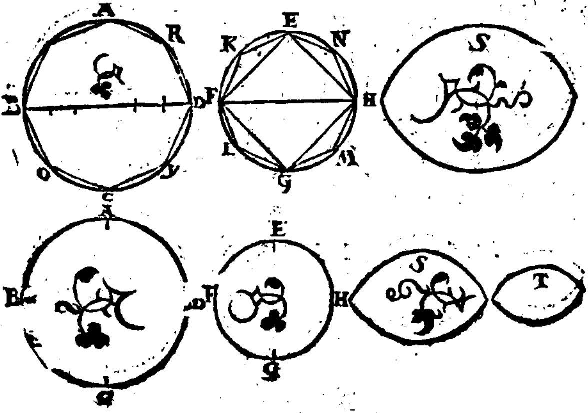
Theorema 1. Propoſitio 1.

Similia, quæ ſunt in circulis polygona, rationē habēt inter ſe, quam deſcripta à diametris quadrata.



Theorema 2. Propoſitio 2.

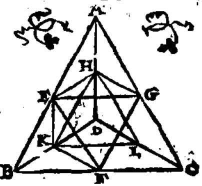
Circuli eam inter ſe rationē habent, quam



descripta à diametris quadrata.

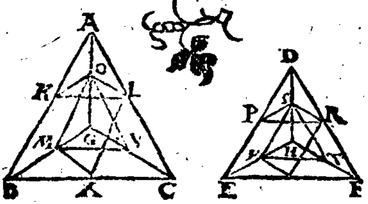
Theorema 3. Propositio 3.

Omnis pyramis trigωnã habens basim, in duas diuiditur pyramidas non tantũ æquales & similes inter se, sed toti etiã pyramidi similes, quarum trigωnæ sunt bases, atq; in duo prismata æqualia, quæ duo prismata dimidio pyramidis totius sunt maiora.



Theorema 4. Propositio 4.

Si duæ eiusdem altitudinis pyramides trigωnas habeãt bases, sit aũt illarum vtraque diuisa & in duas pyramidas inter se æquales totiq; similes, & in duo prismata æqualia, ac eodẽ modo diuidatur vtraq; pyramidũ quæ ex superiore diuisione natæ sunt, idq́ue perpetuò fiat: quemadmodũ se habet vnius pyramidis basis ad alterius pyramidis basim, ita & omnia quæ in vna pyra-

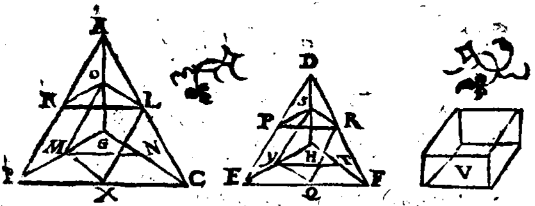


mide

mide priſmata, ad omnia quæ in altera pyramide, priſmata multitudine æqualia.

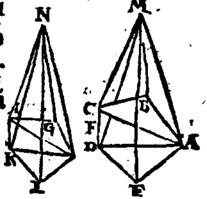
Theorema 5. Propoſitio 5.

Pyramides eiuſdem altitudinis, quarũ trigωnæ ſunt baſes, eam inter ſe rationem habent, quàm ipſæ baſes.



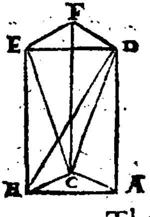
Theorema 6. Propoſitio 6.

Pyramides eiuſdem altitudinis, quarum polygωnæ ſunt baſes, eam inter ſe rationem habent, quã ipſæ baſes.



Theorema 7. Propoſitio 7.

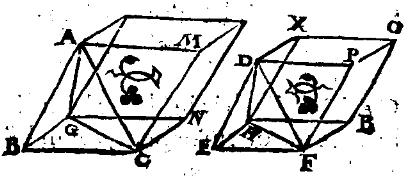
Omne priſma trigωnã habẽs baſim, diuiditur in tres pyramides, inter ſe æquales, quarum trigωnæ ſunt baſes.



Theo-

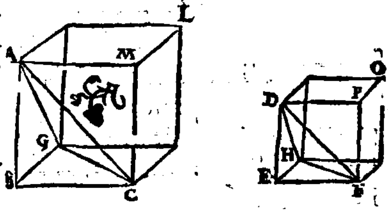
Theorema 8. Propositio 8.

