EUCLIDIS ELEMENTORUM

Libri xv. breviter demonstrati,

Coll Lugd SS Trinit Soc Jesu

Opera

IS. BARROW, *Cantabrigiensis*,

Coll. TRIN. *Soc.*

Καθαρμοὶ ψυχῆς λογικῆς εἰσὶν αἱ μαθηματικαὶ ἐπιστῆμαι. HIEROCL.

LONDINI,

Excudebat R. DANIEL, Impensis
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Cantabrig. CIↃ IↃC LIX.

Nobilissimis & Generosissimis

Adolescentibus,

Dno EDOUARDO CECILIO,

Illustriss. Comitis Sarisburiensis *Filio*;

Dno *IOHANNI KNATCHBUL*,

Et

D. *FRANCIS. WILLOUGHBY*,

ARMIGERIS.

UNicuique vestrum (Optimi Adolescentes) tantum me debere reputo, quantum homo homini debere potest. Mea enim sententia, ultra sincerum amorem non est quod

 quis-

quiſpiam de alio bene mereri poſſit. Hunc autem jamdiu eſt quo ex ſingulari veſtra bonitate mihi indultum experior ; ejuſque ſenſus, intimis animi medullis inhærens, ipſi ardens ſtudium impreſſit quovis honeſto modo reciprocos affectus prodendi. Quandoquidem vero ea fortunarum mearum tenuitas, ea veſtrarum amplitudo, exiſtit, ut nec ego alia quam gratæ alicujus agnitionis ſignificatione uti queam, nec vos aliam admittere velitis; ea propter haud illibenter hanc occaſionem arripio, honoris & benevolentiæ, quibus vos proſequor, publicum hoc & durabile

bile μνημόσυνον edendi. Etsi cum oblati anathematis exilitatem, & libellum vestris nominibus consecratum, quam is longe infra vestrorum meritorum dignitatem subsidat, attentius considero, timor subinde aliquis & dubitatio animum incessant, ne hoc studium erga vos meum vobis dehonestamento sit potius quam ornamento; scilicet memor cum sim, ut malæ causæ, sic & mali libri patrocinium in patroni contumeliam magis quam in gloriam cedere. Sed quum vestrarum virtutum id robur, eam fore soliditatem, recognoscerem, quæ vestrum decus, meo quantumvis labefaſtato, inconcuſſum ſuſtinere poſſint; idcirco non dubitavi vos in aliquatenus commune mecum periculum induere. Virtutes illas intelligo, quibus nemo unquam in veſtra ætate aut in veſtro ordine, ſaltem me judice, majores deprehendit; quæ vos inſigniter gratos omnibus & amabiles reddunt; eximiam modeſtiam, ſobrietatem, benignitatem animi, morum comitatem, prudentiam, magnanimitatem, fidem, præclaram inſuper ingenii indolem, quæ vos ad omnem ingenuam ſcientiam non tantum excellenti captu, ſed & appetitu forti ac ſincero, inſtruxit. Quas veſtras præclariſſimas dotes

prout

prout nemo est fortassis qui me melius novit, aut pro consuetudine, quam jamdudum vobiscum dulcissimam coluisse ex vestro favore mihi contigit, penitus introspexit, ita nemo est qui impensius miratur & suspicit; aut qui ipsas libentius prædicare ac celebrare vellet, si non cum eloquii mei vires supergrederentur, tum etiam quæ in singulis vobis elucent, prolixi alicujus commentarii aut panegyricæ orationis libertatem, potius quam præstitutas hujusmodi salutationibus angustias, exposcerent. Quin potius divinam clementiam imploro, ut vos earundem virtutum sancto tramiti insistere,

ſtere, atque hos egregios fructus vernæ veſtræ ætatis felicibus incrementis matureſcere concedat; vitamque vobis in hoc ſeculo ingenuam, innocentem, jucundam, & in futuro beatam ac ſempiternam tranſigere largiatur. Minime autem dubito, ne pro conſueto veſtro in me candore hoc ultimum fortaſſis quod vobis præſtare potero, benevolentiæ erga vos & obſervantiæ teſtimonium, alacriter accepturi ſitis; quod vobis propenſiſſimo affectu offert

Veſtrî in æternum amantiſſimus,

& obſervantiſſimus,

I. B.

Bene-

Benevolo LECTORI.

SI quid in hac elementorum editione præstitum sit, scire desideras, amice Lector, accipe, pro genio operis, breviter. Ad duos præcipue fines conatus meos direxi. Primum, ut cum requisita perspicuitate summam demonstrationum brevitatem conjungerem, quo eam libello molem compararem, quæ commode absque molestia circumferri posset. Id quod assecutus videor, si absentem Typographi cura non frustretur. Concinnius enim quispiam meliori ingenio aut majori peritia excellens, at nemo forsan brevius plerasque propositiones demonstraverit; præsertim cum in numero & ordine propositionum ipse nihil immutarim, nec licentiam mihi assumpserim quamcunque propositionem Euclideam procul ablegandi tanquam minus necessariam, aut quasdam faciliores in axiomatum censum referendi; quod nonnulli fecerunt: inter quos peritissimus Geometra Andr. Tacquetus, quem ideo etiam nomino, quod quædam ex eo desumpta agnoscere honestum duco; post cujus elegantissimam editionem, ipse nihil atten-

tare

tare voluissem, si non visum fuisset doctissimo viro non nisi octo Euclidis *libros suâ curâ adornatos publico communicare, reliquis septem, tanquam ad elementa Geometriæ minus spectantibus, omnino quasi spretis atque posthabitis. Mihi autem jam ab initio alia provincia demandata fuit, non elementa Geometriæ utcunque pro arbitrio conscribendi, verum* Euclidem *ipsum, eumque totum, quam possem brevissime, demonstrandi. Quod enim quatuor libros spectat, septimum, octavum, nonum, decimum, quamvis illi ad Geometriæ planæ & solidæ elementa, ut sex præcedentes & duo subsequentes, non tam prope pertineant; quod tamen ad res Geometricas admodum utiles sint, tam propter Arithmeticæ & Geometriæ valde propinquam cognationem, quam ob notitiam commensurabilium & incommensurabilium magnitudinum ad figurarum tam planarum quam solidarum intellectum apprime necessariam, nemo est è peritioribus Geometris qui ignorat. Quæ vero in tribus ultimis libris continetur, 5 corporum regularium nobilis contemplatio, illa non nisi injuria prætermitti potuit; quando nempe illius gratia noster* στοιχειωτὴς, *Platonicæ familiæ philosophus, hoc elementorum systema universum condidisse perhibetur;*

uti testis est * Proclus, *iis verbis*, Ὅθεν δὴ καὶ τῆς συμπάσης στοιχειώσεως τέλος προεστήσατο τὴν τῶν καλουμένων πλατωνικῶν σχημάτων σύστασιν. * lib. 2.

Præterea facile in animum induxi ut opinarer, nemini harum scientiarum amanti non futurum esse cordi penes se habere integrum Euclidæum *opus, quale passim ab omnibus citatur & celebratur. Quare nullum librum nullamque propositionem negligere volui earum quæ apud* P. Herigonium *habentur; cujus vestigiis presse insistere necesse habui, quoniam ejusce libri schematismis maxima ex parte uti statutum erat, quod prævideren mihi ad novas describendas tempus non suppetere; etsi nonnunquam id facere praoptassem. Eadem de causa nec alias plerasque quam* Euclidæas *demonstrationes adhibere volui, succinctiori forma expressas, nisi forte in 2, & 13, & parce in 7, 8, 9 libris; ubi ab eo nonnihil deflectere operæ pretium videbatur. Bona igitur spes est saltem in hac parte cum nostris consiliis, tum studiosorum votis, aliquo modo satisfactum iri. Nam quæ adjecta sunt in Scholiis problemata quædam & theoremata, sive ob suum frequentem usum ad naturam elementarem accedentia, sive ad eorum quæ sequuntur expeditam demonstrationem conducentia, seu quæ regula-*

rum

rum practicæ Geometriæ quarundam præcipuarum rationes innuunt ad suos fontes relatas, per ea, ut spero, libellus ultra destinatam molem magnopere non intumescet.

Alter scopus ad quem collineatum est, eorum desideriis consuluit qui demonstrationibus symbolicis potius quam verbalibus delectantur. In quo genere cum plerique apud nos Guilielmi Oughtredi *symbolis assueti sint, ea plerumque usurpare consultius duximus. Nam qui* Euclidem *hâc viâ tradere & interpretari aggressus sit, hactenus, quod ego sciam, præter unum* P. Herigonium, *repertus est nemo. Cujus viri longe doctissimi methodus, sane in multis egregia, ac ejus peculiari proposito admodum accomodata, duplici tamen defectu laborare mihi visa est. Primo, quod cum Propositionum ad unius alicujus theorematis aut problematis probationem adductarum posterior à priori non semper dependeat; quando tamen illæ inter se cohærent, quando non, nec ex ordine singularum, nec ullo alio modo, satis prompte innotescere potest: unde ob defectum conjunctionum & adjectivorum* (ergo, rursus, &c.) *non raro difficultas & dubitandi occasio, præsertim minus exercitatis, inter legendum oboriri solent. Deinde sæpenumero evenit, ut prædicta methodus supervacaneas repetitiones effugere nequeat, à quibus demonstrationes est quando prolixæ, aliquando*

&

& magis intricata, evadunt. Quibus vitiis noster modus facile per verborum signorumque arbitrariam mixturam medetur. Atque hæc de opellæ hujus intentione & methodo dicta sufficiant. Cæterum quæ in laudem Matheseos in genere, aut Geometriæ ipsius; & quæ de historia harum scientiarum, ideoque de Euclide *horum elementorum digestore, dici possent, & reliqua hujusmodi ἐξωτερικά, cui hæc placent, apud alios interpretes consulere potest. Neque nos angustias temporis quod huic operi impendi potuit, nec interpellationes negotiorum, nec adjumentorum ad hæc studia apud nos egestatem, & quædam alia, ut liceret non immerito, in excusationem obtendemus; metu scilicet inducti, ne hæc nostra omnibus minus satisfaciant. Verum quæ ingenui Lectoris usibus elaboravimus, eadem in solidum ipsius censuræ ac judicio submittimus; probanda si utilia sibi compererit; sin omnino secus, rejicienda.*

I. B.

Ad amicissimum Virum, *I. B.* de *EVCLIDE* contracto.

Εὐφημισμός.

FActum bene! didicit Laconice loqui
Senex profundus, & aphorismos induit.
Immensa dudum margo commentarii
Diagramma circuit minutum; utque Insula
Problema breve natabat in vasto mari.
Sed unda jam detumuit; & glossa arctior
Stringit Theoremata: minoris anguli
Lateribus ecce totus Euclides *jacet,*
Inclusus olim velut Homerus in nuce;
Pluteoque sarcina modo qui incubuit, levis
En fit manipulus. Pelle in exigua latet
Ingens Mathesis, matris ut in utero Hercules,
In glande quercus, vel Ithaca Eurus in pila.
Nec mole dum decrescit, usu fit minor;
Quin auctior jam evadit, & cumulatius
Contracta prodest erudita pagina.
Sic ubere magis liquor è presso effluit;
Sic pleniori vasa inundat sanguinis
Torrente cordis Systole; sic fusius
Procurrit aequor ex Abylæ angustiis.
Tantilli operis ars tanta referenda unice est
BAROVIANO nomini, ac solertiæ.
Sublimis euge mentis ingenium potens!
Cui invium nil, arduum esse nil solet.
Sic usque pergas prospero conamine,
Radiusque multum debeat ac abacus tibi;
Sic crescat indies feracior seges,
Simili colonum germine assiduo beans.
Specimen futuræ messis hic stet labor,
Magnæque famæ illustria hæc præludia.
Iuvenis dedit qui tanta, quid dabit senex?

Car. Robotham, *CANTAB.*
Coll. Trin. Sen. Soc.

In novam *Elementorum* *EVCLIDIS* Editionem à D. *IS. BARROW*, *Collegii SS.* TRIN. *Socio*, viro opt. & eruditiſſimo, adornatam.

BEnigne Lector! ſi uſpiam auditam eſt tibi,
Quantus tenella Nix Geometres ſiet;
Quæ mille radiis, mille ludit angulis,
Totumque puro ducit Euclidem ſinu:
Amabis ultro candidiſsimum Virum,
Cui plena nivium eſt indoles, ſed quas tamen
Præclarus ardor mentis urget Enthea;
Et uſque blandis temperat caloribus:
Quo ſuavius nil vivit, & melius nihil.
Is, dum liquentes pectore excutit nives,
Et inde & inde ſpargit, en aliam tibi,
Lector benigne, è nivibus Geomertiam!

G.C. *A.M. C.E.S.*

Notarum explicatio.

$=$ æqualitatem.
$\sqsubset$ majoritatem.
$\sqsupset$ minoritatem.
$+$ plus, vel addendum esse.
$-$ minus, vel subtrahendum esse.
$\div$: differentiam vel excessum; item quantitates omnes, quæ sequuntur, subtrahendas esse, signis non mutatis.
$\times$ multiplicationem, vel ductum lateris rectanguli in aliud latus.
Idem denotat conjunctio literarum, ut $AB = A \times B$.
$\surd$ Latus, vel radicem quadrati, vel cubi, &c.
Q. & q quadratum. C. & c cubum.
Q. Q. rationem quadrati numeri ad quadratum numerum.

significat.

Reliquas, quæ ubicunque occurrunt, vocabulorum abbreviationes ipse Lector per se facile intelliget; exceptis iis, quas tanquam minus generalis usus, suis locis explicandas relinquimus.

LIB. I.

Definitiones.

I. PUnctum est cujus pars nulla est.

II. Linea vero longitudo latitudinis expers.

III. Lineæ autem termini sunt puncta.

IV. Recta linea est, quæ ex æquo sua interjacet puncta.

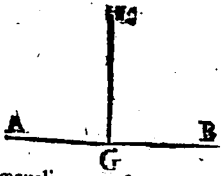
V. Superficies est, quæ longitudinem, latitudinemque tantum habet.

VI. Superficiei autem extrema sunt lineæ.

VII. Plana superficies est, quæ ex æquo suas interjacet lineas.

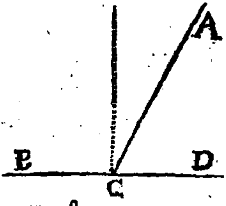
VIII. Planus vero angulus est, duarum linearum in plano se mutuo tangentium, & non in directum jacentium alterius ad alteram inclinatio.

IX. Cum autem quæ angulum continent, lineæ, rectæ fuerint, rectilineus ille angulus appellatur.



X. Cum vero recta linea C G super rectam lineam A B consistens, eos qui sunt deinceps angulos C G A, C G B æquales inter se fecerit, rectus est uterque æqualium angulorum, & quæ insistit recta linea C G, perpendicularis vocatur ejus (A B) cui insistit.

Not. *Cum plures anguli ad unum punctum: (ut ad* G) *exsistunt, designatur quilibet angulus tribus literis, quarum media ad verticem est illius de quo agitur: ut angulus quem rectæ* C G, A G *efficiunt ad partes* A *vocatur* C G A, *vel* A G C.



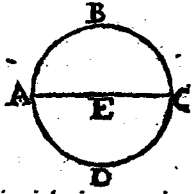
XI. Obtusus angulus est, qui recto major est, ut A C B.

XII. Acutus vero, qui minor est recto, ut A C D.

XIII. Terminus est, quod alicujus extremum est.

XIV. Figura est, quæ sub aliquo, vel aliquibus terminis comprehenditur.

XV. Circulus est figura plana, sub una linea comprehensa, quæ peripheria appellatur, ad quam ab uno puncto eorum, quæ intra figuram sunt posita, cadentes omnes rectæ lineæ inter se sunt æquales.



XVI. Hoc vero punctum centrum circuli appellatur.

XVII. Diameter autem circuli est recta quædam linea per centrum ducta, & ex utraque parte in circuli peripheriam terminata, quæ circulum bifariam secat.

XVIII. Semicirculus vero est figura, quæ continetur sub diametro, & sub ea linea, quæ de circuli peripheria aufertur.

In circulo E A B C D. E *est centrum*, A C *diameter*, A B C *semicirculus*.

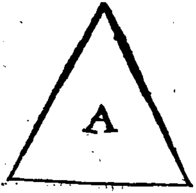
XIX. Rectilineæ figuræ sunt, quæ sub rectis lineis continentur.

XX. Trilateræ quidem, quæ sub tribus.

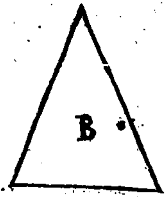
XXI. Quadrilateræ vero, quæ sub quatuor.

XXII. Multilateræ autem, quæ sub pluribus, quam quatuor rectis lineis comprehenduntur.

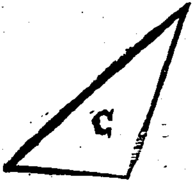
XXIII.



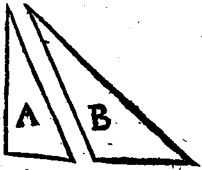
XXIII. Trilaterarum autem figurarum, æquilaterum eſt triangulum, quod tria latera habet æqualia, ut triangulum A.



XXIV. Iſoſceles autem, quod duo tantum æqualia habet latera, ut triangulum B.

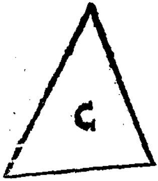


XXV. Scalenum vero, quod tria inæqualia habet latera, ut C.



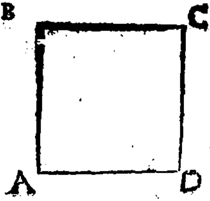
XXVI. Adhæc etjam trilaterarum figurarum, rectangulum quidem triangulum eſt, quod rectum angulum habet, ut triangulum A.

XXVII. Amblygonium autem, quod obtuſum angulum habet, ut B.

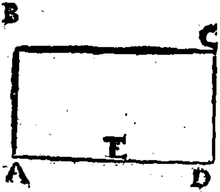


XXVIII. Oxygonium vero, quod tres habet acutos angulos, ut C.

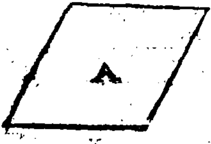
Figura æquiangula est, cujus omnes anguli inter se æquales sunt. Duæ vero figuræ æquiangulæ sunt; si singuli anguli unius singulis angulis alterius sint æquales. Similiter de figuris æquilateris concipe.



XXIX. Quadrilaterarum autem figurarum, quadratum quidem est, quod & æquilaterum, & rectangulum est, ut A B C D.

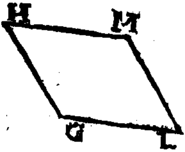


XXX. Altera vero parte longior figura est, quæ rectangula quidem, at æquilatera non est, ut A B C D.

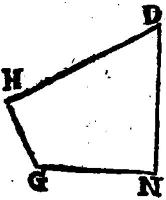


XXXI. Rhombus autem, quæ æquilatera, sed rectangula non est, ut A.

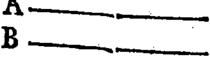
XXXII.



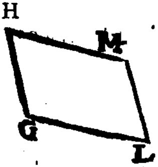
XXXII. Rhomboydes vero, quæ adversa & latera, & angulos habens inter se æquales, neque æquilatera est, neque rectangula, ut G L M H.



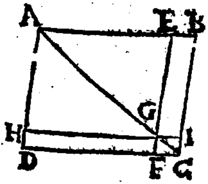
XXXIII. Præter has autem reliquæ quadrilateræ figuræ trapezia appellentur; ut G N D H.



XXXIV. Parallelæ rectæ lineæ sunt, quæ cum in eodem sint plano, & ex utraque parte in infinitum producantur, in neutram sibi mutuo incidunt, ut A, & B.



XXXV. Parallelogrammum est figura quadrilatera, cujus bina opposita latera sunt parallela, seu æquidistantia, ut G L H M.



XXXVI. Cum vero in parallelogrammo A B C D diameter A C ducta fuerit, duæque lineæ E F, H I, lateribus parallelæ secantes diametrum in uno eodemque puncto G, ita ut parallelogrammum ab hisce parallelis in quatuor distribuatur parallelogramma; appellantur duo illa D G, G B, per quæ diameter non transit, Complementa; duo vero reliqua H E, F I, per quæ diameter incedit, circa diametrum consistere dicuntur.

Problema est, *cum proponitur aliquid efficiendum.*

Theorema est, *cum proponitur aliquid demonstrandum.*

Corollarium est *consectarium, quod è facta demonstratione tanquam lucrum aliquod colligitur.*

Lemma est *demonstratio præmissæ alicujus, ut demonstratio quæsiti evadat brevior.*

Postulata.

1. POstuletur, ut à quovis puncto ad quodvis punctum rectam lineam ducere concedatur.

2. Et rectam lineam terminatam in continuum recta producere.

3. Item, quovis centro, & intervallo circulum describere.

Axiomata.

1. Quæ eidem æqualia, & inter se sunt æqualia.

ut A = B = C. ergo A = C, vel ergo omnes A, B, C, æquantur inter se.

Nota, *Cum plures quantitates hoc modo conjunctas invenias, vi hujus axiomatis primam ultimæ & quamlibet earum cuilibet æquari. Quo in casu sæpe, brevitatis causa, ab hoc axiomate citando abstinemus; etsi vis consecutionis ab eo pendeat.*

2. Et si æqualibus æqualia adjecta sunt, tota suntæqualia.

3. Et

3. Et si ab æqualibus æqualia ablata sunt, quæ relinquuntur sunt æqualia.

4. Et si inæqualibus æqualia adjecta sint, tota sunt inæqualia.

5. Et si ab inæqualibus æqualia ablata sint, reliqua sunt inæqualia.

6. Et quæ ejusdem vel æqualium sunt duplicia, inter se sunt æqualia. Idem puta de triplicibus, quadruplicibus, &c.

7. Et quæ ejusdem, vel æqualium sunt dimidia, inter se sunt æqualia. Idem concipe de subtriplis, subquadruplis, &c.

8. Et quæ sibi mutuo congruunt, ea inter se sunt æqualia.

Hoc axioma in rectis lineis, & angulis valet conversum, sed non in figuris, nisi illæ similes fuerint.

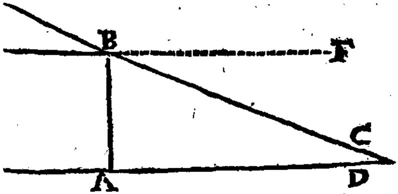
Cæterum, magnitudines congruere dicuntur, quarum partes applicatæ partibus, æqualem vel eundem locum occupant.

9. Et totum sua parte majus est.

10. Duæ rectæ lineæ non habent unum & idem segmentum commune.

11. Duæ rectæ in uno puncto concurrentes, si producantur ambæ, necessario se mutuo in eo puncto intersecabunt.

12. Item omnes anguli recti sunt inter se æquales.



13. Et si in duas rectas lineas AD, CB, altera recta BA incidens, internos ad easdemque partes angu-

angulos B A D; A B C duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ sibi mutuo incident ad eas partes, ubi sunt anguli duobus rectis minores.

14. Duæ rectæ lineæ spatium non comprehendunt.

15. Si æqualibus inæqualia adjiciantur, erit totorum excessus adjunctorum excessui æqualis.

16. Si inæqualibus æqualia adjungantur, erit totorum excessus excessui eorum, quæ à principio, æqualis.

17. Si ab æqualibus inæqualia demantur, erit residuorum excessus, excessui ablatorum æqualis.

18. Si ab inæqualibus æqualia demantur, erit residuorum excessus excessui totorum æqualis.

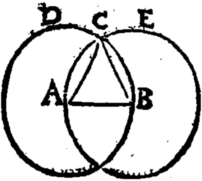
19. Omne totum æquale est omnibus suis partibus simul sumptis.

20. Si totum totius est duplum, & ablatum ablati, erit & reliquum reliqui duplum. Idem de reliquis multiplicibus intellige.

Citationes intellige sic. Cum duo numeri occurrunt, prior designat propositionem, posterior librum. Ut per 4. 1. intelligitur quarta propositio primi libri, atque ita de reliquis. *Cæterum, ax. axioma, post. postulatum, def. definitionem, sch. scholium, cor. corollarium denotant, &c.*

LIB. I.

PROP. I.



Super data recta linea terminata A B, *triangulum æquilaterum* A B C *constituere.*

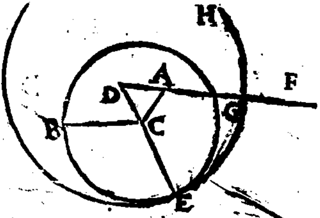
Centris A & B, eodem intervallo A B, vel B A *a* describe duos circulos se intersecantes in puncto C, ex quo *b* duc rectas C A, C B. Erit A C *c* = A B *c* = B C *d* = A C. *e* Quare triangulum A C B est æquilaterum. Quod Erat Faciendum.

a 3.*post.*
b 1.*post.*
c 15.*def.*
d 1.*ax.*
e 23.*def.*

Scholium.

Eodem modo super A B describetur triangulum Isosceles, si intervalla æqualium circulorum majora sumantur, vel minora, quam A B.

PROP. II.



Ad datum punctum A *datæ rectæ lineæ* B C *æqualem rectam lineam* A G *ponere.*

Centro C, intervallo C B *a* describe circulum C B E. *b* Iunge A C, super qua *c* fac triangulum æquilaterum A D C. *d* produc D C ad E. cen-

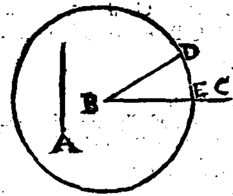
a 3.*post.*
b 1.*po*
c 1.
d 2.*po*

centro D, spatio D E, describe circulum D E H: cujus circumferentiæ occurrat D A *e* protracta
e 2. *post.* ad G. Erit A G = C B.
f 15. *def.* g *constr.* Nam D G *f* = D E, & D A *g* = D C, quare
h 3. *ax.* A G *h* = C E *k* = B C *l* = A G. Q. E. F.
k 15. *def.* Positio puncti A, intra vel extra datam B C,
l 1. *ax.* casus variat, sed ubique similis est constructio, & demonstratio.

Scholium.

Poterat A G circino sumi, sed hoc facere nulli postulato respondet, ut bene innuit Proclus.

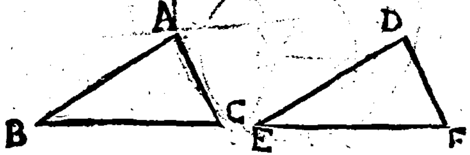
PROP. III.



Duabus datis rectis lineis A, *&* B C, *de majore* BC *minori* A *æqualem rectam lineam* B E *detrahere.*

a 2. 1. Ad punctum B *a* pone rectam B D = A. Circulus centro B, spatio B D descriptus au-
b 15. *def.* feret B E *b* = B D *c* = A *d* = B E. Q. E. F.
c *constr.*
d 1. *ax.*

PROP. IV.



Si duo triangula B A C, E D F *duo latera* B A, A C *duobus lateribus* E D, D F *æqualia habeant, utrumque utrique* (hoc est B A = E D, *&* AC = D F) *habeant vero angulum* A, *angulo* D *æqualem,*

lem, sub æqualibus rectis lineis contentum, & basim B C *basi* E F *æqualem habebunt; eritque triangulum* B A C *triangulo* E D F *æquale, ac reliqui anguli* B, C *reliquis angulis* E, F *æquales erunt, uterque utrique, sub quibus æqualia latera subtenduntur.*

Si punctum D puncto A applicetur, & recta D E rectæ A B superponatur, cadet punctum E in B, quia D E *a* = A B. Item recta D F cadet in A C, quia ang. A *a* = D. Quinetiam punctum F puncto C coincidet, quia A C *a* = D F. Ergo rectæ E F, B C, cum eosdem habeant terminos, *b* congruent, & proinde æquales sunt. Quare triangula B A C, E D F; & anguli B, E; itemque anguli C, F etiam congruunt, & æquantur. Quod erat Demonstrandum.

a *hyp.*
b 14. *ax.*

P R O P. V.

Isoscelium triangulorum ABC *qui ad basim sunt anguli* A B C, A C B *inter se sunt æquales. Et productis æqualibus rectis lineis* A B, A C *qui sub base sunt anguli* C B D, B C E *inter se æquales erunt.*

a Accipe A F = A D, & *b* junge C D, ac B F.

Quoniam in triangulis ACD, ABF, sunt A B *c* = A C, & AF *d* = AD, angulusq; A communis, *e* erit ang. ABF = ACD; & ang. A F B *e* = A D C, & bas. B F *e* = D C; item F C *f* = D B. ergo in triangulis B F C, B D C *g* erit ang. F C B, = D B C. Q.E.D. Item ideo ang. F B C = D C B. atqui ang. ABF *h* = A C D. ergo ang. A B C *k* = A C B. Q. E. D.

a 3. 1.
b 1. *post.*
c *hyp.*
d *constr.*
e 4. 1.
f 3. *ax.*
g 4. 1.
h *pr.*
k 3. *ax.*

Corollarium.

Hinc, Omne triangulum æquilaterum est quoque æquiangulum.

PROP.

PROP. VI.

Si trianguli A B C *duo anguli* A B C, A C B *æquales inter se fuerint, & sub æqualibus angulis subtensa latera* A B, A C *æqualia inter se erunt.*

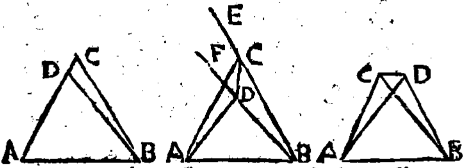
Si fieri poteſt, ſit utravis
a 3. 1. B A ⊏ C A, *a* Fac igitur B D = C A, & *b* duc
b 1. *poſt.* C D.

In triangulis DBC, ACB, quia BD *c* = CA,
c *ſuppoſ.* & latus BC commune eſt, atque ang. DBC *d* =
d *hyp.* A C B, *e* erunt triangula D B C, A C B æqualia
e 4. 1. inter ſe, pars & totum, *f* Quod Fieri Nequit.
f 9. *ax.*

Coroll.

Hinc, Omne triangulum æquiangulum eſt quoque æquilaterum.

PROP. VII.



Super eadem recta linea A B *duabus eiſdem rectis lineis* A C, B C, *aliæ duæ rectæ lineæ æquales* A D, B D, *utraque utrique* (*hoc eſt,* A D = AC, & B D = B C) *non conſtituentur ad aliud punctum* C, *atque aliud* D, *ad eaſdem partes* C, *eoſdemque terminos* A, B *cum duabus initio ductis rectis lineis habentes.*

a 9. *ax.* 1. *Caſ.* Si punctum D ſtatuatur in A C. *a* liquet non eſſe A D = A C.

2. *Caſ.* Si punctum D dicatur intra triangulum A C B, duc C D, & produc B D F, ac B C E.
b 5. 1. Iam vis A D = A C. ergo ang. ADC *b* = ACD;
c *ſuppoſ.* item quia BD *c* = BC, erit ang. FDC *b* = ECD.

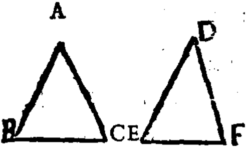
ergo

ergo ang. F D C *d* $\sqsubset$ A C D, id est ang. F D C d 9.ax.
$\sqsubset$ A D C *d* Q. F. N.

3. *Cas.* Sin D cadat extra triangulum A C B, jungatur C D.

Rursus, ang. B C D *e* = BDC, & BCD *e* = e 5.1.
B D C. *f* ergo ang. A C D $\sqsupset$ B D C. & proinde f 9.ax.
multo magis ang. B C D $\sqsupset$ B D C. Sed erat ang. B C D = B D C. Quæ repugnant. Ergo, &c.

P R O P. VIII.



Si duo triangula A B C, D E F *habuerint duo latera* A B, A C *duobus lateribus* D E, D F, *utrumque utrique æqualia; habuerint vero & basim* B C, *basi* E F, *æqualem: angulum* A *sub æqualibus rectis lineis contentum angulo* D *æqualem habebunt.*

Quia B C *a* = E F, si basis B C superponatur a hyp.
basi EF, illæ *b* congruent. ergo, cum AB *c* = DE, b 8.ax.
& AC *c* = D F, cadet punctum A in D. (nam c hyp.
in aliud punctum cadere nequit, per præcedentem) ergo angulorum A, & D latera coincidunt.
d quare anguli illi pares sunt. Q. E. D. d 8.ax.

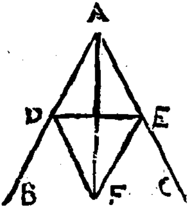
Coroll.

1. Hinc triangula sibi mutuo æquilatera, etiam mutuo *x* æquiangula sunt. x 4.1.

2. Triangula sibi mutuo æquilatera *y* æquentur inter se. y 4.1.

PROP.

PROP. IX.



Datum angulum rectilineum B A C *bifariam secare.*

a 3. 1. *a* Sume A D = A E;
b 1. 1. duc D E, super qua *b* fac triang. æquilat. D F E.

Ducta A F angulum B A C bisecabit.

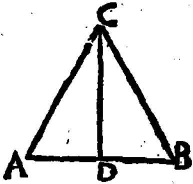
Nam A D *c* = A E,
c constr. & latus A F commune est, & bas. D F *c* = F E.
d 8. 1. *d* ergo ang. D A F = E A F. Q. E. F.

Coroll.

Hinc patet quomodo angulus secari possit in æquales partes 4, 8, 16, &c. Singulos nimirum partes iterum bisecando.

Methodus vero regula & circino angulos secandi in æquales quotcunque hactenus Geometras latuit.

PROP. X.

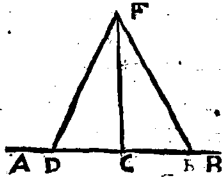


Datam rectam lineam A B *bifariam secare.*

a 1. 1. Super data A B *a* fac triang. æquilat. A B C.
b 9. 1. ejus angulum C *b* biseca recta C D. Eadem datam A B bisecabit.

c constr. Nam A C *c* = B C,
& latus C D est commune; & ang. A C D *c* =
d 4. 1. B C D, *d* ergo A D = B D. Q. E. F. Praxin hujus & præcedentis, constructio primæ hujus libri satis indicat.

PROP. XI.



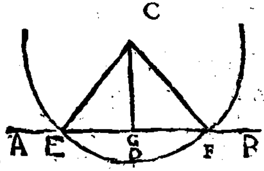
Data recta linea A B, *& puncto in ea dato* C, *rectam lineam* C F *ad angulos rectos excitare.*

a Accipe hinc inde C D = C E. Super D E *b* fac triang. æquilat. D F E. Ducta F C perpendicularis est. a 3. 1. b 1. 1.

Nam triangula D F C, E F C sibi mutuo *c* æquilatera sunt. *d* ergo ang. D C F = E C F. *e* ergo F C perpendicularis est. Q. E. F. c *constr.* d 8. 1. e 10. *def.*

Praxis tam hujus, quam sequentis expeditur facillime ope normæ.

PROP. XII.

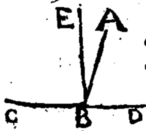


Super datam rectam lineam infinitam A B, *à dato puncto* C *quod in ea non est, perpendicularem rectam* C G *deducere.*

Centro C *a* describe circulum, qui secet datam A B in punctis E & F *b* biseca E F in G. ducta C G perpendicularis est. a 3. *post.* b 10. 1.

Ducantur enim C E, C F. Triangula E G C, F G C, sibi mutuo *c* æquilatera sunt. *d* ergo anguli E G C, F G C, æquales, & *e* proinde recti sunt. Q. E. F. c *constr.* d 8. 1. e 10. *def.*

PROP. XIII.



Cum recta linea A B, *super rectam lineam* C D *consistens, facit angulos* A B C, A B D; *aut duos rectos, aut duobus rectis æquales efficiet.*

Si

a 10 def. Si anguli ABC, ABD pares sint[c] *a* liquet illos
b 11. 1. rectos esse; sin inæquales sint, ex B *b* excitetur
c 19. ax. perpendicularis B E. Quoniam ang. A B C *c* =
d 3. ax. Rect. + ABE; & ang. ABD *d* = Rect. − ABE;
e 2. ax. erit ABC + ABD *e* = 2 Rect. + ABE − ABE = 2 Rect. Q. E. D.

Coroll.

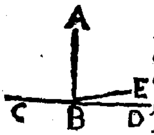
1. Hinc, si unus ang, A B D rectus sit, alter A B C etiam rectus erit; si hic acutus, ille obtusus erit, & contra.

2. Si plures rectæ quam una ad idem punctum eidem rectæ insistant, anguli fient duobus rectis æquales.

3. Duæ rectæ invicem secantes efficiunt angulos quatuor rectis æquales.

4. Omnes anguli circa unum punctum constituti conficiunt quatuor rectos. patet ex Coroll. 2.

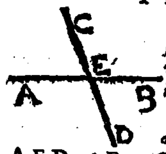
PROP. XIV.



Si ad aliquam rectam lineam A B, *atque ad ejus punctum* B *duæ rectæ lineæ* C B, B D *non ad easdem partes ductæ, eos qui sunt deinceps angulos* A B C, A B D *duobus rectis æquales fecerint, in directum erunt inter se ipsæ rectæ lineæ* C B, B D.

a 13. 1. Si negas, faciant C B, B E unam rectam. ergo
b hyp. ang. A B C + ABE *a* = 2 Rect. *b* = ABC +
c 9. ax. A B D. *c* Quod Est absurdum.

PROP. XV.

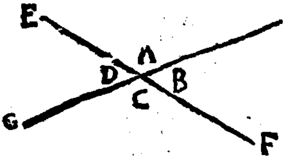


Si duæ rectæ lineæ A B, C D *se mutuo secuerint, angulos ad verticem* C E B, A E D *æquales inter se efficient.*

a 13. 1. Nam ang. A E C + C E B *a* = 2 Rect. *a* = A E C +
b 3. ax. A E D. *b* Ergo C E B = A E D. Q. E. F.

Schol.

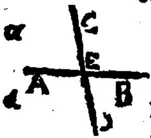
Schol.



Si ad aliquam rectam lineam G H, atque ad ejus punctum, A duæ rectæ lineæ E A, A F non ad easdem partes sumptæ, angulos ad verticem D, & B æquales fecerint, ipsæ rectæ lineæ E A, A F in directum sibi invicem erunt.

Nam 2 Rect. = *a* D + A *a* = B *b* + A. *b* ergo EA, AF sunt in directum sibi invicem. Q.E.D. a 13. 1. b 14. 1.

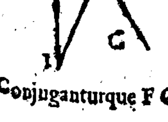
Schol. 2.



Si quatuor rectæ lineæ E A, E B, E C, E D ab uno puncto E exeuntes, angulos oppositos ad verticem æquales inter se fecerint, erunt quælibet duæ lineæ A E, E B, & C E, E D in directum positæ.

Nam quia ang. AEC + AED + CEB + DEB *a* = 4 Rect. erit A E C + A E D *b* = CEB + DEB = 2 Rect. *c* ergo CED, & AEB sunt rectæ lineæ. Q. E. D. a 4 Cor. 13. 1. b 13. 1. & 2. ax. c 14. 1.

P R O P. XVI.



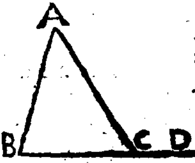
Cujuscunque Trianguli A B C *uno latere* B C *producto, externus angulus* A C D *utrolibet interno & opposito* C A B, C B A, *major est.*

Latera A C, B C *a* bisecent rectæ A H, B E, è quibus productis *b* cape EF = B E, *b* & H I = A H, a 10. 1. & 1. post. b 3. 1.

Conjuganturque F C, I. B Quo-c constr. d 15.1. e 4.1. f 15.1. g 9.ax.

Quoniam C E c = E A, & E F c = E B, & ang. F E C d = BEA; e erit ang. ECF = EAB. Simili argumento ang. ICH (f FCD) = ABH. ergo totus ACD g major est utrovis CAB, & ABC. Q. E. D.

PROP. XVII



Cujuscunque trianguli ABC *duo anguli duobus rectis sunt minores, omnifariam sumpti.*

Producatur latus B C. Quoniam ang. ACD + ACB a = 2 Rect. & ang. ACD b ⊏ A, c erit A + ACB ⊐ 2 Rect. Eodem modo erit ang. B + ACB ⊐ 2 Rect. Denique producto latere A B, erit similiter ang. A + B ⊏ 2 Rect. Quæ E. D.

a 13.1. b 16.1. c 4.ax.

Coroll.

1. Hinc, in omni triangulo, cujus unus angulus fuerit rectus, vel obtusus, reliqui acuti sunt.

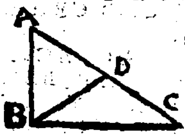
2. Si linea recta A E cum alia recta C D angulos inæquales faciat, unum AED acutum, & alterum A E C obtusum, linea perpendicularis A D ex quovis ejus puncto A ad aliam illam CD demissa, cadet ad partes anguli acuti AED.

Nam si AC ad partes anguli obtusi ducta, dicatur perpendicularis, in triangulo AFC erit ang. AEC + ACE ⊏ 2 Rect. x Q. F. N.

x 17.1.

3. Omnes anguli trianguli æquilateri, & duo anguli trianguli Isoscelis, supra basim, acuti sunt.

PROP. XVIII.



Omnis trianguli A B C *majus latus* A C *majorem angulum* ABC *subtendit.*

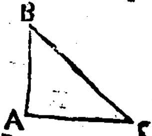
Ex A C a aufer A D = A B, & junge D B. b ergo ang. A D B = A B D. Sed

a 3.1. b 5.1.

c ADB

e ADB $\sqsubset$ C. ergo ABD $\sqsubset$ C. d ergo totus ang. ABC $\sqsubset$ C. Eodem modo erit ABC $\sqsubset$ A. Q. E. D. c 16. 1. d 9 ax.

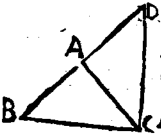
PROP. XIX.



Omnis trianguli A B C *major angulus* A *majori lateri* BC *subtenditur.*

Nam si dicatur A B $=$ BC, a erit ang. A $=$ C. contra Hypoth. & si A B $\sqsubset$ BC, b erit ang. C $\sqsubset$ A, contra hyp. quare potius B C $\sqsubset$ A B, & eodem modo B C $\sqsubset$ A C. Q. E. D. a 5. 1. b 18. 1.

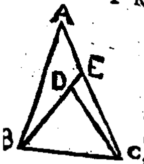
PROP. XX.



Omnis trianguli A B C *duo latera* B A, A C *reliquo* B C *sunt majora quomodocunque sumpta.*

Ex B A producta a cape A D $=$ A C, & duc D C. b ergo ang. D $=$ A C D. c ergo totus B C D $\sqsubset$ D d ergo B D (e B A $+$ A C) $\sqsubset$ B C. Q. E. D. a 3. 1. b 5. 1. c 9. ax. d 19. 1. e constr. & 2. ax.

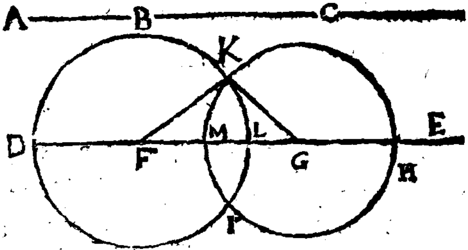
PROP. XXI.



Si super trianguli A B C *uno latere* BC, *ab extremitatibus duæ rectæ lineæ* B D, C D, *interius constitutæ fuerint, hæ constitutæ reliquis trianguli duobus lateribus* B A, C A *minores quidem erunt, majorem vero angulum* BDC *continebunt.*

Producatur BD in E. estque CE $+$ ED a $\sqsubset$ CD adde commune BD, b erit BE $+$ EC $\sqsubset$ BD $+$ DC. Rursus BA $+$ AE a $\sqsubset$ BE; b ergo BA $+$ AC $\sqsubset$ BE $+$ EC. quare B A $+$ AC $\sqsubset$ B D $+$ D C. Q. E. D. 2. Ang. B D C c $\sqsubset$ DEC c $\sqsubset$ A. ergo ang. BDC $\sqsubset$ A, Q. E. D. a 20. 1. b 4. ax. c 16. 1.

PROP. XXII.



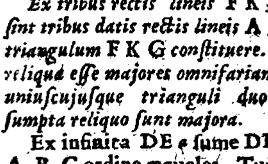
Ex tribus rectis lineis F K, F G, G K, *quæ sint tribus datis rectis lineis* A, B, C, *æquales, triangulum* F K G *constituere. Oportet autem duas reliquâ esse majores omnifariam sumptas; quoniam uniuscujusque trianguli duo latera omnifariam sumpta reliquo sunt majora.*

a 3. 1.
b 3. post.

Ex infinita DE *a* sume DF, FG, GH datis A, B, C ordine æquales. Tum si *b* centris F, & G, intervallis F D, & G H ducantur circuli se intersecantes in K; junctis rectis K F, K G constituetur triangulum F K G, *c* cujus latera F K, F G, G K tribus DF, F G, G H, *d* id est tribus datis A, B, C æquantur. Q. E. F.

c 15. def.
d 1. ax.

PROP. XXIII.



Ad datam rectam lineam A B, *datumque in ea punctum* A, *dato angulo rectilineo* D *æquale angulum rectilineum* A *constituere.*

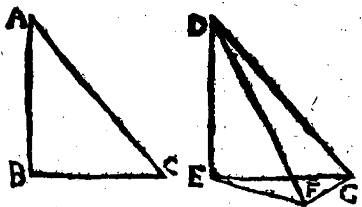
a 1. post.
b 3. 1.
c 22. 1.

a Duc rectam C F secantem dati anguli latera utcunque. *b* Fac AG = CD. Super AG *c* constitue triangulum alteri C D F æquilaterum, ita

ut

ut AH $\doteq$ DF, & GH $\doteq$ CF; & habebis ang.
A [d] $=$ D. Q. E. F. d 8. 1.

PROP. XXIV.



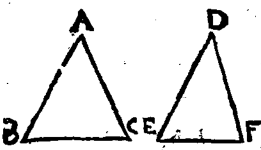
Si duo triangula ABC, DEF *duo latera* AB, AC *duobus lateribus* DE, DF *aequalia habuerint, utrumque utrique; angulum vero* A *angulo* EDF *majorem sub aequalibus rectis lineis contentum, & basim* BC, *basi* EF, *majorem habebunt.*

[a] Fiat ang. EDG $=$ A, & DG [b] $=$ DF [c] $=$ a 23. 1.
AC, connectanturque EG, FG. b 3. 1.
c hyp.
1. *Cas.* Si EG cadit supra EF. Quia AB d hyp.
[d] $=$ DE, & AC $=$ [e] DG, & ang. A [e] $\doteq$ EDG, e constr.
[f] erit BC $=$ EG. Quia vero DF [c] $=$ DG, f 4. 1.
[g] erit ang. DFG $=$ DGF. [h] ergo ang. DFG $\sqsubset$ g 5. 1.
EGF; [h] & proinde ang. EFG $\sqsubset$ EGF. [k] quare h 9. ax.
EG (BC) $\sqsubset$ EF. Q. E. D. k 19. 1.

2. *Cas.* Si basis EF basi EG coincidat; [l] liquet EG (BC) $\sqsubset$ EF. l 9 ax.

3. Sin EG Cadat infra EF. Quoniam
DG + GE [m] $\sqsubset$ DF + FE, si hinc inde au- m 21. 1.
ferantur DG, DF, aequales; manet EG (BC)
[n] $\sqsubset$ EF. Q. E. D. n 5. ax.

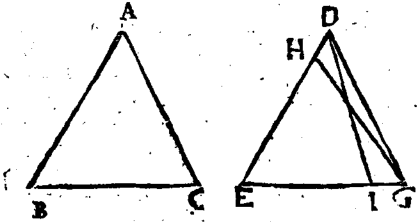
PROP. XXV.



Si duo triangula ABC, DEF *duo latera* AB, AC *duobus lateribus* DE, DF *æqualia habuerint, utrumq; utrique, basim vero* BC *basi* EF *majorem; & angulum* A *sub æqualibus rectis lineis contentum angulo* D *majorem habebunt.*

a 4. 1. Nam si dicatur ang. A = D. *a* erit basis BC = EF, contra Hyp. Sin dicatur ang. A $\sqsupset$ D,
b 24. 1. *b* erit BC $\sqsupset$ EF, etiam contra Hyp. ergo BC $\sqsubset$ EF. Q. E. D.

PROP. XXVI.



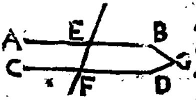
Si duo triangula BAC EDG, *duos angulos* B, C, *duobus angulis* E, DGE, *æquales habuerint, utrumque utrique, unumque latus uni lateri æquale, sive quod æqualibus adjacet angulis, seu quod uni æqualium angulorum subtenditur: reliqua latera reliquis lateribus æqualia, utrumque utrique, & reliquum angulum reliquo angulo æqualem habebunt.*

1. *Hyp.* Sit BC = EG. Dico BA = ED, & AC = DG, & ang. A = EDG. Nam si dicatur
a 3. 1. ED $\sqsubset$ BA, *a* fiat EH = BA, ducaturque GH. Quoniam

Quoniam AB b = HE, & BC c = EG, & ang. B c = E; erit ang. EGH d = C e = DGE. f Q. E. A. ergo AB = ED. Eodem modo AC = DG. d quare etiam ang. A = EDG. b suppos. c hyp. d 4. 1. e hyp. f 9. ax.

2. *Hyp.* Sit AB = ED. Dico BC = EG; & AC = DG & ang. A = EDG. Nam si dicatur EG ⊏ BC, fiat EI = BC, & connectatur DI. Quia AB g = ED, & BC h = EI, & ang B g = E, erit ang. EID k = C m = EGD. n Q. E. A. ergo BC = EG. ergo ut prius, AC = DG, & ang. A = EDG. Q. E. D. g hyp. h suppos. k 4. 1. m hyp. n 16. 1.

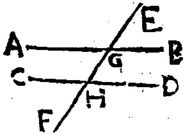
PROP. XXVII.



Si in duas rectas lineas AB, CD recta incidens linea EF alternatim angulos AEF, DFE, æquales inter se fecerit, parallelæ erunt inter se illæ rectæ lineæ AB, CD.

Si AB, CD dicantur non esse parallelæ; conveniant productæ, nempe in G. quo posito angulus externus AEF interno DFE a major erit, cui tamen ponitur æqualis. Quæ repugnant. a 16. 1.

PROP. XXVIII.

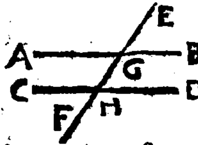


Si in duas rectas lineas AB, CD recta incidens linea EF externum angulum AGE interno & opposito, & ad easdem partes CHG æqualem fecerit, aut internos & ad easdem partes AGH, CHG duobus rectis æquales; parallelæ erunt inter se ipsæ rectæ lineæ AB, CD.

1. *Hyp.* Quia per hyp. ang. AGE = CHG, a erit altern. BGH = GHG. b parallelæ igitur sunt AB, CD. Q. E. D. a 15. 1. b 27. 1.

2. *Hyp.* Quia ex hyp. Ang. AGH + CHG = 2 Rect. a = AGH + BGH, b erit CHG = BGH. Ergo c AB, CD parallelæ sunt. Q. E. D. a 13. 1. b 3. ax. c 27. 1.

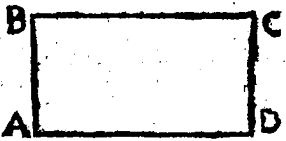
PROP. XXIX.



In parallelas rectas lineas AB, CD, *recta incidens linea* EF, *& alternatim angulos* DHG, AGH *æquales inter se efficit; & externum* BGE *interno, & opposito, & ad easdem partes* DHE *æqualem; & internos & ad easdem partes* AGH, CHG *duobus rectis æquales facit.*

a 13. ax. Liquet AGH, + CHG = 2 Rect. a alias AB, CD non essent parallelæ, contra hyp. Sed
b 13. 1. & ang. DHG + CHG b = 2 Rect. ergo DHG
c 13. ax.
d 15. 1. c = AGH d = BGE. Q. E. D.

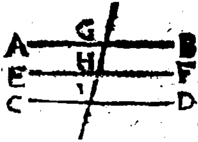
Coroll.



Hinc omne Parallelogrammum AC habens unum angulum rectum A, est rectangulum.

29. 1. Nam A + B a = 2 Rect. ergo cum A re-
a 3. ax. ctus sit, b etiam B rectus erit. Eodem argumento D, & C recti sunt.

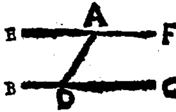
PROP. XXX.



Quæ (AB, CD) *eidem rectæ lineæ* EF *parallelæ, & inter se sunt parallelæ.*

Tres rectas secet utcunque recta GL. Quoniam AB, EF parallelæ sunt,
a 29. 1. a erit ang. AGI = EHI, Item propter CD, EF
b 1. ax. parallelas, a erit ang. EHI = DIG. b ergo ang.
c 27. 1. AGI = DIG. c quare AB, CD parallelæ sunt. Q. E. D.

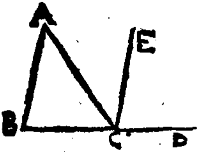
PROP. XXXI.



A dato puncto A datæ rectæ lineæ B C ducere parallelam rectam lineam A E.

Ex A ad datam BC duc rectam utcunque AD. ad quam, ejusque punctum A *a* fac ang. D A E = A D C. *b* erunt A E, BC parallelæ. Q. E. F. a 23. 1. b 27. 1.

PROP. XXXII.



Cujuscunque trianguli A B C *uno latere* B C *producto, externus angulus* A C D *duobus internis, & oppositis,* A B *est æqualis. Et trianguli tres interni anguli,* A, B, A C B *duobus sunt rectis æquales.*

Per C *a* duc C E parall. B A. Ang. A *b* = ACE. & ang. B *b* = ECD. ergo A + B *c* = ACE + ECD *d* = ACD. Q. E. D. Pono ACD + ACB *e* = 2 Rect. *f* ergo A + B + A C B = 2 Rect. Q. E. D. a 31. 1. b 29. 1. c 2. ax. d 19. ax. e 13. 1. f 1. ax.

Corollaria.

1. Tres simul anguli cujusvis trianguli æquales sunt tribus simul cujuscunque alterius. Unde

2. Si in uno triangulo duo anguli (aut singuli, aut simul) æquales sint duobus angulis (aut singulis, aut simul) in altero triangulo, etiam reliquus reliquo æqualis est. Item, si duo triangula unum angulum uni æqualem habeant, reliquorum summæ æquantur.

3. In triangulo si unus angulus rectus sit, reliqui unum rectum conficiunt. Item, angulus, qui duobus reliquis æquatur, rectus est.

4. Cum in Isoscele angulus æquis cruribus contentus rectus est, reliqui ad basim sunt semirecti.

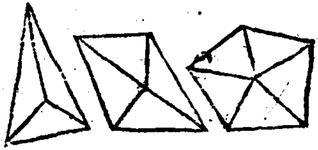
5. Tri-

5. Trianguli æquilateri angulus facit duas tertias unius recti, nam $\frac{1}{3}$ 2 Rect. $=$ $\frac{2}{3}$ Rect.

Schol.

Hujus propositionis beneficio, cujuslibet figuræ rectilineæ tam interni quam externi anguli quot rectos conficiant, innotescet per duo sequentia theoremata.

THEOREMA. I.



Omnes simul anguli cujuscunque figuræ rectilineæ conficiunt bis tot rectos demptis quatuor, quot sunt latera figuræ.

Ex quovis puncto intra figuram ducantur ad omnes figuræ angulos rectæ, quæ figuram resolvent in tot triangula quot habet latera. Quare cum singula triangula conficiant duos rectos, omnia simul conficient bis tot rectos, quot sunt latera. Sed anguli circa dictum punctum conficiunt quatuor rectos. Ergo, si ab omnium triangulorum angulis demas angulos circa id punctum, anguli reliqui qui componunt angulos figuræ conficient bis tot rectos demptis quatuor, quot sunt latera figuræ. Q. E. D.

Hinc *Coroll.* Omnes ejusdem speciei rectilineæ figuræ æquales habent angulorum summas.

THEOREMA 2.

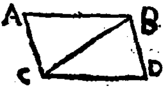
Omnes simul externi anguli cujuscunque figuræ rectilineæ conficiunt quatuor rectos.

Nam singuli figuræ interni anguli cum singulis externis conficiunt duos rectos. Ergo interni

terni simul omnes, cum omnibussimul externis conficiunt bis tot rectos, quot sunt latera figuræ. Sed (ut modo ostensum est,) interni simul omnes etiam cum quatuor rectis efficiunt bis tot rectos, quot sunt latera figuræ. Ergo externi anguli quatuor rectis æquantur. Q. E. D.

Coroll. Omnes cujuscunque speciei rectilineæ figuræ æquales habent externorum angulorum summas.

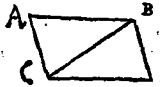
PROP. XXXIII.



Rectæ lineæ AC, BD, *quæ æquales & parallelas lineas* AB, CD, *ad partes easdem conjungunt, & ipsæ æquales ac parallelæ sunt.*

Connectatur C B. Quoniam ob AB, CD parallelas. ang. ABC a = BCD, & per hyp. AB = CD, & latus CB commune est, b erit AC = BD, b & ang. ACB = DBC. c ergo AC, BD etiam parallelæ sunt. Q. E. D. a 29. 1. b 4. 1. c 27. 1.

PROP. XXXIV.



Parallegrammorum spatiorum ABDC *æqualia sunt inter se quæ ex adverso latera* AB, CD; ac AC, BD; *angulique* A, D, & ABD, ACD; *& illa bifariam secat diameter* CB.

Quoniam AB, CD a parallelæ sunt, b erit ang. ABC = BCD. Item ob AC, DB a parallelas, b erit ang. ACB = CBD. c ergo toti anguli ACD, ABD æquantur. Similiter ang. A = D. Porro, cum communi lateri CB adjaceant anguli ABC, ACB, ipsis BCD, CBD pares d, erunt AC = BD, d & AB = CD. adeoque etiam triang. ABC = CBD. Quæ E. D. a hyp. b 29. 1. c 2. ax. d 26. 1.

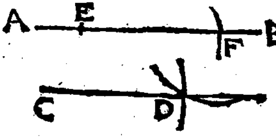
SCHOL.

SCHOL.

Omne quadrilaterium A B D C *habens latera opposita æqualia, est parallelogrammum.*

a 27. 1. Nam per 8. 1. ang. A B C = B C D. a ergo A B, C D parallelæ sunt. Eadem ratione ang. BCA = CBD; a quare A C, B D etiam parallelæ sunt. b Ergo ABDC est parallelogrammum. Q. E. D.

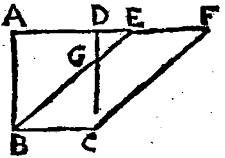
b 35. def. 1.



Hinc expeditius per datum punctum C datæ rectæ A B ducetur parallela C D.

Sume in A B quodvis punctum E. centris E. & C ad quodvis intervallum duc æquales circulos E F, C D. centro vero F, spatio E C duc circulum F D, qui priorem C D secet in D. Erit ducta C D parall. A B. Nam ut modo demonstratum est, C E F D est parallelogrammum.

PROP. XXXV.



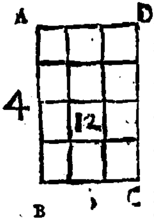
Parallelogramma BCDA, BCFE *super eadem basi* B C, *& in eisdem parallelis* AF, BC *constituta, inter se sunt æqualia.*

a 34. 1.
b 2. ax.
c 29. 1.
d 4. 1.
e 3. ax.
f 2. ax.

Nam A D a = B C a = E F. adde communem DE, b erit AE = DF. Sed & AB a = DC; & ang. A c = C D F. d ergo triang. A B E = D C F. aufer commune D G E, e erit Trapez. ABGD = EGCF. adde commune BGC, f erit Pgr. ABCD = EBCF, Q. E. D. Reliquorum casuum non dissimilis, sed simplicior & facilior est demonstratio.

Scho-

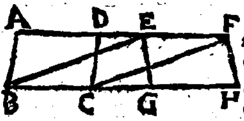
Scholium.



Si latus A B parallelogrammi rectanguli A B C D ferri intelligatur perpendiculariter per totam B C, aut B C per totam A B, producetur eo motu area rectanguli A B C D. Hinc rectangulum fieri dicitur ex ductu seu multiplicatione duorum laterum contiguorum. Sit exempl. gr. B C pedum 3, A B 4. Duc 3 in 4; proveniunt 12 pedes quadrati pro area rectanguli.

Hoc supposito, ex hoc theoremate cujuscunq; parallelogrammi (* EBCF) habetur dimensio. Illius enim area producitur ex altitudine B A ducta in basim B C. Nam area rectanguli A C parallelogrammo E B C F æqualis, fit ex B A in B C. ergo, &c. * v. fig. prop. 35.

Prop. XXXVI.

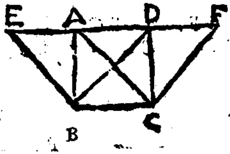


Parallelogramma B C D A, G H F E *super æqualibus basibus* B C, G H, *& in eisdem parallelis* A F, B H *constituta, inter se sunt æqualia.*

Ducantur BE, CF. Quia BC *a* = GH *b* = EF, *c* erit BCFE parallelogrammum. ergo Pgr. BCDA *d* = BCFE *d* GHFE. Q. E. D.

a hyp.
b 34. 1.
c 33. 1.
d 35. 1.

Prop. XXXVII.

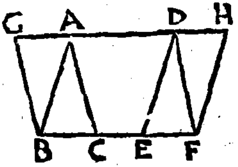


Triangula BCA, B G D *super eadem basi* BC *constituta, & in eisdem parallelis* B C, E F *inter se sunt æqualia.*

a Duc

a 31. 1. a Duc BE parall. CA, a & CF parall. BD.
b 34. 1. Erit triang. BCA b $=\frac{1}{2}$ Pgr. BCAE $=$ c $\frac{1}{2}$
c 35. 1. & 7. ax. BDFC b $=$ BCD. Q.E.D.

PROP. XXXVIII.



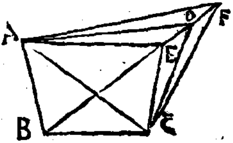
Triangula BCA, EFD *super æqualibus basibus* BC, EF *constituta, & in eisdem parallelis* GH, BF, *inter se sunt æqualia.*

Duc BG parall. CA. & FH parall. ED.
a 34. 1. erit triang. BCA a $=\frac{1}{2}$ Pgr. BCAG b $=\frac{1}{2}$
b 36. 1. & 7. ax. EDHF c $=$ EFD. Q.E.D.
c 34. 1.

Schol.

Si basis BC $\sqsubset$ EF, liquet triang. BAC $\sqsubset$ EDF. & si BC $\sqsupset$ EF, erit BAC $\sqsupset$ EDF.

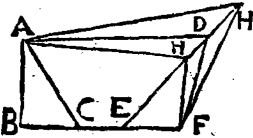
PROP. XXXIX.



Triangula æqualia BCA, BCD, *super eadem basi* BC, *& ad easdem partes constituta, etiam in eisdem sunt parallelis* AD, BC.

Si negas, sit altera AF parall. BC; & ducatur
a 37. 1. CF. ergo triang. CBF a $=$ CBA b $=$ CBD,
b hyp. c Q.E.A.
c 9. ax.

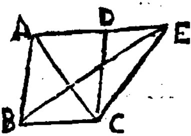
PROP. XL.



Triangula æqualia B C A, E F D *super æqualibus basibus* B C, E F, & *ad easdem partes constituta, & in eisdem sunt parallelis* A D, B F.

Si negas, sit altera AH parall. BF. & ducatur F H. ergo triang. E F H *a* = B C A *b* = EFD. *c* Q. E. A. a 38. 1. b hyp. c 9 ax.

PROP. XLI.



Si parallelogrammum A B C D *cum triangulo* B C E *eandem basim* B C *habuerit, in eisdemque fuerit parallelis* A E, B C, *duplum erit parallelogrammum* ABCD *ipsius trianguli* BCE.

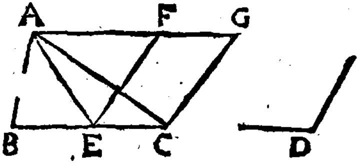
Ducatur AC. Triang. BCA *a* = BCE. ergo Pgr. ABCD *b* = 2 BCA *c* = 2 BCE. Q. E. D. a 37. 1. b 34. 1. c 6 ax.

Scholium.

Hinc habetur area cujuscunq; trianguli BCE. Nam cum area parallelogrammi ABCD producatur ex altitudine in basim ducta; producetur area trianguli ex dimidia altitudine in basim ducta, vel ex dimidia basi in altitudinem. ut si basis B C sit 8, & altitudo 7; erit trianguli B C E area, 28.

PROP.

PROP. XLII.

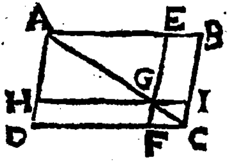


Dato triangulo A B C *æquale parallelogrammum* E C G F *constituere in dato angulo rectilineo* D.

a 31. 1. Per A *a* duc A G parall. B C. *b* fac ang. B C G
b 23. 1. = D. basim B C *c* biseca in E. *a* duc E F parall.
c 10. 1. C G. Dico factum.

Nam ducta A E. erit ex. constr. ang. E C G
d 38. 1. = D, & triang. B A C *d* = 2 A E C *e* = Pgr.
e 41. 1. E C G F. Q. E. F.

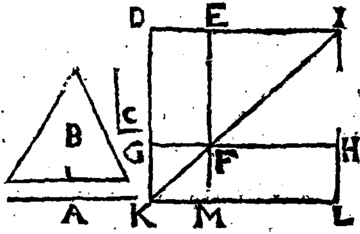
PROP. XLIII.



In omni parallelogrammo ABCD *complementa* D G , G B *eorum quæ circa diametrum* AC *sunt parallelogrammorum* HE, F I *inter se sunt æqualia.*

a 34. 1. Nam Triang. A C D, = *a* A C B. & triang.
b 3. ax. AGH *a* = AGE. & triang. GCF *a* = GCI.
b ergo Pgr. D G = G B. Q. E. D.

PROP. XLIV.



Ad datam rectam lineam A, *dato triangulo* B, *æquale parallelogrammum* FL *applicare in dato angulo rectilineo* C.

a Fac Pgr. FD = triang. B, ita ut ang. GFE = C. & pone lateri GF in directum FH = A. Per H b duc IL parall. EF; cui occurrat DE producta ad I. per IF ductæ rectæ occurrat DG protracta ad K. Per K b duc KL parall. GH; cui occurrant EF, & IH prolongatæ ad M, & L. Erit FL. Pgr. quæsitum. a 43. 1. b 31. 1.

Nam Pgr. FL c = FD = B d & ang. MFH = GFE = C. Q.E.F. c 43. 1. d 15. 1.

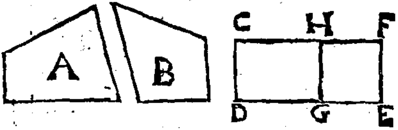
PROP. XLV.

Ad datam rectam lineam FG *dato rectilineo* ABCD *æquale parallelogrammum* FL *constituere, in dato angulo rectilineo* E.

Datum rectilineum resolve in triangula BAD, BCD. a = Fac Pgr. FH = BAD ita ut ang. F = E. producta FI a fac (ad HI) Pgr. a 44. huj.

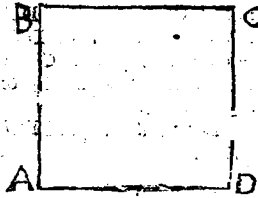
b 19. ax. IL = BCD. erit Pgr. FL = *b* FH + IL *c* =
c constr. ABCD. Q. E. F.

Schol.



Hinc facile invenitur excessus HE, quo rectilineum aliquod A superat rectilineum minus B; nimirum si ad quamvis rectam CD applicentur Pgr. DF = A. & DH = B.

PROP. XLVI.



A data recta linea AD quadratum AC describere.

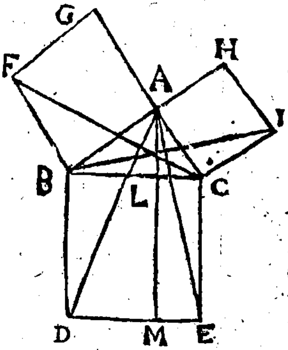
a 11. 1. *a* Erige duas perpendiculares AB,
b 3. 1. DC *b* æquales datæ AD; & junge BC. dico factum.

c constr. Cum enim ang. A + D *c* = 2 Rect. *d* erunt
d 28. 1. AB, DC parallelæ. Sunt vero etiam *e* æquales,
e constr. *f* ergo AD, BC pares etiam sunt, & parallelæ.
f 34. 1. ergo Figura AC est parallelogramma, & æquilatera. Anguli quoque omnes recti sunt, *g* quoni-
g Sch. 29. 1. am unus A est rectus. *h* ergo AC est quadratum.
h 29. def. Q. E. F.

Eodem modo facile describes rectangulum, quod sub datis duabus rectis contineatur.

PROP.

PROP. XLVII.



In rectangulis triangulis BAC *quadratum* BE, *quod à latere* BC *rectum angulum* BAC *subtendente describitur, æquale est eis*, BG, CH, *quæ à lateribus* AB, AC *rectum angulum continentibus describuntur.*

Iunge AE, AD; & duc AM. parall. CE.

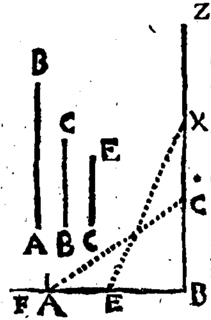
Quoniam ang. DBC *a* = FBA; adde communem ABC, erit ang. ABD = FBC. Sed & AB *b* = FB, & BD *b* = BC. *c* ergo triang. ABD = FBC. atqui Pgr. BM. *d* = 2 ABD; & Pgr. BG *d* = 2 FBC (nam GAC est una recta per hyp. & 14. 1.) *e* ergo Pgr. BM = BG. Simili discursu Pgr. CM = CH. Totum igitur BE = BG + CH. Q. E. D. a 12. ax. b 29. def. c 4. 1. d 41. 1. e 6. ax.

Schol.

Hoc nobilissimum, & utilissimum theorema ab inventore Pythagora, Pythagoricum dici meruit. Ejus beneficio quadratorum additio, & substractio perficitur; quo spectant duo sequentia problemata.

PROBL. 1.

Andr. Tacq.

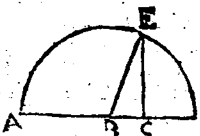


Datis quotcunque quadratis, unum omnibus æquale construere.

Dentur quadrata tria, quorum latera sint A B, B C, C E. *a* Fac ang. rectum F B Z infinita habentem latera, in eaque transfer B A, & B C, & junge A C, *b* erit A Cq = A Bq + B Cq. Tum A C transfer ex B in X; & C E tertium latus datum transfer ex B in E, & junge E X, *b* erit EXq = EBq (CEq) + BXq (ACq) *c* = CEq + ABq + BCq. Q. E. F.

a 11. 1.
b 47. 1.
c 2 ax.

PROBL. 2.

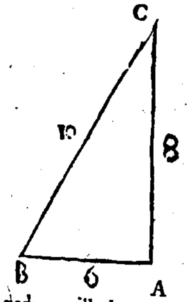


Datis duabus rectis inæqualibus A B, B C, *exhibere quadratum, quo quadratum majoris* A B *excedit quadratum minoris* B C.

Centro B intervallo BA describe circulum. ex C erige perpendicularem C E occurrentem peripheriæ in E. & ducatur B E. *a* Erit B Eq (BAq) = BCq + CEq. *b* ergo BAq − BCq = CEq. Q. E. F.

a 47. 1.
b 3. ax.

PROBL. 3.

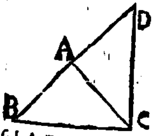


Notis duobus quibuscunque lateribus trigoni rectanguli ABC, reliquum invenire,

Latera rectum angulum ambientia ſint AC, AB, hoc 6. pedum, illud 8. ergo cum ACq + ABq = 64 + 36 = 100 = BCq. erit BC = √100 = 10. 47. 1.

Nota ſint deinde latera AB, BC, hoc 10. pedum, illud 6. ergo cum BCq − ABq = 100 − 36 = 64 = ACq. erit Acq = √64 = 8. 47. 1.

PROP. XLVIII.



Si quadratum quod ab uno latere BG trianguli deſcribitur, æquale ſit eis quæ à reliquis trianguli lateribus AB, AC deſcribuntur quadratis, angulus BAC comprehenſus ſub AB, AC reliquis duobus trianguli lateribus, rectus eſt.

Duc ad AC perpendicularem DA = AB, & junge CD.

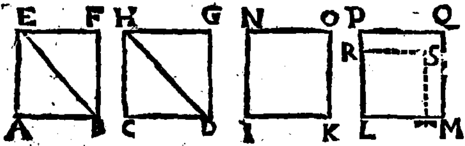
Iam CDq *a* = ADq + ACq = ABq + ACq = BCq. * ergo CD = BC. ergo triangula CAB, CAD, ſibi mutuo æquilatera ſunt; quare ang. CAB *b* = CAD *c* = Rect. Q.E.D.

a 47. 1.
* *Vide ſeq. Theor.*
b 8. 1.
c hyp.

Schol.

Aſſumpſimus exinde quod CDq. = BCq, ſequi CD = BC. Hoc vero manifeſtum fiet ex ſequenti theoremate.

THEOREMA A.



Linearum æqualium A B, C D, *æqualia sunt quadrata* A F, C G; *& quadratorum æqualium* N K, P M *æqualia sunt latera* I K, L M.

a 34. 1.
b 4. 1. & 6. ax.
a 46. 1.
b 1. part.
c hyp.
d 9. ax.

Pro 1 Hyp. Duc diametros E B, H D. Liquet A F $=$ *a* 2 triang. E A B $=$ *b* 2 triang. H C D $=$ *a* C G. Q. E. D.

2. Hyp. Si fieri poteſt, ſit L M $\sqsubset$ I K. fac L T $=$ I K; *a* ſitque L S $=$ L Tq. ergo L S *b* $=$ NK *c* $=$ LQ. *d* Q. E. A. ergo LM $=$ IK.

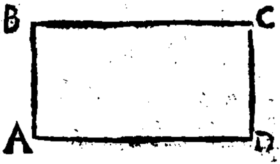
Coroll.

Eodem modo quælibet rectangula inter ſe æquilatera æqualia oſtendentur.

L IB.

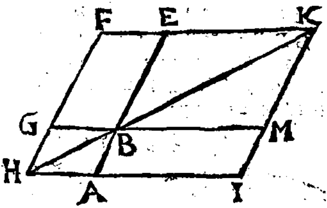
LIB. II.

Definitiones.



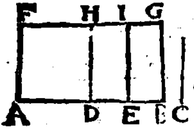
I. Omne parallelogrammum rectangulum A B C D contineri dicitur sub rectis duabus A B, A D, quæ rectum comprehendunt angulum.

Quando igitur dicitur rectangulum sub B A, A D; *vel brevitatis causa, rectangulum* B A D, *vel* B A x AD, (*vel* Z A *pro* Z x A) *designatur rectangulum, quod continetur sub* B A, & A D *ad rectum angulum constitutis.*



II. In omni parallelogrammo spatio FHIK unumquodq; eorum, quæ circa diametrum illius sunt, parallelogrammorum, cum duobus complementis Gnomon vocetur. *ut* Pgr FB + BI + GA (EHM) *est Gnomon. item* Pgr. FB + BI + EM (GKA) *est Gnomon.*

PROP. I.



Si fuerint duæ rectæ lineæ A B, A F, *seceturque ipsarum altera* A B, *in quotcunque segmenta* AD, DE, E B: *rectangulum comprehensum sub illis duabus rectis lineis* A B, A F, *æquale est eis, quæ sub insecta* A F, *& quolibet segmentorum* A D, D E, E B *comprehenduntur rectangulis.*

a 11. 1. *a* Statue A F, perpendicularem ad A B. *a* per F duc infinitam F G perpendicularem ad A F. *a* Ex D, E, B erige perpendiculares D H, E I, B G. erit A G rectangulum sub A F, A B, &

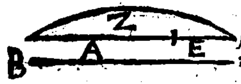
b 19 ax. 1. *b* est æquale rectangulis A H, D I, E G, hoc est

c 34. 1. (quia D H, E I, A F *c* pares sunt) rectangulis sub A F, A D; sub A F, D E; sub A F, E B. Q. E. D.

Schol.

Propositiones decem primæ hujus libri valent etiam in numeris. Reliquas quilibet tyro examinet. pro hac, sit A F 6, & A B 12, sectus in A D 5, D E 3, & E B 4. Estque 6x12 (A G) = 72. 6x5 (A H) = 30. 6 in 3 (D I) = 18. denique 6x4 (E G) = 24. Liquet vero 30 + 18 + 24 = 72.

PROP. II.



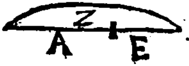
Si recta linea Z *secta sit utcunque; rectangula, quæ sub tota* Z, *& quolibet segmentorum* A, E *comprehenduntur, æqualia sunt ei, quod à tota* Z *fit, quadrato.*

Dico Z A + Z E = Zq. Nam sume B = Z.

a 1. 2. *a* Estque BA + BE = BZ; hoc est (ob B = Z) ZA + ZE = Zq. Q. E. D.

PROP.

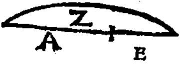
PROP. III.



Si recta linea Z secta sit utcunque; rectangulum sub tota Z, & uno segmentorum E comprehensum, æquale est illi, quod sub segmentis A, E comprehenditur, rectangulo, & illi quod à prædicto segmento E describitur, quadrato.

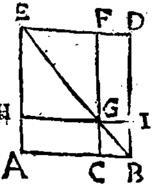
Dico. $ZE = AE + Eq$. a Nam $EZ = EA +$ a 1. 2.
EE.

PROP. IV.



Si recta linea Z secta sit utcunque; Quadratum, quod à tota Z describitur, æquale est, & illis quæ à segmentis A, E describuntur, quadratis, & ei, quod bis sub segmentis A, E comprehenditur, rectangulo.

Dico $Zq = Aq + Eq + 2\,AE$. a Nam $ZA = Aq +$
AE. a & $ZE = Eq + AE$. quum igitur $ZA +$ a 3. 2.
ZE b $= Zq$, c erit $Zq = Aq + Eq + 2\,AE$. b 1. 2.
Q. E. D. c 1. ax.



Aliter. Super A B fac quadratum A D, cujus diameter E B. per divisionis punctum C duc perpendicularem C F; & per G duc H I parall. A B.

Quoniam ang. $EHG = A$ rectus est, & A E B d semirectus, e erit reliquus d 4. Cor. 32. 1.
HGE etiam semirectus. Ergo HE f $=$ HG g $=$ e 32. 1. f 6. 1.
EF g $=$ AC. h proinde HF quadratum est rectæ g 34. 1.
AC. eodem modo C I est C Bq. ergo A G, G D h 29. def. 1. k 19. ax. 1.
rectangula sunt sub A C, C B. Quare totum quadratum A D k $= ACq + CBq + 2\,ACB$.
Q. E. D.

Coroll.

Coroll.

1. Hinc liquet parallelogramma circa diametrum quadrati esse quadrata.

2. Item diametrum cujusvis quadrati ejus angulos bisecare.

3. Si A $= \frac{1}{2}$ Z; erit Zq $=$ 4 Aq, & Aq $= \frac{1}{4}$ Zq. item è contra, si Zq $=$ 4 Aq. erit A $= \frac{1}{2}$ Z.

PROP. V.

Si recta linea A B *secetur in æqualia* A C *b* CB, *& non æqualia* A D, D B, *rectangulum su, inæqualibus segmentis* A D, D B *comprehensum una cum quadrato, quod fit ab intermedia sectionum* C D, *æquale est ei, quod à dimidia* C B *describitur, quadrato.*

Dico C Bq $=$ A D B $+$ C Dq.

Æquantur enim ista { C Bq.
a 4. 2. *a* CDq $+$ CDB $+$ DBq $+$ CDB
b 3. 2. CDq $+$ *b*CBD(*c*AC x BD) $+$ CDB
c hyp. d 1. 2. CDq $+$ *d* ADB.

Scholium.

Si A B aliter dividatur, propius scilicet puncto bisectionis, in E; dico AEB $\sqsubset$ ADB.

a 5. 2 & 3. ax. Nam AEB *a* $=$ CBq $-$ CEq. & ADB *a* $=$ CBq $-$ CDq. ergo quum CDq $\sqsubset$ CEq, erit AEB $\sqsubset$ ADB. Q. E. D.

Coroll.

Hinc ADq $+$ DBq $\sqsubset$ AEq $+$ EBq. Nam
b 4. 2. ADq $+$ DBq $+$ 2 ADB *b* $=$ ABq *b* $=$ AEq $+$ EBq $+$ 2 AEB. ergo quum 2 AEB $\sqsubset$ 2 ADB, erit ADq $+$ DBq $\sqsubset$ AEq $+$ EBq. Q. E. D.

c 3. ax. Unde 2. ADq $+$ DBq $-$ AEq *c* $+$ EBq $=$ 2 AEB $-$ 2 ADB.

PROP. VI.

Si recta linea A *bifariam secetur, & illi recta quæpiam linea* E *in directum adjiciatur; rectangulum comprehensum sub tota cum adjecta* (sub. $A + E$), *& adjecta* E, *una cum quadrato, quod à dimidia* $\frac{1}{2}$ A, *æquale est quadrato à linea, quæ tum ex dimidia, tum ex adjecta componitur, tanquam ab una* $\frac{1}{2} A +$ E *descripto.*

Dico $\frac{1}{4}Aq$ (a $Q.\frac{1}{2}A$) $+ AE + Eq = Q.\frac{1}{2}A + E$. a Nam $Q.\frac{1}{2}A + E = \frac{1}{4}Aq + Eq + AE$. a 4. & 3. Cor. 4 2.

Coroll.

Hinc si tres rectæ E, $E + \frac{1}{2}$ A, $E + A$ sint in proportione Arithmetica, rectangulum sub extremis E, $E + A$ contentum, una cum quadrato excessus $\frac{1}{2}$ A, æquale erit quadrato mediæ $E + \frac{1}{2}A$.

PROP. VII.

Si recta linea Z *secetur utcunque; Quod à tota* Z, *quodque ab uno segmentorum* E, *utraque simul quadrata, æqualia sunt illi, quod bis sub tota* Z, *& dicto segmento* E *comprehenditur, rectangulo, & illi, quod à reliquo segmento* A *fit, quadrato.*

Dico $Zq + Eq = 2\,ZE + Aq$. Nam $Zq \; a\!= Aq + Eq + 2\,AE$. & $2\,ZE \; b\!= 2\,Eq + 2\,AE$. a 4. 2. b 3. 2.

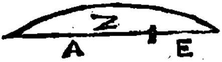
Coroll.

Hinc, quadratum differentiæ duarum quarumcumque linearum Z, E, æquale est quadratis utriusque minus duplo rectangulo sub ipsis.

c Nam $Zq + Eq - 2\,ZE = Aq = Q.\,Z - E$. c 7. 2. & 3. ax.

PROP.

PROP. VIII.



Si recta linea Z secetur utcunque; rectangulum quater comprehensum sub tota Z, & uno segmentorum E, cum eo, quod à reliquo segmento A fit, quadrato, æquale est ei, quod à tota Z, & dicto segmento E, tanquam ab una linea Z+E describitur, quadrato.

a 7. 2. & 3 ax. Dico $4\,ZE + Aq = Q.\,Z+E$. Nam $2\,ZE$ a $= Zq + Eq - Aq$. ergo $4\,ZE + Aq = Zq + Eq + 2\,ZE$ b $= Q.\,Z+E$. Q. E. D.

b 4. 2.

PROP. IX.

Si recta linea AB secetur in æqualia AC, CB, & non æqualia AD, DB. quadrata, quæ ab inæqualibus totius segmentis AD, DB fiunt, simul duplicia sunt, & ejus, quod à dimidia AC, & ejus, quod ab intermedia sectionum CD fit, quadrati.

Dico $ADq + DBq = 2\,ACq + 2\,CDq$. Nam

a 4. 2. $ADq + DBq$ a $= ACq + CDq + 2\,ACD + DBq$.

b hyp. c 7. 2. atqui $2\,ACD$ (b $2\,BCD$) $+ DBq$ c $= Cq$

d 2. ax. (ACq) $+ CDq$. d ergo $ADq + DBq = 2\,ACq + 2\,CDq$. Q. E. D.

PROP. X

Si recta linea A secetur bifariam, adjiciatur autem ei in rectum quæpiam linea; Quod à tota A cum adjuncta E, & quod ab adjuncta E, utraque simul quadrata, duplicia sunt & ejus, quod à dimidia $\frac{1}{2}$ A; & ejus, quod à composita ex dimidia, & adjuncta, tanquam ab una $\frac{1}{2}A + E$, descriptum est, quadrati.

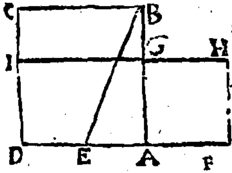
a 4. 2. Dico $Eq + Q.\,A+E$, hoc est a $Aq + 2\,Eq + 2$

b Cor. 4. 2. $AE = 2\,Q.\,\frac{1}{2}A + 2\,Q.\,\frac{1}{2}A+E$. Nam $2\,Q.\,\frac{1}{2}A$ b

c 4. 2. $= \frac{1}{2}Aq$. & $2\,Q.\,\frac{1}{2}A+E$ c $= \frac{1}{2}Aq + 2\,Eq + 2\,AE$.

PROP.

PROP. XI.



Datam rectam lineam AB *secare in* G, *ut comprehensum sub tota* AB, *& altero segmentorum* BG *rectangulum, æquale sit ei, quod à reliquo segmento* AG *fit, quadrato.*

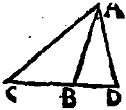
Super AB [a] describe quadratum AC. latus AD [b] biseca in E. duc EB. ex EA producta cape EF = EB. ad AF [a] statue quadratum AH. Erit AH = ABxBG. a 46. 1. b 10. 1.

Nam protracta HG ad I; Rectang. DH + EAq [c] = EFq [d] = EBq [e] = BAq + EAq. ergo DH [f] = BAq [d] = quad. AC. subtrahe commune AI; [f] remanet quad. AH = GC; [d] id est AGq = ABx BG. Q. E. F. c 6. 2. d constr. e 47. 1. f 3. ax.

Scholium.

Hæc Propositio numeris explicari nequit; * neque enim ullus numerus ita secari potest, ut productum ex toto in partem unam æquale sit quadrato partis reliquæ. * vid. 6. 13.

PROP. XII.



In amblygoniis triangulis ABC *quadratum, quod fit à latere* AC *angulum obtusum* ABC *subtendente, majus est quadratis, quæ fiunt à lateribus* AB, BC *obtusum angulum* ABC *comprehendentibus, rectangulo bis comprehenso, & ab* Sub *uno laterum* BC, *quæ sunt circa obtusum angulum* ABC, *in quod, cum protractum fuerit, cadit perpendicularis* AD, *& ab assumpta exterius linea* BD *sub perpendiculari* AD *prope angulum obtusum* ABC.

Dico

Dico $ACq = CBq + ABq + 2\, CB \times BD$.

Nam ista æqualia sunt inter se

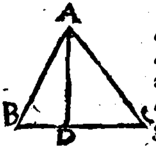
- ACq.
- a 47. 1. — $^{a}\, CDq + ADq$.
- b 4. 2. — $^{b}\, CBq + 2\, CBD + BDq + ADq$
- c 47. 1. — $CBq + 2\, CBD \; ^{c} + ABq$.

Schol.

Hinc, *cognitis lateribus trianguli obtusanguli* ABC, *facile invenientur tum segmentum* BD *inter perpendicularem* A D, *& obtusum angulum* ABC *interceptum, tum ipsa perpendicularis* A D.

Sic; Sit AC 10, AB 7, CB 5; unde ACq 100, ABq 49, CBq 25. Proinde $ABq + CBq = 74$. hunc deme ex 100, manet 26 pro 2 CBD. unde C B D erit 13. hunc divide per C B 5, provenit $2\frac{3}{5}$ pro BD. quare AD invenitur per 47. 1.

PROP. XIII.



In oxygoniis triangulis ABC *quadratum à latere* A B *angulum acutum* A C B *subtendente, minus est quadratis, quæ fiunt à lateribus* A C, C B *acutum angulum* A C B *comprehendentibus, rectangulo bis comprehenso, & ab uno laterum* B C, *quæ sunt circa acutum angulum* ACB, *in quod perpendicularis* AD *cadit, & ab assumpta interius linea* D C *sub perpendiculari* A D, *prope angulum acutum* A C B.

Dico $ACq + BCq = ABq + 2\, BCD$.

Nam æquantur ista

- $ACq + BCq$.
- a 47. 1. — $^{a}\, ADq + DCq + BCq$.
- b 7. 2. — $^{b}\, ADq + BDq + 2\, BCD$.
- c 47. 1. — $^{c}\, ABq + 2\, BCD$.

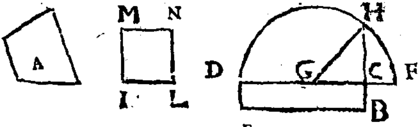
Coroll.

Hinc etiam *cognitis lateribus trianguli* A B C, *invenire est tam segmentum* DC *inter perpendicula-*

rem A D, & acutum angulum A B C interceptum, quam ipsam perpendicularem A B.

Sit AB 13, A C 15, B C 14. Detrahe A Bq (169) ex A Cq + B Cq hoc est ex 225 + 196 = 421; remanet 252 pro 2 BCD; unde BCD erit 126. hunc divide per B C 14, provenit 9 pro D C. unde A D = √: 225 − 81 = 12.

PROP. XIV.



Dato rectilineo A æquale quadratum M L invenire.

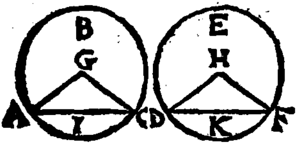
a Fac rectangulum D B = A, cujus majus latus D C produc ad F, ita ut C F = C B. b Biseca D F in G, quo centro ad intervallum G F describe circulum F H D, producatur C B, donec occurrat circumferentiæ in H. Erit C Hq = * M L = A

Ducatur enim G H. Estque A c = D B c = D C F d = G Fq − G Cq e = H Cq c = M L Q. E. F.

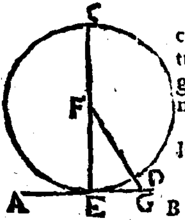
a 45. 1.
b 10. 1.
* 46. 1.
c constr.
d 5. 2. & 3. ax.
e 47. 1. & 3. ax.

LIB.

Definitiones.

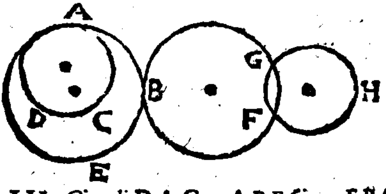


I. Æquales circuli (GABC, HDEF) ſunt, quorum diametri ſunt æquales; vel quorum quæ ex centris rectæ lineæ GA, HD, ſunt æquales.



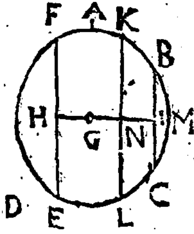
II. Recta linea AB circulum FED tangere dicitur, quæ cum circulum tangat, ſi producatur circulum non ſecat.

Recta FG ſecat circulum FED.

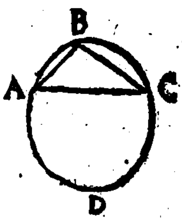


III. Circuli DAC, ABE (item FBG, ABE) ſe mutuo tangere dicuntur, qui ſe mutuo tangentes ſeſe mutuo non ſecant.

Circulus BFG ſecat circulum FGH.



IV. In circulo G A B D æqualiter distare à centro dicuntur rectæ lineæ F E K L, cum perpendiculares GH, GN quæ à centro G in ipsas ducuntur, sunt æquales. Longius autem abesse illa B C dicitur, n quam major perpendicularis G I cadit.

