EUCLIDIS ELEMENTORUM

Libri xv. breviter demonstrati,

Operâ

IS. BARROW, CANTABRIGIENSIS *Coll.* TRIN. *Soc.*

HIEROCL.

Ὄργανα ψυχῆς λογικῆς εἰσιν αἱ μαθηματικαὶ ἐπιστῆμαι.

CANTABRIGIÆ,
Ex Celeberrimæ Academiæ Typographeo.
Impensis Guilielmi Nealand *Bibliopolæ.*
Anno Dom. MDCLV.

Benevolo Lectori.

SI quid in hac elementorum editione præstitum sit, scire desideras, amice Lector, accipe, pro genio operis, breviter. Ad duos præcipuè fines conatus meos direxi. Primùm ut cum requisita perspicuitate summam demonstrationum brevitatem conjungerem, quò eam libello molem compararem, quæ commodè absque molestia circumferri posset. Id quod assecutus videor, si absentem Typographi cura non si[illegible]retur. Concinniùs enim quispiam meliori ingenio, aut majori peritiâ excellens, at nemo forsan breviùs plerásque propositiones demonstraverit, præsertim cùm in numero & ordine propositionum ipse nihil immutârim, nec licentiam mihi assumpserim quamcunque propositionem Euclideam procul ablegandi tanquam minùs necessariam, aut quasdam faciliores in axiomatum censum referendi, quod nonnulli fecerunt; inter quos peritissimus Geometra A. *Tacquetus* C *quem ideò etiam nomino, (quòd quædam ex eo desumpta agnoscere honestum duco) post cujus elegantissimam editionem, ipse nihil atten-*

tare voluissem, si non visum fuisset doctissimo viro non nisi octo Euclidis *libros suâ curâ adornatos publico communicare, reliquis septem, tanquam ad elementa Geometriæ minùs spectantibus, omnino quasi spretis atque posthabitis. Mihi autem jam ab initio alia provincia demandata fuit, non elementa Geometriæ utcunque pro arbitrio conscribendi, verùm* Euclidem *ipsum, eúmque totum, quàm possem brevissimè, demonstrandi. Quod enim quatuor libros spectat, septimum, octavum, nonum, decimum, quamvìs illi ad Geometriæ plana & solidæ elementa, ut sex præcedentes, & duo subsequentes, non tam prope pertineant, quòd tamen ad res Geometricas admodum utiles sint, tam propter Arithmeticæ & Geometriæ valde propinquam cognationem, quàm ob notitiam commensurabilium & incommensurabilium magnitudinum ad figurarum tam planarum, quàm solidarum apprimè necessariam, nemo est è peritioribus Geometris qui ignorat. Quæ verò in tribus ultimis libris continetur, 5 corporum regularium nobilis contemplatio, illa non nisi injuriâ prætermitti potuit, quando nempe illius gratiâ noster στοιχειωτὴς, Platonicæ familiæ philosophus, hoc elementorum systema universum condidisse perhibetur,*

uti

testis est * Proclus, *iis verbis,* Ὅθεν * lib. 2.
καὶ τῆς συμπάσης στοιχειώσεως τέλος προεϛ-
σατο τὴν τῶν καλουμένων πλατωνικῶν σχη-
των σύστασιν. *Præterea facilè in ani-
um induxi ut opinarer, nemini harum
ientiarum amanti non futurum esse
rdi, penes se habere integrum Eucli-
um opus, quale passim ab omnibus ci-
tur, & celebratur. Quare nullum li-
um, nullámque propositionem negligere
olui earum, quæ apud* P. Herigonium
*ibentur, cujus vestigiis pressè insistere
cesse habui, quoniam ejusce libri sche-
atismis maximâ ex parte uti statutum
at, quòd prævidere m mihi ad novas
scribendas tempus non suppetere, etsi
nnunquam id facere præoptâssem. Ea-
m de causa nec alias plerásque quàm
uclideas demonstrationes adhibere vo-
i, succinctiori formâ expressas, nisi
rtè in 2, & 13, & parcè in 7, 8, 9 li-
ris, ubi ab eo nonnihil deflectere operæ
retium videbatur. Bona igitur spes est
altem in hac parte cùm nostris consiliis,
um studiosorum votis aliquo modo satis-
actum iri. Nam quæ adjecta sunt in
Scholiis problemata quædam & theore-
mata, sive ob suum frequentem usum ad
aturam elementarem accedentia, sive ad
orum, quæ sequuntur, expeditam demon-
strationem conducentia, seu quæ regula-*

rum

relatas, per ea, ut spero, libellus ultra destinatam molem magnopere non intumescet.

Alter scopus, ad quem collineatum est, eorum desideriis consuluit, qui demonstrationibus symbolicis potiùs quàm verbalibus delectantur. In quo genere cùm plerique apud nos Gulielmi Oughtredi *symbolis assueti sint, ea plerumque usurpare consultius duximus. Nam qui* Euclidem, *hâc viâ tradere & interpretari aggressus sit, hactenus, quod ego sciam, præter unum* P. Herigonium, *repertus est nemo. Cujus viri longè doctissimi methodus, sanè in multis egregia, ac ejus peculiari proposito admodum accommodata, duplici tamen defectu laborare mihi visa est. Primò, quòd cùm Propositionum ad unius alicujus theorematis aut problematis probationem adductarum, posterior à priori non semper dependeat; quando tamen illæ inter se cohærent, quando non, nec ex ordine singularum, nec ullo alio modo satìs promptè innotescere potest; unde ob defectum conjunctionum, & adjectivorum* ergò, rursus, *&c. non rarò difficultas & dubitandi occasio, præsertim minùs exercitatis, inter legendum oboriri solent. Deinde sæpe evenit, ut prædi-*

cta

cta methodus nimis frequenter supervacaneas repetitiones effugere nequeat, à quibus demonstrationes est quando prolixa, aliquando & magis intricata evadunt. Quibus vitiis noster modus facilè per verborum signorúmque arbitrariam mixturam medetur. Atque hæc de opellæ hujus intentione & methodo dicta sufficiant. Caterùm quæ in laudem Matheseos in genere, aut Geometriæ ipsius; & quæ de historia harum scientiarum, ideóque de Euclide *horum elementorum digestore dici possent, & reliqua hujusmodi* ἐξωτερικὰ*, cui hæc placent, apud alios interpretes consulere potest. Neque nos angustias temporis, quod huic operi impendi potuit, nec interpellationes negotiorum, nec adjumentorum ad hæc studia apud nos egestatem, & quædam alia, ut liceret non immeritò, in excusationem obtendemus, metu scilicet inducti, nè hæc nostra omnibus minùs satisfaciant. Verùm quæ ingenui Lectoris usibus elaboravimus, eadem in solidum ipsius censuræ ac judicio submittimus, probanda si utilia sibi compererit, sin omnino secus, rejicienda.*

J. B.

Ad amicissimum Virum *J. B.* de *EUCLIDE* contracto
'Ευφημισμός.

Factum bene! didicit Laconicè loqui
Senex profundus, & aphorismos induit.
Immensa dudum margo commentarii
Diagramma circuit minutum; útque Insula
Problema breve natabat in vasto mari.
Sed unda jam detumuit; & glossa arctior
Stringit Theoremata: minoris anguli
Lateribus ecce totus Euclides *jacet,*
Inclusus olim velut Homerus in nuce;
Pluteóque sarcina modo qui incubuit, levis
En fit manipulus. Pelle in exigua latet
Ingens Mathésis, matris ut in utero Hercules,
In glande quercus, vel Ithaca Eurus in pila.
Nec mole dum decrescit, usu fit minor,
Quin auctior jam evadit, & cumulatiùs
Contracta prodest erudita pagina.
Sic ubere magìs liquor è presso effluit;
Sic pleniori vasa inundat sanguinis
Torrente cordis Systole; sic fusiùs
Procurrit æquor ex Abylæ angustiis.
Tantilli operis ars tanta referenda unicè est
BAROVIANO nomini, ac solertiæ.
Sublimis euge mentis ingenium potens!
Cui invium nil, arduum esse nil solet.
Sic usque pergas prospero conamine,
Radiúsque multum debeat ac abacus tibi.
Sic crescat indies feracior seges,
Simili colonum germine assiduo beans.
Specimen futuræ messis hic siet labor,
Magnáque famæ illustria hæc prælusia.
Juvenis dedit qui tanta, quid dabit senex?

Car. Robotham, *CANTAB.*
Coll. Trin. Sen. Soc.

novam *Elementorum*
EUCLIDIS
nem, à D. *IS. BARROW*,
legii SS. TRIN. *Socio*,
viro opt. & eruditissimo
adornatam.

e Lector ! si uspiam auditum est tibi,
tus tenella Nix Geometres siet ;
radiis, mille ludit angulis,
puro ducit Euclidem sinu :
ltrò candidissimum Virum,
nivium est indoles, sed quas tamen
ardor mentis urget Enthea,
landis temperat caloribus :
us nil vivit, & melius nihil.
quentes pectore excutit nives,
indè spargit, en aliam tibi,
igne, è nivibus Geometriam !

G. C. *A. M.* C. E. S.

Sam Payne Christ Hospital

$=$ æqualitatem.
$\sqsubset$ majoritatem.
$\sqsupset$ minoritatem.
$+$ plùs, vel addendum esse.
$-$ minùs, vel subtrahendum esse.
$-:$ differentiam vel excessum; item qua
tates omnes, quæ sequuntur, subtrahe
esse, signis non mutatis.
$\times$ multiplicationem, vel ductum laterii
ctanguli in aliud latus.
Idem denotat conjunctio literarum,
$AB = A \times B$.
$\surd$ Latus, vel radicem quadrati, vel cubi,
Q. & q quadratum. C. & c cubum.
Q. Q. rationem quadrati numeri ad
dratum numerum.

Reliquas, quæ ubicunque occurrunt, vo abbreviationes ipse Lector per se facilè intel ceptis iis, quas tanquam minùs generalis usû cis explicandas relinquimus.

CANDIDE LECTOR, Quamvis in hac editi randa multum operæ insumpsimus, caveri tamen omnino it, nè irreperent σφάλματα. *Quorum summæ si sub tum quæ incuriæ & importunitati operarum, tum quæ Au nuscripto calamo festinante exarato debentur; reliqua, si restant, pro nostris libenter agnoscimus. In universum omnia nobis imputes, non ità multa sunt, ut illorum nos a pudeat, aut ut æquus Lector ea & uni, & homini difficulter*

aucula hæc, quæ temere aliquoties pagellas sparsim relege occurrebant, diligenter adnotes velim, aut si placet, calamo

Pag. 5. *lin.* 10. *pro* æquilateræ *lege* quadrilat. *p.* 13. *l.* 10 $\sqsubset$ *pone* $\sqsupset$ *in aliquibus exempl.* *p.* 21. *desunt figuræ pr* *as.* Prop. 24. *p.* 168. *l. penult. pro* Aq. *lege* AB. *p.* 31 *pro* AC. *l.* AD. *p.* 144. & 145 *pro* Lib. VI. *l.* Lib. V *tavo libro* : : *pro* $=$, *sed locum non memini.*

Sam Pryne Christ Hospitall

LIB. I.

Definitiones.

I. PUnctum est cujus pars nulla est.

II. Linea verò longitudo latitudinis expers.

III. Lineæ autem termini sunt puncta.

IV. Recta linea est, quæ ex æquo sua interjacet puncta.

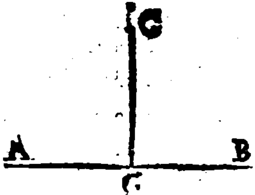
V. Superficies est, quæ longitudinem, latitudinémque tantùm habet.

VI. Superficiei autem extrema sunt lineæ.

VII. Plana superficies est, quæ ex æquo suas interjacet lineas.

VIII. Planus verò angulus est, duarum linearum in plano se mutuò tangentium, & non in directum jacentiũ alterius ad alteram inclinatio.

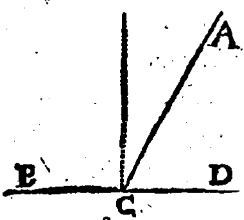
IX. Cùm autem quæ angulum continent lineæ, rectæ fuerint, rectilineus ille angulus appellatur.



X. Cùm verò recta linea C G super rectam lineam A B consistens, eos qui sunt deinceps angulos C G A, C G B æquales inter se fecerit, rectus est uterque æqualium angulorum, & quæ insistit recta linea C G, perpendicularis vocatur ejus (A B) cui insistit.

Not. *Cùm plures anguli ad unum punctum: (ut ad G) existunt, designatur quilibet angulus tribus literis, quarum media ad verticem est illius de quo agitur: ut angulus quem rectæ* CG, AG *efficiunt ad partes* A *vocatur* CGA, *vel* AGC.

B Obtusus



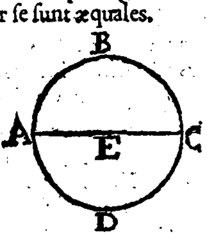
XI. Obtusus angulus est, qui recto major est, ut A C B.

XII. Acutus verò, qui minor est recto, ut A C D.

XIII. Terminus est, quod alicujus extremum est.

XIV. Figura est, quæ sub aliquo, vel aliquibus terminis comprehenditur.

XV. Circulus est figura plana, sub una linea comprehensa, quæ peripheria appellatur, ad quam ab uno puncto eorum, quæ intra figuram sunt posita, cadentes omnes rectæ lineæ inter se sunt æquales.



XVI. Hoc verò punctum centrum circuli appellatur.

XVII. Diameter autem circuli est recta quædam linea per centrum ducta, & ex utraque parte in circuli peripheriam terminata, quæ circulum bifariam secat.

XVIII. Semicirculus verò est figura, quæ continetur sub diametro, & sub ea linea, quæ de circuli peripheria aufertur.

In circulo EABCD. E *est centrum*, AC *diameter*, ABC *semicirculus*.

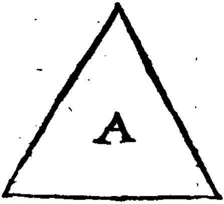
XIX. Rectilineæ figuræ sunt, quæ sub rectis lineis continentur.

XX. Trilateræ quidem, quæ sub tribus.

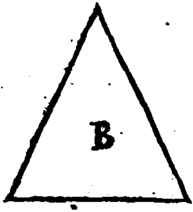
XXI. Quadrilateræ verò, quæ sub quatuor.

XXII. Multilateræ autem, quæ sub pluribus, quàm quatuor rectis lineis comprehenduntur.

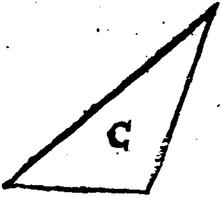
XXIII. Tri-



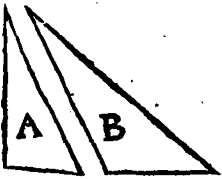
XXIII. Trilaterarum autem figurarum, æquilaterum est triangulum, quod tria latera habet æqualia, ut triangulum A.



XXIV. Isosceles autem, quod duo tantùm æqualia habet latera, ut triangulum B.

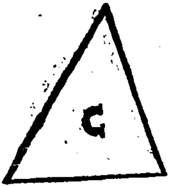


XXV. Scalenum verò, quod tria inæqualia habet latera, ut C.



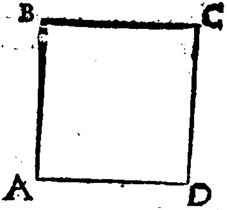
XXVI. Adhæc etiam trilaterarum figurarum, rectangulum quidē triangulum est, quod rectum angulum habet, ut triangulum A.

XXVII. Amblygonium autem, quod obtusum angulum habet, ut B.

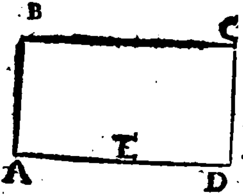


XXVIII. Oxygonium verò, quod tres habet acutos angulos, ut C.

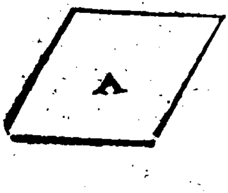
Figurà æquiangula eſt, cujus omnes anguli inter ſe æquales ſunt. Duæ verò figuræ æquiangulæ ſunt; ſi ſinguli anguli unius ſingulis angulis alterius ſint æquales. Similiter de figuris æquilateris concipe.



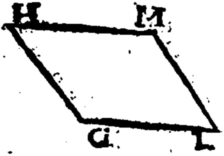
XXIX. Quadrilaterarum autem figurarum, quadratum quidem eſt, quod & æquilaterum, & rectangulum eſt, ut ABCD.



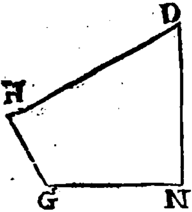
XXX. Alterà verò parte longior figura eſt, quæ rectangula quidem, at æquilatera non eſt, ut ABCD.



XXXI. Rhombus autem, quæ æquilatera, ſed rectangula non eſt, ut A.

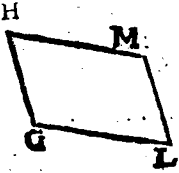


XXXII. Rhomboïdes verò, quæ adversa & latera, & angulos habens inter se æquales, neque æquilatera est, neque rectangula, ut GLMH.

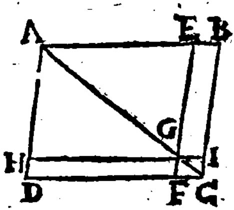


XXXIII. Præter has autem reliquæ æquilateræ figuræ trapezia appellentur; ut GNDH.

XXXIIII. Parallelæ rectæ lineæ sunt, quæ cùm in eodem sint plano, & ex utraque parte in infinitum producantur, in neutram sibi mutuò incidunt, ut A, & B.



XXXV. Parallelogrammum est figura quadrilatera, cujus bina opposita latera sunt parallela, seu æquidistantia, ut GLHM.



XXXVI. Cùm verò in parallelogrammo ABCD diameter AC ducta fuerit, duæq; lineæ EF, HI, lateribus parallelæ secantes diametrum in uno eodémq; puncto G, ita ut parallelogrammum ab hisce parallelis in quatuor distribuatur parallelogramma; appellantur duo illa D G, G B, per quæ diameter non transit, Complementa; duo verò reliqua H E, F I, per quæ diameter incedit, circa diametrum consistere dicuntur.

Problema est, *cùm proponitur aliquid efficiendum.*

Theorema est, *cùm proponitur aliquid demonstrandum.*

Corollarium est *consectarium, quod è facta demonstratione tanquam lucrum aliquod colligitur.*

Lemma est *demonstratio præmissæ alicujus, ut demonstratio quæsiti evadat brevior.*

Postulata.

1. POstuletur, ut à quovis puncto ad quodvis punctum rectam lineam ducere concedatur.

2. Et rectam lineam terminatam in continuum rectâ producere.

3. Item, quovis centro, & intervallo circulum describere.

Axiomata.

1. QUæ eidem æqualia, & inter se sunt æqualia.

Sunt A = B = C. ergò A = C, vel ergò omnes A, B, C æquantur inter se.

Nota, *Cùm plures quantitates hoc modo conjunctas invenias, vi hujus axiomatis primam ultimæ & quamlibet earum cuilibet æquari. Quo in casu sæpe, brevitatis causâ, ab hoc axiomate citando abstinemus; etsi vis consecutionis ab eo pendeat.*

2. Et si æqualibus æqualia adjecta sunt, tota sunt æqualia.

3. Et si ab æqualibus æqualia ablata sunt, quæ relinquuntur sunt æqualia.

4. Et si inæqualibus æqualia adjecta sint, tota sunt inæqualia.

5. Et si ab inæqualibus æqualia ablata sint, reliqua sunt inæqualia.

6. Et quæ ejusdem vel æqualium sunt duplicia, inter se sunt æqualia. Idem puta de triplicibus, quadruplicibus, &c.

7. Et quæ ejusdem, vel æqualium sunt dimidia, inter se sunt æqualia. Idem concipe de subtriplis, subquadruplis, &c.

8. Et quæ sibi mutuò congruunt, ea inter se sunt æqualia.

Hoc axioma in rectis lineis, & angulis valet conversum, sed non in figuris, nisi illæ similes fuerint.

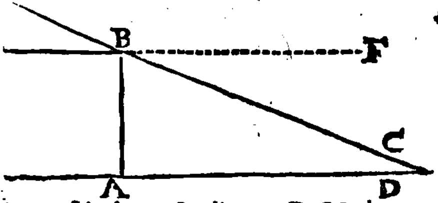
Cæterùm, magnitudines congruere dicuntur, quarum partes applicatæ partibus, æqualem vel eundem locum occupant.

9. Et totum suâ parte majus est.

10. Duæ rectæ lineæ non habent unum & idem segmentum commune.

11. Duæ rectæ in uno puncto concurrentes, si producantur ambæ, necessariò se mutuò in eo puncto intersecabunt.

12. Item omnes anguli recti sunt inter se æquales.



13. Et si in duas rectas lineas AD, CB, altera recta BA incidés, internos ad easdémq; partes angu-

los BAD, ABC duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ sibi mutuò incident ad eas partes, ubi sunt anguli duobus rectis minores.

14. Duæ rectæ lineæ spatium non comprehendunt.

15. Si æqualibus inæqualia adjiciantur, erit totorum excessus adjunctorum excessui æqualis.

16. Si inæqualibus æqualia adjungantur, erit totorum excessus excessui eorum, quæ à principio, æqualis.

17. Si ab æqualibus inæqualia demantur, erit residuorum excessus, excessui ablatorum æqualis.

18. Si ab inæqualibus æqualia demantur, erit residuorum excessus excessui totorum æqualis.

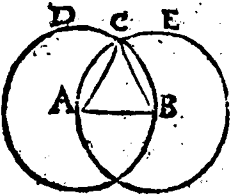
19. Omne totum æquale est omnibus suis partibus simul sumptis.

20. Si totum totius est duplum, & ablatum ablati, erit & reliquum reliqui duplum. Idem de reliquis multiplicibus intellige.

Citationes intellige sic. Cùm duo numeri occurrunt, prior designat propositionem, posterior librum. Ut per 4. 1. intelligitur quarta propositio primi libri, atque ità de reliquis. *Cæterùm, ax. axioma, post. postulatum, def. definitionem, sch. scholium, cor. corollarium denotant, &c.*

LIB. I.

PROP. I.



Super datâ rectâ lineâ terminatâ A B, triangulum æquilaterum A C B constituere.

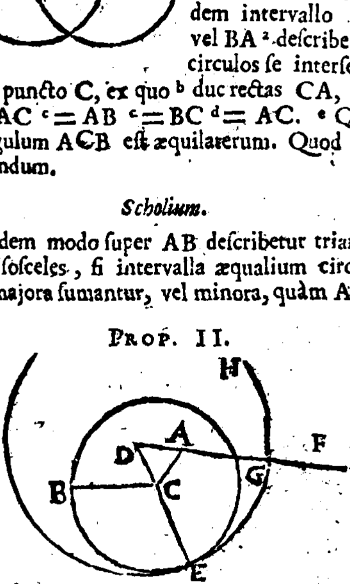
Centris A & B, eodem intervallo AB, vel BA [a] describe duos circulos se intersecantes in puncto C, ex quo [b] duc rectas CA, CB. Erit AC [c] = AB [c] = BC [d] = AC. [e] Quare triangulum ACB est æquilaterum. Quod Erat Faciendum.

a 3 post.
b 1 post.
c 15. def.
d 1. ax.
e 23. def.

Scholium.

Eodem modo super AB describetur triangulum Isosceles, si intervalla æqualium circulorum majora sumantur, vel minora, quàm AB.

PROP. II.



Ad datum punctum A datæ rectæ lineæ BC æqualem rectam lineam AG ponere.

Centro C, intervallo CB [a] describe circulum CBE. [b] Junge AC, super qua [c] fac triangulum æquilaterum ADC. [d] produc DC ad E,

a 3. post.
b 1. post.
c 1. 1.
d 2.

e 2 *post.* f 15. *def.* g *constr.* h 3. *ax.* k 15. *def.* l 1. *ax.*

centro D, spatio DE, describe circulum DEH: cujus circumferentiæ occurrat DA [e] protracta ad G. Erit AG = CB.

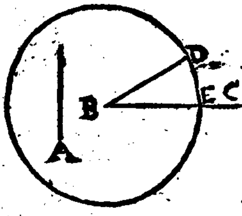
Nam DG [f] = DE, & DA [g] = DC, quare AG [h] = CE [k] = BC [l] = AG. Q. E. F.

Positio puncti A, intrà vel extrà datam BC, casus variat, sed ubique similis est constructio, & demonstratio.

Scholium:

Poterat AG circino sumi, sed hoc facere nulli postulato respondet, ut bene innuit Proclus.

PROP. III.



Duabus datis rectis lineis A, & BC, de majore BC minori A æqualem rectam lineam BE detrahere.

a 2. 1.

Ad punctum B [a] pone rectam BD = A. Circulus centro B, spatio BD descriptus auferet BE [b] = BD [c] = A [d] = BE. Q. E. F.

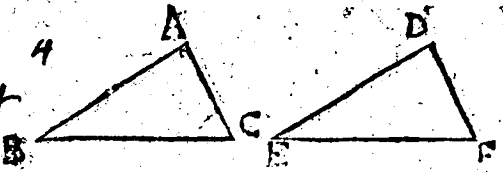
b 15. *def.* c *constr.* d 1. *ax.*

C

PROP. IV.

4

r



Si duo triangula BAC, EDF *duo latera* BA, AC *duobus lateribus* ED, DF *æqualia habeant, utrumque utrique* (hoc est BA = ED, & AC = DF) *habeant verò angulum* A, *angulo* D *æqualem,*

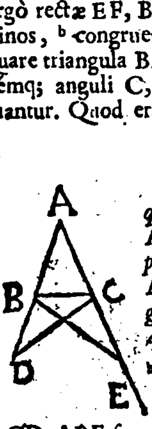
a

lem, sub æqualibus rectis lineis contentum, & basim BC *basi* EF *æqualem habebunt; eritque triangulum* BAC *triangulo* EDF *æquale, ac reliqui anguli* B, C *reliquis angulis* E, F *æquales erunt, uterque utrique, sub quibus æqualia latera subtenduntur.*

Si punctum D puncto A applicetur, & recta DE rectæ AB superponatur, cadet punctum E in B, quia DF[a] = AB. Item recta DF cadet in AC, quia ang. A[a] = D. Quinetiam punctum F puncto C coincidet, quia AC[a] = DF. Ergò rectæ EF, BC, cùm eosdem habeant terminos, [b] congruent, & proinde æquales sunt. Quare triangula BAC, EDF; & anguli B, E; itémq; anguli C, F etiam congruunt, & æquantur. Quod erat Demonstrandum.

a *hyp.*
b 14. *ax.*

Prop. V.



Isoscelium triangulorum ABC *qui ad basim sunt anguli* ABC, ACB *inter sé sunt æquales. Et productis æqualibus rectis lineis* AB, AC *qui sub base sunt anguli* CBD, BCE *inter se æquales erunt.*

[a] Accipe AF = AD, & [b] junge CD, ac BF.

Quoniam in triangulis ACD, ABF, sunt AB[c] = AC, & AF[d] = AD, angulúsq; A communis, [e] erit ang. ABF = ACD; & ang. AFB[e] = ADC, & bas. BF[e] = DC; item FC[f] = DB. ergò in triangulis BFC, BDC[g] erit ang. FCB, = DBC. Q. E. D. Item ideo ang. FBC = DCB. atqui ang. ABF[h] = ACD. ergò ang. ABC[k] = ACB. Q. E. D.

a 3. 1.
b 1. *post.*
c *hyp.*
d *constr.*
e 4. 1.
f 3 *ax.*
g 4. 1.
h *pr.*
k 3. *ax.*

Corollarium.

Hinc, Omne triangulum æquilaterum est quoq; æquiangulum.

PROP.

PROP. VI.

Si trianguli A B C *duo anguli* ABC, ACB *æquales inter se fuerint, & sub æqualibus angulis subtensa latera* AB, AC *æqualia inter se erunt.*

Si fieri potest, sit utravis BA ⊏ CA. [a] Fac igitur BD = CA, & [b] duc CD.

3 1. 1. *post.*

In triangulis DBC, ACB, quia BD [c] = CA, & latus BC commune est, atq; ang. DBC [d] = ACB, [e] erunt triangula DBC, ACB æqualia inter se, pars & totum, [f] Quod Fieri Nequit.

suppos. *hyp.* 4. 1. 9. *ax.*

Coroll.

Hinc, Omne triangulum æquiangulum est quoq; æquilaterum.

PROP. VII.

Super eadem recta linea AB *duabus eisdem rectis lineis* AC, BC, *aliæ duæ rectæ lineæ æquales* AD, BD, *utraque utrique (hoc est,* AD = AC, & BD = BC) *non constituentur ad aliud punctum* C, *atque aliud* D, *ad easdem partes* C, *eosdémque terminos* A, B *cum duabus initio ductis rectis lineis habentes.*

ax.

1. *Cas.* Si punctum D statuatur in AC, [a] liquet non esse AD = AC.

2. *Cas.* Si punctum D dicatur intra triangulum ACB, duc CD, & produc BDF, ac BCE. Jam vis AD = AC. ergò ang. ADC [b] = ACD; item quia BD [c] = BC, erit ang. FDC [b] = ECD. ergò

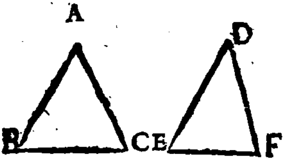
1. *hyp.*

ergò ang. F D C [d] $\sqsubset$ A C D, id est ang. FDC $\sqsubset$ ADC [d] Q. F. N. d 9. ax.

3. *Cas.* Sin D cadat extra triangulum ACB, jungatur CD.

Rursus, ang. BCD [e] = BDC, & BCD [e] = BDC. [f] ergò ang. ACD $\sqsubset$ BDC. & proinde multò magis ang. BCD $\sqsubset$ BDC. Sed erat ang. BCD = BDC. Quæ repugnant. Ergò, &c. e 5. 1. f 9. ax.

Prop. VIII.



Si duo triangula ABC, DEF *habuerint duo latera* AB, AC *duobus lateribus* DE, CE DF, *utrumque utriq; æqualia; habuerint verò & basim* BC, *basi* EF, *æqualem: angulum* A *sub æqualibus rectis lineis contentum angulo* D *æqualem habebunt.*

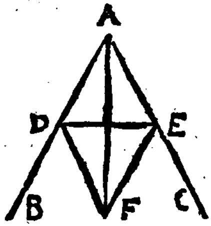
Quia BC [a] = EF, si basis BC superponatur basi EF, illæ [b] congruent. ergò, cùm AB [c] = DE, & AC [c] = D F, cadet punctum A in D. (nam in aliud punctum cadere nequit, per præcedentem) ergò angulorum A, & D latera coincidunt. [d] quare anguli illi pares sunt. Q. E. D. a hyp. b 8. ax. c hyp. d 8. ax

Coroll.

1. Hinc triangula sibi mutuò æquilatera, etiam mutuò [x] æquiangula sunt. x 4.

2. Triangula sibi mutuò æquilatera [y] æquantur inter se. y 4.

Prop. IX.



Datum angulum rectilineum BAC *bifariàm ſecare.*

1\. [a] Sume AD = AE; duc DE, ſuper quâ [b] fac triang. æquilat. DFE.

1\. Ducta A F angulum BAC biſecabit.

Nam AD [c] = AE, & latus A F commune eſt, & baſ. DF [c] = FE.

ſtr. [d] ergò ang. DAF = EAF. Q. E F.

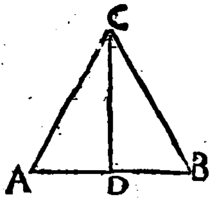
1\.

Coroll.

Hinc patet quomodo angulus ſecari poſſit in æquales partes 4, 8, 16, &c. Singulos nimirum partes iterum biſecando.

Methodus verò regulâ & circino angulos ſecandi in æquales quotcunq; hactenus Geometras latuit.

Prop. X.



Datam rectam lineam AB *bifariàm ſecare.*

1\. Super data AB [a] fac triang. æquilat. ABC. ejus angulum C [b] biſeca rectâ CD. Eadem datam

1\. AB biſecabit.

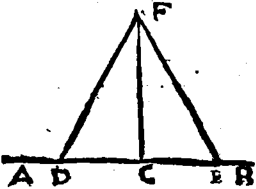
Nam A C [c] = B C, & latus CD eſt commune; & ang. A C D [c] =

ſtr. BCD; [d] ergò A D = B D. Q. E. F. Praxin

1\. hujus & præcedentis, conſtructio primæ hujus libri ſatis indicat.

Prop.

Prop. XI.



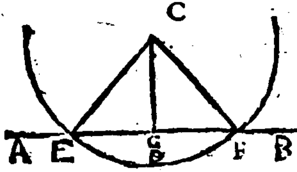
Datâ rectâ lineâ AB, & puncto in ea dato C, rectam lineam CF ad angulos rectos excitare.

