EVCLIDIS ELEMENTORVM LIBRI XV.

ACCESSIT LIBER XVI. DE QVINque solidorum Regularium inter se comparatione.

AD EXEMPLARIA R.P. CHRISTOphori Clauij è Societ. IESV, & aliorum hac postrema editione collati, emendati & aucti.

COLONIAE

Apud Petrum Cholinum

M. DC. XXVII.

Cum gratia & Priuilegio.

MATHESEOS ET GEOMETRIAE DIVISIO.

Mathematicæ disciplinæ, quæ omnes circa quantitatem versantur, nomen acceperunt à Græca dictione μάθημα seu μάθησις, quæ disciplinam, & doctrinam significat; eò quod tum gradatim ascendendo doceantur, & addiscantur; tùm solæ semper ex præcognitis quibusdam, concessis, & probatis, principijs, (Haud ex hypothesibus nondum explicatis) ad conclusiones demonstrandas, quod proprium est doctrinarum & disciplinarum officium, teste Aristotele libr. 1. Poster. procedant.

Pythagoras, & Mathematici vniuersas Mathematicas disciplinas in quatuor partes principes distribuunt, nempe in Arithmeticam, Musicam, Geometriam, & Astronomiam. Cùm enim omnis quantitas, circa quam hæ disciplinæ versantur, sit vel discreta, sub qua omnes numeri continentur; vel continua, sub qua omnes magnitudines comprehenduntur; & vtraque tam secundum ſe, quam comparatione alterius poſſit conſiderari; Viſum eſt illis has quatuor partes inſtituere, quæ vtramque quantitatem pro duplici conſideratione, diligenter contemplarentur. Itaque Arithmetica agit de quantitate diſcreta ſecundum ſe; omneſque numerorum proprietates ac paſſiones inquirit, & accuratè explicat. Muſica tractat eandem quantitatem diſcretam, ſeu numerum cum alio comparatum; quatenus ſonorum Harmoniam, & concentum reſpicit. Geometria de magnitudine, ſeu quantitate continua, ſecundum ſe quoque, vt immobilis exiſtit, diſputat. Aſtronomia denique magnitudinem, vt eſt mobilis, conſiderat; qualia ſunt corpora cœleſtia continuè mota. Ad has autem quatuor ſcientias Mathematicas, tum puras tum mixtas, omnes aliæ de quantitate tractant, vti Perſpectiua Geographia, Stereometria, & cæteræ, facili negotio, tanquam ad ſua capita, & fontes, ex quibus emanant reducuntur

Geometria apud Euclidem diuiditur in planorum (ſuperficierum planarum,) contemplationem, ſeu Geometriam propriè dictam; quæ libris ſex primis abſoluitur; Et in ſolidorum (corporum ſolidorum:) ſpeculationem, ſeu Stereometriam; quæ libris poſtremis pertractatur Prior quidem pars, nempe Geometria, in tres partes ſubdiuidiuiditur. Nam in prioribus quatuor libris agitur de planis absolutè, ita vt eorum ęqualitas, & inæqualitas inuestigetur. In quinto verò libro de rationalibus magnitudinum proportionibus in genere disputatur In sexto denique libro proportiones figurarum planarum discutiuntur. Posterior verò pars, nempe Stereometrica, in tres quoque partes subdiuiditur. In quarum prima, videlicet libris tribus, septimo, octauo, & nono, agitur de numerorum proprietatibus, passionibusque ad linearum (& aliarum magnitudinum:) symetrarum seu commensurabilium, & asymetrarum; seu incommensurabilium tractationem necessarijs In secunda, nimirum libro decimo, satis prolixo, de lineis commensurabilibus, & incommẽsurabilibus, sine quarum notitia, corpora illa quinque solida, regularia, seu platonica perfectè tractari nequeunt, differitur In tertia denique nempe libris sex postremis (qui & ipsi subdiuidi poterunt) de solidis illis acutissimè disputatur, eorumque proprietates inuestigantur. De punctis autem, & lineis in hoc operè nulla ex professo extat cõtemplatio: quoniam Geometria potissimum circa figuras, quibus plana, & solida duntaxat (non puncta, & lineæ:) afficiuntur, versatur.

Demonstratio omnis Mathematicorum ab antiquis scriptoribus diuiditur in Problema,

blema,& Theorema. Problema quidem vocatur ea demonstratio, quæ iubet atque docet aliquid constituere, facere, inuenire, describere, &c. vt super data linea recta terminata triangulum æquilaterum constituere. Theorema verò appellatur ea demonstratio, quæ solum aliquam proprietatem, seu passionem vnius, vel plurium simul quantitatum (multarum vel magnarum iam inuentarum:) perscrutatur, vt in omni triangulo tres angulos esse æquales duobus rectis. Cæterùm tam problema, quàm Theorema dicitur apud Mathematicos propositio; eò quod vtrumque aliquid nobis proponit, vti in exemplis adductis constat, Demonstrationes problematum concluduntur his ferè verbis; Quod faciendum erat: Theorematum verò Demonstrationes, his verbis; Quod ostendendum, vel demonstrandũ erat, habita nimirum ratione vtriusque. Adhæc problemata per modum infinitum, sed Theoremata per modum finitum ferè proferuntur. Lemma (sumptio, vel assumptum latinè) appellatur ea minus principalis, & aliqua tantum declaratione indigens, demõstratio, quæ ad demonstrationem alicuius Problematis, vel Theorematis principalis assumitur, vt illa demonstratio expeditior, ac breuior fiat; vt videre licet libr. 6. propos. 22. & passim

sim Corollarium, seu porisma, denique est, quod protenus ex facta demonstratione tanquam lucrum aliquod additum, seu consectarium accipitur; vt videre est lib. 2. propos. 4. & passim.