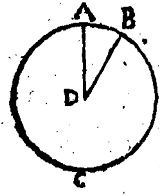


V. Segmentum circuli (A B C) est figura, quæ sub recta linea A C, & circuli peripheria A B C comprehenditur.

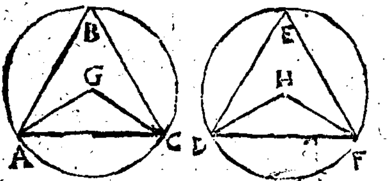
VI. Segmenti autem angulus (C A B) est, ui sub recta linea CA, & circuli peripheria AB ›mprehenditur.

VII. In segmento autem (A B C) angulus \ BC) est, cum in segmenti peripheria sumum fuerit quodpiam punctum B, & ab illo in rminos rectæ ejus lineæ A C, quæ segmenti ısis est, adjunctæ fuerint rectæ lineæ A B, C B, inquam angulus ABC ab adjunctis illis lineis B, C B comprehensus.

VIII. Cum vero comprehendentes angum A B C, rectæ lineæ A B, B C aliquam assuunt peripheriam ADC, illi angulus ABC intere dicitur.

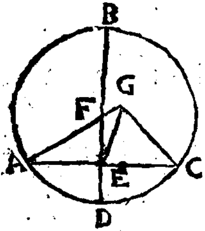


IX. Sector autem circuli (ADB) est, cum ad ipsius circuli centrum D constitutus fuerit angulus A D B; comprehensa nimirum figura A D B. & à rectis lineis A D, B D angulum continentibus, & à peripheria A B ab illis assumpta.



X. Similia circuli segmenta (ABC, DEF) sunt, quæ angulos (A B C, D E F) capiunt æquales; aut in quibus anguli ABC, DEF inter se sunt æquales.

PROP. I.



Dati circuli A B C *centrum F reperire.*

Duc in circulo rectam AC utcunq;, quam biseca in E. per E duc perpendicularẽ DB. hanc biseca in F. erit F centrũ.

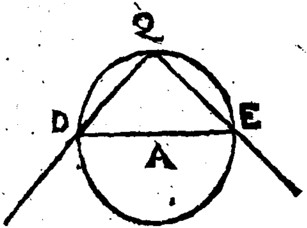
Si negas, centrum esto G, extra rectam DB (nam in ea esse non potest, cum ubique extra F dividatur inæqualiter) ducanturque GA, GC, GE. Vis G centrum esse; *a* ergo GA = GC; & per constr. A E = E C, latus vero G E commune est; *b* ergo anguli GEA, GEC pares, & *c* proinde recti sunt. *d* ergo ang. GEC = FEC rect. *e* Q. E. A.

a 15. *def.* 1.
b 8. 1.
c 10. *def.* 1.
d 12. *ax.*
e 9. *ax.*

Coroll.

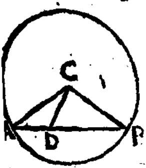
Coroll.

Hinc, si in circulo recta aliqua linea B D aliquam rectam lineam A C bifariam & ad angulos rectos secet, in secante B D erit centrum.



Facillime per normam invenitur centrum vertice Q ad circumferentiam applicato. Si enim recta D E jungens puncta D, & E, in quibus normæ latera Q D, Q E peripheriam secant, bisecetur in A, erit A centrum. Demonstratio pendet ex 31. hujus. *Andr. Tacq.*

PROP. II.



Si in circuli C A B *peripheria duo quælibet puncta*, A, B *accepta fuerint, recta linea* A B, *quæ ad ipsa puncta adjungitur, intra circulum cadet.*

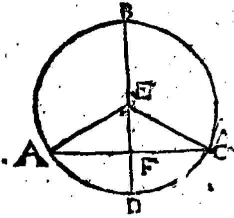
Accipe in recta A B quodvis punctum D, & ex centro C duc C A, C D, C B. & quoniam C A *a* = C B, *b* erit ang. A = B. Sed ang. C D B *c* ⊏ A; ergo ang. C D B ⊏ B. *d* ergo C B ⊏ C D. atqui C B tantum pertingit ex centro ad circumferentiam; ergo C D eousque non pertingit. ergo punctum D est intra circulum. Idemque ostendetur de quovis alio puncto rectæ A B. Tota igitur A B cadit intra circulum. Q. E. D.

a 15. def. 1.
b 5. 1.
c 16. 1.
d 19. 1.

Coroll.

Hinc, Recta circulum tangens, ita ut eum non secet, in unico puncto tangit.

PROP. III.



Si in circulo EABC *recta quædam linea* BD *per centrum extensa quandam* A C *non per centrum extensam bifariam secet, (in* F*) & ad angulos rectos ipsam secabit; & si ad angulos rectos eam secet, bifariam quoque eam secabit.*

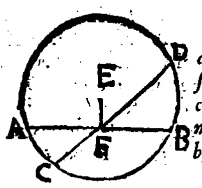
Ex centro E ducantur E A, E C.

a hyp. b 15. def. 1. 1. *Hyp.* Quoniam AF *a* = FC, & EA*b* = EC, latusque EF commune est, *c* erunt anguli EFA,
c 8. 1. EFC pares, & *d* consequenter recti. Q. E. D.
d 10 def 1. e hyp. & 12. ax. 2. *Hyp.* Quoniam ang. EFA *e* = EFC, & ang.
f 5. 1. EAF *f* = ECF, latusque EF commune, *g* erit
g 26. 1. AF = EC. Bisecta est igitur A C. Q. E. D.

Coroll.

Hinc, in triangulo quovis æquilatero & Isosceele linea ab angulo verticis bisecans basim, perpendicularis est basi. & contra perpendicularis ab angulo verticis bisecat basim.

PROP. IV.



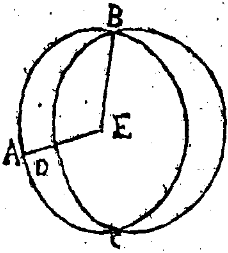
Si in circulo A.CD *duæ rectæ lineæ* A B, CD *sese mutuo secent non per centrum* E *extensæ, sese mutuo bifariam non secabunt.*

Nam si una per centrum

trum transeat, patet hanc non bisecari ab altera, quæ ex hyp. per centrum non transit.

Si neutra per centrum transit, ex E centro duc E F. Si jam ambæ A B, C D forent bisectæ in F, anguli EFB, EFD *a* ambo essent recti, & proinde æquales. *b* Q. E. A. a 3. 3. b 9. ax.

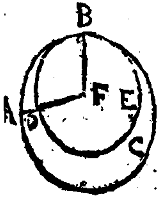
P R O P. V.



Si duo circuli B A C, B D C *sese mutuo secent, non erit illorum idem centrum* E.

Alias enim ductis ex communi centro E rectis E B, E D A, essent E D *a* = E B *a* = E A. *b* Q. E. A. a 15. def. 1. b 9. ax.

P R O P. VI.



Si duo circuli B A C, BDE, *sese mutuo interius tangant* (in B) *eorum non erit idem centrum* F.

Alias ductis ex centro F rectis F B, F D A, essent F D *a* = F B *a* = F A. *b* Q. F. N. a 15. def. 1. b 9. ax.

PROP. VII.

Si in A B *diametro circuli quodpiam sumatur punctum* G, *quod circuli centrum non sit, ab eoque puncto in circulum quædam rectæ lineæ* GC, GD, GE *cadunt; maxima quidem erit ea* (GA) *in qua centrum* F, *minima vero reliqua* GB. *aliarum vero illi, quæ per centrum ducitur, propinquior* G C *remotiore* G D *semper major est. Duæ autem solum rectæ lineæ* GE GH *æquales ab eodem puncto in circulum cadunt, ad utrasque partes minimæ* G B, *vel maximæ* G A.

a 23. 1. Ex centro F duc rectas F C, F D, F E; & *a* fac ang. BFH = BFE.

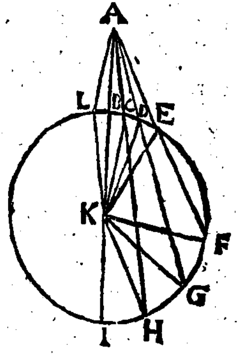
a 20. 1. 1. G F + F C (hoc est G A) *a* $\sqsubset$ G C. Q. E. D.

b 15. def. 1. 2. Latus F G commune est, & FC *b* = F D;
c 9. ax. atque ang. G F C *c* $\sqsubset$ G F D *d* ergo baſ. GC
d 24. 1. $\sqsubset$ G D. Q. E. D.

e 20. 1. 3. F B (F E) *e* $\sqsupset$ G E + G F. ergo ablato
f 5. ax. communi F G *f* remanet B G $\sqsupset$ E G. Q. E. D.

4. Latus F G commune est, & F E = F H;
g constr. atque ang. BFH *g* = BFE. *h* ergo GE = GH.
h 4. 1. Quod vero nulla alia G D ex puncto G æquetur ipsi G E, vel G H, jamjam ostensum est. Q. E. D.

Prop. VIII.



Si extra circulum sumatur punctum quodpiam A, ab eoque puncto ad circulum deducantur quædam lineæ A I, A H, A G, A F, quarum una quidem A I per centrum K protendatur, reliquæ vero ut libet; in cavam peripheriam cadentium rectarum linearum maxima quidem est illa A I, quæ per centrum ducetur, aliarum autem ei quæ per centrum transit propinquior A H remotiore A G semper major est. In convexam vero peripheriam cadentium rectarum linearum minima quidem est illa A B, quæ inter punctum A, & diametrum B I interponitur; aliarum autem ea, quæ est minimæ propinquior A C remotiore A D semper minor est. Duæ autem tantum rectæ lineæ A C, A L æquales ab eo puncto in ipsum circulum cadunt, ad utrasque partes minimæ A B, vel maximæ A I.

Ex centro K duc rectas K H, K G, K F, K C, K D, K E. & fac ang. A K L = A K C.

1. A I (A K + K H) *a* $\sqsubset$ A H. Q. E. D. *a* 20. 1.

2. Latus A K commune est; & K H = K G; atque ang. A K H $\sqsubset$ A K G. *b* ergo baf. A H $\sqsubset$ A G. Q. E. D. *b* 24. 1.

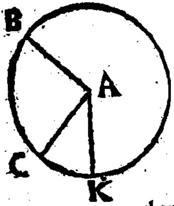
3. K A *c* $\sqsupset$ K C + C A. aufer hinc inde æquales K C, K B, *d* erit A B $\sqsupset$ A C. *c* 20. 1. *d* 5. ax.

4. A C + C K *e* $\sqsupset$ A D + D K. aufer hinc inde æquales C K, D K, *f* erit A C $\sqsupset$ A D. Q. E. D. *e* 21. 1. *f* 5. ax.

5. Latus K A est commune & KL = K C; atque ang. A K L g = A K C; b ergo L A = C A. hisce vero nulla alia æquatur, ex mox ostensis. ergo, &c.

g constr. b 4. 1.

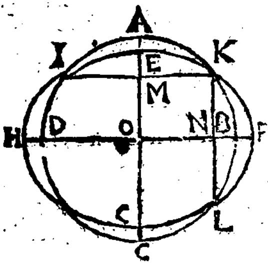
P R O P. IX.



Si in circulo B C K acceptum fuerit punctum aliquod A, & ab eo puncto ad circulum cadant plures, quam duæ rectæ lineæ æquales A B, A C, A K, acceptum punctum A centrum est ipsius circuli.

a 9. 3. Nam a à nullo puncto extra centrum plures quam duæ rectæ lineæ æquales duci possunt ad circumferentiam. Ergo A est centrum. Q. E. D.

P R O P. X.

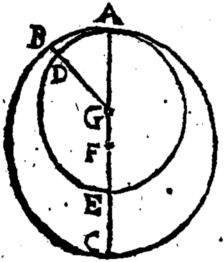


Circulus IAKBL circulum IEKFL in pluribus quam duobus punctis non secat.

Secet, si fieri potest, in tribus punctis I K L. Iunctæ I K K L. bisecentur in M, & N. a Ambo circuli centrum habent in singulis perpendicularibus M C, N H, & proinde in earum intersectione O. ergo secantes circuli idem centrum habent. b Q. F. N.

a cor. 1. 3. b 5. 3.

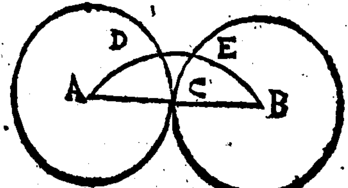
PROP. XI.



Si duo circuli GADE, FABC *sese intus contingant, atque accepta fuerint eorum centra* G, F; *ad eorum centra adjuncta recta linea* F G, *& producta, in* A *contactum circulorum cadet.*

Si fieri potest, recta F G protracta secet circulos extra contactum A, sic ut non F G A, sed FGDB sit recta linea. ducatur G A. Et quia G D *a* = GA, & G B *b* ⊐ GA, (cum recta FGB transeat per F centrum majoris circuli) erit G B ⊐ GD. *c* Q. E. A. a 15. def. 1. b 7. 3. c 9. ax.

PROP. XII.

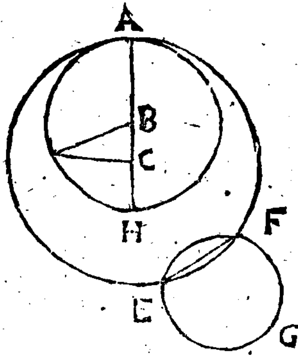


Si duo circuli ACD, BCE *sese exterius contingant, linea recta* A B *quæ ad eorum centra* A, B *adjungitur, per contactum* C *transibit.*

Si fieri potest, sit recta ADEB secans circulos extra contactum C in punctis D, E. Duc A C, CB. erit A D + E B (A C + C B) *a* ⊏ A D-EB. *b* Q. E. A. a 20. 1. b 9. ax.

PROP.

PROP. XIII.



Circulus C A F *circulum* BAH *non tangit in pluribus punctis, quam uno* A, *sive intus, sive extra tangat.*

1. Tangat, si fieri potest, intus in punctis A, H. *a* ergo recta C B centra connectens, si producatur cadet tam in A, quam in H. Quoniam igitur CH *b* = CA, & BH $\sqsubset$ CH. erit BA (*c* BH) $\sqsubset$ CA. Q. E. A.

a 11. 3.
b 15. def. 1.
c 15. def. 1.
d 9. ax.
e 2. 3.

2. Sin dicatur exterius contingere in punctis E & F, *e* ducta recta E F in utroque circulo erit. Circuli igitur se mutuo secant, quod non ponitur.

PROP. XIV.

In circulo E A B C *æquales rectæ lineæ* A C B D, *æqualiter distant à centro* E. & *quæ* A C, B D *æqualiter distant à centro, æquales sunt inter se.*

Ex centro E duc perpendiculares E F,

a 3. 3.
E G: *a* quæ bisecabunt A C, D B. connecte E A E B.

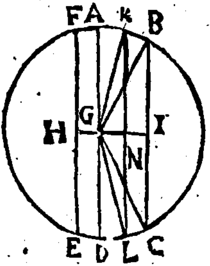
b 7. ax.
1. *Hyp.* AC = BD. ergo A F *b* = B G. sed & E A

EA = EB. ergo FEq c = EAq − AFq = EBq − BGq c = EGq. d ergo FE = EG. Q. E. D.

2. *Hyp.* EF = EG. ergo AFq c = EAq − EFq = EBq − EGq c = GBq. ergo AF d = GB. e proinde AD = BC. Q. E. D.

c 47.1. & 3.ax.
d Schol. 48.1.
e 6.ax.

PROP. XV.



In circulo GABC *maxima quidem linea est diameter* AD; *aliarum autem centro* G *propinquior* FE *remotiore* BC *semper major est.*

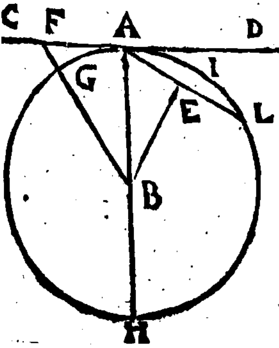
1. Duc GB, GC. Diameter AD (a GB + GC) b ⊏ BC Q. E. D.

a 15.def.1.
b 20.1.

2. Sit distantia GI ⊏ GH. accipe GN = GH. per N duc KL perpend. GI. junge GK, GL. & quia GK = GB, & GL = GC; estque ang. KGL ⊏ BGC, c erit KL (FE) ⊏ BC. Q. E. D.

c 24.1.

PROP. XVI.



Quæ CD *ab extremitate diametri* HA *cujusque circuli* BALH *ad angulos rectos ducitur, extra ipsum circulum cadet, & in locum inter ipsam rectam lineam, & peripheriam comprehen-*

prehensum altera recta linea A L *non cadet*, *& semicirculi quidem angulus* BAI *quovis angulo acuto rectilineo* B A L *major est*; *reliquus autem* D A I *minor*.

1. Ex centro B ad quodvis punctum F in recta A C duc rectam B F. Latus B F subtendens angulum rectum B A F a majus est latere B A, quod opponitur acuto BFA. ergo cum B A (BG) pertingat ad circumferentiam, B F ulterius porrigetur, adeoque punctum F, & eadem ratione quodvis aliud rectæ A C, extra circulum situm erit. Q. E. D.

a 19. 1.

2. Duc BE perpendic. AL. Latus BA oppositum recto angulo B E A b majus est latere B E, quod acutum BAE subtendit: ergo punctum E, adeoque tota E A cadit intra circulum. Q.E.D.

b 19. 1.

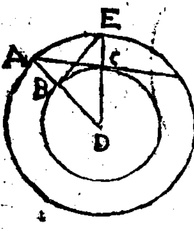
3. Hinc sequitur angulum quemvis acutum, nempe EAD angulo contactus DAI majorem esse. Item angulum quemvis acutum B A L angulo semicirculi BAI minorem esse. Q.E.D.

Coroll.

Hinc, recta à diametri circuli extremitate ad angulos rectos ducta ipsum circulum tangit.

Ex hac propositione paradoxa consequuntur, & mirabilia bene multa, quæ vide apud interpretes.

PROP. XVII.



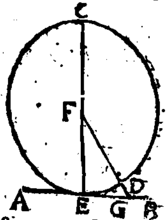
A dato puncto A *rectam lineam* A C *ducere*, *quæ datum circulum* D B C *tangat*.

Ex D dati circuli centro ad datum punctum A ducatur recta D A secans peripheriam in B. Centro D describe per A alium circulum A E;

A E; & ex B duc perpendicularem ad A D, quæ occurrat circulo A E in E. duc E D occurrentem circulo B C in C. ex A ad C ducta recta tanget circulum D B C.

Nam D B *a* = D C, & D E *a* = D A, & ang. D communis est: *b* ergo ang. A C D = E B D, rect. *c* ergo A C tangit circulum C. Q. E. F. a 15. def. 1. b 4. 1. c cor. 16. 3.

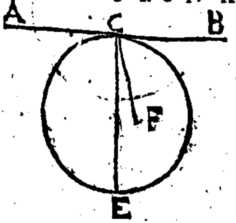
PROP. XVIII.



Si circulum F E D C *tangat recta quæpiam linea* A B, *à centro autem ad contactum* E *adjungatur recta quædam linea* F E; *quæ adjuncta fuerit* F E *ad ipsam contingentem* A B *perpendicularis erit.*

Si negas, sit ex F centro alia quædam F G perpendicularis ad contingentem, *a* secabit ea circulum in D. Quum igitur ang. F G E rectus dicatur *b* erit ang. F E G acutus. *c* ergo F E (F D) $\sqsubset$ F G. *d* Q. E. A. a 2. def. 3. b cor. 17. 1. c 19. 1. d 9 ax.

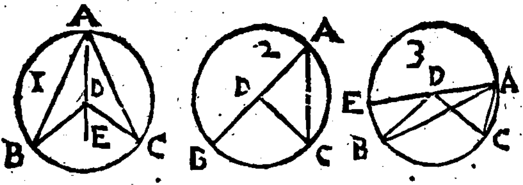
PROP. XIX.



Si circulum tetigerit recta quæpiam linea A B, *à contactu autem* C *recta linea* C E *ad angulos rectos ipsi tangenti excitetur, in excitata* C E *erit centrum circuli.*

Si negas, sit centrum extra C E in F, & ab F ad contactum ducatur F C. Igitur ang. F C B rectus est; & *a* proinde par angulo E C B recto per hypoth. *b* Q. E. A. a 11. ax. b 9. ax.

PROP. XX.



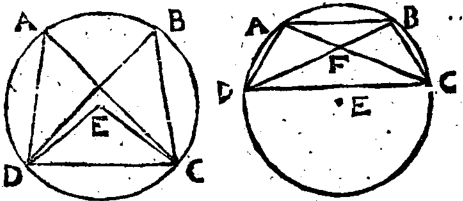
In circulo DABC, *angulus* BDC *ad centrum duplex est anguli* BAC *ad peripheriam, cum fuerit eadem peripheria* BC *basis angulorum.*

Duc diametrum ADE.

a 32. 1. b 5. 1. Externus angulus BDE $a=$ DAB + DBA $b=$ 2 DAB. Similiter ang. EDC $=$ 2 DAC. ergo in primo casu totus BDC $=$ 2 BAC; sed in ter-

c 20. ax. tio casu *c* reliquus angulus BDC $=$ 2 BAC. Q. E. D.

PROP. XXI.



In circulo EDAC *qui in eodem segmento sunt anguli,* DAC *&* DBC *sunt inter se æquales.*

1. *Cas.* Si segmentum DABC semicirculo sit majus, ex centro E, duc ED, EC. Eritque 2 ang.

a 20. 3. A $a=$ E $a=$ 2 B. Q. E. D.

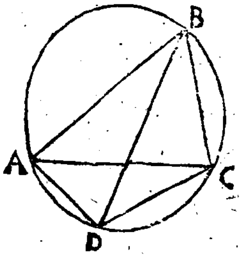
2. *Cas.* Sin segmentum semicirculo majus non fuerit, summa angulorum trianguli ADF æquatur summæ angulorum in triangulo BCF. De-

b 15. 1. mantur hinc inde AFD $b=$ BFC, & ADB $c=$

c per 1. cas. ACB, remanent DAC $=$ DBC. Q. E. D.

PROP.

PROP. XXII.



Quadrilaterorum ABCD *in circulo descriptorum anguli* ADC, ABC, *qui ex adverso, duobus rectis sunt æquales.*

Duc AC, BD. Ang. ABC + BCA + BAC a = 2 Rect. Sed BDA b = BCA, & BDC b = BAC. c ergo ABC + ADC = 2 Rect. Q. E. D.

a 32. 1.
b 21. 3.
c 1. ax.

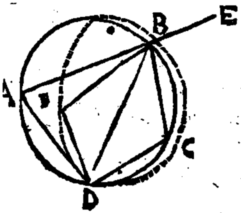
Coroll.

1. Hinc, si * AB unum latus quadrilateri n circulo descripti producatur, erit angulus externus EBC æqualis angulo interno ADC, qui opponitur ei ABC, qui est deinceps externo EBC. ut patet ex 13. 1. & 3. ax.

* vide seq. diagram.

2. Item circa Rhombum circulus describi nequit; quia adversi ejus anguli vel cedunt duobus rectis, vel eos excedunt.

SCHOL.

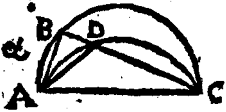


Si in quadrilatero ABCD *anguli* A, & C *qui ex adverso duobus rectis æquantur, circa quadrilaterum circulus describi potest.*

Nam circulus per quoslibet

bet tres angulos B, C, D transibit (ut patebit ex 5.4.) dico eundem per A transire. Nam si neges, transeat per F. ergo ductis rectis BF, FD, BD;
a 22. 3. b hyp. c 3 ax. d 21. 1. ang. C+F *a* = 2 Rect. *b* = C + A *c* quare A = F. *d* Q. E. A.

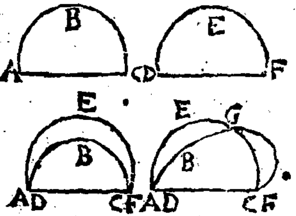
PROP. XXIII.



Super eadem recta linea A C *duo circulorum segmenta* A B C, A D C *similia & inaequalia non constituentur ad easdem partes.*

Nam si dicantur similia, duc C B secantem circumferentias in D, & B, & junge A D, ac
a 10. def. 3. b 16. 1. AB. Quia segmenta ponuntur similia, *a* erit ang. ADC = ABC *b* Q. E. A.

PROP. XXIV.

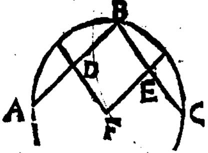


Super aequalibus rectis lineis A C, D F *similia circulorum segmenta* ABC, D E F *sunt inter se aequalia.*

Basis A C superposita basi D F ei congruet, quia A C = D F. ergo segmentum A B C congruet segmento D E F (alias enim
a 23. 3. aut intra cadet, aut extra, *a* atque ita segmenta non erunt similia, contra Hyp. aut saltem partim intra, partim extra, adeoque ipsum in tri-
b 10. 3. c 8. ax. bus punctis secabit. *b* Q. E. A.) *c* proinde segmentum ABC = DEF. Q. E. D.

PROP.

Prop. XXV.

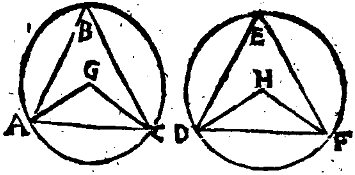


Circuli segmento ABC dato, describere circulum, cujus est segmentum.

Subtendantur utcunque duæ rectæ AB, BC, quas biseca in D, & E. Ex D, & E duc perpendiculares DF, EF occurrentes in puncto F. Hoc erit centrum circuli.

Nam centrum *a* tam in DF, quam in EF exsistit. ergo in communi puncto F. Q. E. F. *a* Cor. 1. 3.

Prop. XXVI.



In æqualibus circulis GABC, HDEF æquales anguli æqualibus peripheriis AC, DF insistunt, sive ad centra G, H, sive ad peripher. B, E constituti insistant.

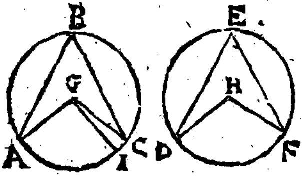
Ob circulorum æqualitatem, est GA = HD, & GC = HF item per hyp. ang. G = H. *a* ergo AC = DF. Sed & ang. B *b* = $\frac{1}{2}$ G = *c* $\frac{1}{2}$ H *b* = E. *d* ergo segmenta ABC, DEF similia, *e* & proinde paria sunt. *f* ergo etiam reliqua segmenta AC, DF æquantur. Q. E. D.

a 4. 1.
b 20. 3.
c hyp.
d 10. def. 3.
e 24. 3.
f 3. ax.

Scholium.

In circulo ABCD, sit arcus AB par arcui DC; erit AD parall. BC. Nam ducta AC, *a* erit ang. ACB = CAD. *a* 26. 3. quare per 27. 1. AD est parall. BC.

PROP. XXVII.

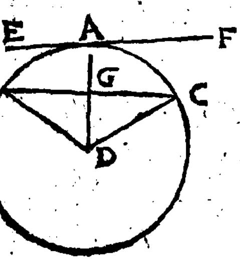


In æqualibus circulis, G A B C, HDEF, *anguli qui æqualibus peripheriis* AC, D F *insistunt, sunt inter se æquales, sive ad centra* G, H, *sive ad peripherias* B, E *constituti insistant.*

Nam si fieri potest, sit alter eorum AGC $\sqsubset$ DHF. fiatque AGI = DHF. ergo arcus AI *a* = DF *b* = AC. *c* Q. E. A.

a 26. 3.
b *hyp.*
c 9. *ax.*

SCHOL.



Linea recta EF, *quæ ducta ex* A *medio puncto peripheriæ alicujus* BC, *circulum tangit, parallela est rectæ lineæ* BC, *quæ peripheriam illam subtendit.*

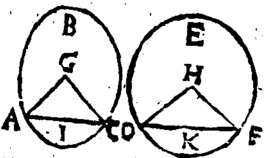
Duc è centro D ad contactum A rectam DA, & connecte DB, DC.

Latus DG commune est; & DB = DC, atque ang. BDA *a* = CDA (ob arcus BA, CA *b* æquales) *c* ergo anguli ad basim DGB, DGC æquales, & *d* proinde recti sunt. Sed interni anguli GAE, GAF *e* etiam recti sunt. *f* ergo BC, EF sunt parallelæ. Q. E. D.

a 27. 3.
b *hyp.*
c 4. 1.
d 10 *def* 1.
e *hyp.*
f 28. 1.

PROP.

PROP. XXVIII.

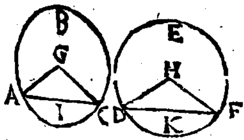


In æqualibus circulis GABC, HDEF, *æquales rectæ lineæ* AC, DF *æquales peripherias auferunt; majorem quidem* ABC *majori* DEF, *minorem autem* AIC *minori* DKF.

E centris G, H, duc GA, GC; & HD, HF. Quoniam GA $=$ HD, & GC $=$ HF, atque AC *a* $=$ DF; *b* erit ang. G $=$ H. *c* ergo arcus AIC$=$DKF. *d* proinde reliquus ABC$=$DEF. Q. E. D. a hyp. b 8. 1. c 26. 3. d 3. ax.

Quod si subtensa AC sit $\sqsubset$ vel $\sqsupset$ DF, erit simili modo arcus AC $\sqsubset$ vel $\sqsupset$ DF.

PROP. XXIX.

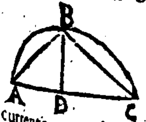


In æqualibus circulis GABC, HDEF, *æquales peripherias* ABC, DEF *æquales rectæ lineæ* AC, DF *subtendunt*.

Duc GA, GC; & HD, HF. Quia GA $=$ HD; & GC $=$ HF; & (ob arcus AC, DF *a* pares) etiam ang. G *b* $=$ H; *c* erit bas. AC$=$DF. Q. E. D. a hyp. b 27. 3. c 4. 1.

Hæc & tres proxime præcedentes intelligantur etiam de eodem circulo.

PROP. XXX.



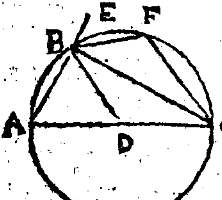
Datam peripheriam ABC *bifariam secare.*

Duc AC; quam bisecca in D. ex D duc perpendicularem DB occurrentem arcui in B. Dico factum.

Iungantur enim A B, C B. Latus D B commune est; & A D *a* = DC; & ang. ADB *b* = CDB. *c* ergo A B = B C. *d* quare arcus A B = B C. Q. E. F.

a const. b 12. ax. c 4. 1. d 28. 3.

PROP. XXXI.



In circulo angulus A B C, *qui in semicirculo, rectus est; qui autem in majore segmento* BAC, *minor recto; qui vero in minore segmento* B F C, *major est recto. Et insuper angulus majoris segmenti recto quidem major est, minoris autem segmenti angulus, minor est recto.*

Ex centro D duc DB. Quia DB = DA, erit ang. A *a* = DBA. pariter ang. DCB *a* = DBC. *b* ergo ang. A B C = A + A C B *c* = E B C, *d* proinde A B C, & E B C recti sunt. Q. E. D. *e* ergo B A C acutus est. Q. E. D. ergo cum B A C + B F C *f* = 2 Rect. erit B F C obtusus. denique angulus sub recta C B, & arcu B A C major est recto A B C. factus vero sub C B, & B F C peripheria minoris segmenti, recto E B C *g* minor est. Q. E. D.

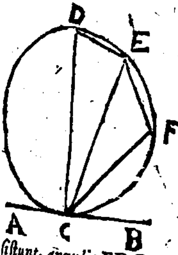
a 5. 1. b 2. ax. c 32. 1. d 10. def. 1. e cor. 17. 1. f 22. 3. g 9. ax.

SCHOLIUM.

In triangulo rectangulo A B C, *si hypotenusa* A C *bisecetur in* D, *circulus centro* D, *per* A *descriptus transibit per* B. ut facile ipse demonstrabis ex hac, & 21 1.

PROP.

PROP. XXXII.

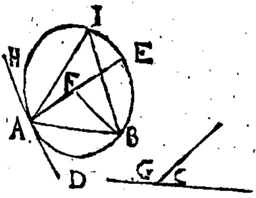


Si circulum tetigerit aliqua recta linea A B, à contactu autem producatur quædam recta linea C E circulum secans: anguli E C B, E C A, quos ad contingentem facit, æquales sunt iis, qui in alternis circuli segmentis consistunt, angulis EDC, EFC.

Sit CD latus anguli EDC perpendiculare ad AB (a perinde enim est) b ergo C D est diameter. c ergo ang. C E D in semicirculo rectus est. d ergo ang. D + DCE = Rect. e = ECB + DCE. f ergo ang. D = ECB. Q. E. D. — a 26. 3. b 19. 3. c 31. 3. d 32. 1. e constr. f 3. ax.

Cum igitur ang. ECB + ECA g = 2 Rect. h = D + F; aufer hinc inde æquales ECB, & D, k remanent ECA = F. Q. E. D. — g 13. 1. h 22. 3. k 3. ax.

PROP. XXXIII.



Super data recta linea A B describere circuli segmentum AIEB, quod capiat angulum AIB æqualem dato angulo rectilineo C.

a Fac ang. B A D = C. per A duc A E perpendicularem ad H D. ad alterum terminum datæ AB fac ang. ABF = BAF. cujus alterum latus secet A E in F. centro F per A describe circulum, quod transibit per B (quia ang. FBA — a 23. 1.

 b =

b constr.
c 6. 1.

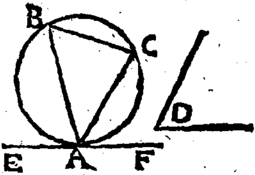
$b = $ F A B, c ideoque F B $=$ F A); segmentum AIB est id quod quæritur.

d cor.16.3.
e 32. 3.
f constr.

Nam quia H D diametro A E perpendicularis est, d tangit HD circulum, quem secat AB. ergo ang. AIB $e =$ BAD $f =$ C. Q. E. F.

PROP. XXXIV.

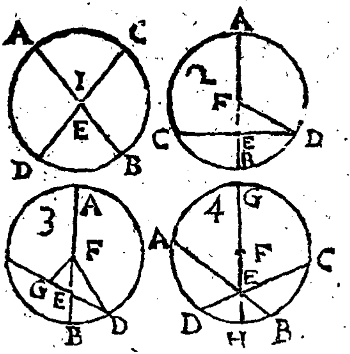
A dato circulo ABC *segmentum* A B C *abscindere capiens angulum* B *æqualem dato angulo rectilineo* D.



a 17. 3.
b 23. 1.
c 32. 3.
d constr.

a Duc rectam E F, quæ tangat datum circulum in A. b ducatur item AC faciens ang. FAC $=$ D. Hæc auferet segmentum ABC capiens angulum B $c =$ CAF $d =$ D. Q. E. F.

PROP. XXXV.



Si in circulo FBCA *duæ rectæ lineæ* A B, D C *sese mutuo secuerint, rectangulum comprehensum sub*

sub segmentis A E, E B *unius*, *æquale est ei quod sub segmentis* C E, E D *alterius comprehenditur, rectangulo.*

Cas. 1. Si rectæ sese in centro secent, res clara est.

2. Si una A B transeat per centrum F, & reliquam C D bisecet, duc F D. Estque Rectang. AEB + FEq [a] = FBq [b] = FDq [c] = EDq + FEq [d] = C E D + FEq. [e] ergo Rectang. AEB = CED. Q. E. D.

a 5. 2. b sch. 48. 1. c 47. 1. d hyp. e 3. ax.

3. Si una A B diameter sit, alteramque C D secet inæqualiter, biseca C D per F G perpendicularem ex centro.

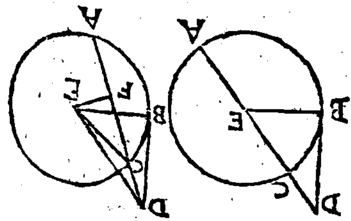
Æquantur ista:
Rectang. AEB + FEq.
[f] FBq (FDq)
[g] FGq + GDq.
FGq + [h] GEq + Rectang. CED.
[k] FEq + CED.

[l] Ergo Rectang. AEB = CED.

f 5. 2. g 47. 1. h 5. 2. k 47. 1. l 3. ax.

4. Si neutra rectarum A B, C D per centrum transeat, per intersectionis punctum E duc diametrum G H. Per modo demonstrata Rectang. AEB = GEH = CED. Q. E. D.

PROP. XXXVI.



Si extra circulum E B C *sumatur punctum aliquod* D, *ab eoque puncto in circulum cadant duæ rectæ lineæ* DA, DB; *quarum altera* DA *circulum* *secet, altera vero* D B *tanget; Quod sub tota secante* D A, *&* *exterius inter punctum* D, *& convexam peripheriam assumpta* D C *comprehenditur rectangulum, æquale erit ei, quod à tangente* D B *describitur, quadrato.*

1. *Cas.* Si secans A D transeat per centrum E, junge E B; a faciet hæc cum D B rectum angulum; quare D Bq + E B Q (E Cq) b = E Dq c = A D x D C + E Cq d ergo A D x D C = D Bq. Q. E. D.

a 18. 3.
b 47. 1.
c 6. 2.
d 3. ax.

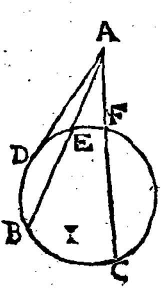
2. *Cas.* Sin A D per centrum non transeat, duc EC, EB, ED; atque EF perpend. A D, quare a bisecta est A C in F.

a 3. 3.

Quoniam igitur BDQ + EBq b = DEq b = EFq + FDq c = EFq + ADC + FCQ d = ADC + C Eq (E Bq); e erit B Dq = A D C. Q. E. D.

b 47. 1.
c 6. 2.
d 47. 1.
e 3. ax.

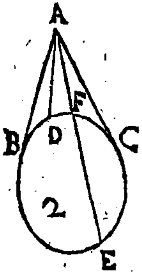
Coroll.



1. Hinc, si à puncto quovis A extra circulum assumpto, plurimæ lineæ rectæ A B, A C circulum secantes ducantur, rectangula comprehensa sub totis lineis A B, A C, & partibus externis A E, A F inter se sunt æqualia. Nam si ducatur tangens AD; erit CAF = ADq a = BAE.

a 36. 3.

2. Con-



2. Constat etiam duas rectas A B, A C ab eodem puncto A ductas, quæ circulum tangant, inter se æquales esse.

Nam si ducatur A E secans circulum; erit A Bq *a* = EAF *b* = ACq. a 36. 3. b 36. 3.

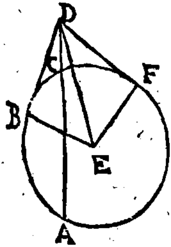
3. Perspicuum quoque est ab eodem puncto A extra circulum assumpto, duci tantum posse duas lineas, A B, A C quæ circulum tangant.

Nam si tertia A D tangere dicatur, erit A D *c* = A B *c* = A C. *d* Q. F. N. c 2. cor. d 8 3.

4. E contra constat, si duæ rectæ æquales A B, A C ex puncto quopiam A in convexam peripheriam incidant, & earum una A B circulum tangat, alteram quoque circulum tangere.

Nam si fieri potest, non A C; sed altera A D circulum tangat. ergo A D *e* = AC *f* = A B. *g* Q. E. A. e 2. cor. f hyp. g 8. 3.

PROP. XXXVII.



Si extra circulum E B F *sumatur punctum* D, *ab eoque in circulum cadant duæ rectæ lineæ* D A, D B; *quarum altera* D A *circulum secet, altera* D B *in eum incidat; sit autem quod sub tota secante* D A, *& exterius inter punctum, & convexam peripheriam assumpta* D C, *comprehenditur rectangulum, æquale ei, quod ab incidente*

D B

DB *describitur quadrato, incidens ipsa* DB *circulum tanget.*

a 17. 3. b hyp. c 36. 3. d 1. ax. & sch. 48, 1. e 8. 1. f 12. ax. g cor. 16. 3.

Ex D *a* ducatur tangens DF; atque ex E centro duc ED, EB, EF. Quia DBq *b* = ADC *c* = DFq, *d* erit DB = DF. Sed EB = EF, & latus ED commune est; *e* ergo ang. EBD = EFD. Sed EFD rectus est, *f* ergo EBD etiam rectus est. *g* ergo DB tangit circulum. Q. E. D.

Coroll.

h 8 1.

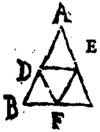
Hinc, *h* ang. EDB = EDF.

LIB.

LIB. IV.

Definitiones.

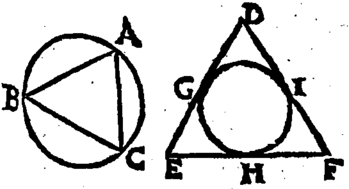
I. Figura rectilinea in figura rectilinea inscribi dicitur, cum singuli ejus figuræ, quæ inscribitur, anguli singula latera ejus in qua inscribitur, tangunt.



Sic triangulum DEF *est inscriptum in triangulo* ABC.

II. Similiter & figura circa figuram describi dicitur, cum singula ejus, quæ circumscribitur, latera singulos ejus figuræ angulos tetigerint, circa quam illa describitur.

Ita triangulum A B C *est descriptum circa triangulum* DEF.



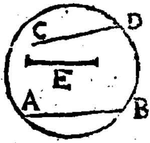
III. Figura rectilinea in circulo inscribi dicitur, cum singuli ejus figuræ, quæ inscribitur, anguli tetigerint circuli peripheriam.

IV. Figura vero rectilinea circa circulum describi dicitur, cum singula latera ejus, quæ circumscribitur, circuli peripheriam tangunt.

V. Similiter & circulus in figura rectilinea inscribi dicitur, cum circuli peripheria singula latera tangit ejus figuræ, cui inscribitur.

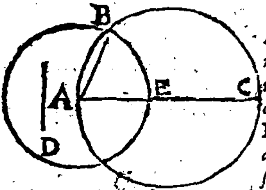
VI. Circulus autem circa figuram describi di-

dicitur, cum circuli peripheria singulos tangit ejus figuræ, quam circumscribit, angulos.



VII. Recta linea in circulo accommodari, seu coaptari dicitur, cum ejus extrema in circuli peripheria fuerint; ut recta linea A B.

PROP. I. *Probl. 1.*

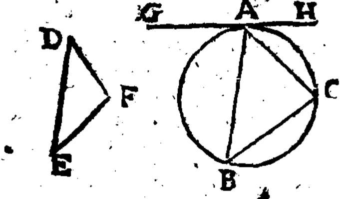


In dato circulo A B C *rectam lineam* A B *accommodare æqualem datæ rectæ lineæ* D, *quæ circuli diametro* A C *non sit major.*

a 3. *post.* & 3. 1. b 15. *def.* 1. c *constr.*

Centro A, spatio AE $=$ D *a* describe circulum dato circulo occurrentem in B. Erit ducta AB *b* $=$ AE *c* $=$ D. Q. E. F.

PROP. II. *Probl. 2.*



In dato circulo A B C *triangulum* ABC *describere dato triangulo* DEF *æquiangulum.*

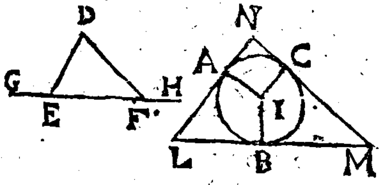
a 17. 3. b 23. 1.

Recta G H circulum datum *a* tangat in A. *b* Fac ang. HAC $=$ E; *b* & ang. GAB $=$ F, & junge BC. Dico factum.

Nam

Nam ang. B c = H A C d = E; & ang. c 32. 3.
C c = GAB d = F; e quare etiam ang. BAC = D. d constr.
ergo triang. B A C circulo inscriptum triangulo e 32. 1.
D E F æquiangulum est. Q. E. F.

PROP. III Probl. 3.



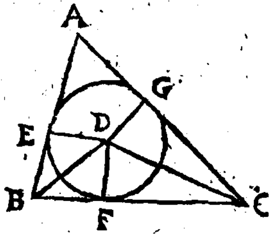
Circa datum circulum IABC *triangulum* LNM *describere, dato triangulo* DEF *æquiangulum.*

Produc latus E F utrinque. *a* Fac ad centrum a 23. 1.
I ang. AIB = DEG. & ang. BIC = DFH.
deinde in punctis A, B, C circulum *b* tangant b 17. 3.
tres rectæ LN, LM, MN. Dico factum.

Nam quod coibunt rectæ L N, L M, M N, atque ita triangulum constituent, patet; *c* quia c 13. ax.
anguli LAI, LBI *d* recti sunt, adeoque ducta d 18. 3.
AB angulos faciet LAB, LBA duobus rectis minores. Quoniam igitur ang. AIB + L *e* = 2 e Schol. 32. 1.
Rect. *f* = DEG + DEF; & AIB *g* = DEG. *h* erit f 13. 1.
ang. L = DEF. Simili argumento ang. M = DFE. g constr. h 3. ax.
k ergo etiam ang. N = D. ergo triang. L N M k 32. 1.
circulo circumscriptum dato E D F est æquiangulum. Q. E. F.

PROP.

PROP. IV. *Probl.* 4.



In dato triangulo ABC *circulum* EFG *inscribere.*

a 9. 1. Duos angulos B, & C a biseca rectis BD, CD coeuntibus in D. Ex

b 12. 1. D b duc perpendiculares DE, DF, DG. circulus centro D per E descriptus transibit per G, & F, tangetque tria latera trianguli.

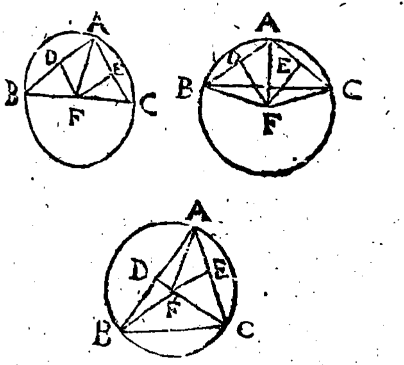
c constr. d 12. ax. e 26. 1. Nam ang. DBE c = DBF; & ang. DEB d = DFB; & latus DB commune est: e ergo DE = DF. Simili argumento DG = DF. Circulus igitur centro D descriptus transit per E, F, G; & cum anguli ad E, F, G sint recti, tangit omnia trianguli latera. Q. E. F.

Scholium.

Petr. Herig. Hinc, *cognitis lateribus trianguli, invenientur eorum segmenta, quæ fiunt à contactibus circuli inscripti.* Sic,

Sit AB 12, AC 18, BC 16. Erit AB + BC = 28. ex quo subduc 18 = AC = AE + FC, remanet 10 = BE + BF. ergo BE, vel BF = 5. proinde FC, vel CG = 11. quare GA, vel AE = 7.

PROP. V. *Probl.* 5.



Circa datum triangulum **ABC** *circulum* **FABC** *describere.*

Latera quævis duo B A, A C a biseca perpendicularibus D F, E F concurrentibus in F. Hoc erit centrum circuli, a 10, & 11. 1.

Nam ducantur rectæ FA, FB, FC. Quoniam AD b = DB; & latus D F commune est; & ang. FDA c = FDB, d erit FB = FA. eodem modo F C = F A, ergo circulus centro F per dati trianguli angulos B, A, C transibit. Q. E. F. b constr. c constr. & 12. ax. d 4. 1.

Coroll.

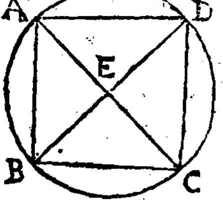
* Hinc, si triangulum fuerit acutangulum, centrum cadet intra triangulum; si rectangulum, in latus recto angulo oppositum; si denique obtusangulum, extra triangulum. * 31. 3.

Schol.

Eadem methodo describetur circulus, qui transeat per data tria puncta, non in una recta linea existentia.

PROP.

PROP. VI. *Probl. 6.*

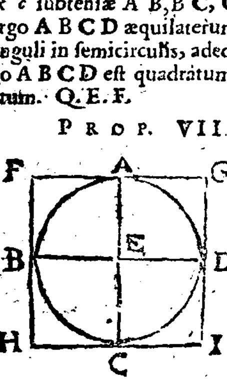


In dato circulo EABCD *quadratum* ABCD *inscribere.*

a 11, 1. *a* Duc diametros AC, BD se mutuo secantes ad angulos rectos in centro E. junge harum terminos rectis AB, BC, CD, DA. Dico factum.

b 26. 3. c 29. 3. Nam quia 4 anguli ad E recti sunt, *b* arcus, & *c* subtensæ AB, BC, CD, DA pares sunt. ergo ABCD æquilaterum est; ejusque omnes d 31. 3. e 29. def. 1. anguli in semicirculis, adeoque *d* recti sunt. *e* ergo ABCD est quadratum, dato circulo inscriptum. Q. E. F.

PROP. VII. *Probl. 7.*



Circa datum circulum EABCD *quadratum* FHIG *describere.*

Duc diametros AC, BD se mutuo secantes perpendiculariter. per a 17. 3. harũ extrema *a* duc tangentes concurrentes in F, H, I, G. Dico factum. Nam ob b 18. 3. c 28. 1. angulos ad A, & C *b* rectos, *c* erit FG parall. HI. eodem modo FH parall. GI. ergo FHIG est parallelogrammum; & quidem rectangulum. sed & æquilaterum, quia FG *d*= HI *d*= BD *e*= d 34. 1. e 15. def. 1. f 29. def. 1. CA *d*= FH *d*= GI. quare FHIG est *f* quadratum, dato circulo circumscriptum. Q. E. F.

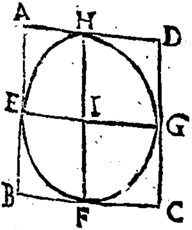
SCHOL.

SCHOL.

Quadratum ABCD circulo circumscriptum, duplum est quadrati EFGH circulo inscripti.

Nam rectang. HB = 2 HEF. & HD = 2 HGF. per 41. 1.

PROP. VIII. *Probl.* 8.

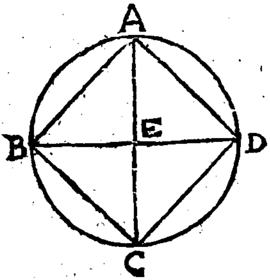


In dato quadrato ABCD *circulum* IEFGH *inscribere.*

Latera quadrati biseca in punctis H, E, F, G; junge HF, EG sese secantes in I. circulus centro I per H descriptus quadrato inscribetur.

Nam quia AH, BF *a* pares ac *b* parallelæ a 7. ax.
sunt, *c* erit AB parall. HE parall. DC. eodem b 34. 1.
modo AD parall. EG parall. BC. ergo IA, c 33. 1.
ID, IB, IC sunt parallelogramma. Ergo
AH*d* = AE*e* = HI = EI = IF = IG. Circulus igi- d 7. ax
tur centro I per H descriptus transibit per e 34. 1.
H, E, F, G, tangetque quadrati latera, cum anguli ad H, E, F, G sint recti. Q. E. F.

PROP. IX. *Probl.* 9.

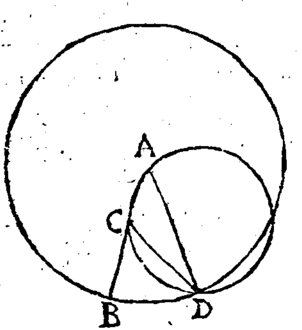


Circa datum quadratum A B C D *circulum* E A-B C D *describere.*

Duc diametros A C, B D secantes in E. centro E per A describe circulum. Is dato quadrato circumscriptus est.

a 4. cor. 32. 1. b 6. 1. 1 Nam anguli ABD, & BAC a semirecti sunt; b ergo EA = EB. eodem modo EA = ED = EC. Circulus igitur centro E descriptus per A, B, C, D dati quadrati angulos transit. Q.E.F.

PROP. X. *Probl.* 10.



Isosceles triangulum A B D *constituere, quod habeat utrumque eorum quæ ad basim sunt angulorum* B *&* A D B *duplum reliqui* A.

Accipe quamvis rectam AB, quam a seca in C, ita ut AB x BC = ACq. Centro A per B describe circulum A B D; in

a 11. 2.

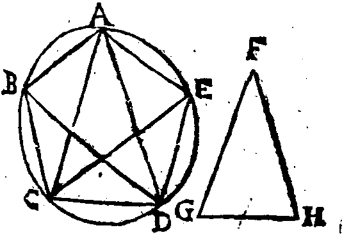
in hoc *b* accommoda BD = AC, & junge AD. b 1. 4.
erit triang. ABD quod quæritur.

Nam duc DC; & per CDA *c* describe circu- c 5. 4.
lum. Quoniam AB x BC = ACq. *d* liquet BD d 37. 3.
tangere circulum ACD, quem secat CD. *e* er- e 32. 3.
go ang. BDC = A. ergo ang. BDC + CDA *f* = f 2. ax.
A + CDA = BCD. sed *g* BDC + CDA = g 32. 1.
BDA *h* = CBD. *k* ergo ang. BCD = CBD. h 5. 1.
ergo DC *l* = DB *m* = AC. *n* quare ang. CDA = k 1. ax.
A = BDC. ergo ADB = 2 A = ABD. l 6. 1. m *constr.* n 5. 1.
Q. E. F.

Coroll.

Cum omnes anguli A, B, D *o* conficiant $\frac{5}{5}$ o 32. 1.
2 Rect. (2 Rect.) liquet A esse $\frac{1}{5}$ 2 Rect.

PROP. XI. *Probl.* 11.



In dato circulo ABCDE *pentagonum æquilaterum & æquiangulum* ABCDE *inscribere.*

a Describe triangulum Isosceles FGH, habens a 10. 4.
utrumque angulorum ad basim duplum anguli
ad verticem. *b* Huic æquiangulum CAD inscri- b 2. 4.
be circulo. Angulos ad basim ACD, & ADC
c biseca rectis DB, CE occurrentibus circumfe- c 9. 1.
rentiæ in B, & E. connecte rectas CB, BA, AE,
ED. Dico factum.

Nam ex constr. liquet quinque angulos CAD, CDB, BDA, DCE, ECA pares esse; quare d arcus e & subtensæ DC, CB, BA, AE, DE æquantur. Pentagonum igitur æquilaterum est. Est vero etiam æquiangulum, f quia ejus anguli BAE, AED, &c. insistunt arcubus g æqualibus BCDE, ABCD, &c.

d 16. 3.
e 29. 3.
f 27. 3.
g 2. ax.

Hujus problematis praxis facilior tradetur ad 10. 13.

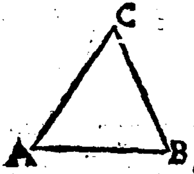
Coroll.

Hinc, angulus pentagoni æquilateri & æquianguli æquatur $\frac{3}{5}$ 2 Rect. vel $\frac{6}{5}$ Rect.

Schol.

Petr. Herig.

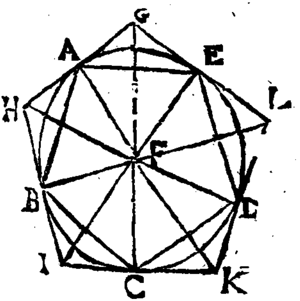
Universaliter figuræ imparium laterum inscribuntur circulo beneficio triangulorum Isoscelium, quorum anguli æquales ad basim multiplices sunt eorum, qui ad verticem sunt, angulorum; parium vero laterum figuræ in circulo inscribuntur ope Isoscelium triangulorum, quorum anguli ad basim multiplices sesquialteri sunt eorum, qui ad verticem sunt, angulorum.



Ut in triangulo Isoscele CAB, si ang. A = 3 C = B; A B erit latus Heptagoni. Si A = 4 C; erit A B latus Enneagoni, &c. Sin vero A = $1\frac{1}{2}$ C, erit A B latus quadrati. Et si A = $2\frac{1}{2}$ C, subtendet AB sextam partem circumferentiæ: pariterque si A = $3\frac{1}{2}$ C; erit AB latus octagoni, &c.

PROP.

PROP. XII. *Probl.* 12.



Circa datum circulum FABCDE *pentagonum æquilaterum & æquiangulum* HIKLG *describere.*

a Inscribe pentagonum ABCDE æquilaterum & æquiangulum; duc è centro rectas FA, FB, FC, FD, FE, iisque totidem perpendiculares GAH, HBI, ICK, KDL, LEG concurrentes in punctis H, I, K, L, G. Dico factum. Nam quia GA, GE ex uno puncto G b tangunt circulum, c erit GA = GE. d ergo ang. GFA = GFE. ergo ang. AFE = 2 GFA. eodem modo ang. AFH = HFB; & proinde ang. AFB = 2 AFH. Sed ang. AFE e = AFB. f ergo ang. GFA = AFH. sed & ang. FAH g = FAG; & latus FA est commune, h ergo HA = AG = GE = EL, &c. k ergo HG, GL, LK, KI, IH latera pentagoni æquantur: sed & anguli etiam, utpote l æqualium AGF, AHF, &c. dupli; ergo, &c.

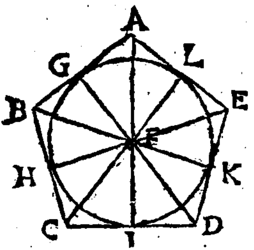
a 11. 4.
b cor. 16. 3.
c 2 cor. 36. 3.
d 8. 1.
e 27. 3.
f 7. ax.
g 12. ax.
h 26. 1.
k 2. ax.
l 32. 1.

Coroll.

Eodem pacto, si in circulo quæcunque figura æquilatera & æquiangula describatur, & ad extrema semidiametrorum ex centro ad angulos ducta-

ductarum, excitentur lineæ perpendiculares, hæ perpendiculares constituent aliam figuram totidem laterum & angulorum æqualium circulo circumscriptam.

P R O P. XIII. *Probl.* 13.



In dato pentagono æquilatero & æquiangulo ABCDE *circulum* FGHK *inscribere.*

Duos pentagoni angulos A, & B *a* biseca rectis AF, BF concurrentibus in F. Ex F duc perpendiculares FG, FH, FI, FK, FL. Circulus centro F per G descriptus tanget omnia pentagoni latera.

a 9. 1.

b hyp. Duc FC, FD, FE. Quoniam BA *b* = BC;
c constr. & latus BF commune est; & ang. FBA *c* =
d 4. 1. FBC, *d* erit AF = FC; & ang. FAB = FCB.
e hyp. Sed ang. FAB *e* = $\frac{1}{2}$ BAE *e* = $\frac{1}{2}$ BCD. ergo ang. FCB = $\frac{1}{2}$ BCD. eodem modo anguli totales C, D, E omnes bisecti sunt. Quum igitur
f 12. ax. ang. FGB *f* = FHB, & ang. FBH = FBG, &
g 26. 1. latus FB sit commune, *g* erit FG = FH. similiter omnes FH, FI, FK, FL, FG æquantur. Ergo circulus centro F per G descriptus transit per
h cor. 16. 3. H, I, K, L; *h* tangitque pentagoni latera, cum anguli ad ea puncta sint recti. Q. E. F.

Coroll.

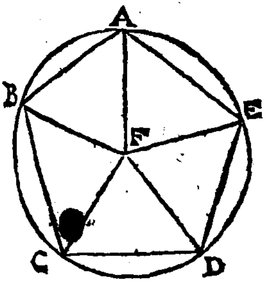
Hinc, si duo anguli proximi figuræ æquilateræ & æquiangulæ bisecentur, & à puncto, in quo coeunt lineæ angulos bisecantes, ducantur rectæ lineæ

lineæ ad reliquos figuræ angulos, omnes anguli figuræ erunt bisecti.

Schol.

Eadem methodo in qualibet figura æquilatera & æquiangula circulus describetur.

PROP. XIV. *Probl.* 14.



Circa datum Pentagonum æquilaterum & æquiangulum ABCDE *circulum* FABCD *describere.*

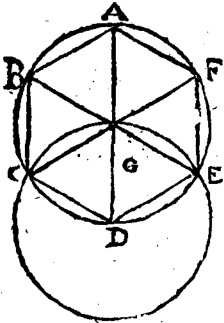
Duos pentagoni angulos biseca rectis AF, BF concurrentibus in F. Circulus centro F per A descriptus pentagono circumscribitur.

Ducantur enim FC, FD, FE. *a* Bisecti itaque sunt anguli C, D, E. *b* ergo FA, FB, FC, FD FE æquantur. ergo circulus centro F descriptus, per A, B, C, D, E, pentagoni angulos transibit. Q. E. F. a cor. 13. 4. b 6. 1.

Schol.

Eadem arte circa quamlibet figuram æquilateram & æquiangulam circulus describetur.

PROP. XV. *Probl.* 15.



In dato circulo G-ABCDEF hexagonum & æquilaterum & æquiangulum ABCDEF *inscribere.*

Duc diametrum AD; centro D per centrum G describe circulum, qui datum secet in C, & E. duc diametros CF, EB. junge AB, BC, CD, DE, EF, FA. Dico factum.

a 32. 1. Nam ang. CGD *a* = $\frac{1}{3}$ 2 Rect. *a* = DGE *b* =
b 15. 1. AGF *b* = AGB. *c* ergo BGC = $\frac{1}{3}$ 2 Rect. = FGE.
c cor. 13. 1. *d* ergo arcus *e* & subtensæ AB, BC, CD, DE,
d 26. 3. EF æquantur. Hexagonum igitur æquilaterum
e 29. 3. est: sed & æquiangulum, *f* quia singuli ejus an-
f 27. 3. guli arcubus insistunt æqualibus. Q. E. F.

Coroll.

1. Hinc latus Hexagoni circulo inscripti semidiametro æquale est.

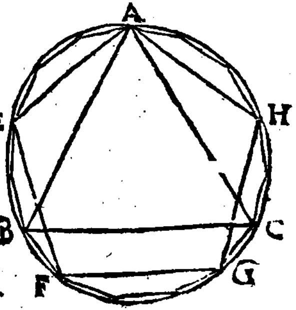
2. Hinc facile triangulum æquilaterum ACE in circulo describetur.

Schol. Probl.

Andr. Tacq. *Hexagonum ordinatum super data recta* C D *ita construes.* *a* Fac triangulum CGD æquilaterum
a 1. 1. super data CD. centro G per C, & D describe circulum. Is capiet Hexagonum super data CD.

PROP.

PROP. XVI. *Probl.* 16.



In dato circulo AEBC *quindecagonum æquilaterum & æquiangulum inscribere.*

Dato circulo *a* inscribe pentagonum æquilaterum AEFGH; *b* itemque triangulum æquilaterum ABC. erit BF latus quindecagoni quæsiti. a 11. 4. b 2. 4.

Nam arcus A B *c* est $\frac{1}{3}$, vel $\frac{5}{15}$ peripheriæ, cujus AF est $\frac{2}{5}$ vel $\frac{6}{15}$. ergo reliquus B F = $\frac{1}{15}$ periph. ergo quindecagonum, cujus latus B F, æquilaterum est; sed & æquiangulum, *d* cum singuli ejus anguli arcubus insistant æqualibus, quorum unusquisque est $\frac{1}{15}$ totius circumferentiæ. ergo, &c. c *constr.* d 27. 3.

Schol.

Circulus dividitur Geometrice in partes $\begin{cases} 4, 8, 16, \&c. \text{ per } 6, 4, \& 9, 1. \\ 3, 6, 12, \&c. \text{ per } 15, 4, \& 9, 1. \\ 5, 10, 20, \&c. \text{ per } 11, 4, \& 9, 1. \\ 15, 30, 60, \&c. \text{ per } 16, 4 \& 9, 1. \end{cases}$

Cæterum divisio circumferentiæ in partes datas etiamnum desideratur; quare pro figurarum quarumcunq; ordinatarum constructionibus sæpe ad mechanica artificia recurrendum est, propter quæ Geometræ practici consulendi sunt.

LIB.

LIB. V.

Definitiones.

I. PArs est magnitudo magnitudinis, minor majoris, cum minor metitur majorem.

II. Multiplex autem est major minoris, cùm minor metitur majorem.

III. Ratio est duarum magnitudinum ejusdem generis mutua quædam secundum quantitatem habitudo.

In omni ratione ea quantitas, quæ ad aliam refertur, dicitur antecedens rationis; ea vero, ad quam alia refertur, consequens rationis dici solet. ut in ratione 6 ad 4; antecedens est 6, & consequens 4.

Nota.

Cujusque rationis quantitas innotescit dividendo antecedentem per consequentem. ut ratio 12 ad 5 effertur per $\frac{12}{5}$. *item quantitas rationis* A *ad* B *est* $\frac{A}{B}$. *Quare non raro brevitatis causa, quantitates rationum sic designamus,* $\frac{A}{B} \sqsubset$, *vel* $=$, *vel* $\sqsupset \frac{C}{D}$; *hoc est, ratio* A *ad* B *major est ratione* C *ad* D, *vel ei æqualis, vel minor. Quod probe animadvertat, quisquis hæc legere volet.*

Rationis, sive proportionis species, ac divisiones vide apud interpretes.

IV. Proportio vero est rationum similitudo.

Rectius quæ hic vertitur proportio, proportionalitas, sive analogia dicitur; nam proportio idem denotat quod ratio, ut plerisque placet.

V. Rationem habere inter se magnitudines dicuntur, quæ possunt multiplicatæ se mutuo superare.

E, 12.	A, 4. B, 6.	G, 24.	
F, 30.	C, 10. D, 15.	H, 60.	

VI. In eadē ratione magnitudines dicuntur esse, prima A ad secundam B, & tertia C ad quartam D, cum primæ A, & tertiæ C æquemultiplicia E, & F à secundæ B, & quartæ D æquemultiplicibus G, & H, qualiscunque sit hæc multiplicatio, utrumque E, F ab utroque G, H, vel una deficiunt, vel una æqualia sunt, vel una excedunt, si ea sumantur E, G; & F, H quæ inter se respondent.

Hujus nota est ::. ut A. B :: C. D. *hoc est* A *ad* B, *&* C *ad* D *in eadem sunt ratione. aliquando sic scribimus* $\frac{A}{B} = \frac{C}{D}$ *id est*, A. B :: C. D.

VII. Eandem autem habentes rationem (A.B :: C.D) proportionales vocentur.

E, 30.	A, 6. B, 4.	G, 28.	
F, 60.	C, 12. D, 9.	H, 63.	

VIII. Cum vero æquemultiplicium, E multiplex primæ magnitudinis A excesserit G multiplicem secundæ B; at F multiplex tertiæ C non excesserit H multiplicem quartæ D; tunc prima A ad secundam B majorem rationem habere dicetur, quam tertia C ad quartam D.

Si $\frac{A}{B} \sqsubset \frac{C}{D}$, *necessarium non est ex hac definitione, ut* E *semper excedat* G; *quum* F *minor est quam* H; *sed conceditur hoc fieri posse.*

IX. Proportio autem in tribus terminis paucissimis consistit. *Quorum secunda est instar duorum.*

X. Cum autem tres magnitudines A, B, C proportionales fuerint, prima A ad tertiam C duplicatam rationem habere dicetur ejus, quam habet ad secundam B: at quum quatuor magnitudines A, B, C, D, proportionales fuerint, prima A ad quartam D triplicatam rationem habere dicetur

dicetur ejus, quam habet ad secundam B; & semper deinceps uno amplius, quamdiu proportio extiterit.

Duplicata ratio exprimitur sic $\frac{A}{C} = \frac{A}{B}$ *bis. Hoc est, ratio* A *ad* C *duplicata est rationis* A *ad* B. *Triplicata autem sic* $\frac{A}{D} = \frac{A}{B}$ *ter.* id est, *ratio* A *ad* D *triplicata est rationis* A *ad* B.

$\div\!\!\div$ *denotat continue proportionales.* ut A, B, C, D; *item* 2, 6, 18, 64 sunt $\div\!\!\div$

XI. Homologæ, seu similes ratione, magnitudines dicuntur, antecedentes quidem antecedentibus, consequentes vero consequentibus.

Ut si A. B :: C. D; *tam* A *&* C, *quam* B *&* D *homologæ magnitudines dicuntur.*

XII. Alterna ratio, est sumptio antecedentis ad antecedentem, & consequentis ad consequentem.

Ut sit A. B :: C. D. *ergo alterne, vel permutando, vel vicissim,* A.C :: B. D. *per* 16. 5.

In hac definitione, & 5. sequentibus imponuntur nomina sex modis argumentandi, quibus mathematici frequenter utuntur; quarum illationum vis innititur propositionibus hujus libri, quæ in explicationibus citantur.

XIII. Inversa ratio, est sumptio consequentis ceu antecedentis, ad antecedentem velut ad consequentem.

Ut A. B :: C. D. *ergo inverse*, B.A :: D. C. *per* cor. 4. 5.

XIV. Compositio rationis, est sumptio antecedentis cum consequente, ceu unius, ad ipsam consequentem.

Ut A. B :: C. D. *ergo componendo*, A+B.B :: C+D.D. *per* 18. 5.

XV. Divisio rationis, est sumptio excessus, quo consequentem superat antecedens, ad ipsam consequentem.

Ut

Vt A. B :: C. D. *ergo dividendo*, A-B. B :: C-D. D. *per* 17. 5.

XVI. Conversio rationis, est sumptio antecedentis ad excessum, quo superat antecedens ipsam consequentem.

Vt A. B :: C. D. *ergo per conversam rationem*, A. A-B :: C. C-D. *per* cor. 19. 5.

XVII. Ex æqualitate ratio est, si plures duabus sint magnitudines, & his aliæ multitudine pares, quæ binæ sumantur, & in eadem ratione; cum ut in primis magnitudinibus prima ad ultimam, sic & in secundis magnitudinibus prima ad ultimam sese habuerit. Vel aliter; sumptio extremorum, per subductionem mediorum.

XVIII. Ordinata proportio est, cum fuerit quemadmodum antecedens ad consequentem, ita antecedens ad consequentem: fuerit etiam ut consequens ad aliud quidpiam, ita consequens ad aliud quidpiam.

Vt si A. B :: D. E. *item* B. C :: E. F. *erit ex equo* A. C :: D. F. *per* 22. 5.

XIX. Perturbata autem proportio est; cum tribus positis magnitudinibus, & aliis, quæ sint his multitudine pares, ut in primis quidem magnitudinibus se habet antecedens ad consequentem, ita in secundis magnitudinibus antecedens ad consequentem: ut autem in primis magnitudinibus consequens ad aliud quidpiam, sic in secundis magnitudinibus aliud quidpiam ad antecedentem.

Vt si A. B :: F. G. *item* B. C :: E. F. *erit ex equo perturbate* A. C :: E. G. *per* 23. 5.

XX. Quotlibet magnitudinibus ordine positis, proportio primæ ad ultimam componitur ex proportionibus primæ ad secundam, & secundæ ad tertiam, & tertiæ ad quartam, & ita deinceps, donec extiterit proportio.

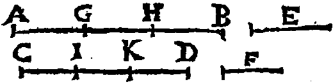
Sint

Sint quotcunque A, B, C, D; ex hac def. $\frac{A}{D} = \frac{A}{B} + \frac{B}{C} + \frac{C}{D}$.

Axioma.

Æquemultiplices eidem multiplici, sunt quoq; inter se æquemultiplices.

P R O P. I.



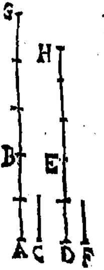
Si sint quotcunque magnitudines AB, CD, *quotcunque magnitudinum* E, F *æqualium numero, singulæ singularum, æquemultiplices; quam multiplex est unius* E *una magnitudo* AB, *tam multiplices erunt & omnes* AB+CD *omnium* E+F.

Sint AG, GH, HB partes quantitatis AB ipsi E æquales. item CI, IK, KD partes quantitatis CD ipsi F pares. Harum numerus illarum numero æqualis ponitur. Quum igitur

a 2. ax. AG+CI *a*=E+F; *a* & GH+IK=E+F; *a* & HB+KD=E+F, liquet AB+CD æque multoties continere E+F, ac una AB unam E continet. Q. E. D.

PROP.

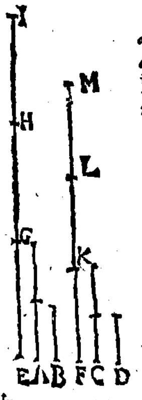
PROP. II.



Si prima A B *secundæ* C *æque fuerit multiplex, atque tertia* D E *quartæ* F; *fuerit autem & quinta* B G *secundæ* C *æque multiplex, atque sexta* E H *quartæ* F, *erit & composita prima cum quinta* (A G) *secundæ* C *æque multiplex, atque tertia cum sexta* (DH) *quartæ* F.

Numerus partium in A B ipsi C æqualium æqualis ponitur numero partium in D E ipsi F æqualium. Item numerus partium in B G ponitur æqualis numero partium in E H. *a* ergo numerus partium in A B+B G æquatur numero partium in DE+EH. hoc est tota AG æquemultiplex est ipsius C, atque tota GH ipsius F. Q. E. D. *a 2. ax.*

PROP. III.



Sit prima A *secundæ* B *æquemultiplex, atque tertia* C *quartæ* D; *sumantur autem* E I, F M *æquemultiplices primæ & tertiæ; erit & ex æquo, sumptarum utraque utriusque æquemultiplex: altera quidem* E I *secundæ* B, *altera autem* F M *quartæ* D.

Sint E G, G H, H I partes multiplicis E I ipsi A pares; item F K, K L, L M partes multiplicis F M ipsi C æquales. *a* Harum numerus illarum numero æquatur. porro A, id est E G, vel G H, vel G I ipsius B ponitur æquemultiplex atque C, vel FK, &c. ipsius D. *a hyp.*

b ergo

b 2. 5. *b* ergo E G + G H æquemultiplex est secundæ
c 2. 5. B, atque F K + K L quartæ D. *c* Simili argumento E I (E H + H I) tam multiplex est ipsius B, quam F M (F L + L N) ipsius D. Q. E. D.

PROP. IV.

Si prima A ad secundam B eandem habuerit rationem, & tertia C ad quartam D; etiam E & F æquemultiplices primæ A, & tertiæ C ad G, & H æquemultiplices secundæ B, & quartæ D, juxta quamvis multiplicationem, eandem habebunt rationem, si prout inter se respondent, ita sumptæ fuerint. (E. G :: F. H.)

Sume I, & K ipsarum E, & F; item L & M ipsarum G, & H æquemultiplices. *a* Erit I ipsius A æquemultiplex atque
a 3. 5. K ipsius C; *a* pariterque L tam multiplex ipsius B quam M ipsius D. Itaque cum sit A. B *b* :: C. D; juxta 6 *def.* si
b hyp. I $\sqsubset$, $=$, $\sqsupset$ L; consequenter pari modo K $\sqsubset$, $=$, $\sqsupset$ M. ergo cum I, & K ipsarum E, & F sumptæ sint æquemultiplices, atque L, & M ipsarum G & H; erit juxta 7. *def.* E. G :: F. H. Q. E. D.

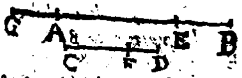
Coroll.

Hinc demonstrari solet inversa ratio.

Nam quoniam A. B :: C. D, si E $\sqsubset$, $=$, $\sqsupset$
c 6 def. 5. G, *c* erit similiter F $\sqsubset$, $=$, $\sqsupset$ H. ergo liquet quod

quod si G $\sqsubset$, $=$, $\sqsupset$ E, esse H $\sqsubset$, $=$, $\sqsupset$ F.
[d] ergo B. A :: D. C. Q. E. D. d 6. def. 5.

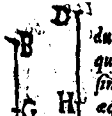
PROP. V.



Si magnitudo AB *magnitudinis* CD *aequè fuerit multiplex, atque ablata* AE *ablata* CF; *etiam reliqua* EB *reliquæ* FD *ita multiplex erit, ut tota* AB *totius* CD.

Accipe aliam quandam GA, quæ reliquæ FD ita sit multiplex, atque tota AB totius CD, vel ablata AE ablatæ CF. [a] ergò tota GA + AE totius CF + FD æquemultiplex est, ac una AE unius CF, hoc est ac AB ipsius CD. [b] ergò GE = AB. [c] proinde, ablata communi AE, manet GA = EB. ergo, &c. a 1. 5. b 6. ax. c 3. ax.

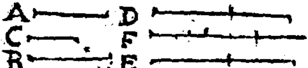
PROP. VI.



Si duæ magnitudines AB, CD *duarum magnitudinum* E, F *sint æquemultiplices; & detractæ quædam sint*, AG, & CH, *earundem* E, & F *æquemultiplices; & reliquæ* GB, HD *eisdem* E, F *aut æquales sunt, aut æque ipsarum multiplices.*

Nam quia numerus partium in AB ipsi E æqualium ponitur æqualis numero partium in CD ipsi F æqualium; item numerus partium in AG æqualis numero partium in CH. si hinc AG, inde CH detrahatur, [a] remanet numerus partium in reliqua GB æqualis numero partium in HD. a 3. ax. ergo si GB sit E semel, erit HD etiam C semel, si GB sit E aliquoties, erit HD etiam C toties accepta. Q. E. D.

PROP. VII.



Æquales A *& B ad eandem* C *eandem habent rationem; & eadem* C *ad æquales* A *&* B.

Sumantur D & E æqualium A & B æquemultiplices, & F utcunque multiplex ipsius C; a erit D $=$ E. quare si D $\sqsubset$, $=$, $\sqsupset$ F, erit similiter E $\sqsubset$, $=$, $\sqsupset$ F. b ergo A. C :: B. C. inverse igitur C. A c :: C. B. Q. E. D.

a 6 ax.
b 6 def. 5.
c cor. 4. 5.

Schol.

Si loco multiplicis F sumantur duæ æquemultiplices, eodem modo ostendetur æquales magnitudines ad alias inter se æquales eandem habere rationem.

PROP. VIII.

Inæqualium magnitudinum A B, C, *major* A B *ad eandem* D *majorem rationem habet, quam minor* C. *Et eadem* D *ad minorem* C *majorem rationem habet, quam ad majorem* AB.

Ex majori AB aufer AE $=$ C. sumatur HG tam multiplex ipsius AE, vel C, quam GF reliquæ FB. Multiplicetur D, donec ejus multiplex IK major evadat quam HG, sed minor quam HF.

Quoniam HG ipsius AE a tam multiplex est, quam GF ipsius EB, b erit tota HF totius AB æquemultiplex, atque una HG unius AE, vel C. ergo cum HF $\sqsubset$ IK (quæ multiplex est ipsius D) sed HG $\sqsupset$ IK, c erit $\frac{AB}{D} \sqsubset \frac{C}{D}$ Q. E. D.

a constr.
b 1. 5.
c 8. def. 5.

Rursus

Rursus quia IK $\sqsubset$ HG, at IK $\sqsupset$ HF (ut prius dictum) *d* erit $\frac{D}{C} \sqsubset \frac{D}{AB}$ Q. E. D.

PROP. IX.

Quæ ad eandem eandem habent rationem, æquales sunt inter se. Et ad quas eadem eandem habet rationem, eæ quoque sunt inter se æquales.

A B C

1. *Hyp.* Sit A. C :: B. C. dico A = B. Nam sit A $\sqsubset$, vel $\sqsupset$ B, *a* erit ideo $\frac{A}{C} \sqsubset$, vel $\sqsupset \frac{B}{C}$. contra Hyp. *a 8. 5.*

2. *Hyp.* Sit C. B :: C. A. dico A = B. nam sit A $\sqsubset$ B. *b* ergo $\frac{C}{B} \sqsubset \frac{C}{A}$. contra Hyp. *b 8. 5.*

PROP. X.

Ad eandem magnitudinem rationem habentium, quæ majorem rationem habet, illa major est: ad quam vero eadem majorem rationem habet, illa minor est.

A B C

1. *Hyp.* Sit $\frac{A}{C} \sqsubset \frac{B}{C}$. Dico A $\sqsubset$ B. Nam si dicatur A = B; *a* erit A. C :: B. C. contra Hyp. Sin A $\sqsupset$ B, *b* erit $\frac{A}{C} \sqsupset \frac{B}{C}$ etiam contra Hyp. *a 7. 5.* *b 8. 5.*

2. *Hyp.* Sit $\frac{C}{B} \sqsubset \frac{C}{A}$. Dico B $\sqsupset$ A. Nam dic B = A. *c* ergo C. B :: C. A. contra Hyp. vel dic B $\sqsubset$ A. *d* ergo $\frac{C}{A} \sqsubset \frac{C}{B}$ etiam contra Hyp. *c 7. 5.* *d 8. 5.*

PROP. XI.

G——— H——— I
A——— C——— E———
B——— D——— F———
K——— L——— M———

Quæ eidem sunt eædem rationes, & inter se sunt eædem.

Sit A.B :: E. F. item C. D :: E.F. dico A, B :: C.D. sume ipsarum A, C, E æquemultiplices G, H, I; atque ipsarum B, D, F æquemultiplices K, L, M. Et quoniam *a* A, B :: E.F. si G ⊏, =, ⊐ K, *b* erit pari modo I ⊏, =, ⊐ M. pariterque quia *a* E. F :: C. D. si I ⊏, =, ⊐ M, *b* erit H similiter ⊏, =, ⊐ L ergo si G ⊏, =, ⊐ K, erit similiter H ⊏, =, ⊐ L, *c* quare A. B :: C. D. Q. E. D.

a hyp.
b 6. def. 5.
c 6. def. 5.

Schol.

Quæ eisdem rationibus sunt eædem rationes, sunt quoque inter se eædem.

PROP. XII.

G——— H——— I———
A——— C——— E———
B——— D——— F———
K——— L——— M———

Si sint magnitudines quotcunque A, & B; C & D; E, & F *proportionales; quemadmodum se habuerit una antecedentium* A *ad unam consequentium* B, *ita se habebunt omnes antecedentes*, A, C, E *ad omnes consequentes*, B, D, F.

Sume antecedentium æquemultiplices G, H, I; & consequentium K, L, M. Quoniam quam multiplex est una G unius A, *a* tam multiplices sunt omnes G, H, I omnium A, C, E; pariterque quam multiplex est una K unius B, *a* tam multiplices sunt omnes K, L, M omnium B, D, F; si G ⊏, =, ⊐ K, erit similiter

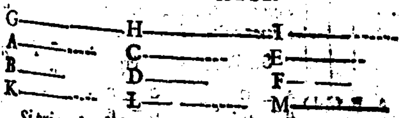
a 1. 5.

$G+H+I \sqsubset, =, \sqsupset K+L+M$. *b* quare A.B :: A+C+E.B+D+F. Q. E. D. b 6. def. 5.

Coroll.

Hinc, si similia proportionalia similibus proportionalibus addantur, tota erunt proportionalia.

PROP. XIII.



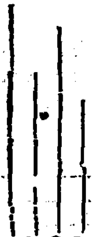
Si prima A ad secundam B eandem habuerit rationem, quam tertia C ad quartam D; tertia vero C ad quartam D majorem habuerit rationem, quam quinta E ad sextam F; prima quoque A ad secundam B majorem rationem habebit, quam quinta E ad sextam F.

Sume ipsarum A, C, E æquemultiplices G, H, I: ipsarumque B, D, F æquemultiplices K, L, M. Quia A.B :: C.D; si $H \sqsubset L$, *a* erit $G \sqsubset K$. Sed quia $\frac{C}{D} \sqsubset \frac{E}{F}$, *b* fieri potest ut sit $H \sqsubset L$, & I non $\sqsubset$ M. ergo fieri potest ut $G \sqsubset K$, & I non $\sqsubset$ M. *c* ergo $\frac{A}{B} \sqsubset \frac{E}{F}$. Q. E. D. a 6. def. 5. b 8. def. 5. c 8 def. 5.

SCHOL.

Quod si $\frac{C}{D} \sqsupset \frac{E}{F}$, erit quoque $\frac{A}{B} \sqsupset \frac{E}{F}$. Item si $\frac{A}{B} \sqsubset \frac{C}{D} \sqsubset \frac{E}{F}$, erit $\frac{A}{B} \sqsubset \frac{E}{F}$. & si $\frac{A}{B} \sqsupset \frac{C}{D} \sqsupset \frac{E}{F}$, erit $\frac{A}{B} \sqsupset \frac{E}{F}$.

PROP. XIV.



Si prima A *ad secundam* B *eandem habuerit rationem, quam tertia* C *ad quartam* D; *prima vero* A, *quam tertia* C *major fuerit; erit & secunda* B *major quam quarta* D. *Quod si prima* A *fuerit æqualis tertiæ* C, *erit & secunda* B *æqualis quartæ* D; *si vero* A *minor, &* B *minor erit.*

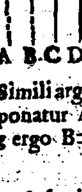
a 8. 5.
b hyp.
c 13. 5.
d 10. 5.
e 7. 5.
f hyp.
g 11. & 9. 5.

Sit A $\sqsubset$ C. *a* ergo $\frac{A}{B} \sqsubset \frac{C}{B}$. *b* sed A B C D $\frac{A}{B} = \frac{C}{D}$. *c* ergo $\frac{C}{D} \sqsubset \frac{C}{B}$. *d* ergo B $\sqsubset$ D. Simili argumento si A $\sqsupset$ C, *d* erit B $\sqsupset$ D. Sin ponatur A = C; ergo C. B *e* :: A. B *f* :: C. D. *g* ergo B = D. Quæ E. D.

SCHOL.

A fortiori, si $\frac{A}{B} \sqsupset \frac{C}{D}$, atque A $\sqsubset$ C, erit B $\sqsubset$ D. Item si A = B, erit C = D. Et si A $\sqsubset$, vel $\sqsupset$ B, erit pariter C $\sqsubset$, vel $\sqsupset$ D.

PROP. XV.

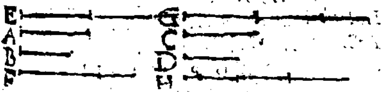


Partes C & F *cum pariter multiplicibus* AB, & DE *in eadem sunt ratione, si prout sibi mutuo respondent, ita sumantur.* (AB. DE :: C. F.)

a hyp.
b 7. 5.
c 12. 5.

Sint AG, GB partes multiplicis AB ipsi C æquales; item DH, HE partes multiplicis DE ipsi F æquales. *a* Harum numerus illarum numero æquatur. ergo quum *b* AG. C :: DH. F; *b* atque GB. C :: HE. F. *c* erit AG + GB (AB.) DH + HE (DE) :: C. F. Q. E. D.

PROP. XVI.



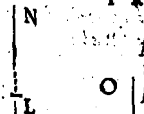
Si quatuor magnitudines A, B, C, D *proportionales fuerint; & vicissim proportionales erunt.* (A.C :: B.D.)

Accipe E & F æquemultiplices ipsarum A & B. ipsarumque C & D æquemultiplices G & H. Itaque E.F *a* :: A.B. *b* :: C.D *a* :: G.H. Quare si E $\sqsubset$, $=$, $\sqsupset$ G, *c* erit similiter F $\sqsubset$, $=$, $\sqsupset$ H. *d* ergo A.C :: B.D. Q. E. D. a 15. 5. b hyp. c 11. 5 & 14. 5. d 6. def. 5.

SCHOL.

Alterna ratio locum tantum habet, quando quantitates ejusdem sunt generis. Nam Heterogeneæ quantitates non comparantur.

PROP. XVII.



Si compositæ magnitudines proportionales fuerint (AB.CB :: DE. FE;) *hæ quoque divisæ proportionales erunt.* (AC.CB :: DF. FE.)

Accipe GH, HL, IK, KM ordine æquemultiplices ipsarum AC, CB, DF, FE. item LN, MO æquemultiplices ipsarum CB, FE. Tota GL totius AB *a* tam multiplex est, quam una GH unius AC, *b* id est quam IK ipsius DF; *c* hoc est quam tota IM totius DE: Item HN (HL + LN) ipsius CB *d* æquemultiplex est, ac KO (KM + MO) ipsius FE. Quum igitur per hyp. AB, BC :: DE. EF. si GL $\sqsubset$, $=$, $\sqsupset$ HN, etiam similiter a 1. 5. b constr. c 1. 5. d 2. 5.

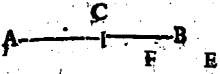
f 6.def.5. militer e erit IM $\sqsubset, =, \sqsupset$ KO, aufer hinc inde
f 5.ax. æquales HL, KM. si reliqua GH $\sqsubset, =, \sqsupset$
g 6.def.5. LN, f erit similiter IK $\sqsubset, =, \sqsupset$ MO. g unde AC. CB :: DF. FE. Q.E.D.

PROP. XVIII.

Si divisæ magnitudines sint proportionales (AB.BC :: DE. EF,) *hæ quoque compositæ proportionales erunt* (AC. CB :: DF. FE.)

Nam si fieri potest, sit AB. CB :: DF. FG $\sqsupset$ FE. a ergo erit divisim
a 17.5. AB. BC :: DG. GF. b hoc est DG.
b hyp. & 11.5. GF :: DE. EF. ergo cùm DG $\sqsubset$ DE,
c 14.5. c erit GF $\sqsubset$ EF. Q.E.A. Simile
d 9.ax. absurdum sequetur, si dicatur AB. CB :: DE.GF $\sqsubset$ FE.

PROP. XIX.



Si quemadmodum totum AB *ad totum* DE, *ita ablatum* AC *se habuerit ad ablatum* DF; *& reliquum* CB *ad reliquum* FE, *ut totum* AB *ad totum* DE, *se habebit*.

a hyp. Quoniam a AB. DE :: AC. DF, b erit per-
b 16.5. mutando AB. AC :: DE. DF. c ergo divisim
c 17.5. AC. CB :: DF. FE. quare rursus b permutando
d hyp. & 11.5. AC.DF :: CB.FE; d hoc est AB.DE :: CB.FE. Q.E.D.

Coroll.

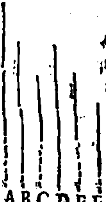
1. Hinc, si similia proportionalia similibus proportionalibus subducantur, residua erunt proportionalia.

2. *Hinc demonstrabitur conversa ratio.*

a 16.5. Sit AB. CB :: DE. FE. Dico AB. AC :: DE.
b 19.5. DF. Nam a permutando AB.DE :: CB. FE. b ergo AB. DE :: AC. DF. quare iterum permutando, AB. AC :: DE. DF. Q.E.D.

PROP.

Prop. XX.



Si sint tres magnitudines A, B, C; *& aliæ* D, E, F *ipsis æquales numero, quæ binæ & in eadem ratione sumantur* (A. B :: D. E, *atque* B. C :: E. F ;) *ex æquo autem prima* A *major fuerit, quam tertia* C; *erit & quarta* D *major quam sexta* F. *Quod si prima* A *tertiæ* C *fuerit æqualis; erit & quarta* D *æqualis sextæ* F. *Sin illa minor, hæc quoque minor erit.*

Hyp. Si A $\sqsubset$ C. quoniam *a* E. F :: B. C. *b* erit inverse F. E :: C. B. *c* Sed $\frac{C}{B} \sqsupset \frac{A}{B}$ *d* ergo $\frac{F}{E} \sqsupset \frac{A}{B}$ vel $\frac{D}{E}$, *e* ergo D $\sqsubset$ F. Q. E. D.

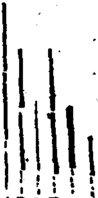
a hyp.
b cor. 4. 5.
c hyp. & 8. 5.
d schol. 13. 5.
e 10. 5.

2. *Hyp.* Simili argumento, si A $\sqsupset$ C, ostendetur D $\sqsupset$ F.

3. *Hyp.* Si A = C. quoniam F. E :: C. B :: *f* A. B :: D. E. *g* erit D = F. Q. E. D.

f 7. 5.
g 11. 5. & 9. 5.

Prop. XXI.



Si sint tres magnitudines A, B, C; *& aliæ* D, E, F *ipsis æquales numero, quæ binæ & in eadem ratione sumantur, fueritque perturbata eorum proportio,* (A. B :: E. F. *atque* B. C :: D. E;) *ex æquo autem prima* A *quam tertia* C *major fuerit; erit & quarta* D *quam sexta* F *major. Quod si prima fuerit tertiæ æqualis, erit & quarta æqualis sextæ: sin illa minor, hæc quoque minor erit.*

1. *Hyp.* A $\sqsubset$ C. Quoniam *a* D. E :: B. C, invertendo erit E. D :: C. B. atqui $\frac{C}{B}$ *b* $\sqsupset$ $\frac{A}{B}$. *c* ergo

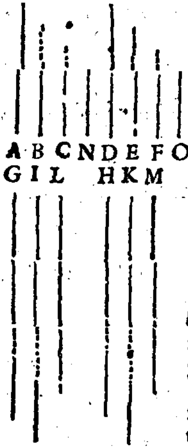
a hyp.
b 8. 5.

a schol. 13. 5. d 10 5. [c] ergo $\frac{E}{D} \sqsupset \frac{A}{B}$, hoc est $\frac{E}{F}$. [d] ergo D $\sqsubset$ F. Q. E. D.