[a] Accipe hinc indè CD = CE. Super DE [b] fac triang. æquilat. DFE. Ducta FC perpendicularis est. a 3. 1. b 1. 1.

Nam triangula DFC, EFC sibi mutuò [c] æquilatera sunt. [d] ergò ang. DCF = ECF. [e] ergò FC perpendicularis est. Q. E. F. c constr. d 8. 1. e 10. def.

Praxis tam hujus, quàm sequentis expeditur facillimè ope normæ.

Prop. XII.

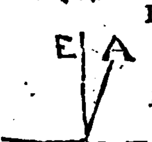


Super datam rectam lineam infinitam AB, à dato puncto C quod in ea non est, perpendicularem rectam CG deducere.

Centro C [a] describe circulum, qui secet datam AB in punctis E & F [b] biseca EF in G. ducta CG perpendicularis est. a 3. post. b 10. 1.

Ducantur enim CE, CF. Triangula EGC, FGC, sibi mutuò [c] æquilatera sunt. [d] ergò anguli EGC, FGC, æquales, & [e] proinde recti sunt. Q. E. F. c constr. d 8. 1. e 10. def.

Prop. XIII.



Cùm recta linea AB, super rectam lineam CD consistens, facit angulos ABC, ABD; aut duos rectos, aut duobus rectis æquales efficiet.

16. def. Si anguli ABC, ABD pares sint, [a] liquet illos rectos esse; sin inæquales sint, ex B [b] excite-
11. 1. tur perpendicularis BE. Quoniam ang. ABC [c] =
19. ax. Rect. + ABE; & ang. ABD [d] = Rect. — ABE;
3. ax. erit ABC + ABD [e] = 2 Rect. + ABE — ABE =
2. ax. 2 Rect. Q. E. D.

Coroll.

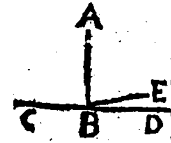
1. Hinc, si unus ang. ABD rectus sit, alter ABC etiam rectus erit; si hic acutus, ille obtusus erit, & contra.

2. Si plures rectæ quàm una ad idem punctum eidem rectæ insistant, anguli fient duobus rectis æquales.

3. Duæ rectæ invicem secantes efficiunt angulos quatuor rectis æquales.

4. Omnes anguli circa unum punctum constituti conficiunt quatuor rectos. patet ex Coroll. 2.

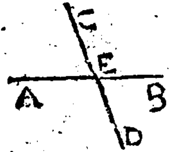
PROP. XIV.



Si ad aliquam rectam lineam AB, *atque ad ejus punctum* B *duæ rectæ lineæ* CB, BD *non ad easdem partes ductæ, eos qui sunt deinceps angulos* ABC, ABD *duobus rectis æquales fecerint, in directum erunt inter se ipsæ rectæ lineæ* CB, BD.

13. 1. Si negas, faciant CB, BE unam rectam, ergò
hyp. ang. ABC + ABE [a] = 2 Rect. [b] = ABC +
ax. ABD. [c] Quod Est absurdum.

PROP. XV.

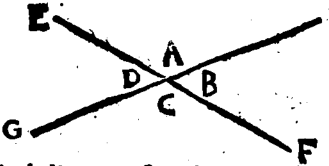


Si duæ rectæ lineæ AB, CD *se mutuò secuerint, angulos ad verticem* CEB, AED *æquales inter se efficient.*

Nam ang. AEC + CEB
. 1. [a] = 2 Rect. [a] = AEC + AED.
ax. [b] Ergò CEB = AED. Q. E. D.

Schol.

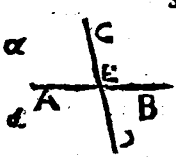
Schol.



Si ad aliquam rectam lineam GH, atque ad ejus punctum, A duæ rectæ lineæ EA, AF non ad easdem partes sumptæ, angulos ad verticem D, & B æquales fecerint, ipsæ rectæ lineæ EA, AF in directum sibi invicem erunt.

Nam 2 Rect. =[a] D + A [a]= B + A. [b] ergò EA, AF sunt in directum sibi invicem. Q. E. D. a 13. 1. b 14. 1.

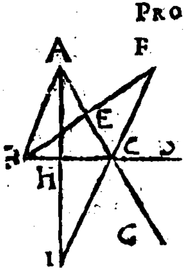
Schol. 2.



Si quatuor rectæ lineæ EA, EB, EC, ED ab uno puncto E exeuntes, angulos oppositos ad verticem æquales inter se fecerint, erunt quælibet duæ lineæ AE, EB, & CE, ED in directum positæ.

Nam quia ang AEC + AED + CEB + DEB [a] = 4 Rect. erit AEC + AED [b] = CEB + DEB = 2 Rect. [c] ergò CED, & AEB sunt rectæ lineæ. Q. E D. a 4 *Cor.* 13. b 13. 1. & 2. *ax.* c 14. 1.

Prop. XVI.



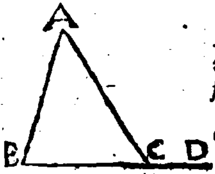
Cujuscunque Trianguli ABC *uno latere* BC *producto, externus angulus* ACD *utrolibet interno & opposito* CAB, CBA, *major est.*

Latera AC, BC [a] bisecent rectæ AH, BE, quibus productis [b] cape EF = BE, [b] & HI = AH, Conjuganturq; FC, I. a 10. 1. & 1. *post.* b 3. 1.

 Quo-

c constr. Quoniam CE [c] = EA, & EF [c] = EB, &
d 15. 1. ang. FEC [d] = BEA; [e] erit ang. ECF = EAB.
e 4. 1. Simili argumento ang. ICH ([f] FCD) = ABH.
f 15. 1. ergò totus ACD [g] major est utrovis CAB, &
g 9. ax. ABC. Q. E. D.

PROP. XVII.

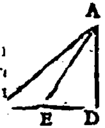


Cujuscunque trianguli ABC *duo anguli duobus rectis sunt minores, omnifariàm sumpti.*

Producatur latus BC. Quoniam ang. ACD +
a 13. 1. ACB [a] = 2 Rect. & ang.
b 16. 1. ACD [b] ⊏ A, [c] erit A + ACB ⊐ 2 Rect. Eo-
c 4 ax. dem modo erit ang. B + ACB ⊐ 2 Rect. Denique producto latere AB, erit similiter ang. A + B ⊐ 2 Rect. Quæ E. D.

Coroll.

1. Hinc, in omni triangulo, cujus unus angulus fuerit rectus, vel obtusus, reliqui acuti sunt.

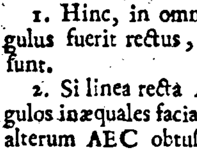


2. Si linea recta AE cùm alia recta CD angulos inæquales faciat, unum AED acutum, & alterum AEC obtusum, linea perpendicularis AD ex quovis ejus puncto A ad aliam illam CD demissa, cadet ad partes anguli acuti AED.

Nam si AC ad partes anguli obtusi ducta, dicatur perpendicularis; in triangulo AFC erit ang.
x 17. 1. AEC + ACB ⊏ 2 Rect. [x] Q. F. N.

3. Omnes anguli trianguli æquilateri, & duo anguli trianguli Isoscelis, supra basim, acuti sunt.

PROP. XVIII.



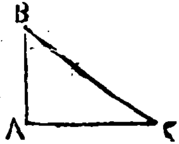
Omnis trianguli ABC *majus latus* AC *majorem angulum* ABC *subtendit.*

a 3. 1. Ex AC [a] aufer AD =
b 5. 1. AB, & junge DB. [b] ergò ang. ADB = ABD. Sed

ADB

ADB $\sqsubset$ C. ergò ABD $\sqsubset$ C. [d] ergò totus c 16. 1.
ang. ABC $\sqsubset$ C. Eodem modo erit ABC $\sqsubset$ A. d 9. ax.
Q. E. D.

Prop. XIX.

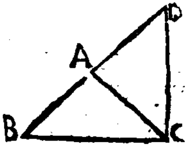


Omnis trianguli ABC *major angulus* A *majori lateri* BC *subtenditur.*

Nam si dicatur AB = BC, [a] erit ang. A = C. contra Hypoth. & si AB $\sqsubset$ BC, [b] erit ang. C $\sqsubset$ A, contra hyp. quare potiùs BC $\sqsubset$ AB. & eodem modo BC $\sqsubset$ AC. Q. E. D.

a 5. 1.
b 18. 1.

Prop. XX.

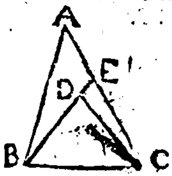


Omnis trianguli ABC *duo latera* BA, AC *reliquo* BC *sunt majora quomodocunque sumpta.*

Ex BA producta [a] cape AD = AC, & duc DC. [b] ergò ang. D = ACD. [c] ergò totus BCD $\sqsubset$ D [d] ergò BD ([e] BA + AC) $\sqsubset$ BC. Q. E. D.

a 3. 1.
b 5. 1.
c 9. ax.
d 19. 1.
e constr. & 2. ax.

Prop. XXI.

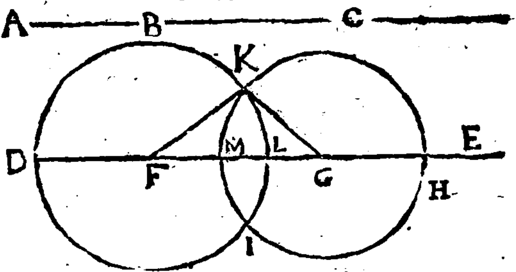


Si super trianguli ABC *uno latere* BC, *ab extremitatibus duæ rectæ lineæ* BD, CD, *interiùs constitutæ fuerint, hæ constitutæ reliquis trianguli duobus lateribus* BA, CA *minores quidem erunt, majorem verò angulum* BDC *continebunt.*

Producatur BD in E. éstq; CE + ED [a] $\sqsubset$ CD adde commune BD, [b] erit BE + EC $\sqsubset$ BD + DC. Rursus BA + AE [a] $\sqsubset$ BE; [b] ergò BA + AC $\sqsubset$ BE + EC. quare BA + AC $\sqsubset$ BD + DC. Q. E. D. 2. Ang. BDC [c] $\sqsubset$ DEC [c] $\sqsubset$ A. ergò ang. BDC $\sqsubset$ A. Q. E. D.

a 20. 1.
b 4. ax.
c 16. 1.

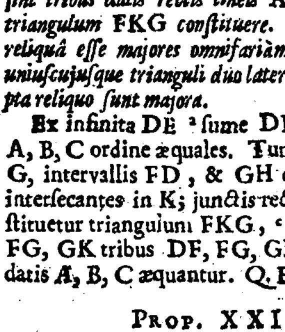
PROP. XXII.



Ex tribus rectis lineis FK, FG, GK, *quæ sint tribus datis rectis lineis* A, B, C *æquales, triangulum* FKG *constituere. Oportet autem duas reliquâ esse majores omnifariàm sumptas; quoniam uniuscujusque trianguli duo latera omnifariàm sumpta reliquo sunt majora.*

a 3. 1. Ex infinita DE [a] sume DF, FG, GH datis
b 3. post. A, B, C ordine æquales. Tum si [b] centris F, & G, intervallis FD, & GH ducantur circuli se intersecantes in K; junctis rectis KF, KG con-
c 15. def. stituetur triangulum FKG, [c] cujus latera FK,
d 1. ax. FG, GK tribus DF, FG, GH, [d] id est tribus datis A, B, C æquantur. Q. E. F.

PROP. XXIII.

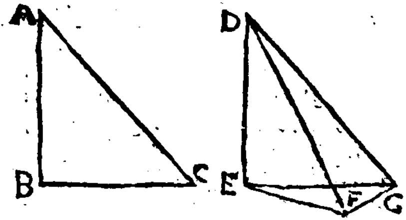


Ad datam rectam lineam AB, *datúmque in ea punctum* A, *dato angulo rectilineo* D *æquale angulum rectilineum* A *constituere.*

a 1. post. [a] Duc rectam CF secantem dati anguli latera
b 3. 1. utcunque. [b] Fac AG = CD. Super AG [c] con-
c 22. 1. stitue triangulum alteri CDF æquilaterum, ità

ut AH $=$ DF, & GH $=$ CF; & habebis ang. A[d] $=$ D. Q. E. F. d 8. 1.

PROP. XXIV.



Si duo triangula ABC, DEF *duo latera* AB, AC *duobus lateribus* DE, DF *æqualia habuerint, utrumque utrique; angulum verò* A *angulo* EDF *majorem sub æqualibus rectis lineis contentum, & basim* BC, *basi* EF, *majorem habebunt.*

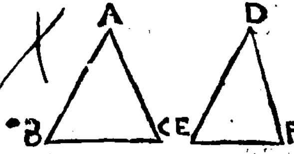
[a] Fiat ang. EDG $=$ A, & DG [b] $=$ DF [c] $=$ AC; connectantúrque EG, FG. a 23. 1. b 3. 1. c hyp.

1. *Cas.* Si EG cadit supra EF. Quia AB [d] $=$ DE, & AC $=$ [e] DG, & ang. A [e] $=$ EDG, [f] erit BC $=$ EG. Quia verò DF [c] $=$ DG, [g] erit ang. DFG $=$ DGF. [h] ergò ang. DFG $\sqsubset$ EGF; [h] & proinde ang. EFG $\sqsubset$ EGF. [k] quare EG (BC) $\sqsubset$ EF. Q. E. D. d hyp. e constr. f 4. 1. g 5. 1. h 9. ax. k 19. 1.

2. *Cas.* Si basis EF basi EG coincidat, [l] liquet EG (BC) $\sqsubset$ EF. l 9. ax.

3. Sin EG cadat infra EF. Quoniam DG + GE [m] $\sqsubset$ DF + FE, si hinc indè auferantur DG, DF, æquales, manet EG (BC) [n] $\sqsubset$ EF. Q. E. D. m 21. 1. n 5. ax.

Prop. XXV.

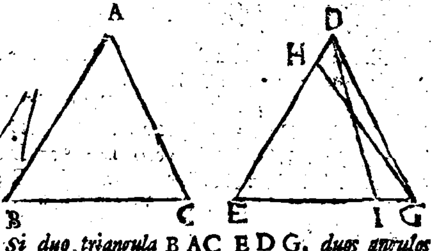


Si duo triangula ABC, DEF duo latera AB, AC duobus lateribus DE, DF æqualia habuerint, utrumq; utrique, basim vero BC basi EF majorem; & angulum A sub æqualibus rectis lineis contentum angulo D majorem habebunt.

a 4. 1. Nam si dicatur ang. A $=$ D. [a] erit basis BC $=$ EF, contra Hyp. Sin dicatur ang. A $\sqsupset$ D,

b 24. 1. [b] erit BC $\sqsupset$ EF, etiam contra Hyp. ergò BC $\sqsubset$ EF. Q. E. D.

Prop. XXVI.



Si duo triangula B AC E D G, *duos angulos* B, C, *duobus angulis* E, DGE, *æquales habuerint, utrumque utrique, unúmque latus uni lateri æquale, sive quod æqualibus adjacet angulis, seu quod uni æqualium angulorum subtenditur: reliqua latera reliquis lateribus æqualia, utrumque utrique, & reliquum angulum reliquo angulo æqualem habebunt.*

1. *Hyp.* Sit BC $=$ EG. Dico BA $=$ ED, & AC $=$ DG, & ang. A $=$ EDG. Nam si dicatur

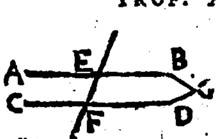
1. ED $\sqsubset$ BA, [a] fiat EH $=$ BA, ducatúrq; G H.

Quoniam

Quoniam AB [b] = HE, & BC [c] = EG, [c] & ang. B [c] = E, erit ang. EGH [d] = C [e] = DGE. [f] Q. E. A. ergò AB = ED. Eodem modo AC = DG. [d] quare etiam ang. A = EDG. — b *suppos.* c *hyp.* d 4. 1. e *hyp.* f 9. *ax.*

2. *Hyp.* Sit AB = ED. Dico BC = EG; & AC = DG & ang. A = EDG. Nam si dicatur EG ⊏ BC, fiat EI = BC, & connectatur DI. Quia AB [g] = ED, & BC [h] = EI; & ang. B [g] = E, erit ang. EID [k] = C [m] = EGD. [n] Q. E. A. ergò BC = EG. ergò ut priùs, AC = DG, & ang. A = EDG. Q. E. D. — g *hyp.* h *sup.* k 4. m *hy.* n 16.

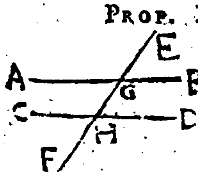
Prop. XXVII.



Si in duas rectas lineas AB, CD *recta incidens linea* EF *alternatim angulos* AEF, DFE, *æquales inter se fecerit, parallelæ erunt inter se illæ rectæ lineæ* AB, CD.

Si AB, CD dicantur non esse parallelæ; conveniant productæ, nempe in G. quo posito angulus externus AEF interno DFE [a] major erit, cui tamen ponitur æqualis. Quæ repugnant. — a 16. 1.

Prop. XXVIII.

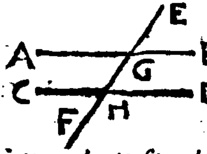


Si in duas rectas lineas AB, CD *recta incidens linea* EF *externum angulum* AGB *interno & opposito, & ad easdem partes* CHG *æqualem fecerit, aut internos & ad easdem partes* AGH, CHG *duobus rectis æquales; parallelæ erunt inter se ipsæ rectæ lineæ* AB, CD.

1. *Hyp.* Quia per hyp. ang. AGE = CHG, [a] erit altern. BGH = CHG. [b] parallelæ igitur sunt AB, CD. Q. E. D. — a 15. 1. b 27. 1.

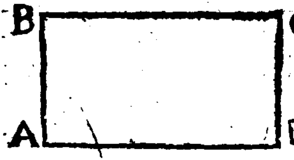
2. *Hyp.* Quia ex hyp. Ang. AGH + CHG = 2 Rect. [a] = AGH + BGH, [b] erit CHG = BGH. Ergò [c] AB, CD parallelæ sunt. Q. E. D. — a 13. 1. b 3. c

PROP. XXIX.



In parallelas rectas lineas AB, CD, *recta incidens linea* EF, *& alternatim angulos* DHG, AGH *æquales inter ſe efficit; & externum* BGE *interno, & oppoſito, & ad eaſdem partes* DHE *æqualem; & internos & ad eaſdem partes* AGH, CHG *duobus rectis æquales facit.*

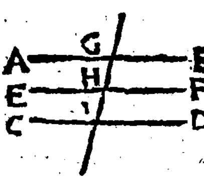
Liquet AGH, + CHG = 2 Rect. a aliàs AB, CD non eſſent parallelæ, contra hyp. Sed
b 13. 1. & ang. DHG + CHG b = 2 Rect. ergò DHG
c 13. ax. c = AGH d = BGE. Q. E. D.
d 15. 1. *Coroll.*



Hinc omne Parallelogrammum AC habens unum angulum rectum A, eſt rectangulum.

a 29. 1. Nam A + B a = 2 Rect. ergò cùm A re-
b 3. ax. ctus ſit, b etiam B rectus erit. Eodem argumento D, & C recti ſunt.

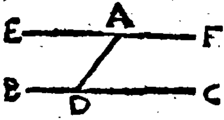
PROP. XXX.



Quæ (AB, CD) eidem rectæ lineæ EF *parallelæ, & inter ſe ſunt parallelæ.*

Tres rectas ſecet utcunq; recta GI. Quoniam AB, EF parallelæ ſunt,
a 29. 1. a erit ang. AGI = EHI, Item propter CD, EF
b 1. ax. parallelas, a erit ang. EHI = DIG. b ergò ang.
c 27. 1. AGI = DIG. c quare AB, CD parallelæ ſunt. Q. E. D.

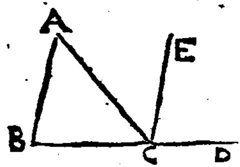
Prop. XXXI.



A dato puncto A *data recta linea* BC *ducere parallelam rectam lineam* AE.

Ex A ad datam BC duc rectam utcunque AD. ad quam, ejúsq; punctum a 23. 1.
A [a] fac ang. DAE = ADC. [b] erunt AE, BC b 27. 1.
parallelæ. Q. E. F.

Prop. XXXII.



Cujuscunque trianguli ABC *uno latere* BC *producto, externus angulus* ACD *duobus internis, & oppositis,* AB *est æqualis. Et trianguli tres interni anguli,* A, B, ACB *duobus sunt rectis æquales.*

Per C [a] duc CE parall. BA. Ang. A [b] = a 31. 1.
ACE. & ang. B [b] = ECD. ergò A + B [c] = b 29. 1.
ACE + ECD [d] = ACD. Q. E. D. Pono c 2. ax.
ACD + ACB [e] = 2 Rect. [f] ergò A + B + d 19. ax.
ACB = 2 Rect. Q. E. D. e 13. 1.
f 1. ax.

Corollaria.

1. Tres simul anguli cujusvis trianguli æquales sunt tribus simul cujuscunque alterius. Unde

2. Si in uno triangulo duo anguli (aut singuli, aut simul) æquales sint duobus angulis (aut singulis, aut simul) in altero triangulo, etiam reliquus reliquo æqualis est. Item, si duo triangula unum angulum uni æqualem habeant, reliquorum summæ æquantur.

3. In triangulo si unus angulus rectus sit, reliqui unum rectum conficiunt. Item, angulus, qui duobus reliquis æquatur, rectus est.

4. Cùm in Isoscele angulus æquis cruribus contentus rectus est, reliqui ad basim sunt semirecti.

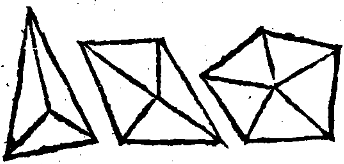
 5. Tri-

5. Trianguli æquilateri angulus facit duas tertias unius recti, nam $\frac{1}{3}$ 2 Rect. = $\frac{2}{3}$ Rect.

Schol.

Hujus propositionis beneficio, cujuslibet figuræ rectilineæ tam interni quàm externi anguli quot rectos conficiant, innotescet per duo sequentia theoremata.

THEOREMA 1.



Omnes simul anguli cujuscunque figuræ rectilineæ conficiunt bis tot rectos demptis quatuor, quot sunt latera figuræ.

Ex quovis puncto intra figuram ducantur ad omnes figuræ angulos rectæ, quæ figuram resolvent in tot triangula quot habet latera. Quare cùm singula triangula conficiant duos rectos, omnia simul conficient bis tot rectos, quot sunt latera. Sed anguli circa dictum punctum conficiunt quatuor rectos. Ergò, si ab omnium triangulorum angulis demas angulos circa id punctum, anguli reliqui qui componunt angulos figuræ conficient bis tot rectos demptis quatuor, quot sunt latera figuræ. Q. E. D.

Hinc *Coroll.* Omnes ejusdem speciei rectilineæ figuræ æquales habent angulorum summas.

THEOREMA 2.

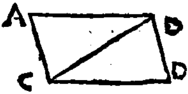
Omnes simul externi anguli cujuscunque figuræ rectilineæ conficiunt quatuor rectos.

Nam singuli figuræ interni anguli cum singulis externis conficiunt duos rectos. Ergò interni

terni ſimul omnes, cum omnibus ſimul externis conficiunt bis tot rectos, quot ſunt latera figuræ. Sed (ut modò oſtenſum eſt,) interni ſimul omnes etiam cum quatuor rectis efficiunt bis tot rectos, quot ſunt latera figuræ. Ergò externi anguli quatuor rectis æquantur. Q. E. D.

Coroll. Omnes cujuſcunque ſpeciei rectilineæ figuræ æquales habent externorum angulorum ſummas.

Prop. XXXIII.



Rectæ lineæ AC, BD, *quæ æquales & parallelas lineas* AB, CD, *ad partes eaſdem conjungunt, & ipſæ æquales ac parallelæ ſunt.*

Connectatur CB. Quoniam ob AB, CD parallelas, ang. ABC [a] = BCD, & per hyp. AB a 29. 1.
= CD, & latus CB commune eſt, [b] erit AC = b 4. 1.
BD, [b] & ang. ACB = DBC. [c] ergò AC, BD c 27. 1.
etiam parallelæ ſunt. Q. E. D.

Prop. XXXIV.

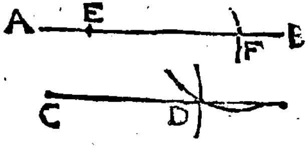
Parallelogrammorum ſpatiorum ABDC *æqualia ſunt inter ſe quæ ex adverſo latera* AB, CD; ac AC, BD; *angulique* A, D, & ABD, ACD; *& illa bifariàm ſecat diameter* CB.

Quoniam AB, CD [a] parallelæ ſunt, [b] erit a hyp.
ang. ABC = BCD. Item ob AC, DB [a] paral- b 29. 1.
lelas, [b] erit ang. ACB = CBD. [c] ergò toti an- c 2. ax.
guli ACD, ABD æquantur. Similiter ang. A = D. Porrò, cùm communi lateri CB adjaceant anguli ABC, ACB, ipſis BCD, CBD pares [d], erunt AC = BD, [d] & AB = CD. ade- d 26. 1.
óq; etiam triang. ABC = CBD. Quæ E. D.

SCHOL.

Omne quadrilaterum ABDC *habens latera opposita æqualia, est parallelogrammum.*

a 27. 1. Nam per 8. 1. ang. ABC = BCD. [a] ergò AB, CD parallelæ ſunt. Eâdem ratione ang. BCA = CBD; [a] quare AC, BD etiam paralle-
b 35. def. 1. læ ſunt. [b] Ergò ABDC eſt parallelogrammum. Q. E. D.



Hinc expeditiùs per datum punctum C datæ rectæ AB ducetur parallela CD.

Sume in AB quodvis punctum E. centris E, & C ad quodvis intervallum duc æquales circulos EF, CD. centro verò F, ſpatio EC duc circulum FD, qui priorem CD ſecet in D. Erit ducta CD parall. AB. Nam ut modò demonſtratum eſt, CEFD eſt parallelogrammum.

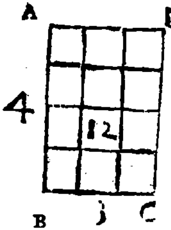
PROP. XXXV.

Parallelogramma BCDA, BCFE *ſuper eâdem baſi* BC, *& in eiſdem parallelis* AF, BC *conſtituta, inter ſe ſunt æqualia.*

a 34. 1. Nam AD [a] = BC [a] = EF. adde commu-
b 2. ax. nem DE. [b] erit AE = DF. Sed & AB [a] = DC;
c 29. 1. & ang. A [c] = CDF, [d] ergò triang. ABE =
d 4. 1. DCF. aufer commune DGE, [e] erit Trapez.
e 3. ax. ABGD = EGCF. adde commune BGC, [f] erit
f 2. ax. Pgr. ABCD = EBCF. Q. E. D. Reliquorum caſuum non diſſimilis, ſed ſimplicior & facilior eſt demonſtratio.

Scholium.

Scholium.

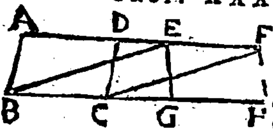


Si latus AB parallelogrammi rectanguli ABCD ferri intelligatur perpendiculariter per totam BC, aut BC per totam AB, producetur eo motu area rectanguli ABCD. Hinc rectangulum fieri dicitur ex ductu seu multiplicatione duorum laterum contiguorum. Sit exempl. gr. BC pedum 3, AB 4. Duc 3 in 4; proveniunt 12 pedes quadrati pro area rectanguli.

Hoc supposito, ex hoc theoremate cujuscunq; parallelogrammi (*EBCF) habetur dimensio. Illius enim area producitur ex altitudine BA ducta in basim BC. Nam area rectanguli AC parallelogrammo EBCF æqualis, fit ex BA in BC. ergò, &c.

* v. fig. præc. prop. 35.

PROP. XXXVI.

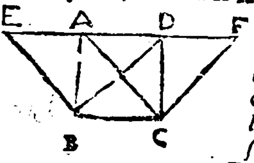


Parallelogramma BCDA, GHFE *super æqualibus basib⁹* BC, GH, *& in eisdem parallelis* AF, BH *constituta, inter se sunt æqualia.*

Ducantur BE, CF. Quia BC [a] = GH [b] = EF, [c] erit BCFE parallelogrammum. ergò Pgr. BCDA [d] = BCFE [d] = GHFE. Q.E.D.

a hyp.
b 34. 1.
c 33. 1.
d 35. 1.

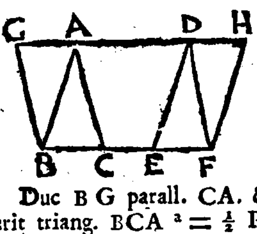
PROP. XXXVII.



Triangula BCA, BCD *super eadem basi* BC *constituta, & in eisdem parallelis* BC, EF, *inter se sunt æqualia.*

31. 1. [a] Duc BE parall. CA, [a] & CF parall. BD.
34. 1. Erit triang. BCA [b] $= \frac{1}{2}$ Pgr. BCAE $=$[c] $\frac{1}{2}$
35. 1. & ax. BDFC [b] $=$ BCD. Q. E. D.

Prop. XXXVIII.



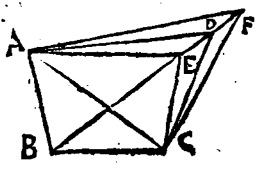
Triangula BCA, EFD *super æqualibus basibus* BC, EF *constituta, & in eisdem parallelis* GH, BF, *inter se sunt æqualia.*

Duc BG parall. CA. & FH parall. ED.
34. 1. erit triang. BCA [a] $= \frac{1}{2}$ Pgr. BCAG [b] $= \frac{1}{2}$
36. 1 & ax. BDHF [c] $=$ EFD. Q. E. D.

34. 1. *Schol.*

Si basis BC ⊏ EF, liquet triang. BAC ⊏ EDF. & si BC ⊐ EF, erit BAC ⊐ EDF.

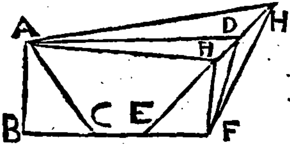
Prop. XXXIX.



Triangula æqualia BCA, BCD, *super eadem basi* BC, *& ad easdem partes constituta, etiam in eisdem sunt parallelis* AD, BC.

Si negas, sit altera AF parall. BC; & ducatur
37. 1. CF. ergò triang. CBF [a] $=$ CBA [b] $=$ CBD.
hyp. 9. ax. [c] Q. E. A.

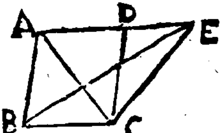
Prop. XL.



Triangula æqualia BCA, EFD *super æqualibus basibus* BC, EF, *&* *ad easdem partes constituta, & in eisdem sunt parallelis* AD, BF.

Si negas, sit altera AH parall. BF. & ducatur FH. ergò triang. EFH [a]= BCA [b]= EFD. [c]Q.E.A. a 38. 1. b *hyp.* c *9. ax.*

Prop. XLI.



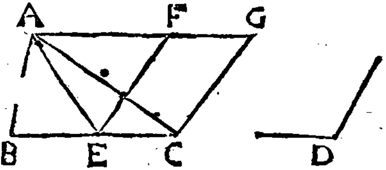
Si parallelogrammum ABCD *cum triangulo* BCE *eandem basim* BC *habuerit, in eisdémque fuerit parallelis* AE, BC, *duplum erit parallelogrammum* ABCD *ipsius trianguli* BCE.

Ducatur AC. Triang. BCA [a]= BCE. ergò Pgr. ABCD [b]= 2 BCA [c]= 2 BCE. Q.E.D. a 37. 1. b 34. 1. c 6. ax.

Scholium.

Hinc habetur area cujuscunq; trianguli BCE. Nam cùm area parallelogrammi ABCD producatur ex altitudine in basim ducta; producetur area trianguli ex dimidia altitudine in basim ducta, vel ex dimidia basi in altitudinem. ut si basis BC sit 8, & altitudo 7; erit trianguli BCE area, 28.

Prop. XLII.

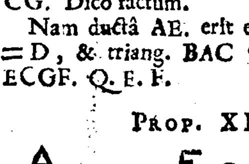


Dato triangulo ABC *æquale parallelogrammum* ECGF *constituere in dato angulo rectilineo* D.

31. 1. Per A [a] duc AG parall. BC. [b] fac ang. BCG
23. 1. = D. basim BC [c] biseca in E. [a] duc EF parall.
10. 1. CG. Dico factum.

Nam ductâ AE, erit ex constr. ang. ECG
38. 1. = D, & triang. BAC [d] = 2 AEC [e] = Pgr.
41. 1. ECGF. Q. E. F.

