EVCLIDIS ELEMENTORVM LIBRI XV.

Musei Mathematici S. J. Pragæ ad S. Clem

Quibus, cùm ad omnem Mathematicæ ſcientiæ partem, tùm ad quamlibet Geometriæ tractationem, facilis comparatur aditus.

Ex libris Jacobi Viti Werner ...

COLONIAE,
Apud Maternum Cholinum.
M. D. LXXXVII.
Cum Gratia & Priuilegio Cæſ. Maieſt.

AD CANDIDVM LECTOREM ST. GRACILIS præfatio.

PERMAGNI *referre sēper existimaui, lector beneuole, quantum quisq; studij & diligentiæ ad percipienda scientiarum elementa adhibeat, quibus nō satis cognitis, aut perperā, intellectis si vel digitum progredi tentes, erroris caliginem animis offundas, non veritatis lucem rebus obscuris adferas. Sed principiorum quanta sint in disciplinis momenta, haud facilè credat, qui rerum naturam ipsa specie, non viribus metiatur. Vt enim corporum quæ oriuntur & intereūt vilissima tenuissimaq; videntur initia: ita rerum æternarum & admirabilium, quibus nobilissimæ artes continentur, elementa ad speciem sunt exilia, ad vires & facultatem quā maxima. Quis nō videt ex fici tātulo grano, vt ait Tullius, aut ex acino vinaceo, aut ex cæterarum frugum aut stirpium minutissimis seminibus tantos truncos ramosq; procreari? Nā Mathematicorū initia illa quidem dictu audituq; perexigua, quantā theorematum syluā nobis pe-*

pererunt? Ex quo intelligi poteſt, vt in ipſis ſenſibus, ſic & in artium principijs ineſſe vim earũ rerum, quæ ex his progignuntur. Præclarè igitur Ariſtoteles, vt alia permulta, μέγιςον ἴσως ἀρχὴ παντὸς, καὶ ὅσῳ κράτιςον τῇ δυνάμει, τοσούτῳ μικρότατον, ὂν τῷ μεγέθει χαλεπόν ἐστιν ὀφθῆναι. Quocirca committendum non eſt, vt non bene prouiſa & diligenter explorata ſcientiarum principia quibus propoſitarum quarumque rerũ veritas ſit demonſtranda, vel conſtituas, vel conſtituta approbes: Cauendum etiam, vt ne tantulum quidem fallaci & captioſa interpretatione turpiter deceptus, à vera principiorum ratione temerè deflectas. Nam qui initio fortè aberrauerit, is vt tandem in maximis verſetur erroribus, neceſſe eſt: cum ex vno erroris capite, denſiores ſenſim tenebræ rebus clariſſimis obducantur. Quid tam varias veterum phyſiologorum ſententias non modò cum rerum veritate pugnantes, ſed vehementer etiam inter ſe diſſentientes nobis inuexit? Equidem haud ſcio, fueritne vlla potior tanti diſſidij cauſa, quàm quòd ex principijs partim falſis, partim non conſentaneis ductas rationes probando adhiberent. Fit enim plerunque, vt qui non rectè de artium rerumq́ elementis ſentiunt, ad præfinitas quaſdam opiniones ſuas omnia reuocare ſtudeant. Pythagorei, vt meminit Ariſtoteles, cum denarij numeri ſummam perfectinem cœlo tribuerent, nec plures tamen quàm nouem ſphæras cernerent, decimam affingere auſi ſunt terræ aduerſam

ſam quam ἀντίχθονα appellarunt. Illi enim vniuerſitatis rerumq́ ſingularum naturam ex numeris ceu principijs æſtimantes, ea protulerũt quæ φαινομένοις congruere nuſquam ſunt cognita Nam ridicula Democriti, Anaximenis, Melißi Anaxagoræ, Anaximandri, & reliquorum id genus phyſiologorum ſomnia ex falſis illa quidem orta natura principijs ſed ad Mathematicum nihil aut parum ſpectantia, ſciens prætereo Nonnullos attingam qui repetitis altius vel aliter ac decuit poſitis rerum initijs, cum phyſicis multa turbarunt, tum Mathematicos oppugnatione principiorum peßimè mulctarũt. Ex planis figuris corpora cõſtituit Timæus: Geometrarum hic quidem principia cuniculis oppugnantur. Nam & ſuperficies ſeu extremitates craßitudinem habebunt, & lineæ latitudinem: denique puncta non erunt indiuidua ſed linearum partes. Prædicant Democritus atque Leucippus illas atomos ſuas, & indiuidua corpuſcula. Concedit Xenocrates impartibiles quaſdã magnitudines. Hic verò Geometriæ fundamẽta apertè petuntur, & funditus euertuntur: quibus dirutis nihil equidẽ aliud video reſtare, quàm vt ampliſſima Mathematicorum theatra repentè concidant. Iacebunt ergo, ſi dijs placet, tot præclara Geometrarum de aſymmetris & alogis magnitudinibus theoremata. Quid enim cauſæ dicas cur indiuidua linea hanc quidem metiatur, illam verò metiri non queat? Siquidem quod minimum in vnoquoque genere reperitur. id communis omnium mensura esse solet. Innumerabilia profectò sunt illa, quæ ex falsis eiusmodi decretis absurda consequuntur: & horum permulta quidem Mathematicus, sed longè plura colligit Physicus. Quid varia ψευδογραφημάτων genera commemorē, quæ ex hoc uno fonte tam longè latéque diffusa fluxisse videntur? Notissimus est Antiphontis tetragonismus, qui Geometrarum & ipse principia non parum labefecit, cùm rectæ lineæ curuam posuit æqualem. Longum esset mihi singula percensere, præsertim ad alia properanti: Hoc ergo certum fixum, & in perpetuum ratum esse oportet, quod sapiēter monet Aristoteles σπουδαστέον ὅπως ὁρισθῶσι καλῶς αἱ ἀρχαὶ μεγάλην γὰρ ἔχουσι ῥοπὴν πρὸς τὰ ἑπόμενα. Nam principiis illa congruere debent, quæ sequuntur. Quòd si tantum perspicitur in istis exilioribus Geometriæ initiis quæ puncto, linea, superficie definiuntur, momentum, ut ne hæc quidem sine summo impendentis ruinæ periculo conuelli aut oppugnari possint, quanta quæso vis putanda est huius στοιχειώσεως, quam collatis tot præstātissimorum artificum inuentis, mira quadam ordinis solertia contexuit Euclides, uniuersæ Matheseos elementa complexu suo coercentem? Ut igitur omnibus rebus instructior & paratior quisque ad hoc studium libentius accedat, & singula vel minutissima exactius secum reputet atque perdiscat, operæprecium censui, in primo institutionis aditu vestibuloq; præcipua quædam capita,

quibus

quibus tota ferè Mathematicæ scientiæ ratio intelligatur, breuiter explicare: tum ea quæ sunt Geometriæ propria, diligẽter persequi: Euclidis deniq; in extruenda hac στοιχειώσει consilium sedulò ac fideliter exponere. Quæ ferè omnia ex Aristotelis potissimum ducta fontibus, nemini inuisa fore confido, qui modò ingenuum animi candorem ad legendum attulerit. Ac de Mathematicæ diuisione primùm dicamus.

Mathematicæ in primis scientiæ studiosos fuisse Pythagoreos, non modò historicorum, sed etiam philosophorum libri declarant. His ergo placuit, vt in partes quatuor vniuersum distribuatur Mathematicæ scientiæ genus, quarum duas περὶ τὸ ποσὸν, reliquas περὶ τὸ πηλίκον versari statuerunt. Nam & τὸ ποσὸν vel sine vlla cõparatione ipsum per se cognosci, vel certa quadam ratione comparatũ spectari: in illo Arithmeticam, in hoc versari Musicam: & τὸ πηλίκον partim quiescere partim moueri quidem: illud Geometriæ propositum esse: quod verò sua sponte motu cietur, Astronomiæ. Sed ne quis falsò putet, Mathematicam scientiam, quòd in vtroque quanti genere cernitur, idcirco inanem videri (si quidem non solum magnitudinis diuisio, sed etiam multitudinis accretio infinitè progredi potest) meminisse decet, τὸ πηλίκον καὶ τὸ ποσὸν, quæ subiecto Mathematicæ generi imposita sunt à Pythagoreis nomina, non cuiuscunque modi quantitatem significare, sed eam demum, quæ tum multitudine tum magnituditudine sit definita, & suis circumscripta terminis. Quis enim vllam infiniti scientiam defendat? Hoc scitum est, quod non semel docet Aristoteles, infinitum ne cogitatione quidem complecti quenquam posse. Itaque ex infinita multitudinis & magnitudinis δυνάμει finitam hac scientia decerpit & amplectitur naturam, quam tractet, & in qua versetur. Nam de vulgari Geometrarum cõsuetudine quid sentiendum sit, cum data interdum magnitudine infinita aut fabricantur aliquid, aut proprias generis subiecti affectiones exquirunt, disertè monet Aristoteles, οὐδὲ νῦν (de Mathematicis loquens) δέονται τοῦ ἀπείρου, οὐδὲ χρῶνται, ἀλλὰ μόνον εἶναι ὅσην ἂν βούλωνται, πεπερασμένην. Quamobrem disputatio ea qua infinitum refellitur, Mathematicorum decretis rationibusq; non aduersatur, nec eorum apodixes labefacit. Etenim tali infinito opus illis nequaquam est, quod exitu nullo peragrari possit, nec talem ponunt infinitam magnitudinẽ: sed quantãcunq; velit aliquis effingere, ea vt suppetat, infinitam præcipiunt. Quinetiã non modò immensa magnitudine opus non habent Mathematici, sed ne maxima quidem: cum instar maxima minima quæque in partes totidem pari ratione diuidi queat. Alteram Mathematicæ diuisionẽ attulit Geminus, vir (quantum ex Proclo cõijcere licet) μαθημάτων laude clarissimus. Eam, quæ superiore plenior & accuratior forte visa est, cũ doctissimè pertractarit sua in decimum Euclidis præfa-

præfatione P. Montaureus vir senatorius, & regiæ bibliothecæ præfectus, leniter attingam. Nam ex duobus rerum velut summis generibus τῶν νοητῶν καὶ τῶν αἰσθητῶν, quæ res sub intelligentiam cadunt, Arithmeticæ & Geometriæ attribuit Geminus: quæ verò in sensus incurrũt, Astrologiæ, Musicæ Supputratici, Opticæ, Geodæsiæ & Mechanicæ adiudicauit. Ad hãc certè diuisionem spectasse videtur Aristoteles, cùm Astrologiam Opticam, harmonicam φυσικωτέρας τῶν μαθημάτων nominat, vt quæ naturalibus & Mathematicis interiectæ sint, ac velut ex vtrisque mixtæ disciplinæ: Siquidem genera subiecta à Phisicis mutuantur, causas verò in demonstrationibus ex superiore aliqua scientia repetunt. Id quod Aristoteles ipse apertissimè testatur, ἐνταῦθα γὰρ, φησὶ, τὸ μὲν ὅτι, τῶν αἰσθητικῶν εἰδέναι τὸ δὲ διότι, τῶν μαθηματικῶν. Sequitur, vt quid Mathematicæ conueniat cum Phisica & prima Philosophia: quid ipsa ab vtraque differat, paucis ostendamus. Illud quidem omnium commune est, quòd in veri contemplatione sunt positæ, ob idq; θεωρητικαὶ à Græcis dicuntur. Nam cùm διάνοια siue ratio & mens omnis sit vel πρακτικὴ, vel θεωρητικὴ, totidem scientiarum sint genera necesse est. Quòd si Physica, Mathematica, & prima Philosophia, nec in agendo, nec in efficiendo sunt occupatæ, hoc certè perspicuum est, eas omnes in cognitione cõtemplationéque necessariò versari. Cùm enim rerum non modò agendarum, sed etiam efficienda-

ciendarum principia in agente vel efficiente cōsistant, illarum quidem προαίρεσις, harum autem vel mens vel ars, vel vis quædam & facultas: rerum profectò naturalium Mathematicarum, atque diuinarum principia in rebus ipsis, non in philosophis inclusa latent. Atque hæc vna in omnes valet ratio, quæ θεωρητικὰς esse colligat. Iam verò Mathematica separatim cū Physica congruit, quòd vtraque versatur in cognitione formarum corpori naturali inhærentiū. Nam Mathematicus plana solida, longitudines & puncta contemplatur, quæ omnia in corpore naturali à naturali quoque philosopho tractantur. Mathematica item & prima philosophia hoc inter se propriè conueniunt, quòd cognitionem vtraque persequitur formarum, quoad immobiles, & à concretione materiæ sunt liberæ. Nam tametsi Mathematicæ formæ re vera per se nō cohærent, cogitatione tamen à materia & motu separantur, οὐδὲ γίνεται ψεῦδος χωριζόντων, vt ait Aristoteles. De cognatione & societate breuiter diximus, iam quid intersit videamus. Vnaquæq; mathematicarū certum quoddam rerum genus propositum habet, in quo versetur, vt Geometria quantitatem & continuationem aliorum in vnam partem, aliorū in duas, quorundam in tres, eorúmque quatenus quanta sunt & continua, affectiones cognoscit. Prima autem philosophia, cùm sit omnium communis, vniuersum Entis genus, quæq; ei accidūt & conueniunt hoc ipso quòd est, consyderat.

Ad

Adhæc, Mathematica eam modò naturã amplectitur, quæ quanquam non mouetur, separari tamen seiungiq; nisi mente & cogitatione à materia non potest, ob eamq; causam, ἐξ ἀφαιρέσεως dici consueuit. Sed Prima philosophia in ijs versatur, quæ & seiuncta, & æterna, & ab omni motu per se soluta sunt ac libera. Cæterùm Physica & Mathematica quanquam subiecto discrepare non videntur, modo tamen rationeq; differunt cognitionis & contemplationis, vnde dissimilitudo quoque scientiarum sequitur. Etenim Mathematicæ species nihil re vera sunt aliud, quàm corporis naturalis extremitates, quas cogitatione ab omni motu & materia separatas Mathematicus contemplatur: sed easdem consectatur Physicorum ars, quatenus cum materia comprehensæ sunt, & corpora motui obnoxia circunscribunt. Ex quo fit vt quæcunque in Mathematicis incommoditates accidunt, eædem etiam in naturalibus rebus videantur accidere. non autem vicissim. Multa enim in naturalibus sequuntur incommoda, quæ nihil ad Mathematicum attinent, διὰ τὸ, inquit Aristoteles, τὰ μὲν ἐξ ἀφαιρέσεως λέγεται, τὰ μαθηματικὰ, τὰ δὲ φυσικὰ ἐκ προσθέσεως. Siquidem res cũ materia deuinctas contemplatur physicus: Mathematicus verò rẽ cognoscit circũscriptis ijs omnibus quæ sensu percipiuntur, vt grauitate leuitate duritie, mollitie, & præterea calore, frigore, aliisque contrariorum paribus, quæ sub sensum subiecta sunt: tantum autem relinquit quantitatem & continuum. Itaq; Mathematicorum ars in iis quæ immobilia sunt, cernitur (τὰ γὰρ μαθηματικὰ τῶν ὄντων ἄνευ κινήσεώς ἐστιν, ἔξω τῶν περὶ τὴν ἀστρολογίαν) quæ verò in naturæ obscuritate posita est, res quidem quæ nec separari nec motu vacare possunt, contemplatur. Idquod in vtroque scientiæ genere perspicuum esse potest, siue res subiectas definias, siue proprietates earum demonstres. Etenim numerus, linea, figura rectum inflexum, æquale, rotundum, vniuersa demique Mathematicus quæ tractat & profitetur, absque motu explicari docerique possunt: χωριστὰ γὰρ τῇ νοήσει κινήσεώς ἐστι; Phisica autem sine motione species nequaquam possunt intelligi. Quis enim hominis planta, ignis, ossiū carnis naturam & proprietates sine motu, qui materiam sequitur, perspiciat? Siquidem tantisper substantia quæque naturalis constare dicitur solet, quoad opus & munus suum, agendo patiēdoque tueri ac sustinere valeat: qua certè amissa δυνάμει, ne nomen quidem nisi ὁμωνύμως retinet. Sed Mathematico ad explicandas circuli aut trianguli proprietates, nullum adferre potest vsum materiæ vt auri, ligni, ferri, in qua insunt, consideratio, quin eò verius eiusmodi rerum, quarum species tanquam materia vacantes efformemus animo, naturam complectemur, quod coniunctione materiæ quasi adulterari dipranarique videntur. Quocirca Mathematicæ species eodem modo quo κοῖλον, siue concauitas, sine motu & subiecto, definitione explicare cognos-

cognoscíq; possunt: naturales verò cum eam vim habeant quam, vt ita dicam simitas, cum materia comprehensæ sunt, nec absque ea separatim possunt intelligi: quibus exemplis quid inter Phisicas & Mathematicas species intersit, haud difficile est animaduertere. Illis certè non semel est vsus Aristoteles. Valeant ergo Protagoræ sophismata, Geometras hoc nomine refellentis, quod circulus normam puncto non attingat. Nã diuina Geometrarum theoremata, qui sensu æstimabit, vix quicquam reperiet quod Geometræ concedendum videatur. Quid enim ex his quæ sensum mouent, ita rectum aut rotundum dici potest, vt à Geometra ponitur? Nec verò absurdum est aut vitiosum, quòd lineas in puluere descriptas pro rectis aut rotundis assumit, quæ nec recta sunt nec rotunda, ac ne latitudinis quidem expertes. Siquidem non iis vtitur Geometra quasi inde vim habeat conclusio, sed eorum quæ discenti intelligenda relinquuntur, rudem ceu imaginem proponit. Nam qui primum instituuntur, hi ductu quodam & velut χειραγωγία sensuum opus habent, vt ad illa quæ sola intelligentia percipiuntur, aditum sibi comparare queant. Sed tamen existimandum non est rebus Mathematicis omnino negari materiam, ac non eam tantùm quæ sensum afficit. Est enim materia alia quæ sub sensum cadit, alia quæ animo & ratione intelligitur. Illam αἰσθητὴν, hanc νοητὴν vocat Aristoteles. Sensu percipitur, vt æs, vt lignum, omnisque materia quæ moueri potest

test. Animo & ratione cernitur ea quæ in rebus sensilibus inest sed nō quatenus sensu percipiuntur, quales sunt res Mathematicorum. Unde ab Aristotele scriptū legimus ἐπὶ τῶν ἐν ἀφαιρέσει ὄντων *rectum se habere ut simum:* μετὰ συνεχοῦς γὰρ: *quasi velit ipsius recti quod Mathematicorum est, suam esse materiam non minus quā simi quod ad Physicos pertinet. Nam licet res Mathematicæ sensili vacent materia, nō sunt tamen indiuiduæ, sed propter continuationem partitioni semper obnoxiæ, cuius ratione dici possunt sua materia non omnino carere: quin aliud videtur* τὸ εἶναι γραμμῇ, *aliud quoad continuationi adiuncta intelligitur linea. Illud enim ceu forma in materia proprietatum causa est quas sine materia percipere non licet. Hæc est societatis & dissidij Mathematicæ cum Physica & prima Philosophia ratio. Nunc autem de nominis etymo & notatione pauca quædam afferamus. Nam si quæ iudicio & ratione imposita sunt rebus nomina, ea certè non temere indita fuisse credendum est, quibus scientias appellare placuit. Sed neque otiosa semper haberi debet illa etymologiæ indagatio cùm ad rei etiam dubiæ fidem sæpe non parum valeat recta nominis interpretatio. Sic enim Aristoteles ducto ex verborum ratione argumento,* αὐτομάτου, μετὰ βουλῆς, αἰθέρος, *aliarumque rerum naturam ex parte confirmauit. Quoniam igitur Pythagoras Mathematicam scientiam nō modò studiosè coluit, sed etiā repetitis à capite principijs, geometricā contem-*

contemplationem in liberalis disciplinæ formã composuit & perspectis absq; materia solius intelligentiæ adminiculo theorematibus, tractationem περὶ τῶν ἀλόγων, & κοσμικῶν σχημάτων cõstitutionem excogitauit: credibile est. Pythagoram, aut certè Pythagoreos, qui et ipsi doctoris sui studia libenter amplexi sunt, huic scientiæ id nomen dedisse, quod cum suis placitis atq; decretis congrueret, rerumq; propositarum naturam quoquo modo declararet. Ita cùm existimarent illi, omnem disciplinam, quæ μάθησις dicitur, ἀνάμνησιν esse quandam. i. recordationem et repetitionem eius scientiæ, cuius antè quam in corpus immigraret compos fuerit anima, quemadmodum Plato quoq; in Menone, Phædone, & alijs aliquot locis videtur astruxisse; animaduerterent autem eiusmodi recordationem, quæ non posset multis ex rebus perspici, ex his potissimum scientijs demonstrari. si quis nimirum, ait Plato, ἐπὶ τὰ διαγράμματα ἄγῃ probabile est equidem Mathematicas à Pythagoreis artes κατ᾽ ἐξοχὴν fuisse nominatas vt ex quibus μάθησις, id est æternarum in anima rationum recordatio διαφερόντως et pcipuè intelligi posset. Cuius etiã rei fidẽ nobis diuinus fecit Plato, q̃ in Menone Socratem induxit hoc argumẽti genere persuadere cupientem discere nihil esse aliud quam suarum ipsius rationum animum recordari. Etenim Socrates pusionem quendam, vt Tullij verbis vtar, interrogat de geometrica dimensione quadrati: ad ea sic ille respondet vt puer, & tamen

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men tam faciles interrogationes sunt, vt gradatim respondens eodem perueniat quò si Geometrica didicisset. Aliam nominis huius rationem Anatolius exposuit, vt est apud Rhodiginum, quòd cùm cæteræ disciplinæ deprehendi vel non docente aliquo possint omnes. Mathematicæ sub nullius cognitionem veniant, nisi preeunte aliquo, cuius solertia succidantur vepreta, vel exurantur, & superciliosa complanentur asprèta. Ita enim Cælius: quod quam vim habeat, nō est huius loci curiosius perscrutari. Equidem M Tullius Mathematicos in magna rerum obscuritate recondita arte multiplicīq; ac subtili versari scribit: sed quis nescit idipsum cum aliis grauioribus scientiis esse commune? Est enim, vel eodem auctore Tullio, omnis cognitio multis obstructa difficultatibus maximaq; est & in ipsis rebus obscuritas, & in iudiciis nostris infirmitas, nec vllus est, modò interius paulò Physica penetrarit, qui non facilè sit expertus, quam multi vndique emergant, rerum naturalium causas inquirentibus inexplicabiles labyrinthi. Sunt qui ex demonstrationum firmitate nominari Mathematicas opinantur: cuius etiam rationis momentum alio seorsim loco expendendum fuerit. Quocirca primam verbi notationem, quam sequutus est Proclus, nobis retinendam censeo. Hactenus de vniuerso Mathematicæ genere, quanta potui & perspicuitate & breuitate dixi. Sequitur vt de Geometria separatim atque ordine ea differam, quæ initio sum pollicitus.

tus. Est autem Geometria, vt definit Proclus scientia quæ versatur in cognitione magnitudinū figurarum & quibus hæ continentur, extremorum, item rationum & affectionum quæ in illis cernuntur ac inhærent: ipsa quidem progrediens à puncto indiuiduo per lineas & superficies, dū ad solida conscendat, variasq; ipsorum differentias patefaciat. Quúmque omnis scientia demonstratiua vt docet Aristoteles, tribus quasi momentis contineatur, genere subiecto, cuius proprietates ipsa scientia exquirit & contemplatur: causis & principiis ex quibus primis demonstrationes conficiuntur: & proprietatibus, quæ de genere subiecto per se enunciantur: Geometriæ quidem subiectum in lineis, triangulis, quadrangulis, circulis, planis, solidis atque omnino figuris & magnitudinibus, earúmque extremitatibus consistit. His autem inhærent diuisiones, rationes, tactus, æqualitates, παραβολαὶ ὑπερβολαὶ ἐλλεί͵ψεισ, *atque alia generis eiusdem propè innumerabilia. Postulata verò & Axiomata ex quibus hæc inesse demonstrantur, eiusmodi ferè sunt: Quouis centro & interuallo circulum describere. Si ab æqualibus æqualia detrahas, quæ relinquuntur esse æqualia, cæteraq; id genus permulta, quæ licet omnium sint cōmunia ac demonstrandum tamen tum sunt accommodata, cùm ad certum quoddam genus traducuntur. Sed cùm præcipua videatur Arithmetica & Geometria inter Mathematicas dignatio; cur Arithmetica sit* ἀκριβεστέρα *&*

 exactior

exactior quam Geometria paucis explicandum arbitror. Hic verò & Aristotelem sequemur ducem, qui scientiam cum scientia ita comparat, vt accuratiorem esse velit eam, quæ rei caussam docet, quam quæ rem esse tantùm declarat, deinde quæ in rebus sub intelligentiam cadentibus versatur, quàm quæ in rebus sensum mouentibus cernitur. Sic enim & Arithmetica quàm Musica, & Geometria quàm Optica & Stereometria quàm Mechanica exactior esse intelligitur. Postremò quæ ex simplicioribus initijs constat, quàm quæ aliqua adiectione compositis vtitur. Atque hac quidem ratione Geometriæ præstat Arithmetica, quòd illius initium ex additione dicatur, huius sit simplicius. Est enim punctum, vt Pythagoreis placet, vnitas quæ sitũ obtinet: vnitas verò punctum est quod situ vacat. Ex quo percipitur, numerorum quàm magnitudinum simplicius esse elementum, numerosque magnitudinibus esse puriores, & à concretione materiæ magis disiunctos. Hæc quanquam nemini sunt dubia, habet & ipsa tamen Geometria quo se plurimum efferat, opibúsque suis ac rerum vbertate multiplici vel cum Arithmetica certet: id quod tute facilè deprehendas cum ad infinitam magnitudinis diuisionem, quam respuit multitudo, animum conuerteris. Nunc quæ sit Arithmeticæ & Geometriæ societas, videamus. Nam theorematum quæ demonstratione illustrantur, quædam sunt vtriusque scientiæ communia, quædam verò singu-

singularum propria. Etenim quòd omnis proportio sit ῥητὸς sive rationalis, Arithmetica soli conuenit, nequaquam Geometriae, in qua sunt etiam ἄῤῥητοι seu irrationales proportiones. item, quadratorum gnωmas minimo definitos esse. Arithmeticae proprium (si quidem in Geometria nihil tale minimum esse potest) sed ad Geometriam propriè spectant situs. qui in numeris locum non habent: tactus, qui quidem à continuis admittuntur: ἄλογον, quoniam vbi diuisio infinitè procedit, ibi etiam τὸ ἄλογον esse solet. Communia porrò vtriusque sunt illa, quae ex sectionibus eueniunt, quas Euclides libro secundo est persequutus: nisi quòd sectio per extremam & mediam rationem in numeris nusquam reperiri potest. Iam verò ex theωrematibus eiusmodi communibus, alia quidem ex Geometria ad Arthmeticam traducuntur: alia contrà ex Arithmetica in Geometriam trāsferuntur: quaedam verò perinde vtrique scientiae conueniunt, vt quae ex vniuersa arte Mathematica in vtranque harum conueniant. Nā & alterna ratio, & rationum conuersiones, compositiones, diuisiones hoc modo communia sunt vtriusque. Quae autem sunt περὶ συμμέτρων, id est, de commensurabilibus, Arithmetica quidem primùm cognoscit & contemplatur: secundo loco Geometria Arithmeticam imitata. Quare & commensurabiles magnitudines illae dicuntur, quae rationem inter se habent quam numerus ad numerum, perinde

inde quasi commensuratio & συμμετρία in numeris primum consistat (Vbi enim numerus, ibi & σύμμετρον cernitur: & vbi σύμμετρον, illic etiam numerus) sed quæ triangulorum sunt & quadrangulorum, à Geometra primùm considerantur: tùm analogia quadam Arithmeticus eadem illa in numeris contemplatur. De Geometriæ diuisione hoc adijciendum putò, quòd Geometriæ pars altera in planis figuris cernitur, quæ solam latitudinem longitudini coniunctam habent: altera verò solidas contemplatur, quæ ad duplex illud interuallum crassitudinem adsciscunt. Illam generali Geometriæ nomine veteres appellarunt: hanc propriè Stereometriam dixerunt. Ita Geometriam cum Optica, & Stereometriam cum Mechanica non rarò comparat Aristoteles. Sed illius cognitio huius inuentionem multis seculis antecessit, si modò Stereometriam ne Socratis quidem ætate vllam fuisse omnino verum est, quemadmodum à Platone scriptum videtur. Ad Geometriæ vtilitatem accedo, quæ quanquam suapte vi & dignitate ipsa per se nititur, nullius vsus aut actionis ministerio mancipata (vt de Mathematicis omnibus scientijs concedit in Politico Socrates) si quid ex ea tamen vtilitatis externæ quæritur, Dij boni quã latos, quàm vberes, quũ varios fructus fundit? Nec verò audiendus est vel Aristippus, vel Sophistarum alius, qui Mathematicorum artes idcirco repudiet, quòd ex fine nihil docere videantur, eiusque quod melius aut

deteri-

deterius nullam habeant rationem Vt enim nihil causæ dicas, cur sit melius triãguli, verbi gratia, tres angulos duobus esse rectis æquales: minimè tamen fuerit consentaneum Geometriæ cognitionem vt inutilem exagitare, criminari, explodere, quasi quæ finem & bonum quò referatur, habeat nullum. Multas haud dubiè solius contemplationis beneficio citra materiæ contagionem adfert Geometria commoditates partim proprias partim cum vniuerso genere communes. Cùm enim Geometria, vt scripsit Plato, eius quod semper est cognitionem profiteatur, ad veritatem excitabit illa quidem animum, & ad ritè philosophandum cuiusque mentem comparabit. Quinetiam ad disciplinas omnes facilius perdiscendas, attigeris necne Geometriam quãti referre censes? Nam vbi cum materia coniũgitur, nónne præstantissimas procreat artes, Geodæsiam, Mechaniam, Opticam, quarum omnium vsu, mortalium vitam summis beneficijs complectitur? Etenim bellica instrumenta, vrbiumq; propugnacula, quibus munitæ vrbes hostium vim propulsarent, his adiutricibus fabricata est: montium ambitus & altitudines, locorumque situs nobis indicauit: dimetiendorum & mari & terra itinerum rationem præscripsit: trutinas & stateras, quibus exacta numerorum æqualitas in ciuitate retineatur, composuit: vniuersi ordinem simulachris expressit, multáque quæ hominum fidem superarent, omnibus persuasit. Vbique extant præclara in eam rem *testimonia. Illud memorabile, quòd Archimedi rex Hiero tribuit. Nam extructo vastæ molis nauigio, quod Hiero Ægyptiorum regi Ptolemæo mitteret, cùm vniuersa Syracusanorum multitudo collectis simul viribus nauem trahere non posset, effecissetq́; Archimedes, vt solus Hiero illam subduceret, admiratus viri scientiā rex ἀπὸ ταύτης, ἔφη, τῆς ἡμέρας περὶ παντὸς ἀρχιμήδει λέγοντι πιστευτέον. Quid? quod Archimedes idem, vt est apud Plutarchum, Hieroni scripsit datis viribus datum pondus moueri posse? fretusq́; demonstrationis robore, illud sæpe iactarit, si terram haberet alteram vbi pedem figeret, ad eam, nostram hanc se transmouere posse? Quid varia, αὐτομάτων machinarumquè genera ad vsus necessarios comparata memorem? Innumerabilia profectò sunt illa, & admiratione dignissima quibus prisci homines incredibili quodam ad philosophandum studio concitati, inopem mortalium vitam artis huius præsidio subleuarunt: tametsi memoriæ sit proditum, Platonem Eudoxo & Archytæ vitio vertisse, quōd Geometrica problemata ad sensilia & organica abducerent. Sic enim corrumpi ab illis & labefieri Geometriæ præstantiam, quæ ab intelligibilibus & incorporeis rebus ad sensiles & corporeas prolaberetur. Quapropter ridicula idem scripsit Plato Geometrarum esse vocabula, quæ quasi ad opus & actionem spectent, ita sonare videntur. Quid enim est quadrare si non opus facere? Quid addere,*

pro-

producere, applicare? Multa quidem sunt eiusmodi nomina, quibus necessariè & tanquam coacti Geometræ vtuntur, quippe cùm alia desint in hoc genere commodiora. Sic ergo censuit Plato, sic Aristoteles, sic denique philosophi omnes, Geometriam ipsam cognitionis gratia exercendam, nec ex aliquo vsu externo sed ex rerum νοητῶν intelligentia æstimandam esse. Exposita breuius quàm res tanta dici possit, vtilitatis ratione, Geometriæ ortum, qui in hac rerum periodo ex historicorum monumentis nobis est cognitus deinceps aperiamus. Geometria apud Ægyptios inuenta, (ne ab Adamo, Setho, Noah, quos cognitione rerum multiplici valuisse constat, eam repetamus) ex terrarum dimensione, vt verbi præ se fert ratio, ortum habuisse dicitur: cùm anniuersaria Nili inundatione & incrementis limo obducti agrorum termini confunderentur. Geometriam enim, sicut & reliquas disciplinas, in vsu quàm in arte prius fuisse aiūt. Quod sanè mirum videri non debet, vt & huius & aliarum scientiarum inuentio ab vsu cœperit ac necessitate. Etenim tempus, rerum vsus, ipsa necessitas ingenium excitat, et ignauiā acuit. Deindè quicquid ortum habuit (vt tradunt Physici) ab inchoato & imperfecto processit ad perfectum. Sic artium & scientiarum principia experientiæ beneficio collecta sunt: experientia verò à memoria fluxit, quæ & ipsa à sensu primùm manauit. Nam quod scribit Aristoteles, Mathematicas artes,

 compa-

comparatis rebus omnibus ad vitam necessariis, in Ægypto fuisse constitutas, quòd ibi sacerdotes omnium concessu in otio degerent: non negat ille adductos necessitate homines ad excogitandam, verbi gratia terræ dimetiendæ rationem, quæ theorematum deinde inuestigationi causam dederit: sed hoc confirmat, præclara eiusmodi theorematum inuenta, quibus extructa Geometriæ disciplina constat, ad vsus vitæ necessarios ab illis non esse expetita. Itaque vetus ipsum Geometriæ nomen ab illa terræ partiundæ finiúmque regundorum ratione posteà recessit, & in certa quadam affectionum magnitudini per se inhærentium scientia propriè remāsit. Quemadmodum igitur in mercium & contractuum gratiam supputandi ratio, quam secuta est accurata numerorum cognitio, à Phœnicibus initium duxit: ita etiam apud Ægyptios, ex ea quam cōmemoraui causa ortū habuit Geometria. Hāc certè, vt id obiter dicam, Thales in Græciam ex Ægypto primùm transtulit: cui non paucæ deinceps à Pythagora, Hippocrate, Chio, Platone, Archyta Tarentino, aliisq. cōpluribus, ad Euclidis tempora factæ sunt rerum magnarum accessiones. Cæterùm de Euclidis ætate id solùm addam, quod à Proclo memoriæ mandatum accepimus. Is enim commemoratis aliquot Platonis tum æqualibus tùm discipulis, subiicit, non multò ætate posteriorem illis fuisse Euclidem eum, qui Elementa conscripsit, & multa ab Eudoxo collecta, in ordinē luculentū

com-

composuit, multaq; à Theæteto inchoata perfecit, queq; mollius ab alijs demonstrata fuerant, ad firmissimas & certissimas apodixes reuocauit. Vixit autem, inquit ille, sub primo Ptolemæo. Etenim ferunt Euclidam à Ptolemæo quõdam interrogatum numqua esset via ad Geometriam magis compendiaria, quàm sit ista στοιχειώσις respondisse, μὴ εἶναι βασιλικὴν ἀτραπὸν ἐπὶ γεωμετρίαν. Deinde subiungit, Euclidem natu quidem esse minorem Platone, maiorem verò Eratosthene & Archimede (hi enim æquales erant) cùm Archimedes Euclidis mentionem faciat. Quòd si quis egregiam Euclidis laudem quam cùm ex alijs scriptionibus accuratissimis, tùm ex hac Geometrica στοιχειώσει cõsequutus est in qua diuinus rerum ordo sapientissimis quibusque hominibus magnæ semper admirationi fuit, is Proclum studiosè legat, quò rei veritatẽ illustriorem reddat grauissimi testis auctoritas. Superest igitur vt finem videamus, quò Euclidis elementa referri, & cuius causa in id studium incumbere oporteat. Et quidem si res quæ tractantur, consyderes: in tota hac tractatione nihil aliud quæri dixeris, quàm vt κοσμικὰ quæ vocantur σχήματα (fuit enim Euclides professione & instituto Platonicus) Cubus, Icosaedrũ, Octaedrum, Pyramis & Dodecaedrum certa quadã suorum & inter se laterum, & ad sphæræ diametrum ratione eidem sphæræ inscripta cõprehendantur. Huc enim pertinet Epigrammation illud vetus, quod in Geometrica Michaelis

Pselli συνόψει scriptum legitur.

