EUCLIDIS ELEMENTORUM

Libri XV. breviter demonstrati,

Operâ

IS. BARROW, *Cantabrigiensis*,
Coll. TRIN. *Soc.*

Et prioribus mendis typographicis nunc demum purgati.

HIEROCL.

Καθαρμοὶ ψυχῆς λογικῆς εἰσιν αἱ μαθηματικαὶ ἐπιστῆμαι.

LONDINI,

Typis *J. Redmayne*: Prostant autem apud *J. Williams* ad Insigne Coronæ in Cœmeterio D. *Pauli*, & *J. Dunmore* ad Insigne Trium Bibliorum in vico vulgò vocato *Ludgate-street*, MDCLXXVIII.

Nobilissimis & Generosissimis

Adolescentibus,

Dno *EDOUARDO CECILIO*,

Illustriss. Comitis Sariesburiensis *Filio*;

Dno *JOHANNI KNATCHBUL*,

E T

D. *FRANCIS. WILLOUGHBY*,

ARMIGERIS.

UNicuique vestrum (Optimi Adolescentes) tantum me debere reputo, quantum homo homini debere potest. Mea enim sententia, ultra sincerum amorem non est quod quispiam de alio bene mereri poſſit. Hunc autem jamdiu eſt quo ex ſingulari veſtra bonitate mihi indultum experior; ejuſque ſenſus, intimis animi medullis inhærens, ipſi ardens ſtudium impreſſit quovis honeſto modo reciprocos affectus prodendi. Quandoquidem vero ea fortunarum mearum tenuitas, ea veſtrarum amplitudo, exiſtit, ut nec ego alia quam gratæ alicujus agnitionis ſignificatione uti queam, nec vos aliam admittere velitis; ea propter haud illibenter hanc occaſionem arripio, honoris & benevolentiæ, quibus vos proſequor, publicum hoc & durabile μνημόσυνον edendi. Etſi cum oblati anathematis exilitatem, & libellum veſtris nominibus conſecratum, quam is longe infra veſtrorum meritorum dignitatem ſubſidat, attentius conſidero, timor ſubinde aliquis & dubitatio animum inceſſant, ne hoc ſtudium erga vos meum vobis

bis dehonestamento sit potius quam ornamento; scilicet memor cum sim, ut malæ causæ, sic & mali libri patrocinium in patroni contumeliam magis quam in gloriam cedere. Sed quum vestrarum virtutum id robur, eam fore soliditatem, recognoscerem, quæ vestrum decus, meo quantumvis labefactato, inconcussum sustinere possint; idcirco non dubitavi vos in aliquatenus commune mecum periculum induere. Virtutes illas intelligo, quibus nemo unquam in vestra ætate aut in vestro ordine, saltem me judice, majores deprehendit; quæ vos insigniter gratos omnibus & amabiles reddunt; eximiam modestiam, sobrietatem, benignitatem animi, morum comitatem, prudentiam, magnanimitatem, fidem, præclaram insuper ingenii indolem, quæ vos ad omnem ingenuam scientiam non tantum excellenti captu, sed & appetitu forti ac sincero, instruxit. Quas vestras præclarissimas dotes prout nemo est fortassis qui me melius novit, aut pro consuetudine, quam jamdudum vobiscum dulcissimam coluisse ex vestro favore mihi contigit, penitus introspexit, ita nemo est qui impensius miratur & suspicit; aut qui ipsas libentius prædicare ac celebrare vellet, si non cum eloquii mei vires supergrederentur, tum etiam quæ in singulis vobis elucent, prolixi alicujus commentarii aut panegyricæ orationis libertatem, potius quam præstitutas hujusmodi salutationibus angustias, exposcerent. Quin potius divinam clementiam imploro, ut vos earundem virtutum sancto tramiti insistere, atque hos egregios fructus vernæ vestræ ætatis felicibus incrementis maturescere concedat; vitamque vobis in hoc seculo ingenuam, innocentem, jucundam, & in futuro beatam

tam ac ſempiternam tranſigere largiatur. Minime autem dubito, ne pro conſueto veſtro in me candore hoc ultimum fortaſſis quod vobis præſtare potero, benevolentiæ erga vos & obſervantiæ teſtimonium, alacriter accepturi ſitis; quod vobis propenſiſſimo affectu offert

Veſtri in æternum amantiſſimus,

& obſervantiſſimus,

I. B.

[illegible]

[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]

[illegible]

[illegible]

[illegible]

Benevolo LECTORI.

SI, quid in hac elementorum editione præstitum sit, scire desideras, amice Lector, accipe, pro genio operis, breviter. Ad duos præcipue fines conatus meos direxi. Primum, ut cum requisita perspicuitate summam demonstrationum brevitatem conjungerem, quo eam libello molem compararem, quæ commode absque molestia circumferri posset. Id quod assecutus videor, si absentem Typographi cura non frustretur. Concinnius enim quispiam meliori ingenio aut majori peritia excellens, at nemo forsan brevius plerasque propositiones demonstraverit; præsertim cum in numero & ordine propositionum ipse nihil immutarim, nec licentiam mihi assumpserim quamcunque propositionem Euclideam procul ablegandi tanquam minus necessariam, aut quasdam faciliores in axiomatum censum referendi; quod nonnulli fecerunt: inter quos peritissimus Geometra Andr. Tacquetus, (quem ideo etiam nomino, quod quædam ex eo desumpta agnoscere honestũ duco,) post cujus elegantissimam editionem, ipse nihil attentare

tare

tare voluissem, si non visum fuisset doctissimo viro non nisi octo Euclidis *libros suâ curâ adornatos publico communicare, reliquis septem, tanquam ad elementa Geometriæ minus spectantibus, omnino quasi spretis atque posthabitis. Mihi autem jam ab initio alia provincia demandata fuit, non elementa Geometriæ utcunque pro arbitrio conscribendi, verum* Euclidem *ipsum, eumque totum, quam possem brevissime, demonstrandi. Quod enim quatuor libros spectat, septimum, octavum, nonum, decimum, quamvis illi ad Geometriæ plana & solida elementa, ut sex præcedentes & duo subsequentes, non tam prope pertineant; quod tamen ad res Geometricas admodum utiles sint, tam propter Arithmeticæ & Geometriæ valde propinquam cognationem, quam ob notitiam commensurabilium & incommensurabilium magnitudinum ad figurarum tam planarum quam solidarum intellectum apprime necessariam, nemo est è peritioribus Geometris qui ignorat. Quæ vero in tribus ultimis libris continetur, 5 corporum regularium nobilis contemplatio, illa non nisi injuria prætermitti potuit; quando nempe illius gratia noster στοιχειώτης, Platonicæ familiæ philosophus, hoc elementorum systema universum condidisse perhibetur;*

uti

uti testis est *Proclus, *iis verbis,* Ὅθεν δὴ καὶ τῆς συμπάσης στοιχειώσεως τέλος προεστήσατο τὴν τῶν καλουμένων πλατωνικῶν σχημάτων σύστασιν. *Præterea facile in animum induxi ut opinarer, nemini harum scientiarum amanti non futurum esse cordi penes se habere integrum* Euclidæum *opus, quale passim ab omnibus citatur & celebratur. Quare nullum librum nullamque propositionem negligere volui earum quæ apud* P. Herigonium *habentur; cujus vestigiis presse insistere necesse habui, quoniam ejusce libri schematismis maxima ex parte uti statutum erat, quod prævidérem mihi ad novas describendas tempus non suppetere; etsi nonnunquam id facere præoptassem. Eadem de causa nec alias plerasque quam* Euclidæas *demonstrationes adhibere volui, succinctiori forma expreßas, nisi forte in 2, & 13, & parce in 7, 8, 9 libris; ubi ab eo nonnihil deflectere operæ pretium videbatur. Bona igitur spes est saltem in hac parte cum nostris consiliis, tum studiosorum votis, aliquo modo satisfactum iri. Nam quæ adjecta sunt in Scholiis problemata quædam & theoremata, sive ob suum frequentem usum ad naturam elementarem accedentia, sive ad eorum quæ sequuntur expeditam demonstrationem conducentia, seu quæ regularum*

* lib. 2.

rum practica Geometria quarundam præcipuarum rationes innuunt ad suos fontes relatas, per ea, ut spero, libellus ultra destinatam molem magnopere non intumescet.

Alter scopus ad quem collineatum est, eorum desideriis consuluit qui demonstrationibus symbolicis potius quam verbalibus delectantur. In quo genere cum plerique apud nos Guilielmi Oughtredi *symbolis assueti sint, ea plerumque usurpare consultius duximus. Nam qui* Euclidem *hâc viâ tradere & interpretari aggressus sit, hactenus, quod ego sciam, præter unum* P. Herigonium, *repertus est nemo. Cujus viri longe doctissimi methodus, sane in multis egregia, ac ejus peculiari proposito admodũ accomodata, duplici tamen defectu laborare mihi visa est. Primo, quod cum Propositionũ ad unius alicujus theorematis aut problematis probationẽ adductarũ posterior à priori non semper dependeat; quando tamen illa inter se cohærent, quando non, nec ex ordine singularũ, nec ullo alio modo, satis prompte innotescere potest: unde ob defectũ conjunctionũ & adjectivorũ (* ergo, rursus, *&c.) non raro difficultas & dubitandi occasio, præsertĩ minus exercitatis, inter legendũ oboriri solent. Deinde sæpenumero evenit, ut prædicta methodus supervacaneas repetitiones effugere nequeat, à quibus demonstrationes est quando proli-*

xa,

xa, aliquando & magis intricata, evadunt. Quibus vitiis noster modus facile per verborum signorumq; arbitrariam mixturam medetur. Atque hæc de opella hujus intentione & methodo dicta sufficiant. Cæterum quæ in laudem Matheseos in genere, aut Geometriæ ipsius; & quæ de historia harum scientiarum, ideoque de Euclide *horum elementorum digestore, dici possent, & reliqua hujusmodi ἐξωτερικά, cui hæc placent, apud alios interpretes consulere potest. Neque nos angustias temporis quod huic operi impendi potuit, nec interpellationes negotiorum, nec adjumentorum ad hæc studia apud nos egestatem, & quædam alia, ut liceret non immerito, in excusationem obtendemus; metu scilicet inducti, ne hæc nostra omnibus minus satisfaciant. Verum quæ ingenui Lectoris usibus elaboravimus, eadem in solidum ipsius censuræ ac judicio submittimus; probanda si utilia sibi compererit; sin omnino secus, rejicienda.*

I. B.

Ad amicissimum Virum, *I. B.* de *EUCLIDE* contracto
Εὐφημισμός.

FActum bene! didicit Laconice loqui
Senex profundus, & aphorismos induit.
Immensa dudum margo commentarii
Diagramma circuit minutum: utque Insula
Problema breve natabat in vasto mari.
Sed unda jam detumuit, & glossa arctior
Stringit Theoremata: minoris anguli
Lateribus ecce totus *Euclides* jacet,
Inclusus olim velut Homerus in nuce;
Pluteoque sarcina modo qui incubuit, levis
En fit manipulus. Pelle in exigua latet
Ingens Mathesis, matris ut in utero Hercules,
In glande quercus, vel Ithaca Eurus in pila.
Nec mole dum decrescit, usu fit minor;
Quin auctior jam evadit, & cumulatius
Contracta prodest erudita pagina.
Sic ubere magis liquor è presso effluit;
Sic pleniori vasa inundat sanguinis
Torrente cordis Systole; sic fusius
Procurrit æquor ex Abylæ angustiis!
Tantilli operis laus tanta referenda unice est
BAROVIANO nomini, ac solertiæ.
Sublimis euge mentis ingenium potens!
Cui invium nil, arduum esse nil solet.
Sic usque pergas prospero conamine,
Radiusque multum debeat ac abacus tibi;
Sic crescat indies feracior seges,
Simili colonum germine assiduo beans.
Specimen futuræ messis hic fiet labor,
Magnæque famæ illustria hæc præludia.
Juvenis dedit qui tanta, quid dabit senex?

Car. Robotham, *CANTAB.*
Coll. Trin. Sen. Soc.

In novam *Elementorum*

EUCLIDIS

Editionem à D. *IS. BARROW*,
Collegii SS. TRIN. *Socio*,
viro opt. & eruditissimo,
adornatam.

BEnigne Lector! si uspiã auditũ est tibi,
Quantus tenella Nix Geometres siet;
Quæ mille radiis, mille ludit angulis,
Totumque puro ducit Euclidem sinu:
Amabis ultro candidissimum Virum,
Cui plena nivium est indoles, sed quas tamẽ
Præclarus ardor mentis urget Enthea;
Et usque blandis temperat caloribus:
Quo suavius nil vivit, & melius nihil.
Is, dum liquentes pectore excutit nives,
Et inde & inde spargit, en aliam tibi,
Lector benigne, è nivibus Geometriam!

G. C. *A. M. C. E. S.*

$=$ æqualitatem.
⊏ majoritatem.
⊐ minoritatem.
$+$ plus, vel addendum esse.
$-$ minus, vel subtrahendum esse.
$-:$ differentiam vel excessum; item quantitates omnes, quæ sequuntur, subtrahendas esse, signis non mutatis.
$\times$ multiplicationem, vel ductum lateris rectanguli in aliud latus.
Idem denotat conjunctio literarum, ut $AB = A \times B$.
$\surd$ Latus, vel radicem quadrati, vel cubi, &c.
Q. & q quadratum. C. & c cubum.
Q. Q. rationem quadrati numeri ad quadratum numerum.

significat.

Reliquas, quæ ubicunque occurrunt, vocabulorum abbreviationes ipse Lector per se facile intelliget; exceptis iis, quas tanquam minus generalis usus, suis locis explicandas relinquimus.

LIB. I.

Definitiones.

I. PUnctum est cujus pars nulla est.

II. Linea vero longitudo latitudinis expers.

III. Lineæ autem termini sunt puncta.

IV. Recta linea est, quæ ex æquo sua interjacet puncta.

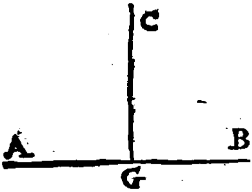
V. Superficies est, quæ longitudinem, latitudinemque tantum habet.

VI. Superficiei autem extrema sunt lineæ.

VII. Plana superficies est, quæ ex equo suas interjacet lineas.

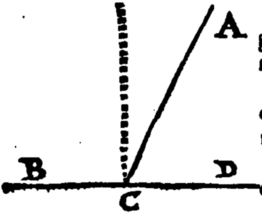
VIII. Planus vero angulus est, duarum linearum in plano se mutuo tangentium, & non in directum jacentium alterius ad alteram inclinatio.

IX. Cum autem quæ angulum continent, lineæ, rectæ fuerint, rectilineus ille angulus appellatur.



X. Cum vero recta linea C G super rectam lineam A B consistens, eos qui sunt deinceps angulos C G A, C G B æquales inter se fecerit, rectus est uterque æqualium angulorum, & quæ insistit recta linea C G, perpendicularis vocatur ejus (A B) cui insistit.

Not. *Cum plures anguli ad unum punctum: (ut ad* G) *exsistunt, designatur quilibet angulus tribus literis, quarum media ad verticem est illius de quo agitur: ut angulus quem rectæ* C G, A G *efficiunt ad partes* A, *vocatur* G G A, *vel* A G C.



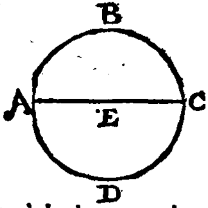
XI. Obtusus angulus est, qui recto major est, ut A C B.

XII. Acutus vero, qui minor est recto, ut A C D.

XIII. Terminus est, quod alicujus extremum est.

XIV. Figura est, quæ sub aliquo, vel aliquibus terminis comprehenditur.

XV. Circulus est figura plana, sub una linea comprehensa, quæ peripheria appellatur, ad quam ab uno puncto eorum, quæ intra figuram sunt posita, cadentes omnes rectæ lineæ inter se sunt æquales.



XVI. Hoc vero punctum centrum circuli appellatur.

XVII. Diameter autem circuli est recta quædam linea per centrum ducta, & ex utraque parte in circuli peripheriam terminata, quæ circulum bifariam secat.

XVIII. Semicirculus vero est figura, quæ continetur sub diametro, & sub ea linea, quæ de circuli peripheria aufertur.

In circulo EABCD. E *est centrum*, A C *diameter*, A B C *semicirculus*.

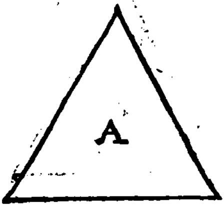
XIX. Recti ineæ figuræ sunt, quæ sub rectis lineis continentur.

XX. Trilateræ quidem, quæ sub tribus.

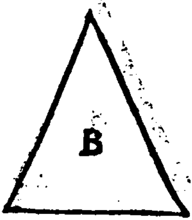
XXI. Quadrilateræ vero, quæ sub quatuor.

XXII. Multilateræ autem, quæ sub pluribus, quam quatuor rectis lineis comprehenduntur.

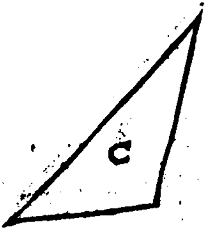
XXIII.



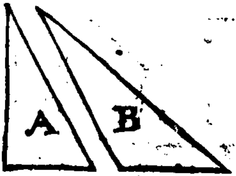
XXIII. Trilaterarum autem figurarum, æquilaterum est triangulum, quod tria latera habet æqualia, ut triangulum A.



XXIV. Isosceles autem, quod duo tantum æqualia habet latera, ut triangulum B.

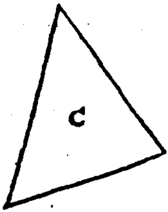


XXV. Scalenum vero, quod tria inæqualia habet latera, ut C.



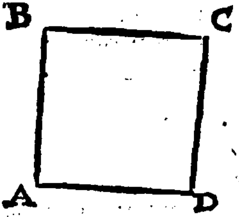
XXVI. Adhæc etiam trilaterarum figurarum, rectangulum quidem triangulum est, quod rectum angulum habet, ut triangulum A.

XXVII. Amblygonium autem, quod obtusum angulum habet, ut B.

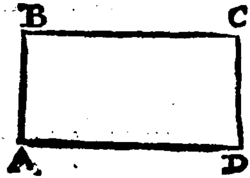


XXVIII. Oxygonium vero, quod tres habet acutos angulos, ut C.

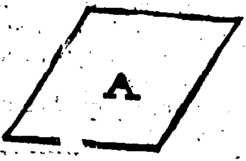
Figura æquiangula eſt, cujus omnes anguli inter ſe æquales ſunt. Duæ vero figuræ æquiangulæ ſunt; ſi ſinguli anguli unius ſingulis angulis alterius ſint æquales. Similiter de figuris æquilateris concipe.



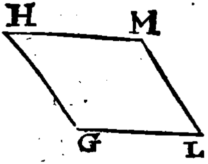
XXIX. Quadrilaterarum autem figurarum, quadratum quidem eſt, quod & æquilaterum, & rectangulum eſt, ut A B C D.



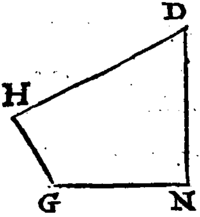
XXX. Altera vero parte longior figura eſt, quæ rectangula quidem, at æquilatera non eſt, ut A B C D.



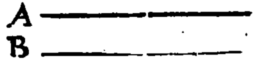
XXXI. Rhombus autem, quæ æquilatera, ſed rectangula non eſt, ut A.



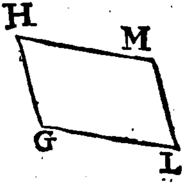
XXXII. Rhomboydes vero, quæ adversa & latera, & angulos habens inter se æquales, neque æquilatera est, neq; rectangula, ut G L M H.



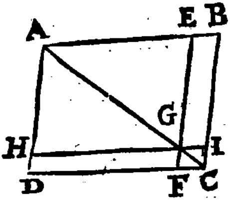
XXXIII. Præter has autem reliquæ quadrilateræ figuræ trapezia appellentur; ut G N D H.



XXXIV. Parallelæ rectæ lineæ sunt, quæ cum in eodem sint plano, & ex utraque parte in infinitum producantur, in neutram sibi mutuo incidunt, ut A, & B.



XXXV. Parallelogrammum est figura quadrilatera, cujus bina opposita latera sunt parallela, seu æquidistantia, ut G L H M.



XXXVI. Cum vero in parallelogrammo A B C D diameter A C ducta fuerit, duæque lineæ E F, H I, lateribus parallelæ secantes diametrum in uno eodemq; puncto G, ita ut parallelogrammum ab hisce pa-

parallelis in quatuor distribuatur parallelogramma; appellantur duo illa D G, G B, per quæ diameter non transit, Complementa; duo vero reliqua H E, F I, per quæ diameter incedit, circa diametrum consistere dicuntur.

Problema est, *cum proponitur aliquid efficiendum.*

Theorema est, *cum proponitur aliquid demonstrandum.*

Corollarium est *consectarium, quod è facta demonstratione tanquam lucrum aliquod colligitur.*

Lemma est *demonstratio præmissa alicujus, ut demonstratio quæsiti evadat brevior.*

Postulata.

1. POstuletur, ut à quovis puncto ad quodvis punctum rectam lineam ducere concedatur.

2. Et rectam lineam terminatam in continuum recta producere.

3. Item, quovis centro, & intervallo circulum describere.

Axiomata.

1. QUæ eidem æqualia, & inter se sunt æqualia.

ut A = B = C. ergo A = C, vel ergo omnes A, B, C, æquantur inter se.

Nota, *Cum plures quantitates hoc modo conjunctas invenias, concipe vi hujus axiomatis primam ultimæ & quamlibet earum cuilibet æquari. Quo in casu sæpe, brevitatis causa, ab hoc axiomate citando abstinemus; etsi vis consecutionis ab eo pendeat.*

2. Et si æqualibus æqualia adjecta sunt, tota sunt æqualia.

3. Et si ab æqualibus æqualia ablata sint, quæ relinquuntur sunt æqualia.

4. Et si inæqualibus æqualia adjecta sint, tota sunt inæqualia.

5. Et si ab inæqualibus æqualia ablata sint, reliqua sunt inæqualia.

6. Et quæ ejusdem vel æqualium sunt duplicia, inter se sunt æqualia. Idem puta de triplicibus, quadruplicibus, &c.

7. Et quæ ejusdem, vel æqualium sunt dimidia, inter se sunt æqualia. Idem concipe de subtriplis, subquadruplis, &c.

8. Et quæ sibi mutuo congruunt, ea inter se sunt æqualia.

Hoc axioma in rectis lineis, & angulis valet conversum, sed non in figuris, nisi illæ similes fuerint.

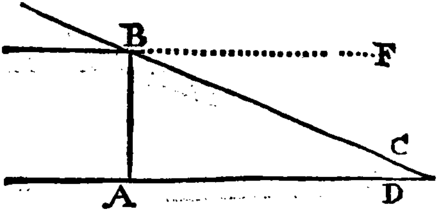
Cæterum, magnitudines congruere dicuntur, quarum partes applicatæ partibus, æqualem vel eundem locum occupant.

9. Et totum sua parte majus est.

10. Duæ rectæ lineæ non habent unum & idem segmentem commune.

11. Duæ rectæ in uno puncto concurrentes, si producantur ambæ, necessario se mutuo in eo puncto intersecabunt.

12. Item omnes anguli recti sunt inter se æquales.



13. Et si in duas rectas lineas A D, C B, in eodem plano jacentes altera recta B A incidens,

 inter-

internos ad easdemque partes angulos BAD, ABC duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ sibi mutuo incident ad eas partes, ubi sunt anguli duobus rectis minores.

14. Duæ rectæ lineæ spatium non comprehendunt.

15. Si æqualibus inæqualia adjiciantur, erit totorum excessus adjunctorum excessui æqualis.

16. Si inæqualibus æqualia adjungantur, erit totorum excessus excessui eorum, quæ à principio, æqualis.

17. Si ab æqualibus inæqualia demantur; erit residuorum excessus, excessui ablatorum æqualis.

18. Si ab inæqualibus æqualia demantur, erit residuorum excessus excessui totorum æqualis.

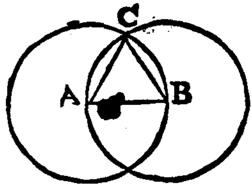
19. Omne totum æquale est omnibus suis partibus simul sumptis.

20. Si totum totius est duplum, & ablatum ablati, erit & reliquum reliqui duplum. Idem de reliquis multiplicibus intellige.

Citationes intellige sic. Cum duo numeri occurrunt, prior designat propositionem, posterior librum. Ut per 4. 1. intelligitur quarta propositio primi libri, atque ita de reliquis. *Cæterum ax. axioma, post. postulatum, def. definitionem, sch. scholium, cor. corollarium denotant, &c.*

LIB. I.

PROP. I.



Super data recta linea terminata A B *triangulum æquilaterum* A B C *constituere.*

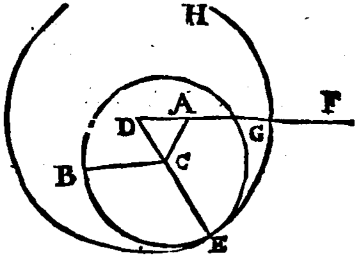
Centris A & B, eodem intervallo A B, vel B A *a* describe duos circulos se intersecantes in puncto C, ex quo *b* duc rectas C A, C B, Erit A C *c* = A B *c* = B C *d* = A C *e* Quare triangulum A C B est æquilaterum. Quod Erat Faciendum.

a 3. *post.*
b 1. *post.*
c 15. *def.*
d 1. *ax.*
e 23. *def.*

Scholium.

Eodem modo super A B describetur triangulum Isosceles, *si intervalla* æqualium *circulorum* majora sumantur, vel minora, quam A B.

PROP. II.



Ad datum punctum A *datæ rectæ lineæ* B C *æqualem rectam lineam* A G *ponere.* a 3. *post.*

Centro C, intervallo C B *a* describe circulum C B E. *b* Junge A C, super qua *c* fac triangulum æquilaterum A D C *d* produc D C ad E. cen-

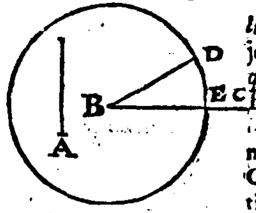
b 1. *post.*
c 1. 1.
d 2. *post.*

centro D, spatio DE, *a* describe circulum DEH:
e 2. *post.* cujus circumferentiæ occurrat D A *e* protracta
f 15. *def.* ad G. Erit A G = C B.
g *constr.* Nam D G *f* = D E, & D A *g* = DC. quare
h 3 *ax.* A G *h* = G E *k* = B C *l* = A G. Q.E.F
k 15. *def.* Positio puncti A, intra vel extra datam B C,
l 1. *ax.* casus variat, sed ubique similis est constructio, & demonstratio.

Scholium.

Poterat A G circino sumi, sed hoc facere nulli postulato respondet, ut bene innuit Proclus.

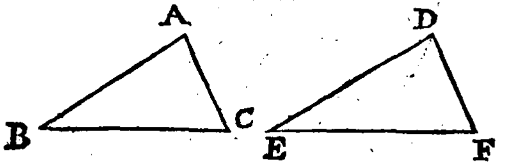
PROP. III.



Duabus datis rectis lineis A, *&* BC, *de majore* B C *minori* A *æqualem rectam lineam* B E *detrahere.*

a 2. 1. Ad punctum B *a* pone rectam B D = A. Circulus centro B, spatio B D descriptus au-
b 15. *def.* feret B E *b* = B D *c* = A *d* = B E. Q.E.F.
c *constr.*
d 1. *ax.*

PROP. IV.

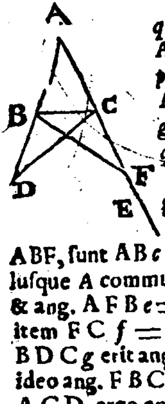


Si duo triangula B A C, E D F *duo latera* B A, A C *duobus lateribus* E D, D F *æqualia habeant, utrumque utrique* (hoc est B A = E D, *&* AC = D F) *habeant vero angulum* A, *angulo* D *æqualem,*

lem, sub æqualibus rectis lineis contentum, & basim BC basi EF æqualem habebunt; eritque triangulum BAC triangulo EDF æquale, ac reliqui anguli B, C reliquis angulis E, F æquales erunt, uterque utrique, sub quibus æqualia latera subtenduntur.

Si punctum D puncto A applicetur, & recta DE rectæ AB superponatur, cadet punctum E in B, quia DE *a* = AB. Item recta DF cadet in AC, quia ang. A *a* = D. Quinetiam punctum F puncto C coincidet, quia AC *a* = DF. Ergo rectæ EF, BC, cum eosdem habeant terminos, *b* congruent, & proinde æquales sunt. Quare triangula BAC, EDF; & anguli B, E; itemque anguli C, F etiam congruunt, & æquantur. Quod erat Demonstrandum. a *hyp.* b 14 *ax.*

PROP. V.



Isoscelium triangulorum ABC qui ad basim sunt anguli ABC, ACB inter se sunt æquales. Et productis æqualibus rectis lineis AB, AC qui sub base sunt anguli CBD, BCE inter se æquales erunt.

a Accipe AF = AD, & *b* junge CD, ac BF. a 3. 1. b 1 *post.*

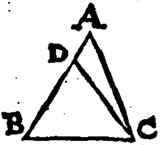
Quoniam in triangulis ACD, ABF, sunt AB *c* = AC, & AF *d* = AD, angulusque A communis, *e* erit ang. ABF = ACD; & ang. AFB *e* = ADC, & bas. BF *e* = DC; item FC *f* = DB. ergo in triangulis BFC, BDC *g* erit ang. FCB, = DBC. Q.E.D. Item ideo ang. FBC = DCB. atqui ang. ABF *h* = ACD. ergo ang. ABC *k* = ACB. Q.E.D. c *hyp.* d *constr.* e 4. 1. f 3 *ax.* g 4. 1. h *pr.* k 3. *ax.*

Corollarium.

Hinc, Omne triangulum æquilaterum est quoque æquiangulum.

PROP.

PROP. VI.



Si triangulis A B C *duo anguli* A B , A C B *æquales inter se fuerint, & sub æqualibus angulis subtensa latera* A B, A C *æqualia inter se erunt.*

Si fieri potest, sit utravis B A ⊏ C A, *a* Fac igitur B D = C A, & *b* duc C D.

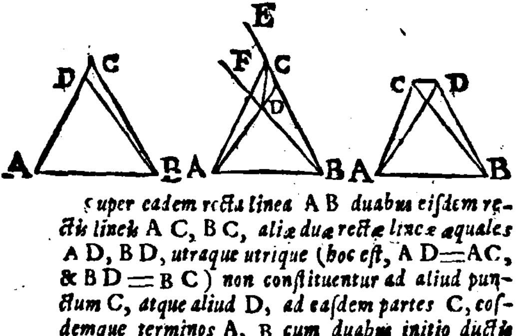
a 3. 1.
b 1. *post.*
c *suppos.*
d *hyp.*
e 4. 1.
f 9 *ax.*

In triangulis D B C, A C B, quia B D *c* = C A, & latus B C commune est; atque ang. D B C *d* = A C B, *e* erunt triangula D B C, A C B æqualia inter se, pars & totum, *f* Quod Fieri Nequit.

Coroll.

Hinc, Omne triangulum æquiangulum est quoque æquilaterum.

PROP VII.



Super eadem recta linea A B *duabus eisdem rectis lineis* A C, B C, *aliæ duæ rectæ lineæ æquales* A D, B D, *utraque utrique (hoc est,* A D = A C, & B D = B C*) non constituentur ad aliud punctum* C, *atque aliud* D, *ad easdem partes* C, *eosdemque terminos* A, B *cum duabus initio ductis rectis lineis habentes.*

a 9. *ax.* 1. *Cas.* Si punctum D statuatur in A C *a* liquet non esse A D = A C.

2. *Cas.* Si punctum D dicatur intra triangulum A C B duc C D, & produc B D F, ac B C E.

b 5. 1. Jam vis A D = A C, ergo ang. A D C *b* = A C D;

c *suppos.* item quia B D, *c* = B C, erit ang. F D C *b* = E C D

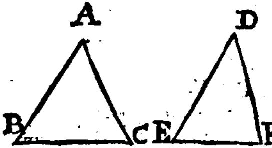
ergo

ergo ang. FDC *d* $\sqsubset$ ACD, id est ang. FDC $\sqsubset$ ADC *d* Q. F. N. d 9. *ax.*

3. *Cas.* Sin D cadat extra triangulum ACB jungatur CD.

Rursus, ang. ACD *e* = ADC, & BDC *e* = BDC *f* ergo ang. ACD $\sqsupset$ BDC, id est ang. ACD $\sqsupset$ BCD. Q. F. N. e 5. 1. f 9. *ax.*

PROP. VIII.



Si duo triangula ABC, DEF *habuerint duo latera* AB, AC *duobus lateribus* DE, DF, *utrumque utrique æqualia; habuerint vero & basim* BC, *basi* EF, *æqualem: angulum* A *sub æqualibus rectis lineis contentum angulo* D *æqualem habebunt.*

Quia BC *a* = EF, si basis BC superponatur basi EF, illæ *b* congruent. ergo, cum AB *c* = DE, & AC *c* = DF, cadet punctum A in D, (nam in aliud punctum cadere nequit, per præcedentem) *d* ergo angulorum A, & D latera coincidunt. *e* quare anguli illi pares sunt. Q. E. D. a *hyp.* b 8. *ax.* c *hyp.* d 14. *ax.* e 8. *ax.*

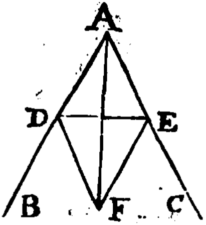
Coroll.

1. Hinc triangula sibi mutuo æquilatera, etiam mutuo *x* æquiangula sunt.

2. Triangula sibi mutuo æquilatera *y* æquentur inter se. x 4. 8. y 4. 1.

PROP.

PROP. IX.



Datum angulum rectilineum BAC *bifariam secare.*

a 3. 1. *a* Sume AD = AE;
b 1. 1. duc DE, ſuper qua *b* fac triang. æquilat. DFE.

Ducta AF angulum BAC biſecabit.

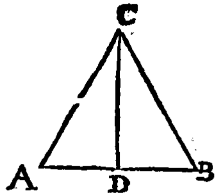
c conſtr. Nam AD *c* = AE, & latus AF commune eſt, & baſ. DF *c* = FE,
d 8. 1. *d* ergo ang. DAF = EAF. Q.E.F.

Coroll.

Hinc patet quomodo angulus ſecari poſſit in æquales partes 4, 8, 16, &c. Singulas nimirum partes iterum biſecando.

Methodus vero regula & circino angulos ſecandi in æquales quotcunque hactenus Geometras latuit.

PROP. X.



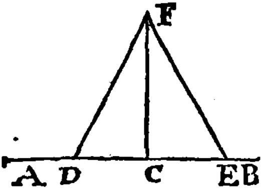
Datam rectam lineam AB *bifariam ſecare.*

a 1. 1. Super data AB *a* fac. triang. æquilat. ABC.
b 9. 1. ejus angulum C *b* biſeca recta CD. Eadem datam AB biſecabit.

c conſtr. Nam AC *c* = BC, & latus CD eſt commune; & ang. ACD *c* = BCD, *d* ergo AD = BD. Q.E.F. Praxin
d 4. 1. hujus & præcedentis, conſtructio primæ hujus libri ſatis indicat.

PROP.

PROP. XI.



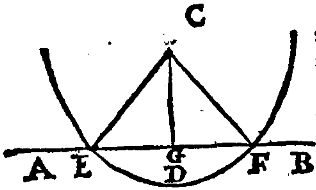
Data recta linea AB, & puncto in ea dato C, rectam lineam CF ad angulos rectos excitare.

a Accipe hinc inde CD = CE. Super DE *b* fac triang æquilat. DFE. Ducta FC perpendicularis est. a 3. 1. b 1. 1.

Nam triangula DFC, EFC sibi mutuo *c* æquilatera sunt. *d* ergo ang. DCF = ECF. *e* ergo FC perpendicularis est. Q. E. F. c *constr.* d 8. 1. e 10. *def.*

Praxis tam hujus, quam sequentis expeditur facillime ope normæ.

PROP. XII.

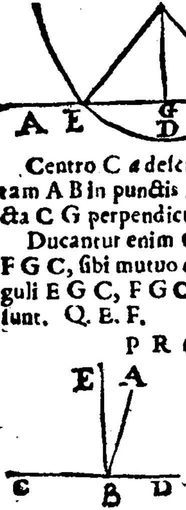


Super datam rectam lineam infinitam AB, à dato puncto C quod in ea non est perpendicularem rectam CG deducere.

Centro C *a* describe circulum, qui secet datam AB in punctis E & F *b* biseca EF in G. ducta CG perpendicularis est. a 3. *post.* b 10. 1.

Ducantur enim CE, CF. Triangula EGC, FGC, sibi mutuo *c* æquilatera sunt. *d* ergo anguli EGC, FGC, æquales, & *e* proinde recti sunt. Q. E. F. c *constr.* d 8. 1. e 10. *def.*

PROP. XIII.



Cum recta linea AB, super rectam lineam CD consistens, facit angulos ABC, ABD, aut duos rectos, aut duobus rectis æquales efficiet.

Si

a 10. *def.* Si anguli A B C, A B D pares sint *a* liquet
b 11. 1. illos rectos esse; sin inæquales sint, ex B *b* exci-
c 19 *ax.* tetur perpendicularis B E. Quoniam ang. A B C
d 3. *ax.* *c* = Rect. + A B E; & ang. A B D *d* = Rect.
e 2. *ax.* − A B E; erit A B C + A B D *e* = 2 Rect. +
A B E − A B E = 2 Rect. Q. E. D.

Coroll.

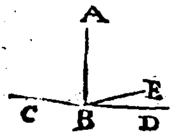
1. Hinc, si unus ang. A B D rectus sit, alter A B C etiam rectus erit; si hic acutus, ille obtusus erit, & contra.

2. Si plures rectæ quam una ad idem punctum eidem rectæ insistant, anguli fient duobus rectis æquales

3. Duæ rectæ invicem secantes efficiunt angulos quatuor rectis æquales.

4. Omnes anguli circa unum punctum constituti conficiunt quatuor rectos, patet ex Coroll. 2.

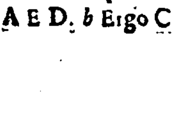
PROP. XIV.



Si ad aliquam rectam lineam A B, *atque ad ejus punctum* B *duæ rectæ lineæ* C B, B D *non ad easdem partes ductæ, eos qui sunt deinceps angulos* A B C, A B D *duobus rectis æquales fecerint, in directum erunt inter se ipsæ rectæ lineæ* C B, B D.

Si negas, faciant C B, B E unam rectam, ergo
a 13. 1. ang. A B C + A B E *a* = 2 Rect. *b* = A B C +
b *hyp.* A B D. *c* Quod est absurdum.
c 9. *ax.*

PROP. XV.

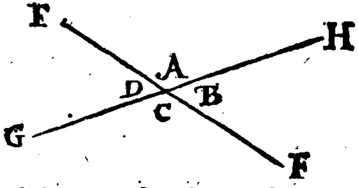


Si duæ rectæ lineæ A B, C D *se mutuo secuerint, angulos ad verticem* C E B, A E D *æquales inter se efficient.*

Nam ang. A E C + C E B
a 13. 1. *a* = 2 Rect. *a* = A E C +
b 3. *ax.* A E D. *b* Ergo C E B = A E D. Q. E. F.

Schol.

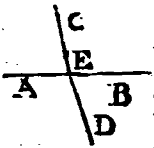
Schol.



Si ad aliquam rectam lineam G H, atque ad ejus punctum, A duæ rectæ lineæ E A, A F non ad eaſdem partes ſumptæ, angulos ad verticem D, & B æquales fecerint, ipſæ rectæ lineæ E A, A F in directum ſibi invicem erunt.

Nam 2 Rect. = *a* D + A *a* = B + A *b* ergo E A, A F ſunt in directum ſibi invicem. Q.E.D. a 13. 1. b 14. 1.

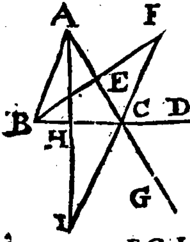
Schol. 2.



Si quatuor rectæ lineæ E A, E B, E C, E D ab uno puncto E exeuntes, angulos oppoſitos ad verticem æquales inter ſe fecerint, erunt quælibet duæ lineæ A E, E B, & C E, E D in directum poſitæ.

Nam quia ang. A E C + A E D + C E B + D E B *a* = 4 Rect. erit A E C + A E D (= *b* C E B + D E B) = 2 Rect. *c* ergo C E D, & A E B ſunt rectæ lineæ. Q. E. D. a 4. *Cor.* 13. 1. b*Hyp.* 2*ax.* c 14. 1.

PROP. XVI.



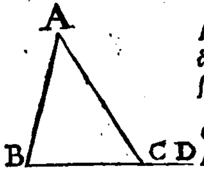
Cujuſcunque Trianguli A B C *uno latere* B C *producto, externus angulus* A C D *utrolibet interno & oppoſito* C A B, C B A, *major eſt.*

Latera A C, B C *a* biſecent rectæ A H, B E, è quibus productis *b* cape E F = B E, *b* & H I = A H, Conjuganturque F C, I C, & producatur A C G. a 10. 1. & 1. *poſt.* b 3. 1.

 Quo-

c *constr.* Quoniam CE *c* = EA, & EF *c* = EB, &
d 15. 1. ang. FEC *d* = BEA, *e* erit ang. ECF = EAB.
e 4. 1. Simili argumento ang. ICH = ABH. ergo
f 15. 1. totus ACD (*f* BCG) *g* major est utrovis CAB,
g 9. *ax.* & ABC. Q. E. D.

PROP. XVII.

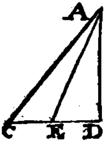


Cujuscunque trianguli ABC *duo anguli duobus rectis sunt minores, omnifariam sumpti.*

Producatur latus BC. Quoniam ang. ACD + ACB *a* = 2 Rect. & ang.
a 13. 1.
b 16. 1. ACD *b* ⊏ A, *c* erit A + ACB ⊐ 2 Rect. Eo-
c 4. *ax.* dem modo erit ang. B + ACB ⊐ 2 Rect. Denique producto latere AB, erit similiter ang. A + B ⊐ 2 Rect. Quæ E. D.

Coroll.

1. Hinc, in omni triangulo, cujus unus angulus fuerit rectus, vel obtusus, reliqui acuti sunt.

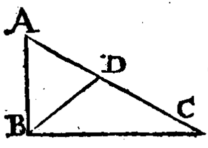


2. Si linea recta AE cum alia recta CD angulos inæquales faciat, unum AED acutum, & alterum AEC obtusum, linea perpendicularis AD ex quovis ejus puncto A ad aliam illam CD demissa, cadet ad partes anguli acuti AED.

Nam si AC ad partes anguli obtusi ducta, dicatur perpendicularis, in triangulo AEC erit
x 17. 1. ang. AEC + ACE ⊏ 2 Rect. *x* Q. E. N.

3. Omnes anguli trianguli æquilateri, & duo anguli trianguli Isoscelis, supra basim, acuti sunt

PROP. XVIII.



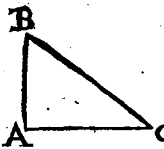
Omnis trianguli ABC *majus latus* AC *majorem angulum* ABC *subtendit.*

Ex AC *a* aufer AD =
a 3. 1.
b 5. 1. AB, & junge DB. *b* ergo ang. ADB = ABD. Sed

c ADB

c ADB ⊏ C. ergo ABD ⊏ C. *d* ergo totus c 16. 1.
ang. ABC ⊏ C. Eodem modo erit ABC ⊏ d 9. *ax.*
A. Q. E D.

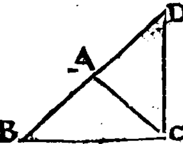
PROP. XIX.



Omnis trianguli ABC *major angulus* A *majori lateri* BC *subtenditur.*

Nam si dicatur AB = BC, *a* erit ang. A = C. contra Hypoth. & si AB ⊏ BC, a 5. 1.
b erit ang. C ⊏ A, contra hyp. quare potius, b 18. 1.
BC ⊏ AB. & eodem modo BC ⊏ AC.
Q. E. D.

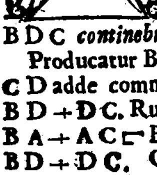
PROP. XX.



Omnis trianguli ABC *duo latera* BA, AC *reliquo* BC *sunt majora quomodocunque sumpta.*

Ex BA producta *a* cape CAD = AC, & duc DC. a 3. 1.
b ergo ang. D = ACD. *c* ergo totus BCD ⊏ b 5. 1.
D *d* ergo BD (*e* BA + AC) ⊏ BC. Q.E.D. c 9. *ax.*
d 19. 1.
e *constr.* & 2. *ax.*

PROP. XXI.

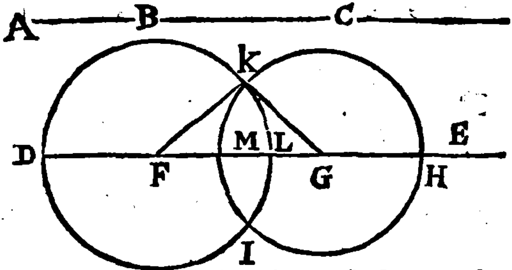


Si super trianguli ABC *uno latere* BC, *ab extremitatibus duæ rectæ lineæ* BD, CD, *interius constitutæ fuerint, hæ constitutæ reliquis trianguli duobus lateribus* BA, CA *minores quidem erunt, majorem vero angulum* BDC *continebunt.*

Producatur BD in E. estque CE + ED *a* ⊏ a 20. 1.
CD adde commune BD, *b* erit BE + EC ⊏ b 4. *ax.*
BD + DC. Rursus BA + AE *a* ⊏ BE; *b* ergo
BA + AC ⊏ BE + EC. quare BA + AC ⊏
BD + DC. Q. E. D. 2. Ang. BDC *c* ⊏ c 16. 1.
DEC *c* ⊏ A, ergo ang. BDC ⊏ A. Q. E. D.

 PROP.

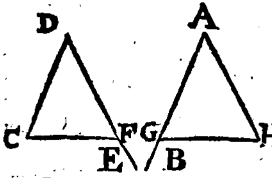
PROP. XXII.



Ex tribus rectis lineis F K, F G, G K, *quæ sint tribus datis rectis lineis* A, B, C, *æquales, triangulum* F K G *constituere. Oportet autem duas reliqua esse majores omnifariam sumptas; quoniam uniuscujusque trianguli duo latera omnifariam sumpta reliquo sunt majora.*

a 3. 1. Et infinita D E *a* sume D F, F G, G H datis
b 3. *post.* A, B, C ordine æquales. Tum si *b* centris F, & G, intervallis F D, & G H ducantur circuli se intersecantes in K; junctis rectis K F, K G con-
c 15. *def.* stituetur triangulum F K G, *c* cujus latera F K,
d 1. *ax.* F G, G K tribus D F, F G, G H, *d* id est tribus datis A, B, C æquantur. Q. E. F.

PROP. XXIII.

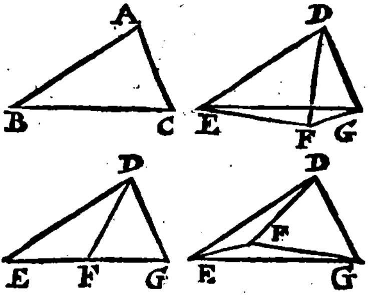


Ad datam rectam lineam A B, *datumque in ea punctum* A, *dato angulo rectilineo* D *æqualem angulum rectilineum* A *constituere.*

a 1. *post.* *a* Duc rectam C F secantem dati anguli latera
b 3. 1. utcunque, *b* Fac A G = C D. Super A G
c 22. 1. *c* constitute triangulum alteri C D F æquilaterum, ita ut A H = D F, & G H = G F; & ha-
d 8. 1. bebis ang. A *d* = D. Q. E. F.

PROP.

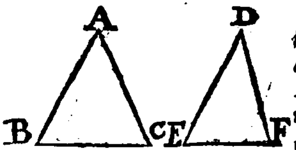
PROP. XXIV.



Si duo triangula ABC, DEF *duo latera* AB, AC *duobus lateribus* DE, DF *æqualia habuerint, utrumque utrique; angulum vero* A *angulo* EDF *majorem sub æqualibus rectis lineis contentum, & basim* BC, *basi* EF, *majorem habebunt.*

a Fiat ang. EDG = A, & DG *b* = DF *c* = a 23. 1.
AC, connectanturque EG, FG. b 3. 1.
1. *Cas.* Si EG cadit supra EF. Quia AB c *hyp.*
d = DE, & AC = *e* DG, & ang. A *e* = EDG, d *hyp.*
f erit BC = EG. Quia vero DF *e* = DG, e *constr.*
g erit ang. DFG = DGF. *h* ergo ang. DFG ⊏ f 4. 1.
EGF; *h* & proinde ang. EFG ⊏ EGF. *k* quare g 5. 1.
EG (BC) ⊏ EF. Q. E. D. h 9. *ax.*
2. *Cas.* Si basis EF basi EG coincidat, *l* li- k 19. 1.
quet EG (BC) ⊏ EF. l 9. *ax.*
3. Sin EG cadat infra EF. Quoniam DG
+ GE *m* ⊏ DF + FE, si hinc inde auferantur m 21. 1.
DG, DF, æquales, manet EG (BC) *n* ⊏ n 5. *ax.*
EF. Q. E. D.

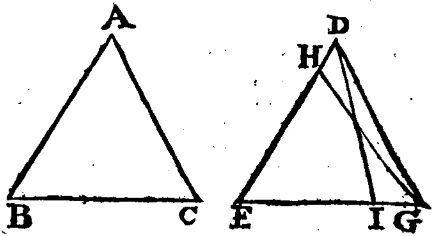
PROP. XXV.



Si duo triangula ABC, DEF *duo latera* AB, AC *duobus lateribus* DE, DF *æqualia habuerint, utrumque utrique, basim vero* BC *basi* EF *majorem; & angulum* A *sub æqualibus rectis lineis contentum angulo* D *majorem habebunt.*

a 4. 1. Nam si dicatur ang. A $=$ D, *a* erit basis BC $=$ EF, contra Hyp. Sin dicatur ang. A $\sqsupset$ D.
b 24. 1. *b* erit BC $\sqsupset$ EF, etiam contra Hyp. ergo BC $\sqsubset$ EF. Q. E. D.

PROP. XXVI.



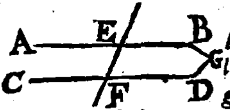
Si duo triangula BAC, EDG, *duos angulos* B, C, *duobus angulis* E, DGE, *æquales habuerint, utrumque utrique, unumque latus uni lateri æquale, sive quod æqualibus adjacet angulis, seu quod uni æqualium angulorum subtenditur: reliqua latera reliquis lateribus æqualia, utrumque utrique, & reliquum angulum reliquo angulo æqualem habebunt.*

1. *Hyp.* Sit BC $=$ EG. Dico BA $=$ ED, & AC $=$ DG, & ang. A $=$ EDG. Nam si dicatur
a 3. 1. ED $\sqsubset$ BA, *a* fiat EH $=$ BA, ducaturq; GH.

Quoniam

Quoniam A B *b* = H E, & B C *c* = E G, & b *suppos.*
ang. B *c* = E, erit ang. E G H *d* = C *e* = D G E. c *hyp.*
f Q. E. A. ergo A B = E D. Eodem modo A C d 4. 1.
= D G. *d* quare etiam ang. A = E D G. e *hyp.*
2. *Hyp.* Sit A B = E D. Dico B C = E G, & f 9. *ax.*
A C = D G & ang. A = E D G. Nam si dicatur
E G $\sqsubset$ B C, fiat E I = B C, & connectatur D I.
Quia A B *g* = E D, & B C *h* = E I, & ang. B g *hyp.*
g = E, erit ang. E I D *k* = C *m* = E G D. *n* Q. h *suppos.*
E. A. ergo B C = E G. ergo ut prius, A C = k 4. 1.
D G, & ang. A = E D G. Q. E. D. m *hyp.*
n 16. 1.

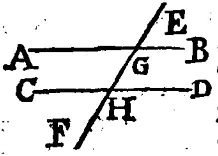
PROP. XXVII.



Si in duas rectas lineas A B, C D *recta incidens linea* E F *alternatim angulos* A E F, D F E, *aequales inter se fecerit, parallelae erunt inter se illae rectae lineae* A B, C D.

Si A B, C D dicantur non esse parallelæ; conveniant productæ, nempe in G. quo posito angulus externus A E F interno D F E *a* major a 16. 1.
erit, cui tamen ponitur æqualis. Quæ repugnant.

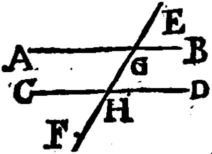
PROP. XXVIII.



Si in duas rectas lineas A B, C D *recta incidens linea* E F *externum angulum* A G E *interno & opposito, & ad easdem partes* C H G *aequalem fecerit, aut internos & ad easdem partes* A G H, C H G *duobus rectis aequales; parallelae erunt inter se ipsae rectae lineae* A B, C D.

1. *Hyp.* Quia per hyp. ang A G E = C H G,
a erit altern. B G H = C H G. *b* parallelæ igi- a 15. 1.
tur sunt A B, C D. Q. E. D. b 17. 1.
2. *Hyp.* Quia ex hyp. Ang. A G H + C H G = a 13. 1.
2 Rect. *a* = A G H + B G H, *b* erit C H G = b 3. *ax.*
B G H. Ergo *c* A B, C D parallelæ sunt. Q. E. D. c 17. 1.

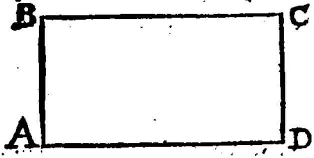
PROP. XXIX.



In parallelas rectas lineas AB, CD, *recta incidens linea* EF, *& alternatim angulos* DHG, AGH *æquales inter se efficit; & externum* BGE *interno, & opposito, & ad easdem partes* DHE *æqualem; & internos & ad easdem partes* AGH, CHG *duobus rectis æquales facit.*

a 13. ax. Liquet AGH, + CHG = 2 Rect. *a* alias AB, CD non essent parallelæ, contra hyp. Sed
b 13. 1. & ang. DHG + CHG *b* = 2 Rect. ergo DHG
c 13. ax. *c* = AGH *d* = BGE. Q. E. D.
d 15 1.

Coroll.



Hinc omne Parallelogrammum AC habens unum angulum rectum
a 29 1. A, est rectangulum.
b 3. ax. Nam A + B *a* = 2 Rect. ergo cum A rectus sit, *b* etiam B rectus erit. Eodem argumento D, & C recti sunt.

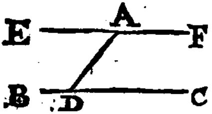
PROP. XXX.

Quæ (AB, CD) *eidem rectæ lineæ* EF *parallelæ, & inter se sunt parallelæ.*

Tres rectas secet utcunque recta GI. Quoniam AB, EF parallelæ sunt, *a* erit ang. AGI
a 29. 1. = EHI, Item propter CD, EF parallelas,
b 1. ax. *a* erit ang. EHI = DIG. *b* ergo ang. AGI =
c 27. 1. DIG. *c* quare AB, CD parallelæ sunt. Q. E. D.

PROP.

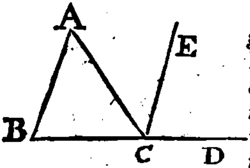
PROP. XXXI.



A dato puncto A datæ rectæ lineæ B C ducere parallelam rectam lineam A E.

Ex A ad datam B C duc rectam utcunque A D. ad quam, ejusque punctum A *a* fac ang. D A E = A D C. *b* erunt a 23. 1.
A E, B C parallelæ. Q. E. F. b 27. 1.

PROP. XXXII.



Cujuscunque trianguli A B C *uno latere* B C *producto, externus angulus* A C D *duobus internis, & oppositis,* A, B *est æqualis. Et trianguli tres interni anguli,* A, B, A C B *duobus sunt rectis æquales.*

Per C *a* duc C E parall. B A. Ang. A *b* = a 31. 1.
A C E. & ang. B *b* = E C D. ergo A + B *c* = b 29. 1.
A C E + E C D *d* = A C D. Q. E. D. Porro c 2. *ax.*
A C D + A C B *e* = 2 Rect. *f* ergo A + B + d 19. *ax.*
A C B = 2 Rect. Q. E. D. e 13. 1.

Corollaria. f 1. *ax.*

1. Tres simul anguli cujusvis trianguli æquales sunt tribus simul cujuscunque alterius. Unde

2. Si in uno triangulo duo anguli (aut singuli, aut simul) æquales sint duobus angulis (aut singulis, aut simul) in altero triangulo, etiam reliquus reliquo æqualis est. Item, si duo triangula unum angulum uni æqualem habeant, reliquorum summæ æquantur.

3. In triangulo si unus angulus rectus sit, reliqui unum rectum conficiunt. Item, angulus, qui duobus reliquis æquatur, rectus est.

4. Cum in Isoscele angulus æquis cruribus contentus rectus est, reliqui ad basim sunt semirecti.

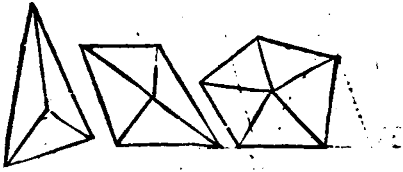
5. Tri-

5. Trianguli æquilateri angulus facit duas tertias unius recti, nam $\frac{1}{3}$ 2 Rect. $=$ $\frac{2}{3}$ Rect.

Schol.

Hujus propositionis beneficio, cujuslibet figuræ rectilineæ tam interni quam externi anguli quot rectos conficiant, innotescet per duo sequentia theoremata.

THEOREMA I.



Omnes simul anguli cujuscunque figuræ rectilineæ conficiunt bis tot rectos demptis quatuor, quot sunt latera figuræ.

Ex quovis puncto intra figuram ducantur ad omnes figuræ angulos rectæ, quæ figuram resolvent in tot triangula quot habet latera. Quare cum singula triangula conficiant duos rectos, omnia simul conficient bis tot rectos, quot sunt latera. Sed anguli circa dictum punctum conficiunt quatuor rectos Ergo, si ab omnium triangulorum angulis demas angulos circa id punctum, anguli reliqui qui componunt angulos figuræ conficient bis tot rect s demptis quatuor, quot sunt latera figuræ. Q. E. D.

Hinc *Coroll.* Omnes ejusdem speciei rectilineæ figuræ æquales habent angulorum summas.

THEOREMA 2.

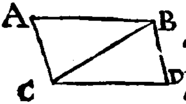
Omnes simul externi anguli cujuscunque figuræ rectilineæ conficiunt quatuor rectos.

Nam singuli figuræ interni anguli cum singulis externis conficiunt duos rectos. Ergo interni

terni simul omnes, cum omnibus simul externis conficiunt bis tot rectos, quot sunt latera figuræ.) Sed (ut modo ostensum est,) interni simul omnes etiam cum quatuor rectis efficiunt bis tot rectos quot sunt latera figuræ. Ergo externi anguli quatuor rectis æquantur. Q. E. D.

Coroll. Omnes cujuscunque speciei rectilineæ figuræ æquales habent externorum angulorum summas.

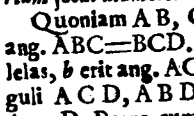
PROP. XXXIII.



Rectæ lineæ A C, B D, *quæ æquales & parallelas lineas* A B, C D, *ad partes easdem conjungunt, & ipsæ æquales ac parallelæ sunt.*

Connectatur C B. Quoniam ob A B, C D parallelas. ang. A B C *a* = BCD, & per hyp. AB a 29. 1.
= C D, & latus C B commune est, *b* erit A C b 4. 1.
= B D, *b* & ang. A C B = D B C. *c* ergo AC, c 27. 1.
B D etiam parallelæ sunt. Q. E. D.

PROP. XXXIV.



Parallelogrammorum spatiorum ABDC *æqualia sunt inter se quæ ex adverso latera* A B, C D; ac A C, B D; *angulique* A, D, & A B D, A C D; *& illa bifariam secat diameter* C B.

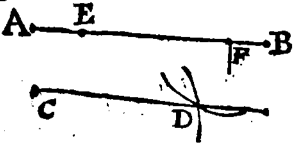
Quoniam A B, C D *a* parallelæ sunt, *b* erit a hyp.
ang. ABC = BCD. Item ob AC, DB *a* paral- b 29. 1.
lelas, *b* erit ang. ACB = CBD. *c* ergo toti an- c 2. ax.
guli A C D, A B D æquantur. Similiter ang. A = D. Porro, cum communi lateri C B adjaceant anguli A B C, A C B, ipsis B C D, C B D
pares, *d* erunt AC = BD, *d* & AB = CD. adeo- d 26. 1.
que etiam triang. A B C = C B D. Q. E. D.

SCHOL.

SCHOL.

Omne quadrilaterum A B DC *habere latera opposita æqualia, est parallelogrammum.*

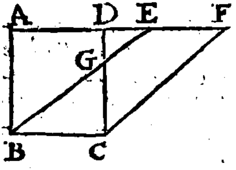
a 27. 1. Nam per 8. 1. ang. A B C = B C D. *a* ergo A B, C D parallelæ sunt. Eadem ratione ang. B C A = C B D; *a* quare A C, B D etiam pa-
b 35. def. 1 rallelæ sunt. *b* Ergo A B D C est parallelogrammum. Q. E. D.



Hinc expeditius per datum punctum C datæ rectæ A B ducetur parallela C D.

Sume in A B quodvis punctum E. centris E. & C ad quodvis intervallum duc æquales circulos E F, C D. centro vero F, spatio E C duc circulum F D, qui priorem C D secet in D. Erit ducta C D parall. A B. Nam ut modo demonstratum est, C E F D est parallelogrammum.

PROP. XXXV.

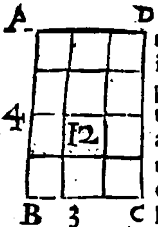


Parallelogramma B C D A, B C F E *super eadem basi* B C, *&* *in eisdem parallelis* AF, B C *constituta, inter se sunt æqualia.*

a 34. 1. Nam A D *a* = BC
b 2. ax. *a* = E F. adde communem D E, *b* erit A E =
c 29. 1. D F. Sed & A B *a* = D C & ang. A *c* = C D F.
d 4. 1. *d* ergo triang. A B E = D C F. aufer commune
e 3. ax. D G E, *e* erit Trapez. A B G D = E G C F.
f 2. ax. adde commune B G C, *f* erit Pgr. A B C D = E B C F, Q. E. D. Reliquorum casuum non dissimilis, sed simplicior & facilior est demonstratio.

Scho-

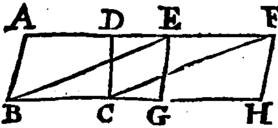
Scholium.



Si latus A B parallelogrammi rectanguli A B C D ferri intelligatur perpendiculariter per totam B C, aut B C per totam A B, producetur eo motu area rectanguli A B C D. Hinc rectangulum fieri dicitur ex ductu seu multiplicatione duorum laterum contiguorum. Sit exempl. gr. B C pedum 3, A B 4. Duc 3. in 4; proveniunt 12. pedes quadrati pro area rectanguli.

Hoc supposito, ex hoc theoremate cujuscunq; parallelogrammi (* E B C F) habetur dimensio. Illius enim area producitur ex altitudine B A ducta in basim B C. Nam area rectanguli A C parallelogrammo E B C F æqualis, fit ex B A in B C, ergo, &c. * *v. fig. propos.* 35.

PROP. XXXVI.

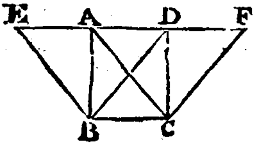


Parallelogramma B C D A, G H F E *super æqualibus basibus* B C G H, *& in eisdem parallelis* A F, B H *constituta, inter se sunt æqualia.*

Ducantur BE, CF. Quia B C *a* = G H *b* = EF, *c* erit BCFE parallelogrammum. ergo Pgr. B C D A *d* = B C F E *d* = G H F E. Q.E.D.

a *hyp.*
b 34. 1.
c 33. 1.
d 35. 1.

PROP. XXXVII.

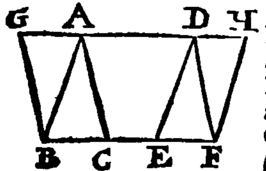


Triangula B C A, B C D *super eadem basi* B C *constituta, & in eisdem parallelis* B C, E F *inter se sunt æqualia.*

a Duc

a 31. 1. *a* Duc BE parall. CA, *a* & CF parall, BD.
b 34. 1. Erit triang. BCA *b* $=\frac{1}{2}$ Pgr. BCAE $=$ *c* $\frac{1}{2}$
c 35. 1. & 7. ax. BDFC *b* $=$ BCD. Q.E.D.

PROP. XXXVIII.



Triangula BCA, EFD *super æqualibus basibus* BC, EF *constitutæ, & in eisdem parallelis* GH, BF, *inter se sunt æqualia.*

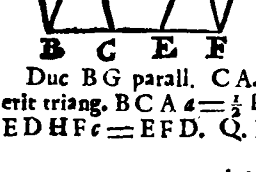
Duc BG parall. CA. & FH parall. ED: erit triang. BCA *a* $=\frac{1}{2}$ Pgr. BCAG *b* $=\frac{1}{2}$ EDHF *c* $=$ EFD. Q.E.D.

a 14. 1.
b 36. 1 & 7. ax.
c 34. 1.

Schol.

Si basis BC $\sqsubset$ EF, liquet triang. BAC $\sqsubset$ EDF. & si BC $\sqsupset$ EF, erit BAC $\sqsupset$ EDF.

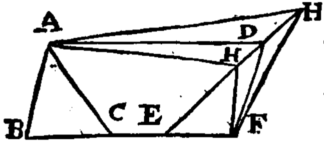
PROP. XXXIX.



Triangula æqualia BCA, BCD, *super eadem basi* BC, *& ad easdem partes constituta, etiam in eisdem sunt parallelis* AD, BC.

a 37. 1. Si negas, sit altera AF parall. BC; & ducatur CF. ergo triang. CBF *a* $=$ CBA *b* $=$ CBD
b *hyp.* *c* Q.E.A.
c 9. ax.

PROP. XL.



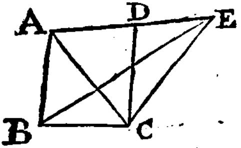
Triangula æqualia BCA, EFD *super æqualibus basibus* BC, EF, *& ad easdem partes constituta, & in eisdem sunt parallelis* AD. BF.

Si negas, sit altera AH parall. BF. & ducatur FH. ergo triang. EFH *a* = BCA *b* = EFD. *c* Q. E. A. a 38. 1.

b *hyp.*

c 9. *ax.*

PROP. XLI.



Si parallelogrammum ABCD *cum triangulo* BCE *eandem basim* BC *habuerit, in eisdemque fuerit parallelis* AE, BC, *duplum erit parallelogrammum* ABCD *ipsius trianguli* BCE.

Ducatur AC. Triang. BCA *a* = BCE. ergo Pgr. ABCD *b* = 2BCA *c* = 2BCE. Q. E. D. a 37. 1.

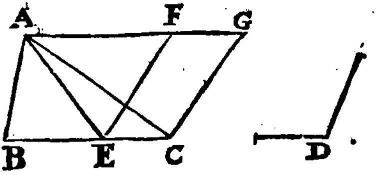
b 34. 1.

c 6. *ax.*

Scholium.

Hinc habetur area cujuscunque trianguli BCE. Nam cum area parallelogrammi ABCD producatur ex altitudine in basim ducta; producetur area trianguli ex dimidia altitudine in basim ducta, vel ex dimidia basi in altitudinem, ut si basis BC sit 8, & altitudo 7; erit trianguli BCE area, 28.

PROP. XLII.

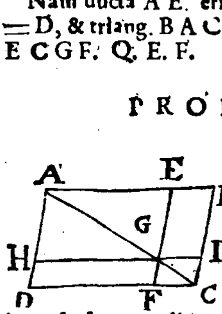


Dato triangulo A B C *æquale parallelogrammum* E C G F *constituere in dato angulo rectilineo* D.

a 31. 1. Per A *a* duc A G parall. B C. *b* fac ang. B C G
b 23. 1. = D. basim B C *c* biseca in E. *a* duc E F parall.
c 10. 1. C G. Dico factum.

Nam ducta A E. erit ex constr. ang. E C G
d 38. 1. = D, & triang. B A C *d* = 2 A E C *e* = Pgr.
e 41. 1. E C G F. Q. E. F.

PROP. XLIII.

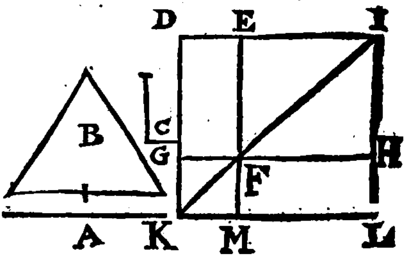


In omni parallelogrammo A B C D *complementa* D G, G B *eorum quæ circa diametrum* A C *sunt parallelogrammorum* H E, F I *inter se sunt æqualia.*

a 34. 1. Nam Triang. A C D, = *a* A C B, & triang. A G H *a* = A G E. & triang. G C F *a* = G C I.
b 3. ax. *b* ergo Pgr. D G = G B. Q. E. D.

PROP.

PROP. XLIV.



Ad datam rectam lineam A, *dato triangulo* B, *æquale parallelogrammum* F L *applicare in dato angulo rectilineo* C.

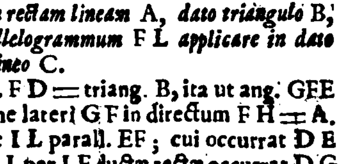
a Fac Pgr. F D = triang. B, ita ut ang. GFE = C. & pone lateri G F in directum F H = A. Per H *b* duc I L parall. EF; cui occurrat D E producta ad I. per I F ductæ rectæ occurrat D G protracta ad K. Per K *b* duc K L parall. G H; cui occurrant E F, & I H prolongatæ ad M, & L. Erit F L. Pgr. quæsitum.

a 42. 1.
b 31. 1.

Nam Pgr. F L *c* = F D = B *d* & ang. M F H = G F E = C. Q. E. F.

c 43. 1.
d 15. 1.

PROP. XLV.



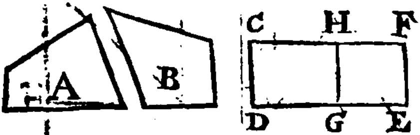
Ad datam rectam lineam F G *dato rectilineo* ABCD *æquale parallelogrammum* F L *constituere, in dato angulo rectilineo* E.

Datum rectilineum resolve in triangula B A D, B C D. *a* Fac. Pgr. F H = BAD ita ut ang. F = E. producta F I, *a* fac (ad H I) Pgr. I L

a 44. 1.

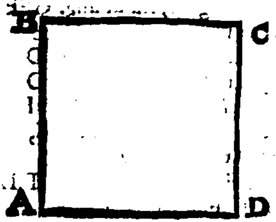
b 19.ax. c Constr. IL = BCD: erit Pgr. FL = *b* FH + IL *c* = ABCD. Q. E. F.

Schol.



Hinc facile invenitur excessus HE, quo rectilineum aliquod A superat rectilineum minus B; nimirum si ad quamvis rectam CD applicentur Pgr. DF = A. & DH = B.

PROP. XLVI.



A data recta linea AD *quadratum* AC *describere.*

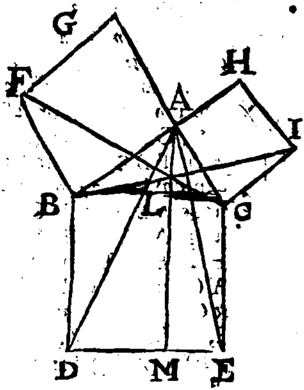
a 11. 1. b 3. 1. *a* Erige duas perpendiculares AB, DC *b* æquales datæ AD; & junge BC. dico factum.

c constr. d 28. 1. e constr. f 33. 1. g Sch. 29. 1 h 29. def. Cum enim ang. A + D *c* = 2 Rect. *d* erunt AB, DC parallelæ. Sunt vero etiam *e* æquales, *f* ergo AD, BC pares etiam sunt, & parallelæ. ergo figura AC est parallelogramma, & æquilatera. Anguli quoque omnes recti sunt, *g* quoniam unus A est rectus, *h* ergo AC est quadratum. Q. E. F.

Eodem modo facile describes rectangulum, quod sub datis duabus rectis continetur.

PROP.

PROP. XLVII.



In rectangulis triangulis BAC quadratum BE, quod à latere BC rectum angulum BAC subtendente describitur, æquale est eis, BG, CH, quæ à lateribus AB, AC rectum angulum continentibus describuntur.

Junge AE; AD; & duc AM parall. CE.

Quoniam ang. DBC *a* = FBA, adde communem ABC; erit ang. ABD = FBC. Sed & AB *b* = FB, & BD *b* = BC. *c* ergo triang. ABD = FBC. atqui Pgr. BM. *d* = 2 ABD; & Pgr. BG *d* = 2 FBC (nam GAC est una recta per hyp. & 14. 1.) *e* ergo Pgr. BM = BG. Similì discursu Pgr. CM = CH. Totum igitur BE = *f* BG + CH. Q. E. D.

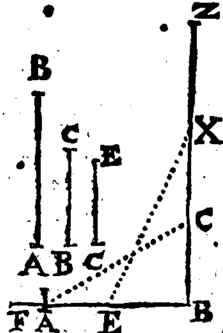
a 12. *ax.*
b 29. *def.*
c 4. 1.
d 41. 1.
e 6. *ax.*
f 2. *ax.*

Schol.

Hoc nobilissimum, & utilissimum theorema ab inventore Pythagora, Pythagoricum dici meruit. Ejus beneficio quadratorum additio, & substractio perficitur; quò spectant duo sequentia problemata.

PROBL. I.

And. Tacq.



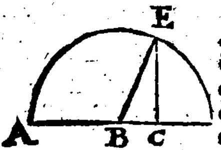
Datis quotcunque quadratis, unum omnibus aequale construere.

a 11. 1. Dentur quadrata tria, quorum latera sint AB, BC, CE. *a* Fac ang. rectum FBZ infinita habentem latera, in eaque transfer BA, & BC, &

b 47. 1. junge AC, *b* erit ACq = ABq + BCq. Tum AC transfer ex B in X; & CE tertium latus datum transfer ex B in E, & junge EX, *b* erit

c 2. ax. EXq = EBq (CEq) + BXq (ACq) *c* = CEq + ABq + BCq. Q. E. F.

PROBL. 2.



Datis duabus rectis inaequalibus AB, BC, *exhibere quadratum, quo quadratum majoris* AB *excedit quadratum minoris* BC.

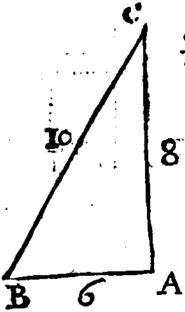
Centro B intervallo BA describe circulum. ex C erige perpendicularem CE occurrentem pe-

a 47. 1. ripheriae in E. & ducatur BE. *a* Erit BEq

b 3. ax. (BAq) = BCq + CEq. *b* ergo BAq − BCq = CEq. Q. E. F.

PROBL.

PROBL. 3.

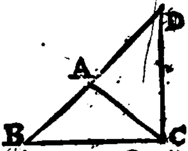


Notis duobus quibuscunque lateribus trigoni rectanguli ABC, *reliquum invenire.*

Latera rectum angulum ambientia sint AC, AB, hoc 6. pedum, illud 8, ergo 47. 1.
cum $ACq + ABq = 64 + 36 = 100 = BCq$. erit $BC = \sqrt{100} = 10$.

Nota sint deinde latera AB, BC, hoc 10. pedum, illud 6. ergo cum $BCq -$
$ABq = 100 - 36 = 64$ 47. 1.
$= ACq$. erit $ACq = \sqrt{64} = 8$.

PROP. XLVIII.



Si quadratum quod ab uno latere BC *trianguli describitur, æquale sit eis quæ à reliquis trianguli lateribus* AB, AC *describuntur quadratis, angulus* BAC *comprehensus sub* AB, AC *reliquis duobus trianguli lateribus, rectus est.*

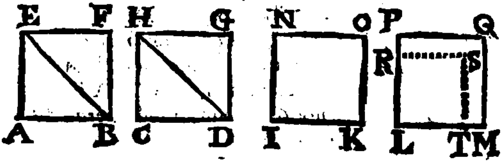
Duc ad AC perpendicularem $DA = AB$, & junge CD.

Jam $CDq \overset{a}{=} ADq + ACq = ABq +$ a 47. 1.
$ACq = BCq$. * ergo $CD = BC$. ergo trian- * *Vide seq.*
gula CAB, CAD, sibi mutuo æquilatera sunt; *Theor.*
quare ang. CAB $\overset{b}{=}$ CAD $\overset{c}{=}$ Rect. Q.E.D. b 8. 1.
c *Hyp.*

Schol.

Assumpsimus exinde quod $CDq. = BCq$, sequi $CD = BC$. Hoc vero manifestum fiet ex sequenti theoremate.

THEOREMA.



Linearum æqualium AB, CD, *æqualia sunt quadrata* AF, CG; & *quadratorum æqualium* NK, PM *æqualia sunt latera* IK, LM.

Pro 1 Hyp. Duc diametros EB, HD. Li-
a 34. 1. quet AF = *a* 2 triang. EAB = *b* 2 triang.
b 4. 1. & HCD = *a* CG. Q.E.D.
6. ax. 2. Hyp. Si fieri potest, sit LM ⊏ IK. fac
a 46. 1. LT = IK; *a* sitque LS = LTq. ergo LS
b 1. part. *b* = NK *c* = LQ, *d* Q.E.A. ergo LM = IK.
c hyp.
d 9. ax.

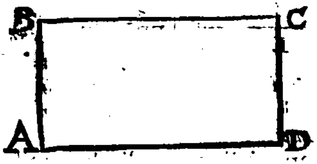
Coroll.

Eodem modo quælibet rectangula inter se æquilatera æqualia ostendentur.

LIB.

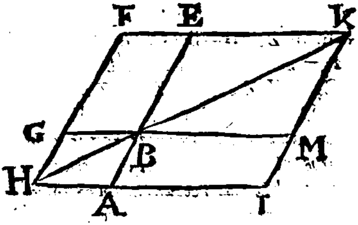
LIB. II.

Definitiones.



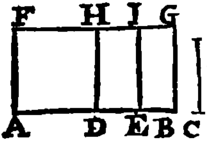
I. Omne parallelogrammum rectangulum ABCD contineri dicitur sub rectis duabus AB, AD, quæ rectum comprehendunt angulum.

Quando igitur dicitur rectangulum sub B A; A D, *vel brevitatis causa;* *rectangulum* B A D, *vel* BA × AD, (*vel* ZA *pro* Z × A;) *designatur rectangulum quod continetur sub* B A, & A D *ad rectum angulum constitutis.*



II. In omni parallelogrammo spatio FHIK unumquodq; eorum, quæ circa diametrum illius sunt, parallelogrammorum, cum duobus complementis Gnomon vocetur, *ut Pgr.* FB+BI+GA (EHM) *est Gnomon, item* Pgr. FB+BI+EM (GKA) *est Gnomon.*

PROP. I.



Si fuerint duæ rectæ lineæ AB, AF, *seceturque ipsarum altera* AB *in quotcunque segmenta* AD, DE, EB: *rectangulum comprehensum sub illis duabus rectis lineis* AB, AF, *æquale est eis, quæ sub insecta* AF, *& quolibet segmentorum* AD, DE, EB *comprehenduntur rectangulis.*

a 11. 1. *a* Statue AF, perpendicularem ad AB. *a* per F duc infinitam FG perpendicularem ad AF. *a* Ex D, E, B erige perpendiculares DH, EI, BG. erit AG rectangulum sub AF, AB, &
b 19.ax.1. *b* est æquale rectangulis AH, DI, EG, hoc est
c 34. 1. (quia DH, EI, AF *c* pares sunt) rectangulis sub AF, AD; sub AF, DE; sub AF, EB. Q. E. D.

Schol.

Imo si fuerint duæ rectæ, secenturque ambæ in quotcunque partes, idem provenit ex ductu totius in totum, & partium in partes.

Nam sit $Z = A + B + C$, & $Y = D + E$; quia
a 1. 2. DZ *a* $= DA + DB + DC$, & EZ *a* $= EA + EB$
b 2. ax. $+ EC$, & YZ *a* $= DZ + EZ$, *b* erit $ZY = DA + DB + DC + EA + EB + EC$. Q.E.D.

Hinc patet ratio ducendi rectas compositas in compositas. Nam omnia partium rectangula accipere oportet, & habetur rectangulum ex totis.

Sin linearum in se ducendarum signis + admisceantur signa —, etiam signorum ratio habenda est. Quippe ex + in — provenit —; at ex — in — provenit +. Nam sit + A ducenda in B — C. & quoniam + A non affirmatur de toto B, sed de ejus parte tantum, qua superat C, debet AC manere negata, quare prodibit AB — AC. Vel sic;

quia

quia B constat partibus C, & B—C, * erit AB * 1. 2.
=AC+A in B—C; aufer utrinque AC, erit AB
—AC=A in B—C. Similiter si —A ducenda
sit in B—C, quoniam ex vi signi — non negatur A de toto B, sed de ejus solummodo excessu supra C, debet AC manere affirmata. proveniet ergo—AB+AC. Vel sic; quia AB*=AC+A in B—C; tolle utrinque omnia, erit—AB=AC—A in B+C; adde AC utrinque, eritque —AB+AC=A in B—C.

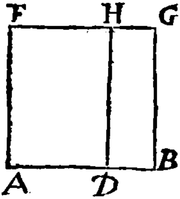
Atque ex his rite perspectis, quæ subsequuntur 9. propositiones, aliæque ejusmodi innumeræ, ex linearum in se ductarum comparatione emergentes (quas apud Vietam, & alios Analystas in numerato habes) nullo negotio demonstrantur, rem plerumque quasi ad simplicem calculum exigendo.

Porro, * liquet productum ex quapiam magnitudine in numeri cujuslibet partes, æquari producto ex eadem in totum numerum. Ut 5 A+7 A=12 A. & 4 A in 5 A+4A in 7 A=4 A in 12 A: quare quæ in hoc loco de rectarum in se ductu dicta sunt, eadem de numerorum in se multiplicatione intelligi possunt. proinde etiam quæ in 9. sequentibus theorematis de lineis affirmantur, eadem valent de numeris accepta; quippe cum istæ omnes ab hac prima immediate dependeant & deducantur. * 19. ax.

Propositiones decem primæ hujus libri valent etiam in numeris. Reliquas quilibet tyro examinet. pro hac, sit A F *6*, & A B 12, sectus in A D 5, D E 3, & E B 4. Estque 6×12 (A G) =72. 6×5 (A H) = 30. *6* in 3 (D I) = 18. denique 6×4 (E G) = 24. Liquet vero 30+18+24=72.

PROP.

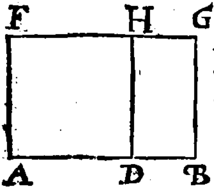
PROP. II.



Si recta linea AB *secta sit utcunque in* D, *rectangula quæ sub tota* AB *& quolibet segmentorum* AD, DB *comprehenduntur, æqualia sunt ei quod à tota* AB *fit quadrato.*

Erige AF perpendicularem & æqualem AB, & erunt
a 1. 2. *a* $AF \times AD + AF \times DB = AF \times AB$; hoc est (ob $AF = AB$) $AB \times AD + AB \times DB = ABq$.

PROP. III.



Si recta linea AB *secta sit utcunque in* D, *rectangulum sub tota* AB *& uno segmentorum* AD *comprehensum, æquale est illi quod sub segmentis* AD, DB *comprehenditur rectangulo, & illi quod à prædicto segmento* AD *describitur quadrato:*

Nam erige AF perpendicularem & æqualem DB, & completis parallelogrammis FD, FB,
a 1. 2. erit $AB \times AF = {}^a AF \times DB + AF \times AD$, hoc est (ob $AF = AD$) $AB \times AD = AD \times DB + ADq$.

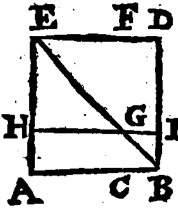
PROP. IV.

Si recta AB *secta sit utcunque in* D, *quadratum quod à tota* AB *describitur, æquale est illis quæ à segmentis* AD, DB *describuntur quadratis, & ei quod bis sub segmentis* AD, DB *comprehenditur rectangulo.*

a 2. 2. Nam $ABq = {}^a AB \times AD + AB \times DB$. Cùm
b 3. 2. ergo *b* $AB \times AD = AD \times DB + ADq$ & *b* $AB \times DB$

$= AD$

$=ADxDB+DBq$, erit *c* $ABq=ADq+DBq$ $+2\,ADxDB$. c 1. ax.



Aliter. Super AB fac quadratum AD, cujus diameter EB. per divisionis punctum C duc perpendicularem CF; & per G duc HI parall. AB.

Quoniam ang. $EHG=A$ rectus est, & AEB *d* semirectus, *e* erit reliquus HGE etiam semirectus. Ergo HE *f* $=$ HG *g* $=$ EF *g* $=$ AC. *h* proinde HF quadratum est rectæ AC. eodem modo CI est CBq. ergo AG. GD rectangula sunt sub AC, CB. Quare totum quadratum AD *k* $=ACq+CBq+2\,ACB$. Q.E.D.

d 4. *Cor.* 32. 1.
e 32. 1.
f 6. 1.
g 34. 1.
h 29. *def.* 1
k 19. *ax.* 1.

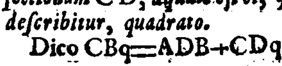
Coroll.

1. Hinc liquet parallelogramma circa diametrum quadrati esse quadrata.

2. Item diametrum cujusvis quadrati ejus angulos bisecare.

3. Si $A=\frac{1}{2}Z$; erit $Zq=4\,Aq$, & $Aq=\frac{1}{4}Zq$. item è contra, si $Zq=4\,Aq$. erit $A=\frac{1}{2}Z$.

PROP. V.



Si recta linea AB *secetur in æqualia* AC *b* CB, *& non æqualia* AD, DB, *rectangulum sub inæqualibus segmentis* AD, DB *comprehensum una cum quadrato, quod fit ab intermedia sectionum* CD, *æquale est ei, quod à dimidia* CB *describitur, quadrato.*

Dico $CBq=ADB+CDq$.

Æquantur

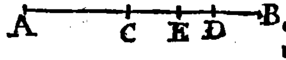
$$\text{Æquantur enim ista}\left\{\begin{array}{l} CBq. \\ {}^{a}\,CDq+CDB+DBq+CDB \\ CDq+{}^{b}\,CBD\,({}^{c}AC\times BD)+CDB \\ CDq+{}^{d}\,ADB. \end{array}\right.$$

a 4. 2. b 3. 2. hyp. d 1. 2.

Hoc Theorema paulo aliter effertur, & facilius demonstratur, sic; Rectangulum ex summa & differentia duarum rectarum A, E, *æquatur differentiæ ex ipsis.*

*sch. 1. 2. Nam si A+E ducatur in A—E, *provenit $Aq - AE + EA - Eq = Aq - Eq$. Q. E. D.

Scholium.



Si A B aliter dividatur, propius scilicet puncto bisectionis, in E; dico $AEB \sqsubset ADB$.

a 5. 2. & 3. ax. Nam $AEB \,{}^{a}= CBq - CEq$. & $ADB \,{}^{a}= CBq - CDq$. ergo quum $CDq \sqsubset CEq$, erit $AEB \sqsubset ADB$. Q. E. D.

Coroll.

Hinc $ADq + DBq \sqsubset AEq + EBq$. Nam
b 4. 2. $ADq + DBq + 2\,ADB \,{}^{b}= ABq \,{}^{b}= AEq + EBq + 2\,AEB$. ergo quum $2\,AEB \sqsubset 2\,ADB$, erit $ADq + DBq \sqsubset AEq + EBq$. Q. E. D.

c 3. ax. Unde $2.\ ADq + DBq - AEq \,{}^{c}- EBq = 2\,AEB - 2\,ADB$.

PROP. VI.

Si recta linea A *bifariam secetur, & illi recta quæpiam linea* E *in directum adjiciatur; rectangulum comprehensum sub tota cum adjecta* (sub. A+E,) *& adjecta* E, *una cum quadrato, quod à dimidia* ($\frac{1}{2}$ A,) *æquale est quadrato à linea, quæ tum ex dimidia, tum ex adjecta componitur, tanquam ab una* ($\frac{1}{2}$A+E) *descripto.*

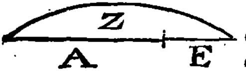
a 4. & 3. Cor. 4. 2. Dico $\frac{1}{4}\,Aq\ ({}^{a}Q.\ \frac{1}{2}A) + AE + Eq = Q.\ \frac{1}{2}A + E$. ^{a} Nam $Q.\ \frac{1}{2}A + E = \frac{1}{4}Aq + Eq + AE$.

Coroll.

Coroll.

Hinc si tres rectæ E, $E + \frac{1}{2} A$, $E + A$ sint in proportione Arithmetica, rectangulum sub extremis E, $E + A$ contentum, una cum quadrato excessus $\frac{1}{2} A$, æquale erit quadrato mediæ $E + \frac{1}{2} A$.

PROP. VII.



Si recta linea Z secetur utcunque; Quod à tota Z, quodque ab uno segmentorum E, utraque simul quadrata, æqualia sunt illi, quod bis sub tota Z, & dicto segmento E comprehenditur, rectangulo, & illi, quod à reliquo segmento A fit, quadrato.

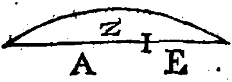
Dico $Zq + Eq = 2\,ZE + Aq$. Nam $Zq \; a = Aq$ a 4. 2.
$+ Eq + 2\,AE$. & $2\,ZE \; b = 2\,Eq + 2\,AE$. b 3. 2.

Coroll.