Similes pyramides qui trigωnas habent ba ses, in tripli- cata sũt ho- molo- gorũ laterũ ratiõe.



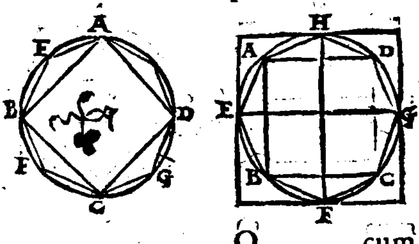
Theorema 9. Propositio 9.

Aequaliũ pyramidum & trigωnas bases ha bentium reciprocantur bases cum altitudi nibus. Et quarũ pyramidum trigωnas ba- ses haben tium reci procãtur bases cũ altitudi- nibus, il- læ sunt æquales.



Theorema 10. Propositio 10.

Om- nis cω n' ter- tia pars est cy- lindri eandẽ

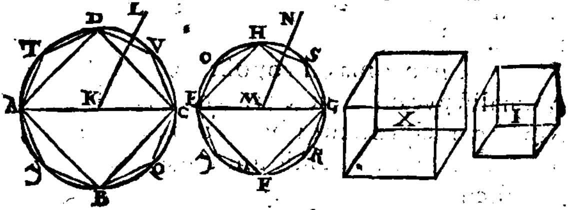


cum

cum ipso cono basim habentis, & altitudinem æqualem.

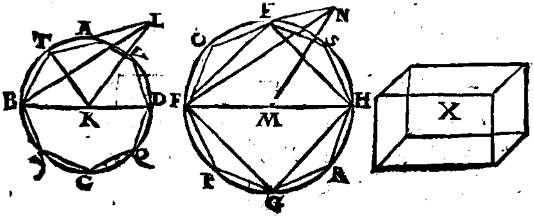
Theorema 11. Propositio 11.

Coni & cylindri eiusdem altitudinis, eam inter se rationem habent, quam bases.



Theorema 12. Propositio 12.

Similes coni & cylindri, triplicatam habēt inter se rationem diametrorum, quæ sunt in basibus.



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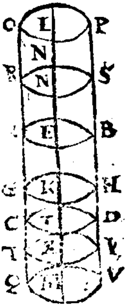
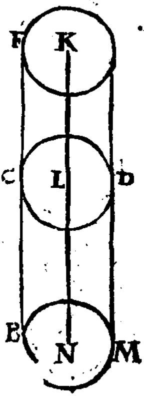
Theorema 13. Propositio 13.

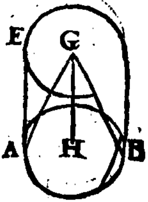


Si cylindrus plano sectus sit ad uersis planis parallelo, erit quẽ admodum cylindrus ad cylindrum, ita axis ad axem.

Theorema 14. Propositio 14.

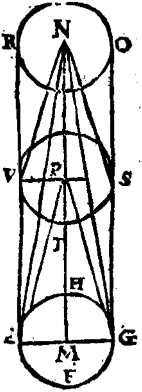
Coni & cylindri qui in æqualibus sunt basibus, eam habent inter se rationem, quam altitudines

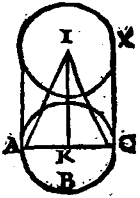




Theorema 15. Propositio 15.

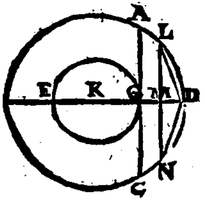
Aequalium conorum & cylindrorũ bases cũ altitudinib⁹ reciprocãtur. Et quorum conorũ & cylindrorũ bases cum altitudinib⁹ reciprocãtur, illi sunt æquales.





Problema 1. Propositio 16.

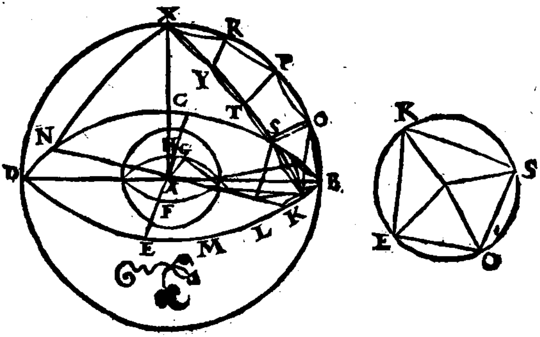
Duobus circulis circũ idem centrum consistentibus, in maiore circulo polygonum æqualium pariumque laterum inscribere, quod minorem circulum nõ tangat.



