g. [redacted].

EVCLIDIS ELEMENTORVM LIBRI XV.

Quibus, cùm ad omnem Mathematicæ ſcientiæ partem, tùm ad quamlibet Geometriæ tractationem, facilis comparatur aditus.

[redacted]

COLONIÆ.

Apud Maternum Cholinum.

M. D. LXXX.

Cum Gratia & Priuilegio Caeſ. Maieſt.

[illegible]

AD CANDIDVM LECTOREM ST. GRACILIS Præfatio.

PERMAGNI *referre ſemper exiſtimaui, lector beneuole, quantum quiſq; ſtudij & diligentiæ ad percipienda ſcientiarum elementa adhibeat, quibus non ſatis cognitis, aut perperam intellectis, ſi vel digitum progredi tentes, erroris caliginem animis offundas, non veritatis lucem rebus obſcuris adferas. Sed principiorum quanta ſint in diſciplinis momenta, haud facilè credat, qui rerum naturam ipſa ſpecie, non viribus metiatur. Vt enim corporum quæ oriuntur & intereunt, viliſſima tenuiſſimaq; videntur initia: ita rerum æternarum & admirabilium, quibus nobiliſſimæ artes continentur, elementa ad ſpeciem ſunt exilia, ad vires & facultatem quàm maxima. Quis non videt ex fici tantulo grano, vt ait Tullius, aut ex acino vinaceo, aut ex cæterarum frugum aut ſtirpium minutiſſimis ſeminibus tantos truncos ramosq; procreari? Nam Mathematicorum initia illa quidem dictu audituq; perexigua, quantam theorematum ſyluam nobis pe-*

pererunt? Ex quo intelligi potest, vt in ipsis seminibus, sic & in artium principijs inesse vim earũ rerum, quæ ex his progignuntur. Præclarè igitur Aristoteles, vt alia permulta, μέγιστον ἴσως ἀρχὴ παντός, καὶ ὅσῳ κράτιστον τῇ δυνάμει, τοσούτῳ μικρότατον ὂν τῷ μεγέθει χαλεπόν ἐστιν ὀφθῆναι. *Quocirca committendum non est, vt non bene promissa & diligenter explorata scientiarum principia, quibus propositarum quarumque rerum veritas sit demonstranda, vel constituas, vel constituta approbes: Cauendum etiam, vt ne tantulum quidem fallaci & captiosa interpretatione turpiter deceptus, à vera principiorum ratione temerè deflectas. Nam qui initio fortè aberrauerit, is vt tandem in maximis verseтur erroribus, necesse est: cum ex vno erroris capite, densiores sensim tenebræ rebus clarissimis obducantur. Quid tam varias veterum physiologorum sententias, non modò cum rerum veritate pugnantes, sed vehemẽter etiam inter se dissentientes nobis inuexit? Equidem haud scio, fueritne vlla potior tanti dissidij causa, quàm quòd ex principijs partim falsis partim non consentaneis ductas rationes probando adhiberent. Fit enim plerunque, vt qui non rectè de artium rerumque elementis sentiunt, ad præfinitas quasdam opiniones suas omnia reuocare studeant. Pythagorei, vt meminit Aristoteles, cum denarij numeri summam perfectionem cœlo tribuerent, nec plures tamen quàm nouem sphæras cernerent, decimam affingere ausi sunt terræ aduersam*

uersam, quam ἀντίχθονα appellarunt. Illi enim uniuersitatis rerumq; singularum naturam ex numeris ceu principijs æstimantes, ea protulerunt, quæ φαινομένοις congruere nusquam sunt cognita. Nam ridicula Democriti, Anaximenis, Melissi, Anaxagoræ, Anaximandri, & reliquorum id genus physiologorum somnia, ex falsis illa quidem orta naturæ principijs, sed ad Mathematicum nihil aut parum spectantia, sciens prætereo. Nonnullos attingam, qui repetitis altius, vel aliter ac decuit positis rerum initijs, cum in physicis multa turbarunt, tum Mathematicos oppugnatione principiorum pessimè mulctarunt. Ex planis figuris corpora constituit Timæus: Geometrarum hîc quidem principia cuniculis oppugnantur. Nam & superficies seu extremitates crassitudinem habebunt, & lineæ latitudinem: denique puncta non erunt indiuidua, sed linearum partes. Prædicant Democritus atque Leucippus illas atomos suas, & indiuidua corpuscula. Concedit Xenocrates impartibiles quasdam magnitudines. Hic verò Geometriæ fundamenta apertè petuntur, & funditus euertuntur: quibus dirutis nihil equidem aliud video restare, quàm vt amplissima Mathematicorum theatra repentè concidant. Iacebunt ergo, si dijs placet, tot præclara Geometrarum de asymmetris & alogis magnitudinibus theoremata. Quid enim causæ dicas cur indiuidua linea hanc quidem metiatur, illam verò metiri non queat? Siquidem quod minimum in vnoquoque *genere reperitur, id communis omnium mensura esse solet. Innumerabilia profectò sunt illa, quæ ex falsis eiusmodi decretis absurda consequuntur: & horum permulta quidem Mathematicus, sed longè plura colligit Phisicus. Quid varia ψευδογραφημάτων genera commemorem, quæ ex hoc vno fonte tam longè lateq́; diffusa fluxisse videntur? Notissimus est Antiphontis tetragωnismus, qui Geometrarum & ipse principia non parum labefecit, cùm rectæ lineæ curuam posuit æqualem. Longum esset mihi singula percensere, præsertim ad alia properanti: Hoc ergo certum fixum & in perpetuum ratum esse oportet, quod sapienter monet Aristoteles, σπουδαστέον ὅπως ὁρισθῶσι καλῶς αἱ ἀρχαί· μεγάλην γὰρ ἔχουσι ῥοπὴν πρὸς ἑπόμενα. Nam principijs illa congruere debent, quæ sequuntur. Quòd si tantum perspicitur in istis exilioribus Geometriæ initijs, quæ puncto, linea, superficie definiuntur, momentum, vt ne hæc quidem sine summo impendentis ruinæ periculo conuelli aut oppugnari possint; quanta quæso vis putanda est huius στοιχειώσεως quam collatis tot præstantissimorum artificum inuentis, mira quadam ordinis solertia contexuit Euclides, vniuersæ Matheseωs elementa complexu suo coërcentem? Vt igitur omnibus rebus instructior & paratior quisque ad hoc studium libentius accedat, & singula vel minutissima exactius secum reputet atq́; perdiscat, operæprecium censui in primo institutionis aditu vestibuloq́; præcipua quædam capita,*

quibus

quibus tota ferè Mathematicæ scientiæ ratio intelligatur, breuiter explicare: tum ea quæ sunt Geometriæ propria, diligenter persequi: Euclidis denique in extruenda hac στοιχειώσεως consilium sedulò ac fideliter exponere. Quæ ferè omnia ex Aristotelis potissimum ducta fontibus, nemini inuisa fore confido, qui modò ingenuum animi candorem ad legendum attulerit. Ac de Mathematicæ diuisione primùm dicamus.

Mathematicæ in primis scientiæ studiosos fuisse Pythagoreos, non modò historicorum, sed etiam philosophorum libri declarant. His ergo placuit, vt in partes quatuor vniuersum distribuatur Mathematicæ scientiæ genus, quarum duas περὶ τὸ ποσὸν, reliquas περὶ τὸ πηλίκον versari statuerunt. Nam & τὸ ποσὸν vel sine vlla comparatione ipsum per se cognosci, vel certa quadam ratione comparatum spectari: in illo Arithmeticam, in hoc versari Musicam: & τὸ πηλίκον partim quiescere, partim moueri quidem: illud Geometriæ propositum esse: quod verò sua sponte motu cietur, Astronomiæ. Sed ne quis falsò putet Mathematicam scientiam, quòd in vtroque quanti genere cernitur, idcirco inanem videri (si quidem non solum magnitudinis diuisio, sed etiam multitudinis accretio infinitè progredi potest) meminisse decet, τὸ πηλίκον καὶ τὸ ποσὸν, quæ subiecto Mathematicæ generi imposita sunt à Pythagoreis nomina, non cuiuscunque modi quantitatem significare, sed eam demum, quæ tum multi-

tudine tum magnitudine ſit definita, & ſuis circũſcripta terminis. Quis enim vllam infiniti ſcientiam defendat? Hoc ſcitum eſt, quod non ſemel docet Ariſtoteles, infinitum ne cogitatione quidem complecti quenquam poſſe. Itaque ex infinita multitudinis & magnitudinis δυνάμει, finitam hæc ſcientia decerpit & amplectitur naturam, quam tractet, & in qua verſetur. Nam de vulgari Geometrarum conſuetudine quid ſentiendum ſit, cũ data interdum magnitudine infinita aut fabricãtur aliquid, aut proprias generis ſubiecti affectiones exquirunt, diſertè monet Ariſtoteles, ὐυδὲ νῦν (de Mathematicis loquens) δέονται τοῦ ἀπείρου, ὀυδὲ χρῶνται, ἀλλὰ μόνον ἔιναι ὅσην ἂν βούλωνται, πεπερασμένην. Quamobrem diſputatio ea qua infinitum refellitur, Mathematicorum decretis rationibusq; non aduerſatur, nec eorum apodixes labefacit. Etenim tali infinito opus illis nequaquã eſt, quod exitu nullo peragrari poſſit, nec talem ponunt infinitam magnitudinem: ſed quantamcunque velit aliquis effingere, ea vt ſuppetat, infinitam præcipiunt. Quinetiam nõ modò immenſa magnitudine opus non habent Mathematici, ſed ne maxima quidem: cum inſtar maximæ minima quæque in partes totidem pari ratione diuidi queat. Alteram Mathematicæ diuiſionem attulit Geminus, vir (quantum ex Proclo conijcere licet) μαθημάτων laude clariſſimus. Eam, quæ ſuperiore plenior & accuratior fortè viſa eſt, cùm doctiſſimè pertractarit ſua in decimum Euclidis

præfatio-

præfatione P. Montaureus vir senatorius, & regiæ bibliothecæ præfectus, leuiter attingam. Nam ex duobus rerum velut summis generibus, τῶν νοητῶν καὶ τῶν αἰσθητῶν, quæ res sub intelligentiam cadunt, Arithmeticæ & Geometriæ attribuit Geminus: quæ verò in sensus incurrunt, Astrologiæ, Musicæ, Supputatrici, Opticæ, Geodæsiæ & Mechanicæ adiudicauit. Ad hanc certè diuisionem spectasse videtur Aristoteles, cùm Astrologiam, Opticam, harmonicam φυσικωτέρας τῶν μαθημάτων nominat, vt quæ naturalibus & Mathematicis interiectæ sint, ac velut ex vtrisque mixtæ disciplinæ: Siquidem genera subiecta à Phisicis mutuantur, causas verò in demonstrationibus ex superiore aliqua scientia repetunt. Id quod Aristoteles ipse apertissimè testatur, ἐνταῦθα γὰρ, φησὶ, τὸ μὲν ὅτι, τῶν αἰσθητικῶν εἰδέναι, τὸ δὲ διότι, τῶν μαθηματικῶν. Sequitur, vt quid Mathematicæ conueniat cum Phisica & prima Philosophia: quid ipsa ab vtraque differat, paucis ostendamus. Illud quidem omnium commune est, quòd in veri contemplatione sunt positæ, ob idque θεωρητικαὶ à Græcis dicuntur. Nam cùm διάνοια siue ratio & mens omnis sit vel πρακτικὴ, vel θεωρητικὴ, totidem scientiarum sint genera necesse est. Quòd si Physica, Mathematica, & prima Philosophia, nec in agendo, nec in efficiendo sunt occupatæ, hoc certè perspicuum est, eas omnes in cognitione contemplationeque necessariò versari. Cùm enim rerum non modò agendarum, sed etiam efficienda-

ciendarum principia in agente vel efficiente consistant, illarum quidem προαίρεσις, harum autem vel mens, vel ars, vel vis quædam & facultas: rerum profectò naturalium, Mathematicarum, atque diuinarum principia in rebus ipsis, non in philosophis inclusa latent. Atque hæc vna in omnes valet ratio, quæ θεωρητικὰς esse colligat. Iam verò Mathematica separatim cum Physica congruit, quòd vtraque versatur in cognitione formarum corpori naturali inhærentium. Nam Mathematicus plana, solida, longitudines & puncta contemplatur, quæ omnia in corpore naturali à naturali quoque philosopho tractantur. Mathematica item & prima philosophia hoc inter se propriè cõueniunt, quòd cognitionem vtraque persequitur formarum, quoad immobiles, & à concretione materiæ sunt liberæ. Nam tametsi Mathematicæ formæ re vera per se non cohærent, cogitatione tamen à materia & motu separantur, οὐδὲ γίνεται ψεῦδος χωριζόντων, vt ait Aristoteles. De cognatione & societate breuiter diximus. iam quid intersit, videamus. Vnaquæque mathematicarum certum quoddam rerum genus propositum habet, in quo versetur, vt Geometria quantitatem & continuationem aliorum in vnam partem, aliorum in duas, quorundam in tres. eorúmque quatenus quanta sunt & continua, affectiones cognoscit. Prima autem philosophia, cùm sit omnium communis, vniuersum Entis genus, quæq; ei accidunt & conueniunt hoc ipso quòd est, considerat.

Adhæc, Mathematica eam modò naturam amplectitur. quæ quanquam non mouetur, separari tamen seiungiq; nisi mente & cogitatione à materia non potest, ob eamq; causam ἐξ ἀφαιρέσεως dici consueuit. Sed Prima philosophia in iis versatur, quæ & seiuncta, & æterna, & ab omni motu per se soluta sunt ac libera. Cæterùm Physica & Mathematica quanquam subiecto discrepare non videntur, modo tamen rationeq; differunt cognitionis & contemplationis, vnde dissimilitudo quoque scientiarum sequitur. Etenim Mathematicæ species nihil re vera sunt aliud, quàm corporis naturalis extremitates, quas cogitatione ab omni motu & materia separatas Mathematicus contemplatur: sed easdem consectatur Physicorum ars, quatenus cum materia comprehensæ sunt, & corpora motui obnoxia circunscribunt. Ex quo fit, vt quæcunque in Mathematicis incommoditates accidunt, eadem etiam in naturalibus rebus videantur accidere, non autem vicissim. Multa enim in naturalibus sequuntur incommoda, quæ nihil ad Mathematicum attinent, διὰ τὸ inquit Aristoteles, τὰ μὲν ἐξ ἀφαιρέσεως λέγεται, τὰ μαθηματικὰ, τὰ ϗ φυσικὰ ἐκ προσθέσεως. Siquidem res cum materia deuinctas contemplatur physicus: Mathematicus verò rem cognoscit circunscriptis iis omnibus quæ sensu percipiuntur, vt grauitate, leuitate, duritie, mollitie, & præterea calore, frigore, aliisque contrariorum paribus quæ sub sensum subiecta sunt: tantùm autem relinquit

quanti-

quantitatem & continuum. Itaque Mathematicorum ars in iis quæ immobilia sunt, cernitur (τὰ γὰρ μαθηματικὰ τῶν ὄντων ἄνευ κινήσεώς ἐστιν, ἔξω τῶν περὶ τὴν ἀστρολογίαν) quæ verò in naturæ obscuritate posita est, res quidem quæ nec separari nec motu vacare possunt, contemplatur. Id quod in vtroque scientiæ genere perspicuum esse potest, siue res subiectas definias, siue proprietates earum demonstres. Etenim numerus, linea, figura, rectum, inflexum, æquale, rotundum, vniuersa denique Mathematicus quæ tractat & profitetur, absque motu explicari docerique possunt: χωριστὰ γὰρ τῇ νοήσει κινήσεώς ἐστι: Phisicæ autem sine motione species nequaquam possunt intelligi. Quis enim hominis, plantæ, ignis, ossium, carnis naturã & proprietates sine motu, qui materiam sequitur, perspiciat? Siquidem tantisper substantia quæque naturalis constare dici solet, quoad opus & munus suum, agendo patiendoque, tueri ac sustinere valeat: qua certè amissa δυνάμει, ne nomen quidẽ nisi ὁμωνύμως retinet. Sed Mathematico ad explicandas circuli aut trianguli proprietates, nullum adferre potest vsum, materiæ vt auri, ligni, ferri, in qua insunt, consideratio. quin eò verius eiusmodi rerum, quarum species tanquam materia vacantes efformemus animo, naturam complectemur, quòd coniunctione materiæ quasi adulterari deprauarique videntur. Quocirca Mathematicæ species eodem modo quo κοιλὸν, siue concauitas, sine motu & subiecto, definitione explicari

cognos-

cognoſciq́; poſſunt: naturales verò cùm eam vim habeant, quam, vt ita dicam, ſimitas, cum materia comprehenſæ ſunt, nec abſque ea ſeparatim poſſunt intelligi: quibus exemplis quid inter Phiſicas & Mathematicas ſpecies interſit, haud difficile eſt animaduertere. Illis certè non ſemel eſt vſus Ariſtoteles. Valeant ergo Protagoræ ſophiſmata, Geometras hoc nomine refellentis, quod circulus normam puncto non attingat. Nam diuina Geometrarum théωremata, qui ſenſu æſtimabit, vix quicquam reperiet quod Geometræ concedendum videatur. Quid enim ex his quæ ſenſum mouent, ita rectum aut rotundum dici poteſt, vt à Geometra ponitur? Nec verò abſurdum eſt aut vitioſum, quòd lineas in puluere deſcriptas pro rectis aut rotundis aſſumit, quæ nec rectæ ſunt nec rotundæ, ac ne latitudinis quidem expertes. Siquidem non iis vtitur Geometra quaſi inde vim habeat concluſio, ſed eorum quæ diſcenti intelligenda relinquuntur, rudem ceu imaginem proponit. Nam qui primùm inſtituuntur, hi ductu quodam & velut χειραγωγίᾳ ſenſuum opus habent, vt ad illa quæ ſola intelligentia percipiuntur, aditum ſibi comparare queant. Sed tamen exiſtimandum non eſt rebus Mathematicis omnino negari materiam, ac non eam tantùm quæ ſenſum afficit. Eſt enim materia alia quæ ſub ſenſum cadit, alia quæ animo & ratione intelligitur. Illam αἰσθητὴν, hanc νοητὴν vocat Ariſtoteles. Senſu percipitur, vt æs, vt lignum, omniſque materia quæ moueri poteſt.

test. Animo & ratione cernitur ea quæ in rebus sensilibus inest, sed non quatenus sensu percipiuntur, quales sunt res Mathematicorum. Vnde ab Aristotele scriptum legimus ἐπὶ τῶν ἐν ἀφαιρέσει ὄντων *rectum se habere vt simum:* μετὰ συνεχοῦς γάρ: *qua si velit ipsius recti, quod Mathematicorum est, suam esse materiam, non minus quàm simi quod ad Physicos pertinet. Nam licet res Mathematicæ sensili vacent materia, non sunt tamen indiuiduæ, sed propter continuationem partitioni semper obnoxiæ, cuius ratione dici possunt sua materia non omnino carere: quin aliud videtur* τὸ εἶναι γραμμῇ, *aliud quoad continuationi adiuncta intelligitur linea. Illud enim ceu forma in materia proprietatum causa est, quas sine materia percipere non licet. Hæc est societatis & dissidij Mathematicæ cum Physica & prima Philosophia ratio. Nunc autem de nominis etymo & notatione pauca quædam afferamus. Nam si quæ iudicio & ratione imposita sunt rebus nomina, ea certè non temere indita fuisse credendum est, quibus scientias appellari placuit. Sed neque otiosa semper haberi debet ista etymologiæ indagatio, cùm ad rei etiam dubiæ fidem sæpe non parum valeat recta nominis interpretatio. Sic enim Aristoteles ducto ex verborum ratione argumento,* αὐτομάτου, μεταβολῆς, αἰθέρος, *aliarumque rerum naturam ex parte confirmauit. Quoniam igitur Pythagoras Mathematicam scientiam non modò studiosè coluit, sed etiam repetitæ à capite principijs, geometricam*

contem-

contemplationem in liberalis disciplinæ formam composuit, & perspectis absque materia, solius intelligentiæ adminiculo theorematibus, tractationem περὶ τῶν ἀλόγων, & κοσμικῶν σχημάτων constitutionem excogitauit: credibile est, Pythagoram, aut certè Pythagoreos, qui & ipsi doctoris sui studia libenter amplexi sunt, huic scientiæ id nomen dedisse, quod cum suis placitis atque decretis congrueret, rerumq́ue propositarum naturam quoquo modo declararet. Ita cùm existimarent illi omnem disciplinam, quæ μάθησις dicitur, ἀνάμνησιν esse quandam. i. recordationem & repetitionem eius scientiæ, cuius antè quam in corpus immigraret, compos fuerit anima, quemadmodum Plato quoque in Menone, Phædone, & alijs aliquot locis videtur astruxisse: animaduerterent autem eiusmodi recordationem, quæ non posset multis ex rebus perspici, ex his potissimum scientijs demonstrari, si quis nimirum, ait Plato, ἐπὶ τὰ διαγράμματα ἄγῃ. probabile est equidem Mathematicas à Pythagoreis artes κατ᾽ ἐξοχὴν fuisse nominatas, vt ex quibus μάθησις, id est æternarum in anima rationum recordatio διαφερόντως & præcipuè intelligi posset. Cuius etiã rei fidẽ nobis diuinus fecit Plato, qui in Menone Socratẽ induxit hoc argumenti genere pſuadere cupientẽ, discere nihil esse aliud quã suarũ ipsius rationũ animũ recordari. Etenim Socrates pusionẽ quẽdã, vt Tullij verbis vtar, interrogat de geometrica dimẽsione quadrati: ad ea sic ille respõdet vt puer, & tamen

men

men tam faciles interrogationes sunt, ut gradatim respondens, eodem perueniat, quò si Geometrica didicisset. Aliam nominis huius rationem Anatolius exposuit, ut est apud Rhodiginum, quòd cùm caeterae disciplinae deprehendi vel non docente aliquo possint omnes, Mathematica sub nullius cognitionem veniant, nisi praeeunte aliquo, cuius solertia succidantur vepreta, vel exurantur, & superciliosa complanentur aspreta. Ita enim Caelius: quod quam vim habeat, non est huius loci curiosius perscrutari. Equidem M. Tullius Mathematicos in magna rerum obscuritate, recondita arte, multiplicique ac subtili versari scribit. sed quis nescit idipsum cum alijs grauioribus scientijs esse commune? Est enim, vel eodem auctore Tullio, omnis cognitio multis obstructa difficultatibus, maximaque est & in ipsis rebus obscuritas, & in iudicijs nostris infirmitas. nec vllus est, modò interius paulò Physica penetrarit, qui non facilè sit expertus, quàm multi vndique emergant, rerum naturalium causas inquirentibus, inexplicabiles labyrinthi. Sunt qui ex demonstrationum firmitate nominari Mathematicas opinantur: cuius etiam rationis momentum alio seorsim loco expendendum fuerit. Quocirca primam verbi notationem, quam sequutus est Proclus, nobis retinendam censeo. Hactenus de vniuerso Mathematicae genere, quanta potui & perspicuitate & breuitate dixi. Sequitur ut de Geometria separatim atque ordine ea disseram, quae initio sum pollicitus.

tus. Est autem Geometria, ut definit Proclus, scientia quæ versatur in cognitione magnitudinum, figurarum, & quibus hæ continentur, extremorum, item rationum & affectionum, quæ in illis cernuntur ac inhærent: ipsa quidem progrediens à puncto indiuiduo per lineas & superficies, dum ad solida conscendat, variasq; ipsorum differentias patefaciat. Quúmque omnis scientia demonstratiua, ut docet Aristoteles, tribus quasi momentis contineatur, genere subiecto, cuius proprietates ipsa scientia exquirit & contemplatur: causis & principijs, ex quibus primis demonstrationes conficiuntur: & proprietatibus, quæ de genere subiecto per se enunciantur: Geometriæ quidem subiectum in lineis, triangulis, quadrangulis, circulis, planis, solidis, atque omnino figuris & magnitudinibus, earúmque extremitatibus consistit. His autem inhærent diuisiones, rationes, tactus, æqualitates, παραβολαὶ ὑπερβολαὶ, ἐλλείψεις, atque alia generis eiusdem propè innumerabilia. Postulata verò & Axiωmata ex quibus hæc inesse demonstrantur, eiusmodi ferè sunt: Quouis centro & interuallo circulum describere. Si ab æqualibus æqualia detrahas, quæ relinquuntur esse æqualia, cæteraq; id genus permulta, quæ licet omnium sint communia, ad demonstrandum tamen tum sunt accommodata, cùm ad certum quoddam genus traducuntur. Sed cùm præcipua videatur Arithmeticæ & Geometriæ inter Mathematicas dignatio, cur Arithmetica sit ἀκριβεστέρα &

exactior quam Geometria paucis explicandum arbitror. Hîc verò & Aristotelem sequemur ducem, qui scientiam cum scientia ita comparat, vt accuratiorem esse velit eam, quæ rei causam docet, quam quæ rem esse tantùm declarat: deinde quæ in rebus sub intelligentiam cadentibus versatur, quàm quæ in rebus sensum mouentibus cernitur. Sic enim & Arithmetica quàm Musica, & Geometria quàm Optica, & Stereometria quàm Mechanica exactior esse intelligitur. Postremò quæ ex simplicioribus initijs constat, quàm quæ aliqua adiectione compositis vtitur. Atque hac quidem ratione Geometriæ præstat Arithmetica, quòd illius initium ex additione dicatur, huius sit simplicius. Est enim punctum, vt Pythagoreis placet, vnitas quæ situm obtinet: vnitas verò punctum est quod situ vacat. Ex quo percipitur, numerorum quàm magnitudinum simplicius esse elementum, numerosq; magnitudinibus esse puriores, & à concretione materiæ magis disiunctos. Hæc quanquam nemini sunt dubia, habet & ipsa tamen Geometria quo se plurimum efferat, opibúsque suis ac rerum vbertate multiplici vel cum Arithmetica certet: id quod tute facilè deprehendas cùm ad infinitam magnitudinis diuisionem, quam respuit multitudo, animum conuerteris. Nunc quæ sit Arithmeticæ & Geometriæ societas, videamus. Nam theorematum quæ demonstratione illustrantur, quædam sunt vtriusque scientiæ communia, quædam verò

singu-

singularum propria. Etenim quòd omnis proportio sit ῥητὸς siue rationalis, Arithmeticæ soli conuenit, nequaquam Geometriæ, in qua sunt etiam ἄρρητοι, seu irrationales proportiones. item, quadratorum gnomas minimo definitos esse, Arithmetica proprium (si quidem in Geometria nihil tale minimum esse potest) sed ad Geometriam propriè spectant situs, qui in numeris locum non habent: tactus, qui quidem à continuis admittuntur: ἄλογον, quoniam vbi diuisio infinitè procedit, ibi etiam τὸ ἄλογον esse solet. Communia porrò vtriusque sunt illa, quæ ex sectionibus eueniunt, quas Euclides libro secundo est persequutus: nisi quòd sectio per extremam & mediam rationem in numeris nusquam reperiri potest. Iam verò ex theorematibus eiusmodi communibus, alia quidem ex Geometria ad Arithmeticam traducuntur: alia contrà ex Arithmetica in Geometriam transferuntur: quædam verò perinde vtrique scientiæ conueniunt, vt quæ ex vniuersa arte Mathematica in vtranque harum conueniant. Nam & alterna ratio, & rationum conuersiones, compositiones, diuisiones hoc modo communia sunt vtriusque. Quæ autem sunt περὶ συμμέτρων, id est, de commensurabilibus, Arithmetica quidem primùm cognoscit & contemplatur: secundo loco Geometria Arithmeticam imitata. Quare & commensurabiles magnitudines illæ dicuntur, quæ rationem inter se habent quam numerus ad numerum, per-

inde quasi commensuratio & συμμετρία in numeris primùm consistat (Vbi enim numerus, ibi & σύμμετρον cernitur: & vbi σύμμετρον, illic etiam numerus) sed quæ triangulorum sunt & quadrangulorum, à Geometra primùm considerantur: tum analogia quadam Arithmeticus eadem illa in numeris contemplatur. De Geometriæ diuisione hoc adijciendum puto, quòd Geometriæ pars altera in planis figuris cernitur, quæ solam latitudinem longitudini coniunctam habent. altera verò solidas contemplatur, quæ ad duplex illud interuallum crassitudinem adsciscunt. Illam generali Geometriæ nomine veteres appellarunt: hanc propriè Stereometriam dixerunt. Ita Geometriam cum Optica, & Stereometriam cum Mechanica non raro comparat Aristoteles. Sed illius cognitio huius inuentionem multis seculis antecessit, si modò Stereometriam ne Socratis quidem ætate vllam fuisse omnino verum est, quemadmodum à Platone scriptum videtur. Ad Geometriæ vtilitatem accedo, quæ quanquam suapte vi & dignitate ipsa per se nititur, nullius vsus aut actionis ministerio mancipata (vt de Mathematicis omnibus scientijs concedit in Politico Socrates) si quid ex ea tamen vtilitatis externæ quæritur, Dij boni quàm latos, quàm vberes, quàm varios fructus fundit? Nec verò audiendus est vel Aristippus, vel Sophistarum alius, qui Mathematicorum artes idcirco repudiet, quòd ex fine nihil docere videantur, eiúsque quod melius aut

deteri-

deterius nullam habeant rationem. Vt enim nihil causæ dicas, cur sit melius, trianguli, verbi gratia, tres angulos duobus esse rectis æquales: minimè tamen fuerit consentaneum, Geometriæ cognitionem ut inutilem exagitare, criminari, explodere, quasi quæ finem & bonum quò referatur, habeat nullum. Multas haud dubiè solius contemplationis beneficio citra materiæ contagionem adfert Geometria commoditates partim proprias, partim cum vniuerso genere communes. Cùm enim Geometria, vt scripsit Plato, eius quod semper est cognitionem profiteatur, ad veritatem excitabit illa quidem animum, & ad ritè philosophandum cuiusque mentem comparabit. Quinetiam ad disciplinas omnes facilius perdiscendas, attigeris nécne Geometriam, quanti referre censes? Nam vbi cum materia coniungitur, nónne præstantissimas procreat artes, Geodæsiam, Mechanicam, Opticam, quarum omnium vsu, mortalium vitam summis beneficijs complectitur? Etenim bellica instrumenta, vrbiúmque propugnacula, quibus munitæ vrbes hostium vim propulsarent, his adiutricibus fabricata est: montium ambitus & altitudines, locorúmque situs nobis indicauit: dimetiendorum & mari & terra itinerum rationem præscripsit: trutinas & stateras, quibus exacta numerorum æqualitas in ciuitate retineatur, composuit: vniuersi ordinem simulachris expressit: multáque quæ hominum fidem superarent, omnibus persuasit. Vbique extant præclara in eam rem *testimonia. Illud memorabile, quòd Archimedi rex Hiero tribuit. Nam extructo vastæ molis nauigio, quod Hiero Ægyptiorum regi Ptolemæo mitteret, cùm vniuersa Syracusanorum multitudo collectis simul viribus nauem trahere non posset, effecissetq; Archimedes, vt solus Hiero illam subduceret, admiratus viri scientiam rex; ἀπὸ ταύτης, ἔφη, τῆς ἡμέρας περὶ παντὸς ἀρχιμήδει λέγοντι πιστευτέον. Quid? quòd Archimedes idem, vt est apud Plutarchum, Hieroni scripsit datis viribus datum pondus moueri posse? fretusq; demonstrationis robore, illud sæpe iactarit, si terram haberet alteram vbi pedem figeret, ad eam, nostram hanc se transmouere posse? Quid varia, αὐτομάτων machinarumq; genera, ad vsus necessarios comparata memorem? Innumerabilia profectò sunt illa, & admiratione dignissima, quibus prisci homines incredibili quodam ad philosophandum studio concitati, inopem mortalium vitam artis huius præsidio subleuarunt: tametsi memoriæ sit proditum, Platonem Eudoxo & Archytæ vitio vertisse, quòd Geometrica problemata ad sensilia & organica abducerent. Sic enim corrumpi ab illis & labefieri Geometriæ præstantiam, quæ ab intelligibilibus & incorporeis rebus ad sensiles & corporeas prolaberetur. Quapropter ridicula idem scripsit Plato Geometrarum esse vocabula, quæ quasi ad opus & actionem spectent, ita sonare videntur. Quid enim est quadrare, si non opus facere? Quid addere,*