Hinc, quadratum differentiæ duarum quarumcunque linearum Z, E, æquale est quadratis utriusque minus duplo rectangulo sub ipsis.

c Nam $Zq + Eq - 2ZE = Aq = Q.\,Z - E$. c 7. 2. & 3. ax.

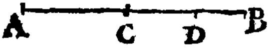
PROP. VIII.



Si recta linea Z secetur utcunque; rectangulum quater comprehensum sub tota Z, & uno segmentorum E, cum eo, quod à reliquo segmento A fit, quadrato, æquale est ei, quod à tota Z, & dicto segmento E, tanquam ab una linea Z + E describitur, quadrato.

Dico $4\,ZE + Aq = Q.\,Z + E$. Nam $2\,ZE \; a =$ a 7. 2. &
$Zq + Eq - Aq$. ergo $4\,ZE + Aq = Zq + Eq + 2$ 3. ax.
$ZE \; b = Q.\,Z + E$. Q.E.D. b 4. 2.

PROP. IX.



Si recta linea AB secetur in æqualia AC, AC, CB, *& non æqualia* AD, DB. *quadrata, quæ ab inæqualibus totius segmentis* AD, DB *fiunt, simul duplicia sunt, & ejus, quod à dimidia* AC, *& ejus, quod ab intermedia sectionum* CD *fit, quadrati.*

Dico ADq + DBq = 2 ACq + 2 CDq. Nam
a 4. 2. ADq + DBq *a* = ACq + CDq + 2 ACD + DBq.
b *hyp.* atqui 2 ACD (*b* 2BCD) + DBq *c* = CBq
c 7. 2. (ACq) + CDq. *d* ergo ADq + DBq = 2 ACq
d 2. *ax.* + 2 CDq. Q.E.D.

Aliter effertur & facilius demonstratur, sic;

Aggregatum quadratorum ex summa, & differentia duarum rectarum A, E, *æquatur duplo quadratorum ex ipsis.*

a 4. 2. Nam Q. A + E *a* = Aq + Eq + 2 AE. & Q. A
b *Cor.* 7. 2. — E *b* = Aq + Eq — 2 AE. Hæc collecta faciunt 2 Aq + 2 Eq. Q.E.D.

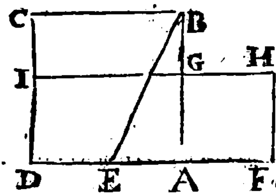
PROP. X.

Si recta linea A *secetur bifariam, adjiciatur autem ei in rectum quæpiam linea; Quod à tota* A *cum adjuncta* E, *& quod ab adjuncta* E, *utraque simul quadrata, duplicia sunt & ejus, quod à dimidia* $\frac{1}{2}$ A; *& ejus, quod à composita ex dimidia, & adjuncta, tanquam ab una* $\frac{1}{2}$ A + E, *descriptum est, quadrati.*

a 4. 2. Dico Eq + Q. A + E, hoc est *a* Aq + 2 Eq + 2
b *Cor.* 4. 2. AE = 2 Q. $\frac{1}{2}$ A + 2 Q. $\frac{1}{2}$ A + E. Nam 2 Q. $\frac{1}{2}$ A *b*
c 4. 2. = $\frac{1}{2}$ Aq & 2 Q. $\frac{1}{2}$ A + E *c* = $\frac{1}{2}$ Aq + 2 Eq + 2 AE.

PROP.

PROP. XI.



Datam rectam lineam AB *secare in* G, *ut comprehensum sub tota* AB, *&* *altero segmentorum* BG *rectangulum, æquale sit ei quod à reliquo segmento* AG, *fit, quadrato.*

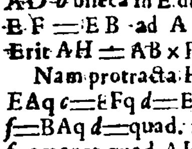
Super AB *a* describe quadratum AC. latus AD *b* biseca in E. duc EB. ex EA producta cape BF = EB ad AF *a* statue quadratum AH. Erit $AH = AB \times BG$. a 46. 1. b 10. 1.

Nam protracta HG ad I; Rectang. DH + EAq *c* = EFq *d* = EBq *e* = BAq + EAq ergo DH *f* = BAq *d* = quad. AC. subtrahe commune AI; *f* remanet quad. AH = GC; *d* id est AGq = AB x BG. Q. E. F. c 6. 2. d constr. e 47. 1. f 3. ax.

Scholium.

Hæc Propositio numeris explicari nequit; *neque enim ullus numerus ita secari potest, ut productum ex toto in partem unam æquale sit quadrato partis reliquæ. *vid 6. 13.

PROP. XII.



In amblygoniis triangulis ABC *quadratum, quod fit à latere* AC *angulum obtusum* ABC *subtendente, majus est quadratis, quæ fiunt à lateribus* AB, BC *obtusum angulum* ABC *comprehendentibus, rectangulo bis comprehenso, & ab uno laterum* BC, *quæ sunt circa obtusum angulum* ABC, *in quod, cum protractum fuerit, cadit perpendicularis* AD, *& ab assumpta exterius linea* BD *sub perpendiculari* AD *prope angulum obtusum* ABC.

Dico

Dico $ACq = CBq + ABq + 2\ CB \times BD$.

a 47.1.
b 4.2.
c 47.1.

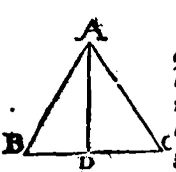
$$\text{Nam ista æqualia sunt inter se}\left\{\begin{array}{l} ACq.\\ {}^{a}\ CDq+ADq.\\ {}^{b}\ CBq+2\,CBD+BDq+ADq\\ CBq+2\,CBD\ {}^{c}+ABq.\end{array}\right.$$

Schol.

Hinc, *cognitis lateribus trianguli obtusanguli* ABC, *facile invenientur tum segmentum* BD *inter perpendicularem* AD, *& obtusum angulum* ABC *interceptum, tum ipsa perpendicularis* AD.

Sic; Sit AC 10, AB 7, CB 5; unde ACq 100, ABq 49, CBq 25. Proinde $ABq + CBq = 74$. hunc deme ex 100, manet 26 pro 2 CBD. unde CBD erit 13. hunc divide per CB 5, provenit $2\frac{3}{5}$ pro BD. quare AD invenitur per 47.1.

PROP. XIII.



In oxygoniis triangulis ABC *quadratum à latere* AB *angulum acutum* ACB *subtendente, minus est quadratis, quæ fiunt à lateribus* AC, CB *acutum angulum* ACB *comprehendentibus, rectangulo bis comprehenso, & ab uno laterum* BC, *quæ sunt circa acutum angulum* ACB, *in quod perpendicularis* AD *cadit, & ab assumpta interius linea* DC *sub perpendiculari* AD, *prope angulum acutum* ACB.

Dico $ACq + BCq = ABq + 2\ BCD$.

a 47.1.
b 7.2.
c 47.1.

$$\text{Nam æquantur ista}\left\{\begin{array}{l} ACq+BCq.\\ {}^{a}\ ADq+DCq+BCq.\\ {}^{b}\ ADq+BDq+2\,BCD.\\ {}^{c}\ ABq+2\,BCD.\end{array}\right.$$

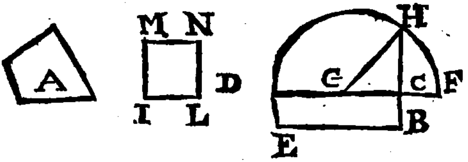
Coroll.

Hinc etiam *cognitis lateribus trianguli* ABC, *invenire est tam segmentum* DC *inter perpendicularem*

rem AD, *&* *acutum angulum* A B C *interceptum*, *quam ipsam perpendicularem* AD.

Sit AB 13. A C 15. BC 14. Detrahe A Bq (169) ex A Cq + B Cq hoc est ex 225 + 196 = 421; remanet 252 pro 2 BCD; unde BCD erit 126. hunc divide per B C 14, provenit 9 pro DC. unde AD = √: 225 — 81 = 12.

PROP. XIV.



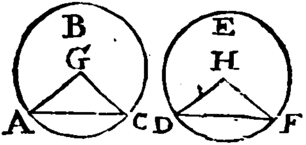
Dato rectilineo A *æquale quadratum* M L *invenire*.

a Fac rectangulum DB = A, cujus majus latus D C produc ad F, ita ut C F = C B. *b* Bi-seca D F in G, quo centro ad intervallum G F describe circulum F H D, producatur CB, donec occurrat circumferentiæ in H. Erit CHq = *ML = A. — a 45. 1. — b 10. 2. — *46. 1.

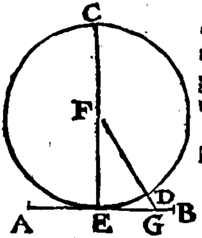
Ducatur enim G H. Estque A *c* = D B *c* = DCF *d* = G Fq — G Cq *e* = H Cq *e* = M L Q. E. F. — c *Constr.* — d 5. 2. & 3. *ax.* — e 47 1. & 3. *ax.*

LIB. III.

Definitiones.

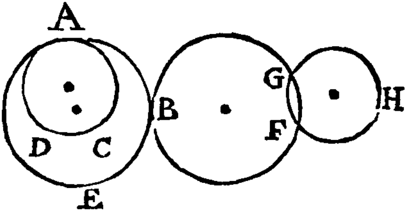


I. Æquales circuli (GABC, HDEF) ſunt, quorum diametri ſunt æquales, vel quorum quæ ex centris rectæ lineæ GA, HD, ſunt æquales.



II. Recta linea AB circulum FED tangere dicitur, quæ cum circulum tangat, ſi producatur circulum non ſecat.

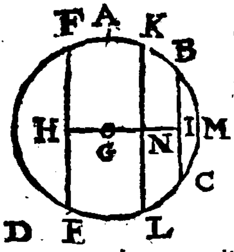
Recta FG ſecat circulum FED.



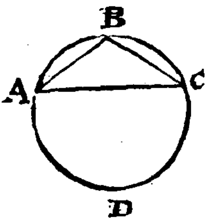
III. Circuli DAC, ABE (item FBG, ABE) ſe mutuo tangere dicuntur, qui ſe mutuo tangentes ſeſe mutuo non ſecant.

Circulus BFG ſecat circulum FGH.

IV. In



IV. In circulo GABD æqualiter distare à centro dicuntur rectæ lineæ FE KL, cum perpendiculares GH, GN, quæ à centro G in ipsas ducuntur, sunt æquales. Longius autem abesse illa BC dicitur, in quam major perpendicularis GI cadit.

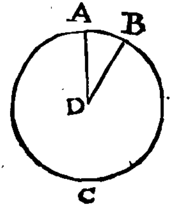


V. Segmentum circuli (ABC) est figura, quæ sub recta linea AC, & circuli peripheria ABC comprehenditur.

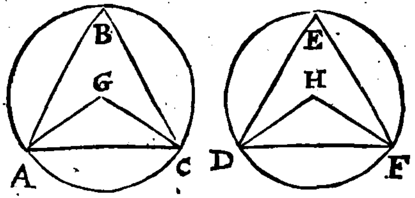
VI. Segmenti autem angulus (CAB) est, qui sub recta linea CA, & circuli peripheria AB comprehenditur.

VII. In segmento autem (ABC) angulus (ABC) est, cum in segmenti peripheria sumptum fuerit quodpiam punctum B, & ab illo in terminos rectæ ejus lineæ AC, quæ segmenti basis est, adjunctæ fuerint rectæ lineæ AB, CB, is inquam angulus ABC ab adjunctis illis lineis AB, CB comprehensus.

VIII. Cum vero comprehendentes angulum ABC, rectæ lineæ AB, BC aliquam assumunt peripheriam ADC, illi angulus ABC insistere dicitur.

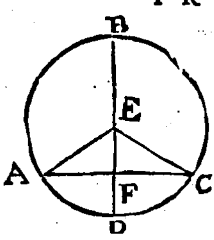


XI. Sector autem circuli (ADB) est, cum ad ipsius circuli centrum D constitutus fuerit angulus ADB; comprehensa nimirum figura ADB. & à rectis lineis AD, BD angulum continentibus, & à peripheria AB ab illis assumpta.



X. Similia circuli segmenta (ABC, DEF) sunt, quæ angulos (ABC, DEF) capiunt æquales; aut in quibus anguli ABC, DEF inter se sunt æquales.

PROP. I.



Dati circuli ABC *centrum* F *reperire.*

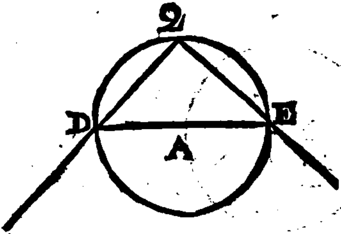
Duc in circulo rectam A C utcunque, quam biseca in E. per E duc perpendicularem D B. hanc biseca in F. erit F centrũ.

Si negas, centrum esto G, extra rectam D B (nam in ea esse non potest, cum ubique extra F dividatur inæqualiter) ducanturque G A,
a 15.def.1. G C, G E. Vis G centrum esse; *a* ergo GA=
b 8. 1. GC; & per constr. A E=E C, latus vero GE
c 10.def.1. commune est; *b* ergo anguli GEA, GEC pares,
d 12. ax. & *c* proinde recti sunt. *d* ergo ang. GEC=FEC
e 9. ax. rect. *e* Q.E.A.

Coroll.

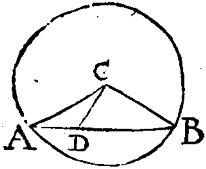
Coroll.

Hinc, si in circulo recta aliqua linea BD aliquam rectam lineam AC bifariam & ad angulos rectos secet, in secante BD erit centrum.



Facillime per normam invenitur centrum vertice Q ad circumferentiam applicato. Si enim recta DE jungens puncta D, & E, in quibus normæ latera QD, QE peripheriam secant, bisecetur in A, erit A centrum. Demonstratio pendet ex 31. hujus. *And. Tacq.*

PROP. II.



Si in circuli CAB *peripheria duo quælibet puncta,* A, B *accepta fuerint, recta linea* AB, *quæ ad ipsa puncta adjungitur, intra circulum cadet.*

Accipe in recta AB quodvis punctum D, & ex centro C duc CA, CD, CB. & quoniam CA *a* = CB, *b* erit ang. A = B. Sed ang. CDB *c* ⊏ A; ergo ang. CDB ⊏ B. *d* ergo CB ⊏ CD. atqui CB tantum pertingit ex centro ad circumferentiam; ergo CD eousque non pertingit. ergo punctum D est intra circulum. Idemque ostendetur de quovis alio puncto rectæ AB. Tota igitur AB cadit intra circulum, Q.E.D.

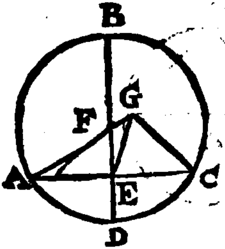
a 15. *def.* 1.
b 5. 1.
c 16. 1.
d 19. 1.

 Coroll.

Coroll.

Hinc, recta circulum tangens, ita ut eum non secet, in unico puncto tangit.

PROP. III.



Si in circulo EABC *recta quædam linea* BD *per centrum extensa quandam* AC *non per centrum extensam bifariam secet,* (*in* F) *&* *ad angulos rectos ipsam secabit; & si ad angulos rectos eam secet, bifariam quoque eam secabit.*

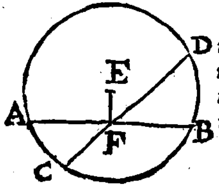
Ex centro E ducantur EA, EC.

a *hyp.* 1. *Hyp.* Quoniam AF *a* = FC, & EA *b* = EC,
b 15 *def.* 1. latusque EF commune est, *c* erunt anguli EFA,
c 8. 1. EFC pares, & *d* consequenter recti. Q.E.D.
d 10. *def.* 1. 2. *Hyp.* Quoniam ang. EFA *e* = EFC, & ang.
e *hyp.* & EAF *f* = ECF, latusque EF commune, *g* erit
12. *ax.* AF = FC. Bisecta est igitur AC. Q.E.D.
f 5. 1.
g 26. 1.

Coroll.

Hinc, in triangulo quovis æquilatero & Isoscele linea ab angulo verticis bisecans basim, perpendicularis est basi. & contra perpendicularis ab angulo verticis bisecat basim.

PROP. IV.

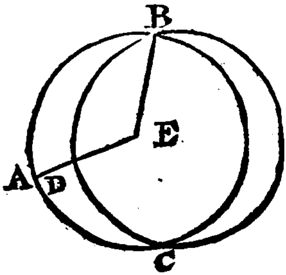


Si in circulo ACD *duæ rectæ lineæ* AB, CD *sese mutuo secent non per centrum* E *extensæ, sese mutuo bifariam non secabunt.*

Nam si una per centrum transeat, patet hanc non nõn bisecari ab altera, quæ ex hyp. per centrum non transit.

Si neutra per centrum transit, ex E centro duc E F. Si jam ambæ A B, C D forent bisectæ in F, anguli E F B, E F D *a* ambo essent recti, & proinde æquales, *b* Q.E.A. a 3. 3. b 9. *ax.*

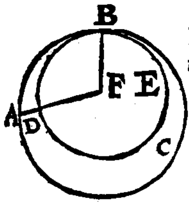
PROP. V.



Si duo circuli B A C, B D C *sese mutuo secent, non erit illorum idem centrum* E.

Alias enim ductis ex communi centro E rectis E B, E D A, essent E D *a* = E B *a* = E A. *b* Q.E.A. a 15. *def.* 1. b 9. *ax.*

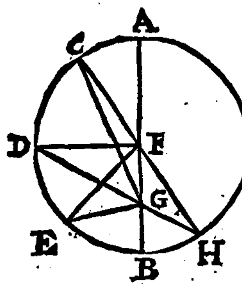
PROP. VI.



Si duo circuli B A C, B D E, *sese mutuo interius tangant* (in B) *eorum non erit idem centrum* F.

Alias ductis ex centro F rectis F B, F D A, essent F D *a* = F B *a* = F A. *b* Q. F. N. a 15. *def.* 1. b 9. *ax.*

PROP. VII.



Si in AB *diametro circuli quodpiam sumatur punctum* G, *quod circuli centrum non sit, ab eoque puncto in circulum quædam rectæ lineæ* GC, GD, GE *cadunt; maxima quidem erit ea* (GA) *in qua centrum* F, *minima vero reliqua* GB, *aliarum vero illi, quæ per centrum ducitur, propinquior* GC *remotiore* GD *semper major est. Duæ autem solum rectæ lineæ* GE GH *æquales ab eodem puncto in circulum cadunt, ad utrasque partes minimæ* GB, *vel maximæ* GA.

a 23. 1. Ex centro F duc rectas FC, FD, FE; & *a* fac ang. BFH $=$ BFE.

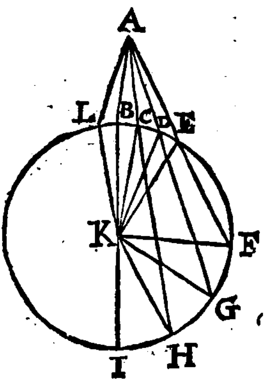
a 20. 1. 1. GF $+$ FC (hoc est GA) *a* $\sqsubset$ GC. Q.E.D.

b 15.*def*.1. 2. Latus FG commune est, & FC *b* $=$ FD,
c 9. *ax*. atque ang. GFC *c* $\sqsubset$ GFD. *d* ergo bas. GC
d 24. 1. $\sqsubset$ GD. Q.E.D.

e 20. 1. 3. FB (FE) *e* $\sqsupset$ GE $+$ GF. ergo ablato
f 5. *ax*. communi FG *f* remanet BG $\sqsupset$ EG. Q.E.D.

4. Latus FG commune est, & FE $=$ FH; atq;
g *constr*. ang. BFH *g* $=$ BFE. *h* ergo GE $=$ GH. Quod
h 4. 1. vero nulla alia GD ex puncto G æquetur ipsi GE, vel GH, jamjam ostensum est. Q.E.D.

PROP. VIII.



Si extra circulum sumatur punctum quodpiam A, *ab eoq; puncto ad circulum deducantur quædam lineæ* AI, AH, AG, AF, *quarum una quidem* AI *per centrum* K *protendatur, reliquæ vero ut libet; in cavam peripheriam cadentium rectarum linearum maxima quidem est illa* AI, *quæ per centrum ducetur, aliarum autem ei quæ per centrum transit propinquior* AH *remotiore* A G *semper major est. In convexam vero peripheriam cadentium rectarum linearum minima quidem est illa* AB, *quæ inter punctum* A, *& diametrum* BI *interponitur; aliarum autem ea, quæ est minimæ propinquior* AC *remotiore* AD *semper minor est. Duæ autem tantum rectæ lineæ* AC, AL *æquales ab eo puncto in ipsum circulum cadunt, ad utrasque partes minimæ* AB, *vel maximæ* AI.

Ex centro K duc rectas KH, KG, KF; KC, KD, KE. & fac ang. AKL=AKC.

1. AI (AK+KH) *a* ⊏ AH. Q.E.D. a 20. 1.

2. Latus A K commune est; & K H=K G; atque ang. A K H ⊏ A K G. *b* ergo baf. A H ⊏ b 24. 1. AG. Q.E.D.

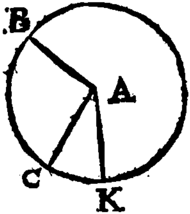
3. KA *c* ⊐ KC+CA. aufer hinc inde æquales c 20. 1. KC, KB, *d* erit AB ⊐ AC. d 5. *ax.*

4. A C + C K *e* ⊐ A D + D K. aufer hinc e 21. 1. inde æquales C K, D K, *f* erit A C ⊐ A D. f 5. *ax.* Q.E.D.

Latus

5. Latus KA est commune & KL = KC;
g *constr.* atque ang. AKL *g* = AKC, *b* ergo LA =
b 4. 1. CA. hisce vero nulla alia æquatur, ex modò ostensis. ergo, &c.

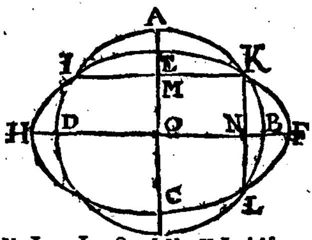
PROP. IX.



Si in circulo BCK *acceptum fuerit punctum aliquod* A, *& ab eo puncto ad circulum cadant plures, quam duæ rectæ lineæ æquales* AB, AC, AK, *acceptum punctum* A *centrum est ipsius circuli.*

a 7. 3. Nam *a* à nullo puncto extra centrum plures quam duæ rectæ lineæ æquales duci possunt ad circumferentiam. Ergo A est centrum. Q.E.D.

PROP. X.

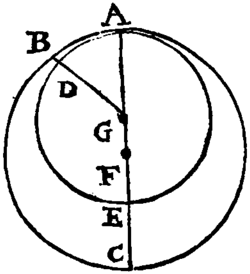


Circulus IAKBL *circulum* IEKFL *in pluribus quam duobus punctis non secat.*

Secet, si fieri potest, in tribus punctis I, K, L. Junctæ IK, KL bisecentur in M & N.
a *Cor.* 1. 3. *a* Ambo circuli centrum habent in singulis perpendicularibus MC, NH, & proinde in earum intersectione O. ergo secantes circuli idem centrum habent. *b* Q.F.N.
b 5. 3.

PROP.

PROP. XI.

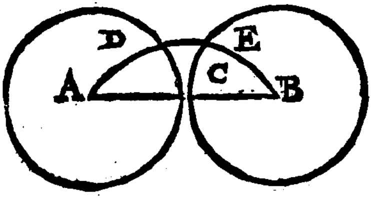


Si duo circuli GADE, FABC *sese intus contingant, atque accepta fuerint eorum centra* G, F; *ad eorum centra adjuncta recta linea* FG, *& producta, in* A *contactum circulorum cadet.*

Si fieri potest, recta FG protracta secet circulos extra contactum A, sic ut non FGA, sed FGDB sit recta linea. Ducatur GA. Et quia GD *a* $=$ GA, & GB *b* $\sqsupset$ GA, (cum recta FGB transeat per F centrum majoris circuli) erit GB $\sqsupset$ GD. *c* Q.E.A.

a 15. *def.* 1.
b 7. 3.
c 9. *ax.*

PROP. XII.



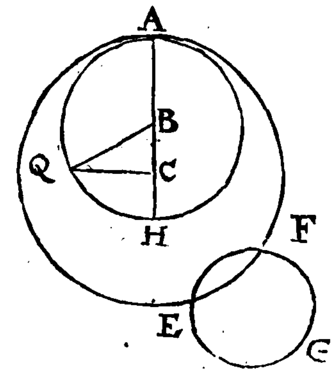
Si duo circuli ACD, BCE *sese exterius contingant, linea recta* AB *quæ ad eorum centra* A, B *adjungitur, per contactum* C *transibit.*

Si fieri potest, sit recta ADEB secans circulos extra contactum C in punctis D, E. Duc AC, CB. erit AD + EB (AC + CB) *a* $\sqsubset$ AD + EB. *b* Q.E.A.

a 20. 1.
b 9. *ax.*

PROP.

PROP. XIII.



Circulus CAF *circulum* BAH *non tangit in pluribus punctis, quam uno* A, *sive intus, sive extra tangat.*

1. Tangat si fieri potest, intus in punctis A, H. *a* ergo recta CB centra connectens, si producatur cadet tam in A,

a 11. 3.

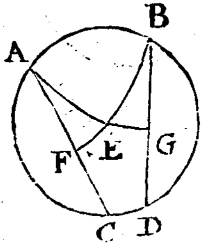
b 15. *def.* 1. quam in H. Quoniam igitur CH *b* = CA, & BH

c 15. *def.* 1. ⊏ CH. erit BA (*c* BH) ⊏ CA. *d* Q.E.A.

d 9. *ax.* 2. Sin dicatur exterius contingere in punctis

e 2. 3. E & F, *e* ducta recta EF in utroque circulo erit. Circuli igitur se mutuo secant, quod non ponitur.

PROP. XIV.



In circulo EABC *æquales rectæ lineæ* AC, BD, *æqualiter distant à centro* E. *& quæ* AC, BD *æqualiter distant à centro, æquales sunt inter se.*

Ex centro E duc perpendiculares EF, EG:

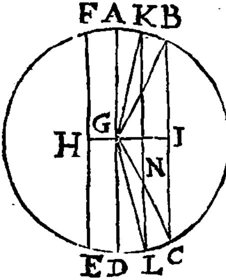
a 3. 3. *a* quæ bisecabunt AC, DB. connecte EA, EB.

b 7. *ax.* 1. *Hyp.* AC = BD. ergo AF *b* = BG. sed & EA = EB, ergo FEq *c* = EAq − AFq = EBq

EBq—BGq *c*=EGq. *d* ergo FE=EG. Q.E.D. c 47. 1. &
2. *Hyp.* EF=EG. ergo AFq *c*=EAq—EFq= 3. *ax.*
EBq—EGq *c*=GBq. ergo AF *d*=GB. d *Schol.*
e proinde AC=BD. Q.E.D. 48. 1.

PROP. XV.

e 6. *ax.*



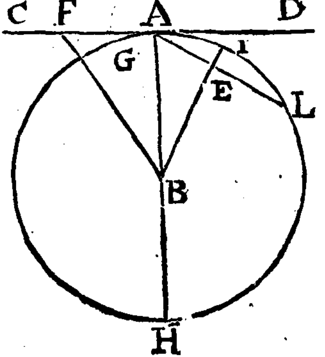
In circulo GABC *maxima quidem linea est diameter* AD; *aliarum autem centro* G *propinquior* FE *remotiore* BC *semper major est.*

1. Duc GB, GC. Diameter AD (*a* = GB + GC) *b* $\sqsubset$ BC. Q.E.D.

a 15. *def.* 1. b 20. 1.

2. Sit distantia GI $\sqsubset$ GH. accipe GN = GH. per N duc KL perpend. GI. junge GK, GL. & quia GK=GB, & GL = GC; estque ang. KGL $\sqsubset$ BGC, *c* erit KL (FE) $\sqsubset$ BC. Q.E.D. c 24. 1.

PROP. XVI.



Quæ CD *ab extremitate diametri* HA *cujusque circuli* BALH *ad angulos rectos ducitur, extra ipsum circulum cadet, & in locum inter ipsam rectam lineam, & peripheriam comprehen-*

prehensum altera recta linea A L *non cadet, & semicirculi quidem angulus* BAI *quovis angulo acuto rectilineo* B A L *major est; reliquus autem* D A I *minor.*

1. Ex centro B ad quodvis punctum F in recta A C duc rectam B F. Latus B F subtendens
a 19. 1. angulum rectum B A F *a* majus est latere B A, quod opponitur acuto BFA. ergo cum BA (BG) pertingat ad circumferentiam, BF ulterius porrigetur, adeoque punctum F; & eadem ratione quodvis aliud rectæ A C, extra circulum situm erit. Q.E.D.

2. Duc BE perpendic. AL. Latus BA opposi-
b 19. 1. tum recto angulo B E A *b* majus est latere B E, quod acutum BAE subtendit: ergo punctum E, adeoque tota EA cadit intra circulum. Q.E.D.

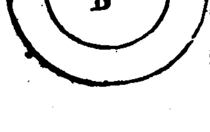
3. Hinc sequitur angulum quemvis acutum, nempe E A D angulo contactus D A I majorem esse. Idem angulum quemvis acutum B A L angulo semicirculi BAI minorem esse. Q.E.D.

Coroll.

Hinc, recta à diametri circuli extremitate ad angulos rectos ducta ipsum circulum tangit.

Ex hac propositione paradoxa consequuntur, & mirabilia bene multa, quæ vide apud interpretes.

PROP. XVII.



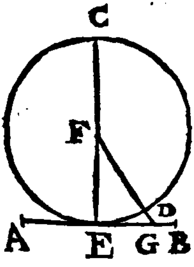
A dato puncto A *rectam lineam* A C *ducere, quæ datum circulum* D B C *tangat.*

Ex D dati circuli centro ad datum punctum A ducatur recta D A secans peripheriam in B. Centro D describe per A alium circulum A E;

AE; & ex B duc perpendicularem ad AD, quæ occurrat circulo A E in E. duc E D occurrentem circulo BC in C. ex A ad C ducta recta tanget circulum D B C.

Nam DB *a* = DC, & DE *a* = DA, & ang. D communis est: *b* ergo ang. A C D = EBD, rect. *c* ergo AC tangit circulum C. Q.E F. a 15. *def.* 1. b 4. 1. c *Cor.* 16. 3.

PROP. XVIII.



Si circulum F E D C *tangat recta quæpiam linea* A B, *à centro autem ad contactum* E *adjungatur recta quædam linea* F E; *quæ adjuncta fuerit* FE *ad ipsam contingentem* A B, *perpendicularis erit.*

Si negas, sit ex F centro alia quædam FG perpendicularis ad contingentem, *a* secabi ea circulum in D. Quum igitur ang. FGE rectus dicatur *b* erit ang. FEG acutus. *c* ergo FE (FD) ⊏ FG. *d* Q.E.A. a 2. *def.* 3. & 16. 3. b *Cor.* 17. 1. c 19. 1. d 9. *ax.*

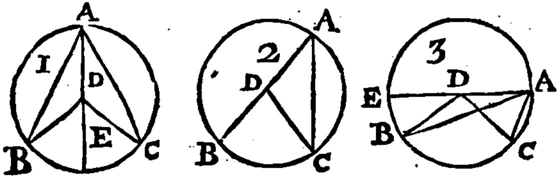
PROP. XIX.

Si circulum tetigerit recta quæpiam linea A B, *à contactu autem* C *recta linea* CE *ad angulos rectos ipsi tangenti excitetur, in excitata* C E *erit centrum circuli.*

Si negas, sit centrum extra CE in F, & ab F ad contactum ducatur FC. Igitur ang. FCB * rectus est; & *a* proinde par angulo ECB recto per hypoth. *b* Q. E. A. * 18. 3. a 11. *ax.* b 9. *ax.*

PROP.

PROP. XX.

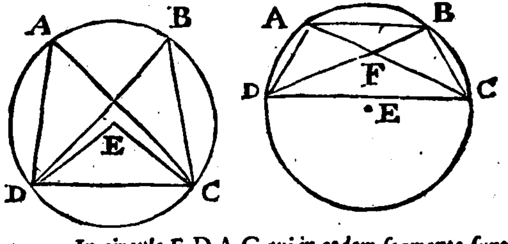


In circulo D A B C, *angulus* BDC *ad centrum duplex est anguli* BAC *ad peripheriam, cum fuerit eadem peripheria* B C *basis angulorum.*

Duc diametrum ADE.

a 32. 1. Externus angulus BDE *a*= DAB + DBA *b*=
b 5. 1. 2 DAB. Similiter ang. EDC = 2 DAC. ergo
c 2. *ax.* in primo casu *c* totus BDC = 2BAC; sed in ter-
d 20. *ax.* tio casu *d* reliquus angulus B D C = 2 B A C. Q. E. D.

PROP. XXI.

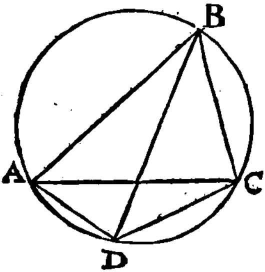


In circulo E D A C *qui in eodem segmento sunt anguli,* DAC *&* DBC *sunt inter se æquales.*

1. *Cas.* Si segmentum DABC semicirculo sit majus, ex centro E, duc ED, EC. Eritque 2 ang.
a 20. 3. A *a*= E *a*= 2 B. Q. E.D.

2. *Cas.* Sin segmentum semicirculo majus non fuerit, summa angulorum trianguli ADF æquatur summæ angulorum in triangulo B C F. De-
b 15. 1. mantur hinc inde AFD *b*= BFC, & ADB *c*=
c *per* 1. *cas.* ACB, remanent DAC = DBC, Q. E.D.

PROP. XXII.



Quadrilaterorum ABCD *in circulo descriptorum anguli* ADC, ABC, *qui ex adverso, duobus rectis sunt æquales.*

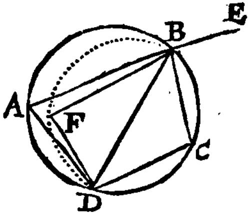
Duc AC, BD. Ang. ABC + BCA + BAC *a* = 2 Rect. Sed a 32. 1.
BDA *b* = BCA, b 21. 1.
& BDC *b* = BAC.
c ergo ABC + ADC = 2 Rect. Q.E.D. c 1. ax.

Coroll.

1. Hinc, si * AB unum latus quadrilateri in circulo descripti producatur, erit angulus externus EBC æqualis angulo interno ADC, qui opponitur ei ABC, qui est deinceps externo EBC ut patet ex 13. 1. & 3. ax. **vide seq. diagram.*

2. Item circa Rhombum circulus describi nequit; quia adversi ejus anguli vel cedunt duobus rectis, vel eos excedunt.

SCHOL.

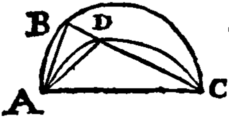


Si in quadrilatero ABCD *anguli* A, *&* C *qui ex adverso duobus rectis æquantur, circa quadrilaterum circulus describi potest.*

Nam circulus per quoslibet tres angulos B, C, D transibit (ut patebit ex 5. 4.) dico eundem per A transire. Nam si neges, transeat per F. ergo ductis rectis

a 22. 3. BF, FD, BD; ang. C+F *a* = 2 Rect. *b* = C+A
b *hyp.* *c* quare A = F. *d* Q.E.A.
c 3. *ax.*
d 21. 1.

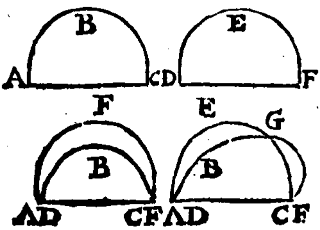
PROP. XXIII.



Super eadem recta linea A C *duo circulorum segmenta* A B C, ADC *similia & inaequalia non constituentur ad easdem partes.*

Nam si dicantur similia, duc C B secantem circumferentias in D, & B, & junge A D, ac
a 10 *def.* 3. AB. Quia segmenta ponuntur similia, *a* erit ang.
b 16. 1. ADC = ABC. *b* Q.E.A.

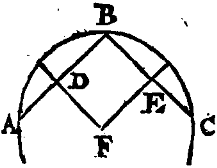
PROP. XXIV.



Super aequalibus rectis lineis AC, DF *similia circulorum segmenta* ABC, DEF *sunt inter se aequalia.*

Basis A C superposita basi DF ei congruet, quia AC = DF. ergo segmentum ABC congruet segmento DEF
a 23. 3. (alias enim aut intra cadet, aut extra, *a* atque ita segmenta non erunt similia, contra Hyp. aut saltem partim intra, partim extra, adeoque ip-
b 10. 3. sum in tribus punctis secabit. *b* Q. E. A.) *c* pro-
c 8. *ax.* inde segmentum ABC = DEF. Q.E D.

PROP. XXV.

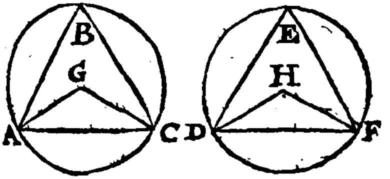


Circuli segmento ABC dato, describere circulum, cujus est segmentum

Subtendantur utcunque duæ rectæ AB, BC, quas biseca in D, & E. Ex D, & E duc perpendiculares DF, EF occurrentes in puncto F. Hoc erit centrum circuli.

Nam centrum *a* tam in DF, quam in EF existit. ergo in communi puncto F. Q.E.F. a *Cor.* 1. 3.

PROP. XXVI.



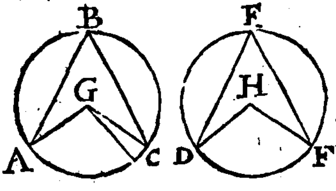
In æqualibus circulis GABC, HDEF *æquales anguli æqualibus peripheriis* AC, DF *insistunt, sive ad centra* G, H, *sive ad peripher.* B, E *constituti insistant.*

Ob circulorum æqualitatem, est GA = HD, & GC = HF, item per hyp. ang. G = H. a 4. 1.
a ergo AC = DF Sed & ang. B *b* = ½ G = *c* ½ b 20. 3.
H *b* = E. *d* ergo segmenta ABC, DEF similia, c *hyp.*
e & proinde paria sunt. *f* ergo etiam reliqua segmenta AC, DF æquantur. Q.E.D. d 10. *def.* 3.
e 24. 3.

Scholium. f 3. *ax.*

In circulo ABCD, sit arcus AB par arcui DC; erit AD parall. BC. Nam ducta AC, *a* erit ang. ACB = CAD. a 26. 3.
quare per 27. 1.

PROP. XXVII.

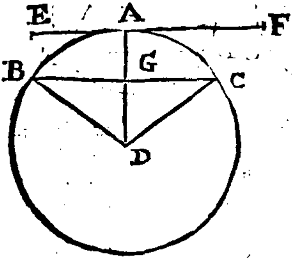


In æqualibus circulis, GABC, HDEF, *anguli qui æqualibus peripheriis* AC, DF *insistunt, sunt inter se æquales, sive ad centra* G, H, *sive ad peripherias* B, E *constituti insistant.*

Nam si fieri potest, sit alter eorum AGC ⊏ DHF. fiatque AGI = DHF. ergo arcus AI *a* = DF *b* = AC. *c* Q.E.A.

a 26. 3.
b *hyp.*
c 9. *ax.*

SCHOL.



Linea recta EF, *quæ ducta ex* A *medio puncto peripheriæ alicujus* BC, *circulum tangit, parallela est rectæ lineæ* BC, *quæ peripheriam illam subtendit.*

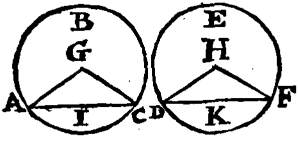
Duc è centro D ad contactum A rectam DA, & connecte DB, DC.

Latus DG commune est; & DB = DC, atque ang. BDA *a* = CDA (ob arcus BA, CA *b* æquales) *c* ergo anguli ad basim DGB, DGC æquales, & *d* proinde recti sunt. Sed interni anguli GAE, GAF *e* etiam recti sunt. *f* ergo BC, EF sunt parallelæ. Q.E.D.

a 27. 3.
b *hyp.*
c 4. 1.
d 10. *def.* 1.
e *hyp.*
f 28. 1.

PROP.

PROP. XXVIII.

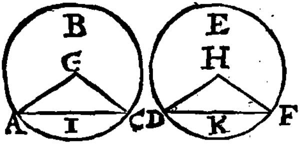


In æqualibus circulis GABC, HDEF, *æquales rectæ lineæ* AC, DF *æquales peripherias auferunt; majorem quidem* ABC *majori* DEF, *minorem autem* AIC *minori* DKF.

E centris G, H, duc GA, GC; & HD, HF. Quoniam GA = HD, & GC = HF, atque AC *a* = DF; *b* erit ang. G = H. *c* ergo arcus AIC = DKF. *d* proinde reliquus ABC = DEF. Q.E.D. a *hyp.* b 8. 1. c 26. 3.

Quod si subtensa AC sit $\sqsubset$ vel $\sqsupset$ DF, erit simili modo arcus AC $\sqsubset$ vel $\sqsupset$ DF. d 3. *ax.*

PROP. XXIX.

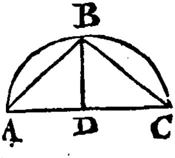


In æqualibus circulis GABC, HDEF, *æquales peripherias* ABC, DEF *æquales rectæ lineæ* AC, DF *subtendunt.*

Duc GA, GC; & HD, HF. Quia GA = HD; & GC = HF; & (ob arcus AC, DF *a* pares) etiam ang. G *b* = H; *c* erit baf. AC = DF. Q.E.D. a *hyp.* b 27. 3.

Hæc & tres proxime præcedentes intelligantur etiam de eodem circulo. c 4. 1.

PROP. XXX.



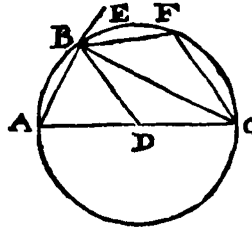
Datam peripheriam ABC *bisariam secare.*

Duc AC; quam biseca in D. ex D duc perpendicularem DB occurrentem arcui in B. Dico factum.

Jungantur enim A B, C B. Latus D B commune est; & AD *a* = DC; & ang. ADB *b* = CDB. *c* ergo A B = B C. *d* quare arcus A B = BC. Q. E. F.

a *const.*
b 12. *ax.*
c 4. 1.
d 28. 3.

PROP. XXXI.



In circulo angulus ABC, *qui in semicirculo, rectus est; qui autem in majore segmento* B A C, *minor recto; qui vero in minore segmento* BFC, *major est recto. Et insuper angulus majoris segmenti recto quidem major est, minoris autem segmenti angulus, minor est recto.*

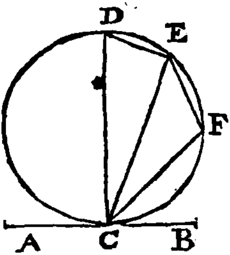
Ex centro D duc DB. Quia DB = DA, erit ang. A *a* = DBA. pariter ang. DCB *a* = DBC. *b* ergo ang. A B C = A + A C B *c* = E B C, *d* proinde ABC, & E B C recti sunt. Q. E. D. *e* ergo B A C acutus est. Q. E. D. ergo cum B A C + B F C *f* = 2 Rect. erit B F C obtusus. denique angulus sub recta C B, & arcu B A C major est recto A B C. factus vero sub C B, & B F C peripheria minoris segmenti, recto EBC *g* minor est. Q.E.D.

a 5. 1.
b 2. *ax.*
c 32. 1.
d 10. *def.* 1.
e *Cor.* 17. 1.
f 22. 3.
g 9. *ax.*

SCHOLIUM.

In triangulo rectangulo A B C, *si hypotenusa* AC *bisecetur in* D, *circulus centro* D, *per* A *descriptus transibit per* B. ut facile ipse demonstrabis ex hac, & 21. 1.

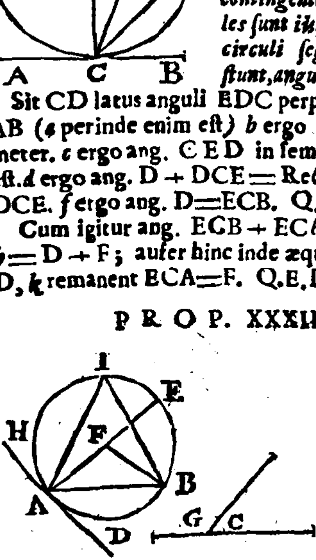
PROP. XXXII.



Si circulum tetigerit aliqua recta linea AB, *à contactu autem producatur quædam recta linea* CE *circulum secans: anguli* ECB, ECA, *quos ad contingentem facit, æquales sunt iis, qui in alternis circuli segmentis consistunt, angulis* EDC, EFC.

Sit CD latus anguli EDC perpendiculare ad
AB (*a* perinde enim est) *b* ergo CD est dia- a 26. 3.
meter. *c* ergo ang. CED in semicirculo rectus b 19. 3.
est. *d* ergo ang. D + DCE = Rect. *e* = ECB + c 31. 3.
DCE. *f* ergo ang. D = ECB. Q.E.D. d 32. 1.
Cum igitur ang. ECB + ECA *g* = 2 Rect. e *Constr.*
h = D + F; aufer hinc inde æquales ECB, & f 3. *ax.*
D, *k* remanent ECA = F. Q.E.D. g 13. 1.
h 22. 3.
k 3. *ax.*

PROP. XXXIII.



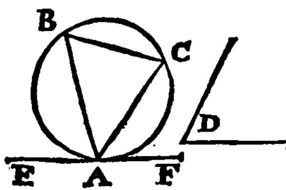
Super data recta linea AB *describere circuli segmentum* AIEB, *quod capiat angulum* AIB *æqualem dato angulo rectilineo* C.

a Fac ang. BAD = C. per A duc AE perpen- a 23. 1.
dicularem ad HD. ad alterum terminum datæ
AB fac ang. ABF = BAF. cujus alterum latus
secet AE in F. centro F per A describe circulum,
quod transibit per B (quia ang. FBA *b* = FAB, b *Constr.*
c ideoque FB = FA;) segmentum AIB est id c 6. 1.
quod quæritur.

 Nam

Nam quia HD diametro AE perpendicularis
d cor.16.3. est, *d* tangit HD circulum, quem secat AB, ergo
e 32. 3. ang. AIB *e*=BAD *f*=C. Q.E.F.
f Constr.

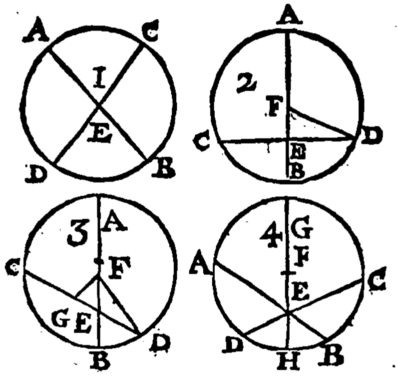
PROP. XXXIV.



A dato circulo ABC segmentum ABC abscindere capiens angulum B aequalem dato angulo rectilineo D.

a 17. 3. *a* Duc rectam EF, quae tangat
b 23. 1. datum circulum in A. *b* ducatur item AC faciens ang. FAC=D. Haec auferet segmentum ABC
c 32. 3. capiens angulum B *c*=CAF *d*=D. Q.E.F.
d Constr.

PROP. XXXV.



Si in circulo FBCA *duae rectae lineae* AB, DC *sese mutuo secuerint, rectangulum comprehensum sub*

sub segmentis AE, EB *unius, æquale est ei quod sub segmentis* CE, ED *alterius comprehenditur, rectangulo.*

Cas. 1. Si rectæ sese in centro secent, res clara est.

2. Si una AB transeat per centrum F, & reliquam CD bisecet, duc FD. Estque Rectang. AEB + FEq *a* = FBq *b* = FDq *c* = EDq + FEq *d* = CED + FEq. *e* ergo Rectang. AEB = CED. Q.E.D.

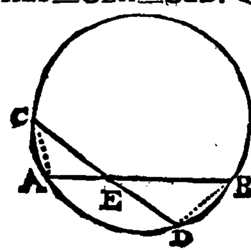
a 5. 2.
b *sch.* 48. 1.
c 47. 1.
d *hyp.*
e 3. *ax.*

3. Si una AB diameter sit, alteramque CD secet inæqualiter, biseca CD per FG perpendicularem ex centro.

Æquantur ista:
- Rectang. AEB + FEq.
- *f* FBq (FDq). — f 5. 2.
- *g* FGq + GDq. — g 47. 1.
- FGq + *h* GEq + Rectang. CED. — h 5. 2.
- *k* FEq + CED. — k 47. 1.

l Ergo Rectang. AEB = CED. — l 3. *ax.*

4 Si neutra rectarum AB, CD per centrum transeat, per intersectionis punctum E duc diametrum GH. Per modo demonstrata Rectang. AEB = GEH = CED. Q.E.D.



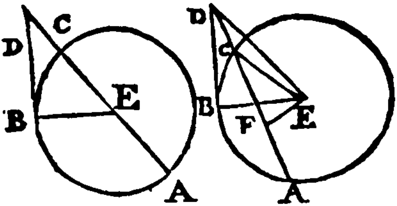
Facilius sic, & universaliter; connecte AC & BD, atque ob angulos *a* CEA, DEB, *b* ipsosque C, B (super eodem arcu AD) pares; trigona CEA, BED, *c* æquiangula sunt. *d* ergo CE. EA :: EB. ED. *e* proinde CE × ED = EA × EB. Q.E.D.

a 15. 1.
b 21. 3.
c *cor.* 32. 1.
d 4. 6.
e 16. 6.

Quæ ex 6. lib. citantur, tam hic quam in seq. ab hac minime pendent; quare iis uti licuit.

PROP.

PROP. XXXVI.

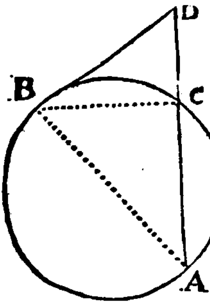


Si extra circulum E B C *sumatur punctum aliquod* D, *ab eoque puncto in circulum cadant duæ rectæ lineæ* DA, DB; *quarum altera* DA *circulum secet, altera vero* DB *tanget; Quod sub tota secante* DA, *& exterius inter punctum* D, *& convexam peripheriam assumpta* DC *comprehenditur rectangulum, æquale erit ei, quod à tangente* DB *describitur, quadrato.*

1. *Cas.* Si secans AD transeat per centrum E,
a 18. 3. junge EB; *a* faciet hæc cum DB rectum angu-
b 47. 1. lum; quare DBq + EBq (ECq) *b* = EDq
c 6. 2. *c* = AD x DC + ECq *d* ergo AD x DC =
d 3. ax. DBq. Q.E.D.

2. *Cas.* Sin AD per centrum non transeat, duc EC, EB, ED; atque EF perpend. AD, quare
a 3. 3. *a* bisecta est AC in F.
b 47. 1. Quoniam igitur BDq + EBq *b* = DEq *b* =
c 6. 2. EFq + FDq *c* = EFq + ADC + FCq *d* =
d 47. 1. ADC + CEq (EBq;) *e* erit BDq = ADC,
e 3. ax. Q.E.D.

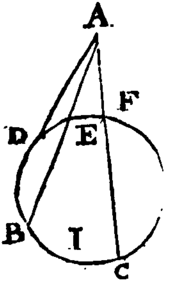
Facilius



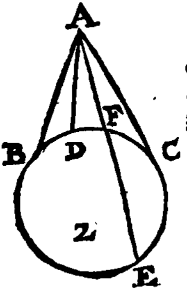
Facilius ac universalius sic;

Duc AB,& BC,ac ob angulos A, DBC *a* pares, & D communem, a 32. 3.
triangula BDC, ADB *b* æquiangula sunt. *c* ergo AD. DB::DB. CD. b 32. 1. c 4. 6.
d quare $AD \times DC =$ d 17. 6.
DBq. Q.E.D.

Coroll.



1. Hinc, si à puncto quovis A extra circulum assumpto, plurimæ lineæ rectæ AB, AC circulum secantes ducantur, rectangula comprehensa sub totis lineis A B, A C, & partibus externis AE, AF inter se sunt æqualia. Nam si ducatur tangens AD; erit CAF
=ADq *a* =BAE. a 36. 3.



2. Constat etiam duas rectas AB,AC ab eodem puncto A ductas, quæ circulum tangant, inter se æquales esse.

Nam si ducatur AE secans circulum; erit ABq *a* = a 36. 3.
EAF=ACq.

3. Per-

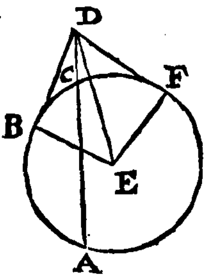
3. Perſpicuum quoque eſt ab eodem puncto A extra circulum aſſumpto, duci tantum poſſe duas lineas, AB, AC quæ circulum tangant.

c 2. cor. Nam ſi tertia AD tangere dicatur, erit AD
d 8. 3. c = AB c = AC. d Q. F. N.

4. E contrà conſtat, ſi duæ rectæ æquales AB, AC ex puncto quopiam A in convexam peripheriam incidant, & earum una AB circulum tangat, alteram quoque circulum tangere.

e 2. cor. Nam ſi fieri poteſt, non AC, ſed altera AD
f hyp. circulum tangat. ergo AD e = AC f = AB.
g 8. 3. g Q. E. A.

PROP. XXXVII.



Si extra circulum EBF *ſumatur punctum* D, *ab eoque in circulum cadant duæ rectæ lineæ* DA, DB; *quarum altera* DA *circulum ſecet, altera* DB *in eum incidat; ſit autem quod ſub tota ſecante* DA, *& exterius inter punctum, & convexam peripheriam aſſumpta* DC, *comprehenditur rectangulum, æquale ei, quod ab incidente* DB *deſcribitur quadrato, incidens ipſa* DB *circulum tanget.*

a 17. 3. Ex D a ducatur tangens DF; atque ex E cen-
b hyp. tro duc ED, EB, EF. Quia DBq b = ADC
c 36. 3. c = DFq, d erit DB = DF. Sed EB = EF,
d 1. ax. & & latus ED commune eſt; e ergo ang. EBD
ſch. 48. 1. = EFD. Sed EFD rectus eſt, f ergo EBD
e 8. 1. etiam rectus eſt. g ergo DB tangit circulum.
f 12. ax. Q. E. D.
g cor. 16. 3.

Coroll.

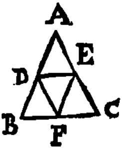
h 8. 1. Hinc, h ang. EDB = EDF.

L I B.

LIB. IV.

Definitiones.

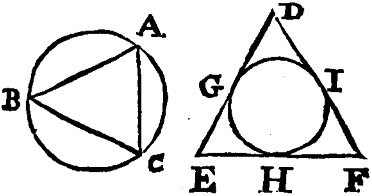
I. Figura rectilinea in figura rectilinea inscribi dicitur, cum singuli ejus figuræ, quæ inscribitur, anguli singula latera ejus in qua inscribitur, tangunt.



Sic triangulum DEF *est inscriptum in triangulo* ABC.

II. Similiter & figura circa figuram describi dicitur, cum singula ejus, quæ circumscribitur, latera singulos ejus figuræ angulos tetigerint, circa quam illa describitur.

Ita triangulum A B C *est descriptum circa triangulum* DEF.



III. Figura rectilinea in circulo inscribi dicitur, cum singuli ejus figuræ, quæ inscribitur, anguli tetigerint circuli peripheriam.

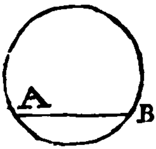
IV. Figura vero rectilinea circa circulum describi dicitur, cum singula latera ejus, quæ circumscribitur, circuli peripheriam tangunt.

V. Similiter & circulus in figura rectilinea inscribi dicitur, cum circuli peripheria singula latera tangit ejus figuræ, cui inscribitur.

VI. Circulus autem circa figuram describi

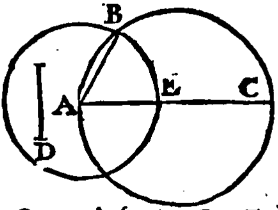
di-

dicitur, cum circuli peripheria singulos tangit ejus figuræ, quam circumscribit, angulos.



VII. Recta linea in circulo accommodari, seu coaptari dicitur, cum ejus extrema in circuli peripheria fuerint; ut recta linea AB.

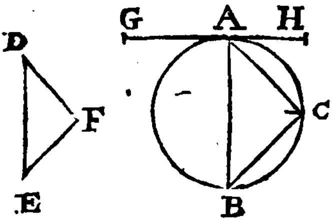
PROP. I. *Probl.* 1.



In dato circulo ABC *rectam lineam* AB *accommodare æqualem datæ rectæ lineæ* D, *quæ circuli diametro* AC *non sit major.*

a 3. *post.* Centro A, spatio AE=D *a* describe circulum
& 3. 1. dato circulo occurrentem in B. Erit ducta AB
b 15. *def.* 1. *b*=AE *c*=D. Q.E.F.
c *constr.*

PROP II. *Probl.* 2.



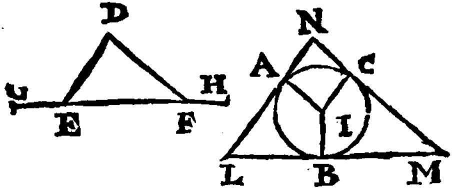
In dato circulo ABC *triangulum* ABC *describere dato triangulo* DEF *æquiangulum*

Recta GH circulum da-
a 17. 3. tum *a* tangat in A. *b* Fac ang. HAC=E; *b* &
b 23. 1. ang. GAB=F, & junge BC. Dico factum.

Nam

Nam ang. B *c* = H A C *d* = E ; & ang. C *c* = GAB *d* = F; *e* quare etiam ang. BAC = D. ergo triang. B A C circulo inscriptum triangulo DEF æquiangulum est. Q.E.F. c 32. 3. d *constr.* e 32. 1.

PROP. III. *Probl.* 3.



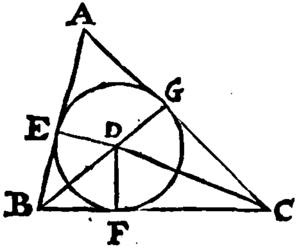
Circa datum circulum IABC *triangulum* LNM *describere, dato triangulo* DEF *æquiangulum.*

Produc latus E F utrinque. *a* Fac ad centrum I ang. A I B = D E G. & ang. B I C = DFH. deinde in punctis A, B, C circulum *b* tangant tres rectæ LN, LM, MN. Dico factum. a 23. 1. b 17. 3.

Nam quod coibunt rectæ L N, L M, M N, atque ita triangulum constituent, patet ; *c* quia anguli LAI, LBI *d* recti sunt, adeoque ducta AB angulos faciet LAB, LBA duobus rectis minores. Quoniam igitur ang. A I B + L *e* = 2 Rect. *f* = DEG + DEF; & AIB *g* = DEG, *h* erit ang. L = DEF. Simili argumento ang. M = DFE. *k* ergo etiam ang. N = D. ergo triang. L N M circulo circumscriptum dato E D F est æquiangulum, Q.E.F. c 13. *ax.* d 18. 3. e *Schol.* 32. 1. f 13. 1. g *constr.* h 3. *ax.* k 32. 1.

PROP.

PROP. IV. *Probl.* 4.



In dato triangulo A B C *circulum* E F G *inscribere.*

a 9. 1. Duos angulos B, & C *a* biseca rectis BD, CD coeuntibus in D.

b 12. 1. Ex D *b* duc perpendiculares DE, DF, DG. circulus centro D per E descriptus transibit per G, & F, tangetque tria latera trianguli.

c constr. Nam ang. DBE *c* = DBF; & ang. DEB *d* =

d 12. ax. DFB; & latus DB commune est: *e* ergo DB =

e 26. 1. DF. Simili argumento DG = DF. Circulus igitur centro D descriptus transit per E, F, G; & cum anguli ad E, F, G sint recti, tangit omnia trianguli latera. Q.E.F.

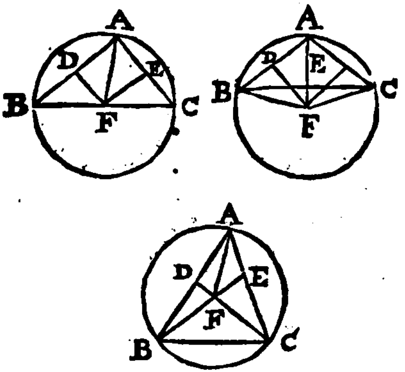
Scholium.

Petr. Herig. Hinc, *cognitis lateribus trianguli, invenientur eorum segmenta, quæ fiunt à contactibus circuli inscripti.* Sic,

Sit A B 12, A C 18, B C 16. Erit A B + B C = 28. ex quo subduc 18 = AC = AE + FC, remanet 10 = BE + BF. ergo BE, vel BF = 5. proinde F C, vel C G = 11. quare G A, vel AE = 7.

PROP.

PROP. V. *Probl.* 5.



Circa datum triangulum ABC *circulum* FABC *deſcribere.*

Latera quævis duo B A, AC *a* biſeca perpendicularibus DF, EF concurrentibus in F. Hoc erit centrum circuli. a 10.& 11. 1.

Nam ducantur rectæ FA,FB, FC. Quoniam AD *b*= DB ; & latus DF commune eſt ; & ang. FDA *c* = FDB, *d* erit FB = FA. eodem modo FC = F A. ergo circulus centro F per dati trianguli angulos B, A, C tranſibit. Q.E.F. b *conſtr.* c *conſtr.* & 12. *ax.* d 4. 1.

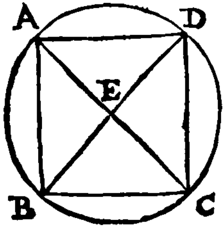
Coroll.

*Hinc, ſi triangulum fuerit acutangulum ; centrum cadet intra triangulum; ſi rectangulum, in latus recto angulo oppoſitum ; ſi denique obtuſangulum, extra triangulum. * 31. 3.

Schol.

Eadem methodo deſcribetur circulus, qui tranſeat per data tria puncta, non in una recta linea exiſtentia.

PROP. IV. *Probl.* 6.

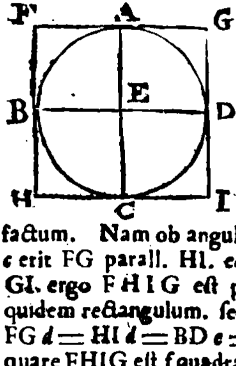


In dato circulo EABCD *quadratum* ABCD *inscribere.*

a 11. 1. *a* Duc diametros AC, BD se mutuo secantes ad angulos rectos in centro E. junge harum terminos rectis AB, BC, CD, DA. Dico factum.

Nam quia 4 anguli ad
b 26. 3. E recti sunt, *b* arcus, & *c* subtensæ AB, BC,
c 29. 3. CD, DA pares sunt. ergo ABCD æquilaterum est; ejusque omnes anguli in semicirculis, adeo-
d 31. 3. que *d* recti sunt, *e* ergo ABCD est quadratum,
e 29 *def*. 1. dato circulo inscriptum. Q.E.F.

PROP. VII. *Probl.* 7.

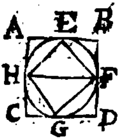


Circa datum circulum EABCD *quadratum* FHIG *describere.*

Duc diametros AC, BD se mutuo secantes perpendiculariter. per
a 17. 3. harum extrema *a* duc tangentes concurrentes in F, H, I, G. Dico
b 18. 3. factum. Nam ob angulos ad A, & C *b* rectos,
c 28. 1. *c* erit FG parall. HI. eodem modo FH parall. GI. ergo FHIG est parallelogrammum; & quidem rectangulum. sed & æquilaterum, quia
d 34. 1. FG *d* = HI *d* = BD *e* = CA *d* = FH *d* = GI.
e 15. *def*. 1. quare FHIG est *f* quadratum, dato circulo cir-
f 29. *def*. 1. cumscriptum. Q.E.F.

SCHOL.

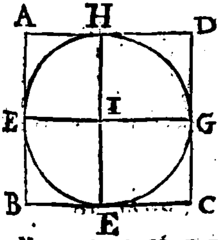
SCHOL.



Quadratum ABCD circulo circumscriptum, duplum est quadrati EFGH circulo inscripti.

Nam rectang. HB = 2 HEF. & HD = 2 HGF. per 41. 1.

PROP. VIII. *Probl.* 8.



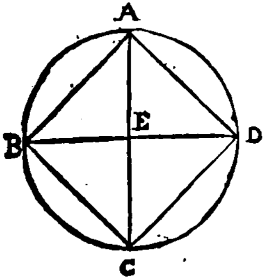
In dato quadrato ABCD *circulum* IEFGH *inscribere.*

Latera quadrati biseca in punctis H, E, F, G; junge HF, EG sese secantes in I. circulus centro I per H descriptus quadrato inscribetur.

Nam quia AH, BF *a* pares ac *b* parallelæ sunt, *c* erit AB parall. HF parall. DC. eodem modo AD parall. EG parall. BC. ergo IA, ID, IB, IC sunt parallelogramma. Ergo AH *d* = AE *e* = HI = EI = IF = IG. Circulus igitur centro I per H descriptus transibit per H, E, F, G, tangetque quadrati latera, cum anguli ad H, E, F, G sint recti. Q.E.F.

a 7. ax.
b 28. 1.
c 33. 1.
d 7. ax.
e 34. 1.

PROP. IX. *Probl. 9.*



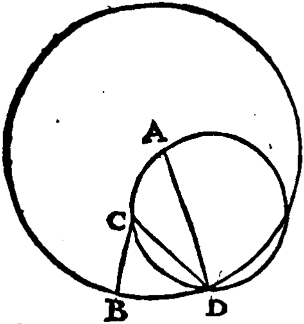
Circa datum quadratum ABCD *circulum* EABCD *describere.*

Duc diametros AC, BD secantes in E. centro E per A describe circulum. Is dato quadrato circumscriptus est.

a 4.cor.32. 1. Nam anguli ABD, & BAC *a* semirecti sunt; *b* ergo EA=EB. eodem modo EA=ED=EC. Circulus

b 6. 1. igitur centro E descriptus per A, B, C, D dati quadrati angulos transit. Q.E.F.

PROP. X. *Probl. 10.*



Isosceles triangulum ABD *constituere, quod habeat utrumque eorum quæ ad basim sunt angulorum B & ADB duplum reliqui A.*

Accipe quamvis rectam AB,

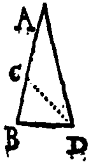
a 11. 2. quam *a* seca in C, ita ut AB×BC = ACq.

Centro A per B describe circulum ABD; in hoc

b 1. 4. *b* accommoda BD=AC, & junge AD. erit triang. ABD quod quæritur.

c 5. 4. Nam duc DC; & per CDA *c* describe circulum.

lum. Quoniam $AB \times BC = ACq$. *d* liquet BD d 37. 3.
tangere circulum ACD, quem secat CD. *e* er- e 32. 3.
go ang. $BDC = A$. ergo ang. $BDC + CDA$ *f* $=$ f 2. *ax.*
$A + CDA =$ *g* BCD. sed $BDC + CDA =$ g 32. 1.
BDA *h* $=$ CBD. *k* ergo ang. $BCD = CBD$. h 5. 1.
ergo DC *l* $=$ DB *m* $=$ AC. *n* quare ang. $CDA =$ k 1. *ax.*
$A = BDC$. ergo $ADB = 2A = ABD$. l 6. 1.
Q. E. F. m *constr.*

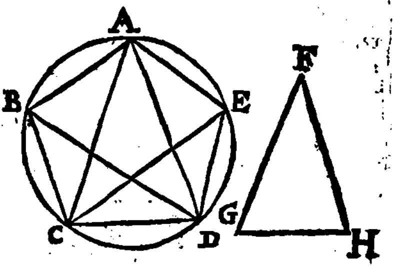


Hæc constructio Analytice indagatur sic; Factum sit; & angulum n 5. 1.
BDA bisecet recta DC. *a* ergo DA. a 3. 6.
DB :: CA. CB. item ob ang. CDA
b $= \frac{1}{2}$ ADB *c* $=$ A, *d* est $CA = DC$. b *constr.*
ac ob ang. DCB *e* $= A + CDA =$ c *hyp.*
2 A *e* $=$ B, *d* erit $DB = DC$. *f* ergo d 6. 1.
$DB = CA$. proinde DA. (*e* BA.) CA :: CA. e 32. 1.
CB. *g* unde $BA \times CB = CAq$. f 1. *ax.*
g 17. 6.

Coroll.

Cum omnes anguli A, B, D *h* conficiant $\frac{5}{5}$ h 32. 1.
2 Rect. (2 Rect.) liquet A esse $\frac{1}{5}$ 2 Rect.

PROP. XI. *Probl.* 11.



In dato circulo ABCDE *pentagonum æquilaterum & æquiangulum* ABCDE *inscribere.*

 a De-

a 10. 4. *a* Describe triangulum Isosceles FGH, habens utrumque angulorum ad basim duplum anguli
b 2. 4. ad verticem. *b* Huic æquiangulum CAD inscribe circulo. Angulos ad basim ACD, & ADC
c 9. 1. *c* biseca rectis DB, CE occurrentibus circumferentiæ in B, & E connecte rectas CB, BA, AE, ED. Dico factum.

Nam ex constr. liquet quinque angulos CAD,
d 26. 3. CDB, BDA, DCB, ECA pares esse; quare *d*
e 29. 3. arcus *e* & subtensæ DC, CB, BA, AE, DE æquantur. Pentagonum igitur æquilaterum est.
f 27. 3. Est vero etiam æquiangulum, *f* quia ejus anguli
g 2. ax. BAE, AED, &c. insistunt arcubus *g* æqualibus BCDE, ABCD, &c.

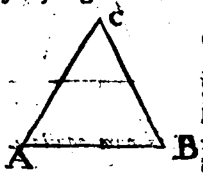
Hujus problematis praxis facilior tradetur ad 10. 13.

Coroll.

Hinc, angulus pentagoni æquilateri & æquianguli æquatur $\frac{1}{5}$ 2 Rect. vel $\frac{6}{5}$ Rect.

Schol.

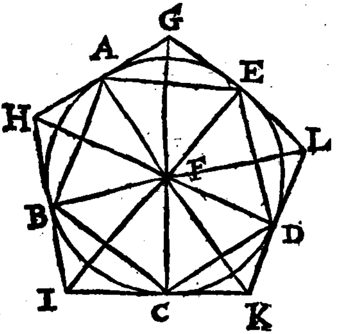
Pet. Herig. *Universaliter figuræ imparium laterum inscribuntur circulo beneficio triangulorum Isoscelium, quorum anguli æquales ad basim multiplices sunt eorum, qui ad verticem sunt, angulorum; parium vero laterum figuræ in circulo inscribuntur ope Isoscelium triangulorum, quorum anguli ad basim multiplices sesquialteri sunt eorum, qui ad verticem sunt, angulorum.*



Ut in triangulo Isoscele CAB, si ang. A = 3 C = B; AB erit latus Heptagoni. Si A = 4 C; erit AB latus Enneagoni, &c. Sin vero A = 1 $\frac{1}{2}$ C, erit AB latus quadrati. Et si A = 2 $\frac{1}{2}$ C, subten-

subtendet A B sextam partem circumferentiæ; paritercque si $A = 3\frac{1}{2}C$; erit A B latus octagoni, &c.

P R O P. XII. *Probl.* 12.



Circa datum circulum FABCDE *pentagonum æquilaterum & æquiangulum* HIKLG *describere.*

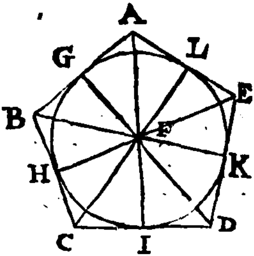
a Inscribe pentagonum ABCDE æquilaterum a 11. 4.
& æquiangulum; duc è centro rectas F A, F B,
FC, FD, FE, iisque totidem perpendiculares
GAH, HBI, ICK, KDL, LEG concurrentes
in punctis H, I, K, L, G. Dico factum. Nam
quia GA, GE ex uno puncto G b tangunt circu- b cor.16.3.
lum, c erit G A = G E. d ergo ang. G F A = c 2.cor.36.
GFE. ergo ang. AFE = 2 G F A. eodem mo- 3.
do ang. AFH = HFB; & proinde ang. AFB = d 8. 1.
2 AFH. Sed ang. A F E e = AFB. f ergo ang. e 27. 3.
GFA = A F H. sed & ang. FAH g = FAG; f 7. ax.
& latus FA est commune, h ergo HA = AG = g 12. ax.
GE = E L, &c. k ergo H G, G L, L K, K I, h 26. 1.
I H latera pentagoni æquantur: sed & anguli k 2 ax.
etiam, utpote l æqualium AGF, AHF, &c. du- l 2.cor.32.1
pli; ergo, &c.

 Coroll.

Coroll.

Eodem pacto, si in circulo quæcunque figura æquilatera & æquiangula describatur, & ad extrema semidiametrorum ex centro ad angulos ductarum, excitentur lineæ perpendiculares, hæ perpendiculares constituent aliam figuram totidem laterum & angulorum æqualium circulo circumscriptam.

PROP. XIII. *Probl.* 13.



In dato pentagono æquilatero & æquiangulo ABCDE *circulum* FGHK *inscribere.*

a 9. 1. Duos pentagoni angulos A, & B *a* biseca rectis AF, BF concurrentibus in F. Ex F duc perpendiculares FG, FH, FI, FK, FL. Circulus centro F per G descriptus tanget omnia pentagoni latera.

b *hyp.* Duc FC, FD, FE. Quoniam BA $b =$ BC;
c *constr.* & latus BF commune est; & ang. FBA $c =$
d 4. 1. FBC, *d* erit AF $=$ FC; & ang. FAB $=$ FCB.
e *hyp.* Sed ang. FAB $e = \frac{1}{2}$ BAE $e = \frac{1}{2}$ BCD. ergo ang. FCB $= \frac{1}{2}$ BCD. eodem modo anguli totales C, D, E omnes bisecti sunt. Quum igitur
f 12. *ax.* ang. FGB $f =$ FHB, & ang. FBH $=$ FBG, &
g 26. 1. latus FB sit commune, *g* erit FG $=$ FH. similiter omnes FH, FI, FK, FL, FG æquantur. Ergo circulus centro F per G descriptus transit per
h *cor.* 16. 3. H, I, K, L; *h* tangitque pentagoni latera, cum anguli ad ea puncta sint recti. Q.E.F.

Coroll.

Hinc, si duo anguli proximi figuræ æquilateræ & æquiangulæ bisecentur, & à puncto, in quo coeunt lineæ angulos bisecantes, ducantur rectæ

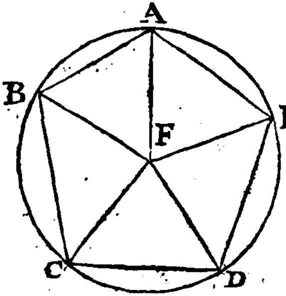
lineæ

lineæ ad reliquos figuræ angulos, omnes anguli figuræ erunt bisecti.

Schol.

Eadem methodo in qualibet figura æquilatera & æquiangula circulus describetur.

P R O P. XIV. *Probl.* 14.



Circa datum Pentagonum æquilaterum & æquiangulum ABCDE *circulum* FABCDE *describere.*

Duos pentagoni angulos biseca rectis AF, BF concurrentibus in F. Circulus centro F per A descriptus pentagono circumscribitur.

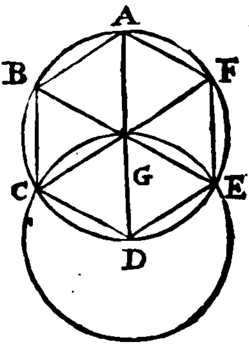
Ducantur enim FC, FD, FE. *a* Bisecti itaque sunt anguli C, D, E. *b* ergo FA, FB, FC, FD, FE æquantur, ergo circulus centro F descriptus, per A, B, C, D, E, pentagoni angulos transibit. Q. E. F. a *cor.*13.4. b 6. 1.

Schol.

Eadem arte circa quamlibet figuram æquilateram & æquiangulam circulus describetur.

P R O P.

PROP. XV. *Probl.* 15.



In dato circulo GABCDEF *hexagonum & æquilaterum & æquiangulum* ABCDEF *inscribere.*

Duc diametrum A D; centro D per centrum G describe circulum, qui datum secet in C, & E. duc diametros C F, E B. junge AB, BC, CD, DE, EF, FA. Dico factum.

a 32. 1. Nam ang. CGD *a* = $\frac{1}{3}$ 2 Rect. *a* = DGE *b* =
b 15. 1. AGF *b* = AGB. *c* ergo BGC = $\frac{1}{3}$ 2 Rect. = FGE.
c *cor.* 13. 1. *d* ergo arcus *e* & subtensæ A B, B C, C D, D E,
d 26. 3. EF æquantur. Hexagonum igitur æquilaterum
e 29. 3. est: sed & æquiangulum *f* quia singuli ejus an-
f 27. 3. guli arcubus insistunt æqualibus. Q.E.F.

Coroll.

1. Hinc latus Hexagoni circulo inscripti semidiametro æquale est.

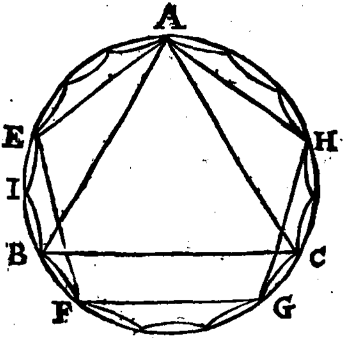
2. Hinc facile triangulum æquilaterum ACE in circulo describetur.

Schol. Probl.

And. Tacq. *Hexagonum ordinatum super data recta* CD *ita*
a 1. 1. *construes.* *a* Fac triangulum CGD æquilaterum super data C D. centro G per C, & D describe circulum. Is capiet Hexagonum super data C D.

PROP.

PROP. XVI. *Probl.* 16.



In dato circulo AEBC *quindecagonum æquilaterum & æquiangulum inscribere.*

Dato circulo *a* inscribe pentagonum æquilaterum AEFGH; *b* itemque triangulum æquilaterum ABC. erit BF latus quindecagoni quæsiti. a 11. 4. b 3. 4.

Nam arcus AB *c* est $\frac{1}{3}$, vel $\frac{5}{15}$ peripheriæ, cujus AF est $\frac{2}{5}$ vel $\frac{6}{15}$ ergo reliquus BF $= \frac{1}{15}$ periph. ergo quindecagonum, cujus latus BF, æquilaterum est; sed & æquiangulum, *d* cum singuli ejus anguli arcubus insistant æqualibus, quorum unusquisque est $\frac{1}{15}$ totius circumferentiæ. ergo, &c. c *constr.* d 27. 3.

Schol.

Circulus dividitur Geometrice in partes
- 4, 8, 16, &c. per 6, 4, & 9, 1.
- 3, 6, 12 &c. per 15, 4, & 9, 1.
- 5, 10, 20, &c. per 11, 4, & 9, 1.
- 15, 30, 60, &c. per 16, 4, & 9, 1.

Cæterum divisio circumferentiæ in partes datas etiamnum desideratur; quare pro figurarum quarumcunq, ordinatarum constructionibus sæpe ad mechanica artificia recurrendum est, propter quæ Geometræ practici consulendi sunt.

LIB.

LIB. V.

Definitiones.

I. PArs est magnitudo magnitudinis, minor majoris, cum minor metitur majorem.

II. Multiplex autem est major minoris, cum minor metitur majorem.

III. Ratio est duarum magnitudinum ejusdem generis mutua quædam secundum quantitatem habitudo.

In omni ratione ea quantitas, quæ ad aliam refertur, dicitur antecedens rationis; ea vero, ad quam alia refertur, consequens rationis dici solet. ut in ratione 6 ad 4; antecedens est 6, & consequens 4.

Nota.

Cujusque rationis quantitas innotescit dividendo antecedentem per consequentem. ut ratio 12 ad 5 effertur per $\frac{12}{5}$ *item quantitas rationis* A *ad* B *est* $\frac{A}{B}$. *Quare non raro brevitatis causa, quantitates rationum sic designamus,* $\frac{A}{B} \sqsubset$, *vel* $=$, *vel* $\sqsupset \frac{C}{D}$; *hoc est, ratio* A *ad* B *major est ratione* C *ad* D, *vel ei æqualis, vel minor. Quod probe animadvertat, quisquis hæc legere volet.*

Rationis, sive proportionis species, ac divisiones vide apud interpretes.

IV. Proportio vero est rationum similitudo.

Rectius quæ hic vertitur proportio, proportionalitas, sive analogia dicitur; nam proportio idem denotat quod ratio, ut plerisque placet.

V. Rationem habere inter se magnitudines dicuntur, quæ possunt multiplicatæ se mutuo superare.

VI. In

E, 12.	A, 4. B, 6.	G, 24.
F, 30.	C, 10. D, 15.	H, 60.

VI. In eadem ratione magnitudines dicuntur esse, prima A ad secundam B, & tertia C ad quartam D, cum primæ A, & tertiæ C. æquemultiplicia E, & F à secundæ B, & quartæ D æquemultiplicibus G, & H, qualiscunque sit hæc multiplicatio, utrumque E, F ab utroque G, H, vel una deficiunt, vel una æqualia sunt, vel una excedunt, si ea sumantur E, G; & F, H quæ inter se respondent.

Hujus nota est ::. ut A. B :: C. D. *hoc est* A *ad* B, *&* C *ad* D *in eadem sunt ratione. aliquando sic scribimus* $\frac{A}{B} = \frac{C}{D}$ *id est*, A.B :: C. D.

VII. Eandem autem habentes rationem (A. B :: C.D) proportionales vocentur.

E, 30.	A, 6. B, 4.	G, 28.
F, 60.	C, 12. D, 9.	H, 63.

VIII. Cum vero æquemultiplicium, E multiplex primæ magnitudinis A excesserit G multiplicem secundæ B; at F multiplex tertiæ C non excesserit H multiplicem quartæ D; tunc prima A ad secundam B majorem rationem habere dicetur, quam tertia C ad quartam D.

Si $\frac{A}{B} \sqsubset \frac{C}{D}$, *necessarium non est ex hac definitione, ut* E *semper excedat* G; *quum* F *minor est quam* H; *sed conceditur hoc fieri posse.*

IX. Proportio autem in tribus terminis paucissimis consistit. *Quorum secundus est instar duorum.*

X. Cum autem tres magnitudines A, B, C proportionales fuerint, prima A ad tertiam C duplicatam rationem habere dicetur ejus, quam habet ad secundam B: at quum quatuor magnitudines A, B, C, D, proportionales fuerint, prima A ad quartam D triplicatam rationem habere dicetur

dicetur ejus, quam habet ad secundam B; & semper deinceps uno amplius, quamdiu proportio extiterit.

Duplicata ratio exprimitur sic $\frac{A}{C} = \frac{A}{B}$ *bis. Hoc est, ratio* A *ad* C *duplicata est rationis* A *ad* B. *Triplicata autem sic* $\frac{A}{D} = \frac{A}{B}$ *ter.* id est, *ratio* A *ad* D *triplicata est rationis* A *ad* B.

$\div$ *denotat continue proportionales.* ut A, B, C, D; *item* 2, 6, 18, 54 sunt $\div$

XI. Homologæ, seu similes ratione, magnitudines dicuntur, antecedentes quidem antecedentibus, consequentes vero consequentibus.

Ut si A.B :: C.D; *tam* A *&* C, *quam* B *&* D *homologa magnitudines dicuntur.*

XII. Alterna ratio, est sumptio antecedentis ad antecedentem, & consequentis ad consequentem.

Ut sit A. B :: C. D. *ergo alterne, vel permutando, vel vicissim,* A.C :: B.D. *per* 16.5.

In hac definitione, & 5. *sequentibus imponuntur nomina sex modis argumentandi, quibus mathematici frequenter utuntur; quarum illationum vis innititur propositionibus hujus libri, quæ in explicationibus citantur.*

XIII. Inversa ratio, est sumptio consequentis ceu antecedentis, ad antecedentem velut ad consequentem.

Ut A. B :: C. D. *ergo inverse,* B. A :: D. C. *per* cor. 4, 5.

XIV. Compositio rationis, est sumptio antecedentis cum consequente, ceu unius, ad ipsam consequentem.

Ut A.B :: C. D. *ergo componendo,* A + B. B :: C+D. D *per* 18. 5.

XV. Divisio rationis, est sumptio excessus, quo consequentem superat antecedens, ad ipsam consequentem.

Ut

Ut A. B :: C. D. *ergo dividendo*, A- B. B :: C- D. D. *per* 17.5.

XVI. Conversio rationis, est sumptio antecedentis ad excessum, quo superat antecedens ipsam consequentem.

Ut A. B :: C. D. *ergo per conversam rationem*, A. A- B :: C. C- D. *per* cor. 19.5.

XVII. Ex æqualitate ratio est, si plures duabus sint magnitudines, & his aliæ multitudine pares, quæ binæ sumantur, & in eadem ratione; cum ut in primis magnitudinibus prima ad ultimam, sic & in secundis magnitudinibus prima ad ultimam sese habuerit. Vel aliter; sumptio extremorum, per subductionem mediorum.

XVIII. Ordinata proportio est, cum fuerit quemadmodum antecedens ad consequentem, ita antecedens ad consequentem: fuerit etiam ut consequens ad aliud quidpiam, ita consequens ad aliud quidpiam.

Ut si A. B :: D. E. *item* B. C :: E. F. *erit ex æquo* A. C :: D. F. *per* 22.5.

XIX. Perturbata autem proportio est; cum tribus positis magnitudinibus, & aliis, quæ sint his multitudine pares, ut in primis quidem magnitudinibus se habet antecedens ad consequentem, ita in secundis magnitudinibus antecedens ad consequentem: ut autem in primis magnitudinibus consequens ad aliud quidpiam, sic in secundis magnitudinibus aliud quidpiam ad antecedentem.

Ut si A. B :: F. G. *item* B. C :: E. F. *erit ex æquo perturbate* A. C :: E. G. *per* 23. 5.

XX. Quotlibet magnitudinibus ordine positis, proportio primæ ad ultimam componitur ex proportionibus primæ ad secundam, & secundæ ad tertiam, & tertiæ ad quartam, & ita deinceps, donec extiterit proportio.

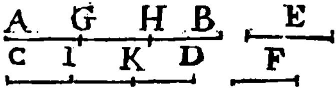
Sint

Sint quotcunque A, B; C, D; ex hac def.
$\frac{A}{D}=\frac{A}{B}+\frac{B}{C}+\frac{C}{D}$.

Axioma.

Æquemultiplices eidem multiplici, sunt quoq; inter se æquemultiplices.

PROP. I.

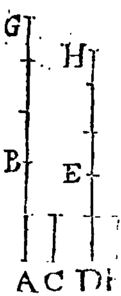


Si sint quotcunque magnitudines AB, CD; *quotcunque magnitudinum* E, F *æqualium numero, singulæ singularum, æquemultiplices; quam multiplex est unius* E *una magnitudo* AB, *tam multiplices erunt & omnes* AB+CD *omnium* E+F.

Sint AG, GH, HB partes quantitatis AB ipsi E æquales. item CI, IK, KD partes quantitatis CD ipsi F pares. Harum numerus illarum numero æqualis ponitur. Quum igitur
a 2. ax. AG+CI a = E+F; a & GH+IK = E+F; a & HB+KD = E+F, liquet AB+CD æque multoties continere E+F, ac una AB unam E continet. Q.E.D.

PROP.

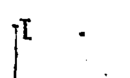
PROP. II.



Si prima AB *secundæ* C *æque fuerit multiplex, atque tertia* DE *quartæ* F; *fuerit autem & quinta* BG *secundæ* C *æque multiplex, atque sexta* EH *quartæ* F, *erit & composita prima cum quinta* (AG) *secundæ* C *æque multiplex, atque tertia cum sexta* (DH) *quartæ* F.

Numerus partium in AB ipsi C æqualium æqualis ponitur numero partium in DE ipsi F æqualium. Item numerus partium in BG ponitur æqualis numero partium in EH. *a* ergo numerus partium in AB+BG æquatur numero partium in DE+EH. hoc est tota AG æquemultiplex est ipsius C, atque tota GH ipsius F. Q.E.D. *a* 2. *ax.*

PROP. III.



Sit prima A *secundæ* B *æquemultiplex, atque tertia* C *quartæ* D; *sumantur autem* EI, FM *æquemultiplices primæ & tertiæ; erit & ex æquo, sumptarum utraque utriusque æquemultiplex: altera quidem* EI *secundæ* B, *altera autem* FM *quartæ* D.

Sint EG, GH, HI partes multiplicis EI ipsi A pares; item FK, KL, LM partes multiplicis FM ipsi C æquales. *a* Harum numerus illarum numero æquatur, Porro A, id est EG, vel GH, vel GI ipsius B ponitur æquemultiplex atque C, vel FK, &c. ipsius D. *a hyp.*

G *b* ergo

b 2. 5. *b* ergo EG+GH æquemultiplex est secundæ
c 2. 5. B, atque FK+KL quartæ D. *c* Simili argumento BI (BH+HI) tam multiplex est ipsius B, quam FM (FL+LN) ipsius D. Q. E. D.

PROP. IV.

Si prima A ad secundam B eandem habuerit rationem, & tertia C ad quartam D; etiam E & F æquemultiplices primæ A, & tertiæ C ad G, & H æquemultiplices secundæ B, & quartæ D, juxta quamvis multiplicationem, eandem habebunt rationem, si prout inter se respondent, ita sumptæ fuerint. (E. G :: F. H.)

Sume I, & K ipsarum E, & F; item L & M ipsarum G, & H æquemultiplices.
a 3. 5. *a* Erit I ipsius A æquemultiplex atque K ipsius C; *a* pariterque L tam multiplex ipsius B quam M ipsius D. Itaque
b hyp. cum sit A. B *b* :: C. D; juxta 6 *def.* si I ⊏, =, ⊐ L; consequenter pari modo K ⊏, =, ⊐ M. ergo cum I, & K ipsarum E, & F sumptæ sint æquemultiplices, atque L, & M ipsarum G & H; erit juxta 7. *def.* E. G :: F. H. Q. E. D.

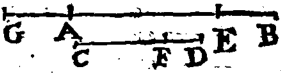
Coroll.

Hinc demonstrari solet inversa ratio.

