ELEMENTORVM EUCLIDIS

Libri XV.

Breviter ac ſuccinctè demonſtrati

Operâ & Studio

Mri Js. BARROW,

Cantabrigienſis,

COLL. TRIN. SOC.

Nunc viciſſim reviſi, & à cunctis erroribus expurgati.

HIEROCL.

Καθαρμοὶ ψυχῆς λογικῆς εἰσιν αἱ μαθηματικαὶ ἐπιστῆμαι.

OSNABRUGI

Apud Johan. Georg. Schwänderum. *Anno* 1676.

Præfatio.

BEnevoli & Candidi Lectores! Hanc noviſſimam, Euclidis reliquorum Mathematicorum facilè Principis, Elementorum editionem, publicis literarum typis impreſſam, prioribus, ſicut mihi ſpes eſt, editionibus longè emendatiorem & correctiorē ſincerâ fronte accipite. Namque extra omne dubium eſt, quod hicce Autor, adhibitâ diligentiſſimâ reviſione & operâ M^{ri}. *Joannis Barrow Cantabrigienſis*, cæteris omnibus tam theoreticis quàm practicis ſit præferendus; ſiquidem non tantum cuilibet geometræ hanc perſcrutari ſcientiam incumbit, verùm etiam nemo non qui in arte fortificatoria, rebusq́ue militaribus verſatur, per hæc ſatis perſpicua elementa, tanquam per aliquod compendiũ, ad optatam metam, ſummasq́ue dignitates, quaſi per alatum Pegaſum ex cruore Meduſæ cretum aſpirare poteſt, ut proinde non tantum ſibi, ſed & proximo & ſuæ Patriæ conſulere queat. Poſtea verò quam animadverti, plurimùm à Mathematicis huncce libellum deſiderari, cujus exemplaria, etiamſi plus ſemel impreſſa, tamen omnia avidiſſimè empta, jam pridem diſtracta ſunt, non potui non gratificari illis, & deſiderata jamdiu exemplaria, prælis meis recuſa, ſtudioſis rurſus affatim ſuppeditare. Nam di-

dignus ſanè eſt Autor qui
crebrò verſetur manibus.
Idcirco, integerrime Lector, velim, ut hanc noſtram bene de te merendi
propenſam voluntatem
æqui conſulas, atque favore tuo proſequâre.

Vossius de scientiis Mathematicis p. 54.
Euclides elementa sua Geometrica commode in
quatuor composuit partes, 1 ἐπιπεδομετρία
planorum doctrina, quae 6 libris absolvitur, quatuor
primis agit de planis absolutè, deinde proportionibus
comparatè, s. de proportionibus, in genere quidem lib. V.
particulariter vero lib. VI. 2 pars est de numerorum
affectionibus lib. VII. IIX, IX, quae Arithmetica vocantur.
3 pars est de lineis συμμέτροις et
ἀσυμμέτροις, unde haec pars συμμετρία vocatur,
in libro X continetur. 4~~ta~~ est στερεομετρία
s. de solidis, quae quinque libris reliquis absolvitur.

Licet

Licet maximâ cum induſtriâ & labore libellus hicce Euclidis correctus atque diligentiſſimè reviſus ſit, nihilo tamen minus pauci ſequentes typographici errores præter ſpem irrepſére, quos, ut *Candidus Lector* confeſtim agnoſcere poſſit, neque per hos in alium ſenſum vel dubium ſeducatur, placuit ſimul ſubjungere. legatur igitur

*Pag.*11.*lin.*6.*in fin.* B C, *pag.*19.*figur. priori deleantur duo a. pag.*29. *lin.*2. *poſt parentheſin leg.* æquales ſint duobus angulis (aut ſingulis aut ſimul) *pag.*39.*lin.*19. *in princ. leg.* & AB *b* = &c. *pag.*50. *lin.*1. *in princ. leg.* quodque, *&* *lin.*26.*ad finem pro* C B, *leg.* D B. *pag.*55. *lin.*6. *leg.* quorum. *pag.*74.*lin.* 25. *leg.* B, E conſtituti &c. *pag.*77.*lin.*19.*in princ.leg.* = E B C. *pag.*79.*lin.*12.*leg.* Q.E.F. *& lin.*20. *pro* rectum *leg.* datum. *pag.* 80. *lin.* 19. *poſt Rectang. leg.* AEB. *pag.*91.*lin.*1.*pro* tanque *leg.*tangetque. *pag.* 93. *lin.* 2.*in princ.leg.*utrumque. *pag.*94. *lin.pen.leg.* ABCDE. *p.*95.*l.*5. *pro* AFA *leg.* GFA. *& lin.* 19.*pro* etiam *leg.*aliam. *p.*103.*l.*10. *in princ.leg.* C – D. *p.*138.*l.*20.*pro* AA *leg.*AB. *p.*140.*l.*24. *poſt* ECD, *leg.* & totus GHB = E F D. Polyg. &c. *p.*155.*l.*14.*in princ.leg.*partes. *p.*182. *l.*26. *ante verb.* ab unitate *inſeratur* : ultimos eſſe totidem continuè proportionalium. *p.* 183. *l.* 13. *pro* æquale *leg.* æquè. *p.*210. *l.*25.*ante m leg.*D. *p.*218. *poſt.lin.* 19. *ponatur* PROP. IX. *p.*258. *addantur figuræ literæ quæ deſunt. p.* 277. *l.* 11. *pro* hot *leg.*hoc. *p.*281.*l.*11.*leg.*apot. *p.*287.*l.*23 *in princ.leg.* CH : : DC.&c. *p.*292.*l.*5.*pro* alius *leg.*illius. *p.*296.*l.*3.*pro* fuit *leg.*ſunt. *p.*303.*l.*18. *pro* HDI *leg.* HEI. *p.* 321. *leg.* PROP. XL. *p.* 329.*l.*10.*leg.*PROP. *p.*364. *l.*24.*leg.* Tetraedri. *& lin.*25.*leg.*Octaedri.

No-

Notarum Explicatio.

⊏ Commenſurabilis
ʿ⊏ Incommenſurabilis
⊏̦ Commenſurabilis potentia
ʿ⊏̦ Incommenſurabilis potentia.
:: Ejusdem rationis.
∺ Continue proportionales.
= Æqualitatem
⊏— Majoritatem
—⊐ Minoritatem
+ Plus, vel addendum eſſe
− Minus, vel ſubtrahendum eſſe
−: Differentiá vel exceſſum; item quantitates omnes, quæ ſequuntur, ſubtrahendas eſſe, ſignis non mutatis.
x Multiplicationem, vel ductum lateris rectanguli in aliud latus
Idem denotat conjunctio literarum, ut AB = AxB.
√ Latus, vel radicem quadrati, vel cubi, &c.
Q. & q quadratum. C. & c cubum.
Q. Q. Rationem quadrati numeri ad quadratum numerum.

ſignificat.

Reliquas, quæ ubicunque occurrunt, vocabulorum abbreviationes ipſe Lector per ſe facile intelliget; exceptis iis, quas tanquam minus generalis uſus, ſuis locis explicandas relinquimus.

LIB.

LIB. I.

DEFINITIONES.

I. PUnctum eſt cujus pars nulla eſt.

II. Linea verò longitudo latitudinis expers.

III. Lineæ autem termini ſunt puncta.

IV. Recta Linea eſt, quæ ex æquo ſua interjacet puncta.

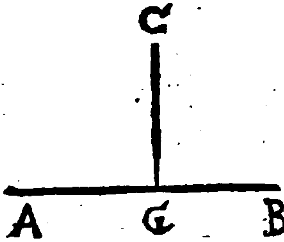
V. Superficies eſt, quæ longitudinem, latitudinemque tantum habet.

VI. Superficiei autem extrema ſunt lineæ.

VII. Plana ſuperficies eſt, quæ ex æquo ſuas interjacet lineas.

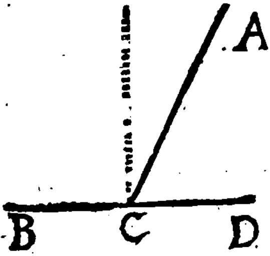
VIII. Planus verò angulus eſt, duarum linearum in plano ſe mutuo tangentium, & non in directum jacentium alterius ad alteram inclinatio.

IX. Cum autem quæ angulum continent, lineæ, rectæ fuerint, rectilineus ille angulus appellatur.



X. Cum vero recta linea C G ſuper rectam lineam A B conſiſtens, eos qui ſunt deinceps angulos CGA, CGB æquales inter ſe fecerit, rectus eſt uterq; æqualium angulorum, & quæ inſiſtit recta linea C G, perpendicularis vocatur ejus (A B) cui inſiſtit.

Not. *Cum plures anguli ad unum punctum (ut ad G) exſiſtunt, deſignatur quilibet angulus* *tribus literis, quarum media ad verticem eſt illius de quo agitur: ut angulus quem recta* CG, AG *efficiunt ad partes* A *vocatur* CGA, *vel* AGC.



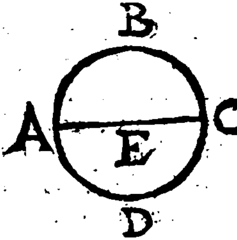
XI. Obtuſus angulus eſt, qui recto major eſt, ut ACB.

XII. Acutus verò, qui minor eſt recto, ut ACD.

XIII. Terminus eſt, quod alicujus extremum eſt.

XIV. Figura eſt, quæ ſub aliquo, vel aliquibus terminis comprehenditur.

XV. Circulus eſt figura plana, ſub una linea comprehenſa, quæ peripheria appellatur, ad quam ab uno puncto eorum, quæ intra figuram ſunt poſita, cadentes omnes rectæ lineæ inter ſe ſunt æquales.



XVI. Hoc vero punctum centrum circuli appellatur.

XVII. Diameter autem circuli eſt recta quędam linea per centrum ducta, & ex utraque parte in circuli peripheriam terminata, quæ circulum bifariam ſecat.

XIIX. Semicirculus verò eſt figura, quæ continetur ſub diametro, & ſub ea linea, quæ de circuli peripheria auſertur.

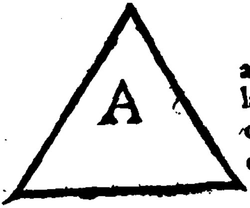
In circulo EABCD. E *eſt centrum*, AC *diameter*; ABC *ſemicirculus*.

XIX. Rectilineæ figuræ ſunt, quæ ſub rectis lineis continentur.

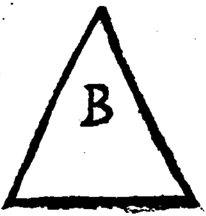
XX. Trilateræ quidem, quæ ſub tribus.

XXI. Quadrilateræ verò, quæ ſub quatuor.

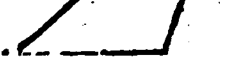
XXII. Multilateræ autem, quæ ſub pluribus, quam quatuor rectis lineis comprehenduntur.



XXIII. Trilaterarum autem figurarum, æquilaterum eſt triangulum, quod tria latera habet æqualia, ut triangulum A.

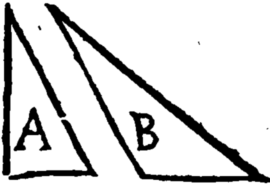


XXIV. Iſoſceles autem, quod duo tantum æqualia habet latera, ut triangulum B.



XXV. Scalenum verò, quod tria inæqualia habet latera, ut C.

XXVI. Adhæc etiam trilaterarum figurarum,

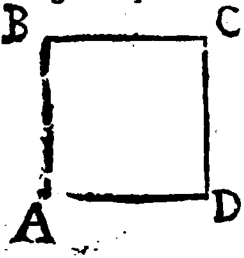


rum, rectangulum quidem triangulum eſt, quod rectum angulum habet, ut triangulum A.

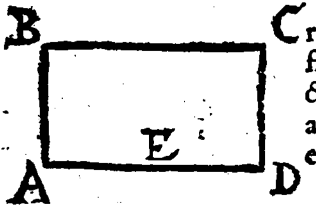
XXVII. Amblygonium autem, quod obtuſum angulum habet, ut B.

XXVIII. Oxygonium verò, quod tres habet acutos angulos, ut C.

Figura æquiangula eſt, cujus omnes anguli inter ſe æquales ſunt. Duæ verò figurę æquiangulæ ſunt; ſi ſinguli anguli unius ſingulis angulis alterius ſint æquales. Similiter de figuris æquilateris concipe.

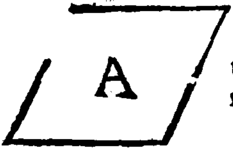


XXIX. Quadrilaterarum autem figurarum, quadratum quidem eſt, quod & æquilaterum, & rectangulum eſt, ut ABCD.

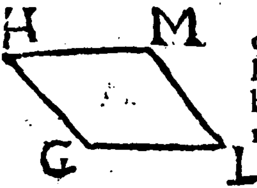


XXX. Altera vero parte longior figura eſt, quæ rectangula quidem, at æquilatera non eſt, ut ABCD.

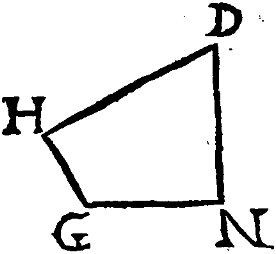
XXXI.



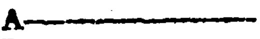
XXXI. Rhombus autem, quæ æquilatera, ſed rectangula non eſt, ut A.



XXXII. Rhomboydes vero, quæ adverſa & latera, & angulos habens inter ſe æquales, neque ęquilatera eſt, neque rectangula, ut G L, M H.

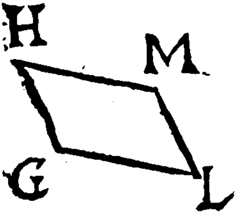


XXXIII. Præter has autem reliquæ quadrilateræ figuræ trapezia appellentur ut G N D H.

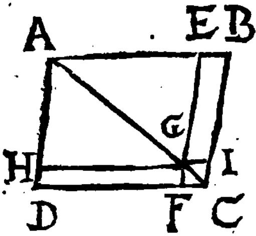


XXXIV. Parallelæ rectæ lineæ ſunt, quæ cum in eodem ſint plano, & ex utraque parte in infinitum producantur, in neutram ſibi mutuo incidunt, ut A, & B.

XXXV. Parallelogrammum eſt figura quadridrilatera, cujus bina oppoſita latera ſunt parallela, ſeu æquidiſtantia, ut G L H M.



XXXVI. Cum vero in parallelogrammo A B C D diameter A C ducta fuerit, duæque lineæ E F, H I, lateribus parallelæ ſecantes diametrum in uno eodemque puncto G, ita ut parallelogrammum ab hisce parallelis in quatuor diſtribuatur parallelogramma; appellantur duo illa D G, G B, per quæ diameter non tranſit, Complementa; duo vero reliqua H E, F I, per quæ diameter incedit, circa diametrum conſiſtere dicuntur.



Problema eſt, *cum proponitur aliquid efficiendum.*

Theorema eſt, *cum proponitur aliquid demonſtrandum.*

Corollarium eſt *conſectarium, quod è facta demonſtratione tanquam lucrum aliquod colligitur.*

Lemma eſt *demonſtratio præmiſſæ alicujus, ut demonſtratio quæſiti evadat brevior.*

Po-

Poſtulata.

1. POſtuletur, ut à quovis puncto ad quodvis punctum rectam lineam ducere, concedatur.

2. Et rectam lineam terminatam in continuum recta producere.

3. Item, quovis centro, & intervallo circulum deſcribere.

Axiomata.

1. QUæ eidem æqualia, & inter ſe ſunt æqualia.

ut $A = B = C$. ergo $A = C$, vel ergo omnes A, B, C, æquantur inter ſe.

Nota, *Cum plures quantitates hoc modo conjunctas invenias, vi hujus axiomatis primam ultimæ & quamlibet earum cuilibet æquari. Quo in caſu ſæpe, brevitatis cauſa, ab hoc axiomate citando abſtinemus; etſi vis conſecutionis ab eo pendeat.*

2. Et ſi æqualibus æqualia adjecta ſunt, tota ſunt æqualia.

3. Et ſi ab æqualibus æqualia ablata ſunt, quæ relinquuntur ſunt æqualia.

4. Et ſi inæqualibus æqualia adjecta ſint, to- ſunt inæqualia.

5. Et ſi ab inæqualibus æqualia ablata ſint, reliqua ſunt inæqualia.

6. Et quæ ejusdem vel æqualium ſunt duplicia, inter ſe ſunt æqualia. Idem puta de triplicibus, quadruplicibus, &c.

7. Et quæ ejusdem, vel æqualium ſunt dimidia,

midia, inter ſe ſunt æqualia. Idem concipe de ſubtriplis, ſubquadruplis, &c.

8. Et quæ ſibi mutuò congruunt, ea inter ſe ſunt æqualia.

Hoc axioma in rectis lineis, & angulis valet converſum, ſed non in figuris, niſi illæ ſimiles fuerint.

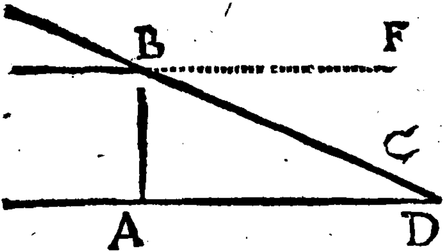
Cæterum, magnitudines congruere dicuntur, quarum partes applicatæ partibus, æqualem vel eundem locum occupant.

9. Et totum ſua parte majus eſt.

10. Duæ rectæ lineæ non habent unum & idem ſegmentum commune.

11. Duæ rectæ in uno puncto concurrentes, ſi producantur ambæ, neceſſario ſe mutuo in eo puncto interſecabunt.

12. Item omnes anguli recti ſunt inter ſe æquales.



13. Et ſi in duas rectas lineas AD, CB, altera recta B A incidens, internos ad eaſdemq; partes, angulos BAD, ABC duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ ſibi mutuò incident ad eas partes, ubi ſunt anguli duobus rectis minores.

14. Duę rectę lineæ ſpatium non comprehendunt.

15. Si æqualibus inęqualia adjiciantur, erit totorum exceſſus adjunctorum exceſſui æqualis.

16. Si inæqualibus æqualia adjungantur, erit totorum exceſſus exceſſui eorum, quæ à principio, æqualis.

17. Si ab æqualibus inæqualia demantur, erit reſiduorum exceſſus, exceſſui ablatorum æqualis.

18. Si ab inæqualibus ęqualia demantur, erit reſiduorum exceſſus exceſſui totorum ęqualis.

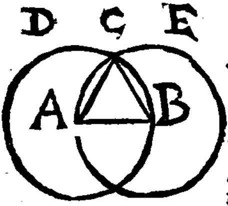
19. Omne totum æquale eſt omnibus ſuis partibus ſimul ſumptis.

20. Si totum totius eſt duplum, & ablatum ablati, erit & reliquum reliqui duplum. Idem de reliquis multiplicibus intellige.

Citationes intellige ſic. Cum duo numeri occurrunt, prior deſignat propoſitionem, poſterior librum. Ut per 4. 1. intelligitur quarta propoſitio primi libri, atque ita de reliquis. *Cæterum, ax. axioma, poſt. poſtulatum, def. definitionem, ſch. ſcholium. cor. corollarium denotant, &c.*

LIB. I.

PROP. I.



Super data recta linea terminata AB, *triangulum æquilaterum* ABC *conſtituere.*

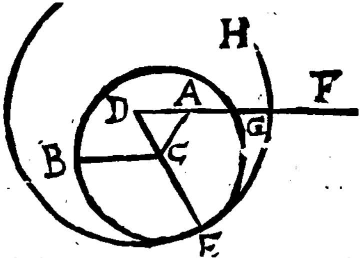
a 3. *poſt.*
b 1. *poſt.*
c 15. *def.*
d 1. *ax.*
e 23. *def.*

Centris A & B, eodem intervallo A B, vel B A, *a* deſcribe duos circulos ſe interſecantes in puncto C, ex quo *b* duc rectas CA, CB. Erit AC *c* = AB *c* = BC *d* = AC. *e* Quare triangulum ACB eſt æquilaterum. Quod erat faciendum.

Scholium.

Eodem modo ſuper A B deſcribetur triangulum Iſoſceles, ſi intervalla æqualium circulorum majora ſumantur, vel minora, quam AB.

PROP. II.



Ad datum punctum A *datæ rectæ lineæ* BC. *æqualem rectam lineam* AG *ponere.*

a 3. *poſt.*
b 1. *poſt.*
c 1. 1.
d 2. *poſt.*

Centro C, intervallo CB *a* deſcribe circulum CBE. *b* Junge AC, ſuper qua *c* fac triangulum æquilaterum ADC *d* produc DC ad E cen-

E. centro D, ſpatio D E, deſcribe circulum D
E H: cujus circumferentiæ occurrat D A *e* e 2. *poſt.*
protracta ad G. Erit AG = CB. f 15. *def.*

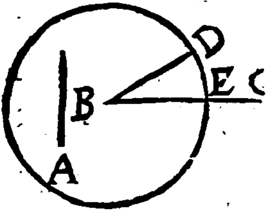
Nam DG *f* = D E, & DA *g* = D C, quare g *conſtr.* h 3. *ax.*
AG *h* = C E *k* = BC *l* = AG. Q.E.F. k 15. *def.*

Poſitio puncti A, intra vel extra datam BC, l 1. *ax.*
caſus variat, ſed ubique ſimilis eſt conſtructio, & demonſtratio.

Scholium.

Poterat A G circino ſumi, ſed hoc facere nulli poſtulato reſpondet, ut benè innuit Proclus.

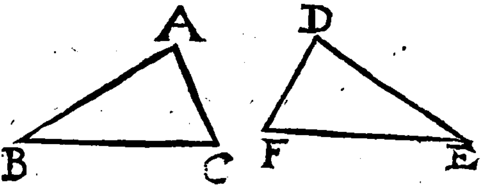
PROP. III.



Duabus datis rectis lineis A, *&* B C, *de majore* B C *minori* A *æqualem rectam lineã* BE *detrahere.*

Ad punctum B *a* pone rectam BD=A. a 2. 1.
Circulus centro B, ſpatio BD deſcriptus auferet BE *b* = BD *c* = A *d* = BE. Q.E.F. b 15. *def.* c *conſtr.* d 1. *ax.*

PROP. IV.



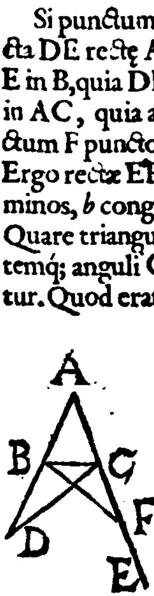
Si duo triangula BAC, EDF *duo latera* BA, AC *duobus lateribus* ED, DF *æqualia habeant,*

utrumq;

utrumque utrique (hoc eſt BA = ED, *&* AC = DF) *habeant vero angulum* A, *angulo* D *æqualem, ſub æqualibus rectis lineis contentum, & baſin* BC *baſi* EF *æqualem habebunt; eritq; triangulum* BAC *triangulo* EDF *æquale, ac reliqui anguli* B,C *reliquis angulis* E,F *æquales erunt, uterque utrique, ſub quibus æqualia latera ſubtenduntur.*

Si punctum D puncto A applicetur, & recta DE rectę AB ſuperponatur, cadet punctum
E in B, quia DF *a* = AB. Item recta DF cadet
a hyp. in AC, quia ang. A *a* = D. Quin etiam punctum F puncto C coincidet, quia AC *a* = DF.
Ergo rectæ EF, BC, cum eoſdem habeant ter-
b 14. ax. minos, *b* congruent, & proinde æquales ſunt.
Quare triangula BAC, EDF; & anguli B,E; itemq; anguli C,F etiam congruunt, & ęquantur. Quod erat demonſtrandum.

PROP. V.



Iſoſcelium triangulorum ABC *qui ad baſim ſunt anguli* ABC, ACB *inter ſe ſunt æquales. Et productis æqualibus rectis lineis* AB, AC *qui ſub baſe ſunt anguli* CBD, BCE *inter ſe æquales erunt.*

a 3. 1. *a* Accipe AF = AD, & *b* junge CD, ac BF.

b 1. poſt.
c hyp. Quoniam in triangulis ACD, ABF, ſunt AB
d conſtr. *c* = AC, & AF *d* = AD, anguluſq; A commu-
e 4. 1. nis, *e* erit ang. ABF = ACD; & ang. AFB *e* = A
f 3. ax. DC, & bas. BF *e* = DC; item FC *f* = DB, ergo
in

in triangulis BFC, BDC *g* erit ang. FCB, = DBC. Q.E.D. Item ideo ang. FBC = DCB. atqui ang. ABF *h* = ACD. ergo ang. ABC *k* = ACB. Q.E.D. g 4. 1. h pr. k 3. ax.

Corollarium.

Hinc, Omne triangulum æquilaterum eſt quoque æquiangulum.

PROP. VI.

Si trianguli A B C *duo anguli* ABC, ACB *æquales inter ſe fuerint, & ſub æqualibus angulis ſubtenſa latera* AB, AC *æqualia inter ſe erunt.*

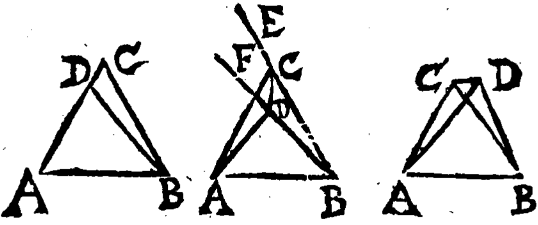
Si fieri poteſt, ſit utravis BA $\sqsubset$ CA, *a* Fac igitur BD = CA, & *b* duc CD. a 3. 1. b 1. poſt.

In triangulis DBC, ACB, quia BD *c* = CA, & latus BC commune eſt, atque ang. DBC *d* = ACB, *e* erunt triangula, DBC, ACB ęqualia inter ſe, pars & totum, *f* Quod Fieri Nequit. c ſuppoſ. d hyp. e 4. 1. f 9. ax.

Coroll.

Hinc, Omne triangulum æquiangulum eſt quoque æquilaterum.

PROP. VII.



Super eadem recta linea A B *duabus eisdem rectis lineis* AC, BC, *aliæ duæ rectæ lineæ æquales* AD , BD , *utraque utrique* (*hoc eſt* , AD= AC, & BD = BC) *non conſtituentur ad aliud punctum* C, *atque aliud* D, *ad easdem partes* C, *eosdemq; terminos* A,B *cum duabus initio ductis rectis lineis habentes.*

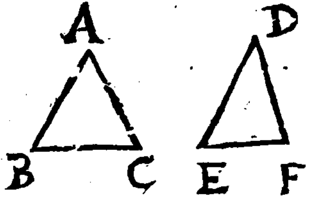
a 9. ax. 1. *Caſ.* Si punctum D ſtatuatur in AC *a* liquet non eſſe AD = AC.

2. *Caſ.* Si punctum D dicatur intra triangulum ACB, duc CD , & produc BDF, ac BC
b 5. 1. E. Jam vis AD=AC. ergo ang. ADC *b* = AC
c ſuppoſ. D ; item quia B D *c* = BC , erit ang. FDC *b* =
d 9. ax. ECD. ergo ang. FDC *d* ⊏ ACD, id eſt ang. F DC ⊏ ADC *d* Q.F.N.

3. *Caſ.* Sin D cadat extra triangulum ACB, jungatur CD.

e 5. 1. Rurſus ang. ACD *e* = ADC, & ang. BCD *e*
f 9. ax. = BDC. *f.* ergo ang. ACD ⊏ BDC. id eſt ang. ADC ⊏ BDC. Q.F.N.

PROP. VIII.



Si duo triangula ABC, DEF *habuerint duo latera* AB, AC *duobus lateribus* DE,DF, *utrumque utrique æqualia; habuerint verò & baſim* BC , *baſi* EF, *æqualem : angulum* A *ſub æqualibus rectis lineis contentum angulo* D *æqualem habebunt.*

Quia

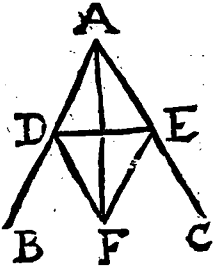
Quia BC *a* = EF, ſi baſis BC ſuperponatur a *hyp.*
baſi EF, illæ *b* congruent. ergo, cum AB *c* = DE, b *8. ax.* c *hyp.*
& AC *c* = DF, cadet punctum A in D. (nam in aliud punctum cadere nequit, per pręcedentem) ergo angulorum A, & D latera coincidunt. *d* quare anguli illi pares ſunt. Q.E.D. d *8. ax.*

Coroll.

1. Hinc triangula ſibi mutuo æquilatera, etiam mutuo *x* æquiangula ſunt. x *4. 1.*

2. Triangula ſibi mutuo æquilatera *y* æquentur inter ſe. y *4. 1.*

PROP. IX.



Datum angulum rectilineum BAC *bifariam ſecare.*

a Sume AD = AE; duc DE, ſuper qua *b* fac triang. æquilat. DFE. a *3. 1.* b *1. 1.*

Ducta AF angulum BAC biſecabit.

Nam AD *c* = AE, & c *conſt.*
& latus AF commune eſt, & baſ. DF *c* = FE. d *8. 1.*
d ergo ang. DAF = EAF. Q.E.F.

Coroll.

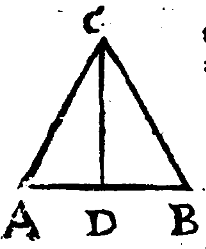
Hinc patet quomodo angulus ſecari poſſit in æquales partes 4,8,16,&c Singulas nimirum partes iterum biſecando.

Methodus verò regula & circino angulos ſecandi in ęquales quotcunque hactenus Geometras latuit.

PROP. X.

Datam rectam lineam AB *bifariam ſecare.*

Super

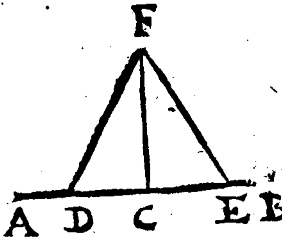


a 1. 1. Super data A B *a* fac triang. æquilat. ABC. ejus b 9. 1. angulum C *b* biſeca recta C D. Eadem datam A B biſecabit.

c conſtr. Nam A C *c* = B C, & latus C D eſt commune; d 4. 1. & ang. ACD *c* = BCD, *d* ergo AD = BD. Q.E.F.

Praxin hujus & præcedentis, conſtructio primæ hujus libri ſatis indicat.

PROP. XI.



Data recta linea AB, & puncto in eo dato C, rectam lineam CF ad angulos rectos excitare.

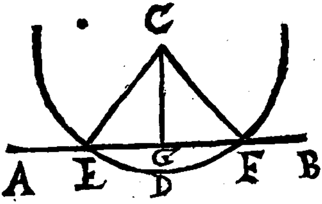
a 3. 1. *a* Accipe hinc inde C D = CE. Su- b 1. 1. per DE *b* fac triangul. æquilat. DFE. Ducta FC perpendicularis eſt.

c conſtr. Nam triangula DFC, EFC ſibi mutuo *c* æquilatera ſunt. d 8. 1. *d* ergo ang. DCF = ECF. *e* ergo e 10. def. FC perpendicularis eſt. Q.E.F.

Praxis tam hujus, quam ſequentis expeditur facillime ope normæ.

PROP. XII.

Super datam rectam lineam infinitam AB, à dato puncto C quod in ea non eſt, perpendicularem rectam CG deducere.



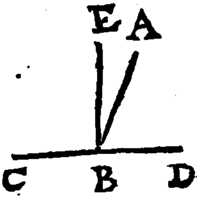
Centro C *a* deſcribe circulum, qui ſecet datam AB in punctis E & F; *b* biſeca EF in G. ducta CG perpendicularis eſt.

a 3.poſt.
b 10.1.

Ducantur enim CE, CF. Triangula EGC, FGC, ſibi mutuo *c* ęquilatera ſunt. *d* ergo anguli EGC, FGC, ęquales, & *e* proinde recti ſunt. Q.E.F.

c conſtr.
d 8.1.
e 10.def.

PROP. XIII.



Cum recta linea AB, *ſuper rectam lineam* CD *conſiſtens, facit angulos* ABC, ABD; *aut duos rectos, aut duobus rectis æquales efficiet.*

Si anguli ABC, ABD pares ſint, *a* liquet illos rectos eſſe; ſin inęquales ſint, ex B *b* excitetur perpendicularis BE. Quoniam ang. ABC *c* = Rect. + ABE; & ang. ABD *d* = Rect. − ABE; erit ABC + ABD *e* = 2. Rect. + ABE − ABE = 2. Rect. Q.E.D.

a 10.def.
b 11.1.
c 19.ax.
d 3.ax.
e 2.ax.

Coroll.

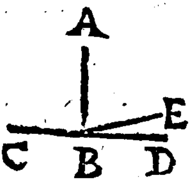
1. Hinc, ſi unus ang. ABD rectus ſit, alter ABC etiam rectus erit; ſi hic acutus, ille obtuſus erit, & contra.

2. Si plures rectę quam una ad idem punctum eidem rectę inſiſtant, anguli fient duobus rectis ęquales.

3. *Duę* rectę invicem ſecantes efficiunt angulos quatuor rectis æquales.

4. Omnes anguli circa unum punctum conſtituti conficiunt quatuor rectos, patet ex Coroll. 2.

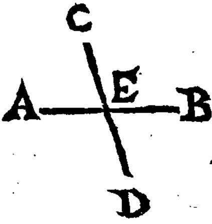
PROP. XIV.



Si ad aliquam rectam lineam A B, atque ad ejus punctum B duæ rectæ lineæ C B, B D non ad eaſdem partes ductæ, eos qui ſunt deinceps angulos A B C, A B D duobus rectis æquales fecerint, in directum erunt inter ſe ipſæ rectæ lineæ CB, BD.

Si negas, faciant CB, BE unam rectam. ergo
a 13. 1. ang. A B C + A B E *a* = 2. Rect. *b* = A B C
b hyp.
c 9. ax. + A B D. *c* Quod eſt abſurdum.

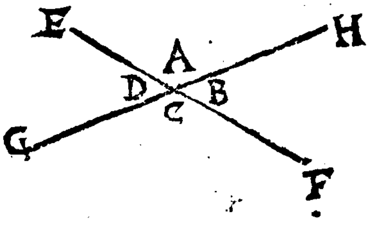
PROP. XV.



Si duæ rectæ lineæ A B, CD ſe mutuo ſecuerint, angulos ad verticem C E B, AED æquales inter ſe efficient.

Nam ang. A E C + C
a 13. 1. E B *a* = 2. Rect. *a* = A E
b 3. ax. C + AED. *b* Ergo CEB = AED. Q.E.F.

Schol.

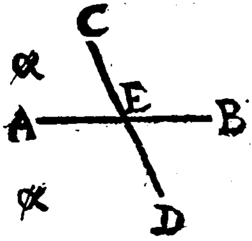


Si

Si ad aliquam rectam lineam G H, atque ad ejus punctum, A duæ rectæ lineæ E A, A F non ad easdem partes ſumptæ, angulos ad verticem D, & B ęquales fecerint, ipſæ rectæ lineæ E A, A F in directum ſibi invicem erunt.

Nam 2.Rect. = *a* D + A *a* = B + A *b* ergo EA, AF ſunt in directum ſibi invicem. Q.E.D. a 13.1. b 14.1.

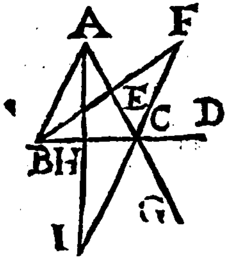
Schol. 2.



Si quatuor rectæ lineæ EA, EB, EC, ED ab uno puncto E exeuntes, angulos oppoſitos ad verticem æquales inter ſe fecerint, erunt quælibet duæ lineæ AE, EB, & CE, ED in directum poſitæ.

Nam quia ang. AEC + AED + CEB + DEB *a* = 4. Rect. erit AEC + AED *b* = CEB + DEB = 2.Rect. *c* ergo CED, & AEB ſunt rectæ lineæ. Q.E.D. a 1.Cor. 13.1. b 13.1. & 2. ax. c 14.1.

PROP. XVI.



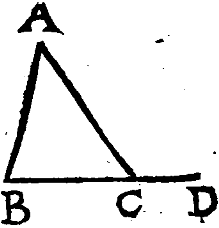
Cujuscunque Trianguli A B C uno latere B C producto, externus angulus A C D utrolibet interno & oppoſito CAB, CBA, major eſt.

Latera A C, BC *a* biſecent rectæ AH, BE, è quibus productis *b* cape E F = BE, *b* & HI = A H, Conjuganturque F C, IC & producatur ACG. a 10.1. & 1.poſt. b 3.1.

Quo-

c *conſtr.* Quoniam CE *c* = EA, & EF *c* = EB, & ang.
d 15.1. FEC *d* = BEA; *e* erit ang. ECF = EAB. Simi-
e 4.1. li argumento ang. ICH = ABH. ergo totus A
f 15.1. CD *f* (BCG) *g* major eſt utroque CAB, &
g 9. ax. ABC. Q.E.D.

PROP. XVII.



Cujuſcunq; trianguli ABC duo anguli duobus rectis ſunt minores, omnifariam ſumpti.

Producatur latus BC. Quoniam ang. ACD +
a 13.1. ACB *a* = 2. Rect. & ang.
b 16.1. ACD *b* $\sqsubset$ A, *c* erit A
c 4. ax. + ACB $\sqsupset$ 2. Rect. Eodem modo erit ang. B + ACB $\sqsupset$ 2. Rect. Denique producto latere AB, erit ſimiliter ang. A + B $\sqsupset$ 2. Rect. Que E.D.

Coroll.

1. Hinc, in omni triangulo, cujus unus angulus fuerit rectus, vel obtuſus, reliqui acuti ſunt.

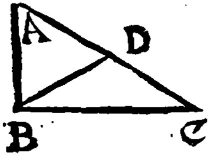
2. Si linea recta AE cum alia recta CD angulos inæquales faciat, unum AED acutum, & alterum AEC obtuſum, linea perpendicularis AD ex quovis ejus puncto A ad aliam illam CD demiſſa, cadet ad partes anguli acuti AED.

Nam ſi AC ad partes anguli obtuſi ducta, dicatur perpendicularis; in triangulo AFC erit

rit angul. AEC + ACE ⊏ 2. Rect. x Q. x 17. 1.
F.N.

3. Omnes anguli trianguli ęquilateri, & duo anguli trianguli Iſoſcelis, ſupra baſim, acuti ſunt.

PROP. XVIII.

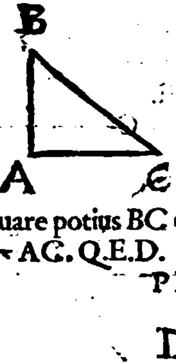


Omnis trianguli ABC majus latus AC majorem angulum ABC ſubtendit.

Ex AC *a* aufer AD = AB, & junge DB. *b*. ergo ang. ADB = ABD. Sed *c* ADB ⊏ C. ergo ABD ⊏ C. *d*. ergo totus ang. ABC ⊏ C. Eodem modo erit ABC ⊏ A. Q.E.D.

a 3. 1.
b 5. 1.
c 16. 1.
d 9. ax.

PROP. XIX.



Omnis trianguli ABC major angulus A majori lateri BC ſubtenditur.

Nam ſi dicatur AB = BC, *a* erit ang. A = C. contra Hypoth. & ſi AB ⊏ BC, *b* erit ang. C ⊏ A, contra Hyp. quare potius BC ⊏ AB. & eodem modo BC ⊏ AC. Q.E.D.

a 5. 1.
b 18. 1.

PROP. XX.

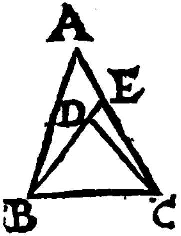
Omnis trianguli ABC duo latera BA, AC reliquo BC ſunt majora quomodocunq; ſumpta.

Ex BA producta *a* cape AD = AC, & duc DC. *b* ergo

a 3. 1.

b 5. 1. *b* ergo ang. D = A C D. *c* ergo totus B C D ⊏
c 9. ax. D *d* ergo B D (*e* BA + AC) ⊏ B C. Q.E.D.
d 19. 1.
e conſtr. & 8. ax.

PROP. XXI.

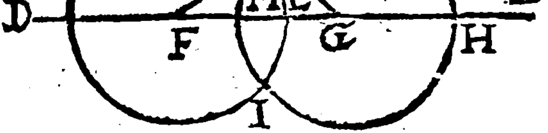


Si ſuper trianguli ABC uno latere BC, ab extremitatibus duæ rectæ lineæ BD, CD, interius conſtitutæ fuerint, hæ conſtitutæ reliquis trianguli duobus lateribus BA, CA minores quidem erunt, majorem vero angulum BDC continebunt.

a 20. 1.
b 4. ax.
c 16. 1.

Producatur BD in E. eſtque CE + ED *a* ⊏ CD adde commune BD, *b* erit BE + EC ⊏ BD + DC. Rurſus BA + AE *a* ⊏ BE; *b* ergo BA + AC ⊏ BE + EC. quare BA + AC ⊏ BD + DC. Q.E.D. 2. Ang. BDC *c* ⊏ DEC *c* ⊏ A. ergo ang. BDC ⊏ A. Q.E.D.

PROP. XXII.

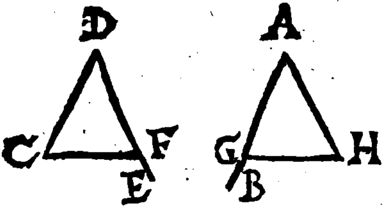


Ex tribus rectis lineis FK, FG, GK, quæ ſint tribus datis rectis lineis A, B, C, æquales, triangulum FKG conſtituere. Oportet autem duas reliqua eſſe majores omnifariam ſumptas; quoniam uniuſcujuſque trianguli duo latera omnifariam ſumpta reliquo ſunt majora.

Ex

Ex infinita D E *a* ſume DF, FG, GH datis a 3. 1.
A, B, C ordine æquales. Tum ſi *b* centris F, & b 3. poſt.
G, intervallis F D, & GH ducantur circuli ſe
interſecantes in K; junctis rectis KF, KG con-
ſtituetur triangulum F K G, *c* cujus latera FK, c 15. def.
FG, GK tribus D F, FG, G H, *d* id eſt tribus d 1. ax.
datis A, B, C æquantur. Q.E.F.

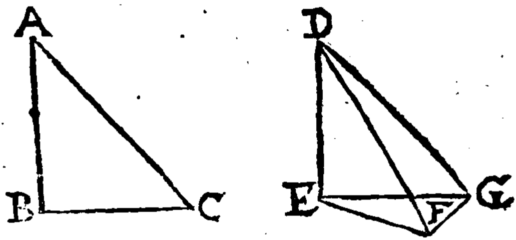
PROP. XXIII.



Ad datā rectam lineam A B, datūq; in ea punctum A, dato angulo rectilineo D æquale angulum rectilineum A conſtituere.

a Duc rectam C F ſecantem dati anguli late- a 1. poſt.
ra utcunque. *b* Fac A G = C D. Super A G, b 3. 1.
c conſtitue triangulum alteri C D F æquilate- c 22. 1.
rum, ita ut AH=DF, & GH = CF; & habebis
ang. A *d* = D. Q.E.F.

PROP. XXIV.



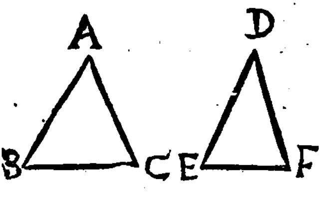
Si duo triangula A B C, D E F duo latera A B, *A C duobus lateribus D E, D F æqualia habuerint, utrumque utrique; angulum vero A angulo E D F majorem ſub æqualibus rectis lineis contentum, & baſim B C, baſi E F, majorem habebunt.*

a 23.1. *a* Fiat ang. EDG = A, & DG *b* = DE *c* = A
b 3.1. C, connectanturque EG, FG.
c hyp.
d hyp. 1. *Caſ.* Si E G cadit ſupra E F: Quia AB *d*
e conſtr. = DE, & AC = *e* DG, & ang. A *e* = EDG, *f*
f 4.1. erit BC = EG Quia vero DF *e* = DG, *g* erit
g 5.1. ang. DFG = DGF. *h* ergo ang. DFG $\sqsubset$ EG
h 9.ax. F; *h* & proinde ang. EFG $\sqsubset$ EGF. *k* quare E
k 19.1. G (BC) $\sqsubset$ EF. Q.E.D.

l 9.ax. 2. *Caſ.* Si baſis EF baſi EG coincidat, *l* liquet EG (BC) $\sqsubset$ EF.

3. Sin EG cadat infra EF. Quoniam DG +
m 21.1. GE *m* $\sqsubset$ D F + F E, ſi hinc inde auferantur
n 5.ax. DG, DF, æquales, manet EG (BC) *n* $\sqsubset$ EF. Q. E. D.

PROP. XXV.

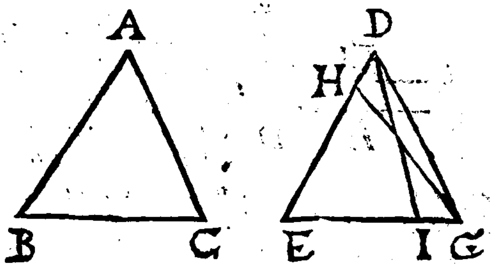


Si duo triangula A B C, D E F duo latera A B, A C duobus lateribus D E, D F æqualia habuerint, utrumque utrique, baſim vero B C baſi E F majorem; & angulum A ſub æqualibus rectis lineis contentum angulo D majorem habebunt.

a 4.1. Nam ſi dicatur ang. A = D. *a* erit baſis BC = EF,

= EF, contra Hyp. Sin dicatur ang. A ⊐ D. *b* erit BC ⊐ EF, etiam contra Hyp. ergo BC ⊏ EF. Q.E.D. b 24.1.

PROP. XXVI.



Si duo triangula BAC, EDG, duos angulos B,C, duobus angulis E, DGE, æquales habuerint, utrumque utrique, unumq; latus uni lateri æquale, ſive quod æqualibus adjacet angulis, ſeu quod uni æqualium angulorum ſubtenditur: reliqua latera reliquis lateribus æqualia, utrumq; utriq;, & reliquum angulum reliquo angulo æqualem habebunt.

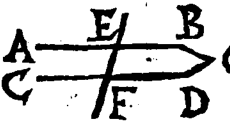
1. *Hyp.* Sit BC = EG. Dico BA = ED, & AC = DG, & ang. A = EDG. Nam ſi dicatur ED ⊏ BA, *a* fiat EH = BA, ducaturque GH.

Quoniam AB *b* = HE, & BC *c* = EG, & ang. B *c* = E, erit ang. EGH *d* = C *e* = DGE. *f* Q.E.A. ergo AB = ED. Eodem modo AC = DG. *d.* quare etiam ang. A = EDG. b ſuppoſ. c hyp. d 4.1. e hyp. f 9. ax.

2. *Hyp.* Sit AB = ED. Dico BC = EG; & AC = DG & ang. A = EDG. Nam ſi dicatur EG ⊏ BC, fiat EI = BC, & connectitur DI. Quia AB *g* = ED, & BC *h* = EI, & ang. g hyp. h ſuppoſ.

k 4. 1. B *g* = E, erit ang. E I *D* *k* = C *m* = E G *D*. *n*
m hyp.
n 16. 1. Q.E.A. ergo B C = E G. ergo ut prius, A C = *D* G, & ang. A = E *D* G. Q.E.*D*.

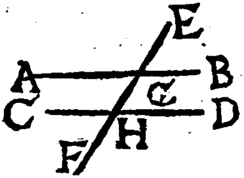
PROP. XXVII.



Si in duas rectas lineas A B, C D recta incidens linea E F alternatim angulos A E F, D F E, æquales inter ſe fecerit, parallelæ erunt inter ſe illæ rectæ lineæ A B, C D.

Si A B, C *D* dicantur non eſſe parallelæ; conveniant productæ, nempe in G. quo poſito
a 16. 1. angulus externus A E F interno *D* F E *a* major erit, cui tamen ponitur æqualis. Quæ repugnant.

PROP. XXVIII.



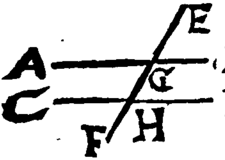
Si in duas rectas lineas A B, C D recta incidens linea E F externum angulum A G E interno & oppoſito, & ad eaſdem partes C H G æqualem fecerit, aut internos & ad eaſdem partes A G H, C H G duobus rectis æquales; parallelæ erunt inter ſe ipſæ rectæ lineæ A B, C D.

1. *Hyp*. Quia per hyp. ang. A G E = C H G,
a 15. 1. *a* erit altern. B G H = C H G. *b* parallelæ igi-
b 17. 1. tur ſunt A B, C *D*. Q.E.*D*.

2. *Hyp*. Quia ex hyp. ang. AGH + CHG =
a 13. 1. 2. Rect. *a* = AGH + BGH, *b* erit C H G = B
b 3. ax. GH. Ergo *c* AB, CD parallelæ ſunt. Q.E.*D*.
c 27. 1.

PROP.

PROP. XXIX.

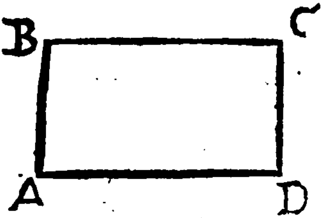


In parallelas rectas lineas AB,CD,recta incidens linea EF,& alternatim angulos DHG,AGH æquales inter ſe efficit; & externum BGE interno, & oppoſito, & ad eaſdem partes DHE æqualem; & internos & ad eaſdem partes AGH,CHG duobus rectis æquales facit.

Liquet $AGH + CHG = 2$. Rect. *a* alias AB,C*D* non eſſent parallelæ,contra hyp. Sed & ang.$DHG + CHG$ *b* $= 2$. Rect. ergo DHG *c* $= AGH$ *d* $= BGE$. Q.E.D.

a 13. ax.
b 13. 1.
c 13. ax.
d 15. 1.

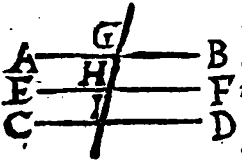
Coroll.



Hinc omne parallelogrammum AC habens unum angulum rectum A, eſt rectangulum.

Nam $A + B$ *a* $=$ 2. Rect. ergo cum A rectus ſit,*b* etiam B rectus erit. Eodem argumento *D*,& C recti ſunt.

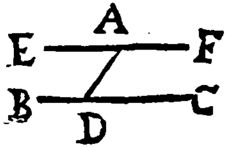
a 29. 1.
3. ax.

PROP. XXX.

Quæ(AB,CD)eidem rectæ lineæ EF parallelæ, & inter ſe ſunt parallelæ.

Tres rectas ſecet utcunq; recta GI. Quoniā AB, EF parallelæ ſunt, *a* crit

a 29.1. *a* erit ang. AGI = EHI, Item propter CD, EF parallelas, *a* erit ang. EHI = DIG. *b* ergo ang. AGI = DIG. *c* quare AB, CD parallelę ſunt. Q.E.D.
b 1.ax.
c 27.1.

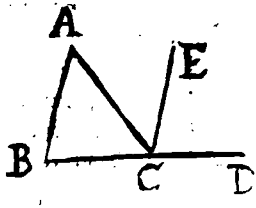
PROP. XXXI.



A dato puncto A data recta linea BC ducere parallelam rectam lineam AE.

Ex A ad datam BC duc rectam utcunque AD. ad quam, ejusque punctum A *a* fac ang. DAE = ADC. *b.* erunt AE, BC parallelæ. Q.E.F.
a 23.1.
b 27.1.

PROP. XXXII.



Cujuſcunque trianguli ABC uno latere BC producto, externus angulus ACD duobus internis, & oppoſitis, AB eſt æqualis. Et trianguli tres interni anguli, A, B, ACB duobus ſunt rectis æquales.

a 31.1. Per C *a* duc CE parall. BA. ang. A *b* = ACE. & ang. B *b* = ECD. ergo A + B *c* = ACE + ECD *d* = ACD. Q.E.D. Pono ACD + ACB *e* = 2. Rect. *f* ergo A + B + ACB = 2. Rect. Q.E.D.
b 29.1.
c 2.ax.
d 19.ax.
e 13.1.
f 1.ax.

Coroll.

1. *Tres ſimul anguli cujuſvis trianguli æquales ſunt tribus ſimul cujuſcunq; alterius. Unde*

2. *Si*

2. *Si in uno triangulo duo anguli (aut ſinguli, aut ſimul) in altero triangulo, etiam reliquus reliquo æqualis eſt. Item, ſi duo triangula unum angulum uni æqualem habeant, reliquorum ſummæ æquantur.*

3. *In triangulo ſi unus angulus rectus ſit, reliqui unum rectum conficiunt. Item, angulus, qui duobus reliquis æquatur, rectus eſt.*

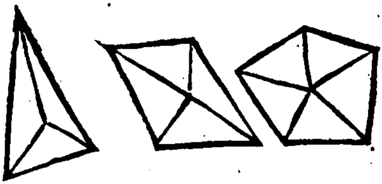
4. *Cum in Iſoſcele angulus æquis cruribus contentus rectus eſt, reliqui ad baſim ſunt ſemirecti.*

5. *Trianguli æquilateri angulus facit duas tertias unius recti, nam* $\frac{1}{3}$ *2. Rect.* = $\frac{2}{3}$ *Rect.*

SCHOL.

Hujus propoſitionis beneficio, cujuſlibet figuræ rectilineæ tam interni quam externi anguli quot rectos conficiant, innoteſcet per duo ſequentia theoremata.

THEOREMA I.



Omnes ſimul anguli cujuſcunque figuræ rectilineæ conficiunt bis tot rectos demptis quatuor, quot ſunt latera figuræ.

Ex quovis puncto intra figuram ducantur ad omnes figurę angulos rectę, quę figuram reſolvent in tot triangula quot habet latera.

Quare cum ſingula triangula conficiant duos rectos, omnia ſimul conficient bis tot rectos, quot ſunt latera. Sed anguli circa dictum punctum conficiunt quatuor rectos. Ergo, ſi ab omnium triangulorum angulis demas angulos circa id punctum, anguli reliqui qui componunt angulos figuræ conficient bis tot rectos demptis quatuor, quot ſunt latera figuræ. Q. E. *D*.

Hinc *Coroll.* Omnes ejusdem ſpeciei rectilineæ figuræ æquales habent angulorum ſummas.

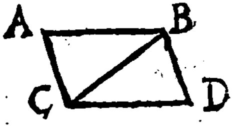
Theorema 2.

Omnes ſimul externi anguli cujuscunque figuræ rectilineæ conficiunt quatuor rectos.

Nam ſinguli figuræ interni anguli cum ſingulis externis conficiunt duos rectos. Ergo interni ſimul omnes, cum omnibus ſimul externis conficiunt bis tot rectos, quot ſunt latera figurę. Sed (ut modo oſtenſum eſt,) interni ſimul omnes etiam cum quatuor rectis efficiunt bis tot rectos, quot ſunt latera figuræ. Ergo externi anguli quatuor rectis æquantur. Q.E.*D*.

Coroll. Omnes cujuscunque ſpeciei rectilineæ figuræ æquales habent externorum angulorum ſummas.

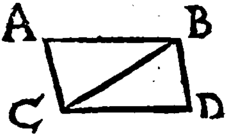
PROP. XXXIII.



Recta lineæ AC, BD, quæ æquales & parallelas lineas AB, CD, ad partes eaſdem conjungunt, & ipſæ æquales ac parallelæ ſunt.

Connectatur CB. Quoniam ob AB, CD parallerallelas. ang. ABC *a* = BCD, & per hyp. AB = CD, & latus CB commune eſt, *b* erit AC = BD, *b* & ang. ACB = DBC. *c* ergo AC, BD etiam parallelæ ſunt. Q.E.D. a 29.1. b 4.1. c 27.1.

PROP. XXXIV.



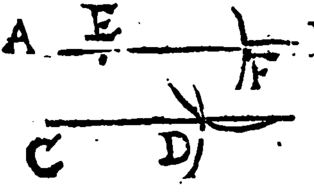
Parallegrammorum ſpatiorum ABDC æqualia ſunt inter ſe quæ ex adverſo latera AB, CD; ac AC, BD; angulíq; A, D, & ABD, ACD; & illa bifariam ſecat diameter CB.

Quoniam AB, CD *a* parallelæ ſunt, *b* erit ang. ABC = BCD: Item ob AC, DB *a* parallelas, *b* erit ang. ACB = CBD. *c* ergo toti anguli ACD, ABD æquantur. Similiter ang. A = D. Porro, cum communi lateri CB adjaceant anguli ABC, ACB, ipſis BCD, CBD pares *d*, erunt AC = BD, *d* & AB = CD. adeoque etiam triang. ABC = CBD. Q.E.D. a hyp. b 29.1 c 2.ax. d 26.1.

SCHOL.

Omne quadrilaterium ABDC habens latera oppoſita æqualia, eſt parallelogrammum.

Nam per 8.1. ang. ABC = BCD. *a* ergo AB, CD parallelæ ſunt. Eadem ratione ang. BCA = CBD; *a* quare AC, BD etiam parallelæ ſunt. *b* ergo ABDC eſt parallelogrammum. Q.E.D. a 27.1. b 35. def.1.

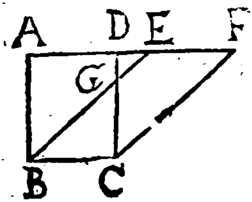


Hinc expeditius per datum punctũ C datæ rectę AB ducetur parallela CD.

Sume

Sume in A B quodvis punctum E. centris E. & C ad quodvis intervallum duc æquales circulos E F; C D. centro vero F, ſpatio E C duc circulum F D, qui priorem C D ſecet in D. Erit ducta C *D* parall. A B. Nam ut modo demonſtratum eſt, C E F *D* eſt parallelogrammum.

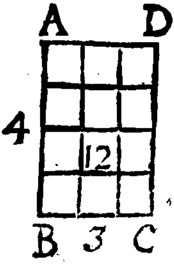
PROP. XXXV.



Perallelogramma B C D A, B C F E ſuper eadem baſi B C, & in eisdem parallelis A F, B C conſtituta, inter ſe ſunt æqualia.

Nam A *D* *a* = B C *a* = EF. adde commu-
a 34.1. nem *D* E, *b* erit AE = *D* F. Sed & A B *a* =
b 2.ax. *D* C; & ang. A *c* = C *D* F. *d* ergo triang. A
c 29.1. B E = *D* C F. auſer commune *D* G E, *e* erit
d 4.1. Trapez. A B G *D* = E G C F, adde commu-
e 3.1. ne B G C, *f* erit Pgr. A B C *D* = E B C F,
f 2.ax. Q. E *D*. Reliquorum caſuum non diſſimilis, ſed ſimplicior & facilior eſt demonſtratio.

Scholium.



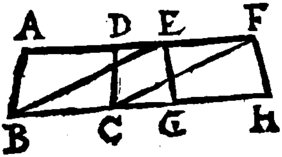
Si latus A B parallelogrammi rectanguli A B C *D* ferri intelligatur perpendiculariter per totam B C, aut B C per totam A B, producetur eo motu area rectanguli A B C *D*. Hinc rectangulum fieri dicitur

citur ex ductu ſeu multiplicatione duorum laterum contiguorum. Sit exempl. grat. B C pedum 3, A B 4. Duc 3 in 4; proveniunt 12. pedes quadrati pro area rectanguli.

Hoc ſuppoſito, ex hoc theoremate cujuscunque parallelogrammi (* EBCF) habetur dimenſio. Illius enim area producitur ex altitudine BA ducta in baſim B C. Nam area rectanguli A C parallelogrammo E B C F ęqualis, fit ex B A in B C. ergo, &c.

* v. fig. propoſ. 35.

PROP. XXXVI.

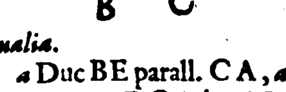


Parallelogramma B C D A, G H F E *ſuper æqualibus baſibus* B C, G H, *& in eiſdem parallelis* AF, BH *conſtituta, inter ſe ſunt æqualia.*

Ducantur B E, CF. Quia B *a* = G H *b* = E F, *c* erit B C F E parallelogrammum. ergo Pgr. B C D A *d* = BCFE *d* G H F E. Q. E. D.

a hyp.
b 34. 1.
c 33. 1.
d 35. 1.

PROP. XXXVII.



Triangula B C A, BCD *ſuper eadem baſi* B C *conſtituta, & in eiſdem parallelis* BC, EF *inter ſe ſunt æqualia.*

a Duc B E parall. C A, *a* & CF parall. BD. Erit triang. B C A *b* = $\frac{1}{2}$ Pgr. B C A E = *c* $\frac{1}{2}$ B D F C *b* = B C D. Q. E. D.

a 31. 1.
b 34. 1.
c 35. 1.
& 7. ax.

PROP. XXXVIII.

Triangula BCA, EFD ſuper æqualibus baſibus BC, EF conſtituta, & in eisdem parallelis GH, BF, inter ſe ſunt æqualia.

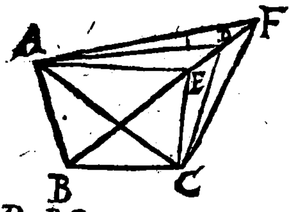
Duc BG parall. CA. & FH parall. ED. erit triang. BCA a = ½ Pgr. BCAG b = ½ EDHF c = EFD. Q.E.D.

a 34.1.
b 36.1.& 7.ax.
c 34.1.

Scholium.

Si baſis BC ⊏ EF, liquet triang. BAC ⊏ EDF. & ſi BC ⊐ EF, erit BAC ⊐ EDF.

PROP. XXXIX.



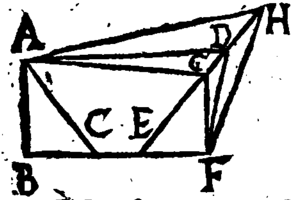
Triangula æqualia BCA, BCD, ſuper eadem baſi BC, & ad easdem partes conſtituta, etiam in eisdem ſunt parallelis AD, BC.

Si negas, ſit altera AF parall. BC; & ducatur CF. ergo triang. CBF a = CBA b = CBD. c Q.E.A.

a 37.1.
b hyp.
c 9.ax.

PROP. XL.

Triangula æqualia BCA, EFD ſuper æqualibus baſibus BC, EF, & ad easdem partes

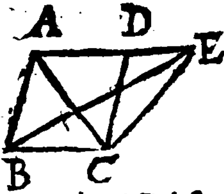


tes conſtituta, & in eiſdem ſunt parallelis A D, B F.

Si negas, ſit altera A H parall. B F. & ducatur FH. ergo triang. E FH a = BCA b = EFD. c Q.E.A.

a 38.1.
b hyp.
c 9. ax.

PROP. XLI.



Si parallelogrammum A B C D cum triangulo B C E eandem baſim B C *habuerit, in eiſdemq́; fuerit parallelis* A E, BC, *duplum erit parallelogrammum* A B C D *ipſius trianguli* B C E.

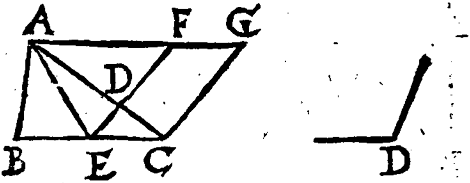
Ducatur A C. Triang. B C A a = B C E. ergo Pgr. A B C D b = 2. B C A c = 2. B C E. Q. E. D.

a 37.1.
b 34.1.
c 6. ax.

Scholium.

Hinc habetur area cujuscunque trianguli B C E. Nam cum area parallelogrammi A B C D producatur ex altitudine in baſim ducta; producetur area trianguli ex dimidia altitudine in baſim ducta, vel ex dimidia baſi in altitudinem. ut ſi baſis B C ſit 8, & altitudo 7; erit trianguli B C E area, 28.

PROP. XLII.

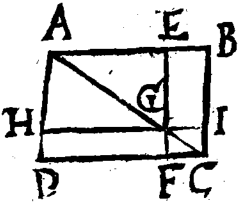


Dato triangulo ABC æquale parallelogrammum E C G F *conſtituere in dato angulo rectilineo* D.

a. 31. 1. Per A *a* duc A G parall. B C. *b* fac ang. BC
b. 23. 1. G = D. baſim BC *c* biſeca in E. *a* duc E F pa-
c. 10. 1. rall. C G. Dico factum.

d. 38. 1. Nam ducta AE. erit ex conſtr. ang. E C G =
e. 41. 1. D, & triang. BAC *d* = 2. A E C *e* = Pgr. E C
G F. Q.E.F.

PROP. XLIII.



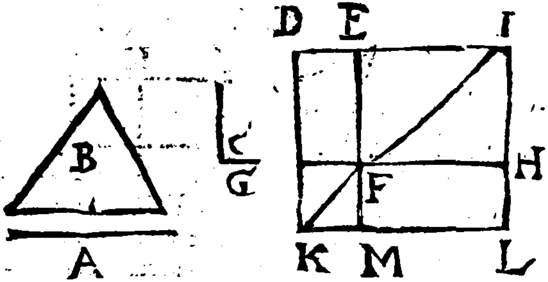
In omni parallelogrammo A B C D *complementa* D G, G B *eorum quæ circa diametrum* A C *ſunt parallelogrammorum* H E, F I *inter ſe ſunt æqualia.*

a 34. 1 Nam Triang. A C D, = *a* A C B. & triang.
A G H *a* = A G E. & triang. G C F *a* = GCI.
3 ax: *b* ergo Pgr. D G = G B. Q.E.D.

PROP. XLIV.

Ad datam rectam lineam A, *dato triangulo* B, *æquale parallelogrammum* F L *applicare in dato angulo rectilineo* C.

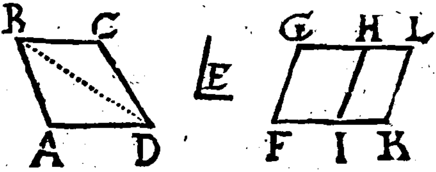
a 42. 1. *a* Fac Pgr. F D = triang. B, ita ut ang. GFE = C.



= C. & pone lateri G F in directum F H = A.
Per H b duc I L parall. E F; cui occurrat D E b 31. 1.
producta ad I. per I F ductę rectę occurrat D
G protracta ad K. Per K b duc K L parall. G
H; cui occurrant E F, & I H prolongatę ad
M, & L. Erit F L. Pgr. quęſitum.

Nam Pgr. F L c = F D = B d & ang. M F H c 43. 1.
= G F E = C. Q.E.F. d 15. 1.

PROP. XLV.



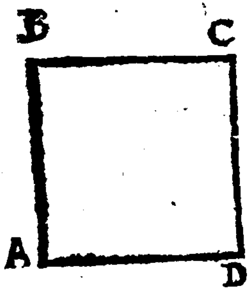
Ad datam rectam lineam F G dato rectilineo A B C D æquale parallelogrammum F L conſtituere, in dato angulo rectilineo E.

Datum rectilineum reſolve in triangula B
A D, B C D. a = Fac Pgr. F H = B A D ita ut
ang. F = E. producta F I a fac (ad H I) Pgr. a 44. 1.
I L = B C D. erit Pgr. F L = b F H + I L c = b 19. ax.
A B C D. Q.E.F. c conſtr.

Schol.

Hinc facile invenitur exceſſus H E, quo rectilineum aliquod A ſuperat rectilineum minus B; nimirum ſi ad quamvis rectam C D applicentur Pgr. D F = A. & D H = B.

PROBL. XLVI.



A data recta linea A D quadratum A C deſcribere.

a 11. 1.
b 3. 1.

a Erige duas perpendiculares A B, D C *b* æquales datæ A D; & junge B C. Dico factum.

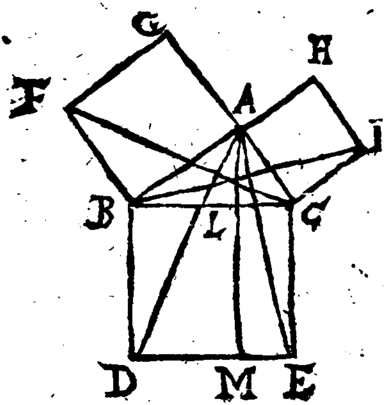
c conſtr.
d 28. 1.
e conſtr.
f 34. 1.
g Sch. 29. 1.
h 29. def.

Cum enim ang. A + D *c* = 2. Rect. *d* erunt A B, D C parallelæ. Sunt vero etiam *e* æquales, *f* ergo A D, B C pares etiam ſunt, & parallelę. ergo Figura A C eſt parallelogramma, & ęquilatera. Anguli quoque omnes recti ſunt, *g* quoniam unus A eſt rectus. *h* ergo A C eſt quadratum. Q.E.F.

Eodem modo facile deſcribes rectangulum, quod ſub datis duabus rectis contineatur.

PROP.

PROP. XLVII.



In rectangulis triangulis B A C quadratum B E, quod à latere B C rectum angulum B A C ſubtendente deſcribitur, æquale eſt eis B G, C H, quæ à lateribus A B, A C rectum angulum continentibus deſcribuntur.

Junge A E, A D; & duc A M, parall. C E.

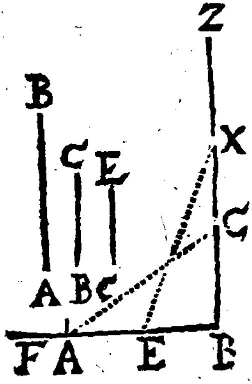
Quoniam ang. DBC *a* = F B A; adde com- a 11. ax.
munem A B C, erit ang. A B D = F B C. Sed
& A B *b* F B, & B D *b* = B C. *c* ergo triang. A b 29. def.
B D = F B C. atqui Pgr. B M *d* = 2 A B D; & c 4. 1.
Pgr. B G *d* = 2 F B C (nam G A C eſt una re- d 4. 11.
cta per hyp. & 14. 1.) *e* ergo Pgr. B M = B G. e 6. ax.
Simili diſcurſu Pgr. C M = C H. Totum igitur
B E = B G + C H. Q. E. D.

Schol.

Hoc nobiliſſimum, & utiliſſimum theorema ab inventore Pythagora, Pythagoricum dici meruit. Ejus beneficio quadratorum additio, & ſubſtractio perficitur; quo ſpectant duo ſequentia problemata.

PROBL. 1.

Andr. Tacq.

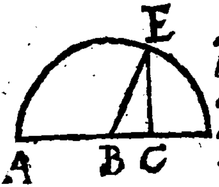


Datis quotcunq; quadratis, unum omnibus æquale conſtruere.

a 11.1. b 47.1. c 2.ax.

Dentur quadrata tria, quorum latera ſint A B, B C, C E. *a* Fac ang. rectum F B Z infinita habentem latera, in eaque transfer B A, & B C, & junge A C; *b* erit A Cq = A Bq + B Cq. Tum A C transfer ex B in X; & C E tertium latus datum transfer ex B in E, & junge E X, *b* erit E Xq = E Bq (C Eq) + B Xq (A Cq) *c* = C Eq + A Bq + B Cq. Q. E. F.

PROBL. 2.



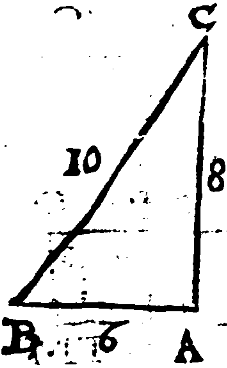
Datis duabus rectis inæqualibus A B, B C, exhibere quadratum, quo quadratum majoris A B excedit quadratum minoris B C.

a 47.1. b 1.ax.

Centro B intervallo B A deſcribe circulum. ex C erige perpendicularem C E occurrentem peripherię in E. & ducatur B E. *a* Erit B Eq (B Aq) = B Cq + C Eq. *b* ergo B Aq − B Cq = C Eq. Q.E.F.

PROBL.

PROBL. 3.

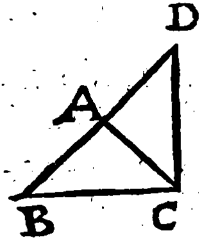


Notis duobus quibuscunq; lateribus trigoni rectanguli A B C, *reliquum invenire.*

Latera rectum angulum ambientia ſint A C, A B, hoc 6. pedum, illud 8. ergo cum $ACq + ABq = 64 + 36 = 100 = BCq$. erit $BC = \sqrt{100} = 10$. 47.1.

Nota ſint deinde latera A B, B C, hoc 10. pedum, illud 6. ergo cum $BCq - ABq = 100 - 36 = 64 = ACq$. erit $ACq = \sqrt{64} = 8$. 47.1.

PROP. XLVIII.



Si quadratum quod ab unò latere B C trianguli deſcribitur, æquale ſit eis quæ à reliquis trianguli lateribus A B, A C deſcribuntur quadratis, angulus B A C comprehenſus ſub A B, A C reliquis duobus trianguli lateribus, rectus eſt.

Duc ad A C perpendicularem $DA = AB$, & junge C D.

Jam $CDq\ a = ADq + ACq = ABq + ACq = BCq$. * ergo $CD = BC$. ergo triangula C A B, C A D, ſibi mutuo æquilatera ſunt; quare ang. $CAB\ b = CAD\ c =$ Rect. Q.E.D.

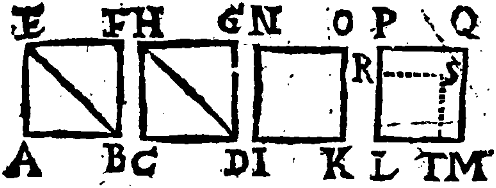
a 47.1.
* Vide ſeq. Theor.
b 8.1.
c hyp.

Schol.

Schol.

Aſſumpſimus exinde quod C Dq. = B Cq. ſequi C D = B C. Hoc verò manifeſtum fiet ex ſequenti theoremate.

THEOREMA.



Linearum æqualium A B, C D, æqualia ſunt quadrata A F, C G; & quadratorum æqualium N K, P M æqualia ſunt latera I K, L M.

Pro 1. Hyp. Duc diametros E B, H D. Liquet A F = *a* 2 triang. E A B = *b* 2 triang. H
a 34. 1. G D = *a* C G. Q. E. D.
b 4. 1. & 6. ax.
a 46. 1. 2. Hyp. Si fieri poteſt, ſit L M $\sqsubset$ I K. fac
b 1. part. L T = I K; *a* ſitque L S = L Tq. ergo L S *b* =
c hyp. N K *c* = L Q. *d* Q. E. A. ergo L M = I K.
d 9. ax.

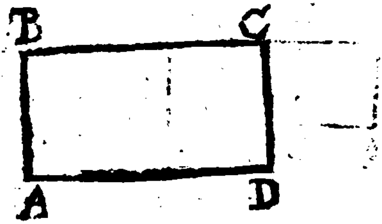
Coroll.

Eodem modo quælibet rectangula inter ſe æquilatera æqualia oſtendentur.

LIB.

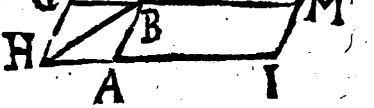
LIB. II.

DEFINITIONES.



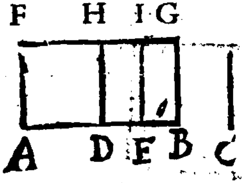
Omne parallelogrammum rectangulum ABCD contineri dicitur ſub rectis duabus AE, AD, quæ rectum comprehendunt angulum.

Quando igitur dicitur rectangulum ſub BA, AD; vel brevitatis cauſa, rectangulum BAD, vel BA x AD, (vel ZA pro Z x A) deſignatur rectangulum, quod continetur ſub BA, & AD ad rectum angulum conſtitutis.



II. In omni parallelogrammo ſpatio FHIK unumquodque eorum, quæ circa diametrum illius

illius ſunt, parallelogrammorum, cum duobus complementis Gnomon vocetur. *ut Pgr.* F B + B I + G A (EHM) *eſt Gnomon. item* Pgr. FB + BI + EM (GKA) *eſt Gnomon.*



PROP. I.

Si fuerint duæ rectæ lineæ AB, AF; *ſeceturq; ipſarum altera* A B *in quotcunque ſegmenta* A D, D E, E B: *rectangulum comprehenſum ſub illis duabus rectis lineis* A B, AF, *æquale eſt eis, quæ ſub inſecta* AF, *& quolibet ſegmentorum* AD, DE, EB *comprehenduntur rectangulis.*

a 11.1. *a* Statue AF, perpendicularem ad AB. *a* per
F duc infinitam F G perpendicularem ad A F.
b 19.ax.1 *a* Ex D, E, B erige perpendiculares DH, EI, B
G. erit A G rectangulum ſub AF, AB, & *b* eſt
æquale rectangulis AH, DI, EG, hoc eſt (quia
c 34.1. DH, EI, AF *c* pares ſunt) rectangulis ſub A F,
AD; ſub AF, DE; ſub AF, EB. Q.E.D.

SCHOL.

Imo ſi fuerint duæ rectæ, ſecenturq; ambæ in quotcunque partes, idem provenit ex ductu totius in totum, & partium in partes.

Nam ſit $Z = A + B + C$; & $Y = D + E$;
a 1.2. quia DZ *a* $= DA + DB + DC$, & EZ *a*
$= EA + EB + EC$, & YZ *a* $= DZ +$
b 2. ax. EZ, *b* erit $ZY = DA + DB + DC + E$
$A + EB + EC$. Q.E.D.

Hinc patet ratio ducendi rectas compoſitas in compoſitas. Nam omnia partium rectangula ac-

accipere oportet, & habetur rectangulum ex totis.

Sin linearum in ſe ducendarum ſignis $+$ admiſceantur ſigna $-$, etiam ſignorum ratio habenda eſt. Quippe ex $+$ in $-$ provenit $-$; at ex $-$ in $-$ provenit $+$. Nam ſit $+ A$ ducenda in $B - C$. & quoniam $+ A$ non affirmatur de toto B, ſed de ejus parte tantum, qua ſuperat C, debet A C manere negata. quare prodibit $AB - AC$. Vel ſic; quia B conſtat partibus C, & $B - C$, * erit $AB = AC + A$ in $B - C$; aufer utrinque A C, erit $AB - AC = A$ in $B - C$. Similiter ſi $- A$ ducenda ſit in $B - C$, quoniam ex vi ſigni $-$ non negatur A de toto B, ſed de ejus ſolummodo exceſſu ſupra C, debet A C manere affirmata. Proveniet ergo $- AB + AC$. Vel ſic; quia AB * $= AC +$ A in $B - C$; tolle utrinque omnia, erit $- AB = - AC - A$ in $B + C$; adde A C utrinque, eritque $- AB + AC = A$ in $B - C$.

* 1. 2

Atque ex his rite perſpectis, quæ ſubſequuntur 9. propoſitiones, aliæque ejusmodi innumeræ, ex linearum in ſe ductarum comparatione emergentes (quas apud Vietam, & alios Analyſtas innumerato habes) nullo negotio demonſtrantur, rem plerumque quaſi ad ſimplicem calculum exigendo.

Porrò, † liquet productum ex quapiam magnitudine in numeri cujuſlibet partes æquari producto ex eadem in totum numerum, Ut $5. A + 7 A = 12. A.$ & $4. A$ in $5. A + 4 A$ in $7 A = 4 A$ in $12 A$. quare quæ in hoc loco de re-

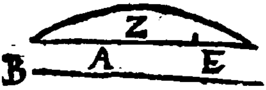
† 19. ax

rectarum in ſe ductu dicta ſunt, eadem de numerorum in ſe multiplicatione intelligi poſſunt. Proinde etiam quæ in 9 ſequentibus theorematis de lineis affirmantur, eadem valent de numeris accepta; quippe cum iſtæ omnes ab hac prima immediate dependeant, & deducantur.

SCHOL.

Propoſitiones decem primæ hujus libri valent etiam in numeris. Reliquas quilibet tyro examinet. pro hac, ſit A F 6, & A B 12, ſectus in A D 5, D E 3, & E B 4. Eſtque $6x12$ (AG) $= 72$. $6x5$ (AH) $= 30$. 6 in 3 (DI) $= 18$. denique $6x4$ (E G) $= 24$. Liquet vero $30 + 18 + 24 = 72$.

PROP. II.



Si recta linea Z ſecta ſit utcunque; rectangula, quæ ſub tota Z, & quolibet ſegmentorum A, E comprehenduntur, æqualia ſunt ei, quod à tota Z fit, quadrato.

Dico $ZA + ZE = Zq$. Nam ſume $B = Z$. a
a 1. 2. Eſtque $BA + BE = BZ$; hoc eſt (ob $B = Z$) $ZA + ZE = Zq$. Q.E.*D.*

PROP. III.

Si recta linea Z ſecta ſit utcunque; rectangulum ſub tota Z, & uno ſegmentorum E comprehenſum, æquale eſt illi, quod ſub ſegmentis A, E comprehenditur, rectangulo, & illi quod à prædicto ſegmento E deſcribitur, quadrato,

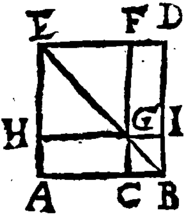
Dico,

Dico $ZE = AE + Eq$. a Nam $EZ = EA + EE$. a 1. 1.

PROP. IV.

Si recta linea Z secta sit utcunque; Quadratum, quod à tota Z deſcribitur, æquale eſt, & illis quæ à ſegmentis A, E deſcribuntur, quadratis, & ei, quod bis ſub ſegmentis A, E comprehenditur, rectangulo.

Dico $Zq = Aq + Eq + 2\,AE$. a Nam $ZA = Aq + AE$. a & $ZE = Eq + AE$. quum igitur $ZA + ZE$ b $= Zq$, c erit $Zq = Aq + Eq + 2\,AE$. Q. E. D. a 3. 2. b 2. 2. c 1. ax.



Aliter. Super AB fac quadratum AD, cujus diameter EB. per diviſionis punctum C duc perpendicularem CF; & per G duc HI parall. AB.

Quoniam ang. EHG = A rectus eſt, & AEB d ſemirectus, e erit reliquus HGE etiam ſemirectus. Ergo HE f = HG g = EF g = AC. h proinde HF quadratum eſt rectæ AC. eodem modo CI eſt CBq. ergo AG, GD rectangula ſunt ſub AC, CB. Quare totum quadratum AD k = ACq + CBq + 2 ACB. Q. E. D. d 4. cor. 32. e 32. 1. f 6. 1. g 34. 1. h 19. def. 1. k 19. ax. 1.

Coroll.

1. Hinc liquet parallelogramma circa diametrum quadrati eſſe quadrata.

2. Item diametrum cujusvis quadrati ejus angulos biſecare.

3. Si

3. Si $A = \frac{1}{2} Z$; erit $Zq = 4\,Aq$, & $Aq = \frac{1}{4}\,Zq$. item è contra, ſi $Zq = 4\,Aq$. erit $A = \frac{1}{2} Z$.

PROP. V.

Si recta linea A B *ſecetur in æqualia* A C, C B, *& non æqualia* A D, D B, *rectangulum ſub inæqualibus ſegmentis* A D, D B *comprehenſum una cum quadrato, quod ſit ab intermedia ſectionum* C D, *æquale eſt ei, quod à dimidia* C B *deſcribitur, quadrato.*

Dico $CBq = ADB + CDq$.

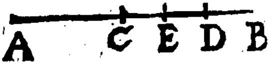
a 4.2. b 3.2. c hyp. d 1.2.

$$\text{Æquantur enim iſta}\begin{cases} CBq. \\ {}^{a}\,CDq + CDB + DBq + CDB \\ CDq + {}^{b}CBD\,({}^{c}AC \times BD) + CDB \\ CDq + {}^{d}\,ADB. \end{cases}$$

Hoc theorema paulo aliter effertur, & facilius demonſtratur, ſic; Rectangulum ex ſumma & differentia duarum rectarum A, E, *æquatur differentiæ ex ipſis.*

† ſch.1.2. Nam ſi $A + E$ ducatur in $A - E$, † provenit $Aq - AE + EA - Eq = Aq - Eq$. Q. E. D.

Scholium.



Si A B aliter dividatur, propius ſcilicet puncto biſectionis, in E; dico $AEB \sqsubset ADB$.

a 5.2. & 9. ax. Nam $AEB\ {}^{a}= CBq - CEq$. & $ADB\ {}^{a}= CBq - CDq$. ergo quum $CDq \sqsubset CEq$, erit $AEB \sqsubset ADB$. Q. E. D.

Coroll.

Corollarium.

Hinc ADq + DBq ⊏ AEq + EBq. Nam ADq + DBq + 2 ADB *b* = ABq *b* = AEq + EBq + 2 AEB. ergo quum 2 AEB ⊏ 2 ADB, erit ADq + DBq ⊏ AEq + EBq. Q. E. D. b 4. 2.

Unde 2. ADq + DBq − AEq *c* + EBq = 2 AEB − 2 ADB. c 3 ax.

PROP. VI.

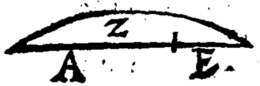
Si recta linea A bifariam secetur, & illi recta quæpiam linea E in directum adjiciatur; rectangulum comprehensum sub tota cum adjecta (sub A + E,) *& adjecta E, una cum quadrato, quod à dimidia $\frac{1}{2}$ A, æquale est quadrato à linea, quæ tum ex dimidia, tum ex adjecta componitur, tanquam ab una $\frac{1}{2}$ A + E descripto,*

Dico $\frac{1}{4}$ Aq (*a* Q. $\frac{1}{2}$ A) + AE + Eq = Q. $\frac{1}{2}$ A + E. *a* Nam Q. $\frac{1}{2}$ A + E = $\frac{1}{4}$ Aq + Eq + AE. a 4. & 1 Cor. 4. 2

Coroll.

Hinc si tres rectæ E, E + $\frac{1}{2}$ A, E + A sint in proportione Arithmetica, rectangulum sub extremis E. E + A contentum, una cum quadrato excessus $\frac{1}{2}$ A, æquale erit quadrato mediæ E + $\frac{1}{2}$ A.

PROP. VII.



Si recta linea Z secetur utcunque; Quod à tota Z, *quoque ab uno ſegmentorum E, utraq́; ſimul quadrata, æqualia ſunt illi, quod bis ſub tota Z, & dicto ſegmento E comprehenditur, rectangulo, & illi, quod à reliquo ſegmento A fit, quadrato.*

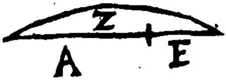
a 4. 2. Dico $Zq + Eq = 2\ ZE + Aq$. Nam $Zq\ a = Aq + Eq + 2\ AE$. & $2\ ZE\ b = 2\ Eq + 2\ AE$.
b 3. 2.

Coroll.

Hinc, quadratum differentię duarum quarumcunque linearum Z, E, ęquale eſt quadratis utriusq́; minus duplo rectangulo ſub ipſis.

c 7. 2. & 3. ax. Nam $Zq + Eq - 2\ ZE = Aq = Q.\ Z - E$.

PROP. VIII.



Si recta linea Z ſecetur utcunque; rectangulum quater comprehenſum ſub tota Z, & uno ſegmentorum E, cum eo, quod à reliquo ſegmento A fit, quadrato, æquale eſt ei, quod à tota Z, & dicto ſegmento E, tanquam ab una linea Z + E deſcribitur, quadrato.

a 7. 2. & 3. ax. Dico $4\ ZE + Aq = Q.\ Z + E$. Nam $2\ ZE\ a = Zq + Eq - Aq$. ergo $4\ ZE + Aq = Zq + Eq + 2\ ZE\ b = Q.\ Z + E$. Q.E.D.
b 4. 2.

PROP. IX.

Si recta linea AB ſecetur in æqualia AC, CB, & non æqualia AD, CB. quadrata, quæ ab inæqualibus totius ſegmentis AD, DB fiunt, ſimul duplicia ſunt, & ejus, quod à dimidia AC, & ejus, quod ab intermedia ſectionum CD fit, quadrati.

a 4. 2. Dico $ADq + DBq = 2\ ACq + 2\ CDq$.

Dq. Nam A Dq + D Bq *a* = A Cq + C Dq + 2 A C D + DBq. atqui 2 ACD (*b* 2 B C *D*) + DBq *c* = CBq (ACq) + C Dq. *d* ergo A Dq + D Bq = 2 A Cq + 2 CDq. Q.E.D.

b hyp.
c 7.2.
d 3 ax.

Aliter effertur & facilius demonſtratur ſic;

Aggregatum quadratorum ex ſumma, & differentia duarum rectarum A, E, aquatur duplo quadratorum ex ipſis.

Nam Q: A + E *a* = Aq + Eq + 2 A E. & Q: A − E *b* = Aq + Eq − 2 A E. Hæc collecta faciunt 2 Aq + 2 Eq. Q.E.D.

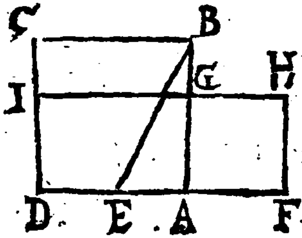
PROP. X.

Si recta linea A ſecetur bifariam, adjiciatur autem ei in rectum quæpiam linea; Quod à tota A cum adjuncta E, & quod ab adjuncta E, utraque ſimul quadrata, duplicia ſunt & ejus, quod à dimidia $\frac{1}{2}$ A; & ejus, quod à compoſita ex dimidia, & adjuncta, tanquam ab una $\frac{1}{2}$ A + E, deſcriptum eſt, quadrati.

Dico Eq + Q. A + E, hoc eſt *a* Aq + 2 Eq + 2 A E = 2 Q. $\frac{1}{2}$ A + 2 Q. $\frac{1}{2}$ A + E. Nam 2 Q. $\frac{1}{2}$ A *b* = $\frac{1}{2}$ Aq. & 2 Q. $\frac{1}{2}$ A + E *c* = $\frac{1}{2}$ Aq + 2 Eq + 2 A E.

a 4.2.
b Cor. 4.2.
c 4.2.

PROP. XI.



Datam rectam lineam A B ſecare in G, ut comprehenſum ſub tota A B, & altero ſegmentorum B G *rectangulum, æquale ſit ei, quod à reliquo ſegmento A G fit, quadrato.*

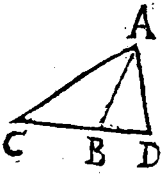
a 46.1. Super A B *a* deſcribe quadratum A C. latus
b 10.1. A D *b* biſeca in E. duc E B. ex E A producta cape $EF = EB$. ad A F *a* ſtatue quadratum A H. Erit $AH = AB \times BG$

Nam protracta H G ad I; Rectang. $DH + EAq$
c 6.2. d conſtr. $c = EFq$ $d = EBq$ $e = BAq + EAq$. ergo
e 47.1. DH $f = BAq$ $d =$ quad. A C. ſubtrahe com-
f 3.ax. mune A I; *f* remanet quad. $AH = GC$; *d* id eſt $AGq = AB \times BG$. Q. E. F.

Scholium.

* vid. 6. 13. Hæc propoſitio numeris explicari nequit; * neque enim ullus numerus ita ſecari poteſt, ut productum ex toto in partem unam æquale ſit quadrato partis reliquæ.

PROP. XII.