produ-

producere, applicare? Multa quidem sunt eiusmodi nomina, quibus necessario & tanquam coacti Geometræ vtuntur, quippe cùm alia desint in hoc genere commodiora. Sic ergo censuit Plato, sic Aristoteles sic denique philosophi omnes, Geometriam ipsam cognitionis gratia exercendam, nec ex aliquo vsu externo, sed ex rerum νοητῶν intelligentia æstimandam esse. Exposita breuius quàm res tanta dici possit, vtilitatis ratione, Geometriæ ortum, qui in hac rerum periodo ex historicorum monumentis nobis est cognitus, deinceps aperiamus. Geometria apud Ægyptios inuenta, (ne ab Adamo, Setho, Noah, quos cognitione rerum multiplici valuisse constat, eam repetamus) ex terrarum dimensione, vt verbi præ se fert ratio, ortum habuisse dicitur: cùm anniuersaria Nili inundatione & incrementis limo obducti agrorum termini confunderentur. Geometriam enim, sicut & reliquas disciplinas, in vsu quàm in arte prius fuisse aiunt. Quod sanè mirum videri non debet, vt & huius & aliarum scientiarum inuentio ab vsu cœperit ac necessitate. Etenim tempus, rerum vsus, ipsa necessitas ingenium excitat, & ignauiam acuit. Deinde quicquid ortum habuit (vt tradunt Physici) ab inchoato & imperfecto processit ad perfectum. Sic artium & scientiarum principia experientiæ beneficio collecta sunt: experientia verò à memoria fluxit, quæ & ipsa à sensu primùm manauit. Nam quod scribit Aristoteles, Mathematicas artes, *comparatis rebus omnibus ad vitam necessariis, in Ægypto fuisse constitutas, quòd ibi sacerdotes omnium concessu in otio degerent: non negat ille adductos necessitate homines ad excogitandam, verbi gratia, terræ dimetiendæ rationem, quæ theorematum deinde inuestigationi causam dederit: sed hoc confirmat, præclara eiusmodi theorematum inuenta, quibus extructa Geometriæ disciplina constat, ad vsus vitæ necessarios ab illis non esse expetita. Itaque vetus ipsum Geometriæ nomen ab illa terræ partiundæ finiúmque regundorum ratione posteà recessit, & in certa quadam affectionum magnitudini per se inhærentium scientia propriè remansit. Quemadmodum igitur in mercium & contractuum gratiam, supputandi ratio, quam secuta est accurrata numerorum cognitio, à Phœnicibus initium duxit: ita etiam apud Ægyptios, ex ea quam commemoraui causa ortum habuit Geometria. Hanc certè, vt id obiter dicam, Thales in Græciam ex Ægypto primùm transtulit: cui non paucæ deinceps à Pythagora, Hippocrate, Chio, Platone, Archyta Tarentino, aliisq; compluribus, ad Euclidis tempora factæ sunt rerum magnarum accessiones. Cæterùm de Euclidis ætate id solùm addam, quod à Proclo memoriæ mandatum accepimus. Is enim commemoratis aliquot Platonis tùm æqualibus tùm discipulis, subiicit, non multò ætate posteriorem illis fuisse Euclidem eum, qui Elementa conscripsit, & multa ab Eudoxo collecta, in ordinem luculentum*

compo-

compoſuit, multaq; à Theateto inchoata perfecit, quæq; mollius ab alijs demonſtrata fuerant, ad firmiſſimas & certiſſimas apodíxes reuocauit. Vixit autem, inquit ille, ſub primo Ptolemæo. Etenim ferunt Euclidem à Ptolemæo quondam interrogatum numqua eſſet via ad Geometriam magis compendiaria, quàm ſit iſta στοιχείωσις reſpondiſſe, μὴ εἶναι βασιλικὴν ἀτραπὸν ἐπὶ γεωμετρίαν. Deinde ſubiungit, Euclidem natu quidem eſſe minorem Platone, maiorem verò Eratoſthene & Archimede (hi enim æquales erant) cùm Archimedes Euclidis mentionem faciat. Quòd ſi quis egregiam Euclidis laudem, quam cùm ex alijs ſcriptionibus accuratiſſimis, tùm ex hac Geometrica στοιχειώσει conſequutus eſt, in qua diuinus rerum ordo ſapientiſſimis quibuſque hominibus magnæ ſemper admirationi fuit, is Proclum ſtudioſè legat, quò rei veritatem illuſtriorem reddat grauiſſimi teſtis auctoritas. Supereſt igitur vt finem videamus, quò Euclidis elementa referri, & cuius cauſa in id ſtudium incumbere oporteat. Et quidem ſi res quæ tractantur, conſyderes: in tota hac tractatione nihil aliud quæri dixeris, quàm vt κοσμικὰ quæ vocantur, σχήματα (fuit enim Euclides profeſſione & inſtituto Platonicus) Cubus, Icoſaëdrum, Octaëdrum, Pyramis & Dodecaëdrum certa quadam ſuorum & inter ſe laterum, & ad ſphæræ diametrum ratione eidem ſphæræ inſcripta cōprehendantur. Huc enim pertinet Epigrammation illud vetus, quod in Geometrica Michaëlis *pselli συνό⸗ scriptum legitur.*

Σχήματα πέντε ὅ πλάτωνος, πυθαγόρας σοφὸς εὗρε,

πυθαγόρας σοφὸς εὗρε, πλάτων δ' ἀρίδηλ' ἐδίδαξεν,

Εὐκλείδης ἐπὶ τοῖσι κλέος περικαλλὲς ἔτευξεν.

Quòd si discentis institutionē spectes, illud certè fuerit propositum, vt huiusmodi elementorum cognitione informatus discentis animus, ad quamlibet non modò Geometriæ, sed & aliarum Mathematicæ partium tractationem idoneus paratusq; accedat. Nam tametsi institutionem hanc solus sibi Geometra vendicare videtur, & tanquam in possessionem suam venerit, alios excludere posse: inde tamen permulta suo quodammodo iure decerpit Arithmeticus, pleraque Musicus, non pauca detrahit Astrologus, Opticus, Logisticus, Mechanicus, itemq; cæteri: nec vllus est denique artifex præclarus, qui in huius se possessionis societatem cupidè non offerat, partemq; sibi concedi postulet. Hinc στοιχείωσις *absolutum operi nomen, &* στοιχειωτὴς *dictus Euclides. Sed quid longius prouehor? Nam quod ad hanc rem attinet, tam copiosè & eruditè scripsit (vt alia complura) eo ipso, quem dixi, loco P. Montaureus, vt nihil desiderio loci reliquerit. Quæ verò ad dicendum nobis erant proposita, hactenus pro ingenij nostri tenuitate omnia mihi perfecisse videor. Nam tametsi & hæc eadem & alia pleraque multo fortè præclariora ab hominibus doctissimis,*

qui

qui tùm acumine ingenij, tùm admirabili quodam lepôre dicendi ſemper floruerunt, grauiùs, ſplendidùs, vberiùs tractari poſſe ſcio: tamen experiri libuit numquid etiam nobis diuino ſit conceſſum munere, quod rudes in hac philoſophiæ parte diſcipulos adiuuare aut certè excitare queat. Huc acceſſit quòd iſta recens elementorum editio, in qua nihil non parum fuiſſet ſtudij, aliquid à nobis efflagitare videbatur, quod eius commendationem adaugeret. Cùm enim vir doctiſſimus Io. Magnienus Mathematicarum artium in hac Parrhiſiorum Academia profeſſor verè regius, noſtrum hunc typographum in excudendis Mathematicorum libris diligentiſſimum, ad hanc Elementorum editionem ſæpè & multum eſſet adhortatus, eiúſque impulſu permulta ſibi iam comparaſſet typographus ad hanc rem neceſſaria, citò interuenit, malùm Ioannis Magnieni mors inſperata, quæ tam graue inflixit Academiæ vulnus, cui ne poſt multos quidem annorum circuitus cicatrix obduci vlla poſſe videatur. Quamobrem amiſſo inſtituti huius operis duce, typographus, qui nec ſumptus antea factos ſibi perire, nec ſtudioſos, quibus id muneris erat pollicitus, ſua ſpe cadere vellet, ad me venit, & impenſè rogauit vt meam propoſitæ editioni operam & ſtudium nauarem. quod cùm denegaret occupatio noſtra, iuberet officij ratio feci equidem rogatus, vt quæ ſub obſcurè vel parum commodè in ſermonem Latinum è Græco tranſlata videban-

videbantur, clariore, aptiore & fideliore interpretatione nostra (quod cuiusque pace dictum volo) lucē acciperent. Id quod in omnibus ferè libris posterioribus tute primo obtutu perspicias. Nam in sex prioribus non tantum temporis quantū in cæteris ponere nobis licuit decimi aūt interpretatio, qua melior nulla potuit adferri, P. Montaureo solida debetur. Atq; vt ad perspicuitatē facilitatēq; nihil tibi deesse querarìs, adscripta sunt propositionibus singulis vel lineares figuræ, vel punctorū tanquā vnitatum notulæ, quæ Theonis apodixin illustrent: illæ quidem magnitudinū, hæ aūt numerorū indices, subscriptis etiam ciphrarum, vt vocant, characteribus, qui propositum quemuis numerū exprimant. ob eamq; causam eiusmodi vnitatum notulæ, quæ pro numeri amplitudine maius paginæ spatiū occuparēt, pauciores sæpius depictæ sunt, aut in lineas etiā commutatæ. Nam literarū, vt a, b, c, characteres non modò numeris & numerorum partibus nominandis sunt accommodati, sed etiā generales esse numerorum, vt magnitudinum affectiones testantur. Adiecta sunt insuper quibusdam locis non pœnitenda Theonis scholia, siue mauis lemmata, quæ quidem longè plura accessissent, si plus otij & temporis vacui nobis fuisset relictum, quod huic studio impartiremus. Hanc igitur operam boni consule, & quæ obuia erunt impressionis vitia, candidus emenda. Vale. Lutetiæ 4. Idus April. 1557.

EVCLIDIS ELEMENTVM PRIMVM.

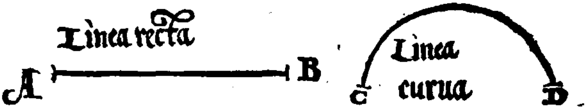
DEFINITIONES.

1

PVnctum est, cuius pars nulla est. Punctum

2

Linea verò, longitudo latitudinis expers.



3

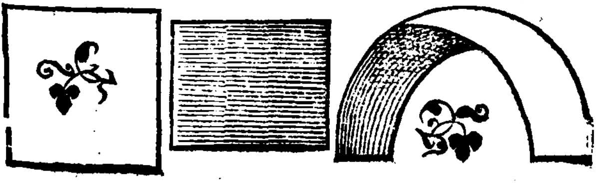
Lineæ autem termini, sunt puncta.

4

Recta linea est, quæ ex æquo sua interiacet puncta.

5

Superficies est, quæ longitudinem latitudinemque tantùm habet.



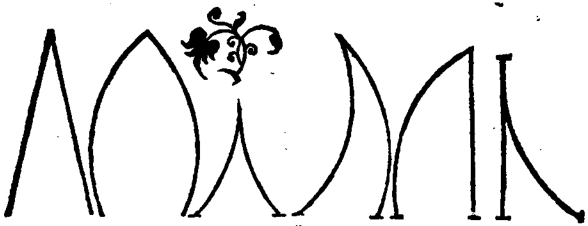
6 Super-

6

Superficiei extrema, ſunt lineæ.

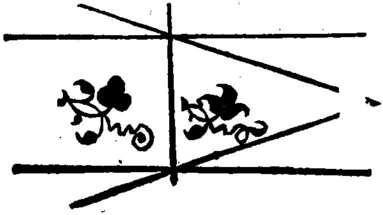
7

Plana ſuperficies eſt, quæ ex æquo ſuas interiacet lineas.



8

Planus angulus eſt duarum linearũ in plano ſe mutuò tangentium, & non in directum



iacentium, alterius ad alteram inclinatio.

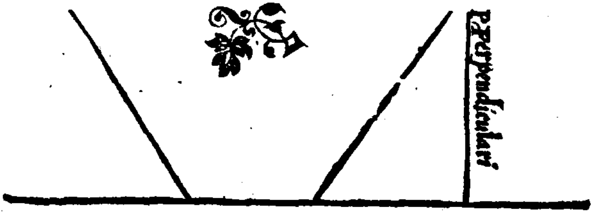
9

Cùm autem quæ angulum continent lineæ, rectæ fuerint, recti lineus ille angulus appellatur.

10

Cùm verò recta linea ſuper rectam conſiſtens lineam, eos qui ſunt deinceps angulos æquales inter ſe fecerit: rectus eſt vterque æqualium

qualium angulorum: & quæ insistit recta linea, perpendicularis vocatur eius, cui insistit.



11

Obtusus angulus est, qui recto maior est.

12

Acutus verò, qui minor est recto.

13

Terminus est, quod alicuius extremum est.

14

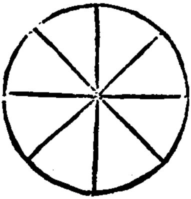
Figura est, quæ sub aliquo, vel aliquibus terminis comprehenditur.

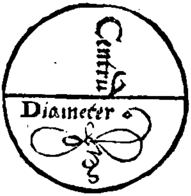
15

Circulus est figura plana sub vna linea comprehensa, quæ peripheria appellatur: ad quam ab vno puncto eorum, quæ intra figuram

sunt

ſunt poſi-
ta, caden-
tes omnes
rectæ li-
neæ inter
ſe ſunt æ-
quales.





16

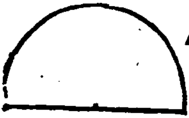
Hoc verò punctum, centrum circuli appellatur.

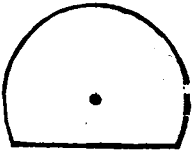
17

Diameter autem circuli, eſt recta quædam linea per centrum ducta, & ex vtraque parte in circuli peripheriam terminata, quæ circulum biſariam ſecat.

18

Semicirculus eſt figura, quæ continentur ſub diametro, & ſub ea linea, quæ de circuli peripheria auſertur.





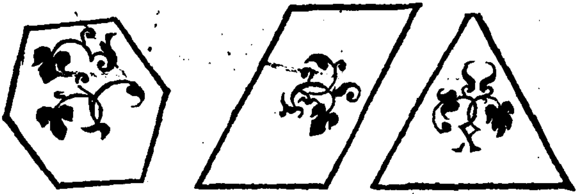
19

Segmentum circuli, eſt figura, quæ ſub recta linea & circuli peripheria continetur.

20 Recti

20

Rectilineæ figuræ, ſunt quæ ſub rectis lineis continentur.



21

Trilateræ quidem, quæ ſub tribus.

22

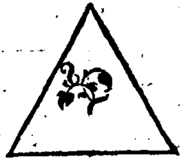
Quadrilateræ, quæ ſub quatuor.

23

Multilateræ verò, quæ ſub pluribus quàm quatuor rectis lineis comprehenduntur.

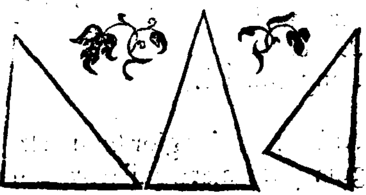
24

Trilaterarum porrò figurarum, æquilaterum eſt triangulum, quod tria latera habet æqualia.



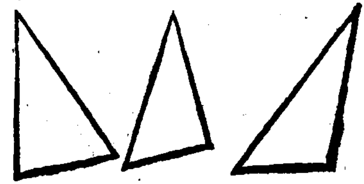
25

Iſoſceles autem, eſt quod duo tantum æqualia habet latera.



26

Scalenum verò, est qđ tria inęqualia habet latera.



27

Ad hæc etiam, trilaterarum figurarum, rectangulum quidem triangulum est, quod rectum angulum habet.

28

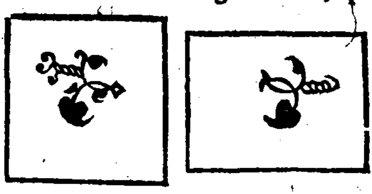
Amblygonium autem, quod obtusum angulum habet.

29

Oxygonium verò, quod tres habet acutos angulos.

30

Quadrilaterarum autem figurarum, quadratũ quidẽ est, qđ & æquilaterum & rectangulum est.

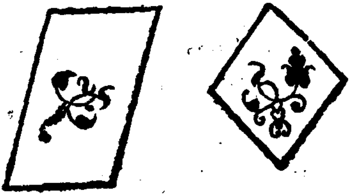


31

Altera parte longior figura est, quæ rectangula quidem, at æquilatera non est.

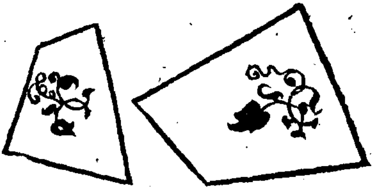
32 Rhom-

32
Rhombus autem, qui æquilaterum & rectangulum eſt.



33
Rhomboides verò, quæ aduerſa & latera & angulos habens inter ſe æqualia, neque æquilatera eſt, neque rectangula.

34
Præter has autem, reliquæ quadrilateræ figuræ, trapezia appellentur.



35
Parallelæ rectæ lineæ ſunt quæ, cùm in eodem ſint plano, & ex vtraque parte in infinitum producantur, in neutram ſibi mutuò incidunt.

Poſtulata.

1
Poſtuletur, vt à quouis puncto in quoduis

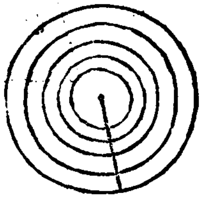
punctum, rectam lineam ducere cócedatur.

2

Et rectam lineam terminatam in continuum recta producere.

3

In quouis centro & interuallo circulum describere.



Communes notiones.

1

Quæ eidem æqualia, & inter se sunt æqualia.

2

Et si æqualibus æqualia adiecta sint, tota sunt æqualia.

3

Et si ab æqualibus æqualia ablata sint, quæ relinquuntur sunt æqualia.

4

Et si inæqualibus æqualia adiecta sint, tota sunt inæqualia.

5

Et si ab inæqualibus æqualia ablata sint, reliqua sunt inæqualia.

6

Quæ eiusdem duplicia sunt, inter se sunt æqualia.

7

Et quæ eiusdem sunt dimidia, inter se æqualia sunt.

8

Et quæ sibi mutuò congruunt, ea inter se sunt æqualia.

9

Totum est sua parte maius.

10

Item, omnes recti anguli sunt inter se æquales.

11

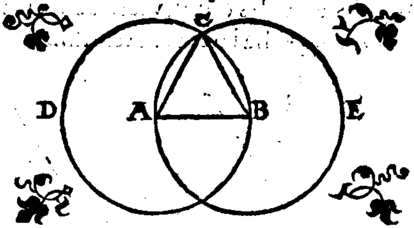
Et si in duas rectas lineas altera recta incidens, internos ad easdemque partes angulos duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ sibi mutuò incident ad eas partes, vbi sunt anguli duobus rectis minores.

12

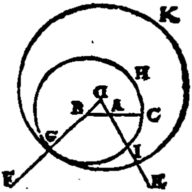
Duæ rectæ lineæ spatium non comprehendunt.

Problema 1. Propositio 1.

Super data recta linea terminata, triangulum equilaterum constituere.

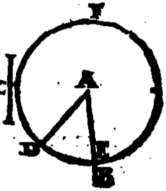


Problema 2. Propositio 2.



Ad datum punctum, datæ rectæ lineæ, æqualem rectam lineam ponere.

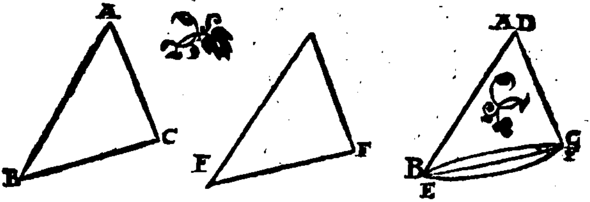
Problema 3. Propositio 3.



Duabus datis rectis lineis inæqualibus, de maiore æqualem minori rectam lineam detrahere.

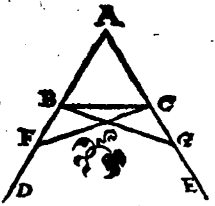
Theorema primum. Propositio 4.

Si duo triangula duo latera duobus lateribus æqualia habeant, vtrunque vtrique, habeant verò & angulum angulo ęqualem sub æqualibus rectis lineis contentum: & basin basi æqualem habebunt, eritq; triangulum triangulo æquale, ac reliqui anguli reliquis angulis æquales erunt, vterque vtrique, sub quibus æqualia latera subtenduntur.



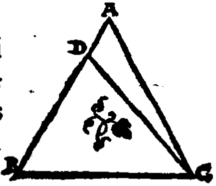
Theore-

Theorema 2. Pro-
poſitio 5.



Iſoſcelium triangulorũ qui ad baſim ſunt anguli, inter ſe ſunt æquales; & ſi vlterius productæ ſint æquales illæ rectæ lineæ, qui ſub baſi ſunt anguli, inter ſe æquales erunt.

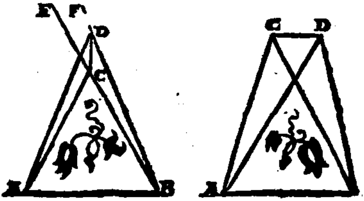
Theorema 3. Pro-
poſitio 6.



Si trianguli duo anguli æquales inter ſe fuerint: & ſub æqualibus angulis ſubtenſa latera æqualia inter ſe erunt.

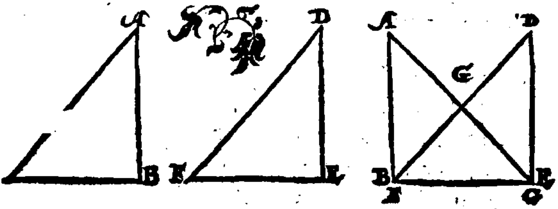
Theorema 4. Propoſitio 7.

Super eadem recta linea, duabus eiſdem rectis lineis aliæ duæ rectæ lineæ æquales, vtraque vtrique, non conſtituentur, ad aliud atque aliud pũctũ, ad eaſdẽ partes, eoſdemq; terminos cum duabus initio ductis rectis lineis habentes.



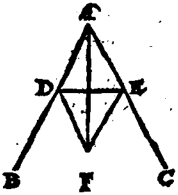
Theorema 5. Propoſitio 8.

Si duo triangula duo latera habuerint duobus lateribus, vtrunque vtrique, æqualia, habuerint verò & baſim baſi æqualem: angulũ quoque ſub æqualibus rectis lineis contentum angulo æqualem habebunt.



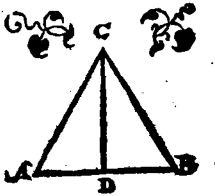
Problema 4. Propoſitio 9.

Datum angulum rectilineum biſariam ſecare.



Problema 5. Propoſitio 10.

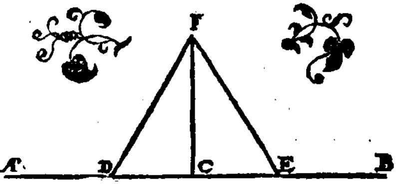
Datam rectam lineam finitam biſariam ſecare.



Proble-

Problema 6. Propositio 11.

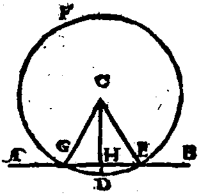
Data recta linea, à puncto in ea dato, re-



ctam lineam ad angulos rectos excitare.

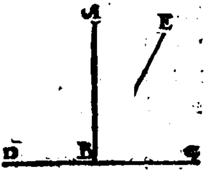
Problema 7. Propositio 12.

Super datam rectam lineam infinitam, à dato puncto quod in ea non est, perpendicularem rectam deducere.



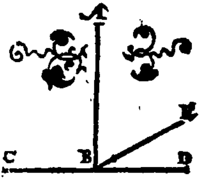
Theorema 6. Propositio 13.

Cùm recta linea super rectam consistens lineã angulos facit, aut duos rectos, aut duobus rectis æquales efficiet.



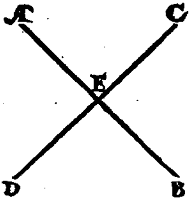
Theorema 7. Propositio 14.

Si ad aliquam rectam lineam, atque ad eius punctú, duæ rectæ lineæ non ad eaſdem partes ductæ, eos qui ſunt deinceps angulos duobus rectis æquales fecerint, in directum erunt inter ſe ipſæ rectæ lineæ.



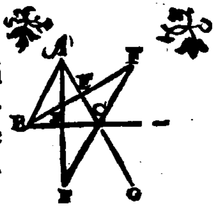
Theorema 8. Propoſitio 15.

Si duæ rectæ lineæ ſe mutuò ſecuerint, angulos qui ad verticem ſunt, æquales inter ſe efficient.



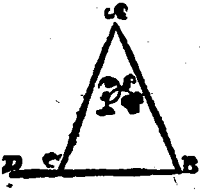
Theorema 9. Propoſitio 16.

Cuiuſcunque trianguli vno latere producto, externus angulus vtroque interno & oppoſito maior eſt.



Theorema 10. Propoſitio 17.

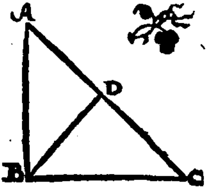
Cuiuſcunque trianguli duo anguli duobus rectis ſunt minores omnifariam ſumpti.



Theore-

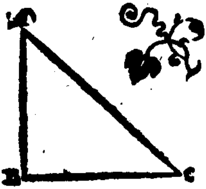
Theorema 11. Propoſitio 18.

Omnis trianguli maius latus maiorem angulũ ſubtendit.



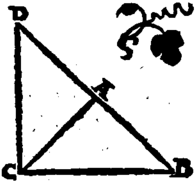
Theorema 12. Propoſitio 19.

Omnis trianguli maior angulus, maiori lateri ſubtenditur.



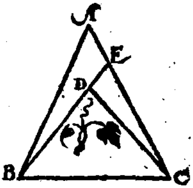
Theorema 13. Propoſitio 20.

Omnis trianguli duo latera reliquo ſunt maiora, quomodocũq; aſſumpta.



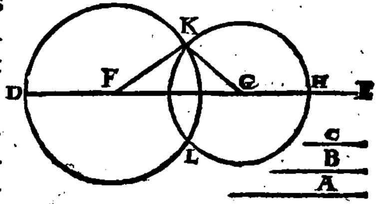
Theorema 14. Propoſitio 21.

Si ſuper trianguli vno latere, ab extremitatibus duæ rectæ lineæ, interius conſtitutæ fuerint, hæ cõſtitutæ reliquis trianguli duobus lateribus minores quidem erunt, minorem verò angulum continebunt.



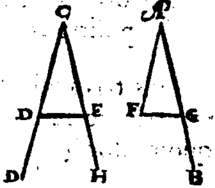
Proble-

Problema 8. Propositio 22.



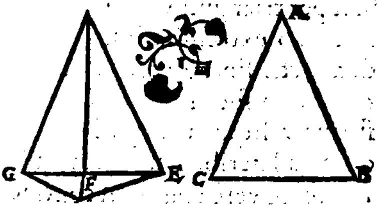
Ex tribus rectis lineis quę sunt tribus datis rectis lineis æquales, triangulum constituere. Oportet autem duas reliqua esse maiores omnifariam sumptas: quoniam vniuscuiusq; trianguli duo latera omnifariam sumpta reliquo sunt maiora.

Problema 9. Propositio 23.



Ad datam rectam lineam datumq; in ea punctum, dato angulo rectilineo æqualem angulum rectilineum constituere.

Theorema 15. Propositio 24.



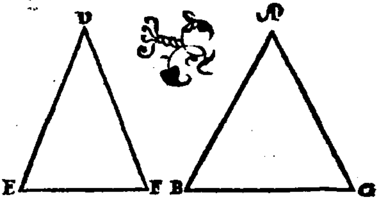
Si duo triangula duo latera duobus lateribus æqualia habuerint, vtrunq; vtriq;, angulum verò angu-

lo

lo maiorem ſub æqualibus rectis lineis contentũm:& baſin baſi maiorem habebunt.

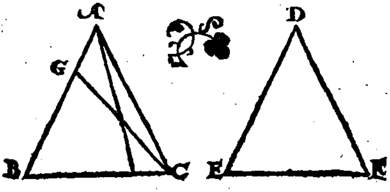
Theorema 16. Propoſitio 25.

Si duo triangula duo latera duobus lateribus æqualia habuerint, vtrunque vtrique, baſin verò baſi maiorẽ: & angulum ſub æqualib' rectis lineis contentum angulo maiorem habebunt.



Theorema 17. Propoſitio 26.

Si duo triangula duos angulos duobus angulis æquales habuerint, vtrunque vtrique, vnumq́ue latus vni lateri æquale, ſiue quod æqualibus adiacet angulis, ſeu quod vni æqualium angulorum ſubtenditur. & reliqua latera reliquis lateríb. ęqualia, vtrumq; vtrique, & reliquum angulum reliquo angulo æqualem habebunt.



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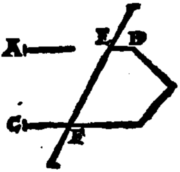
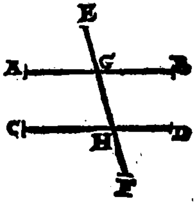
Theorema 18. Propositio 27.



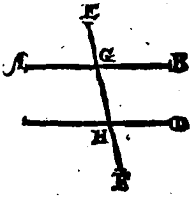
Si in duas rectas lineas recta incidens linea alternatim angulos æquales inter se fecerit: parallelæ erunt inter se illæ rectæ lineæ.

Theorema 19. Propositio 28.

Si in duas rectas lineas recta incidens linea, externum angulum interno, & opposito, & ad easdem partes æqualem fecerit, aut internos & ad easdem partes duobus rectis æquales: parallelæ erunt inter se ipsæ rectæ lineæ.



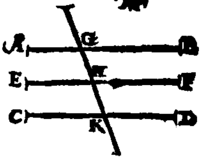
Theorema 20. Propositio 29.



In parallelas rectas lineas recta incidens linea: & alternatim angulos inter se æquales efficit & externũ interno & opposito & ad easdem partes æqualem, & internos & ad easdem partes duobus rectis æquales facit.

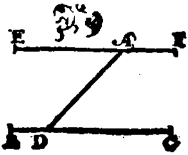
Theore-

Theorema 21. Propositio 30.



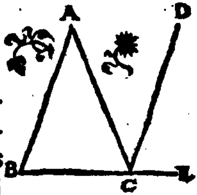
Quæ eidem rectæ lineæ, parallelæ, & inter se sunt parallelæ.

Problema 10. Propositio 31.



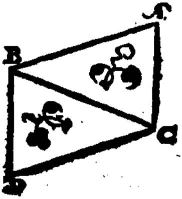
A dato puncto datæ rectæ lineæ parallelam rectam lineam ducere.

Theorema 22. Propositio 32.



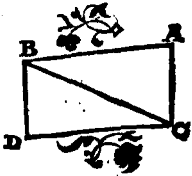
Cuiuscunque trianguli vno latere vlterius producto: externus angulus duobus internis & oppositis est æqualis. Et trianguli tres interni anguli duobus sunt rectis æquales.

Theorema 23. Propositio 33.



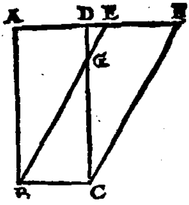
Rectæ lineæ quæ æquales & parallelas lineas ad partes easdem coniungunt, & ipsæ æquales & parallelæ sunt.

Theore-



Theorema 24. Propoſitio 34.

Parallelogrammorũ ſpatiorum æqualia ſunt inter ſe quæ ex aduerſo & latera & anguli: atque illa biſariam ſecat diameter.