Nam quoniam A. B :: C. D, si E ⊏, =, ⊐
c 6.def.5. G, *c* erit similiter F ⊏, =, ⊐ H. ergo liquet, quod si G ⊏, =, ⊐ E, esse H ⊏, =, ⊐ F.
d 6.def.5. *d* ergo B. A :: D. C. Q. E D.

PROP.

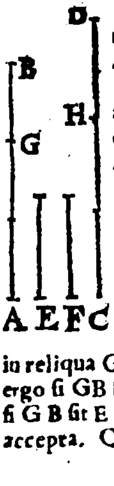
PROP. V.



Si magnitudo AB *magnitudinis* CD *æque fuerit multiplex, atque ablata* AE *ablatæ* CF; *etiam reliqua* EB *reliquæ* FD *ita multiplex erit, ut tota* AB *totius* CD

Accipe aliam quandam GA, quæ reliquæ FD ita sit multiplex, atque tota AB totius CD, vel ablata A E ablatæ CF. *a* ergo tota G A + A B totius C F + F D æquemultiplex est, ac una AE unius CF, hoc est, ac AB ipsius CD. *b* ergo GE = AB *c* proinde, ablata communi AE, manet GA = EB. ergo, &c. a 1. 5. b 6. *ax.* c 3. *ax.*

PROP. VI.



Si duæ magnitudines AB,CD *duarum magnitudinum* E, F *sint æquemultiplices; & detractæ quædam sint,* AG, *&* CH, *earundem* E, *&* F *æquemultiplices; & reliquæ* GB, HD *eisdem* E,F *aut æquales sunt, aut æque ipsarum multiplices.*

Nam quia numerus partium in A B ipsi E æqualium ponitur æqualis numero partium in C D ipsi F æqualium; item numerus partium in A G æqualis numero partium in C H. si hinc A G, inde C H detrahatur, *a* remanet numerus partium in reliqua G B æqualis numero partium in H D. ergo si GB sit E semel, erit HD etiam C semel. si G B sit E aliquoties, erit H D etiam C toties accepta. Q.E.D. a 3. *ax.*

PROP. VII.

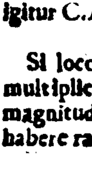
Æquales A *&* B *ad eandem* C *eandem habent rationem; & eadem* C *ad æquales* A *&* B.

Sumantur D & E æqualium A & B æquemultiplices, & F utcunque multiplex ipsius C; a 6. ax. *a* erit D = E. quare si D $\sqsubset$, =, $\sqsupset$ F, erit simi-
b 6. def. 5. liter E $\sqsubset$, =, $\sqsupset$ F. *b* ergo A. C :: B. C. inverse
c cor. 4. 5. igitur C.A *c* :: C.B. Q.E.D.

Schol.

Si loco multiplicis F sumantur duæ æquemultiplices, eodem modo ostendetur æquales magnitudines ad alias inter se æquales eandem habere rationem.

PROP. VIII.



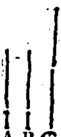
Inæqualium magnitudinum AB, AC, *major* AB *ad eandem* D *majorem rationem habet, quam minor* AC. *Et eadem* D *ad minorem* AC *majorem rationem habet, quam ad majorem* AB.

Sume EF, EG, ipsarum AB, AC, æquemultiplices, ita ut EH ipsius D multiplex, major sit quam EG, at minor quam EF. (Quod facile continget, si utraque EG, GF majores accipiantur ipsa D.) Liquet juxta 8. def. 5. fore $\frac{AB}{D} \sqsubset \frac{AC}{D}$; ac $\frac{D}{AB} \sqsupset \frac{D}{AC}$. Quæ E.D.

Rursus quia IK $\sqsubset$ HG, at IK $\sqsupset$ HF (ut
d 8. def. 5. prius dictum) *d* erit $\frac{D}{C} \sqsubset \frac{D}{AB}$. Q.E.D.

PROP.

PROP. IX.



Quæ ad eandem eandem habent rationem, æquales sunt inter se. Et ad quas eadem eandem habet rationem, eæ quoque sunt inter se æquales.

1. *Hyp.* Sit A. C :: B. C, dico A = B.

A B C Nam sit A $\sqsubset$, vel $\sqsupset$ B, *a* erit ideo a 8. 5.
$\frac{A}{C} \sqsubset$, vel $\sqsupset \frac{B}{C}$. contra Hyp.

2. *Hyp.* Sit C. B :: C. A, dico A = B, nam sit A $\sqsubset$ B, *b* ergo $\frac{C}{B} \sqsubset \frac{C}{A}$. contra Hyp. b 8. 5.

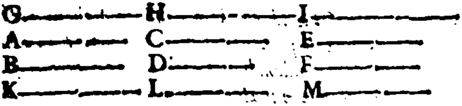
PROP. X.

Ad eandem magnitudinem rationem habentium, quæ majorem rationem habet, illa major est: ad quam vero eadem majorem rationem habet, illa minor est.

A B C 1. *Hyp.* Sit $\frac{A}{C} \sqsubset \frac{B}{C}$. Dico A $\sqsubset$ B. Nam si dicatur A = B, *a* erit A.C :: B.C. contra Hyp. a 7. 5.
Sin A $\sqsupset$ B, *b* erit $\frac{A}{C} \sqsupset \frac{B}{C}$ etiam contra Hyp. b 8. 5.

2. *Hyp.* Sit $\frac{C}{B} \sqsubset \frac{C}{A}$. Dico B $\sqsupset$ A. Nam dic B = A. *c* ergo C.B :: C.A. contra Hyp. vel dic B c 7. 5.
$\sqsubset$ A. *d* ergo $\frac{C}{A} \sqsubset \frac{C}{B}$ etiam contra Hyp. d 8. 5.

PROP. XI.



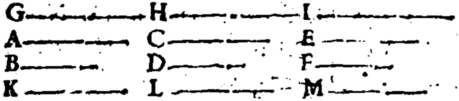
Quæ eidem sunt eædem rationes, & inter se sunt eædem.

Sit A. B :: E. F. item C. D :: E. F. dico A. B :: C. D. sume ipsarum A, C, E æquemultiplices G, H, I; atque ipsarum B, D, F æquemultiplices
a hyp. K, L, M. Et quoniam *a* A. B :: E. F. si G $\sqsubset$, $=$,
b 6. def. 5. $\sqsupset$ K, *b* erit pari modo I $\sqsubset$, $=$, $\sqsupset$ M. pariterque quia *a* E. F :: C. D. si I $\sqsubset$, $=$, $\sqsupset$ M, *b* erit H similiter $\sqsubset$, $=$, $\sqsupset$ L. ergo si G $\sqsubset$, $=$,
c 6. def. 5. $\sqsupset$ K, erit similiter H $\sqsubset$, $=$, $\sqsupset$ L. *c* quare A. B :: C. D. Q. E. D.

Schol.

Quæ eisdem rationibus sunt eædem rationes, sunt quoque inter se eædem.

PROP. XII.



Si sint magnitudines quotcunque A, & B; C & D; E, & F *proportionales; quemadmodum se habuerit una antecedentium* A *ad unam consequentium* B, *ita se habebunt omnes antecedentes*, A, C, E *ad omnes consequentes*, B, D, F.

Sume antecedentium æquemultiplices G, H, I; & consequentium K, L, M. Quoniam quam
a 1. 5. multiplex est una G unius A, *a* tam multiplices sunt omnes G, H, I omnium A, C, E; pariterque quam multiplex est una K unius B, *a* tam multiplices sunt omnes K, L, M omnium B, D, F. Porro
b hyp. ob *b* A. B :: C. D :: E. F. si G $\sqsubset$, $=$, $\sqsupset$ K, erit similiter

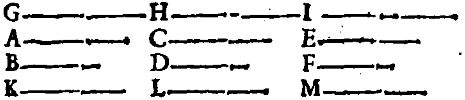
H

H $\sqsubset, =, \sqsupset$ L, & I $\sqsubset, =, \sqsupset$ M, & proinde si G $\sqsubset, =, \sqsupset$ K, erit simili modo G+H+I $\sqsubset, =, \sqsupset$ K+L+M. *c* quare A. B :: A+C+E. B+D+F. *c* 6. *def.* 5.
Q E.D.

Coroll.

Hinc, si similia proportionalia similibus proportionalibus addantur, tota erunt proportionalia.

PROP. XIII.



***Si prima** A **ad secundam** B **eandem habuerit rationem, quam tertia** C **ad quartam** D**; tertia vero** C **ad quartam** D **majorem habuerit rationem, quam quinta** E **ad sextam** F**; prima quoque** A **ad secundam** B **majorem rationem habebit,* quam quinta** E **ad sextam** F.*

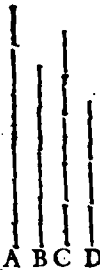
Sume ipsarum A, C, E æquemultiplices G, H, I; ipsarumque B, D, F æquemultiplices K, L, M. Quia A. B :: C. D; si H $\sqsubset$ L, *a* erit G $\sqsubset$ K. Sed quia $\frac{C}{D} \sqsubset \frac{E}{D}$, *b* fieri potest ut sit H $\sqsubset$ L, & I non $\sqsubset$ M. ergo fieri potest ut G $\sqsubset$ K, & I non $\sqsubset$ M *c* ergo $\frac{A}{B} \sqsubset \frac{E}{F}$. Q.E.D.

a 6. *def.* 5.
b 8. *def.* 5.
c 8. *def.* 5.

SCHOL.

Quod si $\frac{C}{D} \sqsupset \frac{E}{F}$, erit quoque $\frac{A}{B} \sqsupset \frac{E}{F}$. Item si $\frac{A}{B} \sqsubset \frac{C}{D} \sqsubset \frac{E}{F}$, erit $\frac{A}{B} \sqsubset \frac{E}{F}$. & si $\frac{A}{B} \sqsupset \frac{C}{D} \sqsupset \frac{E}{F}$, erit $\frac{A}{B} \sqsupset \frac{E}{F}$.

PROP. XIV.



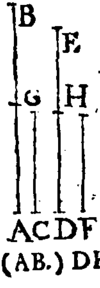
Si prima A *ad secundam* B *eandem habuerit rationem, quam tertia* C *ad quartam* D; *prima vero* A, *quam tertia* C *major fuerit; erit & secunda* B *major quam quarta* D. *Quod si prima* A *fuerit æqualis tertiæ* C, *erit & secunda* B *æqualis quartæ* D; *si vero* A *minor, &* B *minor erit.*

a 8. 5. Sit $A \sqsubset C$. *a* ergo $\frac{A}{B} \sqsubset \frac{C}{B}$. *b* sed
b *hyp.*
c 13. 5. $\frac{A}{B} = \frac{C}{D}$. *c* ergo $\frac{C}{D} \sqsubset \frac{C}{B}$. *d* ergo $B \sqsubset D$.
d 10. 5. Simili argumento si $A \sqsupset C$, *d* erit $B \sqsupset D$. Sin
e 7. 5. ponatur $A = C$; ergo C. B *e* :: A. B *f* :: C. D.
f *hyp.* *g* ergo $B = D$. Quæ E.D.
g 11.&9.5.

SCHOL.

A fortiori, si $\frac{A}{B} \sqsupset \frac{C}{D}$, atque $A \sqsubset C$, erit $B \sqsubset D$. Item si $A = B$, erit $C = D$. Et si $A \sqsubset$, vel $\sqsupset B$, erit pariter $C \sqsubset$, vel $\sqsupset D$.

PROP XV.

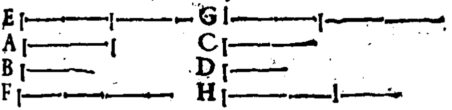


Partes C *&* F *cum pariter multiplicibus* AB, *&* DE *in eadem sunt ratione, si prout sibi mutuo respondent, ita sumantur.* (AB. DE :: C. F.)

Sint AG, GB partes multiplicis AB ipsi C æquales: item DH, HE partes multiplicis DE ipsi F æqua-
a *hyp.* les. *a* Harum numerus illarum nume-
b 7. 5. ro æquatur. ergo quum *b* AG. DH
c 12. 5. :: C. F :: GB. HE. *c* erit AG + GB (AB.) DH + HE (DE) :: C. F. Q.E.D.

PROP.

PROP. XVI.



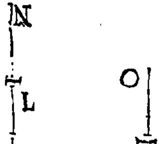
Si quatuor magnitudines A, B, C, D *proportionales fuerint; & vicissim proportionales erunt.* (A.C :: B.D.)

Accipe E & F æquemultiplices ipsarum A & B. ipsarumque C & D æquemultiplices G & H. Itaque E.F *a* :: A.B. *b* :: C.D *a* :: G.H. Quare si E $\sqsubset$, $=$, $\sqsupset$ G, *c* erit similiter F $\sqsubset$, $=$, $\sqsupset$ H. *d* ergo A.C :: B.D. Q.E.D. a 15. 5. b *hyp.* c 11. 5. 14. 5.

SCHOL.

Alterna ratio locum tantum habet, quando quantitates ejusdem sunt generis. Nam Heterogeneæ quantitates non comparantur. d 6. *def.*

P O P. XVII.

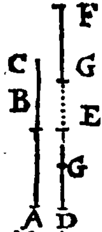


Si compositæ magnitudines proportionales fuerint (AB.CB :: DE FE;) *hæ quoque divisæ proportionales erunt.* (AC.CB :: DF.FE.)

Accipe GH, HL, IK, KM ordine æquemultiplices ipsarum AC, CB, DF, FE. item LN, MO æquemultiplices ipsarum CB, FE. Tota GL totius AB *a* tam multiplex est; quam una GH unius AC, *b* id est quam IK ipsius DF; *c* hoc est quam tota IM totius DE. Item HN (HL+LN) ipsius CB *d* æquemultiplex est, ac KO (KM+MO) ipsius FE. Quum igitur per hyp. AB. BC :: DE. EF. si GL $\sqsubset$, $=$, $\sqsupset$ HN, etiam similiter a 1. 5. b *con.* c 1. 5. d 2. 5.

e 6. def. 5. militer *e* erit IM $\sqsubset, =, \sqsupset$ KO. Itaque ablatis hinc inde communibus HL, KM. si reliqua GH
f 5. ax. $\sqsubset, =, \sqsupset$ LN, *f* erit similiter IK $\sqsubset, =, \sqsupset$ MO.
g 6. def. 5. *g* unde AC.CB :: DF.FE. Q.E.D.

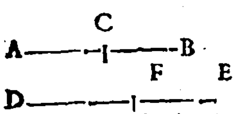
PROP. XVIII.



Si divisæ magnitudines sint proportionales (AB.BC :: DE.EF,) *hæ quoque compositæ proportionales erunt* (AC CB :: DF.FE.)

Nam si fieri potest, sit AC. CB :: DF. FG $\sqsupset$ FE. *a* ergo erit divisim
a 17. 5. AB. BC :: DG. GF. *b* hoc est DG.
b hyp.& 11. 5. GF :: DE. EF. ergo cum DG $\sqsubset$ DE,
c 14. 5. *c* erit GF $\sqsubset$ EF. Q.E.A. Simile
d 9. ax. absurdum *d* sequetur, si dicatur AC.CB :: DF. GF $\sqsubset$ FE.

PROP. XIX.



Si quemadmodum totum AB *ad totum* DE, *ita ablatum* AC *se habuerit ad ablatum* DF, *& reliquum* CB *ad reliquum* FE, *ut totum* AB *ad totum* DE, *se habebit.*

a hyp. Quoniam *a* AB.DE :: AC.DF, *b* erit permu-
b 16. 5. tando AB. AC :: DE. DF. *c* ergo divisim AC.
c 17. 5. CB :: DF. FE. quare rursus *b* permutando AC.
d hyp.& 11. 5. DF::CB.FE; *d* hoc est AB.DE::CB.FE.Q.E.D.

Coroll.

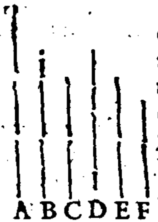
1. Hinc, si similia proportionalia similibus proportionalibus subducantur, residua erunt proportionalia.

2. *Hinc demonstrabitur conversa ratio.*

Sit AB. CB :: DE. FE. Dico AB. AC :: DE. DF. Nam *a* permutando AB.DE :: CB.FE. *b* er-
a 16. 5.
b 19. 5. go AB.DE :: AC.DF. quare iterum permutando, AB.AC :: DE.DF. Q.E.D.

PROP.

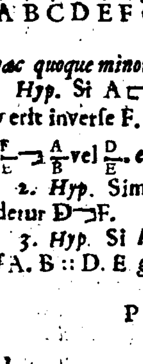
PROP. XX.



Si sint tres magnitudines A, B, C; *& aliæ* D, E, F *ipsis æquales numero, quæ bina & in eadem ratione sumantur* (A.B :: D.E. *atque* B.C :: E.F;) *ex æquo autem prima* A *major fuerit, quam tertia* C; *erit & quarta* D *major quam sexta* F. *Quod si prima* A *tertiæ* C *fuerit æqualis; erit & quarta* D *æqualis sextæ* F. *Sin illa minor, hæc quoque minor erit.*

Hyp. Si A $\sqsubset$ C. quoniam *a* E. F :: B. C. a *hyp.*
b erit inverse F. E :: C. B. *c* Sed $\frac{C}{B} \sqsupset \frac{A}{B}$ *d* ergo b *cor.* 4. 5.
$\frac{F}{E} \sqsupset \frac{A}{B}$ vel $\frac{D}{E}$. *e* ergo D $\sqsubset$ F. Q.E.D. c *hyp &* 8. 5.
2. *Hyp.* Simili argumento, si A $\sqsupset$ C, ostendetur D $\sqsupset$ F. d *schol.* 13. 5.
3. *Hyp.* Si A = C. quoniam F. E :: C. B :: e 10. 5.
f A. B :: D. E *g* erit D = F. Q.E.D. f 7. 5.
g 11. 5. & 9. 5.

PROP. XXI.



Si sint tres magnitudines A, B, C; *& aliæ* D, E, F *ipsis æquales numero, quæ bina & in eadem ratione sumantur, fueritq; perturbata earum proportio,* (A.B :: E F. *atque* B.C :: D. E;) *ex æquo autem prima* A *quam tertia* C *major fuerit; erit & quarta* D *quam sexta* F *major. Quod si prima fuerit tertiæ æqualis, erit & quarta æqualis sextæ: sin illa minor, hæc quoque minor erit.*

1. *Hyp.* A $\sqsubset$ C. Quoniam *a* D. E :: B. C, a *hyp.*
invertendo erit E. D :: C. B. atqui $\frac{C}{B}$ *b* $\sqsupset \frac{A}{B}$. b 8. 5.
c ergo

e schol.13. 5. *c* ergo $\frac{E}{D} \sqsupset \frac{A}{B}$, hoc est $\frac{E}{F}$. *d* ergo D $\sqsubset$ F Q.E.D.

d 10. 5. 2. *Hyp.* Similiter, si A $\sqsupset$ C, erit D $\sqsupset$ F.

3. *Hyp.* Si A = C, quoniam E. D *e* :: C. B ::

e 7. 5. *e* A.B :: *f* E. F. *g* erit D = F. Q.E.D.

f hyp.

g 9. 5.

PROP. XXII.

Si sint quotcunque magnitudines A, B, C; *& aliæ ipsis æquales numero* D, E, F, *quæ binæ & in eadem ratione sumantur* (A. B :: D. E. & B. C :: E. F;) *& ex æqualitate in eadem ratione erunt* (A C :: D. F.)

Accipe G, H ipsarum A, D; & I, K ipsarum B, E; item L, M ipsarum E, F æquemultiplices.

a hyp. Quoniam *a* A. B :: D.

b 4. 5. E. *b* erit G. I :: H. K. eodem modo, erit I. L :: K. M. er-

c 20. 5. go si G $\sqsubset$, =, $\sqsupset$ L, *c* erit

d 6. def. 5. H. $\sqsubset$, =, $\sqsupset$ M; *d* ergo A. C :: D. F. Eodem pacto si ulterius C. N :: F. O, erit ex æquali A. N :: D. O. Q.E.D.

PROP.

PROP. XXIII.

Si sint tres magnitudines A, B, C, *aliæq;* D, E, F *ipsis æquales numero, quæ bina in eadem ratione sumantur; fuerit autem perturbata earum proportio.* (A. B :: E. F. & B. C :: D. E.) *etiam ex æqualitate in eadem ratione erunt* A. C :: D. F.

Sume G, H, I, ipsarum A, B, D; item K, L, M ipsarum C, E, F æquemultiplices. erit G. H *a* :: A. B *b* :: E. F *a* :: L. M. porro quia *b* B. C :: D. E. erit *c* H. I :: K. L. ergo G, H, K; & I, L, M habent se juxta 21. 5. quare si G ⊏, =, ⊐ K, erit similiter I ⊏, =, ⊐ M. *d* proinde A. C :: D. F. Q. E. D.

a 15. 5.
b *hyp.*
c 4. 5.
d 6. *def.* 5.

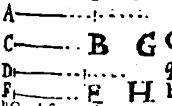
Eodem modo si plures fuerint magnitudinibus tribus, &c.

Coroll.

Ex* his sequitur, rationes ex iisdem rationibus compositas esse inter se easdem. item, earundem rationum easdē partes inter se easdem esse.

*22. & 23. 5. & 20. *def.* 5.

PROP. XXIV.



Si prima A B *ad secundam* C *eandem habuerit rationem quam tertia* DE *ad quartam* F, *habuerit autem & quinta* BG *ad secundam* C *eandem rationem, quam sexta* EH *ad quartam* F; *etiam composita prima cum quinta* (AG) *ad secundam* C *eandem habebit rationem, quam tertia cum sexta* (DH) *ad quartam* F.

Nam quia *a* A B. C :: D E. F. atque ex hyp. & inverse C. BG :: F. EH, erit *b* ex æquali AB. BG :: DE. EH. ergo componendo AG. BG :: DH. EH. *c* item BG. C :: EH. F. *b* ergo rursus ex æquo, AG. C :: DH. F. Q. E. D.

a *hyp.*
b 22. 5.
c *hyp.*

PROP.

PROP. XXV.

Si quatuor magnitudines proportionales fuerint (AB. CD :: E.F.) *maxima* AB *& minima* F *reliquis* CD *&* E *majores erunt.*

Fiant AG = E; & GH = F. Quoniam AB.CD *a* :: E. F *b* :: AG.CH.

a hyp. *c* erit AB. C D :: G B. HD. *d* sed

b 7. 5. AB $\sqsubset$ CD. *e* ergo GB $\sqsubset$ HD. atqui

c 19. 5. AG+F = E + CH. ergo AG + F +

d hyp. GB $\sqsubset$ E + CH + HD, hoc est AB +

e schol. 14. 5. F $\sqsubset$ E + CD. Q.E D.

Quæ sequuntur propositiones non sunt Euclidis; sed ex aliis desumptæ, ob frequentem earum usum Euclidæis subjungi solent.

PROP. XXVI.

A———C——— *Si prima ad secundam*
B——— D——— *habuerit majorem pro-*
E——— *portionem, quam tertia ad*
quartam; habebit convertendo, secunda ad primam minorem proportionem, quam quarta ad tertiam.

Sit $\frac{A}{B} \sqsubset \frac{C}{D}$. Dico $\frac{B}{A} \sqsupset \frac{D}{C}$. Nam concipe

a 13. 5. $\frac{C}{D} = \frac{E}{B}$ *a* ergo $\frac{A}{B} \sqsubset \frac{E}{B}$ *b* quare A $\sqsubset$ E. *c* ergo

b 10. 5. $\frac{B}{A} \sqsupset \frac{B}{E}$, *d* vel $\frac{D}{C}$. Q.E.D.

c 8. 5.

d cor. 4. 5.

PROP. XXVII.

A———C——— *Si prima ad secundam*
B——— D——— *habuerit majorem pro-*
E——— *portionem, quam tertia ad*
quartam; habebit quoque vicissim prima ad tertiam majorem proportionem, quam secunda ad quartam.

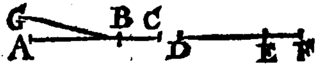
a 10. 5. Sit $\frac{A}{B} \sqsubset \frac{C}{D}$. Dico $\frac{A}{C} \sqsubset \frac{B}{D}$. Nam puta $\frac{E}{B} = \frac{C}{D}$

b 8. 5.

c 16. 5. *a* ergo A $\sqsubset$ E. *b* ergo $\frac{A}{C} \sqsubset \frac{E}{C}$, *c* vel $\frac{B}{D}$. Q.E.D.

PROP.

PROP. XXVIII.

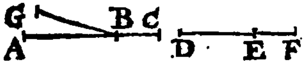


Si prima ad secundam habuerit majorem proportionem, quam tertia ad quartam; habebit quoque composita prima cum secunda ad secundam majorem proportionem, quam composita tertia cum quarta ad quartam.

Sit $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. Dico $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Nam cogita $\frac{GB}{BC} = \frac{DE}{EF}$. *a* ergo AB $\sqsubset$ GB. adde utrinque BC, *b* erit AC $\sqsubset$ GC. *c* ergo $\frac{AC}{BC} \sqsubset \frac{GC}{BC}$. *d* hoc est $\frac{DF}{EF}$. Q.E.D.

a 10. 5.
b 4. ax.
c 8. 5.
d 18. 5.

PROP. XXIX.

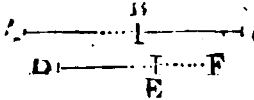


Si composita prima cum secunda ad secundam majorem habuerit proportionem, quam composita tertia cum quarta ad quartam; habebit quoque dividendo prima ad secundam majorem proportionem quam tertia ad quartam.

Sit $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Dico $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. Intellige $\frac{GC}{BC} = \frac{DF}{EF}$. *a* ergo A C $\sqsubset$ G C. aufer commune BC, *b* erit A B $\sqsubset$ GB. *c* ergo $\frac{AB}{BC} \sqsubset \frac{GB}{BC}$ *d* vel $\frac{DE}{EF}$ Q.E.D.

a 10. 5.
b 5. ax.
c 8. 5.
d 17. 5.

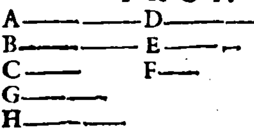
PROP. XXX.



Si composita prima cum secunda ad secundam habuerit majorem proportionem, quàm composita tertia cum quarta ad quartam; habebit, per conversionem rationis, prima cum secunda ad primam minorem rationem, quam tertia cum quarta ad tertiam.

Sit $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Dico $\frac{AC}{AB} \sqsupset \frac{DF}{DE}$. Nam quia
a hyp. $\frac{AC}{BC}$ *a* $\sqsubset \frac{DF}{EF}$ *b* erit dividendo $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. *c* conver-
b 29. 5. tendo igitur $\frac{BC}{AB} \sqsupset \frac{EF}{DE}$. *d* ergo componendo
c 26. 5. $\frac{AC}{AB} \sqsupset \frac{DF}{DE}$. Q.E.D.
d 28. 5.

PROP. XXXI.

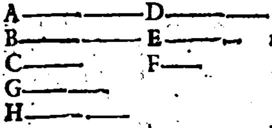


Si sint tres magnitudines A, B, C, *&* *aliæ ipsis æquales numero* D, E, F; *sitque major proportio primæ priorum ad secundam, quam primæ posteriorum ad secundam* ($\frac{A}{B} \sqsubset \frac{D}{E}$;) *item secundæ priorum ad tertiam major, quam secundæ posteriorum ad tertiam* ($\frac{B}{C} \sqsubset \frac{E}{F}$;) *erit quoque ex æqualitate major proportio primæ priorum ad tertiam, quam primæ posteriorum ad tertiam* ($\frac{A}{C} \sqsubset \frac{D}{F}$.)

a 10. 5. Concipe $\frac{G}{C} = \frac{E}{F}$. *a* ergo B $\sqsubset$ G *b* ergo $\frac{A}{G} \sqsubset \frac{A}{B}$:
b 8. 5. Rursus puta $\frac{H}{G} = \frac{D}{E}$, *c* ergo $\frac{H}{G} \sqsupset \frac{A}{B}$; *d* ergo for-
c 13. 5. tius $\frac{H}{G} \sqsupset \frac{A}{G}$. *d* quare A $\sqsubset$ H. *e* proinde $\frac{A}{C} \sqsubset \frac{H}{C}$,
d 10. 5. *f* vel $\frac{D}{F}$. Q.E.D.
e 8. 5.
f 22. 5.

PROP.

PROP. XXXII.

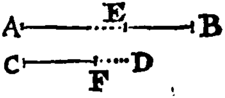


Si sint tres magnitudines A, B, C, *& aliæ ipsis numero æquales* D, E, F; *sitque major proportio primæ priorum ad secundam, quam secundæ posteriorum ad tertiam* ($\frac{A}{B} \sqsubset \frac{E}{F}$;) *item secundæ priorum ad tertiam major, quam primæ posteriorum ad secundam* ($\frac{B}{C} \sqsubset \frac{D}{E}$;) *erit quoque ex æqualitate major proportio primæ priorum ad tertiam, quam primæ posteriorum ad tertiam* ($\frac{A}{C} \sqsubset \frac{D}{F}$.)

Intellige $\frac{G}{C} = DE$. *a* ergo B $\sqsubset$ G. *b* ergo $\frac{A}{G} \sqsubset \frac{A}{B}$. Rursus concipe $\frac{H}{G} = \frac{E}{F}$. *c* ergo $\frac{H}{G} \sqsupset \frac{A}{G}$. æquare A $\sqsubset$ H. *b* proinde $\frac{A}{C} \sqsubset \frac{H}{C}$ *d* vel $\frac{D}{F}$. Q. E. D.

a 10. 5.
b 8. 5.
c *sch.* 13. 5.
d 13. 5.

PROP. XXXIII.



Si fuerit major proportio totius A B *ad totum* CD, *quam ablati* AE *ad ablatum* CF; *erit & reliqui* EB *ad reliquum* FD *major proportio, quam totius* AB *ad totum* CD.

Quoniam $\frac{AB}{CD}$ *a* $\sqsubset$ $\frac{AE}{CF}$ *b* erit permutando $\frac{AB}{AE} \sqsubset \frac{CD}{CF}$. *c* ergo per conversionem rationis $\frac{AB}{EB} \sqsupset \frac{CD}{FD}$. permutando igitur $\frac{AB}{CD} \sqsupset \frac{EB}{FD}$. Q. E. D.

a *hyp.*
b 27. 5.
c 30. 5.

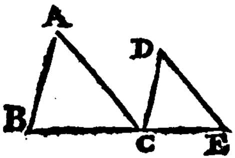
PROP. XXXIV.

Si ſint quotcunque magnitudines, & aliæ ipſis æquales numero, ſitque major proportio primæ priorum ad primam poſteriorum, quam ſecundæ ad ſecundam; & hæc major quam tertiæ ad tertiam, & ſic deinceps: habebunt omnes priores ſimul ad omnes poſteriores ſimul, majorem proportionem, quam omnes priores, relicta prima, ad omnes poſteriores, relicta quoque prima; minorem autem, quam prima priorum ad primam poſteriorum; majorem denique etiam, quam ultima priorum ad ultimam poſteriorum.

Horum demonſtratio eſt penes interpretes; quos adeat, qui eam deſiderat. nos omiſimus, brevitatis ſtudio; & quia illorum nullus uſus in his elementis.

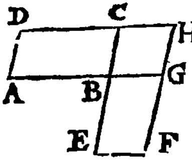
LIB. VI.

Definitiones.

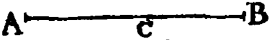


I. SImiles figuræ rectilineæ sunt (ABC, DCE,) quæ & angulos singulos singulis æquales habent; atque etiam latera, quæ circum angulos æquales, proportionalia.

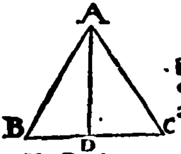
Ang. B = DCE; *&* A B. B C :: D C. CE *item ang.* A = D; *atque* B A. A C :: C D. DE *denique ang.* ACB = E. *atque* BC. CA :: CE. ED.



II. Reciprocæ autem sunt (BD, BF,) cum in utraque figura antecedentes, & consequentes rationum termini fuerint, (hoc est, AB. BG :: EB. BC.)



III. Secundum extremam & mediam rationem recta linea AB secta esse dicitur, cum ut tota AB ad majus segmentum AC, ita majus segmentum AC ad minus CB se habuerit. (AB. AC :: AC. CB.)



IV. Altitudo cujusque figuræ ABC est linea perpendicularis AD, à vertice A ad basim BC deducta.

V. Ratio ex rationibus componi dicitur, cum rationum quantitates inter se multiplicatæ, aliquam effecerint rationem.

Ut ratio A *ad* C, *componitur ex rationibus* A *ad* B, *&* B *ad* C. *nam* $\frac{A}{B} + \frac{B}{C}$ *a* $= \frac{A}{C}$ *b* $= \frac{AB}{BC}$.

a 20 *def.* 5.
b 15. 5.

PROP. I.

Triangula A B C, ACD, *& parallelogramma* BCAE, CDFA, *quorum eadem fuerit altitudo, ita se habent inter se, ut bases* BC, CD.

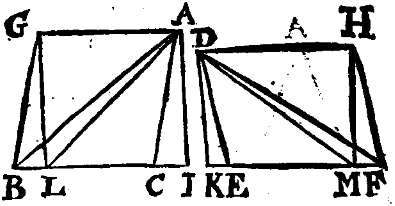
a 3. 1. *a* Accipe quotvis BG, HG, ipsi BC æquales; item DI = CD. & connecte AG, AH, AI.

b 38. 1. *b* Triangula ACB, ABG, AGH æquantur; *b* item triang. ACD = ADI. ergo triangulum ACH tam multiplex est trianguli ACB, quam basis HC basis BC. & æquemultiplex est triang. ACI trianguli ACD, ac basis CI basis CD.

c *sch.* 38. 1. Verum si HC ⊏, =, ⊐ CI, *c* erit similiter
d 6. *def.* 5. triang. AHC ⊏, =, ⊐ ACI. *d* ideoque BC,
e 41. 1. & 15. 5. CD :: triang. ABC. ACD :: *e* Pgr. CE. CF. Q. E. D.

Schol.

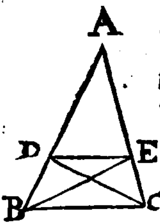
Schol.



Hinc, *triangula* ABC, DEF, *& parallelogramma* AGBC, DEFH, *quorum æquales sunt bases* BC, EF, *ita se habent ut altitudines* AI, DK.

a Sume IL = CB; & KM = EF; ac junge LA, LG, MD, MH. liquet esse triang. ABC. DEF :: *b* ALI. DKM :: *c* AI. DK :: *d* Pgr. AGBC. DEFH. Q.E.D. a 3. 1. b 7. 5. c 1. 6. d 41. 1. & 15. 5.

PROP. II.

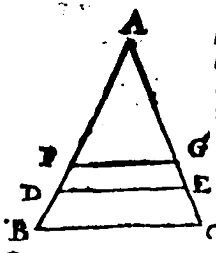


Si ad unum trianguli ABC *latus* BC, *parallela ducta fuerit recta quædam linea* DE, *hæc proportionaliter secabit ipsius trianguli latera* (AD. BD :: AE. EC.) *Et si trianguli latera proportionaliter secta fuerint* (AD. BD :: AE. EC) *quæ ad sectiones* D, E *adjuncta fuerit recta linea* DE, *erit ad reliquum ipsius trianguli latus* BC *parallela.* Ducantur CD, BE.

1. *Hyp.* Quia triang. DEB *a* = DEC; *b* erit triang. ADE. DBE :: ADE. ECD. atqui triang. ADE. DBE *c* :: AD. DB. & triang. ADE. DEC *c* :: AE. EC. *d* ergo AD. DB :: AE. EC. a 37. 1. b 7. 5. c 1. 6. d 11. 5.

2. *Hyp.* Quia AD. DB :: AE. EC. *e* hoc est triang. ADE. DBE :: ADE. ECD; *f* erit triang. DBE = ECD. *g* ergo DE, BC sunt parallelæ. Q. E. D. e 1. 6. f 9. 5. g 39. 1.

Schol.



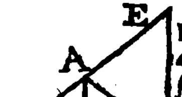
Imo si plures DE, FG; *ad unum latus* BC *parallela fuerint, erunt omnia laterum segmenta proportionalia.*

a 2. 6. Nam DF. FA *a* :: EG. GA; & componendo, invertendoque FA. DA :: GA. EA; *a* ac DA. DB :: EA. EC. ergo ex æquo DF. DB :: EG. EC. Q.E.D.

Coroll.

Si DF. DB :: EG. EC; *a* erunt BC, DE, FG parallelæ.

PROP. III.



Si trianguli BAC *angulus* BAC *bifariam sectus sit, secans autem angulum recta linea* AD *secuerit & basim, basis segmenta eandem habebunt rationem quam reliqua ipsius trianguli latera* (BD. DC :: AB. AC.) *Et si basis segmenta eandem habeant rationem quam reliqua ipsius trianguli latera* (BD. DC :: AB. AC) *recta linea* AD *quæ à vertice* A *ad sectionem* D *ducitur, bifariam secat trianguli ipsius angulum* BAC.

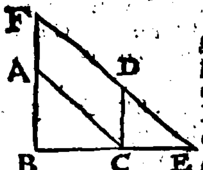
Produc BA; & fac AE = AC. & junge CE.

a 5. 1. b 32. 1. c *hyp.* d 27. 1. e 2. 6. 1. *Hyp.* Quoniam AE = AC, erit ang. ACE *a* = E *b* = $\frac{1}{2}$ BAC *c* = DAC. *d* ergo DA, CE parallelæ sunt. *e* quare BA. AE (AC) :: BD. DC. Q.E.D.

f 2. 6. g 29. 1. h 5. 1. k 1. ax. 2. *Hyp.* Quoniam BA. AC. (AE) :: BD. DC, *f* erunt DA, CE parallelæ: *g* ergo ang. BAD = E; & ang. DAC *g* = ACE *h* = E. *k* erg. ang. BAD = DAC, bisectus igitur est ang. BAC Q.E.D.

PROP.

PROP. IV.



Æquiangulorum triangulorum ABC, DCE *proportionalia sunt latera, quæ circum æquales angulos* B, DCE (AB. BC :: DC. CE, &c.) *& homologa sunt latera* AB, DC, &c. *quæ æqualibus angulis* ACB, E, &c. *subtenduntur.*

Statue latus BC in directum lateri CE, & produc BA, ac ED donec *a* occurrant. a 32. 1. &

Quonjam ang. B *b*=ECD. *c* sunt BF, CD 13. *ax.* parallelæ. Item quia ang. BCA *b*=CED, *c* sunt b *hyp.* CA, EF parallelæ. Figura igitur CAFD est c 28. 1. parallelogramma. *d* ergo AF=CD; *d* & AC =FD. Liquet igitur AB. AF (CD) *e* :: BC. d 34. 1. CE. *f* permutando igitur AB. BC :: CD. CE. e 2. 6. *e* item BC. CE :: FD. (AC) DE. *f* ergo per- f 16. 5. mutando BC. AC :: CE. DE. quare etiam *g* ex æquo AB. AC :: CD. DE. ergo, &c. g 22. 5.

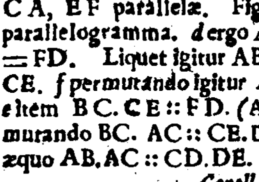
Coroll.

Hinc AB. DC :: BC. CE :: AC. DE.

Schol.

Hinc si in triangulo FBE ducatur uni lateri FE parallela AC; erit triangulum ABC simile toti FBE.

PROP. V.

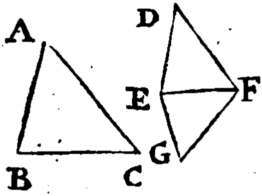


Si duo triangula ABC, DEF *latera proportionalia habeant* (AB. BC :: DE. EF. & AC. BC :: DF. EF. item AB. AC :: DE. DF) *æquiangula erunt triangula, & æquales habebunt eos angulos, sub quibus homologa latera subtenduntur.*

Ad latus EF *a* fac ang. FEG=B; *a* & ang. a 23. 1.

b 32. 1. EFG=C, *b* quare etiam ang. G=A. ergo
c 4. 6. GE. EF *c* :: AB. BC :: *d* DE. EF. *e* ergo
d *hyp.* GE=DE. Item GF. FE *c* :: AC. CB *d* ::
e 11. 5. DF. FE. *e* ergo GF=DF. Triangula igitur
& 9. 5. DEF, GEF sibi mutuo æquilatera sunt. *f* ergo
f 8. 1. ang. D=G=A. *f* & ang. FED=FEG=B.
g 32. 1. *g* proinde & ang. DFE=C. ergo, &c.

PROP. VI.

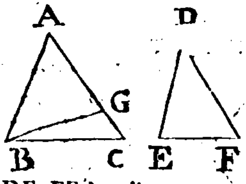


Si duo triangula ABC, DEF *unum angulum* B *uni angulo* DEF *æqualem, & circum æquales angulos* B, DEF *latera proportionalia habuerint* (AB. BC :: DE. EF;) *æquiangula erunt triangula* ABC, DEF; *æqualesque habebunt angulos, sub quibus homologa latera subtenduntur.*

Ad latus EF fac ang. FEG=B, & ang. EFG
a 32. 1. =C. *a* unde & ang. G=A. ergo GE. EF *b* ::
b 4. 6. AB. BC *c* :: DE. EF. *d* ergo DE=GE. atqui
c *hyp.* ang. DEF *e* =B *f* =GEF. *g* ergo ang. D=G
d 9. 5. =A. *h* proinde etiam ang. EFD=C. Q.E.D.

e *hyp.* PROP. VII.

f *constr.*
g 4. 1.
h 32. 1.



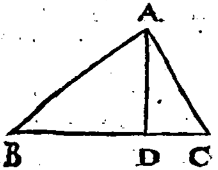
Si duo triangula ABC, DEF *unum angulum* A *uni angulo* D *æqualem, circa autem alios angulos* ABC, E *latera proportionalia habeant* (AB. BC :: DE. EF;) *reliquorum autem simul utrunque* C, F *aut minorem aut non minorem recto, æquiangula erunt triangula* ABC, DEF, *& æquales habebunt eos angulos circum quos proportionalia sunt latera.*

Nam si fieri potest, sit ang. ABC ⊏ E. fac
a *hyp.* igitur ang. ABG=E; ergo cum ang. A *a* =D,
b erit

b erit etiam ang. AGB=F. ergo AB. BG *c* :: DE. EF :: AB. BC. *e* ergo BG=BC. *f* ergo ang. BGC = BCG. *g* ergo ang. BGC. vel C minor est recto; *h* proinde ang. AGB, vel F recto major est. ergo anguli C & F non sunt ejusdem speciei, contra Hyp.

b 32. 1.
c 4. 6.
d *hyp.*
e 9. 5.
f 5. 1.
g *cor.* 17. 1
h *cor.* 13. 1

PROP. VIII.



Si in triangulo rectangulo ABC, *ab angulo recto* BAC *in basin* BC *perpendicularis* AD *ducta est; quæ ad perpendicularem triangula* ADB, ADC, *tum toti triangulo* ABC, *tum ipsa inter se, similia sunt.*

Nam ob angulos BAC, ADB *a* rectos, *b* ideoque æquales, & B communem, trigona BAC, ADB *c* similia sunt. Simili discursu, similia sunt triangula BAC, ADC. *d* proinde ADB, ADC similia erunt. Q.E.D.

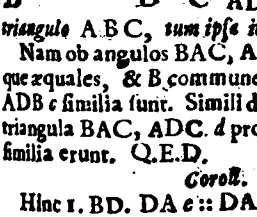
a *hyp.*
b 12. *ax.*
c 32. & 4. 6
d *Vid.* 21. 6.

Coroll.

Hinc 1. BD. DA *e* :: DA. DC.
2. BC. AC :: AC. DC, & CB. BA :: BA. BD.

e 1. *def.* 6.

PROP. IX.



A data recta linea AB *imperatam partem* $\frac{1}{3}$ (AG) *auferre.*

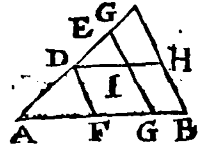
Ex A duc infinitam AC utcunq;, in qua *a* sume tres, AD, DE, EF æquales utcunque,

a 3. 1.

b 31. 1. cunque, junge F B, cui ex D b duc parallelam DG. Dico factum.

c 2. 6. Nam GB. AG c :: FD. AD. ergo d com-
d 18. 5. ponendo AB. AG :: AF. AD. ergo cum AD = $\frac{1}{3}$ AF, erit AG = $\frac{1}{3}$ AB. Q.E.F.

PROP. IX.



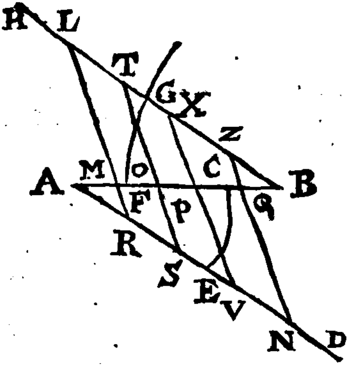
Datam rectam lineam AB *insectam similiter secare (in* F, G,*) ut data altera* AC, *secta fuerit (in* D, E.)

Extremitates sectæ & insectæ jungat recta BC.

a 31. 1. Huic ex punctis E, D a duc parallelas EG, DF rectæ secandæ occurrentes in G, & F. Dico factum.

a Ducatur enim DH parall. AB. Estque AD.
b 2. 6. DE b :: AF. FG, & DE. EC b :: DI. IH c :: FG.
c 34. 1. & GB. Q.E.F.
7. 5.

Scholium.



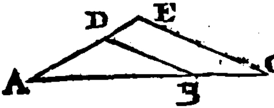
Hinc *discimus rectam datam* A B *in quotvis æquales partes* (puta 5.), *secare.* id quod facilius præstabitur sic;

Duc

Duc infinitam AD, eique parallelam BH etiam infinitam. Ex his cape partes æquales AR, RS, SU, UN; & BZ, ZX, XT, TL; in singulis una pauciores, quam desiderentur in AB; tum rectæ ducantur LR, TS, XV, ZN. hæ quinquisecabunt datam AB. a 31. 1. b constr. c 2. 6.

Nam RL, ST, UX, NZ *a* parallelæ sunt, ergo quum AR, RS, SU, UN *b* æquales sint, *c* erunt AM, MO, OP, PQ æquales. Similiter quia BZ = ZX, erit BQ = QP. ergo AB quinquisecta est. Q.E.F.

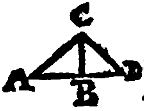
PROP. XI.



Datis duabus rectis lineis AB, AD, *tertiam proportionalem* DE *invenire.*

Junge BD, & ex AB protracta sume BC = AD. per C duc CE parall. BD. cui occurrat AD producta in E. Erit DE expetita. a 2. 6.

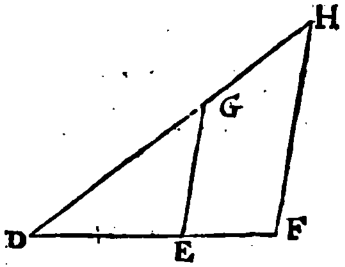
Nam AB. *a* BC (AD) :: AD. DE. Q.E.F. c 1cor. 8. 6.



Vel sic, fac ang. ABC rectum, & ang. ACD etiam rectum. *b* erit AB. BC :: BC. BD.

PROP.

PROP. XII.

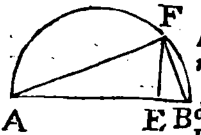


Tribus datis rectis lineis DE, EF, DG, *quartam proportionalem* GH *invenire.*

Connectatur EG. per F, duc FH parall. EG, cui occurrat DG producta ad H. liquet esse DE. EF a :: DG. GH. Q.E.F.

a 2. 6.

PROP. XIII.



Duabus datis rectis lineis AB, EB, *mediam proportionalem* EF *adinvenire.*

Super tota AB diametro describe semicirculum AFB. Ex E erige perpendicularem EF occurrentem peripheriæ in F. Dico AE. EF :: EF. EB. Ducantur enim AF, & FB. Ex trianguli a rectanguli AFB recto angulo deducta est FE basi perpendicularis; b ergo AE. FE :: FE. EB. Q.E.F.

a 31. 3.
b cor. 8. 6.

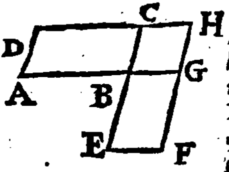
Vel (in eadem figura) sint AB, BF duæ datæ, b liquet est: AB. BF :: BF. BE.

Coroll.

Coroll.

Hinc, linea recta, quæ in circulo à quovis puncto diametri, ipsi diametro perpendicularis ducitur ad circumferentiam usque, media est proportionalis inter duo diametri segmenta.

PROP. XIV.



Æqualium, & unum ABC *uni* EBG *æqualem habentium angulum, parallelogrammorum* BD, BF, *reciproca sunt latera quæ circum æquales angulos.* (AB. BG :: EB. BC.) *Et quorum parallelogrammorum* BD, BF, *unum angulum* ABC *uni angulo* EBG *æqualem habentium, reciproca sunt latera quæ circum æquales angulos, illa sunt æqualia.*

Nam latera AB, BG circa æquales angulos faciant unam rectam: *a* quare EB, BC etiam in directum jacebunt. Producantur FG, DC, donec occurrant. a *sch.* 15. 1.

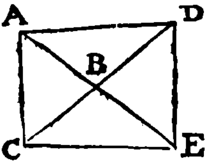
1. *Hyp.* AB, BG *b* :: BD. BH *c* :: BF. BH *d* :: BE. BC. *e* ergo, &c. b 1. 6. c 7. 5.

2. *Hyp.* BD. BH *f* :: AB. BG *g* :: BE. BC *h* :: BF. BH, *k* ergo Pgr. BD = BF. Q. E. D. d 1. 6. e 11. 5.

f 1. 6.
g *hyp.*
h 1. 6.
k 11. & 9. 5

PROP.

PROP. XV.



Æqualium, & unum ABC, *uni* DBE *æqualem habentium angulum triangulorum* ABC, DBE, *reciproca sunt latera, quæ circum æquales angulos* (AB. BE :: DB. BC:) *Et quorum triangulorum* ABC, DBE, *unum angulum* ABC *uni* DBE *æqualem habentium reciproca sunt latera, quæ circum æquales angulos* (AB. BE :: DB. BC.) *illa sunt æqualia.*

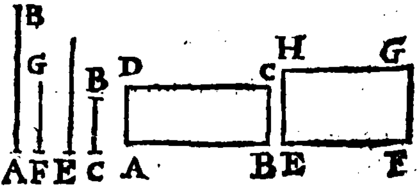
Latera CB, BD circa æquales angulos, statuantur sibi in directum; *a* ergo ABE est recta linea. ducatur CE.

a *sch.* 15. 1.
b 1. 6.
c 7. 5.
d 1. 6.
e 11. 5.
f 1. 6.
g *hyp.*
h 1. 6.
k 11. & 9. 5.

1. *Hyp.* AB. BE *b* :: triang. ABC. CBE *c* :: triang. DBE. CBE. *d* :: DB. BC. *e* ergo, &c.

2. *Hyp.* Triang. ABC. CBE *f* :: AB. BE *g* :: DB. BC *h* :: triang DBE. CBE. *k* ergo triang. ABC=DBE. Q.E.D.

PROP. XVI.



Si quatuor rectæ lineæ proportionales fuerint (AB. FG :: EF. CB,) *quod sub extremis* AB, CB *comprehenditur rectangulum* AC, *æquale est ei, quod sub mediis* EF, FG *comprehenditur, rectangulo* EG. *Et si sub extremis comprehensum rectangulum* AC *æquale fuerit ei, quod sub mediis comprehenditur, rectangulo* EG, *illæ quatuor rectæ lineæ proportionales erunt* (AB. FG :: EF. CB.)

1. *Hyp.*

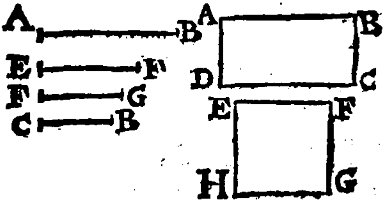
1. *Hyp.* Anguli B & F recti, ac *a* proinde pares a 12.*ax.* sunt; atque ex hyp. AB. FG :: EF. CB. *b* ergo b 14. 6. rectang. AC = EG. Q.E.D.

2. *Hyp. c* Rectang. AC = EG; atque ang. c *hyp.* B = F; *d* ergo AB. FG :: EF. CB. Q.E.D. d 14 6.

Coroll.

Hinc ad datam rectam lineam AB facile est datum rectangulum EG applicare, *e* faciendo e 12. 6. AB. EF :: FG. BC.

PROP. XVII.



Si tres rectæ lineæ sint proportionales (AB. EF :: EF. CB,) *quod sub extremis* AB, CB *comprehenditur rectangulum* AC, *æquale est ei, quod à media* EF *describitur, quadrato* EG. *Et si sub extremis* AB, CB *comprehensum rectangulum* AC, *æquale sit ei, quod à media* EF *describitur, quadrato* EG, *illæ tres rectæ lineæ proportionales erunt* (AB. EF :: EF. CB.)

Accipe FG = EF.

1. *Hyp.* AB. EF *a* :: EF (FG) CB. ergo a *hyp.* Rectang. AC *b* = EG *c* = EFq. Q. E. D. b 16 6.

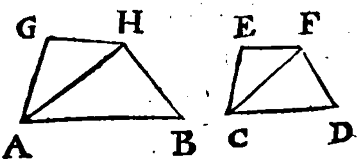
2. *Hyp.* Rectang. AC *d* = quadr. EG = c 29 *def.*1. EFq. *e* ergo AB. EF :: FG (EF.) BC. d *hyp.*

Coroll. e 16. 6.

Sit A in B = Cq. ergo A. C :: C. B.

PROP.

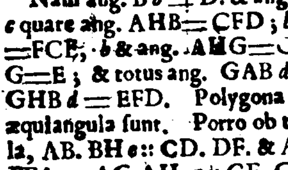
PROP. XVIII.



A data recta linea AB *dato rectilineo* CEFD *simile similiterque positum rectilineum* AGHB *describere.*

Datum rectilineum resolve in triangula. *a* fac
a 23. 1. ang. ABH = D; *a* & ang. BAH = DCE; *a* & ang. AHG = CFE; *a* & ang. HAG = FCE. Rectilineum AGHB est quæsitum.
b constr. Nam ang. B *b* = D. & ang. BAH *b* = DCF.
c 32. 1. *c* quare ang. AHB = CFD; *b* item ang. HAG = FCE; *b* & ang. AHG = CFE. *c* quare ang.
d 2. ax. G = E; & totus ang. GAB *d* = ECD; & totus GHB *d* = EFD. Polygona igitur sibi mutuo æquiangula sunt. Porro ob trigona æquiangula, AB. BH *e* :: CD. DF. & AG. GH. *e* :: CE.
e 4. 6. EF. item AG. AH. *e* :: CE. CF. & AH. AB *e* ::
f 22. 5. CF. CD. *f* unde ex æquo AG. AB :: CE. CD.
g 6. def. 6. eodem modo GH. HB :: EF. FD. *g* ergo polygona ABHG, CDFE similia similiterque posita existunt. Q.E.F.

PROP. XIX.



Similia triangula ABC, DEF *sunt in duplicata ratione laterum homologorum* BC EF.

a 11. 6. *a* Fiat BC. EF :: EF. BG. & ducatur AG.

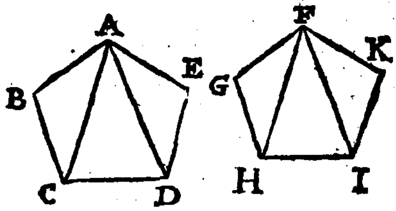
Quia

Quia AB.DB(*b* :: BC.EF)*c* :: EF.BG.& ang. b *cor*.4.6.
B = E; *d* erit triang. ABG = DEF. verum c *constr.*
triang. ABC. ABG *e* :: BC. BG; & *f* $\frac{BC}{BG}$ d 15.6.
= $\frac{BC}{EF}$ bis; ergo triang. $\frac{ABC}{ABG}$ hoc est $\frac{ABC}{DEF}$ *g* = e 1.6.
$\frac{BC}{EF}$ bis. Q.E.D. f 10.*def*.5.
g 11.5.

Coroll.

Hinc, si tres lineæ BC, EF, BG proportionales fuerint; ut est prima ad tertiam, ita est triangulum super primam BC descriptum ad triangulum super secundam EF simile similiterque descriptum. vel ita est triangulum super secundam EF descriptum ad triangulum super tertiam simile similiterque descriptum.

PROP. XX.



Similia polygona ABCDE, FGHIK *in similia triangula* ABC, FGH; *&* ACD, FHI, *&* ADE, FIK *dividuntur, & numero æqualia, & homologa totis.* (ABC. FGH :: ABCDE. FGHIK :: ACD. FHI :: ADE. FIK.) *Et polygona* ABCDE, FGHIK *duplicatam habent tam inter se rationem, quam latus homologum* BC *ad homologum latus* GH.

 1. Nam

a hyp. 1. Nam ang. B *a* = G ; & AB. BC *a* :: FG. GH. *b* ergo triangula ABC, FGH æquiangula sunt. eodem modo, triangula AED, FKI assimilantur. cum igitur ang. BCA *b* = GHF ; & ang. ADE *b* = FIK ; totique anguli BCD, GHI ; atque toti CDE, HIK *c* pares sint, *d* remanent ang. ACD = FHI ; & ang. ADC = FIH ; *e* unde etiam ang. CAD = HFI. ergo triangula ACD, FHI similia sunt. ergo, &c.

b 6. 6.
c hyp.
d 3. ax.
e 32. 1.

2. Quoniam igitur triangula BCA, GHF similia sunt, *f* erit $\frac{BCA}{GHF} = \frac{BC}{GH}$ bis. ob eandem causam $\frac{CAD}{HFI} = \frac{CD}{HI}$ bis. denique triang. $\frac{DEA}{IKF} = \frac{DE}{IK}$ bis. quare cum BC. GH *g* :: CD. HI *g* :: DE. IK, *h* erit triang. BCA. GHF :: CAD. HFI :: DEA. IKF :: *k* polyg. ABCDE. FGHIK :: $\frac{BC}{GH}$ bis.

f 19. 6.
g hyp. & 16. 5.
h cor. 23. 5.
k 12. 5.

Coroll.

I. Hinc, si fuerint tres lineæ rectæ proportionales ; ut est prima ad tertiam, ita erit polygonum super primam descriptum ad polygonum super secundam simile similiterque descriptum. vel ita erit polygonum super secundam descriptum ad polygonum super tertiam simile similiterque descriptum.

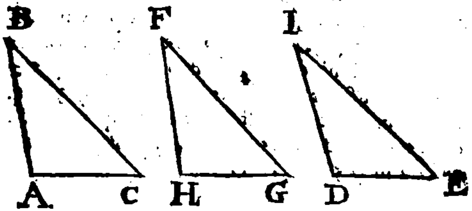
Unde elicitur methodus, *figuram quamvis rectilineam augendi vel minuendi in ratione data. Ut si velis pentagoni, cujus latus* CD, *aliud facere quintuplum, inter* AB, *&* 5 AB *inveni mediam proportionalem. Super hac * construe pentagonum simile dato ; hoc erit quintuplum dati.*

* 18. 6.

II. Hinc etiam, si figurarum similium homologa latera nota fuerint, etiam proportio figurarum innotescet ; nempe inveniendo tertiam proportionalem.

PROP.

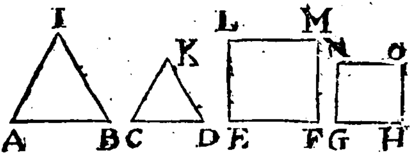
PROP. XXI.



Quæ (ABC, DIE) *eidem rectilineo* HFG *sunt similia, & inter se sunt similia.*

Nam ang. A $\overset{a}{=}$ H $\overset{a}{=}$ D. & ang. C $\overset{a}{=}$ G $\overset{a}{=}$ E; & ang. B $\overset{a}{=}$ F $\overset{a}{=}$ I. *a* item AB. AC :: HF. HG :: DI. DE. *a* & AC. CB :: HG. GF :: DE. EI. & AB. BC :: HF. FG :: DI. IE. *a* ergo ABC, DIE similia sunt. Q.E.D. — a 1. *def.* 6.

PROP. XXII.



Si quatuor rectæ lineæ proportionales fuerint (AB. CD :: EF. GH) *& ab eis rectilinea similia similiterque descripta proportionalia erunt.* (ABI. CDK :: EM. GO.) *Et si à rectis lineis similia similiterque descripta rectilinea proportionalia fuerint* (ABI. CDK :: EM. GO.) *ipsæ etiam rectæ lineæ proportionales erunt.* (AB. CD :: EF. GH)

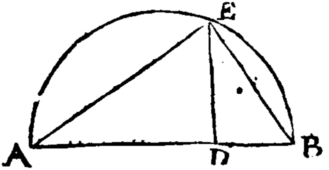
1. *Hyp.* $\frac{ABI}{CDK} \overset{a}{=} \frac{AB}{CD}$ bis $= \frac{EF}{GH}$ bis $\overset{a}{=} \frac{EM}{GO}$. *d* ergo ABI. CDK :: EM. GO. Q.E.D. — a 19. 6.

2. *Hyp.* $\frac{AB}{CD}$ bis $\overset{a}{=} \frac{ABI}{CDK} \overset{b}{=} \frac{EM}{GO} \overset{c}{=} \frac{EF}{GH}$ bis. *d* ergo AB. CD :: EF. GH. Q.E.D. — b *hyp.* c 20. 6. d *cor.* 23. 5.

Schol.

Hinc deducitur, & demonstratur ratio multiplicandi quantitates surdas. ex gr. Sit $\sqrt{5}$ multiplicandus in $\sqrt{3}$. dico provenire $\sqrt{15}$. Nam ex multiplicationis definitione debet esse, 1. $\sqrt{3}$ :: $\sqrt{5}$. product. ergo per hanc, q. 1. q. $\sqrt{3}$ :: q. $\sqrt{5}$. q. product. hoc est. 1. 3 :: 5. q. product. ergo q. product est 15. quare $\sqrt{15}$. est productus ex $\sqrt{3}$ in $\sqrt{5}$. Q. E. D.

THEOR.



Petr. Herig. *Si recta linea* A B *secta sit utcunque in* D, *rectangulum sub partibus* A D, DB *contentum, est medium proportionale inter earum quadrata. Item rectangulum contentum sub tota* A B, *& una parte* AD, vel DB, *est medium proportionale inter quadratum totius* A B, *& quadratum dictæ partis* A D, vel D B.

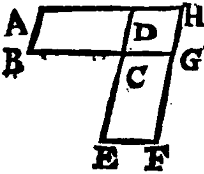
Super diametrum A B describe semicirculum: ex D erige normalem DE occurrentem peripheriæ in E, junge AE, BE.

a cor. 8. 6. Liquet esse A D. D E *a* :: D E. D B. *b* ergo
b 22. 6. A Dq. D Eq :: D Eq. D Bq. *c* hoc est, ADq.
c 17. 6. ADB :: ADB. DBq. Q. E. D.
d cor. 8. 6. Porro, B A. A E *d* :: A E. A D. *e* ergo BAq.
e 22. 6. AEq :: AEq. ADq. *f* hoc est B Aq. B A D ::
f 17. 6. B A D. A Dq. Eodem modo A Bq. A B D :: ABD. BDq. Q. E. D.

a 1. 6. Vel sic; sit Z=A+E. liquet esse Aq. AE :: *a* A. E :: *a* AE. Eq. item Zq. Z A :: *a* Z. A. :: *a* ZA. Aq. & Zq. ZE :: *a* Z, E :: ZE. Eq.

PROP.

PROP. XXIII.



Æquiangula parallelogramma AC,CF *inter se rationem habens eam quæ ex lateribus componitur.* ($\frac{AC}{CF} = \frac{BC}{CG} + \frac{DC}{CE}$.

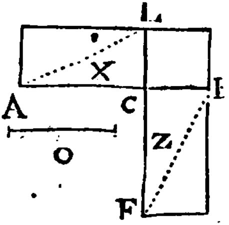
Latera circa æquales angulos C *a* sibi in directum statuantur; & compleatur parallelogrammum CH. a *sch.*15.1.

Ratio $\frac{AC}{CF}$ *b* $= \frac{AC}{CH} + \frac{CH}{CF}$ *c* $= \frac{BC}{CG} + \frac{DC}{CE}$. b 20.*def*.1. c 1. 6.

Q. E. D.

Coroll.

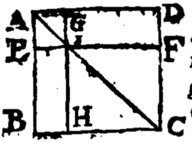
Hinc & ex 34. 1. patet primo, *Triangula, quæ unum angulum* (ad C) *æqualem habent, rationem habere ex rationibus rectarum,* AC ad CB, & LC ad CF, *æqualem angulum continentium.* *And Tacq.* 15. 5.



Patet secundo, *Rectangula ac * proinde & parallelogramma quæcunque rationem inter se habere compositam ex rationibus basis ad basim, & altitudinis ad altitudinem.* Neque aliter de triangulis ratiocinaberis. * 35. 1.

Patet tertio, *Quomodo triangulorum ac parallelogrammorum proportio exhiberi possit.* Sunto parallelogramma X & Z; quorum bases A C, CB; altitudines vero CL, CF. Fiat CL. CF :: CB. O. * erit X, Z :: AC. O. * 14. 6. & 1, 6.

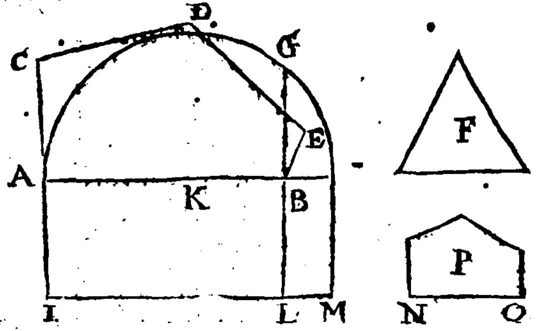
PROP. XXIV.



In omni parallelogrammo ABCD, quæ circa diametrum AC sunt parallelogramma BG, HF, & toti & inter se sunt similia.

Nam parallelogramma EG, HF habent singula unum angulum cum toto communem. a ergo toti & sibi mutuo æqui-
a 29. 1. angula sunt. a Item tam triangula ABC, AEI, IHC, quam triangula ADC, AGI, IFC sunt
b 4. 6. inter se æquiangula. b ergo AE. EI :: AB. BC, b atque AE. AI :: AB. AC; b & AI. AG :: AC.
c 22. 5. AD. c ex æquali igitur, AE. AG :: AB. AD.
d 1. def. 6. d ergo Pgr. EG, BD similia sunt. eodem modo HF, BD similia sunt. ergo, &c.

PROP. XXV.



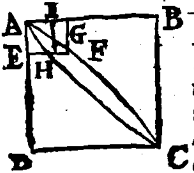
Dato rectilineo ABEDC simile similiterque positum P, idemque alteri dato F æquale, constituere.
a 45. 1. a Fac rectang. AL = ABEDC. b item super
b 44. 1. BL fac triang. BM = F. Inter AB, BH c in-
c 13. 6. veni mediam proportionalem NO, super NO

d fac

d fac polygonum P simile dato ABEDC. Erit hoc æquale dato F. d 18. 6. e cor. 20. 6.

Nam ABEDC (AL.) P :: *e* AB. BH *f* :: AL. BM. ergo P *g* = BM *h* = F. Q. E. F. f 1. 6. g 14. 5. h constr.

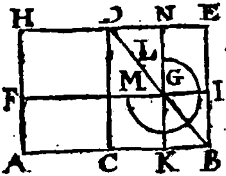
PROP. XXVI.



Si à parallelogrammo ABCD *parallelogrammum* AGFE *ablatum sit, & simile toti, & similiter positum, communem cum eo habens angulum* EAG, *hoc circa eandem cum toto diametrum* AC *consistet.*

Si negas AC esse communem diametrum; esto diameter AHC secans EF in H. & ducatur HI parall. AE. Parallelogramma EI, DB *a* similia sunt. *b* ergo AE. EH :: AD. DC *c* :: AE. EF. *d* proinde EH = EF. *f* Q. E. A. a 24. 6. b 1. def. 6. c hyp. d 9. 5. f 9. ax.

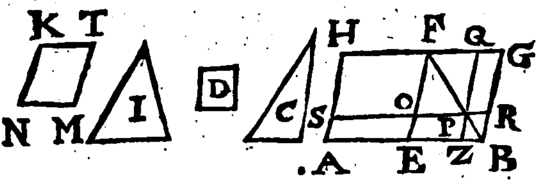
PROP. XXVII.



Omnium parallelogrammorum AD, AG *secundum eandem rectam lineam* AB *applicatorum, deficientiumque figuris parallelogrammis* CE, KI *similibus, similiterque positis, ei* AD, *quod à dimidia describitur, maximum est* AD, *quod ad dimidium est applicatum, simile exsistens defectui* KI.

Nam quia GE *a* = GC, addito communi KI, *b* erit KE = CI *c* = AM. adde commune CG, *d* erit AG = Gnom. MBL. sed Gnom. MBL *e* ⊐ CE (AD.) ergo AG ⊐ AD. Q. E. D. a 43. 1. b 2. ax. c 36. 1. d 2. ax. e 9. ax.

PROP. XXVIII.



Ad datam rectam lineam AB, *dato rectilineo* C *æquale parallelogrammum* AP *applicare deficiens figura parallelogramma* ZR, *quæ similis sit alteri parallelogrammo dato* D. * *Oportet autem datum rectilineum* C, *cui æquale* AP *applicandum est, non majus esse eo* AF, *quod ad dimidiam applicatur, similibus existentibus defectibus, & ejus* AF *quod ad dimidiam applicatur, & ejus* D, *cui simile deesse debet.*

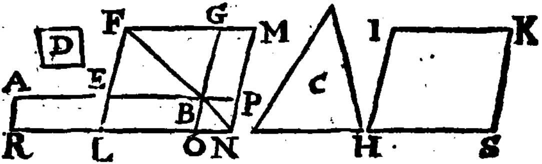
* 27. 6.

a 18. 6. Biseca AB in E. Super EB *a* fac Pgr. EG
b *sch*.45.1. simile dato D. *b* sitque EG = C+I. *c* fac Pgr.
c 25. 6. NT=I, & simile dato D, vel EG, duc diametrum FB. fac FO=KN; & FQ=KT. Per O, & Q duc parallelas SR, QZ. parallelogrammum AP est id quod quæritur.

Nam parallelogramma D, EG, OQ, NT,
d *conft.* & 24. 6. ZR *d* sunt similia inter se. Et Pgr. EG *e* = NT
e *conftr.* + C *e* = OQ + C; *f* quare C = Gnom.
f 3. *ax.* OBQ *g* = AO + PG *h* = AO + EP = AP.
g 2. *ax.* Q. E. F.
h 43. 1.

PROP.

PROP. XXIX.

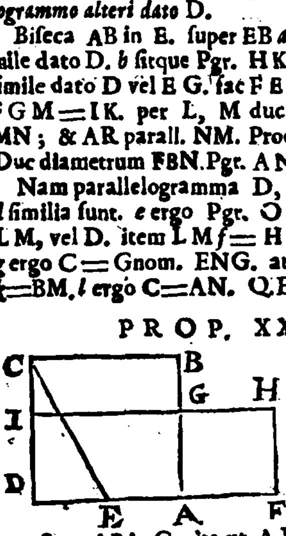


Ad datam rectam lineam AB, *dato rectilineo* C *aequale parallelogrammum* AN *applicare, excedens figura parallelogramma* OP, *quae similis sit parallelogrammo alteri dato* D.

Biseca AB in E. super EB *a* fac Pgr. EG simile dato D. *b* sitque Pgr. $HK = EG + C$, & simile dato D vel EG. fac FEL *c* $= IH$; *c* & $FGM = IK$. per L, M duc parallelas RN, MN; & AR parall. NM. Produc ABP, GBO. Duc diametrum FBN. Pgr. AN est quaesitum. a 18. 6. b 25. 6. c 3. 1.

Nam parallelogramma D, HK, LM, EG *d* similia sunt. *e* ergo Pgr. OP simile est Pgro LM, vel D. item LM *f* $=$ HK *f* $= EG + C$. *g* ergo $C =$ Gnom. ENG. atqui AL *h* $=$ LB *k* $=$ BM, *l* ergo $C = AN$. Q.E.F. d *constr.* e 24. 6. f *constr.* g 3. *ax.* h 36. 1. k 43. 1. l 2. & 1. *ax.*

PROP. XXX.

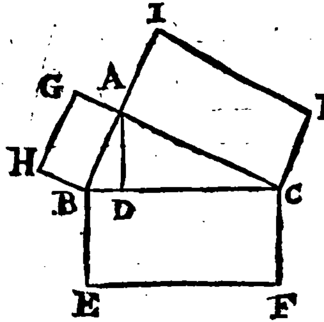


Propositam rectam lineam terminatam AB, *extrema ac media ratione secare.* (AB. AG :: AG. GB.)

a Seca AB in G, ita ut $AB \times BG = AGq$. a 11. 2.
b ergo BA. AG :: AG. GB. Q.E.F. b 17. 6.

PROP.

PROP. XXXI.



In rectangulis triangulis BAG, *figura quævis* BF *à latere* BC *rectum angulum* BAC *subtendente, descripta, æqualis est figuris* BG, AL, *quæ priori illi* BF *similes, & similiter positæ à lateribus* BA, AC *rectum angulum continentibus describuntur.*

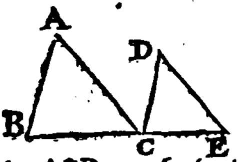
Ab angulo recto BAC demitte perpendicularem AD. Quoniam DC. CA :: *a* CA. CB;
a cor. 8.6. *b* erit AL. BF :: DC. CB. Item ob DB. BA ::
b cor. 20.6. *a* BA. BC, *b* erit BG. BF :: DB. BC. *c* ergo
c 24. 5. AL + BG. BF :: DC + DB (BC.) BC. ergo
d sch. 14. 5. AL+BG=BF. Q.E.D.

Coroll.

Ex hac propositione, addi possunt, & subtrahi figuræ quævis similes, eadem methodo, qua quadrata adduntur & subtrahuntur, in schol. 47. 1.

PROP.

PROP. XXXII.

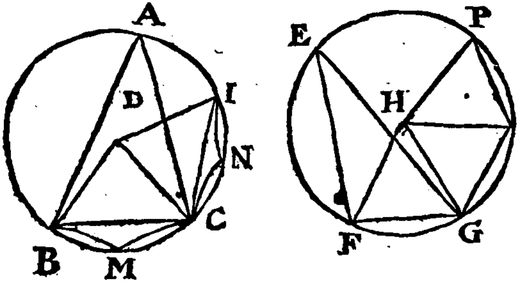


Si duo triangula ABC, DCE, *quæ duo latera duobus lateribus proportionalia habeant* (AB.AC :: DC.DE,) *secundum unum angulum* ACD *composita fuerint, ita ut homologa eorum latera sint etiam parallela* (AB ad DC, & AC ad DE) *tum reliqua illorum triangulorum latera* BC, CE *in rectam lineam collocata reperientur.*

Nam ang. A *a* = ACD *a* = D; & AB. AC *b* :: DC. DE. *c* ergo ang. B = DCE. ergo ang. B+A *d* = ACE. sed ang. B+A+ACB *e* = 2 Rect. *f* ergo ang. ACE+ACB = 2 Rect. *g* ergo BCE est recta linea. Q.E.D.

a 29. 1.
b *hyp.*
c 6. 6.
d 2. *ax.*
e 32. 1.
f 1. *ax.*
g 14. 1.

PROP. XXXIII.



In æqualibus circulis DBCA, HFGP, *anguli* BDC, FHG *eandem habent rationem cum peripheriis* BC, FG, *quibus insistunt; sive ad centra* (ut BDC, FHG,) *sive ad peripherias* A, E *constituti insistant: insuper vero & sectores* BDC, FHG, *quippe qui ad centra consistant.*

Duc

Duc rectas BC, FG. Accommoda CI = CB; & GL = FG = LP; & junge DI, HL, HP.

a 28. 3. Arcus BC a = CI, a item arcus FG, GL, LP

b 27. 3. æquantur. b ergo ang. BDC = CDI b & ang. FHG = GHL = LHP. Ergo arcus BI tam multiplex est arcûs BC, quam ang. BDI anguli BDC. pariterque æquemultiplex est arcus FP arcûs FG, atque ang. FHP anguli FHG. Ve-

c 27. 3. rum si arcus BI ⊏, =, ⊐ FP, c erit similiter

d 6. def. 5. ang. BDI ⊏, =, ⊐ FHP. ergo arc. BC. FG d ::

e 15. 5. ang. BDC. FHG e :: $\frac{BDC}{2}$. $\frac{FHG}{2}$ f :: A. E,

f 20. 3. Q. E. D.

Rursus ang. BMC g = CNI; h atque idcirco

g 27. 3. segm. BCM = CIN. k item triang. BDC =

h 24. 3. CDI. l ergo sector BDCM = CDIN. Simili

k 4. 1. ratione sectores FHG, GHL, LHP æquantur.

l 2. ax. Quum igitur prout arcus BI ⊏, =, ⊐ FGP, ita

m 6. def. 5. similiter sector BDI ⊏, =, ⊐ FHP. m erit sect. BDC. FHG :: arc. BC. FG. Q. E. D.

Coroll.

15. 5. Hinc 1. *Ut sector ad sectorem, sic angulus ad angulum.*

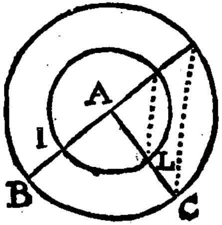
2. *Ang.* BDC *in centro est ad 4 rectos, ut arcus* BC *cui insistit ad totam circumferentiam.*

Nam ut ang. BDC ad rectum, sic arcus BC ad quadrantem. ergo BDC est ad 4 rectos, ut arcus BC ad 4 quadrantes, id est ad totam circumferentiam. item ang. A. 2 Rect :: arc. BC. periph.

Hinc 3. *Inæqualium circulorum arcus* IL, BC, *qui æquales subtendunt angulos, sive ad centra*, ut IAL & BAC, *sive ad peripheriam, sunt similes.*

Nam IL. periph. :: ang. IAL, (BAC.) 4 Rect. item arc. BC. periph :: ang. BAC. 4 Rect.

4 Rect. ergo IL. periph :: BC. periph. proinde arcus IL, & BC sunt similes. Unde



4. *Duæ semidiametri* AB, AC *à concentricis peripheriis arcus auferunt similes* IL, BC.

LIB.

LIB. VII.

Definitiones.

I. UNitas est, secundum quam unumquodque eorum quæ sunt, unum dicitur.

II. Numerus autem est, ex unitatibus composita multitudo.

III. Pars est numerus numeri, minor majoris, quum minor metitur majorem.

Omnis pars ab eo numero nomen sibi sumit, per quem ipsa numerum, cujus est pars, metitur; ut 4 dicitur tertia pars numeri 12, quia metitur 12 per 3.

IV. Partes autem, cum non metitur.

Partes quæcunque nomen accipiunt à duobus illis numeris, per quos maxima communis duorum numerorum mensura utrumque eorum metitur. ut 10 dicitur $\frac{2}{3}$ numeri 15, eo quod maxima communis mensura, nempe 5, metitur 10 per 2, & 15. per 3.

V. Multiplex vero major minoris, cum majorem metitur minor.

VI. Par numerus est, qui bifariam dividitur.

VII. Impar vero numerus, qui bifariam non dividitur; vel, qui unitate differt à pari.

VIII. Pariter par numerus est, quem par numerus metitur per numerum parem.

IX. Pariter autem impar est, quem par numerus metitur per numerum imparem.

X. Impariter vero impar numerus est, quem impar numerus metitur per numerum imparem.

XI. Primus numerus est, quem sola unitas metitur.

XII. Primi inter se numeri sunt, quos sola unitas, communis mensura, metitur.

XIII.

XIII. Compositus numerus est, quem numerus quispiam metitur.

XIV. Compositi autem inter se numeri sunt, quos numerus aliquis, communis mensura, metitur.

In hac definitione & praecedenti unitas non est numerus.

XV. Numerus numerum multiplicare dicitur, cum toties compositus fuerit is qui multiplicatur, quot sunt in ipso multiplicante unitates, & procreatus fuerit aliquis.

Hinc, in omni multiplicatione unitas est ad multiplicatorem ut multiplicatus ad productum.

Nota, quod saepe cum multiplicandi sunt quivis numeri, puta A *in* B, *literarum conjunctio productum denotat. Sic* AB = A *in* B. *item* CDE = C *in* D *in* E.

XVI. Cum autem duo numeri sese multiplicantes aliquem fecerint, qui factus erit, planus appellabitur; Qui vero numeri sese mutuo multiplicarint, latera illius dicentur. *Sic* 2 (C) in 3 (D) = 6 = CD *est numerus planus.*

XVII. Cum vero tres numeri mutuo sese multiplicantes fecerint aliquem, qui procreatus erit, solidus appellabitur; Qui autem numeri mutuo sese multiplicarint, latera illius dicentur. *Sic,* 2 (C) *in* 3 (D) *in* 5 (E) = 30 = CDE *est numerus solidus.*

XVIII. Quadratus numerus est, qui aequaliter aequalis, vel qui sub duobus aequalibus numeris continetur. *Sit* A *latus quadrati; quadratus sic notatur,* AA, *vel* Aq.

XIX. Cubus vero, qui aequaliter aequalis aequaliter, vel qui sub tribus aequalibus numeris continetur. *Sit* A *latus cubi; cubus notatur sic,* AAA, *vel* Ac.

In hac definitione, & tribus praecedentibus, unitas est numerus.

XX. Nu-

XX. Numeri proportionales sunt, cum primus secundi, & tertius quarti æquemultiplex est, vel eadem pars, vel deniq; cum pars primi secundum, & eadem pars tertii æque metitur quartum, vel vice versa. A. B :: C. D. hoc est, 3. 9 :: 5. 15.

XXI. Similes plani, & solidi numeri sunt, qui proportionalia habent latera.

Latera nempe non quælibet, sed quædam.

XXII. Perfectus numerus est, qui suis ipsius partibus est æqualis.

Ut 6. & 28. *Numerus vero qui suis ipsius partibus minor est, abundans appellatur; qui vero major, diminutus. ut 12 est abundans, 15 est diminutus.*

XXIII. Numerus numerum metiri dicitur per illum numerum, quem multiplicans, vel à quo multiplicatus, illum producit.

In divisione, unitas est ad quotientem, ut dividens ad divisum. Nota, quod numerus alteri lineola interjecta subscriptus divisionem denotat. Sic $\frac{A}{B}$ = A divis. per B. *item* $\frac{CA}{B}$ = C *in* A divis. per B.

Termini sive radices proportionis dicuntur duo numeri, quibus in eadem proportione minores sumi nequeunt.

Postulata.

1. POstuletur, cuilibet numero quotlibet sumi posse æquales, vel multiplices.

2. Quolibet numero sumi posse majorem.

3. Additio, subtractio, multiplicatio, divisio, extractionesque radicum, seu laterum, numerorum quadratorum, & cuborum concedantur etiam, tanquam possibilia.

Axio-

Axiomata.

1. Quicquid convenit uni æqualium numerorum, convenit & reliquis æqualibus numeris.

2. Partes eidem parti, vel iisdem partibus, eædem, sunt quoque inter se eædem.

3. Qui numeri æqualium numerorum, vel ejusdem, eædem partes fuerint, æquales inter se sunt.

4. Quorum idem numerus, vel æquales, eædem partes fuerint, æquales inter se sunt.

5. Unitas omnem numerum per unitates, quæ in ipso sunt, hoc est, per ipsummet numerum metitur.

6. Omnis numerus seipsum metitur per unitatem.

7. Si numerus numerum multiplicans, aliquem produxerit, metietur multiplicans productum per multiplicatum, multiplicatus autem eundem per multiplicantem.

Hinc nullus numerus primus planus est aut solidus, quadratus, vel cubus.

8. Si numerus numerum metiatur, & ille per quem metitur, eundem metietur per eas, quæ in metiente sunt, unitates, hoc est, per ipsum numerum metientem.

9. Si numerus numerum metiens, multiplicet eum, per quem metitur, vel ab eo multiplicetur, illum quem metitur, producit.

10. Numerus quotcunque numeros metiens, compositum quoque ex ipsis metitur.

11. Numerus quemcunque numerum metiens, metitur quoque omnem numerum quem ille metitur.

12. Numerus metiens totum & ablatum, metitur & reliquum.

PROP. I.

A....E...G.B 8 5 3
C...F..D $\frac{3}{5}$ $\frac{2}{3}$ $\frac{2}{1}$
H---

Si duobus numeris inaequalibus propositis (AB, CD) *detrahatur semper minor* CD *de majore* AB (& reliquus EB de CD &c.) *alterna quadam detractione, neque reliquus unquam praecedentem metiatur, quoad assumpta sit unitas* GB; *qui principio propositi sunt numeri* AB, CD *primi inter se erunt.*

Si negas, habeant AB, CD communem mensuram, numerum H. Ergo H metiens C D,
a 11.ax.7. *a* etiam AE metitur; proinde & reliquum EB;
b 12.ax.7. *a* ergo & C F, atque *b* idcirco reliquum F D; *a* quare & ipsum EG. sed totum EB metiebatur; *b* ergo & reliquum GB metitur, numerus uni-
c 9.ax.1. tatem. *c* Q. E. A.

PROP. II.

9 6
A........E......B 15 9 6
6 3
C......F...D $\frac{0}{6}\frac{6}{3}\frac{3}{0}$
G---

Duobus numeris datis AB, CD *non primis inter se, maximam eorum communem mensuram* FD *reperire.*

Detrahe minorem numerum C D ex majori
a 6. ax. 7. AB, quoties potes. Si nihil relinquitur, *a* patet ipsum C D esse maximam communem mensuram. Si relinquitur aliquid EB, deme hunc ex CD; & reliquum F D ex EB, & sic deinceps, donec aliquis F D praecedentem EB metiatur.
b 1. 7. (nam *b* hoc fiet antequam ad unitatem perveniatur.) Erit FD maxima communis mensura.
c constr. Nam F D *c* metitur E B, *d* ideoque & CF;
d 11.ax.7. *e* proinde & totum CD, *d* ergo ipsum AE; atque
e 12.ax.7. idcirco totum AB metitur. Liquet igitur F D communem esse mensuram. Si maximam esse negas,

negas, sit major quæpiam G. ergo G metiens CD, *d* metitur AE, *e* & reliquum EB, *d* ipsumque CF. *e* proinde & reliquum FD, *g* major minorem. *h* Q. E. A. *g suppos.* *h 9. ax. 1.*

Coroll.

Hinc, numerus metiens duos numeros, metitur quoq; maximam eorum communem mensuram.

PROP. III.

A 12
B 8
D.... 4
C 6
E .. 2
F --

Tribus numeris datis A, B, C *non primis inter se, maximam eorum communem mensuram* E *reperire.*

Inveni D maximam communem mensuram duorum A, B. Si D metitur tertium C, liquet D maximam esse trium communem mensuram. Si D non metitur C, erunt saltem D, & C compositi inter se, ex coroll. præcedentis. Sit igitur ipsorum D, & C maxima communis mensura E. erit E is quem quæris.

Nam E *a* metitur C, & D; *a* ac D ipsos A, & B metitur; *b* ergo E metitur singulos A, B, C; nec major aliquis (F) eos metietur; nam si hoc affirmas, *c* ergo F metiens A, & B, eorum maximam communem mensuram D metitur. Eodem modo, F metiens D, & C, *c* eorum maximam communem mensuram E, *d* major minorem, metitur. *e* Q. E. A. *a constr.* b 11. *ax.* 7. *c cor.* 2. *d suppos.* e 9. *ax.* 1.

Coroll.

Hinc, numerus metiens tres numeros, maximam quoq; eorum communem mensuram metitur.

PROP. IV.

A...... 6 *Omnis numerus* A, *omnis*
B....... 7 *numeri* B, *minor majoris, aut*
B..................... 18 *pars est, aut partes.*
B......... 9.

Si A & B primi sint inter se, *a* erit A tot partes numeri B, quot sunt in A unitates. (ut $6 = \frac{6}{7}7$.) Sin A metiatur B, *b* liquet A esse partem ipsius B. (ut $6 = \frac{1}{3}18$.) denique si A & B aliter compositi inter se fuerint, *c* maxima communis mensura determinabit, quot partes A conficiat ipsius B; ut $6 = \frac{2}{3}9$.

a 4. def. 7.
b 3. def. 7.
c 4. def. 7.

PROP. V.

A...... 6 D.... 4

6 6 4 4
B...... G...... C 12. E.... H.... F 8

Si numerus A *numeri* BC *pars fuerit, & alter* D *alterius* EF *eadem pars; & simul uterque* (A+D) *utriusque simul* (BC+EF) *eadem pars erit, quæ unus* A *unius* BC.

Nam si BC in suas partes BG, GC ipsi A æquales; atque EF in suas partes FH, HF ipsi D æquales resolvantur; *a* erit numerus partium in BC æqualis numero partium in EF. Quum igitur A + D *b* = BG + EH = GC + HF, erit A + D toties in BC + EF, quoties A in BC. Q. E. D.

a hyp.
b const. & 2. ax. 1.

Vel sic brevius. Sit $a = \frac{x}{2}$ & $b = \frac{y}{2}$. quare $2a = x$ & $2b = y$. *c* Ergo $2a + 2b = x + y$. Ergo $a + b = \frac{x+y}{2}$.

PROP.

PROP. VI.

3 3 4 4
A ... G ... B 6 D .... H .... E 8
C .... 9 F 12

Si numerus AB *numeri* C *partes fuerit; & alter* DE *alterius* F *eædem partes; & simul uterq;* (AB+DE) *utriusq; simul*(C+F) *eadem partes erit, quæ unus* AB *unius* C.