In amblygoniis triangulis A B C quadratum, quod fit à latere A C angulum obtuſum A B C ſubtendente, majus eſt quadratis, quæ fiunt à lateribus A B, B C obtuſum angulum A B C comprehendentibus, rectangulo bis comprehenſo, & ab uno laterum B C, quæ ſunt circa obtuſum angulum A B C, in quod, cum protractum fuerit, cadit perpendicularis A D, & ab aſſumpta exterius linea B D ſub perpendiculari A D prope angulum obtuſum A B C.

Dico $ACq = CBq + ABq + 2\, CB \times BD$.

Nam

$$\text{Nam iſta æqualia ſunt inter ſe}\left\{\begin{array}{l} ACq. \\ {}^{a}\,CDq + ADq. \\ {}^{b}\,CBq + 2\,CBD + BDq + ADq \\ CBq + 2\,CBD\;{}^{c} + ABq. \end{array}\right.$$

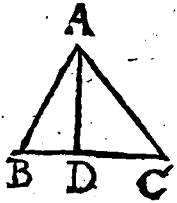
a 47. 1.
b 4. 2.
c 47. 1.

Schol.

Hinc, *cognitis lateribus trianguli obtuſanguli A B C, facile invenientur tum ſegmentum B D inter perpendicularem A D, & obtuſum angulum A B C interceptum, tum ipſa perpendicularis A D.*

Sic; Sit A C 10, A B 7, C B 5; unde A Cq 100, ABq 49. CBq 25. Proinde ABq + CBq = 74. hunc deme ex 100, manet 26 pro 2 CBD. unde C B D erit 13. hunc divide per C B 5, provenit $2\frac{3}{5}$ pro BD. quare A D invenitur per 47. 1.

PROP. XIII.



In oxygoniis triangulis A B C quadratum à latere A B angulum acutum A C B ſubtendente, minus eſt quadratis, quæ ſunt à lateribus A C, C B acutum angulum A C B comprehendentibus, rectangulo bis comprehenſo, & ab uno laterum B C, quæ ſunt circa acutum angulum A C B, in quod perpendicularis A D cadit, & ab aſſumpta interius linea D C ſub perpendiculari A D, prope angulum acutum A C B.

Dico ACq + BCq = ABq + 2 BCD.

a 47. 1.
b 7. 2.
c 47. 1.

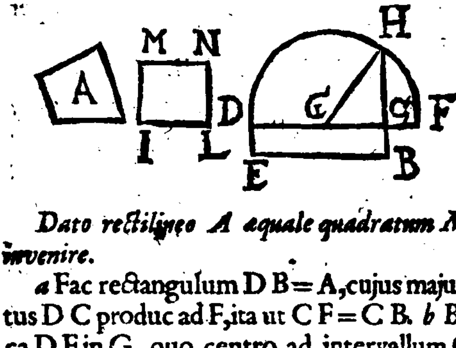
$$\text{Nam æquantur iſta}\begin{cases} ACq + BCq.\\ {}^{a}\, ADq + DCq + BCq.\\ {}^{b}\, ADq + BDq + 2\, BCD.\\ {}^{c}\, ABq + 2\, BCD.\end{cases}$$

Coroll.

Hinc etiam *cognitis lateribus trianguli A B C, invenire eſt tam ſegmentum D C inter perpendicularem A D, & acutum angulum A B C interceptum, quam ipſam perpendicularem A B.*

Sit A B 13, A C 15, B C 14. detrahe A Bq (169) ex A Cq + BCq hoc eſt ex 225 + 196 = 421; remanet 252 pro 2 B C D; unde B C D erit 126. hunc divide per B C 14, provenit 9 pro D C. unde A D = √: 225 − 81 = 12.

PROP. XIV.



Dato rectilineo A æquale quadratum M L invenire.

a 45. 1.
b 10. 2.

a Fac rectangulum D B = A, cujus majus latus D C produc ad F, ita ut C F = C B. *b* Biſeca D F in G, quo centro ad intervallum G F deſcribe circulum F H D, producatur C B. donec occurrat circumferentiæ in H. Erit CHq = * M L = A.

* 46. 1.
c conſtr.
d 5. 2. & 3. ax.
e 47. 1. & 3. ax.

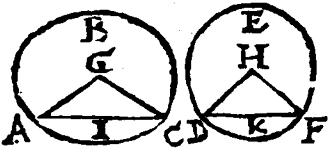
Ducatur enim G H. Eſtq; A *c* = D B *c* = D C F *d* = G Fq − G Cq *e* = H Cq *e* = M L Q. E. F.

LIB.

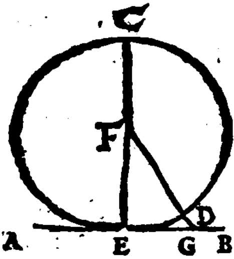
LIB. III.

DEFINITIONES.

I.

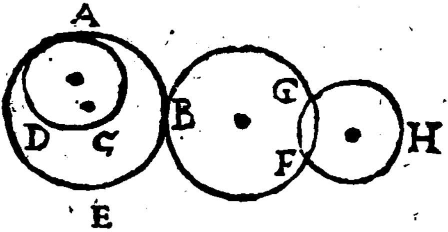


ÆQuales circuli (GABC,HDEF) ſunt, uorum diametri ſunt æquales, vel quorum quæ ex centris rectæ lineæ G A, H D, ſunt æquales.



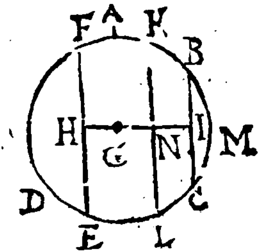
II. Recta linea A B circulum F E D tangere dicitur, quæ cum circulum tangat, ſi producatur circulum non ſecat.

Recta F G ſecat circulum F E D.

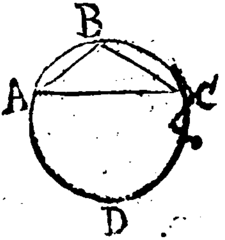


III. Circuli D A C, A B E (item F B G· A B E) ſe mutuo tangere dicuntur, qui ſe mutuo tangentes ſeſe mutuo non ſecant.

Circulus B F G ſecat circulum F G H.



IV. In circulo G A B D ęqualiter diſtare à centro dicuntur rectę lineæ F E, K L, cum perpendiculares G H, G N quæ à centro G in ipſas ducuntur, ſunt æquales. Longius autem abeſſe illa B C dicitur, in quam major perpendicularis G I cadit.



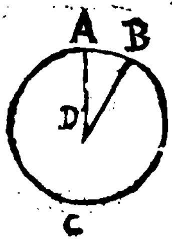
V. Segmentum circuli (ABC) eſt figura, quę ſub recta linea A C, & circuli peripheria A B C comprehenditur.

VI. Segmenti autem angulus (C A B) eſt, qui ſub recta linea C A, & circuli peripheria A B comprehenditur.

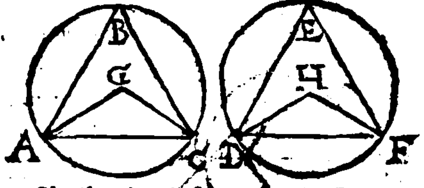
VII. In ſegmento autem (A B C) angulus (ABC) eſt, cum in ſegmenti peripheria ſumptum fuerit quodpiam punctum B, & ab illo in terminos rectę ejus lineæ A C, quæ ſegmenti baſis eſt, adjunctæ fuerint rectæ lineæ AB, CB, is inquam angulus A B C ab adjunctis illis lineis, AB, CB comprehenſus.

VIII. Cum vero comprehendentes angulum A B C, rectæ lineę A B, B C aliquam aſſumunt peripheriam A *D* C, illi angulus A B C inſiſtere dicitur.

IX. Se-



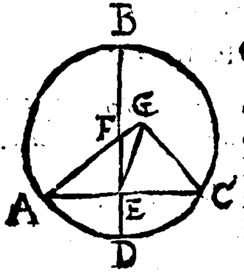
IX. Sector autem circuli (ADB) eſt, cum ad ipſius circuli centrum D conſtitutus fuerit angulus ADB; comprehenſa nimirum figura A D B. & à rectis lineis A D, B D angulum continentibus, & à peripheria A B ab illis aſſumpta.



X. Similia circuli ſegmenta (ABC,DEF) ſunt, quæ angulos (A B C, DEF) capiunt æquales; aut in quibus anguli ABC, DEF inter ſe ſunt æquales.

PROP. I.

Dati circuli A B C centrum F reperire.



Duc in circulo rectam A C utcunque quam biſeca in E. per E duc perpendicularem D B. hanc biſeca in F. erit F. centrum.

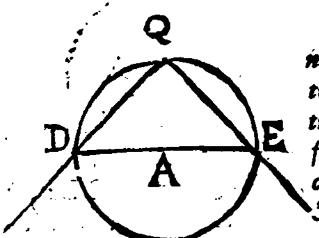
Si negas, centrum eſto G, extra rectam D B (nam in ea eſſe non poteſt, cum ubique extra F dividatur inæqualiter) ducanturque G A, G C, G E. Vis G cen-

a 15.def.1 trum eſſe ; *a* ergo G A = G C; & per conſtr.
b 8.1. A E = E C, latus vero GE commune eſt; *b* er-
c 10.def.1 go anguli GEA, GEC pares, & *c* proinde
d 12.ax. recti ſunt. *d* ergo ang. G E C = F E C rect. *e*
e 9.ax. Q.E.A.

Coroll.

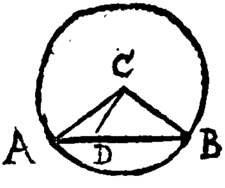
Hinc, *ſi in circulo recta aliqua linea B D aliquam rectam lineam A C bifariam & ad angulos rectos ſecet, in ſecante B D erit centrum.*

Andr. Tacq.



Faciliſſime per normam invenitur centrum vertice Q ad circumferentiam applicato. Si enim recta D E jungens puncta D, & E, in quibus normæ latera Q D, Q E peripheriam ſecant, biſecetur in A, erit A centrum. Demonſtratio pendet ex 31. hujus.

PROP. II.



Si in circuli C A B peripheria duo quælibet puncta, A, B accepta fuerint, recta linea A B, quæ ad ipſa puncta adjungitur, intra circulum cadet.

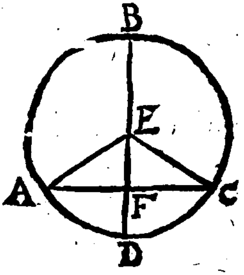
Accipe in recta A B quodvis punctum D, & ex centro C duc C A, C D, C B. & quoniam

C A *a* = C B, *b* erit ang. A = B. Sed ang. C a 15. def. 1.
D B *c* ⊏ A ; ergo ang. C D B ⊏ B. *d* ergo C b 5. 1.
B ⊏ C D. atqui C B tantum pertingit ex centro ad circumferentiam ; ergo C D eouſque non pertingit. ergo punctum D eſt intra circulum. Idemque oſtendetur de quovis alio puncto rectæ A B. Tota igitur A B cadit intra circulum. Q. E. D. c 16. 1. d 19. 1.

Coroll.

Hinc, *Recta circulum tangens, ita ut eum non ſecet, in unico puncto tangit.*

P R O P. III.



Si in circulo E A B C recta quædam linea B D per centrum extenſa quandam A C non per centrum extenſam bifariam ſecet, (in F) & ad angulos rectos ipſam ſecabit; & ſi ad angulos rectos eam ſecet, bifariam quoque eam ſecabit.

Ex centro E ducantur E A, E C,

1. *Hyp.* Quoniam A F *a* = F C, & E A *b* a hyp.
= E C, latuſque E F commune eſt, *c* erunt an- b 15. def. 1.
guli E F A , E F C pares, & *d* conſequenter c 8. 1.
recti. Q. E. D. d 10. def. 1.

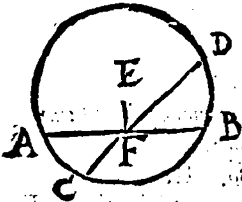
2. *Hyp.* Quoniam ang. E F A *e* = E F C, e hyp. & 12. ax.
& ang. E A F *f* = E C F, latuſque E F com- f 5. 1.
mune, *g* erit A F = F C. Biſecta eſt igitur A g 26. 1.
C. Q. E. D.

Coroll.

Coroll.

Hinc, *in triangulo quovis æquilatero & Iſoſcele linea ab angulo verticis biſecans baſim, perpendicularis eſt baſi. & contra perpendicularis ab angulo verticis biſecat baſim.*

PROP. IV.



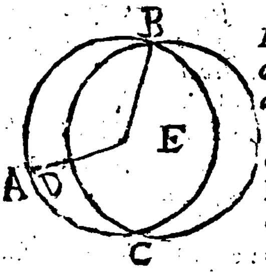
Si in circulo ACD duæ rectæ lineæ AB, CD ſeſe mutuo ſecent non per centrum E extenſæ, ſeſe mutuo bifariam non ſecabunt.

Nam ſi una per centrum tranſeat, patet hanc non biſecari ab altera, quæ ex hyp. per centrum non tranſit.

Si neutra per centrum tranſit, ex E centro duc E F. Si jam ambæ A B, C D forent biſectæ in F, anguli E F B, E F D *a* ambo eſſent recti, & proinde æquales. *b* Q.E.A.

a 3. 3.
b 9. ax.

PROP. V.



Si duo circuli BAC, BDC ſeſe mutuo ſecent, non erit illorum idem centrum E.

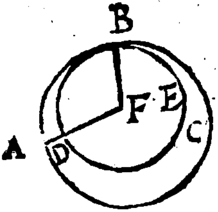
Alias enim ductis ex communi centro E rectis E B, E D A, eſſent E D *a* = E B *a* = E A. *b* Q. E. A.

a 15. def. 1
b 9. ax.

PROP. VI.

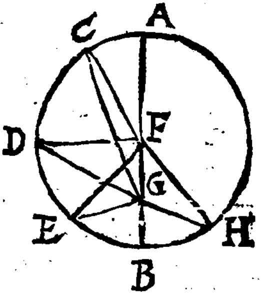
Si duo circuli B A C, B D E, ſeſe mutuo interius tangant (in B) *eorum non erit idem centrum F.*

Alias



Alias ductis ex centro F rectis F B, FDA eſſent FD *a* = FB *a* FA. *b* Q. F. N.

PROP. VII.



Si in A B diametro circuli quodpiā ſumatur punctum G, quod circuli centrum non ſit, ab eoq; puncto in circulum quædam rectæ lineæ GC,GD,GE cadunt; maxima quidem erit ea (G A) in qua centrum F, minima vero reliqua G B. aliarum vero illi, quæ per centrum ducitur, propinquor G C remotiore G D ſemper major eſt. Duæ autem ſolum rectæ lineæ G E,G H æquales ab eodem puncto in circulum cadunt, ad utrasq; partes minimæ G B, vel maximæ G A.

Ex centro F duc rectas F C, F D, F E; & *a* fac ang. B F H = B F E. a 23.1.

1. G F + F C (hoc eſt G A) *a* $\sqsubset$ G C. Q. E. D. a 20.1.

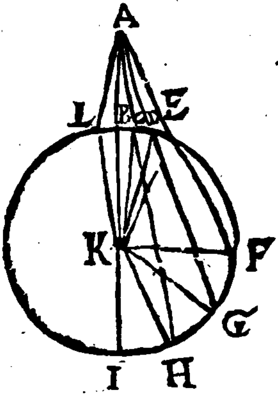
2. Latus F G commune eſt; & F C *b* = F D. atque ang. G F C *c* $\sqsubset$ G F D *d* ergo baſ. G C $\sqsubset$ G D. Q. E. D. b 15. def. 1. c 9. ax. d 24. 1.

3. F B (F E) *e* $\sqsupset$ G E + G F. ergo ablato communi F G *f* remanet B G $\sqsupset$ E G. Q. E. D. e 20. 1. f 5. ax.

4. Latus

4. Latus F G commune eſt, & F E = F H; g conſtr. atque ang. B F H g = B F E. h 4. 1. h ergo G E = G H. Quod vero nulla alia G D ex puncto G æquetur ipſi G E, vel G H, jamjam oſtenſum eſt. Q. E. D.

PROP. VIII.



Si extra circulum ſumatur punctũ quodpiam A, ab eoq; puncto ad circulum deducantur quædam lineæ AI, AH, AG, AF, quarum una quidem A I per centrum K protendatur, reliquæ vero ut libet; in cavam peripheriam cadentium rectarum linearum maxima quidem eſt illa A I, quæ per centrum ducetur, aliarum autem ei quæ per centrum tranſit propinquior A H remotiore A G ſemper major eſt. In convexam vero peripheriam cadentiũ rectarum linearum minima quidem eſt illa A B, quæ inter punctum A, & diametrum B I interponitur; aliarum autem ea, quæ eſt minimæ propinquior A C remotiore A D ſemper minor eſt. Duæ autem tantum rectæ lineæ A C, A L æquales ab eo puncto in ipſum circulum cadunt, ad utraſq; partes minimæ A B, vel maximæ A I.

Ex centro K duc rectas KH, KG, KF; KC, KD, KE & fac ang. AKL = AKC.

1. AI (AK + KH) *a* $\sqsubset$ AH. Q.E.D. a 20.1.

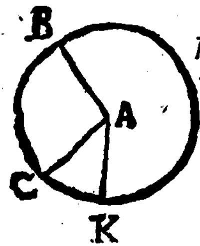
2. Latus AK commune eſt; & KH = KG; atque ang. AKH $\sqsubset$ AKG. *b* ergo baſ. AH b 24.1. $\sqsubset$ AG. Q.E.D.

3. KA *c* $\sqsupset$ KC + CA. aufer hinc inde æquales KC, KB, *d* erit AB $\sqsupset$ AC. c 20.1. d 5.ax.

4 AC + CK *e* $\sqsupset$ AD + DK. aufer hinc inde æquales CK, DK, *f* erit AC $\sqsupset$ AD. Q.E.D. e 21.1. f 5.ax.

5. Latus KA eſt commune & KL = KC; atque ang. AKL *g* = AKC, *h* ergo LA = CA. hisce vero nulla alia ęquatur, ex mox oſtenſis. ergo, &c. g conſt. h 4.1.

PROP. IX.



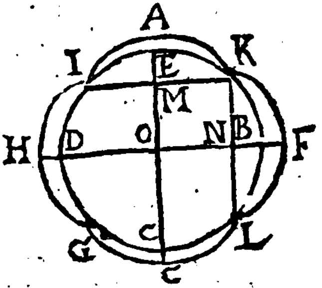
Si in circulo BCK acceptum fuerit punctum aliquod A, & ab eo puncto ad circulum cadant plures, quam duæ rectæ lineæ æquales AB, AC, AK, acceptum punctum A centrum eſt ipſius circuli.

Nam *a* à nullo puncto extra centrum plures quam duæ rectæ lineæ æquales duci poſſunt ad circumferentiam. Ergo A eſt centrum. Q.E.D. a 7.3.

PROP. X.

Circulus IAKBL circulum IEKFL in pluribus quam duobus punctis non ſecat.

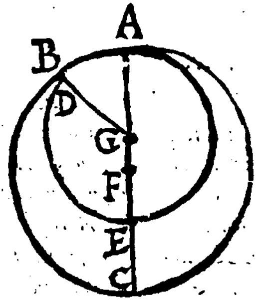
Secet



Secet, ſi fieri poteſt, in tribus punctis I K L. Junctę I K, K L. biſecentur in M & N. *a* Ambo circuli centrum habent in ſingulis perpendicularibus M C, N H, & proinde in earum interſectione O. ergo ſecantes circuli idem centrum habent. *b*. Q. E. N.

a cor. 1. 3.
b 5. 3.

PROP. XI.



Si duo circuli G A D E, F A B C ſeſe intus contingant, atque accepta fuerint eorum centra G, F; ad eorum centra adjuncta recta linea F G & producta, in A contactum circulorum cadet.

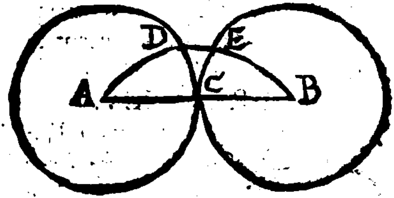
Si fieri poteſt, recta F G protracta ſecet circulos extra contactum A, ſic ut non F G A, ſed F G D B ſit recta linea. ducatur G A. Et quia G D *a* = G A, & G B *b* ¬ G A, (cum recta F G B tranſeat per F centrum majoris circuli) erit G B ¬ G D. *c* Q. E. A.

a 15 def. 1
b 7. 3.
c 9. ax.

PROP. XII.

Si duo circuli A C D, B C E ſeſe exterius con-

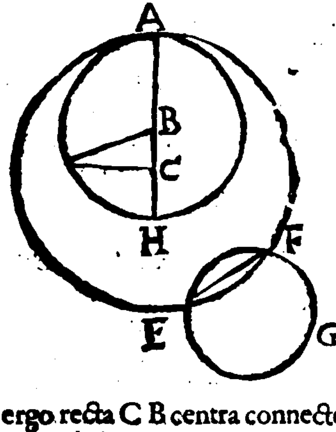
tin-



tingant, linea recta A B quæ ad eorum centra A, B adjungitur, per contactum C transibit.

Si fieri poteſt, ſit recta A D E B ſecans circulos extra contactum C in punctis D, E. Duc A C, C B. erit A D + E B (A C + C B) *a* ⊏ AD − E B. *b* Q. E. A. a 20. 1. b 9. ax.

PROP. XIII.



Circulus C A F circulum B A H non tangit in pluribus punctis, quam uno A, ſive intus, ſive extra tangat.

1. Tangat, ſi fieri poteſt intus in punctis A, H. *a* ergo recta C B centra connectens, ſi producatur cadet tam in A, quam in H. Quoniam igitur C H *b* = C A, & B H ⊏ C H. erit B A (*c* B H) ⊏ C A. Q. E. A. a 11. 3. b 15. def. 1. c 15. def. 1.

2. Sin dicatur exterius contingere in pun- d 9. ax.

c 2.3. ctis E & F, c ducta recta E F in utroque circulo erit. circuli igitur ſe mutuo ſecant, quod non ponitur.

PROP. XIV.

In circulo E A B C æquales rectæ lineæ A C B D, æqualiter diſtant a centro E. & quæ A C, B D æqualiter diſtant à centro, æquales ſunt inter ſe.

Ex centro E duc perpendiculares E F, E G; a quæ biſecabunt A C, D B. connecte E A, E B. a 3.3.

b 7.ax. 1. *Hyp.* A C = B D. ergo A F b = B G. ſed c 47.1. & 3.ax. & E A, = E B. ergo F Eq c = E Aq − A Fq = E Bq − B Gq c = E Gq. d ergo F E = E G. d Schol. 48. 1. Q. E. D.

2. *Hyp.* E F = E G. ergo A Fq c = E Aq − E Fq = E Bq − E Gq c = G Bq. ergo A F d = G B. e proinde A D = B C. Q. E. D.

PROP. XV.

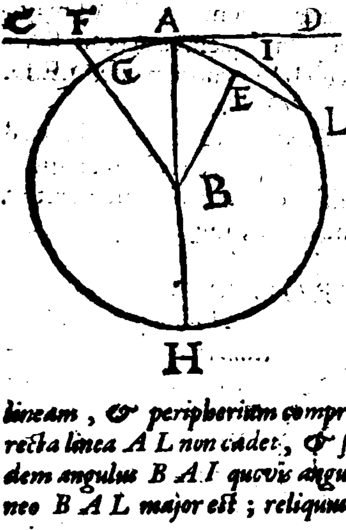
In circulo G A B C maxima quidem linea eſt diameter A D; aliarum autem centro G propinquior F E remotiore B C ſemper major eſt.

a 15. def. 1. 1. Duc G B, G C. Diameter A D (a GB + GC) b ⊏ B C Q. b 20.1. D. E.

2. Sit

2. Sit diſtantia G I ⊏ G H. accipe G N. = G H. per N duc K L perpendic. G I. Junge G K, G L. & quia G K = G B, & G L = G C; eſtque ang. K G L ⊏ B G C, c erit K L (F E) ⊏ B C. Q. E. D. c 24.1.

PROP. XVI.



Quæ C D ab extremitate diametri H A cujuſq; circuli B A L H ad angulos rectos ducitur, extra ipſum circulum cadet, & in locum inter ipſam rectam lineam, & peripheriam comprehenſum altera recta linea A L non cadet; & ſemicirculi quidem angulus B A I quovis angulo acuto rectilineo B A L major eſt; reliquus autem D A I minor.

1. Ex centro B ad quodvis punctum F in recta A C duc rectam B F. Latus B F ſubtendens angulum rectum B A F a majus eſt latere B A, quod opponitur acuto B F A. ergo cum B A (B G) pertingat ad circumſcrentiam, B F ulterius porrigetur, adeoque punctum F, & eadem ratione quodvis aliud rectæ A C, extra circulum ſitum erit. Q. E. D. a 19.1.

2. Duæ

2. Duc B E perpendic A L. Latus B A oppoſitum recto angulo B E A *b* majus eſt latere B E, quod acutum B A E ſubtendit: ergo punctum E, adeoque tota E A cadit intra circulum. Q. E. D. b 19.1.

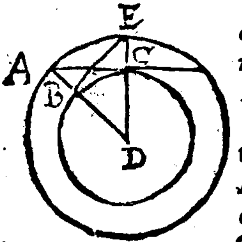
3. Hinc ſequitur angulum quemvis acutum, nempe E A D angulo contactus D A I majorem eſſe. Item angulum quemvis acutum B A L angulo ſemicirculi B A I minorem eſſe. Q. E. D.

Coroll.

Hinc, recta à diametri circuli extremitate ad angulos rectos ducta ipſum circulum tangit.

Ex hac propoſitione paradoxa conſequuntur, & mirabilia bene multa, quę vide apud interpretes.

PROP. XVII.



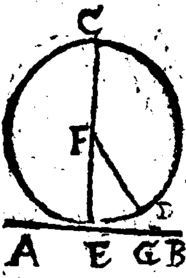
A dato puncto A rectam lineam A C ducere, quæ datum circulum D B C tangat.

Ex D dati circuli centro ad datum punctum A ducatur recta D A ſecans peripheriam in B. Centro D deſcribe per A alium circulum A E; & ex B duc perpendicularem ad A D, quæ occurrat circulo A E in E. duc E D occurrentem circulo B C in C. ex A ad C ducta recta tanget circulum D B C.

a 15.def.1 b 4.1. c cor.16.3 Nam D B *a* = D C, & D E *a* = D A, & ang. D communis eſt: *b* ergo ang. A C *D* = E B D, rect. *c* ergo A C tangit circulum C. Q. E. F.

PROP.

PROP. XVIII.

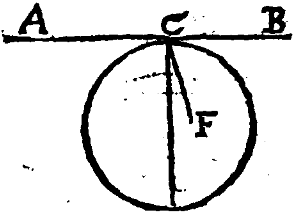


Si circulum FEDC tangat recta quæpiam linea AB, à centro autem ad contactum E adjungatur recta quadam linea FE; quæ adjuncta fuerit FE ad ipſam contingentem AB perpendicularis erit.

Si negas, ſit ex F centro alia quædam FG perpendicularis ad contingentem, *a* ſecabit ea circulum in D. Quum igitur ang. FGE rectus dicatur *b* erit ang. FEG acutus. *c* ergo FE (FD) ⊏ FG. *d* Q.E.A.

a 2.def.3
b cor.17.1
c 19.1.
d 9.ax

PROP. XIX.

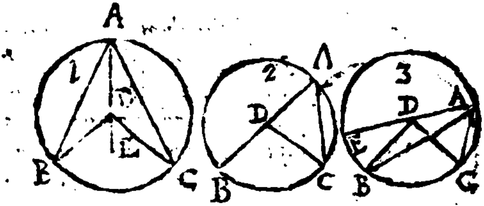


Si circulum tetigerit recta quæpiam linea AB, à contactu autem C recta linea CE ad angulos rectos ipſi tangenti excitetur, in excitata CE erit centrum circuli.

Si negas, ſit centrum extra CE in F, & ab F ad contactum ducatur FC. Igitur ang. FCB rectus eſt; & *a* proinde par angulo ECB recto per hypoth. *b* Q.E.A.

a 12.ax.
b 9.ax.

PROP. XX.

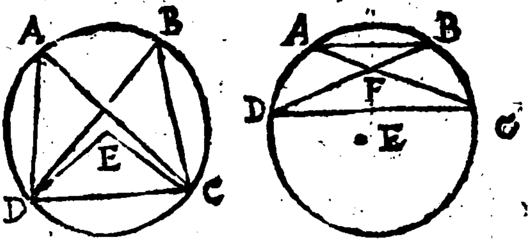


In circulo D. A B C, angulos B D C ad centrum duplex eſt anguli B A C ad peripheriam, cum fuerit eadem peripheria B C baſis angulorum.

Duc diametrum A D E.

a 32.1. b 5.1. Externus angulus B D E *a* = D A B + D B A *b* = 2 D A B. Similiter ang E D C = 2 D A C. ergo in primo caſu totus B D C = 2 B A C; ſed in tertio caſu *c* reliquus angulus B D C = 2 B A C. Q. E. D. c 20.ax.

PROP. XXI.



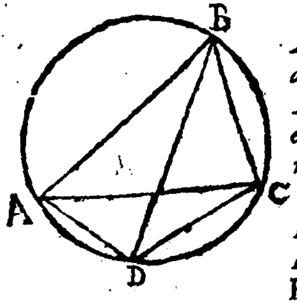
In circulo E D A C qui in eodem ſegmento ſunt anguli, D A C & D B C ſunt inter ſe æquales.

1. *Caſ.* Si ſegmentum D A B C ſemicirculo ſit majus, ex centro E, duc E D, E C. Eritq; 2. ang. A *a* = E *a* = 2. B. Q. E. D. a 20.3.

2. *Caſ.*

2. *Caſ.* Sin ſegmentum ſemicirculo majus non fuerit, ſumma angulorum trianguli A D F æquatur ſummæ angulorum in triangulo BCF. Demantur hinc inde AFD *b* = BFC, & ADB *c* = A C B, remanent D A C = D B C. Q.E.D. b 15. 1. c *per* 1. *caſ.*

PROP. XXII.



Quadrilaterorum A B C D in circulo deſcriptorum anguli A D C, A B C, qui ex adverſo, duobus rectis ſunt æquales.

Duc A C, B D. Ang. A B C + BC A + B A C *a* = 2 Rect. Sed BDA *b* = B C A, & B D C *b* = B A C. *c* ergo A B C + A D C = 2 Rect. Q.E.D. a 32. 1. b 21. 3. c 1. *ax.*

Coroll.

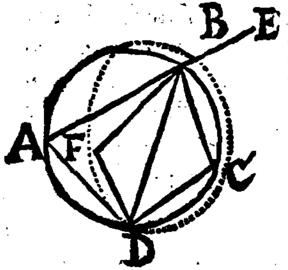
1. Hinc, ſi * A B unum latus quadrilateri in circulo deſcripti producatur, erit angulus externus E B C æqualis angulo interno A D C, qui opponitur ei A B C, qui eſt deinceps externo E B C. ut patet ex 13. 1. & 3. ax. * *vide ſeq. diagram.*

2. Item circa Rhombum circulus deſcribi nequit; quia adverſi ejus anguli vel cedunt duobus rectis, vel eos excedunt.

Scholium.

Si in quadrilatero A B C *D anguli* A, *&* C *qui ex adverſo duobus rectis æquantur, circa quadrilaterum circulus deſcribi poteſt.*

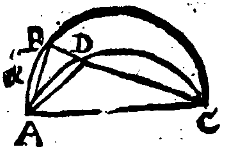
Nam



Nam circulus per quoſlibet tres angulos B,C,D tranſibit (ut patebit ex 5.4.) dico eundem per A tranſire. Nam ſi neges, tranſeat per F. ergo ductis rectis B F,FD, BD; ang. C + F *a* = 2. Rect. *b* = C + A *c* quare A = F. *d* Q.E.A.

a 22.3.
b *hyp.*
c 3.ax.
d 21.1.

PROP. XXIII.

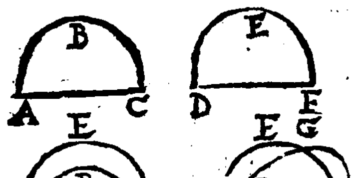


Super eadem recta linea A C duo circulorum ſegmenta A B C, A D C ſimilia & inæqualia non conſtituentur ad eaſdem partes.

Nam ſi dicantur ſimilia, duc C B ſecantem circumferentias in D, & B, & junge A D, ac A B. Quia ſegmenta ponuntur ſimilia, *a* erit ang. A D C = A B C *b* Q. E A.

a 10.def.3
b 16.1.

PROP. XXIV.

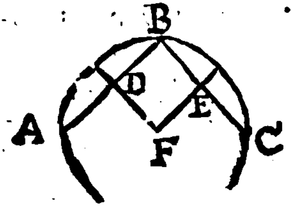


Super æqualibus rectis lineis A C, D F ſimilia circulorum ſegmenta A B C, D E F ſunt inter ſe æqualia.

Baſis A C ſuper-

ſuperpoſita baſi DF ei congruet, quia A C = D F. ergo ſegmentũ ABC congruet ſegmento DEF (alias enim aut intra cadet, aut extra, *a* atq; ita ſegmenta non erunt ſimilia, contra Hyp. aut ſaltem partim intra, partim extra, adeoque ipſum in tribus punctis ſecabit. *b* Q. E. A.) *c* proinde ſegmentum. A B C = D E F. Q.E.D. a 23.3. b 10.3. c 8. ax.

PROP. XXV.



Circuli ſegmento A B C dato, deſcribere circulum, cujus eſt ſegmentum.

Subtendantur utcunque duę rectę AB, BC, quas biſeca in D, & E. Ex D, & E duc perpendiculares D F, E F occurrentes in puncto F. Hoc erit centrum circuli.

Nam centrum *a* tam in D F, quam in E F exſiſtit, ergo in communi puncto F. Q.E.F. a cor. 1. 3

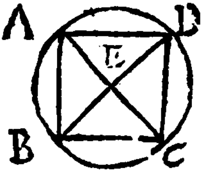
PROP. XXVI.

In æqualibus circulis G A B C, H D E F æquales anguli æqualibus peripheriis A C, D F inſi-

inſiſtunt, ſive ad centra G, H, ſive ad peripher. B, E conſtituti inſiſtant.

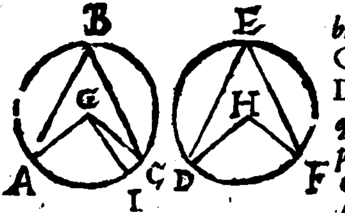
Ob circulorum æqualitatem, eſt G A = H
a 4.1. D, & G C = H F item per hyp. ang. G = H. *a*
b 20.3. ergo A C = D F. Sed & ang. B *b* = $\frac{1}{2}$ G = *c* $\frac{1}{2}$
c hyp. d 10. def. H *b* = E. *d* ergo ſegmenta ABC, DEF ſimilia,
3. e 24.3. *e* & proinde paria ſunt. *f* ergo etiam reliqua ſe-
f 3. ax. gmenta A C, D F æquantur. Q.E.*D*.

Schol.



In circulo A B C D, ſit arcus A B par arcui D C; erit A D parall. B C. Nam ducta A C, *a* erit ang. A C B = C A D. quare per 27. 1.
a 26.3.

PROP. XXVII.

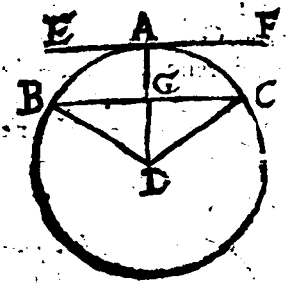


In æqualibus circulis, G A B C, H D E F, anguli qui æqualibus peripheriis A C, D F inſiſtunt, ſunt inter ſe æquales, ſive ad centra G, H, ſive ad peripherias B, E inſiſtant.

Nam ſi fieri poteſt, ſit alter eorum A G C
$\sqsubset$ D H F. fiatque A G I = D H F. ergo arcus
a 26.3. b hyp. A I *a* = D F *b* = A C. *c* Q.E.A.
c ax.

Schol.

Schol.



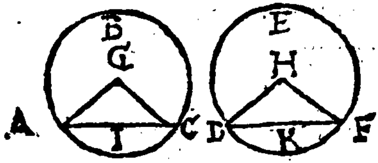
Linea recta E F, quæ ducta ex A medio puncto peripheriæ alicujus B C, circulum tangit, parallela eſt recta linea B C quæ peripheriam illam ſubtendit.

Duc è centro D ad contactum A rectam DA, & connecte D B, DC.

Latus D G commune eſt; & D B = D C, atque ang. BDA *a* = CDA (ob arcus BA, CA *b* æquales) *c* ergo anguli ad baſim DGB, DGC æquales, & *d* proinde recti ſunt. Sed interni anguli G A E, G A F *e* etiam recti ſunt. *f* ergo B C, E F ſunt parallelę. Q.E.D.

a 27. 3.
b *hyp.*
c 4. 1.
d 10. *def.* 1.
e *hyp.*
f 28. 1.

PROP. XXVIII.



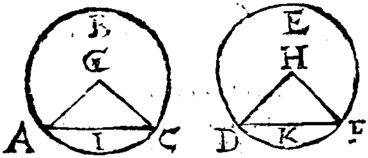
In æqualibus circulis G A B C, H D E F, *æquales rectæ lineæ A* C, *D F æquales peripherias auferunt; majorem quidem A B C majori D E F, minorem autem A I C minori D K F.*

E cen-

E centris G,H, duc G A, G C; & H D, H
F. Quoniam G A = H D, & G C = H F, atq;
a hyp. b 8.1. A C a = D F; b erit ang. G = H. c ergo arcus
c 26.3. A I C = DKF. d proinde reliquus ABC = D
d 3.ax. E F. Q. E. D.

Quod ſi ſubtenſa A C ſit ⊏ vel ⊐ D F, erit ſimili modo arcus A C ⊏ vel ⊐ D F.

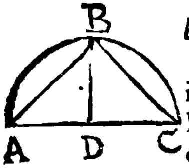
PROP. XXIX.



In æqualibus circulis G A B C, H D E F, æquales peripherias A B C, D E F æquales rectæ lineæ A C, D F ſubtendunt.

Duc G A, G C; & H D, H F. Quia G A
= H D; & G C = H F; & (ob arcus A C,
a hyp. DF a pares) etiam ang. G b = H; erit baſ. A C
b 27.3. = DF. Q.E.D.
c 4.1. Hęc & tres proximè præcedentes intelligantur etiam de eodem circulo.

PROP. XXX.



Datam peripheriam ABC biſariam ſecare.

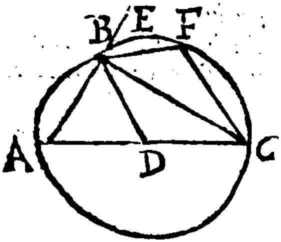
Duc A C; quam biſeca in D. ex D duc perpendicularem D B occurrentem arcui in B. Dico factum.

Jungantur enim A B, C B. Latus D B commune

mune eſt; & AD *a* = DC; & ang. ADB *b* = CDB. *c* ergo AB = BC. *d* quare arcus AB = BC. Q.E.F.

a conſt.
b 12.ax.
c 4.1.
d 28.3.

PROP. XXXI.



In circulo angulus ABC, *qui in ſemicirculo, rectus eſt; qui autem in majore ſegmento* BAC, *minor recto; qui vero in minore ſegmento* BFC, *major eſt recto. Et inſuper angulus majoris ſegmenti recto quidem major eſt, minoris autem ſegmenti angulus, minor eſt recto.*

Ex centro *D* duc DB. Quia DB = DA, erit ang. A *a* = DBA. pariter ang. DCB *a* = DBC. *b* ergo ang. ABC = A + ACB *c* = BBC, *d* proinde ABC, & EBC recti ſunt. Q.E.D. *e* ergo BAC acutus eſt. Q.E.D. ergo cum BAC + BFC *f* = 2. Rect. erit BFC obtuſus. denique angulus ſub recta CB, & arcu BAC major eſt recto ABC. factus vero ſub CB, & BFC peripheria minoris ſegmenti, recto EBC *g* minor eſt. Q.E.D.

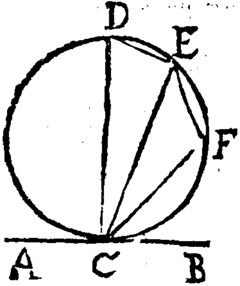
a 5.1.
b 2.ax.
c 32.1.
d 10.def 1.
e cor.17.1
f 22.3.
g 9.ax

Scholium.

In triangulo rectangulo ABC, *ſi hypotenuſa* AC *biſecetur in* D, *circulus centro* D, *per* A *deſcriptus tranſibit per B.* ut facile ipſe demonſtrabis ex hac, & 21.1.

PROP.

PROP. XXXII.

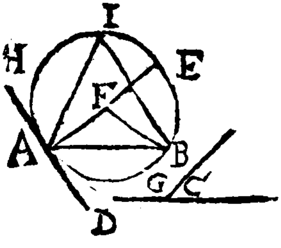


Si circulum tetigerit aliqua recta linea A B, à contactu autem producatur quædam recta linea C E *circulum ſecans : anguli E* C *B*, E C *A, quos ad contingentem facit, æquales ſunt iis, qui in alternis circuli ſegmentis conſiſtunt, angulis E D* C, *E F* C.

Sit C *D* latus anguli E D C perpendiculare
a 26.5. ad A B (*a* perinde enim eſt) *b* ergo C *D* eſt
b 19.3. diameter. *c* ergo angul. C E D in ſemicirculo
c 31.3. d 32.1. rectus eſt. *d* ergo ang. D + D C E = Rect. *e*
e *conſtr.* f 3.ax. = E C B + D C E. *f* ergo angul. D = E C B
Q. E. D.

g 13.1. Cum igitur angul. E C B + E C A *g* = 2
h 22.5. Rect. *h* = *D* + F ; aufer hinc inde æquales
k 3.ax. E C B, & *D*, *k* remanent E C A = F. Q. E. *D*.

PROP. XXXIII.

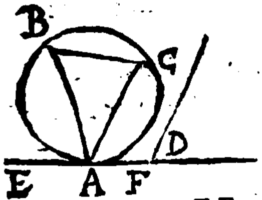


Super data recta linea A B deſcribere circuli ſegmentum A I E B, quod capiat angulum A I B æqualem dato angulo rectilineo C.

a Fac angul. B A *D* = C. per A duc A E perpendicularem ad H *D*. ad alterum terminum datæ A B fac angul. A B F = B A F. cujus alterum latus ſecet A E in F. centro F per A deſcribe circulum, quod tranſibit per B (quia ang. F B A *b* = F A B, *c* ideoque F B = F A;) ſegmentum A I B eſt id quod quæritur. a 23. 1. b *conſtr.* c 6. 1.

Nam quia H *D* diametro A E perpendicularis eſt, *d* tangit H *D* circulum, quem ſecat A B. ergo ang. A I B *e* = B A *D* *f* = C. Q. E. E. d *cor.* 16. 3. e 32. 3. f *conſtr.*

PROP. XXXIV.



A dato circulo A B C ſegmentum A B C abſcindere capiens angulum B æqualem dato angulo rectilineo D.

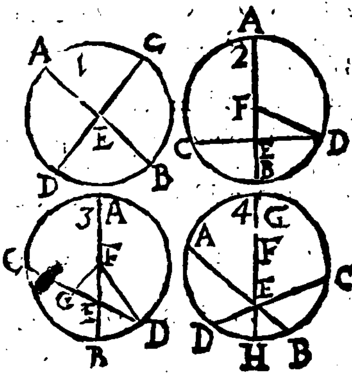
a Duc rectam E F, quę tangat rectum circulum in A. *b* ducatur item A C faciens ang. F A C = *D*. Hæc auferet ſegmentum A B C capiens angulum B *c* = C A F *d* = *D*. Q. E. F. a 17. 3. b 23. 1. c 32. 3. d *conſtr.*

PROP. XXXV.

Si in circulo F B C A duæ rectæ lineæ A B, D C ſeſe mutuo ſecuerint, rectangulum comprehenſum ſub ſegmentis A E, E B unius, æquale eſt ei quod ſub ſegmentis C E, E D alterius comprehenditur, rectangulo.

Caſ. 1. Si rectę ſeſe in centro ſecent, res clara eſt.

2, Si



2 Si una AB tranſeat per centrũ F, & reliquã C D biſecet, duc FD. Eſtque Rectang. A E B + F Eq *a* = FBq *b* = F Dq *c* = E Dq + F Eq *d* = C E D + F Eq. *e* ergo Rectang. A E B = C E D. Q. E. D.

a 5.2.
b ſch.48.1
c 47.1.
d hyp.
e 3.ax.

3. Si una A B diameter ſit, alteramque C D ſecet inæqualiter, biſeca C D per F G perpendicularem ex centro.

Æquantur iſta
Rectang. ABB + FEq.
f 5.2. *f* FBq (FDq)
g 47.1. *g* FGq + GDq.
h 5.2. FGq + *h*GEq + Rectang. CED.
k 47.1. *k*FEq + CED.

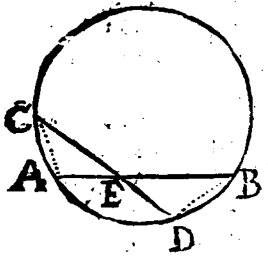
l 3.ax. *l* Ergo Rectang. AEB = CED.

4. Si neutra rectarum A B, C D per centrum tranſeat, per interſectionis punctum E duc diametrum G H. Per modo demonſtrata Rectang. AEB = GEH = CED. Q.E.D.

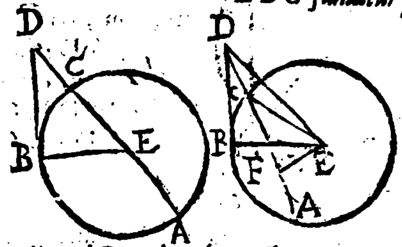
Facilius ſic, & univerſaliter; connecte AC & B D. atque ob angulos *a* CEA, D E B, *b* ipſosque C, B (ſuper eodem arcu A D) pares; trigona CEA EBD, *c* æquiangula ſunt. *d* ergo

a 15.1.
b 21.3.
c cor.32.1
d 4.6.

go C E . E A :: E B . E D. e proinde $CE \times ED = EA \times EB$. Q.E.D. e. 16. 6.



PROP. XXXVI.

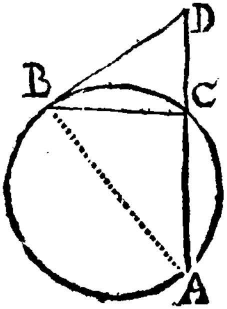


Si extra circulum E B C ſumatur punctum aliquod D, ab eoq; puncto in circulum cadant duæ rectæ lineæ D A, D B; quarum altera D A circulum ſecet, altera vero D B tanget; Quod ſub tota ſecante D A, & exterius inter punctum D, & convexam peripheriam aſſumpta D C comprehenditur rectangulum, æquale erit ei, quod à tangente D B deſcribitur, quadrato.

1. *Caſ.* Si ſecans A D tranſeat per centrum E, junge E B; *a* faciet hæc cum D B rectum angulum; quare $DBq + EBQ$ (ECq) *b*$=$ EDq *c* $= AD \times DC + ECq$ *d* ergo $AD \times DC = DBq$. Q.E.D. a 18. 3. b 47. 1. c 6. 2. d 3. ax.

2. *Caſ.* Sin A D per centrum non tranſeat, duc E C, E B, E D; atque E F perpend. A D, quare *a* biſecta eſt A C in F. a 3. 3.

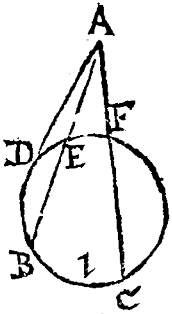
b 47.1. Quoniam igitur B D Q + E Bq *b* = D Eq
c 6.2. *b* = E Fq + F Dq *c* = E Fq + A D C + FCQ
d 47.1. *d* = A D C + C Eq (E B q); *e* erit B Dq = A
e 3.ax. D C. Q. E. D.



Facilius ac univerſalius ſic;

Duc A B, & B C.
a 32.3. **àc òb angulos A, D B**
b 32.1. **C *a* pares, & D com-**
c 4.6. **munem, triangula B**
d 17.6. **D C, A D B *b* ęqui-**
angula ſunt. *c* ergo A
D. D B : : D B. C D *d*
quàre A D x D C = D
Bq. Q. E. D.

Coroll.

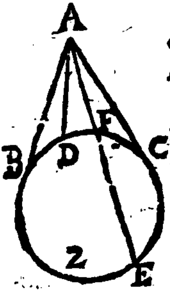


1. Hinc ſi à puncto quovis A extra circulum aſſumpto, plurimæ lineæ rectę A B, A C circulum ſecantes ducantur, rectangula comprehenſa ſub totis lineis A B, A C, & partibus externis A E, A F inter ſe ſunt æqualia. Nam ſi ducatur tangens A D; erit C A F = A Dq *a* = B A E.

a 36.3. 2. Conſtat etiam duas rectas AB, AC ab eodem puncto A ductas, quæ circulum tangant, inter ſe æquales eſſe.

Nam ſi ducatur A E ſecans circulum; erit
a 36.3. A Bq *a* = E A F *b* = A Cq.
b 36.3. 3. Perſpicuum quoque eſt ab eodem puncto A ex-

A extra circulum aſſumpto, duci tantum poſſe duas lineas, AB, AC quæ circulum tangant.

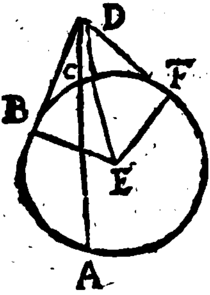


Nam ſi tertia A *D* tangere dicatur, erit A D *c* = A B *c* = A C. *d* Q.F.N. c 2.cor. d 1.8.

4. E contra conſtat, ſi duę rectæ ęquales AB, AC ex puncto quopiam A in convexam peripheriam incidant, & earum una A B circulum tangat, alterm quoque circulum tangere.

Nam ſi fieri poteſt, non A C, ſed altera A D circulum tangat. ergo A D *e* = A C *f* = A B. *g* Q.E.A. e 2.cor. f hyp. g 1.9.

PROP. XXXVII.



Si extra circulum EBF ſumatur punctum D, ab eoq; in circulum cadant duæ recta lineæ D A, D B; quarum altera D A circulum ſecet, altera D B in eum incidat; ſit autem quod ſub tota ſecante D A, & exterius inter punctum, & convexam peripheriam aſſumpta D C, comprehenditur rectangulum, æquale ei, quod ab incidente D B deſcribitur quadrato, incidens ipſa D B circulum tanget.

Ex D *a* ducatur tangens D F; atque ex E centro duc E D, E B, E F. Quia D Bq *b* = A D C *c* = D Fq, *d* erit D B = D F. Sed E B = E F, & latus E D commune eſt; *e* ergo ang. a 17.3. b hyp. c 36.3. d 1.ax. & ſch. ax. 1. e 8.1.

ang. EBD = EFD. Sed EFD rectus eſt, *f* ergo EBD etiam rectus eſt. *g* ergo DB tangit circulum. Q.E.D.

Coroll.

Hinc, *h* ang. EDB = EDF.

LIB. IV.

DEFINITIONES.

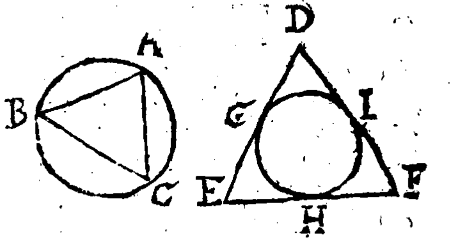
I.

Figura rectilinea in figura rectilinea inſcribi dicitur, cum ſinguli ejus figuræ, quæ inſcribitur, anguli ſingula latera ejus in qua inſcribitur, tangunt.

Sic triangulum DEF eſt inſcriptum in triangulo ABC.

II. Similiter & figura circa figuram deſcribi dicitur, cum ſingula ejus, quæ circumſcribitur, latera ſingulos ejus figuræ angulos tetigerint, circa quam illa deſcribitur.

Ita triangulum ABC eſt deſcriptum circa triangulum DEF.



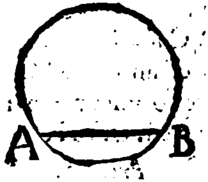
III. Fi-

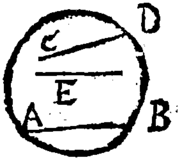
III. Figura rectilinea in circulo inſcribi dicitur, cum ſinguli ejus figurę, quæ inſcribitur, anguli tetigerint circuli peripheriam.

IV. Figura vero rectilinea circa circulum deſcribi dicitur, cum ſingula latera ejus, quæ circumſcribitur, circuli peripheriam tangunt.

V. Similiter & circulus in figura rectilinea inſcribi dicitur, cum circuli peripheria ſingula latera tangit ejus figurę, cui inſcribitur.

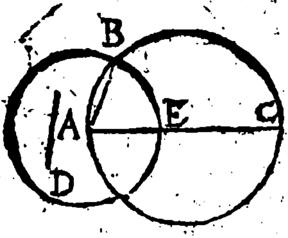
VI. Circulus autem circa figuram deſcribi dicitur, cum circuli peripheria ſingulos tangit ejus figuræ, quam circumſcribit, angulos.





VII. Recta linea in circulo accommodari, ſeu coaptari dicitur, cum ejus extrema in circuli peripheria fuerint; ut recta linea A B.

PROP. I. *Probl.* 1.

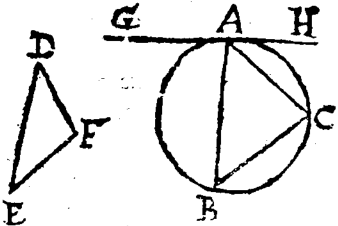


In dato circulo A B C rectam lineam A B accommodare æqualem datæ rectæ lineæ D, *quæ circuli diametro A C non ſit major.*

Centro A, ſpatio A E = D a deſcribe circulum a 3. poſt. & 3. 1.

b 15.def. 1. lum dato circulo occurrentem in B. Erit ducta
A B *b* = A E *c* = D. Q.E.F.
c conſt.

PROP. II. *Probl.* 2.

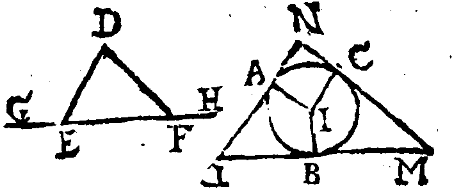


In dato circulo A B C triangulum A B C deſcribere dato triangulo D E F æquiangulum.

a 17.3. Recta G H circulum datum *a* tangat in A. *b*
b 23.1. Fac ang. H A C = E; *b* & ang. G A B = F, &
c 32.3. junge B C. Dico factum.

d conſt. Nam ang. B *c* = H A C *d* = E; & ang. C *c*
e 32.1. = G A B *d* = F; *e* quare etiam ang. BAC = D. ergo triang. B A C circulo inſcriptum triangulo D E F æquiangulum eſt. Q. E. F.

PROP. III. *Probl.* 3.



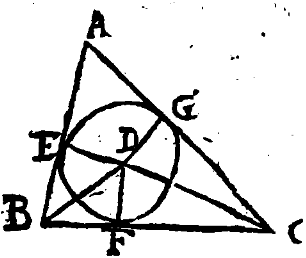
Circa datum circulum I A B C triangulum L N M deſcribere, dato triang. D E F æquiangulũ.

a 23.1. Produc latus EF utrinque. *a* Fac ad centrum
I ang. AIB=DEG. & ang. BIC=DFH. deinde
b 17.3. in punctis A, B, C circulum *b* tangant tres rectæ LN, LM, MN. Dico factum.

Nam

Nam quod coibunt rectę L N, L M, M N,
atque ita triangulum conſtituent, patet; *c* quia c 13. ax.
anguli L A I, L B I *d* recti ſunt, adeoque ducta d 18. 3.
A B angulos faciet L A B, L B A duobus rectis
minores. Quoniam igitur ang. A I B + L *e* = 2 e Schol. 32. 1.
Rect. *f* = D E G + D E F; & A I B *g* = *D* E f 13. 1.
G *h* erit ang. L = DEF. Simili argumento ang. g conſt.
M = D F E. *k* ergo etiam ang. N = D. ergo h 3. ax. k 32. 1.
triang. L N M circulo circumſcriptum dato E
D F eſt æquiangulum. Q.E.F.

PROP. IV. *Probl.* 4.



In dato triangulo A B C circulum E F G inſcribere.

Duos angulos B, & C *a* biſeca rectis B D, a 9. 1.
C D coeuntibus in D. Ex D
b duc perpendiculares D E, D F, D G. circulus centro D b 12. 1.
per E deſcriptus tranſibit per G & F, tangetq;
tria latera trianguli.

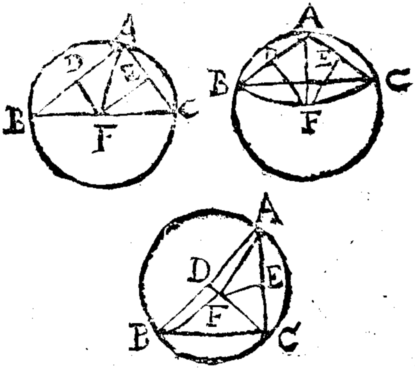
Nam ang. D B E *c* = DBF; & ang. D E B *d* c conſt.
= *D* F B; & latus *D* B commune eſt: *e* ergo d 12. ax. e 26. 1.
DE = *D*F. Simili argumento DG = DF. Circulus igitur centro D deſcriptus tranſit per E,
F, G; & cum anguli ad E, F, G ſint recti, tangit
omnia trianguli latera. Q.E.F.

Schol.

Hinc, *cognitis lateribus trianguli, invenientur* Petr. Herig.
eorum ſegmenta, quæ fiunt à contactibus circuli
inſcripti. Sic,

Sit AB 12, AC 18, BC 16. Erit $AB + BC = 28$, ex quo ſubduc $18 = AC = AE + FC$, remanet $10 = BE + BF$. ergo BE, vel $BF = 5$. proinde FC, vel $CG = 11$. quare GA, vel $AE = 7$.

PROP. V. *Probl.* 5.



Circa datum triangulum A B C circulum F A B C deſcribere.

a 10.& 11.1. Latera quævis duo BA, AC *a* biſeca perpendicularibus DF, EF concurrentibus in F. Hoc erit centrum circuli.

Nam ducantur rectæ FA, FB, FC. Quoniam
b conſtr. AD *b* $= DB$; & latus DF commune eſt, &
c conſtr. & 12.ax. ang. FDA *c* $= FDB$, *d* erit $FB = FA$ eodem
d 4.1. modo $FC = FA$. ergo circulus centro F per dati trianguli angulos B, A, C tranſibit. Q.E.F.

Coroll.

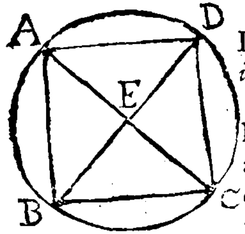
† 3 13. † Hinc, ſi triangulum fuerit acutangulum, centrum cadet intra triangulum; ſi rectangulum

ſum, in latus recto angulo oppoſitum; ſi deniq; obtusangulum, extra triangulum.

Schol.

Eadem methodo deſcribetur circulus, qui tranſeat per data puncta, non in una recta linea exiſtentia.

PROP. *Probl.* 6.



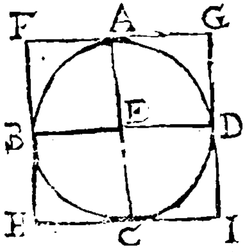
In dato circulo EABCD quadratum ABCD inſcribere.

a Duc diametros A C, BD ſe mutuo ſecantes ad angulos rectos in centro E. junge harum terminos rectis AB,B C, C D,DA. Dico factum. a 11.1.

Nam quia 4 anguli ad E recti ſunt, *b* arcus, & *c* ſubtenſæ A B,BC,CD, DA pares ſunt. ergo A B C D ęquilaterum eſt; ejusque omnes anguli in ſemicirculis, adeoque *d* recti ſunt. *e* ergo A B C D eſt quadratum, dato circulo inſcriptum. Q.E.F.

b 26.3.
c 29.3.
d 31.3.
e 20. *def.* 1.

PROP. VII. *Probl.* 7.



Circa datum circulum EABCD quadratum FHIG deſcribere.

Duc diametros AC, B D ſe mutuo ſecantes perpendiculariter. per harum extrema *a* duc tangentes concurrentes in F,H,I,G. Dico factum. a 17.3.

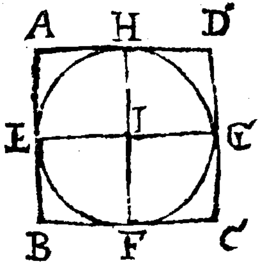
b 18.3. ctum. Nam ob angulos ad A,& C *b* rectos, *c* erit
c 28.1. F G parall. H I. eodem modo F H. parall.
G I. ergo F H I G eſt parallelogrammum ; &
quidem rectangulum. ſed & æquilaterum, quia
d 34. 1. e 15.def. 1. F G *d* = H I *d* = B D *e* = C A *d* = F H *d* = G I.
f 29.def.1 quare FHIG eſt *f* quadratum, dato circulo circumſcriptum. Q.E.F.

Schol.

Quadratum ABCD circulo circumſcriptum, duplum quadrati E F G H circulo inſcripti.

Nam rectang. H B = 2 H E F. & H D = 2 H G F. per 41.1.

PROP. VIII. *Probl.* 8.



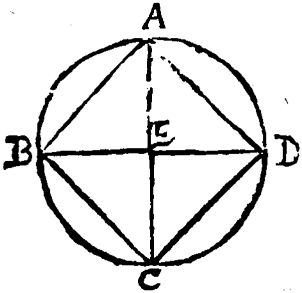
In dato quadrato ABCD circulum I EFGH inſcribere.

Latera quadrati biſeca in punctis H, E, F, G ; junge H F, E G ſeſe ſecantes in I, circulus centro I per H deſcriptus quadrato inſcribetur.

a 7.ax. Nam quia A H , B F *a* pares ac *b* parallelæ
b 34.1. c 33.1. ſunt, *c* erit A B parall. H F parall. D C. eodem
modo A *D* parall. E G parall. BC. ergo I·A,
ID, IB, IC ſunt parallelogramma. Ergo A H *d*
d 7.ax. e 34.1. = A E *e* = H I = E I = I F = I G. Circulus
igitur centro I per H deſcriptus tranſibit per
H,

H,E,F,G,tanque quadrati latera,cum anguli ad H,E,F,G ſunt recti. Q.E.F.

PROP. IX. *Probl. 9.*



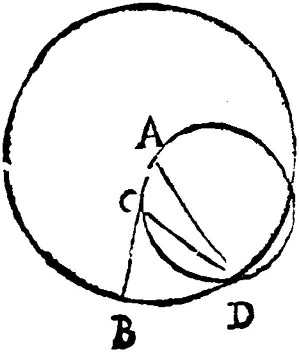
Circa datum quadratum A B C D circulum E A B C D deſcribere.

Duc diametros A C,B *D* ſecantes in E. Centro E per A deſcribe circulum. Is dato quadrato circumſcriptus eſt.

Nam anguli A B D, & B A C *a* ſemirecti ſunt; *b* ergo E A = E B. eodem modo EA = *D D* = E C. Circulus igitur centro E deſcriptus per A,B,C,*D* dati quadrati angulos tranſit. Q.E.F.

a 4. Cor. 32.1.
b 6.1.

PROP. X. *Probl. 10.*

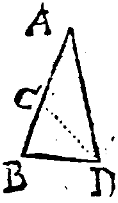


Iſoſceles triangulum A B D conſtituere, quod habeat utrumq; eorum, quæ ad baſim ſunt angulorum B, & A D B duplum reliqui A.

Acci-

Acci-

a 11.2. Accipe quamvis rectam A B, quam a ſeca in C, ita ut A B x B C = ACq. Centro A per B
b 1.4. deſcribe circulum A B D; in hoc b accommoda B D = A C, & junge A D. erit triang. ABD quod quæritur.



Hæc conſtructio Analytice indagatur ſic; Factum ſit; & angulum B D A biſecet recta D C.
a 1.6. a ergo D A. D B :: C A. C B. item
b constr. ob angul. C D A b = $\frac{1}{2}$ A D B c =
c hyp. A, d eſt C A = D C. ac ob ang. D
d 6.1. C B e = A + C D A = 2 A e =
e 32.1. B, d erit D B = D C. f. ergo D B
f 1.ax. = C A. proinde D A. (e B A.) C A :: C A.
g 17.6. C B. g unde B A x C B = C Aq.

Nam duc DC; & per CDA a deſcribe circulum. Quoniam A B x B C = A Cq. b liquet B D tangere circulum. A C D, quem ſecat C D. c ergo ang. B D C = A. ergo ang. BDC + C D A d = A + C D A = B C D. ſed B D C + e C D A = BDA f = C B D g ergo ang. B C D = C B D. ergo D C h = D B k = A C. l quare ang. C D A = A = B D C. ergo A D B = 2 A = A B D. Q. E. F.

a 5.4.
b 37.3.
c 32.3.
d 2.ax.
e 32.1.
f 5.1.
g 1.ax.
h 6.1.
k constr.
l 5.1

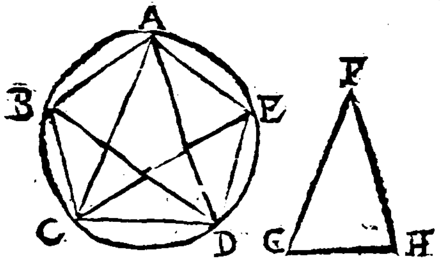
Coroll.

m 32.1. Cum omnes anguli A, B, D m conficiant $\frac{5}{5}$ 2 Rect. (2 Rect.) liquet A eſſe $\frac{1}{5}$ 2 Rect.

PROP. XI. *Probl.* 11.

In dato circulo A B C D E *pentagonum æquilaterum & æquiangulum* A B C D E *inſcribere.*

a De-



a Deſcribe triangulum Iſoſceles F G H, habens utramque angulorum ad baſim duplum anguli ad verticem. *b* Huic æquiangulum C A D inſcribe circulo. Angulos ad baſim A C D, & A D C *c* biſeca rectis D B, C E occurrentibus circumferentiæ in B, & E. connecte rectat C B, B A, A E, E *D*. Dico factum.

a 10.4.
b 2.4.
c 9. 1

Nam ex conſtr. liquet quinque angulos C A *D*, C D B, B *D* A, D C E, E C A pares eſſe; quare *d* arcus *e* & ſubtenſæ D C, C B, BA, A E, D E æquantur. Pentagonum igitur ęquilaterum eſt. Eſt verò etiam ęquiangulum, *f* quia ejus anguli B A E, A E D, &c. inſiſtunt arcubus *g* æqualibus B C D E, A B C D, &c.

d 26.3.
e 29.3.
f 27.3.
g 3.ax.

Hujus problematis praxis facilior tradetur ad 10.13.

Coroll.

Hinc, angulus pentagoni æquilateri & ęquianguli æquatur $\frac{3}{5}$ 2 Rect. vel $\frac{6}{5}$ Rect.

Scholium.

Univerſaliter figuræ imparium laterum inſcribuntur circulo beneficio triangulorum Iſoſcelium, quorum anguli æquales ad baſim multiplices

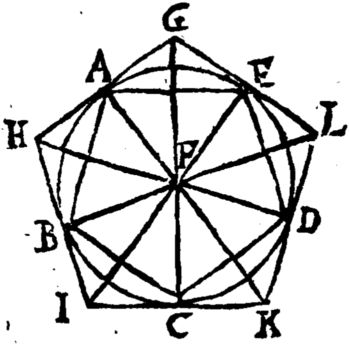
Pet. Herig.

ces

ces ſunt eorum, qui ad verticem ſunt, angulorum; parium vero laterum figurā in circulo inſcribuntur ope Iſoſcelium triangulorum, quorum anguli ad baſim multiplices ſeſquialteri ſunt eorum, qui ad verticem ſunt, angulorum.

Ut in triangulo Iſoſcele C A B, ſi ang. A = 3 G = B; A B erit latus Heptagoni. Si A = 4 C; erit A B latus Enneagoni, &c. Sin vero A = $1\frac{1}{2}$ C, erit A B latus quadrati. Et ſi A = $2\frac{1}{2}$ C, ſubtendet AB ſextam partem circumferentiæ : pariterque ſi A = $3\frac{1}{2}$ G; erit A B latus octagoni, &c.

PROP. XII. *Probl.* 12.



Circa datum circulum FABCDE pentagonum æquilaterū & æquiangulum HIKLG deſcribere.

a 11.4.

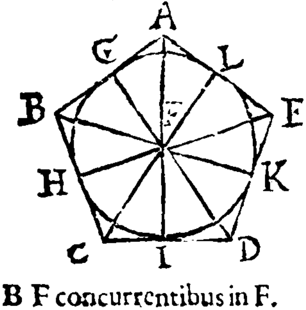
a Inſcribe pentagonum A ACDE æquilaterum & æquiangulum; duc è centro rectas FA, FB, FC, FD, FE, iiſque

toti-

totidem perpendiculares GAH, HBI, ICK, K
DL, LEG concurrentes in punctis H, I, K, L,
G. Dico factum. Nam quia GA, GE ex uno
puncto G *b* tangunt circulum, *c* erit GA = G b cor. 16.
E. *d* ergo ang. AFA = GFE. ergo ang. AF 3.
E = 2 GFA. eodem modo ang. AFH = H c 1. cor.
FB; & proinde ang. AFB = 2 AFH. Sed ang. 36. 3. d 8. 1.
AFE *e* = AFB. *f*. ergo ang. GFA = AFH.
ſed & ang. FAH *g* = FAG; & latus FA eſt e 27. 3. f 7. ax.
commune, *h* ergo HA = AG = GE = EL, g 12. ax.
&c. *k* ergo HG, GL, LK, KI, IH latera h 26. 1. k 2. ax.
pentagoni æquantur: ſed & anguli etiam, ut-
pote *l* æqualium AGF, AHF, &c. dupli; er- l 32. 1.
go, &c. *Coroll.*

Eodem pacto, ſi in circulo quæcunque figura æquilatera & æquiangula deſcribatur, & ad extrema ſemidiametrorum ex centro ad angulos ductarum, excitentur lineæ perpendiculares, hæ perpendiculares conſtituent etiam figuram totidem laterum & angulorum ęqualium circulo circumſcriptam.

PROP. XIII. *Probl.* 13.



In dato pentagono æquilatero & æquiangulo ABCDE circulum FGHK inſcribere.

Duos pentagoni angulos A, & B *a* a 9. 1.
biſeca rectis AF,
BF concurrentibus in F.

Ex F duc perpendiculares F G, F H, F I, F K, F L. Circulus centro F per G deſcriptus tanget omnia pentagoni latera.

b *hyp.* Duc F C, F D, F E. Quoniam B A *b* = B
c *conſt.* C; & latus B F commune eſt; & ang. F B A *c*
d *4. 1.* = F B C, *d* erit A F = F C; & ang. F A B =
e *hyp.* F C B. Sed ang. F A B *e* = ½ B A E *e* = ½ B C
D. ergo ang. F C B = ½ B C D. eodem modo
anguli totales C, D, E omnes biſecti ſunt. Quũ
f *12. ax.* igitur ang. F G B *f* = F H B, & ang. F B H =
g *26. 1.* F B G. & latus F B ſit commune, *g* erit F G
= F H. ſimiliter omnes F H, F I, F K, F L, F
G æquantur. Ergo circulus centro F per G de-
h *cor. 16. 3.* ſcriptus tranſit per H, I, K, L; *h* tangitque pen-
tagoni latera, cum anguli ad ea puncta ſint re-
cti. Q.E.F.

Coroll.

Hinc, ſi duo anguli proximi figuræ æquilaterę & æquiangulæ biſecentur, & à puncto, in quo coeunt lineę angulos biſecantes, ducantur rectæ lineæ ad reliquos figuræ angulos, omnes anguli figuræ erunt biſecti.

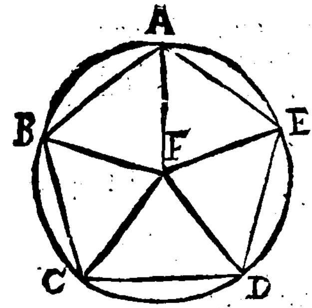
Schol.

Eadem methodo in qualibet figura ęquilatera & æquiangula circulus deſcribetur.

PROP. XIV. *Probl.* 14.

Circa datum pentagonum æquilaterum & æquiangulum A B C D E *circulum* F A B C D *deſcribere.*

Duos pentagoni angulos biſeca rectis A F, B F concurrentibus in F. Circulus centro F per A deſcriptus pentagono circumſcribitur.



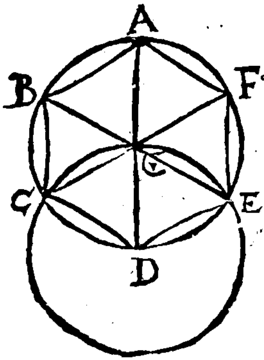
Ducantur enim F C, F D, F E, *a* Biſecti itaq; ſunt anguli C, D, E. *b* ergo F A, F B, F C, F D, F E æquantur. ergò circulus centro F deſcriptus, per A, B, C, D, E, pentagoni angulos tranſibit. Q. E. F. a cor. 13. 4. b 6. 1.

Schol.

Eadem arte circa quamlibet figuram æquilateram & æquiangulam circulus deſcribetur.

PROP. XV. *Probl.* 15.

In dato circulo G A B C D E F hexagonum & æquilaterum & æquiangulum A B C D E F inſcribere.



Duc diametrum A D; centro D per centrum G deſcribe circulum, qui datum ſecet in C, & E. duc diametros C F, E B. junge A B, B C, C D, D E, E F, F A. Dico factum.

Num ang. C G D $a = \frac{1}{3}$ 2 Rect. $a =$ D G E a 32.5.
$b =$ A G F $b =$ AGB. c ergo BGC $= \frac{1}{3}$ 2 Rect. b 15. 1. c cor. 13. 1.
$=$ F G E. d ergo arcus e & ſubtenſæ A B, B C, d 26. 3.
G C D, e 29. 3.

CD, DE, EF æquantur. Hexagonum igitur æquilaterum eſt: ſed & æquiangulum, *f* quia ſinguli ejus anguli arcubus inſiſtunt æqualibus. Q.E.F. *f 27.3.*

Coroll.

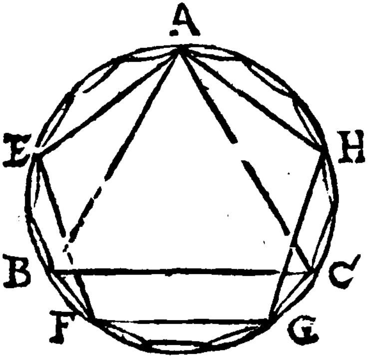
1. Hinc latus Hexagoni circulo inſcripti ſemidiametro æquale eſt.

2. Hinc facile triangulum æquilaterum A C E in circulo deſcribetur.

Schol. Probl.

Andr. Tacq. *Hexagonum ordinatum ſuper data recta C D ita conſtrues.* *a* Fac triangulum C G D æquilaterum ſuper data C D. centro G per C, & D deſcribe circulum. Is capiet Hexagonum ſuper data C D. *a 1. 1.*

PROP. XVI. *Probl.* 16.



In dato circulo AEBC *quindecagonum æquilaterũ & æquiangulum inſcribere.*

Dato circulo *a* inſcribe pentagonũ æquilaterũ AEFGH; *b* itemque triangulum æquilaterũ A B C, erit B F latus quindecagoni quæſiti. *a 11.4.* *b 3.4.*

Nam arcus A B *c* eſt $\frac{1}{3}$ vel $\frac{5}{15}$ peripheriæ, cujus *c constr.*

jus

jus A F eſt $\frac{2}{5}$ vel $\frac{6}{15}$. ergo reliquus B F = $\frac{1}{15}$ periph. ergo quindecagonum, cujus latus B F, æquilaterum eſt; ſed & æquiangulum, *d* cum ſinguli ejus anguli arcubus inſiſtant æqualibus, quorum unusquisque eſt $\frac{13}{15}$ totius circumferentiæ. ergo, &c. d 27.3.

Schol.

Circulus dividitur Geometrice in partes
- 4, 8, 16, &c. per 6, 4, & 9, 1.
- 3, 6, 12, &c. per 15, 4, & 9, 1.
- 5, 10, 20, &c. per 11, 4, & 9, 1.
- 15, 30, 60, &c. per 16, 4 & 9, 1.

Cæterum diviſio circumferentiæ in partes datas etiamnum deſideratur; quare pro figurarũ quarumcunque ordinatarum conſtructionibus ſæpe ad mechanica artificia recurrendum eſt, propter quæ Geometræ practici conſulendi ſunt.

LIB. V.

DEFINITIONES.

I.

PArs eſt magnitudo magnitudinis, minor majoris, cum minor metitur majorem.

II. Multiplex autem eſt major minoris, cum minor metitur majorem.

III. Ratio eſt duarum magnitudinum ejusdem generis mutua quædam ſecundum quantitatem habitudo.

In omni ratione ea quantitas, quæ ad aliam refertur, dicitur antecedens rationis; ea vero, *ad quam alia refertur, conſequens rationis dici ſolet. ut in ratione 6 ad 4; antecedens eſt 6, & conſequens 4.*

Nota.

Cujusque rationis quantitas innoteſcit dividendo antecedentem per conſequentem. ut ratio 12. ad 5. effertur $\frac{12}{5}$. item quantitas rationis A ad B eſt $\frac{A}{B}$. Quare non raro brevitatis cauſa, quantitates rationum ſic deſignamus, $\frac{A}{B} \sqsubset$, vel $=$ vel $\sqsupset \frac{C}{D}$; hoc eſt, ratio A ad B major eſt ratione C ad D, vel ei æqualis, vel minor. Quod probe animadvertat, quisquis hæc legere volet.

Rationis, ſive proportionis ſpecies, ac diviſiones vide apud interpretes.

IV. Proportio vero eſt rationum ſimilitudo.

Rectius quæ hic veſtitur proportio, proportionalitas, ſive analogia dicitur; nam proportio idem denotat quod ratio, ut plerisq; placet.

V. Rationem habere inter ſe magnitudines dicuntur, quæ poſſunt multiplicatæ ſe mutuo ſuperare.

E, 12.	A, 4. B, 6.	G, 24.
F, 30.	C, 10. D, 15.	H, 60.

VI. In eadem ratione magnitudines dicuntur eſſe, prima A ad ſecundam B, & tertia C ad quartam D, cum primę A, & tertiæ C æquemultiplicia E, & F à ſecundæ B, & quartæ D æquemultiplicibus G, & H, qualiscunque

cunque

cunque ſit hęc multiplicatio, utrumque E, F. ab ab utroque G, H, vel una deficiunt, vel una æqualia ſunt, vel una excedunt, ſi ea ſumantur E, G; & F, H quæ inter ſe reſpondent.

Hujus nota eſt ::, ut A. B :: C. D. *hoc eſt A ad B, & C ad D in eadem ſunt ratione. aliquando ſic ſcribimus* $\frac{A}{B} = \frac{C}{D}$ *id eſt*, A.B :: C.D.

VII. Eandem autem habentes rationem (A. B :: C. D) proportionales vocentur.

E,30.	A, 6. B, 4.	G,28.
F,60.	C,12. D,9.	H,63.

VIII. Cum vero æquemultiplicium, E multiplex primæ magnitudinis A exceſſerit G multiplicem ſecundę B; at F multiplex tertię C non exceſſerit H multiplicem quartæ D; tunc prima A ad ſecundam B majorem rationem habere dicetur, quam tertia C ad quartam D.

Si $\frac{A}{B} \sqsubset \frac{C}{D}$, *neceſſarium non eſt ex hac definitione, ut E ſemper excedat G; quum F minor eſt quam H; ſed conceditur hoc fieri poſſe.*

IX. Proportio autem in tribus terminis pauciſſimis conſiſtit. *Quorum ſecunda eſt inſtar duorum.*

X. Cum autem tres magnitudines A, B, C proportionales fuerint, prima A ad tertiam C duplicatā rationem habere dicetur ejus, quam habet ad ſecundam B: at quum quatuor magnitudines A, B, C, D, proportionales fuerint, prima A ad quartam D triplicatam rationem habere dicetur ejus, quam habet ad ſecundam B; & ſemper deinceps uno amplius, quamdiu proportio extiterit.

Duplicata ratio exprimitur ſic $\frac{A}{C} = \frac{A}{B}$ *bis. Hoc eſt, ratio A ad C duplicata eſt rationis A ad B. Triplicata autem ſic* $\frac{A}{D} = \frac{A}{B}$ *ter. id eſt, ratio A ad D triplicata eſt rationis A ad B.*

∺ *Denotat continue proportionales*, ut A, B, C, D; *item*, 2, 6, 18, 64 *ſunt* ∺

XI. Homologæ ſeu ſimiles ratione, magnitudines dicuntur, antecedentes quidem antecedentibus, conſequentes vero conſequentibus.

Ut ſi A. B :: C. D; *tam A & C, quam B & D homologæ magnitudines dicuntur.*

XII. Alterna ratio, eſt ſumptio antecedentis ad antecedentem, & conſequentis ad conſequentem.

Ut ſit A. B :: C. D. ergo alterne, vel permutando, vel viciſſim, A. C :: B. D. per 16. 5.

In hac definitione, & 5 ſequentibus imponuntur nomina ſex modis argumentandi, quibus mathematici frequenter utuntur; quarum illationum vis innititur propoſitionibus huius libri, quæ in explicationibus citantur.

XIII. Inverſa ratio, eſt ſumptio conſequentis ceu antecedentis, ad antecedentem velut ad conſequentem.

Ut A. B :: C. D. ergo inverſe, B. A :: D. C. per cor. 4, 5.

XIV.

XIV. Compoſitio rationis, eſt ſumptio antecedentis cum conſequente, ceu unius, ad ipſam conſequentem,

Ut A. B :: C. D. ergo componendo, A + B. B :: C + D. D, per 18. 5.

XV. Diviſio rationis, eſt ſumptio exceſſus, quo conſequentem ſuperat antecedens, ad ipſam conſequentem.

Ut A. B :: C. D. ergo dividendo, A-B. B :: C-D. D. per 17. 5.

XVI. Converſio rationis, eſt ſumptio antecedentis ad exceſſum, quo ſuperat antecedens ipſam conſequentem.

*Ut A. B :: C. D. ergo per converſam rationem, A. A-B :: C.*C*-D. per* cor. 19. 5.

XVII. Ex æqualitate ratio eſt, ſi plures duabus ſint magnitudines, & his aliæ multitudine pares, quę binæ ſumantur, & in eadem ratione; cum ut in primis magnitudinibus prima ad ultimam, ſic & in ſecundis magnitudinibus prima ad ultimam ſeſe habuerit. Vel aliter; ſumptio extremorum, per ſubductionem mediorum.

XVIII. Ordinata proportio eſt, cum fuerit quemadmodum antecedens ad conſequentem, ita antecedens ad conſequentem: fuerit etiam ut conſequens ad aliud quidpiam, ita conſequens ad aliud quidpiam.

Ut ſi A. B :: D. E. item B. C :: E. F. erit ex æquo A. C :: D. F. per 22. 5.

XIX. Perturbata autem proportio eſt; cum tribus poſitis magnitudinibus, & aliis, quæ ſint his multitudine pares, ut in primis quidem ma-

magnitudinibus ſe habet antecedens ad conſequentem, ita in ſecundis magnitudinibus antecedens ad conſequentem : ut autem in primis magnitudinibus conſequens ad aliud quidpiã, ſic in ſecundis magnitudinibus aliud quidpiam ad antecedentem.

Ut ſi A.B :: F.G. item B.C :: E.F. erit ex æquo perturbate A.C :: E.G. per 23.5.

XX. Quotlibet magnitudinibus ordine poſitis, proportio primæ ad ultimam componitur ex proportionibus primæ ad ſecundam, & ſecundæ ad tertiam, & tertiæ ad quartam, & ita deinceps, donec extiterit proportio.

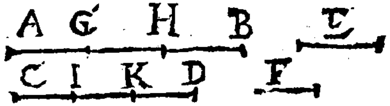
Sint quotcunque A, B, C, D; ex hac def.

$$\frac{A}{D} = \frac{A}{B} + \frac{B}{C} + \frac{C}{D}.$$

Axioma.

Æquemultiplices eidem multiplici, ſunt quoque inter ſe æquemultiplices.

PROP. I.



Si ſint quotcunque magnitudines AB, CD, quotcunque magnitudinum E, F æqualium numero, ſingulæ ſingularum, æquemultiplices; quam multiplex eſt unius E una magnitudo AB, tam multiplices erunt & omnes AB + CD omnium E + F.

Sint

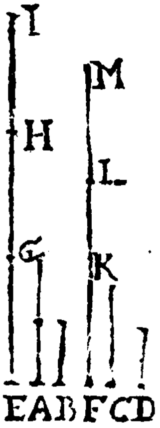
Sint AG, GH, HB partes quantitatis AB ipſi E ęquales: item CI, IK, KD partes quantitatis CD ipſi F pares. Harum numerus illarum numero æqualis ponitur. Quum igitur AG + CI *a* = E + F; *a* & GH + IK = E + F; *a* & HB + KD = E + F, liquet AB + CD ęque multoties continere E + F, ac una AB unam E continet. Q.E.D. *a 2. ax.*

PROP. II.

Si prima A B ſecundæ C æque fuerit multiplex, atque tertia D E quartæ F; fuerit autem & quinta B G ſecundæ C æque multiplex, atque ſexta E H quartæ F, erit & compoſita prima cum quinta (A G) ſecundæ C æque multiplex, atq; tertia cum ſexta (D H) quartæ F.

Numerus partium in AB ipſi C æqualium ęqualis ponitur numero partium in DE ipſi F æqualium. Item numerus partium in BG ponitur æqualis numero partium in EH. *a* ergo numerus partium in AB + BG æquatur numero partium in DE + EH. hoc eſt tota AG æque multiplex eſt ipſius C, atque tota GH ipſius F. Q.E.D. *a 2. ax.*

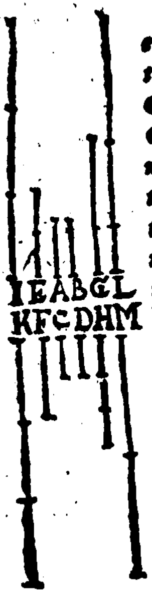
PROP. III.



Sit prima A ſecundæ B æquemultiplex, atque tertia C quartæ D; ſumantur autem E I, F M æquemultiplices primæ & tertiæ; erit & ex æquo, ſumptarum utraque utriusque æquemultiplex: altera quidem E I ſecundæ B, altera autem F M quartæ D.

Sint E G, G H, H I partes multiplicis E I ipſi A pares; item F K, K L, L M partes multiplicis F M ipſi
a hyp. C æquales. *a* Harum numerus illarum numero æquatur. Porro A, id eſt E G, vel G H, vel G I ipſius B ponitur ęquemultiplex atque C, vel F
b 2.5. K, &c. ipſius D. *b* ergo E G + G H ęquemultiplex eſt ſecundæ B, atque F K + K L quartæ
c 2.5. D. *c* Simili argumento E I (E H + H I) tam multiplex eſt ipſius B, quam F M (F L + LN) ipſius D. Q. E. D.

PROP. IV.



Si primæ A ad ſecundam B eandem habuerit rationem, & tertia C ad quartam D; etiam E & F æquemultiplices primæ A, & tertiæ C ad G, & H æquemultiplices ſecundæ B, & quartæ D, juxta quamvis multiplicationem, eandem habebunt rationem, ſi prout inter ſe reſpondent, ita ſumpta fuerint. (E. G :: F. H.)

Sume I, & K ipſarum E, &
F; item L & M ipſarum G, & H
æquemultiplices. a Erit I ipſius a 3.5.
A æquemultiplex atque K ipſius
C; a paritérque L tam multi-
plex ipſius B quam M ipſius D.
Itaque cum ſit A. B b :: C. D; b hyp.
juxta 6. *def.* ſi I ⊏, =, ⊐ L; con-
ſequenter pari modo K ⊏, =, ⊐
M. ergo cum I, & K ipſarum E, & F ſumptæ
ſint æquemultiplices, atque L, & M ipſarum G
& H; erit juxta 7. *def.* E. G :: F. H. Q. E. D.

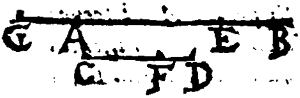
Coroll.

Hinc demonſtrari ſolet inverſa ratio.

Nam quoniam A.B :: C.D, ſi E ⊏, =, ⊐ G,
c erit ſimiliter F ⊏, =, ⊐ H. ergo liquet, quod c 6.def.5
ſi G ⊏, =, ⊐ E, eſſe H ⊏, =, ⊐ F. d ergo B.A :: d 6.def.5
D.C. Q.E.D.

PROP.

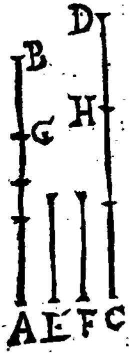
PROP. V.



Si magnitudo A B magnitudinis C D æq; fuerit multiplex atque ablata A E ablatæ C F; etiam reliqua E B reliquæ F D ita multiplex erit, ut tota A B totius C D.

Accipe aliam quandam G A, quæ reliquæ F D ita ſit multiplex, atque tota A B totius C
a 1. 5. D, vel ablata A E ablatæ C F. *a* ergo tota G A + A E totius C F + F D æquemultiplex eſt, ac una A E unius C F, hoc eſt, ac A B ipſius C
b 6. ax. D. *b* ergo G E = A B. *c* proinde, ablata com-
c 3. ax. muni A E, manet G A = E B. ergo, &c.

PROP. VI.

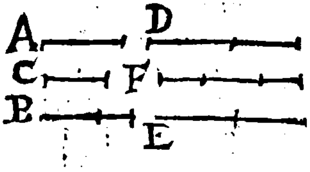


Si duæ magnitudines A B, C D duarum magnitudinum E, F ſint æquemultiplices; & detracta quædam ſint, A G, & C H, earundem E, & F æquemultiplices; & reliquæ G B, H D eisdem E, F aut æquales ſunt, aut æque ipſarum multiplices.

Nam quia numerus partium in A B ipſi E æqualium ponitur æqualis numero partium in C D ipſi F æqualium; item numerus partium in A G æqualis numero partium in C
a 5. ax. H. ſi hinc A G, inde C H detrahatur, *a* remanet numerus partium in reliqua G B æqualis numero partium in H D. ergo ſi G B ſit E ſemel

mel, erit HD etiam C ſemel, ſi GB ſit E aliquoties, erit HD etiam C toties accepta. Q. E. D.

PROP. VII.



Æquales A & B ad eandem C eandem habent rationem; & eadem C ad æquales A & B.

Sumantur D & E æqualium A & B ęquemultiplices, & F utcunque multiplex ipſius C; *a* erit D = E. quare ſi D ⊏, =, ⊐ F, erit ſimiliter E ⊏, =, ⊐ F. *b* ergo A. C :: B. C. inverſe igitur C. A *c* :: C. B. Q. E. D.

a 6. ax.
b 6. def. 5.
c cor. 4. 5

Schol.