Cùm omnis autem doctrina, omnisque disciplina ex præexistente cognitione gignatur, atque ex assumptis, & concessis quibusdam principijs suas demonstrat conclusiones: Nulla autem scientia, teste Aristotele, sua principia demonstret; habebunt & Mathematicæ disciplinæ sua principia, ex quibus positis, & concessis sua Problemata, & Theoremata confirment. Horum autem tria solum genera apud Mathematicos reperiuntur. Quorum primum genus continet definitiones, quas nonnulli Hypotheses appellant, vt punctum est, cuius pars nulla est. Secundum genus complectitur petitiones, seu postulata, quæ per se adeò perspicua sunt, vt nulla confirmatione indigeant, sed auditoris duntaxat assensum exposcant, vt postuletur, vt à quouis puncto in quoduis punctum, rectam lineam ducere concedatur. Tertium denique genus comprehendit Axiomata, seu communes animi notiones, quæ non solum in scientia proposita, sed etiam in alijs omnibus vsque adeo euidentes sunt, vt ab eis nulla ratione dissentire queat is, qui ipsa vocabula rectè perceperit, vt:

Quæ eidem æqualia, inter se sunt æqualia. Verum secundum nonnullos principium formale duplex existit, nempe indemonstrabile seu facilè, vt definitio, postulatum, & axioma; demonstrabile, vt Problema, Theorema, & omnis propositio. Principium verò materiale est punctum, linea, &c. Vid. & P. Christophorus Clauius, nobilissimus Elementorum Euclidis Interpres.. Coloniæ Agrippinæ, anno 1607.
8. Iunij.

EVCLI-

EVCLIDIS ELEMENTVM PRIMVM.

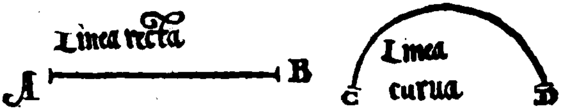
DEFINITIONES

1.

PVunctum est, cuius pars nulla est.

2.

Linea verò, longitudo latitudinis expers.



3.

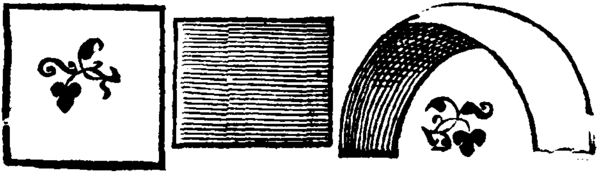
Lineæ autem termini, sunt puncta.

4.

Recta linea est, quæ ex æquo sua interiacet puncta.

5.

Superficies est, quæ longitudinem latitudinemque tantum habet.



6.

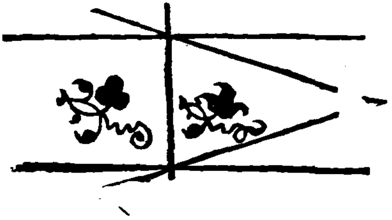
Superficiei extrema ſunt lineæ.

7.

Plana ſuperficies eſt, quæ ex æquo ſuas interiacet lineas.

8.

Planus angulus eſt duarum linearum in planò ſe mutuò tangentium, & non indirectū iacentium alterius ad alterā inclinatio.



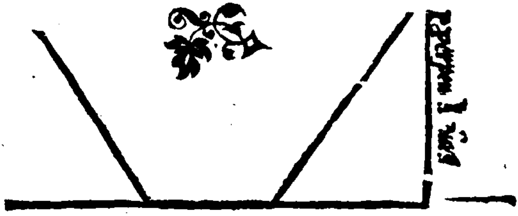
9.

Cùm autem quæ angulum continent lineæ, rectæ fuerint, rectilineus ille angulus appellatur.

10.

Cùm verò recta linea ſuper rectam conſiſtens lineam, eos qui ſunt deinceps, angulos æquales inter ſe fecerit; rectus eſt vterque æqua-

æqualium angulorum: quæ inſiſtit recta linea, perpendicularis vocatur eius, cui inſiſtit.



11.

Obtuſus angulus eſt, qui recto maior eſt.

12.

Acutus verò, qui minor eſt recto.

13.

Terminus eſt, quod alicuius extremum eſt.

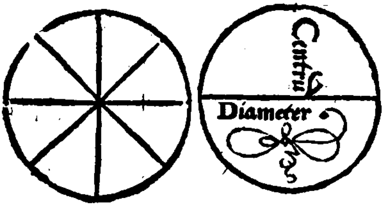
14.

Figura eſt, quæ ſub aliquo, vel aliquibus terminis comprehenditur.

15.

Circulus eſt, figura plana ſub vna linea comprehenſa, quæ peripheria appellatur; ad quã ab vno puncto eorum, quæ intra figuram ſunt

ſunt poſita, cadentes omnes rectæ lineæ inter ſe ſunt æquales.



16.

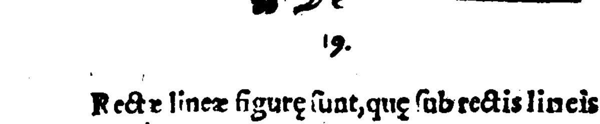
Hoc verò punctum, centrum circuli appellatur.

17.

Diameter autem circuli, eſt recta quædam linea per centrum ducta, & ex vtraque parte in circuli peripheriam terminata, quæ circulum biſariam ſecat.

18.

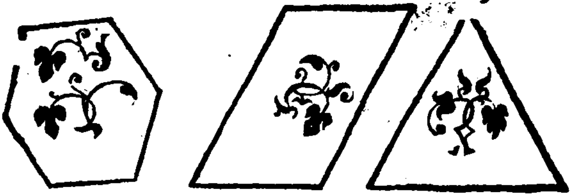
Semicirculus verò eſt figura, quæ continetur ſub diametro & ſub ea linea, quæ de circuli peripheria auſertur.



19.

Rectæ lineæ figuræ ſunt, quæ ſub rectis lineis continentur.

20. Tri-



20. Trilateræ quidem, quæ sub tribus.

21.

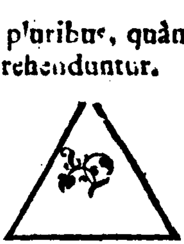
Quadrilateræ, quæ sub quatuor.

22.

Multilateræ verò quæ sub pluribus, quàm quatuor, rectis lineis comprehenduntur.

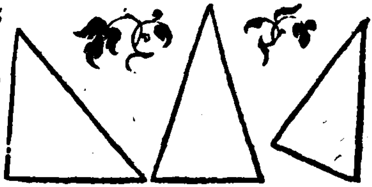
23.

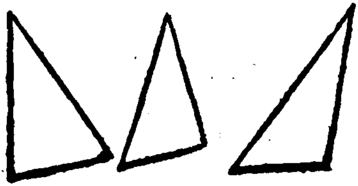
Trilaterarum autem figurarum, æquilaterum est triangulum, quod tria latera habet æqualia.