Divide AB in suas partes AG, GB; & DE in suas DH, HE. Partium in utroque AB, DE æqualis est multitudo, ex hypoth. Quum igitur AG *a* sit eadem pars numeri C, quæ DH numeri F, *b* erit AG+DH eadem pars compositi C+F, quæ unus AG unius C. *b* Eodem modo GB+HE eadem pars est ejusdem C+F, quæ unus GB unius C; *c* ergo AB+DE eædem partes est ipsius C+F, quæ AB ipsius C. Q. E. D.

a *hyp.*
b 5. 7.
c 2. *ax.* 7.

Vel sic. Sit $a = \frac{2}{3}x$, & $b = \frac{2}{3}y$, & $x+y=g$. ob $3a = 2x$, & $3b = 2y$, est $3a+3b = 2x+2y = 2g$. ergo $a+b = \frac{2}{3}g = \frac{2}{3} : x+y$.

PROP. VII.

5 3
A ..... E ... B 8
6 10 6
G ...... C .... F ..... D 16

Si numerus A B *numeri* CD *pars fuerit, qualis ablatus* A E *ablati* CF; *& reliquus* E B *reliqui* FD *eadem pars erit, qualis totus* AB *totius* CD.

a Sit EB eadem pars numeri GC, quæ AB ipsius CD, vel AE ipsius CF. *b* ergo AE+EB eadem est pars ipsius CF+GC, quæ AE ipsius CF, vel AB ipsius CD. *c* ergo GF=CD. aufer communem CF, *d* manet GC=FD. *e* ergo EB eadem est pars reliqui FD (GC) quæ totus AB totius CB. Q.E.D.

a 1. *post.* 7.
b 5. 7.
c 6. *ax.* 1.
d 3. *ax.* 1.
e 2. *ax.* 7.

Vel sic. Sit $a+b=x$, & $c+d=y$; atque tam $x=3y$, quam $a=3c$; dico $b=3d$. Nam $3c+3d$ *f* $=3y=x$ *g* $=a+b$. aufer utrinque $3c$ *g* $=a$, & remanet $3d=b$. Q. E. D.

f 1. 2.
g *hyp.*

PROP. VIII.

6   3   4   2   2
A...... H .. G.... E .. L .. B 16
18   6
C.................... F...... D 24

Si numerus AB *numeri* CD *partes fuerit, quales ablatus* AB *ablati* CF, *& reliquus* EB *reliqui* BD *easdem partes erit, quales totus* AB *totius* CD.

Seca AB in AG, GB partes numeri CD; item AB in AH, HE partes numeri CF; & sume
a 3. ax. 1. GL = AH = HE; *a* quare HG = EL, & quia
b constr. *b* AG = GB, *c* etiam HG = LB. Cum igitur
c 3. ax. 1. totus AG eadem sit pars totius CD, quæ ablatus
d 7. 7. AH ablati CF; *d* erit reliquus HG, vel EL, eadem etiam pars reliqui FD, quæ AG ipsius CD. Eodem pacto, quia GB eadem pars est totius CD, quæ HE, vel GL, ipsius CF, *d* erit reliquus LB eadem pars reliqui FD, quæ GB totius CD; ergo EL + LB (EB) eædem est partes reliqui FD, quæ totus AB totius CD. Q. E. D.

Vel sic facilius. Sit $a + b = x$, & $c + d = y$.
e 9. ax. 7. Item tam $y = \frac{2}{3} x$, quam $c = \frac{2}{3} a$; vel *e* quod idem est, $3y = 2x$; & $3c = 2a$. Dico $d = \frac{2}{3} b$.
f 1. 2. Nam $3c + 3d$ *f* $= 3y = 2x$ *f* $= 2a + 2b$.
g 1. ax. 1. *g* ergo $3c + 3d = 2a + 2b$. aufer utrinque
h hyp. $3c$ *h* $= 2a$; & *k* manet $3d = 2b$. *l* ergo $d = \frac{2}{3} b$.
k 3. ax. 1. Q. E. D.
l 8. ax. 7.

PROP. IX.

A.... 4
4   4
B.... G.... C 8
5 D..... 5
E..... H ..... F 10

Si numerus A *numeri* BC *pars fuerit, & alter* D *alterius* EF *eadem pars; & vicissim quæ pars est, aut partes primus* A *tertii* D, *eadem pars erit, vel eadem partes, & secundus* BC *quarti* EF.

Poni-

Ponitur A⊐D. Sint igitur BG, GC, & EH, HF partes numerorum BC, EF, hæ ipsi A, illæ ipsi D pares. Utrinque multitudo partium æqualis ponitur. Liquet vero BG *a* eandem esse partem, aut easdem partes ipsius EH, quæ GC ipsius HF; *b* quare BC (BG+GC) ipsius EF (EH+HF) eadem pars est aut partes, quæ unus BG (A) unius EH (D.) Q. E. D. a 1. ax. 7. & 4. 7. b 5. vel 6. 7

Vel sic; sit $a = \frac{b}{3}$, & $c = \frac{d}{3}$, vel $3a = b$, & $3c = d$; *c* estque $\frac{c}{a} = (\frac{3c}{3a} =) \frac{b}{d}$. c 5. 15.

PROP. X.

A.. G .. B 4
C......6
5 5
D..... H..... E 10
F............... 15

Si numerus AB *numeri* C *partes fuerit, & alter* DE *alterius* F *eædem partes; & vicissim quæ partes est primus* AB *tertii* DE, *aut pars, eædem partes erit & secundus* C *quarti* F, *aut pars.*

Ponitur AB⊐DE, & C⊐F. Sint AG, GB, & DH, HE partes numerorum C, & F, tot nempe in AB, quot in DE. Constat AG ipsius C eandem esse partem, quæ DH ipsius F. *a* quare vicissim AG ipsius DH, pariterque GB ipsius HE, & *b* proinde conjunctim AB ipsius DE eadem pars erit, aut partes, quæ C ipsius F. Q. E. D. a 9. 7. b 5. & 9. 7.

Vel sic; sit $a = \frac{2}{3} b$, & $c = \frac{2}{3} d$. vel $3a = 2b$, & $3c = 2d$. Est $\frac{c}{a} = \frac{3c}{3a} = \frac{2d}{2b} = \frac{d}{b}$.

PROP. XI.

4 3
A.... E... B 7.
8 6
C........ F ...... D 14

Si fuerit, ut totus AB *ad totum* CD, *ita ablatus* AE *ad ablatum* CF; *& reliquus* EB *ad reliquum*

FD erit, ut totus AB *ad totum* CD.

a 4. 7. Sit primo AB ⊐ CD; *a* ergo AB vel pars
b 20. *def.* 7. est, vel partes numeri CD; *b* eademque pars est,
c 7. vel 8. 7 vel partes ipse AE ipsius CF; *c* ergo reliquus EB reliqui FD eadem pars est, aut partes, quæ totus AB totius CD. *b* ergo AB. CD :: EB. FD. Sin fuerit AB ⊏ CD; eodem modo erit juxta modo ostensa, CD. AB :: FD. EB. ergo invertendo, AB. CD :: EB. FD.

PROP. XII.

A, 4. C, 2. E, 3.
B, 8. D, 4. F, 6.

Si sint quotcunque numeri proportionales (A.B :: C.D :: E.F) *erit quemadmodum unus antecedentium* A *ad unum consequentium* B, *ita omnes antecedentes* (A+C+E) *ad omnes consequentes* (B+D+F.)

Sint primo, A, C, E minores quam B, D, F.
a 20. *def.* 7. ergo (propter easdem rationes) *a* erit A eadem
b 5. & 6. 7. pars aut partes ipsius B, quæ C ipsius D. *b* ergo conjunctim A+C eadem erit pars aut partes ipsius B+D, quæ unus A unius B. Similiter A+C+E eadem pars est, aut partes ipsius
c 20. *def.* 7. B+D+F, quæ A ipsius B. *c* ergo A+C+E. B+D+F :: A. B. Q. E. D. Sin A, C, E, ipsis B, D, F majores ponantur, idem ostendetur invertendo.

PROP. XIII.

A, 3. C, 4.
B, 9. D, 12.

Si quatuor numeri proportionales sint (A. B :: C. D.) *& vicissim proportionales erunt* (A. C :: B. D.)

Sint primo A & C ipsis B & D minores;
a 20. *def.* 7. atque A⊐C. Ob eandem proportionem, *a* erit A eadem pars, aut partes ipsius B, quæ C ipsius
b 9. & 10. 7 D. *b* ergo vicissim A ipsius C eadem pars est, aut partes, quæ B ipsius D. ergo A. C :: B. D. Sin

A⊏

A⊏C; atque A & C majores statuantur, quam B & D, eadem res erit, proportiones invertendo.

PROP. XIV.

A, 9.	D, 6.
B, 6.	E, 4.
C, 3.	F, 2.

Si sint quotcunque numeri A, B, C, *& alii totidem* D, E, F *illis æquales multitudine, qui bini sumantur, & in eadem ratione* (A.B :: D,E. & B.C :: E.F) *etiam ex æqualitate in eadem ratione erunt.* (A.C :: D F.)

Nam quia A.B :: D. E, *a* erit vicissim, A.D :: B. E :: *a* C. F. *a* ergo iterum permutando, A,C :: D. F. Q. E. D. a 13. 7.

PROP. XV.

1.	D.
B...3.	E......6.

Si unitas numerum quempiam B *metiatur; æque autem alter numerus* D *alterum quendam numerum* E *metiatur; & vicissim æque unitas tertium numerum* D *metietur, & secundus* B *quartum* E.

Nam quia 1 est eadem pars ipsius B, quæ D ipsius E, *a* erit vicissim 1 eadem pars ipsius D, quæ B ipsius E. Q. E. D. a 9. 7.

PROP. XVI.

B, 4.	A, 3.
A, 3.	B, 4.
AB, 12.	BA, 12.

Si duo numeri A, B *sese mutuo multiplicantes fecerint aliquos* AB, BA, *geniti ex ipsis* AB, BA *æquales inter se erunt.*

Nam quia AB=A in B, *a* erit 1 in A toties, quoties B in AB. *b* ergo vicissim 1 in B toties erit, quoties A in AB. atqui quoniam BA=B in A, *a* erit 1 in B toties, quoties A in BA. ergo quoties 1 in AB, toties 1 in BA; & *c* proinde AB=BA. Q. E. D. a 15. def. 7. b 15. 7. c 4. ax. 7.

PROP.

PROP. XVII.

A, 3.
B, 2. C, 4.
AB, 6. AC, 12.

Si numerus A *duos numeros* B, C *multiplicans fecerit aliquos* AB, AC; *geniti ex ipsis eandem rationem habebunt, quam multiplicati.* (AB. AC :: B. C.)

a 15.def.7. Nam quia AB = A in B, *a* erit 1 toties in A, quoties B in AB. *a* item quia AC = A in C, erit 1 toties in A, quoties C in AC. ergo quoties B in AB, toties C in AC, quare B. AB ::
b 20.def.7
c 13.7. C. AC. *c* ergo vicissim, B. C :: AB. AC. Q. E. D.

PROP. XVIII.

C, 5. C, 5.
A, 3. B, 9.
$\overline{AC}$, 15. $\overline{BC}$, 45.

Si duo numeri A, B, *numerum quempiam* C *multiplicantes fecerint aliquos* AC, BC; *geniti ex ipsis eandem rationem habebunt, quam multiplicantes.* (A. B :: AC. BC.)

a 16.7. Nam AC *a* = CA; & BC *a* = CB; sic idem C multiplicans A & B producit AC, & BC.
b 17.7. *b* ergo A. B :: AC. BC. Q. E. D.

Schol.

Ex his pendet modus vulgaris reducendi fractiones ($\frac{3}{5}$, $\frac{7}{9}$) ad eandem denominationem. Nam duc 9 tam in 3, quam in 5, proveniunt $\frac{27}{45} = \frac{3}{5}$. quoniam ex his, 3. 5 :: 27. 45. item duc 5 in 7, & 9, prodeunt $\frac{35}{45} = \frac{7}{9}$. quia 7. 9 :: 35. 45.

PROP. XIX.

A, 4. B, 6. C, 8. D, 12.
AD, 48. BC, 48.

Si quatuor numeri proportionales fuerint, (A. B :: C. D;) *qui ex primo & quarto fit numerus* AD, *aequalis est ei, qui ex secundo & tertio fit, numero* BC.

BC. *Et si qui ex primo & quarto fit numerus* AD *æqualis sit ei, qui ex secundo & tertio fit, numero* BC, *ipsi quatuor numeri proportionales erunt.* (A. B :: C. D.)

1. *Hyp.* Nam A C. A D *a* :: C. D *b* :: A. a 17. 7.
B *c* :: AC. BC. *d* ergo AD = BC. Q. E. D. b hyp.

2. *Hyp.* Quoniam *e* AD = BC, erit A C. c 18. 7.
AD *f* :: AC. BC. Sed AC. AD *g* :: C. D. & d 9. 5.
AC. BC *h* :: A.B. *k* ergo C. D :: A. B. Q.E.D. e hyp.
f 7. 5.
g 17. 7.
h 18. 7.
k 11. 5.

PROP XX.

A. B. C.
4. 6. 9.
AC, 36. BB, 36.
D, 6.

Si tres numeri proportionales fuerint (A. B :: B. C.) *qui sub extremis continetur* (A C) *æqualis est ei, qui à medio efficitur* (BB.) *Et si qui sub extremis continetur* (AC) *æqualis fuerit ei* (Bq) *qui sub medio, ipsi tres numeri proportionales erunt* ($\frac{A}{B}$ :: $\frac{B}{C}$.)

1. *Hyp.* Nam sume D = B. *a* ergo A B :: a 1. ax 7.
D (B.) C. *b* quare A C = B D, *a* vel B B. b 19. 7.
Q. E. D.

2. *Hyp.* Quia AC *c* = BD, *d* erit A. B :: D c hyp.
(B.) C. Q.E.D. d 19. 7.

PROP. XXI.

A ... G .. B 5. E .... 10.
C .. H . D 3. F ...... 6.

Numeri A B, C D *minimi omnium eandem cum eis rationem habentium* (E, F) *metiuntur æque numeros* E, F *eandem cum eis rationem habentes, major quidem* AB *majorem* E, *minor vero* CD *minorem* F.

Nam A B. C D *a* :: E. F. *b* ergo vicissim a hyp.
AB. E :: CD. F. *c* ergo AB eadem pars est, b 13. 7.
vel partes ipsius E, quæ CD ipsius F. Non par- c 20. def. 7.
tes; nam si ita, sint AG, GB partes numeri E;
& CH, HD partes numeri F. *e* ergo AG. E ::

CH.

d 13. 7. CH. F; & permutando, AG. CH *d* :: E. F *e* ::
e hyp. AB. CD. ergo AB, CD non sunt minimi in sua ratione, contra hypoth. ergo, &c.

PROP. XXII.

A, 4.	D, 12.
B, 3.	E, 8.
C, 2.	F, 6.

Si fuerint tres numeri A, B, C, & alii ipsis multitudine aequales D, E, F, qui bini sumantur, & in eadem ratione; fuerit autem perturbata eorum proportio (A. B :: E. F & B. C :: D. E;) *etiam ex aequalitate in eadem ratione erunt* (A. C :: D. F.)

a hyp. Nam quia A. B *a* :: E. F, *b* erit AF = BE; &
b 19. 7. quia B. C :: *a* D. E, *b* erit BE = CD. *c* ergo
c 1. ax. 1. AF = CD. *d* quare A. C :: D. F. Q. E. D.
d 19. 7.

PROP. XXIII.

A, 9. B, 4.
C ---- D --
E --

Primi inter se numeri A, B, minimi sunt omnium eandem cum eis rationem habentium.

Si fieri potest, sint C & D minores quam A & B, atque in eadem ratione.
a 21. 7. *a* ergo C metitur A aeque, ac D metitur B, puta per eundem numerum E: quoties igitur
b 23. def 7. 1 in E, *b* toties erit C in A. *c* quare vicissim quo-
c 15. 7. ties 1 in C, toties E in A. simili discursu quoties 1 in D, toties E in B. ergo E utrumque A & B metitur; qui proinde inter se primi non sunt, contra Hypoth.

PROP. XXIV.

A, 9 B, 4.
C ---
D --- E --

Numeri A, B, minimi omnium eandem cum eis rationem habentium, primi inter se sunt.

Si fieri potest, habeant A & B communem mensuram C; is metitur A
a 9. ax. 7. per D, & B per E; *a* ergo CD = A, & CE = B.

b quare

b quare A. B :: D. E. Sed D & E minores sunt quam A & B, utpote eorum partes. Ergo A & B non sunt minimi in sua ratione, contra hypoth. b 17. 7.

PROP. XXV.

A, 9. B, 4.
C, 3. D --

Si duo numeri A, B *primi inter se fuerint, qui unum eorum* A *metitur numerus* C, *ad reliquum* B *primus erit.*

Nam si affirmes aliquem D numeros B & C metiri, *a* ergo D metiens C, metitur A. ergo A & B non sunt primi inter se, contra Hypoth. a 11. ax. 7.

PROP. XXVI.

A, 5. C, 8.
B, 3.
AB, 15. E ----
F ----

Si duo numeri A, B *ad quempiam* C *primi fuerint, etiam ex illis genitus* AB *ad eundem* C *primus erit.*

Si fieri potest, sit ipsorum AB, & C communis mensura numerus E. sitque $\frac{AB}{E} = F$; *a* ergo AB = EF; *b* quare E.A :: B. F. Quia vero A primus est ad C quem E metitur, *c* erunt E & A primi inter se; *d* adeoque in sua proportione minimi, & *e* proinde æque metiuntur B, & F; nempe E ipsum B, & A ipsum F. Quum igitur E utrumque B, C metiatur, non erunt illi primi inter se, contra Hypoth.

a 9. ax. 7.
b 19. 7.
c 25. 7.
d 23. 7.
e 21. 7.

PROP. XXVII.

A, 4. B, 5.
Aq, 16.
D, 4.

Si duo numeri, A, B, *primi inter se fuerint, etiam ex uno eorum genitus* (Aq) *ad reliquum* B *primus erit.*

Sume D = A; ergo *a* singuli D, & A primi sunt ad B. *b* quare A D, vel Aq, ad B primus est. Q. E. D. a 1. ax. 7. b 26. 7.

PROP.

PROP. XXVIII.

A, 5.	C, 4.	*Si duo numeri* A, B *ad duos numeros* C, D, *uterque ad utrumque, primi fuerint, & qui ex eis gignuntur* AB, CD, *primi inter se erunt.*
B, 3.	D, 2.	
$\overline{AB}$, 15.	$\overline{CD}$, 8.	

a 26. 7. Nam quia A & B ad C primi sunt, *a* erit AB ad C primus. Eadem ratione erit AB ad D
b 26. 7. primus. *b* ergo AB ad CD primus est. Q. E. D.

PROP. XXIX.

A, 3.	B, 2.	*Si duo numeri* A, B *primi inter se fuerint, & multiplicans uterque seipsum fecerit aliquem* (Aq, & Bq,) *& geniti ex ipsis* (Aq, Bq) *primi inter se erunt; & si qui in principio* A, B *genitos ipsos* Aq, Bq *multiplicantes fecerint aliquos* (Ac, BC;) *& hi primi inter se erunt: & semper circa extremos hoc eveniet.*
Aq, 9.	Bq, 4.	
Ac, 27.	Bc, 8.	

a 27. 7. Nam quia A primus est ad B, *a* erit Aq ad B primus, & quia Aq primus ad B, *a* erit Aq ad Bq primus. Rursus quia tam A ad B & Bq,
b 28. 7. quam Aq ad eosdem B, & Bq primi sunt, *b* erit A × Aq, id est Ac, ad B × Bq, id est Bc, primus. Et sic porro de reliquis.

PROP. XXX.

8 5
A .... B ..... C 13. D ----

Si duo numeri AB, BC *primi inter se fuerint, etiam uterque simul* (AC) *ad quemlibet illorum* AB, BC *primus erit. Et si uterque simul* AC *ad unum aliquem illorum* AB *primus fuerit, etiam qui in principio numeri* AB, BC *primi inter se erunt.*

1. *Hyp.* Nam si AC, AB compositos velis,
a 12. ax. 7. sit D communis mensura. *a* Is metietur reliquum BC. ergo AB, BC non sunt primi inter se, contra Hypoth.

2. *Hyp.*

2. *Hyp.* Positis AC, AB inter se primis, vis D ipsorum AB, BC communem esse mensuram. *b* Is igitur totum AC metitur. quare AC, AB non sunt primi inter se, contra Hypoth. b 10. *ax.* 7.

Coroll.

Hinc numerus, qui ex duobus compositus, ad unum illorum primus est, ad reliquum quoque primus est.

PROP. XXXI.

A 5, B, 8. *Omnis primus numerus* A *ad omnem numerum* B, *quem non metitur, primus est.*

Nam si commmunis aliqua mensura metiatur utrumque A, B; *a* non erit A primus numerus, contra Hypoth. a 11. *def.* 7.

PROP. XXXII.

A, 4. D, 3.
B, 6. E, 8.
AB, 24.

Si duo numeri A, B, *se mutuo multiplicantes fecerint aliquem* AB; *genitum autem ex ipsis* AB *metiatur aliquis primus numerus* D; *is etiam unum eorum, qui à principio,* A, *vel* B *metietur.*

Pone numerum D non metiri A; sit vero $\frac{AB}{D}$ = E. *a* ergo AB = DE. *b* quare D. A :: B. E. *c* est vero D ad A primus. *d* ergo D & A minimi sunt in sua ratione; *e* proinde D metitur B, æque ac A metitur E. liquet igitur propositum.

a 9. *ax.* 7.
b 19. 7.
c *hyp.* & 31. 7.
d 23. 7.
e 21. 7.

PROP. XXXIII.

A, 12. B, 2. *Omnem compositum numerum* A, *aliquis primus numerus* B *metitur.*

Unus vel plures numeri *a* metiantur A, quorum minimus sit B, is primus erit, nam a 13. *def.* 7

a 13.*def*.7. nam si diceretur compositus, *a* eum minor aliquis
b 11.*ax*.7. metietur, *b* qui proinde ipsum A metietur; quare B non est minimus eorum, qui A metiuntur; contra Hypoth.

PROP. XXXIV.

Omnis numerus A, *aut primus est, aut* A, 9. *eum aliquis primus metitur.*

Nam A necessario vel primus est, vel compositus. Si primus, hoc est quod asseri-
a 33. 7. mus. Si compositus, *a* ergo eum aliquis primus metitur. Q. E. D.

PROP. XXXV.

A, 6. B, 4. C, 8.
D, 2. H -- I -- K ----
E, 3. F, 2. G, 4. L ---

Numeris datis quotcunque A, B, C *reperire minimos omnium* E, F, G *eandem rationem cum eis habentium.*

Si A, B, C primi sint inter se, ipsi in sua rati-
a 23. 7. one minimi *a* erunt. Si compositi sint, *b* esto
b 3. 7. eorum maxima communis mensura D, qui ipsos metiatur per E, F, G. Hi minimi erunt in ratione A, B, C.
c 9. *ax*. 7. Nam D ductus in E, F, G *c* producit ABC.
d 17 7. *d* ergo hi & illi in eadem sunt ratione. Jam puta
e 21. 7. alios H, I, K minimos esse in eadem; *e* qui propterea æque metientur A, B, C mempe per nu-
f 9. *ax*. 7. merum L. *f* ergo L in H, I, K ipsos A, B, C
g 1. *ax*. 1. procreabit. *g* ergo ED = A = HL. *h* unde E.
h 19 7. H :: L. D. Sed E *k* ⊏ H; *l* ergo L ⊏ D. ergo
k *suppos*. D non est maxima communis mensura ipsorum
l 20. *def* 7. A, B, C; contra Hypoth.

Coroll.

Hinc, maxima communis mensura quotlibet nume-

numerorum metitur ipsos per numeros, qui minimi sunt omnium eandem rationem cum ipsis habentium. Ex quo patet methodus vulgaris reducendi fractiones ad minimos terminos.

PROP. XXXVI.

Duobus numeris datis A, B, *reperire, quem illi minimum metiuntur, numerum.*

A, 5. B, 4.
AB, 20.
D------
E---- F----

1. *Cas.* Si A, & B primi sint inter se, est AB quæsitus; Nam liquet A & B metiri AB. Si fieri potest, metiantur A & B aliquem D ⊐ AB; puta per E, & F. *a* ergo AE = D = BF. *b* quare A. B :: F. E. Quia vero A, & B *c* primi sunt inter se, *d* adeoque in sua ratione minimi, *e* æque metientur A ipsum F, ac B ipsum E. Atqui B. E *f* :: AB. AE (D.) *g* ergo AB etiam metietur D, seipso minorem. Q. E. A.

a 9. *ax.* 7. & 1. *ax.* 1.
b 19. 7.
c *hyp.*
d 23. 7.
e 21. 7.
f 17. 7.
g 20. *def.* 7.

A, 6. B, 4. F ----
C, 3. D, 2. G.... H-..
AD, 12.

2. *Cas.* Sin A, & B inter se compositi fuerint, *h* reperiantur C, & D minimi in eadem ratione. *k* ergo A D = BC. Erit A D, vel B C quæsitus.

Nam *l* liquet B, & A ipsum A D, vel B C metiri. Puta A, & B metiri F ⊐ A D, nempe A per G, & B per H. *m* ergo A G = F = B H. *n* unde A. B :: H. G *o* :: C. D. *p* proinde æque metitur C ipsum H, ac D ipsum G. atqui D. G *q* :: A D. A G (F.) ergo A D *r* metitur F, major minorem. Q. E. A.

h 35. 7.
k 19. 7.
l 7. *ax.* 7.
m 9. *ax.* 7.
n 19. 7.
o *constr.*
p 21. 7.
q. 17. 7.
r 20. *def.* 7.

Coroll.

Hinc, si duo numeri multiplicent minimos eandem rationem habentes, major minorem, & minor majorem, producetur numerus minimus, quem illi metiuntur.

PROP. XXXVII.

A, 2. B, 3.
E, ... 6.
C ---- F --- D

Si duo numeri A, B *numerum quempiam* C D *metiantur; etiam minimus* E, *quem illi metiuntur, eundem* CD *metietur.*

Si negas, aufer E ex CD, quoties fieri potest,
a *hyp.* & relinquatur FD ⊐ E. quum igitur A & B *a* me-
b *constr.* tiantur E, *b* & E ipsum CF, *c* etiam A, & B me-
c 11. *ax.* 7. tiuntur CF; *a* metiuntur autem totum CD; *d*
d 12. *ax.* 7. ergo etiam reliquum FD metiuntur. ergo E non est minimus, quem A, & B metiuntur, contra hyp.

PROP. XXXVIII.

A, 3, B, 4, C, 6.
D, 12.

Tribus numeris datis A, B, C, *reperire minimum, quem illi metiuntur.*

a 36. 7. *a* Reperi D minimum, quem duo A, & B metiuntur; quem si tertius C metiatur, patet D esse quæsitum. Quod si C non metiatur D, sit E minimus, quem C, & D metiuntur. Erit E requisitus.

A, 2. B, 3. C, 4.
D, 6. E, 12.
F ---

Nam singulos A, B, C metiri E constat ex 11. ax. 7. Quod vero nullum alium F minorem metiantur,
b 37. 7. facile ostenditur. Nam si affirmas, *b* ergo D metitur F; *b* proinde E eundem F metitur, major minorem. Quod est absurdum.

Coroll.

Hinc, si tres numeri numerum quempiam metiantur; etiam minimus, quem illi metiuntur, eundem metietur.

PROP. XXXIX.

A, 12. *Si numerum* A *quispiam numerus*
B, 4, C, 3. B *metiatur, ille* A *quem* B *metitur, partem habebit* C, *à metiente* B *denominatam.*

Nam quia $\frac{A}{B}$ *a* = C, *b* erit A = BC. *c* ergo $\frac{A}{C}$ = B. Q. E. D. a *hyp.* b 9. *ax.* 7. c 7. *ax.* 7.

PROP. XL.

Si numerus A *partem habuerit*
A, 15. *quamlibet* B, *metietur illum nume-*
B, 3. C, 5. *rus* C, *à quo ipsa pars* B *denominatur.*

Nam quia BC *a* = A, *b* erit $\frac{A}{C}$ = B. Q.E.D. a *hyp.* & 9. *ax.* 7. b 7. *ax.* 7.

PROP. XLI.

$\frac{1}{2}$ G, 12. *Numerum reperire* G, *qui minimus cum sit, habeat datas partes,*
$\frac{1}{3}$ H ---
$\frac{1}{4}$ $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.

a Inveniatur G minimus, quem denominatores 2, 3, 4 metiuntur. *b* Liquet G habere partes, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. Si fieri potest, H ⊐ G habeat easdem partes; *c* ergo 2, 3, 4 metiuntur H, & proinde G non est minimus, quem 2, 3, 4 metiuntur, contra constr. a 38. 7. b 39. 7. c 40. 6.

LIB. VIII.

PROP. I.

A, 8. B, 12. C, 18. D, 27.
E - F -- G --- H ----

SI fuerint quotcunque numeri deinceps proportionales A,B,C, D ; *extremi vero ipsorum* A,D *primi inter se fuerint* ; *ipsi* A,B, C, D *minimi sunt omnium eandem cum eis rationem habentium.*

Nam, si fieri potest, sint alii totidem E,F,G,H
a 14. 7. minores in illa ratione. *a* ergo ex æquali A.D ::
b 23. 7. E. H. ergo A, & D primi numeri, *b* adeoque in
c 21. 7. sua ratione minimi, *c* æque metiuntur E, & H, seipsis minores. Q. E. A.

PROP II.

I.

A, 2. B, 3.
Aq, 4. AB, 6. Bq, 9.
Ac, 8. AqB, 12. ABq, 18. Bc, 27.

Numeros reperire deinceps proportionales minimos, quotcunque jusserit quispiam, in data ratione A ad B.

Sint A, & B minimi in data ratione. Erunt Aq, AB, Bq tres minimi deinceps in ratione A ad B.

a 17. 7. Nam AA. AB *a* :: A. B *a* :: AB. BB. item
b 24. 7. quia A, & B *b* primi sunt inter se, *c* erunt Aq, Bq
c 29. 7. inter se primi; *d* proinde Aq, AB, Bq sunt
d 1. 8. ∺ minimi in ratione A ad B.

Dico porro, Ac, AqB, ABq, Bc in ratione A ad B quatuor esse minimos. Nam AqA,
e 17. 7. AqB *e* :: A. B *e* :: ABA (AqB.) ABB. *e* atque A. B :: ABq. BBq. (Bc) Quum igitur Ac, &

Bc *f* inter se primi sint, *g* erunt Ac, AqB, ABq, Bc quatuor ∺ minimi in ratione A ad B. Eodem modo quotvis proportionales investigabis. Q. E. F. f 29. 7. g 1. 8.

Coroll.

1. Hinc, si tres numeri minimi sunt proportionales, extremi quadrati erunt; si quatuor, cubi.

2. Extremi quotcunque proportionales per hanc propos. inventi in data ratione minimi, inter se primi sunt.

3. Duo numeri, minimi in data ratione, metiuntur omnes medios quotcunque minimorum in eadem ratione; quia scilicet producuntur ex illorum multiplicatione in alios quosdam numeros.

4. Hinc etiam liquet ex constructione, series numerorum 1, A, Aq, Ac; 1, B, Bq, Bc; Ac, AqB, ABq, Bc, constare æquali multitudine numerorum; ac proinde extremos numeros quotcunque minimorum continue proportionalium, esse ultimos totidem continue proportionalium ab unitate. ut extremi Ac, Bc continue proportionalium Ac, AqB, ABq, Bc, sunt ultimi totidem proportionalium ab unitate 1, A, Aq, Ac; & 1, B, Bq, Bc.

5. 1, A, Aq, Ac; & B, BA, BAq; ac Bq, ABq sunt ∺ in ratione 1 ad A. item, B, Bq, Bc; & A, AB, ABq; ac Aq, AqB sunt ∺ in ratione 1 ad B.

PROP. III.

A, 8. B, 12. C, 18. D, 28.

Si sint quotcunque numeri A, B, C, D *deinceps proportionales, minimi omnium eandem cum eis rationem habentium; illorum extremi* A, D *sunt inter se primi.*

Nam

b 2. 8. Nam si *a* inveniantur totidem numeri minimi in ratione A ad B, illi non alii erunt, quam A, B, C, D; ergo juxta 2. coroll. præcedentis extremi A & D primi sunt inter se. Q. E. D.

PROP. IV.

A, 6. B, 5. C, 4. D, 3.
H, 4. F, 24. E, 20. G, 15.
I -- K -- L --

Rationibus datis quotcunque in minimis terminis (A ad B, & C ad D) *reperire numeros deinceps minimos in datis rationibus.*

a 36. 7. *a* Reperi E minimum, quem B, & C metiuntur, & B ipsum E *b* æque metiatur, ac A alterum
b 3. post. 7. F, puta per eundem H. *b* item C ipsum E, ac D alterum G æque metiantur: erunt F, E, G minimi in datis rationibus. Nam AH *c* = F; &
c 9. ax. 7. BH *c* = E. *d* ergo A. B :: AH. BH *e* :: F. E.
d 18. 7. Similiter C. D :: E. G. sunt igitur F, E, G
e 7. 5. deinceps proportionales in datis rationibus. Imò minimi sunt in iisdem: nam puta alios I, K, L minimos esse. *f* ergo A & B ipsos I & K, *f* pariterque C & D ipsos K & L æque metiuntur. ergo
f 21. 7. B, & C eundem K metiuntur. *g* Quare etiam
g 37. 7. E eundem K metitur, seipso minorem. Q. E. A.

A, 6. B, 5. C, 4. D, 3. E, 5. F, 7.
H, 24. G, 20. I, 15. K, 21.

Datis vero tribus rationibus A ad B, & C ad D, ac E ad F. reperi, ut prius, tres H, G, I minimos deinceps in rationibus A ad B, & C ad
h 3. post. 7. D. tunc si E numerum I metiatur, *h* sume alterum K, quem F æque metiatur; erunt quatuor H, G, I, K, deinceps minimi, in datis rationibus, quod non aliter probabimus, quam in priori parte.

A, 6. B, 5. C, 4. D, 3. E, 2. F, 7.
H, 24. G, 29. I, 15.
M, 48. L, 40. K, 30. N, 105.

Sin E non metiatur I, sit K minimus, quem E, & I metiuntur; & quoties I ipsum K, toties G ipsum L, & H ipsum M metiatur. quoties vero E ipsum K, toties F ipsum N metiatur. Erunt M, L, K, N minimi deinceps in datis rationibus; quod demonstrabimus, ut prius.

PROP. V.

Plani numeri C D, E F *rationem habent ex lateribus compositam.*

C, 4. E, 3.
D, 6. F, 16. ED, 18.
$\overline{CD}$, 24. $\overline{EF}$, 48.

$\left(\frac{CD}{EF} = \frac{C}{E} + \frac{D}{F}.\right)$

Nam quia CD. ED *a* :: C. E; *a* & ED. EF :: D. F. atque $\frac{CD}{EF}$ *b* $= \frac{CD}{ED} + \frac{ED}{EF}$, *c* erit ratio $\frac{CD}{EF} = \frac{C}{E} + \frac{D}{F}$. Q. E. D.

a 17. 7.
b 20 *def.* 5.
c 11. 5.

PROP. VI.

A, 16. B, 24. C, 36. D, 54. E, 81.
F, 4. G, 6. H, 9.

Si sint quotcunq; numeri deinceps proportionales A, B, C, D, E; *primus autem* A *secundum* B *non metiatur, neque alius quispiam ullum metietur.*

Quoniam A non metitur B, *a* neque quilibet proxime sequentem metietur, quia A. B :: B. C :: C. D, &c. *b* Accipe tres F, G, H minimos in ratione A ad B. quoniam igitur A non metitur B, *a* neque F metietur G. *c* ergo F non est unitas. *d* sed F, & H inter se primi sunt; ergo quum *e* sit ex æquo A. C :: F. H, & F non metiatur H, neque A ipsum C metietur; proinde nec B ipsum D, nec C ipsum E, &c. quia A. C *e* :: B. D *e* :: C. E, &c. Eodem modo

a 20. *def.* 6.
b 35. 7.
c 5. *ax.* 7.
d 3. 8.
e 14. 7.

sumptis quatuor vel quinque minimis in ratione A ad B, ostendetur A ipsos D, & E; ac B ipsos E, & F non metiri, &c. Quare nullus alium metietur. Q. E. D.

PROP. VII.

A, 3. B, 6. C, 12. D, 24. E, 48.

Si sint quotcunque numeri deinceps proportionales A, B, C, D, E; *primus autem* A *extremum* E *metiatur; & etiam metitur secundum* B.

a 6. 7. Si negas A metiri B, *a* ergo nec ipsum E metietur, contra Hypoth.

PROP. VIII.

A, 24. C, 36. D, 54. B, 81.
G, 8. H, 12. I, 18. K, 27.
E, 32. L, 48. M, 72. F, 108.

Si inter duos numeros A, B *medii continua proportione ceciderint numeri* C, D; *quot inter eos medii continua proportione cadunt numeri, tot & inter alios* E, F *eandem cum illis habentes rationem, medii continua proportione cadent.* (L, M.)

a 35. 7. *a* Sume G, H, I, K minimos ∺ in ratione
b 14. 7. A ad C; *b* erit ex æquali, G. K :: A. B *c* :: E. F.
c *hyp.* Atqui G, & K *d* primi sunt inter se; *e* quare G
d 3. 8. æque metitur E, ac K ipsum F. per eundem nu-
e 21. 7. merum metiatur H ipsum L, & I ipsum M.
f *constr.* *f* itaque E, L, M, F ita se habent ut G, H, I, K; hoc est ut A, B, C, D. Q. E. D.

PROP. IX.

1.
E, 2. F, 3.
G, 4. H, 6. I, 9.
A, 8. C, 12. D, 18. B, 27.

Si duo numeri A, B, *sint inter se primi, & inter eos medii continua proportione ceciderint numeri,* C, D; *quot inter eos medii continua*

tinua proportione ceciderint numeri, totidem (E,G; & F,I) *& inter utrumque eorum ac unitatem medii continua proportione cadent.*

Constat 1, E, G, A; & 1, F,I, B esse ∺; & totidem quot A,C, D, B, nimirum ex 4 coroll. 2. 8. Q.E.D.

PROP. X.

A, 8. I, 12. K, 18. B, 27.
B, 4. DF, 6. G, 9.
D, 2. F, 3.
1.

Si inter duos numeros A,B, *& unitatem continue proportionales ceciderint numeri* (E,D,& F,G,) *quot inter utrumque ipsorum, & unitatem deinceps medii continua proportione cadunt numeri, totidem & inter ipsos medii continua proportione cadent*, I, K.

Nam E, DF, G; & A, DqF (I,) DG (K,) B sunt ∺, per 2. 8. ergo, &c.

PROP. XI.

A, 2. B, 3.
Aq, 4. AB, 6. Bq, 9.

Duorum quadratorum numerorum Aq, Bq *unus medius proportionalis est numerus* AB. *& quadratum* Aq *ad quadratum* Bq, *duplicatam habet lateris* A *ad latus* B *rationem.*

a Liquet Aq, AB, Bq, esse ∺. *b* proinde etiam $\frac{Aq}{Bq} = \frac{A}{B}$ bis. Q.E.D. a 17. 7. b 10. *def.* 5.

PROP.

PROP. XII.

Ac, 27. AqB, 36. ABq, 48. Bc, 64.
A, 3. B, 4.
Aq, 9. AB, 12. Bq, 16.

Duorum cuborum numerorum Ac, Bc *duo medii proportionales sunt numeri* AqB, ABq. *Et cubus* Ac *ad cubum* Bc *triplicatam habet lateris* A *ad latus* B *rationem.*

a 2. 1. *a* Nam Ac, AqB, ABq, Bc sunt ∺ in ratione
b 10. def. 5. A ad B. *b* proinde $\frac{AC}{BC} = \frac{A}{B}$ ter. Q. E. D.

PROP. XIII.

A, 2. B, 4. C, 8.
Aq, 4. AB, 8. Bq, 16. BC, 32. Cq, 64.
Ac, 8. AqB, 16. ABq, 32. Bc, 64. BqC, 128.
BCq, 256. Cc, 512.

Si sint quotlibet numeri deinceps proportionales, A, B, C; *& multiplicans quisque seipsum faciat aliquos; qui ab illis producti fuerint* Aq, Bq, Cq *proportionales erunt: & si numeri primum positi* A, B, C *multiplicantes jam factos* Aq, Bq, Cq, *fecerint aliquos* Ac, Bc, Cc; *ipsi quoque proportionales erunt, & semper circa extremos hoc eveniet.*

a 2. 8. Nam Aq, AB, Bq, BC, Cq *a* sunt ∺ *b* ergo
b 14. 7. ex æquo Aq. Bq :: Bq, Cq. Q. E. D.

a Item Ac, AqB, ABq, Bc, BqC, BCq, Cc sunt ∺, *b* ergo iterum ex æquo, Ac. BC :: Bc. Cc. Q. E. D.

PROP. XIV.

Aq, 4. AB, 12. Bq, 36.
A, 2. B, 6.

Si quadratus numerus Aq *quadratum numerum* Bq *metiatur, & latus unius* (A) *metietur latus alterius* (B:) *& si unius quadrati latus* A *metietur latus alterius* B, *& quadratus* Aq *quadratum* Bq *metietur.*

a 2. & 11. 8 1. *Hyp.* Nam Aq. AB *a* :: AB. Bq; cum igitur ex hyp. Aq metiatur Bq; idem Aq secundum

cundum AB *b* metietur. atqui A q. AB :: A. B. b 7. 8.
c ergo etiam A metitur B. Q. E. D. c 20. *def*. 7.

2. *Hyp.* A metitur B. *c* ergo tam Aq ipsum AB, *c* quam AB ipsum Bq metitur; *d* & proinde Aq metitur Bq. Q. E. D. d 11. *ax*. 7.

PROP. XV.

A, 2. B, 6.
Ac, 8. AqB, 24. ABq, 72. Bc, 216.

Si cubus numerus Ac *cubum numerum* Bc *metiatur, & latus unius* (A) *metietur latus alterius* (B:) *Et si latus* A *unius cubi* Ac *latus* B *alterius* Bc *metiatur, & cubus* Ac *cubum* Bc *metietur.*

1. *Hyp.* Nam Ac, AqB, ABq, Bc *a* sunt ∺. a 2. & 12. 8.
ergo Ac, *b* metiens extremum Bc, *c* etiam secundum AqB metietur. atqui Ac. AqB :: A, B. b *hyp.* c 7. 8.
d ergo etiam A metietur B. Q. E. D.

2. *Hyp.* A metitur B; *d* ergo Ac metitur AqB, d 20. *def*. 7.
isque ABq, & hic Bc; *e* ergo Ac metietur Bc. e 11. *ax*. 7.
Q. E. D.

PROP. XVI.

A, 4. B, 9.
Aq, 16. Bq, 81.

Si quadratus numerus Aq *quadratum numerum* Bq *non metiatur, neque* A *latus unius alterius latus* B *metietur: & si* A *latus unius quadrati* Aq *non metiatur* B *latus alterius* Bq, *neque quadratus* Aq *quadratum* Bq *metietur.*

1. *Hyp.* Nam si affirmes A metiri B, *a* etiam Aq ipsum Bq metietur, contra hyp. a 14. 8.

2. *Hyp.* Vis Aq metiri Bq; *a* ergo A ipsum B metietur, contra hyp.

PROP.

PROP. XVII.

A, 2. B, 3. *Si cubus numerus* Ac *cubum numerum* Bc *non metiatur, neque* A *latus unius latus*
Ac, 8. Bc, 27.

B *alterius metietur. Et si latus* A *unius cubi* Ac *latus* B *alterius* Bc *non metiatur, neque cubus* Ac *cubum* Bc *metietur.*

a 15. 8. 1. *Hyp.* Dic A metiri B; *a* ergo Ac metietur Bc. contra Hypoth.

2. *Hyp.* Dic Ac metiri Bc; *a* ergo A ipsum B metietur. contra Hyp.

PROP. XVIII.

C, 6. D, 2. *Duorum similium planorum numerorum* CD, EF, *unus medius proportionalis est numerus* DE: *&* *planus* CD *ad planum* EF *duplicatam habet lateris* C *ad latus homologum* E *rationem.*

CD, 12.
E, 9. F, 3. DE, 18.
EF, 27.

*21. def. 7. Quoniam * ex hyp. C. D :: E. F; permu-
a 17. 7. tando erit C. E :: D F. atqui C. E *a* :: CD.
b 11. 5. DE; *a* & D. F :: DE. EF. *b* ergo CD. DE ::
DE. EF. Q. E. D.

c 20. def. 5. *c* Ergo ratio CD ad EF duplicata est rationis CD ad DE; hoc est rationis C ad E, vel D ad F.

Coroll.

Hinc perspicuum est, inter duos similes planos cadere unum medium proportionalem, in ratione laterum homologorum.

PROP.

PROP. XIX.

CDE, 30. DEF, 60 FGE, 120. FGH, 240.
CD, 6. DF, 12. FG, 24.
C, 2. D, 3. E, 5. F, 4. G, 6. H, 10.

Duorum similium solidorum CDE, FGH, *duo medii proportionales sunt numeri* DFE, FGE. *Et solidus* CDE *ad solidum* FGH *triplicatam rationem habet lateris homologi* C *ad latus homologum* F.

Quoniam ex * hyp. C. D :: F. G; & D. E :: G. H; erit *a* permutando C. F :: D. G *a* :: E. H. atqui CD. DF *b* :: C. F; & DF. FG *b* :: D. G. *c* quare CD. DF :: DF. FG :: E. H. *d* ergo CDE. DFE :: DFE. FGE :: E. H. :: FGE. FGH. ergo inter CDE, FGH cadunt duo medii proportionales, DFE, FGE. Q.E.D. *e* Liquet igitur rationem CDE ad FGH triplicatam esse rationis CDE ad DFE, vel C ad F. Q. E. D.

* 21. *def.* 7. a 13. 7. b 17. 7. c 11. 5. d 17. 7. e 10. *def.* 1.

Coroll.

Hinc, inter duos similes solidos cadunt duo medii proportionales, in ratione laterum homologorum.

PROP. XX.

A, 12. C, 18. B, 27.
D, 2. E, 3. F, 6. G, 9.

Si inter duos numeros A, B, unus medius proportionalis cadat numerus C. similes plani erunt illi numeri, A, B.

a Accipe D, & E minimos in ratione A ad C, vel C ad B. *b* ergo D æque metitur A, ac E ipsum C, puta per eundem F. *b* item D æque metitur C ac E ipsum B, puta per eundem G. *c* ergo DF=A, & EG=B. *d* quare A, & B plani sunt numeri. Quia vero EF *c* = C *c* = DG; *e* erit D. E :: F. G, & vicissim D. F :: E. G. *f* ergo plani numeri A, & B etiam similes sunt. Q. E. D.

a 35. 7. b 21. 7. c 9. *ax.* 7. d 16. *def.* 7. e 19. 7. f 21. *def.* 7.

PROP.

PROP. XXI.

A 16. C, 24. D, 36. B, 54.
E, 4. F, 6. G, 9.
H, 2. P, 2. M, 4. K, 3. L, 3. N, 6.

Si inter duos numeros A, B *duo medii proportionales cadant numeri* C, D; *similes solidi erunt illi numeri*, A, B.

a 2. 8. *a* Sume E, F, G minimos ∺ in ratione A ad
b 20. 8. C. *b* ergo E, & G sunt numeri plani similes.
c 21. *def.* 7 hujus latera sint H & P; illius K & L: *c* ergo H.
d *cor.* 18. 8. K :: P. L :: *d* E. F. Atqui E, F, G ipsos A, C,
e 21. 7. D *e* æque metiuntur, puta per eundem M; iidemque ipsos, C, D, B æque metiuntur, puta
f 9. *ax.* 7. per eundem N *f* ergo A = EM = HPM, *f* &
g 17. *def.* 7. B = GN = KLN; *g* quare A & B solidi sunt numeri. Quoniam vero C *f* = FM; & D *f* =
h 17. 7. FN, erit M. N *h* :: FM. FN *k* :: C D *l* :: E.
k 7. 5. F :: H. K :: P. L. *m* ergo A, & B sunt numeri
l *constr.* solidi similes. Q. E. D.
m 21. *def.* 7

LEMMA.

AE, BF, CG, DH,
A, B, C, D,
E, F, G, H.

Si proportionales numeri A, B, C, D *proportionales numeros* AE, BF, CG, DH *metiantur per numeros* E, F, G, H, *erunt ei* [E, F, G, H] *proportionales.*

a 19. 7. Nam ob AEDH *a* = BFCG, *a* & AD = BC,
b 1. *ax.* 7. *b* erit $\underbrace{A}_{}E\underbrace{D}_{}H$ = $B\underbrace{F}C\underbrace{G}$, *c* hoc est EH = FG.
c 9. *ax.* 7. AD BC
a ergo E. F :: G. H. Q. E. D.

Coroll.

d 15. *def.* 7. Hinc $\frac{Bq}{Aq} = \frac{B}{A}$ in $\frac{B}{A}$. *d* Nam 1. B :: B. Bq. *d* &
e *lem. præc.* 1. A :: A. Aq. *e* ergo 1. $\frac{B}{A}$ :: $\frac{B}{A}$. $\frac{Bq}{Aq}$. *d* ergo $\frac{Bq}{Aq}$ = $\frac{B}{A} \times \frac{B}{A}$. Similiter $\frac{B}{Ac}$ in $\frac{Bq}{Ac} = \frac{BC}{Acc}$. & sic de reliquis.

PROP.

PROP. XXII.

Aq, B, C. *Si tres numeri*, Aq, B, C
4, 8, 16. *deinceps sint proportionales, primus autem* Aq *sit quadratus*; *& tertius* C *quadratus erit.*

Nam ob AqC *a*$=$Bq, *b* erit C$=\frac{Bq}{Aq}$ *c*$=$Q.$\frac{B}{A}$ a 20. 7. b 7. ax. 7.

Liquet vero $\frac{B}{A}$ esse numerum, *d* ob $\frac{Bq}{Aq}$, vel C numerum, ergo si tres, &c. c cor. lem. præc. d hyp. & 14. 8.

PROP. *XXIII.

Ac, B, C, D. *Si quatuor numeri* Ac,
8, 12, 18. 27. B, C, D *deinceps sint proportionales, primus autem* Ac *sit cubus*; *& quartus* D *cubus erit.*

Nam quia AcD *a*$=$BC, *b* erit D$=\frac{BC}{Ac}$ *c*$=$ a 19. 7. b 7. ax. 7.

$\frac{B}{Ac}\times$C; hoc est (ob Ac C$=$*d* Bq, & *b* proinde c cor. lem. præc. d 20. 7.

C$=\frac{Bq}{Ac}$)D$=\frac{B}{Ac}\times\frac{Bq}{Ac}$ *e*$=\frac{Bc}{Acc}$ *e*$=$C: $\frac{B}{Aq}$ e 15. 8.

e liquet vero ipsum $\frac{B}{Aq}$ esse numerum, quia $\frac{Bc}{Acc}$ vel D numerus ponitur; ergo si quatuor numeri, &c.

PROP. XXIV.

A, 16. 24. B, 36. *Si duo numeri* A, B *ra-*
C, 4. 6. D, 9. *tionem habeant inter se, quam quadratus numerus* C *ad quadratum numerum* D, *primus autem* A *sit quadratus*: *& secundus* B *quadratus erit.*

Inter C, & D numeros quadratos, * adeoque * 8. 8.
inter A, & B eandem rationem habentes, *a* cadit a 11. 8.
unus

b hyp. unus medius proportionalis. Ergo *b* cum A qua-
c 22. 8. dratus sit, *c* etiam B quadratus erit. Q.E.D.

Coroll.

1. Hinc si fuerint duo numeri similes AB, CD (A. B :: C.D) primus autem AB sit quadratus, etiam secundus CD quadratus erit.

* 11.& 18. 8. * Nam AB. CD :: Aq. Cq.

2. Liquet ex his, proportionem cujusvis numeri quadrati ad quemlibet non quadratum, exhiberi nullo modo posse in duobus numeris quadratis. unde non erit, Q. Q :: 1.2. nec 1.5. :: Q. Q, &c.

PROP. XXV.

C, 64. 96, 144. D, 216.
A, 8. 12. 18. B, 27.

Si duo numeri A, B *rationem inter se habeant, quam cubus numerus* C *ad cubum numerum* D, *primus autem* A *sit cubus, & secundus* B *cubus erit.*

a 12. 8. *a* Inter C, & D cubos, *b* adeoque inter A &
b 8. 8. B eandem rationem habentes, cadunt duo me-
c hyp. dii proportionales. ergo propter A *c* cubum,
d 23. 8. *d* etiam B cubus erit. Q. E. D.

Coroll.

1. Hinc etiam si fuerint duo numeri ABC, DEF (A.B :: D.E. & B.C :: E.F;) primus autem ABC cubus fuerit, etiam secundus DEF cubus erit.

* 12.& 19. 8. * Nam ABC. DEF :: Ac=Dc.

2. Patet etiam ex his, proportionem cujusvis numeri cubi ad quemlibet numerum non cubum non posse reperiri in duobus numeris cubis.

PROP. XXVI.

A, 20. C, 30. B, 45.
D, 4. E, 6. F, 9.

Similes plani numeri A, B *rationem inter se habent, quam quadratus numerus ad quadratum numerum.*

a 18. 8. Inter A, & B *a* cadit unus medius proportio-
nalis

nalis C. *b* sume tres D, E, F minimos ∺ in ra- b 2. 8.
tione A ad C: *c* Extremi D, F quadrati erunt: c cor. 2. 8.
atqui ex æquali A. B *d* :: D. F. ergo A. B ∺ d 11. 7.
Q. Q. Q. E. D.

PROP. XXVII.

A, 16. C, 24. D, 36. B, 54.
E, 8. F, 12. G, 18. H, 27.

Similes solidi numeri A, B, rationem habent inter se, quam cubus numerus ad cubum numerum.

a Inter A, & B cadunt duo medii proportionales, puta C & D: *b* sume quatuor E, F, G, H minimos ∺ in ratione A ad C. *b* Extremi E, H cubi sunt. At A. B *c* :: E. H :: C. C. Q. E. D. a 19. 8. b 2. 8. c 14. 7.

Schol.

1. Ex his infertur, nullos numeros habentes proportionem superparticularem, vel superbipartientem, vel duplam, aut aliam quamcunque multiplam non denominatam à numero quadrato, esse similes planos. *Vide Clavium.*

2. Nec duo quivis primi numeri, neque duo quicunque inter se primi, qui quadrati non sint, similes esse possunt.

M LIB.

LIB. IX.

PROP. I.

A, 6. B, 54.
Aq, 36. 108. AB, 324.

SI duo similes plani numeri A, B *multiplicantes se mutuo faciant quendam* AB, *productum* AB *quadratum erit.*

a 17. 7. Nam A.B *a* :: Aq. AB; cum igitur
b 18. 8. inter A, & B *b* cadat unus medius proportiona-
c 8, 8. lis, *c* etiam inter Aq, & AB cadet unus med. pro-
d 22. 8. port. ergo cum primus Aq sit quadratus, *d* etiam tertius AB quadratus erit. Q. E. D.

Vel sic. Sint ab, cd similes plani, nempe a. b ::
x 19. 7. c. d. *x* ergo ad = bc. quare abcd, vel adbc *y* =
y 1. ax 7. adad = Q: ad.

PROP. II.

A, 6. B, 54.
Aq, 36. AB, 324.

Si duo numeri A, B *se mutuo multiplicantes faciant* AB *quadratum, similes plani erunt,* A, B.

a 17. 7. Nam A. B *a* :: Aq. AB; quare cum inter Aq,
b 11. 8. AB *b* cadat unus medius proportionalis, *c* etiam
c 8. 8. unus inter A, & B medius cadet. *d* ergo A, & B
d 20. 8. sunt similes plani. Q. E. D.

PROP. III.

A, 2. Ac, 8. Acc, 64.

Si cubus numerus Ac *seipsum multiplicans procreet aliquem* Acc, *productus* Acc *cubus erit.*

a 15. def. 7. Nam 1. A *a* :: A. Aq *b* :: Aq. Ac. ergo inter 1, &
b 17. 7. Ac cadunt duo medii proportionales. Sed 1. Ac *a*
c 8. 8. :: Ac. Acc. *c* ergo inter Ac, & Acc cadunt etiam

duo

duo medii proportionales. Proinde cum Ac sit cubus, *d* erit Acc cubus. Q. E. D. d 23. 8.

Vel sic; aaa (Ac) in se ductus facit aaaaaa (Acc;) hic cubus est, cujus latus aa.

PROP. IV.

Ac, 8. Bc, 27.
Acc, 64. AcBc, 216.

Si cubus numerus Ac *cubum numerum* Bc *multiplicans, faciat aliquem* AcBc, *factus* AcBc *cubus erit.*

Nam Ac. Bc *a* :: Acc. AcBc. sed inter Ac & Bc *b* cadunt duo medii proportionales; *c* ergo inter Acc, & Ac Bc totidem cadunt. itaque cum Acc sit cubus, *d* erit AcBc etiam cubus. Q.E.D.

a 17. 7.
b 12. 8.
c 8. 8.
d 23. 8.

Vel sic. AcBc = aaabbb (ababab) = C: ab.

PROP. V.

Ac, 8. B, 27.
Acc, 64. AcB, 216.

Si cubus numerus Ac *numerum quendam* B *multiplicans, faciat cubum* AcB; *& multiplicatus* B *cubus erit.*

Nam Acc. AcB *a* :: Ac, B. Sed inter Acc, & AcB *b* cadunt duo medii proportionales. *c* ergo totidem cadent inter Ac, & B. quare cum Ac cubus sit, *d* etiam B cubus erit. Q. E. D.

a 17. 7.
b 12. 8.
c 8. 8.
d 23. 8.

PROP. VI.

A, 8. Aq, 64. Ac, 512.

Si numerus A *seipsum multiplicans faciat* Aq *cubum; & ipse* A *cubus erit.*

Nam quia Aq *a* cubus, & AqA (Ac) *b* cubus, *c* erit A cubus. Q. E. D.

a *hyp.*
b 19. *def.* 7.
c 5. 9.

PROP. VII.

A, 6. B, 11. AB, 66.
D, 2. E, 3.

Si compositus numerus A *numerum quempiam* B *multiplicans, quempiam faciat* AB, *factus* AB *solidus erit.*

a 13.def 7. Quoniam A compositus est, *a* metitur eum aliquis D, puta per E. *b* ergo A=DE; *c* quare
b 9 ax.7.
c 17.def.7. DEB=AB solidus est. Q. E. D.

PROP. VIII.

$1, a, 3. a^2, 9. a^3, 27. a^4, 81. a^5, 243. a^6, 729.$

Si ab unitate quotcunque numeri deinceps proportionales fuerint ($1, a, a^2, a^3, a^4$, &c.) *tertius quidem ab unitate* a^2 *quadratus est; & unum intermittentes omnes* (a^4, $a6$, $a8$, &c.) *quartus autem* a^3 *est cubus; & duos intermittentes omnes* ($a6$, $a9$, &c.) *septimus vero* $a6$, *cubus simul & quadratus; & quinque intermittentes omnes* (a^{12}, $a18$, &c.)

Nam 1. $a^2 = Q. a$. & $a^4 = aaaa = Q. aa$. & $a6 = aaaaaa = Q. aaa$, &c.

2. $a^3 = aaa = C. a$. & $a 6 = aaaaaa = C. aa$. & $aaaaaaaaa = C. aaa$, &c.

3. $a6 = aaaaaa = C. aa = Q. aaa$, ergo, &c.

a hyp. Vel juxta Euclidem; quia $1.a\ a :: a.a^2$, *b* erit
b 20. 7. $a^2 = Q: a$. ergo cum a^2, a^3, a^4 sint $\div$ *c* erit
c 22. 8. tertius a^4 etiam quadratus. pariterq; $a6$, $a8$, &c. Item quia $1. a\ a :: a^2. a^3$. erit a^3 *b* $= a^2$ in $a =$
d 23. 8. C: a *d* ergo quartus ab a^3, nempe $a6$, etiam cubus erit, &c. ergo $a6$ cubus simul & quadratus existit, &c.

PROP. IX.

$1. a, 4. a^2, 16. a^3, 64. a^4, 256$, &c.
$1. a, 8. a^2, 64. a^3, 512. a^4, 4096.$

Si ab unitate quotcunque numeri deinceps proportionales fuerint ($1, a, a^2, a^3$, &c.) *qui vero* (a) *post unitatem sit quadratus; & reliqui omnes,* a^2, a^3, a^4, &c. *quadrati erunt. At si a, qui post unitatem, sit cubus, & reliqui omnes* a^2, a^3, a^4, &c. *cubi erunt.*

1. *Hyp.* Nam $a^2, a^4, a6$, &c. quadrati sunt
a 22. 8. ex præc. item quia a ponitur quadratus, *a* erit tertius a^3 quadratus, pariterque a^5, $a7$, &c. ergo omnes.

2. *Hyp.*

2. *Hyp.* a cubus ponitur, *b* ergo a^4, a^7. a^{10} cubi sunt: atqui ex præced. a^3, a^6, a^9, &c. cubi sunt. denique quia 1. a :: a.aa, *c* erit a^2 = Q. a. cubus autem in se *d* facit cubum; ergo a^2 cubus est, & *e* proinde ab eo quartus a^5, pariterque a^8, a^{11}, &c. cubi sunt, ergo omnes. Q. E. D.

b 23. 8. c 20. 7. d 3. 9. e 23. 8.

Clarius forsitan sic; Sit quadrati a latus b. ergo series a, a^2, a^3, a^4, &c. aliter exprimetur sic, bb, b4, b6, b8, &c. liquet vero hos omnes quadratos esse; & sic etiam exprimi posse; Q:b, Q: bb, Q: bbb, Q: bbbb, &c.

Eodem modo, si b latus fuerit cubi a; series ita nominari potest; b^3, b6, b9, b^{12}, &c. vel C: b, C: b^2, C: b^3, C: b^4, &c.

PROP. X.

1, a, a^2, a^3, a^4, a^5, a6.
1, 2, 4, 8, 16, 32, 64.

Si ab unitate quotcunque numeri deinceps proportionales fuerint (1, a, a^2, a^3, &c.) *qui vero post unitatem* (a) *non sit quadratus, neque alius ullus quadratus erit, præter* a^2 *tertium ab unitate, & unum intermittentes omnes* (a4, a6, a8.) *At si a, qui post unitatem, non sit cubus, neque ullus alius cubus erit præter* a^3 *quartum ab unitate, & duos intermittentes omnes, a6, a9, a12, &c.*

1. *Hyp.* Nam si fieri potest, sit a^5 quadratus numerus. quoniam igitur a.a^2. *a* :: a^4. a^5, atq; inverse a^5. a^4 :: a^2. a; sintque a^5, & a^4 *b* quadrati, primusque a^2 quadratus, *c* erit a etiam quadratus, contra Hyp.

a *hyp.* b *suppos.* & 8. 9. c 24. 8.

2. *Hyp.* Si fieri potest, sit a^4 cubus. quoniam igitur *d* ex æquo a^4. a6 :: a. a^3, atque inverse a6. a^4 :: a^3. a; *b* sintque a6, & a^4 cubi, & primus a^3 cubus, *e* etiam a cubus erit, contra Hypoth.

d 14. 7. e 25. 8.

PROP. XI.

$1, a, a^2, a^3, a^4, a^5, a6.$
$1, 3, 9, 27, 81, 243, 729.$

Si ab unitate quotcunq; numeri deinceps proportionales fuerint ($1, a, a^2, a^3$, &c.) *minor majorem metitur per aliquem eorum qui in proportionalibus sunt numeris.*

a 5. ax. 7. & 20. def. 7. Quoniam $1.a :: a.aa$, *a* erit $\frac{aa}{a} = a = \frac{aaa}{aa}$

b 14. 7. item quia $1.aa$ *b* $:: a.aaa$. *a* erit $\frac{aaa}{a} = aa = \frac{a^4}{aa} = \frac{a^5}{a^3}$ &c. denique quia $1, a^3$ *b* $:: a, a^4$, *a* erit $\frac{a^4}{a} = a^3 = \frac{a^6}{a^3}$ &c.

Coroll.

Hinc, si numerus qui metitur aliquem ex proportionalibus, non sit unus proportionalium, neque numerus per quem metitur, erit aliquis ex proportionalibus.

PROP. XII.

$1, a, a^2, a^3, a^4,$
$1, 6, 36, 216, 1296.$
$B, 3.$

Si ab unitate quotcunq; numeri deinceps proportionales fuerint ($1, a, a^2, a^3, a^4$,) *quicunque primorum numerorum* B *ultimum* a^4 *metiuntur, iidem* (B) *& eum* (a) *qui unitati proximus est, metientur.*

a 31. 7. Dic B non metiri a, *a* ergo B ad a primus est;
b 27. 7. *b* ergo B ad a^2 primus est; & *c* proinde ad a^4
c 26. 7. quem metiri ponitur. Q. E. A.

Coroll.

1. Itaque omnis numerus primus ultimum metiens, metitur quoque omnes alios ultimum præcedentes.

2. Si

2. Si aliquis numerus non metiens proximum unitati, metiatur ultimum, erit numerus compositus.

3. Si proximus unitati fit primus numerus, nullus alius primus numerus ultimum metietur.

PROP. XIII.

$1, a, a^2, a^3, a^4.$

$1, 5, 25, 125, 625.$

H - - G - - F - - E - -

Si ab unitate quotcunque numeri deinceps proportionales fuerint (a, a^2, a^3, *&c.*) *qui vero post unitatem* (a) *primus sit; maximum nullus alius metietur, præter eos qui sunt in numeris proportionalibus.*

Si fieri potest, alius quispiam E metiatur a^4, nempe per F; *a* erit F alius extra a, a^2, a^3. Quia vero E metiens a^4 non metitur a, *b* erit E numerus compositus; *c* ergo eum aliquis primus metitur, *d* qui proinde ipsum a^4 metitur; *e* ideoque alius non est, quam a. ergo a metitur E. Eodem modo ostendetur F compositus numerus, metiens a^4, adeoque a ipsum F metiri. itaque quum EF *f* $= a^4 = a$ in a^3, *g* erit a E :: F. a^3. ergo cum a metiatur E, *h* æque F metietur a^3, puta per eundem G. *k* Nec G erit a, vel a^2. ergo, ut prius, G est numerus compositus, & a eum metitur. quum igitur FG *f* $= a^3 = a^2$ in a, *g* erit a. F :: G. a^2; & proinde, quia A metitur F, *h* æque G metietur a^2, scilicet per eundem H; *k* qui non est a. ergo quum GH $= a^2 = aa$. *l* erit H. a :: a. G. ergo quia a metitur G (ut prius) *m* etiam H metietur a, numerum primum. Q. F. N.

a cor. 12. 9.
b 2 cor. 12. 9.
c 33. 7.
d 11. ax. 7.
e 3 cor. 12. 9.
f 9. ax. 7.
g 19. 7.
h 20. def. 7.
k cor. 11. 9.
l 20. 7.
m 20. def. 7

PROP. XIV.

A, 30. *Si minimum numerum* A
B, 2. C, 3. D, 5. *primi numeri* B, C, D *me-*
E -- F --- *tiantur; nullus alius nume-*
rus primus E *illum metie-*
tur, præter eos, qui à principio metiebantur.

a 9.ax.7. Si fieri potest, sit $\frac{A}{E}$ = F. *a* Ergo A = EF.

b 32. 7. *b* Ergo singuli primi numeri B, C, D ipsorum E, F unum metiuntur; non E, qui primus ponitur; ergo F, minorem scilicet ipso A; contra Hypoth.

PROP. XV.

A, 9. B, 12. C, 16. *Si tres numeri* A, B, C
D, 3. E, 4. *deinceps proportionales, fue-*
rint minimi omnium eandem cum ipsis rationem habentium; duo quilibet compositi, ad reliquum primi erunt.

a 35. 7. *a* Sume D, & E minimos in ratione A ad B.
b 2. 8. *b* ergo A = Dq; *b* & C = Eq; *b* & B = DE. Quia
c 24. 7. vero D ad E *c* primus est, *d* erit D+E primus ad
d 34. 7. singulos D, & E. * ergo D in D+E *e* = Dq+
*. 26. 7. DE (*f* A+B) ad E primus est, *g* ideoque ad C
e 3, 2. vel Eq. Q. E. D. Pari pacto DE+Eq (B+C)
f prius. ad D primus est, & proinde ad A = Dq. Q. E. D.
g 27. 7. Denique quia B ad D+E *h* primus est; is ad
h 26. 7. hujus quadratum *k* Dq+2DE+Eq (A+2
k 4. 2. B+C) primus erit. *l* quare idem B ad A+B+C,
l 30. 7. *l* adeoque ad A+C primus erit. Q. E. D.

PROP.

PROP. XVI.

A, 3. B, 5. C ---. *Si duo numeri* A, B *primi inter se fuerint; non erit ut primus* A *ad secundum* B, *ita secundus* B *ad alium quempiam* C.

Dic A. B :: B. C. ergo quum A & B in sua ratione *a* minimi sint, A *b* metietur B æque ac B ipsum C; sed A *c* seipsum etiam metitur; ergo A & B non sunt primi inter se, contra Hypoth.

a 23. 7. b 21. 7. c 6. *ax*. 7.

PROP. XVII.

A, 8. B, 12. C, 18. D, 27. E ---

Si fuerint quotcunque numeri deinceps proportionales A, B, C, D, *extremi autem ipsorum* A, D *primi inter se sint; non erit ut primus* A *ad secundum* B, *ita ultimus* D *ad alium quempiam* E.

Dic A. B :: D. E. ergo vicissim A. D :: B. E. ergo quum A & D in sua ratione *a* minimi sint, *b* metietur A ipsum B; *c* quare B ipsum C, & C sequentem D, *d* adeoq; A eundem D metietur. Ergo A & D non sunt primi inter se, contra Hypoth.

a 23. 7. & b 21. 7. c 20. *def*. 7. d 11. *ax* 7.

PROP. XVIII.

A, 4. B, 6. C, 9. Bq, 36. *Duobus numeris datis* A, B, *considerare an possit ipsis tertius proportionalis* C *inveniri.*

Si A metiatur Bq per aliquem C, *a* erit A C = Bq. unde *b* liquet esse A. B :: B. C. Q.E.F.

a 9. *ax*. 7. b *per* 20. 7.

A, 6. B, 4. Bq, 16. Sin A non metiatur Bq, non erit aliquis tertius proportionalis.

Nam dic A. B :: B. C. *a* ergo AC = Bq. *c* proinde $\frac{Bq}{A}$ = C. Scilicet A metitur Bq, contra Hypoth.

c 7. *ax*. 7.

PROP.

PROP. XIX.

A, 8. B, 12. C, 18. D, 27.
BC, 216.

Tribus numeris datis A, B, C, *considerare an possit ipsis quartus proportionalis* D *inveniri.*

a 9. ax. 7. Si A metiatur B C per aliquem D, *a* ergo
b ax. 19. 7. AD = BC; *b* constat igitur esse A. B :: C. D.
Q. E. F.

Sin A non metiatur B C, non datur quartus proportionalis; quod ostendetur, prout in præcedenti.

PROP. XX.

A, 2. B, 3. C, 5.
D, 30. G ----

Primi numeri plures sunt omni proposita multitudine primorum numerorum A, B, C.

a 38. 7. *a* Sit D minimus, quem A, B, C metiuntur: si D+1 primus sit, res patet; si compositus,
b 33. 7. *b* ergo aliquis primus, puta G, metitur D+1, qui non est aliquis trium A, B, C; nam si ita,
c suppos. quum is *c* totum D+1, & *d* ablatum D metiatur,
d constr. *e* idem reliquam unitatem metietur. Q. E. A.
e 12. ax. 7. Ergo propositorum primorum numerorum multitudo aucta est per D+1; vel saltem per G.

PROP. XXI.

5 5 3 3 2 2
A ..... E ..... B ... F ... C .. G .. D 20.

Si pares numeri quotcunque AB, BC, CD *componantur, totus* AD *par erit.*

a 6. def. 7. *a* Sume EB = $\frac{1}{2}$ AB & FC = $\frac{1}{2}$ BC, & GD = $\frac{1}{2}$
b 12. 7. CD. *b* liquet EB + FC + GD = $\frac{1}{2}$ AD, *c* ergo
c 6. def. 7. AD est par numerus. Q. E. D.

PROP. XXII.

```
    1        1       1      1
A....... F . B...... G . C..... H . D.. L . E 24.
    9        7       5      3
```

Si impares numeri quotcunque AB, BC, CD, DE *componantur, multitudo autem ipsorum sit par, totus* AE *par erit.*

Detracta unitate ex singulis imparibus, *a* manebunt AF, BG, CH, DL numeri pares, & *b* proinde compositus ex ipsis par erit; adde his *c* parem numerum conflatum ex residuis unitatibus, *d* totus idcirco AE par erit. Q. E. D. — a 7. *def.* 7. b 21. 9. c *hyp.* d 21. 7.

PROP. XXIII.

```
  7     5      1
A ....... B ..... C .. E . D 15.
              3
```

Si impares numeri quotcunque AB, BC, CD *componantur, multitudo autem ipsorum sit impar; & totus* AD *impar erit.*

Nam dempto CD uno imparium, reliquorum aggregatus AC *a* est par numerus. huic adde CD—1; *b* totus AE est etiam par; quare restituta unitate totus AD *c* impar erit. Q. E. D. — a 22. 9. b 21. 9. c 7. *def.* 7.

PROP. XXIV.

```
  4     5     1
A .... B ..... D . C.10.
        6
```

Si à pari numero AC *par* AB *detrahatur, & reliquum* BC *par erit.*

Nam si BD (BC—1) impar fuerit, *a* erit BC (BD+1) par. Q. E. D. Sin BD parem dicas, propter AB *b* parem, *c* erit AD par; *a* ideoque AC (AD+1) impar, contra Hypoth. ergo BC est par. Q. E. D. — a 7. *def.* 7. b *hyp.* c 21. 9.

PROP.

PROP. XXV.

6 : 1 3
A...... D . C ... B 10.
7

Si à pari numero AB *impar* AC *detrahatur, & reliquus* CB *impar erit.*

a 7. *def.* 7. Nam AC—1 (AD) *a* est par. *b* ergo DB
b 24. 9. est par. *c* ergo CB (DB—1) est impar. Q.E.D.
c 7. *def.* 7.

PROP. XXVI.

4 6 1
A .... C ...... D . B 11.
7

Si ab impari numero A B *impar* C B *detrahatur, reliquus* AC *par erit.*

a 7. *def.* 7. Nam AB—1 (AD) & CB—1 (CD) *a* sunt
b 24. 9. pares. *b* ergo AD—CD (AC) est par. Q.E.D.

PROP. XXVII.

1 4 6
A . D .... C ...... B 11.
5

Si ab impari numero AB *par detrahatur* CB, *reliquus* AC *impar erit.*

Nam AB—1 (DB)
a 7. *def* 7. *a* est par ; & CB ponitur par. *b* ergo reliquus
b 24. 9. CD par est. *c* ergo CD+1 (CA) est impar.
c 7. *def* 7. Q. E. D.

PROP. XXVIII.

A, 3.
B, 4.
AB, 12.

Si impar numerus A *parem numerum* B *multiplicans fecerit aliquem* AB, *factus* AB *par erit.*

a *hyp* & 15. Nam AB *a* componitur ex im-
def. 7. pari A toties accepto, quoties unitas continetur
b 21. 9. in B pari. *b* ergo AB est par numerus.

Schol.

Eodem modo, si A sit numerus par, erit AB par.

PROP. XXIX.

$A, 3.$ $B, 5.$ $\overline{AB}, 15.$

Si impar numerus A, imparem numerum B multiplicans fecerit aliquem AB, factus AB impar erit.

Nam AB *a* componitur ex B impari numero toties accepto, quoties unitas includitur in A etiam impari. *b* ergo AB est impar. Q. E. D. a 15. def. 7. b 23. 9.

Scholium.

$B, 12$ (C, 4. $\overline{A}, 3.$

1. *Numerus A impar numerum B parem metiens, per numerum parem C eum metitur.*

Nam si C impar dicatur, quoniam *a* B = AC, a 9. ax. 7.
b erit B impar, contra Hypoth. b 29. 9.

$B, 15$ (C, 5. $\overline{A}, 3$

2. *Numerus A impar numerum B imparem metiens, per numerum C imparem eum metitur.*

Nam si C dicatur par; *a* erit AC, vel B par, contra Hypoth. a 28. 9.

$B, 15$ (C, 5. $\overline{A}, 3$

3 *Omnis numerus (A & C) metiens imparem numerum B, est impar.*

Nam si utervis A, vel C dicatur par, *a* erit B numerus par, contra Hypoth. a 28. 9.

PROP. XXX.

$B, 24$ (C, 8. $D, 12$ E, 4.
$\overline{A}, 3$ $\overline{A}, 3$

Si impar numerus A parem numerum B metiatur, & illius dimidium D metietur. a hyp. b 1. schol. 29. 9.

a Sit $\frac{B}{A} = C$. *b* ergo C est numerus par.
Sit igitur $E = \frac{1}{2}C$, erit B *c* = CA *d* = 2EA *e* = 2D. c 9. ax. 7.
f ergo EA = D; & *g* proinde $\frac{D}{A} = E$. Q. E. D. d 1. 2. e hyp. f 7. ax. 1. g 7. ax. 7.

PROP.

PROP. XXXI.

A, 5. B, 8. C, 16. D --- *Si impar numerus* A *ad aliquem numerum* B *primus sit, & ad illius duplum* C *primus erit.*

a 3. *schol.* 29. 9. b 30. 9. Si fieri potest, aliquis D metiatur A, & C. *a* ergo D metiens imparem A impar erit, *b* ideoque ipsum B paris C semissem metietur. ergo A, & B non sunt primi inter se, contra Hypoth.

Coroll.

Sequitur hinc, numerum imparem, qui ad aliquem numerum progressionis duplæ primus est, primum quoque esse ad omnes numeros illius progressionis.

PROP. XXXII.

1. A, 2. B, 4. C, 8. D, 16. *Numerorum* A, B, C, D, &c. *à binario duplorum unusquisque pariter par est tantum.*

a 6. *def.* 7. b 10. *def.* 7. c 11. 9. d 8. *def.* 7. e 13. 9. Constat omnes A, B, C, D *a* pares esse; atque *b* $\div\!\!\div$ nimirum in ratione dupla, & *c* proinde quemque minorem metiri majorem per aliquem ex illis. *d* Omnes igitur B, C, D sunt pariter pares. Sed quoniam A primus est, *e* nullus extra eos eorum aliquem metietur. Ergo pariter pares sunt tantum. Q. E. D.

PROP. XXXIII.

A, 30. B, 15. *Si numerus* A *dimidium* B
D - - E -- *habeat imparem,* A *pariter impar est tantum.*

a *hyp.* b 9. *def.* 7. c 8. *def.* 7. d 9. *ax.* 7. e 19. 7. Quoniam impar numerus B *a* metitur A per 2 parem, *b* est B pariter impar. Dic etiam pariter parem. *c* ergo eum par aliquis D per parem E metitur. unde 2 B *d* = A *d* = DE. *e* quare 2. E ::

E :: D. B. ergo ut *f* metitur parem E, *g* sic D par imparem B metitur. Q. F. N. f 6. def. 7. g 20. def. 7.

PROP. XXXIV.

A, 24. *Si par numerus* A, *neque à binario duplus sit, neque dimidium habeat imparem; pariter par est, & pariter impar.*

Liquet A esse pariter parem, quia dimidium imparem non habet. Quia vero si A bifarietur, & rursus ejus dimidium, & hoc semper fiat, tandem incidemus in aliquem *a* imparem (quia non in binarium, quoniam A à binario duplus non ponitur) is metietur A per parem numerum (nam *b* alias ipse A impar esset, contra Hypoth.) ergo A est etiam pariter impar. Q. E. D. a 7. def. 7. b 2 sch. 29. 9.

PROP. XXXV.

A .... 8.

4 8

B ...., F .... G 12.

C .... 18.

9 6 4 8

D .... H ...... L .... K .... N 27.

Si sint quotcunque numeri deinceps proportionales A, BG, C, DN, *detrahantur autem* FG *à secundo, &* KN *ab ultimo, æquales ipsi primo* A; *erit ut secundi excessus* BF *ad primum* A, *ita ultimi excessus* DK *ad omnes* A, BG, C *ipsum antecedentes.*

Ex DN deme NL = BG, & NH = C.

Quoniam DN. C. (HN) *a* :: HN. BG. (LN) *a* :: LN. A. (KN.) *b* erit dividendo ubique, DH. HN :: HL. LN :: LK. KN. *c* quare DK. C + BG + A :: LK (*d* BF.) KN. (A.) Q. E. D. a hyp. b 17. 5. c 12. 5. d 3. ax. 1.

Coroll.

Hinc *e* componendo, DN + BG + C. A + BG + C :: BG. A. e 18. 5.

PROP.

PROP. XXXVI.

1. A, 2. B, 4. C, 8. D, 16.
E, 31. G, 62. H, 124. L, 248. F, 496.
M, 31. N, 465.
P ---. Q. --

SI ab unitate quotcunque numeri 1, A, B, C, D, *deinceps exponantur in dupla proportione, quoad totus compositus* E *fiat primus & totus hic* E *in ultimum* D *multiplicatus faciat aliquem* F; *factus* F *erit perfectus.*

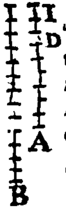
Sume totidem, E, G, H, L etiam in proportione dupla continue; ergo *a* ex æquo A. D ::
a 14. 7. E. L. *b* ergo AL = DE *c* = F. *d* ergo L = $\frac{F}{2}$
b 19. 7. quare E, G, H, L, F sunt ∺ in ratione dupla.
c hyp. Sit G − E = M, & F − E = N. *e* ideo M. E ::
d 7. ax. 7. N E + G + H + L. *f* at M = E. *g* ergo N =
e 35. 9. E + G + H + L. *h* ergo F = 1 + A + B +
f 3. ax. 1. C + D + E + G + H + L = E + N.
g 14. 5. Quinetiam quia D *k* metitur DE (F,) *l* etiam
h 2. ax. 1. singuli 1, A, B, C *m* metientes D, *m* nec non E,
k 7. ax. 7. G, H, L metiuntur F. Porro nullus alius eundem F metitur. Nam si aliquis, sit P, qui metia-
l 11. ax. 7. tur F per Q. *n* ergo PQ = F = DE. *o* ergo
m 11. 9. E. Q :: P. D. ergo cum A primus numerus
n 9. ax. 7. metiatur D, & *p* proinde nullus alius P. eundem
o 19. 7. metiatur, *q* consequenter E non metitur Q. qua-
p 13. 9. re cum E primus ponatur, *r* idem ad Q primus
q 20. def. 7 erit. *s* ergo E & Q in sua ratione minimi sunt,
r 31. 7. & *t* propterea E ipsum P ac Q ipsum D æque
s 23. 7. metiuntur. *u* ergo Q est aliquis ipsorum A, B, C.
t 21. 7. Sit igitur B; ergo cum ex æquo sit B. D :: E. H;
u 13. 9. *x* ideoque BH = DE = F = PQ. *x* adeoque
x 19. 7. Q. B :: H. P. *y* erit H = P. ergo P est etiam
y 14. 5. aliquis ipsorum A, B, C, &c. contra Hypoth.
ergo nullus alius præter numeros prædictos eundem F metietur; *z* proinde F est numerus perfectus. Q. E. D.
z 22. def. 7.

LIB.

LIB. X.

Definitiones.

I. COmmensurabiles magnitudines dicuntur, quas eadem mensura metitur.

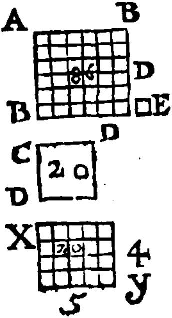


Commensurabilitatis nota est ⌐⊔, *ut* A ⌐⊔ B; *hoc est, linea* A 8 *pedum commensurabilis est linea* B 13 *pedum*; *quia* D *linea unius pedis singulas* A *&* B *metitur. Item* $\sqrt{18}$ ⌐⊔ $\sqrt{50}$; *quia* $\sqrt{2}$ *singulas* $\sqrt{18}$, *&* $\sqrt{50}$ *metitur. Nam* $\sqrt{\frac{18}{2}} = \sqrt{9} = 3$. *&* $\sqrt{\frac{50}{2}} = \sqrt{25} = 5$. *quare* $\sqrt{18}.\sqrt{50} :: 3.5$.

II. Incommensurabiles autem sunt, quorum nullam communem mensuram contingit reperiri.

Incommensurabilitas significatur nota ⌐⊔. *ut* $\sqrt{6}$ ⌐⊔ $\sqrt{25}$ (5;) *hoc est* $\sqrt{6}$ *incommensurabilis est numero* 5, *vel magnitudini hoc numero designatæ; quia harum nulla est communis mensura, ut postea patebit.*

III. Rectæ lineæ potentia commensurabiles sunt, cum quadrata earum idem spatium metitur.



Hujusce commensurabilitatis nota est ᚁ, *ut* AB ᚁ CD; *h.e. linea* AB *sex pedum potentia commensurabilis est linea* C D, *quæ exprimitur per* $\surd$ 20. *quia spatium* E *unius pedis quadrati metitur tam* ABq (36) *quam rectangulum* XY (20,) *cui æquale est quadratum lineæ* CD ($\surd$ 20.) *Eadem nota* ᚁ *nonnunquam valet potentia tantum commensurabilia.*

IV. Incommensurabiles vero potentia, cum quadratis earum nullum spatium, quod sit communis eorum mensura, contingit reperiri.

Hujusmodi incommensurabilitas denotatur sic; 5 ᚁ $v \surd$ 8; *hoc est, numeri vel lineæ* 5, *&* $v \surd$ 8 *sunt incommensurabiles potentia; quia harum quadrata* 25, *&* $\surd$ 8 *sunt incommensurabilia.*

V. Quæ cum ita sint, manifestum est cuicunque rectæ propositæ, rectas lineas multitudine infinitas, & commensurabiles esse, & incommensurabiles; alias quidem longitudine & potentia, alias vero potentia solum. Vocetur autem proposita recta linea Rationalis.

Hujus nota est $\dot{\rho}$.

VI. Et huic commensurabiles, sive longitudine & potentia, sive potentia tantum, Rationales, $\dot{\rho}$.

VII. Huic vero incommensurabiles Irrationales vocentur.

Hæ sic denotantur $\dot{\rho}$.

VIII. Et quadratum, quod à proposita recta fit, dicatur Rationale $\dot{\rho}\nu$.

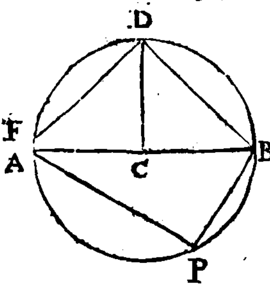
IX. Et huic commensurabilia quidem Rationalia $\dot{\rho}\alpha$.

X. Huic

X. Huic vero incommensurabilia, Irrationalia dicantur, $\dot{\rho}\alpha$.

XI. Et rectæ, quæ ipsa possunt, Irrationales, $\dot{\rho}$.

Schol.



Ut postrema 7 definitiones exemplo aliquo illustrentur; sit circulus ADBP, *cujus semidiameter* CB; *huic inscribantur latera figurarum ordinatarum, Hexagoni quidem* BP, *Trianguli* AP, *quadrati* BD, *pentagoni* FD. *Itaque si juxta 5 defin. semidiameter* CB *sit Rationalis exposita, numero 2. expressa, cui reliqua* BP, AP, BD, FD *comparandæ sunt, a erit* BP $a = $ BC $= 2$. *quare* BP *est* $\acute{\rho}$ ⊓ BC, *juxta 6. def. Item* AP $b = \sqrt{12}$ a cor. 15. 4. b 67. 1. *(nam* ABq (16) — BPq (4) $= 12$) *quare* AP *est* $\acute{\rho}$ ⊓̶ BC, *etiam juxta 6. def. atque* APq (12) *est* $\acute{\rho}\nu$, *per def. 9. Porro* BD $b = \sqrt{}$ DCq + BCq $= \sqrt{8}$; *unde* BD *est* $\acute{\rho}$ ⊓̶ BC; *&* BDq $\acute{\rho}\nu$. *Denique*, FDq $= 10 - \sqrt{20}$ *(ut patebit ex praxi ad 10. 13. tradenda) erit* $\dot{\rho}\nu$, *juxta 10 def. & proinde* FD $= \sqrt{: 10 - \sqrt{20}}$ *est* $\dot{\rho}$, *juxta 11 defin.*

Postulatum.

Postuletur, quamlibet magnitudinem toties posse multiplicari, donec quamlibet magnitudinem ejusdem generis excedat.

Axiomata.

1. MAgnitudo quotcunque magnitudines metiens, compositam quoque ex ipsis metitur.

2. Magnitudo quamcunque magnitudinem metiens, metitur quoque omnem magnitudinem quam illa metitur.

3. Magnitudo metiens totam magnitudinem & ablatam, metitur & reliquam.

PROP. I.

Duabus magnitudinibus inæqualibus AB, C *propositis, si à majore* AB *auferatur majus quam dimidium* (AH) *& ab eo* (HB) *quod reliquum est, rursus detrabatur majus quam dimidium* (HI,) *& hoc semper fiat; relinquetur tandem quædam magnitudo* IB, *quæ minor erit proposita minore magnitudine* C.

a *post.* 10. *a* Accipe C toties, donec ejus multiplex DE proxime excedat AB; sintque DF = FG = GE = C. Deme ex AB plusquam dimidium AH, & à reliquo HB plusquam dimidium HI; & sic deinceps, donec partes AH, HI, IB æque multæ sint partibus DF, FG, GE. Jam liquet FE, quæ non minor est quam $\frac{1}{2}$ DE, majorem esse quam HB, quæ minor est quam $\frac{1}{2}$ AB $\sqsupset$ DE. Pariterque GE quæ non minor est quam $\frac{1}{2}$ FE, major est quam IB $\sqsupset$ $\frac{1}{2}$ HB. ergo C, vel GE $\sqsubset$ IB. Q. E. D.