Σχήματα πέντε τὰ πλάτωνος, πυθαγόρας σοφὸς εὗρε,
πυθαγόρας σοφὸς εὗρε, πλάτων δ' ἀρίδηλ' ἐδίδαξεν,
Εὐκλείδης ἐπὶ τοῖσι κλέος περικαλλὲς ἔτευξεν.

Quòd si discentis institutionem spectes, illud certè fuerit propositum, ut huiusmodi elementorum cognitione informatus discentis animus, ad quamlibet non modò Geometriæ, sed & aliarũ Mathematicæ partium tractationem idoneus paratusq; accedat. Nam tametsi institutionem hanc solus sibi Geometra vendicare videtur, & tanquam in possessionem suam venerit, alios excludere posse: inde tamen per multa suo quodam modo iure decerpit Arithmeticus, pleraque Musicus, non pauca detrahit Astrologus, Opticus, Logisticus, Mechanicus, itemq; cæteri: nec vllus est denique artifex præclarus, qui in huiusce possessionis societatem cupidè nõ offerat, partemq; sibi concedi postulet. Hinc στοιχείωσις *absolutum operi nomen, &* στοιχειωτὴς *dictus Euclides. Sed quid longius prouehor? Nam quod ad hanc rem attinet, tam copiosè & eruditè scripsit (vt alia complura) eo ipso, quo dixi, loco P. Montaureus, ut nihil desiderio loci reliquerit. Quæ verò ad dicendum nobis erant proposita, hactenus pro ingenij nostri tenuitate omnia mihi perfecisse videor. Nam tametsi & hæc eadem & alia pleraque multo fortè præclariora ab hominibus doctissimis,*

quò

qui tùm acumine ingenij, tùm admirabili quodam lepore dicendi semper floruerunt grauiùs, splendidiùs, uberiùs tractari posse scio: tamen experiri libuit. numquid etiam nobis diuino sit concessum munere, quod rudes in hac philosophiæ parte discipulos adiuuare aut certè excitare queat. Huc accessit quòd ista recens elementorum editio, in qua nihil non parum fuisset studij, aliquid à nobis efflagitare videbatur, quod eius commendationem adaugeret. Cùm enim vir doctissimus Io Magnienus Mathematicarum artium in hac Parrhisiorum Academia professor verè regius, nostrum hunc typographum in excudendis Mathematicorum libris diligentissimum, ad hanc Elementorum editionem sæpè & multum esset adhortatus, eiusque impulsu permulta sibi iam comparasset typographus ad hanc rem necessaria, citò interuenit, malum Ioannis Magnieni mors insperata, quæ tam graue inflixit Academiæ vulnus, cui ne post multos quidem annorum circuitus cicatrix obduci vlla posse videatur. Quamobrem amisso instituti huius operis duce, typographus, qui nec sumptus antea factos sibi perire, nec studiosos, quibus id muneris erat pollicitus, sua spe cadere vellet, ad me venit, & impensè rogauit. vt meam propositæ editioni operam & studium nauarem, quod cùm denegaret occupatio nostra, iuberet officij ratio, feci equidem rogatus, vt quæ subobscurè vel parum commode in sermonem Latinum è Græco translata

vi-

videbantur, clariore, aptiore & fideliore interpretatione nostra (quod cuiusq; pace dictum volo) lucem acciperent. Id quod in omnibus ferè libris posterioribus tute primo obtutu perspicias. Nam in sex prioribus nō tantum temporis quātum in cæteros ponere nobis licuit: decimi autem interpretatio, qua melior nulla potuit adferri, P. Montaureo solida debetur. Atq; vt ad perspicuitatem facilitatemque nihil tibi deesse queraris, adscripta sunt propositionibus singulis vel lineares figuræ, vel punctorum tanquam vnitatum notulæ, quæ Theonis apodixin illustrēt: illæ quidem magnitudinum, hæ autem numerorum indices subscriptis etiam ciphrarum, vt vocant, characteribus, qui propositum quemuis numerum exprimant. ob eamq; causam eiusmodi vnitatum notulæ, quæ pro numeri amplitudine maius paginæ spatium occuparent, pauciores sæpius depictæ sunt aut in lineas etiam commutatæ. Nam literarum vt a, b, c characteres non modò numeris & numerorum partibus nominādis sunt accommodati, sed etiam generales esse numerorum, vt magnitudinum affectiones testantur. Adiecta sunt insuper quibusdam locis non pœnitenda Theonis scholia, siue mauis lemmata, quæ quidem longè plura accessissent, si plus otij & temporis vacui nobis fuisset relictū, quod huic studio impartiremus. Hanc igitur operam boni consule, & quæ obuia erunt impressionis vitia, candidè emenda. Vale. Lutetiæ 4. Idus April. 1557.

EVCLIDIS ELEMENTVM PRIMVM.

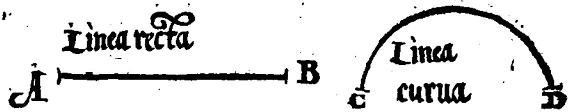
DEFINITIONES.

I

PVnctum est, cuius pars nulla est. Punctum Σημεῖον

2

Linea verò, longitudo latitudinis expers. γραμμὴ



3

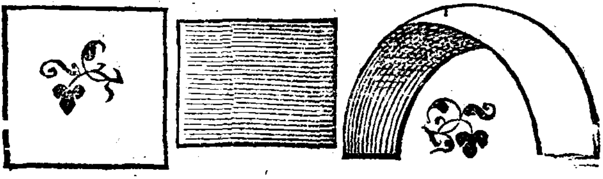
Lineæ autem termini, sunt puncta.

4

Recta linea est, quæ ex æquo sua interiacet puncta.

5

Superficies est, quæ longitudinem latitudinemque tantùm habet. ἐπιφάνεια



6 Super-

6

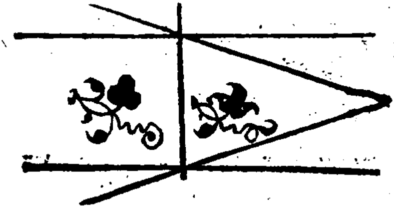
Superficiei extrema, ſunt lineæ.

7

ἐπίπεδος Plana ſuperficies eſt, quæ ex æquo ſuas interiacet lineas.

8

γωνία Planus angulus eſt duarũ linearũ in plano ſe mutuò tangentium, & non in directum



iacétium, alterius ad alteram inclinatio.

9.

Cùm autem quæ angulum continét lineæ, rectæ fuerint, rectilineus ille angulus appellatur.

10

Cùm verò recta linea ſuper rectam conſiſtens lineã, eos qui ſunt deinceps angulos ęquales, inter ſe fecerit: rectus eſt vterq; æquaqualium angulorũ:& quæ inſiſtit recta linea,perpẽdicularis vocatur eiꝰ,cui inſiſtit. γραμμὴ κάθετος.

11

Obtuſus angulus eſt, qui recto maior eſt.

12

Acutus verò,qui minor eſt recto.

13

Terminus eſt,quod alicuius extremũ eſt. ὅρος.

14

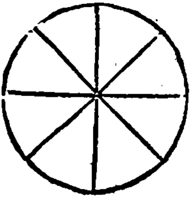
Figura eſt,quæ ſub aliquo , vel aliquibus terminis comprehenditur. κύκλος

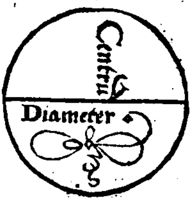
15

Circulus eſt figura plana ſub vna linea cõprehẽſa,quę peripheria appellatur:ad quã ab vno puncto eorum, quæ intra figuram σχῆμα.

ſunt

ſunt poſita, cadentes omnes rectæ lineæ inter ſe ſunt æquales.





16

κέντρον

Hoc verò punctum, centrum circuli appellatur.

17

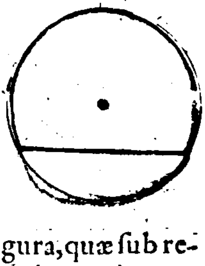
διάμετρος

Diameter autem circuli, eſt recta quædam linea per cẽtrum ducta, & ex vtraque parte in circuli peripheriam terminata, quæ circulum bifariam ſecat.

18

ἡμικύκλιον

Semicirculus eſt figura, quæ cõtinetur ſub diametro, & ſub ea linea, quæ de circuli peripheria aufertur.



19

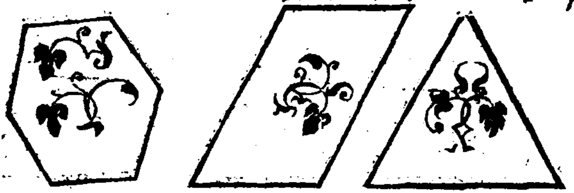
κύκλου τμῆμα ἡμικύκλιον

Segmentum circuli, eſt figura, quæ ſub recta linea & circuli peripheria continetur.

20 Recti

20

Rectilineæ figuræ, sunt quæ sub rectis lineis continentur. σχήματα εὐθύγραμμα



21

Trilateræ quidem, quæ sub tribus. τρίπλευρα

22

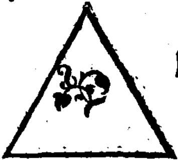
Quadrilateræ, quæ sub quatuor. τετράπλευρα

23

Multilateræ verò, quæ sub pluribus quàm quatuor rectis lineis comprehenduntur. πολύπλευρα

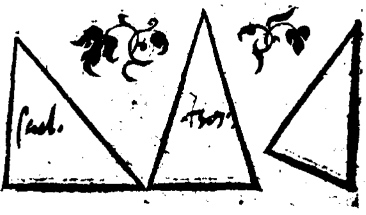
24

Trilateraru porrò figurarum, æquilaterum est triangulum, quod tria latera habet æqualia. ἰσόπλευρον τρίγωνον



25

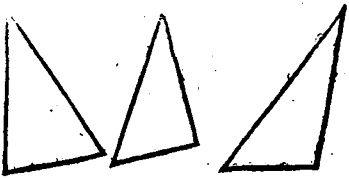
Isosceles autem est quod duo tantum æqualia habet latera. ἰσοσκελὲς



26

σκάληνον

Scalenũ verò, eſt qđ tria inæqualia habet latera



27

ὀρθογώνιον τρίγωνον

Ad hæc etiam, trilaterarum figurarum, rectangulum quidem triangulum eſt, quod rectum angulum habet.

28

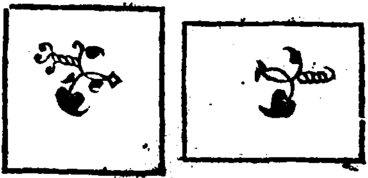
Amblygonium autem, quod obtuſum angulum habet.

29

Oxygonium verò, quod tres habet acutos angulos.

30

Quadrilaterarum autem figurarum, quadratũ quidẽ eſt qđ & æquilaterũ & rectangulum eſt.

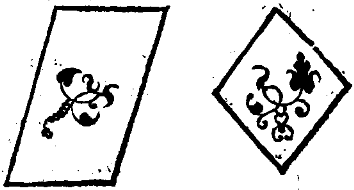


31

Altera parte longior figura eſt, quæ rectãgula quidem, at æquilatera non eſt.

32 Rhom-

32 Rhombus autem, quia æquilaterum & rectangulum est.



33 Rhomboides verò, quæ aduersa & latera & angulos habens inter se æqualia, neque æquilatera est, neque rectangula. ῥομβοειδὲς

34 Preter has autem reliquæ quadrilateræ figuræ, trapezia appellentur. τραπέζια.

35 Parallelæ rectæ lineæ sunt quæ, cùm in eodem sint plano, & ex vtraque parte in infinitum producantur, in neutram sibi mutuò incidunt. παράλληλ.

Postulata.

1 Postuletur, vt à quouis puncto in quoduis

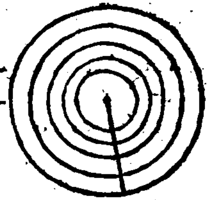
punctum, rectam lineã ducere cõcedatur.

2

Et rectam lineam terminatam in continuum recta producere.

3

In quouis centro & interuallo circulũ describere.



Communes notiones.

1.

Quæ eidẽ æqualia, & inter se sunt æqualia.

2

Et si æqualibus æqualia adiecta sint, tota sunt æqualia.

3

Et si ab æqualibus æqualia ablata sint, quæ relinquuntur sunt æqualia.

4

Et si inæqualibus æqualia adiecta sint, tota sunt inæqualia.

5

Et si ab inęqualibus ęqualia ablata sint, reliqua sunt inæqualia.

6

Quæ eiusdem duplicia sunt, inter se sunt æqualia.

Et

Item Verba. Propria statim propos. ut: LEMMA, casus, Corollarium,
Ins[illegible]

7

Et quæ eiusdem sunt dimidia, inter se æqualia sunt.

8

Et quæ sibi mutuò congruunt, ea inter se sunt æqualia.

9

Totum est sua parte maius.

10

Item, omnes recti anguli sunt inter se ęquales.

11

Et si in duas rectas lineas altera recta incidens, internos ad easdémque partes angulos duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ sibi mutuò incident ad eas partes, vbi sunt anguli duobus rectis minores.

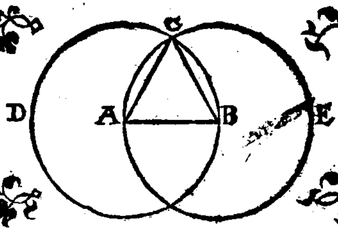
12

Duæ rectæ lineæ spatium non comprehédunt.

Hic 18.
Problematis huius
propria
1 Propositio.
2 Expositio
3 Determinat.
4 Constructio.
5 Demonstrat.
6 Conclusio.

Problema 1. Propositio 1.

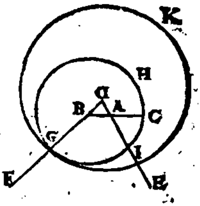
Super data recta linea terminata, triangulũ æquilaterum constituere.



Propria
Datũ
Positione.
Proport.
Magnitud.
Specie.
Quasi

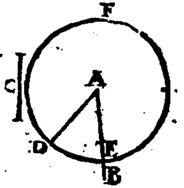
Problema 2. Propoſitio 2.

Ad datum punctum, datæ rectæ lineæ, ęqualem rectam lineam ponere.



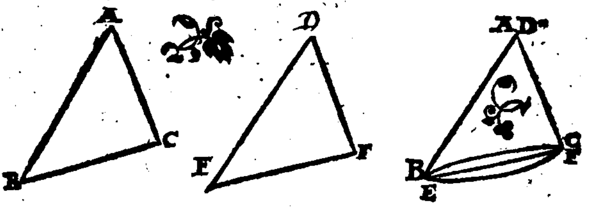
Problema 3. Propoſitio 3.

Duabus datis rectis lineis inęqualibus, de maiore æqualẽ minori rectam lineã detrahere.



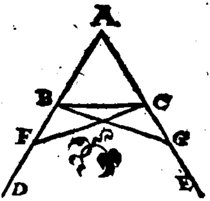
Theorema primum. Propoſitio 4.

Si duo triangula duo latera duobus lateribus æqualia habeant, vtrunque vtriq; , habeant verò & angulum angulo æqualẽ ſub æqualibus rectis lineis contentum: & baſin baſi æqualem habebunt, eritq; triãgulum triãgulo æquale, ac reliqui anguli reliquis angulis æquales erũt, vterque vtrique, ſub quibus æqualia latera ſubtenduntur.



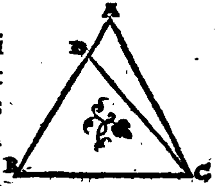
Theore-

Theorema 2. Propoſitio 5.



Iſoſcelium triangulorũ qui ad baſim ſunt anguli, inter ſe ſunt æquales; & ſi vlterius productæ ſint æquales illæ rectæ lineæ, qui ſub baſi ſunt anguli, inter ſe ęquales erunt.

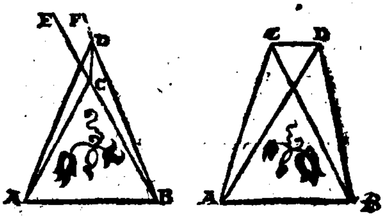
Theorema 3. Propoſitio 6.



Si trianguli duo anguli æquales inter ſe fuerint & ſub ęqualibus angulis ſubtenſa latera æqualia inter ſe erunt.

Theorema 4. Propoiitio 7.

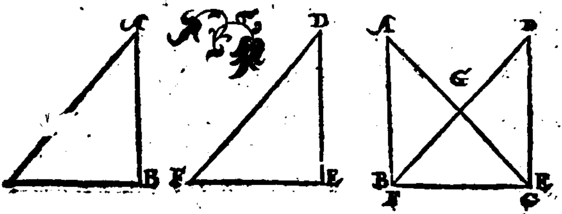
Super eadẽ recta linea, duabus eiſdem rectis lineis alię duę rectę lineę ęquales, vtraque vtrique, nõ conſtituentur, ad aliud atque aliud punctũ, ad eaſdẽ partes eoſdẽ que terminos cum duabus initio ductis rectis lineis habentes.



 Theo-

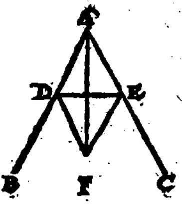
Theorema 5. Propositio 8.

Si duo triãgula duo latera habuerint duobus lateribus, vtrunque vtriq;, æqualia: habuerint verò & basim basi æqualẽ: angulũ quoque sub æqualibus rectis lineis cõtentum angulo æqualem habebunt.



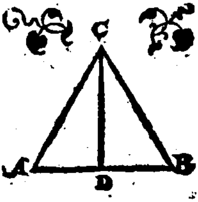
Problema 4. Propositio 9.

Datum angulum rectilineum bifariam secare.



Problema 5. Propositio 10.

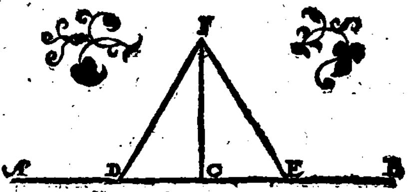
Datam rectam lineã finitam bifariam secare.



Proble-

Problema 6. Propoſitio 11.

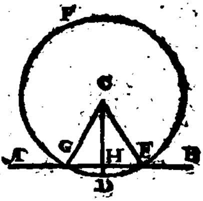
Data recta linea, à puncto in ea dato, re-



ctam lineam ad angulos rectos excitare.

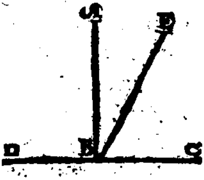
Problema 7. Propoſitio 12.

Super datam rectã lineã infinitam, à dato puncto quod in ea non eſt, perpendicularem rectam deducere.



Theorema 6. Propoſitio 13.

Cùm recta linea ſuper rectã conſiſtens lineã angulos facit, aut duos rectos, aut duobus rectis æquales efficiet.

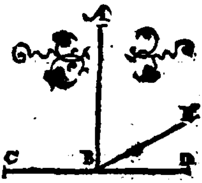


Theorema 7. Propoſitio 14.

Si ad aliquam rectam lineam, atque ad eius

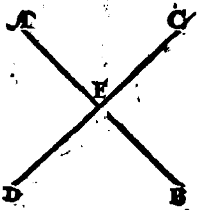
 punctum

punctũ, duę rectæ lineæ nõ ad eaſdem partes du ctæ, ęos qui ſunt deinceps angulos duobus rectis æquales fecerint, in directum erunt inter ſe ipſæ rectæ lineæ.



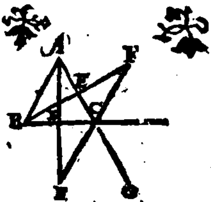
Theorema 8. Propoſitio 15.

Si duę rectę lineę ſe mutuò ſecuerint, angulos qui ad verticem ſunt, æquales inter ſe efficient.



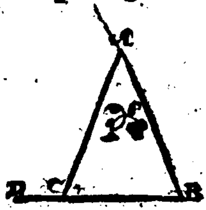
Theorema 9. Propoſitio 16.

Cuiuſcunque trianguli vno latere producto, externus angulus vtroque interno & oppoſito maior eſt.



Theorema 10. Propoſitio 17.

Cuiuſcunque trianguli duo anguli duobus rectis ſunt minores omnifariam ſumpti.



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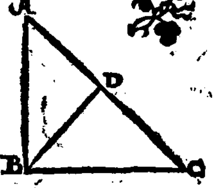
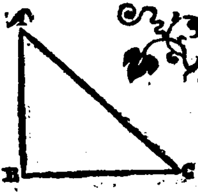
Theorema 11. Propoſitio 18.



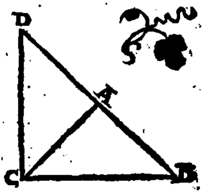
Omnis trianguli maius latus maiorem angulũ ſubtendit.

Theorema 12. Propoſitio 19.



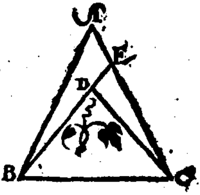
Omnis trianguli maior angulus, maiori lateri ſubtenditur.

Theorema 13. Propoſitio 20.



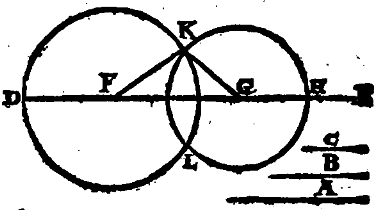
Omnis triãguli duo latera reliquo ſunt maiora, quomodocũq; aſſũpta.

Theorema 14. Propoſitio 21.



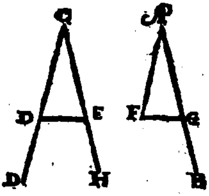
Si ſuper triãguli vno latere, ab extremitatibus duæ rectę lineę, interius cõſtitutæ fuerint, hę cõſtitutæ reliquis triãguli duobus lateribus minores quidem erunt, maiorem verò angulum continebunt.

Problema 8. Propositio 22.



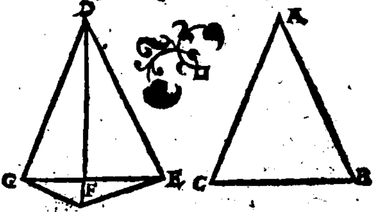
Ex tribus rectis lineis quæ sũt tribus datis rectis lineis æquales, triangulum constituere. Oportet autem duas reliqua esse maiores omnifariam sumptas: quoniã vniuscuiusq; trianguli duo latera omnifariam sumpta reliquo sunt maiora.

Problema 9. Propositio 23.



Ad datam rectam lineã datumq́; in ea punctum dato angulo rectilineo ęqualem angulum rectilineum constituere.

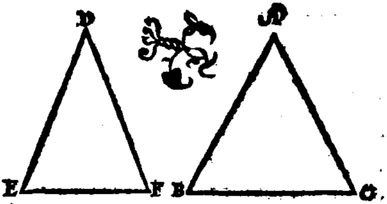
Theorema 15. Propositio 24.



Si duo triãgula duo latera duobus lateribus ęqualia habuerint, vtrũq; vtriq;, angulũ verò angulo lo maiorem ſub æqualibus rectis lineis cōtentum:& baſin baſi maiorem habebunt.

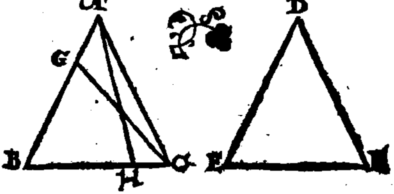
Theorema 16. Propoſitio 25.

Si duo triangula duo latera duobus lateribus æqualia habuerint, vtrunque vtrique, baſin verò baſi maiorē: & angulum ſub ęqualib⁹ rectis lineis contentum angulo maiorē habebunt.



Theorema 17. Propoſitio 26.

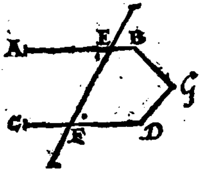
Si duo triangula duos angulos duobus angulis æquales habuerint, vtrūque vtrique, vnumq́; latus vni lateri æquale, ſiue quod æqualibus adiacet angulis, ſeu quod vni ęqualium angulorū ſubtenditur:& reliqua latera reliquis lateribus ęqualia vtrūq; vtriq;, & reliquum angulū reliquo angulo æqualem habebunt.



Theore-

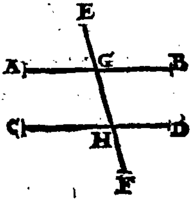
Theorema 18. Propositio 27.

Si in duas rectas lineas recta incidés linea alternatim águlos ęquales inter se fecerit: parallelæ erunt inter se illæ rectæ lineæ.



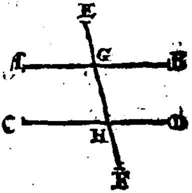
Theorema 19. Propositio 28.

Si in duas rectas lineas recta incidens lineà externum angulũ interno, & opposito, & ad easdem partes æqualé fecerít, aut internos & ad easdé partes duobus rectis æquales: parallelæ erunt inter se ipsæ rectæ lineæ.



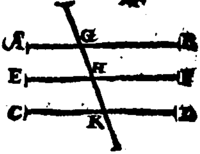
Theorema 20. Propositio 29.

In parallelas rectas lineas recta incidens linea: & alternatim angulos inter se æquales efficit & externum interno & opposito & ad easdem partes æqualem, & internos & ad easdem partes duobus rectis æquales facit.



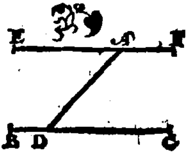
Theore-

Theorema 21. Propoſitio 30.



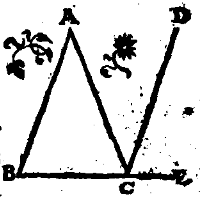
Quæ eidem rectæ lineę, parallelæ, & inter ſe ſunt parallelæ.

Problema 10. Propoſitio 31.



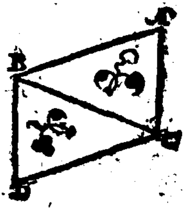
A dato puncto datæ rectæ lineæ parallelam rectam lineam ducere.

Theorema 22. Propoſitio 32.



Cuiuſcunque trianguli vno latere vlterius producto: externus angulꝰ duobus internis & oppoſitis eſt æqualis. Et trianguli tres interni anguli duobus ſunt rectis æquales.

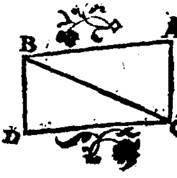
Theorema 23. Propoſitio 33.



Rectæ lineę quę ęquales & parallelas lineas ad partes eaſdem coniungunt, & ipſæ æquales & parallelæ ſunt.

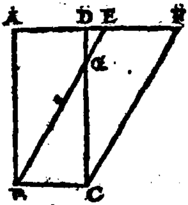
Theore-

Theorema 24. Propositio 34.



Parallelogrammorũ spatiorum æqualia sunt inter se, quæ ex aduerso & latera & anguli: atque illa bifariã secat diameter.

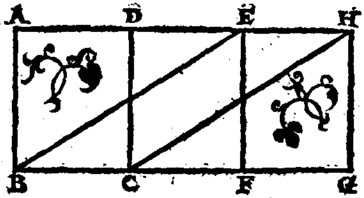
Theorema 25. Propositio 35.



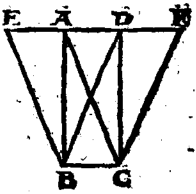
Parallelogramma super eadem basi, & in eisdem parallelis cõstituta, inter se sunt æqualia.

Theorema 26. Propositio 36.

Parallelogrãma super æqualibus basib⁹ & in eisdẽ parallelis cõstituta inter se sunt æqualia.



Theorema 27. Propositio 37.

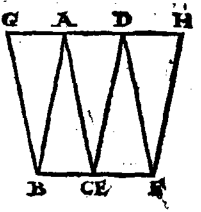


Triãgula super eadẽ basi cõstituta, & in eisdẽ paral lelis, inter se sunt ęqualia.

Theore-

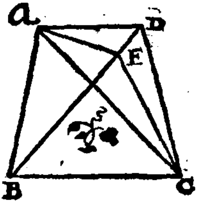
Theorema 28. Propoſitio 38.

Triangula ſuper æqualibus baſibus cõſtituta & in eiſdem parallelis, inter ſe ſunt æqualia.



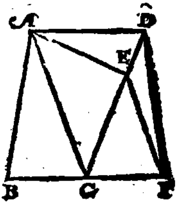
Theorema 29. Propoſitio 39.