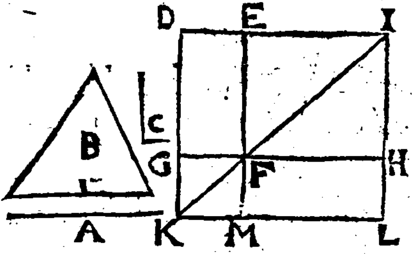
Prop. XLIII.



In omni parallelogrammo ABCD *complementa* DG, GB *eorum quæ circa diametrum* AC *sunt parallelogrammorum* HE, FI *inter se sunt æqualia.*

34. 1. Nam Triang. ACD, = [a] ACB. & triang.
3. ax. AGH [a] = AGB. & triang. GCF [a] = GCI.
[b] ergò Pgr. DG = GB. Q. E. D.

PROP. XLIV.

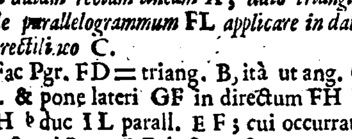


Ad datam rectam lineam A, *dato triangulo* B, *æquale parallelogrammum* FL *applicare in dato angulo rectilineo* C.

[a] Fac Pgr. FD = triang. B, ità ut ang. GFE = C. & pone lateri GF in directum FH = A. Per H [b] duc IL parall. EF; cui occurrat DE producta ad I. per IF ductæ rectæ occurrat DG protracta ad K. Per K [b] duc KL parall. GH; cui occurrant EF, & IH prolongatæ ad M, & L. Erit FL. Pgr. quæsitum. a 42. 1. b 31. 1.

Nam Pgr. FL [c] = FD = B [d] & ang. MFH = GFE = C. Q.E.F. c 43. 1. d 15. 1.

PROP. XLV.

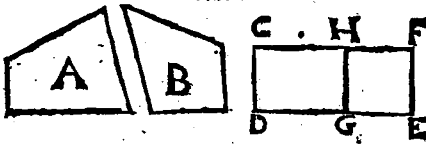


Ad datam rectam lineam FG *dato rectilineo* ABCD *æquale parallelogrammum* FL *constituere, in dato angulo rectilineo* E.

Datum rectilineum resolve in triangula BAD, BCD. [a] = Fac Pgr. FH = BAD. ita ut ang. F = E. productâ FI [a] fac (ad HI) Pgr.

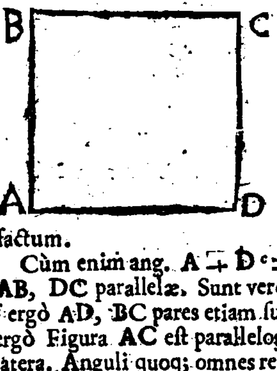
b 19. ax. c constr. IL = BCD. erit Pgr. FL = [b] FH + IL [c] = ABCD. Q. E. F.

Schol.



Hinc facilè invenitur excessus HE, quo rectilineum aliquod A superat rectilineum minus B; nimirum si ad quamvis rectam CD applicentur Pgr. DF = A. & DH = B.

Prop. XLVI.



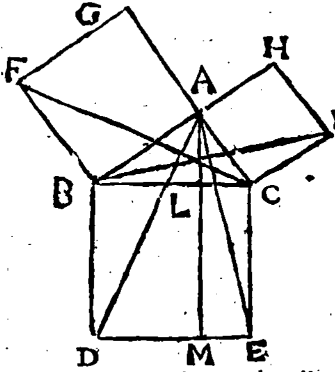
A data recta linea AD quadratum AC describere.

a 11. 1. b 3. 1. [a]Erige duas perpendiculares AB, DC [b] æquales datæ AD; & junge BC. dico factum.

c constr. d 28. 1. e constr. f 34. 1. g Sch. 29. 1. h 29. def. Cùm enim ang. A + D [c] = 2 Rect. [d] erunt AB, DC parallelæ. Sunt verò etiam [e] æquales. [f] ergò AD, BC pares etiam sunt, & parallelæ. ergò Figura AC est parallelogramma, & æquilatera. Anguli quoq; omnes recti sunt, [g] quoniam unus A est rectus. [h] ergò AC est quadratum. Q. E. F.

Eodem modo facilè describes rectangulum, quod sub datis duabus rectis contineatur.

Prop. XLVII.



In rectangulis triangulis B A C *quadratum* BE, *quod à latere* B C *rectum angulum* BAC *subtendente describitur, æquale est eis,* B G, CH, *quæ à lateribus* AB, A C *rectum angulum continentibus describuntur.*

Junge AE, AD; & duc AM. parall. CE.

Quoniam ang. DBC [a] = FBA; adde communem ABC, erit ang. ABD = FBC. Sed & AB [b] = F B, & BD [b] = BC. [c] ergò triang. ABD = FBC. atqui Pgr. B M. [d] = 2 ABD; & Pgr. BG [d] = 2 FBC (nam GAC est una recta per hyp. & 14. 1.) [e] ergò Pgr. BM = BG. Similì discursu Pgr. CM = CH. Totum igitur BE = BG + CH. Q. E. D.

a 12. ax.
b 29. d
c 4. 1.
d 41. 1
e 6. ax

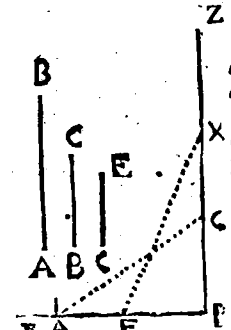
Schol.

Hoc nobilissimum, & utilissimum theorema ab inventore Pythagora, Pythagoricum dici meruit. Ejus beneficio quadratorum additio, & substractio perficitur; quò spectant duo sequentia problemata.

PROBL.

PROBL. 1.

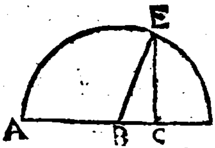
ndr. Tacq.



Datis quotcunque quadratis, unum omnibus æquale construere.

Dentur quadrata tria, quorum latera ſint AB, 11. 1. BC, CE. [a] Fac ang. rectum FBZ infinita habentem latera, in eáque transfer BA, & BC, & 47. 1. junge AC, [b] erit ACq = ABq + BCq. Tum AC transfer ex B in X; & CE tertium latus da- 1. ax. tum transfer ex B in E, & junge EX, [b] erit EXq = EBq (CEq) + BXq (ACq) [c] = CEq + ABq + BCq. Q. E. F.

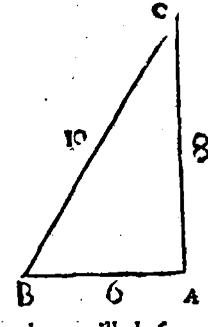
PROBL. 2.



Datis duabus rectis inæqualibus AB, BC, exhibere quadratum, quo quadratum majoris AB excedit quadratum minoris BC.

Centro B intervallo BA deſcribe circulum. ex C erige perpendicularem CE occurrentem pe- 47. 1. ripheriæ in E, & ducatur B E. [a] Erit BBq 1. ax. (BAq) = BCq + CEq. [b] ergò BAq − BCq = CEq. Q. E. F.

PROBL. 3.

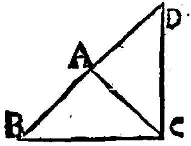


Notis duobus quibuscunque lateribus trigoni rectanguli ABC, reliquum invenire.

Latera rectum angulum ambientia sint AC, AB, hoc 6. pedum, illud 8. ergò cùm $ACq + ABq = 64 + 36 = 100 = BCq$. erit $BC = \sqrt{100} = 10$. 47. 1.

Nota sint deinde latera AB, BC, hoc 10. pedum, illud 6. ergò cùm $BCq - ABq = 100 - 36 = 64 = ACq$. erit $Acq = \sqrt{64} = 8$. 47. 1.

PROP. XLVIII.



Si quadratum quod ab uno latere BC trianguli describitur, æquale sit eis quæ à reliquis trianguli lateribus AB, AC describuntur quadratis, angulus BAC comprehensus sub AB, AC reliquis duobus trianguli lateribus, rectus est.

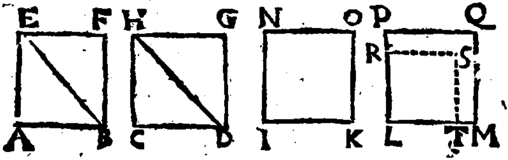
Duc ad AC perpendicularem $DA = AB$, & junge CD.

Jam CDq [a] $= ADq + ACq = ABq + ACq = BCq$. * ergò $CD = BC$. ergò triangula CAB, CAD, sibi mutuò æquilatera sunt; quare ang. CAB [b] $=$ CAD [c] $=$ Rect. Q. E. D. a 47. 1. * Vid. seq. Theor. b 8. 1. c hyp.

Schol.

Assumpsimus exinde quòd $CDq. = BCq$, sequi $CD = BC$. Hoc verò manifestum fiet ex sequenti theoremate.

THEOREMA.



Linearum æqualium AB, CD, *æqualia sunt quadrata* AF, CG; *& quadratorum æqualium* NK, PM *æqualia sunt latera* IK, LM.

Pro 1 Hyp. Duc diametros EB, HD. Liquet AF = [a] 2 triang. EAB = [b] 2 triang. HCD = [a] CG. Q. E. D.

2. Hyp. Si fieri potest, sit LM ⊏ IK. fac LT = IK; [a] sitque LS = LTq. ergò LS [b] = NK [c] = LQ. [d] Q. E. A. ergò LM = IK.

a 34. 1.
b 4. 1. & 6. ax.
a 46. 1.
b 1. part.
c hyp.
d 9. ax.

Coroll.

Eodem modo quælibet rectangula inter se æquilatera æqualia ostendentur.

LIB.

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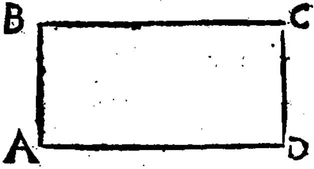
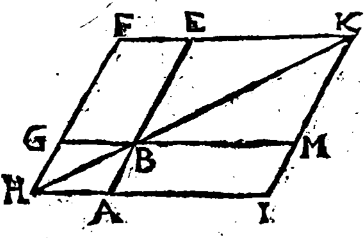
LIB. II. Aprill y 21 - 170

Definitiones.



I. OMne parallelogrammum rectangulum A B C D contineri dicitur sub rectis duabus A B, AD, quæ rectum comprehendunt angulum.

Quando igitur dicitur rectangulum sub B A, A D; *vel brevitatis causâ, rectangulum* B A D, *vel* B A×A D, (*vel* Z A *pro* Z×A) *designatur rectangulum, quod continetur sub* BA, & AD *ad rectum angulum constitutis.*



II. In omni parallelogrammo spatio FHIK unumquodq; eorum, quæ circa diametrum illius sunt, parallelogrammorum, cum duobus complementis Gnomon vocetur. *ut* Pgr. FB+BI+GA (EHM) *est Gnomon. item* Pgr. FB+BI+EM (GKA) *est Gnomon.*

PROP. I.

Si fuerint duæ rectæ lineæ AB, AF, secetúrque ipsarum altera AB in quotcunque segmenta AD, DE, EB: rectangulum comprehensum sub illis duabus rectis lineis AB, AF, æquale est eis, quæ sub insecta AF, & quolibet segmentorum AD, DE, EB comprehenduntur rectangulis.

a 11 1. [a] Statue AF, perpendicularem ad AB. [a] per F duc infinitam FG perpendicularem ad AF. [a] Ex D, E, B erige perpendiculares DH, EI, BG. Erit AG rectangulum sub AF, AB, &

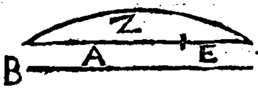
b 19 ax. 1. [b] est æquale rectangulis AH, DI, EG, hoc est

c 34 1. (quia DH, EI, AF [c] pares sunt) rectangulis sub AF, AD; sub AE, DE; sub AF, EB. Q. E. D.

Schol.

Propositiones decem primæ hujus libri valent etiam in numeris. Reliquas quilibet tyro examinet. pro hac, sit AF 6, & AB 12, sectus in AD 5, DE 3, & EB 4. Estque 6×12 (AG) $=72$. 6×5 (AH) $=30$. 6 in 3 (DI) $=18$. denique 6×4 (EG) $=24$. Liquet verò $30+18+24=72$.

PROP. II.

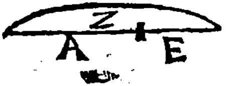


Si recta linea Z secta sit utcunque; rectangula, quæ sub tota Z, & quolibet segmentorum A, E comprehenduntur, æqualia sunt ei, quod à tota Z fit, quadrato.

Dico $ZA+ZE=Zq$. Nam sume $B=Z$. [a] Estque $BA+BE=BZ$; hoc est (ob $B=Z$) $ZA+ZE=Zq$. Q. E. D.

PROP.

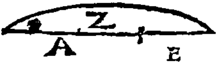
Prop. III.



Si recta linea Z secta sit utcunque; rectangulum sub tota Z, & uno segmentorum E comprehensum, æquale est illi, quod sub segmentis A,E comprehenditur, rectangulo, & illi quod à prædicto segmento E describitur, quadrato.

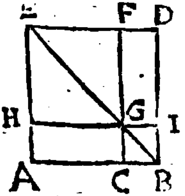
Dico. $ZE = AE + Eq.$ [a] Nam $EZ = EA + EE$. a 1. 2.

Prop. IV.



Si recta linea Z secta sit utcunque; Quadratum, quod à tota Z describitur, æquale est, & illis quæ à segmentis A, E describuntur quadratis, & ei, quod bis sub segmentis A,E comprehenditur, rectangulo.

Dico $Zq = Aq. + Eq. 2\ AE.$ [a] Nam $ZA = Aq + AE.$ [a] & $ZE = Eq + AE.$ quum igitur $ZA + ZE$[b] $= Zq$, [c] erit $Zq = Aq + Eq + 2\ AE$. Q.E.D. a 3. 2. b 2. 2. c 1. ax.



Aliter. Super AB fac quadratum AD, cujus diameter EB. per divisionis punctum C duc perpendicularem CF; & per G duc HI parall. AB.

Quoniam ang. $EHG = A$ rectus est, & AEB [d] semirectus, [e] erit reliquus HGE etiam semirectus. Ergò HE[f] $= HG$[g] $= EF$[g] $= AC$. [h] proinde HF quadratum est rectæ AC. eodem modo CI est CBq. ergò AG, GD rectangula sunt sub AC, CB. Quare totum quadratum AD [k] $= ACq + CBq + 2\ ACB$. Q.E.D.

d 4. *Cor.* 32. 1
e 32. 1.
f 6. 1.
g 34. 1.
h 29. *def.*
k 19. *ax.* 1

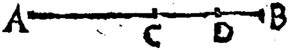
Coroll.

1. Hinc liquet parallelogramma circa diametrum quadrati esse quadrata.

2. Item diametrum cujusvis quadrati ejus angulos bisecare.

3. Si $A = \frac{1}{2} Z$; erit $Zq = 4\, Aq$, & $Aq = \frac{1}{4} Zq$. item è contra, si $Zq = 4\, Aq$. erit $A = \frac{1}{2} Z$.

PROP. V.



Si recta linea AB *secetur in æqualia* A C, CB, *& non æqualia* AD, DB, *rectangulum sub inæqualibus segmentis* A D, D B *comprehensum, unà cum quadrato, quod fit ab intermedia sectionum* CD, *æquale est ei, quod à dimidia* CB *describitur, quadrato.*

Dico $CBq = ADB + CDq$.

a 4. 2. b 3. 2. c hyp. d 1. 2.

$$\text{Æquantur enim ista}\begin{cases} CBq \\ {}^{a}CDq + CDB + DBq + CDB \\ CDq + {}^{b}CBD\,({}^{c}AC \times BD) + CDB \\ CDq + {}^{d}ADB. \end{cases}$$

Scholium.

Si A B aliter dividatur, propiùs scilicet puncto bisectionis, in E; dico $AEB \sqsubset ADB$.

a 5. 2. & ax. Nam $AEB^{a} = CBq - CEq$. & $ADB^{a} = CBq - CDq$. ergò quum $CDq \sqsubset CEq$, erit $AEB \sqsubset ADB$. Q. E. D.

Coroll.

Hinc $ADq + DBq \sqsubset AEq + EBq$. Nam

b 4. 2. $ADq + DBq + 2\, ADB^{b} = ABq\, {}^{b}= AEq + EBq + 2\, AEB$. ergò quum $2\, AEB \sqsubset 2\, ADB$, erit $ADq + DBq \sqsubset AEq + EBq$. Q. E. D.

c ax. Unde $2.\ ADq + DBq - AEq^{c} + EBq = 2\, AEB - 2\, ADB$.

PROP.

PROP. VI.

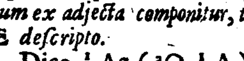
Si recta linea A *bifariam secetur, & illi recta quæpiam linea* B *in directum adjiciatur; rectangulum comprehensum sub tota cum adjecta* (sub. $A+E$), *& adjecta* E, *unà cum quadrato, quod à dimidia* $\frac{1}{2}$ A, *æquale est quadrato à linea, quæ tum ex dimidia, tum ex adjecta componitur, tanquam ab una* $\frac{1}{2} A+E$ *descripto.*

Dico $\frac{1}{4} Aq$ ($^a Q.\frac{1}{2} A$) $+AE+Eq = Q.\frac{1}{2} A$ a 4. & 3.
$+E$. ^{a} Nam $Q.\frac{1}{2} A+E = \frac{1}{4} Aq+Eq+AE$. Cor. 4. 2.

Coroll.

Hinc si tres rectæ E, $E+\frac{1}{2}A$, $E+A$ sint in proportione Arithmetica, rectangulum sub extremis E, $E+A$ contentum, unà cum quadrato excessûs $\frac{1}{2}$ A, æquale erit quadrato mediæ $E+\frac{1}{2} A$.

PROP. VII.



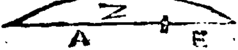
Si recta linea Z *secetur utcunque; Quod à tota* Z, *quódque ab uno segmentorum* E, *utraque simul quadrata, æqualia sunt illi, quod bis sub tota* Z, *& dicto segmento* E *comprehenditur, rectangulo, & illi, quod à reliquo segmento* A *fit, quadrato.*

Dico $Zq+Eq = 2\,ZE+Aq$. Nam $Zq^a = Aq$ a 4. 2.
$+Eq+2\,AE$. & $2\,ZE^b = 2\,Eq+2AE$. b 3. 2.

Coroll.

Hinc, quadratum differentiæ duarum quarumcunque linearum Z, E, æquale est quadratis utriusque minùs duplo rectangulo sub ipsis. c 7. 2. &
^{c} Nam $Zq+Eq-2\,ZE = Aq = Q.\,Z-E$. 3. ax.

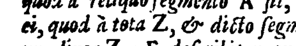
PROP. VIII.



Si recta linea Z secetur utcunque; rectangulum quater comprehensum sub tota Z, & uno segmentorum E, cum eo, quod à reliquo segmento A fit, quadrato, æquale est ei, quod à tota Z, & dicto segmento E, tanquam ab una linea Z+E describitur, quadrato.

a 7. 2. & 3. ax. Dico $4\ ZE + Aq = Q.\ Z+E$. Nam $2\ ZE^a =$
$Zq+Eq-Aq$. ergò $4\ ZE + Aq = Zq+Eq+2$
b 4. 2. $ZE^b = Q.\ Z+E$. Q. E. D.

PROP. IX.



Si recta linea AB secetur in æqualia AC, CB, & non æqualia AD, DB. quadrata, quæ ab inæqualibus totius segmentis AD, DB fiunt, simul duplicia sunt, & ejus, quod à dimidia AC, & ejus, quod ab intermedia sectionum CD fit, quadrati.

Dico $ADq+DBq = 2\ ACq+2\ CDq$. Nam
a 4. 2. $ADq+DBq^a = ACq+CDq+2\ ACD+DBq$.
b hyp. atqui $2\ ACD$ (^{b} $2\ BCD$) $+ DBq^c = CBq$
c 7. 2. (ACq) $+ CDq$.^{d} ergò $ADq+DBq = 2ACq$
d 2. ax. $+2\ CDq$. Q. E. D.

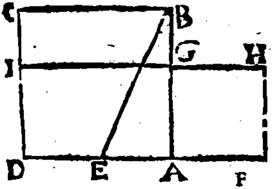
PROP. X.

Si recta linea A secetur bifariam, adjiciatur autem ei in rectum quæpiam linea; Quod à tota A cum adjuncta E, & quod ab adjuncta E, utraque simul quadrata, duplicia sunt & ejus, quod à dimidia ½ A; & ejus, quod à composita ex dimidia, & adjuncta, tanquam ab una ½ A+E descriptum est, quadrati.

4. 2. Dico $Eq+Q.\ A+E$, hoc est^a $Aq+2Eq+2$
Cor. 4. 2. $AE = 2\ Q.\ \frac{1}{2}A+2\ Q.\ \frac{1}{2}A+E$. Nam $2\ Q.\ \frac{1}{2}A^b$
4. 2. $= \frac{1}{2}Aq$. & $2\ Q.\ \frac{1}{2}A+E^c = \frac{1}{2}Aq+2Eq+2\ AE$.

PROP.

PROP. XI.



Datam rectam lineam AB *secare in* G, *ut comprehensum sub tota* AB, *& altero segmentorum* BG *rectangulum, æquale sit ei, quod à reliquo segmento* AG *fit, quadrato.*

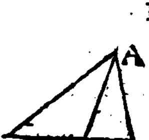
Super A B [a] describe quadratum A C. latus AD [b] biseca in E. duc EB. ex EA producta cape E F=E B. ad AF [a] statue quadratum A H. Erit AH=AB×BG. — a 46. 1. b 10. 1.

Nam protractâ H G ad I; Rectang. DH+EAq [c]=EFq [d]=EBq [c]=BAq+EAq. ergò DH [f]=BAq [d]= quad. AC. subtrahe commune A I; [f] remanet quad. AH=GC; [d] id est AGq=AB×BG. Q. E. F. — c 6. 2. d constr. e 47. 1. f 3 ax.

Scholium.

Hæc Propositio numeris explicari nequit; * neque enim ullus numerus ità secari potest, ut productum ex toto in partem unam æquale sit quadrato partis reliquæ. — * vid. 6. 13.

PROP. XII.



In amblygoniis triangulis ABC *quadratum, quod fit à latere* A C *angulum obtusum* A B C *subtendente, majus est quadratis, quæ fiunt à lateribus* AB, BC *obtusum angulum* A B C *comprehendentibus, rectangulo bis comprehenso, & ab uno laterum* B C, *quæ sunt circa obtusum angulum* ABC, *in quod, cùm protractum fuerit, cadit perpendicularis* AD, *& ab assumpta exterius linea* BD *sub perpendiculari* A D *prope angulum obtusum* ABC.

Dico

Dico $ACq = CBq + ABq + 2\,CB \times BD$.

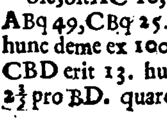
Nam ista — ACq.
a 47. 1. æqualia — $^{a}\, CDq + ADq$.
b 4. 2. sunt inter — $^{b}\, CBq + 2\,CBD + BDq + ADq$.
c 47. 1. ter se — $CBq + 2\,CBD\ ^{c} + ABq$.

Schol.

Hinc, *cognitis lateribus trianguli obtusanguli* ABC, *facilè invenientur tum segmentum* BD *inter perpendicularem* AD, *& obtusum angulum* ABC *interceptum, tum ipsa perpendicularis* AD.

Sic; Sit AC 10, AB 7, CB 5; unde ACq 100, ABq 49, CBq 25. Proinde $ABq + CBq = 74$. hunc deme ex 100, manet 26 pro 2 CBD. unde CBD erit 13. hunc divide per CB 5, provenit $2\frac{3}{5}$ pro BD. quare AD invenitur per 47. 1.

Prop. XIII.



In oxygoniis triangulis ABC *quadratum à latere* AB *angulum acutum* ACB *subtendente, minùs est quadratis, quæ fiunt à lateribus* AC, CB *acutum angulum* ACB *comprehendentibus, rectangulo bis comprehenso, & ab uno laterum* BC, *quæ sunt circa acutum angulum* ACB, *in quod perpendicularis* AD *cadit, & ab assumpta interiùs linea* DC *sub perpendiculari* AD, *prope angulum acutum* ACB.

Dico $ACq + BCq = ABq + 2\,BCD$.

Nam æquantur ista — $ACq + BCq$.
a 47. 1. $^{a}\, ADq + DCq + BCq$.
b 7. 2. $^{b}\, ADq + BDq + 2\,BCD$.
c 47. 1. $^{c}\, ABq + 2\,BCD$.

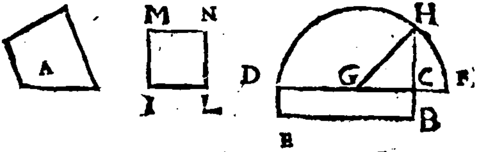
Coroll.

Hinc etiam *cognitis lateribus trianguli* ABC, *invenire est tam segmentum* DC *inter perpendicularem*

rem A D, *&* *acutum angulum* A B C *interceptum, quàm ipsam perpendicularem* A B.

Sit A B 13, AC 15, B C 14. Detrahe A Bq (169) ex A Cq + BCq hoc est ex 225 + 196 = 421; remanet 252 pro 2 BCD; unde BCD erit 126. hunc divide per B C 14, provenit 9 pro D C. unde A D = $\surd$: 225 − 81 = 12.

Prop. XIV.



Dato rectilineo A *æquale quadratum* M L *invenire.*

[a] Fac rectangulum D B = A, cujus majus latus DC produc ad F, ità ut CF = CB. [b] Biseca DF in G, quo centro ad intervallum GF describe circulum F H D, producatur CB, donec occurrat circumferentiæ in H. Erit CHq = * M L = A

Ducatur enim GH. Estque A [c]= D B [c] = D C F [d] = G Fq − G Cq [e] = HCq [c] = ML Q. E. F.

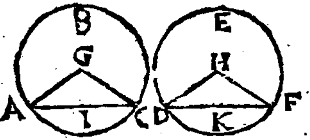
a 45. 1.
b 10. 2.
* 46. 1.
c constr.
d 5. 2. & 3. ax.
e 47. 1. & 3. ax.

LIB.

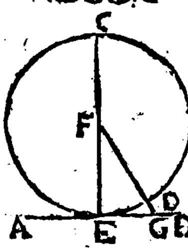
am. Payne Christ Hospitall April 21 - 1707

LIB. III.

Definitiones.

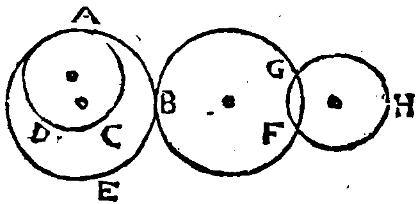


I. ÆQuales circuli (GABC, HDEF) ſunt, quorum diametri ſunt æquales, vel quorum quæ ex centris rectæ lineæ GA, HD, ſunt æquales.



II. Recta linea AB circulum FED tangere dicitur, quæ cùm circulum tangat, ſi producatur circulum non ſecat.

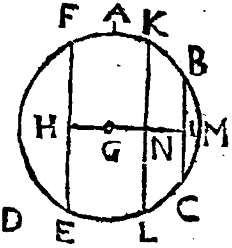
Recta FG ſecat circulum FED.



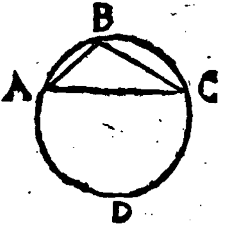
III. Circuli DAC, ABE (item FBG, ABE) ſe mutuò tangere dicuntur, qui ſe mutuò tangentes ſeſe mutuò non ſecant.

Circulus BFG ſecat circulum FGH.

In



IV. In circulo GABD æqualiter distare à centro dicuntur rectæ lineæ FE KL, cùm perpendiculares GH, GN quæ à centro G in ipsas ducuntur, sunt æquales. Longiùs autem abesse illa BC dicitur, in qu ammajor perpendicularis GI cadit.



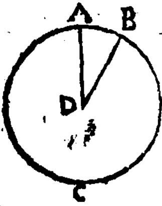
V. Segmentum circuli (ABC) est figura, quæ sub recta linea AC, & circuli peripheria ABC comprehenditur.

VI. Segmenti autem angulus (CAB) est, qui sub recta linea CA, & circuli peripheria AB comprehenditur.

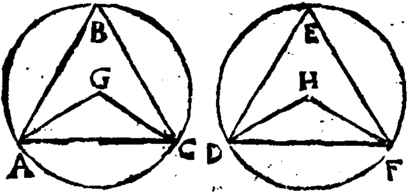
VII. In segmento autem (ABC) angulus (ABC) est, cùm in segmenti peripheria sumptum fuerit quodpiam punctum B, & ab illo in terminos rectæ ejus lineæ AC, quæ segmenti basis est, adjunctæ fuerint rectæ lineæ AB, CB, is inquam angulus ABC ab adjunctis illis lineis AB, CB comprehensus.

VIII. Cùm verò comprehendentes angulum ABC, rectæ lineæ AB, BC aliquam assumunt peripheriam ADC, illi angulus ABC insistere dicitur.

IX. Se-



IX. Sector autem circuli (ADB) est, cùm ad ipsius circuli centrum D constitutus fuerit angulus ADB; comprehensa nimirum figura ADB. & à rectis lineis AD, BD angulum continentibus, & à peripheria AB ab illis assumpta.



X. Similia circuli segmenta (ABC, DEF) sunt, quæ angulos (ABC, DEF) capiunt æquales; aut in quibus anguli ABC, DEF inter se sunt æquales.

PROP. I.

Dati circuli ABC *centrum* F *reperire.*

Duc in circulo rectam AC utcunq;, quam biseca in E. per E duc perpendicularē DB. hanc biseca in F. erit F centrū.

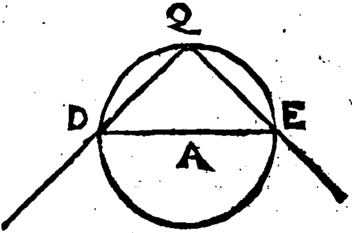
Si negas, centrum esto G, extra rectam DB (nam in ea esse non potest, cùm ubiq; extra F dividatur inæqualiter) ducantúrque GA, GC, GE. Vis G centrum esse; [a] ergò GA = GC; & per constr. AE = EC, latus verò GE commune est; [b] ergò anguli GEA, GEC pares, & [c] proinde recti sunt. [d] ergò ang. GEC = FEC rect. [e] Q. E. A.

a 15. *def.* 1.
b 8 1.
c 10. *def.* 1.
d 12 *ax.*
e 9. *ax.*

Coroll.

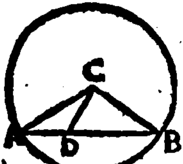
Coroll.

Hinc, si in circulo recta aliqua linea BD aliquam rectam lineam AC bifariàm & ad angulos rectos secet, in secante BD erit centrum.



Facillimè per normam invenitur centrum vertice Q ad circumferentiam applicato. Si enim recta DE jungens puncta D, & E, in quibus normæ latera QD, QE peripheriam secant, bisecetur in A, erit A centrum. Demonstratio pendet ex 31. hujus. *Andr. Tacq.*

PROP. II.



Si in circuli CAB *peripheria duo quælibet puncta*, A, B *accepta fuerint, recta linea* AB, *quæ ad ipsa puncta adjungitur, intra circulum cadet.*

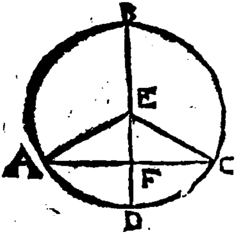
Accipe in recta AB quodvis punctum D, & ex centro C duc CA, CD, CB. & quoniam CA [a] = CB, [b] erit ang. A = B. Sed ang. CDB [c] ⊏ A; ergò ang. CDB ⊏ B. [d] ergò CB ⊏ CD. atqui CB tantùm pertingit ex centro ad circumferentiam; ergò CD eousque non pertingit. ergò punctum D est intra circulum. Idémque ostendetur de quovis alio puncto rectæ AB. Tota igitur AB cadit intra circulum. Q. E. D.

a 15. *def.* 1.
b 5. 1.
c 16. 1.
d 19. 1.

Coroll.

Hinc, Recta circulum tangens, ità ut eum non secet, in unico puncto tangit.

Prop. III.



Si in circulo EABC *recta quædam linea* BD *per centrum ex e sa quandam* AC *non per centrum extensam bifariàm secet, (in* F*) & ad angulos rectos ipsam secabit; & si ad angulos rectos eam secet, bifariàm quoque eam secabit.*

Ex centro E ducantur EA, EC.

a *hyp.*
b 15. *def.* 1.
c 8. 1.
d 10. *def.* 1.
e *hyp.* & 12. *ax.*
f 5. 1.
g 26. 1.