Pro-

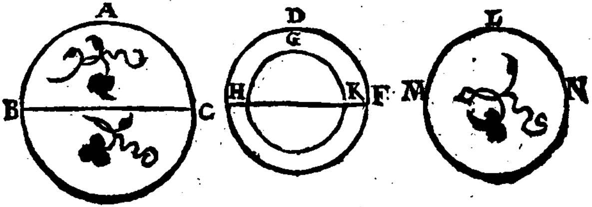
Problema 2. Propositio 17.

Duabus sphæris circum idem centrũ consistentibus, in maiore sphera solidũ polyedrum inscribere, quod minoris sphæræ superficiem non tangat.



Theorema 16. Propositio 18.

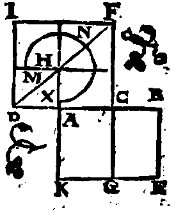
Sphæræ inter se rationem habent suarum diametrorum triplicatam.



ELEMENTI XII. FINIS.

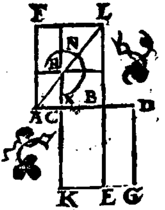
EVCLIDIS ELEMENTVM DECIMVMTERTIVM, ET SOLIDORVM TERTIVM.

Theorema 1. Propositio 1.



Si recta linea per extremã & mediã rationẽ secta sit, maius segmentum quod totius lineæ dimidium assumpserit, quintuplum potest eius quadrati, quod à totius dimidia describitur.

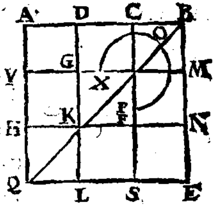
Theorema 2. Propositio 2.



Si recta linea sui ipsius segmenti quintuplũ possit, & dupla segmenti huius linea per extremã & mediam rationẽ secetur maius segmentũ reliqua pars est lineæ primùm positæ.

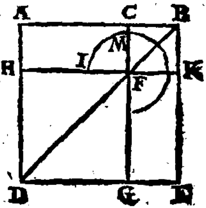
Theo-

Theorema 3. Propositio 3.



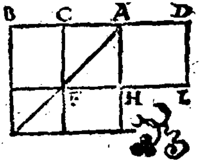
Si recta linea per extremã & mediam rationẽ secta sit, minus segmẽtum quod maioris segmenti dimidium assumpserit, quintuplum potest eius, quòd à maioris segmẽti dimidio describitur, quadrati.

Theorema 4. Propositio 4.



Si recta linea per extremam & mediam rationẽ secta sit, quod à tota, quodque à minore segmẽto simul vtraq; quadrata, tripla sunt eius, quód à maiore segmẽto describitur, quadrati.

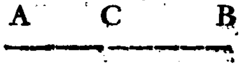
Theorema 5. Propositio 5.



Si ad rectam lineam, quæ per extremam & mediam rationem secetur, adiuncta sit altera segmento maiori æqualis, tota hæc linea recta per extremam & mediam rationẽ secta est, estque maius segmentum linea primùm posita.

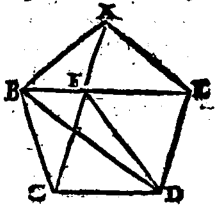
Theorema 6. Propositio 6.

Si recta linea ῥητὴ siue rationalis, per extremam & mediam rationem secta sit, vtrunque segmentorum ἄλογος siue irrationalis est linea, quæ dicitur Residuum.



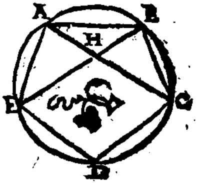
Theorema 7. Propositio 7.

Si pentagωni æquilateri tres sint æquales anguli, siueq; deinceps, siue qui non deinceps sequuntur, illud pétagωnum erit ęquiangulum.



Theorema 8. Propositio 8.

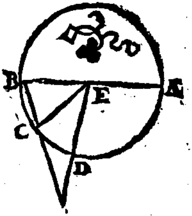
Si pétagωni æquilateri & æquianguli duos qui deinceps sequuntur angulos rectę subtendãt lineæ, illæ per extremã & mediam rationem se mutuò secant, earumq; maiora segmenta, ipsius pentagωni lateri sunt æqualia.