Si loco multiplicis F ſumantur duę æquemultiplices, eodem modo oſtendetur æquales magnitudines ad alias inter ſe æquales eandem habere rationem.

PROP. VIII.

Inæqualium magnitudinum A B, C, major A B ad eandem D majorem rationem habet, quam minor C. Et eadem D ad minorem C majorem rationem habet, quam ad majorem A B.

PROP. ead.

Inæqualium magnitudinum A B, A C, major A B ad eandem D majorem habet rationem, quam minor A C. & eadem D ad minorem A C majorem rationem habet, quam ad majorem A B.

Sume E F, E G, ipſarum A B, A C ęquemultiplices, ita ut E H ipſius D multiplex, major ſit quam E G, at minor quam E F. (Quod facile continget, ſi utraque E G, G F majores accipiantur ipſa D.) Liquet juxta 8. def. 5. fore $\frac{AB}{D} \sqsubset \frac{AC}{D}$; ac $\frac{D}{AB} \sqsupset \frac{D}{AC}$. Quę E. D.

Ex majori A B aufer A E = C. ſumatur H G tam multiplex ipſius A E, vel C, quam G F reliquę F B. Multiplicetur D, donec ejus multiplex I K major evadat quam H G, ſed minor quam H F.

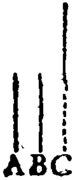
a conſt. b 1.5. Quoniam H G ipſius A E a tam multiplex eſt, quam G F ipſius E B, b erit tota H F totius

A B

A B æquemultiplex, atque una H G unius AE, vel C. ergo cum H F $\sqsubset$ I K (quæ multiplex eſt ipſius D) ſed H G $\sqsupset$ IK, *c* erit $\frac{AB}{D} \sqsubset \frac{C}{D}$ Q. E. D. c 8.def.5

Rurſus quia I K $\sqsubset$ H G, at I K $\sqsupset$ H F (ut prius dictum) *d* erit $\frac{D}{C} \sqsubset \frac{D}{AB}$ Q.E.D.

PROP. IX.

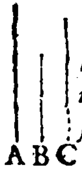


Quæ ad eandem eandem habent rationem, æquales ſunt inter ſe. Et ad quas eadem eandem habet rationem, eæ quoque ſunt inter ſe æquales.

1. *Hyp*. Sit A . C : : B . C. dico A = B. Nam ſit A $\sqsubset$, vel $\sqsupset$ B, *a* erit ideo $\frac{A}{C} \sqsubset$, vel $\sqsupset \frac{B}{C}$ contra Hyp. a 8.5.

2. *Hyp*. Sit C . B : : C . A. dico A = B. nam ſit A $\sqsubset$ B. *b* ergo $\frac{C}{B} \sqsubset \frac{C}{A}$ contra Hyp. b 8.5.

PROP. X



Ad eandem magnitudinem rationem habentium, quæ majorem rationem habet, illa major eſt: ad quam vero eadem majorem rationem habet, illa minor eſt.

1. *Hyp*. Sit $\frac{A}{C} \sqsubset \frac{B}{C}$. Dico A $\sqsubset$ B.

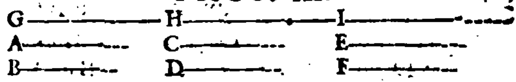
Nam ſi dicatur A = B, *a* erit A . C : : B . C. contra Hyp. Sin A $\sqsupset$ B, *b* erit $\frac{A}{C} \sqsupset \frac{B}{C}$ etiam contra Hyp. a 7.5. b 8.5.

2. *Hyp*. Sit $\frac{C}{B} \sqsubset \frac{C}{A}$. Dico B $\sqsupset$ A. Nam dic

B = A. *c* ergo C.B : : C.A. contra Hyp. vel dic B $\sqsubset$ A. *d* ergo $\frac{C}{A} \sqsubset \frac{C}{B}$ etiam contra Hyp. c 7.5. d 8.5.

PROP.

PROP. XI.



Quæ eidem ſunt eædem rationes, & inter ſe ſunt eædem.

Sit A.B :: E.F. item C.D :: E.F. dico A.B :: C.D. ſume ipſarum A, C, E æquemultiplices G, H, I; atque ipſarum B, D, F æquemultiplices K, L, M. Et quoniam *a* A.B :: E.F. ſi G

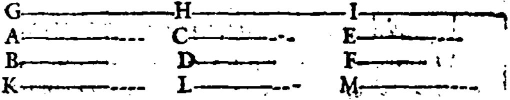
a hyp.
b 6.def.5 $\sqsubset$, $=$, $\sqsupset$ K, *b* erit pari modo I $\sqsubset$, $=$, $\sqsupset$ M. pariterque quia *a* E.F :: C.D. ſi I $\sqsubset$, $=$, $\sqsupset$ M, *b* erit H ſimiliter $\sqsubset$, $=$, $\sqsupset$ L. ergo ſi G $\sqsubset$, $=$,

c 6.def.5 $\sqsupset$ K; erit ſimiliter H $\sqsubset$, $=$, $\sqsupset$ L. *c* quare A.B :: C.D. Q.E.D.

Schol.

Quæ eiſdem rationibus ſunt eædem rationes, ſunt quoque inter ſe eædem.

PROP. XII.



Si ſint magnitudines quotcunque A, & B; C & D; E, & F proportionales; quemadmodum ſe habuerit una antecedentium A ad unam conſequentium B, ita ſe habebunt omnes antecedentes, A, C, E ad omnes conſequentes, B, D, F.

Sume antecedentium æquemultiplices G, H, I; & conſequentium K, L, M. Quoniam quam multiplex eſt una G unius A, *a* tam multiplices ſunt omnes G, H, I omnium A, C, E;

a 1.5. pariterque quam multiplex eſt una K unius B,

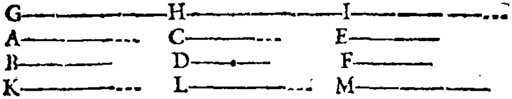
a tam

a tam multiplices ſunt omnes K,L,M omnium B,D,F, Porro ob A.B *b* :: C.D *b* :: E.F, ſi G b *hyp.* $\sqsubset$,=,$\sqsupset$ K, erit ſimiliter H $\sqsubset$,=,$\sqsupset$ L; & I $\sqsubset$, =, $\sqsupset$ M. ac proinde ſi G $\sqsubset$, =, $\sqsupset$ K, erit ſimili modo G + H + I $\sqsubset$,=,$\sqsupset$ K + L + M. *c* quare A.B :: A + C + E. B + D + F. Q. c *6.def.5* E.D.

Coroll.

Hinc, ſi ſimilia proportionalia ſimilibus proportionalibus addantur, tota erunt proportionalia.

PROP. XIII.



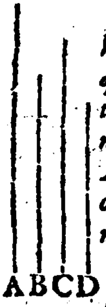
Si prima A ad ſecundam B eandem habuerit rationem, quam tertia C ad quartam D; tertia vero C ad quartam D majorem habuerit rationem, quam quinta E ad ſextam F; prima quoque A ad ſecundam B majorem rationem habebit, quam quinta E ad ſextam F.

Sume pſarum A, C, E æquemultiplices G, H, I: ipſarumque B, D, F æquemultiplices K, L,M. Quia A.B :: C.D; ſi H $\sqsubset$ L, *a* erit G $\sqsubset$ a *6.def.5* K. Sed quia $\frac{C}{D} \sqsubset \frac{E}{F}$, *b* fieri poteſt ut ſit H $\sqsubset$ b *8.def.5* L, & I non $\sqsubset$ M. ergo fieri poteſt ut G $\sqsubset$ K, & I non $\sqsubset$ M. *c* ergo $\frac{A}{B} \sqsubset \frac{E}{F}$. Q.E.D. c *8.def.5*

Schol.

Quod ſi $\frac{C}{D} \sqsupset \frac{E}{F}$, erit quoque $\frac{A}{B} \sqsupset \frac{E}{F}$. Item ſi $\frac{A}{B} \sqsubset \frac{C}{D} \sqsubset \frac{E}{F}$, erit $\frac{A}{B} \sqsubset \frac{E}{F}$. & ſi $\frac{A}{B} \sqsupset \frac{C}{D} \sqsupset \frac{E}{F}$, erit $\frac{A}{B} \sqsupset \frac{E}{F}$.

PROP. XIV.



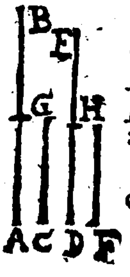
Si prima A ad ſecundam B eandem habuerit rationem, quam tertia C ad quartam D; prima vero A, quam tertia C major fuerit; erit & ſecunda B major quam quarta D. Quod ſi prima A fuerit æqualis tertia C, erit & ſecunda B æqualis quarta D; ſi vero A minor, & B minor erit.

a 8. 5.
b hyp. Sit A $\sqsubset$ C. *a* ergo $\frac{A}{B} \sqsubset \frac{C}{B}$. *b* ſed
c 13. 5.
d 10. 5. $\frac{A}{B} = \frac{C}{D}$. *c* ergo $\frac{C}{D} \sqsubset \frac{C}{B}$. *d* ergo B $\sqsubset$
D. Simili argumento ſi A $\sqsupset$ C, *d* erit B $\sqsupset$ D.
e 7. 5. f hyp Sin ponatur A = C; ergo C. B *e* :: A. B *f* ::
g 11. & 9. 5. C. D. *g* ergo B = D. Quæ E. D.

Schol.

A fortiori, ſi $\frac{A}{B} \sqsupset \frac{C}{D}$, atque A $\sqsubset$ C, erit B $\sqsubset$ D. Item ſi A = B, erit C = D. Et ſi A $\sqsubset$, vel $\sqsupset$ B, erit pariter C $\sqsubset$, vel $\sqsupset$ D.

PROP. XV.



Partes C & F cum pariter multiplicibus A B, & D E in eadem ſunt ratione, ſi prout ſibi mutuo reſpondent, ita ſumantur. (A B. D E :: C. F.)

Sint A G, G B partes multiplicis A B ipſi C æquales: item D H,

HE

HE partes multiplicis DE ipſi F equales. [a] Harum numerus illarum numero æquatur. ergo, quum AG. DH :: C. F :: GB. HE. erit AG + GB (AB.) DH + HE (DE) :: C.F. Q.E.D.

a hyp.

PROP. XVI.

Si quatuor magnitudines A, B, C, D *proportionales fuerint; & viciſſim proportionales erunt.* (A.C :: B.D.)

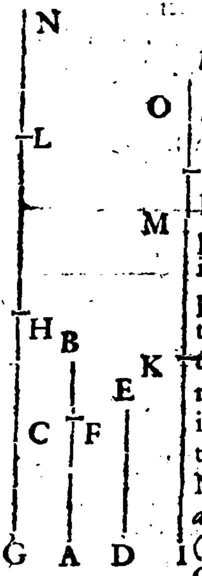
Accipe E & F æquemultiplices ipſarum A & B. ipſarumque C & D æquemultiplices G & H. Itaque E. F [a] :: A. B. [b] :: C. D [a] :: G. H. Quare ſi E ⊏, =, ⊐ G, [c] erit ſimiliter F ⊏, =, ⊐ H. [d] ergo A. C :: B. D. Q.E.D.

a 15.5.
b hyp.
c 14.5.
& 14.5.
d 6. def. 5.

Schol.

Alterna ratio locum tantum habet, quando quantitates ejusdem ſunt generis. Nam Heterogeneæ quantitates non comparantur.

PROP. XVII.



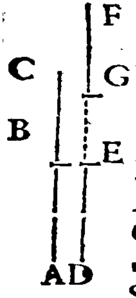
a 1.5.
b conſtr.
c 5.5.
d 2.5.

Si compoſitæ magnitudines proportionales fuerint (A B. C B :: D E. F E;) hæ quoq; diviſæ proportionales erunt. (A C. C B :: D F. F E.)

Accipe G H, H L, I K, K M ordine æquemultiplices ipſarum A C, C B, D F, F E. item L N, M O æquemultiplices ipſarum C B, F E. Tota G L totius A B *a* tam multiplex eſt, quam una G H unius A C, *b* ideſt quam I K ipſius D F; *c* hoc eſt quam tota I M totius D E. Item H N (H L + L N) ipſius C B *d* æquemultiplex eſt, ac K O (K M + M O) ipſis F E. Quum igitur per hyp. A B, B C :: D E. E F. ſi G L $\sqsubset, =, \sqsupset$ H N, etiam ſimiliter *e* erit I M $\sqsubset, =, \sqsupset$ K O. itaque ablatis hinc inde communibus H L, K M ſi reliqua G H $\sqsubset, =, \sqsupset$ L N, *f* erit ſimiliter I K $\sqsubset, =, \sqsupset$ M O. *g* unde A C. C B :: D F. F E. Q. E. D.

e 6.def.5.
f 5.ax.
g 6.def.5.

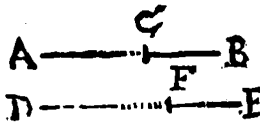
PROP. XVIII.



Si diviſæ magnitudines ſint proportionales (AB. BC :: DE. EF,) hæ quoque compoſitæ proportionales erunt (AC. CB :: DF. FE.)

Nam ſi fieri poteſt, ſit AC. CB ::
DF. FG ⊐ FE. *a* ergo erit diviſim a 17.5.
AB. BC :: DG. GF. *b* hoc eſt D- b hyp.
G. GF :: DE. EF. ergo cum DG & 11.5.
⊏ DE, *c* erit GF ⊏ EF. Q. E. A. c 14.5.
Simile abſurdum ſequetur, ſi dica- d 9.ax.
tur AB. CB :: DE. GF ⊏ FE.

PROP. XIX.



Si quemadmodum totum AB ad totum DE, ita ablatum AC ſe habuerit ad ablatum DF; & reliquum CB ad reliquum FE, ut totum AB ad totum DE, ſe habebit.

Quoniam *a* AB. DE :: AC. DF, *b* erit a hyp.
permutando AB. AC :: DE. DF. *c* ergo di- b 16.5.
viſim AC. CB :: DF. FE. quare rurſus *b* per- c 17.5.
mutando AC. DF :: CB. FE; *d* hoc eſt AB. d hyp.
DE :: CB. FE. Q.E.D. & 11.5.

Coroll.

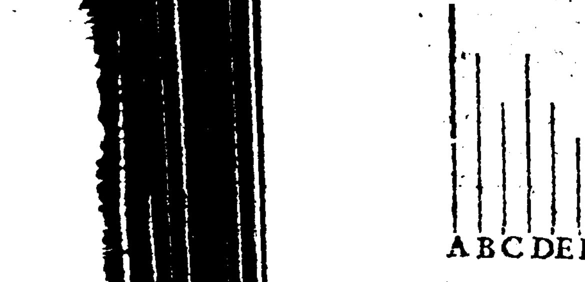
1. Hinc, ſi ſimilia proportionalia ſimilibus proportionalibus ſubducantur, reſidua erunt proportionalia.

2. *Hinc demonſtrabitur converſa ratio.*

Sit AB. CB :: DE. FE. Dico AB. AC ::

D E. D F. Nam *a* permutando A B. D E :: C B. F E. *b* ergo A B. D E :: A C. D F. quare iterum permutando, A B. A C :: D E. D F. Q. E. D.

PROP. XX.

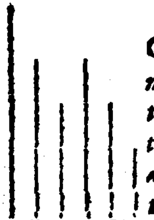


Si ſint tres magnitudines A, B, C; & aliæ D, E, F ipſis æquales numero, quæ bina & in eadem ratione ſumantur (A.B :: D.E, atq; B.C :: E.F;) ex æquo autem prima A major fuerit, quam tertia C; erit & quarta D major quam ſexta F. Quod ſi prima A tertiæ C fuerit æqualis; erit & quarta D æqualis ſextæ F. Sin illa minor, hæc quoque minor erit.

a hyp. *Hyp.* Si $A \sqsubset C$. quoniam *a* E.F :: B.C. *b*
b cor.4.5. erit inverſe F.E :: C.B. *c* Sed $\frac{C}{B} \sqsupset \frac{A}{B}$ *d* ergo
c hyp. & 8.5.
d schol. 13.5. $\frac{F}{E} \sqsupset \frac{A}{B}$ vel $\frac{D}{E}$ *e* ergo $D \sqsubset F$. # Q.E.D.
e 10.5. 2. *Hyp.* Simili argumento, ſi $A \sqsupset C$, oſtendetur $D \sqsupset F$.

f 7.5.
g 11.5. & 9.5. 3. *Hyp.* Si A = C. quoniam F. E :: C.B :: *f* A.B :: D.E *g* erit D = F. Q.E.D.

PROP. XXI.



Si ſint tres magnitudines A, B, C; & aliæ D, E, F ipſis æquales numero, quæ bina & in eadem ratione ſumantur, fueritq; perturbata eorum proportio, (A.B :: E.F atque B.C :: D.E;) ex æquo autem prima A quam tertia C ma-

jo

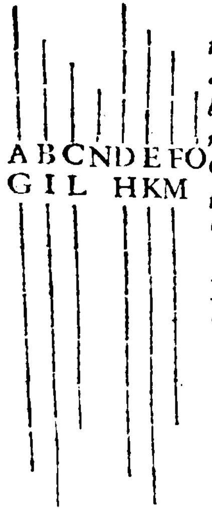
jor fuerit; erit & quarta D quam ſexta F major. Quod ſi prima fuerit tertiæ æqualis, erit & quarta æqualis ſextæ: ſin illa minor, hæc quoq; minor erit.

1. *Hyp.* A $\sqsubset$ C. Quoniam *a* D. E :: B. C, invertendo erit E. D :: C. B. atqui $\frac{C}{B}$ *b* $\sqsupset$ $\frac{A}{B}$. *c* ergo $\frac{E}{D}$ $\sqsupset$ $\frac{A}{B}$, hoc eſt $\frac{E}{F}$. *d* ergo D $\sqsubset$ F. Q. E. D. a *hyp.* b 8. 5. c *ſchol.* 13. 5. d 10. 5.

2. *Hyp.* Similiter, ſi A $\sqsupset$ C, erit D $\sqsupset$ F.

3. *Hyp.* Si A = C. quoniam E. D *e* :: C. B :: *e* A. B :: *f* E. F. *g* erit *D* = F. Q. E. D. e 7. 5. f *hyp.* g 9. 5.

PROP. XXII.

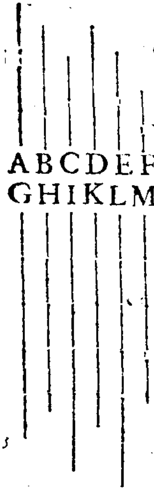


Si ſint quotcunque magnitudines A, B, C; *& aliæ ipſis æquales numero* D, E, F, *quæ bina & in eadem ratione ſumantur* (A. B :: *D.* E. & B. C :: E. F;) *& ex æqualitate in eadem ratione erunt* (A. C :: D. F.)

Accipe G, H ipſarum A, D; & I, K ipſarum B, E; item L, M ipſarum E, F æquemultiplices.

Quoniam *a* A. B :: *D.* E. *b* erit G. I :: H. K. eodem modo, erit I. L :: K. M. ergo ſi G $\sqsubset$, =, $\sqsupset$ L, *c* erit H. $\sqsubset$, =, $\sqsupset$ M; *d* ergo A. C :: D. F. Eodem pacto ſi ulterius C. N :: F. O, erit ex æquali A. N :: D. O. Q. E. D. a *hyp.* b 4. 5. c 20. 5. d 6. *def.* 5.

PROP. XXIII.



Si ſint tres magnitudines A, B, C, aliæq; D, E, F ipſis æquales numero, quæ binæ in eadem ratione ſumantur; fuerit autem perturbata earum proportio. (A.B :: E.F. & B.C :: D. E.) etiam ex æqualitate in eadem ratione erunt.

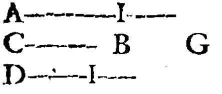
Sume G,H,I, ipſarum A,B,D; item K, L, M ipſarum C, E, F æ-
a 15.5. quemultiplices. erit G.H *a* :: A.
b hyp. B *b* :: E. F *a* :: L. M. porro quia *b*
c 4.5. B. C :: D. E. erit *c* H. K :: I.L.
ergo G,H,K; & I,L,M habent ſe
juxta 21.5. quare ſi G ⊏,=,⊐K,
d 6. def. 5 erit ſimiliter I ⊏,=,⊐ M. *d* proinde A.C :: D. F. Q.E.D.

Eodem modo ſi plures fuerint magnitudinibus tribus, &c.

Coroll.

* 21. & Ex * his ſequitur, rationes ex iiſdem ratio-
23.5. & 20. def. 5 nibus compoſitas eſſe inter ſe eaſdem. item,
earundem rationum eaſdem partes inter ſe eaſdem eſſe.

PROP. XXIV.



Si prima A B *ad ſecundam* C *eandem habuerit rationem quam tertia* D E *ad quartam* F; *habuerit autem & quinta* B G *ad ſecundam* C *eandem rationem, quam ſexta* E H *ad quartam* F;

F; *etiam compoſita prima cum quinta* (AG) *ad ſecundam* C *eandem habebit rationem, quam tertia cum ſexta* (DH) *ad quantam* F.

Nam quia *a* A B. C : : DE. F. atque ex hyp. a *hyp.*
& inverſe C. B G : : F. E H, erit *b* ex æquali A- b 22.5.
B. B G : · D E. E H. ergo componendo A G. c *hyp.*
B G : : DH. E H. *c* item B G. C : : E H. F. *b* ergo rurſus ex æquo, A G. C : : D H. F. Q. E. D.

PROP. XXV.

Si quatuor magnitudines proportionales fuerint (A B. C D : : E. F.) *maxima* A B *&* *minima* F *reliquis* C D *&* E *majores erunt.*

Fiant A G = E ; & C H = F. Quoniam A B. C D *a* : : E. F *b* : : a *hyp.*
A G. C H. *c* erit A B. C D : : G- b 7.5.
B. H D. *d* ſed A B ⊏. C *D*. *e* er- c 18.5. d *hyp.*
go G B ⊏ H D. atqui A G + F e *ſchol.*
= E + C H. ergo A G + F + G B ⊏ E + 14.5.
C H + H D, hoc eſt A B + F ⊏ E + C D. Q. E. *D*.

Quæ ſequuntur propoſitiones non ſunt Euclidis; ſed ex aliis deſumptę, ob frequentem earum uſum Euclidæis ſubjungi ſolent.

PROP. XXVI.

A———C———
B——— D———
E———

Si prima ad ſecundam habuerit majorem proportionem, quam tertia ad quartam; habebit convertendo, ſecunda ad primam minorem proportionē, quam quarta ad tertiam. Sit

Sit $\frac{A}{B} \sqsubset \frac{C}{D}$. Dico $\frac{B}{A} \sqsupset \frac{D}{C}$. Nam concipe

a 13.5. $\frac{C}{D} = \frac{E}{B}$ *a* ergo $\frac{A}{B} \sqsubset \frac{E}{B}$ *b* quare A $\sqsubset$ E. *c* ergo
b 10.5.
c 8.5.
d cor.4.5. $\frac{B}{A} \sqsupset \frac{B}{E}$ *d* vel $\frac{D}{C}$ Q.E.D.

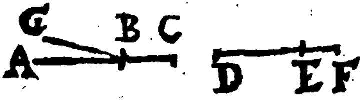
PROP. XXVII.

Si prima ad ſecundam habuerit majorem proportionem, quā tertia ad quartam; habebit quoque viciſſim prima ad tertiam majorem proportionem, quam ſecunda ad quartam.

Sit $\frac{A}{B} \sqsubset \frac{C}{D}$ Dico $\frac{A}{C} \sqsubset \frac{B}{D}$ Nam putâ $\frac{E}{B} = \frac{C}{D}$.

a 10.5. *a* ergo A $\sqsubset$ E. *b* ergo $\frac{A}{C} \sqsubset \frac{E}{C}$ *c* vel $\frac{B}{D}$ Q.E.D.
b 8.5.
c 16.5.

PROP. XXVIII.



Si prima ad ſecundam habuerit majorem proportionem, quam tertia ad quartam; habebit quoque compoſita prima cum ſecunda ad ſecundam majorem proportionem, quam compoſita tertia cum quarta ad quartam.

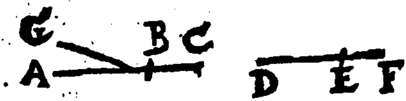
Sit $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. Dico $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Nam cogita

a 10.5. $\frac{GB}{BC} = \frac{DE}{EF}$ *a* ergo AB $\sqsubset$ GB, adde utrinque BC,

b 4. ax. *b* erit AC $\sqsubset$ GC. *c* ergo $\frac{AC}{BC} \sqsubset \frac{GC}{BC}$ *d* hoc eſt $\frac{DF}{FE}$.
c 8.5.
d 18.5. Q.E.D.

PROP.

PROP. XXIX.

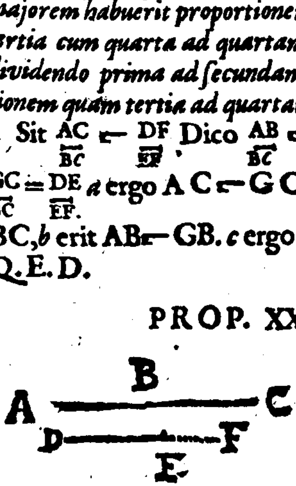


Si compoſita prima cum ſecunda ad ſecundam majorem habuerit proportionem, quam compoſita tertia cum quarta ad quartam; habebit quoq; dividendo prima ad ſecundam majorem proportionem quam tertia ad quartam.

Sit $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$ Dico $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$ Intellige $\frac{GC}{BC} = \frac{DE}{EF}$. *a* ergo A C ⊏ G C. aufer commune BC, *b* erit AB ⊏ GB. *c* ergo $\frac{AB}{BC} \sqsubset \frac{CB}{BC}$ *d* vel $\frac{DE}{EF}$. Q.E.D.

a 1. 5.
b 5. ax.
c 8.5.
d 17.5.

PROP. XXX.



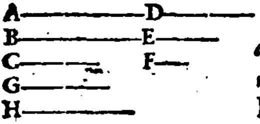
Si compoſita prima cum ſecunda ad ſecundam habuerit majorẽ proportionem, quã compoſita tertia cum quarta ad quartam; habebit, per converſionem rationis, prima cum ſecunda ad primam minorem rationem, quam tertia cum quarta ad tertiam.

Sit $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Dico $\frac{AC}{AB} \sqsupset \frac{DF}{DE}$. Nam quia $\frac{AC}{BC}$ *a* $=$ $\frac{DF}{FE}$, *b* erit dividendo $\frac{AB}{BC} \sqsubset \frac{DF}{EF}$. *c* convertendo igitur $\frac{BC}{AB} \sqsupset \frac{EF}{DE}$. *d* ergo componendo $\frac{AC}{AB} \sqsupset \frac{DE}{DE}$. Q.E.D.

a hyp.
b 29.5.
c 26 5.
d 28.5.

PROP.

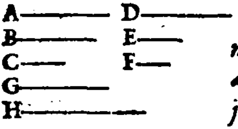
PROP. XXXI.



Si ſint tres magnitudines A, B, C, *& aliæ ipſis æquales numero* D, E, F; *ſitque major proportio primæ priorum ad ſecundam, quam primæ poſteriorum ad ſecundam* $\left(\frac{A}{B} \sqsubset \frac{D}{F}\right)$ *item ſecundæ priorũ ad tertiam major, quam ſecundæ poſteriorum ad tertiam* $\left(\frac{B}{C} \sqsubset \frac{E}{F};\right)$ *erit quoque ex æqualitate major proportio primæ priorum ad tertiam, quam primæ poſteriorum ad tertiam* $\left(\frac{A}{C} \sqsubset \frac{D}{F.}\right)$

a 10.5. Concipe $\frac{G}{C} = \frac{E}{F.}$ *a* ergo E $\sqsubset$ G. : *b* ergo
b 8.5.
c 13.5. $\frac{A}{C} \sqsubset \frac{A}{B}$ Rurſus puta $\frac{H}{G} = \frac{D}{E}$ *c* ergo $\frac{H}{G} \sqsupset \frac{A}{B;}$ *d*
d 10.5. ergo fortius $\frac{H}{G} \sqsupset \frac{A}{G}$ *d* quare A $\sqsubset$ H. *e* proinde
e 8.5.
f 12.5. $\frac{A}{C} \sqsubset \frac{H}{C,}$ vel $\frac{D}{F.}$ Q.E.D.

PROP. XXXII.



Si ſint tres magnitudines A, B.C; *& aliæ ipſis æquales* D,E,F; *ſitq; major proportio primæ priorum ad ſecundam, quam ſecundæ poſteriorum ad tertiam* $\left(\frac{A}{B} \sqsubset \frac{E}{F;}\right)$ *item ſecundæ priorum ad tertiam major quam primæ poſteriorum ad ſecundam* $\left(\frac{B}{C} \sqsubset \frac{D}{E;}\right)$

erit

erit quoque ex æqualitate major proportio primæ priorum ad tertiam, quam primæ poſteriorum ad tertiam $\left(\frac{A}{C} \sqsubset \frac{D}{F};\right)$

Intellige $\frac{G}{C} = DE$. *a* ergo $B \sqsubset G$. *b* ergo $\frac{A}{G} \sqsubset \frac{A}{B}$. Rurſus concipe $\frac{H}{G} = \frac{E}{F}$. *c* ergo $\frac{H}{G} \sqsupset \frac{A}{G}$. *a* quare $A \sqsubset H$. *b* proinde $\frac{A}{C} \sqsubset \frac{H}{C}$ *d* vel $\frac{D}{F}$. Q. E. D.

a 10.5.
b 8.5.
c ſch. 13. 5.
d 13.5.

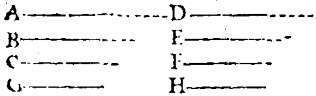
PROP. XXXIII.

Si fuerit major proportio totius AB *ad totum* CD, *quam ablati* AE *ad ablatum* CF; *erit & reliqui* EB *ad reliquum* FD *major proportio, quam totius* AB *ad totum* CD.

Quoniam $\frac{AB}{CD}$ *a* $\sqsubset \frac{AE}{CF}$, *b* erit permutando $\frac{AB}{AE} \sqsubset \frac{CD}{CF}$ *c* ergo per converſionem rationis $\frac{AB}{EB} \sqsupset \frac{CD}{FD}$. permutando igitur $\frac{AB}{CD} \sqsupset \frac{EB}{FD}$. Q. E. D.

a hyp.
b 17.5.
c 30.5.

PROP. XXXIV.



Si ſint quotcunque magnitudines, & aliæ ipſis æquales numero, ſitque major proportio primæ priorum ad primam poſteriorum, quam ſecundæ ad ſecundam; & hæc major quam tertiæ ad tertiam, & ſic deinceps:

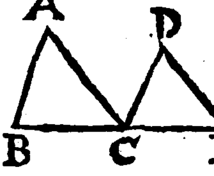
ceps : habebunt omnes priores ſimul ad om
poſteriores ſimul, majorem proportionem, qu
omnes priores, relicta prima, ad omnes poſte
res, relicta quoque prima ; minorem aut
quam prima priorum ad primam poſterioru
majorem denique etiam, quam ultima prior
ad ultimam poſteriorum.

Horum demonſtratio eſt penes interpre
quos adeat, qui eam deſiderat. nos omiſimus, l
vitatis ſtudio ; & quia illorum nullus uſus in
elementis.

LIB. VI.

DEFINITIONES.

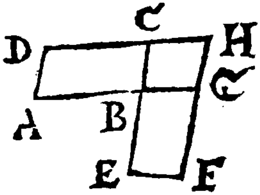
I.



Similes figuræ rectilineæ ſunt (ABC, DCE,) quæ & angulos ſingulos ſingulis æquales habent ; atque etiam latera, quæ circum angulos æquales, prop
tionalia.

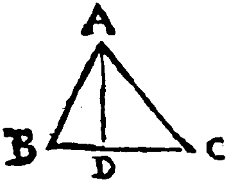
Ang. B=DCE; & AB.BC::DC.
E. *item ang.* A=D ; *atque* BA.AC::C
DE. *denique ang.* ACB=E. *atque* BC.
A::CE.ED.

II.R



II. Reciprocæ autem ſunt (BD, BF,) cum in utraque figura antecedentes, & conſequentes rationum termini fuerint. hoc eſt, A B. B G :: E B. B C.)

III. Secundum extremam & mediam rationẽ recta linea A B ſecta eſſe dicitur, cum ut tota AB ad majus ſegmentum AC, ita majus ſegmentum A C ad minus CB ſe habuerit. (AB. AC :: AC. CB.)



IV. Altitudo cujusque figuræ A B C eſt linea perpendicularis A D, à vertice A ad baſim B C deducta.

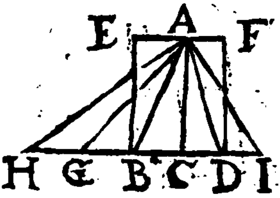
V. Ratio ex rationibus componi dicitur, cum rationum quantitates inter ſe multiplicatæ, aliquam effecerint rationem.

Ut ratio A ad C, componitur ex rationibus A ad B, & B ad C. nam $\frac{A}{B} + \frac{B}{C}$ [a] $\frac{A}{C}$ [b] $= \frac{AB}{BC}$

a 20. def. 5.
b 15. 5.

PROP.

PROP. I.

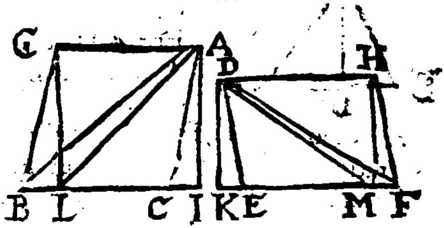


Triangula A B-C, A C D, *& parallelogramma* B-C A E, C D F A, *quorum eadem fuerit altitudo, ita ſe habent inter ſe, ut baſes* B C, C D.

a 3.1. *a* Accipe quotvis B G, H G, ipſi B C æquales; item DI=CD. & connecte A G, A H, AI.

b Triangula A C B, A B G, A G H æquantur; b 38.1. *b* item triang. A C D = A D I. ergo triangulum A C H tam multiplex eſt trianguli A C-B, quam baſis H C baſis B C. & æquemultiplex eſt triang. A C I trianguli A C D, ac baſis C I baſis C D. verum ſi H C $\sqsubset$, $=$, $\sqsupset$ C I, *c* c ſch.38.1. erit ſimiliter triang. A H C $\sqsubset$, $=$, $\sqsupset$ A C I. *d*
6.def.5. ideoque B C, C D :: triang. A B C. A G D :: *e*
d 41.1. & 5.5. pgr. C E. C F. Q.E.D.

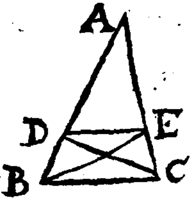
Schol.



Hinc, *triangula* A B C, D E F, *& parallelogramma* AGBC, DEFH, *quorum æquales ſunt baſes* BC, EF, *ita ſe habent ut altitudines* AI, DK.

a Sume I L = C B; & K M = E F; ac junge a 3.1.
L A, L G, M D, M H. liquet eſſe triang. A B C.
D E F :: *b* A L I. D K M :: *c* A I. D K :: *d* pgr. b 7.5.
A G B C. D E F H. Q. E. D. c 1.6.
d 41.1.
& 15.5.

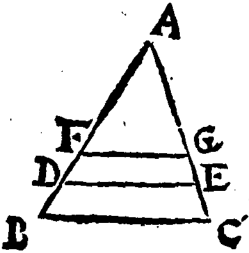
PROP. II.



Si ad unum trianguli A B C latus B C, parallela ducta fuerit recta quædam linea D E, hæc proportionaliter ſecabit ipſius trianguli latera (A D. B D :: A E. E C.) Et ſi trianguli latera proportionaliter ſecta fuerint (A D. B D :: A-E. E C) *quæ ad ſectiones D, E. adjuncta fuerit recta linea D E, erit ad reliquum ipſius trianguli latus B C parallela.* Ducantur C D, B E.

1. *Hyp.* Quia triang. D E B *a* = D E C; *b* a 37.1.
erit triang. A D E. D B E :: A D E. E C D. b 7.5.
atqui triang. A D E. D B E *c* :: A D. D B. & c 1.6.
triang. A D E. D E C *c* :: AE. E C. *d* ergo A- d 11.5.
D. D B :: A E. E C.

2. *Hyp.* Quia A D. D B :: A E. E C. *e* hoc e 1.6.
eſt triang. A D E. D B E :: A D E. E C D;
f erit triang. D B E = E C D. *g* ergo D E, B C f 9.5.
ſunt parallelæ. Q. E. D. g 39.1.



Schol.

Imo ſi plures D E, F G, ad unum latus B-C parallelæ fuerint, erunt omnia laterum ſegmenta proportionalia.

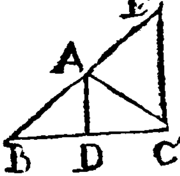
Nam D F. F A *a* ::

a 2.6. E G. G A; & componendo, invertendoque F A. D A :: G A. E A; *a* ac D A. D B :: E A. E C. ergo ex ęquo D F. D B :: E G. E C. Q.E.D.

Coroll.

Si D F. D B :: E G. E C; *a* erunt **B C, DE, F G** parallelæ.

PROP. III.



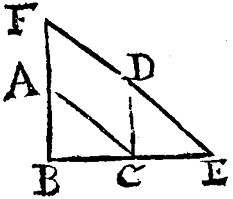
Si trianguli B A C angulus B A C bifariam ſectus ſit, ſecans autem angulum recta linea A D ſecuerit & baſim, baſis ſegmenta eandem habebunt rationem quam reliqua ipſius trianguli latera (**BD. DC :: A B. A C.** *) Et ſi baſis ſegmenta* ***eandem habeant*** *rationem quam reliqua ipſius* ***trianguli latera*** *(B D. D C :: A B. A C.) recta* ***linea A D*** *quæ à vertice A ad ſectionem D* ***ducitur, bifariam*** *ſecat trianguli ipſius angulum* ***B A C.***

Produc B A; & fac A E = **A C. & junge** C E.

1. *Hyp*. Quoniam A E = A **C, erit ang. A-**
a 5.1. C E *a* = E *b* = $\frac{1}{2}$ B A C *c* = D A **C.** *d* ergo D-
b 32.1. A, C E parallelæ ſunt. *e* quare B **A. A E (AC)**
c hyp. :: B D. D C. Q.E.D.
d 27.1.
e 2.6. 2. *Hyp*. Quoniam BA. A C. (**A E**) :: **BD.**
f 2.6. D C. *f* erunt D A, C E parallelæ: *g* **ergo ang.**
g 29.1.
h 5.1. B A D = E; & ang. D A C *g* = **A C E** *h* **E.** *k*
k 1.ax. ergo ang. B A D = D A C. **biſectus igitur eſt** ang. B A C. Q.E.D.

PROP.

PROP. IV.



Æquiangulorum triangulorum A B C, D C E proportionalia ſunt latera, quæ circum æquales angulos B, DCE (A B. B C : : DC. C E, &c.) & homologa ſunt latera A B, D C, &c. quæ æqualibus angulis ACB, E, &c. ſubtenduntur.

Statue latus B C in directum lateri C E, & produc B A, ac E D donec *a* occurrant. a 32.1. & 13.ax.

Quoniam ang. B *b* = E C D, *c* ſunt B F, C D. parallelæ. b hyp. Item quia ang. B C A *b* = CED, c 28.1. *c* ſunt C A, E F parallelæ. Figura igitur C A F D eſt parallelogramma. *d* ergo A F = C D; *d* & A C = F D. Liquet igitur A B. A F (C D) *e* : : B C. C E. d 34.1. e 2.6. *f* permutando igitur A B. B C : : C D. C E. f 16.5. *e* item B C. C E : : F D. (A C) D E. *f* ergo permutando B C. A C : : C E. D E. quare etiam *g* ex æquo A B. A C : : C D. D E. ergo, &c. g 22.5.

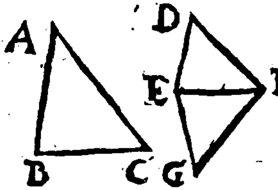
Coroll. Hinc AB. D C : : BC. CE : : AC. DE.

Schol.

Hinc ſi in triangulo F B E ducatur uni lateri F E parallela A C; erit triangulum A B C ſimile toti F B E.

PROP. V.

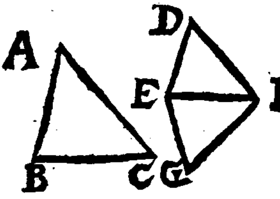
Si duo triangula ABC, DEF latera proportionalia habeant (AB. BC : : DE. EF. & AC. BC : : DF. EF. item AB. AC : : DE. DF) æquiangula erunt triangula, & æquales habebunt eos angulos, ſub quibus homologa latera ſubtenduntur,



a 23.1. Ad latus E F *a* fac ang. F E G = B; *a* & ang. E F G = C, *b* quare etiam ang. G = A. ergo G E. E F *c* :: A B. B C :: *d* D E. E F. *e* ergo G E = D E. Item G F. F E *c* :: A C. C B *d* :: D F. F E. *e* ergo G F = D F. Triangula igitur D E F, G E F ſibi mutuo æquilatera ſunt. *f* ergo ang. D = G = A. *f* & ang. F E D = F E G = B. *g* proinde & ang. D F E = C. ergo, &c.

b 32.1.
c 4.6.
d hyp.
e 11.5.
& 9.5.
f 8.1.
g 32.1.

PROP. VI.



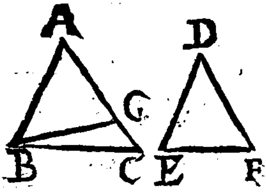
Si duo triangula A B C, D E F unum angulum B uni angulo D E F æqualem, & circum æquales angulos B, D E F latera proportionalia habuerint (A B. B C :: D E. E F,) æquiangula erunt triangula A B C, D E F; æqualesq; habebunt angulos, ſub quibus homologa latera ſubtenduntur.

Ad latus E F fac ang. F E G = B, & ang. E F G = C. *a* unde & ang. G = A. ergo G E. E F *b* :: A B. B C *c* :: D E. E F. *d* ergo D E = G E. atqui ang. D E F *e* = B *f* = G E F. *g* ergo ang. D = G = A. *h* proinde etiam ang. E F D = C. Q.E.D.

a 32.1.
b 4.6.
c hyp
d 9.5.
e hyp.
f conſt.
g 4.1.
h 32.1.

PROP.

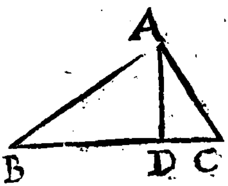
PROP. VII.



Si duo triangula ABC, DEF unum angulum A uni angulo D æqualem, circa autem alios angulos ABC, E latera proportionalia habeant (AB. BC :: DE. EF;) *reliquorum autem ſimul utrunque C, F aut minorem aut non minorem recto, æquiangula erunt triangula ABC, DEF, & æquales habebunt eos angulos circum quos proportionalia ſunt latera.*

Nam ſi fieri poteſt, ſit ang. ABC $\sqsubset$ E. fac
igitur ang. ABG = E; ergo cum ang. A *a* = D, a hyp.
b erit etiam ang. AGB = F. ergo AB. BG *c* b 32.1.
:: DE. EF *d* :: AB. BC. *e* ergo BG = BC. *f* c 4.6.
ergo ang. BGC = BCG. *g* ergo ang. BGC. d hyp.
vel C minor eſt recto; *g* proinde ang. AGB, vel e 9.5.
F recto major eſt. ergo anguli C & F non ſunt f 5.1.
ejuſdem ſpeciei, contra Hyp. g cor. 17.1.

PROP. VIII.



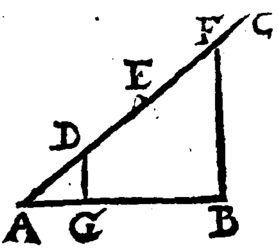
Si in triangulo rectangulo ABC, ab angulo recto BAC in baſin BC perpendicularis AD ducta eſt; quæ ad perpendicularem triangula ADB, ADC, tum toti triangulo ABC, tum ipſa inter ſe, ſimilia ſunt.

a hyp. Nam ob angulos B A C, A D B *a* rectos, *b*
b 12. ax. ideoque æquales, & B communem, trigona B
c 32. & 4.6. A C, A D B *c* ſimilia ſunt. Simili diſcurſu, ſimi-
d vid. 21. 6. lia ſunt triangula B A C, A D C. *d* proinde A
D B, A D C ſimilia erunt. Q.E.D.

Coroll.

e 1 def. 6. Hinc 1. B D. D A *e* :: D A. D C.
2. B C. A C :: A C. D C. & C B. B A :: B A. B D.

PROP. IX

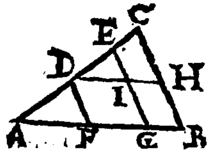


A data recta linea A B imperatam partem $\frac{1}{3}$ (A G) *auferre.*

a 3.1. Ex A duc infinitam A C utcunq;, in qua *a* ſume tres, A D, D E, E F æquales utcunque,
b 31.1. junge F B, cui ex D *b* duc parallelam D G. Dico factum.

c 2.6. Nam G B. A G *c* :: F D. A D. ergo *d* com-
d 18.5. ponendo A B. A G :: A F. A D. ergo cum A D = $\frac{1}{3}$ A F, erit A G = $\frac{1}{3}$ A B. Q.E.F.

PROP. X.



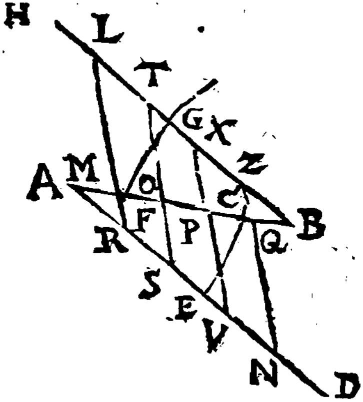
Datam rectam lineam A B inſectam ſimiliter ſecare (in F, G,) ut data altera A C, ſecta fuerit (in D, E.)

Extremitates ſectæ & inſectæ jungat recta B C. Huic ex punctis E, D
a 31.1. *a* duc parallelas E G, D F rectæ ſecandæ occurrentes in G, & F. Dico factum.

a Du-

a Ducatur enim D H parall. A B. Eſtque b 2.6.
A D. D E b :: A F. F G, & D E. E C b :: D I. c 34.1.
I H c :: F G. G B. Q.E.F. d 7.5.

Scholium.

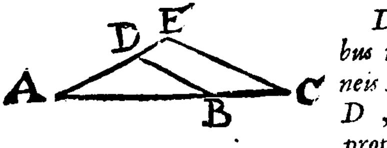


Hinc *diſcimus rectam datam A B in quotvis æquales partes* (puta 5.) *ſecare.* id quod facilius præſtabitur ſic;

Duc infinitam H D, eique parallelam B H etiam infinitam. Ex his cape partes æquales AR, R S, S V, V N; & B Z, Z X, X T, T L; in ſingulis una pauciores, quam deſiderentur in A B; tum rectæ ducantur L R, T S, X V, Z N. hæ quinquiſecabunt datam A B.

Nam RL, ST, VX, NZ *a* parallelæ ſunt. ergo a 33.1.
quum AR, RS, SV, VN *b* æquales ſint, *c* erunt b conſt.
AM, MO, OP, PQ æquales. Similiter quia BZ= c 2.6.
ZX, erit B Q = QP. ergo A B quinquiſecta eſt. Q.E.F.

PROP. XI.



Datis dua bus rectis · li neis A B, D, tertiam proportionalem D E invenire.

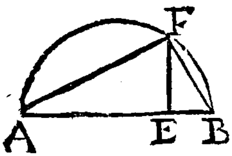
Junge B D, & ex A B protracta ſume B C = A D, per C duc C E parall. B D. cui occurra A D producta in E. Erit D E expetita.

a 2.6. Nam A B. *a* B C. (A D) : : A D. D E. Q E. F.

Vel ſic; Datæ ſint A B, B C; ex quibus fac angulum rectum A B C. duc A C, & huic normalem C D, cui occurrat A B protracta in D. *a*

a cor. 8. 6. eſtque A B. B C : : B C. B D.

PROP. XII.



Duabus datis rectis lineis A E, E B, mediam proportionalem E F adinvenire.

Super tota A B diametro deſcribe ſemicirculum A F B. Ex E erige perpendicularem E F occurrentem peripheriæ in F. Dico A E. E F : : E F. E B. Ducantur enim A F, & F B. Ex

a 31. 3. trianguli *a* rectanguli A F B recto angulo de-

b cor. 8. 6. ducta eſt F E baſi perpendicularis; *b* ergo AE. FE : : FE. EB. Q.E.F.

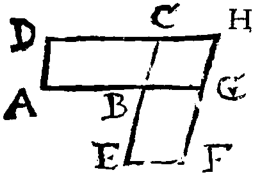
Vel

Vel (in eadem figura) ſint A B, B F duæ datæ, *b* liquet eſſe A B. B F : : B F. B E.

Coroll.

Hinc, linea recta, quæ in circulo à quovis puncto diametri, ipſi diametro perpendicularis ducitur ad circumferentiam uſque, media eſt proportionalis inter duo diametri ſegmenta.

PROP. XIII.



Æqualium, & unum A B C uni E B G æqualem habentium angulum, parallelogrammorum B D, B F, reciproca ſunt latera quæ circum æquales angulos. (AB. B G : : EB. B C.) *Et quorum parallelogrammorum BD, B F, unum angulum A B C uni angulo E B G æqualem habentium, reciproca ſunt latera quæ circum æquales angulos, illa ſunt æqualia.*

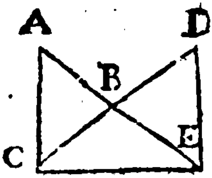
Nam latera A B, B G circa æquales angulos faciant unam rectam *a* quare E B, B C; etiam in directum jacebunt. Producantur F G. D C; donec occurrant. a ſch. 15. 1

1. *Hyp.* A B. B G *b* : : B D. B H *c* : : B F. BH *d* : : B E. B C. *e* ergo, &c. b 1. 6. c 7. 5. d 1. 6.

2. *Hyp.* B D. B H *f* : : A B. B G *g* : : B E. BC *h* : : B F. B H. *k* ergo Pgr. B D = B F. Q. E. D. e 11. 5. f 1. 6. g hyp. h 1. 6. k 11. & 9. 5.

PROP. XIV.

e *Æqualium, & unum A B C, uni D B E æqualem habentium angulum triangulorum A B C, D B E, reciproca ſunt latera, quæ circum æ-*

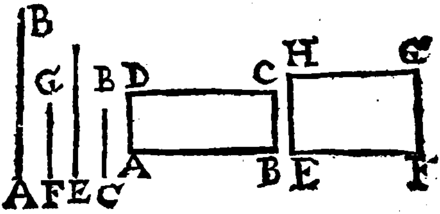


quales angulos (A B. B E :: D B. B C:) *Et quorum triangulorum A B C, D B E, unum angulum A B C uni D B E æqualem habentium reciproca ſunt latera, quæ circum æquales angulos* (A B. B E :: D B. B C.) *illa ſunt æqualia.*

Latera C B, B D circa æquales angulos, ſta-
a ſch.15.1. tuantur ſibi in directum; *a* ergo A B E eſt re-
b 1.6. cta linea. ducatur C E.

1. *Hyp.* A B. B E *b* :: triang. A B C. C B E
c 7.5. *c* :: triang. D B E. C B E. *d* :: D B. B C. *e* er-
d 1.6. go, &c.
e 11.5.
f 1.6. 2. *Hyp.* Triang. A B C. C B E *f* :: A B. B E
g hyp. *g* :: D B. B C *h* :: triang. D B E. C B E. *k* ergo
h 1.6.
k 11.& 9.5. triang. A B C = D B E. Q.E.D.

PROP. XV.



Si quatuor rectæ lineæ proportionales fuerint (A A. F G :: E F. C B) *quod ſub extremis A B, C B comprehenditur rectangulum A C, æquale eſt ei, quod ſub medijs E F, F G comprehenditur, rectangulo E G. Et ſi ſub extremis comprehenſum rectangulum A C æquale fuerit ei, quod ſub me-*

mediis comprehenditur, rectangulo E G. illæ quatuor rectæ lineæ proportionales erunt (A *B*. F G : : E F. C *B*.)

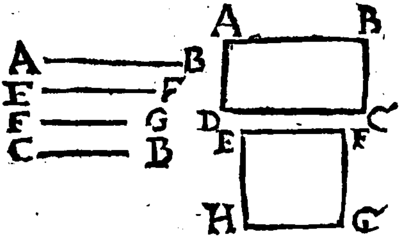
1. *Hyp.* Anguli B & F recti, ac *a* proinde pares ſunt; atque ex hyp. A B. F G : : E F. C *B*. *b* ergo rectang. A C = E G. Q.E.D. a 12. ax. b 14. 6.

2. *Hyp.* *c* Rectang. A C = E G; atque ang. B = F; *d* ergo A B. F G : : E F. C. B. Q.E.D. c hyp. d 14. 6.

Coroll.

Hinc ad datam rectam lineam A B facile eſt datum rectangulum E G applicare, *e* faciendo A B. E F : : F G. B C. e 12. 6.

PROP. XVI.



Si tres rectæ lineæ ſint proportionales (A B. E F : : E F. C B,) *quod ſub extremis A B*, C B *comprehenditur rectangulum A C, æquale eſt ei, quod à media E F deſcribitur, quadrato E G. Et ſi ſub extremis A* B, *C* B *comprehenſum rectangulum A C, æquale ſit ei, quod à media E F deſcribitur, quadrato E G, illæ tres rectæ lineæ proportionales erunt* (A B. E F : : E F. C B.)

Accipe F G = E F.

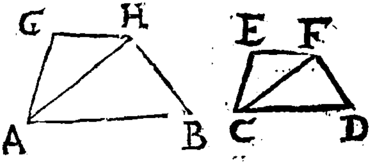
1. *Hyp.*

a hyp. 1. *Hyp*. A B. E F *a* :: E F (F G.) C B. ergo
b 16.6. Rectang. A C *b* = E G *c* = E Fq. Q.E.D.
c 29.def. 1. 2. *Hyp*. Rectang. A C *d* = quadr. E G = E-
d hyp. Fq. *e* ergo A B. E F :: F G (E F.) B C.
e 16.6.

Coroll.

Sit A in B = Cq. ergo A. C :: C. B.

PROP. XVII.



A data recta linea AB dato rectilineo CEFD ſimile ſimiliterq; poſitum rectilineum AGHB deſcribere.

a 23.1. Datum rectilineum reſolve in triangula. *a* fac ang. A B H = D; *a* & ang. B A H = D C-F; *a* & ang. A H G = C F E; *a* & ang. H A-G = F C E. Rectilineum A G H B eſt quæſitum.

b conſt. Nam ang. B *b* = *D*. & ang. B A H *b* = D C-F. *c* quare ang. A H B = C F D; *b* item ang.
c 32.1. H A G = F C E, *b* & ang. A H G = C F E.
d 2.ax. *c* quare ang. G = E; & totus ang. G A B *d* = E C D. Polygona igitur ſibi mutuo æquiangula ſunt. Porro ob trigona æquiangula, A B. B-
e 4.6. H *e* :: C D. D F. & A G. G H. *e* :: C E. E F. item A G. A H. *e* :: C E. C F. & A H. A B *e* :: C E. C D. *f* unde ex æquo A G. A B :: C E.
f 22.5. g 1.def.6. C D. eodem modo G H. H B :: E F. F D. *g* ergo polygona A B H G, C D F E ſimilia ſimiliterque poſita exiſtunt. Q.E.F.

PROP.

PROP. XIIX.

Similia triangula A B C, D E F ſunt in duplicata ratione laterum homologorum B C. *E F.*

a Fiat B C. E F : : E F. B G. & ducatur A-G.

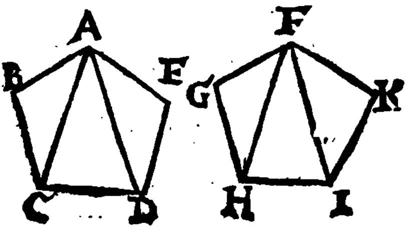
Quia A B. *D* E *b* : : B C. E F *c* : : E F. B G. & ang. B = E; *d* erit triang. A B G = D E F. verum triang. ABC. ABG *e* : : B C. B G; & *f* $\frac{BC}{BG} = \frac{BC}{EF}$ bis; ergo triang. $\frac{ABC}{ABG}$ hoc eſt $\frac{ABC}{DEF}$ *g* = $\frac{BC}{EF}$ bis. Q.E.D.

a 11.6.
b cor.4.6
c conſt.
d 15.6.
e 1.6.
f 10. def.5.
g 11.5.

Coroll.

Hinc ſi tres lineæ B C, E F, B G proportionales fuerint; ut eſt prima ad tertiam, ita eſt triangulum ſuper primam B C deſcriptum ad triangulum ſuper ſecundam E F ſimile ſimiliterque deſcriptum. vel ita eſt triangulum ſuper ſecundam E F deſcriptum ad triangulum ſuper tertiam ſimile ſimiliterque deſcriptum.

PROP. XIX.



Simi-

Similia polygona A B C D E, F G H I K *in ſimilia triangula* A B C, F G H; *&* A C D, F H I, *&* A D E, F I K *dividuntur*, *& numero æqualia*, *& homologa totis*. (A B C, F G H :: A B C D E. F G H I K :: A C D. F H I :: A D E. F I K.) *Et polygona* A B C D E, F G H I K *duplicatam habent eam inter ſe rationem, quam latus homologum* B C *ad homologum latus* G H.

a hyp. 1. Nam ang. B *a* = G; & A B. B C *a* :: F G.
b 6.6. G H. *b* ergo triangula A B C, F G H æquiangula ſunt. eodem modo, triangula A E D, F K I aſſimilantur. cum igitur ang. B C A *b* = G H F; & ang. A D E *b* = F I K; totique anguli B-
c hyp. C D, G H I; atque toti C D E, H I K *c* pa-
d 3.ax. res ſint, *d* remanent ang. A C D = F H I; &
e 32.1. ang. A D C = F I H; *e* unde etiam ang. C A D = H F I. ergo triangula A C D, F H I ſimilia ſunt. ergo, &c.

2. Quoniam igitur triangula B C A, G H F
f 19.6. ſimilia ſunt, *f* erit $\frac{BCA}{GHF} = \frac{BC}{GH}$ bis. ob eandem cauſam $\frac{CAD}{HFI} = \frac{CD}{HI}$ bis. denique triang. $\frac{DEA}{IKF} =$
g hyp. & 16.5. $\frac{DF}{IK}$ bis. quare cum B C. G H. *g* :: C *D*. H I *g*
h sch. 23.5. D E. I K, *h* erit triang. B C A. G H F :: C A D.
k 12.5. H F I :: D E A. I K F; *k* polyg. A B C D E. F G H I K :: $\frac{BC}{GH}$ bis.

Coroll.

I. Hinc, ſi fuerint tres lineæ rectæ proportionales; ut eſt prima ad tertiam, ita erit polygo-

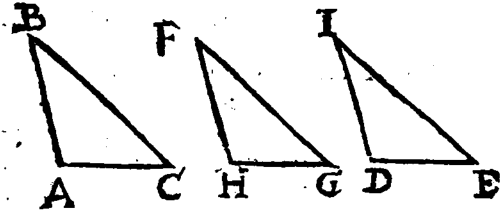
pum

num ſuper primam deſcriptum ad polygnum ſuper ſecundam ſimile ſimiliterque deſcriptum vel ita erit polygonum ſuper ſecundam deſcriptum ad polygonum ſuper tertiam ſimile ſimiliterque deſcriptum.

Unde elicitur methodus *figuram quamvis rectilineam augendi vel minuendi in ratione data. Ut ſi velis pentagoni, cujus latus* C D, *aliud facere quintuplum. inter A B, & 5 A B inveni mediam proportionalem. Super hac * conſtrue pentagonum ſimile dato. hoc erit quintuplum dati.* * 18.6.

II. Hinc etiam, ſi figurarum ſimilium homologa latera nota fuerint, etiam proportio figurarum innoteſcet; nempe inveniendo tertiam proportionalem.

PROP. XX.

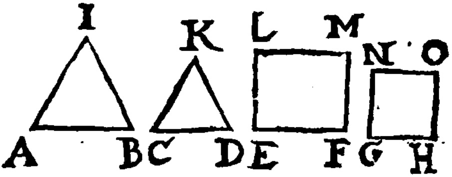


Quæ (A B C, D I E) *eidem rectilineo* H F G *ſunt ſimilia, & inter ſe ſunt ſimilia.*

Nam ang. A [a] = H [a] = D. & ang. C [a] = G [a] = E; & ang. B [a] = F [a] = I. [a] item A B. A C :: H F. H G :: D I. D E. [a] & A C. C B :: H G. G F :: D E. E I. & A B. B C :: H F. F G :: D I. I E. [a] ergo A B C, D I E ſimilia ſunt. Q.E.D. [a] 1.def.6

PROP.

PROP. XXI.



Si quaturor recta lineæ proportionales fuerint (AB.CD :: EF.GH.) *& ab eis rectilinea ſimilia ſimiliterq́ʒ deſcripta proportionalia erunt.* (ABI.CDK :: EM.GO.) *Et ſi à rectis lineis ſimilia ſimiliterq́ʒ deſcripta rectilinea proportionalia fuerint* (ABI.CDK :: EM.GO) *ipſa etiam recta lineæ proportionales erunt.*(AB.CD :: EF.GH.

a 19.6. 1. *Hyp.* $\frac{ABI}{CDK}$ *a* = $\frac{AB}{CD}$ bis = *a* $\frac{EF}{GH}$ bis *a* = $\frac{EM}{GO}$

d ergo ABI.CDK :: EM.GO. Q.E.D.

b hyp. 2. *Hyp.* $\frac{AB}{CD}$ bis = *a* $\frac{ABI}{CDK}$ *b* = $\frac{FM}{GO}$ *c* = $\frac{EF}{GH}$ bis.

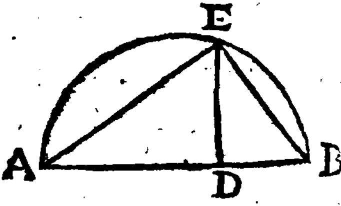
c.20.6.

d cor.23.5. ergo AB.CD :: EF.GH. Q.E.D.

Schol.

Hinc deducitur, & demonſtratur ratio multiplicanai quantitates ſurdas. ex gr. Sit $\sqrt{}5$. multiplicandus in $\sqrt{}3$. dico provenire $\sqrt{}15$. nam ex multiplicationis definitione debet eſſe, 1. $\sqrt{}3$ :: $\sqrt{}5$. product. ergo per hanc, q. 1. q. $\sqrt{}3$ :: $\sqrt{}5$. q. product. hoc eſt. 1. 3 :: 5. q. product. ergo q. product. eſt 15. quare $\sqrt{}15$ eſt productus ex $\sqrt{}3$ in $\sqrt{}5$. Q.E.D.

THEOR.



Si recta linea A B ſecta ſit utcunque in D, rectangulum ſub partibus A D, DB contentum, eſt medium proportionale inter earum quadrata. Item rectangulum contentum ſub tota A B, & una parte A D, vel D B, eſt medium proportionale inter quadratum totius A B, & quadratum dicta partis A D, vel D B.

Petr. Herig.

Super diametrum A B deſcribe ſemicirculum. ex D erige normalem D E occurrentem peripheriæ in E. junge A E, B E.

Liquet eſſe A D. D E *a* : : D E. D B. *b* ergo A Dq. D Eq : : D Eq. D Bq. *c* hoc eſt, A Dq. A D B : : A D B. D Bq. Q.E.D.

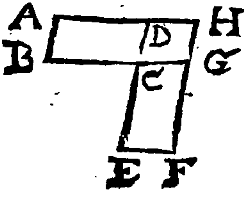
a cor. 8.6. b 22.6. c 16.6.

Porro, B A. A E *d* : : A E. A D. *e* ergo BAq. A Eq : : A Eq. A Dq. *f* hoc eſt B Aq. B A D : : B A D. A Dq. Eodem modo A Bq. A B D : : A B D. B Dq. Q E.D.

d cor. 8.6 e 22.6. f 17.6.

Vel ſic; ſit $Z = A + E$. liquet eſſe Aq. A E : : *a* A. E : : *a* A E. Eq. item Zq. Z A : : *a* Z. A. : : *a* Z A. Aq. & Zq. Z E : : *a* Z. E : : Z E. Eq.

a 1.6.



PROP. XXIII.

Æquiangula, parallelogramma A C, C F inter ſe rationem habens eam quæ ex lateribus

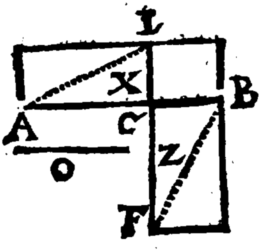
bus componitur. $\left(\frac{AC}{CF} = \frac{BC}{CG} + \frac{DC}{CE}.\right)$

a ſch. 15. Latera circa ęquales angulos C a ſibi in directum ſtatuantur; & compleatur parallelogrammum CH.

b 10. def. 5. c 1. 6. Ratio $\frac{AC}{CF}$ b $= \frac{AC}{CH} + \frac{CH}{CF}$ c $= \frac{BC}{CG} + \frac{DC}{CE}$. Q. E. D.

Coroll.

d Andr. Tacq. 15. 6. Hinc & ex 34. 1. patet primo, *Triangula, quæ unum angulum* (ad C) *æqualem habent, rationem habere ex rationibus rectarum*, A C ad CB, & L C ad C F, *æqualem angulum continentium.*



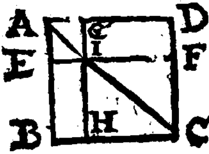
* 35. 1. Patet secundo; *Rectangula ac* proinde ac parallelogramma quæcunque rationem inter ſe habere compoſitam ex rationibus baſis ad baſim, & altitudinis ad altitudinem.* Neque aliter de triangulis ratiocinaberis.

Patet tertio, *Quomodo triangulorum ac parallelogrammorum proportio exhiberi poſſit.* Sunto parallelogramma X & Z; quorum baſes A C, C B; altitudines vero C L, C F. Fiat

† 14. 6. & 1. 6. CL. CF :: CB. O. † erit X. Z :: A C. O.

PROP.

PROP. XXIV.

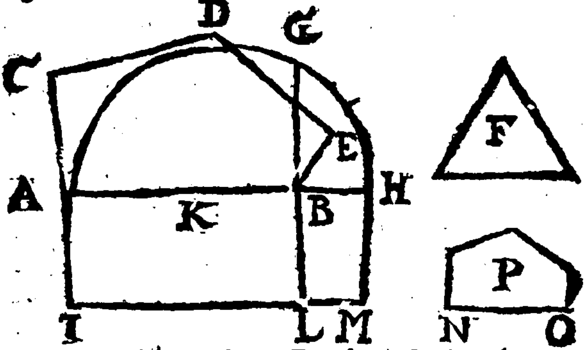


In omni parallelogrammo A B C D, quæ circa diametrum A C ſunt parallelogramma E G, H F, & toti & inter ſe ſunt ſimilia.

Nam parallelogramma E G, H F habent ſingula unum angulum cum toto communem. ergo toti & ſibi mutuo æquiangula ſunt. *a* Item tam triangula A B C, AEI, IHC, quam triangula A D C, A G I, IFC ſunt inter ſe æquiangula. *b* ergo AE. E I :: A B. B C, *b* atque A E. A I :: A B. A C; *b* & AI. A G :: A C. A D. *c* ex æquali igitur, A E. A G :: A B. A D. *d* ergo Pgra. E G. BD ſimilia ſunt eodem modo HF, BD ſimilia ſunt. ergo, &c.

a 29.1.
b 4.6.
c 22.5.
d 1. def. 6.

PROP. XXV.



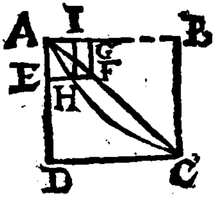
Dato rectilineo A B E D C ſimile ſimiliterque poſitũ P, idemque alteri dato F æquale, conſtituere.

a Fac rectang. A L = ABEDC. *b* item ſuper BL fac triang. B M = F. Inter A B, B H *c* inveni

a 45.1.
b 44.1.
c 13.6.

veni mediam proportionalem N O, ſuper N-
d 17.6. O *d* fac polygonum P ſimile dato A B E D C.
e cor. 19. 6. Erit hoc æquale dato F.
f 1.6. Nam A B E D C (A L.) P : : *e* A B. B H *f*
g 9.5. h conſtr. : : A L. B M. ergo P *g* = B M *h* = F. Q.E.F.

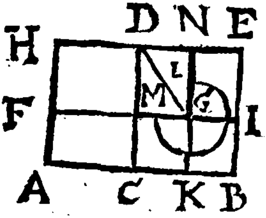
PROP. XXVI.



Si à parallelogrammo A B C D parallelogrammum A G F E ablatum ſit, & ſimile toti, & ſimiliter poſitum, communem cum eo habens angulum E A G, hoc circa eandem cum toto diametrum A C conſiſtet.

Si negas A C eſſe communem diametrum, eſto diameter A H C ſecans E F in H. & ducatur H I parall. A E. Parallelogramma E I, D-
a 24.6. B *a* ſimilia ſunt. *b* ergo A E. E H : : A D. D C
b 1.def.6. *c* : : A E. E F. *d* proinde E H = E F. *f* Q.E.A.
c hyp.
d 9.5.
f 9.ax.

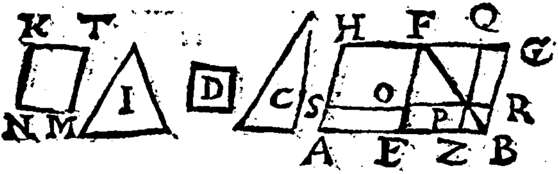
PROP. XXVII.



Omnium parallelogrammorum A D, A G ſecundum eandem rectam lineam A B applicatorum, deficientiumq́; figuris parallelogrammis C E, K I ſimilibus, ſimiliterq́; poſitis, ei A D, quod à dimidia deſcribitur, maximum eſt A D, quod *ad dimidium eſt applicatum, ſimile exiſtens defectui K I.*

Nam quia G E *a* = G C, addito communi K I, *b* erit K E = C I *c* = A M. adde commune C G, *d* erit A G = Gnom. MBL. ſed Gnom. MBL *e* ⊐ C E (A D.) ergo A G ⊐ A D. Q.E.D. a 43.1. b 2.ax. c 36.1. d 2.ax. e 9.ax.

PROP. XXVIII.



Ad datam rectam lineam A B, dato rectilineo C æquale parallelogrammum A P applicare deficiens figura parallelogramma Z R, quæ ſimilis ſit alteri parallelogrammo dato D. † *Oportet autem datum rectilineum C, cui æquale A P applicandum eſt, non majus eſſe eo A F, quod ad dimidiam applicatur, ſimilibus exſiſtentibus defectibus, & ejus A F quod ad dimidiam applicatur, & ejus D, cui ſimile deeſſe debet.* † 27.6.

Biſeca A B in E. Super E B *a* fac Pgr. E G ſimile dato D. *b* ſitque E G = C + I. *c* fac pgr. N T = I, & ſimile dato D., vel E G, duc diametrum F B; fac F O = K N; & F Q = K T. Per O, & Q duc parallelas S R, Q Z, parallelogrammum A P eſt iſt quod quæritur. a 18.6. b ſch. 45.1. c 25.6.

Nam parallelogramma D, E G, O Q, N T,

d conſtr. ZR d ſunt ſimilia inter ſe. Et Pgr. EG e = NT
& 24.6. + C e = OQ + C; f quare C = Gnom. O-
e conſtr. BQ g = AO + PG h = AO + EP = AP.
f 3. ax. Q. E. F.
g 2. ax.
h 43. 1.

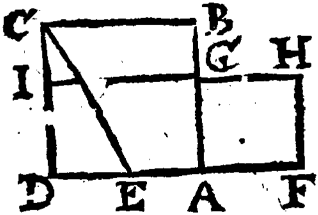
Ad datam rectam lineam A B, dato rectilineo C æquale parallelogrammum A N applicare, excedens figura parallelogramma O P, quæ ſimilis ſit parallelogrammo alteri dato D.

a 18. 6. Biſeca AB in E. ſuper EB a fac Pgr. EG
b 25. 6. ſimile dato D. b ſitque pgr. HK = EG + C,
c 3. 1. & ſimile dato D vel EG. fac FEL c = IH;
c & FGM = IK. per LM duc parallelas RN, MN. & AR parall. NM. Produc ABP, GBO. Duc diametrum FBN. Pgr. AN eſt quæſitum.

d conſtr. Nam parallelogramma D, HK, LM, EG d
e 24. 6. ſimilia ſunt. e ergo parallelogramma OP ſimile eſt parallelogr. LM, vel D. item LM f =
f conſtr. HK f = EG + C. g ergo C = Gnom. E-
g 3. ax. NG. atqui AL h = LB k = BM l ergo C =
h 36. 1. AN. Q. E. F.
k 43. 1.
l 2. & 1. ax.

PROP.

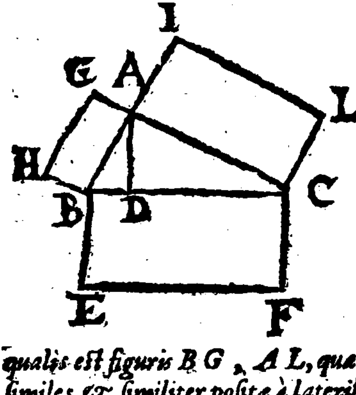
PROP. XXX.



Propoſitam rectam lineam terminatam AB, *extrema ac media ratione ſecare.* (AB.AG :: AG.GB.)

*a*Seca AB in G, ita ut AB x BG = AGq. *a 11.2.*
*b*ergo BA.AG :: AG.GB. Q.E.F. *b 16.6.*

PROP. XXXI.



In rectangulis triangulis BAC, figura quævis BF à latere BC rectum angulum BAC ſubtendente, deſcripta, æqualis eſt figuris BG, AL, quæ priori illi BF ſimiles, & ſimiliter poſitæ à lateribus BA, AC rectum angulum continentibus deſcribuntur.

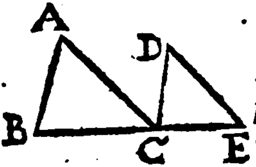
Ab angulo recto BAC demitte perpendi-
cularem AD. Quoniam DC.CA :: *a* CA. *a cor.8.6.*
CB, *b* erit AL.BF :: DC.CB. Item ob D- *b cor.*
B.BA :: *a* BA.BC, *b* erit BG.BF :: DB.BC. *c* *20.6.* *c 24.5.*
ergo AL + BG.BF :: DC + DB (BC.) BC *d ſch.*
ergo AL + BG = BF. Q.E.D. *74.5.*

Coroll.

Coroll.

Ex hac propoſitione addi poſſunt, & ſubtrahi figuræ quævis ſimiles, eadem methodo, qua quadrata adduntur & ſubtrahuntur, in ſchol. 47. 1.

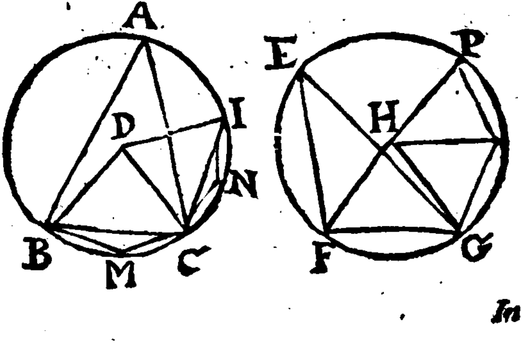
PROP. XXXII.



Si duo triangula ABC, DCE, quæ duo latera duobus lateribus proportionalia habeant (AB. AC :: DC. DE,) *ſecundum unum angulum ACD compoſita fuerint, ita ut homologa eorum latera ſint etiam parallela* (AB ad DC, & AC ad DE) *tum reliqua illorum triangulorum latera BC, CE in rectam lineam collocata reperientur.*

a 29.1. Nam ang. A *a* = ACD *a* = D; & AB. A-
b hyp. C *b* :: DC. DE. *c* ergo ang. B = DCE. ergo
c 6.6. ang. B + A *d* = ACE. ſed ang. B + A + A-
d 2.ax. CB *e* = 2 Rect. *f* ergo ang. ACE + ACB =
e 32.1. 2. Rect. *g* ergo BCE eſt recta linea. Q.E.D.
f 1.ax.
g 14.1.

PROP. XXXIII.



In

In æqualibus circulis DBCA, HFGP, anguli BDC, FHG eandem habent rationem cum peripheriis B C, F G, quibus inſiſtunt; ſive ad centra (ut *B D C, F H G,*) *ſive ad peripherias* A, *E conſtituti inſiſtant : inſuper vero & ſectores BDC, FHG, quippe qui ad centra conſiſtant.*

Duc rectas BC, FG. Accommoda CI = CB; & GL = F G = L P; & junge D I, HL, HP.

Arcus B C *a* = CI, *a* item arcus F G, G L, LP æquantur. *b* ergo ang. B D C = C D I *b* & ang. FGH = GHL = LHP. Ergo arcus BI tam multiplex eſt arcûs BC, quam ang. BDI anguli BDC. pariterque æquemultiplex eſt arcus F P arcûs F G, atque ang. FHP anguli F H G. Verum ſi arcus BI $\sqsubset$, =, $\sqsupset$ F P, *c* erit ſimiliter ang BDI $\sqsubset$, =, $\sqsupset$ FHP. ergo arc. BC. FG *d* : : ang. B *D* C. FHG *e* : : BDC. F H G *f* : : A. E, Q. E. *D*.

a 21.3.
b 27.3.
c 27.3.
d 6. def. 5.
e 15.5.
f 20.3.

Rurſus ang. BMC *g* = CNI; *h* atque idcirco ſegm. BCM = CIN. *k* item triang. BDC = CDI. *l* ergo ſector BDCM = CDIN. Simili ratione ſectores FHG, GHL, LHP æquantur. Quum igitur prout arcus BI $\sqsubset$, =, $\sqsupset$ FGP, ita ſimiliter ſector BDI $\sqsubset$, =, $\sqsupset$ FHP. *m* erit ſect. BDC. FHG : : arc. BC. FG. Q. E. D.

g 27.3.
h 24.3.
k 4.1.
l 2. ax.
m 6. def. 5

Coroll.

Hinc 1. *Ut ſector ad ſectorem, ſic angulus ad angulum.* 11.5.

2. *Ang. BDC in centro eſt ad 4 rectos, ut arcus BC cui inſiſtit ad totam circumferentiam.*

Nam ut ang. BDC ad rectum, ſic arcus BC ad quadrantem. ergo BDC eſt ad 4 rectos, ut arcus B C ad 4 quadrantes, id eſt ad totam circumferentiam. item ang. A, 2. Rect : : arc. B C. periph.

Hinc 3. *Inæqualium circulorum arcus I L, B C; qui æquales ſubtendunt angulos, ſive ad centra,* ut I A L & B A C, *ſive ad peripheriam, ſunt ſimiles.*

Nam I L. periph. : : ang. I A L, (B A C.) 4. Rect. item arc. B C. periph : : ang. B A C. 4. Rect. ergo I L. periph : : B C. periph. proinde arcus I L, & B C ſunt ſimiles. Unde

4. *Duæ ſemidiametri A B, A C à concentricis peripheriis arcus auferunt ſimiles I L, B C.*

LIB. VII.

DEFINITIONES.

I.

UNitas eſt, ſecundum quam unumquodque eorum quæ ſunt, unum dicitur.

II. Numerus autem eſt, ex unitatibus compoſita multitudo.

III. Pars eſt numerus numeri, minor majoris, quum minor metitur majorem.

Omnis pars ab eo numero nomen ſibi ſumit, per quem ipſa numerum, cujus eſt pars, metitur; ut 4 *dicitur tertia pars numeri* 12, *quia metitur* 12 *per* 3.

IV. Partes autem, cum non metitur.

Partes quæcunque nomen accipiunt à duobus illis numeris, per quos maxima communis duorum numerorum menſura utrumque eorum metitur. ut 10. *dicitur* $\frac{2}{3}$ *numeri* 15, *eo quod maxima communis menſura, nempe* 5, *metitur* 10 *per* 2, *&* 15 *per* 3.

V. Multiplex vero major minoris, cum majorem metitur minor.

VI. Par numerus eſt, qui biſariam dividitur.

VII. Impar vero numerus, qui biſariam non dividitur; vel, qui unitate differt à pari.

VIII. Pariter par numerus eſt, quem p[illegible] numerus metitur per numerum parem.

IX. Pariter autem impar eſt, quem par numerus metitur per numerum imparem.

X. Impariter vero impar numerus eſt, quem impar numerus metitur per numerum imparem.

XI. Primus numerus eſt, quem ſola unitas metitur.

XII. Primi inter ſe numeri ſunt, quos ſola unitas, communis menſura, metitur.

XIII. Compoſitus numerus eſt, quem numerus quiſpiam metitur.

XIV. Compoſiti autem inter ſe numeri ſunt, quos numerus aliquis communis menſura metitur.

In hac definitione & præcedenti unitas non eſt numerus.

XV. Numerus numerum multiplicare dicitur, cum toties compoſitus fuerit is qui multiplicatur, quot ſunt in ipſo multiplicante unitates, & procreatus fuerit aliquis.

Hinc, in omni multiplicatione unitas eſt ad multiplicatorem ut multiplicatus ad productum.

Nota, quod ſæpe cum multiplicandi ſunt quivis numeri, puta A *in* B, *literarum conjunctio productum denotat. Sic* $AB = A$ *in* B: *item* $CDE = C$ *in* D *in* E.

XVI. Cum autem duo numeri ſeſe multiplicantes aliquem fecerint, qui factus erit, planus appellabitur; Qui vero numeri ſeſe mutuo multiplicarint, latera illius dicentur. *Sic* 2 (C) in 3 (D) $= 6 =$ CD *eſt numerus planus.*

XVII. Cum vero tres numeri mutuo ſeſe multiplicantes fecerint aliquem, qui procreatus erit, ſolidus appellabitur; Qui autem numeri

meri

meri mutuo ſeſe multiplicarint, latera illius dicentur. *Sic*, 2 (C) *in* 3 (D) *in* 5 (E) = 30 = C-D E *eſt numerus ſolidus.*

XVIII. Quadratus numerus eſt, qui æqualiter æqualis, vel qui ſub duobus æqualibus numeris continetur. *Sit A latus quadrati; quadratus ſic notatur*, AA, *vel* Aq.

XIX. Cubus vero, qui æqualiter æqualis æqualiter, vel qui ſub tribus æqualibus numeris continetur. *Sit* A *latus cubi; cubus notatur ſic*, AAA, *vel* Ac.

In hac definitione, & tribus præcedentibus, unitas eſt numerus.

XX. Numeri proportionales ſunt, cum primus ſecundi, & tertius quarti æquemultiplex eſt, vel eadem pars; vel denique cum pars primi ſecundum, & eadem pars tertii æque metitur quartum, vel vice verſa. A. B :: C. D, hoc eſt, 3.9 :: 5. 15.

XXI. Similes plani, & ſolidi numeri ſunt, qui proportionalia habent latera.

Latera nempe non qualibet, ſed quædam.

XXII. Perfectus numerus eſt, qui ſuis ipſius partibus eſt æqualis.

Ut 6. & 28. *Numerus vero qui ſuis ipſius partibus minor eſt, abundans appellatur: qui vero major, diminutus. ut* 12. *eſt abundans*, 15 *eſt diminutus.*

XXIII. Numerus numerum metiri dicitur per illum numerum, quem multiplicans, vel à quo multiplicatus, illum producit.

In diviſione, unitas eſt ad quotientem, ut dividens

videns ad diviſum. Nota, quod numerus alteri lineolà interjecta ſubſcriptus diviſionem denotat. Sic $\frac{A}{B} =$ A diviſ. per B. *item* $\frac{CA}{B} =$ C *in* A diviſ. per B.

Termini ſive radices proportionis dicuntur duo numeri, quibus in eadem proportione minores ſumi nequeunt.

Poſtulata.

1. Poſtuletur, cuilibet numero quotlibet ſumi poſſe æquales, vel multiplices.

2. Quolibet numero ſumi poſſe majorem.

3. Additio, ſubtractio, multiplicatio, diviſio, extractionesque radicum, ſeu laterum, numerorum quadratorum, & cuborum concedantur etiam, tanquam poſſibilia.

Axiomata.

1. Quicquid convenit uni æqualium numerorum, convenit & reliquis æqualibus numeris.

2. Partes eidem parti, vel iisdem partibus, eædem, ſunt quoque inter ſe eædem.

3. Qui numeri æqualium numerorum, vel ejusdem, eædem partes fuerint, æquales inter ſe ſunt.

4. Quorum idem numerus, vel æquales, eædem partes fuerint, æquales inter ſe ſunt.

5. Unitas omnem numerum per unitates, quæ in ipſo ſunt, hoc eſt, per ipſummet numerum metitur.

6. Omnis numerus ſeipſum metitur per unitatem.

7. Si numerus numerum multiplicans, aliquem

quem

quem produxerit, metietur multiplicans productum per multiplicatum, multiplicatus autem eundem per multiplicantem.

Hinc nullus numerus primus planus eſt aut ſolidus.quadratus,vel cubus.

8. Si numerus numerum metiatur, & ille per quem metitur, eundem metietur per eas, quæ in metiente ſunt, unitates, hoc eſt, per ipſū numerum metientem.

9. Si numerus numerum metiens, multiplicet eum, per quem metitur, vel ab eo multiplicetur, illum quem metitur, producit.

10. Numerus quotcunque numeros metiens, compoſitum quoque ex ipſis metitur.

11. Numerus quemcunque numerum metiens, metitur quoq; omnem numerum quem ille metitur.

12. Numerus metiens totum & ablatum, metitur & reliquum.

PROP. I.

A.....E..G.B   8   5   3
C...F...D $\substack{5\\3}$ $\substack{3\\2}$ $\substack{2\\1}$
H---

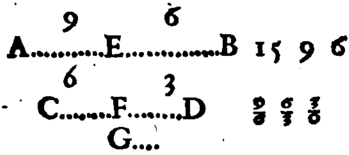
Si duobus numeris inæqualibus propoſitis (AB, CD) *detrahatur ſemper minor CD de majore AB* (& reliquus EB de CD &c.) *alterna quadam detractione, neque reliquus unquam præcedentem metiatur, quoad aſſumpta ſit unitas GB; qui principio propoſiti ſunt numeri AB, CD primi inter ſe erunt.*

Si negas, habeant AB, CD communem menſuram, numerum H. Ergo H metiens CD, a etiam AE metitur; proinde & reliquum FB;

a ergo

a 11.ax.7. *a* ergo & C F, atque *b* idcirco reliquum F D; b 12.ax.7. *a* quare & ipſum E G. ſed totum E B metiebatur; *b* ergo & reliquum G B metitur, numerus unitatem. *c* Q.E.A. c 9.ax.1.

PROP. II.



Duobus numeris datis A B, C D non primis inter ſe, maximam eorum communem menſuram F D reperire.

Detrahe minorem numerum C D ex majori A B, quoties potes. Si nihil relinquitur, a 6.ax.7. *a* patet ipſum C D eſſe maximam communem menſuram. Si relinquitur aliquid E B, deme hunc ex C D; & reliquum F D ex E B, & ſic deinceps, donec aliquis F D præcedentem E B metiatur. b 1.7. (nam *b* hoc fiet antequam ad unitatem perveniatur.) Erit F D maxima communis menſura.

c conſtr. Nam F *D* *c* metitur E B, *d* ideoque & C F; d 11.ax.7 *e* proinde & totum C D; *d* ergo ipſum A E; e 12.ax.7 atque idcirco totum A B metitur. Liquet igitur F D communem eſſe menſuram. Si maximam eſſe negas, ſit major quæpiam G. ergo G metiens C D, *d* metitur A E, *e* & reliquum EB, *d* ipſumque C F. *e* proinde & reliquum F D, *g* g ſuppoſ. major minorem. *h* Q.E.A. h 9.ax.1.

Coroll.

Hinc, numerus metiens duos numeros, metitur quoque maximam eorum communem menſuram.

PROP.

PROP. III.

A.......... 12
B 8
D 4
C 6
E .. 2
F ---

Tribus numeris datis A, B, C non primis inter ſe, maximam eorum communem menſuram E reperire.

Inveni D maximam communem menſuram duorum A, B. Si D metitur tertium C, liquet D maximam eſſe trium communem menſuram. Si D non metitur C, erunt ſaltem D, & C compoſiti inter ſe, ex coroll. præcedentis. Sit igitur ipſorum D, & C maxima communis menſura E. erit E is quem quæris.

Nam E *a* metitur C, & D; *a* ac D ipſos A, & B metitur; *b* ergo E metitur ſingulos A, B, C; nec major aliquis (F) eos metietur; nam ſi hoc affirmas, *c* ergo F metiens A, & B, eorum maximam communem menſuram D metitur. Eodem modo, F metiens D, & C, *c* eorum maximam communem menſuram E, *d* major minorem, metitur. *e* Q.E.A. a *conſtr.* b *11.ax.7* c *cor.1.7* d *ſuppoſ.* e *9.ax.1*

Coroll.

Hinc, numerus metiens tres numeros, maximam quoque eorum communem menſuram metitur.

PROP. IV.

A 6
B 7
B 18
B .... 9.

Omnis numerus A, omnis numeri B. minor majoris, aut pars eſt, aut partes.

Si A & B primi ſint inter ſe, *a* erit A tot partes nu- a *4.def.7* numeri B, quot ſunt in A unitates. (ut $6 = \frac{6}{7}7$.) b 3.def.7. Sin A metiatur B, b liquet A eſſe partem ipſius B. (ut $6 = \frac{1}{3}18$.) denique ſi A & B aliter compoſiti inter ſe fuerint, c 4.def.7. c maxima communis menſura determinabit, quot partes A conficiat ipſius B; ut $6 = \frac{2}{3}9$.

PROP. V.

A ......6 D ....4.

6 6 4 4

B......G......C 12. E....H....F 8

Si numerus A numeri B C pars fuerit, & alter D alterius E F eadem pars; & ſimul uterq; (A + D) utriuſq; ſimul (B C + E F) eadem pars erit, quæ unus A unius B C.

Nam ſi B C in ſuas partes B G, G C ipſi A æquales; atque E F in ſuas partes F H, H F ipſi D æquales reſolvantur; a hyp. a erit numerus partium in B C æqualis numero partium in E F. b conſt. & 2.ax.1. Quum igitur A + D b = B G + E H = G C + H F, erit A + D toties in B C + E F, quoties A in B C. Q.E.D.

Vel ſic; Sit $a = \frac{x}{2}$, & $b = \frac{y}{2}$. quare $2a = x$, & $2b = y$. ergo $2a + 2b = x + y$. ergo $a + b = \frac{x + y}{2}$.

PROP. VI.

3 3 4 4

A...G...B 6. D....H....E 8

C........9 F..........12

Si numeri A B numeri C partes fuerit; & alter D E alterius F eadem par-

partes; & ſimul uterq; (A B + D E) utrinsq; ſimul (C + F) eadem partes erit, quæ unus A B unius C.