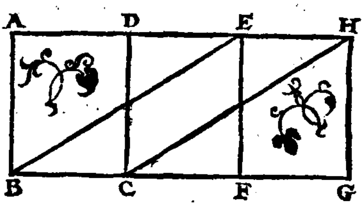


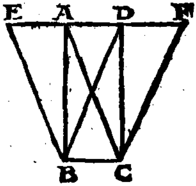
Theorema 25. Propoſitio 35.

Parallelogramma ſuper eadem baſi & in eiſdem parallelis conſtituta, inter ſe ſunt æqualia.

Theorema 26 Propoſitio 36.

Parallelogramma ſuper ęqualibus baſibus & in eiſdẽ parallelis cõſtituta inter ſe ſunt æqualia.



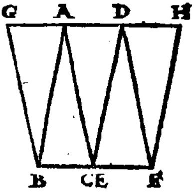


Theorema 27. Proſitio 38.

Triangula ſuper eadẽ baſi cõſtituta, & in eiſdẽ parallelis, inter ſe ſunt æqualia.

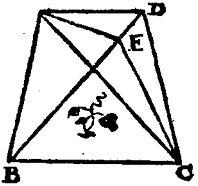
Theore-

Theorema 28. Propositio 38.



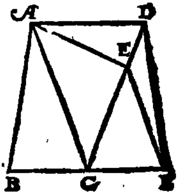
Triangula super æqualibus basibus constituta & in eisdem parallelis, inter se sunt æqualia.

Theorema 29. Propositio 39.



Triangula æqualia super eadem basi & ad easdem partes cōstituta: & in eisdem sunt parallelis.

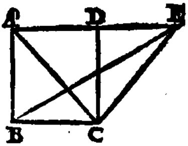
Theorema 30 Propositio 40.



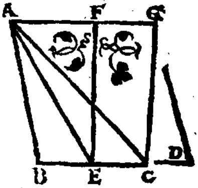
Triangula æqualia super æqualibus basibus & ad easdem partes constituta, & in eisdē sunt parallelis.

Theorema 31. Propositio 41.

Si parallelogrammum cum triangulo eandem basin habuerit non eisdemque fuerit parellelis, duplum erit parallelogrammum ipsius trianguli.

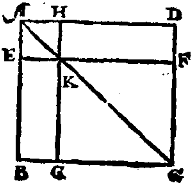


Problema 11. Propositio 42.



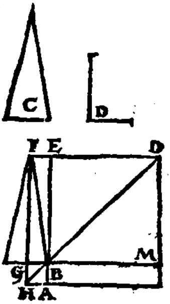
Dato triangulo æquale parallelogrammum constituere in dato angulo rectilineo.

Theorema 32. Propositio 43.



In omni parallelogrammo, complemẽta eorum quæ circa diametrũ sunt parallelogrammorum, inter se sunt æqualia.

Problema 12. Propositio 44.

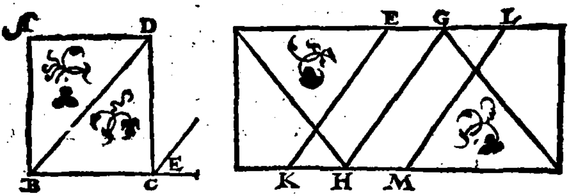


Ad datam rectam lineam, dato triangulo æquale parallelogrammũ applicare in dato angulo rectilineo.

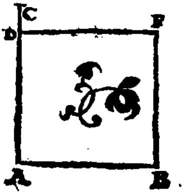
Problema 13. Propositio 45.

Dato rectilineo æquale parallelogrammum constitue-

constituere in dato angulo rectilineo.

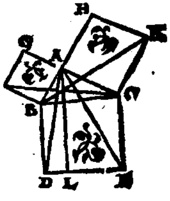


Problema 14. Propositio 46.



A data recta linea quadratum describere.

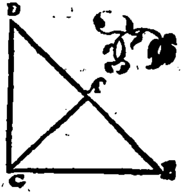
Theorema 33. Propositio 47.



In rectangulis triangulis, quadratum quod à latere rectum angulum subtendente describitur, æquale est eis quæ à lateribus rectum angulum continentibus describuntur, quadratis.

Theorema 34. Propositio 48.

Si quadratum quod ab vno laterum trianguli describitur, æquale sit eis quæ à reliquis trianguli lateribus describuntur, quadratis: angulus comprehensus sub reliquis duobus trianguli lateribus, rectus est.



FINIS ELEMENTI I.

EVCLI-

EVCLIDIS ELEMENTVM SECVNDVM.

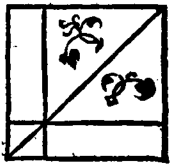
DEFINITIONES.

I

OMne parallelogrammum rectangulũ contineri dicitur ſub rectis duabus lineis, quæ rectum comprehendunt angulum.

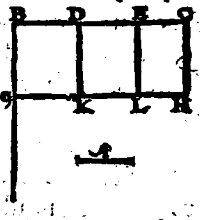
2

In omni parallelogrammo ſpatio, vnumquodlibet eorum quæ circa diametrum illius ſunt parallelogrammorum, cũ duobus complementis, Gnomo vocetur.

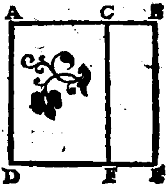


Theorema 1. Propoſitio 1.

Si fuerint duæ rectæ lineæ, ſeceturque ipſarum altera in quotcunque ſegmenta: rectangulum comprehenſum ſub illis duabus rectis lineis, æquale eſt eis rectangulis, quæ ſub inſecta & quolibet ſegmentorum comprehenduntur.



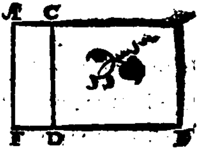
Theorema 2. Propositio 2.



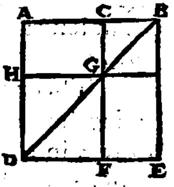
Si recta linea secta sit vtcunq;, rectangula quæ sub tota & quolibet segmentorum comprehenduntur, æqualia sunt ei, quod à tota fit, quadrato.

Theorema 3. Propositio 3.

Si recta linea secta sit vtcunque, rectangulum sub tota & vno segmentorum comprehensum, æquale est & illi quod sub segmentis comprehenditur rectangulo, & illi, quod à prædicto segmẽto describitur, quadrato.



Theorema 4. Propositio 4.

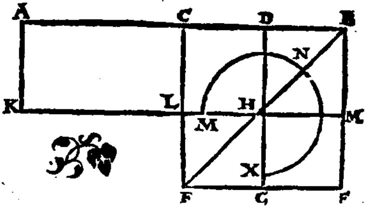


Si recta linea secta sit vtcunque: quadratum quod à tota describitur, æquale est & illis quæ à segmentis describuntur quadratis, & ei quod bis sub segmentis comprehenditur, rectangulo.

Theorema 5. Propositio 5.

Si recta linea secetur in æqualia & non æqualia: rectangulum sub inæqualibus segmentis

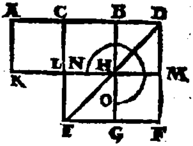
mentis totius comprehẽſum, vnà cum quadrato, quod ab intermedia



ſectionum, æquale eſt ei quod à dimidia deſcribitur, quadrato.

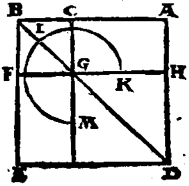
Theorema 6. Propoſitio 6.

Si recta linea bifariam ſecetur, & illi recta quædam linea in rectum adijciatur, rectangulum comprehenſum ſub tota cum adiecta & adiecta ſimul cum quadrato à dimidia, æquale eſt quadrato à linea, quæ tum ex dimidia, tum ex adiecta componitur, tanquam ab vna deſcripto.



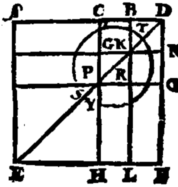
Theorema 7. Propoſitio 7.

Si recta linea ſecetur vtcunquè: quod à tota, quodq́ue ab vno ſegmentorum, vtraque ſimul quadrata, æqualia ſunt & illi quod bis ſub tota & dicto ſegmento comprehenditur, rectangulo, & illi quod à reliquo ſegmento fit, quadrato.



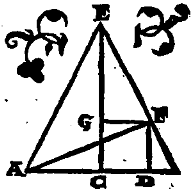
Theorema 8. Propoſitio 8.

Si recta linea ſecetur vtcunque: rectangulum quater comprehenſum ſub tota & vno ſegmentorum, cum eo quod à reliquo ſegmento fit, quadrato, æquale eſt ei quod à tota & dicto ſegmento, tanquam ab vna linea deſcribitur, quadrato.



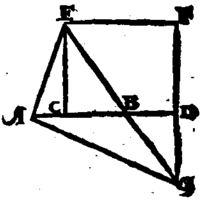
Theorema 9. Propoſitio 9.

Si recta linea ſecetur in æqualia & non æqualia: quadrata quæ ab inæqualibus totius ſegmentis fiunt, duplicia ſunt & eius quod a dimidia, & eius quod ab intermedia ſectionum fit, quadratorum.



Theorema 10. Propoſitio 10.

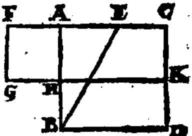
Si recta linea ſecetur bifariam, adijciatur autem ei in rectum quæpiam recta linea: quod à tota cum adiuncta, & quod ab adiuncta, vtraque ſimul quadrata, duplicia ſunt & eius quod à dimidia, & eius quod à compoſita ex dimi-



dimidia & adiuncta, tanquam ab vna descriptum sit, quadratorum.

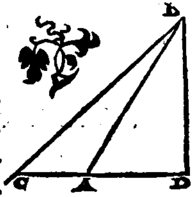
Problema 1. Propositio 11.

Datam rectam lineam secare, vt comprehensum sub tota & altero segmentorum rectangulum, æquale sit ei quod à reliquo segmento fit, quadrato.



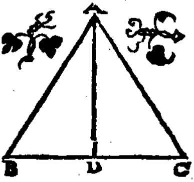
Theorema 11. Propositio 12.

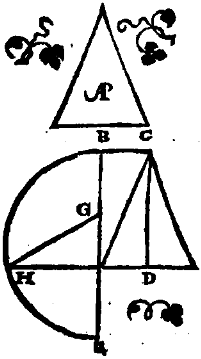
In amblygonijs triangulis, quadratum quod fit à latere angulum obtusum subtendente, maius est quadratis quæ fiunt à lateribus obtusum angulum comprehendentibus, pro quantitate rectanguli bis comprehensi & ab vno laterum quæ sunt circa obtusum angulum, in quod cùm protractum fuerit, cadit perpendicularis, & ab assumpta exterius linea sub perpendiculari prope angulum obtusum.



Theorema 12. Propoſitio 13.

In oxygonijs triangulis, quadratum à latere angulum acutum ſubtendente, minus eſt quadratis quæ fiunt à lateribus acutum angulum comprehendentibus, pro quantitate rectanguli bis comprehenſi, & ab vno laterum, quæ ſunt circa acutum angulum, in quod perpendicularis cadit, & ab aſſumpta interius linea ſub perpendiculari prope acutum angulum.





Problema 2. Propoſitio 14.

Dato rectilineo æquale quadratum conſtituere.

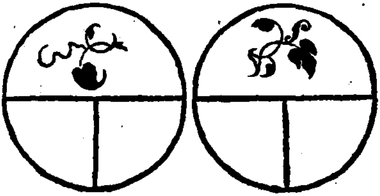
ELEMENTI II. FINIS.

EVCLIDIS ELEMENTVM TERTIVM.

DEFINITIONES.

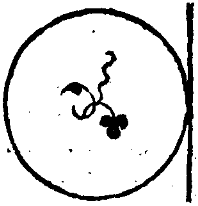
1

Aequales circuli sunt, quorum diametri sunt æquales, vel quorum quę ex cētris rectæ lineæ sunt æquales.



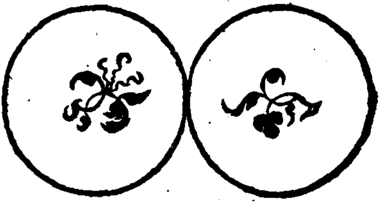
2

Recta linea circulum tangere dicitur, quę cùm circulum tangat, si producatur, circulum non secat.



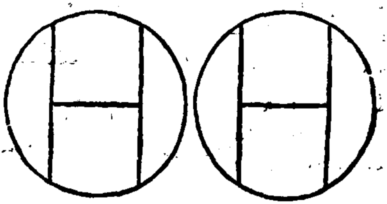
3

Circuli ſeſe mutuò tangere dicuntur: qui ſeſe mutuo tangentes, ſeſe mutuò non ſecant.



4

In circulo æqualiter diſtare à centro rectæ lineæ dicuntur; cùm perpendiculares quæ à centro in ipſas ducuntur, ſunt æquales. Lógius autem abeſſe illa dicitur, in quam maior ꝑpendicularis cadit.



5

Segmentum circuli eſt, figura quæ ſub recta linea & circuli peripheria comprehenditur.

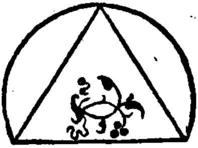
6

Segmenti autem angulus eſt, qui ſub recta linea

nea & circuli peripheria comprehenditur.

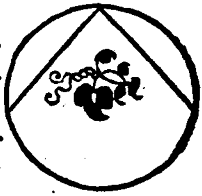
7

In ſegmento autem angulus eſt, cùm in ſegmenti peripheria ſumptum fuerit quodpiam punctum, & ab illo in terminos rectæ eius lineæ, quæ ſegmenti baſis eſt, adiunctæ fuerint rectæ lineæ: is, inquam, angulus ab adiunctis illis lineis comprehenſus.



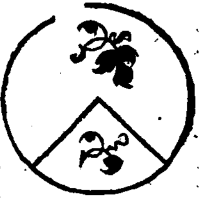
8

Cùm verò comprehendentes angulum rectæ lineæ aliquam aſſumunt peripheriam, illi angulus inſiſtere dicitur.



9

Sector autem circuli eſt, cùm ad ipſius circuli centrum conſtitutus fuerit angulus, comprehenſa nimirum figura & à rectis lineis angulum continentibus, & à peripheria ab illis aſſumpta.

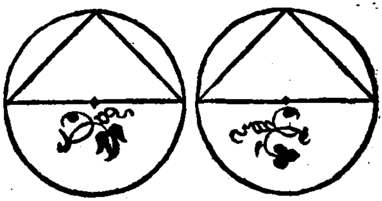


10

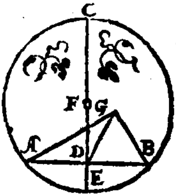
Similia circuli ſegmenta ſunt, quæ angulos

capiunt

capiunt æquales: aut in q̃bus anguli inter se sunt æquales.

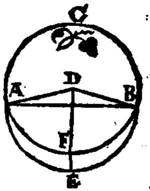


Problema 1. Propositio 1.



Dati circuli centrum reperire.

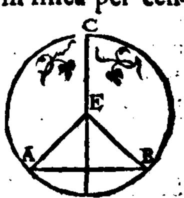
Theorema 1. Propositio 2.



Si in circuli peripheria duo quælibet puncta accepta fuerint, recta linea quæ ad ipsa puucta adiungitur, intra circulum cadet.

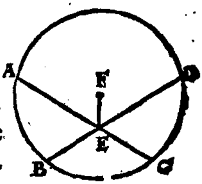
Theoroma 2. Propositio 3.

Si in circulo recta quædam linea per centrum extensa quandam nõ per centrum extensam bifariam secet: & ad angulos rectos ipsam secabit. Et si ad angulos rectos eam secet, bifariam quoque eam secabit.



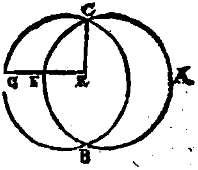
Theore-

Theorema 3. Propo-
ſitio 4.



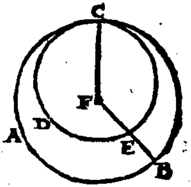
Si in circulo duæ rectæ li-
neæ ſeſe mutuò ſecent non
per centrum extenſæ, ſeſe
mutuò biſariam non ſeca-
bunt.

Theorema 4. Propo-
ſitio 5.



Si duo circuli ſeſe mutuò
ſecent, non erit illorum
idem centrum.

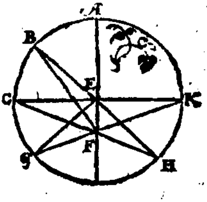
Theorema 5. Propo-
ſitio 6.



Si duo circuli ſeſe mutuò
interius tangant, eorum
non erit idem centrum.

Theorema 6. Propoſitio 7.

Si in diametro circuli quodpiam ſumatur
punctum, quod circuli centrum non ſit, ab
eoq́ue puncto in circulũ
quædam rectæ lineæ ca-
dant: maxima quidem
erit ea in qua centrum, mi
nima verò reliqua: alia-
rum verò propinquior
illi quæ per centrum du-

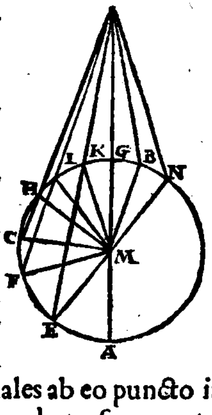


citur

citur, remotiore ſemper maior eſt. Duæ autem ſolùm rectæ lineæ æquales ab eodem puncto in circulum cadunt ad vtraſque partes minimæ.

Theorema 7. Propoſitio 8.

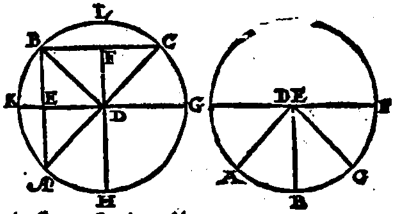
Si extra circulum ſumatur punctum quodpiam, ab eoque puncto ad circulum deducantur rectæ quædam lineæ, quarum vna quidem per centrum protendatur, reliquæ verò vt libet: in cauam peripheriam cadentium rectarum linearum maxima quidem eſt illa, quæ per centrum ducitur: aliarum autem propinquior ei, quæ per centrum tranſit, remotiore ſemper maior eſt. in conuexam verò peripheriam cadẽtium rectarum linearũ, minima quidem eſt illa, quæ inter punctum & diametrum interponitur: aliarum autem, ea quæ propinquior eſt minimę, remotiore ſemper minor eſt. Duæ autem tantùm rectæ lineæ æquales ab eo puncto in ipſum circulum cadunt, ad vtraſque partes minimæ.



Theore-

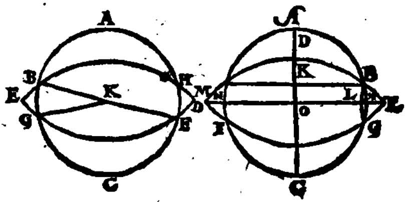
Theorema 8. Propositio 9:

Si in circulo acceptum fuerit punctum aliquod, & ab eo puncto ad circulum cadant plures quã duę rectæ lineæ, æquales, acceptũ pũctum centrum ipsius est circuli.



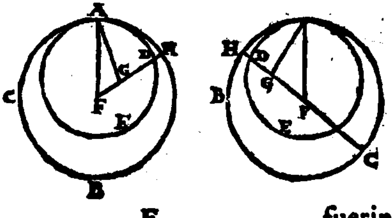
Theorema 9. Propositio 10.

Circulus circulũ in pluribus quàm duob' pũctis non secat.



Theorema 10. Propositio 11.

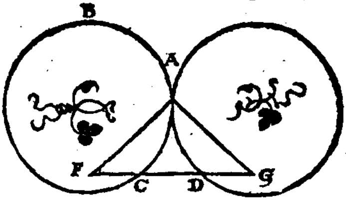
Si duo circuli sese intus contingant, atque accepta



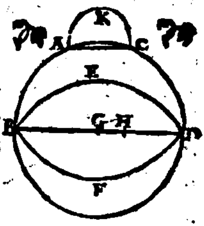
fuerint eorum centra, ad eorum centra adiuncta recta linea & producta in contactum circulorum cadet.

Theorema 11. Propositio 12.

Si duo circuli sese exterius contingant, linea recta q̃ ad cẽtra eorũ adiũgitur, per cõtactũ illũ trãsibit.



Theorema 12. Propositio 13.



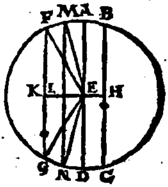
Circulus circulum non tangit in pluribus punctis, quam vno, siue intus siue extra tangat.

Theorema 13. Propositio 14.

In circulo æquales rectæ lineæ æqualiter distant à centro. Et quæ æqualiter distant à centro, æquales sunt inter se.

Theore-

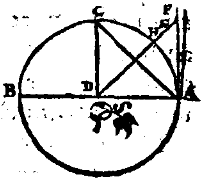
Theorema 14. Propositio 15.



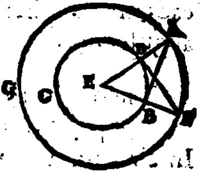
In circulo maxima quidē linea est diameter: aliarū autem propinquior centro, remotiore semper maior.

Theorema 15. Propositio 16.

Quæ ab extremitate diametri cuiusquæ circuli ad angulos rectos ducitur, extra ipsum circulum cadet, & in locum inter ipsam rectam lineam & peripheriam comprehensum, altera recta linea nōn cadet. Et semicirculi quidem angulus, quouis angulo acuto rectilineo maior est, reliquus autem minor.

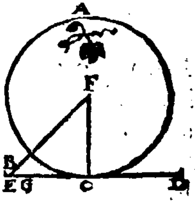


Problema 2. Propositio 17.



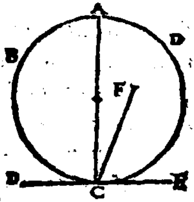
A dato puncto rectam lineam ducere, quæ datum tangat circulum.

Theorema 16. Propositio 18.



Si circulum tangat recta quæpiam linea, à centro autem ad contactum adiungatur recta quædam linea: quæ adiuncta fuerit ad ipsam contingentem perpendicularis erit.

Theorema 17. Propositio 19.



Si circulum tetigerit recta quæpiam linea, à contactu autem recta linea ad angulos rectos ipsi tangenti excitetur, in excitata erit centrum circuli.

Theorema 18. Propositio 20.

In circulo angulus ad centrum duplex est anguli ad peripheriam, cùm fuerit eadem peripheria basis angulorum.

Theorema 19. Propositio 21.

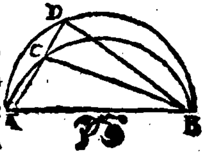
In circulo, qui in eodem segméto sunt anguli, sunt inter se æquales.

Theore-

Theorema 20. Propositio 22.

Quadrilaterorum in circulis descriptorum anguli qui ex aduerso, duobus rectis sunt æquales.

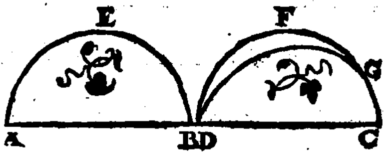
Theorema 21. Propositio 23.



Super eadem recta linea, duo segméta circulorum similia & inæqualia non constituentur ad easdem partes.

Theorema 22. Propositio 24.

Super æqualib. rectis lineis simi lia circulorũ segmenta sunt inter se æqualia.

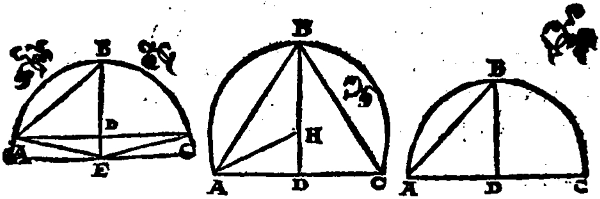


Problema 3. Propositio 25.

Circuli segmento dato, describere circulum,

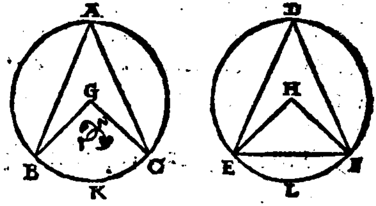
 cuius

cuius eſt ſegmentum.



Theorema 23. Propoſitio 26.

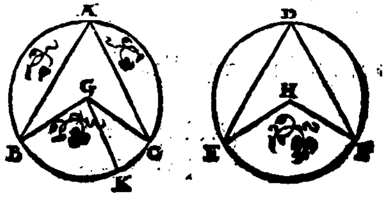
In æqualibus circulis, æquales anguli æqualib. peripherijs inſiſtunt ſiue ad centra, ſiue ad



peripherias conſtituti inſiſtant.

Theorema 24. Propoſitio 27.

In æqualibus circulis, anguli qui æqualibus peripherijs inſiſtũt, ſunt inter ſe æquales ſiue ad centra, ſi

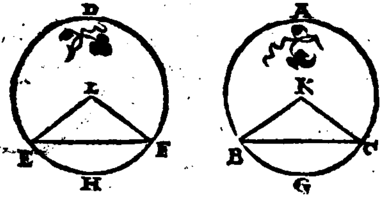


ue ad peripherias conſtituti inſiſtant.

Theore-

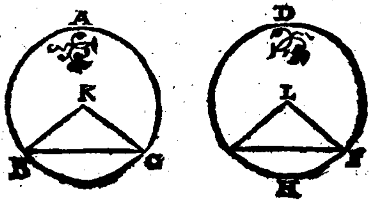
Theorema 25. Propoſitio 28.

In æqualibus circulis æquales rectæ lineæ æquales peripherĩas auferũt, maiorẽ quidem maiori, minorem autem minori.



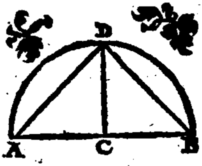
Theorema 26. Propoſitio 29.

In ęqualibus circulis, æquales peripherias æquales rectæ lineæ ſubtendunt.



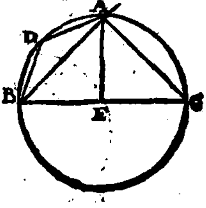
Problema 4. Propoſitio 30.

Datam peripheriam bifariam ſecare.



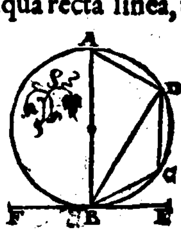
Theorema 27. Propoſitio 13.

In circulo angulus qui in ſemicirculo, rectus eſt: qui autem in maiore ſegmento, minor recto: qui verò in minore ſegmento, maior eſt recto. Et inſuper angulus maioris ſegmenti, recto quidem maior eſt: minoris autem ſegmenti angulus, minor eſt recto.



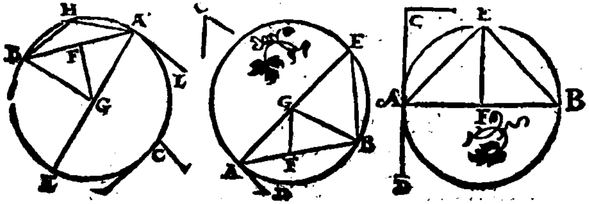
Theorema 28. Propoſitio 32.

Si circulum tetigerit aliqua recta linea, à contactu autem producatur quædam recta linea circulum ſecans: anguli quos ad contingentem facit, æquales ſunt ijs qui in alternis circuli ſegmentis conſiſtunt, angulis.



Problema 5. Propoſitio 33.

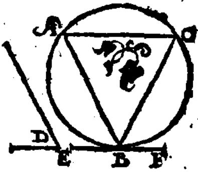
Super data recta linea deſcribere ſegmentū circuli quod capiat angulum æqualem dato angulo rectilineo.



Proble-

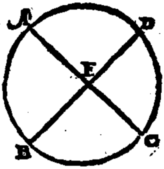
Problema 6. Propositio 34.

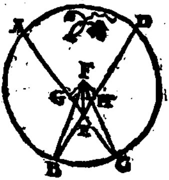
A dato circulo segmentum abscindere capiens angulum æqualem dato angulo rectilineo.



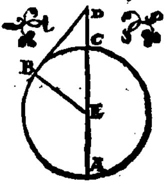
Theorema 29. Propositio 35.

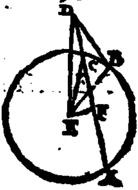
Si in circulo duæ rectæ lineæ sese mutuò secuerint, rectangulum comprehensum sub segmentis vnius, æquale est ei, quod sub segmẽtis alterius cõprehenditur, rectangulo.





Theorema 30. Propositio 36.

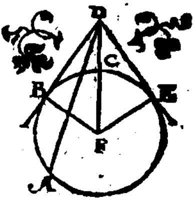




Si extra circulum sumatur pũctũ aliquod, ab eoque in circulum cadant duæ rectę lineæ, quarum altera quidem circulum secet, altera verò tangat: quod ſub tota ſecante & exterius inter punctum & conuexam peripheriam aſſumpta comprehenditur rectangulum, æquale erit ei, quod à tangente deſcribitur, quadrato.

Theorema 31. Propoſitio 37.

Si extra circulum ſumatur punctũ aliquod, ab eoque puncto in circulum cadant duæ rectæ lineæ, quarum altera circulum ſecet, altera in eum incidat, ſit autem quod ſub tota ſecãte & exterius inter punctum & conuexam peripheriã aſſumpta, comprehenditur rectangulũ, æquale ei, quod ab incidente deſcribitur quadrato: incidens ipſa circulum tanget.



ELEMENTI III. FINIS

EVCLI-

EVCLIDIS ELEMENTVM QVARTVM.

DEFINITIONES.

1

FIgura rectilinea in figura rectilinea inſcribi dicitur, cùm ſinguli eius figuræ quæ inſcribitur, anguli ſingula latera eius, in qua inſcribitur, tangunt.

2

Similiter & figura circum figuram deſcribi dicitur, quum ſingula eius quæ circunſcribitur, latera ſingulos eius figurę angulos tetigerint, circum quam illa deſcribitur.

3

Figura rectilinea in circulo inſcribi dicitur, quum ſinguli eius figuræ quæ inſcribitur,

angu-

anguli tetigerint circuli peripheriam.

4

Figura verò rectilinea circa circulum describi dicitur, quum singula latera eius, quæ circum scribitur, circuli peripheriam tangunt.

5

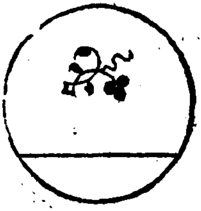
Similiter & circulus in figura rectilinea inscribi dicitur, quum circuli peripheria singula latera tangit eius figuræ, cui inscribitur.

6

Circulus autem circum figuram describi dicitur, quum circuli peripheria singulos tangit eius figuræ, quam circunscribit, angulos.

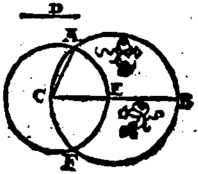
7

Recta linea in circulo accommodari seu coaptari dicitur, quum eius extrema in circuli peripheria fuerint.



Problema 1. Propositio 1.

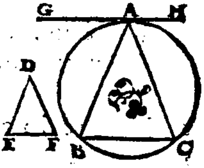
In dato circulo, rectam lineam accommodare æqualem datæ rectæ lineæ, quæ circuli diametro nō sit maior.



Proble-

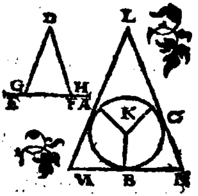
Problema 2. Propositio 2.

In dato circulo, triangulum describere dato triangulo æquiangulum.



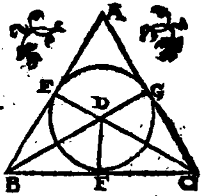
Problema 3. Propositio 3.

Circa datum circulũ triangulũ, describere dato triangulo æquiangulum.



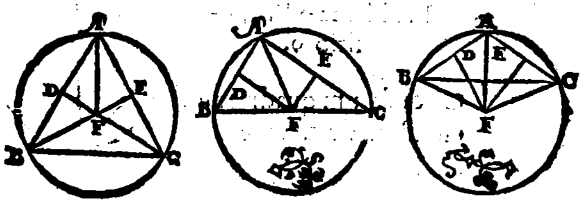
Problema 4. Propositio 4.

In dato triangulo circulum inscribere.

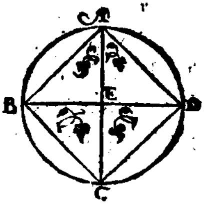


Problema 5. Propositio 5.

Circa datum triangulum, circulum describere.

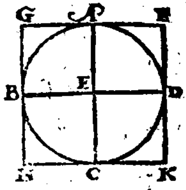


Problema 6. Propositio 6.