Theo-

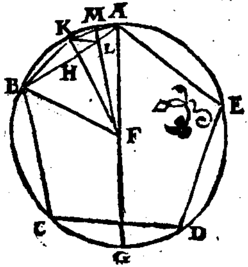
Theorema 9. Propoſitio 9.

Si latus hexagωni & latus decagωni eidem circulo inſcriptorum compoſita ſint, tota recta linea per extremam & mediam rationem ſecta eſt, eiuſque ſegmẽtum maius, eſt hexagωni latus.



Theorema 10. Propoſitio 10.

Si circulo pẽtagωnum æquilaterũ inſcriptum ſit, pẽtagωni lat⁹ poteſt & latus hexagωni & latus decagωni eidem circulo inſcriptorum.

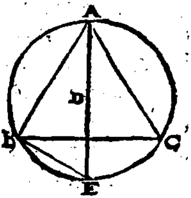


Theorema 11. Propoſitio 11.

Si in circulo ῥητὴν habente diametrum, inſcriptũ ſit pentagωnum æquilaterũ, pentagωni latus irrationalis eſt linea, quæ vocatur Minor.

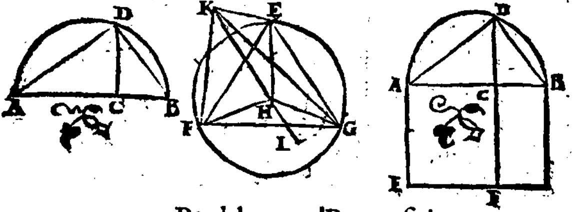
Theorema 12. Propoſitio 12.

Si in circulo inſcriptum ſit triangulum æquilate rum, huius trianguli latus potentia triplum eſt eius lineæ, quæ ex circu li centro ducitur.



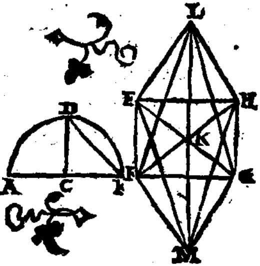
Problema 1. Propoſitio 13.

Pyramidem conſtituere, & data ſphærę cō plecti, atque docere illius ſphæræ diametrum potentia ſeſquialteram eſſe lateris ipſius pyramidis.



Problema 2. Propoſitio 14.

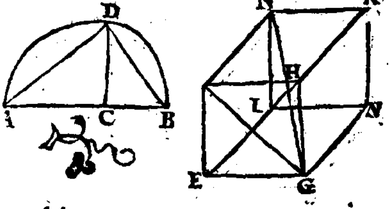
Octaedrum conſtituere, eaq́; ſphęra qua pyramidē complecti, atque ꝓbare illius ſphæræ diametrū potentia duplā eſſe lateris ipſiꝰ octaedri.



Pro-

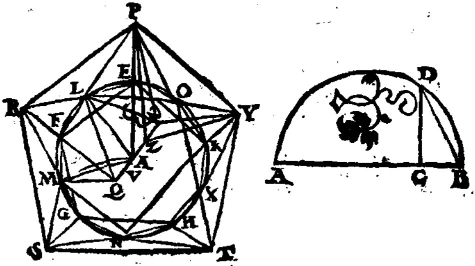
Problema 3. Propositio 15.

Cubum constituere, eaque sphæra qua & superiores figuras complecti, atque docere illius sphæræ diametrum potẽtia triplam esse lateris ipsius cubi.



Problema 4. Propositio 16.

Icosaedrum constituere, eademque sphæra qua & antedictas figuras complecti, atque probare, Icosaedri latus irrationalem esse lineam, quæ vocatur Minor.



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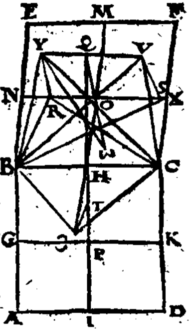
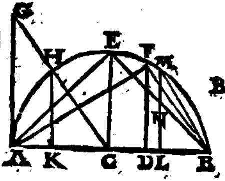
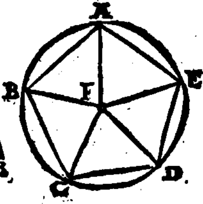


Problema 5. Propositio 17.

Dodecaedrũ cõstituere, eademque sphæra qua & antedictas figuras cõplecti, atque probare dodecaedri latus irrationalem esse lineam, quæ vocatur Residuum.

Theorema 6. Propositio 18.





quinq; figurarũ latera proponere, & inter se comparare.