Triãgula æqualia ſuper eadem baſi, & ad eaſdem partes cõſtituta, & in eiſdem ſunt parallelis.



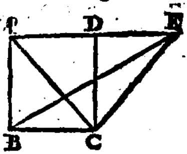
Theorema 30. Propoſitio 40.

Triãgula æqualia ſuper æqualibus baſibus, & ad eaſdem partes cõſtituta, & in eiſdẽ ſunt parallelis.



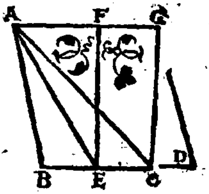
Theorema 31. Propoſitio 41.

Si parallelogrammum cum triangulo eandem baſin habuerit, ~~non~~ in eiſdemq; fuerit parallelis, duplum erit parallelogrammum ipſius triãguli:



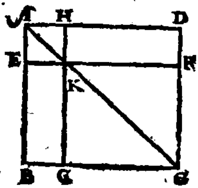
Problema 11. Propoſitio 42.

Dato triangulo æquale parallelogrammũ conſtituere in dato angulo rectilineo.

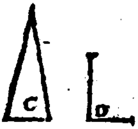


Theorema 32. Propoſitio 43.

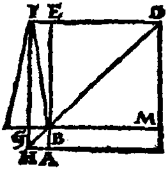
In omni parallelogrammo, complementa eorũ quę circa diametrũ ſunt parallelogrammorum, inter ſe ſunt æqualia.



Problema 12. Propoſitio 44.



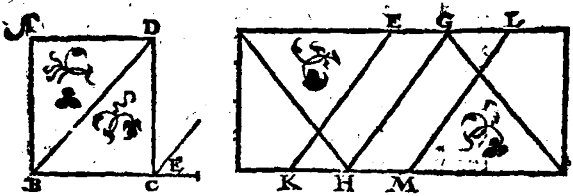
Ad datam rectam lineã, dato triangulo æquale parallelogrammum applicare in dato angulo rectilineo.



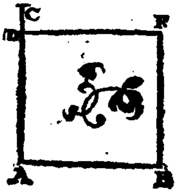
Problema 13. Propopotio 45.

Dato rectilineo æquale parallelogrammũ conſti-

conſtituere in dato angulo rectilineo.



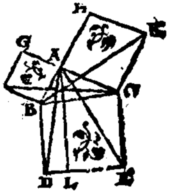
Problema 14. Propoſitio 46.



A data recta linea quadratum deſcribere.

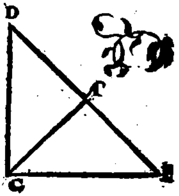
Theorema 33. Propoſitio 47.

In rectangulis triãgulis, quadratum quod à latere rectum angulũ ſubtendẽte deſcribitur, æquale eſt eis, quę à lateribus rectum angulum continentibus deſcribũtur, quadratis.



Theorema 34. Propoſitio 48.

Si quadratum quod ab vno laterum triãguli deſcribitur, æquale ſit eis quæ à reliquis triãguli lateribus deſcribũtur, quadratis: angulus comprehenſus ſub reliquis duobus trianguli lateribus, rectus eſt.



FINIS ELEMENTI I.

EVCLI-

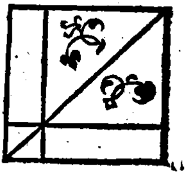
EVCLIDIS ELEMENTVM SECVNDVM.

DEFINITIONES.

I

OMne parallelogrammum rectangulum contineri dicitur ſub rectis duabus lineis, quæ rectum comprehendunt angulum.

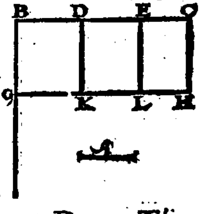
2



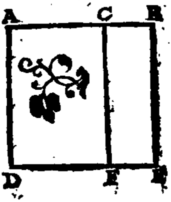
In omni parallelogrammo ſpatio, vnumquodlibet eorũ, quæ circa diametrum illius ſunt parallelogrammorum, cũ duobus complementis, Gnomo vocetur.

Theorema I. Propoſitio I.

Si fuerint duæ rectæ lineæ, ſeceturq; ipſarũ altera in quotcunque ſegmenta: rectangulum comprehenſum ſub illis duab9 rectis lineis, æquale eſt eis rectangulis, quę ſub inſecta & quolibet ſegmentorum comprehenduntur.



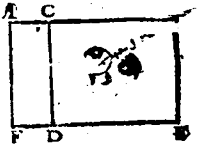
Theorema 2. Propoſitio 2.



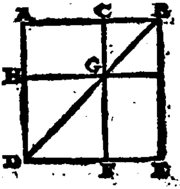
Si recta linea ſecta ſit vtcunq;, rectãgula quæ ſub tota & quolibet ſegmentorum cõprehenduntur æqualia ſunt ei, quod à tota fit, quadrato.

Theorema 3. Propoſitio 3.

Si recta linea ſecta ſit vtcunque, rectangulum ſub tota & vno ſegmẽtorum comprehenſum, ęquale eſt & illi quod ſub ſegmentis cõprehenditur rectãgulo, & illi, quod à prædicto ſegmento deſcribitur, quadrato.



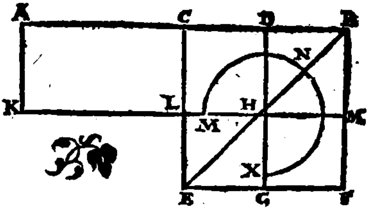
Theorema 4. Propoſitio 4.



Si recta linea ſecta ſit vtcunque: quadratũ quod à tota deſcribitur, ęquale eſt & illis quæ à ſegmẽtis deſcribũtur quadratis, & ei quod bis ſub ſegmentis compreh ẽditur rectangulo.

Theorema 5. Propoſitio 5.

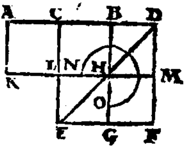
Si recta linea ſecetur in æqualia & non æqualia: rectangulum ſub inæqualibus ſegmentis



mentis totius comprehẽsũ, vnà cum quadrato quod ab intermedia sectionum, æquale est ei quod à dimidia describitur, quadrato.

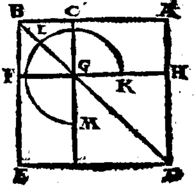
Theorema 6. Propositio 6.

Si recta linea bifariam secetur, & illi recta quædam linea in rectum adijciatur, rectãgulum comprehẽsum sub tota cum adiecta, & adiecta simul cum quadrato à dimidia, æquale est quadrato à linea, quæ tũ ex dimidia, tum ex adiecta componitur, tanquam ab vna descripto.



Theorema 7. Propositio 7.

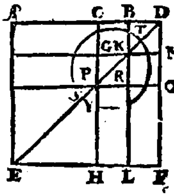
Si recta linea secetur vtcunque: quod à tota, quodque ab vno segmentorum, vtraq; simul quadrata, æqualia sunt & illi quod bis sub tota & dicto segmento comprehenditur, rectãgulo, & illi ꝙ à reliquo segmento fit, quadrato.



 Theo-

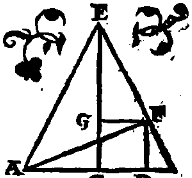
Theorema 8. Propositio 8.

Si recta linea secetur vtcunque rectangulum quater comprehẽsum sub tota & vno segmentorum, cũ eo quod à reliquo segmento fit, quadrato, æquale est ei quod à tota & dicto segmento, tanquam ab vna linea describitur, quadrato.



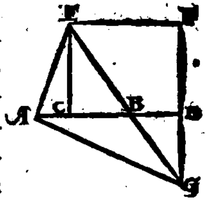
Theorema 9. Propositio 9.

Si recta linea secetur in æqualia & non æqualia: quadrata quæ ab inęqualibus totius segmẽtis fiunt, duplicia sunt & eius quod à dimidia, & eius quod ab intermedia sectionum fit, quadratorum.



Theorema 10. Propositio 10.

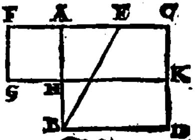
Si recta linea secetur bifariam, adijciatur autem ei in rectum quępiam recta linea: quod à tota cũ adiuncta, & quod ab adiuncta, vtraq; simul quadrata, duplicia sunt, & eius quod à dimidia, & eius quod à cõposita ex dimi-



dimidia & adiuncta, tãquam ab vna descriptum sit, quadratorum.

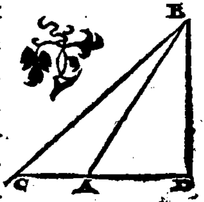
Problema I. Propositio II.

Datam rectam lineã secare, vt comprehensum sub tota & altero segmẽtorum rectangulum, æquale sit ei quod à reliquo segmento fit, quadrato.



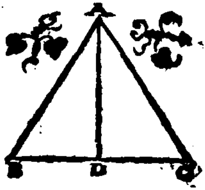
Theorema II. Propositio 12.

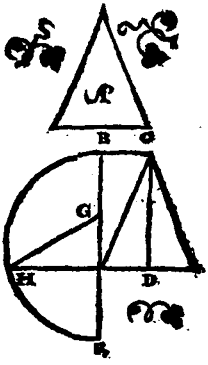
In amblygonijs triangulis, quadratũ quod fit à latere angulum obtusum subtendẽte, maius est quadratis, quæ fiunt à lateribus obtusum angulum comprehendentibus, pro quantitate rectanguli bis comprehẽsi & ab vno laterũ quæ sunt circa obtusum angulũ, in quod cùm protractum fuerit, cadit perpendicularis, & ab assumpta exterius linea sub perpendiculari prope angulũ obtusum.



Theorema 12. Propoſitio 13.

In oxygonijs triangulis, quadratũ à latere angulum acutum ſubtendente, minus eſt quadratis quæ fiunt à lateribus acutũ angulum cõprehendentibus, pro quantitate rectanguli bis comprehenſi, & ab vno late rum, quæ ſunt circa acutum angulum, in quod perpẽdicularis cadit, & ab aſſumpta interius linea ſub perpendiculari prope acutũ angulum.





Problema 2. Propoſitio 14.

Dato rectilineo æquale quadratũ conſtituere.

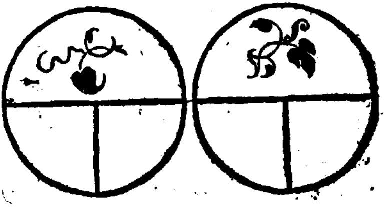
ELEMENTI II FINIS.

EVCLIDIS ELEMENTVM TERTIVM.

DEFINITIONES.

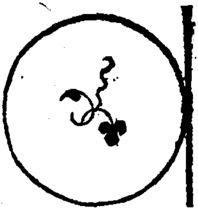
1

Aequales circuli sunt, quorũ diametri sũt ęquales vel quorũ quæ ex centris rectæ lineę sunt æquales.



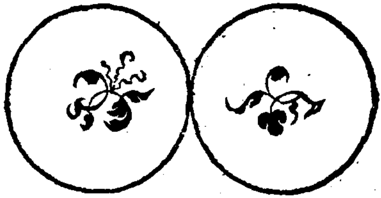
2

Recta linea circulũ tangere dicitur, quæ cùm circulum tangat, si producatur, circulum non secat.



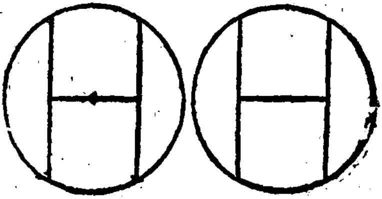
3

Circuli ſeſe mutuò tãgere dicũtur: qui ſeſe mutuo tangentes, ſeſe mutuò non ſecant.



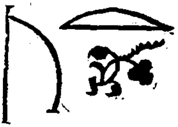
4

In circulo æqualiter diſtare à centro rectæ lineæ dicuntur, cùm perpendiculares, quæ à cẽtro in ipſas ducuntur, ſunt æquales. Lõgius autem abeſſe illa dicitur in quã maior ꝑpẽdicularis cadit.



5

Segmentũ circuli eſt, figura quæ ſub recta linea & circuli peripheria comprehenditur.



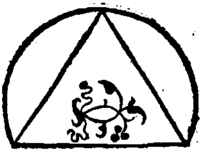
6

Segmenti autẽ angulus eſt, qui ſub recta linea

nea & circuli peripheria comprehēditur.

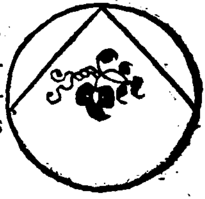
7

In ſegmento autem angulus eſt, cùm in ſegmenti peripheria ſumptū fuerit quodpiam pūctum, & ab illo in terminos rectæ eiꝰ lineę, quę ſegmenti baſis eſt, adiunctę fuerint rectę lineę: is, inquam, angulus ab adiunctis illis lineis comprehenſus.

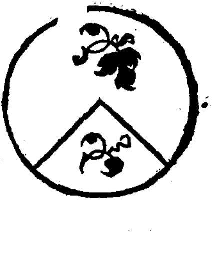


8

Cùm verò comprehendentes angulum rectę lineæ aliquā aſſumunt peripheriam, illi angulus inſiſtere dicitur.



Sector autem circuli eſt cùm ad ipſius circuli cētrum conſtitutus fuerit angulus, cōprehenſa nimirum figura, & à rectis lineis angulū continentibus, & à peripheria ab illis aſſumpta.

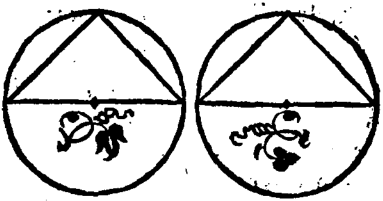


10

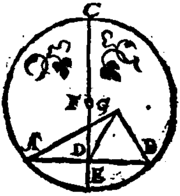
Similia circuli ſegmenta ſunt, quę angulos

capiunt

capiũt
ęquales
aut in q̃
b⁹ angu
li inter
se sunt
ęquales

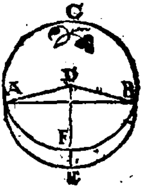


Problema 1. Propositio 1.



Dati circuli cẽtrum reperire.

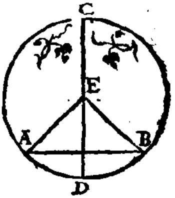
Theorema 1. Propositio 2.



Si in circuli peripheria duo quælibet pũcta accepta fuerint, recta linea quæ ad ipsa puncta adiũgitur, intra circulum cadet.

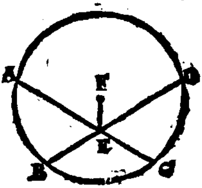
Theorema 2. Propositio 3.

Si in circulo recta quædam linea per centrum extensa quandam non per centrum extensam bifariam secet: & ad angulos rectos ipsam secabit. Et si ad angulos rectos eam secet, bifariam quoque eam secabit.



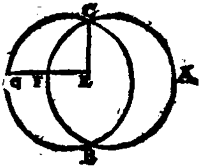
Theore-

Theorema 3. Propositio 4.



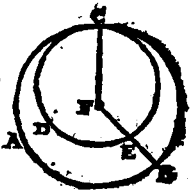
Si in circulo duę rectę lineæ sese mutuò secent non per centrum extẽsæ sese mutuò bifariam nõ secabunt.

Theorema 4. Propositio 5.



Si duo circuli sese mutuò secent, non erit illorum idem centrum.

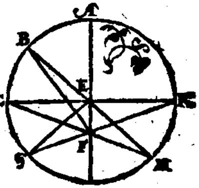
Theorema 5. Propositio 6.



Si duo circuli sese mutuò interius tangant, eorum non erit idem centrum.

Theorema 6. Propositio 7.

Si in diametro circuli quodpiam sumatur punctum, quod circuli cẽtrum non sit, ab eoque pũcto in circulũ quædam rectæ lineæ cadant: maxima quidem erit ea in qua centrũ, minima verò reliqua: aliarum verò propinquior illi quæ per centrũ du-

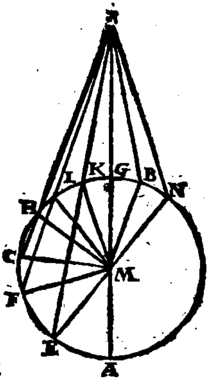


citur

citur,remotiore ſemper maior eſt.Duę aũ tem ſolùm rectæ lineæ æquales ab eodem puncto in circulum cadũt ad vtraſque partes minimæ.

Theorema 7.Propoſitio 8.

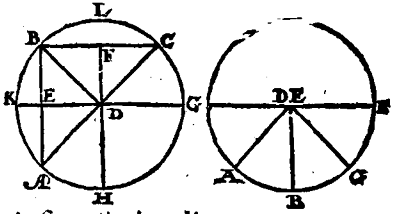
Si extra circulum ſumatur punctũ quodpiam,ab eoq́ue puncto ad circulum deducantur rectæ quædam lineæ, quarum vna quidem per centrum protẽdatur,reliquæ verò vt libet:in cauam peripheriam cadẽtium rectarum linearum maxima quidem eſt illa,quæ per centrum ducitur: aliarum autem propinquior ei, quę per centrum tranſit, remotiore ſemper maior eſt: in cõuexam verò peripheriã cadentium rectarũ linearum minima quidẽ eſt illa, quæ inter punctum & diametrum interponitur: aliarum autem, ea quæ propinquior eſt minimæ, remotiore ſemper minor eſt.Duæ autem tantùm rectæ lineæ æquales ab eo puncto in ipſum circulum cadũt;ad vtraſque partes minimæ.



Theore-

Theorema 8. Propositio 9.

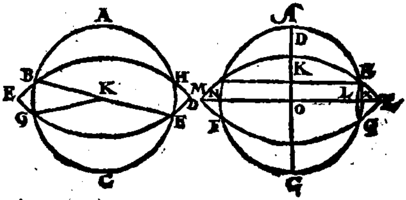
Si in circulo acceptum fuerit pũctum aliquod, & ab eo puncto ad circulum cadant plures quã duę rectę lineæ, æquales, acceptũ punctũ



centrum ipsius est circuli.

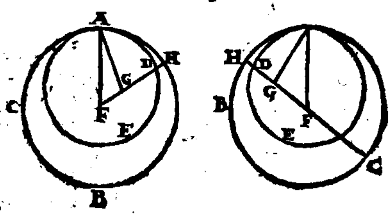
Theorema 9. Propositio 10.

Circulus circulum in pluribus quàm duob⁹ pũctis non secat.



Theorema 10. Propositio 11.

Si duo circuli sese intus cõtingãt, atque accepta

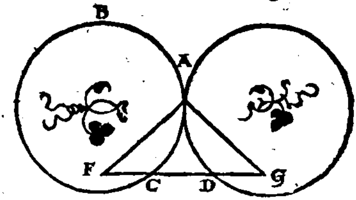


E fuerint

fuerint eorum centra, ad eorum cẽtra adiuncta recta linea & producta in cõtactum circulorum cadet.

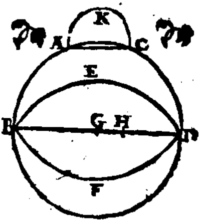
Theorema 11. Propositio 12.

Si duo circuli sese exterius cõtingãt, linea recta q̃ ad cẽtra eorũ ad iũgitur, ꝑ cõtactũ illũ trãsibit.



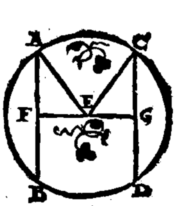
Theorema 12. Propositio 13.

Circulus circulum non tangit in pluribus punctis, quã vno, siue intus siue extra tangat.



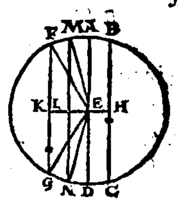
Theorema 13. Propositio 14.

In circulo æquales rectæ lineæ æqualiter distant à centro. Et quæ ęqualiter distant à centro, æquales sunt inter se.



Theore-

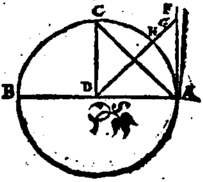
Theorema 14. Propo- ſitio 15.



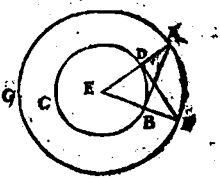
In circulo maxima quidem linea eſt diameter: aliarum autem propinquior centro, remotiore ſemper maior.

Theorema 15. Propoſitio 16.

Quæ ab extremitate diametri cuiuſq; circuli ad angulos rectos ducitur, extra ipsū circulum cadet, & in locum inter ipſam rectam lineam & peripheriam comprehenſum, altera recta linea nō cadet Et ſemicirculi quidem angulus quouis angulo acuto rectilineo maior eſt, reliquus autem minor;

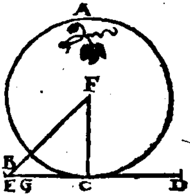


Problema 2. Propo- ſitio 17.



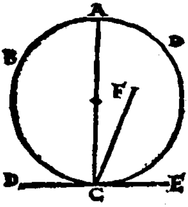
A dato puncto rectam lineam ducere, quæ datum tangat circulum.

Theorema 16. Propositio 18.



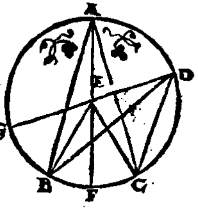
Si circulum tangat recta quæpiam linea, à centro autem ad cõtactum adiungatur recta quædam linea: quæ adiũcta fuerit ad ipsam contingentem perpendicularis erit.

Theorema 17. Propositio 19.



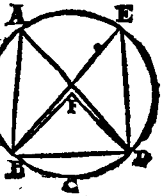
Si circulum tetigerit recta quæpiam linea, à contactu autem recta linea ad angulos rectos ipsi tãgenti excitetur, in excitata erit centrum circuli.

Theorema 18. Propositio 20.



In circulo angulus ad cẽtrum duplex est anguli ad peripheriam, cũ fuerit eadẽ peripheria basis angulorum.

Theorema 19. Propositio 21.



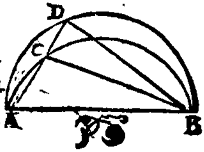
In circulo, qui in eodem segmẽto sunt anguli, sunt inter se æquales.

Theore-

Theorema 20. Propositio 22.

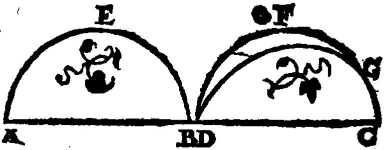
Quadrilaterorum in circulis descriptorũ anguli qui ex aduerso, duob⁹ rectis sunt æquales.

Theorema 21. Propositio 23.



Super eadem recta linea duo segmenta circulorũ similia & inæqualia non constituẽtur ad easdem partes.

Theorema 22. Propositio 24.



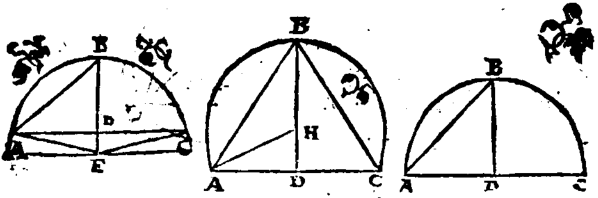
Super ę qualib⁹ rectis lineis similia circulorum segmenta sunt inter se æqualia.

Problema 3. Propositio 25.

Circuli segmento dato, describere circulũ

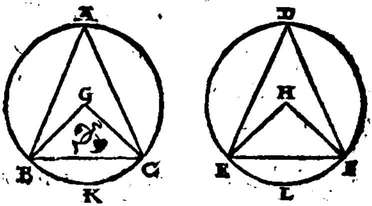
 cuius

cuius est ſegmentum.



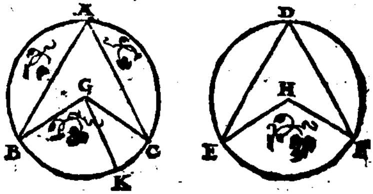
Theorema 23. Propoſitio 26.

In æqualibus circulis, æquales anguli ęqualib. peripherijs inſiſtũt ſiue ad cẽtra, ſiue ad peripherias conſtituti inſiſtant.



Theorema 24. Propoſitio 27.

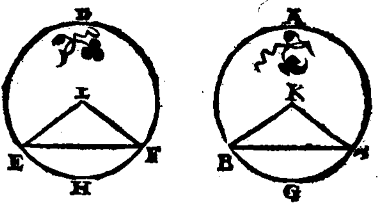
In æqualibus circulis, anguli qui æqualib⁹ peripherijs inſiſtũt ſunt inter ſe ęquales ſiue ad cẽtra, ſiue ad peripherias cõſtituti inſiſtãt.



Theore-

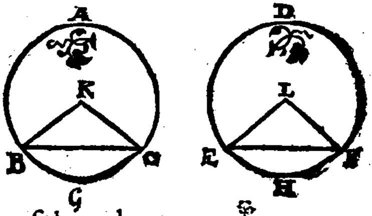
Theorema 25. Propositio 28.

In æqualibus circulis æquales rectæ lineæ equales peripherias auferũt maiorẽ quidẽ maiori, minorem autem minori.



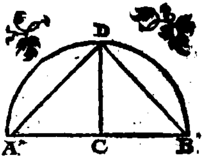
Theorema 26. Propositio 29.

In equalib⁹ circulis, æquales peripherias æquales rectæ lineæ subtendunt.



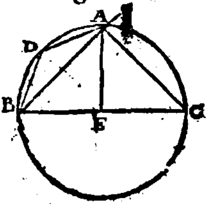
Problema 4. Propositio 30.

Datam peripheriam bifariam secare.



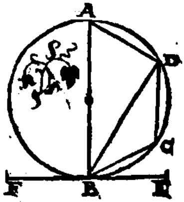
Theorema 27. Propositio 31.

In circulo angulus qui in semicirculo, rectus est: qui autem in maiore segméto, minor recto: qui verò in minore segméto, maior est recto. Et insuper angulus maioris segméti, recto quidé maior est, minoris auté segméti angulus, minor est recto.



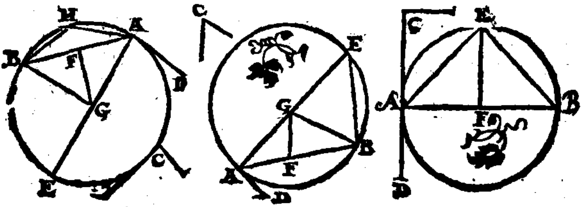
Theorema 28. Propositio 32.

Si circulum tetigerit aliqua recta linea, à contactu auté producatur quædam recta linea circulum secans: anguli quos ad contingentem facit, æquales sunt iis qui in alternis circuli segmétis consistunt, angulis.



Problema 5. Propositio 33.

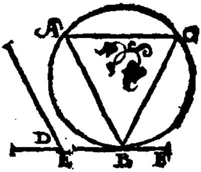
Super data recta linea describere segmentum circuli quod capiat angulum æqualé dato angulo rectilineo.



Proble-

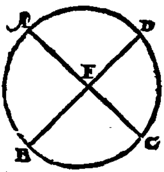
Problema 6. Propositio 34.

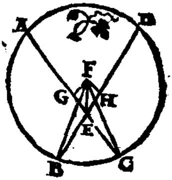
A dato circulo segmentum abscindere capiens angulum æqualem dato angulo rectilineo.



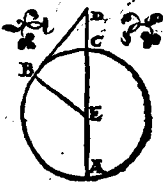
Theorema 29. Propositio 35.

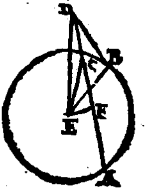
Si in circulo duæ rectæ lineæ sese mutuò secuerint, rectangulum cõprehensum sub segmẽtis vniꝰ, æquale est ei, quod sub segmẽtis alterius comprehenditur, rectangulo.





Theorema 30. Propositio 36.

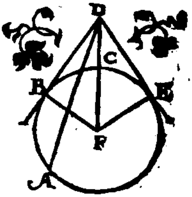




Si extra circulũ sumatur pũctũ aliquod, ab eoque in circulũ cadant duę rectę lineę, quarum altera quidem circulum secet, altera verò tangat: quod ſub tota ſecante & exterius inter punctum & cõuexam peripheriam aſſumpta comprehenditur rectangulum, æquale erit ei, quod à tangente deſcribitur, quadrato.

Theorema 31. Propoſitio 37.

Si extra circulũ ſumatur punctũ aliquod, ab eoq; puncto in circulum cadant duæ rectæ lineæ, quarum altera circulum ſecet, altera in eum incidat, ſit autẽ quod ſub tota ſecante & exterius inter punctũ & cõuexam peripheriam aſſumpta, comprehenditur rectãgulum, æquale ei, quod ab incidẽte deſcribitur quadrato: incidens ipſa circulum tanget.



ELEMENTI III FINIS.

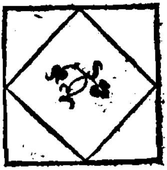
EVCLI-

EVCLIDIS ELEMENTVM QVARTVM.

DEFINITIONES.

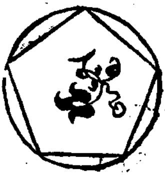
I

Figura rectilinea in figura rectilinea inſcribi dicitur, cũ ſinguli eius figuræ quæ inſcribitur, anguli ſingula latera eius, in qua inſcribitur, tangunt.



2

Similiter & figura cireum figurã deſcribi dicitur, quum ſingula eius quæ circũſcribitur, latera ſingulos eiꝰ figurę angulos tetigerint, circum quam illa deſcribitur.



3

F gura rectilinea in circulo inſcribi dicitur, quũ ſinguli eius figuræ q̃ inſcribitur,

angu-

anguli tetigerint circuli peripheriam.

4

Figura verò rectilinea circa circulũ deſcribi dicitur, quũ ſingula latera eius, quæ circum ſcribitur, circuli peripheriã tangunt.

5.

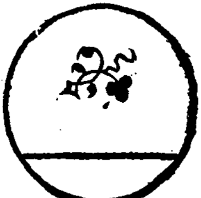
Similiter & circulus in figura rectilinea inſcribi dicitur, quum circuli peripheria ſingula latera tãgit eiꝰ figuræ, cui inſcribitur.

6

Circulus autem circum figurã deſcribi dicitur, quũ circuli peripheria ſingulos tãgit eius figuræ, quam circunſcribit, angulos.

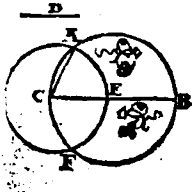
7

Recta linea in circulo accommodari ſeu coaptari dicitur, quum eius extrema in circuli peripheria fuerint.



Problema 1. Propoſitio 1.

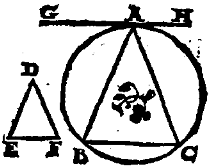
In dato circulo, rectam lineam accommodare ęqualem datæ rectæ lineę quœ circuli diametro nõ ſit maior.



Proble-

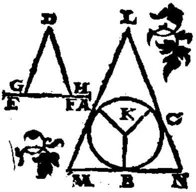
Problema 2. Propoſitio 2.

In dato circulo, triangulū deſcribere dato triangulo æquiangulum.



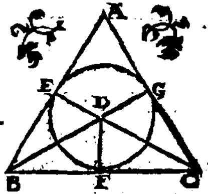
Problema 3. Propoſitio 3.

Circa datum circulū triangulū, deſcribere dato triangulo æquiangulū.



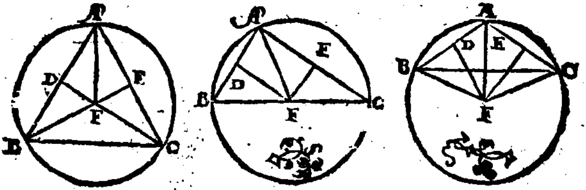
Problema 4. Propoſitio 4.

In dato triangulo circulum inſcribere.



Problema 5. Propoſitio 5.

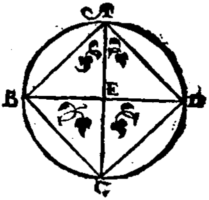
Circa datum triangulum, circulum deſcribere.



Pro-

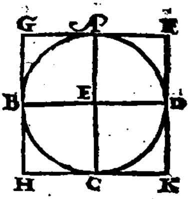
Problema 6. Propositio 6.

In dato circulo quadratum describere.



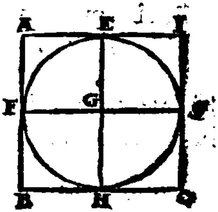
Problema 7. Propositio 7.

Circa datum circulum, quadratum describere.



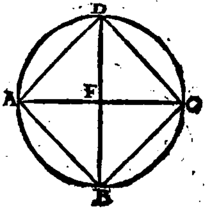
Problema 8. Propositio 8.

In dato quadrato circulum inscribere.



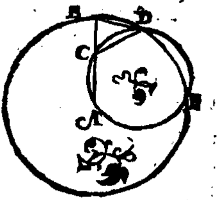
Problema 9. Propositio 9.