24.

Isosceles autem est, quod duo tantùm æqualia habet latera.





25.

Scalenum verò est, quod tria inæ-

inæqualia habet latera.

26.

Adhæc etiam, trilaterarum figurarum, rectangulum quidem triangulum eſt, quod rectum angulum habet.

27.

Amblygonium autem, quod obtuſum angulum habet.

28.

Oxygonium verò, quod tres habet acutos angulos.

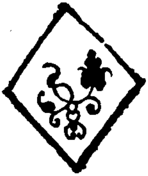
29.

Quadrilaterarum autem figurarum, quadratum quidem eſt, quod & æquilaterum, & rectangulum eſt.

30.

Altera verò parte longior figura eſt, quæ rectangula quidem, at æquilatera non eſt.

31.
Rhombus autem, quæ æquilatera,



sed rectangula non est.

32.
Rhomboides verò, quæ aduersa & latera, & angulos habens inter se æquales, neque æquilatera est, neque rectangula.

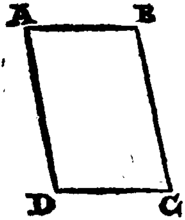
33.
Præter has autem, reliquæ quadri-

lateræ figuræ, trapezia appellentur.

34.
Parallele rectæ lineæ sunt, quæ cùm in eodem sint plano & ex vtraque parte in infinitum producantur, in neutràm sibi mutuò incidunt.

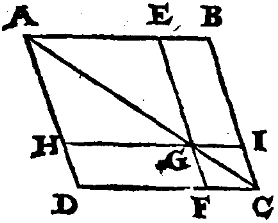
35.
Parallelogrammum est figura, quadrilatera, cuius bina apposita latera sunt parallela, seu æqui distantia.

36. Cùm



36.

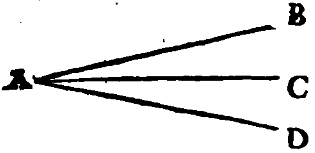
Cùm verò in parallelogrammo diameter ducta fuerit, duæq; lineæ lateribus parallelæ secantes diametrum in vno eodemque puncto, ita vt parallelogrammum ab hisce parallelis in quatuor distribuatur parallelogramma; appellantur duo illa, per quæ diameter non transit, complementa; duo verò reliqua, per quæ diameter incedit, circa diametrum consistere dicuntur.



Petitiones siue Postulata.

I.

Postuletur, vt à quouis puncto in quoduis pun-



punctum, rectam lineam ducere concedatur.

2.

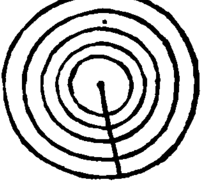
A B C D

. . . .

Et rectam lineam terminatam in continuum recta producere.

3.

Item quouis centro, & interuallo circulum describere.



B C D

4. . . .

Item quacunque magnitudine data, sumi posse aliam magnitudinem vel maiorem, vel minorem.

Communes notiones siue axioma.

1.

Quæ eidem æqualia, & inter se sunt æqualia.

2.

Et, si æqualibus æqualia adiecta sint, tota sunt æqualia.

3.

Et, si æqualibus æqualia adiecta sint, quæ relinquuntur sunt æqualia.

4.

Et, si inæqualibus æqualia adiecta sint, tota sunt inæqualia.

5.

Et, si ab inæqualibus æqualia ablata sint, reliqua sunt inæqualia.

6.

Et, quæ eiusdem duplicia sunt, inter se sunt æqualia.

7.

Et quæ eiusdem sunt dimidia, inter se æqualia sunt.

8.

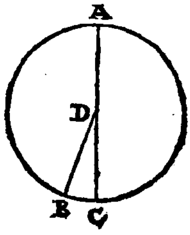
Et quæ sibi mutuò congruunt, ea inter se sunt æqualia.

9.

Et totum sua parte maius est.

10.

Duæ lineæ rectę non habent vnum, & idem segmentum commune.



11.

Duæ lineæ rectæ in vno puncto concurrentes,

rentes, ſi producantur ambæ, neceſſario ſe mutuò in eo puncto interſecabunt.

12.

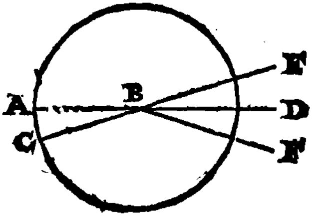
Item, omnes recti anguli ſunt inter ſe æquales.

13.

Et, ſi in duas rectas lineas altera recta incidens, internos, ad eaſdemq́ue partes angulos, duobus rectis minores faciat; duæ illæ rectæ lineæ in infinitum productę ſibi mutuò incident ad eas partes, vbi ſunt anguli duobus rectis minores.

14.

Duæ rectæ lineæ ſpatium non comprehendunt.



15.

Si æqualibus inæqualia adijciantur, erit tòtorum exceſſus, adiectorũ exceſſui æqualis.

16.

Si inæqualibus æqualia adiungantur, erit totorum exceſſus, exceſſui eorum, quæ à principio erant, æqualis.

17.

Si ab æqualibus inæqualia demantur, erit reſiduorum exceſſus, exceſſui ablatorum æqualis.

18.

Si ab inæqualibus æqualia demantur, erit reſiduorum exceſſus, exceſſui totorum æqualis.

19.

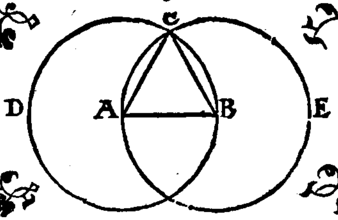
Omne totum æquale eſt omnibus ſuis partibus ſimul ſumptus.

20.

Si totum totius eſt duplum, & ablatum ablati; erit & reliquum reliqui duplum.

Problema 1. Propoſitio 1.

Super data recta linea terminata, triangulum æquilaterum conſtituere.

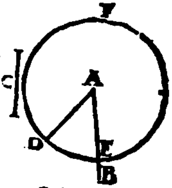


Problema 2. Propoſitio 2.

Ad datum punctum, datæ rectæ lineæ, æqualem rectam lineam ponere.

