EUCLIDIS ELEMENTORUM

Libri xv. breviter demonstrati,

Operâ

IS. BARROW,

CANTABRIGIENSIS

Coll. TRIN. *Soc.*

HIEROCL.

Καθαρμοὶ ψυχῆς λογικῆς εἰσιν αἱ μαθηματικαὶ ἐπιστῆμαι.

CANTABRIGIÆ:
Ex celeberrimæ Academiæ Typographeo.
Impensis Guilielmi Nealand *Bibliopolæ*.
ANN. DOM. MDCLV.

Nobilissimis & Generosissimis Adolescentibus

D^no EDVARDO CECILIO,

Illustriss. Comitis Sarisburiensis *Filio*;

D^no JOHANNI KNATCHBUL,

Et

D. FRANCIS. WILLOUGHBY,

ARMIGERIS.

UNicuique vestrûm (Optimi Adolescentes) tantum me debere reputo, quantum homo homini debere potest. Meâ enim sententiâ, ultra sincerum amorem non est quòd quispiam de alio bene mereri possit. Hunc autem jamdiu est quò ex singulari vestrâ bonitate mihi indultum experior, ejúsque sensus intimis animi medullis inhærens, ipsi ardens studium impressit, quovis honesto modo reciprocos affectus prodendi. Quandoquidem verò ea fortunarum mearum tenuitas, ea vestrarum amplitudo existit, ut nec ego aliâ quâ gratæ alicujus agnitionis significatione uti queam, nec vos aliam admittere velitis, eapropter haud illibenter hanc occasionem atripio, honoris & benevolentiæ, quibus vos

 prosequor;

proſequor, publicum hoc & durabile μνημόσυνον edendi. Etſi cùm oblati anathematis exilitatem, & libellum veſtris nominibus conſecratum, quàm is longè infra veſtrorum meritorum dignitatem ſubſidat, attentiùs conſidero, timor ſubindè aliquis & dubitatio animum inceſſant, nè hoc ſtudium erga vos meum vobis dehoneſtamento ſit potiùs, quàm ornamento; ſcilicet memor cùm ſim, ut malæ cauſæ, ſic & mali libri patrocinium in patroni contumeliam magìs quàm in gloriam cedere. Sed quum veſtrarum virtutum id robur, eam fore ſoliditatem recognoſcerem, quæ veſtrum decus, meo quantumvis labefactato, inconcuſſum ſuſtinere poſſint, idcirco non dubitavi vos in aliquatenus commune mecum periculum inducere Virtutes illas intelligo, quibus nemo unquam in veſtra ætate, aut in veſtro ordine, ſaltem me judice, majores deprehendit, quæ vos inſigniter gratos omnibus & amabiles reddunt, eximiam modeſtiam, ſobrietatem, benignitatem animi, morum comitatem, prudentiam, magnanimitatem, fidem; præclaram inſuper ingenii indolem, quæ vos ad omnem ingenuam ſcientiam non tantùm excellenti captu, ſed & appetitu forti ac ſincero inſtruxit. Quas veſtras præclariſſimas dotes prout nemo eſt fortaſſis, qui me meliùs novit, aut pro conſuetudine, quam jamdudum vobiſcum dulciſſimam coluiſſe ex veſtro favore mihi contigit, penitiùs introſpexerit, ità nemo eſt, qui impenſiùs miratur, & ſuſpicit; aut qui ipſas libentiùs prædicare, ac celebrare

brare vellet, si non cùm eloquii mei vires supergrederentur, tum etiam quæ in singulis vobis elucent, prolixi alicujus commentarii, aut panegyricæ orationis libertatem, potiùs quàm præstitutas hujusmodi salutationibus angustias, exposcerent. Quin potiùs divinam clementiam imploro, ut vos earundem virtutum sancto tramiti insistere, atque hos egregios fructus vernæ vestræ ætatis felicibus incrementis maturescere concedat; vitámque vobis in hoc seculo ingenuam, innocentem, jucundam, & in futuro beatam ac sempiternam transigere largiatur. Minimè autem dubito, nè pro consueto vestro in me candore, hoc ultimum fortassis, quod vobis præstare potero, benevolentiæ erga vos & observantiæ testimonium, alacriter accepturi sitis, quod vobis propensissimo affectu offert

Vestrî in æternum amantissimus,

& observantissimus,

I. B.

Benevolo LECTORI.

SI quid in hac elementorum editione præstitum sit, scire desideras, amice Lector, accipe, pro genio operis, breviter. Ad duos præcipuè fines conatus meos direxi. Primùm ut cum requisita perspicuitate summam demonstrationum brevitatem conjungerem, quò eam libello molem compararem, quæ commodè absque molestia circumferri posset. Id quod assecutus videor, si absentem Typographi cura non frustretur. Concinniùs enim quispiam meliori ingenio, aut majori peritiâ excellens, at nemo forsan breviùs plerásque propositiones demonstraverit, præsertim cùm in numero & ordine propositionum ipse nihil immutârim, nec licentiam mihi assumpserim quamcunque propositionem Euclideam procul ablegandi tanquam minùs necessariam, aut quasdam faciliores in axiomatum censum referendi, quod nonnulli fecerunt; inter quos peritissimus Geometra A. *Tacquetus* C *quem ideò etiam nomino, (quòd quædam ex eo desumpta agnoscere honestum duco) post cujus elegantissimam editionem, ipse nihil atten-*

tare

tare voluissem, si non visum fuisset doctissimo viro non nisi octo Euclidis *libros suâ curâ adornatos publico communicare, reliquis septem, tanquam ad elementa Geometriæ minùs spectantibus, omnino quasi spretis atque posthabitis. Mihi autem jam ab initio alia provincia demandata fuit, non elementa Geometriæ utcunque pro arbitrio conscribendi, verùm* Euclidem *ipsum, eúmque totum, quàm possem brevissimè, demonstrandi. Quod enim quatuor libros spectat, septimum, octavum, nonum, decimum: quamvìs illi ad Geometriæ planæ & solidæ elementa, ut sex præcedentes, & duo subsequentes, non tam prope pertineant, quòd tamen ad res Geometricas admodum utiles sint, tam propter Arithmeticæ & Geometriæ valde propinquam cognationem, quàm ob notitiam commensurabilium & incommensurabilium magnitudinum ad figurarum tam planarum, quàm solidarum apprimè necessariam, nemo est è peritioribus Geometris qui ignorat. Quæ verò in tribus ultimis libris continetur, 5 corporum regularium nobilis contemplatio, illa non nisi injuriâ prætermitti potuit, quando nempe illius gratiâ noster στοιχειωτὴς, Platonicæ familiæ philosophus, hoc elementorum systema universum condidisse perhibetur,*

uti

uti testis est * Proclus, *iis verbis*, "Ὅθεν δὴ καὶ τῆς συμπάσης στοιχειώσεως τέλος προεστήσατο τὴν τῶν καλουμένων πλατωνικῶν σχημάτων σύστασιν". *Præterea facilè in animum induxi ut opinarer, nemini harum scientiarum amanti non futurum esse cordi, penes se habere integrum Euclideum opus, quale passim ab omnibus citatur, & celebratur. Quare nullum librum, nullámque propositionem negligere volui earum, quæ apud* P. Herigonium *habentur, cujus vestigiis pressè insistere necesse habui, quoniam ejusce libri schematismis maximâ ex parte uti statutum erat, quòd prævidcrem mihi ad novas describendas tempus non suppetere, etsi nonnunquam id facere præoptâssem. Eadem de causa nec alias plerásque quàm Euclideas demonstrationes adhibere volui, succinctiori formâ expressas, nisi fortè in 2, & 13, & parcè in 7, 8, 9 libris, ubi ab eo nonnihil deflectere operæ pretium videbatur. Bona igitur spes est saltem in hac parte cùm nostris consiliis, tum studiosorum votis aliquo modo satisfactum iri. Nam quæ adjecta sunt in Scholiis problemata quædam & theoremata, sive ob suum frequentem usum ad naturam elementarem accedentia, sive ad eorum, quæ sequuntur, expeditam demonstrationem conducentia, seu quæ regula-*

* lib. 2.

rum practicæ Geometriæ quarundam præcipuarum rationes innuunt ad suos fontes relatas, per ea, ut spero, libellus ultra destinatam molem magnopere non intumescet.

Alter scopus, ad quem collineatum est, eorum desideriis consuluit, qui demonstrationibus symbolicis potiùs quàm verbalibus delectantur. In quo genere cùm plerique apud nos Gulielmi Oughtredi *symbolis assueti sint, ea plerumque usurpare consultius duximus. Nam qui* Euclidem, *hâc viâ tradere & interpretari aggressus sit, hactenus, quod ego sciam, preter unum* P. Herigonium, *repertus est nemo. Cujus viri longè doctissimi methodus, sanè in multis egregia, ac ejus peculiari proposito admodum accommodata, duplici tamen defectu laborare mihi visa est. Primò, quòd cùm Propositionum ad unius alicujus theorematis aut problematis probationem adductarum, posterior à priori non semper dependeat, quando tamen illæ inter se cohærent, quando non, nec ex ordine singularum, nec ullo alio modo satis promptè innotescere potest; unde ob defectum conjunctionum, & adjectivorum* ergò, rursus, *&c. non rarò difficultas & dubitandi occasio, præsertim minùs exercitatis, inter legendum oboriri solent. Deinde sæpe evenit, ut prædi-*

cta methodus nimìs frequenter supervacaneas repetitiones effugere nequeat, à quibus demonstrationes est quando prolixæ, aliquando & magìs intricatæ evadunt. Quibus vitiis noster modus facilè per verborum signorúmque arbitrariam mixturam medetur. Atque hæc de opellæ hujus intentione & methodo dicta sufficiant. Cæterùm quæ in laudem Matheseos in genere, aut Geometriæ ipsius; & quæ de historia harum scientiarum, ideóque de Euclide *horum elementorum digestore dici possent, & reliqua hujusmodi* ἐξωτερικὰ, *cui hæc placent, apud alios interpretes consulere potest. Neque nos angustias temporis, quod huic operi impendi potuit, nec interpellationes negotiorum, nec adjumentorum ad hæc studia apud nos egestatem, & quædam alia, ut liceret non immeritò, in excusationem obtendemus, metu scilicet inducti, nè hæc nostra omnibus minùs satisfaciant. Verùm quæ ingenui Lectoris usibus elaboravimus, eadem in solidum ipsius censuræ ac judicio submittimus, probanda si utilia sibi compererit, sin omnino secus, rejicienda.*

J. B.

Ad amicissimum Virum *J. B.* de *EUCLIDE* contracto

Ἐυφημισμός.

FActum bene! didicit Laconicè loqui
Senex profundus, & aphorismos induit.
Immensa dudum margo commentarii
Diagramma circuit minutum; útque Insula
Problema breve natabat in vasto mari.
Sed unda jam detumuit; & glossa arctior
Stringit Theoremata: minoris anguli
Lateribus ecce totus Euclides *jacet,*
Inclusus olim velut Homerus in nuce;
Pluteóque sarcina modo qui incubuit, levis
En fit manipulus. Pelle in exigua latet
Ingens Mathésis, matris ut in utero Hercules,
In glande quercus, vel Ithaca Eurus in pila.
Nec mole dum detrescit, usu fit minor,
Quin auctior jam evadit, & cumulatiùs
Contracta prodest erudita pagina.
Sic ubere magìs liquor è presso effluit;
Sic pleniori vasa inundat sanguinis
Torrente cordis Systole; sic fusiùs
Procurrit æquor ex Abylæ angustiis.
Tantilli operis ars tanta referenda unicè est
BAROVIANO nomini, ac solertiæ.
Sublimis euge mentis ingenium potens!
Cui invium nil, arduum esse nil solet.
Sic usque pergas prospero conamine,
Radiúsque multum debeat ac abacus tibi.
Sic crescat indies feracior seges,
Simili colonum germine assiduo beans.
Specimen futuræ messis hic fiet labor,
Magnæ'que famæ illustria hæc prælудia.
Juvenis dedit qui tanta, quid dabit senex?

Car. Robotham, *CANTAB.*
Coll. Trin. Sen. Soc.

In

In novam *Elementorum*
EUCLIDIS
Editionem, à D. *IS. BARROW*,
Collegii SS. TRIN. *Socio*,
viro opt. & eruditissimo
adornatam.

BEnigne Lector! si uspiam auditum est tibi,
Quantus tenella Nix Geometres fiet;
Qua mille radiis, mille ludit angulis,
Totúmque puro ducit Euclidem sinu:
Amabis ultrò candidissimum Virum,
Cui plena nivium est indoles, sed quas tamen
Praeclarus ardor mentis urget Enthea;
Et usque blandis temperat caloribus:
Quo suavius nil vivit, & melius nihil.
Is, dum liquentes pectore excutit nives,
Et indè, & indè spargit, en aliam tibi,
Lector benigne, è nivibus Geometriam!

G. C. *A. M. C. E. S.*

= æqualitatem.
⊏ majoritatem.
⊐ minoritatem.
+ plùs, vel addendum esse.
− minùs, vel subtrahendum esse.
−: differentiam vel excessum; item quantitates omnes, quæ sequuntur, subtrahendas esse, signis non mutatis.
× multiplicationem, vel ductum lateris rectanguli in aliud latus.
Idem denotat conjunctio literarum, ut AB = A×B.
√ Latus, vel radicem quadrati, vel cubi, &c.
Q. & q quadratum. C. & c cubum.
Q. Q. rationem quadrati numeri ad quadratum numerum.

significat.

Reliquas, quæ ubicunque occurrunt, vocabulorum abbreviationes ipse Lector per se facilè intelliget; exceptis iis, quas tanquam minùs generalis usûs, suis locis explicandas relinquimus.

*C*ANDIDE LECTOR, *Quamvis in hac editione accuranda multum operæ insumpsimus, caveri tamen omnino non potuit, nè irreperent σφάλματα· Quorum summæ si subducas ea tum quæ incuriæ & importunitati operarum, tum quæ Autoris manuscripto calamo festinante exarato debentur; reliqua, siqua modò restant, pro nostris libenter agnoscimus. In universum tamen, si omnia nobis imputes, non ità multa sunt, ut illorum nos admodum pudeat, aut ut æquus Lector ea & uni, & homini difficulter ignoscat.*

Paucula hæc, quæ temere aliquoties pagellas sparsim relegenti obiter occurrebant, diligenter adnotes velim, aut si placet, calamo emendes.

Pag. 5. *lin.* 10. *pro* æquilateræ *lege* quadrilat. *p.* 13. *l.* 6, & 7. *pro* ⊏ *pone* ⊐ *in aliquibus exempl. p.* 21. *desunt figuræ pro* 2 & 3 *Cas.* Prop. 24. *p.* 168. *l. penult. pro* Aq. *lege* AB. *p.* 314. *l. ult. pro* AC. *l.* AD. *p.* 144. & 145 *pro* Lib. VI. *l.* Lib. VII. *Est & in octavo libro* ∷ *pro* =, *sed locum non memini.*

LIB. I.

Definitiones.

I. PUnctum est cujus pars nulla est.

II. Linea verò longitudo latitudinis expers.

III. Lineæ autem termini sunt puncta.

IV. Recta linea est, quæ ex æquo sua interjacet puncta.

V. Superficies est, quæ longitudinem, latitudinémque tantùm habet.

VI. Superficiei autem extrema sunt lineæ.

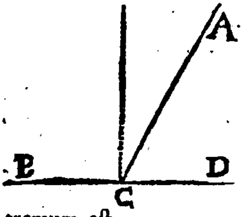
VII. Plana superficies est, quæ ex æquo suas interjacet lineas.

VIII. Planus verò angulus est, duarum linearum in plano se mutuò tangentium, & non in directum jacentiũ alterius ad alteram inclinatio.

IX. Cùm autem quæ angulum continent lineæ, rectæ fuerint, rectilineus ille angulus appellatur.

X. Cùm verò recta linea C G super rectam lineam A B consistens, eos qui sunt deinceps angulos C G A, C G B æquales inter se fecerit, rectus est uterque æqualium angulorum, & quæ insistit recta linea C G, perpendicularis vocatur ejus (A B) cui insistit.

Not. *Cùm plures anguli ad unum punctum: (ut ad* G*) existunt, designatur quilibet angulus tribus literis, quarum media ad verticem est illius de quo agitur: ut angulus quem rectæ* CG, AG *efficiunt ad partes* A *vocatur* CGA, *vel* AGC.



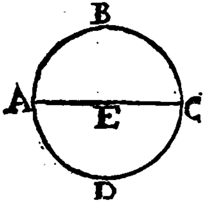
XI. Obtusus angulus est, qui recto major est, ut ACB.

XII. Acutus verò, qui minor est recto, ut ACD.

XIII. Terminus est; quod alicujus extremum est.

XIV. Figura est, quæ sub aliquo, vel aliquibus terminis comprehenditur.

XV. Circulus est figura plana, sub una linea comprehensa, quæ peripheria appellatur, ad quam ab uno puncto eorum, quæ intra figuram sunt posita, cadentes omnes rectæ lineæ inter se sunt æquales.



XVI. Hoc verò punctum centrum circuli appellatur.

XVII. Diameter autem circuli est recta quædam linea per centrum ducta, & ex utraque parte in circuli peripheriam terminata, quæ circulum bifariam secat.

XVIII. Semicirculus verò est figura, quæ continetur sub diametro, & sub ea linea, quæ de circuli peripheria aufertur.

In circulo EABCD. E *est centrum*, AC *diameter*, ABC *semicirculus*.

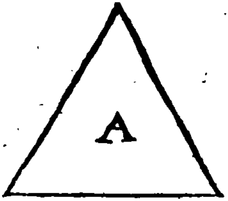
XIX. Rectilineæ figuræ sunt, quæ sub rectis lineis continentur.

XX. Trilateræ quidem, quæ sub tribus.

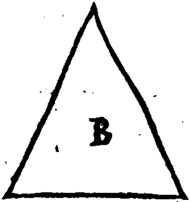
XXI. Quadrilateræ verò, quæ sub quatuor.

XXII. Multilateræ autem, quæ sub pluribus, quàm quatuor rectis lineis comprehenduntur.

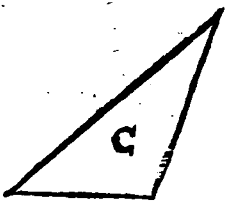
XXIII. Tri-



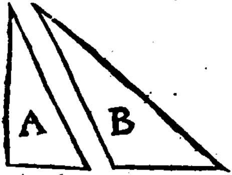
XXIII. Trilaterarum autem figurarum, æquilaterum eſt triangulum, quod tria latera habet æqualia, ut triangulum A.



XXIV. Iſoſceles autem, quod duo tantùm æqualia habet latera, ut triangulum B.

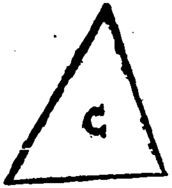


XXV. Scalenum verò, quod tria inæqualia habet latera, ut C.



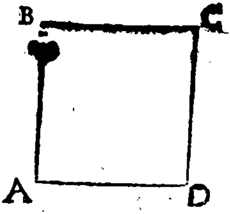
XXVI. Adhæc etiam trilaterarum figurarum, rectangulum quidẽ triangulum eſt, quod rectum angulum habet, ut triangulum A.

XXVII. Amblygonium autem, quod obtuſum angulum habet, ut B.

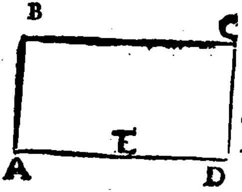


XXVIII. Oxygonium verò, quod tres habet acutos angulos, ut C.

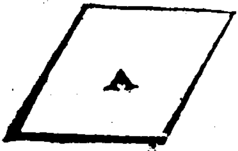
Figura æquiangula est, cujus omnes anguli inter se æquales sunt. Duæ verò figuræ æquiangulæ sunt; si singuli anguli unius singulis angulis alterius sint æquales. Similiter de figuris æquilateris concipe.



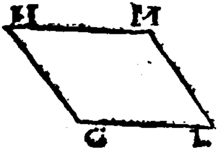
XXIX. Quadrilaterarum autem figurarum, quadratum quidem est, quod & æquilaterum, & rectangulum est, ut ABCD.



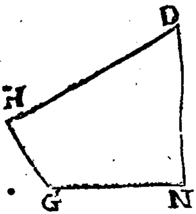
XXX. Alterâ verò parte longior figura est, quæ rectangula quidem, at æquilatera non est, ut ABCD.



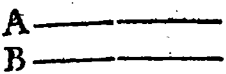
XXXI. Rhombus autem, quæ æquilatera, sed rectangula non est, ut A.



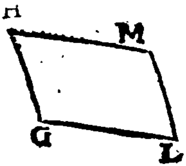
XXXII. Rhomboïdes verò, quæ adversa & latera, & angulos habens inter se æquales, neque æquilatera est, neque rectangula, ut GLMH.



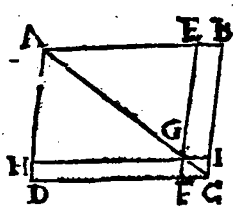
XXXIII. Præter has autem reliquæ æquilateræ figuræ trapezia appellentur; ut GNDH.



XXXIIII. Parallelæ rectæ lineæ sunt, quæ cùm in eodem sint plano, & ex utraque parte in infinitum producantur, in neutram sibi mutuò incidunt, ut A, & B.



XXXV. Parallelogrammum est figura quadrilatera, cujus bina opposita latera sunt parallela, seu æquidistantia, ut GLHM.



XXXVI. Cùm verò in parallelogrammo ABCD diameter AC ducta fuerit, duæq; lineæ EF, HI, lateribus parallelæ secantes diametrum in uno eodémq; puncto G, ita ut parallelogrammum ab hisce

 paralle-

parallelis in quatuor distribuatur parallelogramma; appellantur duo illa D G, G B, per quæ diameter non transit, Complementa; duo verò reliqua H E, F I, per quæ diameter incedit, circa diametrum consistere dicuntur.

Problema est, *cùm proponitur aliquid efficiendum.*

Theorema est, *cùm proponitur aliquid demonstrandum.*

Corollarium est *consectarium, quod è facta demonstratione tanquam lucrum aliquod colligitur.*

Lemma est *demonstratio præmissæ alicujus, ut demonstratio quæsiti evadat brevior.*

Postulata.

1. POstuletur, ut à quovis puncto ad quodvis punctum rectam lineam ducere concedatur.

2. Et rectam lineam terminatam in continuum rectâ producere.

3. Item, quovis centro, & intervallo circulum describere.

Axiomata.

1. QUæ eidem æqualia, & inter se sunt æqualia.

ut A = B = C. ergò A = C, vel ergò omnes A, B, C æquantur inter se.

Nota, *Cùm plures quantitates hoc modo conjunctas invenias, vi hujus axiomatis primam ultimæ & quamlibet earum cuilibet æquari. Quo in casu sæpe, brevitatis causâ, ab hoc axiomate citando abstinemus; etsi vis consecutionis ab eo pendeat.*

2. Et si æqualibus æqualia adjecta sunt, tota sunt æqualia.

3. Et

3. Et si ab æqualibus æqualia ablata sunt, quæ relinquuntur sunt æqualia.

4. Et si inæqualibus æqualia adjecta sint, tota sunt inæqualia.

5. Et si ab inæqualibus æqualia ablata sint, reliqua sunt inæqualia.

6. Et quæ ejusdem vel æqualium sunt duplicia, inter se sunt æqualia. Idem puta de triplicibus, quadruplicibus, &c.

7. Et quæ ejusdem, vel æqualium sunt dimidia, inter se sunt æqualia. Idem concipe de subtriplis, subquadruplis, &c.

8. Et quæ sibi mutuò congruunt, ea inter se sunt æqualia.

Hoc axioma in rectis lineis, & angulis valet conversum, sed non in figuris, nisi illæ similes fuerint.

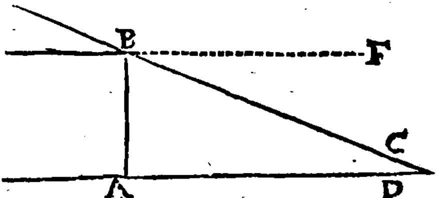
Cæterùm, magnitudines congruere dicuntur, quarum partes applicatæ partibus, æqualem vel eundem locum occupant.

9. Et totum suâ parte majus est.

10. Duæ rectæ lineæ non habent unum & idem segmentum commune.

11. Duæ rectæ in uno puncto concurrentes, si producantur ambæ, necessariò se mutuò in eo puncto intersecabunt.

12. Item omnes anguli recti sunt inter se æquales.



13. Et si in duas rectas lineas AD, CB, altera recta BA incidẽs, internos ad easdémq; partes angu-

 los

los BAD, ABC duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ sibi mutuò incident ad eas partes; ubi sunt anguli duobus rectis minores.

14. Duæ rectæ lineæ spatium non comprehendunt.

15. Si æqualibus inæqualia adjiciantur, erit totorum excessus adjunctorum excessui æqualis.

16. Si inæqualibus æqualia adjungantur, erit totorum excessus excessui eorum, quæ à principio, æqualis.

17. Si ab æqualibus inæqualia demantur, erit residuorum excessus, excessui ablatorum æqualis.

18. Si ab inæqualibus æqualia demantur, erit residuorum excessus excessui totorum æqualis.

19. Omne totum æquale est omnibus suis partibus simul sumptis.

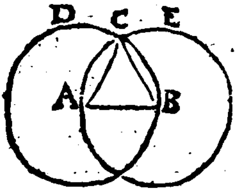
20. Si totum totius est duplum, & ablatum ablati, erit & reliquum reliqui duplum. Idem de reliquis multiplicibus intellige.

Citationes intellige sic. Cùm duo numeri occurrunt, prior designat propositionem, posterior librum. Ut per 4. 1. intelligitur quarta propositio primi libri, atque ità de reliquis. *Cæterùm, ax. axioma, post. postulatum, def. definitionem, sch. scholium, cor. corollarium denotant, &c.*

LIB.

LIB. I.

PROP. I.



Super datâ rectâ lineâ terminatâ A B, *triangulum æquilaterum* A C B *constituere.*

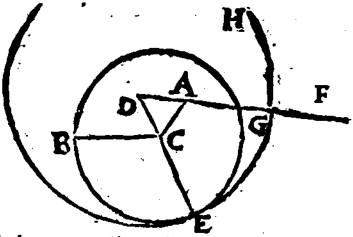
Centris A & B, eodem intervallo AB, vel BA [a] describe duos circulos se intersecantes in puncto C, ex quo [b] duc rectas CA, CB. Erit AC [c] = AB [c] = BC [d] = AC. [e] Quare triangulum ACB est æquilaterum. Quod Erat Faciendum.

a 3 post.
b 1 post.
c 15. def.
d 1. ax.
e 23. def.

Scholium.

Eodem modo super AB describetur triangulum Isosceles, si intervalla æqualium circulorum majora sumantur, vel minora, quàm AB.

PROP. II.



Ad datum punctum A *datæ rectæ lineæ* BC *æqualem rectam lineam* AG *ponere.*

Centro C, intervallo CB [a] describe circulum CBE. [b] Junge AC, super qua [c] fac triangulum æquilaterum ADC. [d] produc DC ad E.

a 3. post.
b 1. post.
c 1. 1.
d 2. post.

B 5 centro

centro D, ſpatio DE, deſcribe circulum DEH: cujus circumferentiæ occurrat DA [e] protracta ad G. Erit AG = CB.

e 2 poſt.
f 15. def.
g conſtr.
h 3. ax.
k 15. def.
l 1. ax.

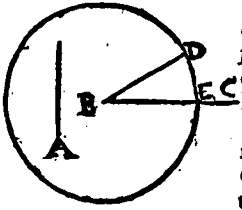
Nam DG [f] = DE, & DA [g] = DC, quare AG [h] = CE [k] = BC [l] = AG. Q. E. F.

Poſitio puncti A, intrà vel extrà datam BC, caſus variat, ſed ubique ſimilis eſt constructio, & demonſtratio.

Scholium.

Poterat AG circino ſumi, ſed hoc facere nulli poſtulato reſpondet, ut bene innuit Proclus.

Prop. III.



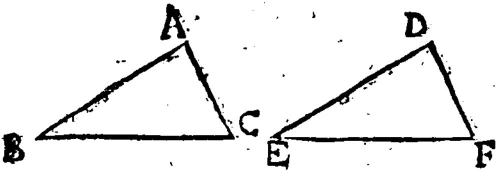
Duabus datis rectis lineis A, & BC, de majore BC minori A æqualem rectam lineam BE detrahere.

a 2. 1.

Ad punctum B [a] pone rectam BD = A. Circulus centro B, ſpatio BD deſcriptus auferet BE [b] = BD [c] = A [d] = BE. Q. E. F.

b 15. def.
c conſtr.
d 1. ax.

Prop. IV.



Si duo triangula BAC, EDF *duo latera* BA, AC *duobus lateribus* ED, DF *æqualia habeant, utrumque utrique* (hoc eſt BA = ED, & AC = DF) *habeant vero angulum* A, *angulo* D *æqualem,*

lem, sub æqualibus rectis lineis contentum, & basim BC *basi* EF *æqualem habebunt; eritque triangulum* BAC *triangulo* EDF *æquale, ac reliqui anguli* B, C *reliquis angulis* E, F *æquales erunt, uterque utrique, sub quibus æqualia latera subtenduntur.*

Si punctum D puncto A applicetur, & recta DE rectæ AB superponatur, cadet punctum E in B, quia DE [a] = AB. Item recta DF cadet in AC, quia ang. A [a] = D. Quinetiam punctum F puncto C coincidet, quia AC [a] = DF. Ergò rectæ EF, BC, cùm eosdem habeant terminos, [b] congruent, & proinde æquales sunt. Quare triangula BAC, EDF; & anguli B, E; itémq; anguli C, F etiam congruunt, & æquantur. Quod erat Demonstrandum.

a hyp.
b 14. ax.

Prop. V.

Isoscelium triangulorum ABC *qui ad basim sunt anguli* ABC, ACB *inter se sunt æquales. Et productis æqualibus rectis lineis* AB, AC *qui sub base sunt anguli* CBD, BCE *inter se æquales erunt.*

[b] [a] Accipe AF = AD, & junge CD, ac BF.

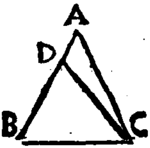
Quoniam in triangulis ACD, ABF, sunt AB [c] = AC, & AF [d] = AD, angulúsq; A communis, [e] erit ang. ABF = ACD; & ang. AFB [e] = ADC, & bas. BF [e] = DC; item FC [f] = DB. ergò in triangulis BFC, BDC [g] erit ang. FCB = DBC. Q. E. D. Item ideo ang. FBC = DCB. atqui ang. ABF [h] = ACD. ergò ang. ABC [k] = ACB. Q. E. D.

a 3. 1.
b 1. post.
c hyp.
d constr.
e 4. 1.
f 3. ax.
g 4. 1.
h pr.
k 3. ax.

Corollarium.

Hinc, Omne triangulum æquilaterum est quoq; æquiangulum.

Prop. VI.



Si trianguli A B C *duo anguli* ABC, ACB *æquales inter se fuerint, & sub æqualibus angulis subtensa latera* AB, AC *æqualia inter se erunt.*

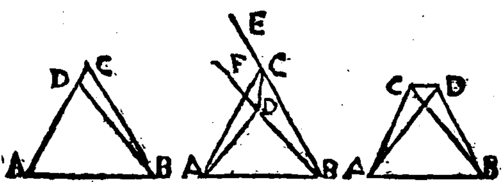
Si fieri potest, sit utravis
a 3 1. b 1. *post.* BA $\sqsubset$ CA. [a] Fac igitur BD = CA, & [b] duc CD.

c *suppos.* d *hyp.* e 4. 1. f 9. *ax.* In triangulis DBC, ACB, quia BD [c] = CA, & latus BC commune est, atq; ang. DBC [d] = ACB, [e] erunt triangula DBC, ACB æqualia inter se, pars & totum, [f] Quod Fieri Nequit.

Coroll.

Hinc, Omne triangulum æquiangulum est quoq; æquilaterum.

Prop. VII.



Super eadem recta linea AB *duabus eisdem rectis lineis* AC, BC, *aliæ duæ rectæ lineæ æquales* AD, BD, *utraque utrique (hoc est,* AD = AC, & BD = BC) *non constituentur ad aliud punctum* C, *atque aliud* D, *ad easdem partes* C, *eosdémque terminos* A, B *cum duabus initio ductis rectis lineis habentes.*

a 9. *ax.* 1. *Cas.* Si punctum D statuatur in AC, [a] liquet non esse AD = AC.

2. *Cas.* Si punctum D dicatur intra triangulum ACB, duc CD, & produc BDF, ac BCE.
b 5 1. Jam vis AD = AC. ergò ang. ADC [b] = ACD;
c *suppos.* item quia BD [c] = BC, erit ang. FDC [b] = BCD.

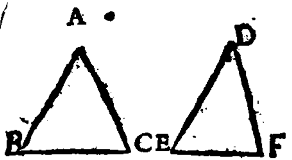
ergò

ergò ang. FDC [d] $\sqsubset$ ACD, id est ang. FDC $\sqsubset$ ADC [d] Q.E.N. d 9. ax.

3. *Cas.* Sin D cadat extra triangulum ACB, jungatur CD.

Rursus, ang. BCD [e] $=$ BDC, & BCD [e] $=$ BDC. [f] ergò ang. ACD $\sqsupset$ BDC. & proinde multò magis ang. BCD $\sqsupset$ BDC. Sed erat ang. BCD $=$ BDC. Quæ repugnant. Ergò, &c. e 5. 1. f 9. ax.

Prop. VIII.



Si duo triangula ABC, DEF *habuerint duo latera* AB, AC *duobus lateribus* DE, DF, *utrumque utriq; æqualia; habuerint verò & basim* BC, *basi* EF, *æqualem: angulum* A *sub æqualibus rectis lineis contentum angulo* D *æqualem habebunt.*

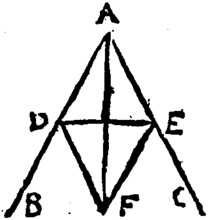
Quia BC [a] $=$ EF, si basis BC superponatur basi EF, illæ [b] congruent. ergò, cùm AB [c] $=$ DE, & AC [c] $=$ DF, cadet punctum A in D. (nam in aliud punctum cadere nequit, per præcedentem) ergò angulorum A, & D latera coincidunt. [d] quare anguli illi pares sunt. Q.E.D. a hyp. b 8. ax. c hyp. d 8. ax.

Coroll.

1. Hinc triangula sibi mutuò æquilatera, etiam mutuò [x] æquiangula sunt. x 4. 1.

2. Triangula sibi mutuò æquilatera [y] æquantur inter se. y 4. 1.

Prop. IX.



Datum angulum rectilineum BAC *bifariàm secare.*

a 3. 1. [a] Sume AD = AE;
b 1. 1. duc DE, super quâ [b] fac triang. æquilat. DFE.

Ducta AF angulum BAC bisecabit.

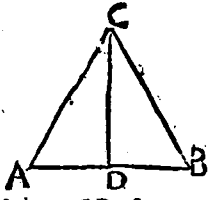
c constr. Nam AD [c] = AE, & latus AF commune est, & baf. DF [c] = FE.
d 8. 1. [d] ergò ang. DAF = EAF. Q. E F.

Coroll.

Hinc patet quomodo angulus secari possit in æquales partes 4, 8, 16, &c. Singulos nimirum partes iterum bisecando.

Methodus verò regulâ & circino angulos secandi in æquales quotcunq; hactenus Geometras latuit.

Prop. X.

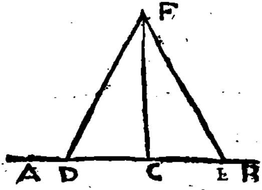


Datam rectam lineam AB *bifariàm secare.*

a 1. 1. Super data AB [a] fac triang. æquilat. ABC.
b 9. 1. ejus angulum C [b] biseca rectâ CD. Eadem datam AB bisecabit.

c constr. Nam AC [c] = BC, & latus CD est commune; & ang. ACD [c] =
d 4. 1. BCD, [d] ergò AD = BD. Q. E. F. Praxin hujus & præcedentis, constructio primæ hujus libri satis indicat.

PROP. XI.



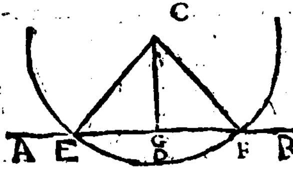
Datâ rectâ lineâ AB, & puncto in ea dato C, rectam lineam CF ad angulos rectos excitare.

[a] Accipe hinc indè CD = CE. Super DE [b] fac triang. æquilat. DFE. Ducta FC perpendicularis est. a 3. 1. b 1. 1.

Nam triangula DFC, EFC sibi mutuò [c] æquilatera sunt. [d] ergò ang. DCF = ECF. [e] ergò FC perpendicularis est. Q. E. F. c constr. d 8. 1. e 10. def.

Praxis tam hujus, quàm sequentis expeditur facillimè ope normæ.

PROP. XII.

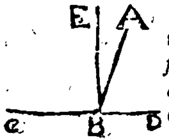


Super datam rectam lineam infinitam AB, à dato puncto C quod in ea non est, perpendicularem rectam CG deducere.

Centro C [a] describe circulum, qui secet datam AB in punctis E & F [b] biseca EF in G. ducta CG perpendicularis est. a 3. post. b 10. 1.

Ducantur enim CE, CF. Triangula EGC, FGC, sibi mutuò [c] æquilatera sunt. [d] ergò anguli EGC, FGC, æquales, & [e] proinde recti sunt. Q. E. F. c constr. d 8. 1. e 10. def.

PROP. XIII.



Cùm recta linea AB, super rectam lineam CD consistens, facit angulos ABC, ABD; aut duos rectos, aut duobus rectis æquales efficiet.

a 10. def. Si anguli ABC, ABD pares sint, [a] liquet illos rectos esse; sin inaequales sint, ex B [b] excitetur perpendicularis BE. Quoniam ang. ABC [c] = Rect. + ABE; & ang. ABD [d] = Rect. — ABE; erit ABC + ABD [e] = 2 Rect. + ABE — ABE = 2 Rect. Q. E. D.

b 11. 1.
c 19. ax.
d 3. ax.
e 2. ax.

Coroll.

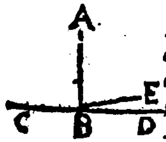
1. Hinc, si unus ang. ABD rectus sit, alter ABC etiam rectus erit; si hic acutus, ille obtusus erit, & contra.

2. Si plures rectae quàm una ad idem punctum eidem rectae insistant, anguli fient duobus rectis aequales.

3. Duae rectae invicem secantes efficiunt angulos quatuor rectis aequales.

4. Omnes anguli circa unum punctum constituti conficiunt quatuor rectos. patet ex Coroll. 2.

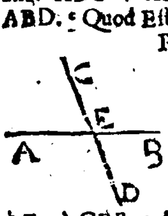
PROP. XIV.



Si ad aliquam rectam lineam AB, atque ad ejus punctum B duae rectae lineae CB, BD non ad easdem partes ductae, eos qui sunt deinceps angulos ABC, ABD duobus rectis aequales fecerint, in directum erunt inter se ipsae rectae lineae CB, BD.

a 13. 1. Si negas, faciant CB, BE unam rectam. ergò
b hyp. ang. ABC + ABE [a] = 2 Rect. [b] = ABC +
c 9. ax. ABD. [c] Quod est absurdum.

PROP. XV.

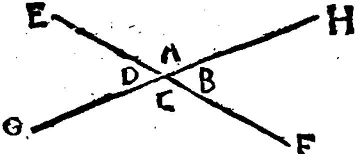


Si duae rectae lineae AB, CD se mutuò secuerint, angulos ad verticem CEB, AED aequales inter se efficient.

Nam ang. AEC + CEB
a 13. 1. [a] = 2 Rect. [b] = AEC + AED.
b 3. ax. [b] Ergò CEB = AED. Q. E. F.

Schol.

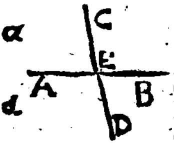
Schol.



Si ad aliquam rectam lineam GH, atque ad ejus punctum, A duæ rectæ lineæ EA, AF non ad easdem partes sumptæ, angulos ad verticem D, & B æquales fecerint, ipsæ rectæ lineæ EA, AF in directum sibi invicem erunt.

Nam 2 Rect. =[a] D + A [a]= B + A. [b] ergò EA, AF sunt in directum sibi invicem. Q. E. D. a 13. 1. b 14. 1.

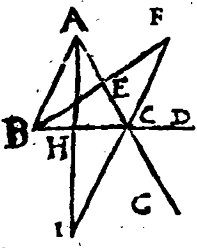
Schol. 2.



Si quatuor rectæ lineæ EA, EB, EC, ED ab uno puncto E exeuntes, angulos oppositos ad verticem æquales inter se fecerint, erunt quælibet duæ lineæ AE, EB, & CE, ED in directum positæ.

Nam quia ang AEC + AED + CEB + DEB [a] = 4 Rect. erit AEC + AED [b] = CEB + DEB = 2 Rect. [c] ergò CED, & AEB sunt rectæ lineæ. Q. E D. a 4 *Cor.* 13. 1. b 15. 1. & 2. *ax.* c 14. 1.

PROP. XVI.

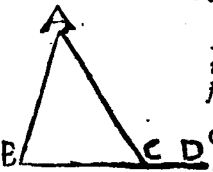


Cujuscunque Trianguli ABC *uno latere* BC *producto, externus angulus* ACD *utrolibet interno & opposito* CAB, CBA, *major est.*

Latera AC, BC [a] bisecent rectæ AH, BE, è quibus productis [b] cape EF = BE, [b] & HI = AH, Conjugantúrq; FC, I. a 10. 1. & 1. *post.* b 3. 1.

c *constr.* Quoniam CE [c] $=$ E A, & EF [c] $=$ EB, &
d 15. 1. ang. FEC [d] $=$ B E A; [e] erit ang. ECF $=$ EAB.
e 4. 1. Simili argumento ang. ICH ([f] FCD) $=$ ABH.
f 15. 1. ergò totus ACD [g] major est utrovis CAB, &
g 9. *ax.* ABC. Q. E. D.

PROP. XVII.

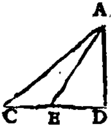


Cujuscunque trianguli ABC duo anguli duobus rectis sunt minores, omnifariàm sumpti.

Producatur latus BC. Quoniam ang. ACD $+$
a 13. 1. ACB [a] $=$ 2 Rect. & ang.
b 16. 1. ACD [b] $\sqsubset$ A, [c] erit A $+$ ACB $\sqsupset$ 2 Rect. Eo-
c 4 *ax.* dem modo erit ang. B $+$ ACB $\sqsupset$ 2 Rect. Denique producto latere AB, erit similiter ang. A $+$ B $\sqsupset$ 2 Rect. Quæ E. D.

Coroll.

1. Hinc, in omni triangulo, cujus unus angulus fuerit rectus, vel obtusus, reliqui acuti sunt.

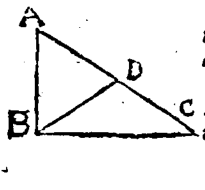


2. Si linea recta AE cùm alia recta CD angulos inæquales faciat, unum AED acutum, & alterum AEC obtusum, linea perpendicularis AD ex quovis puncto A ad aliam illam CD demissa, cadet ad partes anguli acuti AED.

Nam si AC ad partes anguli obtusi ducta, dicatur perpendicularis; in triangulo AFC erit ang.
* 17. 1. AEC $+$ ACB $\sqsubset$ 2 Rect. [x] Q. F. N.

3. Omnes anguli trianguli æquilateri, & duo anguli trianguli Isoscelis, supra basim, acuti sunt.

PROP. XVIII.

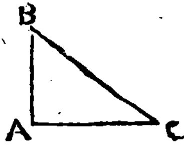


Omnis trianguli ABC majus latus AC majorem angulum ABC subtendit.

a 3. 1. Ex AC [a] aufer AD $=$
b 5. 1. AB, & junge DB. [b] ergò ang. ADB $=$ ABD. Sed ADB

[c] ADB $\sqsubset$ C. ergò ABD $\sqsubset$ C. [d] ergò totus ang. ABC $\sqsubset$ C. Eodem modo erit ABC $\sqsubset$ A. Q. E. D. c 16. 1. d 9. ax.

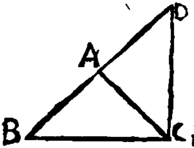
PROP. XIX.



Omnis trianguli ABC *major angulus* A *majori lateri* BC *subtenditur.*

Nam si dicatur AB = BC, [a] erit ang. A = C. contra Hypoth. & si AB $\sqsubset$ BC, [b] erit ang. C $\sqsubset$ A, contra hyp. quare potiùs BC $\sqsubset$ AB. & eodem modo BC $\sqsubset$ AC. Q. E. D. a 5. 1. b 18. 1.

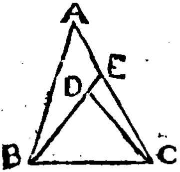
PROP. XX.



Omnis trianguli ABC *duo latera* BA, AC *reliquo* BC *sunt majora quomodocunque sumpta.*

Ex BA producta [a] cape AD = AC, & duc DC. [b] ergò ang. D = ACD. [c] ergò totus BCD $\sqsubset$ D [d] ergò BD ([e] BA + AC) $\sqsubset$ BC. Q. E. D. a 3. 1. b 5. 1. c 9. ax. d 19. 1. e constr. & 2. ax.

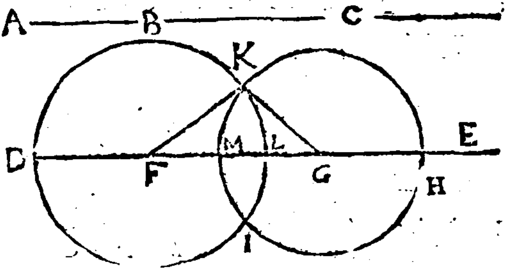
PROP. XXI.



Si super trianguli ABC *uno latere* BC, *ab extremitatibus duæ rectæ lineæ* BD, CD, *interiùs constitutæ fuerint, hæ constitutæ reliquis trianguli duobus lateribus* BA, CA *minores quidem erunt, majorem verò angulum* BDC *continebunt.*

Producatur BD in E. éstq; CE + ED [a] $\sqsubset$ CD adde commune BD, [b] erit BE + EC $\sqsubset$ BD + DC. Rursus BA + AE [a] $\sqsubset$ BE; [b] ergò BA + AC $\sqsubset$ BE + EC. quare BA + AC $\sqsubset$ BD + DC. Q. E. D. 2. Ang. BDC [c] $\sqsubset$ DEC [c] $\sqsubset$ A. ergò ang. BDC $\sqsubset$ A. Q. E. D. a 20. 1. b 4. ax. c 16. 1.

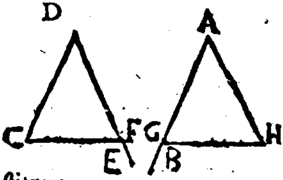
PROP. XXII.



Ex tribus rectis lineis FK, FG, GK, *quæ sint tribus datis rectis lineis* A, B, C *æquales, triangulum* FKG *constituere. Oportet autem duas reliquâ esse majores omnifariàm sumptas; quoniam uniuscujusque trianguli duo latera omnifariàm sumpta reliquo sunt majora.*

a 3. 1. Ex infinita DE [a] sume DF, FG, GH datis
b 3. post. A, B, C ordine æquales. Tum si [b] centris F, & G, intervallis FD, & GH ducantur circuli se intersecantes in K; junctis rectis KF, KG con-
c 15. def. stituetur triangulum FKG, [c] cujus latera FK,
d 1. ax. FG, GK tribus DF, FG, GH, [d] id est tribus datis A, B, C æquantur. Q. E. F.

PROP. XXIII.



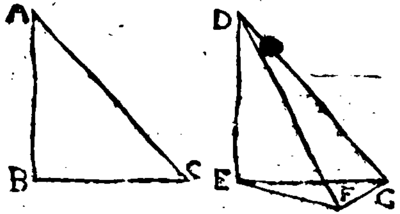
Ad datam rectam lineam AB, *datúmque in ea punctum* A, *dato angulo rectilineo* D *æqualē angulum rectilineum* A *constituere.*

a 1. post. [a] Duc rectam CF secantem dati anguli latera
b 3. 1. utcunque. [b] Fac AG = CD. Super AG [c] con-
c 22. 1. stitue triangulum alteri CDF æquilaterum, ità

ut,

ut AH = DF, & GH = CF; & habebis ang. d 8. 1.
A[d] = D. Q. E. F.

Prop. XXIV.

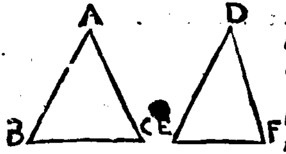


Si duo triangula ABC, DEF *duo latera* AB, AC *duobus lateribus* DE, DF *æqualia habuerint, utrumque utrique; angulum verò* A *angulo* EDF *majorem sub æqualibus rectis lineis contentum, & basim* BC, *basi* EF, *majorem habebunt.*

[a] Fiat ang. EDG = A, & DG [b] = DF [c] = a 23. 1.
AC; connectantúrque EG, FG. b 3. 1.
c *hyp.*
1. *Cas.* Si EG cadit supra EF. Quia AB d *hyp.*
[d] = DE, & AC = [e] DG, & ang. A [e] = EDG, e *constr.*
[f] erit BC = EG. Quia verò DF [c] = DG, f 4. 1.
[g] erit ang. DFG = DGF. [h] ergò ang. DFG $\sqsubset$ g 5. 1.
EGF; [h] & proinde ang. EFG $\sqsubset$ EGF. [k] quare h 9. *ax.* k 19. 1.
EG (BC) $\sqsubset$ EF. Q. E. D.
2. *Cas.* Si basis EF basi EG coincidat, [l] li- l 9. *ax.*
quet EG (BC) $\sqsubset$ EF.
3. Sin EG Cadat infra EF. Quoniam m 21. 1.
DG + GE [m] $\sqsubset$ DF + FE, si hinc indè auferantur DG, DF, æquales, manet EG (BC) n 5. *ax.*
[n] $\sqsubset$ EF. Q. E. D.

Prop.

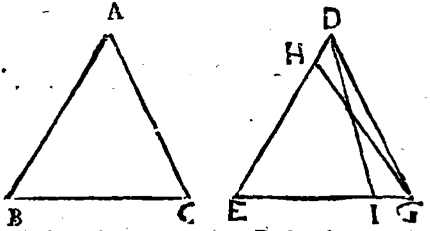
Prop. XXV.



Si duo triangula ABC, DEF *duo latera* AB, AC *duobus lateribus* DE, DF *æqualia habuerint, utrumq; utrique, basim verò* BC *basi* EF *majorem; & angulum* A *sub æqualibus rectis lineis contentum angulo* D *majorem habebunt.*

a 4. 1. Nam si dicatur ang. A $=$ D. [a] erit basis BC $=$ EF, contra Hyp. Sin dicatur ang. A $\sqsupset$ D,
b 24. 1. [b] erit BC $\sqsupset$ EF, etiam contra Hyp. ergò BC $\sqsubset$ EF. Q. E. D.

Prop. XXVI.



Si duo triangula BAC EDG, *duos angulos* B, C, *duobus angulis* E, DGE, *æquales habuerint, utrumque utrique, unúmque latus uni lateri æquale, sive quod æqualibus adjacet angulis, seu quod uni æqualium angulorum subtenditur: reliqua latera reliquis lateribus æqualia, utrumque utrique, & reliquum angulum reliquo angulo æqualem habebunt.*

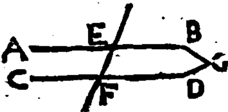
1. *Hyp.* Sit BC $=$ EG. Dico BA $=$ ED, & AC $=$ DG, & ang. A $=$ EDG. Nam si dicatur
a 3. 1. ED $\sqsubset$ BA, [a] fiat EH $=$ BA, ducatúrq; G H.

Quoniam

Quoniam AB [b] = HE, & BC [c] = EG, & ang. B [c] = E, erit ang. EGH [d] = C [e] = DGE. [f] Q. E. A. ergò AB = ED. Eodem modo AC = DG. [d] quare etiam ang. A = EDG.
b suppos. c hyp. d 4. 1. e hyp. f 9. ax.

2. *Hyp.* Sit AB = ED. Dico BC = EG; & AC = DG & ang. A = EDG. Nam si dicatur EG ⊏ BC, fiat BI = BC, & connectatur DI. Quia AB [g] = ED, & BC [h] = EI; & ang. B [g] = E, erit ang. EID [k] = C [m] = EGD. [n] Q. E. A. ergò BC = EG. ergò ut priùs, AC = DG, & ang. A = EDG. Q. E. D.
g hyp. h suppos. k 4. 1. m hyp. n 16. 1.

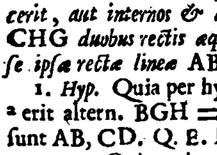
PROP. XXVII.



Si in duas rectas lineas AB, CD *recta incidens linea* EF *alternatim angulos* AEF, DFE, *æquales inter se fecerit, parallelæ erunt inter se illæ rectæ lineæ* AB, CD.

Si AB, CD dicantur non esse parallelæ; conveniant productæ, nempe in G. quo posito angulus externus AEF interno DFB [a] major erit, cui tamen ponitur æqualis. Quæ repugnant.
a 16. 1.

PROP. XXVIII.



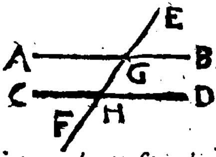
Si in duas rectas lineas AB, CD *recta incidens linea* EF *externum angulum* AGE *interno & opposito, & ad easdem partes* CHG *æqualem fecerit, aut internos & ad easdem partes* AGH, CHG *duobus rectis æquales; parallelæ erunt inter se ipsæ rectæ lineæ* AB, CD.

1. *Hyp.* Quia per hyp. ang. AGE = CHG, [a] erit altern. BGH = CHG. [b] parallelæ igitur sunt AB, CD. Q. E. D.
a 15. 1. b 27. 1.

2. *Hyp.* Quia ex hyp. Ang. AGH + CHG = 2 Rect. [a] = AGH + BGH, [b] erit CHG = BGH. Ergò [c] AB, CD parallelæ sunt. Q. E. D.
a 13. 1. b 3. ax. c 27. 1.

PROP.

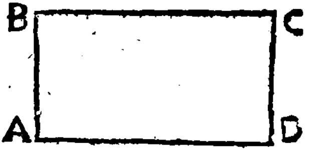
PROP. XXIX.



In parallelas rectas lineas AB, CD, *recta incidens linea* EF, *& alternatim angulos* DHG, AGH *æquales inter se efficit; & externum* BGE *interno, & opposito, & ad easdem partes* DHE *æqualem; & internos & ad easdem partes* AGH, CHG *duobus rectis æquales facit.*

a 13. *ax.* Liquet AGH, + CHG = 2 Rect. [a] aliàs AB, CD non essent parallelæ, contra hyp. Sed
b 13. 1. & ang. DHG + CHG [b] = 2 Rect. ergò DHG
c 13. *ax.* [c] = AGH [d] = BGE. Q. E. D.
d 15. 1.

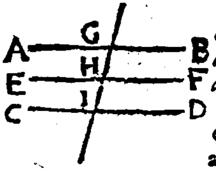
Coroll.



Hinc omne Parallelogrammum AC habens unum angulum rectum A, est rectangulum.

a 29. 1. Nam A + B [a] = 2 Rect. ergò cùm A re-
b 3. *ax.* ctus sit, [b] etiam B rectus erit. Eodem argumento D, & C recti sunt.

PROP. XXX.

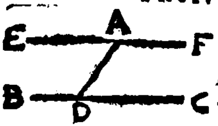


Quæ (AB, CD) *eidem rectæ lineæ* EF *parallelæ, & inter se sunt parallelæ.*

Tres rectas secet utcunq; recta GL Quoniam AB, EF parallelæ sunt,
a 29. 1. [a] erit ang. AGI = EHI, Item propter CD, EF
b 1. *ax.* parallelas, [a] erit ang. EHI = DIG. [b] ergò ang.
c 27. 1. AGI = DIG. [c] quare AB, CD parallelæ sunt. Q. E. D.

PROP.

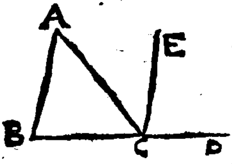
PROP. XXXI.



A dato puncto A *data rectæ lineæ* BC *ducere parallelam rectam lineam* AE.

Ex A ad datam BC duc rectam utcunque AD. ad quam, ejúsq; punctum a 23. 1.
A [a] fac ang. DAE = ADC. [b] erunt AE, BC b 27. 1.
parallelæ. Q. E. F.

PROP. XXXII.



Cujuscunque trianguli ABC *uno latere* BC *producto, externus angulus* ACD *duobus internis, & oppositis,* AB *est æqualis. Et trianguli tres interni anguli,* A, B, ACB *duobus sunt rectis æquales.*

Per C [a] duc CE parall. BA. Ang. A [b] = a 31. 1.
ACE. & ang. B [b] = ECD. ergò A + B [c] = b 29. 1.
ACE + ECD [d] = ACD. Q. E. D. Pono c 2. ax.
ACD + ACB [e] = 2 Rect. [f] ergò A + B + d 19. ax.
ACB = 2 Rect. Q. E. D. e 13. 1.
f 1. ax.

Corollaria.

1. Tres simul anguli cujusvis trianguli æquales sunt tribus simul cujuscunque alterius. Unde

2. Si in uno triangulo duo anguli (aut singuli, aut simul) æquales sint duobus angulis (aut singulis, aut simul) in altero triangulo, etiam reliquus reliquo æqualis est. Item, si duo triangula unum angulum uni æqualem habeant, reliquorum summæ æquantur.

3. In triangulo si unus angulus rectus sit, reliqui unum rectum conficiunt. Item, angulus, qui duobus reliquis æquatur, rectus est.

4. Cùm in Isoscele angulus æquis cruribus contentus rectus est, reliqui ad basim sunt semirecti.

5. Trianguli æquilateri angulus facit duas tertias unius recti, nam $\frac{1}{3}$ 2 Rect. = $\frac{2}{3}$ Rect.

Schol.

Hujus propositionis beneficio, cujuslibet figuræ rectilineæ tam interni quàm externi anguli quot rectos conficiant, innotescet per duo sequentia theoremata.

THEOREMA 1.

Omnes simul anguli cujuscunque figuræ rectilineæ conficiunt bis tot rectos demptis quatuor, quot sunt latera figuræ.

Ex quovis puncto intra figuram ducantur ad omnes figuræ angulos rectæ, quæ figuram resolvent in tot triangula quot habet latera. Quare cùm singula triangula conficiant duos rectos, omnia simul conficient bis tot rectos, quot sunt latera. Sed anguli circa dictum punctum conficiunt quatuor rectos. Ergò, si ab omnium triangulorum angulis demas angulos circa id punctum, anguli reliqui qui componunt angulos figuræ conficient bis tot rectos demptis quatuor, quot sunt latera figuræ. Q.E. D.

Hinc *Coroll.* Omnes ejusdem speciei rectilineæ figuræ æquales habent angulorum summas.

THEOREMA 2.

Omnes simul externi anguli cujuscunque figuræ rectilineæ conficiunt quatuor rectos.

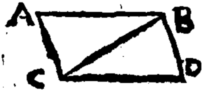
Nam singuli figuræ interni anguli cum singulis externis conficiunt duos rectos. Ergò in-

terni

terni simul omnes, cum omnibus simul externis conficiunt bis tot rectos, quot sunt latera figuræ. Sed (ut modò *ostensum est*,) interni simul omnes etiam cum quatuor rectis efficiunt bis tot rectos, quot sunt latera figuræ. Ergò externi anguli quatuor rectis æquantur. Q. E. D.

Coroll. Omnes cujuscunque speciei rectilineæ figuræ æquales habent externorum angulorum summas.

Prop. XXXIII.

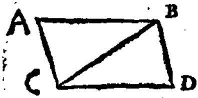


Rectæ lineæ AC, BD, *quæ æquales & parallelas lineas* AB, CD, *ad partes easdem conjungunt, & ipsæ æquales ac parallelæ sunt.*

Connectatur CB. Quoniam ob AB, CD parallelas. ang. ABC [a] = BCD, & per hyp. AB = CD, & latus CB commune est, [b] erit AC = BD, [b] & ang. ACB = DBC. [c] ergò AC, BD etiam parallelæ sunt. Q. E. D.

a 29. 1.
b 4. 1.
c 27. 1.

Prop. XXXIV.



Parallelogrammorum spatiorum ABDC *æqualia sunt inter se quæ ex adverso latera* AB, CD; ac AC, BD; *angulique* A, D, & ABD, ACD; *& illa bifariàm secat diameter* CB.

Quoniam AB, CD [a] parallelæ sunt, [b] erit ang. ABC = BCD. Item ob AC, DB [a] parallelas, [b] erit ang. ACB = CBD. [c] ergò toti anguli ACD, ABD æquantur. Similiter ang. A = D. Porrò, cùm communi lateri CB adjaceant anguli ABC, ACB, ipsis BCD, CBD pares [d], erunt AC = BD, [d] & AB = CD. adeóq; etiam triang. ABC = CBD. Quæ E. D.

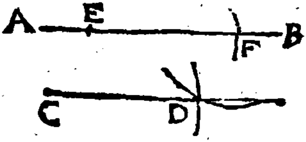
a hyp.
b 29. 1.
c 2. ax.
d 26. 1.

SCHOL.

Omne quadrilaterum ABDC habens latera opposita æqualia, est parallelogrammum.

a 27. 1. Nam per 8. 1. ang. ABC = BCD. [a] ergò AB, CD parallelæ sunt. Eâdem ratione ang. BCA = CBD; [a] quare AC, BD etiam parallelæ sunt. [b] Ergò ABDC est parallelogrammum. Q.E.D.

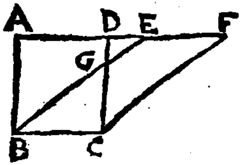
b 35. def. 1.



Hinc expeditiùs per datum punctum C datæ rectæ AB ducetur parallela CD.

Sume in AB quodvis punctum E. centris E, & C ad quodvis intervallum duc æquales circulos EF, CD. centro verò F, spatio EC duc circulum FD, qui priorem CD secet in D. Erit ducta CD parall. AB. Nam ut modò demonstratum est, CEFD est parallelogrammum.

PROP. XXXV.

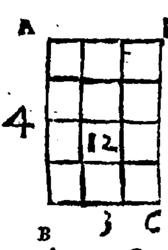


Parallelogramma BCDA, BCFE super eâdem basi BC, & in eisdem parallelis AF, BC constituta, inter se sunt æqualia.

a 34. 1. Nam AD [a] = BC [a] = EF. adde communem DE, [b] erit AE = DF. Sed & AB [a] = DC;
b 2. ax.
c 29. 1. & ang. A [c] = CDF. [d] ergò triang. ABE =
d 4. 1. DCF. aufer commune DGE, [e] erit Trapez.
e 3. ax.
f 2. ax. ABGD = EGCF. adde commune BGC, [f] erit Pgr. ABCD = EBCF. Q.E.D. Reliquorum casuum non dissimilis, sed simplicior & facilior est demonstratio.

Scholium.

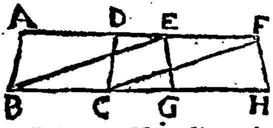
Scholium.



Si latus AB parallelogrammi rectanguli ABCD ferri intelligatur perpendiculariter per totam BC, aut BC per totam AB, producetur eo motu area rectanguli ABCD. Hinc rectangulum fieri dicitur ex ductu seu multiplicatione duorum laterum contiguorum. Sit exempl. gr. BC pedum 3, AB 4. Duc 3 in 4; proveniunt 12 pedes quadrati pro area rectanguli.

Hoc supposito, ex hoc theoremate cujuscunq; parallelogrammi (*EBCF) habetur dimensio. Illius enim area producitur ex altitudine BA ducta in basim BC. Nam area rectanguli AC parallelogrammo EBCF æqualis, fit ex BA in BC. ergò, &c. * *v. fig. prop.* 35.

Prop. XXXVI.

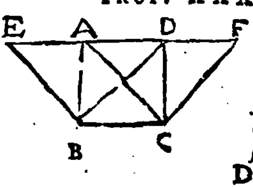


Parallelogramma BCDA, GHFE *super æqualibus basib⁹* BC, GH, *& in eisdem parallelis* AF, BH *constituta, inter se sunt æqualia.*

Ducantur BE, CF. Quia BC [a]= GH [b]= EF, [c] erit BCFE parallelogrammum. ergò Pgr. BCDA [d]= BCFE [d]=GHFE. Q.E.D.

a *hyp.*
b 34. 1.
c 33. 1.
d 35. 1.

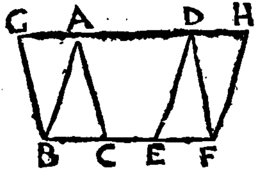
Prop. XXXVII.



Triangula BCA, BCD *super eadem basi* BC *constituta, & in eisdem parallelis* BC, EF, *inter se sunt æqualia.*

a 31. 1. [a] Duc BE parall. CA, [a] & CF parall. BD.
b 34. 1. Erit triang. BCA [b] $= \frac{1}{2}$ Pgr. BCAE $=$ [c] $\frac{1}{2}$
c 35. 1. & BDFC [b] $=$ BCD. Q. E. D.
7. ax.

Prop. XXXVIII.



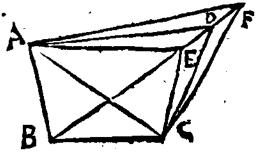
Triangula BCA, EFD *super æqualibus basibus* BC, EF *constituta, & in eisdem parallelis* GH, BF, *inter se sunt æqualia.*

Duc BG parall. CA. & FH parall. ED.
a 34. 1. erit triang. BCA [a] $= \frac{1}{2}$ Pgr. BCAG [b] $= \frac{1}{2}$
b 36. 1 & EDHF [c] $=$ EFD. Q. E. D.
7. ax.
c 34. 1.

Schol.

Si basis BC $\sqsubset$ EF, liquet triang. BAC $\sqsubset$ EDF. & si BC $\sqsupset$ EF, erit BAC $\sqsupset$ EDF.

Prop. XXXIX.

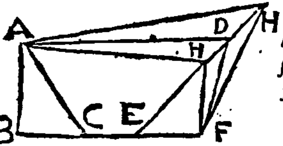


Triangula æqualia BCA, BCD, *super eadem basi* BC, *& ad easdem partes constituta, etiam in eisdem sunt parallelis* AD, BC.

Si negas, sit altera AF parall. BC; & ducatur
a 37. 1. CF. ergò triang. CBF [a] $=$ CBA [b] $=$ CBD.
b hyp. [c] Q. E. A.
c 9. ax.

Prop.

Prop. XL.

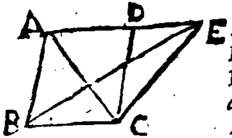


Triangula æqualia BCA, EFD *super æqualibus basibus* BC, EF, *&* *ad easdem partes constituta, & in iisdem sunt parallelis* AD, BF.

Si negas, sit altera AH parall. BF. & ducatur FH. ergò triang. EFH [a] = BCA [b] = EFD. [c] Q. E. A.

a 38. 1.
b hyp.
c 9. ax.

Prop. XLI.



Si parallelogrammum ABCD *cum triangulo* BCE *eandem basim* BC *habuerit, in eisdémque fuerit parallelis* AE, BC, *duplum erit parallelogrammum* ABCD *ipsius trianguli* BCE.

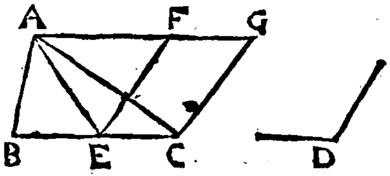
Ducatur AC. Triang. BCA [a] = BCE. ergò Pgr. ABCD [b] = 2 BCA [c] = 2 BCE. Q. E. D.

a 37. 1.
b 34. 1.
c 6. ax.

Scholium.

Hinc habetur area cujuscunq; trianguli BCE. Nam cùm area parallelogrammi ABCD producatur ex altitudine in basim ducta; producetur area trianguli ex dimidia altitudine in basim ducta, vel ex dimidia basi in altitudinem. ut si basis BC sit 8, & altitudo 7; erit trianguli BCE area, 28.

PROP. XLII.

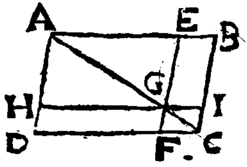


Dato triangulo ABC *æquale parallelogrammum* ECGF *constituere in dato angulo rectilineo* D.

a 31. 1. Per A [a] duc AG parall. BC. [b] fac ang. BCG
b 23. 1. = D. basim BC [c] biseca in E. [a] duc EF parall.
c 10. 1. CG. Dico factum.

Nam ductâ AE. erit ex constr. ang. ECG
d 38. 1. = D, & triang. BAC [d] = 2 ABC [e] = Pgr.
e 41. 1. ECGF. Q. E. F.

PROP. XLIII.

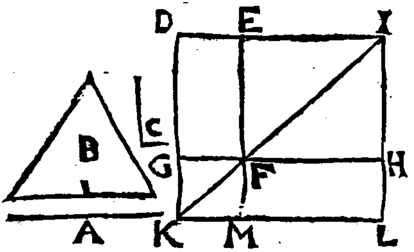


In omni parallelogrammo ABCD *complementa* DG, GB *eorum quæ circa diametrum* AC *sunt parallelogrammorum* HE, FI *inter se sunt æqualia.*

a 34. 1. Nam Triang. ACD, = [a] ACB. & triang.
b 3. ax. AGH [a] = AGB. & triang. GCF [a] = GCI.
[b] ergò Pgr. DG = GB. Q. E. D.

PROP.

PROP. XLIV.



Ad datam rectam lineam A, *dato triangulo* B, *æquale parallelogrammum* FL *applicare in dato angulo rectilineo* C.

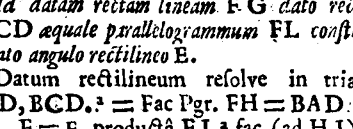
[a] Fac Pgr. FD = triang. B, ità ut ang. GFE = C. & pone lateri GF in directum FH = A. Per H [b] duc IL parall. EF; cui occurrat DE producta ad I. per IF ductæ rectæ occurrat DG protracta ad K. Per K [b] duc KL parall. GH; cui occurrant EF, & IH prolongatæ ad M, & L. Erit FL. Pgr. quæsitum.

a 42. 1.
b 31. 1.

Nam Pgr. FL [c] = FD = B [d] & ang. MFH = GFE = C. Q.E.F.

c 43. 1.
d 15. 1.

PROP. XLV.



Ad datam rectam lineam FG *dato rectilineo* ABCD *æquale parallelogrammum* FL *constituere, in dato angulo rectilineo* E.

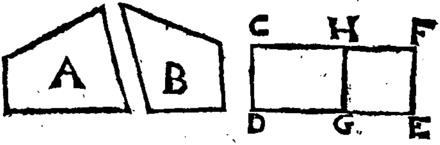
Datum rectilineum resolve in triangula BAD, BCD. [a] Fac Pgr. FH = BAD ità ut ang. F = E. productâ FI [a] fac (ad HI) Pgr.

a 44. 1.

 IL.

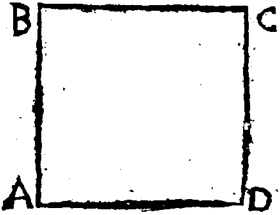
b 19. ax. IL $=$ BCD. erit Pgr. FL $=$ [b] FH $+$ IL [c] $=$
c constr. ABCD. Q. E. F.

Schol.



Hinc facilè invenitur excessus HE, quo rectilineum aliquod A superat rectilineum minus B; nimirum si ad quamvis rectam CD applicentur Pgr. DF $=$ A. & DH $=$ B.

Prop. XLVI.



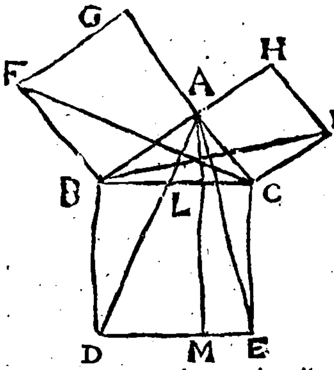
A data recta linea AD quadratum AC describere.

a 11. 1. [a] Erige duas perpendiculares AB,
b 3. 1. DC [b] æquales datæ AD; & junge BC. dico factum.

c constr. Cùm enim ang. A $+$ D [c] $=$ 2 Rect. [d] erunt
d 28. 1. AB, DC parallelæ. Sunt verò etiam [e] æquales.
e constr. [f] ergò AD, BC pares etiam sunt, & parallelæ.
f 34. 1. ergò Figura AC est parallelogramma, & æqui-
g Sch. 29. 1. latera. Anguli quoq; omnes recti sunt, [g] quoni-
h 29. def. am unus A est rectus. [h] ergò AC est quadratum. Q. E. F.

Eodem modo facilè describes rectangulum, quod sub datis duabus rectis contineatur.

PROP. XLVII.



In rectangulis triangulis BAC *quadratum* BE, *quod à latere* BC *rectum angulum* BAC *subtendente describitur, æquale est eis,* BG, CH, *quæ à lateribus* AB, AC *rectum angulum continentibus describuntur.*

Junge AE, AD; & duc AM. parall. CE.

Quoniam ang. DBC [a] = FBA; adde communem ABC, erit ang. ABD = FBC. Sed & AB [b] = FB, & BD [b] = BC. [c] ergò triang. ABD = FBC. atqui Pgr. BM. [d] = 2 ABD; & Pgr. BG [d] = 2 FBC (nam GAC est una recta per hyp. & 14. 1.) [e] ergò Pgr. BM = BG. Simili discursu Pgr. CM = CH. Totum igitur BE = BG + CH. Q. E. D.

a 12. ax.
b 29. d.
c 4. 1.
d 41. 1
e 6. ax.

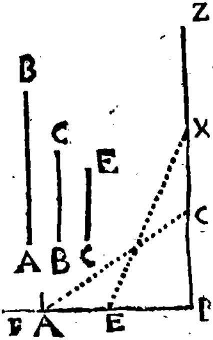
Schol.

Hoc nobilissimum, & utilissimum theorema ab inventore Pythagora, Pythagoricum dici meruit. Ejus beneficio quadratorum additio, & substractio perficitur; quò spectant duo sequentia problemata.

PROBL.

PROBL. 1.

Andr. Tacq.



Datis quotcunque quadratis, unum omnibus æquale construere.

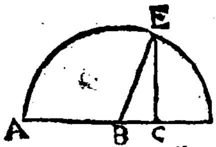
Dentur quadrata tria, quorum latera sint AB, BC, CE. [a] Fac ang. rectum FBZ infinita habentem latera, in eáque transfer BA, & BC, & junge AC, [b] erit ACq = ABq + BCq. Tum AC transfer ex B in X; & CE tertium latus datum transfer ex B in E, & junge EX, [b] erit EXq = EBq (CEq) + BXq (ACq) [c] = CEq + ABq + BCq. Q. E. F.

a 11. 1.

b 47. 1.

c 2. ax.

PROBL. 2.



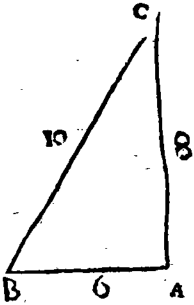
Datis duabus rectis inæqualibus AB, BC, *exhibere quadratum, quo quadratum majoris* AB *excedit quadratum minoris* BC.

Centro B intervallo BA describe circulum. ex C erige perpendicularem CE occurrentem peripheriæ in E. & ducatur BE. [a] Erit BEq (BAq) = BCq + CEq. [b] ergò BAq − BCq = CEq. Q. E. F.

a 47. 1.
b 3. ax.

PROBL. 3.

PROBL. 3.

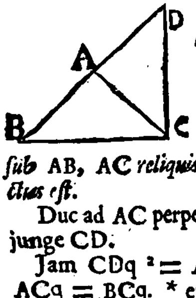


Notis duobus quibuscunque lateribus trigoni rectanguli ABC, *reliquum invenire.*

Latera rectum angulum ambientia sint AC, AB, hoc 6. pedum, illud 8. ergò cùm ACq + ABq = 64 + 36 = 100 = BCq. erit BC = √ 100 = 10. 47. 1.

Nota sint deinde latera AB, BC, hoc 10. pedum; illud 6. ergò cùm BCq — ABq = 100 — 36 = 64 = ACq. erit Acq = √ 64 = 8. 47. 1.

PROP. XLVIII.



Si quadratum quod ab uno latere BC *trianguli describitur, æquale sit eis quæ à reliquis trianguli lateribus* AB, AC *describuntur quadratis, angulus* BAC *comprehensus sub* AB, AC *reliquis duobus trianguli lateribus, rectus est.*

Duc ad AC perpendicularem DA = AB, & junge CD.

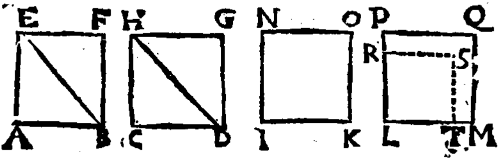
Jam CDq [a] = ADq + ACq = ABq + ACq = BCq. * ergò CD = BC. ergò triangula CAB, CAD, sibi mutuò æquilatera sunt; quare ang. CAB [b] = CAD [c] = Rect. Q. E. D.

a 47. 1.
* *Vid. seq. Theor.*
b 8. 1.
c *hyp.*

Schol.

Assumpsimus exinde quòd CDq. = BCq, sequi CD = BC. Hoc verò manifestum fiet ex sequenti theoremate.

THEOREMA.



Linearum æqualium AB, CD, *æqualia sunt quadrata* AF, CG; *& quadratorum æqualium* NK, PM *æqualia sunt latera* IK, LM.

Pro 1 Hyp. Duc diametros EB, HD. Liquet AF = [a] 2 triang. EAB = [b] 2 triang. HCD = [a] CG. Q. E. D.

2. Hyp. Si fieri potest, sit LM ⊏ IK. fac LT = IK; [a] sitque LS = LTq. ergò LS [b] = NK [c] = LQ. [d] Q. E. A. ergò LM = IK.

a 34. 1.
b 4. 1. & 6. ax.
a 46. 1.
b 1. part.
c *hyp.*
d 9. *ax.*

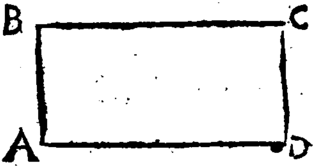
Coroll.

Eodem modo quælibet rectangula inter se æquilatera æqualia ostendentur.

LIB.

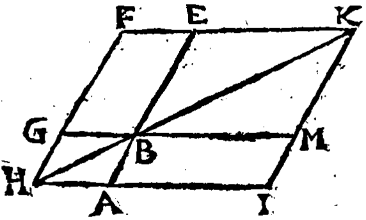
LIB. II.

Definitiones.



I. Omne parallelogrammum rectangulum A B C D contineri dicitur sub rectis duabus A B, AD, quæ rectum comprehendunt angulum.

Quando igitur dicitur rectangulum sub B A, A D; *vel brevitatis causâ, rectangulum* B A D; *vel* B A×A D, (*vel* Z A *pro* Z×A) *designatur rectangulum, quod continetur sub* BA, & AD *ad rectum angulum constitutis.*



I I. In omni parallelogrammo spatio FHIK unumquodq; eorum, quæ circa diametrum illius sunt, parallelogrammorum, cum duobus complementis Gnomon vocetur. *ut Pgr.* FB+BI+GA (EHM) *est Gnomon. item* Pgr. FB+BI+EM (GKA) *est Gnomon.*

PROP. I.