Idem demonstrabitur, si ex AB auferatur dimidium AH, & ex reliquo HB rursus dimidium HI, & ita deinceps.

PROP.

PROP. II.

Si duabus magnitudinibus inæqualibus propositis (AB, CD) *detrabatur semper minor* AB *de majore* CD, *alterna quadam detractione, & reliqua minime præcedentem metiatur; incommensurabiles erunt ipsæ magnitudines.*

Si fieri potest, sit aliqua E communis mensura. Quoniam igitur AB detracta ex CD, quoties fieri potest, relinquit aliquam FD se minorem, & FD ex AB relinquit GB, & sic deinceps, *a* tandem re- a 1. 10.
linquetur aliqua GB ⊐ E. ergo E *b* metiens AB, b *hyp.*
c ideoque CF, *b* & totam CD; *d* etiam reliquam c 2. *ax.* 10.
FD metitur. *c* proinde & AG; *d* ergo & reliquam d 3. *ax.* 10.
GB, seipsa minorem. Q. E. A.

PROP. III.

Duabus magnitudinibus commensurabilibus datis, AB, CD, *maximam earum communem mensuram* FB *reperire.*

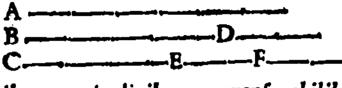
Deme AB ex CD, & reliquum ED ex AB, & FB ex ED, donec FB metiatur ED; (quod tandem fiet, *a* quia per Hyp. a 2. 10.
AB ⊐⊏ CD) erit FB quæsita.

Nam FB *b* metitur ED, *c* ideoque ip- b *constr.*
sam AF; sed & seipsam, *d* ergo etiam c 2. *ax.* 10.
AB, & *c* propterea CE, *d* adeoque & to- d 1. *ax.* 10.
tam CD. Proinde FB communis est mensura ipsarum AB, CD. Dic G communem quoque esse mensuram, hac majorem; ergo G
metiens AB, & CD, *e* metitur CE, & *f* reliquam e 2. *ax.* 10.
ED, *e* ideoque AF, & *f* proinde reliquam FB, f 3. *ax.* 10.
major minorem. Q. E. A.

Coroll.

Hinc, magnitudo metiens duas magnitudines, metitur & maximam earum mensuram communem.

PROP. IV.



Tribus magnitudinibus commensurabilibus datis A, B, C; *maximam earum mensuram communem invenire.*

a 3. 10. *a* Inveni D maximam communem mensuram duarum quarumcunque A, B; *a* item E ipsarum D & C maximam communem mensuram; erit E quæsita.

b *constr.* & 2. *ax.* 10. c *cor.* 3. 10. *a* Nam perspicuum est E metiens D & C *b* metiri tres A, B, C. Puta aliam F hac majorem easdem metiri. *c* ergo F metitur D; *c* proinde & E, ipsorum D, C maximam communem mensuram, major minorem. Q. E. A.

Coroll.

Hinc quoque, magnitudo metiens tres magnitudines, metitur quoque maximam earum communem mensuram.

PROP. V.

Commensurabiles magnitudines A, B *inter se rationem habent, quam numerus ad numerum.*

a 3. 10. *a* Inventa C ipsarum A, B maxima communi mensura; quoties C in A & B, toties 1 contineatur in numeris D & E. *b* ergo C. A :: 1. D;

b 20. *def.* 7. quare inverse A. C :: D. 1. *b* atqui etiam C. B ::

B :: 1. E. *c* ergo ex æquali A.B :: D. E :: N.N. c 22. 5.
Q. E. D.

PROP. VI.

E ——— F.1. *Si duæ magnitudines* A, B
A ———————— C.4.
B —————— D.3. *inter se proportionem habeant, quam numerus* C *ad numerum* D; *commensurabiles erunt magnitudines* A, B.

Qualis pars est 1 numeri C, *a* talis fiat E ipsius A. Quoniam igitur E.A *b* :: 1.C. atq; A. B *c* :: C. D; *d* ex æquo erit E. B :: 1. D. ergo quum 1 *e* metiatur numerum D, *f* etiam E metitur B; sed & ipsum A *g* metitur. *h* ergo A ⊤⊔ B. Q. E. D.

a *sch*.10.6.
b *constr.*
c *hyp.*
d 22. 5.
e 5. *ax*.7.
f 20.*def*.7.
g *constr.*
h 1.*def*.10.

PROP. VII.

A ——————
B —————
Incommensurabiles magnitudines A, B *inter se proportionem non habent, quam numerus ad numerum.*

Dic A. B :: N. N. *a* ergo A ⊤⊔ B, contra Hypoth. a 6. 10.

PROP. VIII.

A ——————
B —————
Si duæ magnitudines A, B *inter se proportionem non habeant, quam numerus ad numerum, incommensurabiles erunt magnitudines.*

Puta A ⊤⊔ B *a* ergo A.B :: N. N, contra Hypoth. a 5. 10.

PROP. IX.

A ———————
B —————
E, 4.
F, 3.

Quæ à rectis lineis longitudine commensurabilibus fiunt quadrata, inter se proportionem habent, quam quadratus numerus ad quadratum numerum: & quadrata inter se proportionem habentia, quam quadratus numerus ad quadratum numerum, & latera habebunt longitudine commensurabilia. Quæ vero à rectis lineis longitudine incommensurabilibus fiunt quadrata, inter se proportionem non habent, quam quadratus numerus ad quadratum numerum: & quadrata inter se proportionem non habentia, quam quadratus numerus ad quadratum numerum, neque latera habebunt longitudine commensurabilia.

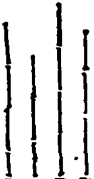
1. *Hyp.* A. ⌝⊔ B. Dico Aq. Bq :: Q. Q.
a per 5.10. Nam *a* sit A. B :: num. E. num. F. ergo
b 20. 6. c sch.23.5. $\frac{Aq}{Bq}$ (*b* $\frac{A}{B}$ bis) *c* = $\frac{E}{F}$ bis. *d* = $\frac{Eq}{Fq}$ *e* ergo Aq.
d 11. 8. Bq :: Eq. Fq :: Q. Q. Q. E. D.
e 11. 5. 2. *Hyp.* Aq. Bq :: Eq. Fq :: Q. Q. Dico A
f 20. 6. ⌝⊔ B. Nam $\frac{A}{B}$ bis (*f* $\frac{Aq}{Bq}$) *g* = $\frac{Eq}{Fq}$ *h* = $\frac{E}{F}$
g hyp. h 11. 8. bis. *i* ergo A. B :: E. F :: N. N. *k* quare A ⌝⊔ B. Q. E. D.
i sch. 23. 5. k 6. 10. 3. *Hyp.* A ⌝⊔ B. Nego esse Aq. Bq :: Q. Q. Nam dic Aq. Bq :: Q. Q. Ergo A ⌝⊔ B, ut modo ostensum est, contra Hypoth.

4. *Hyp.* Non Aq. Bq :: Q. Q. Dico A ⌝⊔ B. Nam puta A ⌝⊔ B; ergo Aq. Bq :: *Q. Q.* ut modo diximus, contra Hypoth.

Coroll.

Lineæ ⌝⊔ sunt etiam ⌝⊐; at non contra. Sed lineæ ⌝⊔ non sunt idcirco ⌝⊐. Lineæ vero ⌝⊐ sunt etiam ⌝⊔.

PROP. X.



Si quatuor magnitudines proportionales fuerint (C. A :: B. D;) *prima vero* C *secundæ* A *fuerit commensurabilis; & tertia* B *quartæ* D *commensurabilis erit. Et si prima* C *secundæ* A *fuerit incommensurabilis, & tertia* B *quartæ* D *incommensurabilis erit.*

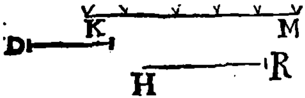
C A B D Si C ⊓ A, *a* ideo erit C. A :: N. a 25. 10.
N :: B. D. *b* ergo B ⊓ D. Sin C b 6. 10.
'⊓ A, ergo *c* non erit C. A :: N. N :: B. D. c 7. 10.
d quare B '⊓ D. Q. E. D. d 8. 10.

LEMMA 1.

Duos numeros planos invenire, qui proportionem non habeant, quam quadratus numerus ad quadratum numerum.

Huic Lemmati satisfacient duo quilibet numeri plani non similes, quales sunt numeri habentes proportionem superparticularem, vel superbipartientem, vel duplam; vel etiam duo quivis numeri primi. vid. Schol. 27. 8.

LEMMA 2.



B, 5.
C, 3.

Invenire lineam HR, *ad quam data recta linea* KM *sit in ratione datorum numerorum* B, C.

a Divide KM in partes æquales æque multas a *sch.*10.6.
unitatibus numeri B. harum tot, quot unitates
sunt in numero C, *b* componant rectam HR. b 3, 1.
liquet esse KM. HR :: B. C.

LEMMA 3.

Invenire lineam D, *ad cujus quadratum datæ rectæ* KM *quadratum sit in ratione datorum numerorum* B, C,

Fac

a 2. lem. 10. Fac B. C *a* :: KM. HR. ac inter KM, & HR
b inveni mediam proportionalem D. Erit KMq.
b 13. 6. Dq *c* :: KM. HR *d* :: B. C.
c 20. 6.
d *constr.*

PROP. XI.

A ———————— B. 20.
E ————— C. 16.
D —————

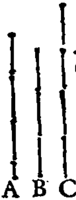
Proposita recta linea A *invenire duas rectas lineas incommensurabiles, alteram quidem* D *longitudine tantum, alteram vero* E *etiam potentia.*

a 1 *lem.* 10. 1. Sume numeros B. C, *a* ita ut non sit B. C ::
Q. Q. *b* fiatque B. C :: Aq. Dq. *c* liquet A $\urcorner\!\sqcup$
b 3 *lem.* 10. D. Sed Aq *d* $\urcorner\!\sqcup$ Dq. Q. E. F.
10. 2. *d* Fac A. E :: B. D. Dico Aq $\urcorner\!\sqcup$ Eq.
c 9. 10. Nam A. D *e* :: Aq Eq. ergo cum A $\urcorner\!\sqcup$ D,
d 8. 10. ut prius, *f* erit Aq $\urcorner\!\sqcup$ Eq. Q. E. F.
d 13. 6.
e 20. 6.

PROP. XII.

f 10. 10.

Quæ (A, B) *eidem magnitudini* C *sunt commensurabiles, & inter se sunt commensurabiles.*



a 5. 10. Quia A $\urcorner\!\sqcup$ C, & C $\urcorner\!\sqcup$ B, *a* sit A. C :: N N :: D. E. atque C. B :: N. N :: F. G. *b* sumantur tres numeri H, I, K minimi $\div$ in rationibus D ad E, & F ad G. Jam
b 4. 8.

D, 18. E, 8.
F, 2. G, 3.
H, 5. I, 4 K, 6.

c *constr.* quia A. C *c* :: D. E *c* :: H. I. ac C. B *c* :: F. G.
d 22. 5. *c* :: I. K. *d* erit ex æquali A. B :: H. K :: N. N.
e 6. 10. *e* ergo A $\urcorner\!\sqcup$ B. Q. E. D.

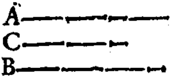
Schol.

Hinc, omnis recta linea rationali lineæ
12. 10. & commensurabilis, est quoque *f* rationalis. Et
def 6. omnes rectæ rationales inter se commensurabiles sunt, saltem potentia. Item, omne spatium rationali spatio commensurabile, est quoque ra-
def 9. tionale; & omnia spatia rationalia inter se com-

men-

menſurabilia ſunt. Magnitudines vero, quarum altera eſt rationalis, altera irrationalis, ſunt inter ſe incommenſurabiles. *def. 7. & 10*

PROP. XIII.



Si ſint duæ magnitudines A, B; *& altera quidem* A *eidem* C *ſit commenſurabilis, altera vero* B *incommenſurabilis, incommenſurabiles erunt magnitudines* A, B.

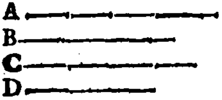
Dic B $\sqcup\!\sqcup$ A. ergo cum C *a* $\sqcup\!\sqcup$ A, *b* erit C $\urcorner\sqcup\!\sqcup$ B, contra Hypoth. a *hyp.* b 12. 10.

PROP. XIV.

Si ſint duæ magnitudines commenſurabiles A, B; *altera autem ipſarum* A *magnitudini cuipiam* C *incommenſurabilis fuerit; & reliqua* B *eidem* C *incommenſurabilis erit.*

Puta B $\sqcup\!\sqcup$ C. ergo cum A $\sqcup\!\sqcup$ *a* B, *b* erit A $\sqcup\!\sqcup$ C, contra Hyp. a *hyp.* b 12. 10.

PROP. XV.

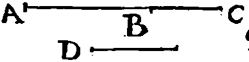


Si quatuor rectæ lineæ proportionales fuerint (A. B :: C. D;) *prima vero* A *tanto plus poſſit quam ſecunda* B, *quantum eſt quadratum rectæ lineæ ſibi commenſurabilis longitudine; & tertia* C *tanto plus poterit, quam quarta* D, *quantum eſt quadratum rectæ lineæ ſibi longitudine commenſurabilis. Quod ſi prima* A *tanto plus poſſit quam ſecunda* B, *quantum eſt quadratum rectæ lineæ ſibi incommenſurabilis longitudine; & tertia* C *tanto plus poterit, quam quarta* D, *quantum eſt quadratum rectæ lineæ ſibi longitudine incommenſurabilis.* a *hyp.*

Nam quia A. B *a* :: C. D. *b* erit Aq. Bq :: Cq. Dq. *c* ergo dividendo Aq—Bq. Bq :: Cq—Dq. b 22. 6. c 17. 5.

d 22. 6. Dq. Dq. *d* quare √: Aq—Bq. B :: √: Cq—Dq.
e cor. 4. 5. D. *e* invertendo igitur B. √ : Aq—Bq :: D. √:
f 22. 5. Cq—Dq. *f* ergo ex æquali A. √ : Aq — Bq ::
C. √ : Cq—Dq. proinde si A ⊏⊐, vel ⊏⊐ √
g 10. 10. Aq — Bq, *g* erit similiter C ⊏⊐, vel ⊏⊐ √:
Cq—Dq. Q. E. D.

PROP. XVI.



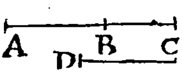
Si duæ magnitudines commensurabiles AB, BC *componantur, & tota magnitudo* AC *utrique ipsarum* AB, BC *commensurabilis erit: quod si tota magnitudo* AC *uni ipsarum* AB, vel BC *commensurabilis fuerit; & quæ à principio magnitudines* AB, BC *commensurabiles erunt.*

a 3. 10. 1. *Hyp.* *a* Sit D ipsarum AB, BC communis
b 1. ax 10. mensura. *b* ergo D metitur AC. *c* ergo AC ⊏⊐
c 1. def. 10. AB, & BC. Q. E. D.
2. *Hyp.* *a* Sit D communis mensura ipsarum
d 3. ax. 10. AC, AB; *d* ergo D metitur AC—AB (BC;)
e proinde AB ⊏⊐ BC. Q. E. D.

Coroll.

Hinc etiam, si tota magnitudo ex duabus composita, commensurabilis sit alteri ipsarum, eadem & reliquæ commensurabilis erit.

PROP. XVII.



Si duæ magnitudines incommensurabiles AB, BC *componantur, & tota magnitudo* AC *utrique ipsarum* AB, BC *incommensurabilis erit: Quod si tota magnitudo* AC *uni ipsarum* AB *incommensurabilis fuerit, & quæ à principio magnitudines* AB, BC *incommensurabiles erunt.*

1. *Hyp.*

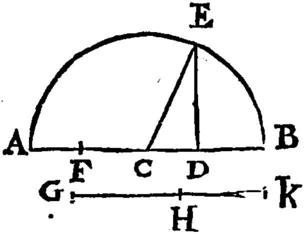
1. *Hyp.* Si fieri potest, sit D ipsarum AC, AB communis mensura. *a* ergo D metitur AC—AB (BC) *b* ergo AB $\llcorner\!\sqcap$ BC, contra Hypoth. a 3.*ax*.10. b 1.*def*.10.

2. *Hyp.* Dic AB $\llcorner\!\sqcap$ BC. *c* ergo AC $\llcorner\!\sqcap$ AB, contra Hypoth. c 16. 10.

Coroll.

Hinc etiam, si tota magnitudo ex duabus composita, incommensurabilis sit alteri ipsarum, eadem & reliquæ incommensurabilis erit.

PROP. XVIII.

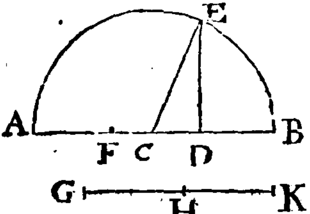


Si fuerint duæ rectæ lineæ inæquales AB, GK; *quartæ autem parti quadrati, quod fit à minori* GK, *æquale parallelogrammũ* ADB *ad majorem* AB *applicetur, deficiens figura quadrata, & in partes* AD, DB *longitudine commensurabiles ipsam dividat; major* AB *tanto plus poterit quam minor* GK, *quantum est quadratum rectæ lineæ* FD *sibi longitudine commensurabilis. Quod si major* AB *tanto plus possit, quam minor* GK, *quantum est quadratum rectæ lineæ* FD *sibi longitudine commensurabilis; quartæ autem parti quadrati, quod fit à minori* GK, *æquale parallelogrammum* ADB *ad majorem* AB *applicetur, deficiens figura quadrata, in partes* AD, DB *longitudine commensurabiles ipsam dividet.* a 10. 1. b 28. 6.

a Biseca GK in H, & *b* fac rectang. ADB = GHq: abscinde AF = DB. Estque ABq *c* = 4 ADB *d* (4 GHq, vel GKq) + FDq. Jam primo c 8. 2. d *constr*. & 4. 2.

e 16 10. primo, Si AD ⊓⊔ DB, erit AB *e* ⊓⊔ DB *e* ⊓⊔
f *constr.* 2 DB *f* (AF + DB, vel AB — FD) *g* ergo
g *cor.* 16. AB ⊓⊔ FD. Q. E. D. Sin secundo, AB ⊓⊔
10. FD, *h* erit ideo AB ⊓⊔ AB — FD (2 DB)
h *cor.* 16. *k* ergo AB ⊓⊔ DB. *l* quare AD ⊓⊔ DB.
10. Q. E. D.
k 12. 10.
l 16. 10.

PROP. XIX.

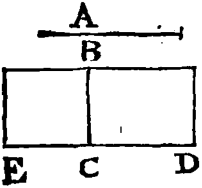


Si fuerint duæ rectæ lineæ inæquales, AB, GK; *quartæ autem parti quadrati, quod fit à minore* GK, *æquale parallelogrammum* ADB *ad majorem* AB *applicetur, deficiens figura quadrata; & in partes incommensurabiles longitudine* AD, DB, *ipsam* AB *dividat; major* AB *tanto plus poterit, quam minor* GK, *quantum est quadratum rectæ lineæ* FD, *sibi longitudine incommensurabilis. Quod si major* AB *tanto plus possit, quam minor* GK, *quantum est quadratum rectæ lineæ* FD *sibi longitudine incommensurabilis; quartæ autem parti quadrati, quod fit à minore* GK, *æquale parallelogrammum* ADB *ad majorem* AB *applicetur, deficiens figura quadrata; in partes longitudine incommensurabiles* AD, DB *ipsam* AB *dividet.*

Facta puta, & dicta eadem, quæ in præce-
a 17. 10. denti. Itaque primo. Si AD ⊓⊔ DB, *a* erit pro-
b 13. 10. pterea AB ⊓⊔ DB: *b* quare AB ⊓⊔ 2 DB
(AB—FD) *c* ergo AB ⊓⊔ FD. Q. E. D.
c *cor.* 17. Secundo, Si AB ⊓⊔ FD; *c* ergo AB ⊓⊔
10. AB — FD (2 BB;) *d* quare AB ⊓⊔ DB, &
d 13. 10. *e* proinde AD ⊓⊔ DB. Q. E. D.
e 17. 10.

PROP.

PROP. XX.



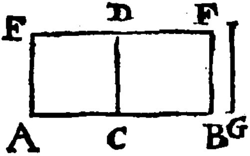
Quod sub rationalibus longitudine commensurabilibus rectis lineis BC, CD, *secundum aliquem prædictorum modorum, continetur rectangulum* BD, *rationale est.*

Exponatur A, ṕ. & *a* describatur BE quadratum ex BC. Quoniam DC. CE (BC) *b* :: BD, BE. & DC *c* ⊔⊔ BC; *d* erit rectang. BD ⊔⊔ quad. BE. ergo quum quad. BE *e* ⊔⊔ Aq; *f* erit BD ⊔⊔ Aq. proinde rectang. BD est ṕν. Q. E. D. — a 46. 1. b 1. 6. c *hyp.* d 10. 10. e *hyp.* & 9. *def.* 10. f 12. 10.

Not. *Tria sunt genera linearum rationalium inter se commensurabilium. Aut enim duarum linearum rationalium longitudine inter se commensurabilium altera æqualis est expositæ rationali; aut neutra rationali expositæ æqualis est, longitudine tamen ei utraque est commensurabilis; aut denique utraque expositæ rationali commensurabilis est solum potentia. Hi sunt modi illi, quos innuit præsens theorema.*

In numeris, sit BC, $\sqrt{8}$ ($2\sqrt{2}$) & CD, $\sqrt{18}$ ($3\sqrt{2}$,) erit rectang. BD $= \sqrt{144} = 12$.

PROP. XXI.



Si rationale DB *ad rationalem* DC *applicetur, latitudinem* CB *efficit rationalem, & ei* DC *ad quam applicatum est* DB, *longitudine commensurabilem.* — a 1. 6.

Exponatur G, ṕ. & describatur DA quadratum ex BC. quoniam BD. DA *a* :: BC. CA; atque, BD. DA *b* sunt ṕα, *c* ideoque ⊔⊔; *d* erit BC — b *hyp.* c *sch.* 12. 10 d 10. 10.

c *sch.* 12. 10 BC ⊔⊔ CA. at CD (CA) *b* est $\overset{\cdot}{\rho}$. *c* ergo BC est $\overset{\cdot}{\rho}$. Q. E. D.

In numeris, sit rectang. DB, 12; & DC, $\sqrt{8}$. erit CB, $\sqrt{18}$. atqui $\sqrt{18} = 3\sqrt{2}$. & $\sqrt{8} = 2\sqrt{2}$.

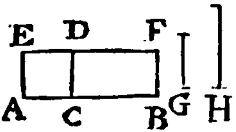
LEMMA.

A ——
B ————
C ——

Duas rectas rationales potentia solum commensurabiles invenire.

a 11. 10. Sit A exposita $\overset{\cdot}{\rho}$. *a* Sume B ⊓ A, *a* & C ⊓ B.
b *sch.* 12. 10 *b* liquet B, & C esse quæsitas.

PROP. XXII.



Quod sub rationalibus DC, CB potentia solum commensurabilibus rectis lineis continetur rectangulum DB, irrationale est; & recta linea H ipsum potens, irrationalis; vocetur autem Media.

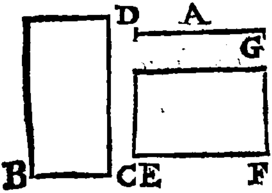
Sit G exposita $\overset{\cdot}{\rho}$. & describatur DA quadratum ex DC; sitque Hq = DB. Quoniam AC:
a 1. 6. CB *a* :: DA. DB. *b* atque AC ⊔⊔ CB, *c* erit
b *hyp.* DA ⊔⊔ DB (Hq.) *d* atqui Gq ⊔⊔ DA. *e* er-
c 10. 10. go Hq ⊔⊔ Gq. *f* ergo H est $\overset{\cdot}{\rho}$. Q. E. D. vo-
d *hyp* & 9. cetur autem Media. quia AC. H :: H. CB.
def. 10. In numeris, sit DC, 3; & CB, $\sqrt{6}$. erit re-
e 13. 10. ctangulum DB (Hq) $\sqrt{54}$. quare H est $v\sqrt{54}$.
f *def.* 11. 10 Mediæ nota est μ, Medii vero $\mu\nu$; pluraliter $\mu\alpha$.

SCHOL.

Omne rectangulum, quod potest contineri sub duabus rectis rationalibus potentia solum commensurabilibus, est Medium; quamvis contineatur sub duabus rectis irrationalibus: atque omne

omne Medium poteſt contineri ſub duabus rectis rationalibus potentia tantum commenſurabilibus, ut exemp. gr. $\sqrt{24}$ eſt $\mu\nu$. quia continetur ſub $\sqrt{3}$, & $\sqrt{8}$, qui ſunt ρ' ⊤⊣. etſi poſſet contineri ſub $v\sqrt{6}$, & $v\sqrt{96}$ irrationalibus; nam $\sqrt{24} = v\sqrt{576} = v\sqrt{6}$ in $v\sqrt{96}$.

PROP. XXIII.

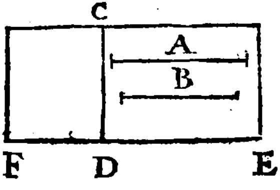


Quod (BD) à media A fit, ad rationalem BC *applicatum, latitudinem* CD *rationalem efficit, & ei* BC, *ad quam applicatum eſt* BD *longitudine incommenſurabilem.*

Quoniam A eſt μ, *a* erit Aq rectangulo alicui (EG) æquale contento ſub EF, & FG ρ' ⊤⊣. *b* ergo BD=EG. *c* quare BC.EF :: FG.CD. *d* ergo BCq. EFq :: FGq. CDq. ſed BCq, & EFq *e* ſunt $\rho\alpha$, *f* ideoque ⊤⊏. *g* ergo FGq ⊤⊏ CDq. Ergo quum FG ſit ρ', *h* erit CD ρ'. Porro, quia EF. FG *k* :: EFq. EG (BD;) ob EF ⊤⊏ FG, *l* erit EFq ⊤⊏ BD. verum EFq *m* ⊤⊏ CDq. *n* ergo rectang. BD ⊤⊏ CDq. quum igitur CDq. BD *o* :: CD. BC. *p* erit CD ⊤⊏ BC. ergo, &c.

a *ſch.*12.10
b 1. *ax.*1.
c 14. *6.*
d 22. *6.*
e *hyp.*
f *ſch.*12.10
g 10. 10.
h *ſch.*12.10
k 1. *6.*
l 10. 10.
m *ſch.* 12. 10.
n 13. 10.
o 1. *6.*
p 10. 10.

PROP. XXIV.



Mediæ A *commenſurabilis* B, *media eſt.*

Ad CD ρ' *a* fac rectang. CE=Aq; *a* & rectang. CF=Bq. Quoniam Aq (CE) eſt $\mu\nu$, *b* & CD ρ', *c* erit latitudo

a 11. *6.*
b *hyp.*
c 23. 10.

d 1.6. DE $\stackrel{s}{\rho}$ ⊣ CD. Quoniam vero CE. CF *d* ::
e *hyp.* ED. DF, & CE *e* ⊐ CF, *f* erit ED ⊐ DF.
f 10. 10. *g* ergo DF est $\stackrel{s}{\rho}$ ⊐ CD. *h* ergo rectang. CF
g 2. & 13. (Bq) est $\mu\nu$ & proinde B est μ. Q. E. D.
10. Nota *quod signum* ⊣ *plerumque valet poten-*
h 22. 10. *tia tantum commensurabile, ut in hac demonstratione, & in praeced. &c. quod intellige, ut ex usu erit, & juxta citationes.*

Coroll.

Hinc liquet spatium medio spatio commensurabile medium esse.

LEMMA.

A——————

B—————

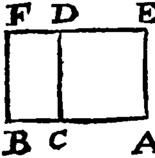
C———

Duas rectas medias A, B *longitudine commensurabiles; item duas* A, C *potentia tantum commensurabiles invenire.*

a *lem.* 22. *a* Sit A μ quaevis; sume *b* B ⊐ A; *c* & C ⊣ A.
10. & 13.6 *d* Factum esse liquet.
b 2. *lem.* 10
10.

PROP. XXV.

c 3. *lem* 10
10.
d *constr.*
& 24. 10.

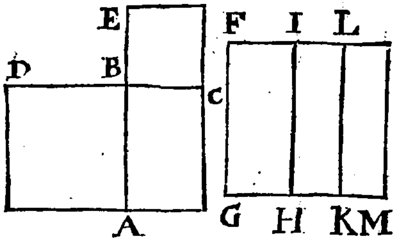


Quod sub DC, CB *mediis longitudine commensurabilibus rectis lineis continetur rectangulum* DB, *medium est.*

Super DC constituatur quadratum DA. Quoniam AC.
a 1. 6. (DC) CB *a* :: DA. DB. & DC
b 10. 10. ⊐ CB; *b* erit DA ⊐ DB. *c* ergo DB est $\mu\nu$.
c 24. 10. Q. E. D.

PROP.

PROP. XXVI.



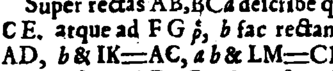
Quod sub mediis potentia tantum commensurabilibus rectis lineis AB, BC *continetur rectangulum* AC, *vel rationale est, vel medium.* a 46. 1.

Super rectas AB, BC *a* describe quadrata AD, CE. atque ad FG ρ̊, *b* fac rectangula FH = AD, *b* & IK = AC, *a b* & LM = CE. b cor. 16. 6.

Quadrata AD, CE, hoc est, rectangula FH, LM *c* sunt μα, & ⌐⊔; ergo eandem habentes rationem GH, KM sunt *d* ρ̊, & *e* ⌐⊔. *f* ergo GH × KM est ρ̊ν. atqui quia AD, AC, CE, hoc est FH, IK, LM *g* sunt ∺; & *h* proinde GH, HK, KM etiam ∺, *k* erit HKq = GH × KM; *l* ergo HK est ρ̊; vel ⌐⊔, vel ⌐⊐ IH (GF;) si ⌐⊔, *m* ergo rectang. IK vel AC est ρ̊ν. Sin ⌐⊐. *n* ergo AC est μν. Q. E. D.

c hyp. & 24. 20. d 23. 10. e 10. 10. f 20. 10. g sch. 22. 6. h 1. 6. k 17. 6. l 12. 10. m 20. 10. n 22. 10.

LEMMA A.



Si A, *&* E *sint* ⌐⊐ *tantum;*

Erunt primo Aq, Eq, Aq+Eq, Aq—Eq *a* ⌐⊔.

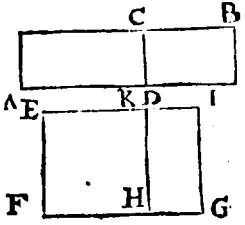
Erunt secundo, Aq, Eq, Aq+Eq, Aq—Eq ⌐⊔ AE, & 2 AE. Nam A. E *b* :: Aq. AE *b* :: AE. Eq. ergo cum A *c* ⌐⊔ E. *d* erit Aq ⌐⊔ AE, *e* & 2 AE. item Eq *d* ⌐⊔ AE, *e* & 2 AE. *c* quare cum Aq+Eq ⌐⊔ Aq, & Eq; & Aq—Eq ⌐⊔ Aq, &

a hyp. & 10. 16. 6. b 1. . c hyp. d 10. 10. e 4. 10.

f 14. 10. Eq, *f* erunt Aq+Eq, *f* & Aq—Eq ⊐⊏ AE, & 2 AE.

Hinc erunt tertio, Aq, Eq, Aq+Eq, Aq—Eq, 2 AE *g* ⊐⊏ Aq+Eq+2 AE ; & Aq+Eq—2 AE.
g 14.10. & *g* & Aq+Eq+2 AE ⊐⊏ Aq+Eq—2 AE.
17. 10. *h* (Q. A—E.)
h *cor*.7.10.

PROP. XXVII.

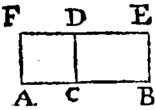


Medium AB *non superat medium* AC *rationali* DB.

a *cor*.16.6. Ad EF ρ', *a* fac EG = AB, *a* & EH = AC. Rectangula AB, AC, hoc est, EG, EH
b *hyp*. *b* sunt μα, *c* ergo FG,
c 23. 10. & FH sunt ρ' ⊐⊏ EF.
d 3. *ax*. 1. itaque si KG, *d* id est DB sit ρ'ν. *e* erit HG ⊐⊏
e 21. 10. HK; *f* quare HG ⊐⊏ FH. *g* ergo FGq ⊐⊏ FHq.
f 13. 10. sed FH est ρ'. *h* ergo FG est ρ̇. verum prius
g *lem*. 26. erat FG ρ̇. Quæ repugnant.
10.

SCHOL.

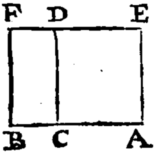
h *sch*.12.10



1. *Rationale* AE *superat rationale* AD *rationali* CE.

a *hyp*. Nam AE *a* ⊐⊏ AD ; *b* ergo
b *cor*. 16. 10. AE ⊐⊏ CE. *c* quare CE est ρ'ν. Q. E. D.

c *sch*.12.10

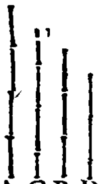


2. *Rationale* AD *cum rationali* CF *facit rationale* AF.

a *sch* 12.10 Nam AD *a* ⊐⊏ CF; *b* quare
b 16. 10. AF ⊐⊏ AD, & CF. *c* proinde
c *sch*.12.10 AF est ρ'ν. Q. E. D.

PROP.

PROP. XXVIII.

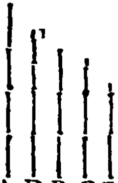


A C B D

Medias invenire (C, & D) qua rationale CD contineant.

a Sume A, & B ρ' ⊤. *b* fac A. C :: a lem. 21.
C. B. *c* atque A. B :: C. D. Dico 10.
factum. Nam AB (Cq) *d* est $\mu\nu$; b 13. 6.
d unde C est μ. quum vero A. B *e* :: c 12. 6.
C. D, *f* erit C ⊤ D. *g* ergo D est μ. d 22. 10.
porro permutando A. C :: B. D. *e* hoc e constr.
est C. B :: B. D. *h* ergo Bq = CD. f 10. 10.
atqui Bq *e* est $\rho'\nu$. *h* ergo CD est $\rho'\nu$. Q.E.F. g 24. 10.
In numeris, sit A, $\sqrt{2}$; & B, $\sqrt{6}$. ergo C est h 17. 6.
$v\sqrt{12}$. fac $\sqrt{2}$. $\sqrt{6}$ :: $v\sqrt{12}$. D. vel $v\sqrt{4}$. $v\sqrt{36}$ :: $v\sqrt{12}$. D. erit D, $v\sqrt{108}$. atqui $v\sqrt{12}$ in h sch. 12. 10
$v\sqrt{108} = v\sqrt{1296} = \sqrt{36} = 6$. ergo CD est 6.
item C. D :: 1. $\sqrt{3}$. quare C ⊤ D.

PROP. XXIX.



A D B C E

Medias invenire potentia tantum commensurabiles D, & E, quæ medium DE contineant.

a Sume A, B, C ρ' ⊤. Fac A. D a lem. 21.
b :: D. B. *c* & B. C :: D. B. Dico 10.
factum. b 13. 6.
Nam AB *d* = Dq & AB *e* est $\mu\nu$; c 12. 6.
ergo D est μ. & B *f* ⊤ C. *g* ergo d 17. 6.
D ⊤ E. *h* ergo E est μ. porro, e 22. 10.
B. C *f* :: D. E, & permutando B. D :: C. E. f constr.
k hoc est D. A :: C. E. *l* ergo DE = AC. Sed g 10. 10.
AC *m* est $\mu\nu$. ergo DE est $\mu\nu$. Q. E. D. h 24. 10.
In numeris sit A, 20; & B, $\sqrt{200}$; & C, $\sqrt{80}$. k constr. &
Ergo D est $\sqrt{\sqrt{80000}}$; & E $v\sqrt{12800}$. Ergo cor. 4. 5.
DE $= \sqrt{\sqrt{1024000000}} = \sqrt{32000}$. & D. E l 16. 6.
:: $\sqrt{10}$. 2. quare D ⊤ E. m 22. 6.

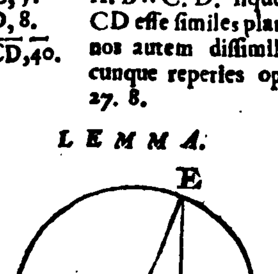
SCHOL.

A, 6.	C, 12.
B, 4.	D, 8.
$\overline{AB}$, 24.	$\overline{CD}$, 96.
A, 6.	C, 5.
B, 4.	D, 8.
$\overline{AB}$, 24.	$\overline{CD}$, 40.

Invenire duos numeros planos similes vel dissimiles.

Sume quoscunque quatuor numeros proportionales, A. B :: C. D. liquet AB, & CD esse similes planos. Planos autem dissimiles quotcunque reperies ope scholii 27. 8.

LEMMA.



1. *Duos numeros quadratos* (DEq & CDq) *invenire, ita ut compositum ex ipsis* (CEq) *quadratum etiam sit.*

Sume AD, DB numeros planos similes (quorum ambo pares sint, vel ambo impares) nimirum AD, 24. & DB, 6. Horum summa, (AB) est 30; differentia (FD) 18, cujus semissis
a 18. 8. (CD) est 9. *a* Habent vero plani similes AD, DB unum medium numerum proportionalem, nempe DE. patet igitur singulos numeros CE,
b 47. 1. CD, DE rationales esse; proinde CEq (*b* CDq + DEq) est numerus quadratus requisitus.

Facile itaque invenientur duo numeri quadrati, quorum excessus sit quadratus, vel non quadratus numerus. nempe ex eadem constructione,
c 3. ax. 1. *c* erit CEq − CDq = DEq.

Quod si AD, DB sint numeri plani dissimiles,

les, non erit media proportionalis (DE) numerus rationalis; proinde quadratorum CEq, CDq excessus (DEq) non erit numerus quadratus.

LEMMA 2.

2. *Duos numeros quadratos* B, C *invenire, ita ut compositus ex ipsis* D, *non sit quadratus. item, quadratum numerum* A *dividere in duos numeros* B, C *non quadratos.*

A, 3. B, 9. C, 36. D, 45.

1. Sume numerum quemlibet quadratum B, sitque C=4 B; & D=B+C. Dico factum.

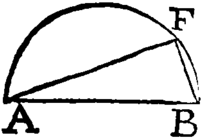
Nam B est *Q.* ex constr. item quia B. C :: 1. 4 :: *Q. Q.* *a* erit C etiam quadratus. Sed quoniam B+C. (D) C :: 5. 4 :: non *Q. Q.* *b* non erit D numerus quadratus. *Q.* E. F. a 24. 8. b *cor.*24.8.

A, 36. B, 24. C, 12. D, 3. E, 2. F, 1.

2. Sit A numerus quivis quadratus. Accipe D, E, F numeros planos dissimiles, sitque D=E+F. fac D. E :: A. B. & D. F. :: A. C. Dico factum.

Nam quia D. E+F :: A. B+C. & D=E+F, *a* erit A=B+C. Jam dic B quadratum esse. *b* ergo A & B, & *c* proinde D & E, sunt numeri plani similes, contra hypoth. idem absurdum sequetur, si C dicatur quadratus. ergo, &c. a 14. 5. b 21.*def.*7. c 26. 8.

PROP. XXX.



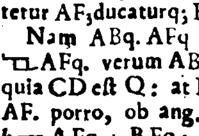
C .... E ..... D

Invenire duas rationales AB, AF *potentia tantum commensurabiles, ita ut major* AB *plus possit, quam minor* AF, *quadrato rectæ lineæ* BF *longitudine sibi commensurabilis.*

a 1. *lem.* Exponatur AB, ρ́. *a* Su-
29. 10. me CD, CE numeros quadratos, ita ut CD − CE
b 3. *lem.* (ED) sit non Q. *b* Fiatque CD. ED ∴ ABq. AFq.
10. 10. In circulo super AB diametrum descripto *c* ap-
c 1. 4. tetur AF, ducaturq; BF. Sunt AB, AF, quas petis.
d *constr.* Nam ABq. AFq *d* :: CD. ED. *e* ergo ABq
e 6. 10. ⊓⊔ AFq. verum AB est ρ́. *f* ergo AF est ρ́. sed
f *sch.* 12. 10 quia CD est Q: at ED non Q: *g* erit AB ⊓⊔
g 9. 10. AF. porro, ob ang. *h* rectum AFB, est ABq
h 31. 3. *k* = AFq + BFq; cum igitur ABq, AFq ::
k 47. 1. CD. ED. per conversionem rationis erit ABq.
l 9. 10. BFq :: CD. CE :: Q. Q. *l* ergo AB ⊓⊔. BF. Q. E. F.

In numeris; sit AB, 6; CD, 9. CE, 4; quare ED, 5. Fac 9. 5 :: 36. (Q. 6) AFq. erit AFq 20. proinde AF √20. ergo BFq = 36 − 20 = 16. quare BF est 4.

PROP. XXXI.



C .... E .... D

Invenire duas rationales AB, AF *potentia tantum commensurabiles, ita ut major* AB *plus possit, quam minor* AF, *quadrato rectæ lineæ* BF *sibi longitudine incommensurabilis.*

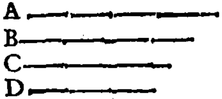
a 2. *lem.* Exponatur AB, ρ́ *a* acci-
29. 10. pe numeros CE, ED quadratos, ita ut CD = CE
+ ED sit non Q. & in reliquis imitare constructionem præcedentis. Dico factum.

Nam,

Nam, ut ibi, AB, AF sunt ῥ ⌐⊥. item ABq. BFq :: C D. E D. ergo cum C D sit non Q. *b* erunt AB, BF ⌐⊏. Q. E. F. b 9. 10.

In numeris, sit AB, 5. C D, 45. CE = 36; E D = 9. Fac 45. 9 :: 25 (ABq.) 5 (AFq.) ergo A F = √5. proinde BFq = 45 − 25 = 20, quare BF = √20.

PROP. XXXII.



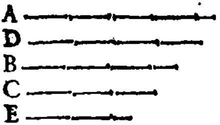
Invenire duas medias C, D *potentia tantum commensurabiles, quæ rationale* CD *contineant, ita ut major* C *plus possit, quam minor* D, *quadrato rectæ lineæ sibi longitudine commensurabilis.*

a Accipe A, & B ῥ ⌐⊥; ita ut √Aq−Bq ⌐⊏ A. *b* Fiatque A. C :: C. B, *c* atque A. B :: C. D. Dico factum. a 30. 10. b 13. 6. c 12. 6.

Nam quia A, & *d* B sunt ῥ ⌐⊥, *e* erit C (*f* √AB) μ. item *g* ideo C ⌐⊥ D. *h* ergo D etiam μ. porro quia A. B *d* :: C. D; & permutatim A. C :: B. D :: C. B; & Bq *d* est ῥν, erit CD *k* (Bq) ῥν. Denique quia √Aq − Bq *d* ⌐⊏ A, *l* erit √Cq − Dq ⌐⊏ C. ergo, &c. Sin √Aq − Bq ⌐⊏ Aq, erit √Cq − Dq ⌐⊏ C. d constr. e 22. 10. f 17. 6. g 10. 10. h 24. 10. k 17. 6. l 15. 10.

In numeris, sit A, 8; B, √48 (√:64 − 16) ergo C = √AB = v√3072. & D = v√1728. quare CD = v√5308416 = √2304.

PROP. XXXIII.

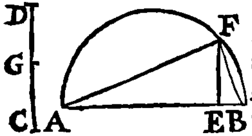


Invenire duas medias D, E *potentia solum commensurabiles, quæ medium* D E *contineant, ita ut major* D *plus possit, quam minor* E, *quadrato rectæ lineæ sibi longitudine commensurabilis.*

Sume

a 30. 10. *a* Sume A, & C $\dot{\rho}$, ⊤⊢; ita ut $\sqrt{}$ Aq — Cq ⊤⊥
b lem. 21. A. *b* sume etiam B ⊤⊢ A, & C; & fac A. D *c* ::
10. D. B *d* :: C. E. Erunt D, & E quæsitæ.
c 13. 6. Nam quoniam A, & C *e* sunt $\dot{\rho}$, *e* & B ⊤⊢
d 12. 6. A & C, *f* erit B $\dot{\rho}$, & D ($\sqrt{}$ AB) *g* erit μ.
e constr. *e* Quia vero A. D :: C. E. erit permutando A.
f sch. 12. 10 C :: D. E. ergo cum A ⊤⊢ C, *h* erit D ⊤⊢ E.
g 21. 10. *k* ergo B est μ. porro, *l* quia D. B :: C. E; *l* &
h 10. 10. BC est μν, etiam DE ei *m* æquale est μν. denique
k 24. 10. propter A. C :: D. E. *e* quia $\sqrt{}$ Aq — Cq ⊤⊥
l 22. 10. A, *n* erit $\sqrt{}$ Dq — Eq ⊤⊥ D. ergo, &c. Sin $\sqrt{}$
m 16. 6. Aq — Cq ⊤⊥ A, erit $\sqrt{}$ Dq — Eq ⊤⊥ Eq.
n 15. 10. In numeris, sit A, 8; C, $\sqrt{}$ 48; B, $\sqrt{}$ 28. erit
D v $\sqrt{}$ 3072; & E v $\sqrt{}$ 588. quare D. E :: 2 $\sqrt{}$ 3.
& DE = $\sqrt{}$ 1344.

PROP. XXXIV.



a 31. 10. b 10. 1. *Invenire duas rectas lineas AF, BF potentia incommensurabiles, quæ faciant compositum quidem ex ipsarum quadratis rationale, rectangulum vero sub ipsis contentum, medium.*

c 28. 6.
d 12. 6. *a* Reperiantur AB, CD $\dot{\rho}$ ⊤⊢; ita ut $\sqrt{}$ ABq —
e cor. 8. 6. CDq ⊤⊥ AB. *b* biseca CD in G. *c* fac rectang.
& 17. 6. ABB = GCq. Super AB diametrum duc se-
f 7. 5. micirculum AFB. erige perpendicularem EF.
g 19. 10. duc AF, BF. Hæ sunt quæ indagandæ erant.
h 10. 10. Nam AE. BE *d* :: BA × AE. AB × BE. Sed
k 31. 3. & BA × AE *e* = AFq; *e* & AB × BE = FBq. *f* ergo
47. 1. AE. EB :: AFq. FBq. ergo cum AE *g* ⊤⊥
l constr. EB, *h* erit AFq ⊤⊥ FBq. Quinetiam ABq
m 1. ax 1. (*k* AFq + FBq) *l* est $\dot{\rho}\nu$. denique EFq *l* =
n 22. 10. AEB *l* = CGq. *m* ergo EF = CG. ergo CD ×
o 24. 10. AB = 2 EF × AB. atqui CD × AB *n* est μν.
p sch. 22. 6. *o* ergo AB × EF, *p* vel AF × FB, est μν. Q. E. D.

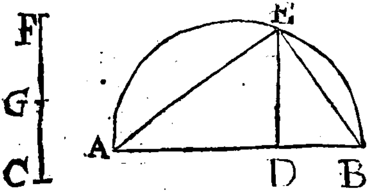
Ex-

Explicatio per numeros.

Sit AB, 6. CD, $\sqrt{12}$. quare CG $= \sqrt{1\frac{3}{4}} - \sqrt{3}$. Est vero AE $= 3 + \sqrt{6}$. & EB $= 3 - \sqrt{6}$. & unde AF erit $\sqrt{:}\ 18 + \sqrt{216}$. Et FB, $\sqrt{:}\ 18 - \sqrt{216}$. item AFq + FBq est 36, & AF $\times$ FB $= \sqrt{108}$.

Cæterum AE invenitur sic. Quia BA (6.) AF :: AF. AB; erit 6 AB = AFq = AEq + 3 (EFq.) ergo 6 AE − AEq = 3. pone 3 + e = AE. ergo 18 + 6e − 9 − 6e − ee, hoc est 9 − ee = 3. vel ee = 6. quare e = $\sqrt{6}$. proinde AE = 3 + $\sqrt{6}$.

PROP. XXXV.



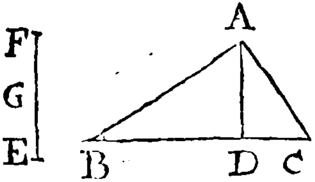
Invenire duas rectas lineas AE, EB potentia incommensurabiles, quæ faciant compositum quidem ex ipsarum quadratis medium, rectangulum vero sub ipsis contentum, rationale.

a Sume AB, & CF μ ⊓⊔, ita ut AB $\times$ CF fit $\dot{\rho}\nu$, atque $\sqrt{}$ ABq − CFq ⊔ AB. & reliqua fiant, ut in præcedenti. erunt AE, EB, quas petis. a 32. 10.

Nam, ut isthic ostensum est, AEq ⊔ EBq: item ABq (AEq + EBq) est $\mu\nu$. & denique AB $\times$ CF *b* est $\dot{\rho}\nu$, idcirco & *c* AB $\times$ DE, *d* hoc est, AE $\times$ EB, est $\dot{\rho}\nu$. ergo, &c. b *constr.* c *schol.* 12. 10. d *sch.* 22. 6.

PROP.

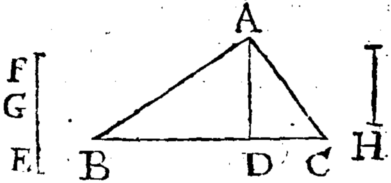
PROP. XXXVI.



Invenire duas rectas lineas BA, AC potentia incommensurabiles, quæ faciant & compositum ex ipsarum quadratis medium, & rectangulum sub ipsis comprehensum medium, incommensurabileque composito ex ipsarum quadratis.

a 33. 10. *a* Accipe BC & EF μ ⌐⊓˯ ; ita ut BC × EF sit
μν. & √ Bcq − EFq ⌐⊔ BC. & reliqua fiant,
ut in præcedentibus. Erunt BA, AC exoptata.
Nam, ut prius, BAq ⌐⊔ ACq ; item BAq +
ACq est μν. & BA × AC est μν. Denique BC
b constr. *b* ⌐⊔ EF, atque *c* ideo BC ⌐⊔ EG; estque BC.
c 13. 10. EG *d* :: BCq. BC × EG, (BC × AD, vel BA
d 1. 6. × AC.) *e* ergo BCq (ABq + ACq) ⌐⊔ BA ×
e 14. 10. AC, ergo, &c.

Schol.



Invenire duas medias longitudine & potentia incommensurabiles.

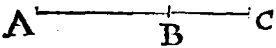
a 36. 10. *a* Sume BC μ. sitque BA × AC μν, & ⌐⊔
b 13. 6. BCq (BAq + ACq.) *b* Fac BA. H :: H.
AC. Sunt BC, & H μ ⌐⊓˯. Nam BC est μ.
c 17. 6. *a* & BA × AC (*c* Hq) est μν. quare H est etiam

μ.

μ. *d* item BA x AC ⊏⊐ BCq; ergo Hq ⊏⊐ BCq. ergo, &c. d 14. 10.

Principium senariorum per compositionem.

PROP. XXXVII.

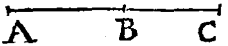


Si duæ rationales AB, BC *potentia tantum commensurabiles componantur, tota* AC *irrationalis est; vocetur autem ex binis nominibus.*

Nam quia AB *a* ⊏⊐ BC, *b* erit ACq ⊏⊐ ABq. Sed AB *a* est $\dot{\rho}$. *c* ergo AC est $\dot{\rho}$. Q.E.D.

a *hyp.*
b *lem.* 26. 10.
c 11. *def.* 10

PROP. XXXVIII.

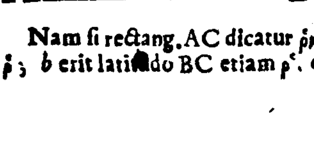


Si duæ mediæ AB, BC *potentia tantum commensurabiles componantur; quæ rationale contineant, tota* AC *irrationalis est; vocetur autem ex binis mediis prima.*

Nam quoniam AB *a* ⊏⊐ BC, *b* erit ACq ⊏⊐ AB x BC, $\dot{\rho}\nu$. *c* ergo AC est $\dot{\rho}$. Q.E.D.

a *hyp.*
b *lem.* 26. 10.
c 11. *def.* 10

LEMMA.



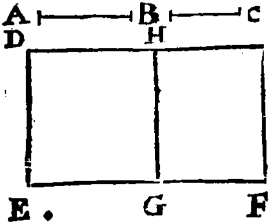
Quod sub linea rationali AB, *& irrationali* BC *continetur rectangulum* AC, *irrationale est.*

Nam si rectang. AC dicatur $\dot{\rho}\nu$; quum AB sit $\dot{\rho}$; *b* erit latitudo BC etiam $\dot{\rho}$. contra Hyp.

a *hyp.*
b 21. 10.

PROP.

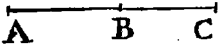
PROP. XXXIX.



Si duæ mediæ AB, BC potentia tantum commensurabiles componantur, quæ medium contineant, tota AC irrationalis erit; vocetur autem ex binis mediis secunda.

Ad expositam
a cor.16.6. DE ρ̇ a fac rectang. DF = ACq; b & DG =
b 47. 1. & ABq + BCq.
11.6. Quoniam ABq c ⊏⊐ BCq, d erit ABq +
c hyp. BCq, hoc est DG ⊏⊐ ABq; sed ABq e est μν.
d 16. 10. e ergo DG est μν. verum rectang. ABC poni-
e 24. 10. tur μν; e ideoque 2 ABC (f HF) est μν; g er-
f 4.2. go EG, & GF sunt ρ̇. quia vero DG h ⊏⊐ HF;
g 23. 10. atque DG. HF :: k EG. GF l erit EG ⊏⊐
h lem. 26. GF. m ergo tota EF est ρ̇. n quare rectang. DF
1. est ρ̇ν. o ergo √DF, id est AC, est ρ̇. Q. E. D.

k 1.6.
PROP. XL.

l 10. 10.


Si duæ rectæ lineæ AB,
m 37.10. *BC potentia tantum com-*
n lem. 38. *mensurabiles componantur, quæ faciant compositum*
10. *quidem ex ipsarum quadratis rationale, quod autem*
o 11.def.10 *sub ipsis continetur, medium; tota recta linea AC, irrationalis erit: vocetur autem major.*

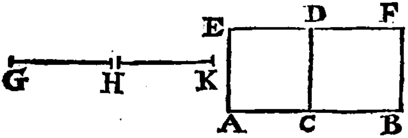
a hyp. Nam quia ABq + BCq a est ρ̇ν, & b ⊏⊐ 2
b sch.12.10 ABC c μν, & proinde ACq (d ABq + BCq +
c hyp.&24. 2 ABC) e ⊏⊐ ABq + BCq ρ̇ν, f erit AC ρ̇.
10. Q. E. D.
d 4. 2.
e 17. 10.
f 11.def.10

PROP. XLI.

Si duæ rectæ lineæ AC, CB *potentia incommensurabiles componantur, quæ faciant compositum quidem ex ipsarum quadratis medium, quod autem sub ipsis continetur, rationale; tota recta linea* AB *irrationalis erit: vocetur autem rationale ac medium potens.*

Nam 2 rectang. ACB, *a* $\rho'\nu$ *b* ⊓⊔ ACq + CBq *c* $\mu\nu$. *d* ergo 2 ACB *d* ⊓⊔ ABq. quare *e* AB est $\dot{\rho}$. Q. E. D. a *hyp.* & *sch.* 12. 10. b *sch.* 12. 10 c *hyp.* d 17. 10. e 11. *def.* 10

PROP. XLII.

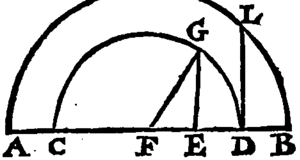


Si duæ rectæ lineæ GH, HK *potentia incommensurabiles componantur, quæ faciant & compositum ex ipsarum quadratis medium, & quod sub ipsis continetur medium, incommensurabileque composito ex quadratis ipsarum; tota recta linea* GK *irrationalis erit: vocetur autem bina media potens.*

Ad expositam FB ρ', fiant rectang. AF = GKq; & CF = GHq + HKq. Quoniam GHq + HKq (CF) *a* est $\mu\nu$; latitudo CB *b* erit ρ'. Item a *hyp.*
quia 2 rectang. GHK (*c* AD) *a* est $\mu\nu$, etiam b 23. 10.
AC *b* erit ρ'. Porro quia rectang AD *a* ⊓⊔ CF, c 4. 2.
d atque AD. CF :: AC. CB, *e* erit AC ⊓⊔ CB d 2. 6.
f Quare AB est *g* $\dot{\rho}$. ergo rectang. AF, id est, e 10. 10.
GKq est $\dot{\rho}\nu$. *h* proinde GK est $\dot{\rho}$. Q. E. D. f 37. 10.
g *lem.* 38. 10.
h 11. *def.* 10

PROP.

PROP. XLIII.



Quæ ex binis nominibus AB, *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

Si fieri potest, binomium AB alibi in E secetur in alia nomina AE, EB. Liquet AB secari utrobique inæqualiter, quia AD ⊐ DB, & AE ⊐ EB.

Quoniam rectangula ADB, AEB *a* sunt $\mu\alpha$;
a 37. 10. *a* & singula ADq, DBq, AEq, EBq sunt $\rho'\alpha$; *b* adeoque
b sch. 27. 10 ADq + DBq, *b* & AEq + EBq etiam $\rho'\alpha$, *b* idcirco ADq + DBq —: AEq + EBq.
c sch. 5. 2. *c* hoc est, 2 AEB — 2 ADB est $\rho'\nu$. *d* ergo AEB
d sch. 12. 10 — ADB $\rho'\nu$. ergo $\mu\nu$ superat $\mu\nu$ per $\rho'\nu$. *e* Q.E.A.
e 27. 10.

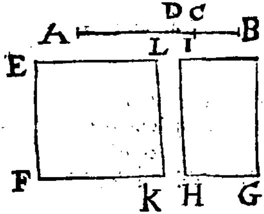
PROP. XLIV.

Quæ ex binis mediis prima AB, *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

Puta AB dividi in alia nomina AE, EB. quo
a 38. 10. posito, singula ADq, DBq, EBq, *a* sunt $\mu\alpha$; *a* &
b sch. 27. 10 rectangula ADB, AEB, eorumque dupla, sunt
c sch. 5. 2. $\rho'\alpha$. *b* ergo 2 AEB — 2 ADB, *c* hoc est ADq
d 27. 10. + DBq —: AEq + EBq est $\rho'\nu$. *d* Q.E.A.

PROP.

PROP. XLV.



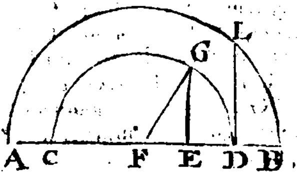
Quæ ex binimediis secunda AB, ad unum duntaxat punctum C dividitur in nomina AC, CB.

Dic alia esse nomina AD, DB. Ad expositam EF $\acute{\rho}$, fac rectang. EG = ABq. & EH = ACq + CBq; item EK = ADq + DBq.

Quoniam ACq, CBq *a* sunt $\mu\alpha$ ⊏⊐; *b* erit ACq + CBq (EH) $\mu\nu$. *c* ergo latitudo FH est $\acute{\rho}$ *a* quin & rectang. ACB, *d* ideoque 2 ACB *e* (IG) est $\mu\nu$: *c* ergo HG, est etiam $\acute{\rho}$. Cum igitur EH *f* ⊏⊐ IG, *g* atque EH. IG :: FH. HG; *h* erunt FH, HG ⊏⊐. *k* ergo FG est binomium; cujus nomina FH, HG. Simili argumento FG est bin. cujus nomina FK, KG, contra 43. hujus.

a 39. 10.
b 16. & 24. 10.
c 23. 10.
d 24. 10.
e 4. 2.
f lem. 26. 10
g 1. 6.
h 10. 10.
k 37. 10.

PROP. XLVI.

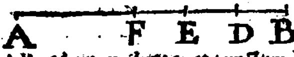


Major AB *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

Concipe alia nomina AE, EB. quo posito rectangula ADB, AEB *a* $\mu\alpha$; *a* & tam ADq + DBq, quam AEq + EBq sunt $\acute{\rho}\alpha$. *b* ergo ADq + DBq — : AEq + EBq, *c* hoc est, 2 AEB — 2 ADB est $\acute{\rho}\nu$. *d* Q. E. N. E. P.

a 40. 10.
b sch. 27. 10
c sch. 5. 2.
d 27. 10.

PROP. XLVII.



Rationale ac medium potens AB, *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

a 41. 10. Dic alia nomina AB, EB. *a* ergo tam AEq + EBq, quam ADq + DBq sunt $\mu\alpha$. *a* & re-

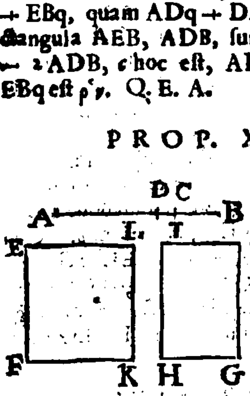
b sch. 27. ctangula AEB, ADB, sunt $\rho\alpha$. *b* ergo 2 AEB

10. + 2 ADB, *c* hoc est, ADq + DBq —: AEq +

c sch. 5. 2. EBq est $\rho\nu$. Q. E. A.

d 27. 10.

PROP. XLVIII.



Bina media potens AB, *ad unum duntaxat punctum* C *dividitur in nomina* AC, CB.

Vis AB dividi in alia nomina AD, DB. Ad expositam BF ρ, fiant rectang. EG = ABq, & EH = ACq + CBq, & EK = ADq + DBq. Quoniam ACq

a 42. 10. + CBq, nempe EH, *a* est $\mu\nu$, *b* erit latitudo FH

b 23. 10. ρ. Item quia 2 ACB, *c* hoc est, IG, est *a* $\mu\nu$,

c 4. 2. *b* erit HG etiam ρ. Ergo cum EH *a* ⊥ IG, sit-

d 1. 6. que EH. IG *d* :: FH. HG, *e* erit FH ⊥ HG.

e 10. 10. *f* ergo FG est bin. cujus nomina FH. HG. Eo-

f 37. 10. dem modo ejusdem nomina erunt FK, KG; contra 43 hujus.

Definitiones secundæ.

Exposita rationali, & quæ ex binis nominibus, divisa in nomina; cujus majus nomen plus possit quam minus, quadrato rectæ lineæ sibi longitudine commensurabilis.

I. Siquidem cujus nomen expositæ rationali

com-

commensurabile sit longitudine, vocetur tota ex binis nominibus prima.

II. Si vero minus nomen expositæ rationali longitudine sit commensurabile, vocetur ex binis nominibus secunda.

III. Quod si neutrum ipsorum nominum sit longitudine commensurabile expositæ rationali, vocetur ex binis nominibus tertia.

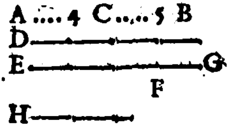
Rursus, si majus nomen plus possit quam minus, quadrato rectæ lineæ sibi longitudine incommensurabilis;

IV. Si quidem majus nomen expositæ rationali commensurabile sit longitudine, vocetur ex binis nominibus quarta.

V. Si vero minus nomen, vocetur quinta.

VI. Quod si neutrum ipsorum nominum, vocetur sexta.

PROP. XLIX.



Invenire ex binis nominibus primam, EG.

a Sume AB, AC numeros quadratos, quorum excessus CB non Q. exponatur D ρ. *b* accipe quamvis EF $\sqcap\!\sqcup$ D. *c* fac AB. CB :: EFq. FGq. erit EG bin. 1.

Nam EF *d* $\sqcap\!\sqcup$ D. *e* ergo EF ρ. *f* item EFq $\sqcap\!\sqcup$ FGq. *g* ergo FG est etiam ρ. item *d* quia EFq. FGq :: AB. CB :: Q. non Q. *h* erit EF $\sqcap\!\sqcup$ FG. denique quia per conversionem rationis EFq. EFq $-$ FGq :: AB. AC :: Q. Q. *k* erit EF $\sqcap\!\sqcup$ $\sqrt{}$ EFq $-$ FGq. *l* ergo EG est bin. 1. Q. E. F.

a *sch.* 29. 10.
b 2. *lem.* 10. 10.
c 3. *lem.* 10. 10.
d *constr.*
e 6. *def.* 10.
f 6. 10.
g *sch.* 12. 10.
h 9. 10.
k 9. 10.
l 1. *def.* 48. 10.

Explicatio per numeros.

Sit D, 8. EF, 6. AB, 9. CB, 5. quare cum 9. 5.

9. 5 :: 36. 20. erit FG, $\sqrt{20}$. proinde BG est 6 + $\sqrt{20}$.

PROP. L.

A .... 4 C ..... 5 B
D ————————
E ————|———— G
F
H ————

Invenire ex binis nominibus secundam, EG.

Accipe AB, & AC numeros quadratos, quorum excessus CB sit non

Proba ut præcedentem. Q. Sit D exposita ρ'. sume FG ⊐⊏ D. Fac CB. AB :: FGq. EFq. Erit EG quæsita.

Nam FG ⊐⊏ D, quare FG est ρ'. item EFq ⊐⊏ FGq. ergo EF est etiam $\dot{\rho}$. item quia FGq. EFq :: CB. AB :: non Q. Q. est FG ⊐⊏ EF. denique quia CB. AB :: FGq. EFq, inverseque AB. CB :: EFq. FGq, erit ut in præcedenti,

a 2. def. 48. 10. EF ⊐⊏ $\sqrt{EFq - FGq}$. *a* è quibus EG est bin. 2. Q. E. F.

In numeris, sit D, 8; FG, 10; AB, 9; CB, 5. erit EF, $\sqrt{180}$. quare EG est 10 + $\sqrt{180}$.

PROP. LI.

A .... 4 C ..... 5 B
L ....., 6
G ————
D ————|———— F
E
H ————————

Invenire ex binis nominibus tertiam, DF.

a sch. 29. 10. *a* Sume numeros AB, AC quadratos, quorum excessus CB non Q. sitq; L numerus non Q, proxime major quam CB, nempe unitate, vel binario. sit G exposita $\dot{\rho}$. *b* Fac L. AB

b 8 lem. 10. 10. :: Gq. DEq. *b* & AB. CB :: DEq. EFq. erit DF bin. 3.

c constr. 6. 10. Nam quia DEq *c* ⊐⊏ Gq, *d* est DE $\dot{\rho}$. item Gq. DEq :: L. AB :: non Q. Q. *e* ergo G ⊐⊏

d sch. 12. 10 DE. item quia DEq *e* ⊐⊏ EFq, *d* etiam EF

e 6. 10. est $\dot{\rho}$ quinetiam quia DEq. EFq :: AB. CB ::

f 9. 10. Q. non Q. *f* est DE ⊐⊏ EF. porro, quia per

constr.

constr. & ex æquali Gq. EFq :: L. CB :: non Q. Q. (nam *g* L, & CB non sunt similes plani numeri) *h* erit G etiam $\urcorner\!\sqcup$ EF. denique ut in præced. $\sqrt{}$DEq $-$ EFq $\urcorner\!\sqcup$ DE. *k* ergo DF est bin. 3. Q. E. F. *g* *sch.* 27.8. *h* 9. 10. *k* 3.*def*.48. 10.

In numeris, ~~sit AB, 9. CB,~~ 5; L, 6; G, 8. erit DE, $\sqrt{96}$ & EF, $\sqrt{\frac{480}{9}}$. quare DF $= \sqrt{96} + \sqrt{\frac{480}{9}}$.

PROP. LII.

A ... 3 C ...... 6 B
G ————
D ——— E ——— F
H ————

Invenire ex binis nominibus quartam, DF.

a Sume quemvis numerum quadratum AB, *a* quē divide in AC, CB non quadratos. sit G exposita $\rho^{\prime}$. *b* accipe DE $\urcorner\!\sqcup$ G. *c* Fac AB.CB :: DEq.EFq. erit DF bin.4. *a* *sch.* 29. 10. *b* 2.*lem*.10 *c* 3.*lem*.10

Nam ut in 49. hujus, DF ostendetur bin. item, quia per constr. & conversionem rationis DEq. DEq $-$ EFq :: AB. AC :: Q. non Q. *d* erit DE $\urcorner\!\sqcup$ $\sqrt{}$DEq $-$ EFq. *e* ergo DF est bin. 4. Q. E. F. 10. *d* 9. 10. *e* 4.*def*. 48 10.

In numeris, sit G, 8; DE, 6. erit EF $\sqrt{24}$. ergo DF est $6 + \sqrt{24}$.

PROP. LIII.

A ... 3 C ...... 6 B
G ————
D ——— E ——— F
HF

Invenire ex binis nominibus quintam, DF.

Accipe quemvis numerum quadratum AB, cujus segmenta AC, CB sint non Q. sit G exposita $\rho^{\prime}$. sume EF $\urcorner\!\sqcup$ G. fac CB. AB :: EFq. DEq. erit DF bin. 5.

Nam ut in 50. hujus, erit DF bin. & quia per constr. & invertendo DEq. EFq :: AB. CB, ideoque per conversionem rationis DEq. DEq $-$ EFq :: AB. AC :: Q. non Q. *a* erit *a* 9. 10. *b* 5.*def*. 48 10.

DB ⊏⊐ $\sqrt{}$ DEq − EFq. *b* ergo DF est bin. 4. Q. E. F.

In numeris, sit G, 7; EF, 6. erit DE $\sqrt{54}$. quare DF est $6 + \sqrt{54}$.

PROP. LIV.

A ..... 5 C .... 7 B

L 9

G———

D————— F

E

H———

Invenire ex binis nominibus sextam.

Accipe AC, CB primos numeros utcunque, sic ut AC + CB (AB) sit non Q. sume etiam

a 3. lem. quemvis L num. Q. sit G expol. ρ. *a* fiatque L.

10. 10. AB :: Gq. DEq. atque AB. CB :: DEq. EFq. erit DF. bin. 6.

Nam ut in 51. hujus, DF ostendetur bin. item quod DE, & EF ⊏⊐ G. denique igitur quia per constr. & conversionem rationis DEq.

b sch. 27. 8. DEq − EFq :: AB. AC :: non Q. Q. (Nam AB primus est ad AC, *b* ideoque ei dissimilis)

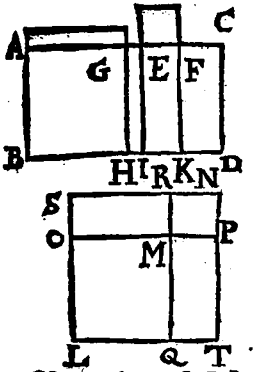
c 9. 10. *c* ergo DE ⊏⊐ $\sqrt{}$ DEq − EFq. *d* ergo DF est

d 6. def. 48. bin. 6. Q. E. F.

10. *In numeris*, sit G, 6; DE $\sqrt{48}$. erit EF $\sqrt{28}$. quare DF est $\sqrt{48} + \sqrt{28}$.

LEM-

LEMMA.



Sit AD *rectangulum, cujus latus* AC *secetur inæqualiter in* E; *bisectumq; sit segmentum minus* EC *in* F; *atque ad* AE, *a fiat rectang.* AGE = EFq; *perque* G, E, F *b ducantur ad* AB *parallelæ* GH, EI, FK. *c Fiat autem quadratum* LM = *rectang.* AH, *atque ad* OMP *productam c fiat quadratum* MN = GI; *rectæque* LOS, LQT, NRS, NPT *producantur.* a 28. 6. b 31. 1. c 14. 2.

Dico 1. MS, MT *sunt rectangula.* Nam ob quadratorum angulos OMQ, RMP rectos, *a* erit QMR recta linea. *b* ergo anguli RMO, QMP recti sunt. quare Pgra MS, MT sunt rectangula. a sch. 15. 1. b 13. 1.

2. *Hinc patet* LS *c* = LT; *& proinde* LN *esse quadratum.* c 2. ax. 2.

3. *Rectangula* SM, MT, EK, FD *æqualia sunt.* Nam quia rectang. AGE *d* = EFq, *e* erit AE. EF :: EF. GE. *f* ideoque AH. EK :: EK. GI. hoc est per constr. LM. EK :: EK. MN. *g* verum LM. SM :: SM. MN. ergo EK *h* = SM *k* = FD *l* = MT. d hyp. e 17. 6. f 1. 6. g sch. 22. 6. h 9. 5. k 36. 1.

4. *Hinc* LN *m* = AD. l 43. 1.

5. *Quia* EC *bisecta est in* F, *n patet* EF, FC, EC ⌐⊔ *esse.* m 2. ax. 1. n 16. 10.

6. Si AE ⌐⊔ EC, & AE ⌐⊔ √ AEq − ECq, *o* erunt AG, GE, AE ⌐⊔. item, quia 10, o 18. & 16.

 AG,

p 10. 10. AG. GE :: AH. GI *p* erunt AH, GI; hoc est LM, MN ⊤⊏. item iisdem positis,

7. OM ⊤⊏ MP. Nam per Hyp. AE, ⊤⊏

q 14. 10. EC, *q* ergo EC ⊤⊏ GE. *q* quare EF ⊤⊏ GE.

r 10. 10. sed EF. GE :: EK. GI. *r* ergo EK ⊤⊏ GI, hoc est SM ⊤⊏ MN. atqui SM. MN :: OM. MP. *r* ergo OM ⊤⊏ MP.

8. Sin ponatur AE ⊤⊏ $\sqrt{}$ AEq − ECq,

s 19. & 17. *s* patet AG, GE, AE esse ⊤⊏. unde LM ⊤⊏

10. MN. nam AG. GE :: AH. GI :: LM. MN.

His bene perspectis, facile sex sequentes Propositiones expediemus.

PROP. LV.

Si spatium AD *contineatur sub rationali* AB, *& ex binis nominibus prima* AC, (AE + EC;) *recta linea* OP *spatium potens irrationalis est, quæ ex binis nominibus appellatur.*

Suppositis iis, quæ in lemmate proxime præcedenti descripta, & demonstrata sunt, liquet rectam OP posse spatium AD. *a* Item AG, GE,

a hyp. & lem. 54. 10 AE sunt ⊤⊏. ergo cum AE *b* sit ρ' ⊤⊏ AB,

b hyp. *c* erunt AG, & GE, ρ' ⊤⊏ AB. *d* ergo rectan-

c sch. 12. 10 gula AH, GI, hoc est quadrata LM, MN sunt

d 20. 10. ρ'α ergo OM, MP sunt ρ' *e* ⊤⊢. *f* proinde OP

e lem. 54. 10. est bin. Q. E. D.

f 37. 10. *In numeris*, sit AB, 5; AC, 4 + $\sqrt{12}$. quare rectang. AD = 20 + $\sqrt{300}$ = quadr. LN. ergo OP est $\sqrt{15} + \sqrt{5}$; nempe bin. 6.

PROP. LVI.

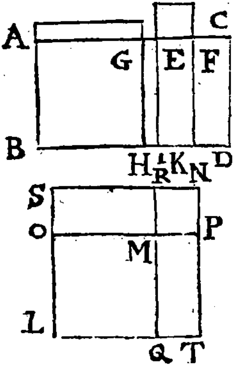
Si spatium* AD *contineatur sub rationali* AB, *& ex binis nominibus secunda* AC (AE + EC;) *recta linea* OP *spatium* AD *potens, irrationalis est, quæ ex binis mediis prima appellatur.

Rursus adhibito lemmate ad 54. hujus, erit $OP = \sqrt{AD}$. *a* item AE, AG, GE sunt $\sqcap$. ergo quum AE *b* sit $\dot{\rho}$, $\sqcap$ AB, *c* erunt AG, GE etiam ρ' $\sqcap$ AB. ergo rectangula AH, GI; hoc est OMq, MPq *d* sunt $\mu\alpha$. *e* quinetiam OM $\sqcap$ MP. denique EF $\sqcap$ EC, & EC *f* $\sqcap$ AB. *f* quare EF est ρ' $\sqcap$ AB. *g* ergo EK; hoc est SM, vel OMP est $\rho'\nu$. *h* Proinde OP est 2 μ prima. Q. E. D. — a hyp. & lem. 54. 10; b hyp.; c sch. 12. 10; d 22. 10.; e lem. 54. 10.; f hyp. 12. 10.

In numeris, sit AB, 5; & AC, $\sqrt{48} : +6$. ergo rectang. $AD = \sqrt{1200} : +30 = OPq$. ergo OP est $v\sqrt{675} + v\sqrt{75}$; nempe bimed. 1. — g 20. 10.; h 38. 10.

Vide Schem. 57.

PROP. LVII.



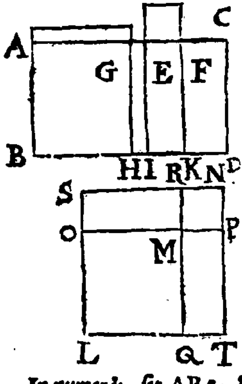
Si spatium* A D *contineatur sub rationali* AB, *& ex binis nominibus tertia* AC (AE+EC;) *recta linea* O P *spatium* AD *potens, irrationalis est, quæ ex binis mediis secunda dicitur.

Ut prius, $OPq = AD$. item rectangula AH, GI, hoc est OMq, MPq sunt $\mu\alpha$. *a* item EK, vel OMP est $\mu\nu$. *b* ergo OP est bimed. 2. — a hyp. & 22. 10.; b 39. 10.

In

In numeris, sit AB, 5; AC, $\sqrt{32} + \sqrt{24}$. quare AD est $\sqrt{800} + \sqrt{600} =$ OPq. proinde OP est $v\sqrt{450} + v\sqrt{50}$; hoc est bimed. 2.

PROP. LVIII.



Si spatium AD *contineatur sub rationali* AB, *& ex binis nominibus quarta* AC (AE + EC;) *recta linea* OP *spatium potens, irrationalis est, quæ vocatur major.*

a lem. 54. 10. Nam iterum OMq ^{a} ⊓⊔ MPq. rectang. vero AI, hoc est OMq + MPq ^{b} est ρ'ν. ^{c} item EK, vel OMP est μν. ^{d} ergo OP ($\sqrt{AD}$) est major. Q. E. D.

b hyp. & 29. 10.

c hyp. & 22. 10.

d 40. 10.

In numeris, sit AB 5; & AC, $4 + \sqrt{8}$. ergo rectang. AD est $20 + \sqrt{200}$. quare OP est $\sqrt{:20 + \sqrt{200}}$.

PROP. LIX.

Si spatium AD *contineatur sub rationali* AB, *& ex binis nominibus quinta* AC; *recta linea* OP *spatium* AD *potens, irrationalis est, quæ rationale & medium potens appellatur.*

Rursus OMP ⊓⊔ MPq. rectang. vero AI, vel OMq + MPq est μν. ^{a} item rectang. EK, vel OMP est ρ'ν. ^{b} ergo OP ($\sqrt{AD}$) est potens ρ'ν, & μν. Q. E. D.

a ut in præc.

b 41. 10.

In numeris, sit AB, 5; & AC $2 + \sqrt{8}$. ergo rectang. AD $= 10 + \sqrt{200} =$ OPq. quare OP est $\sqrt{:10 + \sqrt{200}}$.

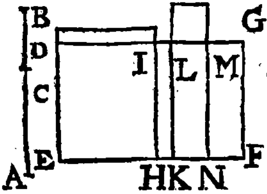
PROP. LX.

Si spatium AD *continеatur sub rationali* AB, *& ex binis nominibus sexta* BC (AE + EC;) *recta linea* OP *spatium* AD *potens, irrationalis est, quæ bina media potens appellatur.*

Ut sæpe prius, OMq ˥⊏ MPq. & OMq + MPq est $\mu\nu$. & rectang. (EK) OMP etiam $\mu\nu$. *a*ergo OP = √AD est potens 2 $\mu\alpha$. Q. E. D. a 42. 10.

In numeris, sit AB, 5; AC, √12 + √8; ergo rectang. AD, vel OPq est √300 + √200, proinde OP est √: √300 + √200.

LEMMA.



Sit recta AB *inæqualiter secta in* C, *sitque* AC *majus segmentum; & cuivis* DE *applicentur rectangula*, DF = ABq, & DH = ACq, & IK = CBq. *sitque* LG *bisecta in* M, *ducaturque* MN *parall.* GF.

Dico 1. *Rectang.* ACB = LN, vel MF. *a*Nam 2 ACB = LF. a 4. 2. & 3.

2. DL ⊏ LG. nam DK (ACq + CBq) ax. 1.
b ⊏ LF (2 ACB) ergo cum DK, LF sint æque b 7. 2.
alta, *c* erit DL ⊏ LG. c 1. 6.

3. Si AC ˥⊐ CB, *d* erit rectang. DK ˥⊏ d 16. 10.
ACq, & CBq.

4 *Item*, DL ˥⊏ LG. nam ACq + CBq
e ˥⊏ 2 ACB: hoc est DK ˥⊏ LF. sed DK. e lem. 26.
LF *e* :: DL. LG. *f* ergo DL ˥⊏ LG. 10.

5. *Ad hæc*, DL ˥⊏ √DLq − LGq. Nam f 10. 10.
ACq. ACB *g* :: ACB. CBq. hoc est DH. g 1. 6.
LN ::

LN :: LN. IK. *c* quare DI. LM :: LM. IL:
h 17. 6. *b* ergo DI × IL = LMq. ergo cum ACq *k* ⊓
k hyp. CBq. hoc est DH ⊓ IK, & *l* proinde DI ⊓
l 10. 10. IL, *m* erit DL ⊓ √DLq−LGq. Q. E. D.
m 18. 10. 6. *Sin ponatur* ACq ⊓ CBq, *n erit* DL ⊓
n 19. 10. √DLq−LGq.

Hoc lemma præparationis vicem subeat pro 6. sequentibus propositionibus.

PROP. LXI.

Quadratum ejus quæ ex binis nominibus (AC +CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus primam.*

Suppositis iis, quæ in lemmate proxime ante-
cedenti descripta & demonstrata sunt. Quoni-
a hyp. am AC, CB *a* sunt ῥ ⊓, *b* erit rectang. DK
b lem. 60. ⊓ ACq; *c* ergo DK est ῥν. *d* ergo DL ⊓
10. DE ῥ. rectang. vero ACB, ideoque 2 ACB
c sch. 12. 10 (LF) *e* est μν. *f* ergo latitudo LG est ῥ ⊓
d 21. 10. DE *g* ergo etiam DL ⊓ LG. *h* item DL ⊓
e 22. & 24. √DLq−LGq. ex quibus *k* sequitur DG esse
10. bin. 1. Q. E. D.
f 23. 10.
g 13. 10.

PROP. LXII.

h lem. 60. *Quadratum ejus, quæ ex binis mediis prima*
10. (AC+CB) *ad rationalem* DE *applicatum, fa-*
k 1. def 48. *cit latitudinem* DG *ex binis nominibus secundam.*
10. Rursus adhibito lemmate proxime præce-
a 24. 10. denti; Rectang. DK ⊓ ACq. *a* ergo DK est
b 23. 10. μν *b* ergo latitudo DK est ῥ ⊓ DE. Quia ve-
c hyp. & ro rectang. ACB, ideoque LF (2 ACB)
sch. 12. 10. *c* est ῥν, *d* erit LG ῥ ⊓ DE. *e* ergo DL,
d 21. 10. LG sunt ⊓. *f* item DL ⊓ √DLq−
e 13. 10. LGq. *g* ex quibus patet DG esse bin. 2. Q. E. D.
f lem. 60.
10.
g 2. def. 48.
81.

PROP.

PROP. LXIII.

Quadratum ejus, quæ ex binis mediis secunda (AC + CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus tertiam.*

Ut in præced. DL est ῥ ⊐⊏ DE. porro quia rectang. ACB, ideoque LF (2 ACB) *a* est *μν*, *b* erit LG ῥ ⊐⊏ DE. *c* quinetiam DL ⊐⊏ LG. *c* itemque DL ⊐⊏ $\surd$ DLq − LGq. *d* ergo DG est bin. 3. Q. E. D.

a *hyp* & 24. 10.
b 23. 10.
c *lem.* 60. 10.
d 3. *def.* 48. 10.

PROP. LXIV.

Quadratum Majoris (AC + CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus quartam.*

Rursus ACq + CBq, hoc est DK *a* est ῥν. *b* ergo DL est ῥ ⊐⊏ DE. item ACB, ideoque LF (2 ACB) *c* est *μν*. *d* ergo LG est ῥ ⊐⊏ DE. *e* proinde etiam DL ⊐⊏ LG. denique quia AC ⊐̸⊏ BC, *f* erit DL ⊐⊏ DLq − LGq. *g* unde DG est bin. 4. Q. E. D.

a *hyp.* & *sch.* 12. 10.
b 21. 10.
c *hyp.* & 24. 10.
d 23. 10.
e 13. 10.
f *lem.* 60. 10.
g 4. *def.* 48. 10.

PROP. LXV.

Quadratum ejus, quæ rationale ac medium potest, (AC + CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus quintam.*

Iterum, DK est *μν*. *a* ergo DL est ῥ ⊐⊏ DE. item LF est ῥν. *b* ergo LG est ῥ ⊐⊏ DE. *c* ergo DL ⊐⊏ LG. *d* item DL ⊐⊏ $\surd$ DLq − LGq. *e* proinde DG est bin. 5.

a 23. 10.
b 21. 10.
c 13. 10.
d *lem.* 60. 10.
e 5. *def.* 48. 10.

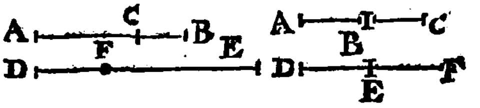
PROP. LXVI.

Quadratum ejus, qua bina media potest (AC + CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus sextam.*

Ut

a *hyp.* Ut prius, DL & LG sunt ρ' ⌐□ DE.
b 14. 10. Quia vero ACq + CBq (DK) *a* ⌐□ ACB,
c 1. 6. *b* ideoque DK ⌐□ LF (2 ACB) estque DK.
d 10. 10. LF *c* :: DL. LG. *d* erit DL ⌐□ LG *e* denique
e *lem*.60.10 DL ⌐□ √ DLq − LGq. *f* ex quibus liquet
f 6. *def*.48. DG esse bin. 6. Q. E. D.
10.

L E M M A.



Sint AB, DE ⌐□; *fiatque* AB. DE :: AC DF.

Dico 1. AC ⌐□ DF. ut patet ex 10. 10.
a 19. 5. item CB ⌐□ FE. *a* quia AB. DE :: CB. FE.

2. AC. CB :: DF. FE. Nam AC. DF :: AB. DE :: CB. FE. ergo permutando AC. CB :: DF. FE.

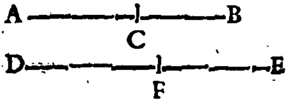
3. *Rectang*. ACB ⌐□ DFE. Nam ACq.
b 1. 6. ACB *b* :: AC. CB *c* :: DF. EF :: DFq. DFE.
c *prius*. quare permutando ACq. DFq :: ACB. DFE.
d 10. 10. ergo cum ACq ⌐□ DFq, *d* erit ACB ⌐□ DFE.

4. ACq + CBq ⌐□ DFq + FEq. Nam
e 22. 6. quia ACq. CBq *e* :: DFq. FEq. erit componendo ACq + CBq. CBq :: DFq + FEq. FEq. ergo
f 10. 10. cum CBq ⌐□ FEq, *f* erit ACq + CBq ⌐□ DFq + FEq.

g 10. 10. 5. Hinc, si AC ⌐⊔, vel ⌐⊔ CB, *g* erit pariter DE ⌐⊔, vel ⌐⊔ EF.

PROP.

PROP. LXVII.



Ei, quæ ex binis nominibus (AC + CB) *longitudine commensurabilis* DE, *& ipsa ex binis nominibus est, atque ordine eadem.*

Fac AB. DE :: AC. DF. *a* sunt AC, DF 'ⵡ; *a* & CB, FE 'ⵡ. quare cum AC, & CB *b* sint ρ' ⊓, *c* erunt DF, FE ρ' ⊓. ergo DE est etiam bin. Quia vero AC. CB *a* :: DF. FE. si AC 'ⵡ, vel ⵡ $\surd$ ACq − BCq, *d* etiam similiter DF 'ⵡ, vel ⵡ $\surd$ DFq − FEq. item si AC 'ⵡ, vel 'ⵡ ρ expos. *e* erit similiter DF 'ⵡ, vel 'ⵡ ρ' expos. at si CB 'ⵡ vel 'ⵡ ρ', *e* erit pariter FE 'ⵡ vel 'ⵡ ρ'. Sin vero utraque AC, CB 'ⵡ ρ', erit utraque etiam DF, FE 'ⵡ ρ'. *g* Hoc est, quodcunque binomium fuerit AB, erit DE ejusdem ordinis. Q. E. D.

a *lem.* 66. 10.
b *hyp.*
c *lem.* 66. 10. *& sch.* 12.10.
d 15.10.
e 12.10. & 14.10.
g *Per def.* 48.10.

PROP. LXVIII.

Ei, quæ ex binis mediis (AC + CB) *longitudine commensurabilis* DE, *& ipsa ex binis mediis est, atque ordine eadem.*

a Fiat AB. DE :: AC. DF. *b* ergo AC ⵡ DF, & CB ⵡ FE. ergo cum AC & CB *c* sint μ, *d* etiam DF, & FE erunt μ. & cum AC *c* ⊓ CB, *e* erit FD ⊓ FE. *f* ergo DE est 2 μ. Si igitur rectang. ACB sit ρ'ν, quia DFE *b* ⵡ ACB, *g* etiam DFE est ρ'ν; & si illud μν, *h* hoc etiam erit μν. *k* Id est, sive AB sit bimed. 1. sive bimed. 2. erit DF ejusdem ordinis. Q. E. D.

a 12. 6.
b *lem.* 66. 10.
c *hyp.*
d 24. 10.
e 10. 16.
f 38. 10.
g *sch.* 12. 10
h 14. 10.
k 38. vel 39. 10.

PROP.

PROP. LXIX.