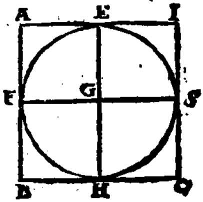
In dato circulo quadratũ describere.

Problema 7. Propositio 7.



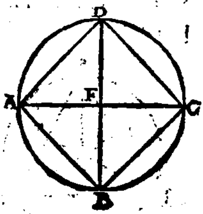
Circa datum circulum, quadratum describere.

Problema 8. Propositio 8



In dato quadrato circulum inscribere.

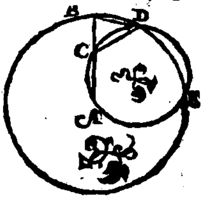
Problema 9. Propositio 9.



Circa datum quadratum, circulum describere.

Proble-

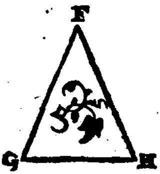
Problema 10. Propositio 10.



Isosceles triangulum constituere, quod habeat vtrunque eorum, qui ad basin sunt, angulorum, duplum reliqui.

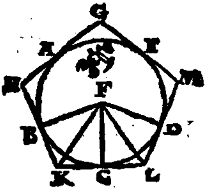
Theorema 11. Propositio 11.

In dato circulo, pentagωnum æquilaterum & æquiangulum inscribere.



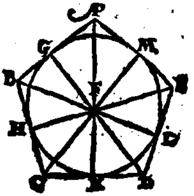
Problema 12. Propositio 12.

Circa datum circulum, pentagωnum æquilaterum & æquiangulum describere.



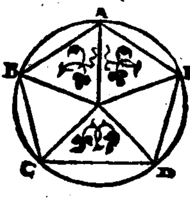
Problema 13. Propositio 13.

In dato pentagωno æquilatero & æquiangulo, circulum inscribere.



Proble-

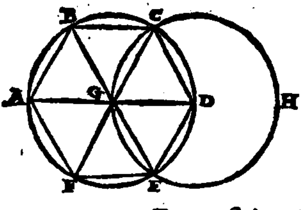
Problema 14. Propositio 14.

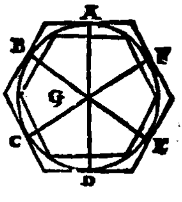


Circa datum pentagonũ, æquilaterum & æquiangulum, circulum describere.

Problema 15. Propositio 15.

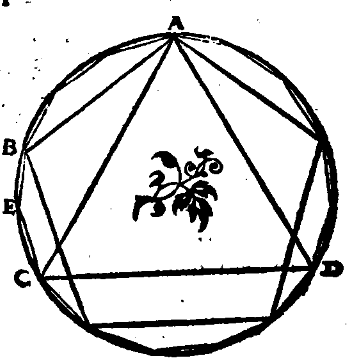
In dato circulo hexagonum & æquilaterum & æquiangulum inscribere.





Propositio 16. Theorema 16.

In dato circulo quintidecagonum & æquilaterũ & æquiangulum describere.



Elementi quarti finis.

EVCLIDIS ELEMENTVM QVINTVM.

DEFINITIONES.

1

PArs est magnitudo magnitudinis minor maioris, quum minor metitur maiorem.

2

Multiplex autem est maior minoris, cùm minor metitur maiorem.

3

Ratio, est duarum magnitudinum eiusdem generis mutua quædam secundum quantitatem habitudo.

4

Proportio verò, est rationum similitudo.

5

Rationem habere inter se magnitudinis dicuntur, quæ possunt multiplicatæ sese mutuò superare.

6

In eadem ratione magnitudines dicuntur esse, prima ad secundam, & tertia ad quartam: cùm primæ & tertiæ æquè multiplicia, à secundæ & quartæ æque multiplicibus, qualiſcunque ſit hæc multiplicatio, vtrunq; ab vtroque: vel vnà deficiunt, vel vnà æqualia ſunt, vel vnà excedunt, ſi ea ſumantur quę inter ſe reſpondent.

7

Eandem autem habentes rationem magnitudines, proportionales vocentur.

8

Cùm verò æquè multiplicium, multiplex primæ magnitudinis exceſſerit multiplicem ſecundæ, at multiplex tertiæ non exceſſerit multiplicem quartæ: tunc prima ad ſecundam, maiorem rationem habere dicetur, quàm tertia ad quartam.

9

Proportio autem in tribus terminis pauciſſimis conſiſtit.

10

Cùm autem tres magnitudines proportionales fuerint, prima ad tertiam, duplicatam rationem habere dicitur eius, quam habet ad ſecundam. At cùm quatuor magnitudines proportionales fuerint, prima ad quartam, triplicatam rationem habere dicitur eius quam habet ad ſecundam: & ſemper deinceps vno amplius, quandiu proportio extiterit.

11

Homologæ, ſeu ſimiles ratione magnitudines dicuntur, antecedentes quidem antecedentidentibus, consequentes verò consequẽtibus.

12

Alterna ratio, est sumptio antecedentis comparati ad antecedentem, & consequentis ad consequentem.

13

Inuersa ratio, est sumptio consequentis, ceu antecedentis, ad antecedentem velut ad consequentem.

14

Compositio rationis, est sumptio antecedentis cum consequente ceu vnius, ad ipsum consequentem.

15

Diuisio rationis, est sumptio excessus quo consequentem superat antecedens ad ipsum consequentem.

16

Conuersio rationis, est sumptio antecedentis ad excessum, quo superat antecedens ipsum consequentem.

17

Ex æqualitate ratio est, si plures duabus sint magnitudines, & his aliæ multitudine pares quæ binæ sumantur, & in eadem ratione: quum vt in primis magnitudinibus prima ad vltimam, sic & in secundis magnitudinibus prima ad vltimam sese habuerit, vel aliter, sumptio extremorum per subductionem mediorum.

18

Ordinata proportio eſt, cùm fuerit quemadmodum antecedens ad conſequentem, ita antecedens ad conſequentem: fuerit etiam vt conſequeus ad aliud quidpiam, ita conſequens ad aliud quidpiam.

19

Perturbata autem proportio eſt, tribus poſitis magnitudinibus, & alijs quæ ſint his multitudine pares, cùm vt in primis quidem magnitudinibus ſe habet antecedens ad cõſequentem, ita in ſecundis magnitudinibus antecedens ad conſequentem: vt autem in primis magnitudinibus conſequens ad aliud quidpiam, ſic in ſecundis magnitudinibus aliud quidpiam ad antecedentem.

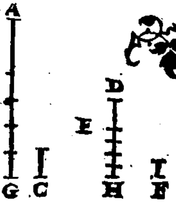
Theorema 1. Propoſitio 1.

Si ſint quotcunque magnitudines quotcunque magnitudinum æqualium numero, ſingulæ ſingularum æquè multiplices, quàm multiplex eſt vnius vna magnitudo, tam multiplices erunt, & omnes omnium.

Theorema 2. Propoſitio 2.

Si prima ſecundæ æquè fuerit multiplex, atque

que tertia quartæ, fuerit autem & quinta ſecundæ æquè multiplex, atque ſexta quartæ: erit & compoſita prima cum quinta, ſecundæ ęquè multiplex, atque tertia cum ſexta, quartæ.



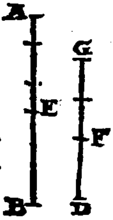
Theorema 3. Propoſitio 3.

Si ſit prima ſecundæ ęquè multiplex atq; tertia quartæ, ſumantur autem æquè multiplices primæ & tertiæ. erit & ex ęquo ſumptarum vtraque vtriuſque æquè multiplex, altera quidem ſecundæ, altera autem quartæ.

Theorema 4. Propoſitio 4.

Si prima ad ſecundam, eandem habuerit rationem, & tertia ad quartam: etiam æquè multiplices primæ & tertiæ, ad æquè multiplices ſecundæ & quartæ iuxta quamuis multiplicationem, eandem habebunt rationem, ſi prout inter ſe

reſpondent, ita ſumptæ fuerint.



Theorema 5. Propoſitio 5.

Si magnitudo magnitudinis æquè fuerit multiplex, atque ablata ablatæ: etiam reliqua reliquæ ita multiplex erit, vt tota totius.

Theorema 6. Propoſitio 6.

Si duæ magnitudines, duarum magnitudinum ſint æquè multiplices, & detractæ quædam ſint earundem æquè multiplices: & reliquæ eiſdem aut æquales ſunt, aut æquè ipſarum multiplices.

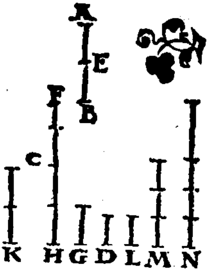
Theorema 7. Propoſitio 7.

Aequales ad eandem, eandem habent rationem: & eadem ad æquales.

Theorema 8. Propoſitio 8.

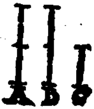
Inæqualium magnitudinum, maior ad eandem

dem maiorem rationem habet, quàm minor. & eadem ad minorem: maiorem rationẽ habet, quàm ad maiorem.



Theorema 9. Propositio 9.

Quæ ad eandem, eandem habent rationem, æquales sunt inter se: & ad quas eadem, eandem habet rationem, eæ quoque sunt inter se æquales.



Theorema 10. Propositio 10.

Ad eandem magnitudinem, rationem habentium, quæ maiorem rationem habet, illa maior est, ad quam autem eadem maiorem rationem habet, illa minor est.

Theorema 11. Propositio 11.

Quæ eidem sunt eædem rationes, & inter se sunt eædem.

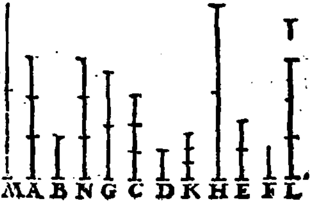
Theorema 12. Propositio 12.

Si sint magnitudines quotcunque proportionales, quemadmodum se habuerit vna antecedentium ad vnã

consequentium, ita se habebunt omnes antecedentes ad omnes consequentes.

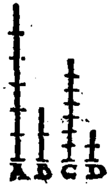
Theoroma 13 Propositio 13.

Si prima ad secundam, eandem habuerit rationem, quam tertia ad quartam, tertia verò ad quartam, maiorem rationem habuerit, quàm quinta ad sextam: prima quoque ad secũdam maiorem rationem habebit, quàm quinta ad sextam.



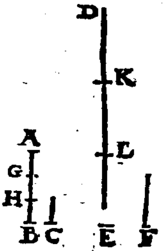
Theorema 14 Propositio 14.

Si prima ad secundam eandem habuerit rationem, quam tertia ad quartam, prima verò quàm tertia maior fuerit: erit & secunda maior quàm quarta. Quòd si prima fuerit æqualis tertiæ, erit



&

& ſecunda æqualis quartæ: ſi verò minor, & minor erit.



Theorema 15. Propoſitio 15.

Partes cum pariter multiplicibus in eadem ſunt ratione, ſi prout ſibi mutuo reſpondent, ita ſumantur.

Theorema 16. Propoſitio 16.

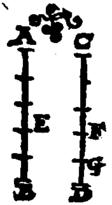
Si quatuor magnitudines proportionales fuerint, & viciſsim proportionales erunt.

Theorema 17. Propoſitio 17.

Si compoſitæ magnitudines proportionales fuerint hæ quoque diuiſæ proportionales erunt.

 Theore-

Thorema 18. Propositio 18.



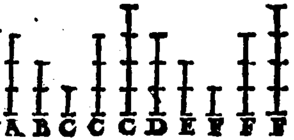
Si diuisæ magnitudines sint proportionales, hæ quoque compositæ proportionales erunt.

Theorema 19. Propositio 19.

Si quemadmodum totum ad totum, ita ablatum se habuerit ad ablatum: & reliquum ad reliquum, vt totum ad totum se habebit.

Theorema 20. Propositio 20.

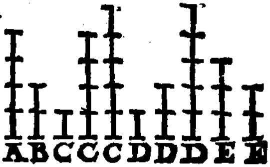
Si sint tres magnitudines, & aliæ ipsis æquales numero, quæ binæ & in eadem ratione sumantur, ex æquo autem prima quàm tertia maior fuerit: erit & quarta, quàm sexta maior. Quòd si prima tertiæ fuerit æqualis, erit & quarta æqualis sextæ: sin illa minor, hæc quoque minor erit.



Theore-

Theorema 21. Propositio 21.

Si sint tres magnitudines, & aliæ ipsis æquales numero quæ binæ & in eadem ratione sumantur, fueritque perturbata earum proportio, ex æquo autem prima quàm tertia maior fuerit, erit & quarta quá sexta maior: quòd si prima tertiæ fuerit æqualis, erit & quarta æqualis sextæ: sin illa minor, hæc quoque minor erit.



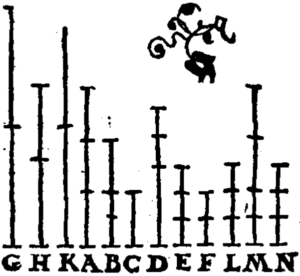
Theorema 22. Propositio 22.

Si sint quotcunq; magnitudines, & aliæ ipsis æquales numero, quæ binæ in eadē ratione sumātur, & ex æqualitate in eadem ratione erunt.

Theorema 23. Propositio 23.

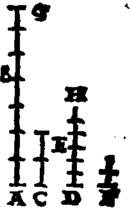
Si sint tres magnitudines, aliæque ipsis ęquales

les numero, quę binæ in eadem ratione ſumantur, fuerit autē, perturbata earū proportio: etiam ex æqualitate in eadem ratione erunt.



Theorema 24. Propoſitio 24.

Si prima ad ſecundam, eandem habuerit rationem, quam tertia ad quartam, habuerit autem & quinta ad ſecundam eandem rationem, quam ſexta ad quartam: etiam cōpoſita prima cum quinta ad ſecundam eandem habebit rationem, quam tertia cum ſexta ad quartá.



Theorema 25. Propoſitio 25.

Si quatuor magnitudines proportionales fuerint, maxima & minima reliquis duabus maiores erunt.

ELEMENTI V. FINIS.

EVCLIDIS ELEMENTVM SEXTVM.

DEFINITIONES.

1

Similes figurę rectilineæ sunt, quæ & angulos singulos singulis æquales habent, atq; etiam latera, quæ circum angulos æquales, proportionalia.

2

Reciprocæ autem figuræ sunt, cùm in vtraque figura antecedentes & consequentes rationum termini fuerint.

3

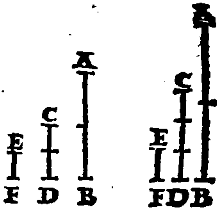
Secundum extremam & mediam rationem recta linea secta esse dicitur, cùm vt tota ad maius segmentum, ita maius ad minus se habuerit.

4

Altitudo cuiusque figuræ, est linea perpendicularis à vertice ad basin deducta.

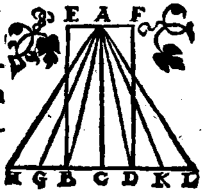
5

Ratio ex rationibus cō-poni dicitur, cùm ratio-num quantitates inter se multiplicatę aliquam ef-fecerint rationem.



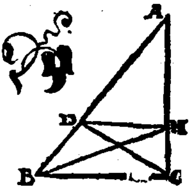
Theorema 1. Propo-sitio 1.

Triangula & parallelo-gramma, quorum eadem fuerit altitudo, ita se ha-bent inter se vt bases.



Theorema 2. Propositio 2.

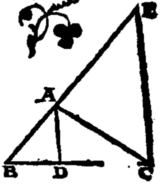
Si ad vnum trianguli latus parallela ducta fuerit recta quædam linea: hęc proportiona-liter secabit, ipsius trian-guli latera. Et si trianguli latera proportionaliter se cta fuerint: quæ ad sectio-nes adiuncta fuerit recta linea, erit ad reliquum ip-sius trianguli latus paral-lela.



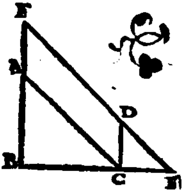
Theorema 3. Propositio 3.

Si trianguli angulus bifariam sectus sit, se-cans autem angulum recta linea secuerit & basim: basis segmenta eandem habebunt ra-tionem,

tionem, quam reliqua ipsius trianguli latera. Et si basis segmenta eandem habeant rationem quam reliqua ipsius trianguli latera, recta linea, quæ à vertice ad sectionem producitur, ea bifariam secat trianguli ipsius angulum.

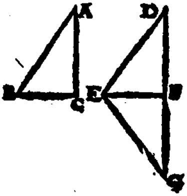


Theorema 4. Propositio 4.



Aequiangulorum triangulorum proportionalia sunt latera, quæ circum æquales angulos, & homologa sunt latera, quæ æqualibus angulis subtenduntur.

Theorema 5. Propositio 5.

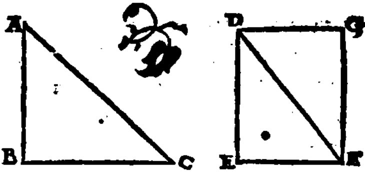


Si duo triangula latera proportionalia habeant, equiangula erunt triangula, & æquales habebunt eos angulos, sub quibus homologa latera subtenduntur.

Theorema 6. Propositio 6.

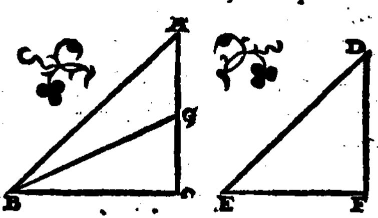
Si duo triangula vnum angulum vni angulo æqualem, & circum equales angulos latera proportionalia habuerint, æquiangula erunt trian-

triangula, æqualesq́; habebunt angulos, ſub quibus homologa latera ſubtenduntur.



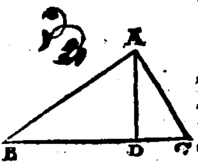
Theorema 7. Propoſitio 7.

Si duo triangula vnum angulum vni angulo æqualem, circum autem alios angulos latera proportionalia habeant, reliquorum verò ſimul vtrunque aut minorem aut non minorē recto: æquiangula erunt triangula, & æquales habebunt eos angulos, circum quos proportionalia ſunt latera.



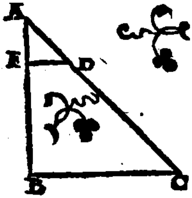
Theorema 8. Propoſitio 8.

Si in triangulo rectangulo, ab angulo recto in baſin perpendicularis ducta ſit, quæ ad perpendicularem triangula, tum toti triangulo, tum ipſa inter ſe ſimilia ſunt.



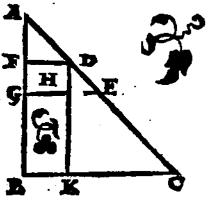
Proble-

Problema 1. Pro-
poſitio 9.



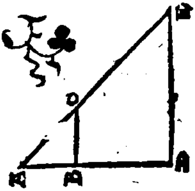
A data recta linea imperatam partem auferre.

Problema 2. Pro-
poſitio 10.



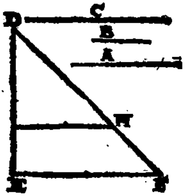
Datam rectam lineam inſectam ſimiliter ſecare, vt data altera recta ſecta fuerit.

Problema 3. Propo-
ſitio 11.



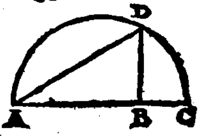
Duabus datis rectis lineis, tertiam proportionalem adinuenire.

Problema 4. Propo-
ſitio 12.



Tribus datis rectis lineis, quartam proportionalẽ adinuenire.

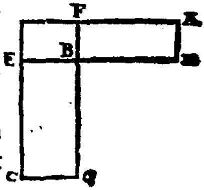
Problema 5. Propoſitio 13.



Duabus datis rectis lineis, mediam proportionalem adinuenire.

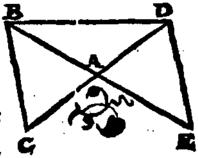
Theorema 8. Propoſitio 14.

Aequalium, & vnum vni æqualem habentium angulum parallelogrammorum reciproca ſunt latera, quæ circum æquales angulos: & quorum parallelogrammorum vnum angulum vni angulo æqualem habentium reciproca ſunt latera, quę circum æquales angulos, illa ſunt æqualia.



Theorema 10. Propoſitio 15.

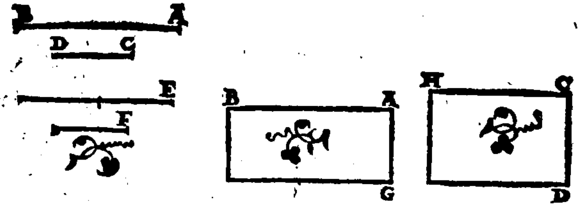
Aequalium, & vnum angulum vni æqualem habentium triangulorum reciproca ſunt latera, quæ circum æquales angulos: & quorum triangulorum vnum angulum vni æqualem habentium reciproca ſunt latera, quæ circum æquales angulos, illa ſunt æqualia.



Theore-

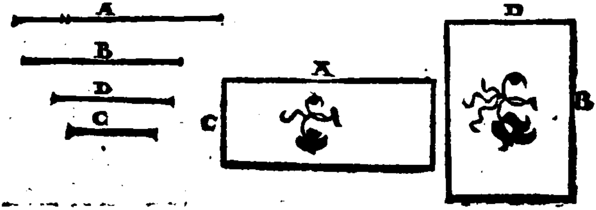
Theorema 11. Propoſitio 16.

Si quatuor rectæ lineæ proportionales fuerint, quod ſub extremis comprehenditur rectangulum, æquale eſt ei, quod ſub medijs comprehenditur rectangulo. Et ſi ſub extremis comprehenſum rectangulum æquale fuerit ei, quod ſub medijs cōtinetur rectangulo, illæ quatuor rectæ lineæ proportionales erunt.



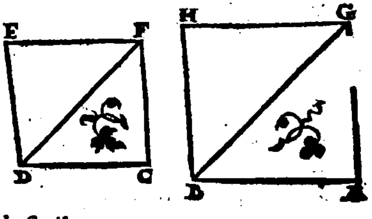
Theorema 12. Propoſitio 17.

Si tres rectæ lineæ ſint proportionales, quod ſub extremis comprehenditur rectangulum æquale eſt ei, quod à media deſcribitur quadrato: & ſi ſub extremis comprehenſum rectangulum æquale ſit ei quod à media deſcribitur quadrato, illæ tres rectæ lineæ proportionales erunt.



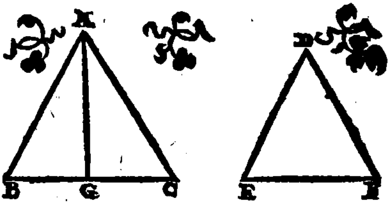
Problema 6. Propoſitio 18.

A data recta linea, dato rectilineo ſimile ſimiliterque poſitum rectilineum deſcribere.



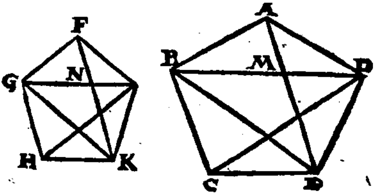
Theorema 13. Propoſitio 19.

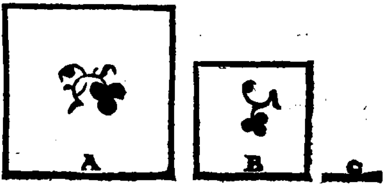
Similia triãgula inter ſe ſũt in duplicata ratiõe laterũ homologorũ.



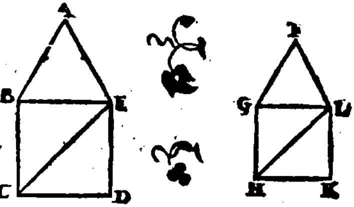
Theore. 14. Propoſitio 20.

Similia polygona in ſimilia triangula diuiduntur, & numero æqualia, & homologa totis. Et po



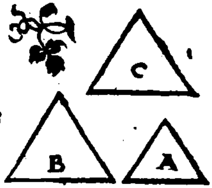


lygona duplicatam habent eã inter ſe rationẽ, quã latus homologum ad homologum latus.



Theorema 15. Propoſitio 21.

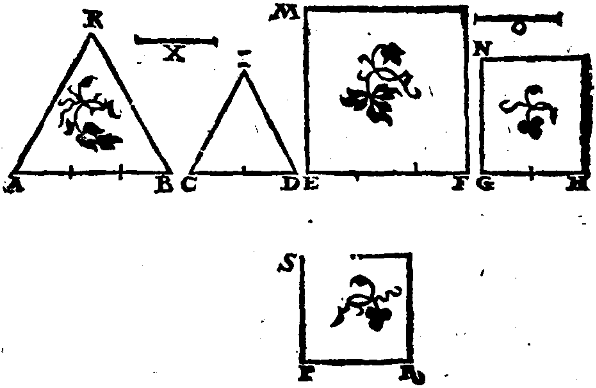
Quæ eidem rectilineo ſunt ſimilia, & inter ſe ſunt ſimilia.



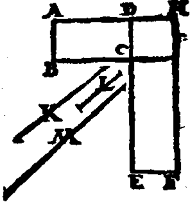
Theorema 16. Propoſitio 22.

Si quatuor rectæ lineæ proportionales fuerint: & ab eis rectilinea ſimilia ſimiliterque deſcripta proportionalia erunt. Et ſi à rectis lineis ſimilia ſimiliterque deſcripta rectilinea proportionalia fuerint, ipſæ etiam re-

&tæ lineæ proportionales erunt.

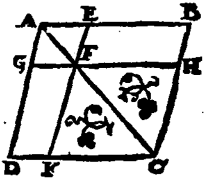


Theorema 17. Propositio 23.



Aequiangula parallelogramma inter se rationẽ habent eam, quæ ex lateribus componitur.

Theorema 18. Propositio 24.

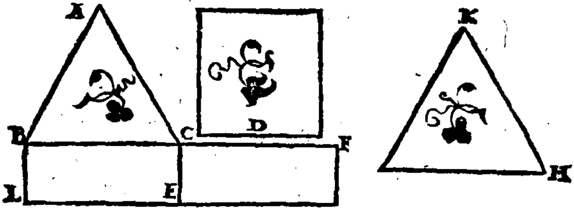


In omni parallelogrammo, quæ circa diametrum sunt parallelogramma, & toti & inter se sunt similia.

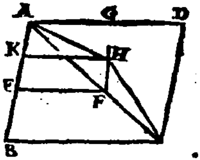
Proble-

Problema 7. Propositio 25.

Dato rectilineo simile, & alteri dato æquale idem constituere.



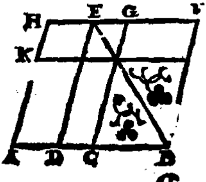
Theorema 19 Propositio 26.

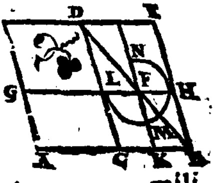


Si à parallelogrammo parallelogrammum ablatũ sit, & simile toti & similiter positum communem cum eo habens angulum, hoc circum eandem cum toto diametrum consistit.

Theorema 20. Propositio 27.

Omnium parallelogrammorum secundum eandem rectam lineam applicatorum deficientiumq; figuris parallelogrammis si-

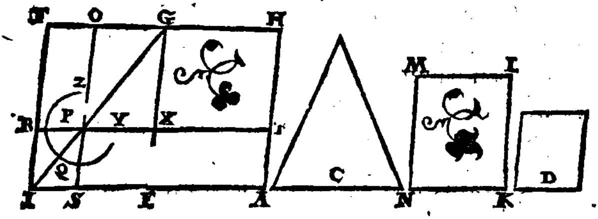




milibus similiterque positis ei, quod à dimidia describitur, maximum, id est, quod ad dimidiam applicatur parallelogrammum simile existens defectui.

Problema 8. Propositio 28.

Ad datam lineam rectam, dato rectilineo æquale parallelogrammum applicare deficiens figura parallelogramma, quæ similis sit alteri rectilineo dato. Oportet autem datum rectilineum, cui æquale applicandum est, non maius esse eo quod ad dimidiam applicatur, cùm similes sint defectus & eius quod à dimidia describitur, & eius cui simile deesse debet.

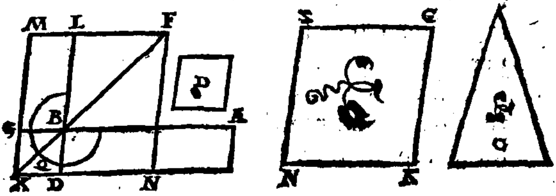


Problema 9. Propositio 29.

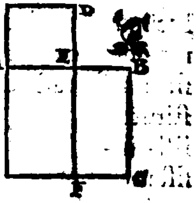
Ad datam rectam lineam, dato rectilineo æquale parallelogrammum applicare, excedens figura parallelogramma, quæ similis sit

paral-

parallelogrammo alteri dato.



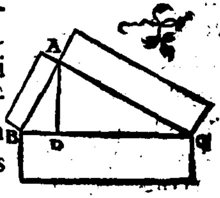
Problema 10. Propositio 30.



Propositam rectam lineam terminatam, extrema ac media ratione secare.

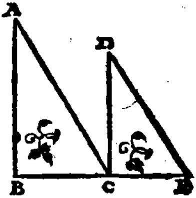
Theorema 21. Propositio 31.

In rectangulis triangulis, figura quæuis à latere rectū angulum subtendente descripta æqualis est figuris, quæ priori illi similes & similiter positæ à lateribus rectum angulum continentibus describuntur.



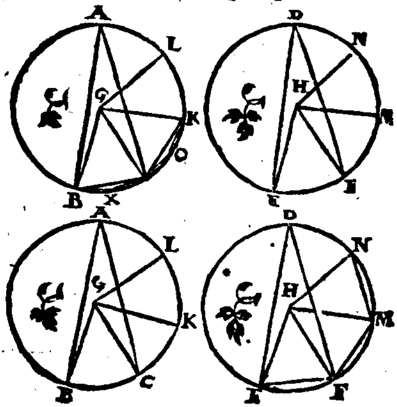
Theorema 22. Propositio 32.

Si duo triangula, quæ duo latera duobus lateribus proportionalia habeant, secundum vnum angulum composita fuerint, ita vt homologa eorum latera ſint etiã parallela, tum reliqua illorum triangulorum latera in rectam lineam col locata reperientur.



Theorema 23. Propoſitio 33.

Inæqualibus circulis anguli eandem habent rationem cum ipſis peripherijs in quibus inſiſtunt, ſiue ad centra, ſiue ad peripherias cõſtituti illis inſiſtant peripherijs. Inſuper verò & ſectores, quippe qui ad centra conſiſtunt.



ELEMENTI VI. FINIS.

EVCLIDIS ELEMENTVM SEPTIMVM.

DEFINITIONES.

1

Vnitas, eſt ſecundum quam entium quodque dicitur vnum.

2

Numerus autem, ex vnitatibus compoſita multitudo.

3

Pars, eſt numerus numeri minori maioris, cùm minor metitur maiorem.

4

Partes autem, cùm non metitur.

5

Multiplex verò, maior minoris, cùm maiorem metitur minor.

6

Par numerus eſt, qui bifariam diuiditur.

7

Impar verò, qui bifariam non diuiditur: vel, qui vnitate differt à pari.

8

Pariter par numerus eſt, quem par numerus metitur per numerum parem.

9 Pari-

9

Pariter autem impar, est quem par numerus metitur per numerum imparem.

10

Impariter verò impar numerus, est quem impar numerus metitur per numerum imparem.

11

Primus numerus, est quem vnitas sola metitur.

12

Primi inter se numeri sunt, quos sola vnitas mensura communis metitur.

13

Compositus numerus est, quem numerus quispiam metitur.

14

Compositi autem inter se numeri sunt, quos numerus aliquis mensura communis metitur.

15

Numerus numerum multiplicare dicitur, cùm toties compositus fuerit is, qui multiplicatur, quot sunt in illo multiplicante vnitates, & procreatus fuerit aliquis.

16

Cùm autem duo numeri mutuò sese multiplicantes quempiam faciunt, qui factus erit planus appellabitur, qui verò numeri mutuò sese multiplicarint, illius latera dicentur.

17 Cùm

17

Cùm verò tres numeri mutuò sese multiplicantes quempiam faciunt, qui procreatus erit solidus appellabitur, qui autem numeri mutuò sese multiplicarint, illius latera dicentur.