Circa datum quadratũ, circulum describere.



Proble-

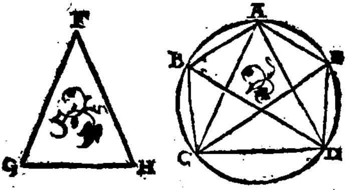
Problema 10. Propoſitio 10.



Iſoſceles triangulum cõſtituere; quod habeat vtrunque eorum, qui ad baſin ſunt, angulorum, duplum reliqui.

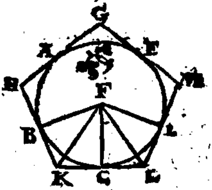
Theorema 11. Propoſitio 11.

In dato circulo, pentagonũ æquilaterũ & æquiãgulum inſcribere.



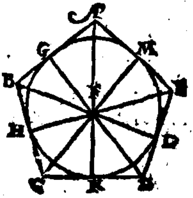
Problema 12. Propoſitio 12.

Circa datum circulum, pentagonum æquilaterum & æquiangulũ deſcribere.



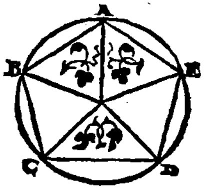
Problema 13. Propoſitio 13.

In dato pẽtagono æquilatero & æquiãgulo, circulum inſcribere.



Proble-

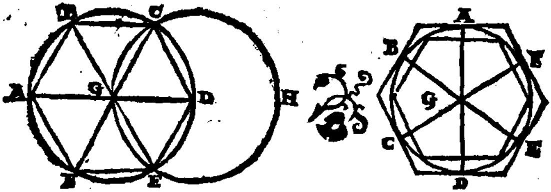
Problema 14. Propositio 14.



Circa datũ pentagonũ, æquilaterum & æquiangulum, circulum deſcribere.

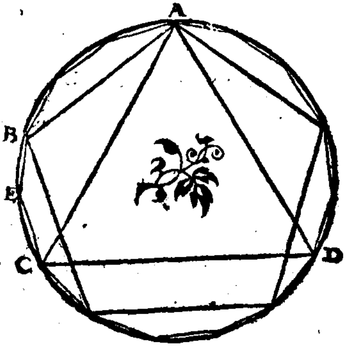
Problema 15. Propoſitio 15.

In dato circulo hexagonum & æquilaterũ & æquiangulum inſcribere.



Propoſitio 16. Theorema 16.

In dato circulo quintidecagonum & æquilaterũ & ęquiangulum deſcribere.



Elementi quarti finis.

EVCLIDIS ELEMENTVM QVINTVM.

DEFINITIONES.

1

PARS est magnitudo magnitudinis minor maioris, quum minor metitur maiorem.

2

Multiplex autem est maior minoris, cùm minor metitur maiorem.

3

Ratio, est duarum magnitudinum eiusdẽ generis mutua quædam secundum quantitatem habitudo.

4

Proportio verò, est rationum similitudo.

5

Rationẽ habere inter se magnitudinis dicuntur, quæ possunt multiplicatæ sese mutuò superare.

6

In eadem ratione magnitudines dicũtur esse, prima ad secundam, & tertia ad quartã: cùm primæ & tertiæ æquè multiplicia à secundæ & quartæ æquè multiplicibus, qualiſcunq; ſit hæc multiplicatio, vtrunq; ab vtroque: vel vnà deficiunt, vel vnà ęqualia ſunt, vel vnà excedũt, ſi ea ſumãtur quæ inter ſe reſpondent.

7

Eandem autem habẽtes rationem magnitudines, proportionales vocentur.

8

Cũm verò æquè multiplicium, multiplex primę magnitudines exceſſerit multiplicẽ ſecũdæ, at multiplex tertiæ non exceſſerit multiplicem quartæ: tunc prima ad ſecundam, maiorem rationem habere dicetur, quàm tertia ad quartam.

9

Proportio autẽ in tribus terminis paucisſimis conſiſtit.

10

Cùm autem tres magnitudines proportionales fuerint, prima ad tertiam, duplicatã rationẽ habere diciturei us, quam habet ad ſecundam. At cùm quatuor magnitudines proportionales fuerint, prima ad quartã, triplicatam rationem habere dicitur eius quam habet ad ſecundam: & ſemper deinceps vno amplius, quandiu proportio extiterit.

11

Homologę, ſeu ſimiles ratione magnitudines dicuntur, antecedentes quidẽ antecedentidentibus, consequẽtes verò cõsequẽtibus.

12

Alterna ratio, est sumptio antecedẽtis cõparati ad antecedentem, & cõsequentis ad consequentem.

13

Inuersa ratio, est sumptio cõsequentis, ceu antecedentis, ad antecedentẽ velut ad cõsequentem.

14

Compositio rationis, est sumptio antecedentis cũ consequente ceu vnius, ad ipsum consequentem.

15

Diuisio rationis, est sumptio excessus quo consequentem superat antecedens ad ipsum consequentem.

16

Conuersio rationis, est sumptio antecedẽtis ad excessum, quo superat antecedẽs ipsum consequentem.

17.

Ex æqualitate ratio est, si plures duab⁹ sint magnitudines, & his alię multitudine pares quæ binæ sumantur, & in eadem ratione: quũ vt in primis magnitudinibus prima ad vltimam, sic & in secundis magnitudinibus prima ad vltimã sese habuerit, vel aliter, sumptio extremorũ per subductionem mediorum.

18

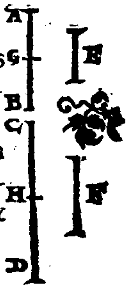
Ordinata proportio eſt, cùm fuerit quem admodum antecedens ad conſequentem, ita antecedens ad conſequentem : fuerit etiam vt conſequẽs ad aliud quidpiam, ità conſequens ad aliud quidpiam.

19

Perturbata autẽ proportio eſt, tribus poſitis magnitudinibus, & alijs quæ ſint his multitudine pares, cùm vt in primis quidẽ magnitudinibus ſe habet antecedẽs ad cõſequentem, ita in ſecũdis magnitudinibus antecedens ad conſequentem: vt autem in primis magnitudinibus conſequens ad aliud quidpiam, ſic in ſecundis magnitudinibus aliud quidpiam ad antecedentem.

Theorema 1. Propoſitio 1.

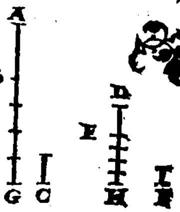
Si ſint quotcũque magnitudines quotcunque magnitudinum æqualium numero ſingulę ſingularum æquè multiplices, quàm multiplex eſt vnius vna magnitudo, tam multiplices erunt, & omnes omnium.



Theorema 2. Propoſitio 2.

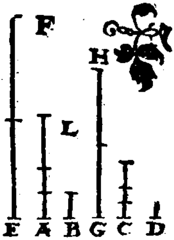
Si prima ſecundæ æquè fuerit multiplex, atque

atque tertia quartę, fuerit autẽ & quinta ſecũdę æquè multiplex, atque ſexta quartæ: erit & cõpoſita prima cũ quinta, ſecũdæ ęquè multiplex atque tertia cum ſexta, quartæ.



Theorema 3. Propoſitio 3.

Si ſit prima ſecũdæ æquè multiplex atque tertia quartæ, ſumãtur autẽ æquè multiplices primę & tertiæ erit & ex æquo ſumptarum vtraque vtriuſque ęquè multiplex, altera quidem ſecundæ, altera autẽ quartæ.



Theorema 4. Propoſitio 4.

Si prima ad ſecundam, eandẽ habuerit rationem, & tertia ad quartam: etiam æque multiplices primę & tertiæ, ad æquè multiplices ſecundæ & quartæ iuxta quãuis multiplicationẽ, eadem habebunt rationem, ſi prout inter ſe

 reſpon-

respondent, ita sumptæ fuerint.

Theorema 5. Propositio 5.

Si magnitudo magnitudinis æquè fuerit multiplex, atque ablata ablatæ: etiam reliqua reliquæ ita multiplex erit, vt tota totius.

Theorema 6. Propositio 6.

Si duæ magnitudines, duarum magnitudinum sint æquè multiplices, & detractę quædam sint earundem æquè multiplices: & reliquæ eisdem aut ęquales sunt, aut æquè ipsarum multiplices.

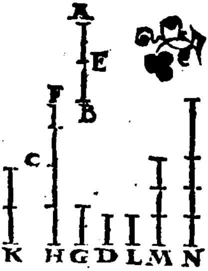
Theorema 7. Propositio 7.

Aequales ad eandem, eādem habent rationem: & eadem ad æquales.

Theorema 8. Propositio 8.

Inæqualium magnitudinū maior, ad eandem

dem maiorem ratioñ habet, quàm minor: & eadem ad minorem: maiorem rationem habet, quàm ad maiorem.



Theorema 9. Propositio 9.

Quæ ad eandem, eandem habẽt rationem, æquales sũt inter se: & ad quas eadem, eandem habet rationem, eæ quoque sunt inter se æquales.

Theorema 10. Propositio 10.

Ad eandem magnitudinẽ, rationem habentium, quæ maiorem rationẽ habet, illa maior est, ad quam autem eadem maiorem rationem habet, illa minor est.

Theorema 11. Propositio 11.

Quæ eidem sunt eædem rationes, & inter se sunt eædem.

Theorema 12. Propositio 12.

Si sint magnitudines quotcunque proportionales, quemadmodũ se habuerit vna antecedentium ad vnā

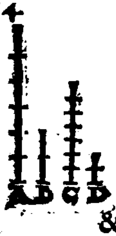
consequentiũ, ita se habebunt omnes antecedentes ad omnes consequentes.

Theorema 13. Propositio 13.

Si prima ad secundam, eandẽ habuerit rationem, quam tertia ad quartã, tertia verò ad quartam, maiorem rationem habuerit, quàm quinta ad sextam, prima quoque ad secũdam maiorem rationem habebit, quàm quinta ad sextam.

Theorema 14. Propositio 14.

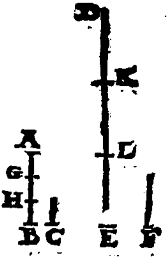
Si prima ad secundam eandem habuerit rationẽ, quam tertia ad quartam, prima verò quàm tertia maior fuerit: erit & secunda maior quàm quarta. Quòd si prima fuerit æqualis tertiæ, erit



& ſecunda æqualis quartæ: ſi verò minor, & minor erit.

Theorema 15. Propoſitio 15.

Partes cum pariter multiplicibus in eadẽ ſunt ratione, ſi prout ſibi mutuo reſpondent, ita ſumantur.

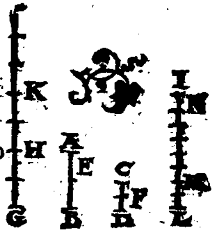


Theorema 16. Propoſitio 16.

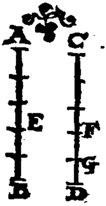
Si quatuor magnitudines proportionales fuerint, & viciſſim proportionales erunt.

Theorema 17. Propoſitio 17.

Si compoſitæ magnitudines proportionales fuerint hæ quoq; diuiſæ proportionales erunt.

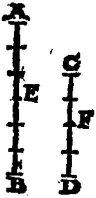


Theorema 18. Propositio 18.



Si diuisæ magnitudines sint proportionales, hæ quoque compositæ proportionales erunt.

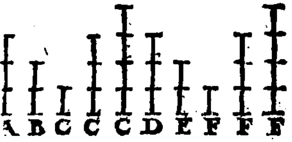
Theorema 19. Propositio 19.



Si quemadmodum totũ ad totum, ita ablatum se habuerit ad ablatum : & reliquum ad reliquum, vt totum ad totum se habebit.

Theorema 20. Propositio 20.

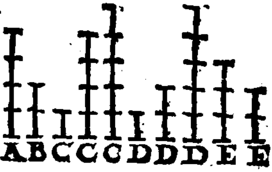
Si sint tres magnitudines, & aliæ ipsis æquales numero, quæ binæ & in eadem ratione sumatur, ex æquo autẽ prima quàm tertia maior fuerit: erit & quarta, quã sexta maior. Quòd si prima tertiæ fuerit æqualis, erit & quarta æqualis sextæ: sin illa minor, hæc quoque minor erit.



Theore-

Theorema 21. Propositio 21.

Si sint tres magnitudines, & alię ipsis æquales numero quæ binæ & in eadem ratione sumãtur, fueritq́ue perturbata earũ proportio ex æquo autem prima quàm tertia maior fuerit



erit & quarta quã sexta maior: quòd si prima tertiæ fuerit æqualis, erit & quarta æqualis sextæ: sin illa minor, hæc quoque minor erit.

Theorema 22. Propositio 22.

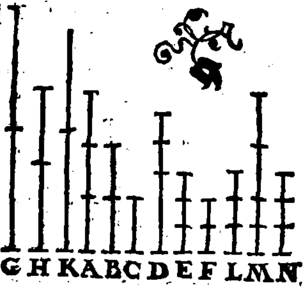
Si sint quotcunq; magnitudines, & alię ipsis ęquales numero, quæ binæ in eadẽ ratione sumantur, et

& ex æqualitate in eadem ratione erunt.

Theorema 23. Propositio 23.

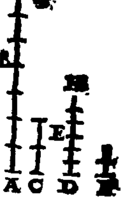
Si sint tres magnitudines, aliæq́; ipsis ęquales

les numero, q̃ binæ in eadem ratione ſumã-tur, fuerit autẽ perturbata ea-rũ proportio: etiã ex æquali-te in eadem ra-tione erunt.



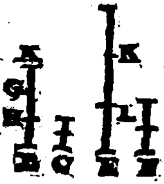
Theorema 24. Propo-ſitio 24.

Si prima ad ſecundam, eandem habuerit rationẽ, quam tertia ad quartam, habuerit autem & quinta ad ſecundam eandẽ ra-tionem, quã ſexta ad quartam: etiam cõpoſita prima cũ quin-ta ad ſecundam eandẽ habebit rationem, quã tertia cum ſexta ad quartã.



Theorema 25. Propo-ſitio 25.

Si quatuor magnitudines proportionales fuerint, ma-xima & minima reliquis duabus maiores erunt.



ELEMENTI V. FINIS.

EVCLIDIS ELEMENTVM SEXTVM.

DEFINITIONES.

1

Similes figuræ rectilineæ sunt, quæ & angulos singulos singulis æquales habẽt, atque etiam latera, quæ circum angulos æquales, proportionalia.

2

Reciprocæ autem figuræ sunt, cùm in vtraque figura antecedentes & consequẽtes rationum termini fuerint.

3

Secundum extremam & mediam rationẽ recta linea secta esse dicitur, cùm vt tota ad maius segmentum, ita maius ad minus se habuerit.

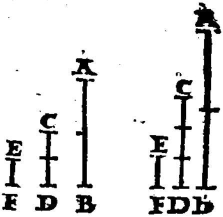
4

Altitudo cuiusque figuræ, est linea perpẽdicularis à vertice ad basin deducta.

5 Ra-

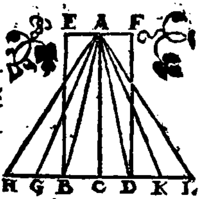
5

Ratio ex rationibus cō poni dicitur, cū rationum quantitates inter se multiplicatæ aliquā effecerint rationem.



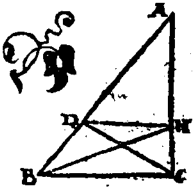
Theorema 1. Propositio 1.

Triangula & parallelogramma, quorum eadē fuerit altitudo, ita se habent inter se vt bases.



Theorema 2. Propositio 2.

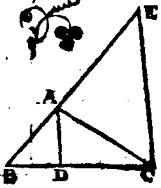
Si ad vnum trianguli latus parallela ducta fuerit recta quædam linea: hęc proportionaliter secabit, ipsius triāguli latera. Et si triāguli latera proportionaliter secta fuerint: quæ ad sectiones adiūcta fuerit recta linea, erit ad reliquum ipsius trianguli latus parallela.



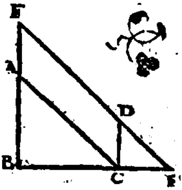
Theorema 3. Propositio 3.

Si trianguli angulus bifariā sectus sit, secās autem angulum recta linea secuerit & basim: basis segmenta eandem habebunt rationem,

tionem, quam reliqua ipsius trianguli latera. Et si basis segmenta eandem habeant rationem quam reliqua ipsius trianguli latera, recta linea, quæ à vertice ad sectionem producitur, ea bifariā secat trianguli ipsius angulum.

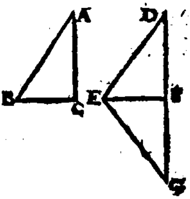


Theorema 4. Propositio 4.



Aequiangulorum triangulorum proportionalia sunt latera, quæ circum ęquales angulos, & homologa sunt latera, quæ æqualib⁹ angulis subtenduntur.

Theorema 5. Propositio 5.

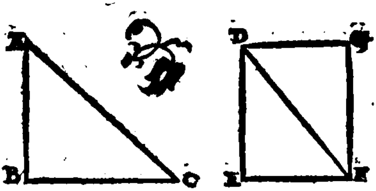


Si duo triāgula latera proportionalia habeant, ęquiangula erunt triangula, & æquales habebunt eos angulos, sub quibus homologa latera subtenduntur.

Theorema 6. Propositio 6.

Si duo triangula vnum angulum vni angulo æqualē, & circū ęquales angulos latera ꝓportionalia habuerint, ęquiangula erūt trian-

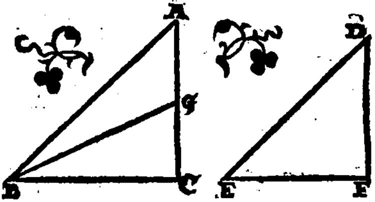
triangu-
la,ęqua-
lesq; ha
bebunt
angulos
ſub qui-
bus ho-



mologa latera ſubtenduntur.

Theorema 7. Propoſitio 7.

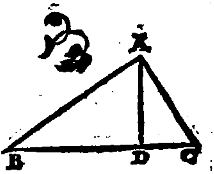
Si duo triangula vnum angulum vni angulo æqualem, circũ autem alios angulos latera proportionalia habeant, reliquorum verò ſimul vtrunq; aut minorem aut non minoré



recto: æquiangula erunt triangula, & ęquales habebunt eos angulos, circum quos ꝓportionalia ſunt latera.

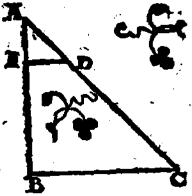
Theorema 8. Propoſitio 8.

Si in triangulo rectangulo, ab angulo recto in baſin perpendicularis ducta ſit, quę ad perpendicularem triangula, tum toti triangulo, tum ipſa inter ſe ſimilia ſunt.



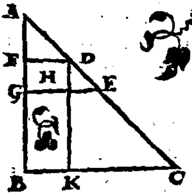
Proble-

Problema 1. Propoſitio 9.



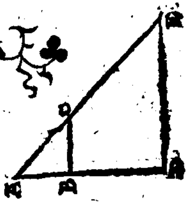
A data recta linea imperatam partem auferre.

Problema 2. Propoſitio 10.



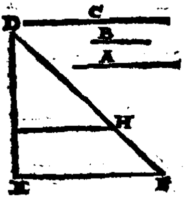
Datam rectam lineã inſectam ſimiliter ſecare, vt data altera recta ſecta fuerit.

Problema 3. Propoſitio 11.



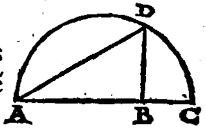
Duabus datis rectis lineis, tertiam proportionalem adinuenire.

Problema 4. Propoſitio 12.



Tribꝰ datis rectis lineis, quartam proportionalem adinuenire.

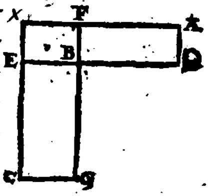
Problema 5. Propoſitio 13.



Duabꝰ datis rectis lineis mediam proportionalẽ adinuenire.

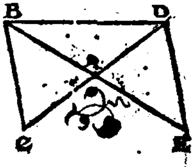
Theorema 9. Propoſitio 14.

Aequalium, & vnũ vni æqualem habentiũ angulum parallelogrammorum reciproca ſunt latera, quæ circum æquales angulos; & quorũ parallelogrammorũ vnũ angulum vni angulo æqualem habentiũ reciproca ſunt latera, quæ circum æquales angulos, illa ſunt æqualia.



Theorema 10. Propoſitio 15.

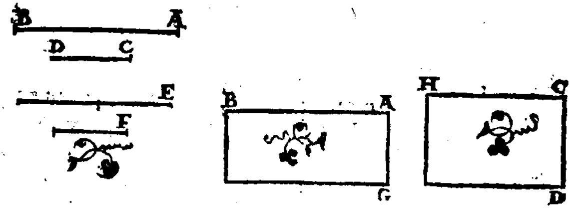
Aequalium, & vnũ angulum vni æqualem habentiũ triangulorũ reciproca ſunt latera, quæ circum æquales angulos: & quorũ triangulorũ vnum angulum vni ęqualem habentium reciproca ſunt latera, q̃ circum ęquales angulos, illa ſunt æqualia.



Theo-

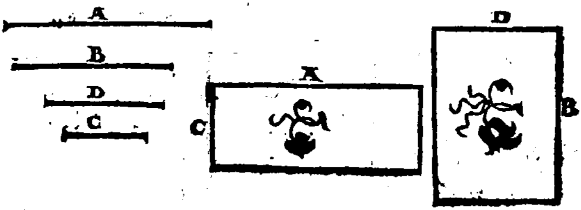
Theorema 11. Propoſitio 16.

Si quatuor rectæ lineæ proportionales fuerint, quod ſub extremis comprehenditur rectangulũ, æquale eſt ei, quod ſub medijs comprehenditur rectãgulo. Et ſi ſub extremis comprehenſum rectangulum æquale fuerit ei, quod ſub medijs cõtinetur rectãgulo, illæ quatuor rectæ lineæ proportionales erunt.



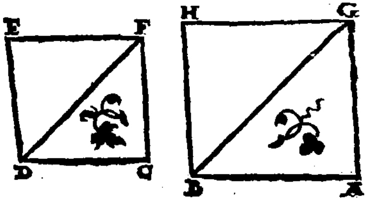
Theorema 12. Propoſitio 17.

Si tres rectæ lineæ ſint proportionales, quod ſub extremis comprehenditur rectangulũ ęquale eſt ei, quod à media deſcribitur quadrato: & ſi ſub extremis comprehẽſum rectangulum æquale ſit ei quod à media deſcribitur quadrato, illæ tres rectæ lineæ proportionales erunt.



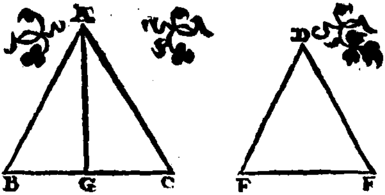
Problema 6. Propoſitio 18.

A data recta linea, dato rectilineo ſimile ſimiliterq; poſitum rectilineum deſcribere.



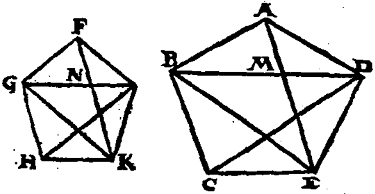
Theorema 13. Propoſitio 19.

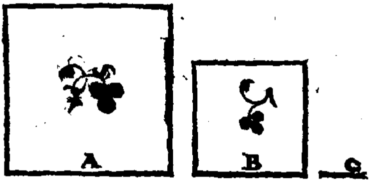
Similia triãgula inter ſe sũt in duplicata ratiõe laterũ homologorũ.



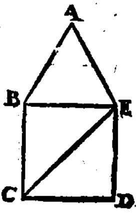
Theore. 14. Propoſitio 20.

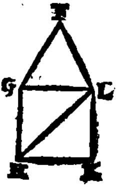
Similia polygona in ſimilia triangula diuiduntur, & numero æqualia, & homologa totis. Et po





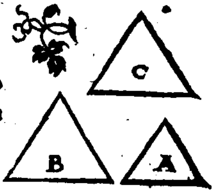
lygona duplicatam habent cã inter ſe rationẽ, quã latus homologum ad homologum latus.





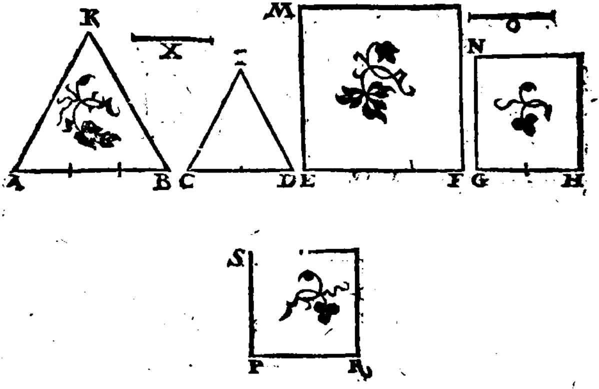
Theorema 15. Propoſitio 21.

Quæ eidem rectilineo ſunt ſimilia, & inter ſe ſunt ſimilia.



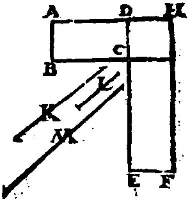
Theorema 16. Propoſitio 22.

Si quatuor rectę lineæ proportionales fuerint: & ab eis rectilinea ſimilia ſimiliterq́; deſcripta proportionalia erũt. Et ſi à rectis lineis ſimilia ſimiliterq́; deſcripta rectilinea proportionalia fuerint, ipſæ etiam rectæ lineæ proportionales erunt.



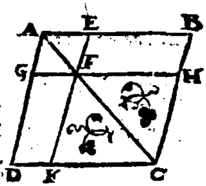
Theorema 17. Propositio 23.

Aequiangula parallelogramma inter se ratione habent eam, quę ex lateribus componitur.



Theorema 18. Propositio 24.

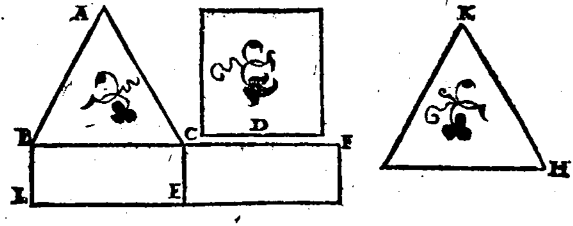
In omni parallelogrammo, quæ circa diametrũ sunt parallelogrãma, & toti & inter se sunt similia.



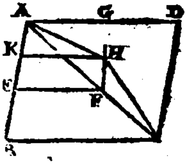
Proble-

Problema 7. Propoſitio 25.

Dato rectilineo ſimile, & alteri dato ęquale idem conſtituere.



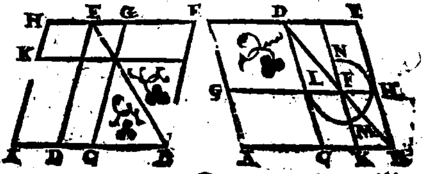
Theorema 19. Propoſitio 26.



Si à parallelogrāmo parallelogrammū ablatū ſit, & ſimile toti & ſimiliter poſitum communē cum eo habens angulum, hoc circum eandem cum toto diametrum conſiſtit.

Theorema 20. Propoſitio 27.

Omnium parallelogrāmorum ſecūdum eandem rectam lineam applicatorū deficiētiumq; figuris parallelogrāmis ſi-

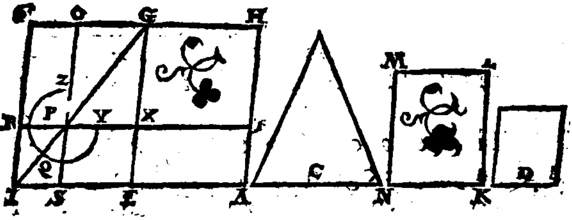


 mili-

milibus similiterque positis ei, quod à dimidia describitur, maximum, id est, quod ad dimidiam applicatur parallelogrammum simile existens defectui.

Problema 8. Propositio 28.

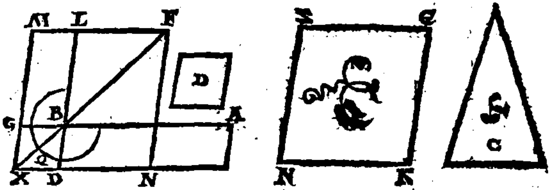
Ad datam lineam rectam, dato rectilineo æquale parallelogrammum applicare deficiens figura parallelogramma, quę similis sit alteri rectilineo dato. Oportet auté datum rectilineum, cui æquale applicandum est, non maius esse eo quod ad dimidiam applicatur, cùm similes sint defectus & eius quod à dimidia describitur, & eius cui simile deesse debet.



Problema 9. Propositio 29.

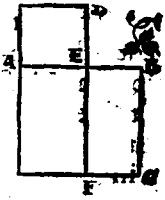
Ad datam rectam lineam, dato rectilineo æquale parallelogrammũ applicare, excedés figura parallelográma, quæ similis sit paral-

parallelogrammo alteri dato.



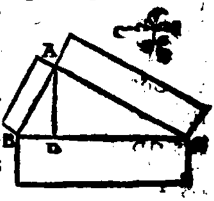
Problema 10. Propoſitio 30.

Propoſitam rectam lineam terminatã, extrema ac media ratione ſecare.



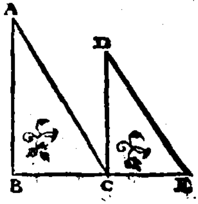
Theorema 21. Propoſitio 31.

In rectangulis triãgulis, figura quęuis à latere rectũ angulũ ſubtendẽte deſcripta ęqualis eſt figuris, quæ priori illi ſimiles & ſimiliter poſitæ à lateribus rectũ angulum cõtinentibus deſcribuntur.



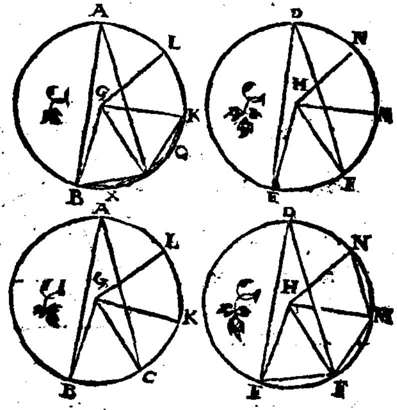
Theorema 22. Propoſitio 32.

Si duo triãgula, quę duo latera duobus lateribus proportionalia habeant, ſecũdum vnum angulū̃ composita fuerint, ita vt homologa eorum latera sint etiam parallela, tū reliqua illorum triangulorum latera in rectam lineam collocata reperiuntur.



Theorema 23. Propositio 33.

Inæqualibus circulis anguli eandē habent rationem cū ipsis peripherijs in quibus insistūt, siue ad cētra, siue ad peripherias cōstituti, illis insistant peripherijs In super verò & sectores qp pe qui ad cētra cōsistūt.



ELEMENTI VI. FINIS.

EVCLIDIS ELEMENTVM SEPTIMVM.

DEFINITIONES.

1

Vnitas, eſt ſecũdum quam entium quodque dicitur vnum.

2

Numerus autem, ex vnitatibus compoſita multitudo.

3

Pars, eſt numerus numeri minor maioris, cùm minor metitur maiorem.

4

Partes autem, cùm non metitur.

5

Multiplex verò, maior minoris, cùm maiorem metitur minor.

6

Par numerus eſt, qui bifariam diuiditur.

7

Impar verò, qui bifariam non diuiditur: vel, qui vnitate differt à pari.

8

Pariter par numerus eſt, quem par numerus metitur per numerum parem.

9 Pari

9

Pariter autem impar, eſt quem par numerus metitur per numerum imparem.

10

Impariter verò impar numerus, eſt quem impar numerus metitur per numerum imparem.

11.

Primus numerus, eſt quem vnitas ſola metitur.

12

Primi inter ſe numeri ſunt, quos ſola vnitas menſura communis metitur.

13

Compoſitus numerus eſt, quem numerus quiſpiam metitur.

14

Compoſiti autẽ inter ſe numeri ſunt, quos numerus aliquis menſura communis metitur.

15

Numerus numerum multiplicare dicitur, cùm toties compoſitus fuerit is, qui multiplicatur, quot ſunt in illo multiplicante vnitates, & procreatus fuerit aliquis.

16

Cùm autẽ duo numeri mutuò ſeſe multiplicantes quempiam faciũt, qui factus erit planus appellabitur, qui verò numeri mutuò ſeſe multiplicarint, illi⁹ latera dicẽtur.

17 Cùm

17

Cùm verò tres numeri mutuò ſeſe multiplicantes quempiã faciunt, qui prócreatus erit ſolidus appellabitur, qui autẽm numeri mutuò ſeſe multiplicarint, illius latera dicentur.