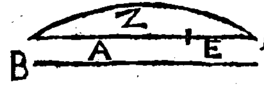
Si fuerint duæ rectæ lineæ AB, AF, *secetúrque ipsarum altera* A B *in quotcunque segmenta* AD, DE, EB: *rectangulum comprehensum sub illis duabus rectis lineis* A B, A F, *æquale est eis, quæ sub insecta* AF, *&* *quolibet segmentorum* A D, D E, E B *comprehenduntur rectangulis.*

a 11 1. [a] Statue AF, perpendicularem ad AB. [a] per F duc infinitam FG perpendicularem ad AF. [a] Ex D, E, B erige perpendiculares DH, EI, B G. Erit AG rectangulum sub AF, AB, &
b 19 ax. 1. [b] est æquale rectangulis AH, DI, EG, hoc est
c 34 1. (quia DH, EI, AF [c] pares sunt) rectangulis sub AF, AD; sub AE, DE; sub AF, EB. Q. E. D.

Schol.

Propositiones decem primæ hujus libri valent etiam in numeris. Reliquas quilibet tyro examinet. pro hac, sit AF 6, & AB 12, sectus in AD 5, DE 3, & EB 4. Estque 6×12 (AG) $=72$. 6×5 (AH) $=30$. 6 in 3 (DI) $=18$. denique 6×4 (E G) $=24$. Liquet verò $30+18+24=72$.

PROP. II.



Si recta linea Z *secta sit utcunque; rectangula, quæ sub tota* Z, *&* *quolibet segmentorum* A, E *comprehenduntur, æqualia sunt ei, quod à tota* Z *fit, quadrato.*

Dico $ZA+ZE=Zq$. Nam sume $B=Z$.
a. 1. 2. [a] Estque $BA+BE=BZ$; hoc est (ob $B=Z$) $ZA+ZE=Zq$. Q. E. D.

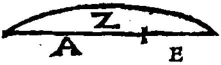
PROP.

Prop. III.

Si recta linea Z secta sit utcunque; rectangulum sub tota Z, & uno segmentorum E comprehensum, æquale est illi, quod sub segmentis A, E comprehenditur, rectangulo, & illi quod à prædicto segmento E describitur, quadrato.

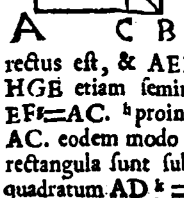
Dico. ZE=AE+Eq. [a] Nam EZ=EA+ a 1. 2.
E E.

Prop. IV.



Si recta linea Z secta sit utcunque; Quadratum, quod à tota Z describitur, æquale est, & illis quæ à segmentis A, E describuntur quadratis, & ei, quod bis sub segmentis A, E comprehenditur, rectangulo.

Dico Zq=Aq.+Eq.2 AE.[a] Nam ZA=Aq+ a 3. 2.
A E.[a] & ZE=Eq+AE. quum igitur ZA+
ZE[b]=Zq, [c] erit Zq=Aq+Eq+2 AE. b 2. 2.
Q.E.D. c 1. ax.



Aliter. Super AB fac quadratum AD, cujus diameter EB. per divisionis punctum C duc perpendicularem CF; & per G duc HI parall. AB.

Quoniam ang. EHG=A rectus est, & AEB [d] semirectus, [e] erit reliquus HGE etiam semirectus. Ergò HE[f]=HG[g]= EF[g]=AC. [h] proinde HF quadratum est rectæ AC. eodem modo CI est CBq. ergò AG, GD rectangula sunt sub AC, CB. Quare totum quadratum AD [k] = ACq + CBq + 2 ACB. Q.E.D.

d 4. Cor. 32. 1.
e 32. 1.
f 6. 1.
g 34. 1.
h 29. def. 1.
k 19. ax. 1.

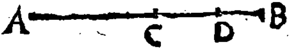
Coroll.

1. Hinc liquet parallelogramma circa diametrum quadrati esse quadrata.

2. Item diametrum cujusvis quadrati ejus angulos bisecare.

3. Si $A = \frac{1}{2} Z$; erit $Zq = 4\,Aq$, & $Aq = \frac{1}{4} Zq$. item è contra, si $Zq = 4\,Aq$. erit $A = \frac{1}{2} Z$.

PROP. V.



Si recta linea AB *secetur in æqualia* A C, CB, *& non æqualia* AD, DB, *rectangulum sub inæqualibus segmentis* A D, D B *comprehensum, unà cum quadrato, quod fit ab intermedia sectionum* CD, *æquale est ei, quod à dimidia* CB *describitur, quadrato.*

Dico $CBq = ADB + CDq$.

Æquantur enim ista $\begin{cases} CBq \\ {}^{a}CDq + CDB + DBq + CDB \\ CDq + {}^{b}CBD\,({}^{c}AC \times BD) + CDB \\ CDq + {}^{d}ADB. \end{cases}$

a 4. 2.
b 3. 2.
c hyp.
d 1. 2.

Scholium.

Si A B aliter dividatur, propiùs scilicet puncto bisectionis, in E; dico $AEB \sqsubset ADB$.

a 5. 2. & 3. ax. Nam $AEB^{a} = CBq - CEq$. & $ADB^{a} = CBq - CDq$. ergò quum $CDq \sqsubset CEq$, erit $AEB \sqsubset ADB$. Q. E. D.

Coroll.

Hinc $ADq + DBq \sqsubset AEq + EBq$. Nam
b 4. 2. $ADq + DBq + 2\,ADB^{b} = ABq\,^{b} = AEq + EBq + 2\,AEB$. ergò quum $2\,AEB \sqsubset 2\,ADB$, erit $ADq + DBq \sqsubset AEq + EBq$. Q. E. D.

c 3. ax. Unde 2. $ADq + DBq - AEq^{c} + EBq = 2\,AEB - 2\,ADB$.

PROP.

PROP. VI.

Si recta linea A bifariam secetur, & illi recta quæpiam linea E in directum adjiciatur; rectangulum comprehensum sub tota cum adjecta (sub. $A+E$), *& adjecta E, unà cum quadrato, quod à dimidia* $\frac{1}{2}$ *A, æquale est quadrato à linea, quæ tum ex dimidia, tum ex adjecta componitur, tanquam ab una* $\frac{1}{2}$ *A+E descripto.*

Dico $\frac{1}{4}$ Aq ([a]Q. $\frac{1}{2}$ A) + AE + Eq = Q. $\frac{1}{2}$ A + E. [a] Nam Q. $\frac{1}{2}$ A + E = $\frac{1}{4}$ Aq + Eq + AE. a 4. & 3. Cor. 4. 2.

Coroll.

Hinc si tres rectæ E, E + $\frac{1}{2}$ A, E + A sint in proportione Arithmetica, rectangulum sub extremis E, E + A contentum, unà cum quadrato excessûs $\frac{1}{2}$ A, æquale erit quadrato mediæ E + $\frac{1}{2}$ A.

PROP. VII.

Si recta linea Z secetur utcunque; Quod à tota Z, quódque ab uno segmentorum E, utraque simul quadrata, æqualia sunt illi, quod bis sub tota Z, & dicto segmento E comprehenditur, rectangulo, & illi, quod à reliquo segmento A fit, quadrato.

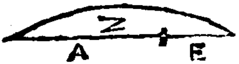
Dico Zq + Eq = 2 ZE + Aq. Nam Zq [a]= Aq + Eq + 2 AE. & 2 ZE [b]= 2 Eq + 2AE. a 4. 2. b 3. 2.

Coroll.

Hinc, quadratum differentiæ duarum quarumcunque linearum Z, E, æquale est quadratis utriusque minùs duplo rectangulo sub ipsis. c 7. 2. & 3. ax.

[c] Nam Zq + Eq − 2 ZE = Aq = Q. Z − E.

PROP. VIII.



Si recta linea Z secetur utcunque; rectangulum quater comprehensum sub tota Z, & uno segmentorum E, cum eo, quod à reliquo segmento A fit, quadrato, æquale est ei, quod à tota Z, & dicto segmento E, tanquam ab una linea Z+E describitur, quadrato.

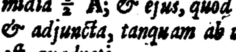
a 7. 2. & 3. ax. Dico 4 ZE+Aq=Q. Z+E. Nam 2 ZE[a]=
b 4. 2. Zq+Eq−Aq. ergò 4 ZE+Aq=Zq+Eq+2 ZE[b]=Q. Z+E. Q. E. D.

PROP. IX.

Si recta linea AB secetur in æqualia AC, CB, & non æqualia AD, DB. quadrata, quæ ab inæqualibus totius segmentis AD, DB fiunt, simul duplicia sunt, & ejus, quod à dimidia AC, & ejus, quod ab intermedia sectionum CD fit, quadrati.

Dico ADq+DBq=2 ACq+2 CDq. Nam
a 4. 2. ADq+DBq[a]=ACq+CDq+2 ACD+DBq.
b hyp. atqui 2 ACD ([b] 2 BCD) + DBq[c]=CBq
c 7. 2. (ACq) + CDq.[d] ergò ADq+DBq=2ACq
d 2. ax. +2 CDq. Q. E. D.

PROP. X.

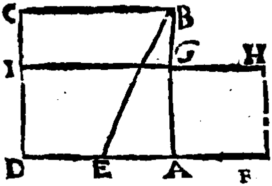


Si recta linea A secetur bifariam, adjiciatur autem ei in rectum quæpiam linea; Quod à tota A cum adjuncta E, & quod ab adjuncta E, utraque simul quadrata, duplicia sunt & ejus, quod à dimidia ½ A; & ejus, quod à composita ex dimidia, & adjuncta, tanquam ab una ½ A+E, descriptum est, quadrati.

a 4. 2. Dico Eq+Q. A+E, hoc est[a] Aq+2Eq+2
b Cor. 4. 2. AE=2 Q. ½ A+2 Q. ½ A+E. Nam 2 Q. ½ A[b]
c 4. 2. =½Aq. & 2 Q. ½ A+E[c]=½Aq+2Eq+2 AE.

PROP.

Prop. XI.



Datam rectam lineam AB *secare in* G, *ut comprehensum sub tota* AB, *& altero segmentorum* BG *rectangulum, æquale sit ei, quod à reliquo segmento* AG *fit, quadrato.*

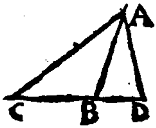
Super A B [a] describe quadratum A C. latus AD [b] biseca in E. duc EB. ex EA producta cape E F=E B. ad AF [a] statue quadratum A H. Erit AH=AB×BG. — a 46. 1. b 10. 1.

Nam protractâ H G ad I; Rectang. DH+EAq [c]=EFq [d]=EBq [e]=BAq+EAq. ergò DH [f]=BAq [d]=quad. AC. subtrahe commune A I; [f] remanet quad. AH=GC; [d] id est AGq=AB×B G. Q. E. F. — c 6. 2. d constr. e 47. 1. f 3 ax.

Scholium.

Hæc Propositio numeris explicari nequit; * neque enim ullus numerus ità secari potest, ut productum ex toto in partem unam æquale sit quadrato partis reliquæ. — * *vid.* 6. 13.

Prop. XII.



In amblygoniis triangulis ABC *quadratum, quod fit à latere* A C *angulum obtusum* A B C *subtendente, majus est quadratis, quæ fiunt à lateribus* AB, BC *obtusum angulum* A B C *comprehendentibus, rectangulo bis comprehenso, & ab uno laterum* B C, *quæ sunt circa obtusum angulum* ABC, *in quod, cùm protractum fuerit, cadit perpendicularis* AD, *& ab assumpta exterius linea* BD *sub perpendiculari* A D *prope angulum obtusum* A B C.

Dico

Dico $ACq = CBq + ABq + 2 CB \times BD$.

a 47. 1. b 4. 2. c 47. 1.

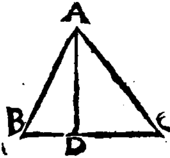
$$\text{Nam ista aequalia sunt inter se}\left\{\begin{array}{l} ACq. \\ ^{a}\ CDq + ADq. \\ ^{b}\ CBq + 2\,CBD + BDq + ADq \\ CBq + 2\,CBD\ ^{c} + ABq. \end{array}\right.$$

Schol.

Hinc, *cognitis lateribus trianguli obtusanguli* ABC, *facilè invenientur tum segmentum* BD *inter perpendicularem* AD, *& obtusum angulum* ABC *interceptum, tum ipsa perpendicularis* AD.

Sic; Sit AC 10, AB 7, CB 5; unde ACq 100, ABq 49, CBq 25. Proinde $ABq + CBq = 74$. hunc deme ex 100, manet 26 pro 2 CBD. unde CBD erit 13. hunc divide per CB 5, provenit $2\frac{3}{5}$ pro BD. quare AD invenitur per 47. 1.

Prop. XIII.



In oxygoniis triangulis ABC *quadratum à latere* AB *angulum acutum* ACB *subtendente, minùs est quadratis, quæ fiunt à lateribus* AC, CB *acutum angulum* ACB *comprehendentibus, rectangulo bis comprehenso, & ab uno laterum* BC, *quæ sunt circa acutum angulum* ACB, *in quod perpendicularis* AD *cadit, & ab assumpta interiùs linea* DC *sub perpendiculari* AD, *prope angulum acutum* ACB.

Dico $ACq + BCq = ABq + 2\,BCD$.

a 47. 1. b 7. 2. c 47. 1.

$$\text{Nam æquantur ista}\left\{\begin{array}{l} ACq + BCq. \\ ^{a}\ ADq + DCq + BCq. \\ ^{b}\ ADq + BDq + 2\,BCD. \\ ^{c}\ ABq + 2\,BCD. \end{array}\right.$$

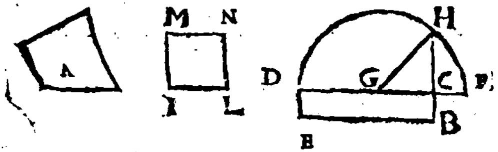
Coroll.

Hinc etiam *cognitis lateribus trianguli* ABC, *invenire est tam segmentum* DC *inter perpendicularem*

rem AD, *&* *acutum angulum* A B C *interceptum, quàm ipsam perpendicularem* AB.

Sit AB 13, AC 15, BC 14. Detrahe ABq (169) ex ACq + BCq hoc est ex 225 + 196 = 421; remanet 252 pro 2 BCD; unde BCD erit 126. hunc divide per BC 14, provenit 9 pro DC. unde AD = √: 225 − 81 = 12.

PROP. XIV.



Dato rectilineo A *æquale quadratum* ML *invenire.*

[a] Fac rectangulum DB = A, cujus majus latus DC produc ad F, ità ut CF = CB. [b] Biseca DF in G, quo centro ad intervallum GF describe circulum FHD, producatur CB, donec occurrat circumferentiæ in H. Erit CHq = * ML = A

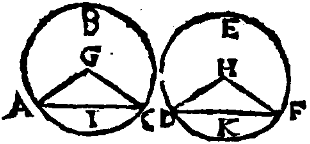
a 45. 1.
b 10. 2.
* 46. 1.
c *constr.*
d 5. 2. & 3. *ax.*
e 47. 1. & 3. *ax.*

Ducatur enim GH. Estque A [c]= DB [c] = DCF [d] = GFq − GCq [e] = HCq [c] = ML Q. E. F.

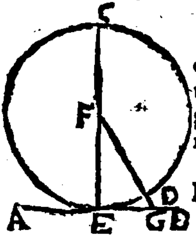
LIB.

LIB. III.

Definitiones.

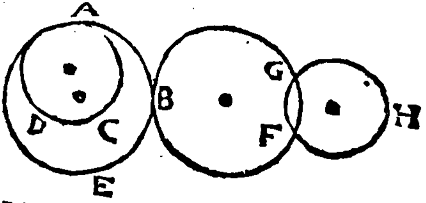


I. Æquales circuli (GABC, HDEF) sunt, quorum diametri sunt æquales, vel quorum quæ ex centris rectæ lineæ GA, HD, sunt æquales.



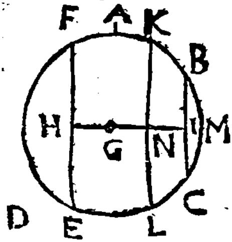
II. Recta linea *AB* circulum FED tangere dicitur, quæ cùm circulum tangat, si producatur circulum non secat.

Recta F G secat circulum FED.

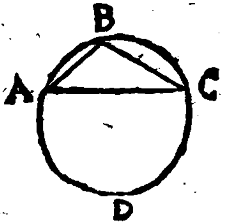


III. Circuli DAC, ABE (item FBG, ABE) se mutuò tangere dicuntur, qui se mutuò tangentes sese mutuò non secant.

Circulus BFG secat circulum FGH.



IV. In circulo GABD æqualiter distare à centro dicuntur rectæ lineæ FE KL, cùm perpendiculares GH, GN quæ à centro G in ipsas ducuntur, sunt æquales. Longiùs autem abesse illa BC dicitur, in qu ammajor perpendicularis GI cadit.

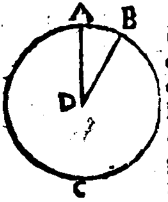


V. Segmentum circuli (ABC) est figura, quæ sub recta linea AC, & circuli peripheria ABC comprehenditur.

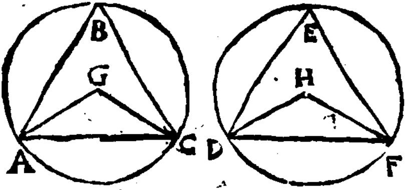
VI. Segmenti autem angulus (CAB) est, qui sub recta linea CA, & circuli peripheria AB comprehenditur.

VII. In segmento autem (ABC) angulus (ABC) est, cùm in segmenti peripheria sumptum fuerit quodpiam punctum B, & ab illo in terminos rectæ ejus lineæ AC, quæ segmenti basis est, adjunctæ fuerint rectæ lineæ AB, CB, is inquam angulus ABC ab adjunctis illis lineis AB, CB comprehensus.

VIII. Cùm verò comprehendentes angulum ABC, rectæ lineæ AB, BC aliquam assumunt peripheriam ADC, illi angulus ABC insistere dicitur.



IX. Sector autem circuli (ADB) est, cùm ad ipsius circuli centrum D constitutus fuerit angulus ADB; comprehensa nimirum figura ADB. & à rectis lineis AD, BD angulum continentibus, & à peripheria AB ab illis assumpta.



X. Similia circuli segmenta (ABC; DEF) sunt, quæ angulos (ABC, DEF) capiunt æquales; aut in quibus anguli ABC, DEF inter se sunt æquales.

PROP. I.

Dati circuli ABC *centrum* F *reperire.*

Duc in circulo rectam AC utcunq;, quam biseca in E. per E duc perpendicularẽ DB. hanc biseca in F. erit F centrũ.

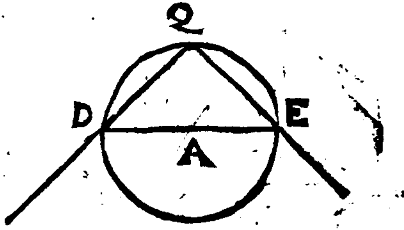
Si negas, centrum esto G, extra rectam DB (nam in ea esse non potest, cùm ubiq; extra F dividatur inæqualiter) ducantúrque GA, GC, GE. Vis G centrum esse; [a] ergò GA = GC; & per constr. AE = EC, latus verò GE commune est; [b] ergò anguli GEA, GEC pares, & [c] proinde recti sunt. [d] ergò ang. GEC = FEC rect. [e] Q. E. A.

a 15. def. 1.
b 8. 1.
c 10. def. 1.
d 12. ax.
e 9. ax.

Coroll.

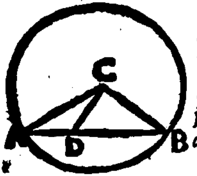
Coroll.

Hinc, si in circulo recta aliqua linea BD aliquam rectam lineam AC bifariàm & ad angulos rectos secet, in secante BD erit centrum.



Facillimè per normam invenitur centrum vertice Q ad circumferentiam applicato. Si enim recta DE jungens puncta D, & E, in quibus normæ latera QD, QE peripheriam secant, bisecetur in A, erit A centrum. Demonstratio pendet ex 31. hujus. *Andr. Tacq.*

PROP. II.



Si in circuli CAB *peripheria duo quælibet puncta,* A, B *accepta fuerint, recta linea* AB, *quæ ad ipsa puncta adjungitur, intra circulum cadet.*

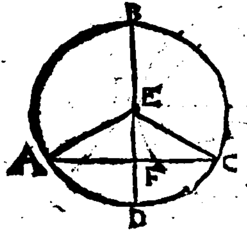
Accipe in recta AB quodvis punctum D, & ex centro C duc CA, CD, CB. & quoniam CA[a] = CB, [b] erit ang. A = B. Sed ang. CDB [c] $\sqsubset$ A; ergò ang. CDB $\sqsubset$ B. [d] ergò CB $\sqsubset$ CD. atqui CB tantùm pertingit ex centro ad circumferentiam; ergò CD eousque non pertingit. ergò punctum D est intra circulum. Idémque ostendetur de quovis alio puncto rectæ AB. Tota igitur AB cadit intra circulum. Q.E.D.

a 15. *def.* 1.
b 5. 1.
c 16. 1.
d 19. 1.

Coroll.

Hinc, Recta circulum tangens, ità ut eum non secet, in unico puncto tangit.

PROP. III.



Si in circulo EABC *recta quædam linea* BD *per centrum extensa quandam* AC *non per centrum extensam bifariàm secet, (in* F*) & ad angulos rectos ipsam secabit; & si ad angulos rectos eam secet, bifariàm quoque eam secabit.*

Ex centro E ducantur EA, EC.

a *hyp.*
b 15. *def.* 1.
c 8. 1.
d 10. *def.* 1.
e *hyp.* & 12. *ax.*
f 5. 1.
g 26. 1.

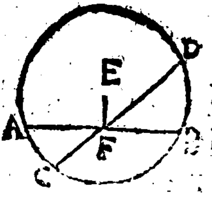
1. *Hyp.* Quoniam AF [a] = FC, & EA [b] = EC, latúsq; EF commune est, [c] erunt anguli EFA, EFC pares, & [d] consequenter recti. Q. E. D.

2. *Hyp.* Quoniam ang. EFA [e] = EFC, & ang. EAF [f] = ECF, latúsque EF commune, [g] erit AF = FC. Bisecta est igitur AC. Q. E. D.

Coroll.

Hinc, in triangulo quovis æquilatero & Isosceele linea ab angulo verticis bisecans basim, perpendicularis est basi. & contrà perpendicularis ab angulo verticis bisecat basim.

PROP. IV.



Si in circulo ACD *duæ rectæ lineæ* AB, CD *sese mutuò secent non per centrum* E *extensæ, sese mutuò bifariàm non secabunt.*

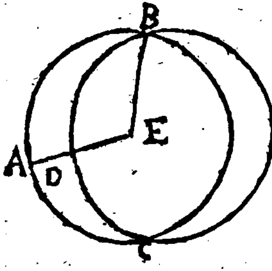
Nam si una per centrum

trum transeat, patet hanc non bisecari ab altera, quæ ex hyp. per centrum non transit.

Si neutra per centrum transit, ex E centro duc EF. Si jam ambæ AB, CD forent bisectæ in F, anguli EFB, EFD [a] ambo essent recti, & proinde æquales. [b] Q. E. A.

a 3. 3.
b 9. ax.

PROP. V.

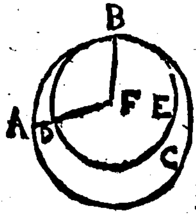


Si duo circuli BAC, BDC *sese mutuò secent, non erit illorum idem centrum* E.

Aliàs enim ductis ex communi centro E rectis EB, EDA, essent ED [a] = EB [a] = EA. [b] Q. E. A.

a 15. def. 1.
b 9. ax.

PROP. VI.

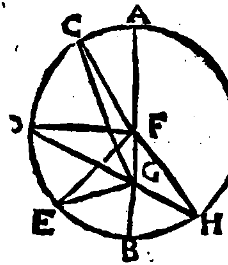


Si duo circuli BAC, BDE, *sese mutuò interiùs tangant* (in B) *eorum non erit idem centrum* F.

Aliàs ductis ex centro F rectis FB, FDA, essent FD [a] = FB [a] = FA. [b] Q. E. N.

a 15. def. 1.
b 9. ax.

Prop. VII.



Si in AB *diametro circuli quodpiam sumatur punctum* G, *quod circuli centrum non sit, ab eóq; puncto in circulum quædam rectæ lineæ* GC, GD, GE *cadunt; maxima quidam erit ea* (GA) *in qua centrum* F, *minima verò reliqua* GB. *aliarum verò illi, quæ per centrum ducitur, propinquior* GC *remotiore* GD *semper major est. Duæ autem solùm rectæ lineæ* GE GH *æquales ab eodem puncto in circulum cadunt, ad utrasque partes minimæ* GB, *vel maximæ* GA.

a 23. 1. Ex centro F duc rectas FC, FD, FE; & [a] fac ang. BFH = BFE.

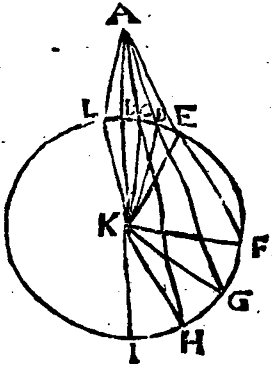
a 20. 1. 1. GF + FC (hoc est GA) [a] $\sqsubset$ GC. Q. E. D.

b 15. *def.* 1. 2. Latus FG commune est, & FC [b] = FD,
c 9. *ax.* atque ang. GFC [c] $\sqsubset$ GFD [d] ergò baf. GC
d 24. 1. $\sqsubset$ GD. Q. E. D.

e 20. 1. 3. FB (FE) [e] $\sqsupset$ GE + GF. ergò ablato
f 5. *ax.* to communi FG [f] remanet BG $\sqsupset$ EG. Q. E. D.

4. Latus FG commune est, & FE = FH;
g *constr.* atque ang. BFH [g] = BFE. [h] ergò GE = GH.
h 4. 1. Quòd verò nulla alia GD ex puncto G æquetur ipsi GE, vel GH, jamjam ostensum est. Q. E. D.

Prop. VIII.



Si extra circulum sumatur punctum quodpiam A, *ab eoque puncto ad circulum deducantur quædam lineæ* AI, AH, AG, AF, *quarum una quidem* AI *per centrum* K *protendatur, reliquæ verò ut libet; in cavam peripheriam cadentium rectarum linearum maxima quidem est illa* AI, *quæ per centrum ducetur, aliarum autem ei quæ per centrum transit propinquior* AH *remotiore* AG *semper major est. In convexam verò peripheriam cadentium rectarum linearum minima quidem est illa* AB, *quæ inter punctum* A, *& diametrum* BI *interponitur; aliarum autem ea, quæ est minimæ propinquior* AC *remotiore* AD *semper minor est. Duæ autem tantùm rectæ lineæ* AC, AL *æquales ab eo puncto in ipsum circulum cadunt, ad utrasque partes minimæ* AB, *vel maximæ* AI.

Ex centro K duc rectas KH, KG, KF; KC, KD, KE. & fac ang. AKL $=$ AKC.

1. AI (AK $+$ KH) [a] $\sqsubset$ AH. Q. E. D. a 20. 1.

2. Latus AK commune est; & KH $=$ KG; atque ang. AKH $\sqsubset$ AKG. [b] ergò baf. AH $\sqsubset$ AG. Q. E. D. b 24. 1.

3. KA [c] $\sqsupset$ KC $+$ CA. aufer hinc indè æquales KC, KB, [d] erit AB $\sqsupset$ AC. c 20. 1. d 5. ax.

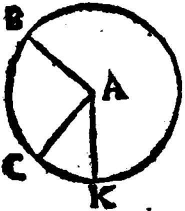
4. AC $+$ CK [e] $\sqsupset$ AD $+$ DK. aufer hinc indè æquales CK, DK, [f] erit AC $\sqsupset$ AD. Q. E. D. e 21. 1. f 5. ax.

 5. Latus

5. Latus KA est commune & KL = KC; atque ang. AKL [g] = AKC, [h] ergò LA = CA. hisce verò nulla alia æquatur, ex mox ostensis. ergò, &c.

g *conft.*
h 4. 1.

PROP. IX.

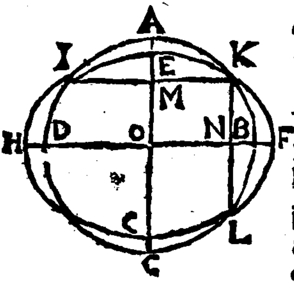


Si in circulo BCK *acceptum fuerit punctum aliquod* A, *& ab eo puncto ad circulum cadant plures, quàm duæ rectæ lineæ æquales* AB, AC, AK, *acceptum punctum* A *centrum est ipsius circuli.*

Nam [a] à nullo puncto extra centrum plures quàm duæ rectæ lineæ æquales duci possunt ad circumferentiam. Ergò A est centrum. Q. E. D.

a 7. 3.

PROP. X.



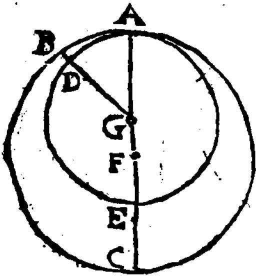
Circulus IAKBL *circulum* IEKFB *in pluribus quàm duobus punctis non secat.*

Secet, si fieri potest, in tribus punctis I K L. Junctæ IK KL. bisecentur in M & N. [a] Ambo circuli centrum habent in singulis perpendicularibus MC, NH, & proinde in earum intersectione O. ergò secantes circuli idem centrum habent. [b] Q. F. N.

a cor. 1. 3.
b 5. 3.

PROP.

PROP. XI.

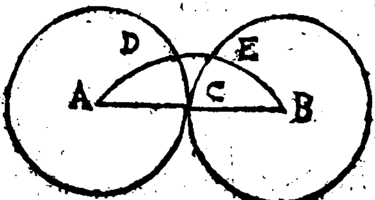


Si duo circuli GADE, FABC *sese intus contingant, atque accepta fuerint eorum centra* G, F; *ad eorum centra adjuncta recta linea* FG, *& producta, in* A *contactum circulorum cadet.*

Si fieri potest, recta F G protracta secet circulos extra contactum A, sic ut non FGA, sed EGDB sit recta linea. ducatur G A. Et quia GD [a] $= GA$, & GB [b] $\sqsupset GA$, (cùm recta FGB transeat per F centrum majoris circuli) erit GB $\sqsupset$ GD. [c] Q. E. A.

a 15. *def.* 1.
b 7. 3.
c 9. ax.

PROP. XII.

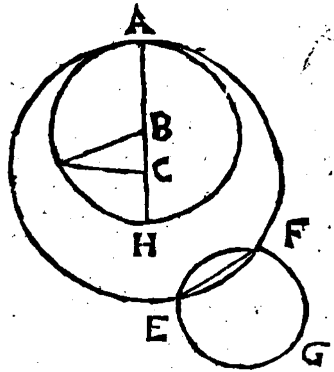


Si duo circuli ACD, BCE *sese exteriùs contingant, linea recta* AB *quæ ad eorum centra* A, B *adjungitur, per contactum* C *transibit.*

Si fieri potest, sit recta ADEB secans circulos extra contactum C in punctis D,E. Duc AC, CB. erit AD + EB (AC + CB) [a] $\sqsubset$ AD + EB. [b] Q. E. A.

a 20. 1.
b 9. ax.

PROP. XIII.



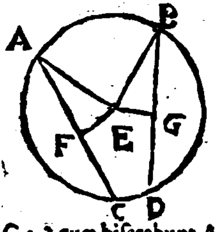
Circulus CAF *circulum* BAH *non tangit in pluribus punctis, quàm uno* A, *sive intus, sive extra tangat*

1. Tangat, si fieri potest, intus in punctis A, H. a 11. 3. [a] ergò recta CB centra connectens, si producatur cadet tam in A, quam in H. Quoniam igitur CH [b] = CA, & BH ⊏ CH. erit BA ([c] BH) ⊏ CA. [d] Q. E. A.

b 15. *def.* 1.
c 15. *def.* 1.
d 9. *ax.*
e 2. 3.

2. Sin dicatur exteriùs contingere in punctis E & F, [e] ducta recta EF in utroque circulo erit. Circuli igitur se mutuò secant, quod non ponitur.

PROP. XIV.



In circulo EABC *æquales rectæ lineæ* AC BD, *æqualiter distant à centro* E. *&* *quæ* AC, BD *æqualiter distant à centro, æquales sunt inter se.*

Ex centro E duc perpendiculares EF, EG: [a] quæ bisecabunt AC, DB. connecte EA EB.

a 3. 3.

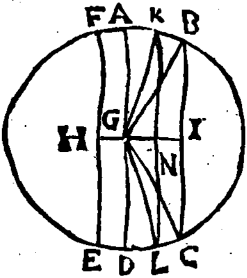
1. *Hyp.* AC = BD, ergò AF [b] = BG. sed & EA

b 7. ax.

$EA = EB$. ergò FEq [c] $= EAq - AFq =$ [c] $EBq - EGq$ [c] $= EGq$. [d] ergò $FE = EG$. Q.E.D. c 47. 1. & 3. ax.

2. *Hyp.* $EF = EG$. ergò AFq [c] $= EAq - EFq =$ [d] $EBq - EGq$ [c] $= GBq$. ergò AF [d] $= GB$. [e] proinde $AD = BC$. Q. E. D. d Schol.48.1. e 6. ax.

PROP. XV.

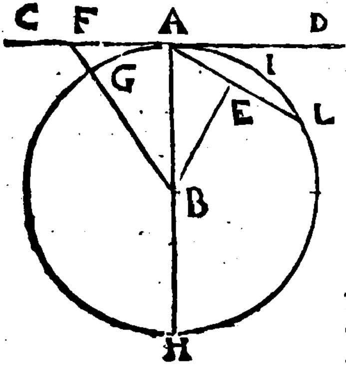


In circulo GABC maxima quidem linea est diameter AD; aliarum autem centro G propinquior FE remotiore BC semper major est.

1. Duc GB, GC. Diameter AD ([a] $GB + GC$) [b] $\sqsubset BC$ Q. E. D a 15. def. 1. b 20. 1.

2. Sit distantia GI $\sqsubset$ GH. accipe GN $=$ GH. per N duc KL perpend. GI. junge GK, GL. & quia GK $=$ GB, & GL $=$ GC; éstque ang. KGL $\sqsubset$ BGC, [c] erit KL (FE) $\sqsubset$ BC. Q. E. D. c 24. 1.

PROP. XVI.



Quæ CD ab extremitate diametri HA cujusq; circuli BALH ad angulos rectos ducitur, extra ipsa circulum cadet, & in locum inter ipsam rectam lineam, & peripheriã compreben-

prehensum altera recta linea AL *non cadet, & semicirculi quidem angulus* BAI *quovis angulo acuto rectilineo* BAL *major est; reliquus autem* DAI *minor.*

1. Ex centro B ad quodvis punctum F in recta AC duc rectam BF. Latus BF subtendens
a 19. 1. angulum rectum BAF [a] majus est latere BA, quod opponitur acutò BFA. ergò cùm BA(BG) pertingat ad circumferentiam, BF ulteriùs porrigetur, adeóque punctum F, & eâdem ratione quodvis aliud rectæ AC, extra circulum situm erit. Q. E. D.

2. Duc BE perpendic. AL. Latus BA opposi-
b 19. 1. tum recto angulo BEA [b] majus est latere BE, quod acutum BAE subtendi:: ergò punctum E, adeóque tota EA cadit intra circulum. Q. E. D.

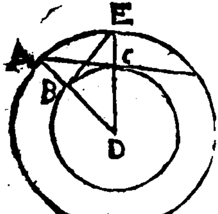
3. Hinc sequitur angulum quemvis acutum, nempe EAD angulo contactûs DAI majorem esse. Item angulum quemvis acutum BAL angulo semicirculi BAI minorem esse. Q. E. D.

Coroll.

Hinc, recta à diametri circuli extremitate ad angulos rectos ducta ipsum circulum tangit.

Ex hac propositione paradoxa consequuntur, & mirabilia bene multa, quæ vide apud interpretes.

PROP. XVII.



A dato puncto A *rectam lineam* AC *ducere, quæ datum circulum* DBC *tangat.*

Ex D dati circuli centro ad datum punctum A ducatur recta DA secans peripheriam in B. Centro D describe per A alium circulum,

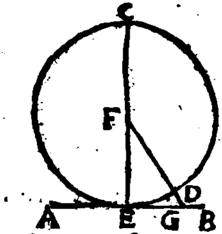
AE.

A E; & ex B duc perpendicularem ad AD, quæ occurrat circulo AE in E. duc ED occurrentem circulo BC in C. ex A ad C ducta recta tanget circulum DBC.

Nam DB [a] = DC, & DE [a] = DA, & ang. D communis est: [b] ergò ang. ACD = EBD, rect. [c] ergò AC tangit circulum C. Q. E. F.

a 15. def. 1.
b 4. 1.
c cor. 16. 3.

Prop. XVIII.

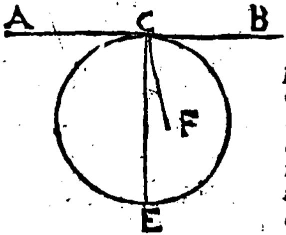


Si circulum FEDC *tangat recta quæpiam linea* AB, *à centro autem ad contactum* E *adjungatur recta quædam linea* FE; *quæ adjuncta fuerit* FE *ad ipsam contingentem* AB *perpendicularis erit.*

Si negas, sit ex F centro alia quædam FG perpendicularis ad contingentem, [a] secabit ea circulum in D. Quum igitur ang. FGE rectus dicatur [b] erit ang. FEG acutus [c]. ergò FE (FD) $\sqsubset$ FG. [d] Q. E. A.

a 2. def. 3.
b cor. 17. 1.
c 19. 1.
d 9. ax.

Prop. XIX.



Si circulum tetigerit recta quæpiam linea AB, *à contactu autem* C *recta linea* CE *ad angulos rectos ipsi tangenti excitetur, in excitata* CE *erit centrum circuli.*

Si negas, sit centrum extra CE in F, & ab F ad contactum ducatur FC. Igitur ang. FCB rectus est; & [a] proinde par angulo ECB recto per hypoth. [b] Q. E. A.

a 12. ax.
b 9. ax.

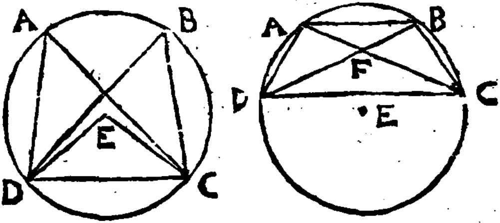
Prop. XX.

In circulo DABC, *angulus* BDC *ad centrum duplex est anguli* BAC *ad peripheriam, cùm fuerit eadem peripheria* BC *basis angulorum.*

Duc diametrum ADE.

Externus angulus BDE [a] = DAB + DBA [b] =
a 32. 1. 2 DAB. Similiter ang. EDC = 2 DAC. ergò
b 5. 1. in primo casu totus BDC = 2 BAC; sed in ter-
c 20. *ax.* tio casu [c] reliquus angulus BDC = 2 BAC.
Q. E. *D.*

Prop. XXI.

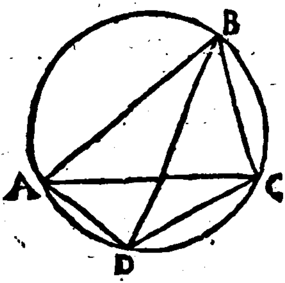


In circulo EDAC *qui in eodem segmento sunt anguli,* DAC *&* DBC *sunt inter se æquales.*

1. *Cas.* Si segmentum DABC semicirculo sit majus, ex centro E, duc ED, EC. Eritq; 2 ang.
a 20. 3. A [a] = E [a] = 2 B. Q. E. D.

2.. *Cas.* Sin segmentum semicirculo majus non fuerit, summa angulorum trianguli ADF æqua-
b 15. 1. tur summæ angulorum in triangulo BCF. *De-*
c *per* 1. *cas.* mantur hinc indè AFD [b] = BFC, & ADB [c] =
ACB, remanent DAC = DBC. Q. E. D.

PROP. XXII.



Quadrilaterorum ABCD in circulo descriptorũ anguli ADC, ABC, qui ex adverso, duobus rectis sunt æquales.

Duc C, BD. Ang. ABC + BCA + BAC[a] = 2 Rect. Sed BDA[b] = BCA, & BDC[b] = BAC. [c]ergò ABC + ADC = 2 Rect. Q. E. D.

a 32. 1.
b 21. 3.
c 1. ax.

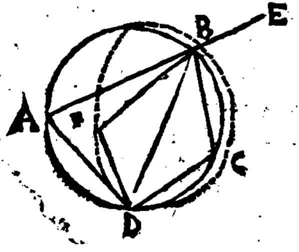
Coroll.

1. Hinc, si * AB unum latus quadrilateri in circulo descripti producatur, erit angulus externus EBC æqualis angulo interno ADC, qui opponitur ei ABC, qui est deinceps externo EBC. ut patet ex 13. 1. & 3. ax.

* *vid. seq. diagram.*

2. Item circa Rhombum circulus describi nequit; quia adversi ejus anguli vel cedunt duobus rectis, vel eos excedunt.

SCHOL.

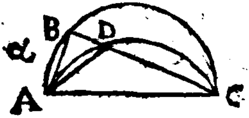


Si in quadrilatero ABCD anguli A, & C, qui ex adverso, duobus rectis æquantur, circa quadrilaterum circulus describi potest.

Nam circulus per quoslibet

bet tres angulos B,C,D transibit (ut patebit ex 5.4.)dico eundem per A transire. Nam si neges,
a 22. 3. transeat per F. ergò ductis rectis BF,FD,BD;
b hyp. ang. C+F [a]= 2 Rect. [b]= C+A [c] quare A=F.
c 3. ax.
d 21. 1. [d] Q. E. A.

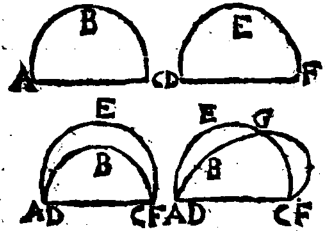
Prop. XXIII.



Super eadem recta linea AC *duo circulorũ segmenta* ABC, ADC *similia & inæqualia non constituentur ad easdem partes.*

Nam si dicantur similia, duc CB secantem circumferentias in D, & B, & junge AD, ac
a 10. def. 3. AB. Quia segmenta ponuntur similia, [a] erit ang.
b 16. 1. ADC=ABC [b] Q. E. A.

Prop. XXIV.

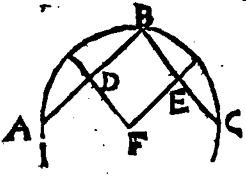


Super æqualib⁹ rectis lineis AC, DF *similiũ circulorũ segmẽta* ABC, DEF *sunt inter se æqualia.*

Basis AC superposita basi DF ei congruet, quia AC=DF. ergò segmentum ABC congruet segmento DEF (alias enim
a. 23. 3. aut intra cadet, aut extra, [a] atque ità segmenta non erunt similia, contra Hyp. aut saltem partim intra, partim extra, adeóque ipsum in tri-
b 10. 3. bus punctis secabit. [b] Q. E. A.) [c] proinde segmentum ABC=DEF. Q. E. D.
c 8. ax.

Prop.

Prop. XXV.

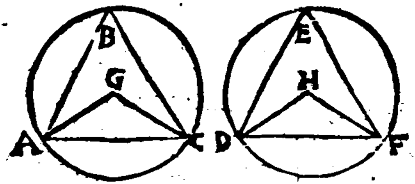


Circuli segmento ABC dato, describere circulum, cujus est segmentum.

Subtendantur utcunque duæ rectæ AB, BC, quas biseca in D, & E. Ex D, & E due perpendiculares DF, EF occurrentes in puncto F. Hoc erit centrum circuli.

Nam centrum a tam in DF, quàm in EF existit. ergò in communi puncto F. Q. E. F. a *Cor.* 1. 3.

Prop. XXVI.

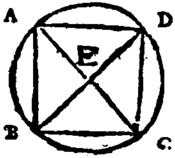


In æqualibus circulis GABC, HDEF *æquales anguli æqualibus peripheriis* AC, DF *insistunt, sive ad centra* G, H, *sive ad peripher.* B, E *constituti insistant.*

Ob circulorum æqualitatem, est GA = HD, & GC = HF item per hyp. ang. G = H. [a] ergò AC = DF. Sed & ang. B [b] = $\frac{1}{2}$ G = [c] $\frac{1}{2}$ H [b] = E. [d] ergò segmenta ABC, DEF similia, [e] & proinde paria sunt. [f] ergò etiam reliqua segmenta AC, DF æquantur. Q. E. D.

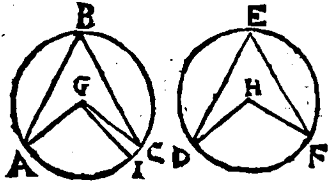
a 4. 1.
b 20. 3.
c *hyp.*
d 10. *def.* 3.
e 24. 3.
f 3. *ax.*

Scholium.



In circulo ABCD, sit arcus AB par arcui DC; erit AD parall. BC. Nam ductâ AC, [a] erit ang. ACB = CAD. quare per 27. 1. & c. a 26. 3.

Prop. XXVII.

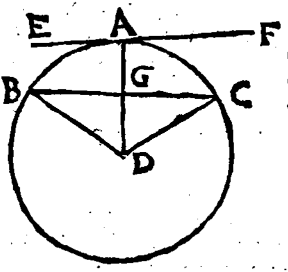


In æqualibus circulis GABC, HDEF, *anguli qui æqualibus peripheriis* AC DF *insistunt, sunt inter se æquales, sive ad centra* G, H, *sive ad peripherias* B, E *constituti insistant.*

Nam si fieri potest, sit alter eorum AGC ⊏ DHF. fiátque AGI = DHF. ergò arcus AI[a] = DF[b] = AC. [c] Q. E. A.

a 26. 3.
b *hyp.*
c 9. *ax.*

SCHOL.



Linea recta EF, *quæ ducta ex* A *medio puncto peripheriæ alicujus* BC *circulum tangit, parallela est rectæ lineæ* BC, *quæ peripheriam illam subtendit.*

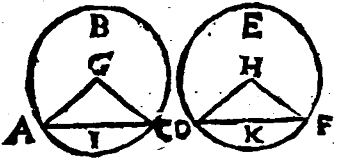
Duc è centro D ad contactum A rectam DA, & connecte DB, DC.

Latus DG commune est; & DB = DC, atq; ang. BDA [a] = CDA (ob arcus BA, CA [b] æquales) [c] ergò anguli ad basim DGB, DGC æquales & [d] proinde recti sunt. Sed interni anguli GAB, GAF [e] etiam recti sunt. [f] ergò BC, EF sunt parallelæ. Q. E. D.

a 27. 3.
b *hyp.*
c 4. 1.
d 10. *def.* 1.
e *hyp.*
f 28. 1.

Prop.

Prop. XXVIII.

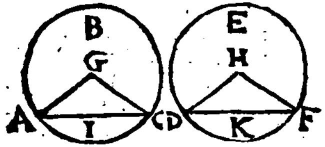


In æqualibus circulis GABC, HDEF *æquales rectæ lineæ* AC, DF *æquales peripherias auferunt, majorem quidem* ABC *majori* DEF, *minorem autem* AIC *minori* DKF.

E centris, G, H duc GA, GC; & HD, HF. Quoniam GA=HD, & GC=HF, atque AC[a] =DF; [b] erit ang. G=H. [c] ergò arcus AIC=DKF. [d] proinde reliquus ABC=DEF. Q. E. D.

a hyp.
b 8. 1.
c 26. 3.
d 3. ax.

Prop. XXIX.



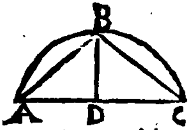
In æqualibus circulis GABC, HDEF *æquales peripherias* ABC, DEF *æquales rectæ lineæ* AC, DF *subtendunt*.

Duc GA, GC; & HD, HF. Quia GA= HD; & GC=HF; & (ob arcus AC, DF [a] pares) etiam ang. G[b]=H; [c] erit bas. AC=DF. Q. E. D.

Hæc & tres proximè præcedentes intelligantur etiam de eodem circulo.

a hyp.
b 27. 3.
c 4. 1.

Prop. XXX.

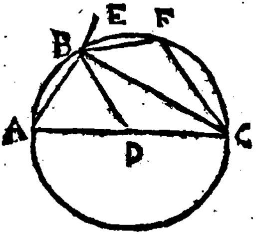


Datam peripheriam ABC *bifariàm secare*.

Duc AC; quam biseca in D. ex D duc perpendicularem DB occurrentem arcui in B. *Dico factum*.

Jungantur enim AB, CB. Latus DB com-
a *constr.* mune est; & AD [a] = DC; & ang. ADB [b] =
b 12. *ax.* CDB. [c] ergò AB = BC. [d] quare arcus AB =
c 4. 1. BC. Q. E. F.
d 28. 3.

Prop. XXXI.



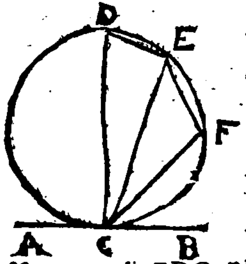
In circulo angulus ABC, *qui in semicirculo, rectus est; qui autem in majore segmento* BAC, *minor recto; qui verò in minore segmento* BFC, *major est recto. Et insuper angulus majoris segmenti recto quidem major est, minoris autem segmenti angulus, minor est recto.*

Ex centro D duc DB. Quia DB = DA, erit
a 5. 1. ang. A [a] = DBA. pariter ang. DCB [a] = DBC.
b 2 *ax.* [b] ergò ang. ABC = A + ACB [c] = EBC,
c 32. 1. [d] proinde ABC, & EBC recti sunt. Q. E. D.
d 10. *def.* 1. [e] ergò BAC acutus est. Q. E. D. ergò cùm
e *cor.* 17. 1. BAC + BFC [f] = 2 Rect. erit BFC obtusus.
f 22. 3. denique angulus sub recta CB, & arcu BAC major est recto ABC. factus verò sub CB, & BFC peripheria minoris segmenti, recto EBC
g 9. *ax.* [g] minor est. Q. E. D.

SCHOLIUM.

In triangulo rectangulo ABC, *si hypotenusa* AC *bisecetur in* D, *circulus centro* D, *per* A *descriptus transibit per* B. ut facilè ipse demonstrabis ex hac, & 21. 1.

PROP. XXXII.



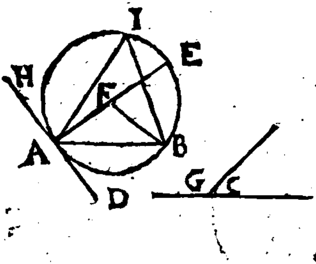
Si circulum tetigerit aliqua recta linea AB, à contactu autē producatur quædam recta linea CE circulum secans: anguli ECB, ECA, quos ad contingentem facit, æquales sunt iis, qui in alternis circuli segmentis consistunt, angulis EDC, EFC.

Sit CD latus anguli EDC perpendiculare ad AB ([a] perinde enim est) [b] ergò CD est diameter. [c] ergò ang. CED in semicirculo rectus est. [d] ergò ang. D + DCF = Rect. [e] = ECB + DCE. [f] ergò ang. D = ECB. Q. E. D.

Cùm igitur ang. ECB + ECA [g] = 2 Rect. [h] = D + F; aufer hinc indè æquales ECB, & D, [k] remanent ECA = F. Q. E. D.

a 26. 3.
b 19. 3.
c 31. 3.
d 32. 1.
e constr.
f 3. ax.
g 13. 1.
h 22. 3.
k 3. ax.

PROP. XXXIII.



Super data recta linea AB describere circuli segmentum AIEB, quod capiat angulū AIB æqualem dato angulo rectilineo C.

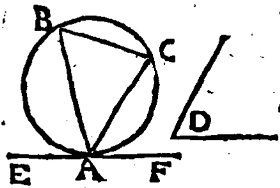
[a] Fac ang. BAD = C. Per A duc AE perpendicularem ad HD. ad alterum terminum datæ AB fac ang. ABF = BAF. cujus alterum latus secet AE in F. centro F per A describe circulum, quod transibit per B (quia ang. FBA [b] = FAB,

a 23. 1.

b constr. $^{b}=$ FAB, ^{c} ideóque FB $=$ FA); segmentum
c 6. 1. AIB est id quod quæritur.

Nam quia HD diametro AE perpendicularis
d cor. 16. 3. est, ^{d} tangit HD circulum, quem secat AB, ergò
e 32. 3. ang. AIB $^{e} =$ BAD $^{f} =$ C. Q. E. F.
f constr.

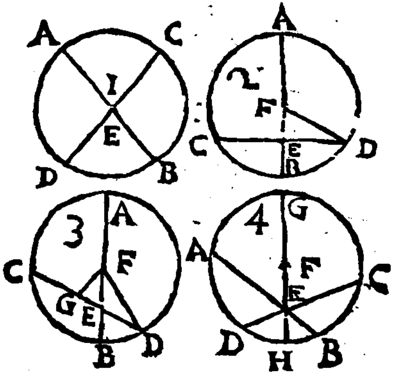
PROP. XXXIV.



A dato circulo ABC *segmentum* ABC *abscindere capiens angulum* B *æqualem dato angulo rectilineo* D.

a 17. 3. ^{a} Duc rectam EF, quæ tangat
datum circulum in A. ^{b} ducatur item AC faciens
b 23. 1. ang. FAC $=$ D. Hæc auferet segmentum ABC
capiens angulum B $^{c} =$ CAF $^{d} =$ D. Q. E. F.
c 32. 3.
d constr.

PROP. XXXV.



Si in circulo FBCA *duæ rectæ lineæ* AB, DC *sese mutuò secuerint, rectangulum comprehensum sub*

sub segmentis AE, EB *unius, æquale est ei quod sub segmentis* CE, ED *alterius comprehenditur, rectangulo.*

Cas. 1. Si rectæ sese in centro secent, res clara est.

2. Si una AB transeat per centrum F, & reliquam CD bisecet, duc FD. Estque Rectang. AEB + FEq [a] = FBq [b] = FDq [c] = EDq + FEq [d] = CED + FEq. [e] ergò Rectang. AEB = CED. Q. E. D.

3. Si una AB diameter sit, alterámque CD secet inæqualiter, biseca CD per FG perpendicularem ex centro.

a 5. 2.
b sch. 48. 1.
c 47. 1.
d hyp.
e 3. ax.

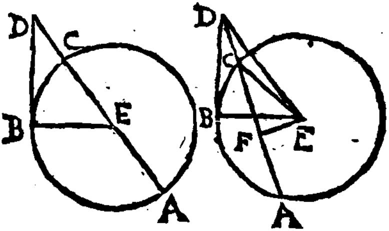
Æquantur ista: Rectang. AEB + FEq.
[f] FBq (FDq)
[g] FGq + GDq.
FGq + [h] GEq + Rectang. CED.
[k] FEq + CED.

[l] Ergò Rectang. AEB = CED.

f 5. 2.
g 47. 1.
h 5. 2.
k 47. 1.
l. 3. ax.

4. Si neutra rectarum AB, CD per centrum transeat; per intersectionis punctum E duc diametrum GH. Per modò demonstrata Rectang. AEB = GEH = CED. Q. E. D.

Prop. XXXVI.



Si extra circulum EBC *sumatur punctum aliquod* D, *ab eóque puncto in circulum cadant duæ rectæ lineæ* DA, DB; *quarum altera* DA *circulum secet,*

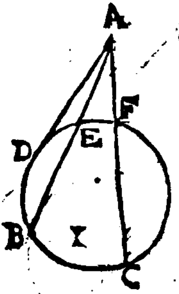
secet, altera vero DB *tanget; Quod sub tota secante* DA, *& exteriùs inter punctum* D, *& convexam peripheriam assumptâ* DC *comprehenditur rectangulum, æquale erit ei, quod à tangente* DB *describitur, quadrato.*

1. *Cas.* Si secans AD transeat per centrum E,
a 18. 3. junge EB; [a] faciet hæc cum DB rectum angu-
b 47. 1. lum; quare DBq + EBQ (ECq) [b] = EDq
c 6. 2. [c] = AD × DC + ECq [d] ergò AD × DC =
d 3. ax. DBq. Q. E. D.

2. *Cas.* Sin AD per centrum non transeat,
a 3. 3. duc EC, EB, ED; atq; EF perpend. AD, quare [a] bisecta est AC in F.

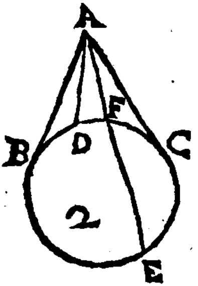
b 47. 1. Quoniam igitur BDQ + EBq [b] = DEq [b] =
c 6. 2. EFq + FDq [c] = EFq + ADC + FCQ [d] =
d 47. 1. ADC + CEq (EBq); [e] erit BDq = ADC.
e 3. ax. Q. E. D.

Coroll.



1. Hinc, si à puncto quovis A extra circulum assumpto, plurimæ lineæ rectæ AB, AC circulum secantes ducantur, rectangula comprehensa sub totis lineis AB, AC, & partibus externis AE, AF inter se sunt æqualia. Nam si ducatur tangens AD; erit CAF
a 36. 3. = ADq [a] = BAE.

2. Constat



2. Constat etiam duas rectas AB, AC ab eodem puncto A ductas, quæ circulum tangant, inter se æquales esse.

Nam si ducatur AE secans circulum; erit ABq [a] = EAF [b] = ACq. a 36. 3. b 36. 3.

3. Perspicuum quoque est ab eodem puncto A extra circulum assumpto, duci tantùm posse duas lineas, AB, AC quæ circulum tangant.

Nam si tertia AD tangere dicatur, erit AD [c] = AB [c] = AC. [d] Q. F. N. c 2 cor. d 8. 3.

4. E contrà constat, si duæ rectæ æquales AB, AC ex puncto quopiam A in convexam peripheriam incidant, & earum una AB circulum tangat, alteram quoq; circulum tangere.

Nam si fieri potest, non AC, sed altera AD circulum tangat. ergò AD [e] = AC [f] = AB. e 2 cor. f hyp.
[g] Q. E. A. g 8. 3.

Prop. XXXVII.

Si extra circulum EBF *sumatur punctum* D, *ab eoque in circulum cadant duæ rectæ lineæ* DA, DB; *quarum altera* DA *circulum secet, altera* DB *in eum incidat; sit autem quod sub tota secante* DA, *& exteriùs inter punctum, & convexam peripheriam assumpta* DC, *comprehenditur rectangulum, æquale ei, quod ab incidente*

DB *describitur quadrato, incidens ipsa* DB *circulum tanget.*

a 17. 3. Ex D [a] ducatur tangens DF; atque ex E cen-
b *hyp.* tro duc ED, EB, EF. Quia DBq [b] = ADC
c 36. 3. [c] = DFq, [d] erit DB = DF. Sed EB = EF,
d 1. *ax.* & & latus ED commune est; [e] ergò ang. EBD
sch. 48. 1. = EFD. Sed EFD rectus est, [f] ergò EBD
e 8. 1. etiam rectus est. [g] ergò DB tangit circulum.
f 12 *ax.*
g *cor.* 16. 3. Q. E. D.

Coroll.

h 8. 1. Hinc, [h] ang. EDB = EDF,

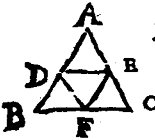
LIB.

LIB. IV.

Definitiones.

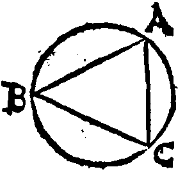
I. Figura rectilinea in figura rectilinea inſcribi dicitur, cùm ſinguli ejus figuræ, quæ inſcribitur, anguli ſingula latera ejus in qua inſcribitur, tangunt.

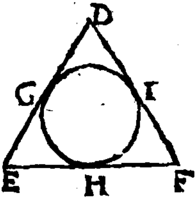
Sic triangulum DEF *eſt inſcriptum in triangulo* ABC.



II. Similiter & figura circa figuram deſcribi dicitur, cùm ſingula ejus, quæ circumſcribitur, latera ſingulos ejus figuræ angulos tetigerint, circa quam illa deſcribitur.

Ità triangulum ABC *eſt deſcriptum circa triangulum* DEF.





III. Figura rectilinea in circulo inſcribi dicitur, cùm ſinguli ejus figuræ, quæ inſcribitur, anguli tetigerint circuli peripheriam.

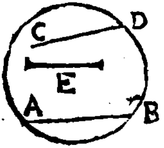
IV. Figura verò rectilinea circa circulum deſcribi dicitur, cùm ſingula latera ejus, quæ circumſcribitur, circuli peripheriam tangunt.

V. Similiter & circulus in figura rectilinea inſcribi dicitur, cùm circuli peripheria ſingula latera tangit ejus figuræ, cui inſcribitur.

VI. Circulus autem circa figuram deſcribi

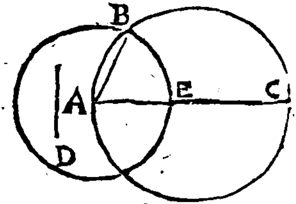
 dicitur,

dicitur, cùm circuli peripheria ſingulcs tangit ejus figuræ, quam circumſcribit, angulos.



VII. Recta linea in circulo accommodari, ſeu coaptari dicitur, cùm ejus extrema in circuli peripheria fuerint; ut recta linea AB.

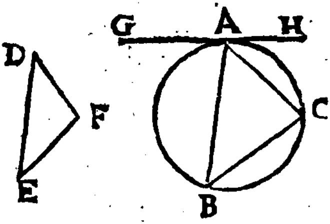
PROP. I. *Probl.* 1.



In dato circulo ABC *rectam lineam* AB *accommodare æqualem datæ rectæ lineæ* D, *quæ circuli diametro* AC *non ſit major.*

a 2. poſt. & 3. 1. Centro A, ſpatio AE=D [a] deſcribe circulum
b 15. def. 1. dato circulo occurrentem in B. Erit ducta
c conſtr. AB[b]=AE [c]=D. Q. E. F.

PROP. II. *Probl.* 2.

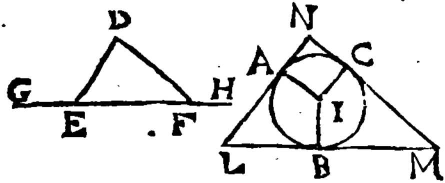


In dato circulo ABC *triangulũ* ABC *deſcribere dato triangulo* DEF *æquiangulum.*

a 17. 3. Recta GH circulum datum [a] tangat in A.
b 23. 1. [b] Fac ang. HAC=E; [b] & ang. GAB=F, & junge BC. Dico factum. Nam

Nam ang. B [c] = HAC [d] = E; & ang. c 22. 3.
C [c] = GAB [d] = F; [e] quare etiam ang. BAC = D. d *constr.*
ergò triang. BAC circulo inſcriptum triangulo e 32. 1.
DEF æquiangulum eſt. Q. E. F.

PROP. III. *Probl.* 3.

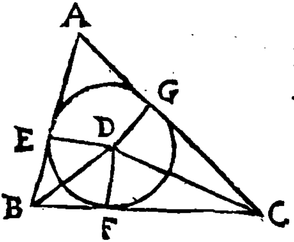


Circa datum circulum IABC *triangulum* LNM *deſcribere, dato triangulo* DEF *æquiangulum.*

Produc latus EF utrinque. [a] Fac ad centrum a 23. 1.
I ang. AIB = DEG. & ang. BIC = DFH.
deinde in punctis A, B, C circulum [b] tangant b 17. 3.
tres rectæ LN, LM, MN. Dico factum.

Nam quòd coibunt rectæ LN, LM, MN,
atque ità triangulum conſtituent, patet; [c] quia c 13. *ax.*
anguli LAI, LBI [d] recti ſunt, adeóque ducta d 18. 3.
AB angulos faciet LAB, LBA duobus rectis minores. Quoniam igitur ang. AIB + L [e] = 2 e *Schol.* 32. 1.
Rect. [f] = DEG + DEF; & AIB [g] = DEG; [h] erit f 13. 1.
ang. L = DEF. Simili argumento ang. M = DFE. g *constr.* h 3. *ax.*
[k] ergò etiam ang. N = D. ergò triang. LNM k 32. 1.
circulo circumſcriptum dato EDF eſt æquiangulum. Q. E. F.

PROP. IV. Probl. 4.



In dato triangulo ABC *circulum* EFG *inſcribere.*

Duos angu-
a 9. 1. los B, & C[a] bi-
ſeca rectis BD,
CD coeunti-
b 12. 1. bus in D. Ex
D[b] duc perpendiculares DE, DF, DG. circulus centro D per E deſcriptus tranſibit per G, & F, tangétque tria latera trianguli.

c conſtr. Nam ang. DBE[c] $=$ DBF; & ang. DEB[d] $=$
d 12. ax. DFB; & latus DB commune eſt, [e] ergò DE $=$
e 26. 1. DF. Simili argumento DG $=$ DF. circulus igitur centro D deſcriptus tranſit per E, F, G; & cùm anguli ad E, F, G ſint recti, tangit omnia trianguli latera. Q.E.F.

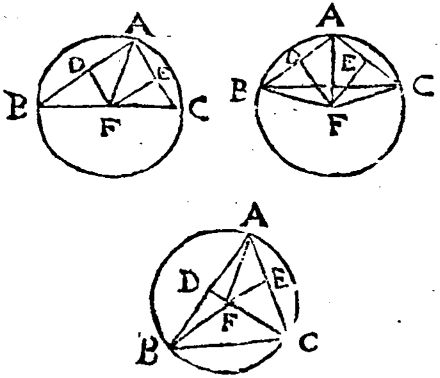
Scholium.

Petr. Herig. Hinc, *cognitis lateribus trianguli, invenientur eorum ſegmenta, quæ fiunt à contactibus circuli inſcripti.* Sic.

Sit AB 12, AC 18, BC 16. Erit AB + BC $=$ 28. ex quo ſubduc 18 $=$ AC $=$ AE + FC, remanet 10 $=$ BE + BF. ergò BE, vel BF $=$ 5. proinde FC, vel CG $=$ 11. quare GA, vel AE $=$ 7.

PROP.

PROP. V. *Probl.* 5.



Circa datum triangulum ABC *circulum* FABC *deſcribere.*

Latera quævis duo BA, AC [a] biſeca perpendicularibus DF, EF concurrentibus in F. Hoc erit centrum circuli. a 10, & 11. 1.

Nam ducantur rectæ FA, FB, FC. Quoniam AD[b] = DB; & latus DF commune eſt; & ang. FDA [c] = FDB, [d] erit FB = FA. eodem modo FC = FA. ergò circulus centro F per dati trianguli angulos B, A, C tranſibit. Q. E. F. b conſtr. c conſtr. & 12. ax. d 4. 1.

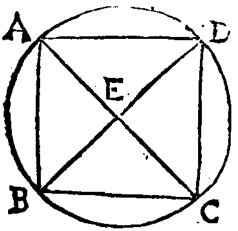
Coroll.

* Hinc, ſi triangulum fuerit acutangulum, centrum cadet intra triangulum, ſi rectangulum, in latus recto angulo oppoſitum; ſi denique obtuſangulum, extra triangulum. * 31. 3.

Schol.

Eâdem methodo deſcribetur circulus, qui tranſeat per data tria puncta, non in una recta linea exiſtentia.

Prop. VI. Probl. 6.

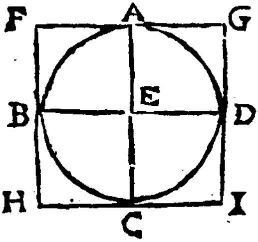


In dato circulo EABCD *quadratum* ABCD *inscribere.*

a 11. 1. [a] Duc diametros AC, BD se mutuò secantes ad angulos rectos in centro E. junge harum terminos rectis AB, BC, CD, DA. Dico factum.

b 26. 3. Nam quia 4 anguli ad E recti sunt, [b] arcus,
c 29. 3. & [c] subtensæ AB, BC, CD, DA pares sunt. ergò ABCD æquilaterum est; ejúsque omnes
d 31. 3. anguli in semicirculis, adeóque [d] recti sunt. [e] er-
e 29. def. 1. gò ABCD est quadratum, dato circulo inscriptum. Q. E. F.

Prop. VII. Probl. 7.



Circa datum circulum EABCD *quadratum* FHIG *describere.*

Duc diametros AC, BD se mutuò secantes perpendiculariter. per
a 17. 3. harũ extrema [a] duc tangentes concurrentes in F, H, I, G. Dico factum. Nam ob
b 18. 3. angulos ad A, & C [b] rectos, [c] erit FG parall.
c 28. 1. HI. eodem modo FH parall. GI. ergò FHIG est parallelogrammum; & quidem rectangulum.
d 34. 1. sed & æquilaterum, quia FG[d]=HI[d]=BD[e]=
e 15. def. 1. CA[d]=FH[d]=GI. quare FHIG est [f] quadra-
f 29. def. 1. tum, dato circulo circumscriptum. Q. E. F.

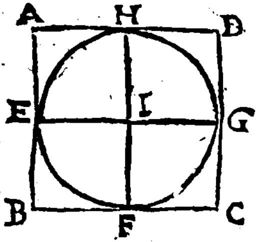
SCHOL.

SCHOL.

Quadratum ABCD circulo circumscriptum duplum est quadrati EFGH circulo inscripti.

Nam rectang. HB = 2 HEF. & HD = 2 HGF. per 41. 1.

Prop. VIII. *Probl.* 8.



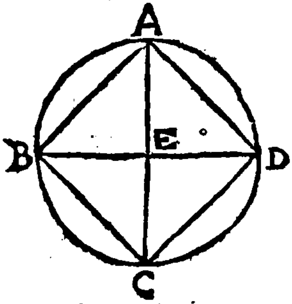
In dato quadrato ABCD *circulum* IEFGH *inscribere.*

Latera quadrati biseca in punctis, H, E, F, G, junge HF, EG sese secantes in I. circulus centro I per H descriptus quadrato inscribetur.

Nam quia AH, BF [a] pares ac [b] parallelæ sunt, [c] erit AB parall. HF parall. DC. eodem modo AD parall. EG parall. BC. ergò IA, ID, IB, IC sunt parallelogramma. Ergò AH[d] = AE[e] = HI = EI = IF = IG. Circulus igitur centro I per H descriptus transibit per H, E, F, G, tangétque quadrati latera, cùm anguli ad H, E, F, G sint recti. Q. E. F.

a 7. ax.
b 34. 1.
c 33. 1.
d 7. ax.
e 34. 1.

PROP. IX. Probl. 9.



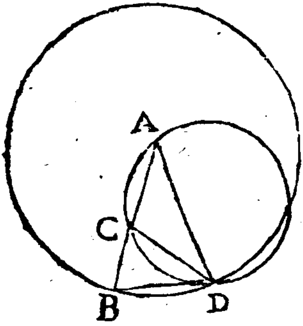
Circa datum quadratum ABCD *circulum* EABCD *describere.*

Duc diametros AC, BD secantes in E. centro E per A describe circulum. Is dato quadrato circumscriptus est.

a 4. cor. 32. 1.
b 6. 1.

Nam anguli ABD, & BAC [a] semirecti sunt; [b] ergò BA = EB. eodem modo EA = ED = EC. Circulus igitur centro E descriptus per A, B, C, D dati quadrati angulos transit. Q.E.F.

PROP. X. Probl. 10.



Isosceles triangulum ABD *constituere, quod habeat utrunq; eorum quæ ad basim sunt angulorum* B *&* ADB *duplum reliqui* A.

a 11. 2.

Accipe quamvis rectam AB, quam [a] seca in C, ità ut AB x BC = ACq. Centro A per B describe circulum ABD

in hoc [b] accommoda BD = AC, & junge AD. b 1. 4.
erit triang. ABD quod quæritur.

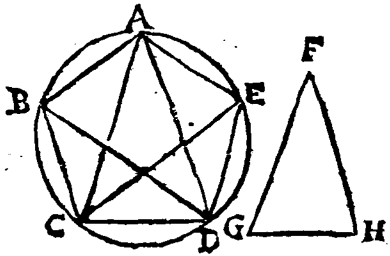
Nam duc DC; & per CDA [c] describe circulum. Quoniam AB × BC = ACq. [d] liquet BD tangere circulum ACD, quem secat CD. [e] ergò ang. BDC = A. ergò ang. BDC + CDA [f] = A + CDA = BCD. sed BDC + CDA = BDA [h] = CBD. [k] ergò ang. BCD = CBD. ergò DC [l] = DB [m] = AC, [n] quare ang CDA = A = BDC . ergò ADB = 2 A = ABD. Q. E. F.

c 5. 4. d 37. 3. e 32. 3. f 2. ax. g 32. 1. h 5. 1. k 1. ax. l 6. 1. m constr. n 5. 1.

Coroll.

Cùm omnes anguli A, B, D [o] conficiant $\frac{5}{5}$ 2 Rect. (2 Rect.) liquet A esse $\frac{1}{5}$ 2 Rect. o 32. 1.

PROP. XI. *Probl.* 11.



In dato circulo ABCDE *pentagonum æquilaterum & æquiangulum* ABCDE *inscribere.*

[a] Describe triangulum Isosceles FGH, habens utrumque angulorum ad basim duplum anguli ad verticem. [b] Huic æquiangulum CAD inscribe circulo. Angulos ad basim ACD, & ADC [c] biseca rectis DB, CE occurrentibus circumferentiæ in B, & E. connecte rectas CB, BA, AE, ED. Dico factum.

a 10. 4. b 2. 4. c 9. 1.

Nam

Nam ex constr. liquet quinque angulos CAD, CDB, BDA, DCE, ECA pares esse; quare [d] arcus [e] & subtensæ DC, CB, BA, AE, DB æquantur. Pentagonum igitur æquilaterum est. Est verò etiam æquiangulum, [f] quia ejus anguli BAE, AED &c. insistunt arcubus [g] æqualibus BCDE, ABCD, &c.

d 26. 3.
e 29. 3.
f 27. 3.
g 2. ax.

Hujus problematis praxis facilior tradetur ad 10, 13.

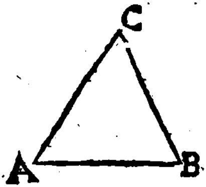
Coroll.

Hinc, angulus pentagoni æquilateri & æquianguli æquatur $\frac{2}{5}$ 2 Rect. vel $\frac{6}{5}$ Rect.

Schol.

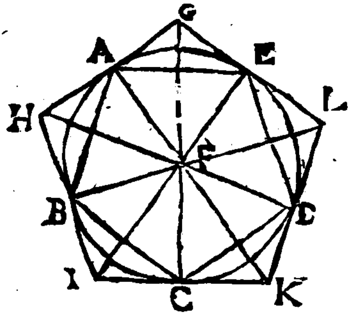
Petr. Herig.

Universaliter figuræ imparium laterum inscribuntur circulo beneficio triangulorum Isoscelium, quorum anguli æquales ad basim multiplices sunt eorum, qui ad verticem sunt angulorum; parium verò laterum figuræ in circulo inscribuntur ope Isoscelium triangulorum, quorum anguli ad basim multiplices sesquialteri sunt eorum, qui ad verticem sunt, angulorum.



Ut in triangulo Isoscele CAB, si ang A = 3 C = B; AB erit latus Heptagoni. Si A = 4 C; erit AB latus Enneagoni, &c. Sin verò A = $1\frac{1}{2}$C, erit AB latus quadrati. Et si A = $2\frac{1}{2}$ C subtendet AB sextam partem circumferentiæ; pariterque si A = $3\frac{1}{2}$ C; erit AB latus octagoni, &c.

PROP. XII. *Probl.* 12.



Circa datum circulum FABCDE *pentagonum æquilaterum & æquiangulum* HIKLG *describere.*

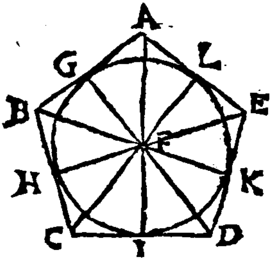
[a] Inscribe pentagonum ABCDE æquilaterum & æquiangulum; duc è centro rectas FA, FB, FC, FD, FE; iisque totidem perpendiculares GAH, HBI, ICK, KDL, LEG concurrentes in punctis H, I, K, L, G. Dico factum. Nam quia GA, GE ex uno puncto G [b] tangunt circulum, [c] erit GA = GE. [d] ergò ang. GFA = GFE. ergò ang. AFE = 2 GFA, eodem modo ang. AFH = HFB; & proinde ang. AFB = 2 AFH. Sed ang. AFE [e] = AFB. [f] ergò ang. GFA = AFH. sed & ang. FAH [g] = FAG; & latus FA est commune, [h] ergo HA = AG = GE = EL, &c. [k] ergò HG, GL, LK, KI, IH latera pentagoni æquantur: sed & anguli etiam, utpote [l] æqualium AGF, AHF, &c. dupli; ergò, &c.

a 11. 4.
b cor. 16. 3.
c 2. cor. 36. 3.
d 8. 1.
e 27. 3.
f 7. ax.
g 12. ax.
h 26. 1.
k 2. ax.
l 32. 1.

Coroll.

Eodem pacto, Si in circulo quæcunque figura æquilatera & æquiangula describatur, & ad extrema semidiametrorum ex centro ad angulos ductarum, excitentur lineæ perpendiculares, hæ perpendiculares constituent aliam figuram totidem laterum & angulorum æqualium circulo circumscriptam.

PROP. XIII. *Probl.* 13.



In dato pentagono æquilatero, & æquiangulo ABCDE *circulum* FGHK *inscribere.*

Duos pentagoni angulos A,
a 9. 1. & B [a] biseca rectis AF, BF cócurrentibus in F. Ex F duc perpendiculares FG, FH, FI, FK, FL. Circulus centro F per G descriptus tanget omnia pentagoni latera.

b *hyp.* Duc FC, FD, FE. Quoniam BA [b] = BC;
c *constr.* & latus BF commune est; & ang. FBA [c] =
d 4. 1. FBC, [d] erit AF = FC; & ang. FAB = FCB.
e *hyp.* Sed ang. FAB [e] = $\frac{1}{2}$ BAE [e] = $\frac{1}{2}$ BCD. ergò ang. FCB = $\frac{1}{2}$ BCD. eodem modo anguli totales C, D, E omnes bisecti sunt. Quum igitur
f 12. *ax.* ang. FGB [f] = FHB; & ang. FBH = FBG, &
g 26. 1. latus FB sit commune, [g] erit FG = FH. similiter omnes FH, FI, FK, FL, FG æquantur. ergò circulus centro F per G descriptus transit per
h *cor.* 16. 3. H, I, K, L; [h] tangítque pentagoni latera, cùm anguli ad ea puncta sint recti. Q. E. F.

Coroll.

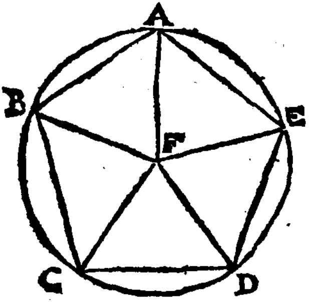
Hinc, si duo anguli proximi figuræ æquilateræ & æquiangulæ bisecentur, & à puncto, in quo coeunt lineæ angulos bisecantes, ducantur rectæ lineæ

lineæ ad reliquos figuræ angulos, omnes anguli figuræ erunt bisecti.