Majori (AC + CB) *commensurabilis* DE, & *ipsa major est.*

Fac AB. DE :: AC. DF. Quoniam AC
a hyp. *a* $\top\!\!\!\sqsubset$ CB, *b* erit DF $\top\!\!\!\sqsubset$ FE. item ACq +
b lem. 66. CBq *a* est $\acute\rho\nu$; proinde cum DFq + FEq *b* $\sqsubset\!\!\sqsupset$
10. ACq + CBq, *c* etiam DFq + FEq est $\acute\rho\nu$. de-
c sch. 12. 10 nique rectang. ACB *a* est $\mu\nu$. *d* ergo rectang.
d 24. 10. DFE est $\mu\nu$ (quia DFE *b* $\sqsubset\!\!\sqsupset$ ACB.) *e* Quare
e 40. 10. DE est major. Q. E. D.

PROP. LXX.

Rationale ac medium potenti (AC + CB) *commensurabilis* DE, *& ipsa rationale ac medium potens est.*

Iterum fac AB. DE :: AC. DF. Quia AC
a hyp. *a* $\top\!\!\!\sqsubset$ CB; *b* etiam DF $\top\!\!\!\sqsubset$ FE. item quia
b lem. 66. ACq + CBq *a* est $\mu\nu$, *c* erit DFq + FEq $\mu\nu$.
10. denique quia rectang. ACB *c* est $\acute\rho\nu$, *d* etiam
c 24 10 DFE est $\acute\rho\nu$. *e* ergo DE est potens $\acute\rho\nu$, ac $\mu\nu$.
d sch. 12. 10 Q. E. D.
e 41. 10.

PROP. LXXI.

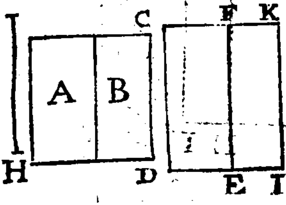
Bina media potenti (AC + CB) *commensurabilis* DE, *& ipsa bina media potens est.*

a hyp. Divide DE, ut in præced. Quia ACq *a* $\sqsubset\!\!\sqsupset$
b lem. 66. CBq, *b* erit DFq $\sqsubset\!\!\sqsupset$ FEq. item quia ACq
10. + CBq *a* est $\mu\nu$, *c* erit DFq + FEq etiam $\mu\nu$.
c 24. 10. pariterque quia ACB *a* est $\mu\nu$, *d* etiam DFE est
d 24. 10. $\mu\nu$. denique quia ACq + CBq $\sqsubset\!\!\sqsupset$ ACB,

e erit

e erit DFq + FEq $\sqcap\!\sqcup$ DFE. *f* è quibus sequitur DE esse potentem 2 $\mu\alpha$. Q. E. D. e 14. 10. f 42. 10.

PROP. LXXII.

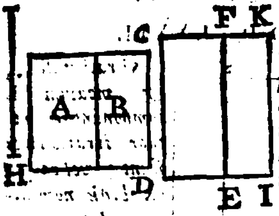


Si rationale A, & medium B componantur, quatuor irrationales fiunt; vel ea quæ ex binis nominibus, vel quæ ex binis mediis prima, vel major, vel rationale ac medium potens.

Nimirum si Hq = A + B, erit H una 4 linearum, quas theorema designat. Nam ad CD expositum $\overset{\circ}{\rho}$, *a* fiat rectang. CE = A; item FI = B; *b* ideoque CI = Hq. Quoniam igitur A est $\overset{\circ}{\rho}\nu$, etiam CE est $\overset{\circ}{\rho}\nu$. *c* ergo latitudo CF est $\overset{\circ}{\rho}$ $\sqcap\!\sqcup$ CD. & quia B est $\mu\nu$, erit FI $\mu\nu$. *d* ergo FK est ρ' $\sqcap\!\sqcup$ CD. *e* ergo CF, FK sunt ρ' $\sqcap\!\sqcup$. Tota igitur CK *f* est bin. Si igitur A $\sqsubset$ B, hoc est CE $\sqsubset$ FI, *g* erit CF $\sqsubset$ FK. ergo si CF $\sqcap\!\sqcup$ $\surd$ CFq − FKq, *h* erit CK bin. 1. & proinde H = $\surd$ CI *k* est bin. Si ponatur CF $\sqcap\!\sqcup$ $\surd$ CFq − FKq, *l* erit CK bin. 4. quare H ($\surd$ CI) *m* est major. Sin A $\sqsupset$ B; *g* erit CF $\sqsupset$ FK; proinde si FK $\sqcap\!\sqcup$ $\surd$ FKq − CFq, *n* erit CK bin. 2. *o* quare H est 2 μ prima. denique si FK $\sqcap\!\sqcup$ $\surd$ FKq − CFq, *p* erit CK bin. 5. *q* unde H erit potens $\overset{\circ}{\rho}\nu$ ac $\mu\nu$. Q. E. D.

a *cor.* 16. 6.
b 2. *ax.* 1.
c 21. 10.
d 23. 10.
e 13. 10.
f 37. 10.
g 1. 6.
h 1. *def.* 48. 10.
k 55. 10.
l 4. *def.* 48. 10.
m 58. 10.
n 2. *def.* 48. 10.
o 56. 10.
p 5. *def.* 48. 10.
q 59. 10.

PROP. LXXIII.



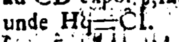
Si duo media A, B, inter se incommensurabilia componantur, duæ reliquæ irrationales fiunt; vel ex binis mediis secunda, vel bina media potens.

Nempe H potens A+B est una dictarum irrationalium. Nam ad CD expos. ρ, fac rectang, CE=A, & FI=B. unde Hq=CI. Quoniam igitur CE, & FI *a* sunt μα, *b* erunt latitudines CF, FK ρ ⊏⊐ CD. item quia CE ⊏⊐ FI; estque CE, FI *c* :: CF. FK, *d* erit CF ⊏⊐ FK *e* ergo CK est bin. 3. nempe, si CF ⊏⊐ √CFq−FKq. unde H = √CI *f* erit 2 μ 2a. Sin vero CF ⊏⊐ √CFq−FKq, *g* erit CK bin. 6. & *h* proinde H est potens 2 μα. Q. E. D.

a hyp.
b 23. 10.
c 1. 6.
d 10. 10.
e 3. def. 48. 10.
f 55. 10.
g 6. def. 48. 10.
h 60. 10.

Principium Senariorum per detractionem.

PROP. LXXIV.



Si à rationali DF rationalis DE auferatur potentia tantum commensurabilis existens toti DF; reliqua EF irrationalis est; vocetur autem apotome.

Nam EFq *a* ⊏⊐ DEq; sed DEq *b* est ρ'ν. *c* ergo EF est ρ. Q. E. D.

In numeris, sit DF, 2; DE, √3. EF erit 2−√3.

a 17. 28.
b hyp.
c 11. def. 10.

PROP.

PROP. LXXV.

D E F *Si à media* DF *media* DE *auferatur, potentia tantum commensurabilis existens toti* DF, *quæ cum tota* DF *rationale contineat; reliqua* EF *irrationalis est; vocetur autem mediæ apotome prima.*

Nam EFq *a* ⊓⊔ rectang. FDE, ergo cum FDE *b* sit ρ'ν, *c* erit EF ῥ. Q. E. D. a *sch.* 26. 10.
b *hyp.*
In numeris, sit DF v √54; & DE v √24, ergo EF est v √54 − v √24. c 20. & 11. *def.* 10.

PROP. LXXVI.

D E F *Si à media* DF *media* DE *auferatur, potentia tantum commensurabilis existens toti* DF, *quæ cum tota* DF *medium contineat; reliqua* EF *irrationalis est; vocetur autem mediæ apotome secunda.*

Quia DFq, & DEq, *a* sunt μα ⊓⊔, a *hyp.*
b erit DFq + DEq ⊓⊔ DEq *c*, quare DFq b 16. 10.
+ DEq est μν. item rectang. FDE, *c* ideoque c 24. 10.
2 FDE *a* est μν, ergo EFq (*d* DFq + DEq − d *cor.* 7. 2.
2 FDE) *e* est ρ̇ν, quare EF est ῥ. Q. E. D. e 27. 10.

In numeris, sit DF, v √18; & DE, v √8. erit EF v √18 − v √8.

PROP. LXXVII.

Si à recta linea AC *recta* A B C *auferatur* AB, *potentia incommensurabilis existens toti* BC, *quæ cum tota* AC *faciat compositum quidem ex ipsarum quadratis rationale, quod autem sub ipsis continetur medium, reliqua* BC *irrationalis est: vocetur autem minor.* a *hyp.*

Nam ACq + ABq *a* est ρ'ν. at rectang. ACB b *sch.* 12. 10
a est μν. *b* ergo 2 CAB ⊓⊔ ACq + ABq c 7. 2.
(*c* 2 CAB + BCq;) *d* ergo ACq + ABq ⊓⊔ d 17. 10.
BCq. *e* ergo BC est ῥ. Q. E. D. e 11. *def.* 10

In numeris, sit AC, $\surd: 18 + \surd 108$. AB $\surd: 18 - \surd 108$. ergo BC est $\surd: 18 + \surd 108. - \surd: 18 - \surd 108$.

PROP. LXXVIII.

D——E——F *Si à recta linea* DF *recta auferatur* DE *potentia incommensurabilis existens toti* DF, *quæ cum tota* DF *faciat compositum quidem ex ipsarum quadratis medium, quod autem sub ipsis continetur, rationale; reliqua* EF *irrationalis est: vocetur autem cum rationali medium totum efficiens.*

a *hyp.* & *sch.* 12.10. Nam 2 FDE *a* est $\acute{\rho}\nu$. *b* & DFq + DEq est $\mu\nu$. *c* ergo 2 FDE ⊏⊐ DFq + DEq *d* (2 FDE

b *hyp.* + EFq) *e* ergo EF est $\dot{\rho}$. Q. E. D.

c *sch.* 12.10 *In numeris*, sit DF, $\surd: \surd 216 + \surd 72$. DE,

d 7 2. $\surd: \surd 216 - \surd 72$. ergo EF est $\surd: \surd 216 +$

e *sch* 12.10 $\surd 72 - \surd: \surd 216 - \surd 72$.

& 11. *def.* 10.

PROP. LXXIX.

D——E F *Si à recta* DF *recta auferatur* DE, *potentia incommensurabilis existens toti* DF, *quæ cum tota faciat & compositum ex ipsarum quadratis, medium; & quod sub ipsis continetur, medium, incommensurabileque composito ex quadratis ipsarum, reliqua irrationalis est: vocetur autem cum medio medium totum efficiens.*

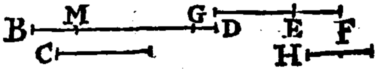
a *hyp.* & 24. 10. Nam 2 FDE, & DFq + DEq *a* sunt $\mu\alpha$; *b* ergo EFq (*c* DFq + DEq − 2 FDE) est $\dot{\rho}\nu$.

b 27. 10. *d* proinde EF est $\dot{\rho}$. Q. E. D.

c *cor.* 7.2. Exempl. gr. sit DF, $\surd: \surd 180 + \surd 60$. DE,

d 11 *def.* 10 $\surd: \surd 180 - \surd 60$. EF erit $\surd: \surd 180 + \surd 60 - \surd: \surd 180 - \surd 60$.

LEMMA.



Si idem sit excessus inter primam magnitudinem BG, *&* *secundam* C (MG) *qui inter tertiam magnitudinem* DF, *&* *quartam* H(EF;) *erit &* *vicissim idem excessus inter primam magnitudinem* BG, *&* *tertiam* DF, *qui inter secundam* C, *&* *quartam* H.

Nam quia *a* æqualibus BM, DE adjectæ sunt inæquales MG, EF, *a* hoc est C, H; erit excessus totorum BG, DF, *b* æqualis excessui adjectorum, C, H. Q. E. D. a *hyp.* b 15, *ax.* 1.

Coroll.

Hinc, quatuor magnitudines Arithmetice proportionales, vicissim erunt Arithmetice proportionales.

PROP. LXXX.

Apotoma AB *una tantum congruit recta linea rationalis* BC, *potentia tantum commensurabilis existens toti* AB.

Si fieri potest, alia BD congruat. *a* ergo rectangula ACB, ADB; *b* ideoque eorum dupla sunt $\mu\alpha$. cum igitur ACq+BCq − 2 ACB *c* = ABq *c* = ADq + DBq − 2 ADB. ergo vicissim ACq + BCq −: ADq + BDq *d* = 2 ACB −: 2 ADB. Sed ACq+BCq −: ADq+BDq *e* est ρ'ν. *f* ergo 2 ACB −: 2 ADB est ρ'ν. Q.E.D.

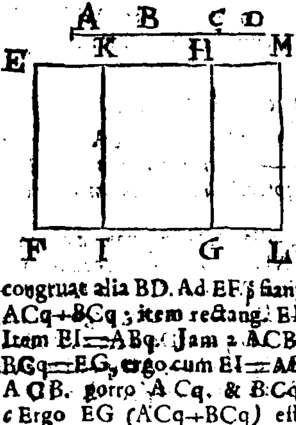
a 22. 10.
b 24 10.
c *cor.* 7. 2.
d *lem.* 79. 10.
e *hyp.* & 29. 10.
f *sch.* 12. 10
g 27. 10.

PROP. LXXXI.

A B D C *Mediæ Apotomæ primæ* AB *una tantum congruit recta linea media* BC, *potentia solum commensurabilis existens toti, & cum tota rationale continens.*

Dic etiam BD congruere. igitur quoniam
a hyp. tam ACq, & BCq; quam ADq, & BDq *a* sunt
b 16. & 24. 10. μα ⊏⊐. *b* etiam ACq+BCq, & ADq+BDq erunt μα. *c* sed rectangula ACB, ADB; *d* adeoq;
c hyp. 2 ACB, & 2 ADB sunt ῥα. *e* ergo 2 ACB
d sch. 12. 10 —: 2 ADB; *f* hoc est ACq+BCq—: ADq
e sch. 27. 10. +BDq est ῥν. *g* Q. E. A.
f 7. 2. & lem. 79. 10.
g 27. 10.

PROP. LXXXII.



Media Apotoma secunda AB *una tantum congruit recta linea media* B C, *potentia solum commensurabilis existens toti, & cum tota medium continens.*

Si fieri potest, congruat alia BD. Ad EF *a* fiant rectang. EG=
a 4. 2. & 3. ax. 1. ACq+BCq; item rectang. EL=ADq+BDq. Item EI=ABq. Jam 2 ACB+ABq=ACq+
b hyp. BCq=EG, ergo cum EI=ABq, *a* erit KG=2
c 24. 10. ACB. porro ACq. & BCq *b* sunt μα ⊏⊐.
d 23. 10. *c* Ergo EG (ACq+BCq) est μν. *d* ergo latitudo EH ῥ ⊏⊐ EF. *e* Quinetiam rectang.
e hyp. ACB; *f* ideoque 2 ACB (KG) est μν. *d* ergo
f 24. 10. KH est etiam ῥ ⊏⊐ EF. denique quia ACq+
g lem. 26. 10. BCq, id est, EG, *g* ⊏⊐ 2 ACB (KG) estque

EG.

E G. K G :: b E H. K H k erit E H $\sqcup\!\sqcup$ K H. h 1. 6.
l ergo E K est apotome, cujus congruens K H. simili k 10. 10.
argumento erit K M ejusdem E K congruens; con- l 74. 10.
tra 80 hujus.

PROP. LXXXIII.

A B D C

Minori AB, *una tantum congruit recta linea* (BC) *potentia incommensurabilis existens toti, & cum tota faciens compositum quidem ex ipsarum quadratis rationale; quod autem sub ipsis continetur medium.*

Puta alium BD congruere. Cum igitur ACq + B Cq, & A Dq + B Dq a sint $\dot{\rho}\alpha$, eorum excessus (2 b ACB —: 2 ADB) c est $\dot{\rho}\nu$, d Q.E.A; quia ACB, & ADB sunt $\mu\alpha$ per hypoth.

a hyp.
b lem. 97. 10.
c sch. 27. 10
d 27. 10.

PROP. LXXXIV.

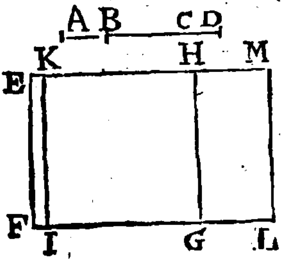
A B D C

Ei (AB,) *quæ cum rationali medium totum facit, una tantum congruit recta linea* BC, *potentia incommensurabilis existens toti, & cum tota faciens compositum quidem ex ipsarum quadratis medium; quod autem sub ipsis continetur, rationale.*

Dic aliam BD etiam congruere. a ergo rectangula ACB, ADB. b ideoque 2 ACB, & 2 ADB sunt $\dot{\rho}\alpha$. ergo 2 ACB —: 2 ADB; c hoc est, A Cq + B Cq —: A Dq + B Dq d est $\dot{\rho}\nu$. Q. E. A: quum A Cq + B Cq, & A Dq + BDq sint $\mu\alpha$ per hypoth.

a hyp.
b sch. 12. 10
c lem. 79. 10.
d sch. 27. 10

PROP. LXXXV.



Ei (AB,) *quæ cum medio medium totum facit una tantum congruit recta linea* BC *potentia incommensurabilis existens toti, & cum tota faciens & compositum ex ipsarum quadratis medium, & quod sub ipsis continetur, medium, incommensurabileque composito ex ipsarum quadratis.*

Suppositis iis quæ *facta* & ostensa sunt in 82 hujus; liquet EH, & KH esse ρ' ⌐□. EF. Porro igitur quia ACq+CBq, hoc est, rectang. EG
a hyp. *a* ⌐□ ACB, *b* ideoque EG ⌐□ 2 ACB (KG)
b 14. 10. estque EG. KG :: *c* EH. KH; erit EH ⌐□
c 1. 6. KH. ergo EK est apotome, cujus congruens KH. Haud aliter KM eidem apotomæ EK. congruere ostendetur; contra 80 hujus.

Definitiones tertiæ.

EXposita rationali, & apotoma, si tota plus possit quam congruens quadrato rectæ lineæ sibi longitudine commensurabilis;

I. Si quidem tota expositæ rationali longitudine sit commensurabilis, vocetur apotome prima.

II. Si vero congruens expositæ rationali longitudine sit commensurabilis, vocetur apotome secunda.

III. Quod si neque tota, neque congruens expositæ rationali sit longitudine commensurabilis, vocetur apotome tertia.

Rur-

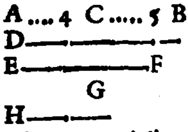
Rursus, si tota plus possit quam congruens quadrato rectæ sibi longitudine incommensurabilis;

IV. Si quidem tota expositæ rationali sit longitudine commensurabilis, vocetur apotome quarta.

V. Si vero congruens expositæ rationali sit longitudine commensurabilis, vocetur apotome quinta.

VI. Quod si neque tota, neque congruens, expositæ rationali sit longitudine commensurabilis, vocetur apotome sexta.

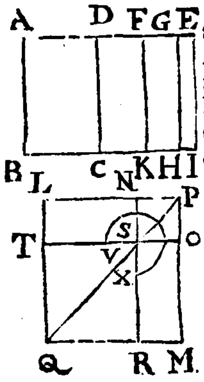
PROP. LXXXVI, 87, 88, 89, 90, 91.



Invenire apotomen primam, secundam, tertiam, quartam, quintam, sextam.

Apotomæ inveniuntur, subductis minoribus binomiorum nominibus ex majoribus. Exemp. gr. Sit $6 + \sqrt{20}$, bin. 1. erit $6 - \sqrt{20}$, apot. 1. &c. Quare de earum inventione plura repetere nihil est necesse.

LEMMA A.



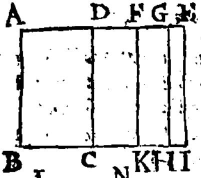
Si rectangulum AC *sub rectis* AB, AD. *producatur* AD *ad* E, *& bisecetur* DE *in* F. *sitque rectang.* AGE = FEq. *& compleantur rectangula* AI, DK, FH. *Fiant vero quadratum* LM = AH, *& quadratum* NO = GI, *producanturque* NSR, OST.

Dico primo, rectangul. AI = LM + NO = TOq + SOq. ut patet ex constr.

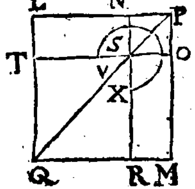
Secundo, *Rectang.* DK = LO. Nam quia
a *constr.* rectang. AGE *a* = FEq, *b* sunt AG, FE, GE
b 17. 6. ∺, *c* adeoque AH, FI, GI ∺ ; *a* hoc est, LM,
c 1. 6. FI, NO ∺. atqui LM, LO, NO *d* sunt ∺ ; ergo
d *sch.* 22. 6. FI = *e* LO *f* = DK = *g* NM.
e 9. 5. Tertio, *Hinc*, AC = AI − DK − FI =
f 35. 1. LM + NO − LO − NM = TR.
g 43. 1. Quarto, *b Liquet* DF, FE, DE esse ⊤□.
h 16. 10. Quinto, *Si* AE ⊤⊏ DE, & AE ⊤□ √ AEq
k 18. 10. & − DEq, *k* erunt AG, GE, AE ⊤□.
10. 10. Sexto, *Item, quia* AE *l* ⊤□ DE, *m erunt* AE,
l *hyp.* FE ⊤□. *n ideoque* AI, FI ; *hoc est*, LM + NO
m 13. 10. & LO *sunt* ⊤□.
n 1. 6. & Septimo, *Item quia* AG * ⊤□ GE, *n erunt* AH
10. 10. GI, *hoc est*, LM, NO ⊤□.
* prius. Octavo, *Sed quia* AE *l* ⊤□ DE, *o erunt* FE,
o 14. 10. GE ⊤□. *n ideoque rectang* FI ⊤□ GI, *hoc est* LO
p 2. 6. ⊤□ NO. *quare cum* LO. NO *p* :: TS. SO *q erunt*
q 10. 10. TS, SO ⊤□.
r 19. 10. Nono, *Sin ponatur* AE ⊤□ √ AEq − DEq ;
& 17. 10. *r erunt* AG, GE, AE ⊤□.
f 1. 6. & 10 Decimo, *f Quare rectang.* AH, GI, *hoc est*
10. TOq, SOq *erunt* ⊤□.

PROP.

PROP. XCII.



Si spatium AC *contineatur sub rationali* AB, *& Apotoma prima* AD (AE—DE;) *recta linea* TS *spatium* AC *potens, apotome est.*



Adhibe lemma proxime antecedens pro præparatione ad demonstrationem hujus. Igitur TS = $\sqrt{}$ AC. item AG, GE, AE sunt ⊔; ergo cum AE ⊔ *a* AB $\acute{\rho}$; *b* erunt AG, & GE ⊔ AB. *c* ergo rectangula AH & GI, hoc est TOq & SOq sunt $\rho^{\prime}\alpha$. *d* item TO, SO sunt $\rho^{\prime}$ ⊓, *e* proinde TS est apotome. Q. E. D.

a hyp.
b 12. 10.
c 20. 10.
d lem. 91. 10.
e 74. 10.

PROP. XCIII.

Vide Schem. præced.

Si spatium AC *contineatur sub rationali* AB, *& apotoma secunda* AD (AE—DE;) *recta linea* TS *spatium* AC *potens; mediæ est apotome prima.*

Rursus juxta lemma antecedens, AG, GE, AE sunt ⊔. cum igitur AE *a* sit $\acute{\rho}$ ⊔ AB, a hyp.
b erunt AE, GE etiam $\rho^{\prime}$ ⊔ AB. *c* ergo rectangula AH, GI, hoc est TOq, SOq, sunt $\mu\alpha$; b 13. 10.
d item TO ⊓ SO. Denique quia DE *e* ⊔ AB. $\rho^{\prime}$. *f* erit rectang. DI, ejusque semissis DK, vel LO, hoc est TOS $\acute{\rho}\nu$ *g* è quibus sequitur TS ($\sqrt{}$ AC) esse mediæ apot. 1. Q. E. D.

c 22. 10.
d lem. 74. 10.
e hyp.
f 20. 10.
g 75. 10.

PROP. XCIV.

Vide idem.

Si spatium AC *contineatur sub rationali* AB, *& apotoma tertia* AD (AE — DE;) *recta linea* TS *spatium* AC *potens, mediæ est apotome secunda.*

Ut in præcedenti TO, & SO sunt μ. Quoniam igitur DE *a* est ρ' ⊓⊔ AB, *b* erit rectang.
a hyp. b 22. 10. DI, *c* ideoque DK; vel TOS $\mu\nu$. *d* ergo TS $=$
c 24. 10. $\sqrt{}$ AC est mediæ apot. 2. Q. E. D.
d 76. 10.

PROP. XCV.

Vide idem.

Si spatium AC *contineatur sub rationali* AB, *& apotoma quarta* AD (AE — DE) *recta linea* TS *spatium* AC *potens, minor est.*

a lem. 91. 10. Rursus TO *a* ⊓⊔ SO. Quoniam igitur AE
b hyp. *b* est ρ' ⊓⊔ AB, *c* erit AI, (TOq+SOq) $\rho'\nu$.
c 20. 10. atqui ut prius rectang. TOS est $\mu\nu$. *d* ergo TS
d 77. 10. $=$ $\sqrt{}$ AC est minor. Q. E. D.

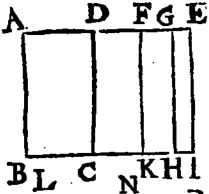
PROP. XCVI.

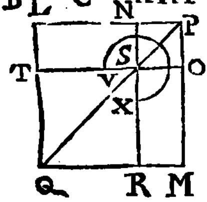
Vide idem.

Si spatium AC *contineatur sub rationali* AB, *& apotoma quinta* AD (AE — DE;) *recta linea* TS *spatium* AC *potens, est quæ cum rationali medium totum efficit.*

Rursus enim TO ⊓⊔ SO. itaque cum AE
a hyp. *a* sit ρ' ⊓⊔ AB, *b* erit AI, hoc est TOq+SOq
b 22. 10. $\mu\nu$. Sed prout in 93 rectang. TOS est $\rho'\nu$. *c* pro-
c 78. 10. inde TS $=$ $\sqrt{}$ AC est quæ cum $\rho'\nu$ facit totum $\mu\nu$. Q. E. D.

PROP. XCVII.





Si spatium AC *contineatur sub rationali* AB, *&* *apotoma sexta* AD (AE — DE;) *recta linea* TS *spatium* AC *potens, est quæ cum medio medium totum efficit.*

Itidem, ut sæpe prius, TO ⊓ SO. item ut in 96, TOq + SOq est μν. rectang. vero TOS est ῥν, ut in 94. *a* denique TOq + SOq ⊓ TOS. *b* ergo TS = √AC est quæ cum μν facit totum μν, Q. E. D. — a *lem.* 91. 10. — b 79. 10.

LEMMA.

Ad rectam quamvis DE * *applicentur rectang.* DF = ABq, *&* DH = ACq, *&* IK = BCq; *&* *sit* GL *bisecta in* M; *ductaque sit* MN *parall.* GF. — * *cor.* 16.6.

Erit primo, *Rectang.* DK = ACq + BCq, *ut constructio indicat.*

Secundo, *Rectang.* ACB = GN, *vel* MK. Nam DK *a* = ACq + BCq *b* = 2ACB + ABq at ABq *a* = DF. ergo GK *c* = 2ACB. & *d* proinde GN, vel MK = ACB. — a *constr.* — b 7. 2. — c 3. *ax.* 1. — d 7. *ax* 1.

Tertio, *Rectang.* DIL = MLq. Nam quia ACq. ACB *e* :: ACB. BCq; hoc est DH. — e 1. 6.

MK

f 17. 6. MK :: MK. IK, *e* erit DI. ML :: ML. IL. *f* ergo DIL = MLq.

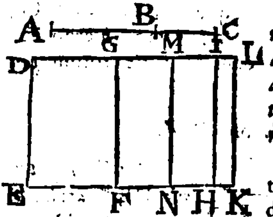
Quarto, Si ponatur AC ⊏⊢ BC, *erit* DK ⊏⊐
g 16. 10. ACq. Nam ACq + BCq (DK) *g* ⊏⊐ ACq.

Quinto, Item, DL ⊏⊐ $\surd$ DLq − GLq.
h 10. 10. Nam quia DH (ACq) ⊏⊐ IK (BCq) *h* erit DI
k 18. 10. ⊏⊐ IL. *k* ergo $\surd$ DLq − GLq − DL.

Sexto, Item DL ⊏⊐ GL. Nam ACq +
l lem 26. 10. BCq ⊏⊐ *l* 2 ACB; hoc est, DK ⊏⊐ GK. *m* ergo DL ⊏⊐ GL.

m 10. 10. *Septimo, Si in ponatur* AC ⊏⊢ BC, *n erit* DL
n 19. 10. ⊏⊐ $\surd$ DLq − GLq.

PROP. XCVIII.



Quadratum apotomæ AB (AC − BC) *ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen primam.*

Fac ut in lemmate proxime præcedenti.

a hyp. Quoniam igitur AC, BC *a* sunt ρ ⊏⊢,
b lem. 97 10 *b* erit DK (ACq + BCq) ⊏⊐ ACq; *c* ergo DK est $\rho\nu$. *d* quare DL est ρ ⊏⊐ DE. *e* item
c sch.12 10 rectang. GK (2 ACB) est $\mu\nu$. *f* ergo GL est ρ
d 21. 10. ⊏⊐ DE. *g* proinde DL ⊏⊐ GL; *h* sed DLq
e 22.& 24. 10. ⊏⊐ GLq *k* ergo DG est apotome, & *l* quidem prima (quia *m* AC ⊏⊢ BC, & propterea DL
f 23. 10. ⊏⊐ $\surd$ DLq − GLq.) Q. E. D.
g 13 10.
h sch 12.10
k 74 10.
l 1. def. 85. 10.
m lem. 97. 10.

PROP.

PROP. XCIX.

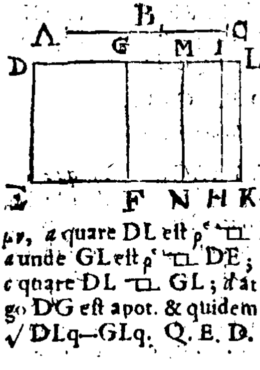
Vide Schema subsequens.

Quadratum mediæ apotomæ primæ AB (AC−BC) *ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen secundam.*

Rursus (supposito lemmate præcedenti) quia AC, & BC *a* sunt $\mu \sqcap$ *b*, erit DK (ACq+BCq) $\sqcap$ ACq; *c* quare DK est $\mu\nu$. *d* ergo DL est ρ' $\sqcap$ DE. *e* item GK (2. ACB) est $\rho'\nu$ *f* ergo GL est ρ' $\sqcap$ DE; *g* quare DL $\sqcap$ GL. *h* Sed DLq $\sqcap$ GLq. *k* ergo DG est apotome. quia vero DL *l* $\sqcap$ $\surd$ DLq−GLq, *m* erit DG apotome secunda. Q.E.D.

a hyp.
b lem. 97. 10.
c 24. 10.
d 23. 10.
e hyp. & sch. 12. 10.
f 21. 10.
g 13. 10.
h sch. 12. 10.
k 74. 10.
l lem. 97. 10.
m 2. def. 85. 10.

PROP. C.



Quadratum mediæ apotomæ secundæ AB (AC−BC) *ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen tertiam.*

Iterum DK est $\mu\nu$, *a* quare DL est ρ' $\sqcap$ DE. item GK est $\mu\nu$. *a* unde GL est ρ' $\sqcap$ DE; *b* item DK $\sqcap$ GK, *c* quare DL $\sqcap$ GL; *d* at DLq $\sqcap$ GLq. *e* ergo DG est apot. & quidem *f* 3a. *g* quia DL $\sqcap$ $\surd$ DLq−GLq. Q. E. D.

a 23. 10.
b lem. 26. 10.
c 1. 6. & 10. 10.
d sch. 12. 10.
e 74. 10.
f 3 def. 85. 10.
g lem 27. 10.

PROP. CI.

Vide Schema præced.

Quadratum minoris AB (AC−BC) *ad rationalem*

tionalem DE *applicatum, facit latitudinem* DG *apotomen quartam.*

Ut prius, ACq + BCq, hoc est DK est $\rho'\nu$;
a 21. 10. *a* ergo DL est ρ' $\rceil\!\sqsubset$ DE. at rectang. ACB, ide-
* hyp. oque GK (2 ACB) * est $\mu\nu$, *b* quare GL est ρ'
b 23. 10. $\rceil\!\sqsubset$ DE. *c* ergo DL $\rceil\!\sqsubset$ GL. *d* at DLq $\rceil\!\sqsubset$
c 13. 10. GLq quia vero * ACq $\rceil\!\sqsubset$ BCq, *e* erit DL $\rceil\!\sqsubset$
d sch. 12. 10 $\sqrt{}$ DLq − GLq: *f* ergo DG conditiones habet
e lem. 97. 10. apotomæ quartæ. Q. E. D.
f 4. def. 85. 10.

PROP. CII.

Vide Schem. præced.

Quadratum ejus AB (AC − BC,) *quæ cum rationali medium totum efficit, ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen quintam.*

a 23 10. Rursus enim, DK est $\mu\nu$, *a* quare DL est ρ'
b 21. 10. $\rceil\!\sqsubset$ DE. item GK est $\rho'\nu$, *b* unde GL est ρ'. $\rceil\!\sqsubset$
c 13. 10. DE. *c* ergo DL $\rceil\!\sqsubset$ GL, *d* sed DLq $\rceil\!\sqsubset$ GLq.
d sch. 12 10 porro, DL *e* $\rceil\!\sqsubset$ $\sqrt{}$ DLq − GLq. ex quibus,
e lem. 97. 10. DG *f* est apot. quinta. Q. E. D.
f 5. def. 85. 10.

PROP. CIII.

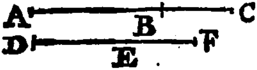
Vide Schema idem.

Quadratum ejus AB (AC − BC,) *quæ cum medio medium totum efficit, ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen sextam.*

Haud aliter, quam antea, DK, & GK sunt
a 23. 10. $\mu\alpha$; *a* quare DL & GL sunt ρ' $\rceil\!\sqsubset$. DE. item
b hyp. & DK *b* $\rceil\!\sqsubset$ GK, *c* quare DL $\rceil\!\sqsubset$ GL. *d* ergo
lem. 97. 10. DG est apot. *b* cum igitur ACq $\rceil\!\sqsubset$ BCq, ideo-
c 10. 10. que DL $\rceil\!\sqsubset$ $\sqrt{}$ DLq − GLq, *e* erit DG. apot.
d 74. 10. sexta. Q. E. D.
e 6. def. 85. 10.

PROP.

PROP. CIV.



Recta linea DE *apotomæ* AB (AC–BC) *longitudine commensurabilis, & ipsa apotome est, atque ordine eadem.*

LEMMA.

Sit AB. DE :: AC. DF. *&* AB ⊔ DE.

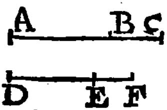
Dico AC + BC ⊔ DF + EF.

Nam AC.BC *a* :: DF.EF. ergo componendo
AC+BC. BC :: DF+EF.EF. ergo permutando
AC+BC. DF+EF :: BC. EF. *a* at BC ⊔ EF. a *lem.* 66.
b ergo AC+BC ⊔ DF+EF. Q.E.D. 10.

a Fac AB. DE :: AC. DF. *b* igitur AC + b 10. 10.
BC ⊔ DF+EF. ergo cum AC+BC *c* binomi- a 12. 6.
um sit, *d* erit DF+EF ejusdem ordinis binomi- b *lem.* 103.
um: *e* quare DF–EF ejusdem ordinis apotome 10.
est, cujus AC–BC. Q.E.D. c *hyp.*
d 61. 10.

PROP. CV.

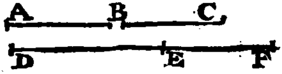
e *Per definitiones ad* 85. 10.



Recta linea DE *mediæ apotomæ* AB (AC–BC) *commensurabilis, & ipsa mediæ apotome est, atque ordine eadem.*

Iterum *a* fac AB. DE :: AC. DF. *b* quare a 12. 6.
AC + BC ⊔ DF + EF. *c* ergo DF + EF est b *lem.* 103.
bimed. ejusdem ordinis, cujus AC + BC. 10.
d proinde & DF–EF mediæ apotome erit ejus- c 68. 10.
dem classis, cujus AC–BC. Q.E.D. d 75 & 76.
10.

PROP. CVI.

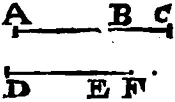


Recta linea DE *Minori* AB (AC — BC) *commensurabilis, & ipsa minor est.*

a *lem.*103. 10. Fiat AB. DE :: AC. DF. *a* estque AC + BC ⊐ DF + EF. atqui AC + BC *b* est Major,
b *hyp.* *c* ergo DF + EF quoque Major est. *d* & proinde
c 69. 10. DF — EF est Minor. Q. E. D.

d 77. 10.

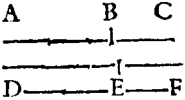
PROP. CVII.



Recta linea DE *commensurabilis ei* AB (AC — BC) *quæ cum rationali medium totum efficit, & ipsa cum rationali medium totum efficiens est.*

Nam ad modum præcedentium ostendemus
a 78, 10. DF + EF esse potentem ῤν, & μν. *a* ergo DF — EF est ut dicitur.

PROP. CVIII.

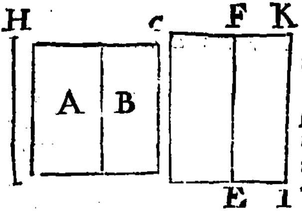


Recta linea DE *commensurabilis ei* AB (AC — BC) *quæ cum medio medium totum efficit, & ipsa cum medio medium totum efficiens est.*

Nam, ad normam præcedentium, erit DF +
a 79. 10. EF potens 2 μα. *a* ergo DF — EF erit ut in propos.

PROP.

PROP. CIX.



Medio B *à rationali* A+B *detracto, recta linea* H, *quæ reliquum spatium* A *potest, una ex duabus irrationalibus fit, vel apotome, vel Minor.*

Ad CD $\mathring{\rho}$, fac rectang. CI = A+B; & FI
= B. quare CE *a* = A: (Hq) Quoniam igitur a 3. ax. 1.
CI *b* est $\mathring{\rho}\nu$, *c* erit CK $\mathring{\rho}$ ⊓⊔ CD. sed quia FI *b* est b hyp. &
$\mu\nu$, *d* erit FK $\mathring{\rho}$ ⊓⊔ CD. *e* unde CK ⊓⊔ FK constr.
f ergo CF est apotome. Si igitur CK ⊓⊔ √ c 21. 10.
CKq − FKq. *g* erit CF apot. prima; *h* quare √ d 23. 10.
CE (H) est apotome. sin CK ⊓⊔ √CKq − e 13. 10.
FKq, *k* erit CF apot. quinta. & proinde H (√ f 74. 10.
CE) *l* erit Minor. Q. E. D. g 1. def. 85. 10.

h 92. 10.
k 4. def. 85. 10.
l 95. 10.

PROP. CX.

Vide Schem. præced.

Rationali B *à medio* A+B *detracto; aliæ duæ irrationales fiunt, vel mediæ apotome prima, vel cum rationali medium totum efficiens.*

Ad CD expos. $\mathring{\rho}$ fiant rectang. CI = A+B; & a 3. ax. 1.
FI = B, *a* unde CE = A = Hq. Quoniam b hyp. &
igitur CI *b* est $\mu\nu$: *c* erit CK $\mathring{\rho}$ ⊓⊔ CD. sed quia constr.
FI *b* est $\rho'\nu$, *d* erit FK ρ' ⊓⊔ CD. *e* unde CK ⊓⊔ c 23. 10.
FK. *f* ergo CF est apot. *g* nempe secunda; si CK d 21. 10.
⊓⊔ √CKq − FKq, *h* quare H (√CE) est me- e 13. 10.
diæ apot. prima. Sin vero CK ⊓⊔ √CKq − f 74. 10.
FKq, *k* erit CF apot. quinta. & proinde H (√ g 2. def. 85. 10.
CE) *l* erit faciens $\mu\nu$ cum $\rho'\nu$. Q. E. D.

h 93. 10.
k 5. def. 85. 10.
l 6. 10.

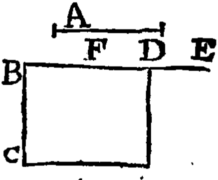
PROP. CXI.

Vide Schema idem.

Media B *à medio* A+B *detracto, quod sit incommensurabile toti* A+B; *reliquæ duæ irrationales fiunt, vel mediæ apotome secunda, vel cum medio medium totum efficiens.*

Ad C D $\dot{\rho}$ fiant rectang. CI = A + B; &
a 3. ax. 1. FI = B, *a* quare CE = A = Hq. Quoniam
b 23. 10. igitur CI est $\mu\nu$. *b* erit CK $\dot{\rho}$ ⊔ CD. eodem
c hyp. modo erit FK $\dot{\rho}$ ⊔ CD. item quia CI *c* ⊔
d 10. 10. FI, *d* erit CK ⊔ FK; *e* quare CF est apotome,
e 74. 10. *f* tertia scilicet, si CK ⊔ $\sqrt{}$ CK − FKq,
f 3. def. 85. *g* unde H ($\sqrt{}$CE) erit mediæ apot. secunda.
10. verum si CK ⊔ $\sqrt{}$CKq − FKq, *h* erit CF
g 94. 10. apot. sexta. *k* quare H erit faciens $\mu\nu$ cum μ.
h 6. def 85. Q. E. D.
10.
k 97. 10.

PROP. CXII.



Apotome A *non est eadem, quæ ex binis nominibus.*

Ad expos. B C $\dot{\rho}$,
a 98. 10. fiat rectang. C D =
b 74. 10. Aq. Ergo cum A sit
c 1. def. 85. apotome, *a* erit B D
10. apot. prima. ejus congruens sit DE. *b* quare BE,
d 37. 10. DE sunt $\dot{\rho}$ ⊤. *c* & BE ⊔ BC. Vis A esse
e 1 def. 48 bin. ergo BD est bin. 1. ejus nomina sint BF,
10. FD; sitque BF ⊏ FD; *d* ergo B F, F D sunt $\dot{\rho}$
f 12. 10. ⊤; & BF *e* ⊔ BC. ergo cum BC ⊔ BE,
g cor. 16. *f* erit BE ⊔ FB. *g* ergo BE ⊔ FE. *h* ergo FE
10 est $\dot{\rho}$. item quia BE ⊔ DE, *k* erit FE ⊔ DE.
h. sch 12. 10 *l* quare FD est apotome, *l* adeoque FD est $\dot{\rho}$. sed
k 14. 10. ostensa est $\dot{\rho}$. quæ repugnant. ergo A male dici-
l 74. 10. tur binomium. Q. E. D.

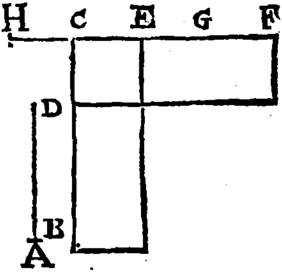
Nomi-

Nomina 13. linearum irrationalium inter se differentium.

1. Media.
2. Ex binis nominibus, cujus 6 species.
3. Ex binis mediis prima.
4. Ex binis mediis secunda.
5. Major.
6. Rationale ac medium potens.
7. Bina media potens.
8. Apotome, cujus etiam 6 species.
9. Mediæ apotome prima.
10. Mediæ apotome secunda.
11. Minor.
12. Cum rationali medium totum efficiens.
13. Cum medio medium totum efficiens.

Cum latitudinum differentiæ arguant differentias rectarum, quarum quadrata sunt applicata ad aliquam rationalem, sitque demonstratum in præcedentibus, latitudines quæ oriuntur ex applicationibus quadratorum harum 13 *linearum inter se differre, perspicue sequitur has* 13 *lineas inter se differre.*

PROP. CXIII.



Quadratum rationalis A *ad eam, quæ ex binis nominibus* BC (BD + DC) *applicatum, latitudinem facit apotomen* EC, *cujus nomina* EH, CH *commensurabilia sunt nominibus* BD, DC *ejus, quæ ex binis nominibus*

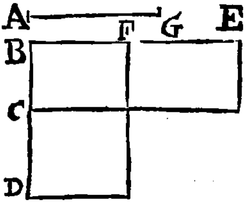
& in eadem proportione (EH. BD :: CH. DC;) *& adhuc, apotome* EC *quæ fit, eundem habet ordinem, quem ea* BC, *quæ ex binis nominibus.*

a *cor.*16.6. Ad DC minus nomen *a* fac rectang. DF =
b 14.6. Aq = BE. quare BC. CD *b* :: FC. CE. ergo dividendo BD. DC :: FE. EC. cum igitur BD
c *hyp.* *c* ⊏ DC. *d* erit FE ⊏ EC. sume EG = EC;
d 14.5. fiatque FG. GE :: EC. CH. Erunt EH, CH, nomina apotomæ EC; quibus conveniunt ea, quæ in theoremate proposita sunt. Nam componendo FE. GE. (EC) :: EH. CH. ergo
e 12.5. FH. EH *e* :: EH. CH *f* :: FE. EC *f* :: BD.
f *Prim.* DC. quare cum BD *g* ⊓ DC, *h* erit EH ⊓
g *hyp.* CH; *h* & FHq ⊔ EHq. ergo, quia FHq.
h 10.10. EHq *k* :: FH. CH. *h* erit FH ⊔ CH, *l* ideoque
k *cor.*20.6. FC ⊔ CH. Porro CD *g* est ρ', & DF (Aq)
l 16.10. *g* est ρ'ν, *m* ergo FC est ρ' ⊔ CD, quare etiam
m 21.10. CH est ρ' ⊔ CD. *n* igitur EH, CH sunt ρ', ac
n *sch.*12.10 ⊓ ut prius. *o* ergo EC est apotome, cui con-
o 74.10. gruit CH. porro EH. CH *f* :: BD. DC, ideo permutando EH. BD :: CH. DC. unde quia CH *f*
p 10.10. ⊔ DC, *p* erit EH ⊔ BD. quinimo pone BD
q 15.10. ⊔ √BDq − DCq; *q* erit ideo EH ⊔ √EHq −
r 12.10. CHq. item si BD ⊔ ρ' expos. *r* erit EH ⊔ ei-
s 1.*def.*48. dem ρ'; *s* hoc est si BC sit bin. 1. *t* erit EC apot.
10. prima. Similiter si DC ⊔ ρ' expos. *t* erit CH
t 1.*def.*85. ⊔ eidem ρ', *u* hoc est si BC sit bin. 2. *x* erit
10. EC apot. 2. & si hæc bin. 3. illa erit apot. 3,
u 2.*def.*48. &c. Sin BD ⊔ √BDq − DCq, *y* erit EH ⊔
10. √EHq − CHq; ~~si igitur BC sit bin.~~ 4, vel 5,
x 1.*def.*85. vel 6. erit EC similiter apot. 4, vel 5, vel 6.
10. Q. E. D.

y 15.10.

PROP.

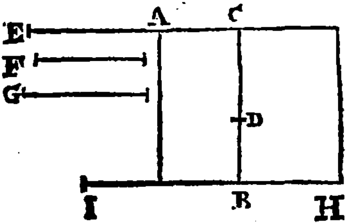
PROP. CXIV.



Quadratum rationalis A *ad apotomen* BC (BD − DC) *applicatum, facit latitudinem* BE *eam, quæ ex binis nominibus; cujus nomina* BE, GE *commensurabilia sint apotomæ* BC *nominibus* BD, DC, *& in eadem proportione; & adhuc, quæ ex binis nominibus fit* (BE,) *eundem habet ordinem, quem ipsa apotome* BC.

a Fac rectang. DF = Aq; & BE, FE *b* :: a *cor.16.6.*
EG. GF. Quoniam igitur DF = Aq = GE, b 12. 6.
c erit BD. BC :: BE. BF. ergo per conversio- c 14. 6.
nem rationis BD. CD :: BE. FE :: EG. GF ::
d BG. EG. sed BD *e* ⊤+ CD. *f* ergo BG ⊤+ d 19. 5.
GE. ergo quia BGq. GEq *g* :: BG. GF. *h* erit e *hyp.*
BG ⊐⊏ GF. *k* ideoque BG ⊐⊏ BF. porro f 10. 10.
BD *e* est ρ^c, & rectang. DF (Aq) *e* est $\rho^c \nu$. *l* er- g *cor.20.6.*
go BF est ρ^c ⊐⊏ BD. *m* ergo etiam BG est ρ^c ⊐⊏ h 10. 10.
BD. *n* ergo BG, GE sunt ρ^c ⊤+. *o* quare BE k *cor.* 16.
est bin. denique igitur quia BD. CD :: BG. 10.
GE; & permutando BD.BG :: CD.GE; sitque l 21. 10.
BD ⊐⊏ BG; *p* erit CD ⊐⊏ GE. ergo si CB sit m 12.10.
apot.prima; erit BE bin.1, &c. ut in anteceden- n *sch.*12.10
ti. ergo, &c. o 37. 10.
p 10. 10.

PROP. CXV.



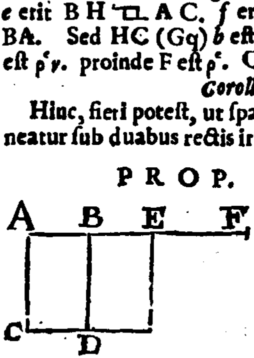
Si spatium A B *contineatur sub apotoma* A C (CE—AE,) *& ea, quæ ex binis nominibus* CB; *cujus nomina* CD, DB *commensurabilia sint apotomæ nominibus* CE, AE, *& in eadem proportione* (CE. AE :: CD. DB;) *recta linea* F *spatium* AB *potens, est rationalis.*

Sit G quævis $\dot{\rho}$; & fiat rectang. CH = Gq.
a 113. 10. *a* erit igitur BH (HI—IB) apotome; & HI
a ⊐⊏ CD *b* ⊐⊏ CE, *a* & BI ⊐⊏ DB; *a* atque
b *hyp.* HI. BI :: CD. DB *b* :: CE, EA. ergo permu-
c 19. 5. tando HI. CE :: BI. EA. *c* ergo BH. AC ::
d 12. 10. HI. CE :: BI. EA. ergo cum HI *d* ⊐⊏ CE,
e 10. 10. *e* erit B H ⊐⊏ A C. *f* ergo rectang. HC ⊐⊏
f 1. 6. & 10 BA. Sed HC (Gq) *b* est $\rho^{c}\nu$. *g* ergo BA (Fq)
10. est $\rho^{c}\nu$. proinde F est ρ^{c}. Q. E. D.
g *sch.* 12. 10

Coroll.

Hinc, fieri potest, ut spatium rationale contineatur sub duabus rectis irrationalibus.

PROP. CXVI.

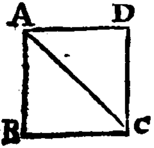


A media AB *fiunt infinitæ irrationales* B E, E F, &c. *& nulla alicui antecedentium est eadem.*

Sit AC expos. ρ^{c}. sit-

ρ^e. fitque AD spatium sub AG, AB. *a* ergo AD est $\dot{\rho}\nu$. Sume BE = $\sqrt{}$ AD. *b* ergo BE est $\dot{\rho}$, nulli priorum eadem. nullum enim quadratum alicujus priorum applicatum ad ρ^e, latitudinem efficit mediam. compleatur rectang. DE; *a* erit DE $\dot{\rho}\nu$; & *b* proinde EF ($\sqrt{}$ DE) erit $\dot{\rho}$; & nulli priorum eadem. nullum enim priorum quadratum ad ρ^e applicatum, latitudinem efficit ipsam BE, ergo, &c. a *lem.* 38. 10. b 11. 10.

PROP. CXVII.



Propositum sit nobis ostendere, in quadratis figuris BD, *diametrum* AC *lateri* AB *incommensurabilem esse.*

Nam ACq. ABq *a* :: 2. 1 *b* :: non Q. Q. *c* ergo AC $\sqcup\!\sqcup$ AB. Q.E.D. a 47. 1. b *cor.* 24. 8. c 9. 10.

Celebratissimum est hoc theorema apud veteres philosophos, adeo ut qui hoc nesciret, eum Plato non hominem esse, sed pecudem diceret.

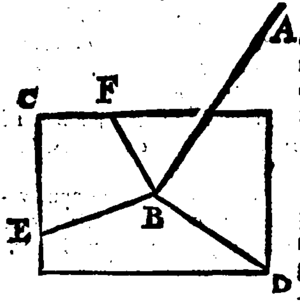
LIB.

LIB. XI.

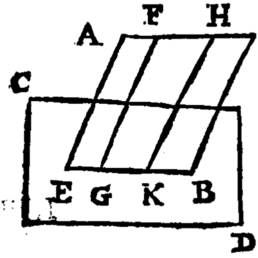
Definitiones.

I. SOlidum eſt, quod longitudinem, latitudinem, & craſſitudinem habet.

II. Solidi autem extremum eſt ſuperficies.

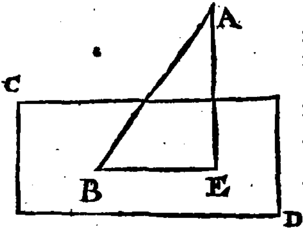


III. Linea recta AB eſt ad planum CD recta, cum ad rectas omnes lineas BD, BE, BF, à quibus illa tangitur, quæque in propoſito ſunt plano, rectos efficit angulos A B D, ABE, ABF.

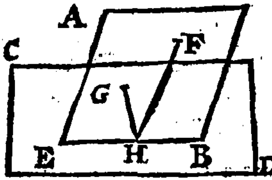


IV. Planum AB ad planum CD rectum eſt, cum rectæ lineæ FG, HK, quæ communi planorum ſectioni EB ad rectos angulos in uno plano AB ducuntur, alteri plano CD ad rectos ſunt angulos.

V. Rectæ



V. Rectæ lineæ AB ad planum CD inclinatio est, cum à sublimi termino A rectæ alius lineæ AB ad planum CD deducta fuerit perpendicularis AE; atque à puncto E, quod perpendicularis AE in ipso plano CD fecerit, ad propositæ illius lineæ extremum B, quod in eodem est plano, altera recta linea EB fuerit adjuncta: est, inquam, angulus acutus ABE insistente linea AB, & adjuncta EB comprehensus.



VI. Plani AB ad planum CD inclinatio, est angulus acutus FHG rectis lineis FH, GH contentus, quæ in utroque planorum AB, CD ad idem communis sectionis BE punctum H ductæ, rectos cum sectione BE efficiunt angulos FHB, GHB.

VII. Planum ad planum similiter inclinatum esse dicitur, atque alterum ad alterum, cum dicti inclinationum anguli inter se fuerint æquales.

VIII. Parallela plana sunt, quæ inter se non conveniunt.

IX. Similes solidæ figuræ sunt, quæ similibus planis continentur, multitudine æqualibus.

X. Æquales & similes solidæ figuræ sunt, quæ similibus planis multitudine & magnitudine æqualibus continentur.

XI. Solidus

XI. Solidus angulus est plurium quam duarum linearum, quæ se mutuo contingunt, nec in eadem sunt superficie, ad omnes lineas inclinatio.

Aliter.

Solidus angulus est, qui pluribus quam duobus planis angulis in eodem non consistentibus plano, sed ad unum punctum constitutis, continetur.

XII. Pyramis est figura solida, planis comprehensa, quæ ab uno plano ad unum punctum constituuntur.

XIII. Prisma est figura solida, quæ planis continetur, quorum adversa duo sunt & æqualia, & similia, & parallela; alia vero parallelogramma.

XIV. Sphæra est, quando semicirculi manente diametro, circumductus semicirculus in seipsum rursus revolvitur unde moveri cœperat, circumassumpta figura.

Coroll.

Hinc radii omnes à centro ad superficiem sphæræ inter se sunt æquales.

XV. Axis autem sphæræ, est quiescens illa recta linea, circum quam semicirculus convertitur.

XVI. Centrum sphæræ est idem quod & semicirculi.

XVII. Diameter autem sphæræ, est recta quædam linea per centrum ducta, & utrinque à sphæræ superficie terminata.

XVIII. Conus est, quando rectanguli trianguli manente uno latere eorum, quæ circa rectum angulum, circumductum triangulum in seipsum rursus revolvitur unde moveri cœperat, circumassumpta figura. Atque si quiescens recta linea

linea æqualis ſit reliquæ, quæ circa rectum angulum continetur, orthogonius erit conus; ſi vero minor, amblygonius; ſi vero major, oxygonius.

XIX. Axis autem coni, eſt quieſcens illa linea, circa quam triangulum vertitur.

XX. Baſis vero coni eſt circulus qui à circumducta recta linea deſcribitur.

XXI. Cylindrus eſt, quando rectanguli parallelogrammi manente uno latere eorum, quæ circa rectum angulum, circumductum parallelogrammum in ſeipſum rurſus revolvitur unde cœperat moveri, circumaſſumpta figura.

XXII. Axis autem cylindri, eſt quieſcens illa recta linea, circum quam parallelogrammum convertitur.

XXIII. Baſes vero cylindri ſunt circuli à duobus adverſis lateribus, quæ circumaguntur, deſcripti.

XXIV. Similes coni & cylindri ſunt, quorum & axes, & baſium diametri proportionales ſunt.

XXV. Cubus eſt figura ſolida ſub ſex quadratis æqualibus contenta.

XXVI. Tetraedrum eſt figura ſolida ſub quatuor triangulis æqualibus & æquilateris contenta.

XXVII. Octaedrum eſt figura ſolida ſub octo triangulis æqualibus & æquilateris contenta.

XXVIII. Dodecaedrum eſt figura ſolida ſub duodecim pentagonis æqualibus & æquilateris & æquiangulis contenta.

XXIX. Icoſaedrum eſt figura ſolida ſub viginti triangulis æqualibus & æquilateris contenta.

XXX. Parallelepipedum eſt figura ſolida ſex figuris quadrilateris, quarum quæ ex adverſo parallelæ ſunt, contenta.

XXXI. Solida figura in solida figura dicitur inscribi, quando omnes anguli figuræ inscriptæ constituuntur vel in angulis, vel in lateribus, vel denique in planis figuræ, cui inscribitur.

XXXII. Solida figura solidæ figuræ vicissim circumscribi dicitur, quando vel anguli, vel latera, vel denique plana figuræ circumscriptæ tangunt omnes angulos figuræ, circum quam describitur.

PROP. I.

Recta lineæ pars quædam AC *non est in subjecto plano, quædam vero* CB *in sublimi.*

Producatur AC in subjecto plano usque ad F. vis CB esse in directum ipsi AC; ergo duæ rectæ AB, AF habent commu-
a 10.ax.1. ne segmentum AG, *a* Q. F. N.

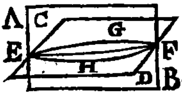
PROP. II.

Si duæ rectæ lineæ AB, CD *se mutuo secent, in uno sunt plano: atque triangulum omne* DEB *in uno est plano.*

Puta enim trianguli DEB partem EFG esse in uno plano, partem vero FDGB in altero. ergo rectæ ED pars EF est in subjecto plano, pars vero FD in sublimi, *a* Q.E.A. ergo triangulum EDB in uno est plano;
a 1. 11. proinde & rectæ ED, EB; *a* quare & totæ AB, DC in uno plano existunt. Q. E. D.

PROP.

PROP. III.

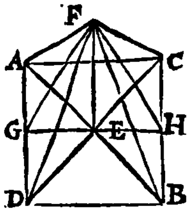


Si duo plana AB, CD *se mutuo secent, communis eorum sectio* EF *est recta linea.*

Si EF communis sectio non est recta linea, *a* ducatur in plano AB recta EGF, *a* & in plano CD recta EHF. duæ igitur rectæ EGF, EHF claudunt spatium. *b* Q.E.A.

a 1. *post.* 1.
b 14. *ax.* 1.

PROP. IV.



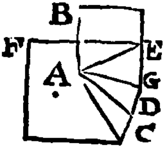
Si recta linea EF *rectis duabus lineis* AB, CD *se mutuo secantibus in communi sectione* E *ad rectos angulos insistat: illa ducto etiam per ipsas plano* ACBD *ad angulos rectos erit.*

Accipe EA, EC, EB, ED æquales, & junge rectas AC, CB, BD, AD. per E ducatur quævis recta GH; junganturque FA, FC, FD, FB, FG, FH. Quoniam AE *a*= EB; & DE *a*= EC; & ang. AED *b*= CEB, *c* erit AD = CB. *c* pariterque AC = DB. *d* ergo AD parall. CB. *d* & AC parall. DB. *e* quare ang. GAE = EBH. *e* & ang. AGE = EHB. sed & AE *f*= EB *g* ergo GE = EH, & *g* AG = BH. quare ob angulos rectos, ex hyp. & proinde pares ad E, *h* bases FA, FC, FB, FD æquantur. Triangula igitur ADF, FBC sibi mutuo æquilatera sunt, *k* quare ang. DAF = CBF. ergo in triangulis AGF, FBH latera FG, FH *l* æquantur; & proinde etiam triangula FEG, FEH sibi mutuo æquilatera sunt. *m* ergo anguli FEG, FEH æquales ac *n* propterea recti sunt. Eodem modo FE cum omni-

a *constr.*
b 15. 1.
c 4. 1.
d *sch.* 34. 1.
e 29. 1.
f *constr.*
g 26. 1.
h 4. 1.
k 8. 1.
l 4. 1.
m 8. 1.
n 10. *def.* 1.

omnibus in plano ADBC per E ductis rectis li-
o 3.def.11. neis rectos angulos constituit, o ideoque eidem
plano recta est. Q. E. D.

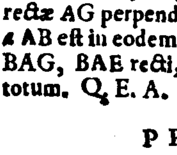
PROP. V.



Si recta linea AB *rectis tribus lineis* AC, AD, AE *se mutuo tangentibus in communi sectione ad rectos angulos insistat; illæ tres rectæ in uno sunt plano.*

Nam AC, AD *a* sunt in
a 2. 11. uno plano FC. *a* item AB, AE
sunt in uno plano BE. vis AE esse extra planum
b 3. 11. FC; sit igitur planorum intersectio *b* recta AG.
Quoniam igitur BA ex hypoth. perpendicularis
c 4. 11. est rectis AC, AD, eadem *c* plano FC, *d* ideoque
d 3.def.11. rectæ AG perpendicularis est, ergo (siquidem &
a AB est in eodem cum AG, AE plano) anguli
BAG, BAE recti, & proinde pares sunt, pars &
totum. Q. E. A.

PROP. VI.



Si duæ rectæ lineæ AB, DC *eidem plano* EF *ad rectos sunt angulos; parallelæ erunt illæ rectæ lineæ* AB, DC.

Ducatur AD, cui in plano EF perpendicularis sit DG=AB; junganturque BD, BG, AG. Quia in triangulis BAD,
a hyp. ADG anguli DAB, ADG *a* recti sunt; atque
b constr. AB *b*=DG; & AD communis est; *c* erit BD
c 4. 1. =AG; quare in triangulis AGB, BGD sibi
d 8. 1. mutuo æquilateris ang. BAG *d* = BDG; quorum BAG rectus cum sit, erit BDG etiam rectus. atqui ang. GDC rectus ponitur; ergo re-
e 5. 11. cta GD tribus DA, DB, CD recta est; *e* quæ
f 2. 11. ideo in uno sunt plano, *f* in quo AB existit;
eum

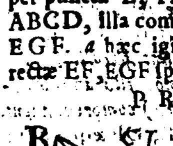
cum igitur AB, & CD sint in uno plano, & anguli interni BAD, CDA recti sint, *g* erunt AB, CD parallelæ. Q. E. D. g 28. 1.

PROP. VII.

Si duæ sint parallelæ rectæ lineæ AB, CD, in quarum utraque sumpta sint qualibet puncta E, F; illa linea EF, quæ ad hæc puncta adjungitur, in eodem est cum parallelis plano ABCD.

Planum in quo AB, CD, secet aliud planum per puncta E, F. si jam EF non est in plano ABCD, illa communis sectio non erit. Sit ergo EGF. *a* hæc igitur recta est linea, duæ ergo a 3. 11.
rectæ EF, EGF spatium claudunt, *b* Q. E. A. b 14. ax. 1.

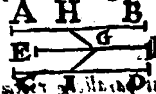
PROP. VIII.



Si duæ sint parallelæ rectæ lineæ AB, CD, *quarum altera* AB *ad rectos eidem plano* EF *sit angulos; & reliqua* CD *eidem plano* EF *ad rectos angulos erit.*

Adscita præparatione & demonstratione sextæ hujus, anguli GDA & GDB recti sunt. *a* ergo GD recta est plano per AD, DB (*b* in quo a 4. 11.
etiam AB, CD, existunt.) *c* ergo GD ipsi CD b 7. 11.
est perpendicularis, atqui ang. CDA etiam *d* re- c 3. def. 11.
ctus est, *e* ergo CD plano EF recta est. Q. E. D. d 29. 1.
e 5. 11.

PROP. IX.



Quæ (AB, CD) *eidem rectæ lineæ* EF *sunt parallelæ, sed non in eodem cum illa plano, hæ quoque sunt inter se parallelæ.*

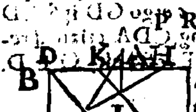
In plano parallelarum AB, EF duc HG perpendicularem ad EF item in plano parallelarum
a 4. 11. EF, CD duc IG perpendicularem ad EF. *a* ergo
b 8. 11. EG recta est plano per HG, GI; eidemque
c 6. 11. plano *b* rectæ sunt AH, & CI. *c* ergo AH, & CI parallelæ sunt. Q. E. D.

PROP. X.

Si duæ rectæ lineæ AB, AC *se mutuo tangentes ad duas rectas* ED, DF *se mutuo tangentes sint parallelæ, non autem in eodem plano, illæ angulos æquales* (BAC, EDF) *comprehendent.*

Sint AB, AC, DE, DF æquales inter se, & ducantur AD, BC, EF, BE,
a *hyp. &* CF. Cum AB, DE *a* sint parallelæ & æquales,
constr. *b* etiam BE, AD parallelæ sunt, & æquales.
b 33. 1. Eodem modo CF, AD parallelæ sunt, & æqua-
c 2. *ax.* 1. les *c* ergo etiam BE, CF sunt parallelæ & æqua-
& 9. 11. les. Æquantur ergo BC, EF. Cum igitur
d 33. 1. triangula BAC, EDF sibi mutuo æquilatera
e 8. 1. sint, anguli BAC, EDF *e* æquales erunt. Q. E. D.

PROP. XI.



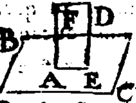
A dato puncto A *in sublimi ad subjectum planum* BC *perpendicularem rectam lineam* AI *ducere.*

In plano BC duc quamvis
a 12. 1. DE, ad quam ex A *a* duc perpendicularem
b 12. 1. AF. ad eandem per F in plano BC *b* duc normalem FH. tum ad FH *a* demitte perpendicularem AI, erit AI recta plano BC.

Nam

Nam per I c duc KIL parall. DE. Quia DE c 31. 1.
d recta est ad AF, & FH, e erit DE recta plano d constr.
IFA; adeoque & KL eidem plano f recta est. e 4. 11.
g ergo ang. KIA rectus est. atqui ang. AIF f 8. 11.
etiam h rectus est. l ergo AI plano BC recta est. g 3. def. 11.
Q. E. D. h constr.
l 4. 11.

PROP. XII.

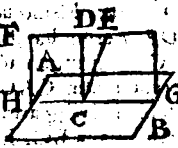


Dato plano BC à puncto A, quod in illo datum est, ad rectos angulos rectam lineam AF excitare.

A quovis extra planum puncto
D a duc DE rectam plano BC; & juncta EA b a 11. 11.
duc AF parall. DE. c perspicuum est AF plano b 31. 1.
BC rectam esse. Q. E. F. c 8. 11.

Practice perficiuntur hoc, & præcedens problema, si duæ normæ ad datum punctum applicentur, ut patet ex 4. 11.

PROP. XIII.

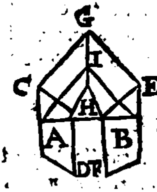


Dato plano AB, à puncto D, quod in illo datum est, duæ rectæ lineæ CD, CE ad rectos angulos non excitabuntur ab eadem parte.

Nam utraque CD, CE
plano AB recta esset, eædemq; a adeo parallelæ a 6. 11.
forent, quod parallelarum definitioni repugnat.

PROP. XIV.

Valet hæc conversa.



Ad quæ plana CD, FE, eadem recta linea AB recta est; illa sunt parallela.

Si nega, plana CD, FE concurrant, ita ut communis sectio sit recta GH; sume in hac quodvis punctum I, ad quod in propositis planis ducantur

a *hyp.* & 3. *def.* 11. rectæ IA, IB. unde in triangulo IAB, duo anguli

IAB, IBA *a* recti sunt. *b* Q. E. A.

b 17. 1.

PROP. XV.

Si duæ rectæ lineæ AB, AC se mutuo tangentes, ad duas rectas DE, DF se mutuo tangentes sint parallelæ, non in eodem consistentes plano; parallela sunt, quæ per illa ducuntur, plana BAC, EDF.

a 11. 11. Ex A *a* duc AG rectam plano EF. *b* Sintque

b 31. 1. GH, GI parallelæ ad DE, DF. *c* erunt hæ pa-

c 9. 11. rallelæ etiam ad AB, AC. Cum igitur anguli

d 3. *def.* 11. IGA, HGA *d* sint recti, *e* erunt etiam CAG,

e 29. 1. BAG recti. *f* ergo GA recta est plano BC; *g* atqui

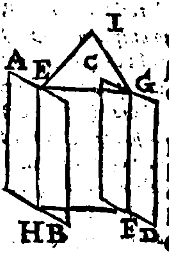
f 4. 11. eadem recta est plano EF. *h* ergo plana BC, EF

g *constr.* sunt parallela. Q. E. D.

h 14. 11.

PROP.

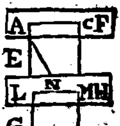
PROP. XVI.



Si duo plana parallela A B, CD, *plano quopiam* H E I G F *secentur, communes illorum sectiones* EH, GF *sunt parallelæ.*

Nam si dicantur non esse parallelæ, cum sint in eodem plano secanti, convenient alicubi, puta in I. quare cum totæ HEI, FGI *a* sint in planis AB, CD productis, etiam hæc convenient, contra hypoth. a 1. 11.

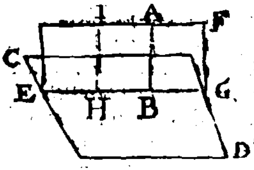
PROP. XVII.



Si duæ rectæ lineæ ALB, CMD *parallelis planis* BF, GH, IK *secentur, in easdem rationes secabuntur* (AL. LB :: CM. MD.)

Ducantur in planis BF, IK rectæ AC, BD. item AD occurrens plano GH in N; junganturque NL, NM. Plana triangulorum ADC, ADB faciunt sectiones BD, LN; & AC, NM *a* parallelas. ergo AL. LB *b* :: AN. ND *b* :: CM. MD. Q. E. D. a 16. 11. b 2. 6.

PROP. XVIII.

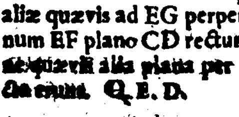


Si recta linea AB *plano cuipiam* CD *ad rectos sit angulos; & omnia, quæ per ipsam* AB *plana* (EF,&c.) *eidem plano* CD *ad rectos angulos erunt.*

Ductum sit per AB planum aliquod EF, faciens cum plano CD sectionem EG; è cujus aliquo puncto H, in plano EF *a* ducatur HI parall. AB. *b* erit HI recta plano CD; pariterque aliæ quævis ad EG perpendiculares. *c* ergo planum EF plano CD rectum est; eademque ratione quævis alia plana per AB ducta plano EF recta erunt. Q. E. D.

a 31. 1.
b 8. 11.
c 4.*def*.11.

PROP. XIX.



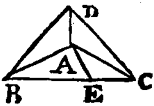
Si duo plana A B, CD, *se mutuo secantia, plano cuidam* G H *ad rectos sint angulos, communis etiam illorum sectio* EF *ad rectos eidem plano*(GH)*angulos erit.*

Quoniam plana AB, CD ponuntur recta plano GH, patet ex 4. def. 11. quod ex puncto F in utroque plano A B, C D duci possit perpendicularis plano GH; quæ *a* unica erit, & propterea eorundem planorum communis sectio. Q. E. D.

a 13. 11.

PROP.

PROP. XX.

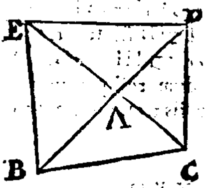


Si solidus angulus ABCD *tribus angulis planis* BAD, DAC, BAC *continetur; ex his duo quilibet, utut assumpti, tertio sunt majores.*

Si tres anguli sunt æquales, patet assertio; si
inæquales, maximus esto BAC. ex quo *a* aufer a 23. 1.
$BAE = BAD$; & fac $AD = AE$; ducanturque
BEC, BD, DC.

Quoniam latus BA commune est, & AD $b =$ b *constr.*
AE; & ang. BAE $b =$ BAD; *c* erit $BE = BD$. c 4. 1.
sed $BD + DC$ $d \sqsubset BC$. *e* ergo $DC \sqsubset EC$. cum d 20. 1.
igitur AD $b = $ AE, & latus AC commune est, e 5. *ax.* 1.
ac $DC \sqsubset EC$ *f*, erit ang. $CAD \sqsubset EAC$. *g* ergo f 25. 1.
ang. $BAD + CAD \sqsubset BAC$. Q. E. D. g 4. *ax.* 1.

PROP. XXI.



Omnis solidus angulus A *sub minoribus quam quatuor rectis angulis planis continetur.*

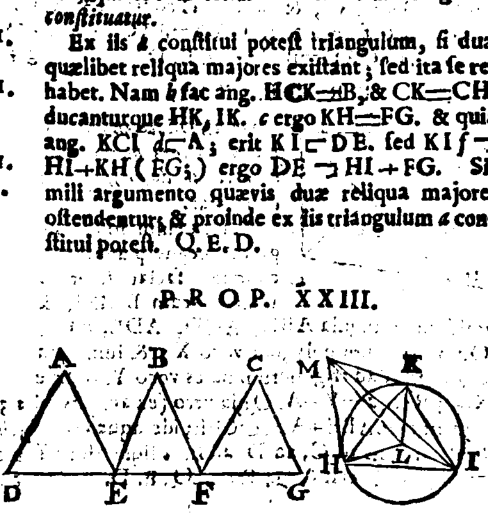
Latera enim solidi anguli A secans planum utcunque faciat figuram multilateram BCDE, &
totidem triangula ABC, ACD, ADE, AEB.
Omnes angulos polygoni voco X; & summam
angulorum ad trigonorum bases voco Y. quare
$X + 4$ Rect. $a = Y + A$. Quia vero (ex angulis ad a 32. 1. &
B) *b* est ang. $ABE + ABC \sqsubset CBE$; idemque verum *sch.* 32. 1.
sit de angulis ad C, ad D, ad E. *c* liquet fore Y b 20. 11.
$\sqsubset X$. proinde erit $A \sqsupset 4$ Rect. Q. E. D. c 5. *ax.* 1.

PROP. XXII.

Si fuerint tres anguli plani A, B, HCI, *quorum duo utlibet assumpti reliquo sint majores; comprehendant autem ipsos rectæ lineæ æquales* AD, AE, FB, &c. *fieri potest, ut ex rectis lineis* DE, FG, HI, *æquales illas rectas connectentibus triangulum constituatur.*

a 22. 1. Ex iis *a* constitui potest triangulum, si duæ quælibet reliqua majores existant; sed ita se res
b 23. 1. habet. Nam *b* fac ang. HCK=B, & CK=CH,
c 4. 1. ducanturque HK, IK. *c* ergo KH=FG. & quia
d *hyp.* ang. KCI *d* ⊏ A; erit KI ⊏ DE. sed KI *f* ⊐
e 24. 1. HI+KH (FG;) ergo DE ⊐ HI + FG. Si-
f 20. 1. mili argumento quævis duæ reliqua majores ostendentur, & proinde ex iis triangulum *a* constitui potest. Q. E. D.

PROP. XXIII.



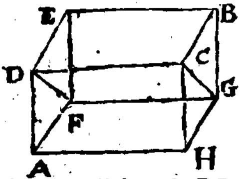
Ex tribus angulis planis A, B, C, *quorum duo quomodocunque assumpti reliquo sunt majores, soli-*
*21.11. *dum angulum* MHIK *constituere.* **Oportet autem illos tres angulos quatuor rectis minores esse.*

Fac

Fac AD, AB, BE, BF, CF, CG æquales
inter se. Ex subtensis DE, EF, FG (hoc est,
ex æqualibus HI, IK, KH) *a* fac triang. HKI, a 22. 11. &
circa quod *b* describatur circulus LHKI. * Quo- 22. 1.
niam vero AD $\sqsubset$ HL; *c* sit ADq = HLq + b 5. 4.
LMq. *d* sitque LM recta plano circuli HKI; & **Vid. Cla-*
ducantur HM, KM, IM. Quoniam igitur ang. *vium.*
HLM *e* rectus est, *f* erit MHq = HLq + LMq c *sch.* 47. 1.
g = ADq. ergo MH = AD. simili argumento d 12. 11.
MK, MI, AD (id est, AE, EB, &c.) æquantur; e 3. *def.* 11.
ergo cum HM = AD, & MI = AE; & DE *h* = f 47. 1.
HI, *k* erit ang. A = HMI; *k* similiter ang. IMK g *constr.*
= B. *k* & ang. HMK = C. Factus est igitur h *constr.*
angulus solidus ad M ex tribus planis datis. k 8. 1.
Q. E. F. Assumptum est fore AD $\sqsubset$ HL.
Hoc autem constat. Nam si AD = vel $\sqsupset$ HL,
erit ang. A *a* =, *b* vel $\sqsubset$ HLI. Eodem modo erit a *constr.*
B =, vel $\sqsubset$ HLK, & C =, vel $\sqsubset$ KLI. quare & 8. 1.
A + B + C *quatuor rectos aut exæquabunt, aut b 21. 1.
excedent, contra hypoth. quin potius sit AD $\sqsubset$ *4. *cor.* 13.
HL. Q. E. D. 1.

PROP.

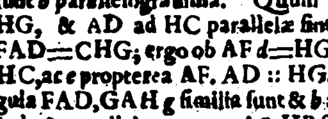
PROP. XXIV.



Si solidum AB *parallelis planis contineatur, adversa illius plana* (AG, DB, &c.) *parallelogramma sunt similia & æqualia.*

Planum AC secans
a 16. 11. plana parallela AG, DB, *a* facit sectiones AH, DC parallelas. Eadem ratione AD, HC parallelæ sunt. Ergo ADCH est parallelogrammum. Simili argumento reliqua parallelepipedi plana
b 35. *def.* 1. sunt *b* parallelogramma. Qum igitur AF ad
c 10. 11. HG, & AD ad HC parallelæ sint, *c* erit ang.
d 34. 1. FAD = CHG; ergo ob AF *d* = HG, & AD *d* =
e 7. 5. HC, ac *e* propterea AF. AD :: HG. HC, trian-
g 6. 6. gula FAD, GAH *g* similia sunt & *h* æqualia; pro-
h 4. 1. inde & parallelogramma AE, HB similia sunt &
k 6. *ax.* 1. *k* æqualia, idemque de reliquis oppositis planis ostendetur, ergo, &c.

PROP. XXV.



Si solidum parallelepipedum ABCD *plano* EF *secetur adversis planis* AD, BC *parallelo, erit quemadmodum basis* AH *ad basim* BH, *ita solidum* AHD *ad solidum* BHC.

Concipe Ppp. ABCD produci utrinque. accipe AI = AE, & BK = EB; & pone plana IQ, KP planis AD, BC parallela. parallelo-
a 36. 1. & gramma IM, AH, *a* & DL, DG, *b* & IQ, AD,
1. *def.* 6. BF, &c. *a* similia ac æqualia sunt; *c* quare Ppp.
b 24. 11. AQ = AF; atque eadem ratione Ppp. BP =
c 10. *def.* 11 BF, ergo solida IF, EP solidorum AF, EC æque-

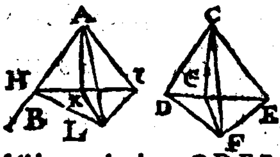
quemultiplicia sunt, ac bases IH, KH basium AH, BH. Quod si basis IH ⊏, =, ⊐ KH, *d* erit similiter solidum IF ⊏, =, ⊐ EP. *e* proinde AH. BH :: AF. EC. Q. E. D. d 24. 11. & 9. def. 11. e 6. def. 5.

Hac eadem omni prismati accommodari possunt; unde

Coroll.

Si prisma quodcunque secetur plano oppositis planis parallelo, sectio erit figura æqualis, & similis planis oppositis.

PROP. XXVI.

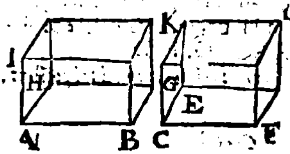


Ad datam rectam lineam AB, *ejusque punctum* A, *constituere angulum solidum* AHIL, *æqualem solido angulo dato* CDEF.

A puncto quovis F *a* demitte FG plano DCE rectam; ducanturque rectæ DF, FE, EG, GD, CG. Fac AH=CD, & ang. HAI=DCE. & AI=CE; atque in plano HAL, fac ang. HAK =DCG; & AK=CG. Tum erige KL rectam plano HAL, & fac KL=GF. ducaturque AL. erit angulus solidus AHIL par dato CDEF. Nam hujus constructio illius constitutionem penitus æmulatur, ut facile patebit examinanti. ergo factum. a 11. 11.

PROP.

PROP. XXVII.

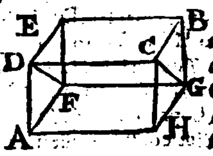


A data recta linea AB, *dato solido parallelepipedo* CD *simile & similiter positum parallelepipedum* AK *describere.*

Ex angulis planis BAH, HAI, BAI, qui æ-
a 26. 11. quales sint ipsis FCE, ECG, FCG, *a* fac angu-
b 12. 6. lum solidum A solido C parem. item *b* fac FC.
c 22. 5. CE :: BA. AH. *b* ac CE.CG :: AH.AI (*c* unde erit ex æquali FC.CG :: BA. AI;) & perficiatur Ppp. AK. erit hoc simile dato.

d 1.*def.* 6. Nam per constr. Pgra *d* BH, FB; *d* & HI,
e 24. 11. EG; & *d* BI, FG similia sunt, & *e* horum ideo opposita illorum oppositis. ergo sex plana solidi
f 9.*def.*11. AK similia sunt sex planis solidi CD. *f* proinde AK, CD similia solida existunt. Q. E. F.

PROP. XXVIII.



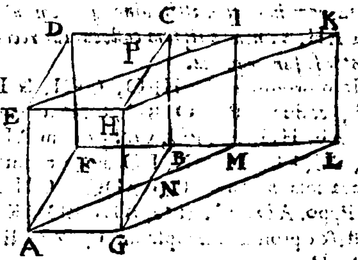
Si solidum parallelepipedum AB *plano* FGCD *secetur per diagonios* DF, CG *adversorum planorum* AE, HB; *bifariam secabitur solidum* AB *ab ipso plano* FGCD.

a 24. 11. Nam quia DC, FG *a* æquales & parallelæ
b 34. 1. sunt, *b* planum FGCD est pgr. & propter *a* pgra AE, HB æqualia, & similia, *b* etiam triangula AFD, HGC, CGB, DFB æqualia & similia sunt. Atqui pgra AC, AG ipsis FB, FD *a* etiam æqualia & similia sunt. ergo prismatis FGCDAH omnia plana æqualia sunt, & simili-
c 9.*def.*11. a planis omnibus prismatis FGCDEB; & *c* proinde hoc prisma illi æquatur. Q. E. D.

O

PROP.

PROP. XXIX.



Solida parallelepipeda A G H E F B C D, A G H E M L K I *super eandem basim* A G H E *constituta, & * in eadem altitudine; quorum insistentes lineæ* AF, AM *in iisdem collocantur rectis lineis* AG, FL, *sunt inter se æqualia.*

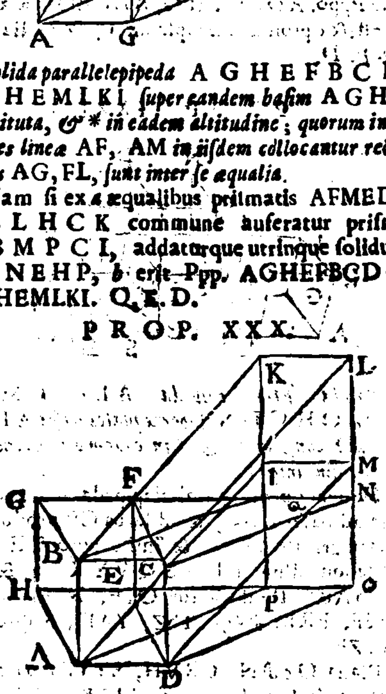
**Id est, inter parallela plana*

Nam si ex *a* æqualibus prismatis AFMEDI, A G H E, FLKD, & G B L H C K commune auferatur prisma N B M P C I, addaturque utrinque solidum AGNEHP, *b* erit Ppp. AGHEFBCD = AGHEMLKI. Q. E. D.

& sic intellige in sequent.

a 10. def. 11. & 35. 1.
b 3. & 2. ax. 1.

PROP. XXX.

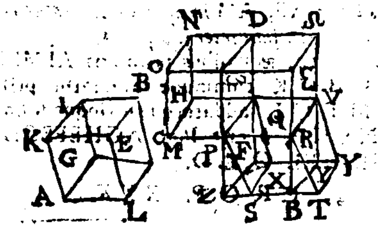


Solida parallelepipeda A D B C H E F G, AD-

ADCBIMLK *super eandem basin* ADCB *constituta, & in eadem altitudine, quorum insistentes lineæ* AH, AI *non in iisdem collocantur rectis lineis, inter se sunt æqualia.*

Nam produc rectas HEO, GFN, & LMO, KIP; & duc AP, DO, BQ, CN. *a* erunt tam
a 34. 1. DC, AB, HG, BF, PQ, ON; quam AD, HE, GF, BC, KL, IM, QN, PO æquales inter sese
b 29. 11. & parallelæ. *b* Quare Ppp. ADCBPONQ utrique Pppo. ADCBHEFG, ADCBIMLK æquale est; & *c* proinde hæc ipsa inter se æqualia sunt.
c 1. ax. 1. Q. E. D.

PROP. XXXI.



Solida parallelepipeda ALEKGMBI, CPωOHQDN *super æquales bases* ALEK, CPωO *constituta, & * in eadem altitudine, æqualia sunt inter se.*

* *Altitudo, est perpendicularis à plano basis ad planum oppositum.*

Habeant primo parallelepipeda AB, CD latera ad bases recta; & ad latus CP productum *a* fiat pgr. PRTS æq. & simile pgro. KELA; *b* adeoque Pop. PRTSQVYX æq. & sim.
a 18. 6. Pppo AB. Producantur OωE, NDʃ, ωPZ,
b 27. 11. & DQF, ERB, ʃVγ, TSZ, YXF; & duc Eʃ,
10. def. 11. Bγ, ZF.

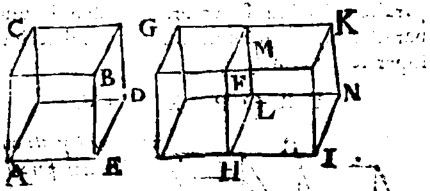
c 30 def. 11 Plana OεʃN, CRVH, STYF *c* parallela
d hyp. & sunt inter se; *d* & pgra ALEK, CPωO,
35. 3. PRTS, PRBZ æqualia sunt. Cum igitur Ppp.

CD

CD. PV $\delta\omega$ e :: pgr. Cω (PRBZ.) Rω :: Ppp. e 25. 11.
PRBZQVγF. PV $\delta\omega$, ferit Ppp. CD f = f 9. 5.
PRBZQVγF g = PRVQSTYX h = AB. g 29. 12.
Q. E. D. h constr.

Sin Pppa AB, CD latera basibus obliqua habeant; super easdem bases, & in eadem altitudine, ponantur parallelepipeda, quorum latera basibus sint recta. k Ea inter se, & obliquis æqualia erunt; m proinde & obliqua AB, CD æquantur. Q. E. D. k 29. 11. m 1. ax. 1.

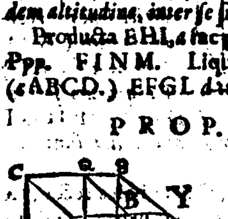
PROP. XXXII.



Solida parallelepipeda ABCD, EFGL sub eadem altitudine, inter se sunt ut bases AB, EF.

Producta EHI, a facpgr. FI = AB, & b comple Ppp. FINM. Liquet esse Ppp. FINM (c ABCD.) EFGL d:: FI. (AB) EF. Q.E.D. a 45. 1. b 31. 5. c 31. 11. d 25. 12.

PROP. XXXIII.



Similia solida parallelepipeda, ABCD, EFGH, inter se sunt in triplicata ratione homologorum laterum AI, EK.

Producantur rectæ AIL, DIO, BIN, & a fiant IL, IO, IN ipsis EK, KH, KF æquales, b adeòque a 31. 1. b 17. 11.

&

c 31. 1. & Ppp. IXMT æq. & sim. Pppo EFGH.
d hyp. c Perficiantur Ppp. a IXPB, DLYQ. Itaque d
e 1. 6. erit AI. IL. (EK) :: DI. IO (HK) :: BI. IN.
f 32. 11. (KF;) hoc est Pgr. AD DL :: DL. IX :: BO. IT; f id est Ppp. ABCD DLQY :: DLQY. IXBP :: IXBP. IXMT. (g EFGH.)
g constr.
h 10. def 5. h ergo ratio ABCD ad EFGH triplicata est rationis ABCD ad DLQY, k vel AI ad EK.
k 1. 6.
Q. E. D.

Coroll.

Hinc, si fuerint quatuor lineæ rectæ continue proportionales, ut est prima ad quartam, ita est parallelepipedum super primam descriptum ad parallelepipedum simile similiterque descriptum super secundam.

PROP. XXXIV.