18

Quadratus numerꝰ eſt, qui ęqualiter æqualis: vel, qui à duobus ęqualibus numeris cõtinetur.

19

Cubus verò, qui æqualiter æqualis æqualiter: vel, qui à tribus æqualibus numeris cõtinetur.

:9

Numeri proportionales ſunt, cùm primus ſecundi, & tertius quarti æque multiplex eſt, vel eadem pars, vel eædem partes.

21

Similes plani & ſolidi numeri ſunt, qui proportionalia habent latera.

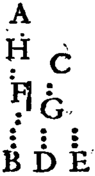
22

Perfectus numerus eſt, qui ſuis ipſius partibus eſt æqualis.

Theorema 1. Propoſitio 1.

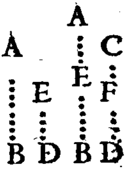
Duobus numeris inæqualibus propoſitis,

tis, si detrahatur semper minor de maiore, alterna, quadam detractione, neque reliquus vnquam metiatur præcedentem quoad assumpta sit vnitas: qui principio propositi sunt, numeri primi inter se erunt.



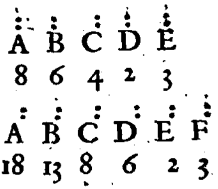
Problema 1. Propositio 2.

Duobus numeris datis non primis inter se, maximã eorum communem mensuram reperire.



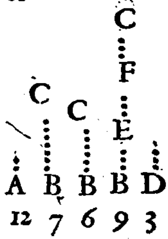
Problema 2. Propositio 3.

Tribus numeris datis non primis inter se, maximam eorum communem mensuram reperire.



Theorema 2. Propositio 4.

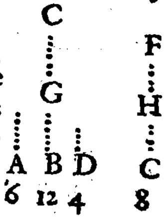
Omnis numerus, cuiusq; numeri minor maioris aut pars est, aut partes.



Theore-

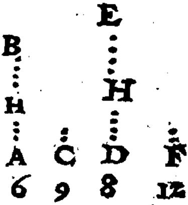
Theorema 3. Propositio 5.

Si numerus numeri pars fuerit, & alter alterius eadẽ pars & simul vterque vtriusque simul eadem pars erit, quæ vnus est vnius.



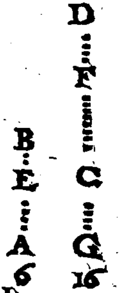
Theor. 4. Propo. 6.

Si numer⁹ sit numeri partes, & alter alterius eędem partes, & simul vterque vtriusque simul eædẽ partes erunt, quæ sunt vnus vnius.



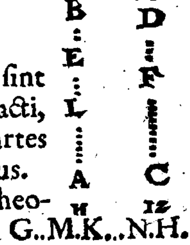
Theorema 5. Propositio 7.

Si numerus numeri eadẽ sit pars quæ detractus detracti, & reliquus reliqui eadẽ pars erit, quæ totus est totius.



Theorema 6. Propositio 8.

Si numerus numeri eædẽ sint partes quæ detractus detracti, & reliquus reliqui eędẽ partes erunt, quæ sunt totus totius.



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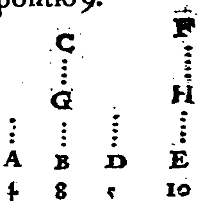
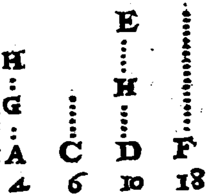
Theorema 7. Propoſitio 9.

Si numerus numeri pars ſit, & alter alterius eadem pars, & viciſsim quæ pars eſt vel partes primus tertij, eadē pars erit vel eędē partes, & ſecūdus quarti.



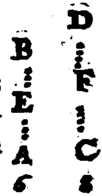
Theorema 8. Propoſitio 10.

Si numerus numeri partes ſint, & alter alterius eædē partes, etiam viciſſim quæ ſunt partes aut pars primus tertij, eædē partes erunt vel pars & ſecundus quarti.



Theorema 9. Propoſitio 11.

Si quemadmodum ſe habet totus ad totum, ita detractus ad detractum, & reliquus ad reliquum ita habebit vt totus ad totum.



Theorema 10. Propoſitio 12.

Si ſint quotcunque numeri proportionales, quemadmodum ſe habet vnus antecedentium ad vnum ſequētium, ita ſe habe-

:	:	:	:
A	B	C	D
9	6	3	2

habebunt omnes antecedentes ad omnes consequentes.

Theorema 11. Propositio 13.

Si quatuor numeri sint proportionales, & vicissim proportionales erunt.

A	B	C	D
12	4	9	3

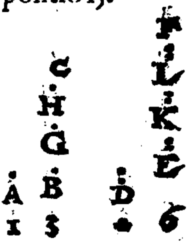
Theorema 12. Propositio 14.

Si sint quotcunque numeri, & alij illis æquales multitudine, qui bini sumantur & in eadem ratione: etiam ex æqualitate in eadem ratione erunt.

A	B	C	D	E	F
12	6	3	8	4	2

Theorema 13. Propositio 15.

Si vnitas numerum quempiã metiatur, alter verò numerus alium quendã numerũ æquè metiatur, & vicissim vnitas tertium numerũ æquè metietur, atque secundis quartum.



Theorema 14. Propositio 16.

Si duo numeri mutuò sese multiplicãtes faciant aliquos qui ex illis geniti fuerint, inter se æquales erunt.

E	A	B	C	D
1	2	4	8	8

Theorema 15. Propositio 17.

Si numerꝰ duos numeros multiplicãs faciat aliquos, qui ex illis procreati erunt, eandem rationem habebunt, quam multiplicati.

I	A	B	C	D	E
1	3	4	5	12	15

Theorema 16. Propositio 18.

Si duo numeri numerum quempiam multiplicantes faciant aliquos, geniti ex illis eandem habebunt rationem, quam qui illum multiplicarunt.

A	B	C	D	E
4	5	3	12	15

Theorema 17. Propositio 19.

Si quatuor numeri sint proportionales, ꝗ ex primo & quarto fit, ęqualis erit ei qui ex secundo & tertio: & si qui ex primo & quarto fit numerus æqualis sit ei qui ex secũdo & tertio, illi quatuor numeri proportionales erunt.

A	B	C	D	E	F	G
6	4	3	2	12	12	18

Theorema 18. Propositio 20.

Si tres numeri sint proportionales, qui ab extremis cõtinetur, ęqualis est ei qui à medio efficitur. Et si qui ab extremis cótinetur æqualis sit ei qui à medio describitur, illi tres nũmeri proportionales erunt.

A	B	C
9	6	4
D		
6		

Theorema 19. Propositio 21.

Minimi numeri omniũ, qui eandem cũ eis rationem habẽt, æqualiter metiũtur numeros eandem rationem habẽtes, maior quidẽ maiorem, minor verò minorem.

D	L		
G	H		
C	E	A	B
4	3	8	6

Theorema 20. Propositio 22.

Si tres sint numeri & alij multitudine illis æquales, qui bini sumantur & in eadem ratione, sit autem perturbata eorũ proportio, etiam ex æqualitate in eadem ratione erunt.

A	B	C	D	E	F
6	4	3	12	8	6

Theorema 21. Propositio 23.

Primi inter se numeri minimi sunt omniũ eandem cum eis rationem habentium.

A	B	E	C	D
5	6	2	4	3

Theorema 22. Propositio 24.

Minimi numeri omnium eandem cum eis rationem habentiũ, primi sunt inter se.

:	:	:	:	:
A	B	C	D	E
8	6	4	3	2

Theorema 23. Propositio 25.

Si duo numeri sint primi inter se, qui alter utrum illorum metitur numerus, is ad reliquũ primus erit.

:	:	:	:
A	B	C	D
6	7	3	4

Theorema 24. Propositio 26.

Si duo numeri ad quẽpiam numerũ primi sint, ad eundẽ primus is quoque futurus est, qui ab illis productus fuerit.

B	A	C	D	E	F
3	5	5	5	3	2

Theorema 25. Propositio 27.

Si duo numeri primi sint inter se, q ab vno eorũ gignitur ad reliquum primus erit.

B	A	C	D
	7	6	3

Theorema 26. Propositio 28.

Si duo numeri ad duos numeros ambo ad vtrunque primi sint, & qui ex eis gignentur, primi inter se erunt.

:	:	:	:	:	:
A	B	E	C	D	F
3	5	15	2	4	8

Theore-

Theorema 27. Propositio 29.

Si duo numeri primi ſint inter ſe, & multiplicãs vterq; ſeipſum procreet aliquẽ, qui ex ijs producti fuerint, primi inter ſe erũt. Quòd ſi numeri initio propoſiti multiplicãtes eos qui producti ſunt, effecerint aliquos, hi quoq; inter ſe primi erũt, & circa extremos idẽ hoc ſemper eueniet.

A	C	E	B	D	F
3	6	27	4	16	63

Theorema 28. Propoſitio 30.

Si duo numeri primi ſint inter ſe, etiam ſimul vterq; ad vtrunq; illorũ primus erit. Et ſi ſimul vterq; ad vnum aliquem eorũ primus ſit, etiam qui initio poſiti ſunt numeri, primi inter ſe erunt.

C

A	B	D
7	5	4

Theorema 29. Propoſitio 31.

Omnis primus numerus ad omnem numerum quem non metitur, primus eſt.

A	B	C
7	10	5

Theorema 30. Propoſitio 32.

Si duo numeri ſeſe mutuò multiplicãtes faciant aliquem, hũc aũt ab illis productum metiatur primus quidam numerus, is alterum etiã metitur eorum qui initio poſiti erant.

A	B	C	D	E
2	6	12	3	4

Theoréma 31. Propositio 33.

Omnem compositum numerum aliquis primus metietur.

A	B	C
27	9	3

Theorema 32. Propositio 34.

Omnis numerus aut primus est, aut eum aliquis primus metitur.

A	A	
3	6	3

Problema 3. Propositio 35.

Numeris datis quotcunque, reperire minimos omnium qui eandem cum illis rationem habeant.

A	B	C	D	E	F	G	H	K	I	M
6	8	12	2	3	4	6	2	3	4	3

Problema 4. Propositio 36.

B				
A	C	D	E	F
7	12	8	4	5

Duobus numeris datis reperire quem illi minimū metiantur numerum.

A					
F	E	C	D	G	H
6	9	12	9	2	3

Theo-

Theorema 33. Propositio 37.

Si duo numeri numerũ quempiam metiantur, & minimus quem illi metiuntur eundem metietur.

F

A B E C

2 3 6 12

Problema 5. Propositio 38.

Tribus numeris datis reperire quem minimum numerũ illi metiantur.

A B C D E

3 4 6 12 8

A B C D E F

3 6 8 12 24 16

Theorema 34. Propositio 39.

Si numerum quiſpiam numerus metiatur, menſus partem habebit metienti cognominem.

A B C D

12 4 3 1

Theorema 35. Propositio 40.

Si numerus partem habuerit quãlibet, illum metietur numerus parti cognominis.

A B C D

8 4 2 1

Problema 6. Propositio 41.

Numerum reperire, qui minimus cùm ſit, datas habeat partes.

A B C G H

2 3 4 12 10

ELEMENTI VII. FINIS.

EVCLIDIS ELEMENTVM OCTAVVM.

Theorema 1. Propositio 1.

Si sint quotcũq; numeri deinceps proportionales, quorum extremi sint inter se, primi, minimi sunt omnium eandem cum eis rationem habentium.

:	:	:	:	:	:	:	:
A	B	C	D	E	F	G	H
8	12	18	27	6	8	12	18

Problema 1. Propositio 1.

Numeros reperire deinceps proportionales minimos, quotcunq; iusserit quispiam in data ratione.

:	:	:	:	:	:	:	:	:
A	B	C	D	E	F	G	H	K
3	4	9	12	16	27	36	49	64

Theorema 2. Propositio 3.

Conuersa primæ.

Si sint quotcũq; numeri deinceps proportionales minimi habentium eandem cum eis rationem, illorum extremi sunt inter se primi.

:	:	:	:	:	:	:	:	:	:	:	:	:
A	B	C	D	E	F	G	H	K	L	M	N	O
27	16	48	64	3	4	9	12	16	27	36	48	64

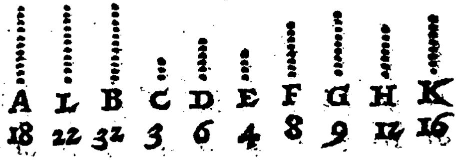
Pro-

Problema 2. Propositio 4.

Rationibus datis quotcunque in minimis numeris reperire numeros deinceps minimos in datis rationibus.

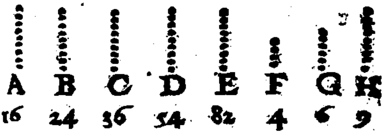
Theorema 3. Propositio 5.

Plani numeri rationem inter se habent ex lateribus compositam.



Theorema 4. Propositio 6.

Si sint quotlibet numeri deinceps



proportionales, primus autem secundum non metiatur, neque alius quispiam vllum metietur.

Theorema 5. Pro-
poſitio 7.

Si ſint quotcunque numeri deinceps proportionales, primus autem extremum metiatur, is etiam ſecundum metietur.

A B C D
4 6 12 24

Theorema 6. Pro-
poſitio 8.

Si inter duos numeros medij continua ꝓportione incidant numeri, quot inter eos medij continua proportione incidunt numeri, tot & inter alios eandem cum illis habentes rationem medij continua proportione incident.

A C D B G H K L C M N F
4 9 27 81 1 3 9 27 2 6 18 54

Theorema 7. Propoſitio 9.

Si duo numeri ſint inter ſe primi, & inter eos medij cõtinua proportione incidãt numeri, quot inter illos medij continua proportione incidunt numeri, totidẽ & inter vtrunque eorum ac vnitatẽ deinceps medij continua proportione incident.

A M H E F N C K X G D L O B
27 27 9 36 3 36 1 12 48 4 48 16 64 64

Theo-

Theorema 8. Propoſitio 10.

Si inter duos numeros & vnitaté continuè proportionales incidãt numeri quot inter vtrunque ipſorum & vnitaté deinceps medij continua proportione incidunt numeri, totidé & inter illos medij continua proportione incident.

A	E	K	D	H	C	L	F	G	B
27	9	36	3	12	1	48	4	16	64

Theorema 9. Propoſitio 11.

Duorum quadratorũ numerorum vnus medius proportionalis eſt numer⁹: & quadratus ad quadratum duplicatam habet lateris ad latus rationem.

A	C	E	D	B
9	3	12	4	16

Theorema 10. Propoſitio 12.

Duorum cuborum numerorũ duo medij proportionales ſunt numeri: & cubus ad cubum triplicatam habet lateris ad latus rationem.

A	H	K	B	C	D	E	F	G
27	36	48	64	3	4	9	12	16

Theo-

Theorema 11. Propositio 13.

Si ſint quotlibet numeri deinceps propor-tionales, & multiplicans quiſq; ſeipſum faciat aliquos, qui ab illis producti fuerint, proportionales erũt: & ſi numeri primùm poſiti, ex ſuo in procreatos ductu faciãt aliquos, ipſi quoque proportionales erunt.

Theorema 12. Propoſitio 14.

Si quadratus numerus quadratum numerũ metiatur, & latus vnius metietur latus alterius. Et ſi vnius quadrati latus metiatur latus alterius, & quadratus quadratum metietur.

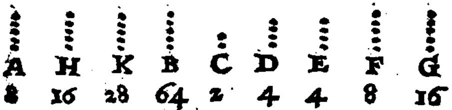
A	E	B	C	D
9	12	16	3	4

Theorema 13. Propoſitio 15.

Si cubus numerus cubum numerũ metiatur, & latus vniᵒ metietur alterius latus. Et ſi latus vnius cubi latus alterius metiatur,

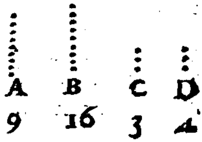
tum

tum cubus cubum metietur.



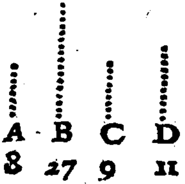
Theorema 14. Propositio 16.

Si quadratus numerus quadratum numerum non metiatur, neque latus vnius metietur alterius latus. Et si latus vni⁹ quadrati nõ metiatur latus alterius, neque quadratus quadratum metietur.



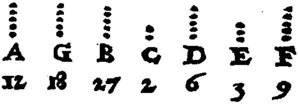
Theorema 15. Propositio 17.

Si cubus numerus cubum numerũ nõ metiatur, neque latus vni⁹ latus alterius metietur. Et si latus cubi alicuius latus alterius nõ metiatur, neque cubus cubũ metietur.



Theorema 16. Propositio 18.

Duorum similium planorum numerorũ vnus medius proportionalis est numerus, & planus ad planũ duplicatam habet literis homologi



logi ad latus homologum rationem.

Theorema 17. Propositio 19.

Inter duos similes numeros solidos, duo medij proportionales incidunt numeri: & solidus ad similem solidum triplicatam rationem habet lateris homologi ad latus homologum.

A	N	X	B	C	D	E	F	G	H	K	M	L
8	12	18	27	2	2	2	3	3	3	4	6	9

Theorema 18. Propositio 20.

Si inter duos numeros vnus medius proportionalis incidat numerus, similes plani erunt illi numeri.

A	C	B	D	E	F	G
18	24	33	3	4	6	8

Theorema 19. Propositio 21.

Si inter duos numeros duo medij proportionales incidant numeri, similes solidi sunt illi numeri.

A	C	D	B	E	F	G	H	K	L	M
27	36	44	64	9	12	16	3	3	3	4

Theo-

Theorema 20. Propositio 22.

Si tres numeri deinceps ſint proportionales, primus autem ſit quadratus, & tertius quadratus erit.

A	B	D
9	15	25

Theorema 21. Propositio 23.

Si quatuor numeri deinceps ſint proportionales, primus autem ſit cubus, & quartus cubus erit.

A	B	C	D
8	12	18	27

Theorema 22. Propoſitio 24.

Si duo numeri rationem habeant inter ſe, quam quadratus numerus ad quadratum numerum, primus autem ſit quadratus, & ſecundus quadratus erit.

A		B	C		D
4	6	9	16	24	36

Theorema 23. Propoſitio 25.

Si numeri duo rationem inter ſe habeant, quam cubus numerus ad cubum numerũ, primus autem cubus ſit, & ſecundus cubus erit.

A	E	F	B	C			D
8	12	18	27	64	95	140	216

Theo-

Theorema 24. Propositio 26.

Similes plani numeri rationem inter se habent, quam quadratus numerus ad quadratum numerum.

A	C	B	D	E	F
18	24	32	9	12	16

Theorema 25. Propositio 27.

Similes solidi numeri rationem habēt inter se, quam cubus numerus ad cubum numerum.

A	C	D	B	E	F	G	H
16	24	36	54	8	12	18	27

ELEMENTI VIII. FINIS.

EVCLI-

EVCLIDIS ELEMENTVM NONVM.

Theorema 1. Propoſitio 1.

Si duo ſimiles plani numeri mutuò ſeſe multiplicã-tes quendã procreent, productus quadratus erit.

A	E	B	D		C
4	6	9	16	24	36

Theorema 2. Propoſitio 2.

Si duo numeri mutuò ſeſe multiplicantes quadratum faciant, illi ſimiles ſunt plani.

A		B	D		C
4	6		9	18	36

Theorema 3. Propoſitio 3.

Si cubus numerus ſeipſum multiplicãs ꝓcreet aliquẽ, productus cubus erit.

Vnitas	D	D	A			B
	3	4	8	16	32	64

Theorema 4. Propositio 4.

Si cubus numerus cubũ	⁝	⁝	⁝	⁝
numerum multiplicans	A	B	D	C
quendam procreet, pro-	8	27	64	216
creatus cubus erit.				

Theorema 5. Propositio 5.

Si cubus numerus numerum quendá mul

tiplicans cubum pro-	⁝	⁝	⁝	⁝
creet, & multiplica-	A	B	D	D
tus cubus erit.	27	64	729	1728

Theorema 6. Propositio 6.

Si numerus ſeipſum	⁝	⁝	⁝
multiplicans cubum	A	B	C
procreet, & ipſe cu-	27	729	19683
bus erit.			

Theorema 7. Propositio 7.

Si compoſitus numerus quẽdam numerũ

multiplicans quem-	:	:	:	:	:
piam procreet, pro-	A	B	C	D	E
ductus ſolidus erit.	6	8	48	2	3

Theorema 8. Propositio 8.

Si ab vnitate quotlibet numeri deinceps ꝓ portionales ſint, tertius ab vnitate quadratus eſt, & vnũ intermittentes omnes: quartus autẽ cubus, & duobus intermiſsis omnes:

nes.ſeptimus verò cubus ſimul & æquadratus, & quinque intermiſſis omnes

Vnitas	A	B	C	D	E	F
	3	9	27	81	243	729

Theorema 9. Propoſitio 9.

Si ab vnitate ſint quotcunque numeri deincéps proportionales, ſit autem quadratus is qui vnitatem ſequitur, & reliqui oẽs quadrati erũt. Quòd ſi qui vnitatem ſequitur cubus ſit, & reliqui omnes cubi erunt.

quadrati		cubi
531441	F	732969
59049	E	531441
6561	D	59049
729	C	6561
81	B	729
9	A	81
	o	
	Vnitas.	

Theorema 10. Propoſitio 10.

Si ab vnitate numeri quotcunque proportionales ſint, non ſit autẽ quadratus is qui vnitatem ſequitur, neq; alius vllus quadratus erit, dẽptis tertio ab vnitate ac omnibus

Vnitas	A	B	C	D	E	F
o	3	9	36	81	243	729

nibus vnum intermittẽtibus. Quòd ſi qui vnitatem ſequitur, cubus non ſit, neque alius vllus cubus erit, demptis quarto ab vnitate ac omnibus duos intermittentibus.

Theorema 11. Propoſitio 11.

¶ Si ab vnitate numeri quotlibet deinceps proportionales ſint, minor maiorẽ metitur per quempiam eorum qui in proportionalibus ſunt numeris.

A	D	C	D	E
1	2	4	8	16

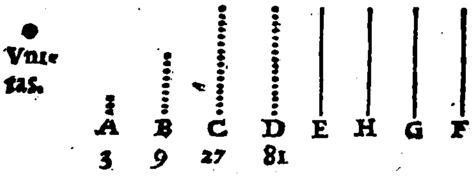
Theorema 12. Propoſitio 12.

Si ab vnitate quotlibet numeri ſint ꝓportionales, quot primorum numerorũ vltimum metiuntur, totidem & eum qui vnitati proximus eſt, metientur.

Vnitas.								
	A	B	C	D	E	H	G	F
	4	16	64	259	2	8	32	128

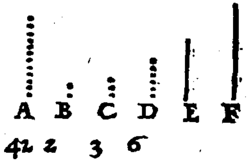
Theorema 13. Propoſitio 13.

Si ab vnitate ſint quotcunq; numeri deinceps proportionales, primus autẽ ſit qui vnitatẽ ſequitur, maximũ nullus alius metietietur, ijs exceptis qui in proportionalib• ſunt numeris.



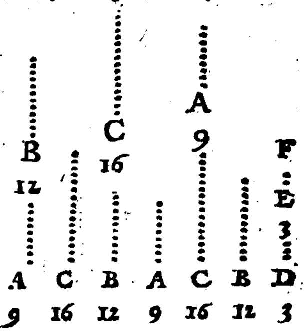
Theorema 14. Propoſitio 14.

Si minimum numerum primi aliquot numeri metiãtur, nullus alius numerus primus illum metietur, ijs exceptis qui primò metiuntur.



Theorema 15. Propoſitio 15.

Si tres numeri deinceps proportionales ſint minimi, eadẽ cum ipſis habẽtium rationẽ, duo quilibet compoſiti ad tertium primi erunt.

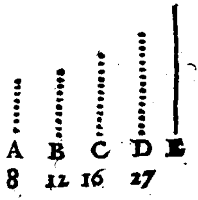


Theorema 16. Propoſitio 16.

Si duo numeri ſint inter ſe primi, non ſe habebit quẽadmodum primus ad ſecũdum, ita ſecundus ad quẽpiam alium.

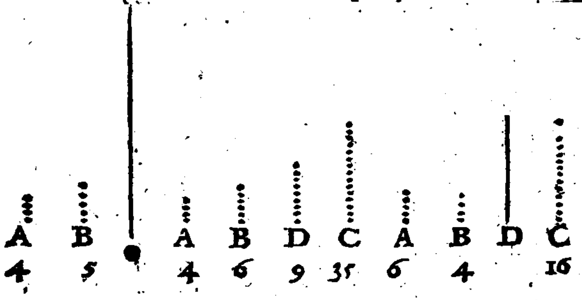
Theorema 17. Propoſitio 17.

Si ſint quotlibet numeri deinceps ꝓportionales, quorum extremi ſint inter ſe primi, non erit quẽadmodum primus ad ſecundum, ita vltimus ad quempiam alium.



Theorema 18. Propoſitio 18.

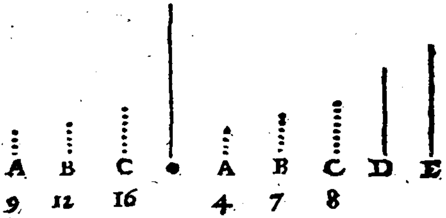
Duob⁹ numeris datis, cõſiderare poſsitne tertius illis inueniri proportionalis.



Theore-

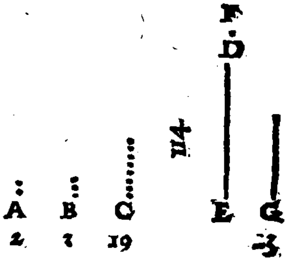
Theorema 19. Propositio 19.

Tribus numeris datis, considerare possitne quartus illis reperiri proportionalis.



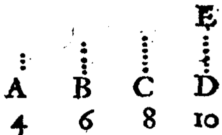
Theorema 20. Propositio 20.

Primi numeri plures sunt quã cũque proposita multitudine primorum numerorum.



Theorema 21. Propositio 21.

Si pares numeri quotlibet compositi sint, totus est par.



Theorema 22. Propositio 22.

Si impares numeri quotlibet compositi sint, sit autem par illorum multitudo, totus par erit.

E
A B C D
5 9 7 5

Theorema 23. Propositio 23.

Si impares numeri quotcunque cõpositi sint, sit autem impar illoru mul titudo, & totus impar erit.

A B C E
5 7 8 1

Theorema 24. Propositio 24.

Si de pari numero par detra ctus sit, & reliquus par erit.

B
A C
6 4

Theorema 25. Propositio 25.

Si de pari numero impar detractus sit, & reliquus impar erit.

B
A C D
8 1 4

Theorema 26. Propositio 26.

Si de impari numero impar detractus sit, & reliquus par erit.

B
A C D
4 6 1

Theo

Theorema 27. Propositio 27.

Si ab impari numero par ablatus sit, reliquus impar erit.

A	D	C
1	4	4

Theorema 28, Propositio 28.

Si impar numerus parem multiplicãs, procreet quẽpiam, procreatus par erit.

A	B	C
3	4	12

Theorema 29. Propositio 29.

Si impar numerus imparẽ numerum multiplicans quemdam procreet, procreatus impar erit.

A	B	C
3	5	15

Theorema 30. Propositio 30.

Si impar numerus parẽ numerum metiatur, & illius dimidium metietur.

A	C	B
3	6	18

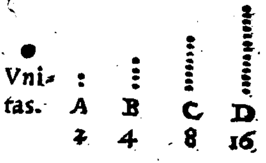
Theorema 31. Propositio 31.

Si impar numerus ad numerum quempiã primus sit, & ad illius duplum primus erit.

A	B	C	D
7	8	16	

Theorema 32. Propositio 32.

Numerorum, qui à binario dupli ſunt, vnuſquiſq; pariter par eſt tantùm.



Theorema 33. Propoſitio 33.

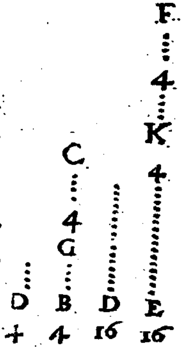
Si numerus dimidiũ impar habeat, pariter impar eſt tantùm.

Theorema 34. Propoſitio 34.

Si par numerus nec ſit duplꝰ à binario, nec dimidium impar habeat, pariter par eſt, & pariter impar.

Theorema 35. Propoſitio 35.

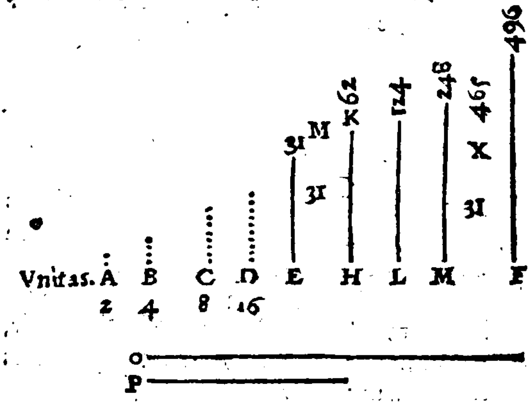
Si ſint quotlibet numeri deinceps proportionales, detrahãtur autẽ de ſecũdo & vltimo æquales ipſi primo, erit quemadmodũ ſecũdi exceſſus ad primũ, ita vltimi exceſſus ad omnes qui vltimum antecedunt.



Theo-

Theorema 36. Propositio 36.

Si ab vnitate numeri quotlibet deinceps expositi sunt in dupla proportione quoad totus compositus primus factus sit, isq; totus in vltimum multiplicatus quempiá procreet, procreatus perfectus erit.



ELEMENTI IX. FINIS.

EVCLIDIS ELEMENTVM DECIMVM.

DEFINITIONES.

1.

Commenſurabiles magnitudines dicuntur illæ, quas eadem menſura metietur.

2

Incommenſurabiles verò magnitudines dicuntur hæ, quarum nullam menſuram communem contingit reperiri.

3

Lineæ rectæ potentia cõmenſurabiles sũt, quarum quadrata vna eadem ſuperficies ſiue area metitur.

4

Incommenſurabiles verò lineæ ſunt, quarum quadrata, quæ metiatur area cõmunis, reperiri nulla poteſt.

5

Hæc cùm ita ſint, oſtendi poteſt quòd quã tacunque linea recta nobis proponatur, exiſtunt etiã aliæ lineæ innumerabiles eidem commenſurabiles, aliæ item incommenſurabiles, hæ quidem longitudine & poten-

potentia:illæ verò potentia tantùm. Vocetur igitur linea recta, quantacunq; proponatur, ῥητὴ, id est rationalis.

6

Lineæ quoq; illi ῥητῇ commensurabiles siue longitudine & potentia, siue potentia tantùm, vocentur & ipsæ ῥηταί, id est rationales.

7

Quæ verò lineæ sunt incommensurabiles illi τῇ ῥητῇ, id est primo loco rationali, vocētur ἄλογοι, id est irrationales.

8

Et quadratum quod à linea proposita describitur, quam ῥητὴν vocari volumus, vocetur ῥητόν.

9

Et quæ sunt huic commensurabilia, vocētur ῥητά.

10

Quæ verò sunt illi quadrato ῥητῷ scilicet incommensurabilia, vocentur ἄλογα, id est surda.

11

Et lineæ quæ illa incommēsurabilia describunt, vocentur ἄλογοι Et quidem si illa incommēsurabilia fuerint quadrata, ipsa eorũ latera vocabũtur ἄλογοι lineæ. quòd si quadrata quidem non fuerint, verũ aliæ quæpiã superficies siue figuræ rectilineæ, tunc

tunc verò lineæ illæ quæ deſcribunt quadrata æqualia figuris rectilineis, vocentur ἄλογοι.

Theorema 1. Propoſitio 1.

Duabus magnitudinibus inæqualibus propoſitis, ſi de maiore detrahatur plus dimidio, & rurſus de reſiduo iterũ detrahatur plus dimidio, idq́; ſemper fiat: relinquetur quædam magnitudo minor altera minore ex duabus propoſitis.

Theorema 2. Propoſitio 2.

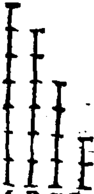
Duabꝰ magnitudinibus propoſitis inæqualibus, ſi detrahatur ſemper minor de maiore, alterna quadam detractione neque reſiduum vnquam metiatur id quod ante ſe metiebatur, incommenſurabiles ſunt illæ magnitudines.

Problema 1. Propoſitio 3.

Duabus magnitudinibus commẽſurabilibus datis, maximam ipſarum communem menſuram reperire.

Proble-

Problema 2. Propositio 4.