Schol.

Eâdem methodo in qualibet figura æquilateră & æquiangula circulus describetur.

PROP. XIV. *Prob.* 14.



Circa datum Pentagonum æquilaterum, & æquiangulum ABCDE *circulum* FABCD *describere.*

Duos pentagoni angulos biseca rectis AF, BF concurrentibus in F. Circulus centro F per A descriptus pentagono circumscribitur.

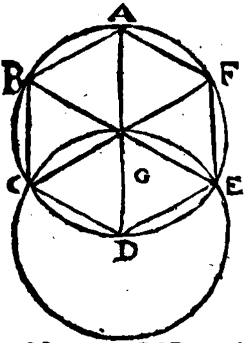
Ducantur enim FC, FD, FE. [a] Bisecti itáq; sunt anguli C, D, E. [b] ergò FA, FB, FC, FD FE æquantur. ergò circulus centro F descriptus, per A, B, C, D, E, pentagoni angulos transibit. Q. E. F.

a *cor.* 13. 4.
b 6. 1.

Schol.

Eâdem arte circa quamlibet figuram æquilateram, & æquiangulam circulus describetur.

PROP. XV. *Probl.* 15.



In dato circulo G-ABCDEF *hexagonum & æquilaterum, & æquiangulum* ABCDEF *inscribere.*

Duc diametrum AD, centro D per centrum G describe circulum, qui datum secet in C, & E. duc diametros CF, EB. junge AB, BC, CD, DE, EF, FA. Dico factum.

a 32. 1. Nam ang. CGD [a] = ⅓ 2 Rect [a] = DGE [b] =
b 15. 1. AGF [b] = AGB. [c] ergò BGC = ⅓ 2 Rect. = FGE.
c cor. 13. 1. [d] ergò arcus [e] & subtensæ AB, BC, CD, DE,
d 26. 3. EF æquantur. Hexagonum igitur æquilaterum
e 29. 3. est: sed & æquiangulum, [f] quia singuli ejus an-
f 27. 3. guli arcubus insistunt æqualibus. Q. E. F.

Coroll.

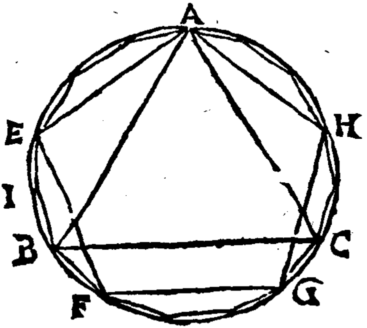
1. Hinc latus Hexagoni circulo inscripti semidiametro æquale est.
2. Hinc facilè triangulum æquilaterum ACE in circulo describetur.

Schol. Probl.

Andr. Tacq. *Hexagonum ordinatum super datâ rectâ* CD *ita*
a. 1. 1. *construes.* [a] Fac triangulum CGD æquilaterum super data CD. centro G per C, & D describe circulum. Is capiet Hexagonum super data CD.

PROP.

PROP. XVI. *Probl.* 16.



In dato circulo AEBC *quindecagonum æquilaterum & æquiangulum inscribere.*

Dato circulo [a] inscribe pentagonum æquilaterum AEFGH; [b] itémque triangulum æquilaterú ABC. erit BF latus quindecagoni quæsiti. a 11. 4. b 3. 4.

Nam arcus AB [c] est $\frac{1}{3}$, vel $\frac{5}{15}$ peripheriæ cujus AF est $\frac{2}{5}$ vel $\frac{6}{15}$, ergò reliquus BF $= \frac{1}{15}$ periph. ergò quindecagonum cujus latus BF, æquilaterum est; sed & æquiangulum, [d] cùm singuli ejus anguli arcubus insistant æqualibus, quorum unusquisque est $\frac{13}{15}$ totius circumferentiæ, ergò, &c. c *constr.* d 27. 3.

Schol.

Circulus dividitur Geometricè in partes	4, 8, 16 &c. per 6, 4, & 9, 1.
	3, 6, 12, &c. per 15, 4, & 9, 1.
	5, 10, 20, &c. prr 11, 4, & 9, 1.
	15, 30, 60, &c. per 16, 4 & 9, 1.

Cæterùm divisio circumferentiæ in partes datas etiamnum desideratur; quare pro figurarum quarumcunq; ordinatarum constructionibus sæpe ad mechanica artificia recurrendum est, propter quæ Geometræ practici consulendi sunt.

LIB. V.

Definitiones.

I. PArs est magnitudo magnitudinis, minor majoris, cùm minor metitur majorem.

II. Multiplex autem est major minoris, cùm minor metitur majorem.

III. Ratio est duarum magnitudinum ejusdem generis mutua quædam secundùm quantitatem habitudo.

In omni ratione ea quantitas, quæ ad aliam refertur, dicitur antecedens rationis; ea verò, ad quam alia refertur, consequens rationis dici solet. ut in ratione 6 ad 4; antecedens est 6, & consequens 4.

Nota.

Cujusque rationis quantitas innotescit dividendo antecedentem per consequentem. ut ratio 12 ad 5 effertur per $\frac{12}{5}$ *item quantitas rationis* A *ad* B *est* $\frac{A}{B}$. *Quare non rarò brevitatis causâ, quantitates rationum sic designamus,* $\frac{A}{B} \sqsubset$, *vel* $=$, *vel* $\sqsupset \frac{C}{D}$; *hoc est ratio* A *ad* B *major est ratione* C *ad* D, *vel ei æqualis, vel minor. Quod probè animadvertat, quisquis hæc legere volet.*

Rationis, sive proportionis species, ac divisiones vide apud interpretes.

IV. Proportio verò est rationum similitudo.

Rectiùs quæ hîc vertitur proportio, proportionalitas, sive analogia dicitur; nam proportio idem denotat quod ratio, ut plerisque placet.

V. Rationem habere inter se magnitudines dicuntur; quæ possunt multiplicatæ se mutuò superare.

E, 12.	A, 4. B. 6.	G,24.
F, 30.	C,10. D,15.	H,60.

VI. In eadē ratione magnitudines dicuntur esse, prima A ad secundum B; & tertia C ad quartum D; cùm primæ A, & tertiæ C æquemultiplicia E, & F à secundæ B, & quartæ D æquemultiplicibus G, & H, qualiscunq; sit hæc multiplicatio, utrumque E, F ab utroq; G, H vel unà deficiunt, vel unà æqualia sunt, vel unà excedunt, si ea sumantur E, G; & F, H quæ inter se respondent.

Hujus nota est :: . ut **A. B :: C. D.** *hoc est* **A** *ad* **B,** *&* **C** *ad* **D** *in eadem sunt ratione. aliquando sic scribimus* $\frac{A}{B} = \frac{\;}{D}$ *id est,* **A.B:: C.D.**

VII. Eandem autem habentes rationem (A.B:: C.D) proportionales vocentur.

E, 30.	A, 6. B, 4.	G,28.
F, 60.	C,12. D, 9.	H,6.3.

VIII. Cùm verò æquemultipliciũ, E multiplex primæ magnitudinis A excesserit G multiplicem secundæ B; at F multiplex tertiæ C non excesserit H multiplicem quartæ D; tunc prima A ad secundam B majorem rationem habere dicetur; quàm tertia C ad quartam D.

Si $\frac{A}{B} \sqsubset \frac{C}{D}$, *necessarium non est ex hac definitione, ut* E *semper excedat* G; *quum* F *minor est quàm* H; *sed conceditur hoc fieri posse.*

IX. Proportio autem in tribus terminis paucissimis consistit. *Quorum secunda est instar duorum.*

X. Cùm autem tres magnitudines A, B, C proportionales fuerint prima A ad tertiam C duplicatam rationem habere dicetur ejus, quam habet ad secundam B: at quum quatuor magnitudines A, B, C, D, proportionales fuerint, prima A ad quartam D triplicatam rationem habere dicetur

dicetur ejus, quam habet ad secundam B; & semper deinceps uno amplius, quamdiu proportio extiterit.

Duplicata ratio exprimitur sic $\frac{A}{C} = \frac{A}{B}$ *bis. Hoc est, ratio* A *ad* C *duplicata est rationis* A *ad* P. *triplicata autem sic* $\frac{A}{D} = \frac{A}{B}$ *ter.* id est, *ratio* A *ad* D *triplicata est rationis* A *ad* B.

$\div$ *denotat continuè proportionales.* ut A, B, C, D; *item* 2, 6, 18, 64 *sunt* $\div$

XI. Homologæ seu similes ratione magnitudines dicuntur, antecedentes quidem antecedentibus, consequentes verò consequentibus.

Ut si A. B :: C. D; *tam* A, *&* C; *quàm* B & D *homologæ magnitudines dicuntur.*

XII. Alterna ratio, est sumptio antecedentis ad antecedentem, & consequentis ad consequentem.

ut sit A. B :: C. D. *ergò alternè, vel permutando, vel vicissim* A. C :: B. D. *per* 16. 5.

In hac definitione, & 5. *sequentibus imponuntur nomina sex modis argumentandi, quibus mathematici frequenter utuntur; quarum illationum vis innititur propositionibus hujus libri, quæ in explicationibus citantur.*

XIII. Inversa ratio, est sumptio consequentis ceu antecedentis, ad antecedentem velut ad consequentem.

Vt A. B :: C. D. *ergò universè*, B. A :: D. C. *per* cor. 4, 5.

XIV. Compositio rationis, est sumptio antecedentis cum consequente, ceu unius, ad ipsam consequentem.

ut A. B :: C. D. *ergò componendo*, A+B. B :: C+D. D. *per* 18. 5.

XV. Divisio rationis, est sumptio excessûs, quo consequentem superat antecedens, ad ipsam consequentem. *ut*

Ut A. B :: C. D. *ergò dividendo*, A-B. B :: C-D. D. *per* 17. 5.

XVI. Conversio rationis, est sumptio antecedentis ad excessum, quo superat antecedens ipsam consequentem.

ut A. B :: C. D. *ergò per conversam rationem*, A. A-B :: C. C-D. *per* cor. 19. 5.

XVII. Ex æqualitate ratio est, si plures duabus sint magnitudines, & his aliæ multitudine pares, quæ binæ sumantur, & in eadem ratione; cùm ut in primis magnitudinibus prima ad ultimam, sic & in secundis magnitudinibus prima ad ultimam sese habuerit. Vel aliter: sumptio extremorum, per subductionem mediorum.

XVIII. Ordinata proportio est, cùm fuerit quemadmodum antecedens ad consequentem, ità antecedens ad consequentem: fuerit etiam ut consequens ad aliud quidpiam, ità consequens ad aliud quidpiam.

Ut si A. B :: D. E. *item* B. C :: E. F. *erit ex æquo* A. C :: D. F. *per* 22. 5.

XIX. Perturbata autem proportio est; cùm tribus positis magnitudinibus, & aliis, quæ sint his multitudine pares, ut in primis quidem magnitudinibus se habet antecedens ad consequentem, ità in secundis magnitudinibus antecedens ad consequentem: ut autem in primis magnitudinibus consequens ad aliud quidpiam, sic in secundis magnitudinibus aliud quidpiam ad antecedentem.

ut si A. B :: F. G. *item* B. C :: E. F. *erit ex æquo perturbatè* A. C :: E. G. *per* 23. 5.

XX. Quotlibet magnitudinibus ordine positis; proportio primæ ad ultimam componitur ex proportionibus primæ ad secundam, & secundæ ad tertiam, & tertiæ ad quartam, & ità deinceps, donec extiterit proportio.

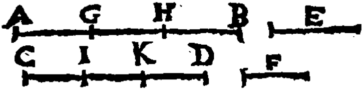
Sint

Sint quotcunque A, B, C, D; ex hac def. $\frac{A}{D}=\frac{A}{B}+\frac{B}{C}+\frac{C}{D}$.

Axioma.

Æquemultiplices eidem multiplici sunt quoque inter se æquemultiplices.

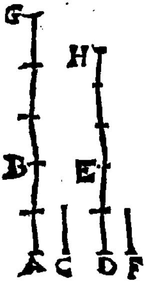
PROP. I.



Si sint quotcunque magnitudines AB, CD *quotcunque magnitudinum* E, F *æqualium numero, singulæ singularum, æquemultiplices; quàm multiplex est unius* E *una magnitudo* AB, *tam multiplices erunt & omnes* AB+CD *omnium* E+F.

Sint AG, GH, HB partes quantitatis AB ipsi E æquales. item CI, IK, KD partes quantitatis CD ipsi F pares. Harum numerus illarum numero æqualis ponitur. Quum igitur
a 2. ax. AG+CI=E+F; [a] & GH+IK=E+F; [a] & HB+KD=E+F, liquet AB+CD æquè multoties continere E+F, ac una AB unam E continet. Q. E. D.

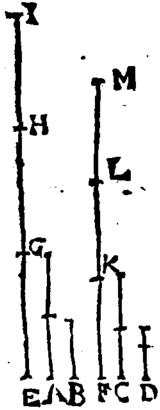
Prop. II.



Si prima AB *secundæ* C *æquè fuerit multiplex, atque tertia* DE *quartæ* F; *fuerit autem & quinta* BG *secundæ* C *æquè multiplex, atque sexta* EH *quartæ* F, *erit & composita prima cum quinta* (AG) *secundæ* C *æquè multiplex, atque tertia cum sexta* (DH) *quartæ* F.

Numerus partium in AB ipsi C æqualium æqualis ponitur numero partium in DE ipsi F æqualium. Item numerus partium in BG ponitur æqualis numero partium in EH. [a] ergò numerus partium in AB+BG æquatur numero partium in DE+EH. hoc est tota AG æquemultiplex est ipsius C, atque tota GH ipsius F. Q. E. D. *a 2. ax.*

Prop. III.



Sit prima A *secundæ* B *æquemultiplex, atque tertia* C *quartæ* D; *sumantur autem* EI FM *æquemultiplices primæ & tertiæ; erit & ex æquo, sumptarum utraque utriusque æquemultiplex: altera quidem* EI *secundæ* B, *altera autem* FM *quartæ* D.

Sint EG, GH, HI partes multiplicis EI ipsi A pares; item FK, KL, LM partes multiplicis FM ipsi C æquales. [a]Harum numerus illarum numero æquatur. porrò A, id est EG, vel GH, vel GI ipsius B ponitur æquemultiplex atque C, vel FK &c. ipsius *D*. *a hyp.*

[b] ergò

b 2.5. [b] ergò EG + GH æquemultiplex est secundæ
c 2.5. B, atque *FK* + KL quartæ D. [c] Simili argumento EI (EH + HI) tam multiplex est ipsius B, quàm *FM* (*FL* + LN) ipsius D. Q. E. D.

Prop. IIII.

Si prima A *ad secundam* B *eandem habuerit rationem, &* *tertia* C *ad quartam* D; *etiam* E *&* F *æquemultiplices primæ* A, *& tertiæ* C, *ad* G, *&* H *æquemultiplices secundæ* B, *& quartæ* D, *juxta quamvis multiplicationem, eandem habebunt rationem, si prout inter se respondent, ità sumptæ fuerint.* (E. G : : F. H.)

Sume I, & K ipsarum E, & F; item L & M ipsarum G, & H æquemultiplices. [a] Erit I
a 3.5. ipsius A æquemultiplex atque K ipsius C; [a] pariterque L tam multiplex ipsius B quàm M ipsius D. Itaque cùm sit A.
b hyp. B [b] : : C. D; juxta 6 def. si I $\sqsubset$, =, $\sqsupset$ L consequenter pari modo K $\sqsubset$, =, $\sqsupset$ M, ergò cùm I, & K ipsarum E, & F sumptæ sint æquemultiplices, atque L, & M ipsarum G & H; erit juxta 7. def. E.G : : F.H. Q.E.D.

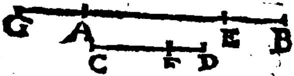
Coroll.

Hinc demonstrari solet inversa ratio.

Nam quoniam A. B : : C. D, si E $\sqsubset$, =, $\sqsupset$
c 9. 6.def.5. G [c], erit similiter F $\sqsubset$, =, $\sqsupset$ H. ergò liquet, quòd

quòd si G $\sqsubset$, $=$, $\sqsupset$ E, esse H $\sqsubset$, $=$, $\sqsupset$ F. [d] ergò B. A : : D. C. Q. E. D. d 6. def. 5.

Prop. V.



Si magnitudo AB *magnitudinis* CD *æquè fuerit multiplex, atq; ablata* AE *ablatæ* CF; *etiam reliqua* EB *reliquæ* FD *ità multiplex erit, ut tota* AB *totius* CD.

Accipe aliam quandam GA, quæ reliquæ FD ità sit multiplex, atque tota AB totius CD, vel ablata AE ablatæ CF. [a] ergò tota GA + AE totius CF; + FD æquemultiplex est, ac una AE unius CF. hoc est, ac AB ipsius CD. [b] ergò GE = AB. [c] proinde, ablatâ communi AE, manet GA = EB. ergò, &c.

a 1. 5.
b 6. ax.
c 3. ax.

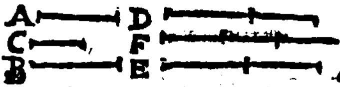
Prop. VI.

Si duæ magnitudines AB CD *duarum magnitudinum* E, F *sint æquemultiplices; & detractæ quædam sint,* AG *&* CH, *earundem* E, *&* F *æquemultiplices, & reliquæ* GB, HD *eisdem* E, F *aut æquales sunt, aut æquè ipsarum multiplices.*

Nam quia numerus partium in AB ipsi E æqualium ponitur æqualis numero partium in CD ipsi F æqualium. Item numerus partium in AG æqualis numero partium in CH. Si hinc AG, indè CH detrahatur, [a] remanet numerus partium in reliqua GB æqualis numero partium in HD. ergò si GB sit E semel, erit HD etiam C semel. Si GB sit E aliquoties, erit HD etiam C aliquoties accepta. Q. E. D.

a 3. ax.

PROP. VII.



Æquales A & B *ad eandem* [c] C *eandem habent rationem, & eadem* C *ad æquales* A & B.

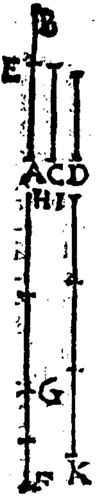
Sumantur D, & E æqualium A, & B æquemultiplices; & F utcunque multiplex ipsius C, [a] erit D $=$ E. quare si D $\sqsubset, =, \sqsupset$ F, erit similiter E $\sqsubset, =, \sqsupset$ F. [b] ergò A. C :: B. C. inversè igitur C. A [c] :: C. B. Q. E. D.

a 6. ax.
b 6. def. 5.
c cor. 4 5.

Schol.

Si loco multiplicis F sumantur duæ æquemultiplices, eodem modo ostendetur æquales magnitudines ad alias inter se æquales eandem habere rationem.

PROP. VIII.



Inæqualium magnitudinum AB, C, *major* AB *ad eandem* D *majorem rationem habet, quàm minor* C. *Et eadem* D *ad minorem* C *majorem rationem habet, quàm ad majorem* AB.

Ex majori AB aufer AE $=$ C. sumatur HG tam multiplex ipsius AE, vel C, quàm GF reliquæ EB. Multiplicetur D, donec ejus multiplex IK major evadat quàm HG, sed minor quàm HF.

Quoniam HG ipsius AE [a] tam multiplex est, quàm GF ipsius EB, [b] erit tota HF totius AB æquemultiplex, atque una HG unius AE, vel C. ergò cùm HF $\sqsubset$ IK (quæ multiplex est ipsius D) sed HG $\sqsupset$ IK, [c] erit $\frac{AB}{D} \sqsubset \frac{C}{D}$ Q. E. D.

a conft.
b 1. 5.
c 8. def. 5.

Rursus

Rursus quia IK $\sqsubset$ HG, at IK $\sqsupset$ HF (ut priùs dictum) [d] erit $\frac{D}{C} \sqsubset \frac{D}{AB}$ Q.E.D.

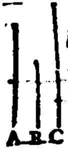
Prop. IX.

Quæ ad eandem eandem habent rationem, æquales sunt inter se. Et ad quas eadem eandem habet rationem, eæ quoque sunt inter se æquales.

1. *Hyp.* Sit A. C : : B. C. dico A = B. Nam sit A $\sqsubset$, vel $\sqsupset$ B, [a] erit ideò $\frac{A}{C} \sqsubset$, vel $\sqsupset \frac{B}{C}$ contra Hyp. a 8. 5.

2. *Hyp.* Sit C. B : : C. A. dico A = B. nam sit A $\sqsubset$ B. [b] ergò $\frac{C}{B} \sqsubset \frac{C}{A}$ contra Hyp. b 8. 5.

Prop. X.

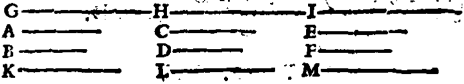


Ad eandem magnitudinem rationem habentium, quæ majorem rationem habet, illa major est: ad quam verò eadem majorem rationem habet, illa minor est.

1. *Hyp.* Sit $\frac{A}{C} \sqsubset \frac{B}{C}$ Dico A $\sqsubset$ B. Nam si dicatur A = B, [a] erit A. C : : B. C. contra Hyp. Sin A $\sqsupset$ B, [b] erit $\frac{A}{C} \sqsupset \frac{B}{C}$ etiam contra (Hyp. a 7. 5. b 8. 5.

2. *Hyp.* Sit $\frac{C}{B} \sqsubset \frac{C}{A}$ Dico B $\sqsupset$ A. Nam dic B = A. [c] ergò C. B : : C. A. contra Hyp. vel dic B $\sqsubset$ A. [d] ergò $\frac{C}{A} \sqsubset \frac{C}{B}$ etiam contra Hyp. c 7. 5. d 8. 5.

PROP. XI.



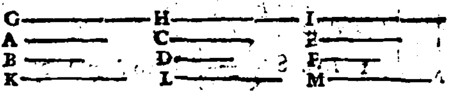
Quæ eidem sunt eadem rationes, & inter se sunt eadem.

Sit A.B :: E.F. item C.D :: E.F. dico A.B :: C.D. sume ipsarum A, C, E æquemultiplices, G, H, I; atque ipsarum B, D, F
a hyp. æquemultiplices K, L, M. Et quoniam
b 6. def. 5. a A.B :: E.F si G $\sqsubset$, =, $\sqsupset$, K. b erit pari modo I $\sqsubset$, =, $\sqsupset$ M, pariterque quia a E.F :: C.D. Si I $\sqsubset$, =, $\sqsupset$ M, b erit H similiter $\sqsubset$, =, $\sqsupset$ L.
c 6. def. 5. ergò si G $\sqsubset$, =, $\sqsupset$ K, erit similiter H $\sqsubset$, =, $\sqsupset$, L. c quare A.B :: C.D. Q.E.D.

Schol.

Quæ eisdem rationibus sunt eædem rationes, sunt quoque inter se eædem.

PROP. XII.



Si sint magnitudines quotcunque A, & B; C & D; E, & F *proportionales; quemadmodum se habuerit una antecedentium* A *ad unam consequentium* B, *ità se habebunt omnes antecedentes*, A, C, E *ad omnes consequentes*. B, D, F.

Sume antecedentium æquemultiplices G, H, I; & consequentium K, L, M. Quoniam quàm
a 1. 5. multiplex est una G unius A, a tam multiplices sunt omnes G, H, I omnium A, C, E; pariterque quàm multiplex est una K unius B, a tam multiplices sunt omnes K, L, M omnium B, D, F; Si G $\sqsubset$, =, $\sqsupset$ K, erit similiter

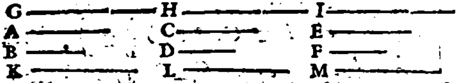
G+

$G+H+I \sqsubset, =, \sqsupset K+L+M$. [b] quare $A.B :: A+C+E.\ B+D+F$. Q.E.D. b 6. de

Coroll.

Hinc, si similia proportionalia similibus proportionalibus addantur, tota erunt proportionalia.

PROP. XIII.



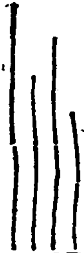
Si prima A ad secundam B eandem habuerit rationem, quam tertia C ad quartam D; tertia verò C ad quartam D majorem habuerit rationem, quàm quinta E ad sextam F; prima quoque A ad secundam B majorem rationem habebit, quàm quinta E ad sextam F.

Sume ipsarum A, C, E æquemultiplices G, H, I: ipsarumque B, D, F æquemultiplices K, L, M. Quia $A.B :: C.D$; Si $H \sqsubset L$, [a] erit $G \sqsubset K$. Sed quia $\frac{C}{D} \sqsubset \frac{E}{F}$, [b] fieri potest ut sit $H \sqsubset L$, & I non $\sqsubset M$. ergò fieri potest ut $G \sqsubset K$, & I non $\sqsubset M$. [c] ergò $\frac{A}{B} \sqsubset \frac{E}{F}$. Q.E.D. a 6. def. 5. b 8. def. 5. c 8. def. 5.

SCHOL.

Quòd si $\frac{C}{D} \sqsupset \frac{E}{F}$, erit quoq; $\frac{A}{B} \sqsupset \frac{E}{F}$. Item si $\frac{A}{B} \sqsubset \frac{C}{D} \sqsubset \frac{E}{F}$. erit $\frac{A}{B} \sqsubset \frac{E}{F}$. & si $\frac{A}{B} \sqsupset \frac{C}{D} \sqsupset \frac{E}{F}$ erit $\frac{A}{B} \sqsupset \frac{E}{F}$.

Prop. XIV.



Si prima A ad secundam B eandem habuerit rationem, quàm tertia C ad quartam D; prima verò A, quàm tertia C major fuerit, erit & secunda B major quàm quarta D. Quòd si prima A fuerit æqualis tertiæ C, erit & secunda B æqualis quartæ D; si verò A minor, & B minor erit.

a 8. 5. b *hyp.* Sit $A \sqsubset C$. [a] ergò $\frac{A}{B} \sqsubset \frac{C}{B}$. [b] sed

c 13. 5. A BCD $\frac{A}{B} = \frac{C}{D}$. [c] ergò $\frac{C}{D} \sqsubset \frac{C}{B}$. [d] ergò $B \sqsubset D$.

d 10. 5. Simili argumento si $A \sqsupset C$, [d] erit $B \sqsupset D$. Sin

e 7. 5. ponatur $A = C$; ergò C. B [e] :: A. B [f] :: C. D.

f *hyp.* [g] ergò $B = D$. Quæ E. D.

g 4. 5. & 9. 5.

SCHOL.

A fortiori, si $\frac{A}{B} \sqsupset \frac{C}{D}$, atque $A \sqsubset C$, erit $B \sqsubset D$. Item si $A = B$, erit $C = D$. Et si $A \sqsubset$, vel $\sqsupset B$, erit pariter $C \sqsubset$, vel $\sqsupset D$.

Prop. XV.

Partes C & F cum pariter multiplicibus AB, & DE in eadem sunt ratione; si prout sibi mutuò respondent, ità sumantur. (AB. DE :: C. F.).

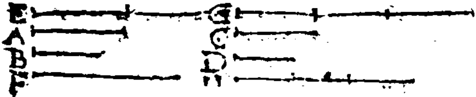
Sint AG, GB partes multiplicis AB ipsi C æquales: item DH, HE partes multiplicis DE ipsi F æquales. [a] Harum numerus illarum numero æquatur. ergò quum [b] AG. C :: DH. F; [b] atq; GB. C :: HE. F. [c] erit AG+GB (AB). DH+HE (DE) :: C. F. Q. E. D.

a *hyp.*
b 7. 5.
c 12. 5.

A C D F

Prop.

Prop. XVI.



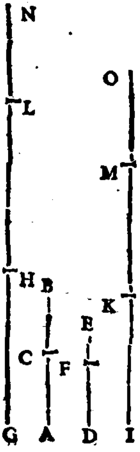
Si quatuor magnitudines A, B, C, D *proportionales fuerint; & vicissim proportionales erunt.* (A. C :: B. D.)

Accipe E, & F æquemultiplices ipsarum A, & B. ipsarúmque C, & D. æquemultiplices G, & H. Itaque E. F[a] :: A. B.[b] :: C.D[a] :: G.H. a 5. 5.
Quare si E $\sqsubset, =, \sqsupset$ G,[c] erit similiter F $\sqsubset, =,$ b hyp.
$\sqsupset$ H. [d] ergò A. C :: B. D. Q. E. D. c 11. 5. & 14. 5.

SCHOL. d 6. def. 5.

Alterna ratio locum tantùm habet, quando quantitates ejusdem sunt generis. Nam Heterogeneæ quantitates non comparantur.

Prop. XVII.

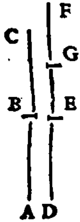


Si compositæ magnitudines proportionales fuerint (AB. CB :: DE. FE) *hæ quoque divisæ proportionales erunt.* (AC. CB :: DF. FE.)

Accipe GH, HL, IK, KM ordine æquemultiplices ipsarum AC, CB, DF, FE, item LN, MO æquemultiplices ipsarum CB, FE. Tota GL totius AB, [a] tam multiplex est, quàm una a 1. 5.
GH unius AC,[b] id est quàm b constr.
IK ipsius DF; [c] hoc est quàm c 1. 5.
tota IM. totius DE; Item HN (HL+LN) ipsius CB [d] æque- d 2. 5.
multiplex est, ac KO (KM+MO) ipsius FE. Cùm igitur per hyp. AB, BC :: DE. EF. Si GL $\sqsubset, =, \sqsupset$ HN, etiam similiter

e 6. *def.* 5. militer [e] erit IM $\sqsubset$, $=$, $\sqsupset$ KQ. aufer hinc inde æquales HL, KM, si reliqua GH $\sqsubset$, $=$, $\sqsupset$
f 5. *ax.* LN, [f] erit similiter IK $\sqsubset$, $=$, $\sqsupset$ MO, [g] unde
g 6 *def.* 5. de AC. CB :: DF. FE. Q. E D.

Prop. XVIII.

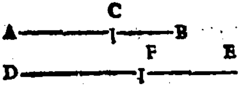


Si divisæ magnitudines sint proportionales (AB. BC :: DE. EF), *hæ quoq; compositæ proportionales erunt* (AC. CB :: DF. FE.).

Nam si fieri potest, sit AB. CB :: DF. FG $\sqsupset$ FE. [a] ergò erit divisim AB. BC :: DG. GF. [b] hoc est DG. GF :: DE. EF. ergò cùm DG $\sqsubset$ DE. [c] erit GF $\sqsubset$ EE. Q. E. A. Simile absurdum sequetur, [d] si dicatur AB. CB :: DE. GF $\sqsubset$ FE.

a 17. 5.
b *hyp.* & 11. 5.
c 14. 5.
d 9. *ax.*

Prop. XIX.



Si quemadmodum totum AB *ad totum* DE *ità ablatum* AC *se habuerit ad ablatum* DF; *& reliquum* CB *ad reliquum* FE, *ut totum* AB *ad totum* DE, *se habebit.*

a *hyp.* Quoniam [a] AB. DE :: AC. DF, [b] erit per-
b 16. 5. mutando AB. AC :: DE. DF. [c] ergò divisim
c 17. 5. AC. CB :: DF. FE. quare rursus [b] permutan-
d *hyp.* & 11. 5. do AC. DF :: CB. FE; [d] Hoc est AB DE :: CB FE. Q. E. D,

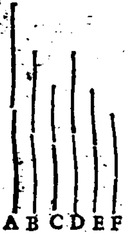
Coroll.

1. Hinc, si similia proportionalia similibus proportionalibus subducantur, residua erunt proportionalia.

2. *Hinc, demonstrabitur conversa ratio.*

Sit AB. CB :: DE. FE. Dico AB. AC ::
a 16. 5. DE. DF. Nam [a] permutando AB DE :: CB.
b 19. 5. FE. [b] ergò AB. DE :: AC. DF. quare iterum permutando, AB. AC :: DE. DF. Q. E. D.

Prop. XX.



Si sint tres magnitudines A,B,C; *& aliæ* D,E,F *ipsis æquales numero, quæ binæ & in eadem ratione sumantur* (A.B :: D.E; *atque* B.C :: E.F); *ex æquo autem prima* A *major fuerit, quàm tertia* C; *erit & quarta* D *major quàm sexta* F. *Quòd si prima* A *tertiæ* C *fuerit æqualis; erit & quarta* D *æqualis sextæ* F. *Sin illa minor, hæc quoque minor erit.*

1. *Hyp.* Si A $\sqsubset$ C. Quoniam [a] E.F :: B.C. [b] erit inversè F.E :: C.B. [c] Sed $\frac{C}{B} \sqsupset \frac{A}{B}$ [d] ergò $\frac{F}{E} \sqsupset \frac{A}{B}$ vel $\frac{D}{E}$. [e] ergò D $\sqsubset$ F. Q.E.D.

2. *Hyp.* Simili argumento, Si A $\sqsupset$ C, ostendetur D $\sqsupset$ F.

3. *Hyp.* Si A = C. Quoniam F.E :: C.B :: [f] A.B :: D.E. [g] erit D = F. Q.E.D.

a *hyp.*
b *cor* 4.5.
c *hyp.* & 8.5.
d *schol* 1.5
e 10.5
f 7.5.
g 11.5. & 9.5.

Prop. XXI.

Si sint tres magnitudines A,B,C; *& aliæ* D,E,F *ipsis æquales numero, quæ binæ & in eadem ratione sumantur, fueritque perturbata eorum proportio,* (A.B :: E.F. *atque* B.C :: D.E.); *ex æquo autem prima* A *quàm tertia* C *major fuerit; erit & quarta* D *quàm sexta* F *major; Quòd si prima fuerit tertiæ æqualis, erit & quarta æqualis sextæ, sin illa minor, hæc quoque minor erit.*

1. *Hyp.* A $\sqsubset$ C. Quoniam [a] D.E :: B.C, invertendo erit E.D :: C.B. atqui $\frac{C}{B}$ [b] $\sqsupset \frac{A}{B}$.

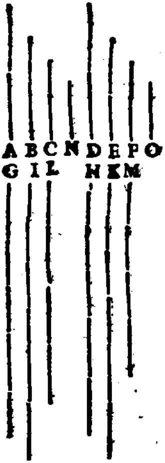
a *hyp.*
b 8.5.

c schol. 13. 5. [c] ergò $\frac{E}{D} \sqsupset \frac{A}{B}$, hoc est $\frac{E}{F}$. [d] ergò D $\sqsubset$ F.
d 10. 5. Q. E. D.

2. *Hyp.* Similiter, Si A $\sqsupset$ C. erit D $\sqsupset$ F.

e 7. 5. 3. *Hyp.* Si A = C. Quoniam E.D [e] :: C.B ::
f hyp. [e] A.B :: [f] E.F. [g] erit D = F. Q. E. D.
g 9. 5.

Prop. XXII.



Si sint quotcunque magnitudines A, B, C; *& aliæ ipsis æquales numero* D, E, F, *quæ binæ & in eadem ratione sumantur* (A.B :: D.E: & B.C :: E.F); *& ex æqualitate in eadem ratione erunt* (A.C :: D.F.).

Accipe G, H ipsarum A, D; & I, K ipsarum B, E; item L, M ipsarum E, F æquemultiplices.

a hyp. Quoniam [a] A. B :: D. E.
b 4. 5. [b] erit G. I :: H. K. eodem modo, erit, I. L :: K. M. ergò si G $\sqsubset$, =, $\sqsupset$ L, [c] erit
c 20. 5.
d 6. def. 5. H. $\sqsubset$, =, $\sqsupset$ M; [d] ergò A. C :: D. F. Eodem pacto si ulteriùs C. N :: F. O; erit ex æquali A. N :: D. O. Q. E. D.

Prop.

PROP. XXIII.

Si sint tres magnitudines A, B, C; *aliaque* D, E, F *ipsis æquales numero, quæ binæ in eadem ratione sumantur, fuerit autem perturbata earum proportio.* (A.B :: E.F. & B.C :: D.E.) *etiam ex æqualitate in eadem ratione erunt.*

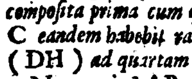
Sume G, H, I ipsarum A, B, D; item K, L, M ipsarum C, E, F æquemultiplices. erit G.H [a] :: A.B [b] :: E.F [a] :: L.M. porrò quia [b] B.C :: D.E. erit [c] H.K :: I.L. ergò G, H, K; & I, L, M habent se juxta 21. 5. quare si G ⊏, =, ⊐ K, erit similiter I ⊏, =, ⊐ M. [d] proinde A.C :: D.F. Q.E.D. — a 15. 5. b. hyp. c 4 5.

Eodem modo si plures fuerint magnitudinibus tribus, &c. — d 6. def 5.

Coroll.

Ex [*] his sequitur, rationes ex iisdem rationibus compositas esse inter se easdem. item, earundem rationum easdem partes inter se easdem esse. — * 22. & 23 5. & 20 def 5.

PROP. XXIIII.



Si prima AB *ad secundam* C *eandem habuerit rationem quàm tertia* DE *ad quartam* F; *habuerit autem & quinta* BG *ad secundam* C *eandem rationem, quàm sexta* EH *ad quartam* F; *etiam composita prima cum quinta* (AG) *ad secundam* C *eandem habebit rationem, quàm tertia cum sexta* (DH) *ad quartam* F.

Nam quia [a] AB. C :: DE. F. atque ex hyp. & inversè C. BG :: F. EH, erit [b] ex æquali AB. BG :: DE. EH. ergò componendo AG. BG :: DH. EH. [c] item BG. C :: EH. F. [b] ergò rursus ex æquo, AG. C :: DH. F. Q.E.D. — a hyp. b 22. 5. c hyp.

PROP.

PROP. XXV.

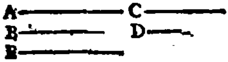
Si quatuor magnitudinis proportionales fuerint (AB. CD :: E. F.) *maxima* AB, *&* *minima* F *reliquis* CD, & E *majores erunt*.

Fiant AG$=$E; & CH$=$F. Quoniam AB. CD[a] :: E. F[b] :: AG. CH. [c]erit AB. CD :: GB. HD. [d]sed AB $\sqsubset$ CD. [e]ergò GB$\sqsubset$HD. atqui AG$+$F$=$E$+$CH. ergò AG$+$F$+$GB$\sqsubset$E$+$CH$+$HD. hoc est AB$+$F$\sqsubset$E$+$CD. Q. E. D.

a hyp.
b 7. 5.
c 19. 5.
d hyp.
e schol. 14. 5.

Quæ sequuntur propositiones non sunt Euclidis; sed ex aliis desumptæ ob frequentem earum usum Euclidæis subjungi solent.

PROP. XXVI.



Si prima ad secundam habuerit majorem proportionem, quàm tertia ad quartam, habebit convertendo, secunda ad primam minorem proportionem, quàm quarta ad tertiam.

Sit $\frac{A}{B} \sqsubset \frac{C}{D}$. Dico $\frac{B}{A} \sqsupset \frac{D}{C}$. Nam concipe $\frac{C}{D} = \frac{E}{B}$. [a]ergò $\frac{A}{B} \sqsubset \frac{E}{B}$. [b]quare A $\sqsubset$. E. [c]ergò $\frac{B}{A} \sqsupset \frac{B}{E}$, [d]vel $\frac{D}{C}$. Q. E. D.

a 13. 5.
b 10. 5.
c 8. 5.
d cor. 4. 5.

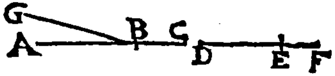
PROP. XXVII.

Si prima ad secundam habuerit majorem proportionem, quàm tertia ad quartam, habebit quoque vicissim prima ad tertiam majorem proportionem, quàm secunda ad quartam.

Sit $\frac{A}{B} \sqsubset \frac{C}{D}$. Dico $\frac{A}{C} \sqsubset \frac{B}{D}$. Nam puta $\frac{E}{B} = \frac{C}{D}$.

[a] ergò $A \sqsubset E$. [b] ergò $\frac{A}{C} \sqsubset \frac{E}{C}$ [c] vel $\frac{B}{D}$. Q.E.D. a 10. 5. b 8. 5. c 16. 5.

Prop. XXVIII.



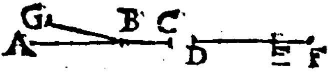
Si prima ad secundam habuerit majorem proportionem, quàm tertia ad quartam; habebit quoque composita prima cum secunda ad secundam majorem proportionem, quàm composita tertia cum quarta ad quartam,

Sit $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. Dico $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Nam cogita $\frac{GB}{BC} = \frac{DE}{EF}$. [a] ergò $AB \sqsubset GB$, adde utrinque BC, a 10. 5.

[b] erit $AC \sqsubset GC$. [c] ergò $\frac{AC}{BC} \sqsubset \frac{GC}{BC}$. [d] hoc est $\frac{DF}{EF}$. b 4. ax. c 8. 5. d 18. 5.

Q. E. D.

Prop. XXIX.



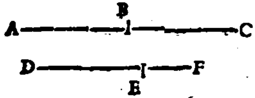
Si composita prima cum secunda ad secundam majorem habuerit proportionem, quàm composita tertia cum quarta ad quartam, habebit quoque dividendo prima ad secundam majorem proportionem quàm tertia ad quartam.

Sit $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Dico $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. Intellige $\frac{GC}{BC} = \frac{DF}{EF}$. [a] ergò $AC \sqsubset GC$. aufer commune a 10. 5.

BC, [b] erit $AB \sqsubset GB$. [c] ergò $\frac{AB}{BC} \sqsubset \frac{GB}{BC}$ [d] vel $\frac{DE}{EF}$. b 5. ax. c 8. 5. d 17. 5.

Q. E. D.

Prop. XXX.



Si composita prima cum secunda ad secundam habuerit majorem proportionem, quàm composita tertia cum quarta ad quartam; Habebit, per conversionem rationis, prima cum secunda ad primam minorem rationem, quàm tertia cum quarta ad tertiam.

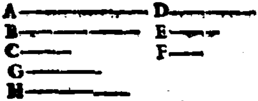
Sit $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Dico $\frac{AC}{AB} \sqsupset \frac{DF}{DE}$. Nam quia

a hyp. $\frac{AC}{BC}$ [a] $\sqsubset \frac{DF}{EF}$. [b] erit dividendo $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. [c] conver-

b 29. 5.
c 26. 5.
d 28. 5.

tendo igitur $\frac{BC}{AB} \sqsupset \frac{EF}{DE}$. [d] ergò componendo $\frac{AC}{AB} \sqsupset \frac{DF}{DE}$. Q. E. D.

Prop. XXXI.



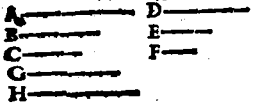
Si sint tres magnitudines A, B, C, & aliæ ipsis æquales numero D, E, F, sitque major proportio primæ priorum ad secundam, quàm primæ posteriorum ad secundam ($\frac{A}{B} \sqsubset \frac{D}{E}$); *item secundæ priorum ad tertiam major, quàm secundæ posteriorum ad tertiam* ($\frac{B}{C} \sqsubset \frac{E}{F}$) *Erit quoque ex æqualitate major proportio primæ priorum ad tertiam, quàm primæ posteriorum ad tertiam* ($\frac{A}{C} \sqsubset \frac{D}{F}$)

a 10. 5.
b 8. 5.
c 13. 5.
d 10. 5.
e 8. 5.
f 22. 5.

Concipe $\frac{G}{C} = \frac{E}{F}$. [a] ergò P $\sqsubset$ G. [b] ergò $\frac{A}{G} \sqsubset \frac{A}{B}$. Rursus puta $\frac{H}{G} = \frac{D}{E}$. [c] ergò $\frac{H}{G} \sqsupset \frac{A}{B}$; [d] ergò fortiùs $\frac{H}{G} \sqsupset \frac{A}{G}$. [d] quare A $\sqsubset$ H. [e] proinde $\frac{A}{C} \sqsupset \frac{H}{G}$, vel $\frac{D}{F}$. Q. E. D.

Prop.

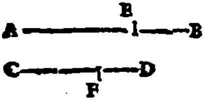
Prop. XXXII.



Si sint tres magnitudines A,B,C; & aliæ ipsis æquales D,E,F, sitque major proportio primæ priorum ad secundam, quàm secundæ posteriorum ad tertiam; ($\frac{A}{B} \sqsubset \frac{E}{F}$) item secundæ priorum ad tertiam major; quàm primæ posteriorum ad secundam; ($\frac{B}{C} \sqsubset \frac{D}{E}$) erit quoque ex æqualitate major proportio primæ priorum ad tertiam, quàm primæ posteriorum ad tertiam. ($\frac{A}{C} \sqsubset \frac{D}{E}$)

Hujusce demonstratio planè similis est demonstrationi præcedentis.

Prop. XXXIII.

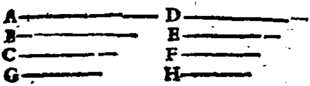


Si fuerit major proportio totius AB ad totum CD, quàm ablati AE ad ablatum CF. Erit & reliqui EB ad reliquum FD major proportio, quàm totius AB ad totum CD. a hyp.

Quoniam $\frac{AB}{CD}$ [a] $\sqsubset \frac{AE}{CF}$, [b] erit permutando $\frac{AB}{AE} \sqsubset \frac{CD}{CF}$. [c] ergò per conversionem rationis $\frac{AB}{EB} \sqsupset \frac{CD}{FD}$. permutando igitur $\frac{AB}{CD} \sqsupset \frac{EB}{FD}$. Q. E. D.

b 27. 5.
c 30. 5.

PROP. XXXIV.

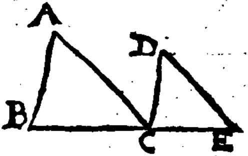


Si sint quocunque magnitudines, & aliæ ipsis æquales numero, sitque major proportio primæ priorum ad primam posteriorum, quàm secunda ad secundam; & hæc major quàm tertiæ ad tertiam, & sic deinceps: habebunt omnes priores simul ad omnes posteriores simul, majorem proportionem, quàm omnes priores, relictâ primâ, ad omnes posteriores, relictâ quoque primâ; minorem autem, quàm prima priorum ad primam posteriorum; majorem deniq; etiam, quàm ultima priorum ad ultimam posteriorum.

Horum demonstratio est penes interpretes. quos adeat, qui eam desiderat. nos omisimus, brevitatis studio; & quia illorum nullus usus in his elementis.

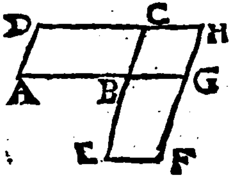
LIB. VI.

Definitiones.



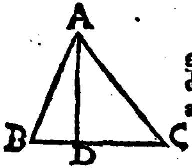
I. Similes figuræ rectilineæ sunt (ABC, DCE), quæ & angulos singulos singulis æquales habent; atque etiam latera, quæ circum angulos æquales, proportionalia.

Ang. B = DCE *&* AB. BC : : DC. CE. *item ang.* A = D; *atque* BA. AC : : CD. DE. *denique ang.* ACB = E. *atque* BC. CA : : CE. ED.



II. Reciprocæ autem sunt (BD, BF), cùm in utraque figura antecedentes, & consequentes rationum termini fuerint. (hoc est AB. BG : : EB. BC.)

III. Secundùm extremam & mediam rationem recta linea AB secta, esse dicitur, cùm ut tota AB ad majus segmentum AC, ità majus segmentum AC ad minus CB se habuerit. (AB. AC : : AC. CB.)

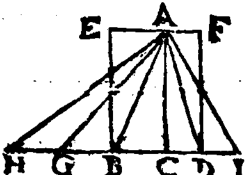


IV. Altitudo cujusq; figuræ ABC est linea perpendicularis AD, à vertice A ad basim BC deducta.

V. Ratio ex rationibus componi dicitur, cùm rationum quantitates inter se multiplicatæ, aliquam effecerint rationem.

a 20. def. 5. b 15. 5. *Ut ratio* A *ad* C, *componitur ex rationibus* A *ad* B, *&* B *ad* C; *nam* $\frac{A}{B} \times \frac{B}{C}$ [a] $= \frac{A}{C}$ [b] $= \frac{AB}{BC}$.

Prop. I.



Triangula ABC, ACD, *&* *parallelogramma* BCAE, CDFA, *quorum eadem fuerit altitudo, ità se habent inter se, ut bases* BC, CD.

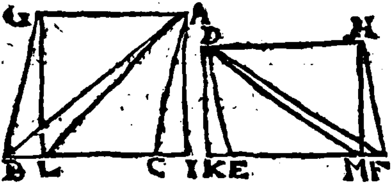
a 3. 1. [a] Accipe quotvis BG, HG ipsi BC æquales; idem DI = CD. & connecte AG, AH, AI.

b 38. 1. [b] Triangula ACB, ABG, AGH æquantur; [b] item triang. ACD = ADI. ergò triangulum ACH tam multiplex est trianguli ACB, quàm basis HC, basis BC. & æquemultiplex est triang. ACI trianguli ACD, ac basis CI basis CD.

c sch. 38. 1. d 6. def. 5. e 41. 1. & 15. 5. cùm igitur si HC ⊏, =, ⊐ CI, [c] erit similiter triang. AHC ⊏, =, ⊐ ACI, [d] ideóque BC. CD : : triang. ABC. ACD : : [e] pgr. CE. CF. Q. E. D.

Schol.

Scholi.

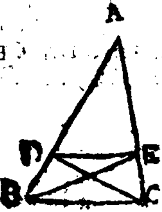


Hinc, *triangula* ABC, DEF, *& parallelogramma* AGBC, DEFH, *quorum æquales sunt bases* BC, EF, *ità se habent ut altitudines* AI, DK.

[a] Sume IL = CB; & KM = EF; ac junge LA, LG, MD, MH. liquet esse triang. ABC. DEF :: [b] ALI. DKM :: [c] AI. DK :: [d] pgr. AGBC. DEFH. Q. E. D.

a 3. 1.
b 7. 5.
c 1. 6.
d 41. 1. & 15. 5.

Prop. II.



Si ad unum trianguli ABC, *latus* BC *parallela ducta fuerit recta quædam linea* DE, *hæc proportionaliter secabit ipsius trianguli latera* (AD. BD :: AE. EC). *Et si trianguli latera proportionaliter secta fuerint* (AD. BD :: AE. EC) *quæ ad sectiones* D, E *adjuncta fuerit recta linea* DE, *erit ad reliquum ipsius trianguli latus* BC *parallela.* Ducantur CD, BE.

1. *Hyp.* Quia triang. DEB [a] = DEC; [b] erit triang. ADE. DBE :: ADE. ECD. atqui triang. ADE. DBE [c] :: AD. DB. & triang. ADE. DEC [c] :: AE. EC. [d] ergò AD. DB :: AE. EC.

a 37. 1.
b 7. 5.
c 1. 6.
d 11. 5.

2. *Hyp.* Quia AD. DB :: AE. EC. [e] hoc est triang. ADE. DBE :: ADE. ECD; [f] erit triang. DBE = ECD. [g] ergò DE, BC sunt parallelæ. Q. E. D.

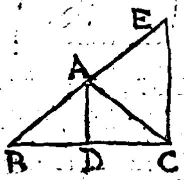
e 1. 6.
f 9. 5.
g 39. 1.

Schol.

Schol.

Imò, si plures ad unum trianguli latus parallelæ ductæ fuerint, erunt omnia laterum segmenta proportionalia, ut facilè deducitur ex hac.

Prop. III.

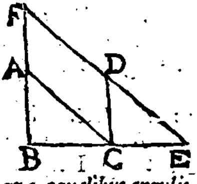


Si trianguli BAC *angulus* BAC *bifariam sectus sit, secans autem angulum recta linea* AD *secuerit & basim, basis segmenta eandem habebunt rationem, quam reliqua ipsius trianguli latera* (BD, DC :: AB. AC.) *Et si basis segmenta eandem habeant rationem quam reliqua ipsius trianguli latera* (BD. DC :: AB. AC.) *recta linea* AD *quæ à vertice* A *ad sectionem* D *ducitur, bifariam secat trianguli ipsius angulum* BAC.

Produc BA; & fac AE $=$ AC. & junge CE.

1. *Hyp.* Quoniam AB $=$ AC, erit ang. ACE
a 5. 1. — [a] $=$ E [b] $= \frac{1}{2}$ BAC [c] $=$ DAC. [d] ergò DA, CE
b 32. 1. — parallelæ sunt. [e] quare BA. AB (AC) :: BD.
c *hyp.* — DC. Q. E. D.
d 27. 1.
e 2. 6. — 2. *Hyp.* Quoniam BA. AC. (AB) :: BD.
f 2. 6. — DC [f] erunt DA, CE parallelæ: [g] ergò ang.
g 29. 1. — BAD $=$ E; & ang. DAC [g] $=$ ACE [h] $=$ E, [k] ergò ang.
h 5. 1. — BAD $=$ DAC. bisectus igitur est ang.
k 1. *ax.* — BAC. Q. E. D.

Prop. IV.



Æquiangulorum triangulorum ABC, DCE *proportionalia sunt latera, quæ circum æquales angulos* B, DCE (AB. BC :: DC. CE, &c.) *& homologa sunt latera* AB, DC &c. *quæ æqualibus angulis* ACB, E &c. *subtenduntur.*

Statue

Statue latus BC in directum lateri CE, &
produc BA, ac ED donec [a] occurrant. a 32. 1. & 13. *ax.*
Quoniam ang. B [b] $=$ ECD, [c] sunt BF, CD b *hyp.*
parallelæ. Item quia ang. BCA [b] $=$ CED [c] sunt c 28. 1.
CA. EF parallelæ. Figura igitur CAFD est
parallelogramma. [d] ergò AF $=$ CD; [d] & AC d 34. 1.
$=$ FD. Liquet igitur AB. AF (CD) [e] :: BC. e 2. 6.
CE. [f] permutando igitur AB. BC :: CD. CE, f 16. 5.
[e] item BC. CE :: FD. (AC) DE [f] ergò per-
mutando BC. AC :: CE. DE. quare etiam [g] ex g 22. 5.
æquo AB. AC :: CD. DE. ergò, &c.

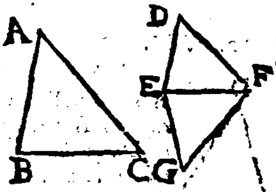
Coroll.

Hinc AB. DC :: BC. CE :: AC. DE.

Schol.

Hinc si in triangulo FBE ducatur uni lateri FE parallela AC; erit triangulum ABC simile toti FBE.

PROP. V.

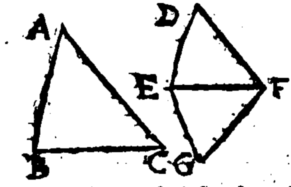


Si duo triangula ABC, DEF *latera proportionalia habeant* (AB. BC :: DE. EF *&* AC. BC :: DF. EF. *item* AB. AC :: DE. DF) *æquiangula erunt triangula, & æquales habebunt eos angulos, sub quibus homologa latera subtenduntur.*

Ad latus EF [a] fac ang. FEG $=$ B; [a] & ang. a 23. 1.
EFG $=$ C, [b] quare etiam ang. G $=$ A. ergò b 32. 1.
GE. EF [c] :: AB. BC :: [d] DE. EF. [e] ergò c 4. 6. d *hyp.*
GE $=$ DE. Item GF. FE [c] :: AC. CB [d] :: e 11. 5.
DF. FE. [e] ergò GF $=$ DF. Triangula igitur & 9. 5.
DEF. GEF sibi mutuò æquilatera sunt. [f] ergò f 8. 1.
ang. D $=$ G $=$ A. [f] & ang. FED $=$ FEG $=$ B.
[g] proinde & ang. DFE $=$ C. ergò &c. g 32. 1.

PROP.

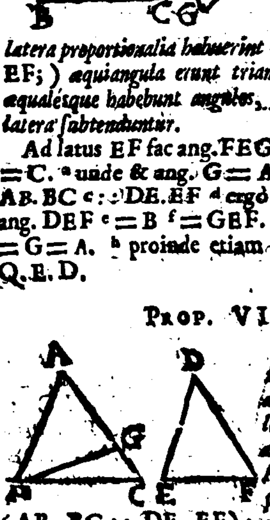
PROP. VI.



Si duo triangula ABC, DEF *unum angulum* B *uni angulo* DEF *æqualem, & circum æquales angulos* B, DEF *latera proportionalia habuerint* (AB. BC : : DE. EF;) *æquiangula erunt triangula* ABC, DEF; *æqualésque habebunt angulos, sub quibus homologa latera subtenduntur.*

a 32. 1. Ad latus EF fac ang. FEG = B; & ang. EFG
b 4. 6. = C. [a] unde & ang. G = A. ergò GE. EF [b] : :
c hyp. AB. BC [c] : : DE. EF [d] ergò DE = GE. atqui
d 9. 5. ang. DEF [e] = B [f] = GEF. [g] ergò ang. D =
e hyp. = G = A. [h] proinde etiam ang. EFD = C.
f constr. Q. E. D.
g 4. 1.
h 32. 1.

PROP. VII.

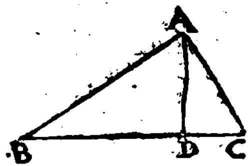


Si duo triangula ABC, DEF *unum angulum* A *uni angulo* D *æqualem, circa autem alios angulos* ABC, E *latera proportionalia habeant* (AB. BC : : DE. EF) ; *reliquorum autem simul utrumque* C, F *aut minorem, aut non minorem recto; æquiangula erunt triangula* ABC, DEF, *& æquales habebunt eos angulos, circum quos proportionalia sunt latera.*

a hyp. Nam si fieri potest, sit ang. ABC ⊏ E. fac
b 32. 1. igitur ang. ABG = E ; ergò cùm ang. A [a] = D,
c 4. 6. [b] erit etiam ang. AGB = F. ergò AB. BG [c] : :
d hyp. DE. EF : : AB. BC. [e] ergò BG = BC. [f] ergò
e 9. 5. ang. BGC : : BCG. [g] ergò ang. BGC. vel C
f 5. 1. minor
g cor. 17. 1.

minor est recto; [g] proinde ang. AGB, vel F recto major est. ergò anguli C, & F non sunt ejusdem speciei, contra Hyp. g cor. 13. 1.

Prop. VIII.



Si in triangulo rectangulo ABC,*ab angulo recto* BAC *in basin* BC *perpendicularis* AD *ducta est; quæ ad perpendicularem triangula* ADB, ADC,*tum toti triangulo* ABC, *tam ipsa inter se similia sunt:*

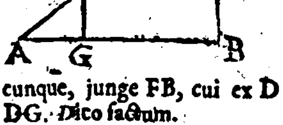
Nam ang. BAC[a] = BDA[a] = CDA. & ang. BAD[b] = C. & CAD[b] = B. ergò per 4. 6. & 1 def. 6. a 12. ax. b 32. 1.

Coroll.

Hinc 1. BD.DA[c] : : DA. DC. c 1. def. 6.

2. BC. AC : : AC. DC. & CB. BA. : : BA. BD.

Prop. IX.



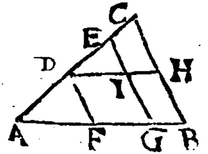
A data recta linea AB *imperatam partem* $\frac{1}{3}$ AG *auferre.*

Ex A duc infinitam AC utcunq;, in qua [a] sume tres, AD, DE, EF æquales utcunque, junge FB, cui ex D [b] duc parallelam DG. *Dico factum.* a 3. 1. b 31. 1.

Nam GB. AG[c] : : FD. AD. ergò [d] componendo AB.AG : : AF.AD.ergò cùm AD = $\frac{1}{3}$ AF, erit AG = $\frac{1}{3}$ AB. Q. E. F. c 2. 6. d 18. 5.

Prop.

PROP. X.



Datam rectam lineam AB *insectam similiter secare* (*in* F, G), *ut data altera* AC, *secta fuerit* (*in* D. E.)

Extremitates sectæ & insectæ jungat recta

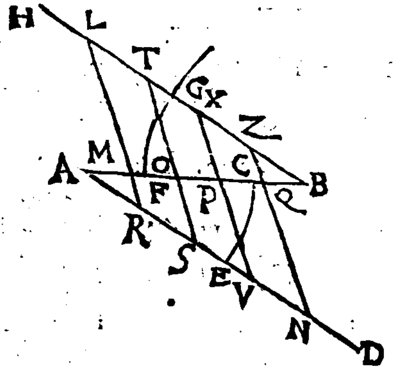
a 31. 1. BC. Huic ex punctis E, D [a] duc parallelas EG, DF rectæ secundæ occurrentes in G, & F. Dico factum.

[a] Ducatur enim DH parall. AB. Estque AD.

b 2. 6. DE [b] :: AF. FG, & DE. EC [b] :: DI. IH [c] ::

c 34. 1. & 7. 5. FG. GB. Q. E. F.

Scholium.



Hinc *discimus rectam datam* AB *in quotvis æquales partes* (puta 5.) *secare*, id quod faciliùs præstabitur sic.

Duc infinitam AD, eiq; parallelam BH etiam infinitam. Ex his cape partes æquales AR, RS, SV, VN; & BZ, ZX, XT, TL; in singulis unâ

pau-

pauciores, quàm desiderentur in AB; tum rectæ ducantur LR, TS, XV, ZN. hæ quinquisecabunt datam AB.

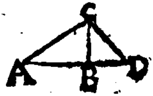
Nam RL, ST, VX, NZ [a] parallelæ sunt. a 33. 1.
ergò quum AR, RS, SV, VN [b] æquales sint, b *constr.*
[c] erunt AM, MO, OP, PQ æquales. Similiter c 2. 6.
quia BZ=ZX, erit BQ=QP. ergò AB. quinquisecta est. Q. E. F.

Prop. XI.

Datis duabus rectis lineis AB AD. *tertiam proportionalem* DE *invenire.*

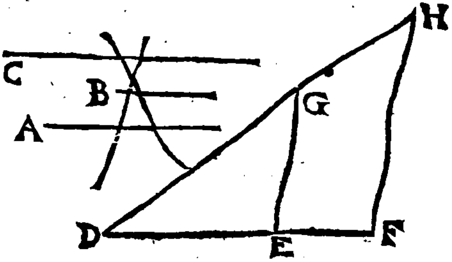
Junge BD, & ex AB protractâ sume BC=AD. per C duc CE parall. BD. cui occurrat AD producta in E. Erit DE expetita.

Nam AB.BC. (AD) :: AD.DE. Q.E.F. a 2. 6.



Vel sic, fac ang. ABC rectum & ang. ACD etiam rectum. [b] erit AB.BC :: BC.BD. b *cor.* 9.

Prop. XII.



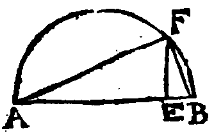
Tribus datis rectis lineis DE, EF, DG, *quartam proportionalem* GH *invenire.*

Connectatur EG. per F duc FH parall. EG, cui occurrat DG producta ad H. liquet esse
a 2. 6. DE. EF[a] :: DG. GH. Q. E. F.

Vel itá. CD=CB+BD adapta circulo. Circino sume AB.
a 35. 3. [a] Erit AB×BE=CB×BD. [b] qua-
b 16. 6. re AB. CB :: BD. BE.

Prop. XIII.

Duabus datis rectis lineis AE,EB, *mediam proportionalem* EF *adinvenire.*



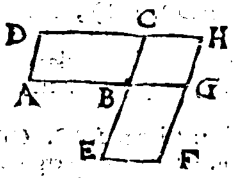
Super tota AB diametro describe semicirculum AFB. Ex E erige perpendicularem EF occurrentem peripheriæ in F. Dico AE. EF :: EF. EB. Du-
a 3. 6. cantur enim AF, & FB. Ex trianguli [a] rectanguli

guli AFB recto angulo, deducta est FE basi perpendicularis; [b] ergo AE. FE :: FE. EB. b *cor.* 8. 6.
Q. E. F.

Coroll.

Hinc, linea recta, quæ in circulo à quovis puncto diametri, ipsi diametro perpendicularis ducitur ad circumferentiam usque, media est proportionalis inter duo diametri segmenta.

Prop. XIV.



Æqualium, & unum ABC *uni* EBG *æqualem habentium angulum; parallelogrammorum* BD, BF *reciproca sunt latera, quæ circum æquales angulos.* (AB. BG :: EB. BC): *Et quorum parallelogrammorum* BD, BF *unum angulum* ABC *uni angulo* EBG *æqualem habentium, reciproca sunt latera, quæ circum æquales angulos, illa sunt æqualia.*

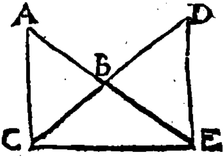
Nam latera AB, BG circa æquales angulos faciant unam rectam, [a] quare EB, BC etiam in directum jacebunt. Producantur FG, DC, donec occurrant. a *sch.* 15. 1.

1. *Hyp.* AB. BG [b] :: BD. BH [c] :: BF. BH [d] :: BE. BC. [e] ergò, &c.

2. *Hyp.* BD. BH [f] :: AB. BG[g] :: BE. BC [h] :: BF. BH. [k] ergo Pgr. BD = BF. Q. E. D.

b 1. 6.
c 7. 5.
d 1. 6.
e 11. 5.
f 1. 6.
g *hyp.*
h 1. 6.
k 11. & 9. 5.

PROP. XV.



Æqualium, & unum ABC, *uni* DBE *æqualem habentiũ angulum triangulorũ* ABC, DBE, *reciproca sunt latera, quæ circum æquales angulos* (AB. BE :: DB. BC): *Et quorum triangulorum* ABC, DBE, *unum angulum* ABC *uni* DBE *æqualem habentium reciproca sunt latera, quæ circum æquales angulos* (AB. BE :: DB. BC), *illa sunt æqualia.*

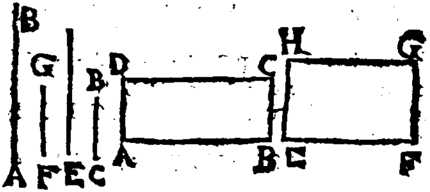
Latera CB, BD circa æquales angulos, statuantur sibi in directum; [a] ergò ABE est recta linea. ducatur CE.

1. *Hyp.* AB. BE [b] :: triang. ABC. CBE [c] :: triang. DBE. CBE. [d] :: DB. BC. [e] ergò, &c.

2. *Hyp.* Triang. ABC. CBE [f] :: AB. BE [g] :: DB. BC [h] :: triang. DBE. CBE. [k] ergò triang. ABC = DBE. Q. E. D.

a *sch.* 15. 1.
b 1. 6.
c 7. 5.
d 1. 6.
e 11. 5.
f 1. 6.
g *hyp.*
h 1. 6.
k 11, & 9. 5.

PROP. XVI.



Si quatuor rectæ lineæ proportionales fuerint (AB. FG :: EF. CB), *quod sub extremis* AB, CB *comprehenditur rectangulum* AC *æquale est ei, quod sub mediis* EF, FG *comprehenditur, rectangulo* EG. *Et si sub extremis comprehensum rectangulum* AC *æquale fuerit ei, quod sub mediis comprehenditur, rectangulo* EG, *illæ quatuor rectæ lineæ proportionales erunt* (AB. FG :: EF. CB.).

1. *Hyp.*

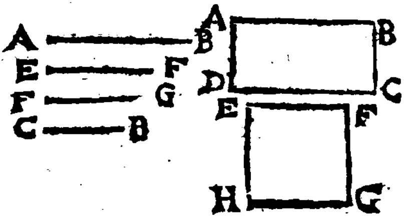
1. *Hyp.* Anguli B, & F recti, ac [a] proinde pares sunt; atque ex hyp. AB. FG :: EF. CB. [b] ergò Rectang. AC = EG. Q. E. D. a 12. *ax.* b 14. 6.

2. *Hyp.* [c] Rectang. AC = EG; atque ang. B = F; [d] ergò AB. FG :: EF. CB. Q. E. D. c *hyp.* d 14. 6.

Coroll.

Hinc ad datam rectam lineam AB facile est datum rectangulum EG applicare, [e] faciendo AB. EF :: FG. BC. e 4, & 12. 6.

Prop. XVII.



Si tres rectæ lineæ sint proportionales (AB. EF :: EF. CB), *quod sub extremis* AB, CB *comprehenditur rectangulum* AC *æquale est ei, quod à media* EF, *describitur, quadrato* EG. *Et si sub extremis* AB, CB *comprehensum rectangulum* AC, *æquale sit ei, quod à media* EF, *describitur, quadrato* EG, *illæ tres rectæ lineæ proportionales erunt* (AB. EF :: EF. CB).

Accipe FG = EF.

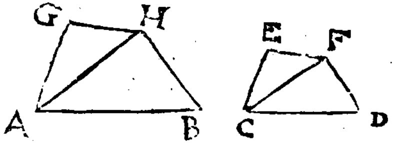
1. *Hyp.* AB. EF [a] :: EF (FG). CB. ergò Rectang. AC [b] = EG [c] = EFq. Q. E. D. a *hyp.* b 16. 6.

2. *Hyp.* Rectang. AC [d] = quadr. EG = EFq. [e] ergò AB. EF :: FG (EF). BC. c 29. *def.* 1. d *hyp.* e 16. 6.

Coroll.

Sit A in B = Cq. ergò A. C :: C. B.

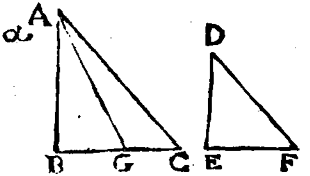
PROP. XVIII.



A data recta linea AB *dato rectilineo* CEFD. *simile similitérque positum rectilineum* AGHB *describere.*

Datum rectilineum resolve in triangula. a fac ang. ABH = D; a & ang. BAH = DCF;
a 23. 1. a & ang. AHG = CFE; a & ang. HAG = FCE. Rectilineum AGHB est quæsitum.
b constr. Nam ang. B b = D. & ang. BAH b = DCF.
c 32. 1. c quare ang. AHB = CFD; b item ang. HAG = FCE, b & ang. AHG = CFE. c quare ang.
d 2. ax. G = E; & totus ang. GAB d ECD; & totus GHB d = EFD. Polygona igitur sibi mutuò æquiangula sunt. Porrò, ob trigona æquiangula,
e 4. 6. AB. BH e :: CD. DF. & AG. GH. e :: CE. EF. item AG. AH. e :: CE. CF. & AH. AB e ::
f 22. 5. CF. CD. f unde ex æquo AG AB :: CE. CD.
g 6. def. 6. eodem modo GH. HB :: EF. FD. g ergò polygona ABHG, CDFE similia similitérque posita existunt. Q. E. F.

PROP. XIX.



Similia triangula ABC, DEF *sunt in duplicata ratione laterum homologorum* BC EF.

a 11. 6. a Fiat BC. EF :: EF. BG. & ducatur AG.

Quia

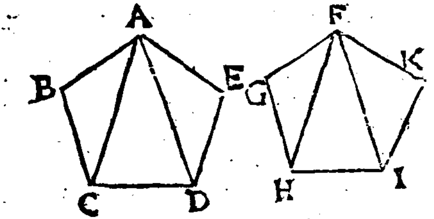
Quia AB.DE [b]:: BC.EF [c] :: EF.BG.& ang. B $=$ E ; [d] erit triang. ABG $=$ DEF. verùm triang. ABC. ABG [e] :: BC. BG; & [f] $\frac{BC}{BG}$ $=$ $\frac{BC}{EF}$ bis; ergò triang. $\frac{ABC}{ABG}$ hoc eſt $\frac{ABC}{DEF}$ [g] $=$ $\frac{BC}{EF}$ bis. Q. E. D.

b *cor.* 4. 6.
c *conſtr.*
d 15. 6.
e 1. 6.
f 10. *def* 5.
g 11. 5.

Coroll.

Hinc, ſi tres lineæ BC, EF, BG proportionales fuerint; ut eſt prima ad tertiam, ità eſt triangulum ſuper primam BC deſcriptum ad triangulum ſuper ſecundam EF ſimile, ſimilitérq; deſcriptum. vel ità eſt triangulum ſuper ſecundam EF deſcriptum ad triangulum ſuper tertiam ſimile ſimilitérque deſcriptum.

PROP. XX.



Similia polygona ABCDE, FGHIK *in ſimilia triangula* ABC, FGH; *&* ACD, FHI, *&* ADE, FIK *dividuntur; & numero æqualia, & homologa totis.* (ABC. FGH :: ABCDE. FGHIK :: ACD. FHI :: ADE. FIK.) *Et polygona* ABCDE, FGHIK *duplicatam habent eam inter ſe rationem, quam latus homologum* BC *ad homologum latus* GH.

M 4 1. Nam

a *hyp.* 1. Nam ang. B [a] $=$ G; & AB. BC [a] :: FG.
b *6. 6.* GH. [b] ergò triangula ABC FGH æquiangula sunt. eodem modo, triangula AED, FKI assimilantur. cùm igitur ang. BCA [b] $=$ GHF; & ang. ADE [b] $=$ FIK; totíque anguli BCD,
c *hyp.* GHI; atque toti CDE, HIK [c] pares sint, [d] re-
d *3. ax.* manent ang. ACD $=$ FHI; & ang. ADC $=$
e *32. 1.* FIH; [e] unde etiam ang. CAD $=$ HFI. ergò triangula ACD, FHI similia sunt: ergò, &c.

2. Quoniam igitur triangula BCA, GHF
f *19. 6.* similia sunt, [f] erit $\frac{BCA}{GHF} = \frac{BC}{GH}$ bis. ob eandem causam $\frac{CAD}{HFI} = \frac{CD}{HI}$ bis. deniq; triang. $\frac{DEA}{IKF} =$
g *hyp. & 16. 5.* $\frac{DE}{IK}$ bis. quare cùm $BC.\ GH$ [g] :: $CD.$ HI [g] ::
h *sch. 23. 5.* DE. IK, [h] erit triang. $BCA.$ GHF :: CAD.
k *12. 5.* HFI :: DEA. IKF :: [k] polyg. $ABCDE.$ FGHIK :: $\frac{BC}{GH}$ bis.

Coroll.

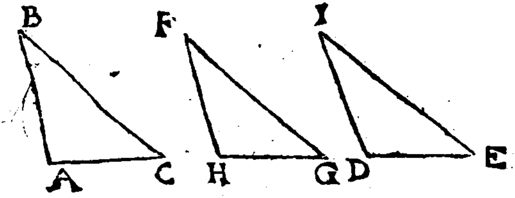
1. Hinc, si fuerint tres lineæ rectæ proportionales; ut est prima ad tertiam, ità erit polygonum super primam descriptum ad polygonum super secundam simile, similitérque descriptum. vel ità erit polygonum super secundam descriptum ad polygonum super tertiam simile similitérque descriptum.

Unde elicitur methodus *figuram quamvis rectilineam augendi vel minuendi in ratione data. ut si velis pentagoni, cujus latus* CD *aliud facere quintuplum. inter* AB, *&* 5 AB *inveni mediam proportionalem. Super hac* * *construe pentagonum simile*
* *18. 6.* *dato. hoc erit quintuplum dati.*

2. Hinc etiam, si figurarum similium homologa latera nota fuerint, etiam proportio figurarum innotescet; nempe inveniendo tertiam proportionalem.

PROP.

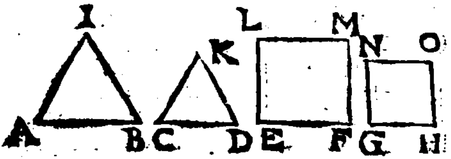
PROP. XXI.



Quæ (ABC, DIE) *eidem rectilineo* HFG *sunt similia, & inter se sunt similia.*

Nam ang. A [a] = H [a] = D. & ang. C [a] = G [a] = E; & ang. B [a] = F [a] = I. [a] item AB. AC :: HF. HG :: DI. DE. [a] & AC. CB :: HG. GF :: DE. EI. & AB. BC :: HF, FG :: DI. IE. [a] ergò ABC, DIE similia sunt. Q. E. D. — a 1. *def.* 6.

PROP. XXII.



Si quatuor rectæ lineæ proportionales fuerint (AB. CD :: EF. GH.) *& ab eis rectilinea similia similitérque descripta proportionalia erunt.* (ABI. CDK :: EM. GO) *Et si à rectis lineis similia similitérque descripta rectilinea proportionalia fuerint* (ABI. CDK :: EM. GO) *ipsæ etiã rectæ lineæ proportionales erunt.* (AB. CD :: EF. GH.)

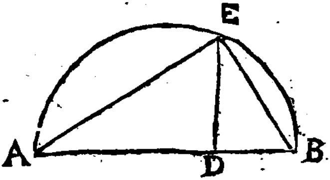
1. *Hyp.* $\frac{ABI}{CDK} \overset{a}{=} \frac{AB}{CD}$ bis $\overset{a}{=} \frac{EF}{GH}$ bis $\overset{c}{=} \frac{EM}{GO}$. — a 19. 6.
[d] ergò ABI. CDK :: EM. GO. Q. E. D.

2. *Hyp.* $\frac{AB}{CD}$ bis $\overset{a}{=} \frac{ABI}{CDK} \overset{b}{=} \frac{EM}{GO} = \frac{EF}{GH}$ — b *hyp.* c 20. 6
bis. ergo AB. CD :: EF. GH. Q. E. D. — d *cor.* 23. 5.

Schol.

Hinc deducitur, & demonstratur ratio multiplicandi quantitates surdas. ex gr. Sit $\sqrt{5}$ multiplicandus in $\sqrt{3}$. dico provenire $\sqrt{15}$. Nam ex multiplicationis definitione debet esse, 1. $\sqrt{3}$ :: $\sqrt{5}$. product. ergò per hanc, q. 1. q. $\sqrt{3}$ :: q. $\sqrt{5}$. q. product. hoc est. 1. 3 :: 5. q. product. ergò q. product. est 15. quare $\sqrt{15}$ est productus ex $\sqrt{3}$ in $\sqrt{5}$. Q. E. D.

THEOR.



Petr. Herig. *Si recta linea* AB *secta sit utcunque in* D. *rectangulum sub partibus* AD, DB *contentum, est medium proportionale inter earum quadrata. Item rectangulum contentum sub tota* AB, *& una parte* AD, vel DB, *est medium proportionale inter quadratum totius* AB, *& quadratum dictæ partis* AD, vel DB.

Super diametrum AB describe semicirculum. ex D erige normalem DE occurrentem peripheriæ in E. junge AE, BE.

a *cor.* 8. 6.
b 22. 6.
c 17. 6.

Liquet esse AD. DE [a] :: DE. DB. [b] ergò ADq. DEq :: DEq. DBq. [c] hoc est ADq. ADB :: ADB. DBq. Q. E. D.

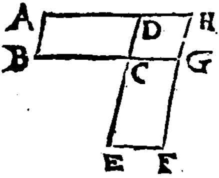
d *cor.* 8. 6.
e 22. 6.
f 17. 6.

Porrò, BA. AE [d] :: AE. AD. [e] ergò BAq. AEq :: AEq. ADq. [f] hoc est BAq. BAD :: BAD. ADq. Eodem modo ABQ. ABD :: ABD. BDq. Q. E. D.

Sic quidem P. Herigonius scitè. Sed facillimè hæc etiam ex 1. 6. & 11. 5. deduci possint.

PROP.

PROP. XXIII.



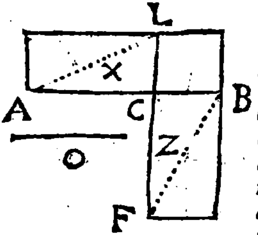
Æquiangula parallelogramma AC, CF *inter se rationem habent eam quæ ex lateribus componitur.* $\left(\frac{AC}{CF} = \frac{BC}{CG} + \frac{DC}{CE}.\right)$

Latera circa æquales angulos C [a] sibi in directum statuantur; & compleatur parallelogrammum CH. a *sch.* 15

Ratio $\frac{AC}{CF}$ [b] $= \frac{AC}{CH} + \frac{CH}{CF}$ [c] $= \frac{BC}{CG} + \frac{DC}{CE}.$ b 20. *def.* 5. c 1. 6.
Q. E. D.