SCHOLIVM.

Aio verò, præter dictas quinque figuras non posse aliam constitui figuram solidam, quæ planis & æquilateris & æquiangulis contineatur, inter se æqualibus. Non enim ex duobus triãgulis, sed neq; ex alijs duabus figuris solidus cõstituetur angulus

Sed

Sed ex tribus triangulis, constat Pyramidis angulus.

Ex quatuor autem, Octaëdri.

Ex quinque verò, Icosaëdri.

Nam ex triangulis sex & æquilateris & æquiangulis ad idem punctum coeuntibus, non fiet angulus solidus. Cùm enim trianguli æquilateri angulus, recti vnius bessem contineat, erunt eiusmodi sex anguli rectis quatuor æquales Quod fieri non potest. Nam solidus omnis angulus minoribus quā rectis quatuor angulis continetur, per 21.11.

Ob easdem sanè causas, neque ex pluribus quā planis sex eiusmodi angulis solidus constat.

Sed ex tribus quadratis, Cubi angulus continetur.

Ex quinque, nullus potest. Rursus enim recti quatuor erunt.

Ex tribus autem pentagonis æquilateris & æquiangulis, Dodecaëdri angulus continetur.

Sed ex quatuor, nullus potest. Cùm enim pentagoni æquilateri angulus rectus sit, et quinta recti pars erunt quatuor anguli rectis quatuor maiores. Quod fieri nequit. Nec sanè ex alijs polygonis figuris solidus angulus continebitur, quòd hinc quoque absurdum sequatur. Quamobrem perspicuum est, præter dictas quinque figuras aliam figuram solidam non posse cónstitui, quæ ex planis æquilateris & æquiangulis contineatur.

ELEMENTI XIII. FINIS.

EVCLIDIS ELEMENTVM DECIMVMQVARTVM, VT quidam arbitrantur, vt alij verò, Hypſiclis Alexandrini, de quinque corporibus.

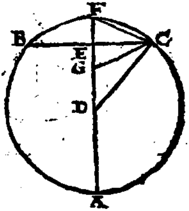
LIBER PRIMVS.

BAſilides Tyrius, Protarche, Alexandriam profectus, patrique noſtrò ob diſciplinæ ſocietatem commendatus, longiſſimo peregrinationis tempore cum eo verſatus eſt. Cúmque aſſererent aliquando deſcripta ab Apollonio cõparatione Dodecaedri & Icoſaedri eidem ſphæræ inſcriptorum, quam hæc inter ſe habeant rationem, cenſuerunt ea non rectè tradidiſſe Apollonium: quæ à ſe emendata, vt de patre audire erat, literis prodiderunt. Ego autem poſtea incidi in alterum librũ ab Apollonio editum, qui demonſtrationem accuratè complecteretur de re propoſita, ex eiuſq; problematis indagatione magnam equidem cepi voluptatem. Illud certè ab omnibus perſpici poteſt, quod ſcripſit Apollonius, cùm ſit in omnium manibus. Quod autẽ diligenti, quantum conijcere licet, ſtudio nos poſtea ſcrip-

scripsisse videmur, id monimentis consignatum tibi nuncupandum duximus, vt qui feliciter cùm in omnibus disciplinis tum vel maximè in Geometria versatus, scitè ac prudẽter iudices ea quæ dicturi sumus ob eam verò, quæ tibi cum patre fuit, vitæ consuetudinem, quaq; nos complecteris, beneuolentiã, tractationem ipsam libenter audias. Sed iam tempus est, vt præmio modum facientes, hanc syntaxim aggrediamur.

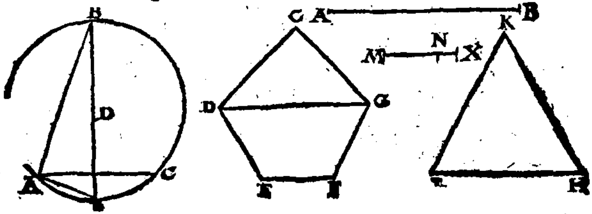
Theorema 1. Propositio 1.

Perpendicularis linea, quæ ex circuli cuiuspiam centro in latus pẽtagωni ipsi circulo inscripti ducitur, dimidia ẽst vtriusq; simul lineæ, & eius quæ ex centro, & lateris decagωni in eodẽ circulo inscripti.



Theorema 2. Propositio 2.