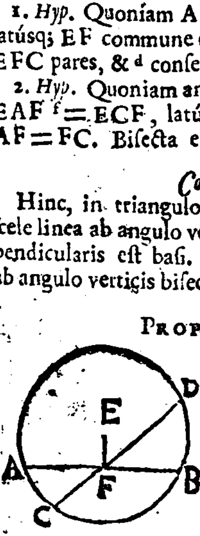
1. *Hyp.* Quoniam AF [a] = FC, & EA [b] = EC, latúsq; EF commune est, [c] erunt anguli EFA, EFC pares, & [d] consequenter recti. Q. E. D.

2. *Hyp.* Quoniam ang. EFA [e] = EFC, & ang. EAF [f] = ECF, latúsque EF commune, [g] erit AF = FC. Bisecta est igitur AC. Q. E. D.

Coroll.

Hinc, in triangulo quovis æquilatero & Isoscele linea ab angulo verticis bisecans basim, perpendicularis est basi. & contrà perpendicularis ab angulo verticis bisecat basim.

Prop. IV.



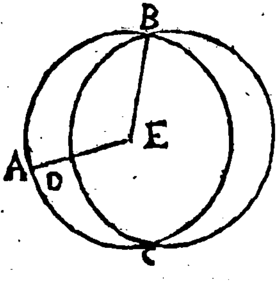
Si in circulo ACD *duæ rectæ lineæ* AB, CD *sese mutuò secent non per centrum* E *extensæ, sese mutuò bifariàm non secabunt.*

Nam si una per centrum

trum transeat, patet hanc non bisecari ab altera, quæ ex hyp. per centrum non transit.

Si neutra per centrum transit, ex E centro duc EF. Si jam ambæ AB, CD forent bisectæ in F, anguli EFB, EFD [a] ambo essent recti, & proinde æquales. [b] Q. E. A. a 3 3. b 9. ax.

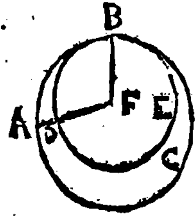
PROP. V.



Si duo circuli BAC, BDC *sese mutuò secent, non erit illorum idem centrum* E.

Aliàs enim ductis ex communi centro E rectis EB, EDA, essent ED [a] = EB [a] = EA. [b] Q. E. A. a 15. def. 1 b 9. ax.

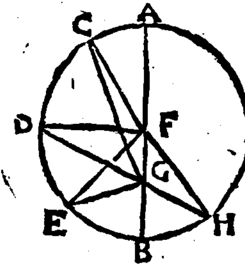
PROP. VI.



Si duo circuli BAC, BDE, *sese mutuò interius tangant* (in B) *eorum non erit idem centrum* F.

Aliàs ductis ex centro F rectis FB, FDA, essent FD [a] = FB [a] = FA. [b] Q. E. N. a 15. def. 1 b 9. ax.

Prop. VII.



Si in AB *diametro circuli quodpiam sumatur punctum* G, *quod circuli centrum non sit, ab eóq; puncto in circulum quædam rectæ lineæ* GC, GD, GE *cadunt; maxima quidam erit ea* (GA) *in qua centrum* F, *minima verò reliqua* GB. *aliarum verò illi, quæ per centrum ducitur, propinquior* GC *remotiore* GD *semper major est. Duæ autem solùm rectæ lineæ* GE GH *æquales ab eodem puncto in circulum cadunt, ad utrasque partes minimæ* GB, *vel maximæ* GA.

23. 1. Ex centro F duc rectas FC, FD, FE; & [a] fac ang. BFH = BFE.

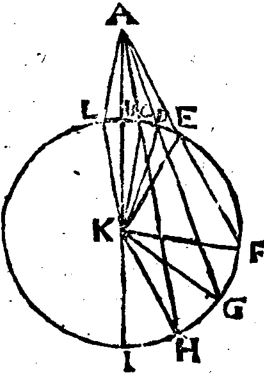
20. 1. 1. GF + FC (hoc est GA) [a] ⊏ GC. Q. E. D.

15. *def.* 1. 2. Latus FG commune est, & FC [b] = FD,
9. *ax.* atque ang. GFC [c] ⊏ GFD [d] ergò bas. GC
24. 1. ⊏ GD. Q. E. D.

20. 1. 3. FB (FE) [e] ⊐ GE + GF. ergò abla-
5. *ax.* to communi. FG [f] remanet BG ⊐ EG. Q. E. D.

4. Latus FG commune est, & FE = FH;
constr. atque ang. BFH [g] = BFE. [h] ergò GE = GH.
4. 1. Quòd verò nulla alia GD ex puncto G æquetur ipsi GE, vel GH, jamjam ostensum est. Q. E. D.

PROP. VIII.



Si extra circulum sumatur punctum quodpiam A, *ab eóque puncto ad circulum deducantur quædam lineæ* AI, AH, AG, AF; *quarum una quidem* AI *per centrum* K *protendatur, reliquæ verò ut libet; in cavam peripheriam cadentium rectarum linearum maxima quidem est illa* AI, *quæ per centrum ducetur, aliarum autem ei quæ per centrum transit propinquior* AH *remotiore* AG *semper major est. In convexam verò peripheriam cadentium rectarum linearum minima quidem est illa* AB, *quæ inter punctum* A, *& diametrum* BI *interponitur; aliarum autem ea, quæ est minimæ propinquior* AC *remotiore* AD *semper minor est. Duæ autem tantùm rectæ lineæ* AC, AL *æquales ab eo puncto in ipsum circulum cadunt, ad utrasque partes minimæ* AB, *vel maximæ* AI.

Ex centro K duc rectas KH, KG, KF; KC, KD, KE. & fac ang. AKL $=$ AKC.

1. AI (AK $+$ KH) [a] $\sqsubset$ AH. Q. E. D. a 20. 1.

2. Latus AK commune est; & KH $=$ KG; atque ang. AKH $\sqsubset$ AKG. [b] ergò baf. AH $\sqsubset$ AG. Q. E. D. b 24. 1.

3. KA [c] $\sqsupset$ KC $+$ CA. aufer hinc indè æquales KC, KB, [d] erit AB $\sqsupset$ AC. c 20. 1. d 5. ax.

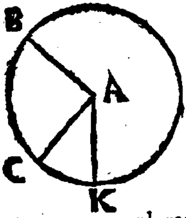
4. AC $+$ CK [e] $\sqsupset$ AD $+$ DK. aufer hinc indè æquales CK, DK, [f] erit AC $\sqsupset$ AD. Q. E. D. e 21. 1. f 5. ax.

 5. Latus

5. Latus KA est commune & KL = KC; atque ang. AKL [g] = AKC, [h] ergò LA = CA. hisce verò nulla alia æquatur, ex mox ostensis. ergò, &c.

g conft.
h 4. 1.

Prop. IX.

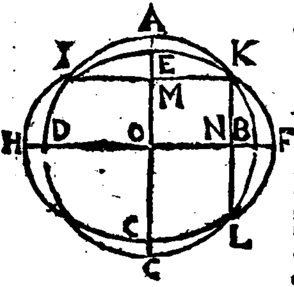


Si in circulo BCK acceptum fuerit punctum aliquod A, & ab eo puncto ad circulum cadant plures, quàm duæ rectæ lineæ æquales AB, AC, AK, acceptum punctum A centrum est ipsius circuli.

Nam [a] à nullo puncto extra centrum plures quàm duæ rectæ lineæ æquales duci possunt ad circumferentiam. Ergò A est centrum. Q. E. D.

a 7. 3.

Prop. X.



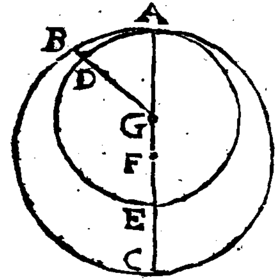
Circulus IAKBL circulum IEKFL in pluribus quàm duobus punctis non secat.

Secet, si fieri potest, in tribus punctis I K L. Junctæ IK KL. bisecentur in M & N. [a] Ambo circuli centrum habent in singulis perpendicularibus MC, NH, & proinde in earum intersectione O. ergò secantes circuli idem centrum habent. [b] Q. F. N.

a cor. 1. 3.
b 5. 3.

Prop.

Prop. XI.

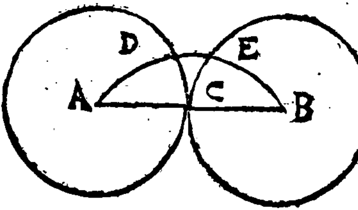


Si duo circuli GADE, FABC *sese intus contingant, atque accepta fuerint eorum centra* G, F; *ad eorum centra adjuncta recta linea* FG, *& producta, in* A *contactum circulorum cadet.*

Si fieri potest, recta F G protracta secet circulos extra contactum A, sic ut non FGA, sed FGDB sit recta linea. ducatur G A. Et quia GD [a] $=$ GA, & GB [b] $\sqsupset$ GA, (cùm recta FGB transeat per F centrum majoris circuli) erit GB $\sqsupset$ GD. [c] Q. E. A.

a 15. *def.*
b 7. 3.
c 9. *ax.*

Prop. XII.

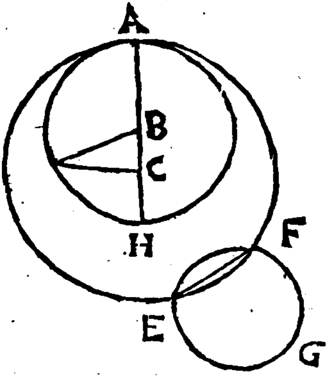


Si duo circuli ACD, BCE *sese exteriùs contingant, linea recta* AB *quæ ad eorum centra* A, B *adjungitur, per contactum* C *transibit.*

Si fieri potest, sit recta ADEB secans circulos extra contactum C in punctis D,E. Duc AC, CB. erit AD + EB (AC + CB). [a] $\sqsubset$ ADEB, [b] Q. E. A.

a 20. 1.
b 9. *ax.*

Prop. XIII.



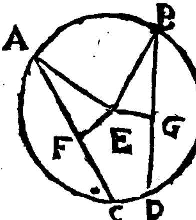
Circulus CAF *circulum* BAH *non tangit in pluribus punctis, quàm uno* A, *sive intus, sive extra tangat*

1. Tangat, si fieri potest, intùs in punctis A,H. [a] ergò recta CB centra connectens, si producatur cadet tam in A, quàm in H. Quoniam igitur CH [b] = CA, & BH ⊏ CH. erit BA ([c] BH) ⊏ CA. [d] Q. E. A.

a 11. 3.
b 15. def. 1.
c 15. def. 1.
d 9. ax.
e 2. 3.

2. Sin dicatur exteriùs contingere in punctis E & F, [e] ducta recta EF in utroque circulo erit. Circuli igitur se mutuò secant, quod non ponitur.

Prop. XIV.



In circulo EABC *æquales rectæ lineæ* AC BD, *æqualiter distant à centro* E. *&* *quæ* AC, BD *æqualiter distant à centro, æquales sunt inter se.*

Ex centro E duc perpendiculares EF, EG: [a] quæ bisecabunt AC, DB. connecte EA EB.

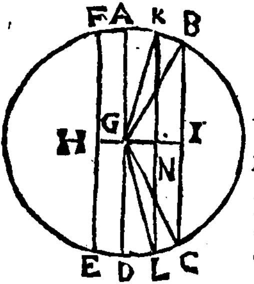
a 3. 3.
b 7. ax.

1. *Hyp.* AC = BD. ergò AF [b] = BG. sed & EA

$EA = EB$. ergò FEq [c] $= EAq - AFq =$ $EBq - EGq$ [c] $= EGq$. [d] ergò $FE = EG$. Q.E.D. — e 47. 1. & 3. ax.

2. *Hyp.* $EF = EG$. ergò AFq [c] $= EAq - EFq =$ $EBq - EGq$ [c] $= GBq$. ergò AF [d] $= GB$. [e] proinde $AD = BC$. Q.E.D. — d Schol. 48. 1. e 6. ax.

PROP. XV.

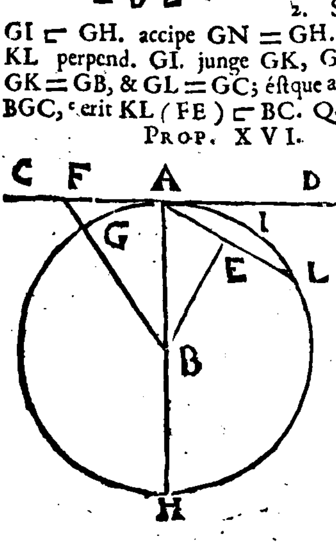


In circulo GABC maxima quidem linea est diameter AD; aliarum autem centro G propinquior FE remotiore BC semper major est.

1. Duc GB, GC. Diameter AD ([a] $= GB + GC$) [b] $\sqsubset BC$ Q.E.D — a 15. def. 1. b 20. 1.

2. Sit distantia $GI \sqsubset GH$. accipe $GN = GH$. per N duc KL perpend. GI. junge GK, GL. & quia $GK = GB$, & $GL = GC$; éstque ang. $KGL \sqsubset BGC$, [c] erit KL (FE) $\sqsubset BC$. Q.E.D. — c 24. 1.

PROP. XVI.



Quæ CD ab extremitate diametri HA cujusq; circuli BALH ad angulos rectos ducitur, extra ipsum circulum cadet, & in locum inter ipsam rectam lineam, & peripheriã compreher-

prehensum altera recta linea AL *non cadet, & semicirculi quidem angulus* BAI *quovis angulo acuto rectilineo* BAL *major est; reliquus autem* DAI *minor.*

1. Ex centro B ad quodvis punctum F in recta AC duc rectam BF. Latus BF subtendens

a 19. 1. angulum rectum BAF [a] majus est latere BA, quod opponitur acutò BFA. ergò cùm BA (BG) pertingat ad circumferentiam, BF ulteriùs porrigetur, adeóque punctum F, & eâdem ratione quodvis aliud rectæ AC, extra circulum situm erit. Q. E. D.

2. Duc BE perpendic. AL. Latus BA oppositum

b 19. 1. recto angulo BEA [b] majus est latere BE, quod acutum BAE subtendit: ergò punctum E, adeóque tota EA cadit intra circulum. Q. E. D.

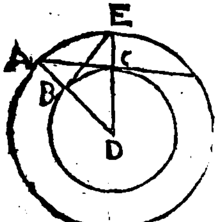
3. Hinc sequitur angulum quemvis acutum, nempe EAD angulo contactûs DAI majorem esse. Item angulum quemvis acutum BAL angulo semicirculi BAI minorem esse. Q. E. D.

Coroll.

Hinc, recta à diametri circuli extremitate ad angulos rectos ducta ipsum circulum tangit.

Ex hac propositione paradoxa consequuntur, & mirabilia bene multa, quæ vide apud interpretes.

PROP. XVII.



A dato puncto A *rectam lineam* AC *ducere, quæ datum circulum* DBC *tangat.*

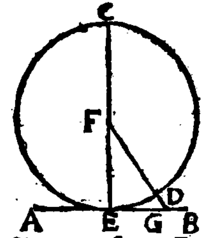
Ex D dati circuli centro ad datum punctum A ducatur recta DA secans peripheriam in B. Centro D describe per A alium circulum

AE;

A E; & ex B duc perpendicularem ad AD, quæ occurrat circulo AE in E. duc ED occurrentem circulo BC in C. ex A ad C ducta recta tanget circulum DBC,

Nam DB [a] = DC, & DE [a] = DA, & ang. D communis est: [b] ergò ang. ACD = EBD, rect. [c] ergò AC tangit circulum C. Q. E. F. a 15. def b 4. 1. c cor. 16.

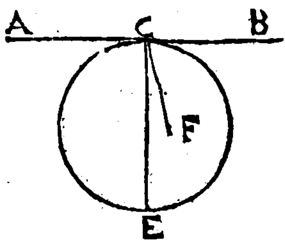
Prop. XVIII.



Si circulum FEDC *tangat recta quæpiam linea* AB, *à centro autem ad contactum* E *adjungatur recta quædam linea* FE; *quæ adjuncta fuerit* FE *ad ipsam contingentem* AB *perpendicularis erit.*

Si negas, sit ex F centro alia quædam FG perpendicularis ad contingentem, [a] secabit ea circulum in D. Quum igitur ang. FGE rectus dicatur [b] erit ang. FEG acutus [c]. ergò FE (FD) ⊏ FG. [d] Q. E. A. a 2. def. b cor. 17 c 19. 1. d 9. ax.

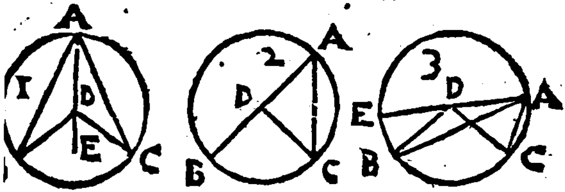
Prop. XIX.



Si circulum tetigerit recta quæpiam linea AB, *à contactu autem* C *recta linea* CE *ad angulos rectos ipsi tangenti excitetur, in excitata* CE *erit centrum circuli.*

Si negas, sit centrum extra CE in F, & ab F ad contactum ducatur FC. Igitur ang. FCB rectus est; & [a] proinde par angulo ECB recto per hypoth. [b] Q. E. A. a 12. ax. b 9. ax.

Prop. XX.

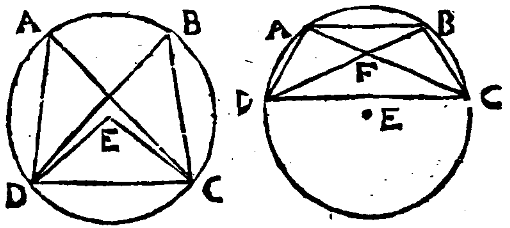


In circulo DABC, angulus BDC ad centrum duplex est anguli BAC ad peripheriam, cùm fuerit eadem peripheria BC basis angulorum.

Duc diametrum ADE.

Externus angulus BDE [a] = DAB + DBA [b] =
32. 1. 2 DAB. Similiter ang. EDC = 2 DAC. ergò
5. 1. in primo casu totus BDC = 2 BAC, sed in ter-
20. *ax.* tio casu [c] reliquus angulus BDC = 2 BAC. Q. E. *D.*

Prop. XXI.

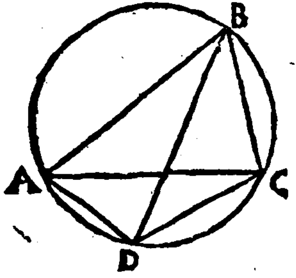


In circulo EDAC qui in eodem segmento sunt anguli, DAC & DBC sunt inter se æquales.

1. *Caſ.* Si ſegmentum DABC ſemicirculo ſit majus, ex centro E, duc ED, EC. Erítq; 2 ang.
10. 3. A [a] = E [a] = 2 B. Q. E. D.

2. *Caſ.* Sin ſegmentum ſemicirculo majus non fuerit, ſumma angulorum trianguli ADF æqua-
5. 1. tur ſummæ angulorum in triangulo BCF. *De-*
ar 1. *caſ.* matur hinc indè AFD [b] = BFC, & ADB [c] = ACB, remanent DAC = DBC. Q. E. D.

Prop. XXII.



Quadrilaterorum ABCD in circulo descriptorū anguli ADC, ABC, qui ex adverso, duobus rectis sunt æquales.

Duc AC, BD. Ang. ABC + BCA + BAC[a] = 2 Rect. Sed BDA[b] = BCA, & BDC[b] = BAC. [c]ergò ABC + ADC = 2 Rect. Q. E. D.

a 32. 1.
b 21. 3.
c 1. ax.

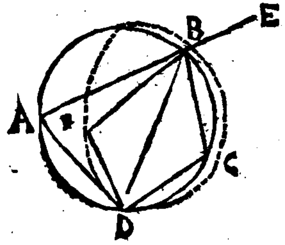
Coroll.

1. Hinc, si * AB unum latus quadrilateri in circulo descripti producatur, erit angulus externus EBC æqualis angulo interno ADC, qui opponitur ei ABC, qui est deinceps externo EBC. ut patet ex 13. 1. & 3. ax.

* vid. seq. diagram.

2. Item circa Rhombum circulus describi nequit; quia adversi ejus anguli vel cedunt duobus rectis, vel eos excedunt.

SCHOL.

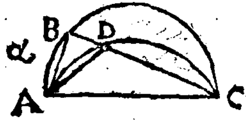


Si in quadrilatera ABCD anguli A, & C, qui ex adverso duobus rectis æquantur, circa quadrilaterum circulus describi potest.

Nam circulus per quosli-

bet tres angulos B, C, D transibit (ut patebit ex 5.4.) dico eundem per A transire. Nam si neges,
a 22. 3. transeat per F. ergò ductis rectis BF, FD, BD;
b hyp. c 3. ax. ang. C+F [a] = 2 Rect. [b] = C+A [c] quare A=F.
d 21. 1. [d] Q. E. A.

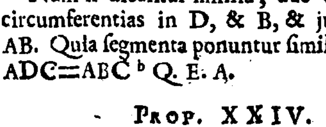
Prop. XXIII.



Super eadem recta linea AC *duo circulorũ segmenta* ABC, ADC *similia & inaequalia non constituentur ad easdem partes.*

Nam si dicantur similia, duc CB secantem circumferentias in D, & B, & junge AD, ac
a 10. def. 3. AB. Quia segmenta ponuntur similia, [a] erit ang.
b 16. 1. ADC=ABC [b] Q. E. A.

Prop. XXIV.

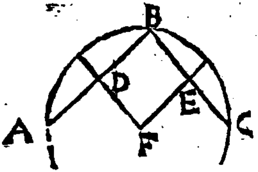


Super aequalib⁹ rectis lineis AC, DF *similia circulorũ segmẽta* ABC, DEF *sunt inter se aequalia.*

Basis AC superposita basi DF ei congruet, quia AC=DF. ergò segmentum ABC congruet segmento DEF (aliàs enim
23. 3. aut intra cadet, aut extra, [a] atque ità segmenta non erunt similia, contra Hyp. aut saltem partim intra, partim extra, adeóque ipsum in tri-
10. 3. bus punctis secabit. [b] Q. E. A.) [c] proinde se-
8. ax. gmentum ABC=DEF. Q. E. D.

PROP.

Prop. XXV.

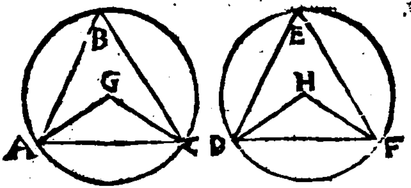


Circuli segmento ABC *dato, describere circulum, cujus est segmentum.*

Subtendantur utcunque duæ rectæ AB, BC, quas biseca in D, & E. Ex D, & E duc perpendiculares DF, EF occurrentes in puncto F. Hoc erit centrum circuli.

Nam centrum [a] tam in DF, quàm in EF existit. ergò in communi puncto F. Q. E. F. a *Cor.* 1. 3.

Prop. XXVI.

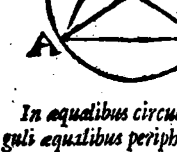


In æqualibus circulis GABC, HDEF *æquales anguli æqualibus peripheriis* AC, DF *insiunt, sive ad centra* G, H, *sive ad peripher.* B, E *constituti insistant.*

Ob circulorum æqualitatem, est GA = HD, & GC = HF item per hyp. ang. G = H. [a] ergò AC = DF. Sed & ang. B [b] = $\frac{1}{2}$ G = [c] $\frac{1}{2}$ H [b] = E. [d] ergò segmenta ABC, DEF similia, [e] & proinde paria sunt. [f] ergò etiam reliqua segmenta AC, DF æquantur. Q. E. D.

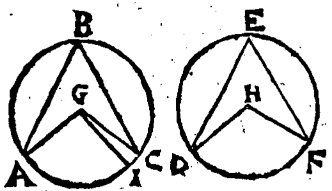
a 4. 1.
b 20. 3.
c *hyp.*
d 10. *def.* 3.
e 24. 3.
f 3. *ax.*

Scholium.



In circulo ABCD, sit arcus AB par arcui DC; erit AD parall. BC. Nam ductâ AC, [a] erit ang. ACB = CAD. quare per 27. 1. a 26. 3.

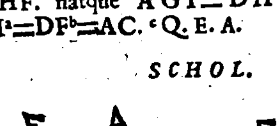
Prop. XXVII.



In æqualibus circulis GABC, HDEF, *anguli qui æqualibus peripheriis* AC DF *insistunt, sunt inter se æquales, sive ad centra* G, H, *sive ad peripherias* B, E *constituti insistant.*

Nam si fieri potest, sit alter eorum AGC ⊏
a 26. 3. DHF. fiátque AGI = DHF. ergò arcus
b *hyp.*
c 9. *ax.* AI[a] = DF[b] = AC. [c] Q. E. A.

SCHOL.



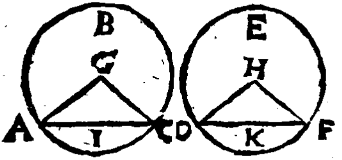
Linea recta EF, *quæ ducta ex* A *medio puncto peripheriæ alicujus* BC *circulum tangit, parallela est rectæ lineæ* BC, *quæ peripheriam illam subtendit.*

Duc è centro D ad contactum A rectam DA, & connecte DB, DC.

Latus DG commune est; & DB = DC, atq;
a 27. 3. ang. BDA [a] = CDA (ob arcus BA, CA [b] æ-
b *hyp.* quales) [c] ergò anguli ad basim DGB, DGC
c 4. 1.
d 10. *def.* 1. æquales & [d] proinde recti sunt. Sed interni an-
e *hyp.* guli GAB, GAF [e] etiam recti sunt. [f] ergò BC,
f 28. 1. EF sunt parallelæ. Q. E. D.

Prop.

Prop. XXVIII.

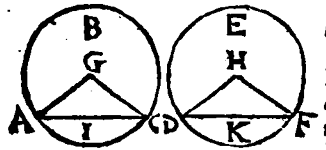


In æqualibus circulis GABC, HDEF *æquales rectæ lineæ* AC, DF *æquales peripherias auferunt, majorem quidem* ABC *majori* DEF, *minorem autem* AIC *minori* DKF.

E centris, G, H duc GA, GC; & HD, HF. Quoniam GA=HD, & GC=HF, atque AC[a] =DF; [b] erit ang. G = H. [c] ergò arcus AIC=DKF. [d] proinde reliquus ABC=DEF. Q. E. D.

a *hyp.*
b 8. 1.
c 26. 3.
d 3. *ax.*

Prop. XXIX.



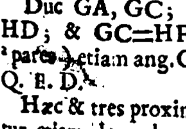
In æqualibus circulis GABC, HDEF *æquales peripherias* ABC, DEF *æquales rectæ lineæ* AC, DF *subtendunt*.

Duc GA, GC; & HD, HF. Quia GA= HD; & GC=HF; & (ob arcus AC, DF [a] pares) etiam ang. G[b]=H; [c]erit bas. AC=DF. Q. E. D.

a *hyp.*
b 27. 3.
c 4. 1.

Hæc & tres proximè præcedentes intelligantur etiam de eodem circulo.

Prop. XXX.

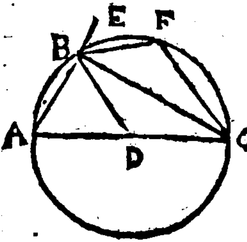


Datam peripheriam ABC *bifariàm secare.*

Duc AC; quam biseca in D. ex D duc perpendicularem DB occurrentem arcui in B. *Dico* factum.

a *conſt*. Jungantur enim AB, CB. Latus DB commune eſt; & AD[a] = DC; & ang. ADB[b] =
b 12. *ax.*
c 4. 1. CDB. [c]ergò AB = BC. [d]quare arcus AB =
d 28. 3. BC. Q. E. F.

Prop. XXXI.



In circulo angulus ABC, *qui in ſemicirculo, rectus eſt; qui autem in majore ſegmento* BAC, *minor recto; qui verò in minore ſegmento* BFC, *major eſt recto. Et inſuper angulus majoris ſegmenti recto quidem major eſt, minoris autem ſegmenti angulus, minor eſt recto.*

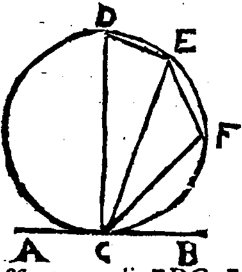
Ex centro D duc DB. Quia DB = DA, erit
a 5. 1. ang. A[a] = DBA. pariter ang. DCB[a] = DBC.
b 2 *ax.* [b]ergò ang. ABC = A + ACB [c] = EBC,
c 32. 1. [d]proinde ABC, & EBC recti ſunt. Q. E. D.
d 10. *def.* 1. [e]ergò BAC acutus eſt. Q. E. D. ergò cùm
e *cor.* 17. 1.
f 22. 3. BAC + BFC [f] = 2 Rect. erit BFC obtuſus. denique angulus ſub recta CB, & arcu BAC major eſt recto ABC. factus verò ſub CB, & BFC peripheria minoris ſegmenti, recto EBC
g 9. *ax.* [g]minor eſt. Q. E. D.

SCHOLIVM.

In triangulo rectangulo ABC, *ſi hypotenuſa* AC *biſecetur in* D, *circulus centro* D, *per* A *deſcriptus tranſibit per* B. ut facilè ipſe demonſtrabis ex hac, & 21. 1.

Prop.

PROP. XXXII.

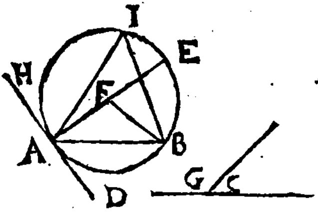


Si circulum tetigerit aliqua recta linea AB, à contactu autē producatur quædam recta linea CE circulum secans: anguli ECB, ECA, quos ad contingentem facit, æquales sunt iis, qui in alternis circuli segmentis consistunt, angulis EDC, EFC.

Sit CD latus anguli EDC perpendiculare ad AB ([a] perinde enim est) [b] ergò CD est diameter. [c] ergò ang. CED in semicirculo rectus est. [d] ergò ang. D + DCE = Rect. [e] = ECB + DCE. [f] ergò ang. D = ECB. Q. E. D. — a 26. 3. b 19. 3. c 31. 3. d 32. 1. e *constr.*

Cùm igitur ang. ECB + ECA [g] = 2 Rect. [h] = D + F; aufer hinc indè æquales ECB, & D, [k] remanent ECA = F. Q. E. D. — f 3. *ax.* g 13. 1. h 22. 3. k 3. *ax.*

PROP. XXXIII.



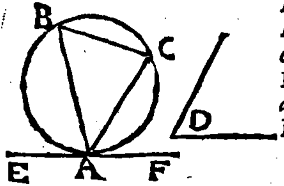
Super data recta linea AB describere circuli segmentum AIEB, quod capiat angulū AIB æqualem dato angulo rectilineo C.

[a] Fac ang. BAD = C. Per A duc AE perpendicularem ad HD. ad alterum terminum datæ AB fac ang. ABF = BAF. cujus alterum latus secet AE in F. centro F per A describe circulum, quod transibit per B (quia ang. FBA [b] = FAB, — a 23. 1

b *constr.* $^{b} =$ FAB, ^{c} ideóque FB $=$ FA); segmentum
c 6. 1. AIB est id quod quæritur.

Nam quia HD diametro AE perpendicularis
d *cor.* 16. 3. est, ^{d} tangit HD circulum, quem secat AB. ergò
e 32. 3. ang. AIB $^{e} =$ BAD $^{f} =$ C. Q. E. F.
f *constr.*

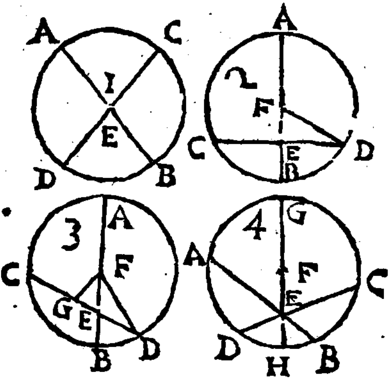
PROP. XXXIV.



A dato circulo ABC *segmentum* ABC *abscindere capiens angulum* B *æqualem dato angulo rectilineo* D.

a 17. 3. ^{a} Duc rectam EF, quæ tangat
datum circulum in A. ^{b} ducatur item AC faciens
b 23. 1. ang. FAC $=$ D. Hæc auferet segmentum ABC
capiens angulum B $^{c} =$ CAF $^{d} =$ D. Q. E. F.
c 32. 3.
d *constr.*

PROP. XXXV.



Si in circulo FBCA *duæ rectæ lineæ* AB, DC *sese mutuò secuerint, rectangulum comprehensum sub*

sub segmentis AF, EB *unius, æquale est ei quod sub segmentis* CE, ED *alterius comprehenditur, rectangulo.*

Cas. 1. Si rectæ sese in centro secent, res clara est.