Æqualium solidorum parallelepipedorum ADCB, EHGF *bases & altitudines reciprocantur* (AD. EH :: EG. AC) *Et quorum solidorum parallelepipedorum* ADCB, EHGF *bases & altitudines reciprocantur, illa sunt æqualia.*

(Sint primo latera CA, GE ad bases recta; si jam solidorum altitudines sint pares, etiam
a 3. 1. bases æquales erunt. & res clara est. Sin altitu-
b 31. 1. dines inæquales sint, a majori BG a detrahe EI
c 32. 11. = AC. & per I b duc planum IK parallelum
d 17. 5. basi EH. itaque
e 1. 6. 1. Hyp. AD. EH c :: Ppp. ADCB. EHIK d ::
f constr. Ppp. EHGF. EHIK e :: GL. IL e :: GE. IE
g 11. 5. & (f AC;) g liquet igitur esse AD. EH :: GE. AC
32. 11. Q. E. D.

2. Hyp

2. *Hyp.* ADCB. EHIK *b* :: AD. EH *k* :: EG. EI *l* :: GL. IL *m* :: Ppp. EHGF. EHIK, *n* quare Ppp ADCB=EHGF. Q. E. D. h 32. 11. k *hyp.* l 1. 6.

Sint deinde latera ad bases obliqua. Erigantur super iisdem basibus, in altitudine eadem, parallelepipeda recta. Erunt obliqua parallelepipeda his æqualia. Quare cum hæc per 1. partem reciprocent bases & altitudines, etiam illa reciprocabunt. Q. E. D. m 32. 11. n 9. 5.

Coroll.

Quæ de parallelepipedis demonstrata sunt Prop. 29, 30, 31, 32, 33, 34. *etiam conveniunt prismatis triangularibus, quæ sunt dimidia parallelepipeda, ut patet ex* Pr. 28. *Igitur,*

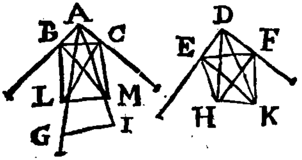
1. Prismata triangularia æque alta sunt ut bases.

2. Si eandem vel æquales habeant bases, & eandem altitudinem, æqualia sunt.

3. Si similia fuerint, eorum proportio triplicata est proportionis homologorum laterum.

4. Si æqualia sunt, reciprocant bases & altitudines, & si reciprocant bases & altitudines, æqualia erunt.

PROP. XXXV.



Si fuerint duo plani anguli BAC, EDF *æquales, quorum verticibus* A, D, *sublimes rectæ lineæ* AG, DH *insistant, quæ cum lineis primo positis angulos contineant æquales, utrumq; utriq;* (ang. GAB=HDB; & GAC=HDF.) *in sublimibus autem lineis* AG, DH *quælibet sumpta fuerint puncta* G, H;

& ab his ad plana BAC, EDF, *in quibus consistunt anguli primum positi* BAC, EDF, *ducta fuerint perpendiculares* GI, HK; *à punctis vero* I, K *quæ in planis à perpendicularibus fiunt, ad angulos primum positos adjunctæ fuerint rectæ lineæ* AI, DK; *hæ cum sublimibus* AG, DH *æquales angulos* GAM, HDK *comprehendent.*

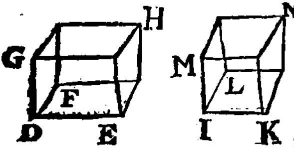
Fiant DH, AL æquales, & GI, LM parallelæ; & MC ad AC, MB ad AB, KF ad DF, KE ad DE perpendiculares, ducanturque rectæ
a 8. 11. BC, LB, LC, atque EF, HF, HE; *a* estque LM
b 3. *def.* 11 recta plano BAC; *b* quare anguli LMC, LMA, LMB; eademque ratione anguli HKF, HKD,
c 47. 1. HKE recti sunt. Ergo ALq *c* = LMq + AMq *c* = LMq + CMq + ACq *c* = LCq + ACq;
d 48. 1. *d* ergo ang. ACL rectus est. Rursus ALq *e* =
e 47. 1. LMq + MAq *e* = LMq + BMq + BAq *e* = BLq + BAq. *d* ergo ang. ABL etiam rectus est. Simili discursu anguli DFH, DEH recti
f 26. 1. sunt, *f* ergo AB = DE; *f* & BL = EH; *f* & AC = DF; & CL = FH. *g* quare etiam BC
g 4. 1. = EF, *g* & ang. ABC = DEF *g* & ang. ACB
h 3. *ax.* 1. = DFE. unde reliqui è rectis anguli CBM,
k 26. 1. BCM reliquis FEK, EFK æquantur. *k* ergo
l 47. 1. CM = FK, *l* ideoque & AM = DK. ergo si
m *constr.* ex LAq *m* = HDq. auferatur AMq = DKq,
n 47. 1. & *n* remanet LMq = HKq. quare trigona LAM,
3. *ax.* HDK sibi mutuo æquilatera sunt. *o* ergo ang.
o 8. 1. LAM = HDK. Q. E. D.

Coroll.

Itaque si fuerint duo anguli plani æquales, quorum verticibus sublimes rectæ lineæ æquales insistant, quæ cum lineis primo positis angulos contineant æquales, utrumque utrique; erunt à punctis extremis linearum sublimium ad plana angulorum primo positorum demissæ perpendiculares inter se æquales; nempe LM = HK.

P R O P.

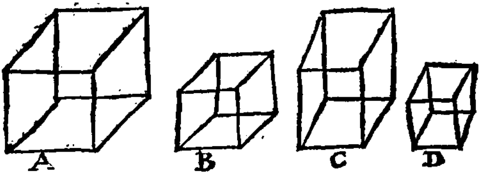
PROP. XXXVI.



Si tres rectæ lineæ DE, DG, DF *proportionales fuerint; quod ex his tribus fit solidum parallelepipedum* DH, *æquale est descripto à media linea* DG (IL) *solido parallelepipedo* IN, *quod æquilaterum quidem sit, æquiangulum vero prædicto* DH.

Quoniam DE. IK *a* :: IL. DF, *b* erit pgr. LK = FE. & propter angulorum planorum ad D & I, ac linearum GD, IM æqualitatem, etiam altitudines parallelepipedorum æquales sunt, ex coroll. præced. *c* ergo ipsa inter se æqualia sunt. Q. E. D. a *hyp.* b 14. 6. c 31. 11.

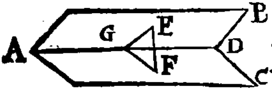
PROP. XXXVII.



Si quatuor rectæ lineæ A, B, C, D *proportionales fuerint, & solida parallelepipeda* A, B, C, D *quæ ab ipsis & similia, & similiter describuntur, proportionalia erunt. Et si solida parallelepipeda, quæ & similia, & similiter describuntur, fuerint proportionalia* (A.B :: C.D.) *& ipsæ rectæ lineæ* A, B, C, D *proportionales erunt.*

Nam rationes parallelepipedorum *a* triplicatæ sunt rationum, quas habent lineæ. ergo si A.B :: C. D. *b* erit Ppp. A. Ppp. B :: Ppp. C. Ppp. D. & vice versa. a 33. 11. b *sch.* 23. 5.

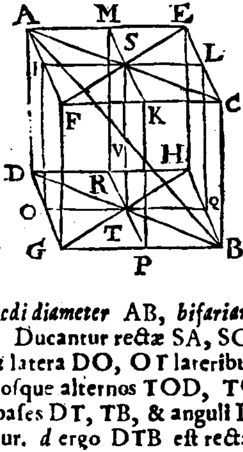
PROP. XXXVIII.



Si planum AB *ad planum* AC *rectum fuerit, & ab aliquo puncto* E *eorum, quæ sunt in uno planorum (* AB *) ad alterum planum* AC *perpendicularis* EF *ducta fuerit, in planorum communem sectionem* AD *cadet ducta perpendicularis* EF.

Si fieri potest, cadat F extra intersectionem
a 12. 1. AD. In plano AC *a* ducatur FG perpendicula-
b 4, & 3. ris ad AD, jungaturque EG. Angulus FGE *b* re-
def. 11. ctus est; & EFG rectus ponitur. ergo in triangu-
c 17. 1. lo EFG sunt duo anguli recti. Q. E. D.

PROP. XXXIX.



Si solidi parallelepipedi A B, *eorum quæ ex adverso planorum* AC, DB *latera* (AE, FC, AF, EC, & DH, GB, DG, HB) *bifariam secta sint; per sectiones autem plana* ILQO, PKMR *sint extensa; planorum communis sectio* ST, *& solidi parallelepipedi diameter* AB, *bifariam se mutuo secabunt.*

a 34. 1. Ducantur rectæ SA, SC, TD, TB. Propter
b 29. 1. *a* latera DO, OT lateribus BQ. QT, *b* angu-
c 4. 1. losque alternos TOD, TQB æquales, *c* etiam
d *sch.* 15. 1. bases DT, TB, & anguli DTO, BTQ æquan-
e 34. 1. tur. *d* ergo DTB est recta linea. eodem modo
f 9. 11. & ASC recta est linea. Porro *e* tam AD ad FG,
1. *ax.* *e* quam FG ad CB; *f* ideoque AD ad CB, *g* ac
g 33. 1. proinde AC ad DB parallelæ & æquales sunt.

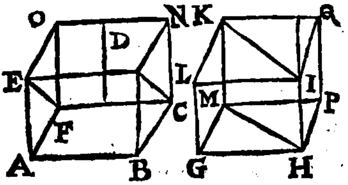
h quare

b quare AB, & ST in eodem plano ABCD exsistunt. Itaque cum anguli AVS, BVT ad verticem, & alterni ASV, BTV æquentur; *k* & AS = BT; erit AV = BV, *l* & SV = VT. Q. E. D. h 7. 11. k 7. *ax.* 1. l 26. 1.

Coroll.

Hinc, in omni parallelepipedo diametri omnes se mutuo bisecant in uno puncto, V.

PROP. XL.



***Si fuerint duo prismata* ABCFED, GHMLIK *æqualis altitudinis, quorum hoc quidem habeat basim* ABCF *parallelogrammum, illud vero* GHM *triangulum; duplum autem fuerit parallelogrammum* ABCF *trianguli* GHM; *æqualia erunt ipsa prismata* ABCFED, GHMLIK.**

Nam si perficiantur parallelepipeda AN, GQ, *a* erunt hæc æqualia ob *b* basium AC, GP, & *c* altitudinum æqualitatem. *d* ergo etiam prismata, *e* horum dimidia, æqualia erunt. Q. E. D. a 31. 11. b 34. 1. & 7. *ax.* c *hyp.* d 28. 11. e 7. *ax.* 1.

Schol. *And. Tar.*

Ex hactenus demonstratis habetur dimensio prismatum triangularium, & quadrangularium, seu parallelepipedorum, si nimirum altitudo ducatur in basim.

Ut si altitudo sit 10 pedum, basis vero pedum quadratorum 100 (mensurabitur autem basis per Sch. 35. 1. vel per 41. 1.) multiplica 100 per 10;

proveniunt 1000 pedes cubici pro soliditate prismatis dati.

Vide schol. 35. 1. Nam quemadmodum rectangulum, ita & parallelepipedum rectum producitur ex altitudine ducta in basim. Ergo quodvis parallelepipedum producitur ex altitudine in basim ducta, ut patet ex 31. hujus.

Deinde cum totum parallelepipedum producatur ex altitudine in totam basim, semissis ejus (hoc est prisma triangulare) producetur ex altitudine ducta in dimidiam basim, nempe triangulum.

Monitum.

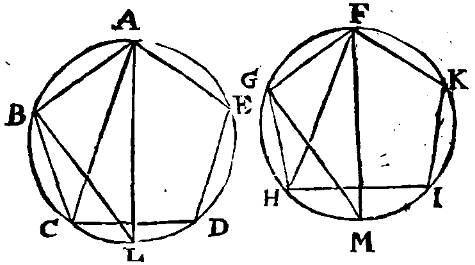
Nota, litterarum quæ designant angulum solidum primam esse semper ad punctum, in quo est angulus; litterarum vero quæ denotant pyramidem, ultimam esse ad verticem pyramidis.

Ex.gr. Angulus solidus ABCD est ad punctum A; pyramidis quoque BCDA vertex est ad punctum A, & basis triangulum BCD.

LIB.

LIB. XII.

PROP. II.



Quæ sunt in circulis ABD, FGI *polygona similia* ABCDE, FGHIK, *inter se sunt, ut quadrata à diametris* AL, FM.

Ducantur AC, BL, FH, GM. Quoniam *a* ang. ABC = FGH, *a* atque AB. BC :: FG. GH, *b* erit ang. ACB (*c* ALB) = FHG (*c* FMG.) anguli autem ABL, FGM *d* recti, ac proinde æquales sunt. *e* ergo triangula ABL, FGM æquiangula sunt. *f* quare AB. FG :: AL. FM. *g* ergo ABCDE. FGHIK :: ALq. FMq.

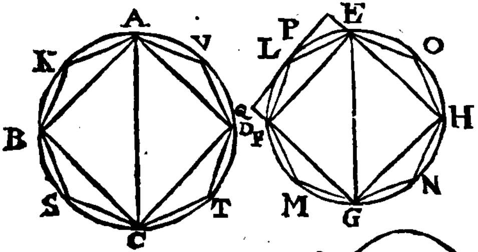
a 1. *def.* 6.
b 6. 6.
c 21. 3.
d 31. 3.
e 32. 3.
f *cor.* 4. 6.
g 22. 6.

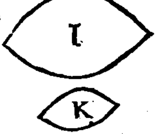
Coroll.

Hinc (quia AB. FG :: AL. FM :: BC. GH, &c.) polygonorum similium circulo inscriptorum *h* ambitus sunt ut diametri.

h 1. 12. & 12. 5.

PROP. II.





Circuli ABT, EFN *inter se sunt, quemadmodum quadrata à diametris* AC, EG.

Ponatur ACq. EGq :: circ. ABT. I. Dico I = circ. EFN.

Nam primo, si fieri potest, sit I ⊐ circ. EFN, sitque excessus K. Circulo EFN inscribatur
a sch. 7. 4. quadratum EFGH, *a* quod dimidium est circumscripti quadrati, adeoque semicirculo majus.
b 30. 3. *b* Biseca arcus EF, FG, GH, HE, & ad puncta bisectionum junge rectas EL, LF, &c. per L
c sch. 27. 3. duc tangentem PQ (*c* quæ ad EF parallela est,) & produc HEP, GFQ; estque triangulum
d 41. 1. ELF *d* dimidium parallelogrammi EPQF, adeoque majus dimidio segmenti ELF; pariterque reliqua triangula ejusmodi reliquorum segmentorum dimidia superant. Et si iterum bisecentur arcus EL, LF, FM, &c. rectæque adjungantur, eodem modo triangula segmentorum semisses excedent. Quare si quadratum EFGH è circulo EFN, & è reliquis segmentis triangula
e 1. 10. detrahantur, & hoc fiat continuo, tandem *e* restabit magnitudo aliqua minor quam K. Eousque perventum sit, nempe ad segmenta EL, LF, FM, &c. minora quam K, simul sumpta.

pta. ergo I (*f* circ. EFN − K) ⊐ polyg. *f hyp. &*
ELFMGNHO (circ. EFN − segm. EL + LF *1. ax.*
&c.) Circulo ABT inscriptum *g* puta simile po- *g 30.3. &*
lygonum AKBSCTDV. itaque quum *1. post. 1.*
AKBSCTDV. ELFMGNHO *h* :: ACq. *h 1. 12.*
EGq *k* :: circ. ABT. I. ac polyg. AKBSCTDV *k hyp.*
l ⊐ circ. ABT. *m* erit polyg. ELFMGNHO *l 9. ax. 1.*
⊐ I. sed prius erat I ⊐ ELFMGNHO. quæ *m 14.5.*
repugnant.

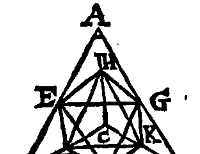
Rursus, si fieri potest, sit I ⊏ circ. EFN.
Quoniam igitur ACq. EGq *n* :: circ. ABT. I; *n hyp.*
inverseque I. circ. ABT :: EGq. ACq. pone L.
circ. ABT :: circ EFN. K. *o* ergo circ. ABT *o 14. 5.*
⊏ K, *p* atque EGq. ACq :: circ. EFN. K. Quæ *p 11. 5.*
repugnare modo ostensum est.

Ergo concludendum est, quod I = circ. EFN,
Q. E. D.

Coroll.

Hinc, ut circulus est ad circulum, ita polygonum in illo descriptum ad simile polygonum in hoc descriptum.

PROP. III.



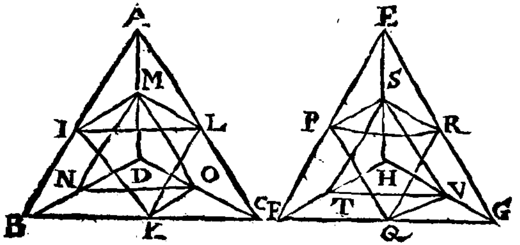
Omnis pyramis ABDC *triangularem habens basim, dividitur in duas pyramides* AEGH, HIKC *æquales & similes inter se, triangulares habentes bases, & similes toti* ABDC; *& in duo prismata æqualia* BFGEIH, FGDIHK; *quæ duo prismata majora sunt dimidio totius pyramidis* ABDC.

Latera pyramidis bisecentur in punctis E, F, G, H, I, K; junganturque rectæ EF, FG, GE, EI, IF, FK, KG, GH, HE. Quoniam latera pyra-

a 2. 6. pyramidis proportionaliter secta sunt, *a* erunt HI, AB; & GF, AB; & IF, DC; atque HG, DC, &c. parallelæ; proinde & HI, FG, & GH, FI parallelæ sunt. liquet igitur triangula ABD,
b 29. 1. AEG, EBF, FDG, HIK *b* æquiangula esse; &
c 26. 1. quatuor ultima *c* æquari. eodem modo triangula ACB, AHE, EIB, HIC, FGK æquiangula sunt; & quatuor postrema inter se æqualia. Similiter triangula BFI, FDK, IKC, EGH; & denuo triangula AHG, GDK, HKC, EFI, similia sunt & æqualia. Quinetiam triang. HIK ad ADB, & EGH ad BDC, & EFI ad ADC, & FGK ad
d 15. 11. ABC *d* parallela sunt. Ex quibus perspicue sequitur primo, pyramides AEGH, HIKC æquales
e 10. *def.* 11 esse; totique ABDC, & inter se *e* similes. deinde solida BFGEIH, FGDIHK prismata esse, & quidem æque alta, nempe sita inter parallela plana ABD, HIK. verum basis BFGE basis FDG
f 2. *ax.* 1. *f* duplex est. *g* quare dicta prismata æqualia sunt.
g 40. 11. quorum alterum BFGEIH pyramide BEFI, hoc est, AEGH majus est, totum sua parte; proinde duo prismata majora sunt duabus pyramidibus, totiusque adeo pyramidis ABDC dimidium excedunt. Q. E. D.

PROP.

PROP. IV.



Si fuerint duæ pyramides ABCD, EFGH *ejusdem altitudinis, triangulares habentes bases* ABC, EFG; *sit autem illarum utraque divisa & in duas pyramides* (AILM, MNOD; & EPRS, STVH) *æquales inter se, & similes toti; & in duo prismata æqualia* (IBKLMN, KLCNMO; & PFQRST, QRGTSV;) *ac eodem modo divisa sit utraque pyramidum, quæ ex superiore divisione natæ sunt, idque semper fiat; erit ut unius pyramidis basis ad alterius pyramidis basim, ita & omnia, quæ in una pyramide, prismata ad omnia, quæ in altera pyramide prismata, multitudine æqualia.*

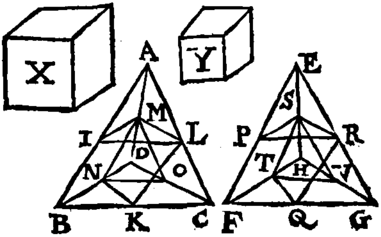
Nam (adhibendo constructionem præcedentis) BC. KC *a* :: FG. QG. *b* ergo triang. ABC est ad simile triang. LKC, ut EFG ad *c* simile RQG. ergo permutando ABC. EFG *d* :: LKC. RQG *e* :: Prism. KLCNMO. QRGTSV (nam hæc æque alta sunt) *f* :: IBKLMN. PFQRST. *g* quare triang. ABC. EFG :: Prism. KLCNMO + IBKLMN, Prism. QRGTSV + PFQRST. Q. E. D.

a 15. 5.
b 22. 6.
c 2. 6. &c.
d 16. 5.
e *sch.* 34. 11
f 7. 5.
g 12. 5.

Sin ulterius simili pacto dividantur pyramides MNOD, AILM; & EPRS, STVH, erunt quatuor nova prismata hic effecta ad quatuor isthic

isthic producta, ut bases MNO & AIL ad bases STV & EPR, hoc est ut LKC ad RQG, vel ut
h 12. 5. ABC ad EFG. *b* quare omnia prismata pyramidis ABCD ad omnia ipsius EFGH ita se habent, ut basis ABC ad basim EFG. Q.E.D.

PROP. V.



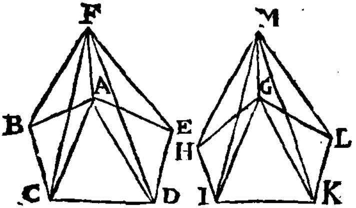
Sub eadem altitudine existentes pyram ABCD, EFGH, *triangulares habentes bases* ABC, EFG, *inter se sunt ut bases* ABC, EFG.

Sit triang. ABC. EFG :: ABCD. X. Dico X = pyr. EFGH. Nam, si possibile est, sit X ⊐ EFGH; sitque Y excessus. Dividatur pyramis EFGH in prismata & pyramides, & reliquæ pyramides similiter,
a 1. 10. *a* donec relictæ pyramides EPRS, STVH minores evadant solido Y. Quum igitur pyr. EFGH = X + Y; liquet reliqua prismata PFQRST, QRGTSV solido X majora esse. Pyramidem ABCD simili-
b 4. 12. ter divisam concipe; *b* eritque prism. IBKLMN + KLCNMO. PFQRST + QRGTSV ::
c hyp. ABC. EFG. *c* :: pyr. ABCD. X. *d* ergo X ⊏
d 14. 5. prism. PFQRST + QRGTSV; quod repugnat prius affirmatis.

Rursus, dic X ⊏ pyr. EFGH. pone pyr.
e hyp. & EFGH. Y :: X. pyr. ABCD *e* :: EFG. ABC.
cor. 4. 5. quia EFGH *f* ⊐ X, *g* erit Y ⊐ pyr. ABCD,
f suppos. quod fieri nequit, ex jam dictis. Concludo igitur,
g 14. 5. quod X = pyr. EFGH. Q.E.D.

PROP.

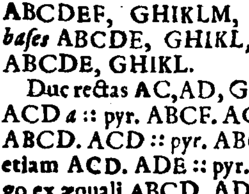
PROP. VI.



Sub eadem altitudine existentes pyramides ABCDEF, GHIKLM, *& polygonas habentes bases* ABCDE, GHIKL, *inter se sunt ut bases* ABCDE, GHIKL.

Duc rectas AC, AD, GI, GK. Est bas. ABC. ACD *a* :: pyr. ABCF. ACDF. *b* ergo composite ABCD. ACD :: pyr. ABCDF. ACDF. *a* atqui etiam ACD. ADE :: pyr. ACDF. ADEF. *c* ergo ex æquali ABCD. ADE :: ABCDF. ADEF. *b* ergo componendo ABCDE. ADE :: pyr. ABCDEF. ADEF. porro ADE. GKL *d* :: pyr. ADEF. GKLM; ac, ut prius, atque inverse GKL. GHIKL :: pyr. GKLM. GHIKLM. *c* ergo iterum ex æqualibus, ABCDE. GHIKL :: pyr. ABCDEF. GHIKLM. Q. E. D.

a 5. 12.
b 18. 5.
c 22. 5.
d 5. 12.

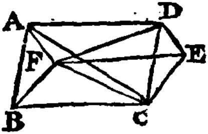


Si bases non habent latera æque multa, demonstratio sic procedet. Bas. ABC. GHI *e* :: pyr. ABCF. GHIK. *e* atque ACD. GHI :: pyr. ACDF. GHIK. *f* ergo bas. ABCD. GHI :: pyr. ABCDF. GHIK. *e* Quinetiam bas. ADE. GHI :: pyr. ADEF. GHIK. *f* ergo bas. ABCDE. GHI :: pyr. ABCDEF. GHIK.

e 5. 12.
f 24. 5.

PROP.

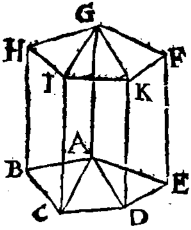
PROP. VII.



Omne prisma ABCDFE *triangularem habens basim, dividitur in tres pyramides* ACBF, ACDF, CDFE *æquales inter se, triangulares bases habentes.*

Ducantur parallelogrammorum diametri
a 34. 1. AC, CF, FD. Triang. ACB *a*= ACD *b* ergo æ-
b 5. 12. que altæ pyramides ACBF, ACDF æquantur. eodem modo pyr. DFAC = pyr. DFEC. atqui ACDF, & DFAC una eademque sunt py-
c 1. ax. 1. ramis. *c* ergo tres pyramides ACBF, ACDF, DFEC, in quas divisum est prisma, inter se æquales sunt. Q. E. D.

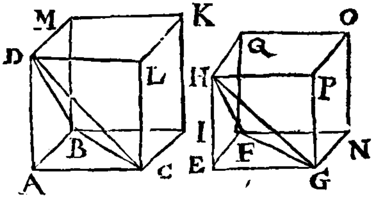
Coroll.



Hinc, quælibet pyramis tertia est pars prismatis eandem cum illa habentis & basim & altitudinem: sive, prisma quodlibet triplum est pyramidis eandem cum ipso habentis basim & altitudinem.

Nam resolve prisma polygonum ABCDEGHIKF in trigona prismata, & pyramidem ABCDEH
a 7. 12. in trigonas pyramides. *a* Erunt singulæ partes prismatis triplæ singularum partium pyramidis.
b 1. 5. *b* proinde totum prisma ABCDEGHIKF totius pyramidis ABCDEH triplum est. Q.E.D.

PROP. VIII.



Similes pyramides ABCD, EFGH, *quæ triangulares habent bases* ABC, EFG, *in triplicata sunt ratione homologorum laterum* AC, EG.

a Perficiantur parallelepipeda ABICDMKL, EFNGHQOP; quæ *b* similia sunt & pyramidum ABCD, EFGH *c* sextupla; *d* ideoque in eadem cum ipsis ratione ad se invicem, *e* hoc est in triplicata homologorum laterum. Q. E. D.

a 27. 11.
b 9. def. 11.
c 28. 11. & 7. 12.
d 15. 5.
e 33. 11.

Coroll.

Hinc, etiam similes polygonæ pyramides rationem habent laterum homologorum triplicatam; ut facile probabitur resolvendo has in trigonas pyramides.

PROP. IX.

Vide Schema præced.

Æqualium pyramidum ABCD, EFGH, *& triangulares bases* ABC, EFG *habentium, reciprocantur bases & altitudines: & quarum pyramidum triangulares bases habentium reciprocantur bases & altitudines, illæ sunt æquales.*

1. *Hyp.* Perfecta parallelepipeda ABICDMKL, EFNGHQOP æqualium pyramidum ABCD, EFGH (utrumque utriusque) *a* sextupla sunt, ac æqualia ideo inter se, ergo alt. (H.)

a 28. 11. & 7. 12.

alt.

b 34. 11. alt. (D) *b* :: ABIC. EFNG *c* :: ABC. EFG.
c 15. 5. Q. E. D.

d *hyp.* 2. *Hyp.* Alt. (H.) alt. (D) *d* :: ABC. EFG *e* ::
e 15. 5. ABIC. EFNG. *f* ergo parallelepipeda ABIC-
f 34. 11. DMKL, EFNGHQOP æquantur; *g* proinde
g 6. *ax.* 1. & pyramides ABCD, EFGH, horum subsextuplæ, pares sunt. Q. E. D.

Eadem polygonis pyramidibus conveniunt: nam hæ ad trigonas reduci possunt.

Coroll.

Quæ de pyramidibus demonstrata sunt Prop. 6, 8, 9. *etiam conveniunt quibuscunque prismatis, cum hæc tripla sint pyramidum eandem basim & altitudinem habentium. itaque* 1. Prismatum æque altorum eadem est proportio, quæ basium.

2. Similium prismatum proportio triplicata est proportionis laterum homologorum.

3. Æqualia prismata reciprocant bases & altitudines; & quæ reciprocant, sunt æquales.

Schol.

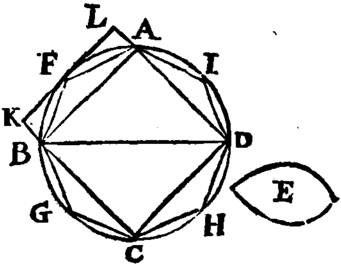
Ex hactenus demonstratis elicitur dimensio quorumcunque prismatum & pyramidum.

a *cor.* 1. *hujus; & sch.* 40. 12. *a* Prismatis soliditas producitur ex altitudine in basim ducta; *b* itaque & pyramidis ex tertia altitudinis parte ducta in basin.

b 7. 12.

PROP.

PROP. X.



Omnis conus tertia pars est cylindri habentis eandem cum ipso basin ABCD, *& altitudinem æqualem.*

Si negas, primo Cylindrus triplum coni superet excessu E. Prisma super quadratum circulo ABCD inscriptum *a* subduplum est prismatis super quadratum eidem circulo circumscriptum sibi & cylindro æque alti. ergo prisma super quadratum ABCD superat cylindri semissem. eodem modo prisma super basim AFB cylindro æque altum segmenti cylindrici AFB *b* dimidio majus est. Continuetur bisectio arcuum, & detrahantur prismata, donec segmenta cylindri relicta, nempe ad AF, FB, &c. minora evadant solido E Itaque cylind. — segment. AF, FB, &c. (prisma ad basim AFBGCHDI) *c* majus est quam cylind. — E (*d* triplum coni.) ergo pyramis dicti prismatis *e* pars tertia (ad eandem basim sita, ejusdemque altitudinis) cono æque alto ad basim ABCD circulum major est, pars toto. Q. E. A.

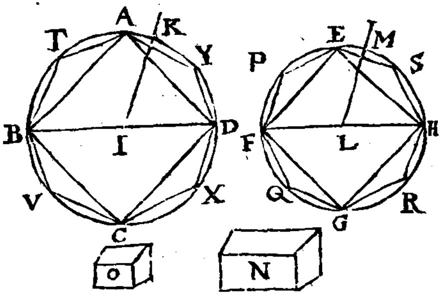
Vide fig. 2. hujus.
a *sch. 7. 4. & cor. 9. 12.*
b *sch. 27. 3. & cor. 9. 12.*
c *5. ax. 1.*
d *hyp.*
e *cor. 7. 12.*

Sin conus tertia parte cylindri major dicatur, sit itidem excessus E. Ex cono detrahe pyramides, ut in priori parte prismata ex cylindro, donec restent coni segmenta aliqua, puta ad AF,

V FB,

FB, BG, &c. minora solido E. ergo con.—B ($f\frac{1}{3}$ cylindr.) ⊐ pyr. AFBGCHDI (con.—
f hyp. segment. AF, FB, &c.) ergo prisma pyramidis triplum (æque altum scilicet atque ad eandem basim) cylindro ad basim ABCD majus est, pars toto. Q.E.A. Quare fatendum est, quod cylindrus triplo cono æquatur. Q. E. D.

PROP. XI.



Sub eadem altitudine existentes cylindri, & coni ABCDK, EFGHM, *inter se sunt ut bases* ABCD, EFGH.

Sit circ. ABCD. circ. EFGH :: con. ABCDK. N. Dico N = con. EFGHM.

Nam si fieri potest, sit N ⊐ con. EFGHM, sitque excessus O. Suppofita præparatione, & argumentatione præcedentis; erit O majus segmentis conicis EP, PF, FQ, &c. ideoque soli-
a 30.3. & dum N ⊐ pyr. EPFQGRHSM. *a* Fiat in cir-
1. post. culo ABCD simile polygonum ATBVCXDY.
b 6 12. Quia pyr. ABVYK. pyr. EFQSM *b* :: polyg.
c cor.2.12. ATBVY. polyg. EPFQS *c* :: circ. ABCD. circ.
d hyp. EFGH *d* :: con. ABCDK. N. *e* erit pyram.
e 14.5. EPFQGRHSM ⊐ N. contra modo dicta.

Rursus dic N ⊏ con. EFGHM. pone con. EFGHM. O :: N. con. ABCDK *f* :: circ. EFGH. ABCD. *g* ergo O ⊐ con. ABCDK, quod

quod absurdum est, ex ostensis in priori parte. f *hyp.* & *invertendo.*
Itaque potius dic, A B C D. EFGH :: con. ABCDK. EFGHM. Q. E. D. g 14. 5.

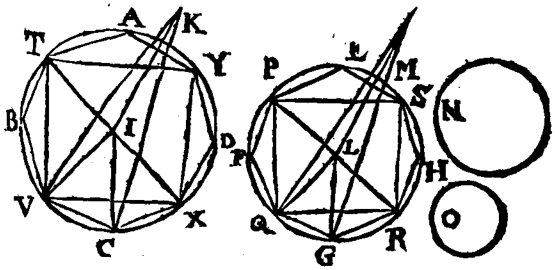
Idem demonstrabitur de cylindris, si conorum & pyramidum loco concipiantur cylindri & prismata. ergo, &c.

SCHOL.

Ex his habetur dimensio cylindrorum & conorum quorumcunque. Cylindri rectæ soliditas producitur ex base circulari (*a* pro cujus dimensione consulendus est Archimedes) ducta in altitudinem. *b* igitur & cujuscunque cylindri. a 1. *Prop. de dimens. circ.*

c Itaque coni soliditas producitur ex tertia parte altitudinis ducta in basim. b 11. 12. c 10. 12.

PROP. XII.



Similes coni & cylindri ABCDK, EFGHM *in triplicata ratione sunt diametrorum* TX, PR, *quæ in basibus* ABCD, EFGH.

Habeat conus A ad aliquod N rationem triplicatam TX ad PR. dico N = con. EFGHM; Nam si fieri potest, sit N ⊐ E F G H M; sitque excessus O. ergo ut in Prioribus, N ⊐ pyr. EPFQGRHSM. Sint axes conorum IK LM, adducanturque rectæ VK, CK, VI, CI; & QM, GM, QL, GL. Quoniam coni similes sunt, *a* est V I. IK :: Q L. L M. anguli vero VIK, QLM *b* recti sunt. *c* ergo trigona VIK, a 24. *def.* 11 b 18 *def.* 11 c 6. 6.

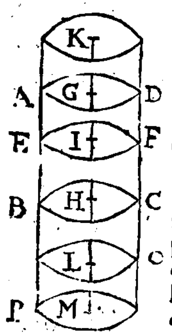
 QLM,

d 4. 6. QLM æquiangula sunt; *d* unde VC, VI :: QG. QL. item VI. VK :: QL. QM. ergo ex æ-
e 7. 5. quali VC. VK :: QG. QM. *e* quinetiam VK. CK :: CM. MG. ergo rursus ex æquo VC.
f 5. 6. CK :: QG. GM. *f* ergo triangula VKC, QMG similia sunt; similique argumento reliqua
g 9 *def.* 11. hujus pyramidis triangula reliquis illius, *g* quare
h *cor.* 8. 12. pyramides ipsæ similes sunt. *h* sunt vero hæ in
k 4. 6. triplicata ratione VC ad QG, *k* hoc est VI ad
l 15. 5. QL, *l* vel TX ad PR. *m* ergo pyr. ATBVC-
m *hyp.* & XDYK. pyr. EPFQGRHSM :: con. ABCDK.
11. 5. N. *n* unde pyr. EPFQGRHSM $\sqsupset$ N; quod
n 14. 5. repugnat prius dictis.

Rursus, dic N. $\sqsubset$ con. EFGHM. sit con.
o *Prim* & EFGHM. O :: N. con. ABCDK *o* :: pyr.
inverse. EPRM. ATCK *p* :: GQ. VC ter :: *q* PR.
p *cor.* 8. 12. TX ter. ergo O *r* $\sqsupset$ ABCDK. quod modo
q 4. 6. repugnare ostensum est. Proinde N = con.
r 14. 5. EFGHM. Q. E. D.

Quoniam vero quam proportionem habent coni, eandem quoque obtinent cylindri, eorum tripli, habebit quoque cylindrus ad cylindrum proportionem diametrorũ in basibus triplicatã.

PROP. XIII.

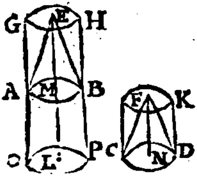


Si cylindrus ABCD *plano* EF *secetur adversis planis* BC, AD *parallelo; erit ut cylindrus* AEFD *ad cylindrum* EBCF, *ita axis* GI *ad axem* IH.

a 3. 1. Producto axe, *a* sume GK = GI, & HL = IH = LM. & concipe per puncta K, L, M, plana duci circulis AD, BC parallela.
b 11. 12. *b* ergo cylind. ED = cyl. AN. & cylin. EC *b* = BO. *b* = OP. itaque cylindrus

drus EN cylindri ED æque multiplex eſt, ac axis IK axis IG. pariterque cylindrus FP æque multiplex eſt cylindri BF, ac axis IM axis IH. prout vero IK $=$, $\sqsubset$, $\sqsupset$ IM, *c* ſic cylindr. EN $=$, $\sqsubset$, $\sqsupset$ EP. *d* ergo cyl. AEFD. cyl. EBCF :: GI. IH. Q. E. D. c 11. 12. d 6. *def.* 5.

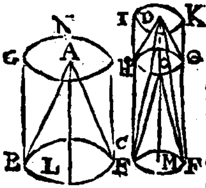
PROP. XIV.



Super æqualibus baſibus AB, CD *exiſtentes coni* AEB, CFD, *& cylindri* AH, CK, *inter ſe ſunt ut altitudines* ME, NF.

Productis cylindro HA & axe EM, ſume ML $=$ FN; & per punctum L ducatur planum baſi AB parallelum. *a* erit cyl. AP $=$ CK. *b* atqui cylind. AH. AP. (CK) :: ME. ML. (NF.) Q. E. D. Idem de conis cylindrorum ſubtriplis dictum puta. * imo de priſmatis & pyramidibus. a 11. 12. b 13. 12. **Adbibe* 9 & 7. 12.

PROP. XV.



Æqualium conórum BAC, EDF, *& cylindrorum* BH, EK, *reciprocantur baſes & altitudines* (BC. EF :: MD. LA:) *& quorum conorum, & cylindrorum reciprocantur baſes & altitudines, illi ſunt æquales.*

Si altitudines pares ſint, etiam baſes pares erunt; & res clara eſt. Sin altitudines ſint impares, aufer MO $=$ LA. a 14. 12. b *conſtr.*

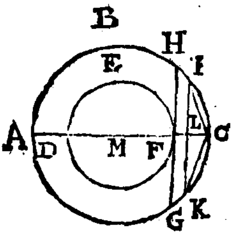
1. *Hyp.* Eſtque MD. MO (*a* LA) *b* :: cyl. EK (*c* BH.) EQ *d* :: circ. BC. EF. Q. E. D c *hyp.* d 11. 12.

 2. *Hyp.*

e hyp. 2. Hyp. BC. EF e :: DM. OM (LA) f :: Cyl. BK. EQ g :: BC. EF h :: BH. EQ. k Ergo cylind. EK = BH. Q. E. D.
f 14. 12.
g 11. 5.
h 11. 12.
k 9. 5.

Simili argumento utere de conis.

PROP. XVI.



Duobus circulis ABCG, DEF *circa idem centrum* M *existentibus, in majori circulo* ABCG *polygonum æquilaterum, & parium laterum inscribere, quod non tangat minorem circulum* DEF.

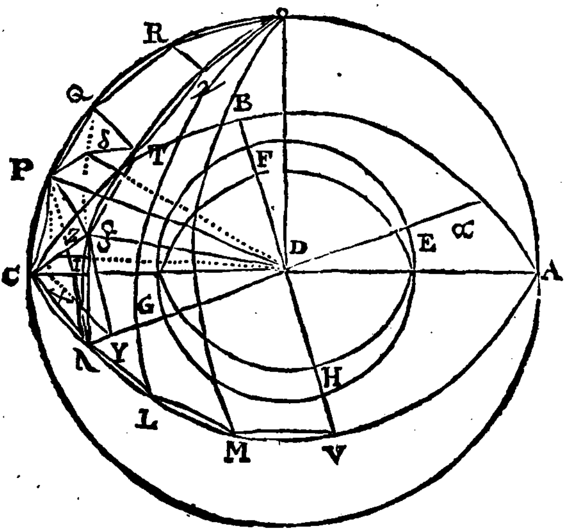
Per centrum M extendatur recta AC secans circulum DEF in F. ex quo erige perpendicularem FH. *a* Biseca semicirculum ABC, ejusque semissem BC, atque ita continuo, *b* donec arcus IC minor evadat arcu HC. ab I demitte perpendicularem IL. Liquet arcum IC totum circulum metiri, numerumque arcuum esse parem, adeoque subtensam IC latus esse *c* polygoni inscriptibilis, quod circulum DEF minime continget. Nam HG *d* tangit circulum DEF; *e* cui parallela est IK, extraque sita, *f* quare IK circulum non tangit, multoque magis CI, CK, & reliqua polygoni latera, longius à centro distantia, circulum DEF non tangunt. Q. E. F.

a 30. 3.
b 1. 10.
c sch. 16. 4.
d cor. 16. 3.
e 28. 1.
f 34. def. 1.

Coroll. Nota, quod IK non tangit circulum DEF.

PROP.

PROP. XVII.



Duabus sphæris ABCV, EFGH *circa idem centrum* D *existentibus, in majori sphæra* ABCV *solidum polyedrum inscribere, quod non tangat superficiem minoris sphæræ* EFGH.

Secentur ambæ sphæræ plano per centrum faciente circulos EFGH, ABCV. ducanturque diametri AC, BV secantes perpendiculariter. Circulo ABCV *a* inscribatur polygonum æquilaterum VMLNC, &c. circulum EFGH minime tangens. ducta diametro Nα, erectaque DO recta ad planum ABC. per DO, perque diametros AC, Nα erigi concipiantur plana DOC, DON, quæ ad circulum ABCV *b* recta erunt, ideoque in superficie sphæræ *c* quadrantes effici- a 16. 11. b 18. 11. c cor. 33. 6.

d 4. 1. efficient DOC, DON. in quibus *d* aptentur
rectæ CP, PQ, QR, RO, NS, ST, Tγ, γO
ipsis CN, NL, &c pares, & æque multæ. In re-
liquis quadrantibus OL, OM, &c. inque tota
sphæra eadem constructio fiat. Dico factum.

A punctis P, S ad planum ABCV demitte
e 38. 11. perpendiculares PX, SY, *e* quæ in sectiones AC,
f 12. ax. Nα cadent. Quoniam igitur tam *f* anguli recti
g 27. 3. PXC, SYN, *g* quam PCX, SNY, *h* æqualibus
peripheriis insistentes, *f* pares sunt, triangula
h 32. 1. PCX, SNY *h* æquiangula sunt. Cum igitur PC
k constr. *k* = SN, *l* etiam PX = SY, *l* & XC = YN;
l 26. 1. *m* quare DX = DY. *n* ergo DX. XC :: DY.
m 3. ax. 1. YN. *o* ergo parallelæ sunt YX, NC. quia vero
n 7. 5. PX, SY pares, & cum eidem plano ABCV re-
o 2. 6. ctæ, etiam *p* parallelæ sunt, *q* erunt YX,
p 6. 11. SP etiam pares & parallelæ. *r* ergo SP, NC
q 33. 1. inter se parallelæ sunt. ergo *s* quadrilaterum
r 9. 11. NCPS, eademque ratione SPQT, TQRG,
s 7. 11. sed & *t* triangulum γRO totidem sunt plana.
t 2. 11. Eodem modo tota sphæra ejusmodi quadrilate-
ris & triangulis repleta ostendetur. quare inscri-
ptum est polyedrum.

u 11. 11. A centro D *u* duc DZ rectum plano NCPS;
& junge ZN, ZC, ZS, ZP. Quoniam DN.
x 4. 6. NC *x* :: DY. YX; est NC *y* ⊏ YX (SP;) pa-
y 14. 5. riterque SP ⊏ TQ, & TQ ⊏ γR. Et quia
z 3. def. 11. anguli DZC, DZN, DZS, DZP, *z* recti sunt,
a 15. def. 1. latera vero DC, DN, DS, DP *a* æqualia, &
b 47. 1. DZ commune, *b* erunt ZC, ZN, ZS, ZP æ-
c 15. def. 1. quales inter se; proinde circa quadrilaterum
d constr. NCPS *c* describi potest circulus, in quo (*b*
e 28. 3. NS, NC, CP *d* æquales, & NC ⊏ SP) NC
f 33. 6. *e* plusquam quadrantem subtendit. *f* ergo ang.
g 12. 2. NZC ad centrum obtusus est. *g* ergo NCq ⊏
h 32. 1. 2 ZCq (ZCq + ZNq.) Sit NI ad AC nor-
k 9. ax. 1. malis. ergo cum ang. ADN (*h* DNC +
l 5. 1. DCN) sit *k* obtusus, *l* erit semissis ejus DCN
recti

recti semisse major; propterea que eo minor est reliquus è recto ang. CNI. *n* unde IN ⊏ IC. n 19. 1.
ergo NCq (NIq + ICq) *o* ⊐ 2 INq. itaque o 47. 1.
IN ⊏ ZC. & consequenter DZ *p* ⊏ DI. atqui p 47. 1.
punctum I est *q* extra sphæram EFGH. ergo q *cor.* 16. 12.
punctum Z potiori jure est extra ipsam. adeoque planum NCPS (cujus *r* proximum centro punctum est Z) sphæram EFGH non contingit. Et r 47. 1.
si ad planum SPQT demittatur perpendicularis Dδ, punctum δ, adeoque & planum SPQT adhuc ulterius à centro elongatur; idemque est de reliquis polyedri planis. ergo polyedrum ORQPCN, &c. majori sphæræ inscriptum, minorem non contingit. Q. E. F.

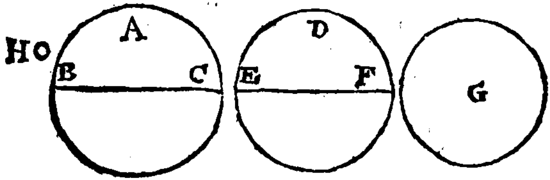
Coroll.

Hinc sequitur, *Si in quavis alia sphæra describatur solidum polyedrum, simile prædicto solido polyedro, proportionem polyedri in una sphæra ad polyedrum in altera esse triplicatam ejus quam habent sphærarum diametri.*

Nam si ex centris sphærarum ad omnes angulos basium dictorum polyedrorum rectæ lineæ ducantur, distribuentur polyedra in pyramides numero æquales & similes, quarum homologa latera sunt semidiametri sphærarum; ut constat, si intelligatur harum sphærarum minor intra majorem circa idem centrum descripta, congruent enim sibi mutuo lineæ rectæ ductæ à centro sphæræ ad basium angulos, ob similitudinem basium, ac propterea pyramides efficientur similes. Quare cum singulæ pyramides in una sphæra, ad singulas pyramides illis similes in altera sphæra
a habeant proportionem triplicatam laterũ homologorum, hoc est, semidiametrorum sphærarum; a *cor.* 8. 12.
sint autem *b* ut una pyramis ad unam pyramidem, ita omnes pyramides, hoc est, solidum polyedrum ex his compositum, ad omnes pyramides, b 12. 5.

mides, id eſt, ad ſolidum polyedrum ex illis conſtitutum; habebit quoque polyedrum unius ſphæræ ad polyedrum alterius ſphæræ proporti-
c 15. 5. onem triplicatam ſemidiametrorum, *c* atque adeo diametrorum.

PROP. XVIII.



Sphæræ BAC, EDF *ſunt in triplicata ratione ſuarum diametrorum* BC EF.

Sit ſphæra BAC ad ſphæram G in triplicata ratione diametri BC ad diametrum EF. Dico $G = EDF$. Nam ſi fieri poteſt, ſit $G \sqsupset EDF$. & cogita ſphæram G concentricam eſſe ipſi EDF.
a 17. 12. Sphæræ EDF *a* polyedrum ſphæram G non tan-
b cor. 17. 12 gens, ſphæræque BAC ſimile polyedrum inſcri-
c *hyp.* batur. *b* Hæc polyedra ſunt in triplicata ratione
d 14. 5. diametrorum BC, EF, *c* id eſt, ſphæræ BAC ad G. *d* Proinde ſphæra G major eſt polyedro ſphæræ EDF inſcripto, pars toto.

Rurſus, ſi fieri poteſt, ſit ſphæra $G \sqsubset EDF$. Sitque ut ſphæra EDF ad aliam ſphæram H, ita
e *hyp. in-* G ad BAC, *e* hoc eſt in triplicata ratione dia-
verſ. metri EF ad BC; cum igitur BAC *f* $\sqsubset$ H, in-
f 14. 5. currimus abſurditatem prioris partis. Quin potius ſphæra $G = EDF$. Q. E. D.

Coroll.

Hinc, ut ſphæra ad ſphæram, ita eſt polyedrum in illa deſcriptum ad polyedrum ſimile in hac deſcriptum.

LIB. XIII.

PROP. I.

SI recta linea z secundum extremam & mediam rationem secetur (z.a::a.e;) majus segmentum a assumens dimidium totius z, quintuplum potest ejus, quod à dimidia totius z describitur, quadrati.

Dico $Q. a + \frac{1}{2} z = 5\ Q. \frac{1}{2} z$. *a* — a 4. 2.
hoc est $aa + \frac{1}{4} zz + za = zz + \frac{1}{4} zz$. *b* vel $aa + za = zz$. Nam $ze + za$ *c* $= zz$. & ze *d* $= aa$. *e* ergo $aa+za = zz$. Q. E. D.

b 3. ax. 1.
c 2. 2.
d hyp & 16 6.
e 2. ax. & 1. ax.

PROP. II.

Si recta linea $\frac{1}{2}$ z + a sui ipsius segmenti $\frac{1}{2}$ z quintuplum possit, dupla prædicti segmenti (z) extrema ac media ratione secta majus segmentum est a, reliqua pars ejus quæ à principio recta $\frac{1}{2}$ z+a.

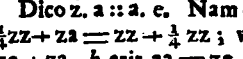
Dico z. a :: a. e. Nam quia per hyp. * $aa + \frac{1}{4}zz + za = zz + \frac{1}{4} zz$; vel $aa + za = zz$ *a* $= ze + za$. *b* erit $aa = ze$. *c* quare z. a :: a. e. Q. E. D.

* 4. 2.
a 2. 2.
b 3. ax. 1.
c 17. 6.

Vide fig. præced.

PROP. III.

Si recta linea z secundum extremam ac mediam rationem secetur (z. a :: a. e;) minus segmentum e assumens dimidium majoris segmenti a, quintuplum potest ejus, quod à dimidia majoris segmenti a describitur, quadrati.



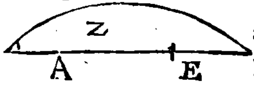
Dico $Q. e + \frac{1}{2} a = 5\ Q. \frac{1}{2} a$. *a* hoc est $ee + \frac{1}{4} aa + ea = aa + \frac{1}{4} aa$. *b* vel $ee + ea = aa$. Nam $ee+ea$ *c* $= ze$ *d* $= aa$. Q. E. D.

a 4. 2.
b 3. ax.
c 3. 2.
d hyp. & 17. 6.

PROP.

PROP. IV.

Si recta linea z ***secundum extremam ac mediam rationem secetur*** (z a :: a. e;) ***quod à tota*** z, ***quodque à minori segmento*** e, ***utraque simul quadrata, tripla sunt ejus, quod à majori segmento*** a ***describitur, quadrati.***



Dico $zz + ee = 3\,aa$. *a* vel $aa + ee + 2\,ae + ee = 3\,aa$. Nam $ae + ee$ *b* $= ze$ *c* $= aa$.
a 4. 1.
b 3. 2.
c 17. 6.
d 2. ax. *d* ergo $aa + 2\,ae + 2\,ee = 3\,aa$. Q. E. D.

PROP. V.

Si recta linea AB ***secundum extremam & mediam rationem secetur in*** C, ***apponaturque ei*** AD ***æqualis majori segmento*** AC; ***tota recta linea*** DB ***secundum extremam ac mediam rationem secatur, & majus segmentum est quæ à principio recta linea*** AB.

a hyp. Nam quia AB. AD *a* :: AC. CB, invertendoque AD. AB :: CB. AC; erit componendo DB. AB :: AB. AC. (AD.) Q. E. D.

Schol.

Quod si fuerit BD. BA :: BA. AD. erit BA. AD :: AD. BA−AD. Nam dividendo est BD −BA (AD) BA :: BA−AD. AD. ergo inverse, BA. AD :: AD. BA−AD. Q. E. D.

PROP. VI.

Si recta linea rationalis AB ***extrema ac media ratione secetur in*** C; ***utrumque segmentorum*** (AC, CB) ***irrationalis est linea, quæ vocatur apotome.***

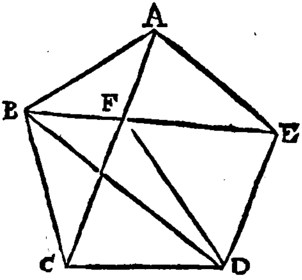
a 3. 1.
b 1. 13. Majori segmento AC *a* adde AD $= \frac{1}{2}$ AB;
c 6. 10. *b* ergo DCq $=5$ DAq. *c* ergo DCq $\top\!\!\perp$ DAq.
d hyp. proinde cùm AB, *e* ideoque ejus semissis DA
e sch. 12. 10 sint $\dot{\rho}$, etiam DC est $\dot{\rho}$. Quia vero 5. 1 :: non

Q.

Q Q f est DC ⊏⊐ DA. *g* ergo DC—AD, id est AC est apotome. Insuper quia ACq *b*=AB × BC, & AB est $\acute{\rho}$, *k* etiam BC est apotome. Q. E. D.

f 9. 10.
g 74. 10.
h 17. 6.
k 98. 10.

PROP. VII.



Si pentagoni æquilateri ABCDE ***tres anguli, sive qui deinceps*** EAB, ABC, BCD, ***sive*** EAB, BCD, CDE ***qui non deinceps sint, æquales fuerint, æquiangulum erit ipsum pentagonum*** ABCDE.

Paribus deinceps angulis subtendantur rectæ BE, AC, BD.

Quoniam latera EA, AB, BC, CD, angulique inclusi *a* æquantur, *b* erunt bases BE, AC, BD, *c* angulique AEB, ABE, BAC, BCA pares. *d* quare BF=FA, & *e* proinde FC=FE. ergo triangula FCD, FED sibi mutuo æquilatera sunt; *f* unde ang. FCD=FED, *g* proinde ang. ABD =BCD Eodem pacto ang. CDE reliquis æquatur. quare pentagonum æquiangulum est. Q. E. D.

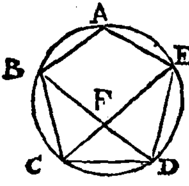
a *hyp.*
b 4. 1.
c 4. & 5. 1.
d 6. 1.
e 3. *ax.* 1.
f 8. 1.
g 2. *ax.* 1.

Sin anguli EAB, BCD, CDE, qui non deinceps, statuantur pares, *h* erit ang. AEB=BDC, & BE=BD, *k* ideoque ang. BED=BDE; *l* totus proinde ang. AED=CDE. ergo propter angulos A, E, D deinceps æquales, ut prius, pentagonum æquiangulum erit. Q. E. D.

h 4. 1.
k 5. 1.
l 2. *ax.*

PROP.

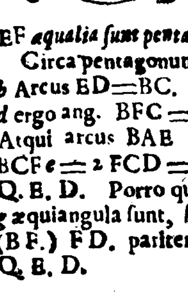
PROP. VIII.



Si pentagoni æquilateri & æquianguli ABCDE *duos angulos* BCD, CDE, *qui deinceps sint, subtendant rectæ lineæ* BD, CE; *hæ extrema ac media ratione se mutuo secant, & majora ipsarum segmenta* BF, *vel* EF *æqualia sunt pentagoni lateri* BC.

a 14. 4. Circa pentagonum *a* describe circulum ABD.
b 28. 3. *b* Arcus ED=BC. *c* ergo ang. FCD=FDC.
c 27. 3. *d* ergo ang. BFC = 2 FCD (FCD + FDC.)
d 32. 1. Atqui arcus BAE *b* = 2 ED, proinde ang.
e 33. 6. BCF *e* = 2 FCD = BFC. *f* quare BF = BC.
f 6. 1. Q. E. D. Porro quia triangula BCD, FCD
g 27. 3. *g* æquiangula sunt, *h* erit BD. DC (BF) :: CD
h 4. 6. (BF.) FD. pariterque EC. EF :: EF. FC.
Q. E. D.

PROP. IX.



Si hexagoni latus BE, *& decagoni* AB, *in eodem circulo* ABC *descriptorum componantur, tota recta linea* AE *extrema ac media ratione secatur,* (AE. BE :: BE. AB) *& majus ejus segmentum est hexagoni latus* BE.

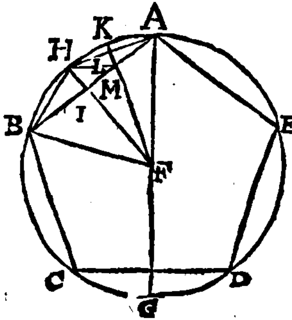
a hyp. & 27. 3. Duc diametrum ABC, & junge rectas DB,
b 32. 1. DE. Quoniam ang. BDC *a*=4 BDA, estque
c 7. ax. 1. ang. BDC *b* = 2 DBA (DAB + DBA,) erit
d 5. 1. DBA (*b* BDE+BED) *c*=2 BDA *d* = 2 BDE.
e 1. ax. 1. proinde ang DBA, vel DAB *e*=ADE. Itaque
f 4. 6. trigona ADE, ADB æquiangula sunt, *f* quare
g cor. 15. 4. AE. AD. (*g* BE) :: AD. (BE.) AB. Q.E.D.

Coroll.

Coroll.

Hinc, si latus hexagoni alicujus circuli secetur extrema ac media ratione; majus illius segmentum erit latus decagoni ejusdem circuli. *sch.* 5. 13.

PROP. X.



Si in circulo ABCE *pentagonum æquilaterum* ABCDE *describatur; pentagoni latus* AB *potest & hexagoni latus* FB, *& decagoni latus* AH, *in* a 28. 3. *& eodem circulo descriptorum.* 3. *ax.*

Duc diametrum AG. Biseca arcum AH in K. b *hyp.* & Et duc FK, FH, FB, BH, HM. 7. *ax.*

Semicirc. AG − arc. AC *a* = AG − AD. c 33. 6. hoc est, arc. CG = GD *b* = AH = HB. ergo d 20. 3. arc. BCG = 2 BHK; *c* adeoque ang. BFG = 2 e 1. *ax.* 1. BFK. *d* sed ang. BFG = 2 BAG. *e* ergo ang. f 32. 1. BFK = BAG. Trigona igitur BFM, FAB *f* æ- g 4. 6. quiangula sunt. *g* quare AB. BF :: BF. BM. h 17. 6. *h* ergo AB x BM = BFq. Rursus ang. AFK *k* = k 27. 3. HFK; & FA = FH; *m* quare AL = LH, *m* & m 4. 1. anguli FLA, FLH pares, ac proinde recti sunt. n 17. 5. ergo ang. LHM *m* = LAM *n* = HBA. Trigo o 32. 1. na igitur AHB, AMH *o* æquiangula sunt. *p* qua p 4. 6.

re

q 17.6. re AB. AH :: AH.AM. *q* ergo AB×AM =
r 2. 2. AHq. Quum igitur ABq *r* = AB×BM + AB
ſ 2. ax. ×AM, *ſ* erit ABq = BFq + AHq. Q. E. D.

Coroll.

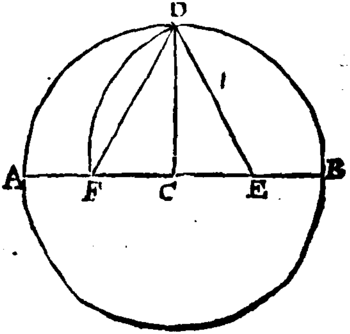
1. Hinc, linea recta (FK) quæ ex centro (F) arcum quempiam (HA) biſecat, etiam rectam (HA) illi arcui ſubtenſam biſecat ad angulos rectos.

2. Diameter circuli (AG) ex angulo quovis (A) pentagoni ducta biſecat & arcum (CD,) quem latus pentagoni illi angulo oppoſitum ſubtendit, & latus ipſum (CD) oppoſitum, idque ad angulos rectos.

Schol.

Hic, ut promiſimus, praxim tradem⁹ expeditam problematis 11. 4.

Problema.



Invenire latus pentagoni circulo ADB *inſcribendi.*

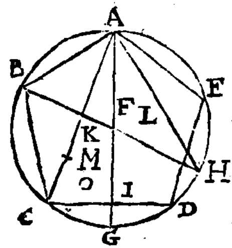
Duc diametrum AB, cui perpendicularem

CD ex centro C erige. Biseca CB in E. Fac EF = ED. erit DF pentagoni latus.

Nam $BF \times FC + ECq$ *a* $= EFq$ *b* $= EDq$ *c* $= DCq + ECq$. *d* ergo $BF \times FC = DCq$, vel BCq. *e* quare BF. BC :: BC. FC. ergo quum BC sit latus hexagoni, *f* erit FC latus decagoni, proinde DF *h* $= \sqrt{DCq + FCq}$ *g* est latus pentagoni. Q. E. F.

a 6. 2.
b *constr.*
c 47. 1.
d 3. *ax.*
e 17. 6.
f 9. 13.
g 10. 13.
h 47. 1.

PROP. XI.



Si in circulo ABCD *rationalem habente diametrum* AG, *pentagonum æquilaterum* ABCDE *describatur; pentagoni latus* AB *irrationalis est linea, quæ vocatur minor.*

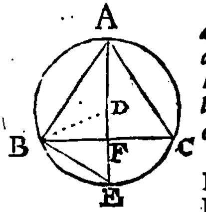
Duc diametrum BFH, rectasque AC, AH; & * fac $FL = \frac{1}{4}$ radii FH, & $CM = \frac{1}{4}$ CA. * 10. 6.

Ob angulos AKF, AIC *a* rectos, & communem CAI, trigona AKF, AIC *b* æquiangula sunt; *c* ergo CI. FK *c* :: CA. FA (FB) *d* :: CM. FL. ergo permutando FK, FL :: CI. CM *d* :: CD. CK (2 CM.) *e* componendo igitur CD + CK. CK :: KL. FL. *f* proinde Q: CD+CK (*g* 5 CKq.) CKq :: KLq. FLq. ergo KLq = 5 FLq. Itaque si BH ($\dot{\rho}$) ponatur 8, erit FH 4; FL 1. & FLq. 1. BL 5. & BLq 25. KLq 5. è quibus liquet BL, & KL esse $\dot{\rho}$ *h* ⊐⊏. *k* ideoque BK esse Apotomen, cujus congruens KL. cum vero $BLq - KLq = 20$, *l* erit BL ⊐⊏ $\sqrt{BLq - KLq}$. * unde BK erit apotome quarta. Quoniam igitur ABq *m* $= HB \times BK$, *n* erit AB minor. Q. E. D.

a *cor.* 10. 13.
b 32. 1.
c 4. 6.
d 15. 5.
e 18. 5.
f 22. 6.
g 1. 13.
h 9. 10.
k 74. 10.
l 9. 10.
*4 *def.* 85. 10.
m *cor.* 8. 6. & 17. 6.
n 95. 10.

X PROP.

PROP. XII.

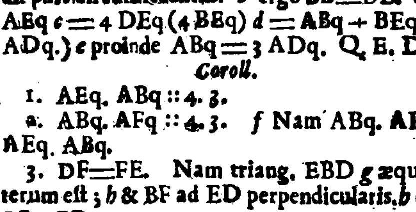


Si in circulo ABEC *triangulum æquilaterum* ABC *describatur, trianguli latus* AB *potentia triplum est ejus lineæ* AD, *quæ ex* D *centro circuli ducitur.*

Protracta diametro ad E, duc BE. Quoniam arcus
a cor. 10. BE *a*=EC, arcus BE sexta
13. est pars circumferentiæ. *b* ergo BE=DE. hinc
b cor. 15. 4. AEq *c*= 4 DEq (4 BEq) *d*= ABq + BEq (+
c 4. 2. ADq.) *e* proinde ABq = 3 ADq. Q. E. D.
d 47. 1. *Coroll.*
e 3. ax. 1. 1. AEq. ABq :: 4. 3.
f cor. 8. 6. 2. ABq. AFq :: 4. 3. *f* Nam ABq. AFq ::
& 21. 6. AEq. ABq.
g cor. 15. 4. 3. DF=FE. Nam triang. EBD *g* æquila-
h cor. 3. 3. terum est; *h* & BF ad ED perpendicularis. *b* ergo EF=FD.

4. Hinc AF=DE+DF=3 DF.

PROP. XIII.



Pyramidem EGFI *constituere, & data sphæra complecti; & demonstrare quod sphæræ diameter* AB

AB potentia sit sesquialtera lateris EF *ipsius pyramidis* EGFI.

Circa AB describe semicirculum ADB. a fitque AC = 2 CB. ex puncto C erige perpendicularem CD; & junge AD, DB. Tum radio HE = CD describe circulum HEFG; cui b inscribe triangulum æquilaterum EFG. ex H c erige IH = CA rectum plano EFG, produc IH ad K; d ita ut IK = AB. rectasque adjunge IE, IF, IG. erit EFGI pyramis expetita. — a 10. 6. b cor. 15. 4. c 12. 11. d 3. 1.

Nam quia anguli ACD, IHE, IHF, IHG e recti sunt; & CD, HE, HF, HG e pares, e atque IH = AC; f erunt AD, IE, IF, IG æquales inter se. Quia vero AC (2 CB.) CB g :: ACq. CDq. erit ACq = 2 CDq. itaque ADq f = ACq + CDq h = 3 CDq = 3 HEq k = EFq. l ergo AD, EF, IE, IF, IG pares sunt, adeoque pyramis EFGI est æquilatera. Quod si punctum C super H collocetur, & AC super HI, rectæ AB, IK m congruent, utpote æquales. quare semicirculus ADB axi AB vel IK circumductus n transibit per puncta, E, F, G, * adeoque pyramis EFGI sphæræ inscripta erit. Q. E. F. liquet vero esse BAq. ADq o :: BA. AC p :: 3. 2. Q. E. D. — e constr. f 47. 1. g 20. 6. h 2. ax. k 12. 13. l 1. ax. 1. m 8. ax. 1. n 15. def. 11. * 31. def. 11. o cor. 8. 6. p constr.

Corollaria.

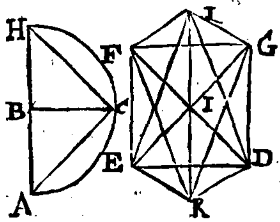
1. ABq. HEq :: 9. 2. Nam si ABq ponatur 9, erit ADq (EFq) 6. q proinde HEq erit 2. — q 12. 13.

2. Si L centrum fuerit, erit AB. LC :: 6. 1. Nam si AB ponatur 6, erit AL, 3; r ideoque AC 4; quare LC erit 1. Hinc — r constr.

3. AB. HI :: 6. 4 :: 3. 2. unde

4. ABq. HIq :: 9. 4.

PROP. XIV.



Octaedrum KEFGDL *constituere, & data sphæra complecti, qua & pyramidem; & demonstrare, quod sphæra diameter* AH *potentia sit dupla lateris* AC *ipsius Octaedri.*

Circa AH describe semicirculum ACH, ex centro B erige perpendicularem BC. duc AC, HC. Super ED = AC *a* fac quadratum EFGD, cujus diametri DF, EG secantes in centro I. ex I duc IL = AB *b* rectam plano EFGD. produc IL, *c* donec IK = IL. Connexis KE, KF, KG, KD, LE, LF, LG, LD; erit KEFGDL octaedrum quæsitum.

a 46. 1.
b 12. 11.
c 3. 1.

Nam AB, BH, FI, IE, &c. æqualium quadratorum semidiametri æquales sunt inter se. *d* quare triangulorum rectangulorum LIE, LIF, FIE, &c. bases LF, LE, FE, &c. æquantur. proinde octo triangula LFE, LFG, LGD, LDE, KEF, KFG, KGD, KDE æquilatera sunt, *e* atque octaedrum constituunt, quod sphæræ cujus centrum I, radius IL, vel AB, inscribi potest. (quoniam AB, IL, IF, IK, &c. *f* æquales sunt.) Q. E. F. porro liquet AHq (LKq) *g* = 2 ACq (2 LDq.) Q. E. D.

d 4. 1.
e 27. *def.* 11
f *constr.*
g 47. 1.

Corollaria.

1. Hinc manifestum est, in Octaedro tres diametros EG, FD, LK se mutuo ad angulos rectos secare in centro sphæræ.

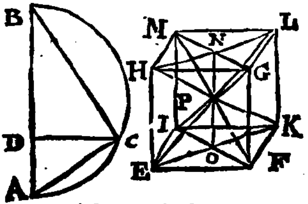
2. Item, tria plana EFGD, LEKG, LEKD esse quadrata, se mutuo ad angulos rectos secantia.

3. Octa-

3. Octaedrum dividitur in duas pyramides similes & æquales EFGDL, & EFGDK, quarum basis communis est quadratum EFGD.

4. Denique, bases octaedri oppositæ, inter se 15. 11.
parallelæ sunt.

PROP. XV.



Cubum EFGHIKLM *constituere, & sphæra complecti, qua & priores figuras; & demonstrare, quod sphæræ diameter* AB *potentia sit tripla lateris* EF *ipsius cubi.*

Super AB describe semicirculum ACB; & *a*
fac AB = 3 DA. ex D erige perpendicularem a 10. 6.
DC, & junge BC ac AC. Tum super EF = AC *b*
construe quadratum EFGH, cujus plano rectæ b 46. 1.
insistant EI, FK, HM, GL ipsi EF pares, quas connecte rectis IK, KL, LM, IM. Solidum EFGHIKLM cubus est, ut satis constat ex constructione.

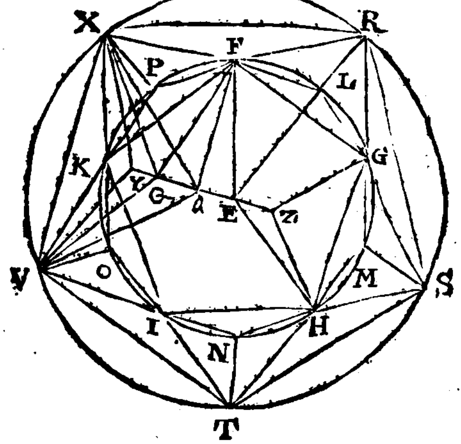
In quadratis oppositis EFKI, HGLM duc diametros EK, FI, HL, MG, per quas ducta plana EKLH, FIMG se intersecent in recta NO.
Hæc diametros cubi EL, FM, GI, HK *c* bisecabit
in P, centro cubi. *d* ergo P centrum erit sphæræ c cor. 39.
per puncta cubi angularia transeuntis. Porro 11.
BLq *e* = BKq + KLq *e* = 3 KLq, *f* vel 3 d 15. def. 1.
ACq. atqui ABq. ACq *g* :: BA. DA *f* :: 3. 1. & 14. def.
g ergo AB = EL. Quare cubum fecimus, &c. 11.
Q. E. F. e 47. 1.

Corol. f constr.

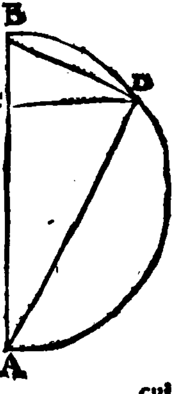
1. Hinc, omnes diametri cubi inter se æqua- g cor. 8. 6.
les sunt, seseque mutuo in centro sphæræ bise- h 14. 5.
cant. Eademque ratione rectæ quæ quadratorum oppositorum centra conjungunt, bisecantur in eodem centro.

k 47. 1. 2. Diameter ſphæræ poteſt latus tetraedri, & l 13. 13. cubi. nempe ABq k= l BCq + m ACq.
m 15. 13.

PROP. XVI.



Icoſaedrum ZGHIKFYVXRST *conſtituere, & ſphæra complecti, qua & antedictas figuras; & demonſtrare, quod icoſaedri latus* FG *irrationalis eſt linea, quæ vocatur minor.*



Super AB diametrum ſphæræ deſcribe ſemicirculum ADB; & *a* fac AB = 5 BC. ex C erige normalem CD, & duc AD ac BD. Ad intervallum EF = BD deſcribe circulum EFKNG;

a 10. 6.

cuj

b cui inscribe pentagonum æquilaterum FKIHG. b 11. 4.
Biseca arcus FG, GH, &c. ac connecte rectas FL, LG, &c. latera nempe decagoni. Tunc *c* erige EQ, LR, MS, NT, OV, PX ipsi EF æquales, rectasque plano FKNG. & connecte RS, ST, TV, VX, XR; item FX, FR, GR, GS, HS, HT, IT, IV, KV, KX. Denique producta EQ, sume QY=FL; & EZ=FL; rectasque duci concipe ZG, ZH, ZI, ZK, ZF; ac YV, YX, YR, YS, YT. Dico factum. c 12. 11.

Nam ob EQ, LR, MS, NT, OV, PX *d* æquales *e* & parallelas, etiam quæ illas jungunt, EL, QR, EM, QS, EN, QT, EO, QV, EP, QX *f* pares & parallelæ sunt. Item ideo LM (vel FG,) RS, MN, ST, &c. æquales sunt inter se. *g* ergo planum per EL, EM, &c. plano per QR, QS, &c. æquidistans, *h* & circulus QXRSTV è centro Q, circulo EPLMNO æqualis est; atque RSTVX est pentagonum æquilaterum. Duci vero intellectis EF, EG, EH, &c. ac QX, QR, QS, &c. quia FRq *k* = FLq + LRq, *l* vel FRq *m* = FGq, *n* erunt FR, FG, adeoque omnes RS, FG, FR, RG, GS, GH, &c. æquales inter se. Proinde 10 triangula RFX, RFG, RGS, &c. æquilatera sunt & æqualia. Rursus ob ang. XQY *o* rectum, erit XYq *p* = QXq + QYq *q* = VXq vel FGq. quare XY, VX, hisque similiter YV, YT, YS, YR, ZG, ZH, &c. æquantur: Ergo alia decem trigona constituta sunt æquilatera, & æqualia, tam sibi mutuo, quam decem prioribus; ac proinde factum est Icosaedrum. d *constr.* e 6. 11. f 33. 1. g 15. 11. h 1. *def.* 3. k 47. 1. l *constr.* m 10. 13. n *sch.* 48. 1. & 1. *ax.* o *cor.* 14. 11 p 47. 1. q 10. 13.

Porro, biseca EQ in *a*, duc rectas *a*F, *a*X, *a*V; & propter QX *r* = QV, & commune latus *a*Q, angulosque EQX, EQV rectos; *s* erit *a*X = *a*V. simlique argumento omnes, *a*X, *a*R, *a*S, *a*T, *a*V, *a*F, *a*G, *a*H, *a*I, *a*K æquantur. r 15. *def.* 1. s 4. 1.

 Quo-

t 9. 13. Quoniam autem ZQ. QE *t* :: QB. ZE, erit
u 3. 13. Zαq *u* = 5 Bαq *x* = EQq (EFq) Eαq *y* = αFq.
x 4. 2. ergo Zα = αF. pari pacto αF = Yα. ergo
y 47. 1. sphæra, cujus centrum α, radius αF, per 12 puncta icosaedri angularia transibit.

z 15. 5. Denique, *z* quia Zα.αE :: ZY. QE; *a* ideoque
a 22. 6. Zαq. αEq :: ZYq. QEq. *b* erit ZYq = 5 QEq,
b 14. 5. vel 5 BDq: atqui ABq. BDq *c* :: AB. BC :: 5.
c *cor.* 8. 6. 1. *d* ergo ZY = AB. Q. E. F.
d 1. *ax.* 1. Itaque si AB ponatur ρ, *e* erit EF = $\sqrt{}$ AB x
e *sch.* 12. 10 BC. etiam ρ; proinde FG pentagoni, idemque
f 11. 13. Icosaedri 5 latus, *f* est minor. Q. E. D.

Coroll.

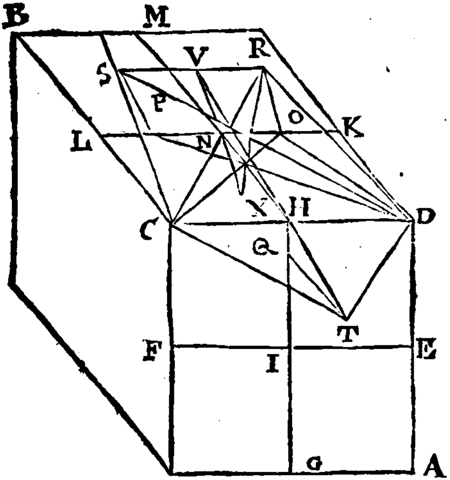
1. Ex dictis infertur, sphæræ diametrum esse potentia quintuplum semidiametri circuli quinque latera icosaedri ambientis.

2. Item manifestum est, sphæræ diametrum esse compositam ex latere hexagoni, hoc est, ex semidiametro, & duobus lateribus decagoni circuli ambientis quinque latera icosaedri.

3. Constat denique latera icosaedri opposita,
a 33. 1. qualia sunt RX, HI, esse parallela. Nam RX *a*
b *sch.* 26. 3. parall. LP. *b* parall. HI.

PROP.

PROP. XVII.



Dodecaedrum constituere, & sphæra complecti, qua & prædictas figuras; & demonstrare, quod dodecaedri latus RS *irrationalis est linea, quæ vocatur apotome.*

Sit AB cubus datæ sphæræ inscriptus, cujus
latera omnia bisecentur in punctis E, H, F, G,
K, L, &c. rectæque adjungantur KL, MH,
HG, EF. *a* Fac HI. IQ :: IQ. QH; & sume a 30. 6.
NO, NP pares ipsi IQ. Erige OR, PS rectas
plano DB, & QT plano AC. sintque OR, PS,
QT ipsis IQ, NO, NP æquales. Connexis DR,
RS, SC, CT, DT, erit DRSCT pentagonum
Dodecaedri expetiti. Nam duc NV parall. OR,
& protracta NV ad occursum cum cubi centro a 47. 1.
X, connecte rectas DS, DO, DP, CR, CP, b 7. ax. 1.
HV, HT, RX. Quia DOq *a* = DKq (*b* KNq) c 4. 13.
+KOq *c* = 3 ONq (3 ORq) *d* erit DRq d 47. 1.
=4

e 4. 2. | $= 4$ ORq $e =$ OPq, vel RSq. ergo DR $=$ RS. Simili argumento DR, RS, SC, CT, TP pares sunt. Quia vero OR $f = g$ & parall. PS,
f constr. 9. 6. 11. | g erunt RS, OP, & h consequenter RS, DC etiam parallelæ; h ergo hæ cum suis conjungentibus DR, CS, VH in uno sunt plano. quinetiam
g 33. 1. | quia HL. IQ k :: IQ (TQ.) QH k :: HN. NV; & tam TQ, HN, quam QH, NV k rectæ eidem plano, l adeoque & parallelæ existunt,
h 9. 1. | m erit THV recta linea. n ergo Trapezium DRSC, & triang. DTS in uno sunt plano per rectas DC, TV extenso. ergo DTCSR est pentagonum, & quidem æquilaterum, ex antedictis.
k 7. 11. | Porro, o quia PK. KN :: KN. NP; & DSq $p =$ DPq $+$ PSq (PNQ) $= p$ DKq $+$ PKq $+$ NPq, q erit DSq $=$ DKq $+ 3$ KNq $= 4$ DKq (4DHq) $r =$ DCq. ergo DS $=$ DC; unde trigona DRS, DCT sibi mutuo æquilatera sunt. f ergo ang. DRS $=$ DTC; & eodem pacto ang. CSR $=$ DTC. ergo * pentagonum DTCSR etiam æquiangum est. Ad hæc, quia AX, DX, CX, &c. sunt cubi semidiametri, t erit XN $=$ IH, vel KN, u adeoque XV $=$ KP. unde ob angulum x rectum RVX, z erit RXq $=$ XVq $+$ RVq (NPq) $=$ KPq $+$ NPq a $= 3$ KNq b $=$ AXq, vel DXq, &c. ergo RX, AX, DX, & eadem ratione XS, XT, AX æquales sunt inter se.

Marginal references: e 4. 2. — f constr. 9. — 6. 11. — g 33. 1. — h 9. 1. — k 7. 11. — k constr. — 16. 11. — m 32. 6. — n 1. & 2. 11 — o 5. 13. — p 47. 1. — q 1. ax. 2. & 4. 13. — r 4. 2. — f 8. 1. — * 7. 13. — t 15. 13. — u 1. ax. 1. — x 29. 1. — z 47. 1. — a 4. 13. — b 15. 13.

Et si eadem methodo, qua constructum est pentagonum DTCSR, fabricentur 12 similia pentagona tangentia duodecim cubi latera, ea Dodecaedrum constituent; ac per eorum puncta angularia transiens sphæra, cujus radius AX, vel RX, Dodecaedrum complectetur. Q. E. F.

c constr. — d 15. 5. — e 15. 13. — f sch. 12. 10 — g 6. 13.

Denique, quia KN. NO c :: NO. OK, d erit KL. OP :: OP. OK $+$ PL. Itaque si sphæræ diameter AB ponatur $\acute{\rho}$, erit KL $e = \sqrt{}$ ABq f etiam $\acute{\rho}$. g unde OP, vel RS latus dodecaedri apotome erit. $\overline{3}$ Q. E. D.

Coroll.

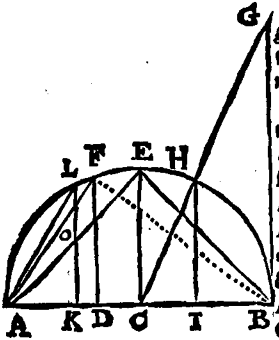
Coroll.

1. Hinc, si latus cubi secetur *extrema ac media ratione*, majus segmentum erit latus dodecaedri in eadem sphæra descripti.

2. Si rectæ lineæ sectæ extrema ac media ratione, minus segmentum sit latus dodecaedri, majus segmentũ erit latus cubi ejusdem sphæræ.

3. Liquet etiam latus cubi æquale esse lineæ rectæ subtendenti angulum pentagoni dodecaedri eadem sphæra comprehensi.

PROP. XVIII.



Latera quinq; figurarum exponere, & inter se comparare.

Sit AB diameter sphæræ, ac AEB semicirculus. sitque AC *a* = $\frac{1}{2}$ AB, & AD *b* = $\frac{1}{3}$ AB. Erige perpendiculares CE, DF, & BG = AB. junge AF, AE, BE, BF, CG, ex H demitte perpendicularem HI, & sumpta CK = CI, ex K erige perpendicularem KL, & connecte AL. Denique *c* fac AF. AO :: AO. OF.

a 10. 1.
b 10. 6.
c 30. 6.

Itaque 3. 2 *d* :: AB. BD *e* :: ABq. BFq, latus Tetraedri. & 2.1 :: *a* AB. AC :: ABq. BEq, *f* latus Octaedri.

d *constr.*
e *cor.* 8. 6.
f 14. 13.

Item 3. 1 *d* :: AB. AD *e* :: ABq. AFq, *g* latus Hexaedri.

g 15. 13.
h *constr.*

Porro, quia AF. AO *h* :: AO. OF. *k* erit AO

k *cor.* 17. 13.

l 4. 6. AO latus Dodecaedri. denique BG (2 BC.)
m 24 5. BC *l* :: HI. IC. *m* ergo HI = 2 CI *n* = KI. ergo
n *constr.* HIq *o* = 4 CIq. proinde CHq = *p* 5 CIq *q* ergo
o 4. 2. ABq = 5 KIq. *r* itaque KI, vel HI, est radius cir-
p 47. 1. culi circumscribentis pentagonum icosaedri; &
q 15. 5. AK, vel IB, *r* est latus decagoni eidem circulo in-
r *cor.* 16. 13 scripti. unde AL *s* erit latus pentagoni, *t* idemque
s 10. 13. Icosaedri latus. Ex quibus liquet BF, BE, AF
t 16. 13. esse ρ̇ ⊐. & AL, AO esse ρ̇ ⊐; atque BF
⊏ BE; & BE ⊏ AF; ac AF ⊏ AO. Quia
u 1. 6. vero 3 AFq = ABq *u* = 5 KLq. ac AF × AO
x 4. *ax.* 1. ⊏ AF × OF, *x* ideoque AF × AO + AF × OF
y 1. 2. ⊏ 2 AF × OF, *y* hoc est AFq ⊏ *z* 2 AOq. *a* e-
z 17. 6. rit 3 AFq (5 KLq) ⊏ 6 AOq. proinde KL
a 47. 1. ⊏ AO; & fortius, AL ⊏ AO.

Jam vero ut hæc latera numeris exprimamus, si AB ponatur $\sqrt{60}$, erit ex jam dictis ad calculum exactis, BF $= \sqrt{40}$. & BE $= \sqrt{30}$. & AF $= \sqrt{20}$. item AL $= \sqrt{:30 - \sqrt{180}}$ (nam AK $= \sqrt{15} - \sqrt{3}$. & KL (HI) $= \sqrt{12}$.) denique AO $= \sqrt{:30 - \sqrt{500}}$ ($\sqrt{25} - \sqrt{5}$.)

SCHOL.

SCHOL.

Præter jam dictas figuras nullam dari posse figuram solidam regularem (nempe quæ figuris planis ordinatis & æqualibus contineatur) admodum perspicuum est. Nam ad anguli solidi constitutionem requiruntur ad minimum tres anguli plani; *a* hique omnes simul 4 rectis minores esse debent. a 21. 11.
b Atqui 6 anguli trigoni æquilateri, 4 quadratici, & 3 hexagonici, sigillatim 4 rectos exæquant; b *Vid. schol.* 32. 1.
quatuor vero pentagonici, 3 heptagonici, 3 octagonici, &c. 4 rectos excedunt. ergo solummodo ex 3, 4, vel 5 triangulis æquilateris, ex 3 quadratis, vel 3 pentagonis, effici potest angulus solidus. Proinde, præter quinque prædicta, nulla existere possunt corpora regularia.

Ex P. Herigonio.

Proportiones sphæræ, & 5 figurarum regularium eidem inscriptarum.

Sit diameter sphæræ 2. Erunt

Peripheria circuli majoris, 6 | 28318.

Superficies circuli majoris, 3 | 14159.

Superficies sphæræ, 12 | 56637.

Soliditas sphæræ, 4 | 18859.

Latus tetraedri, 1 | 63299.

Latus

Superficies tetraedri, 4 | 6188.

Soliditas tetraedri, 0 | 15132.

Latus hexaedri, 1 | 1547.

Superficies hexaedri, 8.

Soliditas hexaedri, 1 | 5396.

Latus octaedri, 1 | 41421.

Superficies octaedri, 6 | 9282.

Soliditas octaedri, 1 | 33333.

Latus dodecaedri, 0 | 71364.

Superficies dodecaedri, 10 | 51462.

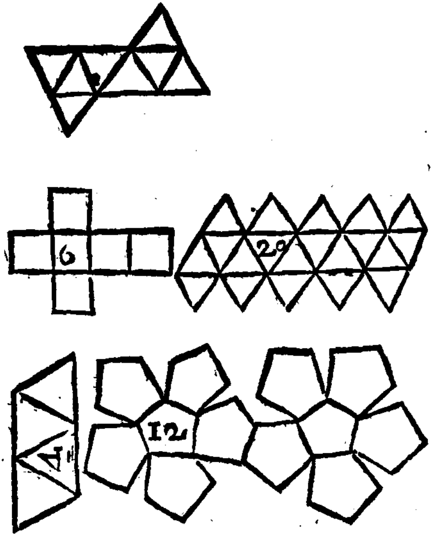
Soliditas dodecaedri, 2 | 78516.

Latus Icosaedri, 1 | 05146.

Superficies Icosaedri, 9 | 57454.

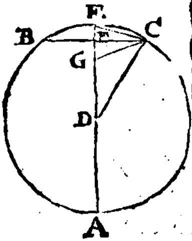
Soliditas Icosaedri, 2 | 53615.

Quod si ex charta conficiantur quinque figuræ æquilateræ & æquiangulæ similes his quæ sunt in subiecta figura, componentur quinque figuræ solidæ, si rite complicentur.



LIB. XIV.

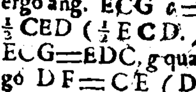
PROP. I.



Quæ ex D *centro circuli cujuspiam* ABC *in pentagoni eidem circulo inscripti latus* BC *ducitur perpendicularis* DF, *dimidia est utriusque lineæ simul, & lateris hexagoni* DE, *& lateris decagoni* EC *eidem circulo* ABC *inscripti.*

a 4. 1. Sume FG=FE, & duc CG. *a* Estque CE
b 5. 1. =CG. ergo ang. CGE *b*= CEG *b*= ECD.
c 32. 1. ergo ang. ECG *c*= EDC *d*= $\frac{1}{4}$ ADC *e*=
d *hyp.* & $\frac{1}{2}$ CED ($\frac{1}{2}$ ECD.) proinde ang. GCD =
33. 6. ECG=EDC, *g* quare DG=GC (CE.) er-
e 20. 3. go DF = CE (DG) + EF = $\frac{DE+CE}{2}$.
f 7. *ax.* Q. E. D.
g 5. 1.

PROP. II.



Si binæ rectæ lineæ AB, DE *extrema ac media ratione secentur* (AB. AG :: AG. GB, *&* DE. DH :: DH. HE;) *ipsæ similiter secabuntur, in easdem scilicet proportiones.* (AG. GB :: DH. HE.)