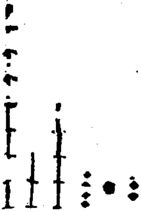


A B C D

Tribus magnitudinibus cõmensurabilibus datis, maximam ipsarũ communẽ mensuram reperire.

Theorema 3. Propositio 5.

Cõmensurabiles magnitudines inter se proportionẽ eam habent, quam habet numerus ad numerum.

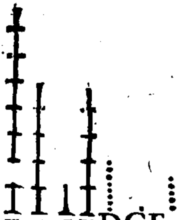


ACBD E

3 4

Theorema 4. Propositio 6.

Si duæ magnitudines proportionem eam habent inter se quam numerus ad numerum, commensurabiles sunt illæ magnitudines.

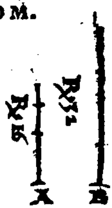


A B C F D G E

8 1 5

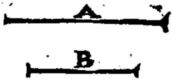
Theo-

Theorema 5. Propoſitio 7.



Incommenſurabiles magnitudines inter ſe proportionem non habét, quam numerus ad numerum.

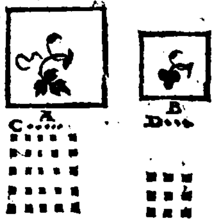
Theorema 6. Propoſitio 8.



Si duæ magnitudines inter ſe proportionem nó habét quam numerus ad numerum, incommenſurabiles illæ ſunt magnitudines.

Theorema 7. Propoſitio 9.

Quadrata, quæ deſcribũtur à rectis lineis lõgitudine cõmenſurabilibus, inter ſe proportionem habent quã numerus quadratus ad alium numerum quadratũ. Et quadrata habentia proportioné inter ſe quá quadratus numerus ad numerum quadratũ, habent quoque latera longitudine comméſurabilia. Quadrata verò

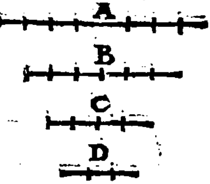


quæ

quæ deſcribuntur à lineis longitudine incõmenſurabilibus, proportioně nõ habět inter ſe, quam quadratus numerus ad numerum alium quadratum. Et quadrata nõ habentia proportionem inter ſe quam numerus quadratus ad numerum quadratũ, neque latera habebunt longitudine commenſurabilia.

Theorema 8. Propoſitio 10.

Si quatuor magnitudines fuerint proportionales, prima verò ſecũdæ fuerit cõmenſurabilis, tertia quoque quartę commẽſurabilis erit. quòd ſi prima ſecũdæ fuerit incommẽſurabilis, tertia quoq; quartæ incommenſurabilis erit.



Problema 3. Propoſitio 11.

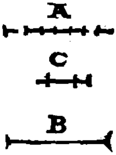
Propoſitę lineæ rectæ (quam ῥητὴν vocari diximus) reperire duas lineas rectas incõmenſurabiles, hanc quidem longitudine tantùm, illam verò non longitudine tantùm, ſed etiam potentia incommẽſurabilem.

Theorema 9. Propoſitio 12.

Magnitudines quæ eidē magnitudini ſunt cō menſurabiles, inter ſe quoque ſunt commenſurabiles.

6 D...... 4 F..
4 E.... 8 G..
3 H...
2 K..
4 L...

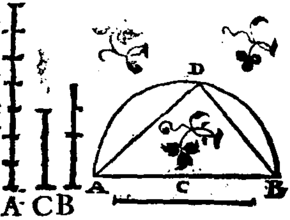
Theorema 10. Propoſitio 3.



Si ex duabus magnitudinibꝰ hæc quidem cōmenſurabilis ſit tertię magnitudini, illa verò eidem incommenſurabilis, incōmenſurabiles ſunt illæ duæ magnitudines.

Theorema 11. Propoſitio 14.

Si duarum magnitudinum commenſurabiliū altera fuerit incōmenſurabilis magnitudini alteri cuipiam tertiæ, reliqua quoque magnitudo eidē tertiæ incommēſurabilis erit.



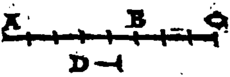
Theorema 12. Propoſitio 15.

Si quatuor rectæ proportionales fuerint, poſſit

posſit autem prima plusquã ſecunda tãto quantum eſt quadratum lineæ ſibi commẽſurabilis longitudine: tertia quoq; poterit plusquàm quarta tanto quantum eſt quadratũ lineæ ſibi commenſurabilis lõgitudine. Quòd ſi prima poſsit plusquã ſecũda quadrato lineę ſibi longitudine incommẽſurabilis: tertia quoque poterit plusquã quarta quadrato lineæ ſibi incõmẽſurabilis lõgitudine.

Theorema 13. Propoſitio 16.

Si duæ magnitudines commenſurabiles componantur, tota magnitudo cõpoſita ſingulis partibus commenſurabilis erit, quòd ſi tota magnitudo cõpoſita alterutri parti commenſurabilis fuerit, illæ duæ quoque partes cõmẽſurabiles erunt.

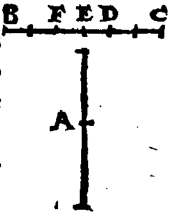


Theorema 14. Propoſitio 17.

Si duæ magnitudines incõmenſurabiles cõponantur, ipſa quoque tota magnitudo ſingulis partibus componentibus incommẽſurabilis erit. Quòd ſi tota alteri parti incõmenſurabilis fuerit, illæ quoque primę magnitudines inter ſe incommẽſurabiles erunt.

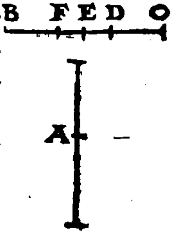
Theorema 15. Propositio 18.

Si fuerint duæ rectæ lineæ inæquales, & quartæ parti quadrati quod describitur à minore, æquale parallelogrammũ applicetur secundũ maiorem, ex qua maiore tãtum excurrat extra latus parallelogrãmi, quantum est alterum latus ipsius parallelogrammi: si præterea parallelogrammũ sui applicatione diuidat lineã illam in partes inter se commẽsurabiles longitudine, illá maior linea tanto plus potest quã minor, quantũ est quadratum lineæ sibi commensurabilis longitudine. Quòd si maior plus posſit quàm minor, tanto quantũ est quadratum lineæ sibi cõmensurabilis lõgitudine, & престереà quartæ parti quadrati lineæ minoris ęquale parallelogrãmum applicetur secundum maiorẽ, ex qua maiore tantum excurrat extra latus parallelogrãmi, quantum est alterum latus ipsius parallelogrammi, parallelogrammum sui applicatione diuidit maiorẽ in partes inter se longitudine commẽsurabiles.

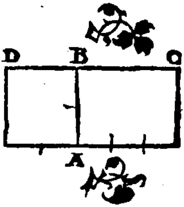


Theorema 16. Propositio 19.

Si fuerint duæ rectæ inæquales, quartæ autem parti quadrati lineæ minoris ęquale le parallelogrammorum ſecundum lineá maiorem applicetur, ex qua linea tantum excurrat extra latus parallelogrãmi, quãtum eſt alterum latus eiuſdem parallelogrammi: ſi parallelogrammũ præterea ſui applicatione diuidat lineã in partes inter ſe longitudine incommẽſurabiles, maior illa linea tantò plus poteſt quàm minor, quantum eſt quadratum lineæ ſibi maiori incommenſurabilis longitudine. Quòd ſi maior linea tãtò plus poſsit quàm minor, quantum eſt quadratum lineæ incõmenſurabilis ſibi lõgitudine: & preterea quartæ parti quadrati lineę minoris ęquale parallelogrammũ applicetur ſecũdum maiorem, ex qua tantũ excurrat extra latus parallelogrãmi, quantum eſt alterum latus ipſius: parallelogrammum ſui applicatione diuidit maiorem in partes inter ſe incommenſurabiles lõgitudine.



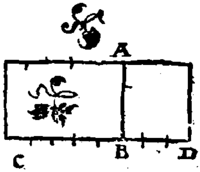
Theorema 17. Propoſitio 20.



Superficies rectangula contẽta ex lineis rectis rationalib⁹ lõgitudine commenſurabilibus ſecundum vnum aliquem modū ex antedictis, rationalis eſt.

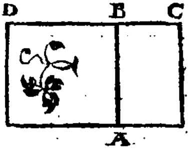
Theorema 18. Propoſitio 21.

Si rationale ſecundū lineam rationalem applicetur, habebit alterū latus lineā rationalem & cōmenſurabilem longitudine lineæ cui rationale parallelogrammū applicatur.



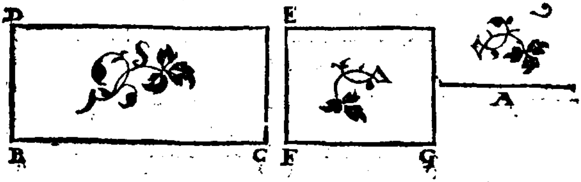
Theorema 19. Propoſitio 22.

Superficies rectangula cōtenta duabus lineis rectis rationalibus potentia tantum cōmēſurabilibus, irrationalis eſt. Linea autem quæ illam ſuperficiem poteſt, irrationalis & ipſa eſt: vocetur verò medialis.



Theorema 20. Propoſitio 23.

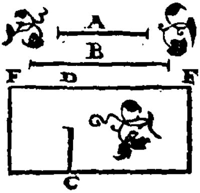
Quadrati lineæ medialis applicati ſecun-



dum

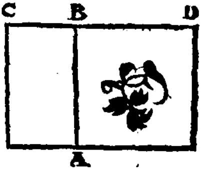
dum lineam rationalem, alterum latus eſt linea rationalis, & incōmenſurabilis lōgitudine lineæ ſecundum quam applicatur.

Theorema 21. Propoſitio 24.



Linea recta mediali cōmēſurabilis, eſt ipſa quoque medialis.

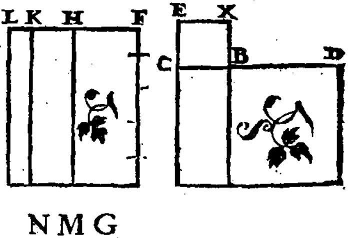
Theorema 22. Propoſitio 25.



Parallelogrammū rectāgulum cōtentum ex lineis medialibus longitudine cōmenſurabilibus, mediale eſt.

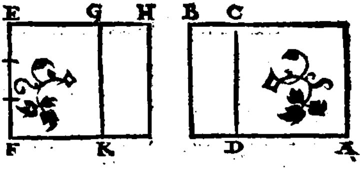
Theorema 23. Propoſitio 26.

Parallelogrammum rectangulum cōprehenſum duabus lineis medialibus potentia tātùm cōmenſurabilib⁹, vel rationale eſt, vel mediale.



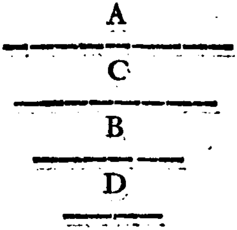
Theorema 24. Propoſitio 27.

Mediale nó eſt maius quàm mediale ſuperficie rationali.



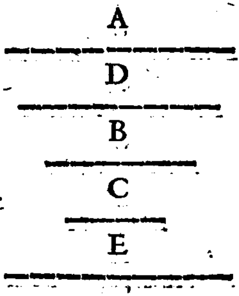
Proble.4. Propoſitio 28.

Mediales lineas inuenire potentia tantùm commenſurabiles rationale comprehendentes.



Problema 5. Propoſitio 29.

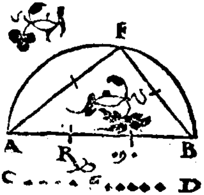
Mediales lineas inuenire potentia tantùm commenſurabiles mediale comprehendendentes.



Pro-

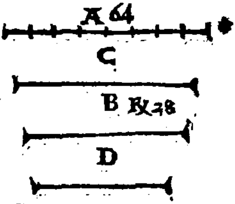
Problema 6. Propositio 30.

Reperire duas rationales potẽtia tantùm commẽsurabiles huiusmodi, vt maior ex illis possit plus quàm minor quadrato lineæ sibi commensurabilis longitudine.



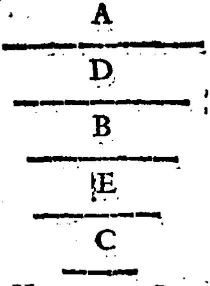
Problema 7. Propositio 31.

Reperire duas lineas mediales potentia tãtùm commensurabiles rationalem superficiẽ cõtinentes, tales inquã vt maior possit plus quàm minor quadrato lineæ sibi commẽsurabilis longitudine.



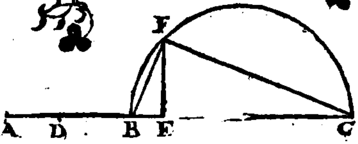
Problema 8. Propositio 32.

Reperire duas lineas mediales potentia tantùm commensurabiles medialem superficiem continentes, huiusmodi vt maior plus possit quàm minor quadrato lineæ sibi commensurabilis longitudine.



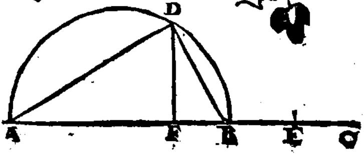
Problema 9. Propoſitio 33.

Reperire duas rectas potẽtia incommẽſurabiles, quarum quadrata ſimul addita faciant ſuperficiẽ rationalẽ, parallelogrãmum verò ex ipſis contentum ſit mediale.



Problema 10. Propoſitio 34.

Reperire lineas duas rectas potentia incõmenſurabiles, conficientes compoſitũ ex ipſarum quadratis mediale, parallelogrammũ verò ex ipſis contentum rationale.



Problema 11. Propoſitio 35.

Reperire duas lineas rectas potentia incõmenſurabiles, cõficientes id quod ex ipſarum quadratis cõponitur mediale, ſimul-

que

que parallelogrammũ ex ipſis contẽtum, mediale, quod præterea parallelogrãmu ſit incõmenſurabile cõpoſito ex quadratis ipſarum.

PRINCIPIVM SENARIOrum per compoſitionem.

Theorema 25. Propoſitio 36.

Si duæ rationales potẽtia tantùm commẽſurabiles cõponantur, tota linea erit irrationalis. Vocetur autem Binomium.

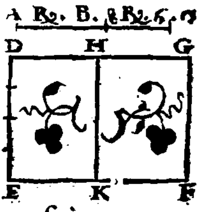
Theorema 26. Propoſitio 37.

Si duæ mediales potentia tantùm cõmenſurabiles rationale continentes cõponantur, tota linea eſt irrationalis, vocetur autem Bimediale prius.

Theo-

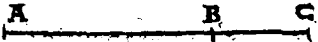
Theorema 27. Propositio 38.

Si duæ mediales potẽtia tantùm cõmensurabiles mediale continentes cõponãtur, tota linea est irrationalis: vocetur autẽ Bimediale secundum.



Theorema 28. Propositio 39.

Si duæ rectæ potentia incommẽsurabiles cõponantur, conficientes compositum ex quadratis ipsarũ rationale, parallelogrammum verò ex ipsis contentũ mediale, tota linea recta est irrationalis. Vocetur autẽ linea maior.



Theorema 29. Propositio 40.

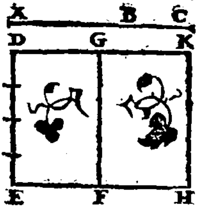
Si duæ rectæ potentia incõmensurabiles componantur, conficientes compositũ ex ipsarũ quadratis mediale, id verò ꝙ fit ex ipsis, rationale, tota linea est irrationalis. Vocetur autem potens rationale & mediale.

Theorema 30. Propositio 41.

Si duæ rectę potentia incommensurabiles componantur, conficientes cõpositum ex quadratis ipsarum mediale, & quod cõti-

netur

netur ex ipſis, mediale, & præterea incōmenſurabile cōpoſito ex quadratis ipſarū, tota linea eſt irrationalis. Vocetur autem potens duo medialia.



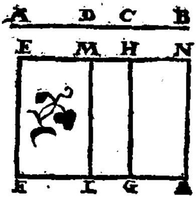
Theorema 31. Propoſitio 42.

Binomium in vnico tantùm pūcto diuiditur in ſua nomina, id eſt in lineas ex quibus componitur.

Theorema 32. Propoſitio 43.

Bimediale prius in vnico tantū puncto diuiditur in ſua nomina.

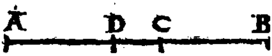
Problema 33. Propoſitio 44.



Bimediale ſecundum in vnico tantū puncto diuiditur in ſua nomina.

Problema 34. Propoſitio 45.

Linea maior in vnico tātùm in pūctō diuiditur in ſua nomina.



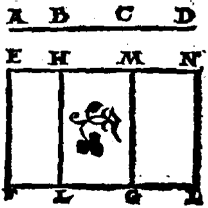
Theo-

Theorema 35. Propoſitio 46.

Linea potẽs rationale & mediale in vnico tãtùm pũcto diuiditur in ſua nomina.

Theorema 36. Propoſitio 47.

Linea potẽs duo medialia in vnico tantùm pũcto diuiditur in ſua nomina.



DEFINITIONES ſecundæ.

Propoſita linea rationali, & binomio diuiſo in ſua nomina, cuius binomij maius nomen, id eſt, maior portio poſsit pluſquam minus nomen quadrato lineæ ſibi, maiori inquam nomini, commenſurabilis longitudine.

1

Si quidẽ maius nomen fuerit cõmenſurabile longitudine propoſitæ lineę rationali, vocetur tota linea Binomium primum.

2

Si verò minus nomen, id eſt minor portio Binomij, fuerit cõmenſurabile lõgitudine ne propoſitæ lineæ rationali, vocetur tota linea Binomium ſecundum.

3

Si verò neutrum nomen fuerit commẽſurabile longitudine propoſitæ lineæ rationali, vocetur Binomium tertium.

Rurſus ſi maius nomen poſſit pluſquã minus nomen quadrato lineæ ſibi incommenſurabilis longitudine.

4

Si quidem maius nomen eſt commenſurabile lõgitudine propoſitæ lineæ rationali, vocetur tota linea Binomium quartum.

5

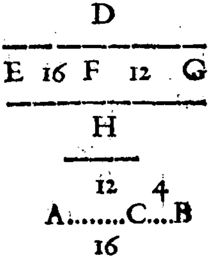
Si verò minus nomen fuerit cõmenſurabile longitudine lineæ rationali, vocetur Binomium quintum.

6

Si verò neutrum nomen fuerit longitudine commenſurabile lineæ rationali, vocetur illa Binomium ſextum.

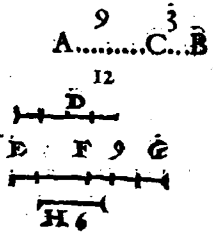
Proble.12. Propoſitio 48.

Reperire Binomium primum.



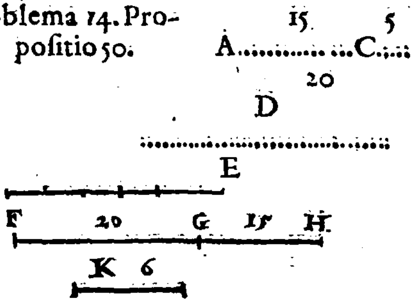
Proble-

Problema 13. Propositio 49.



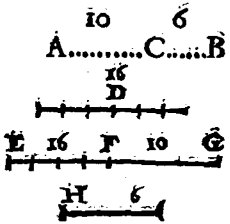
Reperire Binomium secundum.

Problema 14. Propositio 50.



Rereri re Binomium tertiũ.

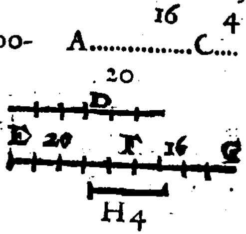
Problema 15. Propositio 51.



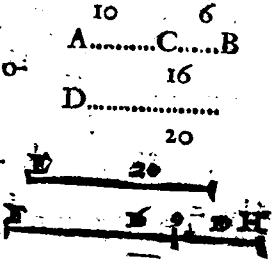
Reperire Binomium quartum.

Proble-

Problema 16. Propositio 52.



Reperire Binomii quintum.

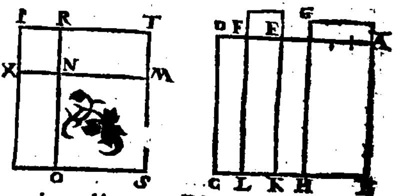


Probl. 17. Propositio 53.

Reperire Binonimum sextum.

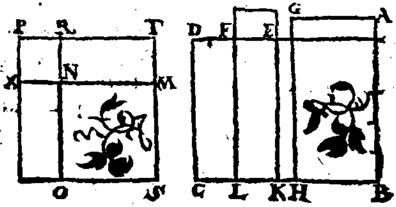
Theorema 37. Propositio 54.

Si superficies contēta fuerit ex rationali & Binomio primo, linea quę illā superficiem potest, est irrationalis, quæ Binomiū vocatur.



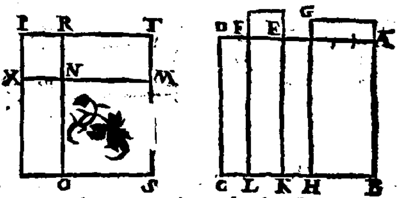
Theorema 38. Propositio 55.

Si superficies cõtenta fuerit ex linea rationali & Binomio secundo, linea potens illã superficiẽ est irrationalis, quę Bimediale primum vocatur.



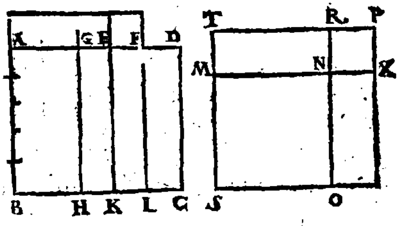
Theorema 39. Propositio 56.

Si superficies cõtineatur ex rationali & Binomio tertio, linea q̃ illã superficiem potest, est irrationalis q̃ dicitur Bimediale secundũ.



Theorema 40. Propositio 57.

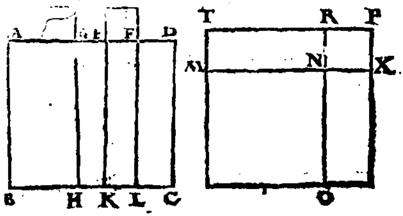
Si superficies cõtineatur ex rationali & Bino- mio



mio quarto, linea potens superficiem illã, est irrationalis, quæ dicitur maior.

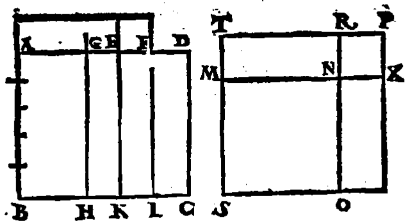
Theorema 41. Propositio 58.

Si superficies cõtineatur ex rationali & Binomio quinto, linea quæ illam superficiẽ potest est irrationalis, quę dicitur potẽs rationale & mediale.

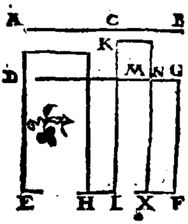


Theorema 42. Propositio 59.

Si superficies contineatur ex rationali & Binomio sexto, linea quæ illam superficiem potest est irrationalis, quæ dicitur potens duo mediala.

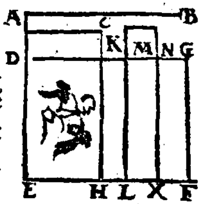


Theoremá 43. Propoſitio 60.



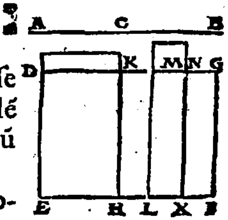
Quadratum Binomij ſecundum lineam rationalem applicatum, facit alterum latus Binomium primum.

Theorema 44. Propoſitio 61.



Quadratū Bimedialis primi ſecundum rationalē lineam applicatum, facit alterum latus Binomiū ſecundum.

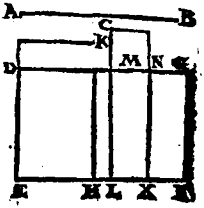
Theorema 45. Propoſitio 62.



Quadratū Bimedialis ſecūdi ſecundū rationalē applicatum, facit alterū latus Binominū tertiū.

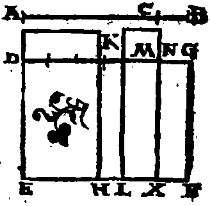
Theo-

Theorema 46. Propoſitio 63.



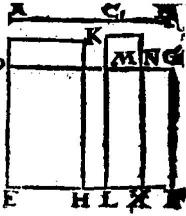
Quadratum lineæ maioris ſecundum lineam rationalem applicatum, facit alterum latus Binomium quartum.

Theorema 47. Propoſitio 64.



Quadratum lineæ potẽtis rationale & mediale ſecundum rationalẽ applicatũ, facit alterum latus Binomium quintũ.

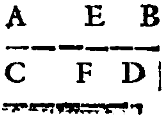
Theorema 48. Propoſitio 65.



Quadratum lineę potentis duo medialia ſecundũ rationalem applicatum, facit alterum latus Binomium ſextum.

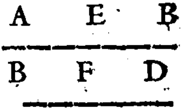
Theorema 49. Propoſitio 66.

Linea longitudine commenſurabilis Binomio eſt & ipſa Binomium eiuſdẽ ordinis.



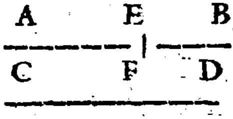
Theorema 50. Propoſitio 67.

Linea longitudine commenſurabilis alteri bimedialium, eſt & ipſa bimediale etiam eiuſdē ordinis.



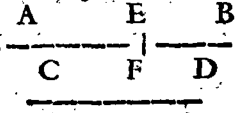
Theorema 51. Propoſitio 68.

Linea cōmēſurabilis lineæ maiori, eſt & ipſa maior.



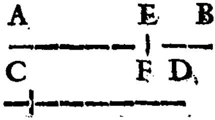
Teorema 52. Propoſitio 69.

Linea commēſurabilis lineæ potenti rationale & mediale, eſt & ipſa linea potens rationale & mediale.



Theorema 53. Propoſitio 70.

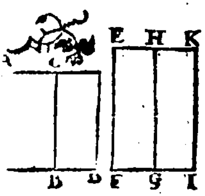
Linea commenſurabilis lineæ potenti duo medialia, eſt & ipſa linea potens duo medialia.



Theorema 54. Propoſitio 71.

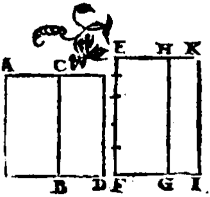
Si duæ ſuperficies rationalis & medialis ſimul componantur, linea quę totā ſuperficiem

ciem compoſitã poteſt, eſt vna ex quatuor irrationalibus, vel ea quæ dicitur Binomium, vel bimediale primum, vel linea maior, vel linea potẽs rationale & mediale.



Theorema 55. Propoſitio 72.

Si duæ ſuperficies mediales incõmenſurabiles ſimul componãtur, fiunt reliquæ duæ lineę irrationales, vel bimediale ſecundum, vel linea potẽs duo medialia



SCHOLIVM.

Binomium & cæteræ conſequentes lineæ irrationales, neque ſunt eadem cum linea mediali, neque ipſa inter ſe.

Nam quadratum lineæ medialis applicatum ſecundum lineam rationalem, facit alterum latus lineam rationalem, & longitudine incommenſurabilem lineæ ſecundum quam applicatur, hoc eſt, lineæ rationali, per 23.

Quadratum verò Binomij ſecundum rationalem applicatum, facit alterum latus Binomium primum, per 60.

Quadratum verò Bimedialis primi secundum rationalem applicatum, facit alterum latus Binomiũ secundum, per 61.
Quadratum verò Bimedialis secundi secundum rationalem applicatum, facit alterum latus Binomiũ tertium, per 62.
Quadratum verò lineæ maioris secundum rationalem applicatum, facit alterum latus Binomiũ quartum, per 63.
Quadratum verò lineæ potẽtis rationale & mediale secundum rationalem applicatum, facit alterũ latus Binomium quintum, per 64.
Quadratum verò lineæ potentis duo medialia secũdum rationalem applicatum, facit alterum latus Binomium sextum, per 65.
Cùm igitur dicta latera, quæ latitudines vocantur, differant & à prima latitudine, quoniam est rationalis, cùm inter se quoque differant, eo quia sunt Binomia diuersorum ordinum: manifestum est ipsas lineas irrationales, differentes esse inter se.

SECVN-

SECVNDVS ORDO ALterius sermonis, qui est de detractione.

Principium senariorum per detractionẽ.

Theorema 56. Propositio 73.

Si de linea rationali detrahatur rationalis potentia tantùm commẽsurabilis ipsi toti, residua est irrationalis, vocetur autẽ Residuum.

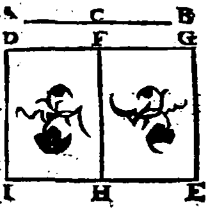
Theorema 57. Propositio 74.

Si de linea mediali detrahatur medialis potentia tantũ cõmensurabilis toti lineæ, quæ verò detracta est cũ tota contineat superficiem rationalem, residua est irrationalis. Vocetur autem Residuũ mediale primum.

Theorema 58. Propositio 75.

Si de linea mediali detrahatur medialis

tentia tantùm commensurabilis toti, quæ verò detracta est, cùm tota cõtineat superficiẽ medialẽ, reliqua est irrationalis. Vocetur autem Residuũ mediale secundum.



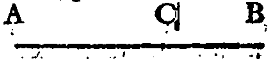
Theorema 57. Propositio 76.

Si de linea recta detrahatur recta potẽtia incommensurabilis toti, compositum autem ex quadratis totius lineæ & lineæ detractæ sit rationale, parallelogrammũ verò ex ijsdem contentum sit mediale, reliqua linea erit irrationalis. Vocetur autem linea minor.

Theorema 58. Propositio 77.

Si de linea recta detrahatur recta potentia

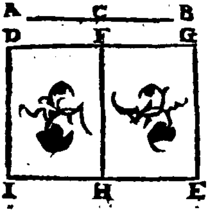
tia incommensurabilis toti lineæ, composítũ autem ex quadratis totius & lineæ detractæ ſit mediale, parallelogrammum verò bis ex eiſdem contentum ſit rationale, reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie rationali totam ſuperficiem medialem.



Theorema 59. Propoſitio 78.

Si de linea recta detrahatur recta potentia incommenſurabilis toti lineæ, compoſitum autem ex quadratis totius & lineæ detractæ ſit mediale, parallelogrammum verò bis ex ijſdem ſit etiam mediale: prætereà ſint quadrata ipſarum incommenſurabilia parallelogrammo bis ex ijſdem cõtento, reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie

ficie mediali totam superficiem medialem.

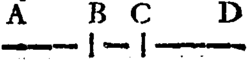


Theorema 60. Propositio 79.

Residuo vnica tantùm linea recta cõiungitur rationalis, potentia tantùm commensurabilis toti lineæ.

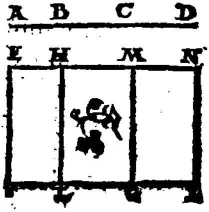
Theorema 61. Propositio 80.

Residuo mediali primo vnica tãtùm linea coniungitur medialis, potentia tantùm cõmensurabilis toti, ipsa cum tota continẽs rationale.



Theorema 62. Propositio 81.

Residuo mediali secundo vnica tantùm cõiungitur medialis, potentia tantùm commẽsurabilia toti ipsa cum tota continens mediale.



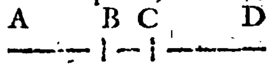
Theorema 63. Propositio 82.

Lineæ minori vnica tantùm recta cõiungitur

tur potentia incommenſurabilis toti, faciens cum tota compoſitum ex quadratis ipſarum rationale, id verò parallelogrammum, quod bis ex ipſis fit, mediale.

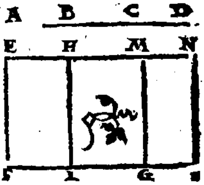
Theorema 64. Propoſitio 83.

Lineæ facienti cum ſuperficie rationali totam ſuperficiem medialem, vnica tantùm coniungitur linea recta potentia incommenſurabilis toti, faciens autem cum tota compoſitum ex quadratis ipſarũ, mediale, id verò quod fit bis ex ipſis, rationale.



Theorema 65. Propoſitio 84.

Lineæ cum mediali ſuperficie facienti totam ſuperficiem medialem, vnica tantùm coniũgitur linea potẽtia toti incommenſurabilis, faciens cum tota compoſitũ ex quadratis ipſarũ mediale, id verò quod fit bis ex ipſis etiam mediale, & prætereà faciens compoſitum ex quadratis ipſarum incommenſurabile ei quod fit bis ex ipſis.



DE-

DEFINITIONES TERTIAE.

Proposita linea rationali & residuo.

I

Si quidem tota, nempe composita ex ipso residuo & linea illi cōiuncta, plus potest quàm coniuncta, quadrato lineæ sibi cōmensurabilis longitudine, fueritq; tota longitudine commēsurabilis lineæ propositæ rationali, residuū ipsum vocetur Residuum primum.