Coroll.

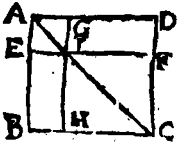
Hinc & ex 34. 1. patet primò, *Triangula, quæ unum angulum* (ad C) *æqualem habent, rationem habere ex rationibus rectarum,* AC ad CB, & LC ad CF, *æqualem angulum continentium.* *Andr. Tacqu.* 15. 5.



Patet secundò, *Rectangula ac *proinde & parallelogramma quæcunque rationem inter se habere compositam ex rationibus basis ad basim, & altitudinis ad altitudinem.* Neque aliter de triangulis ratiocinaberis. * 35. 1.

Patet tertiò, *Quomodo triangulorum ac parallelogrammorum proportio exhiberi possit.* Sunto parallelogramma X & Z; quorum bases AC, CB; altitudines verò CL, CF. Fiat CL. CF :: CB. O. * erit X. Z :: AC. O. * 14. 6. & 1. 6.

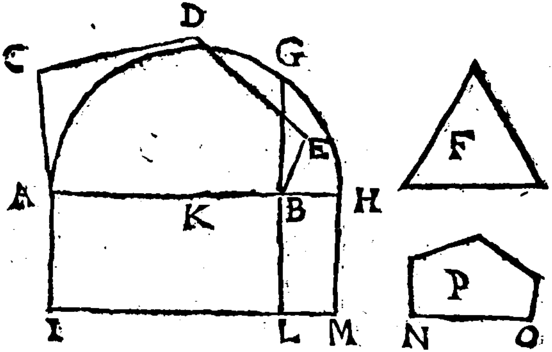
PROP. XXIV.



In omni parallelogrammo ABCD, *quæ circa diametrum* AC *sunt parallelogramma* EG, HF, *& toti & inter se sunt similia.*

Nam parallelogramma EG, HF habent singula unum angulum cum
a 29. 1. toto communem. [a] ergò toti & sibi mutuò æquiangula sunt. [a] Item tam triangula ABC, AEI, IHC, quàm triangula ADC, AGI, IFC sunt
b 4. 6. inter se æquiangula. [b] ergò AE. EI :: AB. BC, [b] atque AE. AI :: AB. AC; [b] & AI. AG .: AC.
c 22. 5. AD. [c] ex æquali igitur, AE. AG :: AB. AD.
d 1. def. 6. [d] ergò Pgra. EG, BD similia sunt. eodem modo HF, BD similia sunt, ergò, &c.

PROP. XXV.



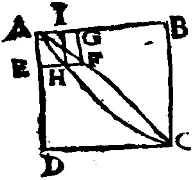
Dato rectilineo ABEDC *simile, similitérque positum* P, *idémque alteri dato* F *æquale, constituere.*

a 45: 1. [a] Fac rectang. AL = ABEDC. [b] item super
b 44. 1. BL fac rectang. BM = F. Inter AB, BH [c] in-
c 13. 6. veni mediam proportionalem NO, super NO

[d] fac

[d] fac polygonum P simile dato ABEDC. Erit d 18. 6.
hoc æquale dato F. e cor. 20. 6.
Nam ABEDC (AL). P :: [e] AB. BH [f] :: f 1. 6.
AL. BM. ergò P [g] = BM [h] = F. Q. E. F. g 14. 5. h constr.

Prop. XXVI.

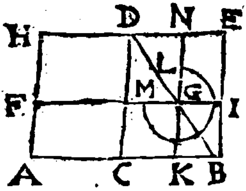


Si à parallelogrammo ABCD *parallelogrammum* AGFE *ablatum sit, & simile toti, & similiter positum, communem cum eo habens angulum* EAG, *hoc circa eandem cum toto diametrum* AC *consistet.*

Si negas AC esse communem diametrum, esto diameter AHC secans EF in H. & ducatur HI parall. AE. Parallelogramma EI, DB [a] similia sunt. [b] ergò AE. EH :: AD. DC [c] :: AE. EF. [d] proinde EH = EF. [f] Q. E. A.

a 24. 6.
b 1. def. 6.
c hyp.
d 9. 5.
f 9. ax.

Prop. XXVII.

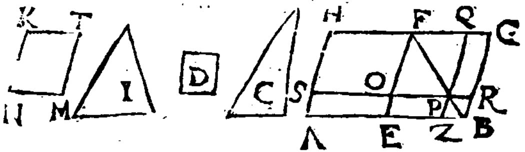


Omnium parallelogrammorum AD, AG *secundùm eandem rectam lineam* AB *applicatorum, deficientiúmque figuris parallelogrammis* CE, KI *similibus, similitérq; positis, ei* AD, *quod à dimidia describitur, maximum est* AD, *quod ad dimidium est applicatum, simile existens defectui* KI.

Nam quia GE [a] = GC, addito communi KI, [b] erit KE = CI [c] = AM. adde commune CG, [d] erit AG = Gnom. MBL. sed Gnom. MBL [e] ⊐ CE (AD). ergò AG ⊐ AD. Q. E. D.

a 43. 1.
b 2. ax.
c 36. 1.
d 2. ax.
e 9. ax.

Prop. XXVIII.



Ad datam rectam lineam AB, dato rectilineo C æquale parallelogrammum AP applicare deficiens figurâ parallelogrammâ ZR, quæ similis sit alteri parallelogrammo dato D. * *Oportet autem datum rectilineum C, cui æquale AP applicandum est, non majus esse eo AF, quod ad dimidiam applicatur, similibus existentibus defectibus, & ejus AF quod ad dimidiam applicatur; & ejus D, cui simile deesse debet.*

* 27. 6.

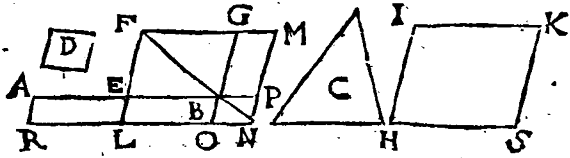
Biseca AB in E. Super EB [a] fac Pgr. EG. simile dato D. [b] sitque EG = C + I. [c] fac pgr. NT = I; & simile dato D, vel EG. duc diametrum FB. fac FO = KN; & FQ = KT. Per O, & Q duc parallelas SR, QZ. parallelogrammum AP est id quod quæritur.

a 18. 6.
b sch. 45. 1.
c 25. 6.

Nam parallelogramma D, EG, OQ, NT, ZR [d] sunt similia inter se. Et Pgr. EG [e] = NT + C [e] = OQ + C; [f] quare C = Gnom. OBQ [g] = AO + PG [h] = AO + EP = AP. Q. E. F.

d constr. & 24. 6.
e constr.
f 3. ax.
g 2. ax.
h 43. 1.

PROP. XXIX.

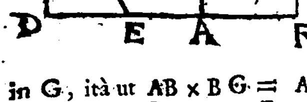


Ad datam rectam lineam AB, *dato rectilineo* C *æquale parallelogrammum* AN *applicare, excedens figurâ parallelogrammâ* OP, *quæ similis sit parallelogrammo alteri dato* D.

Biseca AB in E. super EB [a] fac Pgr. FG simile dato D. [b] sítq; pgr. HK = EG + C; & simile dato D, vel EG. fac FEL. [c] = IH; [c] & FGM = IK. per L, M duc parallelas RN, MN. & AR parall. NM. Produc ABP, GBO. Duc diametrum FBN. Pgr. AN est quæsitum. a 18. 6. b 25. 6. c 3. 1.

Nam parallelogramma D, HK, LM, EG [d] similia sunt, [e] ergò pgr. OP simile est pgro LM, vel D. item LM [f] = HK [f] = EG + C. [g] ergò C = Gnom. ENG. atqui AL [h] = LB [k] = BM. [l] ergò C. = AN. Q. E. F. d constr. e 24. 6. f constr. g 3. ax. h 36. 1. k 43. 1. l 2. & 1. ax.

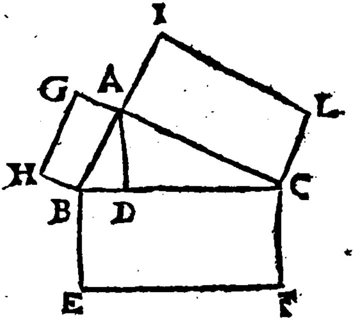
PROP. XXX.



Propositâ rectam lineam terminatam AB, *extremâ, ac mediâ ratione secare.* (AB. AG :: AG. GB.)

[a] Seca AB in G, ità ut AB × BG = AGq. [b] ergò BA. AG :: AG. GB. Q. E. F. a 11. 2. b 17. 6.

PROP. XXXI.



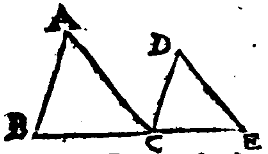
In rectangulis triangulis BAC, *figura quævis* BF *à latere* BC *rectum angulum* BAC *subtendente, descripta, æqualis est figuris* BG, AL, *quæ priori illi* BF *similes, & similiter positæ à lateribus* BA, AC *rectum angulum continentibus describuntur.*

Ab angulo recto BAC demitte perpendicu-
a Cor. 8. 6. larem AD. Quoniam CB. CA [a] :: CA. DC.
b Cor. 20. 6. [b] erit BF. AL :: CB. DC; inverséque AL.
BF :: DC. CB. Item quia BC, BA [a] :: BA. DB.
[b] erit BF. BG :: BC, DB; ac invertendo, BG.
c 24. 5. BF :: DB. BC. [c] ergò AL+BG. BF :: DC+
d schol. 14. 5. DB. BC. [d] ergò AL+BG=BF. Q. E. D.
e 22. 6. Vel sic. BG. BF [e] :: BAq. BCq. [e] & AL. BF ::
f 24. 5. ACq. BCq. [f] ergò BG+AL. BF :: BAq+
g sch. 14. 5. ACq. BCq. [g] ergò cùm BAq+ACq [h] =BCq.
h 47. 1. [h] erit BG+AL=BF. Q. E. D.

Coroll.

Ex hac propositione, addi possunt, & subtrahi figuræ quævis similes, eâdem methodo, quâ quadrata adduntur & subtrahuntur, in schol. 47. 1.

PROP. XXXII.

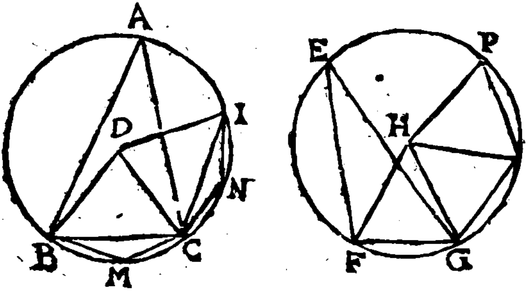


Si duo triangula ABC, DCE, *quæ duo latera duobus lateribus proportionalia habeant* (AB. AC :: DC. DE), *secundùm unum angulum* ACD *composita fuerint, ità ut homologa eorum latera sint etiam parallela* (AB ad DC, & AC ad DE): *tum reliqua illorum triangulorum latera* BC, CE *in rectam linem collocata reperientur.*

Nam ang. A [a] = ACD [a] = D; & AB. AC [b] :: DC. DE. [c] ergò ang. E = DCE. ergò ang. B + A [d] = ACE. sed ang. B + A + ACB [e] = 2 Rect. [f] ergò ang. ACE + ACB = 2 Rect. [g] ergò BCE est recta linea. Q. E. D.

a 29. 1.
b hyp.
c 6. 6.
d 2. ax.
e 32. 1.
f 1. ax.
g 14. 1.

PROP. XXXIII.



In æqualibus circulis DBCA, HFGP, *anguli* BDC, FHG *eandem habent rationem cum peripheriis* BC, FG, *quibus insistunt; sive ad centra* (ut BDC, FHG), *sive ad peripherias* A, E *constituti insistant: insuper verò & sectores* BDC, FHG, *quippe qui ad centra consistant.*

 Duc

Duc rectas BC. FG. Accommoda CI = CB; & GL = FG = LP; & junge DI, HL, HP.

a 28. 3. Arcus BC [a] = CI, [a] item arcus FG, GL, LP
b 27. 3. æquantur, [b] ergò ang. BDC = CDI. [b] & ang. FGH = GHL = LHP. Ergò arcus BI tam multiplex est arcûs BC, quàm ang. BDI anguli BDC. pariterque æquemultiplex est arcus FP arcûs FG; atque ang. FHP anguli FHG. Verùm si arcus BI $\sqsubset$, =, $\sqsupset$ FP, [c] erit similiter
c 27. 3. ang. BDI $\sqsubset$, =, $\sqsupset$ FHP. ergò arc. BC. FG [d] ::
d 6. def. 5. ang. BDC. FHG [e] :: $\frac{BDC}{2}$. $\frac{FHG}{2}$ [f] :: A. E.
e 15. 5.
f 20. 3.

Q. E. D.

g 27. 3. Rursus ang. BMC [g] = CNI; [h] atque idcirco
h 24. 3. segm. BCM = CIN. [k] item triang. BDC =
k 4. 1. CDI. [l] ergò sector BDCM = CDIN. Simili
l 2. ax. ratione sectores FHG, GHL, LHP æquantur. Quum igitur prout arcus BI $\sqsubset$, =, $\sqsupset$ FGP, ità
m 6. def. 5. similiter sector BDI $\sqsubset$, =, $\sqsupset$ FHP. [m] erit sect. BDC. FHG :: arc. BC. FG. Q. E. D.

Coroll.

11. 5. Hinc 1. *ut sector ad sectorem, sic angulus ad angulum.*

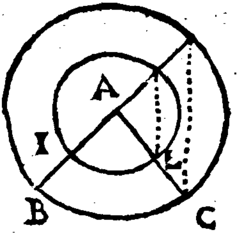
2. *Ang.* BDC *in centro est ad* 4 *rectos, ut arcus* BC *cui insistit ad totam circumferentiam.*

Nam ut ang. BDC ad rectum; sic arcus BC ad quadrantem. ergò BDC est ad 4 Rectos, ut arcus BC ad 4 quadrantes, id est ad totam circumferentiam. item ang. A. 2 Rect :: arc. BC. periph.

Hinc 3. *Inæqualium circulorum arcus* IL, BC, *qui æquales subtendunt angulos, sive ad centra,* ut IAL & BAC, *sive ad peripheriam, sunt similes.*

Nam IL. periph. :: ang. IAL, (BAC). 4 Rect. item arc. BC. periph :: ang. BAC. 4 Rect.

4. Rect. ergò IL. periph :: BC. periph. proinde arcus IL, & BC sunt similes. Unde



4. *Duæ semidiametri* AB, AC *à concentricis peripheriis arcus auferunt similes* IL, BC.

LIB. VII.

Definitiones.

I. Unitas est, secundùm quam unumquodq; eorum quæ sunt, unum dicitur.

II. Numerus autem est, ex unitatibus composita multitudo.

III. Pars est numerus numeri, minor majoris, quum minor metitur majorem.

Omnis pars ab eo numero nomen sibi sumit, per quem ipsa numerum, cujus est pars, metitur; ut 4 dicitur tertia pars numeri 12; quia metitur 12 per 3.

IV. Partes autem, cùm non metitur.

Partes quæcunque nomen accipiunt à duobus illis numeris, per quos maxima communis duorum numerorum mensura utrumque eorum metitur. ut 10 dicitur $\frac{2}{3}$ numeri 12, eò quòd maxima communis mensura, nempe 5, metitur 10 per 2, & 15. per 3.

V. Multiplex verò major minoris, cùm majorem metitur minor.

VI. Par numerus est, qui bifariam dividitur.

VII. Impar verò numerus, qui bifariam non dividitur, vel qui unitate differt à pari.

VIII. Pariter par numerus est, quem par numerus metitur per numerum parem.

IX. Pariter autem impar est, quem par numerus metitur per numerum imparem.

X. Impariter verò impar numerus est; quem impar numerus metitur per numerum imparem.

XI. Primus numerus est, quem sola unitas metitur.

XII. Primi inter se numeri sunt, quos sola unitas, communis mensura metitur.

XIII. Com-

XIII. Compositus numerus est, quem numerus quispiam metitur.

XIV. Compositi autem inter se numeri sunt, quos numerus aliquis communis mensura metitur.

In hac definitione & praecedenti unitas non est numerus.

XV. Numerus numerum multiplicare dicitur, cùm toties compositus fuerit is, qui multiplicatur, quot sunt in ipso multiplicante unitates, & procreatus fuerit aliquis.

Hinc, in omni multiplicatione unitas est ad multiplicatorem ut multiplicatus ad productum.

Nota, quòd saepe cùm multiplicandi sunt quivis numeri, puta A *in* B, *literarum conjunctio productum denotat. Sic* AB = A *in* B. *item* CDE = C *in* D *in* E.

XVI. Cùm autem duo numeri sese multiplicantes aliquem fecerint, qui factus erit, planus appellabitur. Qui verò numeri sese mutuò multiplicârint, latera illius dicentur. *Sic* 2 (C) *in* 3 (D) = 6 = CD, *est numerus planus.*

XVII. Cùm verò tres numeri mutuò sese multiplicantes fecerint aliquem, qui procreatus erit, solidus appellabitur; qui autem numeri mutuò sese multiplicârint, latera illius dicentur, *Sic*, 2 (C) *in* 3 (D) *in* 5 (E) = 30 = CDE *est numerus solidus.*

XVIII. Quadratus numerus est, qui aequaliter aequalis, vel qui sub duobus aequalibus numeris continetur. *Sit* A *latus quadrati; quadratus sic notatur*, AA, *vel* Aq.

XIX. Cubus verò, qui aequaliter aequalis aequaliter, vel qui sub tribus aequalibus numeris continetur. *Sit* A *latus cubi, cubus notatur sic*, AAA, *vel* AC.

In hac definitione, & tribus praecedentibus, unitas est numerus.

XX. Nu-

XX. Numeri proportionales sunt, cùm primus secundi, & tertius quarti æquemultiplex est, vel eadem pars; vel deniq; cùm pars primi secundum, & eadem pars tertii æquè metitur quartum, vel vice versâ. A. B :: C. D. hoc est 3. 9 :: 5. 15.

XXI. Similes plani, & solidi numeri sunt, qui proportionalia habent latera.

Latera nempe non qualibet, sed quædam.

XXII. Perfectus numerus est, qui suis ipsius partibus est æqualis.

Ut 6. & 28. *Numerus verò qui suis ipsius partibus minor est, abundans appellatur, qui verò major, diminutus. ut* 12 *est abundans,* 15 *est diminutus.*

XXIII. Numerus numerum metiri dicitur per illum numerum, quem multiplicans, vel à quo multiplicatus, illum producit.

In divisione, unitas est ad quotientem, ut dividens ad divisum. Nota, quòd numerus alteri lineolâ interjectâ subscriptus divisionem denotat, Sic $\frac{A}{B}$ = A divis. per B. *item* $\frac{CA}{B}$ = C *in* A divis. per B.

Termini, sive radices proportionis dicuntur duo numeri, quibus in eâdem proportione minores sumi nequeunt.

Postulata.

1. POstuletur, cuilibet numero quotlibet sumi posse æquales, vel multiplices.

2. Quolibet numero sumi posse majorem.

3. Additio, subtractio, multiplicatio, divisio, extractionésque radicum, seu laterum numerorum quadratorum, & cuborum conceduntur etiam, tanquam possibilia.

Axiomata

Axiomata.

1. Quicquid convenit uni æqualium numerorum, convenit & reliquis æqualibus numeris.

2. Partes eidem parti, vel iisdem partibus eædem, sunt quoque inter se eædem.

3. Qui numeri æqualium numerorum, vel ejusdem, eædem partes fuerint, æquales inter se sunt.

4. Quorum idem numerus, vel æquales, eædem partes fuerint, æquales inter se sunt.

5. Unitas omnem numerum per unitates, quæ in ipso sunt, hoc est per ipsummet numerum metitur.

6. Omnis numerus seipsum metitur per unitatem.

7. Si numerus numerum multiplicans, aliquem produxerit, metietur multiplicans productum per multiplicatum, multiplicatus autem eundem per multiplicantem.

Hinc nullus numerus primus planus est aut solidus, quadratus, vel cubus.

8. Si numerus numerum metiatur, & ille per quem metitur, eundem metietur per eas, quæ in metiente sunt, unitates, hoc est per ipsum numerum metientem.

9. Si numerus numerum metiens, multiplicet eum, per quem metitur, vel ab eo multiplicetur, illum quem metitur, producit.

10. Numerus quotcunque numeros metiens, compositum quoque ex ipsis metitur.

11. Numerus quemcunque numerum metiens, metitur quoque omnem numerum, quem ille metitur.

12. Numerus metiens totum, & ablatum, metitur & reliquum.

PROP.

PROP. I.

A ..... E .. G . B 8 5 3
C ... F .. D $\frac{5}{3}$ $\frac{3}{2}$ $\frac{2}{1}$
H - - -

Si duobus numeris inæqualibus propositis (AB, CD) *detrahatur semper minor* CD *de majore* AB (& reliquus EB de CD &c.) *alternâ quâdam detractione, neque reliquus unquam præcedentem metiatur, quoad assumpta sit unitas* GB; *qui principio propositi sunt numeri* AB, CD *primi inter se erant.*

Si negas, habeant AB, CD communem mensuram, numerum H. Ergò H metiens CD,
a 11. *ax.* 7. [a] etiam AE metitur; proinde & reliquum EB;
b 12. *ax.* 7. [a] ergò & CF, atque [b] idcirco reliquum FD; [a] quare & ipsum EG; sed totum EB metiebatur; [b] ergò & reliquum GB metitur, numerus unitatem.
c 9. *ax.* 1. [c] Q. E. A.

PROP. II.

9 6
A .... E ...... B 15 9 6
6 3
C ...... F ... D $\frac{2}{6}$ $\frac{6}{3}$ $\frac{3}{0}$
G - - -

Duobus numeris datis AB, CD *non primis inter se, maximam eorum communem mensuram* FD *reperire.*

Detrahe minorem numerum CD ex majori AB, quoties potes. Si nihil relinquitur,
a 6. *ax.* 7. [a] patet ipsum CD esse maximam communem mensuram. Si relinquitur aliquid EB, deme hunc ex CD; & reliquum FD ex EB, & sic deinceps donec aliquis FD præcedentem EB metiatur.
b 1. 7. (nam [b] hoc fiet antequam ad unitatem perveniatur) Erit FD maxima communis mensura.

c *constr.* Nam FD [c] metitur EB, [d] ideóque & CF;
d 11. *ax.* 7. [e] proinde & totum CD; [d] ergò ipsum AE; atque
e 12. *ax.* 7. idcircò totum AB metitur. Liquet igitur FD communem esse mensuram, Si maximam esse negas,

gas, sit major quæpiam G. ergò G metiens CD, [d] metitur AE, [e] & reliquum EB, [d] ipsúmque CF, [e] proinde & reliquum FD, [g] major minorem. [h] Q. E. A. *g suppos.* *h 9. ax. 1.*

Coroll.

Hinc, numerus metiens duos numeros, metitur quoque maximam eorum communem mensuram.

Prop. III.

A 12
B 8
D 4
C 6
E .. 2
F - - -

Tribus numeris datis A,B,C non primis inter se, maximam eorum communem mensuram E reperire.

Inveni D maximam communem mensuram duorũ A,B. Si D metitur tertium C; liquet D maximam esse trium communem mensuram. Si D non metitur C, erunt saltem D, & C compositi inter se, ex coroll. præcedentis. Sit igitur ipsorum D, & C maxima communis mensura E. erit E is, quem quæris.

Nam E [a] metitur C, & D; [a] ac D ipsos A, & B metitur; [b] ergò E metitur singulos A, B, C; nec major aliquis (F) eos metietur; nam si hoc affirmas, [c] ergò F metiens A, & B, eorum maximam communem mensuram D metitur. Eodem modo, F metiens D, & C, [c] eorum maximam communem mensuram E, [d] major minorem, metitur. [e] Q. E. A. *a constr.* *b 11. ax. 7.* *c cor. 1. 7.* *d suppos.* *e 9. ax. 1.*

Coroll.

Hinc, numerus metiens tres numeros, maximam quoque eorum communem mensuram metitur.

PROP. IV.

A 6
B 7
B 18
B .... 9.

Omnis numerus A, *omnis numeri* B, *minor majoris, aut pars est, aut partes.*

Si A, & B primi sint inter se, [a] erit A tot partes numeri B, quot sunt in A unitates. (ut $6 = \frac{6}{7}\ 7.$) Sin A metiatur B, [b] liquet A esse partem ipsius B. (ut $6 = \frac{1}{3}\ 18.$) denique si A, & B aliter compositi inter se fuerint, [c] maxima communis mensura determinabit, quot partes A conficiat ipsius B; ut $6 = \frac{2}{3}\ 9.$

a 4. def. 7.
b 3. def. 7.
c 4 def 7.

PROP. V.

A 6   D ..., 4
6   6   4   4
B ..... G ..... C 12.   E .... H .... F 8

Si numerus A *numeri* BC *pars fuerit, & alter* D *alterius* EF *eadem pars; & simul uterque* (A+D) *utriusque simul* (BC+EF) *eadem pars erit, quæ unus* A *unius* BC.

Nam si BC in suas partes BG, GC ipsi A æquales; atque EF in suas partes FH, HF ipsi D æquales resolvantur; [a] erit numerus partium in BC æqualis numero partium in EF. Quum igitur A+D [b] = BG+EH = GC+HF, erit A+D toties in BC+EF, quoties A in BC. Q. E. D.

Vel sic breviús. Sit $a = \frac{x}{2}$ & $b = \frac{y}{2}$. [c] ergò

$$a+b = \frac{x}{2} + \frac{y}{2} = \frac{x+y}{2}.$$ Q. E. D.

a hyp.
b const. & 2. ax. 1.
c 2. ax. 1.

PROP.

Prop. VI.

3 3 4 4

A ... G ... B 6 D .... H .... E 8

C .... 9 F 12

Si numerus AB *numeri* C *partes fuerit; & alter* DE *alterius* F *eædem partes & simul uterq;* (AB + DE) *utriusq; simul* (C + F) *eadem partes erit, quæ unus* AB *unius* C.

Divide AB in suas partes AG, GB; & DE in suas DH, HE. Partium in utroque AB, DE æqualis est multitudo, ex hypoth. Quum igitur AG [a] sit eadem pars numeri C, a *hyp.*
quæ DH numeri F, [b] erit AG + DH eadem b 5. 7.
pars compositi C + F, quæ unus AG unius C. [b] Eodem modo GB + HE eadem pars est ejusdem C + F, quæ unus GB unius C; [c] ergò c 2. *ax.* 7.
AB + DE eædem partes est ipsius C + F, quæ AB ipsius C. Q. E. D.

Vel sic. Sit $a = \frac{2}{3}x$. & $b = \frac{2}{3}y$. [d] ergò $a + b =$ a 2. *ax.* 1.
$\frac{2}{3}x + \frac{2}{3}y = \frac{2}{3}\ y + x$. Q. E. D.

Prop. VII.

5 3

A ..... E ... B 8

6 10 6

G ...... C .... F ...... D 16

Si numerus AB *numeri* CD *pars fuerit, qualis ablatus* AE *ablati* CF; *& reliquus* EB *reliqui* FD *eadem pars erit, qualis totus* AB *totius* CD.

[a] Sit EB eadem pars numeri GC, quæ AB a 1. *post.* 7.
ipsius CD, vel AE ipsius CF. [b] ergò AE + EB b 5. 7.
eadem est pars ipsius CF + GC, quæ AE ipsius CF, vel AB ipsius CD. [c] ergò GF = CD. au- c 6. *ax.* 1.
fer communem CF, [d] manet GC = FD. [e] ergò d 3. *ax.* 1.
EB eadem est pars reliqui FD (GC) quæ totus e 2. *ax.* 7.
AB totius CB. Q. E. D.

Vel sic. Sit $a + b = x$; & $c + d = y$; atque tam $x = 3\ y$; quàm $a = 3\ c$; dico $b = 3\ d$. Nam
$3\ c + 3\ d$ [f] $= 3\ y = x$ [g] $= a + b$. aufer utrinq; f 1. 2.
$3\ c$ [g] $= a$ & [h] remanet $3\ d = b$. Q. E. D. g *hyp.*

Prop. VIII.

```
      6    2   4    2   2
A ...... H .. G .... E .. L .. B 16
          18         6
C ................ F ...... D 24
```

Si numerus AB *numeri* CD *partes fuerit, quales ablatus* AE *ablati* CF; *& reliquus* EB *reliqui* FD *eadem partes erit, quales totus* AB *totius* CD.

Seca AB in AG, GB partes numeri CD; item AE in AH, HE partes numeri CF; & sume GL = AH = HE; [a] quare HG = EL. &
h 3 ax. 1. quia [b] AG = GB, [c] etiam HG = LB. Cùm i-
a 3. ax. 1. gitur totus AG eadem sit pars totius CD, quæ
b constr. ablatus AH ablati CF; [d] erit reliquus HG,
c 3. ax. 1. vel EL eadem etiam pars reliqui FD, quæ
d 7. 7. AG ipsius CD. Eodem pacto, quia GB eadem pars est totius CD, quæ HE, vel GL ipsius CF, [d] erit reliquus LB eadem pars reliqui FD, quæ GB totius CD; ergò EL + LB (EB) eædem est partes reliqui FD, quæ totus AB totius CD. Q. E. D.

Vel sic faciliùs. Sit $a + b = x$. & $c + d = y$.
e 9 ax. 7. Item tam $y = \frac{2}{3} x$; quàm $c = \frac{2}{3} a$; vel [e] quod idem est, $3 y = 2 x$; & $3 c = 2 a$. Dico $d = \frac{2}{3} b$.
f 1. 2. Nam $3 c + 3 d$ [f] $= 3 y = 2 x$ [f] $= 2 a + 2 b$.
g 1. ax. 1. [g] ergò $3 c + 3 d = 2 a + 2 b$. aufer utrinque
h hyp. $3 c$ [h] $= 2 a$; & [k] manet $3 d = 2 b$. [l] ergò $d = \frac{2}{3} b$.
k 3 ax. 1.
l 8. ax. 7. Q. E. D.

Prop. IX.

```
     A .... 4
     4    4
B .... G .... C 8
     5 D ..... 5
E ..... H ..... F 10
```

Si numerus A *numeri* BC *pars fuerit, & alter* D *alterius* EF *eadem pars, & vicissim qua pars est, aut partes primus* A *tertii* D, *eadem pars erit, vel eadem partes & secundus* BC *quarti* EF.

Poni-

Ponitur A $\sqsupset$ D. Sint igitur BG, GC, & EH, HF partes numerorum BC, EF, hæ ipsi A, illæ ipsi D pares. Utrinque multitudo partium æqualis ponitur. Liquet verò BG [a] eandem esse partem, aut easdem partes ipsius EH, quæ GC ipsius HF; [b] quare BC (BG + GC) ipsius EF (EH + HF) eadem pars est aut partes; quæ unus BG (A) unius EH (D) Q. E. D. a 1. ax. 7. & 4. 7. b 5, vel 6. 7.

Vel sic; Sit $a = \frac{b}{3}$. & $c \sqsupset \frac{d}{3}$. dico a 1. ax 7.

$\frac{c}{a} = \frac{d}{b}$. Nam $\frac{c}{a}$ [a] $= 3\frac{d}{3b} = \frac{d}{b}$.

Prop. X.

A .. G .. B 4
C ....., 6
5 5
D ..... H ..... E 10
F 15

Si numerus AB *numeri* C *partes fuerit, & alter* DE *alterius* F *eadem partes: Et vicissim quæ partes est primus* AB *tertii* DE, *aut pars: Eædem partes erit & secundus* C *quarti* F, *aut pars.*

Ponitur AB $\sqsupset$ DE, & C $\sqsupset$ F. Sint AG, GB, & DH, HE partes numerorum C, & F, tot nempe in AB, quot in DE. Constat AG ipsius C eandem esse partem, quæ DH ipsius F. [a] quare vicissim AG ipsius DH, pariterque GB ipsius HE, & [b] proinde conjunctim AB ipsius DE eadem pars erit, aut partes, quæ C ipsius F. Q. E. D. a 9. 7. b 5. & 9. 7.

Applicare potes secundam præcedentis demonstrationem etiam huic.

Prop. XI.

4 3
A .... E ... B 7
8 6
C .... F ...... D 14

Si fuerit, ut totus AB *ad totum* CD, *ità ablatus* AE *ad ablatam* CF; & *reliquus* EB *ad reliquum*

FD *erit, ut totus* AB *ad totum* CD.

a 4.7. Sit primò AB ⊐ CD, [a] ergò AB vel pars
b 20. *def.* eſt, vel partes numeri CD; [b] eadémque pars eſt,
c 7, vel 8.7. vel partes ipſe AE ipſius CF; [c] ergò reliquus EB
reliqui FD eadem pars eſt, aut partes, quæ totus
AB totius CD. [b] ergò AB. CD :: EB. FD.
Sin fuerit AB ⊏ CD; eodem modo erit juxta
modò oſtenſa, CD. AB :: FD. EB. ergò invertendo AB. CD :: EB. FD.

Prop. XII.

A, 4. C, 2. E, 3. B, 8. D, 4. F, 6.

Si ſint quotcunq; numeri proportionales (A. B :: C. D :: E. F) *erit quemadmodum unus antecedentium* A *ad unum conſequentium* B, *ità omnes antecedentes* (A + C + E) *ad omnes conſequentes* (B + D + F).

Sint primò, A, C, E minores, quàm B, D, F.
a 20. *def.* 7. ergò (propter eaſdem rationes) [a] erit A eadem
b 5, & 6. 7. pars aut partes ipſius B, quæ C ipſius D, [b] ergò
conjunctim A + C eadem erit pars aut partes
ipſius B + D; quæ unus A unius B. Similiter
A + C + E eadem pars eſt, aut partes ipſius
c 20. *def.* 7. B + D + F, quæ A ipſius B. [c] ergò A + C +
E. B + D + F :: A. B. Q.E.D. Sin A, CE,
ipſis B, D, F majores ponantur, idem oſtendetur invertendo.

Prop. XIII.

A, 3. C, 4. B, 5. D, 12.

Si quatuor numeri proportionales ſint (A. B :: C. D. *& viciſſim proportionales erunt* (A. C :: B. D.)

Sint primò A, & C ipſis B, & D minores,
a 20. *def.* 7. atque A ⊐ C. Ob eandem proportionem, [a] erit
b 9. & 10. 7. A eadem pars, aut partes ipſius B, quæ C ipſius
D, [b] ergò viciſſim A ipſius C eadem pars eſt, aut
partes, quæ B ipſius D. ergò A. C :: B. D. Sin

A ⊏

A ⊏ C; atque A, & C majores statuantur, quàm B, & D, eadem res erit, proportiones invertendo.

Prop. XIV.

A, 9. D, 6.
B, 6. E, 4.
C, 3. F, 2.

Si sint quotcunque numeri A, B, C, & alii totidem D, E, F illis æquales multitudine, qui bini sumantur, & in eadem ratione (A. B :: D. E. & B. C :: E. F) *etiam ex æqualitate in eadem ratione erunt.* (A. C :: D F)

Nam quia A. B :: D. E, [a] erit vicissim, A. D :: B. E :: [a] C. F. [a] ergò iterum permutando A. C :: D. F. Q. E. D. a 13. 7.

Prop. XV.

1 . D..
B ... 3. E ...... 6.

Si unitas numerum quempiam B metiatur; æquè autem alter numerus D alterum quendam numerum E metiatur; & vicissim æquè unitas tertium numerum D metietur, & secundus B quartum E.

Nam quia 1 est eadem pars ipsius B, quæ D ipsius E, [a] erit vicissim 1 eadem pars ipsius D, quæ B ipsius E. Q. E. D. a 9. 7.

Prop. XVI.

B, 4. A, 3.
A, 3. B, 4.
AB, 12. BA, 12.

Si duo numeri A, B sese mutuò multiplicantes fecerint aliquos AB, BA, geniti ex ipsis AB, BA æquales inter se erunt.

Nam quia AB = A in B, [a] erit 1 in A toties quoties B in AB. [b] ergò vicissim 1 in B toties erit, quoties A in AB. atqui quoniam BA = B in A, [a] erit 1 in B toties, quoties A in BA. ergò quoties 1 in AB, toties 1 in BA; & [c] proinde AB = BA. Q. E. D.

a 15. def. 7.
b 15. 7.
c 4. ax. 7.

PROP. XVII.

A, 3.
B, 2. C. 4.
AB, 6. AC, 12.

Si numerus A duos numeros B, C multiplicans fecerit aliquos AB, AC; geniti ex ipsis eandem rationem habebunt, quam multiplicati. (AB. AC :: B. C.)

a 15. def. 7. Nam quia AB = A in B, [a] erit 1 toties, in A, quoties B in AB. [a] item quia AC = A in C. erit 1 toties in A, quoties C in AC. ergò quo-
b 20. def. 7. ties B in AB, toties C in AC. quare B. AB ::
c 13. 7. C. AC. [c] ergò vicissim, B. C :: AB. AC. Q. E. D.

PROP. XVIII.

C, 5. C, 5.
A, 3. B, 9.
$\overline{AC}$, 15. $\overline{BC}$, 45.

Si duo numeri A, B, numerum quempiam C multiplicantes fecerint aliquos AC, BC; geniti ex ipsis eandem rationem habebunt, quam multiplicantes. (A. B :: AC. BC.)

a 16. 7. Nam AC [a] = CA; & BC [a] = CB; sic idem
b 17. 7. C multiplicans A, & B producit AC, & BC. [b] ergò A. B :: AC. BC. Q. E. D.

Schol.

Ex his pendet modus vulgaris reducendi fractiones ($\frac{3}{5}$, $\frac{7}{9}$) ad eandem denominationem. Nam duc 9 tam in 3, quàm in 5, proveniunt $\frac{27}{45} = \frac{3}{5}$ quoniam ex his, 3. 5 :: 27. 45. item duc 5 in 7, & 9, prodeunt $\frac{35}{45} = \frac{7}{9}$. quia 7. 9 :: 35. 45.

PROP. XIX.

A, 4. B, 6. C, 8. D, 12.
AD, 48. BC, 48.

Si quatuor numeri proportionales fuerint, (A B :: C. D) *; qui ex primo & quarto fit numerus AD, aequalis est ei, qui ex secundo & tertio fit, numero* BC.

BC. *Et si qui ex primo & quarto fit numerus* AD, *aequalis sit ei, qui ex secundo & tertio fit, numero* BC, *ipsi quatuor numeri proportionales erunt.* (A. B :: C. D.) a 17. 7.

1. *Hyp.* Nam AC. AD [a] :: C. D [b] :: A. B [c] :: AC. BC. [d] ergò AD = BC. Q. E. D. b hyp. c 18. 7. d 9. 5.

2. *Hyp.* Quoniam [e] AD = BC, erit AC. AD [f] :: AC. BC. Sed AC. AD [g] :: C. D. & AC. BC [h] :: A. B. [k] ergò C. D :: A. B. Q. E. D. e hyp. f 7. 5. g 17. 7. h 18. 7. k 11. 5.

Prop. XX.

A.	B.	C.
4.	6.	9.
AC, 36.	BB, 36.	
	D, 6.	

Si tres numeri proportionales fuerint (A. B :: B. C.) *qui sub extremis continetur* (AC), *aequalis est ei, qui à medio efficitur* (BB). *Et si qui sub extremis continetur* (AC) *aequalis fuerit ei* (Bq), *qui sub medio, ipsi tres numeri proportionales erunt* $\left(\frac{A}{B} :: \frac{B}{C}\right)$. a 1. ax. 7.

1. *Hyp.* Nam sume D = B. [a] ergò A. B :: D (B). C. [b] quare AC = BD, [a] vel BB. Q. E. D. b 19. 7.

2. *Hyp.* Quia AC [c] = BD, [d] erit A. B :: D (B). C. Q. E. D. c hyp. d 19. 7.

Prop. XXI.

A ... G .. B 5. E .... 10.
C .. H. D 3. F ...... 6.

Numeri AB, CD *minimi omnium eandem cum eis rationem habentium* (E, F) *metiuntur aequè numeros* E, F *eandem cum eis rationem habentes, major quidem* AB *majorem* E, *minor verò* CD *minorem* F.

Nam AB. CD [a] :: E. F. [b] ergò vicissim AB. E :: CD. F. [c] ergò AB eadem pars est, vel partes ipsius E, quae CD ipsius F. Non partes, nam si ità, sint AG, GB partes numeri E; & CH, HD partes numeri F. [c] ergò AG. E :: a hyp. b 13. 7. c 20. def. 7.

d 13. 7. e hyp. CH. F; & permutando AG. CH [d] :: E. F [e] :: AB. CD. ergò AB, CD non sunt minimi in suâ ratione, contra hypoth. ergò, &c.

PROP. XXII.

A, 4.	D, 12.	
B, 3.	E, 8.	
C, 2.	F, 6.	

Si fuerint tres numeri A, B, C; *& alii ipsis multitudine æquales* D, E, F; *qui bini sumantur, & in eadem ratione; fuerit autem perturbata eorum proportio* (A.B :: E.F & B.C :: D.E); *etiam ex æqualitate in eadem ratione erunt* (A. C :: D. F.)

a hyp. b 19. 7. c 1. ax. 1. d 19. 7. Nam quia A. B [a] :: E. F, erit AF = BE; & quia B. C :: [a] D. E, [b] erit BE = CD. [c] ergò AF = CD. [d] quare A. C :: D. F. Q. E. D.

PROP. XXIII.

A, 9.	B, 4.
C ----	D ---
E --	

Primi inter se numeri A, B, *minimi sunt omnium eandem cum eis rationem habentium.*

a 21. 7. b 23. def. 7. c 15. 7. Si fieri potest, sint C, & D minores, quàm A, & B, atque in eadem ratione. [a] ergò C metitur A æquè, ac D metitur B, putà per eundem numerum E: quoties igitur 1 in E, [b] toties erit C in A. [c] quare vicissim quoties 1 in C toties E in A. simili discursu quoties 1 in D, toties E in B. ergò E utrumque A, & B metitur; qui proinde inter se primi non sunt. contra Hypoth.

PROP. XXIV.

A, 9.	B, 4.
C ---	
D ---	E --

Numeri A, B, *minimi omnium eandem cum eis rationem habentium, primi inter se sunt.*

a 9. ax. 7. b 17. 7. Si fieri potest habeant A, & B communem mensuram C; is metiatur A per D; & B per E; [a] ergò CD = A, [b] & CE = B. [b] quare

[b] quare A. B :: D. E. Sed D, & E minores sunt, quàm A, & B, utpote eorum partes. Ergò A, & B non sunt minimi in sua ratione, contra hypoth. b 17. 7.

Prop. XXV.

A. 9. B. 4.
C 3. D. -

Si duo numeri A, B *primi inter se fuerint, qui unum eorum* A *metitur numerus* C, *ad reliquum* B *primus erit.*

Nam si affirmes aliquem D numeros B, & C metiri, [a] ergò D metiens C, metitur A. ergò A & B non sunt primi inter se, contra Hypoth. a 11. ax. 7.

Prop. XXVI.

A, 5. C, 8.
B, 3.
AB, 15. E - - - -
F - - - -

Si duo numeri A, B *ad quempiam* C *primi fuerint, etiam ex illis genitus* AB *ad eundem* C *primus erit.*

Si fieri potest, sit ipsorum AB, & C communis mensura, numerus E. sítque $\frac{AB}{E} = F$; [a] ergò $AB = EF$; [b] quare E. A :: B. F Quia verò A primus est ad C quem E metitur, [c] erunt E & A primi inter se, [d] adeóque in sua proportione minimi, & [e] proinde æquè metiuntur B, & F; nempe E ipsum B, & A ipsum F. Quum igitur E utrumque B, C. metiatur, non erunt illi primi inter se, contra Hypoth.

a 9. ax. 7.
b 19. 7.
c 25. 7.
d 23. 7.
e 21. 7.

Prop. XXVII.

A, 4. B, 5.
Aq, 16.
D, 4.

Si duo numeri, A, B, primi inter se fuerint, etiam ex uno eorum genitus (Aq) ad reliquum B primus erit.

Sume D = A; ergò [a] singuli D, & A primi sunt ad B. [b] quare AD, vel Aq, ad B primus est. Q. E. D. a 1. ax. 7. b 26. 7.

Prop.

Prop. XXVIII.

A, 5.	C, 4.	*Si duo numeri* A, B *ad*
B, 3.	D, 2.	*duos numeros* C, D, *u-*
AB, 15.	CD 8.	*terque ad utrumque primi*

fuerint, & qui ex eis gignentur AB, CD, *primi inter se erunt.*

a 26. 7. Nam quia A & B ad C primi sunt, [a] erit AB ad C primus. Eâdem ratione erit AB ad D
b 26. 7. primus. [b] ergò AB ad CD primus est. Q. E. D.

Prop. XXIX.

A, 3.	B, 2.	*Si duo numeri* A, B *primi*
Aq, 9.	Bq, 4.	*inter se fuerint; & multipli-*
Ac, 27.	Bc, 8.	*cans uterque seipsum fecerit a-*

liquem (Aq, & Bq); *& geniti ex ipsis* (Aq, Bq) *primi inter se erunt; & si qui in principio* A, B *genitos ipsos* Aq, Bq *multiplicantes fecerint aliquos* (Ac, Bc); *& hi primi inter se erunt: & semper circa extremos hoc eveniet.*

a 27. 7. Nam quia A primus est ad B, [a] erit Aq ad B primus. & quia Aq primus ad B, [a] erit Aq ad Bq primus. Rursus quia tam A ad B, & Bq;
b 28. 7. quàm Aq ad eosdem B, & Bq primi sunt, [b] erit A x Aq, id est Ac, ad B x Bq, id est Bc, primus. Et sic porrò de reliquis.

Prop. XXX.

8 5	*Si duo numeri*
A .... B ..... C 13. D ----	AB, BC *primi*
	inter se fuerint,

etiam uterque simul (AC) *ad quemlibet illorum* AB, BC *primus erit. Et si uterque simul* AC *ad unum aliquem illorum* AB *primus fuerit, etiam qui in principio numeri* AB, BC *primi inter se erunt.*

1. *Hyp.* Nam si AC, AB compositos velis,
a 12. ax. 7. sit D communis mensura. [a] Is metietur reliquum BC. ergò AB, BC non sunt primi inter se, contra Hypoth.

2. Hyp.

2. *Hyp.* Positis AC, AB inter se primis, vis D ipsorum AB, BC communem esse mensuram. [b] Is igitur totum AC metitur. quare AC, AB non sunt primi inter se, contra Hypoth. b 10. ax. 7.

Coroll.

Hinc numerus, qui ex duobus compositus, ad unum illorum primus est, ad reliquum quoque primus est.

Prop. XXXI.

A 5, B, 8. *Omnis primus numerus A ad omnem numerum B, quem non metitur, primus est.*

Nam si communis aliqua mensura metiatur utrumque A, B; [a] non erit A primus numerus, contra Hypoth. a 11. def. 7.

Prop. XXXII.

A, 4. D, 3.
B, 6. E, 8.
$\overline{AB}$, 24.

Si duo numeri AB, se mutuò multiplicantes fecerint aliquem AB; genitum autem ex ipsis AB metiatur aliquis primus numerus D, is etiam unum eorum, qui à principio, A, vel B metietur.

Pone numerum D non metiri A; sit verò $\frac{AB}{D} = E$. [a] ergò AB = DE. [b] quare D. A :: B. E. [c] est verò D ad A primus. [d] ergò D, & A minimi sunt in suâ ratione; [e] proinde D metitur B, æquè ac A metitur E. liquet igitur propositum.

a 9. ax. 7.
b 19. 7.
c hyp. & 31. 7.
d 23. 7.
e 21. 7.

Prop. XXXIII.

A, 12. *Omnem compositum numerum A, aliquis primus numerus B metitur.*
B, 2.

Unus vel plures numeri [a] metiantur A, quorum minimus sit B. is primus erit. a 13. def. 7.

a 13. def. 7. nam si dicetur compositus, [a] eum minor aliquis
b 11. ax. 7. metietur, [b] qui proinde ipsum A metietur. quare B non est minimus eorum, qui A metiuntur; contra Hypoth.

PROP. XXXIV.

Omnis numerus A aut primus est, aut
A, 9. *eum aliquis primus metitur.*

Nam A necessariò vel primus est, vel compositus. Si primus hoc est quod asserimus.
a 33. 7. Si compositus, [a] ergò eum aliquis primus metitur. Q. E. D.

PROP. XXXV.

A, 6. B, 4. C, 8. H - - I - - K - - - -
D, 2. L - - -
E, 3. F, 2. G, 4.

Numeris datis quotcunque A, B, C *reperire minimos omnium* E, F, G *eandem rationem cum eis habentium.*

Si A, B, C primi sint inter se, ipsi in sua ra-
a 23. 7. tione minimi [a] erunt. Si compositi sint, [b] esto
b 3. 7. eorum maxima communis mensura D, qui ipsos metiatur per E, F, G. Hi minimi erunt in ratione A, B, C.
c 9. ax. 7. Nam D ductus in E, F, G [c] producit ABC,
d 17. 7. [d] ergò hi & illi in eadem sunt ratione. Jam puta
e 21. 7. alios H, I, K minimos esse in eadem; [e] qui propterea æquè metientur A, B, C, nempe per nu-
f 9. ax. 7. merum L. [f] ergò L in H, I, K ipsos A, B, C
g 1. ax. 1. procreabit. [g] ergò ED = A = HL. [h] unde E.
h 19. 7. H :: L. D. Sed E [k] ⊏ H; [l] ergò L ⊏ D. ergò
k suppos. D non est maxima communis mensura ipsorum
l 20. def. 7. A, B, C; contra Hypoth.

Coroll.

Hinc maxima communis mensura quotlibet numerorum

numerorum metitur ipsos per numeros, qui minimi sunt omnium eandem rationem cum ipsis habentium. Ex quo patet methodus vulgaris reducendi fractiones ad minimos terminos.

Prop. XXXVI.

Duobus numeris datis A, B; *reperire quem illi minimum metiuntur, numerum.*

A, 5. B, 4.
AB, 20.
D - - - - - -
E - - - F - - -

1. *Cas.* Si A, & B primi sint inter se, est AB quæsitus. Nam liquet A, & B metiri AB. Si fieri potest, metiantur A & B aliquem D ⊐ AB; puta per E, & F. [a] ergò AE = D = BF. [b] quare A. B :: F. E. Quia verò A, & B [c] primi sunt inter se, [d] adeóque in sua ratione minimi, [e] æquè metientur A ipsum F, ac B ipsum E. Atqui B. E [f] :: AB. AE (D). [g] ergò AB etiam metietur D, seipso minorem. Q. E. A.

a 9. ax. 7. & 1. ax. 1.
b 19. 7.
c hyp.
d 23. 7.
e 21. 7.
f 17. 7.
g 20. def. 7.

A, 6. B, 4. F - - - -
C, 3. D, 2. G - - - H - - -
AD, 12

2. *Cas.* Sin A, & B inter se compositi fuerint, [h] reperiantur C, & D minimi in eadem ratione. [k] ergò AD = BC. Erit AD, vel BC quæsitus.

Nam [l] liquet B, & A ipsum AD, vel BC metiri. Puta A, & B metiri F ⊐ AD, nempe A per G, & B per H. [m] ergò AG = F = BH. [n] unde A. B :: H. G [o] :: C. D. [p] proinde æquè metitur C ipsum H, ac D ipsum G. atqui D. G [q] :: AD. AG (F). ergò AD [r] metitur F, major minorem. Q. E. A.

h 35. 7.
k 19. 7.
l 7. ax. 7.
m 9. ax. 7.
n 19. 7.
o constr.
p 21. 7.
q 17. 7.
r 20. def. 7.

Coroll.

Hinc, si duo numeri multiplicent minimos eandem rationem habentes, major minorem, & minor majorem, producetur numerus minimus, quem illi metiuntur.

PROP. XXXVII.

A, 2. B, 3.
E, ... 6.
C - - - - F - - - D

Si duo numeri A, B numerum quempiam CD metiantur; etiam minimus E, quem illi metiuntur, eundem CD metietur.

Si negas, aufer E ex CD, quoties fieri potest, & relinquatur FD ⊐ E. quum igitur A & B [a] metiantur E, [b] & E ipsum CF, [c] etiam A, & B metiuntur CF; [a] metiuntur autem totum CD; [d] ergò etiam reliquum FD metiuntur. ergò E non est minimus, quem A, & B metiuntur, contra hyp.

a hyp.
b constr.
c 11. ax. 7.
d 12. ax. 7.

PROP. XXXVIII.

A, 3, B, 4, C, 6.
D, 12.

Tribus numeris datis A, B, C reperire minimum, quem illi metiuntur.

a 36. 7.

[a] Reperi D minimum, quem duo A, & B metiuntur, quem si tertius C metiatur, patet D esse quæsitum. Quòd si C non metiatur D, sit E minimus, quem C, & D metiuntur. Erit E requisitus.

A, 2. B, 3. C, 4.
D, 6. E, 12.
F - - -

Nam singulos A, B, C metiri E constat ex 11. ax. 7. Quòd verò nullum alium F minorem metiantur, facilè ostenditur. Nam si affirmas, [b] ergò D metitur F; [b] proinde E eundem F metitur, major minorem, Quod est absurdum.

b 37. 7.

Coroll.

Hinc, si tres numeri numerum quempiam metiantur, etiam minimus, quem illi metiuntur, eundem metietur.

Prop. XXXIX.

A, 12. B, 4, C, 3.

Si numerum A quispiam numerus B metiatur, ille A quem B metitur, partem habebit C, à metiente B denominatam.

Nam quia $\frac{A}{B}$ [a] $= C$, [b] erit $A = BC$. [c] ergò $\frac{A}{C} = B$. Q. E. D.

a *hyp.*
b *9. ax. 7.*
c *7. ax. 7.*

Prop. XL.

A, 15. B, 3. C, 5.

Si numerus A partem habuerit quamlibet B, metietur illum numerus C, à quo ipsa pars B denominatur.

Nam quia BC [a] $= A$, [b] erit $\frac{A}{C} = B$. Q.E.D.

a *hyp.* & *9. ax. 7.*
b *7. ax. 7.*

Prop. XLI.

$\frac{1}{2}$ G, 12.
$\frac{1}{3}$ H - - -
$\frac{1}{4}$

Numerum reperire G, qui minimus cùm sit, habeat datas partes, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.

[a] Inveniatur G minimus, quem denominatores 2, 3, 4 metiuntur. [b] Liquet G habere partes, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. Si fieri potest $H \sqsupset G$ habeat easdem partes; [c] ergò 2, 3, 4 metiuntur H, & proinde G non est minimus, quem 2, 3, 4 metiuntur. contra constr.

a 38. 7.
b 39. 7.
c 40. 7.

LIB. VIII.

Prop. I.

A, 8. B, 12. C, 18. D, 27.
E - F -- G --- H ----

Si fuerint quotcunque numeri deinceps proportionales A, B, C, D *extremi verò ipsorum* A, D *primi inter se fuerint, ipsi* A, B, C, D *minimi sunt omnium eandem cum eis rationem habentium.*

Nam, si fieri potest, sint alii totidem E, F, G, H
a 14.7. minores in illa ratione. [a] ergò ex æquali A.D : :
b 23.7. E.H. ergò A, & D primi numeri, [b] adeóque in
c 21.7. sua ratione minimi [c] æquè metiuntur E, & H seipsis minores. Q. E. A.

Prop. II.

1.
A, 2. B, 3.
Aq, 4. AB, 6. Bq, 9.
Ac, 8. AqB, 12. ABq, 18. Bc, 27.

Numeros reperire deinceps proportionales minimos, quotcunque jusserit quispiam, in data ratione A *ad* B.

Sint A, & B minimi in data ratione. Erunt Aq, AB, Bq tres minimi deinceps in ratione A ad B.

a 17.7. Nam AA. AB [a] : : A. B [a] : : AB. BB.
b 24.7. item quia A, & B [b] primi sunt inter se, [c] erunt
c 29.7. Aq, Bq inter se primi; [d] proinde Aq, AB, Bq
d 1.8. sunt ∺ minimi in ratione A ad B.

Dico porrò, Ac, AqB, ABq, Bc in ratione A ad B quatuor esse minimos. Nam AqA,
e 17.7. AqB[e] : : A.B [e] : : ABA (AqB) ABB. [e] atq;
A.B : : ABq. BBq. (Bc) Quum igitur Ac, &

Bc

& Bc [f] inter se primi sint, [g] erunt Ac, AqB, ABq, Bc quatuor ∺ minimi in ratione A ad B. Eodem modo quotvis proportionales investigabis. Q. E. F. f 29. 7. g 1. 8.

Coroll.

1. Hinc, si tres numeri minimi sunt proportionales, extremi quadrati erunt; si quatuor, cubi.

2. Extremi quotcunque proportionales per hanc propos. inventi in data ratione minimi, inter se primi sunt.

3. Duo numeri, minimi in data ratione, metiuntur omnes medios quotcunque minimorum in eadem ratione, quia scilicet producuntur ex illorum multiplicatione, in alios quosdam numeros.

4. Hinc etiam liquet ex constructione, series numerorum 1, A, Aq, Ac; 1, B, Bq, Bc; Ac, AqB, ABq, Bc. constare æquali multitudine numerorum, ac proinde extremos numeros quotcunque minimorum continuè proportionalium, esse ultimos totidem continuè proportionalium ab unitate. ut extremi Ac, Bc continuè proportionalium Ac, AqB, ABq, Bc sunt ultimi totidem proportionalium ab unitate 1, A, Aq, Ac; & 1, B, Bq, Bc.

5. 1, A, Aq, Ac; & B, BA, BAq; ac Bq, ABq sunt ∺ in ratione 1 ad A. item, B, Bq, Bc; & A, AB, ABq; ac Aq, AqB sunt ∺ in ratione 1 ad B.

PROP. III.

A, 8. B, 12. C, 18. D, 28. *Si sint quotcunque numeri* A, B, C, D *deinceps proportionales, minimi omnium eandem cum eis rationem habentium; illorum extremi* A, D *sunt inter se primi.*

Nam

a 2. 8. Nam si [a] inveniantur totidem numeri minimi in ratione A ad B, illi non alii erunt, quàm A, B, C, D; ergò juxta 2. coroll. præcedentis extremi A & D primi sunt inter se. Q. E. D.

PROP. IV.

A, 6, B, 5. C, 4. D, 3.
H, 4. F, 24, E, 20. G, 15.
I - - K - - L - - -

Rationibus datis quotcunq; in minimis terminis, (A ad B, & C ad D) *reperire numeros deinceps minimos in datis rationibus.*

a 36. 7. [a] Reperi E minimum, quem B, & C metiuntur; & B ipsum E [b] æquè metiatur, ac A alterum
b 3. post. 7. F, puta per eundem H. [b] item C ipsum E, ac D alterum G æquè metiantur, erunt F, E, G minimi in datis rationibus. Nam AH [c] = F; &
c 9. ax. 7. BH [c] = E. [d] ergò A. B : : AH. BH [e] : : F. E.
d 18. 7. Similiter C. D : : E. G. sunt igitur F, E, G deinceps proportionales in datis rationibus. Imò minimi sunt in iisdem: nam puta alios I, K, L minimos esse. [f] ergò A, & B ipsos I, & K, [f] pariterque C & D ipsos K & L æquè metiuntur. ergò B, & C eundem K metiuntur. [g] Quare etiam E eundem K metitur, seipso minorem. Q. E A.
e 7. 5.
f 21. 7.
g 37. 7.

A, 6. B, 5. C, 4. D, 3. E, 5. F, 7.
H, 24, G, 20. I, 15. K, 21.

Datis verò tribus rationibus A ad B, & C ad D; ac E ad F. Reperi, ut priùs, tres H, G, I minimos deinceps in rationibus A ad B, & C ad D. tunc si E numerum I metiatur,

h 3. post. 7. [h] Sume alterum K, quem F æquè metiatur; erunt quatuor H, G, I, K deinceps minimi, in datis rationibus, quod non aliter probabimus, quàm in priori parte.

A, 6.

A, 6. B, 5. C, 4. D, 3. E, 2. F, 7.
H, 24. G, 29. I, 15.
M, 48. L, 40. K, 30. N, 105.

S in E non metiatur I, sit K minimus, quem E, & I metiuntur; & quoties I ipsum K, toties G ipsum L, & H ipsum M metiatur. quoties verò E ipsum K, toties F ipsum N metiatur, Erunt M, L, K, N minimi deinceps in datis rationibus, quod demonstrabimus, ut priùs.

Prop. V.

C, 4. E, 3.
D, 6 F, 16 ED, 18.
$\overline{CD}$, 24. $\overline{EF}$, 48.

Plani numeri CD, EF rationem habent ex lateribus compositam.

$$\left(\frac{\overline{CD}}{\overline{EF}} = \frac{C}{E} + \frac{D}{F}.\right)$$

Nam quia CD.ED [a] :: C. E; [a] & ED.EF :: D. F. atque $\frac{\overline{CD}}{\overline{EF}}$ [b] $= \frac{\overline{CD}}{\overline{ED}} + \frac{\overline{ED}}{\overline{EF}}$, [c] erit ratio $\frac{\overline{CD}}{\overline{EF}} = \frac{C}{E} + \frac{D}{F}$. Q. E. D.

a 17. 7.
b 20. *def.* 5.
c 11. 5.

Prop. VI.

A, 16. B, 24. C, 36. D, 54. E, 81.
F, 4. G, 6. H, 9.

Si sint quotcunque numeri deinceps proportionales A, B, C, D, E: *primus autem* A *secundum* B *non metiatur, neque alius quispiam ullum metietur.*

Quoniam A non metitur B, [a] neque quilibet proximè sequentem metietur; quia A.B :: B.C :: C. D, &c. [b] Accipe tres F, G, H minimos in ratione A ad B. quoniam igitur A non metitur B, [a] neque F metietur G. [c] ergò F non est unitas. sed F, & H inter se primi sunt; ergò quum [e] sit ex æquo A. C :: F. H, & F non metiatur H, [a] neque A ipsum C metietur; proinde nec B ipsum D, nec C ipsum E, &c. quia A. C [e] :: B. D [e] :: C. E, &c. Eodem modo sumptis

a 20. *def.* 7.
b 35. 7.
c 5. *ax.* 7.
d 3. 3.
e 14. 7.

ſumptis quatuor vel quinque minimis in ratione A ad B, oſtendetur A ipſos D, & E; ac B ipſos E, & F non metiri, &c. Quare nullus alium metietur. Q. E. D.

PROP. VII.

A, 3. B, 6. C, 12. D, 24. E, 48.

Si ſint quotcunque numeri deinceps proportionales A, B, C, D, E; primus autem A extremum E metiatur, is etiam metitur ſecundum B.

a 6. 7. Si negas A metiri B, [a] ergò nec ipſum E metietur, contra Hypoth.

PROP. VIII.

A, 24. C, 36. D, 54. B, 81.
G, 8. H, 12. I, 18. K, 27.
E, 32. L, 48. M. 72. F, 108.

Si inter duos numeros A, B medii continuâ proportione ceciderint numeri C, D; quot inter eos medii continuâ proportione cadunt numeri, tot & inter alios E, F eandem cum illis habentes rationem medii continuâ proportione cadent. (L, M.)

a 35. 7. [a] Sume G, H, I, K minimos ∺ in ratione
b 14. 7. A ad C; [b] erit ex æquali, G. K :: A. B [c] :: E. F.
c hyp. Atqui G, & K [d] primi ſunt inter ſe; [e] quare G
d 3. 8. æquè metitur E, ac K ipſum F. per eundem nu-
e 21. 7. merum metiatur H ipſum L, & I ipſum M.
f conſtr. [f] itaque E, L, M, F ità ſe habent ut G, H, I, K; hoc eſt ut A, B, C, D. Q. E. D.

PROP. IX.

1.
E, 2. F, 3.
G, 4. H, 6. I, 9.
A, 8. C, 12. D, 18. B, 27.

Si duo numeri A, B ſint inter ſe primi, & inter eos medii continuâ proportione ceciderint numeri, C, D; quot inter eos medii continuâ

tinuâ proportione ceciderint numeri, totidem (E,G; & F, I) *& inter utrumque eorum ac unitatem medii continuâ proportione cadent.*

Conſtat 1, E, G, A; & 1, F, I, B eſſe ∺; & totidem quot A, C, D, B, nimirum ex 4 coroll. 2. 8. Q. E. D.

Prop. X.

A, 8. I, 12. K, 18. B, 27.
E, 4. DF, 6. G, 9.
D, 2. F, 3.
1.

Si inter duos numeros A, B, *& unitatem continuè proportionales ceciderint numeri* (E, D; & F, G,) *quot inter utrumque ipſorum, & unitatem deinceps medii continuâ proportione cadunt numeri, totidem & inter ipſos medii continuâ proportione cadent*, I, K.

Nam E, DF, G; & A,DqF (I), DG (K), B ſunt ∺, per 2. 8. ergò, &c.

Prop. XI.

A, 2. B, 3.
Aq, 4. AB, 6. Bq, 9.

Duorum quadratorum numerorum Aq, Bq *unus medius proportionalis eſt numerus* AB. *& quadratum* Aq *ad quadratum* Bq, *duplicatam habet lateris* A *ad latus* B *rationem.*

[a] Liquet Aq, AB, Bq. eſſe ∺. [b] proinde etiam $\frac{Aq}{Bq} = \frac{A}{B}$ bis. Q. E. D. a 17. 7. b 10. *def.* 5.

Prop.

Prop. XII.

Ac,27. AqB,36. ABq,48. Bc,64.
A, 3. B, 4
Aq, 9. AB, 12. Bq, 16.

Duorum cuborum numerorum Ac, Bc *duo medii proportionales sunt numeri* AqB, ABq. *Et cubus* Ac *ad cubum* Bc *triplicatam habet lateris* A *ad latus* B *rationem.*

a 2. 8. [a] Nam Ac, AqB, ABq, Bc sunt ∺ in ratio-
b 10. *def.* 5. ne A ad B. proinde $\frac{Ac}{Bc} = \frac{A}{B}$ ter. Q. E. D.

Prop. XIII.

A. 2. B, 4. C, 8.
Aq, 4, AB. 8, Bq, 16. BC, 32. Cq. 64.
Ac,8,AqB,16,ABq.32.Bc,64,BqC,128,BCq,256. Cc,512.

Si sint quotlibet numeri deinceps proportionales, A, B, C; *& multiplicans quisque seipsum faciat aliquos; qui ab illis producti fuerint* Aq, Bq, Cq *proportionales erunt; & si numeri primùm positi* A, B, C *multiplicantes jam factos* Aq, Bq, Cq, *fecerint aliquos* Ac, Bc, Cc; *ipsi quoq; proportionales erunt. & semper circa extremos hoc eveniet.*

a 2. 8. Nam Aq. AB, Bq, BC, Cq [a] sunt ∺. [b] ergò
b 14. 7. ex æquo Aq. Bq :: Bq Cq. Q. E. D.

[a] Item Ac, AqB, ABq, Bc, BqC, BCq, Cc sunt ∺, [b] ergò iterum ex æquo, Ac. Bc :: Bc. Cc. Q. E. D.

Prop. XIV.

Aq, 4. AB, 12. Bq, 36.
A. 2. B, 6.

Si quadratus numerus Aq *quadratum numerum* Bq *metiatur, & latus unius* (A) *metietur latus alterius* (B) : *& si unius quadrati latus* A *metietur latus alterius* B, *& quadratus* Aq *quadratum* Bq *metietur.*

a 2 & 11. 8. 1. *Hyp.* Nam Aq. AB [a] :: Aq. Bq; cùm igitur ex hyp. Aq metiatur Bq; idem Aq secundùm cundum AB [b] metietur. atqui Aq AB :: A. B. [c] ergò etiam A metitur B. Q. E. D. b 7. 8. c 20. *def.* 7.

2. *Hyp.* A metitur B. [c] ergò tam Aq ipsum AB, [c] quàm AB ipsum Bq metitur; [d] & proinde Aq metitur Bq. Q. E. D. d 11. *ax.* 7.

Prop. XV.

A, 2. B, 6.
Ac, 8. AqB, 24. ABq, 72. Bc, 216.

Si cubus numerus Ac, *cubum numerum* Bc *metiatur, & latus unius* (A) *metietur latus alterius* (B): *Et si latus* A *unius cubi* Ac *latus* B *alterius* Bc *metiatur; & cubus* Ac *cubum* Bc *metietur.*

1. *Hyp.* Nam Ac, AqB, ABq, Bc [a] sunt ∺, ergò Ac, [b] metiens extremum Bc, [c] etiam secundum AqB metietur. atqui Ac. AqB :: A. B. [d] ergò etiam A metietur B. Q. E. D. a 2. & 12. 8. b *hyp.* c 7. 8. d 20. *def.* 7.

2. *Hyp.* A metitur B; [d] ergò Ac metitur AqB, ísque ABq, & hic Bc; [e] ergò Ac metietur Bc. Q. E. D. e 11. *ax.* 7.

Prop. XVI.

A, 4. B, 9.
Aq, 16. Bq, 81.

Si quadratus numerus Aq *quadratum numerum* Bq *non metiatur; neq; A latus unius alterius latus* B *metietur; & si* A *latus unius quadrati* Aq *non metiatur* B *latus alterius* Bq, *neq; quadratus* Aq *quadratum* Bq *metietur.*

1. *Hyp.* Nam si affirmes A metiri B, [a] etiam Aq ipsum Bq metietur, contra hyp. a 14. 8.

2. *Hyp.* Vis Aq metiri Bq; [a] ergò A ipsum B metietur, contra Hyp.

PROP. XVII.

A, 2. B, 3. Ac, 8. Bc, 27. *Si cubus numerus* Ac *cubum numerum* Bc *non metiatur, neque A latus unius latus* B *alterius metietur. Et si latus* A *unius cubi* Ac *latus* B *alterius* Bc *non metiatur, neque cubus* Ac *cubum* Bc *metietur.*

a 15. 8. 1. *Hyp.* Dic A metiri B; [a] ergò Ac metietur Bc. contra Hypoth.

2. *Hyp.* Dic Ac metiri Bc; [a] ergò A ipsum B metietur. contra Hyp.

PROP. XVIII.

C, 6. D, 2. CD, 12. E, 9. F, 3. DE, 18. EF, 27. *Duorũ similium planorum numerorum* CD, EF, *unus medius proportionalis est numerus* DE: *& planus* CD *ad planum* EF *duplicatam habet lateris* C *ad latus homologum* E *rationem.*

* 21. def. 7. Quoniam * ex hyp. C. D :: E. F; permu-
a 17. 7. tando erit C. E :: D. F. atqui C. E [a] :: CD.
b 11. 5. DE; [a] & D. F :: DE. EF. [b] ergò CD. DE :: DE. EF. Q. E. D.

c 10. def. 5. [c] Ergò ratio CD ad EF duplicata est rationis CD ad DE; hoc est rationis C ad E, vel D ad F.

Coroll.

Hinc perspicuum est, inter duos similes planos cadere unum medium proportionalem, in ratione laterum homologorum.

PROP.

PROP. XIX.

CDE, 30. DFE, 60. FGE, 120. FGH, 240.
CD, 6. DF, 12. FG, 24.
C, 2. D, 3. E, 5. F, 4. G, 6. H, 10.

Duorum similium solidorum CDE, FGH, *duo medii proportionales sunt numeri* DFE, FGE. *Et solidus* CDE *ad solidum* FGH *triplicatam rationem habet lateris homologi* C *ad latus homologum* F.

Quoniam ex * hyp. C. D :: F. G; & D. E :: G. H; erit [a] permutando C. F :: D. G [a] :: E. H. atqui CD. DF [b] :: C. F; & DF. FG [b] :: D. G. [c] quare CD. DF :: DF. FG :: E. H. [d] ergò CDE. DFE :: DFE. FGE :: E. H :: FGE. FGH. ergò inter CDE. FGH cadunt duo medii proportionales, DFE, FGE. Q. E. D. [e] liquet igitur rationem CDE ad FGH triplicatam esse rationis CDE ad DFE, vel C ad F. Q. E. D.

* 21. def. 7.
a 13. 7.
b 17. 7.
c 11. 5.
d 17. 7.
e 10. def. 5.

Corell.

Hinc, inter duos similes solidos cadunt duo medii proportionales, in ratione laterum homologorum.

PROP. XX.

A, 12. C, 18. B, 27.
D, 2. E, 3. F, 6. G, 9.

Si inter duos numeros A, B, *unus medius proportionalis cadat numerus* C, *similes plani erunt illi numeri*, A, B.

[a] Accipe D, & E minimos in ratione A ad C, vel C ad B. [b] ergò D æquè metitur A, ac E ipsum C, puta per eundem F. [b] item D æquè metitur C ac E ipsum B, puta per eundem G. [c] ergò DF = A, & EG = B. [d] quare A, & B plani sunt numeri. Quia verò EF [c] = C [c] = DG; [e] erit D. E :: F. G, & vicissim D. F :: E. G. [f] ergò plani numeri A, & B etiam similes sunt. Q. E. D.

a 35. 7.
b 21. 7.
c 9. ax. 7.
d 16. def. 7.
e 19. 7.
f 21. def. 7.

PROP. XXI.

A, 16. C, 24. D, 36. B, 54.
E, 4. F, 6. G, 9.
H, 2. P, 2. M, 4. K, 3. L, 3. N, 6.

Si inter duos numeros A, B duo medii proportionales cadant numeri C, D; similes solidi erunt illi numeri, A, B.

a 2. 8. [a] Sume E, F, G minimos ∴ in ratione A ad
b 20. 8. C. [b] ergò E, & G sunt numeri plani similes;
c 21. *def.* 7. hujus latera sint H & P; illius K, & L: [c] ergò H.
d *cor.* 18. 8. K :: P. L :: [d] E. F. Atqui E, F, G ipsos A, C,
e 21. 7. D [e] æquè metiuntur; puta per eundem M; iidémque ipsos, C, D, B æquè metiuntur, puta
f 9. *ax.* 7. per eundem N. [f] ergò A = EM = HPM [f] &
g 17. *def.* 7. B = GN = KLN; [g] quare A & B solidi sunt numeri. *Quoniam verò* C [f] = FM; & D [f] =
h 17. 7. FN, erit M. N [h] :: FM. FN [k] :: C. D [l] :: E.
k 7. 5. F :: H. K :: P. L. [m] ergò A, & B sunt numeri
l *constr.*
m 21. *def.* 7. solidi similes. Q. E. D.

PROP. XXII.

A, 4. B, 6. C, 9.

Si tres numeri A, B, C deinceps sint proportionales, primus autem A sit quadratus, & tertius C quadratus erit.

Inter A, & C cadit medius proportionalis.
a 20. 8. [a] ergò A, & C sunt similes plani; quare [b] cùm A
b *hyp.* quadratus sit, erit C etiam quadratus. Q. E. D.

PROP. XXIII.

A, 8. B, 12. C, 18. D, 27.

Si quatuor numeri A, B, C, D deinceps sint proportionales; primus autem A sit cubus, & quartus D cubus erit.

a 21. 8. Nam A, & D [a] similes solidi sunt; ergò
b *hyp.* [b] cùm A cubus sit, erit D cubus. Q. E. D.

PROP.

Prop. XXIV.

A, 16. 24. B, 36.
C, 4. 6. D, 9.

Si duo numeri A, B *rationem habeant inter se, quam quadratus numerus* C *ad quadratum numerum* D, *primus autem* A *sit quadratus; & secundus* B *quadratus erit.*

Inter C, & D numeros quadratos, * adeóque inter A, & B eandem rationem habentes [a] cadit unus medius proportionalis. Ergò [b] cùm A quadratus sit, [c] etiam B quadratus erit. Q. E. D.

* 8. 8.
a 11. 8.
b hyp.
c 22. 8.

Coroll.

Liquet ex his, proportionem cujusvis numeri quadrati ad quemlibet non quadratum, exhiberi nullo modo posse in duobus numeris quadratis. unde non erit, Q. Q :: 1. 2. nec 1. 5 :: Q. Q. &c.

Prop. XXV.

C, 64. 96. 144. D, 216.
A, 8. 12. 18. B, 27.

Si duo numeri A, B *rationem inter se habeant, quam cubus numerus* C *ad cubum numerum* D, *primus autem* A *sit cubus, & secundus* B *cubus erit.*

[a] Inter C, & D cubos, [b] adeóque inter A & B eandem rationem habentes, cadunt duo medii proportionales. ergò propter A [c] cubum, [d] etiam B cubus erit. Q. E. D.

a 12. 8.
b 8. 8.
c hyp.
d 23. 8.

Coroll.

Patet etiam ex his proportionem cujusvis numeri cubi ad quemlibet numerum non cubum non posse reperiri in duobus numeris cubis.

PROP. XXVI.

A, 20. C, 30. B, 45.
D, 4. E, 6. F, 9.

Similes plani numeri A, B rationem inter se habeant, quam quadratus numerus ad quadratum numerum.

a 18. 8. Inter A, & B [a] cadit unus medius proportionalis C, [b] sume tres D, E, F minimos ∺ in ra-
b 2. 8. tione A ad C. Extremi D, F [b] quadrati erunt.
c ≡ 14. 7. atqui ex æquali A. B [c] :: D. F. ergò A. B ∺ Q. Q. Q. E. D.

PROP. XXVII.

A, 16. C, 24. D, 36. B, 54.
E, 8. F, 12. G, 18. H, 27.

Similes solidi numeri A, B, rationem habent inter se, quam cubus numerus ad cubum numerum.

a 19. 8. [a] Inter A, & B cadunt duo medii proportio-
b 2. 8. nales, puta C & D : [b] sume quatuor E, F, G, H minimos ∺ in ratione A ad C. [b] Extremi E,
c 14. 7. H cubi sunt. At A. B [c] :: E. H :: C. C. Q. E. D.

Schol.

Vide Clavium. 1. Ex his infertur, nullos numeros habentes proportionem superparticularem, vel superbipartientem, vel duplam, aut aliam quamcunque multiplam non denominatam à numero quadrato esse similes planos.

2. Nec duo quivis primi numeri, neque duo quicunque inter se primi, qui quadrati non sint, similes esse possunt.

LIB.

LIB. IX.

PROP. I.

A, 6. B, 54.
Aq. 36. 108. AB, 324.

SI duo similes plani numeri A, B *multiplicantes se mutuò faciant quendam* AB, *productus* AB *quadratus erit.*

Nam A. B [a] :: Aq. AB; cùm igitur inter A, & B [b] cadat unus medius proportionalis, [c] etiam inter Aq, & AB cadet unus med. proport. ergò cùm primus Aq sit quadratus, [d] etiam tertius AB quadratus erit. Q. E. D.

a 17. 7.
b 18. 8.
c 8. 8.
d 22. 8.

Vel sic. Sint ab, cd similes plani, nempe a. b :: c.d. [x] ergò a d = bc. quare abcd, vel adbc [y] = adad = Q: ad.

x 19. 7
y 1. *ax.*

PROP. II.

A, 6. B 54.
Aq, 36. AB, 324.

Si duo numeri A, B *se mutuò multiplicantes faciant* AB *quadratum, similes plani erunt,* A, B.