Divide A B in ſuas partes A G, G B; & D E in ſuas D H, H E. Partium in utroque A B, D E æqualis eſt multitudo, ex hypoth. Quum igitur A G *a* ſit eadem pars numeri C, quæ D H numeri F, *b* erit A G + D H eadem pars compoſiti C + F, quæ unus A G unius C. *b* Eodem modo G B + H E eadem pars eſt ejusdem C + F, quæ unus G B unius C; *c* ergo A B + D E eædem partes eſt ipſius C + F, quę A B ipſius C. Q.E.D.

a hyp.
b 5. 7.
c 2. ax. 7

Vel ſic; Sit $a = \frac{2x}{3}$, & $b = \frac{2y}{3}$, & $x + y = g$. ob $3a = 2x$, & $3b = 2y$, eſt $3a + 3b = 2x + 2y = 2g$. ergo $a + b = \frac{2}{3} g. = \frac{2}{3} : x + y$.

PROP. VII.

```
    5   3
  A.....E...B 8
   6     10      6
G......C.........F......D 16
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Si numerus A B numeri C D pars fuerit, qualis ablatus A E ablati C F; & reliquus E B reliqui F D eadem pars erit, qualis totus A B totius C D.

a Sit E B eadem pars numeri G C, quæ A B ipſius C D, vel A E ipſius C F. *b* ergo A E + E B eadem eſt pars ipſius C F + G C. quæ A E ipſius C F, vel A B ipſius C D. *c* ergo G F = C D. aufer communem C F, *d* manet G C = F D. *e* ergo E B eadem eſt pars reliqui F *D* (G C) quæ totus A B totius C B. Q.E.D.

a 1. poſt. 7
b 5. 7.
c 6. ax. 1.
d 3. ax. 1
e 2. ax. 7

Vel ſic. Sit $a + b = x$, & $c + d = y$; atque tam $x = 3y$, quam $a = 3c$; dico $b = 3d$. Nam
f 1.2. $3c + 3d$ *f* $= 3y = x$ *g* $= a + b$. aufer utrinq;
g hyp. $3c$ *g* $= a$, & *h* remanet $3d = b$. Q.E.D.

PROP. VIII.

6 2 4 2 2
A......H..G....E..L..B 16
18 6
C..............F.....D 24

Si numerus A B *numeri* C D *partes fuerit, quales ablatus* AE *ablati* CF; *& reliquus* E B *reliqui* E D *eædem partes erit, quales totus* A B *totius* C D.

Seca AB in AG, GB partes numeri C D; item A E in A H, H E partes numeri G F; &
a 3.ax.1. ſume GL = AH = HE; *a* quare HG = EL, &
b conſtr. quia *b* A G = GB, *c* etiam HG = LB. Cum igi-
c 3.ax.1. tur totus A G eadem ſit pars totius C D, quæ
d 7.7. ablatus AH ablati CF; *d* erit reliquus HG, vel EL, eadem etiam pars reliqui F D, quæ A G ipſius C *D*. Eodem pacto, quia G B eadem pars eſt totius C*D*, quæ HE, vel GL, ipſius CF, *d* erit reliquus LB eadem pars reliqui FD, quæ GB totius C D; ergo E L + L B (E B) eædem eſt partes reliqui F D, quæ totus A B totius CD. Q.E.D.

Vel ſic facilius. Sit $a + b = x$. & $c + d = y$.
e 9.ax.7. Item tam $y = \frac{2}{3}x$, quam $c = \frac{2}{3}a$; vel *e* quod idem eſt, $3y = 2x$; & $3c = 2a$. Dico $d = \frac{2}{3}b$.
f 1.2. Nam $3c + 3d$ *f* $= 3y = 2x$ *f* $= 2a + 2b$. *g*
g 1.ax.1. ergo $3c + 3d = 2a + 2b$. aufer utrinque 3
h hyp.
k 3.ax.1. c *h* $= 2a$; & *k* manet $3d = 2b$. *l* ergo $d = \frac{2}{3}b$.
l 1.ax.7. Q.E.D.

PROP.

PROP. IX.

A.... 4

4 4

B.... G.... C 8

5 D 5

E..... H..... F 10

Si numerus A numeri B C pars fuerit, & alter D alterius E F eadem pars; & viciſſim quæ pars eſt, aut partes primus A tertij D, eadem pars erit, vel eædem partes, & ſecundus B C quarti E F.

Ponitur $A \sqsupset D$. Sint igitur B G, G C, & E H, H F partes numerorum B C, E F, hæ ipſi A, illæ ipſi D pares. Utrinque multitudo partium æqualis ponitur. Liquet vero B G *a* eandem eſſe partem, aut eaſdem partes ipſius E H, quæ G C ipſius H F; *b* quare B C (B G + G C) ipſius E F (E H + H F) eadem pars eſt aut partes, quæ unus B G (A) unius E H (D.) Q. E. D. a 1.ax.7. & 4.7. b 5.vel 7.6.

Vel ſic; ſit $a = \frac{b}{3}$, & $c = \frac{d}{3}$, vel $3a = b$, & $\frac{3c}{a} = d$, eſtque $\frac{c}{a} * = \frac{3c}{3a} = \frac{d}{b}$. * 5.15.

PROP. X.

A ... G .. B 4

C 6

5 5

D ..... H ..... E 10

F 15

Si numerus A B numeri C partes fuerit, & alter D E alterius F eadem partes; & viciſſim quæ partes eſt primus A B tertij D E, aut pars, eadem partes erit & ſecundus C quarti F, aut pars.

Ponitur $A B \sqsupset D E$, & $C \dot{\sqsupset} F$. Sint A G, G B, G B, & D H, H E partes numerorum C, & F, tot nempe in A B, quot in D E. Conſtat A G ipſius C eandem eſſe partem, quæ DH ipſius F. a 9.7. *a* quare viciſſim A G ipſius D H, pariterq; G B b 5, & 9, 7 ipſius H E, & *b* proinde conjunctim A B ipſius D E eadem pars erit, aut partes, quæ C ipſius F. Q.E.D.

Vel ſic; Sit $a = \frac{2b}{3}$, & $c = \frac{2d}{3}$. vel $3\,a = 2b$, & $3\,c = 2\,d$. Eſt $\frac{c}{a} = \frac{3c}{3a} = \frac{2d}{2b} = \frac{d}{b}$

LEMMA.

AE, BF, CG, DH,
A, B, C, D,
E, F, G. H.

Si proportionales numeri A, B, C, D proportionales numeros A E, B F, C G, D H metiantur per numeros E, F, G, H, erunt ei (E, F, G, H) proportionales.

a 19.7. Nam ob A E D H *a* = B F C G, *a* & A D b 1.ax.7. = B C, *b* erit $\frac{AEDH}{AD} = \frac{BFCG}{BC}$, c 9.ax.7. *c* hoc eſt E H = = F G. *a* ergo E. F : : G. H. Q.E.*D*.

Coroll.

d 15.def. 7. Hinc $\frac{Bq}{Aq} = \frac{B}{A}$ in $\frac{B}{A}$. *d* Nam 1. B : : B. Bq. *d* & e lem.præ. 1. A : : A. Aq. *e* ergo $\frac{1}{A}$. B $\frac{::}{A}$ B. $\frac{Bq}{Aq}$. *d* ergo $\frac{Bq}{Aq} = \frac{B}{A} \times \frac{B}{A}$. Similiter $\frac{B}{AC}$ in $\frac{Bq}{AC} = \frac{BC}{ACC}$. & ſic de reliquis.

PROP. XI.

$\overline{4}$ $\overline{3}$
A....E...B 7
8 6
C........F......*D* 14

Si fuerit, ut totus A B ad totum CD, ita ablatus A E ad ablatum C F; & reliquus E B ad reliquum F D erit, ut totus A B ad totum C D.

Sit primo A B ⊐ C D; *a* ergo A B vel pars a 4.7.
eſt, vel partes numeri C D ; *b* eademque pars b 20.def.
eſt, vel partes ipſe A E ipſius C F; *c* ergo reli- c 7. vel
quus E B reliqui F D eadem pars eſt, aut par- 8.7.
tes, quæ totus A B totius C D. *b* ergo A B. C D : : E B. F D. Sin fuerit A B ⊏ C D; eodem modo erit juxta modo oſtenſa, C D. A B : : F D. E B. ergo invertendo, AB. C D : : E B. FD.

PROP. XII.

A,4. C,2. E,3.
B,8. D,4. F,6.

Si ſint quotcunque numeri proportionales (A.B : : C.D : : E. F *erit quemadmodum unus antecedentium A ad unum conſequentium B, ita omnes antecedentes* (A + C + E) *ad omnes conſequentes* (B + D + F.)

Sint primo, A, C, E minores quam B, D, F. a 20.def.
ergo (propter eaſdem rationes) *a* erit A ea- 7.
dem pars aut partes ipſius B, quæ C ipſius D. *b* b 5. & 6.7
ergo conjunctim A + C eadem erit pars aut partes ipſius B + D, quæ unus A unius B. Similiter A + C + E eadem pars eſt, aut partes ipſius B + D + F, quæ A ipſius B. *e* ergo A +
C + E. B + D + F : : A. B. Q.E.D. Sin A, C, c 20.def.
E, ipſis B, D, F majores ponantur, idem oſten- 7.
detur invertendo.

PROP. XIII.

A, 3. C, 4.
B, 5. D, 12.

Si quatuor numeri proportionales ſint (A.B : : C. D. *& viciſſim proportionales erunt* (A.C : : B.D.)

Sint primo A & C ipſis B & D minores, atq; A ⊐ C. Ob eandem proportionem, *a* erit A eadem pars, aut partes ipſius B, quæ C ipſius D. *b* ergo viciſſim A ipſius C eadem pars eſt, aut partes, quæ B ipſius D. ergo A. C : : B. D. Sin A ⊏ C ; atque A & C majores ſtatuantur, quam B & D, eadem res erit, proportiones invertendo.

a 10.def. 7.
b 9 & 10 7.

PROP. XIV.

A, 9. D, 6.
B, 6. E, 4.
C, 3. F, 2.

Si ſint quotcunque numeri A, B, C, & alij totidem D, E, F illis æquales multitudine, qui bini ſumantur, & in eadem ratione (A.B : : D. E. & B. C : : E. F) *etiam ex æqualitate in eadem ratione erunt.* (A.C : : D F.)

Nam quia A.B : : D. E, *a* erit viciſſim, A. D : : B. E : : *a* C. F. *a* ergo iterum permutando, A.C : : D. F. Q.E.D.

a 13.7.

PROP. XV.

1. D.
B... 3. E...... 6.

Si unitas numerum quempiam B metiatur; æque autem alter numerus D alterum quendam numerum E metiatur; & viciſſim æque unitas tertium numerum D metietur, & ſecundus B quartum E.

Nam quia 1 eſt eadem pars ipſius B, quæ D ipſius E. *a* erit viciſſim 1 eadem pars ipſius D, quæ B ipſius E. Q.E.D. a 9a.

PROP. XVI.

B, 4. A, 3.
A, 3. B, 4.
AB, 12. BA, 12.

Si duo numeri A, B ſeſe mutuo multiplicantes fecerint aliquos A B, B A, geniti ex ipſis A B, B A æquales inter ſe erunt.

Nam quia A B = A in B, *a* erit 1 in A toties, quoties B in AB. *b* ergo viciſſim 1 in B toties erit, quoties A in A B. atqui quoniam BA = B in A, *a* erit 1 in B toties, quoties A in BA. ergo quoties 1 in A B, toties 1 in BA; & *c* proinde A B = B A. Q.E.D. a 15. def. 7. b 15. 7. c 4. ax. 7

PROP. XVII.

A, 3.
B, 2. C, 4.
AB, 6. AC, 12.

Si numerus A duos numeros B, C multiplicans fecerit aliquos AB, AC; geniti ex ipſis eandem rationem habebunt, quam multiplicati. (A B. A C : : B.C.)

Nam quia A B = A in B, *a* erit 1 toties in A, quoties B. in A B. *a* item quia A C = A in C, erit 1. toties in A, quoties C in A C. ergo quoties B in AB, toties C in A C. quare B. AB : : C.AC. *c* ergo viciſſim, B. C : : A B. A C. Q.E.D. a 15. def. 7. b 20. def. 7. c 13. 7.

PROP. XVIII.

C, 5. C, 5.
A, 3. B, 9.
AC, 15. BC, 45.

Si duo numeri A, B, numerum quempiam C multiplicantes fecerint aliquos AC, BC; geniti ex ipſis eandem rationem habebunt, quam multiplicantes. (A.B : : AC.BC.)

Nam

a 16.7. Nam A C *a* = C A ; & B C *a* = C B ; ſic idem C multiplicans A & B producit A C, &
b 17.7. B C. *b* ergo A. B :: A C. B C. Q.E.D.

Schol.

Ex his pendet modus vulgaris reducendi fractiones ($\frac{3}{5}$, $\frac{7}{9}$) ad eandem denominationem. Nam duc 9 tam in 3, quam in 5, proveniunt $\frac{27}{45} = \frac{3}{5}$. quoniam ex his, 3.5 :: 27. 45. item duc 5 in 7, & 9, prodeunt $\frac{35}{45} = \frac{7}{9}$. quia 7. 9 :: 35. 45.

PROP. XIX.

A,4.B,6. C,8.D,12.
AD,48.BC,48.

Si quatuor numeri proportionales fuerint, (A B :: C. D ;) *qui ex primo & quarto ſit numerus* A D, *æqualis eſt ei, qui ex ſecundo & tertio ſit, numero BC. Et ſi qui ex primo & quarto ſit numerus* A D, *æqualis ſit ei, qui ex ſecundo & tertio ſit, numero B C, ipſi quatuor numeri proportionales erunt.* (A.B :: C. D.)

a 17.7. b hyp. 1. *Hyp.* Nam A C. A D *a* :: C. D *b* :: A. B
c 18.7. *c* :: A C. B C. *d* ergo A D = B C. Q.E.D.
d 9.5. e hyp. 2. *Hyp.* Quoniam *e* A D = B C, erit A C.
f 7.5. A D *f* :: A C. B C. Sed A C. A D *g* :: C.D. &
g 7.7. h 18.7. AC.BC *h* :: A.B. *k* ergo C. D :: A.B. Q.E.D.
k 11.5.

PROP. XX.

A. B. C.
4. 6. 9.
AC, 36. BB, 36.
D, 6.

Si tres numeri proportionales fuerint (A.B :: B.C.) *qui ſub extremis continetur* (A C) *æqualis eſt ei, qui à medio efficitur* (*B B.*) *Et ſi qui*

ſub

ſub extremis continetur (A C) æqualis fuerit ei (Bq) qui ſub medio, ipſi tres numeri proportionales erunt $\left(\frac{A}{B} :: \frac{B}{C}\right)$

1. *Hyp*. Nam ſume D = B. *a* ergo A.B :: D a 1. ax. 7.
(B.) C. *b* quare A C = B D, *a* vel BB. Q.E.D. b 19. 7.
2. *Hyp*. Quia A C *c* = B D, *d* erit A. B :: D c hyp.
(B.) C. Q.E.D. d 19. 7.

PROP. XXI.

A ... G .. B 5. E .... 10.
C .. H . D 3. F ...... 6.

Numeri AB. CD minimi omnium eandem cum eis rationem habentium (E, F) metiuntur æque numeros E, F eandem cum eis rationem habentes, major quidem AB majorem E; minor vero C D mino[illegible] *F.*

Nam A B. C D *a* :: E. F. *b* ergo viciſſim A- a hyp.
B. E :: C D. F. *c* ergo A B eadem pars eſt, vel b 13. 7.
partes ipſius E, quæ C D ipſius F. Non par- c 20. def. 7.
tes; nam ſi ita, ſint A G, G B partes numeri E;
& C H, H D partes numeri F. *c* ergo A G. E
:: C H. F; & permutando, A G. C H *d* :: E. d 13. 7.
F *e* :: AB. C D. ergo AB, C *D* non ſunt mini- e hyp.
mi in ſua ratione, contra hypoth. ergo, &c.

PROP. XXII.

A, 4. D, 12.
B, 3. E, 8.
C, 2. F, 6.

Si fuerint tres numeri A, B, C, & alij ipſis multitudine æquales D, E, F, qui bini ſumantur, & in eadem ratione; fuerit autem perturbata eorum proportio (A. B :: E.F & B. C :: D.E;) etiam ex æqualitate in eadem ratione erunt (A.C :: D.F.)

Nam

a hyp. Nam quia A.B *a*::E.F, erit AF = BE; &
b 19.7. quia B.C:: *a* D.E, *b* erit BE = CD. *c* ergo
c 1.ax.1. *d 19.7.* AF = CD. *d* quare A.C::D.F. Q.E.D.

Vel ſic;

PROP. eand.

Aq, B, C.
4, 8, 16.

Si tres numeri Aq, B, C *deinceps ſint proportionales, primus autem* Aq *ſit quadratus; & tertius* C *quadratus erit.*

a 20.7. *b 7.ax.7.* Nam ob AqC *a* = Bq, *b* erit C = $\frac{Bq}{Aq}$ *c* = Q.
c cor. lem. præ. $\frac{B}{A}$. Liquet vero $\frac{B}{A}$ eſſe numerum, *d* ob $\frac{Bq}{Aq}$, vel C
d hyp. & 14.8. numerum. ergo ſi tres, &c.

PROP. XXIII.

A,9. B,4.
C.... D...
E--

Primi inter ſe numeri A, B, *minimi ſunt omnium eandem cum eis rationem habentium.*

Si fieri poteſt, ſint C & D minores quam A & B, atque in eadem ratione.
a 21.7. *a* ergo C metitur A æque, ac D metitur B, puta per eundem numerum E: quoties igitur 1
b 23.def. 7. in E, *b* toties erit C in A. *c* quare viciſſim quo-
c 15.7. ties 1 in C, toties E in A. ſimili diſcurſu quoties 1 in D, toties E in B. ergo E utrumque A & B metitur; qui proinde inter ſe primi non ſunt, contra Hypoth.

Vel ſic;

Ac, B, C, D.
8, 12, 18, 27.

Si quatuor numeri Ac, B, C, D *deinceps ſint proportionales, primus autem* Ac *ſit cubus; & quartus* D *cubus erit.*

a 19.7. *b 7.ax.7.* Nam quia AcD *a* = BC, *b* erit D = $\frac{BC}{AC}$

c =

e $= \frac{B}{\overline{AC}} \times C$; hoc eſt (ob A *c* C = *d* Bq, & *b* proinde C = $\frac{Bq}{\overline{AC}}$) D = $\frac{B}{\overline{AC}} \times \frac{Bq}{\overline{AC}}$ *e* = $\frac{BC}{\overline{ACC}}$ *e* = C : $\frac{B}{\overline{AC}}$ *e* liquet vero ipſum $\frac{B}{\overline{AC}}$ eſſe numerum, quia $\frac{BC}{\overline{ACC}}$, vel D numerus ponitur ; ergo ſi quatuor numeri, &c.

c com. *lem. pr 4.*
d 20.7.
e 13.8.

PROP. XXIV.

A,9. B,4.
C...
D... E..

Numeri A, B, *minimi omnium eandem cum eis rationem habentium, primi inter ſe ſunt.*

Si fieri poteſt, habeant A & B communem menſuram C; is metiatur A per D, & B per E; *a* ergo C D = A, *b* & C E = B. *b* quare A. B :: D. E. Sed D & E minores ſunt quam A & B, utpote eorum partes. Ergo A & B non ſunt minimi in ſua ratione, contra hypoth.

a *9. ax. 7*
b *17. 7.*

PROP. XXV.

A,9. B,4.
C,3. D—

Si duo numeri A, B *primi inter ſe fuerint, qui unum eorum* A *metitur numerus* C, *ad reliquum* B *primus erit.*

Nam ſi affirmes aliquem D numeros B & C metiri, *a* ergo D metiens C, metitur A. ergo A & B non ſunt primi inter ſe, contra Hyp.

a *11. ax. 7.*

PROP. XXVI.

A,5. C,8.
B,3.

AB,15. E....
F....

Si duo numeri, A, B *ad quempiam* C *primi fuerint, etiam ex illis genitus* A B *ad eundem* C *primus erit.*

Si fieri poteſt, ſit ipſorum A *B*, & C communis menſura numerus E. ſitque $\frac{AB}{E} = F$; *a*
a 9.ax.7. ergo A *B* = E F; *b* quare E. A :: B. F. Quia
b 19.7. vero A primus eſt ad C quem E metitur, *c* e-
c 25.7. runt E & A primi inter ſe; *d* adeoque in ſua
d 23.7. proportione minimi, & *e* proinde æque me-
e 21.7. tiuntur *B*, & F; nempe E ipſum *B*. & A ipſum F. Quum igitur E utrumque *B*, C metiatur, non erunt illi primi inter ſe, contra hypoth.

PROP. XXVII.

A,4. *B*,5.
Aq,16.
D,4

Si duo numeri, A, B, primi inter ſe fuerint, etiam ex uno eorum genitus (Aq) *ad reliquum B primus erit.*

a 1.ax.7. Sume D = A; ergo *a* ſinguli D, & A primi
b 16.7. ſunt ad *B*. *b* quare A D, vel Aq, ad *B* primus eſt. Q.E.D.

PROP. XXVIII.

A,5. C,4.
B,3. D,2.
AB,15. CD,8.

Si duo numeri A, B ad duos numeros C, D, uterque ad utrumque, primi fuerint, & qui ex eis gignentur AB, CD, primi inter ſe erunt.

a 16.7. Nam quia A & *B* ad C primi ſunt, *a* erit A-
b 26.7. B ad C primus. Eadem ratione erit AB ad D primus. *b* ergo A B ad CD primus eſt. Q.E.D.

PROP. XXIX.

A,3 B,2,
Aq,9. Bq,4.
Ac,27. Bc,8.

Si duo numeri A, B primi inter ſe fuerint, & multiplicans uterque ſeipſum fecerit ali-

aliquem (Aq, & Bq;) & geniti ex ipſis (Aq, Bq) primi inter ſe erunt; & ſi qui in principio A, B genitos ipſos Aq., Bq multiplicantes fecerint aliquos (Ac, Bc;) & hi primi inter ſe erunt: & ſemper circa extremos hoc eveniet.

Nam quia *A* primus eſt ad *B*, *a* erit Aq ad B primus. & quia Aq primus ad B, *a* erit Aq ad Bq primus. Rurſus quia tam A ad B, & Bq; quam *A*q ad eosdem *B*, & Bq primi ſunt, *b* erit *A* x *A*q, id eſt *A*c, ad B x Bq, id eſt Bc, primus. Et ſic porro de reliquis. a 27.7. b 28.7.

PROP. XXX.

8 5
A .... B ..... C 13. D ----

Si duo numeri A B, B C primi inter ſe fuerint, etiam uterque ſimul (A C) ad quemlibet illorum A B, B C primus erit. Et ſi uterque ſimul A C ad unum aliquem illorum A B primus fuerit, etiam qui in principio numeri A B, B C primi inter ſe erunt.

1. *Hyp*. Nam ſi *A* C, *A B* compoſitos velis, ſit D communis menſura. *a* Is metietur reliquum B C. ergo *A* B, B C non ſunt primi inter ſe, contra Hypoth. a 12.ax.7

2. *Hyp*. Poſitis *A* C, AB inter ſe primis, vis D ipſorum A B, B C communem eſſe menſuram. *b* Is igitur totum *A* C metitur. quare A C, *A*B non ſunt primi inter ſe, contra Hyp. b 10.ax.7

Coroll.

Hinc numerus, qui ex duobus compoſitus, ad unum illorum primus eſt, ad reliquum quoque primus eſt.

PROP.

PROP. XXXI.

A 5, B, 8. *Omnis primus numerus* A *ad omnem numerum* B, *quem non metitur, primus eſt.*

Nam ſi communis aliqua menſura metiatur utrumque A, B; *a* non erit A primus numerus, contra Hypoth.

a 11. def. 7.

PROP. XXXII.

A, 4. D, 3.
B, 6. E, 8.

AB, 24.

Si duo numeri A, B, *ſe mutuo multiplicantes fecerint aliquem AB; genitum autem ex ipſis AB metiatur aliquis primus numerus D; is etiam unum eorum, qui à principio,* A, vel B *metietur.*

Pone numerum D non metiri A; ſit vero $\frac{AB}{D} = E$. *a* ergo AB = DE. *b* quare D. A :: B. E. *c* eſt vero D ad A primus. *d* ergo D, & A minimi ſunt in ſua ratione; *e* proinde D metitur B, æque ac A metitur E. liquet igitur propoſitum.

a 9. ax. 7.
b 19. 7.
c hyp. & 31. 7.
d 23. 7.
e 21. 7.

PROP. XXXIII.

A, 12.
B, 2.

Omnem compoſitum numerum A, *aliquis primus numerus B metitur.*

Unus vel plures numeri *a* metiantur A, quorum minimus ſit B. is primus erit. nam ſi dicetur compoſitus, *a* eum minor aliquis metietur *b* qui proinde ipſum A metietur; quare B non eſt minimus eorum, qui A metiuntur; contra Hypoth.

a 13. def. 7.
b 11. ax. 7.

PROP.

PROP. XXXIV.

Omnis numerus A, aut primus eſt, aut
A, 9. *eum aliquis primus metitur.*

Nam A neceſſario vel primus eſt, vel compoſitus. Si primus, hoc eſt quod aſſerimus. Si compoſitus, *a* ergo eum aliquis primus metitur. Q.E.D. a 33.7.

PROP. XXXV.

A, 6. B, 4. C, 8.
D, 2. H--I--K----
E; 3. F, 2. G, 4. L---

Numeris datis quotcunque A, B, C reperire minimos omnium E, F, G eandem rationem cum eis habentium.

Si A, B, C primi ſint inter ſe, ipſi in ſua ratione minimi *a* erunt. Si compoſiti ſint, *b* eſto eorum maxima communis menſura D, qui ipſos metiatur per E, F, G. Hi minimi erunt in ratione A, B, C. a 23.7. b 3.7.

Nam D ductus in E, F, G. *c* producit ABC. *d* ergo hi & illi in eadem ſunt ratione. Jam puta alios H, I, K minimos eſſe in eadem; *e* qui propterea æque metientur A, B, C nempe per numerum I. *f* ergo L in H, I, K ipſos A, B, C procreabit. *g* ergo $ED = A = HL$. *h* unde E. H :: L. D. Sed E *k* $\sqsubset$ H; *l* ergo L $\sqsubset$ D. ergo D non eſt maxima communis menſura ipſorum A, B, C; contra Hypoth. c 9.ax.7 d 17.7. e 21.7. f 9.ax.7. g 1.ax.1 h 19.7. k ſuppoſ. l 20. def.7.

Coroll.

Hinc, maxima communis menſura quotlibet

bet numerorum metitur ipſos per numeros, qui minimi ſunt omnium eandem rationem cum ipſis habentium. Ex quo patet methodus vulgaris reducendi fractiones ad minimos terminos.

PROP. XXXVI.

Duobus numeris datis A, B, reperire, quem illi minimum metiuntur, numerum.

A, 5. B,4.
AB, 20.
D ------
E ---F ---

1. *Caſ.* Si A, & B primi ſint inter ſe, eſt A *B* quæſitus. Nam liquet A & B metiri AB. Si fieri poteſt, metiantur A & B aliquem D ⊐ AB; puta per
a 9.ax.7. E, & F. *a* ergo A E = D = B F. *b* quare A. B : :
& 1.ax.1. F. E. Quia vero A, & B *c* primi ſunt inter ſe,
b 19.7.
c hyp. *d* adeoque in ſua ratione minimi, *e* æque me-
d 23.7. tientur A ipſum F, ac *B* ipſum E. Atqui B. E *f*
e 21.7.
f 17.7. : : A *B*. A E (D.) *g* ergo A B etiam metietur
g 20 def. 7. D, ſeipſo minorem. Q.E.A.

A,6. B,4.
C,3. D,2.
AD,12.
F ----
G ---H ---

2. *Caſ.* Sin A, & B inter ſe compoſiti fuerint, *h* re-
h 35.7.
k 19.7.
periantur C, & D minimi in eadem ratione. *k* ergo AD = BC. Erit AD, vel BC quæſitus.
l 7.ax.7. Nam *l* liquet B, & A ipſum A D, vel B C
metiri. Puta A, & B metiri F ⊐ A D, nempe A
m 9.ax.7. per G, & B per H. *m* ergo A G = F = B H. *n*
n 19.7. unde A. B : : H. G *o* : : C. D. *p* proinde æque
o conſtr. metitur C ipſum H, ac D ipſum G. atqui D.G
p 21.7.
q 17.7.
r 10.def. 7. *q* : : AD.AG (F.) ergo AD *r* metitur F, major minorem. Q.E.A.

Coroll.

Coroll.

Hinc, ſi duo numeri multiplicent minimos eandem rationem habentes, major minorem, & minor majorem, producetur numerus minimus, quem illi metiuntur.

PROP. XXXVII.

A,2. *B*,3.
E,......6.
C----F---D

Si duo numeri A, B numerum quempiam C D metiantur; etiam minimus E, quem illi metiuntur, eundem C D metietur.

Si negas, aufer E ex C D, quoties fieri poteſt, & relinquatur F D ⊐ E. quum igitur A & *B* *a* metiantur E, *b* & E ipſum CF, *c* etiam A, & B metiuntur C F; *a* metiuntur autem totum C D; *d* ergo etiam reliquum F D metiuntur. ergo E non eſt minimus, quem A, & B metiuntur, contra hyp.

a *hyp.*
b *conſt.*
c *11. ax. 7.*
d *12. ax. 7.*

PROP. XXXVIII.

A,3.B,4.C,6.
D,12.

Tribus numeris datis A, B, C, reperire minimum, quem illi metiuntur.

a Reperi D minimum, quem duo A, & *B* metiuntur; quem ſi tertius C metiatur, patet D eſſe quæſitum. Quod ſi C non metiatur D, ſit E minimus, quem C, & D metiuntur. Erit E requiſitus.

a *36.7.*

A,2.B,3.C,4.
D,6.E,12.
F---

Nam ſingulos A, B, C metiri E conſtat ex 11. ax. 7. Quod vero nullum alium F minorem metiantur, facile oſtenditur. Nam ſi affirmas, *b* ergo D

b *37.7.*

b 11.7. metitur F; *b* proinde E eundem F metitur, major minorem. Quod eſt abſurdum.

Coroll.

Hinc, ſi tres numeri numerum quempiam metiantur; etiam minimus, quem illi metiuntur, eundem metietur.

PROP. XXXIX.

A, 12.
B, 4. C, 3.

Si numerum A quiſpiam numerus B metiatur, ille A quem B metitur, partem habebit C, à metiente B denominatam.

a hyp.
b 9.ax.7.
c 7.ax.7.

Nam quia $\frac{A}{B}$ *a* = C, *b* erit A = BC. *c* ergo $\frac{A}{C}$ = B. Q.E.D.

PROP. XL.

A, 15.
B, 3. C, 5.

Si numerus A partem habuerit quamlibet B, metietur illum numerus C, à quo ipſa pars B denominatur.

a hyp. & 9.ax.7.
b 7.ax.7.

Nam quia B C *a* = A, *b* erit $\frac{A}{C}$ = B. Q.E.D.

PROP. XLI.

$\frac{1}{2}$ G, 12.
$\frac{1}{3}$ H---
$\frac{1}{4}$

Numerum reperire G, qui minimus cum ſit, habeat datas partes, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.

a 38.7.
b 39.7.
c 40.7.

a Inveniatur G minimus, quem denominatores 2,3,4 metiuntur. *b* Liquet G habere partes, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. Si fieri poteſt, $H \sqsupset G$ habeat easdem partes; *c* ergo 2, 3, 4 metiuntur H, & proinde G non eſt minimus, quem 2,3,4 metiuntur. contra conſtr.

LIB.

LIB. VIII.

PROP. I.

A,8. B.12. C,18. D,27.
E-F--G---H----

SI fuerint quotcunque numeri deinceps proportionales A,B,C,D; extremi vero ipsorum A, D primi inter se fuerint; ipsi A, B,C, D minimi ſunt omnium eandem cum eis rationem habentium.

Nam, ſi fieri poteſt, ſint alii totidem E, F, G,
H minores in illa ratione. *a* ergo ex æquali A. a 14.7.
D :: E.H. ergo A, & D primi numeri, *b* adeoq; b 23.7.
in ſua ratione minimi, *c* æque metiuntur E, & c 21.7.
H, ſeipſis minores. Q.E.A.

PROP. II.

I.

A,2. B,3.
Aq,4. AB,6. Bq,9.
Ac,8. AqB,12. ABq,18. Bc,27.

Numeros reperire deinceps proportionales minimos, quotcunque juſſerit quiſpiam, in data ratione A ad B.

Sint A, & B minimi in data ratione. Erunt Aq, A B, Bq tres minimi deinceps in ratione A ad B.

Nam AA. AB *a* :: A. B *a* :: A B. B B: item a 17.7.
quia A, & *B b* primi ſunt inter ſe, *c* erunt Aq, b 24.7.
Bq inter ſe primi; *d* proinde Aq, AB, Bq ſunt c 29.7.
÷ minimi in ratione A ad B. d 1.8.

Dico porro, Ac, AqB, ABq, *B*c in ratione A ad B quatuor eſſe minimos. Nam AqA, AqB *e* :: A.*B* *e* :: ABA (AqB.) ABB. *e* atque A. B :: ABq. BBq. (Bc) Quum igitur A C, & Bc *f* inter ſe primi ſint, *g* erunt A c, AqB, ABq, Bc quatuor ∺ minimi in ratione A ad B. Eodem modo quotvis proportionales inveſtigabis. Q. E. F.

e 17.7.

f 19, 7.

g 1.8.

Coroll.

1. Hinc, ſi tres numeri minimi ſunt proportionales, extremi quadrati erunt; ſi quatuor, cubi.

2. Extremi quotcunque proportionales per hanc prop. inventi in data ratione minimi, inter ſe primi ſunt.

3. Duo numeri, minimi in data ratione, metiuntur omnes medios quotcunque minimorum in eadem ratione; quia ſcilicet producuntur ex illorum multiplicatione in alios quoſdam numeros.

4. Hinc etiam liquet ex conſtructione, ſeries numerorum 1, A, Aq, Ac; 1, B, Bq, Bc; Ac, Aq*B*, ABq, Bc, conſtare æquali multitudine numerorum; ac proinde extremos numeros quotcunque minimorum continue proportionalium ab unitate. ut extremi Ac, Bc continue proportionalium Ac, AqB, ABq, Bc, ſunt ultimi totidem proportionalium ab unitate 1, A, Aq, Ac; & 1, B, Bq, *B*c.

5. 1, A, Aq, Ac; & B, BA, BAq; ac Bq, ABq ſunt ∺ in ratione 1 ad A. item, B, Bq, Bc; &

& A, AB, ABq; ac Aq, AqB ſunt ∺ in ratione 1 ad B.

PROP. III.

A, 8. B, 12. C, 18. D, 28.

Si ſint quotcunque numeri A, B, C, D deinceps proportionales, minimi omnium eandem cum eis rationem habentium; illorum extremi A, D ſunt inter ſe primi.

Nam ſi *a* inveniantur totidem numeri minimi in ratione A ad B, illi non alii erunt, quam A, B, C, D; ergo juxta 2. coroll. præcedentis extremi A & D primi ſunt inter ſe. Q.E.D. a 2.8.

PROP. IV.

A, 6. B, 5. C, 4. D, 3.
H, 4. F, 24. E, 20. G, 15.
I - - K - - L - - -

Rationibus datis quotcunque in minimis terminis, (A ad B, & C ad D) *reperire numeros deinceps minimos in datis rationibus.*

a Reperi E minimum, quem B, & C metiuntur; & B ipſum E *b* æque metiatur, ac A alterum F, puta per eundem H. *b* item C ipſum E, ac D alterum G æquale metiantur: erunt F, E, G minimi in datis rationibus. Nam A H *e* = F; & B H *c* = E. *d* ergo A. B :: A H. B H *e* :: F. E. Similiter C. D :: E. G. ſunt igitur F, E, G deinceps proportionales in datis rationibus. Imo minimi ſunt in iiſdem: nam puta alios I, K, L minimos eſſe. *f* ergo A & B ipſos I & K, *f* pariterque C & D ipſos K & L æque metiuntur. ergo B, & C eundem K metiuntur. *g* Quare etiam E eundem K metitur, ſeipſo minorem. Q.E.A.

a 36.7.
b 3. poſt. 7.
c 9. ax. 7.
d 18. 7.
e 7. 5.
f 21. 7.
g 37. 7.

A, 6.

A, 6. B, 5. C, 4. D, 3. E, 5. F, 7.
H, 24. G, 20. I, 15. K, 21.

Datis vero tribus rationibus A ad B, & C ad D, ac E ad F. reperi, ut prius, tres H, G, I minimos deinceps in rationibus A ad B, & C ad D. tunc ſi E numerum I metiatur.

b 3 poſt. 7 *b* Sume alterum K, quem F æque metiatur; erunt quatuor H, G, I, K, deinceps minimi, in datis rationibus; quod non aliter probabimus, quam in priori parte.

A, 6. B, 5. C, 4. D, 3. E, 2. F, 7.
H, 24. G, 20 I, 15.
M, 48. L, 40. K, 30. N, 105.

Sin E non metiatur I, ſit K minimus, quem E, & I metiuntur; & quoties I ipſum K, toties G ipſum L, & H ipſum M metiatur, quoties vero E ipſum K, toties F ipſum N metiatur. Erunt M, L, K, N minimi deinceps in datis rationibus; quod demonſtrabimus, ut prius.

PROP. V.

Plani numeri C D, E F *rationem habent ex lateribus compoſitã.*

$$\left(\frac{CD}{EF} = \frac{C}{E} + \frac{D}{E.} \right)$$

C, 4. E, 3.
D, 6 F, 16 E D, 18.

CD, 24 EF, 48.

a 17.7. Nam quia C D. E D *a* : : C. E; *a* & E D. E-
b 20. def. 5. F : : D. F. atque $\frac{CD}{EF.}$ *b* $= \frac{CD}{ED} + \frac{ED}{EF,}$ *c* erit ratio
c 23.5. $\frac{CD}{EF} = \frac{C}{E} + \frac{D}{F.}$ Q.E.D.

PROP. VI.

A, 16. B, 24. C, 36. D, 54. E, 81.
F.4. G,6. H,9.

Si ſint quotcunque numeri deinceps proportionales A,B,C,D, E; primus autem A ſecundum B non metiatur, neque alius quiſpiam ullum metietur.

Quoniam A non metitur B, *a* neque quilibet proximè ſequentem metietur; quia A.B : : B. C : : C. D, &c. *b* Accipe tres F, G, H minimos in ratione A ad B. quoniam igitur A non metitur B, *a* neque F metietur G. *c* ergo F non eſt unitas. ſed F, & H inter ſe primi ſunt; ergo quum *e* ſit ex æquo A. C : : F. H, & F non metiatur H, *d* neque A ipſum C metietur; proinde nec B ipſum D, nec C ipſum E, &c. quia A. C *e* : : B. D *e* : : C. E, &c. Eodem modo ſumptis quatuor vel quinque minimis in ratione A ad B, oſtendetur A ipſos D, & E; ac B ipſos E, & F non metiri, &c. Quare nullus alium metietur. Q.E.D.

a 20. def. 7. b 35.7. c 5. ax. 7 d 3.3. e 14.7.

PROP. VII.

A, 3. B, 6. C, 12. D, 24. E, 48.

Si ſint quotcunque numeri deinceps proportionales A,B,C,D,E; primus autem A extremum E metiatur; is etiam metitur ſecundum B.

Si negas A metiri B, *a* ergo nec ipſum E metietur, contra Hypoth.

a 6.8.

PROP. VII.

A,24. C,36. D,54. B,81.
G,8. H,12. I,18. K, 27.
E,32. L,48. M,72, F,108.

Si inter duos numeros A,B *medii continua pro-*

portione ceciderint numeri C, D; quot inter eos medij continua proportione cadunt numeri, tot & inter alios E, F eandem cum illis habentes rationem, medij continua proportione cadent. (L, M.)

a 35.7. *a* Sume G, H, I, K minimos ∺ in ratione A ad C; *b* erit ex æquali, G. K : : A.B *c* : : E. F.
b 14.7.
c hyp.
d 3.8. Atqui G, & K *d* primi ſunt inter ſe; *e* quare G
e 21.7. æque metitur E, ac K ipſum F. per eundem numerum metictur H ipſum L, & I ipſum M:
f constr. *f* itaque E, L, M, F ita ſe habent ut G, H, I, K; hoc eſt ut A, B, C, D. Q.E.D.

PROP. IX.

1.
E, 2. F, 3.
G, 4. H, 6. I, 9.
A, 8. C, 12. D, 18. B, 27.

Si duo numeri A; B; ſint inter ſe primi, & inter eos medij continua proportione ceciderint numeri, C, D; quot inter eos medij continua proportione ceciderint numeri, totidem (E, G, & F, I) & inter utrumq; eorum ac unitatem medij continua proportione cadent.

Conſtat 1, E, G, A; & 1, F, I, B eſſe ∺; & totidem quot A, C, D, B, nimirum ex 4. coroll. 2.8. Q.E.D.

PROP. X.

A, 8. I, 12. K, 18. B, 27.
E, 4. D F, 6. G, 9.
D, 2. F, 3.
1.

Si inter duos numeros A, B, & unitatem continue proportionales ceciderint numeri (E, D, & F, G,) quot inter utrumq;

trumq; ipſorum, & unitatem deinceps medij continua proportione cadunt numeri, totidem & inter ipſos medij continua propotione cadent, I, K

Nam E, DF, G; & A, DqF (I,) DG (K,) B ſunt ∺ per 2.8. ergo, &c.

PROP. XI.

A, 2. B, 3.
Aq, 4. AB, 6. Bq, 9.

Duorum quadratorum numerorum Aq, Bq unus medius proportionalis eſt numerus A B. & quadratum Aq ad quadratum Bq, duplicatam habet lateris A ad latus B rationem.

a Liquet Aq, AB, Bq, eſſe ∺ . *b* proinde etiam $\frac{Aq}{Bq} = \frac{A}{B}$ bis. Q.E.D.

a 17.7.
b 10. def. 5.

PROP. XII.

Ac, 27. AqB, 36. ABq, 48. BC, 64.
A, 3. B, 4.
Aq, 9. AB, 12. Bq, 16.

Duorum cuborum numerorum Ac, Bc duo medij proportionales ſunt numeri AqB, ABq. Et cubus Ac ad cubum Bc triplicatam habet lateris A ad latus B rationem.

a Nam Ac. AqB, ABq, Bc ſunt ∺ in ratione A ad B. *b* proinde $\frac{AC}{BC} = \frac{A}{B}$ ter. Q.E.D.

a 2.8.
b 10. def. 5.

PROP. XIII.

A, 2. B, 4. C, 8.
Aq, 4. AB, 8. Bq, 16. BC, 32. Cq, 64.
Ac, 8. AqB, 16. ABq, 32. Bc, 64. BqC, 128. BCq, 256. Cc, 512.

Si ſint quotlibet numeri deinceps proportionales, A; B, C; & multiplicans quisque ſeipſum

fa-

faciat aliquos ; qui ab illis producti fuerint Aq, Bq, Cq *proportionales erunt : & ſi numeri primum poſiti* A, B, C *multiplicantes jam factos* Aq, Bq, Cq, *fecerint aliquos* Ac, Bc, Cc ; *ipſi quoque proportionales erunt. & ſemper circa extremos hoc eveniet.*

a 2.8. Nam Aq, AB, Bq, BC, Cq *a* ſunt ∺ *b* ergo
b 14.7. ex æquo Aq.Bq : : Bq Cq. Q.E.D.

a Item Ac, AqB, ABq, Bc, BqC, BCq, Cc ſunt ∺ , *b* ergo iterum ex æquo, Ac.Bc : : Bc. Cc. Q.E.D.

PROP. XIV.

Aq,4. AB,12. Bq,36.
A,2. B,6.

Si quadratus numerus Aq *quadratum numerum* Bq *metiatur; & latus unius* (A) *metietur latus alterius* (B:) *& ſi unius quadrati latus* A *metietur latus alterius* B, *& quadratus* Aq *quadratum* Bq *metietur.*

a 2.& 11.8. 1. *Hyp.* Nam Aq. AB *a* : : A B. Bq ; cum i-
b 7.8. gitur ex hyp. Aq metiatur Bq ; idem Aq ſecun-
c 20.def.7. dum A B *b* metietur. atqui Aq AB : : A. B. *c*
ergo etiam A metitur B. Q.E.D.

2. *Hyp.* A metitur B. *c* ergo tam Aq ipſum
d 11.ax.7 AB, *c* quam A B ipſum Bq metitur ; *d* & proinde Aq metitur Bq. Q.E.D.

PROP. XV.

A, 2. B, 6.
Ac,8. AqB,24. ABq,72. Bc,216.

Si cubus numerus Ac *cubum numerum* Bc *metiatur, & latus unius* (A) *metietur latus al-*

alterius (B:) *Et ſi latus* A *unius cubi* Ac *latus* B *alterius* Bc *metiatur, & cubus* Ac *cubum* Bc *metietur.*

1. *Hyp.* Nam Ac, AqB, ABq, Bc *a* ſunt ∺. a 2. & 12.8.
ergo Ac, *b* metiens extremum Bc, *c* etiam ſecundum AqB metietur. atqui Ac, AqB :: A. B. *d* ergo etiam A metietur B. Q.E.D. b *hyp.* c 7.8.

2. *Hyp.* A metitur B; *d* ergo Ac metitur AqB, isque ABq, & hic Bc; *e* ergo Ac metietur Bc. Q.E.D. d 20. *def.* 7. e 11. *ax.* 7.

PROP. XVI.

A,4. B,9.
Aq,16. Bq,81.

Si quadratus numerus Aq *quadratum numerum* Bq *non metiatur, neque* A *latus unius alterius latus* B *metietur: & ſi* A *latus unius quadrati* Aq *non metiatur B latus alterius* Bq, *neque quadratus* Aq *quadratum* Bq *metietur.*

1. *Hyp.* Nam ſi affirmes A metiri B, *a* etiam Aq ipſum Bq metietur, contra hyp.

2. *Hyp.* Vis Aq metiri Bq; *a* ergo A ipſum *B* metietur, contra hyp. a 14.8.

PROP. XVII.

A,2. B,3.
Ac,8. B,27.

Si cubus numerus A c cubum numerum Bc *non metiatur, neque A latus unius latus B alterius metietur. Et ſi latus A unius cubi* Ac *latus B alterius* Bc *non metiatur, neque cubus A*c *cubum* Bc *metietur.*

1. *Hyp.* Dic A metiri B; *a* ergo Ac metietur Bc. contra Hyp.

2. *Hyp.* Dic Ac metiri *Bc*; *a* ergo A ipſum *B* metietur. contra Hyp. a 15.8.

PROP.

PROP. XVIII.

C,6. D,2.
CD,12.
E,9. F,3. DE,18.
EF,27.

Duorum ſimilium planorum numerorum C D, E F, unus medius proportionalis eſt numerus DE: & planus C D ad planum E F duplicatam habet lateris C ad latus homologum E rationem.

* 21.def.7 Quoniam * ex hyp. C. *D* :: E. F; permutando erit C.E :: D.F. atqui C.E *a* :: C D. DE; *a* & D.F :: D E. E F. *b* ergo C *D*. D E :: D E. E F. Q.E.D.

a 17.7.
b 11.5.

c 10.def. 5. *c* Ergo ratio C D ad E F duplicata eſt rationis C D ad D E; hoc eſt rationis C ad E, vel D ad F.

Coroll.

Hinc perſpicuum eſt, inter duos ſimiles planos cadere unum medium proportionalem, in ratione laterum homologorum.

PROP. XIX.

CDE, 30. DEF, 60. FGE, 120. FGH, 240.
CD, 6. DF, 12. F G, 24.
C,2. D,3. E,5. F,4. G,6. H,10.

Duorum ſimilium ſolidorum CDE, FGH, duo medij proportionales ſunt numeri DFE, FGE. Et ſolidus CDE ad ſolidum FGH triplicatam rationem habet lateris homologi C ad latus homologum F.

* 12.def.7 Quoniam ex * hyp. C. D :: F.G; & D. E :: G.H, erit *a* permutando C. F :: D.G *a* :: E. H. atqui C D. D F *b* :: C. F; & D F. F G *b* :: D. G. *c* quare C D. D F :: D F. F G :: E.H. *d* ergo

a 13.7.
b 17.7.
c 11.5.
d 7.1.

go C D E. D F E :: D F E. F G E :: E. H :: F G E. F G H. ergo inter C D E, F G H cadunt duo medii proportionales, DFE, FGE. Q. E. *D.* e Liquet igitur rationem C D E ad F G H triplicatam eſſe rationis C D E ad DFE, vel C ad F. Q.E.D. e 10. def.5.

Coroll.

Hinc, inter duos ſimiles ſolidos cadunt duo medii proportionales, in ratione laterum homologorum.

PROP. XX.

A,12. C,18. B,27.
D, 2. E,3. F, 6. G,9.

Si inter duos numeros A,B. unus medius proportionalis cadat numerus C, ſimiles plani erunt illi numeri A, B.

a Accipe D, & E minimos in ratione A ad C, vel C ad B. *b* ergo D æque metitur A, ac E ipſum C, puta per eundem F. *b* item D æque metitur C ac E ipſum B, puta per Eundem G. *c* ergo DF=A, & EG=*B*. *d* quare A, & *B* plani ſunt numeri. Quia vero EF *c*=C *c*=DG; *e* erit D.E :: F.G, & viciſſim D.F :: E.G. *f* ergo plani numeri A, & *B* etiam ſimiles ſunt, Q.E.D.

a 35.7.
b 21.7.
c 9.ax.7.
d 16. def.7.
e 19.7.
f 21. def.7.

PROP. XXI.

A, 16. C, 24. D, 36. *B*, 54.
E, 4. F, 6. G, 9.
H, 2. P, 2. M, 4. K, 3. L, 3. N, 6.

Si inter duos numeros A, B duo medij proportionales cadant numeri C, D; ſimiles ſolidi erunt illi numeri, A, B.

a Sume E, F, G minimos ∻ in ratione *A* ad C. *b* ergo E, & G ſunt numeri plani ſimiles. hujus latera ſunt H & P; illius K & L: *c* ergo H.

a 2.8.
b 20.[illegible]
c 22 def.7.

d cor.18.8. K :: P. L :: *d* E. F. Atqui E, F, G ipſos A, C, D
e 21.7. *e* æque metiuntur, puta per eundem M; iidemque ipſos, C, D, B æque metiuntur, puta per eundem N.
f 9.ax.7. *f* ergo A = E M = H P M, *f* & B =
g 17.def.7. G N = K L N; *g* quare A & B ſolidi ſunt numeri.
h 17.7. Quoniam vero C *f* = F M; & D *f* = F N, erit M. N *h* :: F M. F N
k 7.5. *k* :: C. D *l* :: E.
l constr. F :: H. K :: P. L, *m* ergo A, & B ſunt numeri
m 21.def.7. ſolidi ſimiles. Q.E.D.

PROP. XXII.

A,4. B,6. C,9.

Si tres numeri A,B,C deinceps ſint proportionales, primus autem A ſit quadratus, & tertius C quadratus erit.

a 20.8. Inter A, & C cadit medius proportionalis.
b hyp. *a* ergo *A*, & C ſunt ſimiles plani; quare *b* cum A quadratus ſit, erit C etiam quadratus. Q.E.D.

PROP. XXIII.

A,8. B,12. C,18. D,27.

Si quatuor numeri A,B,C,D *deinceps ſint proportionales; primus autem* A *ſit cubus, & quartus D cubus erit.*

a 21.8. Nam A, & D *a* ſimiles ſolidi ſunt; ergo *b*
b hyp. cum *A* cubus ſit, erit D cubus. Q.E.D.

PROP. XXIV.

A,16. 24. B. 36.
C,4. 6. D,9.

Si duo numeri A, B rationem habeant inter ſe, quam quadratus numerus C ad quadratum numerum D, primus autem A ſit quadratus; & ſecundus B *quadratus erit.*

Inter

Inter C, & D numeros quadratos,† adeoque inter A,& B eandem rationem habentes, *a* cadit unus medius proportionalis. Ergo *b* cum A quadratus ſit, *c* etiam B quadratus erit. Q. E. D. † 1. 8. a 11. 8. b *hyp.* c 22. 8.

Coroll.

1. Hinc ſi fuerint duo numero ſimiles A B, C D (A.B : : C. *D*) primus autem A B ſit quadratus,etiam ſecundus C D quadratus erit.

† Nam A B. C D : : Aq. Cq. † 11. 8. 11. 8.

2. Liquet ex his,proportionem cujusvis numeri quadrati ad quemlibet non quadratum, exhiberi nullo modo poſſe in duobus numeris quadratis. unde non erit, Q. Q : : 1.2.nec 1.5. : : Q. Q. &c.

PROP. XXV.

C,64. 96. 144. D,216.
A, 8. 12. 18. B, 27.

Si duo numeri A, B rationem inter ſe habeant, quam cubus numerus C ad cubum numerum D, primus autem A ſit cubus, & ſecundus B cubus erit.

a Inter C,& D cubos, *b* adeoque inter A & B eandem rationem habentes,cadunt duo medii proportionales. ergo propter A *c* cubum, *d* etiam B cubus erit. Q.E.D. a 12. 8. b 8. 8. c *hyp.* d 23. 8.

Coroll.

1. Hinc etiam ſi fuerint duo numeri A B C, DEF (A. B : : D. E. & B. C : : E. F ;) primus autem ABC cubus fuerit,etiam ſecundus DEF cubus erit.

*12.& * Nam ABC.DEF : : Ac = Dc.

13.8. 2. Patet etiam ex his, proportionem cujusvis numeri cubi ad quemlibet numerum non cubum non poſſe reperiri in duobus numeris cubis.

PROP. XXVI.

A,20. C,30. B,45.
D,4. E,6. F,9.

Similes plani numeri A, B rationem inter ſe habent, quam quadratus numerus ad quadratum numerum.

a *18.8.* Inter A, & B *a* cadit unus medius proportionalis C. *b* ſume tres D,E,F minimos ∺ in ratione A ad C. Extremi D, F *b* quadrati erunt. atqui ex æquali A. B *c* : : D. F. ergo A. B : : Q. Q. Q.E.D.

b *2.8.*

c *14.7.*

PROP. XXVII.

A,16. C,24. D,36. B,54.
E,8. F,12. G,18. H,27.

Similes ſolidi numeri A, B, rationem habent inter ſe, quam cubus numerus ad cubum numerum.

a *19.8.* *a* Inter A, & B cadunt duo medii proportionales, puta C & D : *b* ſume quatuor E, F, G, H minimos ∺ in ratione A ad C. *b* Extremi E, H cubi ſunt. At A. B *c* : : E. H : : C. C. Q.E.D.

b *2.8.*

c *14.7.*

SCHOL.

Vide Clavium. 1. Ex his infertur, nullos numeros habentes proportionem ſuperparticularem, vel ſuperbipartientem, vel duplam, aut aliam quamcunq; multiplam non denominatam à numero quadrato, eſſe ſimiles planos.

2. Nec duo quivis primi numeri, neque duo quicunque inter ſe primi, qui quadrati non ſint, ſimiles eſſe poſſunt.

LIB.

LIB. IX.

PROP. I.

A,6. B,54.
Aq,36. 108. AB, 324.

SI duo ſimiles plani numeri A, B multiplicantes ſe mutuo faciant quendam A B, productus A B quadratus erit.

Nam A. B *a* :: Aq. AB; cum igitur inter A, & B *b* cadat unus medius proportionalis, *c* etiam inter Aq, & A B cadet unus medius proportionalis. ergo cum primus Aq ſit quadratus, *d* etiam tertius AB quadratus erit. Q.E.D. a 17.7. b 18.8. c 8.8. d 22.8.

Vel ſic; Sint ab,cd ſimiles plani, nempe a. b :: c.d. *x* ergo a d = b c. quare a b c d, vel adbc *y* = adad = Q: ad. x 19.7. y 1.ax.1.

PROP. II.

A, 6. B,54.
Aq, 36. AE,324.

Si duo numeri A, B ſe mutuo multiplicantes faciant A B quadratum, ſimiles plani erunt, A, B.

Nam A.B *a* :: Aq. AB; quare cum inter Aq, AB *b* cadat unus medius proportionalis, *c* etiam unus inter A,& B medius cadet. *d* ergo A,& B ſunt ſimiles plani. Q.E.D. a 17.7. b 11.8. c 8.8. d 20.8.

PROP. III.

A,2. Ac,8 Acc,64.

Si cubus numerus Ac ſeipſum multiplicans procreet aliquem Acc, productus Acc cubus erit.

Nam 1 A *a* :: A. Aq *b* :: Aq. Ac. ergo inter 1,& Ac cadunt duo medii proport. Sed 1. Ac *a* :: Ac. Acc. *c* ergo inter Ac,& Acc cadunt et- a 15. def.7. b 17.7. c 8.8.

iam duo medii proportionales. Proinde cum
d 23.8. Ac ſit cubus, *d* erit Acc cubus. Q.E.D.

Vel ſic; aaa (Ac) in ſe ductus facit aaaaaa. (Acc;) hic cubus eſt, cujus latus aa.

PROP. IV.

Ac, 8. Bc, 27.
Acc, 64. AcBc, 216.

Si cubus numerus Ac *cubum numerum* Bc *multiplicans, faciat aliquem* AcBc, *factus* AcBc *cubus erit.*

a 17.7. Nam Ac. Bc *a* :: Acc. AcBc. ſed inter Ac
b 12.8. & Bc *b* cadunt duo medii proportionales; *c*
c 8.8. ergo inter Acc, & AcBc totidem cadunt. itaq;
d 23.8. cum Acc ſit cubus, *d* erit AcBc etiam cubus. Q.E.D.

Vel ſic; AcBc = aaabbb (ababab) = C:ab.

PROP. V.

Ac, 8. B, 27.
Acc, 64. AcB, 216.

Si cubus numerus Ac *numerum quendam B multiplicans, faciat cubum* AcB; *& multiplicatus B cubus erit.*

a 17.7. Nam Acc. AcB *a* :: Ac. B. Sed inter Acc, &
b 12.8. AcB *b* cadunt duo medii proportionales. *c* er-
c 8.8. go totidem cadent inter Ac, & B. quare cum
d 23.8. Ac cubus ſit *d* etiam B cubus erit. Q.E.D.

PROP. VI.

A, 8. Aq, 64. Ac, 512.

Si numerus A ſeipſum multiplicans faciat Aq cubum; & ipſe A cubus erit.

a hyp. Namquia Aq *a* cubus, & AqA (Ac) *b* cu-
b 19.def. 7. bus, *c* erit A cubus. Q.E.D.
c 5.8.

PROP. VII.

A, 6. B, 11. AB, 66.
D, 2. E, 3.

Si compoſitus numerus A numerum quempiam B multiplicans, quempiam faciat AB, factus AB ſolidus erit.

Quoniam A compoſitus eſt, *a* metiur eum aliquis D, puta per E. *b* ergo A = DE; *c* quare DEB = AB ſolidus eſt. Q.E.D.

a 13. def.
b 9. ax. 7.
c 17 def. 7

PROP. VIII.

1. a, 3. a2, 9. a3, 27. A4, 81. a5, 243. a6, 729.

Si ab unitate quotcunque numeri deinceps proportionales fuerint (1, a, a2, a3, a4, &c.) *tertius quidem ab unitate* a2 *quadratus eſt; & unum intermittentes, omnes* (a4, a6, a8, &c.): *quartus autem* a3 *eſt cubus; & duos intermittentes omnes* (a6, a9, &c.) *ſeptimus vero* a6, *cubus ſimul & quadratus; & quinque intermittentes omnes* (a12, a18, &c.)

Nam 1. a2 = Q. a. & a4 = aaaa = Q. aa. & a6 = aaaaaa = Q. aaa, &c.

2. a3 = aaa = C. a. & a6 = aaaaaa = C. aa. & aaaaaaaaa = C. aaa, &c.

3. a6 = aaaaaa = C. aa = Q. aaa. ergo, &c.

Vel juxta Euclidem; quia 1. a *a* :: a. a2, *b* erit a2 = Q: a, ergo cum a2, a3, a4 ſint ∺ *c* erit tertius a4 etiam quadratus pariterq; a6, a8, &c. Item quia 1. a *a* :: a2. a3. erit a3 *b* = a2 in a = C: a. *d* ergo quartus ab a3, nempe a6, etiam cubus erit, &c. ergo a6 cubus ſimul & quadratus exiſtit, &c.

a hy.
b 20. 7.
c 22. 8.
d 21. 8.

PROP. IX.

1. a, 4. a^{2}, 16. a^{3}, 64. a^{4}, 256, &c.
1. a, 8. a^{2}, 64. a^{3}, 512. a^{4}, 4096.

Si ab unitate quotcunque numeri deinceps proportionales fuerint (1, a, a^{2}, a^{3} &c.); *qui vero* (a) *poſt unitatem ſit quadratus; & reliqui omnes*, a^{2}, a^{3}, a^{4} &c. *quadrati erunt. At ſi* a, *qui poſt unitatem, ſit cubus, & reliqui omnes* a^{2}, a^{3}, a^{4}, &c. *cubi erunt.*

1. *Hyp.* Nam a^{2}, a^{4}, a^{6}, &c. quadrati ſunt
a 22.8. ex præc. item quia a ponitur quadratus, *a* erit tertius a^{3} quadratus, pariterque a^{5}, a^{7}, &c. ergo omnes.

b 23.8. 2. *Hyp.* a cubus ponitur, *b* ergo a^{4}, a^{7}, a^{10} cubi ſunt: atqui ex præced. a^{3}, a^{6}, a^{9}, &c. cu-
c 20.7. bi ſunt. denique quia 1. a :: a. aa, *c* erit a^{2} = Q:
d 3.9. a. cubus autem in ſe *d* facit cubum; ergo a^{2} cu-
e 23.8. bus eſt, & *e* proinde ab eo quartus a^{5}, pariterque a^{8}, a^{11}, &c. cubi ſunt. ergo omnes. Q. E. D.

Clarius forſitan ſic; Sit quadrati a latus b ergo ſeries a, a^{2}, a^{3}, a^{4}, &c. aliter exprimetur ſic, bb, b^{4}, b^{6}, b^{8}, &c. liquet vero hos omnes quadratos eſſe; & ſic etiam exprimi poſſe; Q: b, Q: bb, Q: bbb, Q: bbbb. &c.

Eodem modo, ſi b latus fuerit cubi a, ſeries ita nominari poteſt; b^{3}, b^{6}, b^{9}, b^{12}, &c. vel C: b, C: b^{2}, C: b^{3}, C: b^{4}, &c.

PROP. X.

1, a, a^{2}, a^{3}, a^{4}, a^{5}, a^{6}.
1, 2, 4, 8, 16, 32, 64.

Si ab unitate quotcunque numeri deinceps

ceps proportionales fuerint (1, a, a^2, a^3, &c.); *qui vero poſt unitatem* (a) *non ſit quadratus neq; alius ullus quadratus erit. præter* a^2 *tertium ab unitate, & unum intermittentes omnes* (a^4, a^6, a^8.) *At ſi* a, *qui poſt unitatem, non ſit cubus, neque ullus alius cubus erit præter* a^3 *quartum ab unitate, & duos intermittentes omnes,* a^6, a^9, a^{12}, *&c.*

1. *Hyp.* Nam ſi fieri poteſt, ſit a^5 quadratus numerus. quoniam igitur a. a^2 *a* :: a^4. a^5, atque inverſe a^5. a^4 :: a^2. a; ſintque a^5, & a^4 *b* quadrati, primusque a^2 quadratus, *c* erit a etiam quadratus, contra Hyp. a *hyp.* b *ſuppoſ. & 8.9.* c *24.8.*

2. *Hyp.* Si fieri poteſt, ſit a^4 cubus. quoniam igitur *d* ex æquo a^4. a^6 :: a. a^3, atque inverſe a^6. a^4 :: a^3. a; *b* ſintque a^6, & a^4 cubi, & primus a^3 cubus, *e* etiam a cubus erit, contra Hypoth. d *14.7.* e *25.8.*

PROP. XI.

1, a, a^2, a^3, a^4, a^5, a^6.
1, 3, 9, 27, 81, 243, 729.

Si ab unitate quotcunque numeri deinceps proportionales fuerint(1, a, a^2, a^3, &c.) *minor majorem metitur per aliquem eorum qui in proportionalibus ſunt numeris.*

Quoniam 1. a :: a. aa, *a* erit $\frac{aa}{a} = a = \frac{aaa}{aa}$. item quia 1. aa *b* :: a. aaa. *a* erit $\frac{aaa}{a} = aa = \frac{a4}{aa} = \frac{a5}{a3}$ &c. denique quia 1. a^3 *b* :: a. a^4, *a* erit $\frac{a4}{a} = a^3 = \frac{a^6}{a6}$ &c. a *9. ax. 7 & 20. def. 7.* b *14.7.*

Coroll.

Coroll.

Hinc, ſi numerus qui metitur aliquem ex proportionalibus, non ſit unus proportionalium, neque numerus per quem metitur, erit aliquis ex proportionalibus.

PROP. XII.

1, a, a2, a3, a4,
1, 6, 36, 216, 1296.
B, 3.

Si ab unitate quotcunq; numeri deinceps proportionales fuerint (1, a, a2, a3, a4,); *quicunque primorum numerorum B ultimum* a4 *metiuntur, iidem* (*B*) *& eum* (a) *qui unitati proximus eſt, metientur.*

a 31.7. Dic B non metiri a, *a* ergo *B* ad a primus
b 27.7. eſt; *b* ergo B ad a2 primus eſt; & *c* proinde ad
c 26.7. a4 quem metiri ponitur Q.E.A.

Coroll.

1. Itaque omnis numerus primus ultimum metiens, metitur quoque omnes alios ultimum præcedentes.

2. Si aliquis numerus non metiens proximum unitati, metiatur ultimum, erit numerus compoſitus.

3. Si proximus unitati ſit primus numerus, nullus alius primus numerus ultimum metietur.

PROP. XIII.

1, *a*, *a2*, *a3*, *a4*,
1. 5, 25, 125, 625.
H--G--F--E--

Si ab unitate quotcunque numeri deinceps proportionales fuerint (a, a2, a3, *&c.*), *qui vero poſt unitatem* (a) *primus ſit;*

ma-

maximum nullus alius metietur, præter eos qui ſunt in numeris proportionalibus.

Si fieri poteſt, alius quiſpiam E metiatur *a*4, nempe per F; *a* erit F alius extra *a*, *a*2, *a*3. Quia *a* cor. 12. 9 vero E metiens *a*4 non metietur *a*, *b* erit E *b* 2. cor. 12. 9. numerus compoſitus; *c* ergo eum aliquis pri- *c* 33. 7. mus metietur, *d* qui proinde ipſum *a*4 meti- *d* 11. ax. 7 tur; *e* ideoque alius non eſt, quam *a*. ergo *a* *e* 3. cor. 12. 9. metitur E. Eodem modo oſtendetur F compoſitus numerus, metiens *a*4, adeoque *a* ipſum F metiri. itaque quum EF *f* = *a*4 = *a* in *a*3, *g* *f* 9. ax. 7. *g* 19. 7. erit *a*. E : : F. *a*3. ergo cum *a* metiatur E, *h* æ- *h* 20. def. 7. que F metietur *a*3, puta per eundem G. *k* Nec *k* cor. 11. 9. G erit *a*, vel *a*2. ergo, ut prius, G eſt numerus compoſitus, & *a* eum metitur. quum igitur FG *f* = *a*3 = *a*2 in *a*, *g* erit *a*. F : : G. *a*2; & proinde, quia A metitur F, *h* æque G metietur *a*2, ſcilicet per eundem H; *k* qui non eſt *a*. ergo quum GH = *a*2 = *aa*. *l* erit H. *a* : : *a*. G. ergo *l* 20. 7. quia *a* metitur G (ut prius) *m* etiam H metie- *m* 20. def. 7. tur *a*, numerum primum. Q.F.N.

PROP. XIV.

A, 30.
B, 2. C, 3. D, 5.
E - - F - - -

Si minimum numerum A primi numeri B, C, D metiantur; nullus alius numerus primus E illum metietur, præter eos, qui à principio metiebantur.

Si fieri poteſt, ſit $\frac{A}{E}$ = F. *a* Ergo A = E F. *b* *a* 9. ax. 7. *b* 31. 7. Ergo ſinguli primi numeri B, C, D ipſorum E, F unum metiuntur; non E, qui primus ponitur; ergo F, minorem ſcilicet ipſo A; contra Hyp.

PROP. XV.

A,9. B,12. C,16.
D,3. E,4.

Si tres numeri A, B, C deinceps proportionales, fuerint minimi omnium eandem cum ipſis rationem habentium; duo quilibet compoſiti, ad reliquum primi erunt.

a Sume D, & E minimos in ratione A ad B.
a 35.7. *b* ergo A = Dq; *b* & C = Eq; *b* & B = D E.
b 2.8. Quia vero D ad E *c* primus eſt, *d* erit D + E
c 24.7. primus ad ſingulos D, & E. * ergo D in D +
d 30.7. E *e* = Dq + D E (*f* A + B) ad E primus eſt,
* 26.7. ideoque ad C vel Eq. Q.E.D. Pari pacto D-
e 3.2. E + Eq (B + C) ad D primus eſt, & proinde
f prius. ad A = Dq. Q.E.D. Denique quia B ad D +
g 27.7. E *h* primus eſt; is ad hujus quadratum *k* Dq +
h 26.7. 2 D E + Eq (A + 2 B + C) primus erit. *l*
k 4.2. quare idem B ad A + B + C, *l* adeoque ad
l 30.7. A + C primus erit. Q.E.D.

PROP. XVI.

A, 3. B, 5. C---

Si duo numeri A, B primi inter ſe fuerint; non erit ut primus A ad ſecundum B, ita ſecundus B ad alium quempiam C.

Dic A. B : : B. C. ergo quum A & B in ſua
a 23.7. ratione *a* minimi ſint, A *b* metietur B æque ac
b 21.7. B ipſum C; ſed A *c* ſeipſum etiam metitur; er-
c 6. ax. 7. go A & B non ſunt primi inter ſe, contra Hyp.

PROP. XVII.

A, 8. B, 12. C, 18. D, 27. E---

Si fuerint quotcunque numeri deinceps proportionales A, B, C, D, extremi autem ipſorum A, D

A,D *primi inter ſe ſint; non erit ut primus* A *ad ſecundum B, ita ultimus D ad alium quempiam E.*

Dic A. B : : D. E. ergo viciſſim A. D : : B. E. ergo quum A & D in ſua ratione *a* minimi ſint, *b* metietur A ipſum B; *c* quare B ipſum C, & C ſequentem D, *d* adeoque A eundem D metietur. Ergo A & D non ſunt primi inter ſe, contra Hypoth. a 23.7. b 21.7. c 10. def. 7. d 11.ax. 7.

PROP. XVIII.

A, 4. B, 6. C, 9.
Bq, 16.
veniri.

Duobus numeris datis A, B, conſiderare an poſſit ipſis tertius proportionalis C inveniri.

Si A metiatur Bq per aliquem C, *a* erit A C = Bq. unde *b* liquet eſſe A. B : : B. C. Q.E.F. a 9.ax. 7 b per 20, 7.

A, 6. B, 4. Bq, 16. Sin A non metiatur Bq, non erit aliquis tertius proportionalis. Nam dic A. B : : B. C. *a* ergo A C = Bq. *c* proinde $\frac{Bq}{A}$ = C. Scilicet A metitur Bq, contra Hypoth. c 7.ax. 7

PROP. XIX.

A, 8. B, 12, C, 18. D, 27.
BC, 216.

Tribus numeris datis A, B, C, conſiderare an poſſit ipſis quartus proportionalis D inveniri.

Si A metiatur BC per aliquem D, *a* ergo A D = B C; *b* conſtat igitur eſſe A.B : : C. D. Q.E.F. a 9.ax. 7 b ax. 19. 7.

Sin A non metiatur BC, non datur quartus proportionalis; quod oſtendetur, prout in præcedenti.

PROP.

PROP. XX.

A, 2. B, 3. C, 5. *Primi numeri plures ſunt omni propoſita multitudine*
D, 30. G - - - - *primorum numerorum A, B, C.*

a 38.7. *a* Sit D minimus, quem A,B,C metiuntur. ſi D + 1 primus ſit, res patet; ſi compoſitus,
b 33.7. *b* ergo aliquis primus, puta G, metitur D + 1,
c ſuppoſ. qui non eſt aliquis trium A, B, C; nam ſi ita,
d conſtr. quum is *c* totum D + 1, & *d* ablatum D me-
e 12.ax.7 tiatur, *e* idem reliquam unitatem metietur. Q. E. A. Ergo propoſitorum primorum numerorum multitudo aucta eſt per D + 1; vel ſaltem per G.

PROP. XXI.

5 5 3 3 2 2
A.....E..... B...F...C..G..D 20.

Si pares numeri quotcunque AB, BC, CD componantur, totus AD par erit.

a 6.def.7. *a* Sume EB = $\frac{1}{2}$ AB & FC = $\frac{1}{2}$ BC, & GD
b 12.7. = $\frac{1}{2}$ CD. *b* liquet EB + FC + GD = $\frac{1}{2}$ AD.
c 6.def.7. *c* ergo AD eſt par numerus. Q.E.D.

PROP. XXII.

1 1 1
A.......F.B......G.C....H.D..L.E 22.
9 7 5 3

Si impares numeri quotcunque AB, BC, CD, DE componantur, multitudo autem ipſorum ſit
a 7.def.7. *par, totus AE par erit.*

b 21.9. Detracta unitate ex ſingulis imparibus, *a* ma-
c hyp. nebunt AF, BG, CH, DL numeri pares, & *b* proinde compoſitus ex ipſis par erit; adde his *c*

parem

parem numerum conflatum ex reſiduis unitatibus, *d* totus idcirco AE par erit. Q.E.D. d 21.9.

PROP. XXIII.

7 5 1
A……. *B* .….C . . E . D 15.
3

Si impares numeri quotcunque AB, BC, CD componantur, multitudo autem ipſorum ſit impar; & totus AD impar erit.

Nam dempto CD uno imparium, reliquórum aggregatus AC *a* eſt par numerus. huic adde CD − 1; *b* totus AE eſt etiam par; quare reſtituta unitate totus AD *c* impar erit. Q.E.D.

a 22.9.
b 21.3.
c 7.def.7

PROP. XXIV.

4 5 1
A…. B ,…. D. C 10.
6

Si à pari numero AC par AB detrahatur, & reliquus BC par erit.

Nam ſi BD (BC − 1) impar fuerit, *a* erit BC (BD + 1) par. Q.E.D. Sin *B* D parem dicas, propter AB *b* parem, *c* erit AD par; *a* ideoque AC (AD + 1) impar, contra Hypoth. ergo BC eſt par. Q.E.D.

a 7.def.7
b hyp.
c 21.9.

PROP. XXV.

6 1 3
A……D . C … B 10.
7

Si à pari numero AB impar AC detrahatur, & reliquus CB impar erit.

Nam AC − 1 (AD) *a* eſt par. *b* ergo DB eſt par. *c* ergo CB (DB − 1) eſt impar. Q.E.D.

a 7.def.7
b 24.9.
c 7.def.7

PROP.

PROP. XXVI.

4 6 1
A....C......D.B 11.
7

Si ab impari numero AB impar CB detrahatur, reliquus AC par erit.

a 7.def.7. Nam AB − 1 (AD) & CB - 1 (CD) *a*
b 24.9. ſunt pares. *b* ergo AD − CD (AC) eſt par. Q.E.D.

PROP. XXVII.

1 4 6
A .D....C......B 11.
5

Si ab impari numero AB par detrahatur CB, reliquus AC impar erit.

a 7.def.7. Nam AB − 1 (DB) *a* eſt par; & CB po-
b 24.9. nitur par. *b* ergo reliquus CD par eſt. *c* ergo
c 7.def.7. CD + 1 (CA) eſt impar. Q.E.D.

PROP. XXVIII.

A, 3.
B, 4.
— —
AB, 12.

Si impar numerus A parem numerum B multiplicans fecerit aliquem AB, factus AB par erit.

a hyp. & Nam AB *a* componitur ex impari A toties
15.def.7 accepto, quoties unitas continetur in B pari, *b*
b 21.9. ergo AB eſt par numerus.

Schol.

Eodem modo, ſi A ſit numerus par, erit AB par.

PROP. XXIX.

A, 3.
B, 5.
— —
AB, 15.

Si impar numerus A, imparem numerum B multiplicans fecerit aliquem AB, factus AB impar erit.

Nam

Nam A B *a* componitur ex B impari numero toties accepto, quoties unitas includitur in A etiam impari. *b* ergo A B eſt impar. Q.E.D. a 25. def. 7. b 13.9.

Schol.

$\frac{B}{A}$, $\frac{12}{3}$ (C,4.

1. *Numerus A impar numerum B parem metiens, per numerum parem C eum metitur.*

Nam ſi C impar dicatur, quoniam *a* B = A C, erit B impar, contra Hypoth. a 9.ax.9 b 29.9.

$\frac{B}{A}$, $\frac{15}{3}$ (C,5.

2. *Numerus A impar numerum B imparem metiens, per numerum C imparem eum metitur.*

Nam ſi C dicatur par; *a* erit A C, vel B par, contra Hypoth. a 28.9.

$\frac{B}{A}$, $\frac{15}{3}$ (C,5.

3. *Omnis numerus (A & C) metiens imparem numerum B, eſt impar.*

Nam ſi utervis A, vel C dicatur par, *a* erit B numerus par, contra Hypoth. a 28.9.

PROP. XXX.

$\frac{B}{A}$, $\frac{24}{3}$ (C,8. $\frac{D}{A}$, $\frac{12}{3}$ (E,4.

Si impar numerus A parem numerum B metiatur, & illius dimidium D metietur.

a Sit $\frac{B}{A}$ = C *b* ergo C eſt numerus par. Sit igitur E = $\frac{1}{2}$ C, erit B *c* = C A *d* = 2 E A *e* = 2 *D*. *f* ergo E A = D; & *g* proinde $\frac{D}{A}$ = E. Q.E.D. a hyp. b 1 ſcho. 19. 9. c 9.ax. d 1.2. e hyp. f 7.ax.1. g 7.ax.3

PROP. XXXI.

A,5. B,8. C,16. D - - -

Si impar numerus A ad aliquem nu-

merum B primus ſit; & ad illius duplum C primus erit.

a 3.ſchol. 29.9. Si fieri poteſt, aliquis D metiatur A, & C. *a*
b 30.9. ergo D metiens imparem A impar erit, *b* ideoque ipſum B paris C ſemiſſem metietur. ergo A, & B non ſunt primi inter ſe, contra Hyp.

Coroll.

Sequitur hinc, numerum imparem, qui ad aliquem numerum progreſſionis duplæ primus eſt, primum quoque eſſe ad omnes numeros illius progreſſionis.

PROP. XXXII.

1. A, 2. B, 4. C, 8. D, 16.

Numerorum A, B, C, D, &c. à binario duplorum unusquisque pariter par eſt tantum.

a 6.def.7. Conſtat omnes 1, A, B, C, D *a* pares eſſe;
b 20.def.7. atque *b* ∺ nimirum in ratione dupla, & *c* pro-
c 11.9. inde quemque minorem metiri majorem per
d 8.def.7 aliquem ex illis. *d* Omnes igitur ſunt pariter
e 13.9. pares. Sed quoniam A primus eſt, *e* nullus extra eos eorum aliquem metietur. Ergo pariter pares ſunt tantum. Q.E.D.

PROP. XXXIII.

A, 30. B, 15.
D---E--

Si numerus A dimidium B habeat imparem, A pariter impar eſt tantum.

Quoniam impar numerus B *a* metitur A
a hyp. b 9 def.7 per 2 parem, *b* eſt B pariter impar. Dic etiam
c 8.def.7 pariter parem. *c* ergo eum par aliquis D per
d 9.ax.7. e 19.7. parem E metitur. unde 2 B *d* = A *d* = D E. *e*
quare

quare 2 E : : D.B. ergo ut 2 *f* metitur parem, *g* *f 16.def.7*
ſic D par imparem B metitur. Q. F. N. *g 10.def. 7.*

PROP. XXXIV.

A,24. *Si par numerus A, neque à binario duplus ſit, neque dimidium habeat imparem; pariter par eſt, & pariter impar.*

Liquet A eſſe pariter parem, quia dimidium imparem non habet. Quia vero ſi A biſarietur, & rurſus ejus dimidium, & hoc ſemper fiat, tandem incidemus in aliquem *a* imparem (quia *a 7.def.7* non in binarium, quoniam A à binario duplus non ponitur) is metietur A per parem numerum (nam *b* alias ipſe A impar eſſet, contra *b 1.ſch. 29.9.* Hypoth.) ergo A eſt etiam pariter impar. Q. E. D.

PROP. XXXV.

A .... 8.

4 8
B .... F .... G 12.

C .... 18.

9 6 4 8
D .... H ...... L .... K .... N 27.

Si ſint quotcunque numeri deinceps proportionales A, B G, C, D N, detrahantur autem F G à ſecundo, & K N ab ultimo, æquales ipſi primo A; erit ut ſecundi exceſſus B F ad primum A, ita ultimi exceſſus D K ad omnes A, B G, C ipſum antecedentes.

Ex D N deme N L = B G, & N H = C.

Quoniam D N. C. (H N) *a* : : H N. B G. *a hyp.* (L N) *a* : : L N. (B G) A. (K N.) *b* erit divi- *b 17.5.*

O dendo

dendo ubique, D H. H N : : H L. L N : : L K.
c 18.5. d 3.ax.1. K N. *c* quare D K. C + B G + A : : L K (*d* B F.) K N. (A.) Q.E.D.

Coroll.