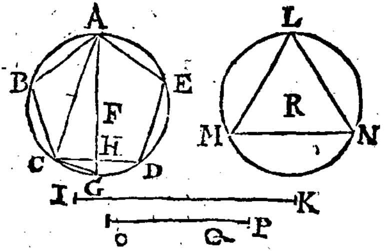
a 17. 6. Accipe BC = BG & EF = EH. Estque
b 8. 2. AB × BG *a*= AGq. quare ACq *b*= 4 ABG
c 1. *ax.* 1. + AGq *c*= 5 AGq. Similiter erit DFq =
d 22. 5. & 5 DHq. *d* ergo AC. AG :: DF. DH. compo-
22. 6. nendo igitur AC + AG, AG :: DF + DH.

DH,

DH. hoc est ½ AB. AG :: 2 DE. DH. *e* proinde AB. AG :: DE. DH. unde *f* dividendo AG. GB :: DH. HE. Q. E. D.

e 22. 5.
f 17. 5.

PROP. III.

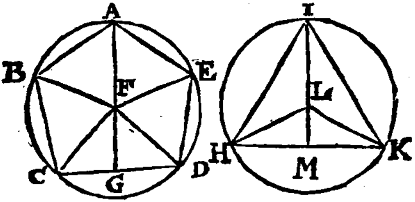


Idem circulus ABD *comprehendit & Dodecaedri pentagonum* ABCDE, *& Icosaedri triangulum* LMN, *eidem sphæræ inscriptorum.*

Duc diametrum AG, rectasque AC, CG. Sitque IK diameter sphæræ, *a* & IKq = 5 OPq. *b* fiatque OP. OQ :: OQ. QP. Quia ACq + CGq *c* = AGq *d* = 4 FGq; & ABq *e* = FGq + CGq. *f* erit ACq + ABq = 5 FGq. porro, quia CA. AB *g* :: AB. CA − AB; ac OP. OQ :: OQ. QP. *h* ideoque CA. OP :: AB. OQ. *k* erit 3 ACq (*l* IKq.) 5. OPq (*m* IKq) :: 3 ABq. 5 OQq. ergo 3 ABq = 5 OQq. Verum ob ML *n* latus pentagoni circulo inscripti, cujus radius OP, erunt 15 RMq *o* = 5 MLq *p* = 5 OPq + 5 OQq = * 3 ACq + 3 ABq *q* = 15 FGq. *r* ergo RM = FG. *s* proinde circ. ABD = circ. LMN. Q. E. D.

a *sch.* 47. 1.
b 30. 6.
c 47. 1.
d 4. 2.
e 10. 13.
f 2. & 3. *ax.*
g 8. 13.
h 2. 13. & 16. 5.
k 22. 6. & 4. 5.
l 15. 13.
m *constr.*
n *cor.* 16. 13.
o 12. 13.
p 10. 13.
p 15. 5. & supra.
* *Prim.*
r 1. *ax.* 1. & *sch.* 48. 1.
s 1. *def.* 3.

PROP. IV.



Si ex F *centro circuli pentagonum dodecaedri* ABCDE *circumscribentis ducatur perpendicularis* FG *ad pentagoni unum latus* CD; *erit quod sub dicto latere* CD, *&* *perpendiculari* FG *comprehenditur rectangulum trigesies sumptum, icosaedri superficiei æquale.* item,

Si ex centro L *circuli triangulum icosaedri* HIK *circumscribentis, perpendicularis* LM *ducatur ad trianguli unum latus* HK; *erit quod sub dicto latere* HK, *&* *perpendiculari* LM *comprehenditur rectangulum trigesies sumptum, icosaedri superficiei æquale.*

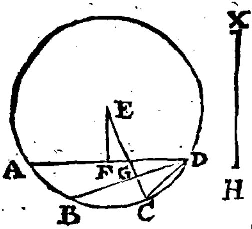
a 8. 1. Duc FA, FB, FC, FD, FE. *a* Erunt triangula CFD, DFE, EFA. AFB, BFC æqualia. at-
b 41. 1. qui CD×FG *b* = 2 triang. CFD. ergo 30 CD
c 15. 5. ×GF *c* = 60 CFD *d* = 12 pentag. ABCDE *e* =
d 6. ax. superf. dodecaedri. Q. E. D.
e 17. 3.
f 41. 1. Duc LI, LH, IK. estque HK×LM *f* = 2
g 15. 5. triang. LHK. ergo 30 HK×LM *g* = 60 HLK
h 16. 13. = 20 HIK *h* = superfic. icosaedri. Q. E. D.

Coroll.

k 15. 5. CD×FG. HK×LM *k* :: superfic. dodecaed. ad superf. icosaedri.

PROP.

PROP. V.



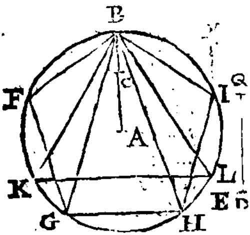
Superficies dodecaedri ad superficiem icosaedri in eadem sphæra descripti eandem proportionem habet, quam H latus cubi ad AD latus icosaedri.

Circulus ABCD *a* circumscribat tam dodecaedri pentagonum, quam icosaedri triangulum; quorum latera BD, AD; ad quæ demittantur ex E centro perpendiculares EF, EGC. & connectatur CD. a 3. 14.

Quoniam EC+CD. EC *b* :: EC. CD. erit EG (*c* $\frac{1}{2}$ EC+$\frac{1}{2}$ CD.) EF (*d* $\frac{1}{2}$ EC) *e* :: EF. EG—EF ($\frac{1}{2}$ CD.) atqui H. BD *f* :: BD. H—BD. *g* ergo H. BD :: EG. EF. proinde H × EF = BD × EG. quum igitur H. AD *h* :: H × EF. AD × EF. erit H. AD :: BD × EG. AD × EF :: *l* superfic. dodecaedri ad superfic. icosaedri. Q. E. D.

b 9. 13.
c 1. 14.
d *cor.* 12. 13.
e 15. 5.
f *cor.* 17. 13.
g 2. 14.
h 1. 6.
k 7. 5.
l *cor.* 4. 14.

PROP. VII.



Si recta linea AB *secetur extrema ac media ratione; erit ut recta* BF *potens id, quod à tota* AB, *& id quod à majori segmento* AC, *ad rectam* E, *potentem id quod à tota* AB, *& id quod à minori segmento* BC; *ita latus cubi* BG *ad latus icosaedri* BK *eidem sphæræ cum cubo inscripti.*

Circulo, cujus semidiameter AB, inscribantur
a cor. 17. 13. dodecaedri pentagonum BFGHI, & icosaedri triangulum BKL. *a* quare BG latus cubi erit ei-
b 12. 13. dem sphæræ inscripti. igitur BKq *b* = 3 ABq;
c 4. 13. & Eq *c* = 3 ACq. ergo BKq. Eq *d* :: ABq. ACq
d 15. 5. *e* :: BGq. BFq. permutando igitur BGq, BKq ::
e 2. 14. BFq. Eq *f* unde BG. BK :: BF. E. Q. E. D.
f 22. 6

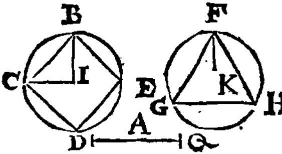
PROP. VII.

Dodecaedrum est ad Icosaedrum, ut cubi latus ad latus Icosaedri, in una eademque sphæra inscripti.

a 3. 14. Quoniam *a* idem circulus comprehendit & dodecaedri pentagonum & icosaedri triangulum,
b 47. 1. *b* erunt perpendiculares à centro sphæræ ad plana pentagoni & trianguli ductæ inter se æquales. itaque si dodecaedrum & icosaedrum intelligantur esse divisa in pyramides, ductis rectis à centro sphæræ ad omnes angulos, omnium pyramidum altitudines erunt inter se æquales. Cum igitur pyramides æque altæ *c* sint ut bases,
c 5, & 6. 12. & superficies dodecaedri sit æqualis 12 pentagonis, superficies vero icosaedri 20 triangulis; erit

erit dodecaedrum ad icosaedrum; ut superficies dodecaedri ad superficiem icosaedri, *d* hoc est, ut latus cubi ad latus icosaedri. d 5. 14.

PROP. VIII.



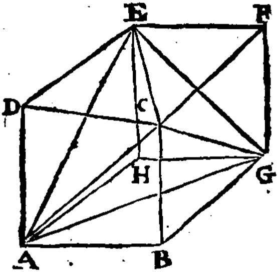
Idem circulus BCDE *comprehendit & cubi quadratū* BCDE *& octaedri triangulum* FGH, *ejusdem sphæræ.*

Sit A diameter sphæræ. Quoniam Aq *a* = 3 BCq *b* = 6 BIq; itemque Aq *c* = 2 GFq *d* = 6 KFq; erit BI = KF. *e* ergo circulus CBED = GFH. Q. E. D.

a 15. 13.
b 47. 1.
c 14. 13.
d 12. 13.
e 2. *def.* 1.

LIB. XV.

PROP. I.

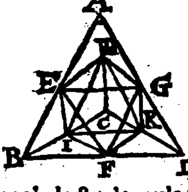


IN dato cubo ABGHDCFE *pyramidem* AGEC *describere.*

Ab angulo C duc diametros CA,CG, CE; Easque connecte diametris AG,GE,EA. Hæ omnes inter se *a* æquales sunt,utpote æqualium quadratorum diametri.ergo triangula CAG,CGE, CEA,EAG æquilatera sunt,ac æqualia: proinde AGEC est pyramis, quæ cubi angulis insistit, eique idcirco *b* inscribitur. Q. E. F.

a 47. 1.
b 31.*def*.11

PROP. II.



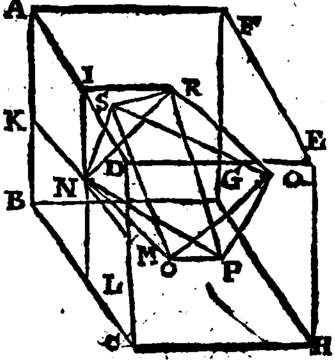
In data pyramide ABDC *octaedrum* EGKIFH *describere.*

a Biseca latera pyramidis in punctis E, I, F, K,G, H; quæ connecte 12 rectis EF,FG,GE,&c.Hæ omnes *b* æquales sunt inter se. proinde 8 triangula EHI, IHK, &c. æquilatera sunt & æqualia,adeoq; constituunt *c* octaedrum *d* in data pyramide descriptum. Q. E. F.

a 10. 1.
b 4. 1.
c 27.*def*.11
d 31.*def*.11

PROP.

PROP. III.



In dato cubo CHGBDEFA *octaedrum* NPQSOR *describere.*

Connecte quadratorum * centra N, P, Q,S, O,R, 12 rectis NP, PQ, QS, &c. quæ *a* æqualia sunt inter se, ideoque 8 triangula efficiunt æquilatera & æqualia. proinde *b* inscriptum est cubo *b* Octaedrum NPQSOR. Q. E. F.

* 8. 4.
a 4. 1.
b 31. & 27. def. 11.

PROP. IV.

In dato octaedro ABCDEF *cubum inscribere.*

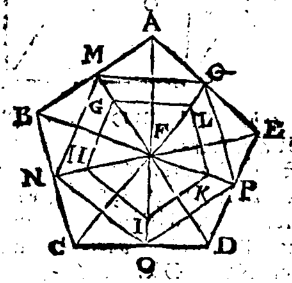
Latera pyramidis EABCD, cujus basis quadratum ABCD, bisecentur rectis LM, MN, NO, OL; quæ *a* æquales sunt & *b* parallelæ lateribus quadrati ABCD *c* ergo quadrilaterum LMNO est quadratum.

a 4. 1.
b 2. 6.
c 29. def. 1.

Eodem modo, si latera quadrati LMNO bisecentur

centur in punctis G, H, K, I, & connectantur GH, HK, KI, IG, erit GHKI quadratum. Quod si eadem arte in reliquis 5 pyramidibus octaedri centra triangulorum rectis conjungantur, describentur quadrata similia & æqualia quadrato GHKI. quare sex hujusmodi quadrata cubum constituent, quiquidem intra octaedrum descri-
d 31. def. 11 ptus erit, *d* cum octo ejus anguli tangant octo octaedri bases in earum centris. Q. E. F.

PROP. V.



In dato Icosaedro Dodecaedrum inscribere.

Sit ABCDEF pyramis Icosaedri, cujus basis pentagonum ABCDE; centra autem tri-
* 5. 4. angulorum G, H, I, K, L; quæ connectantur rectis GH, HK, IK, KL, LG. Erit GHIKL pentagonum dodecaedri inscribendi.

Nam rectæ FM, FN, FO, FP, FQ, per cen-
a cor. 5. 3. tra triangulorum transeuntes, *a* bisecant bases.
b 4. 1. *b* ergo rectæ MN, NO, OP, PQ, QM æquales sunt inter se. quinetiam FM, FN, FO, FP, FQ
c 4. 1. *c* pares sunt. *d* ergo anguli MFN, NFO, OFP,
d 8. 1. PFQ, QFM æquantur. pentagonum igitur GHIKL æquilangulum est; *e* proinde &
e 4. 1. æquilaterum, cum FG, FH, FI, FK, FL *f* pares
f 12. 13. sunt. Quod si eadem arte in reliquis undecim

pyra-

pyramidibus icoſaedri, centra triangulorum rectis lineis connectantur, deſcribentur pentagona æqualia & ſimilia pentagono GHIKL, quamobrem 12 hujuſmodi pentagona dodecaedrum conſtituent; quod quidem in icoſaedro erit deſcriptum, cum viginti anguli dodecaedri in centris viginti baſium icoſaedri conſiſtant. Quapropter in dato icoſaedro dodecaedrum deſcripſimus, Q. E. F.

F I N I S.

EUCLIDIS
DATA

succincte demonstrata;

Una cum Emendationibus quibusdam & Additionibus ad ELEMENTA

EUCLIDIS

nuper opera.

Opera

Mri. IS. BARROW, *Cantabrigiensis*, Coll. *Trin.* Soc.

LONDINI,
Excudebat *J. Redmayne*, 1678.

Ornatissimo viro
D. JACOBO STOCK,
Amico suo & patrono singulari.

NEc publica, nec tui nominis luce dignum censeo hunc paucorum dierum partum pusillum & præmaturum. Qui quidem quod se mundo, quodque Tibi, spectandum obtulerit, duplici nomine arrogantiæ speciem incurrit. Sed utrinque parata est excusatio qualiscunque. Nam amico obtemperatum oportuit jubenti mitterem hunc libellum Euclidæis (quæ cognatione proxima attingit) Elementis subjungendum. In eum quicquid est in publicum aut peccati aut meriti protinus rejicio, facti cujus author fuit, rationem redditurum. In te autem delictum quod maxime aggravat, idem potenter extenuat, Tibi tantum debere. Nam cum iis, qui Diis ipsis sacrificia, ac modica magnis Regibus donaria offerre non dubitarunt, satius esse credo, etiam pro immensis beneficiis parum, quam nihil rependere. Sufficiat igitur regessisse, me Tibi multis magnisque nominibus obstrictum fore; vices, quas potuero maximas, referre debere; ultra vota & grates nihil posse; illa privatim, has publice persolutas præcellere; quibus

bus agendis, quam jamdiu spe & studio aucupor, occasionem nondum comparere; præstare hanc oblatam prehendere, quamvis exilem, quam elapsam nequicquam pœnitentia prosequi. Esto igitur hæc oblatio pignus quoddam & præludium futuræ amplioris, in qua meritorum in me Tuorum historia uberior ac distinctior commemoranda occurret. Quæ simpliciter agnoscere, non aut fuse describere, aut digne prædicare, præsentis est instituti. Ac revera jam brevis sum ἑκὼν ἀέκοντι γε θυμῷ, necessitate potius coactus, quam inductus consilio. Nam me vela ventis turgentia alio avocant; ac vereor ne hæc pene currenti calamo exequentem, quæ hæc ad te perferet, amica manus, importuna patientia præstoletur. Quid superest igitur, nisi ut te domi studiis ac rebus honestis animum intendentem salutari præsentia tutetur, eum exorem venerandi ac ἀῤῥήτου nominis; quem tantæ beneficentia benignum remuneratorem jugibus votis exopto; idemque me extemplo super Tyrrhenos, Ionios, Ægeosque fluctus longinquam profectionem suscepturũ comitetur. Obtestor autem, ne tenuis opellæ patrociniũ respuas, quod ultro impertire dignatus es

Tibi devinctissimo
& obsequentissimo,

I. B.

E U-

EUCLIDIS Data.

Definitiones.

I. DAta magnitudine dicuntur ſpatia, lineæ, anguli, quibus æqualia poſſumus invenire.

II. Ratio dari dicitur, cui poſſumus eandem invenire.

III. Rectilineæ figuræ ſpecie dari dicuntur, quarum & ſinguli anguli dati ſunt, & laterum rationes ad invicem datæ ſunt.

Hinc, datæ ſunt ſpecie figuræ, quibus ſimiles inveniri poſſunt.

IV. Poſitione dari dicuntur puncta, lineæ, angulique, quæ eundem ſitum ſemper obtinent.

V. Circulus magnitudine dari dicitur, cujus ea quæ ex centro datur magnitudine.

VI. Poſitione & magnitudine dari dicitur circulus, cujus datur centrum poſitione, & ea quæ ex centro magnitudine.

VII. Circuli ſegmenta magnitudine dari dicuntur, in quibus dati ſunt magnitudine anguli & ſegmentorum baſes.

VIII. Poſitione & magnitudine dari dicuntur circuli ſegmenta, in quibus anguli magnitudine dati ſunt, & ſegmentorum baſes poſitione & magnitudine.

IX. Magnitudo magnitudine major eſt data, quando ablata data, reliqua eidem æqualis eſt.

X. Magnitudo magnitudine minor eſt data; quando adjuncta data, tota eidem æqualis eſt.

Ut ſi A data ſit, erit $A + B \sqsubset B$ data. At $B \sqsupset A + B$ data.

XI. Magnitudo magnitudine major eſt data quam in ratione, quando ablata data, reliqua ad eandem habet rationem datam.

XII. Magnitudo magnitudine minor est data quam in ratione, quando adjuncta data tota ad eandem rationem habet datam.

Ut si A data sit, & $\frac{B}{C}$ detur, erit $A+B \sqsubset C$; data q. in r. sin $\frac{A+B}{C}$ detur, erit $B \sqsupset C$ data q. in r.

PROP. I.

A. B. *Datarum magnitudinum* A, B,
a. b. *ad invicem datur ratio.*

Nam quia A*datur, a inveniri potest aliqua a = A. Eodem jure sume b = B. b estque a. b :: A. B; c quare ratio $\frac{A}{B}$ data est. Q. E. D.

*hyp. a 1. def. b sch. 7. 5. c 2. def.

PROP. II.

A. B. *Si data magnitudo* A *ad aliam*
a. b. *aliquam* B *habeat rationem datam, datur etiam hæc alia magnitudine.*

Nam ob A*datam, a sume a = A; ac ob $\frac{A}{B}$ *datam, b sit $\frac{a}{b} = \frac{A}{B}$. c ergo b = B. a quare B datur. Q. E. D.

*hyp. a 1. def. d. b 2. def. d. c 9. 5.

PROP. III.

A. B. *Si quotlibet datæ magnitudines*
a b. A, B *componantur, etiam ea* A+B *quæ ex his componitur, data erit.*

Nam a cape a = A, & b = B; b estque a+b = A+B. a quare A+B datur. Q. E. D.

a 1. def. b 2. ax. 1.

PROP. IV.

A. B. *Si à data magnitudine* A *auferatur*
a. b. *tur data magnitudo* B, *etiam reliqua* A—B *dabitur.*

a Sint enim a = A, & b = B. ergo A—B = a—b, a proinde A—B datur. Q. E. D.

a 1. def. d. b 3. ax. 1.

PROP. V.

A. B. C. D. *Si magnitudo A ad sui-ipsius aliquam partem B habeat rationem datam, etiam ad reliquam A—B habebit rationem datam.*

a hyp. b 2. def. d. c cor. 9. 5.

Nam, quia $\frac{A}{B}$ a data est, b sit A. B :: C. D. c ergo A. A—B :: C. C—D. b proinde $\frac{A}{A-B}$ datur. Q. E. D.

PROP. VI.

A. B. C. D. *Si componantur duæ magnitudines A, B, habentes ad invicem rationem datam, etiam quæ ex his componitur magnitudo A+B, habebit ad utramque A & B rationem datam.*

a 2. def. d. b 18. 5. c 2. def. d.

Nam a sit A. B :: C. D. b ergo A+B. B :: C+D. D. c quare $\frac{A+B}{B}$ datur. Similiter $\frac{B+A}{}$ datur. Q. E. D.

PROP. VII.

A. B. *Si data magnitudo A+B data ratione secetur, utrumque segmentorum A, & B datum est.*

* hyp. a 6. dat. b 2. dat.

Nam ob $\frac{A}{B}$ * datam, a erit $\frac{A+B}{A}$ data. b ergo A datur. Eodem modo B datur. Q. E. D.

PROP. VIII.

A. C. B. D. E. F. *Quæ A, B ad idem C rationem habent datam, habebunt ad invicem rationem datam.*

a 1. def. d.

Nam a sit A. C :: D. E. a & C. B :: E. F. quare ex æquali A. B :: D. F. a ergo $\frac{A}{B}$ datur. Q. E. D.

Coroll.

Rationes ex datis rationibus compositæ, datæ sunt. Ut $\frac{A}{B}$ fit ex $\frac{A}{C}$, & $\frac{C}{B}$ datis.

PROP.

PROP. IX.

A. B. C. *Si duæ, pluresve magnitudines*
D. E. F. A,B,C *ad invicem habeant rationem datam, habeant autem illæ magnitudines* A,B,C *ad alias quasdam* D,E,F *rationes datas, etsi non easdem; illæ aliæ magnitudines* D, E, F *etiam ad invicem habent rationes datas.*

Nam ratio $\frac{D}{E}$ *a* fit ex *b* datis $\frac{D}{A}$, $\frac{A}{B}$, $\frac{B}{E}$; *c* ergo $\frac{D}{E}$ datur. Eadem de causa datur $\frac{E}{F}$. Q.E.D. a 20. *def.* 5. b *hyp.* c *cor.* 8. *dat.*

PROP. X.

A. B. C. *Si magnitudo magnitudine major fuerit data, quam in ratione; & simul utraque illa eadem major erit data quam in ratione. Sin autem simul utraq; magnitudo eadem magnitudine major fuerit data, quam in ratione; & reliqua illa eadem major erit data quam in ratione; aut reliqua data est cum consequente, ad quam habet altera magnitudo rationem datam.*

1. Sint A, & $\frac{B}{C}$ datæ. *a* erit $\frac{B+C}{C}$ data. *b* ergo a 6. *dat.* b 11. *def. d.*
A+B+C ⊏C data q. in r. Q. E. D.

2. Sint A, & $\frac{B+C}{C}$ datæ: *c* ergo $\frac{B}{C}$ datur. c 17. 5.
proinde A+B ⊏C data q. in r. Q. E. D.

3. Sint A+B, & $\frac{C}{B+C}$ datæ. *d* Liquet $\frac{B}{B+C}$ dari. d 5. *dat.*
Q. E. D.

PROP. XI.

A. B. C. *Si magnitudo magnitudine major sit data quam in ratione, eadem simul utraque major erit data quam in ratione. Et si eadem simul utraque major sit data quam in ratione, eadem reliqua magnitudine major erit data quam in ratione.*

a 6. *dat.* 1. A, & $\frac{B}{C}$ dantur. *a* ergo $\frac{B}{B+C}$ datur. proinde
b 11. *def d.* *b* $A+B \sqsubset B+C$ data q. in r. Q. E. D.
c 5. *dat.*

2. A, & $\frac{B}{B+C}$ dantur. *c* ergo $\frac{B}{C}$ datur. proinde *b* $A+B \sqsubset C$ data q. in r. Q. E. D.

PROP. XII.

A. B. C. *Si fuerint tres magnitudines* A, B, C, *& prima cum secunda* (A + B) *data sit, secunda quoque cum tertia* (B+C) *data sit; aut prima* A *tertiæ* C *æqualis est, aut altera altera major data.*

a 4. *ax.* 1. Nam si A+B, & B+C pares sint, *b* liquet
b 4. *dat.* A & C æquari; sin istæ impares fuerint, *b* liquet excessum A—C, vel C—A dari. Q. E. D.

PROP. XIII.

D, A + B, C.
E

Si fuerint tres magnitudines D, A+B, C, *& earum prima* D *ad secundam* A+B *habeat rationem datam; secunda autem* A + B *tertia* C *major sit data quam in ratione; prima quoque* D *major erit tertia* C *data quam in ratione.*

a 2. *def. d.* Sint A, & $\frac{B,}{C}$ ac $\frac{D}{A+B}$ datæ; *a* fitque A+B.
b 19. 5.
c 2. *dat.* D :: A. E *b* :: B. D — E. ergo *c* E, *d* & $\frac{B}{D-E}$
d 2 *def. d.*
e 8. *dat.* & (ob $\frac{B,}{C}$ datam, *e* $\frac{C}{D-E}$ dantur. *f* quare D(E+:
f 11. *def. d.* D—E) $\sqsubset$ C data q. in r. Q. E. D.

PROP. XIV.

A. C.
B. D.
E.

Si duæ magnitudines A *&* C *ad invicem habeant rationem datam, utrique autem illarum adjiciatur data magnitudo* B *&* D; *totæ* A+B, C+D, *aut habent rationem datam, aut altera* A + B *altera* C+D *major erit data quam in ratione.*

Nam

Nam si A. C :: B. D *a* :: A+B. C+D — a 13. 5.
ob $\frac{A}{C}$ *b* datam, *c* liquet $\frac{A+B}{C+D}$ dari. — b *hyp.* — c 2. *def. d.*

Saltem *d* sit A. C :: E. D. *a* :: A+E. C+D. — d 2. *def. d.*
Ergo *e* $\frac{A+E}{C+D}$, ac *e* B, *f* ideoque B—E dantur. — e 2. *dat.* — f 4. *dat.*
g proinde A+B (A+E : +B—E) ⊏ C+D data q. in r. Q. E D. — g 11. *def. d.*

PROP. XV.

A. C.
B. D.
E.

Si duæ magnitudines A *&* C *habeant ad invicem rationem datam, & ab utraque harum auferatur data magnitudo* E *&* D; *reliquæ magnitudines* A—B, C—D *ad invicem habebunt aut rationem datam, aut altera* A—B, *altera* C—D *major erit data quam in ratione.*

b Nam si A. C :: B. D *a* :: A—B. C—D. — a 19. 5.
ob $\frac{A}{C}$ datam, *c* liquet $\frac{A-B}{A-C}$ dari. — b *hyp.* — c 2. *def. d.*

Saltem *d* sit A. C :: E. D *a* :: A—E. C—D. — d 2. *def. 2.*
Ergo *e* $\frac{A-E}{C-D}$, & *e* E, ac *f* ideo E—B dantur. — e 2. *dat.* — f 4. *dat.*
g proinde A—B (A—E : +E—B) ⊏ C—D data q. in r. Q. E. D. — g 11. *def. d.*

PROP. XVI.

B. C.
A. D.
E.

Si duæ magnitudines B, C *habeant rationem datam, & ab una quidem illarum* C *auferatur data magnitudo* D, *alteri autem* B *adjiciatur data magnitudo* A; *tota* A+B *residua* C—D *major erit data quam in ratione.*

Sit enim C. B *a* :: D. E *b* :: C—D. B—E. ergo *c* $\frac{C-D}{B-E}$, & *d* E, ac *e* ideo E+A dantur. *f* proinde B+A (E+A : +B—E) ⊏ C—D data q. in r. Q. E. D. — a 2. *def. d.* — b 19. 5. — c 2. *def. d.* — d 2. *dat.* — e 3. *dat.* — f 11. *def. d.*

PROP. XVII.

A+B. D+E.
C.

Si fuerint tres magnitudines A+B, C, D+E; *& prima quidem* A+B *secunda* C *major sit data quam in ratione, tertia quoque* D+E *eadem secunda* C *major sit data quam in ratione; prima* A+B *ad tertiam* D+E *aut rationem habebit datam, aut altera altera major erit data quam in ratione.*

a *hyp.* Nam ob A, D, & $\frac{B}{C}$, $\frac{E}{C}$ *a* datas, *b* erit $\frac{B}{E}$
b 8. *dat.* data. ergo per 14. hujus.

PROP. XVIII.

A+C. B. G.
B+D. F. H.

Si fuerint tres magnitudines, atque ex his una utraque reliquarum major sit data quam in ratione; reliquæ duæ aut datam rationem habebunt ad invicem, aut altera altera major erit data quam in ratione.

Datæ sint A, B, $\frac{C}{E}$, $\frac{D}{F}$; ac sit A+C = B+D.

a 2. *def. d.* Sitque C. C *a* :: A. G *b* :: C+A. E+G. itemque
b 12. 5. D. F *a* :: B. H *b* :: D+B. F+H. *c* ergo
c 2. *def. d.* $\frac{C+A}{E+G}$, *d* hoc est $\frac{B+D}{E+G}$, *c* & $\frac{B+D}{F+H}$ ac *e* idcirco
d 7. 5.
e 8. 5. $\frac{E+G}{F+H}$; quin & G ac H *f* dantur. ergo per 15.
f 2. *dat.* (hujus.

PROP. XIX.

A+B. E.
C+D. F.

Si fuerint tres magnitudines, & prima quidem magnitudo secunda magnitudine major sit data quam in ratione, sit quoque secunda major tertia data quam in ratione; prima magnitudo tertia magnitudine major erit data quam in ratione.

Sint A, C, & $\frac{C+D}{B}$, $\frac{D}{E}$ datæ; dico A+B ⊏ E data q. in r.

Nam sit C+D. B *a* :: C. F *b* :: D. B—F. er- a 2. *def. d.*
go *c* $\frac{C}{F}$ & *d* F, ac *e* ideo F+A, & *c* $\frac{D}{B-F}$ *f* ideoque b 19. 5.
c 2. *def. d.*
$\frac{E}{B-F}$ dantur. *g* proinde A+B (F+A : +B—F) d 2. *dat.*
e 3. *dat.*
f 8. *dat.*
⊏ E data q. in r. Q. E. D. g 11. *def. d.*

PROP. XX.

A. C. E.
B. D.

Si datæ fuerint duæ magnitudines A, C; *& auferantur ab ipsis magnitudines* B, D *habentes ad invicem rationem datam; residuæ magnitudines* A—B, C—D *aut habebunt ad invicem rationem datam, aut altera* A—B *altera* C—D *major erit data quam in ratione.*

Nam si A. C :: B. D *a* :: A—B. C—D ; *b* li- a 19. 5.
quet $\frac{A-B}{C-D}$ dari. b 2. *def. d.*

Saltem sit D. B *b* :: C. E *a* :: C—D. E—B.
ergo *b* $\frac{C}{E}$ & *c* E, ac *d* propterea A—E, *b* itemque c 2. *dat.*
$\frac{C-D}{E-B}$ datæ sunt. *e* ergo A—B (A—E : +E d 4. *dat.*
—B) ⊏ C—D data q. in r. Q. E. D. e 11. *def. d.*

PROP. XXI.

A. C. E.
B. D.

Si datæ fuerint duæ magnitudines A, C; *& adjiciantur ipsis aliæ magnitudines* B, D *habentes ad invicem rationem datam, totæ* A+B, C+D *aut habebunt ad invicem rationem datam, aut altera* A+B *altera* C+D *major erit data quam in ratione.*

Nam si B. D :: AC *a* :: A+B. C+D, *b* li- a 12. 5.
quet $\frac{A+B}{C-D}$ dari. b 2. *def. d.*

Saltem sit B. D *b* :: E. C *a* :: B+E. D+C.
ergo *c* B, *d* ideoque A—E, & *b* $\frac{B+E}{D+C}$ dantur. c 2. *dat.*
d 4. *dat.*

e 11. *def.* *e* ergo $A+B\ (B+E :: +A-E) \sqsubset C+D$ data q. in r. Q. E. D.

PROP. XXII.

A. C.
B.

Si duæ magnitudines A, B *ad aliam aliquam magnitudinem* C *habeant rationem datam, & simul utraque* A+B *ad eandem* C *habebit rationem datam.*

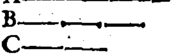
a *hyp.* Nam ob $\frac{A}{C}, \frac{B}{C}$ datas, *b* erit $\frac{A}{B}$ data. *c* quare
b 8. *d.* $\frac{A+B}{B}$ *b* ideoque $\frac{A+B}{C}$ data est. Q. E D.
c 6. *d.*

PROP. XXIII.

Si totum AB *ad totum* CD *habeat rationem datam, habeant autem & partes* AE, EB *ad partes* CF, FD *rationes datas (etsi non easdem;) habebunt omnia ad omnia rationes datas.*

Nam sit AE. CF *a* :: AG. CD *b* :: GE.FD.
a *def. d.* *a* ergo $\frac{GE}{FD}$ datur. quare (ob $\frac{EB}{FD}$ *c* datam) *d* erit
b 19. 5. $\frac{GE}{EB}$ ac *e* ideo $\frac{EB}{GB}$ data. ergo quum *c* $\frac{AB}{CD}$, &
c *hyp.* *a* $\frac{AG}{CD}$, *d* ideoque $\frac{AB}{AG}$, ac proinde *e* $\frac{AB}{GB}$, dentur,
d 8. *dat.* *d* erit $\frac{EB}{AB}$, data. Quare *e* $\frac{AB}{AE}$, & *d* $\frac{AE}{EB}$, & *e* $\frac{EB}{CF}$,
e 5. *dat.* dantur. Q. E. D.

PROP. XXIV.



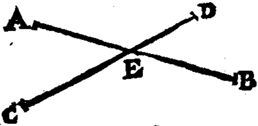
Si tres rectæ lineæ, A, B, C, *proportionales fuerint; prima autem* A *ad tertiam* C *habeat rationem datam; & ad secundam* B *habebit rationem datam.*

Nam

Nam A. C *a* :: Aq. Bq. *b* ergo $\frac{Aq}{Bq}$ data est, proinde $\frac{A}{B}$ *c* datur. Q. E. D

a cor. 20. 6.
b 2. def. d.
c 1. d.

PROP. XXV.



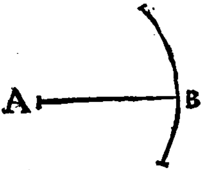
Si duæ rectæ lineæ, A B, C D *positione datæ se mutuo secuerint, punctum* E, *in quo se invicem secant, positione datum est.*

a Nam hæ lineæ alibi quam in E, neutrius situ mutuo, sese intersecare nequeunt. a 4 def. d.

Schol.

a Idem patet de quibuscunque lineis positione datis, seque in unico puncto intersecantibus: ut de circuli arcu, & recta, &c.

PROP. XXVI.

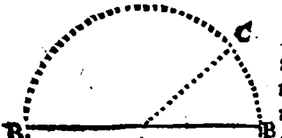


Si recta linea A B *extremitates* A, B, *positione datæ sint, recta* A B *positione & magnitudine data est.*

Positione quidem, *a* quia inter eosdem terminos unica recta duci potest: & magnitudine, *b* quia si centro A per B ducatur circulus, hujus omnes radii ipsi AB æquantur.

a 14 ex.
b 1. def. d.

PROP. XXVII.



Si rectæ lineæ A B *positione & magnitudine datæ, data fuerit una extremitas* A; *& altera extremitas* B *data erit.*

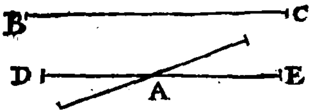
Nam

a 1.*def.d.* Nam si centro A, spatio AC *a* = AB *b* ducatur circulus, qui data recta *c* occurrat in B, *d* erit
b 3.*post.*
c 2.*post.* extremitas B data.
d *cor.* 25.

Schol.

Vides partes puncti B determinandas esse.

PROP. XXVIII.

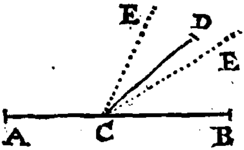


Si per datum punctum A *contra datam positione rectam* BC *agatur recta linea* DE, *acta recta* DE *positione data est.*

a 4. *def.d.* Nam *a* dic alteram per A ad BC fore paralle-
b 30. 1. lam. Hæc idcirco ad DE *b* parallela erit. *c* Quod
c 34. *def.*1. repugnat.

Nota, Vocabulum *contra* in hoc libro parallelismum significare.

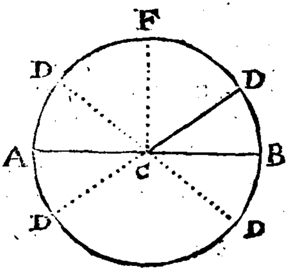
PROP. XXIX.



Si ad positione datam rectam AB, *datumque in ea punctum* C, *agatur recta linea* CD, *quæ faciat angulum* DCB *datum; acta recta* CD *positione data erit.*

a 4.*def.d.* *a* Nam quævis alia CE angulum *b* efficiet
b 9. *ax*, 1. majorem, vel minorem dato BCD.

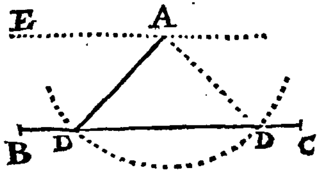
Schol.



Determinari debet situs anguli dati tam respectu perpendicularis CF, quam ipsius AB, ut cernis in apposita figura.

PROP.

PROP. XXX.



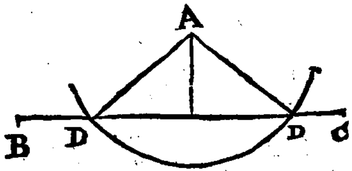
Si à dato puncto A *in datam positione rectam* B C *agatur recta linea* A D, *quæ faciat angulum* A D C *datum, acta linea* A D *positione data est.*

Nam per A duc A E ad B C parallelam. *a* Hæc positione datur. Item ang. DAE par dato alterno ADC *b* datus est. *c* ergo recta AD positione data est, Q. E. D. a 28. *dat.* b 1. *def. d.* c 29 *dat.*

Schol.

Hinc praxim discimus à dato puncto ducendi rectam, quæ cum data positione recta datum angulum efficit.

PROP. XXXI.

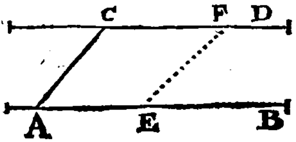


Si à dato puncto A *in datam positione rectam* BC *data magnitudine recta* AD *ducatur, positione quoque data erit.*

Nam puncta D, per quæ transit circulus centro A, *a* spatio AD descriptus, *b* data sunt. *c* ergo AD positione data est. Q. E. D. a 1. *def. d.* b *sch.* 25. *d.* c 26. *d.*

PROP.

PROP. XXXII.

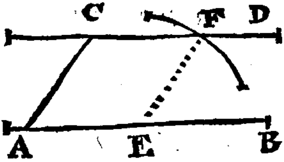


Si in datas positione parallelas rectas AB, CD *agatur recta linea* AC, *quæ faciat angulos datos* BAC, ACD, *acta recta* A C *magnitudine data est.*

Nam ad E (quodvis punctum in AB) fac ang. BEF = *a* BAC. liquet rectas BF, AC *b* parallelas, & *c* pares fore. *d* quare A C data est. Q. E. D.

a 1. *def.d.*
b 29. 1.
c 34. 1.
d 2. *def.d.*

PROP. XXXIII.



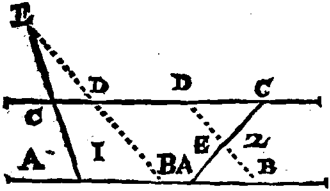
Si in datas positione parallelas rectas AB, CD *agatur magnitudine data recta* A C, *faciet angulos* BAC, ACD *datos.*

Nam ex quovis puncto E in A B, spatio E F *a* = A C describe circulum occurrentem rectæ CD in F. *b* Liquet E F, & A C parallelas esse posse. *c* ergo.

a 1. *def.d.*
b 34. 1.
c 29. 1.

PROP.

PROP. XXXIV.



Si in datas positione parallelas rectas A B, C D *à dato puncto* E *agatur recta linea* E C A, *secabitur data ratione.*

Nam ab E duc rectam E B utcunque parallelis occurrentem in D, & B, *a* liquet esse EC.CA :: ED. DB, *b* quare $\frac{EC}{CA}$ datur. Q. E. D. a 2. 6. b 2. *def. d.*

PROP. XXXV.

Si à dato puncto E *in datam positione rectam* AB *agatur recta linea* EA, *seceturque data ratione; agatur autem per punctum sectionis* C *contra datam positione rectam* A B *recta linea* C D; *acta linea* CD *positione data est.*

Recta enim E B ducta ab E utcunque in A B, *a* secetur sic ut ED.DB :: EC. CA. ob punctum D datum, *b* erit CD positione data. Q. E. D. a 10. 6. b 28. *dat.*

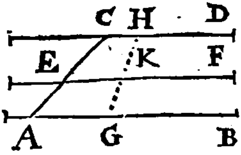
PROP. XXXVI.

Si à dato puncto E *in datam positione rectam lineam* AB *agatur recta linea* EA; *adjiciatur autem ipsi aliqua recta* EC, *quæ ad illam* (E A) *habeat rationem datam; per extremitatem autem* C *adjectæ lineæ* EC *agatur contra datam positione rectam* AB *recta linea* CD; *acta linea* CD *positione data est.*

Demonstratio parum differt à præcedenti. *Vide fig.* 2.

PROP.

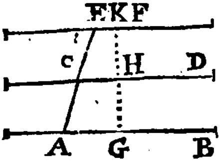
PROP. XXXVII.



Si in datas positione parallelas rectas AB, CD, *agatur recta linea* AC, *& secetur ratione data; agatur autem per sectionis punctum* E *contra datas positione rectas* AB, CD *linea recta* EF; *acta recta* EF *positione data est*.

a 2. def. d. b 28. dat. & sch. 2. 6. Nam duc rectam GH utcunque occurrentem parallelis. Hæc *a* secta sit in K ita ut GK, KH :: AB, EC. *b* Punctum K parallelæ (EF) situm determinat. Q. E. F.

PROP. XXXVIII.

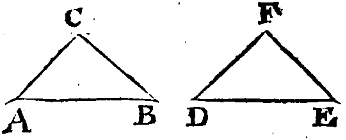


Si in datas positione rectas parallelas AB, CD *agatur recta linea* AC; *adjiciatur autem ipsi quædam recta* CE, *quæ ad actam* AE *habeat rationem datam; per extremitatem autem* E *adjectæ* CE *agatur contra datas positione parallelas* AB, CD *recta linea* EF; *acta recta linea* EF *est data positione*.

Demonstratio persimilis est præcedenti. Cerne & compara figuras.

PROP.

PROP. XXXIX.



Si trianguli ABC *singula latera* AB, BC, AC *magnitudine data sint, triangulum* ABC *specie datum est.*

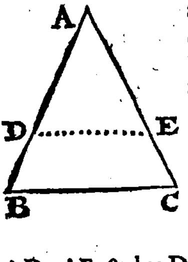
Nam *a* fac triang. DEF ipsi ABC æquilaterum. Hoc eidem *b* æquiangulum erit. *c* ergo ABC specie datum est. Q. E. D. a 22. 1. b 5. 6. c 3. *def. d.*

PROP. XL.

Si trianguli ABC *singuli anguli*, A, B, C *magnitudine dati sint, triangulum* ABC *specie datum est.*

Nam ad quamvis DE *a* fac triang. DEF ipsi ABC æquiangulum. *b* Hoc eidem simile erit. *c* proinde trigonum ABC specie datum est. Q. E. D. a 23. 1. b 4. 6. c 3. *def. d.*

PROP. XLI.



Si triangulum ABC *unum angulum* A *datum habeat; circa datum autem angulum* A *duo latera* AB, AC *ad invicem habeant rationem datam; triangulum* ABC *specie datum est.*

Nam in uno latere dati anguli sume quampiam AD; & *a* fit AB. AC :: AD. AE. & duc DE. *b* Liquet trigonum ADE ipsi ABC simile fore. *c* Quare ABC specie datum est. Q. E. D. a 1. *def. d.* b 6. 6. c 3. *def. d.*

PROP.

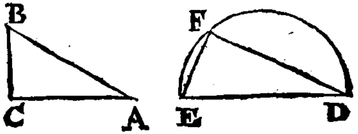
PROP. XLII.

Si trianguli ABC *latera ad invicem habeant rationem datam, triangulum* ABC *specie datum est.*

a 12.6. Nam *a* fac A B. BC :: DE. EF. *a* & B C. CA
b 5.6. :: EF. FD. *b* Liquet trigonum DEF trigono ABC
c 3.def.d. assimilari. *c* quare A B C specie datum est. Q. E D.

Vide fig. 39.

PROP. XLIII.

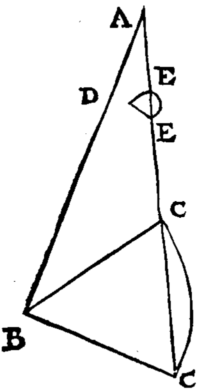


Si trianguli rectanguli ACB *circa unum acutorum angulorum* A *latera* AB, AC *ad invicem rationem habeant datam, triangulum* A C B *specie datum est.*

a 12.5. Nam esto D E F semicirculus utcunque; &
b 1.4. *a* fac A B. A C :: D B. D F. inventamque D F
c 32.1.& *b* adapta in semicirculo; & duc EF. *c* Liquet tri-
4.6. ang. D F E ipsi A C B assimilari; & *d* proinde
d 3.def.d. ipsum ACB specie dari. Q. E. D.

PROP.

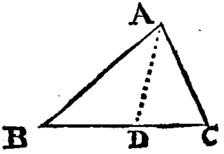
PROP. XLIV.



Si triangulum A B C *habeat unum angulum* A *datum; circa alium autem angulum* A B C *latera* A B, B C *ad invicem habeant rationem datam; triangulum* A B C *specie datum est.*

Nam in crure dati anguli sume quamlibet A D. & *a* fac A B. B C :: A D. D E. centro D spatio D E describe circulum, qui secet alterum dati anguli latus in E. *b* Eritque triang. A D E ipsi A B C simile. *c* quare datur specie triang. A B C. Q. E. D. a 2. *def. d.* b 7. 6. c 3. *def. d.*

PROP. XLV.



Si triangulum B A C *unum angulum* BAC *datum habeat; circa datum autem angulum* BAC *latera simul utraque tanquam unum* (BA + AC) *ad reliquum latus* (B C) *rationem habeant datam; triangulum* BAC *specie datum est.*

Datum angulum B A C *a* bisecet recta A D. a 9. 1.
b ergo B A. A C :: B D. D C. & componendo b 3. 6.
BA + AC. AC :: BC. DC. permutando igitur BA + AC. BC :: AC. DC. ergo ob $\frac{BA+AC}{BC}$ c hyp.
c datam, *d* erit $\frac{AC}{DC}$ data, item ang. DAC sub- d 2. *def. d.*
duplus

c 2.dat. duplus dati B A C *e* datur. *f* ergo ang. C datur;
f 44.dat. *g* proinde trigonum ABC specie datum est.
g 40.dat.

Coroll.

Hinc in triangulo, datis uno latere AB, uno angulo B A C, & ratione aggregati laterum ad basim (R ad S;) datur triangulum. Nam datum angulum biseca, & fac R. S :: AB. BD. & centro B spatio B D duc circulum occurrentem rectæ bisecanti in D; & produc BDC. habes triangulum.

PROP. XLVI.

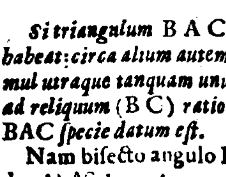
Si triangulum B A C *unum angulum* C *datum habeat: circa alium autem angulum* BAC *latera simul utraque tanquam unum* (BA + AC) *habeant ad reliquum* (B C) *rationem datam; triangulum* BAC *specie datum est.*

Nam bisecto angulo BAC, erit (ut in præcedenti) $\frac{AC}{DC}$ data. item ang. C *a* datus est. ergo ang. D A C, *b* proinde & duplus B A C datur. *c* quare triang. BAC specie datur. Q. E. D.

a hyp.
b 2. dat.
c 40. dat.

Deducetur ab hac corollarium simile præcedenti.

PROP. XLVII.



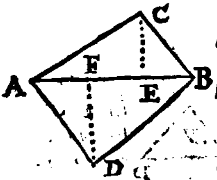
Data specie recti linea ABCDE *in data specie triangula* BAE, CDE BCE *dividuntur.*

Nam ob ang. B, & $\frac{BA}{AE}$ *a* dat. *b* erit triang. B A E specie datum. Simili discursu triang. CDE specie datur. *c* quare ang. D C E datus est; Hunc deme ex dato BCD, *d* estque reliquus BCE datus. Similiter ang. CBE datur. *e* ergo triang. BCE etiam specie datum est. Q. E. D

a hyp. & 3.def.d
b 41. dat.
c 3.def.d.
d 4.dat.
e 40.dat.

PROP.

PROP. XLVIII.

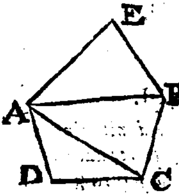


Si ab eadem recta AB describantur triangula ACB, ADB data specie, habebunt ad invicem rationem datam.

Duc enim perpendiculares CE, DF. Liquet angulos trianguli rectanguli CEB, *a* proinde & $\frac{CE}{CB}$ dari. ergo (quum $\frac{AB}{CB}$ *b* data sit) *c* erit $\frac{CE}{AB}$ data. Simili discursu datur $\frac{DF}{AB}$; *c* quare $\frac{CE}{DF}$; *d* hoc est triang. $\frac{ACB}{ADB}$ datur. Q. E. D.

a 40. *d.* b *hyp.* c 8. *d.* d *sch.* 1. 6.

PROP. XLIX.

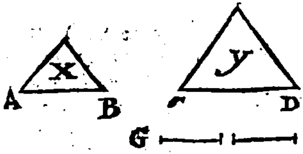


Si ab eadem recta linea AB duo rectilinea qualibet BA ABCD, AEB data specie describantur, habebunt ad invicem rationem datam.

Nam rectilineum ABCD resolvatur in triangula. *a* haec specie data sunt. ergo ob communem basim AC, *b* ratio ADC ad ACB & *c* proinde totius ABCD ad ACB datur. *b* item ratio AEB ad ACB, *d* proinde & ABCD ad AEB datur. Q. E. D.

a 47. *d.* b 48. *d.* c 6. *d.* d 8. *d.*

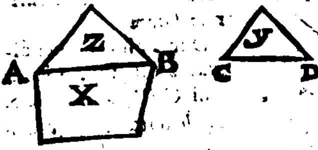
PROP. L.



Si duae rectae lineae AB CD ad invicem habeant rationem datam; & ab illis similia, similiterque descripta rectilinea X, Y habebunt ad invicem rationem datam.

a 11. 6. Nam sit AB. CD :: *a* CD. G. *d* liquet AB ad
b 8. *d.* G, *e* hoc est X ad Y dari. Q. E. D.
c *cor.* 20. 6.

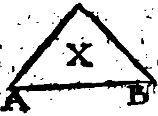
PROP. LI.



Si duæ rectæ lineæ AB, CD *habeant ad invicem rationem datam; & ab illis rectilinea quacunque* X, Y *specie data describantur; habebunt ad invicem rationem datam.*

a 18. 6. Nam *a* fac Z simile ipsi Y. Ac ob *b* $\frac{Z}{X}$, *c* & $\frac{Z}{Y}$
b 49. *d.*
c 50. *d.* datas, *d* liquet $\frac{X}{Y}$ dari. Q. E. D.
d 8. *d.*

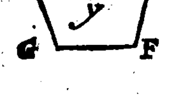
PROP. LII.



Si à data magnitudine recta AB *figura* X *specie data describatur, descripta figura* X *magnitudine data est.*

Nam ABq *a* datur specie, &
a 3. & 1. *def. d.* magnitudine; & *b* $\frac{ABq}{X}$ datur. *c* ergo X datur.
b 49. *d.*
c 2. *d.*

PROP. LIII.



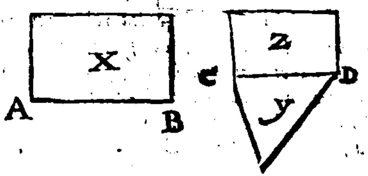
Si duæ figuræ X, Y *speciæ datæ fuerint; & unum latus unius* BC *ad unum latus alterius* DE *habuerit rationem datam; reliqua quoque latera* AB *ad reliqua* FG *habebunt rationem datam.*

Nam

Nam { a $\frac{AB}{BC}$, b $\frac{BC}{DE}$, a $\frac{DE}{EF}$, $\frac{EF}{FG}$ } datur. a &c. ergo per 8. dat.

a 3. def. d.
b hyp.

PROP. LIV.

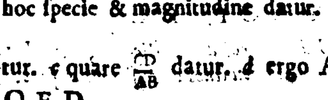


Si duæ figuræ X, Y *specie datæ ad invicem habuerint rationem datam, etiam latera* (AB, CD, &c.) *habebunt ad invicem rationem datam.*

Nam ad CD *a* fiat Z ipsi X similis. *b* Hæc specie datur. *c* ergo $\frac{Y}{Z}$ datur. Proinde ob $\frac{Y}{X}$ *d* datam, *e* datur $\frac{X}{Z}$. *f* ergo $\frac{AB}{CD}$ datur. ergo per præcedentē.

a 18. 6.
b 3. *def. d.*
c 49. *dat.*
d *hyp.*
e 8. *dat.*
f *cor.* 20. 6. & 24. *dat.*

PROP. LV.

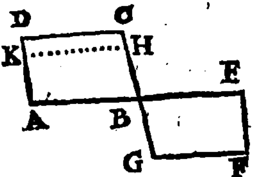


Si spatium X *magnitudine & specie datum fuerit, ejus latera* (AB, &c.) *magnitudine data erunt.*

Nam ad quamvis CD *a* fac Y simile ipsi X, hoc specie & magnitudine datur. *b* ergo $\frac{Y}{X}$ datur. *c* quare $\frac{CD}{AB}$ datur. *d* ergo AB data est. Q. E. D.

a 18. 6.
b 1. *dat.*
c 54. *dat.*
d 2. *dat.*

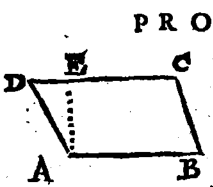
PROP. LVI.



Si duo æquiangula parallelogramma AC, BF *habuerint ad invicem rationem datam, est ut primi latus* AB *ad secundi latus* BE, *ita reliquum secundi latus* BG *ad eam* BH, *ad quam alterum primi latus* BC *habet rationem datam, quam habet parallelogrammum* AC *ad parallelogrammum* BF.

a 1. 6. Nam duc HK parall. AB. Liquet esse BC.
b 14. 6. BH *a* :: AC. AH *b* :: AC. BF. Q. E. D.
& 7. 5.

PROP. LVII.



Si datum spatium AC *ad datam rectam* AB *applicatum fuerit, in angulo* BAD *dato, datur applicationis altitudo* AD.

a 11. 1. *a* Erige perpendicularem AE, estque AB. AE
b 1. 6. *b* :: ABq. AB × AE *c* :: ABq. pgr. AC. *d* ergo
c. 35. 1. AE datur. quare per E duc parallelam DC, *e*
d 1. & 2. hæc abscindet quæsitam AD. Q. E. F.
dat.
e 28. & 25.
dat.

PROP. LVIII.

Si datum ad datam rectam applicetur, deficiens data specie figura, latitudines defectus data sunt.

Non differt à vigesima octava sextæ.

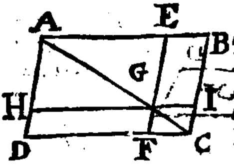
PROP. LIX.

Si datum ad datam rectam applicetur, excedens data specie figura, latitudines excessus datæ sunt.

Eadem est cum vigesima nona sextæ.

PROP.

PROP. LX.

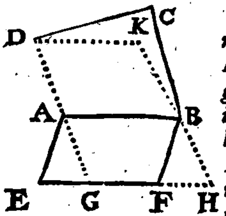


Si datum specie parallelogrammum (H E, *vel* DB) *dato gnomone* HCB *augeatur, vel minuatur; latitudines gnomonis* HD, EB *data sunt.*

1. *Hyp.* Liquet totum DB tam *a* magnitudine, quam *b* specie dari, *c* proinde & latitudines AB, AD; è quibus aufer *d* datas AE, AH, *e* manent EB, HD datæ. Q.E.D. a 3. *d.* b 24. 6. c 55. *d.*

2. *Hyp.* Liquet HE *b* specie, & *a* magn. *c* dari, *c* quare & latera AE, AH; hæc deme ex *d* datis AB, AD: *e* remanent EB, HD datæ. Q. E. D. d *hyp.* e 4. *d.*

PROP. LXI.

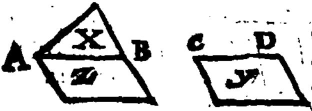


Si ad data specie figuræ ABCD *unum latus* AB *applicetur parallelogrammum spatium* AF *in angulo* BAE *dato; habeat autem data figura* AC *ad parallelogrammum* AF *rationem datam; parallelogrammum* AF *specie datum est.*

Ad DAG protractam duc (per B) parallelam, cui occurrant EFH, & DK parall. AB. Ac ob $\frac{AD}{AB}$, & ang. BAD *a* dat. *a* liquet pgr. AK specie dari. *b* ergo $\frac{AK}{AC}$ & *c* proinde $\frac{AK}{AF}$ *d* vel $\frac{AK}{AH}$; *e* hoc est $\frac{AD}{AG}$ datur. *e* ergo $\frac{AB}{AG}$ datur. Item ob angulos E, & GAE *f* notos, *g* datur $\frac{AE}{AG}$; *c* ergo $\frac{AB}{AE}$ datur. *h* unde pgr. AF specie datur. Q. E. D.

a 3. *def. d.* b 49. *d.* c 8. *d.* d 35. 1. e 1. 6. f *hyp.* & 4. *d.* g 40. *d.* h 3. *def. d.*

PROP. LXII.



Si duæ rectæ AB, CD *ad invicem habeant rationem datam; & ab una quidem data specie figura* X *descripta sit, ab altera autem spatium parallelogrammum* Y *in angulo dato; habeat autem figura* X *ad parallelogrammum* Y *rationem datam; parallelogrammum* Y *specie datum est.*

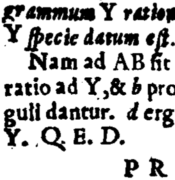
a 50. *dat.* Nam ad AB fit pgr. Z simile ipsi Y. *a* Hujus
b 8. *dat.* ratio ad Y, & *b* proinde ad X datur. *c* ejusque an-
c *hyp.* guli dantur. *d* ergo Z specie datur. *e* proinde &
d 61. *dat.* Y. Q. E. D.
e 3. *def. d.*

PROP. LXIII.

Si triangulum specie datum sit, quod ab unoquoq; laterum describitur quadratum, ad triangulum habebit rationem datam.

Sequitur ex 49. hujus.

PROP. LXIV.



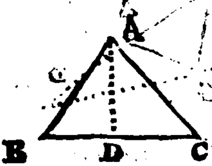
Si triangulum ABC *angulum obtusum* ABC *datum habeat; illud spatium, quo latus* AC *obtusum angulum subtendens magis potest quam latera* AB, CB *obtusum angulum* ABC *ambientia, ad triangulum* ABC *habebit rationem datam.*

a 4. *dat.* Nam demittatur AD perpendicularis productæ CBD. atque ob angulos *a* ABD, & D da-
b 40. *dat.* tos, *b* datur $\frac{BD}{AD}$, *c* hoc est $\frac{BD \times CB}{AD \times CB}$. *d* ergo
c 1. 6.
d 8. *dat.*

$\frac{2\,BD \times CB}{\frac{1}{2}\,AD \times CB}$, hoc est, *e* $\frac{ACq - ABq - CBq}{f\ \text{triang.}\ ABC}$ datur. e 12. 2. f 41. 1.

Q. E. D.

PROP. LXV.



Si triangulum ACB *angulum acutum* C *datum habeat; illud spatium, quo latus* AB *angulum* C *subtendens minus potest, quam latera* AC, CB *angulum acutum* C *ambientia, habebit ad triangulum* ACB *rationem datam.*

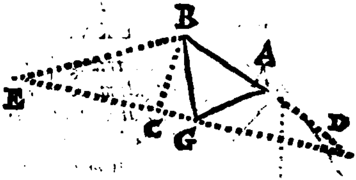
Nam duc perpendicularem AD. Datur *a* $\frac{CD}{AD}$, *b* hoc est $\frac{CD \times BC}{AD \times BC}$. *c* ergo $\frac{2\,CD \times BC}{\frac{1}{2}\,AD \times BC}$, hoc est *d* $\frac{ACq + BCq - ABq}{e\ \text{triang.}\ ACB}$ datur. Q. E. D. a 40. *d.* b 1. 6. c 8. *d.* d 13. 1. e 41. 1.

PROP. LXVI.

Si triangulum ACB *habuerit angulum* C *datum; quod sub rectis* AC, CB *datum angulum* C *comprehendentibus, continetur rectangulum, habebit ad triangulum* ACB *rationem datam.*

Nam in figura præcedentis, est *a* $\frac{AC}{AD}$, *b* hoc est, $\frac{AC \times BC}{AD \times BC}$, *c* hoc est $\frac{AC \times BC}{2\ \text{triang.}\ ACB}$ data. *d* ergo $\frac{AC \times BC}{\text{triang.}\ ACB}$ datur. Q. E. D. a 40. *d.* b 1. 6. c 41. 1. d 8. *d.*

PROP. LXVII.

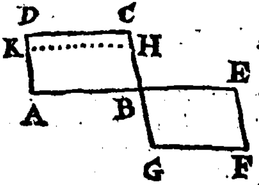


Si triangulum ABG *habuerit datum angulum* BAG; *illud spatium, quo duo datum angulum* BAG *comprehendentia latera tanquam una recta* BA+AG, *plus possunt, quam quadratum à reliquo latere* BG, *ad triangulum* ABG *habebit rationem datam.*

Produc BA ita ut AD=AG. per B duc BE parall. AG; cui occurrat DGE. denique duc normalem BC.

a 5.1. Liquet ang. D *a*=AGD *b*=E. *c* quare BE=
b 29.1. BD, ideoque EC=CD. *e* ergo EG×GD+
c 6.1. CGq=CDq. proinde BDq *f* (CDq+BCq)
d *cor.* 3.3. *g*=EG×GD+CGq+BCq=EG×GD*+
e 5.2. BGq. Jam ob angulos AGD, & D *h* subduplos
f 47.1. dati BAG, liquet *k* $\frac{AD}{DG}$, ideoq; $\frac{ADq}{DGq}$ dari. Cum
g 2. *ax.* 1.
* 47.1. igitur BA×AD. ADq *l* :: BA. AD *m* :: EG.
h 32.1. GD :: *l* EG×GD. GDq, & permutando BA×AD.
k 40. *d.* EG×GD :: ADq. GDq; *n* erit $\frac{BA\times AD}{EG\times GD}$; *o* hoc
l 1.6.
m 2.6.
n 2. *def. d.* est $\frac{BA\times AG}{EG\times GD}$ data. *p* Atqui $\frac{BA\times AG}{triang.\ AGB}$ datur; *q* er-
o *constr.*
p 66. *d.* go $\frac{EG\times GD}{triang.\ AGB}$ datur. Q. E. D.
q 8. *d.*

PROP. LXVIII.

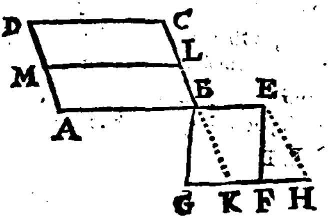


Si duo parallelogramma æquiangula AC, BF *habeant ad invicem rationem datam, & unum latus* AB *ad unum latus* BE *habeat rationem datam; & reliquum latus* BC *ad reliquum latus* BG *habebit rationem datam.*

Nam sit AB. BE :: BG. BH. *a* ergo $\frac{BG}{BH}$ datur. *b* item $\frac{BC}{BH}$ datur. *c* ergo $\frac{BC}{BG}$ datur.

a 2. def. d.
b 56. d.
c 8. d.

PROP. LXIX.



Si duo parallelogramma AC, BF *datos angulos habeant, & ad invicem rationem datam; habeat autem & unum latus* AB *ad unum latus* BE *rationem datam; & reliquum latus* BC *ad reliquum latus* BG *habebit rationem datam.*

Latera AB, BE jaceant in directum. produc CBK, ac GFH ad occursum cum EH parall. CK.

Ob *a* ang. KBE (ABC) & pgr. *a* $\frac{AC}{BF}$, vel *a hyp.*

AC

b 35. 1. $\frac{AC}{BH}$ & *a* $\frac{AB}{BE}$ datas, *c* liquet $\frac{KB}{BC}$ dari. item ob ang.
c 68. *d.* G, & GBK *d* datos, *e* datur $\frac{KB}{BG}$. *f* quare $\frac{BC}{BG}$ datur.
d *hyp.* & 4. *d.* Q. E. D.
e 40. *d.*
f 8. *d.*

PROP. LXX.

Si duorum parallelogrammorum (AC, BH, *vel* BF) *circa æquales angulos* (ABC, KBE) *aut circa inæquales quidem* (ABC, GBE) *datos tamen, latera* (AB, BE, & BC, BK, & BC, BG) *ad invicem habeant rationem datam; & ipsa parallelogramma* (AC, BH, & AC, BF) *habebunt ad invicem rationem datam.*

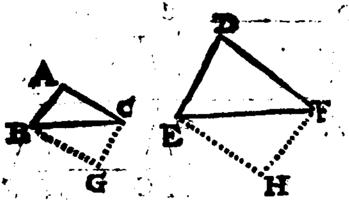
Nam (in fig. præced.) fit AB. BE :: KB. BL. & duc LM parall. BA.

a *hyp.* Primo, Quia *a* $\frac{AB}{BE}$ *b* id est $\frac{KB}{BL}$ *a* ac $\frac{KB}{CB}$ datæ
b *constr.* sunt, *c* erit $\frac{CB}{BL}$, *d* hoc est $\frac{AC}{AL}$ *e* vel pgr. $\frac{AC}{BH}$ data.
c 8. *d.*
d 1. 6.
e 14. 6. Q. E. D.

f *hyp.* & 4. *d.* Secundo, Ob angulos G, & GBK *f* datos, *g* datur $\frac{BK}{BG}$; item *b* $\frac{CB}{BG}$ data est, *c* ergo $\frac{CB}{BK}$ da-
g 40. *d.*
h 35. 1. tur. proinde, ut prius, $\frac{AC}{BH}$, hoc est pgr. $\frac{AC}{BF}$ datur. Q. E. D.

PROP.

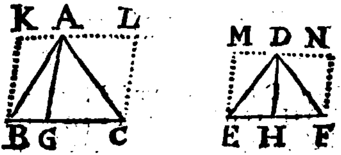
PROP. LXXI.



Si duorum triangulorum ABC, DEF, *circa æquales angulos, aut circa inæquales quidem, datos tamen* (A, & D) *latera* AB, DE, & AC, DF *ad invicem habeant rationem datam; & ipsa triangula* ABC, DEF *habebunt ad invicem rationem datam.*

Nam compleantur pgra. AG, DH. *a* hæc datam habent rationem, *b* proinde & trigona ABC, DEF illorum *c* subdupla. Q. E. D. a 70. *d.* b 15. 5. c 34. 1.

PROP. LXXII.

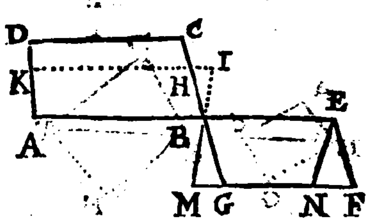


Si duorum triangulorum ABC, DEF & *bases* BC, EF *fuerint in ratione data, & ductæ ab angulis ad bases* (AG, DH,) *quæ faciant ang.* AGC, DHF *æquales, aut inæquales quidem, sed tamen datos, habeant ad invicem rationem datam; & ipsa triangula* ABC, DEF *habebunt ad invicem rationem datam.*

Nam duc BK ad AG, ac EM ad DH parallelas, & comple pgra. CK, FM. Hæc se habent juxta 70. hujus; quare triangula eorum * subdupla ABC, DEF rationem habent datam. Q. E. D. * 34. 1.

PROP.

PROP. LXXIII.



Si duorum parallelogrammorum (AC, BF, *vel* AC, BN) *circa æquales angulos, aut circa inæquales quidem, sed tamen datos, latera ad invicem ita se habeant, ut sit quemadmodum primi latus* AB *ad secundi latus* BE, *ita reliquum secundi latus* (BG, *vel* BM) *ad aliam aliquam rectam* (BH, *vel* BI;) *habeat autem & reliquum primi latus* BC *ad eandem rectam* (BH *vel* BI) *rationem datam; & ipsa parallelogramma* (AC, BF, *vel* AC, BN) *habebunt ad invicem rationem datam*

a hyp. b 2.6. c 14.6. Nam 1. Hyp. liquet *a* $\frac{CB}{BH}$ *b* id est $\frac{AC}{AH}$ *c* $\left(\frac{\ }{BF}\right)$ dari. Q. E. D.

a hyp & 4. dat. b 40. dat. c 8. dat. d 35.1. 2. Hyp. Duc parallelam IHK. *a* Liquet angulos IBH (GBM) & BHI (ABH) dari. *b* ergo $\frac{BH}{BI}$ datur. item $\frac{CB}{BI}$ *a* data est. *c* proinde $\frac{CB}{BH}$, hoc est pgr. $\frac{AC}{BF}$ *d* vel $\frac{AC}{BN}$, datur. Q. E. D.

PROP. LXXIV.

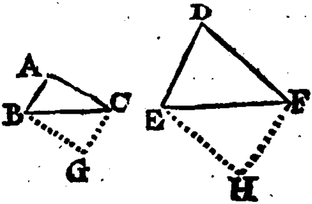
Si duo parallelogramma datam rationem habeant, aut in æqualibus angulis (*ut* AC, BF) *aut inæqualibus quidem, sed tamen datis* (*ut* AC, BN;) *erit ut primi latus* AB *ad secundi latus* BE, *ita alterum secundi latus* (BG, *vel* BM) *ad eam* (BH, *vel* BI) *ad quam reliquum primi latus* BC *rationem habet datam.*

Nam

Nam in fig. præcedentis. 1. Hyp. a Liquet $\frac{CB}{BH}$ dari. Q. E. D. a 56. dat.

2. Hyp. ut in præcedenti, datur $\frac{BI}{BH}$, ac ex hyp. $\frac{AC}{BF(BN)}$ item AB.BE :: a *MB.BI b :: GB. BH. * hyp. b 4. 6.
c quare $\frac{CB}{BH}$ etiam datur. c ergo $\frac{CB}{BI}$ data est. c 8. dat.
Q. E D.

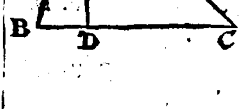
PROP. LXXV.



Si duo triangula ABC, DEF *ad invicem habeant rationem datam, aut in angulis*(A, D) *æqualibus, aut inæqualibus quidem sed tamen datis, erit ut primi latus* A B *ad secundi latus* D E, *ita alterum secundi latus* DF *ad eam rectam, ad quam reliquum primi latus* AC *habet rationem datam.*

Nam compleantur pgr. AG, DH. Ergo per præcedentem.

PROP. LXXVI.



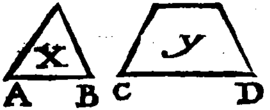
Si à trianguli ABC *specie dati vertice* A *linea perpendicularis* A D *agatur ad basim* B C, *acta linea* A D *ad basim* B C *habebit ationum datam.*

Nam

* *hyp.* & 3. *def. d.* a 40. *dat.* b 8. *dat.*

Nam ob angulos, *B, & ADB datos, *a* datur $\frac{AB}{AD}$, *a* item $\frac{AB}{BC}$ datur. *b* Ergo $\frac{AD}{BC}$ datur. Q.E.D.

PROP. LXXVII.

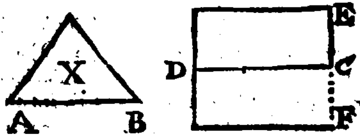


Si data figura specie X, Y *ad invicem habeant rationem datam, & quodlibet latus unius* AB *ad quodlibet alterius latus* CD *habebit rationem datam.*

a 49. *dat.* b *hyp.* c 8. *dat.*

Nam *a* $\frac{ABq}{X}$, & *b* $\frac{Y}{X}$, ac *c* proinde $\frac{ABq}{Y}$ datur; item $\frac{CDq}{Y}$ datur. *c* ergo $\frac{ABq}{CDq}$, ac ideo $\frac{AB}{CD}$ datur. Q. E. D.

PROP. LXXVIII.



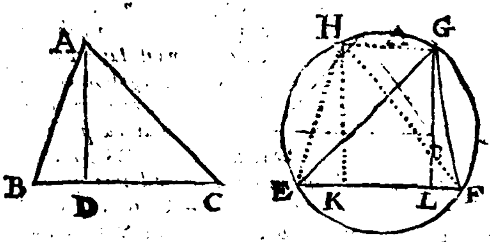
Si data figura specie X *ad aliquod rectangulum* DCE *habeat rationem datam, habeat autem & unum latus* AB *ad unum latus* DC *rationem datam; rectangulum* DCE *specie datum est.*

a 8. *dat.* 3. b 49. *dat.* c *hyp.* d 17. 6. e 1. 6. f 3. *def. d.*

Sit DC. AB :: AB. CF. *a* ergo $\frac{DC}{CF}$ datur. Item ob *b* $\frac{X}{ABq}$, & *c* $\frac{X}{DCE}$ datas, *a* erit $\frac{ABq}{DCE}$, *d* hoc est $\frac{DC\times CF}{DC\times CE}$, vel *e* $\frac{GF}{CE}$ data. proinde *e* $\frac{DC}{CE}$ datur. *f* quare rectang. DCE specie datur. Q. E. D.

PROP.

PROP. LXXIX.



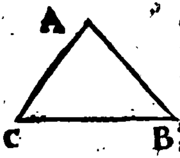
Si duo triangula ABC, GEF *unum angulum* BAC *uni angulo* EGF *æqualem habeant; ab æqualibus autem angulis* BAC, EGF *ad bases* BC, EF *perpendiculares agantur* AD, GL; *sitque ut primi trianguli basis ad perpendicularem, ita & alterius trianguli basis ad perpendicularem* (BC.AD::EF.GL;) *illa triangula* ABC, EGF *æquiangula sunt.*

Circa triang. GEF describe circulum. Fac ang. FEH = B. Connecte HF, HG; & demitte perpendicularem HK.

Liquet triangula ABC, HEF, & ABD, HEK, a 4. 6.
ac ACD, HFK æquiangula fore. Proinde EK. b 24. 5.
KH :: BD. DA. *a* & FK. KH :: CD. DA. c *hyp.*
b quare EF. KH :: BC. DA :: *c* EF. LG. d 9. 5.
d quare KH = LG. *e* ergo HG parall. KL. *f* un- e 33. 1.
de ang. EGH = GEF. *g* ergo arcus EH, FG, f 29. 1.
h ideoque anguli EFH, GEF æquantur. *k* Item g 26. 3.
ang. EHF = EGF. *l* ergo trigona EHF, EGF; h 27. 3.
m proinde & trigona EGF, ABC sibi mutuo æ- k 21. 3.
quiangula sunt. Q. E. D. l 32. 1.
m 21. 6.

PROP.

PROP. LXXX.



Si triangulum ABC *unum angulum* A *datum habuerit; quod autem sub lateribus* AB, AC *datum angulum comprehendentibus continetur rectangulum, habeat ad quadratum reliqui lateris* BC *rationem datam; triangulum* ABC *specie datum est.*

Nam Q: AC + AB: − CBq vocetur X.

a 67. d. *a* ergo $\frac{X}{\text{triang. } ABC}$; *b* & $\frac{ACxAB}{\text{triang. } ABC}$; & *c* propterea
b 66. d.
c 8 d. $\frac{X}{ACxAB}$ data est, *d* item $\frac{ACxAB}{CBq}$ datur. *e* ergo
d hyp.
e 6. d. $\frac{X}{CBq}$ *e* ideoque $\frac{X + CBq}{CBq}$, *f* hoc est $\frac{Q: AC + AB}{CBq}$
f hyp.
g 46. d. datur. *g* proinde triang. ABC specie datur. Q.E.D

PROP. LXXXI.

A. D. *Si tres rectæ proportionales*
B. E. A, B, C *tribus rectis proportio-*
C. F. *nalibus* D, E, F *extremas* A, D, & C, F *habuerint in ratione data; medias quoque* B, E *habebunt in ratione data. Et si extrema* A *ad extremam* D, & *media* B *ad mediam* E *habeat rationem datam;* & *reliqua* C *ad reliquam* F *habebit rationem datam.*

a 70. d. Nam primo, ob $\frac{A}{D}$ & $\frac{C}{F}$ datas, *a* datur $\frac{AC}{DF}$,
b 17. 6. *b* hoc est, $\frac{Bq.}{Eq}$ ergo $\frac{B}{E}$ datur. Q. E. D.

c hyp. Secundo, ob *c* $\frac{Bq}{Eq}$, *b* hoc est $\frac{AC}{DF}$ datam, & *c* $\frac{A}{D}$
d 68. d. datam, *d* datur $\frac{C}{F}$. Q. E. D.

PROP. LXXXII.

A. B :: D. E.
B. C :: E. F.

Si quatuor rectæ proportionales fuerint (A.B :: D, E) *erit ut prima* A *ad eam* C, *ad quam secunda* B *rationem habet datam, ita tertia* D *ad eam* F, *ad quam quarta* E *rationem habet datam.*

Nam quia B. C :: *a* E. F. & *a* $\frac{B}{C}$ data est; *b* erit $\frac{E}{F}$ data, atqui ex æquali A. C :: D. F. ergo, &c. a *hyp.* b 2. *def. d.*

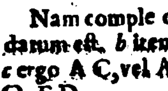
PROP. LXXXIII.

A. B. C. D.
F. E.

Si quatuor rectæ A, B, C, D *ita ad invicem se habeant, ut tribus ex iis, quibuscunque sumptis* A, B, C, *& quarta ipsis proportionali accepta* E, *ad quam reliqua* D *ex quatuor rectis proportionem habet datam; erit ut quarta* D *ad tertiam* C, *ita secunda* B *ad eam* F, *ad quam habet prima* A *rationem datam.*

Nam A E *a* = B C *b* = D F. & datur *b* $\frac{D}{E}$ *c* hoc est $\frac{AD}{AE}$, *d* vel $\frac{AD}{DF}$, *e* vel $\frac{A}{F}$. ergo, &c. a 16. 6. b *hyp.* c 1. 6. d 7. 5.

PROP. LXXXIV.



Si duæ rectæ A B, A C *datum spatium comprehendant in angulo* A *dato; sit autem altera* A B *altera* A C *major data* DB; *etiam unaquæque ipsarum* AB, AC *data erit.*

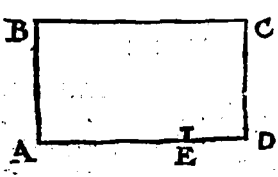
Nam comple quadratum A E. *a* Hoc specie datum est. *b* item pgr. CB, & recta DB datur. *c* ergo A C, vel AD, & tota *d* proinde AB datur. Q. E D. a 3. *def. d.* b *hyp.* c 59. *dat.* q 3. *dat.*

PROP. LXXXV.

Si duæ rectæ BD, DE *datum spatium comprehendant in angulo* BDE *dato, sit autem simul utraq;* (BD+DE) *data; & earum quoque unaquaque* BD, *&* DE *data erit.*

Nam sume DA=DE, & comple quad. DC. Hoc specie datur; item pgr. BE, & recta BA
a hyp. *a* dantur. *b* ergo AD (DE) & *c* reliqua DB dan-
b 58. d. tur. Q. E. D.
c 4. d.

PROP. LXXXVI.



Si duæ rectæ AB, AD *datum spatium* BD *comprehendant in angulo dato; quadratum autem unius* AD *quadrato alterius* AB *majus sit dato quam in ratione* (nempe ut sit
* 2. 2. AD×AB datum, & * reliqui AD×ED ad ABq ratio data;) *& utraque ipsarum* AB, AD *data erit.*

a hyp. Nam ob BD, & DA × AE *a* data, *b* datur
b 1. d. $\frac{BD}{DA \times AE}$. *c* ergo $\frac{AB}{AE}$ *d* ideoque $\frac{ABq}{AEq}$ datur. *e* item
c 69. d.
d 51. d. $\frac{ABq}{AD \times ED}$ datur. *f* ergo $\frac{AEq}{AD \times ED}$ ideoque $\frac{ABq}{4 AD \times ED}$,
e hyp.
f 8. d. *g* & $\frac{AEq}{4 AD \times ED + AEq}$ *b* hoc est $\frac{ABq}{Q{:}AD+ED}$ datur.
g 6. d.
h 8. 2.
k 54. d. *k* ergo $\frac{AB}{AD \times ED}$; & *l* componendo $\frac{AE}{2 AD}$, * ideoq;
l 6. d.
* 8. d. $\frac{AE}{AD}$, *m* hoc est $\frac{AEq}{AD \times AE}$ datur. denique igitur ob
m 1. 6.
n 2. d. datum AD×AE, *n* erit AEq data. *o* ergo AE,
o 55. d. & *p* proinde AD, ac AB datæ sunt. Q. E. D.
p 57. d.

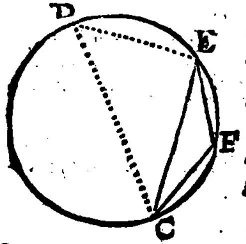
PROP. LXXXVII.

Si duæ rectæ AB, AD *datum spatium comprehendant in angulo dato, quadratum autem unius* AD *quadrato alterius* AB *majus sit dato* (AD×AE;) *earum utraque* AB, AD *data erit.*

Nam ob $\frac{BA \times AE}{BD}$ *a* datum, *b* erit $\frac{AE}{AB}$, ideoque $\frac{AEq}{ABq}$, *c* hoc est $\frac{AEq}{AD \times ED}$, *d* ac idcirco $\frac{AEq}{AEq + 4AD \times ED}$, *e* hoc est $\frac{AEq}{Q{:}AD+ED}$, ac proinde $\frac{AE}{AD+ED}$, & *d* componendo $\frac{AE}{2AD}$, *e* ac ideo $\frac{AE}{AD}$, *e* hoc est $\frac{AEq}{AD \times AE}$ data. ergo ob AD×AE *f* datum, dantur *g* AEq, & *h* AE, ac *k* ideo AD, ac AB. Q. E. D.

a 2. *d.*
b 69. *d.*
c *hyp.* & 1. 2.
d 8. & 6. *d.*
e 8. 2.
d 6. *d.*
e 1. 6.
f *hyp.*
g 2. *d.*
h 55. *d.*
k 57. *d.*

PROP. LXXXVIII.



Si in circulum CFED *magnitudine datum acta sit recta linea* CE, *quæ segmentum auferat, quod datum angulum* F *comprehendat; acta recta linea* CE *magnitudine data est.*

Nam ducatur diameter CD; & connectatur ED. Acob ang. F *a* datum, *b* erit ang. D (reliquus è 2 rectis) datus. item rectus CED datur. *c* quare $\frac{CE}{CD}$ datur. ergo ob *d* datam CD, *e* erit CE data. Q. E. D.

a *hyp.*
b 4 *d.*
c 40. *d.*
d *hyp* & 5. *def. d.*
e 2. *d.*

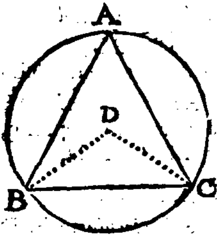
PROP. LXXXIX.

Si in datum magnitudine circulum CFED *data magnitudine recta* CE *acta fuerit, auferet segmentum quod angulum* (GFB) *datum comprehendet.*

Nam (in fig. praecedentis) quia $\frac{CE}{CD}$, & ang. CED dantur, *a* erit ang. D datus. *b* ergo ang. F *c* (1 Rect. — D) datus erit. Q. E. D.

a 43. *dat.*
b 4. *dat.*
c 22. 3.

PROP. XC.



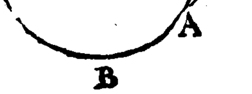
Si in circuli positione dati circumferentia BAC *datum fuerit punctum* B, *ab eo autem puncto* B *ad circumferentiam circuli inflexa fuerit recta* BAC *quae datum angulum* A *efficiat; inflexa rectae altera extremitas* C *data erit*

a 1. 3.
b 2. *dat.*
c 20. 3.
d 26. *dat.*
e 29. *dat.*
f *sch.* 25. *d.*

Ad *a* centrum D duc BD, & CD; *b* datusque est ang. D dati A *c* duplus, quare ob B D *d* datam, *e* erit DC data. *f* ergo punctum C datum est. Q. E. D.

Si ang. A obtusus fuerit; sume reliquum è 2 rectis acutum; ejus subsidio punctum C invenies, juxta dicta.

PROP. XCI.



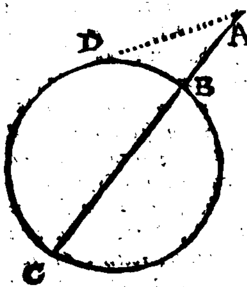
Si à dato puncto G *acta fuerit recta* G A, *quae datum positione circulum* B E A *contingat; acta linea* G A *positione & magnitudine data est.*

Nam centrum D & punctum G

G connectat recta D G. super qua descriptus sit semicirculus D A G circulo priori occurrens in a 31.3.
A. Ob ang. DAG *a* rectum, GA circulum *b* tangit. *c* ergo G A situ & magnitudine datur. Q. E. D. b cor. 16.3. c 26. dat.

Hinc modus dicitur à dato puncto tangentem ducendi, eo nonnunquam expeditior qui habetur ad 17.3.

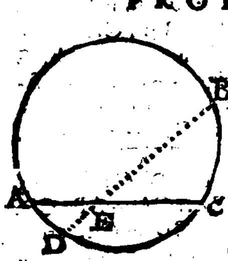
PROP. XCII.



Si extra circulum positione datam BCD *accipiatur aliquod punctum* A, *à dato autem puncto* A *in circulum producatur quadam recta* A C; *datum est id quod sub acta linea* AC, *&* *ea* AB, *quæ inter punctum* A *&* *convexam peripheriam* B *comprehenditur rectangulum* CAB.

a Nam duc tangentem A D, *b* eritque ADq (hoc est C A x A B) datum. Q. E. D. a 91. dat. b 36.3.

PROP. XCIII.

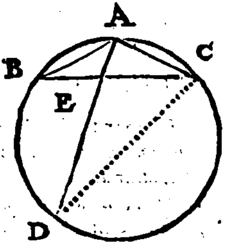


Si intra datum positione circulum A B C D *sumatur aliquod punctum* E, *per punctum autem* E *agatur in circulum aliqua recta* A E C; *quod sub segmentis* A E, E C *acta recta linea comprehenditur rectangulum, datum est.*

Nam

Nam per E duc rectam DEB utcunque occur-
a 35. 1. rentem circulo in B, & D. estque rectang. DEB
b 1. def. d. = *a* AEC. *b* ergo ABC datur. Q. E. D.

PROP. XCIV.



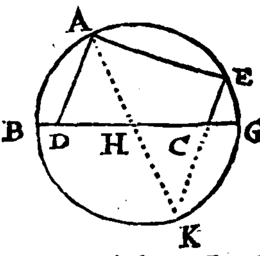
Si in circulum BACD *magnitudine datum agatur recta linea* BC, *quæ segmentum auferat, quod angulum* BAC *datum comprehendat; angulum autem* BAC, *qui in segmento consistit, bifariam secetur; simul utraque rectarum* BA, AC *quæ angulum datum* BAC *comprehendunt, ad lineam* AD, *quæ angulum bifariam secat, habebit rationem datam & quod sub simul utrisque* BA, AC, *quæ datum angulum* BAC *comprehendunt, rectis; & inferno abscissa* (ED) *ab ea* AD, *quæ angulum* BAC *in circumferentia datum bifariam secat, rectangulum datum erit.*

a 88 dat. Duc CD; & primo ob angulos BAC, CAD
* 1. dat. datos, *a* dantur subtensæ BC, CD, * ideoque $\frac{CB}{BC}$
b 3. 6. datur. Cum igitur CA. AB :: *b* CE. EB, & per-
c 12. 5. mutando CA. CE :: AB. EB :: (CA + AB.
* 4. 6. CB ::) * AD. DC. (Nam * ob ang. BAE
d 2. def. d. = CAD; & D = B; trigona ABE, ADC similia sunt) ac rursus permutando CA + AB. AD :: CB. DC. *d* erit $\frac{CA + AB}{AD}$ data. Q. E. D.

e 21. 3. Secundo, ob triangula AEB, DEC *e* similia;
1 4. 6. *b* erit CD, DE :: AB. BE *c* :: CA + AB. CB.
c prius. *d* ergo CA + AB in DE = CD in CB. atqui
d 16. 6. CD × CB *e* datur. *f* ergo CA + AB in DE da-
e 52. dat. tum est. Q. E. D.
f 1. def. d.

PROP.

PROP. XCV.



Si in circuli BAG *positione dati diametro* BG *sumatur datum punctum* D; *à puncto autem* D *in circulum producatur quædam recta* DA, *& agatur à sectione* A *ad rectos angulos in productam rectam* DA *linea* AE; *per punctum autem* E, *in quo linea* AE, *quæ ad rectos angulos consistit, occurrit circumferentiæ circuli, agatur parallela* (BCK) *producta rectæ* DA; *datum est illud punctum* C, *in quo parallela* EK *occurrit ipsi diametro* BG; *& quod sub parallelis lineis* AD, EC *comprehenditur rectangulum, datum est.* a 31. 3.

Nam connectatur AK. *a* estque AK (ob angulum E, vel DAE rectum) diameter. ergo interfectio H est centrum. *b* ergo DH datur. Atqui ob $KH . HA$ *c* $::$ $CH . HD$, *d* est $CH = HD$. *e* ergo CH datur. *f* ergo punctum C datur. Q. E. D. *g* ergo $KC \times CE$, hoc est *d* $AD \times CE$ datur. Q. E. D.

b 26. d.
c 4. 6.
d 9. 5.
e 1. def. d.
f 27. d.
g 93. dat.

F I N I S.