2

Si verò coniuncta fuerit longitudine commensurabilis rationali, ipsa autem tota plus possit quam coniuncta, quadrato lineæ sibi longitudine commensurabilis, residuum vocetur Residuum secundū.

3

Si verò neutra linearū fuerit longitudine commensurabilis rationali, possit autem ipsa tota plusquam coniuncta, quadrato lineæ sibi longitudine commēsurabilis, vocetur Residuum tertium.

Rursus si tota possit plus quàm coniuncta, quadrato lineæ sibi longitudine incōmensurabilis.

4

Et quidem si tota fuerit longitudine commenmensurabilis ipsi rationali, vocetur Residuum quartum.

5

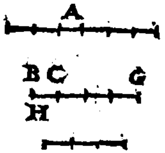
Si verò coniuncta fuerit lõgitudine commensurabilis rationali,& tota plus possit quàm coniuncta,quadrato lineæ sibi longitudine incõmensurabilis, vocetur Residuum quintum.

6

Si verò neutra linearum fuerit cõmensurabilis lõgitudine ipsi rationali, fueritque tota potentior quã coniuncta,quadrato lineæ sibi lõgitudine incommẽsurabilis,vocetur Residuum sextum.

Problema 18. Propositio 85.

Reperire primum Residuum.



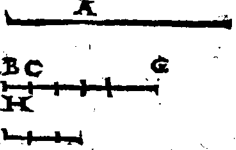
16

D.......F.........E

7 9

Problema 19. Propositio 86.

Reperire secundum Residuum.



D.........F.........E

27 9

Proble-

E..............

Problema 20. Propositio 87.

12

B.........D........C

9 7

A

Reperire tertium Residuum.

F H G

K

Probl. 21. Propositio 88.

A

B C G

H

Reperire quartū Residuum.

D...............F....E

16 4

Problema 22. Propositio 89.

A

B C G

H

Reperire quintum Residuum.

D.......F....E

25 7

Problema 23. Propopositio 90.

A

F H G

K

Reperire sextum Residuum.

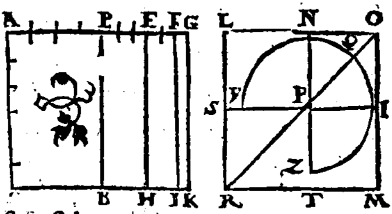
E...............

15

Theo-B.........D......C

Theorema 66. Propositio 91.

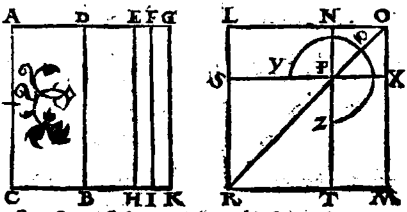
Si superficies cõtineatur ex linea rationali & resi-duo primo linea, quæ illã sup-ficie m



potest, est residuum.

Theorema 67. Propositio 92.

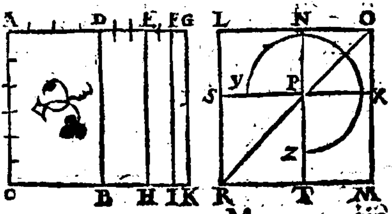
Si superficies cõtineatur ex linea rationali & resi duo secũ-do, li-nea q̃ illam supfi-



ciẽ potest, est residuum mediale primum:

Theorema 68. Propositio 93.

Si sup-ficies cõ tinea-tur ex linea ra tionali & resi-duo ter-

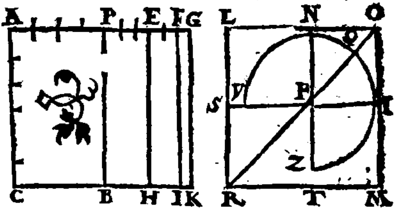


M tio

tio, linea quæ illam ſuperficiem poteſt, eſt reſiduum mediale ſecundum.

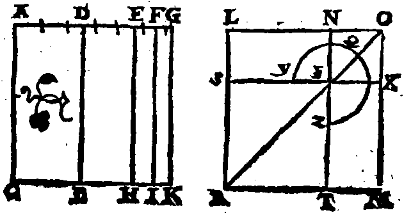
Theorema 69. Propoſitio 94.

Si ſuperficies cõtineatur ex linea rationali & reſiduo quarto linea q̃ illã ſuperficiem poteſt, eſt linea minor.



Theorema 70. Propoſitio 95.

Si ſuperficies cõtineatur ex linea rationali & reſiduo quinto, linea quæ illã ſuperficiẽ poteſt, eſt ea quæ dicitur cum rationali ſuperficie faciens totã medialem.

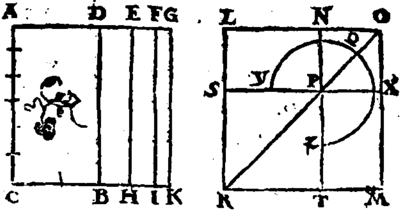


Theorema 71. Propoſitio 96.

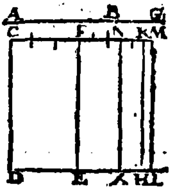
Si ſuperficies cõtineatur ex linea rationali

&

& residuo sexto, linea quæ illam superficiē pōt, est ea quę dicitur faciēs cum mediali superficie totam medialem.

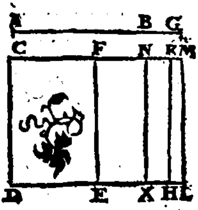


Theorema 72. Propositio 97.



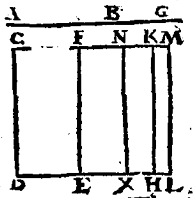
Quadratum residui secundum lineam rationalem applicatum, facit alterum latꝰ residuūm primum.

Theorema 73. Propositio 98.



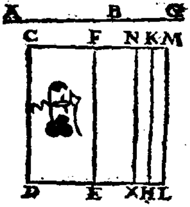
Quadratum residui medialis primi secundū rationalem applicatū, facit alterū latus residuū secundum.

Theorema 74. Propositio 99.



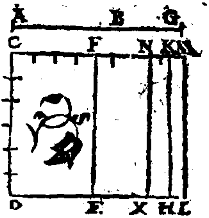
Quadratū residui medialis secūdi secundum rationalē applicatū, facit alterū latus residuū tertium.

Theorema 75. Propoſitio 100.



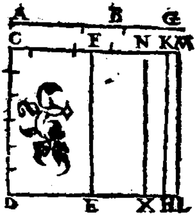
Quadratũ lineæ minoris ſecundum rationalẽ applicatum, facit alterũ latus reſiduum quartũ.

Theorema 76. Propoſitio 101.



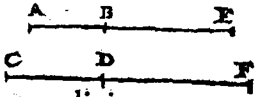
Quadratũ lineæ cum rationali ſuperficie faciẽtis totam medialem, ſecũdum rationalem applicatum, facit alterum latus reſiduum quintum.

Theorema 77. Propoſitio 102.



Quadratum lineæ cum mediali ſuperficie faciẽtis totam medialem, ſecundum rationalem applicatum, facit alterum latus reſiduum ſextum.

Theorema 78. Propoſitio 103.

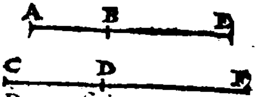


Linea reſiduo cõmẽſurabilis lõgitudine, eſt & ipſa reſiduum, & eiuſdem ordinis.

Theorema 79. Propoſitio 104.

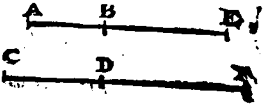
Linea cõmenſurabilis reſiduo mediali, eſt &

& ipſa reſiduũ mediale, & eiuſdẽ ordinis.



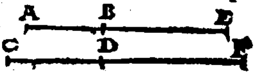
Theorema 80. Propoſitio 105.

Linea commẽſurabilis lineæ minori, eſt & ipſa linea minor.



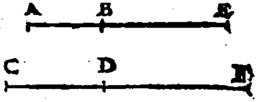
Theorema 81. Propoſitio 106.

Linea commenſurabilis lineæ cũ rationali ſuperficie facienti totã medialẽ, eſt & ipſa linea cum rationali ſuperficie faciens totam medialem.



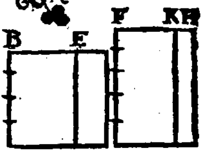
Theorema 82. Propoſitio 107.

Linea commẽſurabilis lineæ cũ mediali ſuperficie facienti totam medialem, eſt & ipſa cum mediali ſuperficie faciens totam medialem.

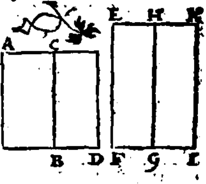


Theorema 83. Propoſitio 108.

Si de ſuperficie rationali detrahatur ſuperficies medialis, linea quæ reliquã ſuperficiem poteſt, eſt alterutra ex duabus irrationalibus, aut reſiduũ, aut linea minor.

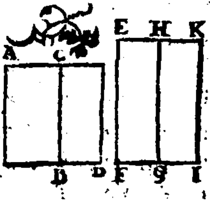


Theorema 84. Propositio 109.



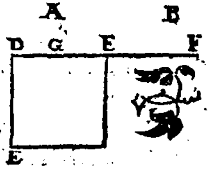
Si de ſuperficie mediali detrahatur ſuperficies rationalis, aliæ duæ irrationales fiũt, aut reſiduum mediale primũ aut cũ rationali ſuperficiem faciens totam medialem.

Theorema 85. Propoſitio 110.



Si de ſuperficie mediali detrahatur ſuperficies medialis q̃ ſit incommẽſurabilis toti, reliquæ duæ fiunt irrationales, aut reſiduum mediale ſecundum, aut cũ mediali ſuperficie faciens totam medialem.

Theorema 86. Propoſitio 111.



Linea quæ Reſiduum dicitur, non eſt eadem cum ea quæ dicitur Binomium.

SCHO-

SCHOLIVM.

Linea quæ Residuum dicitur, & cætera quinque eã consequentes irrationales, neque lineæ mediali neque sibi ipsæ inter se sunt eadem. Nam quadratum lineæ medialis secundum rationalem applicatum, facit alterum latus, rationalem lineam longitudine incommensurabilem ei, secundum quam applicatur, per 23.

Quadratum verò residui secundum rationalem applicatum, facit alterum latus residuum primum per 97.

Quadratum verò residui medialis primi secundum rationalem applicatum, facit alterum latus residuum secundum, per 98.

Quadratum verò residui medialis secundi, facit alterum latus residuum tertium, per 99.

Quadratum verò lineæ minoris, facit alterum latus residuum quartum, per 100.

Quadratum verò lineæ cum rationali superficie facientis totam medialem, facit alterum latus residuum quintum, per 101.

Quadratum verò lineæ cum mediali superficie facientis totam medialem, secundum rationalem applicatum, facit alterum latus residuum sextum, per 102.

Cùm igitur dicta latera, quæ sunt latitudines cuiusque parallelogrammi vnicuique quadrato æqualis & secundum rationalem applicati, differant & à primo latere, & ipsa inter se (nam à primo differunt: quoniam sunt resi-

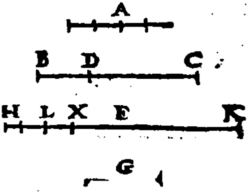
dua non eiusdem ordinis) constat ipsas quoque lineas irrationales inter se differentes esse. Et quoniam demonstratum est, Residuum non esse idem quod Binomium, quadrata autem residui & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex residuis eiusdem ordinis cuius sunt & residua, quorum quadrata applicantur rationali similiter & quadrata Binomij & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex Binomijs eiusdem ordinis, cuius sunt & Binomia, quorum quadrata applicantur rationali. Ergo lineæ irrationales quæ cõsequuntur Binomium, & quæ consequuntur residuum, sunt inter se differentes. Quare dictæ lineæ omnes irrationales sunt numero 13.

1 *Medialis.*
2 *Binomium.*
3 *Bimediale primum.*
4 *Bimediale secundum.*
5 *Maior.*
6 *Potens rationale & mediale.*
7 *Potẽs duo medialia.*
8 *Residuum.*
9 *Residuum mediale primum.*
10 *Residuum mediale secundum.*
11 *Minor.*
12 *Faciens cum rationali superficie totam medialem.*
13 *Faciens cum mediali superficie totam medialem.*

Theo-

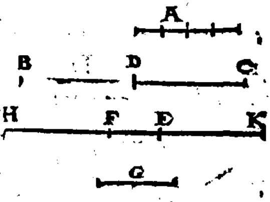
Theorema 87. Propoſitio 112.

Quadratũ lineæ rationalis ſecundum Binomiũ applicatũ, facit alterũ latus reſiduũ, cuius nomina ſunt cõmenſurabilia Binomij nominibus, & in eadẽ proportione præterea id quod fit Reſiduum, eundem ordinem retinet quem Binomium.



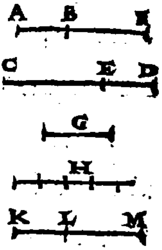
Theorema 88. Propoſitio 113.

Quadratũ lineæ rationalis ſecũdum reſiduum applicatũ, facit alterum latus Binomium, cuius nomina ſunt cõmenſurabilia nominibus reſidui & in eadẽ proportione: præterea id quod fit Binomiũ, eſt eiuſdem ordinis, cuius & Reſiduum.



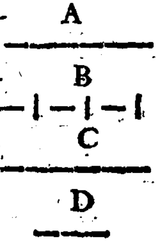
Theorema 89. Propoſitio 114.

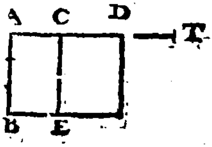
Si parallelogrammũ contineatur ex reſidúo & Binomio, cuius nomina sunt cõmensurabilia nominibus residui & in eadẽ proportione, linea quæ illam superficiem potest, est rationalis.



Theorema 90. Propositio 115.

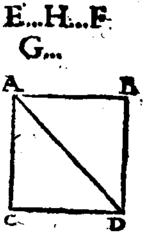
Ex linea mediali nascũtur lineę irrationales innumerabiles, quarum nulla vlli ante dictarum earundem sit.





Propositio 116.

Propositũ nobis esto demõstrare in figuris quadratis diametrũ esse longitudine incõmensurabilem ipsi lateri.



ELEMENTI X. FINIS.

EVCLIDIS ELEMENTVM VNDECIMVM, ET SOLIDORVM *primum.*

DEFINITIONES.

1

Solidum, est quod longitudinem, latitudinem, & crassitudinem habet.

2

Solidi autem extremum est superficies.

3

Linea recta est ad planum recta, cùm ad rectas oēs lineas, à quibus illa tangitur, quæque in proposito sunt plano, rectos angulos efficit.

4

Planum ad planum rectum est, cùm rectæ lineæ, quæ cōmuni planorum sectioni ad rectos angulos in vno planorum ducūtur, alteri plano ad rectos sunt angulos.

5

Rectæ lineę ad planū inclinatio, acutus est angulus, ipsa insistente linea & adiūcta altera comprehēsus, cùm à sublimi rectę illius lineę termino deducta fuerit ꝑpēdicularis,

laris, atq; à puncto quod perpendicularis in ipso plano fecerit, ad propositę illius lineæ extremum, quod in eodē est plano, altera recta linea fuerit adiuncta.

6

Plani ad planū inclinatio, acutus est angulus rectis lineis contentus, quæ in vtroque planorum ad idem cōmunis sectionis pūctū ductæ, rectos ipsi sectioni angulos efficiunt.

7

Planum similiter inclinatum esse ad planum, atq; alterum ad alterum dicitur, cū dicti inclinationum anguli inter se sunt ęquales.

8

Parallela plana, sunt quæ eodem non incidunt, nec concurrunt.

9

Similes figuræ solidæ, sunt quæ similibus planis, multitudine æqualibus cōtinētur.

10

Aequales & similes figurę solidę sunt, quę similibus planis, multitudine & magnitudine æqualibus continentur.

11

Solidus angulus, est plurium quàm duarū linearum, quæ se mutuò contingant, nec in eadem sint superficie, ad omnes lineas inclinatio.

Aliter.

Aliter.

Solidus angulus, est qui pluribus quã duobus planis angulis in eodem non cõsistentibus plano, sed ad vnũ punctũ collectis, continetur.

12

Pyramis, est figura solida quæ planis cõtinetur, ab vno plano ad vnum punctũ collecta.

13

Prisma, figura est solida quæ planis cõtinetur, quorum aduersa duo sunt & æqualia & similia & parallela, alia verò parallelogramma.

14

Sphęra est figura, quæ cõuerso circum quiescentem diametrum semicirculo cõtinetur, cùm in eundem rursus locũ restitutus fuerit, vnde moueri cœperat.

15

Axis autem sphæræ, est quiescẽs illa linea circum quam semicirculus conuertitur.

16

Centrum verò Sphæræ est idẽ, quod & semicirculi.

17

Diameter autem Sphæræ, est recta quædã linea per centrum ducta, & vtrinq; à sphęræ superficie terminata.

Conu-

18

Conus est figura, quæ conuerso circũquiescens alterum latus eorum quæ rectũ angulum continent, orthogonio triangulo continetur, cùm in eundem rursus locum illud triangulum restitutum fuerit, vnde moueri cœperat. Atq; si quiescens recta linea æqualis sit alteri, quę circum rectũ angulum conuertitur, rectangulus erit Conus: sin minor, amblygonius: si verò maior, oxygonius.

19

Axis autem Coni, est quiescens illa linea, circum quam triangulum vertitur.

20

Basis verò Coni, circulus est, qui à circũducta linea recta describitur.

21

Cylindrus figura est, quæ conuerso circũ quiescens alterum latus eorum quæ rectũ angulum continẽt, parallelogrammo orthogonio comprehenditur, cùm in eundẽ rursus locum restitutum fuerit illud parallelogrammũ, vnde moueri cœperat.

22

Axis autem Cylindri, est quiescens illa recta linea, circũ quam parallelogrammum vertitur.

23

Bases verò cylindri, sunt circuli à duobus ad-

aduersus lateribus quæ circumaguntur, descripti.

24

Similes coni & cylindri, sunt quorũ & axes & basiũ diametri proportionales sunt.

25

Cubus est figura solida, quæ sex quadratis æqualibus continetur.

26

Tetraedum est figura, quæ triãgulis quatuor æqualibus & æquilateris continetur.

27

Octaedrũ figura est solida, quæ octo triangulis æqualibus & æquilateris cõtinetur.

28

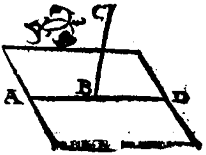
Dedecaedrum figura est solida, quæ duodecim pentagonis æqualibus, æquilateris, & æquiangulis continetur.

29

Eicosaedrum figura est solida, quę triãgulis viginti æqualibus & æquilateris continetur.

Theorema 1. Propositio 1.

Quædã rectæ lineę pars in subiecto quidem nõ est plano, quædã verò in sublimi.



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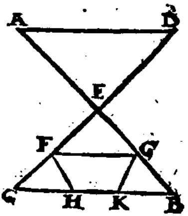
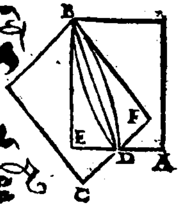
Theorema 2. Propoſitio 2.



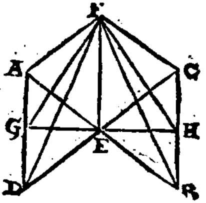
Si duę rectę lineę ſe mutuò ſecēt, in vno ſūt plano: atque triangulum omne in vno eſt plano.

Theorema 3. Propoſitio 3.



Si duo plana ſe mutuò ſecent, communis eorū ſectio eſt recta linea.

Theorema 4. Propoſitio 4.



Si recta linea rectis duabus lineis ſe mutuò ſecantibus, in cōmuni ſectione ad rectos angulos inſiſtat illa ducto etiam per ipſas plano ad angulos rectos erit.

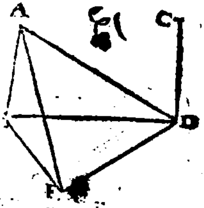
Theorema 5. Propoſitio 5.

Si recta linea rectis tribus lineis ſe mutuò tangentibus, in communi ſectione ad rectos angulos inſiſtat, illæ tres rectę in vno ſunt plano.

Theo-

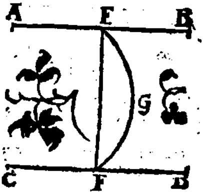
Theorema 6. Propositio 6.

Si duæ rectæ lineæ eidē plano ad rectos sint angulos, parallelæ erūt illæ rectæ lineæ.



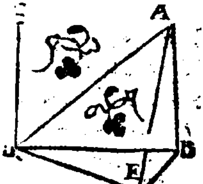
Theorema 7. Propositio 7.

Si duæ sint parallelæ rectæ lineæ, in quarū vtraque sumpta sint quę libet pūcta, illa linea quę ad hæc puncta adiungitur, in eodem est cū parallelis plano.



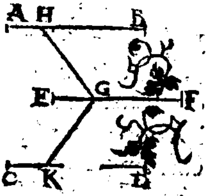
Theorema 8. Propositio 8.

Si duæ sint parallelæ rectæ lineæ, quarum altera ad rectos cuidam plano sit angulos, & reliqua eidē plano ad rectos angulos erit.



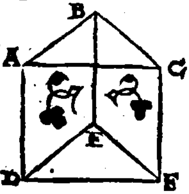
Theorema 9. Propositio 9.

Quæ eidem rectæ lineæ sunt parallelæ, sed nō in eodem cū illa plano, hæ quoque sunt inter se parallelæ.



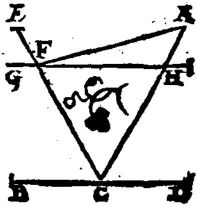
Theorema 10. Propoſitio 10.

Si duę rectę lineæ ſe mutuò tangẽtes ad duas rectas ſe mutuò tangentes ſint parallelæ, non autẽ in eodem plano, illę angulos æquales comprehendent.



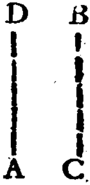
Proble. 1. Propoſitio 11.

A dato ſublimi puncto, in ſubiectum planũ perpendicularem rectã lineam ducere.



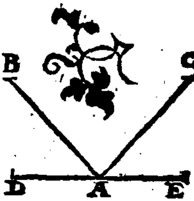
Problema 2. Propoſitio 12.

Dato plano, à puncto quod in illo datum eſt, ad rectos angulos rectam lineam excitare.



Theorema 11. Propoſitio 13.

Dato plano, à pũcto qđ in illo datum eſt, duæ rectæ lineæ ad rectos angulos non excitabuntur ad eaſdem partes.



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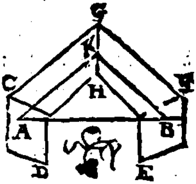
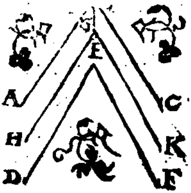
Theorema 12. Propositio 14.



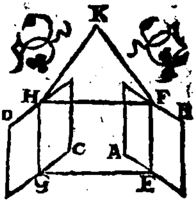
Ad quæ plana, eadem recta linea recta eſt, illa sũt parallela.

Theorema 13. Propoſitio 15.



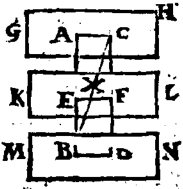
Si duæ rectæ lineæ ſe mutuò tangentes ad duas rectas ſe mutuò tangentes ſint parallelæ, non in eodem conſiſtentes plano, parallela ſunt quæ per illas ducuntur plana.

Theorema 14. Propoſitio 16.



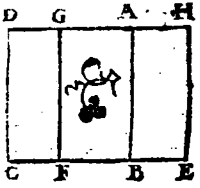
Si duo plana parallela plano quopiam ſecẽtur, communes illorum ſectiones ſunt parallelæ.

Theorema 15. Propoſitio 17.



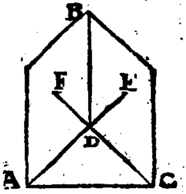
Si duæ rectæ lineę parallelis planis ſecẽtur, in eaſdem rationes ſecabũtur.

Theorema 16. Propoſitio 18.



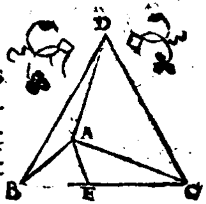
Si recta linea plano cuipiam ad rectos ſit angulos, illa etiam omnia quę per ipſam plana, ad rectos eidem plano angulos erunt.

Theorema 17. Propoſitio 19.



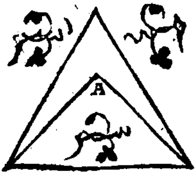
Si duo plana ſe mutuò ſecantia plano cuidã ad rectos ſint angulos, cómunis etiam illorum ſectio ad rectos eidẽ plano angulos erit.

Theorema 18. Propoſitio 20.



Si angulus ſolidus planis tribus angulis contineatur, ex his duo quilibet vtut aſſumpti tertio ſunt maiores.

Theorema 19. Propoſitio 21.

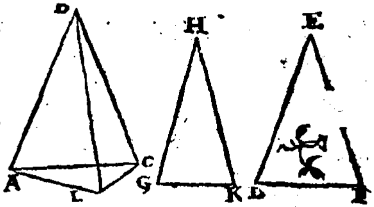


Solidus omnis angulus minoribus continetur, quàm rectis quatuor angulis planis.

Theo-

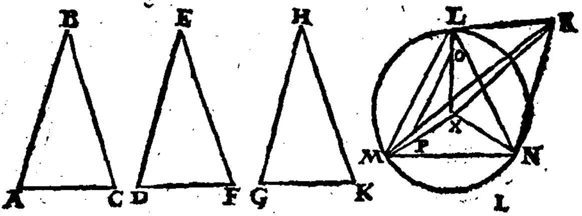
Theorema 20. Propositio 22.

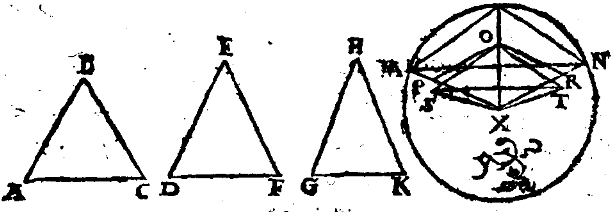
Si plani tres anguli ęqualibus rectis cótinoantur lineis, quorū duo vt libet assumpti, tertio sint maiores, triangulum constitui potest ex lineis æquales, illas rectas coniungétibus.



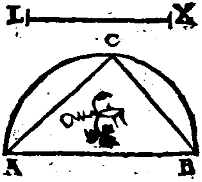
Problema 3. Propositio 23.

Ex planis tribus angulis, quorū duo vt libet assumpti tertio sint maiores, solidū angulum constituere. Decet autem illos tres angulos rectis quatuor esse minores.

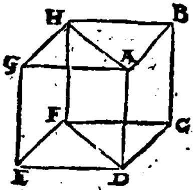




Theorema 21. Propoſitio 24.

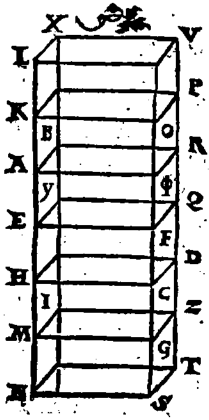


Si ſolidum parallelis planis contineatur, aduerſa illius plana & æqualia ſunt & parallelogramma.



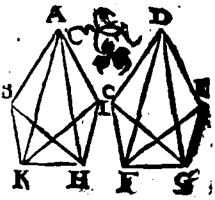
Theorema 22. Propoſitio 25.

Si ſolidum parallelis planis contentũ plano ſecetur aduerſis planis parallelo, erit quẽadmodum baſis ad baſim, ita ſolidum ad ſolidum.



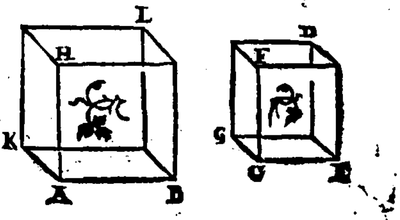
Problema 4. Propositio 26.

Ad datam rectam lineã eiusque punctũ, angulum solidum constituere solido angulo dato æqualem.



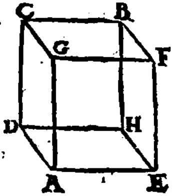
Problema 5. Propositio 27.

A data recta, dato solido parallelis planis comprehenso simile & similiter positum solidũ parallelis planis contẽtum describere.

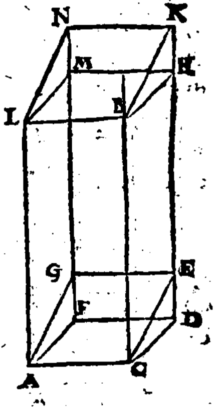


Theorema 23. Propositio 28.

Si solidũ parallelis planis comprehẽsum, ducto per aduersorum planorum diagonios plano sectũ sit, illud solidũ ab hoc plano bifariàm secabitur.

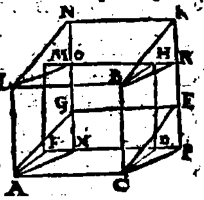


Theorema 34. Propositio 29.



Solida parallelis planis comprehẽsa, quę super eandem basim, & in eadẽ sunt altitudine, quorum insistẽtes lineæ in ijsdem collocantur rectis lineis, illa sũt inter se æqualia.

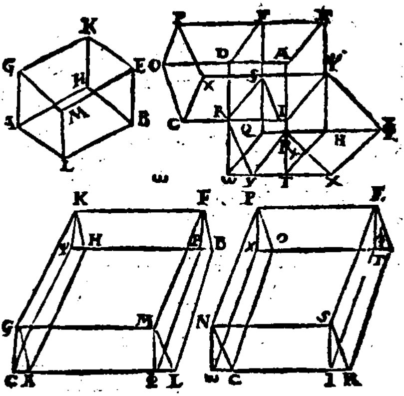
Theorema 25. Propositio 30.



Solida parallelis planis circunscripta, quæ super eandẽ basim & in eadẽ sũt altitudine, quorum insistentes lineæ nõ in ijsdem reperiuntur rectis lineis, illa sunt inter se æqualia.

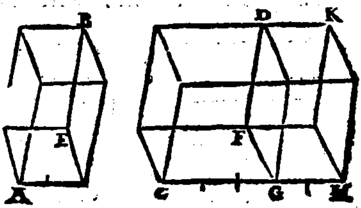
Theorema 26. Propositio 31.

Solida parallelis planis circunscripta, quę in in eadē sunt altitudine, æqualia sunt inter se.

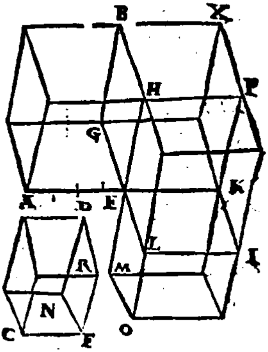


Theorema 27. Propositio 32.

Solida parallelis planis circunſcripta quæ eiuſdem ſunt altitudinis, eam habēt inter ſe rationē, quam baſes.

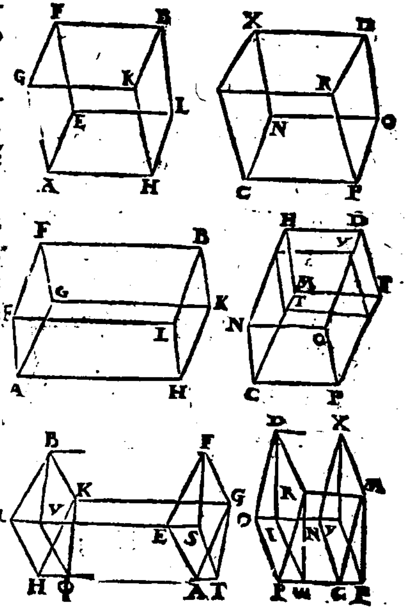


Theor.28.Propositio 33.



Similia solida parallelis planis circunscripta habẽt inter se rationẽ homologorum laterum triplicatam.

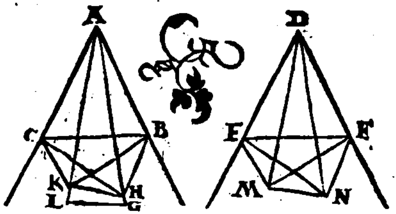
Theor.29.Propositio 34.



Aequalium solidorũ parallelis planis cõtentorum bases cũ altitudinibus reciprocãtur. Et solida parallelis planis cõtenta, quorũ bases cum altitudinibus reciprocantur, illa sunt æqualia.

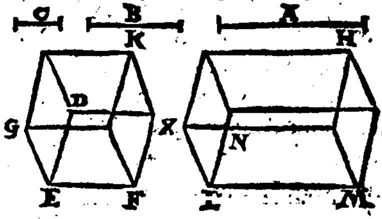
Theorema 30. Propositio 35.