2. *Hyp.* Similiter, si A $\sqsupset$ C, erit D $\sqsupset$ F.

e 7. 5. f hyp. 3 g 9 5. 3. *Hyp.* Si A = C. quoniam E. D [e] :: C. B :: [e] A. B :: [f] E. F. [g] erit D = F. Q. E. D.

PROP. XXII.



Si sint quotcunque magnitudines A, B, C; & aliæ ipsis æquales numero D, E, F, quæ binæ & in eadem ratione sumantur (A. B :: D. E. & B. C :: E. F;) *& ex æqualitate in eadem ratione erunt* (A. C :: D. F.)

Accipe G, H ipsarum A, D; & I, K ipsarum B, E; item L, M ipsarum E, F æquemultiplices.

a hyp. b 4 5. Quoniam [a] A. B :: D. E. [b] erit G. I :: H. K. eodem modo, erit I. L :: K. M. ergo si G $\sqsubset$, =, $\sqsupset$ L, [c] erit H. $\sqsubset$, =, $\sqsupset$ M; [d] ergo A. C :: D. F. Eodem pacto si ulterius C. N :: F. O, erit ex æquali A. N :: D. O. Q. E. D.

c 20. 5. d 6. def 5.

PROP. XXIII.

Si sint tres magnitudines A, B, C, *aliaeque* D, E, F *ipsis aequales numero, quae binae in eadem ratione sumantur; fuerit autem perturbata earum proportio.* (A. B :: E. F. & B. C :: D. E.) *etiam ex aequalitate in eadem ratione erunt.*

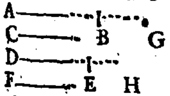
Sume G, H, I, ipsarum A, B, D; item K, L, M ipsarum C, E, F aequemultiplices. erit G. H a :: A. B b :: E. F a :: L. M. porro quia b B. C :: D. E. erit c H. K :: I. L. ergo G, H, K; & I, L, M, habent se juxta 21. 5. quare si G $\sqsubset$, $=$, $\sqsupset$ K, erit similiter I $\sqsubset$, $=$, $\sqsupset$ M. d proinde A. C :: D. F. Q. E. D. a 15. 5. b hyp c 4. 5. d 6. def. 5.

Eodem modo si plures fuerint magnitudinibus tribus, &c.

Coroll.

Ex *his sequitur, rationes ex iisdem rationibus compositas esse inter se easdem. item, earundem rationum easdem partes inter se easdem esse. * 21 & 13. 5. & 20. def. 1.

PROP. XXIV.

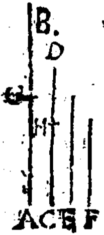


Si prima AB *ad secundam* C *eandem habuerit rationem quam tertia* DE *ad quartam* F; *habuerit autem & quinta* BG *ad secundam* C *eandem rationem, quam sexta* EH *ad quartam* F; *etiam composita prima cum quinta* (AG) *ad secundam* C *eandem habebit rationem, quam tertia cum sexta* (DH) *ad quartam* F.

Nam quia a AB. C :: DE. F. atque ex hyp. & inverse C. BG :: F. EH, erit b ex aequali AB. BG :: DE. EH. ergo componendo AG. BG :: DH. EH. c item BG. C :: EH. F. b ergo rursus ex aequo, AG. C :: DH. F. Q. E. D. a hyp. b 22. 5. c hyp.

PROP.

PROP. XXV.



Si quatuor magnitudines proportionales fuerint (AB. CD :: E. F.) *maxima* AB *&* *minima* F *reliquis* CD *&* E *majores erunt.*

Fiant A G $=$ E ; & C H $=$ F.
a hyp. Quoniam AB. CD a :: E. F b ::
b 7.5. AG. CH. c erit AB. CD :: GB.
c 19.5. HD. d sed AB $\sqsubset$ CD. e ergo
d hyp. GB $\sqsubset$ HD. atqui AG + F $=$ E +
e schol. 14.5. CH. ergo AG + F + GB $\sqsubset$ E + CH + HD, hoc est AB + F $\sqsubset$ E + CD. Q. E. D.

Quæ sequuntur propositiones non sunt Euclidis; sed ex aliis desumptæ, ob frequentem earum usum Euclidæis subjungi solent.

PROP. XXVI.

A———C———
B——— D———
E———

Si prima ad secundam habuerit majorem proportionem, quam tertia ad quartam; habebit convertendo, secunda ad primam minorem proportionem, quam quarta ad tertiam.

Sit $\frac{A}{B} \sqsubset \frac{C}{D}$. Dico $\frac{B}{A} \sqsupset \frac{D}{C}$. Nam concipe
a 13.5. $\frac{C}{D} = \frac{E}{B}$. a ergo $\frac{A}{B} \sqsubset \frac{E}{B}$. b quare A $\sqsubset$ E. c ergo
b 10.5.
c 8.5
d cor 4.5. $\frac{B}{A} \sqsupset \frac{B}{E}$, d vel $\frac{D}{C}$. Q. E. D.

PROP. XXVII.

A———C———
B——— D———
E———

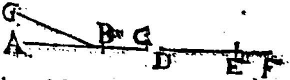
Si prima ad secundam habuerit majorem proportionem, quam tertia ad quartam; habebit quoque vicissim prima ad tertiam majorem proportionem, quam secunda ad quartam.

Sit

Sit $\frac{A}{B} \sqsubset \frac{C}{D}$. Dico $\frac{A}{C} \sqsubset \frac{B}{D}$. Nam puta $\frac{E}{B} = \frac{C}{D}$.

a ergo $A \sqsubset E$, *b* ergo $\frac{A}{C} \sqsubset \frac{E}{C}$, *c* vel $\frac{B}{D}$. Q.E.D. a 10. 5. b 8. 5. c 16. 5.

PROP. XXVIII.

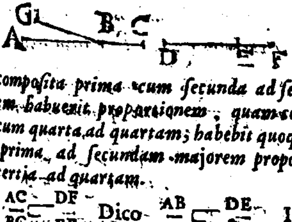


Si prima ad secundam habuerit majorem proportionem, quam tertia ad quartam; habebit quoque composita prima cum secunda ad secundam majorem proportionem, quam composita tertia cum quarta ad quartam.

Sit $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. Dico $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Nam cogita $\frac{GB}{BC} = \frac{DE}{EF}$. *a* ergo $AB \sqsubset GB$. adde utrinque BC, a 10. 5.

b erit $AC \sqsubset GC$. *c* ergo $\frac{AC}{BC} \sqsubset \frac{GC}{BC}$, *d* hoc est $\frac{DF}{FE}$. b 4. ax. c 8. 5. d 18. 5.
Q.E.D.

PROP. XXIX.



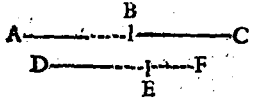
Si composita prima cum secunda ad secundam majorem habuerit proportionem, quam composita tertia cum quarta ad quartam; habebit quoque dividenda prima, ad secundam majorem proportionem quam tertia ad quartam.

Sit $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Dico $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. Intellige $\frac{GC}{BC} = \frac{DE}{EF}$. *a* ergo $AC \sqsubset GC$. aufer commune a 10. 5.

BC, *b* erit $AB \sqsubset GB$. *c* ergo $\frac{AB}{BC} \sqsubset \frac{CB}{BC}$ *d* vel $\frac{DE}{EF}$. b 5. ax. c 8. 5. d 17. 5.
Q.E.D.

PROP.

PROP. XXX.



Si composita prima cum secunda ad secundam habuerit majorem proportionem, quam composita tertia cum quarta ad quartam; habebit, per conversionem rationis, prima cum secunda ad primam minorem rationem, quam tertia cum quarta ad tertiam.

Sit $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Dico $\frac{AC}{AB} \sqsupset \frac{DF}{DE}$. Nam quia

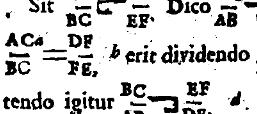
a hyp. $\frac{AC}{BC}$ *a* $= \frac{DF}{FE}$, *b* erit dividendo $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. *c* conver-

b 19. 5.

c 16 5.

d 28. 5. tendo igitur $\frac{BC}{AB} \sqsupset \frac{EF}{DE}$. *d* ergo componendo $\frac{AC}{AB} \sqsupset \frac{DF}{DE}$. Q. E. D.

PROP. XXXI.



Si sint tres magnitudines A, B, C, & aliæ ipsis æquales numero D, E, F; sitque major proportio primæ priorum ad secundam, quam primæ posteriorum ad secundam $\left(\frac{A}{B} \sqsubset \frac{D}{F};\right)$ *item secundæ priorum ad tertiam major, quam secundæ posteriorum ad tertiam* $\left(\frac{B}{C} \sqsubset \frac{E}{F};\right)$ *erit quoque ex æqualitate major proportio primæ priorum ad tertiam, quam primæ posteriorum ad tertiam* $\left(\frac{A}{C} \sqsubset \frac{D}{F}.\right)$

a 10. 5. Concipe $\frac{G}{C} = \frac{E}{F}$. *a* ergo $E \sqsubset G$. *b* ergo $\frac{A}{C} \sqsubset \frac{A}{B}$.

b 8 5.

c 13. 5.

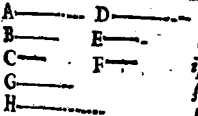
d 10 5. Rursus puta $\frac{H}{G} = \frac{D}{E}$ *c* ergo $\frac{H}{G} \sqsupset \frac{A}{B}$; *d* ergo fortius

e 8 5.

f 12. 5. $\frac{H}{G} \sqsupset \frac{A}{G}$. *d* quare $A \sqsubset H$. *e* proinde $\frac{A}{C} \sqsubset \frac{H}{C}$, vel $\frac{D}{F}$.

Q. E. D.

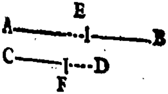
PROP. XXXII.



Si sint tres magnitudines A,B.C *; & aliæ ipsis æquales* D, E, F*; sitque major proportio primæ priorum ad secundam, quam secundæ posteriorum ad tertiam* $\left(\frac{A}{B} \sqsubset \frac{E}{F};\right)$ *item secundæ priorum ad tertiam major quam primæ posteriorum ad secundam* $\left(\frac{B}{C} \sqsubset \frac{D}{E};\right)$ *erit quoque ex æqualitate major proportio primæ priorum ad tertiã, quam primæ posteriorum ad tertiam* $\left(\frac{A}{C} \sqsubset \frac{D}{F}.\right)$

Hujusce demonstratio plane similis est demonstrationi præcedentis.

PROP. XXXIII.



Si fuerit major proportio totius AB *ad totum* CD, *quam ablati* AE *ad ablatum* CF *; erit & reliqui* EB *ad reliquum* FD *major proportio, quam totius* AB *ad totum* CD.

Quoniam $\frac{AB}{CD}$ a $\sqsubset \frac{AE}{CF}$, b erit permutando $\frac{AB}{AE} \sqsubset \frac{CD}{CF}$. c ergo per conversionem rationis $\frac{AB}{EB} \sqsupset \frac{CD}{FD}$. permutando igitur $\frac{AB}{CD} \sqsupset \frac{EB}{FD}$. Q. E. D.

a hyp.
b 17. 5.
c 30. 5.

PROP. XXXIV.

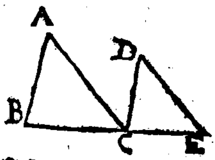
A——————D——————
B———— E————
C———— F————
G———— H————

Si sint quotcunque magnitudines, & aliæ ipsis æquales numero, sitque major proportio prima priorum ad primam posteriorum, quam secundæ ad secundam; & hæc major quam tertiæ ad tertiam, & sic deinceps: habebunt omnes priores simul ad omnes posteriores simul, majorem proportionem, quam omnes priores, relicta prima, ad omnes posteriores, relicta quoque prima; minorem autem, quam prima priorum ad primam posteriorum; majorem denique etiam, quam ultima priorum ad ultimam posteriorum.

Horum demonstratio est penes interpretes, quos adeat, qui eam desiderat. nos omisimus; brevitatis studio; & quia illorum nullus usus in his elementis.

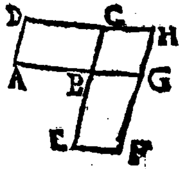
LIB. VI.

Definitiones.

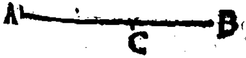


I. Similes figuræ rectilineæ sunt (A B C, D C E,) quæ & angulos singulos singulis æquales habent; atque etiam latera, quæ circum angulos æquales, proportionalia.

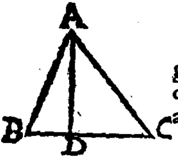
Ang. B = D C E; *&* A B. B C :: D C. C E. *item ang.* A = D; *atque* BA. AC :: CD. DE. *denique ang.* A C B = E. *atque* B C. C A :: C E. E D.



II. Reciprocæ autem sunt (B D, B F,) cum in utraque figura antecedentes, & consequentes rationum termini fuerint. (hoc est, AB. BG :: EB. BC.)



III. Secundum extremam & mediam rationem recta linea A B secta esse dicitur, cum ut tota A B ad majus segmentum AC, ita majus segmentum A C ad minus C B se habuerit. (A B. AC :: A C. C B.)

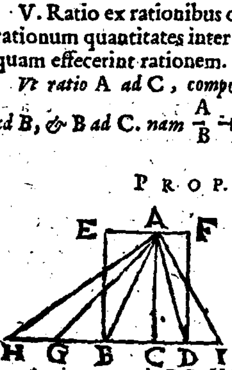


IV. Altitudo cujusque figuræ ABC est linea perpendicularis A D, à vertice A ad basim B C deducta.

V. Ratio ex rationibus componi dicitur, cum rationum quantitates inter se multiplicatæ, aliquam effecerint rationem.

Vt ratio A *ad* C, *componitur ex rationibus* A
a 20.def.5. *ad* B, *&* B *ad* C. *nam* $\frac{A}{B} + \frac{B}{C}$ [a] $= \frac{A}{C}$ [b] $= \frac{AB}{BC}$.
b 15.5.

PROP. I.

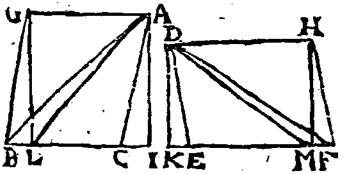


Triangula A B C, ACD, *&* *parallelogramma* B C A E, CDFA, *quorum eadem fuerit altitudo, ita se habent inter se, ut bases* BC, C D.

5.1. *a* Accipe quotvis BG, HG, ipsi BC æquales; item DI = CD. & connecte AG, AH, AI.

b 38.1. *b* Triangula ACB, ABG, AGH æquantur; *b* item triang. A C D = A D I. ergo triangulum ACH tam multiplex est trianguli ACB, quam basis H C basis B C. & æquemultiplex est triang. ACI trianguli ACD, ac basis CI basis CD.
c sch.38 1. 6.def.5. cum igitur si HC $\sqsubset, =, \sqsupset$ CI, *c* erit similiter
41.1.& triang. AHC $\sqsubset, =, \sqsupset$ ACI, *d* ideoque BC,
1.5. C D :: triang. A B C. A C D :: *e* pgr. C E. C F.
Q. E. D.

Schol.



Hinc, *triangula* A B C, D E F, *& parallelogramma* A G B C, D E F H, *quorum æquales sunt bases* BC, EF, *ita se habent ut altitudines* AI, DK.

aSume IL = CB; & KM = EF; ac junge LA, LG, MD, MH. liquet esse triang. ABC. DEF :: bALI. DKM :: cAI. DK :: d pgr. AGBC. DEFH. Q. E. D. a 3. 1. b 7. 5. c 1. 6. d 41. 1. & 15. 5.

PROP. II.

Si ad unum trianguli A B C *latus* B C, *parallela ducta fuerit recta quædam linea* D E, *hæc proportionaliter secabit ipsius trianguli latera* (A D. B D :: A E. E C.) *Et si trianguli latera proportionaliter secta fuerint* (A D. B D :: A E. E C) *quæ ad sectiones* D, E adjuncta *fuerit recta linea* D E, *erit ad reliquum ipsius trianguli latus* BC *parallela*. Ducantur CD, BE.

1. *Hyp.* Quia triang. DEB a = DEC; b erit triang. A D E. D B E :: A D E. E C D. atqui triang. A D E. D B E c :: A D. D B. & triang. ADE. DEC c :: A E. E C. d ergo A D. D B :: AE. EC. a 37. 1. b 7. 5. c 1. 6. d 11. 5.

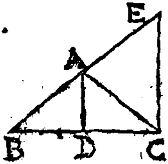
2. *Hyp.* Quia A D. D B :: A E. E C. e hoc est triang. A D E. D B E :: A D E. E C D; f erit triang. D B E = E C D. g ergo D E, B C sunt parallelæ. Q. E. D. e 1. 6. f 9. 5. g 39. 1.

 Schol.

Schol.

Imo, si plures ad unum trianguli latus parallelæ ductæ fuerint, erunt omnia laterum segmenta proportionalia, ut facile deducitur ex hac.

PROP. III.



Si trianguli BAC angulus BAC bifariam sectus sit, secans autem angulum recta linea A D *secuerit & basim, basis segmenta eandem habebunt rationem quam reliqua ipsius trianguli latera* (BD. DC :: AB. AC.) *Et si basis segmenta eandem habeant rationem quam reliqua ipsius trianguli latera* (B D. D C :: A B. A C.) *recta linea* A D *quæ à vertice* A *ad sectionem* D *ducitur, bifariam secat trianguli ipsius angulum* BAC.

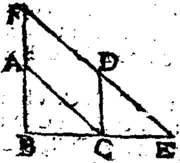
Produc BA; & fac AE = AC. & junge CE.

1. *Hyp.* Quoniam AE = AC, erit ang. ACE *a* = E *b* = $\frac{1}{2}$ BAC *c* = DAC. *d* ergo DA, CE parallelæ sunt. *e* quare B A. A E (A C) :: B D. D C. Q. E. D.

2. *Hyp.* Quoniam B A. A C. (A E) :: B D. DC. *f* erunt D A, C E parallelæ: *g* ergo ang. BAD = E; & ang. DAC *g* = ACE *h* = E. *k* ergo ang. BAD = DAC. bisectus igitur est ang. BAC. Q. E. D.

a 5. 1.
b 32. 1.
c *hyp.*
d 27. 1.
e 2. 6.
f 2. 6.
g 29. 1.
h 5. 1.
k 1. *ax.*

PROP. IV.



Æquiangulorum triangulorum A B C, D C E *proportionalia sunt latera, quæ circum æquales angulos* B, DCE (AB. BC :: DC. CE, &c.) *& homologa sunt latera* A B, D C, &c. *quæ æqualibus angulis* ACB, E, &c. *subtenduntur.*

Statue

Statue latus B C in directum lateri C E, & produc B A, ac E D donec *a* occurrant. a 32. 1.

Quoniam ang. B *b* = ECD, *c* sunt B F, C D parallelæ. Item quia ang BCA *b* = CED, *c* sunt C A, E F parallelæ. Figura igitur C A F D est parallelogramma. *d* ergo A F = C D; *d* & A C = F D. Liquet igitur A B. A F (C D) *e* :: B C. CE. *f* permutando igitur AB. BC :: CD. CE. *e* item B C. C E :: F D. (A C) D E. *f* ergo permutando BC. AC :: CE. DE. quare etiam *g* ex æquo AB. AC :: CD. DE. ergo, &c.

& 13. ax. b hyp. c 28. 1. d 34. 1. e 2. 6. f 16. 5. g 22. 5.

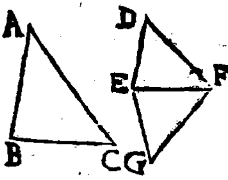
Coroll.

Hinc AB. DC :: BC. CE :: AC. DE.

Schol.

Hinc si in triangulo FBE ducatur uni lateri FE parallela AC; erit triangulum ABC simile toti FBE.

PROP. V.

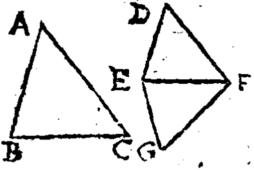


Si duo triangula A B C, D E F *latera proportionalia habeant.* (A B. B C :: D E. E F. & A C. B C :: D F. E F. item A B. A C :: D E D F) *æquiangula erunt triangula, & æquales habebunt eos angulos, sub quibus homologa latera subtenduntur.*

Ad latus E F *a* fac ang. F E G = B; *a* & ang. EFG = C, *b* quare etiam ang. G = A. ergo G E. E F *c* :: A B. B C :: *d* D E. E F. *e* ergo G E = D E. Item G F F E *c* :: A C. C B *d* :: DF. FE. *e* ergo GF = DF. Triangula igitur DEF, GEF sibi mutuo æquilatera sunt. *f* ergo ang. D = G = A. *f* & ang. FED = FEG = B. *g* proinde & ang. DFE = C. ergo, &c.

a 23. 1. b 32. 1. c 4. 6. d hyp. e 11. 5. & 9. 5. f 8. 1. g 32. 1.

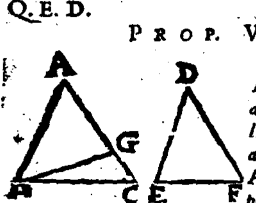
PROP. VI.



Si duo triangula A B C, D E F *unum angulum* B *uni angulo* D E F *æqualem, & circum æquales angulos* B, D E F *latera proportionalia habuerint* (A B. B C :: D E. E F;) *æquiangula erunt triangula* A B C, D E F; *æqualesque habebunt angulos, sub quibus homologa latera subtenduntur.*

a 32. 1. Ad latus E F fac ang. F E G = B, & ang. E F G
b 4. 6. = C. a unde & ang. G = A. ergo G E. E F b ::
c hyp. AB. BC c :: DE. EF. d ergo DE = GE. atqui
d 9. 5. e hyp. f constr. ang. D E F e = B f = G E F. g ergo ang. D
g 4. 1. h 32. 1. = G = A. h proinde etiam ang. E F D = C.
Q. E. D.

PROP. VII.



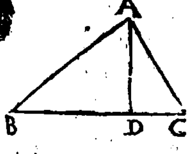
Si duo triangula ABC, DEF *unum angulum* A *uni angulo* D *æqualem, circa autem alios angulos* A B C, E *latera proportionalia habeant* (A B. B C :: D E. E F;) *reliquorum autem simul utrunque* C, F *aut minorem aut non minorem recto, æquiangula erunt triangula* ABC, DEF, *& æquales habebunt eos angulos circum quos proportionalia sunt latera.*

a hyp. Nam si fieri potest, sit ang. ABC $\sqsubset$ E. fac
b 32. 1. igitur ang. ABG = E; ergo cum ang. A a = D,
c 4. 6. b erit etiam ang. AGB = F. ergo AB. BG c ::
d hyp. e 9. 5. DE. EF :: AB. BC. e ergo BG = BC. f ergo
f 5. 1. g cor. 17. 1. ang. BGC = BCG. g ergo ang. BGC. vel C

minor

minor est recto; g proinde ang. AGB, vel F re- g cor. 13. 1.
cto major est. ergo anguli C & F non sunt e-
jusdem speciei, contra Hyp.

PROP. VIII.



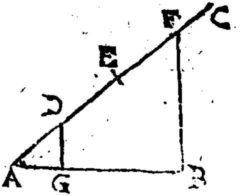
Si in triangulo rectangulo ABC, ab angulo recto BAC in basin BC perpendicularis AD ducta est; quæ ad perpendicularem triangula ADB, ADC, tum toti triangulo ABC, tum ipsa inter se, similia sunt.

Nam ang. BAC a = BDA a = CDA. & ang. BAD b = C. & CAD b = B. ergo per 4. 6. & 1 def. 6. (a 12. ax. b 32. 1.)

Coroll.

Hinc 1. BD. DA c :: DA. DC. (c 1 def. 6.)
2. BC. AC :: AC. DC. & CB. BA :: BA. BD.

PROP. IX.

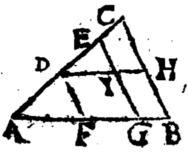


A data recta linea AB imperatam partem $\frac{1}{3}$ *(AG) auferre.*

Ex A duc infinitam AC utcunq; in qua a sume tres AD, DE, EF æquales utcunque, junge FB, cui ex D b duc parallelam DG. Dico factum. (a 3. 1. b 31. 1.)

Nam GB. AG c :: FD. AD. ergo d componendo AB. AG :: AF. AD. ergo cum AD = $\frac{1}{3}$ AF, erit AG = $\frac{1}{3}$ AB. Q. E. F. (c 2. 6. d 18. 5.)

PROP. X.



Datam rectam lineam A B *insectam similiter secare* (in F, G,) *ut data altera* A C, *secta fuerit* (*in* D, E.)

Extremitates se[illegible] & insectæ jungat recta

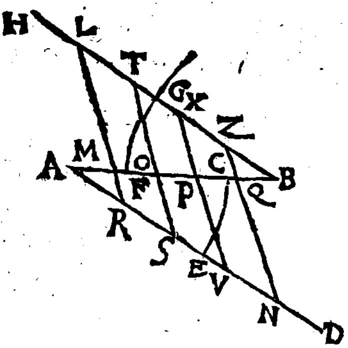
a 31. 1. B C. Huic ex punctis E, D *a* duc parallelas E G, D F rectæ secandæ occurrentes in G, & F. Dico factum.

a Ducatur enim D H parall. A B. Estque AD.

b 2. 6. D E *b* :: A F. F G, & D E. E C *b* :: D I. I H *c* ::

c 34. 1. & 7. 5. F G. G B. Q. E. F.

Scholium.



Hinc *distimus rectam datam* A B *in quotvis æquales partes* (puta 5.) *secare*. id quod facilius præstabitur sic;

Duc infinitam AD, eique parallelam BH etiam infinitam. Ex his cape partes æquales AR, RS, SV, VN; & BZ, ZX, XT, TL; in singulis una pau-

pauciores, quam desiderentur in AB; tum rectæ ducantur LR, TS, XV, ZN. hæ quinquisecabunt datam AB.

Nam RL, ST, VX, NZ *a* parallelæ sunt. ergo quum AR, RS, SV, VN *b* æquales sint, *c* erunt AM, MO, OP, PQ æquales. Similiter quia BZ = ZX, erit BQ = QP. ergo AB quinquisecta est. Q. E. F. a 33. 1. b constr. c 2. 6.

PROP. XI.

Datis duabus rectis lineis AB, AD, *tertiam proportionalem* DE *invenire.*

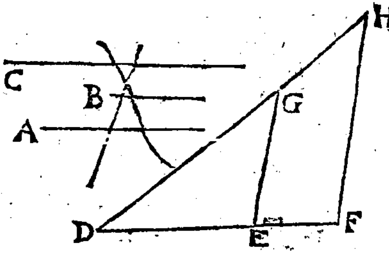
Junge BD, & ex AB protracta sume BC = AD. per C duc CE parall. BD. cui occurrat AD producta in E. Erit DE expetita.

Nam AB. *a* BC. (AD) :: AD. DE. Q. E. F. a 2. 6.

Vel sic, fac ang. ABC rectum, & ang. ACD etiam rectum. *b* erit AB. BC :: BC. BD. b cor. 8.

PROP.

PROP. XII.



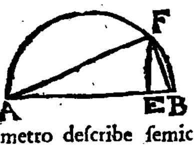
Tribus datis rectis lineis D E, E F, D G, *quartam proportionalem* GH *invenire*.

Connectatur E G. per F duc F H parall. E G, cui occurrat D G producta ad H. liquet esse
a 2. 6. DE. EF [a] :: DG. GH. Q. E. F.

Vel ita. $CD = CB + BD$ adapta [a] circulo. Circino sume A B.
a 35. 3. Erit $AB \times BE = CB \times BD$. [b] qua-
b 16. 6. re AB. CB :: BD. BE.

PROP. XIII.

Duabus datis rectis lineis A E, E B, *mediam proportionalem* E F *adinvenire*.



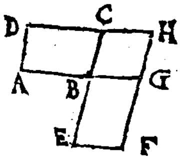
Super tota A B diametro describe semicirculum A F B. Ex E erige perpendicularem E F occurrentem peripheriæ in F. Dico A E. E F :: E F. E B. Du-
a 31. 3. cantur enim A F, & F B. Ex trianguli [a] rectanguli

guli A F B recto angulo deducta est F E basi perpendicularis ; *b* ergo A E. F E :: F E. E B. Q. E. F. b cor. 8. 6.

Coroll.

Hinc, linea recta, quæ in circulo à quovis puncto diametri, ipsi diametro perpendicularis ducitur ad circumferentiam usque, media est proportionalis inter duo diametri segmenta.

PROP. XIV.



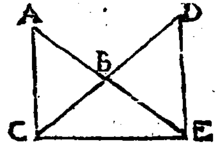
Æqualium, & unum A B C *uni* E B G *æqualem habentium angulum, parallelogrammorum* B D, B F, *reciproca sunt latera quæ circum æquales angulos.* (A B. B G :: E B. B C:) *Et quorum parallelogrammorum* B D, B F, *unum angulum* A B C *uni angulo* E B G *æqualem habentium, reciproca sunt latera quæ circum æquales angulos, illa sunt æqualia.*

Nam latera A B, B G circa æquales angulos faciant unam rectam: *a* quare E B, B C etiam in directum jacebunt. Producantur F G, D C; donec occurrant. a sch. 15. 1.

1. *Hyp.* AB. BG *b* :: BD. BH *c* :: BF. BH *d* :: BE. BC. *e* ergo, &c. b 1. 6. c 7. 5. d 1. 6.

2. *Hyp.* BD. BH *f* :: AB. BG *g* :: BE. BC *h* :: BF. BH, *k* ergo Pgr. BD = BF. Q. E. D. e 11. 5. f 1. 6. g hyp. h 1. 6. k 11. & 9. 5.

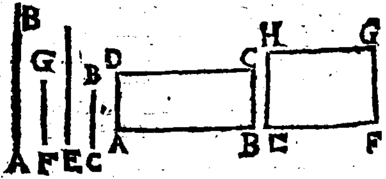
PROP. XV.



Æqualium, & unum ABC, *uni* DBE *æqualem habentium angulum triangulorum* ABC, DBE, *reciproca sunt latera, quæ circum æquales angulos* (AB. BE :: DB. BC) : *Et quorum triangulorum* ABC, DBE, *unum angulum* ABC *uni* DBE *æqualem habentium reciproca sunt latera, quæ circum æquales angulos* (AB. BE :: DB. BC.) *illa sunt æqualia.*

Latera C-B, BD circa æquales angulos, sta-
a sch. 15. 1. tuantur sibi in directum; *a* ergo ABE est recta
b 1. 6. linea. ducatur CE.
c 7. 5. 1. *Hyp.* AB. BE *b* :: triang. ABC. CBE
d 1. 6. *c* :: triang. DBE. CBE. *d* :: DB. BC. *e* ergo, &c.
e 11. 5. 2. *Hyp.* Triang. ABC. CBE *f* :: AB. BE *g* ::
f 1. 6. DB. BC *h* :: triang. DBE. CBE. *k* ergo triang.
g hyp. ABC = DBE. Q. E. D.
h 1. 6.
k 11, & 9. 5.

PROP. XVI.



Si quatuor rectæ lineæ proportionales fuerint (AB. FG :: EF. CB,) *quod sub extremis* AB, CB *comprehenditur rectangulum* AC, *æquale est ei, quod sub mediis* EF, FG *comprehenditur, rectangulo* EG. *Et si sub extremis comprehensum rectangulum* AC *æquale fuerit ei, quod sub mediis comprehenditur, rectangulo* EG, *illæ quatuor rectæ lineæ proportionales erunt* (AB. FG :: EF. CB.)

1. *Hyp.*

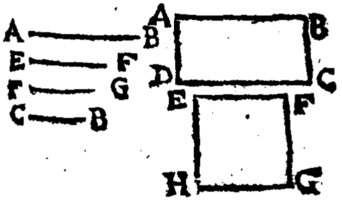
1. *Hyp.* Anguli B & F recti, ac *a* proinde pares sunt; atque ex hyp. AB. FG :: EF. CB. *b* ergo rectang. AC = EG. Q. E. D. a 12. ax.

2. *Hyp.* *c* Rectang. A C = E G; atque ang. B = F; *d* ergo AB. FG :: EF. CB. Q. E. D. b 14. 6. c hyp. d 14. 6.

Coroll.

Hinc ad datam rectam lineam A B facile est datum rectangulum E G applicare, *e* faciendo AB. EF :: FG. EC. e 12. 6.

PROP. XVII.



Si tres rectæ lineæ sint proportionales (AB. EF :: EF. CB,) *quod sub extremis* AB, CB *comprehenditur rectangulum* AC, *æquale est ei, quod à media* EF *describitur, quadrato* EG. *Et si sub extremis* AB, CB *comprehensum rectangulum* AC, *æquale sit ei, quod à media* EF *describitur, quadrato* EG, *illæ tres rectæ lineæ proportionales erunt* (AB. EF :: EF. CB.)

Accipe FG = EF.

1. *Hyp.* AB. EF *a* :: EF (FG.) CB. ergo Rectang. AC *b* = EG *c* = EFq. Q. E. D. a hyp. b 16. 6.

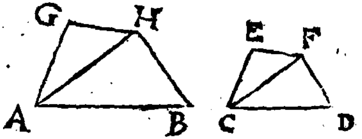
2. *Hyp.* Rectang. AC *d* = quadr. EG = EFq. *e* ergo AB. EF :: FG (EF.) BC. c 29. def. 1. d hyp. e 16. 6.

Coroll.

Sit A in B = Cq. ergo A. C :: C. B.

PROP.

PROP. XVIII.

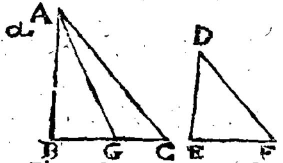


A data recta linea AB *dato rectilineo* CEFD *simile similiterque positum rectilineum* AGHB *describere.*

Datum rectilineum resolve in triangula. *a* fac ang. ABH = D; *a* & ang. BAH = DCF;
a 23. 1. *a* & ang. A H G = C F E ; *a* & ang. H A G = FCE. Rectilineum AGHB est quæsitum.

b constr. Nam ang. B *b* = D. & ang. BAH *b* = DCF.
c 32. 1. *c* quare ang. AHB = CFD ; *b* item ang. HAG = FCE, *b* & ang. AHG = CFE. *c* quare ang.
d 2. ax. G = E ; & totus ang. GAB *d* = ECD; & totus GHB *d* = EFD. Polygona igitur sibi mutuo æquiangula sunt. Porro ob trigona æquiangula,
e 4. 6. AB. BH *e* :: CD. DF. & AG. GH. *e* :: CE. EF. item AG. AH. *e* :: CE. CF. & AH. AB *e* ::
f 22. 5. CF. CD. *f* unde ex æquo AG. AB :: CE. CD.
g 6. def. 6. eodem modo GH HB :: EF. FD. *g* ergo polygona ABHG, CDFE similia similiterque posita existunt. Q. E. F.

PROP. XIX.



Similia triangula A B C, D E F *sunt in duplicata ratione laterum homologorum* B C E F.

a 11. 6. *a* Fiat B C. E F :: E F. B G. & ducatur A G.

Quia

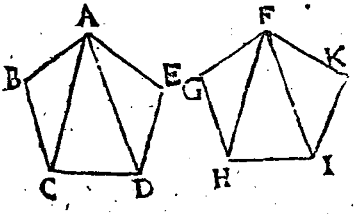
Quia AB. DE b :: BC. EF c :: EF. BG. & ang. B = E; d erit triang. ABG = DEF. verum triang. ABC. ABG e :: BC. BG; & f $\frac{BC}{BG} = \frac{BC}{EF}$ bis; ergo triang. $\frac{ABC}{ABG}$ hoc est $\frac{ABC}{DEF}$ g = $\frac{BC}{EF}$ bis. Q. E. D.

b cor. 4. 6. c constr. d 15. 6. e 1. 6. f 10. def. 5. g 11. 5.

Coroll.

Hinc, si tres lineæ BC, EF, BG proportionales fuerint; ut est prima ad tertiam, ita est triangulum super primam BC descriptum ad triangulum super secundam EF simile similiterque descriptum. vel ita est triangulum super secundam EF descriptum ad triangulum super tertiam simile similiterque descriptum.

PROP. XX.



Similia polygona ABCDE, FGHIK *in similia triangula* ABC, FGH; & ACD, FHI; & ADE, FIK *dividuntur*, & *numero æqualia*, & *homologa totis*. (ABC. FGH :: ABCDE. FGHIK :: ACD. FHI :: ADE. FIK.) *Et polygona* ABCDE, FGHIK *duplicatam habent eam inter se rationem, quam latus homologum* BC *ad homologum latus* GH.

1. Nam

a hyp. b 6.6. 1. Nam ang. B $a =$ G; & A B. BC a :: F G. G H. b ergo triangula ABC, FGH æquiangula sunt. eodem modo, triangula A E D, F K I assimilantur. cum igitur ang. BCA $b =$ GHF; & ang. ADE $b =$ FIK; totique anguli BCD, GHI; atque toti CDE, HIK c pares sint, d remanent ang. ACD $=$ FHI; & ang. ADC $=$ FIH; e unde etiam ang. CAD $=$ HFI. ergo triangula ACD, FHI similia sunt. ergo, &c.

c hyp. d 3.ax. e 32. 1.

f 19 6. 2. Quoniam igitur triangula B C A, G H F similia sunt; f erit $\frac{BCA}{GHF} = \frac{BC}{GH}$ bis. ob eandem causam $\frac{CAD}{HFI} = \frac{CD}{HI}$ bis. denique triang. $\frac{DEA}{IKF} = \frac{DE}{IK}$ bis. quare cum B C. G H g :: C D. H I g :: DE. IK, h erit triang. B C A. G H F :: C A D. H F I :: D E A. I K F :: k polyg. A B C D E. FGHIK :: $\frac{BC}{GH}$ bis.

g hyp. & 16. 5. h sch. 23 5. k 12. 5.

Coroll.

I. Hinc, si fuerint tres lineæ rectæ proportionales; ut est prima ad tertiam, ita erit polygonum super primam descriptum ad polygonum super secundam simile similiterque descriptum. vel ita erit polygonum super secundam descriptum ad polygonum super tertiam simile similiterque descriptum.

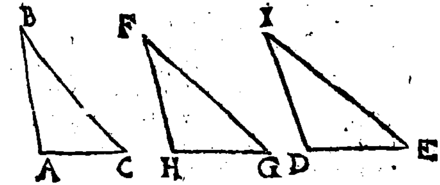
Unde elicitur methodus *figuram quamvis rectilineam augendi vel minuendi in ratione data. Ut si velis pentagoni, cujus latus* C D, *aliud facere quintuplum. inter* AB, *&* 5 AB *inveni mediam proportionalem. Super hac* * *construe pentagonum simile dato. hoc erit quintuplum dati.*

* 18. 6.

II. Hinc etiam, si figurarum similium homologa latera nota fuerint, etiam proportio figurarum innotescet; nempe inveniendo tertiam proportionalem.

PROP.

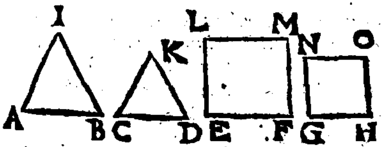
PROP. XXI.



Quæ (ABC, DIE) *eidem rectilineo* HFG *sunt similia, & inter se sunt similia.*

Nam ang. A $\overset{a}{=}$ H $\overset{a}{=}$ D. & ang. C $\overset{a}{=}$ G $\overset{a}{=}$ E; & ang. B $\overset{a}{=}$ F $\overset{a}{=}$ I. a item AB. AC :: HF. HG :: DI. DE. a & AC. CB :: HG. GF :: DE. EI. & AB. BC :: HF. FG :: DI. IE. a ergo ABC, DIE similia sunt. Q.E.D. a 1. def. 6.

PROP. XXII.



Si quatuor rectæ lineæ proportionales fuerint (AB. CD :: EF. GH.) *& ab eis rectilinea similia similiterque descripta proportionalia erunt.* (ABI. CDK :: EM. GO.) *Et si à rectis lineis similia similiterque descripta rectilinea proportionalia fuerint* (ABI. CDK :: EM. GO) *ipsæ etiam rectæ lineæ proportionales erunt.* (AB. CD :: EF. GH.)

1. *Hyp.* $\frac{ABI}{CDK} \overset{a}{=} \frac{AB}{CD}$ bis $\overset{a}{=} \frac{EF}{GH}$ bis $\overset{a}{=} \frac{EM}{GO}$. a 19. 6.

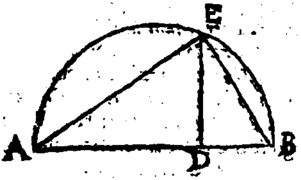
d ergo ABI. CDK :: EM. GO. Q.E.D.

2. *Hyp.* $\frac{AB}{CD}$ bis $\overset{a}{=} \frac{ABI}{CDK} \overset{b}{=} \frac{FM}{GO} \overset{c}{=} \frac{EF}{GH}$ bis. ergo AB. CD :: EF. GH. Q.E.D. b hyp. c 20. 6. d cor. 23. 5.

Schol.

Hinc deducitur, & demonstratur ratio multiplicandi quantitates surdas. ex gr. Sit $\sqrt{}$ 5 multiplicandus in $\sqrt{}$ 3. dico provenire $\sqrt{}$ 15. Nam ex multiplicationis definitione debet esse, 1. $\sqrt{}$ 3 :: $\sqrt{}$ 5. product. ergo per hanc, q. 1. q. $\sqrt{}$ 3 :: q. $\sqrt{}$ 5. q. product. hoc est. 1. 3 :: 5. q. product. ergo q. product. est 15. quare $\sqrt{}$ 15 est productus ex $\sqrt{}$ 3 in $\sqrt{}$ 5. Q. E. D.

THEOR.



Petr. Herig. *Si recta linea* A B *secta sit utcunque in* D, *rectangulum sub partibus* A D, D B *contentum, est medium proportionale inter earum quadrata. Item rectangulum contentum sub tota* A B, *& una parte* A D, vel D B, *est medium proportionale inter quadratum totius* A B, *& quadratum dictæ partis* A D, vel D B.

Super diametrum A B describe semicirculum. ex D erige normalem DE occurrentem peripheriæ in E. junge AE, BE.

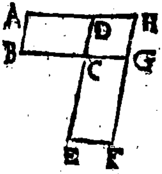
a cor. 8 6. b 22. 6 c 17 6. Liquet esse A D. D E a :: D E. D B. b ergo A Dq. D Eq :: D Eq. D Bq. c hoc est, A Dq. ADB :: ADB. DBq. Q. E. D.

d cor. 8. 6. e 22. 6. f 17. 6. Porro, B A. A E d :: A E. A D. e ergo BAq. AEq :: AEq. ADq. f hoc est B Aq. BA D :: B A D. ADq. Eodem modo ABq. ABD :: ABD. BDq. Q. E. D.

a 1. 6. Vel sic; sit Z = A + E. liquet esse Aq. AE :: a A. E :: a AE. Eq. item Zq. ZA :: a Z. A. :: a ZA. Aq. & Zq. ZE :: a Z. E :: ZE. Eq.

PROP.

PROP. XXIII.



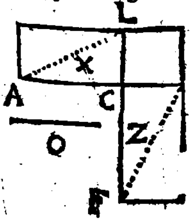
Æquiangula parallelogramma AC, CF *inter se rationem habent eam quæ ex lateribus componitur.* $\left(\frac{AC}{CF} = \frac{BC}{CG} + \frac{DC}{CE}.\right)$

Latera circa æquales angulos C *a* sibi in directum statuantur; & compleatur parallelogrammum CH. *a sch. 15.*

Ratio $\frac{AC}{CF}$ *b* $= \frac{AC}{CH} + \frac{CH}{CF}$ *c* $= \frac{BC}{CG} + \frac{DC}{CE}$. Q.E.D. *b 20 def. 5. c 1. 6.*

Coroll.

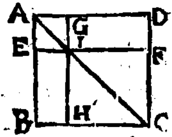
Hinc & ex 34. 1. patet primo, *Triangula, quæ unum angulum* (ad C) *æqualem habent, rationem habere ex rationibus rectarum*, AC ad CB, & LC ad CF, *equalem angulum continentium.* *Andr. Tacq. 15. 5.*



Patet secundo, *Rectangula ac * proinde & parallelogramma quæcunque rationem inter se habere compositam ex rationibus basis ad basim, & altitudinis ad altitudinem.* Neque aliter de triangulis ratiocinaberis. ** 35. 1.*

Patet tertio; *quomodo triangulorum ac parallelogrammorum proportio exhiberi possit.* Sunto parallelogramma X & Z; quorum bases AC, CB; altitudines vero CL, CF. Fiat CL. CF :: CB. Q. * erit X. Z :: AC. Q. ** 14. 6. & 1. 6.*

PROP. XXIV.

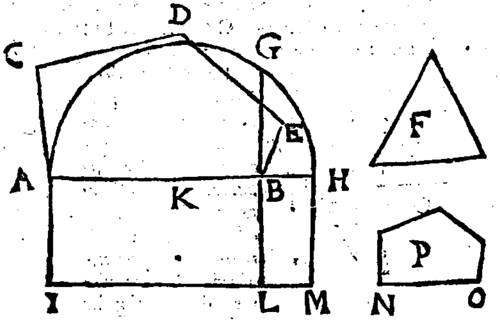


In omni parallelogrammo ABCD, *quæ circa diametrum* AC *sunt parallelogramma* EG, HF, *& toti & inter se sunt similia.*

Nam parallelogramma EG, HF habent singula unum angulum cum toto communem. *a* ergo toti & sibi mutuo æquiangula sunt. *a* Item tam triangula ABC, AEI, IHC, quam triangula ADC, AGI, IFC sunt inter se æquiangula. *b* ergo AE. EI :: AB. BC, *b* atque AE. AI :: AB. AC; *b* & AI. AG :: AC. AD. *c* ex æquali igitur, AE. AG :: AB. AD. *d* ergo Pgra. EG, BD similia sunt. eodem modo HF, BD similia sunt. ergo, &c.

a 29. 1.
b 4. 6.
c 22. 5.
d 1 def. 6.

PROP. XXV.



Dato rectilineo ABEDC *simile similiterque positum* P, *idemque alteri dato* F *æquale, constituere.*

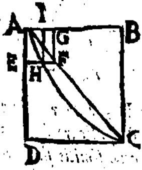
a Fac rectang. AL = ABEDC. *b* item super BL fac triang. BM = F. Inter AB, BH *c* inveni mediam proportionalem NO. super NO *d* fac

a 45. 1.
b 44. 1.
c 13. 6.

^d fac polygonum P simile dato ABEDC. Erit hoc æquale dato F, d 18 6. e cor. 20. 6.

Nam ABEDC (AL.) P :: ^e AB. BH ^f :: AL. BM. ergo P ^g = BM ^h = F. Q. E. F. f 1. 6. g 14. 5. h constr.

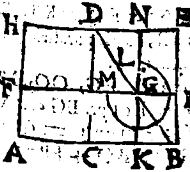
Prop. XXVI.



Si à parallelogrammo ABCD *parallelogrammum* AGFE *ablatum sit, & simile toti, & similiter positum, communem cum eo habens angulum* EAG, *hoc circa eandem cum toto diametrum* AC *consistet.*

Si negas AC esse communem diametrum, esto diameter AHC secans EF in H. & ducatur HI parall. AE. Parallelogramma EI, DB ^a similia sunt. ^b ergo AE, EH :: AD. DC ^c :: AE. EF. ^d proinde EH = EF. ^f Q. E. A. a 24. 6. b 1. def. 6. c hyp. d 9. 5. f 9. ax.

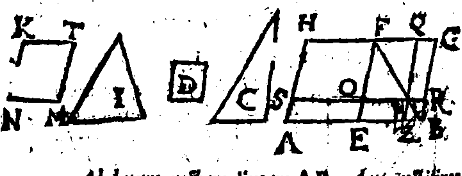
Prop. XXVII.



Omnium parallelogrammorum AD, AG *secundum eandem rectam lineam* AB *applicatorum, deficientiumque figuris parallelogrammis* CE, KI *similibus, similiterque positis, ei* AD, *quod à dimidia describitur, maximum est* AD, *quod ad dimidium est applicatum, simile exsistens defectui* KI.

Nam quia GE ^a = GC, addito communi KI, ^b erit KE = CI ^c = AM. adde commune CG, ^d erit AG = Gnom. MBL. sed Gnom. MBL ^e $\sqsupset$ CE (AD.) ergo AG $\sqsupset$ AD. Q. E. D. a 43. 1. b 2. ax. c 36. 1. d 2. ax. e 9. ax.

PROP. XXVIII.



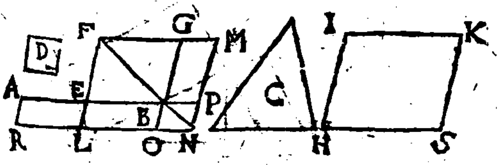
Ad datum rectam lineam AB, *dato rectilineo* C *æquale parallelogrammum* AP *applicare deficiens figura parallelogramma* ZR, *quæ ſimilis ſit alteri parallelogrammo dato* D. * *Oportet autem datum*
27.6. *rectilineum* C, *cui æquale* AP *applicandum eſt, non majus eſſe eo* AF, *quod ad dimidiam applicatur, ſimilibus exſiſtentibus defectibus, & ejus* AF *quod ad dimidiam applicatur, & ejus* D, *cui ſimile deeſſe debet.*

a 28.6. Biſeca AB in E. Super EB *a* fac Pgr. EG
b ſch.45.1. ſimile dato D. *b* ſitque EG = C + I. *c* fac pgr.
c 25.6. NT = I, & ſimile dato D, vel EG. duc diametrum FB. fac FO = KN; & FQ = KT. Per O, & Q duc parallelas SR, QZ. parallelogrammum AP eſt id quod quæritur.

Nam parallelogramma D, EG, OQ, NT,
d conſtr. & 24.6 ZR *d* ſunt ſimilia inter ſe. Et Pgr. EG *e* = NT
e conſtr. + C *e* = OQ + C; *f* quare C = Gnom.
f 3.ax. OBQ *g* = AO + PG *h* = AO + EP = AP.
g 2.ax.
h 43.1. Q. E. F.

PROP.

PROP. XXIX.

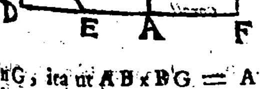


Ad datam rectam lineam AB, *dato rectilineo* C *æquale parallelogrammum* AN *applicare, excedens figura parallelogramma* OP, *quæ similis sit parallelogrammo alteri dato* D.

Biseca AB in E. super EB *a* fac Pgr. EG simile dato D. *b* sitque pgr. HK = EG + C, & simile dato D vel EG. fac FEL *c* = IH; *c* & FGM = IK. per L M duc parallelas RN, MN: & AR parall. NM. Produc ABP, GBO. Duc diametrum FBN. Pgr. AN est quæsitum. a 18 6. b 25. 6. c 3. 1.

Nam parallelogramma D, HK, LM, EG *d* similia sunt. *e* ergo pgr. OP simile est pgro LM, vel D. item LM *f* = HK *f* = EG + C. *g* ergo C = Gnom. ENG. atqui AL *h* = LB *k* = BM. *l* ergo C = AN. Q. E. F. d constr. e 24. 6. f constr. g 3 ax. h 36. 1 k 43. 1. l 2. & 1. ax

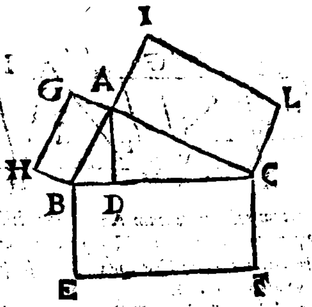
PROP. XXX.



Propositam rectam lineam terminatam AB, *extrema ac media ratione secare.* (AB. AG :: AG. GB.)

a Seca AB in G, ita ut AB x BG = AGq. *b* ergo BA. AG :: AG. GB. Q. E. F. a 11. 2. b 17. 6.

PROP. XXXI.



In rectangulis triangulis B A C, *figura quævis* B F *à latere* B C *rectam angulum* B A C *subtendente, descripta, æqualis est figuris* BG, AL, *quæ priori illi* B F *similes, & similiter positæ à lateribus* B A, A C *rectum angulum continentibus describuntur.*

Ab angulo recto B A C demitte perpendicularem A D. Quoniam C B. C A *a* :: C A. D C *b* erit B F. A L :: C B. D C; inverseque A L. BF :: DC. CB. Item quia BC, BA *a* :: BA. DB *b* erit BF. BG :: BC, DB; ac invertendo, BG. BF :: DB. BC. *c* ergo AL + BG. BF :: DC + DB. BC. *d* ergo AL + BG = BF. Q. E. D.

a cor. 8. 6.
b cor. 20. 6.
c 24. 5.
d schol. 14. 5.
e 22. 6.

Vel sic. BG. BF *e* :: BAq. BCq. *e* & AL. BF :: ACq. BCq. *f* ergo B G + A L. B F :: BAq + ACq. BCq. *g* ergo cum BAq + ACq *h* = BCq. *h* erit BG + AL = BF. Q. E. D.

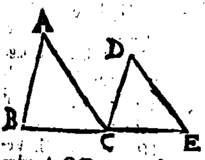
f 24. 5.
g sch. 14. 5.
h 47. 1.

Coroll.

Ex hac propositione, addi possunt, & subtrahi figuræ quævis similes, eadem methodo, qua quadrata adduntur & subtrahuntur, in schol. 47. 1.

PROP.

PROP. XXXII.

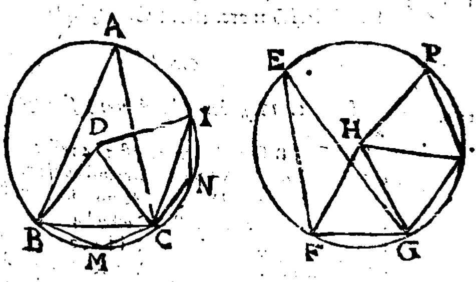


Si duo triangula ABC, DCE, *quæ duo latera duobus lateribus proportionalia habeant* (AB. AC :: DC. DE,) *secundum unum angulum* ACD *composita fuerint, ita ut homologa eorum latera sint etiam parallela* (AB ad DC, & AC ad DE) *tum reliqua illorum triangulorum latera* BC, CE *in rectam lineam collocata reperientur.*

Nam ang. A a = ACD a = D; & AB. AC b :: DC. DE. c ergo ang. B = DCE. ergo ang. B + A d = ACE. sed ang. B + A + ACB e = 2 Rect. f ergo ang. ACE + ACB = 2 Rect. g ergo BCE est recta linea. Q. E. D.

a 19. 1.
b hyp.
c 6. 6.
d 2. ax.
e 32. 1.
f 1. ax.
g 14. 1.

PROP. XXXIII.



In æqualibus circulis DBCA, HFGP, *anguli* BDC, FHG *eandem habent rationem cum peripheriis* BC, FG, *quibus insistunt; sive ad centra* (ut BDC, FHG,) *sive ad peripherias* A, E *constituti insistant: insuper vero & sectores* BDC, FHG, *quippe qui ad centra consistant.*

Duc

Duc rectas BC, FG. Accommoda CI = CB; & GL = FG = LP; & junge DI, HL, HP.

Arcus BC *a* = C I, *a* item arcus FG, GL, LP
a 28. 3. b 17. 3. æquantur. *b* ergo ang. BDC = CDI, *b* & ang. FGH = GHL = LHP. Ergo arcus BI tam multiplex est arcus BC, quam ang. BDI anguli BDC. pariterque æquemultiplex est arcus FP arcus FG, atque ang. FHP anguli FHG. Verum si arcus BI ⊏, =, ⊐ FP, *c* erit similiter
c 27. 3. d 6. def. 5. e 15. 5. f 20. 3. ang. BDI ⊏, =, ⊐ FHP. ergo arc. BC. FG *d* :: ang. BDC. FHG *e* :: BDC. FHG *f* :: A. E. Q. E. D.

Rursus ang. BMC *g* = CNI; *h* atque idcirco
g 27. 3. h 24. 3. k 4. 1. l 2 ax. segm. BCM = CIN. *k* item triang. BDC = CDI. *l* ergo sector BDCM = CDIN. Simili ratione sectores FHG, GHL, LHP æquantur. Quum igitur prout arcus BI ⊏, =, ⊐ FGP, ita similiter sector BDI ⊏, =, ⊐ FHP. *m* erit sect.
m 6. def. 5. BDC. FHG :: arc. BC. FG. Q. E. D.

Coroll.

Hinc 1. *Vt sector ad sectorem, sic angulus ad angulum.*

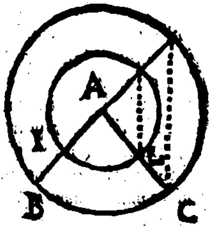
2. *Ang.* B D C *in centro est ad 4 rectos, ut arcus* BC *cui insistit ad totam circumferentiam.*

Nam ut ang. B D C ad rectum, sic arcus B C ad quadrantem. ergo BDC est ad 4 rectos, ut arcus B C ad 4 quadrantes, id est ad totam circumferentiam. item ang. A. 2 Rect :: arc. B C. periph.

Hinc 3. *Inæqualium circulorum arcus* IL, BC, *qui æquales subtendunt angulos, sive ad centra, ut* IAL & BAC, *sive ad peripheriam, sunt similes.*

Nam I L. periph. :: ang. IAL, (BAC.) 4 Rect. item arc. B C. periph :: ang. B A C. 4. Rect.

4. Rest. ergo IL. periph :: BC, periph. proinde arcus IL, & BC sunt similes. Unde



4. *Duæ semidiametri* A B , A C *à concentricis peripheriis arcus auferunt similes* IL, BC.

LIB.

LIB. VII.

Definitiones.

I. Unitas est, secundum quam unumquodque eorum quæ sunt, unum dicitur.

II. Numerus autem est, ex unitatibus composita multitudo.

III. Pars est numerus numeri, minor majoris, quum minor metitur majorem.

Omnis pars ab eo numero nomen sibi sumit, per quem ipsa numerum, cujus est pars, metitur; ut 4 dicitur tertia pars numeri 12, quia metitur 12 per 3.

IV. Partes autem, cum non metitur.

Partes quæcunque nomen accipiunt à duobus illis numeris, per quos maxima communis duorum numerorum mensura utrumque eorum metitur. ut 10 dicitur $\frac{2}{3}$ numeri 15, eo quod maxima communis mensura, nempe 5, metitur 10 per 2, & 15 per 3.

V. Multiplex vero major minoris, cum majorem metitur minor.

VI. Par numerus est, qui bifariam dividitur.

VII. Impar vero numerus, qui bifariam non dividitur; vel, qui unitate differt à pari.

VIII. Pariter par numerus est, quem par numerus metitur per numerum parem.

IX. Pariter autem impar est, quem par numerus metitur per numerum imparem.

X. Impariter vero impar numerus est, quem impar numerus metitur per numerum imparem.

XI. Primus numerus est, quem sola unitas metitur.

XII. Primi inter se numeri sunt, quos sola unitas, communis mensura, metitur.

XIII.

XIII. Compositus numerus est, quem numerus quispiam metitur.

XIV. Compositi autem inter se numeri sunt, quos numerus aliquis communis mensura metitur.

In hac definitione & praecedenti unitas non est numerus.

XV. Numerus numerum multiplicare dicitur, cum toties compositus fuerit is qui multiplicatur, quot sunt in ipso multiplicante unitates, & procreatus fuerit aliquis.

Hinc, in omni multiplicatione unitas est ad multiplicatorem ut multiplicatus ad productum.

Nota, quod saepe cum multiplicandi sunt quivis numeri, puta A *in* B, *literarum conjunctio productum denotat. Sic* AB = A *in* B. *item* CDE = C *in* D *in* E.

XVI. Cum autem duo numeri sese multiplicantes aliquem fecerint, qui factus erit, planus appellabitur; Qui vero numeri sese mutuo multiplicarint, latera illius dicentur. *Sic* 2 (C) in 3 (D) = 6 = CD *est numerus planus.*

XVII. Cum vero tres numeri mutuo sese multiplicantes fecerint aliquem, qui procreatus erit, solidus appellabitur; Qui autem numeri mutuo sese multiplicarint, latera illius dicentur. *Sic,* 2 (C) *in* 3 (D) *in* 5 (E) = 30 = CDE *est numerus solidus.*

XVIII. Quadratus numerus est, qui aequaliter aequalis, vel qui sub duobus aequalibus numeris continetur. *Sit* A *latus quadrati; quadratus sic notatur*, AA, *vel* Aq.

XIX. Cubus vero, qui aequaliter aequalis aequaliter, vel qui sub tribus aequalibus numeris continetur. *Sit* A *latus cubi; cubus notatur sic*, AAA, *vel* Ac.

In hac definitione, & tribus praecedentibus, unitas est numerus.

XX. Nu-

XX. Numeri proportionales sunt, cum primus secundi, & tertius quarti æquemultiplex est, vel eadem pars; vel deniq; cum pars primi secundum, & eadem pars tertii æque metitur quartum, vel vice versa. A. B :: C. D. hoc est, 3. 9 :: 5. 15.

XXI. Similes plani, & solidi numeri sunt, qui proportionalia habent latera.

Latera nempe non quælibet, sed quædam.

XXII. Perfectus numerus est, qui suis ipsius partibus est æqualis.

Ut 6. & 28. *Numerus vero qui suis ipsius partibus minor est, abundans appellatur; qui vero major, diminutus. ut 12 est abundans, 15 est diminutus.*

XXIII. Numerus numerum metiri dicitur per illum numerum, quem multiplicans, vel à quo multiplicatus, illum producit.

In divisione, unitas est ad quotientem, ut dividens ad divisum. Nota, quod numerus alteri lineola interjecta subscriptus divisionem denotat. Sic $\frac{A}{B}$ = A divis. per B. *item* $\frac{CA}{B}$ = C *in* A divis. per B.

Termini sive radices proportionis dicuntur duo numeri, quibus in eadem proportione minores sumi nequeunt.

Postulata.

1. POstuletur, cuilibet numero quotlibet sumi posse æquales, vel multiplices.

2. Quolibet numero sumi posse majorem.

3 Additio, subtractio, multiplicatio, divisio, extractionesque radicum, seu laterum, numerorum quadratorum, & cuborum concedantur etiam, tanquam possibilia.

Axiomata.

1. Quicquid convenit uni æqualium numerorum, convenit & reliquis æqualibus numeris.

2. Partes eidem parti, vel iisdem partibus, eædem, sunt quoque inter se eædem.

3. Qui numeri æqualium numerorum, vel ejusdem, eædem partes fuerint, æquales inter se sunt.

4. Quorum idem numerus, vel æquales, eædem partes fuerint, æquales inter se sunt.

5. Unitas omnem numerum per unitates, quæ in ipso sunt, hoc est, per ipsummet numerum metitur.

6. Omnis numerus seipsum metitur per unitatem.

7. Si numerus numerum multiplicans, aliquem produxerit, metietur multiplicans productum per multiplicatum, multiplicatus autem eundem per multiplicantem.

Hinc nullus numerus primus planus est aut solidus, quadratus, vel cubus.

8. Si numerus numerum metiatur, & ille per quem metitur, eundem metietur per eas, quæ in metiente sunt, unitates, hoc est, per ipsum numerum metientem.

9. Si numerus numerum metiens, multiplicet eum, per quem metitur, vel ab eo multiplicetur, illum quem metitur, producit.

10. Numerus quotcunque numeros metiens, compositum quoque ex ipsis metitur.

11. Numerus quemcunque numerum metiens, metitur quoque omnem numerum quem ille metitur.

12. Numerus metiens totum & ablatum, metitur & reliquum.

PROP.

PROP. I.

A.....E..G.B 8 5 3
C...F..D $\frac{5}{3}$ $\frac{3}{2}$ $\frac{2}{1}$
H---

Si duobus numeris inæqualibus propositis (AB, CD) *detrahatur semper minor* CD *de majore* AB (& reliquus EB de CD &c.) *alterna quadam detractione, neque reliquus unquam præcedentem metiatur, quoad assumpta sit unitas* GB; *qui principio propositi sunt numeri* AB, CD *primi inter se erunt.*

Si negas, habeant AB, CD communem mensuram, numerum H. Ergo H metiens CD;
a 11 ax.7. *a* etiam AE metitur; proinde & reliquum FB;
b 12.ax.7. *a* ergo & CF, atque *b* idcirco reliquum FD; *a* quare & ipsum EG. sed totum EB metiebatur; *b* ergo & reliquum GB metitur, numerus unitatem.
c 9 ax.1. *c* Q.E.A.

PROP. II.

9 6
A......... E....... B 15 9 6
6 3
C...... F... D $\frac{9}{6}$ $\frac{6}{3}$ $\frac{3}{6}$
G---

Duobus numeris datis AB, CD *non primis inter se, maximam eorum communem mensuram* FD *reperire.*

Detrahe minorem numerum CD ex majori
a 6.ax 7. AB, quoties potes. Si nihil relinquitur, *a* patet ipsum CD esse maximam communem mensuram. Si relinquitur aliquid EB, deme hunc ex CD; & reliquum FD ex EB, & sic deinceps, donec aliquis FD præcedentem EB metiatur.
b 1.7. (nam *b* hoc fiet antequam ad unitatem perveniatur.) Erit FD maxima communis mensura.
c constr. Nam FD *c* metitur EB, *d* ideoque & CF;
d 11.ax 7.
e 12.ax.7. *e* proinde & totum CD; *d* ergo ipsum AE; atque idcirco totum AB metitur. Liquet igitur FD communem esse mensuram. Si maximam esse negas,

gas, sit major quæpiam G. ergo G metiens CD, *d* metitur A E, *e* & reliquum E B, *d* ipsumque C F. *e* proinde & reliquum F D, *g* major minorem. *h* Q. E. A. *g suppos.* *h 9 ax. 1.*

Coroll.

Hinc, numerus metiens duos numeros, metitur quoque maximam eorum communem mensuram.

PROP. III.

A 12
B 8
D 4
C 6
E ... 2
F - - -

Tribus numeris datis A, B, C *non primis inter se, maximam eorum communem mensuram* E *reperire.*

Inveni D maximam communem mensuram duorū A, B. Si D metitur tertium C, liquet D maximam esse trium communem mensuram. Si D non metitur C, erunt saltem D, & C compositi inter se, ex coroll. præcedentis. Sit igitur ipsorum D, & C maxima communis mensura E. erit E is quem quæris.

Nam E *a* metitur C, & D; *a* ac D ipsos A, & B metitur; *b* ergo E metitur singulos A, B, C; nec major aliquis (F) eos metietur; nam si hoc affirmas, *c* ergo F metiens A, & B, eorum maximam communem mensuram D metitur. Eodem modo, F metiens D, & C, *c* eorum maximam communem mensuram E, *d* major minorem, metitur. *e* Q. E. A. *a constr.* *b 11. ax. 7.* *c cor. 1. 7.* *d suppos.* *e 9 ax. 1.*

Coroll.

Hinc, numerus metiens tres numeros, maximam quoque eorum communem mensuram metitur.

PROP. IV.

A 6
B 7
B 18
B .... 9.

Omnis numerus A; omnis numeri B minor majoris, aut pars est, aut partes.

a 4 def. 7.
b 3. def. 7.
c 4. def. 7.

Si A & B primi sint inter se, *a* erit A tot partes numeri B, quot sunt in A unitates. (ut $6 = \frac{6}{7} 7$.) Sin A metiatur B, *b* liquet A esse partem ipsius B. (ut $6 = \frac{1}{3} 18$.) denique si A & B aliter compositi inter se fuerint, *c* maxima communis mensura determinabit, quot partes A conficiat ipsius B; ut $6 = \frac{2}{3} 9$.

PROP. V.

A ...... 6 D 4
6 6 4 4
B ...... G ...... C 12. E .... H .... F 8

Si numerus A numeri B C pars fuerit, & alter D alterius E F eadem pars; & simul uterque (A + D) utriusque simul (B C + E F) eadem pars erit, quæ unus A unius B C.

a hyp.
b conft & 2. ax. 1.
c 2. ax. 1.

Nam si B C in suas partes B G, G C ipsi A æquales; atque E F in suas partes F H, H F ipsi D æquales resolvantur; *a* erit numerus partium in B C æqualis numero partium in E F. Quum igitur A + D *b* = BG + EH = GC + HF, erit A + D toties in B C + E F, quoties A in B C. Q. E. D.

Vel sic brevius. Sit $a = \frac{x}{2}$ & $b = \frac{y}{2}$. *c* ergo

$$a + b = \frac{x}{2} + \frac{y}{2} = \frac{x + y}{2}. \text{ Q. E. D.}$$

Prop. VI.

3 3 4 4

A...G...B 6. D.... H.... E 8

C........ 9 F............ 12

Si numerus AB *numeri* C *partes fuerit; & alter* DE *alterius* F *eædem partes; & simul uterq; (AB+DE) utriusq; simul (C+F) eædem partes erit, quæ unus* AB *unius* C.

Divide AB in suas partes AG, GB; & DE in suas DH, HE. Partium in utroque AB, DE æqualis est multitudo, ex hypoth. Quum igitur AG *a* sit eadem pars numeri C, quæ DH numeri F, *b* erit AG + DH eadem pars compositi C + F, quæ unus AG unius C. *b* Eodem modo GB + HE eadem pars est ejusdem C + F, quæ unus GB unius C; *c* ergo AB + DE eædem partes est ipsius C + F, quæ AB ipsius C. Q. E. D.

a hyp. b 5.7. c 2.ax 7.

Vel sic. Sit $a = \frac{2}{3}x$. & $b = \frac{2}{3}y$. *d* ergo $a + b = \frac{2}{3}x + \frac{2}{3}y = \frac{2}{3}y + \frac{2}{3}x$. Q. E. D.

d 2.ax. 1.

Prop. VII.

5 3

A..... E... B 8

6 10 6

G...... C........ F..... D 16

Si numerus AB *numeri* CD *pars fuerit, qualis ablatus* AE *ablati* CF; *& reliquus* EB *reliqui* FD *eadem pars erit, qualis totus* AB *totius* CD.

a Sit EB eadem pars numeri GC, quæ AB ipsius CD, vel AE ipsius CF. *b* ergo AE + EB eadem est pars ipsius CF + GC, quæ AE ipsius CF, vel AB ipsius CD. *c* ergo GF = CD. aufer communem CF, *d* manet GC = FD. *e* ergo EB eadem est pars reliqui FD (GC) quæ totus AB totius CB. Q. E. D.

a 1. post.7. b 5.7. c 6 ax 1. d 3.ax.1. e 2.ax 7.

Vel sic. Sit $a + b = x$, & $c + d = y$; atque tam $x = 3y$, quam $a = 3c$; dico $b = 3d$. Nam $3c + 3d$ *f* $= 3y = x$ *g* $= a + b$. aufer utrinq; $3c$ *g* $= a$, & *h* remanet $3d = b$. Q. E. D.

f 1. 2. g hyp.

PROP. VIII.

6 2 4 2 2
A...... H.. G.... E.. L.. B 16
18 6
C................ F...... D 24

Si numerus AB *numeri* CD *partes fuerit, quales ablatus* AE *ablati* CF; *& reliquus* EB *reliqui* FD *eadem partes erit, quales totus* AB *totius* CD.

Seca AB in AG, GB partes numeri CD; item AE in AH, HE partes numeri CF; & sume GL = AH = HE; *a* quare HG = EL. & quia *b* AG = GB, *c* etiam HG = LB. Cum igitur totus AG eadem sit pars totius CD, quæ ablatus AH ablati CF; *d* erit reliquus HG, vel EL, eadem etiam pars reliqui FD, quæ AG ipsius CD. Eodem pacto, quia GB eadem pars est totius CD, quæ HE, vel GL, ipsius CF, *d* erit reliquus LB eadem pars reliqui FD, quæ GB totius CD; ergo EL + LB (EB) eædem est partes reliqui FD, quæ totus AB totius CD. Q. E. D.

a 3. ax. 1.
b constr.
c 3. ax. 1.
d 7. 7.

Vel sic facilius. Sit $a + b = x$. & $c + d = y$. Item tam $y = \frac{2}{3} x$, quam $c = \frac{2}{3} a$; vel *e* quod idem est, $3\ y = 2x$; & $3\ c = 2\ a$. Dico $d = \frac{2}{3} b$. Nam $3\ c + 3\ d$ *f* $= 3\ y = 2\ x$ *f* $= 2\ a + 2\ b$. *g* ergo $3\ c + 3\ d = 2\ a + 2\ b$. aufer utrinque $3\ c$ *h* $= 2\ a$; & *k* manet $3\ d = 2\ b$. *l* ergo $d = \frac{2}{3} b$. Q. E. D.

e 9. ax. 7.
f 1. 2.
g 1. ax. 1.
h hyp.
k 3. ax. 1.
l 8. ax. 7.

PROP. IX.

A 4
4 4
B..... G..... C 8
5 D 5
E..... H..... F 10

Si numerus A *numeri* BC *pars fuerit, & alter* D *alterius* EF *eadem pars; & vicissim quæ pars est, aut partes primus* A *tertii* D, *eadem pars erit, vel eædem partes, & secundus* BC *quarti* EF.

Posi-

Ponitur A ⊐ D. Sint igitur BG, GC, & EH, HF partes numerorum BC, EF, hæ ipsi A, illæ ipsi D pares. Utrinque multitudo partium æqualis ponitur. Liquet vero BG a eandem esse partem, aut easdem partes ipsius EH, quæ GC ipsius HF; b quare BC (BG + GC) ipsius EF (EH + HF) eadem pars est aut partes, quæ unus BG (A) unius EH (D.) Q. E. D. a 1. ax. 7. & 4 7. b 5, vel 6. 7.

Vel sic; Sit $a = \frac{b}{3}$. & $c = \frac{d}{3}$. dico $\frac{c}{a} = \frac{d}{b}$. Nam $\frac{c}{a} = \frac{3d}{3b} = \frac{d}{b}$. a 1. ax. 7.

PROP. X.

A .. G .. B 4	
C...... 6	
5 5	
D..... H..... E 10	
F.............. 15	

Si numerus AB numeri C partes fuerit, & alter DE alterius F eædem partes; & vicissim quæ partes est primus AB tertii DE, aut pars, eædem partes erit & secundus C quarti F, aut pars.

Ponitur AB ⊐ DE, & C ⊐ F. Sint AG, GB, & DH, HE partes numerorum C, & F, tot nempe in AB, quot in DE. Constat AG ipsius C eandem esse partem, quæ DH ipsius F. a quare vicissim AG ipsius DH, pariterque GB ipsius HE, & b proinde conjunctim AB ipsius DE eadem pars erit, aut partes, quæ C ipsius F. Q. E. D. a 9. 7. b 5, & 9. 7.

Applicare potes secundam præcedentis demonstrationem etiam huic.

PROP. XI.

4 3	
A.... E ... B 7	
8 6	
C........ F...... D 14	

Si fuerit, ut totus AB ad totum CD, ita ablatus AE ad ablatum CF; & reliquus EB ad reliquum

F D *erit, ut totus* A B *ad totum* C D.

a 4. 7. Sit primo A B ⊐ C D; *a* ergo A B vel pars
b 10. def. est, vel partes numeri C D; *b* eademque pars est,
c 7. vel 8. 7. vel partes ipse AE ipsius CF; *c* ergo reliquus EB reliqui F D eadem pars est, aut partes, quæ totus AB totius C D. *b* ergo A B. C D :: E B. F D. Sin fuerit A B ⊏ C D; eodem modo erit juxta modo ostensa, C D. A B :: F D. E B. ergo invertendo, AB. CD :: EB. FD.

PROP. XII.

A, 4. C, 2. E, 3.
B, 8. D, 4. F, 6.

Si sint quotcunque numeri proportionales (A. B :: C. D :: E. F) *erit quemadmodum unus antecedentium* A *ad unum consequentium* B, *ita omnes antecedentes* (A + C + E) *ad omnes consequentes* (B + D + F.)

Sint primo, A, C, E minores quam B, D, F.
a 20. def. 7. ergo (propter easdem rationes) *a* erit A eadem
b 5, & 6 7. pars aut partes ipsius B, quæ C ipsius D. *b* ergo conjunctim A + C eadem erit pars aut partes ipsius B + D, quæ unus A unius B. Similiter A + C + E eadem pars est, aut partes ipsius
c 20. def. 7. B + D + F, quæ A ipsius B. *c* ergo A + C + E. B + D + F :: A. B. Q. E. D. Sin A, C, E, ipsis B, D, F majores ponantur, idem ostendetur invertendo.

PROP. XIII.

A, 3. C, 4.
B, 5. D, 12.

Si quatuor numeri proportionales sint (A. B :: C. D. *& vicissim proportionales erunt* (A. C :: B. D.)

Sint primo A & C ipsis B & D minores,
a 20. def. 7. atque A ⊐ C. Ob eandem proportionem, *a* erit A eadem pars, aut partes ipsius B, quæ C ipsius
b 9. & 10. 7. D. *b* ergo vicissim A ipsius C eadem pars est, aut partes, quæ B ipsius D. ergo A. C :: B. D. Sin

A ⊏

A ⊏ C; atque A & C majores statuantur, quam B & D, eadem res erit, proportiones invertendo.

PROP. XIV.

A, 9.	D, 6.
B, 6.	E, 4.
C, 3.	F, 2.

Si sint quotcunque numeri A, B, C, *& alii totidem* D, E, F *illis æquales multitudine, qui bini sumantur, & in eadem ratione* (A. B :: D. E. & B. C :: E. F) *etiam ex æqualitate in eadem ratione erunt.* (A. C :: D F.)

Nam quia A. B :: D. E, *a* erit vicissim, A. D :: B. E :: *a* C. F. *a* ergo iterum permutando, A. C :: D. F. Q. E. D. a 13. 7.

PROP. XV.

1.	D.
B ... 3.	E ...... 6.

Si unitas numerum quempiam B *metiatur; æque autem alter numerus* D *alterum quendam numerum* E *metiatur; & vicissim æque unitas tertium numerum* D *metietur, & secundus* B *quartum* E.

Nam quia 1 est eadem pars ipsius B, quæ D ipsius E, *a* erit vicissim 1 eadem pars ipsius D, quæ B ipsius E. Q. E. D. a 9. 7.

PROP. XVI.

B, 4.	A, 3.
A, 3.	B, 4.
AB, 12.	BA, 12.

Si duo numeri A, B *sese mutuo multiplicantes fecerint aliquos* AB, BA, *geniti ex ipsis* AB, BA *æquales inter se erunt.*

Nam quia AB = A in B, *a* erit 1 in A toties, quoties B in AB. *b* ergo vicissim 1 in B toties erit, quoties A in AB. atqui quoniam BA = B in A, *a* erit 1 in B toties, quoties A in BA. ergo quoties 1 in AB, toties 1 in BA; & *c* proinde AB = BA. Q. E. D. a 15. def. 7. b 15. 7. c 4. ax. 7.

PROP. XVII.

A, 3.
B, 2. C, 4.
AB, 6. AC, 12.

Si numerus A *duos numeros* B,C *multiplicans fecerit aliquos* AB, AC; *geniti ex ipsis eandem rationem habebunt, quam multiplicati.* (AB. AC :: B. C.)

a 15. def. 7. Nam quia AB = A in B, *a* erit 1 toties in A, quoties B in AB. *a* item quia AC = A in C, erit 1 toties in A, quoties C in AC. ergo quo-
b 20. def. 7. ties B in AB, toties C in AC. quare B. AB ::
c 13. 7. C. AC. *c* ergo vicissim, B. C :: AB. AC. Q. E. D.

PROP. XVIII.

C, 5. C, 5.
A, 3. B, 9.
AC, 15. BC, 45.

Si duo numeri A, B, *numerum quempiam* C *multiplicantes fecerint aliquos* AC, BC; *geniti ex ipsis eandem rationem habebunt, quam multiplicantes.* (A. B :: AC. BC.)

a 16. 7. Nam AC *a* = CA; & BC *a* = CB; sic idem C multiplicans A & B producit AC, & BC.
b 17. 7. *b* ergo A. B :: AC. BC. Q. E. D.

Schol.

Ex his pendet modus vulgaris reducendi fractiones ($\frac{3}{5}$, $\frac{7}{9}$) ad eandem denominationem. Nam duc 9 tam in 3, quam in 5, proveniunt $\frac{27}{45} = \frac{3}{5}$. quoniam ex his, 3. 5 :: 27. 45. item duc 5 in 7, & 9, prodeunt $\frac{35}{45} = \frac{7}{9}$. quia 7. 9 :: 35. 45.

PROP. XIX.

A, 4. B, 6. C, 8. D, 12.
AD, 48. BC, 48.

Si quatuor numeri proportionales fuerint, (A. B :: C. D;) *qui ex primo & quarto fit numerus* AD, *æqualis est ei, qui ex secundo & tertio fit, numero* BC.

BC. *Et si qui ex primo & quarto fit numerus* AD, *æqualis sit ei, qui ex secundo & tertio fit, numero* BC, *ipsi quatuor numeri proportionales erunt.* (A. B :: C. D.)

1. *Hyp.* Nam AC. AD *a* :: C. D *b* :: A. B *c* :: AC. BC. *d* ergo AD = BC. Q. E. D. a 17. 7. b hyp. c 18. 7. d 9. 5.

2. *Hyp.* Quoniam *e* AD = BC, erit AC. AD *f* :: AC. BC. Sed AC. AD *g* :: C. D. & AC. BC *h* :: A. B. *k* ergo C. D. :: A. B. Q. E. D. e hyp. f 7. 5. g 7. 7. h 18. 7. k 11. 5.

PROP. XX.

A. B. C.
4. 6. 9.
AC, 36. BB, 36.
D, 6.

Si tres numeri proportionales fuerint (A. B :: B. C.) *qui sub extremis continetur* (AC) *æqualis est ei, qui à medio efficitur* (BB.) *Et si qui sub extremis continetur* (AC) *æqualis fuerit ei* (Bq) *qui sub medio, ipsi tres numeri proportionales erunt* $\left(\frac{A}{B} :: \frac{B}{C}\right)$

1. *Hyp.* Nam sume D = B. *a* ergo A. B :: D (B.) C. *b* quare AC = BD, *a* vel BB. Q. E. D. a 1. ax. 7. b 19. 7.

2. *Hyp.* Quia AC *c* = BD, *d* erit A. B :: D (B.) C. Q. E. D. c hyp. d 19. 7.

PROP. XXI.

A.. G.. B 5. E .... 10.
C.. H. D 3. F ...... 6.

Numeri AB. CD *minimi omnium eandem cum eis rationem habentium* (E, F) *metiuntur æque numeros* E, F *eandem cum eis rationem habentes, major quidem* AB *majorem* E, *minor vero* CD *minorem* F.

Nam AB. CD *a* :: E. F. *b* ergo vicissim AB. E :: CD. F. *c* ergo AB eadem pars est, vel partes ipsius E, quæ CD ipsius F. Non partes; nam si ita, sint AG, GB partes numeri E; & CH, HD partes numeri F. *c* ergo AG. E :: CH. a hyp. b 13. 7. c 20. def. 7.

d 13. 7. e hyp. C H. F; & permutando, A G. C H d :: E. F e :: AB. CD. ergo AB, CD non sunt minimi in sua ratione, contra hypoth. ergo, &c.

PROP. XXII.

A, 4. D, 12.
B, 3. E, 8.
C, 2. F, 6.

Si fuerint tres numeri A, B, C, *& alii ipsis multitudine æquales* D, E, F, *qui bini sumantur, & in eadem ratione; fuerit autem perturbata eorum proportio*(A.B :: E.F & B.C :: D.E;) *etiam ex æqualitate in eadem ratione erunt* (A.C :: D.F.)

a hyp. b 19. 7. c 1. ax. 1. d 19. 7. Nam quia A. B a :: E. F, erit AF = BE; & quia B. C :: a D. E, b erit BE = CD. c ergo AF = CD. d quare A. C :: D. F. Q. E. D.

PROP. XXIII.

A, 9. B, 4.
C---- D---
E--

Primi inter se numeri A, B, *minimi sunt omnium eandem cum eis rationem habentium.*

a 21. 7. b 23 def. 7. c 15. 7. Si fieri potest, sint C & D minores quam A & B, atque in eadem ratione. a ergo C metitur A æque, ac D metitur B, puta per eundem numerum E: quoties igitur 1 in E, b toties erit C in A. c quare vicissim quoties 1 in C, toties E in A. simili discursu quoties 1 in D, toties E in B. ergo E utrumque A & B metitur; qui proinde inter se primi non sunt, contra Hypoth.

PROP. XXIV.

A, 9. B, 4.
C---
D--- E--

Numeri A, B, *minimi omnium eandem cum eis rationem habentium, primi inter se sunt.*

a 9 ax. 7. b 17. 7. Si fieri potest, habeant A & B communem mensuram C; is metiatur A per D, & B per E; a ergo CD = A, b & CE = B.

b quare

b quare A. B :: D. E. Sed D & E minores sunt quam A & B, utpote eorum partes. Ergo A & B non sunt minimi in sua ratione, contra hypoth. b 17. 7.

PROP. XXV.

A, 9. B, 4.
C, 3. D - -

Si duo numeri A, B primi inter se fuerint, qui unum eorum A metitur numerus C, ad reliquum B primus erit.

Nam si affirmes aliquem D numeros B & C metiri, *a* ergo D metiens C, metitur A. ergo A & B non sunt primi inter se, contra Hypoth. a 11. ax. 7.

PROP. XXVI.

A, 5. C, 8.
B, 3.
$\overline{AB}$, 15. E - - - -
F - - - -

Si duo numeri A, B ad quempiam C primi fuerint, etiam ex illis genitus AB ad eundem C primus erit.

Si fieri potest, sit ipsorum AB. & C communis mensura numerus E. sitque $\frac{AB}{E} = F$; *a* ergo AB = EF; *b* quare E. A :: B. F. Quia vero A primus est ad C quem E metitur, *c* erunt E & A primi inter se; *d* adeoque in sua proportione minimi, & *e* proinde æque metiuntur B, & F; nempe E ipsum B. & A ipsum F. Quum igitur E utrumque B, C metiatur, non erunt illi primi inter se, contra Hypoth. a 9. ax. 7. b 19. 7. c 25. 7. d 23. 7. e 21. 7.

PROP. XXVII.

A, 4. B, 5.
Aq, 16.
D, 4.

Si duo numeri, A, B, primi inter se fuerint, etiam ex uno eorum genitus (Aq) ad reliquum B primus erit.

Sume D = A; ergo *a* singuli D, & A primi sunt ad B. *b* quare A D, vel Aq, ad B primus est. Q. E. D. a 1. ax. 7. b 26. 7.

PROP.

PROP. XXVIII.

A, 5. C, 4.
B, 3. D, 2.
AB, 15. CD, 8.

Si duo numeri A, B *ad duos numeros* C, D, *uterque ad utrumque, primi fuerint, & qui ex eis gignentur* AB, CD, *primi inter se erunt.*

a 16. 7. Nam quia A & B ad C primi sunt, *a* erit AB ad C primus. Eadem ratione erit AB ad D
b 26. 7. primus. *b* ergo AB ad CD primus est. Q.E.D.

PROP. XXIX.

A, 3. B, 2.
Aq, 9. Bq, 4.
Ac, 27. Bc, 8.

Si duo numeri A, B *primi inter se fuerint, & multiplicans uterque seipsum fecerit aliquem* (Aq, & Bq;) *& geniti ex ipsis* (Aq, Bq) *primi inter se erunt; & si qui in principio* A, B *genitos ipsos* Aq, Bq *multiplicantes fecerint aliquos* (Ac, Bc;) *& hi primi inter se erunt: & semper circa extremos hoc eveniet.*

a 27. 7. Nam quia A primus est ad B, *a* erit Aq ad B primus. & quia Aq primus ad B, *a* erit Aq ad Bq primus. Rursus quia tam A ad B, & Bq;
b 28. 7. quam Aq ad eosdem B, & Bq primi sunt, *b* erit A x Aq, id est Ac, ad B x Bq, id est Bc, primus. Et sic porro de reliquis.

PROP. XXX.

8 5
A .... B ..... C 13. D ----

Si duo numeri AB, BC *primi inter se fuerint, etiam uterque simul* (AC) *ad quemlibet illorum* AB, BC *primus erit. Et si uterque simul* AC *ad unum aliquem illorum* AB *primus fuerit, etiam qui in principio numeri* AB, BC *primi inter se erunt.*

1. *Hyp.* Nam si AC, AB compositos velis,
a 12. ax. 7. sit D communis mensura. *a* Is metietur reliquum BC. ergo AB, BC non sunt primi inter se, contra Hypoth.

2. *Hyp.*

2. *Hyp.* Positis AC, AB inter se primis, vis D ipsorum AB, BC communem esse mensuram. *b* Is igitur totum AC metitur. quare AC, AB non sunt primi inter se, contra Hypoth. b 10.ax.7.

Coroll.

Hinc numerus, qui ex duobus compositus, ad unum illorum primus est, ad reliquum quoque primus est.

Prop. XXXI.

A 5, B, 8. *Omnis primus numerus A ad omnem numerum B, quem non metitur, primus est.*

Nam si communis aliqua mensura metiatur utrumque A, B; *a* non erit A primus numerus, contra Hypoth. a 11.def.7.

Prop. XXXII.

A, 4. D, 3.
B, 6. E, 8.
$\overline{AB}$, 24.

Si duo numeri A, B, se mutuo multiplicantes fecerint aliquem AB; genitum autem ex ipsis AB metiatur aliquis primus numerus D; is etiam unum eorum, qui à principio, A, vel B metietur.

Pone numerum D non metiri A; sit vero $\frac{AB}{D} = E$. *a* ergo AB = DE. *b* quare D. A :: B. E. c est vero D ad A primus. *d* ergo D, & A minimi sunt in sua ratione; *e* proinde D metitur B, æque ac A metitur E. liquet igitur propositum. a 9.ax.7. b 19.7. c hyp. & 31.7. d 23.7. e 21.7.

Prop. XXXIII.

A, 12. B, 2. *Omnem compositum numerum A, aliquis primus numerus B metitur.*

Unus vel plures numeri *a* metiantur A, quorum minimus sit B. is primus erit. a 13.def.7.

nam

a 13. def 7. b 11. ax. 7. nam si dicetur compositus, a eum minor aliquis metietur, b qui proinde ipsum A metietur; quare B non est minimus eorum, qui A metiuntur; contra Hypoth.

PROP. XXXIV.

Omnis numerus A, *aut primus est, aut eum aliquis primus metitur.*

A, 9.

Nam A necessario vel primus est, vel compositus. Si primus, hoc est quod asserimus. Si compositus, a ergo eum aliquis primus metitur. Q. E. D.

a 33. 7.

PROP. XXXV.

A, 6. B, 4. C, 8.
D, 2.
E, 3. F, 2. G, 4.

H - - I - - K - - - -
L - - -

Numeris datis quotcunque A, B, C *reperire minimos omnium* E, F, G *eandem rationem cum eis habentium.*

a 23. 7. b 3. 7. Si A, B, C primi sint inter se, ipsi in sua ratione minimi a erunt. Si compositi sint, b esto eorum maxima communis mensura D, qui ipsos metiatur per E, F, G. Hi minimi erunt in ratione A, B, C.

c 9. ax 7. d 17. 7. e 21. 7. Nam D ductus in E, F, G c producit A B C. d ergo hi & illi in eadem sunt ratione. Iam puta alios H, I, K minimos esse in eadem; e qui propterea æque metientur A, B, C nempe per numerum L. f ergo L in H, I, K ipsos A, B, C procreabit. g ergo ED = A = HL. h unde E. H :: L. D. Sed E k ⊏ H; l ergo L ⊏ D. ergo D non est maxima communis mensura ipsorum A, B, C; contra Hypoth.

f 9. ax. 7. g 1. ax. 1. h 19. 7. k suppos. l 10. def 7.

Coroll.