e 18.5. Hinc *e* componendo, D N + B G + C. A + B G + C : : B G A.

PROP. XXXVI.

1. A, 2. B, 4. C, 8. D, 16.
E, 31. G, 62. H, 124. L, 248. F, 496.
M, 31. N, 465.
P - - - - Q - - -

Si ab unitate quotcunq; numeri 1, *A,B,C,D, deinceps exponantur in dupla proportione, quoad totus compoſitus E fiat primus, & totus hic E in ultimum D multiplicatus faciat aliquem F; factus F erit perfectus.*

Sume totidem, E, G, H, L etiam in propor-
a 14.7. tione dupla continue; ergo *a* ex æquo A. D : :
b 19.7. c hyp. d 7.ax.7. E. L. *b* ergo A L = D E *c* = F. *d* ergo L = $\frac{E}{2}$
quare E,G,H,L,F ſunt ∺ in ratione dupla. Sit
e 35.9. G − E = M, & F − E = N. *e* ideo M. E : : N. E
f 3.ax.1. + G + H + L. *f* at M = E. *g* ergo N = E +
g 14.5. h 2.ax.1. G + H + L. ergo F = 1 + B + C + D + E
+ G + H + L = E + N. Quin etiam quia D
k 7.ax.7. l 11.ax.7. *k* metitur D E (F,) *l* etiam ſinguli 1, A, B, C *m*
m 11.9. nec non E,G,H,L metiuntur F. Porro nullus
alius eundem F metitur. Nam ſi aliquis, ſit P, qui
n 9.ax.7. metiatur F per Q. *n* ergo P Q = F = D E. *o* er-
o 19.7. go E. Q : : P. D. ergo cum A primus numerus
p 13.9. metiatur D, & *p* proinde nullus alius P eundem
q 20.def.7. metiatur, *q* conſequenter E non metitur Q.
r 31.7. quare cum E primus ponatur, *r* idem ad Q pri-

mus

mus erit. *s* ergo E & Q in ſua ratione minimi ſunt, & *t* propterea E in ipſum P ac Q ipſum D æque metiuntur. *u* ergo Q eſt aliquis ipſorum A, B, C. Sit igitur B; ergo cum ex æquo ſit B.D :: E.H; *x* ideoque BH = DE = F = PQ. *x* adeoque Q.B :: H.P. *y* erit H = P. ergo P eſt etiam aliquis ipſorum A, B, C, &c. contra Hyp. ergo nullus alius præter numeros prædictos eundem F metietur: *z* proinde F eſt numerus perfectus. Q.E.D.

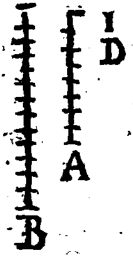
s 23.7.
t 21.7.
u 13.7.
x 19.7.
y 14.5.
z 20. *def.7.*

LIB. X.

DEFINITIONES.

I.

COmmenſurabiles magnitudines dicuntur, quas eadem menſura metitur.

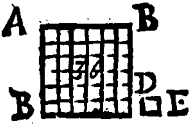


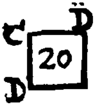
Commenſurabilitatis nota eſt ⊏⊐, *A* ⊏⊐ *B; hoc eſt, linea A 8 pedum commenſurabilis eſt linea B* 13 *pedum; quia D linea unius pedis ſingulas A & B metitur. Item* $\sqrt{18}$ ⊏⊐ $\sqrt{50}$; *quia* $\sqrt{2}$ *ſingulas* $\sqrt{18}$, & $\sqrt{50}$ *metitur. Nam* $\sqrt{\frac{18}{2}} = \sqrt{9} = 3$, & $\sqrt{\frac{50}{2}} = \sqrt{25} = 5$. *quare* $\sqrt{18}.\sqrt{50} :: 3.5$.

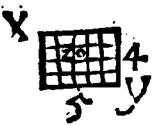
II. Incommenſurabiles autem ſunt, quorum nullam communẽ menſurã contingit reperiri.

Incommenſurabilitas ſignificatur nota ⊏⊐. *ut* $\sqrt{6}$ ⊏⊐ $\sqrt{25}$ (5;) *hoc eſt* $\sqrt{6}$ *incommenſurabilis eſt numero* 5, *vel magnitudini hoc numero deſignatæ; quia harum nulla eſt communis menſura, ut poſtea patebit.*

III. [Rectæ lineæ potentia commenſurabiles ſunt, cum quadrata earum idem ſpatium metitur.







Hujusce commenſurabilitatis nota eſt ⊐, ut A B ⊐ CD; h.e. linea AB ſex pedum potentia commenſurabilis eſt linea C D, qua exprimitur per √ 20. quia ſpatium E unius pedis quadrati metitur tam ABq (36) *quam rectangulum X-Y* (20,) *cui æquale eſt quadratum lineæ C D* (√ 20.) *Eadem nota ⊐ nonnunquam valet potentia tantum commenſurabilis.*

IV. Incommenſurabiles vero potentia, cum quadratis earum nullum ſpatiũ, quod ſit communis eorum menſura, contingit reperiri.

Hujusmodi incommenſurabilitas denotatur ſic; 5 ⊐ v√ 8; *hoc eſt, numeri vel lineæ* 5, *&* v√ 8 *ſunt incommenſurabiles potentia; quia harum quadrata* 25, *&* √ 8 *ſunt incommenſurabilia.*

V. Quæ cum ita ſint, manifeſtum eſt cuicunque rectæ propoſitæ, rectas lineas multitudine infinitas, & commenſurabiles eſſe, & incommenſurabiles; alias quidem longitudine & potentia, alias vero potentia ſolum. Vocetur autem propoſita recta linea Rationalis.

Hujus nota eſt ρ̇.

VI. Et huic commenſurabiles, ſive longitudine & potentia, ſive potentia tantum, Rationales ρ̇.

VII. Huic vero incommenſurabiles Irrationales vocentur.

Hæ ſic denotantur ρ.

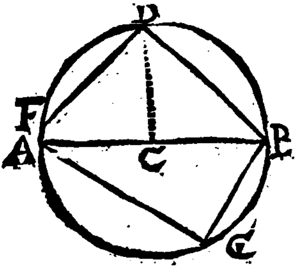
VIII. Et quadratum, quod à propoſita recta fit, dicatur rationale ρ̇ν.

IX. Et huic commenſurabilia quidem Rationalia ρ̇α.

X. Huic vero incommenſurabilia, Irrationalia dicantur ρα.

XI. Et rectæ, quæ ipſa poſſunt, Irrationales, ρ.

Schol.



Vt poſtrema 7 definitiones exemplo aliquo illuſtrentur, ſit circulus A D B P, cujus ſemidiameter C B; huic inſcribantur latera figurarum ordinatarum, Hexagoni quidem B P, Trianguli A P, quadrati B D, pentagoni F D. Itaque ſi juxta 5 defin. ſemidiameter C B ſit rationalis expoſita, numero 2. expreſſa, cui reliquæ B P, A P, B D, F D comparandæ ſunt, [a] erit B P a = B C = 2. quare B P eſt ρ̇ ⊐ B C, juxta 6. def. Item A P [b] = $\surd$ 12 (nam ABq (16) − BPq (4) = 12) quare A P eſt ρ̇. ⊐ BC, etiam juxta 6. def. atq;

a cor. 15. 4.
b 67. 1.

APq (12) *eſt ṗr, per def.* 9. *Porro B D b* = √ *DCq* + *BCq* = √ 8; *unde B D eſt ṗ ⊐ B C;* & *BDq ṗr. Denique, FDq* = 10 -- √ 20 (*ut patebit ex praxi ad* 10. 13. *tradenda*) *erit ṗr, juxta* 10. *def.* & *proinde F D* = √ : 10 − √ 20 *eſt ṗ, juxta* 11. *defin.*

Poſtulatum.

Poſtuletur, quamlibet magnitudinem toties poſſe multiplicari, donec quamlibet magnitudinem ejusdem generis excedat.

Axiomata.

1. Magnitudo quotcunque magnitudines metiens, compoſitam quoque ex ipſis metitur.

2. Magnitudo quamcunque magnitudinem metiens, metitur quoque omnem magnitudinem quam illa metitur.

3. Magnitudo metiens totam magnitudinem & ablatam, metitur & reliquam.

PROP. I.

Duabus magnitudinibus inæqualibus A B, C propoſitis, ſi à majore AB auferatur majus quam dimidium (AH) & ab eo (HB) quod reliquum eſt, rurſus detrahatur majus quam dimidium (HI,) & hoc ſemper fiat; relinquetur tandem quædam magnitudo I B, quæ minor erit propoſita minore magnitudine C.

a poſ. 1. *a* Accipe C toties, donec ejus multiplex DE proxime excedat A B; ſintque D F = F G = G E = C. Deme ex A B plusquam dimidium A H, A H, & à reliquo H B plusquam dimidium H I; & ſic deinceps, donec partes A H, H I, I B æque multæ ſint partibus D F, F G, G E. Jam liquet F E, quæ non minor eſt quam $\frac{1}{2}$ D E, majorem eſſe quam H B, quæ minor eſt quam $\frac{1}{2}$ A B $\sqsupset$ D E. Pariterque G E quæ non minor eſt quam $\frac{1}{2}$ F E, major eſt quam I B $\sqsupset$ $\frac{1}{2}$ H B. ergo C, vel G E $\sqsubset$ I B. Q.E.D.

Idem demonſtrabitur, ſi ex A B auferatur dimidium A H, & ex reliquo H B rurſus dimidium H I, & ita deinceps.

PROP. II.

Si duabus magnitudinibus inæqualibus propoſitis (A B, C D) detrahatur ſemper minor A B de majore C D, alterna quadam detractione, & reliqua minime præcedentem metiatur; incommenſurabiles erunt ipſæ magnitudines.

Si fieri poteſt, ſit aliqua E communis menſura. Quoniam igitur A B detracta ex C D, quoties fieri poteſt, relinquit aliquam F D ſe minorem, & F D ex A B relinquit G B, & ſic deinceps, *a* tandem relinquetur aliqua G B $\sqsupset$ E. ergo E *b* metiens A B, *c* ideoque C F, *b* & totam C D; *d* etiam reliquam F D, metitur. *c* proinde & A G; *d* ergo & reliquam G B, ſeipſa minorem. Q.E.A.

a 1. 10.
b hyp.
c 2. ax. 10.
d 3. ax. 10.

PROP.

PROP, III.

Duabus magnitudinibus commenſurabilibus datis, A B, C D, maximam earum communem menſuram F B reperire.

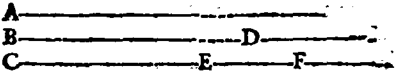
Deme A B ex C D, & reliquum E D ex A B, & F B ex E D, donec F B metiatur E D; (quod tandem
a 2.10. fiet, *a* quia per Hyp. A B ⊤ C D) erit F B quæſita.

b *conſtr.* Nam F B *b* metitur E D, *c* ideoque
c 2.*ax.*10 ipſam A F; ſed & ſeipſam, *d* er-
d 1.*ax.*10 go etiam A B, & *c* propterea C E, *d* adeoque & totam C D. Proinde F B communis eſt menſura ipſarum A B, C D. Dic G communem quoque eſſe menſuram, hac majorem;
e 2.*ax.*10 ergo G metiens A B, & C D, *e* metitur C E, &
f 3.*ax.*10. *f* reliquam E D, *e* ideoque A F, & *f* proinde reliquam F B, major minorem. Q.E.A.

Corell.

Hinc, magnitudo metiens duas magnitudines, metitur & maximam earum menſuram communem.

PROP. IV.



Tribus magnitudinibus commenſurabilibus datis A, B, C; maximam earum menſuram communem invenire.

a 3.10. *a* Inveni D maximam communem menſuram duarum quarumcunque A, B; *a* item E ipſa-

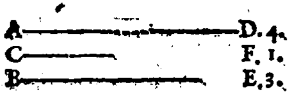
ipſarum D & C maximam communem menſuram; erit E quæſita.

a Nam perſpicuum eſt E metiens D & C *b* metiri tres A, B, C. Puta aliam F hac majorem eaſdem metiri. *c* ergo F metitur D; *c* proinde & E, ipſorum D, C maximam communem menſuram, major minorem. Q.E.A. b conſtr. & 2. ax. 10. c cor. 3. 10.

Coroll.

Hinc quoque, magnitudo metiens tres magnitudines, metitur quoque maximam earum communem menſuram.

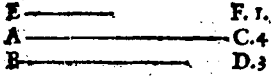
PROP. V.



Commenſurabiles magnitudines A, B inter ſe rationem habent, quam numerus ad numerum.

a Inventa C ipſarum A, B maxima communi menſura; quoties C in A & B, toties 1 contineatur in numeris D & E. *b* ergo C. A :: 1. D; quare inverſe A. C :: D. 1. *b* atqui etiam C. B :: 1. E. *c* ergo ex æquali A. B :: D. E :: N. Q.E.D. a 3. 10. b 20. def. 7. c 22. 5.

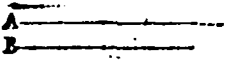
PROP. VI.



Si duæ magnitudines A, B inter ſe proportionem habeant, quam numerus C ad numerum D; commenſurabiles erunt magnitudines A, B.

a ſch.10.6 Qualis pars eſt 1 numeri C, *a* talis fiat E i-
b conſtr. *c hyp.* pſius A. Quoniam igitur E. A *b* :: 1. C. atque
d 22.5. A. B *c* :: C. D; *d* ex æquo erit E. B :: 1. D. er-
e 5.ax.7. go quum 1 *e* metiatur numerum D, *f* etiam E
f 20.def.7 *g conſtr.* metitur B; ſed & ipſum A *g* metitur. *h* ergo A
h 1.def. 10. ⊏ B. Q.E.D

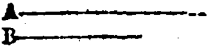
PROP. VII.



Incommenſurabiles magnitudines, A, B inter ſe proportionem non habent, quam numerus ad numerum.

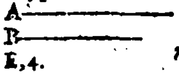
a 6.10. Dic A. B :: N. N. *a* ergo A ⊏ B, contra Hypoth.

PROP. VIII.



Si duæ magnitudines A, B inter ſe proportionem non habeant, quam numerus ad numerum, incommenſurabiles erunt magnitudines.

a 5.10. Puta A ⊏ B *a* ergo A. B :: N. N, contra Hypoth.



Quæ à rectis lineis longitudine commenſurabilibus fiunt quadrata, inter ſe proportionem habent, quam quadratus numerus ad quadratum numerum: & quadrata inter ſe proportionem habentia, quam quadratus numerus ad quadratum numerum, & latera habebunt longitudine commenſurabilia. Quæ vero à rectis lineis longitudine incommenſurabilibus fiunt quadrata, inter ſe proportionem non habent, quam quadratus numerus ad quadratum numerum:

rum : & quadrata inter ſe proportionem non habentia, quam quadratus numerus ad quadratum numerum, neque latera habebunt longitudine commenſurabilia.

1. *Hyp.* A. ▭ B. Dico Aq. Bq :: Q.Q.

Nam *a* ſit A. B :: num. E. num. F. ergo a per 5. 10.
$\frac{Aq}{Bq}$ (*b* $\frac{A}{B}$ bis) *c* = $\frac{E}{F}$ bis. *d* = $\frac{Eq}{Eq}$ *e* ergo Aq Bq :: b 20.6. c ſch. 23. 5.
E.q. Fq :: Q.Q. Q.E.D. d 11.8.

2. *Hyp.* Aq.Bq :: Eq. Fq :: Q. Q. Dico A ▭ e 11.5.
B. Nam $\frac{A}{B}$ bis (*f* $\frac{Aq}{Bq}$) *g* = $\frac{Eq}{Fq}$ *h* = $\frac{E}{F}$ bis. *i* ergo f 20.6. g hyp. h 11.8.
A. B :: E. F :: N. N *k* quare A ▭ B. Q.E.D. i ſch. 23. 5.

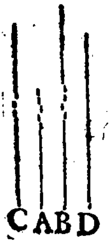
3. *Hyp.* A ▭̸ B. Nego eſſe Aq. Bq :: Q. Q. k 6.10.
Nam dic Aq. Bq :: Q.Q. Ergo A ▭ B, ut modo oſtenſum eſt, contra Hypoth.

4. *Hyp.* Non Aq. Bq :: Q. Q. Dico A ▭̸ B. Nam puta A ▭ B ; ergo Aq. Bq :: Q. Q. ut modo diximus, contra Hypoth.

Coroll.

Lineæ ▭ ſunt etiam ⊓; at non contra. Sed lineæ ▭̸ non ſunt idcirco ⊓̸. Lineæ vero ⊓̸ ſunt etiam ▭̸.

PROP. X.



Si quatuor magnitudines proportionales fuerint (C. A :: *B*.D ;) *prima vero* C *ſecunda A fuerit commenſurabilis ; & tertia B quartæ D commenſurabilis erit. Et ſi prima* C *ſecunda* A *fuerit incommenſurabilis, & tertia* B *quartæ D incommenſurabilis erit.*

Si

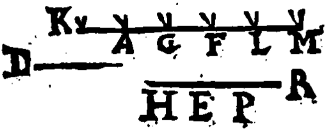
a 5.10. Si C ⊏⊐ A, *a* ideo erit C. A : : N. N *b* : : B.
b 6.10. D. *b* ergo B ⊏⊐ D. Sin C ⊏⊐ A, ergo *c* non erit
c 7.10. C. A : : N. N : : B. D. *d* quare B ⊏⊐ D. Q.E.D.
d 8.10.

LEMMA I.

Duos numeros planos invenire, qui proportionem non habeant, quam quadratus numerus ad quadratum numerum.

Hinc Lemmati ſatisfacient duo quilibet numeri plani non ſimiles, quales ſunt numeri habentes proportionem ſuperparticularem, vel ſuperbipartientem, vel duplam; vel etiam duo quivis numeri primi. vid. Schol. 27.8.

LEMMA 2.



Invenire lineam H R, ad quam data recta linea K M ſit in ratione datorum numerorum B, C.

a ſch.106. *a* Divide K M in partes æquales æque mul-
tas unitatibus numeri B, harum tot, quot unita-
tes ſunt in numero C *b* componant rectam H-
b 3.1. R. liquet eſſe KM. H R : : B. C.

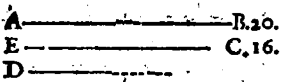
LEMMA 3.

Invenire lineam D, ad cujus quadratum data recta K M quadratum ſit in ratione datorum numerorum B, C.

a 2.lem. 10.10. Fac B. C *a* : : K M. H R ac inter K M, & H-
b 13.6. R *b* inveni mediam proportionalem D. Erit K-
c 20.6. Mq. Dq *c* : : K M. H R *d* : : B. C.
d conſtr.

PROP.

PROP. XI.

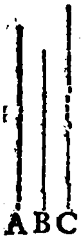


Propoſita recta linea A invenire duas rectas lineas incommenſurabiles; alteram quidem D longitudine tantum, alteram vero E etiam potentia.

1. Sume numeros B, C, *a* ita ut non ſit B. C :: Q.Q. *b* fiatque B. C :: Aq.Dq. *c* liquet A ⊏⊐ D. Sed Aq *a* ⊏⊐ Dq. Q.E.F. a 2. lem. 10. b 3 lem. 10. c 9.10.

2. *d* Fac A. E :: E. D. Dico Aq ⊏⊐ Eq. Nam A. D *e* :: Aq. Eq. ergo cum A ⊏⊐ D, ut prius, *f* erit Aq ⊏⊐ Eq. Q.E.F. d 6.10. e 20.6. f 10.10.

PROP. XII.



Quæ (A, B) eidem magnitudini C ſunt commenſurabiles, & inter ſe ſunt commenſurabiles.

Quia A ⊏⊐ C, & C ⊏⊐ B, *a* ſit A. C :: N. N :: D. E. atque C. B :: N. N :: F. G. *b* ſumantur tres numeri H, I, K minimi ∺ in rationibus D ad E, & F ad G. Jam quia A. C *c* :: D. E, *c* :: H. I. ac C. B *c* :: F. G. *c* :: I. K. *d* erit ex æquali A.B :: H. K :: N.N. *e* ergo A ⊏⊐ B. Q.E.D. a 5.10. b 4.8. c conſtr. d 22.5. e 6.10.

D, 18. E, 8.
F, 2. G, 3.
H, 5. I, 4. K, 6.

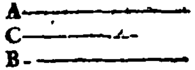
Schol.

Hinc, omnis recta linea rationali lineæ commenſurabilis, eſt quoque ρ rationalis. Et omnes rectę rationales inter ſe commenſurabiles ſunt, ſaltem potentia. Item, omne ſpatium rationali ſpatio commenſurabile, eſt quoque rationale; 12.10. & def. 6. def. 9.

& o-

& omnia ſpatia rationalia inter ſe commenſurabilia ſunt. Magnitudines vero, quarum altera eſt rationalis, altera irrationalis, ſunt inter ſe incommenſurabiles. def.7 & 10.

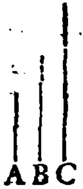
PROP. XIII.



Si ſint duæ magnitudines A, B; & altera quædam A eidem C ſit commenſurabilis, altera vero B incommenſurabilis; incommenſurabiles erunt magnitudines A, B.

a hyp. b 12.10 Dic B ⊏⊐ A. ergo cum C *a* ⊏⊐ A, *b* erit C ⊏⊐ B, contra Hypoth.

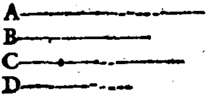
PROP. XIV.



Si ſint duæ magnitudines commenſurabiles A, B; altera autem ipſarum A magnitudini cuipiam C incommenſurabilis fuerit; & reliqua B eidem C incommenſurabilis erit.

a hyp. b 12.10. Puta B ⊏⊐ C. ergo cum A ⊏⊐ *a* B, *b* erit A ⊏⊐ C, contra Hyp.

PROP. XV.



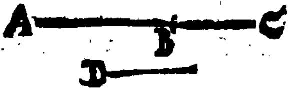
Si quatuor rectæ lineæ proportionales fuerint (A. B : : C. D;) *prima vero* A *tanto plus poſſit quam ſecunda* B, *quantum eſt quadratum rectæ lineæ ſibi commenſurabilis longitudine; & tertia* C *tanto plus poterit, quam quarta* D, *quantum eſt quadratum rectæ lineæ ſibi longitudine commenſurabilis. Quod ſi prima* A *tanto plus poſſit quam ſecunda* B, *quantum eſt quadratum rectæ lineæ ſibi incommenſurabilis longitudine; & tertia* C

tan-

tanto plus poterit, quam quarta D, *quantum eſt quadratum rectæ lineæ ſibi longitudine incommenſurabilis.*

Nam quia A. B *a* :: C. D. *b* erit Aq. Bq :: a hyp.
Cq. Dq. *c* ergo dividendo Aq -- Bq. Bq :: Cq -- b 22.6.
Dq. Dq. *d* quare √: Aq -- Bq. B :: √: Cq -- c 17.5.
Dq. D. *e* invertendo igitur B. √: Aq -- Bq :: d 22.6.
D. √: Cq -- Dq. *f* ergo ex æquali A. √: Aq -- e cor. 4.5
Bq :: C. √: Cq -- Dq. proinde ſi A ▭, vel ▭ f 22.5.
√: Aq -- Bq; *g* erit ſimiliter C ▭, vel ▭ √: g 10.10.
Cq -- Dq. Q.E.D.

PROP. XVI.



Si duæ magnitudines commenſurabiles AB, BC *componantur, & tota magnitudo* A C *utriq; ipſarum* A B, B C *commenſurabilis erit: quod ſi tota magnitudo* A C *uni ipſarum* A B. *vel* B C *commenſurabilis fuerit; & quæ à principio magnitudines* A B, B C *commenſurabiles erunt.*

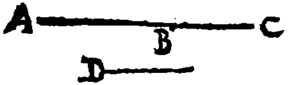
1. *Hyp. a* Sit D ipſarum A B, B C communis menſura. *b* ergo D metitur A C. *c* ergo A C ▭ A B, & B C. Q.E.D. a 3.10. b 1.ax. 10.

2. *Hyp. a* Sit D communis menſura ipſarum A C, A B; *d* ergo D metitur A C -- A B (B C;) *c* proinde A B ▭ B C. Q.E.D. c 1.def. 10. d 3.ax. 10.

Coroll.

Hinc etiam, ſi tota magnitudo ex duabus compoſita, commenſurabilis ſit alteri ipſarum, eadem & reliquæ commenſurabilis erit.

PROP. XVII.



Si duæ magnitudines incommenſurabiles AB, BC componantur, & tota magnitudo A C utrique ipſarum AB, BC incommenſurabilis erit: Quod ſi tota magnitudo A C uni ipſarum A B *incommenſurabilis fuerit, & quæ à principio magnitudines A* B, B *C incommenſurabiles erunt.*

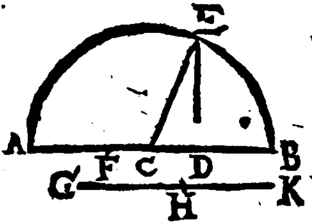
1. *Hyp*. Si fieri poteſt, ſit D ipſarum A C, A-
a 3.ax.10 B communis menſura. *a* ergo D metitur A-
b 1.def. 10. C — A B (B C.) *b* ergo A B ⊏⊐ B C, contra Hypoth.

c 16.10. 2. *Hyp*. Dic A B ⊏⊐ B C. *c* ergo A C ⊏⊐ A-*B*, contra Hypoth.

Coroll.

Hinc etiam, ſi tota magnitudo ex duabus compoſita, incommenſurabilis ſit alteri ipſarum, eadem & reliquę incommenſurabilis erit.

PROP. XVIII.



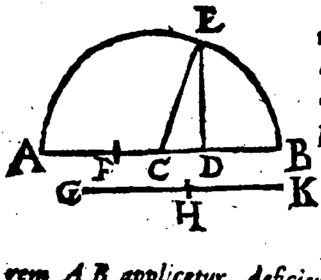
Si fuerint duæ rectæ lineæ inæquales A B, *GK; quartæ autem parti quadrati, quod ſit à minori GK, æquale parallelogrammum A D B ad majorem A*B *applicetur, deficiens figura quadrata, & in partes* A D, D B *longitudine commenſurabiles ipſam dividat; major* A B *tanto plus*

plus poterit quam minor G K, quantum eſt quadratum recta linea F D ſibi longitudine commenſurabilis. Quod ſi major A B tanto plus poſſit, quam minor G K, quantum eſt quadratum recta linea F D ſibi longitudine commenſurabilis; quarta autem parti quadrati, quod fit à minori G K, aquale parallelogrammum A D B ad majorem A B applicetur, deficiens figura quadrata, in partes A D, D B longitudine commenſurabiles ipſum dividet.

a Biſeca GK in H; & *b* fac rectang. ADB= GHq: abſcinde AF=DB. Eſtque ABq *c* = 4. ADB *d* (4 GHq, vel GKq) + FDq. Jam primo, Si A D ⊏⊐ D B, erit A B *e* ⊏⊐ D B *e* ⊏⊐ 2 DB *f* (A F + D B, vel A B -- F D) *g* ergo AB ⊏⊐ F D. Q.E.D. Si ſecundo, A B ⊏⊐ FD,, *h* erit ideo A B ⊏⊐ A B -- F D (2 D B) *k* ergo A B ⊏⊐ D B. *l* quare A D ⊏⊐ D B. Q.E.D.

a *10.1.*
b *28.6.*
c *1.2.*
d *conſtr.* & *4.2.*
e *16.10.*
f *conſtr.*
g *cor. 16. 10.*
h *cor. 16.10.*
k *12.10*
l *16.10.*

PROP. XIX.



Si fuerint dua recta linea inaquales, A B, G K; quarta autem parti quadrati, quod fit à minore G K, aquale parallelogrammum A D B ad majorem A B applicetur, deficiens figura quadrata; & in partes incommenſurabiles longitudine

P AD,

AD, DB, ipſam AB dividat; major AB tanto plus poterit, quam minor GK, quantum eſt quadratum rectæ lineæ FD, ſibi longitudine incommenſurabilis. Quod ſi major AB tanto plus poſſit, quam minor GK, quantum eſt quadratum rectæ lineæ FD ſibi longitudine incommenſurabilis; quartæ autem parti quadrati, quod fit à minore GK, æquale parallelogrammum ADB ad majorem AB applicetur, deficiens figura quadrata; in partes longitudine incommenſurabiles AD, DB ipſam AB dividet.

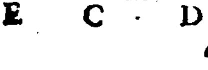
Facta puta, & dicta eadem, quæ in præcedenti. Itaque primò, Si AD ⊏ DB, *a* erit propterea AB ⊏ DB: *b* quare AB ⊏ 2 DB (AB — FD) *c* ergo AB ⊏ FD. Q.E.D.

a 17.10.
b 13.10.
c cor. 17. 10.

Secundo, Si AB ⊏ FD; *c* ergo AB ⊏ AB — FD (2 DB:) *d* quare AB ⊏ DB, & *e* proinde AD ⊏ DB. Q.E.D.

d 13.10.
e 17.10.

PROP. XX.



Quod ſub rationalibus longitudine commenſurabilibus rectis lineis BC, CD, ſecundum aliquem prædictorum modorum, continetur rectangulum BD, rationale eſt.

a 46.1.
b 1.6.
c hyp.
d 10.10.

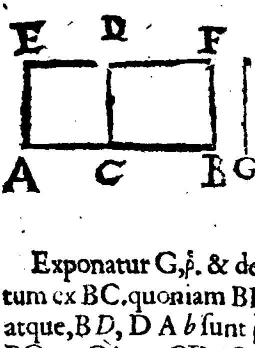
Exponatur A, $\dot{\rho}$. & *a* deſcribatur BE quadratum ex BC. Quoniam DC. CE (BC) *b* :: BD. BE. & DC *c* ⊏ BC; *d* erit rectang. BD

BD ⊏ quad. BE. ergo quum quad. BE *e* ⊏ Aq; *f* erit BD ⊏ Aq. proinde rectang. BD est ρ̇. Q.E.D. e hyp 9.def.10 f 12.10.

Not. *Tria ſunt genera linearum rationalium inter ſe commenſurabilium. Aut enim duarum linearum rationalium longitudine inter ſe commenſurabilium altera æqualis eſt expoſitæ rationali; aut neutra rationali expoſitæ æqualis eſt, longitudine tamen ei utraque eſt commenſurabilis; aut denique utraque expoſitæ rationali commenſurabilis eſt ſolum potentia. Hi ſunt modi illi, quos innuit præſens theorema.*

In numeris, ſit BC, $\surd 8$ ($2\surd 2$) & CD, $\surd 18$ ($3\surd 2$,) erit rectang. BD $= \surd 1.4 = 12$.

PROP. XXI.



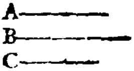
Si rationale DB ad rationalem DC applicetur, latitudinem CB efficit rationalem, & ei DC ad quam applicatum eſt DB, longitudine commenſurabilem.

Exponatur G, ρ̇. & deſcribatur DA quadratum ex BC, quoniam BD. DA *a* :: BC. CA; atque, BD, DA *b* ſunt ρ̇α, *c* ideoque ⊏; *d* erit BG ⊏ CA. at CD (CA) *b* eſt ρ̇. *e* ergo BC eſt ρ̇. Q.E.D. a 1.6. b hyp. c ſch.12.10 d 10.10 e ſch.12.10.

In numeris, ſit rectang. DB, 12; & DC, $\surd 8$. erit CB, $\surd 18$. atqui $\surd 18 = 3\surd 2$, & $\surd 8 = 2 \times \surd 2$.

LEM-

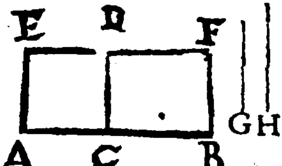
LEMMA



Duas rectas rationales potentia ſolum commenſurabiles invenire.

a 11.10. Sit A expoſita ρ̇. a Sume B ⊤ A, a & C ⊤
b ſch.12. 10. B. b liquet B, & C eſſe quæſitas.

PROP. XXII.



Quod ſub rationalibus D C, C B potentia ſolum commenſurabilibus rectis lineis continetur rectangulum D B, irrationale eſt; & recta linea H ipſum potens, irrationalis; vocetur autem Media.

Sit G expoſita ρ̇. & deſcribatur D A quadratum ex D C; ſitque Hq = DB. Quoniam
a 1.6. AC. CB a :: D A. D B. b atque A C ⊏ CB,
b hyp. c erit D A ⊏ D B (Hq.) d atqui Gq ⊏ D-
c 10.10. A. e ergo Hq ⊏ Gq. f ergo H eſt ρ̇. Q.
d hyp & 9.def.10. E. D. vocetur autem Media. quia A C. H ::
e 13.10. H. C B.
f 11.10.

In numeris, ſit D C, 3; & C B, √6. erit rectangulum D B (Hq) √54. quare H eſt • √54.

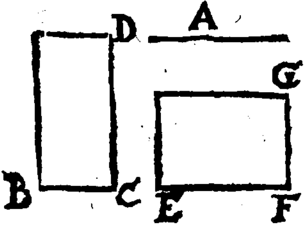
Mediæ nota eſt μ, Medii vero μν; pluraliter μα

Schol.

Omne rectangulum, quod poteſt contineri ſub duabus rectis rationalibus potentia ſolum commenſurabilibus, eſt Medium; quamvis con-

contineatur ſub duabus rectis irrationalibus: atq; omne Medium poteſt contineri ſub duabus rectis rationalibus potentia tantum commenſurabilibus, ut exempli grat. √24 eſt μν, quia continetur ſub √3, & √8, qui ſunt ρ ╤. etſi poſſet contineri ſub $^{u}\surd$6, & $^{u}\surd$96 irrationalibus; nam √94 = $^{u}\surd$576 = $^{u}\surd$6 in $^{u}\surd$96.

PROP. XXIII.



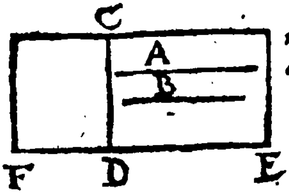
Quod (BD) à media A fit, ad rationalem BC applicatum, latitudinem CD rationalem efficit, & ei BC, ad quam applicatum eſt, BD longitudine incommenſurabilem.

Quoniam A eſt μ, *a* erit Aq rectangulo alicui (EG) æquale contento ſub EF, & FG ρ ╤. *b* ergo BD = EG. *c* quare BC. EF :: FG. CD. *d* ergo BCq. EFq :: FGq. CDq. ſed BCq, & EFq *e* ſunt ρα, *f* ideoque ⊏. *g* ergo FGq ⊏ CDq. Ergo quum FG ſit ρ, *h* erit CD ρ. Porro, quia EF. FG *k* :: EFq. EG (BD;) ob EF ⊏ FG, *l* erit EFq ⊏ BD. verum EFq *m* ⊏ CDq. *n* ergo rectang BD ⊏ CDq. quum igitur CDq. BD *o* :: CD. BC. *p* erit CD ⊏ BC. ergo, &c.

a ſch. 21. 10.
b 1. ax. 1.
c 14. 6.
d 22. 6.
e hyp.
f ſch. 12. 10.
g 10. 10.
h ſch. 12. 10.
k 1. 6.
l 10. 10.
m ſch. 12. 10.
n 13. 10.
o 1. 6.
p 10. 10.

P 3 PROP.

PROP. XXIV.



Media A commenſurabilis B, media eſt.

a 11.6. Ad CD ρ̇ *a* fac rectang. CE = Aq; *a* & rectang. CF =

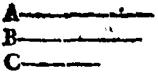
b hyp. Bq. Quoniam Aq (CE) eſt μν, *b* & CD ρ̇, *c* erit
c 23.10. latitudo DE ρ̇ ⊐̩. CD. Quoniam vero CE. CF
d 1,6. *d* : : ED. DF, & CE *e* ⊐ CF, *f* erit ED ⊐
e hyp. DF. *g* ergo DF eſt ρ̇ ⊐ CD. *h* ergo rectang.
f 10.10. CF (Bq) eſt μν & proinde B eſt μ. Q.E.D.
g 12.& 13.10.
h 22.10.

Nota *quod ſignum* ⊐̩ *plerumque valet potentia tantum commenſurabile, ut in hac demonſtratione, & in præced. &c. quod intellige, ut ex uſu erit, & juxta citationes.*

Coroll.

Hinc liquet ſpatium medio ſpatio commenſurabile medium eſſe.

LEMMA



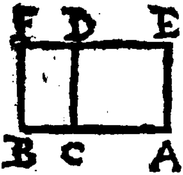
Duas rectas medias A, B *longitudine commenſurabiles; item duas* A, C *potentia tantum commenſurabiles invenire.*

a lem. 22. 10. & 13.6. *a* Sit A μ quævis; ſume B ⊐ A; *c* & C ⊐̩ A. *d* Factum eſſe liquet.

b 1. lem. 10.10.
c 3 lem.
d conſtr.
& 24.10.

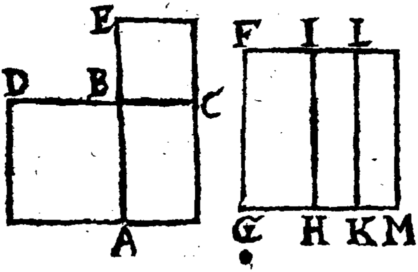
PROP.

PROP. XXV.



Quod ſub D C, C B mediis longitudine commenſurabilibus rectis lineis continetur rectangulum D B, medium eſt.

Super D C conſtruatur quadratum D A. Quoniam A C (D C) C B *a* :: D A. D B. & D C ⊏⊐ C B; *b* erit D A ⊏⊐ D B. *c* ergo D B eſt μν. Q.E.D. a 1.6. b 10.10 c 24.10



Quod ſub mediis potentia tantum commenſurabilibus rectis lineis A B, B C continetur rectangulum A C, vel rationale eſt, vel medium.

Super rectas A B, B C *a* deſcribe quadrata A D, C E. atque ad F G ρ̇, *b* fac rectangula F H = A D, *b* & I K = A C, *a b* & L M = C E. a 46.1 b cor.16.6.

Quadrata A D, C E, hoc eſt, rectangula F H, L M *c* ſunt μα, & ⊏⊐; ergo eandem habentes rationem G H, K M ſunt *d* ρ̇, & *e* ⊏⊐, *f* ergo G H c hyp.& 24.10. d 23.10 e 10.10 f 20.10.

GH$\times$KM eſt ρ̇ν. atqui quia AD, AC, CE, hoc
g ſch.22.6 eſt FH, IK, LM *g* ſunt ∺; & *h* proinde GH,
h 1.6. HK, KM etiam ∺, *k* erit HKq = GH$\times$KM;
k 17.6. *l* ergo HK eſt ρ̇, vel ⊏⊐, vel ⊏⊥⊐ IH (GF); ſi ⊏⊐,
l 12.10. m 20.10. *m* ergo rectang. IK vel AC eſt ρ̇ν. Sin ⊏⊥⊐ *n* ergo AC eſt μν. Q.E.D.
n 22.10.

LEMMA.

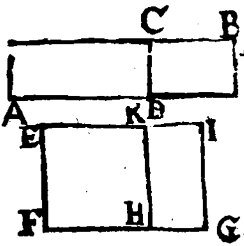
Si A, & E ſint ⊏⊥⊐ tantum;

a hyp. & 16.10. *Erunt primo,* Aq, Eq, Aq + Eq, Aq − Eq *a* ⊏⊐.

Erunt ſecundo, Aq, Eq, Aq + Eq, Aq − Eq
b 1.6. ⊏⊐ AE, & 2 AE. Nam A. E *b* :: Aq. AE *b* ::
c hyp. AE. Eq. ergo cum A *c* ⊏⊐ E. *d* erit Aq ⊏⊐ AE,
d 10.10. e 14.10. *e* & 2 AE. item Eq. *d* ⊏⊐ AE, *e* & 2 AE. *c* quare cum Aq + Eq ⊏⊐ Aq, & Eq; & Aq − Eq
f 14.10. ⊏⊐ Aq & Eq, *f* erunt Aq + Eq, *f* & Aq − Eq ⊏⊐ AE, & 2 AE.

Hinc erunt tertio, Aq, Eq, Aq + Eq, Aq − Eq,
g 14.16. & 17.10. 2 AE *g* ⊏⊐ Aq + Eq + 2 AE; & Aq + Eq − 2 AE. *g* & Aq + Eq + 2 AE ⊏⊐ Aq + Eq −
h cor. 7. 20. 2. AE. *h* (Q. A − E.)

PROP. XXVII.



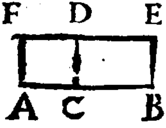
Medium AB non ſuperat medium AC rationali DB.

a cor 16.6 Ad EF ρ̇, *a* fac EG = AB, *a* & EH = AC. Rectangula AB, AC, hoc eſt, EG, EH *b*
b hyp. ſunt

ſunt $\mu\alpha$, *c* ergo FG. & FH ſunt $\dot{\rho}$ $\sqsubset\!\!\!\top$ E F. itaque ſi KG, *d* id eſt DB ſi $\dot{\rho}\nu$. *e* erit HG $\sqsubset$ HK; *f* quare HG $\sqsubset\!\!\!\top$ FH. *g* ergo FGq $\sqsubset\!\!\!\top$ FHq. ſed FH eſt $\dot{\rho}$. *h* ergo FG eſt $\dot{\rho}$. verum prius erat FG $\dot{\rho}$. Quæ repugnant.

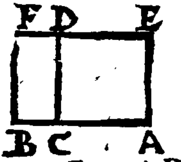
c 23.10.
d 5.ax1.
e 21.10.
f 13.10.
g lem.16.10.
h ſch.12.10.

SCHOL.



1. *Rationale A E ſuperat rationale A D rationali C E.*

Nam AE *a* $\sqsubset$ AD; *b* ergo AE $\sqsubset$ CE. *c* quare CE eſt $\dot{\rho}\nu$. Q.E.D.



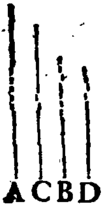
2. *Rationale AD cum rationali C F facit rationale A F.*

Nam AD *a* $\sqsubset$ CF; *b* quare AF $\sqsubset$ AD, & CF. *c* proinde AF eſt $\dot{\rho}\nu$. Q.E.D.

a hyp.
b cor.16.10.
c ſch.12.10.

a ſch.12.10.
b 16.10.
c ſch.12.10.

PROP. XXVIII.



Medias invenire (C, & D) quæ rationale C D contineant.

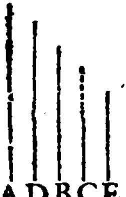
a Sume A, & B $\dot{\rho}$ $\sqsubset\!\!\!\top$. *b* fac A. C :: C.B. *c* atque A.B :: C.D. Dico factum. Nam A B (Cq) *d* eſt $\mu\nu$; *d* unde C eſt μ. quum vero A.B *e* :: C. D, *f* erit C $\sqsubset\!\!\!\top$ D. *g* ergo D eſt μ. porró permutando A. C :: B.D. *e* hoc eſt C. B :: B. D. *h* ergo Bq = C D. atqui Bq *c* eſt $\dot{\rho}\nu$. *h* ergo CD eſt $\dot{\rho}\nu$. Q.E.F.

a lem.1.10.
b 13.6.
c 12.6.
d 22.10.
e conſt.
f 10.10.
g 24.10.
h 17.6.
k ſch.12.10.

In numeris, ſit A, $\surd 2$; & B, $\surd 6$. ergo C eſt $\nu\surd 12$. fac $\surd 2 . \surd 6 :: \nu\surd 12$. D. vel $\nu\surd 4 . \nu\surd 36 :: \nu\surd 12$. D. erit D, $\nu\surd 108$.

atqui u√12 in u√108 = u√1296 = √36 = 6. ergo C D eſt 6. item C. D :: 1. √3. quare C ⊐ D.

PROP. XXIX.



Medias invenire potentia tantum commenſurabiles D, & E, quæ medium D E contineant.

a lem.21.10. a Sume A,B,C ρ̇ ⊐. Fac A. D b :: D.B. c & B.C :: D E. Dico factum.

b 13.6. c 11.6. d 17.6. e 22.10. f conſtr. Nam A B d = Dq & A B e eſt μν; ergo D eſt μ. & B f ⊐ C. g

g 10.10. ergo D ⊐ E. h ergo E eſt μ. porro, B.C f ::

h 24.10. D. E, & permutando B. D :: C. E. k hoc eſt

k conſtr. & cor.4.5 D.A :: C.E. l ergo D E = A C. Sed AC m eſt

l 16.6. m 22.6. μν. ergo D E eſt μν. Q.E.D.

In numeris ſit A,20; & B,√200; & C,√80. Ergo D eſt √√80000; & E u√12800. Ergo DE = √√1024000000 = √32000. & D. E :: √10. 2. quare D ⊐ E.

Schol.

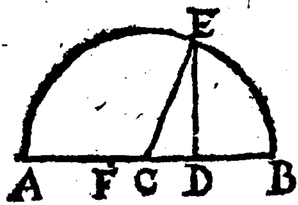
A, 6. C, 12.
$\underline{B, 4.}$ $\underline{D, 8.}$
$\overline{AB}$-24. $\overline{C}\,\overline{D}$96.

A, 6. C, 5.
$\underline{B, 4.}$ $\underline{D, 8.}$
$\overline{AB}$,24. $\overline{CD}$,40.

Invenire duos numeros planos ſimiles & diſſimiles.

Sume quoſcunque quatuor numeros proportionales, A. B :: C. D. liquet AB, & CD eſſe ſimiles planos. Planos autem diſſimiles quotcunque reperies ope ſcholii 27.8.

LEMMA.



1. *Duos numeros quadratos DEq & CDq) invenire, ita ut compoſitus ex ipſis (CEq) quadratus etiam ſit.*

Sume AD, DB numeros planos ſimiles (quorum ambo pares ſint, vel ambo impares) nimirum AD, 24. & DB, 6. Horum ſumma (AB) eſt 30; differentia (FD) 18, cujus ſemiſſis (CD) eſt 9. *a* Habent vero plani ſimiles A *D*, DB unum medium numerum proportionalem, nempe DE. patet igitur ſingulos numeros CE, CD, DE rationales eſſe; proinde CEq (*b* CDq + DEq) eſt numerus quadratus requiſitus. a 18.8. b 47.1.

Facile itaque invenientur duo numeri quadrati, quorum exceſſus ſit quadratus, vel non quadratus numerus. nempe ex eadem conſtructione, *c* erit CEq − C*D*q = DEq. c 3.ax.1.

Quod ſi AD, DB ſint numeri plani diſſimiles, non erit media proportionalis (DE) numerus rationalis; proinde quadratorum CEq, CDq exceſſus (DEq) non erit numerus quadratus.

LEMMA 2.

2. *Duos numeros quadratos B, C invenire, ita ut compoſitus ex ipſis D, non ſit quadratus. item, quadratum numerum A dividere in duos numeros B, C non quadratos.*

A, 3. B, 9. C, 36. D, 45.

1. Sume numerum quemlibet quadratum B, ſitque $C = 4B$; & $D = B + C$. Dico factum.

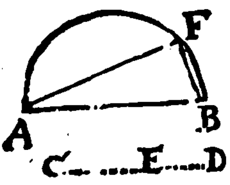
Nam B eſt Q. ex conſtr. item quia B. C : : 1.
a 24. 8. 4 : : Q. Q. *a* erit C etiam quadratus. Sed quo-
b cor. 24. 8. niam $B + C$. (D) C : : 5. 4 : : non Q. Q. *b* non erit D numerus quadratus. Q.E.F.

A, 36. B, 24. C, 12. D, 3. E, 2. F, 1.

2. Sit A numerus quivis quadratus. Accipe D, E, F numeros planos diſſimiles, ſitque $D = E + F$. fac D.E : : A.B. & D.F : : A. C. Dico factum.

Nam quia D. $E + F$: : A. $B + C$: & $D = E$
a 14. 5. $+ F$, *a* erit $A = B + C$. Jam dic B quadratum
b 21. def. 7. eſſe. *b* ergo A & B, & *c* proinde D & E, ſunt
c 26. 8. numeri plani ſimiles, contra Hypoth. idem abſurdum ſequetur, ſi C dicatur quadratus. ergo, &c.

PROP. XXX.



Invenire duas rationales AB, A F potentia tantum commenſurabiles, ita ut major AB plus poſſit, quam minor A F, quadrato recta linea BF longitudine ſibi commenſurabilis.

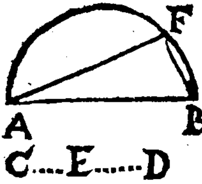
a 1. lem. 29. 10. Exponatur AB, ρ. *a* Sume C D, CE numeros quadratos, ita ut $CD - CE$ (ED) ſit non
b 3. lem. 29. 10. Q. *b* Fiatque CD. ED : : ABq. AFq. in circulo ſuper

ſuper A B diametrum deſcripto *c* aptetur A F, c 1.4.
ducaturque BF. Sunt AB, AF, quas petis.

Nam ABq. AFq *d* :: C D. ED. *e* ergo ABq d conſtr.
⊏ AFq. verum AB eſt $\dot{\rho}$. *f* ergo AF eſt $\dot{\rho}$. ſed e 6.10.
quia C D eſt Q: at E D non Q: *g* erit A B ⊏ f ſch.12.10.
AF. porro, ob ang. *h* rectum A F B, eſt ABq *k* g 9.10.
= AFq + BFq; cum igitur ABq. AFq :: C D. h 31.3.
E D. per converſionem rationis erit ABq. BFq k 47.1.
:: CD. CE :: Q.Q. *l* ergo AB ⊏ BF. Q.E.F. l 9.10.

In numeris; ſit AB, 6; CD, 9, CE, 4; quare ED, 5. Fac 9. 5 :: 36. (Q: 6) AFq. erit AFq 20. proinde AF √ 20. ergo BFq = 36 − 20 = 16. quare BF eſt 4.

PROP. XXXI.



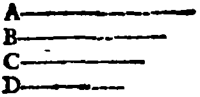
Invenire duas rationales AB, AF potentia tantum commenſurabiles, ita ut major AB plus poſſit, quam minor AF, quadrato rectæ lineæ BF ſibi longitudine incommenſurabiles.

Exponatur AB, $\dot{\rho}$. *a* accipe numeros CE, E- a 1. lem.
D quadratos, ita ut C D = C E + ED ſit non 29.10.
Q. & in reliquis imitare conſtitutionem præcedentis. Dico factum.

Nam, ut ibi, AB, AF ſunt $\dot{\rho}$ ⊏. item ABq. B-
Fq :: CD. E D. ergo cum C D ſit non Q. *b* e- b 9.10.
runt AB, BF ⊏. Q.E.F.

In numeris, ſit AB, 5. CD, 45. CE = 36; ED = 9. Fac 45. 9 :: 25 (ABq.) 5 (AFq.) ergo AF = √ 5. proinde BFq = 45 − 25 = 20. quare BF = √ 20.

PROR. XXXII.

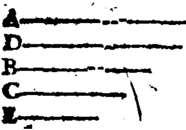


Invenire duas medias C, D potentia tantum commenſurabiles, quæ rationale CD contineant, ita ut major C plus poſſit, quam minor D, quadrato rectæ lineæ ſibi longitudine commenſurabilis.

a 30.10. *a* Accipe A, & B ρ̇ ⊤; ita ut √ Aq − Bq ⊐
b 13.6. A. *b* Fiatque A. C :: C. B, *c* atque A. B :: C.
c 11.6. D. *D*ico factum.
d conſtr.
e 22.10. Nam quia A, & *d* B ſunt ρ̇ ⊤, *e* erit C (*f* √
f 17.6. AB) μ. item *g* ideo C ⊤ D. *h* ergo D etiam
g 10.10.
h 24.10. μ. porro quia A. B *d* :: C. D; & permutatim A. C :: B D :: C. B; & Bq *d* eſt ρ̇ν, erit C D
k 17.6. *k* (Bq) ρ̇ν. Denique quia √ Aq − Bq *d*
l 15.10. ⊐ A, *l* erit √ Cq − Dq ⊐ C. ergo, &c. Sin √ Aq − Bq ⊐ Aq, erit √ Cq − *D*q ⊐ C.

In numeris, ſit A, 8; B, √ 48 (√: 64 − 16) ergo C = √ AB = υ√ 3072. & D = υ√ 1728. quare CD = υ√ 5308416 = √ 2304.

PROP. XXXIII.



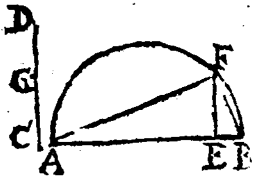
Invenire duas medias D, E potentia ſolum commenſurabiles, quæ medium D E contineant, ita ut major D plus poſſit quam minor E, quadrato rectæ lineæ ſibi longitudine commenſurabilis.

a Sume A, & C ρ̇, ⊓; ita ut √ Aq — Cq ⊏ A. *b* ſume etiam B ⊓ A, & C; & fac A. D a 30.10.
c : : D. B *d* : : C. E. Erunt D, & E quæſitæ. b lem.28 10.

Nam quoniam A, & C *e* ſunt ρ̇, *e* & B ⊓ A & C, *f* erit B ρ̇, & D (√ AB) *g* erit μ. *e* Quia vero A. D : : C. E. erit permutando A. C : : D. E. ergo cum A ⊓ C, *h* erit D ⊓ E. *k* ergo E eſt μ. porro, *l* quia D. B : : C. E; *l* & B C eſt μν, etiam *D* E ei *m* æquale eſt μν. deniq; propter A. C : : *D*. E. *e* quia √ Aq — Cq ⊏ A, *n* erit √ *D*q — Eq ⊏ *D*. ergo, &c. Sin √ Aq — Cq ⊏̸ A, erit √ *D*q — Eq ⊏̸ Eq. c 19.6. d 12.6. e conſtr. f ſch.12.10. g 22.10. h 10.10 k 24.10 l 22.10. m 16.6. n 15.10

In numeris, ſit A, 8; C, √ 48; B, √ 28. erit *D* υ √ 3072; & E υ √ 588. quare *D*. E : : 2. √ 3. & *D* E = √ 1344.

PROP. XXXIV.



Invenire duas rectas lineas A F, B F potentia incommenſurabiles, quæ faciant compoſitum quidem ex ipſarum quadratis rationale, rectangulum vero ſub ipſis contentum, medium.

a Re-

a 32.10. *a* Reperiantur A B, C*D* ρ' ⊓; ita ut √ A-
b 10.1. Bq − C*D*q ⊓ AB. *b* biſeca CD in G. *c* fac re-
c 28.6. ctang. AEB = GCq. Super A B diametrum duc ſemicirculum A F B. Erige perpendicularem E F. duc A F, BF. Hæ ſunt quæ indagandæ erant

Nam A E. BE *d* :: B A x A E. A B x B E.
d 1.6. Sed B A x A E *e* = A F q; *e* & A B x B E = F-
e cor 8.6 & 17.6. Bq *f* ergo A E. E B :: A F q. F B q. ergo cum
f 7.5. A E. *g* ⊏ E B, *h* erit A Fq ⊏ FBq. Quin et-
g 19.10. h 10.10. iam ABq (*k* A F q + F B q) *l* eſt ρ'ν. denique
k 31.3. & 47.1. EFq *l* = A E B *l* = CGq. *m* ergo E F = C G.
l conſtr. ergo C *D* x A B = 2 EFxAB. atqui C*D*xAB *n*
m 1 ax 1. n 22.10. eſt μν. *o* ergo ABxEF, *p* vel AFxFB eſt μν.
o 24.10. p ſch. 28. 6. Q.E.*D*.

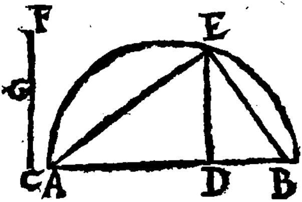
Explicatio per numeros.

Sit AB, 6. C*D*, √ 12. quare C G = √ $\frac{12}{4}$ = √ 3. Eſt vero A E = 3 + √ 6. & E*B* = 3 − √ 6. & unde *A* F erit √ : 18. + 216. Et F *B*, √ : 18 − √ 216. item *A*Fq + F *B*q eſt 36, & *A* F x F *B* = √ 108.

Cæterum *A* E invenitur ſic. Quia *B* *A* (6) *A* F :: *A* F. *A* E; erit 6 *A* E = *A* F q = *A*Eq + 3 (EFq.) ergo 6 *A* E − *A*Eq = 3. pone 3 + e = A E. ergo 18 + 6 e − 9 − 6 e − ee, hoc eſt 9 − ee = 3. vel ee = 6. quare e = √ 6. proinde *A* E = 3 + √ 6.

PROP.

PROP. XXXV.



Invenire duas rectas lineas A E, E B potentia incommenſurabiles, quæ faciant compoſitum quidem ex ipſarum quadratis medium, rectangulum vero ſub ipſis contentum, rationale. a 32.10.

a Sume A B, & C F μ ⊐⊤, ita ut A B x C F ſit $\dot{\rho}\nu$, atque $\surd$ ABq − CFq ⊏⊐ A B. & reliqua fiant, ut in præcedenti. erunt A E, E B, quas petis.

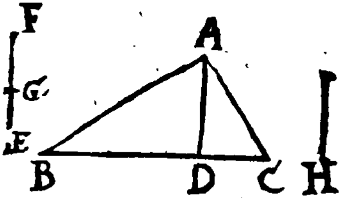
Nam, ut iſthic oſtenſum eſt, AEq ⊏⊐ EBq: item ABq (AEq + EBq) eſt $\mu\nu$. & denique A B x C F b eſt $\dot{\rho}\nu$, idcirco & A B x D E, d hoc eſt, A E x E B, eſt $\dot{\rho}\nu$, ergo, &c. b conſtr. c ſch.12.10. d ſchol. 22.6.

PROP. XXXVI.

Invenire duas rectas lineas B A, A C potentia incommenſurabiles, quæ faciant & compoſitum ex ipſarum quadratis medium, & rectangulum ſub ipſis comprehenſum medium, incommenſurabileq; compoſito ex ipſarum quadratis.

a 33.10. *a* Accipe B C & E F μ ⊤; ita ut BCxEF ſit μν. & √ Bcq − EFq ⊏ BC. & reliqua fiant, ut in præcedentibus. Erunt BA, AC exoptata. Nam, ut prius, BAq ⊏ ACq; item BAq + b conſtr. ACq eſt μν. & BAxAC eſt μν. Denique BC *b* c 13.10. d 1.6 ⊏ EF, atque *c* ideo B C ⊏ E G; eſtque B C: E G *d* : : BCq. BCxEG, (BCxAD, vel BAx e 14.10. AC.) *e* ergo BCq (ABq + ACq) ⊏ BAxAC. ergo, &c.

Schol.



Invenire duas medias longitudine & potentia incommenſurabiles.

a 36.10. b 13.6. *a* Sume B C μ. ſitque BAxAC μν, & ⊏ BCq (BAq + ACq.) *b* Fac BA. H : : H. A C. Sunt BC, & H μ ⊤. Nam BC eſt μ. *a* & BA c 17.6. xAC (*c* Hq) eſt μν. quare H eſt etiam μ. *d* i- d 14.10. tem B A x A C ⊏ B Cq; ergo Hq ⊏ B Cq. ergo, &c.

Principium ſenariorum per compoſitionem.

PROP. XXXVII.

Si duæ rationales AB, BC potentia tantum commenſurabiles com-

componantur, tota A C irrationalis eſt; vocetur autem ex binis nominibus.

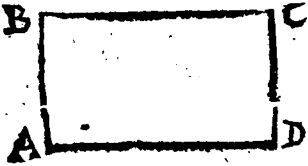
Nam quia AB *a*, ▭ BC, *b* erit ACq ▭ ABq. a hyp. b lem.
Sed AB *a* eſt ρ̇, *c* ergo AC eſt ῥ. Q.E.D. 26.10. c 11.def. 10.

PROP. XXXVIII.

Si duæ mediæ A B, B C potentia tantum commenſurabiles componantur, quæ rationale contineant, tota A C irrationalis eſt; vocetur autem ex binis mediis prima.

Nam quoniam AB *a* ▭ BC, *b* erit ACq ▭ a hyp. b lem.
ABxBC, ρ̇ν. *c* ergo AC eſt ῥ. Q.E.D. 26.10. c 11. def. 10.

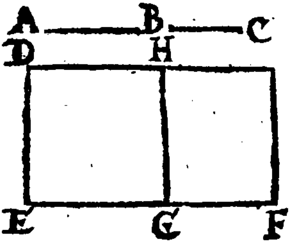
LEMMA.



Quod ſub linea rationali AB, & irrationali BC continetur rectangulum AC, irrationale eſt.

Nam ſi rectang. A C dicatur ρ̇ν; quum A B a hyp.
ſit ρ̇; *b* erit latitudo BC etiam ρ̇. contra Hyp. b 21.10.

PROP. XXXIX.



Si duæ mediæ AB, BC potentia tantum commenſurabiles componantur, quæ medium contineant, tota A C irrationalis erit; vocetur autem ex binis mediis ſecunda.

a cor. 26. 6 Ad expoſitam D E ρ̇ *a* fac rectangul. DF =
b 47.1. & 11.6. ACq; *b* & DG = ABq + BCq.
c hyp. Quoniam ABq *c* ⊏ BCq, *d* erit ABq +
d 16.10. BCq, hoc eſt D G. ⊏ ABq ; ſed ABq *e* eſt μν.
e 24.10. *e* ergo DG eſt μν. verum rectang. ABC poni-
f 4.2. tur μν; *e* ideoque 2 ABC (*f* H F) eſt μν; *g* er-
g 23.10. go E G, & G F ſunt ρ̇. quia vero D G *h* ⊏̸
h lem. 26. 10. H F; atque D G. H F :: *e* E G. G F *l* erit E G
k 1.6. ⊏ G F. *m* ergo tota E F eſt ρ̇. *n* quare rectang.
l 10.10. D F eſt ρ̇ν. *o* ergo √ D F, id eſt A C, eſt ρ̇.
m 37.10 Q.E.D.
n lem. 38. 10.
o 11. def. 10.

PROP. XL.

Si duæ rectæ lineæ AB, BC potentia tantum commenſurabiles componantur, quæ faciant compoſitum quidem ex ipſarum quadratis rationale, quod autem ſub ipſis continetur, medium; tota recta linea A C, irrationalis erit: vocetur autem major.

a hyp. Nam quia ABq + BCq *a* eſt ρ̇ν, & *b* ⊏ 2
b sch. 12. 10. ABC *c* μν, & proinde ACq (*d* A Bq + B Cq
c hyp. & 24.10. + 2 ABC) *e* ⊏ ABq + BCq ρ̇ν, *f* erit A C ρ̇.
d 4.2. Q.E.D.
e 17.10.
f 11. def. 10:

PROP. XLI.

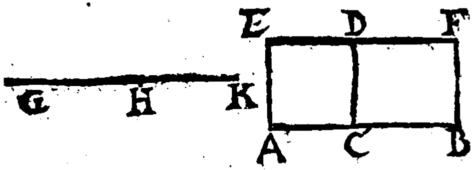
Si duæ rectæ lineæ AC, CB potentia incommenſurabiles componantur, quæ faciant compoſitum quidem ex ipſarum quadratis medium, quod autem ſub ipſis con-

continetur, rationale; tota recta linea A B irrationalis erit: vocetur autem rationale ac medium potens.

Nam 2 rectang. A C B, *a* $\dot{\rho}\nu$ *b* ⊏⊐ A C q + C Bq *c* $\mu\nu$. *d* ergo 2 A C B *d* ⊏⊐ ABq. quare *e* AB eſt $\dot{\rho}$. Q.E.D.

a *hyp.* & *ſch.* 12. 10.
b *ſch.* 12. 10
c *hyp.*
d 17.10.
e 11. *def.* 10.

PROR. XLII.

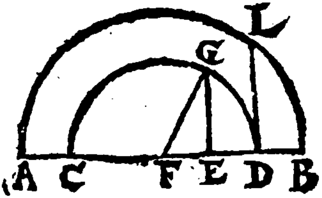


Si duæ rectæ lineæ G H, H K potentia incommenſurabiles componantur, quæ faciant & compoſitum ex ipſarum quadratis medium, & quod ſub ipſis continetur medium, incommenſurabileque compoſito ex quadratis ipſarum; tota recta linea G K irrationalis erit: vocetur autem bina media potens,

Ad expoſitam F B $\dot{\rho}$, fiant rectang. A F = G K q, & C F = G Hq + H K q. Quoniam GHq + H K q (CF) *a* eſt $\mu\nu$; latitudo C B *b* erit $\dot{\rho}$. Item quia 2. rectang. G H K (*c* A *D*) *a* eſt $\mu\nu$, etiam A C *b* erit $\dot{\rho}$. Porro quia rectangul. A D *a* ⊏⊐ C F, *d* atque A D. C F :: A C. C B, *e* erit A C ⊏⊐ C B. *f* Quare A B eſt *g* $\dot{\rho}$. ergo rectang. A F, id eſt, GKq eſt $\dot{\rho}\nu$. *h* proinde G K eſt $\dot{\rho}$. Q.E.*D*.

a *hyp.*
b 23.10
c 4.2.
d 2.6.
e 10,10.
f 37.10.
g *lem.* 38.10.
h 11. *def.* 10.

PROP. XLIII.



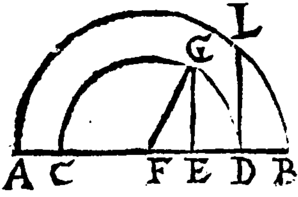
Quæ ex binis nominibus AB, ad unum duntaxat punctum D dividitur in nomina AD, DB.

Si fieri poteſt, binomiūm AB alibi in E ſecetur in alia nomina AE, EB. Liquet AB ſecari utrobique inæqualiter, quia AD ⊏ DB, & AE ⊏ EB.

a 37.10. b ſch. 27. 10. Quoniam rectangula ADB, AEB *a* ſunt μα; *a* & ſingula ADq, DBq, AEq, EBq ſunt ῥα; *b* adeoque ADq + DBq, *b* & AEq + EBq etiam ῥα, *b* idcirco ADq + DBq —: AEq + EBq.

c ſch. 5. 2. d ſch. 12. 10. e 27.10. *c* hoc eſt, 2 AEB — 2 ADB eſt ῥν. *d* ergo AEB — ADB ῥν. ergo μν ſuperat μν per ῥν. *e* Q.E.A.

PROP. XLIV.



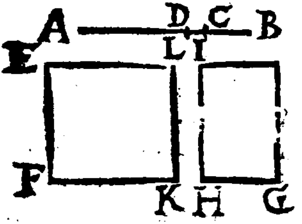
Quæ ex binis medius prima AB, ad unū duntaxat punctum D dividitur in nomina AD, DB.

Puta AB dividi in alia nomina AE, EB, quo poſito, ſingula ADq,

DBq,

DBq,EBq, *a* ſunt μα; *a* & rectangula A D B, a 38. 10
AEB,eorumque dupla,ſunt ρ'α. *b* ergo 2 AEB. b ſch. 27.
— 2 ADB, *c* hoc eſt ADq + DBq —: AEq + 10.
EBq eſt ρ'ν. *d* Q.E.A. c ſch. 5. 2 d 27. 10

PROR. XLV.



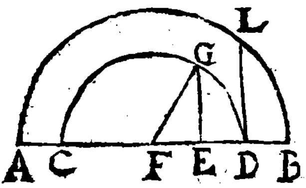
Quæ ex binis medius ſecunda A B, ad unum duntaxat punctū C dividitur in nomina AC,CB.

Dic alia eſſe nomina A *D*,DB.

Ad expoſitam EF ρ', fac rectang. E G = ABq. & E H = ACq + CBq; item EK = ADq + DBq.

Quoniam ACq, CBq *a* ſunt μα ▭; *b* erit a 39. 10
ACq + CBq (E H) μν. *c* ergo latitudo F H b 16. & 24. 10.
eſt ρ'. *a* quin & rectang. A C B, *d* ideoque 2 c 23. 10
ACB *e* (IG) eſt μν: *c* ergo HG, eſt etiam ρ'. d 24. 10.
Cum igitur E H *f* ▭ I G, *g* atque E H. I G : : e 4. 2. f lem. 26. 10.
F H. H G; *h* erunt FH,HG ▭. *k* ergo FG eſt g 1. 6.
binomium; cujus nomina F H, H G. Simili ar- h 10. 10.
gumento F G eſt bin. cujus nomina F K, K G, k 37. 10.
contra 43. hujus.

PROP. XLVI.

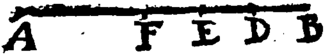


Ma-

Major A B ad unum duntaxat punctum D dividitur in nomina A D, DB.

Concipe alia nomina A E, E B. quo poſito
a 40.10. rectangula A D B, AE B *a* μα; *a* & tam A Dq
b ſch.27. 10. + DBq, quam AEq + EBq + ſunt ρ̀α. *b* er-
c ſch.5.2 go ADq + DBq −: AEq + EBq, *c* hoc eſt, 2
d 27.10. AEB − 2 ADB eſt ρ̀ν. *d* Q.F.N.

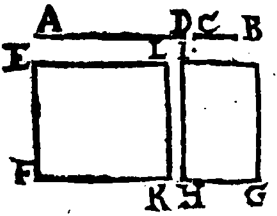
PROP. XLVII.



Rationale ac medium potens A B, ad unum duntaxat punctum D dividitur in nomina A D, D B.

a 41.10. Dic alia nomina A E, EB. *a* ergo tam AEq
+ EBq, quam ADq + DBq ſunt μα. *a* & re-
b ſch.27. 10. ctangula AEB, ADB, ſunt ρ̀α. *b* ergo 2 AEB.
c ſch.5.2. − 2 ADB. *c* hoc eſt, ADq + DBq −: AEq +
d 27.10. EBq eſt ρ̀ν. Q.E.A.

PROP. XLVIII.



Bina media potens A B, ad unum duntaxat punctum C dividitur in nomina AC, CB.

Vis AB dividi in alia nomina AD, DB. Ad expoſitam E F ρ̀, fiant rectang. E G = A B q, & EH = ACq

ACq + CBq, & EK = ADq + DBq. Quoniam ACq + CBq, nempe EH, *a* eſt μν, *b* erit latitudo FH $\dot{\rho}$. Item quia 2 ACB, *c* hoc eſt, IG, eſt *a* μν, *b* erit HG etiam $\dot{\rho}$. Ergo cum EH *a* ⊏⊐ IG, ſitque EH. IG *d* :: FH. HG, *e* erit FH ⊏⊐ HG. *f* ergo FG eſt bin. cujus nomina FH. HG. Eodem modo ejusdem nomina erunt FK, KG; contra 43. hujus.

a 42. 10.
b 23. 10.
c 4. 2.
d 1. 6.
e 10. 10.
f 37. 10.

Definitiones ſecundæ.

EXpoſita rationali, & quæ ex binis nominibus, diviſa in nomina; cujus majus nomen plus poſſit quam minus, quadrato rectæ lineæ ſibi longitudine commenſurabilis;

I. Siquidem majus nomen expoſitæ rationali commenſurabile ſit longitudine, vocetur tota ex binis nominibus prima.

II. Si vero minus nomen expoſitę rationali longitudine ſit commenſurabile, vocetur ex binis nominibus ſecunda.

III. Quod ſi neutrum ipſorum nominum ſit longitudine commenſurabile expoſitæ rationali, vocetur ex binis nominibus tertia.

Rurſus ſi majus nomen plus poſſit quam minus, quadrato rectę lineæ ſibi longitudine incommenſurabilis;

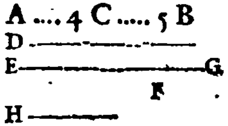
IV. Si quidem majus nomen expoſitæ rationali commenſurabile ſit longitudine, vocetur ex binis nominibus quarta.

V. Si vero minus nomen, vocetur quinta.

VI. Quod ſi neutrum ipſorum nominum, vocetur ſexta.

PROP.

PROP. XLIX.



Invenire ex binis nominibus primam, EG.

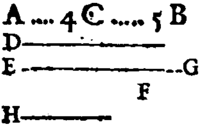
a ſch.29.10. *a* Sume A B, A C numeros quadratos, quorum exceſſus C.B non
b 2.lem 10.10. Q. exponatur D ρ̊. *b* accipe quamvis E F ▭ D.
c 3.lem. 10.10. *c* fac AB.CB :: EFq. FGq. erit EG bin. 1.

d conſtr. Nam E F *d* ▭ D. *e* ergo E F ρ̊. *f* item EFq
e 6.def. 10. ▭ F Gq. *g* ergo F G eſt etiam ρ̊. item *d* quia
f 6.10. EFq. FGq :: AB. CB :: Q. non Q. *h* erit E F
g ſch.12.10. ▭ F G. denique quia per converſionem ratio-
h 9.10. nis EFq. EFq − FGq :: AB. AC :: Q.Q. *k* erit
k 9.10. EF ▭ √ EFq − FGq. *l* ergo E G eſt bin. 1.
l 1.def. 48.10. Q.E.F.

Explicatio per numeros.

Sit D,8. EF,6. AB,9. CB,5. quare cum 9. 5 :: 36. 20. erit FG, √ 20. proinde EG eſt 6 + √ 20.

PROP. L.



Invenire ex binis nominibus ſecundam, EG.

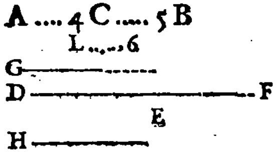
Probant præcedentem. Accipe AB; & AC numeros quadratos, quorum exceſſus C B ſit non Q. Sit D expoſita ρ̊. ſume F G ▭ D. Fac C B. AB :: FGq. EFq. Erit E G quæſita.

Nam F G ▭ D, quare F G eſt ρ̊. item EFq ▭ FGq. ergo EF eſt etiam ρ̊. item quia FGq. EFq

EFq : : CB. AB : : non Q. Q. eſt FG ⊏⊐ EF. denique quia CB.AB : : FGq.EFq, inverſeque AB. CB : : EFq.FGq, erit ut in præcedenti, EF ⊏⊐ √ EFq -- FGq. *a* è quibus EG eſt bin. 2. Q.E.F. a 2.*def.* 48.10.

In numeris, ſit D,8 ; FG 10 ; AB,9 ; CB, 5. erit EF, √ 180. quare EG eſt 10 + 180.

PROP. LI.



Invenire ex binis nominibus tertiam, DF.

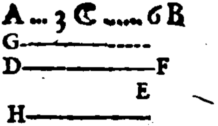
a Sume numeros AB, AC quadratos, quorum exceſſus CB non Q. Sitque L numerus non Q, proximè major quam CB, nempe unitate, vel binario. ſit G expoſita ῥ. *b* Fac L. AB : : Gq. DEq. *b* & AB. CB : : DEq. EFq. erit DF bin. 3. a *ſch.*29.10. b 3.*lem.* 10.10.

Nam quia DEq *c* ⊏⊐ Gq, *d* eſt DE ῥ. item Gq. DEq : : L. AB : : non Q. Q. *e* ergo G ⊏⊐ DE. item quia DEq *e* ⊏⊐ EFq, *d* etiam EF eſt ῥ. quin etiam quia DEq. EFq : : AB. CB : : Q. non Q. *f* eſt DE ⊏⊐ EF. porro, quia per conſtr. & ex æquali Gq. EFq : : L. CB : : non Q. Q. (nam *g* L, & CB non ſunt ſimiles plani numeri) *h* erit G etiam ⊏⊐ EF. denique ut in præced. √ DEq -- EFq ⊏⊐ DE. *k* ergo DE eſt bin. 3. Q.E.F. c *conſt.* 6.10. d *ſch.*12.10. e 6.10. f 9.10. g *ſch.*27.8. h 9.10. k 3 *def.* 48.10.

In

In numeris, ſit A B, 9; C B, 5; L, 6; G, 8. erit D E, $\surd 96$. & E F, $\surd \frac{480}{9}$. quare D F = $\surd 96 + \surd \frac{480}{9}$.

PROP. LII.



Invenire ex binis nominibus quartam, D F.

a ſch. 29. 10. *a* Sume quemvis numerum quadratum A B, *a* quem divide in A C, C B non quadratos. ſit G expoſita ρ. *b* accipe

b 2. lem. 10 D E. ⊏⊐ G. *c* Fac A B. C B : : D E q. E F q. erit

c 3. lem. 10. 10. D F bin. 4.

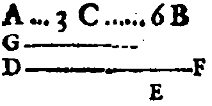
Nam ut in 49. hujus, D F oſtendetur bin. item, quia per conſtr. & converſionem rationis

d 9. 10. D E q. D E q − E F q : : A B. A C : : Q. non Q. *d*

e 4. def. 48. 10. erit D E ⊏⊐ $\surd$ D E q − E F q. *e* ergo D F eſt bin. 4. Q. E. F.

In numeris, ſit G, 8; D E, 6. erit E F $\surd 24$. ergo D F eſt $6 + \surd 24$.

PROP. LIII.



Invenire ex binis nominibus quintam, D F.

Accipe quemvis numerum quadratum A B, cujus ſegmenta A C, C B ſint non Q. ſit G expoſita ρ. ſume E F ⊏⊐ G. Fac C B. A B : : E F q. D E q. erit D F bin. 5.

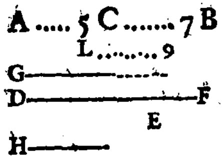
Nam ut in 50. hujus, erit D F bin. & quia per conſtr.

conſtr. & invertendo DEq. EFq :: AB. CB. ideoque per converſionem rationis DEq. DEq − EFq :: AB. AC :: Q. non Q. *a* erit DE ⊏⊐ √ DEq − EFq. *b* ergo DF eſt bin. 4. Q.E.F.

a 9. 10.
b 5. *def.* 48. 10.

In numeris, ſit G, 7; EF, 6. erit DE √ 54. quare DF eſt 6 + √ 54.

PROP. LIV.



Invenire ex binis nominibus ſextam.

Accipe AC, CB primos numeros utcunque, ſic ut AC + CB (AB) ſit non Q. ſume etiam quemvis L num. Q. ſit G expoſ. ρ. *a* fiatque L. AB :: Gq. DEq. atque AB. CB :: DEq. EFq. erit DF. bin. 6.

a 3. *lem.* 50. 10.

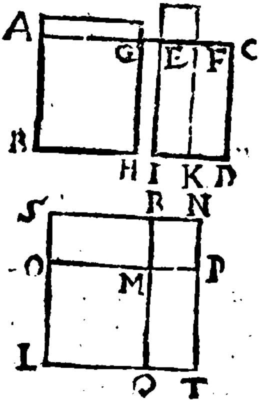
Nam ut in 51. hujus, DF oſtendetur bin. item quod DE, & EF ⊏⊐ G. denique igitur quia per conſtr. & converſionem rationis DEq. DEq − EFq :: AB. AC :: non Q. Q. (Nam AB primus eſt ad AC, *b* ideoque ei diſſimilis) *c* ergo DE ⊏⊐ √ DEq − EFq. *d* ergo DF eſt bin. 6. Q.E.F.

b *ſch.* 27. 8.
c 9. 10.
d 6. *def.* 48. 10.

In numeris, ſit G, 6; DE √ 48. erit EE √ 28. quare DF eſt √ 48 + √ 28.

LEM-

LEMMA.



Sit A D rectangulum, cujus latus A C ſecetur inæqualiter in E ; biſectumq; ſit ſegmentum minus EC in
a 28,6. *F ; atque ad A E, a fiat*
b 31.1. *rectang. AGE=EFq; perque G, E, F* b *ducantur ad A B paral-*
c 14.2. *lela G H, E I, F K. c Fiat autem quadratum L M = rectang. A H, atque ad O M P productam* c *fiat quadratum M N = G I; rectaq; LOS, LQT, NRS, NPT producantur.*

Dico 1. MS, MT *ſunt rectangula.* Nam ob quadratorum angulos OMQ, RMP rectos, *a*
a ſch.15.1 erit QMR recta linea. *b* ergo anguli R M O,
b 13.1. QMP recti ſunt. quare pgra MS, MT ſunt re-ctangula.

2. *Hinc patet L S e = L T; & proinde L N*
c 2.ax 2 *eſſe quadratum.*

3. *Rectangula SM, MT, EK, FD æqualia ſunt.* Nam quia rectang. AGE *d* = E Fq, *e* erit
d hyp. AE.EF :: EF. G E. *f* ideoque AH.EK :: E K.
e 17.6. GI. hoc eſt per conſtr LM. EK :: EK. M N. *g*
f 1.6. verum LM.SM :: SM. MN. ergo EK *h* = SM *k*
g ſch.22.6
h 9.5.
k 36.1. = FD *l* = MT.
l 23.1.

m 16.10. 4 *Hinc* LN *m* = AD.

5. *Quia* EC *biſecta eſt in* F, *n patet* E F, FC, EC ⊏ eſſe.

6. Si AE $\underset{|}{\sqsubset}$ EC, & AE ⊏ $\surd$ AEq − ECq,

o erunt AG, GE, AE ⊏⊐. item, quia AG. GE :: AH.GI *p* erunt AH, GI; hoc eſt LM, MN ⊏⊐. item iisdem poſitis. *o* 18. & 16. 10. *p* 10. 10.

7. OM ⊏⊐ MP. Nam per Hyp. AE, ⊏⊐ EC, *q* ergo EC ⊏⊐ GE. *q* quare EF ⊏⊐ GE. ſed EF. GE :: EK.GI. *r* ergo EK ⊏⊐ GI, hoc eſt SM ⊏⊐ MN. atqui SM.MN :: OM. MP. *r* ergo OM ⊏⊐ MP. q 14. 10. r 10. 10.

8. Sin ponatur AE ⊏⊐ √AEq − ECq, *ſ* patet AG, GE, AE eſſe ⊏⊐; unde LM ⊏⊐ MN. nam AG.GE :: AH.GI :: LM.MN. ſ 19. & 17.

His bene perſpectis, facile ſex ſequentes Propoſitiones expediemus.

PROP. LV.

Si ſpatium AD *contineatur ſub rationali* AB, *& ex binis nominibus prima* AC, (AE + EC;) *recta linea* OP *ſpatium potens irrationalis eſt, quæ ex binis nominibus appellatur.*

Suppoſitis iis, quæ in lemmate proxime præcedenti deſcripta, & demonſtrata ſunt, liquet rectam OP poſſe ſpatium AD. *a* item AG, GE, AE ſunt ⊏⊐. ergo cum AE *b* ſit ρ́ ⊏⊐ AB. *c* erunt AG, & GE, ρ́ ⊏⊐ AB. *d* ergo rectangula AH, GI, hoc eſt quadrata LM, MN ſunt ρ́α. ergo OM, MP ſunt ρ́ *e* ⊏⊐̸. *f* proinde OP eſt bin. Q.E.*D*. a hyp. & lem. 54. 10. b hyp. c ſch 12. 10. d 20. 10. e lem. 54. 10. f 37. 10.

In numeris, ſit AB, 5; AC, 4 + √12. quare rectang. AD = 20 + √300 = quadr. LN. ergo OP eſt √15 + √5; nempe bin. 6.

PROP. LVI.

Si ſpatium AD *contineatur ſub rationali* AB, *& ex binis nominibus ſecunda* AC (AE + EC;) *recta linea* OP *ſpatiū* AD *potens, irrationalis eſt, quæ ex binis mediis prima appellatur.* Rur-

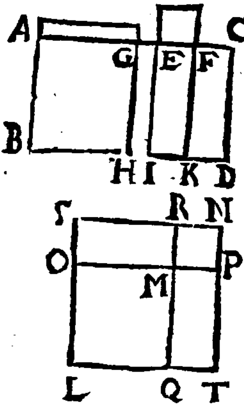
Rurſus adhibito lemmate ad 54. hujus, erit OP = √ A D. *a* item A E, AG, GE ſunt ▭. ergo quum AE *b* ſit ρ̇, ▭̸ AB, *c* erunt AG, GE etiam ρ̇ ▭ AB. ergo rectangula A H, G I; hoc eſt OMq, MPq *d* ſunt μα. *e* quinetiam OM ▭̸ MP. denique E F ▭ E C, & EC *f* ▭ AB. *g* quare EF eſt ρ̇ ▭ AB. *g* ergo EK; hoc eſt S M, vel O M P eſt ṗr. *h* Proinde O P eſt 2 μ prima. Q.E.D.

a hyp. & lem. 54. 10.
b hyp.
c ſch. 12. 10.
d 22. 10.
e lem. 54. 10.
f hyp. 12. 10.
g 20. 10.
h 38. 10.

In numeris, ſit AB, 5; & A C, √ 48 : + 6 ergo rectang. A D = √: 1200 + 30 = OPq. ergo OP eſt υ√ 675 + υ√ 75; nempe bimed. 1.

Vide ſchem. 57.

PROP. LVII.



Si ſpatium A D contineatur ſub rationali A B, & ex binis nominibus tertia A C (A E + E C;) recta linea OP ſpatium A D potens, irrationalis eſt, quæ ex binis mediis ſecunda dicitur.

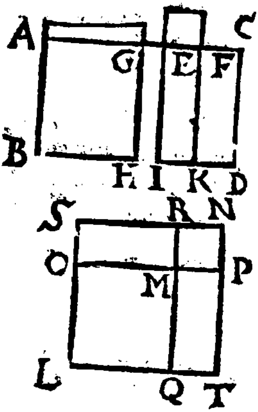
Ut prius, OPq = A D. item rectangula AH, GI, hoc eſt OMP MPq ſunt μα. *a* item EK, vel OMP eſt μṙ: *b* ergo O P eſt bimed. 2.

a hyp. & 22. 10.
b 39. 10.

In

In numeris, ſit AB, 5; AC, $\sqrt{32} + \sqrt{34}$ quare AD eſt $\sqrt{800} + \sqrt{600}$ = OPq. proinde OP eſt $u\sqrt{450} + u\sqrt{50}$; hoc eſt bimed. 2.

PROP. LVIII.



Si ſpatium AD contineatur ſub rationali AB, & ex binis nominibus quarta AC (AE + EC;) recta linea OP ſpatium potens, irrationalis eſt, quæ vocatur major.

Nam iterum, OMq *a* ▭ MPQ. rectang. vero AI, hoc eſt OMq + MPq *b* eſt ρ̇ν. *c* item EK, vel OMP eſt μν. *d* ergo OP ($\sqrt{}$ AD) eſt major. Q. E. D.

a lem. 54. 10.
b hyp. & 20. 10.
c hyp. & 22. 10.
d 40. 10.

In numeris, ſit AB 5; & AC, + 8. ergo rectang. AD eſt $20 + \sqrt{200}$. quare OP eſt $\sqrt{}$: $20 + \sqrt{200}$.

PROP. LIX.

Si ſpatium AD contineatur ſub rationali AB, & ex binis nominibus quinta AC; recta linea OP ſpatium AD potens, irrationalis eſt, quæ rationale & medium potens appellatur.

Rurſus OMP ▭ MPq. rectang. vero AI, vel OMq + MPq eſt μν. *a* item rectang. EK, vel OMP eſt ρ̇ν. *b* ergo OP ($\sqrt{}$ AD) eſt potens ρ̇ν, & μν. Q.E.D.

a ut in præced.
b 41. 10

In numeris, ſit AB, 5; & AC $2 + \sqrt{8}$. erg
rectang. AD = $10 + \sqrt{200}$ = OPq. quare O
eſt $\sqrt{}: 10 \mp \sqrt{200}$.

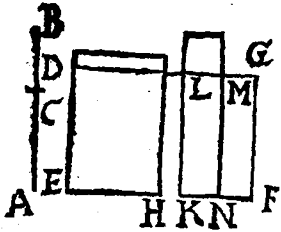
PROP. LX.

Si ſpatium AD contineatur ſub rationali AB,
& ex binis nominibus ſexta BC (AE + EC;)
recta linea OP ſpatium AD potens, irrationalis
eſt, quæ bina media potens appellatur.

Ut ſæpe prius, OMq ⊏ MPq. & OMq +
MPq eſt μν. & rectang. (EK) OMP etiam μν.
a 42.10. *a* ergo OP = $\sqrt{}$ AD eſt potens 2 μα. Q.E.D.

In numeris, ſit AB, 5; A C, $\sqrt{12} + \sqrt{8}$;
ergo rectang. A D, vel OPq eſt $\sqrt{300} + \sqrt{200}$. proinde OP eſt $\sqrt{}: \sqrt{300} + \sqrt{200}$.

LEMMA.



Sit recta AB inæqualiter ſecta in C, ſitq; AC majus ſegmentum; & cuivis DE applicentur rectangula, DF = ABq, & DH = ACq, & IK = CBq. ſitque LG biſecta in M, ducaturq; MN parall. GF.

a 43.1 & 3. ax. 1. Dico 1. *Rectang.* ACB = LN, vel MF. *a* Nam 2 ACB = LF.

b 7.2. 2. DL ⊏− LG. nam DK (ACq + CBq) *b* ⊏− LF (2 ACB) ergo cum DK, LF ſint æque
c 1.6. alta, *c* erit DL ⊏− LG.

3. Si

3. Si AC ⊤ CB, *d* erit rectang. DK ⊓ A- d 16. 10
Cq, & CBq.

4. *Item*, DL ⊓̀ LG. nam ACq + CBq *e* ⊓̀
2 ACB: hoc eſt DK ⊓̀ LF. ſed DK. LF *e* :: e lem. 26 10
DL. LG. *f* ergo DL ⊓̀ LG. f 10. 10.

5. *Ad hæc*, DL ⊓ √ DLq − LGq. Nam
ACq. ACB *g* :: ACB. CBq. hoc eſt DH. LN g 1. 6.
:: LN. IK. *c* quare DI. LM :: LM. IL. *h* ergo h 17. 6.
DI×IL = LMq. ergo cum ACq *k* ⊓ CBq. hoc k hyp.
eſt DH ⊓ IK, & *l* proinde DI ⊓ IL, *m* erit l 10. 10.
DL ⊓ √ DLq − LGq. Q.E.D. m 18. 10

6. *Sin ponatur* ACq ⊓̀ CBq, *n erit* DL ⊓̀ n 19. 10.
√ DLq − LGq.

Hoc lemma præparationis vicem ſubeat pro 6 ſequentibus propoſitionibus.

PROP. LXI.

Quadratum ejus quæ ex binis nominibus (AC + CB) *ad rationalem DE applicatum, facit latitudinem DG ex binis nominibus primam.*

Suppoſitis iis, quæ in lemmate proxime antecedenti deſcripta & demonſtrata ſunt. Quoniam AC, CB *a* ſunt ρ̇ ⊤, *b* erit rectang. DK a hyp.
⊓ ACq; *c* ergo DK eſt ρ̇ν. *d* ergo DL ⊓ DE b lem. 60. 10.
ρ̇. rectang. vero ACB, ideoque 2 ACB (LF) c ſch. 12. 10.
e eſt μν. *f* ergo latitudo LG eſt ρ̇ ⊓ DE. *g* d 21. 10.
ergo etiam DL ⊓̀ LG. *h* itèm DL ⊓ √ DLq e 22. &
− LGq. ex quibus *k* ſequitur DG eſſe bin. 1. & 34. 10
Q.E.D. f 23. 10.
g 13. 10.
h lem. 60. 10.
k 1. def. 48. 10.

PROP. LXII.

Quadratum ejus, quæ ex binis mediis prim
(A C + C B) *ad rationalem DE applicatun*
facit latitudinem D G ex binis nominibus secun
dam.

Rurſus adhibito lemmate proximè præce-
a 24.10. denti; Rectang. D K ⊏⊐ ACq. *a* ergo D K eſ
b 23.10. *μν*. *b* ergo latitudo DK eſt ρ̇ ⊏⊐ DE. Quia vero
c hyp. & ſch. 22. 10. rectang. A CB, ideoque LF (2 ACB) *c* eſt ρ̇ν,
d 21.10. erit LG ρ̇ ⊏⊐ DE. *e* ergo DL, LG ſunt ⊏⊐. *f*
e 13.10. tem D L ⊏⊐ √ DLq − LGq. *g* ex quibus pa
f lem. 60. 10. tet DG eſſe bin. 2. Q.E.D.
g 2. def. 48.10.

PROP. LXIII.

Quadratum ejus, quæ ex binis mediis secun
(AC + CB) *ad rationalem DE applicatum, f*
cit latitudinem D G ex binis nominibus tertian

Ut in præced. DL eſt ρ̇ ⊏⊐ D E. porro qu
a hyp. & 24.10. rectang. ACB. ideoque LF (2 ACB) *a* eſt μν
b 23.10. *b* erit LG ρ̇ ⊏⊐ D E. *c* quin etiam DL ⊏⊐ LG.
c lem. 60. 10. itemque DL ⊏⊐ √ DLq − LGq. *d* ergo DG
d 3. def. 48.10. eſt bin. 3. Q.E.D.

PROP. LXIV.

Quadratum Majoris (AC + CB) *ad ratio*
nalem DE applicatum, facit latitudinem DG e
binis nominibus quartam.

Rurſus ACq + CBq, hoc eſt DK *a* eſt ρ̇ν.
a hyp. & ſch. 22.10. ergo DL eſt ρ̇ ⊏⊐ DE. item ACB, ideoque L