Nam A. B [a] :: Aq. AB; quare cùm inter Aq, AB [b] cadat unus medius proportionalis, [c] etiam unus inter A, & B medius cadet. [d] ergò A, & B sunt similes plani. Q. E. D.

a 17. 7.
b 11. 8.
c 8. 8.
d 20. 8.

PROP. III.

A, 2. Ac, 8. Acc, 64.

Si cubus numerus Ac *seipsum multiplicans procreet aliquem* Acc, *productus* Acc *cubus erit.*

Nam 1. A [a] :: A. Aq [b] :: Aq. Ac. ergò inter 1, & Ac cadunt duo medii proportionales. Sed 1. Ac [b] :: Ac. Acc. [c] ergò inter Ac, & Acc cadunt etiam duo

a 15. *def.*
b 17. 7.
c 8. 8.

 medii

medii proportionales. Proinde cùm Ac sit cubus, [d] erit Acc cubus. Q. E. D.

d 23. 8.

Vel sic; aaa (Ac) in se ductus facit aaaaaa. (Acc); hic cubus est, cujus latus aa.

PROP. IV.

Ac, 8. Bc, 27.
Acc, 64. AcBc, 216.

Si cubus numerus Ac *cubum numerum* Bc *multiplicans, faciat aliquem* AcBc, *factus* AcBc *cubus erit.*

a 17. 7. Nam Ac. Bc [a] :: Acc. AcBc. sed inter Ac,
b 12. 8. & Bc [b] cadunt duo medii proportionales; [c] ergò
c 8. 8. inter Acc, & AcBc totidem cadunt. itáque cùm
d 23. 8. Acc sit cubus, [d] erit AcBc etiam cubus. Q. E. D.

Vel sic. AcBc = aaabbb (ababab) = C: ab.

PROP. V.

Ac, 8. B, 27.
Acc, 64. AcB, 216.

Si cubus numerus Ac *numerum quendã* B *multiplicans, faciat cubum* AcB; *& multiplicatus* B *cubus erit.*

a 17. 7. Nam Acc. AcB [a] :: Ac. B. Sed inter Acc, &
b 12. 8. AcB [b] cadunt duo medii proportionales. [c] ergò
c 8. 8. totidem cadent inter Ac, & B. quare cùm Ac cu-
d 23. 8. bus sit, [d] etiam B cubus erit. Q. E. D.

PROP. VI.

A, 8. Aq, 64. Ac, 512.

Si numerus A *seipsum multiplicans faciat* Aq *cubum; & ipse* A *cubus erit.*

a hyp. Nam quia Aq [a] cubus, & AqA (Ac) [b] cu-
b 19. def. 7. bus, [c] erit A cubus. Q. E. D.
c 5. 9.

PROP. VII.

A, 6. B, 11. AB, 66.
D, 2. E, 3.

Si compositus numerus A *numerum quempiam* B *multiplicans quempiam faciat* AB, *factus* AB *solidus erit.*

Quoniam

Quoniam A compositus est, [a] metitur eum a- liquis D, puta per E. [b] ergò $A = DE$; [c] quare $DEB = AB$ solidus est. Q. E. D. a 13. *def* 7. b 9. *ax.* 7. c 17. *def.* 7.

Prop. VIII.

$1. a, 3. a^2, 9. a^3, 27. a^4, 81. a^5, 243. a^6, 729.$

Si ab unitate quotcunque numeri deinceps proportionales fuerint ($1, a, a^2, a^3, a^4$, &c.): *tertius quidem ab unitate* a^2 *quadratus est*; *& unum intermittentes omnes* (a^4, a^6, a^8, &c.) : *quartus autem* a^3 *est cubus, & duos intermittentes omnes* (a^6, a^9, &c.) *septimus verò* a^6, *cubus simul & quadratus, & quinq; intermittentes omnes* (a^{12}, a^{18}, &c.)

Nam 1. $a^2 = Q.\, a.$ & $a^4 = aaaa = Q.\, aa.$ & $a^6 = aaaaaa = Q.\, aaa$, &c.

2. $a^3 = aaa = C.\, a.$ & $a^6 = aaaaaa = C.\, aa.$ & $aaaaaaaaa = C.\, aaa$, &c.

3. $a^6 = aaaaaa = C.\, aa = Q.\, aaa.$ ergò, &c. a *hyp.*

Vel juxta Euclidem; quia $1. a :: a. a^2$, [b] erit b 20 7.
$a^2 = Q: a.$ ergò cùm a^2, a^3, a^4 sint ∺ [c] erit c 22. 8.
tertius a^4 etiam quadratus. paritérq; a^6, a^8, &c.
Item quia $1. a^2 :: a^2. a^3$. erit a^3 [b] $= a^2$ in $a =$ d 23 8.
$C: a.$ [d] ergò quartus ab a^3, nempe a^6, etiam cubus erit, &c. ergò a^6 cubus simul & quadratus existit, &c.

Prop. IX.

$1. a, 4. a^2, 16. a^3, 64. a^4, 256.$ &c.
$1. a, 8. a^2, 64. a^3, 512. a^4, 4096.$

Si ab unitate quotcunque numeri deinceps proportionales fuerint ($1, a, a^2, a^3$, &c.) ; *qui verò* (a) *post unitatem sit quadratus*, *& reliqui omnes*, a^2, a^3, a^4, &c. *quadrati erunt. At si* a, *qui post unitatem sit cubus, & reliqui omnes* a^2, a^3, a^4, &c. *cubi erunt.*

1. *Hyp.* Nam a^2, a^4, a^6, &c. quadrati sunt ex præc. item quia a ponitur quadratus, [a] erit tertius a^3 quadratus, paritérque a^5, a^7, &c. ergò omnes. a 22. 8.

b 23. 8. c 20. 7. d 3. 9. e 23. 8.

2. *Hyp.* a cubus ponitur, [b] ergò a^4, a^7, a^{10} cubi sunt: atqui ex præced. a^3, a^6, a^9, &c. cubi sunt. denique quia 1. a :: a. aa [c], erit $a^2 = Q$: a. cubus autem in se [d] *facit cubum*; ergò a^2 cubus est, & [e] proinde ab eo quartus a^5, paritérq; a^8, a^{11}, &c. cubi sunt, ergò omnes. Q. E. D.

Clariùs forsitan sic; Sit quadrati a latus *b*, ergò series a, a^2, a^3, a^4, &c. aliter exprimetur sic, bb, b^4, b^6, b^8, &c. liquet verò hos omnes quadratos esse; & sic etiam exprimi posse; Q: b, Q: *bb*, *Q: bbb*, *Q: bbbb*, *&c.*

Eodem modo, si b latus fuerit cubi a. series ità nominari potest: b^3, b^6, b^9, b^{12}, &c. vel C: b, C: b^2, C: b^3, C: b^4, &c.

Prop. X.

1, a, a^2, a^3, a^4, a^5, a^6.
1, 2, 4, 8, 16, 32, 64.

Si ab unitate quotcunque numeri deinceps proportionales fuerint (1, a, a^2, a^3, &c.); *qui verò post unitatem* (a) *non sit quadratus, neque alius ullus quadratus erit, præter* a^2 *tertium ab unitate, & unum intermittentes omnes* (a^4, a^6, a^8, &c.) *At si* a, *qui post unitatem non sit cubus, neque ullus alius cubus erit præter* a^3 *quartum ab unitate, & duos intermittentes omnes*, a^6, a^9, a^{12}, &c.

a hyp. b suppos. & 8. 9. c 24. 8.

1. *Hyp.* Nam si fieri potest, sit a^5 quadratus numerus. quoniam igitur a. a^2 [a] :: a^4. a^5, atq; inversè a^5. a^4 :: a^2. a; síntque a^5, & a^4 [b] quadrati, primúsque a^2 quadratus, [c] erit a etiam quadratus, contra Hyp.

d 14. 7. e 25. 8.

2. *Hyp.* Si fieri potest, sit a^4 cubus. quoniam igitur [d] ex æquo a^4. a^6 :: a. a^3, atque inversè a^6. a^4 :: a^3. a; [b] síntque a^6, & a^4 cubi, & primus a^3 cubus, [e] etiam a cubus erit, contra Hypoth.

PROP. XI.

1, a, a^2, a^3, a^4, a^5, a^6.
1, 3, 9, 27, 81, 243, 729.

Si ab unitate quotcunq; numeri deinceps proportionales fuerint (1, a, a^2, a^3, &c.) *minor majorem metitur per aliquem eorum, qui in proportionalibus sunt numeris.*

Quoniam 1. a :: a. aa. [a] erit $\frac{aa}{a} = a = \frac{aaa}{aa}$ a 5. ax. 7. & 20. def. 7.

item quia 1. aa [b] :: a. aaa. [a] erit $\frac{aaa}{a} = aa =$ b 14. 7.

$\frac{a^4}{aa} = \frac{a^5}{a^3}$ &c. denique quia 1. a^3. [b] :: a. a^4, [a] erit $\frac{a^4}{a} = a^3 = \frac{a^6}{a^3}$ &c.

Coroll.

Hinc, si numerus qui metitur aliquem ex proportionalibus, non sit unus proportionalium, neque numerus per quem metitur, erit aliquis ex proportionalibus.

PROP. XII.

1, a, a^2, a^3, a^4.
1, 6, 36, 216, 1296.
B, 3.

Si ab unitate quotcunq; numeri deinceps proportionales fuerint (1, a, a^2, a^3, a^4,) ; *quicunque primorum numerorum* B *ultimum* a^4 *metiuntur, iidem* (B) *& eum* (a) *qui unitati proximus est, metientur.*

Dic B non metiri a, [a] ergò B ad a primus est; [b] ergò B ad a^2 primus est ; & [c] proinde ad a^4, quem metiri ponitur Q. E. A.

a 31. 7.
b 27. 7.
c 26. 7.

Coroll.

1. Itáq; omnis numerus primus ultimum metiens, metitur quoq; omnes alios ultimum præcedentes.

2. Si

2. Si aliquis numerus non metiens proximum unitati, metiatur ultimum, erit numerus compositus.

3. Si proximus unitati sit primus numerus, nullus alius primus numerus ultimum metietur.

PROP. XIII.

1,	a,	a^2,	a^3,	a^4,	
1.	5,	25,	125,	625.	
H --	G --	F --	E --		

Si ab unitate quotcunque numeri deinceps proportionales fuerint (a, a^2, a^3, &c.), qui verò post unitatem (a) primus sit; maximum nullus alius metietur, præter eos qui sunt in numeris proportionalibus.

Si fieri potest, alius quispiam E metiatur a^4, nempe per F; [a] erit F alius extra a, a^2, a^3. Quia verò E metiens a^4 non metitur a, [b] erit E numerus compositus; [c] ergò eum aliquis primus metitur, [d] qui proinde ipsum a^4 metitur; [e] ideóque alius non est, quàm a. ergò a metitur E. Eodem modo ostendetur F compositus numerus, metiens a^4, adeóque a ipsum F metiri. itaque quum EF [f] $= a^4 = a$ in a^3, [g] erit a. E :: F. a^3. ergò cùm a metiatur E, [h] æquè F metietur a^3; puta per eundem G. [k] Nec G erit a, vel a^2. ergò, ut priùs, G est numerus compositus, & a eum metitur. quum igitur FG [f] $= a^3 = a^2$ in a, [g] erit a. F :: G. a^2; & proinde, quia A metitur F, [h] æquè G metietur a^2, scilicet per eundem H; [k] qui non est a. ergò quum GH [x] $= a^2 = aa$. [l] erit H. a :: a. G. ergò quia a metitur G (ut priùs); [m] etiam H metietur a, numerum primum. Q. F. N.

Sic Euclides satìs prolixé; breviùs sic; Nam quia a primus est numerus [x] nullus alius primus ultimum a^4 metietur; proinde nec compositus; [y] quia omnem compositum aliquis primus metitur. ergò, &c.

a cor. 11. 9.
b 2 cor. 12. 9.
c 33. 7.
d 11. ax. 7.
e 3 cor. 12. 9.
f 9. ax. 7.
g 19. 7.
h 20. def. 7.
k cor. 11. 9.
l 20. 7.
m 20. def. 7.
x 3 cor. 12. 9.
y 33. 7.
& 11. ax. 7.

PROP. XIV.

A, 30.
B, 2. C, 3. D, 5.
E -- F ---

Si minimum numerum A primi numeri B, C, D metiantur; nullus alius numerus primus E illum metietur, præter eos, qui à principio metiebantur.

Si fieri potest, sit $\frac{A}{E} = F$. [a] Ergò $A = EF$. a 9. ax. 7.
[b] Ergò singuli primi numeri B, C, D ipsorum E, F unum metiuntur; non E, qui primus ponitur; ergò F, minorem scilicet ipso A; contra Hypoth. b 32. 7.

PROP. XV.

A, 9. B, 12. C, 16.
D, 3. E, 4.

Si tres numeri A, B, C deinceps proportionales, fuerint minimi omnium eandem cum ipsis rationem habentium; duo quilibet compositi, ad reliquum primi erunt.

[a] Sume, D, & E minimos in ratione A ad B. a 35. 7.
[b] ergò $A = Dq$; [b] & $C = Eq$; [b] & $B = DE$. Quia b 2. 8.
verò D ad E [c] primus est, [d] erit $D + E$ primus ad c 24. 7.
singulos D, & E. * ergò D in $D + E$ [e] $= Dq +$ d 30. 7.
DE ([f] $A + B$) ad E primus est, ideóque ad C * 26. 7.
vel Eq. Q. E. D. Pari pacto $DE + Eq$ ($B + C$) e 3. 2.
ad D primus est, & proinde ad $A = Dq$. Q. E. D. f priùs.
Denique quia B ad $D + E$ [h] primus est; is ad g 27. 7.
hujus quadratum [k] $Dq + 2 DE + Eq$ ($A + 2$ h 26. 7.
$B + C$) primus erit; [l] quare idem B ad $A + B + C$; k 4. 2.
[l] adeóque ad $A + C$ primus erit. Q. E. D. l 30. 7.

In hac demonstratione nonnulla in numeris assumpsimus, quæ in secundo libro de lineis demonstrata sunt; id quod brevitatis studio fecimus, cùm alioqui eadem in numeris demonstrata habeas apud Clavium, &c.

PROP. XVI.

A, 3. B, 5. C --- *Si duo numeri* A, B *primi inter se fuerint; non erit ut primus* A *ad secundum* B, *ità secundus* B *ad alium quempiam* C.

Dic A. B :: B. C. ergò quum A, & B in sua
a 23. 7. ratione [a] minimi sint, A [b] metietur B æquè ac B
b 21. 7. ipsum C; sed A [c] seipsum etiam metitur; ergò
c 6. ax. 7. A, & B non sunt primi inter se, contra Hypoth.

PROP. XVII.

A, 8. B, 12. C, 18. D, 27. E ---

Si fuerint quotcunque numeri deinceps proportionales A, B, C, D, *extremi autem ipsorum* A, D *primi inter se sint; non erit ut primus* A *ad secundum* B, *ità ultimus* D *ad alium quempiam* E.

Dic A. B :: D. E. ergò vicissim A. D :: B. E.
a 23. 7. ergò quum A, & D in sua ratione [a] minimi sint,
b 21. 7. [b] metietur A ipsum B; [c] quare B ipsum C, & C
c 20. def. 7. sequentem D, [d] adeóque A eundem D metietur.
d 11. ax. 7. Ergò A, & D non sunt primi inter se, contra Hypoth.

PROP. XVIII.

A, 4. B, 6. C, 9. Bq, 16. *Duobus numeris datis* A, B; *considerare an possit ipsis tertius proportionalis* C *inveniri*.

Si A metiatur Bq per aliquem C, [a] erit AC.
a 9. ax. 7. = Bq. unde [b] liquet esse A. B :: B. C. Q. E. F.
b per 20. 7. A, 6. B, 4. Bq, 16. Sin A non metiatur Bq, non erit aliquis tertius proportionalis:
c 7. ax. 7. Nam dic A. B :: B. C. [a] ergò AC = Bq. [c] proinde $\frac{Bq}{A} = C$. Scilicet A metitur Bq, contra Hypoth.

Prop. XIX.

A, 8. B, 12. C, 18. D, 27.
BC, 216.

Tribus numeris datis A,B,C, considerare an possit ipsis quartus proportionalis D *inveniri.*

Si A metiatur BC per aliquem D, [a] ergò AD=BC; [b] constat igitur esse A. B :: C. D. Q. E. F. — a *9. ax. 7.* b *ex 19. 7.*

Sin A non metiatur BC, non datur quartus proportionalis, quod ostendetur, prout in præcedenti.

Prop. XX.

A, 2. B, 3. C, 5.
D, 30. G - - - -

Primi numeri plures sunt omni propositâ multitudine primorum numerorum A, B, C.

[a] Sit D minimus, quem A, B, C metiuntur, si D+1 primus sit, res patet; si compositus, [b] ergò aliquis primus, puta G, metitur D+1, qui non est aliquis trium A, B, C; nam si ità, quum is [c] totum D+1, & [d] ablatum D metiatur, [e] idem reliquam unitatem metietur. Q. E. A. Ergò propositorum primorum numerorum multitudo aucta est per D+1; vel saltem per G. — a *38. 7.* b *33. 7.* c *suppos.* d *constr.* e *12. ax. 7.*

Prop. XXI.

A ..... E ..... B ... F ... C .. G .. D 20 (5, 5, 3, 3, 2, 2)

Si pares numeri quotcunque AB, BC, CD *componantur, totus* AD *par erit.*

[a] Sume EB=$\frac{1}{2}$ AB, & FC=$\frac{1}{2}$ BC; & GD=$\frac{1}{2}$ CD. [b] liquet EB+FC+GD=$\frac{1}{2}$ AD. [c] ergò AD est par numerus. Q. E. D. — a *6. def. 7.* b *12. 7.* c *6. def. 7.*

PROP. XXII.

$$\begin{array}{l} \quad\quad\quad\quad\quad\quad 1 \quad\quad 1 \quad\quad 1 \\ A \ldots\ldots F.B \ldots\ldots G.C \ldots H.D..L.E.\ 22. \\ \quad 9 \quad\quad 7 \quad\quad 5 \quad\quad 3 \end{array}$$

Si impares numeri quotcunque AB, BC, CD, DE *componantur, multitudo autem ipsorum sit par, totus* AE *par erit.*

a 7. def. 7. Detractâ unitate ex singulis imparibus,[a] manebunt AF, BG, CH, DL numeri pares, &
b 21. 9. [b] proinde compositus ex ipsis par erit; adde his
c hyp. [c] parem numerum conflatum ex residuis unitatibus,
d 21. 9. [d] totus idcircò AE par erit. Q. E. D.

PROP. XXIII.

$$\begin{array}{l} \quad 7 \quad\quad 5 \quad\quad 1 \\ A \ldots\ldots B \ldots\ldots C..E.D\ 15. \\ \quad\quad\quad\quad\quad\quad 3 \end{array}$$

Si impares numeri quotcunque AB, BC, CD *componantur, multitudo autem ipsorum sit impar; & totus* AD *impar erit.*

Nam dempto CD uno imparium, reliquorum
a 21. 9. aggregatus AC [a] est par numerus. huic adde
b 21. 3. CD—1; [b] totus AE est etiam par; quare restitutâ
c 7. def. 7. unitate totus AD [c] impar erit. Q. E. D.

PROP. XXIV.

$$\begin{array}{l} \quad 4 \quad\quad 5 \quad\quad 1 \\ A \ldots B \ldots\ldots D.C\ 10. \\ \quad\quad\quad 6 \end{array}$$

Si à pari numero AC *par* AB *detrahatur, & reliquus* BC *par erit.*

Nam si BD (BC—
a 7. def. 7. 1) impar fuerit,[a] erit BC (BD+1) par. Q. E. D.
b hyp. Sin BD parem dicas, propter AB [b] parem,[c] erit
c 21. 9. AD par; [a] ideóque AC (AD+1) impar, contra Hypoth. ergò BC est par. Q. E. D.

PROP. XXV.

6 1 3
A......D.C...B 10.
7

Si à pari numero AB impar AC detrahatur, & reliquus CB impar erit.

Nam AC—1 (AD) [a] est par. [b] ergò DB est par. [c] ergò CB (DB—1) est impar. Q.E.D. a 7. def. 7. b 24. 9. c 7. def. 7.

PROP. XXVI.

4 6 1
A....C......D.B 11.
7

Si ab impari numero AB impar CB detrahatur; reliquus AC par erit.

Nam AB—1 (AD) & CB—1 (CD) [a] sunt pares. [b] ergò AD—CD (AC) est par. Q.E.D. a 7. def. 7. b 24. 9.

PROP. XXVII.

1 4 6
A.D....C......B 11.
5

Si ab impari numero AB par detrahatur CB, reliquus AC impar erit.

Nam AB—1 (DB) [a] est par; & CB ponitur par. [b] ergò reliquus CD par est. [c] ergò CD+1 (CA) est impar. Q.E.D. a 7. def. 7. b 24. 9. c 7. def. 7.

PROP. XXVIII.

A, 3.
B, 4.
AB, 12.

Si impar numerus A parem numerum B multiplicans fecerit aliquem AB, factus AB par erit.

Nam AB [a] componitur ex impari A toties accepto, quoties unitas continetur in B pari. [b] ergò AB est par numerus. a hyp. & 15. def. 7. b 21. 9.

Schol.

Eodem modo, si A sit numerus par, erit AB par.

PROP. XXIX.

$A, 3.$
$\underline{B, 5.}$
$AB, 15.$

Si impar numerus A, imparem numerum B multiplicans, fecerit aliquem AB; factus AB impar erit.

a 15. def. 7. Nam AB[a] componitur ex B impari numero toties accepto, quoties unitas includitur in A etiam impari. [b] ergò AB est impar. Q. E. D.

b 23. 9.

Scholium.

$\underline{B, 12}$ (C, 4.
$A, 3$

1. *Numerus A impar numerum B parem metiens, per numerum parem C eum metitur.*

a 9. ax. 7. Nam si C impar dicatur, quoniam[a] B=AC,
b 29. 9. erit B impar; contra Hypoth.

$\underline{B, 15}$ (C, 5.
$A, 3$

2. *Numerus A impar numerum B imparem metiens, per numerum C imparem eum metitur.*

a 28. 9. Nam si C dicatur par; [a] erit AC, vel B par, contra Hypoth.

$\underline{B, 15}$ (C, 5.
$A, 3$

3. *Omnis numerus (A & C) metiens imparem numerum B est impar.*

a 28. 9. Nam si utervis A, vel C dicatur par, [a] erit B numerus par, contra Hypoth.

PROP. XXX.

$\underline{B, 24}$ (C, 8. $\underline{D, 12}$ (E, 4.
$A, 3$ $A, 3$

a hyp.
b 1. Schol. 29 9.
c 9. ax. 7.
d 1. 2.
e hyp.
f 7. ax. 1.
g 7. ax. 7.

Si impar numerus A parem numerum B metiatur, & illius dimidium D metietur.

[a] Sit $\frac{B}{A} = C$. [b] ergò C est numerus par.

Sit igitur $E = \frac{1}{2}C$, erit B[c]$= CA$[d]$= 2EA$[e]$= 2D$.

[f] ergò $EA = D$; & [g] proinde $\frac{D}{A} = E$. Q.E.D.

PROP.

PROP. XXXI.

A, 5. B, 8. C, 16. D--- *Si impar numerus A ad aliquem numerum B primus sit, & ad illius duplum C primus erit.*

Si fieri potest, aliquis D metiatur A, & C, [a] ergò D metiens imparem A impar erit, [b] ideóque ipsum B paris C semissem metietur. ergò A, & B non sunt primi inter se, contra Hypoth. a 3. *schol.* 29. 9. b 30. 9.

Coroll.

Sequitur hinc, numerum imparem qui ad aliquem numerum progressionis duplæ primus est, primum quoque esse ad omnes numeros illius progressionis.

PROP. XXXII.

1. A, 2. B, 4. C, 8. D, 16. *Numerorum* A, B, C, D, &c. *à binario duplorum unusquisque pariter par est tantùm.*

Constat omnes 1, *A*, B, C, D [a] pares esse; atque b ∸∸ nimirum in ratione dupla, & [c] proinde quemque minorem metiri majorem per aliquem ex illis. [d] Omnes igitur sunt pariter pares. Sed quoniam *A* primus est, [e] nullus extra eos eorum aliquem metietur. Ergò pariter pares sunt tantùm. Q. E. D. a 6. *def.* 7. b 20. *def.* 7. c 11. 9. d 8. *def.* 7. e 13. 9.

PROP. XXXIII.

A, 30. B, 15. *Si numerus* A *dimidium* B
D--- E-- *habeat imparem, A pariter impar est tantùm.*

Quoniam impar numerus B [a] metitur *A* per 2 parem, b est B pariter impar; Dic etiam pariter parem. [c] ergò cum par aliquis D per parem E metitur. unde 2 B [d] = *A* [d] = DE. [e] quare 2. a *hyp.* b 9. *def.* 7. c 8. *def.* 7. d 9. *ax.* 7. e 19. 7.

f 6. def. 7. E :: D. B. ergò ut 2 [f] metitur parem E, [g] sic D
g 20. def. 7. par imparem B metitur. Q. F. N.

Prop. XXXIV.

A, 24. *Si par numerus A, neque à binario duplus sit, neque dimidium habeat imparem; pariter par est, & pariter impar.*

Liquet A esse pariter parem, quia dimidium imparem non habet. Quia verò si A bifarietur, & rursus ejus dimidium, & hoc semper fiat, tandem incidemus in aliquem [a] imparem, (quia non in binarium, quoniam A à binario duplus non ponitur) is metietur A per parem numerum (nam [b] aliàs ipse A impar esset, contra Hypoth.) ergò A est etiam pariter impar. Q. E. D.

a 7. def. 7.
b 1. sch. 29. 9.

Prop. XXXV.

A .... 8.

4 8

B .... F .... G 12.

C .... 18.

9 6 4 8

D .... H ...... L .... K .... N 27.

Si sint quotcunque numeri deinceps proportionales A, BG, C, DN, detrahantur autem FG à secundo, & KN ab ultimo æquales ipsi primo A, erit ut secundi excessus BF ad primum A, ità ultimi excessus DK ad omnes A, BG, C ipsum antecedentes.

Ex DN deme NL=BG; & NH=C.

a hyp. Quoniam DN. C. (HN) [a] :: HN. BG.
b 17. 5. (LN) [a] :: LN. (BG) A. (KN.) [b] erit dividendo ubique, DH. HN :: HL. LN :: LK.
c 12. 5. KN. [c] quare DK. C+BG+A :: LK ([d] BF.)
d 3. ax. 1. KN. (A.) Q. E. D.
e 18. 5.

Coroll.

Hinc [e] componendo, DN+BG+C. A+BG+C :: BG. A.

Prop. XXXVI.

1. A, 2. B, 4. C, 8. D, 16.
E, 31. G, 62. H, 124. L, 248. F, 496.
M, 31. N, 465.
P---- Q---

Si ab unitate quotcunque numeri 1, A, B, C, D *deinceps exponantur in dupla proportione, quoad totus compositus* E *fiat primus, & totus hic* E *in ultimum* D *multiplicatus faciat aliquem* F; *factus* F *erit perfectus.*

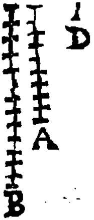
Sume totidem, E, G, H, L etiam in proportione dupla continuè; ergò [a] ex æquo A. D : : E. L. [b] ergò AL = DE [c] = F. [d] ergò L = $\frac{F}{2}$ quare E, G, H, L, F sunt ∺ in ratione dupla. Sit G — E = M, & F — E = N. [e] ideò M. E : : N. E + G + H + L. [f] at M = E. [g] ergò N = E + G + H + L. ergò F = 1 + B + C + D + E + G + H + L = E + N. Quinetiam quia D [k] metitur DE (F), [l] etiam singuli 1, A, B, C [m] metientes D, [m] nec non E, G, H, L metiuntur F. Porrò nullus alius eundem F metitur. Nam si aliquis, sit P; qui metiatur F per Q. [n] ergò P Q = F = DE. [o] ergò E. Q : : P. D. ergò cùm A primus numerus metiatur D, & [p] proinde nullus alius P eundem metiatur, [q] consequenter E non metitur Q. quare cùm E primus ponatur, [r] idem ad Q primus erit. [s] ergò E, & Q in sua ratione minimi sunt, & [t] propterea E ipsum P, ac Q ipsum D æquè metiuntur. [u] ergò Q est aliquis ipsorum A, B, C. Sit igitur B; ergò cùm ex æquo sit B. D : : E. H; [x] ideóque B H = DE = F = PQ, [x] adeóque Q. B : : H. P. [y] erit H = P. ergò P est etiam aliquis ipsorum A, B, C, &c. contra Hypoth. ergò nullus alius præter numeros prædictos eundem F metietur: [z] proinde F est numerus perfectus. Q. E. D.

a 14. 7.
b 19. 7.
c *hyp.*
d 7. *ax.* 7.
e 35. 9.
f 3. *ax.* 1.
g 14. 5.
h 2. *ax.* 1.
k 7. *ax.* 7.
l 11. *ax.* 7.
m 11. 9.
n 9. *ax.* 7.
o 19. 7.
p 13. 9.
q 20. *def.* 7.
r 31. 7.
s 23. 7.
t 21. 7.
u 13. 7.
x 19. 7.
y 14. 5.
z 22. *def.* 7.

LIB. X.

Definitiones.

I. Commensurabiles magnitudines dicuntur, quas eadem mensura metitur.



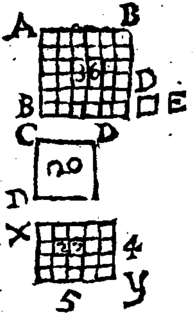
Commensurabilitatis nota est $\sqcap$, *ut* A $\sqcap$ B; *hoc est linea* A 8 *pedum. commensurabilis est lineæ* B 13 *pedum; quia* D *linea unius pedis singulas* A, & B *metitur. Item* $\sqrt{18} \sqcap \sqrt{50}$; *quia* $\sqrt{2}$ *singulas* $\sqrt{18}$, & $\sqrt{50}$ *metitur. Nam* $\sqrt{\frac{18}{2}} = \sqrt{9} = 3.$ & $\sqrt{\frac{50}{2}} = \sqrt{25} = 5.$ *quare* $\sqrt{18}. \sqrt{50} :: 3. 5.$

II. Incommensurabiles autem sunt, quorum nullam communem mensuram contingit reperiri.

Incommensurabilitas significatur notâ $\not\sqcap$ *ut* $\sqrt{6} \not\sqcap \sqrt{25}$ (5) *hoc est* $\sqrt{6}$ *incommensurabilis est numero* 5; *vel magnitudini hoc numero designatæ; quia harum nulla est communis mensura, ut postea patebit.*

III. Rectæ lineæ potentiâ commensurabiles sunt, cùm quadrata earum idem spatium metitur.

Hujusce



Hujusce commensurabilitatis nota est ⊤ *ut* AB ⊤ CD; *h. e. linea* AB *sex pedũ potentiâ commensurabilis est lineæ* CD, *quæ exprimitur per* √ 20. *quia spatium* E *unius pedis quadrati metitur tam* ABq (36) *quàm rectangulum* XY (20), *cui æquale est quadratum lineæ* CD(√20) *Eadem nota* ⊤ *nonnunquam valet potentiâ tantùm commensurabilis:*

IV. Incommensurabiles verò potentiâ, cùm quadratis earum nullum spatium, quod sit communis eorum mensura, contingit reperiri.

Hujusmodi incommensurabilitas denotatur sic; 5 ⊤ *v*√ 8; *hoc est numeri vel lineæ* 5, *&* *v*√ 8 *sunt incommensurabiles potentiâ, quia harum quadrata* 25, *&* √ 8 *sunt incommensurabilia.*

V. Quæ cùm ità sint, manifestum est cuicunque rectæ propositæ, rectas lineas multitudine infinitas, & commensurabiles esse, & incommensurabiles, alias quidem longitudine & potentiâ, alias verò potentiâ solúm. Vocetur autem proposita recta linea Rationalis.

Hujus nota est ῥ.

VI. Et huic commensurabiles, sive longitudine & potentia, sive potentià tantùm, Rationales, ῥ.

VII. Huic verò incommensurabiles Irrationales vocentur.

Hæ sic denotantur ʼϱ.

VIII. Et quadratum, quod à proposita rectâ fit, dicatur Rationale, ῥν.

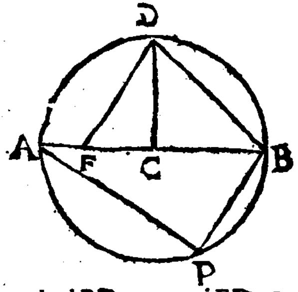
IX, Et huic commensurabilia quidem Rationalia ῥα.

X. Huic

X. Huic verò incommensurabilia, Irrationalia dicantur; $\rho\alpha$.

XI. Et rectæ, quæ ipsa possunt, Irrationales, $\dot{\rho}$.

Schol.



Ut postremæ 7 definitiones exemplo aliquo illustrentur, sit circulus ADBP, *cujus semidiameter* CB; *huic inscribantur latera figurarum ordinatarum, Hexagoni quidem* BP, *Trianguli* AP, *quadrati* BD, *pentagoni* FD. *Itaque si juxta 5 defin. semidiameter* CB *sit Rationalis exposita, numero* 2. *expressa, cui reliquæ* BP, AP, BD, FD *comparandæ sunt* [a]; *erit* BP [a] = BC = 2. *quare* BP *est*

a cor. 15. 4.

b 67. 1.

$\dot{\rho}$ ⊤ BC, *juxta 6. def. Item* AP [b] $= \sqrt{12}$ (*nam* ABq (16) — BPq (4) = 12) *quare* AP *est* $\dot{\rho}$ ⊤ BC, *etiam juxta 6. def. atque* APq (12) *est* $\dot{\rho}\nu$, *per def.* 9. *Porrò* BD [b] $= \sqrt{}$ DCq + BCq $= \sqrt{8}$; *unde* BD *est* $\dot{\rho}$ ⊤ BC; *&* BDq $\dot{\rho}\nu$. *Denique*, FDq $= 10 - \sqrt{20}$. (*ut patebit ex praxi ad* 10. 13. *tradendâ*) *erit* $\dot{\rho}\nu$, *juxta* 10 *def. & proinde* FD $= \sqrt{10 - \sqrt{20}}$ *est* $\dot{\rho}$, *juxta* 11 *defin.*

Postulatum.

POstuletur, quamlibet magnitudinem toties posse multiplicari, donec quamlibet magnitudinem ejusdem generis excedat.

Axiomata.

Axiomata.

1. **M**Agnitudo quotcunque magnitudines metiens, compositam quoque ex ipsis metitur.

2. Magnitudo quamcunque magnitudinem metiens, metitur quoque omnem magnitudinem quam illa metitur.

3. Magnitudo metiens totam magnitudinem & ablatam, metitur & reliquam.

Prop. I.

Duabus magnitudinibus inæqualibus AB, C *propositis, si à majore* AB *auferatur majus quàm dimidium,* (AH) *& ab eo* (HB), *quod reliquum est, rursus detrahatur majus quàm dimidium* (HI), *& hoc semper fiat; relinquetur tandem quædam magnitudo* IB, *quæ minor erit propositâ minore magnitudine* C.

Accipe C toties, donec ejus multiplex DE proximè excedat AB; síntque DF = FG = GE = C. Deme ex AB plusquam dimidium AH, & à reliquo HB plusquam dimidium HI, & sic deinceps, donec partes AH, HI, IB æquè multæ sint partibus DF, FG, GE. Jam liquet FE, quæ non minor est quàm $\frac{1}{2}$ DE, majorem esse, quàm HB, quæ minor est, quàm $\frac{1}{2}$ AB $\sqsupset$ DE. Pariterque GE quæ non minor est quàm $\frac{1}{2}$ FE, major est quàm IB $\sqsupset \frac{1}{2}$ HB. ergò C, vel GE $\sqsubset$ IB. Q. E. D.

Idem demonstrabitur, si ex AB auferatur dimidium AH, & ex reliquo HB rursus dimidium HI, & ità deinceps.

PROP. II.

Si duabus magnitudinibus inæqualibus propositis (AB, CD) detrahatur semper minor AB de majore CD, alternâ quâdam detractione, & reliqua minimè præcedentem metiatur; incommensurabiles erunt ipsæ magnitudines.

Si fieri potest, sit aliqua E communis mensura. Quoniam igitur AB detracta ex CD, quoties fieri potest, relinquit aliquam FD se minorem, & FD ex AB relinquit GB, & sic deinceps, [a] tandem relinquetur aliqua GB ⊐ E. ergò E [b] metiens AB [c], ideóq; CF, [b] & totam CD; [d] etiam reliquam FD metietur, [c] proinde & AG; [d] ergò & reliquam GB, seipsâ minorem. Q. E. A.

a 1. 10.
b *hyp.*
c 2. *ax.* 10.
d 3. *ax.* 10.

PROP. III.

Duabus magnitudinibus commensurabilibus datis, AB, CD, *maximam earum communem mensuram* FB *reperire.*

Deme AB ex CD, & reliquum ED ex AB, & FB ex ED, donec FB metiatur ED; (quod tandem fiet, [a] quia per Hyp. AB ⊐⊏ CD) erit FB quæsita.

Nam FB [b] metitur ED, [c] ideóque ipsam AF; sed & seipsam, [d] ergò etiam AB, & [c] propterea CE, [d] adeóque & totam CD. Proinde FB communis est mensura ipsarum AB, CD. Dic G communem quoq; esse mensuram, hâc majorem; ergò G metiens AB, & CD [e] metitur CE, & [f] reliquam ED, [e] idcóque AF, & [f] proinde reliquam FB, major minorem. Q. E. A.

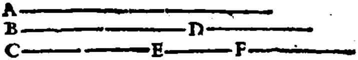
a 2. 10.
b *constr.*
c 2. *ax.* 10.
d 1. *ax.* 10.
e 2. *ax.* 10.
f 3. *ax.* 10.

Coroll.

Coroll.

Hinc, magnitudo metiens duas magnitudines, metitur & maximam earum mensuram communem.

Prop. IV.



Tribus magnitudinibus commensurabilibus datis A, B, C; *maximam earum mensuram communem invenire.*

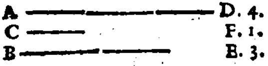
[a] Inveni D maximam communem mensuram duarum quarumcunque A, B, [a] item E ipsarum D, & C maximam communem mensuram; erit E quæsita. a 3. 10.

Nam perspicuum est E metiens D, & C [b] metiri tres A, B, C. Puta aliam F hâc majorem easdem metiri. [c] ergò F metitur D; [c] proinde & E, ipsorum D, C maximam communem mensuram, major minorem. Q. E. A. b. *constr.* & 2. *ax.* 10. c *cor.* 3.10.

Coroll.

Hinc quoque Magnitudo metiens tres magnitudines, metitur quoque maximam earum communem mensuram.

Prop. V.



Commensurabiles magnitudines A, *B inter se rationem habent, quàm numerus ad numerum.*

[a] Inventâ C ipsarum A, B maximâ communi mensurâ; quoties C in A, & B, toties 1 contineatur in numeris D & E. [b] ergò C. A : : 1. *D*; quare inversè A. C : : *D*. 1. [b] atqui etiam C, a 3. 10. b 20. *def.* 7.

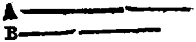
c 22. 5. B : : 1. E. [c] ergò ex æquali A. B : : D. E :: N. N. Q. E. D.

Prop. VI.

Si duæ magnitudines A, B *inter ſe proportionem habeant, quàm numerus* C *ad numerum* D; *commenſurabiles erunt magnitudines* A, B.

a *ſch.* 10. 6. Qualis pars eſt 1 numeri C, [a] talis fiat E ip-
b *conſtr.* ſius A. Quoniam igitur E. A [b] : : 1. C. atque
c *hyp.* A. B [c] : : C. D; [d] ex æquo erit E. B : : 1. D.
d 22. 5. ergò quum 1 [e] metiatur numerum D, [f] etiam
e 5. *ax.* 7. E metitur B. ſed & ipſum A [g] metitur. [h] ergò
f 20. *def.* 7. A ⊓⊔ B. Q. E. D.
g *conſtr.*
h 1. *def.* 10.

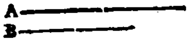
Prop. VII.



Incommenſurabiles magnitudines A, B *inter ſe proportionem non habent, quàm numerus ad numerum.*

a 6. 10. Dic A. B : : N. N. [a] ergò A ⊓⊔ B. contra Hypoth.

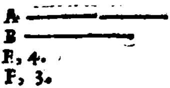
Prop. VIII.



Si duæ magnitudines A, B *inter ſe proportionem non habeant, quam numerus ad numerum, incommenſurabiles erunt magnitudines.*

a 5. 10. Puta A ⊓⊔ B. [a] ergò A. B : : N. N, contra Hypoth.

Prop. IX.



Quæ à rectis lineis longitudine commensurabilibus fiunt quadrata, inter se proportionem habent, quam quadratus numerus ad quadratum numerum: & quadrata inter se proportionem habentia, quam quadratus numerus ad quadratum numerum, & latera habebunt longitudine commensurabilia. Quæ verò à rectis lineis longitudine incommensurabilibus fiunt quadrata, inter se proportionem non habent, quam quadratus numerus ad quadratum numerum: & quadrata inter se proportionem non habentia, quam quadratus numerus ad quadratum numerum, neq; latera habebunt longitudine commensurabilia.

1. *Hyp.* A. $\top\!\bot$ B. Dico Aq. Bq :: Q. Q.

Nam [a] sit A. B :: num. E. num. F. ergò $\frac{Aq}{Bq}$ $\left({}^{b}\frac{A}{B}\text{ bis.}\right)$ [c] $= \frac{E}{F}$ bis. [d] $= \frac{Eq}{Fq}$ [e] ergò Aq. Bq :: Eq. Fq :: Q. Q. Q. E. D.

2. *Hyp.* Aq. Bq :: Eq. Fq :: Q. Q. Dico A $\top\!\bot$ B. Nam $\frac{A}{B}$ bis. $\left({}^{f}\frac{Aq}{Bq}\right)$ [g] $= \frac{Eq}{Fq}$ [h] $= \frac{E}{F}$ bis. [i] ergò A. B :: E. F :: N. N. [k] quare A $\top\!\bot$ B. Q. E. D.

3. *Hyp.* A $\top\!\bot$ B. Nego esse Aq. Bq :: Q. Q. Nam dic Aq. Bq :: Q. Q. Ergò A $\top\!\bot$ B, ut modò ostensum est, contra Hypoth.

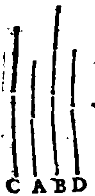
4. *Hyp.* Non Aq. Bq :: Q. Q. Dico A $\top\!\bot$ B. Nam puta A $\top\!\bot$ B; ergò Aq. Bq :: Q. Q; ut modò diximus, contra Hypoth.

a per. 5. 10.
b 20. 6.
c sch. 23. 5.
d 11. 8.
e 11. 5.
f 20. 6.
g hyp.
h 11. 8.
i sch. 23. 5.
k 6. 10.

Coroll.

Lineæ $\top\!\bot$ sunt etiam $\top\!\rfloor$; at non contra. Sed lineæ $\top\!\bot$ non sunt idcirco $\top\!\rfloor$. lineæ verò $\top\!\rfloor$ sunt etiam $\top\!\bot$.

PROP. X.



Si quatuor magnitudines proportionales fuerint (C. A :: B. D); *prima verò* C *secundæ* A *fuerit commensurabilis*; *& tertia* B *quartæ* D *commensurabilis erit. Et si prima* C *secundæ* A *fuerit incommensurabilis, & tertia* B *quartæ* D *incommensurabilis erit.*

Si C ⊓⊔ A,[a] ideò erit C. A:: N. N [b] :: B. D. [b] ergò B ‾⊓⊔ D. Sin C ‾⊓⊔ A, ergò [c] non erit C. A :: N. N :: B. D. [d] quare B ‾⊓⊔ D. Q. E. D.

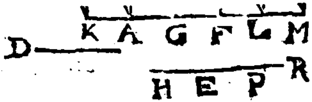
a 5. 10.
b 6. 10.
c 7. 10.
d 8. 10.

LEMMA 1.

Duos numeros planos invenire, qui proportionem non habeant, quam quadratus numerus ad quadratum numerum.

Huic Lemmati satisfacient duo quilibet numeri plani non similes, quales sunt numeri habentes proportionem superparticularem, vel superbipartientem, vel duplam, vel etiam duo quivis numeri primi. vid. Schol. 27. 8.

LEMMA 2.



B, 5, C, 3.

Invenire lineam HR, *ad quam data recta linea* KM *sit in ratione datorum numerorum* B, C.

a sch. 10. 6. [a] Divide KM in partes æquales æquè multas unitatibus numeri B. harum tot, quot unitates sunt in numero C, [b] componant rectam HR. liquet esse, KM. HR :: B. C.

b 3. 1.

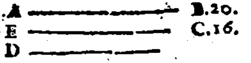
LEMMA 3.

Invenire lineam D, *ad cujus quadratum datæ rectæ* KM *quadratum sit in ratione datorum numerorum* B, C.

Fac

Fac B. C [a] :: KM. HR. ac inter KM, & HR [b] inveni mediam proportionalem D. Erit KMq. Dq [c] :: KM. HR [d] :: B. C. a 2. *lem.* 10. 10. b 13. 6. c 20. 6. d *constr.*

PROP. XI.

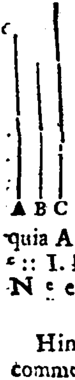


Propositæ rectæ lineæ A *invenire duas rectas lineas incommensurabiles; alteram quidem* D *longitudine tantùm, alteram verò* E *etiam potentiâ.*

1. Sume numeros B, C,[a] ità ut non sit B.C :: Q.Q. [b] fiatque B. C :: Aq. Dq. [c] liquet A $\not\sqcap\!\!\!\sqcup$ D. Sed Aq [d] $\sqcap\!\!\!\sqcup$ Dq. Q. E. F. a 1 *lem.* 10. 10. b 3. *lem.* 10. 10 c 9. 10. d 6 10

2. [d] Fac A. E :: E. D. Dico Aq $\not\sqcap\!\!\!\sqcup$ Eq. Nam A. D [e] :: Aq. Eq. ergò cùm A $\not\sqcap\!\!\!\sqcup$ D, ut priùs, [f] erit Aq $\not\sqcap\!\!\!\sqcup$ Eq. Q. E. F. d 13. 6. e 20. 6. f 10. 10.

PROP. XII.



Quæ (A, B) *eidem magnitudini* C *sunt commensurabiles, & inter se sunt commensurabiles.*

Quia A $\sqcap\!\!\!\sqcup$ C; & C $\sqcap\!\!\!\sqcup$ B, [a] sit A. C :: N. N :: D. E. ätq; C. B :: N. N :: F. G. [b] sumantur tres numeri H, I, K minimi ∺ in rationibus D ad E, & F ad G. Jam quia A. C [c] :: D. E [c] :: H. I. ac C. B [c] :: F. G [c] :: I. K. [d] erit ex æquali A. B :: H. K :: N. N [e] ergò A $\sqcap\!\!\!\sqcup$ B. Q. E. D. a 5. 10. b 4. 8. c *constr.* d 22. 5. e 6. 10.

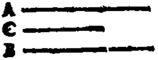
D, 18. E, 8. F, 2. G, 3. H, 5. I, 4. K, 6.

Schol.

Hinc, omnis recta linea rationali lineæ commensurabilis, est quoque ρ̂ rationalis. Et omnes rectæ rationales inter se commensurabiles sunt, saltem potentiâ. Item, omne spatium rationali spatio commensurabile, est quoque rationale; & omnia spatia rationalia inter se commensura- 12. 10. & *def.* 6. *def.* 9.

mensurabilia sunt. Magnitudines verò, quarum
ᵃ def. 7. & 10. altera est rationalis, altera irrationalis, sunt inter se incommensurabiles.

Prop. XIII.

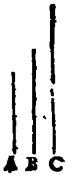


Si sint duæ magnitudines A, B; & altera quidem A eidem C sit commensurabilis, altera verò B incommensurabilis; incommensurabiles erunt magnitudines A, B.

a hyp.
b 12. 10.

Dic B ⊓⊔ A, ergò cùm C [a] ⊓⊔ A, [b] erit C ⊓⊔ B, contra Hypoth.

Prop. XIV.

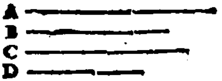


Si sint duæ magnitudines commensurabiles A, B; altera autem ipsarum A magnitudini cuipiam C incommensurabilis fuerit, & reliqua B eidem C incommensurabilis erit.

a hyp.
b 12. 10.

Puta B ⊓⊔ C. ergò cùm A ⊓⊔ [a] B, [b] erit A ⊓⊔ C, contra Hyp.

Prop. XV.



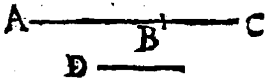
Si quatuor rectæ lineæ proportionales fuerint (A. B :: C. D); prima verò A tanto plùs possit quàm secunda B, quantum est quadratum rectæ lineæ sibi commensurabilis longitudine: & tertia C tanto plùs poterit, quàm quarta D, quantum est quadratum rectæ lineæ sibi longitudine commensurabilis. Quòd si prima A, tanto plùs possit quàm secunda B, quantum est quadratum rectæ lineæ sibi incommensurabilis longitudine; & tertia C tanto plùs poterit, quàm quarta D, quantum est quadratum rectæ lineæ sibi longitudine incommensurabilis.

a hyp.
b 22. 6.
c 17. 5.

Nam quia A. B [a] :: C. D. [b] erit Aq. Bq :: Cq. Dq. [c] ergò dividendo Aq—Bq. Bq :: Cq—Dq.

Dq. Dq. [d] quare $\surd$: Aq—Bq. B :: $\surd$ Cq—Dq. d 22. 6.
D. [e] invertendo igitur B. $\surd$: Aq—Bq :: D. $\surd$: e cor. 4 5.
Cq — Dq. [f] ergò ex æquali A. $\surd$: Aq — Bq :: f 22. 5.
C. $\surd$: Cq — Dq. proinde si A ⊓⊔, vel ⊓⊔. $\surd$
Aq — Bq, [g] erit similiter C ⊓⊔, vel ⊓⊔. $\surd$: g 10. 10.
Cq — Dq. Q. E. D.

Prop. XVI.



Si duæ magnitudines commensurabiles AB, BC *componantur, & tota magnitudo* AC *utriq; ipsarum* AB, BC *commensurabilis erit: quòd si tota magnitudo* AC *uni ipsarum* AB, *vel* BC *commensurabilis fuerit, & quæ à principio magnitudines* AB, BC *commensurabiles erunt.*

1. *Hyp.* [a] Sit D ipsarum AB, BC communis a 3. 10.
mensura. [b] ergò D metitur AC. [c] ergò AC ⊓⊔ b 1. ax. 10.
AB, & BC. Q. E. D. c 1. def. 10.

2. *Hyp.* [a] Sit D communis mensura ipsarum
AC, AB; [d] ergò D metitur AC — AB (BC); d 3. ax. 10.
[c] proinde AB ⊓⊔ BC. Q. E. D.

Coroll.

Hinc etiam, si tota magnitudo ex duabus composita commensurabilis sit alteri ipsarum, eadem & reliquæ commensurabilis erit.

Prop. XVII.

Si duæ magnitudines incommensurabiles AB, BC *componantur, & tota magnitudo* AC *utrique ipsarum* AB, BC *incommensurabilis erit: Quòd si tota magnitudo* AC *uni ipsarum* AB *incommensurabilis fuerit, & quæ à principio magnitudines* AB, BC *incommensurabiles erunt.*

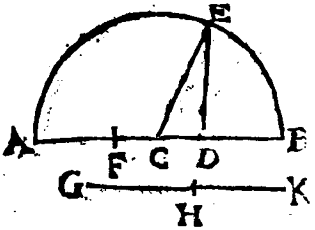
1. *Hyp.* Si fieri poteſt, ſit D ipſarum AC,
a 3. *ax.* 10. AB communis menſura. [a] ergò D metitur
b 1. *def.* 10. AC — AB (BC). [b] ergò AB $\not\sqcup$ BC, contra Hypoth.

c 16. 10. 2. *Hyp.* Dic AB $\not\sqcup$ BC. [c] ergò AC $\not\sqcup$ AB, contra Hypoth.

Coroll.

Hinc etiam, ſi tota magnitudo ex duabus compoſita, incommenſurabilis ſit alteri ipſarum, eadem & reliquæ incommenſurabilis erit.

Prop. XVIII.



Si fuerint duæ rectæ lineæ inequales AB, GK; *quartæ autem parti quadrati, quod fit à minori* GK, *æquale parallelogrammum* ADB *ad majorem* AB *applicetur, deficiens figurâ quadratâ, & in partes* AD, DB *longitudine commenſurabiles ipſam dividat, major* AB *tanto plùs poterit quàm minor* GK, *quantum eſt quadratum rectæ lineæ* FD *ſibi longitudine commenſurabilis: Quòd ſi major* AB *tanto plus poſſit, quàm minor* GK, *quantum eſt quadratum rectæ lineæ* FD *ſibi longitudine commenſurabilis; quartæ autem parti quadrati, quod fit à minori* GK, *æquale parallelogrammum* ADB *ad majorem* AB *applicetur, deficiens figurâ quadratâ, in partes* AD, DB *longitudine commenſurabiles ipſam dividet.*

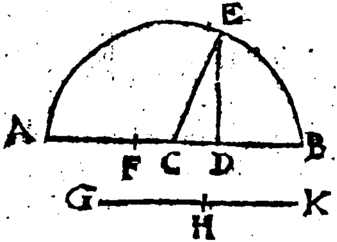
a 10. 1.
b 28. 6.
c 8. 2.
d *conſtr.* & 4. 2.

[a] Biſeca GK in H; & [b] fac rectang. ADB = GHq: abſcinde AF = DB. Eſtque ABq [c] = 4 ADB [d] (4 GHq, vel HKq) + FDq. Jam

primò, Si AD ⊤⊤ DB, erit AB[e] ⊤⊤ DB[e] ⊤⊤ 2 DB[f] (AF + DB, vel AB − FD) [g] ergò AB ⊤⊤ FD. Q. E. D. Sin secundò, AB ⊤⊤ FD, [h] erit ideò AB ⊤⊤ AB − FD (2 DB) [k] ergò AB ⊤⊤ DB. [l] quare AD ⊤⊤ DB. Q. E. D.

e 16. 10.
f constr.
g cor. 16. 10.
h cor. 16. 10.
k 12 1. a.
l 16. 10.

Prop. XIX.



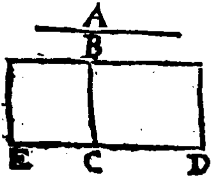
Si fuerint duæ rectæ lineæ inæquales, AB, GK, quartæ autem parti quadrati, quod fit à minore GK, æquale parallelogrammum ADB ad majorem AB applicetur deficiens figurâ quadratâ; & in partes incommensurabiles longitudine AD, DB, ipsam AB dividat; major AB tanto plus poterit, quàm minor GK, quantum est quadratum rectæ lineæ FD, sibi longitudine incommensurabilis: Quòd si major AB tanto plus possit, quàm minor GK, quantum est quadratum rectæ lineæ FD sibi longitudine incommensurabilis; quartæ autem parti quadrati, quod fit à minore GK, æquale parallelogrammum ADB ad majorem AB applicetur, deficiens figurâ quadratâ, in partes longitudine incommensurabiles AD, DB ipsam AB dividet.

Facta puta, & dicta eadem, quæ in præcedenti. Itáq; primò, Si AD ⊤⊤ DB, [a] erit propterea AB ⊤⊤ DB; [b] quare AB ⊤⊤ 2 DB (AB − FD) [c] ergò AB ⊤⊤ FD. Q. E. D.

Secundò, Si AB ⊤⊤ FD; [c] ergò AB ⊤⊤ AB − FD (2 DB); [d] quare AB ⊤⊤ DB, & [e] proinde AD ⊤⊤ DB. Q. E. D.

a 17. 10.
b 13. 10.
c cor. 17. 10.
d 13. 10.
e 17. 10.

Prop. XX.



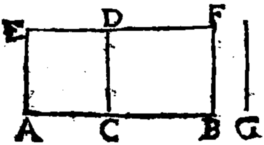
Quod sub rationalibus longitudine commensurabilibus rectis lineis BC, CD, *secundùm aliquem prædictorum modorum, continetur rectangulum* BD, *rationale est.*

a 46. 1. Exponatur A, $\dot{\rho}$. & [a] describatur BE quadra-
b 1. 6. tum ex BC. Quoniam DC. CE (BC) [b] ::
c *hyp.* BD. BE. & DC [c] $\sqcap$ BC; [d] erit rectang.
d 10. 10. BD $\sqcap$ quad. BE. ergò quum quad. BE [e] $\sqcap$ Aq;
e *hyp.* & 9. *def.* 10. [f] erit BD $\sqcap$ Aq. proinde rectang. BD est $\dot{\rho}$.
f 12. 10. Q. E. D.

Not. *Tria sunt genera linearum rationalium inter se commensurabilium. Aut enim duarum linearum rationalium longitudine inter se commensurabilium altera æqualis est expositæ rationali, aut neutra rationali expositæ æqualis est, longitudine tamen ei utraque est commensurabilis; aut denique utraque expositæ rationali commensurabilis est solùm potentiâ. Hi sunt modi illi, quos innuit præsens theorema.*

In numeris, sit BC, $\sqrt{8}$ ($2\sqrt{2}$) & CD, $\sqrt{18}$ ($3\sqrt{2}$), erit rectang. BD $= \sqrt{144} = 12$.

Prop. XXI.



Si rationale DB *ad rationalem* DC *applicetur, latitudinem* CB *efficit rationalem, & ei* DC *ad quam applicatum est* DB, *longitudine commensurabilem.*

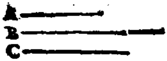
a 1. 6. Exponatur G, $\dot{\rho}$. & describatur DA quadra-
b *hyp.* tum ex BC. quoniam BD. DA [a] :: BC. CA;
c *sch.* 12. 10. atque BD: DA [b] sunt $\dot{\rho}\alpha$, [c] ideóque $\sqcap$; [d] erit
d 10. 10.

BC

BC ⊤⊥ CA. at CD (CA) [b] est $\dot{\rho}$. [e] ergò BC est $\dot{\rho}$. Q. E. D. e sch. 12.10.

In numeris, sit rectang. DB, 12; & DC, $\sqrt{8}$. erit CB, $\sqrt{18}$. atqui $\sqrt{18} = 3\sqrt{2}$. & $\sqrt{8} = 2 \times \sqrt{2}$.

LEMMA.

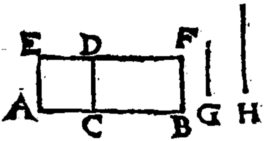


Duas rectas rationales potentiâ solùm commensurabiles invenire.

Sit A exposita $\dot{\rho}$. [a] Sume B ⊤⊥ A. [a] & C ⊤⊥ B. [b] liquet B, & C esse quæsitas.

a 11. 10.
b sch. 12. 10.

PROP. XXII.



Quod sub rationalibus DC, CB *potentiâ solùm commensurabilibus rectis lineis continetur rectangulum* DB, *irrationale est; & recta linea* H *ipsum potens, irrationalis; vocetur autem Media.*

Sit G exposita $\dot{\rho}$. & describatur DA quadratum ex DC; sitque Hq = DB. Quoniam AC. CB [a] :: DA, DB. [b] atque AC ⊤⊥ CB, [c] erit DA ⊤⊥ DB (Hq). [d] atqui Gq ⊤⊥ DA. [e] ergò Hq ⊤⊥ Gq. [f] ergò H est $\dot{\rho}$. Q. E. D. vocetur autem Media, quia AC. H :: H. CB.

a 1. 6.
b hyp.
c 10. 10
d hyp. & 9 def. 10.
e 13. 10.
f 11. 10.

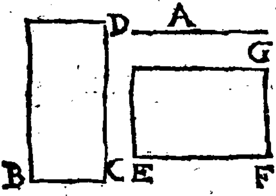
In numeris, sit DC, 3; & CB, $\sqrt{6}$. erit rectangulum DB (Hq) $\sqrt{54}$. quare H est $v\sqrt{54}$.

Mediæ nota est μ; Medii verò $\mu\nu$; pluraliter $\mu\alpha$.

SCHOL.

Omne rectangulum, quod potest contineri sub duabus rectis rationalibus potentiâ solùm commensurabilibus, est Medium; quamvis contineatur sub duabus rectis irrationalibus; atque omne Medium potest contineri sub duabus rectis rationalibus potentiâ tantùm commensurabilibus. ut exemp. gr. $\sqrt{24}$ est $\mu\nu$. quia continetur sub $\sqrt{3}$, & $\sqrt{8}$, qui sunt $\dot{\rho}$, $\sqcap\!\sqcup$. etsi possit contineri sub $\nu\sqrt{6}$, & $\nu\sqrt{96}$ irrationalibus. nam $\sqrt{24} = \nu\sqrt{576} = \nu\sqrt{6}$ in $\nu\sqrt{96}$.

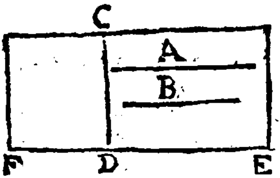
Prop. XXIII.



Quod (BD) *à media* A *fit, ad rationalem* B C *applicatum, latitudinem* CD *rationalem efficit, & ei* B C, *ad quam applicatum est* BD, *longitudine incommensurabilem.*

a sch. 22. 10. Quoniam A est μ, [a] erit Aq rectangulo alicui
b 1. ax. 1. (EG) æquale, contento sub EF, & FG
c 14. 6. $\dot{\rho}$ $\sqcap\!\sqcup$. [b] ergò BD=EG. [c] quare BC. EF :: FG.
d 22. 6. CD. [d] ergò BCq.EFq :: FGq. CDq. sed BCq,
e hyp. & EFq. [e] sunt $\dot{\rho}\alpha$, [f] ideóque $\sqcap\!\sqcup$. [g] ergò FGq $\sqcap\!\sqcup$
f sch. 12. 10.
g 10. 10. CDq. Ergò quum FG sit $\dot{\rho}$, [h] erit CD $\dot{\rho}$. Por-
h sch. 12. 10.
k 1. 6. rò, quia EF. FG [k] :: EFq. EG (BD); ob
l 10 10. EF $\sqcap\!\sqcup$ FG, [l] erit EFq $\sqcap\!\sqcup$ BD. verùm EFq
m sch. 12. 10. [m] $\sqcap\!\sqcup$ CDq. [n] ergò rectang. BD $\sqcap\!\sqcup$ CDq.
n 13. 10.
o 1. 6. quum igitur CDq. BD [o] :: CD.BC. [p] erit CD
p 10. 10. $\sqcap\!\sqcup$ BC. ergò, &c.

Prop. XXIV.



Mediæ A *commensurabilis* B, *media est.*

a 11. 6. Ad CD, $\dot{\rho}$ [a] fac rectang. CE=Aq; [a] & rectang. CF= Bq. Quoniam
b hyp.
c 23. 10. Aq (CE) est $\mu\nu$, [b] & CD $\dot{\rho}$, [c] erit latitudo

DE

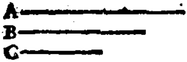
DB $\dot{\rho}$ ┬ CD. quoniam verò CE. CF [d] :: ED. DF, & CE [e] ┬┬ CF, [f] erit ED ┬┬ DF. d 1. 6. e hyp. f 10. 10.
[g] ergò DE est $\dot{\rho}$ ┬┬ CD. [h] ergò rectang. CF (B ꓶ) est $\mu\nu$, & proinde B est μ. Q. E. D. g 12, & 13. 10. h 22. 10.

Nota *quòd signum* ┬ *plerumque valet potentiâ tantùm commensurabile, ut in hac demonstratione, & in praeced. &c. quod intellige, ut ex usu erit, & juxta citationes.*

Coroll.

Hinc liquet spatium medio spatio commensurabile medium esse.

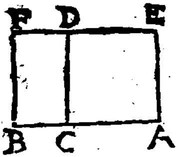
LEMMA.



Duas rectas medias A, B longitudine commensurabiles; item duas A, C potentiâ tantùm commensurabiles invenire.

[a] Sit A μ quævis; [b] sume B ┬┬ A; [c] & C ┬ A. [d] Factum esse liquet. a lem. 21. 10. & 13. 6. b 2. lem. 10. 10. c 3. lem. 10. 10. d constr. & 24. 10.

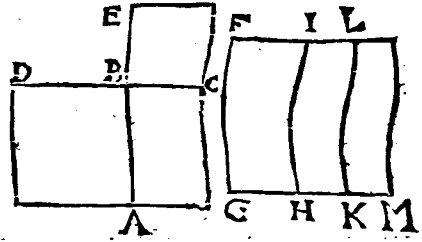
PROP. XXV.



Quod sub DC, CB mediis longitudine commensurabilibus rectis lineis continetur rectangulum DB, medium est.

Super DC construatur quadratum DA. Quoniam AC. (DC) CB [a] :: DA. DB & DC ┬┬ CB; [b] erit DA ┬┬ DB. [c] ergò DB est $\mu\nu$. Q. E. D. a 1. 6. b 10. 10. c 24. 10.

PROP. XXVI.



Quod sub mediis potentiâ tantùm commensurabilibus rectis lineis AB, BC *continetur rectangulum* AC, *vel rationale est, vel medium.*

a 46. 1. b 11 & 12. 6.

Super rectas AB, BC [a] describe quadrata AD, CE; atque ad FG $\acute{\rho}$, [b] fac rectangula FH= AD, [b] & IK=AC, [b] & LM=CE.

c hyp. & 24. 10. d 23. 10. e 10. 10. f 20. 10. g sch. 22. 6. h 2. 6. k 17. 6. l 12. 10. m 20. 10. m 22. 10.

Quadrata AD, CE, hoc est rectangula FH, LM [c] sunt $\mu\alpha$, & ⊓⊔; ergò eandem habentes rationem GH, KM sunt [d] $\acute{\rho}$, & [e] ⊓⊔. [f] ergò GH×KM est $\acute{\rho}\nu$. atqui quia AD, AC, CE, hoc est FH, IK, LM [g] sunt ∺; & [h] proinde GH, HK, KM etiam ∺, [k] erit HKq=GH× KM; [l] ergò HK est $\acute{\rho}$; vel ⊓⊔, vel ⊓⊢ IH (GF); si ⊓⊔, [m] ergò rectang. IK, vel AC est $\acute{\rho}\nu$. Sin ⊓⊢, [m] ergò AC est $\mu\nu$. Q. E. D.

LEMMA.

Si A, *&* E *sint* ⊓⊢ *tantùm;*

a hyp. & 16. 10. b 1. 6. c hyp. d 10. 10. e 14. 10.

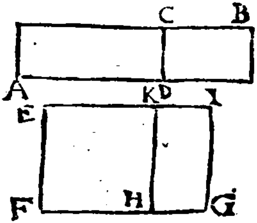
Erunt primò, Aq, Eq, Aq+Bq, Aq−Eq [a] ⊓⊔

Erunt secundò, Aq, Eq, Aq+Eq, Aq−Eq ⊓⊔ AE, & 2 Æ. Nam A. E [b] :: Aq. AE [b] :: AE. Eq. ergò cùm A [c] ⊓⊔ E, [d] erit Aq ⊓⊔ AE, [e] & 2 AE. item Eq [d] ⊓⊔ Æ, [e] & 2 Æ. quare cùm Aq+Eq ⊓⊔ Aq, & Eq; & Aq−Eq ⊓⊔ Aq, & Eq;

Eq, [f] erunt Aq+Eq, [f] & Aq−Eq '⊔⊔ AE, & f 14. 10.
2 AE.

Hinc erunt tertiò, Aq, Eq, Aq+Eq, Aq−Eq, 2 Æ [g] '⊔⊔ Aq+Eq+2 Æ; & Aq+Eq−2 Æ. g 14, 16, & 17. 10.
[g] & Aq + Eq + 2 AE '⊔⊔ Aq + Eq − 2 Æ. h cor. 7. 2.
[h] (Q: A−E.)

PROP. XXVII.



Medium AB *non ſuperat medium* AC *rationali* DB.

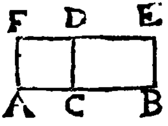
Ad EF ρ̇, [a] fac EG=AB, [a] & EH =AC. Rectangula AB, AC, hoc eſt EG, EH [b] ſunt μα, [c] ergò FG, & FH ſunt ρ̇ ⊔⊔ EF.

a cor. 16. 6.
b hyp.
c 23. 10.

itaque ſi KG, [d] id eſt DB ſit ρ̇ν, [e] erit HG ⊔⊔ HK; [f] quare HG ⊔⊔ FH. [g] ergò FGq '⊔⊔ FHq. ſed FH eſt ρ̇. [h] ergò FG eſt ρ̇. verùm priùs erat FG ρ̇. Quæ repugnant.

d 3. ax. 1.
e 21. 10.
f 13. 10.
g lem. 26. 10.
h ſch. 12. 10.

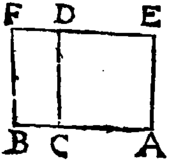
SCHOL.



1. *Rationale* AE *ſuperat rationale* AD *rationali* CE.

Nam [a] AE [a] ⊔⊔ AD; [b] ergò AE ⊔⊔ CE. [c] quare CE eſt ρ̇ν. Q. E. D.

a hyp.
b cor. 16. 10.
c ſch. 12. 10.



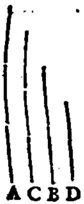
2. *Rationale* AD *cum rationali* C F *facit rationale* AF.

Nam A D [a] ⊔⊔ C F; [b] quare AF ⊔⊔ AD, & CF, [c] proinde AF eſt ρ̇ν. Q. E. D.

a ſch. 12. 10.
b 16. 10.
c ſch. 12. 10.

Prop. XXVIII.

Medias invenire (C, & D), quæ rationale CD contineant.



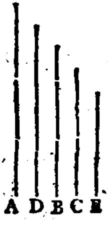
a lem. 2. 10.
b 13. 6.
c 12. 6.
d 22. 10.
e *conft.*
f 10. 10.
g 24. 10.
h 17. 6.
h *fch.* 12. 10.

[a] Sume A, & B ρ́ ⊐. [b] fac A.C :: C.B. [c] atque A.B :: C.D. Dico factum. Nam AB (Cq) [d] eſt μν; [d] unde C eſt μ. quum verò A.B [e] :: C.D, [f] erit C ⊐ D. [g] ergò D eſt μ. porro permutando A.C :: B.D. [e] hoc eſt C.B :: B.D. [h] ergò Bq = CD. atqui Bq [e] eſt ρ́ν. [h] ergò CD eſt ρ́ν. Q.E.F.

In numeris, ſit A, √2; & B, √6. ergò C, eſt v√12. fac √2.√6 :: v√12. D. vel v√4 v√36 :: v√12. D. erit D, v√108. atqui v√12 in v√108 = v√1296 = √36 = 6. ergò CD. eſt 6. item C.D :: 1.√3. quare C ⊐ D.

Prop. XXIX.

Medias invenire potentiâ tantùm commenſurabiles D, & E, quæ medium DE contineant.



a lem. 21. 10.
b 13. 6.
c 12. 6.
d 17. 6.
e 22. 10.
f *conſtr.*
g 10. 10.
h 24. 10.
k *conſtr.* & *cor.* 4. 5.
l 16. 6.
m 22. 6.

[a] Sume A,B,C ρ́ ⊐. Fac A.D [b] :: D.B. [c] & B.C :: D.E. Dico factum.

Nam AB [d] = Dq & AB [e] eſt μν; ergò D eſt μ. & B [f] ⊐ C, [g] ergò D ⊐ E. [h] ergò E eſt μ. porrò, B.C [f] :: D.E, & permutando B.D :: C.E [k] hoc eſt D.A :: C.E. [l] ergò DE = AC. Sed AC [m] eſt μν. ergò DE eſt μν. Q.E.D.

In numeris, ſit A, 20; & B, √200, & C, √80. Ergò D, eſt √√80000; & E v√12800. Ergò DE = √√1024000000 = √32000. & D.E :: √10.2, quare D ⊐ E.

Schol.

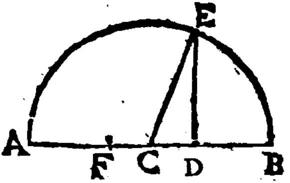
SCHOL.

A, 6.	C, 12.	
B, 4.	D, 8.	
$\overline{A}\overline{B}$, 24.	$\overline{C}\overline{D}$, 96.	
A, 6.	C, 5.	
B, 4.	D, 8.	
$\overline{A}\overline{B}$, 24.	$\overline{C}\overline{D}$, 50.	

Invenire duos numeros planos similes vel dissimiles.

Sume quoscunque quatuor numeros proportionales, A. B :: C. D. liquet AB, & CD esse similes planos. Sin latera ipsorum AB, CD non proportionalia accipias, erunt AB, CD numeri plani dissimiles.

LEMMA.



1. *Duos numeros quadratos* (DEq, & CDq) *invenire, ità ut compositus ex ipsis* (CEq) *quadratus etiam sit.*

Sume AD, DB numeros planos similes (quorum ambo pares sint, vel ambo impares) nimirum AD, 24, & DB, 6. Horum summa, (AB) est 30; differentia (FD) 18, cujus semissis (CD) est 9. [a] Habent verò plani similes AD, DB unum medium numerum proportionalem, nempe DE. patet igitur singulos numeros CE, CD, DE rationales esse; proinde CEq ([b] CDq + DEq) est numerus quadratus requisitus. a 18. 8. b 47. 1.

Facilè itaque invenientur duo numeri quadrati, quorum excessus sit quadratus, vel non quadratus numerus. nempe ex eadem constructione, [c] erit CEq—CDq=DEq. c 3. ax. 1.

Quòd si AD, DB sint numeri plani dissimi-

 les,

les, non erit media proportionalis (*DE*) numerus rationalis, proinde quadratorum CEq, CDq excessus (*D*Eq) non erit numerus quadratus.

LEMMA. 2.

2. *Duos numeros quadratos* B, C *invenire, ità ut compositus ex ipsis* D, *non sit quadratus. item quadratum numerum* A *dividere in duos numeros* B, C *non quadratos.*

A, 3. B, 9. C, 36. *D*, 45.

1. Sume numerum quemlibet quadratum B, sítq; C=4 B; & *D*=B+C. *D*ico factum.

Nam B est Q. ex constr. item quia B. C :: 1. 4 :: Q. Q. [a] erit C etiam quadratus. Sed quoniam B+C. (*D*) C :: 5. 4 :: non Q.Q. [b] non erit *D* numerus quadratus. Q. E. F.

a 24. 8.
b cor. 24. 8.

A, 36. B, 24. C, 12. *D*, 3. E, 2. F, 1.

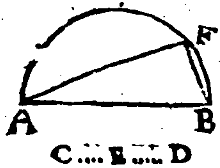
2. Sit A numerus quivis quadratus. Accipe *D*, E, F numeros planos dissimiles, sítque *D*=E+F. fac *D*. E :: A. B. & *D*. F :: A. C. *D*ico factum.

Nam quia D. E+F :: A. B+C. & D=E+F, [a] erit A=B+C. Jam dic B quadratum esse. [b] ergò A & B; & [c] proinde D & E sunt numeri plani similes, contra Hypoth. idem absurdum sequetur, si C dicatur quadratus. ergò, &c.

a 14. 5.
b 21. def. 7.
c 26. 8.

PROP.

Prop. XXX.



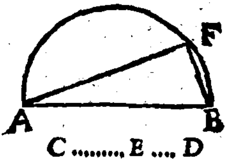
Invenire duas rationales AB, AF *potentiâ tantùm commensurabiles, itâ ut major* AB *plùs possit, quàm minor* AF, *quadrato rectæ lineæ* BF *longitudine sibi commensurabilis.*

Exponatur AB, $\dot{\rho}$. [a] Sume CD, CE numeros quadratos, ità ut CD — CE (ED) sit non Q. [b] Fiátque CD. ED :: ABq. AFq. In circulo super AB diametrum descripto [c] aptetur AF; ducatúrq; BF. Sunt AB, AF, quas petis. a 1. *lem.* 29. 10. b 3. *lem.* 10. 10. c 1. 4.

Nam ABq. AFq [d] :: CD. ED. [e] ergò ABq $\top\!\!\perp$ AFq. verùm AB est $\dot{\rho}$. [f] ergò AF est $\dot{\rho}$. sed quia CD est Q: at ED non Q: [g] erit AB '$\top\!\!\perp$ AF. porrò, ob ang. [h] rectum AFB, est ABq [k] = AFq + BFq; cùm igitur ABq. AFq :: CD. ED. per conversionem rationis erit ABq. BFq :: CD.CE :: Q.Q. [l] ergò AB $\top\!\!\perp$ BF. Q.E.F. d *constr.* e 6. 10. f *sch.* 12. 10. g 9. 10. h 31. 3. k 47. 1. l 9. 10.

In numeris; sit AB, 6; CD, 9; CE, 4; quare ED, 5. Fac 9. 5 :: 36. (Q: 6) AFq. erit AFq 20. proinde AF $\surd$ 20. ergò BFq = 36 — 20 = 16. quare BF est 4.

Prop. XXXI.



Invenire duas rationales AB, AF *potentiâ tantùm commensurabiles, itâ ut major* AB, *plùs possit, quàm minor* AF *quadrato rectæ lineæ* BF *sibi longitudine incommensurabilis.*

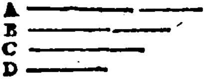
Exponatur AB, $\dot{\rho}$. [a] accipe numeros CE, ED quadratos, ità ut CD = CE + ED sit non Q. & in reliquis imitare constructionem præcedentis. Dico factum. a 2. *lem.* 29. 10.

Nam,

Nam, ut ibi, AB, AF sunt ῥ ⊔. item ABq. BFq :: CD. ED. ergò cùm CD sit non Q.
b 9. 10. [b] erunt AB, BF ⊔. Q. E. F.

In numeris, sit AB, 5. CD, 45. CE = 36; ED = 9. Fac 45. 9 :: 25 (ABq). 5 (AFq). ergò AF = √5. proinde BFq = 45 − 25 = 20. quare BF = √20.

Prop. XXXII.



Invenire duas medias C, D potentiâ tantùm commensurabiles, quæ rationale CD contineant, ità ut major C plùs possit, quàm minor D, quadrato rectæ lineæ sibi longitudine commensurabilis.

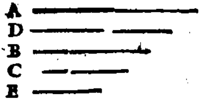
[a] Accipe A, & B ῥ ⊔; ità ut √ Aq − Bq ⊔ A. [b] Fiátque A. C :: C. B. [c] atque A. B :: C. D. Dico factum.

a 30. 10.
b 13. 6.
c 12. 6.
d constr.
e 22. 10.
f 17. 6.
g 10. 10.
h 24. 10.
k 17. 6.
l 15. 10.

Nam quia A, & [d] B sunt ῥ ⊔, [e] erit C ([f] √ AB) μ. item [g] ideò C ⊔ D. [h] ergò D etiam μ. porrò quia A. B [d] :: C. D; & permutatim A. C :: B. D :: C. B; & Bq [d] est ῥν, erit CD [k] (Bq) ῥν. Denique quia √ Aq − Bq [d] ⊔ A, [l] erit √ Cq − Dq ⊔ C. ergò, &c. Sin √ Aq − Bq ⊓ Aq, erit √ Cq − Dq ⊓ C.