Hinc, maxima communis mensura quotlibet nume-

numerorum metitur ipsos per numeros, qui minimi sunt omnium eandem rationem cum ipsis habentium. Ex quo patet methodus vulgaris reducendi fractiones ad minimos terminos.

PROP. XXXVI.

Duobus numeris datis A, B, *reperire, quem illi minimum metiuntur, numerum.*

A, 5. B, 4.
AB, 20.
D - - - - - -
E - - - F - - -

1. *Caſ.* Si A, & B primi ſint inter ſe, eſt AB quæſitus. Nam liquet A & B metiri AB. Si fieri poteſt, metiantur A & B aliquem D ⊐ AB; *a* 9 ax 7.
puta per E, & F. *a* ergo AE = D = BF. *b* quare *a* 1.ax.1.
A. B :: F. E. Quia vero A, & B *c* primi ſunt b 19.7.
inter ſe, *d* adeoque in ſua ratione minimi, *e* æque c hyp.
metientur A ipſum F, ac B ipſum E. Atqui d 23.7.
B. E *f* :: A B. A E (D.) *g* ergo A B etiam me- e 21.7.
tietur D, ſeipſo minorem. Q. E. A. f 17.7. g 20.def 7.

A, 6. B, 4. F - - - -
C, 3. D, 2. G - - - H - - -
AD, 12.

2. *Caſ.* Sin A, & B inter ſe compoſiti fuerint, *h* reperiantur C, & D minimi in eadem ratione. *k* ergo h 35.7. k 19.7.
AD = BC. Erit AD, vel BC quæſitus. l 7.ax.7.
Nam *l* liquet B, & A ipſum A D, vel B C metiri. Puta A, & B metiri F ⊐ AD, nempe
A per G, & B per H. *m* ergo AG = F = BH. m 9.ax.7.
n unde A. B :: H. G *o* :: C. D. *p* proinde æque n 19 7.
metitur C ipſum H, ac D ipſum G. atqui D. G o conſtr.
q :: AD. AG (F.) ergo AD *r* metitur F, major p 21.7. q 17.7.
minorem. Q. E. A. r 20 def.7.

Coroll.

Hinc, ſi duo numeri multiplicent minimos eandem rationem habentes, major minorem, & minor majorem, producetur numerus minimus, quem illi metiuntur.

PROP.

PROP. XXXVII.

A, 2. B, 3. E, ... 6. C----F---D

Si duo numeri A, B numerum quempiam C D metiantur; etiam minimus E, quem illi metiuntur, eundem C D metietur.

Si negas, aufer E ex C D, quoties fieri potest, & relinquatur F D ⊐ E. quum igitur A & B a metiantur E, b & E ipsum C F, c etiam A, & B metiuntur C F; a metiuntur autem totum C D; d ergo etiam reliquum F D metiuntur. ergo E non est minimus, quem A, & B metiuntur, contra hyp.

a hyp.
b constr.
c 11. ax. 7.
d 12. ax. 7.

PROP. XXXVIII.

A, 3, B, 4, C, 6. D, 12.

Tribus numeris datis A, B, C reperire minimum, quem illi metiantur.

a Reperi D minimum, quem duo A, & B metiuntur; quem si tertius C metiatur, patet D esse quæsitum. Quod si C non metiatur D, sit E minimus, quem C, & D metiuntur. Erit E requisitus.

a 36. 7.

A, 2. B, 3. C, 4. D, 6. E, 12. F---

Nam singulos A, B, C metiri E constat ex 11. ax. 7. Quod vero nullum alium F minorem metiantur, facile ostenditur. Nam si affirmas, b ergo D metitur F; b proinde E eundem F metitur, major minorem. Quod est absurdum.

b 37. 7.

Coroll.

Hinc, si tres numeri numerum quempiam metiantur; etiam minimus, quem illi metiuntur, eundem metietur.

PROP.

PROP. XXXIX.

A, 12. B, 4, C, 3.

Si numerum A quispiam numerus B metiatur, ille A quem B metitur, partem habebit C, à metiente B denominatam.

Nam quia A a = C, b erit A = BC. c ergo $\frac{A}{B}$ = B. Q.E.D. a hyp. b 9. ax. 7. c 7. ax. 7.
$\frac{A}{C}$

PROP. XL.

A, 15. B, 3. C, 5.

Si numerus A partem habuerit quamlibet B, metietur illum numerus C, à quo ipsa pars B denominatur.

Nam quia BC a = A, b erit $\frac{A}{C}$ = B. Q.E.D. a hyp. & 9. ax. 7. b 7. ax. 7.

PROP. XLI.

$\frac{1}{2}$ G, 12. $\frac{1}{3}$ H.... $\frac{1}{4}$

Numerum reperire G, qui minimus cum sit, habeat datas partes, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.

a Inveniatur G minimus, quem denominatores 2, 3, 4 metiuntur. b Liquet G habere partes, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. Si fieri potest, H ⊐ G habeat easdem partes; c ergo 2, 3, 4 metiuntur H. & proinde G non est minimus, quem 2, 3, 4 metiuntur. contra constr. a 38. 7. b 39. 7. c 40. 7.

LIB. VIII.

PROP. I.

A, 8. B, 12. C, 18. D, 27.
E-- F-- G--- H----

SI fuerint quotcunque numeri deinceps proportionales A, B, C, D; extremi vero ipsorum A, D primi inter se fuerint; ipsi A, B, C, D minimi sunt omnium eandem cum eis rationem habentium.

Nam, si fieri potest, sint alii totidem E, F, G, H
a 14. 7. minores in illa ratione. *a* ergo ex æquali A.D ::
b 23. 7. E. H. ergo A, & D primi numeri, *b* adeoque in
c 21. 7. sua ratione minimi, *c* æque metiuntur E, & H, seipsis minores. Q. E. A.

PROP. II.

I.

A, 2. B, 3.
Aq, 4. AB, 6. Bq, 9.
Ac, 8. AqB, 12. ABq, 18. Bc, 27.

Numeros reperire deinceps proportionales minimos, quotcunque jusserit quispiam, in data ratione A ad B.

Sint A, & B minimi in data ratione. Erunt Aq, AB, Bq tres minimi deinceps in ratione A ad B.

a 17. 7. Nam AA. AB *a* :: A. B *a* :: AB. BB. item
b 24. 7. quia A, & B *b* primi sunt inter se, *c* erunt
c 29. 7. Aq, Bq inter se primi; *d* proinde Aq, AB, Bq
d 1. 8. sunt $\div$ minimi in ratione A ad B.

Dico porro, Ac, AqB, ABq, Bc in ratione A ad B quatuor esse minimos. Nam AqA.
e 17. 7. AqB *e* :: A. B *e* :: ABA (AqB.) ABB. *e* atque A. B :: ABq. BBq. (Bc) Quum igitur Ac, &

Bc

Bc f inter se primi sint, g erunt Ac, AqB, ABq, Bc quatuor ∺ minimi in ratione A ad B. Eodem modo quotvis proportionales investigabis. Q. E. F. f 29. 7. g 1. 8.

Coroll.

1. Hinc, si tres numeri minimi sunt proportionales; extremi quadrati erunt; si quatuor, cubi.

2. Extremi quotcunque proportionales per hanc propos. inventi in data ratione minimi, inter se primi sunt.

3. Duo numeri, minimi in data ratione, metiuntur omnes medios quotcunque minimorum in eadem ratione; quia scilicet producuntur ex illorum multiplicatione in alios quosdam numeros.

4. Hinc etiam liquet ex constructione, series numerorum 1, A, Aq, Ac; 1, B, Bq, Bc; Ac, AqB, ABq, Bc, constare æquali multitudine numerorum; ac proinde extremos numeros quotcunque minimorum continue proportionalium, esse ultimos totidem continue proportionalium ab unitate. ut extremi Ac, Bc continue proportionalium Ac, AqB, ABq, Bc, sunt ultimi totidem proportionalium ab unitate 1, A, Aq, Ac; & 1, B, Bq, Bc.

5. 1, A, Aq, Ac; & B, BA, BAq; ac Bq, ABq sunt ∺ in ratione 1 ad A. item, B, Bq, Bc; & A, AB, ABq; ac Aq, AqB sunt ∺ in ratione 1 ad B.

PROP. III.

A, 8. B, 12. C, 18. D, 28. *Si sint quotcunque numeri* A, B, C, D *deinceps proportionales, minimi omnium eandem cum eis rationem habentium; illorum extremi* A, D *sunt inter se primi.*

a 2.8. Nam si *a* inveniantur totidem numeri minimi in ratione A ad B, illi non alii erunt, quam A, B, C, D; ergo juxta 2. coroll. præcedentis extremi A & D primi sunt inter se. Q. E. D.

PROP. IV.

A, 6. B, 5. C, 4. D, 3.
H, 4. F, 24. E, 20. G, 15.
I - - K - - L - - -

Rationibus datis quotcunque in minimis terminis, (A ad B; & C ad D) *reperire numeros deinceps minimos in datis rationibus.*

a 36.7. *a* Reperi E minimum, quem B, & C metiantur; & B ipsum E *b* æque metiatur, ac A alterum
b 3.post.7. F, puta per eundem H. *b* item C ipsum E, ac D alterum G æque metiantur: erunt F, E, G minimi in datis rationibus. Nam A H *c* = F; &
c 9 ax 7. B H *c* = E. *d* ergo A. B :: A H. B H *e* :: F. E.
d 18.7.
e 7.5. Similiter C. D :: E. G. sunt igitur F, E, G deinceps proportionales in datis rationibus. Imo minimi sunt in iisdem: nam puta alios I, K, L
f 21.7. minimos esse. *f* ergo A & B ipsos I & K, *f* pariterque C & D ipsos K & L æque metiuntur.
g 37.7. ergo B, & C eundem K metiuntur. *g* Quare etiam E eundem K metitur, seipso minorem. Q. E. A.

A, 6. B, 5. C, 4. D, 3. E, 5. F, 7.
H, 24. G, 20. I, 15. K, 21.

Datis vero tribus rationibus A ad B, & C ad D, ac E ad F: reperi, ut prius, tres H, G, I minimos deinceps in rationibus A ad B, & C ad D. tunc si E numerum I metiatur,

h 5.post.7. *h* Sume alterum K, quem F æque metiatur; erunt quatuor H, G, I, K, deinceps minimi, in datis rationibus; quod non aliter probabimus quam in priori parte.

A, 6

A, 6. B, 5. C, 4. D, 3. E, 2. F, 7.
H, 24. G, 29. I, 15.
M, 48. L, 40. K, 30. N, 105.

Sin E non metiatur I, sit K minimus, quem E, & I metiuntur; & quoties I ipsum K, toties G ipsum L, & H ipsum M metiatur. quoties vero E ipsum K, toties F ipsum N metiatur. Erunt M, L. K, N minimi deinceps in datis rationibus; quod demonstrabimus, ut prius.

PROP. V.

C, 4. E, 3,
D, 6 F, 16 ED, 18.
CD, 24 EF, 48.

Plani numeri CD, EF *rationem habent ex lateribus compositam.* $(\frac{CD}{EF} = \frac{C}{E} + \frac{D}{F})$

Nam quia CD. ED *a* :: C. E; *a* & ED. EF :: D. F. atque $\frac{CD}{EF}$ *b* $= \frac{CD}{ED} + \frac{ED}{EF}$, *c* erit ratio $\frac{CD}{EF} = \frac{C}{E} + \frac{D}{F}$. Q. E. D.

a 17. 7.
b 20. def. 5.
c 11. 5.

PROP. VI.

A, 16. B, 24. C, 36. D, 54. E, 81.
F, 4. G, 6. H, 9.

Si sint quotcunque numeri deinceps proportionales A, B, C, D, E; *primus autem* A *secundum* B *non metiatur, neque alius quispiam ullum metietur.*

Quoniam A non metitur B, *a* neque quilibet proxime sequentem metietur; quia A.B :: B.C :: C.D, &c. *b* Accipe tres F, G, H minimos in ratione A ad B. quoniam igitur A non metitur B, *a* neque F metietur G. *c* ergo F non est unitas. sed F, & H inter se primi sunt; ergo quum *e* sit ex æquo A. C :: F. H, & F non metiatur H, *d* neque A ipsum C metietur; proinde nec B ipsum D, nec C ipsum E, &c. quia A. C *e* :: B. D *e* :: C. E, &c. Eodem modo

a 20. def. 8.
b 35. 7.
c 5 ax. 7.
d 3. 3.
e 14. 7.

 sumptis

sumptis quatuor vel quinque minimis in ratione A ad B, ostendetur A ipsos D, & E; ac B ipsos E, & F non metiri, &c. Quare nullus alium metietur. Q. E. D.

PROP. VII.

A, 3. B, 6. C, 12. D, 24. E, 48.

Si sint quotcunque numeri deinceps proportionales A, B, C, D, E; *primus autem* A *extremum* E *metiatur; is etiam metitur secundum* B.

a 6. 7. Si negas A metiri B, a ergo nec ipsum E metietur, contra Hypoth.

PROP. VIII.

A, 24.	C, 36.	D, 54.	B, 81.
G, 8.	H, 12.	I, 18.	K, 27.
E, 32.	L, 48.	M, 72.	F, 108.

Si inter duos numeros A, B *medii continua proportione ceciderint numeri* C, D; *quot inter eos medii continua proportione cadunt numeri, tot & inter alios* E, F *eandem cum illis habentes rationem, medii continua proportione cadent.* (L, M.)

a 35. 7.
b 14. 7.
c hyp.
d 3. 8.
e 21. 7.
a Sume G, H, I, K minimos ∺ in ratione A ad C; b erit ex æquali, G. K :: A. B c :: E. F. Atqui G, & K d primi sunt inter se; e quare G æque metitur E, ac K ipsum F. per eundem numerum metiatur H ipsum L, & I ipsum M. f itaque E, L, M, F ita se habent ut G, H, I, K; hoc est ut A, B, C, D. Q. E. D.

f constr.

PROP. IX.

	I.		
	E, 2.	F, 3.	
	G, 4.	H, 6.	I, 9.
A, 8.	C, 12.	D, 18.	B, 27.

Si duo numeri A, B, *sint inter se primi, & inter eos medii continua proportione ceciderint numeri,* C, D; *quot inter eos medii continua*

tinua proportione cecidesint numeri, totidem (E, G, & F, I) *& inter utrumque eorum ac unitatem medii continua proportione cadent.*

Constat 1, E, G, A; & 1, F, I, B esse ∺; & totidem quot A, C, D, B, nimirum ex 4 coroll. 2. 8. Q. E. D.

PROP. X.

A, 8. I, 12. K, 18. B, 27.
E, 4. DF, 6. G, 9.
D, 2. F, 3.
1.

Si inter duos numeros A, B, & unitatem continue proportionales ceciderint numeri (E, D, & F, G,) *quot inter utrumque ipsorum, & unitatem deinceps medii continua proportione cadunt numeri, totidem & inter ipsos medii continua proportione cadent,* I, K.

Nam E, D F, G; & A, DꟼF (I,) D G (K,) B sunt ∺, per 2. 8. ergo, &c.

PROP. XI.

A, 2. B, 3.
Aq, 4. AB, 6. Bq, 9.

Duorum quadratorum numerorum Aq, Bq *unus medius proportionalis est numerus* AB. *& quadratum* Aq *ad quadratum* Bq, *duplicatam habet lateris* A *ad latus* B *rationem.*

[a] Liquet Aq, AB, Bq, esse ∺. [b] proinde etiam $\frac{Aq}{Bq} = \frac{A}{B}$ bis. Q. E. D. a 17 7. b 10 def. 5.

PROP. XII.

Ac, 27. AqB, 36. ABq, 48. Bc, 64.
A, 3. B, 4.
Aq, 9. AB, 12. Bq, 16.

Duorum cuborum numerorum Ac, Bc *duo medii proportionales sunt numeri* AqB, ABq. *Et cubus* Ac *ad cubum* Bc *triplicatam habet lateris* A *ad latus* B *rationem.*

a 2. 1.
b 10 def. 5.

a Nam Ac, AqB, ABq, Bc sunt ∵ in ratione A ad B. b proinde $\frac{Ac}{Bc} = \frac{A}{B}$ ter. Q. E. D.

PROP. XIII.

A, 2. B, 4. C, 8.
Aq, 4. AB, 8. Bq, 16. BC, 32. Cq, 64.
Ac, 8. AqB, 16. ABq, 32. Bc, 64. BqC, 128. BCq, 256. Cc, 512.

Si sint quotlibet numeri deinceps proportionales, A, B, C; *& multiplicans quisque seipsum faciat aliquos; qui ab illis producti fuerint* Aq, Bq, Cq *proportionales erunt: & si numeri primum positi* A, B, C *multiplicantes jam factos* Aq, Bq, Cq, *fecerint aliquos* Ac, Bc, Cc; *ipsi quoque proportionales erunt. & semper circa extremos hoc eveniet.*

a 2. 8.
b 14. 7.

Nam Aq, AB, Bq, BC, Cq a sunt ∵. b ergo ex æquo Aq. Bq :: Bq. Cq. Q. E. D.

a Item Ac, AqB, ABq, Bc, BqC, BCq, Cc sunt ∵, b ergo iterum ex æquo, Ac, Bc :: Bc, Cc. Q. E. D.

PROP. XIV.

Aq, 4. AB, 12. Bq, 36.
A, 2. B, 6.

Si quadratus numerus Aq *quadratum numerum* Bq *metiatur, & latus unius* (A) *metietur latus alterius* (B:) *& si unius quadrati latus* A *metietur latus alterius* B, *& quadratus* Aq *quadratum* Bq *metietur.*

a 2. & 11. 8.

1. *Hyp.* Nam Aq. AB a :: AB. Bq; cum igitur ex hyp. Aq metiatur Bq; idem Aq secundum

cundum A B *b* metietur. atqui Aq A B :: A. B. *c* ergo etiam A metitur B. Q. E. D. b 7. 8 c 10. *def.* 7.

2. *Hyp.* A metitur B. *c* ergo tam Aq ipsum A B, *c* quam A B ipsum Bq metitur; *d* & proinde Aq metitur Bq. Q. E. D. d 11. *ax.* 7.

PROP. XV.

A, 2. B, 6.
Ac, 8. AqB, 24. ABq, 72. Bc, 216.

Si cubus numerus Ac *cubum numerum* Bc *metiatur, & latus unius* (A) *metietur latus alterius* (B:) *Et si latus* A *unius cubi* Ac *latus* B *alterius* Bc *metiatur, & cubus* Ac *cubum* Bc *metietur.*

1. *Hyp.* Nam Ac, AqB, ABq, Bc *a* sunt ∺. ergo Ac, *b* metiens extremum Bc, *c* etiam secundum AqB metietur. atqui Ac. AqB :: A. B. *d* ergo etiam A metietur B. Q. E. D. a 2. & 12. 8. b *hyp.* c 7. 8.

2. *Hyp.* A metitur B; *d* ergo Ac metitur AqB, isque ABq, & hic Bc; *e* ergo Ac metietur Bc. Q. E. D. d 20. *def.* 7. e 11. *ax.* 7.

PROP. XVI.

A, 4. B, 9.
Aq, 16. Bq, 81.

Si quadratus numerus Aq *quadratum numerum* Bq *non metiatur, neque* A *latus unius alterius latus* B *metietur: & si* A *latus unius quadrati* Aq *non metiatur* B *latus alterius* Bq, *neque quadratus* Aq *quadratum* Bq *metietur.*

1. *Hyp.* Nam si affirmes A metiri B, *a* etiam Aq ipsum Bq metietur, contra hyp. a 14. 8.

2. *Hyp.* Vis Aq metiri Bq; *a* ergo A ipsum B metietur, contra hyp.

PROP.

PROP. XVII.

A, 2. B, 3.
Ac, 8. B, 27.

Si cubus numerus Ac *cubum numerum* Bc *non metiatur, neque* A *latus unius latus* B *alterius metietur. Et si latus* A *unius cubi* Ac *latus* B *alterius* Bc *non metiatur, neque cubus* Ac *cubum* Bc *metietur.*

a 15. 8. 1. *Hyp.* Dic A metiri B; *a* ergo Ac metietur Bc. contra Hypoth.

2. *Hyp.* Dic Ac metiri *B*c; *a* ergo A ipsum B metietur. contra Hyp.

PROP. XVIII.

C, 6. D, 2.
CD, 12.
E, 9. F, 3. DE, 18.
EF, 27.

Duorum similium planorum numerorum CD, EF, *unus medius proportionalis est numerus* DE: *& planus* CD *ad planum* EF *duplicatam habet lateris* C *ad latus homologum* E *rationem.*

* 21. def. 7. Quoniam * ex hyp. C. D :: E. F; permu-
a 17. 7. tando erit C. E :: D. F. atqui C. E *a* :: CD.
b 11. 5. DE; *a* & D. F :: DE. EF. *b* ergo CD. DE :: DE. EF. Q. E. D.

c 10. def. 5. *c* Ergo ratio CD ad EF duplicata est rationis CD ad DE; hoc est rationis C ad E, vel D ad F.

Coroll.

Hinc perspicuum est, inter duos similes planos cadere unum medium proportionalem, in ratione laterum homologorum.

PROP.

PROP. XIX.

CDE, 30. DEF, 60. FGE, 120. FGH, 240.
CD, 6. DF, 12. FG, 24.
C, 2. D, 3. E, 5. F, 4. G, 6. H, 10.

Duorum similium solidorum CDE, FGH, *duo medii proportionales sunt numeri* DFE, FGE. *Et solidus* CDE *ad solidum* FGH *triplicatam rationem habet lateris homologi* C *ad latus homologum* F.

Quoniam ex* hyp. C. D :: F. G ; & D. E :: G. H ; erit *a* permutando C. F :: D. G *a* :: E. H. atqui CD. DF *b* :: C. F ; & DF. FG *b* :: D. G. *c* quare CD. DF :: DF. FG :: E. H. *d* ergo CDE. DFE :: DFE. FGE :: E. H :: FGE. FGH. ergo inter CDE, FGH cadunt duo medii proportionales, DFE, FGE. Q. E. D. *e* Liquet igitur rationem CDE ad FGH triplicatam esse rationis CDE ad DFE ; vel C ad F. Q. E. D.

* 21. def. 7. a 13. 7. b 17. 7. c 11. 5. d 17. 7. e 10 def. 5.

Coroll.

Hinc, inter duos similes solidos cadunt duo medii proportionales, in ratione laterum homologorum.

PROP. XX.

A, 12. C, 18. B, 27.
D, 2. E, 3. F, 6. G, 9.

Si inter duos numeros A, B, *unus medius proportionalis cadat numerus* C, *similes plani erunt illi numeri*, A, B.

a Accipe D, & E minimos in ratione A ad C, vel C ad B. *b* ergo D æque metitur A, ac E ipsum C, puta per eundem F. *b* item D æque metitur C ac E ipsum B, puta per eundem G. *c* ergo DF = A, & EG = B. *d* quare A, & B plani sunt numeri. Quia vero EF *c* = C *c* = DG ; *e* erit D. E :: F. G, & vicissim D. F :: E. G. *f* ergo plani numeri A, & B etiam similes sunt. Q. E. D.

a 35. 7. b 21. 7. c 9. ax. 7. d 16. def. 7. e 19. 7. f 21. def. 7.

PROP. XXI.

A, 16. C, 24. D, 36. B, 54.
E, 4. F, 6. G, 9.
H, 2. P, 2. M, 4. K, 3. L, 3. N, 6.

Si inter duos numeros A, B duo medii proportionales cadant numeri C, D; similes solidi erunt illi numeri, A, B.

a 2. 8. *a* Sume E, F, G minimos ∺ in ratione A ad
b 20 8. C. *b* ergo E, & G sunt numeri plani similes.
c 21. def. 7. hujus latera sint H & P; illius K & L: *c* ergo H.
d cor. 2. 8. K :: P. L :: *d* E. F. Atqui E, F, G ipsos A, C,
e 21. 7. D *e* æque metiuntur, puta per eundem M; iidemque ipsos, C, D, B æque metiuntur, puta
f 9. ax. 7. per eundem N. *f* ergo A = E M = H P M, *f* &
g 17 def. 7. B = G N = K L N; *g* quare A & B solidi sunt numeri. Quoniam vero C *f* = FM; & D *f* =
h 17. 7. F N, erit M. N *h* :: F M. F N *k* :: C. D *l* :: E.
k 7. 5. l constr. F :: H, K :: P. L. *m* ergo A, & B sunt numeri
m 21. def. 7. solidi similes. Q. E. D.

PROP. XXII.

A, 4. B, 6. C, 9.

Si tres numeri A, B, C deinceps sint proportionales, primus autem A sit quadratus, & tertius C quadratus erit.

Inter A, & C cadit medius proportionalis.
a 20. 8. *a* ergo A, & C sunt similes plani; quare *b* cum A
b hyp. quadratus sit, erit C etiam quadratus. Q. E. D.

PROP. XXIII.

A, 8. B, 12. C, 18. D, 27.

Si quatuor numeri A, B, C, D deinceps sint proportionales; primus autem A sit cubus, & quartus D cubus erit.

a 21. 8. Nam A, & D *a* similes solidi sunt; ergo
b hyp. *b* cum A cubus sit, erit D cubus. Q. E. D.

PROP. XXIV.

A, 16. 24. B, 36.
C, 4. 6. D, 9.

Si duo numeri A, B rationem habeant inter se, quam quadratus numerus C ad quadratum numerum D, primus autem A sit quadratus; & secundus B quadratus erit.

Inter C, & D numeros quadratos, * adeoque inter A, & B eandem rationem habentes, *a* cadit unus medius proportionalis. Ergo *b* cum A quadratus sit, *c* etiam B quadratus erit. Q.E.D. * 8. 8. a 11. 8. b hyp. c 22. 8.

Coroll.

1. Hinc si fuerint duo numero similes AB, CD (A. B :: C. D) primus autem AB sit quadratus, etiam secundus CD quadratus erit. * 11. & 8. 8

* Nam AB. CD :: Aq. Cq.

2. Liquet ex his, proportionem cujusvis numeri quadrati ad quemlibet non quadratum, exhiberi nullo modo posse in duobus numeris quadratis. unde non erit, Q. Q :: 1. 2. nec 1. 5. :: Q. Q. &c.

PROP. XXV.

C, 64. 96. 144. D, 216.
A, 8. 12. 18. B, 27.

Si duo numeri A, B rationem inter se habeant, quam cubus numerus C ad cubum numerum D, primus autem A sit cubus, & secundus B cubus erit.

a Inter C, & D cubos, *b* adeoque inter A & B eandem rationem habentes, cadunt duo medii proportionales. ergo propter A *c* cubum, *d* etiam B cubus erit. Q. E. D. a 11. 8. b 8. 8. c hyp. d 23. 8.

Coroll.

1. Hinc etiam si fuerint duo numeri ABC, DEF (A. B :: D. E. & B. C :: E. F;) primus autem ABC cubus fuerit, etiam secundus DEF cubus erit. * 12, & 19. 8.

* Nam ABC. DEF :: Ac = Dc.

2. Patet etiam ex his, proportionem cujusvis nu-

numeri cubi ad quemlibet numerum non cubum non posse reperiri in duobus numeris cubis.

PROP. XXVI.

A, 20. C, 30. B, 45.
D, 4. E, 6. F, 9.

Similes plani numeri A, B *rationem inter se habent, quam quadratus numerus ad quadratum numerum.*

a 18. 8. b 2. 8. Inter A, & B *a* cadit unus medius proportionalis C. *b* sume tres D, E, F minimos ∺ in ratione A ad C. Extremi D, F *b* quadrati erunt: atqui ex æquali A. B *c* :: D. F. ergo A. B :: Q. Q. Q. E. D.

c 14. 7.

PROP. XXVII.

A, 16. C, 24. D, 36. B, 54.
E, 8. F, 12. G, 18. H, 27.

Similes solidi numeri A, B, *rationem habent inter se, quam cubus numerus ad cubum numerum.*

a 19. 8. b 2. 8. *a* Inter A, & B cadunt duo medii proportionales, puta C & D. *b* sume quatuor E, F, G, H minimos ∺ in ratione A ad C. *b* Extremi E, H cubi sunt. At A. B *c* :: E. H :: C. C. Q. E. D.

c 14. 7.

Schol.

8 Vide Clavium. 2. 1. Ex his infertur, nullos numeros habentes proportionem superparticularem, vel superbipartientem, vel duplam, aut aliam quamcunque multiplam non denominatam à numero quadrato, esse similes planos.

2. Nec duo quivis primi numeri, neque duo quicunque inter se primi, qui quadrati non sint, similes esse possunt.

LIB.

LIB. IX.

PROP. I.

A, 6. B, 54.
Aq, 36. 108. AB, 324.

SI duo similes plani numeri A, B *multiplicantes se mutuo faciant quendam* A B, *productus* AB *quadratus erit.*

Nam A. B a :: Aq. AB; cum igitur inter A, & B b cadat unus medius proportionalis, c etiam inter Aq, & AB cadet unus med. proport. ergo cum primus Aq sit quadratus, d etiam tertius AB quadratus erit. Q. E. D. a 17. 7. b 18. 8. c 8. 8. d 22. 8

Vel sic. Sint ab, cd similes plani, nempe a. b :: c. d. x ergo a d = bc. quare abcd, vel adbc y = adad = Q: ad. x 19 7. y 1. ax. 7.

PROP. II.

A, 6. B, 54.
Aq, 36. AB, 324.

Si duo numeri A, B *se mutuo multiplicantes faciant* AB *quadratum, similes plani erunt*, A, B.

Nam A. B a :: Aq. AB; quare cum inter Aq, AB b cadat unus medius proportionalis, c etiam unus inter A, & B medius cadet. d ergo A, & B sunt similes plani. Q. E. D. a 17. 7. b 11. 8. c 8. 8. d 20. 8.

PROP. III.

A, 2. Ac, 8. Acc, 64.

Si cubus numerus Ac *seipsum multiplicans procreet aliquem* Acc, *productus* Acc *cubus erit.*

Nam 1. A a :: A. Aq b :: Aq. Ac. ergo inter 1, & Ac cadunt duo medii proportionales. Sed 1. Ac a :: Ac. Acc. c ergo inter Ac, & Acc cadunt etiam duo medii a 15. def. 7. b 17. 7. c 8. 8.

medii proportionales. Proinde cum Ac sit cubus,
d 23. 8. *d* erit Acc cubus. Q. E. D.

Vel sic; aaa (Ac) in se ductus facit aaaaaa. (Acc;) hic cubus est, cujus latus aa.

PROP. IV.

Ac, 8. Bc, 27.
Acc, 64. AcBc, 216.

Si cubus numerus Ac *cubum numerum* Bc *multiplicans, faciat aliquem* AcBc, *factus* AcBc *cubus erit.*

a 17. 7. Nam Ac. Bc *a* :: Acc. AcBc. sed inter Ac
b 11. 8. & BC *b* cadunt duo medii proportionales; *c* ergo
c 8. 8. inter Acc, & AcBc totidem cadunt. itaque cum
d 23. 8. Acc sit cubus, *d* erit AcBc etiam cubus. Q.E.D.

Vel sic. AcBc = aaabbb (ababab) = C: ab.

PROP. V.

Ac, 8. B, 27.
Acc, 64. AcB, 216.

Si cubus numerus Ac *numerum quendam* B *multiplicans, faciat cubum* AcB; *& multiplicatus* B *cubus erit.*

Nam Acc. AcB *a* :: Ac. B. Sed inter Acc, &
a 17. 7. AcB *b* cadunt duo medii proportionales. *c* ergo
b 12. 8. totidem cadent inter Ac, & B. quare cum Ac cu-
c 8. 8. bus sit, *d* etiam B cubus erit. Q. E. D.
d 23. 8.

PROP. VI.

A, 8. Aq, 64. Ac, 512.

Si numerus A *seipsum multiplicans faciat* Aq *cubum; & ipse* A *cubus erit.*

a hyp. Nam quia Aq *a* cubus, & AqA (Ac) *b* cu-
b 29 def. 7. bus, *c* erit A cubus. Q. E. D.
c 5. 9.

PROP. VII.

A, 6. B, 11. AB, 66.
D, 2. E, 3.

Si compositus numerus A *numerum quempiam* B *multiplicans, quempiam faciat* AB, *factus* AB *solidus erit.*

Quoniam

Quoniam A compositus est, *a* metitur eum aliquis D, puta per E. *b* ergo A = DE; *c* quare DEB = AB solidus est. Q. E. D. a 13. def. b 9 ax. 7. c 17. def. 7.

PROP. VIII.

1. a, 3. a^2, 9. a^3, 27. a^4, 81. a^5, 243. a^6, 729.

Si ab unitate quotcunque numeri deinceps proportionales fuerint (1, a, a^2, a^3, a^4, &c.) *tertius quidem ab unitate* a^2 *quadratus est; & unum intermittentes, omnes* (a^4, a^6, a^8, &c.); *quartus autem* a^3 *est cubus; & duos intermittentes omnes* (a^6, a^9, &c.) *septimus vero* a^6, *cubus simul & quadratus; & quinque intermittentes omnes* (a^{12}, a^{18}, &c.)

Nam 1. a^2 = Q. a. & a^4 = aaaa = Q. aa. & a^6 = aaaaaa = Q. aaa, &c.

2. a^3 = aaa = C. a. & a^6 = aaaaaa = C. aa. & aaaaaaaaa = C. aaa, &c.

3. a^6 = aaaaaa = C. aa = Q. aaa. ergo, &c.

Vel juxta Euclidem; quia 1. a *a* :: a. a^2, *b* erit a^2 = Q: a. ergo cum a^2, a^3, a^4 sint $\div$ *c* erit tertius a^4 etiam quadratus. pariterq; a^6, a^8, &c. Item quia 1. a *a* :: a^2. a^3. erit a^3 *b* = a^2 in a = C: a. *d* ergo quartus ab a^3, nempe a^6, etiam cubus erit, &c. ergo a^6 cubus simul & quadratus existit, &c. a hyp. b 20. 7. c 22. 8. d 23. 8.

PROP. IX.

1. a, 4. a^2, 16. a^3, 64. a^4, 256, &c.

1. a, 8. a^2, 64. a^3, 512. a^4, 4096.

Si ab unitate quotcunque numeri deinceps proportionales fuerint (1, a, a^2, a^3, &c.); *qui vero* (a) *post unitatem sit quadratus; & reliqui omnes,* a^2, a^3, a^4, &c. *quadrati erunt. At si* a, *qui post unitatem, sit cubus, & reliqui omnes* a^2, a^3, a^4, &c. *cubi erunt.*

1. *Hyp.* Nam a^2, a^4, a^6, &c. quadrati sunt ex præc. item quia a ponitur quadratus, *a* erit tertius a^3 quadratus, pariterque a^5, a^7, &c. ergo omnes. a 22. 8.

 2. *Hyp.*

b 13. 8. c 20. 7. d 5. 9. e 23. 8.

2. *Hyp.* a cubus ponitur, b ergò a^4, a^7, a^{10} cubi sunt; atqui ex præced. a^3, a^6, a^9, &c. cubi sunt. denique quia 1. a :: a. aa, c erit a^2 = Q: a. cubus autem in se d facit cubum; ergo a^2, cubus est, & e proinde ab eo quartus a^5, pariterq; a^8, a^{11}, &c. cubi sunt. ergo omnes. Q. E. D.

Clarius forsitan sic; Sit quadrati a latus b. ergo series a, a^2, a^3, a^4, &c. aliter exprimetur sic, bb, b_4, b6, b8, &c. liquet vero hos omnes quadratos esse; & sic etiam exprimi posse; Q: b, Q: bb, Q: bbb, Q: bbbb, &c.

Eodem modo, si b latus fuerit cubi a, series ita nominari potest; b3, b^6, b_9, b^{12}, &c. vel C: b, C: b2, C: b3, C: b4, &c.

PROP. X.

1, a, a^2, a^3, a^4, a^5, a^6.
1, 2, 4, 8, 16, 32, 64.

Si ab unitate quotcunque numeri deinceps proportionales fuerint (1, a, a^2, a^3, &c.); *qui vero post unitatem* (a) *non sit quadratus, neque alius ullus quadratus erit, præter* a^2 *tertium ab unitate, & unum intermittentes omnes* (a^4, a^6, a^8.) *At si* a, *qui post unitatem, non sit cubus, neque ullus alius cubus erit præter* a^3 *quartum ab unitate, & duos intermittentes omnes,* a^6, a^9, a^{12}, *&c.*

a hyp. b suppos. & 8. 9. c 24. 8.

1. *Hyp.* Nam si fieri potest, sit a^5 quadratus numerus. quoniam igitur a. a_2 a :: a4. a_5, atq; inverse a5. a_4 :: a2. a; sintque a5, & a_4 b quadrati, primusque a_2 quadratus, c erit a etiam quadratus, contra Hyp.

d 14. 7. e 25. 8.

2. *Hyp.* Si fieri potest, sit a^4 cubus. quoniam igitur d ex æquo a4. a^6 :: a. a3, atque inverse a_6. a_4 :: a_3. a; b sintque a6, & a_4 cubi, & primus a_3 cubus, e etiam a cubus erit, contra Hypoth.

PROP. XI

1, a, a2, a3, a4, a5, a6.
1, 3, 9, 27, 81, 243, 729.

a Si ab unitate quotcunq; numeri deinceps proportionales fuerint (1, a, a_2, a3, &c.) *minor majorem metitur per aliquem eorum qui in proportionalibus sunt numeris.*

Quoniam 1. a :: a. aa, *a* erit $\frac{aa}{a} = a = \frac{aaa}{aa}$ — a 5. ax. 7. & 20. def. 7.
item quia 1. aa *b* :: a. aaa. *a* erit $\frac{aaa}{a} = aa =$ — b 14 7.
$\frac{a4}{aa} = \frac{a5}{a3}$ &c. denique quia 1. a3 *b* :: a. a4, *a* erit $\frac{a4}{a} = a5 = \frac{a6}{a3}$ &c.

Coroll.

Hinc, si numerus qui metitur aliquem ex proportionalibus, non sit unus proportionalium, neque numerus per quem metitur, erit aliquis ex proportionalibus.

PROP. XII.

1, a, a2, a3, a4,
1, 6, 36, 216, 1296.
B, 3.

Si ab unitate quotcunq; numeri deinceps proportionales fuerint (1, a, a2, a3, a4,); *quicunque primorum numerorum* B *ultimum* a4 *metiuntur, iidem* (B) *& eum* (a) *qui unitati proximus est, metientur.*

Dic B non metiri a, *a* ergo B ad a primus est; *b* ergo B ad a2 primus est; & *c* proinde ad a4 quem metiri ponitur Q. E. A. — a 31. 7. b 27. 7. c 26. 7.

Coroll.

1. Itaq; omnis numerus primus ultimum metiens, metitur quoq; omnes alios ultimum præcedentes.

 2. Si

2. Si aliquis numerus non metiens proximum unitati, metiatur ultimum, erit numerus compositus.

3. Si proximus unitati sit primus numerus, nullus alius primus numerus ultimum metietur.

PROP. XIII.

1,	a,	a^2,	a^3,	a^4,
1.	5,	25,	125,	625.
H --	G --	F --	E --	

Si ab unitate quotcunque numeri deinceps proportionales fuerint (a, a^2, a^3, &c.), qui vero post unitatem (a) primus sit; maximum nullus alius metietur, præter eos qui sunt in numeris proportionalibus.

Si fieri potest, alius quispiam E metiatur $a4$,
a cor. 12.9. nempe per F; a erit F alius extra a, a^2, a^3.
b 2 cor 12.9 Quia vero E metiens $a4$ non metitur a, b erit
c 33.7 E numerus compositus; c ergo eum aliquis pri-
d 11. ax. 7. mus metitur, d qui proinde ipsum $a4$ metitur;
e 3 cor 12.9. e ideoque alius non est, quam a. ergo a metitur E. Eodem modo ostendetur F compositus numerus, metiens $a4$, adeoque a ipsum F metiri.
f 9 ax. 7. itaque quum EF f $= a4 = a$ in $a3$. g erit a. E :: F.
g 19.7. $a3$. ergo cum a metiatur E, h æque F metietur
h 20. def 7. a^3, puta per eundem G. k Nec G erit a, vel a^2.
k cor. 11.9. ergo, ut prius, G est numerus compositus, & a eum metitur. quum igitur FG f $= a^3 = a^2$ in a, g erit a. F :: G. a^2; & proinde, quia A metitur F, h æque G metietur a^2, scilicet per eundem H; k qui non est a. ergo quum GH $= a^2 = aa$.
l 10.7. l erit H. a :: a. G. ergo quia a metitur G (ut
m 10 def. 7. prius) m etiam H metietur a, numerum primum. Q.E.N.

PROP.

PROP. XIV.

A, 30.
B, 2. C, 3. D, 5.
E -- F -- .

Si minimum numerum A primi numeri B, C, D metiantur; nullus alius numerus primus E illum metietur, præter eos, qui à principio metiebantur.

Si fieri potest, sit $\frac{A}{E} = F$. a Ergo $A = EF$. b Ergo singuli primi numeri B, C, D ipsorum E, F unum metiuntur; non E, qui primus ponitur; ergo F, minorem scilicet ipso A; contra Hypoth.

a 9 ax. 7.
b 31. 7.

PROP. XV.

A, 9. B, 12. C, 16.
D, 3. E, 4.

Si tres numeri A, B, C deinceps proportionales, fuerint minimi omnium eandem cum ipsis rationem habentium; duo quilibet compositi, ad reliquum primi erunt.

a Sume D, & E minimos in ratione A ad B. b ergo $A = Dq$; b & $C = Eq$; b & $B = DE$. Quia vero D ad E c primus est, d erit $D + E$ primus ad singulos D, & E. * ergo D in $D + E$ e $= D$ q $+ DE$ (f $A + B$) ad E primus est, ideoque ad C vel Eq. Q. E. D. Pari pacto $DE + Eq$ ($B + C$) g ad D primus est, & proinde ad $A = Dq$. Q. E. D. Denique quia B ad $D + E$ h primus est; is ad hujus quadratum k $Dq + 2 DE + Eq$ ($A + 2 B + C$) primus erit. l quare idem B ad $A + B + C$, l adeoque ad $A + C$ primus erit. Q. E. D.

a 35. 7.
b 2. 8.
c 24 7.
d 30 7.
* 26 7.
e 3. 2.
f prius.
g 27. 7.
h 26 7.
k 4. 2.
l 30 7.

PROP. XVI.

A, 3. B, 5. C--- *Si duo numeri A, B primi inter se fuerint; non erit ut primus A ad secundum B, ita secundus B ad alium quempiam C.*

Dic A. B :: B. C. ergo quum A & B in sua
a 23.7. ratione *a* minimi sint, A *b* metietur B æque ac B
b 21.7. c 6. ax.7. ipsum C; sed A *c* seipsum etiam metitur; ergo A & B non sunt primi inter se, contra Hypoth.

PROP. XVII.

A, 8. B, 12. C, 18. D, 27. E---

Si fuerint quotcunque numeri deinceps proportionales A, B, C, D, *extremi autem ipsorum* A, D *primi inter se sint; non erit ut primus* A *ad secundum* B, *ita ultimus* D *ad alium quempiam* E.

Dic A. B :: D. E. ergo vicissim A. D :: B. E.
a 23.7. ergo quum A & D in sua ratione *a* minimi sint,
b 21.7. *b* metietur A ipsum B; *c* quare B ipsum C, & C
c 20. def. 7. d 11. ax.7. sequentem D, *d* adeoque A eundem D metietur. Ergo A & D non sunt primi inter se, contra Hypoth.

PROP. XVIII.

A, 4. B, 6. C, 9. Bq, 16. *Duobus numeris datis* A, B; *considerare an possit ipsis tertius proportionalis* C *inveniri.*

a 9. ax.7. Si A metiatur Bq per aliquem C, *a* erit AC
b per 20.7. = Bq. unde *b* liquet esse A.B :: B.C. Q. E. F.

A, 6. B, 4. Bq, 16. Sin A non metiatur Bq, non erit aliquis tertius proportionalis.

e 7. ax.7. Nam dic A.B :: B.C. *a* ergo AC = Bq. *c* proinde $\frac{Bq}{A}$ = C. Scilicet A metitur Bq, contra Hypoth.

PROP.

PROP. XIX.

A, 8. B, 12. C, 18. D, 27. *Tribus numeris datis A, B, C, considerare an*
E C, 216.

posit ipsis quartus proportionalis D *inveniri.*

Si A metiatur B C per aliquem D, *a* ergo AD = B C; *b* constat igitur esse A. B :: C. D. Q. E. F. a 9. ax. 7. b ex 19. 7.

Sin A non metiatur B C, non datur quartus proportionalis; quod ostendetur, prout in praecedenti.

PROP. XX.

A, 2. B, 3. C, 5. *Primi numeri plures sunt omni proposita multitudine primorum numerorum* A, B, C.
D, 30. G - - - -

a Sit D minimus, quem A, B, C metiuntur. si D+1 primus sit, res patet; si compositus, *b* ergo aliquis primus, puta G, metitur D+1, qui non est aliquis trium A, B, C; nam si ita, quum is *c* totum D+1, & *d* ablatum D metiatur, *e* idem reliquam unitatem metietur. Q. E. A. Ergo propositorum primorum numerorum multitudo aucta est per D+1; vel saltem per G. a 38. 7. b 33. 7. c suppos. d constr. e 12. ax. 7.

PROP. XXI.

A

5 5 3 3 2 2
A ..... E ..... B ... F ... C .. G .. D 20.

Si pares numeri quotcunque AB, BC, CD *componantur, totus* AD *par erit.*

a Sume EB = $\frac{1}{2}$ AB & FC = $\frac{1}{2}$ BC, & GD = $\frac{1}{2}$ CD. *b* liquet EB + FC + GD = $\frac{1}{2}$ AD. *c* ergo AD est par numerus. Q. E. D. a 6. def. 7. b 12. 7. c 6. def. 7.

PROP. XXII.

1 1 1 1
A........F.B......G.C....H.D..L.E 22.
9 7 5 3

Si impares numeri quotcunque AB, BC, CD, DE *componantur, multitudo autem ipsorum sit par; totus* AE *par erit.*

a 7. def. 7. Detracta unitate ex singulis imparibus, *a* manebunt AF, BG, CH, DL numeri pares, & b 21. 9. *b* proinde compositus ex ipsis par erit; adde his c hyp. *c* parem numerum conflatum ex residuis unitati- d 21. 9. bus, *d* totus idcirco AE par erit. Q. E. D.

PROP. XXIII.

7 5 1
A.......B.....C..E.D 15.
3

Si impares numeri quotcunque AB, BC, CD *componantur, multitudo autem ipsorum sit impar; & totus* AD *impar erit.*

Nam dempto CD uno imparium, reliquorum a 22. 9. aggregatus AC *a* est par numerus. huic adde b 21. 9. CD − 1; *b* totus AE est etiam par; quare resti- c 7. def. 7. tuta unitate totus AD *c* impar erit. Q. E. D.

PROP. XXIV.

4 5 1
A....B.....D. C 10.
6

Si à pari numero AC *par* AB *detrahatur, & reliquus* BC *par erit.*

Nam si BD (BC − a 7. def. 7. 1) impar fuerit, *a* erit BC (BD + 1) par. Q. E. D. b hyp. Sin BD parem dicas, propter AB *b* parem, *c* erit c 21. 9. AD par; *a* ideoque AC (AD + 1) impar, contra Hypoth. ergo BC est par. Q. E. D.

PROP.

PROP. XXV.

6 1 3
A...... D . C ... B 10.
7

Si à pari numero AB impar AC detrahatur, & reliquus CB impar erit.

Nam AC − 1 (AD) a est par. b ergo DB est par. c ergo CB (DB − 1) est impar. Q.E.D. a 7. def. 7. b 24. 9. c 7. def. 7.

PROP. XXVI.

4 6 1
A.... C ...... D . B 11.
7

Si ab impari numero AB impar CB detrahatur, reliquus AC par erit.

Nam AB − 1 (AD) & CB − 1 (CD) a sunt pares. b ergo AD − CD (AC) est par. Q.E.D. a 7. def. 7. b 24. 9.

PROP. XXVII.

1 4 6
A . D .... C ...... B 11.
5

Si ab impari numero AB par detrahatur CB, reliquus AC impar erit.

Nam AB − 1 (DB) a est par; & CB ponitur par. b ergo reliquus CD par est. c ergo CD + 1 (CA) est impar. Q.E.D. a 7. def. 7. b 24. 9. c 7. def. 7.

PROP. XXVIII.

A, 3.
B, 4.
AB, 12.

Si impar numerus A parem numerum B multiplicans fecerit aliquem AB, factus AB par erit.

Nam AB a componitur ex impari A toties accepto, quoties unitas continetur in B pari. b ergo AB est par numerus. a hyp. & 15. def. 7. b 21. 9.

Schol.

Eodem modo, si A sit numerus par, erit AB par.

PROP.

PROP. XXIX.

$A, 3.$
$B, 5.$
$\overline{A}\overline{B}, 15.$

Si impar numerus A, imparem numerum B multiplicans fecerit aliquem AB; factus AB impar erit.

a 15. def. 7. b 23. 9.

Nam AB a componitur ex B impari numero toties accepto, quoties unitas includitur in A etiam impari. b ergo AB est impar. Q. E. D.

Scholium.

$B, 12$ (C, 4.
$\overline{A}, 3.$

1. *Numerus A impar numerum B parem metiens, per numerum parem C eum metitur.*

a 9. ax 7. b 29. 9.

Nam si C impar dicatur, quoniam a $B = AC$, erit B impar, contra Hypoth.

$B, 15$ (C, 5.
$\overline{A}, 3$

2. *Numerus A impar numerum B imparem metiens, per numerum C imparem eum metitur.*

a 28 9.

Nam si C dicatur par; a erit A C, vel B par, contra Hypoth.

$B, 15$ (C, 5.
$A, 3$

3. *Omnis numerus (A & C) metiens imparem numerum B, est impar.*

a 28. 9.

Nam si utervis A, vel C dicatur par, a erit B numerus par, contra Hypoth.

PROP. XXX.

$B, 24$ (C, 8. $D, 12$ (E, 4.
$\overline{A}, 3$ $\overline{A}, 3$

a hyp. b 1. Schol. 29 9. c 9. ax. 7. d 1. 2. e hyp. f 7. ax. 1. g 7. ax. 7.

Si impar numerus A parem numerum B metiatur, & illius dimidium D metietur.

a Sit $\frac{B}{A} = C$. b ergo C est numerus par. Sit igitur $E = \frac{1}{2}C$, erit B c $= CA$ d $= 2EA$ e $= 2D$. f ergo $EA = D$; & g proinde $\frac{D}{A} = E$. Q.E.D.

PROP.

Prop. XXXI.

A, 5. B, 8. C, 16. D - - - *Si impar numerus* A *ad aliquem numerum* B *primus sit; & ad illius duplum* C *primus erit.*

Si fieri potest, aliquis D metiatur A, & C. a ergo D metiens imparem A impar erit, b ideoque ipsum B paris C semissem metietur. ergo A, & B non sunt primi inter se, contra Hypoth. a 3. schol. 29.9 b 30.9.

Coroll.

Sequitur hinc, numerum imparem, qui ad aliquem numerum progressionis duplæ primus est, primum quoque esse ad omnes numeros illius progressionis.

Prop. XXXII.

1. A, 2. B, 4. C, 8. D, 16. *Numerorum* A, B, C, D, &c. *à bindrio duplorum unusquisque pariter par est tantum.*

Constat omnes 1, A, B, C, D a pares esse; a 6 def. 7. atque b ∺ nimirum in ratione dupla, & c proinde quemque minorem metiri majorem per aliquem ex illis. b 10 def. 7. c 11. 9. d Omnes igitur sunt pariter pares. d 8. def. 7. Sed quoniam A primus est, e nullus extra eos eorum aliquem metietur. e 13. 9. Ergo pariter pares sunt tantum. Q. E. D.

Prop. XXXIII.

A, 30. B, 15. D - - - E - - *Si numerus* A *dimidium* B *habeat imparem,* A *pariter impar est tantum.*

Quoniam impar numerus B a metitur A per 2 parem, b est B pariter impar. a hyp. b 9. def. 7. Dic etiam pariter parem. c ergo eum par aliquis D per parem E metitur. c 8. def. 7. d 9. ax. 7. unde 2 B d = A d = D E. e quare 2. E :: e 19. 7.

16.def.7 g 20.def.7. E :: D. B. ergo ut $2f$ metitur parem E, g sic D par imparem B metitur. Q. E. D.

PROP. XXXIV.

A, 24. *Si par numerus A, neque à binario duplus sit, neque dimidium habeat imparem; pariter par est, & pariter impar.*

Liquet A esse pariter parem, quia dimidium imparem non habet. Quia vero si A bisarietur, & rursus ejus dimidium, & hoc semper fiat, tandem incidemus in aliquem *a* imparem (quia non in binarium, quoniam A à binario duplus non ponitur) is metietur A per parem numerum (nam *b* alias ipse A impar esset, contra Hypoth.) ergo A est etiam pariter impar. Q. E. D.

a 7. def. 7 b 1 sch. 29. 9.

PROP. XXXV.

A........ 8.

4 8

B..... F .... G 12.

C. .... 18.

9 6 4 8

D........, H...... L..... K........ N 27.

Si sint quotcunque numeri deinceps proportionales A, B G, C, D N, detrahantur autem F G à secundo, & K N ab ultimo, æquales ipsi primo A; erit ut secundi excessus B F ad primum A, ita ultimi excessus DK ad omnes A, BG, C ipsum antecedentes.

Ex DN deme NL = BG, & NH = C.

Quoniam DN. C. (HN) *a* :: HN. BG. (LN) *a* :: LN. (BG) A. (KN.) *b* erit dividendo ubique, D H. HN :: H L. L N :: LK. KN. *c* quare DK. C + BG + A :: LK (*d* BF.) KN. (A.) Q. E. D.

a hyp. b 17. 5. c 12. 5. d 3. ax. 1.

Coroll.

e 18. 5. Hinc *e* componendo, DN + BG + C. A + BG + C :: BG. A.

PROP.

PROP. XXXVI.

1. A, 2. B, 4. C, 8. D, 16.
E, 31. G, 62. H, 124. L, 248. F, 496.
M, 31. N, 465.
P---- Q---

Si ab unitate quotcunque numeri 1, *A*, *B*, *C*, *D*, *deinceps exponantur in dupla proportione, quoad totus compositus* E *fiat primus, & totus hic* E *in ultimum* D *multiplicatus faciat aliquem* F; *factus* F *erit perfectus.*

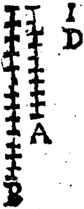
Sume totidem, E, G, H, L etiam in proportione dupla continuè; ergo a ex æquo A. D ∷ E. L. b ergo AL = DE c = F. d ergo L = F/2 quare E, G, H, L, F sunt ∺ in ratione dupla. Sit G − E = M, & F − E = N. e ideo M. E ∷ N. E + G + H + L. f at M = E. g ergo N = E + G + H + L. ergo F = 1 + B + C + D + E + G + H + L = E + N. Quinetiam quia D k metitur DE (F,) l etiam singuli 1, A, B, C m metientes D, m nec non E, G, H, L metiuntur F. Porro nullus alius eundem F metitur. Nam si aliquis, sit P, qui metiatur F per Q. n ergo PQ = F = DE. o ergo E. Q ∷ P. D. ergo cum A primus numerus metiatur D, & p proinde nullus alius P eundem metiatur, q consequenter E non metitur Q. quare cum E primus ponatur, r idem ad Q primus erit. s ergo E & Q in sua ratione minimi sunt, & t propterea E ipsum P ac Q ipsum D æque metiuntur. u ergo Q est aliquis ipsorum A, B, C. Sit igitur B; ergo cum ex æquo sit B. D ∷ E. H; x ideoque BH = DE = F = PQ. x adeoque Q. B ∷ H. P. y erit H = P. ergo P est etiam aliquis ipsorum A, B, C, &c. contra Hypoth. ergo nullus alius præter numeros prædictos eundem F metietur: z proinde F est numerus perfectus. Q. E. D.

a 14. 7.
b 19. 7.
c hyp.
d 7. ax. 7.
e 35. 9.
f 3. ax. 1.
g 4. 5.
h 2. ax. 1.
k 7. ax. 7.
l 11. ax. 7.
m 14. 9.
n 9. ax. 7.
o 19. 7.
p 13. 9.
q 20 def. 7.
r 31. 7.
s 23. 7.
t 21. 7.
u 13. 7.
x 19. 7.
y 14. 5.
z 22. def. 7.

LIB. X.

Definitiones.

I. COmmensurabiles magnitudines dicuntur, quas eadem mensura metitur.



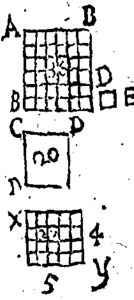
Commensurabilitatis nota est $\sqcap$, *ut* A $\sqcap$ B; *hoc est*, *linea* A 8 *pedum commensurabilis est lineæ* B 13 *pedum*; *quia* D *linea unius pedis singulas* A *&* B *metitur*. *Item* $\sqrt{18} \sqcap \sqrt{50}$; *quia* $\sqrt{2}$ *singulas* $\sqrt{18}$, *&* $\sqrt{50}$ *metitur*. *Nam* $\sqrt{\frac{18}{2}} = \sqrt{9} = 3$. *&* $\sqrt{\frac{50}{2}} = \sqrt{25} = 5$. *quare*. $\sqrt{18}$. $\sqrt{50}$ $::$ 3. 5.

II. Incommensurabiles autem sunt, quorum nullam communem mensuram contingit reperiri.

Incommensurabilitas significatur nota $\sqcap$. *ut* $\sqrt{6}$ $\sqcap$ $\sqrt{25}$ (5;) *hoc est* $\sqrt{6}$ *incommensurabilis est numero* 5, *vel magnitudini hoc numero designatæ*; *quia harum nulla est communis mensura*, *ut postea patebit*.

III. Rectæ lineæ potentia commensurabiles sunt, cum quadrata earum idem spatium metitur.

Au-



Hujusce commensurabilitatis nota est ꟻ, *ut* AB ꟻ CD; *h.e. linea* AB *sex pedum potentia commensurabilis est lineæ* CD, *quæ exprimitur per* $\surd$ 20. *quia spatium* E *unius pedis quadrati metitur tam* ABq (36) *quam rectangulum* XY (20,) *cui æquale est quadratum lineæ* CD($\surd$20.) *Eadem nota* ꟻ *nonnunquam valet potentia tantum commensurabilis.*

IV. Incommensurabiles vero potentia, cum quadratis earum nullum spatium, quod sit communis eorum mensura, contingit reperiri.

Hujusmodi incommensurabilitas denotatur sic; 5 ꟻ $v\surd$ 8; *hoc est, numeri vel lineæ* 5, *&* $v\surd$ 8 *sunt incommensurabiles potentia; quia harum quadrata* 25, *&* $\surd$ 8 *sunt incommensurabilia.*

V. Quæ cum ita sint, manifestum est cuicunque rectæ propositæ, rectas lineas multitudine infinitas, & commensurabiles esse, & incommensurabiles; alias quidem longitudine & potentia, alias vero potentia solum. Vocetur autem proposita recta linea Rationalis.

Hujus nota est ῥ.

VI. Et huic commensurabiles, sive longitudine & potentia, sive potentia tantum, Rationales, ῥ.

VII. Huic vero incommensurabiles Irrationales vocentur.

Hæ sic denotantur ἀ.

VIII. Et quadratum, quod à proposita recta fit, dicatur Rationale ῥν.

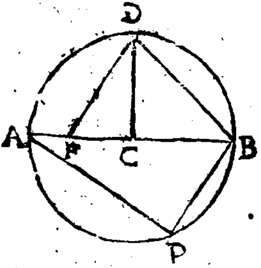
IX. Et huic commensurabilia, quidem Rationalia ῥα.

X. Huic

X. Huic vero incommensurabilia, Irrationalia dicantur, ἄλ.

XI. Et rectæ, quæ ipsa possunt, Irrationales, ἄ.

Schol.



Vt postremæ 7 definitiones exemplo aliquo illustrentur, sit circulus ADBP, *cujus semidiameter* CB; *huic inscribantur latera figurarum ordinatarum; Hexagoni quidem* BP, *Trianguli* AP, *quadrati* BD, *pentagoni* FD. *Itaque si juxta 5 defin. semidiameter* CB *sit Rationalis exposita, numero* 2. *expressa, cui reliquæ* BP, AP, BD, FD *comparandæ sunt,* [a] *erit* BP $=$ BC $=$ 2. *quare* BP *est* ῥ ⊓ BC, *juxta 6. def. Item* AP [b] $= \sqrt{12}$ (*nam* ABq (16) $-$ BPq (4) $= 12$) *quare* AP *est* ῥ ⫟ BC, *etiam juxta 6. def. atque* APq (12) *est* ῥν, *per def. 9. Porro* BD [b] $= \sqrt{}$ DCq $+$ BCq $= \sqrt{8}$; *unde* BD *est* ῥ ⫟ BC; *&* BDq ῥν. *Denique,* FDq $= 10 - \sqrt{20}$ (*ut patebit ex praxi ad* 10. 13. *tradenda*) *erit* ῥν, *juxta* 10 *def. & proinde* FD $= \sqrt{:10 - \sqrt{20}}$ *est* ῥ, *juxta* 11 *defin.*

a cor. 15 4.
b 67. 1.

Postulatum.

POstuletur, quamlibet magnitudinem toties posse multiplicari, donec quamlibet magnitudinem ejusdem generis excedat.

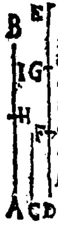
Axiomata.

1. MAgnitudo quotcunque magnitudines metiens, compositam quoque ex ipsis metitur.

2. Magnitudo quamcunque magnitudinem metiens, metitur quoque omnem magnitudinem quam illa metitur.

3. Magnitudo metiens totam magnitudinem & ablatam, metitur & reliquam.

P R O P. I.



Duabus magnitudinibus inæqualibus A B, C *propositis, si à majore* A B *auferatur majus quam dimidium* (A H) *& ab eo* (HB) *quod reliquum est, rursus detrahatur majus quam dimidium* (H I,) *& hoc semper fiat; relinquetur tandem quædam magnitudo* I B, *quæ minor erit proposita minore magnitudine* C.

a Accipe C toties, donec ejus multiplex DE proxime excedat AB; sintque DF $=$ FG $=$ GE $=$ C. Deme ex A B plusquam dimidium AH, & à reliquo HB plusquam dimidium HI; & sic deinceps, donec partes AH, HI, IB æque multæ sint partibus DF, FG, GE. Iam liquet FE, quæ non minor est quam $\frac{1}{2}$ DE, majorem esse quam HB, quæ minor est quam $\frac{1}{2}$ AB $\sqsupset$ DE. Pariterque GE quæ non minor est quam $\frac{1}{2}$ FE, major est quam IB $\sqsupset$ $\frac{1}{2}$ HB. ergo C, vel GE $\sqsubset$ IB. Q. E. D. a post. 10.

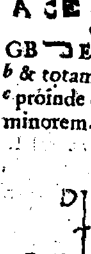
Idem demonstrabitur, si ex AB auferatur dimidium AH, & ex reliquo HB rursus dimidium HI, & ita deinceps.

PROP. II.

Si duabus magnitudinibus inæqualibus propositis (AB, CD) *detrahatur semper minor* AB *de majore* CD, *alterna quadam detractione, & reliqua minime præcedentem metiatur; incommensurabiles erunt ipsæ magnitudines.*

Si fieri potest, sit aliqua E communis mensura. Quopiam igitur A B detracta ex CD, quoties fieri potest, relinquit aliquam F D se minorem, & FD ex A B relinquit G B, & sic deinceps,
a 1. 10. *a* tandem relinquetur aliqua
b hyp. GB ⊐ E. ergo E *b* metiens AB, *c* ideoque CF, *b* & totam C D; *d* etiam reliquam FD, metitur,
c 2 ax. 10.
d 3. ax. 10. *c* proinde & AG; *d* ergo & reliquam GB, seipsa minorem. Q. E. A.

PROP. III.



Duabus magnitudinibus commensurabilibus datis, A B, C D, *maximam earum communem mensuram* F B *reperire.*

Deme A B ex C D, & reliquum ED ex A B, & F B ex E D, donec FB metiatur ED; (quod tandem fiet,
a 2. 10. *a* quia per Hyp. AB ⊤⊥ CD) erit FB quæsita.

b constr.
c 1 ax. 10.
d 1 ax. 10. Nam FB *b* metitur ED, *c* ideoque ipsam AF; sed & seipsam, *d* ergo etiam AB, & *c* propterea CE, *d* adeoque & totam C D. Proinde F B communis est mensura ipsarum AB, CD. Dic G communem quoq; esse mensuram, hac majorem; ergo G metiens AB, & CD, *e* metitur CE, & *f* reliquam
e 2. ax. 10.
f 3. ax. 10. E D, *e* ideoque A F, & *f* proinde reliquam F B, major minorem. Q. E. A.

Coroll.

Coroll.

Hinc, magnitudo metiens duas magnitudines, metitur & maximam earum mensuram communem.

PROP. IV.

A ————————————
B ——————————D——————
C ——————E————F——————

Tribus magnitudinibus commensurabilibus datis A, B, C; *maximam earum mensuram communem invenire.*

a Inveni D maximam communem mensuram duarum quarumcunque A, B; *a* item E ipsarum D & C maximam communem mensuram; erit E quæsita. a 3. 10.

b Nam perspicuum est E metiens D & C *b* metiri tres A, B, C. Puta aliam F hac majorem easdem metiri. *c* ergo F metitur D; *c* proinde & E, ipsorum D, C maximam communem mensuram, major minorem. Q. E. A. b constr. & 2. ax. 10. c cor. 3. 10.

Coroll.

Hinc quoque, magnitudo metiens tres magnitudines, metitur quoque maximam earum communem mensuram.

PROP. V.

A ———————— D. 4.
C ——— F. 1.
B —————— E. 3.

Commensurabiles magnitudines A, B *inter se rationem habent, quam numerus ad numerum.*

a Inventa C ipsarum A, B maxima communi mensura; quoties C in A & B, toties 1 continetur in numeris D & E. *b* ergo C. A :: 1. D; quare inverse A. C :: D. 1. *b* atqui etiam C. a 3. 10. b 20. def. 7.

c 22. 5. B :: 1. E. c ergo ex æquali A. B :: D. E :: N. N. Q. E. D.

Prop. VI.

E ——— F. 1.
A ——————— C. 4.
B —————— D. 3.

Si duæ magnitudines A, B inter se proportionem habeant, quam numerus C ad numerum D; commensurabiles erunt magnitudines A, B.

a sch. 10. 6. b constr. c hyp. d 22. 5. e 5 ax. 7. f 20. def. 7. g constr. h 1. def. 10.

Qualis pars est 1 numeri C, a talis fiat E ipsius A. Quoniam igitur E. A b :: 1. C. atque A. B c :: C. D; d ex æquo erit E. B :: 1. D. ergo quum 1 e metiatur numerum D, f etiam E metitur B; sed & ipsum A g metitur. h ergo A ⊓⊔ B. Q. E. D.

Prop. VII.

A ———————
B ——————

Incommensurabiles magnitudines A, B inter se proportionem non habent, quam numerus ad numerum.

a 6. 10. Dic A. B :: N. N. a ergo A ⊓⊔ B, contra Hypoth.

Prop. VIII.

A ————
B ————

Si duæ magnitudines A, B inter se proportionem non habeant, quam numerus ad numerum, incommensurabiles erunt magnitudines.

a 5. 10. Puta A ⊓⊔ B a ergo A. B :: N. N, contra Hypoth.

PROP. IX.

A ——————
B ————
E, 4.
F, 3.

Quæ à rectis lineis longitudine commensurabilibus fiunt quadrata, inter se proportionem habent, quam quadratus numerus ad quadratum numerum; & quadrata inter se proportionem habentia, quam quadratus numerus ad quadratum numerum, & latera habebunt longitudine commensurabilia. Quæ vero à rectis lineis longitudine incommensurabilibus fiunt quadrata, inter se proportionem non habent, quam quadratus numerus ad quadratum numerum: & quadrata inter se proportionem non habentia, quam quadratus numerus ad quadratum numerum, neq; latera habebunt longitudine commensurabilia.

1. *Hyp.* A. ⊓⊔ B. Dico Aq. Bq :: Q. Q. Nam [a] sit A. B :: num. E. num. F. ergo $\frac{Aq}{Bq}\left(^{b}\ \frac{A}{B}\text{bis}\right)^{c} = \frac{E}{F}\text{bis.} \overset{d}{=} \frac{Eq}{Fq}$ [e] ergo Aq. Bq :: Eq. Fq :: Q. Q. Q. E. D. — a per 9. 10. b 20. 6. c sch. 23. 5. d 11. 8. e 11. 5.

2. *Hyp.* Aq. Bq :: Eq. Fq :: Q. Q. Dico A ⊓⊔ B. Nam $\frac{A}{B}$ bis $\left(^{f}\frac{Aq}{Bq}\right)^{g} = \frac{Eq}{Fq} \overset{h}{=} \frac{E}{F}$ bis. [i] ergo A. B :: E. F :: N. N. [k] quare A ⊓⊔ B. Q. E. D. — f 20. 6. g hyp. h 11. 8. i sch. 23. 5. k 6. 10.

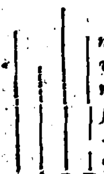
3. *Hyp.* A ⊓̸⊔ B. Nego esse Aq. Bq :: Q. Q. Nam dic Aq. Bq :: Q. Q. Ergo A ⊓⊔ B, ut modo ostensum est, contra Hypoth.

4. *Hyp.* Non Aq. Bq :: Q. Q. Dico A ⊓̸⊔ B. Nam puta A ⊓⊔ B; ergo Aq. Bq :: Q. Q. ut modo diximus, contra Hypoth.

Coroll.

Lineæ ⊓⊔ sunt etiam ⊓⊔̲; at non contra. Sed lineæ ⊓̸⊔ non sunt idcirco ⊓̸⊔̲. Lineæ vero ⊓̸⊔̲ sunt etiam ⊓̸⊔.

PROP. X.



Si quatuor magnitudines proportionales fuerint (C. A :: B. D;) prima vero C *secundæ* A *fuerit commensurabilis; & tertia* B *quartæ* D *commensurabilis erit. Et si prima* C *secundæ* A *fuerit incommensurabilis, & tertia* B *quartæ* D *incommensurabilis erit.*

Si C ⊤⊥ A, *a* ideo erit C. A :: N. N *b* :: B. D. *b* ergo B ⊤⊥ D. Sin C ⊤⊥ A, ergo *c* non erit C. A :: N.N :: B. D. *d* quare B ⊤⊥ D. Q. E. D.

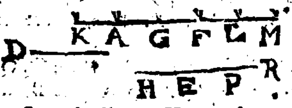
a 5. 10.
b 6. 10.
c 7. 10.
d 8. 10.

LEMMA 1.

Duos numeros planos invenire, qui proportionem non habeant, quam quadratus numerus ad quadratum numerum.

Huic Lemmati satisfacient duo quilibet numeri plani non similes, quales sunt numeri habentes proportionem superparticularem, vel superbipartientem, vel duplam; vel etiam duo quivis numeri primi. vid. Schol. 27. 8.

LEMMA 2.



B, 5, C, 3.

Invenire lineam HR, *ad quam data recta linea* KM *sit in ratione datorum numerorum* B, C.

a sch. 10. 6. *a* Divide K M in partes æquales æque multas unitatibus numeri B. harum tot, quot unitates sunt in numero C, *b* componant rectam H R. liquet esse KM. HR :: B. C.

b 3. 1.

LEMMA 3.

Invenire lineam D, *ad cujus quadratum datæ rectæ* K M *quadratum sit in ratione datorum numerorum* B, C.

Fac

Fac *B.* C *a* :: KM. HR. ac inter KM, & HR *b* inveni mediam proportionalem D. Erit KMq. Dq *c* :: KM. HR *d* :: B. C. a 2. *lem.* 10. 10. b 13. 6. c 20. 6. d *constr.*

PROP. XI.

A ——————— B. 20.
E —————— C. 16.
D ——————

Propositæ rectæ lineæ A invenire duas rectas lineas incommensurabiles; alteram quidem D longitudine tantum, alteram vero E etiam potentia.

1. Sume numeros B, C, *a* ita ut non sit B. C :: Q. Q. *b* fiatque B. C :: Aq. Dq. *c* liquet A ⊐⊏ D. Sed Aq *d* ⊐⊏ Dq. Q. E. F. a 2 *lem.* 10. 10. b 3. *lem.* 10. 10. c 9. 10. d 6. 10

2. *d* Fac A. E :: E. D. Dico Aq ⊐⊏ Eq. Nam A. D *e* :: Aq. Eq. ergo cum A ⊐⊏ D, ut prius, *f* erit Aq ⊐⊏ Eq. Q. E. F. d 13. 6. e 20. 6. f 10. 10.

PROP. XII.

Quæ (A, B) eidem magnitudini C sunt commensurabiles, & inter se sunt commensurabiles.

Quia A ⊐⊏ C, & C ⊐⊏ B, *a* sit A. C :: N. N :: D. E. atque C. *B* :: N. N :: F. G. *b* sumantur tres numeri H, I, K minimi ∺ in rationibus D ad E, & F ad G. Iam quia A. C *c* :: D. E *c* :: H. I. ac C. B *c* :: F. G. *c* :: I. K. *d* erit ex æquali A. B :: H. K :: N. N. *e* ergo A ⊐⊏ B. Q. E. D. a 5. 10. b 4 8. c *constr.* d 22. 5. e 6. 10.

D, 18. E, 8.
F, 2. G, 3.
H, 5. I, 4. K, 6.
A B C

Schol.

Hinc, omnis recta linea rationali lineæ commensurabilis, est quoque ρ rationalis. Et omnes rectæ rationales inter se commensurabiles sunt, saltem potentia. Item, omne spatium rationali spatio commensurabile, est quoque rationale; & omnia spatia rationalia inter se com- 12. 10 & *def.* 6. *def.* 9.

mensurabilia sunt. Magnitudines vero, quarum altera est rationalis, altera irrationalis, sunt inter se incommensurabiles. *def. 7. & 10.*

PROP. XIII.

A ——————
C ————
B ——————

Si sint duæ magnitudines A, B; *& altera quidam* A *eidem* C *sit commensurabilis, altera vero* B *incommensurabilis; incommensurabiles erunt magnitudines* A, B.

a hyp. *b 22. 10.* Dic B ⊤⊔ A. ergo cum C *a* ⊤⊔ A, *b* erit C ⊤⊔ B, contra Hypoth.

PROP. XIV.

Si sint duæ magnitudines commensurabiles A, B; *altera autem ipsarum* A *magnitudini cuipiam* C *incommensurabilis fuerit; & reliqua* B *eidem* C *incommensurabilis erit.*

a hyp. *b 22. 10.* A B C Puta B ⊤⊔ C. ergo cum A ⊤⊔ *a* B, *b* erit A ⊤⊔ C, contra Hyp.

PROP. XV.

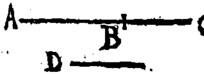
A ——————
B ——————
C ——————
D ————

Si quatuor rectæ lineæ proportionales fuerint (A. B :: C. D;) *prima vero* A *tanto plus possit quam secunda* B, *quantum est quadratum rectæ lineæ sibi commensurabilis longitudine; & tertia* C *tanto plus poterit, quam quarta* D, *quantum est quadratum rectæ lineæ sibi longitudine commensurabilis. Quod si prima* A *tanto plus possit quam secunda* B, *quantum est quadratum rectæ lineæ sibi incommensurabilis longitudine; & tertia* C *tanto plus poterit, quam quarta* D, *quantum est quadratum rectæ lineæ sibi longitudine incommensurabilis.*

a hyp. *b 22. 6.* *c 17. 5.* Nam quia A. B *a* :: C. D. *b* erit Aq. Bq :: Cq. Dq. *c* ergo dividendo Aq − Bq. Bq :: Cq − Dq.

Dq. Dq. *d* quare √ : Aq − Bq. B :: √ : Cq − Dq. d 22.6.
D. *e* invertendo igitur B. √ : Aq − Bq :: D. √ : e cor.4.5. f 22.5.
Cq − Dq. *f* ergo ex æquali A. √ : Aq − Bq ::
C. √ : Cq − Dq. proinde si A ⊓⊔, vel ⊓⊔ √
Aq − Bq, *g* erit similiter C ⊓⊔, vel ⊓⊔ √ : g 10.10.
Cq − Dq. Q. E. D.

PROP. XVI.



Si duæ magnitudines commensurabiles AB, BC *componantur; & tota magnitudo* AC *utrique ipsarum* AB, BC *commensurabilis erit: quod si tota magnitudo* AC *uni ipsarum* AB, *vel* BC *commensurabilis fuerit; & quæ à principio magnitudines* AB, BC *commensurabiles erunt.*

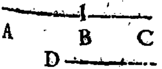
1. *Hyp.* *a* Sit D ipsarum AB, BC communis mensura, *b* ergo D metitur AC. *c* ergo AC ⊓⊔ AB, & BC. Q. E. D. a 3.10. b 1.ax.10. c 1.def.10.

2. *Hyp.* *a* Sit D communis mensura ipsarum AC, AB; *d* ergo D metitur AC − AB (BC;) *c* proinde AB ⊓⊔ BC. Q. E. D. d 3.ax.10.

Coroll.

Hinc etiam, si tota magnitudo ex duabus composita, commensurabilis sit alteri ipsarum, eadem & reliquæ commensurabilis erit.

PROP. XVII.



Si duæ magnitudines incommensurabiles AB, BC *componantur, & tota magnitudo* AC *utrique ipsarum* AB, BC *incommensurabilis erit: Quod si tota magnitudo* AC *uni ipsarum* AB *incommensurabilis fuerit, & quæ à principio magnitudines* AB, BC *incommensurabiles erunt.*

1. *Hyp.*

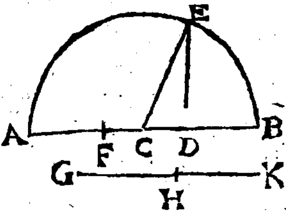
a 3 ax.10. b 1.def.10. 1. *Hyp.* Si fieri potest, sit D ipsarum AC, AB communis mensura. *a* ergo D metitur AC − AB (BC.) *b* ergo AB $\top\!\!\perp$ BC, contra Hypoth.

c 16, 10. 2. *Hyp.* Dic AB $\top\!\!\perp$ BC. *c* ergo AC $\top\!\!\perp$ AB, contra Hypoth.

Coroll.

Hinc etiam, si tota magnitudo ex duabus composita, incommensurabilis sit alteri ipsarum, eadem & reliquæ incommensurabilis erit.

PROP. XVIII.

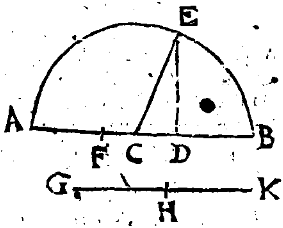


Si fuerint duæ rectæ lineæ inæquales AB, GK; *quartæ autem parti quadrati, quod fit à minori* GK, *æquale parallelogrammum* ADB *ad majorem* AB *applicetur, deficiens figura quadrata, & in partes* AD, DB *longitudine commensurabiles ipsam dividat; major* AB *tanto plus poterit quam minor* GK, *quantum est quadratum rectæ lineæ* FD *sibi longitudine commensurabilis. Quod si major* AB *tanto plus possit, quam minor* GK, *quantum est quadratum rectæ lineæ* FD *sibi longitudine commensurabilis; quartæ autem parti quadrati, quod fit à minori* GK, *æquale parallelogrammum* ADB *ad majorem* AB *applicetur, deficiens figura quadrata, in partes* AD, DB *longitudine commensurabiles ipsum dividet.*

a 10. 1. b 28.6 c 8 2. d constr. & 4 2. *a* Biseca GK in H; & *b* fac rectang. ADB = GHq: abscinde AF = DB. Estque ABq *c* = 4 ADB *d* (4 GHq, vel GKq) + FDq. Iam primo

primo, Si AD ⊓⊔ DB, erit AB e ⊓⊔ DB e ⊓⊔ e 16. 10.
2 DB f (AF + DB, vel AB − FD) g ergo f constr. g cor. 16 10
AB ⊓⊔ FD. Q. E. D. Sin secundo, AB ⊓⊔ h cor. 16. 10.
FD, h erit ideo AB ⊓⊔ AB − FD (2 DB) k 12. 10. l 16 10.
k ergo AB ⊓⊔ DB. l quare AD ⊓⊔ DB.
Q. E. D.

Prop. XIX.



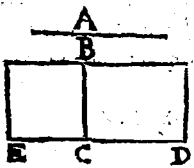
Si fuerint duæ rectæ lineæ inæquales, AB, GK; quartæ autem parti quadrati, quod fit à minore GK, æquale parallelogrammum ADB ad majorem AB applicetur, deficiens figura quadrata; & in partes incommensurabiles longitudine AD, DB, ipsam AB dividat; major AB tanto plus poterit, quam minor GK, quantum est quadratum rectæ lineæ FD, sibi longitudine incommensurabilis. Quod si major AB tanto plus possit, quam minor GK, quantum est quadratum rectæ lineæ FD sibi longitudine incommensurabilis; quartæ autem parti quadrati, quod fit à minore GK, æquale parallelogrammum ADB ad majorem AB applicetur, deficiens figura quadrata; in partes longitudine incommensurabiles AD, DB ipsam AB dividet.

Facta putas, & dicta eadem, quæ in præcedenti. Itaque primo, Si AD ⊓⊔ DB, a erit propterea AB ⊓⊔ DB; b quare AB ⊓⊔ 2 DB (AB − FD) c ergo AB ⊓⊔ FD. Q. E. D. a 17. 10. b 13. 10. c cor. 17 10.

Secundo, Si AB ⊓⊔ FD; c ergo AB ⊓⊔ AB − FD (2 DB;) d quare AB ⊓⊔ DB, & e proinde AD ⊓⊔ DB. Q. E. D. d 13. 10. e 17. 10.

Prop.

PROP. XX.



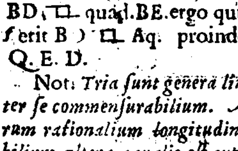
Quod sub rationalibus longitudine commensurabilibus rectis lineis B C, C D, *secundum aliquem prædictorum modorum, continetur rectangulum* B D, *rationale est.*

a 46 1. Exponatur A, ρ̊. & a describatur BE quadra-
b 1. 6. tum ex B C. Quoniam D C. C E (B C) b ::
c hyp. B D. B E. & D C c ⊓ B C; d erit rectang.
d 10. 10. BD ⊓ quad. BE. ergo quum quad. BE e ⊓ Aq;
e hyp & 9. def. 10. f erit B) ⊓ Aq. proinde rectang. BD est ρ̊.
f 12. 10. Q. E. D.

Not. *Tria sunt genera linearum rationalium inter se commensurabilium. Aut enim duarum linearum rationalium longitudine inter se commensurabilium altera æqualis est expositæ rationali; aut neutra rationali expositæ æqualis est, longitudine tamen ei utraque est commensurabilis; aut denique utraque expositæ rationali commensurabilis est solum potentia. Hi sunt modi illi, quos innuit præsens theorema.*

In numeris, sit BC, $\sqrt{8}$ ($2\sqrt{2}$) & CD, $\sqrt{18}$ ($3\sqrt{2}$), erit rectang. BD $= \sqrt{144} = 12$.

PROP. XXI.



Si rationale D B *ad rationalem* D C *applicetur, latitudinem* G B *efficit rationalem, & ei* D C *ad quam applicatum est* DB, *longitudine commensurabilem.*

a 1. 6. Exponatur G; ρ̊. & describatur D A quadra-
b hyp. tum ex B C. quoniam B D. D A a :: B C. C A;
c sch. 12. 10. & 10. 10. atque, BD DA b sunt ρ̊a, c ideoque ⊓; d erit

BC

BC ⊔⊔ CA. at CD (CA) ^{b} est $\overset{\circ}{\rho}$. ^{e} ergo BC est $\overset{\circ}{\rho}$. Q. E. D. e *sch.* 12. 10.

In numeris, sit rectang. DB, 12; & DC, $\sqrt{8}$. erit CB, $\sqrt{18}$. atqui $\sqrt{18} = 3\sqrt{2}$, & $\sqrt{8} = 2 \times \sqrt{2}$.

LEMMA.

A———

B————

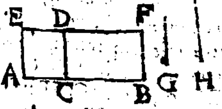
C———

Duas rectas rationales potentia solum commensurabiles invenire.

Sit A exposita $\overset{\circ}{\rho}$. ^{a} Sume B ⊓⊔ A, ^{a}& C ⊓⊔ B. ^{b} liquet B, & C esse quæsitas.

a 11. 10.
b *sch.* 12. 10.

PROP. XXII.



Quod sub rationalibus DC, CB potentia solum commensurabilibus rectis lineis continetur rectangulum DB, irrationale est; & recta linea H ipsum potens, irrationalis; vocetur autem Media.

Sit G exposita $\overset{\circ}{\rho}$. & describatur DA quadratum ex DC; sitque Hq = DB. Quoniam AC. CBq :: DA. DB. ^{b} atque AC ⊔⊔ CB, ^{c} erit DA ⊔⊔ DB (Hq). ^{d} atqui Gq ⊓⊔ DA. ^{e} ergo Hq ⊔⊔ Gq. ^{f} ergo H est $\dot{\rho}$. Q. E. D. Vocetur autem Media. quia AC. H :: H. CB.

a 1. 6.
b *hyp.*
c 10. 10.
d *hyp.* & 9. *def.* 10.
e 13. 10.
f 11. 10.

In numeris, sit DC, 5; & CB, $\sqrt{6}$. erit rectangulum DB (Hq) $\sqrt{54}$. quare H est $v\sqrt{54}$.

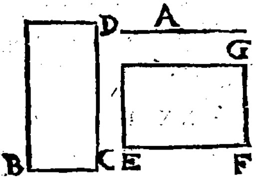
Mediæ nota est μ, Medii vero $\mu\nu$; pluraliter $\mu\alpha$.

SCHOL.

Omne rectangulum, quod potest contineri sub duabus rectis rationalibus potentia solum commensurabilibus, est Medium; quamvis contineatur sub duabus rectis irrationalibus; atque omne

omne Medium poteſt contineri ſub duabus rectis rationalibus potentia tantum commenſurabilibus, ut exemp. gr. $\sqrt{24}$ eſt $\mu\nu$. quia continetur ſub $\sqrt{3}$, & $\sqrt{8}$, qui ſunt $\acute{\rho}$ ⊓⊔. etſi poſſet contineri ſub $\nu\sqrt{6}$, & $\nu\sqrt{96}$ irrationalibus; nam $\sqrt{94} = \nu\sqrt{576} = \nu\sqrt{6}$ in $\nu\sqrt{96}$.

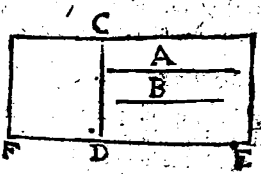
PROP. XXIII.



Quod (BD) *à media* A *fit, ad rationalem* BC *applicatum, latitudinem* CD *rationalem efficit, & ei* BC, *ad quam applicatum eſt* BD *longitudine incommenſurabilem.*

a ſch 22.10. Quoniam A eſt μ, a erit Aq rectangulo ali-
b 1 ax. 1. cui (EG) æquale contento ſub EF, & FG
c 14 6. $\acute{\rho}$ ⊓⊔. b ergo BD = EG. c quare BC. EF :: FG.
d 22. 6. GD. d ergo BCq. EFq :: FGq. CDq. ſed BCq,
e hyp. & EFq e ſunt $\acute{\rho}\alpha$, f ideoque ⊓⊔. g ergo FGq ⊓⊔
f ſch. 12.10. CDq. Ergo quum FG ſit $\acute{\rho}$, h erit CD $\acute{\rho}$. Por-
g 10. 10. ro, quia EF. FG k :: EFq. EG (BD); ob
h ſch. 12.10. EF ⊓⊔ FG, l erit EFq ⊓⊔ BD. verum EFq
k 1. 6. m ⊓⊔ CDq, n ergo rectang. BD ⊓⊔ CDq.
l 10. 10. quum igitur CDq. BD o :: CD. BC. p erit CD
m ſch. 12.10. ⊓⊔ BC. ergo, &c.
n 13. 10.
o 1. 6.
p 10. 10.

PROP. XXIV.



Mediæ A *commenſurabilis* B, *media eſt.*

a 11. 6. Ad CD $\acute{\rho}$ a fac rectang. CE = Aq; a & rectang. CF = Bq. Quoniam Aq (CE) eſt $\mu\nu$, b & CD $\acute{\rho}$, c erit latitudo DE

b hyp.
c 23. 10

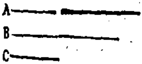
DE ρ́ ⊔ CD. Quoniam vero CE. CF d :: ED. DF, & CE e ⊔ CF, f erit ED ⊔ DF. g ergo DF est ρ́ ⊔ CD. h ergo rectang. CF (Bq) est μν. & proinde B est μ. Q. E. D. d 1. 6. e hyp. f 10 10. g 2. & 13 10. h 23. 10.

Nota quod signum ⊔ plerumque valet potentia tantum commensurabile, ut in hac demonstratione, & in præced. &c. quod intellige, ut ex usu erit, & juxta citatione.

Coroll.

Hinc liquet spatium medio spatio commensurabile medium esse.

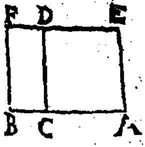
LEMMA.



Duas rectas medias A, B *longitudine commensurabiles ; item duas* A, C *potentia tantum commensurabiles invenire.*

a Sit A μ quævis; sume B ⊔ A; c & C ⊔ A. d Factum esse liquet. a 1 lem 24 10. & 13. 6 b 2. lem. 10. 10. c 3. lem. 10. 10. d constr. & 24 10.

PROP. XXV.

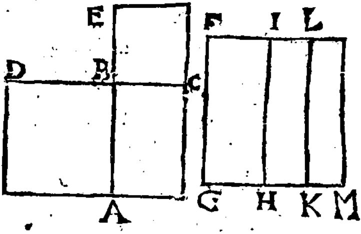


Quod sub DC, CB *mediis longitudine commensurabilibus rectis lineis continetur rectangulum* DB, *medium est.*

Super DC construatur quadratum DA. Quoniam AC. (DC) CB a :: DA. DB. & DC ⊔ CB; b erit DA ⊔ DB. c ergo DB est μν. Q. E. D. a 1. 6 b 10. 10. c 24. 10.

PROP.

PROP. XXVI.



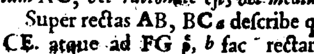
Quod sub mediis potentia tantum commensurabilibus rectis lineis A B, BC *continetur rectangulum* AC, *vel rationale est, vel medium.*

Super rectas AB, BC *a* describe quadrata AD, CE. atque ad FG ρ̂, *b* fac rectangula FH= AD, *b* & IK=AC, *a b* & LM=CE.
a 46 1.
b cor. 16. 6.

Quadrata AD, CE, hoc est, rectangula FH, LM *c* sunt μα, & ⊓; ergo eandem habentes rationem GH, KM sunt *d* ρ̂, & *e* ⊓. *f* ergo GH×KM est ρυ. atqui quia AD, AC, CE, hoc est FH, IK, LM *g* sunt ∺; & *h* proinde GH, HK, KM etiam ∺, *k* erit HKq=GH×KM; *l* ergo HK est ρ̂; vel ⊓, vel ⊔ IH (GF); si ⊓, *m* ergo rectang. IK vel AC est ρυ. Sin ⊔, *m* ergo AC est μυ. Q. E. D.
c cap. & 24. 20.
d 13. 10.
e 10. 10.
f 20. 10.
g sch. 23. 6.
h 1. 6.
k 17. 6.
l 12. 10.
m 20. 10
m 22. 10.

LEMMA.