b 21.10. (2 A C B) *c* eſt μν. *d* ergo LG eſt ρ̇ ⊏⊐ D E.
c hyp. & 24.10. proinde etiam D L ⊏⊐ L G. denique quia A C
d 23.10.
e 13.10.

⊏⊐ BC, *f* erit DL ⊏⊐ DLq — LGq. *g* unde DG. est bin. 4. Q.E.D. f lem. 60. 10. g 4. def. 48. 10.

PROP. LXV.

Quadratum ejus, quæ rationale ac medium poteſt, (AC + CB) ad rationalem DE applicatum, facit latitudinem DG ex binis nominibus quintam.

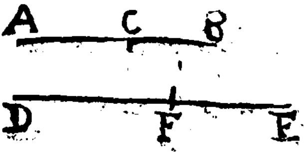
Iterum, DK eſt μν. *a* ergo DL eſt ρ̇ ⊏⊐ DE. item LF eſt ρ̇ν. *b* ergo LG eſt ρ ⊏⊐ DE. *c* ergo DL ⊏⊐ LG. *d* item DL ⊏⊐ √ DLq — LGq. *e* proinde DG eſt bin. 5. a 13. 10. b 22. 10. c 13. 10. d lem. 60. 10. e 5. def. 48. 10.

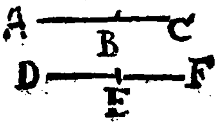
PROP. LXVI.

Quadratum ejus, quæ bina media poteſt (AC + CB) ad rationalem DE applicatum, facit latitudinem DG ex binis nominibus ſextam.

Ut prius, DL & LG ſunt ρ̇ ⊏⊐ DE. Quia vero ACq + CBq (DK) *a* ⊏⊐ ACB, *b* ideoque DK ⊏⊐ LF (2 ACB) eſtque DK. LF *c* :: DL. LG. *d* erit DL ⊏⊐ LG. *e* deniq; DL ⊏⊐ √ DLq — LGq. *f* ex quibus liquet DG eſſe bin. 6. Q.E.D. a hyp. b 14. 10. c 1. 6. d 10. 10. e lem. 60. 10. f 6. def. 48. 10.

LEMMA.





Sint AB, DE ⊏⊐; fiatq; AB. DE :: AC DF.

Dico 1. AC ⊏⊐ DF ut patet ex 10. 10. item CB ⊏⊐ FE. *a* quia AB. DE :: CB. FE. a 19. 5.

2. AC. CB :: DF. FE. Nam AC. DF :: AB. DE :: CB. FE. ergo permutando AC. CB :: DF. FE.

b 1.6. c prius. 3. *Rectang.* ACB ▭ DFE. Nam ACq. ACB *b* :: AC. CB *c* :: DF. EF :: DFq. DFE. quare permutando ACq. DFq :: ACB. DFE. ergo

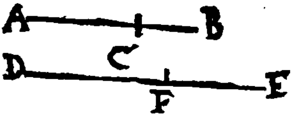
d 10.10. cum ACq ▭ DFq, *d* erit ACB ▭ DFE.

e 22.6. 4. ACq + CBq ▭ DFq + FEq. Nam quia ACq. CBq *e* :: DFq. FEq. erit componendo ACq + CBq. CBq :: DFq + FEq. FEq. ergo

f 10.10. cum CBq ▭ FEq, *f* erit ACq + CBq ▭ DFq + FEq.

g 10.10. 5. Hinc, ſi AC ⊤, vel ⊤ CB, *g* erit pariter DE ⊤, vel ⊤ EF.

PROP. LXVII.



Ei, quæ ex binis nominibus (AC + CB) *longitudine commenſurabilis* DE, *& ipſa ex binis nominibus eſt, atque ordine eadem.*

a lem.66.10. Fac AB. DE :: AC. DF. *a* ſunt AC, DF ▭; *a* & CB, FE ▭. quare cum AC, & CB *b* ſint ρ̇

b hyp. ⊤, *c* erunt DF, FE ρ̇ ⊤. ergo DE eſt etiam

c lem.66. 1?. & ſch.12.10. bin. Quia vero AC. CB *a* :: DF. FE. ſi AC ▭, vel ▭ √ ACq − BCq, *d* etiam ſimiliter DF ▭, vel ▭ √ DFq -- FEq. item ſi AC ▭,

d 15.10. e 13.10. & 14.10. vel ▭ ρ̇ expoſ *e* erit ſimiliter *D*F ▭, vel ▭ ρ̇ expoſ. at ſi CB ▭ vel ▭ ρ̇, *e* erit pariter FE ▭ vel ▭ ρ̇. Sin vero utraque AC. CB ▭ ρ̇, erit u-

g Per def. 41.10. traque etiam DF, FE ▭ ρ̇. *g* Hoc eſt, quodcunq; bino-

binomium fuerit AB, erit DE ejuſdem ordinis. Q.E.D.

PROP. LXVIII.

Ei, qua ex binis mediis (AC + CB) *longitudine commenſurabilis DE, & ipſa ex binis mediis eſt, atque ordine eadem.*

a Fiat AB.DE :: AC.DF. *b* ergo AC ⊐ DE, & CB ⊐ FE. ergo cum AC & CB *c* ſint μ, *d* etiam DF, & FE erunt μ. & cum AC *e* ⊤ CB, *e* erit FD ⊤ FE. *f* ergo DE eſt 2 μ. Si igitur rectang. ACB ſit ρ̇ν; quia DFE *b* ⊐ ACB, *g* etiam DFE eſt ρ̇ν; & ſi illud μν, *h* hoc etiam erit μν. *k* Id eſt, ſive AB ſit bimed. 1. ſive bimed. 2. erit DF ejusdem ordinis. Q.E.D.

a 12.64. b lem. 66.10. c hyp. d 24.10. e 10.10. f 38.10. g ſch. 12. 10. h 24.10. k 38. vel 30. 10.

PROP. LXIX.

Majori (AC + CB) *commenſurabilis DE, & ipſa Major eſt.*

Fac AB.DE :: AC.DF. Quoniam AC *a* ⊤ CB, *b* erit DF ⊤ FE. item ACq + CBq *a* eſt ρ̇ν; proinde cum DFq + FEq *b* ⊐ ACq + CBq, *c* etiam DFq + FEq eſt ρ̇ν. denique rectang. ACB *a* eſt μν *d* ergo rectang. DFE eſt μν (quia DFE *b* ⊐ ACB.) *e* Quare DE eſt major. Q.E.D.

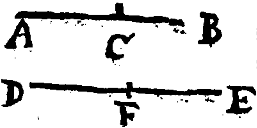
a hyp. b lem. 66.10. c ſch. 12.10. d 24.10. e 40.10.

PROP. LXX.

Rationale ac medium potenti (AC + CB) *commenſurabilis DE, & ipſa rationale ac medium potens eſt.*

a *hyp.* Iterum ſac AB. DE : : AC. DF. Quia AC *a*
b *lem.66.* *10.* ╤ CB, *b* etiam DF ╤ FE. item quia ACq +
c *24.10.* CBq *a* eſt *μν*, *c* erit DFq + FEq *μν*. denique
d *ſch 22.* *10.* quia rectang. A C B *c* eſt *ρν*, *d* etiam D F E eſt
e *41.10.* *ρν*. *e* ergo DE eſt potens *ρν*, ac *μν*. Q.E.D.

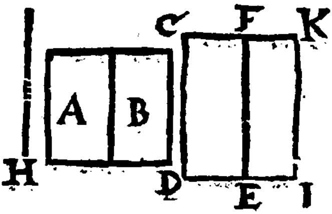
PROP. LXXI.



Bina media potenti (AC + CB) commenſurabilis D E, & ipſa bina media potens eſt.

a *hyp.* Divide D E, ut in præced. Quia ACq *a* ⊏
b *lem 66.* *10.* CBq, *b* erit DFq ⊏ FEq. item quia ACq +
c *24.10.* CBq *a* eſt *μν*, *c* erit DFq + FEq etiam *μν*. pa-
d *24.10.* riterque quia A C B *a* eſt *μν*, *d* etiam DFE eſt
e *14 10.* *μν*. denique quia ACq + CBq ⊏ ACB. *e* erit
f *48.10.* DFq + FEq ⊏ DFE. *f* è quibus ſequitur DE
eſſe potentem 2 *μα*. Q.E.D.

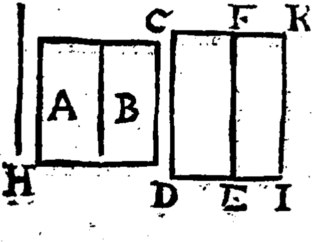
PROP. LXXII.



Si rationale A, & medium B componantur, quatuor irrationales fiunt; vel ea quæ ex binis nominibus, vel quæ ex binis mediis prima, vel major, vel rationale ac medium potens.

Nimirum ſi Hq = A + B, erit H una 4 linearum, quas theorema deſignat. Nam ad C D
expoſitum ρ̇, *a* fiat rectang. C E = A; item F I a cor. 16.
= B; *b* ideoque C I = Hq. Quoniam igitur A 6.
eſt ρ̇ν, etiam CE eſt ρ̇ν. *c* ergo latitudo C F eſt b 2. ax. 1
ρ̇ ⊏⊐ C D. & quia B eſt μν, erit FI μν. *d* ergo c 21.10.
F K eſt ρ̇ ⊏⊐ C D. *e* ergo C F, F K ſunt ρ̇ ⊏⊐. d 23.10.
Tota igitur C K *f* eſt bin. Si igitur A ⊏ B, e 13.10.
hoc eſt C E ⊏ F I, *g* erit C F ⊏ F K. ergo ſi f 37.10.
C F ⊏⊐ √ CFq − F Kq, *h* erit C K bin. 1, & g 1.6.
proinde H = √ C I *k* eſt bin. Si ponatur C F h 1. def. 48.10.
⊏⊐ √ C Fq − F Kq, *l* erit C K bin. 4. quare k 55.10
H (√ C I) *m* eſt major. Sin A ⊐ B; *g* erit l 4. def. 48.10.
C F ⊐ F K; proinde ſi F K ⊏⊐ √ F Kq − CFq, *n* erit C K bin. 2. *o* quare H eſt 2 μ prima. m 58.10
denique ſi F K ⊏⊐ √ F Kq − C F q, *p* erit n 2 def. 48.10.
C K bin. 5. *q* unde H erit potens ρ̇ν ac μν. o 56.10 p 5. def. 48.10.
Q.E.D. q 59.10.

PROP. LXXIII.



Si duo media A, B, inter ſe incommenſurabilia componantur, duæ reliquæ irrationales fiunt; vel ex binis mediis ſecunda, vel bina media potens.

Nempe H potens A + B eſt una dictarum irrationalium. Nam ad C D expoſ. ρ̇, fac rectang.

ctang. CE = A, & FI = B. unde Hq = CF.
a hyp. Quoniam igitur CE, & FI *a* sunt μα, *b* erunt
b 63.10. latitudines CF, FK ρ̇ ⊏ CD. item quia CE *a*
c 1.6. ⊏ FI; eſtque CE. FI *c* :: CF. FK, *d* erit CF
d 10.10. ⊏ FK. *e* ergo CK eſt bin. 3. nempe, ſi CF ⊏
e 3.def. 48.10. √ CFq − FKq. unde H = √ CI *f* erit 2 μ-
f 57.10. 2a. Sin vero CF ⊏ √ CFq − FKq, *g* erit
g 6.def. 48.10. CK bin. 6. & *h* proinde H eſt potens 2 μα.
h 60 10. Q.E.D.

Principium Senariorum per detractionem.

PROP. LXXIV.

Si à rationali DF rationalis DE auferatur, potentia tantum commenſurabilis exiſtens toti DF; reliqua EF irrationalis eſt: vocetur autem apotome.

a lem. 26. 10. Nam EFq *a* ⊏ DEq; ſed DEq *b* eſt ρ̇ν. *c* ergo EF eſt ρ̇. Q.E.D.
b hyp.
c 10. & 11 def. 10. In numeris, ſit DF, 2; DE, √ 3. EF erit 2 − √ 3.

PROP. LXXV.

Si à media DF media DE auferatur, potentia tantum commenſurabilis exiſtens toti DF, quæ cum tota DF rationale contineat; reliqua EF irrationalis eſt; vocetur autem media apotome prima.

a ſch. 26. 10. Nam EFq *a* ⊏ rectang. FDE. ergo cum F-
b hyp.
c 20. & 11. def. 10. DE *b* ſit ρ̇ν, *c* erit EF ρ̇. Q.E.D.

In

In numeris, ſit DF υ√ 54; & DE υ√ 24. ergo EF eſt υ√ 54 − υ√ 24.

PROP. LXXVI.

Si à media D F media DE auferatur, potentia tantum commenſurabilis exiſtens toti D F, quæ cum tota D F medium contineat; reliqua E F irrationalis eſt; vocetur autem mediæ apotome ſecunda.

Quia DFq, & DEq ᵃ ſunt μα ⊐⊏, ᵇ erit DFq + D E q ⊐⊏ D E q. ᶜ quare DFq + DEq eſt μν. item rectang. FDE, ᶜ ideoque 2 FDE ᵃ eſt μν. ergo EFq (ᵈ DFq + DEq − 2 FDE) ᵉ eſt ρ̇ν. quare EF eſt ρ̇. Q.E.D. a hyp. b 16. 10. c 24. 10. d cor. 7. 2. e 27. 10.

In numeris, ſit DF, υ√ 18; & DE, υ√ 8. erit EF υ√ 18 − υ√ 8.

PROP. LXXVII.

Si à recta linea A C recta auferatur A B, potentia incommenſurabilis exiſtens toti B C, quæ cum tota A C faciat compoſitum quidem ex ipſarum quadratis rationale, quod autem ſub ipſis continetur medium; reliqua BC irrationalis eſt: vocetur autem minor.

Nam Acq + ABq ᵃ eſt ρ'ν. at rectang. ACB ᵃ eſt μν. ᵇ ergo 2 CAB ⊐⊏ ACq + ABq (2 ᶜ CAB + BCq;) ᵈ ergo ACq + ABq ⊐⊏ BCq. ᵉ ergo BC eſt ρ̇. Q.E.D. a hyp. 1 b ſch. 12. 10. c 7. 2. d 17. 10. e 11. def. 10.

In numeris, ſit AC, √: 18. + √ 108. A B √: 18 − √ 108. ergo BC eſt √: 18 + √: 108. − √: 18 − √ 108.

PROP.

PROP. LXXVIII.

Si à recta linea D F recta auferatur D E potentia incommenſurabilis exiſtens toti D F, quæ cum tota D F faciat compoſitum quidem ex ipſarum quadratis medium, quod autem ſub ipſis continetur, rationale; reliqua E F irrationalis eſt: vocetur autem cum rationali medium totum efficiens.

a hyp. & ſch. 12. 10.
b hyp.
c ſch. 12. 10.
d 7. 2.
e ſch. 12. 10. & 11. def. 10.

Nam 2 FDE *a* eſt ῥν. *b* & D Fq + DEq eſt μν. *c* ergo 2 FDE ▭ DFq + DEq *d* (2 FDE + EFq) *e* ergo E F eſt ῥ. Q.E.*D*.

In numeris ſit D F, √: √ 216 + √ 72. DE, √: √ 216 − √ 72. ergo E F eſt √: √ 216 + √ 72 − √: √ 216 − √ 72.

PROP. LXXIX.

Si à recta D F recta auferatur D E, potentia incommenſurabilis exiſtens toti D F, quæ cum tota faciat & compoſitum ex ipſarum quadratis, medium; & quod ſub ipſis continetur, medium, incommenſurabileq; compoſito ex quadratis ipſarum, reliqua irrationalis eſt: vocetur autem cum medio medium totum efficiens.

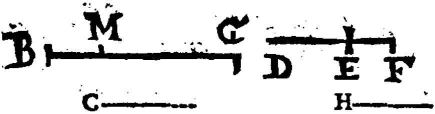
a hyp. & 24. 10.
b 27. 10.
c cor. 7. 2.
d 11. def. 10.

Nam 2 FDE, & DFq + DEq *a* ſunt μα; *b* ergo EFq (*c* DFq + DEq − 2 EDE) eſt ῥν. *d* proinde EF eſt ῥ. Q.E.D.

Exempl. gr. ſit DF, √: √ 180. + √ 60. D E, √: √ 180 − √ 60. E F erit √: √ 180. + √ 60 − √: √ 180. − √ 60.

LEM-

LEMMA.



Si idem ſit exceſſus inter primam magnitudinem B G, & ſecundam C (M G) qui inter tertiam magnitudinem D F, & quartam H (E F;) erit & viciſſim idem exceſſus inter primam magnitudinem B G, & tertiam D F, qui inter ſecundam C, & quartam H.

Nam quia *a* æqualibus BM, DE adjectæ ſunt æquales MG, E F, *a* hoc eſt C, H; erit exceſſus totorum B G, D F, *b* æqualis exceſſui adjectorum, C, H. Q.E.D. a hyp. b 3. ax. 1

Coroll.

Hinc; quatuor magnitudines Arithmeticè proportionales; viciſſim erunt Arithmeticè proportionales.

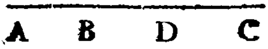
PROP. LXXX.

Apotome A B una tantum congruit recta linea rationalis B C, potentia tantum commenſurabilis exiſtens toti A B.

Si fieri poteſt; alia B D congruat. *a* ergo rectangula ACB, ADB; *b* ideoque eorum dupla ſunt μα cum igitur ACq + B Cq − 2 A C B *c* = ABq *c* = ADq + DBq − 2 A D B. ergo viciſſim A Cq + B Cq − : A Dq + B D q *d* = 2 ACB a 22. 10. b 24. 10. c cor. 7. 2. d lem. 79. 10.

e hyp. & 27. 10. ACB −: 2 ADB. Sed ACq + BCq −: ADq + BDq *e* eſt ς́ν. *f* ergo 2 ACB −: 2 ADB eſt ρ́ν. Q.E.A.
f ſch. 12. 10.
g 27. 10.

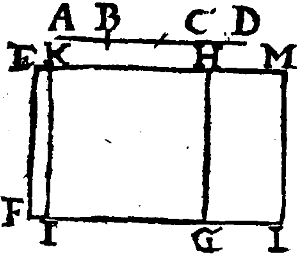
PROP. LXXXI.



Mediæ Apotomæ primæ AB una tantum congruit recta linea media BC, potentia ſolum commenſurabilis exiſtens toti, & cum tota rationale continens.

a hyp. Dic etiam BD congruere. igitur quoniam
b 16. & 24. 10. tam ACq, & BCq; quam ADq, & BDq *a* ſunt
c hyp. μα ▭. *b* etiam ACq + BCq, & ADq + BDq
d ſch. 12. 10. erunt μα. *c* ſed rectangula ACB, ADB; *d* adeoque 2 ACB, & 2 ADB ſunt ς́α. *e* ergo 2 A
e ſch. 27. 10. CB −: 2 ADB; *f* hoc eſt ACq + BCq −:
f 7. 2. & lem. 79. 10. ADq + BDq eſt ρ́ν. *g* Q.E.A.
g 27. 10.

PROP. LXXXII.



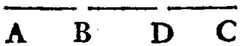
Mediæ Apotomæ ſecundæ AB una tantum congruit recta linea media BC, potentia ſolum commenſurabilis exiſtens toti, & cum tota medium continens.

Si fieri poteſt, congruat alia BD. Ad EF ρ́ fiant rectang. EG = ACq + BCq; item rectang. EL = ADq + BDq. Item EI = ABq. Jam 2 ACB + ABq = ACq + BCq = EG, ergo

ergo cum EI = ABq, *a* erit KG = 2 ACB. porro ACq, & BCq *b* ſunt μα ⊐. *c* Ergo EG (ACq + BCq) eſt μν. *d* ergo latitudo EH ρ' ⊐ EF. *e* Quinetiam rectang. ACB; *f* ideoque 2 ACB (KG) eſt μν. *d* ergo KH eſt etiam ρ' ⊐ EF. denique quia ACq + BCq, id eſt, EG, *g* ⊐ 2 ACB (KG) eſtque EG. KG :: *h* EH. KH *k* erit EH ⊐ KH. *l* ergo EK eſt apotome, cujus congruens KH. ſimili argumento erit KM ejusdem EK congruens; contra 80. hujus.

a 4.2. & 3.ax.1.
b hyp.
c 24.10
d 23.10.
c hyp.
f 24.10.
g lem. 16.10.
h 1.6.
k 10.10.
l 74.10.

PROP. LXXXIII.

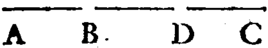


Minori AB, una tantum congruit recta linea (BC) potentia incommenſurabilis exiſtens toti, & cum tota faciens compoſitum quidem ex ipſarum quadratis rationale; quod autem ſub ipſis continetur medium.

Puta alium BD congruere. Cum igitur ACq + BCq, & ADq + BDq *a* ſint ρ'α, eorum exceſſus (2 *b* ACB —: 2 ADB) *c* eſt ρ'ν, *d* Q.E.A; quia ACB, & ADB ſunt μα per hypoth.

a hyp.
b lem. 97.10.
c ſch.27. 10
d 27.10.

PROP. LXXXIV.



Ei (AB,) quæ cum rationali medium totum facit, una tantum congruit recta linea BC. potentia incommenſurabilis exiſtens toti, & cum tota faciens compoſitum quidem ex ipſarum quadratis medium; quod autem ſub ipſis continetur, rationale.

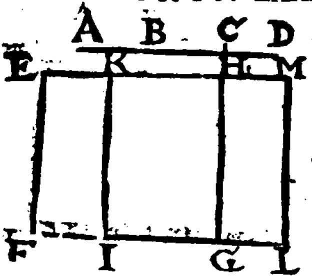
Dic aliam BD etiam congruere. *a* ergo rectangula ACB, ADB. *b* ideoque 2 ACB, & 2

ADB

a hyp.
b ſch.1 10.

c lem. 17. 10. ADB ſunt ϱ'α. ergo 2 ACB --: 2 ADB; c hoc eſt, ACq + BCq --: ADq + BDq d eſt ϱ'ν. d ſch. 17. 10. Q.E.A: quum ACq + BCq, & ADq + BDq ſunt μα per hypoth.

PROP. LXXXV.



Ei (A B,) quæ cum medio medium totum facit una tantum congruit recta linea B C potentia incommenſurabilis exiſtens toti, & cum tota faciens & compoſitum ex ipſarum quadratis medium, & quod ſub ipſis continetur, medium, incommenſurabileq; compoſito ex ipſarum quadratis.

Suppoſitis iis quæ facta & oſtenſa ſunt in 82. hujus; liquet EH, [illegible] KH eſſe ϱ' ⊏⊐ EF. Porro igitur quia ACq + CBq, hoc eſt, rectang. EG a ⊏⊐ ACB, b ideoque EG ⊏⊐ 2 ACB (KG) eſtque EG. KG :: c EH. KH; erit EH ⊏⊐ KH. ergo EK eſt apotome, cujus congruens KH. Haud aliter KM eidem apotomę EK congruere oſtendetur; contra 80. hujus.

a hyp.
b 14. 10.
c 1. 6.

Definitiones tertiæ.

Expoſita rationali, & apotoma, ſi tota plus poſſit quam congruens quadrato rectæ lineæ ſibi longitudine commenſurabilis;

I. Si quidem tota expoſitæ rationali longitudine ſit commenſurabilis, vocetur apotome prima.

II. Si

II. Si vero congruens expoſitæ rationali longitudine ſit commenſurabilis, vocetur apotome ſecunda.

III. Quod ſi neque tota, neque congruens expoſitæ rationali ſit longitudine commenſurabilis, vocetur apotome tertia.

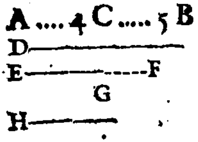
Rurſus, ſi tota plus poſſit quam congruens quadrato rectæ ſibi longitudine incommenſurabilis;

IV. Si quidem tota expoſitæ rationali ſit longitudine commenſurabilis, vocetur apotome quarta.

V. Si vero congruens expoſitæ rationali ſit longitudine commenſurabilis, vocetur apotome quinta.

VI. Quod ſi neque tota, neque congruens, expoſitæ rationali ſit longitudine commenſurabilis, vocetur apotome ſexta.

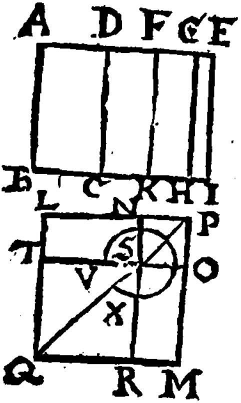
PROP. LXXXVI. 87,88,89,90,91.



Invenire apotomen primam, ſecundam, tertiam, quartam, quintam, ſextam.

Apotomę inveniuntur, ſubductis minoribus binomiorum nominibus ex majoribus. Exempli gr. Sit $6 + \sqrt{20}$, bin. 1. erit $6 - \sqrt{20}$, apot. 1. &c. Quare de earum inventione plura repetere nihil eſt neceſſe.

LEMMA.



Sit rectangulum A-C ſub rectis AB, AD. producatur A D ad E, & biſecetur D E in F. ſitq; rectang. A-GE = FEq. & compleantur rectangula AI, DK, FH. Fiant vero quadratum L M = A H, & quadratum NO = G I, producantur N S R, O-S T.

Dico primo, rectangul. A I = L M + NO = TOq + SOq. ut patet ex conſtr.

Secundo, *Rectang.* DK = LO. Nam quia re-
a conſtr. ctang. AGE *a* = FEq, *b* ſunt AG, FE, GE ∺, *c*
b 17.6. adeoque AH, FI, GI ∺ ; *a* hoc eſt, LM, FI, NO
c 1.6. ∺. atqui LM, LO, N O *d* ſunt ∺ ; ergo F I =
d ſch. 22. 6. *e* LO *f* = DK = *g* NM.
e 9.5.
f 30.1. Tertio, *Hinc*, AC = A I — D K — F I =
g 43.1. LM + NO — LO — NM = TR.

h 16 10. Quarto, *h Liquet* DF, FE, DE eſſe ⊏⊐.

Quinto, *Si* AE ⊏⊐ DE, & A E ⊏⊐ √ A Eq
k 18.10. & 10.10 — DEq, *k erunt* AG, GE, AE ⊏⊐.

l hyp. Sexto, *Item, quia* A E *l* ⊏⊐ DE, *m erunt* A E,
m 13.10. FE ⊏⊐, *n ideoque* AI, FI; *hoc eſt*, LM + NO &
n 1.6. & 10.10. LO *ſunt* ⊏⊐.

* prius. Septimo, *Item quia* A G * ⊏⊐ G E, *n erunt* AHGI, *hoc eſt*, LM. NO ⊏⊐.

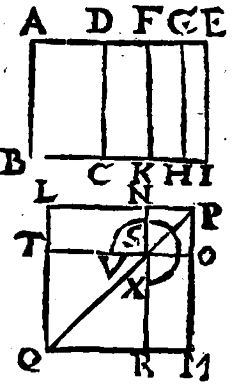
Octa-

Octavo, *Sed quia* A E *l* ⊏⊐ D E, *o erunt* F E, E ⊏⊐. *n ideoque rectang.* FI ⊏⊐ GI, *hoc eſt* LO ⊏⊐ NO. *quare cum* L O. N O *p* :: TS, SO. *q erunt* TS, SO ⊏⊐. o 14.10. p 2.6. q 10.10.

Nono, *Sin ponatur* AE ⊏⊐ √ AEq — Dq; *r erunt* AG, GE, AE ⊏⊐. r 19.10. & 17.10

Decimo, *ſ Quare rectang.* AH, GI, *hoc eſt* Oq, SOq *erunt* ⊏⊐. ſ 1.6. & 20.10.

PROP. XCII.



Si ſpatium AC contineatur ſub rationali AB, & Apotome prima AD (AE — DE;) recta linea TS ſpatium AC potens, apotome eſt.

Adhibe lemma proxime antecedens pro præparatione ad demonſtrationem hujus. Igitur TS = √ AC item AG, GE, E ſunt ⊏⊐; ergo cum AE ⊏⊐ *a* AB ρ̇; *b* ent AG, & GE ⊏⊐ AB. *c* ergo rectangula H & GI, hoc eſt TOq & SOq ſunt ρα. *d* em TO, SO ſunt ρ ⊏⊐, *e* proinde TS eſt apome. Q.E.D. a hyp. b 12.10 c 20.10. d lem. 91 10. e 74.10.

PROP. XCIII.

Vide Schema praeced.

Si ſpatium AC contineatur ſub rationali A
& apotoma ſecunda AD (AE — DE;) r
ſta linea TS ſpatium AC potens; media eſt
potome prima.

Rurſus juxta lemma antecedens, AG, GE
a hyp. AE ſunt ⊏⊐. cum igitur AE *a* ſit ρ̇ ⊏⊐ AB, *b*
b 13.10. runt AE, GE etiam ρ̇ ⊏⊐ AB. *c* ergo rectang
c 22.10. la AH, GI, hoc eſt TOq, SOq, ſunt μὰ; *d* iter
d lem.74. 10. TO ⊏⊤ SO. Denique quia DE *e* ⊏⊐ AB. ρ̇.
e hyp. f 20.10. erit rectang. DI, ejusque ſemiſſis DK, vel LO
g 75.10. hoc eſt TOS ρ̇ν *g* è quibus ſequitur TS (√
AC) eſſe mediæ apot. 1. Q.E.D.

PROP. XCIV.

Vide idem.

Si ſpatium AC contineatur ſub rationali AB
& apotoma tertia AD (AE — DE;) rect.
linea TS ſpatium AC potens, mediæ eſt apotom
ſecunda.

Ut in præcedenti TO, & SO ſunt μ. Quo
a hyp. b 22.10. niam igitur DE *a* eſt ρ̇ ⊏⊐ AB, *b* erit rectang
c 24.10. DI, *c* ideoque DK; vel TOS μν. *d* ergo TS =
d 76.10. √ AC eſt mediæ apot. 2. Q.E.D.

PROP. XCV.

Vide idem.

Si ſpatium AC contineatur ſub rationali AB
& apotoma quarta AD (AE — DE) rect.
linea TS ſpatium AC potens, minor eſt.

a lem.91. 10. Rurſus TO *a* ⊏⊤ SO. Quoniam igitur AE
b eſt

eſt ρ̇ ⊏ AB; *c* erit AI, (TOq + SOq) ρ̇ν. atqui ut prius rectang. TOS eſt μν. *d* ergo TS = √ AC eſt minor. Q.E.D. b hyp. c 20.10. d 77.10.

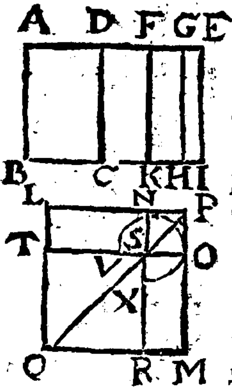
PROP. XCVI.

Vide idem.

Si ſpatium AC contineatur ſub rationali AB, & apotoma quinta AD (AE — DE;) recta linea TS ſpatium AC potens, eſt quæ cum rationali medium totum efficit.

Rurſus enim TO ⊏̸ SO. itaque cum AE *a* ſit ρ̇ ⊏ AB, *b* erit AI, hoc eſt TOq + SOq μν. Sed prout in 93 rectang. TOS eſt ρ̇ν. *c* proinde TS = √ AC eſt quę cum ρ̇ν facit totum μν. Q.E.D. a hyp. b 22.10 c 61.10.

PROP. XCVII.

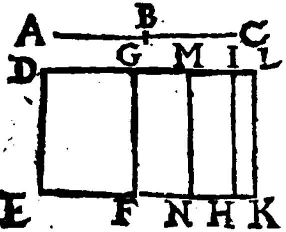


Si ſpatium AC contineatur ſub rationali AB, & apotoma ſexta AD (AE — DE;) recta linea TS ſpatium AC potens, eſt quæ cum media medium totum efficit.

Itidem, ut ſæpe prius, TO ⊏̸ SO. item ut in 96, TOq + SOq eſt μν. rectang. vero TOS eſt ρ̇ν, ut in 94. *a* denique TOq + SOq ⊏ TOS. *b* ergo TS = √ AC eſt quæ cum μν facit totum μν. Q.E.D. a lem. 91.10. b 79.10.

LEMMA.

† cor. 16. 6



Ad rectam quamvis DE † applicentur rectang. DF = ABq, & DH = ACq, & IK = BCq; & ſit GL biſecta in M; ductaque ſit MN parall. GF.

Erit primo, *Rectang.* DK = ACq + BCq, *ut conſtructio indicat.*

Secundo, *Rectang.* ACB = GN, *vel* MK.
a conſtr. Nam DK *a* = ACq + BCq *b* = 2 ACB + A-
b 7. 2. c 3. ax. 1. Bq. at ABq *a* = DF. ergo GK *c* = 2 ACB. &
d 7. ax. 1. *d* proinde GN, vel MK = ACB.

Tertio, *Rectang.* DIL = MLq. Nam quia
e 1. 6. ACq. ACB *e* :: ACB. BCq; hoc eſt DH. MK
f 17. 6. :: MK. IK, *e* erit DI. ML :: ML. IL. *f* ergo DIL = MLq.

g 16. 10. Quarto, *Si ponatur* AC ⊑ BC, *erit* DK ⊏ ACq. Nam ACq + BCq (DK) *g* ⊏ ACq.

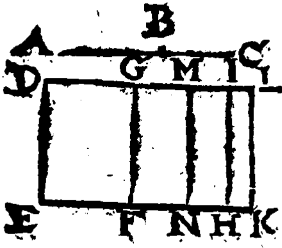
Quinto, *Item*, DL ⊏ √DLq — GLq.
h 10. 10. Nam quia DH (ACq) ⊏ IK (BCq) *h* erit
k 11. 10. DI ⊏ IL. *k* ergo √DLq — GLq ⊏ DL.

Sexto, *Item* DL ⊏ GL. Nam ACq + BCq
l lem. 26. 10. ⊏ *l* 2 ACB; hoc eſt, DK ⊏ GK. *m* ergo DL ⊏ GL.

m 18. 10. n 19. 10. Septimo, *Sin ponatur* AC ⊑ BC, *n erit* DL ⊏ √DLq — GLq.

PROP.

PROP. XCVIII.



Quadratum apotomæ AB (AC — BC) ad rationalem DE applicatum, facit latitudinem DG apotomen primam.

Fac ut in lemmate proxime præcedenti.

Quoniam igitur AC, BC *a* ſunt ρ̇ ⊐⊏, *b* erit DK (ACq + BCq) ⊏⊐ ACq; *c* ergo DK eſt ρ̇ν. *d* quare DL eſt ρ̇ ⊏⊐ DE. *e* item rectang. GK (2 ACB) eſt μν. *f* ergo GL eſt ρ̇ ⊏⊐ DE. *g* proinde DL ⊏⊐ GL; *h* ſed DLq ⊏⊐ GLq. *k* ergo DG eſt apotome, & *l* quidem prima (quia *m* AC ⊐⊏ BC, & propterea DL ⊏⊐ √ DLq — GLq.) Q.E.D.

a hyp.
b lem. 97.10.
c ſch.12.10.
d 21.10.
e 22.& 24.10.
f 23.10.
g 13.10.
h ſch.12.10.
k 74.10
l 1.def. 85.10.
m lem. 97.10.

PROP. XCIX.

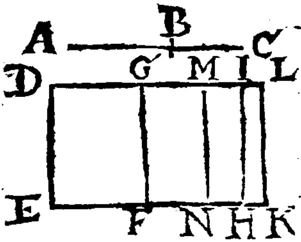
Vide Schema ſubſequens.

Quadratum mediæ apotomæ primæ AB (AC — BC) ad rationalem DE applicatum, facit latitudinem DG apotomen ſecundam.

Rurſus (ſuppoſito lemmate præcedenti) quia AC, & BC *a* ſunt μ ⊐⊏ *b*, erit DK (ACq + BCq) ⊏⊐ ACq; *c* quare DK eſt μν. *d* ergo DL eſt ρ̇ ⊏⊐ DE. *e* item GK (2 ACB) eſt ρ̇ν. *f* ergo GL eſt ρ̇ ⊏⊐ DE; *g* quare DL ⊏⊐ GL. *h* Sed DLq ⊏⊐ GLq. *k* ergo DG eſt apotome, quia vero DL *l* ⊏⊐ √ DLq — GLq, *m* erit DG apotome ſecunda. Q.E.D.

a hyp.
b lem. 97.10.
c 24.10.
d 23.10.
e hyp.& ſch.12.10.
f 21.10.
g 13.10.
h ſch.12.10.
k 74.10.
l lem. 97.10.
m 2.def. 85.10.

PROP. C.



Quadratum mediæ apotomæ secundæ A B (A C — B C) *ad rationalem D E applicatum, facit latitudinem D G apotomen tertiam.*

Iterum D K eſt
a 23.10. *μν*, *a* quare DL eſt ρ̇ ⊏ DE. item GK eſt *μν*. *a*
b lem.26.10. unde GL eſt ρ̇ ⊏ D E; *b* item D K ⊏ G K, *c*
c 1.6. & 10.10. quare D L ⊏ G L ; *d* at DLq ⊏ GLq. *e* ergo
d ſch.12.10. DG eſt apot. & quidem *f* 3a. *g* quia DL ⊏ √
e 74.10. DLq — GLq. Q.E.D.
f 3.def. 85.10.
g lem.97.10.

PROP. CI.

Vide Schema præced.

Quadratum minoris A B (A C — B C) *ad rationalem D E applicatum, facit latitudinem DG apotomen quartam.*

a 21.10. * hyp. Ut prius, ACq + BCq, hoc eſt D K eſt *ρν*; *a*
b 23.10. ergo DL eſt ρ̇ ⊏ DE. at rectang. ACB, ideoq;
c 13.10. GK (2 A C B) * eſt *μν*, *b* quare G L eſt ρ̇ ⊏
d ſch.12.10. D E. *c* ergo D L ⊏ G L. *d* at DLq ⊏ G Lq.
e lem.97.10. quia vero * ACq ⊏ BCq, *e* erit DL ⊏ √ D-
f 4.def. 85.10. Lq — GLq : *f* ergo D G conditiones habet apotomę quartæ. Q.E.D.

PROP.

PROP. CII.

Vide Schem. præced.

*Quadratum ejus AB (*AC — BC,*) quæ cum rationali medium totum efficit, ad rationalem DE applicatum, facit latitudinem DG apotomen quintam.*

Rurſus enim, DK eſt μν, *a* quare DL eſt ρ̊ ⊏⊐ DE. item GK eſt ρν, *b* unde GL eſt ρ̊. ⊏⊐ DE. *c* ergo DL ⊏⊐ GL, *d* ſed DLq. ⊏⊐ GLq. porro, DL *e* ⊏⊐ √ DLq — GLq. ex quibus, DG *f* eſt epot. quinta. Q.E.D.

a 23.10.
b 21.10.
c 13.10.
d ſch.12. 10.
e lem.97 10.
f 5.def. 10.
g 5.10.

PROP. CIII.

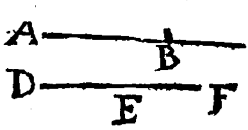
Vide Schema idem.

*Quadratum ejus AB (*AC — BC,*) quæ cum medio medium totum efficit, ad rationalem DE applicatum, facit latitudinem DG apotomen ſextam.*

Haud aliter, quam antea, DK, & GK ſunt μα; *a* quare DL & GL ſunt ρ̊ ⊏⊐ DE. item DK *b* ⊏⊐ GK, *c* quare DL ⊏⊐ GL. *d* ergo DG eſt apot. *b* cum igitur ACq ⊏⊐ BCq, ideoque DL ⊏⊐ √ DLq — GLq, *e* erit DG, apot. ſexta. Q.E.D.

a 13.10.
b hyp. & lem.97. 10.
c 10.10.
d 74.10.
e 6.def.
g 5.10.

PROP. CIV.



*Recta linea DE apotomæ AB (*AC — BC*) longitudine commenſurabilis, & ipſa apotome eſt, atque ordine eadem.*

LEM-

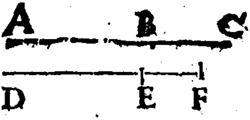
LEMMA.

Sit AB. DE : : AC.DF. *&* AB ⊏ DE.
Dico AC + BC ⊏ DF + EF.

Nam AC.BC *a* : : DF.EF. ergo componendo AC + BC. BC : : DF + EF. EF. ergo per-
a l.a.66.10. mutando AC + BC.DF + EF : : BC. EF. *a* at
BC ⊏ EF. *b* ergo AC + BC ⊏ DF + EF.
b 20.10. Q.E.D.

a 12.6. *a* Fac AB. DE : : AC. DF. *b* igitur AC +
b lem.103.10 BC ⊏ DF + EF. ergo cum AC + BC *c* bi-
c hyp. nomium ſit, *d* erit DF + EF ejusdem ordinis
d 61.10. binomium : *e* quare DF — EF ejusdem ordi-
e Per definitionem ad 85.10. nis apotome eſt, cujus AC — BC. Q.E.D.

PROP. CV.

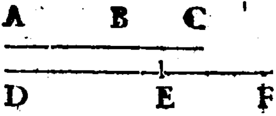


Recta linea DE mediæ apotomæ AB (AC — BC) commenſurabilis, & ipſa mediæ apotome eſt, atque ordine eadem.

a 11.6. Iterum *a* fac AB. DE : : AC. DF. *b* qua-
b lem. 103.10. re AC + BC ⊏ DF + EF. *c* ergo DF +
c 68.10. EF eſt bimed. ejusdem ordinis, cujus AC +
d 75. & 96.10. BC. *d* proinde & DF — EF mediæ apotome
erit ejusdem claſſis, cujus AC — BC.
Q.E.D.

PROP.

PROP. CVI.

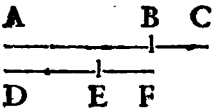


Recta linea DE Minori AB (AC — BC) commenſurabilis, & ipſa minor eſt.

Fiat AB.DE :: AC. DF. *a* eſtque AC + BC ⊏ DF + EF. atqui AC + BC *b* eſt Major, *c* ergo *D*F + EF quoque Major eſt. *d* & proinde DF — EF eſt Minor. Q.E.D.

a lem. 103.
b hyp.
c 69.10.
d 77.10

PROP. CVII.

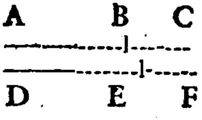


Recta linea DE commenſurabilis ei AB (AC — BC) quæ cum rationali medium totum efficit, & ipſa cum rationali medium totum efficiens eſt.

Nam ad modum precedentium oſtendemus DF + EF eſſe potentem ῥτ, & μν. *a* ergo DF — EF eſt ut dicitur.

a 78. 10

PROP. CVIII.



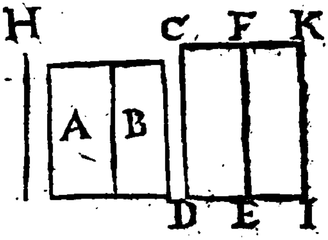
Recta linea DE commenſurabilis ei AB (AC — BC) quæ cum medio medium totum efficit, & ipſa cum medio medium totum efficiens eſt.

Nam, ad normam precedentium, erit *D*F + EF potens 2 μα. *a* ergo DF — EF erit ut in propoſ.

a 79.10.

PROP.

PROP. CIX.



Medio B à rationali A + B detracto, recta linea H, quæ reliquum ſpatium A poteſt, una ex duabus irrationalibus ſit, vel apotome, vel Minor.

Ad C D ρ', fac rectang. C I = A + B; & FI
a 3.ax.1 = B. quare C E *a* = A: (Hq) Quoniam igitur
b hyp. & conſtr. C I *b* eſt ρ'ν, *c* erit CK ρ' ⊏⊐ C D. ſed quia F I *b*
c 21.10. eſt μν, *d* erit FK ρ' ⊏⊐ C D. *e* unde CK ⊏⊐ F K
d 23.10. *f* ergo CF eſt apotome. Si igitur CK ⊏⊐ √ C-
e 13.10. *f* 74.10. Kq — FKq. *g* erit CF apot. prima; *h* quare √
g 1.def. 85.10. CE (H) eſt apotome. ſin C K ⊏⊐ √ CKq —
h 92.10. FKq, *k* erit CF apot. quinta. & proinde H (√
k 4.def. 85.10. CE) *l* erit Minor. Q.E.D.
l 85.10.

PROP. CX.

Vide Schem. præced.

Rationali B à medio A + B detracto; aliæ duæ irrationales ſiunt, vel mediæ apotome prima, vel cum rationali medium totum efficiens.

Ad CD expoſ. ρ' fiant rectang. CI = A + B;
a 3.ax.1. & FI = B, *a* unde CE = A = Hq. Quoniam igitur
b hyp & conſtr. CI *b* eſt μν: *c* erit CK ρ' ⊏⊐ CD. ſed quia F I *b*
c 23.10. *d* 21.10. eſt ρ'ν, *d* erit IK ρ' ⊏⊐ CD. *e* unde CK ⊏⊐ F K. *f*
e 13.10. ergo CF eſt apot. *g* nempe ſecunda; ſi C K ⊏⊐
f 74.10. *g* 2.def. √ CKq — FKq, *h* quare H (√ CE) eſt me-
85.10. *h* 93.10. diæ apot. prima. Sin vero CK ⊏⊐ √ CKq —
FKq,

FKq, *k* erit CE apot.quinta. & proinde H (√ CE) erit faciens *μν* cum ρ'ν. Q.E.D. k 5.*def.* 85 10. 96.10.

PROP. CXI.

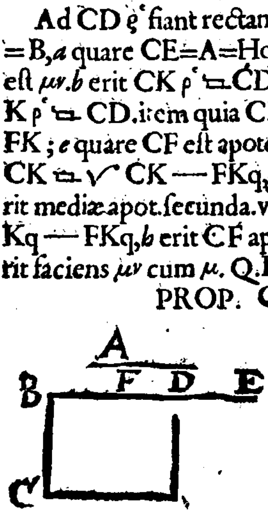
Vide Schema idem.

Medio B à medio A + B detracto, quod ſit incommenſurabile toti A + B; reliqua duæ irrationales fiunt, vel media apotome ſecunda, vel cum medio medium totum efficiens.

Ad CD ϱ' fiant rectang. CI = A + B; & FI = B, *a* quare CE = A = Hq. Quoniam igitur CI eſt *μν*. *b* erit CK ρ' ⊏⊐ CD. eodem modo erit FK ρ' ⊏⊐ CD. item quia CI *c* ⊏⊐ FI, *d* erit CK ⊏⊐ FK; *e* quare CF eſt apotome, *f* tertia, ſcilicet, ſi CK ⊏⊐ √ CK — FKq, *g* unde H (√ CE) erit mediæ apot. ſecunda. verum ſi CK ⊏⊐ √ CKq — FKq, *h* erit CF apot. ſexta. *k* quare H erit faciens *μν* cum *μ*. Q.E.D.

a 3 *ax.* 1
b 23.10.
c *hyp.*
d 10.10
e 74.10.
f 3.*def.* 85.10.
g 94.10
h 6.*def.* 85.10.
k 97.10.

PROP. CXII.



Apotome A non eſt eadem, quæ ex binis nominibus.

Ad expoſ. BC ϱ', fiat rectang. CD = Aq Ergo cum A ſit apotome, *a* erit BD apot. prima. ejus congruens ſit DE. *b* quare BE, DE ſunt ρ' ⊏⊐. *c* & BE ⊏⊐ BC. Vis A eſſe bin. ergo BD eſt bin. 1. ejus nomina ſint BF, FD; ſitq; BF ⊏ FD; *d* ergo BF, FD ſunt ρ' ⊏⊐; & BF *e* ⊏⊐ BC. ergo cum BC ⊏⊐ BE, *f* erit BE ⊏⊐ FB. *g* ergo BE ⊏⊐ FE. *h* ergo FE eſt ϱ'. item quia BE ⊏⊐ DE, *k* erit FE ⊏⊐ DE. *l* quare FD eſt apot. *l* adeoque FD eſt ρ̇. ſed oſtenſa eſt ϱ'. quæ repugnant. ergo A male dicitur binomiũ. Q.E.D. *No-*

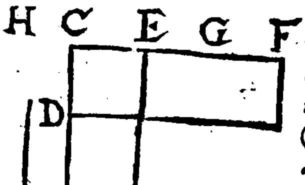
a 98.10.
b 74.10.
c 1.*def.* 85.10.
d 37.10
e 1.*def.* 48.10.
f 12.10.
g *cor.* 16. 10.
h *ſch.* 12. 10.
k 14.10
l 74.10.

Nomina 13. linearum irrationalium inter ſe differentium.

1. Media.
2. Ex binis nominibus, cujus 6 ſpecies.
3. Ex binis mediis prima.
4. Ex binis mediis ſecunda.
5. Major.
6. Rationale ac medium potens.
7. Bina media potens.
8. Apotome, cujus etiam ſex ſpecies.
9. Mediæ apotome prima.
10. Mediæ apotome ſecunda.
11. Minor.
12. Cum rationali medium totum efficiens.
13. Cum medio medium totum efficiens.

Cum latitudinum differentiæ arguant differentias rectarum, quarum quadrata ſunt applicata ad aliquam rationalem, ſitq́ demonſtratum in præcedentibus, latitudines quæ oriuntur ex applicationibus quadratorum harum 13. linearum inter ſe differre, perſpicuè ſequitur has 13. lineas inter ſe differre.

PROP. XCIII.



Quadratum rationalis A ad eam, quæ ex binis nominibus B C (BD + DC) applicatum, latitudinem facit apotomen EC, cujus nomina EH, CH

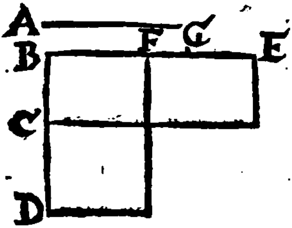
CH commenſurabilia ſunt nominibus B D, DC ejus, quæ ex binis nominibus & in eadem proportione (EH.BD : : CH.DC ;) *& adhæc, apotome E C quæ ſit, eundem habet ordinem, quem ea BC, quæ ex binis nominibus.*

Ad DC minus nomen *a* fac rectang. DF =
Aq = BE. quare BC. C D *b* : : F C. C E. ergo a cor.16.
dividendo BD. DC : : FE. E C. cum igitur BD 6.
c ⊏– DC, *d* erit FE ⊏– E C. ſume E G = E C ; b 14.6.
fiatque FG. GE : : EC. CH. Erunt EH, C H c hyp.
nomina apotomæ E C ; quibus conveniunt ea, d 14.5.
quæ in theoremate propoſita ſunt. Nam componendo F E. G E. (E C) : : E H. G H. ergo
F H. E H *e* : : E H. C H *f* : : FE. EC *f* : : BD. e 12.5.
D C. quare cum B D *g* ⊐ D C, *h* erit EH ⊐ f Prius.
C H ; *h* & F Hq ▭ EHq. ergo, quia FHq. E- g hyp.
Hq *k* : : F H. C H. *h* erit F H ▭ CH. *l* ideoque h 10.10.
F C ▭ C H. Porro C D *g* eſt ρ̇, & D F (Aq) k cor.20.6.
g eſt ρ̇ν, *m* ergo F C eſt ρ́ ▭ C D, quare etiam l 16.10.
CH eſt ρ́ ▭ CD. *n* igitur EH, CH ſunt ρ̇. ac ⊐ m 21.10
ut prius. *o* ergo E C eſt apot. cui congruit CH. n ſch.12.10.
porro EH. CH *f* : : BD. DC, ideo permutando o 74.10.
E H. B *D* : : C H. *D* C. unde quia C H *f* ▭
*D*C, *p* erit EH ▭ B *D*. quinimo pone B*D* ▭ p 10.10.
√ B*D*q — *D*Cq ; *q* erit ideo EH ▭ √ EHq q 15.10.
— CHq. item ſi B*D* ▭ ρ̇ expoſ. erit EH ▭ ei- r 12.10.
dem ρ̇ ; *s* hoc eſt ſi BC ſit bin. 1. *t* erit E C apot. s 1. def.
prima. Similiter ſi *D* C ▭ ρ́ expoſ. *t* erit C H 48.10.
▭ eidem ρ̇. *u* hoc eſt ſi BC ſit bin. 2. *x* erit E C t 1. def.
apotom. 2. & ſi hæc bin. 3. illa erit apot. 3. &c. 85.10.
Sin B *D* ▭ √ B *D*q — *D* Cq, *y* erit E H ▭ u 2. def.
√ EHq 48.10.

x 2. def.
85.10.
y 15.10.

√ EHq — CHq; ſi igitur BC ſit bin.4, vel 5, vel 6. erit EC ſimiliter apot. 4, vel 5, vel 6. Q.E.D.

PROP. CXIV.

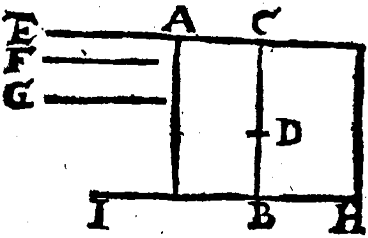


Quadratum rationalis A ad apotomen BC (BD — DC) *applicatum, facit latitudinem BE eam, quæ ex binis nominibus; cujus nomina BE, GE commenſurabilia ſint apotomæ BG nominibus BDDC, & in eadem proportione; & adhuc, quæ ex binis nominibus ſit (BE,) eundem habet ordinem, quem ipſa apotome BC.*

a cor.16.6 *a* Fac rectang. DF = Aq; & BE. FE *b* :: EG.
b 12.6. GF. Quoniam igitur DF = Aq = CE, *c* erit B-
c 14.6. D. BC :: BE. BF. ergo per converſionem ra-
d 19.5. e hyp. tionis BD.CD :: BE. FE :: EG. GF :: *d* BG.
f 10.10. g cor.20.6. EG. ſed BD *e* ⊏⊐̨ CD. *f* ergo BG ⊏⊐̨ GE. ergo
h 10.10. quia BGq. GEq *g* :: BG. GF. *h* erit BG ⊏⊐ G-
k cor.16.10. F. *k* ideoque BG ⊏⊐ BF. porro BD *e* eſt ῥ, & re-
l 21.10. ctang. DF (Aq) *e* eſt ῥν. *l* ergo BF eſt ῥ ⊏⊐ BD.
m 12.10. *m* ergo etiam BG eſt ῥ ⊏⊐ BD *n* ergo BG, GE
n ſch.12.10. ſunt ῥ ⊏⊐̨. *o* quare BE eſt bin. denique igitur
o 37.10. quia BD. CD :: BG. GE; & permutando
p 10.10. BD. BG :: CD. GE; ſitque BD ⊏⊐ BG; *p* erit CD ⊏⊐ GE. ergo ſi CB ſit apot. prima; erit BE bin.1. &c. ut in antecedenti. ergo, &c.

PROP.

PROP. CXV.



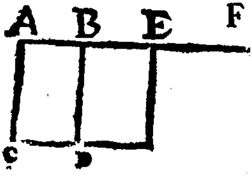
Si ſpatium A B contineatur ſub apotoma AC (CE − AE,) & ea, quæ ex binis nominibus CB; cujus nomina C D, D B commenſurabilia ſint apotomæ nominibus C E, A E, & in eadem proportione (CE. AE : : C D. D B. ;) recta linea F ſpatium AB potens, eſt rationalis.

Sit G quævis $\grave{\rho}$; & fiat rectang. CH = Gq. *a*
erit igitur BH (HI − IB) apotome; & HI *a* ⊏⊐
CD *b* ⊏⊐ CE, *a* & BI ⊏⊐ DB; *a* atque HI. BI : : a 113. 10.
CD. DB *b* : : C E, E A. ergo permutando H I. b *hyp.*
CE : : BI. E A. *c* ergo B H. A C : : H I. C E : : c *19. 5.*
B I. E A ergo cum HI *d* ⊏⊐ C E, *e* erit BH ⊏⊐ d *12. 10.*
A C. *f* ergo rectang. H C ⊏⊐ B A. Sed H C e *10. 10.*
(Gq) *b* eſt $\grave{\rho}$v. *g* ergo BA (Fq) eſt $\grave{\rho}$v. proinde F f *1. 6. &* 10. 10.
eſt $\grave{\rho}$. Q.E.D. g *ſch. 12.* 10.

Coroll.

Hinc, fieri poteſt, ut ſpatium rationale contineatur ſub duabus rectis irrationalibus.

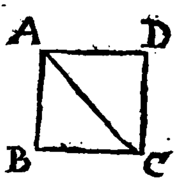
PROP. CXVI.



A media A B fiunt infinitæ irrationales BE, EF, &c. & nulla alicui antecedentium eſt eadem.

a lem. 31. 10. Sit A C expoſ. ῥ. ſitque A D ſpatium ſub A C, A B. *a* ergo A D eſt ῥν. Sume BE = √
b 11. 10. AD. *b* ergo BE eſt ῥ, nulli priorum eadem. nullum enim quadratum alicujus priorum applicatum ad ῥ, latitudinem efficit mediam. compleatur rectang. D E; *a* erit D E ῥν; & *b* proinde E F (√ D E) erit ῥ; & nulli priorum eadem. nullum enim priorum quadratum ad ῥ applicatum, latitudinem efficit ipſam BE. ergo, &c.

PROP. CXVII.



Propoſitum ſit nobis oſtendere, in quadratis figuris B D, diametrum A C lateri A B incommenſurabilem eſſe.

a 47. 1.
b cor. 24. 8.
c 9. 10.
Nam A Cq. A Bq *a* :: 2. 1 *b* :: non Q. Q. *c* ergo A C ⊏ A B. Q.E.D.

Celebratiſſimum eſt hoc theorema apud veteres Philoſophos, adeo ut qui hoc neſciret, eum Plato non hominem eſſe, ſed pecudem diceret.

LIB.

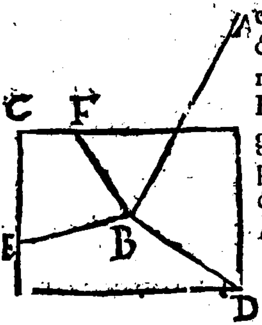
LIB. XI.

DEFINITIONES.

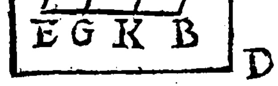
I.

SOlidum eſt, quod longitudinem, latitudinem, & craſſitudinem habet.

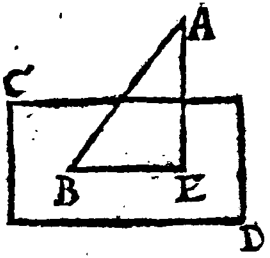
II. **Solidi autem extremum** eſt ſuperficies.



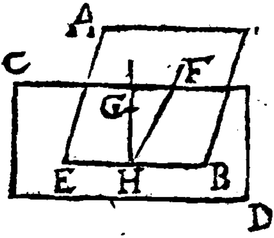
III. Linea recta A B eſt ad planum C D recta, cum ad rectas omnes lineas B D, B E, B F, à quibus illa tangitur, quæque in propoſito ſunt plano, rectos efficit angulos ABD, ABE, ABF.



IV. Planum A B ad planum C D rectum eſt, cum rectæ lineæ FG, HK, quæ communi planorum ſectioni E B ad rectos angulos in uno plano A B ducuntur, alteri plano C D ad rectos ſunt angulos.



V. Rectæ lineæ A B ad planum CD inclinatio eſt, cum à ſublimi termino A rectæ alius lineæ AB ad planum CD deducta fuerit perpendicularis A E; atque à puncto E, quod perpendicularis A E in ipſo plano C D fecerit, ad propoſitæ illius lineę extremum B, quod in eodem eſt plano, altera recta linea E B fuerit adjuncta: eſt, inquam, angulus acutus ABE inſiſtente linea AB,& adjunctá EB comprehenſus.



VI. Plani AB ad planum CD inclinatio, eſt angulus acutuş FHG rectis lineis FH, GH contentus, quę in utroque planorum A B, C D ad idem communis ſectionis B E punctum H ductæ, rectos cum ſectione BE efficiunt angulos FHB,GHB.

VII. Planum ad planum ſimiliter inclinatum eſſe dicitur,atque alterum ad alterum, cum dicti inclinationum anguli inter ſe fuerint æquales.

VIII. Parallela plana ſunt, quæ inter ſe non conveniunt.

IX.Si-

IX. Similes ſolidæ figuræ ſunt, quæ ſimilibus planis continentur, multitudine æqualibus.

X. Æquales & ſimiles ſolidæ figuræ ſunt, quę ſimilibus planis multitudine & magnitudine æqualibus continentur.

XI. Solidus angulus eſt plurium quam duarum linearum, quæ ſe mutuo contingunt, nec in eadem ſunt ſuperficie, ad omnes lineas inclinatio.

Aliter.

Solidus angulus eſt, qui pluribus quam duobus planis angulis in eodem non conſiſtentibus plano, ſed ad unum punctum conſtitutis, continetur.

XII. Pyramis eſt figura ſolida, planis comprehenſa, quæ ab uno plano ad unum punctum conſtituuntur.

XIII. Priſma eſt figura ſolida, quæ planis continetur, quorum adverſa duo ſunt & æqualia, & ſimilia, & parallela; alia vero parallelogramma.

XIV. Sphæra eſt, quando ſemicirculi manente diametro, circumductus ſemicirculus in ſeipſum rurſus revolvitur unde moveri cœperat, circumaſſumpta figura.

Coroll.

Hinc radii omnes à centro ad ſuperficiem ſphęræ inter ſe ſunt ęquales.

XV. Axis autem ſphæræ, eſt quieſcens illa recta linea, circũ quam ſemicirculus convertitur.

XVI. Centrum ſphæræ eſt idem quod & ſemicirculi.

XVII. Diameter autem ſphæræ, eſt recta quædam linea per centrum ducta, & utrinque à ſphæræ ſuperficie terminata.

XVIII. Conus eſt, quando rectanguli trianguli manente uno latere eorum, quę circa rectum angulum, circumductum triangulum in ſeipſum rurſus revolvitur unde moveri cœperat, circumaſſumpta figura. Atque ſi quieſcens recta linea æqualis ſit reliquæ, quę circa rectum angulum continetur, orthogonius erit conus; ſi vero minor, amblygonius; ſi vero major, oxygonius.

XIX. Axis autem coni, eſt quieſcens illa linea, circa quam triangulum vertitur.

XX. Baſis vero coni eſt circulus qui à circumducta recta linea deſcribitur.

XXI. Cylindrus eſt, quando rectanguli parallelogrammi manente uno latere eorum, quę circa rectum angulum, circumductum parallelogrammum in ſe ipſum rurſus revolvitur unde cœperat moveri, circumaſſumpta figura.

XXII. Axis autem cylindri, eſt quieſcens illa recta linea, circum quam parallelogrammum convertitur.

XXIII. Baſes vero cylindri ſunt circuli à duobus adverſis lateribus, quæ circumaguntur, deſcripti.

XXIV. Similes coni & cylindri ſunt, quorum & axes, & baſium diametri proportionales ſunt.

XXV. Cubus eſt figura ſolida ſub ſex quadratis æqualibus contenta.

XXVI. Tetraedrum eſt figura ſolida ſub quatuor triangulis æqualibus & æquilateris contenta.

XXVII.

XXVII. Octaedrum eſt figura ſolida ſub octo triangulis æqualibus & æquilateris contenta.

XXVIII. Dodecaedrum eſt figura ſolida ſub duodecim pentagonis æqualibus & æquilateris & æquiangulis contenta.

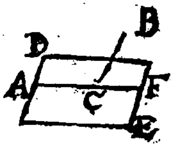
XXIX. Icoſaedrum eſt figura ſolida ſub viginti triangulis æqualibus & æquilateris contenta.

XXX. Parallelepipedum eſt figura ſolida ſex figuris quadrilateris, quarum quę ex adverſo parallelæ ſunt, contenta.

XXXI. Solida figura in ſolida figura dicitur inſcribi, quando omnes anguli figuræ inſcriptæ conſtituuntur vel in angulis, vel in lateribus, vel denique in planis figuræ, cui inſcribitur.

XXXII. Solida figura ſolidæ figuræ viciſſim circumſcribi dicitur, quando vel anguli, vel latera, vel denique plana figurę circumſcriptæ tangunt omnes angulos figuræ, circum quam deſcribitur.

PROP. I.



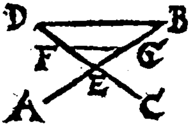
Recta linea pars quadam, A C non eſt in ſubjecto plano, quadam vero CB in ſublimi.

Producatur A C in ſubjecto plano uſque ad F. vis C B eſſe in directum ipſi A C; ergo duæ rectæ A B, A F habent commune ſegmentum A C. a Q.F.N.

a 10. ax. 1.

PROP.

PROP. II.

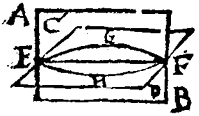


Si duæ rectæ lineæ A B, C D ſe mutuo ſecent, in una ſunt plano: atque triangulum omne D E B in uno eſt plano.

Puta enim trianguli DEB partem EFG eſſe in uno plano, partem vero FDGB in altero. ergo rectæ E D pars E F eſt in ſubjecto plano, pars vero F D in ſublimi, *a* Q.E.A. ergo triangulum E D B in uno eſt plano; proinde & rectę ED, EB; *a* quarę & totæ A B, D C in uno plano exiſtunt. Q.E.D.

a ax. 11.

PROP. III.

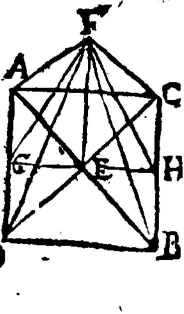


Si duo plana A B, C D ſe mutuo ſecent, communis eorum ſectio E F eſt recta linea.

Si E F communis ſectio non eſt recta linea, *a* ducatur in plano A B recta EGF, *a* & in plano CD recta E H F, duę igitur rectæ E G F, E H F claudunt ſpatium. *b* Q.E.A.

a 1.poſt.1.
b 14.ax.1.

PROP. IV.



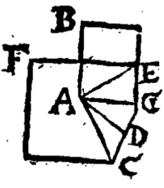
Si recta linea E F rectis duabus lineis A B, C D ſe mutuo ſecantibus in communi ſectione E ad rectos angulos inſiſtat: illa ducto etiam per ipſas plano A C B D ad angulos rectos erit.

Acci-

Accipe E A, E C, E B, E D æquales, & junge rectas AC, CB, BD, AD. per E ducatur quævis recta G H; junganturq; F A, FC, FD, FB, FG, FH. Quoniam AE a = E B; & D E a = E C; & ang. A E D b = C E B, c erit A D. = C B. c pariterque A C = DB. d ergo A D parall. C B. d & A C parall. D B. e quare ang. GAE = EBH. e & ang. AGE = E H B. ſed & A E f = EB g ergo GE = EH, & g AG = BH. quare ob angulos rectos, ex hyp, & proinde pares ad E, h baſes F A, FC, F B. F D æquantur. Triangula igitur ADF. F B C ſibi mutuo æquilatera ſunt, k quare ang. DAF = CBF. ergo in triangulis AGF, FBH latera FG, FH l æquantur; & proinde etiam triangula FEG, FEH ſibi mutuo æquilatera ſunt. m ergo anguli FEG, FEH æquales ac n propterea recti ſunt. Eodem modo FE cum omnibus in plano A D B C per E ductis rectis lineis rectos angulos conſtituit, o ideoque eidem plano recta eſt. Q.E.D.

a conſtr.
b 15.1.
c 4.1.
d ſch.34.1.
e 29.1.
f conſtr.
g 26.1.
h 4.1.
k 8.1.
l 4.1.
m 8.1.
n 10. def.1.
o 3. def. 11.

PROP. V.



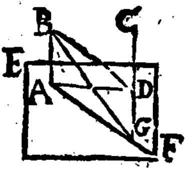
Si recta linea A B rectis tribus lineis AC, AD, AE ſe mutuo tangentibus in communi ſectione ad rectos angulos inſiſtat; illæ tres rectæ in uno ſunt plano.

Nam AC, AD a ſunt in uno plano FC. a item AD, AE ſunt in uno plano BE. vis diverſa eſſe hæc plana; ſit igitur eorum

a 2. 11.

b 3.11. rum interſectio *b* recta A G. Quoniam igitur B A ex hypoth. perpendicularis eſt rectis c 4.11. A C, A D, eadem *c* plano F C, *d* ideoque re- d 3 def. 11. ctæ A G perpendicularis eſt. ergo (ſiquidem & *a* A B eſt in eodem cum A C, A E plano) anguli B A G, B A E recti, & proinde pares ſunt, *e* pars & totum. Q.E.A.

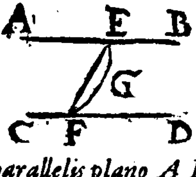
PROP. VI.



Si duæ rectæ lineæ A B, D C eidem plano E F ad rectos ſint angulos; parallelæ erunt illæ rectæ lineæ A B, D C.

Ducatur A D, cui in plano E F perpendicularis ſit D G = A B; junganturque B D, B G, A G. Quia in triangulis B A D, A D G an- a hyp. b conſtr. guli D A B, A D G *a* recti ſunt; atque A B *b* c 4.1. = D G; & A D communis eſt; *c* erit B D = A G; quare in triangulis A G B, B G D ſibi d 8.1. mutuo æquilateris angul. B A G *d* = B D G; quorum B A G rectus cum ſit, erit B D G etiam rectus. atqui ang. G D C rectus ponitur; ergo recta G D tribus D A, D B, C D recta e 5.11. eſt; *e* quæ ideo in uno ſunt plano, *f* in quo A B f 2.11. exiſtit; cum igitur A B, & C D ſint in uno plano, & anguli interni BAD, CDA recti ſint, *g* e- g 28.1. runt A B, C D parallelæ. Q.E.D.

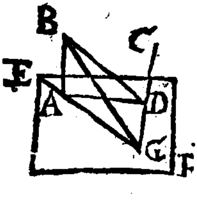
PROP. VII.



Si duæ ſint parallelæ rectæ lineæ A B, C D, in quarum utraque ſumpta ſint quælibet puncta E, F; illa linea E F, quæ ad hæc puncta adjungitur, in eodem eſt cum parallelis plano A B C D.

Planum in quo A B, C D, ſecet aliud planum per puncta E, F. ſi jam E F non eſt in plano A B C D, illa communis ſectio non erit.
Sit ergo E G F, *a* hæc igitur recta eſt linea. duæ a 3. 11.
ergo rectæ E F, E G F ſpatium claudunt. *b* b 14. ax.
Q.E.A. 1.

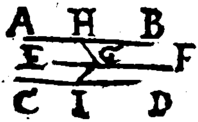
PROP. VIII.



Si duæ ſint parallelæ rectæ lineæ A B, C D, quarum altera A B ad rectos cuidam plano E F ſit angulos; & reliqua C D eidem plano E F ad rectos angulos erit.

Adſcita præparatione & demonſtratione
ſextæ hujus; anguli G D A & G D B recti ſunt,
a ergo G D recta eſt plano per A D, D B (*b* in a 4. 11.
quo etiam A B, C D exiſtunt.) *c* ergo G D i- b 7. 11.
pſi C D eſt perpendicularis; atqui ang. C D A c 3. def. 11. d 29. 1.
etiam *d* rectus eſt. *e* ergo C D plano E F recta e 4. 11.
eſt. Q.E.D.

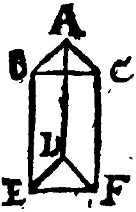
PROP. IX.



Quæ (A B, CD) eidem rectæ lineæ E F ſunt parallelæ, ſed non in eodem cum illa plano, hæ quoque ſunt inter ſe parallelæ.

In plano parallelarum
A B, E F duc H G perpendicularem ad E F. i-
tem in plano parallelarum E F, C D duc IG
a 4.11. perpendicularem ad EF. *a* ergo E G recta eſt
b 8.11. plano per H G, GI, eidemque plano *b* rectæ
c 6.11. ſunt A H, & C I *c* ergo A H, & C I parallelæ
ſunt. Q.E.D.

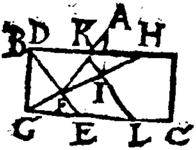
PROP. X.



Si duæ rectæ lineæ AB, AC ſe mutuo tangentes ad duas rectas E D, D *F ſe mutuo tangentes ſint parallelæ, non autem in eodem plano, illæ angulos æquales (B A C, EDF) comprehendent.*

Sint AB, AC, DE, DF æqua-
les inter ſe, & ducantur A D, BC,
a *hyp. & conſtr.* EF, BE, CF. Cum AB, DE *a* ſunt parallelæ & æ-
b 33.1. quales, *b* etiam B E, A D parallelæ ſunt, & æ-
quales. Eodem modo C F, A D parallelæ ſunt,
c 2.ax.1. & 30.1. & æquales. *c* ergo etiam BE, FC ſunt parallelæ
d 33.1. & æquales. Æquantur ergo BC, EF. Cum igi-
e 8.1. tur trianguli *B*AC, EDF ſibi mutuo æquilatera
ſint, anguli B A C, E D F *e* æquales erunt.
Q. E. D.

PROP. XI.



A dato puncto A *in ſublimi ad ſubjectum planum* B C *perpendicularem rectam lineam* AI *ducere.*

In plano B C duc quamvis D E, ad quam ex
A *a* duc perpendicularem A F. ad eandem per a 12.1.
F in plano B C *b* duc normalem F H. tum ad b 11.1.
F H *a* demitte perpendicularem AI. erit AI
recta plano B C.

Nam per I *c* duc KIL parall. DE. Quia DE c 31 1.
d recta eſt ad AF, & F H, *e* erit DE recta plano d *conſt.*
IFA; adeoque & KL eidem plano *f* recta eſt. e 4.11.
g ergo ang. K I A rectus eſt. atqui ang. AIF et- f 8.11.
iam *h* rectus eſt. *k* ergo AI plano B C recta eſt. g 3. *def.* 11.
Q. E. D. h *conſtr.* k 4.11.

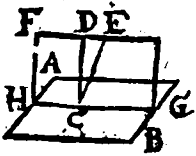
PROP. XII.

Dato plano B C *à puncto* A, *quod in illo datum eſt, ad rectos angulos rectam lineam* A F *excitare.*

A quovis extra planum
puncto D *a* duc D E re-
ctam plano B C; & juncta E A *b* duc A F pa- a 11.11. b 31.1.
rall. D E. *c* perſpicuum eſt A F plano B C re- c 8.11.
ctam eſſe. Q. E. F.

Practicè perficiuntur hoc, & pręcedens problema, ſi duæ normæ ad datum punctum applicentur, ut patet ex 4.11.

PROP. XIII.



Dato plano A B, à puncto D, quod in illo datum eſt, duæ rectæ lineæ C D, C E ad rectos angulos non excitabuntur ab eadem parte.

a 6.11.

Nam utraque C D, C E plano A B *a* recta eſſet, eædemque adeo parallelæ forent, quod parallelarum definitioni repugnat.

PROP. XIV.

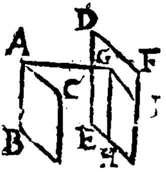
valet hæc conversa.

Ad quæ plana C D, F E, eadem recta linea A B recta eſt; illa ſunt parallela.

Si negas, plana CD, FE concurrant, ita ut communis ſectio ſit recta G H; ſume in hac quodvis punctum I, ad quod in propoſitis planis ducantur rectæ IA, IB. unde in triangulo I A B, duo anguli I A B, I B A *a* recti ſunt. *b* Q.E.A.

a hyp. & 3. def.11.
b 17.1.

PROP. XV.



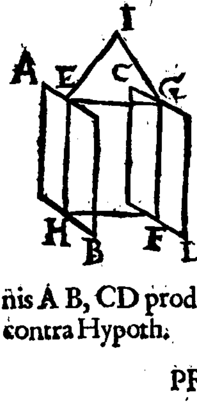
Si duæ rectæ lineæ A B, A C ſe mutuo tangentes ad duas rectas D E, D F ſe mutuo tangentes ſint parallelæ non in eodem conſiſtentes plano; parallela ſunt, quæ per illa ducuntur, plana B A C, E D F.

Ex

Ex A *a* duc A G rectam plano E F. *b* Sintq; G H, G I parallelæ ad D E, D F. *c* erunt hæ parallelæ etiam ad A B, A C. Cum igitur anguli I G A, H G A *d* ſint recti, *e* erunt etiam CAG, BAG recti. *f* ergo G A recta eſt plano BC; atqui eadem recta eſt plano E F. *b* ergo plana B C, E F ſunt parallela. Q.E.D.

a 11.11.
b 31.1.
c 30.11.
d 3.def. 11.
e 29.1.
f 4.11.
g conſtr.
h 14.11.

PROP. XVI.

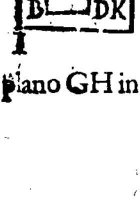


Si duo plana parallelæ A B, C D, plano quopiam HEIGF ſecentur, communes illorum ſectiones E H, G F ſunt parallelæ.

Nam ſi dicantur non eſſe parallelæ, cum ſint in eodem plano ſecanti, convenient alicubi, puta in I. quare cum totæ HDI, FGI *a* ſint in planis A B, CD productis, etiam hæc convenient; contra Hypoth.

a 1.11.

PROP. XVII.

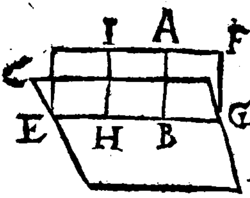


Si duæ rectæ lineæ A L B, CMD parallelis planis EF, GH, I K ſecentur, in eaſdem rationes ſecabuntur (A L. L B :: C M. M D.)

Ducantur in planis EF, I K rectæ AC, BD. item A D occurrens plano GH in N; junganturque N L, N M. Pla-

na triangulorum A D C, A D B faciunt ſectiones
a 16.11. BD, LN; & A C, NM *a* parallelas. ergo A L.
b 2.6 LB *b* :: AN. ND *b* :: CM. MD. Q.E.D.

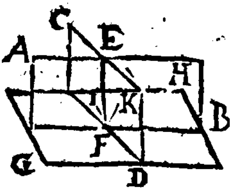
PROP. XVIII.



Si recta linea A B plano cuipiam C D ad rectos ſit angulos; & omnia, quæ per ipſam AB plana (EF, &c.) *eidem plano* CD *ad rectos angulos erunt.*

Ductum ſit per A B planum aliquod E F, faciens cum plano C D ſectionem E G; è cujus
a 31.1. aliquo puncto H, in plano EF *a* ducatur HI pa-
b 8.11. rall. AB. *b* erit HI recta plano C D; pariterque
c 4.def.11 aliæ quævis ad EG perpendiculares. *c* ergo planum EF plano CD rectum eſt; eademque ratione quævis alia plana per AB ducta plano E F recta erunt. Q.E.D.

PROP. XIX.



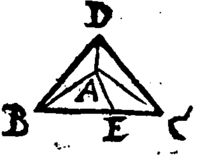
Si duo plana A B, C D, *ſe mutuo ſecantia, plano cuidam* G H *ad rectos ſint angulos, communis etiam illorum ſectio* E F *ad rectos eidem plano*(GH)*angulos erit.*

Quoniam plana A B, C D ponuntur recta plano GH, patet ex 4. def. 11. quod ex puncto

F in

F in utroque plano A B, C D duci poſſit perpendicularis plano G H; quæ *a* unica erit, & propterea eorundem planorum communis ſectio. Q.E.D. a 13.11.

PROP. XX.

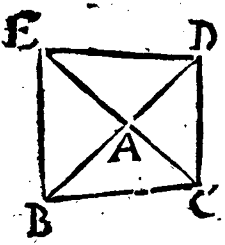


Si ſolidus angulus A B C D tribus angulis planis B A D, D A C, B A C contineatur; ex his duo quilibet, utut aſſumpti, tertio ſunt majores.

Si tres anguli ſunt æquales, patet aſſertio; ſi inæquales, maximus eſto B A C. ex quo *a* aufer B A E = BAD; & fac A D = A E; ducanturque BEC, BD, DC. a 23.1.

Quoniam latus B A commune eſt, & A D *b* = A E; & ang. BAE *b* = BAD; *c* erit B E = B D. ſed BD + DC *d* $\sqsubset$ B C. *e* ergo D C $\sqsubset$ E C. cum igitur A D *b* = A E, & latus A C commune eſt, ac D C $\sqsubset$ E C *f*, erit ang. CAD $\sqsubset$ EAC. *g* ergo ang. BAD + CAD $\sqsubset$ BAC. Q.E.D. b conſtr. c 4.1. d 20.1. e 5.ax.1 f 25.1. g 4.ax.1

PROP. XXI.



Omnis ſolidus angulus A ſub minoribus quam quatuor rectis angulis planis continetur.

Latera enim ſolidi anguli A ſecans planum utcunque faciat figuram multilateram B C D E, & totidem triangula ABC, A C D,

U ADE,

ADE, AEB. Omnes angulos polygoni voco X; & ſummá angulorum ad trigonorum baſes voco Y. quare X + 4 Rect. a = Y + A. Quia vero (ex angulis ad B) b eſt ang. ABE + ABC $\sqsubset$ CBE; idemque verum ſit de angulis ad C, ad D, ad E. c liquet fore Y $\sqsubset$ X, proinde erit A $\sqsupset$ 4 Rect. Q.E.D.

a 32.1. & ſch. 32.1
b 20.11.
c 5. ax.1.

PROP. XXII.

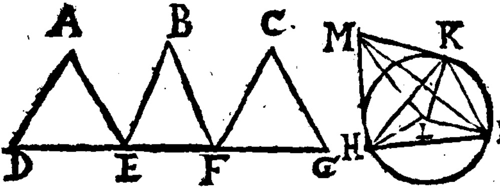
Si fuerint tres anguli plani A, B, HCI, quorum duo utlibet aſſumpti reliquo ſint majores; comprehendant autem ipſos rectæ lineæ æquales AD, AE, FB, &c. fieri poteſt, ut ex rectis lineis DE, FG, HI, æquales illas rectas connectentibu triangulum conſtituatur.

Ex iis a conſtitui poteſt triangulum, ſi duæ quælibet reliqua majores exiſtant; ſed ita ſe res habet. Nam b fac ang. HCK = B, & CK = CH, ducanturque HK, IK. c ergo KH = FG. & quia ang. KCI d $\sqsubset$ A; erit KI $\sqsubset$ DE. ſed KI f $\sqsupset$ HI + KH (FG;) ergo DE $\sqsupset$ HI + FG. Simili argumento quævis duæ reliqua majores oſtendentur; & proinde ex iis triangulum a conſtitui poteſt. Q.E.D.

a 22.1.
b 23.1.
c 4.1.
d hyp.
e 24.1.
f 20.1.

PROP,

PROP. XXIII.

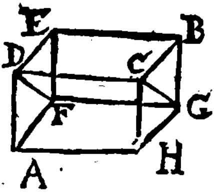


Ex tribus angulis planis A, B, C, quorum duo quomodocunque aſſumpti reliquo ſunt majores, ſolidum angulum M H I K conſtituere. * *Oportet autem illos tres angulos quatuor rectis minores eſſe.* * 21.11.

Fac AD, AE, BE, BF, CF, CG æquales inter
ſe. Ex ſubtenſis DE, EF, FG (hoc eſt, ex æqua-
libus HI, IK, KH) *a* fac triang. HKI. circa quod
b deſcribatur circulus LHKI. * Quoniam ve- a 22.11. & 22.1.
ro $AD \sqsubset HL$; *c* ſit $ADq = HLq + LMq$. *d* b 5. 4.
ſitque LM recta plano circuli HKI; & ducan- * vide Clavium
tur HM, KM, IM. Quoniam igitur ang. HLM c ſch. 47. 1.
e rectus eſt, *f* erit $MHq = HLq + LMq$ *g* $=$ d 12. 11.
ADq. ergo $MH = AD$. ſimili argumento MK, e 3. def. 11.
MI, AD (id eſt, AE, EB, &c.) æquantur; ergo f 47. 1.
cum $HM = AD$, & $MI = AE$; & DE *h* $= HI$, *k* g conſtr. h conſtr.
erit ang. $A = HMI$; *k* ſimiliter ang. $IMK = B$. *k* & k 8. 1.
ang. $HMK = C$. Factus eſt igitur angulus ſolidus
ad M ex tribus planis datis. Q.E.F. Aſſumptum
eſt fore $AD \sqsubset HL$. Hoc autem conſtat. Nam ſi
$AD =$ vel $\sqsupset HL$, erit angul. A *a* $=$, *b* vel $\sqsubset$
HLI. Eodem modo erit $B =$, vel $\sqsubset HLK$, a conſtr. & 1.
& $C =$, vel $\sqsubset KLI$. quare $A + B + C$ b 21. 1.
† quatuor rectos aut exæquabunt, aut excedent, † 4 cor. 23. 1.

contra hypoth. quin potius ſit A D ⊏ HI
Q.E.D.

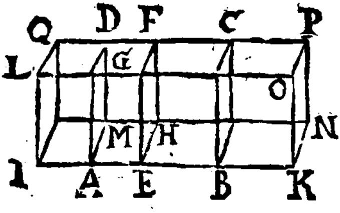
PROP. XXIV.



Si ſolidum A B paral
lelis planis contineatu
adverſa illius plana (AG
DB, &c.) parallelogram
ma ſunt ſimilia & æqua
lia.

Planum A C ſecan
a 16.11. plana parallela AG, D B, *a* facit ſectiones A H
DC parallelas. Eadem ratione A D, H C pa
rallelæ ſunt. Ergo A D C H eſt parallelogram
mum. Simili argumento reliqua parallelepipe
b 35.def. 1. di plana ſunt *b* parallelogramma. Quum ig
tur A F ad H G, & A D ad H C parallelæ ſin
c 10.11. *c* erit ang. F A D = C H G; ergo ob A F *d*
d 34.1. HG, & A D *d* = H C, ac *e* propterea AF. AI
e 7.5. : : H G. H C, triangula FAD, G A H *g* ſimili
g 6.6. h 4.1. ſunt & *h* æqualia; proinde & parallelogram
k 6.ax.1. ma A E, HB ſimilia ſunt & *k* æqualia. idemqu
de reliquis oppoſitis planis oſtendetur. ergo
&c.

PROP. XXV.



Si ſolidur
parallelepi-
pedū ABC
plano E F ſ
cetur adve
ſis planis
D, B C pa
ral

allelo, erit quemadmodum baſis AH ad baſim H, ita ſolidum AHD ad ſolidum BHC.

Concipe Ppp. ABCD produci utrinque, ac-pe AI = AE, & BK = EB; & pone plana Q, KP planis AD, BC parallela. parallelo-ramma IM, AH, *a* & DL, DG, *b* & IQ, D, EF, &c. *a* ſimilia ac æqualia ſunt; *c* qua- Ppp. AQ = AF; atque eadem ratione Ppp. P = BF. ergo ſolida IF, EP ſolidorum AF, C æquemultiplicia ſunt, ac baſes IH, KH aſium AH, BH. Quod ſi baſis IH ⊏, =, ⊐ H, *d* erit ſimiliter ſolidum IF ⊏, =, ⊐ EP. proinde AH. BH :: AF. EC. Q.E.D.

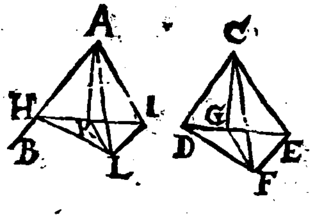
a 36.1. & 1.def. 6.
b 24.11
c 10.def. 11.
d 24.11. & 9.def. 11.
e 6.def. 5.

Hæc eadem omni priſmati accommodari poſ-int; unde

Coroll.

Si priſma quodcunque ſecetur plano oppo- tis planis parallelo, ſectio erit figura æqualis, & milis planis oppoſitis.

PROP. XXVI.



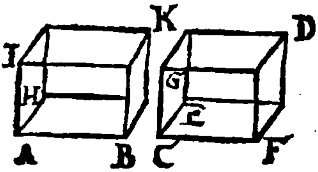
Ad datam rectam lineā AB, ejusque punctum A, conſtituere angulum ſolidum AHIL, æqualem ſolido angulo dato CDEF.

A puncto quovis F *a* demitte FG plano DCE rectam; ducanturque rectæ DF, FE,

a 11.11.

EG,GD,CG. Fac A H = CD.& ang. HAI = DCE. & A I = CE; atque in plano HAI, fac ang. HAK = DCG,& AK = CG. Tum erige KL rectam plano HAI, & ſit KL = G F. ducaturque AL. erit angulus ſolidus AHIL par dato C D E F. Nam hujus conſtructio illius conſtitutionem penitus æmulatur, ut facile patebit examinanti. ergo factum.

PROP. XXVII.

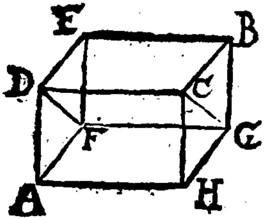


A data recta linea A B; dato ſolido parallelepipedo C D ſimile & ſimiliter poſitum parallepipedum A K deſcribere.

Ex angulis planis BAH, HAI, BAI,qui æ- a 26.11.
quales ſint ipſis FCE, ECG, FCG, *a* fac angu- b 12.6.
lum ſolidum A ſolido C parem. item *b* fac FC. c 22.5.
CE : : BA.AH. *b* ac CE.CG : : AH.AI (*c* unde erit ex æquali FC.CG : : BA.AI ;) & perficiatur Ppp. AK. erit hoc ſimile dato. d 1.def.6.

Nam per conſtr. Pgra *d* BH , F E ; *d* & HI, E G ; & *d* BI, F G ſimilia ſunt,& *e* horum ideo e 24.11.
oppoſita illorum oppoſitis. ergo ſex plana ſoli- f 9.def.11.
di A K ſimilia ſunt ſex planis ſolidi C D. *f* proinde A K, C D ſimilia ſolida exiſtunt. Q.E.F.

PROP. XXVIII.

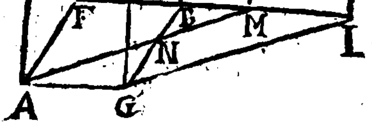


Si ſolidum parallelepipedum AB plano FGCD ſecetur per diagonios DF, CG adverſorum planorum AE, HB, bifariam ſecabitur ſolidum AB ab ipſo plano FGCD.

Nam quia DC, FG *a* æquales & parallelæ ſunt, *b* planum FGCD eſt Pgr. & propter *a* Pgra AE, HB æqualia, & ſimilia, *b* etiam triangula AFD, HGC, CGB, DFE æqualia & ſimilia ſunt. Atqui Pgra AC, AG ipſis FB, ED *a* etiam æqualia & ſimilia ſunt. ergo priſmatis FGCDAH omnia plana æqualia ſunt, & ſimilia planis omnibus priſmatis FGCDEB; & *c* proinde hoc priſma illi æquatur. Q.E.D.

a 24. 11.
b 34. 1.
c 9. def. 11.

PROP. XXIX.



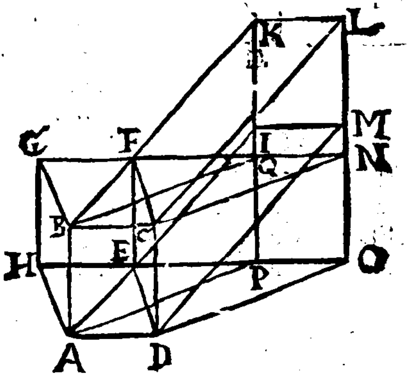
Solida parallelepipeda AGHEFBCD, AGHEMLKI ſuper eandê baſim AGHE conſtituta, & † *in eadem altitudine; quorum inſiſtentes lineæ AF, AM in iiſdem collocantur rectis lineis AG, FL, ſunt inter ſe æqualia.*

Nam

† *Id eſt, inter parallela plana AGHE, FLKD. & ſic intellige in ſequent.*

a 10.def. 11. & 35.1. Nam ſi ex *a* æqualibus priſmatis AFMEDI, GBLHCK commune auferatur priſma
b 3 & 2. NBMPCI, addaturque utrinque ſolidum
ax.1. AGNEHP, *b* erit Ppp. AGHEFBCD = AGHEMLKI. Q.E.D.

PROP. XXX.

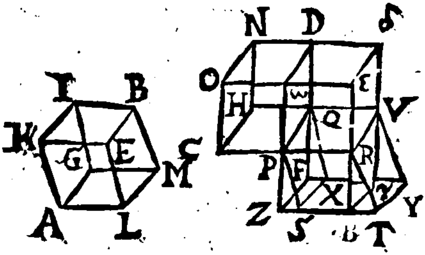


Solida Parallelepipeda ADBCHEFG, ADCBIMLK ſuper eandem baſim ADCB conſtituta, & in eadem altitudine, quorum inſiſtentes lineæ AH, AI non in iiſdem collocantur rectis lineis, inter ſe ſunt æqualia.

Nam produc rectas HEO, GFN; & LMO,
a 34.1. KIP; & duc AP, DO, BQ, CN. *a* erunt tam DC, AB, HG, EF, PQ, ON; quam AD, HE, GF, BC, KL, IM, QN, PO ęquales inter ſeſe &
b 29.11. parallelæ. *b* Quare Ppp. ADCBPONQ utriq; Pppo. ADCBHEFG, ADCBIMLK æquale
c 1.ax.1. eſt; & *c* proinde hęc ipſa inter ſe æqualia ſunt. Q.E.D.

PROP.

PROP. XXXI.



*Solida parallelepipeda ALEKGMBI, CPωOHQDN ſuper æquales baſes ALEK, CPωO conſtituta, & * in eadem altitudine, æqualia ſunt inter ſe.*

Habeant primo parallelepipeda AB, CD latera ad baſes recta; & ad latus CP productum *a* fiat pgr. PRTS æq. & ſimile pgro KELA; *b* adeoque Ppp. PRTSQVYX æq. & ſim. Pppo AB. Producantur OωE, NDδ, ωPZ, DQF, ERB, δVγ, TSZ, YXF; & duc Eδ, Bγ, ZF.

Plana Oεδ N, CRVH, ZTYF *c* parallela ſunt inter ſe; *d* & pgra ALEK, CPωO, PRTS, PRBZ æqualia ſunt. Cum igitur Ppp. CDPVδω *e* :: pgr. Gω (PRBZ.) Pε *e* :: Ppp. PRBZQVγF. PVδω, *f* erit Ppp. CD *f* = PRBZQVγF *g* = PRVQSTYX *h* = AB. Q.E.D.