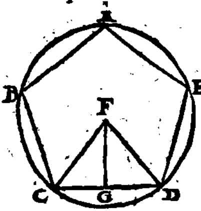
Idem circulus cõprehendit & dωdecaedri pentagωnum & icosaëdri triangulũ, eidem sphæræ inscriptorum.

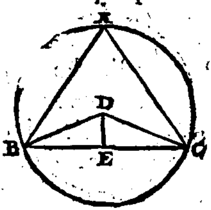


Theo

Theorema 3. Propositio 3.

Si pentagωno & æquilatero & æquiangulo circunſcriptus ſit circulus, ex cuius centro in vnum pentagωni latus ducta ſit perpendicularis: quod vno laterum & perpẽdiculari trigeſies continetur, illud



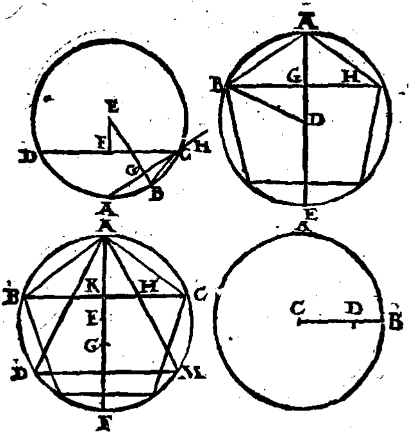


æquale eſt dωdecaedri ſuperficiei:

Theorema 4. Propositio 4.

Hoc perſpicuum cùm ſit, probandum eſt; quemadmodum ſe habet dωdecaedri ſuperficies ad icoſaedri ſuperficiem, ita ſe habere cubi latus ad icoſaedri latus.

Cubi



Cubi latus.

E——————

Dodecaedri.

F——————

Icosaedri.

G——————

SCHOLIVM.

Nunc autem probandum est, quemadmodum se habet cubi latus ad icosaedri latus, ita se habere solidum dodecaedri ad icosaedri solidum. Cùm enim aequales circuli comprehendant & dodecaedri pẽtagonum & Icosaedri triangulum, eidem sphaerae in-

ſcriptorum: in ſphæris autem æquales circuli æquali interuallo diſtent à centro (ſiquidem perpendiculares à ſphæræ centro ad circulorum plana ductæ & æquales ſunt, & ad circulorum centra cadunt) idcirco lineæ, hoc eſt perpendiculares, quæ à ſphæræ cẽtro ducuntur ad centrum circuli comprehendẽtis & triangulum Icoſaëdri, & pentagonũ in dodecaëdri, ſunt æquales. Sũt igitur æqualis altitudinis Pyramides, quæ baſes habent ipſa dodecaëdri pentagona, et quæ Icoſaëdri triangula. At æqualis altitudinis pyramides rationem inter ſe habent eam quam baſes, ex 5. & 6. 11. Quemadmodum igitur pentagonũ ad triangulum, ita pyramis, cuius baſis quidem eſt dodecaëdri pentagonum, vertex autem ſphæræ centrum ad pyramida, cuius baſis quidem eſt Icoſaëdri triangulum, vertex autem, ſphæræ centrum. Quamobrem vt ſe habent duodecim pentagona ad viginti triangula, ita duodecim pyramides, quorum pẽtagona ſint baſes, ad viginti pyramidas, quæ trigonas habeant baſes. At pentagona duodecim ſunt dodecaëdri ſuperficies, viginti autem triangula, Icoſaëdri. Eſt igitur vt dodecaëdri ſuperficies ad Icoſaëdri ſuperficiem, ita duodecim pyramides, quæ pentagonas habeant baſes, ad viginti pyramidas, quarum trigonæ ſunt baſes. Sunt autem duodecim quidem pyramides, quæ pentagonas habeant baſes, ſolidũ dodecaëdri: viginti autem pyramides, quæ trigonas habeant baſes, Icoſaëdri ſolidum. Quare ex 11. 5. vt dodecaëdri ſuperficies ad Icoſaëdri ſuperficiem, ita ſolidum dodecaëdri ad Icoſaëdri ſolidum. Vt autem

tem

rem dodecaëdri superficies ad Icosaëdri superficiem, ita probatum est cubi latus ad Icosaëdri latus. Quemadmodum igitur cubi latus ad Icosaëdri latus, ita se habet solidum dodecaëdri ad Icosaëdri solidum.