Si A, *&* E *sint* ⊔ *tantum;*



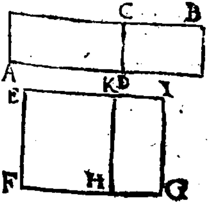
Erunt primo, Aq, Eq, Aq+Eq, Aq—Eq ⊓.
Erunt secundo, Aq, Eq, Aq+Eq, Aq—Eq ⊓, AE, & 2 AE. Nam A. E *b* :: Aq. AE *b* :: AE. Eq, ergo cum A *c* ⊓ E. *d* erit Aq ⊓ AE, *e* & 2 AE. item Eq *d* ⊓ A E, *e* & 2 AE. quare cum Aq+Eq ⊓ Aq, & Eq; & Aq—Eq ⊓ Aq, & Eq,

a hyp. & 16. 10.
b 1, 6.
c hyp.
d 10. 10.
e 14. 10.

Eq, *f*erunt Aq+Eq, *f* & Aq−Eq ⊔⊓ AE, & 2 AE. *f* 14. 10.

Hinc erunt tertio, Aq, Eq, Aq+Eq, Aq−Eq, 2 AE *g* ⊔⊓ Aq+Eq+2 AE; & Aq+Eq−2 AE. *g* & Aq+Eq+2 AE ⊔⊓ Aq+Eq−2 AE. *h* (Q. A−E.) *g* 14, 16, & 17 10. *h* cor. 7. 10.

P R O P. XXVII.

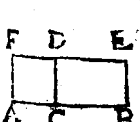


Medium A B *non superat medium* AC *rationali* DB.

Ad EF ρ^c, *a* fac EG = AB, *a* & EH = AC. Rectangula AB, AC, hoc est, EG, EH *b* sunt $\mu\alpha$, *c* ergo FG, & FH sunt ρ^c ⊓⊥ EF. itaque si KG, *d* id est DB sit $\rho^c v$, *e* erit HG ⊔⊓ HK; *f* quare HG ⊓⊥ FH. *g* ergo FGq ⊔⊓ FHq. sed FH est ρ^c. *h* ergo FG est ρ'. verum prius erat FG ρ^c. Quæ repugnant.

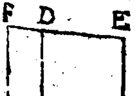
a cor. 16. 6. *b* hyp. *c* 23. 10. *d* 3. ax. 1. *e* 21. 10. *f* 13. 10. *g* lem. 26 10. *h* sch. 12. 10.

S C H O L.



1. *Rationale* A E *superat rationale* AD *rationali* CE.

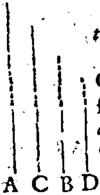
Nam A E *a* ⊔⊓ AD; *b* ergo AE ⊔⊓ CE. *c* quare CE est $\rho^c v$. Q. E. D. *a* hyp. *b* cor. 16. 10 *c* sch. 12. 10



2. *Rationale* AD *cum rationali* C F. *facit rationale* AF.

Nam AD *a* ⊔⊓ CF; *b* quare A F ⊔⊓ AD, & CF. *c* proinde A F est $\rho^c v$. Q. E. D. *a* sch 12. 10. *b* 16 10. *c* sch. 12. 10.

PROP. XXVIII.



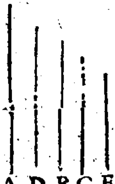
Medias invenire (C, & D) quæ rationale CD contineant.

a lem.21.10.
b 13.6.
c 12.6.
d 22.10.
e constr.
f 10.10.
g 24.10.
h 17.6.
h sch.12.10.

a Sume A, & B $\dot{\rho}$ ⊓. *b* fac A. C :: C. B. *c* atque A, B :: C. D. Dico factum. Nam AB (Cq) *d* est $\mu\nu$; *d* unde C est μ. quum vero A. B *e* :: C. D, *f* erit C ⊓ D. *g* ergo D est μ. porro permutando A. C :: B. D. *e* hoc est C. B :: B. D. *h* ergo Bq = CD. atqui Bq *e* est $\dot{\rho}\nu$. *h* ergo CD est $\dot{\rho}\nu$. Q. E. F.

In numeris, sit A, $\sqrt{2}$; & B, $\sqrt{6}$. ergo C est $\nu\sqrt{12}$. fac $\sqrt{2}.\sqrt{6} :: \nu\sqrt{12}$. D. vel $\nu\sqrt{4}.\nu\sqrt{36} :: \nu\sqrt{12}$. D. erit D, $\nu\sqrt{108}$. atqui $\nu\sqrt{12}$ in $\nu\sqrt{108} = \nu\sqrt{1296} = \sqrt{36} = 6$. ergo CD est 6. item C. D :: 1. $\sqrt{3}$. quare C ⊓ D.

PROP. XXIX.



Medias invenire potentia tantum commensurabiles D, & E, quæ medium DE contineant.

a lem.21.10.
b 13.6.
c 12.6.
d 17.6.
e 22.10.
f constr.
g 10.10.
h 24.10
k constr. & cor.4.5.
l 16.6.
m 22.6.

a Sume A, B, C $\dot{\rho}$ ⊓. Fac A. D *b* :: D. B. *c* & B. C :: D. E. Dico factum.

Nam AB *d* = Dq & AB *e* est $\mu\nu$; ergo D est μ. & B *f* ⊓ C. *g* ergo D ⊓ E. *h* ergo E est μ. porro, B. C *f* :: D. E, & permutando B. D :: C. E. *k* hoc est D. A :: C. E. *l* ergo DE = AC. Sed AC *m* est $\mu\nu$. ergo DE est $\mu\nu$. Q. E. D.

In numeris sit A, 20; & B, $\sqrt{200}$; & C, $\sqrt{80}$. Ergo D est $\sqrt{\sqrt{80000}}$; & E $\nu\sqrt{12800}$. Ergo DE = $\sqrt{\sqrt{1024000000}} = \sqrt{32000}$. & D. E :: $\sqrt{10}$. 2. quare D ⊓ E.

Schol.

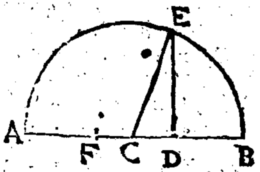
SCHOL.

A, 6.	C, 12.
B, 4.	D, 8.
AB, 24.	CD, 96.
A, 6.	C, 5.
B, 4.	D, 8.
AB, 24.	CD, 40.

Invenire duos numeros planos similes vel dissimiles.

Sume quoscunque quatuor numeros proportionales, A.B :: C. D. liquet AB, & CD esse similes planos. Planos autem dissimiles quotcunque reperies ope scholii 27. 8.

LEMMA.



1. *Duos numeros quadratos* (DEq & CDq) *invenire, ita ut compositus ex ipsis* (CEq) *quadratus etiam sit.*

Sume AD, DB numeros planos similes (quorum ambo pares sint, vel ambo impares) nimirum AD, 24. & DB, 6. Horum summa, (AB) est 30; differentia (F D) 18, cujus semissis (CD) est 9. *a* Habent vero plani similes A D, DB unum medium numerum proportionalem, nempe DE. patet igitur singulos numeros CE, CD, DE rationales esse; proinde CEq (*b* CDq + DEq) est numerus quadratus requisitus. a 18. 8. b 47. 1.

Facile itaque invenientur duo numeri quadrati, quorum excessus sit quadratus, vel non quadratus numerus. nempe ex eadem constructione, *c* erit CEq − CDq = DEq. c 3 ax. 1.

Quod si A D, D B sint numeri plani dissimiles, non erit media proportionalis (DE) numerus rationalis; proinde quadratorum CEq, CDq excessus (DEq) non erit numerus quadratus.

LEMMA 2.

2. *Duos numeros quadratos* B, C *invenire, ita ut compositus ex ipsis* D, *non sit quadratus. item, quadratum numerum* A *dividere in duos numeros* B, C *non quadratos.*

A, 3. B, 9. C, 36. D, 45.

1. Sume numerum quemlibet quadratum B, sitque C=4B; & D=B+C. Dico factum.

Nam B est Q. ex constr. item quia B. C :: 1. 4 :: Q. Q. *a* erit C etiam quadratus. Sed quoniam B+C. (D) C :: 5.4 :: non Q. Q. *b* non erit D numerus quadratus. Q. E. F.

a 24. 8.
b cor. 24 8.

A, 36. B, 24. C, 12. D, 3. E, 2. F, 1.

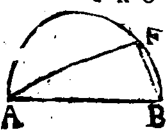
2. Sit A numerus quivis quadratus. Accipe D, E, F numeros planos dissimiles, sitque D=E+F. fac D. E :: A. B. & D. F :: A. C. Dico factum.

Nam quia D. E+F :: A. B+C. & D=E+F, *a* erit A=B+C. Iam dic B quadratum esse. *b* ergo A & B, & *c* proinde D & E, sunt numeri plani similes, contra Hypoth. idem absurdum sequetur, si C dicatur quadratus. ergo, &c.

a 14. 5.
b 21. def. 7.
c 26. 8.

PROP.

PROP. XXX.



C.... E..... D

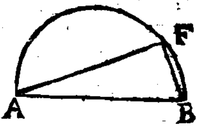
Invenire duas rationales A B, A F *potentia tantum commensurabiles, ita ut major* A B *plus possit, quam minor* A F, *quadrato rectæ lineæ* B F *longitudine sibi commensurabilis.*

Exponatur AB, ρ'. *a* Sume CD, CE numeros quadratos, ita ut CD – CE (ED) sit non Q. *b* Fiatque CD. ED :: ABq. AFq. In circulo super AB diametrum descripto *c* aptetur A F, ducaturque BF. Sunt AB, AF, quas petis. *a* 1. *lem.* 29. 10. *b* 3. *lem.* 10. 10. *c* 1. 4.

Nam ABq. AFq *d* :: CD. ED. *e* ergo ABq $\sqcap\!\sqcup$ AFq. verum AB est ρ'. *f* ergo AF est ρ'. sed quia CD est Q: at ED non Q: *g* erit A B $\sqcap\!\sqcup$ AF. porro, ob ang. *h* rectum AFB, est ABq *k* = AFq + BFq; cum igitur ABq. AFq :: CD. ED. per conversionem rationis erit ABq. BFq :: CD. CE :: Q. Q. *l* ergo AB $\sqcap\!\sqcup$ BF. Q. E. F. *d* *constr.* *e* 6. 10. *f* *sch.* 12. 10. *g* 9. 10. *h* 31. 3. *k* 47. 1. *l* 9. 10.

In numeris; sit AB, 6; CD, 9, CE, 4; quare ED, 5. Fac 9. 5 :: 36. (Q: 6) AFq. erit AFq 20. proinde AF $\sqrt{20}$. ergo BFq = 36 – 20 = 16. quare BF est 4.

PROP. XXXI.



C......... E.... D

Invenire duas rationales AB, AF *potentia tantum commensurabiles, ita ut major* A B *plus possit, quam minor* AF, *quadrato rectæ lineæ* B F *sibi longitudine incommensurabilis.*

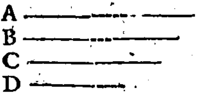
Exponatur AB, ρ'. *a* accipe numeros CE, ED quadratos, ita ut CD = CE + ED sit non Q. & in reliquis imitare constructionem præcedentis. Dico factum. *a* 2. *lem.* 29. 10.

 Nam,

Nam, ut ibi, AB, AF sunt $\dot{\rho}$ ⊐┘. item ABq. BFq :: CD. ED. ergo cum CD sit non Q.
b 9. 10. *b* erunt AB, BF ⊓⊔. Q. E. F.

In numeris, sit AB, 5. CD, 45. CE = 36; ED = 9. Fac 45. 9 :: 25 (ABq.) 5 (AFq.) ergo AF = $\sqrt{}$ 5. proinde BFq = 45 − 25 = 20. quare BF = $\sqrt{}$ 20.

PROP. XXXII.

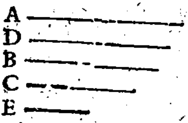


Invenire duas medias C, D potentia tantum commensurabiles, quæ rationale CD contineant, ita ut major C plus possit, quam minor D, quadrato rectæ lineæ sibi longitudine commensurabilis.

a 30. 10. *a* Accipe A, & B $\dot{\rho}$ ⊐┘; ita ut $\sqrt{}$Aq − Bq ⊓⊔
b 13. 6. A. *b* Fiatque A. C :: C. B, *c* atque A. B :: C.
c 12. 6. D. Dico factum.
d constr.
e 21. 10. Nam quia A, & *d* B sunt $\dot{\rho}$ ⊐┘, *e* erit C (*f* $\sqrt{}$
f 17. 6. AB) μ. item *g* ideo C ⊐┘ D. *h* ergo D etiam
g 10. 10. μ. porro quia A. B *d* :: C. D; & permutatim A.
h 24. 10. C :: B. D :: C. B; & Bq *d* est $\dot{\rho}\nu$, erit CD
k 17. 6. *k* (Bq) $\dot{\rho}\nu$. Denique quia $\sqrt{}$ Aq − Bq *d* ⊓⊔
l 15. 10. A, *l* erit $\sqrt{}$ Cq − Dq ⊓⊔ C. ergo, &c. Sin $\sqrt{}$ Aq − Bq ⊓⊔ Aq, erit $\sqrt{}$ Cq − Dq ⊓⊔ C.

In numeris, sit A, 8; B. $\sqrt{}$ 48 ($\sqrt{}$: 64 − 16) ergo C = $\sqrt{}$ AB = $v\sqrt{}$ 3072. & D = $v\sqrt{}$ 1728. quare CD = $v\sqrt{}$ 5308416 = $\sqrt{}$ 2304.

PROP. XXXIII.



Invenire duas medias D, E potentia solum commensurabiles, quæ medium DE contineant, ita ut major D plus possit, quam minor E, quadrato rectæ lineæ sibi longitudine commensurabilis.

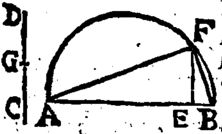
Sume

^a Sume A, & C ρ', ⊤⊥, ita ut √ Aq − Cq ⊓⊥ A. ^b sume etiam B ⊤⊥ A, & C; & fac A.D ^c :: D.B ^d :: C.E. Erunt D, & E quæsitæ. a 10. 10. b lem. 21. 10. c 13. 6. d 11. 6.

Nam quoniam A, & C ^e sunt ρ', ^e & B ⊤⊥ A & C, ^f erit B ρ', & D (√ AB) ^g erit μ. ^h Quia vero A. D :: C. E. erit permutando A. C :: D. E. ergo cum A ⊤⊥ C, ^h erit D ⊤⊥ E. ^k ergo E est μ. porro, ^l quia D. B :: C. E; ^l & BC est μν, etiam DE ei ^m æquale est μν. deniq; propter A. C :: D. E. ^e quia √ Aq − Cq ⊓⊥ A, ^n erit √ Dq − Eq ⊓⊥ D. ergo, &c. Sin √ Aq − Cq ⊓⊥ A, erit √ Dq − Eq ⊓⊥ Eq. e constr. f sch. 12. 10. g 21. 10. h 10. 10. k 24. 10. l 21. 10. m 16. 6 n 15. 10.

In numeris, sit A, 8; C, √ 48; B, √ 28. erit D ^v √ 3072; & E ^v √ 588. quare D. E :: 2. √ 3. & DE = √ 1344.

Prop. XXXIV.



Invenire duas rectas lineas A F, B F *potentia incommensurabiles, quæ faciant compositum quidem ex ipsarum quadratis rationale, rectangulum vero sub ipsis contentum, medium.*

^a Reperiantur AB, CD ρ', ⊤⊥, ita ut √ ABq − CDq ⊓⊥ AB. ^b biseca CD in G. ^c fac rectang. AEB = GCq. Super AB diametrum duc ^d semicirculum A F B. erige perpendicularem E F. ^e duc AF, BF. Hæ sunt quæ indagandæ erant. a 31. 10. b 10. 1. c 28. 6. d 12. 6. e cor. 8. 6 & 17. 6.

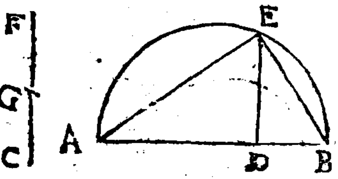
Nam AE. BE ^d :: BA x AE. AB x BE. Sed BA x AE ^e = AFq; ^e & AB x BE = FBq. ^f ergo AE. EB :: AFq. FBq. ergo cum AE ^g ⊓⊥ EB, ^h erit AFq ⊓⊥ FBq. Quinetiam ABq (^k AFq + FBq) ^l est ρ'ν. denique EFq ^l = AEB ^l = CGq. ^m ergo EF = CG. ergo CD x AB = 2 EF x AB. atqui CD x AB ^n est μν. ^o ergo AB x EF, ^p vel AF x FB, est μν. Q. E. D. f 7. 5. g 19. 10. h 10. 10. k 31. 3. & 47. 1. l constr. m 1. ax. 1. n 21. 10. o 24. 10. p sch. 22. 6.

Explicatio per numeros.

Sit AB, 6. CD, $\sqrt{12}$. quare CG $= \sqrt{\frac{12}{4}} = \sqrt{3}$. Est vero AE $= 3 + \sqrt{6}$. & EB $= 3 - \sqrt{6}$. & unde AF erit $\sqrt{:18 + 216}$. Et FB, $\sqrt{:18 - \sqrt{216}}$. item AFq + FBq est 36, & AF x FB $= \sqrt{108}$.

Cæterum AE invenitur sic. Quia BA (6.) AF :: AF. AE; erit 6 AE = AFq = AEq + 3 (EFq.) ergo 6 AE − AEq = 3. pone 3 + e = AE, ergo 18 + 6e − 9 − 6e − ee, hoc est 9 − ee = 3. vel ee = 6. quare e = $\sqrt{6}$. proinde AE = 3 + $\sqrt{6}$.

PROP. XXXV.



Invenire duas rectas lineas AE, EB *potentia incommensurabiles, quæ faciant compositum quidem ex ipsarum quadratis medium, rectangulum vero sub ipsis contentum, rationale.*

a 32. 10. *a* Sume AB, & CF μ ⊥, ita ut AB x CF sit ρ'ν, atque $\sqrt{}$ ABq − CFq ⊥ AB. & reliqua fiant, ut in præcedenti. erunt AE, EB, quas petis.

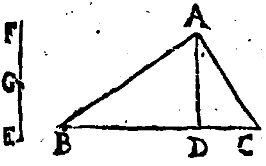
Nam, ut isthic ostensum est, AEq ⊥ EBq: item ABq (AEq + EBq) est μν. & denique

b constr. AB x CF *b* est ρ'ν, idcirco & AB x DE, *d* hoc.

c schol. 12. 10 est, AE x EB, est ρ'ν. ergo, &c.

d schol. 22. 6.

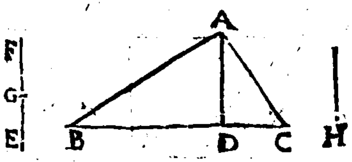
PROP. XXXVI.



Invenire duas rectas lineas BA, AC potentia incommensurabiles, quæ faciant & compositum ex ipsarum quadratis medium, & rectangulum sub ipsis comprehensum medium, incommensurabileque composito ex ipsarum quadratis.

a Accipe BC & EF μ ⊓; ita ut BC x EF sit $\mu\nu$. & $\sqrt{}$ Bcq – EFq ⊓ BC. & reliqua fiant, ut in præcedentibus. Erunt BA, AC exoptata: Nam, ut prius, BAq ⊓ ACq; item BAq + ACq est $\mu\nu$. & BA x AC est $\mu\nu$. Denique BC b ⊓ EF, atque c ideo BC ⊓ EG; estque BC. EG d :: BCq. BC x EG, (BC x AD, vel BA x AC.) e ergo BCq (ABq + ACq) ⊓ BA x AC. ergo, &c. a 33. 10. b constr. c 13. 10. d 1. 6. e 14. 10.

Schol.



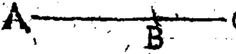
Invenire duas medias longitudine & potentia incommensurabiles.

a Sume BC μ. sitque BA x AC $\mu\nu$, & ⊓ BCq (BAq + ACq.) b Fac BA. H :: H. AC. Sunt BC, & H μ ⊓. Nam BC est μ. & BA x AC (c Hq) est $\mu\nu$. quare H est etiam μ. a 36. 10. b 13. 6. c 17. 6.

d 14.10. μ *d* item B A x A C $\sqcup\!\sqcup$ B Cq; ergo Hq $\sqcup\!\sqcup$ B Cq. ergo, &c.

Principium senariorum per compositionem.

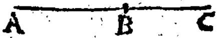
PROP. XXXVII.



Si duæ rationales A B, B C *potentia tantum commensurabiles componantur, tota* A C *irrationalis est; vocetur autem ex binis nominibus.*

a hyp.
b lem. 16.10. Nam quia A B *a* $\sqcup\!\sqcup$ B C, *b* erit A Cq $\sqcup\!\sqcup$
c 11. def. 10. ABq. Sed AB *a* est ρ'. *c* ergo AC est ρ'. Q. E. D.

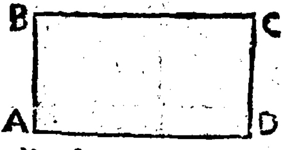
PROP. XXXVIII.



Si duæ mediæ A B, B C *potentia tantum commensurabiles componantur, quæ rationale contineant, tota* A C *irrationalis est; vocetur autem ex binis mediis prima.*

a hyp.
b lem. 16.10 Nam quoniam AB *a* $\sqcup\!\sqcup$ BC, *b* erit ACq $\sqcup\!\sqcup$
c 11. def. 10. AB x BC, ρ'. *c* ergo AC est ρ'. Q. E. D.

LEMMA.

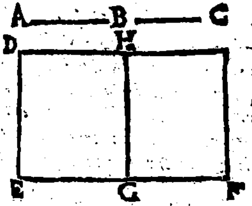


Quod sub linea rationali AB, *&* *irrationali* BC *continetur rectangulum* AC, *irrationale est.*

a hyp.
b 21. 10. Nam si rectang. A C dicatur ρ'; quum AB sit ρ'; *b* erit latitudo B C etiam ρ'. contra Hyp.

PROP.

PROP. XXXIX.



Si duæ mediæ AB, BC potentia tantum commensurabiles componantur, quæ medium continent, tota AC irrationalis erit; vocetur autem ex binis mediis secunda.

Ad expositam DE ρ' *a* fac rectang. DF $=$ ACq; *b* & DG $=$ ABq $+$ BCq. a cor 16.6. b 47.1. & 11.6.

Quoniam ABq *c* $\sqcap\!\sqcup$ BCq; *d* erit ABq $+$ BCq, hoc est DG $\sqcap\!\sqcup$ ABq; sed ABq *e* est $\mu\nu$. *e* ergo DG est $\mu\nu$. verum rectang. ABC ponitur $\mu\nu$; *e* ideoque 2ABC (*f* HF) est $\mu\nu$; *g* ergo EG, & GF sunt ρ'. quia vero DG *h* $\sqcap\!\sqcup$ HF; atque DG. HF :: *k* EG. GF *l* erit EG $\sqcap\!\sqcup$ GF. *m* ergo tota EF est ρ'. *n* quare rectang. DF est $\rho'\nu$. *o* ergo $\sqrt{DF}$, id est AC, est ρ'. Q.E.D. c hyp. d 16.10. e 14.10. f 4.2. g 23.10. h lem.26.10. k 1.6. l 10.10. m 37.10. n lem.38.10. o 11.def.10.

PROP. XL.

Si duæ rectæ lineæ AB, BC potentia tantum commensurabiles componantur, quæ faciant compositum quidem ex ipsarum quadratis rationale, quod autem sub ipsis continetur, medium; tota recta linea AC, irrationalis erit: vocetur autem major.

Nam quia ABq $+$ BCq *a* est $\rho'\nu$, & *b* $\sqcap\!\sqcup$ 2ABC *c* $\mu\nu$, & proinde ACq (*d* ABq $+$ BCq $+$ 2ABC) *e* $\sqcap\!\sqcup$ ABq $+$ BCq $\rho'\nu$, *f* erit AC ρ'. Q.E.D. a hyp. b sch.12.10. c hyp. & 24.10. d 4.2. e 17.10. f 11.def.10.

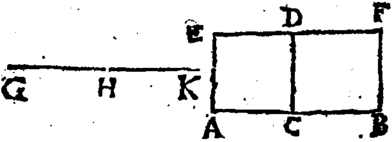
PROP. XLI.

Si duæ rectæ lineæ A C, C B *potentia incommensurabiles componantur, quæ faciant compositum quidem ex ipsarum quadratis medium, quod autem sub ipsis continetur, rationale; tota recta linea* AB *irrationalis erit: vocetur autem rationale ac medium potens.*

a hyp. & sch. 12. 10. b sch. 12. 10. c hyp. d 17. 10. e 11. def. 10.

Nam 2 rectang. A C B, a ρ'ν b ⊏⊐ A Cq + CB q c μν. d ergo 2 ACB d ⊏⊐ ABq. quare e AB est ρ'. Q. E. D.

PROP. XLII.



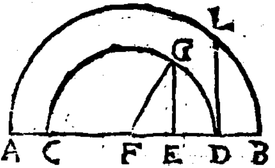
Si duæ rectæ lineæ GH, HK *potentia incommensurabiles componantur, quæ faciant & compositum ex ipsarum quadratis medium, & quod sub ipsis continetur medium, incommensurabileque composito ex quadratis ipsarum; tota recta linea* GK *irrationalis erit: vocetur autem bina media potens.*

a hyp. b 23. 10. c 4. 2. d 1. 6. e 10. 10. f 37. 10. g lem 38. 10. h 11. def. 10.

Ad expositam FB ρ', fiant rectang. AF = GKq, & CF = GHq + HKq. Quoniam GHq + HKq (CF) a est μν; latitudo CB b erit ρ'. Item quia 2 rectang. GHK (c AD) a est μν, etiam AC b erit ρ'. Porro quia rectang. AD a ⊏⊐ CF, d atque AD. CF :: AC. CB, e erit AC ⊏⊐ CB. f Quare AB est g ρ'. ergo rectang. AF, id est, GKq est ρ'ν. h proinde GK est ρ'. Q. E. D.

PROP.

PROP. XLIII.



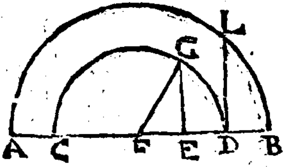
Quæ ex binis nominibus A B, *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

Si fieri potest, binomium A B alibi in E secetur in alia nomina A E, E B. Liquet A B secari utrobique inæqualiter, quia AD ⊐⊏ DB, & AE ⊐⊏ EB.

Quoniam rectangula ADB, AEB *a* sunt μα; *a* & singula ADq, DBq, AEq, EBq sunt ρ'α; *b* adeoque ADq + DBq, *b* & AEq + EBq etiam ρ'α, *b* idcirco ADq + DBq −: AEq + EBq. *c* hoc est, 2 AEB − 2 ADB est ρ'ν. *d* ergo AEB − ADB ρ'ν. ergo μν superat μν per ρ'ν. *e* Q.E.A.

a 37. 10.
b sch. 27. 10
c sch. 9. 2.
d sch. 12. 10
e 27. 10.

PROP. XLIV.



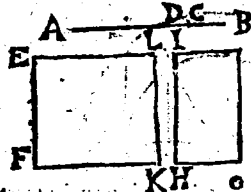
Quæ ex binis mediis prima A B, *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

Puta AB dividi in alia nomina AE, EB. quo posito, singula ADq, DBq, EBq, *a* sunt μα; *a* & rectangula ADB, AEB, eorumque dupla, sunt ρ'α. *b* ergo 2 AEB − 2 ADB, *c* hoc est ADq + DBq −: AEq + EBq est ρ'ν. *d* Q. E. A.

a 38. 10.
b sch. 27. 10.
c sch. 9. 2.
d 27. 10.

PROP.

PROP. XLV.



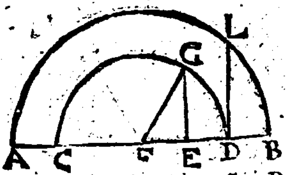
Quae ex binis mediis secunda AB, *ad unum duntaxat punctum* C *dividitur in nomina* AC, CB.

Dic alia esse nomina AD, DB. Ad expositam EF ρ, fac rectang. EG = ABq. & EH = ACq + CBq; item EK = ADq + DBq.

a 39. 10. b 16 & 24. 10. c 22. 10. d 24. 10. e 4. 2. f lem. 26. 10. g 1. 6. h 10. 10. k 37. 10.

Quoniam ACq, CBq *a* sunt $\mu\epsilon$ ⊔⊔; *b* erit ACq + CBq (EH) $\mu\nu$. *c* ergo latitudo FH est ρ', *a* quin & rectang. ACB, *d* ideoque 2 ACB *e* (IG) est $\mu\nu$: *c* ergo HG, est etiam ρ'. Cum igitur EH *f* ⊔⊔ IG, *g* atque EH. IG :: FH. HG; *h* erunt FH, HG ⊔⊔. *k* ergo FG est binomium; cujus nomina FH, HG. Simili argumento FG est bin. cujus nomina FK, KG, contra 43. hujus.

PROP. XLVI.



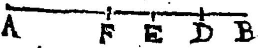
Major AB *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

a 40. 10. b sch. 27. 10. c sch. 5. 2. d 27. 10.

Concipe alia nomina AE, EB. quo posito rectangula ADB, AEB *a* $\mu\epsilon$; *a* & tam ADq + DBq, quam AEq + EBq sunt $\rho\alpha$. *b* ergo ADq + DBq — : AEq + EBq, *c* hoc est, 2 AEB — 2 ADB est $\rho'\nu$. *d* Q. E. N.

PROP.

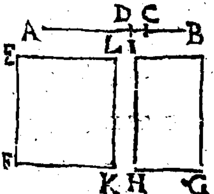
Prop. XLVII.



Rationale ac medium potens AB, *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

Dic alia nomina AE, EB. *a* ergo tam AEq + EBq, quam ADq + DBq sunt μα. *a* & rectangula AEB, ADB, sunt ρ'α. *b* ergo 2 AEB − 2 ADB, *c* hoc est, ADq + DBq − : AEq + EBq est ρ'ν. Q.E.A. a 41. 10. b sch. 27. 10. c sch. 5. 2. d 27. 10.

Prop. XLVIII.



Bina media potens AB, *ad unum duntaxat punctum* C *dividitur in nomina* AC, CB.

Vis AB dividi in alia nomina AD, DB. Ad expositam EF ρ', fiant rectang. EG = ABq, & EH = ACq + CBq, & EK = ADq + DBq. Quoniam ACq + CBq, nempe EH, *a* est μν, *b* erit latitudo FH ρ'. Item quia 2 ACB, *c* hoc est, IG, est *a* μν, *b* erit HG etiam ρ'. Ergo cum EH ⊏⊐ IG, itque EH. IG *d* :: FH. HG, *e* erit FH ⊏⊐ HG. *f* ergo FG est bin. cujus nomina FH. HG. Eodem modo ejusdem nomina eruat FK, KG; contra 43 hujus.

a 40. 10. b 23. 10. c 4. 2. d 1. 6. e 10. 10. f 37. 10.

Definitiones secundæ.

Exposita rationali, & quæ ex binis nominibus, divisa in nomina; cujus majus nomen plus possit quam minus, quadrato rectæ lineæ sibi longitudine commensurabilis;

I. Siquidem majus nomen expositæ rationali com-

commensurabile sit longitudine, vocetur tota ex binis nominibus prima.

II. Si vero minus nomen expositæ rationali longitudine sit commensurabile, vocetur ex binis nominibus secunda.

III. Quod si neutrum ipsorum nominum sit longitudine commensurabile expositæ rationali, vocetur ex binis nominibus tertia.

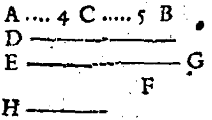
Rursus, si majus nomen plus possit quam minus, quadrato rectæ lineæ sibi longitudine incommensurabilis;

IV. Si quidem majus nomen expositæ rationali commensurabile sit longitudine, vocetur ex binis nominibus quarta.

V. Si vero minus nomen, vocetur quinta.

VI. Quod si neutrum ipsorum nominum, vocetur sexta.

PROP. XLIX.



Invenire ex binis nominibus primam, E G.

a sch. 29. 10. b 2. lem. 29. 10. c 3 lem. 30. 10.

a Sume AB, AC numeros quadratos, quorum excessus CB non Q. exponatur D ρ́. *b* accipe quamvis E F ⊓⊔ D. *c* fac AB. CB :: EFq. FGq. erit EG bin. 1.

d constr. e 6. def 10. f 6. 10. g sch. 12. 10 h 9. 10. k 9. 10. l 1. def. 48. 10.

Nam E F *d* ⊓⊔ D. *e* ergo E F ρ́. *f* item E Fq ⊓⊔ F Gq. *g* ergo F G est etiam ρ́. item *d* quia EFq. FGq :: AB. CB :: Q. non Q. *h* erit EF ⊓⊔ F G. denique quia per conversionem rationis EFq. EFq − FGq :: AB. AC :: Q. Q. *k* erit E F ⊓⊔ √ EFq − FGq. *l* ergo E G est bin. 1. Q. E. F.

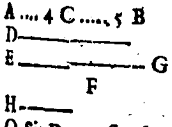
Explicatio per numeros.

Sit D, 8. E F, 6. A B, 9. C B, 5. quare cum

9. 5.

9. 5 :: 36. 20. erit FG, $\sqrt{}$ 20. proinde EG est 6 + $\sqrt{}$ 20.

P R O P. L.



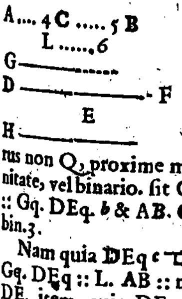
Invenire ex binis nominibus secundam, E G.

Accipe AB, & AC numeros quadratos, quorum excessus CB sit non Q. Sit D exposita ρ. sume FG $\sqcup$ D. Fac CB. AB :: FGq. EFq. Erit EG quæsita. *Proba ut præcedentem*

Nam FG $\sqcup$ D, quare FG est ρ. item EFq $\sqcup$ FGq. ergo EF est etiam ρ. item quia FGq. EFq :: CB. AB :: non Q. Q. est FG $\sqcup$ EF. denique quia CB. AB :: FGq. EFq, inverseque AB. CB :: EFq. FGq, erit ut in præcedenti, EF $\sqcup$ $\sqrt{}$ EFq − FGq. *a* è quibus EG est bin. 2. Q. E. F. *a 2. def. 48. 10.*

In numeris, sit D, 8; FG 10; AB, 9; CB, 5. erit EF, $\sqrt{}$ 180. quare EG est 10 + 180.

P R O P. LI.



Invenire ex binis nominibus tertiã, DF.

a Sume numeros AB, AC quadratos, quorum excessus CB non Q. Sitq; L numerus non Q, proxime major quam CB, nempe unitate, vel binario. sit G exposita ρ. *b* Fac L. AB :: Gq. DEq. *b* & AB. CB :: DEq. EFq. erit DF bin. 3. *a sch. 29. 10.* *b 3. lem. 10. 10.*

Nam quia DEq *c* $\sqcup$ Gq, *d* est DE ρ. item Gq. DEq :: L. AB :: non Q. Q. *e* ergo G $\sqcup$ DE. item quia DEq *e* $\sqcup$ EFq, *d* etiam EF est ρ. quinetiam quia DEq. EFq :: AB. CB :: Q. non Q. *f* est DE $\sqcup$ EF. porro, quia per constr. *c constr. 6. 10.* *d sch. 12. 10.* *e 6. 10.* *f 9. 10.*

constr. & ex æquali Gq. EFq :: L.CB :: non Q.
g sch.27.8. Q. (nam g L, & CB non sunt similes plani numeri) b erit G etiam $\sqcup\!\sqcup$ EF. denique ut in
h 9 10. k 5 def.48. 10. præced. $\surd$ DEq $-$ EFq $\sqcup\!\sqcup$ DE. k ergo DF est bin.3. Q. E. F.

In numeris, sit AB, 9; CB, 5; L, 6; G, 8. erit DE, $\surd 96$ & EF, $\surd \frac{480}{9}$. quare DF $= \surd 96 + \surd \frac{480}{9}$.

PROP. LII.

A ... 3 C ..... 6 B

G ———

D ——— F

E

H ———

Invenire ex binis nominibus quartam, DF.

a sch.29.10. a Sume quemvis numerum quadratum AB, a quem divide in A C, CB non quadratos. sit G exposita ρ'. b accipe DE $\sqcup\!\sqcup$
b 1.lem.10. c 3.lem.10. G. c Fac AB. CB :: DEq. EFq. erit DF bin.4.
10. Nam ut in 49. hujus, DF ostendetur bin. item, quia per constr. & conversionem rationis DEq. DEq $-$ EFq :: AB. AC :: Q. non Q.
d 9. 10. d erit DE $\sqcup\!\sqcup$ $\surd$ DEq $-$ EFq. e ergo DF est
e 4 def. 48 10. bin. 4. Q. E. F.

In numeris, sit G, 8; DE, 6, erit EF $\surd 24$. ergo DF est $6 + \surd 24$.

PROP. LIII.

A ... 3 C ...... 6 B

G ———

D ——— F

E

H F

Invenire ex binis nominibus quintam, D F.

Accipe quemvis numerum quadratum AB, cujus segmenta AC, CB sint non Q. sit G exposita ρ'. sume EF $\sqcup\!\sqcup$ G. fac CB. AB :: EFq. DEq. erit DF bin.5.

Nam ut in 50. hujus, erit DF bin. & quia per constr. & invertendo D Eq. E Fq :: AB.
a 9. 10 b 5 def.48. 10. CB, ideoque per conversionem rationis DEq. DEq $-$ EFq :: AB. AC :: Q. non Q. a erit DE

DE ⊐⊏ $\surd$ DEq − EFq. *b* ergo DF est bin. 4. Q. E. F.

In numeris, sit G, 7; EF, 6. erit DE $\surd$ 54. quare DF est 6 + $\surd$ 54.

PROP. LIV.

A ..... 5 C .... 7 B
L 9
G ————
D —————— F
E
H ———

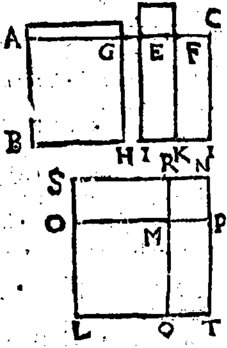
Invenire ex binis nominibus sextam.

Accipe AC, CB primos numeros utcunque, sic ut AC + CB (AB) sit non Q. sume etiam quemvis L num. Q. sit G expos. ρ. *a* fiatque L. AB :: Gq. DEq. atque AB. CB :: DEq. EFq. erit DF. bin. 6. a 3. lem. 10. 10.

Nam ut in 51. hujus, DF ostendetur bin. item quod DE, & EF ⊐⊏ G. denique igitur quia per constr. & conversionem rationis DEq. DEq − EFq :: AB. AC :: non Q. Q. (Nam AB primus est ad AC, *b* ideoque ei dissimilis) *c* ergo DE ⊐⊏ $\surd$ DEq − EFq. *d* ergo DF est bin. 6. Q. E. F. b sch. 17. 8. c 9. 10. d 6. def. 48. 10.

In numeris, sit G, 6; DE $\surd$ 48. erit EF $\surd$ 28. quare DF est $\surd$ 48 + $\surd$ 28.

LEMMA A.



Sit AD *rectangulum*, *cujus latus* AC *secetur inaequaliter in* E; *bisectumque sit segmentum minus* EC *in* F; *atque ad*

a 28. 6. AE, *a fiat rectang.* AGE = EFq; *perque*

b 31. 1. G, E, F *b ducantur ad* AB *parallela* GH, EI, FK. *c Fiat autem*

c 14. 2. *quadratum* LM = *rectang.* AH, *atque ad* OMP *productam c fiat quadratum* MN = GI; *rectaeque* LOS, LQT, NRS, NPT *producantur*.

Dico 1. MS, MT *sunt rectangula*. Nam ob quadratorum angulos OMQ, RMP rectos,

a sch. 15. 6. *a* erit QMR recta linea. *b* ergo anguli RMO,

b 13. 1. QMP recti sunt. quare pgra MS, MT sunt rectangula.

c 2. ax. 1. 2. *Hinc patet* LS *c*= LT; *& proinde* LN *esse quadratum*.

3. *Rectangula* SM, MT, EK, FD *aequalia*

d hyp. *sunt*. Nam quia rectang. AGE *d* = EFq. *e* erit

e 17. 6. AE. EF :: EF. GE. *f* ideoque AH. EK :: EK.

f 1. 6. GI. hoc est per constr. LM. EK :: EK. MN.

g sch. 21. 6. *g* verum LM. SM :: SM. MN. ergo EK *h* =

h 9. 5. SM *k* = FD *l* = MT.

k 36. 1.

l 43. 1.

m 2. ax. 1. 4. *Hinc* LN *m* = AD.

n 16. 10. 5. *Quia* EC *bisecta est in* F, *n patet* EF, FC, EC ⊓ *esse*.

o 18 & 6. Si AE ⊓ EC, & AE ⊓ √ AEq − ECq, *o* erunt AG, GE, AE ⊓. item, quia

16. 10. AG,

AG. GE :: AH. GI ^{p} erunt AH, GI; hoc est LM, MN $\sqcup\!\sqcup$. item iisdem positis, p 10. 10.

7. OM $\sqcup\!\sqcup$ MP. Nam per Hyp. AE, $\sqcup\!\sqcup$ EC, ^{q} ergo EC $\sqcup\!\sqcup$ GE. ^{q} quare EF $\sqcup\!\sqcup$ GE. sed EF. GE :: EK. GI. ^{r} ergo EK $\sqcup\!\sqcup$ GI, hoc est SM $\sqcup\!\sqcup$ MN. atqui SM. MN :: OM. MP. ^{r} ergo OM $\sqcup\!\sqcup$ MP. q 14. 10. r 10. 10.

8. Si ponatur AE $\sqcup\!\sqcup$ $\sqrt{AEq - ECq}$, ^{s} patet AG, GE, AE esse $\sqcup\!\sqcup$. unde LM $\sqcup\!\sqcup$ MN. nam AG. GE :: AH. GI :: LM. MN. s 19. & 17. 10.

His bene perspectis, facile sex sequentes Propositiones expediemus.

PROP. LV.

Si spatium AD contineatur sub rationali AB, & ex binis nominibus prima AC, (AE + EC) recta linea OP spatium potens irrationalis est, quæ ex binis nominibus appellatur.

Suppositis iis, quæ in lemmate proxime præcedenti descripta, & demonstrata sunt, liquet rectam OP posse spatium AD. ^{a} item AG, GE, AE sunt $\sqcup\!\sqcup$. ergo cum AE ^{b} sit ρ^c $\sqcup\!\sqcup$ AB, ^{c} erunt AG, & GE, ρ^c $\sqcup\!\sqcup$ AB. ^{d} ergo rectangula AH, GI, hoc est quadrata LM, MN sunt $\rho\tau$. ergo OM, MP sunt ρ^c ^{e} $\sqcup\!\sqcup$. ^{f} proinde OP est bin. Q. E. D. a hyp. & lem. 54. 10. b hyp. c sch. 12. 10. d 20. 10. e lem. 54. 10. f 37. 10.

In numeris, sit AB, 5; AC, $4 + \sqrt{12}$. quare rectang. AD $= 20 + \sqrt{300} =$ quadr. LN. ergo OP est $\sqrt{15} + \sqrt{5}$; nempe bin. 6.

PROP. LVI.

Si spatium AD *contineatur sub rationali* AB, *& ex binis nominibus secunda* AC (AE + EC;) *recta linea* OP *spatium* AD *potens, irrationalis est, quæ ex binis mediis prima appellatur.*

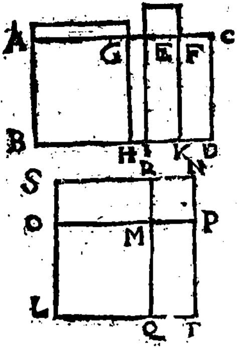
Rursus adhibito lemmate ad 54. hujus, erit OP = √AD. a item AE, AG, GE sunt ⊓⊔. ergo quum AE b sit ῥ, ⊓⊔ AB, c erunt AG, GE etiam ῥ ⊓⊔ AB. ergo rectangula AH, GI; hoc est OMq, MPq d sunt μα. e quinetiam OM ⊓⊔ MP. denique EF ⊓⊔ EC, & EC f ⊓⊔ AB. g quare EF est ῥ ⊓⊔ AB. g ergo EK; hoc est SM, vel OMP est ῥν. h Proinde OP est 2 μ prima. Q. E. D.

a hyp. & lem. 54. 10.
b hyp.
c sch. 12. 10.
d 22. 10.
e lem. 54. 10.
f hyp. 12. 10.
g 20. 10.
h 38. 10.

In numeris, sit AB, 5; & AC, √48 : + 6. ergo rectang. AD = √ : 1200 + 30 = OPq. ergo OP est υ√675 + υ√75; nempe bimed. 1.

Vide Schem. 57.

PROP. LVII.



Si spatium A D *contineatur sub rationali* AB, *& ex binis nominibus tertia* AC (AE + EC;) *recta linea* OP *spatium* AD *potens, irrationalis est, quæ ex binis mediis secunda dicitur.*

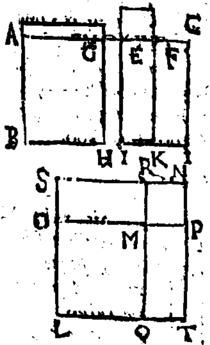
Ut prius, OPq = AD. item rectangula AH, GI, hoc est OMP, MPq sunt μα. a item EK, vel OMP est μν. b ergo OP est bimed. 2.

a hyp. & 22. 10.
b 39. 10.

In

In numeris, sit AB, 5; AC, $\sqrt{32} + \sqrt{34}$. quare AD est $\sqrt{800} + \sqrt{600}$ = OPq. proinde OP est $v\sqrt{450} + v\sqrt{50}$; hoc est bimed. 2.

PROP. LVIII.



Si spatium AD contineatur sub rationali AB, & ex binis nominibus quarta AC (AE + EC;) recta linea OP spatium potens, irrationalis est, quæ vocatur major.

Nam iterum, OMq ᵃ ⊏⊐ MPq. a lem. 54. 10.
rectang. vero AI, hoc est OMq + MPq ᵇ est ρ'ν. ᶜ item EK, vel OMP est μν. b hyp. & 20. 10. c hyp. & 22. 10.
ᵈ ergo OP ($\sqrt{AD}$) est major. Q. E. D. d 40. 10.

In numeris, sit AB 5; & AC, 4 + $\sqrt{8}$, ergo rectang. AD est 20 + $\sqrt{200}$. quare OP est $\sqrt{:}\ 20 + \sqrt{200}$.

PROP. LIX.

Si spatium AD contineatur sub rationali AB, & ex binis nominibus quinta AC; recta linea OP spatium AD potens, irrationalis est, quæ rationale & medium potens appellatur.

Rursus OMP ⊏⊐ MPq. rectang. vero AI; a ut in præc.
vel OMq + MPq est μν. ᵃ item rectang. EK, vel OMP est ρ'ν. ᵇ ergo OP ($\sqrt{AD}$) est potens ρ'ν, & μν. Q. E. D. b 41. 10.

In numeris, sit AB, 5; & AC 2 + $\sqrt{8}$. ergo rectang. AD = 10 + $\sqrt{200}$ = OPq. quare OP est $\sqrt{:}\ 10 + \sqrt{200}$

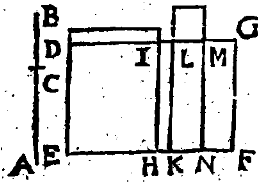
PROP. LX.

Si spatium A D *contineatur sub rationali* A B, *& ex binis nominibus sexta* BC (AE + EC;) *recta linea* OP *spatium* AD *potens, irrationalis est, quæ bina media potens appellatur.*

Ut sæpe prius, OMq $\urcorner\!\sqcup$ MPq. & OMq + MPq est $\mu\nu$. & rectang. (EK) OMP etiam $\mu\nu$.
a 42. 10. *a* ergo OP = $\sqrt{}$ AD est potens 2 $\mu\alpha$. Q. E. D.

In numeris, sit AB, 5; AC, $\sqrt{12} + \sqrt{8}$; ergo rectang. AD, vel OPq est $\sqrt{300} + \sqrt{200}$. proinde OP est $\sqrt{:\sqrt{300} + \sqrt{200}}$.

LEMMA.



Sit recta A B *inæqualiter secta in* C, *sitque* A C *majus segmentum; & cuivis* D E *applicentur rectangula*, DF = ABq, & D H = ACq, & I K = CBq. *sitque* LG *bisecta in* M, *ducaturque* M N *parall.* GF.

Dico 1. *Rectang.* A C B = L N, vel M F.

a 4 2. & 3. ax. 1. *a* Nam 2 ACB = L F.

b 7. 2. 2. D L $\sqsubset$ LG. nam DK (ACq + CBq)

c 1. 6. *b* $\sqsubset$ LF (2 ACB) ergo cum DK, LF sint æque alta, *c* erit DL $\sqsubset$ LG.

d 16. 10. 3. Si AC $\urcorner\!\sqcup$ CB, *d* erit rectang. DK $\urcorner\!\sqcup$ ACq, & CBq.

4. *Item*, D L $\urcorner\!\sqcup$ L G. nam ACq + CBq *e* $\urcorner\!\sqcup$ 2 A C B; hoc est D K $\urcorner\!\sqcup$ LF. sed DK.

e lem. 26. 10. f 10. 10. LF *e* :: DL. LG. *f* ergo DL $\urcorner\!\sqcup$ LG.

5. *Ad hæc*, DL $\urcorner\!\sqcup$ $\sqrt{}$ DLq − LGq. Nam

g 1. 6. ACq. A C B *g* :: A C B. C Bq. hoc est D H. L N ::

LN :: LN. IK. *c* quare DI. LM :: LM. IL. *b* ergo DI x IL = LMq. ergo cum ACq *k* ⊐⊏ C*b*q. hoc est DH ⊐⊏ IK, & *l* proinde DI ⊐⊏ IL, *m* erit DL ⊐⊏ $\sqrt{}$ DLq − LGq. Q. E. D. h 17. 6. k hyp. l 10. 10. m 18. 10.

6. *Sin ponatur* ACq ⊐⊏ CBq, *n erit* DL ⊐⊏ $\sqrt{}$ DLq − LGq. n 19. 10.

Hoc lemma præparationis vicem subeat pro 6. sequentibus propositionibus.

PROP. LXI.

Quadratum ejus quæ ex binis nominibus (AC + CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus primam.*

Suppositis iis, quæ in lemmate proxime antecedenti descripta & demonstrata sunt. Quoniam AC, CB *a* sunt ῥ ⊐⊏, *b* erit rectang. DK ⊐⊏ ACq; *c* ergo DK est ῥν. *d* ergo DL ⊐⊏ DE ῥ'. rectang. vero ACB, ideoque 2 ACB (LF) *e* est μν. *f* ergo latitudo LG est ῥ' ⊐⊏ DE. *g* ergo etiam DL ⊐⊏ LG. *h* item DL ⊐⊏ $\sqrt{}$ DLq − LGq. ex quibus *k* sequitur DG esse bin. 1. Q. E. D. a hyp. b lem 60. 10. c sch. 12. 10. d 21. 10. e 22. & 24. 10. f 23. 10. g 13. 10. h lem 60. 10. k 1. def. 48. 10.

PROP. LXII.

Quadratum ejus, quæ ex binis mediis prima (AC + CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus secundam.*

Rursus adhibito lemmate proxime præcedenti; Rectang. DK ⊐⊏ ACq. *a* ergo DK est μν. *b* ergo latitudo DK est ῥ' ⊐⊏ DE. Quia vero rectang. ACB, ideoque LF (2 ACB) *c* est ῥν, *d* erit LG ῥ' ⊐⊏ DE. *e* ergo DL, LG sunt ῥ ⊐⊏. *f* item DL ⊐⊏ $\sqrt{}$ DLq − LGq. *g* ex quibus patet DG esse bin. 2. Q. E. D. a 24. 10. b 23. 10. c hyp. & sch. 12. 10. d 21. 10. e 13. 10. f lem. 60. 10. g 2 def. 48. 10.

PROP.

PROP. LXIII.

Quadratum ejus, quæ ex binis mediis secunda (AC+CB) ad rationalem DE applicatum, facit latitudinem DG ex binis nominibus tertiam.

Ut in præced. DL est $\acute{\rho}$ $\sqcap\!\sqcup$ DE. porrò quia
a hyp. & 24. 10. rectang. ACB, ideoque LF (2 ACB) [a] est
b 23. 10. $\mu\nu$, [b] erit LG $\acute{\rho}$ $\sqcap\!\sqcup$ DE. [c] quinetiam DL $\sqcap\!\sqcup$
c lem. 60. 10. LG. [c] itemque DL $\sqcap\!\sqcup$ $\sqrt{}$ DLq − LGq. [d] er-
d 3. def. 48. 10. go DG est bin. 3. Q. E. D.

PROP. LXIV.

Quadratum Majoris (AG+CB) ad rationalem DE applicatum, facit latitudinem DG ex binis nominibus quartum.

a hyp. & sch. 12. 10. Rursus ACq + CBq, hoc est DK [a] est $\acute{\rho}\nu$.
b 21. 10. [b] ergo DL est $\acute{\rho}$ $\sqcap\!\sqcup$ DE. item ACB, ideoque
c hyp. & 24. 10. LF (2 ACB) [c] est $\mu\nu$. [d] ergo LG est $\acute{\rho}$ $\sqcap\!\sqcup$
d 23. 10. DE. [e] proinde etiam DL $\sqcap\!\sqcup$ LG. denique
e 13, 10. quia AC $\sqcap\!\sqcup$ BC, [f] erit DL $\sqcap\!\sqcup$ DLq −
f lem. 60. 10. LGq. [g] unde DG est bin. 4. Q. E. D.
g 4. def. 48. 10.

PROP. LXV.

Quadratum ejus, quæ rationale ac medium potest, (AG+CB) ad rationalem DE applicatum, facit latitudinem DG ex binis nominibus quintam.

a 13. 10. Iterum, DK est $\mu\nu$. [a] ergo DL est $\acute{\rho}$ $\sqcap\!\sqcup$
b 21. 10. DE. item LF est $\acute{\rho}\nu$. [b] ergo LG est $\acute{\rho}$ $\sqcap\!\sqcup$ DE.
c 13. 10. [c] ergo DL $\sqcap\!\sqcup$ LG. [d] item DL $\sqcap\!\sqcup$ $\sqrt{}$ DLq −
d lem. 60. 10. LGq. [e] proinde DG est bin. 5.
e 5. def. 48. 10.

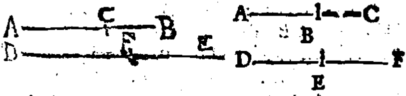
PROP. LXVI.

Quadratum ejus, quæ bina media potest (AG + CB) ad rationalem DE applicatum, facit latitudinem DG ex binis nominibus sextam.

Ut

Ut prius, DL & LG sunt $\dot{\rho}$ ⊓⊔ DE. a hyp.
Quia vero ACq + CBq (DK) [b]⊓⊔ ACB, b 14. 10.
[c]ideoque DK ⊓⊔ LF (2 ACB) estque DK. c 1. 6.
LF [e]:: DL. LG. [d]erit DL ⊓⊔ LG. [e]denique d 10. 10. e lem. 60. 10.
DL ⊓⊔ $\surd$ DLq − LGq. [f]ex quibus liquet f 6. def. 48. 10.
DG esse bin. 6. Q. E. D.

LEMMA.



Sint AB, DE ⊓⊔; *fiatque* AB. DE :: AC DF.

Dico 1. AC ⊓⊔ DF. ut patet ex 10. 10.
item CB ⊓⊔ FE. [a]quia AB. DE :: CB. FE. a 19. 5.

2. AC. CB :: DF. FE. Nam AC. DF :: AB. DE :: CB. FE. ergo permutando AC. CB :: DF. FE.

3. *Rectang.* ACB ⊓⊔ DFE. Nam ACq. ACB [b]:: AC. CB [c]:: DF. EF :: DFq. DFE. quare permutando ACq. DFq :: ACB. DFE. ergo cum ACq ⊓⊔ DFq, [d]erit ACB ⊓⊔ DFE. b 1. 6. c prius. d 10. 10.

4. ACq + CBq ⊓⊔ DFq + FEq. Nam quia ACq. CBq [e]:: DFq. FEq. erit componendo ACq + CBq. CBq :: DFq + FEq. FEq. ergo cum CBq ⊓⊔ FEq, [f]erit ACq + CBq ⊓⊔ DFq + FEq. e 22. 6. f 10. 10.

5. Hinc, si AC ⊓⊔, vel ⊓⊔ CB, [g]erit pariter DE ⊓⊔, vel ⊓⊔ EF. g 10. 10.

PROP.

PROP. LXVII.

Ei, quæ ex binis nominibus (AC + CB) *longitudine commensurabilis* DE, *& ipsa ex binis nominibus est, atq; ordine eadem.*

Fac AB. DE :: AC. DF. *a* sunt AC, DF
a lem. 66. 10. ⊓⊥ ; *a* & CB, FE ⊓⊥. quare cum AC, & CB
b hyp.
c lem. 66. 10. *b* sint ρ ⊓⊢, *c* erunt DF, FE ρ ⊓⊢. ergo DE
& sch. 12. 10. est etiam bin. Quia vero AC. CB *a* :: DF.
FE. si AC ⊓⊥, vel ⊓⊥ √ ACq − BCq,
d 15. 10. *d* etiam similiter DF ⊓⊥, vel ⊓⊥ √ DFq −
e 12. 10. & 14. 10. FEq. item si AC ⊓⊥, vel ⊓⊥ ρ expos. *e* erit similiter DF ⊓⊥, vel ⊓⊥ ρ expos. at si CB ⊓⊥ vel ⊓⊥ ρ, *e* erit pariter FE ⊓⊥ vel ⊓⊥ ρ. Sin
g Per def. 48. 10. vero utraque AC, CB ⊓⊥ ρ, erit utraq; etiam DF, FE ⊓⊥ ρ. *g* Hoc est, quodcunque binomium fuerit AB, erit DE ejusdem ordinis. Q. E. D.

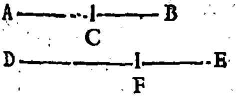
PROP. LXVIII.

Ei, quæ ex binis mediis (AC + CB) *longitudine commensurabilis* DE, *& ipsa ex binis mediis est, atque ordine eadem.*

a 12. 6. *a* Fiat AB. DE :: AC. DF. *b* ergo AC ⊓⊥
b lem 66. 10. DF, & CB ⊓⊥ FE. ergo cum AC & CB
c hyp. *c* sint μ, *d* etiam DF, & FE erunt μ. & cum
d 24 10. AC *c* ⊓⊢ CB, *e* erit FD ⊓⊢ FE. *f* ergo DE
e 10. 10.
f 38. 10. est 2μ. Si igitur rectang. ACB sit ρν, quia
g sch. 12. 10. DFE *b* ⊓⊥ ACB, *g* etiam DFE est ρν; et si
h 24 10.
k 38. vel 39. 10. illud μν, *h* hoc etiam erit μν. *k* Id est, sive AB sit bimed. 1. sive bimed. 2. erit DF ejusdem ordinis. Q. E. D.

PROP.

PROP. LXIX.



Majori (AC + CB) *commensurabilis* DE, & *ipsa Major est.*

Fac AB.DE :: AC.DF. Quoniam AC [a] $\not\sqcup$ CB, [b] erit DF $\not\sqcup$ FE. item ACq + CBq [a] est $\rho\nu$; proinde cum DFq + FEq [b] $\sqcup$ ACq + CBq, [c] etiam DFq + FEq est $\rho\nu$. denique rectang. ACB [a] est $\mu\nu$. [d] ergo rectang. DFE est $\mu\nu$ (quia DFE [b] $\sqcup$ ACB.) [e] Quare DE est major. Q. E. D.

a hyp.
b lem.66.10.
c sch.12.10.
d 24.10.
e 40.10.

PROP. LXX.

Rationale ac medium potenti (AC + CB) *commensurabilis* DE, *& ipsa rationale ac medium potens est.*

Iterum fac AB. DE :: AC. DF. Quia AC [a] $\not\sqcup$ CB, [b] etiam DF $\not\sqcup$ FE. item quia ACq + CBq [a] est $\mu\nu$, [c] erit DFq + FEq $\mu\nu$. denique quia rectang. ACB [c] est $\rho\nu$, [d] etiam DFE est $\rho\nu$. [e] ergo DE est potens $\rho\nu$, ac $\mu\nu$. Q. E. D.

a hyp
b lem 66.10.
c 24 10.
d sch.12.10.
e 41. 10.

PROP. LXXI.

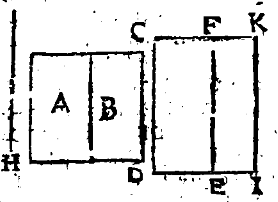
Bina media potenti (AC + CB) *commensurabilis* DE, *& ipsa bina media potens est.*

Divide DE, ut in præced. Quia ACq [a] $\sqcup$ CBq, [b] erit DFq $\sqcup$ FEq. item quia ACq + CBq [a] est $\mu\nu$, [c] erit DFq + FEq etiam $\mu\nu$. pariterque quia ACB [a] est $\mu\nu$, [d] etiam DFE est $\mu\nu$. denique quia ACq + CBq $\sqcup$ ACB. [e] erit

a hyp
b lem 66.10.
c 24 10.
d 24. 10.

e 14. 10. f 41. 10. *e* erit DFq + FEq ⊐⊏ DFE. *f* è quibus sequitur DE esse potentem 2 μσ. Q. E. D.

PROP. LXXII.

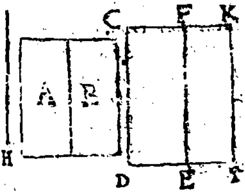


Si rationale A, & medium B componantur, quatuor irrationales fiunt; vel ea quæ ex binis nominibus, vel quæ ex binis mediis prima, vel major, vel rationale ac medium potens.

Nimirum si Hq = A + B, erit H una 4 linearum, quas theorema designat. Nam ad CD
a cor. 16. 6. expositum ρ́, *a* fiat rectang. CE = A; item FI
b 2. ax. 1. = B; *b* ideoque CI = Hq. Quoniam igitur A
c 21. 10. est ρ́ν, etiam CE est ρ́ν. *c* ergo latitudo CF
est ρ́ ⊐⊏ CD. & quia B est μν, erit FI μν.
d 23. 10. *d* ergo FK est ρ́ ⊐⊏ CD. *e* ergo CF, FK sunt
e 13. 10. ρ́ ⊐⊏. Tota igitur CK *f* est bin. Si igitur A
f 38. 10. ⊏ B, hoc est CE ⊏ FI, *g* erit CF ⊏ FK. er-
g 1. 6. go si CF ⊐⊏ √ CFq − FKq, *h* erit CK bin.
h 1. def. 48. 10. 1. & proinde H = √ CI *k* est bin. Si ponatur
k 55. 10. CF ⊐⊏ √ CFq − FKq, *l* erit CK bin. 4.
l 4. def. 48. 10. quare H (√ CI) *m* est major. Sin A ⊐ B;
m 58. 10. *g* erit CF ⊐ FK; proinde si FK ⊐⊏ √ FKq −
n 2. def. 48. 10. CFq, *n* erit CK bin. 2. *o* quare H est 2 μ prima. denique si FK ⊐⊏ √ FKq − CFq, *p* erit
o 56. 10. p 5 def. 48. 10. CK bin. 5. *q* unde H erit potens ρ́ν ac μν.
q 59. 10. Q. E. D.

PROP.

PROP. LXXIII.



Si duo media A, B, inter se incommensurabilia componantur, duæ reliquæ irrationales fiunt; vel ex binis mediis secunda, vel bina media potens.

Nempe H potens A + B est una dictarum irrationalium. Nam ad CD expos. ρ, fac rectang. CE = A, & FI = B. unde Hq = CI. Quoniam igitur CE, & FI [a] sunt μα, [b] erunt latitudines CF, FK ρ ⊓ CD. item quia CE [c] ⊓ FI; estque CE. FI [c] :: CF. FK, [d] erit CF ⊓ FK. [e] ergo CK est bin 3. nempe, si CF ⊓ √ CFq − FKq. unde H = √ CI [f] erit 2 μ 2a. Sin vero CF ⊓ √ CFq − FKq, [g] erit CK bin. 6. & [h] proinde H est potens 2 μα. Q. E. D. a hyp. b 23. 10. c 1. 6. d 10. 10. e 3. def. 48. 10. f 57. 10. g 6. def. 48. 10 h 60. 10.

Principium Senariorum per detractionem.

PROP. LXXIV.

D E F

Si à rationali DF rationalis DE auferatur, potentia tantum commensurabilis existens toti DF; reliqua EF irrationalis est: vocetur autem apotome.

Nam EFq [a] ⊓ DEq; sed DEq [b] est ρν. [c] ergo EF est ρ'. Q. E. D. a lem. 26. 10. b hyp. c 10. & 11. def. 10.

In numeris, sit DF, 2; DE, √ 3. EF erit 2 − √ 3.

PROP. LXXV.

D E F *Si à media* DF *media* DE *auferatur, potentia tantum commensurabilis existens toti* DF, *quæ cum tota* DF *rationale contineat; reliqua* EF *irrationalis est; vocetur autem mediæ apotome prima.*

a sch. 16 10. Nam EFq ^{a} $\sqsupset\!\sqsubset$ rectang. FDE. ergo cum
b hyp. FDE ^{b} sit $\rho\nu$, ^{c} erit EF $\dot{\rho}$. Q. E. D.
c 20. & 11. def. 10. *In numeris*, sit DF $v\sqrt{54}$; & DE $v\sqrt{24}$. ergo EF est $v\sqrt{54} - v\sqrt{24}$.

PROP. LXXVI.

D E F *Si à media* DF *media* DE *auferatur, potentia tantum commensurabilis existens toti* DF, *quæ cum tota* DF *medium contineat; reliqua* EF *irrationalis est; vocetur autem mediæ apotome secunda.*

a hyp. Quia DFq, & DEq ^{a} sunt $\mu\alpha$ $\sqsupset\!\sqsubset$,
b 16. 10. ^{b} erit DFq + DEq $\sqsupset\!\sqsubset$ DEq. ^{c} quare DFq
c 24. 10. + DEq est $\mu\nu$. item rectang. FDE, ^{c} ideoque
d cor. 7. 2. 2 FDE ^{e} est $\mu\nu$. ergo EFq (^{d} DFq + DEq − 2 FDE) ^{e} est $\rho'\nu$. quare EF est ρ'. Q. E. D.
e 27. 10. *In numeris*, sit DF, $v\sqrt{18}$; & DE, $v\sqrt{8}$. erit EF $v\sqrt{18} - v\sqrt{8}$.

PROP. LXXVII.

Si à recta linea AC *recta*

A B C *auferatur* AB, *potentia incommensurabilis existens toti* BC, *quæ cum tota* AC *faciat compositum quidem ex ipsarum quadratis rationale, quod autem sub ipsis continetur medium; reliqua* BC *irrationalis est: vocetur autem minor.*

a hyp. Nam ACq + ABq ^{a} est $\rho'\nu$. at rectang. ACB
b sch. 12. 10. ^{a} est $\mu\nu$. ^{b} ergo 2 CAB $\sqsupset\!\sqsubset$ ACq + ABq
c 7. 2. (2 ^{c} CAB + BCq;) ^{d} ergo ACq + ABq $\sqsupset\!\sqsubset$
d 17. 10. e 11. def. 10. BCq. ^{e} ergo BC est $'\rho$. Q. E. D.

In

In numeris, sit AC, $\sqrt{:}\,18. + \sqrt{108}$. AB $\sqrt{:}\,18 - \sqrt{108}$. ergo BC est $\sqrt{:}\,18 + \sqrt{:}\,108. - \sqrt{:}\,18 - \sqrt{108}$.

PROP. LXXVIII.

D———E———F *Si à recta linea* DF *recta auferatur* DE *potentia incommensurabilis existens toti* DF, *quæ cum tota* DF *faciat compositum quidem ex ipsarum quadratis medium, quod autem sub ipsis continetur, rationale; reliqua* EF *irrationalis est: vocetur autem cum rationali medium totum efficiens.*

Nam 2 FDE *a* est ῥν. *b* & DFq + DEq est μν. *c* ergo 2 FDE ⊏ DFq + DEq *d* (2 FDE + EFq) *e* ergo EF est ᾱ'. Q. E. D.

In numeris sit DF, $\sqrt{:}\,\sqrt{216} + \sqrt{72}$. DE, $\sqrt{:}\,\sqrt{216} - \sqrt{72}$. ergo EF est $\sqrt{:}\,\sqrt{216} + \sqrt{72} - \sqrt{:}\,\sqrt{216} - \sqrt{72}$.

a *hyp.* & *sch.* 12.10.
b *hyp.*
c *sch.* 12.10.
d 7. 2.
e *sch.* 12.10. & 11. *def.* 10.

PROP. LXXIX.

D E F

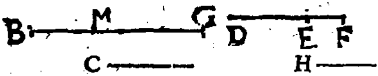
Si à recta DF *recta auferatur* DE, *potentia incommensurabilis existens toti* DF, *quæ cum tota faciat & compositum ex ipsarum quadratis, medium; & quod sub ipsis continetur, medium, incommensurabileque composito ex quadratis ipsarum, reliqua irrationalis est: vocetur autem cum medio medium totum efficiens.*

Nam 2 FDE, & DFq + DEq *a* sunt μα; *b* ergo EFq (*c* DFq + DEq − 2 FDE) est ᾱ'ν. *d* proinde EF est ᾱ'. Q. E. D.

Exempl. gr. sit DF, $\sqrt{:}\,\sqrt{180} + \sqrt{60}$. DE, $\sqrt{:}\,\sqrt{180} - \sqrt{60}$. EF erit $\sqrt{:}\,\sqrt{180} + \sqrt{60} - \sqrt{:}\,\sqrt{180} - \sqrt{60}$.

a *hyp.* & 24. 10.
b 27. 10.
c *cor.* 7. 2.
d 11. *def.* 10.

LEMMA.



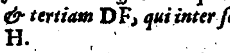
Si idem sit excessus inter primam magnitudinem BG, *& secundam* C (MG) *qui inter tertiam magnitudinem* DF, *& quartam* H (EF;) *erit & vicissim idem excessus inter primam magnitudinem* BG, *& tertiam* DF, *qui inter secundam* C, *& quartam* H.

Nam quia *a* æqualibus BM, DE adjectæ sunt
a hyp. æquales MG, EF, *a* hoc est C, H; erit excessus to-
a 15.ax,1. torum BG, DF, *b* æqualis excessui adjectorum, C, H. Q. E. D.

Coroll.

Hinc, quatuor magnitudines Arithmetice proportionales, vicissim erunt Arithmetice proportionales.

PROP. LXXX.



Apotomæ AB *una tantum congruit recta linea rationalis* BC, *potentia tantum commensurabilis existens toti* AB.

Si fieri potest, alia BD congruat. *a* ergo re-
a 22. 10. ctangula ACB, ADB; *b* ideoq; eorum dupla sunt
b 24. 10. μέσα. cum igitur ACq + BCq − 2 ACB *c* = ABq
c cor.7. 2. *c* = ADq + DBq − 2 ADB. ergo vicissim ACq
d lem.79.10. + BCq −: ADq + BDq *d* = 2 ACB −: 2
e hyp. & 27. 10. ADB. Sed ACq + BCq −: ADq + BDq *e* est
f sch.12. 10. ῥητ. *f* ergo 2 ACB −: 2 ADB est ῥητ.
g 27. 10. Q. E. A.

PROP.

PROP. LXXXI.

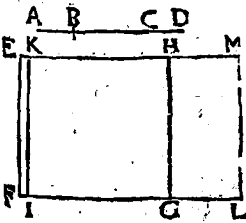
A B D C

Mediæ Apotomæ primæ A B *una tantum congruit recta linea media* B C, *potentia solum commensurabilis existens toti, & cum tota rationale continens.*

Dic etiam B D congruere. igitur quoniam tam ACq, & BCq; quam ADq, & BDq *a* sunt μα ⌐⊥. *b* etiam ACq + BCq, & ADq + BDq erunt μα. *c* sed rectangula ACB, ADB; *d* adeoque 2 ACB, & 2 ADB sunt ρ᾽α. *e* ergo 2 ACB –: 2 ADB; *f* hoc est ACq + BCq –: ADq + BDq est ρ᾽ν. *g* Q. E. A.

a hyp.
b 16. & 24. 10.
c hyp.
d sch. 12. 10.
e sch. 27. 10.
f 7. 2. & lem. 79. 10.
g 27. 10.

PROP. LXXXII.



Mediæ Apotomæ secundæ A B *una tantum congruit recta linea media* B C, *potentia solum commensurabilis existens toti, & cum tota medium continens.*

Si fieri potest, congruat alia BD. Ad EF ρ᾽ fiant rectang. E G = ACq + BCq; item rectang. EL = ADq + BDq. Item EI = ABq. Jam 2 ACB + ABq = ACq + BCq = EG, ergo cum EI = ABq, *a* erit KG = 2 ACB. porro ACq, & BCq *b* sunt μα ⌐⊥. *c* Ergo EG (ACq + BCq) est μν. *d* ergo latitudo E H ρ᾽ ⌐⊥ E F. *e* Quinetiam rectang. ACB; *f* ideoque 2 ACB (KG) est μν. *d* ergo KH est etiam ρ᾽ ⌐⊥ EF. denique quia ACq + BCq, id est, EG, *g* ⌐⊥ 2 ACB (KG) estque

a 4. ax. & 3. ax. 1.
b hyp.
c 24. 10.
d 23. 10.
e hyp.
f 24. 10.
g lem. 26. 10.

h 1.6. k 10.10. l 74.10.

EG.KG :: [h] EH. KH [k] erit EH ⊔⊔ KH. [l] ergo EK est apotome, cujus congruens KH. simili argumento erit KM ejusdem EK congruens; contra 80 hujus.

PROP. LXXXIII.

A B D C

Minori AB, *una tantum congruit recta linea* (BC) *potentia incommensurabilis existens toti, & cum tota faciens compositum quidem ex ipsarum quadratis rationale; quod autem sub ipsis continetur medium.*

a hyp. b lem.97.10. c sch.27.10. d 27.10.

Puta alium BD congruere. Cum igitur ACq + BCq, & ADq + BDq [a] sint $\rho\alpha$, eorum excessus (2 [b] ACB −: 2 ADB) [c] est $\rho\nu$, [d] Q.E.A; quia ACB, & ADB sunt $\mu\alpha$ per hypoth.

PROP. LXXXIV.

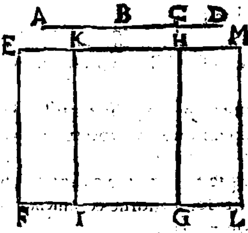
A B D C

Ei (AB,) *quæ cum rationali medium totum facit, una tantum congruit recta linea* BC, *potentia incommensurabilis existens toti, & cum tota faciens compositum quidem ex ipsarum quadratis medium; quod autem sub ipsis continetur, rationale.*

a hyp. b sch.12.10. c lem 79.10. d sch.27.10.

Dic aliam BD etiam congruere. [a] ergo rectangula ACB, ADB. [b] ideoque 2 ACB, & 2 ADB sunt $\rho\alpha$. ergo 2 ACB −: 2 ADB; [c] hoc est, ACq + BCq −: ADq + BDq [d] est $\rho\nu$. Q.E.A: quum ACq + BCq, & ADq + BDq sunt $\mu\alpha$ per hypoth.

PROP. LXXXV.



Ei (AB,) quæ cum media medium totum facit una tantum congruit recta linea BC potentia incommensurabilis existens toti, & cum tota faciens & compositum ex ipsarum quadratis medium, & quod sub ipsis continetur, medium, incommensurabileque composito ex ipsarum quadratis.

Suppositis iis quæ facta & ostensa sunt in 82 hujus; liquet EH, & KH esse ρ ⊔⊔ EF. Porro igitur quia ACq + CBq, hoc est, rectang. EG [a] ⊔⊔ ACB, [b] ideoque EG ⊔⊔ 2 ACB (KG) estque EG. KG :: [c] EH. KH; erit EH ⊔⊔ KH. ergo EK est apotome, cujus congruens KH. Haud aliter KM eidem apotomæ EK. congruere ostendetur; contra 80 hujus.

a hyp.
b 14. 10.
c 1. 6.

Definitiones tertia.

Exposita rationali, & apotoma, si tota plus possit quam congruens quadrato rectæ lineæ sibi longitudine commensurabilis;

I. Si quidem tota expositæ rationali longitudine sit commensurabilis, vocetur apotome prima.

II. Si vero congruens expositæ rationali longitudine sit commensurabilis, vocetur apotome secunda.

III. Quod si neque tota, neque congruens expositæ rationali sit longitudine commensurabilis, vocetur apotome tertia.

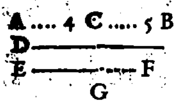
Rursus, si tota plus possit quam congruens quadrato rectæ sibi longitudine incommensurabilis;

IV. Si quidem tota expositæ rationali sit longitudine commensurabilis, vocetur apotome quarta.

V. Si vero congruens expositæ rationali sit longitudine commensurabilis, vocetur apotome quinta.

VI. Quod si neque tota, neque congruens, expositæ rationali sit longitudine commensurabilis, vocetur apotome sexta.

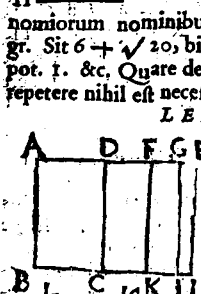
PROP. LXXXVI, 87, 88, 89, 90, 91.



Invenire apotomen primam, secundam, tertiam, quartam, quintam, sextam.

Apotomæ inveniuntur, subductis minoribus binomiorum nominibus ex majoribus. Exemp. gr. Sit $6+\sqrt{20}$, bin. 1. erit $6-\sqrt{20}$, apot. 1. &c. Quare de earum inventione plura repetere nihil est necesse.

LEMMA.



Sit rectangulum AC *sub rectis* AB, AD. *producatur* AD *ad* E, & *bisecetur* DE *in* F. *sitque rectang.* AGE = FEq. & *compleantur rectangula* AI, DK, FH. *Fiant vero quadratum* LM = AH, & *quadratum* NO = GI, *producanturque* NSR, OST.

Dico primo, rectangul. AI = LM + NO = TOq + SOq. ut patet ex constr.

Se-

Secundo, *Rectang.* DK = LO. Nam quia rectang. AGE *a* = FEq, *b* sunt AG, FE, GE ∺, *c* adeoque AH, FI, GI ∺; *a* hoc est, LM, FI, NO ∺. atqui LM, LO, NO *d* sunt ∺; ergo FI = *e* LO *f* = DK = *g* NM. a constr. b 17. 6. c 1. 6. d sch. 22. 6. e 9. 5. f 36. 1.

Tertio, *Hinc*, AC = AI − DK − FI = LM + NO − LO − NM = TR. g 43. 1.

Quarto, *h Liquet* DF, FE, DE esse ⊓⊔. h 16. 10.

Quinto, *Si* AE ⊓⊔ DE, & AE ⊓⊔ √ AEq − DEq, *k erunt* AG, GE, AE ⊓⊔. k 18. 10. & 10. 10.

Sexto, *Item, quia* AE *l* ⊓⊔ DE, *m erunt* AE, FE ⊓⊔. *n ideoque* AI, FI; *hoc est*, LM + NO & LO *sunt* ⊓⊔. l hyp. m 13. 10.

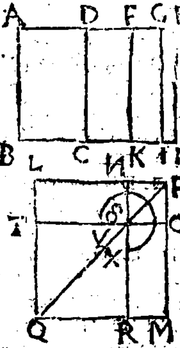
Septimo, *Item quia* AG * ⊓⊔ GE, *n erunt* AH GI, *hoc est*, LM, NO ⊓⊔. n 1. 6. & 10. 10. * prius.

Octavo, *Sed quia* AE *l* ⊓⊔ DE, *o erunt* FE, GE ⊓⊔. *n ideoque rectang.* FI ⊓⊔ GI, *hoc est* LO ⊓⊔ NO. *quare cum* LO. NO *p* :: TS, SO. *q erunt* TS, SO ⊓⊔. o 14. 10. p 2. 6. q 10. 10.

Nono, *Sin ponatur* AE ⊓⊔ √ AEq − DEq; *r erunt* AG, GE, AE ⊓⊔. r 19. 10. & 17. 10.

Decimo, *s Quare rectang.* AH, GI, *hoc est* TOq, SOq *erunt* ⊓⊔. s 1. 6. & 10. 10.

PROP. XCII.



Si spatium AC contineatur sub rationali AB, & Apotoma prima AD (AE − DE;) recta linea TS spatium AC potens, apotome est.

Adhibe lemma proxime antecedens pro præparatione ad demonstrationem hujus. Igitur TS = $\sqrt{}$ AC. item AG, GE, AE sunt $\sqcap$; ergo cum AE $\sqcap$ a AB ρ'; b erunt AG, & GE $\sqcap$ AB. c ergo rectangula AH & GI, hoc est TOq & SOq sunt $\rho'\alpha$. d item TO, SO sunt ρ' $\not\sqcap$, e proinde TS est apotome. Q. E. D.

a hyp.
b 12. 10.
c 20. 10.
d lem. 91. 10.
e 74. 10.

PROP. XCIII.

Vide Schem. præced.

Si spatium AC contineatur sub rationali AB, & apotoma secunda AD (AE − DE;) recta linea TS spatium AC potens; mediæ est apotome prima.

Rursus juxta lemma antecedens, AG, GE, AE sunt $\sqcap$. cum igitur AE a sit ρ' $\sqcap$ AB, b erunt AE, GE etiam ρ' $\sqcap$ AB. c ergo rectangula AH, GI, hoc est TOq, SOq, sunt $\mu\alpha$; d item TO $\not\sqcap$ SO. Denique quia DE e $\sqcap$ AB. ρ'. f erit rectang. DI, ejusque semissis DK, vel LO, hoc est TOS $\rho'\nu$ g è quibus sequitur TS ($\sqrt{}$ AC) esse mediæ apot. 1. Q. E. D.

a hyp.
b 13. 10.
c 22. 10.
d lem. 74 10.
e hyp.
f 20. 10.
g 75. 10.

PROP. XCIV.

Vide idem.

Si spatium AC *contineatur sub rationali* AB, *& apotoma tertia* AD (AE $-$ DE;) *recta linea* TS *spatium* AC *potens, mediæ est apotome secunda.*

Ut in præcedenti TO, & SO sunt μ. Quoniam igitur DE *a* est ρ^{c} ⊐⊏ AB, *b* erit rectang.	a hyp.
DI, *c* ideoque DK; vel TOS $\mu\nu$. *d* ergo TS	b 22. 10.
	c 14. 10.
$=\surd$ AC est mediæ apot. 2. Q. E. D.	d 76. 10.

PROP. XCV.

Vide idem.

Si spatium AC *contineatur sub rationali* AB, *& apotoma quarta* AD (AE $-$ DE) *recta linea* TS *spatium* AC *potens, minor est.*

Rursus TO *a* ⊐⊏ SO. Quoniam igitur AE	a lem 91. 10.
b est ρ^{c} ⊐⊏ AB, *c* erit AI, (TOq + SOq) $\rho^{c}\nu$.	b hyp.
	c 20. 10.
atqui ut prius rectang. TOS est $\mu\nu$. *d* ergo TS	d 77. 10.
$=\surd$ AC est minor. Q. E. D.	

PROP. XCVI.

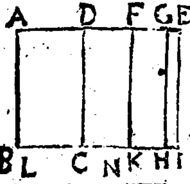
Vide idem.

Si spatium AC *contineatur sub rationali* AB, *& apotoma quinta* AD (AE $-$ DE;) *recta linea* TS *spatium* AC *potens, est quæ cum rationali medium totum efficit.*

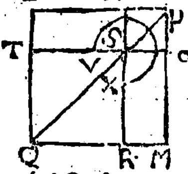
Rursus enim TO ⊐⊏ SO. itaque cum AE *a* sit ρ^{c} ⊐⊏ AB, *b* erit AI, hoc est TOq + SOq	
$\mu\nu$. Sed prout in 93 rectang. TOS est $\rho^{c}\nu$. *c* proinde TS $=\surd$ AC est quæ cum $\rho^{c}\nu$ facit totum	a hyp.
	b 22. 10.
$\mu\nu$. Q. E. D.	c 78. 10.

PROP.

PROP. XCVII.



Si spatium AC *contineatur sub rationali* AF, *& apotoma sexta* AD (AE − DE;) *recta linea* TS *spatium* AC *potens, est quæ cum medio medium totum efficit.*



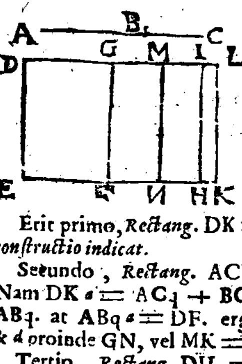
Itidem, ut sæpe prius, TO ⊓ SO. item ut in 96, TOq + SOq est $\mu\nu$. rectang. vero TOS est $\acute{\rho}\nu$, ut in 94. a denique TOq + SOq ⊓ TOS. b ergo TS = √. AC est quæ cum $\mu\nu$ facit totum $\mu\nu$.

a lem 91. 10.
b 79. 10.

Q. E. D.

LEMMA.

* cor. 16. 6.



Ad rectam quamvis DE * *applicentur rectang.* DF = ABq, *&* DH = ACq, *&* IK = BCq; *& sit* GL *bisecta in* M; *ductaque sit* MN *parall.* GF.

Erit primo, *Rectang.* DK = ACq + BCq, *ut constructio indicat.*

Secundo, *Rectang.* ACB = GN, *vel* MK. Nam DK a = ACq + BCq b = 2 ACB + ABq. at ABq a = DF. ergo GK c = 2 ACB. & d proinde GN, vel MK = ACB.

a constr.
b 7. 2.
c 3. ax. 1.
d 7. ax. 1.

Tertio, *Rectang.* DIL = MLq. Nam quia ACq. ACB e :: ACB. BCq; hoc est DH. MK

e 1. 6

MK :: MK. IK, *e* erit DI. ML :: ML. IL. *f* ergo DIL = MLq.

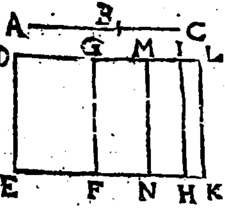
Quarto, *Si ponatur* AC ⊤⊥ BC, *erit* DK ⊏⊐ ACq. Nam ACq + BCq (DK) *g* ⊏⊐ ACq. f 17. 6. g 16. 10.

Quinto, *Item*, DL ⊏⊐ √ DLq − GLq. Nam quia DH (ACq) ⊏⊐ IK (BCq) *h* erit DI ⊏⊐ IL. *k* ergo √ DLq − GLq ⊏⊐ DL. h 10. 10. k 18. 10.

Sexto, *Item* DL ⊏⊐ GL. Nam ACq + BCq ⊏⊐ *l* 2 ACB; hoc est, DK ⊏⊐ GK. *m* ergo DL ⊏⊐ GL. l lem. 26. 10. m 10. 10.

Septimo, *Si non ponatur* AC ⊤⊥ BC, *n erit* DL ⊏⊐ √ DLq − GLq. n 19. 10.

PROP. XCVIII.



Quadratum apotomæ AB (AC − BC) *ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen primam.*

Fac ut in lemmate proxime præcedenti.

Quoniam igitur AC, BC *a* sunt ῥ' ⊤⊥, *b* erit DK (ACq + BCq) ⊏⊐ ACq; *c* ergo DK est ῥ'ν. *d* quare DL est ῥ' ⊏⊐ DE. *e* item rectang. GK (2 ACB) est μν. *f* ergo GL est ῥ' ⊏⊐ DE. *g* proinde DL ⊏⊐ GL; *h* sed DLq ⊏⊐ GLq. *k* ergo DG est apotome, & *l* quidem prima (quia *m* AC ⊤⊥ BC, & propterea DL ⊏⊐ √ DLq − GLq.) Q. E. D. a hyp. b lem 97. 10. c sch 12. 10. d 21. 10. e 22. & 24. 10. f 23. 10. g 13. 10. h sch. 12. 10. k 74 10. l 1 def. 85. 10. m lem. 97. 10.

PROP.

PROP. XCIX.

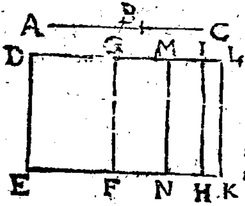
Vide Schema subsequens.

Quadratum mediæ apotomæ primæ AB (AC — BC) *ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen secundam.*

a hyp. b lem. 97. 10. c 24. 10. d 23. 10. e hyp. & sch. 12. 10. f 21. 10. g 13. 10. h sch. 12. 10. k 74. 10. l lem. 97. 10. m 2. def. & 85. 10.

Rursus (supposito lemmate præcedenti) quia AC, & BC *a* sunt μ ⊐⊏ *b*, erit DK (ACq + BCq) ⊐⊏ ACq; *c* quare DK est μν. *d* ergo DL est ῥ ⊐⊏ DE. *e* item GK (2. ACB) est μν. *f* ergo GL est ῥ ⊐⊏ DE; *g* quare DL ⊐⊏ GL. *h* Sed DLq ⊐⊏ GLq. *k* ergo DG est apotome. quia vero DL *l* ⊐⊏ √ DLq — GLq, *m* erit DG apotome secunda. Q. E. D.

PROP. C.



Quadratum mediæ apotomæ secundæ AB (AC — BC) *ad rationalem* D E *applicatum, facit latitudinem* DG *apotomen tertiam.*

a 23. 10. b lem. 16. 10. c 1. 6. & 10. 10. d sch. 12. 10. e 74. 10. f 3. def. 85. 10. g lem. 97. 10.

Iterum DK est μν, *a* quare DL est ῥ ⊐⊏ DE. item GK est μν. *a* unde GL est ῥ ⊐⊏ DE; *b* item DK ⊐⊏ GK, *c* quare DL ⊐⊏ GL; *d* at DLq ⊐⊏ GLq. *e* ergo DG est apot. & quidem *f* 3ª. *g* quia DL ⊐⊏ √ DLq — GLq. Q. E. D.

PROP. CI.

Vide Schema præced.

Quadratum minoris AB (AC — BC) *ad rationalem*

tionalem DE *applicatum, facit latitudinem* DG *apotomen quartam.*

Ut prius, ACq + BCq, hoc est DK est ρης; *a* ergo DL est ῥ ⊓⊔ DE. at rectang. ACB, ideoque GK (2 ACB) * est μν, *b* quare GL est ῥ ⊓⊔ DE. *c* ergo DL ⊓⊔ GL. *d* at DLq ⊓⊔ GLq. quia vero * ACq ⊓⊔ BCq, *e* erit DL ⊓⊔ √DLq − GLq: *f* ergo DG conditiones habet apotomæ quartæ. Q. E. D. — *a* 21.10. * hyp. *b* 13.10. *c* 13.10. *d* sch.12.10. *e* lem.97.10 *f* 4.def.85.10.

Prop. CII.

Vide Schem. præced.

Quadratum ejus AB (AC − BC,) *quæ cum rationali medium totum efficit, ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen quintam.*

Rursus enim, DK est μν, *a* quare DL est ῥ ⊓⊔ DE. item GK est ῥν, *b* unde GL est ῥ ⊓⊔ DE. *c* ergo DL ⊓⊔ GL, *d* sed DLq ⊓⊔ GLq. porro, DL *e* ⊓⊔ √DLq − GLq. ex quibus, DG *f* est apot. quinta. Q. E. D. — *a* 23.10. *b* 21.10. *c* 13.10. *d* sch.12.10. *e* lem.97.10. *f* 5.def.85.10.

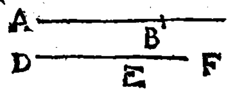
Prop. CIII.

Vide Schema idem.

Quadratum ejus AB (AC − BC,) *quæ cum medio medium totum efficit, ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen sextam.*

Haud aliter, quam antea, DK, & GK sunt μν; *a* quare DL & GL sunt ῥ ⊓⊔. DE. item DK *b* ⊓⊔ GK, *c* quare DL ⊓⊔ GL. *d* ergo DG est apot. *b* cum igitur ACq ⊓⊔ BCq, ideoque DL ⊓⊔ √DLq − GLq, *e* erit DG. apot. sexta. Q. E. D. — *a* 23.10. *b* hyp. & lem. 97.10. *c* 10.10. *d* 74.10. *e* 6.def.85.10.