In numeris, sit A, 8; B, √48 (√ : 64 − 16) ergò C = √ AB = u√ 3072. & D = u√ 1728. quare CD = u√ 5308416 = √ 2304.

Prop. XXXIII.



Invenire duas medias D, E potentiâ solùm commensurabiles, quæ medium DE contineant, ità ut major D plus possit, quàm minor E, quadrato rectæ lineæ sibi longitudine commensurabilis.

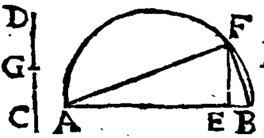
[a] Sume

[a] Sume A, & C $\acute{\rho}$, ⊤ ità ut $\sqrt{Aq - Cq}$ ⊏⊐ A. [b] sume etiam B ⊤ A, & C; & fac A. D [c] :: D. B [d] :: C. E. Erunt D, & E quæsitæ.
a 30. 10.
b *lem.* 21. 10.
c 13. 6.
d 12. 6.

Nam quoniam A, & C [e] sunt $\acute{\rho}$, [e] & B ⊤ A, & C, [f] erit B $\acute{\rho}$ & D ($\sqrt{AB}$) [g] erit μ. [e] Quia verò A. D :: C. E. erit permutando A. C :: D. E. ergò cùm A ⊤ C, [h] erit D ⊤ E, [k] ergò E est μ. porrò, [l] quia D. B :: C. E; [l] & BC est $\mu\nu$, etiam DE ei [m] æquale est $\mu\nu$. deniq; propter A. C :: D. E. [e] quia $\sqrt{Aq - Cq}$ ⊏⊐ A, [n] erit $\sqrt{Dq - Eq}$ ⊏⊐ D. ergò, &c. Sin $\sqrt{Aq - Cq}$ ⊏⊐ A. erit $\sqrt{Dq - Eq}$ ⊏⊐ Eq.
e *constr.*
f *sch.* 12. 10.
g 22. 10.
h 10. 10.
k 24. 10.
l 22. 10.
m 16. 6.
n 15. 10.

In numeris, sit A, 8; C, $\sqrt{48}$; B, $\sqrt{28}$. erit D $\upsilon\sqrt{3072}$; & E $\upsilon\sqrt{588}$. quare D. E :: 2. $\sqrt{3}$. & DE = $\sqrt{1344}$.

PROP. XXXIV.



Invenire duas rectas lineas AF, BF potentiâ incommensurabiles, quæ faciant compositum quidem ex ipsarum quadratis rationale; rectangulum verò sub ipsis contentum, medium.

[a] Reperiantur AB, CD $\acute{\rho}$ ⊤; ità ut $\sqrt{ABq - CDq}$ ⊏⊐ AB. [b] biseca CD in G. [c] fac rectang. AEB = GCq. Super AB diametrum duc semicirculum AFB. erige perpendicularem EF. duc AF, BF. Hæ sunt quæ indagandæ erant.
a 31. 10.
b 10. 1.
c 28. 6.
d 1. 6.
e *cor.* 8. 6. & 17. 6.

Nam AE. BE [d] :: BA × AE. AB × BE. Sed BA × AE [e] = AFq; [e] & AB × BE = FBq. [f] ergò AE. EB :: AFq. FBq. ergò cùm AE [g] ⊏⊐ EB, [h] erit AFq ⊏⊐ FBq. Quinetiam ABq ([k] AFq + FBq) [l] est $\acute{\rho}\nu$. denique EFq [l] = AEB [l] = CGq. [m] ergò EF = CG. ergò CD × AB = 2 EF × AB. atqui CD × AB [n] est $\mu\nu$. [o] ergò AB × EF, [p] vel AF × FB est $\mu\nu$. Q. E. D.
f 7. 5.
g 19. 10.
h 10. 10.
k 31. 3. & 47. 1.
l *constr.*
m 1. *ax.* 1.
n 22. 10.
o 24. 10.
p *sch.* 22. 6.

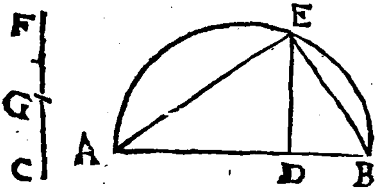
Explicatio

Explicatio per numeros.

Sit *AB*, *6.* CD, $\sqrt{12}$, quare CG $= \sqrt{\frac{12}{4}} = \sqrt{3}$. Est verò AE $= 3 + \sqrt{6}$. & EB $= 3 - \sqrt{6}$. & unde *AF* erit $\sqrt{: 18 + 216}$. Et FB, $\sqrt{: 18 - \sqrt{216}}$. item AFq + FBq est 36, & AF × FB $= \sqrt{108}$.

Cæterùm AE invenitur sic. Quia BA (6). AF : : AF. AE; erit 6 AE = AFq = AEq + 3 (EFq) ergò 6 AE − AEq = 3. pone 3 + e = AE. ergò 18 + 6 e − 9 − 6 e − ee, hoc est 9 − ee = 3. vel ee = 6. quare e $= \sqrt{6}$. proinde AE $= 3 + \sqrt{6}$.

Prop. XXXV.



Invenire duas rectas lineas AE, EB *potentiâ incommensurabiles; quæ faciant compositum quidem ex ipsarum quadratis, medium, rectangulum verò sub ipsis contentum, rationale.*

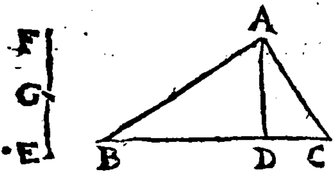
a 32. 10. [a] Sume AB, & CF μ ⊔, ità ut *AB* × *CF* sit ʹρν, atque $\sqrt{ABq - CFq}$ ⊔ AB. & reliqua fiant, ut in præcedenti. erunt AE, EB quas petis.

Nam, ut isthic ostensum est, AEq ⊔ EBq: item ABq (AEq + EBq) est μν. & denique AB × CF [b] est ʹρν, [c] idcirco & AB × DE, hoc est, AE × EB, est ʹρν. ergò &c.

b *constr.*
c *schol.* 12. 10
d *schol.* 22. 6.

Prop.

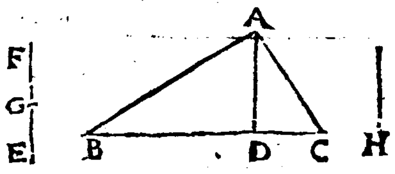
PROP. XXXVI.



Invenire duas rectas lineas BA, AC *potentiâ incommensurabiles; quæ faciant & compositum ex ipsarum quadratis medium; & rectangulum sub ipsis comprehensum medium, incommensurabiléque composito ex ipsarum quadratis.*

[a] Accipe BC & EF μ ⊓; ità ut BC $\times$ EF sit $\mu\nu$. & $\sqrt{}$ Bcq — EF q ⊓ BC. & reliqua fiant, ut in præcedentibus. Erunt BA, AC exoptata. Nam, ut priùs, BAq ⊓ ACq; item BAq + ACq est $\mu\nu$. & BA $\times$ AC est $\mu\nu$. Denique BC [b] ⊓ EF, atque [c] ideò BC ⊓ EG; ést]; BC. EG [d] :: BCq. BC $\times$ EG, (BC $\times$ AD, vel BA $\times$ AC). [e] ergò BCq (BAq + ACq) ⊓ BA $\times$ AC. ergò &c. — a 33. 10. b *constr.* c 13. 10. d 1. 6. e 14. 10.

Schol.



Invenire duas medias longitudine, & potentiâ incommensurabiles.

[a] Sume BC μ. sítque BA $\times$ AC $\mu\nu$; & ⊓ BCq (BAq + ACq). [b] Fac BA. H :: H. AC. Sunt BC, & H μ ⊓. Nam BC est μ. & BA $\times$ AC ([c] Hq) est $\mu\nu$. quare H est etiam μ. — a 36. 10. b 13. 6. c 17. 6.

d 14.10. μ.[1] [d] Item BA × AC ⊓ BCq; ergò Hq ⊓ BCq. ergò &c.

Principium senariorum per compositionẽ.

PROP. XXXVII.

Si duæ rationales AB, BC potentiâ tantùm commensurabiles componantur, tota AC irrationalis est; vocetur autem ex binis nominibus.

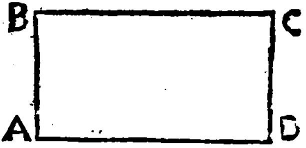
Nam quia AB [a] ⊓ BC, [b] erit ACq ⊓
a hyp.
b lem.26.10. ABq. Sed AB [a] est $\mathring{\rho}$. [c] ergò AC est $\acute{\rho}$. Q.E.D.
c 11.def.10.

PROP. XXXVIII.

Si duæ mediæ AB, BC potentiâ tantùm commensurabiles componantur, quæ rationale contineant, tota AC irrationalis est; vocetur autem ex binis mediis prima.

Nam quoniam AB [a] ⊓ BC, [b] erit ACq ⊓
a hyp.
b lem.26.10. AB × BC, $\mathring{\rho}\nu$. [c] ergò AC est $\acute{\rho}$. Q.E.D.
c 11.def. 10.

LEMMA.

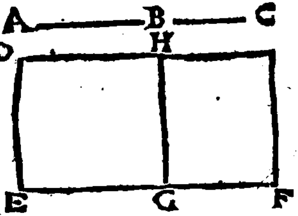


Quod sub linea rationali AB, & irrationali BC continetur rectangulum AC, irrationale est.

a hyp. Nam si rectang. AC dicatur $\mathring{\rho}\nu$; quum AB [a]
b 21. 10. sit $\mathring{\rho}$; [b] erit latitudo BC etiam $\mathring{\rho}$. contra Hyp.

PROP.

Prop. XXXIX.



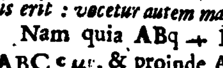
Si duæ mediæ AB, BC *potentiâ tantùm commensurabiles cõponantur, quæ medium contineant, tota* AC *irrationalis erit; vocetur autem ex binis mediis secunda.*

Ad expositam DE, $\dot{\rho}$ [a] fac rectang. DF = ACq; [b] & DG =, ABq + BCq.

Quoniam ABq [c] ⊓⊔ BCq, [d] erit ABQ + BCq, hoc est DG ⊓⊔ ABq; sed ABq [e] est $\mu\nu$. [e] ergò DG est $\mu\nu$. verùm rectang. ABC ponitur $\mu\nu$; [e] ideóque 2 ABC ([f] HF) est $\mu\nu$; [g] ergò EG, & GF sunt $\dot{\rho}$. quia verò DG [h] ⊓⊔ HF; atque DG. HF :: [k] EG. GF [l] erit EG ⊓⊔ GF. [m] ergò tota EF est $\dot{\rho}$. [n] quare rectang. DF est $\dot{\rho}\nu$. [o] ergò $\sqrt{}$ DF, id est AC, est $\dot{\rho}$. Q. E. D.

a *cor.* 16. 6.
b 47. 1. & 11. 6.
c *hyp.*
d 16. 10.
e 24. 10.
f 4. 2.
g 23. 10.
h *lem.* 26. 10.
k 1. 6.
l 10. 10.
m 37. 10.
n *lem.* 38. 10
o 11. *def.* 10.

Prop. XL.

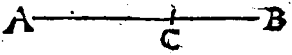


Si duæ rectæ lineæ AB, BC *potentiâ tantùm commensurabiles componantur, quæ faciant compositum quidem ex ipsarum quadratis rationale, quod autem sub ipsis continetur, medium; tota recta linea* AC, *irrationalis erit: vocetur autem major.*

Nam quia ABq + BCq [a] est $\dot{\rho}\nu$, & [b] ⊓⊔ 2 ABC [c] $\mu\nu$, & proinde ACq ([d] ABq + BCq + 2 ABC) [e] ⊓⊔ ABq + BCq $\dot{\rho}\nu$, [f] erit AC $\dot{\rho}$. Q. E. D.

a *hyp.*
b *sch.* 12. 10.
c *hyp.* & 24. 10.
d 4. 2.
e 17. 10.
f 11. *def.* 10.

PROP. XLI.

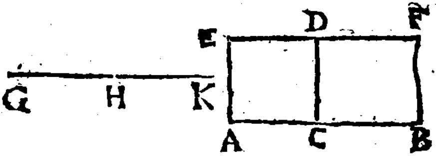


Si duæ rectæ lineæ AC, CB *potentiâ incommensurabiles componantur; quæ faciant compositum quidem ex ipsarum quadratis medium, quod autem sub ipsis continetur, rationale; tota recta linea* AB *irrationalis erit: vocetur autem rationale ac medium potens.*

a hyp. & sch. 12.10.
b sch. 12.10.
c hyp.
d 17.10.
e 11.def.10.

Nam 2 rectang. ACB, [a] ρ̂ν [b] ⊓ ACq + CBq [c] μν. [d] ergò 2 ACB [d] ⊓ ABq. quare [e] AB est ρ̂. Q.E.D.

PROP. XLII.



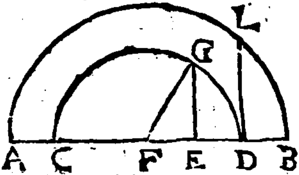
Si duæ rectæ lineæ GH, HK *potentiâ incommensurabiles componantur, quæ faciant & compositum ex ipsarum quadratis medium, & quod sub ipsis continetur medium, incommensurabileq; composito ex quadratis ipsarum; tota recta linea* GK *irrationalis erit: vocetur autem bina media potens.*

a hyp.
b 23.10.
c 4.2.
d 2.6.
e 10.10.
f 37.10.
g lem.38.10.
h 11.def.10.

Ad expositam FB ρ̂, fiant rectang. AF = GKq, & CF = GHq + HKq. Quoniam GHq + HKq (CF) [a] est μν; latitudo CB [b] erit ρ̂. Item quia 2 rectang. GHK ([c] AD) [a] est μν, etiam AC [b] erit ρ̂. Porrò quia rectang. AD [a] ⊓ CF, [d] atque AD. CF : : AC. CB, [e] erit AC ⊓ CB. [f] Quare AB est [g] ρ̂. ergò rectang. AF, id est, GKq est ρ̂ν. [h] proinde GK est ρ̂. Q.E.D.

PROP.

Prop. XLIII.

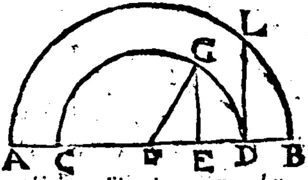


Quæ ex binis nominibus AB *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

Si fieri poteſt, binomium AB alibi in E ſecetur in alia nomina AE, EB. Liquet AB ſecari utrobique inæqualiter, quia AD ⊐ DB, & AE ⊐ EB.

Quoniam rectangula ADB, AEB [a] ſunt μα; a 37. 10
[a] & ſingula ADq, DBq, AEq, EBq ſunt ῥα; [b] a- b ſcb 27. 10.
deóque ADq + DBq, [b] & AEq + EBq etiam
ῥα, [b] idcirco ADq + DBq —: AEq + EBq.
[c] hoc eſt, 2 AEB — 2 ADB eſt ῥν. [d] ergò AEB c ſch 5. 2.
— ADB ῥν. ergò μν ſuperat μν per ῥν. [e] Q.E.A. d ſch. 12 10.
e 27. 10.

Prop. XLIV.

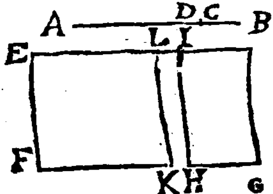


Quæ ex binis mediis primo A B *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

Puta AB dividi in alia nomina AE, EB. quo
poſito, ſingula ADq, DBq, EBq, [a] ſunt μα; [a] & a 38. 10.
rectangula ADB, AEB, eorúmque dupla, ſunt c ſcb 27 10.
[c] ῥα. [b] ergò 2 AEB — 2 ADB, [c] hoc eſt ADq d ſcb. 5 2.
+ DBq —: AEq + EBq eſt [c] ῥν. [d] Q. E. A. q 27. 10.

 Prop.

PROP. XLV.

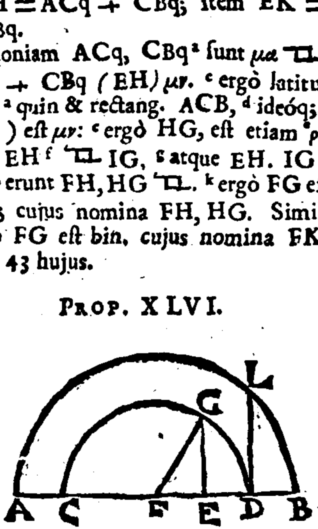


Quæ ex binis mediis secunda AB, *ad unum duntaxat punctum* C *dividitur in nomina* AC, CB.

Dic alia esse nomina AD, DB. Ad expositam EF ῥ; fac rectang. EG = ABq. & EH = ACq + CBq; item EK = ADq + DBq.

a 39. 10. Quoniam ACq, CBq [a] sunt μα ⊏⊐; [b] erit
b 16. & 24. 10. ACq + CBq (EH) μν. [c] ergò latitudo FH est ῥ. [a] quin & rectang. ACB, [d] ideóq; [a] ACB
c 23. 10. [e] (IG) est μν: [c] ergò HG, est etiam ῥ. Cùm
d 24. 10. igitur EH [f] ⊏⊐ IG, [g] atque EH. IG :: FH.
e 4. 2. HG; [h] erunt FH, HG ⊏⊐. [k] ergò FG est bino-
f lem. 26. 10. mium; cujus nomina FH, HG. Simili argu-
g 1. 6. mento FG est bin. cujus nomina FK, KG,
h 10. 10. contra 43 hujus.
k 37. 10.

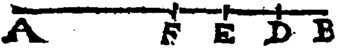
PROP. XLVI.



Major AB *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

a 40. 10. Concipe alia nomina AE, EB. quo posito re-
b sch. 27. 10. ctangula ADB, AEB [a] μα; [a] & tam ADq +
c sch. 5. 2. DBq, quàm AEq + EBq sunt ῥα. [b] ergò ADq
d 27. 10. + DBq — AEq + EBq, [c] hoc est, 2 AEB — 2 ADB est ῥν. [d] Q. F. N.

Prop. XLVII.

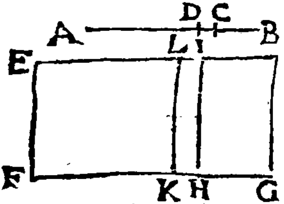


Rationale ac medium potens AB, *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

Dic alia nomina AE, EB. [a] ergò tam AEq + EBq; quàm ADq + DBq sunt μα. [a] & rectangula AEB, ADB, sunt ρα. [b] ergò 2 AEB — 2 ADB, [c] hoc est, ADq + DBq — AEq + EBq est [d] ρτ. Q. E. A.

a 41. 10.
b sch. 27. 10.
c sch. 5. 2.
d 27. 10.

Prop. XLVIII.



Bina media potens AB *ad unum duntaxat punctum* C *dividitur in nomina* AC, CB.

Vis AB dividi in alia nomina AD, DB. Ad expositam EF ρ, fiant rectang. EG = ABq, & EH = ACq + CBq, & EK = ADq + DBq. Quoniam ACq + CBq, nempe EH [a] est μν, [b] erit latitudo FH ρ. Item quia 2 ACB, [c] hoc est, IG, est [a] μν, [b] erit HG etiam ρ. Ergò cùm EH [a] ⊏⊐ IG, sitque EH. IG [d] : : FH. HG, [e] erit FH ⊏⊐ HG. [f] ergò FG est bin. cujus nomina FH. HG. Eodem modo ejusdem nomina erunt FK, KG; contra 43 hujus.

a 42. 10.
b 23. 10.
c 4. 2.
d 1. 6.
e 10. 10.
f 37. 10.

Definitiones secundæ.

Expositâ rationali, & quæ ex binis nominibus; divisa in nomina; cujus majus nomen plus possit quàm minus, quadrato rectæ lineæ sibi longitudine commensurabilis;

I. Siquidem majus nomen expositæ rationali commensurabile sit longitudine; vocetur tota ex binis nominibus prima.

II. Si verò minus nomen expositæ rationali longitudine sit commensurabile, vocetur ex binis nominibus secunda.

III. Quòd si neutrum ipsorum nominum sit longitudine commensurabile expositæ rationali, vocetur ex binis nominibus tertia.

Rursus, si majus nomen plus possit quàm minus, quadrato rectæ lineæ sibi longitudine incommensurabilis;

IV. Si quidem majus nomen expositæ rationali commensurabile sit longitudine; vocetur ex binis nominibus quarta.

V. Si verò minus nomen; vocetur quinta.

VI. Quòd si neutrum ipsorum nominum, vocetur sexta.

Prop. XLIX.

A .... 4 C ..... 5 B
D————————
E——————————G
F
H————

Invenire ex binis nominibus primam, E G.

a *sch.* 29.10. — [a] Sume AB, AC numeros quadratos, quorum excessus CB non Q. exponatur D ρ.

b 2. *lem.* 10. 10. — [b] accipe quamvis EF $\sqcup\!\sqcup$ D. [c] fac AB. CB :: EFq. FGq. erit EG bin. 1.

c 3. *lem.* 10. 10.

d *constr.* — Nam EF [d] $\sqcup\!\sqcup$ D. [e] ergò EF ρ. [f] item

e 6. *def.* 10.

f 6. 10. — EFq $\sqcup\!\sqcup$ FGq. [g] ergò FG est etiam ρ. item

g *sch.* 12. 10. — [g] quia EFq. FGq :: AB. CB :: Q. non Q; [h] erit

h 9. 10. — EF $\sqcup\!\sqcup$ FG. denique quia per conversionem rationis EFq. EFq — FGq :: AB. AC :: Q. Q.

k 9. 10. — [k] erit EF $\sqcup\!\sqcup$ $\sqrt{}$ EFq — FGq. [l] ergò EG est

l 1. *def.* 48. 10. — bin. 1. Q. E. F.

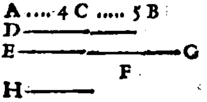
Explicatio per numeros.

Sit D, 8. EF, 6. AB, 9. CB, 5. quare cùm

9. 5

9. 5 :: 36. 20. erit FG, $\sqrt{20}$. proinde EG est $6 + \sqrt{20}$.

PROP. L.



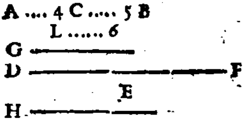
Invenire ex binis nominibus secundam, EG.

Accipe AB, & AC numeros quadratos, quorum excessus CB sit non Q. Sit D exposita $\dot{\rho}$. sume FG ⊐⊏ D. Fac CB. AB :: FGq. EFq. Erit EG quæsita. Proba ut præcedentem.

Nam FG ⊐⊏ D, quare FG est $\dot{\rho}$. item EFq ⊐⊏ FGq. ergò EF est etiam $\dot{\rho}$. item quia FGq. EFq :: CB. AB :: non Q. Q. est FG ⊐⊏ EF. denique quia CB. AB :: FGq. EFq, inverséque AB. CB :: EFq. FGq, erit ut in præcedenti, EF ⊐⊏ $\sqrt{EFq - FGq}$. [a] è quibus EG est bin. 2. Q. E. F. a 2 def. 48. 10.

In numeris, sit D, 8. FG, 10. AB, 9. CB, 5. erit EF, $\sqrt{180}$. quare EG est $10. + \sqrt{180}$.

PROP. LI.



Invenire ex binis nominibus tertiam, DF.

[a] Sume numeros AB, AC quadratos, quorum excessus CB non Q. Sitque L numerus non Q, proximè major quàm CB, nempe unitate, vel binario. sit G exposita $\dot{\rho}$. [b] Fac L. AB :: Gq. DEq. [b] & AB. CB :: DEq. EFq. erit DF bin. 3. a sch. 29. 10. b 3 lem 10. 10.

Nam quia DEq [c] ⊐⊏ Gq, [d] est DE $\dot{\rho}$, item Gq. DEq :: L, AB :: non Q. Q. [e] ergò G ⊐⊏ DE. item quia DEq [c] ⊐⊏ EFq, [d] etiam EF est $\dot{\rho}$. quinetiam quia DEq. EFq :: AB. CB :: Q. non Q. [f] est DE ⊐⊏ EF. porrò, quia per c constr. 6. 10. d sch. 12 10 e 6. 10. f 9. 10.

 constr.

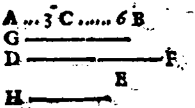
g sch. 27. 8. constr. & ex æquali Gq. EFq :: L. CB :: non Q. Q. (nam g L, & CB non sunt similes plani numeri) h erit G etiam ⊓ EF. denique ut in

h 9. 10.

k 3 def. 48. 10. præced. $\surd$ DEq − EFq ⊓ DE. k ergò DF est bin. 3. Q. E. F.

In numeris, sit AB, 9; GB, 5; L, 6. G, 8. erit DE, $\surd 96$ & EF, $\surd \frac{480}{9}$ quare DF = $\surd 96 + \surd \frac{480}{9}$.

Prop. LII.



Invenire ex binis nominibus quartam DF.

a sch. 29. 10. a Sume quemvis numerum quadratum AB, a quem divide in AC, CB non quadratos. sit G exposita ρ. b accipe DE ⊓ G. c Fac AB. CB :: DEq. EFq. erit DF bin. 4.

b 2 lem. 10. 10.

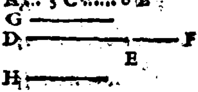
c 3 lem. 10. 10. Nam ut in 49. hujus, DF ostendetur bin. item, quia per constr. & conversionem rationis DEq. DEq − EFq :: AB. AC :: Q. non Q.

d 9. 10. d erit DE ⊓ $\surd$ DEq − EFq. e ergò DF est bin. 4. Q. E. F.

e 4. def. 48. 10.

In numeris, sit G, 8. DE, 6. erit EF $\surd 24$. ergò DF est $6 + \surd 24$.

Prop. LIII.



Invenire ex binis nominibus quintam, DF.

Accipe quemvis numerum quadratum AB, cujus segmenta AC, CB sint non Q. sit G exposita ρ. sume EF ⊓ G. fac CB. AB :: EFq. DEq. erit DF bin. 5.

Nam ut in 50 hujus, erit DF bin. & quia per constr. & invertendo DEq. EFq :: AB.

a 9. 10. CB, ideóque per conversionem rationis DEq.

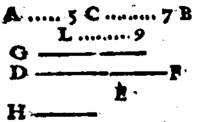
b 5. def. 48. 10. DEq − EFq :: AB. AC :: Q. non Q. a erit

DE.

DE $\sqcup$ $\surd$ DEq $-$ EFq. [b] ergò DF est bin. 4. Q. E. F.

In numeris sit G, 7. EF, 6. erit DE $\surd$ 54. quare DF est 6 $+$ $\surd$ 54.

Prop. LIV.



Invenire ex binis nominibus sextam.

Accipe AC, CB primos numeros utcunque, sic ut AC $+$ CB (AB) sit non Q. sume etiam quemvis L num. Q. sit G expos. ρ. [a] fiátque L. AB :: Gq. DEq. atque AB. CB :: DEq. EFq. erit DF. bin. 6.

a 3. *lem.* 10. 10.

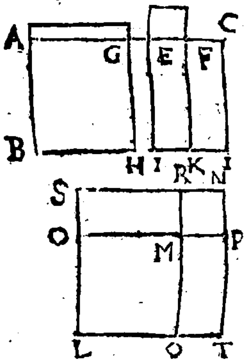
Nam, ut in 51. hujus, DF ostendetur bin. item quòd DE, & EF $\sqcup$ G. denique igitur quia per constr. & conversionem rationis DEq. DEq $-$ EFq :: AB. AC :: non Q. Q. (Nam AB primus est ad AC, [b] ideóque ei dissimilis). [c] ergò DE $\sqcup$ $\surd$ DEq $-$ EFq. [e] ergò DF est bin. 6. Q. E. F.

b *sch.* 27. 8.
c 9. 10.
c 6. *def.* 48. 10.

In numeris sit G, 6. DE $\surd$ 48. erit EF $\surd$ 28. quare DF est $\surd$ 48 $+$ $\surd$ 28.

LEMMA.

LEMMA.



Sit AD *rectangulum, cujus latus* AC *secetur inæqualiter in* E; *bisectúmq; sit segmentum minus* EC *in* F; *atque ad* AE, [a] *fiat rectang.* AGE = EFq. *pérq;* G, E, F [b] *ducantur ad* AB *parallela* GH, EI, FK. [c] *Fiat autem quadratum* LM = *rectang* AH, *atq; ad* OMP *productam* [c] *fiat quadratum* MN = GI; *rectaque* LOS, LQT, NRS, NPT *producantur.*

a 28. 6.
b 31. 1.
c 14. 2.

Dico 1. MS, MT *sunt rectangula.* Nam ob quadratorum angulos OMQ, RMP rectos,
a sch. 15. 1. [a] erit QMR recta linea. [b] ergò anguli RMO,
b 13. 1. QMP recti sunt. quare pgra MS, MT sunt rectangula.

c 2. ax. 1. 2. *Hinc patet* LS [c] = LT; *& proinde* LN *esse quadratum.*

3. *Rectangula* SM, MT, EK, FD *æqualia*
d hyp. *sunt.* Nam quia rectang. AGE [d] = EFq. [e] erit
e 17. 6. AE. EF :: EF. GE. [f] ideóque AH. EK :: EK.
f 1. 6. GI. hoc est per constr. LM. EK :: EK. MN.
g sch. 22. 6. [g] verùm LM. SM :: SM. MN. ergò EK [h] =
h 9. 5. SM [k] = FD [l] = MT.
k 36. 1.
l 43. 1. 4. *Hinc* LN [m] = AD.
m 2. ax. 1. 5. *Quia* EC *bisecta est in* F, [n] *patet* EF, FC,
n 16. 10. EC ⊏⊐ *esse.*

6. Si AE ⊤⊥ EC; & AE ⊏⊐ √ AEq —
o 18. & 16. 10. ECq, [o] erunt AG, GE, AE ⊏⊐. item, quia

AG,

AG, GE :: AH. GI [p] erunt AH, GI; hoc eſt p 10. 10.
LM, MN ⊓. item iiſdem poſitis.

7. OM ⊓ MP. Nam per Hyp. AE. ⊓ EC, [q] ergò EC ⊓̸ GE. [q] quare EF ⊓ GE. q 14. 10.
ſed EF. GE :: EK. GI. [r] ergò EK ⊓ GI, r 10. 10.
hoc eſt SM ⊓ MN. atqui SM. MN :: OM. MP. [r] ergò OM ⊓ MP.

8. Sin ponatur AE ⊓ $\surd$ AEq — ECq, [f] patet AG, GE, AE eſſe ⊓. unde LM ⊓ MN. nam AG. GE :: AH. GI :: LM. MN. f 19, & 17. 10.

His bene perſpectis, facilè ſex ſequentes Propoſitiones expediemus.

Prop. LV.

Si ſpatium AD *contineatur ſub rationali* AB, *& ex binis nominibus primâ* AC, (AE + EC) *recta linea* OP *ſpatium potens irrationalis eſt, quæ ex binis nominibus appellatur.*

Suppoſitis iis, quæ in lemmate proximè præcedenti deſcripta, & demonſtrata ſunt, liquet rectam OP poſſe ſpatium AD. [a] item AG, GE, AE ſunt ⊓. ergò cùm AE [b] ſit ῥ ⊓ AB, [c] erunt AG, & GE, ῥ ⊓ AB. [d] ergò rectangula AH, GI, hoc eſt quadrata LM, MN ſunt [e] ῥ. ergò OM, MP ſunt ῥ [e] ⊓̸. [f] proinde OP eſt bin. Q. E. D.

a *hyp. & lem.* 54. 10.
b *hyp.*
c *ſch.* 12. 10.
d 20. 10.
e *lem.* 54. 10.
f 37. 10.

In numeris ſit AB, 5. AC, 4 + $\surd$ 12. quare rectang. AD = 20 + $\surd$ 300 = quadr. LN. ergò OP eſt $\surd$ 15 + $\surd$ 5; nempe bin. 6.

Prop. LVI.

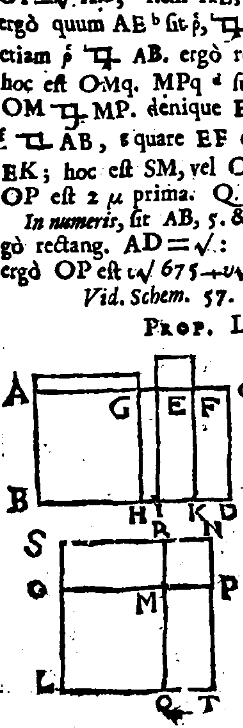
Si spatium AD contineatur sub rationali AB, & ex binis nominibus secundâ AC (AE+EC); recta linea OP spatium AD potens, irrationalis est, quæ ex binis mediis prima appellatur.

Rursus adhibito lemmate ad 54 hujus, erit
a hyp. & lem. 54. 10. OP=√.AD, [a] item AE, AG, GE sunt ⊓⊔.
ergò quum AE [b] sit ῥ, ⊓⊔ AB, [c] erunt AG, GE
b hyp.
c sch. 12. 10. etiam ῥ ⊓⊔ AB. ergò rectangula AH, GI;
d 22. 10. hoc est OMq. MPq [d] sunt μα. [e] quinetiam
e lem. 54. 10. OM ⊓⊔ MP. denique EF ⊓⊔ EC, & EC
f hyp. 12. 10. [f] ⊓⊔ AB, [g] quare EF est ῥ ⊓⊔ AB. [g] ergò
g 20. 10. EK; hoc est SM, vel OMP est ῥr. [h] Proinde
h 38. 10. OP est 2 μ prima. Q. E. D.

In numeris, sit AB, 5. & AC, √48: + 6. ergò rectang. AD = √: 1200 + 30 = OPq. ergò OP est ν√ 675 + ν√ 75; nempe bimed. 1.

Vid. Schem. 57.

Prop. LVII.



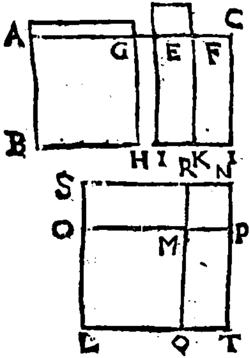
Si spatium AD contineatur sub rationali AB, & ex binis nominibus tertia AC (AE+EC); recta linea OP spatium AD potens, irrationalis est, quæ ex binis mediis secunda dicitur.

Ut priùs, OPq = AD. item rectangula AH, GI, hoc est
a hyp. & 22. 10. OMP, MPq sunt μα. [a] item EK, vel
b 38. 10. OMP est μr. [b] ergò OP est bimed. 2.

In-

In numeris, sit AB, 5. AC, $\sqrt{32}+\sqrt{34}$. quare AD est $\sqrt{800}+\sqrt{600}=$ OPq. proinde OP est $v\sqrt{450}+v\sqrt{50}$; hoc est bimed. 2.

Prop. LVIII.



Si spatium AD contineatur sub rationali AB, & ex binis nominibus quarta AC; (AE + EG) recta linea OP spatium potens, irrationalis est, quæ vocatur major.

Nam iterum, OMq [a] ⊓ MPq. rectang. verò AI, hoc est OMq+MPq [b] est $\dot{\rho}\nu$, [c] item EK, vel OMP est $\mu\nu$. [d] ergò OP($\sqrt{}$AD) est major. Q. E. D

a 1 lem. 54. 10.
b hyp. & 20. 10.
c hyp. & 22. 10.
d 40. 10.

In numeris sit AB, 5. & AC, $4+\sqrt{8}$. ergò rectang. AD est $20+\sqrt{200}$. quare OP est $\sqrt{:}\,20+\sqrt{200}$.

Prop. LIX.

Si spatium AD contineatur sub rationali AB, & ex binis nominibus quinta AC; recta linea OP spatium AD potens, irrationalis est, quæ rationale & medium potens appellatur.

Rursus OMP ⊓ MPq. rectang. verò AI, vel OMq+MPq est $\mu\nu$. [a] item rectang. EK, vel OMP est $\dot{\rho}\nu$. [b] ergò OP ($\sqrt{}$AD) est potens $\dot{\rho}\nu$, & $\mu\nu$. Q. E. D.

a ut in præced.
b 41. 10.

In numeris, sit AB, 5. & AC, $2+\sqrt{8}$. ergò rectang. AD $=10+\sqrt{200}=$ OPq. quare OP est $\sqrt{:}\,10+\sqrt{200}$.

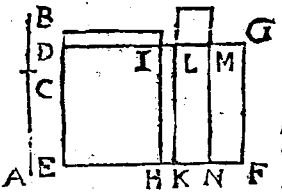
PROP. LX.

Si spatium AD *contineatur sub rationali* AB, *& ex binis nominibus sexta* AC (AE + EC); *recta linea* OP *spatium* AD *potens, irrationalis est, quæ bina media potens appellatur.*

Ut sæpe priùs, OMq ⌐⊔ MPq. & OMq + MPq est μr. & rectang. (EK) OMP etiam
a 42. 10. μr. [a] ergò OP = √AD est potens 2 μa. Q.E.D.

In numeris, sit AB, 5. AC, √12 + √8; ergò rectang. [b] AD, vel OPq est √300 + √200. proinde OP est √:√300 + √200.

LEMMA.



Sit recta AB *inæqualiter secta in* C, *sitque* AC *majus segmentum; & cuivis* DE *applicentur rectangula*, DF = ABq, & DH = ACq, & IK = CBq. *sitque* LG *bisecta in* M, *ducatúrque* MN. *parall.* GF.

Dico 1. *Rectang.* ACB = LN, vel MF.
a 4 2. & 3. [a] Nam 2 ACB = LF.
ax. 1. 2. DL ⊏ LG. nam DK (ACq + CBq)
b 7. 2. [b] ⊏ LF (2 ACB) ergò cùm DK, LF sint æ-
c 1. 6. què alta, [c] erit DL ⊏ LG.
d 16. 10. 3. Si AC ꜔⊔ CB, [d] erit rectang. DK ꜔⊔ ACq, & CBq.

4. *Item*, DL ꜔⊔ LG. nam ACq + CBq
e lem. 26. 10. [e] ꜔⊔ 2 ACB: hoc est DK ꜔⊔ LF. sed DK.
f 10. 10. LF [c] :: DL. LG. [f] ergò DL ꜔⊔ LG.

5. *Ad hæc* DL ꜔⊔ √DLq − LGq. Nam
g 1. 6. ACq. ACB [g] :: ACB. CBq. hoc est DH LN ::

LN :: LN. IK. [g] quare DI. LM :: LM. IL.
[h] ergò DI × IL = LMq. ergò cùm ACq [k] ⊐⊏ h 17. 6.
CBq. hoc est DH ⊐⊏ IK, & [l] proinde DI ⊐⊏ k *hyp.*
IL, [m] erit DL ⊐⊏ √ DLq − LGq. Q. E. D. l 10. 10. m 18. 10.

6. Sin ponatur ACq '⊐⊏ CBq, [n] *erit* DL '⊐⊏ n 19. 10.
√ DLq − LGq.

Hoc lemma præparationis vicem subeat pro 6 sequentibus propositionibus.

PROP. LXI.

Quadratum ejus quæ ex binis nominibus (AC + CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus primam.*

Suppositis iis, quæ in lemmate proximè antecedenti descripta & demonstrata sunt, Quoniam AC, CB [a] sunt ῥ ⊐⊏, [b] erit rectang. DK ⊐⊏ ACq; [c] ergò DK est ῥr. [d] ergò DL ⊐⊏ DE ῥ. rectang. verò ACB, ideóque 2 ACB (LF) [e] est μv. [f] ergò latitudo LG est ῥ '⊐⊏ DE. [g] ergò etiam DL '⊐⊏ LG. [h] item DL ⊐⊏ √ DLq − LGq. ex quibus, [k] sequitur DG esse bin. 1. Q. E. D.

a *hyp.* b *lem.* 60. 10. c *sch.* 12. 10. d 21. 10. e 22. & 24. 10. f 23. 10. g 13. 10. h *lem.* 60. 10. k 1. *def.* 48. 10.

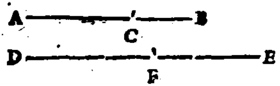
PROP. LXII.

Quadratum ejus, quæ ex binis mediis prima (AC + CB) *ad rationalem* DE *applicatum facit latitudinem* DG *ex binis nominibus secundam.*

Rursus adhibito lemmate proximè præcedenti; Rectang. DK ⊐⊏ ACq. [a] ergò DK est μv. [b] ergò latitudo DK est ῥ ⊐⊏ DE. Quia verò rectang. ACB, ideóque LF (2 ACB) [c] est ῥr, [d] erit LG ῥ ⊐⊏ DE. [e] ergò DL, LG sunt '⊐⊏. [f] item DL ⊐⊏ √ DLq − LGq. [g] ex quibus patet DG esse bin. 2. Q. E. D.

a 24. 10. b 23. 10. c *hyp.* & *sch.* 12. 10. d 21. 10. e 13. 10. f *lem.* 60. 10. g 2 *def.* 48. 10.

Prop. LXVII.



Ei, quæ ex binis nominibus (AC + CB), *longitudine commensurabilis* DE, *& ipsa ex binis nominibus est, atque ordine eadem.*

Fac AB. DE :: AC. DF. [a] sunt AC, DF ⊤⊔; [a] & CB, FE ⊤⊔. quare cùm AC, & CB [b] sint ʻρ ⊤⊐, [c] erunt DF, FE ʻρ ⊤⊐. ergò DE est etiam bin. Quia verò AC. CB [a] :: DF. FE. Si AC ⊤⊔, vel ʻ⊤⊔ √ ACq — BCq. [d] etiam similiter DF ⊤⊔, vel ʻ⊤⊔ √ DFq — FEq. item si AC ⊤⊔, vel ʻ⊤⊔ ʻρ expos. [e] erit similiter DF ⊤⊔, vel ʻ⊤⊔ ʻρ expos. at si CB ⊤⊔ vel ʻ⊤⊔ ʻρ [e] erit pariter FE ⊤⊔ vel ʻ⊤⊔ ʻρ. Sin verò utraque AC, CB ʻ⊤⊔ ʻρ, [f] erit utraq; etiam DF, FE ʻ⊤⊔ ʻρ. [g] Hoc est quodcunque binomium fuerit AB, erit DE ejusdem ordinis. Q. E. D.

a lem. 66. 10.
b hyp.
c lem. 66. 10. & sch. 12. 10.
d 15. 10.
e 12. 10. & 14. 10.
f 14. 10.
g Per def. 48. 10.

Prop. LXVIII.

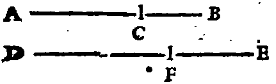
Ei, quæ ex binis mediis (AC + CB), *longitudine commensurabilis* DE, *& ipsa ex binis mediis est, atq; ordine eadem.*

[a] Fiat AB. DE :: AC. DF. [b] ergò AC ⊤⊔ DF. & CB ⊤⊔ FE. ergò cùm AC & CB [c] sint μ. [d] etiam DF, & FE erunt μ. & cùm AC [c] ⊤⊐ CB, [e] erit FD ⊤⊐ FE. [f] ergò DE est 2 μ. Si igitur rectang. ACB sit ʻρν, quia DFE [b] ⊤⊔ ACB, [g] etiam DFE est ʻρν; et si illud $\mu\nu$, [h] hoc etiam erit $\mu\nu$. [k] Id est, sive AB sit bimed. 1. sive bimed. 2. erit DE ejusdem ordinis. Q. E. D.

a 12. 6.
b lem. 66. 10.
c hyp.
d 24. 10.
e 10. 10.
f 38. 10.
g sch. 12. 10.
h 24. 10.
k 38, vel 39. 10.

Prop.

Prop. LXIX.



Majori (AC + CB) *commensurabilis* DE, *& ipsa Major e'*.

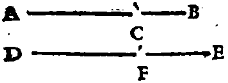
Fac AB. DE :: AC. DF. Quoniam AC
[a] ⊤ CB, [b] erit DF ⊤ FE. item ACq + [a] *hyp.*
CBq [a] est 'ϱν; proinde cùm DFq + FEq [b] ⊓ [b] *lem. 66.10.*
ACq + CBq, [c] etiam DFq + FEq est 'ϱν. de- [c] *sch. 12.10.*
nique rectang. ACB [a] est μν. [d] ergò rectang. [d] 24. 10.
DFE est μν. (quia DFE [b] ⊓ ACB) [e] Quare [e] 40. 10.
DE est major Q. E. D.

Prop. LXX.

Rationale ac medium potenti (AC + CB) *commensurabilis* DE. *& ipsa rationale ac medium potens est.*

Iterum fac AB. DE :: AC. DF. Quia AC
[a] ⊤ CB; [b] etiam DF ⊤ FE. item quia [a] *hyp.* [b] *lem. 66.10.*
ACq + CBq [a] est μν, [c] erit DFq + FEq μν. [c] 24. 10.
denique quia rectang. ACB [a] est 'ϱν. [d] etiam [d] *sch. 12.10.*
DFE est 'ϱν. [e] ergò DE est potens 'ϱν, ac μν. [e] 41. 10.
Q. E. D.

Prop. LXXI.

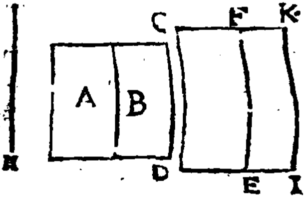


Bina media potenti (AC + CB) *commensurabilis* DE, *& ipsa bina media potens est.*

Divide DE, ut in præced. Quia ACq [a] ⊓ [a] *hyp.*
CBq, [b] erit DFq ⊓ FEq. item quia ACq [b] *lem. 66.10.*
+ CBq [a] est μν; [c] erit DFq + FEq etiam μν. [c] 24. 10.
pariterque quia ACB [a] est μν, [d] etiam DFE est [d] 24.10.
μν. denique quia ACq + CBq ⊓ ACB,
[e] erit

e 14. 10. [e] erit $DFq + FEq$ '$\sqcap\!\!\perp$ DFE.[f] è quibus sequitur
f 42. 10. DE esse potentem 2 μα. Q. E. D.

Prop. LXXII.

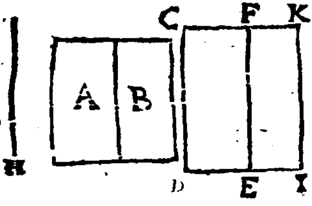


Si rationale A, & medium B componantur, quatuor irrationales fiunt; vel ea quæ ex binis nominibus, vel quæ ex binis mediis prima, vel major, vel rationale ac medium potens.

Nimirum si $Hq = A + B$, erit H una 4 linearum, quas theorema designat. Nam ad CD
a cor. 16. 6. expositam 'ρ, [a] fiat rectang. $CE = A$; item FI
b 2. ax. 1. $= B$; [b] ideóque $CI = Hq$. Quoniam igitur A
c 21. 10. est 'ρν, etiam CE est 'ρν, [c] ergò latitudo CF
est 'ρ $\sqcap\!\!\perp$ CD. & quia B est μν, erit FI μν.
d 23. 10. [d] ergò FK est 'ρ '$\sqcap\!\!\perp$ CD. [e] ergò CF, FK sunt
e 13. 10. 'ρ '$\sqcap\!\!\perp$. Tota igitur CK [f] est bin. Si igitur A
f 37. 10. $\sqsubset$ B, hoc est CE $\sqsubset$ FI, [g] erit CF $\sqsubset$ FK. er-
g 1. 6. gò si CF $\sqcap\!\!\perp$ $\sqrt{CFq - FKq}$, [h] erit CK bin.
h 1. def. 41. 10. 1. & proinde $H = \sqrt{CI}$ [k] est bin. *Si ponatur*
k 55. 10. CF '$\sqcap\!\!\perp$, $\sqrt{CFq - FKq}$, [l] erit CK bin. 4.
l 4. def. 48. 10. quare H ($\sqrt{CI}$) [m] est major. Sin A $\sqsupset$ B;
m 58. 10. [g] erit CF $\sqsupset$ FK; proinde si FK '$\sqcap\!\!\perp$ $\sqrt{FKq -}$
n 2. def. 48. 10. CFq: [n] erit CK bin. 2. [o] quare H est 2 μ prima. denique si FK '$\sqcap\!\!\perp$ $\sqrt{FKq - CFq}$, [p] erit
o 56. 10. CK bin. 5. [q] unde H erit potens 'ρν, ac μν.
p 5. def. 48. 10. Q. E. D.
q 59. 10.

Prop.

Prop. LXXIII.



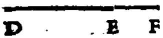
Si duo media A, B inter se incommensurabilia componantur, duæ reliquæ irrationales fiunt, vel ex binis mediis secunda, vel bina media potens.

Nempe H potens $A + B$ eſt una dictarum irrationalium. Nam ad CD expoſ. $'\rho$, fac rectang. $CE = A$, & $FI = B$. unde $Hq = CI$. Quoniam igitur CE, & FI [a] ſunt $\mu\alpha$, [b] erunt latitudines CF, FK $'\rho$ $\top\!\!\!\sqcup$ CD. item quia CE [a] $\top\!\!\!\sqcup$ FI; éſtque CE. FI [c] :: CF. FK, [d] erit CF $\top\!\!\!\sqcup$ FK. [e] ergò CK eſt bin 3. nempe, ſi CF $\top\!\!\!\sqcup$ $\sqrt{CFq - FKq}$. unde $H = \sqrt{CI}$ [f] erit $2\,\mu\,2^{a}$. Sin verò CF $\top\!\!\!\sqcup$ $\sqrt{CFq - FKq}$, [g] erit CK bin.6. & [h] proinde H eſt potens $2\,\mu\alpha$. Q. E. D.

a *hyp.*
b 23. 10.
c 1. 6.
d 10. 10.
e 3. *def.* 48. 10.
f 57. 10.
g 6. *def.* 48. 10.
h 60. 10.

Principium Senariorum per detractionem.

Prop. LXXIV.



Si à rationali DF *rationalis* DE *auferatur potentiâ tantùm commenſurabilis exiſtens toti* DF: *reliqua* EF *irrationalis eſt; vocetur autem apotome.*

Nam EFq [a] $\top\!\!\!\sqcup$ DEq; ſed DEq [b] eſt $'\rho v$. [c] ergò EF. eſt ρ'. Q. E. D.

In numeris, ſit DF, 2. DE, $\sqrt{3}$. EF erit $2 - \sqrt{3}$.

a lem.26.10.
b *hyp.*
c 10. & 11. *def.* 10.

PROP. LXXV.

D ——— E ——— F

Si à media DF *media* DE *auferatur, potentiâ tantùm commensurabilis existens toti* DF, *quæ cum tota* DF *rationale contineat; reliqua* EF *irrationalis est; vocetur autem mediæ apotome prima.*

a *sch.* 26.10. Nam EFq [a] ⊐⊏ rectang. FDE. ergò cùm
b *hyp.*
c 20. & 11. FDE [b] sit 'ρν, [c] erit EF 'ρ. Q. E. D.
def. 10. *In numeris*, sit DF $\nu\surd$ 54. & DE $\nu\surd$ 24. ergò EF est $\nu\surd 54 - \nu\surd 24$.

PROP. LXXVI.

D ——— E ——— F

Si à media DF *media* DE *auferatur, potentiâ tantum commensurabilis existens toti* DF, *quæ cum tota* DF *medium contineat, reliqua* EF *irrationalis est: vocetur autem mediæ apotome secunda.*

a *hyp.* Quia DFq, & DEq [a] sunt μα ⊐⊏,
b 16. 10. [b] erit DFq + DEq ⊐⊏ DEq. [c] quare DFq
c 24. 10. + DEq est μν. item rectang. FDE, [c] ideóque
d *cor.* 7. 2. 2 FDE [a] est μν. ergò EFq ([d] DFq + DEq —
e 27. 10. 2 FDE) [e] est 'ρν. quare EF est 'ρ. Q. E. D.

In numeris, sit DF, $\nu\surd$ 18; & DE, $\nu\surd$ 8. erit EF $\nu\surd 18 - \nu\surd 8$.

PROP. LXXVII.

A ——— B ——— C

Si à recta linea AC *recta auferatur* AB *potentiâ incommensurabilis existens toti* BC, *quæ cum tota* AC *faciat compositum quidem ex ipsarum quadratis rationale, quod autem sub ipsis continetur medium, reliqua* BC *irrationalis est; vocetur autem minor.*

a *hyp.* Nam ACq + ABq [a] est 'ρν. at rectang. ACB
b *sch.* 12.10. [a] est μν. [b] ergò 2 CAB ⊐⊏ ACq + ABq
c 7. 2. (2 [c] CAB + BCq); [d] ergò ACq + ABq ⊐⊏
d 17. 10.
e 11. *def.* 10. BCq. [e] ergò BC est 'ρ Q. E. D.

In

In numeris, sit AC, √: 18. + √108. AB √: 18 − √ 108. ergò BC est √: 18 + √: 108. − √: 18 − √ 108.

PROP. LXXVIII.

D————————E————F

Si à recta linea DF *recta auferatur* DE *potentiâ incommensurabilis existens toti* DF, *quæ cum tota* DF *faciat compositum quidem ex ipsarum quadratis medium, quod autem sub ipsis continetur, rationale; reliqua* EF *irrationalis est: vocetur autem cum rationali medium totum efficiens.*

Nam 2 FDE [a] est 'ρτ. [b] & DFq + DEq est μτ. [c] ergò 2 FDE ⊔⊔ DFq + DEq [d] (2 FDE + EFq) [e] ergò EF est 'ρ. Q. E. D.

In numeris sit DF, √: √ 216 + √ 72. DE, √: √ 216 − √ 72. ergò EF est √: √ 216 + √ 72 − √: √ 216 − √ 72.

a hyp. & sch. 12. 10.
b hyp.
c sch. 12. 10.
d 7. 2.
e sch. 12. 10. & 11. def. 10

PROP. LXXIX.

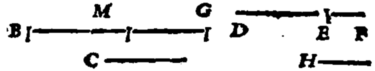
Si à recta DF *recta auferatur* DE, *potentiâ incommensurabilis existens toti* DF, *quæ cum tota faciat & compositum ex ipsarum quadratis, medium; & quod sub ipsis continetur, medium, incommensurabileque composito ex quadratis ipsarum, reliqua irrationalis est: vocetur autem cum medio medium totum efficiens.*

Nam 2 FDE, & DFq + DEq [a] sunt μα; [b] ergò EFq ([c] DFq + DEq − 2 FDE) est 'ρτ. [d] proinde EF est 'ρ. Q. E. D.

Exempl. gr. sit DF, √: √ 180. + √ 60. DE, √: √ 180 − √ 60. EF erit √: √ 180. + √ 60 − √: √ 180. − √ 60.

a hyp. & 24. 10.
b 27. 10.
c cor. 7. 2.
d 11. def. 10.

LEMMA.



Si idem sit excessus inter primam magnitudinem BG, *& secundam* C (MG) *qui inter tertiam magnitudinē* DF, *& quartam* H (EF); *erit & vicissim idem excessus inter primam magnitudinem* BG, *& tertiam* DF; *qui inter secundam* C, *& quartam* H.

a hyp. — Nam quia [a] æqualibus BM, DE adjectæ sunt MG, EF, [a] hoc est C, H; erit excessus totorum

b 15. ax. 1. — BG, DF, [b] æqualis excessui adjectorum C, H. Q. E. D.

Coroll.

Hinc, quatuor magnitudines Arithmeticè proportionales, vicissim erunt Arithmeticè proportionales.

Prop. LXXX.

Apotomæ AB *una tantùm congruit recta linea rationalis* BC *potentiâ, tantùm commensurabilis existens toti* AB.

a 22. 10. — Si fieri potest, alia BD congruat. [a] ergò rectangula ACB, ADB; [b] ideóq; eorum dupla sunt

b 24. 10.

c cor. 7. 2. — $\mu\alpha$. cùm igitur ACq + BCq − 2 ACB [c] = ABq; [c] = ADq + DBq − 2 ADB. ergò vicissim ACq

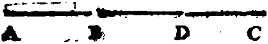
d lem. 79. 10. — + BCq − ADq + BDq [d] = 2 ACB. −: 2 ADB. Sed ACq + BCq −: ADq + BDq [e] est

e hyp. & 27. 10. — [e] pr. [f] ergò 2. ACB −: 2 ADB est [g] pr.

f sch. 12. 10. — Q. E. A.

g 27. 10.

Prop. LXXXI.

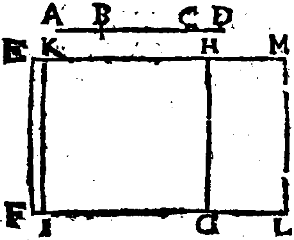


Mediæ Apotomæ primæ AB *una tantùm congruit recta linea media* BC, *potentiâ solùm commensurabilis existens toti, & cum tota rationale continens.*

Dic etiam BD congruere, igitur quoniam tam ACq, & BCq; quàm ADq, & BDq [a] sunt $\mu\alpha$ ⊤⊥. [b] etiam ACq + BCq, & ADq + BDq erunt $\mu\alpha$. [c] sed rectangula ACB, ADB; [d] adeóq; 2 ACB, & 2 ADB sunt [e] $\rho\alpha$. [e] ergò 2 ACB —: 2 ADB; [f] hoc est ACq + BCq —: ADq + BDq est [e] $\rho\nu$. [g] Q. E. A.

a *hyp.*
b 16 & 24. 10.
c *hyp.*
d *sch.* 12. 10.
e *sch.* 27. 10.
f 7. 2. & *lem.* 79. 10.
g 27. 10.

Prop. LXXXII.



Mediæ Apotomæ secundæ AB *una tantùm congruit recta linea media* BC, *potentiâ solùm commensurabilis existens toti, & cum tota medium continens.*

Si fieri potest, congruat alia BD. Ad EF ρ fiant rectang. EG = ACq + BCq; item rectang. EL = ADq + BDq. Item EI = ABq. Jam 2 ACB + ABq = ACq + BCq = EG, ergò cùm EI = ABq; [a] erit KG = 2 ACB. porrò ACq, & BCq [b] sunt $\mu\alpha$ ⊤⊥. [c] Ergò EG (ACq + BCq) est $\mu\nu$. [d] ergò latitudo EH ρ ⊤⊥. EF. [e] Quinetiam rectang. ACB; [f] ideóque 2 ACB (KG) est $\mu\nu$. [d] ergò KH est etiam ρ ⊤⊥. EF. denique quia ACq + BCq, id est, EG [g] ⊤⊥. 2 ACB (KG) éstque

a 4. 2. & *ax.* 1.
b *hyp.*
c 24. 10.
d 23. 10.
e *hyp.*
f 24. 10.
lem. 26. 10

 EG.

h 1. 6. EG. KG :: [h] EH. KH [k] erit EH ⊏⊐ KH.
k 10. 10. [l] ergò EK est aptome, cujus congruens KH. si-
l 74. 10. mili argumento erit *KM* ejusdem EK congruens; contra 80 hujus.

Prop. LXXXIII.

A B D C

Minori AB, *una tantùm congruit recta linea* (BC) *potentiâ incommensurabilis existens toti, & cum tota faciens compositum quidem ex ipsarum quadratis rationale; quod autem sub ipsis continetur medium.*

Puta alium BD congruere. Cùm igitur ACq
a hyp. + BCq, & ADq + BDq [a] sint $\mu\alpha$, eorum ex-
b lem 97. 10. cessus (2 [b] ACB —: 2 ADB) [c] est $\rho\nu$, [d] Q. E. A;
c sch. 27. 10. quia ACB, & ADB sunt $\mu\alpha$ per hypoth.
d 27. 10.

Prop. LXXXIV.

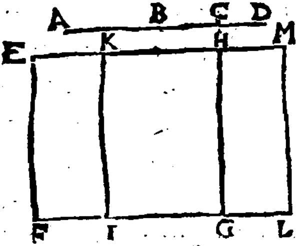
A B D C

Ei (AB), *quæ cum rationali medium totum facit, una tantùm congruit recta linea* BC, *potentiâ incommensurabilis existens toti, & cum tota faciens compositum quidem ex ipsarum quadratis medium; quod autem sub ipsis continetur, rationale.*

Dic aliam BD etiam congruere. [a] ergò re-
a hyp. ctangula ACB, ADB. [b] ideóque 2 ACB, & 2
b sch. 12. 10. ADB sunt $\rho\tilde{\alpha}$. ergò 2 ACB —: 2 ADB; [c] hoc
c lem. 79. 10. est, ACq + BCq —: ADq + BDq [d] est $\rho\nu$.
d scho. 27. 10 Q. E. A: quum ACq + BCq, & ADq + BDq sint $\mu\alpha$ per hypoth.

Prop.

PROP. LXXXV.



Ei (AB), quæ cum medio medium totum facit una tantùm congruit recta linea BC *potentiâ incommensurabilis existens toti, & cum tota faciens & compositum ex ipsarum quadratis medium, & quod sub ipsis continetur, medium, incommensurabiléque composito ex ipsarum quadratis.*

Suppositis iis quæ facta & ostensa sunt in 82 hujus; liquet EH, & KH esse ρ ⌐⊏ EF. Porrò igitur quia ACq + CBq, hoc est, rectang. EG [a] ⌐⊏ ACB, [b] ideóque EG ⌐⊏ 2 ACB (KG) éstque EG. KG :: [c] EH. KH; erit EH ⌐⊏ KH. ergò EK est apotome, cujus congruens KH. Haud aliter KM eidem apotomæ EK congruere ostendetur; contra 80 hujus.

a *hyp.*
b 14. 10.
c 1. 6.

Definitiones tertiæ.

EXpositâ rationali, & apotomâ, si tota plus possit quàm congruens quadrato rectæ lineæ sibi longitudine commensurabilis;

I. Si quidem tota expositæ rationali longitudine sit commensurabilis, vocetur apotome prima.

II. Si verò congruens expositæ rationali longitudine sit commensurabilis, vocetur apotome secunda.

III. Quòd si neque tota, neque congruens expositæ rationali sit longitudine commensurabilis, vocetur apotome tertia.

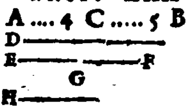
Rursus si tota plus possit quàm congruens quadrato rectæ sibi longitudine incommensurabilis;

IV. *Si quidem tota expositæ rationali sit longitudine commensurabilis, vocetur apotome quarta.*

V. Si verò congruens expositæ rationali sit longitudine commensurabilis, vocetur apotome quinta.

VI. Quòd si neque tota neque congruens expositæ rationali sit longitudine commensurabilis, vocetur apotome sexta.

PROP. LXXXVI, 87, 88, 89, 90, 91.



Invenire apotomen primam, secundam, tertiam, quartam, quintam, sextam.

Apotomæ inveniuntur, subductis minoribus binomiorum nominibus ex majoribus. Exemp. gr. Sit $6 + \sqrt{20}$. bin. 1. erit $6 - \sqrt{20}$, apot. 1. &c. Quare de earum inventione plura repetere nihil est necesse.

LEMMA.

Sit rectangulum AC *sub rectis* AB, AD. *producatur* AD *ad* E, *& bisecetur* DE *in* F. *sitq; rectang.* AGE = FEq *& compleantur rectangula* AI, DK, FH. *Fiant verò quadratum* LM = AH; *& quadratum* NO = GI, *producantúrque* NSR, OST.

Dico primò rectangul. AI = LM + NO = TOq + SOq. ut patet ex constr. Secun-

Secundò, *Rectang.* DK = LO. Nam quia rectang. AGE [a] = FEq, [b] sunt AG, FE, GE ∺, [c] adeóque AH, FI, GI ∺; [a] hoc est, LM, FI, NO ∺; atqui LM, LO, NO [d] sunt ∺; ergò FI = [e] LO [f] = DK = [g] NM. — a *constr.* b 17.6. c 1.6. d *sch.* 22.6. e 9.5.

Tertiò, *Hinc*, AC = AI — DK — FI = LM + NO — LO — NM = TR. — f 36.1. g 43.1.

Quartò, [h] *Liquet* DF, FE, DE esse ⊓⊔. — h 16.10.

Quintò, *Si* AE ⊓⊔ DE, & AE ⊓⊔ √ AEq — DEq, [k] *erunt* AG, GE, AE ⊓⊔. — k 18.10. & 10.10.

Sextò, *Item, quia* AE [l] ⊓⊔ DE, [m] *erunt* AE, FE ⊓⊔. [n] *ideòque* AI, FI; *hoc est*, LM + NO & LO *sunt* ⊓⊔. — l *hyp.* m 13.10.

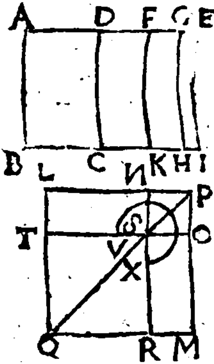
Septimò. *Item quia* AG [*] ⊓⊔ GE [n] *erunt* AH GI, *hoc est*, LM, NO ⊓⊔. — n 1.6 & 10.10. * prius.

Octavò, *Sed quia* AE [l] ⊓⊔ DE, [o] *erunt* FE, GE ⊓⊔, [n] *ideóque rectang.* FI ⊓⊔ GI, *hoc est* LO ⊓⊔ NO. *quare cùm* LO. NO [p] ∷ TS. SO. [q] *erunt* TS, SO ⊓⊔. — o 14.10. p 2.6. q 10.10.

Nonò, *Sin ponatur* AE ⊓⊔ √ AEq — DEq; [r] *erunt* AG, GE, AE ⊓⊔. — r 19.10. & 17.10

Decimo, [f] *Quare rectang.* AH, GI, *hoc est* TOq, SOq *erunt* ⊓⊔. — f 1.6. & 10.10.

Prop. XCII.



Si spatium AC *contineatur sub rationali* AB, *& Apotoma prima* AD (AE — DE); *recta linea* TS *spatium* AC *potens, apotome est.*

Adhibe lemma proximè antecedens pro præparatione ad demonstrationem hujus. Igitur TS = √ AC. item AG, GE, AE sunt ⊓; ergò cùm AE ⊓ [a] AB $\dot{\rho}$ [b] erunt AG, & GE ⊓

a hyp. b 12. 10.

c 20. 10. AB. [c] ergò rectangula AH & GI, hoc est TOq

d lem. 91. 10 & SOq sunt $\dot{\rho}\alpha$. [d] item TO, SO sunt $\dot{\rho}$ ⊓̸,

e 74. 10. [e] proinde TS est apotome. Q. E. D.

Prop. XCIII.

Vide Schem. præced.

Si spatium AC *contineatur sub rationali* AB, *& apotoma secunda* AD (AE — DE) ; *recta linea* TS *spatium* AC *potens; mediæ est apotome prima.*

Rursus juxta lemma antecedens, AG, GE,

a hyp. AE sunt ⊓. cùm igitur AE [a] sit $\dot{\rho}$ ⊓ AB

b 13. 10. [b] erunt AE, GE etiam $\dot{\rho}$ ⊓ AB. [c] ergò rect-

c 22. 10. angula AH, GI, hoc est TOq, SOq, sunt $\mu\alpha$;

d lem. 74. 10 [d] item TO ⊓̸ SO. Denique quia DE [e] ⊓

e hyp. f 20. 10. AB $\dot{\rho}$. [f] erit rectang. DI, ejúsque semissis DK,

g 75. 10. vel LO, hoc est TOS $\dot{\rho}\nu$ [g] è quibus sequitur TS (√ AC) esse mediæ apot. 1. Q. E. D.

Prop. XCIV.

Vide idem.

Si spatium AC *contineatur sub rationali* AB, *&* *apotoma tertia* AD (AE — DE) ; *recta linea* TS *spatium* AC *potens, mediæ est apotome secunda.*

Ut in præcedenti TO, & SO sunt μ. Quoniam igitur DE [a] est $\dot{\rho}$ ⊓ AB; [b] erit rectang. DI, [c] ideóque DK; vel TOS $\mu\nu$. [d] ergò TS $= \sqrt{}$AC est mediæ apot. 2. Q. E. D.

a hyp.
b 22. 10.
c 24. 10.
d 76. 10.

Prop. XCV.

Vide idem.

Si spatium AC *contineatur sub rationali* AB, *& apotoma quarta* AD (AE — DE) *recta linea.* TS *spatium* AC *potens, minor est.*

Rursus TO [a] ⊓ SO. Quoniam igitur AE [b] est $\dot{\rho}$ ⊓ AB, [c] erit AI; (TOq + SOq) $\dot{\rho}\nu$. atqui ut priùs rectang. TOS est $\mu\nu$. [d] ergò TS $= \sqrt{}$ AC est minor. Q. E. D.

a lem, 91. 10.
b hyp.
c 20. 10.
d 77. 10.

Prop. XCVI.

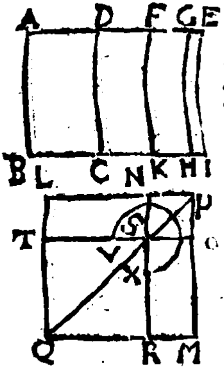
Vide idem.

Si spatium AC *contineatur sub rationali* AB, *& apotoma quinta* AD (AE — DE) ; *recta linea* TS *spatium* AC *potens, est quæ cum rationali medium totum efficit.*

Rursus enim TO ⊓ SO. itaque cùm AE [a] sit $\dot{\rho}$ ⊓ AB, [b] erit AI, hoc est TOq + SOq $\mu\nu$. Sed prout in 93 rectang. TOS est $\dot{\rho}\nu$. [c] proinde TS $= \sqrt{}$ AC est quæ cum $\dot{\rho}\nu$ facit totum $\mu\nu$. Q. E. D.

a hyp.
b 22. 10.
c 78. 10.

PROP. XCVII.



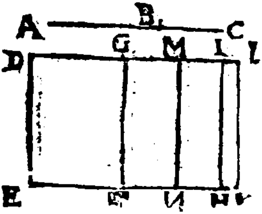
Si spatium AC *contineatur sub rationali* AB, *& apotoma sexta* AD (AE − DE); *recta linea* TS *spatium* AC *potens, est quæ cum medio medium totum efficit.*

Itidem, ut sæpe priùs, TO ⊔ SO. item ut in 96, TOq + SOq est μν. rectang. verò TOS est ῥν, ut in 94. [a] deniq; TOq + SOq [b] ⊓ TOS. [b] ergò TS = √ AC est quæ cum μν facit totum μν. Q. E. D.

a lem. 91. 10.
b 79. 10.

LEMMA.

* cor. 16. 6.



Ad rectā quamvis DE * *applicentur rectang.* DF = ABq; & DH = ACq, & IK = BCq, *& sit* GL *bisecta in* M; *ductáque sit* MN *parall.* GF.

Erit primò, *Rectang.* DK = ACq + BCq, *ut constructio indicat.*

Secundò, *Rectang.* ACB = GN, *vel* MK. Nam DK [a] = ACq + BCq [b] = 2 ACB + ABq. at ABq [a] = DF. ergò GK [c] = 2 ACB. & [d] proinde GN, vel MK = ACB.

Tertiò, *Rectang.* DIL = MLq. Nam quia ACq, ACB [e] :: ACB, BCq; hoc est DH MK.

a constr.
b 7. 2.
c 3. ax. 1.
d 7. ax. 1.
e 1. 6.

MK :: MK. IK, [e] erit DI. ML :: ML. IL.
[f] ergò DIL = MLq. f 17. 6.

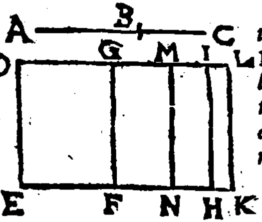
Quartò, *Si ponatur* AC ⊐⊐ BC, *erit* DK ⊐⊏ ACq. Nam ACq + BCq (DK) [g] ⊐⊏ ACq. g 16. 10.

Quintò, *Item*, DL ⊐⊏ √ DLq − GLq. Nam quia DH (ACq) ⊐⊏ IK (BCq) [h] erit DI ⊐⊏ IL. [k] ergò √ DLq − GLq ⊐⊏ DL. h 10. 10. k 18. 10.

Sextò, *Item* DL '⊐⊏ GL. Nam ACq + BCq '⊐⊏ [l] 2 ACB; hoc est DK '⊐⊏ GK. [m] ergò DL '⊐⊏ GL. l *lem.* 26. 10. m 10. 10.

Septimò, *Sin ponatur* AC '⊐⊐ BC, [n] *erit* DL '⊐⊏ √ DLq − GLq. n 19. 10.

Prop. XCVIII.



Quadratū apotomæ AB (AC − BC) *ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen primam.*

Fac ut in lemmate proximè præcedenti:

Quoniam igitur AC, BC [a] sunt ρ́ ⊐⊐, [b] erit DK (ACq + BCq) ⊐⊏ ACq; [c] ergò DK est ρ́ν. [d] quare DL est ρ́ ⊐⊏ DE. [e] item rectang. GK (2 ACB) est μν. [f] ergò GL est ρ́ ⊐⊏ DE. [g] proinde DL '⊐⊏ GL; [h] sed DLq ⊐⊏ GLq. [k] ergò DG est apotome, & [l] quidem prima (quia [m] AC ⊐⊐ BC, & propterea DL ⊐⊏ √ DLq − GLq.) Q. E. D.

a *hyp.*
b *lem.* 97. 10.
c *sch.* 12. 10.
d 21. 10.
e 22, & 24. 10.
f 23. 10.
g 13. 10.
h *sch.* 12. 10.
k 74. 10.
l 1. *def.* 85. 10.
m *lem.* 97. 10.

Prop. XCIX.

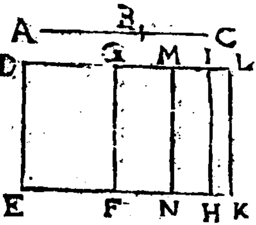
Vide Schema subsequens.

Quadratum mediæ apotomæ primæ AB (AC — BC) *ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen secundam.*

a hyp.
b lem 97. 10.
c 24. 10.
d 23. 10.
e hyp. & sch. 12. 10.
f 21. 10.
g 13. 10.
h sch. 12. 10.
k 74. 10.
l lem. 97 10.
m 2 def. 85. 10.

Rursus (supposito lemmate præcedenti) quia AC, & BC [a] sunt μ ⊓⊔ [b], erit DK (ACq + BCq) ⊓⊔ ACq; [c] quare DK est μν. [d] ergò DL est [e] ρ ⊓⊔ DE. [e] item GK (2 ACB) est [e] μν. [f] ergò GL est [e] ρ ⊓⊔ DE; [g] quare DL ⊓⊔ GL. [h] Sed DLq ⊓⊔ GLq. [k] ergò DG est apotome. quia verò DL [l] ⊓⊔ √ DLq — GLq, [m] erit DG apotome secunda. Q. E. D.

Prop. C.



Quadratū mediæ apotomæ secundæ AB (AC — BC) *ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen tertiam.*

a 23. 10.
b lem. 26. 10.
c 1. 6. & 10. 10.
d sch. 12. 10.
e 74. 10.
f 3. def. 85. 10.
g lem 97. 10.

Iterum DK est μν. [a] quare DL est ρ̇ ⊓⊔ DE. item GK est μν. [a] unde GL est ρ̇ ⊓⊔ DE; [b] item DK ⊓⊔ GK, [c] quare DL ⊓⊔ GL; [d] at DLq ⊓⊔ GLq. [e] ergò DG est apot. & quidem [f] 3ª. [g] quia DL ⊓⊔ √ DLq — GLq. Q. E. D.

Prop. CI.

Vide Schema præced.

Quadratum minoris AB (AC — BC) *ad rationalem*

tionalem DE *applicatum, facit latitudinem* DG *apotomen quartam.*

Ut priùs, ACq + BCq, hoc est DK est $\dot{\rho}\nu$;
[a] ergò DL est $\dot{\rho}$ $\urcorner\!\sqcap$ DE. at rectang. ACB, ide- a 21. 10.
óque GK (2 ACB) * est $\mu\nu$, [b] quare GL est $\dot{\rho}$ * hyp.
$\urcorner\!\sqcap$ DE. [c] ergò DL $\urcorner\!\sqcap$ GL. [d] at DLq $\urcorner\!\sqcap$ b 23. 10.
GLq. quia verò * ACq $\urcorner\!\sqcap$ BCq, [e] erit DL $\urcorner\!\sqcap$ c 13. 10.
$\sqrt{}$ DLq — GLq: [f] ergò DG conditiones habet d *sch.* 12. 10. e *lem.* 97. 10.
apotomæ quartæ. Q. E. D. f 4. *def.* 85. 10.

PROP. CII.

Vide Schem. præced.

Quadratum ejus AB (AC — BC), *quæ cum rationali medium totum efficit, ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen quintam.*

Rursus enim, DK est $\mu\nu$, [a] quare DL est $\dot{\rho}$ a 23. 10.
$\urcorner\!\sqcap$ DE. item GK est $\dot{\rho}\nu$, [b] unde GL est $\dot{\rho}$. $\urcorner\!\sqcap$ b 21. 10.
DE. [c] ergò DL $\urcorner\!\sqcap$ GL, [d] sed DLq $\urcorner\!\sqcap$ GLq. c 13. 10. d *sch.* 12. 10.
porrò, DL [e] $\urcorner\!\sqcap$ $\sqrt{}$ DLq — GLq. ex quibus, e *lem.* 97. 10.
DG [f] est apot. quinta. Q. E. D. f 5. *def.* 85. 10.

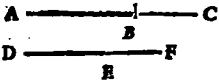
PROP. CIII.

Vide Schema idem.

Quadratum ejus AB (AC — BC), *quæ cum medio medium totum efficit, ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen sextam.*

Haud aliter, quàm antea, DK, & GK sunt a 23. 10.
$\mu\alpha$; [a] quare DL & GL sunt $\dot{\rho}$ $\urcorner\!\sqcap$ DE. item b *hyp* & *lem.* 97. 10.
DK [b] $\urcorner\!\sqcap$ GK, [c] quare DL $\urcorner\!\sqcap$ GL. [d] ergò c 10. 10.
DG est apot. [b] cùm igitur ACq $\urcorner\!\sqcap$ BCq, ide- d 74. 10.
óq; DL $\urcorner\!\sqcap$ $\sqrt{}$ DLq — GLq, [e] erit DG. e 6. *def.* 85. 10.
apot. sexta. Q. E. D.

PROP. CIV.



Recta linea DE *apotome* AB (AC—BC) *longitudine commensurabilis; & ipsa apotome est, atque ordine eadem.*

LEMMA

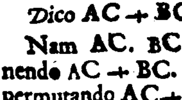
Sit AB. DE :: AC. DF. *&* AB $\sqsupset\sqsubset$ DE. *Dico* AC + BC $\sqsupset\sqsubset$ DF + EF.

Nam AC. BC [a] :: DF. EF. ergò componendo AC + BC. BC :: DF + EF. EF. ergò permutando AC + BC. DF + EF :: BC. EF.

a *lem.* 66.10. [a] at BC $\sqsupset\sqsubset$ EF. [b] ergò AC + BC $\sqsupset\sqsubset$ DF
b 10. 10. + EF; Q. E. D.

a 12. 6. [a] Fac AB. DE :: AC. DF. [b] igitur AC +
b *lem.* 103. 10. BC $\sqsupset\sqsubset$ DF + EF. ergò cùm AC + BC [c] binomium sit, [d] erit *D*F + EF ejusdem ordinis bi-
c *hyp.* nomium: [e] quare *D*F — EF ejusdem ordinis
d 67. 10. apotome est, cujus AC — BC. Q. E. *D*.
e *Per definitiones ad* 85. 10.

PROP. CV.