2. Si una AB transeat per centrum F, & reliquam CD bisecet, duc FD. Estque Rectang. AEB + FEq [a] = FBq [b] = FDq [c] = EDq + FEq [d] = CED + FEq. [e] ergò Rectang. AEB = CED. Q. E. D.

a 5. 2.
b *sch.* 48. 1.
c 47. 1.
d *hyp.*
e 3. *ax.*

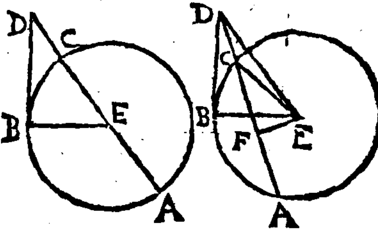
3. Si una AB diameter sit, alterámque CD secet inæqualiter, biseca CD per FG perpendicularem ex centro.

Æquantur ista:
- Rectang. AEB + FEq.
- [f] FBq (FDq) — f 5. 2.
- [g] FGq + GDq. — g 47. 1.
- FGq + [h] GEq + Rectang. CED. — h 5. 2.
- [k] FEq + CED. — k 47. 1.

[l] Ergò Rectang. AEB = CED. — l. 3. *ax.*

4. Si neutra rectarum AB, CD per centrum transeat; per intersectionis punctum E duc diametrum GH. Per modò demonstrata Rectang. AEB = GEH = CED. Q. E. D.

Prop. XXXVI.



Si extra circulum EBC *sumatur punctum aliquod* D, *ab eoque puncto in circulum cadant duæ rectæ lineæ* DA, DB; *quarum altera* DA *circulum secet,*

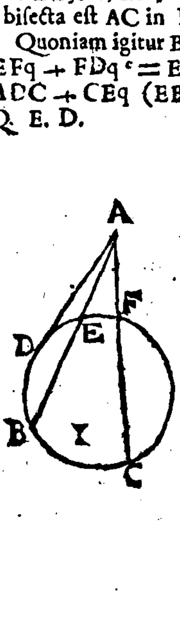
secet, altera verò DB *tanget; Quod sub tota secante* DA, *& exteriùs inter punctum* D, *& convexam peripheriam assumptâ* DC *comprehenditur rectangulum, æquale erit ei, quod à tangente* DB *describitur, quadrato.*

1. *Cas.* Si secans AD transeat per centrum E,
a 18. 3. junge EB; [a] faciet hæc cum DB rectum angu-
b 47. 1. lum; quare DBq + EBQ (ECq) [b] = EDq
c 6. 2. [c] = AD × DC + ECq [d] ergò AD × DC =
d 3. ax. DBq. Q. E. D.

2. *Cas.* Sin AD per centrum non transeat
a 3. 3. duc EC, EB, ED; atq; EF perpend. AD, quare
[a] bisecta est AC in F.

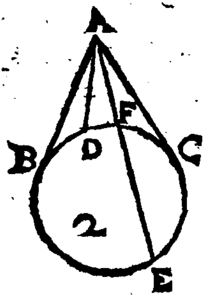
Quoniam igitur BDQ + EBq [b] = DEq [b] =
b 47. 1. EFq + FDq [c] = EFq + ADC + FCQ [d] =
c 6. 2. ADC + CEq (EBq); [e] erit BDq = ADC.
d 47. 1. Q. E. D.
e 3. ax.

Coroll.



1. Hinc, si à puncto quovis A extra circulum assumpto, plurimæ lineæ rectæ AB, AC circulum secantes ducantur, rectangula comprehensa sub totis lineis AB, AC, & partibus externis AE, AF inter se sunt æqualia. Nam si ducatur tangens AD; erit CAF
a 36. 3. = ADq [a] = BAE.

2. Constat



2. Constat etiam duas rectas AB, AC ab eodem puncto A ductas, quæ circulum tangant, inter se æquales esse.

Nam si ducatur AE secans circulum; erit ABq [a] = EAF [b] = ACq. a 36. 3. b 36. 3.

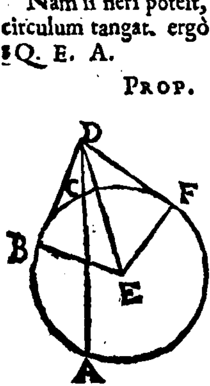
3. Perspicuum quoque est ab eodem puncto A extra circulum assumpto, duci tantùm posse duas lineas, AB, AC quæ circulum tangant.

Nam si tertia AD tangere dicatur, erit AD [c] = AB [c] = AC. [d] Q. F. N. c 2 *cor.* d 8. 3.

4. E contrà constat, si duæ rectæ æquales AB, AC ex puncto quopiam A in convexam peripheriam incidant, & earum una AB circulum tangat, alteram quoq; circulum tangere.

Nam si fieri potest, non AC, sed altera AD circulum tangat. ergò AD [e] = AC [f] = AB. [g] Q. E. A. e 2 *cor.* f *hyp.* g 8. 3.

Prop. XXXVII.



Si extra circulum EBF *sumatur punctum* D, *ab eoque in circulum cadant duæ rectæ lineæ* DA, DB; *quarum altera* DA *circulum secet, altera* DB *in eum incidat; sit autem quod sub tota secante* DA, *& exteriùs inter punctum, & convexam peripheriam assumpta* DC, *comprehenditur rectangulum, æquale ei, quod ab incidente*

DB *describitur quadrato, incidens ipsa* DB *circulum tanget.*

a 17. 3. Ex D [a] ducatur tangens DF; atque ex E cen-
b *hyp.* tro duc ED, EB, EF. Quia DBq [b] = ADC
c 36. 3. [c] = DFq, [d] erit DB = DF. Sed EB = EF,
d 1. *ax.* & *sch.* 48. 1. & latus ED commune est; [e] ergò ang. EBD
e 8. 1. = EFD. Sed EFD rectus est, [f] ergò EBD
f 12. *ax.* etiam rectus est. [g] ergò DB tangit circulum.
g *cor.* 16. 3. Q. E. D.

Coroll.

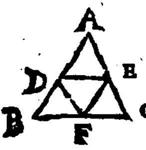
h 8. 1. Hinc, [h] ang. EDB = EDF.

LIB.

LIB. IV.

Definitiones.

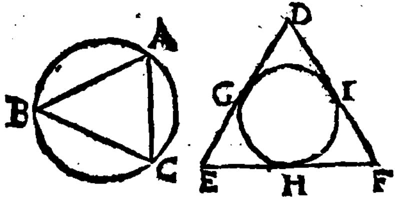
I. Figura rectilinea in figura rectilinea inscribi dicitur, cùm singuli ejus figuræ, quæ inscribitur, anguli singula latera ejus in qua inscribitur, tangunt.



Sic triangulum DEF *est inscriptum in triangulo* ABC.

II. Similiter & figura circa figuram describi dicitur, cùm singula ejus, quæ circumscribitur, latera singulos ejus figuræ angulos tetigerint, circa quam illa describitur.

Ità triangulum ABC *est descriptum circa triangulum* DEF.

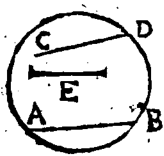


III. Figura rectilinea in circulo inscribi dicitur, cùm singuli ejus figuræ, quæ inscribitur, anguli tetigerint circuli peripheriam.

IV. Figura verò rectilinea circa circulum describi dicitur, cùm singula latera ejus, quæ circumscribitur, circuli peripheriam tangunt.

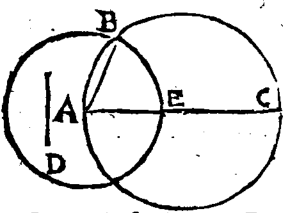
V. Similiter & circulus in figura rectilinea inscribi dicitur, cùm circuli peripheria singula latera tangit ejus figuræ, cui inscribitur.

VI. Circulus autem circa figuram describi dicitur, cùm circuli peripheria ſingulos tangit ejus figuræ, quam circumſcribit, angulos.



VII. Recta linea in circulo accommodari, ſeu coaptari dicitur, cùm ejus extrema in circuli peripheria fuerint; ut recta linea AB.

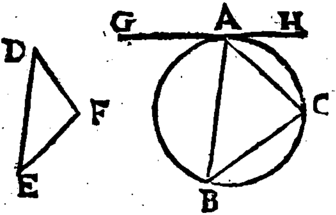
PROP. I. *Probl.* 1.



In dato circulo ABC *rectam lineam* AB *accommodare æqualem datæ rectæ lineæ* D, *quæ circuli diametro* AC *non ſit major.*

a 3. *poſt.* & 3. 1. Centro A, ſpatio AE=D [a] deſcribe circulum
b 15. *def.* 1. dato circulo occurrentem in B. Erit ducta
c *conſtr.* AB [b] = AE [c] = D. Q. E. F.

PROP. II. *Probl.* 2.

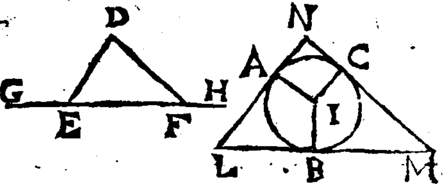


In dato circulo ABC *triangulũ* ABC *deſcribere dato triangulo* DEF *æquiangulum.*

a 17. 3. Recta GH circulum datum [a] tangat in A.
b 23. 1. [b] Fac ang. HAC=E; [b] & ang. GAB=F, & junge BC. Dico factum. Nam

Nam ang. B[c]=HAC[d]=E; & ang. C[c]=GAB[d]=F; [e] quare etiam ang. BAC=D. ergò triang. BAC circulo inscriptum triangulo DEF æquiangulum est. Q. E. F. c 32. 3. d constr. e 32. 1.

PROP. III. *Probl.* 3.

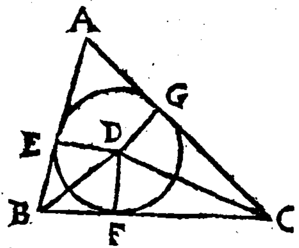


Circa datum circulum* IABC *triangulum* LNM *describere, dato triangulo* DEF *æquiangulum.

Produc latus EF utrinque. [a] Fac ad centrum I ang. AIB=DEG. & ang. BIC=DFH. deinde in punctis A, B, C circulum [b] tangant tres rectæ LN, LM, MN. Dico factum. a 23. 1. b 17. 3.

Nam quòd coibunt rectæ LN, LM, MN, atque ità triangulum constituent, patet; [c] quia anguli LAI, LBI [d] recti sunt, adeóque ducta AB angulos faciet LAB, LBA duobus rectis minores. Quoniam igitur ang. AIB + L [e]= 2 Rect. [f]= DEG + DEF; & AIB [g]= DEG; [h] erit ang. L=DEF. Simili argumento ang. M=DFE. [k] ergò etiam ang. N=D. ergò triang. LNM circulo circumscriptum dato EDF est æquiangulum. Q. E. F. c 13. ax. d 18. 3. e Schol. 32. 1. f 13. 1. g constr. h 3. ax. k 32. 1.

PROP. IV. Probl. 4.



In dato triangulo ABC *circulum* EFG *inscribere.*

a 9. 1. Duos angulos B, & C[a] biseca rectis BD, CD coeuntibus in D. Ex

b 12. 1. D[b] duc perpendiculares DE, DF, DG. circulus centro D per E descriptus transibit per G, & F, tangétque tria latera trianguli.

c constr. d 12. ax. e 26. 1. Nam ang. DBE[c]=DBF; & ang. DEB[d]= DFB; & latus DB commune est, [e] ergò DE= DF. Simili argumento DG=DF. circulus igitur centro D descriptus transit per E, F, G; & cùm anguli ad E, F, G sint recti, tangit omnia trianguli latera. Q.E.F.

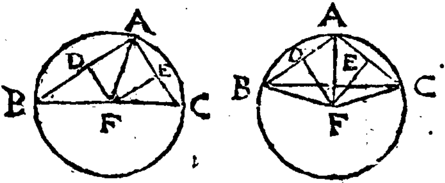
Scholium.

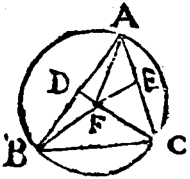
Peir. Herig. Hinc, *cognitis lateribus trianguli, invenientur eorum segmenta, quæ fiunt à contactibus circuli inscripti.* Sic.

Sit AB 12, AC 18, BC 16. Erit AB+ BC=28. ex quo subduc 18=AC=AE+FC, remanet 10=BE+BF. ergò BE, vel BF=5. proinde FC, vel CG=11. quare GA, vel AE=7.

PROP.

PROP. V. Probl. 5.





Circa datum triangulum ABC *circulum* FABC *describere.*

Latera quævis duo BA, AC [a] biseca perpendicularibus DF, EF concurrentibus in F. Hoc erit centrum circuli. a 10, & 11. 1.

Nam ducantur rectæ FA, FB, FC. Quoniam AD[b]=DB; & latus DF commune est; & ang. FDA [c]=FDB, [d] erit FB=FA. eodem modo FC=FA. ergò circulus centro F per dati trianguli angulos B, A, C transibit. Q. E. F. b *constr.* c *constr.* & 12. *ax.* d 4. 1.

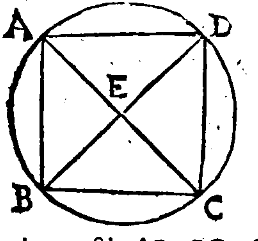
Coroll.

* Hinc, si triangulum fuerit acutangulum, centrum cadet intra triangulum, si rectangulum, in latus recto angulo oppositum; si denique obtusangulum, extra triangulum. * 31. 3.

Schol.

Eâdem methodo describetur circulus, qui transeat per data tria puncta, non in una recta linea existentia.

PROP. VI. *Probl.* 6.

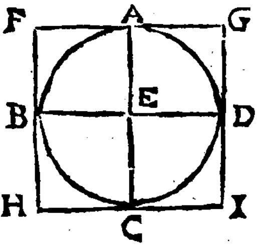


In dato circulo EABCD *quadratum* ABCD *inscribere.*

a 11. 1. [a] Duc diametros AC, BD se mutuò secantes ad angulos rectos in centro E. junge harum terminos rectis AB, BC, CD, DA. Dico factum.

b 26. 3.
c 29. 3.
Nam quia 4. anguli ad E recti sunt; [b] arcus, & [c] subtensæ AB, BC, CD, DA pares sunt. ergò ABCD æquilaterum est; ejúsque omnes
d 31. 3. anguli in semicirculis, adeóque [d] recti sunt. [e] er-
e 29. def. 1. gò ABCD est quadratum, dato circulo inscriptum. Q. E. F.

PROP. VII. *Probl.* 7.



Circa datum circulum EABCD *quadratum* FHIG *describere.*

Duc diametros AC, BD se mutuò secantes perpendiculariter. per
a 17. 3. harū extrema [a] duc tangentes concurrentes in F, H, I, G. Dico factum. Nam ob
b 18. 3. angulos ad A, & C [b] rectos, [c] erit FG parall.
c 28. 1. HI. eodem modo FH parall. GI. ergò FHIG est parallelogrammum; & quidem rectangulum.
d 34. 1. sed & æquilaterum, quia FG [d] = HI [d] = BD [e] =
e 15. def. 1. CA [d] = FH [d] = GI. quare FHIG est [f] quadra-
f 29. def. 1. tum, dato circulo circumscriptum. Q. E. F.

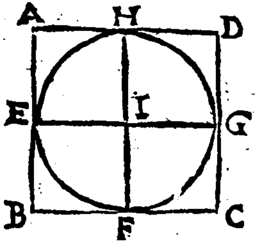
SCHOL.

SCHOL.

Quadratum ABCD circulo circumſcriptum duplum eſt quadrati EFGH circulo inſcripti.

Nam rectang. HB = 2 HEF. & HD = 2 HGF. per 41. 1.

PROP. VIII. *Probl.* 8.



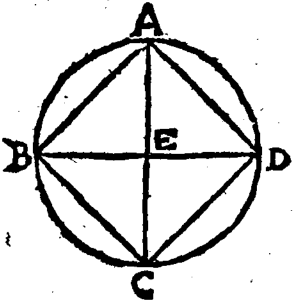
In dato quadrato ABCD *circulum* IEFGH *inſcribere.*

Latera quadrati biſeca in punctis, H, E, F, G. junge HF, EG ſeſe ſecantes in I. circulus centro I per H deſcriptus quadrato inſcribetur.

Nam quia AH, BF [a] pares ac [b] parallelæ ſunt, [c] erit AB parall. HF parall. DC. eodem modo AD parall. EG parall. BC. ergò IA, ID, IB, IC ſunt parallelogramma. Ergò AH [d] = AE [e] = HI = EI = IF = IG. Circulus igitur centro I per H deſcriptus tranſibit per H, E, F, G, tangétque quadrati latera, cùm anguli ad H, E, F, G ſint recti. Q. E. F.

a 7. ax.
b 34. 1.
c 33. 1.
d 7. ax.
e 34. 1.

PROP. IX. Probl. 9.



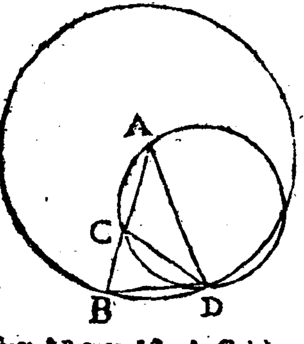
Circa datum quadratum ABCD *circulum* EABCD *describere.*

Duc diametros AC, BD secantes in E. centro E per A describe circulum. Is dato quadrato circumscriptus est.

a 4.cor.32.1. b 6.1.

Nam anguli ABD, & BAC [a] semirecti sunt; [b] ergò EA = EB. eodem modo EA = ED = EC. Circulus igitur centro E descriptus per A, B, C, D dati quadrati angulos transit. Q.E.F.

PROP. X. Probl. 10.



Isosceles triangulum ABD *constituere, quod habeat utrunque eorum quæ ad basim sunt angulorum* B *&* ADB *duplum reliqui* A.

a 11.2.

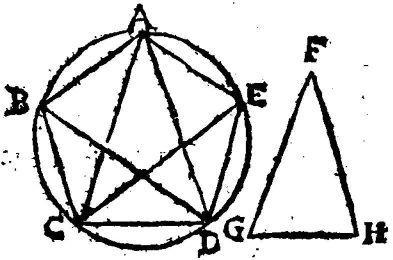
Accipe quamvis rectam AB, quam [a] seca in C, ità ut AB × BC = ACq. Centro A per B describe circulum ABD in hoc [b] accommoda BD = AC, & junge AD. erit triang. ABD quod quæritur. b 1. 4.

Nam duc DC; & per CDA [c] describe circulum. Quoniam AB × BC = ACq. [d] liquet BD tangere circulum ACD, quem secat CD. [e] ergò ang. BDC = A. ergò ang. BDC + CDA [f] = A + CDA = BCD. sed BDC + CDA = BDA [h] = CBD. [k] ergò ang. BCD = CBD. ergò DC [l] = DB [m] = AC, [n] quare ang. CDA = A = BDC. ergò ADB = 2 A = ABD. Q. E. F. c 5. 4. d 37. 3. e 32. 3. f 2. ax. g 32. 1. h 5. 1. k 1. ax. l 6. 1. m constr. n 5. 1.

Coroll.

Cùm omnes anguli A, B, D [o] conficiant $\frac{5}{2}$ 2 Rect. (2 Rect.) liquet A esse $\frac{1}{5}$ 2 Rect. o 32. 1.

PROP. XI. *Probl.* 11.



In dato circulo ABCDE *pentagonum æquilaterum & æquiangulum* ABCDE *inscribere.*

[a] Describe triangulum Isosceles EGH, habens utrumque angulorum ad basim duplum anguli ad verticem. [b] Huic æquiangulum CAD inscribe circulo. Angulos ad basim ACD, & ADC [c] biseca rectis DB, CE occurrentibus circumferentiæ in B, & E. connecte rectas CB, BA, AE, ED. Dico factum. a 10. 4. b 2. 4. c 9. 1.

Nam

Nam ex constr. liquet quinque angulos CAD, CDB, BDA, DCE, ECA pares esse; quare [d] arcus [e] & subtensæ DC, CB, BA, AE, DE æquantur. Pentagonum igitur æquilaterum est. Est verò etiam æquiangulum, [f] quia ejus anguli BAE, AED &c. insistunt arcubus [g] æqualibus BCDE, ABCD, &c.

d 26. 3.
e 29. 3.
f 27. 3.
g 2. ax.

Hujus problematis praxis facilior tradetur ad 10, 13.

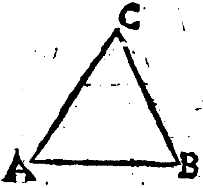
Coroll.

Hinc, angulus pentagoni æquilateri & æquianguli æquatur $\frac{3}{5}$ 2 Rect. vel $\frac{6}{5}$ Rect.

Schol.

Petr. Herig.

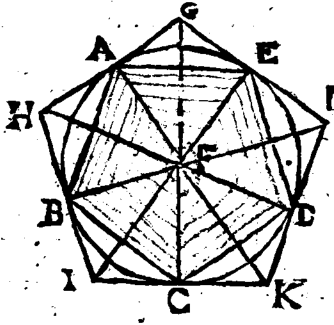
Universaliter figuræ imparium laterum inscribuntur circulo beneficio triangulorum Isoscelium, quorum anguli æquales ad basim multiplices sunt eorum, qui ad verticem sunt angulorum; parium verò laterum figuræ in circulo inscribuntur ope Isoscelium triangulorum, quorum anguli ad basim multiplices sesquialteri sunt eorum, qui ad verticem sunt, angulorum.



Ut in triangulo Isoscele CAB, si ang A = 3 C = B; AB erit latus Heptagoni. Si A = 4 C; erit AB latus Enneagoni, &c. Sin verò A = $1\frac{1}{2}$C, erit AB latus quadrati. Et si A = $2\frac{1}{2}$ C subtendet AB sextam partem circumferentiæ; pariterque si A = $3\frac{1}{2}$ C; erit AB latus octagoni, &c.

PROP.

PROP. XII. *Probl.* 12.



Circa datum circulum FABCDE *pentagonum æquilaterum & æquiangulum* HIKLG *describere.*

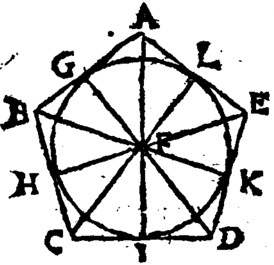
[a] Inscribe pentagonum ABCDE æquilaterum & æquiangulum; duc è centro rectas FA, FB, FC, FD, FE; iisque totidem perpendiculares GAH, HBI, ICK, KDL, LEG concurrentes in punctis H, I, K, L, G. Dico factum; Nam quia GA, GE ex uno puncto G [b] tangunt circulum, [c] erit GA = GE. [d] ergò ang. GFA = GFE. ergò ang. AFE = 2 GFA, eodem modo ang. AFH = HFB; & proinde ang. AFB = 2 AFH. Sed ang. AFE [e] = AFB. [f] ergò ang. GFA = AFH. sed & ang. FAH [g] = FAG; & latus FA est commune, [h] ergo HA = AG = GE = EL, &c. [k] ergò HG, GL, LK, KI, IH latera pentagoni æquantur: sed & anguli etiam, utpote [l] æqualium AGF, AHF, &c. dupli; ergò, &c.

a 11. 4.
b cor. 16.
c 2. cor. 36
d 8. 1.
e 27. 3.
f 7. ax.
g 12. ax.
h 26. 1.
k 2. ax.
l 32. 1.

Coroll.

Eodem pacto, Si in circulo quæcunque figura æquilatera & æquiangula describatur, & ad extrema semidiametrorum ex centro ad angulos ductarum, excitentur lineæ perpendiculares, hæ perpendiculares constituent aliam figuram totidem laterum & angulorum æqualium circulo circumscriptam.

Prop. XIII. Probl. 13.



In dato pentagono æquilatero, & æquiangulo ABCDE *circulum* FGHK *inscribere.*

Duos pentagoni angulos A, & B [a] biseca rectis AF, BF cōcurrentibus in F.

). 1. Ex F duc perpendiculares FG, FH, FI, FK, FL. Circulus centro F per G descriptus tanget omnia pentagoni latera.

hyp. Duc FC, FD, FE. Quoniam BA [b] = BC;
constr. & latus BF commune est; & ang. FBA [c] =
4. 1. FBC, [d] erit AF = FC; & ang. FAB = FCB.
hyp. Sed ang. FAB [e] = ½ BAE [e] = ½ BCD. ergò ang. FCB = ½ BCD. eodem modo anguli totales C, D, E omnes bisecti sunt. Quum igitur
2. ax. ang. FGB [f] = FHB; & ang. FBH = FBG, &
26. 1. latus FB sit commune, [g] erit FG = FH. similiter omnes FH, FI, FK, FL, FG æquantur. ergò circulus centro F per G descriptus transit per
cor. 16. 3. H, I, K, E; [h] tangitque pentagoni latera, cùm anguli ad ea puncta sint recti. Q. E. F.

Coroll.

Hinc, si duo anguli proximi figuræ æquilateræ & æquiangulæ bisecentur, & à puncto, in quo coeunt lineæ angulos bisecantes, ducantur rectæ

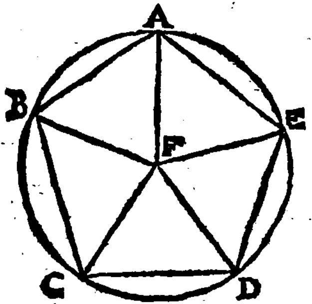
lineæ

lineæ ad reliquos figuræ angulos, omnes anguli figuræ erunt bisecti.

Schol.

Eâdem methodo in qualibet figura æquilatera & æquiangula circulus describetur.

Prop. XIV. *Prob.* 14.



Circa datum Pentagonum æquilaterum, & æquiangulum ABCDE *circulum* FABCD *describere.*

Duos pentagoni angulos biseca rectis AF, BF concurrentibus in F. Circulus centro F per A descriptus pentagono circumscribitur.

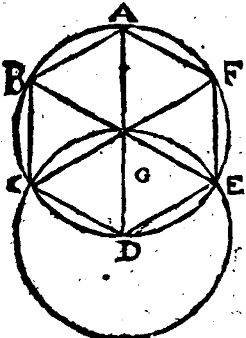
Ducantur enim FC, FD, FE. [a] Bisecti itáq; sunt anguli C, D, E. [b] ergò FA, FB, FC, FD FE æquantur. ergò circulus centro F descriptus, per A, B, C, D, E, pentagoni angulos transibit. Q. E. F.

a cor. 13. 4
b 6. 1.

Schol.

Eâdem arte circa quamlibet figuram æquilateram, & æquiangulam circulus describetur.

Prop. XV. *Probl.* 15.



In dato circulo G-ABCDEF *hexagonum & æquilaterum, & æquiangulum* ABCDEF *inſcribere.*

Duc diametrum AD; centro D per centrum G deſcribe circulum, qui datum ſecet in C, & E. duc diametros CF, EB. junge AB, BC, CD, DE, EF, FA. Dico factum.

32. 1. Nam ang. CGD [a] = $\frac{1}{3}$ 2 Rect [a] = DGE [b] =
15. 1. AGF [b] = AGB. [c] ergò BGC = $\frac{1}{3}$ 2 Rect. = FGE.
cor. 13. 1. [d] ergò arcus [e] & ſubtenſæ AB, BC, CD, DE,
26. 3. EF æquantur. Hexagonum igitur æquilaterum
29. 3. eſt: ſed & æquiangulum, [f] quia ſinguli ejus an-
27. 3. guli arcubus inſiſtunt æqualibus. Q. E. F.

Coroll.

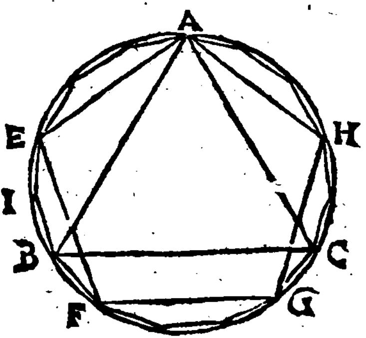
1. Hinc latus Hexagoni circulo inſcripti ſemidiametro æquale eſt.

2. Hinc facilè triangulum æquilaterum ACE in circulo deſcribetur.

Schol. Probl.

Andr. Tacq. *Hexagonum ordinatum ſuper datâ rectâ* CD *ita*
1. 1. *conſtrues.* [a] Fac triangulum CGD æquilaterum ſuper data CD. centro G per C, & D deſcribe circulum. Is capiet Hexagonum ſuper data CD.

PROP. XVI. *Probl.* 16.



In dato circulo AEBC *quindecagonum æquilaterum & æquiangulum inscribere.*

Dato circulo [a] inscribe pentagonum æquilaterum AEFGH; [b] itémque triangulum æquilaterũ ABC. erit BF latus quindecagoni quæsiti. a 11. 4. b 3. 4.

Nam arcus AB [c] est $\frac{1}{3}$, vel $\frac{5}{15}$ peripheriæ cujus AF est $\frac{2}{5}$ vel $\frac{6}{15}$, ergò reliquus BF $= \frac{1}{15}$ periph. ergò quindecagonum cujus latus BF, æquilaterum est; sed & æquiangulum, [d] cùm singuli ejus anguli arcubus insistant æqualibus, quorum unusquisque est $\frac{13}{15}$ totius circumferentiæ, ergò, &c. c constr. d 27. 3.

Schol.

Circulus dividitur Geometricè in partes
- 4, 8, 16 &c. per 6, 4, & 9, 1.
- 3, 6, 12, &c. per 15, 4, & 9, 1.
- 5, 10, 20, &c. per 11, 4, & 9, 1.
- 15, 30, 60, &c. per 16, 4 & 9, 1.

Cæterùm divisio circumferentiæ in partes datas etiamnum desideratur; quare pro figurarum quarumcunq; ordinatarum constructionibus sæpe ad mechanica artificia recurrendum est, propter quæ Geometræ practici consulendi sunt.

Payne: Chesse: Rospt: Boug: 90: 6: prety 21-1707

LIB. V.

Definitiones.

I. PArs est magnitudo magnitudinis, minor majoris, cùm minor metitur majorem.

II. Multiplex autem est major minoris, cùm minor metitur majorem.

III. Ratio est duarum magnitudinum ejusdem generis mutua quædam secundùm quantitatem habitudo.

In omni ratione ea quantitas, quæ ad aliam refertur, dicitur antecedens rationis; ea verò, ad quam alia refertur, consequens rationis dici solet. ut in ratione 6 ad 4; antecedens est 6, & consequens 4.

Nota.

Cujusque rationis quantitas innotescit dividendo antecedentem per consequentem. ut ratio 12 ad 5 effertur per $\frac{12}{5}$ item quantitas rationis A ad B est $\frac{A}{B}$. Quare non rarò brevitatis causâ, quantitates rationum sic designamus, $\frac{A}{B}$ ⊏, vel =, vel ⊐ $\frac{C}{D}$; hoc est ratio A ad B major est ratione C ad D, vel ei æqualis, vel minor. Quod probè animadvertat, quisquis hæc legere volet.

Rationis, sive proportionis species, ac divisiones vide apud interpretes.

IV. Proportio verò est rationum similitudo.

Rectiùs quæ hîc vertitur proportio, proportionalitas, sive analogia dicitur; nam proportio idem denotat quod ratio, ut plerisque placet.

V. Rationem habere inter se magnitudines dicuntur; quæ possunt multiplicatæ se mutuò superare.

VI. In

E, 12.	A, 4. B. 6.	G, 24.	
F, 30.	C, 10. D, 15.	H, 60.	

VI. In eadẽ ratione magnitudines dicuntur esse, prima A ad secundum B; & tertia C ad quartum D; cùm primæ A, & tertiæ C æquemultiplicia E, & F à secundæ B, & quartæ D æquemultiplicibus G, & H, qualiscunq; sit hæc multiplicatio, utrumque E, F ab utroq; G, H vel unà deficiunt, vel unà æqualia sunt, vel unà excedunt, si ea sumantur E, G; & F, H quæ inter se respondent.

Hujus nota est :: . ut A. B :: C. D. *hoc est* A *ad* B, *&* C *ad* D *in eadem sunt ratione. aliquando sic scribimus* $\frac{A}{B} = \frac{C}{D}$ *id est*, A.B:: C.D.

VII. Eandem autem habentes rationem (A.B:: C.D) proportionales vocentur.

E, 30.	A, 6. B, 4.	G, 28.	
F, 60.	C, 12. D, 9.	H, 63.	

VIII. Cùm verò æquemultipliciũ, E multiplex primæ magnitudinis A excesserit G multiplicem secundæ B; at F multiplex tertiæ C non excesserit H multiplicem quartæ D; tunc prima A ad secundam B majorem rationem habere dicetur; quàm tertia C ad quartam D.