PROP. CIV.



Recta linea DE *apotomæ* AB (AC — BC) *longitudine commensurabilis, & ipsa apotome est, atque ordine eadem.*

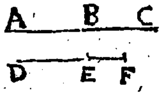
LEMMA.

Sit AB. DE :: AC. DF. *&* AB ⊓⊔ DE.
Dico AC + BC ⊓⊔ DF + EF.

Nam AC. BC *a* :: DF. EF. ergo componendo AC + BC. BC :: DF + EF. EF. ergo per-
a lem 66 10. mutando AC + BC. DF + EF :: BC. EF. *a* at
b 10.10. BC ⊓⊔ EF. *b* ergo AC + BC ⊓⊔ DF + EF. Q. E. D.

a 12 6 *a* Fac AB. DE :: AC. DF. *b* igitur AC +
b lem. 103. 10. BC ⊓⊔ DF + EF. ergo cum AC + BC *c* bi-
c hyp. nomium sit, *d* erit DF + EF ejusdem ordinis bi-
d 6. 10. nomium; *e* quare DF — EF ejusdem ordinis a-
e Praefinitione, ad 85. 10. potome est, cujus AC — BC. Q. E. D.

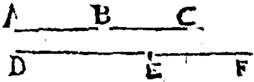
PROP. CV.



Recta linea DE *mediæ apotomæ* AB (AC — BC) *commensurabilis, & ipsa mediæ apotome est, atque ordine eadem.*

a 12. 6 Iterum *a* fac AB. DE :: AC. DF. *b* quare
b lem. 103. 10. AC + BC ⊓⊔ DF + EF. *c* ergo DF + EF
c 68 10 est bimed. ejusdem ordinis, cujus AC + BC.
d 75, & 76. 10. *d* proinde & DF — EF mediæ apotome erit ejusdem classis, cujus AC — BC. Q. E. D.

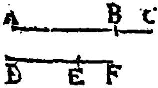
PROP. CVI.



Recta linea DE *Minori* AB (AC − BC) *commensurabilis, & ipsa minor est.*

Fiat AB. DE :: AC. DF. *a* estque AC + BC ⊓ DF + EF. atqui AC + BC *b* est Major, *c* ergo DF + EF quoque Major est. *d* & proinde DF − EF est Minor. Q. E. D. a lem. 103. 10. b hyp. c 69. 10. d 77. 10.

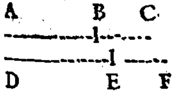
PROP. CVII.



Recta linea DE *commensurabilis ei* AB(AC − BC) *quæ cum rationali medium totum efficit, & ipsa cum rationali medium totum efficiens est.*

Nam ad modum præcedentium ostendemus DF + EF esse potentem $\rho\nu$, & $\mu\nu$. *a* ergo DF − EF est ut dicitur. a 78. 10.

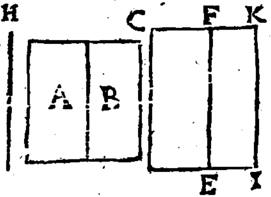
PROP. CVIII.



Recta linea DE *commensurabilis ei* AB (AC − BC) *quæ cum medio medium totum efficit, & ipsa cum medio medium totum efficiens est.*

Nam, ad normam præcedentium, erit DF + EF potens 2 $\mu\alpha$. *a* ergo DF − EF erit ut in propos. a 79. 10.

PROP. CIX.



Medio B à rationali A + B detracto, recta linea H, quæ reliquum spatium A potest, una ex duabus irrationalibus fit, vel apotome, vel Minor.

a 3. ax. 1.
b hyp. & constr.
c 21. 10.
d 23. 10.
e 13. 10.
f 74. 10.
g 1. def. 85. 10.
h 92. 10.
k 4. def. 85. 10.
l 95. 10.

Ad CD ρ', fac rectang. CI = A + B; & FI = B. quare CE *a* = A: (Hq) Quoniam igitur CI *b* est $\dot{\rho}\nu$, *c* erit CK ρ' $\sqcup$ CD. sed quia FI *b* est $\mu\nu$, *d* erit FK ρ' $\sqcup$ CD. *e* unde CK $\sqcup$ FK *f* ergo CF est apotome. Si igitur CK $\sqcup$ $\sqrt{}$ CKq − FKq. *g* erit CF apot. prima; *h* quare $\sqrt{}$ CE (H) est apotome. sin CK $\sqcup$ $\sqrt{}$ CKq − FKq, *k* erit CF apot. quinta. & proinde H ($\sqrt{}$ CE) *l* erit Minor. Q. E. D.

PROP. CX.

Vide Schem. præced.

Rationali B à medio A + B detracto; aliæ duæ irrationales fiunt, vel mediæ apotome prima, vel cum rationali medium totum efficiens.

a 3. ax. 1.
b hyp. & constr.
c 23. 10.
d 21. 10.
e 13. 10.
f 74. 10.
g 2. def. 85. 10.
h 93. 10.
k 5. def. 85. 10.
l 96. 10.

Ad CD expos. $\dot{\rho}$ fiant rectang. CI = A + B; & FI = B, *a* unde CE = A = Hq. Quoniam igitur CI *b* est $\mu\nu$: *c* erit CK ρ' $\sqcup$ CD. sed quia FI *b* est $\dot{\rho}\nu$, *d* erit FK ρ' $\sqcup$ CD. *e* unde CK $\sqcup$ FK. *f* ergo CF est apot. *g* nempe secunda; si CK $\sqcup$ $\sqrt{}$ CKq − FKq, *h* quare H ($\sqrt{}$ CE) est mediæ apot. prima. Sin vero CK $\sqcup$ $\sqrt{}$ CKq − FKq, *k* erit CF apot. quinta. & proinde H ($\sqrt{}$ CE) erit faciens $\mu\nu$ cum $\dot{\rho}\nu$. Q. E. D.

PROP.

Prop. CXI.

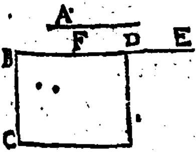
Vide Schema idem.

Medio B à medio A + B detracto, quod sit incommensurabile toti A + B; reliquæ duæ irrationales sunt, vel mediæ apotome secunda, vel cum medio medium totum efficiens.

Ad CD ρ' fiant rectang. CI = A + B; & FI = B, a quare CE = A = Hq. Quoniam igitur CI est μν. b erit CK ρ' ⊤⊏ CD. eodem modo erit FK ρ' ⊤⊏ CD. item quia CI c ⊤⊏ FI, d erit CK ⊤⊏ FK; e quare CF est apotome, f tertia scilicet, si CK ⊤⊏ √CK − FKq, g unde H (√CE) erit mediæ apot: secunda. verum si CK ⊤⊏ √CKq − FKq, h erit CF apot. sexta. k quare H erit faciens μν cum μ. Q. E. D.

a 3.ax.1.
b 23.10.
c hyp.
d 10.10.
e 74.10.
f 3 def.85.10.
g 94.10.
h 6 def.85.10.
k 97.10.

Prop. CXII.



Apotome A non est eadem, quæ ex binis nominibus.

Ad expos. BC ρ, fiat rectang. CD = Aq. Ergo cum A sit apotome, a erit BD apot. prima. ejus congruens sit DE. b quare BE, DE sunt ρ' ⊤⊏; c & BE ⊤⊏ BC. Vis A esse bin. ergo BD est bin. 1. ejus nomina sint BF, FD; sitque BF ⊏ FD; d ergo BF, FD sunt ρ' ⊤⊏; & BF e ⊤⊏ BC. ergo cum BC ⊤⊏ BE, f erit BE ⊤⊏ FB. g ergo BE ⊤⊏ FE. h ergo FE est ρ. item quia BE ⊤⊏ DE, k erit FE ⊤⊏ DE. l quare FD est apotome, l adeoque FD est ρ̄: sed ostensa est ρ. quæ repugnant. ergo A male dicitur binomium. Q. E. D.

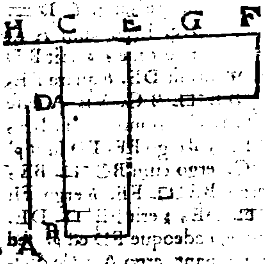
a 98.10.
b 74.10.
c 1.def. 85.10.
d 37.10.
e 1.def.48.10
f 12.10.
g cor.16.10.
h sch.12.10.
k 14.10.
l 74.10.

Nomina 13 linearum irrationalium inter se differentium.

1. Media.
2. Ex binis nominibus, cujus 6 species.
3. Ex binis mediis prima.
4. Ex binis mediis secunda.
5. Major.
6. Rationale ac medium potens.
7. Bina media potens.
8. Apotome, cujus etiam 6 species.
9. Mediæ apotome prima.
10. Mediæ apotome secunda.
11. Minor.
12. Cum rationali medium totum efficiens.
13. Cum medio medium totum efficiens.

Cum latitudinum differentiæ arguant differentias rectarum, quarum quadrata sunt applicata ad aliquam rationalem, sitque demonstratum in præcedentibus, latitudines quæ oriuntur ex applicationibus quadratorum harum 13 linearum inter se differre, perspicue sequitur has 13 lineas inter se differre.

PROP. CXIII.



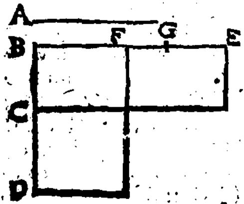
Quadratum rationalis A *ad eam, quæ ex binis nominibus* BC (BD + DC) *applicatum, latitudinem facit apotomen* EC, *cujus nomina* EH, CH *commensurabilia sunt nominibus* BD, DC *ejus, quæ ex binis nominibus &*

& in eadem proportione (EH. BD :: CH. DCs) *& adhæc, apotome* EC *quæ fit, eundem habet ordinem, quem ea* BC, *quæ ex binis nominibus.*

Ad DC minus nomen *a* fac rectang. DF = Aq = BE; quare BC. CD *b* :: FC. CE. ergo dividendo BD. DC :: FE. EC. cum igitur BD *c* ⊏ DC, *d* erit FE ⊏ EC. sume EG = ES; fiatque FG. GE :: EC. CH. Erunt EH, CH nomina apotomæ EC; quibus conveniunt ea, quæ in theoremate proposita sunt. Nam componendo FE. GE. (EC) :: EH. CH. ergo FH. EH *e* :: EH. CH *f* :: FE. EC *f* :: BD. DC. quare cum BD *g* ⊐⊏ DC, *h* erit EH ⊐⊏ CH; *h* & FHq ⊐⊏ EHq. ergo, quia FHq. EHq *k* :: FH. CH. *h* erit FH ⊐⊏ CH, *l* ideoque FC ⊐⊏ CH. Porro CD *g* est ρ, & DF (Aq) *g* est ρv, *m* ergo FC est ρ ⊐⊏ CD, quare etiam CH est ρ ⊐⊏ CD. *n* igitur EH CH sunt ρ, ac ⊐⊏ ut prius. *o* ergo EC est apotome, cui congruit CH. porro EH. CH *f* :: BD. DC, ideo permutando EH. BD :: CH. DC. unde quia CH *f* ⊐⊏ DC, *p* erit EH ⊐⊏ BD. quinimo pone BD ⊐⊏ √BDq − DCq; *q* erit ideo EH ⊐⊏ √EHq − CHq. item si BD ⊐⊏ ρ expos. erit EH ⊐⊏ eidem ρ; *s* hoc est si BC sit bin. 1. *t* erit EC apot. prima. Similiter si DC ⊐⊏ ρ expos. *t* erit CH ⊐⊏ eidem ρ. *u* hoc est si BC sit bin. 2. *a* erit EC apot. 2. & si hæc bin. 3. illa erit apot. 3, &c. Sin BD ⊐⊏ √BDq − DCq, *y* erit EH ⊐⊏ √EHq − CHq; si igitur BC sit bin. 4, vel 5, vel 6. erit EC similiter apot. 4, vel 5, vel 6. Q.E.D.

a cor. 16. 6.
b 14. 6.
c hyp.
d 14. 5.
e 12. 5.
f Prius.
g hyp.
h 10. 10.
k cor. 20. 6.
l 16. 10.
m 21. 10.
n sch. 12. 10.
o 74. 10.
p 10. 10.
q 15. 10.
r 12. 10.
s 1. def.
48. 10.
t 1. def.
85. 10.
u 2. def.
48. 10.
x 2. def.
85. 10.
y 15. 10.

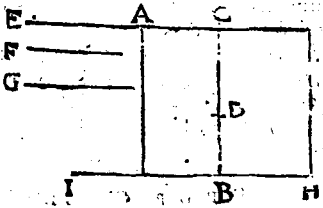
PROP. CXIV.



Quadratum rationalis A *ad apotomen* BC (BD—DC) *applicatum, facit latitudinem* BE *eam, quæ ex binis nominibus; cujus nomina* BE, GE *commensurabilia sint apotomæ* BC *nominibus* BD DC, *& in eadem proportione; & adhuc, quæ ex binis nominibus fit* (BE,) *eundem habet ordinem, quem ipsa apotome* BC.

a cor.16.6. a Fac rectang. DF = Aq; & BE. FE b ::
b 12.6. EG. GF. Quoniam igitur DF = Aq = CE,
c 14.6. c erit BD. BC :: BE. BF. ergo per conversionem rationis BD. CD :: BE. FE :: EG. GF :: d BG. EG. sed BD e ⊐⊏ CD. f ergo BG ⊐⊏
d 19.5.
e hyp. GE. ergo quia BGq. GEq g :: BG. GF. h erit
f 10. 10.
g cor.20.6. B G ⊓⊔ G F. k ideoque B G ⊓⊔ B F. porro
h 10. 10. BD e est ῥ́, & rectang. DF (Aq) e est ῥ́ν. l er-
k cor.16. 10. go BF est ῥ́ ⊓⊔ BD. m ergo etiam BG est ῥ́ ⊓⊔
l 21. 10.
m 12. 10. BD. n ergo BG, GE sunt ῥ́ ⊐⊏. o quare BE
n sch.12.10. est bin. denique igitur quia BD. CD :: BG.
o 37. 10. GE; & permutando BD. BG :: CD. GE; sitque BD ⊓⊔ BG; p erit CD ⊓⊔ GE. ergo si CB sit
p 10. 10. apot. prima; erit BE bin. 1. &c ut in antecedenti. ergo, &c.

PROP. CXV.



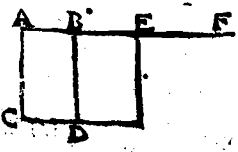
Si spatium AB *contineatur sub apotoma* AC (CE − AE,) *& ea, quæ ex binis nominibus* CB; *cujus nomina* CD, DB *commensurabilia sint apotome nominibus* CE, AE, *& in eadem proportione* (CE. AE :: CD. DB.;) *recta linea* F *spatium* AB *potens, est rationalis.*

Sit G quævis ρ; & fiat rectang. CH = Gq. *a* erit igitur BH (HI − IB) apotome; & HI *a* 113.10.
a ⊔⊔ CD *b* ⊔⊔ CE, *a* & BI ⊔⊔ DB; *a* atque
HI. BI :: CD. DB *b* :: CE, EA. ergo permu- b hyp.
tando HI. CE :: BI. EA. *c* ergo BH. AC :: c 19.5
HI. CE :: BI. EA. ergo cum HI *d* ⊔⊔ CE, d 12.10.
e erit BH ⊔⊔ AC. *f* ergo rectang. HC ⊔⊔ e 10.10.
BA. Sed HC (Gq) *b* est $\rho\nu$. *g* ergo BA (Fq) f 1.6. & 10
est $\rho\nu$, proinde F est ρ. Q. E. D. g sch. 12.10.

Coroll.

Hinc, fieri potest, ut spatium rationale contineatur sub duabus rectis irrationalibus.

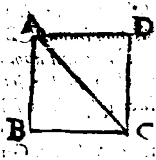
PROP. CXVI.



A media AB *fiunt infinitæ irrationales* BE, EF, &c. *& nulla alicui antecedentium est eadem.*

Sit AC expos. ρ. sita lem 38. 10. b. 11. 10. β. sitque AD spatium sub AC, AB. *a* ergo AD est $\rho'\nu$. Sume BE $=\sqrt{}$ AD. *b* ergo BE est ρ', nulli priorum eadem. nullum enim quadratum alicujus priorum applicatum ad ρ', latitudinem efficit mediam. compleatur rectang. DE; *a* erit DE $\rho'\nu$; & *b* proinde EF ($\sqrt{}$ DE) erit ρ'; & nulli priorum eadem. nullum enim priorum quadratum ad ρ' applicatum, latitudinem efficit ipsam BE. ergo, &c.

PROP. CXVII.



Propositum sit nobis ostendere, in quadratis figuris BD, diametrum AC lateri AB incommensurabilem esse.

a 47. 1. b cor. 24. 8. c 9. 10. Nam ACq. ABq *a* :: 2. 1 *b* :: non Q. Q. *c* ergo AC ⊥ AB. Q. E. D.

Celebratissimum est hoc theorema apud veteres philosophos, adeo ut qui hoc nesciret, eum Plato non hominem esse, sed pecudem diceret.

LIB. XI.

Definitiones.

I. SOlidum est, quod longitudinem, latitudinem, & crassitudinem habet.

II. Solidi autem extremum est superficies.

III. Linea recta est ad planum recta, cum ad rectas omnes lineas, à quibus illa tangitur, quæque in proposito sunt plano, rectos angulos efficit.

IV. Planum ad planum rectum est, cum rectæ lineæ, quæ communi planorum sectioni ad rectos angulos in uno plano ducuntur, alteri plano ad rectos sunt angulos.

V. Rectæ lineæ ad planum inclinatio est, cùm à sublimi termino rectæ illius lineæ ad planum deducta fuerit perpendicularis; atque à puncto quod perpendicularis in ipso plano effecerit, ad propositæ illius lineæ extremum, quod in eodem est plano, altera recta linea fuerit adjuncta; est, inquam, angulus acutus insistente linea, & adjuncta comprehensus.

VI. Plani ad planum inclinatio, est angulus acutus rectis lineis contentus, quæ in utroque planorum ad idem communis sectionis punctum ductæ, rectos cum sectione angulos efficiunt.

VII. Planum ad planum similiter inclinatum esse dicitur, atque alterum ad alterum, cum dicti inclinationum anguli inter se fuerint æquales.

VIII. Parallela plana sunt, quæ inter se non conveniunt.

IX. Similes solidæ figuræ sunt, quæ similibus planis continentur, multitudine æqualibus.

X. Æquales & similes solidæ figuræ sunt, quæ similibus planis multitudine & magnitudine æqualibus continentur.

X I. Solidus angulus est plurium quam duarum linearum, quæ se mutuo contingunt, nec in eadem sunt superficie, ad omnes lineas inclinatio.

Aliter.

Solidus angulus est, qui pluribus quam duobus planis angulis in eodem non consistentibus plano, sed ad unum punctum constitutis, continetur.

X I I. Pyramis est figura solida, planis comprehensa, quæ ab uno plano ad unum punctum constituuntur.

X I I I. Prisma est figura solida, quæ planis continetur, quorum adversa duo sunt & æqualia, & similia, & parallela; alia vero parallelogramma.

X I V. Sphæra est, quando semicirculi manente diametro, circumductus semicirculus in seipsum rursus revolvitur unde moveri cœperat, circumassumpta figura.

Coroll.

Hinc radii omnes à centro ad superficiem sphæræ inter se sunt æquales.

X V. Axis autem sphæræ, est quiescens illa recta linea, circum quam semicirculus convertitur.

X V I. Centrum sphæræ est idem quod & semicirculi.

X V I I. Diameter autem sphæræ, est recta quædam linea per centrum ducta, & utrinque à sphæræ superficie terminata.

X V I I I. Conus est, quando rectanguli trianguli manente uno latere eorum, quæ circa rectum angulum, circumductum triangulum in seipsum rursus revolvitur unde moveri cœperat, circumassumpta figura. Atque si quiescens recta lineæ linea æqualis sit reliquæ, quæ circa rectum angulum continetur, orthogonius erit conus; si vero minor, amblygonius; si vero major, oxygonius.

XIX. Axis autem coni, est quiescens illa linea, circa quam triangulum vertitur.

XX. Basis vero coni est circulus qui à circumducta recta linea describitur.

XXI. Cylindrus est, quando rectanguli parallelogrammi manente uno latere eorum, quæ circa rectum angulum, circumductum parallelogrammum in seipsum rursus revolvitur unde cœperat moveri, circumassumpta figura.

XXII. Axis autem cylindri, est quiescens illa recta linea, circum quam parallelogrammum convertitur.

XXIII. Bases vero cylindri sunt circuli à duobus adversis lateribus, quæ circumaguntur, descripti.

XXIV. Similes coni & cylindri sunt, quórum & axes, & basium diametri proportionales sunt.

XXV. Cubus est figura solida sub sex quadratis æqualibus contenta.

XXVI. Tetraedrum est figura solida sub quatuor triangulis æqualibus & æquilateris contenta.

XXVII. Octaedrum est figura solida sub octo triangulis æqualibus & æquilateris conteta.

XXVIII. Dodecaedrum est figura solida sub duodecim pentagonis æqualibus & æquilateris & æquiangulis contenta.

XXIX. Icosaedrum est figura solida sub viginti triangulis æqualibus & æquilateris contenta.

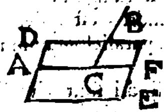
XXX. Parallelepipedum est figura solida sex figuris quadrilateris, quarum quæ ex adverso parallelæ sunt, contenta.

XXXI. So-

XXXI. Solida figura in solida figura dicitur inscribi, quando omnes anguli figuræ inscriptæ constituuntur vel in angulis, vel in lateribus, vel denique in planis figuræ, cui inscribitur.

XXXII. Solida figura solidæ figuræ vicissim circumscribi dicitur, quando vel anguli, vel latera, vel denique plana figuræ circumscriptæ tangunt omnes angulos figuræ, circum quam describitur.

PROP. I.

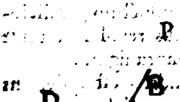


Rectæ lineæ pars quedam A C non est in subjecto plano, quædam vero CB in sublimi.

Producatur A C in subjecto plano usque ad F. vis CB esse in directum ipsi AC; ergo duæ rectæ A B, A F habent commune segmentum A C. a Q. F. N.

a 10. ax. 1.

PROP. II.



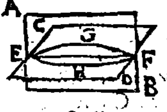
Si duæ rectæ lineæ A B, C D se mutuo secent, in uno sunt plano; atque triangulum omne DEB in uno est plano.

Puta enim trianguli DEB partem EFG esse in uno plano, partem vero FDGB in altero. ergo rectæ ED pars EF est in subjecto plano, pars vero FD in sublimi, a Q. E. A. ergo triangulum EDB in uno est plano; proinde & rectæ ED, EB; a quare & totæ A B, D C in uno plano existunt. Q. E. D.

a 1. 11.

PROP.

PROP. III.

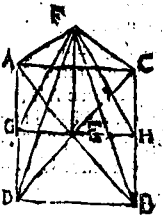


Si duo plana A B, C D *se mutuo secent, communis eorum sectio* E F *est recta linea.*

Si E F communis sectio non est recta linea, a ducatur in plano A B recta EGF, a & in plano CD recta EHF. duæ igitur rectæ EGF, EHF claudunt spatium. b Q. E. A.

a 1. post. 1.
b 14. ax. 1.

PROP. IV.



Si recta linea E F *rectis duabus lineis* A B, C D *se mutuo secantibus in communi sectione* E *ad rectos angulos insistat: illa ducto etiam per ipsas plano* A C B D *ad angulos rectos erit.*

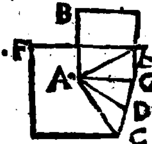
Accipe EA, EC, EB, ED æquales, & junge rectas A C, C B, B D, A D. per E ducatur quævis recta G H; Junganturque FA, FC, FD, FB, FG, FH. Quoniam A E a = EB; & D E a = E C; & ang. A E D b = CEB, c erit A D. = CB. c pariterque AC = DB. d ergo A D parall. C B, & A C parall. DB; e quare ang. GAE = EBH. e & ang. AGE = EHB. sed & AE f = EB g ergo GE = EH, & g AG = BH. quare ob angulos rectos, ex hyp. & proinde pares ad E, h bases FA, FC, FB, FD æquantur. Triangula igitur ADF, FBC sibi mutuo æquilatera sunt, k quare ang. DAF = CBF. ergo in triangulis AGF, FBH latera FG, FH l æquantur; & proinde etiam triangula FEG, FEH sibi mutuo æquilatera sunt. m ergo anguli FEG, FEH æquales ac n propterea recti sunt. Eodem modo F E cum omni-

a constr.
b 15. 1.
c 4. 1.
d sch 34. 1.
e 29. 1.
f constr.
g 26. 1.
h 4. 1.
k 8. 1.
l 4. 1.
m 8. 1.
n 10. def. 1.

omnibus in plano A D B C per E ductis rectis lineis rectos angulos constituit, *o* ideoque eidem plano recta est. Q. E, D.

o 3. def. 11.

Prop. V.

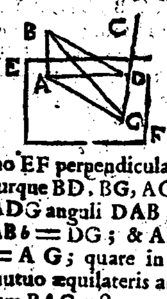


Si recta linea AB rectis tribus lineis AC, AD, AE se mutuo tangentibus in communi sectione ad rectos angulos insistat; illæ tres rectæ in uno sunt plano.

Nam A C, A D *a* sunt in uno plano FC. *a* item AD, AE sunt in uno plano BE. vis diversa esse hæc plana; sit igitur eorum intersectio *b* recta A G. Quoniam igitur B A ex hypoth. perpendicularis est rectis A C, AD, eadem *c* plano FC, *d* ideoque rectæ AG perpendicularis est. ergo (siquidem & *a* AB est in eodem cum AG, AE plano) anguli BAG, BAE recti, & proinde pares sunt, pars & totum. Q. E. A.

a 2. 11.
b 3. 11.
c 4. 11.
d 3. def. 11.

Prop. VI.



Si duæ rectæ lineæ A B, DC eidem plano EF ad rectos sint angulos; parallelæ erunt illæ rectæ lineæ A B, D C.

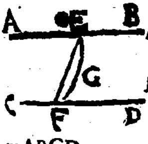
Ducatur AD, cui in plano EF perpendicularis sit DG = AB; junganturque BD, BG, AG. Quia in triangulis BAD, ADG anguli DAB, ADG *a* recti sunt; atque AB *b* = DG; & AD communis est; *c* erit BD = A G; quare in triangulis AGB, BGD sibi mutuo æquilateris ang. BAG *d* = BDG; quorum BAG rectus cum sit, erit BDG etiam rectus. atqui ang. GDC rectus ponitur; ergo recta GD tribus DA, DB, CD recta est; *e* quæ, ideo in uno sunt plano, *f* in quo A B existit;

a hyp.
b constr.
c 4. 1.
d 8. 1.
e 5. 11.
f 2. 11.

cum

cum igitur AB, & CD sint in uno plano, & anguli interni BAD, CDA recti sint, g erunt AB, CD parallelæ. Q. E. D. g 28. 1.

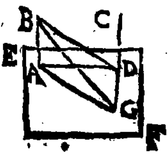
PROP. VII.



Si duæ sint parallelæ rectæ lineæ AB, CD, *in quarum utraque sumpta sint qualibet puncta* E, F; *illa linea* E F, *quæ ad hæc puncta adjungitur, in eodem est cum parallelis plano* ABCD.

Planum in quo A B, C D, secet aliud planum per puncta E, F. si jam E F non est in plano ABCD, illa communis sectio non erit. Sit ergo E G F. a hæc igitur recta est linea. duæ ergo rectæ EF, EGF spatium claudunt. b Q. E. A. a 3. 11. b 14 ax. 1.

PROP. VIII.



Si duæ sint parallelæ rectæ lineæ A B, C D, *quarum altera* A B *ad rectos cuidam plano* EF *sit angulos; & reliqua* C D *eidem plano* E F *ad rectos angulos erit.*

Adscita præparatione & demonstratione sextæ hujus; anguli G D A & G D B recti sunt. a ergo GD recta est plano per AD, DB (b in quo etiam A B, C D existunt.) c ergo G D ipsi C D est perpendicularis; atqui ang. CDA etiam d rectus est. e ergo CD plano E F recta est. Q. E. D. a 4. 11. b 7. 11. c 3 def. 11. d 29. 1. e 4. 11.

PROP.

PROP. IX.

Quæ (AB, CD) eidem rectæ lineæ EF sunt parallelæ, sed non in eodem cum illa plano, hæ quoque sunt inter se parallelæ.

a In plano parallelarum AB, EF duc HG perpendicularem ad EF. item in plano parallelarum EF, CD duc IG perpendicularem ad EF. a ergo EG recta est plano per HG, GI; eidemque plano b rectæ sunt AH, & CI. c ergo AH, & CI parallelæ sunt. Q. E. D.

a 4. 11.
b 8. 11.
c 6. 11

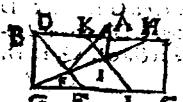
PROP. X.

Si duæ rectæ lineæ AB, AC se mutuo tangentes ad duas rectas ED, DF se mutuo tangentes sint parallelæ, non autem in eodem plano, illæ angulos æquales (BAC, EDF) comprehendent.

Sint AB, AC, DE, DF æquales inter se, & ducantur AD, BC, EF, BE, CF. Cum AB, DE a sint parallelæ & æquales, b etiam BE, AD parallelæ sunt, & æquales. Eodem modo CF, AD parallelæ sunt, & æquales; c ergo etiam BE, FC sunt parallelæ & æquales. d Æquantur ergo BC, EF. Cum igitur triangula BAC, EDF sibi mutuo æquilatera sint, anguli BAC, EDF e æquales erunt. Q. E. D.

a hyp. & constr.
b 33. 1.
c 2. ax. 1. & 30. 1.
d 33. 1.
e 8. 1.

PROP. XI.



A dato puncto A in sublimi ad subjectum planum BC perpendicularem rectam lineam AI ducere.

In plano BC duc quamvis DE, ad quam ex A a duc perpendicularem AF. ad eandem per F in

F in plano BC *b* duc normalem FH. tum ad FH *c* demitte perpendicularem AI. erit AI recta plano BC. a 12. 1. b 11. 1.

Nam per I *c* duc KIL parall. DE. Quia DE *d* recta est ad AF, & FH, *e* erit DE recta plano IFA; adeoque & KL eidem plano *f* recta est. *g* ergo ang. KIA rectus est. atqui ang. AIF etiam *h* rectus est. *l* ergo AI plano BC recta est. Q.E.D. c 31. 1. d constr. e 4. 11. f 8 11. g 3. def. 11. h constr. l 4. 11.

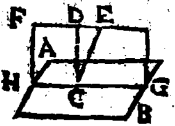
PROP. XII.

Dato plano BC, à puncto A, quod in illo datum est, ad rectos angulos rectam lineam AF extitare.

A quovis extra planum puncto D *a* duc DE rectam plano BC; & juncta EA *b* duc AF parall. DE. *c* perspicuum est AF plano BC rectam esse. Q.E.F. a 11. 11. b 31. 1. c 8. 11.

Practice perficiuntur hoc, & præcedens problema, si duæ normæ ad datum punctum applicentur, ut patet ex 4. 11.

PROP. XIII.



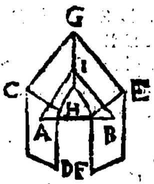
Dato plano AB, à puncto D, quod in illo datum est, duæ rectæ lineæ CD, CE ad rectos angulos non excitabuntur ab eadem parte.

Nam utraque CD, CE plano AB *a* recta esset, eædemque adeo parallelæ forent, quod parallelarum definitioni repugnat. a 6. 11.

PROP.

PROP. XIV.

vide: hac conversa.



Ad quæ plana C D, F E, *eadem recta linea* A B *recta est, illa sunt parallela.*

Si negas, plana C D, F E concurrant, ita ut communis sectio sit recta G H; sume in hac quodvis punctum I, ad quod in propositis planis ducantur rectæ I A, I B. unde in triangulo I A B, duo anguli IAB, IBA *a* recti sunt. *b* Q. E. A.

a hyp. & 3. def. 11.
b 17. 1.

PROP. XV.

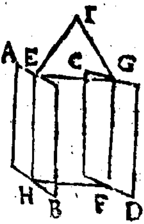
Si duæ rectæ lineæ A B, A C *se mutuo tangentes, ad duas rectas* D E, D F *se mutuo tangentes sint parallelæ, non in eodem consistentes plano; parallela sunt, quæ per* *a* *illa ducuntur, plana* BAC, EDF.

Ex A *a* duc A G rectam plano E F. *b* Sintque G H, G I parallelæ ad D E, D F. *c* erunt hæ parallelæ etiam ad A B, A C. Cum igitur anguli IGA, HGA *d* sint recti, *e* erunt etiam CAG, BAG recti. *f* ergo GA recta est plano BC; atqui eadem recta est plano E F. *h* ergo plana BC, EF sunt parallela. Q. E. D.

a 11. 11.
b 31. 1.
c 30. 1.
d 3. def. 11.
e 29. 1.
f 4. 11.
g constr.
h 14. 11.

PROP.

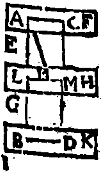
Prop. XVI.



Si duo plana parallela AB, CD, *plano quopiam* HEIGF *secentur, communes illorum sectiones* EH, GF *sunt parallelæ.*

Nam si dicantur non esse parallelæ, cum sint in eodem plano secanti, convenient alicubi, puta in *I.* quare cum totæ HEI, FGI *a* sint in planis AB, CD productis, etiam hæc convenient, contra hypoth. a 1. 11.

Prop. XVII.



Si duæ rectæ lineæ ALB, CMD *parallelis planis* EF, GH, IK *secentur, in easdem rationes secabuntur* (AL. LB :: CM. MD.)

Ducantur in planis EF, IK rectæ AC, BD. item AD occurrens plano GH in N; junganturque NL, NM. Plana triangulorum ADC, ADB faciunt sectiones BD, LN; & AC, NM *a* parallelas. ergo AL. LB *b* :: AN. ND *b* :: CM. MD. Q. E. D. a 16. 11. b 2. 6.

quæ ſimilibus planis multitudine & magnitudine æqualibus continentur.

XI. Solidus angulus eſt plurium quam duarum linearum, quæ ſe mutuo contingunt, nec in eadem ſunt ſuperficie, ad omnes lineas inclinatio.

Aliter.

Solidus angulus eſt, qui pluribus quam duobus planis angulis in eodem non conſiſtentibus plano, ſed ad unum punctum conſtitutis, continetur.

XII. Pyramis eſt figura ſolida, planis comprehenſa, quæ ab uno plano ad unum punctum conſtituuntur.

XIII. Priſma eſt figura ſolida, quæ planis continetur, quorum adverſa duo ſunt & æqualia, & ſimilia, & parallela; alia vero parallelogramma.

XIV. Sphæra eſt, quando ſemicirculi manente diametro, circumductus ſemicirculus in ſeipſum rurſus revolvitur unde moveri cœperat, circumaſſumpta figura.

Coroll.

Hinc radii omnes à centro ad ſuperficiem ſphæræ inter ſe ſunt æquales.

XV. Axis autem ſphæræ, eſt quieſcens illa recta linea, circum quam ſemicirculus convertitur.

XVI. Centrum ſphæræ eſt idem quod & ſemicirculi.

XVII. Diameter autem ſphæræ, eſt recta quædam linea per centrum ducta, & utrinque à ſphæræ ſuperficie terminata.

XVIII. Conus eſt, quando rectanguli trianguli manente uno latere eorum, quæ circa rectum angulum, circumductum triangulum in ſeipſum rurſus revolvitur unde moveri cœperat, circumaſſumpta figura. Atque ſi quieſcens recta

lineæ

linea æqualis sit reliquæ, quæ circa rectum angulum continetur, orthogonius erit conus; si vero minor, amblygonius; si vero major, oxygonius.

XIX. Axis autem coni, est quiescens illa linea, circa quam triangulum vertitur.

XX. Basis vero coni est circulus qui à circumducta recta linea describitur.

XXI. Cylindrus est, quando rectanguli parallelogrammi manente uno latere eorum, quæ circa rectum angulum, circumductum parallelogrammum in seipsum rursus revolvitur unde cœperat moveri, circumassumpta figura.

XXII. Axis autem cylindri, est quiescens illa recta linea, circum quam parallelogrammum convertitur.

XXIII. Bases vero cylindri sunt circuli à duobus adversis lateribus, quæ circumaguntur, descripti.

XXIV. Similes coni & cylindri sunt, quórum & axes, & basium diametri proportionales sunt.

XXV. Cubus est figura solida sub sex quadratis æqualibus contenta.

XXVI. Tetraedrum est figura solida sub quatuor triangulis æqualibus & æquilateris contenta.

XXVII. Octaedrum est figura solida sub octo triangulis æqualibus & æquilateris contéta.

XXVIII. Dodecaedrum est figura solida sub duodecim pentagonis æqualibus & æquilateris & æquiangulis contenta.

XXIX. Icosaedrum est figura solida sub viginti triangulis æqualibus & æquilateris contenta.

XXX. Parallelepipedum est figura solida sex figuris quadrilateris, quarum quæ ex adverso parallelæ sunt, contenta.

XXXI. So-

XXXI. Solida figura in solida figura dicitur inscribi, quando omnes anguli figuræ inscriptæ constituuntur vel in angulis, vel in lateribus, vel denique in planis figuræ, cui inscribitur.

XXXII. Solida figura solidæ figuræ vicissim circumscribi dicitur, quando vel anguli, vel latera, vel denique plana figuræ circumscriptæ tangunt omnes angulos figuræ, circum quam describitur.

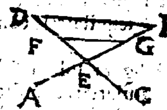
PROP. I.

Recta lineæ pars quædam A C *non est in subjecto plano, quædam vero* CB *in sublimi.*

Producatur A C in subjecto plano usque ad F. vis CB esse in directum ipsi AC; ergo duæ rectæ A B, A F habent commune segmentum A C.

a 10. ax. 1. a Q. F. N.

PROP. II.



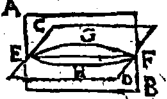
Si duæ rectæ lineæ A B, C D *se mutuo secent, in uno sunt plano: atque triangulum omne* DEB *in uno est plano.*

Puta enim trianguli DEB partem EFG esse in uno plano, partem vero FDGB in altero. ergo rectæ ED pars EF est in subjecto plano, pars vero FD in sublimi, a Q. E. A. ergo triangulum EDB in uno est plano; proinde & rectæ ED, EB;

a 2. 11. a quare & totæ A B, D C in uno plano existunt. Q. E. D.

PROP.

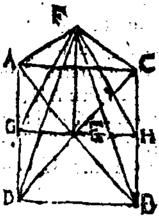
PROP. III.



Si duo plana A B, C D se mutuo secent, communis eorum sectio E F est recta linea.

Si E F communis sectio non est recta linea, a ducatur in plano A B recta EGF, a & in plano CD recta EHF. duæ igitur rectæ EGF, EHF claudunt spatium. b Q. E. A. a 1.post.1. b 14.ax.1.

PROP. IV.



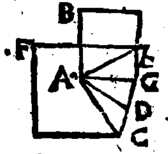
Si recta linea E F rectis duabus lineis A B, C D se mutuo secantibus in communi sectione E ad rectos angulos insistat: illa ducto etiam per ipsas plano A C B D ad angulos rectos erit.

Accipe E A, E C, E B, E D æquales, & junge rectas A C, C B, B D, A D. per E ducatur quævis recta G H; Junganturque FA, FC, FD, FB, FG, FH. Quoniam A E a = BE; & D E a = E C; & ang. A E D b = CEB; c erit A D = C B. c pariterque AC = DB. d ergo A D parall. C B. & A C parall. DB. e quare ang. GAE = EBH. e & ang. AGE = EHB. sed & AE f = EB g ergo GE = EH, & g AG = BH. quare ob angulos rectos, ex hyp. & proinde pares ad E, h bases FA, FC, FB, FD æquantur. Triangula igitur ADF, FBC sibi mutuo æquilatera sunt, k quare ang. DAF = CBF. ergo in triangulis AGF, FBH latera FG, FH l æquantur; & proinde etiam triangula FEG, FEH sibi mutuo æquilatera sunt. m ergo anguli FEG, FEH æquales ac n propterea recti sunt. Eodem modo F E cum omni-

a constr. b 15. 1. c 4. 1. d sch 34.1. e 29. 1. f constr. g 26. 1. h 4. 1. k 8. 1. l 4. 1. m 8. 1. n 10.def.1.

omnibus in plano A D B C per E ductis rectis
o 3. def. 11. lineis rectos angulos constituit, o ideoque eidem plano recta est. Q. E. D.

PROP. V.

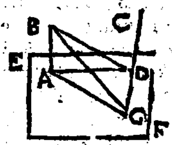


Si recta linea AB rectis tribus lineis AC, AD, AE se mutuo tangentibus in communi sectione ad rectos angulos insistat; illæ tres rectæ in uno sunt plano.

Nam A C, A D *a* sunt in uno plano FC. *a* item AD, AE sunt in uno plano BE. vis diversa esse hæc plana; sit igitur eorum intersectio *b* recta A G. Quoniam igitur B A ex hypoth. perpendicularis est rectis A C, AD, eadem *c* plano FC, *d* ideoque rectæ AG perpendicularis est. ergo (siquidem & *a* AB est in eodem cum AG, AE plano) anguli BAG, BAE recti, & proinde pares sunt, pars & totum. Q. E. A.

a 2. 11.
b 3. 11.
c 4. 11.
d 3. def. 11.

PROP. VI.



Si duæ rectæ lineæ AB, DC eidem plano EF ad rectos sint angulos; parallelæ erunt illæ rectæ lineæ AB, DC.

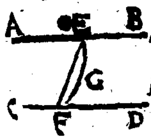
Ducatur AD, cui in plano EF perpendicularis sit DG = AB; junganturque BD, BG, AG. Quia in triangulis BAD, ADG anguli DAB, ADG *a* recti sunt; atque AB *b* = DG; & AD communis est; *c* erit BD = A G; quare in triangulis AGB, BGD sibi mutuo æquilateris ang. BAG *d* = BDG; quorum BAG rectus cum sit, erit BDG etiam rectus. atqui ang. GDC rectus ponitur; ergo recta GD tribus DA, DB, CD recta est; *e* quæ ideo in uno sunt plano, *f* in quo A B existit;

a hyp.
b constr.
c 4. 1.
d 8. 1.
e 5. 11.
f 2. 11.

cum

cum igitur AB, & CD sint in uno plano, & anguli interni BAD, CDA recti sint, g erunt AB, CD parallelæ. Q. E. D. g 28. 1.

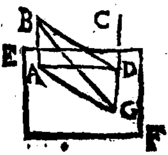
PROP. VII.



Si duæ sint parallelæ rectæ lineæ AB, CD, *in quarum utraque sumpta sint quælibet puncta* E, F; *illa linea* EF, *quæ ad hæc puncta adjungitur, in eodem est cum parallelis plano* ABCD.

Planum in quo AB, CD, secet aliud planum per puncta E, F. si jam EF non est in plano ABCD, illa communis sectio non erit. Sit ergo EGF. a hæc igitur recta est linea. duæ ergo rectæ EF, EGF spatium claudunt. b Q. E. A. a 3. 11. b 14 ax. 1.

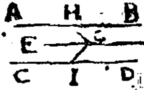
PROP. VIII.



Si duæ sint parallelæ rectæ lineæ AB, CD, *quarum altera* AB *ad rectos cuidam plano* EF *sit angulos; & reliqua* CD *eidem plano* EF *ad rectos angulos erit.*

Adscita præparatione & demonstratione sextæ hujus; anguli GDA & GDB recti sunt. a ergo GD recta est plano per AD, DB (b in quo etiam AB, CD existunt.) c ergo GD ipsi CD est perpendicularis; atqui ang. CDA etiam d rectus est. e ergo CD plano EF recta est. Q. E. D. a 4. 11. b 7. 11. c 3 def. 11. d 29. 1. e 4. 11.

PROP. IX.



Quæ (AB, CD) *eidem rectæ lineæ* EF *sunt parallelæ, sed non in eodem cum illa plano, hæ quoque sunt inter se parallelæ.*

a In plano parallelarum AB, EF duc HG perpendicularem ad EF. item in plano parallelarum EF, CD duc IG perpendicularem ad EF. *a* ergo EG recta est plano per HG, GI; eidemque plano *b* rectæ sunt AH, & CI. *c* ergo AH, & CI parallelæ sunt. Q. E. D.

a 4. 11.
b 8. 11.
c 6. 11

PROP. X.

Si duæ rectæ lineæ AB, AC *se mutuo tangentes ad duas rectas* ED, DF *se mutuo tangentes sint parallelæ, non autem in eodem plano, illæ angulos æquales* (BAC, EDF) *comprehendent.*

Sint AB, AC, DE, DF æquales inter se, & ducantur AD, BC, EF, BE, CF. Cum AB, DE *a* sint parallelæ & æquales, *b* etiam BE, AD parallelæ sunt, & æquales. Eodem modo CF, AD parallelæ sunt, & æquales, *c* ergo etiam BE, FC sunt parallelæ & æquales. Æquantur ergo BC, EF. Cum igitur triangula BAC, EDF sibi mutuo æquilatera sint, anguli BAC, EDF *e* æquales erunt. Q. E. D.

a hyp. & constr.
b 33. 1.
c 2. ax. 1. & 30. 1.
d 33. 1.
e 8. 1.

PROP. XI.

A dato puncto A *in sublime ad subjectum planum* BC *perpendicularem rectam lineam* AI *ducere.*

In plano BC duc quamvis DE, ad quam ex A *a* duc perpendicularem AF. ad eandem per

F in

F in plano BC *b* duc normalem FH. tum ad FH *c* demitte perpendicularem AI. erit AI recta plano BC. a 12. 1. b 11. 1.

Nam per I *c* duc KIL parall. DE. Quia DE *d* recta est ad AF, & FH, *e* erit DE recta plano IFA; adeoque & KL eidem plano *f* recta est. *g* ergo ang. KIA rectus est. atqui ang. AIF etiam *h* rectus est. *l* ergo AI plano BC recta est. Q.E.D. c 31. 1. d constr. e 4. 11. f 8. 11. g 3. def. 11. h constr. l 4. 11.

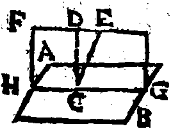
Prop. XII.

Dato plano BC, à puncto A, quod in illo datum est, ad rectos angulos rectam lineam AF extitare.

A quovis extra planum puncto D *a* duc DE rectam plano BC; & juncta EA *b* duc AF parall. DE. *c* perspicuum est AF plano BC rectam esse. Q.E.F. a 11. 11. b 31. 1. c 8. 11.

Practice perficiuntur hoc, & præcedens problema, si duæ normæ ad datum punctum applicentur, ut patet ex 4. 11.

Prop. XIII.

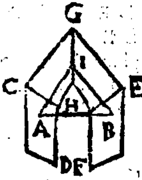


Dato plano AB, à puncto D, quod in illo datum est, duæ rectæ lineæ CD, CE ad rectos angulos non excitabuntur ab eadem parte.

Nam utraque CD, CE plano AB *a* recta esset, eædemque adeo parallelæ forent, quod parallelarum definitioni repugnat. a 6. 11.

PROP. XIV.

vel et hæc conversa.



Ad quæ plana C D, F E, *eadem recta linea* A B *recta est, illa sunt parallela.*

Si negas, plana C D, F E concurrant, ita ut communis sectio sit recta G H; sume in hac quodvis punctum I, ad quod in propositis planis ducantur rectæ

a hyp. & 3. def. 11.
b 17. 1.

I A, I B. unde in triangulo I A B, duo anguli IAB, IBA *a* recti sunt. *b* Q. E. A.

PROP. XV.

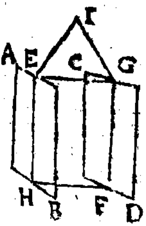
Si duæ rectæ lineæ A B, A C *se mutuo tangentes, ad duas rectas* D E, D F *se mutuo tangentes sint parallelæ, non in eodem consistentes plano; parallela sunt, quæ per* *a* *illa ducuntur, plana* BAC, EDF.

a 11. 11.
b 31. 1.
c 30. 1.
d 3. def. 11.
e 29. 1.
f 4. 11.
g constr.
h 14. 11.

Ex A *a* duc A G rectam plano E F. *b* Sintque G H, G I parallelæ ad D E, D F. *c* erunt hæ parallelæ etiam ad A B, A C. Cum igitur anguli IGA, HGA *d* sint recti, *e* erunt etiam CAG, BAG recti. *f* ergo GA recta est plano BC; atqui eadem recta est plano E F. *h* ergo plana BC, EF sunt parallela. Q. E. D.

PROP.

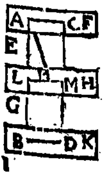
PROP. XVI.



Si duo plana parallela AB, CD, *plano quopiam* HEIGF *secentur, communes illorum sectiones* EH, GF *sunt parallelæ.*

Nam si dicantur non esse parallelæ, cum sint in eodem plano secanti, convenient alicubi, puta in *I*. quare cum totæ HEI, FGI *a* sint in planis AB, CD productis, etiam hæc convenient, contra hypoth. a 1. 11.

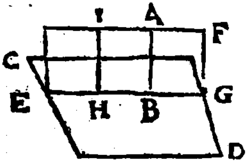
PROP. XVII.



Si duæ rectæ lineæ ALB, CMD *parallelis planis* EF, GH, IK *secentur, in easdem rationes secabuntur* (AL. LB :: CM. MD.)

Ducantur in planis EF, IK rectæ AC, BD. item AD occurrens plano GH in N; junganturque NL, NM. Plana triangulorum ADC, ADB faciunt sectiones BD, LN; & AC, NM *a* parallelas. ergo AL. LB *b* :: AN. ND *b* :: CM. MD. Q.E.D. a 16. 11. b 2. 6.

PROP. XVIII.

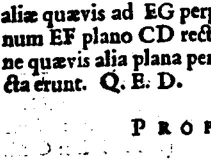


Si recta linea AB *plano cuipiam* CD *ad rectos sit angulos; & omnia, quæ per ipsam* AB *plana* (EF, &c.) *eidem plano* CD *ad rectos angulos erunt.*

Ductum sit per AB planum aliquod EF, faciens cum plano C D sectionem E G; è cujus aliquo puncto H, in plano EF *a* ducatur HI parall. AB. *b* erit HI recta plano CD; pariterque aliæ quævis ad EG perpendiculares. *c* ergo planum EF plano CD rectum est; eademque ratione quævis alia plana per AB ducta plano EF recta erunt. Q. E. D.

a 31. 1. *b* 8. 11. *c* 4. def. 11.

PROP. XIX.



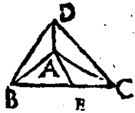
Si duo plana A B, C D, *se mutuo secantia, plano cuidam* G H *ad rectos sint angulos, communis etiam illorum sectio* E F *ad rectos eidem plano* (GH) *angulos erit.*

Quoniam plana A B, C D ponuntur recta plano GH, patet ex 4. def. 11. quod ex puncto F in utroque plano A B, C D duci possit perpendicularis plano GH; quæ *a* unica erit, & propterea eorundem planorum communis sectio. Q. E. D.

a 13. 11.

PROP.

PROP. XX.



Si solidus angulus ABCD *tribus angulis planis* BAD, DAC, BAC *contineatur; ex his duo quilibet, utut assumpti, tertio sunt majores.*

Si tres anguli sunt æquales, patet assertio; si inæquales, maximus esto BAC. ex quo *a* aufer BAE $=$ BAD; & fac AD $=$ AE; ducanturque BEC, BD, DC. a 23. 1.

Quoniam latus BA commune est, & AD *b* $=$ AE; & ang. BAE *b* $=$ BAD; *c* erit BE $=$ BD. sed BD $+$ DC *d* $\sqsubset$ BC. *e* ergò DC $\sqsubset$ EC. cum igitur AD *b* $=$ AE, & latus AC commune est, ac DC $\sqsubset$ EC *f*, erit ang. CAD $\sqsubset$ EAC. *g* ergo ang. BAD $+$ CAD $\sqsubset$ BAC. Q. E. D. b constr. c 4. 1. d 20. 1. e 5. ax. 1. f 25. 1. g 4. ax. 1.

PROP. XXI.

Omnis solidus angulus sub minoribus, quam quatuor rectis angulis planis, continetur.

Esto solidus angulus A; planis angulis illum componentibus subtendantur rectæ BC, CD, DE, EF, FB in uno plano existentes. Quo facto constituitur pyramis, cujus basis est polygonum BCDEF, vertex A, totque cincta triangulis quot plani anguli componunt solidum A. Jam vero quia duo anguli ABF, ABC *a* majores sunt uno FBC, *a* & duo ACB, ACD majores uno BCD, & sic deinceps, erunt triangulorum G, H, I, K, L circa basim anguli simul sumpti omnibus simul angulis basis B, C, D, E, F majores. *b* sed anguli baseos una cum quatuor rectis faciunt bis tot rectos, quot sunt latera, sive quot triangula. *c* Ergo omnes triangulorum circa basim anguli una a 20. 11. b sch. 32. 1. c 4. ax. 1.

cum 4 rectis conficiunt amplius quam bis tot rectos quot sunt triangula. sed iidem anguli circa basim una cum angulis qui componunt solidum,
d 32. 1. componunt d bis tot rectos quot sunt triangula. liquet ergo angulos solidum angulum A componentes quatuor rectis esse *minores*. Q. E. D.

PROP. XXII.

Si fuerint tres anguli plani A, B, HCI, *quorum duo utlibet assumpti reliquo sint majores; comprehendant autem ipsos rectæ lineæ æquales* AD, AE, FB, &c. *fieri potest, ut ex rectis lineis* DE, FG, HI, *æquales illas rectas connectentibus triangulum constituatur.*

a 22. 1. Ex iis a constitui potest triangulum, si duæ qualibet reliqua majores existant; sed ita se res
b 23. 1. habet. Nam b fac ang. HCK = B, & CK = CH,
c 4. 1. ducanturque HK, IK. c ergo KH = FG. & quia
d hyp. ang. KCI d $\sqsubset$ A; erit KI $\sqsubset$ DE. sed KI f $\sqsupset$
e 24. 1.
f 20. 1. HI + KH (FG;) ergo DE $\sqsupset$ HI + FG. Simili argumento quævis duæ reliqua majores ostendentur; & proinde ex iis triangulum a constitui potest. Q. E. D.

PROP.

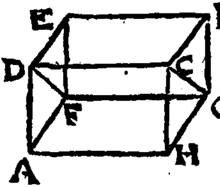
Prop. XXIII.

Ex tribus angulis planis A, B, C, *quorum duo quomodocunque assumpti reliquo sunt majores, solidum angulum* MHIK *constituere.* * *Oportet autem illos tres angulos quatuor rectis minores esse.* * 21. huj.

Fac AD, AE, BE, BF, CF, CG æquales inter se. Ex subtensis DE, EF, FG (hoc est, ex æqualibus HI, IK, KH) *a* fac triang. HKI. circa quod *b* describatur circulus LHKI. * Quoniam vero AD $\sqsubset$ HL; *c* sit ADq $=$ HLq $+$ LMq. *d* sitque LM recta plano circuli HKI; & ducantur HM, KM, IM. Quoniam igitur ang. HLM *e* rectus est, *f* erit MHq $=$ HLq $+$ LMq *g* $=$ ADq. ergo MH $=$ AD. simili argumento MK, MI, AD (id est, AE, EB, &c.) æquantur; ergo cum HM $=$ AD, & MI $=$ AE; & DE *h* $=$ HI, *k* erit ang. A $=$ HMI; *k* similiter ang. IMK $=$ B. *k* & ang. HMK $=$ C. Factus est igitur angulus solidus ad M ex tribus planis datis. Q. E. F. Brevitatis causa assumptum est, esse AD $\sqsubset$ HL, id quod in variis casibus demonstratum vide apud Clavium.

a 22. 11. & 23. 1.
b 5. 4. vid. Clavium.
c sch 47. 1.
d 12. 11.
e 3. def. 11.
f 47. 1.
g constr.
h constr.
k 8. 1.

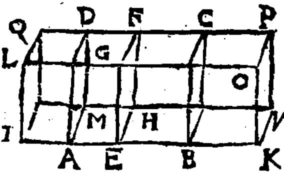
PROP. XXIV.



Si solidum AB *parallelis planis contineatur, adversa illius plana* (AG, DB, &c.) *parallelogramma sunt similia & æqualia.*

Planum AC secans plana paral-
a 16. 11. lela AG, DB, a facit sectiones AH, DC paralle-
las. Eadem ratione AD, HC parallelæ sunt. Er-
go ADCH est parallelogrammum. Simili argu-
mento reliqua parallelepipedi plana sunt b pa-
b 35. def. 1. rallelogramma. Quum igitur AF ad HG, & AD
ad HC parallelæ sint, c erit ang. FAD = CHG;
c 10. 11. ergo ob AF d = HG, & AD d = HC, ac e
d 34. 1. propterea AF. AD :: HG. HC, triangula FAD,
e 7. 5. GAH g similia sunt & h æqualia; proinde & pa-
g 6. 6. rallelogramma AE, HB similia sunt & k æqua-
h 4. 1. lia. idemque de reliquis oppositis planis ostende-
k 6. ax. 1. tur. ergo, &c.

PROP. XXV.



Si solidum parallelepipedum ABCD *plano* EF *secetur adversis planis* AD, BC *parallelo, erit quemadmodum basis* AH *ad basim* BH, *ita solidum* AHD *ad solidum* BHC.

Concipe Ppp. ABCD produci utrinque. accipe AI = AE, & BK = EB; & pone plana IQ, KP planis AD, BC parallela. parallelogramma

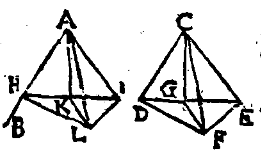
gramma IM, AH, a & DL, DG, b & IQ, AD, EF, &c. a similia ac æqualia sunt; c quare Ppp. AQ = AF; atque eadem ratione Ppp. BP = BF. ergo solida IF, EP solidorum AF, EC æquemultiplicia sunt, ac bases IH, KH basium AH, BH. Quod si basis IH ⊏, =, ⊐ KH, d erit similiter solidum IF ⊏, =, ⊐ EP. e proinde AH. BH :: AF. EC. Q. E. D. a 34. 1. & 1. def. 6. b 24. 11. c 10. def. 11. d 14. 11. & 9. def. 11. e 6. def. 5.

Hæc eadem omni prismati accommodari possunt; unde

Coroll.

Si prisma quodcunque secetur plano oppositis planis parallelo, sectio erit figura æqualis, & similis planis oppositis.

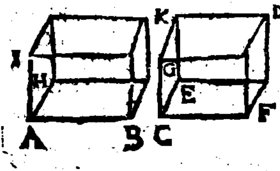
PROP. XXVI.



Ad datam rectam lineam A B, *ejusque punctum* A, *constituere angulum solidum* AHIL, *æqualem solido angulo dato* CDEF.

A puncto quovis F a demitte FG plano DCE rectam; ducanturque rectæ DF, FE, EG, GD, CG. Fac AH = CD, & ang. HAI = DCE. & AI = CE; atque in plano HAI, fac ang. HAK = DCG, & AK = CG. Tum erige KL rectam plano HAI, & sit KL = GF. ducaturque AL. erit angulus solidus AHIL par dato CDEF. Nam hujus constructio illius constitutionem penitus æmulatur, ut facile patebit examinanti. ergo factum. a 11. 11.

PROP. XXVII.

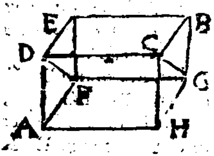


A data recta linea AB, *dato solido parallelepipedo* CD *simile & similiter positum parallelepipedum* AK *describere.*

Ex angulis planis BAH, HAI, BAI, qui æquales sint ipsis FCE, ECG, FCG, *a* fac angu-
a 26. 11. lum solidum A solido C parem. item *b* fac FC.
b 12. 6. CE :: BA. AH. *b* ac CE. CG :: AH. AI (*c* unde
c 22. 5. erit ex æquali FC.CG :: BA. AI;) & perficiatur Ppp. AK. erit hoc simile dato.

Nam per constr. Pgra *d* BH, FE; *d* & HI,
d 1. def. 6. EG; & *d* BI, FG similia sunt, & *e* horum ideo
e 24. 11. opposita illorum oppositis. ergo sex plana solidi AK similia sunt sex planis solidi CD. *f* proinde
f 9. def. 11. AK, CD similia solida existunt. Q. E. F.

PROP. XXVIII.

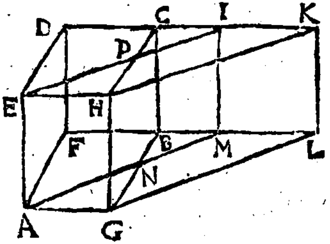


Si solidum parallelepipedum AB *plano* FGCD *secetur per diagonios* DF, CG *adversorum planorum* AE, HB, *bifariam secabitur solidum* AB *ab ipso plano* FGCD.

Nam quia DC, FG *a* æquales & parallelæ
a 24. 11. sunt, *b* planum FGCD est Pgr. & propter
b 34. 1. *a* Pgra AE, HB æqualia, & similia, *b* etiam triangula AFD, HGC, CGB, DFE æqualia & similia sunt. Atqui Pgra AC, AG ipsis FB, FD *a* etiam æqualia & similia sunt. ergo prismatis FGCDAH omnia plana æqualia sunt, & simi-
c 9. def. 11. lia planis omnibus prismatis FGCDEB; & *c* proinde hoc prisma illi æquatur. Q. E. D.

PROP.

PROP. XXIX.



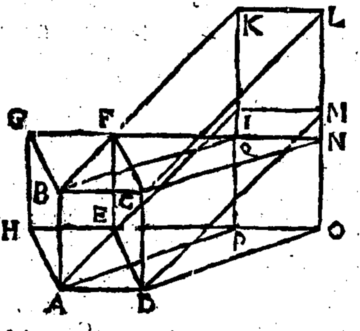
Solida parallelepipeda AGHEFBCD, AGHEMLKI *super eandem basim* AGHE *constituta, &* * *in eadem altitudine; quorum insistentes lineæ* AF, AM *in iisdem collocantur rectis lineis* AG, FL, *sunt inter se æqualia.*

* *Id est, inter parallela plana* AGH F, FLKD. *& sic intellige in sequent.*

Nam si ex a æqualibus prismatis AFMEDI, GBLHCK commune auferatur prisma NBMPCI, addaturque utrinque solidum AGNEHP, b erit Ppp. AGHEFBCD = AGHEMLKI. Q. E D.

a 10. *def.* 11. & 35. 1.
b 3. & 2. *ax.* 2.

PROP. XXX.



Solida parallelepipeda ADBCHEFG, AD-

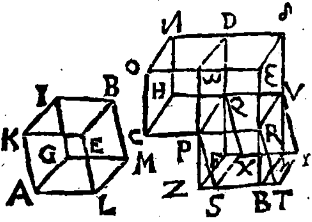
ADCBIMLK *ſuper eandem baſim* ADCB *conſtituta, & in eadem altitudine, quorum inſiſtentes lineæ* AH, AI *non in iiſdem collocantur rectis lineis, inter ſe ſunt æqualia.*

a 34. 1. Nam produc rectas HEO, GFN; & LMO, KIP; & duc AP, DO, BQ, CN. *a* erunt tam DC, AB, HG, EF, PQ, ON; quam AD, HE, GF, BC, KL, IM, QN, PO æquales inter ſeſe

b 29. 11. & parallelæ. *b* Quare Ppp. ADCBPONQ utrique Pppº. ADCBHEFG, ADCBIMLK æ-

c 1. ax. 1. quale eſt; & *c* proinde hæc ipſa inter ſe æqualia ſunt. Q. E. D.

PROP. XXXI.



Solida parallelepipeda ALEKGMBI, CPωOHQDN *ſuper æquales baſes* ALEK, CPωO *conſtituta, & * in eadem altitudine, æqualia ſunt inter ſe.*

* Altitudo eſt perpendicularis à plano baſis ad planum oppoſitum.

Habeant primo parallelepipeda AB, CD latera ad baſes recta; & ad latus CP productum

a 18 6. *a* fiat pgr. PRTS æq. & ſimile pgrº KELA;

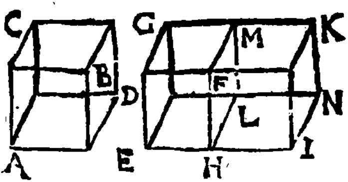
b 27 11. & 10. def. 11. *b* adeoque Ppp. PRTSQVYX æq. & ſim. Pppº AB. Producantur Oω E, ND δ, ω P Z, DQF, ERB, δVγ, TSZ, YXF; & duc Eδ, Bγ, ZF.

c 30 def. 11. d hyp. & 35 1. Plana Oε δ N, CRVH, ZTYF *c* parallela ſunt inter ſe; *d* & pgrª ALEK, CPωO, PRTS, PRBZ æqualia ſunt. Cum igitur Ppp. CD.

CD. PV$\delta\omega$ ^{e} :: pgr. Cω (PRBZ.) P$\iota\epsilon$ :: Ppp. e 25. 11.
PRBZQVγF. PV$\delta\omega$, ^{f} erit Ppp. CD ^{f} = f 9. 5.
PRBZQVγF ^{g} = PRVQSTYX ^{h} = AB. g 29 11. h constr.
Q. E. D.

Sin Ppp[a] AB, CD latera basibus obliqua habeant; super easdem bases, & in eadem altitudine, ponantur parallelepipeda, quorum latera basibus sint recta. ^{k} Ea inter se, & obliquis æqualia erunt; ^{m} proinde & obliqua AB, CD æquantur. Q. E. D. k 29 11. m 1. ax. 1.

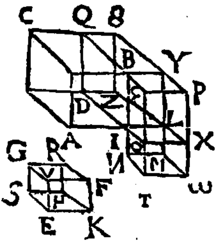
Prop. XXXII.



Solida parallelepipeda ABCD, EFGL *sub eadem altitudine, inter se sunt ut bases* AB, EF.

Producta EHI, ^{a} fac pgr. FI = AB, & ^{b} comple a 45 1.
Ppp. FINM. Liquet esse Ppp. FINM. b 31. 1.
(^{c}ABCD.) EFGL ^{d} :: FI. (AB) EF. Q. E. D. c 31. 11. d 25. 11.

Prop. XXXIII



Similia solida parallelepipeda, ABCD, EFGH, *inter se sunt in triplicata ratione homologorum laterum* AI, EK.

Producantur rectæ AIL, DIO, BIN. & ^{a} fiant IL, IO, a 3. 1.
IN ipsis EK, KH, KF æquales, ^{b} adeoque b 17. 11.

&

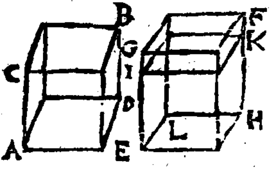
& Ppp. IXMT æq. & sim. Ppp° EFGH. c31.1. ᶜ Perficiantur Ppp ᵃ IXPB, DLYQ. Itaque ᵈ erit AI. IL. (EK) :: DI. IO (HK) :: BI. IN. (KF;) hoc est Pgr. AD. DL :: DL. IX :: BO. IT; ᶠ id est Ppp. ABCD. DLQY :: DLQY. IXBP :: IXBP. IXMT. (ᵍ EFGH.) ʰ ergo ratio ABCD ad EFGH triplicata est rationis ABCD ad DLQY, ᵏ vel AI ad EK. Q. E. D.

c 31. 1. d hyp. e 1. 6 f 32. 11. g constr. & h 10 def. 5. k 1. 6.

Coroll.

Hinc, si fuerint quatuor lineæ rectæ continue proportionales, ut est prima ad quartam, ita est parallelepipedum super primam descriptum ad parallelepipedum simile similiterque descriptum super secundam.

P R O P. XXXIV.



Æqualium solidorum parallelepipedorũ ADCB, EHGF *bases & altitudines reciprocantur* (AD. EH :: EG. AC.) *Et quorum solidorum parallelepipedorum* ADCB, EHGF *bases & altitudines reciprocantur, illa sunt æqualia.*

Sint primo latera CA, GE ad bases recta; si jam solidorum altitudines sint pares, etiam bases æquales erunt. & res clara est. Sin altitudines inæquales sint, à majori EG ᵃ detrahe EI = AC. & per I ᵇ duc planum IK parallelum basi EH. itaque

a 3. 1. b 31. 1. c 32. 11. d 17. 5. e 1. 6. f constr. g 11. 5.

1. *Hyp.* AD. EH ᶜ :: Ppp. ADCB. EHIK ᵈ :: Ppp. EHGF. EHIK ᵉ :: GL. IL ᵉ :: GE. IE. (ᶠ AC;) ᵍ liquet igitur esse AD. EH :: GE. AC. Q. E. D.

2. *Hyp.*

2. *Hyp.* ADCB. EHIK *h* :: AD. EH *i* : : EG. EI *l* : : *GL. IL* *m* : : Ppp. EHGF. EHIK, *n* quare Ppp. ADCB = EHGF. Q. E. D. h 32. 11. k hyp. l 1. 6. m 32. 11. n 9. 5.

Sint deinde latera ad bases obliqua. Erigantur super iisdem basibus, in altitudine eadem, parallelepipeda recta. Erunt obliqua parallelepipeda his æqualia. Quare cum hæc per 1. partem reciprocent bases & altitudines, etiam illa reciprocabunt. Q. E. D.

Coroll.

Quæ de parallelepipedis demonstrata sunt Prop. 29, 30, 31, 32, 33, 34. *etiam conveniunt prismatis triangularibus, quæ sunt dimidia parallelepipeda, ut patet ex* Pr. 28. *Igitur,*

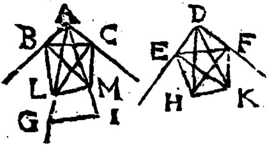
1. Prismata triangularia æque alta sunt ut bases.

2. Si eandem vel æquales habeant bases, & eandem altitudinem, æqualia sunt.

3. Si similia fuerint, eorum proportio triplicata est proportionis homologorum laterum.

4. Si æqualia sunt, reciprocant bases & altitudines. & si reciprocant bases & altitudines, æqualia erunt.

PROP. XXXV.



Si fuerint duo plani anguli BAC, EDF *æquales, quorum verticibus* A, D, *sublimes rectæ lineæ* AG, DH *insistant, quæ cum lineis primo positis angulos contineant æquales, utrumq; utriq; (ang.* GAB = HDE; & GAC = HDF.) *in sublimibus autem lineis* AG, DH *qualibet sumpta fuerint puncta* G, H; &

& ab his ad plana BAC, EDF, *in quibus consistunt anguli primum positi* BAC, EDF, *ductæ fuerint perpendiculares* GI, HK; *à punctis vero* I, K *quæ in planis à perpendicularibus fiunt, ad angulos primum positos adjunctæ fuerint rectæ lineæ* AI, DK; *hæ cum sublimibus* AG, DH *æquales angulos* GAM, HDK *comprehendent*,

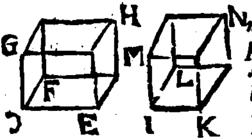
Fiant DH, AL æquales, & GI, LM parallelæ; & MC ad AC, MB ad AB, KF ad DF, KE ad DE perpendiculares, ducanturque rectæ
a 8. 11. BC, LB, LC, atque EF, HF, HE; *a* estque LM
b 3. def. 11. recta plano BAC; *b* quare anguli LMC, LMA, LMB; eademque ratione anguli HKF, HKD,
c 47. 1. HKE recti sunt. Ergo ALq *c* = LMq + AMq *c* = LMq + CMq + ACq *c* = LCq + ACq;
d 48. 1. *d* ergo ang. ACL rectus est. Rursus ALq *e* =
e 47. 1. LMq + MAq *e* = LMq + BMq + BAq *e* = BLq + BAq. *d* ergo ang. ABL etiam rectus est. Simili discursu anguli DFH, DEH recti
f 26. 1. sunt; *f* ergo AB = DE; *f* & BL = EH; *f* & AC = DF; & CL = FH. *g* quare etiam BC
g 4. 1. = EF, *g* & ang. ABC = DEF, *g* & ang. ACB
h 3. ax. 1. = DFE. unde reliqui è rectis anguli CBM,
k 26. 1. BCM reliquis FEK, EFK æquantur. *k* ergo
l 47. 1 CM = FK, *l* ideoque & AM = DK. ergo si
m constr. ex LAq *m* = HDq. auferatur AMq = DKq,
n 47. 1. & 3. ax. *n* remanet LMq = HKq quare trigona LAM,
o 8. 1. HDK sibi mutuo æquilatera sunt. *o* ergo ang. LAM = HDK. Q.E.D.

Coroll.

Itaque si fuerint duo anguli plani æquales, quorum verticibus sublimes rectæ lineæ æquales insistant, quæ cum lineis primo positis angulos contineant æquales, utrumque utrique; erunt à punctis extremis linearum sublimium ad plana angulorum primo positorum demissæ perpendiculares inter se æquales; nempe LM = HK.

PROP.

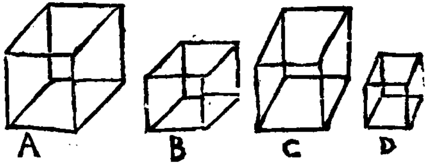
PROP. XXXVI.



Si tres rectæ lineæ DE, DG, DF *proportionales fuerint; quod ex his tribus fit solidum parallelepipedum* DH, *æquale est descripto à media linea* DG (IL) *solido parallelepipedo* IN, *quod æquilaterum quidem sit, æquiangulum vero prædicto* DH.

Quoniam DE. IK *a* :: IL. DF, *b* erit pgr. LK = FE. & propter angulorum planorum ad E & I, ac linearum GD, IM æqualitatem, etiam altitudines parallelepipedorum æquales sunt, ex coroll. præced. *c* ergo ipsa inter se æqualia sunt. Q.E.D. a hyp. b 14. 6. c 31. 11.

PROP. XXXVII.



Si quatuor rectæ lineæ A, B, C, D *proportionales fuerint, & solida parallelepipeda* A, B, C, D *quæ ab ipsis & similia, & similiter describuntur, proportionalia erunt. Et si solida parallelepipeda, quæ & similia, & similiter describuntur, fuerint proportionalia* (A.B :: C. D.) *& ipsæ rectæ lineæ* A, B, C, D *proportionales erunt.*

Nam rationes parallelepipedorum *a* triplicatæ sunt rationum, quas habent lineæ. ergo si A.B :: C. D. *b* erit Ppp. A. Ppp. B :: Ppp. C. Ppp. D. & vice versa. a 33. 11. b sch 23. 5.

PROP.

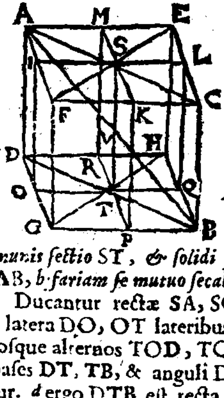
PROP. XXXVIII.

Si planum AB ad planum AC rectum fuerit, & ab aliquo puncto E eorum, quæ sunt in uno planorum (AB) ad alterum planum AC perpendicularis EF ducta fuerit, in planorum communem sectionem AD cadet ducta perpendicularis EF.

a 12. 11. b 4, & 3. def. 11. c 17. 1.

Si fieri potest, cadat F extra intersectionem AD. In plano AC a ducatur FG perpendicularis ad AD, jungaturque EG. Angulus FGE b rectus est; & EFG rectus ponitur. ergo in triangulo EFG sunt duo anguli recti. Q. E. A.

PROP. XXXIX.



Si solidi parallelepipedi AB, eorum quæ ex adverso planorum AC, DB latera (AE, FC, AF, EC, & DH, GB, DG, HB) bifariam secta sint; per sectiones autem plana ILQO, PKMR sint extensa; planorum communis sectio ST, & solidi parallelepipedi diameter AB, bifariam se mutuo secabunt.

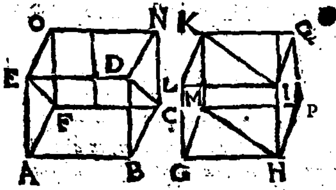
a 34. 1. b 29. 1. c 4. 1. d sch. 15. 1. e 34. 1 f 9. 11. & 1 ax. g 33. 1.

Ducantur rectæ SA, SC, TD, TB. Propter a latera DO, OT lateribus BQ, QT, b angulosque alternos TOD, TQB æquales, c etiam bases DT, TB, & anguli DTO, BTQ æquantur. d ergo DTB est recta linea. eodem modo ASC recta est linea. Porro e tam AD ad FG, e quam FG ad CB; f ideoque AD ad CB, g ac proinde AC ad DB parallelæ & æquales sunt. h quare *b* quare A B, & S T in eodem plano ABCD exsistunt. Itaque cum anguli AVS, BVT ad verticem, & alterni ASV, BTV æquentur; *k* & AS = BT; erit AV = BV, *l* & SV = VT. Q.E.D. b 7. 11. k 7. ax. 1. l 26. 1.

Coroll.

Hinc, in omni parallelepipedo diametri omnes se mutuo bisecant in uno puncto, V.

PROP. XL.



Si fuerint duo prismata ABCFED, GHMLIK *equalis altitudinis, quorum hoc quidem habeat basim* ABCF *parallelogrammum, illud vero* GHM *triangulum; duplum autem fuerit parallelogrammum* ABCF *trianguli* GHM; *æqualia erunt ipsa prismata* ABCFED, GHMLIK.

Nam si perficiantur parallelepipeda AN, GQ, *a* erunt hæc æqualia ob *b* basium AC, GP, & *c* altitudinum æqualitatem, *d* ergo etiam prismata, *e* horum dimidia, æqualia erunt. Q.E.D. a 31. 11. b 34. 1. & 7. ax. c hyp. d 28. 11. e 7. ax. 1.

Schol.

Ex hactenus demonstratis habetur dimensio prismatum triangularium, & quadrangularium, seu parallelepipedorum, si nimirum altitudo ducatur in basim. Andr. Tacq.

Ut si altitudo sit 10 pedum, basis vero pedum quadratorum 100 (mensurabitur autem basis per ch. 35. 1. vel per 41. 1.) multiplica 100 per 10;

T pro-

proveniunt 2000 pedes cubici pro soliditate prismatis dati.

Vide schol. 35. 1. Nam quemadmodum rectangulum, ita & parallelepipedum rectum producitur ex altitudine ducta in basim. Ergo quodvis parallelepipedum producitur ex altitudine in basim ducta, ut patet ex 30. hujus.

Deinde cum totum parallelepipedum producatur ex altitudine in totam basim, semissis ejus (hoc est prisma triangulare) producetur ex altitudine ducta in dimidiam basim, nempe triangulum.

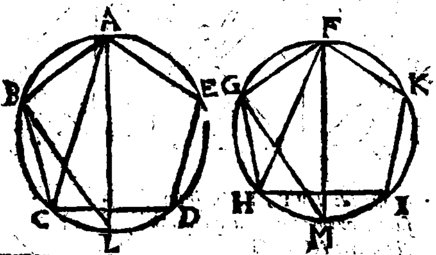
Monitum.

Nota, litterarum quæ designant angulum solidum primam esse semper ad punctum, in quo est angulus; litterarum vero quæ denotant pyramidem, ultimam esse ad verticem pyramidis.

Ex. gr. Angulus solidus ABCD est ad punctum A; pyramidis quoque BCDA vertex est ad punctum A, & basis triangulum BCD.

LIB. XII.

PROP. I.



Quæ sunt in circulis ABD, FGI *polygona similia* ABCDE, FGHIK, *inter se sunt, ut quadrata à diametris* AL, FM.

Ducantur AC, BL, FH, GM. Quoniam *a* ang. ABC = FGH, *a* atque AB. BC :: FG. GH, *b* erit ang. ACB (*c* ALB) = FHG (*c* FMG.) anguli autem ABL, FGM *d* recti, ac proinde æquales sunt. *e* ergo triangula ABL, FGM æquiangula sunt. *f* quare AB. FG :: AL. FM. *g* ergo ABCDE. FGHIK :: ALq. FMq.

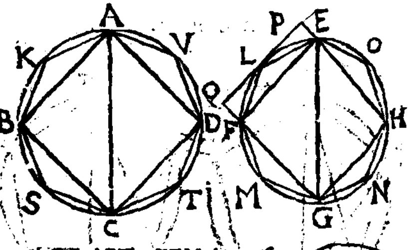
a 1. def. 6. b 6. 6. c 21. 3. d 31. 3. e 32. 3. f 4. 6. g 22. 6.

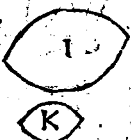
Coroll.

Hinc (quia AB. FG :: AL. FM :: BC. GH, &c.) polygonorum similium circulo inscriptorum *h* ambitus sunt ut diametri.

h 1. 12. & 12. 5.

PROP. II.





Circuli ABT, EFN *inter se sunt, quemadmodum quadrata à diametris* AC, EG.

Ponatur ACq. EGq :: circ. ABT. I. Dico I = circ. EFN.

Nam primo, si fieri potest, sit I $\sqsupset$ circ. EFN, sitque excessus K. Circulo EFN inscribatur
a sch. 7. 4. quadratum EFGH, *a* quod dimidium est circumscripti quadrati, adeoque semicirculo majus.
b 30. 3. *b* Biseca arcus EF, FG, GH, HE, & ad puncta bisectionum junge rectas EL, LF, &c. per L
c sch. 17. 3. duc tangentem PQ (*c* quæ ad EF parallela est,) & produc HEP, GFQ; estque triangulum
d 41. 1 ELF *d* dimidium parallelogrammi EPQF, adeoque majus dimidio segmenti ELF; pariterque reliqua triangula ejusmodi reliquorum segmentorum dimidia superant. Et si iterum bisecentur arcus EL, LF, FM, &c. rectæque adjungantur, eodem modo triangula segmentorum semisses excedent. Quare si quadratum EFGH è circulo EFN, & è reliquis segmentis triangula
e 1. 10. detrahantur, & hoc fiat continuo, tandem *e* restabit magnitudo aliqua minor quam K. Eousque perventum sit, nempe ad segmenta EL, LF, FM, &c. minora quam K, simul sumpta.

pta. ergo I (f circ. EFN $-$ K) $\sqsupset$ polyg. ELFMGNHO (circ. EFN $-$ segm. EL $+$ LF &c.) Circulo ABT inscriptum g puta simile polygonum AKBSCTDV. itaque quum AKBSCTDV. ELFMGNHO h :: ACq. EGq k :: circ. ABT. I, ac polyg. AKBSCTDV l $\sqsubset$ circ. ABT. m erit polyg. ELFMGNHO $\sqsubset$ I. sed prius erat I $\sqsupset$ ELFMGNHO. quæ repugnant.

f hyp. & 3 ax. — g 30. 3. & 1 post. — h 1. 12. — k hyp. — l 9 ax. 1. — m 14. 5.

Rursus, si fieri potest, sit I $\sqsubset$ circ. EFN. Quoniam igitur ACq. EGq n :: circ. ABT. I; inverseque I. circ. ABT :: EGq. ACq. pone I. circ. ABT :: circ. EFN. K. o ergo circ. ABT $\sqsubset$ K. p atque EGq. ACq :: circ. EFN. K. Quæ repugnare modo ostensum est.

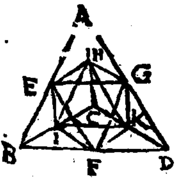
n hyp. — o 14. 5. — p 11. 5.

Ergo concludendum est, quod I $=$ circ. EFN. Q. E. D.

Coroll.

Hinc, ut circulus est ad circulum, ita polygonum in illo descriptum ad simile polygonum in hoc descriptum.

PROP. III.

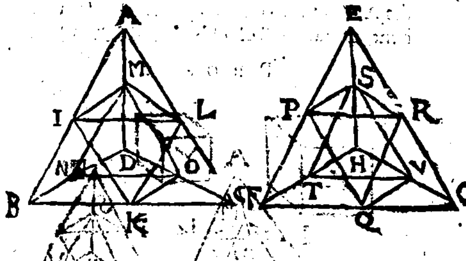


Omnis pyramis ABDC *triangularem habens basim, dividitur in duas pyramides* AEGH, HIKC *æquales & similes inter se, triangulares habentes bases, & similes toti* ABDC; *& in duo prismata æqualia* BFGEIH, EGDIHK; *quæ duo prismata majora sunt dimidio totius pyramidis* ABDC.

Latera pyramidis bisecentur in punctis E, F, G, H, I, K; junganturque rectæ EF, FG, GE, EI, IF, FK, KG, GH, HE. Quoniam latera

pyramidis proportionaliter secta sunt, *a* erunt
a 2. 6. HI, AB; & GF, AB; & IF, DC; atque HG,
DC, &c. parallelæ; proinde & HI, FG, & GH,
FI parallelæ sunt. liquet igitur triangula ABD,
b 29. 1. AEG, EBF, FDG, HIK *b* æquiangula esse; &
quatuor ultima *c* æquari. eodem modo triangula
c 26. 1. ACB, AHE, EIB, HIC, FGK æquiangula sunt;
& quatuor postrema inter se æqualia. similiter
triangula BFI, FDK, IKC, EGH; & denuo
triangula AHG, GDK, HKC, EFI, similia sunt
& æqualia. Quinetiam triang. HIK ad ADB, &
EGH ad BDC, & EFI ad ADC, & FGK ad
ABC *d* parallela sunt. Ex quibus perspicue sequi-
d 15. 11. tur primo, pyramides AEGH, HIKC æquales
e 10. def. 11. esse; totique ABDC, & inter se *e* similes. deinde
solida BFGEIH, FGDIHK prismata esse, &
quidem æque alta, nempe sita inter parallela pla-
na ABD, HIK. verum basis BFGE basis FDG
f 2. ax. 1. *f* duplex est. quare dicta prismata æqualia sunt.
g 40. 11. quorum alterum BFGEIH pyramide BEFI, hoc
est, AEGH majus est, totum sua parte; proinde
duo prismata majora sunt duabus pyramidibus,
totiusque adeo pyramidis ABDC dimidium ex-
cedunt. Q. E. D.

PROP. IV.



Si fuerint duæ pyramides ABCD, EFGH *ejusdem altitudinis, triangulares habentes bases* ABC, EFG; *sit autem illarum utraque divisa & in duas pyramides* (AILM, MNOD; & EPRS, STVH) *æquales inter se, & similes totis & in duo prismata æqualia* (IBKLMN, KLCNMO; & PFQRST, QRGTSV;) *ac eodem modo divisa sit utraque pyramidum, quæ ex superiore divisione natæ sunt, idque semper fiat; erit ut unius pyramidis basis ad alterius pyramidis basim, ita omnia, quæ in una pyramide, prismata ad omnia, quæ in altera pyramide prismata, multitudine æqualia.*

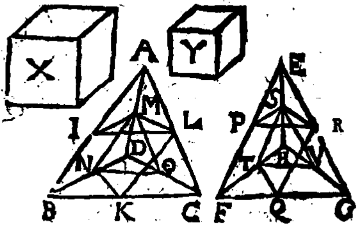
Nam (adhibendo constructionem præcedentis) BC. KC a :: FG. QG. b ergo triang. ABC est ad simile triang. LKC, ut EFG ad c simile RQG. ergo permutando ABC. EFG d :: LKC. RQG e :: Prism. KLCNMO. QRGTSV (nam hæc æque alta sunt) f :: IBKLMN. PFQRST. g quare triang. ABC. EFG :: Prism. KLCNMO + IBKLMN. Prism. QRGTSV + PFQRST. Q. E. D.

a 15. 5.
b 22. 6.
c 2. 6. &c.
d 16. 5.
e sch. 34. 11.
f 7. 5.
g 12. 5.

Sin ulterius simili pacto dividantur pyramides MNOD, AILM; & EDRS, STVH, erunt quatuor nova prismata hic effecta ad quatuor isthic producta, ut bases MNO & AIL ad bases STV & EPR, hoc est ut LKC ad RQG, vel ut ABC ad EFG. *b* quare omnia prismata pyramidis ABCD ad omnia ipsius EFGH ita se habent, ut basis ABC ad basim EFG. Q.E.D.

h 12. 5.

PROP. V.



Sub eadem altitudine existentes pyramides ABCD, EFGH, *triangulares habentes bases* ABC, EFG, *inter se sunt ut bases* ABC, EFG.

Sit triang. ABC. EFG :: ABCD. X. Dico X = pyr. EFGH. Nam, si possibile est, sit X ⊐ EFGH; sitque Y excessus. Dividatur pyramis EFGH in prismata & pyramides, & reliquae pyramides similiter, *a* donec relictae pyramides EPRS, STVH minores evadant solido Y. Quum igitur pyr. EFGH = X + Y; liquet reliqua prismata PFQRST, QRGTSV solido X majora esse. Pyramidem ABCD similiter divisam concipe; *b* eritque prism. IBKLMN + KLCNMO. PFQRST + QRGTSV :: ABC. EFG. *c* :: pyr. ABCD. X. *d* ergo X ⊏ prism. PFQRST + QRGTSV; quod repugnat prius affirmatis.

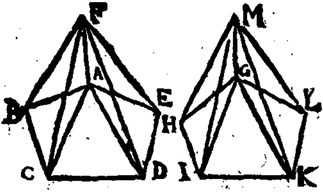
a 1. 10.
b 4. 12.
c hyp.
d 14. 5.

Rursus, dic X ⊏ pyr. EFGH. pone pyr. EFGH. Y :: X. pyr. ABCD *e* :: EFG. ABC. quia EFGH *f* ⊐ X, *g* erit Y ⊐ pyr. ABCD, quod fieri nequit, ex jam dictis. Concludo igitur, quod X = pyr. EFGH. Q.E.D.

e hyp. & cor. 4. 5.
f suppos.
g 14. 5.

PROP.

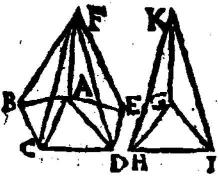
PROP. VI.



Sub eadem altitudine existentes pyramides ABCDEF, GHIKLM, *& polygonas habentes bases* ABCDE, GHIKL, *inter se sunt ut bases* ABCDE, GHIKL.

Duc rectas AC, AD, GI, GK. Est bas. ABC. ACD *a* :: pyr. ABCF. ACDF. *b* ergo composite ABCD. ACD :: pyr. ABCDF. ACDF. *a* atqui etiam ACD. ADE :: pyr. ACDF. ADEF. *c* ergo ex æquali ABCD. ADE :: ABCDF. ADEF. *b* ergo componendo ABCDE. ADE :: pyr. ABCDEF. ADEF. porro ADE. GKL *d* :: pyr. ADEF. GKLM; ac, ut prius, atque inverse GKL. GHIKL :: pyr. GKLM. GHIKLM. *c* ergo iterum ex æqualibus, ABCDE. GHIKL :: Pyr. ABCDEF. GHIKLM. Q. E. D.

a 5. 12.
b 18. 5.
c 22. 5.
d 5. 12.

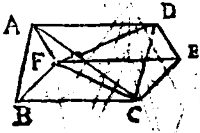


Si bases non habent latera æque multa, demonstratio sic procedet. Bas. ABC. GHI *e* :: pyr. ABCF. GHIK. *e* atque ACD. GHI :: pyr. ACDF. GHIK. *f* ergo bas. ABCD. GHI :: pyr. ABCDF. GHIK. *e* Quinetiam bas. ADE. GHI :: pyr. ADEF. GHIK. *f* ergo bas. ABCDE. GHI :: pyr. ABCDEF. GHIK.

e 5. 12.
f 24. 5.

PROP.

PROP. VII.

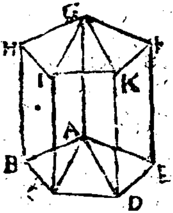


Omne prisma ABCDFE triangularem habens basim, dividitur in tres pyramides ACBF, ACDF, CDFE æquales inter se, triangulares bases habentes.