18

Quadratus numerus est, qui æqualiter æqualis: vel, qui à duobus æqualibus numeris continetur.

19

Cubus verò, qui æqualiter æqualis æqualiter. vel, qui à tribus æqualibus numeris continetur,

20

Numeri proportionales sunt, cùm primus secundi, & tertius quarti æquè multiplex est, vel eadem pars, vel eædem partes.

21

Similes plani & solidi numeri sunt, qui proportionalia habent latera.

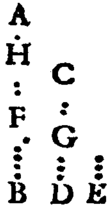
22

Perfectus numerus est, qui suis ipsius partibus est æqualis.

Theorema 1. Propositio 1.

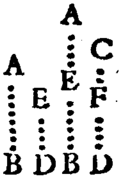
Duobus numeris inæqualibus propositis,

tis, si detrahatur semper minor de maiore, alterna quadam detractione, neque reliquus vnquam metiatur præcedentem quoad assumpta sit vnitas: qui principio propositi sunt numeri primi inter se erunt.

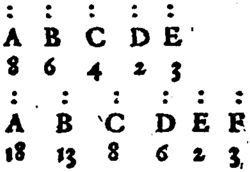


Problema 1. Propositio 2.

Duobus numeris datis non primis inter se, maximam eorum communem mensuram reperire.



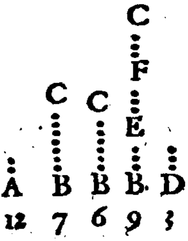
Problema 2. Prop. 2.



Tribus numeris datis non primis inter se, maximã eorum communem mensuram reperire.

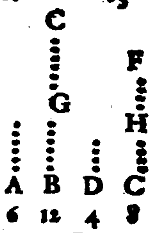
Theorema 2. Propositio 4.

Omnis numerus, cuiusq; numeri minor maioris aut pars est, aut partes.



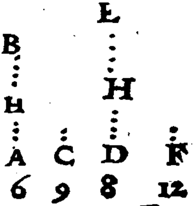
Theore-

Theorema 3. Propositio 5.



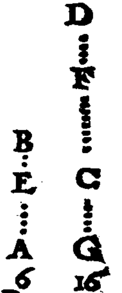
Si numerus numeri pars fuerit, & alter alterius eadem pars, & simul vterque vtriusque simul eadem pars erit, quæ vnus est vnius.

Theor. 4. Propo. 6.



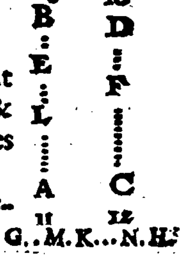
Si numerus sit numeri partes, & alter alterius eædem partes, & simul vterque vtriusque simul eædem partes erunt, quæ sunt vnus vnius.

Theorema 5. Propositio 7.



Si numerus numeri eadẽ sit pars quæ detractus detracti, & reliquus reliqui eadem pars erit, quæ totus est totius.

Theorema 6. Propositio 8.

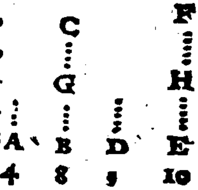


Si numerus numeri eædem sint partes quæ detractus detracti, & reliquus reliqui eædem partes erunt, quæ sunt totus totius.

Theore-

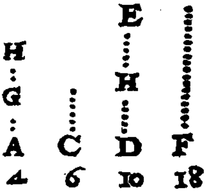
Theorema 7. Propoſitio 9.

Si numerus numeri pars ſit, & alter alterius eadem pars, & viciſsim quæ pars eſt vel partes primus tertij, eadem pars erit vel eædem partes & ſecundus quarti.



Theorema 8. Propoſitio 10.

Si numerus numeri partes ſint, & alter alterius eędem partes, etiam viciſſim quæ ſunt partes aut pars primus tertij, eædem partes erunt vel pars & ſecundus quarti.

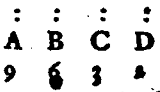


Theorema 9. Propoſitio 11.

Si quemadmodum ſe habet totus ad totum, ita detractus ad detractũ, & reliquus ad reliquum ita habebit vt totus ad totum.

Theorema 10. Propoſitio 12.

Si ſint quotcunque numeri proportionales, quemadmodum ſe habet vnus antecedentium ad vnum ſequentium, ita ſe habe



habebunt omnes antecedentes ad omnes consequentes.

Theorema 11. Propositio 13.

Si quatuor numeri sint proportionales, & vicissim proportionales erunt.

A	B	C	D
12	4	9	3

Theorema 12. Propositio 14.

Si sint quotcunque numeri & alij illis æquales

A	B	C	D	E	F
12	6	3	8	4	2

multitudine, qui bini sumantur & in eadem ratione: etiam ex æqualitate in eadem ratione erunt.

Theorema 13. Propositio 15.

Si vnitas numerum quempiam metiatur, aliter verò numerus alium quendam numerum æquè metiatur, & vicissim vnitas tertium numerum æquè metietur, atque secundus quartum.

C H G A B D F L K E

1 3 2 6

Theorema 14. Propositio 16.

Si duo numeri mutuò sese multiplicantes faciant aliquos. qui ex illis geniti fuerint, inter se æquales erunt.

E	A	B	C	D
1	2	4	8	8

Theorema 15. Propositio 17.

Si numerus duos numeros multiplicans faciat aliquos, qui ex illis procreati erunt, eandem rationem habebunt, quam multiplicati.

I	A	B	C	D	E
1	3	4	5	12	15

Theorema 16. Propositio 18.

Si duo numeri numerum quempiam multiplicantes faciant aliquos, geniti ex illis eandem habebunt rationem, quam qui illum multiplicarunt.

A	B	C	D	E
4	5	3	12	15

Theorema 17. Propositio 19.

Si quatuor numeri sint proportionales, qui ex primo & quarto fit, æqualis erit ei qui ex secundo & tertio: & si qui ex primo & quarto fit numerus æqualis sit ei qui ex secundo & tertio, illi quatuor numeri proportionales erunt.

A	B	C	D	E	F	G
6	4	3	2	12	12	18

Theorema 18. Propositio 20.

Si tres numeri sint proportionales, qui ab extremis continetur æqualis est ei qui à medio

dio

dio efficitur. Et si qui ab extremis continetur æqualis sit ei qui à medio describitur, illi tres numeri proportionales erunt.

:	:	:
A	B	C
9	6	4
:		
D		
6		

Theorema 19. Propositio 21.

Minimi numeri omnium, qui eandem cum eis rationem habent, æqualiter metiuntur numeros eandem rationem habentes, maior quidem maiorem, minor verò minorem.

D	L		
:	.		
G	H		
:	:	:	:
C	E	A	B
4	3	8	6

Theorema 20. Propositio 22.

Si tres sint numeri & alij multitudine illis æquales, qui bini sumantur & in eadem ratione, sit autem perturbata eorum proportio, etiam ex æqualitate in eadem ratione erunt.

:	:	:	:	:	:
A	B	C	D	E	F
6	4	3	12	8	6

Theorema 21. Propositio 23.

Primi inter se numeri minimi sunt omnium eandem cum eis rationem habentiũ.

:	:	:	:	:
A	B	E	C	D
5	6	2	4	3
H	2			

Theo-

Theorema 22. Propositio 24.

Minimi numeri omnium eandem cum eis rationem habētium, primi sunt inter se.

A	B	C	D	E
8	6	4	3	2

Theorema 23. Propositio 25.

Si duo nùmeri sint primi inter se, qui alterutrum illorum metitur numerus, is ad reliquū primus erit.

A	B	C	D
6	7	3	4

Theorema 24. Propositio 26.

Si duo numeri ad quempiam numerum primi sint, ad eundem primus is quoque futurus est, qui ab illis productus fuerit.

B	A	C	D	E	F
3	5	5	5	3	2

Theorema 25. Propositio 27.

Si duo numeri primi sint inter se, qui ab vno eorum gignitur, ad reliquum primus erit.

A	B	C	D
7		6	3

Theorema 26. Propositio 28.

Si duo numeri ad duos numeros ambo ad vtrunque primi sint, & qui ex eis gignentur, primi inter se erunt.

A	B	E	C	D	F
3	5	15	2	4	8

Theore-

Theorema 27. Propositio 29.

Si duo numeri primi ſint inter ſe, & multiplicans vterque ſeipſum procreet aliquem, qui ex ijs producti fuerint, primi inter ſe erunt. Quòd ſi numeri initio propoſiti multiplicantes eos qui producti ſunt, effecerint aliquos, hi quoq; inter ſe primi erunt, & circa extremos idē hoc ſemper eueniet.

A	C	E	B	D	F
3	6	27	4	16	63

Theorema 28. Propoſitio 30.

Si duo numeri primi ſint inter ſe, etiam ſimul vterq; ad vtrunq; illorum primus erit. Et ſi ſimul vterq; ad vnum aliquem eorum primus ſit, etiam qui initio poſiti ſunt numeri, primi inter ſe erunt.

	C	
A	B	D
7	5	4

Theorema 29. Propoſitio 31.

Omnis primus numerus ad omnem numerum quem non metitur, primus eſt.

A	B	C
7	10	5

Theorema 30. Propoſitio 32.

Si duo numeri ſeſe mutuò multiplicantes faciant aliquem, hunc aūt ab illis productum metiatur primus quidam numerus, is alterum etiam metitur eorum qui initio poſiti erant.

A	B	C	D	E
2	6	12	3	4

Theorema 31. Propositio 33.

Omnem compositum numerum aliquis primus metietur.

A	B	C
27	9	3

Theorema 32. Propositio 34.

Omnis numerus aut primus est, aut eum aliquis primus metitur.

A	A	
3	6	3

Problema 3. Propositio 35.

Numeris datis quotcunque, reperire minimos omnium qui eandem cum illis rationem habeant.

A	B	C	D	E	F	G	H	K	I	M
6	8	12	2	3	4	6	2	3	4	3

Problema 4. Propositio 36.

B				
A	C	D	E	F
7	12	8	4	5

Duobus numeris datis, reperire quẽ quem illi minimũ metiantur numerum.

A					
F	E	C	D	G	H
6	9	12	9	8	5

Theo-

Theorema 33. Propositio 37.

Si duo numeri numerum quempiam metiantur, & minimus quem illi metiuntur eundem metietur.

				F
A	B	E	C	
2	3	6	12	

Problema 5. Propositio 38.

Tribus numeris datis reperire quem minimum numerú illi metiantur.

A	B	C	D	E	
3	4	6	12	8	
A	B	C	D	E	F
3	6	8	12	24	16

Theorema 34. Propositio 39.

Si numerum quispiam numerus metiatur, mensus partem habebit metienti cognominem.

A	B	C	D
12	4	3	1

Theorema 35. Propositio 40.

Si numerus partem habuerit quamlibet, illum metietur numerus parti cognominis.

A	B	C	D
8	4	2	1

Problema 6. Propositio 41.

Numerum reperire, qui minimus cùm sit, datas habeat partes.

A	B	C	G	H
2	3	4	12	10

ELEMENTI VII. FINIS.

EVCLIDIS ELEMENTVM OCTAVVM.

Theorema 1. Propositio 1.

Si sint quotcunq; numeri deinceps proportionales, quorum extremi sint inter se primi, minimi sunt omnium eandem cum eis rationem habentium.

	:	:	:	:	:	:	:	:
	A	B	C	D	E	F	G	H
	8	12	18	27	6	8	12	18

Problema 1. Propositio 2.

Numeros reperire deinceps proportionales minimos, quotcunque iusserit quispiam in data ratione.

:	:	:	:	:	:	:	:	:
A	B	C	D	E	F	G	H	K
3	4	9	12	16	27	36	49	64

Theorema 2. Propositio 3.

Conuersa primæ.

Si sint quotcunque numeri deinceps proportionales minimi habentium eandem cum eis rationem, illorum extremi sunt inter se primi.

:	:	:	:	:	:	:	:	:	:	:	:	:
A	B	C	D	E	F	G	H	K	L	M	N	O
27	16	48	64	3	4	9	12	16	27	36	48	64

Proble-

Problema 2. Propositio 4.

Rationibus datis quotcunque in minimis numeris reperire numeros deinceps minimos in datis rationibus.

A	B	C	D	E	F	H	G	K	L	N	X	M	O
3	4	2	3	4	5	6	8	12	15	4	6	10	12

Theorema 3. Propositio 5.

Plani numeri rationem inter se habent ex lateribus compositam.

A	L	B	C	D	E	F	G	H	K
18	22	32	3	6	4	8	9	12	16

Theorema 4. Propositio 6.

A	B	C	D	E	F	G	H
16	24	36	54	81	4	6	9

Si sint quotlibet numeri deinceps proportionales, primus autem secundum non metiatur, neque alius quispiam vllum metietur.

Theorema 5. Propositio 7.

Si sint quotcunque numeri deinceps proportionales, primus autem extremum metiatur, is etiam secundũ metietur.

A B C D
4 6 12 24

Theorema 6. Propositio 8.

Si inter duos numeros medij continua proportione incidant numeri, quot inter eos medij continua proportione incidunt numeri, tot & inter alios eandem cum illis habentes rationem, medij continua proportione incident.

A C D B G H K L C M N F
4 9 27 81 1 3 9 27 2 6 18 54

Theorema 7. Propositio 9.

Si duo numeri sint inter se primi, & inter eos medij continua proportione incidant numeri, quot inter illos medij continua proportione incidunt numeri, totidem & inter vtrunque eorum ac vnitatem deinceps medij continua proportione incident.

A M H E F N C K X G D L O E
27 27 9 36 3 36 1 12 48 4 48 16 64 64

Theo-

Theorema 8. Propositio 10.

Si inter duos numeros & vnitatem continuè proportionales incidant numeri, quot inter vtrunque ipsorum & vnitatem deinceps medij continua proportione incidunt numeri: totidem & inter illos medij continua proportione incident.

A	E	K	D	H	C	L	F	G	B
27	9	36	3	12	1	48	4	16	64

Theorema 9. Propositio 11.

Duorum quadratorum numerorum vnus medius proportionalis est numerus: & quadratus ad quadratum duplicatam habet lateris ad latus rationem.

A	C	E	D	B
9	3	12	4	16

Theorema 10. Propositio 12.

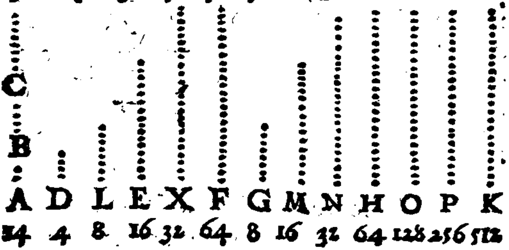
Duorum cuborum numerorum duo medij proportionales sunt numeri: & cubus ad cubum triplicatam habet lateris ad latus rationem.

A	H	K	B	C	D	E	F	G
27	36	48	64	3	4	9	12	16

Theo-

Theorema 11. Propositio 13.

Si sint quotlibet numeri deinceps proportionales,& multiplicans quisque seipsum faciat aliquos, qui ab illis producti fuerint, proportionales erunt: & si numeri primùm positi, ex suo in procreatos ductu faciant aliquos, ipsi quoque proportionales erunt.



Theorema 12. Propositio 14.

Si quadratus numerus quadratum numerum metiatur,& latus vnius metietur latus alterius. Et si vnius quadrati latus metiatur latus alterius,& quadratus quadratum metietur.

A	E	B	C	D
9	12	16	3	4

Theorema 13. Propositio 15.

Si cubus numerus cubum numerum metiatur,& latus vnius metietur alterius latus. Et si latus vnius cubi latus alterius metiatur,

tum

tum cubus cubum metietur.

A	H	K	B	C	D	E	F	G
8	16	28	64	2	4	4	8	16

Theorema 14. Propositio 16.

Si quadratus numerus quadratum numerum non metiatur, neque latus vnius metietur alterius latus. Et si latus vnius quadrati non metiatur latus alterius, neque quadratus quadratum metietur.

A	B	C	D
9	16	3	4

Theorema 15. Propositio 17.

Si cubus numerus cubum numerum non metiatur, neque latus vnius latus alterius metietur. Et si latus cubi alicuius latus alterius non metiatur. neque cubus cubum metietur.

A	B	C	D
8	27	9	11

Theorema 16. Propositio 18.

Duorum similium planorum numerorum vnus medius proportionalis est numerus: & planus ad planum duplicatam habet literis homologi

A	G	B	C	D	E	F
12	18	27	2	6	3	9

logi ad latus homologum rationem.

Theorema 17. Propositio 19.

Inter duos similes numeros solidos, duo medij proportionales incidunt numeri: & solidus ad similem solidum triplicatam rationem habet lateris homologi ad latus homologum.

A	N	X	B	C	D	E	F	G	H	K	M	L
8	12	18	27	2	2	2	3	3	3	4	6	9

Theorema 18. Propositio 20.

Si inter duos numeros vnus medius proportionalis incidat numerus, similes plani erunt illi numeri.

A	C	B	D	E	F	G
18	24	33	3	4	6	8

Theorema 19. Propositio 21.

Si inter duos numeros duo medij proportionales incidant numeri, similes solidi sunt illi numeri.

A	C	D	B	E	F	G	H	K	L	M
27	36	44	64	9	12	16	3	3	3	4

Theo-

Theorema 20. Propositio 22.

Si tres numeri deinceps sint proportionales, primus autem sit quadratus, & tertius quadratus erit.

A	B	D
9	15	25

Theorema 21. Propositio 23.

Si quatuor numeri deinceps sint proportionales, primus autem sit cubus, & quartus cubus erit.

A	B	C	D
8	12	18	27

Theorema 22. Propositio 24.

Si duo numeri rationem habeant inter se, quam quadratus numerus ad quadratum numerum, primus autem sit quadratus, & secundus quadratus erit.

A		B	C		D
4	6	9	16	24	36

Theorema 23. Propositio 25.

Si numeri duo rationem inter se habeant, quam cubus numerus ad cubum numerum, primus autem cubus sit, & secundus cubus erit.

A	E	F	B	C			D
8	12	18	27	64	95	140	216

Theo-

Theorema 24. Propositio 26.

Similes plani numeri rationem inter se habent, quam quadratus numerus ad quadratum numerum.

A	C	B	D	E	F
18	24	32	9	12	16

Theorema 25. Propositio 27.

Similes solidi numeri rationem habent inter se, quam cubus numerus ad cubum numerum.

A	C	D	B	E	F	G	H
36	24	16	54	8	12	18	27

ELEMENTI VIII. FINIS.

EVCLIDIS ELEMENTVM NONVM.

Theorema 1. Propositio 1.

Si duo similes plani numeri mutuò sese multiplicantes quendam procreent, productus quadratus erit.

A	E	B	D		C
4	6	9	16	24	36

Theorema 2. Propositio 2.

Si duo numeri mutuò sese multiplicantes quadratum faciant, illi similes sunt plani.

A		B	D		C
4	6		9	18	36

Theorema 3. Propositio 3.

Si cubus numerus seipsum multiplicans procreet aliquem, productus cubus erit.

Vnitas	D	D	A			B
	3	4	8	16	32	64

Theorema 4. Propositio 4.

Si cubus numerus cubũ numerum multiplicans quendam procreet, procreatus cubus erit.

A	B	D	C
8	27	64	216

Theorema 5. Propositio 5.

Si cubus numerus numerum quendam multiplicans cubum procreet, & multiplicatus cubus erit.

A	B	D	C
27	64	729	1728

Theorema 6. Propositio 6.

Si numerus seipsum multiplicans cubum procreet, & ipse cubus erit.

A	B	C
27	729	19683

Theorema 7. Propositio 7.

Si compositus numerus quendam numerum multiplicans quempiam procreet, productus solidus erit.

A	B	C	D	E
6	8	48	2	3

Theorema 8. Propositio 8.

Si ab vnitate quotlibet numeri deinceps proportionales sint, tertius ab vnitate quadratus est, & vnum intermittentes omnes: quartus autem cubus, & duobus intermissis omnes:

nes: septimus verò cubus simul & quadratus, & quinque intermissis omnes.

Vnitas	A	B	C	D	E	F
●	3	9	27	81	243	729

Theorema 9. Propositio 9.

Si ab vnitate sint quotcunque numeri deinceps proportionales, sit autem quadratus is qui vnitatem sequitur, & reliqui omnes quadrati erunt. Quòd si qui vnitatem sequitur cubus sit, & reliqui omnes cubi erunt.

quadrati.		cubi
531441	F	732969
59049	E	531441
6561	D	59049
729	C	6561
81	B	729
9	A	81
	●	
	Vnitas.	

Theorema 10. Propositio 10.

Si ab vnitate numeri quotcunque proportionales sint, non sit autem quadratus is qui vnitatem sequitur, neq; alius vllus quadratus erit, demptis tertio ab vnitate ac om-

Vnitas.	A	B	C	D	E	F
●	3	9	36	81	243	729

nibus vnum intermittentibus. Quòd si qui vnitatem sequitur, cubus non sit, neque alius vllus cubus erit, demptis quarto ab vnitate ac omnibus duos intermittentibus.

Theorema 11. Propositio 11.

Si ab vnitate numeri quotlibet deinceps proportionales sint, minor maiorem metitur per quempiam eorum qui in proportionalibus sunt numeris.

A	D	C	D	E
1	2	4	8	16

Theorema 12. Propositio 12.

Si ab vnitate quotlibet numeri sint proportionales, quot primorum numerorum vltimum metiuntur, totidem & eum qui vnitati proximus est, metientur.

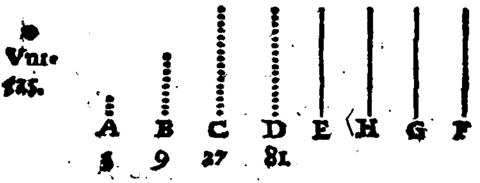
Vnitas.	A	B	C	D	E	H	G	F
	4	16	64	259	2	8	32	128

Theorema 13. Propositio 13.

Si ab vnitate sint quotcunque numeri deinceps proportionales, primus autem sit qui vnitatem sequitur, maximum nullus alius metie-

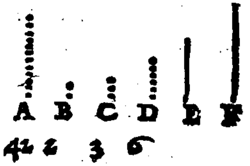
tie-

tietur, ijs exceptis qui in proportionalibus ſunt numeris.



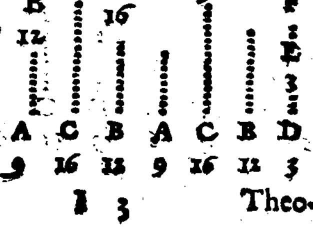
Theorema 14. Propoſitio 14.

Si minimum numerum primi aliquot numeri metiantur, nullus alius numerus primus illum metietur, ijs exceptis qui primò metiuntur.



Theorema 15. Propoſitio 15.

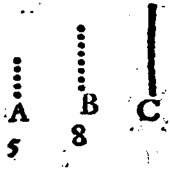
Si tres numeri deinceps proportionalis ſint minimi, eadem eam ipſis habentium rationem, duo quilibet compoſiti ad tertium primi erunt.



Theo-

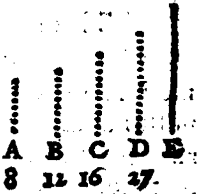
Theorema 16. Propositio 16.

Si duo numeri sint inter se primi, non se habebit quemadmodum primus ad secundum, ita secundus ad quempiam alium.



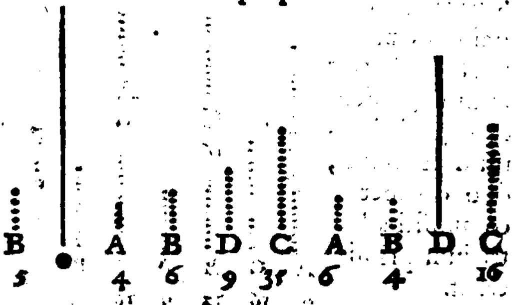
Theorema 17. Propositio 17.

Si sint quotlibet numeri deinceps proportionales, quorum extremi sint inter se primi, non erit quemadmodum primus ad secundum, ita vltimus ad quempiam alium.



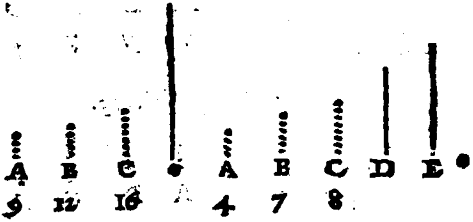
Theorema 18. Propositio 18.

Duobus numeris datis, considerare possítne tertius illis inueniri proportionalis.



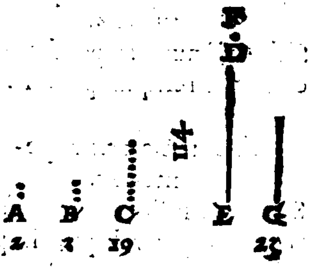
Theorema 19. Propositio 19.

Tribus numeris datis, considerare possítne quartus illis reperiri proportionalis.



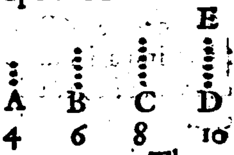
Theorema 20. Propositio 20.

Primi numeri plures sunt quacunque proposita multitudine primorum numerorum.



Theorema 21. Propositio 21.

Si pares numeri quotlibet compositi sint, totus est par.



Theorema 22 Propositio 22.

Si impares numeri quotlibet compositi sint, sit autẽ par illorum multitudo, totus par erit.

A B C D E
5 9 7 3

Theorema 23. Propositio 23.

Si impares numeri quotcunque compositi sint, sit autem impar illorũ multitudo, & totus impar erit.

A B C E
5 7 8 1

Theorema 24. Propositio 24.

Si de pari numero par detractus sit, & reliquus par erit.

B
A C
6 4

Theorema 25. Propositio 25.

Si de pari numero impar detractus sit, & reliquus impar erit.

B
A C D
8 1 4

Theorema 26. Propositio 26.

Si de impari numero impar detractus sit, & reliquus par erit.

B
A C D
4 6 1

Theorema 27. Propositio 27.

Si ab impari numero par ablatus sit, reliquus impar erit.

B
A D C
1 4 4

Theorema 28. Propositio 28.

Si impar numerus parem multiplicans, procreet quempiam, procreatus par erit.

A B C
3 4 12

Theorema 29. Propositio 29.

Si impar numerus imparem numerum multiplicãs quẽdam procreet, procreatus impar erit.

A B C
3 5 15

Theorema 30. Propositio 30.

Si impar numerus parem numerum metiatur, & illius dimidium metietur.

A C B
3 6 18

Theorema 31. Propositio 31.

Si impar numerus ad numerum quempiam primus sit, & ad illius duplum primus erit.

A B C D
7 8 16

Theorema 32. Propositio 32.

Numerorum, qui à binario dupli sunt, vnusquisque pariter par est tantùm.

Vnitas. A B C D
2 4 8 16

Theorema 33. Propositio 33.

Si numerus dimidium impar habeat, pariter impar est tantùm.

A
20

Theorema 34. Propositio 34.

Si par numerus nec sit duplus à binario, nec dimidium impar habeat, pariter par est, & pariter impar.

A
20

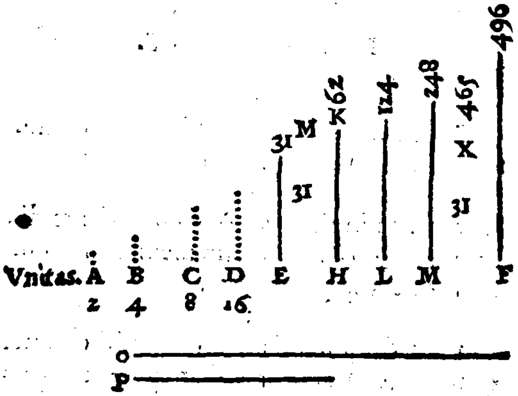
Theorema 35. Propositio 35.

Si sint quotlibet numeri deinceps proportionales, detrahantur autem de secundo & vltimo æquales ipsi primo, erit quemadmodum secundi excessus ad primum, ita vltimi excessus ad omnes qui vltimum antecedunt.

F
4
K
4
C
4
G
D B D E
4 4 16 16

Theorema 36. Propositio 36.

Si ab vnitate numeri quotlibet deinceps expositi sunt in duplici proportione quoad totus compositus primus factus sit, isque totus in vltimum multiplicatus quempiam procreet, procreatus perfectus erit.



ELEMENTI IX. FINIS.

EVCLI.

EVCLIDIS ELEMENTVM DECIMVM.

DEFINITIONES.

1

Commensurabiles magnitudines dicuntur illæ, quas eadem mensura metietur.

2

Incommensurabiles verò magnitudines dicuntur hæ, quarum nullam mensuram communem contingit reperiri.

3

Lineæ rectæ potentia commẽsurabiles sunt, quarum quadrata vna eadem superficies siue area metitur.

4

Incommensurabiles verò lineæ sunt, quarũ quadrata, quæ metiatur area communis, reperiri nulla potest.

5

Hæc cùm ita sint, ostendi potest quòd quantacunque linea recta nobis proponatur, existunt etiam aliæ lineæ innumerabiles eidem commensurabiles, aliæ item incommensurabiles, hæ quidem longitudine & poten-

potentia: illæ verò potentia tantùm. Vocetur igitur linea recta, quantacunque proponatur, ῥητὴ, id est rationalis.

6

Lineæ quoque illi ῥητῇ commensurabiles siue longitudine & potentia, siue potentia tantùm, vocentur & ipsæ ῥηταί, id est rationales.

7

Quæ verò lineæ sunt incommensurabiles illi τῇ ῥητῇ, id est primo loco rationali, vocentur ἄλογοι, id est irrationales.

8

Et quadratum quod à linea proposita describitur, quam ῥητὴν vocari volumus, vocetur ῥητόν.

9

Et quæ sunt huic commensurabilia, vocentur ῥητά.

10

Quæ verò sunt illi quadrato ῥητῷ scilicet incommensurabilia, vocentur ἄλογα, id est surda.

11

Et lineæ quæ illa incommensurabilia describunt, vocentur ἄλογοι. Et quidem si illa incommesurabilia fuerint quadrata, ipsa eorum latera vocabũtur ἄλογοι lineæ. quòd si quadrata quidem non fuerint, verùm aliæ quæpiam superficies siue figuræ rectilineæ, tunc

tunc verò lineæ illæ quæ describunt quadrata æqualia figuris rectilineis, vocentur ἄλογοι.

Theorema 1. Propositio 1.

Duabus magnitudinibus inæqualibus propositis, si de maiore detrahatur plus dimidio, & rursus de residuo iterum detrahatur plus dimidio, idque semper fiat: relinquetur quædam magnitudo minor altera minore ex duabus propositis.

Theorema 2. Propositio 2.

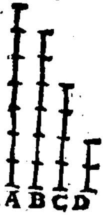
Duabus magnitudinibus propositis inæqualibus, si detrahatur semper minor de maiore, alterna quadam detractione, neque residuum vnquam metiatur id quod ante se metiebatur, incommensurabiles sunt illæ magnitudines.

Problema 1. Propositio 3.

Duabus magnitudinibus commensurabilibus datis, maximam ipsarum communem mensuram reperire.

Proble-

Problema 2. Propositio 4.



Tribus magnitudinibus commensurabilibus datis, maximam ipsarũ communem mẽsuram reperire.

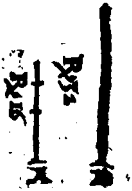
Theorema 3. Propositio 5.

Commensurabiles magnitudines inter se proportionem eam habent, quam habet numerus ad numerum.

Theorema 4. Propositio 6.

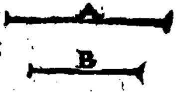
Si duæ magnitudines proportionem eam habent inter se quam numerus ad numerum, commensurabiles sunt illæ magnitudines.

Theorema 5. Propoſitio 7.



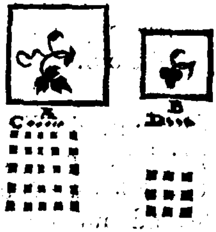
Incommenſurabiles magnitudines inter ſe proportionem non habent, quam numerus ad numerum.

Theorema 6. Propoſitio 8.



Si duæ magnitudines inter ſe proportionem non habent quam numerus ad numerũ, incommenſurabiles illæ ſunt magnitudines.