Si $\frac{A}{B} \sqsubset \frac{C}{D}$, *necessarium non est ex hac definitione, ut* E *semper excedat* G; *quum* F *minor est quàm* H; *sed conceditur hoc fieri posse.*

IX. Proportio autem in tribus terminis paucissimis consistit. *Quorum secunda est instar duorum.*

X. Cùm autem tres magnitudines A, B, C proportionales fuerint prima A ad tertiam C duplicatam rationem habere dicetur ejus, quam habet ad secundam B: at quum quatuor magnitudines A, B, C, D, proportionales fuerint, prima A ad quartam D triplicatam rationem habere dicetur

dicetur ejus, quam habet ad ſecundam B; & ſemper deinceps uno amplius, quamdiu proportio extiterit.

Duplicata ratio exprimitur ſic $\frac{A}{C} = \frac{A}{B}$ *bis. Hoc eſt, ratio* A *ad* C *duplicata eſt rationis* A *ad* B. *triplicata autem ſic* $\frac{A}{D} = \frac{A}{B}$ *ter.* id eſt, *ratio* A *ad* D *triplicata eſt rationis* A *ad* B.

∺ *denotat continuè proportionales*.ut A,B,C,D∺ *item* 2, 6, 18, 64 *ſunt* ∺.

XI. Homologæ ſeu ſimiles ratione magnitudines dicuntur, antecedentes quidem antecedentibus, conſequentes verò conſequentibus.

Ut ſi A. B :: C. D; *tam* A, *&* C; *quàm* B & D *homologæ magnitudines dicuntur.*

XII. Alterna ratio, eſt ſumptio antecedentis ad antecedentem, & conſequentis ad conſequentem.

ut ſit A. B :: C. D. *ergò alternè, vel permutando, vel viciſſim* A. C :: B. D. *per* 16. 5.

In hac definitione, & 5. *ſequentibus imponuntur nomina ſex modis argumentandi, quibus mathematici frequenter utuntur; quarum illationum vis innititur propoſitionibus hujus libri, quæ in explicationibus citantur.*

XIII. Inverſa ratio, eſt ſumptio conſequentis ceu antecedentis, ad antecedentem velut ad conſequentem.

Ut A. B :: C. D. *ergò universè*, B. A :: D. C. *per* cor. 4, 5.

XIV. Compoſitio rationis, eſt ſumptio antecedentis cum conſequente, ceu unius, ad ipſam conſequentem.

ut A. B :: C. D. *ergò componendo*, A + B. B :: C + D. D. *per* 18. 5.

XV. Diviſio rationis, eſt ſumptio exceſsûs, quo conſequentem ſuperat antecedens, ad ipſam conſequentem.

ut

Ut A. B :: C. D. *ergò dividendo*, A-B. B :: C-D. D. *per* 17. 5.

XVI. Conversio rationis, est sumptio antecedentis ad excessum, quo superat antecedens ipsam consequentem.

Ut A. B :: C. D. *ergò per conversam rationem*, A. A-B :: C. C-D. *per* cor. 19. 5.

XVII. Ex æqualitate ratio est, si plures duabus sint magnitudines, & his aliæ multitudine pares, quæ binæ sumantur, & in eadem ratione; cùm ut in primis magnitudinibus prima ad ultimam, sic & in secundis magnitudinibus prima ad ultimam sese habuerit. Vel aliter: sumptio extremorum, per subductionem mediorum.

XVIII. Ordinata proportio est, cùm fuerit quemadmodum antecedens ad consequentem, ità antecedens ad consequentem: fuerit etiam ut consequens ad aliud quidpiam, ità consequens ad aliud quidpiam.

Ut si A. B :: D. E. *item* B. C :: E. F. *erit ex æquo* A. C :: D. F. *per* 22. 5.

XIX. Perturbata autem proportio est; cùm tribus positis magnitudinibus, & aliis, quæ sint his multitudine pares, ut in primis quidem magnitudinibus se habet antecedens ad consequentem, ità in secundis magnitudinibus antecedens ad consequentem: ut autem in primis magnitudinibus consequens ad aliud quidpiam, sic in secundis magnitudinibus aliud quidpiam ad antecedentem.

Ut si A. B :: F. G. *item* B. C :: E. F. *erit ex æquo perturbatè* A. C :: E. G. *per* 23. 5.

XX. Quotlibet magnitudinibus ordine positis; proportio primæ ad ultimam componitur ex proportionibus primæ ad secundam, & secundæ ad tertiam, & tertiæ ad quartam, & ità deinceps, donec extiterit proportio.

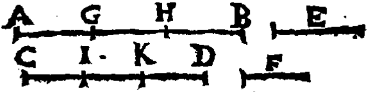
Sint

Sint quotcunque A, B, C, D; ex hac def. $\frac{A}{D} = \frac{A}{B} + \frac{B}{C} + \frac{C}{D}$.

Axioma.

Æquemultiplices eidem multiplici sunt quoq; inter se æquemultiplices.

Prop. I.

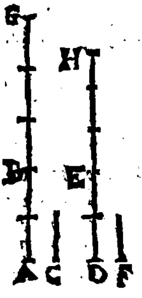


Si sint quotcunque magnitudines AB, CD *quotcunque magnitudinum* E, F *æqualium numero, singulæ singularum, æquemultiplices; quàm multiplex est unius* E *una magnitudo* AB, *tam multiplices erunt & omnes* AB+CD *omnium* E+F.

Sint AG, GH, HB partes quantitatis AB ipsi E æquales. item CI, IK, KD partes quantitatis CD ipsi F pares. Harum numerus illarum numero æqualis ponitur. Quum igitur
a 2. ax. AG+CI[a]=E+F;[a] & GH+IK=E+F;[a] & HB+KD=E+F, liquet AB+CD æquè multoties continere E+F, ac una AB unam E continet. Q. E. D.

PROP.

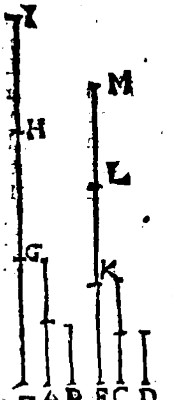
PROP. II.



Si prima AB secunda C æquè fuerit multiplex, atque tertia DE quartæ F; fuerit autem & quinta BG secundæ C æquè multiplex, atque sexta EH quartæ F, erit & composita prima cum quinta (AG) secundæ C æquè multiplex, atque tertia cum sexta (DH) quartæ F.

Numerus partium in AB ipsi C æqualium æqualis ponitur numero partium in DE ipsi F æqualium. Item numerus partium in BG ponitur æqualis numero partium in EH. [a] ergò numerus partium in AB+BG æquatur numero partium in DE+EH. hoc est tota AG æquemultiplex est ipsius C, atque tota GH ipsius F. Q. E. D. a 2. ax.

PROP. III.



Sit prima A secundæ B æquemultiplex, atque tertia C quartæ D; sumantur autem EI FM æquemultiplices primæ & tertiæ; erit & ex æquo, sumptarum utraque utriusque æquemultiplex: altera quidem EI secundæ B, altera autem FM quartæ D.

Sint EG, GH, HI partes multiplicis EI ipsi A pares; item FK, KL, LM partes multiplicis FM ipsi C æquales. [a]Harum numerus illarum numero æquatur. porrò A, id est EG, vel GH, vel GI ipsius B ponitur æquemultiplex atque C, vel FK &c. ipsius D. a hyp.

[b] ergò

b 2.5. [b] ergò EG + GH æquemultiplex est secundæ B, atque FK + KL quartæ D. [c] Simili argumento EI (EH + HI) tam multiplex est
c 2.5. ipsius B, quàm FM (FL + LN.) ipsius D. Q. E. D.

Prop. IIII.

Si prima A ad secundam B eandem habuerit rationem, & tertia C ad quartam D; etiam E & F æquemultiplices primæ A, & tertiæ C, ad G, & H æquemultiplices secundæ B, & quartæ D, juxta quamvis multiplicationem, eandem habebunt rationem, si prout inter se respondent, ità sumptæ fuerint. (E. G : : F. H.)

Sume I, & K ipsarum E, & F; item L & M ipsarum G, & H æquemultiplices. [a] Erit I
a 3.5. ipsius A æquemultiplex atque K ipsius C; [a] pariterque L tam multiplex ipsius B quàm M ipsius D. Itaque cùm sit A.
b hyp. B [b] : : C. D; juxta 6 def. si I $\sqsubset$, $=$, $\sqsupset$ L consequenter pari modo K $\sqsubset$, $=$, $\sqsupset$ M, ergò cùm I, & K ipsarum E, & F sumptæ sint æquemultiplices, atque L, & M ipsarum G & H; erit juxta 7. def. E.G : : F.H. Q.E.D.

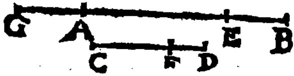
Coroll.

Hinc demonstrari solet inversa ratio.

Nam quoniam A. B : : C. D, si E $\sqsubset$, $=$, $\sqsupset$
c o. 6.def.5. G [c], erit similiter F $\sqsubset$, $=$, $\sqsupset$ H. ergò liquet, quòd

quòd si G ⊏, =, ⊐ E, esse H ⊏, =, ⊐ F.
[d] ergò B. A : : D. C. Q. E. D. d 6. *def.* 5.

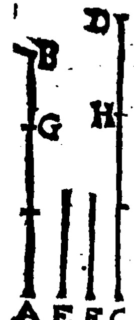
PROP. V.



Si magnitudo AB *magnitudinis* CD *æquè fuerit multiplex, atq; ablata* AE *ablatæ* CF; *etiam reliqua* EB *reliquæ* FD *ità multiplex erit, ut tota* AB *totius* CD.

Accipe aliam quandam GA, quæ reliquæ FD ità sit multiplex, atque tota AB totius CD, vel ablata AE ablatæ CF. [a] ergò tota GA + AE, totius CF; + FD æquemultiplex est, ac una AE unius CF. hoc est, ac AB ipsius CD. [b] ergò GE = AB. [c] proinde, ablatâ communi AB, manet GA = EB. ergò, &c. a 1. 5. b 6. *ax.* c 3. *ax.*

PROP. VI.



Si duæ magnitudines AB CD *duarum magnitudinum* E, F *sint æquemultiplices; & detractæ quædam sint*, AG & CH, *earundem* E, & F *æquemultiplices, & reliquæ* GB, HD *eisdem* E, F *aut æquales sunt, aut æquè ipsarum multiplices.*

Nam quia numerus partium in AB ipsi E æqualium ponitur æqualis numero partium in CD ipsi F æqualium. Item numerus partium in AG æqualis numero partium in CH. Si hinc AG, indè CH detrahatur, [a] remanet numerus partium in reliqua GB æqualis numero partium in HD. ergò si GB sit E semel, erit HD etiam C semel. Si GB sit E aliquoties, erit HD etiam C aliquòties accepta. Q. E. D. a 3. *ax.*

Prop. VII.

Æquales A & B ad eandem C eandem habent rationem; & eadem C ad æquales A & B.

Sumantur D, & E æqualium A, & B æquemultiplices; & F utcunque multiplex ipsius C,

a 6. ax. [a] erit D = E. quare si D $\sqsubset$, =, $\sqsupset$ F, erit simili-

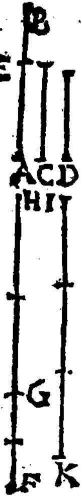
b 6. def. 5. ter E $\sqsubset$, =, $\sqsupset$ F. [b] ergò A. C : : B. C. invert.

c cor. 4 5. igitur C. A [c] : : C. B. Q. E. D.

Schol.

Si loco multiplicis F sumantur duæ æquemultiplices, eodem modo ostendetur æquales magnitudines ad alias inter se æquales eandem habere rationem.

Prop. VIII.



Inæqualium magnitudinum AB, C, *major* AB *ad eandem* D *majorem rationem habet, quàm minor* C. *Et eadem* D *ad minorem* C *majorem rationem habet, quàm ad majorem* AB.

Ex majori AB aufer AE = C. sumatur HG tam multiplex ipsius AE, vel C, quàm GF reliquæ EB. Multiplicetur D, donec ejus multiplex IK major evadat quàm HG, sed minor quàm HF.

a const. Quoniam HG ipsius AE [a] tam multiplex est, quàm GF ipsius EB, [b] erit

b 1. 5. tota HF totius AB æquemultiplex, atque una HG unius AE, vel C. ergò cùm HF $\sqsubset$ IK (quæ multiplex est

c 8. def. 5. ipsius D) sed HG $\sqsupset$ IK, [c] erit

$$\frac{AB}{D} \sqsubset \frac{C}{D}$$ Q. E. D.

Rursus

Rursus quia IK $\sqsubset$ HG, at IK $\sqsupset$ HF (ut priùs dictum) [d] erit $\frac{D}{C} \sqsubset \frac{D}{AB}$ Q. E. D.

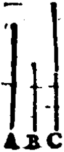
Prop. IX.

Quæ ad eandem eandem habent rationem, æquales sunt inter se. Et ad quas eadem eandem habet rationem, eæ quoque sunt inter se æquales.

1. *Hyp.* Sit A. C :: B. C. dico A = B. Nam sit A $\sqsubset$, vel $\sqsupset$ B, [a] erit ideò $\frac{A}{C} \sqsubset$, vel $\sqsupset \frac{B}{C}$ contra Hyp. a 8. 5.

2. *Hyp.* Sit C. B :: C. A. dico A = B. nam sit A $\sqsubset$ B. [b] ergò $\frac{C}{B} \sqsubset \frac{C}{A}$ contra Hyp. b 8. 5.

Prop. X.

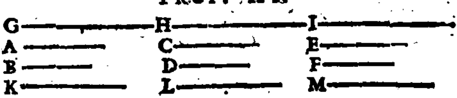


Ad eandem magnitudinem rationem habentium, quæ majorem rationem habet, illa major est: ad quam verò eadem majorem rationem habet, illa minor est.

1. *Hyp.* Sit $\frac{A}{C} \sqsubset \frac{B}{C}$. Dico A $\sqsubset$ B. Nam si dicatur A = B, [a] erit A. C :: B. C. contra Hyp. Sin A $\sqsupset$ B, [b] erit $\frac{A}{C} \sqsupset \frac{B}{C}$ etiam contra (Hyp. a 7. 5. b 8. 5.

2. *Hyp.* Sit $\frac{C}{B} \sqsubset \frac{C}{A}$ Dico B $\sqsupset$ A. Nam dic B = A. [c] ergò C. B :: C. A. contra Hyp. vel dic B $\sqsubset$ A. [d] ergò $\frac{C}{A} \sqsubset \frac{C}{B}$ etiam contra Hyp. c 7. 5. d 8. 5.

Prop. XI.



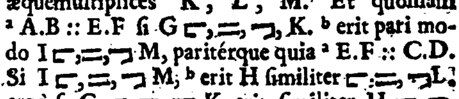
Quæ eidem sunt eædem rationes, & inter se sunt eædem.

Sit A.B :: E.F. item C.D :: E.F. dico A.B :: C.D. sume ipsarum A, C, E æquemultiplices, G, H, I; atque ipsarum B, D, F
a hyp. æquemultiplices K, L, M. Et quoniam
b 6. def. 5. [a] A.B :: E.F si G $\sqsubset$, =, $\sqsupset$, K. [b] erit pari modo I $\sqsubset$, =, $\sqsupset$ M, pariterque quia [a] E.F :: C.D. Si I $\sqsubset$, =, $\sqsupset$ M; [b] erit H similiter $\sqsubset$, =, $\sqsupset$ L;
c 6. def. 5. ergò si G $\sqsubset$, =, $\sqsupset$ K, erit similiter H $\sqsubset$, =, $\sqsupset$, L. [c] quare A.B :: C.D. Q.E.D.

Schol.

Quæ eisdem rationibus sunt eædem rationes, sunt quoque inter se eædem.

Prop. XII.



Si sint magnitudines quotcunque A, & B; C & D; E, & F *proportionales; quemadmodum se habuerit una antecedentium* A *ad unam consequentium* B, *ità se habebunt omnes antecedentes*, A, C, E *ad omnes consequentes*. B, D, F.

Sume antecedentium æquemultiplices G, H, I; & consequentium K, L, M. Quoniam quàm
a 1. 5. multiplex est una G unius A, [a] tam multiplices sunt omnes G, H, I omnium A, C, E; pariterque quàm multiplex est una K unius B, [a] tam multiplices sunt omnes K, L, M omnium B, D, F; Si G $\sqsubset$, =, $\sqsupset$ K, erit similiter

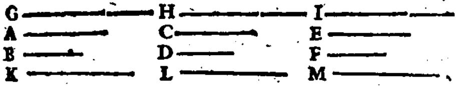
G+

$G+H+I \sqsubset, =, \sqsupset K+L+M$. [b] quare b *6. de* 5.
$A.B :: A+C+E.\ B+D+F$. Q. E. D.

Coroll.

Hinc, si similia proportionalia similibus proportionalibus addantur, tota erunt proportionalia.

PROP. XIII.



Si prima A *ad secundam* B *eandem habuerit rationem, quàm tertia* C *ad quartam* D; *tertia verò* C *ad quartam* D *majorem habuerit rationem; quàm quinta* E *ad sextam* F; *prima quoque* A *ad secundam* B *majorem rationem habebit, quàm quinta* E *ad sextam* F.

Sume ipsarum A, C, E æquemultiplices G, H, I: ipsarúmque B, D, F æquemultiplices K, L, M. Quia $A.B :: C.D$; Si $H \sqsubset L$, [a] erit a *6. def* 5.
$G \sqsubset K$. Sed quia $\frac{C}{D} \sqsubset \frac{E}{F}$, [b] fieri potest ut sit b *8. def.* 5
$H \sqsubset L$, & I non $\sqsubset M$. ergò fieri potest ut $G \sqsubset K$, & I non $\sqsubset M$. [c] ergò $\frac{A}{B} \sqsubset \frac{E}{F}$. Q.E.D. c *8. def.* 5.

SCHOL.

Quòd si $\frac{C}{D} \sqsupset \frac{E}{F}$, erit quoq; $\frac{A}{B} \sqsupset \frac{E}{F}$. Item si $\frac{A}{B} \sqsubset \frac{C}{D} \sqsubset \frac{E}{F}$. erit $\frac{A}{B} \sqsubset \frac{E}{F}$. & si $\frac{A}{B} \sqsupset \frac{C}{D} \sqsupset \frac{E}{F}$ erit $\frac{A}{B} \sqsupset \frac{E}{F}$.

PROP. XIV.

Si prima A *ad secundam* B *eandem habuerit rationem, quàm tertia* C *ad quartam* D; *prima verò* A, *quàm tertia* C *major fuerit, erit & secunda* B *major quàm quarta* D. *Quòd si prima* A *fuerit æqualis tertiæ* C, *erit & secunda* B *æqualis quartæ* D; *si verò* A *minor, &* B *minor erit.*

a 8.5.
b *hyp.*
c 13.5.
d 10.5.
e 7.5.
f *hyp.*
g 4.5. & 9.5.

Sit A $\sqsubset$ C. [a] ergò $\frac{A}{B} \sqsubset \frac{C}{B}$. [b] sed A BCD $\frac{A}{B} = \frac{C}{D}$. [c] ergò $\frac{C}{D} \sqsubset \frac{C}{B}$. [d] ergò B $\sqsubset$ D. Simili argumento si A $\sqsupset$ C, [d] erit B $\sqsupset$ D. Sin ponatur A $=$ C; ergò C. B [e] :: A. B [f] :: C. D. [g] ergò B $=$ D. Quæ E. D.

SCHOL.

A fortiori, si $\frac{A}{B} \sqsupset \frac{C}{D}$, atque A $\sqsubset$ C, erit B $\sqsubset$ D. Item si A $=$ B, erit C $=$ D. Et si A $\sqsubset$, vel $\sqsupset$ B, erit pariter C $\sqsubset$, vel $\sqsupset$ D.

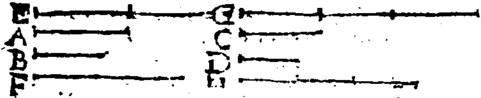
PROP. XV.

Partes C & F *cum pariter multiplicibus* AB, & DE *in eadem sunt ratione; si prout sibi mutuò respondent, ità sumantur.* (AB. DE :: C. F.).

a *hyp.*
b 7.5.
c 12.5.

Sint AG, GB partes multiplicis AB ipsi C æquales: item DH, HE partes multiplicis DE ipsi F æquales. [a] Harum numerus illarum numero æquatur. ergò quum [b] AG. C :: ACDF DH. F; [b] atq; GB. C :: HE. F. [c] erit AG + GB (AB). DH + HE (DE) :: C. F. Q. E. D.

PROP. XVI.



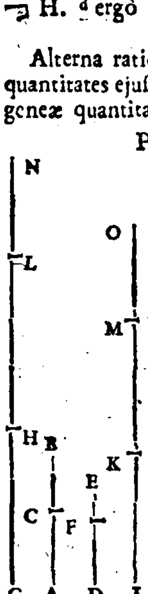
Si quatuor magnitudines A, B, C, D *proportionales fuerint; & vicissim proportionales erunt.* (A. C :: B. D.)

Accipe E, & F æquemultiplices ipsarum A, & B. ipsarúmque C, & D. æquemultiplices G, & H. Itaque E. F [a] :: A. B. [b] :: C.D [a] :: G.H. a 5. 5. b hyp.
Quare si E ⊏, =, ⊐ G, [c] erit similiter F ⊏, =, ⊐ H. [d] ergò A. C :: B. D. Q. E. D. c 11. 5. & 14. 5. d 6. def. 5.

SCHOL.

Alterna ratio locum tantùm habet, quando quantitates ejusdem sunt generis. Nam Heterogeneæ quantitates non comparantur.

PROP. XVII.

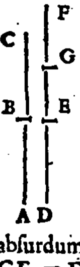


Si compositæ magnitudines proportionales fuerint (AB. CB :: DE. FE) *hæ quoque divisæ proportionales erunt.* (AC. CB :: DF. FE.)

Accipe GH, HL, IK, KM ordine æquemultiplices ipsarum AC, CB, DF, FE, item LN, MO æquemultiplices ipsarum CB, FE. Tota GL totius AB, [a] tam multiplex est, quàm una GH unius AC, [b] id est quàm IK ipsius DF; [c] hoc est quàm tota IM. totius DE: Item HN (HL+LN) ipsius CB [d] æquemultiplex est, ac KO (KM+MO) ipsius FE. Quum igitur per hyp. AB. BC :: DE. EF. Si GL ⊏, =, ⊐ HN, etiam similiter a 1. 5. b constr. c 1. 5. d 2. 5.

. def. 5. militer [e] erit IM ⊏, =, ⊐ KO. aufer hinc inde æquales HL, KM, si reliqua GH ⊏, =, ⊐
. ax. LN, [f] erit similiter IK ⊏, =, ⊐ MO, [g] un-
ƒ def. 5. de AC. CB :: DF. FE. Q. E D.

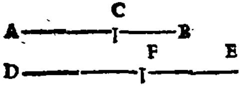
Prop. XVIII.



Si divisæ magnitudines sint proportionales (AB. BC :: DE. EF), *hæ quoq; compositæ proportionales erunt* (AC. CB :: DF. FE.).

7. 5. Nam si fieri potest, sit AB. CB :: DF. FG ⊐ FE. [a] ergò erit divisim
yp. & AB. BC :: DG. GF. [b] hoc est DG.
5. GF :: DE. EF. ergò cùm DG ⊏ DE.
4. 5. [c] erit GF ⊏ EF. Q. E. A. Simile
). ax. absurdum sequetur, si dicatur AB. CB :: DE. GF ⊏ FE.

Prop. XIX.



Si quemadmodum totum AB *ad totum* DE *ità ablatum* AC *se habuerit ad ablatum* DF; *& reliquum* CB *ad reliquum* FE, *ut totum* AB *ad totum* DE, *se habebit.*

yp. Quoniam [a] AB. DE :: AC. DF, [b] erit per-
6. 5. mutando AB. AC :: DE. DF. [c] ergò divisim
7. 5. AC. CB :: DF. FE. quare rursus [b] permutan-
yp. & do AC. DF :: CB. FE; [d] Hoc est AB DE ::
5. CB FE. Q. E. D,

Coroll.

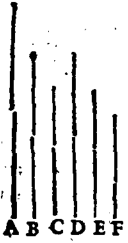
1. Hinc, si similia proportionalia similibus proportionalibus subducantur, residua erunt proportionalia.

2. *Hinc, demonstrabitur conversa ratio.*

Sit AB. CB :: DE. FE. Dico AB. AC ::
6. 5. DE. DF. Nam [a] permutando AB DE :: CB.
9. 5. FE. [b] ergò AB. DE :: AC. DF. quare iterum permutando, AB. AC :: DE. DF. Q. E. D.

Prop.

Prop. XX.



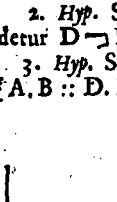
Si sint tres magnitudines A, B, C; *& aliæ* D, E, F *ipsis æquales numero, quæ binæ & in eadem ratione sumantur* (A.B :: D.E; *atque* B.C :: E.F); *ex æquo autem prima* A *major fuerit, quàm tertia* C; *erit & quarta* D *major quàm sexta* F. *Quòd si prima* A *tertiæ* C *fuerit æqualis; erit & quarta* D *æqualis sextæ* F. *Sin illa minor, hæc quoque minor erit.*

1. *Hyp.* Si A $\sqsubset$ C. Quoniam [a] E.F :: B.C. [b] erit inversè F.E :: C.B. [c] Sed $\frac{C}{B} \sqsupset \frac{A}{B}$ [d] ergò $\frac{F}{E} \sqsupset \frac{A}{B}$ vel $\frac{D}{E}$. [e] ergò D $\sqsubset$. F. Q.E.D. a *hyp.* b *cor* 4.5. c *hyp* & 8.5. d *schol* 1 e 10.5

2. *Hyp.* Simili argumento, Si A $\sqsupset$ C, ostendetur D $\sqsupset$ F.

3. *Hyp.* Si A = C. Quoniam F.E :: C.B :: [f] A.B :: D.E. [g] erit D = F. Q.E.D. f 7.5. g 11.5. & 9.5.

Prop. XXI.



Si sint tres magnitudines A, B, C; *& aliæ* D, E, F *ipsis æquales numero, quæ binæ & in eadem ratione sumantur, fueritque perturbata eorum proportio,* (A.B :: E.F. *atque* B.C :: D.E.); *ex æquo autem prima* A *quàm tertia* C *major fuerit; erit & quarta* D *quàm sexta* F *major; Quòd si prima fuerit tertiæ æqualis, erit & quarta æqualis sextæ; sin illa minor, hæc quoque minor erit.*

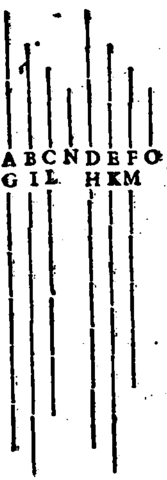
1. *Hyp.* A $\sqsubset$ C. Quoniam [a] D.E :: B.C, invertendo erit E.D :: C.B. atqui $\frac{C}{B}$ [b] $\sqsupset \frac{A}{B}$. [c] ergò a *hyp.* b 8.5

c *schol.* 13. 5. [c] ergò $\frac{E}{D} \sqsupset \frac{A}{B}$; hoc est $\frac{E}{F}$. [d] ergò $D \sqsubset F$.
d 10. 5. Q. E. D.

2. *Hyp.* Similiter, Si $A \sqsupset C$. erit $D \sqsupset F$.

e 7. 5. 3. *Hyp.* Si $A = C$. Quoniam E.D [e] :: C.B ::
f *hyp.* [e] A.B :: [f] E.F. [g] erit $D = F$. Q. E. D.
g 9. 5.

Prop. XXII.



Si sint quotcunque magnitudines A, B, C; *& aliæ ipsis æquales numero* D, E, F, *quæ binæ & in eadem ratione sumantur* (A.B :: D.E: & B.C :: E.F); *& ex æqualitate in eadem ratione erunt* (A.C :: D.F.).

Accipe G, H ipsarum A, D; & I, K ipsarum B, E; item L, M ipsarum E, F æquemultiplices.

a *hyp.* Quoniam [a] A. B :: D. E.
b 4. 5. [b] erit G. I :: H. K. eodem modo, erit, I. L :: K. M. ergò si $G \sqsubset, =, \sqsupset L$, [c] erit
c 20. 5. $H \sqsubset, =, \sqsupset M$; [d] ergò A.C ::
d 6. *def.* 5. D. F. Eodem pacto si ulteriùs C. N :: F. O; erit ex æquali A. N :: D. O. Q. E. D.

PROP. XXIII.

Si sint tres magnitudines A,B,C; *aliæque* D, E, F *ipsis æquales numero, quæ binæ in eadem ratione sumantur, fuerit autem perturbata earum proportio.* (A.B :: E.F. & B.C :: D.E.) *etiam ex æqualitate in eadem ratione erunt.*

Sume G,H,I ipsarum A,B,D; item K, L, M ipsarum C, E, F æquemultiplices. erit G.H [a] :: A.B [b] :: E.F [a] :: L.M. porrò quia [b] B.C :: D.E, erit [c] H.K :: I.L. ergò G,H,K; & I,L,M habent se juxta 21. 5. quare si G ⊏, =, ⊐ K, erit similiter I ⊏, =, ⊐ M. [d] proinde A.C :: D.F. Q.E.D.

a 15. 5.
b hyp.
c 4. 5.

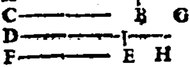
Eodem modo si plures fuerint magnitudinibus tribus, &c. — d 6. def. 5.

Coroll.

Ex* his sequitur, rationes ex iisdem rationibus compositas esse inter se easdem. item, earundem rationum easdem partes inter se easdem esse.

* 22. & 23. 5 & 20. def. 1.

PROP. XXIIII.



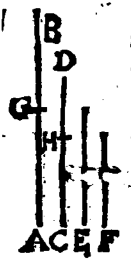
Si prima AB *ad secundam* C *eandem habuerit rationem quàm tertia* DE *ad quartam* F; *habuerit autem & quinta* BG *ad secundam* C *eandem rationem, quàm sexta* EH *ad quartam* F; *etiam composita prima cum quinta* (AG) *ad secundam* C *eandem habebit rationem, quàm tertia cum sexta* (DH) *ad quartam* F.

Nam quia [a] AB. C :: DE. F. atque ex hyp. & inversè C. BG :: F. EH, erit [b] ex æquali AB. BG :: DE. EH. ergò componendo AG. BG :: DH. EH. [c] item BG. C :: EH. F. [b] ergò rursus ex æquo, AG. C :: DH. F. Q.E.D.

a hyp.
b 22. 5.
c hyp.

PROP.

PROP. XXV.



Si quatuor magnitudinis proportionales fuerint (AB. CD :: E. F.); *maxima* AB, *& minima* F *reliquis* CD, & E *majores erunt.*

Fiant AG $=$ E ; & CH $=$ F.
Quoniam A B. C D [a] :: E. F [b] ::
AG. CH. [c] erit AB. CD :: GB.
hyp. HD. [d] sed A B $\sqsubset$ C D. [e] ergò
7. 5. GB $\sqsubset$ HD. atqui AG $+$ F $=$ E $+$
19. 5. CH. ergò AG $+$ F $+$ GB $\sqsubset$ E $+$
hyp. CH $+$ HD. hoc est AB $+$ F $\sqsubset$ E $+$
schol. 14. 5. CD. Q. E. D.

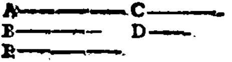
Quæ sequuntur propositiones non sunt Euclidis ; sed ex aliis desumptæ ob frequentem earum usum Euclidæis subjungi solent.

PROP. XXVI.

Si prima ad secundam habuerit majorem proportionem, quàm tertia ad quartam, habebit convertendo, secunda ad primam minorem proportionem, quàm quarta ad tertiam.

Sit $\frac{A}{B} \sqsubset \frac{C}{D}$. Dico $\frac{B}{A} \sqsupset \frac{D}{C}$. Nam concipe
13. 5. $\frac{C}{D} = \frac{E}{B}$. [a] ergò $\frac{A}{B} \sqsubset \frac{E}{B}$. [b] quare A $\sqsubset$. E. [c] ergò
10. 5. $\frac{B}{A} \sqsupset \frac{B}{E}$, [d] vel $\frac{D}{C}$. Q. E. D.
8. 5.
cor. 4. 5.

PROP. XXVII.



Si prima ad secundam habuerit majorem proportionem, quàm tertia ad quartam, habebit quoque vicissim prima ad tertiam majorem proportionem, quàm secunda ad quartam.

Sit

Sit $\frac{A}{B} \sqsubset \frac{C}{D}$. Dico $\frac{A}{C} \sqsubset \frac{B}{D}$. Nam puta $\frac{E}{B} = \frac{C}{D}$.