Pro-

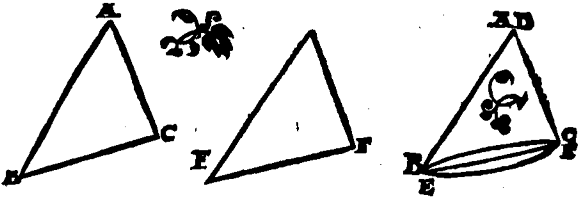
Problema 3. Propositio 3.



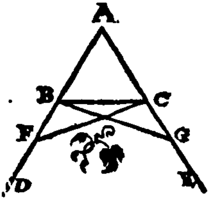
Duabus datis rectis lineis inæqualibus, de maiore æqualem minori rectam lineam detrahere.

Theorema 1. Propositio 4.

Si duo triangula duo latera duobus lateribus æqualia habeant, vtrunq; vtrique; habeant verò & angulum angulo æqualem sub æqualibus rectis lineis contentum: & basin basi æqualem habebunt; eritq; triangulum triangulo æquale, ac reliqui anguli reliquis angulis æquales erunt, vterque vtrique, sub quibus æqualia latera subtenduntur.



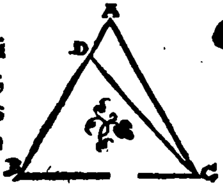
Theorema 2. Propositio 5.



Isosceleum triangulorum, qui ad basim sunt anguli, inter se sũt æquales; & si vlterius productæ sint æquales illæ rectæ lineæ, qui ſub baſi ſunt anguli, inter ſe æquales erunt.

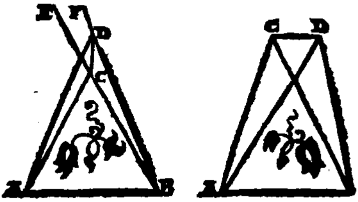
Problema 3. Propoſitio 6.

Si trianguli duo anguli æquales inter ſe fuerint; & ſub æqualibus angulis ſubtenſa latera æqualia inter ſe erunt.



Theorema 4. Propoſitio 7.

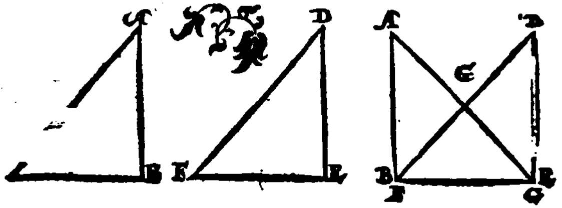
Super eadem recta linea, duabus eiſdem rectis lineis aliæ duæ rectæ lineæ æquales, vtraque vtrique, non conſtituentur, ad aliud atque aliud punctum, ad eaſdem partes, eoſdemque terminos cum duabus initio ductis rectis lineis habentes.



Theorema 5. Propoſitio 8.

Si duo triangula duo latera habuerunt duobus lateribus, vtrunque vtrique, æqualia: habuerunt verò & baſim baſi æqualem: angulum quoque ſub æqualibus rectis lineis contentum angulo æqualem habebunt.

Pro-

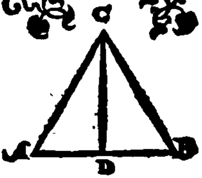


Problema 4. Propoſitio 9.

Datum angulum rectilineum bifariam ſecare.

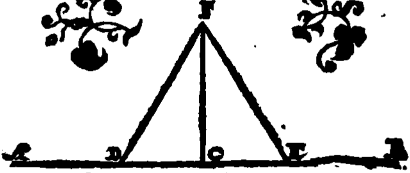
Problema 5. Propoſitio 10.

Datam rectaml ineam finitam bifariam ſecare.

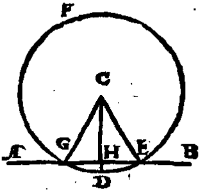


Problema 6. Propoſitio 11.

Data recta linea, à puncto in eo dato, rectam lineam ad angulos rectos excitare.

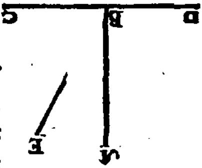


Problema 7. Propoſitio 12.



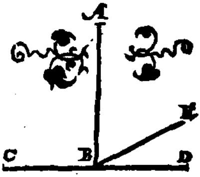
Super datam rectam lineam infinitam, à dato puncto, quod in ea non eſt, perpendicularem rectam deducere.

Theorema 6. Propoſitio 13.



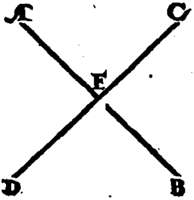
Cùm recta linea ſuper rectam cõſiſtens lineam angulos facit, aut duos rectos, aut duobus rectis æquales efficiet.

Theorema 7. Propoſitio 14.



Si ad aliquam rectam lineam, atq; ad eius punctum, duæ rectæ lineæ non ad eaſdem partes ductæ, eos, qui ſunt deinceps, angulos duobus rectis æquales fecerint, indirectum erũt inter ſe ipſe rectæ lineæ.

Theorema 8. Propoſitio 15.

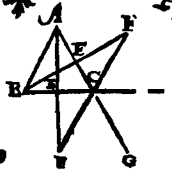


Si duę rectæ lineę ſe mutuò ſecuerint, angulos, qui ad verticem ſunt, æquales in ter ſe efficient.

Theo-

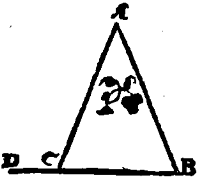
Theorema 9. Propositio 16.

Cuiuscunque trianguli vno latere producto, externus angulus vtroque interno & opposito maior est.



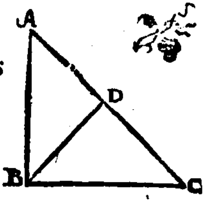
Theorema 10. Propositio 17.

Cuiuscunque trianguli duo anguli duobus rectis sunt minores, omnifariam sumpti.



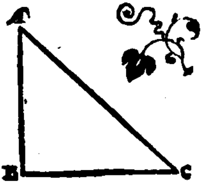
Theorema 11. Propositio. 18.

Omnis trianguli maius latus maiorem angulum subtendit.

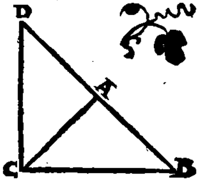


Theorema 12. Propositio 19.