Theorema 7. Propoſitio 9.



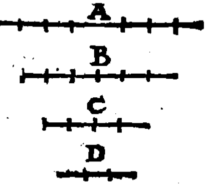
Quadrata, quæ deſcribuntur à rectis lineis lõgitudine commẽſurabilibus, inter ſe proportionem habent quam numerus quadratus ad alium numerum quadratum. Et quadrata habentia proportionẽ inter ſe quam quadratus numerus ad numerum quadratum, habent quoque latera longitudine commenſurabilia. Quadrata verò

quæ

quæ deſcribuntur à lineis longitudine incõmenſurabilibus, proportionẽ non habent inter ſe, quam quadratus numerus ad numerum alium quadratum. Et quadrata non habentia proportionem inter ſe quam numerus quadratus ad numerum quadratum, neque latera habebunt longitudine commenſurabilia.

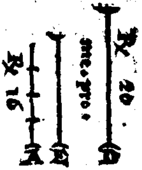
Theorema 8. Propoſitio 10.

Si quatuor magnitudines fuerint proportionales, prima verò ſecundæ fuerit commenſurabilis, tertia quoque quartæ commenſurabilis erit. quòd ſi prima ſecundæ fuerit incommenſurabilis, tertia quoque quartæ incommenſurabilis erit.



Problema 3. Propoſitio 11.

Propoſitæ lineæ rectæ (quam ῥητὴν vocari diximus) reperire duas lineas rectas incommenſurabiles, hanc quidem longitudine tantùm, illam verò non longitudine tantùm, ſed etiam potentia incommenſurabilem.

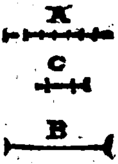


Theorema 9. Propositio 12.

Magnitudines quæ eidẽ magnitudini sunt commensurabiles, inter se quoque sunt cõmensurabiles.

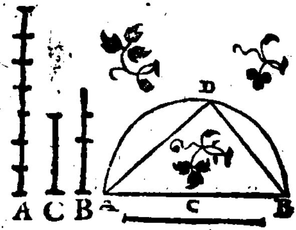
Theorema 10. Propositio 13.

Si ex duabus magnitudinibus hæc quidem commensurabilis sit tertiæ magnitudini, illa verò eidem incommensurabilis, incommensurabiles sunt illæ duæ magnitudines.



Theorema 11. Propositio 14.

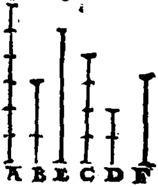
Si duarum magnitudinum commensurabilium altera fuerit incommensurabilis magnitudini alteri cuipiam tertiæ, reliqua quoque magnitudo eidẽ tertiæ incõmensurabilis erit.



Theorema 12. Propositio 15.

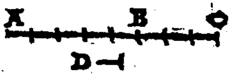
Si quatuor rectæ proportionales fuerint, possit

posſit autem prima pluſquam ſecunda tanto quantum eſt quadratum lineæ ſibi commenſurabilis longitudine: tertia quoque poterit pluſquàm quarta tanto quantum eſt quadratum lineæ ſibi commenſurabilis longitudine. Quòd ſi prima poſsit pluſquam ſecunda quadrato lineæ ſibi longitudine incommenſurabilis: tertia quoque poterit pluſquam quarta quadrato lineæ ſibi incõmenſurabilis longitudine.



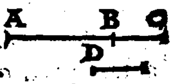
Theorema 13. Propoſitio 16.

Si duæ magnitudines commenſurabiles componantur, tota magnitudo compoſita ſingulis partibus commenſurabilis erit. quòd ſi tota magnitudo compoſita alterutri parti commenſurabilis fuerit, illæ duæ quoque partes commenſurabiles erunt.



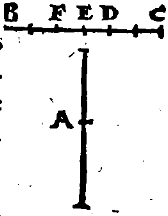
Theorema 14 Propoſitio 17.

Si duæ magnitudines incommenſurabiles componantur, ipſa quoque tota magnitudo ſingulis partibus componentibus incommenſurabilis erit. Quòd ſi tota alteri parti incommenſurabilis fuerit, illæ quoque primæ magnitudines inter ſe incommenſurabiles erunt.



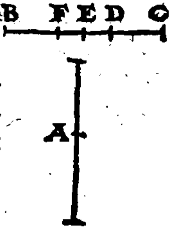
Theorema 15. Propositio 18.

Si fuerint duæ rectæ lineæ inæquales, & quartæ parti quadrati quod describitur à minore, æquale parallelogrammum applicetur secundum maiorem, ex qua maiore tantum excurrat extra latus parallelogrammi, quantum est alterum latus ipsius parallelogrammi: si præterea parallelogrammum sui applicatione diuidat lineam illam in partes inter se commensurabiles longitudine, illa maior linea tanto plus potest quàm minor, quantum est quadratum lineæ sibi commensurabilis longitudine. Quòd si maior plus possit quàm minor, tanto quantum est quadratum lineæ sibi commensurabilis longitudine, & prætereà quartæ parti quadrati lineæ minoris æquale parallelogrammum applicetur secundum maiorem, ex qua maiore tantum excurrat extra latus parallelogrammi, quantum est alterum latus ipsius parallelogrammi, parallelogrammum sui applicatione diuidit maiorem in partes inter se longitudine commensurabiles.

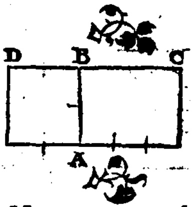


Theorema 16. Propositio 19.

Si fuerint duæ rectæ inæquales, quartæ autem parti quadrati lineæ minoris æquale le parallelogrammum ſecundum lineam maiorem applicetur, ex qua linea tantum excurrat extra latus parallelogrammi, quantum eſt alterum latus eiuſdem parallelogrammi: ſi parallelogrammum præterea ſui applicatione diuidat lineam in partes inter ſe longitudine incommenſurabiles, maior illa linea tantò plus poteſt quàm minor, quantum eſt quadratum lineæ ſibi maiori incommenſurabilis longitudine. Quòd ſi maior linea tantò plus poſsit quàm minor, quantum eſt quadratum lineæ incommenſurabilis ſibi longitudine: & præterea quartæ parti quadrati lineæ minoris æquale parallelogrammum applicetur ſecundum maiorem, ex qua tantum excurrat extra latus parallelogrammi, quantum eſt alterum latus ipſius: parallelogrammum ſui applicatione diuidit maiorem in partes inter ſe incommenſurabiles longitudine.



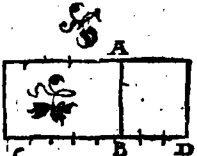
Theorema 17. Propoſitio 20.



Superficies rectangula contenta ex lineis rectis rationalibus longitudine commenſurabilibus ſecundum vnum aliquem modum ex antedictis, rationalis eſt.

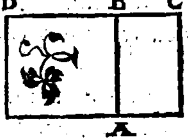
Theorema 18. Propoſitio 21.

Si rationale ſecundum lineam rationalem applicetur, habebit alterum latus lineam rationalem & commenſurabilem longitudine lineæ cui rationale parallelogrammum applicatur.



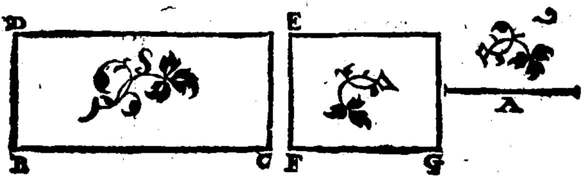
Theorema 19. Propoſitio 22.

Superficies rectangula contenta duabus lineis rectis rationalibus potentia tantum commẽſurabilibus, irrationalis eſt. Linea autem quæ illam ſuperficiem poteſt, irrationalis & ipſa eſt: vocetur verò medialis.



Theorema 20. Propoſitio 23.

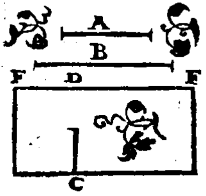
Quadrati lineæ medialis applicati ſecun-



dum

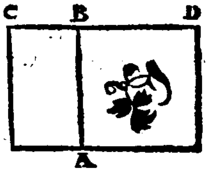
dum lineam rationalem, alterum latus est linea rationalis, & incommensurabilis longitudine lineæ secundum quam applicatur.

Theorema 21. Propositio 24.



Linea recta mediali commẽsurabilis, est ipsa quoque medialis.

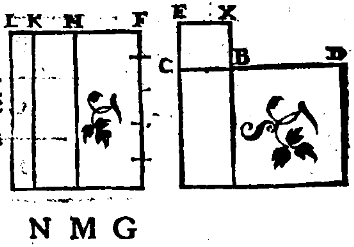
Theorema 22. Propositio 25.



Parallelogrammũ rectangulum contentum ex lineis medialibus longitudine commẽsurabilibus, mediale est.

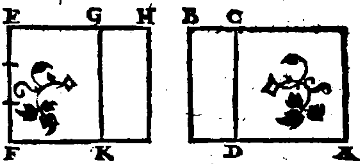
Theorema 23. Propositio 26.

Parallelogrammum rectangulum comprehensum duabus lineis medialibus potẽtia tantùm commensurabilibus, vel rationale est, vel mediale.



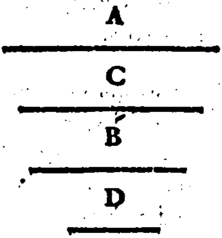
Theorema 24. Propositio 27.

Mediale nõ est maius quàm mediale superficie rationali



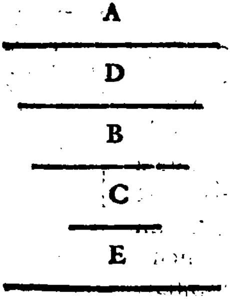
Proble. 4. Propositio 28.

Mediales lineas inuenire potentia tantùm commensurabiles rationale comprehendentes.



Problema 5. Propositio 29.

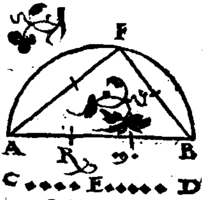
Mediales lineas inuenire potentia tantùm commensurabiles mediale comprehendendentes.



Proble-

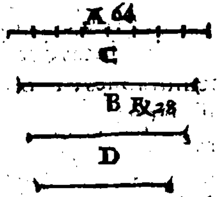
Problema 6. Propoſitio 30.

Reperire duas rationales potentia tantùm commē-ſurabiles huiuſmodi, vt maior ex illis poſsit plus quàm minor quadrato lineæ ſibi commenſurabilis longitudine.



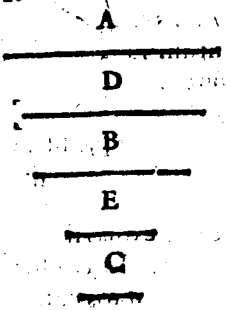
Problema 7. Propoſitio 31.

Reperire duas lineas mediales potentia tantùm commenſurabiles rationalem ſuperficiem continentes, tales inquã, vt maior poſsit plus quàm minor quadrato lineæ ſibi commenſurabilis longitudine.



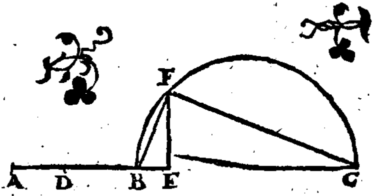
Problema 8. Propoſitio 32.

Reperire duas lineas mediales potentia tantùm commenſurabiles medialem ſuperficiem continentes, huiuſmodi vt maior plus poſsit quàm minor quadrato lineæ ſibi cō-menſurabilis longitudine.



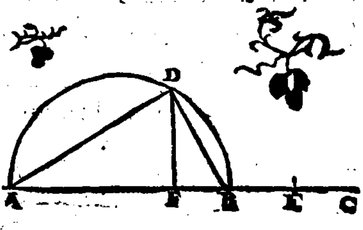
Problema 9. Propositio 33.

Reperire duas rectas potentia incommensurabiles, quarum quadrata simul addita, faciant superficiem rationalem, parallelogrammú verò ex ipsis contentum sit mediale.



Problema 10. Propositio 34

Reperire lineas duas rectas potentia incommensurabiles, conficientes compositum ex ipsarũ quadratis mediale, parallelogrã mum verò ex ipsis cõtentum rationale.



Problema 11. Propositio 35.

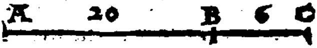
Reperire duas lineas rectas potentia incommensurabiles, conficientes id quod ex ipsarum quadratis componitur mediale, simulque

que parallelogrammum ex ipsis contentum, mędiale, quod pręterea parallelogrammum sit incõmensurabile cõposito ex quadratis ipsarum

PRINCIPIVM SENARIO-rum per compositionem.

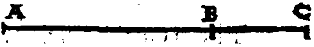
Theorema 25. Propositio 36.

Si duæ rationales potentia tantùm commensurabiles componantur, tota linea erit irrationalis. Vocetur autem Binomium



Theorema 26. Propositio 37.

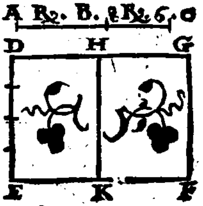
Si duæ mędiales potentia tantùm commensurabiles rationale continentes componantur, tota linea est irrationalis, vocetur autem Bimediale prius.



Theo-

Theorema 27. Propositio 38.

Si duæ mediales potentia tantùm commenſurabiles mediale continentes componantur, tota linea eſt irrationalis. vocetur autem Bimediale ſecundum.



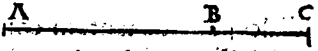
Theorema 28. Propoſitio 39.

Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex quadratis ipſarum rationale, parallelogrammum verò ex ipſis contentum mediale, tota linea recta eſt

irrationalis. Vocetur autem linea maior.

Theorema 29. Propoſitio 40.

Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex ipſarum quadratis mediale, id verò quod fit ex ipſis, rationale, tota linea eſt irrationalis. Vocetur autem

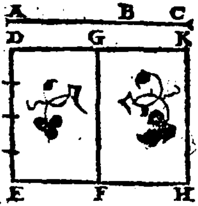


potens rationale & mediale.

Theorema 30. Propoſitio 41.

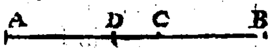
Si duæ rectæ potentia incommenſurabiles componantur, conficientes compoſitum ex quadratis ipſarum mediale, & quod continetur

netur ex ipsis, mediale, & præterea incommmensurabile composito ex quadratis ipsarum, tota linea est irrationalis. Vocetur autem potens duo medialia.



Theorema 31. Propositio 42.

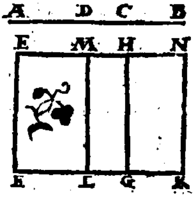
Binomium in vnico tantùm puncto diuiditur in sua nomina, id est in lineas ex quibus componitur.



Theorema 32. Propositio 43.

Bimediale prius in vnico tantùm puncto diuiditur in sua nomina.

Theorema 33. Propositio 44.



Bimediale secundum in vnico tantùm puncto diuiditur in sua nomina.

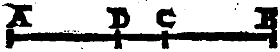
Theorema 34. Propositio 45.

Linea maior in vnico tantùm puncto diuiditur in sua nomina.

Theo-

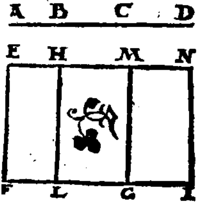
Theorema 35. Propositio 46.

Linea potens rationale & mediale in vnico tantùm puncto diuiditur in sua nomina.



Theorema 36. Propositio 47.

Linea potens duo medialia in vnico tantùm puncto diuiditur in sua nomina.



DEFINITIONES secundæ.

Proposita linea rationali, & binomio diuiso in sua nomina, cuius binomij maius nomen, id est, maior portio possit plusquam minus nomen quadrato lineæ sibi, maiori inquam nomini, commensurabilis longitudine:

1

Si quidem maius nomen fuerit commensurabile longitudine propositæ lineæ rationali, vocetur tota linea Binomium primum.

2

Si verò minus nomen, id est minor portio Binomij, fuerit commensurabile longitudine ne propositæ lineæ rationali, vocetur tota linea Binomium secundum:

3

Si verò neutrum nomen fuerit commensurabile longitudine propositæ lineæ rationali, vocetur Binomium tertium.

Rursus si maius nomen posit plusquam minus nomen quadrato lineæ sibi incommensurabilis longitudine:

4

Si quidem maius nomen est commensurabile longitudine propositæ lineæ rationali, vocetur tota linea Binomium quartum:

5

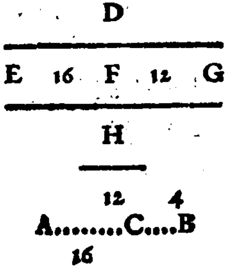
Si verò minus nomen fuerit commensurabile longitudine lineæ rationali, vocetur Binomium quintum:

6

Si verò neutrum nomen fuerit longitudine commensurabile lineæ rationali, vocetur illa Binomium sextum.

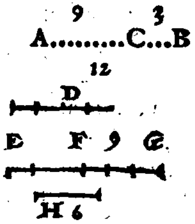
Problema 12. Propositio 48.

Reperire Binomium primum.



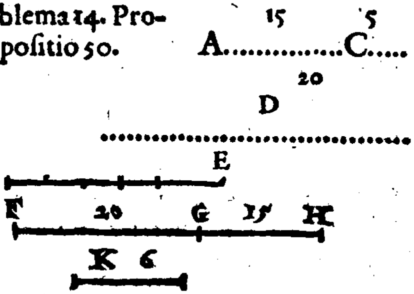
Proble-

Problema 13. Propositio 49.



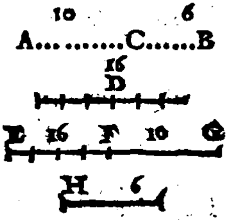
Reperire Binomium secundum.

Problema 14. Propositio 50.



Reperi reBino mium tertiū.

Problema 15. Propositio 51.

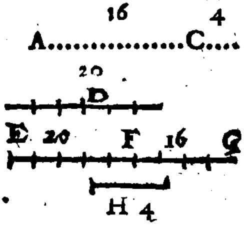


Reperire Binomium quartum.

Proble-

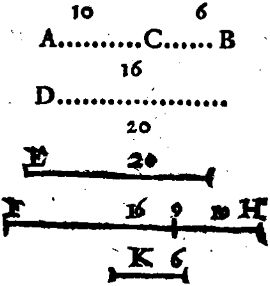
Problema 16. Propositio 52.

Reperire Binomiun quintum.



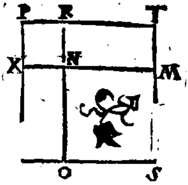
Probl. 17. Propositio 53.

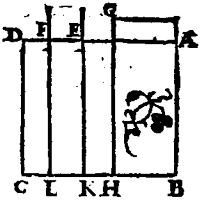
Reperire Binonimũ sextum.



Theorema 37. Propositio 54.

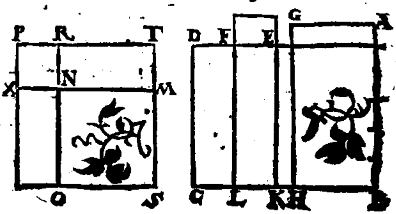
Si superficies contenta fuerit ex rationali & Binomio primõ, linea quæ illam superficiem potest, est irrationalis, quæ Binómium vocatur.





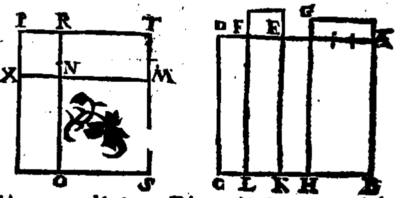
Theorema 38. Propositio 55.

Si superficies contenta fuerit ex linea rationali & Binomio secundo, linea potens illam superficiem est irrationalis, quæ Bimediale primum vocatur.



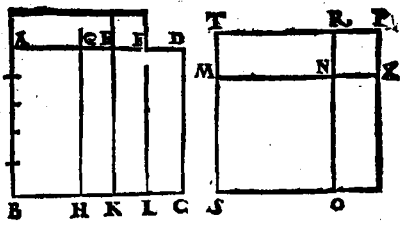
Theorema 39. Propositio 56.

Si superficies contineatur ex rationali & Binomio tertio, linea quæ illam superficiem potest, est irrationalis, quæ dicitur Bimediale secundũ.



Theorema 40. Propositio 57.

Si superficies contineatur ex rationali & Bino

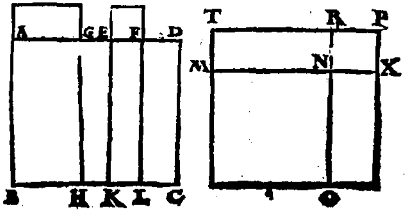


mio

mio quarto, linea potens superficiem illam, est irrationalis, quæ dicitur maior.

Theorema 41. Propositio 58.

Si superficies contineatur ex rationali & Binomio quinto, linea quæ illam superficiem potest



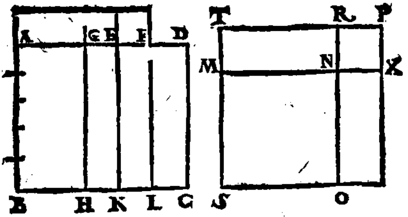
est irrationalis, quę dicitur potés rationale & mediale.

Theorema 42 Propositio 59.

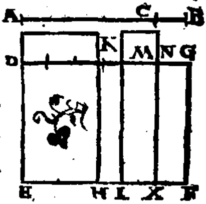
Si superficies contineatur ex rationali & Binomio sexto, linea quæ illam superficiem potest est irrationalis, quæ dicitur

 potens

potens duo mediala.

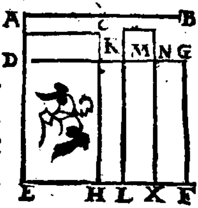


Theorema 43. Propoſitio 60.



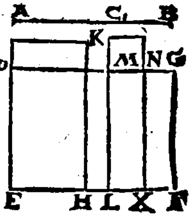
Quadratum Binomij ſecundum lineam rationalem applicatum, facit alterum latus Binomium primum.

Theorema 44. Propoſitio 61.



Quadratũ Bimedialis primi ſecundum rationalem lineam applicatum, facit alterum latus Binomium ſecundum.

Theorema 45. Propoſitio 62.

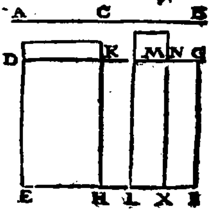


Quadratum Bimedialis ſecundi ſecundum rationalem applicatũ, facit alterũ latus Binomiũ tertium.

Theo-

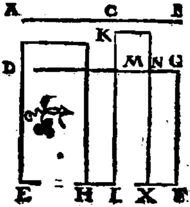
Theorema 46. Propositio 63.

Quadratum lineæ maioris secundum lineam rationalem applicatum, facit alterum latus Binomium quartum.



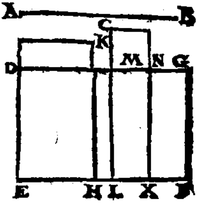
Theorema 47. Propositio 64.

Quadratum lineæ potentis rationale & mediale secundum rationalem applicatum, facit alterum latus Binomium quintum.



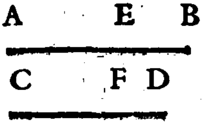
Theorema 48. Propositio 65.

Quadratum lineæ potentis duo medialia secundũ rationalem applicatum, facit alterum latus Binomiũ sextum.



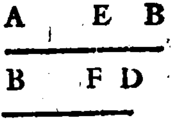
Theorema 49. Propositio 66.

Linea longitudine commensurabilis Binomio est & ipsa Binomium eiusdẽ ordinis.



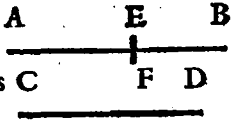
Theorema 50. Propoſitio 67.

Linea longitudine commenſurabilis alteri bimedialium, eſt & ipſa bimediale etiam eiuſdem ordinis.



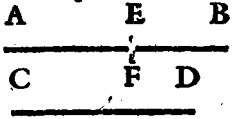
Theorema 51. Propoſitio 68.

Linea commenſurabilis lineæ maiori, eſt & ipſa maior.



Theorema 52. Propoſitio 69.

Linea commenſurabilis lineæ potenti rationale & mediale, eſt & ipſa linea potens rationale & mediale.



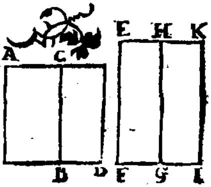
Theorema 53. Propoſitio 70.

Linea commenſurabilis lineæ potenti duo medialia, eſt & ipſa linea potens duo medialia.

Theorema 54. Propoſitio 71.

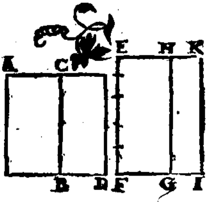
Si duæ ſuperficies rationalis & medialis ſimul componantur, linea quæ totam ſuperficiem

ciem compositam potest, est vna ex quatuor irrationalibus, vel ea quæ dicitur Binomium, vel bimediale primum, vel linea maior, vel linea potens rationale & mediale.



Theorema 55. Propositio 72.

Si duæ superficies mediales incommensurabiles simul componantur, fiunt reliquæ duæ lineæ irrationales, vel bimediale secundum, vel linea potens duo medialia.



SCHOLIVM.

Binomium & cætera consequentes lineæ irrationales, neque sunt eadem cum linea mediali, neque ipsæ inter se.

Nam quadratum lineæ medialis applicatum secundum lineam rationalem, facit alterum latus lineam rationalem, & longitudine incommensurabilem lineæ secundum quam applicatur, hoc est, lineæ rationali, per 23.

Quadratum verò Binomij secundum rationalem applicatum, facit alterum latus Binomium primum, per 60.

Quadratum verò Bimedialis primi secundum rationalem applicatum, facit alterum latus Binomium secundum per 61.

Quadratum verò Bimedialis secundi secundum rationalem applicatum, facit alterum latus Binomium tertium, per 62.

Quadratum verò lineæ maioris secundum rationalem applicatum, facit alterum latus Binomium quartum, per 63.

Quadratum verò lineæ potentis rationale & mediale secundum rationalem applicatum, facit alterum latus Binomium quintum, per 64.

Quadratum verò lineæ potentis duo medialia secundum rationalem applicatum, facit alterum latus Binomium sextum, per 65.

Cùm igitur dicta latera, quæ latitudines vocantur, differant & à prima latitudine, quoniam est rationalis, cùm inter se quoque differant, eo quia sunt Binomia diuersorum ordinum: manifestum est ipsas lineas irrationales, differentes esse inter se.

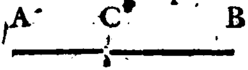
SECVN.

SECVNDVS ORDO ALterius ſermonis, qui eſt de detractione.

Principium ſenatorium per detractionem.

Theorema 56. Propoſitio 73.

Si de linea rationali detrahatur rationalis potentia tantùm commenſurabilis ipſi toti, reſidua eſt irrationalis, vocetur autem Reſiduum.

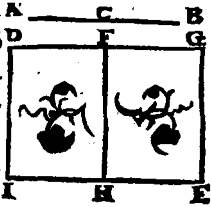


Theorema 57. Propoſitio 74.

Si de linea mediali detrahatur medialis potentia tantùm commenſurabilis toti lineæ, quæ verò detracta eſt cum tota contineat ſuperficiem rationalem, reſidua eſt irrationalis. Vocetur autem Reſiduum mediale primum.

Theorema 58. Propoſitio 75.

Si de linea mediali detrahatur medialis potentia tantùm commensurabilis toti, quæ verò detracta est, cùm tota contineat superficiem medialem, reliqua est irrationalis. Vocetur autem Residuum mediale secundũ.



Theorema 57. Propositio 76.

Si de linea recta detrahatur recta potentia incommensurabilis toti, compositum autem ex quadratis totius lineæ & lineæ detractæ sit rationale, parallelogrammum verò ex iisdem contentum sit mediale, reliqua linea erit irrationalis. Vocetur autem linea minor.

Theorema 58. Propositio 77.

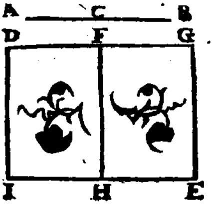
Si de linea recta detrahatur recta potentia

tia incommenſurabilis toti lineæ, compoſitum autem ex quadratis totius & lineæ detractæ ſit mediale, parallelogrammum verò bis ex eiſdem contentum ſit rationale, reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie rationali totam ſuperficiem medialem.

Theorema 59. Propoſitio 78.

Si de linea recta detrahatur recta potentia incommenſurabilis toti lineæ, compoſitum autem ex quadratis totius & lineæ detractæ ſit mediale, parallelogrammum verò bis ex ijſdem ſit etiam mediale: præterea ſint quadrata ipſarum incommenſurabilia parallelogrammo bis ex ijſdem contento, reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie

ficie mediali totam superficiem medi-lem.

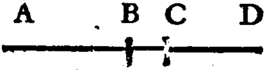


Theorema 60. Propositio 79.

Residuo vnica tantùm linea recta coniungitur rationalis, potentia tantùm commensurabilis toti lineæ.

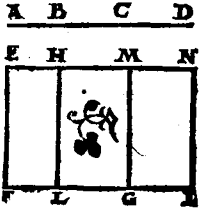
Theorema 61. Propositio 80.

Residuo mediali primo vnica tantùm linea coniungitur medialis, potentia tantùm commensurabilis toti, ipsa cum tota continens rationale.



Theorema 62. Propositio 81.

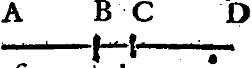
Residuo mediali secundo vnica tantùm coniungitur medialis, potenti- tantùm commensurabilia toti ipsa cum tota continens mediale.



Theorema 63. Propositio 82.

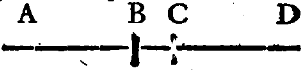
Lineæ minori vnica tantùm recta coniungitur

tur potentia incommensurabilis toti, faciens cum tota compositum ex quadratis ipsarum rationale, id verò parallelogrammum, quod bis ex ipsis fit, mediale.



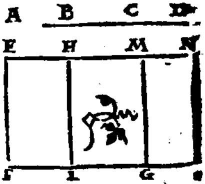
Theorema 64. Propositio 83.

Lineæ facienti cum superficie rationali totam superficiem medialem, vnica tantùm coniungitur linea recta potentia incommensurabilis toti, faciens autem cum tota compositum ex quadratis ipsarum, mediale, id verò quod fit bis ex ipsis, rationale.



Theorema 65. Propositio 84.

Lineæ cum mediali superficie facienti totam superficiem medialem, vnica tantùm coniungitur linea potentia toti incommensurabilis, faciens cum tota compositum ex quadratis ipsarũ mediale, id verò quod fit bis ex ipsis etiam mediale, & prætereà faciens compositum ex quadratis ipsarum incõmensurabile ei quod fit bis ex ipsis.



DEFINI-

DEEINITIONES TERTIÆ.

Propoſita linea rationali & reſiduo.

1

Si quidem tota, nempe compoſita ex ipſo reſiduo & linea illi coniuncta, plus poteſt quàm coniuncta, quadrato lineæ ſibi cõmenſurabilis longitudine, fueritque tota longitudine commenſurabilis lineæ propoſitæ rationali, reſiduum ipſum vocetur Reſiduum primum.

2

Si verò coniuncta fuerit longitudine commenſurabilis rationali, ipſa autem tota plus poſsit quam coniuncta, quadrato lineæ ſibi longitudine commenſurabilis, reſiduum vocetur Reſiduum ſecundum.

3

Si verò neutra linearum fuerit longitudine commenſurabilis rationali, poſsit autem ipſa tota plusquam coniuncta, quadrato lineæ ſibi longitudine commenſurabilis, vocetur Reſiduum tertium.

Rurſus ſi tota poſsit plus quàm coniuncta, quadrato lineæ ſibi longitudine incommenſurabilis.

4

Et quidem ſi tota fuerit longitudine commenſumensurabilis ipsi rationali, vocetur Residuum quartum.

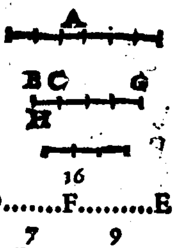
5

Si verò coniuncta fuerit longitudine commensurabilis rationali, & tota plus possit quàm coniuncta, quadrato lineæ sibi longitudine incommensurabilis, vocetur Residuum quintum.