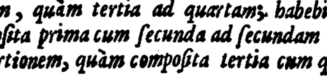
[a] ergò $A \sqsubset E$. [b] ergò $\frac{A}{C} \sqsubset \frac{E}{C}$ [c] vel $\frac{B}{D}$. Q.E.D. a 10. 5. b 8. 5. c 16. 5.

Prop. XXVIII.

Si prima ad secundam habuerit majorem proportionem, quàm tertia ad quartam; habebit quoque composita prima cum secunda ad secundam majorem proportionem, quàm composita tertia cum quarta ad quartam,

Sit $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. Dico $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Nam cogita $\frac{GB}{BC} = \frac{DE}{EF}$. [a] ergò $AB \sqsubset GB$. adde utrinque BC, a 10. 5.
[b] erit $AC \sqsubset GC$. [c] ergò $\frac{AC}{BC} \sqsubset \frac{GC}{BC}$ [d] hoc est $\frac{DF}{FE}$. b 4. ax. c 8. 5. d 18. 5.
Q. E. D.

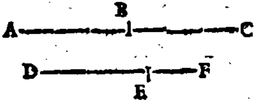
Prop. XXIX.



Si composita prima cum secunda ad secundam majorem habuerit proportionem, quàm composita tertia cum quarta ad quartam, habebit quoque dividendo prima ad secundam majorem proportionem quàm tertia ad quartam.

Sit $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Dico $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. Intellige $\frac{GC}{BC} = \frac{DE}{EF}$. [a] ergò $AC \sqsubset GC$. aufer commune a 10. 5.
BC, [b] erit $AB \sqsubset GB$. [c] ergò $\frac{AB}{BC} \sqsubset \frac{GB}{BC}$ [d] vel $\frac{DE}{EF}$. b 5. ax. c 8. 5. d 17. 5.
Q. E. D.

Prop. XXX.



Si composita prima cum secunda ad secundam habuerit majorem proportionem, quàm composita tertia cum quarta ad quartam; Habebit, per conversionem rationis, prima cum secunda ad primam minorem rationem, quàm tertia cum quarta ad tertiam.

Sit $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Dico $\frac{AC}{AB} \sqsupset \frac{DF}{DE}$. Nam quia

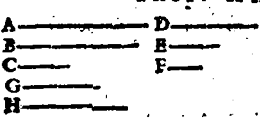
hyp. $\frac{AC}{BC}$ [a] $= \frac{DF}{EF}$. [b] erit dividendo $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. [c] conver-

29. 5. tendo igitur $\frac{BC}{AB} \sqsupset \frac{EF}{DE}$. [d] ergò componendo

26. 5.

28. 5. $\frac{AC}{AB} \sqsupset \frac{DF}{DE}$. Q. E. D.

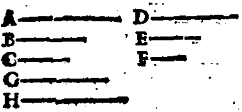
Prop. XXXI.



Si sint tres magnitudines A, B, C, & aliæ ipsis æquales numero D, E, F, sitque major proportio primæ priorum ad secundam, quàm primæ posteriorum ad secundam ($\frac{A}{B} \sqsubset \frac{D}{E}$); *item secundæ priorum ad tertiam major, quàm secundæ posteriorum ad tertiam* ($\frac{B}{C} \sqsubset \frac{E}{F}$) *Erit quoque ex æqualitate major proportio primæ priorum ad tertiam, quàm primæ posteriorum ad tertiam* ($\frac{A}{C} \sqsubset \frac{D}{F}$)

10. 5.
8. 5.
13. 5. Concipe $\frac{G}{C} = \frac{E}{F}$. [a] ergò $E \sqsubset G$. [b] ergò $\frac{A}{G} \sqsubset \frac{A}{E}$.
10. 5.
8. 5. Rursus puta $\frac{H}{G} = \frac{D}{E}$. [c] ergò $\frac{H}{G} \sqsupset \frac{A}{E}$; [d] ergò fortiùs
22. 5. $\frac{H}{G} \sqsupset \frac{A}{G}$. [d] quare $A \sqsubset H$. [e] proinde $\frac{A}{C} \sqsupset \frac{H}{C}$; vel $\frac{D}{F}$. Q. E. D.

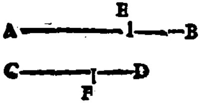
Prop. XXXII.



Si sint tres magnitudines A,B,C; *& aliæ ipsis æquales* D,E,F, *sitque major proportio primæ priorum ad secundam, quàm secundæ posteriorum ad tertiam;* $\left(\frac{A}{B} \sqsubset \frac{E}{F}\right)$ *item secundæ priorum ad tertiam major; quàm primæ posteriorum ad secundam;* $\left(\frac{B}{C} \sqsubset \frac{D}{E}.\right)$ *erit quoque ex æqualitate major proportio primæ priorum ad tertiam, quàm primæ posteriorum ad tertiam.* $\left(\frac{A}{C} \sqsubset \frac{D}{F}.\right)$

Hujusce demonstratio planè similis est demonstrationi præcedentis.

Prop. XXXIII.



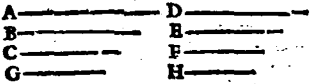
Si fuerit major proportio totius AB *ad totum* CD, *quàm ablati* AE *ad ablatum* CF. *Erit & reliqui* EB *ad reliquum* FD *major proportio, quàm totius* AB *ad totum* CD. a *hyp.*

Quoniam $\frac{AB}{CD}$ [a] $\sqsubset \frac{AE}{CF}$, [b] erit permutando b 27. 5. c 30. 5.

$\frac{AB}{AE} \sqsubset \frac{CD}{CF}$. [c] ergò per conversionem rationis $\frac{AB}{EB} \sqsupset \frac{CD}{FD}$. permutando igitur $\frac{AB}{CD} \sqsupset \frac{EB}{FD}$.

Q. E. D.

Prop. XXXIV.

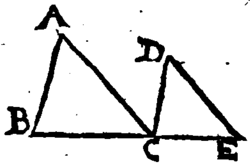


Si sint quocunque magnitudines, & aliæ ipsis æquales numero, sitque major proportio primæ priorum ad primam posteriorum, quàm secundæ ad secundam, & hæc major quàm tertiæ ad tertiam, & sic deinceps: habebunt omnes priores simul ad omnes posteriores simul, majorem proportionem, quàm omnes priores, relictâ primâ, ad omnes posteriores, relictâ quoque primâ; minorem autem, quàm prima priorum ad primam posteriorum; majorem deniq; etiam, quàm ultima priorum ad ultimam posteriorum.

Horum demonstratio est penes interpretes. quos adeat, qui eam desiderat. nos omisimus, brevitatis studio, & quia illorum nullus usus in his elementis.

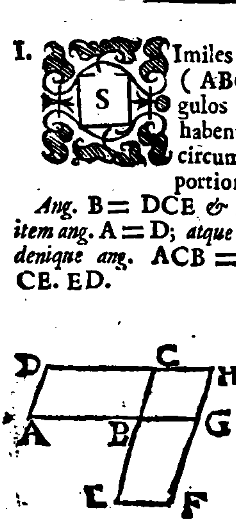
LIB. VI.

Definitiones.

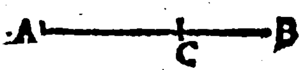


I. Similes figuræ rectilineæ sunt (ABC, DCE), quæ & angulos singulos singulis æquales habent; atque etiam latera, quæ circum angulos æquales, proportionalia.

Ang. B = DCE *&* AB. BC : : DC. CE. *item ang.* A = D; *atque* BA. AC : : CD. DE. *denique ang.* ACB = E. *atque* BC. CA : : CE. ED.

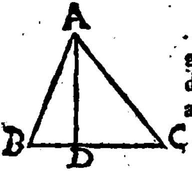


II. Reciprocæ autem sunt (BD, BF), cùm in utraque figura antecedentes, & consequentes rationum termini fuerint. (hoc est AB. BG : : EB. BC.)



III. Secundùm extremam & mediam rationem recta linea AB secta, esse dicitur, cùm ut tota AB ad majus segmentum AC, ità majus segmentum AC ad minus CB se habuerit. (AB. AC : : AC. CB.)

I. 4 IV. Alti-

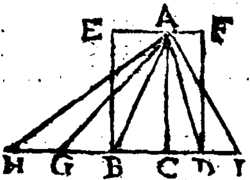


IV. Altitudo cujusq; figuræ ABC est linea perpendicularis AD, à vertice A ad basim BC deducta.

V. Ratio ex rationibus componi dicitur, cùm rationum quantitates inter se multiplicatæ, aliquam effecerint rationem.

Ut ratio A *ad* C, *componitur ex rationibus* A
a 20. *def.* 5. *ad* B, *&* B *ad* C. *nam* $\frac{A}{B} + \frac{B}{C}$ [a] $= \frac{A}{C}$ [b] $= \frac{AB}{BC}$.
b 15.5.

PROP. I.



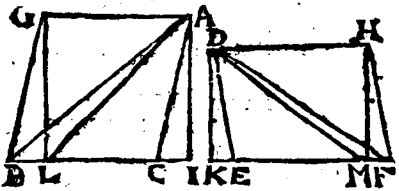
Triangula ABC, ACD, *&* *parallelogramma* BCAE, CDFA, *quorum eadem fuerit altitudo, ità se habent inter se, ut bases* BC, CD.

a 3. 1. [a] Accipe quotvis BG, HG ipsi BC æquales; idem DI = CD. & connecte AG, AH, AI.

b 38. 1. [b] Triangula ACB, ABG, AGH æquantur; [b] item triang. ACD = ADI. ergò triangulum ACH tam multiplex est trianguli ACB, quàm basis HC, basis BC. & æquemultiplex est triang. ACI trianguli ACD, ac basis CI basis CD.
c sch. 38. 1. cùm igitur si HC $\sqsubset, =, \sqsupset$ CI, [c] erit similiter
d 6. def. 5. triang. AHC $\sqsubset, =, \sqsupset$ ACI, [d] ideóque BC.
e 41. 1. & CD : : triang. ABC. ACD : : [e] pgr. CE. CF.
15. 5. Q. E. D.

Schol.

Scholi.

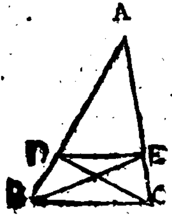


Hine, *triangula* ABC, DEF, *&* *parallelogramma* AGBC, DEFH, *quorum æquales sunt bases* BC, EF, *ità se habent ut altitudines* AI, DK.

[a] Sume IL = CB; & KM = EF; ac junge LA, LG, MD, MH. liquet esse triang. ABC. DEF :: [b] ALI. DKM :: [c] AI. DK :: [d] pgr. AGBC. DEFH. Q. E. D.

a 3. 1.
b 7. 5.
c 1. 6.
d 41. 1. & 15. 5.

Prop. II.



Si ad unum trianguli ABC, *latus* BC *parallela ducta fuerit recta quædam linea* DE, *hæc proportionaliter secabit ipsius trianguli latera* (AD. BD :: AE. EC). *Et si trianguli latera proportionaliter secta fuerint* (AD. BD :: AE. EC) *quæ ad sectiones* D, E *adjuncta fuerit recta linea* DE, *erit ad reliquum ipsius trianguli latus* BC *parallela.* Ducantur CD, BE.

1. *Hyp.* Quia triang. DEB [a] = DEC; [b] erit triang. ADE. DBE :: ADE. ECD. atqui triang. ADE. DBE [c] :: AD. DB. & triang. ADE. DEC [c] :: AE. EC. [d] ergò AD. DB :: AE. EC.

a 37. 1.
b 7. 5.
c 1. 6.
d 11. 5.

2. *Hyp.* Quia AD. DB :: AE. EC. [e] hoc est triang. ADE. DBE :: ADE. ECD; [f] erit triang. DBE = ECD. [g] ergò DE, BC sunt parallelæ. Q. E. D.

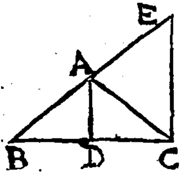
e 1. 6.
f 9. 5.
g 39. 1.

Schol.

Schol.

Imò, si plures ad unum trianguli latus parallelæ ductæ fuerint, erunt omnia laterum segmenta proportionalia, ut facilè deducitur ex hac.

PROP. III.



Si trianguli BAC *angulus* BAC *bifariam sectus sit, secans autem angulum recta linea* AD *secuerit & basim, basis segmenta eandem habebunt rationem, quam reliqua ipsius trianguli latera* (BD, DC : : AB. AC.) *Et si basis segmenta eandem habeant rationem quam reliqua ipsius trianguli latera* (BD. DC : : AB. AC.) *recta linea* AD *quæ à vertice* A *ad sectionem* D *ducitur, bifariam secat trianguli ipsius angulum* BAC.

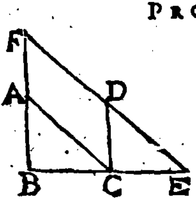
Produc BA; & fac AE = AC. & junge CE.

1. *Hyp.* Quoniam AE = AC, erit ang. ACE [a] = E [b] = ½ BAC [c] = DAC. [d] ergò DA, CE parallelæ sunt. [e] quare BA. AE (AC) : : BD. DC. Q.E.D.

2. *Hyp.* Quoniam BA. AC. (AE) : : BD. DC [f] erunt DA, CE parallelæ: [g] ergò ang. BAD = E; & ang. DAC [g] = ACE [h] = E, [k] ergò ang. BAD = DAC. bisectus igitur est ang. BAC. Q.E.D.

a 5. 1.
b 32. 1.
c *hyp.*
d 27. 1.
e 2. 6.
f 2. 6.
g 29. 1.
h 5. 1.
k 1. *ax.*

PROP. IV.



Æquiangulorum triangulorum ABC, DCE *proportionalia sunt latera, quæ circum æquales angulos* B, DCE (AB. BC : : DC. CE, &c.) *& homologa sunt latera* AB, DC &c. *quæ æqualibus angulis* ACB, E &c. *subtenduntur.*

Statue

Statue latus BC in directum lateri CE, & produc BA, ac ED donec [a] occurrant. a 32. 1. & 13. ax.

Quoniam ang. B [b] = ECD, [c] sunt BF, CD parallelæ. Item quia ang. BCA [b] = CED [c] sunt CA. EF parallelæ. Figura igitur CAFD est parallelogramma. [d] ergò AF = CD; [d] & AC = FD. Liquet igitur AB. AF (CD) [e] : : BC. CE. [f] permutando igitur AB. BC : : CD. CE, [e] item BC. CE : : FD. (AC) DE [f] ergò permutando BC. AC : : CE. DE. quare etiam [g] ex æquo AB. AC : : CD. DE. ergò, &c. b hyp. c 28. 1. d 34. 1. e 2. 6. f 16. 5. g 22. 5.

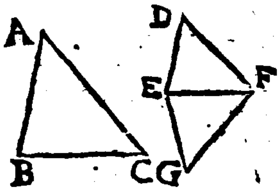
Coroll.

Hinc AB. DC : : BC. CE : : AC. DE.

Schol.

Hinc si in triangulo FBE ducatur uni lateri FB parallela AC; erit triangulum ABC simile toti FBE.

PROP. V.

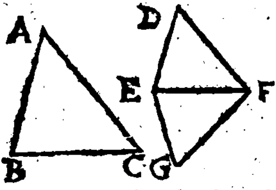


Si duo triangula ABC, DEF *latera proportionalia habeant* (AB. BC : : DE. EF *&* AC BC : : DF. EF. *item* AB.AC : : DE DF) *æquiangula erunt triangula, & æquales habebunt eos angulos, sub quibus homologa latera subtenduntur.*

Ad latus EF [a] fac ang. FEG = B; [a] & ang. EFG = C, [b] quare etiam ang. G = A. ergò GE. EF [c] : : AB. BC : : [d] DE. EF [e] ergò GE = DE. Item GF. FE [c] : : AC. CB [d] : : DF. FE. [e] ergò GF = DF. Triangula igitur DEF. GEF sibi mutuò æquilatera sunt. [f] ergò ang. D = G = A. [f] & ang. FED = FEG = B. [g] proinde & ang. DFE = C. ergò &c. a 23. 1. b 32. 1. c 4. 6. d hyp. e 11. 5. & 9. 5. f 8. 1. g 32. 1.

PROP.

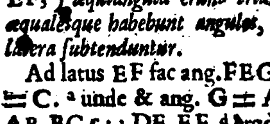
PROP. VI.



Si duo triangula ABC, DEF *unum angulum* B *uni angulo* DEF *æqualem, & circum æquales angulos* B, DEF *latera proportionalia habuerint* (AB. BC : : DE. EF;) *æquiangula erunt triangula* ABC, DEF; *æqualesque habebunt angulos, sub quibus homologa latera subtenduntur.*

a 32. 1. Ad latus EF fac ang. FEG = B; & ang. EFG
b 4. 6. = C. [a] unde & ang. G = A. ergò GE. EF[b] : :
c hyp. AB. BC[c] : : DE. EF [d] ergò DE = GE. atqui
d 9. 5. ang. DEF [e] = B [f] = GEF. [g] ergò ang. D =
e hyp. = G = A. [h] proinde etiam ang. EFD = C.
f constr. Q. E. D.
g 4. 1.
h 32. 1.

PROP. VII.



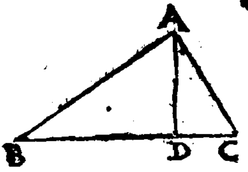
Si duo triangula ABC, DEF *unum angulum* A *uni angulo* D *æqualem, circa autem alios angulos* ABC, E *latera proportionalia habeant* (AB. BC : : DE. EF); *reliquorum autem simul utrumque* C, F *aut minorem, aut non minorem recto; æquiangula erunt triangula* ABC, DEF, *& æquales habebunt eos angulos, circum quos proportionalia sunt latera.*

hyp. Nam si fieri potest, sit ang. ABC ⊏ E. fac
32. 1. igitur ang. ABG = E ; ergò cùm ang. A [a] = D,
4. 6. [b] erit etiam ang. AGB = F. ergò AB. BG[c] : :
hyp. DE. EF : : AB. BC. [e] ergò BG = BC. [f] ergò
9. 5. ang. BGC : : BCG. [g] ergò ang. BGC. vel C
5. 1.
cor. 17. 1.

minor

mainor est recto; [g] proinde ang. AGB, vel F recto major est. ergò anguli C, & F non sunt ejusdem speciei, contra Hyp. g cor. 13. 1.

Prop. VIII.



Si in triangulo rectangulo ABC ab angulo recto BAC in basin BC perpendicularis AD ducta est; quæ ad perpendicularem triangula ADB, ADC, tum toti triangulo ABC, tum ipsa inter se similia sunt.

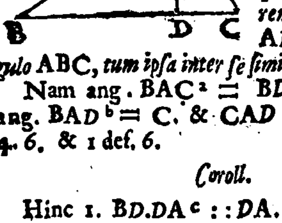
Nam ang. BAC [a] = BDA [a] = CDA. & ang. BAD [b] = C. & CAD [b] = B. ergò per 4. 6. & 1 def. 6. a 12. ax. b 32. 1.

Coroll.

Hinc 1. BD.DA [c] :: DA. DC. c 1. def. 6.

2. BC. AC :: AC. DC. & CB. BA. :: BA. BD.

Prop. IX.



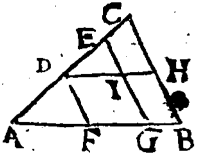
A data recta linea AB imperatam partem $\frac{1}{3}$ AG auferre.

Ex A duc infinitam AC utcunq;, in qua [a] sume tres, AD, DE, EF æquales utcunque, junge FB, cui ex D [b] duc parallelam DG. Dico factum. a 3. 1. b 31. 1.

Nam GB. AG [c] :: FD. AD. ergò [d] componendo AB.AG :: AF.AD. ergò cùm AD = $\frac{1}{3}$ AF, erit AG = $\frac{1}{3}$ AB. Q. E. F. c 2. 6. d 18. 5.

Prop.

Prop. X.

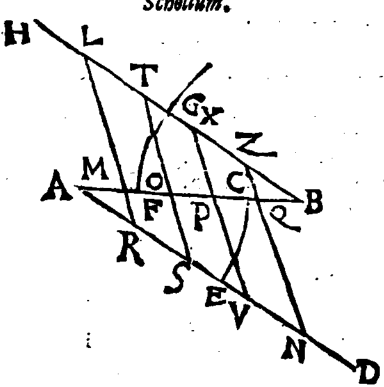


Datam rectam lineam AB *insectam similiter secare* (*in* F, G), *ut data altera* AC, *secta fuerit* (*in* D. E.)

Extremitates sectæ & insectæ jungat recta
a 31. 1. BC. Huic ex punctis E, D [a] duc parallelas EG, DF rectæ secundæ occurrentes in G, & F. Dico factum.

[a] Ducatur enim DH parall. AB. Estque AD.
b 2. 6. DE [b] :: AF. FG, & DE. EC [b] :: DI. IH [c] ::
c 34. 1. & 7. 5. FG. GB. Q. E. F.

Scholium.



Hinc *discimus rectam datam* AB *in quotvis æquales partes* (puta 5.) *secare*. id quod faciliùs præstabitur sic.

Duc infinitam AD, eiq; parallelam BH etiam infinitam. Ex his cape partes æquales AR, RS, SV, VN; & BZ, ZX, XT, TL; in singulis unâ

pau-

pauciores, quàm desiderentur in AB; tum rectæ ducantur LR, TS, XV, ZN. hæ quinquisecabunt datam AB.

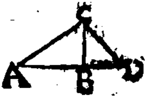
Nam RL, ST, VX, NZ [a] parallelæ sunt. ergò quum AR, RS, SV, VN [b] æquales sint, [c] erunt AM, MO, OP, PQ æquales. Similiter quia BZ=ZX, erit BQ=QP. ergò AB. quinquisecta est. Q. E. F. a 33. 1. b constr. c 2. 6.

Prop. XI.

Datis duabus rectis lineis AB AD. *tertiam proportionalem* DE *invenire.*

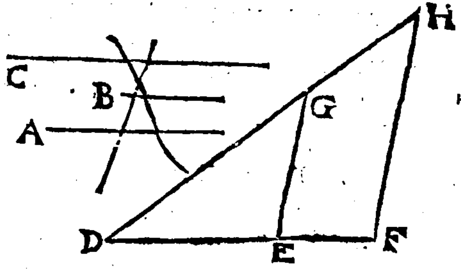
Junge BD, & ex AB protractâ sume BC = AD. per C duc CE parall. BD. cui occurrat AD producta in E. Erit DE expetita.

Nam AB. BC. (AD) :: AD. DE. Q. E. F. a 2. 6.



Vel sic, fac ang. ABC rectum & ang. ACD etiam rectum. [b] erit AB. BC :: BC. BD. b cor. 9.

PROP. XII.



Tribus datis rectis lineis DE, EF, DG, *quartam proportionalem* GH *invenire.*

Connectatur EG. per F duc FH parall. EG, cui occurrat DG producta ad H. liquet esse
a 2. 6. DE. EF [a] :: DG. GH. Q. E. F.

Vel itá. CD=CB+BD adapta circulo. Circino sume AB.
a 35. 3. [a] Erit AB×BE=CB×BD. [b] quare AB. CB :: BD. BE.
b 16. 6.

PROP. XIII.

Duabus datis rectis lineis AE, EB *mediam proportionalem* EF *adinvenire.*

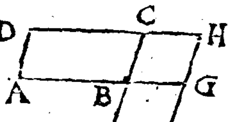
Super tota AB diametro describe semicirculum AFB. Ex E erige perpendicularem EF occurrentem peripheriæ in F. Dico AE. EF :: EF. EB. Ducantur enim AF, & FB. Ex trianguli [a] rectanguli

guli AFB recto angulo, deducta est FE basi perpendicularis; [b] ergo AE. FE :: FE. EB. *b cor. 8. 6.*
⌒. E. F.

Coroll.

Hinc, linea recta, quæ in circulo à quovis puncto diametri, ipsi diametro perpendicularis ducitur ad circumferentiam usque, media est proportionalis inter duo diametri segmenta.

Prop. XIV.



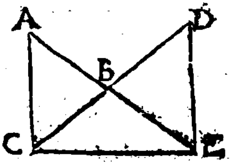
Æqualium, & unum ABC *uni* EBG *æqualem habentium angulum, parallelogrammorum* BD, BF *reciproca sunt latera, quæ circum æquales angulos.* (AB. BG :: EB. BC): *Et quorum parallelogrammorum* BD, BF *unum angulum* ABC *uni angulo* EBG *æqualem habentium, reciproca sunt latera, quæ circum æquales angulos, illa sunt æqualia.*

Nam latera AB, BG circa æquales angulos faciant unam rectam, [a] quare EB, BC etiam in directum jacebunt. Producantur FG, DC; donec occurrant. *a sch. 15. 1.*

1. *Hyp.* AB. BG [b] :: BD. BH [c] :: BF. BH [d] :: EB. BC. [e] ergò, &c. *b 1. 6. c 7. 5. d 1. 6. e 11. 5.*

2. *Hyp.* BD. BH [f] :: AB. BG[g] :: BE. BC [h] :: BF. BH. [k] ergò Pgr. BD = BF. Q. E. D. *f 1. 6. g hyp. h 1. 6. k 11. & 9. 5.*

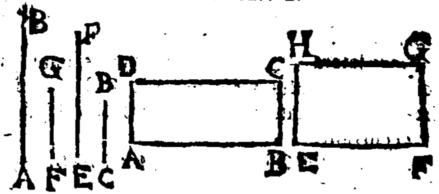
PROP. XV.



Æqualium, & unum ABC, *uni* DBE *æqualem habentiũ angulum triangulorũ* ABC, DBE, *reciproca sunt latera, quæ circum æquales angulos* (AB. BE :: DB. BC): *Et quorum triangulorum* ABC, DBE, *unum angulum* ABC *uni* DBE *æqualem habentium reciproca sunt latera, quæ circum æquales angulos* (AB. BE :: DB. BC), *illa sunt æqualia.*

Latera CB, BD circa æquales angulos, statuantur sibi in directum; [a] ergò ABE est recta
a *sch.* 15. 1. linea. ducatur CE.
b 1. 6. 1. *Hyp.* AB. BE [b] :: triang. ABC. CBE
c 7. 5. [c] :: triang. DBE. CBE. [d] :: DB. BC. [e] ergò, &c.
d 1. 6. 2. *Hyp.* Triang. ABC. CBE [f] :: AB. BE [g] ::
e 11. 5. DB. BC [h] :: triang. DBE. CBE. [k] ergò triang.
f 1. 6. ABC = DBE. Q. E. D.
g *hyp.*
h 1. 6.
k 11, & 9. 5.

PROP. XVI.



Si quatuor rectæ lineæ proportionales fuerint (AB. FG :: EF. CB), *quod sub extremis* AB, CB *comprehenditur rectangulum* AC *æquale est ei, quod sub mediis* EF, FG *comprehenditur, rectangulo* EG. *Et si sub extremis comprehensum rectangulum* AC *æquale fuerit ei, quod sub mediis comprehenditur, rectangulo* EG, *illæ quatuor rectæ lineæ proportionales erunt* (AB. FG :: EF. CB.).

1. *Hyp.*

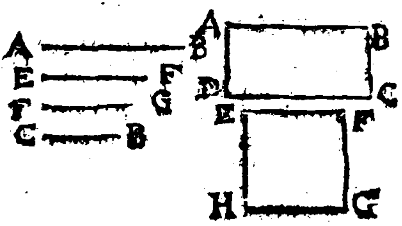
1. *Hyp.* Anguli B, & F recti, ac [a] proinde pares sunt; atque ex hyp. AB. FG :: EF. CB. [b] ergò Rectang. AC = EG. Q. E. D. — a 12. ax. — b 14. 6.

2. *Hyp.* [c] Rectang. AC = EG; atque ang. B = F; [d] ergò AB. FG :: EF. CB. Q. E. D. — c hyp. — d 14. 6.

Coroll.

Hinc ad datam rectam lineam AB facile est datum rectangulum EG applicare, [e] faciendo AB. EF :: FG. BC. — e 4, & 10. 6

PROP. XVII.



Si tres rectæ lineæ sint proportionales (AB. EF :: EF. CB), *quod sub extremis* AB, CB *comprehenditur rectangulum* AC *æquale est ei, quod à media* EF, *describitur, quadrato* EG. *Et si sub extremis* AB, CB *comprehensum rectangulum* AC, *æquale sit ei, quod à media* EF, *describitur, quadrato* EG, *illæ tres rectæ lineæ proportionales erunt* (AB. EF :: EF. CB).

Accipe FG = EF.

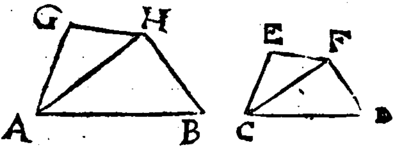
1. *Hyp.* AB. EF [a] :: EF (FG). CB. ergò Rectang. AC [b] = EG [c] = EFq. Q. E. D. — a hyp. — b 16. 6. — c 29. def. 1.

2. *Hyp.* Rectang. AC [d] = quadr. EG = EFq. [e] ergò AB. EF :: FG (EF). BC. — d hyp. — e 16. 6.

Coroll.

Sit A in B = Cq. ergò A. C :: C. B.

PROP. XVIII.

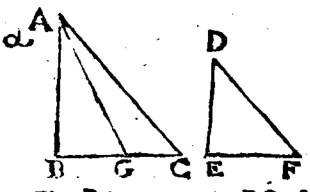


A data recta linea AB *dato rectilineo* CEFD. *simile similitérque positum rectilineum* AGHB *describere.*

Datum rectilineum resolve in triangula. [a] fac ang. ABH = D; [a] & ang. BAH = DCF;
23. 1. [a] & ang. AHG = CFE; [a] & ang. HAG = FCE. Rectilineum AGHB est quæsitum.

constr. Nam ang. B [b] = D. & ang. BAH [b] = DCF.
32. 1. [c] quare ang. AHB = CFD; [b] item ang. HAG = FCE, [b] & ang. AHG = CFE.. [c] quare ang.
3. ax. G = E; & totus ang. GAB [d] ECD; & totus GHB [d] = EFD. Polygona igitur sibi mutuò æquiangula sunt. Porrò, ob trigona æquiangu-
4. 6. la, AB. BH [e] :: CD. DF. & AG. GH. [e] :: CE. EF. item AG. AH. [e] :: CE. CF. & AH. AB [e] ::
22. 5. CF. CD. [f] unde ex æquo AG AB :: CE. CD.
6. def. 6. eodem modo GH. HB :: EF. FD. ergò polygona ABHG, CDFE similia similitérque posita existunt. Q. E. F.

PROP. XIX.



Similia triangula ABC, DEF *sunt in duplicata ratione laterum homologorum* BC EF.

11. 6. [a] Fiat BC. EF :: EF. BG. & ducatur AG.

Quia

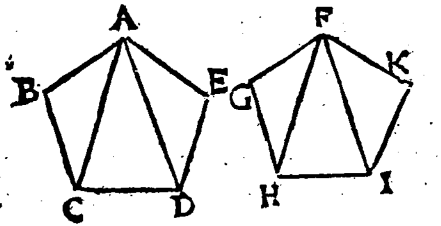
Quia AB.DE [b]:: BC.EF [c] :: EF.BG.& ang. B = E ; [d] erit triang. ABG = DEF. verùm triang. ABC. ABG [e] :: BC. BG ; & [f] $\frac{BC}{BG} = \frac{BC}{EF}$ bis; ergò triang. $\frac{ABC}{ABG}$ hoc est $\frac{ABC}{DEF}$ [g] = $\frac{BC}{EF}$ bis. Q. E. D.

b *cor.* 4. 6.
c *constr.*
d 15. 6.
e 1. 6.
f 10. *def* 5.
g 11. 5.

Coroll.

Hinc, si tres lineæ BC, EF, BG proportionales fuerint; ut est prima ad tertiam, ità est triangulum super primam BC descriptum ad triangulum super secundam EF simile, similitérque descriptum. vel ità est triangulum super secundam EF descriptum ad triangulum super tertiam simile similitérque descriptum.

PROP. XX.



Similia polygona ABCDE, FGHIK *in similia triangula* ABC, FGH ; *&* ACD, FHI, *&* ADE, FIK *dividuntur*; *& numero æqualia, & homologa totis.* (ABC. FGH :: ABCDE. FGHIK :: ACD. FHI :: ADE. FIK.) *Et polygona* ABCDE, FGHIK *duplicatam habent eam inter se rationem, quam latus homologum* BC *ad homologum latus* GH.