Omnis trianguli maior angulus maiori lateri subtenditur.

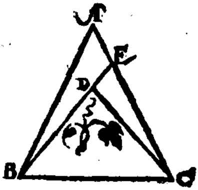


Theorema 13. Propositio 20.



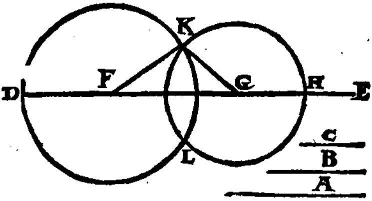
Omnis trianguli duo latera reliquo ſunt maiora, quomodocunque aſſumpta.

Theorema 14: Propoſitio 21.



Si ſuper trianguli vno latere, ab extremitatibus duæ rectæ lineæ, interius conſtitutæ fuerint; hæ conſtitutæ reliquis trianguli duobus lateribus minores quidem erunt; maiorem verò angulum continebũt.

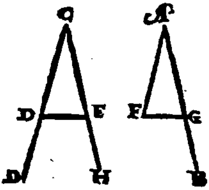
Problema 8. Propoſitio 22.



Ex tribus rectis lineis, quæ ſunt trib. datis rectis lineis æquales, triangulum conſtituere. Oportet autem duas reliqua eſſe maiores omnifariam ſumptas: quoniam vnuſcuiuſque trianguli duo latera omnifariam ſumpta, reliquo ſunt maiora.

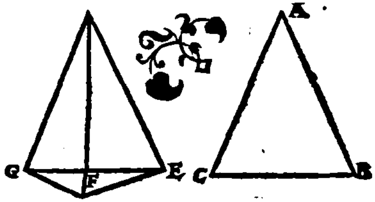
Pro-

Problema 9. Propositio 23.



Ad datam rectam lineã, datumq́; in ea punctum, dato angulo rectilineo æqualem angulum rectilineum constituere.

Theorema 15. Propositio 24.



Si duo triangula duo latera duobus lateribus ęqualia habuerint, vtrumq; vtriq;, angulum verò angulo maiorem sub ęqualib. rectis lineis cõtentũ: & basin basi maiorẽ habebũt.

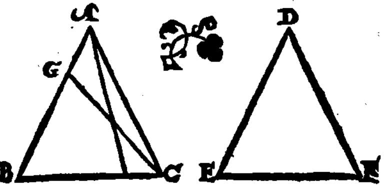
Theorema 16. Propositio 25.

Si duo triangula duo latera duobus lateribus æqualia habuerint, vtrunque vtrique; basin verò basi maiorẽ: & angulum sub æqualibus rectis lineis contentum angulo maiorem habebunt.

Theo-

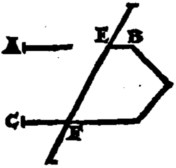
Theorema 17. Propositio 26.

Si duo triangula duos angulos duobus angulis æquales habuerint, vtrunque vtrique; vnumque latus vni lateri æquale, siue quod æqualibus adiacet angulis, seu quod vni æqualium angulorum subtenditur: & reliqua latera reliquis lateribus æqualia, vtrunq; vtriq; & reliquum angulum reliquo angulo ęqualem habebunt.



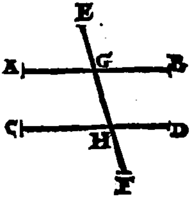
Theorema 18. Propositio 27.

Si in duas rectas lineas recta incidens linea alternatim angulos æquales inter se fecerit; parallelæ erunt inter se illæ rectæ lineæ.



Theorema 19. Propropositio 28.

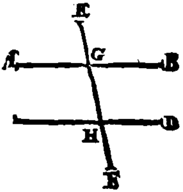
Si in duas rectas lineas recta incidens linea, externum angulum interno & opposito, & ad easdem partes, æqualem fecerit,



cerit,

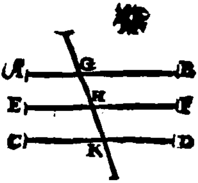
cerit, aut internos, & ad easdem partes duobus rectis æquales: parallelæ erunt inter se ipsæ rectæ lineæ.

Theorema 20. Propositio 29.



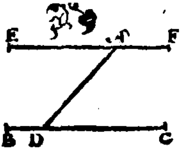
In parallelas rectas lineas recta incidens linea; & alternatim angulos inter se æquales efficit, & externum interno & opposito, & ad easdem partes æqualem, & internos, & ad easdem partes duobus rectis æquales facit.

Theorema 21. Propositio 30.



Quæ eidem rectæ lineæ, parallelæ, & inter se sunt parallelæ.

Problema 10. Propositio 31.

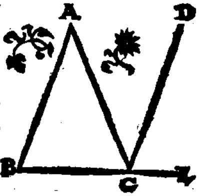


A dato puncto datæ rectæ lineæ parallelam rectam lineam ducere.

Theorema 22. Propositio 31.

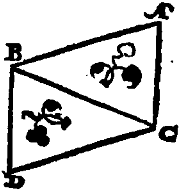
Cuiuscunque trianguli vno latere alterius pro-

producto: externus angulus duobus internis & oppoſitis eſt æqualis. Et trianguli tres interni anguli duobus ſunt rectis æquales.



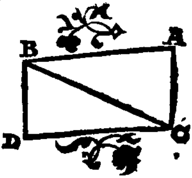
Theorema 23. Propoſitio 33.

Rectæ lineæ quæ æquales & parallelas lineas ad partes eaſdem coniungunt, & ipſæ æquales & parallelæ ſunt.



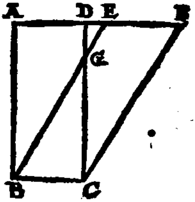
Theorema 24. Propoſitio 34.

Parallelogrammorum ſpatiorum æqualia ſunt inter ſe, quæ ex aduerſo, & latera, & anguli: atque illa bifariam ſecat diameter.



Theorema 25. Propoſitio 35.

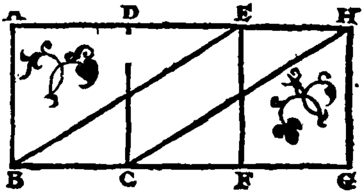
Parallelogramma ſuper eadem baſi, & in eiſdem parallelis conſtituta inter ſe ſunt æqualia.