Si duo plani sint anguli æquales, quorũ verticibus sublimes rectæ lineæ insistant, quæ cum lineis primò positis angulos cõtineant æquales, vtrunq; vtriq;, in sublimi bus autem lineis quælibet sumpta sint pũcta, & ab his ad plana in quibus consistunt anguli primũ positi, ductæ sint perpendiculares, ab earũ verò punctis, quæ in planis signata fuerint, ad angulos primũ positos ad



iũctæ sint rectæ lineę, hæ cũ sublimibus æquales angulos comprehendent.

Theorema 31. Propositio 36.



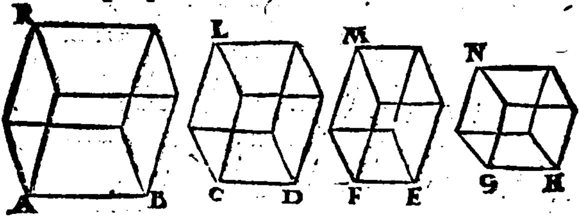
Si rectæ tres lineæ sint proporionales, quod

ex

ex his tribus fit ſolidum parallelis planis contentum, æquale eſt deſcripto à media linea ſolido parallelis planis compreheſo, quod æquilaterum quidem ſit, ſed antedicto æquiangulum.

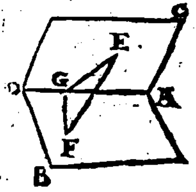
Theorema 32. Propoſitio 37.

Si rectæ quatuor lineæ ſint proportionales, illa quoque ſolida parallelis planis cõtenta, quæ ab ipſis lineis & ſimilia & ſimiliter deſcribuntur, proportionalia erũt. Et ſi ſolida parallelis planis cõprehenſa, quæ & ſimilia & ſimiliter deſcribuntur, ſint proportionalia, illæ quoque rectæ lineæ proportionales erunt.



Theorema 33. Propoſitio 38.

Si planum ad planũ rectum ſit, & à quodã puncto eorũ quæ in vno ſunt planorũ perpendicularis ad alterum ducta ſit, illa quæ ducitur perpẽdicularis, in cõmunẽ cadet planorũ ſectionẽ.



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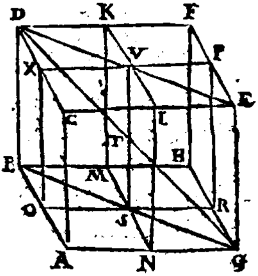
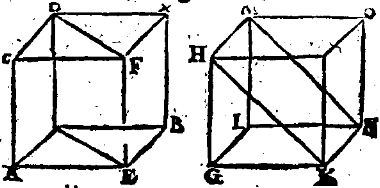
Theorema 34. Propositio 39.

Si in solido parallelis planis circūscripto, aduersorum planorum lateribus bifariā sectis, educta sint per sectiones plana, cōmunis illa planorū sectio & solidi parallelis plani circūscripti diameter, se mutuò bifariam secant.



Theorema 35. Propositio 40.

Si duo sint æqualis altitudinis prismata, quorū hoc quidē basim habeat parallelogrammū, illud verò triāgulum, sit autem parallelogrammū triāguli duplū, illa prismata erunt æqualia.

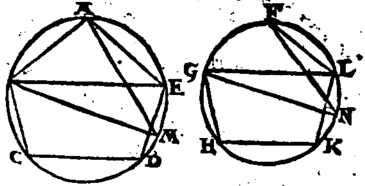


ELEMENTI XI. FINIS.

EVCLIDIS ELEMENTVM DVODECIMVM. ET SOLIDORVM ſecundum.

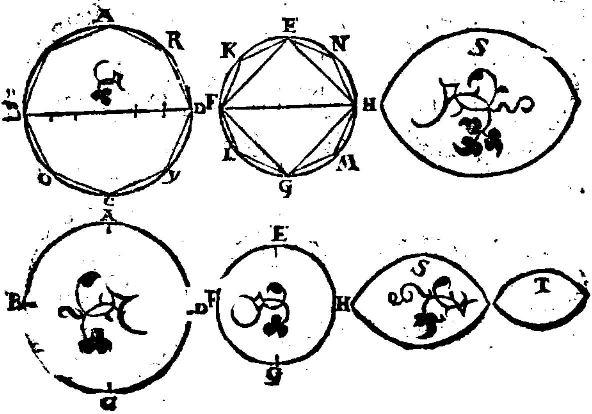
Theorema 1. Propoſitio 1.

Similia, quæ ſunt in circulis polygona, rationẽ habẽt inter ſe, quam deſcripta à diametris quadrata.



Theorema 2. Propoſitio 2.

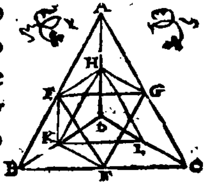
Circuli eam inter ſe rationẽ habent, quam



descripta à diametris quadrata.

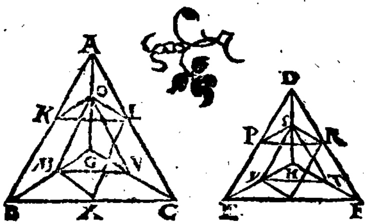
Theorema 3. Propositio 3.

Omnis pyramis trigωnã habens basim, in duas diuiditur pyramidas non tantũ æquales & similes inter se, sed toti etiã pyramidi similes, quarum trigωnæ sunt bases, atq; in duo prismata æqualia, quæ duo prismata dimidio pyramidis totius sunt maiora.



Theorema 4. Propositio 4.

Si duæ eiusdem altitudinis pyramides trigωnas habeãt bases, sit aũt illarum vtraque diuisa & in duas pyramidas inter se æquales totiq; similes, & in duo prismata æqualia, ac eodẽ modo diuidatur vtraq; pyramidũ quæ ex superiore diuisione natæ sunt, idque perpetuò fiat: quemadmodũ se habet vnius pyramidis basis ad alterius pyramidis basim, ita & omnia quæ in vna pyra-

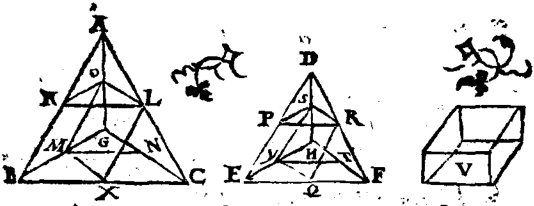


mide

mide prismata, ad omnia quæ in altera pyramide, prismata multitudine æqualia.

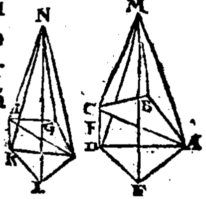
Theorema 5. Propositio 5.

Pyramides eiusdem altitudinis, quarũ trigωnæ sunt bases, eam inter se rationem habent, quàm ipsæ bases.



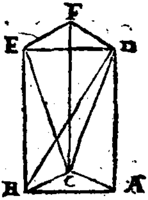
Theorema 6. Propositio 6.

Pyramides eiusdem altitudinis, quarum polygωnæ sunt bases, eam inter se rationem habent, quã ipsæ bases.



Theorema 7. Propositio 7.

Omne prisma trigωnã habẽs basim, diuiditur in tres pyramides inter se æquales, quarum trigωnæ sunt bases.



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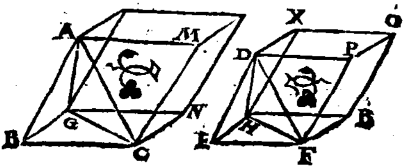
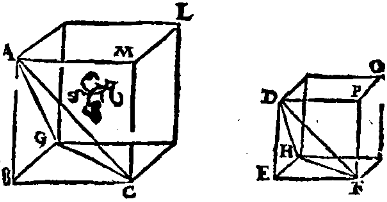
Theorema 8. Propositio 8.

Similes pyramides qui trigωnas habent bases, in triplicata sũt homologorũ laterũ ratiõe.



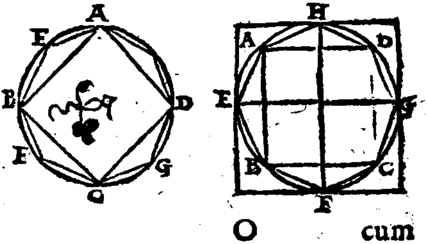
Theorema 9. Propositio 9.

Aequaliũ pyramidum & trigωnas bases habentium reciprocantur bases cum altitudinibus. Et quarũ pyramidum trigωnas bases habentium recipro cãtur bases cũ altitudinibus, illæ sunt æquales.



Theorema 10. Propositio 10.

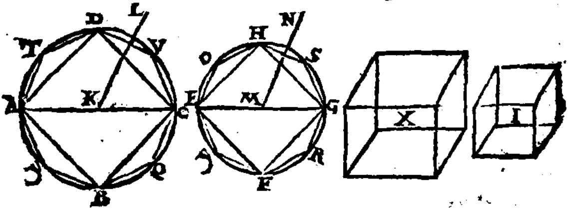
Omnis cωnꝰ tertia pars est cylindri eandẽ



cum ipſo cωno baſim habentis,& altitudinem æqualem.

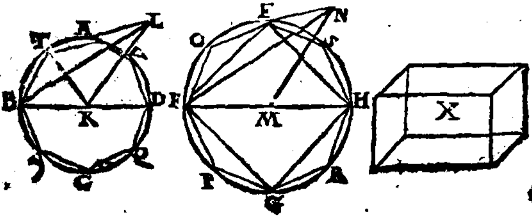
Theorema 11. Propoſitio 11.

Cωni & cylindri eiuſdem altitudinis, eam inter ſe rationem habent, quam baſes.



Theorema 12. Propoſitio 12.

Similes cωni & cylindri, triplicatam habẽt inter ſe rationem diametrorum, quæ ſunt in baſibus.



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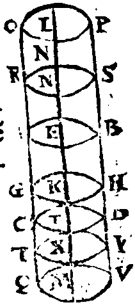
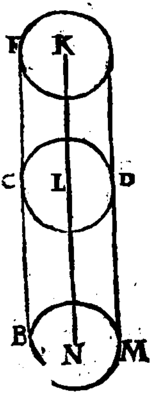
Theorema 13. Propositio 13.

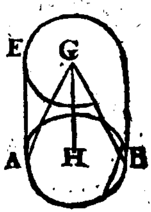


Si cylindrus plano sectus sit ad uersis planis parallelo, erit quē admodum cylindrus ad cylindrum, ita axis ad axem.

Theorema 14. Propositio 14.

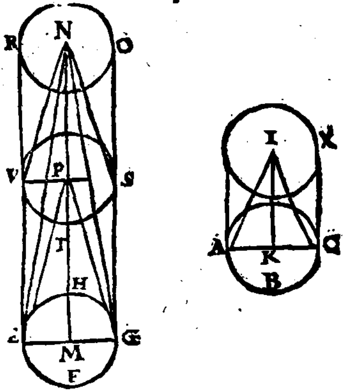
Coni & cylindri qui in æqualibus sunt basibus, eam habent inter se rationem, quam altitudines





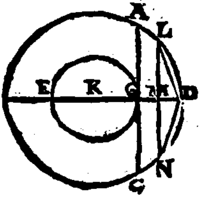
Theorema 15. Propoſitio 15.

Aequalium conorum & cylindrorũ baſes cũ altitudinib⁹ reciprocãtur. Et quorum conorũ & cylindrorũ baſes cum altitudinib⁹ reciprocãtur, illi ſunt æquales.



Problema 1. Propoſitio 16.

Duobus circulis circũ idem centrum conſiſtentibus, in maiore circulo polygonum æqualium pariumque laterum inſcribere, quod minorem circulum nõ tangat.



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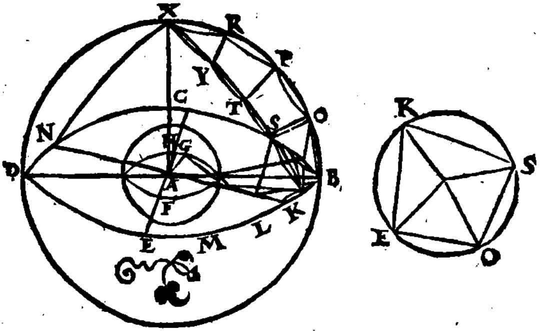
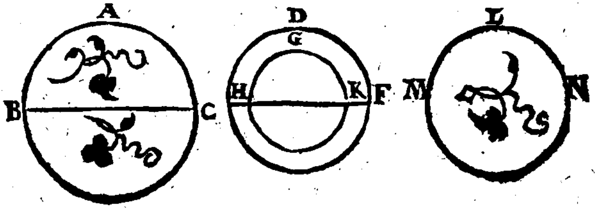
Problema 2. Propositio 17.

Duabus sphæris circum idem centrũ consistentibus, in maiore sphæra solidũ polyedrum inscribere, quod minoris sphæræ superficiem non tangat.



Theorema 16. Propositio 18.

Sphæræ inter se rationem habent suarum diametrorum triplicatam.

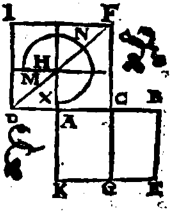


ELEMENTI XII. FINIS.

EVCLIDIS ELEMENTVM DEDECIMVMTERTIVM, ET SOLIDORVM TERTIVM.

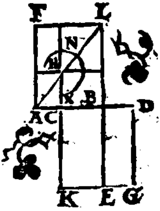
Theorema 1. Propositio 1.

Si recta linea per extremã & mediã rationẽ secta sit, maius segmentum quod totius lineæ dimidium assumpserit, quintuplum potest eius quadrati, quod à totius dimidia describitur.



Theorema 2. Propositio 2.

Si recta linea sui ipsius segmenti quintuplũ possit, & dupla segmenti huius linea per extremã & mediam rationẽ secetur maius segmentũ reliqua pars est lineæ primum positæ.



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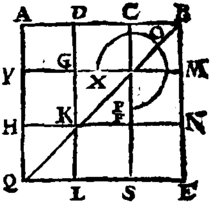
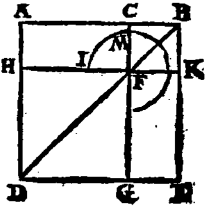
Theorema 3. Propoſitio 3.



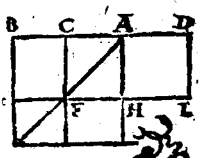
Si recta linea per extremā & mediam rationē ſecta ſit, minus ſegmētum quod maioris ſegmenti dimidium aſſumpſerit, quintuplum poteſt eius, quòd à maioris ſegmēti dimidio deſcribitur, quadrati.

Theorema 4. Propoſitio 4.



Si recta linea per extremam & mediam rationē ſecta ſit, quod à tota, quodque à minore ſegmēto ſimul vtraq; quadrata, tripla ſunt eius, quod à maiore ſegmēto deſcribitur, quadrati.

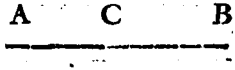
Theorema 5. Propoſitio 5.



Si ad rectam lineam, quæ per extremam & mediam rationem ſecetur, adiuncta ſit altera ſegmento maiori æqualis, tota hæc linea recta per extremam & mediam rationē ſecta est, estque maius segmentum linea primùm posita.

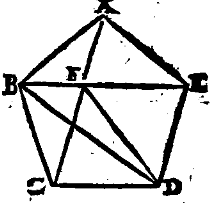
Theorema 6. Propositio 6.

Si recta linea ῥητὴ siue rationalis, per extremam & mediam rationem secta sit, vtrunque segmentorum ἄλογος siue irrationalis est linea, quæ dicitur Residuum.



Theorema 7. Propositio 7.

Si pentagωni æquilateri tres sint æquales anguli, siue q; deinceps, siue qui non deinceps sequuntur, illud pétagωnum erit ęquiangulum.

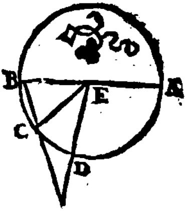


Theorema 8. Propositio 8.

Si pétagωni æquilateri & æquianguli duos qui deinceps sequuntur angulos rectę subtendãt lineæ, illæ per extremã & mediam rationem se mutuò secant, earumq; maiora segmenta, ipsius pentagωni lateri sunt æqualia.

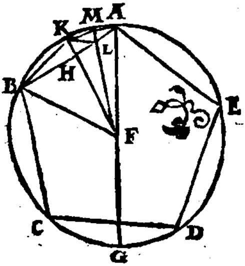
Theo-

Theorema 9. Propositio 9.



Si latus hexagωni & latus decagωni eidem circulo inſcriptorum compoſita ſint, tota recta linea per extremam & mediam rationem ſecta eſt, eiuſque ſegmẽtum maius, eſt hexagωni latus.

Theorema 10. Propoſitio 10.



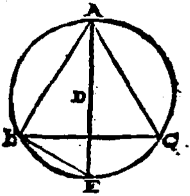
Si circulo pẽtagωnum æquilaterũ inſcriptum ſit, pẽtagωni lat⁹ poteſt & latus hexagωni & latus decagωni eidem circulo inſcriptorum.

Theorema 11. Propoſitio 11.

Si in circulo ῥητὴν habente diametrum, inſcriptũ ſit pentagωnum æquilaterũ, pentagωni latus irrationalis eſt linea, quæ vocatur Minor.

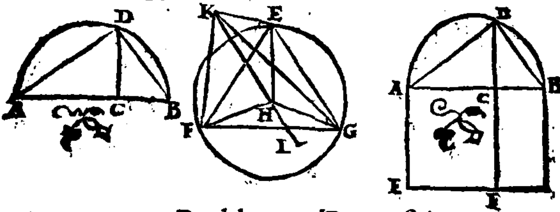
Theorema 12. Propoſitio 12.

Si in circulo inſcriptum ſit triangulum æquilaterũm, huius trianguli latus potentia triplum eſt eius lineæ, quæ ex circuli centro ducitur.



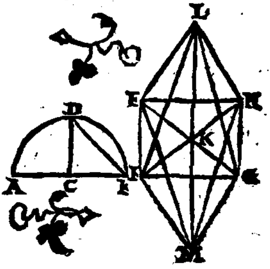
Problema 1. Propoſitio 13.

Pyramidem conſtituere,& data ſphærę cō plecti, atque docere illius ſphæræ diametrum potentia ſeſquialteram eſſę lateris ipſius pyramidis.



Problema 2. Propoſitio 14.

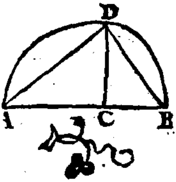
Octaedrum conſtituere, eaq; ſphęra qua pyramidē complecti, atque ꝓbare illius ſphæræ diametrũ potentia duplā eſſe lateris ipſi⁹ octaedri.

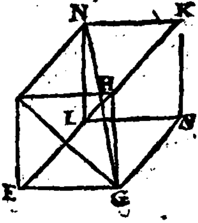


Pro-

Problema 3. Propositio 15.

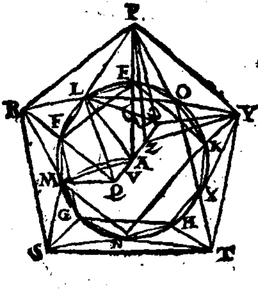
Cubum constituere, eaque sphæra qua & superiores figuras complecti, atque docere illius sphæræ diametrum potẽtia triplam esse lateris ipsius cubi.

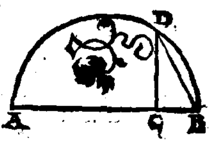




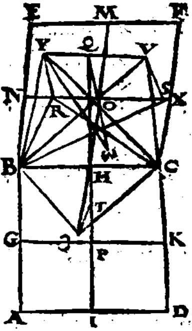
Problema 4. Propositio 16.

Icosaedrum constituere, eademque sphæra qua & antedictas figuras complecti, atque probare, Icosaedri latus irrationalem esse lineam, quæ vocatur Minor.





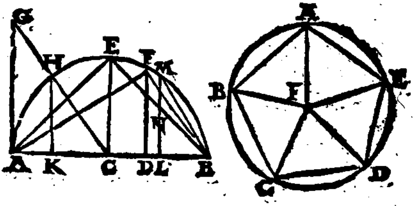
Problema 5. Propositio 17.



Dodecaedrū cōstituere, eademque sphæra qua & antedictas figuras cōplecti, atque probare dodecaedri latus irrationalem esse lineam, quæ vocatur Residuum.

Theorema 6. Propositio 18.

quinq; figurarū latera proponere, & inter se comparare.



SCHOLIVM.

Aio verò, præter dictas quinque figuras non posse aliam constitui figuram solidam, quæ planis & æquilateris & æquiangulis contineatur, inter se æqualibus. Non enim ex duobus triāgulis, sed neq; ex alijs duabus figuris solidus cōstituetur angulus

Sed

Sed ex tribus triangulis, constat Pyramidis angulus.

Ex quatuor autem, Octaëdri.

Ex quinque verò, Icosaëdri.

Nam ex triangulis sex & æquilateris & æquiangulis ad idem punctum coeuntibus, non fiet angulus solidus. Cùm enim trianguli æquilateri angulus, recti vnius bessem contineat, erunt eiusmodi sex anguli rectis quatuor æquales Quod fieri non potest. Nam solidus omnis angulus minoribus quã rectis quatuor angulis continetur, per 21.11.

Ob easdem sanè causas, neque ex pluribus quã planis sex eiusmodi angulis solidus constat.

Sed ex tribus quadratis, Cubi angulus continetur.

Ex quinque, nullus potest: Rursus enim recti quatuor erunt.

Ex tribus autem pentagonis æquilateris & æquiangulis, Dodecaëdri angulus continetur.

Sed ex quatuor, nullus potest. Cùm enim pentagoni æquilateri angulus rectus sit, et quinta recti pars erunt quatuor anguli rectis quatuor maiores. Quod fieri nequit. Nec sanè ex alijs polygonis figuris solidus angulus continebitur, quòd hinc quoque absurdum sequatur. Quamobrem perspicuum est, præter dictas quinque figuras aliam figuram solidam non posse constitui, quæ ex planis æquilateris & æquiangulis contineatur.

ELEMENTI XIII. FINIS.

EVCLIDIS ELEMENTVM DECIMVMQVARTVM, VT quidam arbitrantur, vt alij verò, Hypsiclis Alexandrini, de quinque corporibus.

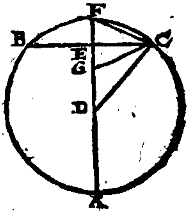
LIBER PRIMVS.

Basilidès Tyrius, Protarche, Alexandriam profectus, patrique nostro ob disciplinæ societatem commendatus, longissimo peregrinationis tempore cum eo versatus est. Cúmque disserent aliquando de scripta ab Apollonio cōparatione Dodecaëdri & Icosaëdri eidem sphæræ inscriptorum, quam hæc inter se habeant rationem, censuerunt ea non rectè tradidisse Apollonium: quæ à se emendata, vt de patre audire erat, literis prodiderunt. Ego autem postea incidi in alterum librū ab Apollonio editum, qui demonstrationem accuratè complecteretur de re proposita, ex eiusq; problematis indagatione magnam equidem cepi voluptatem. Illud certè ab omnibus perspici potest, quòd scripsit Apollonius, cùm sit in omnium manibus. Quod autē diligenti, quantum conijcere licet, studio nos postea scrip-

scripsisse videmur, id monimentis consignatum tibi nuncupandum duximus, vt qui feliciter cùm in omnibus disciplinis tum vel maximè in Geometria versatus, scitè ac prudēter iudices ea quæ dicturi sumus ob eam verò, quæ tibi cum patre fuit, vitæ consuetudinem, quaq; nos complecteris, beneuolentiā, tractationem ipsam libenter audias. Sed iam tempus est, vt præmio modum facientes, hanc syntaxim aggrediamur.

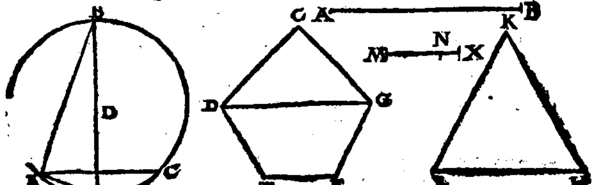
Theorema 1. Propositio 1.

Perpendicularis linea, quæ ex circuli cuiuspiam centro in latus pētagωni ipsi circulo inscripti ducitur, dimidia est vtriusq; simul lineæ, & eius quæ ex centro, & lateris decagωni in eodē circulo inscripti.



Theorema 2. Propositio 2.

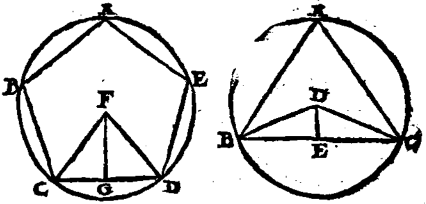
Idem circulus cōprehendit & dωdecaedri pentagωnum & icosaedri triangulū, eidem sphæræ inscriptorum.



Theo

Theorema 3. Propositio 3.

Si pentagωno & æquilatero & æquiangulo circunſcriptus ſit circulus, ex cuius centro in vnum pentagωni latus ducta ſit perpendicularis: quod vno laterum & perpẽdicu lari trigeſies continetur, illud

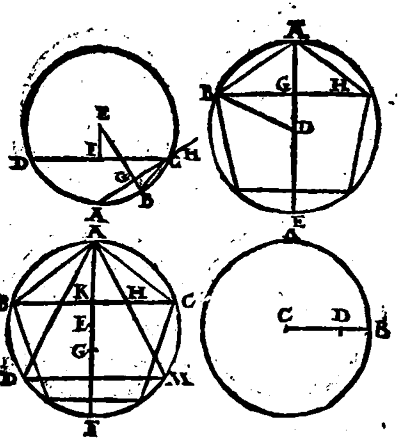


æquale eſt dωdecaedri ſuperficiei:

Theorema 4. Propoſitio 4.

Hoc perſpicuum cùm ſit, probandum eſt, quemadmodum ſe habet dωdecaedri ſuperficies ad icoſaedri ſuperficiem, ita ſe habere cubi latus ad icoſaedri latus.

Cubi



Cubi latus.

E————————

Dodecaedri.

F————————

Icoſaedri.

G————————

SCHOLIVM.

Nunc autem probandum eſt, quemadmodum ſe habet cubi latus ad icoſaedri latus, ita ſe habere ſolidum dodecaedri ad icoſaedri ſolidum. Cùm enim æquales circuli comprehendant & dodecaedri pẽtagonum & Icoſaedri triangulum, eidem ſphæra *scriptorum: in sphæris autem æquales circuli æquali interuallo distent à centro (siquidem perpendiculares à sphæra centro ad circulorum plana ductæ & æquales sunt, & ad circulorum centra cadunt) idcirco lineæ, hoc est perpendiculares, quæ à sphæra cētro ducuntur ad centrum circuli comprehendētis & triangulum Icosaëdri, & pentagonum dodecaëdri, sunt æquales. Sūt igitur æqualis altitudinis Pyramides, quæ bases habent ipsa dodecaëdri pentagonā, et quæ Icosaëdri triangula. At æqualis altitudinis pyramides rationem inter se habent eam quam bases, ex 5. & 6. 11. Quemadmodum igitur pentagonū ad triangulum, ita pyramis, cuius basis quidem est dodecaëdri pentagonum, vertex autem sphæræ centrum ad pyramida, cuius basis quidem est Icosaëdri triangulum, vertex autem sphæræ centrum. Quamobrem vt se habent duodecim pentagona ad viginti triangula, ita duodecim pyramides, quorum pētagona sint bases, ad viginti pyramidas, quæ trigonas habeant bases. At pentagona duodecim sunt dodecaëdri superficies, viginti autem triangula, Icosaëdri. Est igitur vt dodecaëdri superficies ad Icosaëdri superficiem, ita duodecim pyramides, quæ pentagonas habeant bases, ad viginti pyramidas, quarum trigonæ sunt bases. Sunt autem duodecim quidem pyramides, quæ pentagonas habeant bases, solidū dodecaëdri: viginti autem pyramides, quæ trigonas habeant bases, Icosaëdri solidum. Quare ex 11. 5. vt dodecaëdri superficies ad Icosaëdri superficiem, ita solidum dodecaëdri ad Icosaëdri solidum. Vt autem*

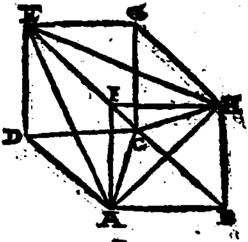
tem dodecaëdri ſuperficies ad Icoſaëdri ſuperficiem, ita probatum eſt cubi latus ad Icoſaëdri latus. Quemadmodum igitur cubi latus ad Icoſaedri latus, ita ſe habet ſolidum dodecaëdri ad Icoſaëdri ſolidum.

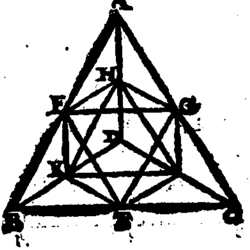
EVCLIDIS ELEMENTVM DECIMVMQVINTVM, ET Solidorum quintum, vt nonnulli putant, vt autem alij, Hypsiclis Alexandrini, de quinque corporibus,

LIBER II.

Problema 1. Propositio 1.

In dato cubo pyramida inscribere.





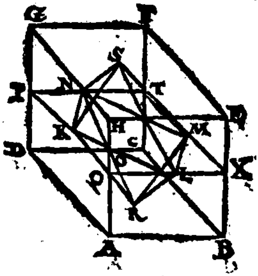
Problema 2. Propositio 2.

In data pyramide octaedrum inscribere.

Pro-

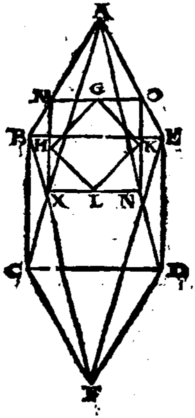
Problema 3. Propositio 3.

In dato cubo octaedrum inscribere.



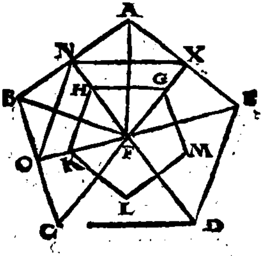
Problema 4. Propositio 4.

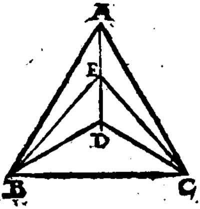
In dato octaedro cubũ inscribere.

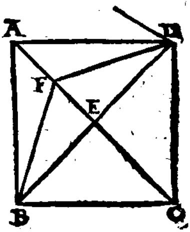


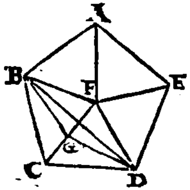
Problema 5. Propositio 5.

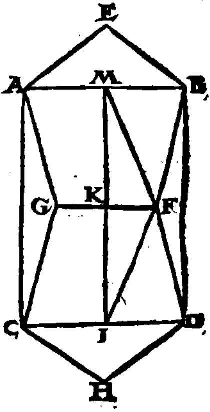
In dato Icosaedro dodecaedrum inscribere.

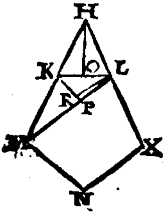












SCHO-

SCHOLIVM.

Meminiſſe decet, ſi quis nos roget, quot Icoſaëdrũ habeat latera, ita reſpondendum eſſe: Patet Icoſaëdrum viginti contineri triangulis, quodlibet verò triangulum rectis tribus conſtare lineis. Quare multiplicanda ſunt nobis viginti triangula in trianguli vnius latera, ſiuntq́; ſexaginta, quorum dimidium eſt triginta. Ad eundem modum & in dodecaëdro. Cũ enim rurſus duodecim pentagona dodecaëdrum cõprehendant, itemq; pentagonum quoduis rectis quinque conſtet lineis, quinq; duodecies multiplicamus, fiunt ſexaginta, quorum rurſus dimidium eſt triginta. Sed cur dimidiũ capimus? Quoniam vnumquodq; latus ſiue ſit trianguli, ſiue pentagoni, ſiue quadrati, vt in cubo, iteratò ſumitur. Similiter autem eadem via & in cubo & in pyramide & in octaëdro latera inuenies. Quòd ſi item velis ſingularum quoq; figurarum angelos reperire, facta eadem multiplicatione numerum procreatum partire in numerum planorũ, quæ vnum ſolidum angulum includunt: vt quoniã triangula quinq; vnum Icoſaëdri angulum continet; partire 60. in quinque, naſcũtur duodecim anguli Icoſaëdri. In dodecaëdro autem tria pentagona angulum comprehendunt. partire ergo 60. in tria, & habebis dodecaëdri angulos viginti. Atq; ſimili ratione in reliquis figuris angulos reperies.

Finis Elementorum Euclidis.