 1. Nam

a hyp. b 6. 6. 1. Nam ang. B [a] = G; & AB. BC [a] :: FG. GH. [b] ergò triangula ABC FGH æquiangula sunt. eodem modo, triangula AED, FKI assimilantur. cùm igitur ang. BCA [b] = GHF; & ang. ADE [b] = FIK; totíque anguli BCD, GHI; atque toti CDE, HIK [c] pares sint, [d] remanent ang. ACD = FHI; & ang. ADC = FIH; [e] unde etiam ang. CAD = HFI. ergò triangula ACD, FHI similia sunt. ergò, &c.

c hyp. d 3. ax. e 32. 1.

f 19. 6. 2. Quoniam igitur triangula BCA, GHF similia sunt, [f] erit $\frac{BCA}{GHF} = \frac{BC}{GH}$ bis. ob eandem causam $\frac{CAD}{HFI} = \frac{CD}{HI}$ bis. denique triang. $\frac{DEA}{IKF} = \frac{DE}{IK}$ bis. quare cùm BC. GH [g] :: CD. HI [g] :: DE. IK, [h] erit triang. BCA. GHF :: CAD. HFI :: DEA. IKF :: [k] polyg. ABCDE. FGHIK :: $\frac{BC}{GH}$ bis.

g hyp. & 16. 5. h sch. 23. 5. k 12. 5.

Coroll.

I. Hinc, si fuerint tres lineæ rectæ proportionales; ut est prima ad tertiam, ità erit polygonum super primam descriptum ad polygonum super secundam simile, similitérque descriptum. vel ità erit polygonum super secundam descriptum ad polygonum super tertiam simile similitérque descriptum.

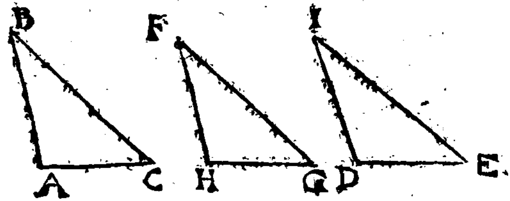
Unde elicitur methodus *figuram quamvis rectilineam augendi vel minuendi in ratione data. ut si velis pentagoni, cujus latus CD aliud facere quintuplum. inter AB, & 5 AB inveni mediam proportionalem. Super hac* [*] *construe pentagonum simile dato. hoc erit quintuplum dati.*

* 18. 6.

2. Hinc etiam, si figurarum similium homologa latera nota fuerint, etiam proportio figurarum innotescet; nempe inveniendo tertiam proportionalem.

PROP.

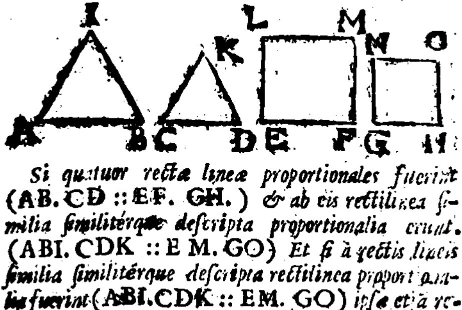
PROP. XXI.



Quæ (ABC, DIE) *eidem rectilineo* HFG *sunt similia, & inter se sunt similia.*

Nam ang. A [a] = H [a] = D. & ang. C [a] = G [a] = E; & ang. B [a] = F [a] = I. [a] item AB. AC :: HF. HG :: DI. DE. [a] & AC. CB :: HG. GF :: DE. EI. & AB. BC :: HF. FG :: DI. IE. [a] ergò ABC, DIE similia sunt. Q. E. D. a 1. def. 6

PROP. XXII.



Si quatuor rectæ lineæ proportionales fuerint (AB. CD :: EF. GH.) *& ab eis rectilinea similia similitérque descripta proportionalia erunt.* (ABI. CDK :: EM. GO.) *Et si à rectis lineis similia similitérque descripta rectilinea proportionalia fuerint* (ABI. CDK :: EM. GO) *ipsæ etiam rectæ lineæ proportionales erunt.* (AB. CD :: EF. GH.)

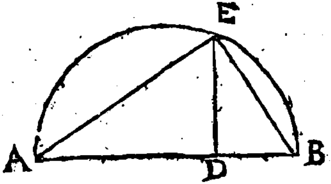
1. *Hyp.* $\frac{ABI}{CDK} \overset{a}{=} \frac{AB}{CD}$ bis $\overset{b}{=} \frac{EF}{GH}$ bis $\overset{c}{=} \frac{EM}{GO}$. [d] ergò ABI. CDK :: EM. GO. Q. E. D. a 19. 6.

2. *Hyp.* $\frac{AB}{CD}$ bis $\overset{a}{=} \frac{ABI}{CDK} \overset{b}{=} \frac{EM}{GO} \overset{c}{=} \frac{EF}{GH}$ bis. ergo AB. CD :: EF. GH. Q. E. D. b hyp. c 20 d cor

Schol.

Hinc deducitur, & demonstratur ratio multiplicandi quantitates surdas. ex gr. Sit $\sqrt{5}$ multiplicandus in $\sqrt{3}$. dico provenire $\sqrt{15}$. Nam ex multiplicationis definitione debet esse, 1. $\sqrt{3}$ :: $\sqrt{5}$. product. ergò per hanc, q. 1. q. $\sqrt{3}$ :: q. $\sqrt{5}$. q. product. hoc est. 1. 3 :: 5. q. product. ergò q. product. est 15. quare $\sqrt{15}$ est productus ex $\sqrt{3}$ in $\sqrt{5}$. Q. E. D.

THEOR.



ctr. Herig. *Si recta linea* AB *secta sit utcunque in* D. *rectangulum sub partibus* AD, DB *contentum, est medium proportionale inter earum quadrata. Item rectangulum contentum sub tota* AB, *& una parte* AD, vel DB, *est medium proportionale inter quadratum totius* AB, *& quadratum dictæ partis* AD, vel DB.

Super diametrum AB describe semicirculum. ex D erige normalem DE occurrentem peripheriæ in E. junge AE, BE.

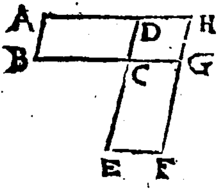
cor. 8. 6. Liquet esse AD. DE [a] :: DE. DB. [b] ergò
22. 6. ADq. DEq :: DEq. DBq. [c] hoc est ADq.
17. 6. ADB :: ADB. DBq. Q. E. D.

cor. 8. 6. Porrò, BA. AE [d] :: AE. AD. [e] ergò BAq.
22. 6. AEq :: AEq. ADq. [f] hoc est BAq. BAD ::
17. 6. BAD. ADq. Eodem modo ABQ. ABD :: ABD. BDq. Q. E. D.

Sic quidem P. Herigonius scitè. Sed facillimè hæc etiam ex 1. 6. & 11. 5. deduci possunt.

PROP.

Prop. XXIII.



Æquiangula parallelogramma AC, CF inter se rationem habent eam quæ ex lateribus componitur. $\left(\frac{AC}{CF} = \frac{BC}{CG} + \frac{DC}{CE.}\right)$

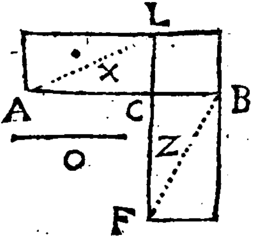
Latera circa æquales angulos C [a] sibi in directum statuantur; & compleatur parallelogrammum CH. a *sch.* 15

Ratio $\frac{AC}{CF}$ [b] $= \frac{AC}{CH} + \frac{CH}{CF}$ [c] $= \frac{BC}{CG} + \frac{DC}{CE.}$ b 20. *def.* c 1. 6.

Q. E. D.

Coroll.

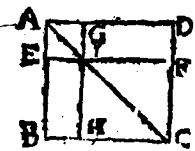
Hinc & ex 34. 1. patet primò, *Triangula, quæ unum angulum* (ad C) *æqualem habent, rationem habere ex rationibus rectarum*, AC ad CB, & LC ad CF, *æqualem angulum continentium.* *Adr. Tacq* 15. 5.



Patet secundò, *Rectangula ac *proinde & parallelogramma quæcunque rationem inter se habere compositam ex rationibus basis ad basim, & altitudinis ad altitudinem.* Neque aliter de triangulis ratiocinaberis. * 35. 1.

Patet tertiò, *Quomodo triangulorum ac parallelogrammorum proportio exhiberi possit.* Sunto parallelogramma X & Z; quorum bases AC, CB; altitudines verò CL, CF. Fiat CL. CF :: CB. O. * erit X. Z :: A.C. O.. * 14. 6. & 1. 6.

Prop. XXIV.

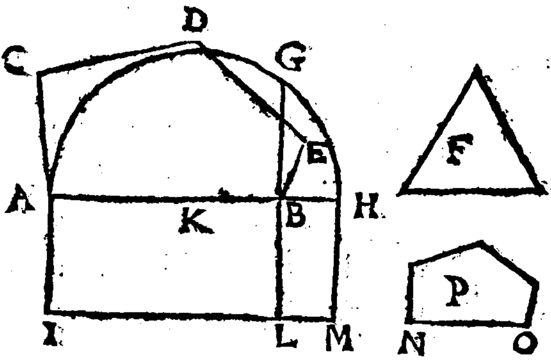


In omni parallelogrammo ABCD, *quæ circa diametrum* AC *sunt parallelogramma* EG, HF, *& toti & inter se sunt similia.*

Nam parallelogramma EG, HF habent singula unum angulum cum toto communem. [a] ergò toti & sibi mutuò æquiangula sunt. [a] Item tam triangula ABC, AEI, IHC, quàm triangula ADC, AGI, IFC sunt inter se æquiangula. [b] ergò AE. EI :: AB. BC, [b] atque AE. AI :: AB. AC; [b] & AI. AG .: AC. AD. [c] ex æquali igitur, AE. AG :: AB. AD. [d] ergò Pgra. EG, BD similia sunt. eodem modo HF, BD similia sunt, ergò, &c.

a 29. 1.
b 4. 6.
c 22. 5.
d 1. def. 6.

Prop. XXV.



Dato rectilineo ABEDC *simile, similitérque positum P, idémque alteri dato* F *æquale, constituere.*

[a] Fac rectang. AL = ABEDC. [b] item super BL fac rectang. BM = F. Inter AB, BH [c] inveni mediam proportionalem NO. super NO [d] fac

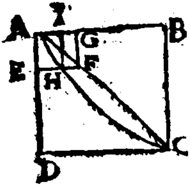
a 45. 1.
b 44. 1.
c 13. 6.

[d]fac polygonum P ſimile dato ABEDC. Erit hoc æquale dato F. d 18. 6

Nam ABEDC (AL). P :: [e] AB. BH [f] :: AL. BM, ergò P [g] = BM [h] = F. Q. E. F.

e cor. 20
f 1. 6.
g 14. 5
h conſtr.

Prop. XXVI.

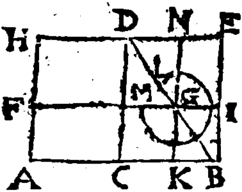


Si à parallelogrammo ABCD parallelogrammum AGFE ablatum ſit, & ſimile toti, & ſimiliter poſitum, communem cum eo habens angulum EAG, hoc circa eandem cum toto diametrum AC conſiſtet.

Si negas AC eſſe communem diametrum, eſto diameter AHC ſecans EF in H. & ducatur HI parall. AE. Parallelogramma EI, DB [a] ſimilia ſunt. [b] ergò AE. EH :: AD. DC [c] :: AE. EF. [d] proinde EH = EF. [f] Q. E. A.

a 24. 6
b 1. def
c hyp.
d 9. 5.
f 9. ax.

Prop. XXVII.

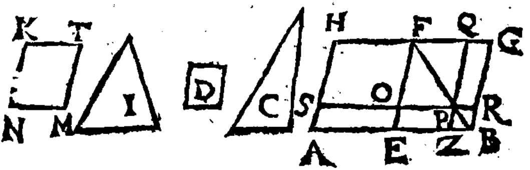


Omnium parallelogrammorum AD, AG ſecundùm eandem rectam lineam AB applicatorum, deficientiúmque figuris parallelogrammis CE, KI ſimilibus, ſimiliterq; poſitis, ei AD, quod à dimidia deſcribitur, maximum eſt AD, quod ad dimidium eſt applicatum, ſimile exiſtens defectui KI.

Nam quia GE [a] = GC, addito communi KI, [b] erit KE = CI [c] = AM. adde commune CG, [d] erit AG = Gnom. MBL. ſed Gnom. MBL [e] ⊐ CE (AD). ergò AG ⊐ AD. Q. E. D.

a 43. 1.
b 2. ax.
c 36. 1.
d 2. ax.
e 9. ax.

PROP. XXVIII.



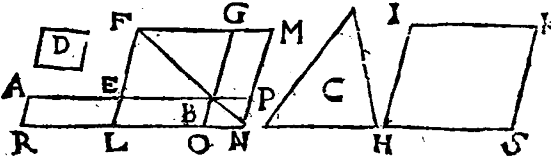
Ad datam rectam lineam AB, *dato rectilineo* C *æquale parallelogrammum* AP *applicare deficiens figurâ parallelogrammâ* ZR, *quæ similis sit alteri*
7. 6. *parallelogrammo dato* D. * *Oportet autem datum rectilineum* C, *cui æquale* AP *applicandum est, non majus esse eo* AF, *quod ad dimidiam applicatur, similibus existentibus defectibus, & ejus* AF *quod ad dimidiam applicatur; & ejus* D, *cui simile deesse debet.*

8. 6. Biseca AB in E. Super EB [a] fac Pgr. EG.
cb. 45. 1. simile dato D. [b] sitque EG = C + I. [c] fac pgr.
5. 6. NT = I; & simile dato D, vel EG. duc diametrum FB. fac FO = KN; & FQ = KT. Per O, & Q duc parallelas SR, QZ. parallelogrammum AP est id quod quæritur.

Nam parallelogramma D, EG, OQ, NT,
onstr. & ZR [d] sunt similia inter se. Et Pgr. EG [e] = NT
6. + C [e] = OQ + C; [f] quare C = Gnom.
onstr. OBQ [g] = AO + FG [h] = AO + EP = AP.
. ax. Q. E. F.
. ax.
43. 1.

PROP.

PROP. XXIX.

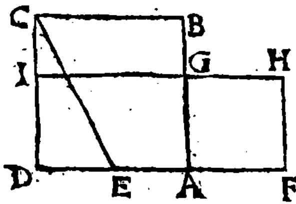


Ad datam rectam lineam AB; *dato rectilineo* C *æquale parallelogrammum* AN *applicare, excedens figurâ parallelogrammâ* OP, *quæ similis sit parallelogrammo alteri dato* D.

Biseca AB in E. super EB [a] fac Pgr. EG simile dato D. [b] sitq; pgr. HK = EG + C; & simile dato D, vel EG. fac FEL [c] = IH, [c] & FGM = IK. per L, M duc parallelas RN, MN. & AR parall. NM. Produc ABP, GBO. Duc diametrum FBN. Pgr. AN est quæsitum. a 18. 6. b 25. 6. c 3. 1.

Nam parallelogramma D, HK, LM, EG [d] similia sunt, [e] ergò pgr. OP simile est pgro LM, vel D. item LM [f] = HK [f] = EG + C. [g] ergò C = Gnom. BNG. atqui AL [h] = LB [k] = BM. [l] ergò C = AN. Q. E. F. d *constr.* e 24. 6. f *constr.* g 3. ax. h 36. 1. k 43. 1. l 2. & 1. ax.

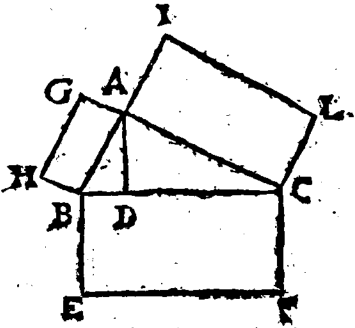
PROP. XXX.



Propositâ rectam lineam terminatam AB, *extremâ, ac mediâ ratione secare.* (AB. AG :: AG. GB.)

[a] Seca AB in G, ità ut AB × BG = AGq. [b] ergò BA. AG :: AG. GB. Q. E. F. a 11. 2. b 17. 6.

PROP. XXXI.



In rectangulis triangulis BAC, figura quævis BF à latere BC rectum angulum BAC subtendente, descripta, æqualis est figuris BG, AL, quæ priori illi BF similes, & similiter positæ à lateribus BA, AC rectum angulum continentibus describuntur.

Ab angulo recto BAC demitte perpendicularem AD. Quoniam CB. CA [a] :: CA. DC. [b] erit BF. AL :: CB. DC; inverséque AL. BF :: DC. CB. Item quia BC, BA [a] :: BA. DB. [b] erit BF. BG :: BC, DB; ac invertendo, BG. BF :: DB. BC. [c] ergò AL+BG. BF :: DC+DB, BC. [d] ergò AL+BG=BF. Q. E. D.

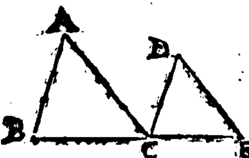
a Cor. 8. 6.
b Cor. 20. 6.
c 24. 5.
d schol. 14. 5.
e 22. 6.
f 24. 5.
g sch. 14. 5.
h 47. 1.

Vel sic. BG.BF [e] :: BAq.BCq. [e] & AL.BF :: ACq. BCq. [f] ergò BG+AL. BF :: BAq+ACq. BCq. [g] ergò cùm BAq+ACq [h] =BCq. [h] erit BG+AL=BF. Q. E. D.

Coroll.

Ex hac propositione, addi possunt, & subtrahi figuræ quævis similes, eâdem methodo, quâ quadrata adduntur & subtrahuntur, in schol. 47. 1.

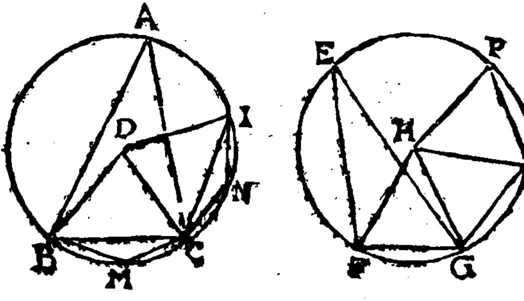
Prop. XXXII.



Si duo triangula ABC, DCE, *quæ duo latera duobus lateribus proportionalia habeant* (AB. AC :: DC. DE), *secundùm unum angulum* ACD *composita fuerint, ità ut homologa eorum latera sint etiam parallela* (AB *ad* DC, & AC *ad* DE); *tum reliqua illorum triangulorum latera* BC, CE *in rectam lineam collocata reperientur.*

Nam ang. A [a] = ACD [a] = D; & AB. a 29. 1.
AC [b] :: DC. DE. [c] ergò ang. B = DCE. ergò b hyp.
ang. B + A [d] = ACE. sed ang. B + A + ACB [e] = 2 c 6. 6.
Rect. [f] ergò ang. ACE + ACB = 2 Rect. [g] ergò d 2. ax.
BCE est recta linea. Q. E. D. e 32. 1.
f 1. ax.
g 14. 1.

Prop. XXXIII.



In æqualibus circulis DBCA, HFGP, *anguli* BDC, FHG *eandem habent rationem cum peripheriis* BC, FG, *quibus insistunt; sive ad centra* (ut BDC, FHG), *sive ad peripherias* A, E *constituti insistant: insuper verò & sectores* BDC, FHG, *quippe qui ad centra consistunt.*

Duc rectas BC, FG. Accommoda CI=CB; & GL=FG=LP; & junge DI, HL, HP.

28. 3. Arcus BC [a]=CI, [a] item arcus FG, GL, LP
27. 3. æquantur, [b] ergò ang. BDC=CDI. [b] & ang. FGH=GHL=LHP. Ergò arcus BI tam multiplex est arcûs BC, quàm ang. BDI anguli BDC. pariterque æquemultiplex est arcus FP arcûs FG; atque ang. FHP anguli FHG. Verùm si arcus BI $\sqsubset, =, \sqsupset$ FP, [c] erit similiter
27. 3. ang. BDI $\sqsubset, =, \sqsupset$ FHP. ergò arc. BC. FG [d] ::
6. def. 5. ang. BDC. FHG [e] :: $\frac{BDC}{2}$. $\frac{FHG}{2}$ [f] :: A. E.
15. 5.
20. 3.

Q. E. D.

27. 3. Rursus ang. BMC [g] = CNI; [h] atque idcirco
24. 3. segm. BCM = CIN. [k] item triang. BDC=
4. 1. CDI. [l] ergò sector BDCM=CDIN. Simili
2. ax. ratione sectores FHG, GHL, LHP æquantur. Quum igitur prout arcus BI $\sqsubset, =, \sqsupset$ FGP, ità
6. def. 5. similiter sector BDI $\sqsubset, =, \sqsupset$ FHP. [m] erit sect. BDC. FHG :: arc. BC. FG. Q. E. D.

Coroll.

[...]. 5. Hinc 1. *ut sector ad sectorem, sic angulus ad angulum.*

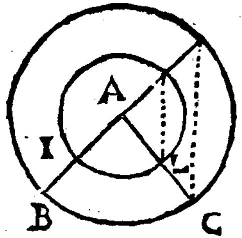
2. *Ang.* BDC *in centro est ad 4 rectos, ut arcus* BC *cui insistit ad totam circumferentiam.*

Nam ut ang. BDC ad rectum; sic arcus BC ad quadrantem. ergò BDC est ad 4 Rectos, ut arcus BC ad 4 quadrantes, id est ad totam circumferentiam. item ang. A. 2 Rect :: arc. BC. periph.

Hinc 3. *Inæqualium circulorum arcus* IL, BC, *qui æquales subtendunt angulos, sive ad centra*, ut IAL & BAC, *sive ad peripheriam, sunt similes.*

Nam IL. periph. :: ang. IAL, (BAC.). 4 Rect. item arc. BC. periph :: ang. BAC. 4. Rect.

4. Rect. ergò IL. periph :: BC. periph. proinde arcus IL, & BC sunt similes. Unde



4. *Duæ semidiametri* AB, AC *à concentricis peripheriis arcus auferunt similes* IL, BC.

Sam. Payne Christ Hospitall Aprill 1705

LIB. VII.

Definitiones.

I. Unitas est, secundùm quam unumquodq; eorum quæ sunt, unum dicitur.

II. Numerus autem est, ex unitatibus composita multitudo.

III. Pars est numerus numeri, minor majoris, quum minor metitur majorem.

Omnis pars ab eo numero nomen sibi sumit, per quem ipsa numerum, cujus est pars, metitur; ut 4 dicitur tertia pars numeri 12; quia metitur 12 per 3.

IV. Partes autem, cùm non metitur.

Partes quæcunque nomen accipiunt à duobus illis numeris, per quos maxima communis duorum numerorum mensura utrumque eorum metitur. ut 10 dicitur $\frac{2}{3}$ numeri 12, eo quòd maxima communis mensura, nempe 5, metitur 10 per 2, & 15. per 3.

V. Multiplex verò major minoris, cùm majorem metitur minor.

VI. Par numerus est, qui bifariam dividitur.

VII. Impar verò numerus, qui bifariam non dividitur, vel qui unitate differt à pari.

VIII. Pariter par numerus est, quem par numerus metitur per numerum parem.

IX. Pariter autem impar est, quem par numerus metitur per numerum imparem.

X. Impariter verò impar numerus est; quem impar numerus metitur per numerum imparem.

XI. Primus numerus est, quem sola unitas metitur.

XII. Primi inter se numeri sunt, quos sola unitas, communis mensura metitur.

XIII. Com-

XIII. Compositus numerus est, quem numerus quispiam metitur.

XIV. Compositi autem inter se numeri sunt, quos numerus aliquis communis mensura metitur.

In hac definitione & praecedenti unitas non est numerus.

XV. Numerus numerum multiplicare dicitur, cùm toties compositus fuerit is, qui multiplicatur, quot sunt in ipso multiplicante unitates, & procreatus fuerit aliquis.

Hinc, in omni multiplicatione unitas est ad multiplicatorem ut multiplicatus ad productum.

Nota, quòd saepe cùm multiplicandi sunt quivis numeri, puta A *in* B, *literarum conjunctio productum denotat. Sic* AB = A *in* B. *item* CDE = C *in* D *in* E.

XVI. Cùm autem duo numeri sese multiplicantes aliquem fecerint, qui factus erit, planus appellabitur. Qui verò numeri sese mutuò multiplicârint, latera illius dicentur. *Sic* 2 (C) *in* 3 (D) = 6 = CD, *est numerus planus.*

XVII. Cùm verò tres numeri mutuò sese multiplicantes fecerint aliquem, qui procreatus erit, solidus appellabitur; qui autem numeri mutuò sese multiplicârint, latera illius dicentur, *Sic*, 2 (C) *in* 3 (D) *in* 5 (E) = 30 = CDE *est numerus solidus.*

XVIII. Quadratus numerus est, qui aequaliter aequalis, vel qui sub duobus aequalibus numeris continetur. *Sit* A *latus quadrati; quadratus sic notatur*, AA, *vel* Aq.

XIX. Cubus verò, qui aequaliter aequalis aequaliter, vel qui sub tribus aequalibus numeris continetur. *Sit* A *latus cubi, cubus notatur sic*, AAA, *vel* AC.

In hac definitione, & tribus praecedentibus, unitas est numerus.

XX. Nu-

XX. Numeri proportionales sunt, cùm primus secundi, & tertius quarti æquemultiplex est, vel eadem pars; vel deniq; cùm pars primi secundum, & eadem pars tertii æquè metitur quartum, vel vice versâ. A. B :: C. D. hoc est 3. 9 :: 5. 15.

XXI. Similes plani, & solidi numeri sunt, qui proportionalia habent latera.

Latera nempe non quælibet, sed quædam.

XXII. Perfectus numerus est, qui suis ipsius partibus est æqualis.

Ut 6. & 28. *Numerus verò qui suis ipsius partibus minor est, abundans appellatur, qui verò major, diminutus. ut* 12 *est abundans*, 15 *est diminutus.*

XXIII. Numerus numerum metiri dicitur per illum numerum, quem multiplicans, vel à quo multiplicatus, illum producit.

In divisione, unitas est ad quotientem, ut dividens ad divisum. Nota, quòd numerus alteri lineolâ interjectâ subscriptus divisionem denotat, Sic $\frac{A}{B}$ = A divis. per B. *item* $\frac{CA}{B}$ = C *in* A divis. per B.

Termini, sive radices proportionis dicuntur duo numeri, quibus in eadem proportione minores sumi nequeunt.

Postulata.

1. POstuletur, cuilibet numero quotlibet sumi posse æquales, vel multiplices.

2. Quolibet numero sumi posse majorem.

3. Additio, subtractio, multiplicatio, divisio, extractionésque radicum, seu laterum numerorum quadratorum, & cuborum conceduntur etiam, tanquam possibilia.

Axiomata

Axiomata.

1. Quicquid convenit uni æqualium numerorum, convenit & reliquis æqualibus numeris.

2. Partes eidem parti, vel iisdem partibus eædem, sunt quoque inter se eædem.

3. Qui numeri æqualium numerorum, vel eiusdem, eædem partes fuerint, æquales inter se sunt.

4. Quorum idem numerus, vel æquales, eædem partes fuerint, æquales inter se sunt.

5. Unitas omnem numerum per unitates, quæ in ipso sunt, hoc est per ipsummet numerum metitur.

6. Omnis numerus seipsum metitur per unitatem.

7. Si numerus numerum multiplicans, aliquem produxerit, metietur multiplicans productum per multiplicatum, multiplicatus autem eundem per multiplicantem.

Hinc nullus numerus primus planus est aut solidus, quadratus, vel cubus.

8. Si numerus numerum metiatur, & ille per quem metitur, eundem metietur per eas, quæ in metiente sunt, unitates, hoc est per ipsum numerum metientem.

9. Si numerus numerum metiens, multiplicet eum, per quem metitur, vel ab eo multiplicetur, illum quem metitur, producit.

10. Numerus quotcunque numeros metiens, compositum quoque ex ipsis metitur.

11. Numerus quemcunque numerum metiens, metitur quoque omnem numerum, quem ille metitur.

12. Numerus metiens totum, & ablatum, metitur & reliquum.

Prop. I.

A.....E..G.B 8 5 3
C...F..D $\frac{5}{3}$ $\frac{3}{2}$ $\frac{2}{1}$
H---

Si duobus numeris inæqualibus propositis (AB, CD) *detrahatur semper minor* CD *de majore* AB (& reliquus EB de CD &c.) *alternâ quâdam detractione, neque reliquus unquam præcedentem metiatur, quoad assumpta sit unitas* GB; *qui principio propositi sunt numeri* AB, CD *primi inter se erunt.*

Si negas, habeant AB, CD communem mensuram, numerum H. Ergò H metiens CD,
a 11. ax. 7. [a] etiam AE metitur; proinde & reliquum EB;
b 12. ax. 7. [a] ergò & CF, atque [b] idcirco reliquum FD;
[a] quare & ipsum EG; sed totum EB metiebatur;
[b] ergò & reliquum GB metitur, numerus uni-
c 9. ax. 1. tatem. [c] Q. E. A.

Prop. II.

  9 6
A........E.....B 15 9 6
  6 3
C......F...D $\frac{9}{6}$ $\frac{6}{3}$ $\frac{3}{0}$
G---

Duobus numeris datis AB, CD *non primis inter se, maximam eorum communem mensuram* FD *reperire.*

Detrahe minorem numerum CD ex majori
a 6. ax. 7. AB, quoties potes. Si nihil relinquitur, [a] patet ipsum CD esse maximam communem mensuram. Si relinquitur aliquid EB, deme hunc ex CD, & reliquum FD ex EB, & sic deinceps donec aliquis FD præcedentem EB metiatur.
b 1. 7. (nam [b] hoc fiet antequam ad unitatem perveniatur.) Erit FD maxima communis mensura.
c constr. Nam FD [c] metitur EB, [d] ideóque & CF;
d 11. ax. 7.
e 12. ax. 7. [e] proinde & totum CD; [d] ergò ipsum AE; atq; idcircò totum AB metitur. Liquet igitur FD communem esse mensuram, Si maximam esse ne-

gas,

gas, sit major quæpiam G. ergò G metiens CD, [f] metitur AE, [e] & reliquum EB, [d] ipsúmque CF, [e] proinde & reliquum FD, [g] major minorem. [h] Q. E. A. *g suppos.* h *9. ax. 1.*

Coroll.

Hinc, numerus metiens duos numeros, metitur quoque maximam eorum communem mensuram.

Prop. III.

A 12
B 8
D 4
C 6
E .. 2
F - - -

Tribus numeris datis A,B,C *non primis inter se, maximam eorum communem mensuram* E *reperire.*

Inveni D maximam communem mensuram duorũ A,B. Si D metitur tertium C; liquet D maximam esse trium communem mensuram. Si D non metitur C, erunt saltem D,& C compositi inter se, ex coroll. præcedentis. Sit igitur ipsorum D, & C maxima communis mensura E. erit E is, quem quæris.

Nam E [a] metitur C, & D; [a] ac D ipsos A, & B metitur; [b] ergò E metitur singulos A, B, C; nec major aliquis (F) eos metietur; nam si hoc affirmas, [c] ergò F metiens A, & B, eorum maximam communem mensuram D metitur. Eodem modo, F metiens D, & C, [c] eorum maximam communem mensuram E, [d] major minorem, metitur. [e] Q. E. A.

a *constr.*
b *11. ax. 7.*
c *cor. 1. 7.*
d *suppos.*
e *9. ax. 1.*

Coroll.

Hinc, numerus metiens tres numeros, maximam quoque eorum communem mensuram metitur.

Prop. IV.

A 6
B 7
B 18
B .... 9.

Omnis numerus A, *numeri* B, *minor maj*[e] *pars est, aut partes.*

Si A, & B pri[mi] inter se, [a] erit A [tot par]tes numeri B, quot sunt in A unitat[es] ($6 = \frac{6}{7}$ 7.) Sin A metiatur B, [b] liquet A [...] te ipsius B. (ut $6 = \frac{1}{3}$ 18.) denique [si A &] B aliter compositi inter se fuerint, [c] m[axima] communis mensura determinabit, quot p[artes] conficiat ipsius B; ut $6 = \frac{2}{3}$ 9.

a 4. def. 7.
b 3. def. 7.
c 4 def. 7.

Prop. V.

A 6   D 4
6   6   4   4
B .... G ..... C 12.   E .... H .... F

Si numerus A *numeri* BC *pars fuerit, &* D *alterius* EF *eadem pars; & simul* (A+D) *utriusque simul* (BC+EF) *pars erit, quæ un*[us] A *unius* BC

Nam si BC in suas partes BG, GC [...] æquales; atque EF in suas partes FH, H[E ...] D æquales resolvantur; [a] erit numerus p[artium] in BC æqualis numero partium in EF [...] igitur A+D [b] = BG+EH = GC+HF; [...] A+D toties in BC+EF, quoties A i[n BC]. Q. E. D.

Vel sic breviùs. Sit $a = \frac{x}{2}$ & $b = \frac{y}{2}$.

$$a+b = \frac{x}{2} + \frac{y}{2} = \frac{x+y}{2}. \text{ Q. E. D.}$$

a hyp.
b const.
& 2. ax. 1.
c 2. ax. 1.

P[rop.]