Theo-

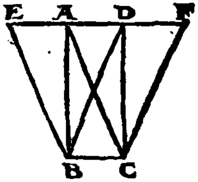
Theorema 26. Propositio 36.

Parallelogramma super æqualibus basibus, in eisdẽ parallelis cõstituta inter se sunt æqualia.



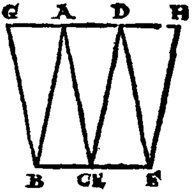
Theorema 27. Propositio 37.

Triangula super eadem basi constituta, & in eisdem parallelis, inter se sunt æqualia.



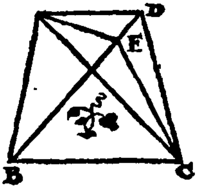
Theorema 28. Propositio 38.

Triangula super æqualibus basibus constituta, & in eisdem parallelis, inter se sunt æqualia.



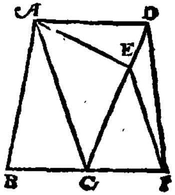
Theorema 22. Propositio 39.

Triangula æqualia super eadem basi, & ad easdem partes constituta: & in eisdem sunt parallelis.



Theo-

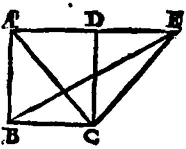
Theorema 30. Propo-
ſitio 40.



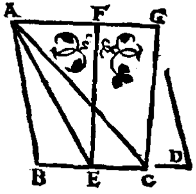
Triangula ęqualia ſuper æqualibus baſibus, & ad eaſdem partes conſtituta, & in eiſdem ſunt parallelis.

Theorema 31. Propo-
ſitio 41.

Si parallelogrammum cum triangulo eandem baſin habuerit, in eiſdemque fuerit parallelis, duplum erit parallelo grammum ipſius trianguli.



Problema 11. Propo-
ſitio 42.

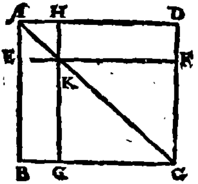


Dato triangulo æquale parallelogrammum conſtituere in dato angulo rectilineo.

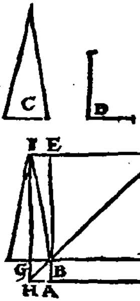
Theorema 32. Propo-
ſitio 34.

In omni parallelogrammo, complimenta eorum,

eorum, quæ circa diametrum sunt, parallelogrammorum, inter se sunt æqualia.



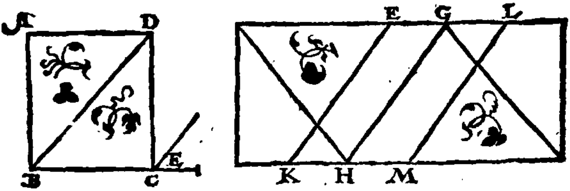
Problema 13. Propositio 44.



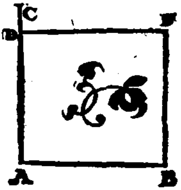
Ad datam rectam lineam, dato, triangulo, æquale parallelogrammum applicare, in dato angulo rectilineo.

Problema 13. Propositio 45.

Dato rectilineo, æquale parallelogrammum constituere in dato angulo rectilineo.



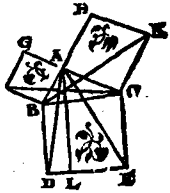
Problema 14. Propositio 46.



A data recta linea quadratum describere.

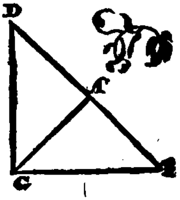
Theorema 33. Propositio 47.

In rectangulis triangulis, quadratum, quod à latere rectum angulum subtendente describitur, æquale est eis, quæ à lateribus rectum angulum continentib. describuntur, quadratis.



Theorema 34. Propositio 48.

Si quadratum, quod ab vno laterum trianguli describitur, æquale sit eis, qui à reliquis trianguli lateribus describuntur, quadratis: angulus compræhensus sub reliquis duobus trianguli lateribus, rectus est.



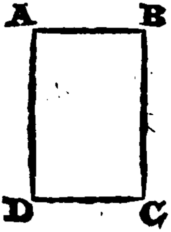
FINIS ELEMENTI I.

EVCLI-

EVCLIDIS ELEMENTVM SECVNDVM.

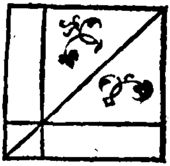
DEFINITIONES.

1.



Omne parallelogrammum rectangulum contineri dicitur sub rectis duabus lineis, quæ rectum comprehendunt angulum.

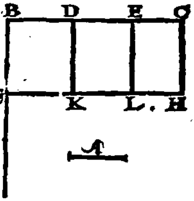
2.



In omni parallelogrammo spatio, vnum quodlibet eorum, quæ circa diametrum illius sunt, parallelogrammorum, cum duobus complementis, *Gnomon* vocetur.

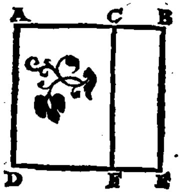
Theorema 1. Propositio 1.

Si fuerint duæ rectæ lineæ, seceturq; ipsarũ altera in quotcunque segmenta rectangulũ comprehensum sub illis duabus rectis lineis, æquale est eis, quæ sub insecta & quolibet segmẽtorum compræhenduntur, rectangulis.



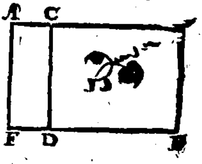
Theorema 2. Propositio 2.

Si recta linea secta sit vtcunque: rectangula, quæ sub tota, & quolibet segmentorum compræhenduntur, æqualia sunt ei, quod à toto fit, quadrato.



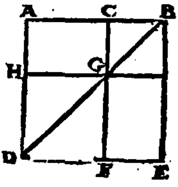
Theorema 3. Propositio 3.

Si recta linea secta sit vtcunque rectangulum sub tota, & vno segmentorum compræhensum, æquale est illi, quod sub segmentis cõprehenditur triangulo, & illi, quòd à prædicto segmento describitur, quadrato.



Theorema 4. Propositio 4.