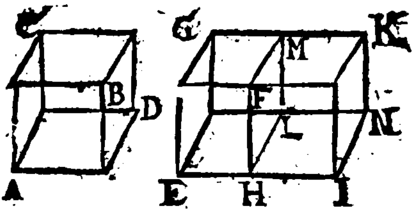
* *Altitudo eſt perpendicularis à plana baſis ad planum oppoſitum.*

a 18.6.
b 27.11. & 10. def. 11.
c 30. def 11.
d hyp. & 35.1.
e 25.11.
f 9.5.
g 29.11.
h conſtr.

Sin Pppa AB, CD latera baſibus obliqua habeant; ſuper easdem baſes, & in eadem altitudine, ponantur parallelepipeda, quorum latera ba-

k 29.11. baſibus ſint recta. *k* Ea inter ſe, & obliquis æ-
m 1.ax.1. qualia erunt; *m* proinde & obliqua A B, C D æquantur. Q.E.D.

PROP. XXXII.

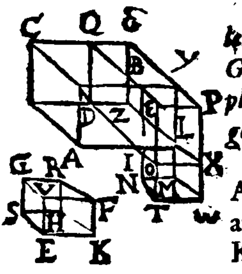


Solida parallelepipeda ABCD, EFGL ſub eadem altitudine, inter ſe ſunt ut baſes AB, EF.

a 45.1. Producta EHI, *a* fac pgr. FI = AB, & *b* com-
b 31.1. ple Ppp. F I N M. Liquet eſſe Ppp. FINM. (*c*
c 31.11. ABCD.) EFGL *d* :: F I. (AB) E F. Q.E.D.
d 25.11.

PROP. XXXIII.

Similia ſolida parallelepipeda, ABCD, EFGH, inter ſe ſunt in triplicata ratione homologorum laterum AI, EK.



Producantur rectæ AIL, DIO, BIN. & *a* fi-
a 3.1. ant IL, IO, IN ipſis EK,
b 27.11. KH, KF ęquales, *b* adeoque & Ppp. IXMT æq. &
c 31.11. ſim. Pppo EFGH. *c* Perficiantur Ppp. *a* IXPB,
d hyp. D L Y Q. Itaque *d* erit AI. IL. (EK) :: DI.
e 1.6.

IO

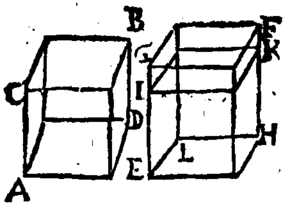
IO (HK) : : BI. IN. (KF;) hoc eſt Pgr. A D. DL : : DL. IX : : BO. IT; *f* id eſt Ppp. ABCD. DLQY : : DLQY. IXBP : : IXBP. IXMT. (*g* EFGH.) *h* ergo ratio A B C D ad EFGH triplicata eſt rationis ABCD ad DLQY, *k* vel AI ad EK. Q.E.D.

f 32. 11.
g *conſtr.*
h 10. *def.* 5.
k 1. 6.

Coroll.

Hinc, ſi fuerint quatuor lineæ rectæ continue proportionales, ut eſt prima ad quartam, ita eſt parallelepipedum ſuper primam deſcriptum ad parrallelepipedum ſimile ſimiliterq; deſcriptum ſuper ſecundam.

PROP. XXXIV.



Æqualium ſolidorum parallelepipedorum ADCB, EHGF baſes & altitudines reciprocantur (AD. EH : : EG. AC.) Et quorum ſolidorum parallelepipedorum A D C B, EHGF baſes & altitudines reciprocantur, illa ſunt æqualia.

Sint primo latera CA, GE ad baſes recta; ſi jam ſolidorum altitudines ſint pares, etiam baſes æquales erunt, & res clara eſt. Sin altitudines inæquales ſint, à majori EG *a* detrahe EI = A C. & per I *b* duc planum I K parallelum baſi E H. itaque

a 3. 1.
b 31. 11.

1. *Hyp.*

c 32.11. 1. *Hyp.* AD. EH *c* :: Ppp. ADCB. EHIK *d*
d 11.5. :: Ppp. EHGF. EHIK *e* :: GL. IL *e* :: GE.
e 1.6. IE. (*f* AC;) *g* liquet igitur eſſe AD. EH ::
f *conſtr.* GE. AC. Q.E.D.
g 11.5.

h 32.11. 2. *Hyp.* ADCB. EHIK *h* :: AD. EH *k* ::
k *hyp.* EG. EI *l* :: GL. IL *m* :: Ppp. EHGF. RHIK,
l 1.6.
m 32.11. *n* quare Ppp. ADCB = EHGF. Q.E.D.
n 9.5.

Sint deinde latera ad baſes obliqua. Erigantur ſuper iisdem baſibus, in altitudine eadem, parallelepipeda recta. Erunt obliqua parallelepipeda his æqualia. Quare cum hæc per 1. partem reciprocent baſes & altitudines, etiam illa reciprocabunt, Q.E.D.

Coroll.

Quæ de parallelepipedis demonſtrata ſunt Prop. 29, 30, 31, 32, 33, 34. *etiam conveniunt priſmatis triangularibus, quæ ſunt dimidia parallelepipeda, ut patet ex* Pr. 28. *Igitur,*

1. Priſmata triangularia æque alta ſunt ut baſes.

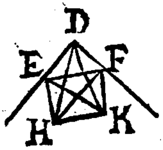
2. Si eandem vel æquales habeant baſes, & eandem altitudinem, æqualia ſunt.

3. Si ſimilia fuerint, eorum proportio triplicata eſt proportionis homologorum laterum.

4. Si æqualia ſunt, reciprocant baſes & altitudines. & ſi reciprocant baſes & altitudines, æqualia erunt.

PROP.

PROP. XXXV.



Si fuerint duo plani anguli B A C, E D F æquales, quorum verticibus A, D, ſublimes rectæ lineæ A G, D H inſiſtant, quæ cum lineis primo poſitis angulos contineant æquales, utrumque utrique (ang. G A B = HDE; & G A C = H D F.) *in ſublimibus autem lineis A G, D H qualibet ſumpta fuerint puncta G, H; & ab his ad plana B A C, E D F, in quibus conſiſtunt anguli primum poſiti B A C, E D F, ductæ fuerint perpendiculares G I, H K; à punctis vero I, K quæ in planis à perpendicularibus ſiunt, ad angulos primum poſitos adjunctæ fuerint rectæ lineæ A I, D K; hæ cum ſublimibus A G, D H æquales angulos G A M, H D K comprehendent.*

Fiunt DH, AL æquales, & G I, L M parallelæ; & M C ad A C, M B ad A B, K F ad D F, KE ad D E perpendiculares, ducanturque rectæ B C, L B, L C, atque E F, H F, H E; *a* eſtque L M recta plano B A C; *b* quare anguli L M C, L M A, L M B; eademque ratione anguli H K F, H K D, H K E recti ſunt. Ergo ALq *c* = L Mq + A Mq *c* = L Mq + CMq + ACq *c* = LCq + ACq; *d* ergo ang. A C L rectus eſt. Rurſus A Lq *e* = L Mq + M Aq *e* = LMq + BMq + BAq *e* = BLq + BAq. *d* ergo ang.

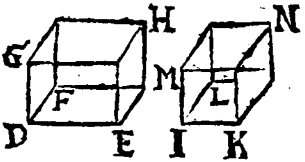
a 1.11.
b 3.def. 11.
c 47.1.
d 48.1.

ang. ABL etiam rectus eſt. Simili diſcurſu anguli DFH, DEH recti ſunt; *f* ergo AB = DE; *f* 26.1. *f* & BL = EH; *f* & AC = DF; & CL = FH. *g* 4.1. *h* 3.ax.1. *g* quare etiam BC = EF, *g* & ang. ABC = DEF, *g* & ang. ACB = DFE. unde reliqui è rectis anguli CBM, BCM reliquis FEK, EFK æquantur. *k* 26.1. *k* ergo CM = FK, *l* ideoque & AM = DK. *l* 47.1. *m* conſtr. ergo ſi ex LAq *m* = HDq. auferatur AMq = *n* 47.1. & 3.ax. DKq, *n* remanet LMq = HKq quare trigona LAM, HDK ſibi mutuo æquilatera ſunt. *o* 8.1. *o* ergo ang. LAM = HDK. Q.E.D.

Coroll.

Itaque ſi fuerint duo anguli plani æquales, quorum verticibus ſublimes rectę lineæ æquales inſiſtant, quæ cum lineis primo poſitis angulos contineant æquales, utrumque utrique; erunt à punctis extremis linearum ſublimium ad plana angulorum primo poſitorum demiſſæ perpendiculares inter ſe æquales; nempe LM = HK.

PROP. XXXVI.



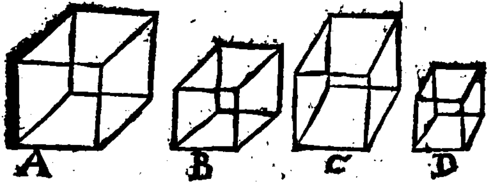
Si tres rectæ lineæ DE, DG, DF proportionales fuerint; quod ex his tribus fit ſolidum parallelepipedum DH, æquale eſt deſcripto à media linea DG (IL) ſolido parallelepipedo IN, quod æquilaterum quidem ſit, æquiangulum vero prædicto DH.

Quo-

Quoniam DE.IK *a* : : IL.DF, *b* erit pgr.LK = FE.& propter angulorum planorum ad E & I,ac linearum GD,IM æqualitatem, etiam altitudines parallelepipedorum æquales ſunt, ex coroll.præced. *c* ergo ipſa inter ſe æqualia ſunt. Q.E.D.

a *hyp.*
b 24.6.
c 31.11.

PROP. XXXVII.

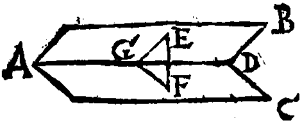


Si quatuor rectæ lineæ A, B, C, D proportionales fuerint, & ſolida parallelepipeda A,B,C, D quæ ab ipſis & ſimilia, & ſimiliter deſcribuntur, proportionalia erunt. Et ſi ſolida parallelepipeda, quæ & ſimilia, & ſimiliter deſcribuntur, fuerint proportionalia (A.B : : C. D.) & ipſæ rectæ lineæ A, B, C, D proportionales erunt.

Nam rationes parallelepipedorum *a* triplicatę ſunt rationum, quas habent lineæ. ergo ſi A.B : : C.D. *b* erit Ppp. A. Ppp.B : : Ppp. C. Ppp. D. & vice verſa.

a 33.11.
b *ſch.33.* 1.

PROP. XXXVIII.

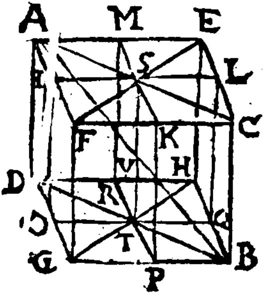


Si planum AB ad planum AC rectum fuerit, & ab aliquo puncto E eorum, quæ ſunt

ſunt in uno planorum (AB) ad alterum planum AC perpendicularis EF ducta fuerit, in planorum communem ſectionem AD cadet ducta perpendicularis E F.

Si fieri poteſt, cadat F extra interſectionem
a 12. 1. A D. In plano A C *a* ducatur F G perpendicularis ad A D, jungaturque E G. Angulus
b 4. & 3. def. 11. F G E *b* rectus eſt; & E F G rectus ponitur.
c 17. 1. ergo in triangulo E F G ſunt duo anguli recti. Q.E.A.

PROP. XXXIX.



Si ſolidi parallelepipedi A B, eorum quæ ex adverſo planorum A C, D B latera (AE, FC, AF, EC, & DH, GB, DG, HB) bifariam ſecta ſint; per ſectiones autem plana ILQO, P K M R ſint extenſa; planorum communis ſectio S T, & ſolidi parallelepipedi diameter A B, bifariam ſe mutuo ſecabunt.

Ducantur rectæ S A, SC, TD, TB. Propter
a 34. 1. *a* latera DO, OT lateribus, B Q, QT, *b* angu-
b 19. 1 losque alternos TOD, T QB æquales, *c* etiam
c 4. 1 d ſch. 15. 1. baſes DT, TB, & anguli DTO, BTQ æquan-
e 34. 1. tur. *d* ergo DTB eſt recta linea. eodem modo ASC recta eſt linea. Porro *e* tam A D ad F G, *e* quam

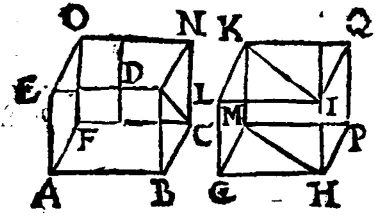
e quam FG ad CB; *f* ideoque AD ad CB, *g* ac proinde AC ad DB parallelæ & æquales ſunt. *h* quare AB, & ST in eodem plano ABCD exſiſtunt. Itaque cum anguli AVS, BVT ad verticem, & alterni ASV, BTV æquentur; *k* & AS = BT; erit AV = BV, *l* & SV = VT. Q.E.D.

e 34.1.
f 9.11, & 1 ax.
g 33.1.
h 7.11.
k 17.ax.1
l 26.1.

Coroll.

Hinc, in omni parallelepipedo diametri omnes ſe mutuo biſecant in uno puncto, V.

PROP. LX.



Si fuerint duo priſmata ABCFED, GHMLIK æqualis altitudinis, quorum hoc quidem habeat baſim ABCF parallelogrammum, illud vero GHM triangulum; duplum autem fuerit parallelogrammum ABCF trianguli GHM; æqualia erunt ipſa priſmata ABCFED, GHMLIK.

Nam ſi perficiantur parallelepipeda AN, GQ, *a* erunt hæc æqualia ob *b* baſium AC, GP, & *c* altitudinum æqualitatem. *d* ergo etiam priſmata, *e* horũ dimidia, æqualia erunt. Q.E.D.

a 31.11.
b 34.1. & 7.ax.
c hyp.
d 28 11.
e 7.ax.1

Schol.

Ex hactenus demonſtratis habetur dimenſio priſmatum triangularium, & quadrangularium, ſeu parallelepipedorum, ſi nimirum altitudo ducatur in baſim.

Ut ſi altitudo ſit 10.pedum,baſis vero pedum quadratorum 100 (menſurabitur autem baſis per ſch.35.1.vel per 41.1.) multiplica 100.per 10; proveniunt 1000 pedes cubici pro ſoliditate priſmatis dati.

vide ſchol. 35.1. Nam quemadmodum rectangulum, ita & parallelepipedum rectum producitur ex altitudine ducta in baſim. Ergo quodvis parallelepipedum producitur ex altitudine in baſim ducta,ut patet ex 31.hujus.

Deinde cum totum parallelepipedum producatur ex altitudine in totam baſim, ſemiſſis ejus (hoc eſt priſma triangulare) producetur ex altitudine ducta in dimidiam baſim, nempe triangulum.

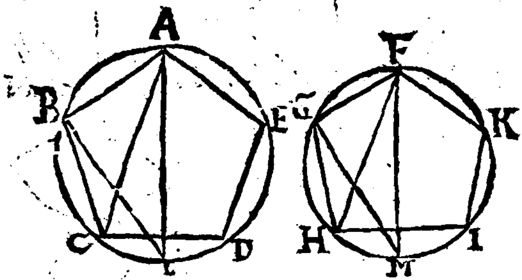
Monitum.

Nota, litterarum quæ deſignant angulum ſolidum primam eſſe ſemper ad punctum,in quo eſt angulus; litterarum vero quæ denotant pyramidem,ultimam eſſe ad verticem pyramidis.

Ex. gr. Angulus ſolidus ABCD eſt ad punctum A; pyramidis quoque BCDA vertex eſt ad punctum A,& baſis triangulum BCD.

LIB. XII.

PROP. I.



QVæ ſunt in circulis ABD, FGI polygona ſimilia ABCDE, FGHIK, inter ſe ſunt, ut quadrata à diametris AL, FM.

Ducantur AC, BL, FH, GM. Quoniam *a* ang. ABC = FGH, *a* atque AB.BC :: FG. GH, *b* erit angul. ACB (*c* ALB) = FHG (*c* FMG.) anguli autem ABL, FGM *d* recti, ac proinde æquales ſunt. *e* ergo triangula ABL, FGM æquiangula ſunt. *f* quare AB. FG :: AL. FM. *g* ergo ABCDE. FGHIK :: ALq. FMq.

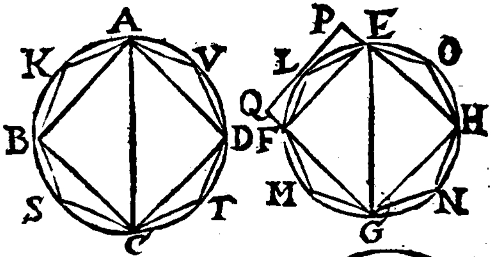
a 1.def.6
b 6.6.
c 31.3.
d 31.3.
e 32.3.
f cor.4.6
g 22.6.

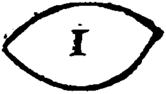
Coroll.

Hinc (quia AB. FG :: AL. FM :: BC.GH, &c.) polygonorum ſimilium circulo inſcriptorum *h* ambitus ſunt ut diametri.

h 1.12.
& 12.5.

PROP. II.





Circuli ABT, EFN inter ſe ſunt, quemadmodum quadrata à diametris AC, EG.

Ponatur ACq. EGq :: circ. ABT. I. Dico I = circ. EFN.

Nam primo, ſi fieri poteſt, ſit I ⊐ circ. EFN, ſitque exceſſus K. Circulo EFN inſcribatur quadratum EFGH, *a* quod dimidium eſt circumſcripti quadrati, adeoque ſemicirculo majus. *b* Biſeca arcus EF, FG, GH, HE, & ad puncta biſectionum junge rectas EL, LF, &c. per L duc tangentem PQ (*c* quæ ad EF parallela eſt,) & produc HEP, GFQ; eſtque triangulum ELF *d* dimidiũ parallelogrammi EPQF, adeoque majus dimidio ſegmenti ELF; pariterque reliqua triangula ejusmodi reliquorũ ſegmentorum dimidia ſuperant. Et ſi iterum biſeçentur arçus EL, LF, FM, &c. rectæque adjungantur, eodem modo triangula ſegmentorum ſemiſſes excedent. Quare ſi quadratum EFGH è circulo EFN, & è reliquis ſegmentis triangula detrahantur, & hoc fiat continuo, tandem *e* reſtabit

a ſch. 7.4

b 30.3.

c ſch. 27.3

d 41.1.

e 1.10.

ſtabit

ſtabit magnitudo aliqua minor quam K. Eouſq; perventum ſit, nempe ad ſegmenta EL, LF, FM, &c. minora quam K, ſimul ſumpta. ergo I (*f* cir. EFN − K) $\sqsupset$ polyg. ELFMGNHO (circ. EFN − ſegm. EL + LF &c.) Circulo ABT inſcriptum *g* puta ſimile polygonum AKBSCTDV. itaque quum AKBSCTDV. ELFMGNHO *h* :: ACq. EGq *k* :: circ. ABT. I ac polyg. AKBSCTDV *l* $\sqsubset$ circ. ABT. *m* erit polygon. ELFMGNHO $\sqsubset$ I. ſed prius erat I $\sqsupset$ ELFMGNHO. quæ repugnant.

f hyp. & 3. ax.
g 30. 1. & 1. poſt. 4
h 1. 12.
k hyp.
l 9. ax. 1.
m 14. 5.

Rurſus, ſi fieri poteſt, ſit I $\sqsubset$ circ. EFN. Quoniam igitur ACq. EGq *n* :: circ. ABT. I; inverſeque I. circ. ABT :: EGq. ACq. pone I. circ. ABT :: circ. EFN. K. *o* ergo circ. ABT $\sqsubset$ K. *p* atque EGq. ACq :: circ. EFN. K. Quę repugnare modo oſtenſum eſt.

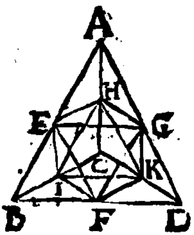
n hyp.
o 14. 5.
p 11. 5.

Ergo concludendum eſt, quod I = circ. EFN. Q.E.D.

Coroll.

Hinc, ut circulus eſt ad circulum, ita polygonum in illo deſcriptum ad ſimile polygonum in hoc deſcriptum.

PROP. III.



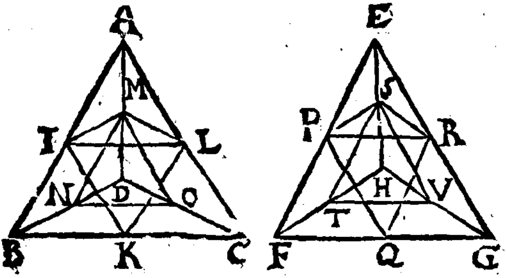
Omnis pyramis ABDC triangularem habens baſim, dividitur in duas pyramides AEGH, HIKC æquales & ſimiles inter ſe, triangulares habentes baſes, & ſimiles toti

ABDC; & in duo priſmata æqualia BFGE IH, EGDIHK; quæ duo priſmata majora ſunt dimidio totius pyramidis ABDC.

Latera pyramidis biſecentur in punctis E, F, G, H, I, K; junganturque rectæ EF, FG, GE, EI, IF, FK, KG, GH, HE. Quoniam latera py-
a 2.6. ramidis proportionaliter ſecta ſunt, *a* erunt HI, AB; & GF, AB; & IF, DC; atque HG, DC, &c. parallelæ; proinde & HI, FG, & GH, FI parallelæ ſunt. liquet igitur triangula ABD,
b 29.1 AEG, EBF, FDG, HIK *b* æquiangula eſſe; &
c 26.1. quatuor ultima *c* æquari. eodem modo triangula ACB, AHE, EIB, HIC, FGK ęquiangula ſunt; & quatuor poſtrema inter ſe æqualia. ſimiliter triangula BFI, FDK, IKC, EGH; & denuo triangula AHG, GDK, HKC, EFI, ſimilia ſunt & æqualia. Quin etiam triang. HIK ad ADB, & EGH ad BDC, & EFI ad ADC,
d 15.11 & FGK ad ABC *d* parallela ſunt. Ex quibus perſpicuè ſequitur primò, pyramides AEGH, HIKC æquales eſſe; totique ABDC, & inter
e 10.def. 11. ſe *e* ſimiles. deinde ſolida BFGEIH, FGDIHK priſmata eſſe, & quidem ęque alta, nempe ſita inter parallela plana ABD, HIK. verum baſis
f 2.ax.1. BFGE baſis FDG *f* duplex eſt. quare dicta pri-
g 40.11. ſmata ęqualia ſunt. quorum alterum BFGEIH pyramide BEFI, hoc eſt, AEGH majus eſt, totum ſua parte; proinde duo priſmata majora ſunt duabus pyramidibus, totiusque adeo pyramidis ABDC dimidium excedunt. Q.E.D.

PROP.

PROP. IV.



Si fuerint duæ pyramides ABCD, EFGH ejuſdem altitudinis, triangulares habentes baſes ABC, EFG; ſit autem illarum utraque diviſa & in duas pyramides (AILM, MNOD; & EPRS, STVH) æquales inter ſe, & ſimiles toti; & in duo priſmata æqualia (IBKLMN, KLCNMO; & PFQRST, QRGTSV;) ac eodem modo diviſa ſit utraque pyramidum, quæ ex ſuperiore diviſione natæ ſunt, idq́ ſemper fiat; erit ut unius pyramidis baſis ad alterius pyramidis baſim, ita & omnia, quæ in una pyramide, priſmata ad omnia, quæ in altera pyramide priſmata, multitudine æqualia.

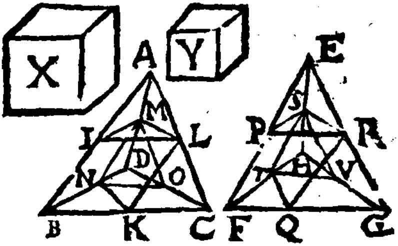
Nam (adhibendo conſtructionem præcedentis) BC.KC *a* : : FG. QG. *b* ergo triang. ABC eſt ad ſimile triang. L K C, ut EFG ad *c* ſimile RQG, ergo permutando ABC.EFG *d* :: LKC. RQG *e* :: Priſm. KLCNMO. QRGTSV (nam hęc æque alta ſunt) *f* : : IBKLMN. PFQRST. *g* quare triang. ABC. EFG : : Priſm. KLCNMO + IBKLMN. Priſm. QRGTSV + PFQRST. Q.E.D.

a 15.5.
b 22,6.
c 2.6. &c.
d 16.5.
e ſch.34.11.
f 7.5.
g 12.5.

Sin-

Sin ulterius ſimili pacto dividantur pyramides MNOD, AILM; & EPRS, STVH, erunt quatuor nova priſmata hic effecta ad quatuor iſthic producta, ut baſes M N O & AIL ad baſes STV, & EPR, hoc eſt ut L K C ad RQG,
b 11.5. vel ut A B C ad EFG. *b* quare omnia priſmata pyramidis A B C D ad omnia ipſius E F G H ita ſe habent, ut baſis A B C ad baſim E F G. Q. E. D.

PROP. V.



Sub eadem altitudine exiſtentes pyramides ABCD, EFGH, triangulares habentes baſes ABC, EFG, inter ſe ſunt ut baſes ABC, EFG.

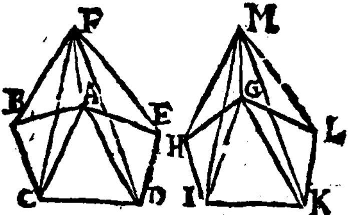
Sit triang. ABC. EFG : : ABCD. X. Dico X. = pyr. EFGH. Nam, ſi poſſibile eſt, ſit X ¬ EFGH; ſitque Y exceſſus. Dividatur pyramis E F G H in priſmata & pyramides, & reliquæ
a 1.10. pyramides ſimiliter, *a* donec relictæ pyramides E P R S, S T V H minores evadant ſolido Y. Quum igitur pyr. EFGH = X + Y; liquet reliqua priſmata PFQRST, QRGTSV ſolido X majora eſſe. Pyramidem A B C D ſimiliter
b 4.12. diviſam concipe; *b* eritque priſm. IBKLMN

+ K-

+ KLCNMO. PFQRST + QRGTSV :: ABC. EFG. *c* :: pyr. ABCD. X. *d* ergo X ⊏ priſm. PFQRST + QRGTSV; quod repugnat prius affirmatis. c hyp. d 14.5.

Rurſus, dic X ⊏ pyr. EFGH. pone pyr. EFGH. Y :: X. pyr. ABCD *e* :: EFG. ABC. quia EFGH *f* ⊐ X, *g* erit Y ⊐ pyr. ABCD, quod fieri nequit, ex jam dictis. Concludo igitur, quod X = pyr. EFGH. Q.E.D. e hyp. & cor. 4. 5. f ſuppoſ. g 14.5.

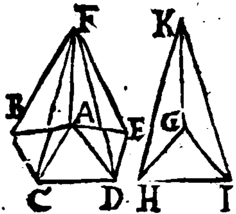
PROP. VI.



Sub eadem altitudine exiſtentes pyramides ABCDEF, GHIKLM, & polygonas habentes baſes ABCDE, GHIKL, inter ſe ſunt ut baſes ABCDE, GHIKL.

Duc rectas AC, AD, GI, GK. Eſt baſ. ABC. ACD *a* :: pyr. ABCF. ACDF. ergo compoſite ABCD. ACD :: pyr. ABCDF. ACDF. *a* atqui etiam ACD. ADE :: pyr. ACDF. ADEF. *c* ergo ex æquali ABCD. ADE :: ABCDF. ADEF. *b* ergo componendo ABCDE. ADE :: pyr. ABCDEF. ADEF. porro ADE. GKL *d* :: pyr. ADEF. GKLM; ac, ut prius, atque inverſe GKL. GHIKL :: pyr. GKLM. GHIKLM *c* ergo iterum ex equalibus, ABCDE. GHIKL :: Pyr. ABCDEF. GHIKLM. Q.E.D. a 5.12. b 18.5. c 21.5. d 5.12.

Si

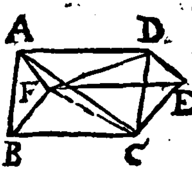


Si baſes non habent latera ęque multa, demonſtratio ſic procedet. Baſ. A B C. GHI e :: pyr. ABCF. GHIK. e atque A C D. GHI :: pyr. ACDF. G H I K. f ergo baſ. ABCD. G HI :: pyram. A B C D F. G H I K. e Quin etiam baſ. A D E. G H I :: pyram. A D E F. G H I K. f ergo baſ. ABCDE. GHI :: pyr. A B-CDEF. GHIK.

e 5.12.

f 24.5.

PROP. VII.



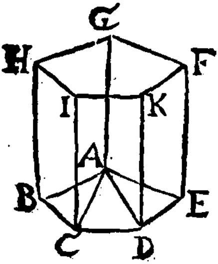
Omne priſma A B C-D F E triangularem habens baſim, dividitur in tres pyramides A C B F, A C D F, C D F E, æquales inter ſe, triangulares baſes habentes.

Ducantur parallelogrammorum diametri AC, CF, FD. Triang. ACB a = ACD. b ergo æque altę pyramides ACBF, ACDF ęquantur. eodem modo pyr. D F A C = pyr. DFEC. atqui ACDF, & D F A C una eademque ſunt pyramis. c ergo tres pyramides ACBF, ACDF, DFEC, in quas diviſum eſt priſma, inter ſe æquales ſunt. Q.E.D.

a 34.1.
b 5.12.
c 1.ax.1.

Coroll.

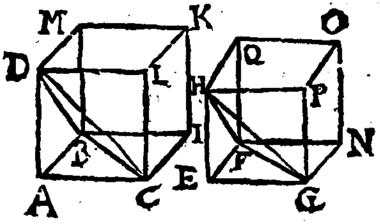
Coroll.



Hinc, quælibet pyramis tertia eſt pars priſmatis eandem cum illa habentis & baſim & altitudinem: ſive, priſma quodlibet triplum eſt pyramidis eandem cum ipſo habentis baſim & altitudinem.

Nam reſolve priſma polygonum ABCDEGHIKF in trigona priſmata, & pyramidem
ABCDEH in trigonas pyramides. *a* Erunt a 7.12.
ſingulæ partes priſmatis triplæ ſingularum partium pyramidis. *b* proinde totum priſma AB- b 1.5.
CDEGHIKF totius pyramidis ABCDEH triplum eſt. Q.E.D.

PROP. VIII.



Similes pyramides ABCD, EFGH, quæ triangulares habent baſes ABC, EFG, in triplicata ſunt ratione homologorum laterum AC, EG.

a Perficiantur parallelepipeda ABICDM-
KL, EFNGHQOP; quæ *b* ſimilia ſunt & py- a 27.11. b 9.def.
rami- 11.

c 31.11. ramidum ABCD, EFGH *c* ſextupla; *d* ideoq;
& 7.12. in eadem cum ipſis ratione ad ſe invicem, *e* hoc
d 15.5.
e 33.11. eſt in triplicata homologorum laterum. Q.E.D.

Coroll.

Hinc, etiam ſimiles polygonæ pyramides rationem habent laterum homologorum triplicatam; ut facile probabitur reſolvendo has in trigonas pyramides.

PROP. IX.

Vide Schema praeced.

Æqualium pyramidum ABCD, EFGH, & triangulares baſes ABC, EFG habentium, reciprocantur baſes & altitudines, & quarum pyramidum triangulares baſes habentium reciprocantur baſes & altitudines, illa ſunt aquales.

1. *Hyp.* Perfecta parallelepipeda ABICDMKL, EFNGHQOP æqualium pyramidum,
a 28.11. ABCD, EFGH (utrumque utriusque) *a* ſex-
& 7.12. tupla ſunt, ac æqualia ideo inter ſe, ergo alt.(H.)
b 34.11. alt.(D) *b* :: ABIC. EFNG. *c* :: ABC. EFG.
c 15.5. Q.E.D.

d hyp. 2. *Hyp.* Alt.(H.) alt.(D) *d* :: ABC.EFG *e* ::
e 15.5. ABIC.EFNG. *f* ergo parallelepipeda ABIC-
f 34.11. DMKL, EFNGHQOP æquantur; *g* proinde
g 6.ax.1. & pyramides ABCD, EFGH, horum ſubſextuplæ, pares ſunt. Q.E.D.

Eadem polygonis pyramidibus conveniunt: nam ha ad trigonas reduci poſſunt.

Coroll.

Qua de pyramidibus demonſtrata ſunt Prop. 6, 8, 9. etiam conveniunt quibuscunque priſmatis, cum hac tripla ſint pyramidum eandem baſim & alti-

altitudinem habentium. itaque 1. Priſmatum æque altorum eadem eſt proportio, quæ baſium.

2. Similium priſmatum proportio triplicata eſt proportionis laterum homologorum.

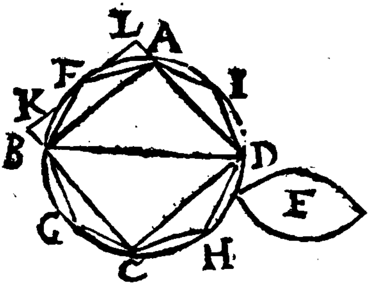
3. Æqualia priſmata reciprocant baſes & altitudines; & quæ reciprocant, ſunt æquales.

Schol.

Ex hactenus demonſtratis elicitur dimenſio quorumcunque priſmatum & pyramidum.

a Priſmatis ſoliditas producitur ex altitudine in baſim ducta; *b* itaque & pyramidis ex tertia altitudinis parte ducta in baſim. *a cor. 7. 1. hujus & ſch. 40. 11. b 9. 12.*

PROP. X.



Omnis conus tertia pars eſt cylindri habentis eandem cum ipſo baſin ABCD, & altitudinem æqualem.

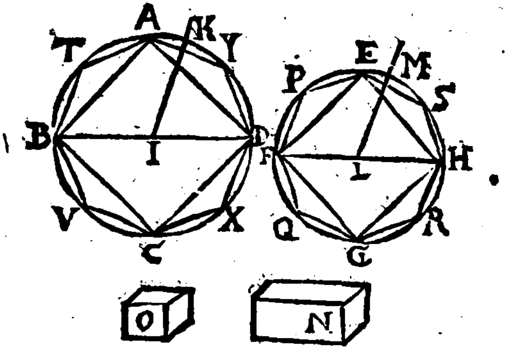
Si negas, primo Cylindrus triplum coni ſuperet exceſſu E. Priſma ſuper quadratum circulo ABCD inſcriptum *a* ſubduplum eſt priſmatis ſuper quadratū eidem circulo circumſcriptum ſibi & cylindro æque alti. ergo priſma ſuper qua- *Vide fig. 2. hujus a ſch. 7. & cor. 9. 12.*

quadratum A B C D ſuperat cylindri ſemiſſem.
eodem modo priſma ſuper baſim AFB cylindro
b ſch. 27. 5. & cor. ęque altum ſegmenti cylindrici AFB *b* dimidio
9. 12. majus eſt. Continuetur biſectio arcuum, & de-
trahantur priſmata, donec ſegmenta cylindri
relicta, nempe ad A F, F B, &c. minora evadant
ſolido E. Itaque cylind. -- ſegment. A F, F B,
c 5. ax. 1. &c. (priſma ad baſim AFBGCHDI) *c* majus
d hyp. eſt quam cylind — E (*d* triplum coni.) ergo
e cor. 7. 12 pyramis dicti priſmatis *e* pars tertia (ad eandem
baſim ſita, ejusdemque altitudinis) cono ęque
alto ad baſim ABCD circulum major eſt, pars
toto. Q.E.A.

Sin conus tertia parte cylindri major dica-
tur, ſit itidem exceſſus E. Ex cono detrahe py-
ramides, ut in priori parte priſmata ex cylindro,
donec reſtent coni ſegmenta aliqua, puta ad
AF, FB, BG, &c. minora ſolido E. ergo con. —
f hyp. E (*f* $\frac{1}{3}$ cylindr..) ⊐ pyr. AFBGCHDI (con. —
ſegment. A F, FB, &c.) ergo priſma pyramidis
triplum (æque altum ſcilicet atque ad eandem
baſim) cylindro ad baſim A B C D majus eſt,
pars toto. Q. E. A. Quare fatendum eſt, quod
cylindrus triplo cono æquatur. Q. E. D.

PROP.

PROP. XI.



Sub eadem altitudine exiſtentes cylindri, & coni ABCDK, EFGHM, inter ſe ſunt ut baſes ABCD, EFGH.

Sit circ. ABCD. circ. EFGH : : con. ABCDK. N. Dico N = con. EFGHM.

Nam ſi fieri poteſt, ſit N ⊐ con. EFGHM, ſitque exceſſus O. Suppoſita præparatione, & argumentatione præcedentis; erit O majus ſegmentis conicis EP, PF, FQ, &c. ideoque ſolidum N ⊐ pyr. EPFQGRHSM. *a* Fiat in circulo ABCD ſimile polygonum ATBVCXDY. Quia pyr. ABVYK. pyr. EFQSM *b* : : polyg. ATBVY. polyg. EPFQS *c* : : circ. ABCD. circ. EFGH *d* : : con. ABCDK. N. *e* erit pyr. EPFQGRHSM ⊐ N. contra modo dicta.

a 30.3. & 1.poſt.
b 6.12.
c cor.2. 12.
d hyp.
e 14.5.

Rurſus dic N ⊏ con. EFGHM. pone con. EFGHM. O : : N. con. ABCDK *f* : : circ. EFGH. ABCD. *g* ergo O ⊐ con. ABCDK, quod

quod abſurdum eſt, ex oſtenſis in priori parte. f hyp. & invertendo.

Itaque potius dic, ABCD. EFGH :: con. ABCDK. EFGHM. Q.E.D. g 14.5.

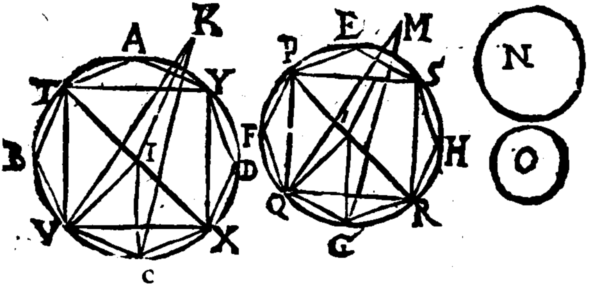
Idem demonſtrabitur de cylindris, ſi conorum & pyramidum loco concipiantur cylindri & priſmata. ergo, &c.

Schol.

Ex his habetur dimenſio cylindrorum & conorum quorumcunque. Cylindri rectæ ſoliditas producitur ex baſe circulari (a pro cujus dimenſione conſulendus eſt Archimedes) ducta in altitudinem. b igitur & cujuſcunq; cylindri. c Itaque coni ſoliditas producitur ex tertia parte altitudinis ducta in baſim.

a 1. Prop. de dimenſ. circ.
b 11. 12.
c 10. 12.

PROP. XII.



Similes coni & cylindri ABCDK, EFGHM, in triplicata ratione ſunt diametrorum TX, PR, quæ in baſibus ABCD, EFGH.

Habeat conus A ad aliquod N rationem triplicatam TX ad PR. dico N = con. EFGHM: Nam ſi fieri poteſt, ſit N ⊐ EFGHM; ſitque exceſſus O. ergo ut in Prioribus, N ⊐ pyr.

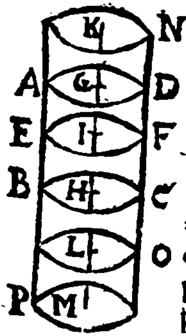
EPFQGRHSM. Sint axes conorum IKLM,
adducanturque rectæ VK, CK, VI, CI; & QM,
GM, QL, GL. Quoniam coni ſimiles ſunt, *a* eſt — a 24.
VI. IK :: QL. LM. anguli vero VIK, QLM *b* — def 11.
recti ſunt. *c* ergo trigona VIK, QLM æquian- — b 18. def. 11.
gula ſunt; *d* unde VC. VI :: QG. QL. item — c 6.6.
VI. VK :: QL. QM. ergo ex æquali VC. VK — d 4.6.
:: QG. QM. *e* quin etiam VK. CK :: QM. — e 7.5.
MG. ergo rurſus ex æquo VC. CK :: QG.
GM. *f* ergo triangula VKC, QMG ſimilia — f 5.6.
ſunt; ſimilique argumento reliqua hujus pyra-
midis triangula reliquis illius. *g* quare pyrami-
des ipſæ ſimiles ſunt. *h* ſunt vero hæ in triplicata — g 9. def. 11.
ratione VC ad QG, *k* hoc eſt VI ad RL, *l* vel — h cor. 8. 12.
TX ad PR. *m* ergo Pyr. AIBVCXDYK. pyr. — k 4.6.
EPFQGRHSM :: con. ABCDKN. *n* unde — l 15.5.
pyr. EPFQGRHSM $\sqsupset$ N; quod repugnat — m hyp. & 11.5.
prius dictis. — n 14.5.

Rurſus, dic N. $\sqsubset$ con. EFGHM. ſit con. E-
FGHM. O :: N. con. ABCDK *o* :: pyr. EPR- — o Prius, & inverſe.
M. ATCK *p* :: GQ. VC ter :: *q* PR. TX
ter. verum O *r* $\sqsupset$ ABCDK. quod modo re- — p cor. 8. 12.
pugnare oſtenſum eſt. Proinde N = con. EF- — q 4.6.
GHM. Q.E.D. — r 14.5.

Y PROP.

Quoniam vero quam proportionẽ habent coni, eandem quoque obtinent cylindri, eorum tripli, habebit quoque cylindrus ad cylindrum proportionem diametrorũ in baſibus triplicatã.

PROP. XIII.



Si cylindrus ABCD plano EF ſecetur adverſis planis BC, AD parallelo; erit ut cylindrus AEFD ad cylindrum EBCF, ita axis GI ad axem IH.

Producto axe, *a* ſume GK = GI, & HL = IH = LM. & concipe per puncta K, L, M, plana duci circulis AD, BC parallela. *b* ergo cylind. ED = cylind. AN. & cylin. EC *b* = BO *b* = OP. itaque cylindrus EN cylindri ED æque multiplex eſt, ac axis IK axis IG. pariterque cylindrus FP æque multiplex eſt cylindri BF, ac axis IM axis IH. prout vero IK =, ⊏, ⊐ IM, *c* ſic cylindr. EN =, ⊏, ⊐ EP. *d* ergo cylindr. AEFD. cylind. EBCF ;: GI. IH. Q.E.D.

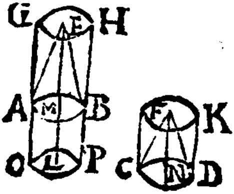
a 3. 1.

b 11. 11.

c 11. 12.

d 6. def. 5

PROP. XIV.



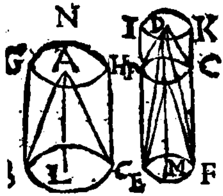
Super æqualibus baſibus AB, CD exiſtentes coni AEB, CFD, & cylindri AH, CK, inter ſe ſunt ut altitudines ME, NF.

Productis cylindro HA & axe EM, ſume MI

MI = FN; & per punctum L ducatur planum aſi A B parallelum. a erit cylindr. A P = CK. atqui cylind. A H. AP. (CK) : : M E. M L. NF.) Q.E.D. Idem de conis cylindrorum ubtriplis dictum puta. * imo de priſmatis & yramidibus.

a 11. 12.
b 13. 12
* Adhibe 9. & 7. 12.

PROP. XV.



Æqualium conorum BAC, EDF, & cylindrorum BH, EK, reciprocantur baſes & altitudines (BC. EF : : MD. L A :) *& quorum conorum, & cylindrorum eciprocantur baſes & altitudines, illi ſunt æquales.*

Si altitudines pares ſint, etiam baſes pares eſunt; & res clara eſt. Sin altitudines ſunt impaes, aufer MO = LA,

1. *Hyp.* Eſtque M D. M O (a LA) b : : cylindr. E K (c B H.) E Q d : : circ. B C. E F. Q.E.D.

2. *Hyp.* B C. E F e : : D M. O M (L A) f : : Cyl. E K. EQ g : : BC. EF h : : BH. E Q. k Ergo cylind. EK = BH. Q.E.D.

a 14. 12.
b conſtr.
c hyp.
d 11. 12
e hyp.
f 11. 12.
g 11. 5.
h 11. 12.
k 9. 5.

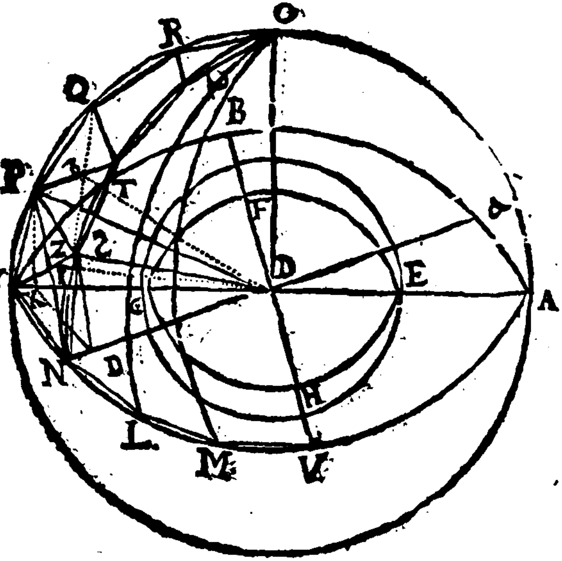
Simili argumento utere de conis.

PROP. XVI.

Duobus circulis ABCG, DEF. circa idem centrum M exiſtentibus, in majori circulo ABCG polygonum æquilaterum, & parium laterum inſcribere, quod non tangat minorem circulum DEF.

Per centrum M extendatur recta AC ſecans circulum DEF in F. ex quo erige perpendicularem FH. a 30.3. *a* Biſeca ſemicirculum ABC, & b 1.10. jusque ſemiſſem BC, atque ita continuo, *b* donec arcus IC minor evadat arcu HC. ab I demitte perpendicularem IL. Liquet arcum IC totum circulum metiri, numerumque arcuum c ſch,16.4 eſſe parem, adeoque ſubtenſam IC latus eſſe *c* polygoni inſcriptibilis, quod circulum DEF minime continget. Nam HG *d* tangit circu- d cor,16.3, lum DEF; *e* cui parallela eſt IK, extraque ſi- e 28.1, f 34.def.1 ta, *f* quare IK circulum non tangit, multoque magis CI, CK, & reliqua polygoni latera, longius à centro diſtantia, circulum DEF non tangunt. Q.E.F. Coroll. Nota, quod IK non tangit circulum DEF.

PROP. XVII.



Duabus ſphæris ABCV, EFGH circa idem centrum D exiſtentibus, in majori ſphæra ABCV ſolidum polyedrum inſcribere, quod non tangat ſuperficiem minoris ſphæra EFGH.

Secentur ambæ ſphæræ plano per centrum faciente circulos EFGH, ABCV. ducanturque diametri AC, BV ſecantes perpendiculariter. Circulo ABCV *a* inſcribatur polygonum æquilaterum VMLNC, &c. circulum EFGH minime tangens. ducta diametro Nα, erectaque *a 16.12.*

DO recta ad planum ABC. per DO, perq; diametros AC, Nα erigi concipiantur plana DOC,
b 18.11. DO N, quæ ad circulum A B C V *b* rectæ erunt,
c cor.33.6. ideoque in ſuperficie ſphæræ *c* quadrantes
d 4.1. efficient D O C, D O N. in quibus *d* aptentur
rectæ CP, PQ, QR, RO, NS, ST, Tγ, γO ipſis
CN, NL, &c. pares, & æque multæ. In reliquis
quadrantibus OL, OM, &c. inque tota ſphæra
eadem conſtructio fiat. Dico factum.

A punctis P, S ad planum A B C V demitte
e 38.11. perpendiculares PX, SY, *e* quę in ſectiones AC,
f 12.ax. Nα cadent. Quoniam igitur tam *f* anguli recti
g 27.3. PXC, SYN, *g* quam PCX, SNY *h* ęqualibus peripheriis
inſiſtentes, *f* pares ſunt, triangula PCX,
h 32.1. SNY *h* æquiangula ſunt. Cum igitur PC *k*
k constr.
l 26.1. = S N, *l* etiam P X = S Y, *l* & X C = Y N; *m*
m 3.ax. quare DX = DY. *n* ergo DX.XC :: DY.YN.
n 7.5.
o 2.6. ergo parallelæ ſunt YX, N C. quia vero PX, SY
pares, & cum eidem plano ABCV rectæ, etiam
p 6.11. *p* parallelæ ſunt. *q* erunt YX, SP etiam pares &
q 33.1. parallelæ. *r* ergo SP, NC inter ſe parallelæ ſunt.
r 9.11.
s 7.11. ergo *ſ* quadrilaterum NCPS, eademque ratione
t 2.11. SPQT, TQRG, ſed & *t* triangulum γ R O
totidem ſunt plana. Eodem modo tota ſphæra
ejusmodi quadrilateris & triangulis repleta oſtendetur,
quare inſcriptum eſt polyedrum.

u 11.11. A centro D *u* duc D Z rectum plano N CPS;
& junge ZN, ZC, ZS, ZP. Quoniam DN.
x 4.6. NC *x* :: DY.YX; eſt NC *y* ⊏ Y X (SP;) pa-
y 14.5. riterque SP ⊏ TQ, & TQ ⊏ γR. Et quia an-
z 3.def.11. guli DZC, DZN, DZS, DZP, *z* recti ſunt, la-

tera

ra vero DC, DN, DS, DP *a* æqualia, & DZ a 15. def.
ommune, *b* erunt ZC, ZN, ZS, ZP æquales 1.
ter ſe; proinde circa quadrilaterum NCPS b 47. 1.
c deſcribi poteſt circulus, in quo (ob NS, NC, c 15. def. 1.
P *d* æquales, & $NC \sqsubset SP$) NC *e* plus quam d conſtr.
uadrantem ſubtendit. *f* ergo angul. NZC ad e 28. 3.
entrum obtuſus eſt. *g* ergo $NCq \sqsubset 2\,ZCq$ f 33. 6.
(ZCq + ZNq.) Sit NI ad AC normalis. ergo g 12. 2.
cum ang. ADN (*h* DNC + DCN) ſit *k* ob-
tuſus, *l* erit ſemiſſis ejus DCN recti ſemiſſe h 32. 1.
major; propterea que eo minor eſt reliquus è k 9. ax. 1.
recto ang. CNI. *n* unde $IN \sqsubset IC$. ergo NCq l 5. 1.
(NIq + ICq) *o*. $\sqsupset 2\,INq$. itaque IN n 19. 1.
$\sqsubset ZC$. & conſequenter DZ *p* $\sqsubset$ DI. at- o 47. 1.
qui punctum I eſt *q* extra ſphæram EFGH. p 47. 1.
ergo punctum Z potiori jure eſt extra ipſam. q cor. 16. 12.
adeoque planum NCPS (cujus *r* proximum
centro punctum eſt Z) ſphæram EFGH non r 47. 1.
contingit. Et ſi ad planum SPQT demitta-
tur perpendicularis Dδ, punctum δ, adeoque
& planum SPQT adhuc ulterius à centro e-
longatur; idemque eſt de reliquis polyedri pla-
nis. ergo polyedrum ORQPCN, &c. ma-
jori ſphæræ inſcriptum, minorem non contin-
git. Q.E.F.

Coroll.

Hinc ſequitur, *Si in quavis alia ſphæra deſcribatur ſolidum polyedrum, ſimile prædicto ſolido polyedro, proportionem polyedri in una ſphæra*

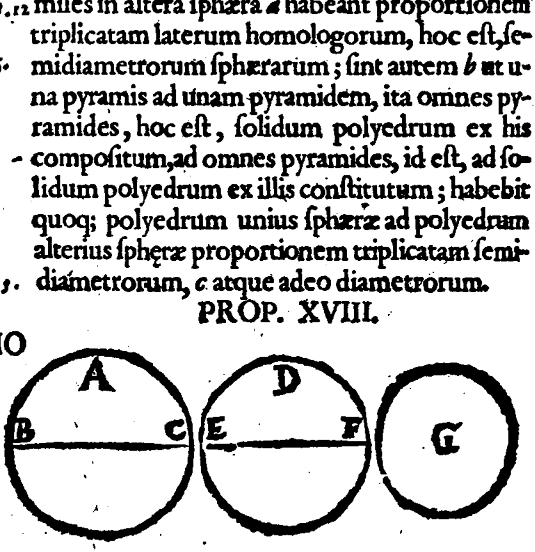
ad

ad polyedrum in altera eſſe triplicatam eius quam habent ſphærarum diametri.

Nam ſi ex centris ſphęrarum ad omnes angulos baſium dictorum polyedrorum rectæ lineæ ducantur, diſtribuentur polyedra in pyramides numero ęquales & ſimiles, quarum homologa latera ſunt ſemidiametri ſphęrarum; ut conſtat, ſi intelligatur harum ſphærarum minor intra majorem circa idem centrum deſcripta. congruent enim ſibi mutuo lineæ rectę ductæ à centro ſphęræ ad baſium angulos, ob ſimilitudinem baſium, ac propterea pyramides efficientur ſimiles. Quare cum ſingulæ pyramides in una ſphæra, ad ſingulas pyramides illis ſi-
a cor 8.12 miles in altera ſphæra *a* habeant proportionem triplicatam laterum homologorum, hoc eſt, ſe-
b 12.5. midiametrorum ſphærarum; ſint autem *b* ut una pyramis ad unam pyramidem, ita omnes pyramides, hoc eſt, ſolidum polyedrum ex his compoſitum, ad omnes pyramides, id eſt, ad ſolidum polyedrum ex illis conſtitutum; habebit quoq; polyedrum unius ſphæræ ad polyedrum alterius ſphęræ proportionem triplicatam ſemi-
c 15.5. diametrorum, *c* atque adeo diametrorum.

PROP. XVIII.

HO



Sphæra BAC, EDF ſunt in triplicata ratione ſuarum diametrorum BCEF.

Sit ſphæra B A C ad ſphæram G in triplicata ratione diametri B C ad diametrum EF. Dico $G = EDF$. Nam ſi fieri poteſt, ſit $G \sqsupset EDF$. & cogita ſphæram G concentricam eſſe ipſi
E D F. Sphæræ E D F *a* polyedrum ſphęram G a 17.12.
non tangens, ſphæræque B A C ſimile polyedrum b cor.17.12.
inſcribatur. *b* Hæc polyedra ſunt in triplicata
ratione diametrorum B C, E F, *c* id eſt, c hyp.
ſphæræ BAC ad G. *d* Proinde ſphæra G major d 14.5.
eſt polyedro ſpærę EDF inſcripto, pars toto.

Rurſus, ſi fieri poteſt, ſit ſphæra $G \sqsubset EDF$. Sitque ut ſphæra EDF ad aliam ſphæram H, ita G ad B A C, *e* hoc eſt in triplicata ratione diametri E F ad BC; cum igitur BAC *f* $\sqsubset$ H, in- e hyp. inverſe
currimus abſurditatem prioris partis. Quin po- f 14.5.
tius ſphæra $G = EDF$. Q.E.D.

Coroll.

Hinc, ut ſphæra ad ſphæram, ita eſt polyedrum in illa deſcriptum ad polyedrum ſimile in hac deſcriptum.

LIB. XIII.

PROP. I.

SI recta linea Z ſecundum extremam & mediam rationem ſecetur (z. a : : a. e;) *majus ſegmentum* a *aſſumens dimidium totius z, quintuplum poteſt ejus, quod à dimidia totius z deſcribitur, quadrati.*

Dico Q. a $-$ $\frac{1}{2}$z = 5 Q: $\frac{1}{2}$z. *a*
a 4.2. hoc eſt aa + $\frac{1}{4}$zz
b 3.ax.1. + za = zz + $\frac{1}{4}$ zz. *b* vel aa + za = zz. Nam ze
+ za *c* = zz & ze *d* = aa. *e* ergo aa + za = zz.
c 2.2.
d hyp. & 16.6. Q.E.D.
e 2 ax. & 1.ax.

PROP. II.

Si recta linea $\frac{1}{2}$z + a ſui ipſius ſegmenti $\frac{1}{2}$z quintuplum poſſit, duplæ prædicti ſegmenti (z) *extrema ac media ratione ſectæ majus ſegmentum eſt* a, *reliqua pars ejus quæ à principio rectæ* $\frac{1}{2}$z + a.

Dico z. a : : a. e. Nam quia per hyp. *aa +
a 4.2. $\frac{1}{4}$zz + za = zz + $\frac{1}{4}$zz ; vel aa + za = zz *a* =
a 2.2.
b 3.ax.1. ze + za. *b* erit aa = ze. *c* quare z. a : : a. e.
c 17.6. Q.E.D.

Vid. fig. præced.

PROP. III.

Si recta linea z *ſecundum extremam ac mediam rationem ſecetur* (z. a : : a. e ;) *minus ſegmentum* e *aſſumens dimidium majoris ſegmenti* a, *quintuplum poteſt ejus, quod à dimidia majoris ſegmenti* a *deſcribitur, quadrati.*

Dico Q: e + $\frac{1}{2}$a =
a 4.2. 5 Q: $\frac{1}{2}$ a. *a* hoc eſt ee
+ $\frac{1}{4}$ aa + ea = aa
+ $\frac{1}{4}$

+ $\frac{1}{4}$aa. *b* vel ee + ea = aa. Nam ee + ea *c* = b 3.ax.
ae *d* = aa. Q.E.D. c 3.2. d hyp. & 6.

PROP. IV.

Si recta linea z ſecundum extremam ac mediam rationem ſecetur (z.a::a.e;) quod à tota z, quodque à minori ſegmento e, utraque ſimul quadrata, tripla ſunt ejus, quod à majori ſegmento a deſcribitur, quadrati.

Dico zz + ee = 3 aa. *a* vel aa + ee + 2 ae + ee = 3 aa. Nam ae + ee *b* = ze *c* = aa. *d* ergo aa + 2 ae + 2 ee = 3 aa. Q.E.D.

a 4.2. b 3.2. c 17.6. d 2.ax.

PROP. V.

Si recta linea A B ſecundum extremam & mediam rationem ſecetur in C, apponaturque ei A D æqualis majori ſegmento A C; tota recta linea D B ſecundum extremam ac mediam rationem ſecatur, & majus ſegmentum eſt quæ à principio recta linea A B.

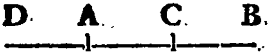
Nam quia AB.AD *a* :: AC.CB, invertendoque AD.AB :: CB.AC; erit componendo DB.AB :: AB.AC.(AD.) Q.E.D. a hyp.

SCHOL.

Quod ſi fuerit BD.BA::BA.AD. erit BA.AD::AD.BA—AD. Nam dividendo eſt BD—BA (AD) BA :: BA—AD.AD. ergo inverſe, BA.AD :: AD.BA—AD. Q.E.D.

PROP.

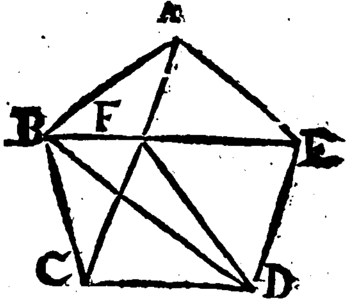
PROP. VI.



Si recta linea rationalis AB extrema ac media ratione ſecetur in C; utrumq; ſegmentorum (AC, CB) irrationalis eſt linea, quæ vocatur apotome.

a 3.1. b 1.13. Majori ſegmento AC a adde AD = $\frac{1}{2}$ AB; b ergo DCq = 5 DAq. c ergo DCq ⊏ DAq. c sch.12.10. proinde cum AB, e ideoque ejus ſemiſſis DA ſint ϱ, etiam DC eſt ϱ. Quia vero 5, 1 ∷ non f 9.10. Q. Q. f eſt DC ⊏ DA. g ergo DC — AD, g 74.10. h 17.6. id eſt AC eſt apotome. Inſuper quia ACq h = k 98.10. ABxBC, & AB eſt ϱ, k etiam BC eſt apotome. Q.E.D.

PROP. VII.



Si pentagoni æquilateri ABCDE tres anguli, ſive qui deinceps EAB, ABC, BCD, ſive EAB, BCD, CDE qui non deinceps ſint, æquales fuerint, æquiangulum erit ipſum pentagonum ABCDE.

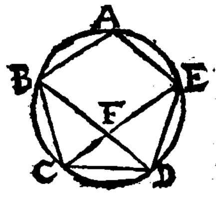
Pari-

Paribus deinceps angulis ſubtendantur rectæ BE, AC, BD.

Quoniam latera EA, AB, BC, CD, anguliq;
incluſi *a* æquantur, *b* erunt baſes BE, AC, BD, a hyp.
c angulique AEB, ABE, BAC, BCA pares. *d* b 4.1.
quare BF = FA, & *e* proinde FC = FE. ergo c 4.& 5.1. d 6.1.
triangula FCD, FED ſibi mutuo æquilatera e 3.ax.1.
ſunt; *f* unde ang. FCD = FED, *g* proinde ang. f 8.1. g 2.ax.1.
AED = BCD. Eodem pacto ang. CDE reliquis æquatur. quare pentagonum æquiangulum eſt. Q.E.D.

Sin anguli EAB, BCD, CDE, qui non deinceps, ſtatuantur pares, *h* erit ang. AEB = BDC,
& BE = BD, *k* ideoque ang. BED = BDE; *l* h 4.1. k 5.1.
totus proinde ang. AED = CDE. ergo pro- l 2.ax.
pter angulos A, E, D deinceps æquales, ut prius, pentagonum æquiangulum erit. Q.E.D.

PROP. VIII.

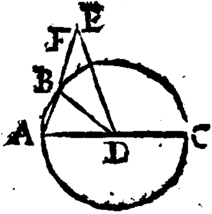


Si pentagoni æquilateri & æquianguli ABCDE duos angulos BCD, CDE, qui deinceps ſint, ſubtendant rectæ lineæ BD, CE; hæ extrema ac media ratione ſe mutuo ſecant, & majora ipſarum ſegmenta BF, vel EF æqualia ſunt pentagoni lateri BC.

Circa pentagonum *a* deſcribe circulum A-
BD. *b* Arcus ED = BC. *c* ergo ang. FCD = F- a 14.4. b 28.3.
DC. *d* ergo ang. BFC = 2 FCD (FCD + F- c 27.3.
DC.) Atqui arcus BAE *b* = 2 ED, proinde ang.
BCF

BCF *e* = 2 FCD = BFG *f* quare BF = BC.
e 33.6. Q.E.D. Porro quia triangula BCD, FCD *g* æ-
f 6.1. quiangula ſunt, *h* erit BD. DC (BF) : : CD (B
g 27.3. h 4.6. F.) FD. pariterque EC. EF : : EF. FC. Q.E.D.

PROP. IX.



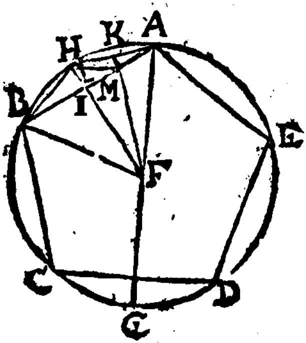
Si hexagoni latus BE, & decagoni AB, in eodem circulo ABC deſcriptorum componantur, tota recta linea AE extrema ac media ratione ſecatur, (AE. BE : : BE. AB.) *& majus ejus ſegmentum eſt hexagoni latus BE.*

Duc diametum ADC, & junge rectas DB,
a pr.p.& 31.3. DE. Quoniam ang. BDC *a* = 4. BDA, eſtque
b 32.1. ang. BDC *b* = 2 DBA (DAB + DBA,) erit
c 7.ax.1. DBA (*b* BDE + BED) *c* = 2 BDA *d* = 2 B-
d 5.1. e 1.ax.1. DE. proinde ang. DBA, vel DAB *e* = ADE.
Itaque trigona ADE, ADB equiangula ſunt,
f 4.6. g cor.15.4. *f* quare AE. AD. (*g* BE) : : AD. (BE.) AB.
Q.E.D.

Coroll.

Hinc, ſi latus hexagoni alicujus circuli ſece-
h 5.13. tur extrema ac media ratione; majus illius ſegmentum erit latus decagoni ejusdem circuli.

PROP. X.



Si in circulo A B C E pentagonum æquilaterum ABCDE deſcribatur; pentagoni latus AB poteſt & hexagoni latus FB, & decagoni latus AH, in eodem circulo deſcriptorum.

*D*uc diametrum A G. Biſeca arcum A H in K. Et duc FK, FH, FB, BH, HM.

Semicirc. AG −− arc. AC *a* = A G—AD. a 28.3.
hoc eſt arc. C G $\frac{1}{2}$ = G D *b* = A H = H B. ergo & 3.ax.
arc. BCG = 2 BHK; *c* adeoque ang. B F G = b hyp. & 7.ax.
2 BFK. *d* ſed ang. BFG = 2 BAG. *e* ergo ang. c 33.6.
BFK = BAG. Trigona igitur BFM, F A B *f* æ- d 20.3.
quiangula ſunt. *g* quare AB. BF : : BF. BM. *h* e 1.ax.1.
ergo ABxBM = BFq. Rurſus ang. A F K *k* = f 32.1.
HFK; & FA = FH; *m* quare AL = L H, *m* & g 4.6.
anguli FLA, FLH pares, ac proinde recti ſunt. h 17.6.
ergo ang. LHM *m* = LAM *n* = HBA. Trigona k 27.3.
igitur AHB, AMH *o* æquiangula ſunt. *p* quare m 4.1.
AB.AH : : AH. AM. *q* ergo ABxAM = AHq. n 27.3.
Quum igitur ABq *r* = ABxBM + ABxAM, *ſ* o 32.1.
erit ABq = BFq + AHq. Q.E.D. p 4.6 q 17.6. r 2.2. ſ 2.ax.

Co-

Coroll.

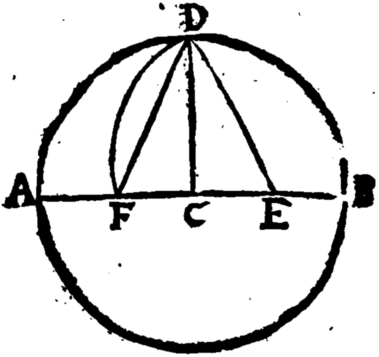
1. Hinc, linea recta (FK) quæ ex centro (F) arcum quempiam (HA) biſecat, etiam rectam (HA) illi arcui ſubtenſam biſecat ad angulos rectos.

2. Diameter circuli (A G) ex angulo quovis (A) pentagoni ducta biſecat & arcum (CD,) quem latus pentagoni illi angulo oppoſitum ſubtendit, & latus ipſum (C *D*) oppoſitum, idque ad angulos rectos.

Schol.

Hic, ut promiſimus, praxin trademus expeditam problematis 11.4.

Problema.



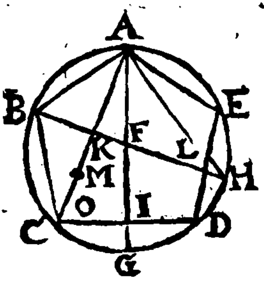
Invenire latus pentagoni circulo A D B inſcribendi.

Duc diametrum A B. cui perpendicularem C *D* ex centro C erige. Biſeca C B in E. Fac EF = E*D*. Erit *D* F pentagoni latus.

Nam

Nam BF x FC + ECq *a* = EFq *b* = EDq *c* a 6.2.
= DCq + ECq. *d* ergo BF x FC = DCq, vel b conſtr.
BCq. *e* quare BF. BC :: BC. FC. ergo quum c 47.1.
BC ſit latus hexagoni, *f* erit FC latus decagoni, d 3 ax.
proinde DF *h* = √ DCq + FCq *g* eſt latus e 17.6.
pentagoni. Q.E.F. f 9.13.
g 10.13.
h 47.2.

PROP. XI.

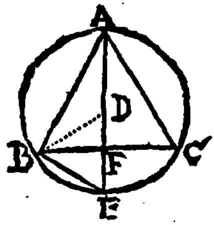


Si in circulo ABCD rationalem habente diametrum AG, pentagonum æquilaterum ABCDE deſcribatur; pentagoni latus AB irrationalis eſt linea, quæ vocatur minor.

Duc diametrum
BFH, rectasque AC, AH; & * fac FL = $\frac{1}{4}$ radii FH, & CM = $\frac{1}{4}$ CA. * 10. 6.

Ob angulos AKF, AIC *a* rectos, & com- a cor. 10.
munem CAI, trigona AKF, AIC *b* æquiangu- 13.
la ſunt; *c* ergo CI. FK *c* :: CA. FA (FB) *d* :: b 32.1.
CM. FL. ergo permutando FK. FL :: CI. CM c 4.6.
d :: CD. CK (2 CM.) *e* componendo igitur d 15 5.
CD + CK. CK :: KL. FL. *f* proinde Q : CD e 25.5.
+ CK (*g* 5 CKq.) CKq :: KLq. ergo KLq = f 22.6.
5 FLq. Itaque ſi BH (ρ^{l}) ponatur 8, erit FH g 1.13.
4; FL 1. & FLq. 1. BL 5. & BLq 25. KLq 5. è
quibus liquet BL, & KL eſſe ρ^{l} *h* ⊏⊐. *k* ideoque h 9.10.
BK eſſe Apotomen; cujus congruens KL cum k 74 10
vero BLq — KLq = 20, *l* erit BL ⊏⊐ √ BLq l 9.10
— KLq. *m* unde BK erit apot. quarta. Quo- m cor. 8.
niam igitur ABq *m* = HBxBK, *n* erit AB mi- 6. & 17.6
nor. Q.E.D. n 95.10.

PROP. XII.



Si in circulo ABEC triangulum æquilaterum ABC deſcribatur, trianguli latus AB potenti triplum eſt ejus lineæ AD, quæ ex D centro circul ducitur.

Protracta diametro a

a cor. 16. 13. E, duc BE. Quoniam arcus BE *a* $=$ E C, arcus
b cor. 15. 4 BE ſexta eſt pars circumferentiæ. *b* ergo BE $=$
c 4. 2. DE. hinc AEq *c* $= 4$ DEq (4 BEq) *d* $=$ ABq
d 47. 1.
e 3. ax. 1. $+$ BEq ($+$ ADq.) *e* proinde ABq $= 3$ ADq.
Q.E.D.

Coroll.

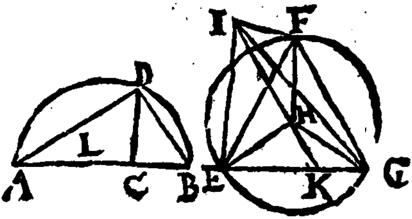
1. AEq. ABq :: 4. 3.

f cor. 8. 6. & 22. 6. 2. ABq. AFq :: 4. 3. *f* Nam ABq. AFq :: AEq. ABq.

g cor. 15.
h cor. 3. 3. 3. DF $=$ FE. Nam triang. EBD *g* æquilaterum eſt; *h* & BF ad ED perpendicularis. *h* ergo EF $=$ FD.

4. Hinc AF $=$ DE $+$ DF $= 3$ DF.

PROP. XIII.



Pyramidem EGFI conſtituere, & data ſphæra complecti; & demonſtrare quod ſphæræ diameter AB potentia ſit ſeſquilatera lateris EF ipſius pyramidis EGFI.

Circa AB deſcribe ſemicirculum ADB. *a* a 10. 6.
ſitque AC = 2 CB. ex puncto C erige perpendicularem
CD; & junge AD, DB. Tum radio
HE = CD deſcribe circulum HEFG; cui *b* b cor. 15. 4.
inſcribe triangulum æquilaterum EFG. ex H *c* c 12. 11.
erige IH = CA rectum plano EFG, produc IH d 3. 1.
ad K; *d* ita ut IK = AB. rectaſque adjunge
IE, IF, IG. erit EFGI pyramis expetita.

Nam quia anguli ACD, IHE, IHF, IHG *e* e conſtr.
recti ſunt; & CD, HE, HF, HG *e* pares, *e* atque
IH = AC; *f* erunt AD, IE, IF, IG æquales inter f 47. 1.
ſe. Quia vero AC (2 CB.) CB *g* :: ACq. g 20. 6.
CDq. erit ACq = 2 CDq. itaque ADq *f* = h 2. ax.
ACq + CDq *h* = 3 CDq = 3 HEq *k* = EFq. k 12. 13.
l ergo AD, EF, IE, IF, IG pares ſunt, adeoque l 1. ax. 1.
pyramis EFGI eſt æquilatera. Quod ſi punctum
C ſuper H collocetur, & AC ſuper HI,
rectæ AB IH *m* congruent, utpote æquales. quare m 8. ax. 1
ſemicirculus ADB axi AB vel IK circumductus n 7 3.
n tranſibit per puncta, E, F, G, * adeoq; def. 1.
pyramis EFGI ſphæræ inſcripta erit. Q.E.F. liquet * 31. def. 11.
vero eſſe BAq. ADq *o* :: BA. AC *p* :: 3. 2. o cor. 8. 6 p conſtr.
Q.E.D.

Coroll.

1. ABq. HEq :: 9. 2. Nam ſi ABq ponatur q 12. 13.
9, erit ACq (EFq) 6. *q* proinde HEq erit 2.

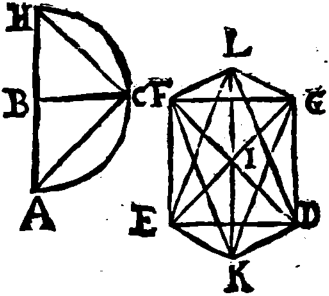
2. Si L centrum fuerit, erit AB. LC ::6. 1

r conſtr. Nam ſi AB ponatur 6. erit AL, 3; r ideoqu
AC 4; quare LC erit 1. Hinc

3. AB.HI ::6. 4 :: 3.2. unde

4. ABq.HIq ::9. 4.

PROP. XIV.



Octaedru KEFGDL conſtituere, & data ſphæra cõplecti, qua & pyramidē; & demonſtrare, quod ſphæra di-ameter AH potentia ſit du-pla lateris AC ipſius octaedri.

Circa AH deſcribe ſemicirculum ACH. ex centro B erige perpendicularem BC. duc AC,

a 46. 1. HC. Super ED = AC a fac quadratum EFGD, cujus diametri DF, EG ſecantes in centro L

b 12. 11. ex I duc IL = AB b rectam plano EFGD. pro-

c 3. 1. duc IL, c donec IK = IL. Connexis KE, KF, KG, KD, LE, LF, LG, LD; erit KEFG*D*L octaedrum quæſitum.

Nam AB. BH, FI, FE, &c. æqualium quadra-

d 4. 1. torū ſemidiametri æquales ſunt inter ſe. d quare triangulorum rectangulorum LIE, LIF, FIE, &c. baſes LF, LE, FE, &c. ęquantur. proinde octo triangula LFE, LFG, LGD, LDE, KEF, K-

e 27. def. 11. FG, KGD, KDE ęquilatera ſunt, e atque octaedrum

drum conſtituunt,quod ſphæræ cujus centrum radius I L, vel A B, inſcribi poteſt. (quoniam B, IL, IF, IK, &c. *f* æquales ſunt.) Q.E.F. porro liquet AHq (LKq) *g* = 2 ACq (2 LDq.) Q.E.D. f conſtr. g 47.1.

Coroll.

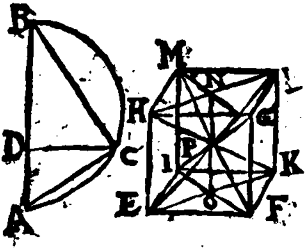
1. Hinc manifeſtum eſt, in Octaedro tres diametros E G, F D, L K ſe mutuo ad angulos rectos ſecare in centro ſphæræ.

2. Item, tria plana EFGD, LEKG, LFKD ſſe quadrata, ſe mutuo ad angulos rectos ſecantia.

3. Octaedrum dividitur in duas pyramides ſimiles & æquales EFGDL, & EFGDK, quarum baſis communis eſt quadratum EFGD.

4. Denique, baſes octaedri oppoſitæ, inter ſe parallelæ ſunt. 15.11.

PROP. XV.



Cubum EFGHIKLM conſtituere, & ſphæra complecti, qua & priores figuras; & demonſtrare, quod ſphæræ diameter A B potentia ſit tripla lateris EF ipſius cubi.

Super A B deſcribe ſemicirculum ACB; & *a* fac AB = 3 DA. ex D erige perpendicularem DC, & junge BC ac AC. Tum ſuper EF = AC *b* conſtrue quadratum EFGH, cujus plano recte inſiſtant EI, FK, HM, GL ipſi EF pares, quas connecte rectis IK, KL, LM, IM. Solidũ EFGHIKLM cubus eſt, ut ſatis conſtat ex conſtructione. a 10.6. b 46.1.

In

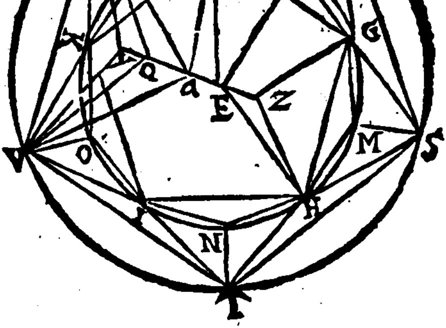
In quadratis oppoſitis E F K I , H G L M duc diametros EK, FI, HL, MG, per quas ducta plana EKLH, FIMG ſe interſecent in recta N O.
c cor. 39. 11. Hæc diametros cubi EL, FM, GI, HK *c* biſecabit in P, centro cubi. *d* ergo P centrum erit
d 15. def. 11. & 14. def. 11. ſphæræ per puncta cubi angularia tranſeuntis.
Porro ELq *e* = EKq + KLq *e* = 3 KLq, *f* vel
e 47. 1. f conſtr. 3 ACq. a qui ABq. ACq *g* : : BA. DA *f* : : 3. 1.
g cor. 8. 6. *h* ergo AB = EL. Quare cubum fecimus, &c.
h 14. 5. Q. E. F.

Coroll.

1. Hinc, omnes diametri cubi inter ſe æquales ſunt, ſeſeque mutuo in centro ſphęræ biſecant. Eademque ratione rectæ quę quadratorum oppoſitorum centra conjungunt, biſecantur in eodem centro.

2. Diameter ſphæræ poteſt latus tetraedri, & cubi, nempe ABq *k* = *l* BCq + *m* ACq.
k 47. 1. l 13. 13. m 15. 13.

PROP. XVI.



Icoſaedrum ZGHIKFYVXRST conſtituere, & ſphæra complecti, qua & antedictas figuras; & demonſtrare, quod icoſaedri latus FG irrationalis eſt linea, quæ vocatur minor.

Super AB diametrum ſphæræ deſcribe ſemicirculum ADB; & a fac AB = 5 BC. ex C erige normalem CD, & duc AD ac BD. Ad intervallum EF = BD deſcribe circulum EFKNG; b cui inſcribe pentagonum æquilaterum FKIHG. Biſeca arcus FG, GH, &c. ac connecte rectas FL, LG, &c. latera nempe decagoni. Tunc c erige EQ, LR, MS, NT, OV, PX ipſi EF æquales, rectasque plano FKNG. & connecte RS, ST, TV, VX, XR; item FX, FR, GR, GS, HS, ST, HT, IT, IV, KV, KX. Denique producta EQ, ſume QY = FL; & EZ = FL; rectasque duci concipe ZG, ZH, ZI, ZK, ZF; ac YV, YX, YR, YS, YT. Dico factum.

a 10.6.
b 11.4.
c 12.11.

Nam ob EQ, LR, MS, NT, OV, PX d æquales e & parallelas, etiam quæ illas jungunt, EL, QR, EM, QS, EN, QT, EO, QV, EP, QX f pares & parallelæ ſunt. Item ideo LM (vel FG,) RS, MN, ST, &c. æquales ſunt inter

d conſtr.
e 6.11.
f 33.1.

g 15.11. inter ſe. *g* ergo planum per EL, EM, &c. plano
h 1.def.3 per QR, QS, &c. æquidiſtans, *h* & circulus Q
XRSTV è centro Q, circulo E P L M N O æ-
qualis eſt; atque RSTVX eſt pentagonum æ-
quilaterum. Duci vero intellectis EF, EG, EH,
k 47.1. &c. ac QX, QR, QS, &c. quia FRQ *k* = FLq
l conſtr. + LRq, *l* vel EFq *m* = FGq, *n* erunt FR, FG,
m 10.13. adeoque omnes RS, FG. FR, RG, GS, GH, &c.
n ſch.48. 1.& 1.ax. equales inter ſe. Proinde 10 triangula RFX,
RFG, RGS, &c. æquilatera ſunt & equalia.
o cor.14. 11. Rurſus ob ang. XQY *o* rectum, erit XYq *p* =
p 47.1. QXq + QYq *q* = V Xq vel F Gq. quare XY,
q 10.13. VX. hisque ſimiliter YV, YT, YS, YR, ZG, ZH,
&c. æquantur: Ergo alia decem trigona conſti-
tuta ſunt æquilatera, & equalia, tam ſibi mutuo,
quam decem prioribus; ac proinde factum eſt
Icoſaetrum.

Porro, biſecta EQ in α, duc rectas αF, αX,
r 15.def.1 αV; & propter QX *r* = QV, & commune la-
ſ 4.1. tus αQ, angulosque EQX, EQV rectos; *ſ* erit
αX = αV. ſimilique argumento omnes, αX, αR,
αS, αT, αV, αF, αG, αH, αI, αK æquantur. Quo-
t 9.13. u 3.13. niam autem ZQ.QE *t* :: QE.ZE, erit Zαq *u* =
x 4.2. 5 Eαq *x* = EQq (EFq) + Eαq *y* = αFq ergo
y 47.1. z 15.5. Zα = αF. *z* pari pacto αF = Yα. ergo ſphæra, cu-
jus centrum α, radius αF, per 12 puncta icoſae-
dri angularia tranſibit.

a 22.6. Denique, quia Zα. αE :: ZY. QE; *a* ideoque
b 14.5. Zαq. αEq :: ZYq. QEq. *b* erit ZYq = 5 QEq,
c cor.8.6. d 1.ax.1. vel 5 BDq: atqui ABq. BDq *c* :: A B. B C :: 5.
1 *d* ergo ZY = AB. Q.E.F.

Itaque ſi AB ponatur ρ', *e* erit EF $=\sqrt{}$ ABq' etiam ρ'; proinde FG pentagoni, idemque Icoſaedri 5 latus, *f* eſt minor. Q.E.D. *e ſch. 19. 10.* *f 11. 23.*

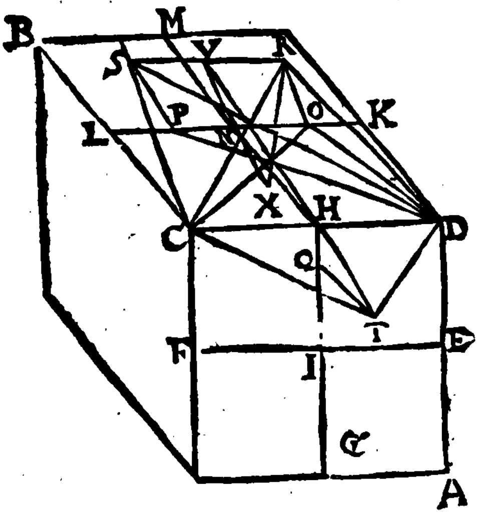
Coroll.

1. Ex dictis infertur, ſphæræ diametrum eſſe potentia quintuplū ſemidiametri circuli quinque latera icoſaedri ambientis.

2. Item manifeſtum eſt, ſphæræ diametrum eſſe compoſitam ex latere hexagoni, hoc eſt, ex ſemidiametro, & duobus lateribus decagoni circuli ambientis quinque latera icoſaedri.

3. Conſtat denique latera icoſaedri oppoſita, qualia ſunt RX, HI, eſſe parallela. Nam RX *a* parall. LP, *b* parall. HI. *a 33. 1.* *b ſch. 26. 3.*

PROP. XVII.



Dodecaedrum conſtituere, & ſphæra complecti, qua & prædictas figuras; & demonſtrare, quod dodecaedri latus RS irrationalis eſt linea, quæ vocatur apotome.

Sit A B cubus datæ ſphęrę inſcriptus, cujus
latera omnia biſecentur in punctis E, H, F, G,
K, L, &c. rectæque adjungantur KL, MH, HG,
a 30.6. EF. *a* Fac H I. I Q :: I Q. Q H; & ſume N O,
NP pares ipſi IQ. Erige OR, PS rectas plano
DB, & QT plano AC. ſintque O R, P S, Q T
ipſis IQ, N O, N P æquales. Connexis D R,
RS, SC, CT, DT, erit DRSCT pentagonum
Dodecaedri expetiti. Nam duc N V parall.
O R, & protracta N V ad occurſum cum cubi
centro X, connecte rectas DS, DO; D P, C R,
a 47.1. CP, HV, HT, RX. Quia DOq *a* = D K q (*b*
b 7.ax.1. K Nq) + K Oq *c* = 3. O N q (3 O Rq) *d* erit
c 4.13. d 47.1. DRq = 4 ORq *e* = OPq, vel RSq. ergo DR =
e 4.2. RS. Simili argumento DR, RS, SC, CT, TP pares
f conſtr. g 6.11. ſunt. Quia vero O R *f* = *g* & parall. PS, *g*
g 33.1. erunt RS, OP. & *h* conſequenter RS, DC etiam
h 9.1. parallelę; *h* ergo hæ cum ſuis conjungentibus
D K, C S, V H in uno ſunt plano. quin etiam
k 7.12. k conſtr. quia HI. IQ *k* :: IQ (TQ.) QH *k* :: H N. NV;
& tam TQ, HN, quam QH, NV *k* rectæ
l 6.11. eidem plano, *l* adeoque & parallelæ exiſtunt, *m*
m 32.6. erit THV recta linea. *n* ergo Trapezium DR-
n 1. & 2. 11. S C, & triang. D T S in uno ſunt plano per re-
o 5.13. ctas DC, TV extenſo. ergo DTCSR eſt pentagonum,
& quidem æquilaterum, ex antedictis.
Porro, quia PK. KN :: K N. N P; & DSq

p =

p = DPq + PSq (PNQ) = *p* DKq + PKq + NPq, *q* erit DSq = DKq + 3 KNq = 4 D Kq (4 D Hq) *r* = D Cq. ergo D S = D C; unde trigona DRS. DCT ſibi mutuo ęquilatera ſunt. *ſ* ergo ang. DRS = DTC; & eodem pacto angul. CSR = DCT. ergo pentagonum DTCSR etiam æquiangulum eſt. Ad hæc, quia AX, DX, CX, &c. ſunt cubi ſemidiametri, *t* erit XN = IH, vel KN, *u* adeoque XV = K P. unde ob angulum *x* rectum RVX, *z* erit RXq = XVq + RVq (NPq) = KPq + NPq *a* = 3 KNq *b* = AXq, vel DXq, &c. ergo RX, AX, DX & eadē ratione XS, X T, AX ęquales ſunt inter ſe. Et ſi eadem methodo, qua conſtructum eſt pentagonum DTCSR, fabricentur 12 ſimilia pentagona tangentia duodecim cubi latera, ea Dodecaedrum conſtituent; ac per eorum puncta angularia tranſiens ſphæra, cujus radius A X, vel RX, Dodecaedrum complectetur. Q.E.F.

p 47.1.
q 1 ax. 2 & 4.13.
r 4.2.
ſ 8.1.
t 15.13.
u 1. ax. 1
x 29.1.
z 47.1.
a 4.13.
b 15.13.

Denique, quia KN. NO *c* : : NO. OK, *d* erit KL. O P : : OP. OK + PL. Itaque ſi ſphæræ diameter AB ponatur $\dot{\rho}$, erit K L *e* = $\sqrt{}$ A B *f* etiam $\dot{\rho}$. *g*. unde O P, vel RS latus dodeca- $\overline{3}$ ædri apotome erit. Q.E.D.

c conſtr.
d 15.5.
e 15.13.
f ſch. 13. 10.
g 6.13.

Coroll.

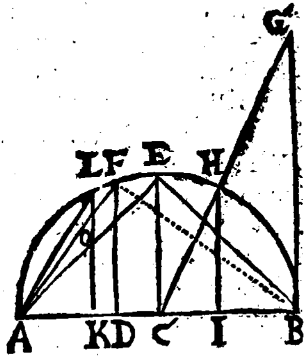
1. Hinc, ſi latus cubi ſecetur extrema ac media ratione, majus ſegmentum erit latus dodecaedri in eadem ſphæra deſcripti.

2. Si rectæ lineæ ſectę extrema ac media ratione, minus ſegmentum ſit latus dodecaedri,

dri, majus ſegmentum erit latus cubi ejuſdem ſphærę.

3. Liquet etiam latus cubi æquale eſſe lineæ rectæ ſubtendenti angulum pentagoni dodecaedri eadem ſphæra comprehenſi.

PROP. XVIII.



Latera quinq; figurarum exponere, & inter ſe comparare.

Sit AB diameter
ſphæræ, ac AEB
ſemicirculus.
a 10.1. ſitque AC *a*
b 10.6. = $\frac{1}{2}$ AB, & AD *b*
= $\frac{1}{3}$ AB. Erige
perpendiculares
CE, DF, & BG
= AB. junge AF, AE, BE, BF, CG, ex H demitte perpendicularem HI, & ſumpta CK =
CI, ex K eripe perpendicularem KL, & connecte
c 30.6. AL. Denique *c* fac AF. AO :: AO. OF.

d conſtr. Itaque 3. 2 *d* :: AB. BD *e* :: ABq. BFq, latus
e cor. 8. 6 Tetraedi, & 2. 1 :: *a* AB. AC :: ABq. BEq, *f* latus
f 14. 13. Oaedri.

g 15. 13. Item 3. 1 *d* :: AB. AD *e* :: ABq. AFq. *g* latus
Hexaedri.

h conſtr. Porro, quia AF. AO *h* :: AO. OF. *k* erit AO
k cor. 17. 13. latus Dodecaedri. Denique BG (2 BC.) BC
l 4. 6. *l* :: HL. IC. *m* ergo HI = 2 CI *n* = KL. ergo
m 24. 5.
n conſtr. HIq

HIq *o* = 4 CIq. proinde CHq = *p* 5 CIq. *q* ergo ABq = 5 KIq. *r* itaque KI, vel HI, er radius circuli circumſcribentis pentagonum icoſaedri, & A K, vel I B, *ſ* eſt latus decagoni eidem circulo inſcripti. unde AL erit latus pentagoni, idemque Icoſaedri latus. Ex quibus liquet B F; BE, AF eſſe ę̓ ⊐ & AL, A O eſſe ῥ ⊐; atque BF ⊏ BE; & BE ⊏ AF; ac AF ⊏ AO. Quia vero 3 AFq = ABq *u* = 5 KLq. ac AFxAO ⊏ AFxOF, *x* ideoque AFxAO + AFxOF ⊏ 2 AFxOF, *y* hoc eſt AFq ⊏ *z* 2 AOq. *a* erit 3 AFq (5 KLq) ⊏ 6 AOq. proinde KL ⊏ AO; & fortius, AL ⊏ AO.

o 4.2.
p 47.1.
q 15.5.
r cor.16.13.
ſ 10.13.
t 16.13.
u 1.6.
x 4.ax.
y 1.2.
z 17.6.
a 47 1.

Jam vero ut hęc latera numeris exprimamus, ſi AB ponatur $\sqrt{}$ 60, erit ex jam dictis ad calculum exactis, BF = $\sqrt{}$ 40. & BE = $\sqrt{}$ 30. & AF = $\sqrt{}$ 20. item A L = $\sqrt{}$: 30 — $\sqrt{}$ 18. (nam AK = $\sqrt{}$ 15 — $\sqrt{}$ 3. & KL (HI) = $\sqrt{}$ 12.) denique A O = $\sqrt{}$: 30 — $\sqrt{}$ 500 ($\sqrt{}$ 25 — $\sqrt{}$ 5.)

SCHOL.

Præter jam dictas figuras nullam dari poſſe figuram ſolidam regularem (nempe quæ figuris planis ordinatis & æqualibus contineatur) admodum perſpicuum eſt. Nam ad anguli ſolidi conſtitutionem requiruntur ad minimum tres anguli plani; *a* hique omnes ſimul 4 rectis minores eſſe debent. *b* Atqui ſex anguli trigoni æquilateri, 4 quadratici, & 3 hexagonici, ſigillatim 4 rectos exęquant; quatuor vero pentagonici, 3 heptagonici, 3 octagonici, &c. 4 rectos exce-

a 21. 11.
b Vid. ſchol. 32. 1.

excedunt.ergo ſolummodo ex 3,4, vel 5 triangulis æquilateris,ex 3.quadratis, vel 3 pentagonis,effici poteſt angulus ſolidus. Proinde,præter quinque prędicta,nulla exiſtere poſſunt corpora regularia.

Ex P.Herigonio.

Proportiones ſphæræ, & 5.figurarum regularium eidem inſcriptarum.

Sit diameter ſphæræ 2. Erunt

Peripheria circuli majoris,	6 ⌊28318.
Superficies circuli majoris,	3 ⌊14159.
Superficies ſphęræ,	12 ⌊56637.
Soliditas ſphærę,	4 ⌊1879.
Latus tetraedri,	1 ⌊62299.
Superficies tetraedri,	4 ⌊6188.
Soliditas tetraedri,	0 ⌊15132.
Latus hexaedri,	1 ⌊1547.
Superficies hexaedri,	8.
Soliditas hexaedri,	1 ⌊5396.
Latus octaedri,	1 ⌊41421.
Superficies octaedri,	6 ⌊9282.
Soliditas octaedri,	1 ⌊33333.

Latus dodecaedri, 0 | 71364.

Superficies dodecaedri, 10 | 51462.

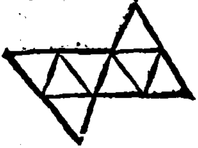
Soliditas dodecaedri, 2 | 78516.

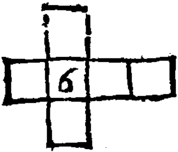
Latus Icoſaedri, 10 | 5146.

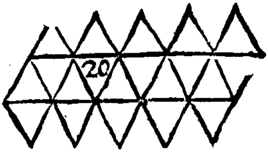
Superficies Icoſaedri, 9 | 57454.

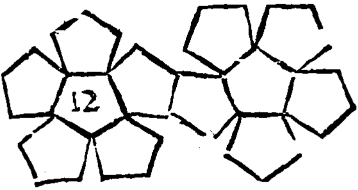
Soliditas Icoſaedri, 2 | 43615.

Quod ſi ex charta conficiantur quinque figuræ æquilateræ & æquiangulæ ſimiles his quæ ſunt in ſubiecta figura, componentur quinque figuræ ſolidæ, ſi rite complicentur.









Hypsiclis Alexandri [illegible]
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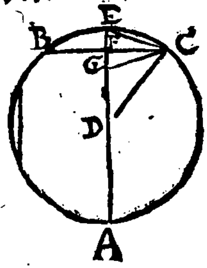
LIB. XIV.

PROP. I.

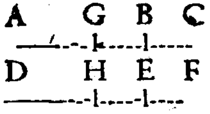


QUæ ex D centro circuli cujuſpiam ABC in pentagoni eidem circulo inſcripti latus BC ducitur perpendicularis DF, dimidia eſt utriusq; lineæ ſimul; & lateris hexagoni DE, & lateris decagoni EC eidem circulo ABC inſcripti.

a 4.1. Sume FG = FE, & duc CG. *a* Eſtque CE
b 5.1. = CG. ergo ang. CGE *b* = CEG *b* = ECD.