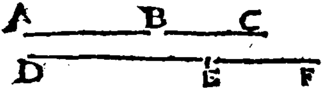


Recta linea DE *mediæ apotomæ* AB (AC — *B*C) *commensurabilis; & ipsa mediæ apotome est, atque ordine eadem.*

a 12. 6. Iterum [a] fac AB. *D*E :: AC. *D*F. [b] quare
b *lem.* 103. 10. AC + BC $\sqsupset\sqsubset$ *D*F + EF. [c] ergò DF + EF est bimed. ejusdem ordinis, cujus AC + BC.
c 68. 10. [d] proinde & *D*F — EF mediæ apotome erit e-
d 75, & 79. 10. jusdem classis, cujus AC — BC. Q. E. *D*.

PROP.

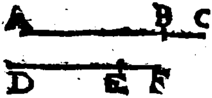
Prop. CVI.



Recta linea DE *Minori* AB (AC — BC) *commensurabilis, & ipsa minor est.*

Fiat AB. DE :: AC. DF. [a] estq; AC + BC ⊓⊔ DF + EF. atqui AC + BC [b] est Major, [c] ergò DF + EF quoq; Major est. [d] & proinde DF — EF est Minor. Q. E. D. a lem. 103. 10. b hyp. c 69. 10. d 77. 10.

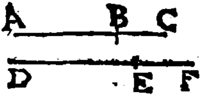
Prop. CVII.



Recta linea DE *commensurabilis ei* AB (AC — BC) *quæ cum rationali medium totum efficit, & ipsa cum rationali medium totum efficiens est.*

Nam ad modum præcedentium ostendemus DF + EF esse potentem ρν, & μν. [a] ergò DF — EF est ut dicitur. a 78. 10.

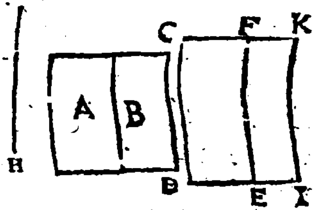
Prop. CVIII.



Recta linea DE *commensurabilis ei* AB (AC — BC) *quæ cum medio medium totum efficit, & ipsa cum medio medium totum efficiens est.*

Nam, ad normam præcedentium, erit DF + EF potens 2μα. [a] ergò DF — EF erit ut in propos. a 79. 10.

Prop. CIX.



Medio B à rationali A + B detracto, recta linea H, quæ reliquum spatiũ A potest, una ex duabus irrationalibus fit vel apotome, vel Minor.

a 3. ax. 1. b hyp. & constr. c 21. 10. d 23. 10. e 13. 10. f 74. 10. g 1 def. 85 10. h 92. 10. k 4 def. 85. 10. l 95. 10.

Ad CD ρ̇, fac rectang. CI = A + B; & FI = B. quare CE [a] = A: (Hq) Quoniam igitur CI [b] est ρ̇ν, [c] erit CK ρ̇ ⊓⊔ CD. sed quia FI [b] est μν, [d] erit FK ρ̇ ⊓⊔ CD. [e] unde CK ⊓⊔ FK [f] ergò CF est apotome. Si igitur CK ⊓⊔ √ CKq — FKq, [g] erit CF apot. prima; [h] quare √ CE (H) est apotome. sin CK ⊓⊔ √ CKq — FKq, [k] erit CF apot. quinta. & proinde H (√ CE) [l] erit Minor. Q. E. D.

Prop. CX.

Vide Schem. præced.

Rationali B à medio A + B detracto; aliæ duæ irrationales fiunt, vel mediæ apotome prima, vel cum rationali medium totum efficiens.

a 3. ax. 1. b hyp. & constr. c 23. 10. d 21. 10. e 13. 10. f 74. 10. g 2 def. 85. 10. h 93. 10. k 5. def. 85. 10. l 96. 10.

Ad CD expos. ρ̇ fiant rectang. CI = A + B; & FI = B, [a] unde CE = A = Hq. Quoniam igitur CI [b] est μν; [c] erit CK ρ̇ ⊓⊔ CD. sed quia FI [b] est ρ̇ν, [d] erit FK ρ̇ ⊓⊔ CD. [e] unde CK ⊓⊔ FK. [f] ergò CF est apot. [g] nempe secunda; si CK ⊓⊔ √ CKq — FKq, [h] quare H (√ CE) est mediæ apot. prima. Sin verò CK ⊓⊔ √ CKq — FKq, [k] erit CF apot. quinta. & proinde H (√ CE) erit faciens μν cum ρ̇ν. Q. E. D.

Prop. CXI.

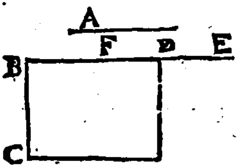
Vide Schema idem.

Medio B *à medio* A + B *detracto, quod sit incommensurabile toti* A + B, *reliquæ duæ irrationales sunt, vel media apotome secunda; vel cum medio medium totum efficiens.*

Ad CD $\dot{\rho}$ fiant rectang. CI = A + B; & FI = B, [a] quare CE = A = Hq. Quoniam igitur CI est $\mu\nu$. [b] erit CK $\dot{\rho}$ ⊐⊏ CD. eodem modo erit FK $\dot{\rho}$ ⊐⊏ CD. item quia CI [c] ⊐⊏ FI, [d] erit CK ⊐⊏ FK; [e] quare CF est apotome, [f] tertia scilicet, si CK ⊐⊏ $\sqrt{CKq - FKq}$, [g] unde H ($\sqrt{CE}$) erit mediæ apot. secunda. verùm si CK ⊐⊏ $\sqrt{CKq - FKq}$, [h] erit CF apot. sexta. [k] quare H erit faciens $\mu\nu$ cum μ. Q. E. D.

a 3. *ax.* 1.
b 23. 10.
c *hyp.*
d 10. 10.
e 74. 10.
f 3. *def.* 85. 10.
g 94. 10.
h 6. *def.* 85. 10.
k 97. 10.

Prop. CXII.



Apotome A *non est eadem, quæ ex binis nominibus.*

Ad expos. BC $\dot{\rho}$, fiat rectang. CD = Aq. Ergò cùm A sit apotome, [a] erit BD apot. prima; ejus congruens sit DE. [b] quare BE, DE sunt $\dot{\rho}$ ⊐⊏. [c] & BE ⊐⊏ BC. Vis A esse bin. ergò BD est bin. 1. ejus nomina sint BF, FD; sítque BF ⊏ FD; [d] ergò BF, FD sunt $\dot{\rho}$ ⊐⊏; & BF [e] ⊐⊏ BC. ergò cùm BC ⊐⊏ BE, [f] erit BE ⊐⊏ BF [g] ergò BE ⊐⊏ FE. [h] ergò FE est $\dot{\rho}$. item quia BE ⊐⊏ DE, [k] erit FE ⊐⊏ DE. [l] quare FD est apotome, [l] adeóque FD est $\dot{\rho}$. sed ostensa est $\dot{\rho}$. quæ repugnant. ergò A malè dicitur binomium. Q. E. D.

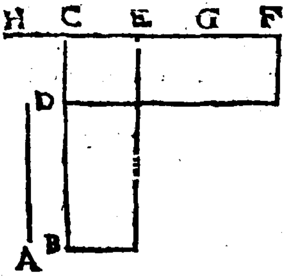
a 98. 10.
b 74. 10.
c 1. *def.* 85. 10.
d 37. 10.
e 1 *def.* 48. 10.
f 12. 10.
g *cor.* 16. 10.
h *sch.* 12. 10.
k 14. 10.
l 74. 10.

Nomina 13. linearum irrationalium inter se differentium.

1. Mediæ
2. Ex binis nominibus, cujus 6 species
3. Ex binis mediis prima.
4. Ex binis mediis secunda.
5. Major.
6. Rationale ac medium potens.
7. Bina media potens.
8. Apotome, cujus etiam 6 species.
9. Mediæ apotome prima.
10. Mediæ apotome secunda.
11. Minor.
12. Cum rationali medium totum efficiens.
13. Cum medio medium totum efficiens.

Cùm latitudinum differentiæ arguant differentias rectarum, quarum quadrata sunt applicata ad aliquam rationalem, sitque demonstratum in præcedentibus, latitudines quæ oriuntur ex applicationibus quadratorum harum 13 linearum inter se differre, perspicuè sequitur has 13 lineas inter se differre.

PROP. CXIII.



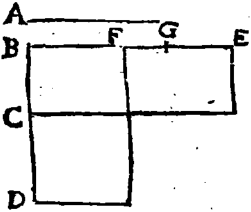
Quadratum rationalis A *ad eam, quæ ex binis nominibus* BC (BD + DC) *applicatam, latitudinem facit apotomen* EC, *cujus nomina* EH, CH *commensurabilia sunt nominibus* BD, DC *ejus, quæ ex binis nominibus &*

& in eadem proportione (EH. BD :: CH. DC); *& adhæc, apotome* E C *quæ sit, eundem habet ordinem, quem ea* BC, *quæ ex binis nominibus.*

Ad DC minus nomen [a] fac rectang. DF = Aq = BE. quare BC. CD [b] :: FC. CE. ergò dividendo BD. DC :: FE. EC. cùm igitur BD [c] ⊏ DC, [d] erit FE ⊏ EC. sume EG = EC; fiátque FG. GE :: BC. CH. Erunt EH, CH nomina apotomæ EC; quibus conveniunt ea, quæ in theoremate proposita sunt. Nam componendo FE. GE. (EC) :: EH. CH. ergò FH. EH [e] :: EH. CH [f] :: FE. EC [f] :: BD. DC. quare cùm BD [g] ⊓⊔ DC, [h] erit EH ⊓⊔ CH; [h] & FHq ⊓⊔ EHq. ergò, quia FHq. EHq [k] :: FH. CH. [h] erit FH ⊓⊔ CH, [l] ideóq; FC ⊓⊔ CH. Porrò CD [g] est $\acute{\rho}$, & DF (Aq) [g] est $\acute{\rho}\nu$, [m] ergò FC est $\acute{\rho}$ ⊓⊔ CD. quare etiã CH est $\acute{\rho}$ ⊓⊔ CD. [n] igitur EH CH sunt $\acute{\rho}$, ac ⊓⊔ ut priús. [o] ergò EC est apotome; cui congruit CH. porrò EH. CH [f] :: BD. DC, ideò permutando EH. BD :: CH. DC. unde quia CH [f] ⊓⊔ DC, [p] erit EH ⊓⊔ BD. quinimo pone BD ⊓⊔ $\surd$ BDq — DCq; [q] erit ideò EH ⊓⊔ $\surd$ EHq — CHq. item si BD ⊓⊔ $\acute{\rho}$ expos. erit EH ⊓⊔ eidem $\acute{\rho}$; [s] hoc est si BC sit bin. 1. [t] erit EC apot. prima. Similiter si DC ⊓⊔ $\acute{\rho}$ expos. [r] erit CH ⊓⊔ eidem $\acute{\rho}$. [u] hoc est si BC sit bin. 2. [x] erit EC apot. 2. & si hæc bin. 3. illa erit apot. 3, &c. Sin BD ⊓⊔ $\surd$ BDq — DCq, [y] erit EH ⊓⊔ $\surd$ EHq — CHq; si igitur BC sit bin. 4, vel 5, vel 6. erit EC similiter apot. 4, vel 5, vel 6. Q. E. D.

a cor. 16. 6.
b 14. 6.
c hyp.
d 14. 5.
e 12. 5.
f Priùs.
g hyp.
h 10. 10.
k cor. 20. 6.
l 16. 10.
m 21. 10.
n sch. 12. 10.
o 74. 10.
p 10. 10.
q 15. 10.
r 12. 10.
s 1. def. 48. 10.
t 1. def. 85. 10.
u 2 def. 48. 10.
x 2 def. 85. 10.
y 15. 10.

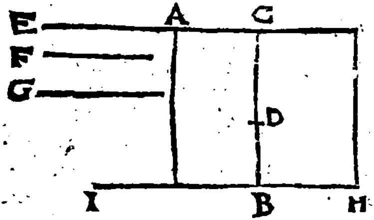
Prop. CXIV.



Quadratum rationalis A *ad apotomen* BC (BD—DC) *applicatum, facit latitudinem* BE *eam, quæ ex binis nominibus; cujus nomina* BE, GE *commensurabilia sint apotomæ* BC *nominibus* BD DC, *& in eadem proportione; & adhuc, quæ ex binis nominibus fit* (BE), *eundem habet ordinem, quem ipsa apotome* BC.

a *cor.* 16. 6. [a] Fac rectang. DF = Aq; & BE. FE [b] ::
b 18. 6. EG. GF. Quoniam igitur DF = Aq = CB,
c 14. 6. [c] erit BD. BC :: BE. BF. ergò per conversionem rationis BD. CD :: BE. FE :: EG. GF ::
d 19. 5. [d] BG. EG. sed BD [e] ⊤⌡ CD. [f] ergò BG ⊤⌡
e *hyp.*
f 10. 10. GE. ergò quia BGq. GEq [g] :: BG. GF. [h] erit
g *cor.* 20. 6. BG ⊤⊥ GF [k][l] ideóque BG ⊤⊥ BF. porrò
h 10. 10. BD [e] est ῥ, & rectang. DF (Aq) [e] est ῥτ. [l] ergò BF est ῥ ⊤⊥ BD. [m] ergò etiam BG est ῥ ⊤⊥
k *cor.* 16. 10.
l 21. 10.
m 12. 10. BD. [n] ergò BG, GE sunt ῥ ⊤⌡. [o] quare BE
n *sch.* 12. 10. est bin. denique igitur quia BD. CD :: BG.
o 37. 10. GE; & permutando BD. EG :: CD. GE; sitq;
p 10. 10. BD ⊤⊥ BG, [p] erit CD ⊤⊥ GE. ergò si CB sit apot. prima; erit BE bin. 1. &c. ut in antecedenti. ergò, &c.

PROP. CXV.



Si spatium AB *contineatur sub apotoma* AC (CE — AE), *& ea, qua ex binis nominibus* CB; *cujus nomina* CD, DB *commensurabilia sint apotomæ nominibus* CE, AE; *& in eadem proportione* (CE. AE :: CD. DB.); *recta linea F spatium* AB *potens, est rationalis.*

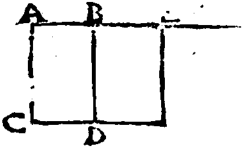
Sit G quævis ῥη & fiat rectang. CH = Gq. [a] erit igitur BH (HI — IB) apotome; & HI [a] ⊓⊔ CD [b] ⊓⊔ CE. [a] & BI ⊓⊔ DB; [a] atque HI. BI :: CD. DB [b] :: CE. EA. ergò permutando HI. CE :: BI. EA. [c] ergò BH. AC :: HI. CE :: BI. EA. ergò cùm HI [d] ⊓⊔ CE, [e] erit BH ⊓⊔ AC. [f] ergò rectang. HC ⊓⊔ BA. Sed HC (Gq) [b] est ῥν. [g] ergò BA (Fq) est ῥν. proinde F est ῥ. Q. E. D.

a 113. 10.
b hyp.
c 19. 5.
d 12. 10.
e 10. 10.
f 1. 6. & 10. 10.
g sch. 12. 10.

Coroll.

Hinc, fieri potest, ut spatium rationale contineatur sub duabus rectis irrationalibus.

PROP. CXVI.

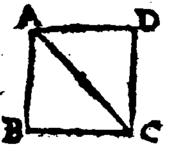


A media AE *fiunt infinitæ irrationales* BE, EF, &c. *& nulla alicui antecedentium est eadem.*

Sit AC expos. ῥ. sitque

a lem. 38.10. ῥ. sitque AD spatium sub AC, AB. [a] ergò AD
b 11.10. est ῥν. Sume BE = √AD. [b] ergò BE est ῥ, nulli priorum eadem. nullum enim quadratum alicujus priorum applicatum ad ῥ, latitudinem efficit mediam. compleatur rectang. DE; [a] erit DE ῥν; & [b] proinde EF (√DE) erit ῥ; & nulli priorum eadem. nullum enim priorum quadratum ad ῥ applicatum, latitudinem efficit ipsam BE. ergò, &c.

Prop. CXVII.



Propositum sit nobis ostendere, in quadratis figuris BD, diametrum AC lateri AB incommensurabilem esse.

a 47. 1. Nam ACq. ABq [a] :: 2.
b cor. 24. 8. 1 [b] :: non Q. Q. [c] ergò AC
c 9. 10. ⊥⊥ AB. Q. E. D.

Celebratissimum est hoc theorema apud veteres philosophos, adeò ut qui hoc nesciret, eum Plato non hominem esse, sed pecudem diceret.

LIB.

LIB. XI.

Definitiones.

I. SOlidum est, quod longitudinem, latitudinem, & crassitudinem habet.

II. Solidi autem extremum est superficies.

III. Linea recta est ad planum recta, cùm ad rectas omnes lineas, à quibus illa tangitur, quæque in proposito sunt plano, rectos angulos efficit.

IV. Planum ad planum rectum est, cùm rectæ lineæ, quæ communi planorum sectioni ad rectos angulos in uno plano ducuntur, alteri plano ad rectos sunt angulos.

V. Rectæ lineæ ad planum inclinatio est, cùm à sublimi termino rectæ illius lineæ ad planum deducta fuerit perpendicularis; atque à puncto quod perpendicularis in ipso plano effecerit, ad propositæ illius lineæ extremum, quod in eodem est plano, altera recta linea fuerit adjuncta; est, inquam, angulus acutus insistente lineâ, & adjunctâ comprehensus.

VI. Plani ad planum inclinatio, est angulus acutus rectis lineis contentus, quæ in utroque planorum ad idem communis sectionis punctum ductæ, rectos cum sectione angulos efficiunt.

VII. Planum ad planum similiter inclinatum esse dicitur, atque alterum ad alterum, cùm dicti inclinationum anguli inter se fuerint æquales.

VIII. Parallela plana sunt, quæ inter se non conveniunt.

IX. Similes solidæ figuræ sunt, quæ similibus planis continentur, multitudine æqualibus.

X. Æquales & similes solidæ figuræ sunt, quæ

quæ similibus planis multitudine, & magnitudine æqualibus continentur.

XI. *Solidus angulus est plurium quàm duarum linearum, quæ se mutuò contingunt, nec in eadem sunt superficie, ad omnes lineas inclinatio.*

Aliter.

Solidus angulus est, qui pluribus quàm duobus planis angulis in eodem non consistentibus plano, sed ad unum punctum constitutis continetur.

XII. Pyramis est figura solida, planis comprehensa, quæ ab uno plano ad unum punctum constituuntur.

XIII. Prisma est figura solida, quæ planis continetur, quorum adversa duo sunt & æqualia, & similia, & parallela; alia verò parallelogramma.

XIV. Sphæra est, quando semicirculi manente diametro, circumductus semicirculus in seipsum rursus revolvitur unde moveri cœperat, circumassumpta figura.

Coroll.

Hinc radii omnes à centro ad superficiem sphæræ inter se sunt æquales.

XV. Axis autem sphæræ, est quiescens illa recta linea, circum quam semicirculus convertitur.

XVI. Centrum sphæræ est idem quod & semicirculi.

XVII. Diameter autem sphæræ, est recta quædam linea per centrum ducta, & utrinque à sphæræ superficie terminata.

XVIII. Conus est, quando rectanguli trianguli manente uno latere eorum, quæ circa rectum angulum, circumductum triangulum in seipsum rursus revolvitur, unde moveri cœperat, circumassumpta figura. Atque si quiescens recta linea

linea æqualis ſit reliquæ, quæ circa rectum angulum continetur, orthogonius erit conus: ſi verò minor, amblygonius: ſi verò major, oxygonius.

XIX. Axis autem coni, eſt quieſcens illa linea, circa quam triangulum vertitur.

XX. Baſis veri coni eſt circulus qui à circumducta recta linea deſcribitur.

XXI. Cylindrus eſt, quando rectanguli parallelogrammi manente uno latere eorum, quæ circa rectum angulum, circumductum parallelogrammum in ſeipſum rurſus revolvitur, unde cœperat moveri, circumaſſumpta figura.

XXII. Axis autem cylindri, eſt quieſcens illa recta linea, circum quam parallelogrammum convertitur.

XXIII. Baſes verò cylindri ſunt circuli à duobus adverſis lateribus, quæ circumaguntur, deſcripti.

XXIV. Similes coni & cylindri ſunt, quorum & axes, & baſium diametri proportionales ſunt.

XXV. Cubus eſt figura ſolida ſub ſex quadratis æqualibus contenta.

XXVI. Tetraedrum eſt figura ſolida ſub quatuor triangulis æqualibus & æquilateris contenta.

XXVII. Octaedrum eſt figura ſolida ſub octo triangulis æqualib9 & æquilateris contenta.

XXVIII. Dodecaedrum eſt figura ſolida ſub duodecim pentagonis æqualibus, & æquilateris, & æquiangulis contenta.

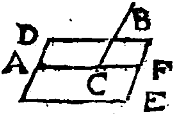
XXIX. Icoſaedrum eſt figura ſolida ſub viginti triangulis æqualibus & æquilateris contenta.

XXX. Parallelepipedum eſt figura ſolida ſex figuris quadrilateris, quarum quæ ex adverſo parallelæ ſunt, contenta.

XXXI. Solida figura in solida figura dicitur *inscribi*, quando omnes anguli figuræ inscriptæ *constituuntur vel in angulis*, vel in lateribus, *vel denique in planis figuræ*, cui inscribitur.

XXXII. Solida figura solidæ *figuræ vicissim* circumscribi dicitur, quando vel anguli, vel latera, vel denique plana figuræ circumscriptæ tangunt omnes angulos figuræ, circum quam describitur.

PROP. I.

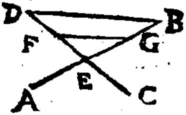


Rectæ lineæ pars quadam AC *non est in subjecto plano, quadam verò* CB *in sublimi.*

Producatur AC in subjecto plano usque ad F, vis CB esse in directum ipsi AC; ergò duæ rectæ AB, AF habent commune segmentum AC. [a] Q. F. N.

a 10. ax. 1.

PROP. II.



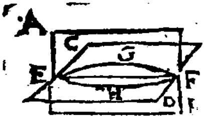
Si duæ rectæ lineæ AB, CD *se mutuò secent, in uno sunt plano: atque triangulum omne* DEB *in uno est plano.*

Puta enim trianguli DEB partem EFG esse in uno plano, partem verò FDGB in altero. ergò rectæ ED pars EF est in subjecto plano, pars verò FD in sublimi,[a] Q. E. A. ergò triangulum EDB in uno est plano, proinde & rectæ ED, EB;[a] quare & totæ AB, DC in uno plano existunt. Q. E. D.

a 1. 11.

PROP.

PROP. III.

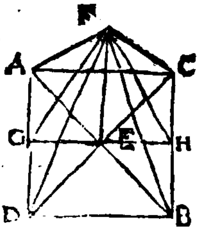


Si duo plana AB, CD *se mutuò secent, communis eorum sectio* EF *est recta linea.*

Si EF communis sectio non est recta linea, [a] ducatur in plano AB recta EGF, [a] & in plano CD recta EHF. duæ igitur rectæ EGF, EHF claudunt spatium. [b] Q.E.A.

a 1. *post.* 1.
b 14. *ax.* 1.

PROP. IV.



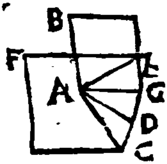
Si recta linea EF *rectis duabus lineis* AB, CD *se mutuò secantibus in communi sectione* E *ad rectos angulos insistat: illa ducto etiam per ipsas plano* ACBD *ad angulos rectos erit.*

Accipe EA, EC, EB, ED æquales, & junge rectas AC, CB, BD, AD. per E ducatur quævis recta GH; junganturque FA, FC, FD, FB, FG, FH. Quoniam AE [a] = EB; & DE [a] = EC; & ang. AED [b] = CEB, [c] erit AD = CB. [c] paritérque AC = DB. [d] ergò AD. parall. CB. [d] & AC parall. DB. [e] quare ang. GAE = EBH. [e] & ang. AGE = EHB. sed & AE [f] = EB [g] ergò GE = EH, & [g] AG = BH. quare ob angulos rectos, ex hyp. & proinde pares ad E, [h] bases FA, FC, FB, FD æquantur. Triangula igitur ADF, FBC sibi mutuò æquilatera sunt, [k] quare ang. DAF = CBF. ergò in triangulis AGF, FBH latera FG, FH [l] æquantur; & proinde etiam triangula FEG, FEH sibi mutuò æquilatera sunt. [m] ergò anguli FEG, FEH æquales ac [n] propterea recti sunt. Eodem modo FE cum

a *constr.*
b 15. 1.
c 4. 1.
d *sch.* 34. 1.
e 29. 1.
f *constr.*
g 26. 1.
h 4. 1.
k 8. 1.
l 4. 1.
m 8. 1.
n 10. *def.* 1.

omnibus in plano ADBC per E ductis rectis
◆ 3. def. 11. lineis rectos angulos constituit ◆, ideóque eidem plano recta est. Q. E. D.

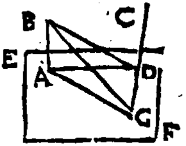
Prop. V.



Si recta linea AB rectis tribus lineis AC, AD, AE se mutuò tangentibus in communi sectione ad rectos angulos insistat; illæ tres rectæ in uno sunt plano.

Nam AC, AD [a] sunt in
a 2. 11. uno plano FC. [a] item AD, AE sunt in uno plano BE. vis diversa esse hæc plana; sit igitur eo-
b 3. 11. rum intersectio [b] recta AG. Quoniam igitur BA ex hypoth. perpendicularis est rectis AC,
c 4. 11. AD, eadem [c] plano FC; [d] ideóq; rectæ AG per-
d 3. def. 11. pendicularis est. ergò (siquidem & [a] AB est in eodem cum AG, AE plano) anguli BAG, BAE recti, & proinde pares sunt, pars & totum. Q.E.A.

Prop. VI.

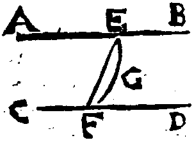


Si duæ rectæ lineæ AB, DC eidem plano EF ad rectos sint angulos; parallelæ erunt illæ rectæ lineæ AB, DC.

Ducatur AD, *cui in plano* EF *perpendicularis sit* DG = *AB; junganturq;* BD, BG, AG. Quia in triangulis BAD,
a hyp. ADG anguli DAB, ADG [a] recti sunt; atque
b constr. AB [b] = DG; & AD communis est, [c] erit BD
c 4. 1. = AG; quare in triangulis AGB, BGD sibi
d 8. 1. mutuò æquilateris ang. BAG [d] = BDG; quorum BAG rectus cùm sit, erit BDG etiam rectus. atqui ang. GDC rectus ponitur; ergò re-
e 5. 11. cta GD tribus DA, DB, CD recta est; [e] quæ
f 2. 11. ideò in uno sunt plano; [f] in quo AB existit;
cum

cùm igitur AB, & CD ſint in uno plano, & anguli interni BAD, CDA recti ſint, [g] erunt AB, CD parallelæ. Q. E. D. g 28. 1.

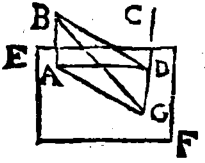
Prop. VII.



Si duæ ſint parallelæ rectæ lineæ AB, CD, *in quarum utraque ſumpta ſint qualibet puncta* E, F; *illa linea* EF, *quæ ad hæc puncta adjungitur, in eodem eſt cum parallelis plano* ABCD.

Planum in quo AB, CD ſecet aliud planum per puncta E, F. ſi jam EF non eſt in plano ABCD, illa communis ſectio non erit. Sit ergò EGF. [a] hæc igitur recta eſt linea. duæ ergò a 3. 11.
rectæ EF, EGF ſpatium claudunt [b]. Q. E. A. b 14. ax. 1.

Prop. VIII.



Si duæ ſint parallelæ rectæ lineæ AB, CD, *quarum altera* AB *ad rectos cuidam plano* EF *ſit angulos; & reliqua* CD *eidem plano* EF *ad rectos angulos erit.*

Adſcitâ præparatione & demonſtratione ſextæ hujus; anguli GDA, & GDB recti ſunt,
[a] ergò GD recta eſt plano per AD, DB ([b] in quo a 4. 11.
etiam AB, CD exiſtunt). [c] ergò GD ipſi CD b 7. 11.
eſt perpendicularis; atqui ang. CDA etiam [d] re- c 3. def. 11.
ctus eſt. [e] ergò CD plano EF recta eſt. Q. E. D. d 29. 1. e 4. 11.

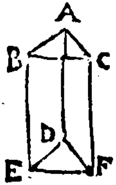
PROP. IX.

Quæ (AB,CD) *eidem rectæ lineæ* EF *sunt parallelæ, sed non in eodem cum illa plano, hæ quoq; sunt inter se parallelæ.*

In plano parallelarum AB, EF duc HG perpendicularem ad EF. item in plano parallelarum EF, CD duc IG perpendicularem ad EF. [a] ergò EG recta est plano per HG, GI; eidémque plano [b] rectæ sunt AH, & CI. [c] ergò AH, & CI parallelæ sunt. Q.E.D.

a 4. 11.
b 8. 11.
c 6. 11.

PROP. X.

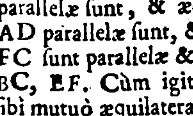


Si duæ rectæ lineæ AB, AC *se mutuò tangentes ad duas rectas* ED, DF *se mutuò tangentes sint parallelæ, non autem in eodem plano, illæ angulos æquales* (BAC, EDF) *comprehendent.*

Sint AB, AC, DE, DF æquales inter se, & ducantur AD, BC, EF, BE, CF. Cùm AB, DE [a] sint parallelæ & æquales, [b] etiam BE, AD parallelæ sunt, & æquales. Eodem modo CF, AD parallelæ sunt, & æquales. [c] ergò etiam BE, FC sunt parallelæ & æquales. Æquantur ergò BC, EF. Cùm igitur triangula BAC, EDF sibi mutuò æquilatera sint, anguli BAC, EDF [e] æquales erunt. Q. E. D.

a *hyp. & constr.*
b 33. 1.
c 1. *ax.* 1.
& 30. 1.
d 33. 1.
e 8. 1.

PROP. XI.



A dato puncto A *in sublimi ad subjectum planum* BC *perpendicularem rectam lineam* AI *ducere.*

In plano BC duc quamvis DE, ad quam ex A [a] duc perpendicularem AF. ad eandem per

F in

F in plano BC [b] duc normalem FH. tum ad FH a 12. 1.
[i] demitte perpendicularem AI. erit AI recta b 11. 1.
plano BC.

Nam per I [c] duc KIL parall. DE. Quia DE c 31. 1.
[d] recta est ad AF, & FH, [e] erit DE recta plano d *constr.*
IFA; adeóque & KL eidem plano [f] recta est. e 4. 11.
[g] ergò ang. KIA rectus est. atqui ang. AIF f 8. 11.
etiam [h] rectus est. [l] ergò AI plano BC recta g 3. *def.* 11. h *constr.*
est. Q. E. D. l 4. 11.

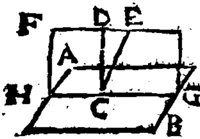
Prop. XII.

Dato plano BC *à puncto* A, *quod in illo datum est, ad rectos angulos rectam lineam* AF *excitare.*

A quovis extra planum puncto D [a] duc DE rectam plano BC; & junctâ EA [b] duc AF parall. DE. [c] perspicuum est AF plano BC rectam esse. Q. E. F. a 11. 11. b 31. 1. c 8. 11.

Practicè perficiuntur hoc, & præcedens problema, si duæ normæ ad datum punctum applicentur, ut patet ex 4. 11.

Prop. XIII.

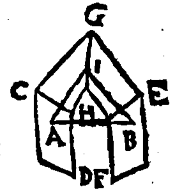


Dato plano AB, *à puncto* D, *quod in illo datum est, duæ rectæ lineæ* CD, CE *ad rectos angulos non excitabuntur, ab eadem parte.*

Nam utraque CD, CE plano AB [a] recta esset, eædémque adeò parallelæ forent, quod parallelarum definitioni repugnat. a 6. 11.

PROP. XIV.

valet haec conversa.

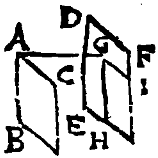


Ad quae plana CD, FE, *eadem recta linea* AB *recta est; illa sunt parallela.*

Si negas, plana *GD*, *FE* concurrant, ità ut communis sectio sit recta GH; sume in hac quodvis punctum I, ad quod in propositis planis ducantur rectae IA, IB. unde in triangulo IAB, duo anguli IAB, IBA [a] recti sunt. [b] Q. E. A.

a *hyp.* & 3. *def.* 11.
b 17. 1.

PROP. XV.



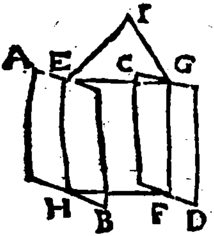
Si duae rectae lineae AB, AC *se mutuò tangentes ad duas rectas* DE, DF *se mutuò tangentes sint parallelae, non in eodem consistentes plano, parallela sunt, quae per illa ducuntur, plana* BAC, EDF.

Ex A [a] duc AG rectam plano EF. [b] Síntq; GH, GI parallelae ad DE, DF. [c] erunt hae parallelae etiam ad AB, AC. Cùm igitur anguli IGA, HGA [d] sint recti, [e] erunt etiam CAG, BAG recti. [f] ergò GA recta est plano BC; atqui eadem recta est, plano EF. [h] ergò plana BC, EF sunt parallela. Q. E. D.

a 11. 11.
b 31. 1.
c 30. 1.
d 3. *def.* 11.
e 29. 1.
f 4. 11.
g *constr.*
h 14. 11.

PROP.

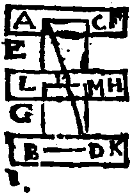
PROP. XVI.



Si duo plana parallela AB, CD *plano quopiam* HEIGF *secentur, communes illorum sectiones* EH, GF *sunt parallelæ.*

Nam si dicantur non esse parallelæ, cùm sint in eodem plano secanti, convenient alicubi, puta in I. quare cùm totæ HEI, FGI [a] sint in planis AB, CD productis, etiam hæc convenient; contra hypoth. a 1. th.

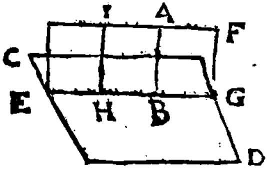
PROP. XVII.



Si duæ rectæ lineæ ALB, CMD *parallelis planis* EF, GH, IK *secentur, in easdem rationes secabuntur* (AL. LB :: CM. MD).

Ducantur in planis EF, IK rectæ AC, BD. item AD occurrens plano GH in N; junganturque NL, NM. Plana triangulorum ADC, ADB faciunt sectiones BD, LN; & AC, NM [a] parallelas. ergò AL. LB [b] :: AN. ND [b] :: CM. MD. Q.E.D. a 16. 11. b 2. 6.

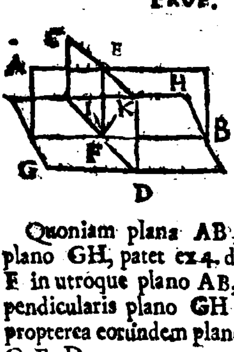
PROP. XVIII.



Si recta linea AB plano cuipiam CD ad rectos sit angulos; & omnia, quæ per ipsam AB plana (EF &c), eidem plano CD ad rectos angulos erunt.

Ductum sit per AB planum aliquod EF, faciens cum plano CD sectionem EG; è cujus aliquo puncto H, in plano EF [a] ducatur HI parall. AB. [b] erit HI recta plano CD; paritérque
a. 31. 1.
b. 8. 11.
c. 4. def. 11. aliæ quævis ad EG perpendiculares. [c] ergò planum EF plano CD rectum est; eâdemque ratione quævis alia plana per AB ducta plano EF recta erunt. Q. E. D.

PROP. XIX.



Si duo plana AB, CD se mutuò secantia plano cuidam GH ad rectos sint angulos, communis etiam illorum sectio EF ad rectos eidem plano (GH) angulos erit.

Quoniam plana AB, CD ponuntur recta plano GH, patet ex 4. def. 11. quòd ex puncto F in utroque plano AB, CD duci possit per-
a. 13. 11. pendicularis plano GH; quæ [a] unica erit, & propterea eorundem planorum communis sectio. Q. E. D.

PROP.

Prop. XX.

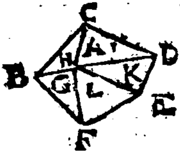
Si solidus angulus ABCD *tribus angulis planis* BAD, DAC, BAC *contineatur; ex his duo quilibet, utut assumpti, tertio sunt majores.*

Si tres anguli sunt æquales, patet assertio; si inæquales, maximus esto BAC. ex quo [a] aufer BAF = BAD; & fac AD = AE; ducantúrque BEC, BD, DC. a 23. 1.

Quoniam latus BA commune est, & AD [b] = AE; & ang. BAE [b] = BAD; [c] erit BE = BD. sed BD + DC [d] $\sqsubset$ BC. [e] ergò DC $\sqsubset$ EC cùm igitur AD [b] = AE, & latus AC commune est, ac DC $\sqsubset$ BC [f], erit ang. CAD $\sqsubset$ EAC. [g] ergò ang. BAD + CAD $\sqsubset$ BAC. Q. E. D.

b constr.
c 4. 1.
d 20. 1.
e 5. ax. 1.
f 25. 1.
g 4. ax. 1.

Prop. XXI.



Omnis solidus angulus sub minoribus, quàm quatuor rectis angulis planis, continetur.

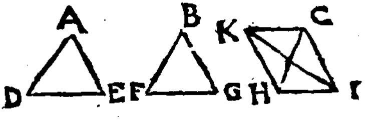
Esto solidus angulus A; planis angulis illum componentibus subtendantur rectæ BC, CD, DE, EF, FB in uno plano existentes. Quo facto constituitur pyramis, cujus basis est polygonum BCDEF; vertex A, tótque cincta triangulis, quot plani anguli componunt solidum A. Jam verò quia duo anguli ABF, ABC [a] majores sunt uno FBC, [a] & duo ACB, ACD majores uno BCD, & sic deinceps, erunt triangulorum G, H, I, K, L circa basim anguli simul sumpti omnibus simul angulis basis B, C, D, E, F majores. [b] sed anguli baseos unà cum quatuor rectis faciunt bis tot rectos, quot sunt latera, sive quot triangula. [c] Ergò omnes triangulorum circa basim anguli unà

a 20. 11.
b sch. 32. 1.
c 4. ax. 1.

cum

cum 4 rectis conficiunt ampliùs, quàm bis tot rectos, quot sunt triangula. sed iidem anguli circa basim unà cum angulis, qui componunt solidum,
d 32. 1. componunt [d] *bis tot rectos quot sunt triangula. liquet ergò angulos solidum angulum* A *componentes* quatuor rectis esse minores. *Q. E. D.*

Prop. XXII.

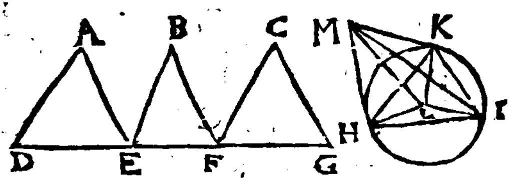


Si fuerint tres anguli plani A, B, HCI, *quorum duo utlibet assumpti reliquo sint majores; comprehendant autem ipsos rectæ lineæ æquales* AD, AE, FB &c. *fieri potest, ut ex rectis lineis* DE, FG, HI *æquales illas rectas connectentibus triangulum constituatur.*

a 22. 1. Ex iis [a] constitui potest triangulum, si duæ quælibet reliquâ majores existant; sed ità se res
b 23. 1. habet. Nam [b] fac ang. HCK $=$ B, & CK $=$ CH,
c 4. 1. ducantúrque HK, IK. [c] ergò KH $=$ FG. & quia
d hyp. ang. KCI [d] $\sqsubset$ A; erit KI $\sqsubset$ DE. sed KI [f] $\sqsupset$
e 24. 1. HI $+$ KH (FG); ergò DE $\sqsupset$ HI $+$ FG.
f 20. 1. Simili argumento quævis duæ reliquâ majores ostendentur, & proinde ex iis triangulum [a] constitui potest. Q. E. D.

Prop.

Prop. XXIII.

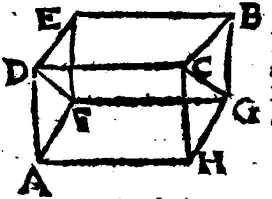


Ex tribus angulis planis A, B, C *quorum duo quomodocunque assumpti reliquo sunt majores, solidum angulum* MHIK *constituere. * Oportet autem illos tres angulos quatuor rectis minores esse.* * 21. 11.

Fac AD, AE, BE, BF, CF, CG æquales inter se. Ex subtensis DE, EF, FG (hoc est ex æqualibus HI, IK, KH) [a] fac triang. HKI. circa quod [b] describatur circulus LHKI. *Quoniam verò AD $\sqsubset$ HL; [c] sit ADq = HLq + LMq. [d] sitque LM recta plano circuli HKI; & ducantur HM, KM, IM. Quoniam igitur ang. HLM [e] rectus est, [f] erit MHq = HLq + LMq [g] = ADq. ergò MH = AD. simili argumento MK, MI, AD (id est AE, EB &c.) æquantur. ergò cùm HM=AD, & MI=AE; & DE [h]= HI, [k] erit ang. A = HMI; [k] similiter ang. IMK = B. [k] & ang. HMK = C. Factus est igitur angulus solidus ad M ex tribus planis datis. Q. E. F. Brevitatis causâ assumptum est, esse AD $\sqsubset$ HL, id quod in variis casibus demonstratum vide apud Clavium.

a 22. 11. & 22. 1.
b 5. 4.
vid. Clavium.
c sch. 47. 1.
d 12. 11.
e 3. def. 11.
f 47. 1.
g constr.
h constr.
k 8. 1.

Prop. XXIV.

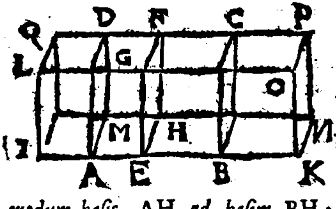


Si solidum AB parallelis planis contineatur, adversa illius plana (AG, DB &c.) parallelogramma sunt similia & æqualia.

Planum AC secans plana parallela AG, DB [a] facit sectiones AH, DC parallelas. Eâdem ratione AD, HC parallelæ sunt. Ergò ADCH est parallelogrammum. Simili argumento reliqua parallelepipedi plana sunt [b] parallelogramma. Quum igitur AF ad HG, & AD ad HC parallelæ sint, [c] erit ang. FAD = CHG; ergò ob AF [d] = HG, & AD [d] = HC, ac [e] propterea AF. AD :: HG. HC, triangula FAD, GAH [g] similia sunt & [h] æqualia; proinde & parallelogramma AE, HB similia sunt, & [k] æqualia. idémque de reliquis oppositis planis ostendetur. ergò &c.

a 16. 11.
b 35. def. 1.
c 10. 11.
d 34. 1.
e 7. 5.
g 6. 6.
h 4. 1.
k 6. ax. 1.

Prop. XXV.



Si solidum parallelepipedum ABCD plano EF secetur adversis planis AD, BC parallelo, erit quemadmodum basis AH ad basim BH, ità solidum AHD ad solidum BHC.

Concipe Ppp. ABCD produci utrinque. accipe AI = AE, & BK = EB, & pone plana IQ, KP planis AD, BC parallela. parallelogramma.

gramma IM, [a]AH; [a] & DL, DG, [b] & IQ, AD, EF, &c. [a] similia ac æqualia sunt; [c] quare Ppp. AQ = AF; atque eâdem ratione Ppp. BP = BF. ergò solida IF, EP solidorum AF, EC æquemultiplicia sunt, ac bases IH, KH basium AH, BH. Quòd si basis IH ⊏, =, ⊐ KH, [d] erit similiter solidum IF ⊏, =, ⊐ EP. [e] proinde AH. BH :: AF. EC. Q. E. D.

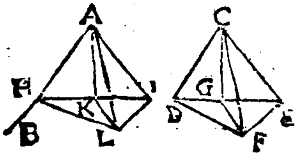
a 36. 1 & 1. def. 6.
b 24. 11.
c 10. def. 11.
d 24. 11. & 9. def. 11.
e 6. def. 5.

Hæc eadem omni prismati accommodari possunt; unde

Coroll.

Si prisma quodcunque secetur plano oppositis planis parallelo, sectio erit figura æqualis, & similis planis oppositis.

Prop. XXVI.

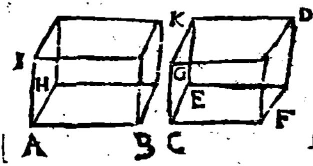


Ad datam rectam lineam AB, *ejúsq; punctum* A *constituere angulum solidum* AHIL, *æqualem solido angulo dato* CDEF.

A puncto quovis F [a] demitte FG plano DCE rectam; ducantúrque rectæ DF, FE, EG, GD, CG. Fac AH = CD, & ang. HAI = DCE. & AI = CE; atque in plano HAI, fac ang. HAK = DCG, & AK = CG. Tum erige KL rectam plano HAI, & sit KL = GF. ducatúrque AL. erit angulus solidus AHIL par dato CDEF. Nam hujus constructio illius constitutionem penitus æmulatur, ut facilè patebit examinanti. ergò factum.

a 11. 11.

PROP. XXVII.



A data recta linea AB, dato solido parallelepipedo CD simile & similiter positum parallelepipedum AK describere.

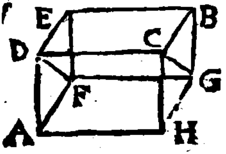
Ex angulis planis BAH, HAI, BAI, qui æquales sint ipsis FCE, ECG, FCG, [a] fac angulum solidum A solido C parem. item [b] fac FC. CE :: BA. AH. [b] ac CE. CG :: AH, AI ([c] unde erit ex æquali FC. CG :: BA. AI); & perficiatur Ppp. AK. erit hoc simile dato.

a 26. 11.
b 12. 6.
c 22. 5.

Nam per constr. pgr[a]. [d] BH, FE; [d] & *HI*, EG; & [d] BI, *FG* similia sunt, & [e] horum ideò opposita illorum oppositis. ergò sex plana solidi AK similia sunt sex planis solidi CD, [f] proinde AK, CD similia solida existunt. Q. E. F.

d 1. def. 6.
e 24. 11.
f 9. def. 11.

PROP. XXVIII.



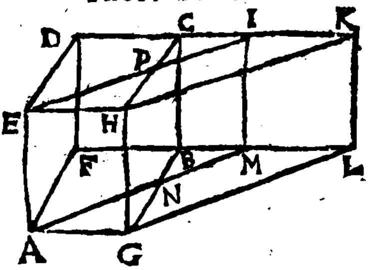
Si solidum parallelepipedum AB *plano* FGCD *secetur per diagonios* DF, CG *adversorum planorum* AE, HB, *bifariam secabitur solidum* AB *ab ipso plano* FGCD.

Nam quia DC, FG [a] æquales & parallelæ sunt, [b] planum FGCD est pgr. & propter [a] pgr[a] AE, HB æqualia, & similia, [b] etiam triangula AFD, HGC, CGB, DFE æqualia & similia sunt. Atqui Pgr[a] AC, AG ipsis FB, FD [a] etiam æqualia & similia sunt. ergò prismatis FGCDAH omnia plana æqualia sunt, & similia planis omnibus prismatis FGCDEB, & [c] proinde hoc prisma illi æquatur. Q.E.D.

a 24. 11.
b 34. 1.
c 10. def. 11.

PROP.

PROP. XXIX.



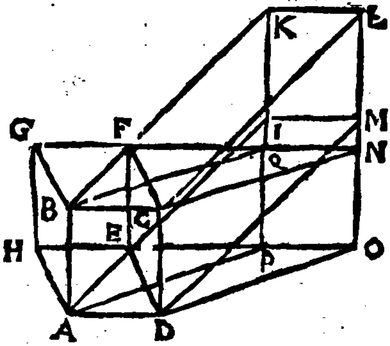
Solida parallelepipeda AGHEFBCD, AGHEMLKI *super eandem basim* AGHE *constituta, &* * *in eadem altitudine; quorum insistentes lineæ* AF, AM *in iisdem collocantur rectis lineis* AG, FL, *sunt inter se æqualia.*

* *Id est, inter parallela plana* AGHE, FLKD, *& sic intellige in sequent.*

Nam si ex [a] æqualibus prismatis AFMEDI, GBLHCK commune auferatur prisma NBMPCI, addatúrque utrinque solidum AGNEHP, [b] erit Ppp. AGHEFBCD = AGHEMLKI. Q. E. D.

a 10. *def.* 11. & 35. 1.
b 3, & 2. *ax.* 1.

PROP. XXX.

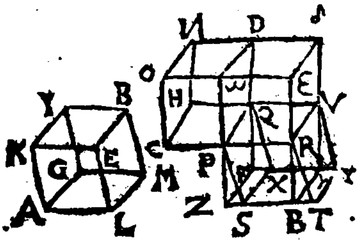


Solida parallelepipeda ADBCHEFG,

ADCBIMLK *super eandem basim* ADCB *constituta, & in eadem altitudine, quorum insistentes lineæ* AH, AI *non in iisdem collocantur rectis lineis, inter se sunt æqualia.*

Nam produc rectas HEO, GFN; & LMO,
a 34. 1. KIP; & duc AP, DO, BQ, CN. [a] erunt tam DC, AB, HG, EF, PQ, ON; quàm AD, HE, GF, BC, KL, IM, QN, PO æquales inter se-
b 29. 11. se, & parallelæ. [b] Quare Ppp. ADCBPONQ utrique Pppº. ADCBHEFG, ADCBIMLK
c 1. ax. 1. æquale est; & [c] proinde hæc ipsa inter se æqualia sunt. Q. E. D.

Prop. XXXI.



Solida parallelepipeda ALEKGMBI, CPωOHQDN *super æquales bases* ALEK, CPωO *constituta, & * in eadem altitudine, æqualia sunt inter se.*

* *Altitudo, est perpendicularis à plano basis ad planum oppositum.*

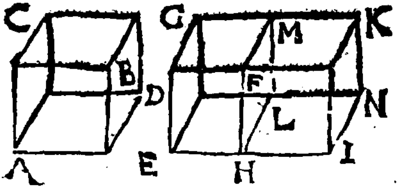
Habeant primò parallelepipeda AB, CD latera ad bases recta; & ad latus CP productum
a 18. 6. [a] fiat pgr. PRTS æq. & simile pgrº KELA;
b 27. 11. & 10. def. 11. [b] adeóque Ppp. PRTSQVYX æq. & sim. Pppº AB. Producantur OωE, NDƒ, ωPZ, DQF, ERB, ƒVγ, TSZ, YXF; & duc Eƒ, Bγ, ZF.

c 30. def. 11. Plana OωƒN, CRVH, ZTYF [c] parallela
d hyp. & 35. 1. sunt inter se; [d] & pgrª ALEK, CPωO, PRTS, PRBZ æqualia sunt. Cùm igitur Ppp.

CD.PVδω, [e] :: pgr. Cω (PRBZ).Pε [e] :: Ppp. e 25. 11.
PRBZQVγF. PVδω, [f] erit Ppp. CD [f] = f 9. 5.
PRBZQVγF [g] = PRVQSTYX [h] = AB. g 29. 11. h *constr.*
Q. E. D.

Sin Ppp[a] *AB*, CD latera basibus obliqua habeant; super easdem bases, & in eadem altitudine ponantur parallelepipeda, quorum latera basibus sint recta. [k] Ea inter se, & obliquis æqualia k 29. 11.
erunt; [m] proinde & obliqua AB, CD æquantur. Q. E. D. m 1. ax. 1.

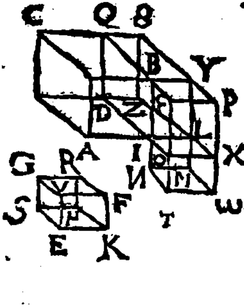
PROP. XXXII.



Solida parallelepipeda ABCD, EFGL *sub eadem altitudine, inter se sunt ut bases* AB, EF.

Productâ EHI, [a] fac pgr. FI=AB, & [b] comple a 45. 1.
Ppp. FINM. Liquet esse Ppp. FINM. b 31. 1.
([c] ABCD). EFGL [d] :: FI. (AB) EF. Q. E. D. c 31. 11. d 25. 11.

PROP. XXXIII.



Similia solida parallelepipeda, ABCD, EFGH, *inter se sunt in triplicata ratione homologorum laterum* AI, EK.

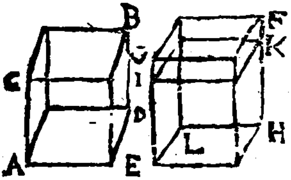
Producantur rectæ AIL, DIO, BIN. & [a] fiant IL, IO, a 3. 1.
IN ipsis EK. KH, KF æquales, [b] adeóq; b 27. 11.

& Ppp. IXMT æq. & ſim. Pppº EFGH.
c 31. 1. ᶜ Perficiantur Pppª IXPB, DLYQ. Itaque ᵈ e-
d hyp. rit AI. IL. (EK) :: DI. IO (HK) :: BI. IN.
e 1. 6. (KF); ᵉ hoc eſt pgr. AD. DL :: DL. IX ::
f 32. 11. BO. IT; ᶠ id eſt Ppp. ABCD. DLQY ::
DLQY. IXBP :: IXBP. IXMT. (ᵍ EFGH).
g conſtr. ʰ ergò ratio ABCD ad EFGH triplicata eſt
h 10. def. 5. rationis ABCD ad DLQY, ᵏ vel AI ad EK.
k 1. 6. Q. E. D.

Coroll.

Hinc ſi fuerint quatuor lineæ rectæ continuè proportionales, ut eſt prima ad quartam, ità eſt parallelepipedum ſuper primam deſcriptum ad parallelepipedum ſimile, ſimilitérque deſcriptum ſuper ſecundam.

PROP. XXXIV.



Æqualium ſolidorũ parallelepipedorũ ADCB EHGF *baſes, & altitudines reciprocantur* (AD. EH :: EG. AC) : *Et quorum ſolidorum parallelepipedorum* ADCB, EHGF *baſes, & altitudines reciprocantur, illa ſunt æqualia.*

Sint primò latera CA, GE ad baſes recta, ſi jam ſolidorum altitudines ſint pares, etiam baſes æquales erunt, & res clara eſt. Sin altitudines inæquales ſint, à majori EG ᵃ detrahe EI
a 3. 1.
b 31. 1. = AC. & per I ᵇ duc planum IK parallelum
c 32. 11. baſi EH, itaque
d 17. 5. 1. *Hyp.* AD. EH ᶜ :: Ppp. ADCB. EHIK ᵈ ::
e 1. 6.
f conſtr. Ppp. EHGF. EHIK ᵉ :: GL. IL ᵉ :: GE. IE.
g 11. 5. (ᶠ AC) ᵍ liquet igitur eſſe AD. EH :: GE. AC.
Q. E. D.

2. *Hyp.*

2. *Hyp.* ADCB. EHIK [h] :: AD. EH [k] :: EG. EI [l] :: GL. IL [m] :: Ppp. EHGF. EHIK, [n] quare Ppp. ADCB = EHGF. Q. E. D. h 32. 11. k *hyp.* l 1. 6. m 32. 11. n 9. 5.

Sint deinde latera ad bases obliqua. Erigantur super iisdem basibus, in altitudine eâdem parallelepipeda recta. Erunt obliqua parallelepipeda his æqualia. Quare cùm hæc per 1. partem reciprocent bases & altitudines, etiam illa reciprocabunt. Q. E. D.

Coroll.

Quæ de parallelepipedis demonstrata sunt Prop. 29, 30, 31, 32, 33, 34. *etiam conveniunt prismatis triangularibus, quæ sunt dimidia parallelepipeda, ut patet ex* Pr. 28. *Igitur,*

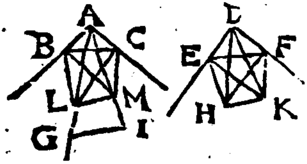
1. Prismata triangularia æquè alta sunt ut bases.

2. Si eandem vel æquales habeant bases, & eandem altitudinem, æqualia sunt.

3. Si similia fuerint, eorum proportio triplicata est proportionis homologorum laterum.

4. Si æqualia sunt, reciprocant bases, & altitudines; & si reciprocant bases & altitudines, æqualia erunt.

Prop. XXXV.



Si fuerint duo plani anguli BAC, EDF *æquales, quorum verticibus* A, D *sublimes rectæ lineæ* AG, DH *insistant, quæ cum lineis primò positis angulos contineant æquales, utrumq; utriq;* (ang. GAB = HDE; & GAC = HDF) *in sublimibus autem lineis* AG, DH *qualibet sumpta fuerint puncta* G, H; *&*

& ab his ad plana BAC, EDF, *in quibus consistunt anguli primùm positi* BAC, EDF, *ductæ fuerint perpendiculares* GI, HK; *à punctis verò* I, K *quæ in planis à perpendicularibus fiunt, ad angulos primùm positos adjunctæ fuerint rectæ lineæ* AI, DK, *hæ cum sublimibus* AG, DH *æquales angulos* GAM, HDK *comprehendent.*

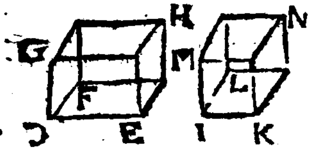
Fiant DH, AL æquales, & GI, LM parallelæ; & MC ad AC, MB ad AB, KF ad DF, KE ad DE perpendiculares, ducantúrque rectæ BC, LB, LC, atq; EF, HF, HE; [a] éstq; LM
a 8. 11. recta plano BAC; [b] quare anguli LMC, LMA,
b 3. def. 11. LMB; eâdémque ratione anguli HKF, HKD, HKE recti sunt. Ergò ALq [c] = LMq + AMq
c 47. 1. [c] = LMq + CMq + ACq [c] = LCq + ACq;
d 48. 1. [d] ergò ang. ACL rectus est. Rursus ALq [e] = LMq + MAq [e] = LMq + BMq + BAq [e] =
e 47. 1. BLq + BAq. [d] ergò ang. ABL etiam rectus est. Simili discursu anguli DFH, DEH recti
f 26. 1. sunt; [f] ergò AB = DE; [f] & BL = EH; [f] & AC = DF; & CL = FH. [g] quare etiam BC
g 4. 1. = EF, [g] & ang. ABC = DEF, [g] & ang. ACB
h 3. ax. 1. = DFE. unde reliqui è rectis anguli CBM, BCM reliquis FEK, EFK æquantur. [k] ergò
k 26. 1. CM = FK, [l] ideóque & AM = DK. ergò si
l 47. 1. ex LAq [m] = HDq. auferatur AMq = DKq,
m constr. [n] remanet LMq = HKq. quare trigona LAM,
n 47.1. & 3. ax. HDK sibi mutuò æquilatera sunt. [o] ergò ang.
o 8. 1. LAM = HDK. Q. E. D.

Coroll.

Itaque si fuerint duo anguli plani æquales, quorum verticibus sublimes rectæ lineæ æquales insistant, quæ cum lineis primò positis angulos contineant æquales, utrumque utrique; erunt à punctis extremis linearum sublimium ad plana angulorum primò positorum demissæ perpendiculares inter se æquales; nempe LM = HK.

Prop.

PROP. XXXVI.

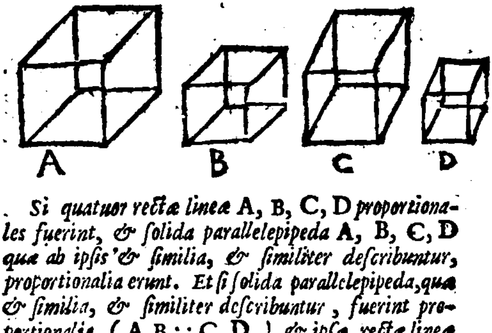


Si tres rectæ lineæ DE, DG, DF *proportionales fuerint; quod ex his tribus fit solidum parallelepipedum* DH, *æquale est descripto à media linea* DG (IL) *solido parallelepipedo* IN, *quod æquilaterum quidem sit, æquiangulum verò prædicto* DH.

Quoniam DE. IK[a] : : IL. DF, [b] erit pgr. LK = FE. & propter angulorum planorum ad E, & I, ac linearum GD, IM æqualitatem, etiam altitudines parallelepipedorum æquales sunt, ex coroll. præced. [h] ergò ipsa inter se æqualia sunt. Q. E. D.

a *hyp.*
b 14. 6.
h 31. 11.

PROP. XXXVII.



Si quatuor rectæ lineæ A, B, C, D *proportionales fuerint, & solida parallelepipeda* A, B, C, D *quæ ab ipsis & similia, & similiter describuntur, proportionalia erunt. Et si solida parallelepipeda, quæ & similia, & similiter describuntur, fuerint proportionalia* (A. B : : C. D.) *& ipsæ rectæ lineæ* A, B, C, D *proportionales erunt.*

Nam rationes parallelepipedorum [a] triplicatæ sunt rationum, quas habent lineæ. ergò si A. B : : C. D. [b] erit Ppp. A. Ppp. B. : : Ppp. C. Ppp. D. & vice versâ.

a 33. 11.
b *sch.* 23. 5.

PROP.

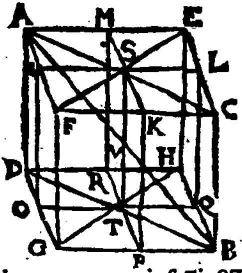
PROP. XXXVIII.

Si planum AB ad planum AC rectum fuerit, & ab aliquo puncto E eorum, quæ sunt in uno planorum (AB) ad alterum planum AC perpendicularis EF ducta fuerit, in planorum communem sectionem AD cadet ducta perpendicularis EF.

Si fieri potest, cadat F extra intersectionem AD. In plano AC [a] ducatur FG perpendicularis ad AD, jungatúrque EG. Angulus FGE [b] rectus est; & EFG rectus ponitur. ergò in triangulo EFG sunt duo anguli recti. Q.E.A.

a 12. 1.
b 4, & 3. def. 11.
c 17. 1.

PROP. XXXIX.



Si solidi parallelepipedi AB, eorum quæ ex adverso planorum AC, DB latera (AB, FC, AF, EC. & DH, GB, DG, HB) bifariam secta sint; per sectiones autem plana ILQO, PKMR sint extensa, planorum communis sectio ST, & solidi parallelepipedi diameter AB, bifariam se mutuò secabunt.

Ducantur rectæ SA, SC, TD, TB. Propter [a] latera DO, OT lateribus BQ, QT, [b] angulósque alternos TOD, TQB æquales, [c] etiam bases DT, TB, & anguli DTO, BTQ æquantur [d] ergò DTB est recta linea. eodem modo ASC recta est linea. Porrò [e] tam AD ad FG, [e] quàm FG ad CB; [f] ideóque AD ad CB, [g] ac proinde AC ad DB parallelæ, & æquales sunt, [h] quare

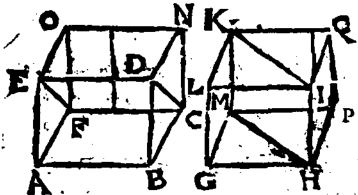
a 34. 1.
b 29. 1.
c 4. 1.
d sch. 15. 1.
e 34. 1.
f 9. 11. & 1. ax.
g 33. 1.

[h] quare AB, & ST in eodem plano ABCD existunt. Itaque cùm anguli AVS, BVT ad verticem, & alterni ASV, BTV æquentur; [k] & AS = BT; erit AV = BV, [l] & SV = VT. Q. E. D. h 7. 11. k 7. ax. 1. l 26. 1.

Coroll.

Hinc, In omni parallelepipedo diametri omnes se mutuò bisecant in uno puncto, V.

PROP. XL.



Si fuerint duo prismata ABCFED, GHMLIK *æqualis altitudinis, quorum hoc quidem habeat basim* ABCF *parallelogrammum; illud verò* GHM *triangulum; duplum autem fuerit parallelogrammum* ABCF *trianguli* GHM, *æqualia erunt ipsa prismata* ABCFED, GHMLIK.

Nam si perficiantur parallelepipeda AN, GQ, [a] erunt hæc æqualia ob [b] basium AC, GP, & [c] altitudinum æqualitatem. [e] ergò etiam prismata, [d] horum dimidia, æqualia erunt. Q. E. D. a 31. 11. b 34. 1. & 7. ax. c hyp. d 28. 11. e 7. ax. 1.

Schol.

Ex hactenus demonstratis habetur dimensio prismatum triangularium, & quadrangularium, seu parallelepipedorum, si nimirum altitudo ducatur in basim. Andr. Tacq

Ut si altitudo sit 10 pedum, basis verò pedum quadratorum 100 (mensurabitur autem basis per sch. 35. 1. vel per 41. 1.) multiplica 100 per 10;

proveniunt 1000 pedes cubici pro soliditate prismatis dati.

Vide schol. 35. 1. Nam quemadmodum rectangulum, ità & parallelepipedum rectum producitur ex altitudine ducta in basim. Ergò quodvis *parallelepipedum* producitur ex altitudine in basim ducta, ut patet ex 31 hujus.

Deinde cùm totum parallelepipedum producatur ex altitudine in totam basim, semissis ejus (hoc est prisma triangulare) producetur ex altitudine ducta in dimidiam basim, nempe triangulum.

Monitum.

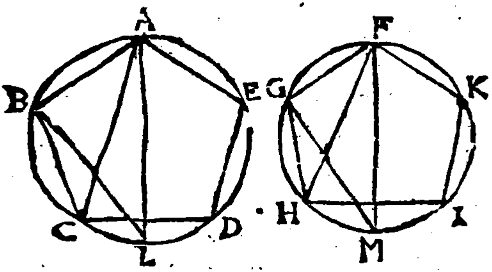
Nota, litterarum quæ designant angulum solidum primam esse semper ad punctum, in quo est angulus, Litterarum verò quæ denotant pyramidem, ultimam esse ad verticem pyramidis.

Ex. gr. Angulus solidus ABCD est ad punctum A, pyramidis quoque BCDA, vertex est ad punctum A, & basis triangulum BCD.

LIB.

LIB. XII.

PROP. I.



Quæ sunt in circulis ABD, FGI *polygona similia* ABCDE, FGHIK *inter se sunt, ut quadrata à diametris* AL, FM.

Ducantur AC, BL, FH, GM.

Quoniam [a] ang. ABC = FGH, [a] atque AB. BC :: FG. GH, [b] erit ang. ACB ([c] ALB) = FHG ([c] FMG). anguli autem ABL, FGM [d] recti, ac proinde æquales sunt. [e] ergò triangula ABL, FGM æquiangula sunt. [f] quare AB. FG :: AL. FM. [g] ergò ABCDE. FGHIK :: ALq. FMq.

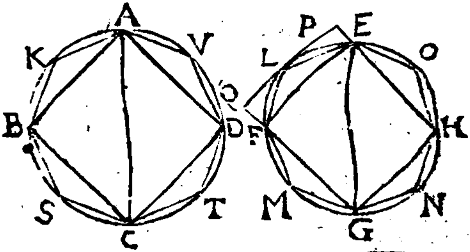
a 1. *def.* 6.
b 6. 6.
c 21. 3.
d 31. 3.
e 32. 3.
f *cor.* 4. 6.
g 22. 6.

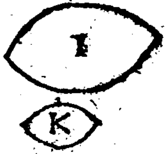
Coroll.

Hinc, quia (AB. FG :: AL. FM :: BC. GH &c.) polygonorum similium circulo inscriptorum [h] ambitus sunt ut diametri.

h 1. 12. & 12. 5.

PROP. II.





Circuli ABT, EFN *inter se sunt, quemadmodum quadrata à diametris* AC, EG.

Ponatur ACq. EGq :: circ. ABT. I. Dico I = circ. EFN.

Nam primò, si fieri potest, sit I ⊐ circ. EFN, sitque excessus K. Circulo EFN inscribatur

a *sch.* 7. 4. quadratum EFGH, [a] quod dimidium est circumscripti quadrati, adeóque semicirculo majus.

b 30. 3. [b] Biseca arcus EF, FG, GH, HE, & ad puncta bisectionum junge rectas EL, LF &c. per L

c *sch.* 27. 3. duc tangentem PQ ([c] quæ ad EF parallela est), & produc HEP, GFQ; éstque *triangulum*

d 41. 1. ELF [d] dimidium parallelogrammi EPQF, adeóque majus dimidio segmenti ELF; paritérque reliqua triangula ejusmodi reliquorum segmentorum dimidia superant. Et si iterum bisecentur arcus EL, LF, FM &c. rectæque adjungantur, eodem modo triangula segmentorum semisses excedent. Quare si quadratum EFGH è circulo EFN, & è reliquis segmentis triangula

e 1. 10. detrahantur, & hoc fiat continuò, tandem [e] restabit magnitudo aliqua minor quàm K. Eoúsque perventum sit, nempe ad segmenta EL, LF, FM &c. minora quàm K, simul sumpta,

pta. ergò I ([f] circ. EFN $-$ K) $\sqsupset$ polyg. ELFMGNHO (circ. EFN $-$ segm. EL $+$ LF &c.) Circulo ABT inscriptum [g] puta simile polygonum AKBSCTDV. itaque quum AKBSCTDV. ELFMGNHO [h] :: ACq. EGq [k] :: circ. ABT. I. ac polyg. AKBSCTDV [l] $\sqsubset$ circ. ABT. [m] erit polyg. ELFMGNHO $\sqsubset$ I. sed priùs erat I $\sqsupset$ ELFMGNHO. quæ repugnant.

f *hyp. &* 3. *ax.*
g 30. 3. & 1. *post.* 1.
h 1. 12.
k *hyp.*
l 9. *ax.* 1.
m 14. 5.

Rursus, si fieri potest, sit I $\sqsubset$ circ. EFN. Quoniam igitur ACq. EGq [n] :: circ. ABT. I; inversèque I. circ. ABT :: EGq. ACq. pone I. circ. ABT :: circ. EFN. K. [o] ergò circ. ABT $\sqsubset$ K. [p] atque EGq. ACq :: circ. EFN. K. Quæ repugnare modò ostensum est.

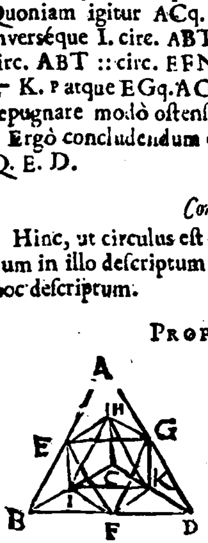
n *hyp.*
o 14. 5.
p 11. 5.

Ergò concludendum est, quòd I $=$ circ. EFN. Q. E. D.

Coroll.

Hinc, ut circulus est ad circulum, ità polygonum in illo descriptum ad simile polygonum in hoc descriptum.

PROP. III.



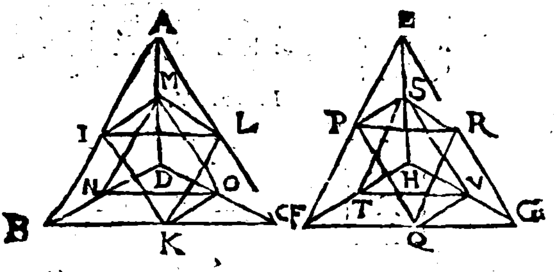
Omnis pyramis ABDC *triangularem habens basim, dividitur in duas pyramides* AEGH, HIKC *æquales & similes inter se, triangulares habentes bases, & similes toti* ABDC; *& in duo prismata æqualia* BFGEIH, EGDIHK; *quæ duo prismata majora sunt dimidio totius pyramidis* ABDC.

Latera pyramidis bisecentur in punctis E, F, G, H, I, K; junganturque rectæ EF, FG, GE, EI, IF, FK, KG, GH, HE. Quoniam latera

a 2. 6. pyramidis proportionaliter secta sunt, [a] orunt
HI, AB; & GF, AB; & IF, DC, atque HG,
DC &c. parallelæ; proinde & HI, FG, & GH,
FI parallelæ sunt. liquet igitur triangula ABD,
b 29. 1. AEG, EBF, FDG, HIK [b] æquiangula esse; &
c 26. 1. quatuor ultima [c] æquari; eodém modo triangula
ACB, AHE, EIB, HIC, FGK æquiangula
sunt, & quatuor postrema inter se æqualia; similiter triangula BFI, FDK, IKC, EGH; & denuò triangula AHG, GDK, HKC, EFI, similia sunt & æqualia. Quinetiam triang. HIK
ad ADB, & EGH ad BDC, & EFI ad ADC,
d 15. 11. & FGK ad ABC [d] parallela sunt. Ex quibus
perspicuè sequitur primò, pyramides AEGH,
HIKC æquales esse; totíque ABDC, & inter
e 10. def 11. se [e] similes. deinde solida BFGEIH, FGDIHK
prismata esse, & quidem æquè alta, nempe sita
inter parallela plana ABD, HIK. verùm basis
f 2. ax. 1. BFGE basis FDG [f] duplex est. quare dicta
g 40. 11. prismata æqualia sunt. quorũ alterum BFGEIH
pyramide BEFI, hoc est AEGH majus est,
totum suâ parte; proinde duo prismata majora
sunt duabus pyramidibus, totiúsque adeò pyramidis ABDC dimidium excedunt. Q. E. D.

PROP.

PROP. IV.



Si fuerint duæ pyramides ABCD, EFGH *ejusdem altitudinis, triangulares habentes bases* ABC, EFG; *sit autem illarum utraque divisa & in duas pyramides* (AILM, MNOD; & EPRS, STVH) *æquales inter se, & similes toti; & in duo prismata æqualia* (IBKLMN, KLCNMO; & PFQRST, QRGTSV); *ac eodem modo divisa sit utraque pyramidum, quæ ex superiore divisione natæ sunt, idque semper fiat; erit ut unius pyramidis basis ad alterius pyramidis basim; ita & omnia, quæ in una pyramide, prismata ad omnia, quæ in altera pyramide prismata, multitudine æqualia.*

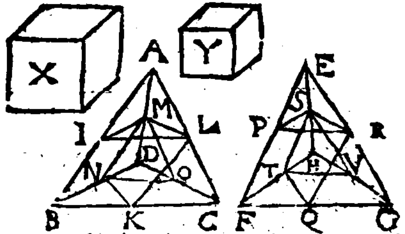
Nam (adhibendo constructionem præcedentis) BC. KC[a] :: FG. QG. [b] ergò triang. ABC est ad simile triang. LKC, ut EFG ad [c] simile RQG. ergò permutando ABC.EFG [d] :: LKC. RQG [e] :: Prism.KLCNMO.QRGTSV (nam hæc æquè alta sunt)[f] :: IBKLMN. PFQRST. [g] quare triang. ABC. EFG :: Prism.KLCNMO +IBKLMN. Prism. QRGTSV+PFQRST. Q. E. D.

a 15. 5.
b 22. 6.
c 2. 6. & c.
d 16. 5.
e sch. 34 11.
f 7. 5.
g 12. 5.

Sin ulteriùs simili pacto dividantur pyramides MNOD, AILM; & EPRS, STVH, erunt quatuor nova prismata hîc effecta ad quatuor

isthic producta, ut bases MNO, & AIL ad bases STV, & EPR, hoc est ut LKC ad RQG,
h 12. 5. vel ut *ABC* ad *EFG*. [h] quare omnia prismata pyramidis *ABCD* ad *omnia ipsius* EFGH ita se habent, ut *basis ABC ad basim* EFG. Q. E. D.

PROP. V.

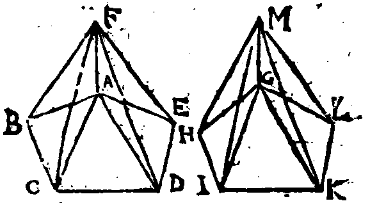


Sub eadem altitudine existentes pyramides ABCD, EFGH *triangulares habentes bases* ABC, EFG, *inter se sunt ut bases*. ABC, EFG.

Sit triang. ABC. EFG :: ABCD. X. Dico X = pyr. EFGH. Nam, si possibile est, sit X ⊐ EFGH; sitque Y excessus. Dividatur pyramis EFGH in prismata & pyramides, & reli-
a 1. 10. quæ pyramides similiter, [a] donec relictæ pyramides EPRS, STVH minores evadant *solido* Y. Quum igitur pyr. EFGH = X + Y; liquet reliqua prismata PFQRST, QRGTSV solido X majora esse. Pyramidem *ABCD* simili-
b 4. 12. ter divisam concipe; [b] Eritq; prism. IBKLMN + KLCNMO. PFQRST + QRGTSV ::
c *hyp.* ABC. EFG. [c] :: pyr. ABCD. X. [d] ergo X ⊏
d 14. 5. prism. PFQRST + QRGTSV; quod repugnat prius affirmatis.

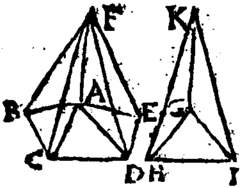
e *hyp.* & Rursus, dic X ⊏ pyr. EFGH. pone pyr.
cor. 4. 5. EFGH. Y :: X. pyr. ABCD [e] :: EFG. ABC.
f *suppos.* quia EFGH [f] ⊐ X, [g] erit Y ⊐ pyr. ABCD,
g 14. 5. quod fieri nequit, ex jam dictis. Concludo igitur, quod X = pyr. EFGH. Q. E. D.

PROP. VI.



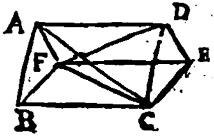
Sub eadem altitudine existentes pyramides ABCDEF, GHIKLM, *& polygonas habentes bases* ABCDE, GHIKL, *inter se sunt ut bases* ABCDE, GHIKL.

Duc rectas AC, AD, GI, GK. Est bas. ABC. ACD [a] :: pyr. ABCF. ACDF. [b] ergò compositè ABCD. ACD :: pyr. ABCDF. ACDF. [a] atqui etiam ACD. ADE :: pyr. ACDF. ADEF. [c] ergò ex æquali ABCD. ADE :: ABCDF. ADEF. [b] ergò componendo ABCDE. ADE :: pyr. ABCDEF. ADEF. a 5.12.
porrò ADE. GKL [d] :: pyr. ADEF. GKLM; b 18.5.
ac, ut priùs, atque inversè GKL. GHIKL :: pyr.
GKLM GHIKLM. [c] ergò iterum ex æquali- c 22.5.
bus. ABCDE. GHIKL :: Pyr. ABCDEF. GHIKLM. Q. E. D.



Si bases non ha- d 5.12.
bent latera æquè multa, demonstratio sic procedet. Bas. ABC. GHI [e] :: pyr. ABCF. GHIK. [e] atque
ACD.GHI :: pyr. e 5.12.
ACDF. GHIK. f 24.5.
[f] ergò bas. ABCD. GHI :: pyr. ABCDF. GHIK. [e] Quinetiam bas. ADE. GHI :: pyr. ADEF. GHIK. [f] ergò bas. ABCDE. GHI :: pyr. ABCDEF. GHIK.

Prop. VII.

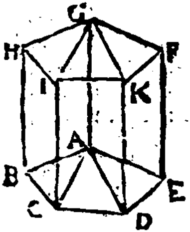


Omne prisma ABCDFE *triangularem habens basim, dividitur in tres pyramides* ACBF, ACDF, CDFE *æquales inter se, triangulares bases habentes.*

Ducantur parallelogrammorum diametri AC, CF, FD. Triang. ACB [a] = ACD. [b] ergò æquè altæ pyramides ACBF, ACDF æquantur. eodem modo pyr. DFAC = pyr. DFEC. atqui ACDF, & DFAC una eadémque sunt pyramis. [c] ergò tres pyramides ACBF, ACDF, DFEC, in quos divisum est prisma, inter se æquales sunt. Q. E. D.

a 34. 1.
b 5. 12.
c 1. ax. 1.