6

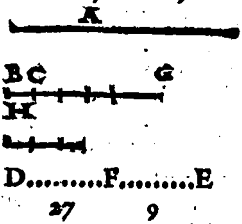
Si verò neutra linearum fuerit commensurabilis longitudine ipsi rationali, fueritque tota potentior quàm coniuncta, quadrato lineæ sibi longitudine incommensurabilis, vocetur Residuum sextum.

Problema 18. Propositio 85.



Reperire primum Residuum.

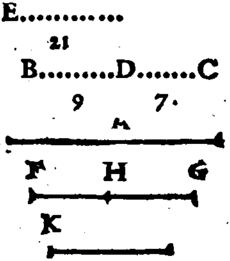
Problema 19. Propositio 86.



Reperire secundum Residuum.

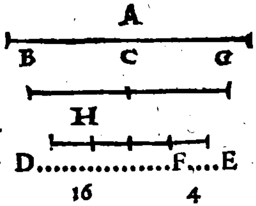
Proble-

Problema 20. Propositio 87.



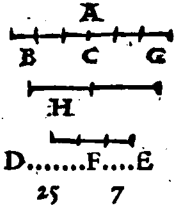
Reperire tertium Residuum.

Probl. 21. Propositio 88.



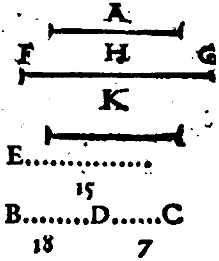
Reperire quartum Residuum.

Problema 22. Propositio 89.



Reperire quintum Residuum.

Problema 23. Propositio 90.

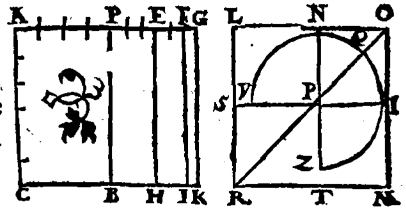


Reperire sextum Residuum.

Theore-

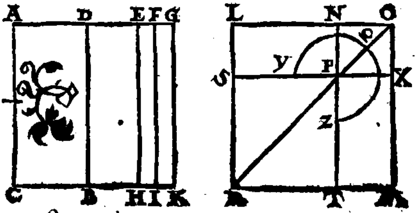
Theorema 66. Propositio 91.

Si superficies contineatur ex linea rationali & residuo primo, linea quæ illam superficiem potest, est residuum.



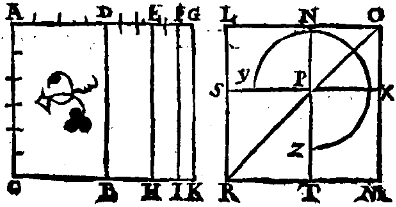
Theorema 67. Propositio 92.

Si superficies contineatur ex linea rationali & residuo secundo, linea quæ illam superficiem potest, est residuum mediale primum.



Theorema 68. Propositio 93.

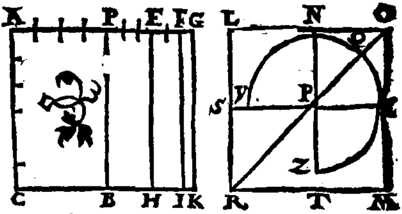
Si superficies contineatur ex linea rationali & residuo ter-



tio, linea quæ illam superficiem potest est residuum mediale secundum.

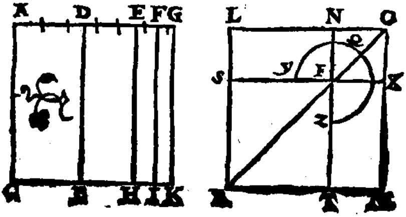
Theorema 69. Propositio 94.

Si superficies contineatur ex linea rationali & residuo quarto, linea quę illam superficiem potest, est linea minor.



Theorema 70. Propositio 95.

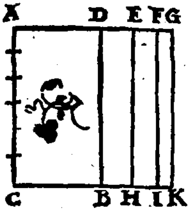
Si superficies contineatur ex linea rationali & residuo quinto, linea quæ illam superficiem potest est ea quæ dicitur cum rationali superficie faciens totam medialem.

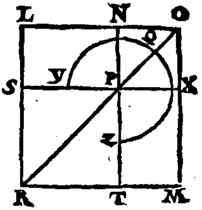


Theorema 71. Propositio 96.

Si superficies contineatur ex linea rationali &

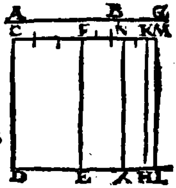
& residuo sexto, linea quæ illam superficiem potest, est ea quę dicitur faciēs cum mediali superficie totam medialem.





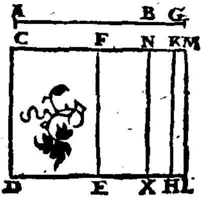
Theorema 72. Propositio 97.

Quadratum residui secundum lineam rationalem applicatum, facit alterum latus residuum primum.



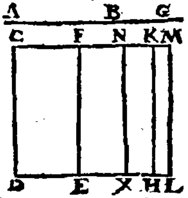
Theorema 73. Propositio 98.

Quadratum residui medialis primi secundum rationalem applicatum, facit alterum latus residuū secundum.

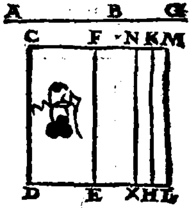


Theorema 74. Propositio 99.

Quadratū residui medialis secundi secundum rationalem applicatum, facit alterum latus residuū tertium

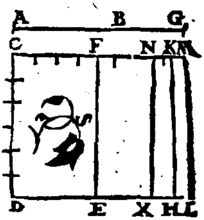


Theorema 75. Propositio 100.



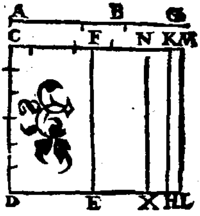
Quadratum lineæ minoris ſecundum rationalem applicatum, facit alterum latus reſiduum quartum.

Theorema 76. Propoſitio 101.



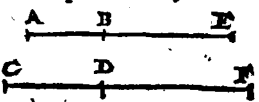
Quadratum lineę cum rationali ſuperficie facientis totam medialem, ſecundum rationalem applicatum, facit alterum latus reſiduum quintum.

Theorema 77. Propoſitio 102.



Quadratum lineæ cum mediali ſuperficie facientis totam medialem, ſecundum rationalem applicatum, facit alterum latus reſiduum ſextum.

Theorema 78. Propoſitio 103.



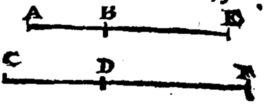
Linea reſiduo commenſurabilis longitudine, eſt & ipſa reſiduum, & eiuſdem ordinis.

Theorema 79. Propoſitio 104.

Linea commenſurabilis reſiduo mediali, eſt

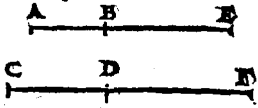
&

& ipſa reſiduum mediale, & eiuſdem ordinis.



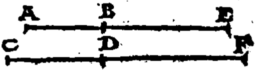
Theorema 80. Propoſitio 105.

Linea commenſurabilis lineæ minori, eſt & ipſa linea minor.



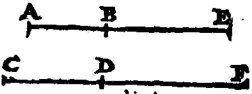
Theorema 81. Propoſitio 106.

Linea commenſurabilis lineæ cum rationali ſuperficie facienti totam medialem, eſt & ipſa linea cum rationali ſuperficie faciens totam medialem.



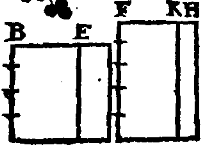
Theorema 82. Propoſitio 107.

Linea commenſurabilis lineæ cum mediali ſuperficie facienti totam medialem, eſt & ipſa cum mediali ſuperficie faciens totam medialem.



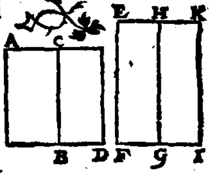
Theorema 83. Propoſitio 108.

Si de ſuperficie rationali detrahatur ſuperficies medialis, linea quæ reliquam ſuperficiem poteſt, eſt alterutra ex duabus irrationalibus, aut reſiduum, aut linea minor.



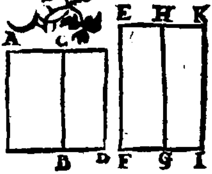
Theorema 84. Propoſitio 109.

Si de ſuperficie mediali detrahatur ſuperficies rationalis, aliæ duæ irrationales fiunt, aut reſiduum mediale primum, aut cū rationali ſuperficiem faciens totam medialem.



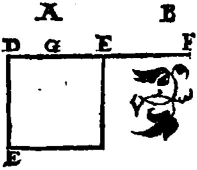
Theorema 85. Propoſitio 110.

Si de ſuperficie mediali detrahatur ſuperficies medialis quæ ſit incommenſurabilis toti, reliquæ duæ fiunt irrationales, aut reſiduum mediale ſecundum, aut cū mediali ſuperficie faciēs totam medialem.



Theorema 86. Propoſitio 111.

Linea quæ Reſiduum dicitur, non eſt eadem cum ea quæ dicitur Binomiū.



SCHO-

SCHOLIVM.

Linea quæ Residuum dicitur, & cæteræ quinque eam consequentes irrationales, neque lineæ mediali neque sibi ipsæ inter se sunt eædem. Nam quadratum lineæ medialis secundum rationalem applicatum, facit alterum latus, rationalem lineam longitudine incommensurabilem ei, secundum quam applicatur, per 23

Quadratum verò residui secundum rationalem applicatum, facit alterum latus residuum primum. per 97.

Quadratum verò residui medialis primi secundum rationalem applicatum, facit alterum latus residuum secundum, per 98.

Quadratum verò residui medialis secundi, facit alterum latus residuum tertium. per 99.

Quadratum verò lineæ minoris, facit alterum latus residuum quartum, per 100.

Quadratum verò lineæ cum rationali superficie facientis totam medialem, facit alterum latus residuum quintum, per 101.

Quadratum verò lineæ cum mediali superficie facientis totam medialem, secundum rationalem applicatum, facit alterum latus residuum sextum, per 102.

Cùm igitur dicta latera, quæ sunt latitudines cuiusque parallelogrammi vnicuique quadrato æqualis & secundum rationalem applicati, differant & à primo latere, & ipsa inter se (nam à primo differunt, quoniam sunt resi-

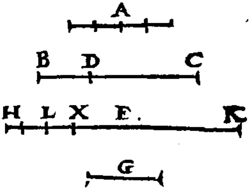
dua non eiusdam ordinis) constat ipsas quoque lineas irrationales inter se differentes esse. Et quoniam demonstratum est, Residuum non esse idem quod Binomium, quadrata autem residui & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex residuis eiusdem ordinis cuius sunt & residua, quorum quadrata applicantur rationali. similiter & quadrata Binomij & quinque linearum irrationalium illud consequentium, secundum rationalem applicata, faciunt altera latera ex Binomijs eiusdem ordinis, cuius sunt & Binomia, quorum quadrata applicantur rationali. Ergo lineæ irrationales quæ consequuntur Binomium, & quæ consequuntur residuum, sunt inter se differentes. Quare dictæ lineæ omnes irrationales sunt numero 13.

1 *Medialis.*
2 *Binomium.*
3 *Bimediale primum.*
4 *Bimediale secundũ.*
5 *Maior.*
6 *Potens rationale & mediale.*
7 *Potẽs duo medialia.*
8 *Residuum.*
9 *Residuum mediale primum.*
10 *Residuum mediale secundum.*
11 *Minor.*
12 *Faciens cum rationali superficie totam medialem.*
13 *Faciens cum mediali superficie totam medialem.*

Theore-

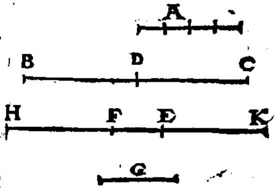
Theorema 87. Propositio 112.

Quadratum lineæ rationalis secundum Binomium applicatū, facit alterum latus residuum, cuius nomina sunt commensurabilia Binomij nominibus, & in eadem proportione: præterea id quod fit Residuum, eundem ordinem retinet quem Binomium.



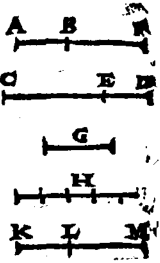
Theorema 88. Propositio 113.

Quadratum lineæ rationalis secundum residuum applicatum, facit alterum latus Binomium, cuius nomina sunt commensurabilia nominibus residui & in eadem proportione: præterea id quod fit Binomium, est eiusdē ordinis, cuius & Residuum.



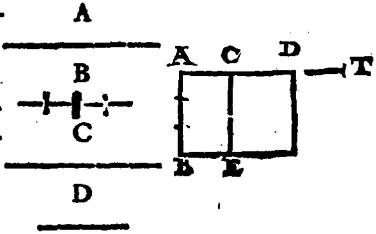
Theorema 89. Propositio 114.

Si parallelogrammum contineatur ex residuo & Binomio, cuius nomina ſunt commenſurabilis nominibus reſidui & in eadem proportione, linea quæ illam ſuperficiem poteſt, eſt rationalis.



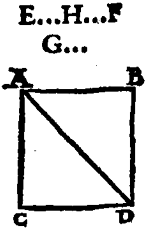
Theorema 90. Propoſitio 115.

Ex linea mediali naſcuntur lineæ irrationales innumerabiles, quarum nulla vlli ante dictarum eadem ſit.



Propoſitio 116.

Propoſitum nobis eſto demonſtrare in figuris quadratis diametrum eſſe longitudine incommenſurabilem ipſi lateri.



ELEMENTI X. FINIS.

EVCLIDIS ELEMENTVM VNDECIMVM, ET SOLIDORVM *primum*.

DEFINITIONES.

1

Solidum, eſt quod longitudinem, latitudinem, & craſsitudinem habet.

2

Solidi autem extremum eſt ſuperficies.

3

Linea recta eſt ad planum recta, cùm ad rectas omnes lineas, à quibus illa tangitur, quęque in propoſitio ſunt plano, rectos angulos efficit.

4

Planum ad planum rectum eſt, cùm rectę lineæ, quæ communi planorum ſectioni ad rectos angulos in vno planorum ducuntur, alteri plano ad rectos ſunt angulos.

5

Rectæ lineæ ad planum inclinatio, acutus eſt angulus ipſa inſiſtente linea & adiuncta altera comprehenſus, cùm à ſublimi rectæ illius lineæ termino deducta fuerit perpendicularis,

laris, atque à puncto quod perpendicularis in ipso plano fecerit, ad propositæ illius lineę extremum, quod in eodem est plano, altera recta linea fuerit adiuncta.

6

Plani ad planum inclinatio, acutus est angulus rectis lineis contentus, quæ in vtroque planorum ad idem communis sectionis punctum ductæ, rectos ipsi sectioni angulos efficiunt.

7

Planum similiter inclinatum esse ad planum, atque alterum ad alterum dicitur, cùm dicti inclinationum anguli inter se sunt æquales.

8

Parallela plana, sunt quæ eodem non incidunt, nec concurrunt.

9

Similes figuræ solidæ, sunt quæ similibus planis, multitudine æqualibus continentur.

10

Aequales & similes figuræ solidæ sunt, quæ similibus planis, multitudine & magnitudine æqualibus continentur.

11

Solidus angulus, est plurium quàm duarum linearum, quæ se mutuò contingant, nec in eadem sint superficie, ad omnes lineas inclinatio.

Aliter.

Aliter.

Solidus angulus, eſt qui pluribus quàm duobus planis angulis in eodem non conſiſtentibus plano, ſed ad vnum punctum collectis, continetur.

12

Pyramis, eſt figura ſolida quæ planis continetur, ab vno plano ad vnum punctum collecta.

13

Priſma, figura eſt ſolida quæ planis continetur, quorum aduerſa duo ſunt & æqualia & ſimilia & parallela, alia verò parallelogramma.

14

Sphæra eſt figura, quæ conuerſo circumquieſcentem diametrum ſemicirculo continetur, cùm in eundem rurſus locum reſtitutus fuerit, vnde moueri cœperat.

15

Axis autem ſphæræ, eſt quieſcens illa linea circum quam ſemicirculus conuertitur.

16

Centrum verò Sphæræ eſt idem, quod & ſemicirculi.

17

Diameter autem Sphæræ, eſt recta quædam linea per centrum ducta, & vtrinque à ſphæræ ſuperficie terminata.

Conus

18

Conus est figura, quæ conuerso circumquiescens alterum latus eorum quæ rectum angulum continent, orthogonio triangulo continetur, cùm in eundem rursus locum illud triangulum restitutum fuerit, vnde moueri cœperat. Atque si quiescens recta linea æqualis sit alteri, quæ circum rectum angulum conuertitur, rectangulus erit Conus: sin minor, amblygonius: si verò maior oxygonius.

19

Axis autem Coni, est quiescens illa linea, circum quam triangulum vertitur.

20

Basis verò Coni, circulus est, qui à circunducta linea recta describitur.

21

Cylindrus figura est, quæ conuerso circum quiescens alterum latus eorum quæ rectum angulum continent, parallelogrammo orthogonio comprehenditur, cùm in eundem rursus locum restitutum fuerit illud parallelogrammum, vnde moueri cœperat.

22

Axis autem Cylindri, est quiescens illa recta linea, circum quam parallelogrammũ vertitur.

23

Bases verò cylindri, sunt circuli à duobus

aduer-

aduerſis lateribus quæ circumaguntur, deſcripti.

24

Similes coni & cylindri, ſunt quorum & axes & baſium diametri proportionales ſunt.

25

Cubus eſt figura ſolida, quæ ſex quadratis æqualibus continetur.

26

Tetraëdum eſt figura, quæ triangulis quatuor æqualibus & æquilateris continetur.

27

Octaëdrum figura eſt ſolida, quæ octo triangulis æqualibus & æquilateris continetur.

28

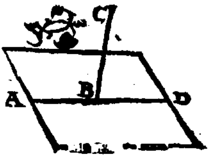
Dodecaëdrum figura eſt ſolida, quæ duodecim pentagonis æqualibus, æquilateris, & æquiangulis continetur.

29

Eicoſaëdrum figura eſt ſolida, quæ triangulis viginti æqualibus & æquilateris continetur.

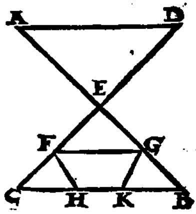
Theorema 1. Propoſitio 1.

Quædam rectæ lineæ pars in ſubiecto quidem non eſt plano, quædam verò in ſublimi.



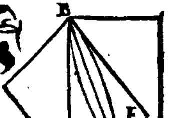
Theore-

Theorema 2. Propoſitio 2.



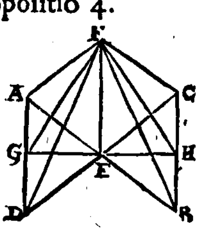
Si duæ rectæ lineæ ſe mutuò ſecēt, in vno ſunt plano: atque triangulū omne in vno eſt plano.

Theorema 3. Propoſitio 3.



Si duo plana ſe mutuò ſecēt, communis eorum ſectio eſt recta linea.

Theorema 4. Propoſitio 4.

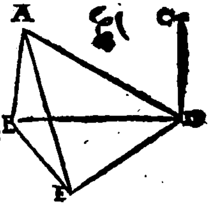


Si recta linea rectis duabus lineis ſe mutuò ſecantibus, in communi ſectione ad rectos angulos inſiſtat illa, ducto etiam per ipſas plano ad angulos rectos erit.

Theorema 5. Propoſitio 5.

Si recta linea rectis tribus lineis ſe mutuò tangentibus, in communi ſectione ad rectos angulos inſiſtat, illæ tres rectæ in vno ſunt plano.

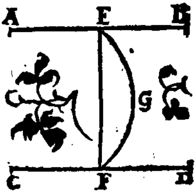
Theorema 6. Propo-
sitio 6.



Si duæ rectæ lineæ eidem plano ad rectos sint angulos, parallelæ erunt illæ rectæ lineæ.

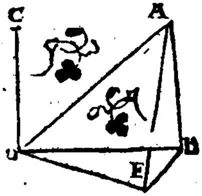
Theorema 7. Propositio 7.

Si duæ sint parallelæ rectæ lineæ, in quarum vtraque sumpta sint quælibet puncta, illa linea quę ad hæc puncta adiungitur, in eodem est cum parallelis plano.



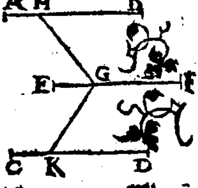
Theorema 8. Pro-
positio 8.

Si duæ sint parallelæ rectæ lineæ, quarum altera ad rectos cuidam plano sit engulos, & reliqua eidem plano ad rectos angulos erit.



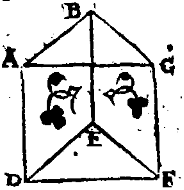
Theorema 9. Propo-
sitio 9.

Quæ eidem rectæ lineæ sunt parallelæ, sed non in eodem cum illa plano, hæ quoque sunt inter se parallelæ.



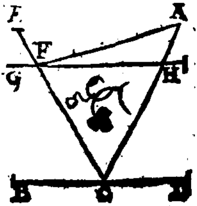
Theorema 10. Propositio 10.

Si duæ rectæ lineæ se mutuò tangentes ad duas rectas se mutuò tangentes sint parallelæ, non autem in eodem plano, illæ angulos æquales comprehendent.



Problema 1. Propositio 11.

A dato sublimi puncto, in subiectum planum perpendicularem rectam lineam ducere.

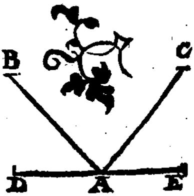


Problema 2. Propositio 12.

Dato plano, à puncto quod in illo datum est, ad rectos angulos rectam lineam excitare.

Theorema 11. Propositio 13.

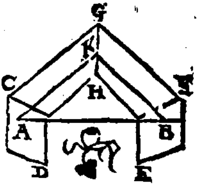
Dato plano, à puncto quod in illo datum est, duæ rectæ lineæ ad rectos angulos non excitabuntur ad easdem partes.



Theo-

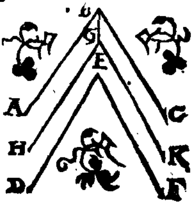
Theorema 12. Propositio 14.

Ad quæ plana, eadem recta linea recta est, illa sunt parallela.



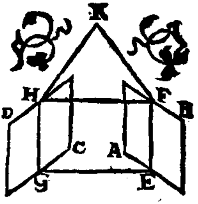
Theorema 13. Propositio 15.

Si duæ rectæ lineæ se mutuò tangentes ad duas rectas se mutuò tangentes sint parallelæ, non in eodem consistentes plano, parallela sunt quæ per illas ducuntur plana.



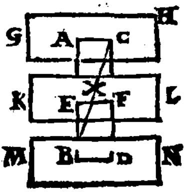
Theorema 14. Propositio 16.

Si duo plana parallela plano quopiam secentur, cómunes illorum sectiones sunt parallelæ.

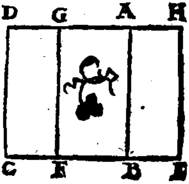


Theorema 15. Propositio 17.

Si duæ rectæ lineæ parallelis planis secétur, in easdem rationes secabuntur.

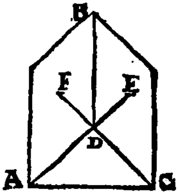


Theorema 16. Propositio 18.



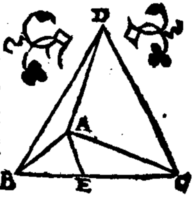
Si recta linea plano cuipiam ad rectos sit angulos, illa etiam omnia quæ per ipsam plana, ad rectos eidem plano angulos erunt.

Theorema 17. Propositio 19.



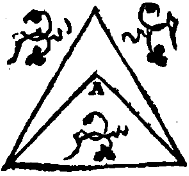
Si duo plana se mutuò secantia plano cuidam ad rectos sint angulos, communis etiam illorum sectio ad rectos eidem plano angulos erit.

Theorema 18 Propositio 20.



Si angulus solidus planis tribus angulis contineatur, ex his duo quilibet vtut assumpti tertio sunt maiores.

Theorema 19. Propositio 21.

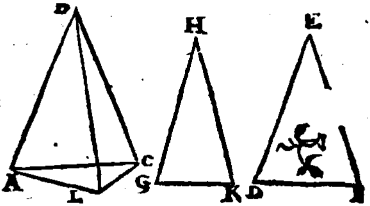


Solidus omnis angulus minoribus continetur, quàm rectis quatuor angulis planis.

Theo-

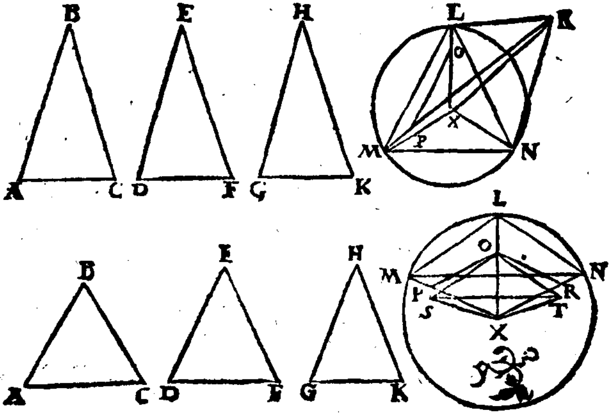
Theorema 20. Propositio 22.

Si plani tres anguli æqualibus rectis contineantur lineis, quorum duo vt libet assumpti, tertio sint maiores, triangulum constitui potest ex lineis æquales, illas rectas coniungentibus.

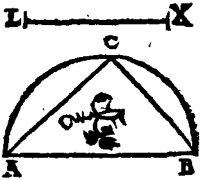


Problema 3. Propositio 23.

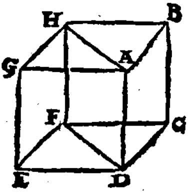
Ex planis tribus angulis, quorum duo vt libet assumpti tertio sint maiores, solidum angulum constituere. Decet autem illos tres angulos rectis quatuor esse minores.



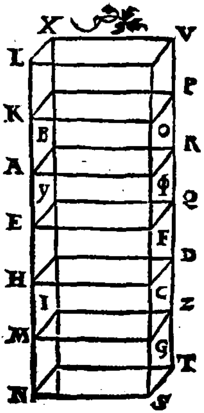
Theorema 21. Propositio 24.



Si ſolidum parallelis planis contineatur, aduerſa illius plana & æqualia ſunt & parallelogramma.



Theorema 22. Propositio 25.

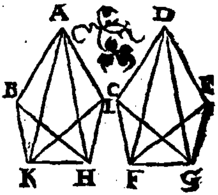


Si ſolidum parallelis planis contentum plano ſecetur aduerſis planis parallelo, erit quemadmodum baſis ad baſim, ita ſolidum ad ſolidum.

Proble-

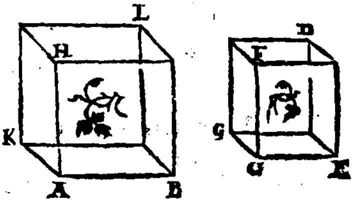
Problema 4. Propositio 26.

Ad datam rectam lineam eiusque punctum, angulum solidum constituere solido angulo dato æqualem.



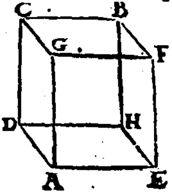
Problema 5. Propositio 27.

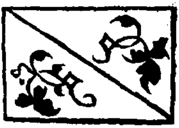
A data recta, dato solido parallelis planis comprehenso simile & similiter positum solidum parallelis planis contentum describere.



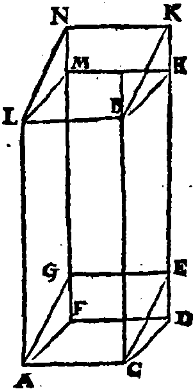
Theorema 23. Propositio 28.

Si solidum parallelis planis comprehensum, ducto per aduersorum planorum diagonios plano sectum sit, illud solidū ab hoc plano bifariàm secabitur.





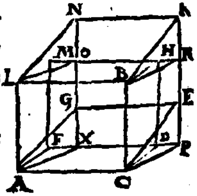
Theorema 34. Propositio 29.



Solida parallelis planis comprehensa, quæ super eandem basim, & in eadē sunt altitudine, quorum insistentes lineæ in ijsdem collocantur rectis lineis, illa sunt inter se æqualia.

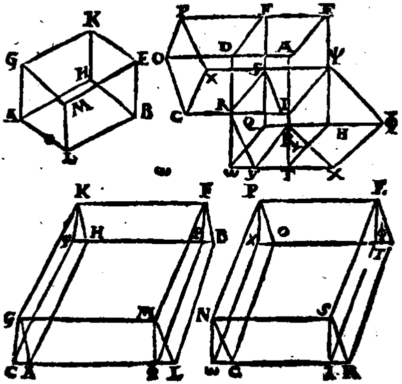
Theorema 25. Propositio 30.

Solida parallelis planis circunscripta, quæ super eandem basim & in eadem sunt altitudine, quorum insistentes lineæ non in ijsdem reperiuntur rectis lineis, illa sunt inter se æqualia.



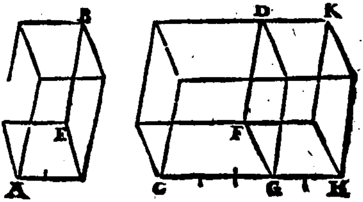
Theorema 26. Propositio 31.

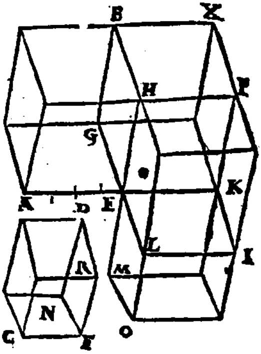
Solida parallelis planis circunscripta, quæ in in eadē ſunt altitudine, æqualia ſunt inter ſe.



Theorema 27. Propoſitio 32.

Solida parallelis planis circunſcripta quæ eiuſdem ſunt altitudinis, eam habent inter ſe rationē, quam baſes.



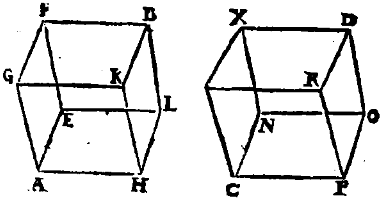


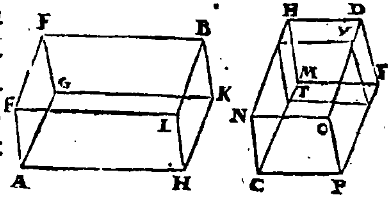
Theor. 28. Propoſitio 33.

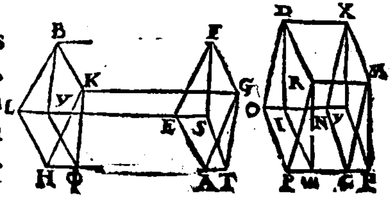
Similia ſolida parallelis planis circunſcripta habent inter ſe rationem homologorum laterum triplicatam.

Theor. 29. Propoſitio 34.

Aequalium ſolidorum parallelis planis contentorum baſes cũ altitudinibus reciprocãtur. Et ſolida parallelis planis contẽta, quorum baſes cũ altitudi-



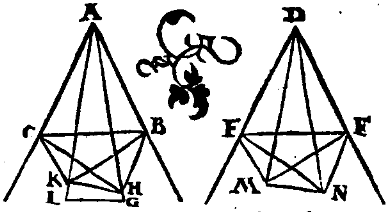




nibus reciprocantur, illa sunt æqualia.

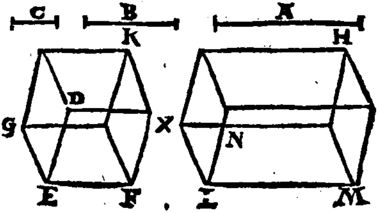
Teorema 30. Propositio 35.

Si duo plani sint anguli æquales, quorum verticibus sublimes rectæ lineæ insistant, quæ cum lineis primò positis angulos contineant æquales, vtrunque vtrique, in sublimibus autem lineis quælibet sumpta sint puncta, & ab his ad plana in quibus consistũt anguli primùm positi, ductæ, sint perpendiculares, ab earum verò punctis, quæ in planis signata fuerint, ad angulos primum positos adiunctæ sint rectæ lineæ, hæ cũ sublimibus æquales angulos comprehendent.