Si recta linea secta sit vtcunq; quadratum, quod à tota describitur, æquale est & illis, quæ à segmen-

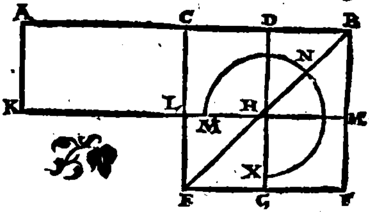


tis

tis deſcribuntur, quadratis; & ei quod bis ſub ſegmentis comprehenditur, rectangulo.

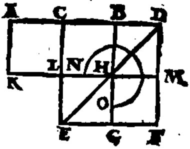
Theorema 5. Propoſitio 5.

Si recta linea ſecetur inæqualia, & non æqualia: rectangulum ſub inæqualibus ſegmentis totius comprehenſum, vnà cũ quadrato, quod ab intermedia ſectionum, æquale eſt ei, quod à dimidia deſcribitur, quadrato.



Theorema 6. Propoſitio 6.

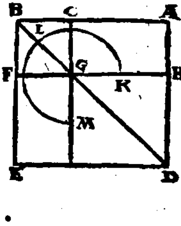
Si recta linea bifariam ſecetur, & illi recta quædam linea in rectum adijciatur: rectangulum comprehenſum ſub tota cum adiecta, & adiecta, vnà cum quadrato à dimidia, æquale eſt quadrato à linea, quæ tum ex dimidia, tum ex adiecta componitur, tanquam ab vna deſcripto.



Theorema 7. Propoſitio 7.

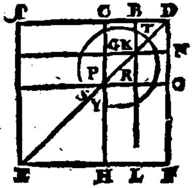
Si recta linea ſecetur vtcunque; quod à tota, quod-

quodq́ue ab vno ſegmentorum, vtraq; ſimul quadrata, æqualia ſunt & illi, quod bis ſub tota, & dicto ſegmento comprehenditur, rectãgulo; & illi quod à reliquo ſegmento fit, quadrato.



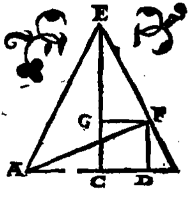
Theorema 8. Propoſitio 8.

Si recta linea ſecetur vtcunque; rectangulũ quater comprehenſum ſub tota, & vno ſegmentorum, cum eo quod à reliquo ſegmẽto fit, quadrato, ęquale eſt ei, quod à totá, & dicto ſegmento, tanquam ab vna linea deſcribitur, quadrato.



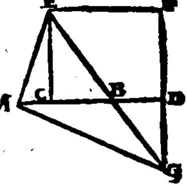
Theorema 9. Propoſitio 9.

Si recta linea ſecetur in æqualia & non æqualia: quadrata, quæ ab inæqualibus totius ſegmentis fiunt, ſimul duplicia ſunt, & eius, quod à dimidia, & eius, quod ab inter media ſectionum fit, quadratorum.



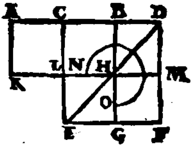
Theorema 10. Propoſitio 10.

Si recta linea ſecetur bifariá; adijciatur aũt ei in ei in rectū quæpiam recta linea: quod à tota cū adiuncta, & quod ab adiuncta, vtraque ſimul quadrata; duplicia ſunt, & eius, quod à dimidia, & eius, quod à compoſita ex dimidia & adiuncta, tanquam ab vna, deſcriptum ſit, quadratorum.



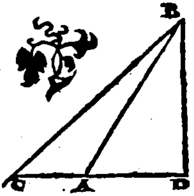
Problema 1. Propoſitio 11.

Datam rectam lineā ſecare, vt compræhēſum ſub tota, & altero ſegmentorum rectangulum, æquale ſit ei, quod à reliquo ſegmento fit quadrato.



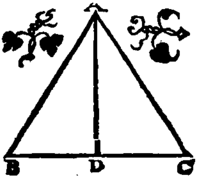
Theorema 11. Propoſitio 12.

In amblygonijs triangulis, quadratū, quod fit à latere angulum obtuſum ſubtendente, maius eſt quadratis, quæ fiunt à lateribus obtuſum angulum comprehendentibus; rectangulo bis compræhenſo, & ab vno laterum, quæ ſunt circa obtuſum angulum, in quod eum protractum fuerit, cadit perpendicularis, & ab aſſumpta exterius linea ſub perpendiculari prope angulum obtuſum.



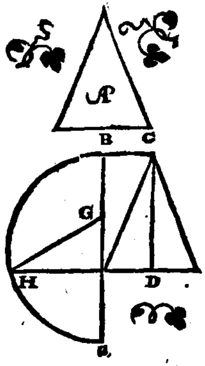
Theorema 12. Propositio 28.

In oxygonijs triangulis, quadratum à latere angulum acutum ſubtendente, minus eſt quadratis, quæ fiunt à lateribus acutum angulum comprehendentibus; rectangulo bis comprehenſo; & vno laterum, quæ ſunt circa acutum angulum, in quod perpendicularis cadit; & ab aſſumpta interius linea ſub perpendiculari prope acutum angulum.



Problema 2. Propoſitio 14.

Dato rectilineo æquale quadratum conſtituere.



FINIS ELEMENTI II.

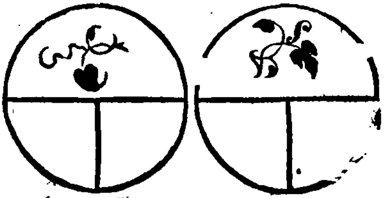
EVCLI-

EVCLIDIS ELEMENTVM TERTIVM.

DEFINITIONES.

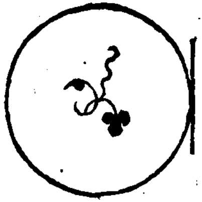
1.

Aequales circuli ſunt, quorum diametri ſunt æquales; vel quorum, quæ ex centris, rectæ lineæ ſunt æquales.

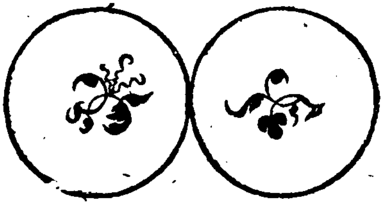


2.

Recta linea circulum tangere dicitur, quæ cùm circulum tangat; ſi producatur, circulum non ſecat.