Coroll.

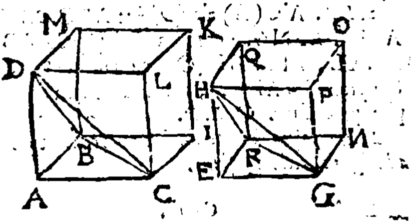


Hinc, quælibet pyramis tertia est pars prismatis eandem cum illa habentis & basim & altitudinem: sive prisma quodlibet triplum est pyramidis, eandem cum ipso habentis basim & altitudinem.

Nam resolve prisma polygonum ABCDEGHIKF in trigona prismata; & pyramidem ABCDEH in trigonas pyramides. [a] Erunt singulæ partes prismatis triplæ singularum partium pyramidis. [b] proinde totum prisma ABCDEGHIKF totius pyramidis ABCDEH triplum est Q. E. D.

a 7. 12.
b 1. 5.

Prop. VIII.



Similes pyramides ABCD, EFGH, *quæ triangulares habent bases* ABC, EFG, *in triplicata sunt ratione homologorum laterum* AC, EG.

[a] Perficiantur parallelepipeda ABICDMKL, EFNGHQOP; quæ [b] similia sunt & pyramidum ABCD, EFGH [d] sextupla; [e] ideóq; in eadem cum ipsis ratione ad se invicem, [f] hoc est in triplicata homologorum laterum. Q. E. D.

a 27. 11.
b 9. def. 11.
d 28. 11 & 7. 12.
e 15. 5.
f 33. 11.

Coroll.

Hinc, etiam similes polygonæ pyramides rationem habent laterum homologorum triplicatam; ut facilè probabitur resolvendo has in trigonas pyramides.

Prop. IX.

Vide Schema praced.

Æqualium pyramidum ABCD, EFGH, *&* *triangulares bases* ABC, EFG *habentium, reciprocantur bases, & altitudines. & quarum pyramidum triangulares bases habentium reciprocantur bases & altitudines, illæ sunt æquales.*

1. *Hyp.* Perfecta parallelepipeda ABICDMKL, EFNGHQOP æqualium pyramidum ABCD, EFGH (utrumque utriusque) [a] sextupla sunt, ac æqualia ideo inter se, ergò alt. (H.)

a 28. 11. & 7. 12.

alt.

b 34. 11. c 15. 5. alt. (D) [b] :: ABIC. EFNG [c] :: ABC. EFG. Q. E. D.

d hyp. e 15. 5. f 34. 11. g 6. ax. 1. 2. Hyp. Alt. (H) alt. (D) [d] :: ABC. EFG [e] :: ABIC. EFNG. [f] ergò parallelepipeda ABICDMKL, EFNGHQOP æquantur; [g] proinde & pyramides ABCD, EFGH horum subsextuplæ pares sunt. Q. E. D.

Eadem polygoniis pyramidibus conveniunt: nam hæ ad trigonas reduci possunt.

Corell.

Quæ de pyramidibus demonstrata sunt, Prop. 6, 8, 9. etiam conveniunt quibuscunque prismatis, cùm hæc tripla sint pyramidum eandem basim & altitudinem habentium. itaque 1. prismatum æquè altorum eadem est proportio, quæ basium.

2. Similium prismatum proportio triplicata est proportionis laterum homologorum.

3. Æqualia prismata reciprocant bases & altitudines, & quæ reciprocant, sunt æquales.

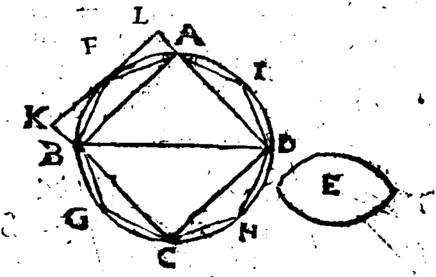
Schol.

Ex hactenus demonstratis elicitur dimensio quorumcunq; prismatum & pyramidum.

a cor. 1. hujus; & sch. 40. 11. b 7. 12. [a] Prismatis soliditas producitur ex altitudine in basim ducta; [b] itaq; & pyramidis ex tertia altitudinis parte ducta in basin.

PROP.

PROP. X.



Omnis conus tertia pars est cylindri habentis eandem cum ipso basin ABCD, *& altitudinem æqualem.*

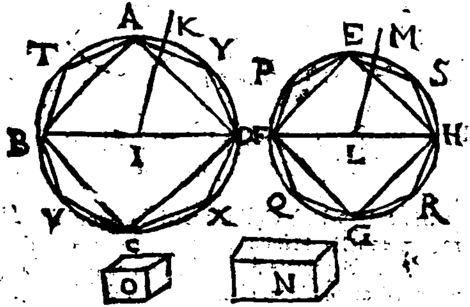
Si negas, primò Cylindrus triplum coni superet excessu E. Prisma super quadratum circulo ABCD inscriptum [a] subduplum est prismatis super quadratum eidem circulo circumscriptum sibi & cylindro æquè alti. ergò prisma super quadratum ABCD superat cylindri semissem. eodem modo prisma super basim AFB cylindro æquè altum segmenti cylindrici AFB [b] dimidio majus est. Continuetur bisectio arcuum, & detrahantur prismata, donec segmenta cylindri relicta, nempe ad AF, FB, &c. minora evadant solido E. Itaque cylind. — segment. AF, FB, &c. (prisma ad basim AFBGCHDI.) [c] majus est, quàm cylind. — E ([d] triplum coni). ergò pyramis dicti prismatis [e] pars tertia (ad eandem basim sita, ejusdémque altitudinis) cono æquè alto ad basim ABCD circulum major est, pars toto. Q. E. A.

Vide fig. 11 hujus.
a *sch. 7. 4. & cor. 9. 12.*
b *sch. 27. 3. & cor. 9. 12.*
c *5. ax. 1.*
d *hyp.*
e *cor 7. 12.*

Sin conus tertiâ parte cylindri major dicatur, sit itidem excessus E. Ex cono detrahe pyramides, ut in priori parte prismata ex cylindro, donec restent coni segmenta aliqua, puta ad AF,

FB, BG, &c. minora solido E. ergò con. ⊏ E
f hyp. (f $\frac{1}{3}$ cylindr.) ⊐ pyr. AFBGCHDI (con. ⊏ segment. AF, FB, &c.); ergò prisma pyramidis triplum (æquè altum scilicet atque ad eandem basim) cylindro ad basim ABCD majus est, pars toto. Q. E. A. Quare fatendum est, quòd cylindrus triplo cono æquatur. Q. E. D.

PROP. XI.



Sub eadem altitudine existentes cylindri, & coni ABCDK, EFGHM inter se sunt ut bases ABCD, EFGH.

Sit circ. ABCD. circ. EFGH :: con. ABCDK. N. Dico N = con. EFGHM.

Nam si fieri potest, sit N ⊐ con. EFGHM, sitque excessus O. Suppositâ præparatione, & argumentatione præcedentis; erit O majus segmentis conicis EP, PF, FQ, &c. ideóque solidum N ⊐ pyr. EPFQGRHSM. a Fiat in circulo ABCD simile polygonum ATBVCXDY.

a. 30. 3, & 1. post. Quia pyr. ABVYK. pyr. EFQSM b :: polyg. ATBVY. polyg. EPFQS. c :: circ. ABCD. circ.
b 6. 12. EFGH d :: con. ABCDK. N. e erit pyr.
c. cor. 2. 12. EPFQGRHSM ⊐ N. contra modò dicta.
d hyp. Rursus dic N ⊏ con. EFGHM. pone con.
e. 14. 5. EFGHM. O :: N. con. ABCDK f :: circ. EFGH. ABCD. g ergò O ⊐ con. ABCDK

quod

quod absurdum est, ex ostensis in priori parte. f hyp. & invertendo.

Itaque potiùs dic, ABCD. EFGH :: con. ABCDK. EFGHM. Q. E. D. g 14. 5.

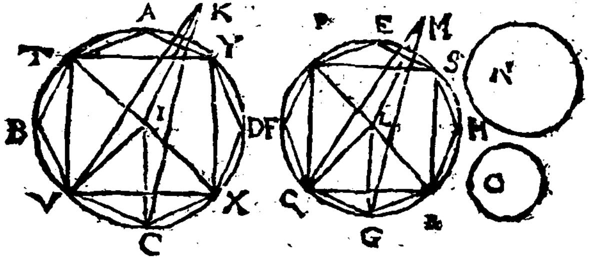
Idem demonstrabitur de cylindris, si conorum, & pyramidum loco concipiantur cylindri & prismata, ergò, &c.

SCHOL.

Ex his habetur dimensio cylindrorum, & conorum quorumcunque. Cylindri rectæ soliditas producitur ex base circulari ([a] pro cujus dimensione consulendus est Archimedes) ductâ in altitudinem, [b] igitur & cujuscunq; cylindri. a 1. Prop. de dimens. circ. b 11. 12.

[c] Itaq; coni soliditas producitur ex tertia parte altitudinis ducta in basim. c 10. 12.

PROP. XII.



Similes coni & cylindri ABCDK, EFGHM *in triplicata ratione sunt diametrorum* TX, PR, *quæ in basibus* ABCD, EFGH.

Habeat conus A ad aliquod N rationem triplicatam TX ad PR, dico N = con. EFGHM: Nam si fieri potest, sit N ⊐ EFGHM; sitque excessus O, ergò ut in Prioribus, N ⊐ pyr. EPFQGRHSM. Sint axes conorum IK LM, ad ducantúrque rectæ VK, CK, VI, CI; & QM, GM, QL, GL. Quoniam coni similes sunt, [a] est VI. IK :: QL. LM. anguli verò VIK, QLM [b] recti sunt. [c] ergò trigona VIK, a 24 def 11. b 18. def. 11. c 6. 6.

Dd 2 QLM.

d 4. 6. QLM æquiangula sunt; [d] unde VC. VI :: QG. QL. item VI. VK :: QL. QM. ergò ex æquali VC. VK :: QG. QM. [e] quinetiam VK. CK :: QM. MG. ergò rursus ex æquo VC. CK :: QG. GM. [f] ergò triangula VKC, QMG similia sunt; similíque argumento reliqua hujus pyramidis triangula reliquis illius. [g] quare pyramides ipsæ similes sunt. [h] sunt verò hæ in triplicata ratione VC ad QG, [k] hoc est VI ad RL, [l] vel TX ad PR. [m] ergò Pyr. AIBVCXDYK. pyr. EPFQGRHSM :: con. ABCDK. N. [n] unde pyr. EPFQGRHSM $\sqsupset$ N; quod repugnat priùs dictis.

e 7. 5.

f 5. 6.

g 9. def. 11.

h cor. 8. 12.

k 4. 6.

l 15. 5.

m hyp. & 11. 5.

n 14. 5.

Rursus, dic N $\sqsubset$ con. EFGHM. fit con. EFGHM. O :: N. con ABCDK [o] :: pyr. EPRM. ATCK [p] :: GQ. VC ter :: [q] PR. TX ter. verùm [r] O [r] $\sqsupset$ ABCDK. quod modò repugnare ostensum est. Proinde N $=$ con. EFGHM. Q. E. D.

o Priùs & inversè.

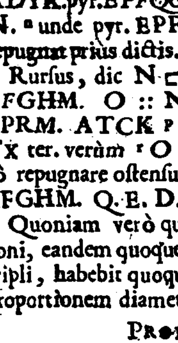
p cor. 8. 12.

q 4. 6.

r 14. 5.

Quoniam verò quam proportionem habent coni, eandem quoque obtinent cylindri, eorum tripli, habebit quoque cylindrus ad cylindrum proportionem diametrorum in basibus triplicatam.

PROP. XIII.



Si cylindrus ABCD *plano* EF *secetur adversis planis* BC, AD *parallelo: erit ut cylindrus* AEFD *ad cylindrum* EBCF, *ità axis* GI *ad axem* IH.

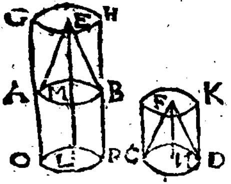
Producto axe, [a] sume GK $=$ GI, & HL $=$ IH $=$ LM. & concipe per puncta K, L, M plana duci circulis AD, BC parallela. [b] ergò cylind. ED $=$ cyl. AN. & cyl. EC [b] $=$ BO [b] $=$ OP. itaque cylindrus

a 3. 1.

b 11. 12.

drus EN cylindri ED æquè multiplex est, ac axis IK axis IG. paritérq; cylindrus FP æquè multiplex est cylindri BF, ac axis IM axis IH. prout verò IK =, ⊏, ⊐ IM, [c] sic cylindr. EN =, ⊏, ⊐ FP. [d] ergò cyl. AEFD. cyl. EBCF :: GI. IH. Q. E. D. c 11. 12. d 6. def. 5.

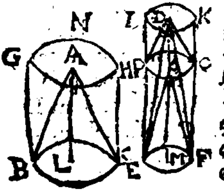
Prop. XIV.



Super æqualibus basibus AB, CD *existentes coni* AEB, CFD, *& cylindri* AH, CK, *inter se sunt ut altitudines* ME, NF.

Productis cylindro HA, & axe EM, sume ML = FN; & per punctum L ducatur planum basi AB parallelum. [a] erit cyl. AP = CK. [b] atqui cyl. AH. AP. (CK) :: ME. ML (NF) Q. E. D. Idem de conis cylindrorum subtriplis dictum puta. * imò de prismatis & pyramidibus. a 11. 12. b 13. 12. * *Adhibe* 9. & 7. 12.

Prop. XV.



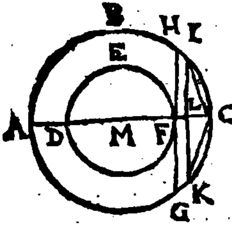
Æqualium conorũ BAC, EDF, *& cylindrorum* BH, EK *reciprocantur bases, & altitudines* (BC. EF :: MD. LA): *& quorum conorum, & cylindrorum reciprocantur bases & altitudines, illi sunt æquales.*

Si altitudines pares sint, etiam bases pares erunt, & res clara est. Sin altitudines sint impares, aufer MO = LA. a 14. 12.

1. *Hyp.* Estque MD. MO ([b] LA) [a] :: cyl. EK ([c] BH) EQ [d] :: circ. BC. EF. Q. E. D. b *constr.* c *hyp.* d 11. 12.

e hyp. 2. *Hyp.* BC.EF [e]:: DM. OM (LA) [f] ::
f 11. 12. Cyl. EK. EQ [g] :: BC. EF [h] :: BH. EQ
g 11. 5. [k] *Ergò cylind.* EK = BH. Q. E. D.
h 11. 12. *Simili argumento utere de conis.*
k 9. 5.

PROP. XVI.

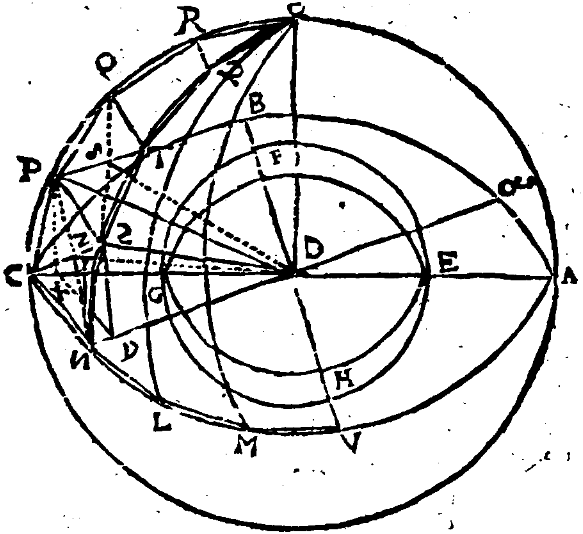


Duobus circulis ABCG. DEF *circa idem centrum* M *existentibus, in majori circulo* ABCG *polygonum æquilaterum, & parium laterum inscribere, quod non tangat minorem circulum* DEF.

Per centrum M extendatur recta AC secans circulum DEF in
a 30. 3. F. ex quo erige perpendicularem FH. [a] Biseca semicirculum ABC, ejúsque semissem AC, atq;
b 1. 10. ità continuò, [b] donec arcus IC minor evadat arcu HC. ab I demitte perpendicularem IL. Liquet arcum IC totum circulum metiri, numerúmque arcuum esse parem, adeóque subtensam
c sch. 16. 4. IC latus esse [c] polygoni inscriptibilis, quod cir-
d cor. 16. 3. culum DEF minimè continget. Nam HG
e 28. 1. [d] tangit circulum DEF; [e] cui parallela est IK,
f 34. def. 1. extráque sita, [f] quare IK circulum non tangit, multóque magìs CI, CK, & reliqua polygoni latera, longiùs à centro distantia, circulum DEF non tangunt. Q. E. F. Coroll. Nota, quòd IK non tangit circulum DEF.

PROP.

PROP. XVII.



Duabus sphæris ABCV, EFGH *circa idem centrum* D *existentibus, in majori sphæra* ABCV *solidum polyedrum inscribere, quod non tangat superficiem minoris sphæræ* EFGH.

Secentur ambæ sphæræ plano per centrum faciente circulos EFGH, ABCV, ducantúrque diametri AC, BV secantes perpendiculariter. Circulo ABCV [a] inscribatur polygonum æquilaterum VMLNC, &c. circulum EFGH minimè tangens. ductâ diametro Nα, erectâque DO rectâ ad planum ABC. per DO, pérq; diametros AC, Nα erigi concipiantur plana DOC, DON, quæ ad circulum ABCV [b] rectæ erunt, ideóque in superficie sphæræ [c] quadrantes effici-

a 16. 12.
b 18. 11.
c cor. 33. 6.

d 4. 1. efficient DOC, DON. in quibus [d] aptentur rectæ CP, PQ, QR, RO, NS, ST, Tγ, γO ipsis CN, NL &c. pares, & æquè multæ. In reliquis quadrantibus OL, OM, &c. inque tota sphæra eadem constructio fiat. *Dico factum.*

A punctis P, S ad planum ABCV *demitte*
e 38. 11. perpendiculares PX, SY, [e] quæ in sectiones AC,
f 12. ax. Nα cadent. Quoniam igitur tam [f] anguli recti
g 27. 3. PXC, SYN, [g] quàm PCX SNY [h] æqualibus peripheriis insistentes, [f] *pares sunt*, triangula
h 32. 1. PCX, SNY [h] æquiangula sunt. Cùm igitur PC
k *constr.* [k] = SN, [l] etiam PX = SY, [l] & XC = YN;
l 26. 1. [m] quare DX = DY. [n] ergò DX. XC :: DY.
m 3. ax. 1. YN. [o] ergò parallelæ sunt YX, NC. quia verò
n 7. 5. PX. SY pares, & cùm eidem plano ABCV re-
o 2. 6. ctæ, etiam [p] parallelæ sunt, [q] erunt YX,
p 6. 11. SP etiam pares & parallelæ. [r] ergò, SP, NC
q 33. 1. inter se parallelæ sunt. ergò [s] quadrilaterum
r 9. 11. NCPS, eâdémque ratione SPQT, TQRG,
s 7. 11. sed & [t] triangulum γRO totidē sunt plana. Eo-
t 2. 11. dem modo tota sphæra ejusmodi quadrilateris, & triangulis repleta ostendetur. quare inscriptum est polyedrum.

u 11. 11. A centro D [u] duc DZ rectum plano NCPS; & junge ZN, ZC, ZS, ZP. Quoniam DN.
x 4. 6. NC [x] :: DY. YX; est NC [y] ⊏ YX (*SP*); pa-
y 14. 5. ritérque SP ⊏ TQ, & TQ ⊏ γR. Et quia
z 3. *def.* 11. anguli DZC, DZN, DZS, DZP, [z] recti sunt,
a 15. *def.* 1. latera verò DC, DN, DS, DP [a] æqualia, &
b 47. 1. DZ commune, [b] erunt ZC, ZN, ZS, ZP æquales inter se; proinde circa quadrilaterum
c 15. *def.* 1. NC PS [c] describi potest circulus, in quo (*ob*
d *constr.* NS, NC, CP [d] æquales, & NC ⊏ *SP*) NC
e 28. 3. [e] plusquam quadrantem subtendit. [f] ergò ang.
f 33. 6. NZC ad centrum obtusus est. [g] ergò NCq ⊏
g 12. 2. 2 ZCq (ZCq + ZNq). Sit NI ad AC nor-
h 32. 1. malis. ergò cùm ang. ADN ([h] DNC +
k 9. *ax.* 1. DCN) sit [k] obtusus, [l] erit semissis ejus DCN
l 5. 1. recti

recti semisse major; proptereáque eo minor est reliquus è recto ang. CNI. [n] unde IN $\sqsubset$ IC. n 19. 1.
ergò NCq (NIq + ICq) [o] $\sqsupset$ 2 INq. itaq; o 47. 1.
IN $\sqsubset$ ZC. & consequenter DZ [p] $\sqsubset$ DI. atqui p 47. 1.
punctum I est [q] extra sphæram EFGH. ergò q cor. 16.12.
punctum Z potiori jure est extra ipsam. adeóq; planum NCPS (cujus [r] proximum centro pun- r 47. 1.
ctum est Z) sphæram EFGH non contingit. Et si ad planum SPQT demittatur perpendicularis DЉ, punctum Љ, adeóque & planum SPQT adhuc ulteriùs à centro elongatur, idémque est de reliquis polyedri planis. ergò polyedrum ORQPCN &c. majori sphæræ inscriptum, minorem non contingit. Q. E. F.

Coroll.

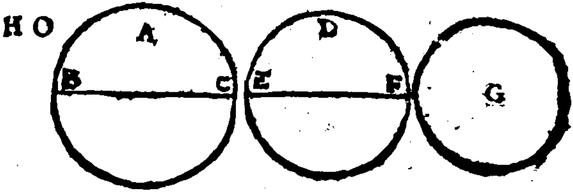
Hinc sequitur, *Si in quavis alia sphæra describatur solidum polyedrum simile prædicto solido polyedro, proportionem polyedri in una sphæra ad polyedrum in altera esse triplicatam ejus, quam habent sphærarum diametri.*

Nam si ex centris sphærarum ad omnes angulos basium dictorum polyedrorum rectæ lineæ ducantur, distribuentur polyedra in pyramides numero æquales & similes, quarum homologa latera sunt semidiametri sphærarum, ut constat, si intelligatur harum sphærarum minor intra majorem circa idem centrum descripta. congruent enim sibi mutuò lineæ rectæ ductæ à centro sphæræ ad basium angulos, ob similitudinem basium, ac propterea pyramides efficientur similes. Quare cùm singulæ pyramides in una sphæra, ad singulas pyramides illis similes in altera sphæra [a] habeant proportionem triplicatam laterum homologorum, hoc est, semidiametrorum sphærarum: sint autem [b] ut una pyramis ad unam pyramidem, ità omnes pyramides, hoc est, solidum polyedrum ex his compositum, ad omnes pyramides,

a cor. 8.12.
b 12. 5.

mides, id est, ad solidum polyedrum ex illis constitutum; habebit quoq; polyedrum unius sphæræ ad polyedrum alterius sphæræ proportionem
c 15.5. triplicatam semidiametrorum, [c] atque adeò diametrorum.

Prop. XVIII.



Sphæræ BAC, EDF *sunt in triplicata ratione suarum diametrorum* BC, EF.

Sit sphæra BAC ad sphæram G in triplicata ratione diametri BC ad diametrum EF. Dico G $=$ EDF. Nam si fieri potest sit G $\sqsupset$ EDF. & cogita sphæram G concentricam esse ipsi EDF
a 17.12. Sphæræ EDF [a] polyedrum sphæram G non tangens, sphæræque BAC simile polyedrum in-
b cor. 17.12. scribatur. [b] Hæc polyedra sunt in triplicata rati-
c hyp. one diametrorum BC, EF, [c] id est, sphæræ
d 14.5. BAC ad G. [d] Proinde sphæra G major est polyedro sphæræ EDF inscripto, pars toto.

Rursus, si fieri potest, sit sphæra G $\sqsubset$ *EDF*. Sitque ut sphæra EDF ad aliam sphæram *H*, ità
e hyp. invers. G ad BAC, [e] hoc est in triplicata ratione dia-
f 14.5. metri EF ad BC; cùm igitur BAC [f] $\sqsubset$ H, incurrimus absurditatem prioris partis. Quin potiùs sphæra G $=$ EDF, Q. E. D.

Coroll.

Hinc, ut sphæra ad sphæram, ità est polyedrum in illa descriptum ad polyedrum simile in hac descriptum.

LIB. XIII.

Prop. I.

Si recta linea z secundùm extremam & mediam rationem secetur (z. a :: a. e), *majus segmentum* a *assumens dimidium totius* z, *quintuplum potest ejus, quod à dimidia totius* z *describitur, quadrati.*

Dico Q. a $+ \frac{1}{2}$ z $=$ 5 Q: $\frac{1}{2}$ z. [a] hoc est aa $+ \frac{1}{4}$ zz $+$ za $=$ zz $+ \frac{1}{4}$ zz. [b] vel aa $+$ za $=$ zz. Nam ze $+$ za [c] $=$ zz. & ze [d] $=$ aa. [e] ergò aa $+$ za $=$ zz. Q.E.*D.*

a 4. 2.
b 3. ax. 1.
c 2. 2.
d hyp. & 16 6.
e 2. ax. & 1. ax.

Prop. II.

Si recta linea $\frac{1}{2}$ z $+$ a *sui ipsius segmenti* $\frac{1}{2}$ z *quintuplum possit, duplæ prædicti segmenti* (z) *extremâ ac mediâ ratione sectæ majus segmentum est* a, *reliqua pars ejus quæ à principio rectæ* $\frac{1}{2}$ z $+$ a.

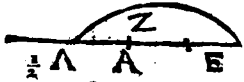
Dico z. a :: a. e. Nam quia per hyp. * aa $+ \frac{1}{4}$ zz $+$ za $=$ zz $+ \frac{1}{4}$ zz; vel aa $+$ za $=$ zz [a] $=$ ze $+$ za. [b] erit aa $=$ ze. [c] quare z. a :: a. e. Q.E.*D.*

* 4. 2.
a 2. 2.
b 3. ax. 1.
c 17. 6.

Vid. fig. præced.

Prop. III.

Si recta linea z *secundùm extremam ac mediam rationem secetur* (z. a :: a. e); *minus segmentum* e *assumens dimidiam majoris segmenti* a, *quintuplum potest ejus, quod à dimidia majoris segmenti* a *describitur, quadrati.*

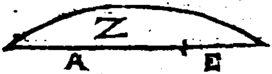


Dico Q: e $+ \frac{1}{2}$ a $=$ 5 Q: $\frac{1}{2}$ a. [a] hoc est ee $+ \frac{1}{4}$ aa $+$ ea $=$ aa $+ \frac{1}{4}$ aa. [b] vel ee $+$ ea $=$ aa. Nam ee $+$ ea [c] $=$ ze [d] $=$ aa. Q.E.*D.*

a 4. 2.
b 3. ax.
c 3. 2.
d hyp. & 17. 6.

PROP. IV.

Si recta linea z secundum extremam ac mediam rationem secetur (z. a :: a. e), *quod à tota* z, *quódque à minori segmento* e *utraque simul quadrata, tripla sunt ejus, quod à majori segmento* a *describitur, quadrati.*



Dico zz + ee = 3 aa. ª vel aa + ee + 2 ae + ee = 3 aa.

a 4. 2. Nam ae + ee ᵇ =

b 3. 2. ze ᶜ = aa. ᵈ ergò aa + 2 ae + ee = 3 aa.

c 17. 6.

d 1. ax. Q. E. D.

PROP. V.

Si recta linea AB secundùm extremam & mediam rationem secetur in C, apponatúrque ei AD æqualis majori segmento AC; tota recta linea DB secundùm extremam ac mediam rationem secatur, & majus segmentum est quæ à principio recta linea AB.

a hyp. Nam quia AB. AD ª :: AC. CB, invertendóque AD. AB :: CB. AC, erit componendo DB. AB :: AB. AC. (AD). Q. E. D.

PROP. VI.

Si recta linea rationalis AB extremâ ac mediâ ratione secetur in C; utrúmque segmentorum (AC, CB) irrationalis est linea, quæ vocatur apotome.

a 3. 1. Majori segmento AC ª adde AD = ½ AB;

b 1. 13. ᵇ ergò DCq = 5 DAq. ᶜ ergò DCq ⊓⊔ DAq.

c 6. 10. proinde cùm AB, ᵉ ideóque ejus semissis DA

d hyp.

e sch. 12.10. sint ρ́. etiam DC est ρ́. Quia verò 5. 1 :: non

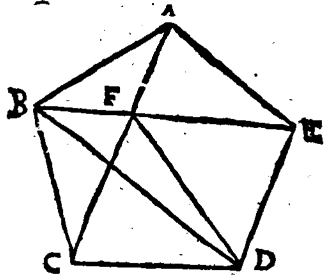
f 9. 10. Q. Q. ᶠ est DC ⊓⊔ DA. ᵍ ergò DC – AD, id

g 74. 10. est AC est apotome. Insuper quia ACq ʰ = AB

h 17. 6. × BC. & AB est ρ́, ᵏ etiam BC est apotome.

k 98. 10. Q. E. D.

PROP. VII.



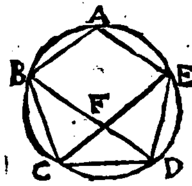
Si pentagoni æquilateri ABCDE *tres anguli, sive qui deinceps* EAB, ABC, BCD, *sive* EAB, BCD, CDE *qui non deinceps sint, æquales fuerint, æquiangulum erit ipsum pentagonum* ABCDE.

Paribus deinceps angulis subtendantur rectæ BE, AC, BD.

Quoniam latera EA, AB, BC, CD, angulíque inclusi [a] æquantur, [b] erunt bases BE, AC, a *hyp.*
BD, [c] angulíque AEB, ABE, BAC, BCA pa- b 4. 1.
res. [d] quare BF=FA, & [e] proinde FC=FE. c 4. & 5. 1.
d 6. 1.
ergò triangula FCD, FED sibi mutuò æqui- e 3. *ax.*
latera sunt; [f] unde ang. FCD=FED, [g] proin- f 8. 1.
de ang. AED=BCD. Eodem pacto ang. CDE g 2. *ax.* 1.
reliquis æquatur. quare pentagonum æquiangulum est. Q. E. D.

Sin anguli EAB, BCD, CDE, qui non deinceps, statuantur pares, [h] erit ang. AEB=BDC. h 4. 1.
& BE=BD, [k] ideóque ang. BED=BDE; k 5. 1.
[l] totus proinde ang. AED=CDE. ergò propter l 2. *ax.*
angulos A, E, D deinceps æquales, ut priùs, pentagonum æquiangulum erit. Q. E. D.

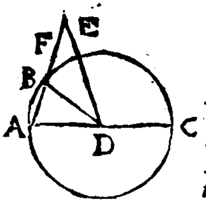
Prop. VIII.



Si pentagoni æquilateri & æquianguli ABCDE *duos angulos* BCD, CDE, *qui deinceps sint, subtendant rectæ lineæ* BD, CE: *hæ extremâ ac mediâ ratione se mutuò secant, & majora ipsarum segmenta* BF, *vel* EF *æqualia sunt pentagoni lateri* BC.

a 14. 4. Circa pentagonum [a] describe circulum ABD.
b 28. 3. [b] Arcus ED = BC, [c] ergò ang. FCD = FDC.
c 27. 3. [d] ergò ang. BFC = 2 FCD (FCD + FDC).
d 32. 1. Atqui arcus BAE [b] = 2 ED, proinde ang.
e 33. 6. BCF [e] = 2 FCD = BFC. [f] quare BF = BC.
f 6. 1. Q. E. D. Porrò quia triangula BCD, FCD
g 27. 3. [g] æquiangula sunt, [h] erit BD. DC. (BF) :: CD.
h 4. 6. (BF) FD. paritérque EC. EF :: EF. FC
Q. E. D.

Prop. IX.



Si hexagoni latus BE, *& decagoni* AB *in eodem circulo* ABC *descriptorum componantur, tota recta linea* AE *extremâ ac mediâ ratione secatur,* (AE. BE :: BE. AB.) *& majus ejus segmentum est hexagoni latus* BE.

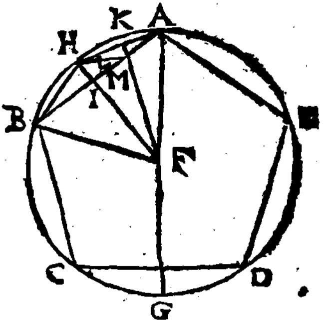
Duc diametrum ADC, & junge rectas DB, DE. Quoniam ang. BDC
a hyp. & 27. 3. [a] = 4 BDA, éstque ang. BDC [b] = 2 DBA
b 32. 1. (DAB + DBA), erit DBA ([b] BDE + BED)
c 7. ax. 1. [c] = 2 BDA [d] = 2 BDE. proinde ang. DBA, vel
d 5. 1. DAB [e] = ADE. Itaque trigona ADE, ADB
e 1 ax. 1. æquiangula sunt, [f] quare AE. AD. ([g] BE).
f 4. 6.
g cor. 15. 4. :: AC. (BE) AB. Q. E. D.

Coroll.

Coroll.

Hinc, si latus hexagoni alicujus circuli secetur extremâ ac mediâ ratione; majus illius segmentum erit latus decagoni ejusdem circuli.

Prop. X.



Si in circulo ABCE *pentagonum æquilaterum* ABCDE *describatur; pentagoni latus* AB *potest & hexagoni latus* FB, *& decagoni latus* AH, *in eodem circulo descriptorum.*

Duc diametrum AG. Biseca arcum AH in K. Et duc FK, FH, FB, BH, HM. a 28. 3. & 3. ax.

Semicirc. AG — arc. AC [a] = AG — AD. hoc est, arc. CG = GD [b] = AH = HB. ergò arc. BCG = 2 BHK; [c] adeóque ang. BFG = 2 BFK. [d] sed ang. BFG = 2 BAG. [e] ergò ang. BFK = BAG. Trigona igitur BFM, FAB [f] æquiangula sunt. [g] quare AB. BF :: BF. BM. [h] ergò AB x BM = BFq. Rursus ang. AFK [k] = HFK; & FA = FH; [m] quare *A*L = LH, [m] & anguli FL*A*, FLH pares ac proinde recti sunt. ergò ang. LHM [m] = L*A*M [n] = HB*A*. Trigona igitur *A*HB, *A*MH [o] æquiangula sunt, [p] quare

b hyp. & 7. ax.
c 33. 6.
d 20. 3.
e 1. ax. 1.
f 32. 1.
g 4. 6.
h 17. 6.
k 27. 3.
m 4. 1.
n 27. 3.
o 32. 1.
p 4. 6.

q 17. 6. re $AB.AH :: AH.AM$ [q] ergò $AB \times AM =$
r 2. 2. AHq. Quum igitur ABq [r] $= AB \times BM + AB$
f 2. ax. $\times AM$, [f] erit $ABq = BFq + AHq$. Q. E. D.

Coroll.

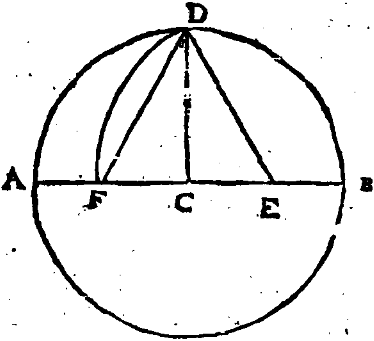
1. Hinc, linea recta, (FK) quæ ex centro (F) arcum quempiam (HA) bisecat, etiam rectam (HA) illi arcui subtensam bisecat, ad angulos rectos.

2. Diameter circuli (AG) ex angulo quovis (A) pentagoni ducta bisecat & arcum (CD), quem latus pentagoni illi angulo oppositum subtendit, & latus ipsum (CD) oppositum, ídque ad angulos rectos.

Schol.

Hic, ut promisimus, praxim tradenus expeditam problematis 11. 4.

Problema.



Invenire latus pentagoni circulo ADB inscribendi.

Duc diametrum AB. cui perpendicularem

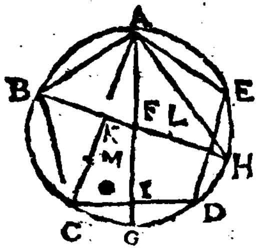
CD.

CD ex centro C erige. Biseca CB in E. Fac EF=ED. Erit DF pentagoni latus.

Nam BF x FC + ECq. [a] = EFq [b] = EDq [c] = DCq + ECq [d] ergò BF x FC = DCq; vel BCq. [e] quare BF. BC :: BC. FC. ergò quum BC sit latus hexagoni, [f] erit FC latus decagoni. proinde DF [h] = √ DCq + FCq, [g] est latus pentagoni. Q.E.F.

a 6. 2.
b constr.
c 47. 1.
d 3. ax.
e 17. 6.
f 9. 13.
g 10. 13.
h 47. 1.

Prop. XI.



Si in circulo ABCD *rationalem habente diametrum* AG, *pentagonum æquilaterũ* ABCDE *describatur; pentagoni latus* AB *irrationalis est linea, quæ vocatur minor.*

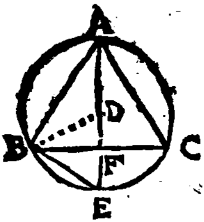
Duc diametrum BFH, rectásque AC, AH; & * fac FL = ¼ radii FH; & CM = ¼ CA.

* 10. 6.

Ob angulos AKF, AIC [a] rectos, & communem CAI, trigona AKF, AIC [b] æquiangula sunt; [c] ergo CI. FK [c] :: CA. FA (FB) [d] :: CM. FL. ergò permutando FK. FL :: CI. CM [d] :: CD. CK (2 CM). [e] componendo igitur CD+CK. CK :: KL. FL. [f] proinde Q: CD+EK. ([g] 5 CKq). CKq :: KLq. FLq. ergò KLq = 5 FLq. Itaque si BH (ρ) ponatur 8, erit FH, 4; FL. 1, & FLq. 1. BL, 5. & BLq, 25. KLq, 5. è quibus liquet BL, & KL esse ρ [h] ⊓⊢. [k] ideóque BK esse Apotomen; cujus congruens KL cùm verò BLq — KLq = 20, [l] erit BL ⊓⊢ √ BLq—KLq. [m] unde BK erit apotome quarta. Quoniam igitur ABq [m] = HB x BK, [n] erit AB minor. Q. E. D.

a cor. 10. 13.
b 32. 1.
c 4. 6.
d 15. 5.
e 18. 5.
f 22. 6.
g 1. 13.
h 9. 10.
k 74. 10.
l 9. 10.
m cor. 8. 6. & 17. 6.
n 95. 10.

Prop. XII.



Si in circulo ABEC *triangulum æquilaterum* ABC *describatur, trianguli latus* AB *potentiâ triplum est ejus lineæ* AD, *quæ ex* D *centro circuli ducitur.*

Protractâ diametro ad E, duc BE. Quoniam arcus BE [a] = EC, arcus BE sexta est pars circumferentiæ. [b] ergò BE = DE. hinc AEq [c] = 4·DEq (4 BEq) [d] = ABq. + BEq (+ ADq). [e] proinde ABq = 3 ADq. Q.E.D.

a cor. 10.13.
b cor. 15.4.
c 4.2.
d 47.1.
e 3. ax. 1.

Coroll.

1. AEq. ABq :: 4. 3.

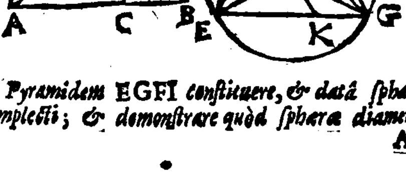
2. ABq. AFq :: 4. 3. [f] Nam ABq. AFq :: AEq. ABq.

3. DF = FE. Nam triang. EBD [g] æquilaterum est; [h] & BF ad ED perpendicularis. [h] ergò EF = FD.

4. Hinc AF = DE + DF = 3 DF.

f cor. 8. 6. & 22. 6.
g cor. 15. 4.
h cor 3. 3.

Prop. XIII.



Pyramidem EGFI *constituere, & datâ sphærâ complecti; & demonstrare quòd sphæræ diameter*

AB

AB *potentiâ sit sesquialtera lateris* EF *ipsius pyramidis* EGFI.

Circa AB describe semicirculum ADB. [a] sítque AC = 2 CB. ex puncto C erige perpendicularem CD; & junge AD, DB. Tum radio HE = CD describe circulum HEFG; cui [b] inscribe triangulum æquilaterum EFG. ex H [c] erige IH = CA rectum plano EFG. produc IH ad K; [d] ità ut IK = AB, rectásque adjunge IE, IF, IG. erit EFGI pyramis expetita.

a 10. 6.
b cor. 15. 4.
c 12. 11.
d 3. 1.

Nam quia anguli ACD, IHE, IHF, IHG [e] recti sunt; & CD, HE, HF, HG [e] pares, [e] atq; IH = AC; [f] erunt AD, IE, IF, IG æquales inter se. Quia verò AC. (2 CB) CB [g] :: ACq. CDq. erit ACq = 2. CDq. itaque ADq [f] = ACq + CDq [h] = 3 CDq = 3 HEq [k] = EFq. [l] ergò AD, EF, IE, IF, IG pares sunt, adeóque pyramis EFGI est æquilatera. Quòd si punctum C super H collocetur, & AC super HI, rectæ AB, IH [m] congruent, utpote æquales. quare semicirculus ADB axi AB, vel IK circumductus [n] transibit per puncta, E, F, G, [*] adeóque pyramis EFGI sphæræ inscripta erit. Q. E. F.

liquet verò esse BAq. ADq [o] :: BA. AC [p] :: 3. 2. Q. E. D.

e constr.
f 47. 1.
g 20. 6.
h 2. ax.
k 12. 13.
l 1. ax. 1.
m 8. ax.
n 15. def. 1.
* 31. def. 11.
o cor 8 6.
p constr.

Corollaria.

1. ABq. HEq :: 9. 2. Nam si ABq ponatur 9, erit ACq (EFq) 6. [q] proinde HEq erit 2.

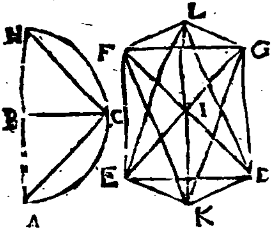
2. AB. LC :: 6. 1. Nam si AB ponatur 6 erit AL, 3; [r] ideóque AC 4; quare LC erit 1. Hinc

q 12. 13.
r constr.

3. AB. HI :: 6. 4 :: 3. 2. unde

4. ABq. HIq :: 9. 4.

PROP. XIV.



Octaedrum KEFGDL constituere, & datâ sphærâ complecti, quâ & pyramidem; & demonstrare, quòd sphæræ diameter AH potentiâ sit dupla lateris AC ipsius Octaedri.

Circa *A*H describe semicirculum *A*CH. ex centro B erige perpendicularem BC. duc *A*C,

a 46. 1. HC. Super *E*D = *A*C [a] fac quadratum *E*FGD, cujus diametri D*F*, *E*G secantes in centro I. ex

b 12. 11. I duc IL = *A*B [b] rectam plano *EF*GD. produc

c 3. 1. IL, [c] donec IK = I L. Connexis K*E*, K*F*, KG, KD, L*E*, L*F*, LG, LD; erit K*EF*GDL octaedrum quæsitum.

Nam *A*B, BH, *F*I, I*E*, &c. æqualium quadratorum semidiametri æquales sunt inter se.

d 4. 1. [d] quare triangulorum rectangulorm LI*E*, LI*F*, *F*I*E*, &c. bases L*F*, L*E*, *FE*, &c. æquantur. proinde octo triangula L*FE*, L*F*G, LG*D*, LD*E*, K*EF*, K*F*G, KGD, KD*E* æquilatera

e 27. def. 11. sunt, [e] atque octaedrum constituunt, quod sphæræ cujus centrum I, radius IL, vel *A*B inscribi

constr. potest. (quoniam AB, IL, I*F*, IK, &c. [f] æquales sunt) Q. *E*. *F*. porrò, liquet *A*Hq. (LKq)

g 47. 1. [g] = 2 *A*Cq (2 LDq). Q. E. D.

Corollaria.

1. Hinc manifestum est, in Octaedro tres diametros *E*G, *F*D, LK se mutuò ad angulos rectos secare in centro sphæræ.

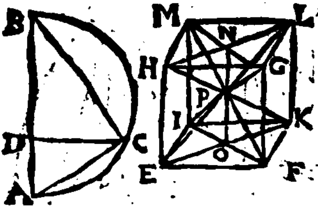
2. Item, tria plana EFGD, LEKG, LFKD esse quadrata, se mutuò ad angulos rectos secantia.

3. Octa-

3. Octaedrum dividitur in duas pyramides similes & æquales EFGDL, & EFGDK, quarum basis communis est quadratum EFGD.

4. Denique, bases octaedri oppositæ inter se parallelæ sunt. 15. 11.

PROP. XV.



Cubum EFGHIKLM *constituere, & sphærâ complecti, quâ & priores figuras; & demonstrare, quòd sphæræ diameter* AB *potentiâ sit tripla lateris* EF *ipsius cubi.*

Super AB describe semicirculum ACB; & [a] fac AB = 3 DA. ex D erige perpendicularem DC, & junge BC ac AC. Tum super EF = AC [b] construe quadratum EFGH, cujus plano rectæ insistant EI, FK, HM, GL ipsi EF pares, quas connecte rectis IK, KL, LM, IM. Solidum EFGHIKLM cubus est, ut satis constat ex constructione.

a 10. 6.

b 46. 1.

In quadratis oppositis EFKI, HGLM duc diametros EK, FI, HL, MG, per quas ducta plana EKLH, FIMG se intersecent in recta NO. Hæc diametros cubi EL, FM, GI, HK [c] bisecabit in P, centro cubi. [d] ergò P centrum erit sphæræ per puncta cubi angularia transeuntis. Porrò ELq [e] = EKq + KLq [e] = 3 KLq, [f] vel 3 ACq. atqui ABq. ACq [g] :: BA. DA [f] :: 3. 1. [h] ergò AB = EL. Quare cubum fecimus, &c. Q. E. F.

c cor. 39. 11.

d 15. def 1. & 14. def. 11.

e 47. 1.

f constr.

g cor. 8. 6.

h 14. 5.

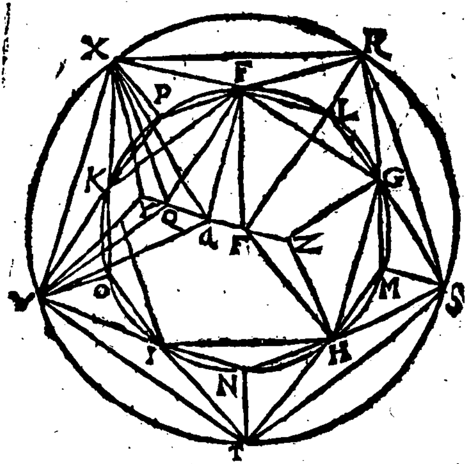
Coroll.

1. Hinc, omnes diametri cubi inter se æquales sunt, seséque mutuò in centro sphæræ bisecant. Eâdémque ratione rectæ quæ quadratorum oppositorum centra conjungunt, bisecantur in eodem centro.

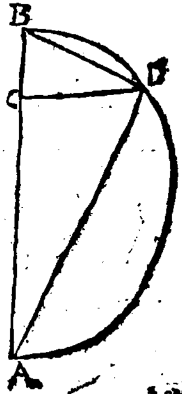
2 Dia-

k 47. 1. l 13. 15. m 15. 13. 2. Diameter sphæræ potest latus tetraedri, & cubi, nempe $ABq\ ^{k}= {}^{l}BCq + {}^{m}ACq$.

PROP. XVI.



Icosaedrum ZGHIKF-YVXRST *constituere, & sphærâ complecti, quâ & antedictas figuras, & demonstrare, quòd icosaedri latus* FG *irrationalis est linea, quæ vocatur minor.*



Super AB diametrum sphæræ describe semicirculum ADB; & [a] fac AB = 5 BC. ex C erige normalem CD, & duc AD, ac BD. Ad intervallum EF = BD describe circulũ EFKNG;

a 10. 6.

[b] cui inscribe pentagonum æquilaterum FKIHG b 11. 4.
Biseca arcus FG, GH &c. ac connecte rectas FL, LG, &c. latera nempe decagoni. Tunc [c] erige EQ, LR, MS, NT, OV, PX ipsi EF c 12. 11.
æquales, rectásque plano FKNG. & connecte RS, ST, TV, VX, XR; item FX, FR, GR, GS, HS, ST, HT, IT, IV, KV, KX. Denique productâ EQ, sume QY = FL; & EZ = FL; rectásque duci concipe ZG, ZH, ZI, ZK, ZF; ac YV, YX, YR, YS, YT. Dico factum.

Nam ob EQ, LR, MS, NT, OV, PX [d] æquales [e] & parallelas; etiam quæ illas jungunt, EL, QR, EM, QS, EN, QT, EO, QV, EP, QX [f] pares & parallelæ sunt. Itera ideò LM (vel FG), RS, MN, ST, &c. æquales sunt inter se. [g] ergò planum per EL, EM &c. plano per QR, QS, &c. æquidistans, [h] & circulus QXRSTV è centro Q, circulo EPLMNO æqualis est; atque RSTVX est pentagonum æquilaterum. Duci verò intellectis EF, EG, EH, &c. ac QX, QR, QS, &c. quia FRQ [k] = FLq + LRq [l] vel EFq [m] = FGq, [n] erunt FR, FG, adeóque omnes RS, FG, FR, RG, GS, GH, &c. æquales inter se. Proinde 10 triangula RFX, RFG, RGS, &c. æquilatera sunt & æqualia. Rursus ob ang. XQY [o] rectum, erit XYq [p] = QXq + QYq [q] = VXq vel FGq. quare XY, VX, hísque similiter YV, YT, YS, YR, ZG, ZH, &c. æquantur: Ergò alia decem trigona constituta sunt æquilatera, & æqualia tam sibi mutuò, quàm decem prioribus; ac proinde factum est Icosaedrum.

d constr. e 6. 11. f 33. 1. g 15. 11. h 1. def. 3. k 47. 1. l constr. m 10. 13. n sch. 48. 1. & 1. ax. o cor. 14. 11. p 47. 1. q 10. 13.

Porrò, bisectâ EQ in α, duc rectus αF, αX, αV; & propter QX [r] = QV, & commune latus αQ, angulósq; EQX, EQV rectos; [s] erit αX = αV, similíque argumento omnes, αX, αR, αS, αT, αV, αF, αG, αH, αI, αK æquantur.

r 15. def. 1. s 4. 1.

Quo-

t 9. 13. Quoniam autem ZQ, QE [t] : : QE, ZE, erit
u 3. 13. Zzq [u]= 5 Eaq [x]= EQq(EFq)+Eaq [y]= aFq.
x 4. 2. ergò Zz = aFz pari pacto aF = Ya. ergò
y 47. 1. sphæra, cujus Centrum a radius aF per 12 puncta icosaedri angularia transibit.

z 15. 5. Denique, quia Za. aE : : ZY. QE; [z] ideóq;
a 22. 6. Zaq, aEq : : ZYq. QEq. [b] erit ZYq = 5
b 14. 5. QEq, vel 5 BDq = : atqui ABq. BDq [c] : : AB
c cor. 8. 6. BC : : 5. 1. [d] ergò ZY = AB. Q. E. F.
d 1. ax. 1. Itaque si AB ponatur $\acute{\rho}$, [e] erit EF = √ ABq
e sch. 12. 10. etiam $\acute{\rho}$; proinde FG pentagoni, idémque Ico-
f 11. 13. saedri 5 latus, [f] est minor. Q. E. D.

Coroll.

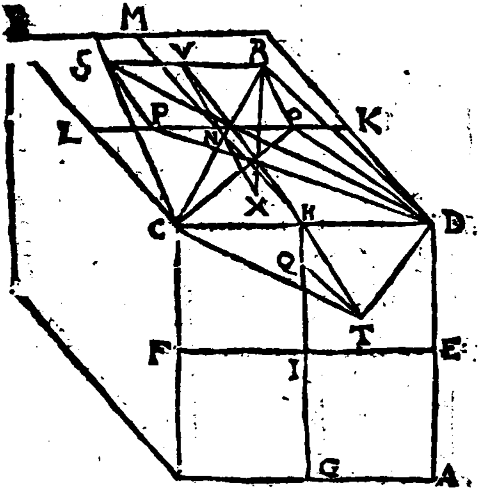
1. Ex dictis infertur, sphæræ diametrum esse potentiâ quintuplum semidiametri circuli quinq; latera icosaedri ambientis.

2. Item manifestum est, sphæræ diametrum esse compositam ex latere hexagoni, hoc est, ex semidiametro, & duobus lateribus decagoni circuli ambientis quinque latera icosaedri.

3. Constat denique latera icosaedri opposita,
a 33. 1. qualia sunt RX, HI esse parallela. Nam RX [a] pa-
b sch. 26. 3. rall, LP. [b] parall. HI.

PROP.

PROP. XVII.



Dodecaedrum constituere, & sphærâ complecti, quâ & prædictas figuras; & demonstrare, quòd dodecaedri latus RS *irrationalis est linea, quæ vocatur apotome.*

Sit AB cubus datæ sphæræ inscriptus, cujus latera omnia bisecentur in punctis E, H, F, G, K, L, &c. rectæque adjungantur KL, MH, HG, EF. [a] Fac HL. IQ : : IQ. QH; & sume NO, NP pares ipsi IQ. Erige OR, PS rectas plano DB, & QT plano AC, síntque OR, PS, QT ipsis IQ, NO, NP æquales. Connexis DR, RS, SC, CT, DT, erit DRSCT pentagonum Dodecaedri expetiti. Nam duc NV parall. OR, & protractâ NV ad occursum cum cubi centro X, connecte rectas DS, DO, DP, CR, CP, HV, HT, RX. Quia DOq [a] = DKQ ([b] KNq) + KOq [c] = 3 ONq (3 ORq) [d] erit DRq [d]

a 30. 6.

a 47. 1.
b 7. ax. 1.
c 4. 13.
d 47. 1.

e 4. 2. = 4 ORq [e] = OPq, vel RSq. ergò DR = RS. Simili argumento DR, RS, SC, CT, TP pa-
f constr. 9. 6. 11. res sunt. Quia verò OR [f] = [g] & parall. PS,
g 33. 1. [g] erunt RS, OP, & [h] consequenter RS, DC et-
h 9. 1. iam parallelæ; [h] ergò hæ cum suis conjungenti-
k 7. 11. bus DK, CS, VH in uno sunt plano. quinetiam
k constr. quia HI. IQ [k] : : IQ (TQ). QH [k] : : HN. NV; & tam TQ, HN, quàm QH, NV [k] re-
l 6. 11. ctæ eidem plano, [l] adeóq; & parallelæ existant,
m 32. 6. [m] erit THV recta linea. [n] ergò Trapezium
n 1. & 2. 11. DRSC, & triang. DTS in uno sunt plano per rectas DC, TV extenso. ergò DTCSR est pentagonum, & quidem æquilaterum ex antedi-
o 5. 13. ctis. Porrò, quia PK. KN : : KN. NP; &
p 47. 1. DSq [p] = DPq + PSq (PNQ) = [p] DKq + PKq
q 1. ax 2. + NPq, [q] erit DSq = DKq + 3 KNq = 4 DKq
& 4. 13. (4 DHq) [r] = DCq. ergò DS = DC; unde tri-
r 4. 2. gona DKS, DCT sibi mutuò æquilatera sunt.
s 8. 1. [s] ergò ang. DRS = DTC; & eodem pacto ang. CSR = DCT. ergò pentagonum DTCSR etiam æquiangulum est. Ad hæc, quia AX, DX, CX &c sunt cubi semidiametri, [t] erit XN =
t 15. 13.
u 1. ax. 1. IH, vel KN, [u] adeóq; XV = KP, unde ob angu-
x 29. 1. lum [x] rectum RVX, [z] erit RXq = XVq + RVq.
z 47. 1. (NPq) = KPq + NPq [a] = 3 KNq [b] =
a 4. 13. AXq, vel DXq &c. ergò RX, AX, DX, & ea-
b 15. 13. dem ratione XS XT, AX æquales sunt inter se. Et si eâdem methodo, quâ constructum est pentagonum DTCSR, fabricentur 12 similia pentagona tangentia duodecim cubi latera, ea Dodecaedrum constituent; ac per eorum puncta angularia transiens sphæra, cujus radius AX, uel RX Dodecaedrum complectetur. Q. E. F.

c constr. Denique, quia KN. NO [c] : : NO. OK, [d]
d 15. 5. erit KL. OP : : OP. OK + PL. Itaque si
e 15. 13. sphæræ diameter AB ponatur ρ, erit KL [e] = $\sqrt{\frac{AB}{3}}$
f sch. 12. 10. [f] etiam ρ. [g] unde OP, vel RS latus dodeca-
g 6. 13. edri apotome erit. Q. E. D.

Coroll.

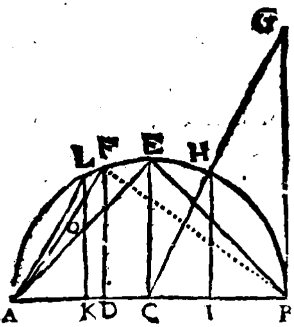
Coroll.

1. Hinc, si latus cubi secetur extremâ ,ac mediá ratione, majus segmentum erit latus dodecaedri in eadem sphæra descripti.

2. Si rectæ lineæ sectæ extremâ ac mediâ ratione, minus segmentum sit latus dodecaedri, majus segmentum erit latus cubi ejusdem sphæræ.

3. Liquet etiam latus cubi æquale esse lineæ rectæ subtendenti angulum pentagoni dodecaedri eâdem sphærâ comprehensi.

Prop. XVIII.



Latera quinq; figurarum exponere, & inter se comparare.

Sit AB diameter sphæræ, ac AEB semicirculus, sitq; AC [a] $= \frac{1}{2}$ AB & AD [b] $= \frac{1}{3}$ AB. Erige perpendiculares CE, DF, & BG = AB. junge AF, AE, BE, BF, CG ex H demitte perpendicularem HI, & sumptâ CK = CI, ex K erige perpendicularem KL, & connecte AL. Denique [c] fac AF. AO : : AO. OF.

a 10. 1.
b 10. 6.

Itaque 3. 2 [d] : : AB. BD [e] : : ABq. BFq, latus Tetraedri. & 2. 1 : : [a] AB AC : : ABq. BEq [f] latus Octaedri.

Item 3. 1. [d] : : AB. AD [e] : : ABq. AFq. [g] latus Hexaedri.

Porrò quia AF. AQ [h] : : AO. OF. [k] erit

c 30 6.
d constr.
e cor. 8. 6.
f 14. 13.
g 15. 13.
h constr.
k cor. 16. 13.

AO

l 4. 6. AO latus Dodecaedri. denique BG (2 BC).
m 14. 5. BC [l] :: HI. IC. [m] ergò HI = 2 CI [n] = KI.
n constr. ergò HIq [o] = 4 CIq. proinde CHq [p] = 5
o 4. 2. CIq. [q] ergò ABq = 5 KIq. [r] itaque KI, vel HI
p 47. 1. est *radius* circuli circumscribentis pentagonum
q 15. 5. icosaedri, & AK, vel IB [s] est latus *decagoni* ei-
r cor. 16. 13. dem circulo inscripti. unde AL erit latus pen-
f 10. 13. tagoni, [t] idémque Icosaedri latus. Ex quibus li-
t 16. 13. quet BF, BE, AF esse ρ̂ ⊤⊤. & AL, AO esse ρ̂. ⊤⊤; atque BF $\sqsubset$ BE; & BE $\sqsubset$ AF; ac AF $\sqsubset$
u 1. 6. AO. Quia verò 3 AFq = [u] ABq [x] = 5 KLq. ac
x 4. ax. 1. AF × AO $\sqsubset$ AF × OF, [x] ideóque AF × AO
y 2. 2. + AF × OF $\sqsubset$ 2 AF × OF, [y] hoc est AFq
z 17. 6. $\sqsubset$ [z] 2 AOq. [a] erit 3 AFq (5 KLq) $\sqsubset$ 6 AOq.
a 47. 1. proinde KL $\sqsubset$ AO; & fortiùs AL $\sqsubset$ AO.

Jam verò ut hæc latera numeris exprimamus, Si AB ponatur $\sqrt{60}$, erit ex jam dictis ad calculum exactis. BF $= \sqrt{40}$. & BE $= \sqrt{30}$. & AF $= \sqrt{20}$. item AL $= \sqrt{:}\ 30 - \sqrt{180}$ (nam AK $= \sqrt{15} - \sqrt{3}$. & KL (HI) $= \sqrt{12}$) denique AO $= \sqrt{:}\ 30 - \sqrt{500}$ ($\sqrt{25} - \sqrt{5}$.).

SCHOL.

SCHOL.

Præter jam dictas figuras nullam dari posse figuram solidam regularem (nempe quæ figuris planis ordinatis & æqualibus contineatur) admodum perspicuum est. Nam ad anguli solidi constitutionem requiruntur ad minimum tres anguli plani;[a] hiq; omnes simul 4 rectis minores esse debent.[b] Atqui 6 anguli trigoni æquilateri, 4 quadratici, & 3 hexagonici sigillatim 4 rectos exæquant; quatuor verò pentagonici, 3 heptagonici, 3 octagonici, &c. 4 rectos excedunt. ergò solummodo ex 3, 4, vel 5 triangulis æquilateris, ex 3 quadratis, vel 3 pentagonis effici potest angulus solidus. Proinde præter quinque prædicta, nulla existere possunt corpora regularia.

a 21. 11.
b *Vid. schol.* 32. 1.

Ex P. Herigonio.

Proportiones sphæræ, & 5 figurarum regularium eidem inscriptarum.

Sit diameter sphæræ 2, Erunt

Area circuli majoris, 6.|28318.

Superficies circuli majoris, 3|14159.

Superficies sphæræ, 12|56637.

Soliditas sphæræ, 4|1879.

Latus tetraedri, 1|62299.

Superficies tetraedri, 4 | 6188.

Soliditas tetraedri, 0 | 15132.

Latus hexaedri, 1 | 1547.

Superficies hexaedri, 8.

Soliditas hexaedri, 1 | 5396.

Latus octaedri, 1 | 41421.

Superficies octaedri, 6 | 9282.

Soliditas octaedri, 1 | 33333.

Latus dodecaedri, 0 | 71364.

Superficies dodecaedri, 10 | 51462.

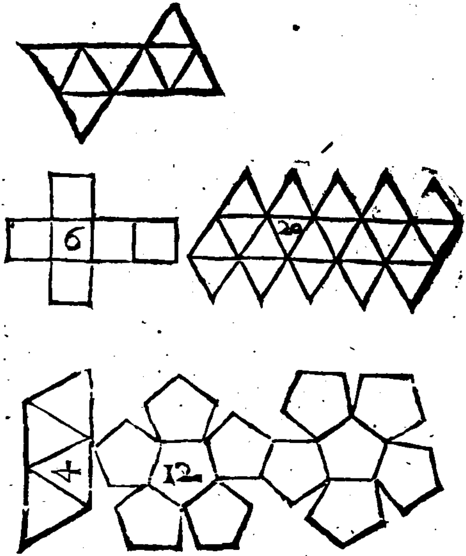
Soliditas dodecaedri, 2 | 78516.

Latus Icosaedri, 1 | 05146.

Superficies Icosaedri, 9 | 57454.

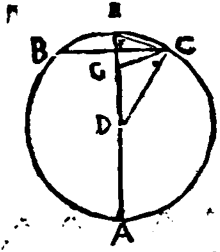
Soliditas Icosaedri, 2 | 53615.

Quòd ſi ex charta conficiantur quinque figuræ æquilateræ & æquiangulæ ſimiles his, quæ ſunt in ſubjecta figura, componentur quinque figuræ ſolidæ, ſi ritè complicentur.



LIB. XIV.

Prop. I.

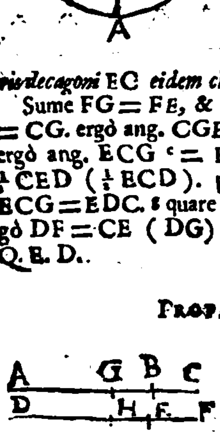


Quæ ex D centro circuli cujuspiã ABC in pentagoni eidem circulo inscripti latus BC ducitur perpendicularis DF, dimidia est utriusque lineæ simul, & lateris hexagoni DE, & lateris decagoni EC eidem circulo ABC inscripti.

a 4. 1.
b 5. 1.
c 32. 1.
d *hyp.* & 33. 6.
e 20. 3.
f 7. *ax.*
g 6. 1.

Sume $FG = FE$, & duc CG. [a] Estque $CE = CG$. ergò ang. CGE [b] $= CEG$ [b] $= ECD$. ergò ang. ECG [c] $= EDC$ [d] $= \frac{1}{4} ADC$ [e] $= \frac{1}{2} CED$ ($\frac{1}{2} ECD$). proinde ang. $GCD = ECG = EDC$. [g] quare $DG = GC$ (CE). ergò $DF = CE$ (DG) $+ EF = \frac{DE + CE}{2}$. Q.E.D.

Prop. II.



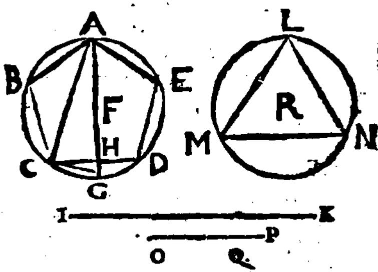
Si binæ rectæ lineæ AB, DE *extremâ ac mediâ ratione secentur* (AB. AG :: AG. GB. & DE. DH :: DH. HE) *ipsæ similiter secabuntur, in easdem scilicet proportiones.* (AG. GB :: DH. HE.)

a 17. 6.
b 8. 2.
c 2. *ax.* 1.
d 22. 5. & 22. 6.

Accipe $BC = BG$; & $EF = EH$. Estque $AB \times BG$ [a] $= AGq$. quare ACq [b] $= 4\, ABG + AGq$ [c] $= 5\, AGq$. Similiter erit $DFq = 5\, DHq$. [d] ergò $AC.\ AG :: DF.\ DH$, componendo igitur $AC + AG.\ AG :: DF + DH.\ DH$.

DH. hoc est 2 AB. AG :: 2 DE. DH. [e] pro- e 22. 5.
inde AB. AG :: DE. DH. unde [f] dividendo f 17. 5.
AG. GB :: DH. HE. Q. E. D.

Prop. III.



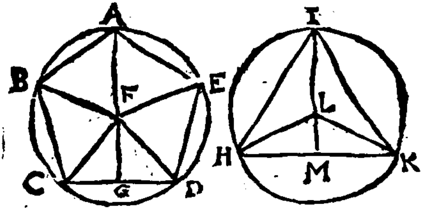
Idem circulus ABD *comprehendit & Dodecaedri pentagonum* ABCDE, *& Icosaedri triangulum* LMN, *eidem sphæræ inscriptorum.* a *sch.* 47. 1.

Duc diametrum AG, rectásque AC, CG. b 30. 6.
Sítque IK diameter sphæræ,[a] & IKq = 5 OPq. c 47. 1.
[b] fiátque OP. OQ :: OQ. QP. Quia ACq d 4. 2.
+ CGq [c] = AGq [d] = 4 FGq; & ABq [e] = e 10. 13.
FGq + CGq. [f] erit ACq + ABq = 5 FGq. f 2, & 3. *ax.*
porrò, quia CA. AB [g] :: AB. CA − AB; ac g 8. 13.
OP. OQ :: OQ. QP. [h] ideóque CA. OP :: h 2. 13. & 16. 5.
AB. OQ. [k] erit 3 ACq ([l] IKq). 5 OPq k 22. 6. & 4. 5.
([m] IKq) :: 3 ABq. 5 OQq. ergò 3 ABq = 5 l 15. 13.
OQq. Verùm ob ML [n] latus pentagoni circu- m *constr.*
lo inscripti, cujus radius OP, erunt [o] 15 RMq n *cor.* 16. 13.
[p] = 5 MLq [p] = 5 OPq + 5 OQq = [*] 3 o 12. 13.
ACq + 3 ABq [q] = 15 FGq. [r] ergò RM p 10. 13.
= FG. [s] proinde circ. ABD = circ. LMN. q 15. 5.
Q. E. D. * *Prius.*

r 1. *ax.* 1. & *sch.* 48. 1.
s 1. *def.* 3.

Prop.

PROP. IV.



Si ex F centro circuli pentagonum dodecaedri ABCDE *circumscribentis ducatur perpendicularis* FG *ad pentagoni unum latus* CD; *Erit quod sub dicto latere* CD, *& perpendiculari* FG *comprehenditur rectangulum trigesies sumptum, icosaedri superficiei aequale.* item,

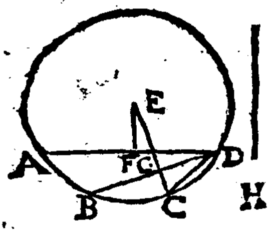
Si ex centro L *circuli triangulum icosaedri* HIK *circumscribentis, perpendicularis* LM *ducatur ad trianguli unum latus* HK, *erit quod sub dicto latere* HK; *& perpendiculari* LM *comprehenditur rectangulum trigesies sumptum, icosaedri superficiei aequale.*

a 8. 1. Duc FA, FB, FC, FD, FE. [a] Erunt triangula CFD, DFE, EFA, AFB, BFC aequalia.
b 41. 1. atqui $CD \times FG$ [b] $=$ 2 triang. CFD. ergò 30
c 15. 5. $CD \times GF$ [c] $=$ 60 CFD [d] $=$ 12 pentag. ABCDE [e] $=$
d 6. ax. superf. dodecaedri. Q. E. D.
e 17. 3.
f 41. 1. Duc LI, LH, LK. éstque $HK \times LM$ [f] $=$ 2
g 15. 5. triang. LHK. ergò 30 $HK \times LM$ [g] $=$ 60 HLK
h 16. 13. $=$ 20 HIK [h] $=$ superfic. icosaedri. Q. E. D.

Coroll.

k 15. 5. $CD \times FG . HK \times LM$ [k] :: superfic. dodecaed. ad superf. icosaedri.

Prop. V.



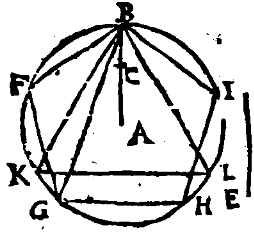
Superficies dodecaedri ad superficiem icosaedri in eadem sphæra descripti eandem proportionem habet, quam H *latus cubi ad* AD *latus icosaedri.*

Circulus ABCD [a] circumscribat tam dodecaedri pentagonum, quàm icosaedri triangulum; quorum latera BD, AD; ad quæ demittantur ex E centro perpendiculares EF, EGC. & connectatur CD. a 3.14.

Quoniam $EC + CD. EC$ [b] :: $EC.CD$. erit b 9.13.
EG ([c] $\frac{1}{2}$ $EC + CD$). EF. ([d] $\frac{1}{2}$ EC) [e] :: EF. c 1.14.
$EG - EF$ ($\frac{1}{2}$ CD). atqui $H. BD$ [f] :: $BD.H -$ d cor.12.13.
BD. [g] ergò $H. BD$ :: $EG. EF$. proinde $H \times EF$ e 15.5.
$= BD \times EG$. quum igitur $H. AD$ [h] :: $H \times EF$. f cor. 17.13.
$AD \times EF$. erit $H. AD$ :: $BD \times EG$. $AD \times EF$ g 2.14. h 1.6.
:: [l] superfic. dodecaedri ad superfic. icosaedri. k 7.5.
Q.E.D. l cor. 4.14.

Prop.

PROP. VI.



Si recta linea AB *secetur extremâ ac mediâ ratione; erit ut recta* BF *potens id, quod à tota* AB, *& id quod à majori segmento* AC *ad rectam* E, *potentem id quod à tota* AB, *& id quod à minori segmento* BC; *ità latus cubi* BG *ad latus icosaedri* BK *eidem sphæra cum cubo inscripti.*

Circulo, cujus semidiameter AB, inscribantur dodecaedri pentagonum BFGHI, & icosaedri
a cor. 17.13. triangulum BKL. [a] quare BG latus cubi erit ei-
b 12.13. dem sphæræ inscripti. igitur BKq [b] $=$ 3 ABq;
c 4.13. & Eq [c] $=$ 3 ACq. ergò BKq. Eq [d] :: ABq. ACq
d 15.5. [e] :: BGq. BFq. permutando igitur BGq. BKq ::
e 2.14. BFq. Eq. [f] unde BG. BK :: BF. E. Q.E.D.
f 22.6.

PROP. VII.

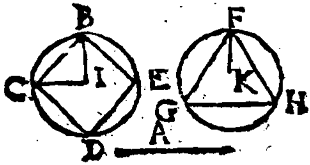
Dodecaedrum est ad Icosaedrum, ut cubi latus ad latus Icosaedri, in una eadémque sphæra inscripti.

a 3.14. Quoniam [a] idem circulus comprehendit & dodecaedri pentagonum & icosaedri triangulum,
b 47.1. [b] erunt perpendiculares à centro sphæræ ad plana pentagoni & trianguli ductæ inter se æquales. itaque si dodecaedrum & icosaedrum intelligantur esse divisa in pyramides, ductis rectis à centro sphæræ ad omnes angulos, omnium pyramidum altitudines erunt inter se æquales.
c 5 & 6.12. Cùm igitur pyramides æquè altæ [c] sint ut bases, & superficies dodecaedri sit æqualis 12 pentagonis, superficies verò icosaedri 20 triangulis;

erit

erit dodecaedrum ad icosaedrum, ut superficies dodecaedri ad superficiem icosaedri, [d] hoc est, ut latus cubi ad latus icosaedri. d 5. 14.

Prop. VIII.

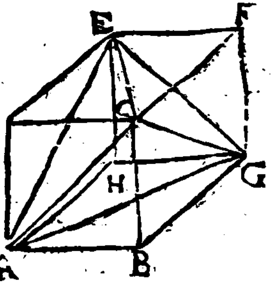


Idem circulus BCDE *comprehendit & cubi quadratũ* BCDE *& octaedri triangulũ* FGH, *ejusdem sphæræ.*

Sit A diameter sphæræ. Quoniam Aq [a] $= 3$ BCq [b] $= 6$ BIq; itémque Aq [c] $= 2$ GFq. [d] $= 6$ KFq; erit $BI = KF$. [e] ergò circulus $CBED = GFH$. Q. E. D.

a 15. 13.
b 47. 1.
c 14. 13.
d 12. 13.
e 2. def. 3.

PROP. I.



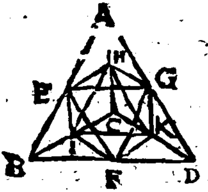
IN dato cubo ABGHDCFE *pyramidem* AGBC *describere.*

Ab angulo C duc diametros CA, CG, CE; Easque connecte diametris AG, GE, EA. Hæ omnes inter se [a] æquales sunt, utpote æqualium quadratorum diametri. ergò triangula CAG, CGE, CEA, EAG æquilatera sunt, ac æqualia: proinde AGEC est pyramis, quæ cubi angulis insistit, eique idcirco [b] inscribitur. Q. E. F.

a 47. 1.

b 31. def. 11.

PROP. II.

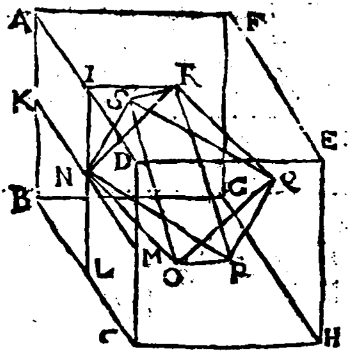


In data pyramide ABDC *octaedrum* EGKIFH *describere.*

[a] Biseca latera pyramidis in punctis E, I; F, K, G, H quæ connecte 12 rectis EF, FG, GE &c. Hæ omnes [b] æquales sunt inter se. proinde 8 triangula EHI, IHK, &c. æquilatera sunt & æqualia, adeóque constituunt [c] octaedrum [d] in data pyramide descriptum. Q. E. F.

a 10. 1.
b 4. 1.
c 27. def. 11.
d. 31. def. 11.

PROP. III.

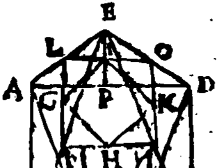


In dato cubo CHGBDEFA *octaedrum* NPQSOR *describere.*

Connecte quadratorum * centra N, P, Q, S, O, R, 12 rectis NP, PQ, QS &c. quæ [a] æqualia sunt inter se, ideóque 8 triangula efficiunt æquilatera & æqualia. proinde [b] inscriptum est cubo [b] Octaedrum NPQSOR. Q. E. F.

* 8. 4.
a 4. 1.
b 31. & 27. def. 11.

Prop. IV.



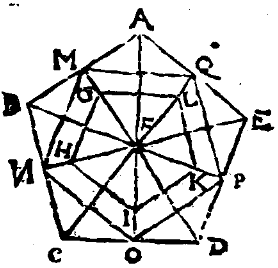
In dato Octaedro ABCDEF *cubum inscribere.*

Latera pyramidis E-ABCD, *cujus basis.* quadratum ABCD, bisecentur rectis LM, MN, NO, OL; quæ [a] æquales sunt, & [b] parallelæ lateribus quadrati ABCD [c] ergò quadrilaterum LMNO est quadratum. Eodem modo, si latera quadrati LMNO bisecentur in punctis G, H, K, I, & connectantur GH, HK, KI, IG, erit GHKI quadratum. Quòd si eadem arte in reliquis 5 pyramidibus octaedri centra triangulorum rectis conjungantur, describentur quadrata similia & æqualia quadrato GHKI. quare sex hujusmodi quadrata cubum constituent, qui quidem intra octaedrum descriptus erit, [d] cùm octo ejus anguli tangant octo octaedri bases in earum centris. Q. E. F.

a 4. 1.
b 2. 6.
c 29. def. 1.
d 31. def. 11.

Prop.

PROP. V.



In dato Icosaedro Octaedrum inscribere.

Sit ABCDEF pyramis Icosaedri, cujus basis pentagonum ABCDE; centra autem triangulorum G, H, I, K, L; quæ connectantur rectis GH, HI, IK, KL, LG. Erit GHIKL pentagonum dodecaedri inscribendi. * 5. 4.

Nam rectæ FM, FN, FO, FP, FQ per centra triangulorum transeuntes [a] bisecant bases. [b] ergò rectæ MN, NO, OP, PQ, QM æquales sunt inter se. quinetiam FM, FN, FO, FP, FQ [c] pares sunt. [d] ergò anguli MFN, NFO, OFP, PFQ, QFM æquantur. pentagonum igitur GHIKL æquiangulum est; [e] proinde & æquilaterum, cùm FG, FH, FI, FK, FL [f] pares sint. Quòd si eâdem arte in reliquis undecim pyramidibus icosaedri, centra triangulorum rectis lineis connectantur, describentur pentagona æqualia & similia pentagono GHIKL. quamobrem 12 hujusmodi pentagona dodecaedrum

a cor. 3. 3.
b 4. 1.
c 4. 1.
d 8. 1.
e 4. 1.
f 12. 13.

constituent; quod quidem in icosaedro erit descriptum, cùm viginti anguli dodecaedri in centris viginti basium icosaedri consistant. Quapropter in dato icosaedro dodecaedrum descripsimus. Q. E. F.

FINIS.