Ducantur parallelogrammorum diametri AC, CF, FD. Triang. ACB *a* = ACD. *b* ergo æquæ altæ pyramides ACBF, ACDF æquantur. eodem modo pyr. DFAC = pyr. DFEC. atqui ACDF, & DFAC una eademque sunt pyramis. *c* ergo tres pyramides ACBF, ACDF, DFEC, in quos divisum est prisma, inter se æquales sunt. Q. E. D.

a 34. 1.
b 5. 12.
c 1. ax. 1.

Coroll.



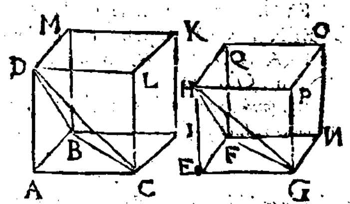
Hinc, quælibet pyramis tertia est pars prismatis eandem cum illa habentis & basim & altitudinem: sive, prisma quodlibet triplum est pyramidis eandem cum ipso habentis basim & altitudinem.

Nam resolve prisma polygonum ABCDEGHIKF in trigona prismata, & pyramidem ABCDEH in trigonas pyramides. *a* Erunt singulæ partes prismatis triplæ singularum partium pyramidis. *b* proinde totum prisma ABCDEGHIKF totius pyramidis ABCDEH triplum est Q. E. D.

a 7. 12.
b 1. 5.

PROP.

Prop. VIII.



Similes pyramides ABCD, EFGH, *quæ triangulares habent bases* ABC, EFG, *in triplicata sunt ratione homologorum laterum* AC, EG.

a Perficiantur parallelepipeda ABICDMKL, EFNGHQOP; quæ *b* similia sunt & pyramidum ABCD, EFGH *c* sextupla; *d* ideoque in eadem cum ipsis ratione ad se invicem, *e* hoc est in triplicata homologorum laterum. Q. E. D. — a 17. 11. b 9. def 11. c 28. 11. & 7. 12. d 15. 5. e 33. 11.

Coroll.

Hinc, etiam similes polygonæ pyramides rationem habent laterum homologorum triplicatam; ut facile probabitur resolvendo has in trigonas pyramides.

Prop. IX.

Vide Schema præced.

Æqualium pyramidum ABCD, EFGH, *& triangulares bases* ABC, EFG *habentium, reciprocantur bases & altitudines. & quarum pyramidum triangulares bases habentium reciprocantur bases & altitudines, illæ sunt æquales.*

1. *Hyp.* Perfecta parallelepipeda ABICDMKL, EFNGHQOP æqualium pyramidum ABCD, EFGH (utrumque utriusque) *a* sextupla sunt, ac æqualia ideo inter se, ergo alt. (H.) — a 28. 11. & 7. 12.

alt.

b 34. 11. c 15. 5. alt. (D) *b* :: ABIC. EFNG *c* :: ABC. EFG. Q. E. D.

d hyp. e 15. 5. f 34. 11. g 6. ax. 1. 2. *Hyp.* Alt. (H) alt. (D) *d* :: ABC. EFG *e* :: ABIC. EFNG. *f* ergo parallelepipeda ABIC-DMKL, EFNGHQOP æquantur; *g* proinde & pyramides ABCD, EFGH, horum subsextuplæ, pares sunt. Q. E. D.

Eadem polygonis pyramidibus conveniunt: nam hæ ad trigonas reduci possunt.

Coroll.

Quæ de pyramidibus demonstrata sunt Prop. 6, 8, 9. etiam conveniunt quibuscunque prismatis, cum hæc tripla sint pyramidum eandem basim & altitudinem habentium. itaque 1. Prismatum æque altorum eadem est proportio, quæ basium.

2. Similium prismatum proportio triplicata est proportionis laterum homologorum.

3. Æqualia prismata reciprocant bases & altitudines; & quæ reciprocant, sunt æquales.

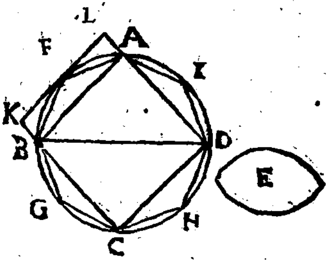
Schol.

Ex hactenus demonstratis elicitur dimensio quorumcunque prismatum & pyramidum.

a cor. 1. hujus; & sch. 40. 11. b 7. 12. *a* Prismatis soliditas producitur ex altitudine in basim ducta; *b* itaque & pyramidis ex tertia altitudinis parte ducta in basin.

PROP.

PROP. X.



Omnis conus tertia pars est cylindri habentis eandem cum ipso basin ABCD, *& altitudinem æqualem.*

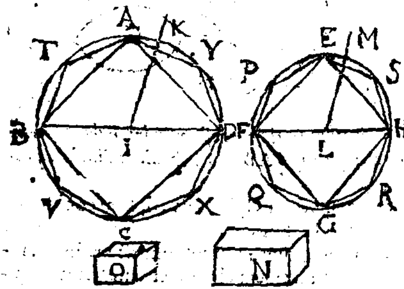
Si negas, primo Cylindrus triplum coni superet excessu E. Prisma super quadratum circulo ABCD inscriptum *a* subduplum est prismatis super quadratum eidem circulo circumscriptum sibi & cylindro æque alti. ergo prisma super quadratum ABCD superat cylindri semissem. eodem modo prisma super basim AFB cylindro æque altum segmenti cylindrici AFB *b* dimidio majus est. Continuetur bisectio arcuum, & detrahantur prismata, donec segmenta cylindri relicta, nempe ad AF, FB, &c. minora evadant solido E. Itaque cylind. — segment. AF, FB, &c. (prisma ad basim AFBGCHDI) *c* majus est quam cylind. — E (*d* triplum coni.) ergo pyramis dicti prismatis *e* pars tertia (ad eandem basim sita, ejusdemque altitudinis) cono æque alto ad basim ABCD circulum major est, pars toto. Q. E. A. *Vide sch. 2. hujus.* *a sch. 7. 4. & cor. 9. 12.* *b sch. 27. 3. & cor. 9. 12.* *c 5 ax. 1.* *d hyp.* *e cor. 7. 12.*

Sin conus tertia parte cylindri major dicatur, sit itidem excessus E. Ex cono detrahe pyramides, ut in priori parte prismata ex cylindro, donec restent coni segmenta aliqua, puta ad AF,

FB,

FB, BG, &c. minora solido E. ergo con. — E
f hyp. ($f\frac{1}{3}$ cylindr.) $\sqsupset$ pyr. AFBGCHDI (con. — segment. AF, FB, &c.) ergo prisma pyramidis triplum (æque altum scilicet atque ad eandem basim) cylindro ad basim ABCD majus est, pars toto. Q. E. A. Quare fatendum est, quod cylindrus triplo cono æquatur. Q. E. D.

P R O P. XI.



Sub eadem altitudine existentes cylindri, & coni ABCDK, EFGHM, *inter se sunt ut bases* ABCD, EFGH.

Sit circ. ABCD. circ. EFGH : : con. ABCDK. N. Dico N — con. EFGHM.

Nam si fieri potest, sit N $\sqsupset$ con. EFGHM, sitque excessus O. Supposita præparatione, & argumentatione præcedentis; erit O majus segmentis conicis EP, PF, FQ, &c. ideoque solidum N $\sqsupset$ pyr. EPFQGRHSM. *a* Fiat in circulo ABCD simile polygonum ATBVCXDY.
a 30. 5. & 11. post.
b 6 12. Quia pyr. ABVYK. pyr. EFQSM *b* : : polyg.
c cor 2. 12. ATBVY. polyg. EPFQS *c* : : circ. ABCD. circ.
d hyp. EFGH *d* : : con. ABCDK. N. *e* erit pyram.
e 14 5. EPFQGRHSM $\sqsupset$ N. contra modo dicta.

Rursus dic N $\sqsubset$ con. EFGHM. pone con. EFGHM. O : : N. con. ABCDK *f* : : circ. EFGH. ABCD. *g* ergo O $\sqsupset$ con. ABCDK, quod

quod absurdum est, ex ostensis in priori parte. f hyp. & invertendo.

Itaque potius dic, ABCD. EFGH :: con. g 14. 5.
ABCDK. EFGHM. Q. E. D.

Idem demonstrabitur de cylindris, si conorum & pyramidum loco concipiantur cylindri & prismata. ergo, &c.

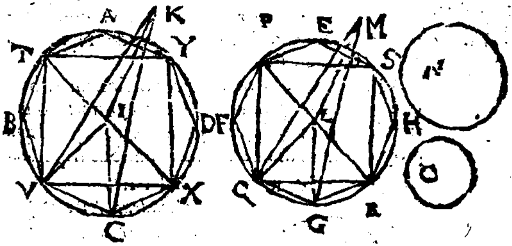
SCHOL.

Ex his habetur dimensio cylindrorum & conorum quorumcunque. Cylindri rectæ soliditas producitur ex base circulari (a pro cujus dimensione consulendus est Archimedes) ducta in altitudinem. b igitur & cujuscunque cylindri.

a 1. Prop. de dimens. circ.
b 11. 12.

c Itaque coni soliditas producitur ex tertia parte altitudinis ducta in basim. c 10. 12.

PROP. XII.



Similes coni & cylindri ABCDK, EFGHM, *in triplicata ratione sunt diametrorum* TX, PR, *quæ in basibus* ABCD, EFGH.

Habeat conus A ad aliquod N rationem triplicatam TX ad PR. dico N = con. EFGHM: Nam si fieri potest, sit N $\sqsupset$ EFGHM; sitque excessus O: ergo ut in Prioribus, N $\sqsupset$ pyr. EPFQGRHSM. Sint axes conorum IK LM, adducanturque rectæ VK, CK, VI, CI; & QM, GM, QL, GL. Quoniam coni similes sunt, a est VI. IK :: QL. LM. anguli vero VIK, QLM b recti sunt. c ergo trigona VIK, QLM,

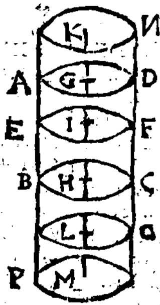
a 24. def. 11.
b 18. def. 11.
c 6. 6.

d 4.6. QLM æquiangula sunt; *d* unde VC. VI :: QG. QL. item VI. VK :: QL. QM. ergo ex æ-
e 7.5. quali VC. VK :: QG.QM. *e* quinetiam VK. CK :: QM. MG. ergo rursus ex æquo VC.
f 5.6. CK :: QG. GM. *f* ergo triangula *VKC*, QMG similia sunt; similique argumento *reliqua*
g 9. def. 11. hujus *pyramidis* triangula reliquis illius. *g* quare
h cor. 8. 12. pyramides ipsæ similes sunt. *h* sunt vero hæ in
k 4.6. triplicata ratione VC ad QG, *k* hoc est VI ad
l 15.5. RL, *l* vel TX ad PR. *m* ergo Pyr. AIBVC-
m hyp. & 11.5. XDYK. pyr. EPFQGRHS*M* :: con. ABCD*K*.
n 14.5. N. *n* unde pyr. EPFQGRHS*M* ⊐ N; quod repugnat prius dictis.

Rursus, dic N. ⊏ con. EFG*HM*. sit con.
o Prius, & inversè. EFGHM. O :: N. con. ABCD*K* *o* :: pyr.
EPR*M*. ATC*K* *p* :: GQ. VC ter :: *q* PR.
p cor. 8. 12. TX ter. verum O *r* ⊐ ABCD*K*. quod mo-
q 4.6.
r 14.5. do repugnare ostensum est. *Proinde* N = con. EFGH*M*. Q. E. D.

Quoniam vero quam proportionem habent coni, eandem quoque obtinent cylindri, eorum tripli, habebit quoque cylindrus ad cylindrum proportionem diametrorum in basibus triplicatã.

PROP. XIII.

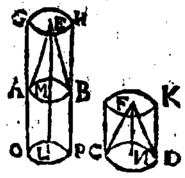


Si cylindrus ABCD *plano* EF *secetur adversis planis* BC; AD *parallelo; erit ut cylindrus* AEFD *ad cylindrum* EBCF, *ita axis* GI *ad axem* IH.

a 3.1. *Producto axe*, *a* sume G*K* = GI, & HL = I*H* = L*M*. & concipe per puncta *K*, L, *M*, plana duci circulis AD, BC parallela. *b* ergo cylind. ED =
b 11.12. cyl. AN. & cylin. EC *b* = BO *b* = OP. itaque cylindrus

drus EN cylindri ED æque multiplex est, ac axis IK axis IG. pariterque cylindrus FP æque multiplex est cylindri BF, ac axis IM axis IH. prout vero IK $=$, $\sqsubset$, $\sqsupset$ IM, c sic cylindr. EN $=$, $\sqsubset$, $\sqsupset$ EP. d ergo cyl. AEFD. cyl. EBCF :: GI. IH. Q. E. D. c 11. 12. d 6. def. 5.

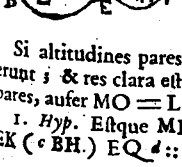
Prop. XIV.



Super æqualibus basibus AB, CD *existentes coni* AEB, CFD, *& cylindri* AH, CK, *inter se sunt ut altitudines* ME, NF.

Productis cylindro HA & axe EM, sume MI $=$ FN; & per punctum L ducatur planum basi AB parallelum. a erit cyl. AP $=$ CK. b atqui cylind. AH. AP. (CK) :: ME. ML. (NF.) Q. E. D. Idem de conis cylindrorum subtriplis dictum puta, * imo de prismatis & pyramidibus. a 11. 12. b 13. 12. * Adhibe 9. & 7. 12.

Prop. XV.



Æqualium conorum BAC, EDF, *& cylindrorum* BH, EK, *reciprocantur bases & altitudines* (BC. EF :: MD. LA:) *& quorum conorum, & cylindrorum reciprocantur bases & altitudines, illi sunt æquales.*

Si altitudines pares sint, etiam bases pares erunt; & res clara est. Sin altitudines sint impares, aufer MO $=$ LA. a 14. 12.

1. *Hyp.* Estque MD. MO (a LA) b :: cyl. EK (c BH.) EQ d :: circ. BC. EF. Q. E. D. b constr. c hyp. d 11. 12.

2. *Hyp.*

a hyp. 2. Hyp. BC. EF e :: DM. OM (L A) f ::
f 11. 12. Cyl. EK. EQ g :: BC. EF h :: BH. EQ. k Ergo
g 11. 5. cylind. EK = BH. Q. E. D.
h 11. 12.
k 9. 5.

Simili argumento utere de conis.

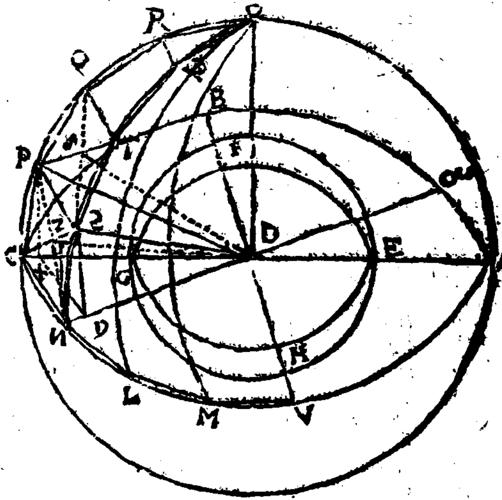
PROP. XVI.

Duobus circulis ABCG, DEF circa idem centrum M existentibus, in majori circulo ABCG polygonum æquilaterum, & parium laterum inscribere, quod non tangat minorem circulum DEF.

Per centrum M extendatur recta A C secans circulum D E F in F. ex quo erige perpendicularem FH. a Biseca
a 30. 3. semicirculum ABC, ejusque semissem BC, atque
b 1. 10. ita continuo, b donec arcus IC minor evadat arcu HC. ab I demitte perpendicularem IL. Liquet arcum IC totum circulum metiri, numerumque arcuum esse parem, adeoque subtensam
c sch. 16. 4. IC latus esse c polygoni inscriptibilis, quod circulum DEF minime continget. Nam HG d tan-
d cor. 16. 3. git circulum DEF; e cui parallela est IK, ex-
e 28. 1.
f 34. def. 1. traque sita; f quare IK circulum non tangit; multoque magis CI, CK, & reliqua polygoni latera, longius à centro distantia, circulum DEF non tangunt. Q. E. F. Coroll. Nota, quod IK non tangit circulum D E F.

PROP.

PROP. XVII.



Duabus sphæris ABCV, EFGH *circa idem centrum* D *existentibus, in majori sphæra* ABCV *solidum polyedrum inscribere, quod non tangat superficiem minoris sphæræ* EFGH.

Secentur ambæ sphæræ plano per centrum faciente circulos EFGH, ABCV. ducanturque diametri AC, BV secantes perpendiculariter. Circulo ABCV *a* inscribatur polygonum æquilaterum VMLNC, &c. circulum EFGH minime tangens. ducta diametro N*a*, erectaque DO recta ad planum ABC. per DO, perq; diametros AC, N*a* erigi concipiantur plana DOC, DON, quæ ad circulum ABCV *b* recta erunt, ideoque in superficie sphæræ *c* quadrantes a 16. 12. b 18. 11. c 10. 11.

V 2 effici-d 4. 1. efficient DOC, DON. in quibus d aptentur rectæ CP, PQ, QR, RO, NS, ST, Tγ, γO ipsis CN, NL, &c. pares, & æque multæ. In reliquis quadrantibus OL, OM, &c. inque tota sphæra eadem constructio fiat. Dico factum.

A punctis P, S ad planum A B C V demitte e 38. 11. perpendiculares PX, SY, e quæ in sectiones AC, f 12 ax. NC cadent. Quoniam igitur tam f anguli recti g 27. 3. PXC, SYN, g quam P C X, S N Y h æqualibus peripheriis insistentes, f pares sunt, triangula h 32. 1. PCX, SNY h æquiangula sunt. Cum igitur PC k constr. k = SN, l 26. 1. l etiam PX = SY, l & XC = YN; m 3. ax. 1. m quare DX = DY. n 7. 5. n ergo DX. XC :: DY. YN. o 2. 6. o ergo parallelæ sunt YX, NC. quia vero p 6. 11. PX, SY pares, & cum eidem plano ABCV rectæ, etiam p parallelæ sunt, q 33. 1. q erunt YX, r 9. 11. SP etiam pares & parallelæ. r ergo SP, NC s 7. 11. inter se parallelæ sunt. ergo s quadrilaterum t 2. 11. NCPS, eademque ratione S P Q T, T Q R G, sed & t triangulum γRO totidem sunt plana. Eodem modo tota sphæra ejusmodi quadrilateris & triangulis repleta ostendetur. quare inscriptum est polyedrum.

u 11. 11. A centro D u duc DZ rectum plano NCPS; & junge ZN, ZC, ZS, ZP. Quoniam D N. x 4. 6. NC x :: DY. YX; est NC y ⊏ YX (SP;) pariy 14. 5. terque SP ⊏ TQ, & TQ ⊏ γR. Et quia z 3. def. 11. anguli DZC, DZN, DZS, DZP, z recti sunt, a 15. def. 1. latera vero DC, DN, DS, DP a æqualia, & b 47. 1. DZ commune, b erunt ZC, ZN, ZS, ZP æquales inter se; proinde circa quadrilaterum c 25. def. 1. NCPS c describi potest circulus, in quo (ob NS, NC, CP d æquales, & NC ⊏ SP) NC d constr. e plusquam quadrantem subtendit. f ergo ang. e 28. 3. NZC ad centrum obtusus est. g ergo NCq ⊏ f 33. 6. 2 ZCq (ZCq + ZNq.) Sit NI ad AC norg 12. 2. malis. ergo cum ang. A D N (h D N C + h 32. 1. DCN) sit k obtusus, l erit semissis ejus D C N k 9. ax. 1. l 8. 1.

rectí

recti semisse major; propterea que eo minor est reliquus è recto ang. CNI. [n] unde IN ⊏ IC. [n 19. 1.] ergo NCq (NIq + ICq) [o] ⊐ 2 INq. itaque [p] [o 47. 1.] [p 47 1.] IN ⊏ ZC. & consequenter DZ [p] ⊏ DI. atqui [q] [q cor. 16. 12.] punctum I est [q] extra sphæram EFGH. ergo punctum Z potiori jure est extra ipsam. adeoque planum NCPS (cujus [r] proximum centro punctum est Z) sphæram EFGH non contingit. Et si ad planum SPQT demittatur perpendicularis Dδ, punctum δ, adeoque & planum SPQT adhuc ulterius à centro elongatur; idemque est de reliquis polyedri planis. ergo polyedrum ORQPCN, &c. majori sphæræ inscriptum, minorem non contingit. Q. E. F. [r 47. 1.]

Coroll.

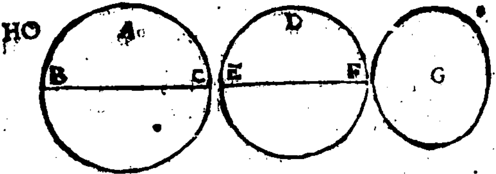
Hinc sequitur, *Si in quavis alia sphæra describatur solidum polyedrum, simile prædicto solido polyedro, proportionem polyedri in una sphæra ad polyedrum in altera esse triplicatam ejus quam habent sphærarum diametri.*

Nam si ex centris sphærarum ad omnes angulos basium dictorum polyedrorum rectæ lineæ ducantur, distribuentur polyedra in pyramides numero æquales & similes, quarum homologa latera sunt semidiametri sphærarum; ut constat, si intelligatur harum sphærarum minor intra majorem circa idem centrum descripta. congruent enim sibi mutuo lineæ rectæ ductæ à centro sphæræ ad basium angulos, ob similitudinem basium, ac propterea pyramides efficientur similes. Quare cum singulæ pyramides in una sphæra, ad singulas pyramides illis similes in altera sphæra [a] habeant proportionem triplicatam laterum homologorum, hoc est, semidiametrorum sphærarum; sint autem [b] ut una pyramis ad unam pyramydem, ita omnes pyramides, hoc est, solidum polyedrum ex his compositum, ad omnes pyra- [a cor. 8. 12.] [b 12. 5.]

mides, id est, ad solidum polyedrum ex illis constitutum; habebit quoque polyedrum unius sphæræ ad polyedrum alterius sphæræ proportionem triplicatam semidiametrorum, *c* atque adeo diametrorum.

c 33. 8.

PROP. XVIII.



Sphæræ BAC, EDF *sunt in triplicata ratione suarum diametrorum* BC EF.

Sit sphæra BAC ad sphæram G in triplicata ratione diametri BC ad diametrum EF. Dico G = EDF. Nam si fieri potest, sit G ⊐ EDF. & cogita sphæram G concentricam esse ipsi EDF. Sphæræ EDF *a* polyedrum sphæram G non tangens, sphæræque BAC simile polyedrum inscribatur. *b* Hæc polyedra sunt in triplicata ratione diametrorum BC, EF, *c* id est, sphæræ BAC ad G. *d* Proinde sphæra G major est polyedro sphæræ EDF inscripto, pars toto.

a 17. 12. *b* cor. 17. 12. *c* hyp. *d* 14. 5.

Rursus, si fieri potest, sit sphæra G ⊏ EDF. Sitque ut sphæra EDF ad aliam sphæram H, ita G ad BAC, *e* hoc est in triplicata ratione diametri EF ad BC; cum igitur BAC *f* ⊏ H, incurrimus absurditatem prioris partis. Quin potius sphæra G = EDF. Q. E. D.

e hyp. invers. *f* 14. 5.

Coroll.

Hinc, ut sphæra ad sphæram, ita est polyedrum in illa descriptum ad polyedrum simile in hac descriptum.

LIB.

LIB. XIII.

PROP. I.

SI recta linea z *secundum extremam & mediam rationem secetur* (z.a :: a.e;) *majus segmentum* a *assumens dimidium totius* z, *quintuplum potest ejus, quod à dimidia totius* z *describitur, quadrati.*

Dico Q. $a + \frac{1}{2}z = 5$ Q. $\frac{1}{2}z$. a hoc est b $aa + \frac{1}{4}zz + c\, za = zz + \frac{1}{4}zz$. b vel $aa + za = zz$. Nam $ze + za$ c $= zz$. & ze d $= aa$. e ergo $aa + za = zz$. Q.E.D.

a 4. 2. b 3. ax. 1. c 2. 2. d hyp. & 16. 6. e 1. ax. & 1. ax.

PROP. II.

Si recta linea $\frac{1}{2}z + a$ *sui ipsius segmenti* $\frac{1}{2}z$ *quintuplum possit, duplæ prædicti segmenti* (z) *extrema ac media ratione sectæ majus segmentum est* a, *reliqua pars ejus quæ à principio rectæ* $\frac{1}{2}z + a$.

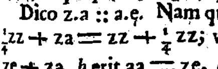
Dico z.a :: a.e. Nam quia per hyp. * $aa + \frac{1}{4}zz + za = zz + \frac{1}{4}zz$; vel $aa + za = zz$ a $= ze + za$, b erit $aa = ze$. c quare z. a :: a. e. Q.E.D.

* 4. 2. a 2. 2. b 3 ax. 1. c 17. 6.

Vide fig. præced.

PROP. III.

Si recta linea z *secundum extremam ac mediam rationem secetur* (z. a :: a.e;) *minus segmentum* e *assumens dimidium majoris segmenti* a , *quintuplum potest ejus , quod à dimidia majoris segmenti* a *describitur, quadrati.*

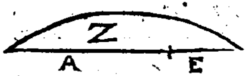


Dico Q: $e + \frac{1}{2}a = 5$ Q: $\frac{1}{2}a$. a hoc est b $ee + \frac{1}{4}aa + ea = aa + \frac{1}{4}aa$. b vel $ee + ea = aa$. Nam $ee + ea$ c $= ze$ d $= aa$. Q.E.D.

a 4. 2. b 3. ax. c 3. 2. d hyp. & 17. 6.

PROP. IV.

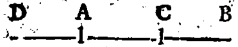
Si recta linea z secundum extremam ac mediam rationem secetur (z. a :: a. e;) quod à tota z, quodque à minori segmento e, utraque simul quadrata, tripla sunt ejus, quod a majori segmento a describitur, quadrati.



a 4. 2.
b 3. 2.
c 17. 6.
d 2. ax.

Dico zz + ee = 3 aa. ᵃ vel aa + ee + 2 ae + ee = 3 aa. Nam ae + ee = ze ᵇ = aa. ᵈ ergo aa + 2 ae + 2 ee = 3 aa. Q. E. D.

PROP. V.



Si recta linea AB *secundum extremam & mediam rationem secetur in* C*, apponaturque ei* AD *æqualis majori segmento* AC*; tota recta linea* DB *secundum extremam ac mediam rationem secatur, & majus segmentum est quæ à principio recta linea* AB.

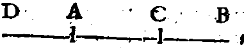
a hyp.

Nam quia AB. AD ᵃ :: AC. CB, invertendoque AD. AB :: CB. AC; erit componendo DB. AB :: AB. AC. (AD.) Q. E. D.

Schol.

Quod si fuerit BD. BA :: BA. AD. erit BA. AD :: AD. BA − AD. Nam dividendo est BD − BA (AD) BA :: BA − AD. AD. ergo inverse, BA. AD :: AD. BA − AD. Q. E. D.

PROP. VI.



Si recta linea rationalis AB *extrema ac media ratione secetur in* C*; utrumque segmentorum* (AC, CB) *irrationalis est linea, quæ vocatur apotome.*

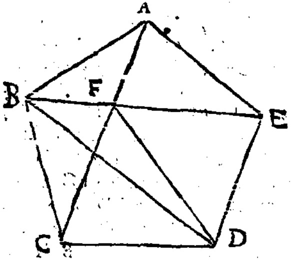
a 3. 1.
b 1. 13.
c 6. 10.
d hyp.
e sch. 12. 10.

Majori segmento AC ᵃ adde AD = $\frac{1}{2}$ AB; ᵇ ergo DCq = 5 DAq. ᶜ ergo DCq $\sqcap^{2}$ DAq. proinde cum AB ᵈ, ᵉ ideoque ejus semissis DA sint ρ', etiam DC est ρ'. Quia vero 5. 1 :: non

Q.

Q.Q. *f* est DC ⊔⊔ DA. *g* ergo DC − AD, id est AC est apotome. Insuper quia ACq *h* = AB × BC, & AB est ρ', *k* etiam BC est apotome. Q.E.D. f 9. 10. g 74. 10. h 17. 6. k 98. 10.

PROP. VII.



Si pentagoni æquilateri ABCDE *tres anguli, sive qui deinceps* EAB, ABC, BCD, *sive* EAB, BCD, CDE *qui non deinceps sint, æquales fuerint, æquiangulum erit ipsum pentagonum* ABCDE.

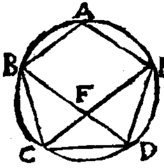
Paribus deinceps angulis subtendantur rectæ BE, AC, BD.

Quoniam latera EA, AB, BC, CD, angulique inclusi *a* æquantur, *b* erunt bases BE, AC, BD, *c* angulique AEB, ABE, BAC, BCA pares. *d* quare BF = FA, & *e* proinde FC = FE. ergo triangula FCD, FED sibi mutuo æquilatera sunt; *f* unde ang. FCD = FED, *g* proinde ang. AED = BCD. Eodem pacto ang. CDE reliquis æquatur. quare pentagonum æquiangulum est. Q.E.D. a hyp. b 4. 1. c 4. & 5. 1. d 6. 1. e 3. ax. 1. f 8. 1. g 2. ax. 2.

Sin anguli EAB, BCD, CDE, qui non deinceps, statuantur pares, *h* erit ang. AEB = BDC, & BE = BD, *k* ideoque ang. BED = BDE; *l* totus proinde ang. AED = CDE. ergo propter angulos A, E, D deinceps æquales, ut prius, pentagonum æquiangulum erit. Q.E.D. h 4. 1. k 5. 1. l 2. ax.

PROP.

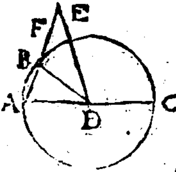
PROP. VIII.



Si pentagoni æquilateri & æquianguli ABCDE *duos angulos* BCD, CDE, *qui deinceps sint, subtendant rectæ lineæ* BD, CE; *hæ extrema ac media ratione se mutuo secant, & majora ipsarum segmenta* BF, *vel* EF *æqualia sunt pentagoni lateri* BC.

a 14. 4. Circa pentagonum *a* describe circulum ABD.
b 28. 3. *b* Arcus ED = BC. *c* ergo ang. FCD = FDC.
c 27. 3. *d* ergo ang. BFC = 2 FCD (FCD + FDC.)
d 32. 1. Atqui arcus BAE *b* = 2 ED, proinde ang.
e 33. 6. BCF *e* = 2 FCD = BFC. *f* quare BF = BC.
f 6. 1. Q. E. D. Porro quia triangula BCD, FCD
g 27. 3. *g* æquiangula sunt, *h* erit BD. DC (BF) :: CD
h 4. 6. (BF.) FD. pariterque EC. EF :: EF. FC. Q. E. D.

PROP. IX.



Si hexagoni latus BE, *& decagoni* AB, *in eodem circulo* ABC *descriptorum componantur, tota recta linea* AE *extrema ac media ratione secatur*, (AE. BE :: BE. AB.) *& majus ejus segmentum est hexagoni latus* BE.

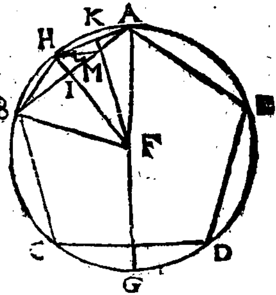
a hyp. & 37. 3. Duc diametrum ADC, & junge rectas DB, DE. Quoniam ang. BDC *a* = 4 BDA, estque
b 32. 1. ang. BDC *b* = 2 DBA (DAB + DBA,) erit
c 7. ax. 1. DBA (*b* BDE + BED) *c* = 2 BDA *d* = 2 BDE.
d 5. 1. proinde ang. DBA, vel DAB *e* = ADE. Itaque
e 1. ax. 1. trigona ADE, ADB æquiangula sunt, *f* quare
f 4. 6. AE. AD. (*g* BE) :: AD. (BE.) AB. Q. E. D.
g cor. 15. 4.

Coroll.

Coroll.

Hinc, si latus hexagoni alicujus circuli secetur extrema ac media ratione; majus illius segmentum erit latus decagoni ejusdem circuli. *sch* 5. 13.

Prop. X.



Si in circulo ABCE *pentagonum æquilaterum* ABCDE *describatur; pentagoni latus* AB *potest & hexagoni latus* FB, *& decagoni latus* AH, *in eodem circulo descriptorum.*

Duc diametrum AG. Biseca arcum AH in K. Et duc FK, FH, FB, BH, HM.

Semicirc. AG − arc. AC *a* = AG − AD. a 28. 3. &
hoc est, arc. CG = GD *b* = AH = HB. ergo 3. *ax.*
arc. BCG = 2 BHK; *c* adeoque ang. BFG = 2 b *hyp.* &
BF*K*. *d* sed ang. BFG = 2 BAG. *e* ergo ang. 7. *ax.* c 33. 6.
BFK = BAG. Trigona igitur BFM, FAB *f* æ- d 20. 3.
quiangula sunt. *g* quare AB. BF :: BF. BM. e 1. *ax.* 1. f 32. 1.
h ergo AB x BM = BFq. Rursus ang. AF*K* *k* = g 4. 6.
HF*K*; & FA = FH; *m* quare AL = LH, *m* & h 17. 6. k 27. 3.
anguli FLA, FLH pares, ac proinde recti sunt. m 4. 1.
ergo ang. LHM *m* = LAM *n* = HBA. Trigo- n 27. 3.
na igitur AHB, AMH *o* æquiangula sunt. *p* qua- o 32. 1. p 4. 6.

re

q 17. 6. te AB. AH :: AH. AM. q ergo AB x AM =
r 2. 2. AHq. Quum igitur ABq r = AB x BM + AB
f 2. ax. x AM, f erit ABq=BFq + AHq. Q. E. D.

Coroll.

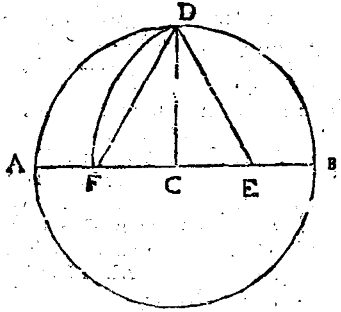
1. Hinc, linea recta (FK) quæ ex centro (F) arcum quempiam (H A) bisecat, etiam rectam (HA) illi arcui subtensam bisecat ad angulos rectos.

2. Diameter circuli (AG) ex angulo quovis (A) pentagoni ducta bisecat & arcum (CD,) quem latus pentagoni illi angulo oppositum subtendit, & latus ipsum (CD) oppositum, idque ad angulos rectos.

Schol.

Hic, ut promisimus, praxim tradémus expeditam problematis 11. 4.

Problema.



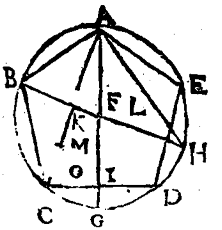
Invenire latus pentagoni circulo ADB *inscribendi.*

Duc diametrum A B. cui perpendicularem C D

C D ex centro C erige. Biſeca CB in E. Fac EF = ED. Erit DF pentagoni latus.

Nam BF x FC + ECq *a* = EFq *b* = EDq *c* = DCq + ECq. *d* ergo BF x FC = DCq, vel BCq. *e* quare BF. BC :: BC. FC. ergo quum BC *d* ſit latus hexagoni, *f* erit FC latus decagoni, proinde DF *h* = √DCq + FCq *g* eſt latus pentagoni. Q. E. F. a 6. 2. b conſtr. c 47. 1. d 3. ax. e 17. 6. f 9. 13. g 10. 13. h 47. 1.

Prop. XI.



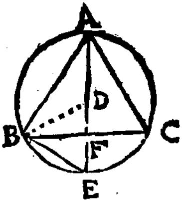
Si in circulo ABCD *rationalem habente diametrum* AG, *pentagonu● æquilaterum* ABCDE *deſcribatur; pentagoni latus* AB *irrationalis eſt linea, quæ vocatur minor.*

Duc diametrum BFH, rectaſque AC, AH; & * fac FL = $\frac{1}{4}$ radii FH, & CM = $\frac{1}{4}$ CA. * 10. 6.

Ob angulos AKF, AIC *a* rectos, & communem CAI, trigona AKF, AIC *b* æquiangula ſunt; *c* ergo CI. FK *c* :: CA. FA (FB) *d* :: CM. FL. ergo permutando FK. FL :: CI. CM *d* :: CD. CK (2 CM.) *e* componendo igitur CD + CK. CK :: KL. FL. *f* proinde Q: CD + CK (*g* 5 CKq.) CKq :: KLq. FLq. ergo KLq = 5 FLq. Itaque ſi BH (ρ̇) ponatur 8, erit FH 4; FL 1. & FLq. 1. BL 5. & BLq 25. KLq 5. è quibus liquet BL, & KL eſſe ρ̇ *h* ⊐. : *k* ideoque BK eſſe Apotomen; cujus congruens KL. cum vero BLq − KLq = 20, *l* erit BL ⊓ √BLq − KLq. *m* unde BK erit apotome quarta. Quoniam igitur ABq *m* = HB x BK, *n* erit AB minor. Q. E. D. a cor. 10. 13. b 32. 1. c 4. 6. d 15. 5. e 18. 5. f 22. 6. g 1. 13. h 9. 10. k 74. 10. l 9. 10. m cor. 8. 6 & 17. 6. n 95. 10.

Prop.

PROP. XII.



Si in circulo ABEC *triangulum æquilaterum* ABC *describatur, trianguli latus* A B *potentia triplum est ejus lineæ* AD, *quæ ex* D *centro circuli ducitur.*

Protracta diametro ad E, duc B E. Quoniam arcus BE a = EC, arcus BE sexta est pars circumferentiæ. b ergo BE = DE. hinc AEq c = 4 DEq (4 BEq) d = ABq + BEq (+ ADq.) e proinde ABq = 3 ADq. Q. E. D.

a cor. 10. 13.
b cor. 15. 4.
c 4. 2.
d 47. 1.
e 3. ax. 1.

Coroll.

1. AEq. ABq :: 4. 3.

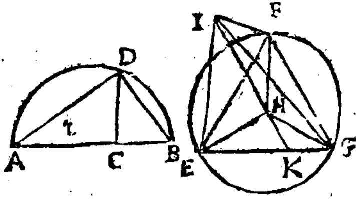
2. ABq. AFq :: 4. 3. f Nam ABq. AFq :: AEq. ABq.

3. DF = FE. Nam triang. EBD g æquilaterum est; h & BF ad ED perpendicularis. h ergo EF = FD.

4. Hinc AF = DE + DF = 3 DF.

f cor 8. 6. & 21. 6.
g cor. 15. 4.
h cor. 3. 3.

PROP. XIII.



Pyramidem EGFI *constituere, & data sphæra complecti; & demonstrare quod sphæræ diameter*

AB

AB potentia sit sesquilatera lateris E F *ipsius pyramidis* EGFI.

Circa A B describe semicirculum A D B. [a] sitque AC = 2 CB. ex puncto C erige perpendicularem CD; & junge AD, DB. Tum radio HE = CD describe circulum H E F G; cui [b] inscribe triangulum æquilaterum EFG. ex H [c] erige I H = C A rectum plano E F G, produc IH ad K; [d] ita ut IK = AB. rectasque adjunge IE, IF, IG. erit EFGI pyramis expetita.

a 10. 6.
b cor. 15. 4.
c 12. 11.
d 3. 1.

Nam quia anguli ACD, IHE, IHF, IHG [e] recti sunt; & CD, HE, HF, HG [e] pares, [e] atque IH = AC; [f] erunt AD, IE, IF, IG æquales inter se. Quia vero AC (2 CB.) CB [g] :: ACq. CDq. erit ACq = 2 CDq. itaque ADq [f] = ACq + CDq [h] = 3 CDq = 3 HEq [k] = EFq. [l] ergo AD, EF, IE, IF, IG pares sunt, adeoque pyramis EFGI est æquilatera. Quod si punctum C super H collocetur, & AC super H I, rectæ AB, IH [m] congruent, utpote æquales. quare semicirculus ADB axi AB vel IK circumductus [n] transibit per puncta, E, F, G, * adeoque pyramis EFGI sphæræ inscripta erit. Q. E. F. liquet vero esse BAq. ADq [o] :: BA. AC [p] :: 3. 2. Q. E. D.

e constr.
f 47. 1.
g 20. 6.
h 2. ax.
k 12. 13.
l 1. ax. 1.
m 8 ax. 1.
n 15. def. 1.
* 31. def. 11
o cor. 8 6.
p constr.

Corollaria.

1. ABq. HEq :: 9. 2. Nam si ABq ponatur 9, erit ACq (EFq) 6. [q] proinde HEq erit 2.

2. Si L centrum fuerit, erit AB. LC :: 6. 1. Nam si AB ponatur 6, erit AL, 3; [r] ideoque AC 4; quare LC erit 1. Hinc

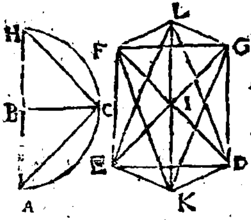
3. AB. HI :: 6. 4 :: 3. 2. unde

4. ABq. HIq :: 9. 4.

q 12. 13.
r constr.

PROP.

PROP. XIV.



Octaedrum KEFGDL *constituere, & data sphæra complecti, qua & pyramidem; & demonstrare, quod sphæræ diameter* AH *potentia sit dupla lateri* AC *ipsius Octaedri.*

Circa AH describe semicirculum ACH. ex centro B erige perpendicularem BC. duc AC, HC. Super ED = AC *a* fac quadratum EFGD, cujus diametri DF, EG secantes in centro I. ex I duc IL = AB *b* rectam plano EFGD. produc IL, *c* donec IK = IL. Connexis KE, KF, KG, KD, LE, LF, LG, LD; erit KEFGDL octaedrum quæsitum.

a 46. 1. *b* 12. 11. *c* 3. 1.

Nam AB, BH, FI, IE, &c. æqualium quadratorum semidiametri æquales sunt inter se. *d* quare triangulorum rectangulorum LIE, LIF, FIE, &c. bases LF, LE, FE, &c. æquantur. proinde octo triangula LFE, LFG, LGD, LDE, KEF, KFG, KGD, KDE æquilatera sunt, *e* atque octaedrum constituunt, quod sphæræ cujus centrum I, radius IL, vel AB, inscribi potest. (quoniam AB, IL, IF, IK, &c. *f* æquales sunt.) Q. E. F. porro liquet AHq (LKq) *g* = 2 ACq (2 LDq.) Q. E. D.

d 4. 1. *e* 27. def. 11. *f* constr. *g* 47. 1.

Corollaria.

1. Hinc manifestum est, in Octaedro tres diametros EG, FD, LK se mutuo ad angulos rectos secare in centro sphæræ.

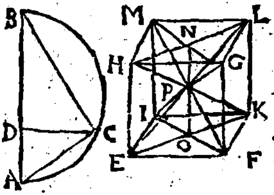
2. Item, tria plana EFGD, LEKG, LFKD esse quadrata, se mutuo ad angulos rectos secantia.

3. Octa-

3. Octaedrum dividitur in duas pyramides similes & æquales EFGDL, & EFGDK, quarum basis communis est quadratum EFGD.

4. Denique, bases octaedri oppositæ, inter se parallelæ sunt. 15. 11.

PROP. XV.



Cubum EFGHIKLM *constituere, & sphæra complecti, qua & priores figuras; & demonstrare, quod sphæræ diameter* AB *potentia sit tripla lateris* EF *ipsius cubi.*

Super AB describe semicirculum ACB; & *a* fac AB = 3 DA. ex D erige perpendicularem DC, & junge BC ac AC. Tum super EF = AC *b* construe quadratũ EFGH, cujus plano rectæ insistant EI, FK, HM, GL ipsi EF pares, quas connecte rectis IK, KL, LM, IM. Solidum EFGHIKLM cubus est, ut satis constat ex constructione. a 10. 6. b 46. 1.

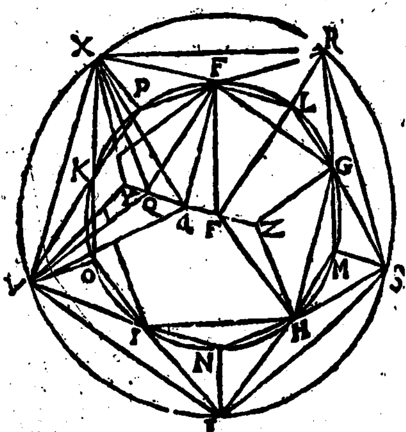
In quadratis oppositis EFKI, HGLM duc diametros EK, FI, HL, MG, per quas ducta plana EKLH, FIMG se intersecent in recta NO. Hæc diametros cubi EL, FM, GI, HK *c* bisecabit in P, centro cubi. *d* ergo P centrum erit sphæræ per puncta cubi angularia transeuntis. Porro ELq *e* = EKq + KLq *e* = 3 KLq, *f* vel 3 ACq. atqui ABq. ACq *g* :: BA. DA *g* :: 3. 1. *h* ergo AB = EL. Quare cubum fecimus, &c. Q. E. F. c cor. 39. 11. d 15. def. 1. & 14. def. 11. e 47. 1. f constr. g cor. 8. 6. h 14. 5.

Coroll.

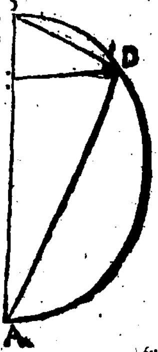
1. Hinc, omnes diametri cubi inter se æquales sunt, seseque mutuo in centro sphæræ bisecant. Eademque ratione rectæ quæ quadratorum oppositorum centra conjungunt, bisecantur in eodem centro.

X 2 Diak 47. 1. l 13. 13. m 15. 13. 2. Diameter sphæræ potest latus tetraedri, & cubi. nempe ABq k= l BCq + m ACq.

PROP. XVI.



Icosaedrum ZGHIKFYVXRST *constituere, & sphæra complecti, qua & antedictas figuras; & demonstrare, quod icosaedri latus* F G *irrationalis est linea, qua vocatur minor.*



Super AB diametrum sphæræ describe semicirculum ADB; & a fac AB = 5 BC. ex C erige normalem CD, & duc AD ac BD. Ad intervallum EF = BD describe circulum EFKNG; cui

a 10. 6.

b cui inscribe pentagonum æquilaterum FKIHG. Biseca arcus FG, GH, &c. ac connecte rectas FL, LG; &c. latera nempe decagoni. Tunc c erige EQ, LR, MS, NT, OV, PX ipsi EF æquales, rectasque plano FKNG. & connecte RS, ST, TV, VX, XR; item FX, FR, GR, GS, HS, ST, HT, IT, IV, KV, KX. Denique producta EQ, sume QY = FL; & EZ = FL; rectasque duci concipe ZG, ZH, ZI, ZK, ZF; ac YV, YX, YR, YS, YT. Dico factum. b 11. 4. c 12. 11.

Nam ob EQ, LR, MS, NT, OV, PX d æquales e & parallelas, etiam quæ illas jungunt, EL, QR, EM, QS, EN, QT, EO, QV, EP, QX f pares & parallelæ sunt. Item ideo LM (vel FG,) RS, MN, ST, &c. æquales sunt inter se. g ergo planum per EL, EM, &c. plano per QR, QS, &c. æquidistans, h & circulus QXRSTV è centro Q, circulo EPLMNO æqualis est; atque RSTVX est pentagonum æquilaterum. Duci vero intellectis EF, EG, EH, &c. ac QX, QR, QS, &c. quia FRQ k = FLq + LRq, l vel EFq m = FGq, n erunt FR, FG, adeoque omnes RS, FG, FR, RG, GS, GH, &c. æquales inter se. Proinde 10 triangula RFX, RFG, RGS, &c. æquilatera sunt & æqualia. Rursus ob ang. XQY o rectum, erit XYq p = QXq + QYq q = VXq vel FGq. quare XY, VX, hisque similiter YV, YT, YS, YR, ZG, ZH, &c. æquantur. Ergo alia decem trigona constituta sunt æquilatera, & æqualia, tam sibi mutuo, quam decem prioribus; ac proinde factum est Icosaedrum. d constr. e 6. 11. f 33. 1. g 15. 11. h 1. def. 3. k 47. 1. l constr. m 10. 13. n sch. 48. 1 & 1. ax. o cor. 14. 11. p 47. 1. q 10. 13.

Porro, biseca EQ in a, duc rectas aF, aX, aV; & propter QX e = QV, & commune latus aQ, angulosque EQX, EQV rectos; f erit aX = aV. similique argumento omnes, aX, aR, aS, aT, aV, aF, aG, aH, aI, aK æquantur. e 15. def. 1. f 4. 1.

t 9. 13. Quoniam autem ZQ. QE t :: QE. ZE, u erit
u 3. 13. Zαq u = 5 Eαq x = EQq (EFq) + Eαq, y = αFq.
x 4. 2. ergo Zα = αF. z pari pacto αF = Yα. ergo
y 47. 1. sphæra, cujus centrum α, radius αF, per 12 puncta icosaedri angularia transibit.

Denique, quia Zα. αE :: ZY. QE; a ideoque
z 15. 5. Zαq, αEq :: ZYq. QEq. b erit ZYq = 5 QEq,
a 22. 6. vel 5 BDq c atqui ABq. BDq c :: AB. BC :: 5.
b 14. 5. 1. d ergo ZY = AB. Q. E. F.
c cor. 8. 6.
d 1. ax. 1.
e sch. 12. 10. Itaque si AB ponatur ρ, e erit EF = $\sqrt{}$ ABq
f 11. 13 etiam ρ; proinde FG pentagoni, idemque icosaedri 5 latus, f est minor. Q. E. D.

Coroll.

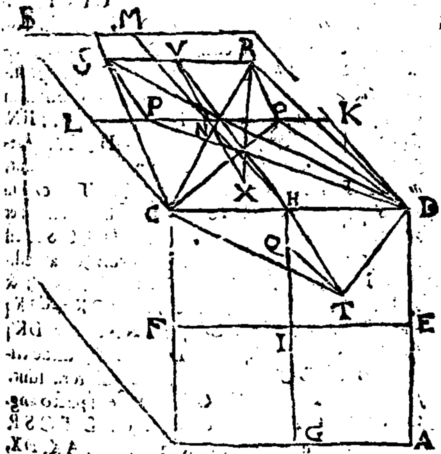
1. Ex dictis infertur, sphæræ diametrum esse potentia quintuplum semidiametri circuli quinque latera icosaedri ambientis.

2. Item manifestum est, sphæræ diametrum esse compositam ex latere hexagoni, hoc est, ex semidiametro, & duobus lateribus decagoni circuli ambientis quinque latera icosaedri.

3. Constat denique latera icosaedri opposita,
a 33. 1. qualia sunt RX, HI, esse parallela: Nam RX a pa-
b sch. 26. 3. rall. LP. b parall. HI.

PROP.

PROP. XVII.



Dodecaedrum constituere; & sphæra complecti, qua & prædictas figuras; & demonstrare, quod dodecaedri latus RS irrationalis est linea, quæ vocatur apotome.

Sit AB cubus datæ sphæræ inscriptus, cujus latera omnia bisecentur in punctis E, H, F, G, K, L, &c. rectæque adjungantur KL, MH, HG, EF. a Fat HI. IQ :: IQ. QH; & sume a 30. 6.
NO, NP pares ipsi TQ. Erige OR, PS rectas plano DB, & QT plano AC. sintque OR, PS, QT ipsis IQ, NO, NP æquales. Connexis DR, RS, SC, CT, DT, erit DRSCT pentagonum Dodecaedri expetiti. Nam duc NV parall. OR, & protracta NV ad occursum cum cubi centro X, connecte rectas DS, DO, DP, CR, CP, a 47. 1.
HV, HT, RX. Quia DOq a = DKQ (b KNq) b 7. ax. 1.
+ KOq c = 3 ONq (3 ORq) d erit DRq c 4. 13. d 47. 1.

d 4. ax. 1 $= 4$ ORq $e =$ OPq, vel RSq. ergo DR $=$ RS. Simili argumento DR, RS, SC, CT, TP pares sunt. Quia vero OR $f = g$ & parall. PS,
f constr. 9. 6. 21. g erunt RS, OP, & h consequenter RS, DC etiam parallelæ; h ergo hæ cum suis conjungentibus DK, CS, VH in uno sunt plano. quinetiam
g 33. 1. quia HI. IQ k:: IQ (TQ) QH k:: HN.
h 9. 11. NV; & tam TQ, HN, quam QH, NV k rectæ eidem plano, l adeoque & parallelæ existunt,
i 7. 11. m erit THV recta linea. n ergo Trapezium
k constr. DRSC, & triang DTS in uno sunt plano per
l 6. 11. rectas DC, TV extensa. ergo DTCSR est
m 32. 6. pentagonum, & quidem æquilaterum, ex antedictis.
n 2. ax. 11. Porro, quia PK. KN :: KN. NP; &
o 5. 13. DSq $p =$ DPq + PSq (PNQ) $= p$ DKq + PKq
p 47. 1. + NPq, q erit DSq $=$ DKq $+ 3$ KNq $= 4$ DKq
q 1. ax. 2. (4DHq) $r =$ DCq. ergo DS $=$ DC; unde trigona DRS, DCT sibi mutuo æquilatera sunt.
r 4. 13. f ergo ang. DRS $=$ DTC; & eodem pacto ang.
s 4. 2. CSR $=$ DCT. ergo pentagonum DTCSR
t 8. 1. etiam æquiangulum est. Ad hæc, quia AX, DX,
x 15. 13. CX, &c. sunt cubi semidiametri, y erit XN $=$
y 1. ax. 1. IH, vel KN, u adeoque XV $=$ KP. unde ob angu-
u 29. 1. lum x rectum RVX, z erit RXq $=$ XVq + RVq
z 47. 1. (NPq) $=$ KPq + NPq $a = 3$ KNq $b =$
a 4. 13. AXq, vel DXq, &c. ergo RX, AX, DX, & ea-
b 15. 13. dem ratione XS, XT, AX æquales sunt inter se.

Et si eadem methodo, qua constructum est pentagonum DTCSR, fabricentur 12 similia pentagona tangentia duodecim cubi latera, ea Dodecaedrum constituent; ac per eorum puncta angularia transiens sphæra, cujus radius AX, vel RX, Dodecaedrum complectetur. Q. E. F.

c constr. Denique, quia KN. NO c:: NO. OK. d
d 15. 5. erit KL. OP :: OP. OK + PL. Itaque si
e 15. 13. sphæræ diameter AB ponatur ρ, erit KL $e = \sqrt{\frac{AB}{3}}$
f sch. 12. 10. f etiam ρ. g unde OP, vel RS latus dodeca-
g 6. 13. edri apotome erit. Q. E. D.

Coroll.

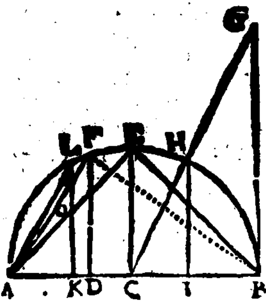
Coroll.

1. Hinc, si latus cubi secetur extrema ac media ratione, majus segmentum erit latus dodecaedri, in eadem sphæra descripti.

2. Si rectæ lineæ sectæ extrema ac media ratione, minus segmentum sit latus dodecaedri, majus segmentum erit latus cubi ejusdem sphęrę.

3. Liquet etiam latus cubi æquale esse lineæ rectæ subtendenti angulum pentagoni dodecaedri eadem sphæra comprehensi.

PROP. XVIII.



Latera quinque figurarum exponere, & inter se comparare.

Sit AB diameter sphæræ, ac AEB semicirculus. sitq; AC [a] $= \frac{1}{2}$AB, & AD [b] $= \frac{1}{3}$ AB. Erige perpendiculares CE, DF, & BG = AB. junge AF, AE, BE, BF, CG. ex H demitte perpendicularem HI, & sumpta CK = CI, ex K erige perpendicularem KL, & connecte AL. Denique [c] fac AF. AO :: AO. OF.

Itaque 3. 2 [d] :: AB. BD [e] :: ABq. BFq, latus Tetraedri. & 2. 1 :: [a] AB. AC :: ABq. BEq, [f] latus Octaedri.

Item 3. 1 [d] :: AB. AD [e] :: ABq. AFq, [g] latus Hexaedri.

Porro, quia AF. AO [h] :: AO. OF. [k] erit AO

a 10. 1.
b 10. 6.
c 30. 6.
d constr.
e cor. 8. 6.
f 14. 13.
g 15. 13.
h constr.
k cor. 17. 1.

l 4.6. AO latus Dodecaedri. denique BG (2 BC.) BC [l] :: HI.IC. [m] ergo HI = 2CI [n] = KI. ergo
m 24.5. n constr. o 4.2. HIq [o] = 4 CIq. proinde CHq = [p] 5 CIq. [q] ergo
p 47.1. q 15.5. ABq = 5 KIq. [r] itaque KI, vel HI, est radius circuli circumscribentis pentagonum icosaedri; &
r cor.16.13. s 10.13. AK, vel IB, [s] est latus decagoni eidem circulo inscripti. unde AL erit latus pentagoni, [t] idemque
t 16.13. Icosaedri latus. Ex quibus liquet BF, BE, AF esse ρ' ⊐. & AL, AO esse ρ' ⊐; atque BF $\sqsubset$ BE; & BE $\sqsubset$ AF; ac AF $\sqsubset$ AO. Quia vero 3 AFq = ABq [u] = 5 KLq. ac AF x AO

u 1.6. $\sqsubset$ AF x OF, [x] ideoque AF x AO + AF x OF
x 4.ax.1. $\sqsubset$ 2 AF x OF, [y] hoc est AFq $\sqsubset$ [z] 2 AOq. [a] e-
y 1.2. z 17.6. rit 3 AFq (5 KLq) $\sqsubset$ 6 AOq. proinde KL
a 47.1. $\sqsubset$ AO; & fortius, AL $\sqsubset$ AO.

Jam vero ut hæc latera numeris exprimamus, si AB ponatur $\sqrt{60}$, erit ex jam dictis ad calculum exactis, BF = $\sqrt{40}$. & BE = $\sqrt{30}$. & AF = $\sqrt{20}$. item AL = $\sqrt{:30 - \sqrt{180}}$ (nam AK = $\sqrt{15} - \sqrt{3}$. & KL (HL) = $\sqrt{12}$.) denique AO = $\sqrt{:30 - \sqrt{500}}$ ($\sqrt{25} - \sqrt{5}$.)

SCHOL.

SCHOL.

Præter jam dictas figuras nullam dari posse figuram solidam regularem (nempe quæ figuris planis ordinatis & æqualibus contineatur) admodum perspicuum est. Nam ad anguli solidi constitutionem requiruntur ad minimum tres anguli plani; a hique omnes simul 4 rectis minores esse debent. b Atqui 6 anguli trigoni æquilateri, 4 quadratici, & 3 hexagonici, sigillatim 4 rectos exæquant; quatuor vero pentagonici, 3 heptagonici, 3 octagonici, &c. 4 rectos excedunt. ergo solummodo ex 3, 4, vel 5 triangulis æquilateris, ex 3 quadratis, vel 3 pentagonis, effici potest angulus solidus. Proinde, præter quinque prædicta, nulla existere possunt corpora regularia.

a 21. 11.
b Vid. schol. 32. 1.

Ex P. Herigonio.

Proportiones sphæræ, & 5 figurarum regularium eidem inscriptarum.

Sit diameter sphæræ 2. Erunt

Peripheria circuli majoris, 6 28318.

Superficies circuli majoris, 3 14159.

Superficies sphæræ, 12 56637.

Soliditas sphæræ, 4 1879.

Latus tetraedri, 1 62299.

Latus

Superficies tetraedri, 4 | 6188.

Soliditas tetraedri, 0 | 15132.

Latus hexaedri, 1 | 1547.

Superficies hexaedri, 8.

Soliditas hexaedri, 1 | 5396.

Latus octaedri, 1 | 41421.

Superficies octaedri, 6 | 9282.

Soliditas octaedri, 1 | 33333.

Latus dodecaedri, 0 | 71364.

Superficies dodecaedri, 10 | 51462.

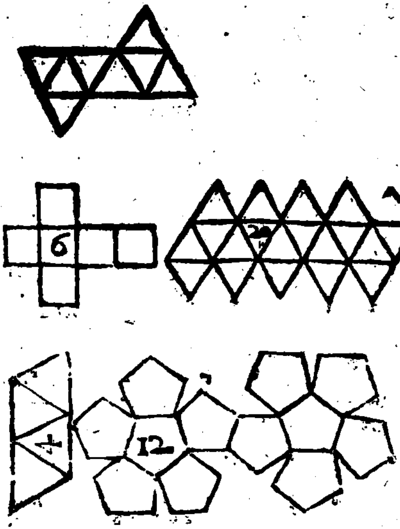
Soliditas dodecaedri, 2 | 78516.

Latus Icosaedri, 1 | 05146.

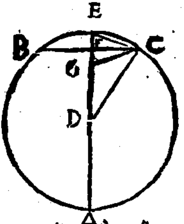
Superficies Icosaedri, 9 | 57454.

Soliditas Icosaedri, 2 | 53615.

Quod si ex charta conficiantur quinque figuræ æquilateræ & æquiangulæ similes his quæ sunt in subjecta figura, componentur quinque figuræ solidæ, si rite complicentur.



P R O P. I.



Quæ ex D centro circuli cujuspiam ABC in pentagoni eidem circulo inscripti latus BC ducitur perpendicularis DF, dimidia est utriusque lineæ simul, & lateris hexagoni DE, & lateris decagoni EC eidem circulo ABC inscripti.

a 4. 1. b 5. 1. c 32. 1. d hyp. & 33. 6. e 20. 13. f 7. ax. g 6. 1.

Sume FG = FE, & duc CG. a Estque CE = CG, ergo ang. CGE b = CEG b = ECD. ergo ang. ECG c = EDC d = $\frac{1}{2}$ ADC e = $\frac{1}{2}$ CED ($\frac{1}{2}$ ECD.) proinde ang. f GCD = ECG = EDC. g quare DG = GC (CE.) ergo DF = CE (DG) + EF = DE + CE. Q. E. D.

P R O P. II.

Si binæ rectæ lineæ AB, DE *extrema ac media ratione secentur* (AB. AG :: AG. GB. *&* DE. DH :: DH. HE;) *ipsæ similiter secabuntur, in easdem scilicet proportiones.* (AG. GB :: DH. HE.)

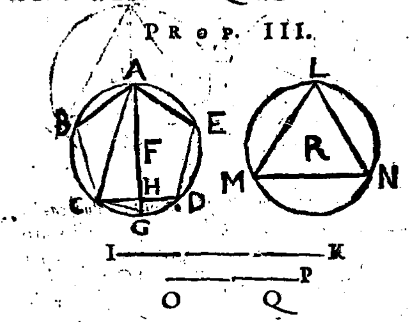
a 17. 6 b 8. 2. c 2. ax. 1. d 22. 5 & 22. 6.

Accipe BC = BG & EF = EH. Estque AB x BG a = AGq. quare ACq b = 4 ABG + AGq c = 5 AGq. Similiter erit DFq = 5 DHq. d ergo AC. AG :: DF. DH. componendo igitur AC + AG. AG :: DF + DH. DH.

DH.

DH. hoc est 2 AB. AG :: 2 DE. DH. *e* proinde AB. AG :: DE. DH. unde *f* dividendo AG. GB :: DH. HE. Q. E. D. e 22. 5. f 17. 5.

PROP. III.



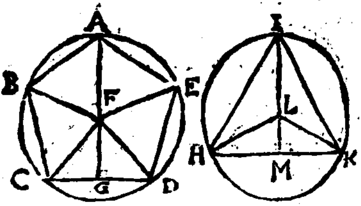
Idem circulus ABD *comprehendit & Dodecaedri pentagonum* ABCDE, *& Icosaedri triangulum* LMN, *eidem sphæræ inscriptorum.*

Duc diametrum AG, rectasque AC, CG. Sitque IK diameter sphæræ, *a* & IKq = 5 OPq. *b* fiatque OP. OQ :: OQ. QP. Quia ACq + CGq *c* = AGq *d* = 4 FGq; & ABq *e* = FGq = CGq. *f* erit ACq + ABq = 5 FGq. porro, quia CA. AB *g* :: AB. CA − AB; ac OP. OQ :: OQ. QP. *h* ideoque CA. OP :: AB. OQ. *k* erit 3 ACq (*l* IKq.) 5 OPq (*m* IKq) :: 3 ABq. 5 OQq. ergo 3 ABq = 5 OQq. Verum ob ML *n* latus pentagoni circulo inscripti, cujus radius OP, erunt 15 RMq *o* = 5 MLq *p* = 5 OPq + 5 OQq = * 3 ACq + 3 ABq *q* = 15 FGq. *r* ergo RM = FG. *s* proinde circ. ABD = circ. LMN. Q. E. D.

a *sch.* 47. 1.
b 30. 6.
c 47. 1.
d 4. 2.
e 10. 13.
f 1, & 3. *ax.*
g 8. 13.
h 2. 13. & 16. 5.
k 22 6 & 4. 5
l 15. 13.
m *constr.*
n *cor.* 16. 13.
o 12. 13.
p 10. 13.
q 15. 5.
* *Prius.*
r 1. *ax.* 1. & *sch.* 48 1.
s 1. *def.* 3.

PROP.

PROP. IV.



Si ex F *centro circuli pentagonum dodecaedri* ABCDE *circumscribentis ducatur perpendicularis* FG *ad pentagoni unum latus* CD; *erit quod sub dicto latere* CD, *& perpendiculari* FG *comprehenditur rectangulum trigesies sumptum, icosaedri superficiei æquale.* item,

Si ex centro L *circuli triangulum icosaedri* HIK *circumscribentis, perpendicularis* LM *ducatur ad trianguli unum latus* HK; *erit quod sub dicto latere* HK, *& perpendiculari* LM *comprehenditur rectangulum trigesies sumptum, icosaedri superficiei æquale.*

a 8. 1. Duc FA, FB, FC, FD, FE. a Erunt triangula CFD, DFE, EFA, AFB, BFC æqualia. atqui CDxFG b = 2 triang. CFD. ergo 30 CD x GF c = 60 CFD d = 12 pentag. ABCDE e = superf. dodecaedri. Q. E. D.

b 41. 1. c 15. 5. d 6. ax. e 17. 3. f 41. 1. g 15. 5. h 16. 13.

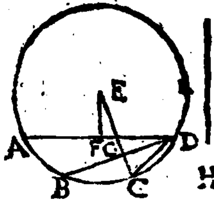
Duc LI, LH, LK. estque HKxLM f = 2 triang. LHK. ergo 30 HKxLM g = 60 HLK = 20 HIK h = superfic. icosaedri. Q. E. D.

Coroll.

k 15. 5. CDxFG. HKxLM k :: superfic. dodecaed. ad superf. icosaedri.

PROP.

Prop. V.



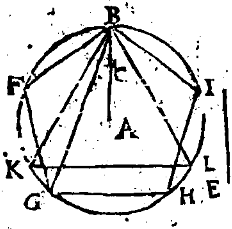
Superficies dodecaedri ad superficiem icosaedri in eadem sphæra descripti eandem proportionem habet, quam H *latus cubi ad* A D *latus icosaedri.*

Circulus A B C D [a] circumscribat tam dodecaedri pentagonum, quam icosaedri triangulum; quorum latera BD, AD; ad quæ demittantur ex E centro perpendiculares E F, E G C; & connectatur C D. [a 8. 14.]

Quoniam EC+CD. EC [b] :: EC. CD. erit E G ([c] $\frac{1}{2}$ E C + C D.) E F ([d] $\frac{1}{2}$ E C) [e] :: E F. EG − EF ($\frac{1}{2}$ CD.) atqui H. BD [f] :: BD. H − BD. [g] ergo [h] H. BD :: EG. EF. proinde H x EF = BD x EG. quum igitur H.AD [k] :: H x EF. AD x EF. erit H. AD :: BD x EG. AD x EF :: [l] superfic. dodecaedri ad superfic. icosaedri. Q. E. D. [b 9. 13.] [c 1. 14.] [d cor. 12. 13.] [e 15. 5.] [f cor. 17. 13.] [g 2. 14.] [h 1. 6.] [k 17. 5.] [l cor. 4. 14.]

Prop.

PROP. VI.



Si recta linea AB *secetur extrema ac media ratione; erit ut recta* BF *potens id, quod à tota* AB, *& id quod à majori segmento* AC, *ad rectam* E, *potentem id quod à tota* AB, *& id quod à minori segmento* BC; *ita latus cubi* BG *ad latus icosaedri* BK *eidem sphæræ cum cubo inscripti.*

Circulo, cujus semidiameter AB, inscribantur dodecaedri pentagonum BFGHI, & icosaedri triangulum BKL. *a* quare BG latus cubi erit ei- a cor. 17. 13.
dem sphæræ inscripti. igitur BKq *b* = 3 ABq; b 12. 13.
& Eq *c* = 3 ACq. ergo BKq. Eq *d* :: ABq. ACq c 4. 13.
e :: BGq. BFq. permutando igitur BGq. BKq :: d 15. 5.
BFq. Eq. *f* unde BG. BK :: BF. E. Q. E. D. e 22. 14. f 22. 6

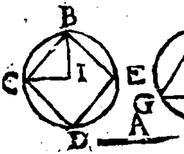
PROP. VII.

Dodecaedrum est ad Icosaedrum, ut cubi latus ad latus Icosaedri, in una eademque sphæra inscripti.

a 3. 14. Quoniam *a* idem circulus comprehendit & dodecaedri pentagonum & icosaedri triangulum,
b 47. 1. *b* erunt perpendiculares à centro sphæræ ad plana pentagoni & trianguli ductæ inter se æquales. itaque si dodecaedrum & icosaedrum intelligantur esse divisa in pyramides, ductis rectis à centro sphæræ ad omnes angulos, omnium pyramidum altitudines erunt inter se æquales.
c 5, & 6. 12. Cum igitur pyramides æque altæ *c* sint ut bases, & superficies dodecaedri sit æqualis 12 pentagonis, superficies vero icosaedri 20 triangulis; erit

erit dodecaedrum ad icosaedrum, ut superficies dodecaedri ad superficiem icosaedri, *d* hoc est, ut latus cubi ad latus icosaedri. d 5. 14.

PROP. VIII.



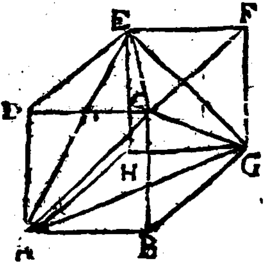
Idem circulus BCDE *comprehendit & cubi quadratum* BCDE *& octaedri triangulum* FGH, *ejusdem sphæræ.*

Sit A diameter sphæræ. Quoniam Aq *a* = 3 BCq *b* = 6 BIq; itemque Aq *c* = 2 GFq *d* = 6 KFq; erit BI = KF. *e* ergo circulus CBED = GFH. Q. E. D.

a 15. 13. b 47. 1. c 14. 13. d 12. 13. e 2. def. 3.

LIB. XV.

PROP. I.



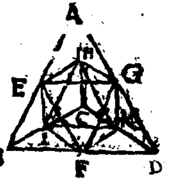
IN *dato cubo* ABGHDCFE *pyramidem* AGEC *describere.*

Ab angulo C duc diametros CA, CG, CE; Easque connecte diametris AG,GE,EA. Hæ omnes

a 47. 1. inter se *a* æquales sunt, utpote æqualium quadratorum diametri. ergo triangula CAG,CGE, CEA, EAG æquilatera sunt,ac æqualia: proinde AGEC est pyramis,quæ cubi angulis insistit,

b 31. *def.* 11. eique idcirco *b* inscribitur. Q. E. F.

PROP. II.

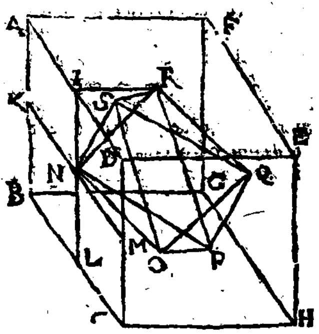


In data pyramide ABDC *octaedrum* EGKIFH *describere.*

a Bifeca latera pyramidis in punctis E, I, F, K, G, H; quæ connecte 12 rectis EF, FG, GE, &c. Hæ omnes b æquales sunt inter se. proinde 8 triangula EHI, IHK, &c. æquilatera sunt & æqualia, adeoque constituunt c octaedrum d in data pyramide descriptum. Q. E. F.

a 10. 1.
b 4. 1.
c 27. def. 11.
d 31. def. 11.

PROP. III.

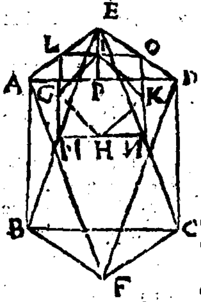


In dato cubo CHGBDEFA *octaedrum* NPQSOR *describere.*

Connecte quadratorum * centra N, P, Q, S, O, R, 12 rectis NP, PQ, QS, &c. quæ a æqualia sunt inter se, ideoque 8 triangula efficiunt æquilatera & æqualia. proinde b inscriptum est cubo Octaedrum NPQSOR. Q. E. F.

* 4. 4.
a 4. 1.
b 31. & 27. def. 11.

PROP. IV.



In dato octaedro ABCDEF *cubum inscribere.*

Latera pyramidis EABCD, cujus basis quadratum ABCD, bisecentur rectis LM, MN, NO, OL; quæ *a* æquales sunt & *b* parallelæ lateribus quadrati ABCD. *c* ergo quadrilaterum LMNO est quadratum.

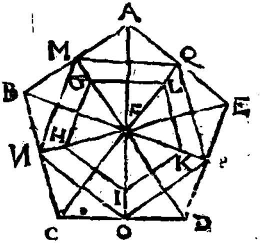
a 4. 1.
b 2. 6.
c 29 def. 1.

Eodem modo, si latera quadrati LMNO bisecentur in punctis G, H, K, I, & connectantur GH, HK, KI, IG, erit GHKI quadratum. Quod si eadem arte in reliquis 5 pyramidibus octaedri centra triangulorum rectis conjungantur, describentur quadrata similia & æqualia quadrato GHKI. quare sex hujusmodi quadrata cubum constituent, qui quidem intra octaedrum descriptus erit, *d* cum octo ejus anguli tangant octo octaedri bases in earum centris. Q. E. F.

d 31. def. 11.

PROP.

PROP. V.



In dato Icosaedro Dodecaedrum inscribere.

Sit A B C D E F pyramis Icosaedri, cujus basis pentagonum ABCDE; centra autem triangulorum G, H, I, *K*, L; quæ con- nectantur rectis GH, HI, IK, KL, LG. Erit GHIKL pentagonum dodecaedri inscribendi. *a* 5. 4.

Nam rectæ FM, FN, FO, FP, FQ, per centra triangulorum transeuntes, *a* bisecant bases. *b* ergo rectæ MN, NO, OP, PQ, QM æquales sunt inter se. quinetiam FM, FN, FO, FP, FQ *c* pares sunt. *d* ergo anguli MFN, NFO, OFP, PFQ, QFM æquantur. pentagonum igitur GHIKL æquiangulum est; *e* proinde & æquilaterum, cum FG, FH, FI, FK, FL *f* pares sint. Quod si eadem arte in reliquis undecim pyramidibus icosaedri, centra triangulorum rectis lineis connectantur, describentur pentagona æqualia & similia pentagono GHIKL. quamobrem 12 hujusmodi pentagona dodecaedrum

a cor. 3. 3. b 4. 1. c 4. 1. d 8. 1. e 4. 1. f 12. 13.

conſtituent; quod quidem in icoſaedro erit deſcriptum, cum viginti anguli dodecaedri in centris viginti baſium icoſaedri conſiſtant. Quapropter in dato icoſaedro dodecaedrum deſcripſimus. Q. E. F.

FINIS,

Annotationes in Elementa Euclidis nuper edita, in quibus obſcura illuſtrantur, errata emendantur, plurimaque quæ conducant ad Geometriæ rudimenta facilius percipienda adjiciuntur.

*pag.*13.*lin.*5. ſcribe, Rurſus ang. ACD e = e 5. 1.
ADC; & ang. BCD e = BDC, f ergo ang. ACD 19 ax.
⊏ BDC, id eſt ang. ADC ⊏ BDC. Q. F. N.

*p.*17.*l.*ult. ſcribe, conjunganturque FC, IC, & producatur ACG.

*p.*18. *l.*3. ſcribe, ſimili argumento ang. ICH = ABH. ergo totus ACD, f (BCG) g major eſt utroque CAB, & ABC. Q. E. D.

*p.*21. apponantur figuræ quæ deſunt.

*p.*40. *lin.*18. ſcribe, *Schol.*

Imo ſi fuerint duæ rectæ, ſecenturque ambæ in quotcunque partes, idem provenit ex ductu totius in totum, & partium in partes.

Nam ſit Z = A + B + C, & Y = D + E; quia
D Z a = DA + DB + DC, & E Z a = EA + EB a 1. 2.
+ EC, & YZ a = DZ + EZ, b erit ZY = DA b 2. ax.
+ DB + DC + EA + EB + EC. Q. E. D.

Hinc patet ratio ducendi rectas compoſitas in compoſitas. Nam omnia partium rectangula accipere oportet, & habetur rectangulum ex totis.

Sin linearum in ſe ducendarum ſignis + admiſceantur ſigna −, etiam ſignorum ratio habenda eſt. Quippe ex + in − provenit − ; at ex − in − provenit +. Nam ſit + A ducenda in B − C. & quoniam + A non affirmatur de toto B, ſed de ejus parte tantum, qua ſuperat C, debet AC manere negata. quare prodibit AB − AC. Vel ſic; quia B conſtat partibus C, & B − C, * erit AB * 1. 2.
= AC + A in B − C; aufer utrinque AC, erit AB − AC = A in B − C. Similiter ſi − A ducenda ſit in B − C, quoniam ex vi ſigni − non nega-

tur A de toto B, sed de ejus solummodo excessu supra C, debet AC manere affirmata. proveniet ergo $-AB+AC$. Vel sic; quia AB *$=AC+A$ in $B-C$; tolle utrinque omnia, erit $-AB=AC-A$ in $B+C$; adde AC utrinque, eritq; $-AB+AC=A$ in $B-C$.

Atque ex his rite perspectis, quæ subsequuntur 9. propositiones, aliæque ejusmodi innumeræ, ex linearum in se ductarum comparatione emergentes (quas apud Vietam, & alios Analystas innumerato habes) nullo negotio demonstrantur, rem plerumque quasi ad simplicem calculum exigendo.

* 19. ex. Porro,* liquet productum ex quapiam magnitudine in numeri cujuslibet partes æquari producto ex eadem in totum numerum. Ut $5A+7A=12A$. & $4A$ in $5A+4A$ in $7A=4A$ in $12A$. quare quæ in hoc loco de rectarum in se ductu dicta sunt, eadem de numerorum in se multiplicatione intelligi possunt. proinde etiam quæ in 9 sequentibus theorematis de lineis affirmantur, eadem valent de numeris accepta; quippe cum istæ omnes ab hac prima immediate dependeant, & deducantur.

p. 42. inter demonstr. & Schol. propositionis quintæ, scribe.

Hoc theorema paulo aliter effertur, & facilius demonstratur, sic; Rectangulum ex summa & differentia duarum rectarum A, E, *æquatur differentiæ ex ipsis.*

* sch. 1.2. Nam si $A+E$ ducatur in $A-E$, *provenit $Aq-AE+EA-Eq=Aq-Eq$. Q. E. D.

p. 44. post demonstrationem prop. 9. scribe,

Aliter effertur & facilius demonstratur, sic;

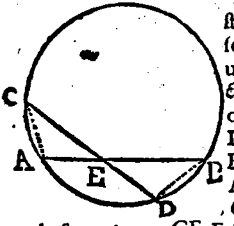
Aggregatum quadratorum ex summa, & differentia duarum rectarum A, E, *æquatur duplo quadratorum ex ipsis.*

a 4. 2. Nam Q: $A+E$ a$=Aq+Eq+2AE$. & Q: A

b sch. 7. 2. $-E$ b$=Aq+Eq-2AE$. Hæc collecta faciunt $2Aq+2Eq$. Q. E. D.

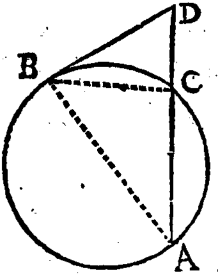
p. 67.

p.67. post demonstrationem prop.28 scribe; Quod si subtensa AC $\sqsubset$ vel $\sqsupset$ DF, erit simili modo a cus AC $\sqsubset$, vel $\sqsupset$ DF.



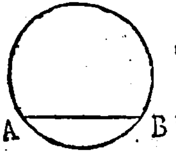
p.71. post demonstrationem prop. 35. scribe, Facilius sic, & universaliter; connecte AC & BD. atque *a* 15 1.
ob angulos *a* CEA, *b* 21. 3.
DEB, *b* ipsosque C, B (super eodem arcu AD) pares; trigona *c* cor.32.1.
CEA, BBD, *c* æqui- *d* 4 6
angula sunt. *d* ergo CE.EA :: EB. ED. *e* proinde *e* 16. 6.
CE x ED = EA x EB. Q. E. D.

Quæ ex 6.lib.citantur, tam hic quam in seq. ab hac minime pendent; quare iis uti licuit.



p. 71. Inter demonstr. & coroll. prop.36. scribe, Facilius ac universalius sic;

Duc AB, & EC. ac ob angulos A, *a* 32. 3.
DBC *a* pares, & D *b* 32. 1.
communem, trian- *c* 4 6.
gula BDC, ADB *d* 17 6.
b æquiangula sunt. *c* ergo AD. DB :: DB. CD. *d* quare AD x DC = DBq. Q. E. D.



p.76. ad def.7. 4. substitue figuram hanc.

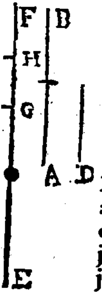
pag.82. post demonstrationem propos. 10. 4. scribe sic.

a 3 6.
b constr.
c hyp.
d 6. 1.
e 32. 1.
f 2 ax.
g 17. 6.

Hæc constructio Analytice indagatur sic; Factum sit; & angulum BDA bisecet recta DC. a ergo DA. DB :: CA. CB. item ob ang. CDA b $= \frac{1}{2}$ ADB c $=$ A, d est CA $=$ DC. ac ob ang. DCB e $=$ A $+$ CDA $=$ 2A c $=$ B, d erit DB $=$ DC. f ergo DB $=$ CA. proinde DA. (c BA.) CA :: CA. CB. g unde BA x CB $=$ CAq.

p. 98. scribe Prop. 8. 5. sic.

PROP. 8.



Inæqualium magnitudinum AB, AC, *major* AB *ad eandem* D *majorem habet rationem, quam minor* AC: *& eadem* D *ad minorem* AC *majorem rationem habet, quam ad majorem* AB.

Sume EF, EG, ipsarum AB, AC æquemultiplices, ita ut EH ipsius D multiplex, major sit quam EG, at minor quam EF. (Quod facile continget, si utraque EG, GF majores accipiantur ipsa D.) Liquet juxta 8 def. 5. fore $\frac{AB}{D} \sqsubset \frac{AC}{D}$; ac $\frac{D}{AB} \sqsupset \frac{D}{AC}$. Quæ E. D.

b hyp.
c 6 def. 5

p. 100 *lin.* ult. post B, D, F. scribe, Porro ob A.B b :: C.D b :: E.F, c si G $\sqsubset, =, \sqsupset$ *K*, erit similiter H $\sqsubset, =, \sqsupset$ L; & I $\sqsubset, =, \sqsupset$ M. ac proinde si G $\sqsubset, =, \sqsupset$ *K*, erit simili modo G $+$ H $+$ I $\sqsubset, =, \sqsupset$ K $+$ L $+$ M. c quare A.B :: A $+$ C $+$ E. B $+$ D $+$ F. Q. E. D.

pag. 102. circa 23 *lin.* post (æquatur) scribe, Ergo, quum AG. DH :: C. F :: GB. HE. erit, &c. ut sequitur ibi.

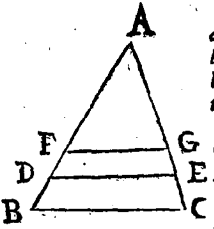
p. 104. *lin.* 1. post KO scribe, Itaque ablatis hinc inde communibus HL, KM, &c. ut ibi sequitur.

p. 111. *l*. 12. dele, Hujuſce demonſtratio, &c. & ſcribe, Intellige $\frac{G}{C} = DE$. [a] ergo $B \sqsubset G$. [b] ergo $\frac{A}{G} \sqsubset \frac{A}{B}$. Rurſus concipe $\frac{H}{G} = \frac{E}{F}$. [c] ergo $\frac{H}{G} \sqsupset \frac{A}{G}$. [a] quare $A \sqsubset H$. [b] proinde $\frac{A}{C} \sqsubset \frac{H}{C}$ [d] vel $\frac{D}{F}$. Q. E. D. a 10. 5. b 8. 5. c ſch. 13 5. d 13 5.

p. 114. circa 25. *lin*. dele, cum igitur, & ſcribe, Verum ſi HC, &c. ut ſequitur.

p. 116. *l*. 2. dele Imo ſi plures, &c. & ſcribe ſic.

Schol.



Imo ſi plures DE, FG, *ad unum latus* BC *parallelæ fuerint, erunt omnia laterum ſegmenta proportionalia.*

Nam DF. FA [a] :: EG. GA; & componendo, invertendoque FA. DA :: GA. EA; [a] ac DA. DB :: EA. EC. ergo ex æquo DF. DB :: EG. EC. Q. E. D. a 2. 6.

Coroll.

*S*i DF. DB :: EG. EC; [a] erunt BC, DB, FG parallelæ.

p. 119. Prop. 8. demonſtretur ſic.

Nam ob angulos BAC, ADB [a] rectos, [b] ideoque æquales, & B communem, trigona BAC, ADB [c] ſimilia ſunt. Simili diſcurſu, ſimilia ſunt triangula BAC, ADC. [d] proinde ADB, ADC ſimilia erunt. Q. E. D. a hyp. b 12. ax. c 32 & 4 6. d vid. 21. 6.

Coroll. &c. ut ſequitur.

pag. 121. *lin. antepen*. ſcribe, Vel ſic; Datæ ſint AB, BC; ex quibus fac angulum rectum ABC. duc AC, & huic normalem CD, cui occurrat AB protracta in D. [a] eſtque AB. BC :: BC. BD. a cor. 8. 6.

pag. 122. dele figuram iſtam furciferam.

ibid.

ibid. lin. 6. dele, vel ita; CD = CB. & quæ seq. cum sua figura.

pag. 123. post *lin.* 3. scribe, Vel (in eadem figura) sint AB, BF duæ datæ, b liquet esse AB. BF :: BF. BE.

p. 136. Propos. 31. a demonstretur sic.

a cor. 8. 6. b cor. 20. 6. c 24. 5. d sch. 14. 5.

Ab angulo recto BAC demitte perpendicularem AD. Quoniam DC. CA :: a CA. CB, b erit AL. BF :: DC. CB. Item ob DB. BA :: a BA. BC, b erit BG. BF :: DB. BC. c ergo AL + BG. BF :: DC + DB (BC.) BC. ergo AL + BG = BF. Q. E. D.

pag. 146. *lin.* penult. scribe, vel sic, sit $a = \frac{x}{2}$, & $b = \frac{y}{2}$. quare $2a = x$, & $2b = y$. ergo $2a + 2b = x + y$. ergo $a + b = \frac{x+y}{2}$.

p. 147. *lin.* 17. scribe, Vel sic, sit $a = \frac{2x}{3}$, & $b = \frac{2y}{3}$, & $x + y = g$. ob $3a = 2x$, & $3b = 2y$, est $3a + 3b = 2x + 2y = 2g$. ergo $a + b = \frac{2}{3}g = \frac{2}{3} : x + y$.

p. 149. *l.* 9. scribe, Vel sic; sit $a = \frac{b}{3}$, & $c = \frac{d}{3}$,

* 5. 15. vel $3a = b$, & $3c = d$, estque $\frac{c}{a}$ * $= \frac{3c}{3a} = \frac{d}{b}$.

ibid. lin. 27. dele, Applicare potes, &c. & scribe, Vel sic; sit $a = \frac{2b}{3}$, & $c = \frac{2d}{3}$. vel $3a = 2b$, & $3c = 2d$. Est $\frac{c}{a} = \frac{3c}{3a} = \frac{2d}{2b} = \frac{d}{b}$.

LEMMA.

AE,	BF,	CG,	DH,	*Si proportionales*
A,	B,	C,	D,	*numeri* A, B, C, D
E,	F,	G,	H.	*proportionales numeros* AE, BF, CG, DH

DH *metiantur per numeros* E, F, G, H, *erunt ei* [E, F, G, H] *proportionales.*

Nam ob AEDH a = BFCG, a & AD = BC, b erit $\frac{AEDH}{AD} = \frac{BFCG}{BC}$, c hoc est EH = FG. a 19. 7. b 1. ax. 7. c 9. ax. 7.

a ergo E. F :: G. H. Q. E. D.

Coroll.

Hinc $\frac{Bq}{Aq} = \frac{B}{A}$ in $\frac{B}{A}$. d Nam 1. B :: B. Bq. d & 1. A :: A. Aq. e ergo $\frac{1}{A}$. $\frac{B}{A}$ :: $\frac{B}{A}$. $\frac{Bq}{Aq}$. d ergo $\frac{Bq}{Aq} = \frac{B}{A} \times \frac{B}{A}$. *Similiter* $\frac{B}{Ac}$ in $\frac{Bq}{Ac} = \frac{BC}{Ace}$. & sic de reliquis. d 15. def. 7. e lem. praec.

PROP. 22.

Aq, B, C.
4, 8, 16.

Si tres numeri , Aq, B, C *deinceps sint proportionales, primus autem* Aq *sit quadratus; & tertius* C *quadratus erit.*

Nam ob AqC a = Bq, b erit C = $\frac{Bq}{Aq}$ c = Q. $\frac{B}{A}$. a 10 7. b 7. ax 7. c cor. lem. praec.

Liquet vero $\frac{B}{A}$ esse numerum, d ob $\frac{Bq}{Aq}$, vel C numerum. ergo si tres, &c. d hyp. & 14. 8.

PROP. 23.

Ac, B, C, D.
8, 12, 18, 27.

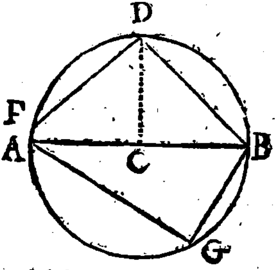
Si quatuor numeri Ac, B, C, D *deinceps sint proportionales, primus autem* Ac *sit cubus; & quartus* D *cubus erit.*

Nam quia AcD a = BC, b erit D = $\frac{BC}{Ac}$ c = $\frac{B}{Ac} \times$ C; hoc est (ob AcC = d Bq, & b proinde C = $\frac{Bq}{Ac}$) D = $\frac{B}{Ac} \times \frac{Bq}{Ac}$ c = $\frac{BC}{Acc}$ c = C: $\frac{B}{Ac}$. a 19. 7. b 7. ax. 7. c cor. lem. praec. d 20. 7.

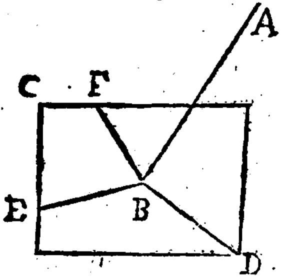
e liquet vero ipsum $\frac{B}{Ac}$ esse numerum, quia $\frac{BC}{Acc}$, vel D numerus ponitur; ergo si quatuor numeri, &c. e 15. 8

p. 192.

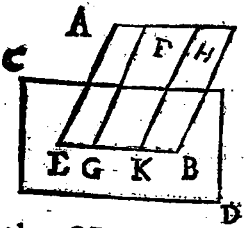
p.192. ſubſtitue hanc figuram.



p. 263. ad def. 3. ſcribe ſic.