Theorema 31. Propositio 36.

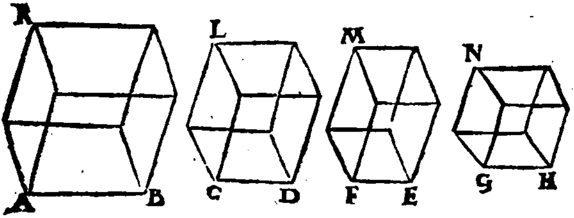
Si rectę tres lineæ sint ꝓportionales, quod



ex his tribus fit solidum parallelis planis contentum, æquale est descripto à media linea solido parallelis planis comprehenso, quod æquilaterum quidem sit, sed antedicto æquiangulum.

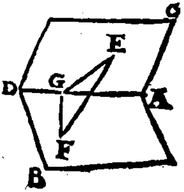
Theorema 32. Propositio 37.

Si rectæ quatuor lineæ sint proportionales, illa quoque solida parallelis planis contenta, quæ ab ipsis lineis & similia & similiter describuntur, proportionalia erunt. Et si solida parallelis planis comprehensa, quæ & similia & similiter describuntur, sint proportionalia, illæ quoque rectæ lineæ proportionales erunt.



Theorema 33. Propositio 38.

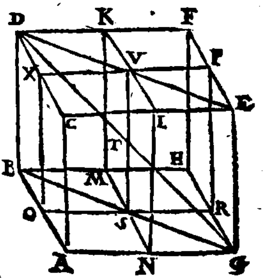
Si planum ad planum rectum sit, & à quodã puncto eorum quæ in vno sunt planorum perpendicularis ad alterum ducta sit, illa quæ ducitur perpẽdicularis, in communem cadet planorũ sectionem.



Theo-

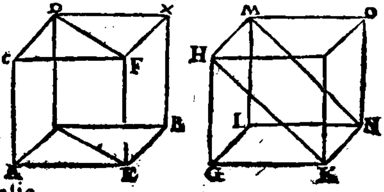
Theorema 34. Propoſitio 39.

Si in ſolido parallelis planis circunſcripto, aduerſorum planorum lateribus biſariàm ſectis, educta ſint per ſectiones plana, communis illa planorum ſectio & ſolidi parallelis plani circũſcripti diameter, ſe mutuò biſariàm ſecant.



Theorema 35. Propoſitio 40.

Si duo ſint æqualis altitudinis priſmata, quorum hoc quidem baſim habeat parallelogrammum, illud verò triangulum, ſit autem parallelogrammũ trianguli duplum, illa priſmata erunt æqualia.

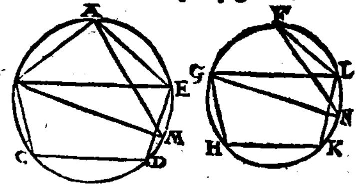


ELEMENTI XI. FINIS.

EVCLIDIS ELEMENTVM DVODECIMVM, ET SOLIDORVM *secundum*.

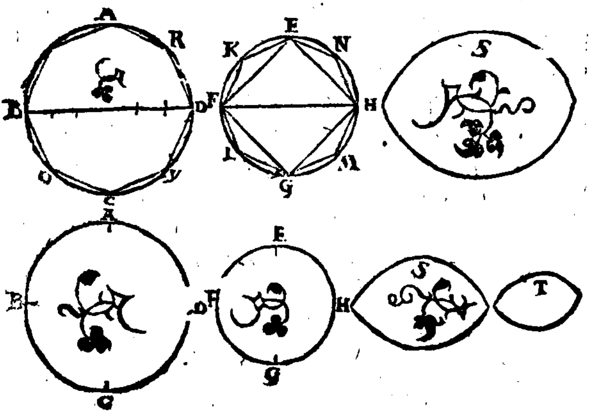
Theorema 1. Propositio 1.

Similia, quæ sunt in circulis polygona, rationem habent inter se, quam descripta à diametris quadrata.



Theorema 2. Propositio 2.

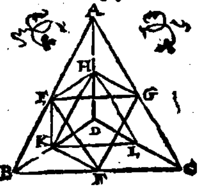
Circuli eam inter se rationem habent, quam



descripta à diametris quadrata.

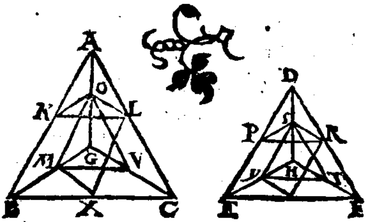
Theorema 3. Propositio 3.

Omnis pyramis trigonam habens basim, in duas diuiditur pyramidas non tantùm æquàles & similes inter se, sed toti etiam pyramidi similes, quarum trigonæ sunt bases, atq; in duo prismata æqualia, quæ duo prismata dimidio pyramidis totius sunt maiora.



Theorema 4. Propositio 4.

Si duæ eiusdem altitudinis pyramides trigonas habeant bases, sit autem illarum vtraque diuisa & in duas pyramidas inter se æquales totique similes, & in duo prismata æqualia, ac eodem modo diuidatur vtraque pyramidum quæ ex superiore diuisione natæ sunt, idque perpetuò fiat: quemadmodum se habet vnius pyramidis basis ad alterius pyramidis basim, ita & omnia quæ in vna pyra-

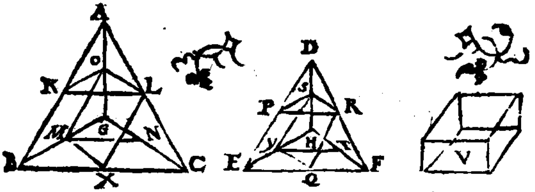


mide

mide priſmata, ad omnia quæ in altera pyramide, priſmata multitudine æqualia.

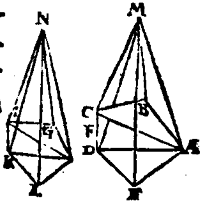
Theorema 5. Propoſitio 5.

Pyramides eiuſdem altitudinis, quarum trigonæ ſunt baſes, eam inter ſe rationem habent, quam ipſæ baſes.



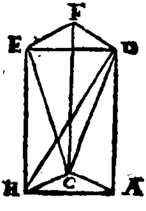
Theorema 6. Propoſitio 6.

Pyramides eiuſdem altitudinis, quarum polygonæ ſunt baſes, eam inter ſe rationem habēt, quam ipſæ baſes.



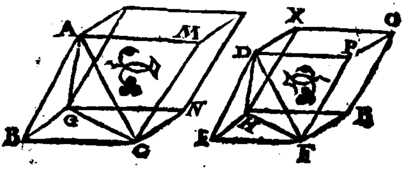
Theorema 7. Propoſitio 7.

Omne priſma trigonam habens baſim, diuiditur in tres pyramidas inter ſe æquales, quarum trigonæ ſunt baſes.



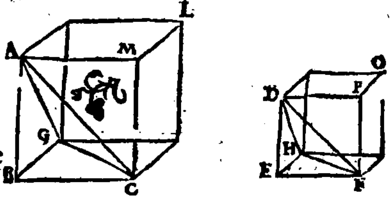
Theorema 8. Propositio 8.

Similes pyramides qui trigωnas habent bases, in triplicata sunt homologorum laterũ ratione



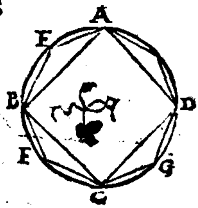
Theorema 9. Propositio 9.

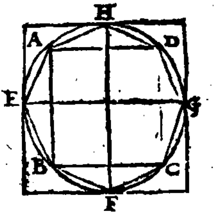
Aequalium pyramidum & trigωnas bases habentium reciprocantur bases cum altitudinibus. Et quarum pyramidum trigωnas bases habentium reciprocãtur bases cũ altitudinibus, illę sunt æquales.



Theorema 10. Propositio 10.

Omnis cωnus tertia pars est cylindri eandẽ



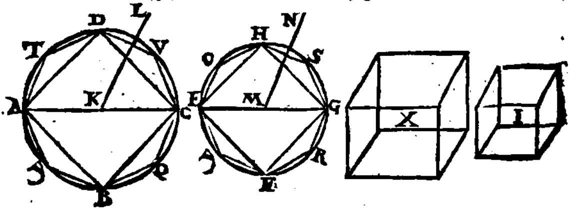


O cum

cum ipso cono basim habentis, & altitudinem æqualem.

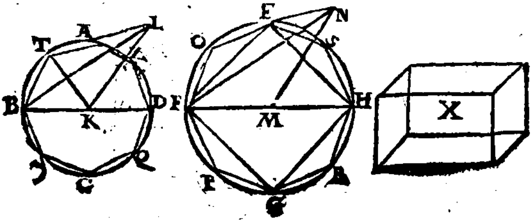
Theorema 11. Propositio 11.

Coni & cylindri eiusdem altitudinis, eam inter se rationem habent, quam bases.



Theorema 12. Propositio 12.

Similes coni & cylindri, triplicatam habent inter se rationem diametrorum, quæ sunt in basibus.



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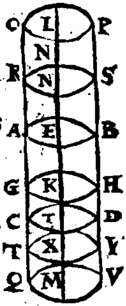
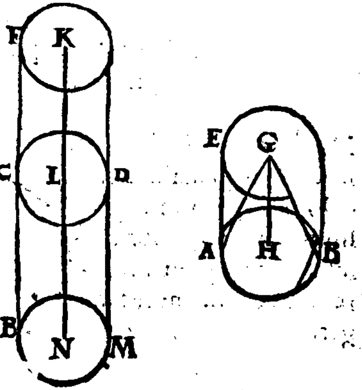
Theorema 13. Propositio 13.



Si cylindrus plano sectus sit aduersis planis parallelo, erit quemadmodum cylindrus ad cylindrum, ita axis ad axem.

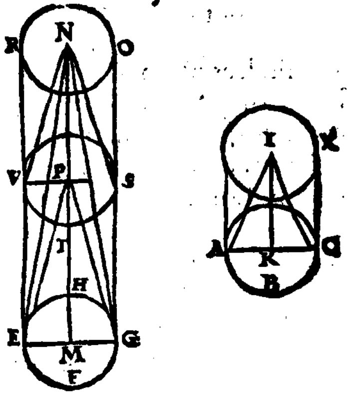
Theorema 14. Propositio 14.



coni & ylindri qui in æqualibus sunt basibus, eam habent inter se rationem, quam altitudines.

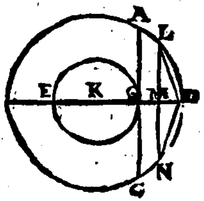
Theorema 15. Propositio 15.

Aequalium conorum & cylindrorum bases cum altitudinibus reciprocantur. Et quorum conorum & cylindrorum bases cum altitudinibus reciprocantur, illi sunt æquales.



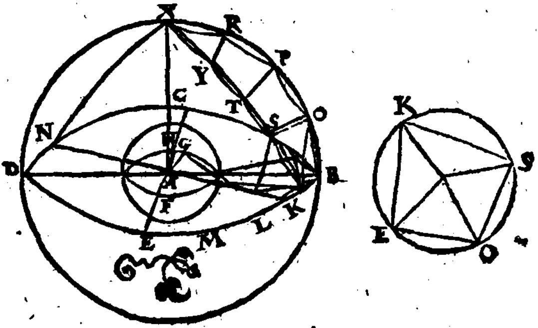
Problema 1. Propositio 16.

Duobus circulis circum idem centrum consistentibus, in maiore circulo polygonum æqualium pariumq; laterum inscribere, quod minorem circulum non tangat.



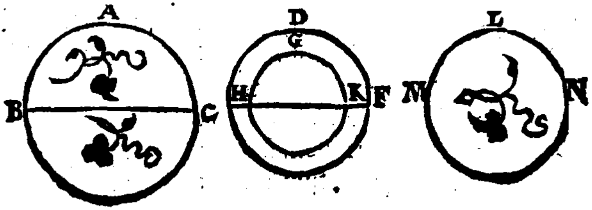
Problema 2. Propoſitio 17.

Duabus ſphæris circum idem centrum conſiſtentibus, in maiore ſphera ſolidum polyedrum inſcribere, quod minoris ſphæræ ſuperficiem non tangat.



Theorema 16. Propoſitio 18.

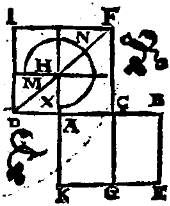
Sphæræ inter ſe rationem habent ſuarum diametrorum triplicatam.



ELEMENTI XII. FINIS.

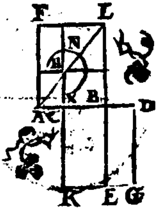
EVCLIDIS ELEMENTVM DECIMVMTERTIVM, ET SOLIDOrum tertium.

Theorema 1. Propositio 1.



Si recta linea per extremã & mediam rationem secta sit, maius segmentũ quod totius lineæ dimidium assumpserit, quintuplum potest eius quadrati, quod à totius dimidia describitur.

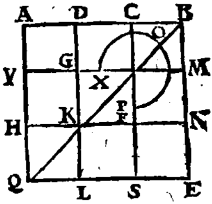
Theorema 2. Propositio 2.



Si recta linea sui ipsius segmenti quintuplum possit, & dupla segmenti huius linea per extremam & mediam rationem secetur, maius segmentum reliqua pars est lineæ primùm positæ.

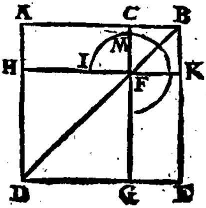
Theo-

Theorema 3. Propositio 3.



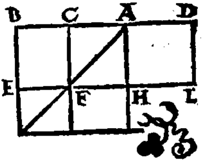
Si recta linea per extremam & mediam rationem secta sit, minus segmentum quod maioris segmenti dimidium assumpserit, quintuplum potest eius, quod à maioris segmenti dimidio describitur, quadrati.

Theorema 4. Propositio 4.



Si recta linea per extremam & mediam rationē secta sit, quod à tota, quodque à minore segmento simul vtraq; quadrata, tripla sunt eius, quod à maiore segmento describitur, quadrati.

Theorema 5. Propositio 5.



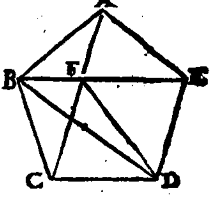
Si ad rectam lineam, quæ per extremam & mediam rationem secetur, adiuncta sit altera segmento maiori æqualis, tota hæc linea recta per extremam & mediam rationem secta est, estque maius segmentum linea primùm posita.

Theorema 6. Propositio 6.

Si recta linea ῥητὴ siue rationalis, per extremam & mediam rationem secta sit, vtrunque segmentorum ἄλογος siue irrationalis est linea, quæ dicitur Residuum.

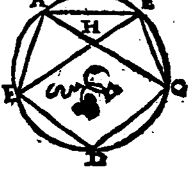
Theorema 7. Propositio 7.

Si pentagωni æquilateri tres sint æquales anguli, siue qui deinceps, siue qui non deinceps sequuntur, illud pentagωnum erit æquiangulum.



Theorema 8. Propositio 8.

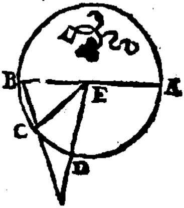
Si pentagωni æquilateri & æquianguli duos qui deinceps sequuntur angulos rectæ subtendant lineæ, illæ per extremam & mediam rationem se mutuò secant, earumque maiora segmenta, ipsius pentagωni lateri sunt æqualia.



Theo-

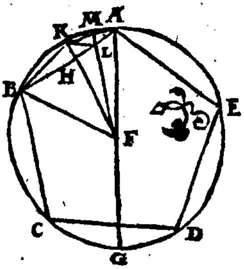
Theorema 9. Propositio 9.

Si latus hexagωni & latus decagωni eidem circulo inscriptorum composita sint, tota recta linea per extremam & mediam rationem secta est, eiusque segmentum maius, est hexagωni latus.



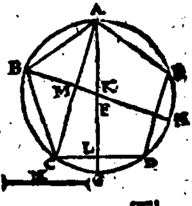
Theorema 10. Propositio 10.

Si circulo pentagωnum æquilaterum inscriptum sit, pentagωni latus potest & latus hexagωni & latus decagωni eidem circulo inscriptorum.



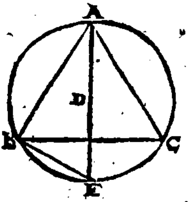
Theorema 11. Propositio 11.

Si in circulo ῥητὴν habente diametrum, inscriptum sit pentagωnum æquilaterum, pentagωni latus irrationalis est linea, quæ vocatur Minor.



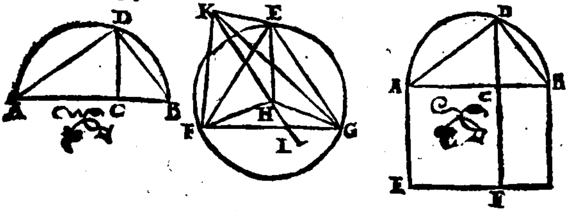
Theorema 12. Propoſitio 12.

Si in circulo inſcriptum ſit triangulum æquilaterum, huius trianguli latus potentia triplum eſt eius lineæ, quæ ex circuli centro ducitur.



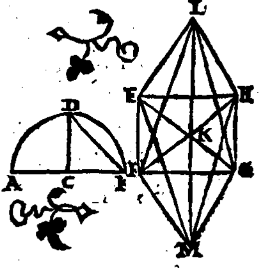
Problema 1. Propoſitio 13.

Pyramidem conſtituere, & data ſphæra cõplecti, atque docere illius ſphæræ diametrum potentia ſeſquialteram eſſe lateris ipſius pyramidis.



Problema 2. Propoſitio 14.

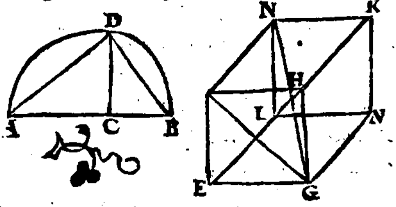
Octaëdrum conſtituere, eaq́; ſphęra qua pyramidē complecti, atque probare illius ſphęræ diametrum potentia duplam eſſe lateris ipſius octaëdri.



Proble-

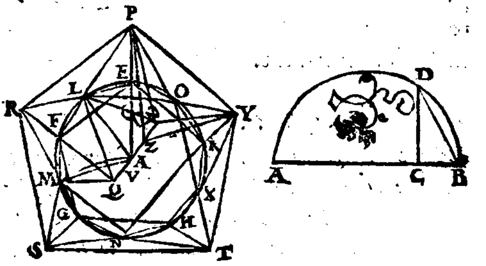
Problema 3. Propositio 15.

Cubum constituere, eaque sphæra qua & superiores figuras complecti, atque docere illius sphæræ diametrum potentia triplam esse lateris ipsius cubi.



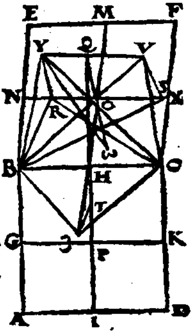
Problema 4. Propositio 16.

Icosaëdrum constituere, eademque sphæra qua & antedictas figuras complecti, atque probare, Icosaëdri latus irrationalem esse lineam, quæ vocatur Minor.



Proble-

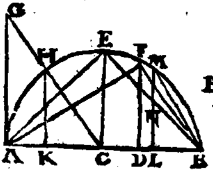
Problema 5. Propositio 17.

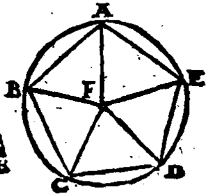


Dodecaëdrum constituere, eademque sphæra qua & antedictas figuras complecti, atque probare dodecaëdri latus irrationalem esse lineam, quæ vocatur Residuum.

Theorema 6. Propositio 18.

Quinque figurarũ latera proponere, & inter se comparare.





SCHOLIVM.

Aio verò, præter dictas quinque figuras non posse aliam constitui figuram solidam, quæ planis & æquilateris & æquiangulis contineatur, inter se æqualibus. Non enim ex duobus triangulis, sed neque ex alijs duabus figuris solidus

dus constituetur angulus.

Sed ex tribus triangulis, constat Pyramidis angulus.

Ex quatuor autem, Octaëdri.

Ex quinque verò, Icosaëdri.

Nam ex triangulis sex & æquilateris & æquiangulis ad idem punctum coëuntibus, non fiet angulus solidus. Cùm enim trianguli æquilateri angulus, recti vnius bessem contineat, erunt eiusmodi sex anguli rectis quatuor æquales. Quod fieri non potest. Nam solidus omnis angulus, minoribus quàm rectis quatuor angulis continetur, per 21.11.

Ob easdem sanè causas, neque ex pluribus quàm planis sex eiusmodi angulis solidus constat.

Sed ex tribus quadratis, Cubi angulus cõtinetur.

Ex quinque, nullus potest. Rursus enim recti quatuor erunt.

Ex tribus autem pentagonis æquilateris & æquiangulis, Dodecaëdri angulus continetur.

Sed ex quatuor, nullus potest. Cùm enim pentagoni æquilateri angulus rectus sit, & quinta recti pars, erunt quatuor anguli rectis quatuor maiores. Quod fieri nequit. Nec sanè ex alijs polygonis figuris solidus angulus continebitur, quòd hinc quoque absurdum sequatur. Quamobrẽ perspicuũ est, præter dictas quinque figuras aliam figuram solidam non posse constitui, quæ ex planis æquilateris & æquiangulis contineatur.

ELEMENTI XIII. FINIS.

EVCLIDIS ELEMENTVM DECIMVMQVARTVM, VT quidam arbitrantur, vt alij verò, Hypsiclis Alexandrini, de quinque corporibus,

LIEER PRIMVS.

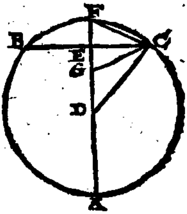
BAsilides Tyrius, Protarche, Alexandriam profectus, patriq́ue nostro ob disciplinæ societatem commendatus, longissimo peregrinationis tempore cũ eo versatus est. Cumq́ue different aliquando de scripta ab Apollonio comparatione Dodecaëdri & Icosaëdri eidem sphæræ inscriptorum, quam hæc inter se habeant rationem, censuerunt ea non rectè tradidisse Apollonium: quæ à se emendata, vt de patre audire erat, literis prodiderunt. Ego autem postea incidi in alterum librum ab Apollonio editum, qui demonstrationem accuratè complecteretur de re proposita, ex eiusq́ue problematis indagatione magnam equidem cepi voluptatem. Illud certè ab omnibus perspici potest, quod scripsit Apollonius, cum sit in omnium manibus. Quod autem diligenti, quantum conycere licet, studio nos postea

scrip-

scripsisse videmur, id monimentis consignatum tibi nuncupandum duximus, ut qui feliciter cùm in omnibus disciplinis tum vel maximè in Geometria versatus, scitè ac prudenter iudices ea quæ dicturi sumus. ob eam verò, quæ tibi cum patre fuit, vitæ consuetudinem, quaq; nos complecteris, beneuolentiam, tractationem ipsam libenter audias. Sed iam tempus est, ut proœmio modum facientes, hanc syntaxim aggrediamur.

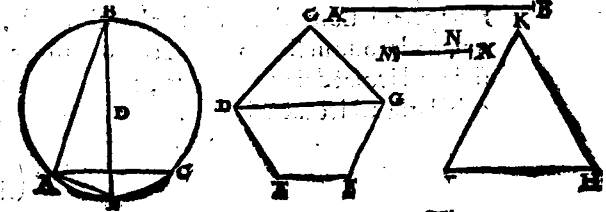
Theorema 1. Propositio 1.

Perpendicularis linea, quæ ex circuli cuiuspiam centro in latus pentagoni ipsi circulo inscripti ducitur, dimidia est utriusque simul lineæ, & eius quæ ex centro, & lateris decagoni in eodem circulo inscripti.



Theorema 2. Propositio 2.

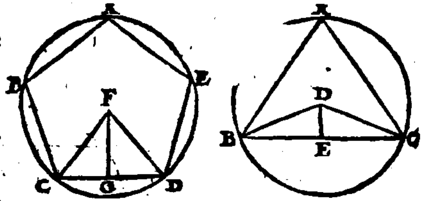
Idem circulus comprehendit & dodecaëdri pentagonum & icosaëdri triangulum, eidem sphæræ inscriptorum.



Theo-

Theorema 3. Propoſitio 3.

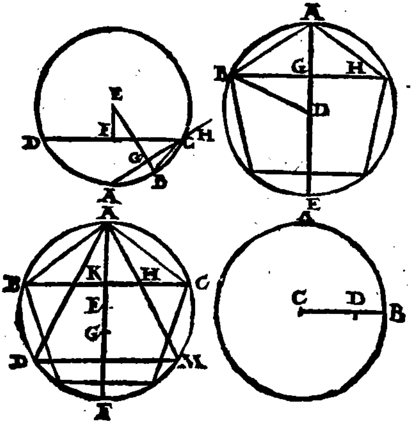
Si pentagono & æquilatero & æquiangulo circumſcriptus ſit circulus, ex cuius centro in vnum pentagoni latus ducta ſit perpendicularis: quod vno laterum & perpendiculari trigeſies continetur, illud



æquale eſt dodecaëdri ſuperficiei.

Theorema 4. Propoſitio 4.

Hoc perſpicuum cùm ſit, probandum eſt, quemadmodum ſe habet dodecaëdri ſuperficies ad icoſaëdri ſuperficiem, ita ſe habere cubi latus ad icoſaëdri latus.



Cubi latus.

E

Dωdecaëdri.

F

Icωsaëdri.

G

SCHOLIVM.

Nunc autem probandum est, quemadmodum se habet cubi latus ad icosaëdri latus, ita se habere solidum dωdecaëdri ad Icosaëdri solidum. Cùm enim æquales circuli comprehendant & dωdecaëdri pentagωnum & Icosaëdri triangulum, eidem sphæræ inscriptorum: in sphæris autem *æquales circuli æquali interuallo distent à centro (siquidem perpendiculares à sphæræ centro ad circulorum plana ducta & æquales sunt, & ad circulorum centra cadunt) idcirco lineæ, hoc est perpendiculares, quæ à sphæræ centro ducuntur ad centrum circuli comprehendentis & triangulum Icosaëdri & pentagonum dodecaëdri, sunt æquales. Sunt igitur æqualis altitudinis Pyramides, quæ bases habent ipsa dodecaëdri pentagona, & quæ Icosaëdri triangula. At æqualis altitudinis pyramides rationem inter se habent eam quam bases, ex 5. & 6.11. Quemadmodum igitur pentagonum ad triangulum, ita pyramis, cuius basis quidem est dodecaëdri pentagonum, vertex autem sphæræ centrum, ad pyramida cuius basis quidem est Icosaëdri triangulum, vertex autem, sphæræ centrum. Quamobrem ut se habent duodecim pentagona ad viginti triangula, ita duodecim pyramides quorum pentagona sint bases, ad viginti pyramidas, quæ trigonas habeant bases. At pentagona duodecim sunt dodecaëdri superficies, viginti autem triangula, Icosaëdri. Est igitur ut dodecaëdri superficies ad Icosaëdri superficiem, ita duodecim pyramides, quæ pentagonas habeant bases, ad viginti pyramidas, quarum trigona sunt bases. Sunt autem duodecim quidem pyramides, quæ pentagonas habeant bases, solidum dodecaëdri: viginti autem pyramides, quæ trigonas habeant bases, Icosaëdri solidum. Quare ex 11. 5. ut dodecaë-*

decaëdri ſuperficies ad Icoſaedri ſuperficiem, ita ſolidum dodecaëdri ad Icoſaëdri ſolidum. Ut autem dodecaëdri ſuperficies ad Icoſaëdri ſuperficiem, ita probatum eſt cubi latus ad Icoſaëdri latus. Quemadmodum igitur cubi latus ad Icoſaëdri latus, ita ſe habet ſolidum dodecaëdri ad Icoſaëdri ſolidum.

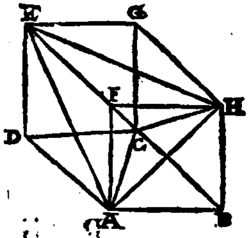
ELEMENTI XIIII. FINIS.

EVCLIDIS ELEMENTVM DECIMVMQVINTVM, ET Solidorum quintum, vt nonnulli putant, vt autem alij, Hypſiclis Alexandrini, de quinque corporibus,

LIBER II.

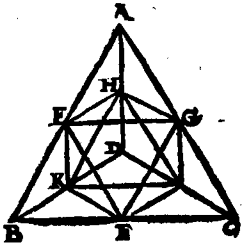
Problema 1. Propoſitio 1.

In dato cubo pyramida inſcribere.



Problema 2. Propoſitio 2.

In data pyramide octaëdrum inſcribere.



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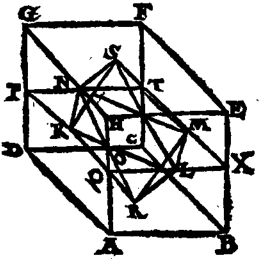
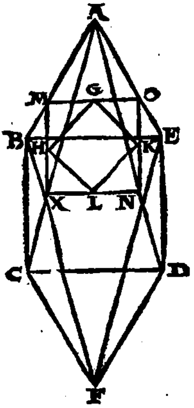
Problema 3. Propoſitio 3.

In dato cubo octaëdrum inſcribere.



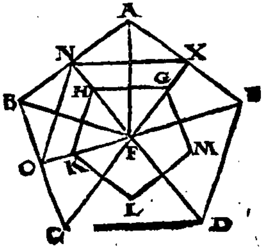
Problema 4. Propoſitio 4.

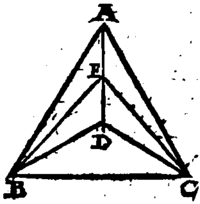
In dato octaëdro cubum inſcribere.

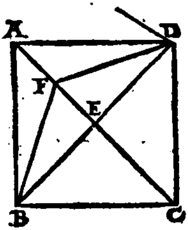


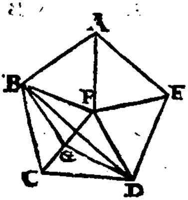
Problema 5. Propoſitio 5.

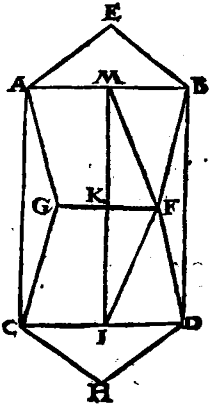
In dato Icoſaëdro dωdecaedrum inſcribere

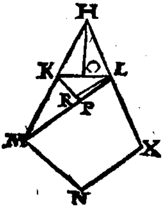












SCHO-

SCHOLIVM.

Meminiſſe decet, ſi quis nos roget quot Icoſaëdrum habeat latera, ita reſpondendum eſſe: Patet Icoſaëdrum viginti contineri triangulis, quodlibet verò triangulum rectis tribus conſtare lineis. Quare multiplicanda ſunt nobis viginti triangula in trianguli vnius latera, fiuntq́; ſexaginta, quorum dimidium eſt triginta. Ad eundem modum & in dodecaëdro. Cùm enim rurſus duodecim pentagωna dωdecaëdrum comprehendant, itemque pentagωnum quoduis rectis quinque conſtet lineis, quinque duodecies multiplicamus, fiunt ſexaginta, quorum rurſus dimidium eſt triginta. Sed cur dimidium capimus? Quoniam vnumquodque latus ſiue ſit trianguli ſiue pentagωni, ſiue quadrati, vt in cubo, iteratò ſumitur. Similiter autem eadem via & in cubo & in pyramide & in octaëdro latera inuenies. Quòd ſi item velis ſingularum quoque figurarum angulos reperire, facta eadem multiplicatione numerum procreatum partire in numerum planorum, quæ vnum ſolidum angulum includunt: vt quoniam triangula quinque vnum Icoſaëdri angulum continent, partire 60. in quinque, naſcuntur duodecim anguli Icoſaëdri. In dωdecaëdro autem tria pentagωna angulum comprehendunt. partire ergo 60. in tria, & habebis dωdecaëdri angulos viginti. Atque ſimili ratione in reliquis figuris angulos reperies.

Finis Elementorum Euclidis.