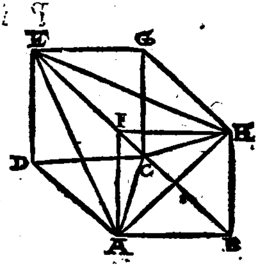
EVCLIDIS ELEMENTVM DECIMVMQVINTVM, ET

Solidorum quintum, vt nonnulli putant, vt autem alij, Hypsiclis Alexandrini, de quinque corporibus,

LIBER II.

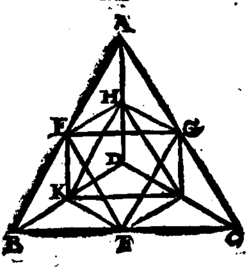
Problema 1. Propositio 1.

In dato cubo pyramida inscribere.



Problema 2. Propositio 2.

In data pyramide octaedrum inscribere.



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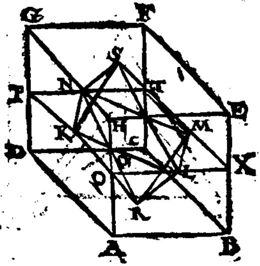
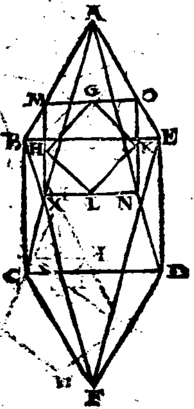
Problema 3. Propositio 3.

In dato cubo octaedrum inscribere.



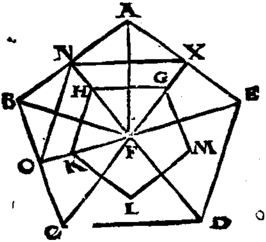
Problema 4. Propositio 4.

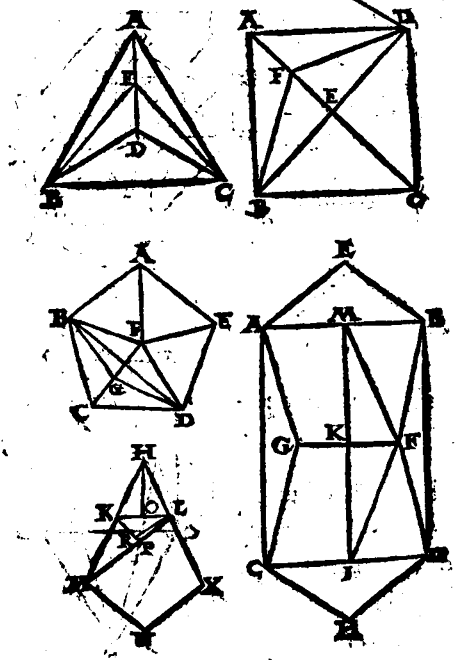
In dato octaedro cubū inscribere.



Problema 5. Propositio 5.

In dato Icosaedro dωdecaedrum inscribere.





SCHO

SCHOLIVM.

Meminisse decet, si quis nos roget, quot Icosaëdrũ habeat latera, ita respondendum esse: Patet Icosaëdrum viginti contineri triangulis, quodlibet verò triangulum rectis tribus constare lineis. Quare multiplicanda sunt nobis viginti triangula in trianguli vnius latera, fiuntq́ sexaginta, quorum dimidum est triginta. Ad eundem modum & in dodecaëdro. Cũ enim rursus duodecim pentagona dodecaëdrum cõprehendant, itemq; pentagonum quoduis rectis quinque constet lineis, quinq; duodecies multiplicamus, fiunt sexaginta, quorum rursus dimidium est triginta. Sed cur dimidiũ capimus? Quoniam vnumquodq; latus siue sit trianguli, siue pentagoni, siue quadrati, vt in cubo, iteratò sumitur. Similiter autem eadem via & in cubo & in pyramide & in octaëdro latera inuenies. Quòd si item velis singularum quoq; figurarum angelos reperire, facta eadem multiplicatione numerum procreatum partire in numerum planorũ, quæ vnum solidum angulum includunt: vt quoniã triangula quinq; vnum Icosaëdri angulum continet, partire 60. in quinque, nascũtur duodecim anguli Icosaëdri. In dodecaëdro autem tria pentagona angulum comprehendunt. partire ergo 60. in tria, & habebis dodecaëdri angulos viginti. Atq; simili ratione in reliquis figuris angulos reperies.

Finis Elementorum Euc