c 32.1. ergo ang. ECG *c* = EDC *d* = $\frac{1}{4}$ ADC *e* = $\frac{1}{2}$
d hyp. & 33.6. CED ($\frac{1}{2}$ ECD.) proinde ang. GCD = ECG
e 10.3. = EDC. *g* quare DG = GC (CE.) er-
f 7.ax. go DF = CE (DG) + EF = DE + CE.
g 6.1 Q.E.D.

2

PROP. II.



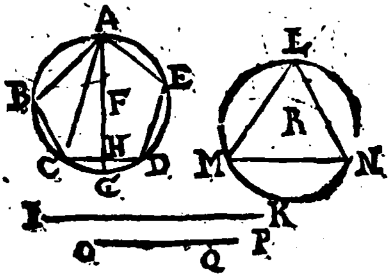
Si binæ rectæ lineæ AB. DE extrema ac media ratione ſecentur (AB.AG :: AG.GB. & DE.DH :: DH.HE;) *ipſæ ſimiliter ſecabuntur, in eaſdem ſcilicet proportiones*. (AG.GB :: DH.HE.

Acci-

Accipe BC = BG & EF = EH. Eſtque ABx BG *a* = AGq. quare ACq *b* = 4 ABG + AGq *c* = 5 AGq. Similiter erit DFq = 5 DHq. *d* ergo AC. AG :: DF. DH. componendo igitur AC + AG. AG :: DF + DH. DH. hoc eſt 2 AB. AG :: 2 DE. DH. *e* proinde AB. AG :: DE. DH. unde *f* dividendo AG. GB :: DH. HE. Q.E.D.

a 17.6.
b 8.2.
c 2.ax.1.
d 22.5.
& 22.6.
e 22.5.
f 17.5.

PROP. III.



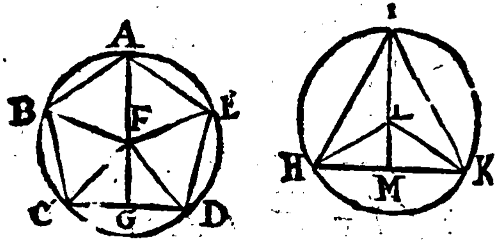
Idem circulus A B D comprehendit & Dodecaedri pentagonum A B C D E, & Icoſaedri triangulum LMN, eidem ſphæræ inſcriptorum.

Duc diametrum AG, rectasque AC, CG. Sitque IK diameter ſphæræ, *a* & IKq = 5 OPq. *b* fiatque OP, OQ :: OQ. OP. Quia ACq + CGq *c* = A Gq *d* = 4 F Gq; & A Bq *e* = F Gq = CGq. *f* erit ACq + A Bq = 5 F Gq. porro, quia CA. AB *g* :: AB. CA — AB; ac OP. OQ :: OQ. QP. *h* ideoque CA. OP :: AB. OQ. *k* erit 3 ACq (*l* IKq.) 5 OPq (*m* IKq) :: 3 ABq. 5 OQq. ergo 3 ABq = 5 OQq. Verum ob ML *n* latus pentagoni circulo inſcripti, cujus radius OP, erunt 15 RMq *o* = 5 MLq *p* = 5 OPq + 5

a ſch. 47. 1.
b 30.6.
c 47.1.
d 4.2.
e 10.13.
f 2. & 3. ax.
g 8.13.
h 2.13. & 16.5.
k 22.6. & 4.5.
l 15.13.
m conſtr.
n cor. 16. 13.
o 12.13.
p 15.5.

q 15.5. * Prius. + 5 OQq = * 3 ACq + 3 ABq *q* = 15 FGq.
r 1.ax.1. & ſch.48.1. *r* ergo RM = FG. *ſ* proinde circ. ABD = circ.
LMN. Q.E.D.
ſ 1.def.3.

PROP. IV.



*Si ex F centro circuli pentagonium dodecaedri ABCDE circumſcribentis ducatur perpendicularis F G ad pentagoni unum latus C D; erit quod ſub dicto latere CD, & perpendiculari FG comprehenditur rectangulum trigeſies ſumptum, icoſaedri ſuperficiei æquale.*item,

Si ex centro L circuli triangulum icoſaedri HIK circumſcribentis, perpendic. LM ducatur ad trianguli unum latus HK; erit quod ſub dicto latere HK, & perpendiculari L M comprehenditur rectangulum trigeſies ſumptum, icoſaedri ſuperficiei æquale.

a 8.1. Duc FA, FB, FC, FD, FE. *a* Erunt triangula
b 41.1. CFD, DFE, EFA, AFB, BFC æqualia. atqui C-
c 5.5. DxFG *b* = 2.triang. CFD.ergo 30 CDxGF *c*
d 6.ax. = 60 CFD *d* = 12. pentag. ABCDE *e* = ſuperf.
e 17.3. dodecaedri. Q.E.D.

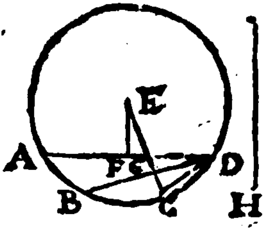
f 41.5. Duc LI, LH, LK eſtque HKxLM *f* = 2 tri-
g 15.5. angLHK.ergo 30 HKxLM *g* = 60 HLK = 20
h 16.13. HIK *h* = ſuperfic. icoſaedri. Q.E.D.

Co-

Coroll.

CDxFG. HKxLM *k* :: ſuperfic. dodecaed. ad ſuperfic. icoſaeeri.

PROP. V.



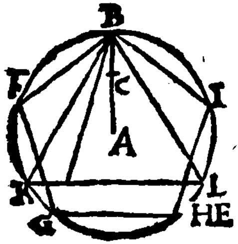
Superficies dodecaedri ad ſuperficiem icoſaedri in eadem ſpara deſcripti eandē proportionem habet, quam H latus cubi ad A D latus icoſaedri.

Circulus A B C D *a* circumſcribat tam dodecaedri pentagonum, quam icoſaedri triangulum; quorum latera B D; A D; ad quæ demittantur ex E centro perpendiculares E F, E G C. & connectatur C D. a 5.14.

Quoniam E C + CD. E C. *b* :: E C. C D. erit b 9.13.
EG (*c* $\frac{1}{2}$ EC + CD.) E F (*d* $\frac{1}{2}$ EC) *e* :: EF. EG − c 1.14.
EF $\frac{1}{2}$ CD.) atqui H. BD *f* :: BD. H − BD. *g* er- d cor. 12. 13.
go H. BD :: EG. EF proinde HxEF = BDxEG. e 15.5.
quum igitur H. AD *h* :: HxEF. ADxEF. erit f cor. 17. 13.
H. AD *k* :: BDxEG. ADxEF :: *l* ſuperfic. do- g 2.14.
decaedri ad ſuperfic. icoſaedri. Q.E.D. h 1.6. k 7.5.

PROP. VI.

l cor. 4. 14.



Si recta linea AB ſecetur extrema ac media ratione; erit ut recta BF potens id, quod à tota AB, & id quod à majori ſegmento AC, ad rectam E, potentem *id quod à tota AB, & id quod à minori ſegmento BC; ita latus cubi B G ad latus icoſaedri BK eidem ſphæra cum cubo inſcripti.*

Circulo, cujus ſemidiameter AB, inſcribantur dodecaedri pentagonum BFGHI, & icoſaedri triangulum B K L. *a* quare B G latus cubi erit eidem ſphæræ inſcripti. igitur BKq *b* = 3 ABq; & Eq *c* = 3 ACq. ergo BKq. Eq *d* :: ABq. ACq *e* :: BGq. BFq. permutando igitur BGq. BKq :: BFq. Eq. *f* unde BG.BK :: BF.E. Q.E.D.

a cor.17.13.
b 12.13.
c 4.13.
d 15.5.
e 2.14.
f 22.6.

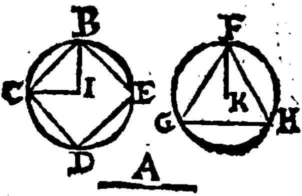
PROP. VII.

Dodecaedrum eſt ad Icoſaedrū, ut cubi latus ad latus Icoſaedri, in una eademq; ſphæra inſcripti.

Quonian *a* idem circulus comprehendit & dodecaedri pentagonum & icoſaedri triangulum, *b* erunt perpendiculares à centro ſphæræ ad plana pentagoni & trianguli ductæ inter ſe æquales: itaque ſi dodecaedrum & icoſaedrum intelligantur eſſe diviſa in pyramides, ductis rectis à centro ſphæræ ad omnes angulos, omniū pyramidum altitudines erunt inter ſe æquales. Cum igitur pyramides ęque altæ *c* ſint ut baſes, & ſuperficies dodecaedri ſit æqualis 12 pentagonis, ſuperficies vero icoſaedri 20. triangulis; erit dodecaedrum ad icoſaedrum, ut ſuperficies dodecaedri ad ſuperficiem icoſaedri, *d* hoc eſt, ut latus cubi ad latus icoſaedri.

a 3.14.
b 47.1.
c 5, & 6. 12.
d 5.14.

PROP. VIII.



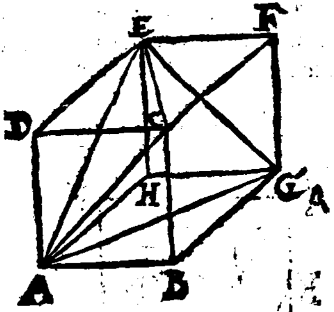
Idem circulus BCDE comprehendit & cubi quadratum BCDE

DE & octaedri triangulum FGH, *ejuſdem ſphæra.* a 15.13.

Sit A diameter ſphærę. Quoniam Aq *a* = 3 b 47.1.
BCq *c* = 6 BIq; itemque Aq *c* = 2 GFq *d* = 6 c 14.13. d 12.13
KFq; erit BI = KF. *e* ergo circulus CBED = e 2.def.8
GFH. Q.E.D.

LIB. XV.

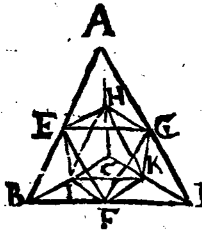
PROP. I.



IN dato cubo ABGHDCFE pyramidem AGEC deſcribere.

Ab angulo C duc diametros CA, CG, CE; a 47.1.
Easque connecte diametris AG, GE, EA. Hæ omnes inter ſe *a* ęquales ſunt, utpote æqualium quadratorum diametri. ergo triangula CAG, b 31. def.11.
CGE, CEA, EAG æquilatera ſunt, ac æqualia: proinde AGEC eſt pyramis, quæ cubi angulis inſiſtit, eique idcirco *b* inſcribitur. Q.E.F.

PROP. II.



In data pyramide ABDC *octaedrum* EGKIFH *deſcribere*.

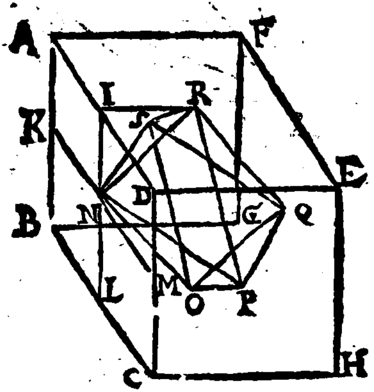
a 10. 1. *a* Biſeca latera pyramidis in punctis E, I, F, K, G, H; quæ connecte 12 rectis E F, F G, GE, &c. Hæ omnes *b* æquales ſunt inter ſe. proinde 8 triangula EHI, IHK, &c. æquilatera ſunt & æqualia, adeoque conſtituunt *c* octaedrum *d* in data pyramide deſcriptum. Q.E.F.

b 4. 1.

c 27. def. 11.

d 31. def. 11.

PROP. III.



In dato cubo CHGBDEFA *octaedrum* NPQSOR *deſcribere*.

* 8. 4.

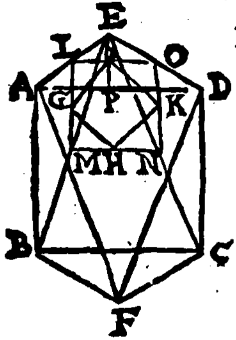
a 1. 4.

Connecte quadratorum * centra N, P, Q, S, O, R, 12 rectis NP, PQ, QS, &c. quæ *a* æqualia

ſunt

ſunt inter ſe, ideoque 8 triangula efficiunt ęqui- b 31. &
latera & ęqualia. proinde *b* inſcriptum eſt cubo 27 *def.*
b Octaedrum NPQSOR. Q.E.F. *11.*

PROP. IV.



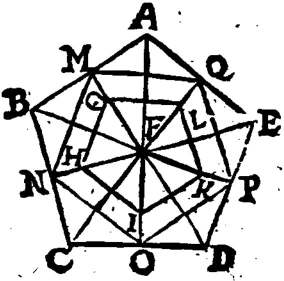
In dato octaedro A-
BCDEF *cubum inſcri-*
bere.

Latera pyramidis E-
ABCD, cujus baſis
quadratum ABCD,
biſecentur rectis LM, a 4. 1.
MN, NO, OL; quæ *a* b 2. 6.
æquales ſunt & *b* paral-
lelę lateribus quadrati
ABCD. *c* ergo quadri- c 29. *def. 1.*
laterum LMNO eſt
quadratum.

Eodem modo, ſi latera quadrati LMNO bi-
ſecentur in punctis G, H, K, I, & connectantur
GH, HK, KI, IG, erit GHKI quadratum.
Quod ſi eadem arte in reliquis 5 pyramidibus
octaedri centra triangulorum rectis conjungan-
tur, deſcribentur quadrata ſimilia & æqualia
quadrato GHKI. quare ſex hujusmodi qua-
drata cubum conſtituent, qui quidem intra o- d 31.
ctaedrum deſcriptus erit, *d* cum octo ejus an- *def. 11.*
guli tangant octo octaędri baſes in earum cen-
tris. Q.E.F.

PROP.

PROR. V.



In dato Icoſaedro dodecaedrum inſcribere.

Sit ABCDEF pyramis Icoſaedri, cujus baſis pentagonum ABCDE; centra autem triangulorum G, H, I, K, L; quæ conne-
* 5.4. ctantur rectis GH, HI, IK, KL, LG. Erit GHIKL pentagonum dodecaedri inſcribendi.

Nam rectæ FM, FN, FO, FP, FQ, per centra
a cor.3.3. b 4.1. triangulorum tranſeuntes, *a* biſecant baſes. *b* ergo rectę MN, NO, OP, PQ, QM ęquales ſunt
c 4.1. d 8.1. inter ſe. quin etiam FM, FN, FO, FP, FQ *c* pares ſunt. *d* ergo anguli MFN, NFO, OFP, PFQ, QFM æquantur. pentagonum igitur GH-
e 4.1. IKL æquiangulum eſt; *e* proinde & æquilaterum, cum FG, FH, FI, FK, FL *f* pares ſint. Quod
f 12.13. ſi eadem arte in reliquis undecim pyramidibus icoſaedri, centra triangulorum rectis lineis connectantur, deſcribentur pentagona æqualia & ſimilia pentagono G H I K L. quamobrem 12 hujusmodi pentagona dodecaedrum conſtituent; quod quidem in icoſaedro erit deſcriptum, cum viginti anguli dodecaedri in centris viginti baſium icoſaedri conſiſtant. Quapropter in dato icoſaedro dodecaedrum deſcripſimus. Q.E.F.

FINIS.

EVCLIDIS DATA

Succinctè demonſtrata,
Una cum Emendationibus & Additionibus
Ad
Elementa Euclidis,
Edita, operâ
M^{ri} Js. BARROW,
Cantabrigienſis,
COLL. TRIN. SOC.
Nunc denuo reviſa, & à cunctis erroribus expurgata.

OSNABRUGI

Apud Johan.-Georg. Schwänderum. *Anno* 1675.

L. S.

DATA hęc Euclidis antea opera Js. BARROW Cantabrigienſis demonſtrata placuit elementis ipſius Euclidis adjungere; quippe quæ proximè ad ea accedunt, & quaſi concatenatè cum eis cohærent. Licet autem breviter Autor maximo cum labore & ſtudio propoſuerit; attamen nemo non qui animum ad hocce ſtudium, Κρίσιν ſubtiliſſimam requirens, applicat, tantum commodi ex eis, quantum deſiderari poteſt, percipiet. Commendatione ſanè nulla indigent, cum proba merx facilè inveniet emptorem.

EUCLIDIS Data.

DEFINITIONES.

I.

DAta magnitudine dicuntur ſpatia, lineę, anguli, quibus æqualia poſſumus invenire.

II. Ratio dari dicitur, cui poſſumus eandem invenire.

III. Rectilineæ figuræ ſpecie dari dicuntur, quarum & ſinguli anguli dati ſunt, & laterum rationes ad invicem datæ ſunt.

Hinc, datæ ſunt ſpecie figuræ, quibus ſimiles inveniri poſſunt.

IV. Poſitione dari dicuntur puncta, lineę, angulique, quæ eundem ſitum ſemper obtinent.

V. Circulus magnitudine dari dicitur, cujus ea quæ ex centro datur magnitudine.

VI. Poſitione & magnitudine dari dicitur circulus, cujus datur centrum poſitione, & ea quæ ex centro magnitudine.

VII. Circuli ſegmenta magnitudine dari dicuntur, in quibus dati ſunt magnitudine anguli & ſegmentorum baſes.

VIII. Poſitione & magnitudine dari dicuntur circuli ſegmenta, in quibus anguli magnitudine dati ſunt, & ſegmentorum baſes poſitione & magnitudine.

IX. Magnitudo magnitudine major eſt data, quando ablata data, reliqua eidem æqualis eſt.

X. Magnitudo magnitudine minor eſt data, quando adjuncta data, tota eidem æqualis eſt.

Ut ſi A data ſit, erit $A + B \sqsubset$ data. At $B \sqsupset A + B$ data.

XI. Magnitudo magnitudine major eſt data quam in ratione, quando ablata data, reliqua ad eandem habent rationem datam.

XII. Magnitudo magnitudine minor eſt data quam in ratione, quando adjuncta data tota ad eandem rationem habent datam.

Ut ſi A data ſit, & $\frac{B}{C}$ detur, erit $A + B \sqsubset C$, data q. in r. ſin $\frac{A + B}{C}$ detur, erit $B \sqsupset C$ data q. in r.

PROP. I.

A. B.

a. b.

Datarum magnitudinum A, B, ad invicem datur ratio.

* hyp.
a 1.def.
b ſch 7. 5.
c 1.def.

Nam quia A * datur, *a* inveniri poteſt aliqua a = A. Eodem jure ſume b = B. *b* eſtque a. b :: A.B. *c* quare ratio $\frac{A}{B}$ data eſt. Q.E.D.

PROP. II.

A. B.

a. b.

Si data magnitudo A ad aliam aliquam B habeat rationem datam, datur etiam hac alia magnitudine.

* hyp.
a 1.def.d.
b 2.def.d.
c 9.5.

Nam ob A * datam, *a* ſume a = A; ac ob $\frac{A}{B}$ * datam, *b* ſit $\frac{a}{b} = \frac{A}{B}$. *c* ergo b = B. *a* quare B datur. Q.E.D.

PROP. 3.

A. B.

a. b.

Si quotlibet datæ magnitudines A, B componantur, etiam ea A + B

+ B quæ ex his componitur, data erit.

Nam *a* cape a = A, & b = B; *b* eſtque a + *b* + B. *a* quare A + B datur. Q.E.D. a 1.def. b 2.ax.1

PROP. 4.

B. *Si à data magnitudine A au-*
b. *feratur data magnitudo B, etiam reliqua A − B dabitur.*

a Sint enim a = A, & b = B. ergo A − B = b. *a* proinde A − B datur. Q.E.D. a 1.def.d b 3.ax.1

PROP. 5.

B. *Si magnitudo A ad ſui-ipſius*
D. *aliquam partem B habeat rationem datam, etiam ad reliquam − B habebit rationem datam.*

Nam, quia $\frac{A}{B}$ *a* data eſt, *b* ſit A B :: C. D. *c* ergo A . A − B :: C . C − D. *b* proinde $\frac{A}{A-B}$ datur. Q.E.D. a hyp. b 2.def.d c cor.19.5

PROP. 6.

B. *Si componantur duæ magnitu-*
D. *dines, A, B, habentes ad invicem rationem datam, etiam quæ ex his componitur magnitudo A + B, habebit ad utramque A & B rationem datam.*

Nam *a* ſit A . B :: C . D. *b* ergo A + B . B :: C + D . D. *c* quare $\frac{A+B}{B}$ datur. Similiter B + A datur. Q.E.D. a 2.def.d b 18.5. c 2.def.d

PROP. 7.

A. B. *Si data magnitudo A + B data ratione ſecetur, utrumque ſegmentorum A & B datum eſt.*

† hyp. a 6. dat. b 2. dat. Nam ob $\frac{A}{B}$ † datam, a erit $\frac{A \mp B}{A}$ data. b ergo $\frac{A}{B}$ datur. Eodem modo B datur. Q.E.D.

PROP. 8.

A. C. B.
D. E. F.

Quæ A, B ad idem C rationem habent datam, habebunt ad invicem rationem datam.

a 1. def. d. Nam a ſit A.C::D.E. a & C.B::E.F. quare ex æquali A.B::D.F. a ergo $\frac{A}{B}$ datur. Q.E.D.

Coroll.

Rationes ex datis rationibus compoſitæ, datæ ſunt. Ut $\frac{A}{B}$ ſit ex $\frac{A}{C}$, & $\frac{C}{B}$ datis.

PROP. 9.

A. B. C.
D. E. F.

Si duæ, pluresve magnitudines A, B, C ad invicem habeant rationem datam, habeant autem illæ magnitudines A, B, C ad alias quasdam D, E, F rationes datas, etſi non easdem; illæ aliæ magnitudines D, E, F etiam ad invicem habent rationes datas.

a 20. def. 5. b hyp. c cor. 8. dat. Nam ratio $\frac{D}{E}$ a ſit ex b datis $\frac{D}{A}$, $\frac{A}{B}$, $\frac{B}{E}$; c ergo $\frac{D}{E}$ datur. Eadem de cauſa datur $\frac{E}{F}$. Q.E.D.

PROP. 10.

A. B. C.

Si magnitudo magnitudine major fuerit data, quam in ratione; & ſimul utraque illa eadem major erit data quam in ratione. Sin autem ſimul utraque magnitudo eadē magnitudine major fuerit data, quam in ratione; & reliqua illa eadem major erit da-

quam in ratione; aut reliqua data eſt cum conſequente, ad quam habet altera magnitudo rationem datam.

1. Sint A, & $\frac{B}{C}$ datæ. *a* erit $\frac{B+C}{C}$ data. *b* ergo $A+B+C \sqsubset C$ data q. in r. Q.E.D. a 6. dat. b 11. def. d.

2. Sint A, & $\frac{B+C}{C}$ datæ: *c* ergo $\frac{B}{C}$ datur. proinde $A+B \sqsubset C$ data q. in r. Q.E.D. c 17. 5.

3. Sint $A+B$, & $\frac{C}{B+C}$ datæ. *d* Liquet $\frac{B}{B+C}$ dari. Q.E.D. d 5. 6.

PROP. 11.

A. B. C. *Si magnitudo magnitudine major ſit data quam in ratione, eadem ſimul utraque major erit data quam in ratione. Et ſi eadem ſimul utraque major ſit data quam in ratione, eadem reliqua magnitudine major erit data quam in ratione.*

1. A, & $\frac{B}{C}$ dantur. *a* ergo $\frac{B}{B+C}$ datur. proinde *b* $A+B \sqsubset B+C$ data q. in r. Q.E.D. a 6. dat. b 11. def. d.

2. A, & $\frac{B}{B+C}$ dantur. *c* ergo $\frac{B}{C}$ datur, proinde *b* $A+B \sqsubset C$ data q. in r. Q.E.D. c 5. dat.

PROR. 12.

A. B. C. *Si fuerint tres magnitudines A, B, C, & prima cum ſecunda* (A+B) *data ſit, ſecunda quoque cum tertia* (B+C) *data ſit; aut prima A tertiæ C æqualis eſt, aut altera altera major data.* a 4. ax. 1. b 4. dat.

Nam ſi A + B, & B + C pares ſint, *b* liquet A & C æquari; ſin iſtæ impares fuerint, *b* liquet exceſſum A − C, vel C − A dari. Q.E.D.

PROP. 13.

D, A+B, C.
E

Si fuerint tres magnitudines D, A+B, C, & earum prima D ad ſecundam A+B habeat rationem datam; ſecunda autem A+B tertia C major ſit data quam in ratione; prima quoque D major erit tertia C data quam in ratione.

a 2 def.d. b 29.5. c 2.dat. d 2.def.d. Sint A, & $\frac{B}{C}$, ac $\frac{D}{A+B}$ datæ; *a* ſitque A+B.D :: A. E *b* :: B. D−E. ergo *c* E, *d* & $\frac{B}{D-E}$ & (ob

e 1.dat. f 11.def.d. $\frac{B}{C}$ datam) *e* $\frac{C}{D-E}$ dantur. *f* quare D (E + : D−E) ⌐ C data q. in r. Q.E.D.

PROP. 14.

A. C.
B. D.
E.

Si duæ magnitudines A & C ad invicem habeant rationem datam, utrique autem illarum adjiciatur data magnitudo B & D; totæ A+B, C+D, aut habent rationem datam, aut altera A+B altera C+D major erit data quam in ratione.

a 12.5. b hyp. c 2.def.d. Nam ſi A. C :: B. D *a* :: A+B. C+D ob $\frac{A}{C}$ *b* datam, *c* liquet $\frac{A+B}{C+D}$ dari.

d 2.def.d. e 2.dat. f 4.dat. g 11.def.d. Saltem *d* ſit A. C :: E.D. *a* :: A+E.C+D. Ergo *c* $\frac{A+E}{C+D}$ ac *e* E, *f* ideoque B−E dantur. *g* proinde A+B (A+E : + B−E) ⌐ C+D data q. in r. Q.E.D.

PROP,

PROP. 15.

A. C.
B. D.
E.

Si duæ magnitudines A & C habeant ad invicem rationem datam, & ab utraque harum auferatur data magnitudo B & D; reliquæ magnitudines A - B, C - D *ad invicem habebunt aut rationem Datam, aut altera* A − B, *altera* C − D *major erit data quam in ratione.*

b Nam ſi A.C :: B.D *a* :: A − B. C − D. ob $\frac{A}{C}$ datam, *c* liquet $\frac{A - B}{A - C}$ dari. a 19.5. b hyp. c 2.def.6

Saltem *d* ſit A. C :: E. D *a* :: A − E. C − D. Ergo *e* $\frac{A - E}{C - D}$, & *e* E, ac *f* ideo E − B dantur. d 2.def. e 2.dat. f 4.dat.

g proinde A − B (A − E : + E − B) ⌐ C − D data q. in r. Q.E.D. g 13.def.d

PROP. 16.

B. C.
A. D.
E.

Si duæ magnitudines B, C habeant rationem datam, & ab una quidem illarum C *auferatur data magnitudo* D, *alteri autem B adjiciatur data magnitudo* A; *tota* A + B *reſidua* C − D *major erit data quam in ratione.*

Sit enim C. B *a* :: D. E *b* :: C − D. B − E. ergo *c* $\frac{C - D}{B - E}$, & *d* E, ac *e* ideo E + A dantur. a 2.def.6 b 19.5. c 2.def.d d 2.dat.

f proinde B + A (E + A : + B − E) ⌐ C − D data q. in r. Q.E.D. e 5.dat. f 11.def.d

PROP.

PROP. 17.

A + B. D + E. *Si fuerint tres magnitudines* A + B, C, D + E; *& prima quidem* A + B *ſecunda C major ſit data quam in ratione, tertia quoque* D + E *eadem ſecunda C major ſit data quam in ratione; prima* A + B *ad tertiam* D + E *aut rationem habebit datam, aut altera altera major erit data quã in ratione.*

C.

a hyp.
b 1.dat.

Nam ob A, D, & B E *a* datas, *b* erit B data,
$\overline{C}$, $\overline{D}$ $\overline{E}$
ergo per 14. hujus.

PROP. 18.

A + C. E. G.
B + D. F. H.

Si fuerint tres magnitudines, atque ex his una utraque reliquarum major ſit data quam in ratione; reliquæ duæ aut datam rationem habebunt ad invicem, aut altera altera major erit data quam in ratione.

Datæ ſint A, B, C D ac ſit A + C = B + D.
$\overline{E}$, $\overline{F}$;

a 2.def.d.
b 12.5.
c 2.def.d.
d 7.5.
e 8.5.
f 3.dat.

Sitque C. E *a* :: A. G *b* :: C + A. E + G. itemque D. F *a* :: B. H *b* :: D + B. F + H. *c* ergo C + A *d* hoc eſt B + D, *c* & B + D, ac *e* idcirco E + G quin & G ac H *f* dantur. ergo per
$\overline{E}+\overline{G}$, $\overline{E}+\overline{G}$, $\overline{F}+\overline{H}$
$\overline{F}+\overline{H}$; (15. hujus.

PROP. 19.

A + B. E.
C + D. F.

Si fuerint tres magnitudines, & prima quidem magnitudo ſecunda magnitudine major ſit data quam in ratione, ſit quoque ſecunda major tertia data quam in ratione; prima magnitudo ter-

tertia magnitudine major erit data quam in ratione.

Sint A, C, & $\frac{C+D}{B}$, $\frac{D}{E}$ datæ; dico $A \pm B$ ⊏ E data q.in r.

Nam ſit C + D. B *a* :: C. F *b* :: D. B − F. ergo *c* $\frac{C}{F}$ & *d* F, ac *e* ideo F + A, & *c* $\frac{D}{B-F}$ *f* ideoque $\frac{E}{B-F}$ dantur. *g* proinde A + B (F + A : + B − F) ⊏ E data q.in r. Q.E.D.

a 1.def.4
b 19.5.
c 2.def.4
d 2.dat.
e 3.dat.
f 1 dat.
g 11.def.4

PROP. 20.

A. C. E.
B. D.

Si datæ fuerint duæ magnitudines A, C; & auferantur ab ipſis magnitudines B, D habentes ad invicem rationem datam; reſiduæ magnitudines A − B, C − D aut habebunt ad invicem rationem datam; aut altera A − B altera C − D major erit data quam in ratione.

Nam ſi A. C :: B. D *a* :: A − B. C − D; *b* liquet $\frac{A-B}{C-D}$ dari.

a 19.5.
b 2.def.4

Saltem ſit D. B *b* :: C. E *a* :: C − D. E − B. ergo *b* $\frac{C}{E}$ & *c* E, ac *d* propterea A − E, *b* itemq; $\frac{C-D}{E-B}$ datæ ſunt. *e* ergo A − B (A − E : + E − B) ⊏ C − D data q.in r. Q.E.D.

c 1.dat.
d 4 dat.
e 11 def. 4.

PROP. 21.

A. C. E.
B. D.

Si datæ fuerint duæ magnitudines A, C; & adjiciantur ipſis aliæ magnitudines B, D habentes ad invicem rationem datam; totæ A + B, C + D aut habebunt ad invicem rationem datam, aut altera

A + B

$A + B$ *altera* $C + D$ *major erit data quam in ratione.*

a 12.5. Nam ſi B. D :: A. C *a* :: A + B. C + D, *b* li-
b 2 def.d. quet $\frac{A+B}{C-D}$ dari.

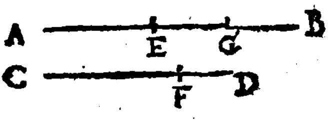
Saltem ſit B. D *b* :: E. C *a* :: B + E. D + C.
c 2.dat ergo *c* E, *d* ideoque A − E, & *b* $\frac{B+E}{D+C}$ dantur,
d 4.dat.
e 11 def. ergo $A + B\ (B + E :: + A - E) \sqsubset C + D$ data q. in r. Q.E.D.

PROP. 22.

A.
B. C.

Si duæ magnitudines A, B *ad aliam aliquam magnitudinem* C *habeant rationem datam, & ſimul utraque* A + B
a hyp. *ad eandem* C *habebit rationem datam.*
b 8.d.
c 6.d. Nam ob $\frac{A}{C}, \frac{B}{C}$ *a* datas, *b* erit $\frac{A}{B}$ data. *c* quare $\frac{A+B}{B}$, *b* ideoque $\frac{A+B}{C}$, data eſt. Q.E.D.

PROP. 23.



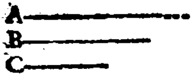
Si totum AB *ad totum* CD *habeat rationẽ datam, habeant autem & partes* AE, EB *ad partes* CF, FD *rationes datas* (*etſi non eaſdem*;) *habebunt omnia ad omnia rationes datas.*

a def.d. Nam ſit AE. CF *a* :: AG, CD *b* :: GE. F-
b 19.5
c hyp. D. *a* ergo $\frac{GE}{ED}$ datur. quare (ob $\frac{EB}{FD}$ *c* datam)
d 8.dat.
e 5.dat. *d* erit $\frac{GE}{EB}$ ac *e* ideo $\frac{EB}{GB}$ data. ergo quum *c* $\frac{AB}{CD}$, & *a* $\frac{AG}{GD}$, *d* ideoque $\frac{AB}{AG}$, ac proinde *e* $\frac{AB}{GB}$, den-

tur,

tur, *d* erit $\frac{EB}{AB}$ data. Quare *e* $\frac{AB}{AE}$, & *d* $\frac{AE}{EB}$ & *e* $\frac{EB}{CF}$ dantur. Q.E.D.

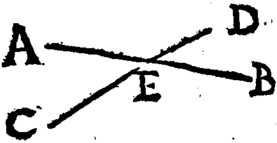
PROP. 24.



Si tres rectæ lineæ, A, B, C, *proportionales fuerint; prima autem* A *ad tertiam* C *habeat rationem datam; & ad ſecundam* B *habebit rationem datam.*

Nam A. C *a* :: Aq. Bq. *b* ergo $\frac{Aq}{Bq}$ data eſt. *a cor. 20. 6.*
proinde $\frac{A}{B}$ *c* datur. Q.E.D. *b 2. def. d* *c 5. d.*

PROP. 25.



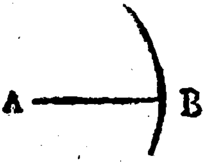
Si duæ rectæ lineæ, AB, CD *poſitione datæ ſe mutuo ſecuerint, punctum* E, *in quo ſe invicem ſecant, poſitione datum eſt.*

a Nam hæ lineæ alibi quam in E, neutrius ſitu mutuo, ſeſe interſecare nequeunt. *a 4. def. d*

Schol.

a Idem patet de quibuscunque lineis poſitione datis, ſeque in unico puncto interſecantibus: ut de circuli arcu, & recta, &c.

PROP. 26.

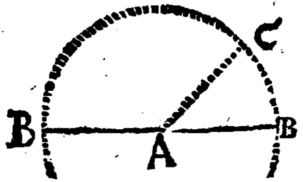


Si rectæ lineæ A B *extremitates* A, B, *poſitione datæ ſint, recta* A B *poſitione*

tione & magnitudine data eſt.

a 3.ax. Poſitione quidem, *a* quia inter eosdem terminos unica recta duci poteſt: & magnitudine, b 1.def.d. *b* quia ſi centro A per B ducatur circulus, hujus omnes radii ipſi A B æquantur.

PROP. 27.



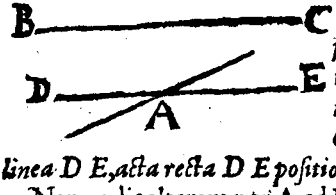
Si recta linea A B poſitione & magnitudine data, data fuerit una extremitas A; & altera extremitas B data erit.

a 1.def.d. Nam ſi centro A, ſpatio A C *a* = A B *b* ducatur circulus, b 3.poſt. cui data recta *c* occurrat in B, *d* c 2.poſt. erit extremitas B data. d cor. 15.

SCHOL.

Vides partes puncti B determinandas eſſe.

PROP. 28.



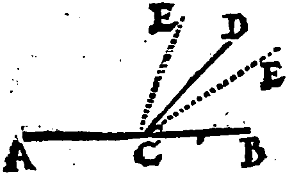
Si per datum punctum A contra datam poſitione rectam B C agatur recta linea D E, acta recta D E poſitione data eſt.

a 4.def.d. Nam *a* dic alteram per A ad B C fore parallelam. b 30.1. Hæc idcirco ad D E *b* parallela erit. *c* c 34.def. Quod repugnat.

Nota Vocabulum *contra* in hoc libro parallelisſmum ſingnificare.

PROP.

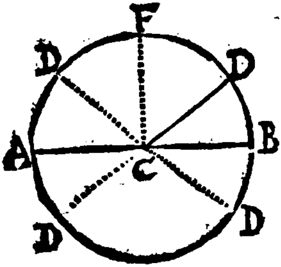
PROP. 29.



Si ad poſitione datam rectam A B, datumq́; in ea punctum C, agatur recta linea C D, quæ faciat angulum D C B datum; acta recta C D poſitione data erit.

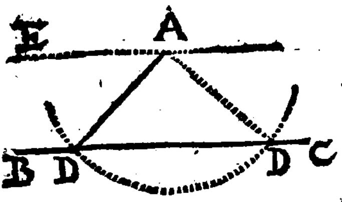
a Nam quævis alia C E angulum B efficiet majorem, vel minorum dato B C D. a 4.def.d b 9.ax.1.

Schol.



Determinari debet ſitus anguli dati tam reſpectu perpendicularis C F, quam ipſius A B, ut cernis in appoſita figura.

PROP. 30.



Si à dato puncto A in datam poſitione rectam B C agatur recta linea A D, quæ faciat angulum A D C datum, acta linea A D poſitione data eſt.

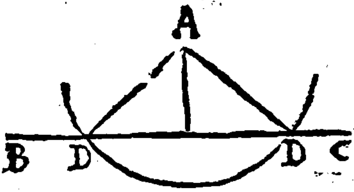
Nam per A duc AE ad B C parallelam. *a* Hæc poſitione datur. Item ang. DAE par dato alternō ADC *b* datus eſt. *c* ergo recta A D poſitione data eſt. Q.E.D. a 28.dat. b 1 def.d c 29.dat.

Schol.

Schol.

Hinc praxin diſcimus à dato puncto ducendi rectam, quæ cum data poſitione recta datum angulum efficit.

PROP. 31.

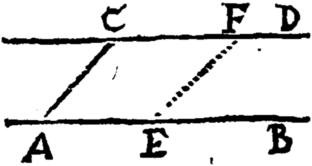


Si à dato puncto A in datam poſitione rectam BC data magnitudine recta AD ducatur, poſitione quoque data erit.

Nam puncta D, per quę tranſit circulus centro A, *a* ſpatio AD deſcriptus, *b* data ſunt. *c* ergo AD poſitione data eſt. Q.E.D.

a 1.def.d.
b ſch.25.d.
c 26.d.

PROP. 32.



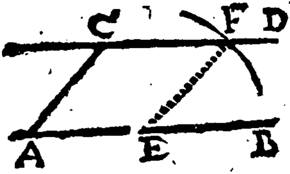
Si in datas poſitione parallelas rectas AB, CD agatur recta linea AC, quæ faciat angulos datos BAC, ACD, *acta recta* AC *magnitudine data eſt.*

Nam ad E (quodvis punctum in AB) fac ang. BEF = *a* BAC. liquet rectas EF, AC *b* parallelas. & *c* pares fore. *d* quare AC data eſt. Q.E.D.

a 1.def.d.
b 29.1.
c 34.1.
d 2.def.d.

PROP.

PROP. 33.

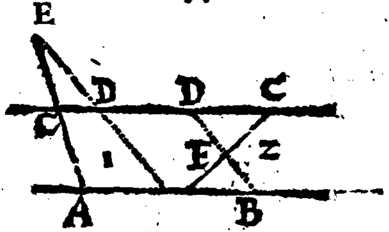


Si in datas poſitione parallelas rectas A B, C D *agatur magnitudine data recta AC, faciet angulos B-A C, A* C D *datos.*

Nam ex quovis puncto E in A B, ſpatio E F *a* = A C deſcribe circulum occurrentem rectę C D in F. *b* Liquet E F, & A C parallelas eſſe poſſe. *c* ergo.

a 1.def.d.
b 34.1.
c 2 p.1.

PROP. 34.



Si in datas poſitione parallelas rectas A B, C-D *à dato puncto E agatur recta linea E C A, ſecabitur data ratione.*

Nam ab E duc rectam E B utcunque parallelis occurrentem in D, & B. *a* liquet eſſe E C. C A :: E D. D B, *b* quare $\frac{\text{F C}}{\text{C A}}$ datur. Q.E.D.

a 2.6.
b 2.def.d.

PROP. 35.

Si à dato puncto E in datam poſitione rectam A B agatur recta linea E A, ſeceturq; data ratione; agatur autem per punctum ſectionis C *contra datam poſitione rectam A B recta linea C D; acta linea C D poſitione data eſt.*

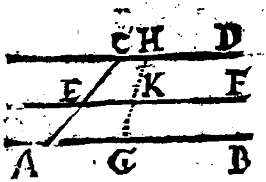
Recta enim E B ducta ab E uteunque in A
a 10. 6. b 28 dat. B, *a* ſecetur ſic ut E D. D B :: E C. C A. ob punctum D datum, *b* erit C D poſitione data. Q.E.D.

PROP. 36.

Si à dato puncto E in datam poſitione rectam lineam A B agatur recta linea E A; adjiciatur autem ipſi aliqua recta E C, quæ ad illam (E A) habeat rationem datam; per extremitatem autem C adjectæ lineæ E C agatur contra datam poſitione rectam A B recta linea C D; acta linea C D poſitione data eſt.

Demonſtratio parum differt à præcedenti *Vide fig.* 2.

PROP. 37.

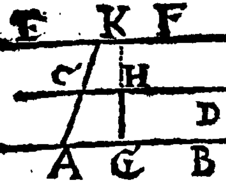


Si in datas poſitione parallelas rectas A B, C D, agatur recta linea A C, & ſecetur ratione data; agatur autem per ſectionis punctum E contra datas poſitione rectas A B, C D linea recta E F; acta recta E F poſitione data eſt.

Nam duc rectam G H utcunque occurrentem parallelis. Hæc *a* ſecta ſit in K ita ut G K.
a 2. def. d. b 28. dat. & ſch. 2. 6 K H :: A E. E C. *b* Punctum K parallelæ (EF) ſitum determinat. Q.E.F.

PROP.

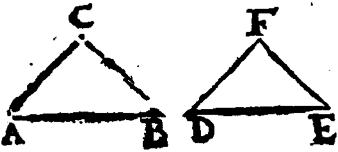
PROP. 38.



Si in datas poſitione rectas parallelas AB, C D agatur recta linea A C; adjiciatur autem ipſi quædam recta C E, quæ ad actam A E habeat rationem datam; per ex-*tremitatem autem E adjectæ C E agatur contra datas poſitione parallelas* AB, CD *recta linea E*-*F; acta recta linea E F eſt data poſitione.*

Demonſtratio perſimilis eſt præcedenti. Cerne & compara figuras.

PROP. 39.



Si trianguli A B C ſingula latera A B, B-C, AC *magnitudine data ſint, triangulum* ABC *ſpecie datum eſt.*

Nam *a* fac triang. D E F ipſi A B C æquilaterum. Hoc eidem *b* æquiangulum erit. *c* ergo ABC ſpecie datum eſt. Q.E.D.

a 22.3.
b 5.6.
c 3.def.A

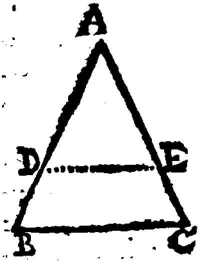
PROP. 40.

Si trianguli ABC *ſinguli anguli*, A,B,C *magnitudine dati ſint, triangulũ* ABC *ſpecie datum eſt.*

Nam ad quamvis D E *a* fac triang. DEF ipſi ABC æquiangulum. *b* Hoc eidem ſimile erit. *c* proinde trigonum ABC ſpecie datum eſt. Q.E.D.

a 23.1.
b 4.6.
c 3.def.A

PROP. 41.



Si triangulum ABC unum angulum A datum habeat; circa datum autem angulum A duo latera A B, A C ad invicem habeant rationem datam; triangulum A B C ſpecie datum eſt.

Nam in uno latere dati anguli ſume quampiam a 1.def.d. A D; & *a* ſit A B.A C :: A D. AE. & duc D- b 6.6. E. *b* Liquet trigonum A D E ipſi A B C ſimile c 3.def.d. fore. *c* Quare A B C ſpecie datum eſt. Q.E.D.

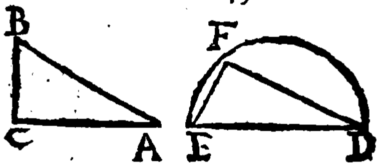
PROP. 42.

Si trianguli A B C latera ad invicem habeant rationem datam, triangulum A B C ſpecie datum eſt.

a 12.6. Nam *a* fac A B. B C :: D E. E F. *a* & B C. b 5.6 C A :: E F. F D. *b* Liquet trigonum D E F tri- c 3.def.d. gono A B C aſſimilari. *c* quare A B C ſpecie datum eſt. Q.E.D.

Vide fig. 39.

PROP. 43.



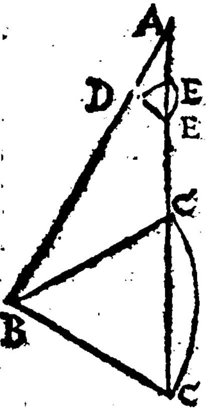
Si trianguli rectanguli A C B circa unum acutorum angulorum A latera A B, A C ad invi-

icem rationem habeant datam, triangulum A-C B ſpecie datum eſt.

Nam eſto D E F ſemicirculus utcunque; & fac A B. A C :: D E. D F. inventamq; D F *b* dapta in ſemicirculo; & duc EF. *c* Liquet tri-ng. D F E ipſi A C B aſſimilari; & *d* proinde pſum A C B ſpecie dari. Q.E.D.

a 18.5.
b 1 4.
c 32.1. & 4.6.
d 3.def.d.

PROP. 44.

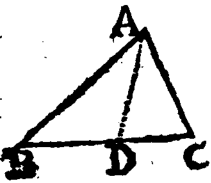


Si triangulum A B C habeat unum angulum A datum; circa alium autem angulum A B C latera A B, BC ad invicem habeant rationem datam; triangulum ABC ſpecie datum eſt.

Nam in crure dati anguli ſume quamlibet A D. & *a* fac A B. B C :: A D. D E. centro D ſpatio D E deſcribe circulum, qui ſecet alterum dati anguli latus in E. *b* Eritque triang. A D E ipſi A B C ſimile. *c* quare datur ſpecie riang. ABC. Q.E.D.

a 2.def.d.
b 7.6.
c 3.def.d.

PROP. 45.



*Si triangulum B A C unum angulum B A C datum habeat; circa datum autem angulum B A C latera ſimul utraq; tanquam unum (*BA + AC*) ad reliquum latus (*B C*) rationem*

nem habeant datam; triangulum BAC *ſpecie datum eſt.*

a 9. 1. Datum angulum BAC a biſecet recta AD.
b 3. 6. b ergo BA. AC :: BD. DC. & componendo
BA + AC. AC :: BC. DC, permutando i-
gitur BA + AC. BC :: AC. DC. ergo ob
c hyp. d 2. def. d. $\frac{BA + AC}{BC}$ c datam, d erit $\frac{AC}{DC}$ data. item ang.
e 2. dat. DAC ſubduplus dati BAC e datur. f ergo an-
f 44. dat. gul. C datur. g proinde trigonum ABC ſpecie
g 40. dat. datum eſt.

Coroll.

Hinc in triangulo, datis uno latere AB, uno angulo BAC, & ratione aggregati laterum ad baſim (R ad S;) datur triangulum. Nam datum angulum biſeca, & fac R. S :: AB. BD. & centro B ſpatio BD duc circulum occurrentem rectæ biſecanti in D; & produc BDC. habes triangulum.

PROP. 46.

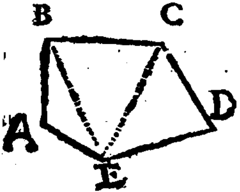
Si triangulum BAC *unum angulum* C *datum habeat; circa alium autem angulum* BAC *latera ſimul utraque tanquam unum* (BA + AC) *habeant ad reliquum* (BC) *rationem datam; triangulum* BAC *ſpecie datum eſt.*

Nam biſecto angulo BAC, erit (ut in præ-
a hyp. cedenti) $\frac{AC}{DC}$ data. item ang. C a datus eſt. er-
b 2 dat. go ang. DAC, b proinde & duplus BAC da-
c 40. dat. tur. c quare triang. BAC ſpecie datur. Q.E.D.

Deducetur ab hac corollarium ſimile præcedenti.

PROP.

PROP. 47.

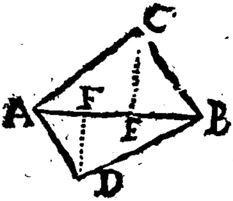


Data ſpecie rectilinea ABCDE *in data ſpecie triangula* BAE, CDE, BCE *dividuntur.*

Nam ob ang. B, & $\frac{BA}{AE}$ *a* dat, *b* erit triang. BAE ſpecie datum. Simili diſcurſu triang. CDE ſpecie datur, *c* quare ang. DCE datus eſt; Hunc deme ex dato BCD, *d* eſtque reliquus BCE datus. Similiter ang. CBE datur. *e* ergo triang. BCE etiam ſpecie datum eſt. Q.E.D.

a hyp. &
b 3. def. d.
b 41. dat.
c 3. def. d.
d 4. dat.
e 40. dat.

PROP. 48.

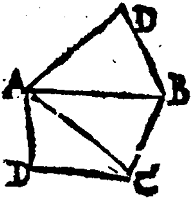


Si ab eadem recta AB *deſcribantur triangula* ACB, ADB *data ſpecie, habebunt ad invicem rationem datam.*

Duc enim perpendiculares CE, DF. Liquet angulos trianguli rectanguli CEB, *a* proinde & $\frac{CE}{CB}$ dari. ergo (quum $\frac{AB}{CB}$ *b* data ſit) *c* erit $\frac{CE}{AB}$ data. Simili diſcurſu datur $\frac{DF}{AB}$; *c* quare $\frac{CE}{DF}$, *d* hoc eſt triang. $\frac{ACB}{ADB}$ datur. Q.E.D.

a 40. dat.
b hyp.
c 8. dat.
d ſch. 1. 6

PROP. 49.

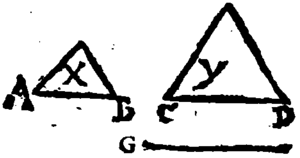


Si ab eadem recta linea AB duo rectilinea quælibet ABCD, AEB data ſpecie deſcribantur, habebunt ad invicem rationem datam.

a 47.dat.
b 48.dat
c 6.dat.
d 8.dat.

Nam rectilineum ABCD reſolvatur in triangula. *a* hæc ſpecie data ſunt. ergo ob communem baſim AC, *b* ratio ADC ad ACB & *c* proinde totius ABCD ad ACB datur. *b* item ratio AEB ad ACB. *d* proinde & ABCD ad AEB datur. Q.E.D.

PROP. 50.

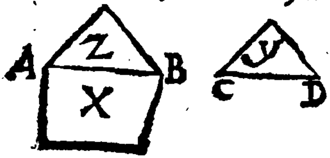


Si duæ rectæ lineæ ABCD ad invicem habeant aut rationem datam; & ab illis ſimilia, ſimiliterq́; deſcripta rectilinea X, Y habebunt ad invicem rationem datam.

a 11.6.
b 1.dat.
c cor. 20. 6.

Nam ſit AB.CD :: *a* CD.G. *b* liquet AB ad G, *c* hoc eſt X ad Y dari. Q.E.D.

PROP. 51.



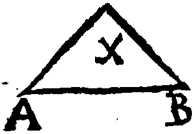
Si duæ rectæ lineæ AB, CD habeant ad invicem rationem datam; & ab illis rectilinea

ctilinea quæcunque X,Y ſpecie data deſcribantur; habebunt ad invicem rationem datam.

Nam *a* fac Z ſimile ipſi Y. Ac ob *b* $\frac{Z}{X}$, *c* & $\frac{Z}{Y}$ datas, *d* liquet $\frac{X}{Y}$ dari. Q.E.D.

a 18.6.
b 49.dat.
c 50.dat.
d 8.dat.

PROP. 52.

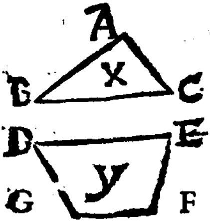


Si à data magnitudine recta A B *figura* X *ſpecie data deſcribatur, deſcripta figura* X *magnitudine data eſt.*

Nam ABq *a* datur ſpecie, & magnitudine; & *b* $\frac{ABq}{X}$ datur. *c* ergo X datur.

a 3.& 4. def.d.
b 49.dat.
c 2.dat.

PROP. 53.

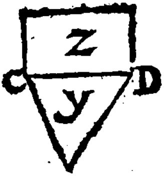


Si duæ figuræ X,Y ſpecie datæ fuerint; & unum latus unius B C *ad unum latus alterius* D E *habuerit rationem datam; reliqua quoque latera* A B *ad reliqua* F G *habebunt rationem datam.*

$$\text{Nam}\left\{\begin{array}{l} a\ AB \\ b\ \overline{BC} \\ a\ \overline{DE} \\ \quad \overline{EF} \\ a\ \overline{FG} \end{array}\right. \begin{array}{l} \\ \\ \text{dantur.} \\ \\ \&c,\ \text{ergo per 8.dat.} \end{array}$$

a 3.def.d.
b hyp.

PROP. 54.



Si duæ figuræ X, Y ſpecie datæ ad invicem habuerint rationem datam, etiam latera (AB, CD, &c.) *habebunt ad invicem rationem datam.*

a 18.6.
b 3.def.d.
c 49.dat.
d hyp.
e 8.dat.
f cor.20.6
g 24.dat.

Nam ad CD *a* fiat Z ipſi X ſimilis. *b* Hæc ſpecie datur. *c* ergo $\frac{Y}{Z}$ datur. Proinde ob $\frac{Y}{X}$ *d* datam, *e* datur $\frac{X}{Z}$. *f* ergo $\frac{AB}{CD}$ datur. ergo per præcedentem.

PROP. 55.

Si ſpatium X magnitudine & ſpecie datum fuerit, ejus latera (AB &c.) *magnitudine data erunt.*

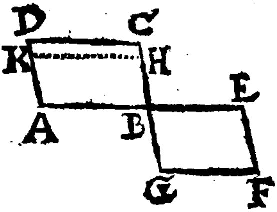
a 18.6.
b 1 dat.
c 54.dat.
d 2.dat.

Nam ad quamvis CD *a* fac Y ſimile ipſi X. hoc ſpecie & magnitudine datur. *b* ergo $\frac{Y}{X}$ datur. *c* quare $\frac{CD}{AB}$ datur. *d* ergo AB data eſt.

Q.E.D.

PROP.

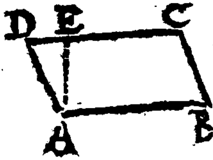
PROP. 56.



Si duo æquiangula parallelogrãma A C, B F habuerint ad invicem rationem datam, eſt ut primi latus A B ad ſecundi latus B-E, ita reliquum ſecundi latus B G ad eam B H, ad quam alterum primi latus B C habet rationem datam, quam habet parallelogrammum A C ad parallelogrammum B F.

Nam duc H K parall. A B. Liquet eſſe B C. BH *a* :: A C. A H *b* :: A C. B F. Q.E.D. a 1.6. b 14.6. c 7.5.

PROP. 57.



Si datum ſpatium A C ad datam rectam A B applicatum fuerit, in angulo B A D dato, datur applicationis altitudo A D.

a Erige perpendicularem A E. eſtque A B. A E *b* :: A Bq. A B x A E *c* :: A Bq. pgr. A C. *d* ergo A E datur, quare per E duc parallelam D C, *e* hæc abſcindet quæſitam A D. Q.E.F. a 11.1. b 1.6. c 35.1. d 1. & 2. dat. e 28. & 35 dat.

PROP. 58.

Si datum ad datam rectam applicetur, deficiens data ſpecie figura, latitudines defectus datæ ſunt.

Non differt à vigeſima octava ſextæ.

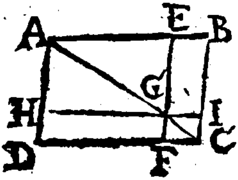
PROP.

PROP. 59.

Si datum ad datam rectam applicetur, excedens data ſpecie figura, latitudines exceſſus data ſunt.

Eadem eſt cum vigeſima nona ſextæ.

PROP. 60.



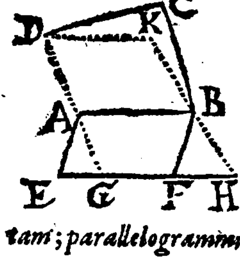
Si datum ſpecie parallelogrammum (H E, vel DB) dato gnomone H C E augeatur, vel minuatur; latitudines gnomonis H D, E B data ſunt.

a 3. dat.
b 24. 6.
c 55. dat.
d hyp.
e 4. dat.

1. *Hyp.* Liquet totum D B tam *a* magnitudine, quam *b* ſpecie dari, *c* proinde & latitudines AB, AD; è quibus aufer *d* datas A E, A H, *e* manent E B, H D datæ. Q.E.D.

2, *Hyp.* Liquet HE *b* ſpecie, & *a* magn. *c* dari, *c* quare & latera AE, AH; hæc deme ex *d* datis A B, A D: *e* remanent E B, H D datæ. Q.E.D.

PROP. 61.



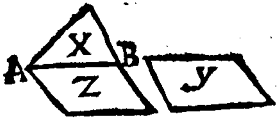
Si ad datæ ſpecie figuræ ABCD *unum latus* AB *applicetur parallelogrammum ſpatium A F in angulo B A E dato; habeat autem data figura* AC *ad parallelogrammum A F rationem datam; parallelogrammum A F datum eſt.*

Ad

Ad DAG protractam duc (per B) parallelam, cui occurrant EFH, & DK parall. AB. Ac ob $\frac{AD}{AB}$, & ang. BAD a dat. a liquet pgr. AK ſpecie dari. b ergo $\frac{AK}{AC}$ & c proinde $\frac{AK}{AF}$, d vel $\frac{AK}{AH}$, e hoc eſt $\frac{AD}{AG}$ dantur. e ergo $\frac{AB}{AG}$ datur. Item ob angulos E, & GAE f notos, g datur $\frac{AE}{AG}$; c ergo $\frac{AB}{AE}$ datur. h unde pgr. AF ſpecie datur. Q.E.D.

a 3.def.B
b 49.dat.
c 8.dat.
d 35.1.
e 1.6.
f hyp. & 4.dat.
g 40.dat.
h 3.def.B

PROP. 62.



Si duæ rectæ AB, CD ad invicem habeant rationem datam; & ab una quidem data ſpecie figura X deſcripta ſit, ab altera autem ſpatium parallelogrammum Y in angulo dato; habeat autem figura X ad parallelogrammum Y rationem datam; parallelogrammum Y ſpecie datum eſt.

Nam ad AB ſit pgr. Z ſimile ipſi Y. a Hujus ratio ad Y, & b proinde ad X datur. c ejuſq; anguli dantur. d ergo Z ſpecie datur. e proinde & Y. Q.E.D.

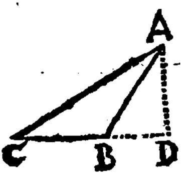
a 50.dat.
b 8.dat.
c hyp.
d 61.dat.
e 3.def.B

PROP. 63.

Si triangulum ſpecie datum ſit, quod ab unoquoque laterum deſcribitur quadratum, ad triangulum habebit rationem datam.

Sequitur ex 49. hujus.

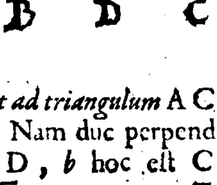
PROP. 64.



Si triangulum A B C *angulum obtuſum A B C datum habeat; illud ſpatium, quo latus A C obtuſum angulum ſubtendens magis poteſt quam latera A B, C B obtuſum angulum A B C ambientia, ad triangulum A B C habebit rationem datam.*

Nam demittatur A D perpendicularis pro-
a 4. dat. ductæ C B D. atque ob angulos *a* A B D, & D
b 40. dat. datos, *b* datur $\frac{BD}{AD}$, *c* hoc eſt $\frac{BD \times CB}{AD \times CB}$. *d* ergo
* 1. 6.
d 8 dat.
e 12. 2. $\frac{2\,BD \times CB}{\frac{1}{2}AD \times CB}$, hoc eſt, *e* $\frac{ACq - ABq \div CBq}{f\ \text{triangul.}\ ABC}$ da-
f 41. 1. tur. Q.E.D.

PROP. 65.



Si triangulum A C B *angulum acutum* C *datum habeat; illud ſpatium, quo latus A B angulum C ſubtendens minus poteſt, quam latera* A C, C B *angulum acutum* C *ambientia, habebit ad triangulum* A C B *rationem datam.*

a 40 dat. Nam duc perpendicularem A D. Datur *a*
b 1. 6. $\frac{CD}{AD}$, *b* hoc eſt $\frac{CD \times BC}{AD \times BC}$. *c* ergo $\frac{2}{\frac{1}{2}}$
c 8. dat

CD-

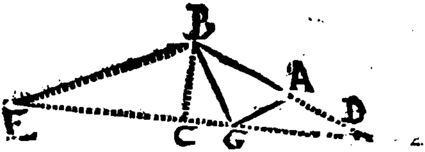
$\frac{CD \times BC}{AD \times BC}$, hoc eſt *d* $\frac{ACq + BCq - ABq}{2 \text{ triang. } ACB}$ datur. Q.E.D. d 13.2. c 11.1.

PROP. 66.

Si triangulum ACB *habuerit angulum* C *datum; quod ſub rectis* AC, CB *datum angulum* C *comprehendentibus, continetur rectangulum, habebit ad triangulum* ACB *rationem datam.*

Nam in figura præcedentis, eſt *a* $\frac{AC}{AD}$, *b* hoc eſt, $\frac{AC \times BC}{AD \times BC}$, *c* hoc eſt $\frac{AC \times BC}{2 \text{ triang. } ACB}$ data. *d* ergo $\frac{AC \times BC}{\text{triang. } ACB}$ datur. Q.E.D. a 40.dat. b 1.6. c 41.1. d 1.dat.

PROP. 67.



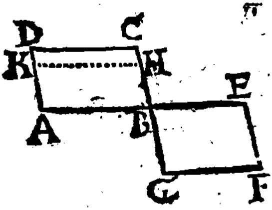
Si triangulum ABG *habuerit datum angulum* BAG; *illud ſpatium, quo duo datum angulum* BAG *comprehendentia latera tanquam una recta* BA + AG, *plus poſſunt, quam quadratum à reliquo latere* BG, *ad triangulum* ABG *habebit rationem datam.*

Produc BA ita ut AD = AG. per B duc BE parall. AG; cui occurrat DGE. denique duc normalem BC.

Liquet

a 5.1. Liquet ang. D *a* = AGD *b* = E. *c* quare BE =
b 29.1. BD, ideoque EC = CD. *e* ergo EG x GD +
c 6.1.
d cor.3.3. CGq = CDq. proinde BDq *f* (CDq + BCq)
e 5.2. *g* = EG xGD + CGq + BCq = EGxGD * +
f 47.1.
g 2.ax.1. BGq. Jam ob angulos AGD, & D *h* ſubduplos
* 47.1. dati BAG, liquet *k* $\frac{AD}{DG}$, ideoque $\frac{ADq}{DGq}$ dari.
h 32.1.
k 40.dat.
l 1.6. Cum igitur BA x AD. ADq *l* : : BA. AD *m* : :
m 2.6. EG. GD : : *l* EGxGD. GDq, & permutando
n 2.def.d. BA x AD. EG x GD : : ADq. GDq; *n* erit
o conſtr. $\frac{BAxAD}{EGxGD}$; *o* hoc eſt $\frac{BAxAG}{EGxGD}$ data. *p* Atqui
p 66.dat.
q 1.dat. $\frac{BAxAG}{triang.\ AGB}$ datur; *q* ergo $\frac{EGxGD}{triang.\ AGB}$ datur.
Q.E.D.

PROP. 68.

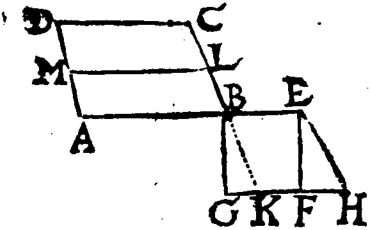


π *Si duo parallelogramma æquiangula* AC; BF *habeant ad invicem rationem datam, & unum latus* AB *ad unum latus* BE *habeat rationem datam; & reliquum latus* BC *ad reliquum latus* BG *habebit rationem datam.*

a 2.def.d. Nam ſit AB. BE : : BG, BH. *a* ergo $\frac{BG}{BH}$ da-
b 56 dat. tur. *b* item $\frac{BC}{BH}$ datur. *c* ergo $\frac{BC}{BG}$ datur.
c 8.dat.

PROP.

PROP. 69.



Si duo parallelogramma A C, B F datos angulos habeant, & ad invicem rationem datam; habeat autem & unum latus A B ad unum latus B E, rationem datam; & reliquum latus B C ad reliquum latus B G habebit rationem datam.

Latera AB, BE jaceant in directum. produc CBK, ac G F H ad occurſum cum E H parall. C K.

Ob *a* ang. KBE (ABC) & pgr. *a* $\frac{AC}{BF}$, vel $\frac{AC}{BH}$ & *a* $\frac{AB}{BE}$ datas, *e* liquet $\frac{KB}{BC}$ dari, item ob ang. G, & GBK *d* datos, *e* datur $\frac{KB}{BG}$. *f* quare $\frac{BC}{BG}$ datur. Q.E.D.

a *hyp.*
b 35. 1.
c 61. *dat.*
d *hyp.* & 4. *dat.*
c 40, *dat.*
f 8. *dat.*

PROP. 70.

Si duorum parallelogrammorum (A C, B H, vel B F) circa æquales angulos (A B C, K B E) aut circa inæquales quidem (A B C, G B E) datos tamen, latera (A B, B E, & B C, B K, & B C, B G) ad invicem habeant rationem datam; & ipsa parallelogramma (A C, B H, & A C, B F) habebunt ad invicem.

Nam (in fig. præced.) ſit AB. BE :: KB. BL & duc LM parall. BA.

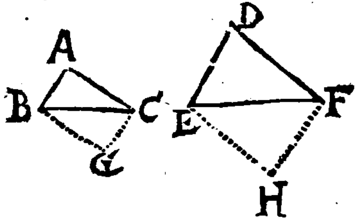
a hyp. b conſtr c 8.dat. d 1.6. e 14.6.

Primo, Quia a $\frac{AB}{BE}$ b id eſt $\frac{KB}{BL}$, a ac $\frac{KB}{CB}$ datæ ſunt, c erit $\frac{CB}{BL}$, d hoc eſt $\frac{AC}{AL}$, e vel $\frac{\text{pgr. AC}}{BH}$ data. Q.E.D.

f hyp. & 4.dat. g 40.d. h 35.1.

Secundo, ob angulos G, & GBK f datos, g datur $\frac{BK}{BG}$; item h $\frac{CB}{BG}$ data eſt. c ergo $\frac{CB}{BK}$ datur. proinde, ut prius, $\frac{AC}{BH}$, hoc eſt $\frac{\text{pgr. AC}}{BF}$ datur. Q.E.D.

PROP. 71.



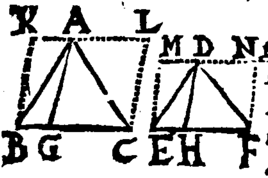
Si duorum triangulorum ABC, DEF, circa æquales angulos, aut circa inæquales quidem, datos tamen (A, & D) latera AB, DE, & AC, DF ad invicem habeant rationem datam; & ipſa triangula ABC, DEF habebunt ad invicem rationem datam.

a 70.dat, b 15.5. c 34.1.

Nam compleantur pgra. AG, DH. a hæc datam habent rationem, b proinde & trigona ABC, DEF illorum c ſubdupla. Q.E.D.

PROP.

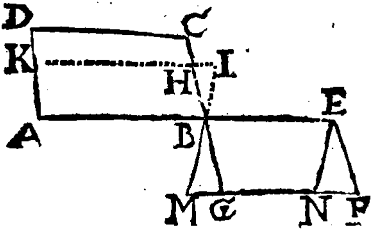
PROP. 72.



Si duorum triangulorum ABC, DEF & baſes BC, EF fuerint in ratione data, & actæ ab angulis ad baſes (AG, DH,) quæ faciant ang. AGC, DHF æquales, aut inæquales quidem, ſed tamen datos, habeant ad invicem rationem datam; & ipſa triangula ABC, DEF habebunt ad invicem rationem datam.

Nam duc BK ad A G, ac EM ad DH parallelas, & comple pgra. CK, EM. Hæc ſe habent juxta 70. hujus; quare triangula eorum * ſubdupla A B C, D E F rationem habent datam. Q.E.D. * 34.1.

PROP. 73.



Si duorum parallelogrammorum (AC, BF, vel AC, BN) circa æquales angulos, aut circa inæquales quidem, ſed tamen datos, latera ad invicẽ ita ſe habeant, ut ſit quemadmodũ primi latus AB ad ſecundi latus BE, ita reliquũ ſecundi latus (BG, vel BM) ad aliã aliquam rectam) BH, vel BI;)

BI;) habeat autem & reliquum primi latus BC ad eandem rectam (BH vel BI) rationem dati; & ipſa parallelogramma (AC, BF, vel AC, BN) habebunt ad invicem rationem datam.

a hyp. b 2.6. c 14.6. Nam 1. Hyp. liquet *a* $\frac{CB}{BH}$, *b* id eſt $\frac{AC}{AH}$ *c* $(\overline{BF})$ dari. Q.E.D.

a hyp & 4. dat. b 40. dat. c 8 dat d 35.1. 2. Hyp. Duc parallelam IHK. *a* Liquet angulos IBH (GBM) & BHI (ABH) dari. *b* ergo $\frac{BH}{BI}$ datur. item $\frac{CB}{BI}$ *a* data eſt. *c* proinde $\frac{CB}{BH}$, hoc eſt pgr. $\frac{AC}{BF}$ *d* vel $\frac{AC}{BN}$, datur. Q.E.D.

PROP. 74.

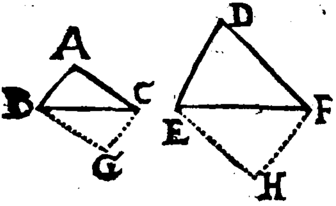
Si duo parallelogramma datam rationem habeant, aut in æqualibus angulis (ut AC, BF) aut inæqualibus quidem, ſed tamen datis (ut AC, BN;) erit ut primi latus AB ad ſecundi latus BE, ita alterum ſecundi latus (BG, vel BM) ad eam (BH, vel BI) ad quam reliquum primi latus BC rationem habet datam.

a 56. dat. Nam in fig. præcedentis 1. Hyp. *a* Liquet $\frac{CB}{BH}$ dari. Q.E.D.

* hyp. b 4.6. c 8. dat. 2. Hyp. ut in præcedenti, datur $\frac{BI}{BH}$, ac ex hyp. $\frac{AC}{BF}$ $(\overline{BN})$ item AB. BE :: *a* * MB. BI *b* :: $\frac{GB.}{BH.}$ *a* quare $\frac{CB}{BH}$ etiam datur. *c* ergo $\frac{CB}{BI}$ data eſt. Q.E.D.

PROP.

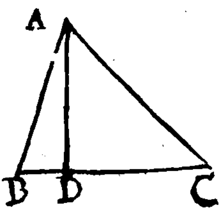
PROP. 75.



Si duo triangula ABC, DEF ad invicem habeant rationem datam, aut in angulis (A, D) æqualibus, aut inæqualibus quidem ſed tamen datis, erit ut primi latus AB *ad ſecundi latus* DE, *ita alterum ſecundi latus* DF *ad eam rectam, ad quam reliquum primi latus* AC *habet rationem datam.*

Nam compleantur pgr. AG, DH. Ergo per præcedentem.

PROP. 76.

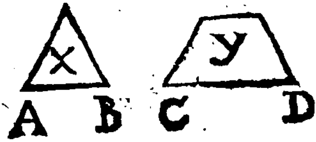


Si à trianguli ABC *ſpecie dati vertice* A *linea perpendicularis* AD *agatur ad baſim* BC, *acta linea* AD *ad baſim* BC *habebit rationem datam.*

Nam ob angulos, † B, & ADB datos, *a* datur $\frac{AB}{AD}$; *a* item $\frac{AB}{BC}$ datur. *b* Ergo $\frac{AD}{BC}$ datur. Q.E.D.

† hyp. & 3. def. d.
a 40 dat.
b 8. dat.

PROP. 77.

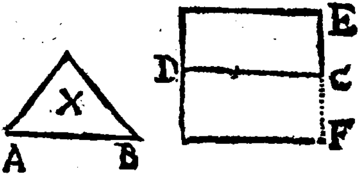


Si duæ figuræ ſpecie X, Y ad invicem habeant rationem datam, & quodlibet latus unius A B ad quodlibet alterius latus C D habebit rationem datam.

a 49. dat.
b hyp.
c 8. dat.

Nam *a* $\frac{ABq}{X}$, & *b* $\frac{Y}{X}$, ac *c* proinde $\frac{ABq}{Z}$ datur; item $\frac{CDq}{Y}$ datur. *c* ergo $\frac{ABq}{CDq}$, ac ideo $\frac{AB}{CD}$ datur. Q.E.D.

PROP. 78.



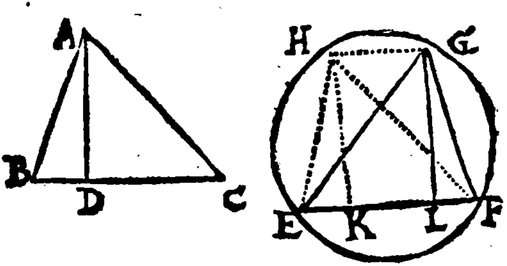
Si data figura ſpecie X ad aliquod rectangulum DCE *habeat rationem datam; habeat autem & unum latus* AB *ad unum latus* DC *rationem datam; rectangulum* DCE *ſpecie datum eſt.*

a 8. dat.
b 49. dat.
c hyp.
d 17. 6.
e 1. 6.
f 3. def. d.

Sit DC. AB : : AB. CF. *a* ergo $\frac{DC}{CF}$ datur. Item ob *b* $\frac{X}{ABq}$, & *c* $\frac{X}{DCE}$ datas. *a* erit $\frac{ABq}{DCE}$, *d* hoc eſt $\frac{DC \times CF}{f\,DCE}$, vel *e* $\frac{CF}{CE}$ data. proinde *e* $\frac{DC}{CE}$ datur. quare rectang. DCE ſpecie datur, Q.E.D.

PROP.

PROP. 79.

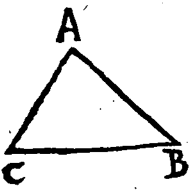


Si duo triangula ABC, GEF *unum angulum* BAC *uni angulo* EGF *æqualem habeant; ab æqualibus autem angulis* BAC, EGF *ad baſes* BC, *EF perpendiculares agantur A D,GL; ſitque ut primi trianguli baſis ad perpendicularem, ita & alterius trianguli baſis ad perpendicularem* (BC.AD :: EF.GL;) *illa triangula* ABC,EGF *æquiangula ſunt.*

Circa triang. G E F deſcribe circulum. Fac ang.FEH = B. Connecte HF, HG; & demitte perpendicularem HK.

Liquet triangula ABC,HEF,& ABD, HEK,ac ACD,HEK æquiangula fore. Proinde
EK. KH :: BD. DA. *a* & FK. KH :: C D. D- a 4.6.
A. *b* quare EF. KH :: BC. DA :: *c* E F.L G. *d* b 24.5.
c hyp.
quare KH = LG. *e* ergo HG parall.KL. *f* un- d 9.5.
de ang.EGH = GEF. *g* ergo arcus EH, F G, *h* e 33.1.
f 29.1.
ideoque anguli EFH, GEF æquantur. *k* Item g 26.3.
ang. EHF = EGF. *l* ergo trigona EHF,EGF h 27.3.
k 21.3.
m proinde & trigona E G F, ABC ſibi mutuo l 32.1.
æquiangula ſunt. Q.E.D. m 21.6.

PROP. 80.



Si triangulum ABC *unum angulum* A *datum habuerit; quod autem ſub lateribus* AB, AC *datum angulum comprehendentibus continetur rectangulũ, habeat ad quadratum reliqui lateris* BC *rationem datam; triangulum* ABC *ſpecie datum eſt.*

a 67.dat Nam Q: AC + AB: − CBq vocetur X. *a*
b 66.dat. c 8.dat. ergo $\frac{X}{\text{triang. ABC}}$; *b* & $\frac{AC \text{x} AB}{\text{triang. ABC}}$; & *c* propterea
d hyp. e 6.dat. $\frac{X}{AC \text{x} AB}$ data eſt. *d* item $\frac{AC \text{x} AB}{CBq}$ datur. *e* ergo
f hyp. $\frac{X\ e}{CBq}$ ideoque $\frac{X + CBq}{CBq}$, *f* hoc eſt $\frac{Q: AC + AB.}{CBq}$
g 46.dat. datur. *g* proinde triang. ABC ſpecie datur. Q.E.D.

PROP. 81.

A. D.
B. E.
C. F.

Si tres rectæ proportionales A, B, C *tribus rectis proportionalibus* D, E, F *extremas* A, D, *&* C, F *habuerint in ratione data; medias quoque* B, E *habebunt in ratione data. Et ſi extrema* A *ad extremam* D, *& media* B *ad mediam* E *habeat rationem datam; & reliqua C ad reliquam F habebit rationem datã.*

a 70.dat Nam primo, ob $\frac{A}{D}$ & $\frac{C}{F}$ datas, *a* datur $\frac{AC}{DF}$,
b 17.6. *b* hoc eſt, $\frac{Bq}{Eq}$. ergo $\frac{B}{E}$ datur. Q.E.D.

Se-

Secundo, ob *c* $\frac{Bq}{Eq}$, *b* hoc eſt $\frac{AC}{DF}$ datam, & *c* $\frac{A}{D}$ datam, *d* datur $\frac{C}{F}$. Q.E.D. c hyp. d 61. dat.

PROP. 82.

A. B :: D. E.
B. C :: E. F.

Si quatuor rectæ proportionales fuerint (A. B :: D. E) *erit ut prima* A *ad eam* C, *ad quam ſecunda* B *rationem habet datam, ita tertia D ad eam F, ad quam quarta E rationem habet datam.*

Nam quia B. C :: *a* E.F. & *a* $\frac{B}{C}$ data eſt; *b* erit $\frac{E}{F}$ data. atqui ex ęquali A. C :: D.F. ergo. a hyp. b 2.def.[illegible]

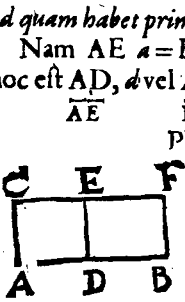
PROP. 83.

A. B. C. D.
F. E. D

Si quatuor rectæ A,B,C, D *ita ad invicem ſe habeant, ut tribus ex iis quibuscunque ſumptis* A, B, C, *& quarta ipſis proportionali accepta E, ad quam reliqua D ex quatuor rectis proportionem habet datam; erit ut quarta D ad tertiam* C, *ita ſecunda B ad eam F, ad quam habet prima* A *rationem datam.*

Nam AE *a* = BC *b* = DF. & datur *b* $\frac{D}{E}$, *c* hoc eſt $\frac{AD}{AE}$, *d* vel $\frac{AD}{DF}$, *e* vel. $\frac{A}{F}$. ergo. a 16.6. b hyp. c 1.6. d 7.5.

PROP. 84.



Si duæ rectæ AB, AC *datum ſpatium comprehendant in angulo A dato; ſit autem altera* AB *altera* AC *major data*

data DB; *etiam unaquaque ipſarum* AB, AC *data erit.*

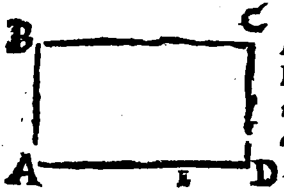
a 3.def.d. Nam comple quadratum A E. *a* Hoc ſpecie
b hyp. datum eſt. *b* item pgr. CB, & recta DB dantur.
c 59.d. *c* ergo AC, vel AD. & tota *d* proinde AB da-
d 3.dat. tur. Q.E.D.

PROP. 85.

Si duæ rectæ BD, DE *datum ſpatium comprehendant in angulo BDE dato, ſit autem ſimul utraque* (BD + D E) *data* ; *& earum quoque unaquaque* BD, *&* DE *data erit.*

Nam ſume DA=DE, & comple quad. DC.
Hoc ſpecie datur; item pgr. BE, & recta B A
a hyp. *a* dantur. *b* ergo AD (DE) & *c* reliqua DB
b 58.dat c 4.dat. dantur. Q.E.D.

PROP. 86.



Si duæ rectæ AB, AD *datum ſpatium* BD *comprehendant in angulo dato; quadratum autem unius* AD *quadrato alterius* AB *majus ſit dato quam in ratione* (nempe ut ſit AD x AE da-
* 8.2. tum, & * reliqui ADxED ad ABq ratio data;)
& utraque ipſarum AB, AD data erit.

a hyp. Nam ob BD, & DAxAE *a* data, *b* datur
b 1.dat. c 69.dat. d 51.dat. $\frac{BD}{DAxAE}$. *c* ergo $\frac{AB}{AE}$ *d* ideoque $\frac{ABq}{AEq}$ datur. *e* item
e hyp. f 1.dat. $\frac{ABq}{ADxED}$ datur. *f* ergo $\frac{AEq}{ADxED}$ ideoque $\frac{AEq}{4ADxED}$,

g &

g & $\frac{AEq}{4AD \times ED + AEq}$ *h* hoc eſt $\frac{AEq}{Q:AD + ED}$ datur. g 6.dat. h 1.2.
k ergo $\frac{AE}{AD\ ED;}$ & *l* componendo $\frac{AE}{2AD,}$ † ideoq; k 54.dat. l 6.dat. † 1.dat.
$\frac{AE}{AD,}$ *m* hoc eſt $\frac{AEq}{AD \times AE}$ datur. denique igitur ob m 1.6.
e datum AD x AE, *n* erit AEq data. *o* ergo AE, & *p* proinde AD, ac AB datæ ſunt. n 2.dat. o 55.dat. p 57.dat.
Q.E.D.

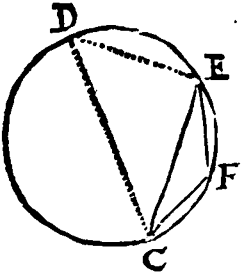
PROP. 87.

Si duæ rectæ AB, AD *datum ſpatium comprehendant in angulo dato, quadratum autem unius* AD *quadrato alterius* AB *majus ſit dato* (AD x AE;) *earum utraque* AB, AD *data erit.*

Nam ob $\frac{BA \times AE}{BD}$ *a* datum, *b* erit $\frac{AE}{AB,}$ ideoq; a 1.dat. b 69.dat.
$\frac{AEq}{ABq.}$ *c* hoc eſt $\frac{AEq.}{AD \times ED,}$ *d* ac idcirco $\frac{AEq}{AEq + 4AD \times ED,}$ c hyp.& 1.2. d 8.& 6.dat.
e hoc eſt $\frac{AEq}{Q:AD + ED,}$ ac proinde $\frac{AEq}{AD + ED,}$ & *d* componendo $\frac{AE}{2AD,}$ *e* ac ideo $\frac{AE}{AD,}$ *e* hoc eſt $\frac{AEq}{ADA \times E}$ e 1.2. d 6.dat. e 1.6.
data. ergo ob AD x AE *f* datum, dantur *g* AEq, & *h* AE, ac *k* ideo AD, ac AB. Q.E.D. f hyp. g 2.dat. h 55.dat. k 57.dat.

PROP.

PROP. 88.



Si in circulum CFED *magnitudine datum acta ſit recta linea* C E,*quæ ſegmẽtum auferat,quod datum angulum F comprehendat; acta recta linea* C E *magnitudine data eſt.*

Nam ducatur diameter C D ; & connectatur E D. Ac ob ang.F

a hyp. *a* datum,*b* erit ang.D (reliquus è 2 rectis) datus.

b 4.dat.

c 40.dat. item rectus CED datur. *c* quare $\frac{CE}{CD}$ datur. ergo ob *d* datam CD, *e* erit C E data. Q.E.D.

d hyp. & 5 def. d.

e 2.dat.

PROP. 89.

Si in datum magnitudine circulum C F E D *data magnitudine recta* CE *acta fuerit, auferet ſegmentum quod angulum* (CFE) *datum comprehendet.*

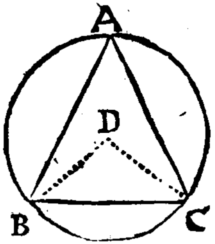
Nam (in fig.præcedentis) quia $\frac{CE}{CD}$, & ang.

a 43.dat. C E D dantur, *a* erit Angulus D datus. *b* ergo

b 4.dat.

c 32.3. angul. F *c* (1 Rect. – D) datus erit. Quod Erat Demonſtrandum.

PROP. 90.

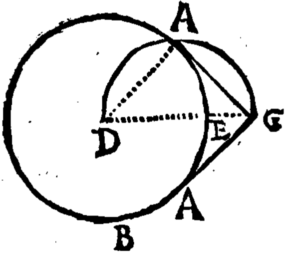


Si in circuli poſitione dati circumferentia BAC datum fuerit punctum B, ab eo autem puncto B ad circumferentiam circuli inflexa fuerit recta BAC quæ datum angulum A efficiat; inflexa rectæ altera extremitas C data erit.

Ad *a* centrum D duc BD, & CD; *b* datusq́; a 1.3.
eſt ang. D dati A *c* duplus. quare ob BD *d* da- b 1.dat.
tam, *e* erit DC data. *f* ergo punctum C datum c 20.3. d 26.dat.
eſt. Q.E.D. e 29.dat. f ſch.23.d

Si ang. A obtuſus fuerit; ſume reliquum è 2 rectis acutum; ejus ſubſidio punctum C invenies, juxta dicta.

PROP. 91.



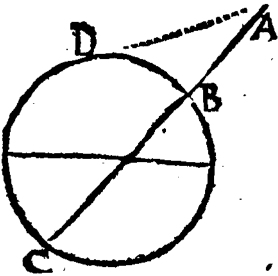
Si à dato puncto G acta fuerit recta GA, quæ datum poſitione circulum BEA contingat; acta linea GA poſitione & magnitudine data eſt.

Nam centrum D & punctum G connectat recta DG. ſuper qua deſcriptus ſit ſemicirculus DAG circulo priori occurrens in A. Ob ang.
DAG *a* rectum, GA circulum *b* tangit. *c* ergo a 31.3.
GA ſitu & magnitudine datur. Q.E.D. b cor.16.3 c 26.dat.

Hinc

Hinc modus diſcitur à dato puncto tangentem ducendi, eo nonnunquam expeditior qui habetur ad 17.3.

PROP. 92.

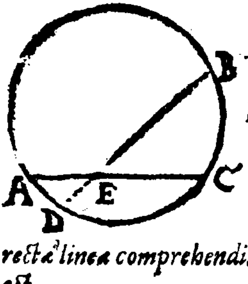


Si extra circulum poſitione datum BCD *accipiatur aliquod punctum* A, *à dato autem puncto* A *in circulum producatur quadam recta* A C; *datum eſt id quod ſub acta linea* AC, *& ea* AB, *quæ inter punctum* A *& convexam peripheriam* B *comprehenditur rectangulum* CAB.

a 91. dat.
b 36. 3.

a Nám duc tangentem AD, *b* eritque ADq (hoc eſt CAxAB) datum. Q.E.D.

PROP. 93.

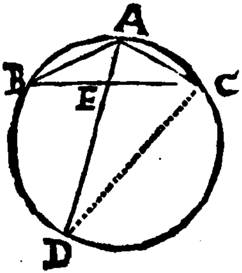


Si intra datum poſitione circulum A B C D *ſumatur aliquod punctum* E; *per punctum autem* E *agatur in circulum aliqua recta* AEC; *quod ſub ſegmentis* AE, EC *actæ rectæ lineæ comprehenditur rectangulum, datum eſt.*

Nam

Nam per E duc rectam DEB utcunque occurrentem circulo in B, & D. eſtque rectang. DEB = *a* AEC. *b* ergo AEC datur. Q.E.D. a 35. 3. b 1. def. 6

PROP. 94.



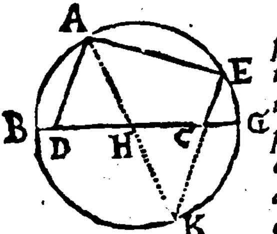
Si in circulum BACD magnitudine datum agatur recta linea BC, quæ ſegmentum auferat, quod angulum BAC datum comprehendat; angulus autem BC, qui in ſegmento conſiſtit, bifariam ſecetur; ſimul utraque rectarum BA, AC quæ angulum datum BAC comprehendunt, ad lineam AD, quæ angulum bifariam ſecat, habebit rationem datam: & quod ſub ſimul utriſque BA, AC, quæ datum angulum BAC comprehendunt, rectis: & inferne abſciſſa (ED) *ab ea* AD, *quæ angulum BAC in circumferentia datum bifariam ſecat, rectangulum datum erit.*

Duc CD; & primo ob angulos BAC, CAD datos, *a* dantur ſubtenſæ BC, CD, † ideoq; $\frac{CB}{BC}$ datur. Cum igitur CA.AB :: *b* CE.EB, & permutando CA.CE :: AB.EB :: (CA + AB.CB ::) † AD.DC. (Nam * ob ang. BAE = CAD; & D = BD; trigona ABE, ADC ſimilia a 11. dat. † 1. dat. b 3. 6. c 16. 5. † 4. 6. * 2. def. 6.

milia ſunt) ac rurſus permutando C A + AB. A D :: CB. D C. *d* erit $\frac{CA + AB}{AD}$ data. Q.E.D.

e 21.3. Secundo, ob triangula AEB, DEC *e* ſimilia;
b 4.6. *b* erit CD, DE :: AB. BE *c* :: CA + AB. CB. *d*
c prius. ergo CA + AB in DE = CD in CB. atqui C-
d 16.6. D x CB *e* datur. *f* ergo C A + AB in D E da-
e 52. dat. tum eſt, Q.E.D.
f 1. def. d.

PROP. 95.



Si in circuli B A G *poſitione dati diametro* B G *ſumatur datum punctum* D; *à puncto autem* D *in circulū producatur quadam recta* D A, *& agatur à ſectione* A *ad rectos angulos in productam rectam* D A *linea* AE; *per punctum autem E, in quo linea A E, qua ad rectos angulos conſiſtit, occurrit circumferentia circuli, agatur parallela* (E CK) *producta recta* D A; *datum eſt illud punctum C, in quo parallela* E K *occurrit ipſi diametro* BG; *& quod ſub parallelis lineis* AD, E C *comprehenditur rectangulum, datum eſt.*

a 31.3. Nam connectatur AK. *a* eſtque AB (ob angulum E, vel D A E rectum) diameter. ergo
b 26. dat. inſectio H eſt centrum. *b* ergo DH datur. At-
c 4.6. qui ob KH. HA *c* :: CH. HD, *d* eſt CH = HD.
d 9.5. *e* ergo C H datur. *f* ergo punctum C datur.
e 1. def. d. Q.E.D. *g* ergo KC x CE, hoc eſt *d* AD x CE
f 27. dat. datur. Q.E.D.
g 83. dat.

FINIS.