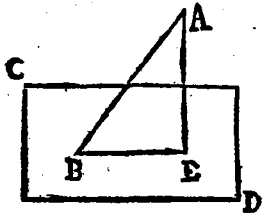


3. Linea recta AB eſt ad planum C D recta, cum ad rectas omnes lineas BD,BE, B F, à quibus illa tangitur, quæque in propoſito ſunt plano, rectos efficit angulos ABD, ABE, ABF.

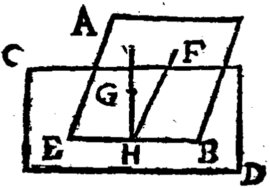


4. Planum AB ad planum C D rectum eſt, cum rectæ lineæ FG, HK, quæ communi planorum ſectioni E B ad rectos angulos in uno plano A B ducuntur, alteri plano C D ad rectos ſunt angulos.

5. Rectæ



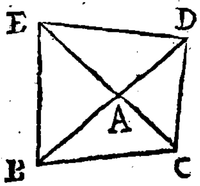
5. Rectæ lineæ A B ad planum C D inclinatio est, cum à sublimi termino A rectæ alius lineæ AB ad planum C D deducta fuerit perpendicularis A E; atque à puncto E, quod perpendicularis AE in ipso plano CD fecerit, ad propositæ illius lineæ extremum B, quod in eodem est plano, altera recta linea EB fuerit adjuncta: est, inquam, angulus acutus ABE insistente linea AB, & adjuncta EB comprehensus.



6. Plani AB ad planum CD inclinatio, est angulus acutus FHG rectis lineis FH, GH contentus, quæ in utroque planorum AB, CD ad idem communis sectionis BE punctum H ductæ, rectos cum sectione BE efficiunt angulos FHB, GHB.

pag. 275. Propositio 21 scribatur sic.

PROP. 21.



Omnis solidus angulus A sub minoribus quam quatuor rectis angulis planis continetur.

Latera enim solidi anguli A secans planum utcunque faciat figuram multilateram BCDE, & totidem triangula ABC, ACD, ADE, AEB. Omnes angulos polygoni voco X; & summam angulorum ad trigonorum bases voco Y. quare X + 4 Rect. a $=$ Y + A. Quia vero (ex angulis ad B) b est ang. ABE + ABC $\sqsubset$ CBE; idemque verum sit de angulis ad C, ad D, ad E. c liquet fore Y $\sqsubset$ X. proinde erit A $\sqsupset$ 4 *Rect.* Q. E. D.

a 32. 1. & sch 32. 1.
b 20. 11.
c 5. ax. 1.

p. 277. *lin.* antepen. dele Brevitatis causa ass. &c. & scribe sic; Assumptum est fore AD $\sqsubset$ HL. Hoc autem constat. Nam si AD $=$ vel $\sqsupset$ HL, erit ang. A a $=$, b vel $\sqsubset$ HLI. Eodem modo erit B $=$, vel $\sqsubset$ HLK, & C $=$, vel $\sqsubset$ KLI. quare A + B + C * quatuor rectos aut exæquabunt, aut excedent, contra hypoth. quin potius sit AD $\sqsubset$ HL. Q. E. D.

a constr. & 8. 1.
b 21. 1.
* 4 cor. 15. 1.

FINIS.

EUCLIDIS DATA

succinctè demonstrata;

Una cum Emendationibus quibusdam & Additionibus ad ELEMENTA.

EUCLIDIS

nuper edita.

Opera

Mri. Is. BARROW, *Cantabrigiensis;*
Coll, *Trin.* Soc.

LONDINI
Excudebat R. Daniel, 165[illegible]

Ornatissimo viro

D. IACOBO STOCK,

amico suo & patrono singulari.

NEc publica, nec tui nominis luce dignum censeo hunc paucorum dierum partum pusillum & prœmaturum: Qui quidem quod se mundo, quodque Tibi, spectandum obtulerit, duplici nomine arrogantiæ speciem incurrit. Sed utrinque parata est excusatio qualiscunque. Nam amico obtemperatum oportuit jubenti mitterem hunc libellum Euclideis (quæ cognatione proxima attingit) Elementis subjungendum. In eum quicquid est in publicum aut peccati aut meriti protinus reficio, facti cujus author fuit, rationem redditurum. In Te autem delictum quod maxime aggravat, idem potenter extenuat, Tibi tantum debere. Nam cum iis, qui Diis ipsis sacrificia, ac modica magnis Regibus donaria offerre non dubitarunt, satius esse credo, etiam pro immensis beneficiis parum, quam nihil rependere. Sufficiat igitur regessisse, me Tibi multis magnisque nominibus obstrictum fore; vices, quas potuero maximas, referre debere; ultra vota & grates nihil posse; illa privatim, has publice persolutas præcellere; quibus agendis, quam jamdiu spe & studio aucupor, occasionem nondum comparere; præstare hanc

 oblatam

oblatam prahendere, quamvis exilem, quam elapsam nequicquam pœnitentia prosequi. Esto igitur hæc oblatio pignus quoddam & præludium futuræ amplioris, in qua meritorum in me Tuorum historia uberior ac distinctior commemoranda occurret. Quæ simpliciter agnoscere, non aut fuse describere, aut digne prædicare, præsentis est instituti. Ac revera jam brevis sum ἑκὼν ἀέκοντί γε θυμῷ, necessitate potius coactus, quam inductus consilio. Nam me vela ventis turgentia alio avocant; ac vereor ne hæc pene currenti calamo exequentem, quæ hæc ad te perferet, amica manus, importuna patientia præstoletur. Quid superest igitur, nisi ut te domi studiis ac rebus honestis animum intendentem salutari præsentia tuetur, eum exorem venerandi ac ἀῤῥήτου nominis; quem tantæ beneficentiæ benignum remuneratorem jugibus votis exopto; idemque me extemplo super Tyrrhenos, Ionios, Ægeosque fluctus longinquam profectionem suscepturum comitetur. Obtestor autem, ne tenuis opellæ patrocinium respuas, quod ultro impertire dignatus es

Tibi devinctissimo,

& obsequentissimo,

I. B.

Ev.

EVCLIDIS Data.

Definitiones.

I. Data magnitudine dicuntur ſpatia, lineæ, anguli, quibus æqualia poſſumus invenire.

II. Ratio dari dicitur, cui poſſumus eandem invenire.

III. Rectilineæ figuræ ſpecie dari dicuntur, quarum & ſinguli anguli dati ſunt, & laterum rationes ad invicem datæ ſunt.

Hinc, datæ ſunt ſpecie figuræ, quibus ſimiles inveniri poſſunt.

IV. Poſitione dari dicuntur puncta, lineæ, anguliqu e, quæ eundem ſitum ſemper obtinent.

V. Circulus magnitudine dari dicitur, cujus ea quæ ex centro datur magnitudine.

VI. Poſitione & magnitudine dari dicitur circulus, cujus datur centrum poſitione, & ea quæ ex centro magnitudine.

VII. Circuli ſegmenta magnitudine dari dicuntur, in quibus dati ſunt magnitudine anguli & ſegmentorum baſes.

VIII. Poſitione & magnitudine dari dicuntur circuli ſegmenta, in quibus anguli magnitudine dati ſunt, & ſegmentorum baſes poſitione & magnitudine.

IX. Magnitudo magnitudine major eſt data, quando ablata data, reliqua eidem æqualis eſt.

X. Magnitudo magnitudine minor eſt data, quando adjuncta data, tota eidem æqualis eſt.

Ut ſi A data ſit, erit A + B ⊏ B data. At B ⊐ A + B data.

XI. Magnitudo magnitudine major eſt data quam in ratione, quando ablata data; reliqua ad eandem habet rationem datam.

XII. Magnitudo magnitudine minor est data quam in ratione, quando adjuncta data tota ad eandem rationem habet datam.

Ut si A data sit, & B detur, erit $\frac{A+B \sqsubset C}{G}$, data q. in r. sin $\frac{A+B}{C}$ detur, erit $B \sqsupset C$ data q. in r.

PROP. 1.

A. B. *Datarum magnitudinum A, B,*
a. b. *ad invicem datur ratio.*

* hyp. a 1.def. b sch.7.5. c 1.def. Nam quia A * datur, *a* inveniri potest aliqua a = A. Eodem jure sume b = B. *b* estque a. b :: A. B. *c* quare ratio $\frac{A}{B}$ data est. Q. E. D.

PROP. 2.

A. B. *Si data magnitudo A ad aliam*
a. b. *aliquam B habeat rationem datam, datur etiam hæc alia magnitudine.*

* hyp. a 1.def.d. b 2.def.d. c 9.5. Nam ob A * datam, *a* sume a = A; ac ob $\frac{A}{B}$ * datam, *b* sit $\frac{a}{b} = \frac{A}{B}$. cergo b = B. *a* quare B datur. Q.E.D.

PROP. 3.

A. B. *Si quotlibet datæ magnitudines*
a. b. *A, B componantur, etiam ea A+B quæ ex his componitur, data erit.*

a 1.def. b 2.ax.1. Nam *a* cape a = A, & b = B; *b* estque a + b = A+B. *a* quare A+B datur. Q. E. D

PROP. 4.

A. B. *Si à data magnitudine A auferatur*
a. b. *data magnitudo B, etiam reliqua A−B dabitur.*

a 1.def.d. b 3.ax.1. *a* Sint enim a = A, & b = B. ergo A−B = a−b. *a* proinde A−B datur. Q. E. D.

PROP.

PROP. 5.

A. B. C. D. *Si magnitudo A ad sui-ipsius aliquam partem B habeat rationem datam, etiam ad reliquam A − B habebit rationem datam.*

Nam, quia $\frac{A}{B}$ a data est, b sit A. B :: C. D. c ergo A. A − B :: C. C − D. b proinde $\frac{A}{A-B}$ datur. Q. E. D.

a hyp.
b 2. def. d.
c cor. 9. 5.

PROP. 6.

A. B. C. D. *Si componantur duæ magnitudines A, B, habentes ad invicem rationem datam, etiam quæ ex his componitur magnitudo A + B, habebit ad utramque A & B rationem datam.*

Nam a sit A. B :: C. D. b ergo A + B. B :: C + D. D. c quare $\frac{A+B}{B}$ datur. Similiter $\frac{B+A}{A}$ datur. Q. E. D.

a 2. def. d.
b 18. 5.
c 2. def. d.

PROP. 7.

A. B. *Si data magnitudo A + B data ratione secetur, utrumque segmentorum A, & B datum est.*

Nam ob $\frac{A}{B}$ * datam, a erit $\frac{A+B}{A}$ data. b ergo A datur. Eodem modo B datur. Q. E. D.

* hyp.
a 6 dat.
b 2. dat.

PROP. 8.

A. C. B. D. E. F. *Quæ A, B ad idem C rationem habent datam, habebunt ad invicem rationem datam.*

Nam a sit A. C :: D. E. a & C. B :: E. F. quare ex æquali A. B :: D. F. a ergo $\frac{A}{B}$ datur. Q. E. D.

a 1. def. d.

Coroll.

Rationes ex datis rationibus compositæ, datæ sunt. Ut $\frac{A}{B}$ fit ex $\frac{A}{C}$, & $\frac{C}{B}$ datis.

Prop. 9.

A. B. C. *Si duæ, pluresve magnitudines*
D. E. F. A, B, C *ad invicem habeant rationem datam, habeant autem illæ magnitudines* A, B, C *ad alias quasdam* D, E, F *rationes datas, etsi non easdem; illæ aliæ magnitudines* D, E, F *etiam ad invicem habent rationes datas.*

a [illegible] def. 5. b [illegible] c [illegible] Nam ratio $\frac{D}{E}$ a fit ex b datis $\frac{D}{A}$, $\frac{A}{B}$, $\frac{B}{E}$; c ergo $\frac{D}{F}$ datur. Eadem de causa datur $\frac{E}{F}$. Q. E. D.

Prop. 10.

A. B. C. *Si magnitudo magnitudine major fuerit data, quam in ratione; & simul utraque illa eadem major erit data quam in ratione. Sin autem simul utraq; magnitudo eadem magnitudine major fuerit data, quam in ratione; & reliqua illa eadem major erit data quam in ratione; aut reliqua data est cum consequente, ad quam habet altera magnitudo rationem datam.*

a 8. dat. b 11. def. 4. 1. Sint A, & $\frac{B}{C}$ datæ. a erit $\frac{B+C}{C}$ data. b ergo $A+B+C \sqsubset C$ data q. in r. Q. E. D.

c [illegible] 5. 2. Sint A, & $\frac{B+C}{C}$ datæ: c ergo $\frac{B}{C}$ datur. proinde $A+B \sqsubset C$ data q. in r. Q. E. D.

d 5. 4. 3. Sint $A+B$; & $\frac{C}{B+C}$ datæ. d Liquet B dari. $\frac{B}{B+C}$ Q. E. D.

Prop. 11.

A. B. C. *Si magnitudo magnitudine major sit data quam in ratione, eadem simul utraque major erit data quam in ratione. Et si eadem simul utraque major sit data quam in ratione, eadem reliqua magnitudine major erit data quam in ratione.*

1. A

1. A, & $\frac{B}{C}$ dantur. [a] ergo $\frac{B}{B+C}$ datur. proinde [b] $A + B \sqsubset B + C$ data q. in r. Q. E. D.

2. A, & $\frac{B}{B+C}$ dantur. [c] ergo $\frac{B}{C}$ datur, proinde [b] $A + B \sqsubset C$ data q. in r. Q. E. D.

a 6.dat.
b 11.def.d.
c 5.dat.

PROP. 12.

A. B. C. *Si fuerint tres magnitudines* A, B, C, *& prima cum secunda* (A + B) *data sit, secunda quoque cum tertia* (B + C) *data sit; aut prima* A *tertiæ* C *æqualis est, aut altera altera major data.*

Nam si A + B, & B + C pares sint, [a] liquet A & C æquari; sin istæ impares fuerint, [b] liquet excessum A — C, vel C — A dari. Q. E. D.

a 4 ax 1.
b 4.dat.

PROP. 13.

D, A + B, C. E. *Si fuerint tres magnitudines* D, A + B, C, *& earum prima* D *ad secundam* A + B *habeat rationem datam; secunda autem* A + B *tertia* C *major sit data quam in ratione; prima quoque* D *major erit tertia* C *data quam in ratione.*

Sint A, & $\frac{B}{C}$, ac $\frac{D}{A+B}$ datæ; [a] sitque A + B. D :: A. E [b] :: B. D − E. ergo [c] E, [d] & $\frac{B}{D-E}$ & (ob $\frac{B}{C}$ datam) [e] $\frac{C}{D-E}$ dantur. [f] quare D (E + D − E) $\sqsubset$ C data q. in r. Q. E. D.

a 2.def.d.
b 19.5.
c 2.dat.
d 2.def.d.
e 8.dat.
f 11.def.d.

PROP. 14.

A. C. B. D. E. *Si duæ magnitudines* A *&* C *ad invicem habeant rationem datam, utrique autem illarum adjiciatur data magnitudo* B *&* D; *totæ* A + B, C + D, *aut habent rationem datam, aut altera* A + B *altera* C + D *major erit data quam in ratione.*

Nam

a 12. 5. b hyp. c 2. def. d. Nam ſi A. C :: B. D ^{a} :: A + B. C + D ob $\frac{A}{C}$ ^{b} datam, ^{c} liquet $\frac{A+B}{C+D}$ dari.

d 2. def. d. e 2. dat. f 4. dat. g 11. def. d. Saltem ^{d} ſit A. C :: E. D. ^{a} :: A + E. C + D. Ergo ^{e} $\frac{A+E}{C+D}$ ac ^{e} E, ^{f} ideoque B ∸ E dantur. ^{g} proinde A + B (A + E : + B ∸ E) ⊏ C + D data q. in r. Q. E. D.

P R O P. 15.

A.	C.
B.	D.
E.	

Si duæ magnitudines A & C habeant ad invicem rationem datam, & ab utraque harum auferatur data magnitudo B & D; reliquæ magnitudines A − B, C − D ad invicem habebunt aut rationem datam, aut altera A − B, altera C − D major erit data quam in ratione.

a 19. 5. b hyp. c 2 def. d. d 2. def. 2. e 2. dat. f 4. dat. g 11. def. d. ^{b} Nam ſi A. C :: B. D ^{a} :: A − B. C − D; ob $\frac{A}{C}$ datam, ^{c} liquet $\frac{A-B}{A-C}$ dari.

Saltem ^{d} ſit A. C :: E. D ^{a} :: A − E. C − D. Ergo ^{e} $\frac{A-E}{C-D}$, & ^{e} E, ac ^{f} ideo E − B dantur. ^{g} proinde A − B (A − E : + E − B) ⊏ C − D data q. in r. Q. E. D.

P R O P. 16.

B,	C.
A.	D.
E.	

Si duæ magnitudines B, C habeant rationem datam, & ab una quidem illarum C auferatur data magnitudo D, alteri autem B adjiciatur data magnitudo A; tota A + B reſidua C − D major erit data quam in ratione.

a 2. def. d. b 19. 5. c 2. def. d. d 2. dat. e 3. dat. f 11. def. d. Sit enim C. B ^{a} :: D. E ^{b} :: C − D. B − E. ergo ^{c} $\frac{C-D}{B-E}$ & ^{d} E, ac ^{e} ideo E + A dantur. ^{f} proinde B + A (E + A : + B ∸ E) ⊏ C − D data q. in r. Q. E. D.

PROB

PROP. 17.

A+B. D+E. *Si fuerint tres magnitudi-*
C. *nes* A+B, C, D+E; *&*
prima quidem A + B *secun-*
da C *major sit data quam in ratione, tertia quoque* D+E *eadem secunda* C *major sit data quam in ratione; prima* A+B *ad tertiam* D+E *aut rationem habebit datam, aut altera altera major erit data quam in ratione.*

Nam ob A, D, & B E *a* datas, *b* erit B data. a hyp. b 8. dat.
C, C E
ergo per 14. hujus.

PROP. 18.

A+C. E. G. *Si fuerint tres magni-*
B+D. F. H. *tudines, atque ex his una*
utraque reliquarum major
sit data quam in ratione; reliquæ duæ aut datam rationem habebunt ad invicem, aut altera altera major erit data quam in ratione.

Datæ sint A, B, C D ac sit A+C=B+D.
E, F;

Sitque C.E *a* :: A.G *b* :: C+A. E+G. itemque a 1. def. d.
D. F *a* :: B. H *b* :: D + B. F + H. *c* ergo b 12. 5.
C+A *d* hoc est B+D, *e* & B + D, ac *e* idcirco c 1. def. d.
E+G, E+G, F+H d 7. 5.
E+G quin & G ac H *f* dantur. ergo per 15. e 8. 5.
F+H. (hujus. f 2 dat.

PROP. 19.

A+B. E. *Si fuerint tres magnitudines, &*
C+D. F. *prima quidem magnitudo secunda*
magnitudine major sit data quam in ratione, sit quoque secunda major tertia data quam in ratione; prima magnitudo tertia magnitudine major erit data quam in ratione.

Sint A, C, & C + D, D datæ; dico A + B
B E
⊏ E data q. in r.

Nam

a 2. def. d. b 19. 5. c 2. def. d. d 2. dat. e 3. dat. f 8. dat. g 11. def. d.

Nam sit C+D.B *a* :: C.F *b* :: D. B − F. ergo *c* $\frac{C}{F}$ & *d* F, ac *e* ideo F + A, & *c* $\frac{D}{B-F}$ *f* ideoque $\frac{E}{B-F}$ dantur. *g* proinde A+B (F+A : +B − F) ⊏ E data q. in r. Q. E. D.

PROP. 20.

A. C. E.
B. D.

Si datæ fuerint duæ magnitudines A, C; & auferantur ab ipsis magnitudines B, D habentes ad invicem rationem datam; residuæ magnitudines A − B, C − D aut habebunt ad invicem rationem datam, aut altera A − B altera C − D major erit data quam in ratione.

a 19. 5. b 2. def. d.

Nam si A.C :: B. D *a* :: A − B. C − D; *b* liquet $\frac{A-B}{C-D}$ dari.

c 2. dat. d 4. dat. e 11. def. d.

Saltem sit D. B *b* :: C. E *a* :: C − D. E − B. ergo *b* $\frac{C}{E}$ & *c* E, ac *d* propterea A − E, *b* itemque $\frac{C-D}{E-B}$ datæ sunt. *e* ergo A − B (A − E : + E − B) ⊏ C − D data q. in r. Q. E. D.

PROP. 21.

A. C. E.
B. D.

Si datæ fuerint duæ magnitudines A, C; & adjiciantur ipsis aliæ magnitudines B, D habentes ad invicem rationem datam; totæ A + B, C + D aut habebunt ad invicem rationem datam, aut altera A + B altera C + D major erit data quam in ratione.

a 12. 5. b 2. def. d.

Nam si B. D :: A. C *a* :: A + B. C + D, *b* liquet $\frac{A+B}{C+D}$ dari.

c 2. dat. d 4. dat.

Saltem sit B.D *b* :: E.C *a* :: B + E. D + C. ergo *c* E, *d* ideoque A − E, & *b* $\frac{B+E}{D+C}$ dantur.

e ergo

e ergo A+B (B+E :: +A −E) ⊏ C+D da- e 11. def.
ta q. in r. Q. E. D.

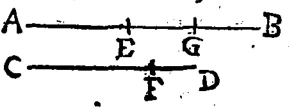
PROP. 22.

A. C. B.

Si duæ magnitudines A, B *ad aliam aliquam magnitudinem* C *habeant rationem datam, & simul utraque* A + B *ad eandem* C *habebit rationem datam.*

Nam ob $\frac{A}{C}, \frac{B}{C}$ a datas, b erit $\frac{A}{B}$ data. c quare $\frac{A+B}{B}$ b ideoque $\frac{A+B}{C}$ data est. Q. E. D. a hyp. b 8 d. c 6 d.

PROP. 23.



a *Si totum* AB *ad totum* CD *habeat rationem datam, habeant autem & partes* AE, EB *ad partes* CF, FD *rationes datas (etsi non easdem;) habebunt omnia ad omnia rationes datas.*

Nam sit AE. CF a :: AG. CD b :: GE. FD. a def. d.
a ergo $\frac{GE}{FD}$ datur. quare (ob $\frac{EB}{FD}$ c datam) d erit b 19. 5. c hyp.
$\frac{GE}{EB}$ ac e ideo $\frac{EB}{GB}$ data. ergo quum c $\frac{AB}{CD}$, & d 8. dat. e 5. dat.
a $\frac{AG}{CD}$, d ideoque $\frac{AB}{AG}$, ac proinde e $\frac{AB}{GB}$, dentur,
d erit $\frac{EB}{AB}$, data. Quare e $\frac{AB}{AE}$, & d $\frac{AE}{EB}$, & e $\frac{EB}{CF}$,
dantur. Q.E.D.

PROP. 24.

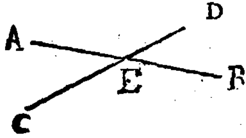
Si tres rectæ lineæ, A, B, C, *proportionales fuerint; prima autem* A *ad tertiam* C *habeat rationem datam; & ad secundam* B *habebit rationem datam.*

Nam

a cor. 20. 6. b 2. def. d. c 2. d.

Nam A. C a :: Aq. Bq. b ergo $\frac{Aq}{Bq}$ data est. proinde $\frac{A}{B}$ c datur. Q. E. D.

PROP. 25.



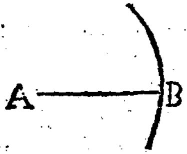
Si duæ rectæ lineæ, AB, CD positione datæ se mutuo secuerint, punctum E, in quo se invicem secant, positione datum est.

a 4. def. d. a Nam hæ lineæ alibi quam in E, neutrius situ mutuo, sese intersecare nequeunt.

Schol.

a Idem patet de quibuscunque lineis positione datis, seque in unico puncto intersecantibus: ut de circuli arcu, & recta, &c.

PROP. 26.

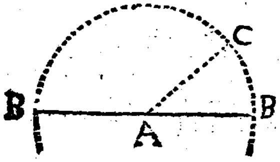


Si rectæ lineæ AB extremitates A, B, positione datæ sint, recta AB positione & magnitudine data est.

a 14 ex. Positione quidem, a quia inter eosdem terminos unica recta duci potest: &

b 2. def. d. magnitudine, b quia si centro A per B ducatur circulus, hujus omnes radii ipsi AB æquantur.

PROP. 27.



Si recta linea AB positione & magnitudine datæ, data fuerit una extremitas A; & altera extremitas B data erit.

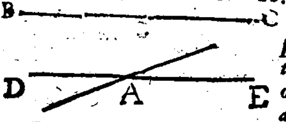
Nam

Nam si centro A, spatio AC *a* = AB *b* ducatur circulus, cui data recta *c* occurrat in B, *d* erit extremitas B data. a 1. def. d. b 3 post. c 2. post. d cor, 25.

Schol.

Vides partes puncti B determinandas esse.

PROP. 28.



Si per datum punctum A contra datam positione rectam B C agatur recta linea DE, acta recta DE positione data est,

Nam *a* dic alteram per A ad BC fore parallelam. Hæc idcirco ad DE *b* parallela erit. *c* Quod repugnat. a 4. def. d. b 30. 1. c 34. def. 1.

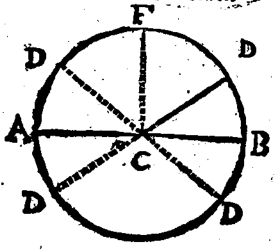
Nota, Vocabulum *contra* in hoc libro parallelismum significare.

PROP, 29.

Si ad positione datam rectam AB, datumque in ea punctum C, agatur recta linea C D, quæ faciat angulum DCB *datum; acta recta C D positione data erit.*

a Nam quævis alia CE angulum *b* efficiet majorem, vel minorum dato BCD. a 4. def. d. b 9. ax. 1.

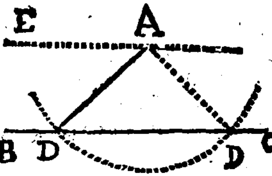
Schol.



Determinari debet situs anguli dati tam respectuperpendicularis C F, quam ipsius AB, ut cernis in apposita figura.

PROP.

PROP. 30.



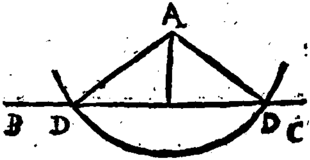
Si à dato puncto A in datam positione rectam B C agatur recta linea A D a que faciat angulum ADC datum, recta linea A D positione data est.

a 28. dat. b 1. def. d. c 29 dat. Nam per A duc A E ad BC parallelam. [a] Hęc positione datur. Item ang. DAE par dato alterno ADC [b] datus est. [c] ergo recta AD positione data est. Q. E. D.

Schol.

Hinc praxim discimus à dato puncto ducendi rectam, quæ cum data positione recta datum angulum efficit.

PROP. 31.



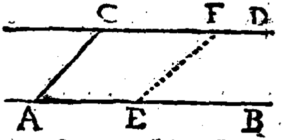
Si à dato puncto A in datam positione rectam BC data magnitudine recta A D ducatur, positione quoque data erit.

Nam puncta D, per quæ transit circulus centro A, a spatio A D descriptus, [b] data sunt. [c] ergo A D positione data est. Q. E. D.

a 1. def. d. b sch. 25. d. c 26. d.

PROP

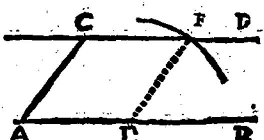
PROP. 32.



Si in datas positione parallelas rectas AB, CD *agatur recta linea* AC, *quæ faciat angulos datos* BAC, ACD, *acta recta* AC *magnitudine data est.*

Nam ad E (quodvis punctum in AB) fac ang. BEF = *a* BAC. liquet rectas EF, AC *b* parallelas, & *c* pares fore. *d* quare AC data est. Q. E. D. a 1. def. d. b 29. 1. c 34. 1. d 2. def. d.

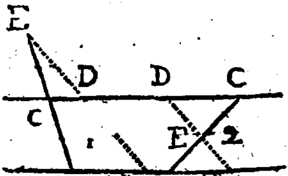
PROP. 33.



Si in datas positione parallelas rectas AB, CD *agatur magnitudine data recta* AC, *faciet angulos* BAC, ACD *datos.*

Nam ex quovis puncto E in AB, spatio EF *a* = AC describe circulum occurrentem rectæ CD in F. *b* Liquet EF, & AC parallelas esse posse. *c* ergo. a 1. def. d. b 34. 1. c 29. 1.

PROP. 34.



Si in datas positione parallelas rectas AB, CD *à dato puncto* E *agatur recta linea* ECA, *secabitur data ratione.*

a 2. 6. Nam ab E duc rectam E B utcunque parallelis occurrentem in D, & B. *a* liquet esse EC. CA

b [illegible] :: ED. DB. *b* quare EC/CA datur. Q. E. D.

PROP. 35.

Si à dato puncto E *in datam positione rectam* AB *agatur recta linea* EA, *seceturque data ratione; agatur autem per punctum sectionis* C *contra datam positione rectam* A B *recta linea* C D; *acta linea* CD *positione data est.*

Recta enim E B ducta ab E utcunque in A B,

a 10. 6. *a* secetur sic ut ED. DB :: EC. CA. ob punctum

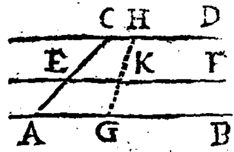
b 28. *dat.* D datum, *b* erit CD positione data. Q. E. D.

PROP. 36.

Si à dato puncto E *in datam positione rectam lineam* AB *agatur recta linea* EA; *adjiciatur autem ipsi aliqua recta* EC, *quæ ad illam* (EA) *habeat rationem datam; per extremitatem autem* C *adjectæ lineæ* EC *agatur contra datam positione rectam* AB *recta linea* CD; *acta linea* CD *positione data est.*

Demonstratio parum differt à præcedenti. *Vide fig.* 2.

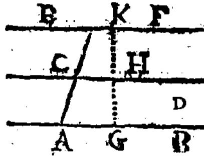
PROP. 37.



Si in datas positione parallelas rectas AB, CD, *agatur recta linea* AC, *& secetur ratione data; agatur autem per sectionis punctum* E *contra datas positione rectas* AB, CD *linea recta* EF; *acta recta* EF *positione data est.*

Nam duc rectam GH utcunque occurrentem parallelis. Hæc [a] secta sit in K ita ut GK. KH :: AE. EC. [b] Punctum K parallelæ (EF) situm determinat. Q. E. F. [a] 2. def. dat. [b] 28. dat. & sch. 2. h.

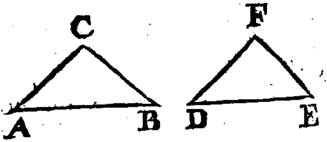
PROP. 38.



Si in datas positione rectas parallelas AB, CD *agatur recta linea* AC; *adjiciatur autem ipsi quædam recta* CE, *quæ ad actam* AE *habeat rationem datam; per extremitatem autem* E *adjecta* CE *agatur contra datas positione parallelas* AB, CD *recta linea* EF; *acta recta linea* EF *est data positione.*

Demonstratio persimilis est præcedenti. Cerne & compara figuras.

PROP. 74.



Si trianguli ABC *singula latera* AB,BC,AC *magnitudine data sint, triangulum* ABC *specie datum est.*

a 22. 1. Nam *a* fac triang. D E F ipsi A B C æquilaterum. Hoc eidem *b* æquiangulum erit. *c* ergo ABC specie datum est. Q. E. D.

b 5. 6.

c 3. def. d.

PROP. 40.

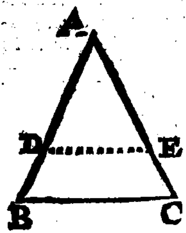
Si trianguli A B C *singuli anguli*, A, B, C *magnitudine dati sint, triangulum* ABC *specie datum est.*

a 23. 1. Nam ad quamvis DE *a* fac triang. DEF ipsi ABC æquiangulum. *b* Hoc eidem simile erit. *c* proinde trigonum A B C specie datum est. Q. E. D.

b 4. 6.

c 3. def. d.

PROP. 41.



Si triangulum A B C *unum angulum* A *datum habeat; circa datum autem angulum* A *duo latera* AB, AC *ad invicem habeant rationem datam; triangulum* A B C *specie datum est.*

Nam in uno latere dati anguli sume quampiam AD; & *a* fit A B. A C :: AD.AE.& duc DE. *b* Liquet trigonum A D E ipsi A B C simile fore. *c* Quare ABC specie datum est. Q. E. D.

a 1. def. d.

b 6. 6.

c 3. def. d.

PROP. 42.

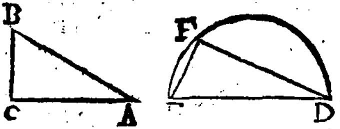
Si trianguli ABC *latera ad invicem habeant rationem datam, triangulum* ABC *specie datum est.*

Nam *a* fac AB. BC :: DE. EF. *a* & BC.CA :: EF.FD. *b* Liquet trigonum DEF trigono ABC assimilari. *c* quare ABC specie datum est. Q. E. D.

a 12.6.
b 5.6.
c 3.def.d.

Vide fig.39.

PROP. 43.

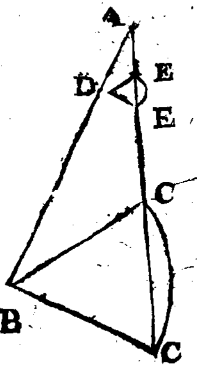


Si trianguli rectanguli ACB *circa unum acutorum angulorum* A *latera* AB, AC *ad invicem rationem habeant datam, triangulum* ACB *specie datum est.*

Nam esto DEF semicirculus utcunque; & *a* fac AB. AC :: DE. DF. inventamque DF *b* adapta in semicirculo; & duc EF. *c* Liquet triang. DFE ipsi ACB assimilari; & *d* proinde ipsum ACB specie dari. Q. E. D.

a 12.5.
b 1.4.
c 32.1. & 4.6.
d 3.def.d.

PROP. 44.

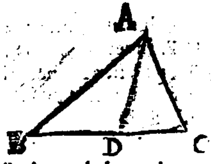


Si triangulum ABC habeat unum angulum A datum; circa alium autem angulum ABC latera AB,BC ad invicem habeant rationem datam; triangulum ABC specie datum est.

Nam in crure dati anguli sume quamlibet AD. & a fac AB. BC :: AD.DE. centro D spatio DE describe circulum, qui secet alterum dati anguli latus in E. b Eritque triang. ADE ipsi ABC simile. c quare datur specie triang. ABC. Q.E.D.

a 2. def. d.
b 7. 6.
c 3. def. d.

PROP. 45.



Si triangulum BAC unum angulum BAC datum habeat; circa datum autem angulum BAC latera simul utraque tanquam unam (BA+AC) *ad reliquum latus* (BC) *rationem habeant datam; triangulum BAC specie datum est.*

Datum angulum BAC a bisecet recta AD. b ergo BA. AC :: BD. DC. & componendo BA + AC. AC :: BC. DC. permutando igitur BA + AC. BC :: AC. DC. ergo ob $\frac{BA + AC}{BC}$ c datam, d erit $\frac{AC}{DC}$ data. item ang. DAC sub-

a 9. 1.
b 3. 6.
c hyp.
d 2. def. d.

duplus

duplus dati B A C *e* datur. *f* ergo ang. C datur. *e* 2. *dat.* *f* 44. *dat.*
g proinde trigonum ABC specie datum est. *g* 40. *dat.*

Coroll.

Hinc in triangulo, datis uno latere AB, uno angulo BAC, & ratione aggregati laterum ad basim (R ad S;) datur triangulum. Nam datum angulum biseca, & fac R.S :: AB. BD. & centro B spatio B D duc circulum occurrentem rectæ bisecanti in D; & produc BDC. habes triangulum.

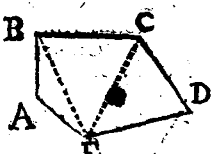
P R O P. 46.

Si triangulum B A C *unum angulum* C *datum habeat; circa alium autem angulum* BAC *latera simul utraque tanquam unum* (BA+AC) *habeant ad reliquum* (B C) *rationem datam; triangulum* BAC *specie datum est.*

Nam bisecto angulo BAC, erit (ut in præcedenti) $\overline{\text{AC}}$ data. item ang. C *a* datus est. ergo *a hyp.*
$\overline{\text{DC}}$
ang. D A C, *b* proinde & duplus B A C datur. *b* 2. *dat.*
c quare triang. BAC specie datur. Q. E. D. *c* 40. *dat.*

Deducetur ab hac corollarium simile præcedenti.

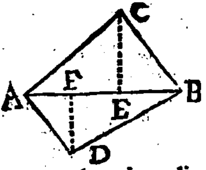
P R O P. 47.



Data specie rectilinea ABCDE *in data specie triangula* BAE, CDE BCE *dividuntur.*

Nam ob ang. B, & BA *a* dat. *b* erit triang. $\overline{\text{AE}}$ BAE specie datum. Simili discursu triang. CDE specie datur. *c* quare ang. DCE datus est, Hunc deme ex dato BCD, *d* estque reliquus BCE datus. Similiter ang. CBE datur. *e* ergo triang. BCE etiam specie datum est. Q. E. D. *a hyp. &* 3. *def. d.* *b* 41. *dat.* *c* 3. *def. d.* *d* 4 *dat.* *e* 40. *dat.*

PROP. 48.

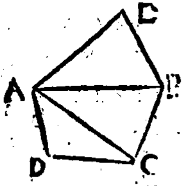


Si ab eadem recta AB describantur triangula ACB, ADB data specie, habebunt ad invicem rationem datam.

Duc enim perpendiculares CE, DF. Liquet angulos trianguli rectanguli CEB, a proinde & CE dari. ergo (quum AB b data sit) c erit $\overset{\overline{CB}}{\underset{\overline{AB}}{CE}}$ data. Simili discursu datur $\overset{\overline{CB}}{\underset{\overline{AB}}{DF}}$; c quare $\frac{CE}{DF}$, d hoc est triang. $\frac{ACB}{\overline{ADB}}$ datur. Q. E. D.

a 40. dat.
b sup.
c 8. dat.
d 1. 6.

PROP. 49.



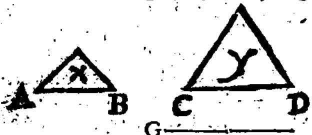
Si ab eadem recta linea AB duo rectilinea qualibet ABCD, AEB data specie describantur, habebunt ad invicem rationem datam.

Nam rectilineum ABCD resolvatur in triangula. a hæc specie data sunt. ergo ob communem basim AC, b ratio ADC ad ACB & c proinde totius ABCD ad ACB datur. b item ratio AEB ad ACB. d proinde & ABCD ad AEB datur. Q. E. D.

a 47. dat.
b 48. dat.
c 6. dat.
d 8. dat.

PROP. 50.

Si duæ rectæ lineæ AB CD ad invicem habeant rationem datam; & ab

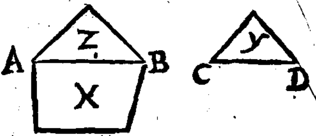


illis similia, similiterque descripta rectilinea X, Y habebunt ad invicem rationem datam.

Nam

Nam sit AB. CD :: [a] CD. G. [b] liquet AB ad G, [c] hoc est X ad Y dari. Q. E. D. a 11. 6. b 8. dat. c cor 20. 6.

PROP. 51.



Si duæ rectæ lineæ AB, CD habeant ad invicem rationem datam; & ab illis rectilinea quæcunque X, Y specie data describantur; habebunt ad invicem rationem datam.

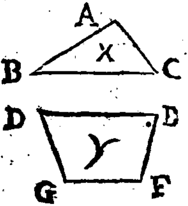
Nam [a] fac Z simile ipsi Y. Ac ob [b] $\frac{Z}{X}$, [c] & $\frac{Z}{Y}$ datas, [d] liquet $\frac{X}{Y}$ dari. Q. E. D. a 18. 6. b 49. dat. c 50. dat. d 8. dat.

PROP. 52.

Si à data magnitudine recta AB figura X specie data describatur, descripta figura X magnitudine data est.

Nam ABq [a] datur specie, & magnitudine; & [b] $\frac{ABq}{X}$ datur. [c] ergo X datur. a & 1. dat. b 1. dat. c 2. dat.

PROP. 53.



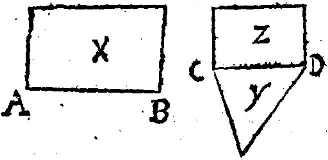
Si duæ figuræ X, Y specie datæ fuerint; & unum latus unius BC ad unum latus alterius DE habuerit rationem datam; reliqua quoque latera AB ad reliqua EG habebunt rationem datam.

Nam

a 3. def. d.
b hyp.

Nam ${}^a\frac{AB}{BC}$, ${}^a\frac{DE}{EF}$, ${}^a\frac{}{FG}$ dantur. &c. ergo per 8. dat.

PROP. 54.

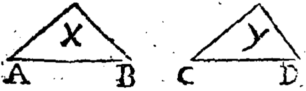


Si duæ figuræ X, Y *speciè datæ ad invicem habuerint rationem datam, etiam latera* (AB, CD, &c.) *habebunt ad invicem rationem datam.*

a 18. 6.
b 3. def. d.
c 49. dat.
d hyp.
e 8. dat.
f cor 10. 6 & 24. dat.

Nam ad CD ^{a} fiat Z ipsi X similis. ^{b} Hæc specie datur. ^{c} ergo $\frac{Y}{Z}$ datur. Proinde ob $\frac{Y}{X}$ ^{d} datam, ^{e} datur $\frac{X}{Z}$. ^{f} ergo $\frac{AB}{CD}$ datur. ergo per præcedentem.

PROP. 55.



Si spatium X *magnitudine & specie datum fuerit, ejus latera* (AB &c.) *magnitudine data erunt.*

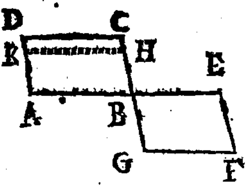
a 18. 6.
b 1. dat.
c 54. dat.
d 2. dat.

Nam ad quamvis CD ^{a} fac Y simile ipsi X. hoc specie & magnitudine datur. ^{b} ergo $\frac{Y}{X}$ datur. ^{c} quare $\frac{CD}{AB}$ datur. ^{d} ergo AB data est.

Q. E. D.

PROP.

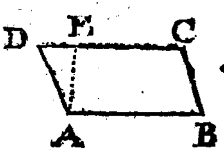
PROP. 56.



Si duo æquiangula parallelogramma AC, BF habuerint ad invicem rationem datam, est ut primi latus AB ad secundi latus BE, ita reliquum secundi latus BG ad eam BH, ad quam alterum primi latus BC habet rationem datam, quam habet parallelogrammum AC ad parallelogrammum BF.

Nam duc HK parall. AB. Liquet esse BC. a 1 6.
BH a :: AC. AH b :: AC. BF. Q. E. D. b 14 6. & 7. 5.

PROP. 57.



Si datum spatium AC ad datam rectam AB applicatum fuerit, in angulo BAD dato, datur applicationis altitudo AD.

a Erige perpendicularem AE. estque AB. AE b :: ABq. AB x AE c :: ABq. pgr. AC. d ergo AE datur. quare per E duc parallelam DC, e hæc abscindet quæsitam AD. Q. E. F.

a 11. 1.
b 1. 6.
c 35. 1.
d 1. & 2. dat.
e 28. & 25. dat.

PROP. 58.

Si datum ad datam rectam applicetur, deficiens data specie figura, latitudines defectus data sunt.

Non differt à vigesima octava sextæ.

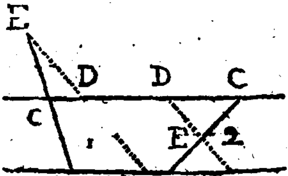
PROP. 59.

Si datum ad datam rectam applicetur, excedens data specie figura, latitudines excessus data sunt.

Eadem est cum vigesima nona sextæ.

PROP.

PROP. 34.



Si in datas positione parallelas rectas AB, CD à dato puncto E agatur recta linea ECA, secabitur data ratione.

Nam ab E duc rectam E B utcunque parallelis occurrentem in D, & B. a liquet esse EC. CA :: ED. DB. b quare $\frac{EC}{CA}$ datur. Q. E. D.

a 2. 6.
b 22. dat.

PROP. 35.

Si à dato puncto E in datam positione rectam AB agatur recta linea EA, seceturque data ratione; agatur autem per punctum sectionis C contra datam positione rectam AB recta linea CD; acta linea CD positione data est.

Recta enim E B ducta ab E utcunque in A B, a secetur sic ut ED. DB :: EC. CA. ob punctum D datum, b erit CD positione data. Q. E. D.

a 10. 6.
b 28. dat.

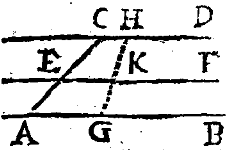
PROP. 36.

Si à dato puncto E in datam positione rectam lineam AB agatur recta linea EA; adjiciatur autem ipsi aliqua recta EC, quæ ad illam (EA) habeat rationem datam; per extremitatem autem C adjectæ lineæ EC agatur contra datam positione rectam AB recta linea CD; acta linea CD positione data est.

Demonstratio parum differt à præcedenti. *Vide fig. 2.*

PROP.

PROP. 37.



Si in datas positione parallelas rectas AB, CD, *agatur recta linea* AC, *& secetur ratione data; agatur autem per sectionis punctum* E *contra datas positione rectas* AB, CD *linea recta* EF; *acta recta* EF *positione data est.*

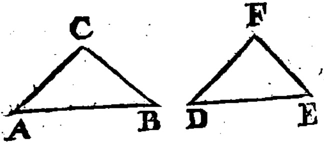
Nam duc rectam GH utcunque occurrentem parallelis. Hæc *a* secta sit in K ita ut GK. KH :: AE. EC. *b* Punctum K parallelæ (EF) situm determinat. Q. E. F. a 2. def. d b 2. dat. & sch. 2. 6.

PROP. 38.

Si in datas positione rectas parallelas AB, CD *agatur recta linea* AC; *adjiciatur autem ipsi quædam recta* CE, *quæ ad actam* AE *habeat rationem datam; per extremitatem autem* E *adjecta* CE *agatur contra datas positione parallelas* AB, CD *recta linea* EF; *acta recta linea* EF *est data positione.*

Demonstratio persimilis est præcedenti. Cerne & compara figuras.

PROP. 74.



Si trianguli ABC *singula latera* AB, BC, AC *magnitudine data sint, triangulum* ABC *specie datum est.*

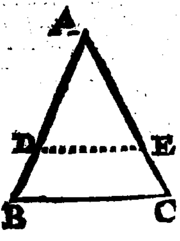
a 22. 1. b 5. 6. c 3. def. d. Nam *a* fac triang. D E F ipsi A B C æquilaterum. Hoc eidem *b* æquiangulum erit. *c* ergo ABC specie datum est. Q. E. D.

PROP. 40.

Si trianguli A B C *singuli anguli,* A, B, C *magnitudine dati sint, triangulum* ABC *specie datum est.*

a 23. 1. b 4. 6. c 3. def. d. Nam ad quamvis DE *a* fac triang. DEF ipsi ABC æquiangulum. *b* Hoc eidem simile erit. *c* proinde trigonum A B C specie datum est. Q. E. D.

PROP. 41.



Si triangulum A B C *unum angulum* A *datum habeat; circa datum autem angulum* A *duo latera* AB, AC *ad invicem habeant rationem datam; triangulum* A B C *specie datum est.*

a 1. def. d. b 6. 6. c 3. def. d. Nam in uno latere dati anguli sume quampiam A D; & *a* fit A B. A C :: AD. AE. & duc DE. *b* Liquet trigonum A D E ipsi A B C simile fore. *c* Quare ABC specie datum est. Q. E. D.

PROP.

PROP. 42.

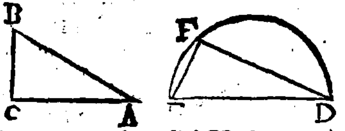
Si trianguli ABC *latera ad invicem habeant rationem datam, triangulum* ABC *specie datum est.*

Nam *a* fac AB. BC :: DE. EF. *a* & BC. CA :: EF. FD. *b* Liquet trigonum DEF trigono ABC assimilari. *c* quare ABC specie datum est. Q. E. D.

a 12. 6.
b 5. 6.
c 3. def. d.

Vide fig. 39.

PROP. 43.

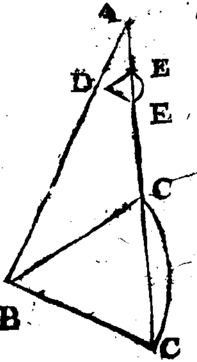


Si trianguli rectanguli ACB *circa unum acutorum angulorum* A *latera* AB, AC *ad invicem rationem habeant datam, triangulum* ACB *specie datum est.*

Nam esto DEF semicirculus utcunque; & *a* fac AB. AC :: DE. DF. inventamque DF *b* adapta in semicirculo; & duc EF. *c* Liquet triang. DFE ipsi ACB assimilari; & *d* proinde ipsum ACB specie dari. Q. E. D.

a 12. 5.
b 1. 4.
c 32. 1. & 4. 6.
d 3. def. d.

PROP. 44.

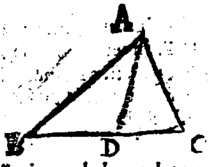


Si triangulum ABC *habeat unum angulum* A *datum; circa alium autem angulum* ABC *latera* AB,BC *ad invicem habeant rationem datam; triangulum* ABC *specie datum est.*

Nam in crure dati anguli sume quamlibet A D. & *a* fac A B. BC :: AD.DE. centro D spatio D E describe circulum, qui secet alterum dati anguli latus in E. *b* Eritque triang. ADE ipsi ABC simile. *c* quare datur specie triang. ABC. Q.E.D.

a 2. def. d.
b 7. 6.
c 3. def. d.

PROP. 45.



Si triangulum B A C *unum angulum* BAC *datum habeat; circa datum autem angulum* BAC *latera simul utraque tanquam unum* (BA+AC) *ad reliquum latus* (B C) *rationem habeant datam; triangulum* BAC *specie datum est.*

Datum angulum B A C *a* bisecet recta A D. *b* ergo B A. A C :: B D. D C. & componendo BA + AC. AC :: BC. DC. permutando igitur BA + AC. BC :: AC. DC. ergo ob $\frac{BA + AC}{BC}$ *c* datam, *d* erit $\frac{AC}{DC}$ data. item ang. LAC sub-

a 9. 1.
b 3. 6.
c hyp.
d 2. def. d.

duplus

duplus dati B A C e datur. f ergo ang. C datur. g proinde trigonum ABC specie datum est. e 2. dat. f 44. dat. g 40. dat.

Coroll.

Hinc in triangulo, datis uno latere AB, uno angulo BAC; & ratione aggregati laterum ad basim (R ad S;) datur triangulum. Nam datum angulum biseca, & fac R.S :: AB. BD. & centro B spatio B D duc circulum occurrentem rectæ bisecanti in D; & produc BDC. habes triangulum.

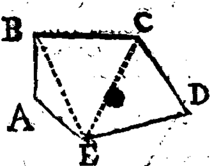
PROP. 46.

Si triangulum B A C *unum angulum* C *datum habeat; circa alium autem angulum* BAC *latera simul utraque tanquam unum* (BA+AC) *habeant ad reliquum* (B C) *rationem datam; triangulum* BAC *specie datum est.*

Nam bisecto angulo BAC, erit (ut in præcedenti) $\frac{AC}{DC}$ data. item ang. C a datus est. ergo a hyp.
ang. D A C, b proinde & duplus B A C datur. b 2. dat.
c quare triang. BAC specie datur. Q. E. D. c 40. dat.

Deducetur ab hac corollarium simile præcedenti.

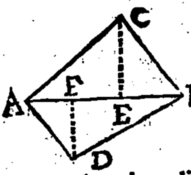
PROP. 47.



Data specie rectilinea ABCDE *in data specie triangula* BAE, CDE BCE *dividuntur.*

Nam ob ang. B, & $\frac{BA}{AE}$ a dat. b erit triang. BAE specie datum. Simili discursu triang. CDE specie datur. c quare ang. DCE datus est; Hunc deme ex dato BCD, d estque reliquus BCE datus. Similiter ang. CBE datur, e ergo triang. BCE etiam specie datum est. Q. E. D. a hyp. & 3. def. d. b 41. dat. c 3. def. d. d 4. dat. e 40. dat.

PROP. 48.

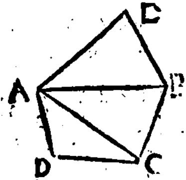


Si ab eadem recta AB describantur triangula ACB, ADB data specie, habebunt ad invicem rationem datam.

Duc enim perpendiculares CE, DF. Liquet angulos trianguli rectanguli CEB, a proinde & CE dari. ergo (quum AB b data sit) c erit $\overset{\overline{CB}}{\underset{\overline{AB}}{CE}}$ data. Simili discursu datur $\overset{\overline{CB}}{\underset{\overline{AB}}{DF}}$; c quare $\frac{CE,}{\overline{DF}}$ d hoc est triang. $\frac{ACB}{\overline{ADB}}$ datur. Q. E. D.

a 40. dat.
b sup.
c 8. dat.
d def. 1. 6.

PROP. 49.

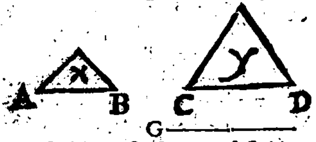


Si ab eadem recta linea AB duo rectilinea qualibet ABCD, AEB data specie describantur, habebunt ad invicem rationem datam.

Nam rectilineum ABCD resolvatur in triangula. a hæc specie data sunt. ergo ob communem basim AC, b ratio ADC ad ACB & c proinde totius ABCD ad ACB datur. b item ratio AEB ad ACB. d proinde & ABCD ad AEB datur. Q. E. D.

a 47. dat.
b 48. dat.
c 6. dat.
d 8. dat.

PROP. 50.

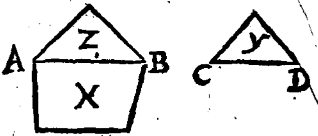


Si duæ rectæ lineæ AB CD ad invicem habeant rationem datam; & ab illis similia, similiterque descripta rectilinea X, Y habebunt ad invicem rationem datam.

Nam

Nam sit AB. CD :: [a] CD. G. [d] liquet AB ad G, [c] hoc est X ad Y dari. Q. E. D. a 11. 6. b 8. dat. c cor 20. 6.

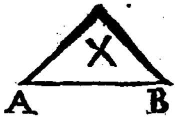
PROP. 51.



Si duæ rectæ lineæ AB, CD habeant ad invicem rationem datam; & ab illis rectilinea quæcunque X, Y specie data describantur; habebunt ad invicem rationem datam.

Nam [a] fac Z simile ipsi Y. Ac ob [b] $\frac{Z}{X}$, [c] & $\frac{Z}{Y}$ datas, [d] liquet $\frac{X}{Y}$ dari. Q. E. D. a 18. 6. b 49. dat. c 50. dat. d 8. dat.

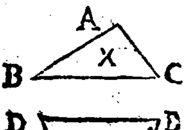
PROP. 52.



Si à data magnitudine recta AB figura X specie data describatur, descripta figura X magnitudine data est.

Nam ABq [a] datur specie, & magnitudine; & [b] $\frac{ABq}{X}$ datur. [c] ergo X datur. a. & 1. dat. b ... dat. c 2 dat.

PROP. 53.



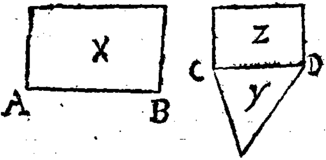
Si duæ figuræ X, Y specie datæ fuerint; & unum latus unius BC ad unum latus alterius DE habuerit rationem datam; reliqua quoque latera AB ad reliqua FG habebunt rationem datam.

Nam

a 3.def.d.
b hyp.

Nam $\left\{ \begin{array}{l} ^{a}\,AB \\ ^{b}\,\overline{BC} \\ ^{a}\,\overline{DE} \\ \overline{EF} \\ ^{a}\,\overline{FG} \end{array} \right.$ dantur. &c. ergo per 8. dat.

P R O P. 54.



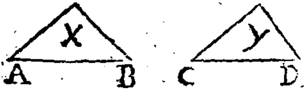
Si duæ figuræ X, Y specie datæ ad invicem habuerint rationem datam, etiam latera (AB, CD, &c.) habebunt ad invicem rationem datam.

a 18. 6.
b 3. def d.
c 40. dat.
d hyp.
e 8. dat.
f cor 10. 6 & 24. dat.

Nam ad CD ᵃ fiat Z ipsi X similis. ᵇ Hæc specie datur. ᶜ ergo $\frac{Y}{Z}$ datur. Proinde ob $\frac{Y}{X}$ ᵈ datam, ᵉ datur $\frac{X}{Z}$. ᶠ ergo $\frac{AB}{CD}$ datur. ergo per præcedentem.

P R O P. 55.

Si spatium X magnitudine & specie datum fuerit, ejus latera (AB &c.) magnitudine data erunt.

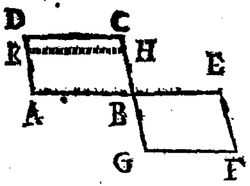


a 18. 6.
b 1. dat.
c 54. dat.
d 2. dat.

Nam ad quamvis CD ᵃ fac Y simile ipsi X. hoc specie & magnitudine datur. ᵇ ergo $\frac{Y}{X}$ datur. ᶜ quare $\frac{CD}{AB}$ datur. ᵈ ergo AB data est. Q. E. D.

P R O P.

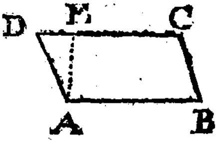
PROP. 56.



Si duo æquiangula parallelogramma AC, BF habuerint ad invicem rationem datam, est ut primi latus AB ad secundi latus BE, ita reliquum secundi latus BG ad eam BH, ad quam alterum primi latus BC habet rationem datam, quam habet parallelogrammum AC ad parallelogrammum BF.

Nam duc HK parall. AB. Liquet esse BC. BH a :: AC. AH b :: AC. BF. Q. E. D. a 1 6. b 14 6. & 7. 5.

PROP. 57.



Si datum spatium AC ad datam rectam AB applicatum fuerit, in angulo BAD dato, datur applicationis altitudo AD.

a Erige perpendicularem AE. estque AB. AE b :: ABq. AB x AE c :: ABq. pgr. AC. d ergo AE datur. quare per E duc parallelam DC, e hæc abscindet quæsitam AD. Q. E. F. a 11. 1. b 1. 6. c 35. 1. d 1. & 2. dat. e 28. & 25. dat.

PROP. 58.

Si datum ad datam rectam applicetur, deficiens data specie figura, latitudines defectus datæ sunt.

Non differt à vigesima octava sextæ.

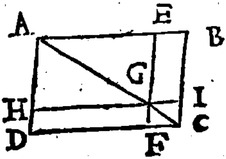
PROP. 59.

Si datum ad datam rectam applicetur, excedens data specie figura, latitudines excessus datæ sunt.

Eadem est cum vigesima nona sextæ.

PROP.

PROP. 60.



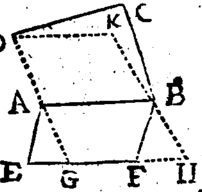
Si datum specie parallelogrammum (HE, *vel* DB) *dato gnomone* HCE *augeatur, vel minuatur; latitudines gnomonis* HD, EB *datæ sunt.*

a 3. dat. b 24. 6. c 55. dat. d hyp. e 4 dat.

1. *Hyp.* Liquet totum DB tam *a* magnitudine, quam *b* specie dari, *c* proinde & latitudines AB, AD; è quibus aufer *d* datas AE, AH, *e* manent EB, HD datæ. Q. E. D.

2. *Hyp.* Liquet HE *b* specie, & *a* magn. *c* dari, *c* quare & latera AE, AH; hæc deme ex *d* datis AB, AD: *e* remanent EB, HD datæ. Q. E. D.

PROP. 61.



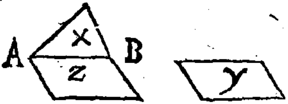
Si ad datæ specie figura ABCD *unum latus* AB *applicetur parallelogrammum spatium* AF *in angulo* BAE *dato; habeat autem data figura* AC *ad parallelogrammum* AF *rationem datam; parallelogrammum* AF *specie datum est.*

a 3. def. d. b 49. dat. c 8. dat. d 35. 1. e 1 6. f hyp. & 4. dat. g 40. dat. h 3 def. d.

Ad DAG protractam duc (per B) parallelam, cui occurrant EFH, & DK parall. AB. Ac ob $\frac{AD}{AB}$, & ang. BAD *a* dat. *a* liquet pgr. AK speciẹ dari. *b* ergo $\frac{AK}{AC}$ & *c* proinde $\frac{AK}{AF}$, *d* vel $\frac{AK}{AH}$, *e* hoc est $\frac{AD}{AG}$ dantur. *e* ergo $\frac{AB}{AG}$ datur. Item ob angulos E, & GAE *f* notos, *g* datur $\frac{AE}{AC}$; *e* ergo $\frac{AB}{AE}$ datur. *h* unde pgr. AF specie datur. Q. E. D.

PROP.

PROP. 62.



Si duæ rectæ AB, CD *ad invicem habeant rationem datam; & ab una quidem data specie figura* X *descripta sit, ab altera autem spatium parallelogrammum* Y *in angulo dato; habeat autem figura* X *ad parallelogrammum* Y *rationem datam; parallelogrammum* Y *specie datum est.*

Nam ad AB sit pgr. Z simile ipsi Y. *a* Hujus ratio ad Y, & *b* proinde ad X datur. *c* ejusque anguli dantur. *d* ergo Z specie datur. *e* proinde & Y. Q. E. D.

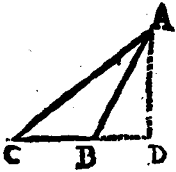
a 50. dat.
b 8. dat.
c hyp.
d 61. dat.
e 3 def. d.

PROP. 63.

Si triangulum specie datum sit, quod ab unoquoq; laterum describitur quadratum, ad triangulum habebit rationem datam.

Sequitur ex 49. hujus.

PROP. 64.



Si triangulum ABC *angulum obtusum* ABC *datum habeat; illud spatium, quo latus* AC *obtusum angulum subtendens magis potest quam latera* AB, CB *obtusum angulum* ABC *ambientia, ad triangulum* ABC *habebit rationem datam.*

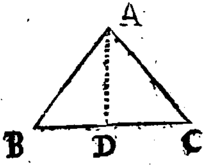
Nam demittatur AD perpendicularis productæ CBD. atque ob angulos *a* ABD, & D datos, *b* datur $\frac{BD}{AD}$, *c* hoc est $\frac{BD \times CB}{AD \times CB}$. *d* ergo

a 4. dat.
b 40. dat.
c 1. 6.
d 8. dat.

2 BD

e 12. 2. f 41. 1. $\frac{2\,BD \times CB}{\frac{1}{2}\,AD \times CB}$, hoc est, $^{e}\frac{ACq - ABq - CBq}{{}^{f}\,\text{triang. } ABC}$ datur. Q. E. D.

PROP. 65.



Si triangulum ACB *angulum acutum* C *datum habeat; illud spatium, quo latus* AB *angulum* C *subtendens minus potest, quam latera* AC, CB *angulum acutum* C *ambientia; habebit ad triangulum* ACB *rationem datam.*

a 40. dat. Nam duc perpendicularem AD. Datur $^{a}\frac{CD}{AD}$,

b 1. 6. c 8. dat. ^{b}hoc est $\frac{CD \times BC}{AD \times BC}$. ^{c}ergo $\frac{2\,CD \times BC}{\frac{1}{2}\,AD \times BC}$, hoc

d 13. 2. e 41. 1. est $^{d}\frac{ACq + BCq - ABq}{{}^{e}\,\text{triang. } ACB}$ datur. Q. E. D.

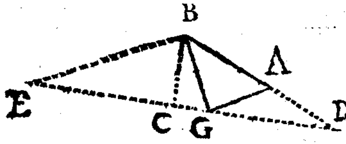
PROP. 66.

Si triangulum ACB *habuerit angulum* C *datum; quod sub rectis* AC, CB *datum angulum* C *comprehendentibus, continetur rectangulum, habebit ad triangulum* ACB *rationem datam.*

a 40. dat. b 1. 6 c 41. 1. d 8. dat. Nam in figura præcedentis, est $^{a}\frac{AC}{AD}$, ^{b}hoc est, $\frac{AC \times BC}{AD \times BC}$, ^{c}hoc est $\frac{AC \times BC}{2\,\text{triang. } ACB}$ data. ^{d}ergo $\frac{AC \times BC}{\text{triang. } ACB}$ datur. Q. E. D.

PROP.

PROP. 67.



Si triangulum ABG *habuerit datum angulum* BAG; *illud spatium, quo duo datum angulum* BAG *comprehendentia latera tanquam una recta* BA + AG, *plus possunt, quam quadratum à reliquo latere* BG; *ad triangulum* ABG *habebit rationem datam.*

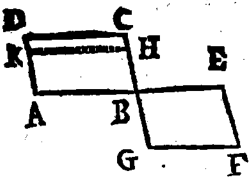
Produc BA ita ut AD = AG. per B duc BE parall. AG; cui occurrat DGE. denique duc normalem BC.

Liquet ang. D *a* = AGD *b* = E. *c* quare BE = BD, ideoque EC = CD. *e* ergo EG x GD + CGq = CDq. proinde BDq *f* (CDq + BCq) *g* = EG x GD + CGq + BCq = EG x GD * + BGq. Iam ob angulos AGD, & D *h* subduplos dati BAG, liquet *k* $\frac{AD}{DG}$, ideoq; $\frac{ADq}{DGq}$ dari. Cum igitur BA x AD. ADq *l* :: BA. AD *m* :: EG. GD :: *l* EGxGD. GDq, & permutando BAxAD. EGxGD :: ADq. GDq; *n* erit $\frac{BA \times AD}{EG \times GD}$; *o* hoc est $\frac{BA \times AG}{EG \times GD}$ data. *p* Atqui $\frac{BA \times AG}{triang.\ AGB}$ datur; *q* ergo $\frac{EG \times GD}{triang.\ AGB}$ datur. Q. E. D.

a 5. 1.
b 29. 1.
c 6. 1.
d cor. 3. 3.
e 5. 2.
f 47. 1.
g 2. ax. 1.
* 47. 1.
h 32. 1.
k 40. dat.
l 1. 6.
m 2. 6.
n 2. def. dat.
o constr.
p 66 dat.
q 8. dat.

PROP.

PROP. 68.

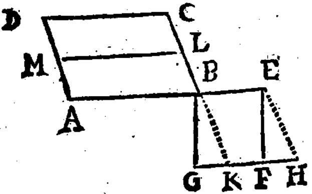


Si duo parallelogramma æquiangula AC, BF *habeant ad invicem rationem datam, & unum latus* AB *ad unum latus* BE *habeat rationem datam; & reliquum latus* BC *ad reliquum latus* BG *habebit rationem datam.*

a 2 def. d.
b 56. dat.
c 8. dat.

Nam sit AB. BE :: BG. BH. *a* ergo $\frac{BG}{BH}$ datur. *b* item $\frac{BC}{BH}$ datur. *c* ergo $\frac{BC}{BG}$ datur.

PROP. 69.



Si duo parallelogramma AC, BF *datos angulos habeant, & ad invicem rationem datam; habeat autem & unum latus* AB *ad unum latus* BE *rationem datam; & reliquum latus* B C *ad reliquum latus* BG *habebit rationem datam.*

Latera AB, BE jaceant in directum. produc C B K, ac G F H ad occursum cum EH parall. CK.

a hyp.

Ob *a* ang. KBE (ABC) & pgr. *a* $\frac{AC}{BF}$, vel

AC

$\frac{AC}{BH}$ & a $\frac{AB}{BF}$ datas, c liquet $\frac{KB}{BC}$ dari. item ob ang. G, & GBK d datos, e datur $\frac{KB}{BG}$. f quare $\frac{BC}{BG}$ datur. Q. E. D.

b 35. 1.
c 68 dat.
d hyp. & 4 dat.
e 40. dat.
f 8. dat.

PROP. 70.

Si duorum parallelogrammorum (AC, BH, *vel* BF) *circa æquales angulos* (ABC, KBE) *aut circa inæquales quidem* (ABC, GBE) *datos tamen, latera* (AB, BE, & BC, BK, & BC, BG) *ad invicem habeant rationem datam; & ipsa parallelogramma* (AC, BH, & AC, BF) *habebunt ad invicem rationem datam.*

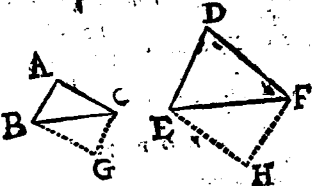
Nam (in fig. præced.) fit AB. BE :: KB. BL. & duc LM parall. BA.

Primo, Quia a $\frac{AB}{BE}$ b id est $\frac{KB}{BL}$, c ac $\frac{KB}{CB}$ datæ sunt, c erit $\frac{CB}{BL}$, d hoc est $\frac{AC}{AL}$, e vel pgr. $\frac{AC}{BH}$ data. Q. E. D.

Secundo, Ob angulos G, & GBK f datos, g datur $\frac{BK}{BG}$; item h $\frac{CB}{BG}$ data est. c ergo $\frac{CB}{BK}$ datur. proinde, ut prius, $\frac{AC}{BH}$, hoc est pgr. $\frac{AC}{BF}$ datur. Q. E. D.

a hyp.
b constr.
c 8. dat.
d 1. 6.
e 14. 6.
f hyp. & 4. dat.
g 40. d.
h 35. 1.

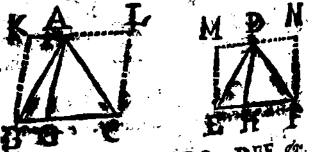
PROP. 71.



Si duorum triangulorum ABC, DEF, *circa æquales angulos, aut circa inæquales quidem, datos tamen* (A, & D) *latera* AB, DE, & AC, DF *ad invicem habeant rationem datam; & ipsa triangula* ABC, DEF *habebunt ad invicem rationem datam.*

a 70. dat. b 15. 5 c 34. 1. Nam compleantur pgra. AG, DH. a hæc datam habent rationem, b proinde & trigona ABC, DEF illorum c subdupla. Q. E. D.

PROP. 72.



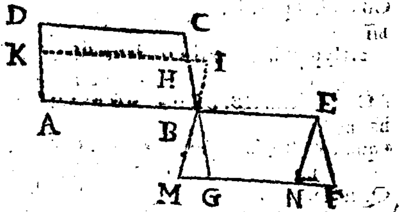
Si duorum triangulorum ABC, DEF *&c. bases* BC, EF *fuerint in ratione data, & actæ ab angulis ad bases* (AG, DH,) *quæ faciant ang.* AGC, DHF *æquales, aut inæquales quidem, sed tamen datos, habeant ad invicem rationem datam; & ipsa triangula* ABC, DEF *habebunt ad invicem rationem datam.*

Nam duc BK ad AG, ac EM ad DH parallelas, & comple pgra. CK, FM. Hæc se habent juxta 70. hujus; quare triangula eorum

* 34. 1. * subdupla ABC, DEF rationem habent datam. Q. E. D.

PROP.

PROP. 73.



Si duorum parallelogrammorum (AC, BF, *vel* AC, BN) *circa æquales angulos, aut circa inæquales quidem, sed tamen datos, latera ad invicem ita se habeant, ut sit quemadmodum primi latus* AB *ad secundi latus* BE, *ita reliquum secundi latus* (BG, *vel* BM) *ad aliam aliquam rectam* (BH, *vel* BI;) *habeat autem & reliquum primi latus* BC *ad eandem rectam* (BH *vel* BI) *rationem datam; & ipsa parallelogramma* (AC, BF, *vel* AC, BN) *habebunt ad invicem rationem datam.*

Nam 1. Hyp. liquet a $\frac{CB}{BH}$, b id est $\frac{AC}{AH}$ c (BF) dari. Q. E. D. — a hyp. b 1. 6. c 14. 6.

2. Hyp. Duc parallelam IHK. a Liquet angulos IBH (GBM) & BHI (ABH) dari. b ergo $\frac{BH}{BI}$ datur. item $\frac{CB}{BI}$ a data est. c proinde $\frac{CB}{BH}$, hoc est pgr. $\frac{AC}{BF}$ d vel $\frac{AC}{BN}$ datur. Q. E. D. — a hyp. & 4 dat. b 40. dat. c 8 dat. d 35. 6.

PROP. 74.

Si duo parallelogramma datam rationem habeant, aut in æqualibus angulis (ut AC, BF) *aut inæqualibus quidem, sed tamen datis* (ut AC, BN;) *erit ut primi latus* AB *ad secundi latus* BE, *ita alterum secundi latus* (BG, *vel* BM) *ad eam* (BH, *vel* BI) *ad quam reliquum primi latus* BC *rationem habet datam.*

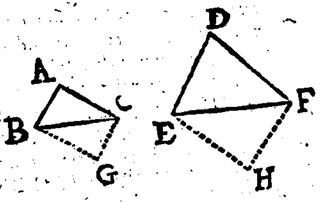
 Nam

a 56. dat. Nam in fig. præcedentis. 1. Hyp. a Liquet $\frac{CB}{BH}$ dari. Q. E. D.

2. Hyp. ut in præcedenti, datur $\frac{BI}{BH}$, ac ex hyp. $\frac{AC}{BF\,(BN)}$ item AB. BE :: a *MB. BI b :: GB. BH. a quare $\frac{CB}{BH}$ etiam datur. c ergo $\frac{CB}{BI}$ data est. Q. E. D.

* hyp. b 4. 6. c 8. dat.

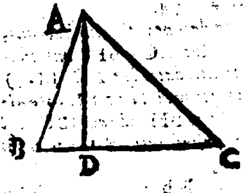
PROP. 75.



Si duo triangula ABC, DEF *ad invicem habeant rationem datam, aut in angulis* (A, D) *æqualibus, aut inæqualibus quidem sed tamen datis, erit ut primi latus* AB *ad secundi latus* DE, *ita alterum secundi latus* DF *ad eam rectam, ad quam reliquum primi latus* AC *habet rationem datam.*

Nam compleantur pgr. AG, DH. Ergo per præcedentem.

PROP. 76.

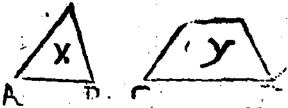


Si à trianguli ABC *specie dati vertice* A *linea perpendicularis* AD *agatur ad basim* BC, *acta linea* AD *ad basim* BC *habebit rationem datam.*

Nam

Nam ob angulos, * B, & ADB datos, a datur $\frac{AB}{AD}$; a item $\frac{AB}{BC}$ datur. b Ergo $\frac{AD}{BC}$ datur. Q. E. D. * hyp. & 3. def d. a 40. dat. b 8 dat.

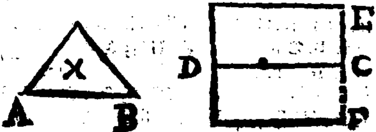
PROP. 77.



Si datæ figuræ specie X, Y *ad invicem habeant rationem datam, & quodlibet latus unius* AB *ad quodlibet alterius latus* CD *habebit rationem datam.*

Nam a $\frac{ABq}{X}$, & b $\frac{X}{Y}$, ac c proinde $\frac{ABq}{Y}$ datur; item $\frac{CDq}{Y}$ datur. c ergo $\frac{ABq}{CDq}$, ac ideo $\frac{AB}{CD}$ datur. Q. E. D. a 49. dat. b hyp. c 8. dat.

PROP. 78.

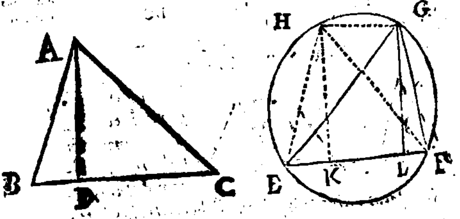


Si data figura specie X *ad aliquod rectangulum* DCE *habeat rationem datam; habeat autem & unum latus* AB *ad unum latus* DC *rationem datam; rectangulum* DCE *specie datum est.*

Sit DC, AB :: AB. CF. a ergo $\frac{DC}{CF}$ datur.

Item ob b $\frac{X}{ABq}$, & c $\frac{X}{DCE}$ datas. d erit $\frac{ABq}{DCE}$, d hoc est $\frac{DC \times CF}{DCE}$, vel e $\frac{CF}{CE}$ data. proinde f $\frac{DC}{CE}$ datur. quare rectang. DCE specie datur. Q. E. D. a 8. dat. b 49 dat. c hyp. d 17. 6. e 1. 6. f 3. def. d.

PROP. 79.



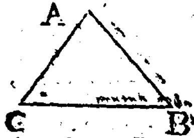
Si duo triangula ABC, GEF *unum angulum* BAC *uni angulo* EGF *æqualem habeant; ab æqualibus autem angulis* BAC, EGF *ad bases* BC, EF *perpendiculares agantur* AD, GL; *sitque ut primi trianguli basis ad perpendicularem, ita & alterius trianguli basis ad perpendicularem* (BC. AD :: EF. GL;) *illa triangula* ABC, EGF *æquiangula sunt.*

Circa triang. GEF describe circulum. Fac ang. FEH = B. Connecte HF, HG; & demitte perpendicularem HK.

Liquet triangula ABC, HEF, & ABD, HEK, ac ACD, HFK æquiangula fore. Proinde FK. KH :: CD. DA. [a] & FK. KH :: BD. DA. [b] quare EF. KH :: BC. DA :: [c] EF. LG. [d] quare KH = LG. [e] ergo HG parall. KL. [f] unde ang. EGH = GEF. [g] ergo arcus EH; FG, [h] ideoque anguli EFH, GEF æquantur. [k] Item ang. EHF = EGF. [l] ergo trigona EHF, EGF; [m] proinde & trigona EGF, ABC sibi mutuo æquiangula sunt. Q. E. D.

a 4. 6.
b 24. 5.
c hyp.
d 9. 5.
e 33. 1.
f 29. 1.
g 26. 3.
h 27. 3.
k 21. 3.
l 32. 1.
m 21. 6.

PROP. 80



Si triangulum ABC *unum angulum* A *datum habuerit; quod autem sub lateribus* AB, AC *datum angulum comprehendentibus continetur rectangulum, habeat ad quadratum reliqui lateris* BC *rationem datam; triangulum* ABC *specie datum est.*

Nam Q: AC + AB : − CBq vocetur X. a ergo $\frac{X}{\text{triang.ABC}}$; b & $\frac{AC \times AB}{\text{triang.ABC}}$; & c propterea $\frac{X}{AC \times AB}$, data est. d item $\frac{AC \times AB}{CBq}$ datur. e ergo $\frac{X}{CBq}$, ideoq; $\frac{X + CBq}{CBq}$, f hoc est $\frac{Q: AC + AB}{CBq}$, datur. g proinde triang. ABC specie datur. Q.E.D.

a 67. dat. b 66. dat. c 8. dat. d hyp. e 6. dat. f hyp. g 46. dat.

PROP. 81.

A.	D.
B.	E.
C.	F.

Si tres rectæ proportionales A, B, C *tribus rectis proportionalibus* D, E, F *extremas* A, D, & C, F *habuerint in ratione data; medias quoque* B, E *habebunt in ratione data. Et si extrema* A *ad extremam* D, *&* *media* B *ad mediam* E *habeat rationem datam; & reliqua* C *ad reliquam* F *habebit rationem datam.*

Nam primo, ob $\frac{A}{D}$ & $\frac{C}{F}$ datas, a datur $\frac{AC}{DF}$ a 70. dat.

b hoc est, $\frac{Bq}{Eq}$. ergo $\frac{B}{E}$ datur. Q.E.D. b 17. 6.

Secundo, ob c $\frac{Bq}{Eq}$, b hoc est $\frac{AC}{DF}$ datam, & c $\frac{A}{D}$ c hyp.

datam, d datur $\frac{C}{F}$. Q.E.D. d 68. dat.

PROP. 82.

A. B :: D. E.
B. C :: E. F.

Si quatuor rectæ proportionales fuerint (A. B :: D. E) *erit ut prima* A *ad eam* C, *ad quam secunda* B *rationem habet datam, ita tertia* D *ad eam* F, *ad quam quarta* E *rationem habet datam.*

a hyp.
b 2. def. d.

Nam quia B. C :: a E. F. & a $\frac{B}{C}$ data est; b erit $\frac{E}{F}$ data. atqui ex æquali A. C :: D. F. ergo, &c.

PROP. 83.

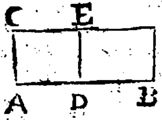
A. B. C. D.
F. E.

Si quatuor rectæ A,B,C,D *ita ad invicem se habeant, ut tribus ex iis, quibuscunque sumptis* A, B, C, *& quarta ipsis proportionali accepta* E, *ad quam reliqua* D *ex quatuor rectis proportionem habet datam; erit ut quarta* D *ad tertiam* C, *ita secunda* B *ad eam* F, *ad quam habet prima* A *rationem datam.*

a 16. 6.
b hyp.
c 1. 6.
d 7. 5.

Nam AE a = BC b = DF. & datur b $\frac{D}{E}$, c hoc est $\frac{AD}{AE}$, d vel $\frac{AD}{DF}$, e vel $\frac{A}{F}$. ergo, &c.

PROP. 84.



Si duæ rectæ A B, A C *datum spatium comprehendant in angulo* A *dato; sit autem altera* A B *altera* A C *major data* DB; *etiam unaquæque ipsarum* AB, AC *data erit.*

a 3. def. d.
b hyp.
c 59. d.
d 3. dat.

Nam comple quadratum A E. a Hoc specie datum est. b item pgr. CB, & recta DB dantur. c ergo AC, vel AD, & tota d proinde AB datur. Q. E. D.

PROP.

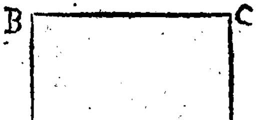
PROP. 85.

Si duæ rectæ BD, DE *datum spatium comprehendant in angulo* BDE *dato, sit autem simul utraq;* (BD+DE) *data; & earum quoque unaquæque* BD, *&* DE *data erit.*

Nam sume DA=DE, & comple quad. DC. Hoc specie datur; item pgr. BE, & recta BA *a* dantur. *b* ergo AD (DE) & *c* reliqua DB dantur. Q. E. D.

a hyp.
b 58. dat.
c 4. dat.

PROP. 86.



Si duæ rectæ AB, AD *datum spatium* BD *comprehendant in angulo dato; quadratum autem unius* AD *quadrato alterius* AB *majus sit dato quam in ratione* (nempe ut sit ADxAE datum, & * reliqui ADxED ad ABq ratio data;) *& utraque ipsarum* AB, AD *data erit.* * 2. 2.

Nam ob BD, & DAxAE *a* data, *b* datur $\frac{BD.}{DAxAE}$ *c* ergo $\frac{AB}{AB}$ *d* ideoque $\frac{ABq}{AEq}$ datur. *e* item $\frac{ABq}{ADxED}$, datur. *f* ergo $\frac{AEq}{ADxED}$, ideoque $\frac{AEq}{4ADxED}$, *g* & $\frac{AEq}{4ADxED+AEq}$ *h* hoc est $\frac{AEq}{Q:AD+ED}$ datur. *k* ergo $\frac{AE}{AD\;ED}$; & *l* componendo $\frac{AE}{2AD}$, * ideoq; $\frac{AE}{AD}$, *m* hoc est $\frac{AEq}{ADxAE}$ datur. denique igitur ob *e* datum ADxAE, *n* erit AEq data. *o* ergo AE, & *p* proinde AD, ac AB datæ sunt. Q.E.D.

a hyp.
b 1. dat.
c 69 dat.
d 51. dat.
e hyp.
f 8. dat.
g 6. dat.
h 8. 2.
k 54 dat.
l 6 dat.
* 8. dat.
m 1. 6.
n 2 dat.
o 55. dat.
p 57 dat.

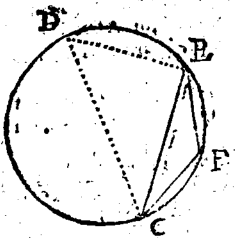
PROP.

PROP. 87.

Si duæ rectæ AB, AD *datum spatium comprehendant in angulo dato, quadratum autem unius* AD *quadrato alterius* AB *majus sit dato* (AD × AE;) *earum utraque* AB, AD *data erit.*

a 2. dat. Nam ob $\frac{BA \times AE}{BD}$ a datum, b erit $\frac{AE}{AB}$, ideoq;
b 69. dat.
c hyp. & 2. 2. $\frac{AEq}{ABq}$ c hoc est $\frac{AEq}{AD \times ED}$, d ac idcirco $\frac{AEq}{AEq + 4AD \times ED}$
d 8. & 6. dat.
e 8. 1. e hoc est $\frac{AEq}{Q{:}AD + ED}$, ac proinde $\frac{AE}{AD + ED}$, & d com-
d 6 dat.
e 1 6. ponendo $\frac{AE}{2AD}$, e ac ideo $\frac{AE}{AD}$, e hoc est $\frac{AEq}{AD \times AE}$
f hyp.
g 2. dat. data: ergo ob AD × AE f datum, dantur g AEq,
h 55. dat.
k 57. dat. & h AE, ac k ideo AD, ac AB. Q. E. D.

PROP. 88.



Si in circulum CFED *magnitudine datum acta sit recta linea* CE, *quæ segmentum auferat, quod datum angulum* F *comprehendat; acta recta linea* CE *magnitudine data est.*

Nam ducatur diameter CD; & connectatur ED. Ac ob ang. F a datum, b erit ang. D (reliquus è 2 rectis) datus. item rectus CED datur. c quare $\frac{CE}{CD}$ datur. ergo ob d datam CD, e erit CE data. Q. E. D.

a hyp.
b 4 dat.
c 40. dat.
d hyp. & 5. def. dat.
e 2. dat.

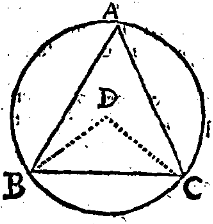
PROP.

PROP. 89.

Si in datum magnitudine circulum CFED *data magnitudine recta* CE *acta fuerit, auferet segmentum quod angulum* (CFE) *datum comprehendet.*

Nam (in fig. præcedentis) quia CE, & ang. CED dantur, *a* erit ang. D datus. *b* ergo ang. F *c* (1 Rect. — D) datus erit. Q. E. D. a 41. dat. b 4 dat. c 22. 3.

PROP. 90.



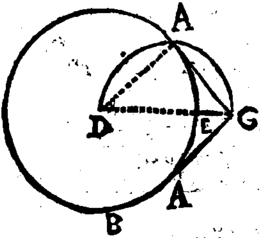
Si in circuli positione dati circumferentia BAC *datum fuerit punctum* B, *ab eo autem puncto* B *ad circumferentiam circuli inflexa fuerit recta* BAC *quæ datum angulum* A *efficiat; inflexa rectæ altera extremitas* C *data erit.*

Ad *a* centrum D duc BD, & CD; *b* datusque est ang. D dati A *c* duplus: quare ob BD *d* datam, *e* erit DC data. *f* ergo punctum C datum est. Q. E. D. a 1. 3 b 2. dat. c 20. 3. d 26. dat. e 19. dat. f sch. 29. d.

Si ang. A obtusus fuerit; sume reliquum è 2 rectis acutum; ejus subsidio punctum C invenies, juxta dicta.

PROP.

PROP. 91.



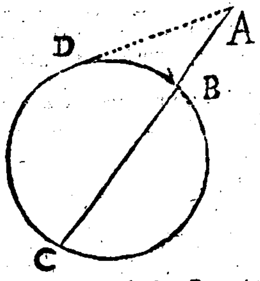
Si à dato puncto G acta fuerit recta GA, quæ datum positione circulum B E A *contingat; acta linea* GA *positione & magnitudine data est.*

Nam centrum D & punctum G connectat recta D G. super qua descriptus sit semicirculus D A G circulo priori occurrens in A. Ob ang. D G *a* rectum, GA circulum *b* tangit. *c* ergo G A situ & magnitudine datur. Q. E. D.

a 31. 3.
b cor. 16. 3.
c 26. dat.

Hinc modus discitur à dato puncto tangentem ducendi, eo nonnunquam expeditior qui habetur ad 17. 3.

PROP. 92.

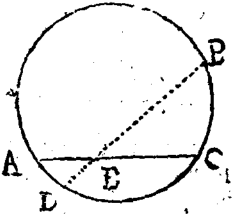


Si extra circulum positione datum BCD *accipiatur aliquod punctum* A, *à dato autem puncto* A *in circulum producatur quædam recta* AC; *datum est id quod sub acta linea* AC, *&* *ea* A B, *quæ inter punctum* A *&* *convexam peripheriam* B *comprehenditur rectangulum* CAB.

Nam

a Nam duc tangentem AD, *b* eritque ADq (hoc est CA x AE) datum. Q. E. D. a 91. dat. b 36. 3.

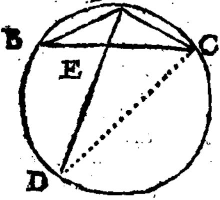
PROP. 93.



Si intra datum positione circulum ABCD sumatur aliquod punctum E; per punctum autem E agatur in circulum aliqua recta AEC; quod sub segmentis AE, EC actæ rectæ lineæ comprehenditur rectangulum, datum est.

Nam per E duc rectam DEB utcunque occurrentem circulo in B, & D. estque rectang. DEB = *a* AEC. *b* ergo AEC datur. Q. E. D. a 35. 3. b 2. def.

PROP. 94.



Si in circulum BACD magnitudine datum agatur recta linea BC, quæ segmentum auferat, quod angulum BAC datum comprehendat; angulus autem B C, qui in segmento consistit, bifariam secetur; simul utraque rectarum BA, AC quæ angulum datum BAC comprehendunt, ad lineam AD, quæ angulum bifariam secat, habebit rationem datam: & quod sub simul utrisque BA, AC, quæ datum angulum BAC comprehendunt, rectis: & inferne abscissa (ED) ab ea AD, quæ angulum BAC in circumferentia datum bifariam secat, rectangulum datum erit.

Duc

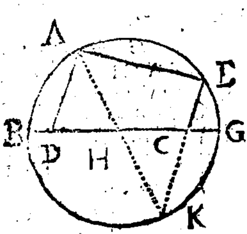
a 88. dat. * 1. dat. b 3. 6. c 11. 5. * 4. 6. d 2. def. d.

Duc CD; & primo ob angulos BAC, CAD datos, a dantur subtensæ BC, CD, * ideoque $\frac{CB}{BC}$ datur. Cum igitur CA. AB :: b CE. EB, & permutando CA. CE :: AB. EB :: (CA + AB. CB ::) * AD. DC. (Nam * ob ang. BAE = CAD & D = BD; trigona ABE, ADC similia sunt) ac rursus permutando CA + AB. AD :: CB. DC. d erit $\frac{CA + AB}{AD}$ data. Q. E. D.

a 21. 3. b 4. 6. c prius. d 16. 6. e 52. dat. f 1. def. d.

Secundo, ob triangula AEB, DEC a similia; b erit CD, DE :: AB. BE c :: CA + AB. CB. d ergo CA + AB in DE = CD in CB. atqui CD x CB e datur f ergo CA + AB in DE datum est. Q. E. D.

PROP. 95.



Si in circuli BAG positione dati diametro BG sumatur datum punctum D; à puncto autem D in circulum producatur quædam recta DA, & agatur à sectione A ad rectos angulos in productam rectam DA linea AE; per punctum autem E, in quo linea AE, quæ ad rectos angulos consistit, occurrit circumferentiæ circuli, agatur parallela (ECK) productæ rectæ DA; datum est illud punctum C, in quo parallela EK occurrit ipsi diametro BG; & quod sub parallelis lineis AD, EC comprehenditur rectangulum, datum est.

a 31. 3.

Nam connectatur AK. a estque AB (ob angulum E, vel DAE rectum) diameter. ergo in-

interſectio H eſt centrum. [b] ergo DH datur. Atqui ob KH. HA [c] :: CH. HD, [d] eſt CH = HD. [e] ergo CH datur. [f] ergo punctum C datur. Q. E. D. [g] ergo KC x CE, hoc eſt [d] AD x CE datur. Q. E. D.

b 36. dat. c 4 6. d 9. 5. e 1. def. d. f 27 dat. g 93. dat.

F I N I S.