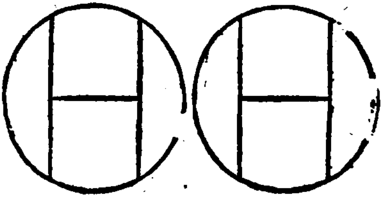
3.



Circuli ſeſe mutuò tangere dicuntur: qui ſeſe mutuò tangentes, ſeſe mutuò non ſecant.

4.

In circulo æqualiter diſtare à centro rectæ lineæ dicuntur, cùm perpendiculares, quæ à centro in ipſas ducuntur, ſunt æquales. Lõgius autem abeſſe illa dicitur, in quã maior perpendicularis cadit.



5.

Segmentum circuli eſt figura, quæ ſub recta linea, & circuli peripheria cõprehenditur.

6.

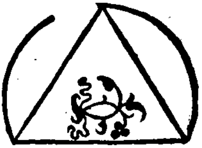
Segmenti autem angulus eſt, qui ſua recta

linea

linea, & circuli peripheria comprehenditur.

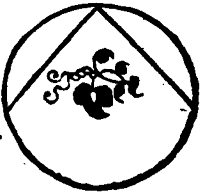
7.

In ſegmento autem angulus eſt, cùm in ſegmenti peripheria ſumptum fuerit quodpiam punctum, & ab illo in terminos rectæ eius lineæ quæ ſegmenti baſis eſt, adiunctæ fuerint, rectæ lineæ: is, inquam, angulus ab adiunctis illis lineis comprehenſus.



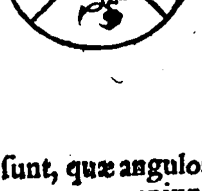
8.

Cùm verò comprehendentes angulum rectæ lineæ aliquam aſſumát peripheriam, illi angulus inſiſtere dicitur.



9.

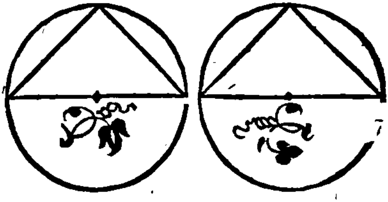
Sector autem circuli eſt, cùm ad ipſius circuli cétrum conſtitutus fuerit angulus, comprehenſa nimirum figura, & à rectis lineis angulum continentibus, & à peripheria ab illis aſſumpta.



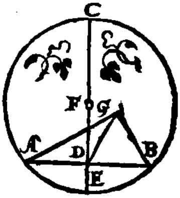
10.

Similia circuli ſegmenta ſunt, quæ angulos capiunt

capiunt æquales aut in quibus anguli inter ſe ſunt æquales.

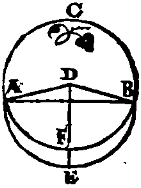


Problema 1. Propoſitio 1.



Dati circuli centrum reperire.

Theorema 1. Propoſitio 2.



Si in circuli peripheria duo quælibet puncta accepta fuerint; recta linea, quæ ad ipſa puncta adiungitur, intra circulum cadet.

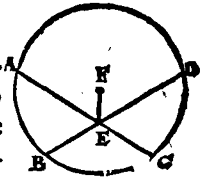
Theorema 2. Propoſitio 3.

Si in circulo recta quædam linea per centrum extenſa quandam non per centrum extenſam bifariam ſecet: & ad angulos rectos ipſam ſecabit: Et ſi ad angulos rectos eam ſecet, bifariam quoque eam ſecabit.

Theo-

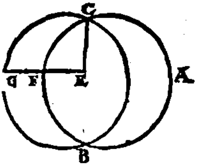
Theorema 3. Propositio 4.

Si in circulo duæ rectæ lineæ sese mutuò secent, nõ per centrum extensæ; sese mutuò bifariam non secabunt.



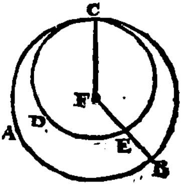
Theorema 4. Propositio 5.

Si duo circuli sese mutuò secent; non erit illorum idem centrum.



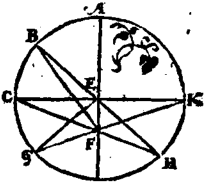
Theorema 5. Propositio 6.

Si duo circuli sese mutuò interius tangent, eorum non erit idem centrum.



Theorema 6. Propositio 7.

Si in diametro circuli quodpiam sumatur punctum, quod circuli centrum non sit, ab eoq; puncto in circulum quædam rectæ lineæ cadant; maxima quidem erit ea in qua centrũ; minima verò reliqua; aliarum verò propinquior illi, quæ per centrum du-

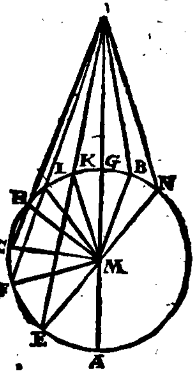


citur

citur, remotiore ſemper maior eſt. Duæ autem ſolùm rectæ lineæ æquales ab eodem puncto in circulum cadunt, ad vtraſq; partes minimæ, vel maximæ.

Theorema 7. Propoſitio 8.

Si extra circulum ſumatur punctum quodpiam, ab eoque puncto ad circulum deducantur rectæ quædam lineæ, quarum vna quidem per centrum protendatur, reliquæ verò vt libet: in cauam peripheriam cadentium rectarum linearum minima quidem eſt illa, quæ per centrum ducitur: aliarum autem propinquior ei, quæ per centrum tranſit, remotiore ſemper maior eſt: in conuexam verò peripheriã cadentium rectarum linearũ minima quidem eſt illa, quæ inter punctum, & diametrum interponitur: aliarum autem, ea, quæ propinquior eſt minimæ, remotiore ſemper minor eſt. Duæ

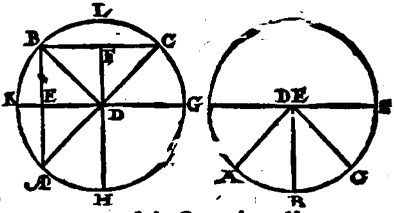


autem tantùm rectæ lineæ æquales ab eo puncto in ipſum circulum cadunt, ad vtraſque partes minimæ, vel maximæ.

Theo-

Theorema 8. Propoſitio 9.

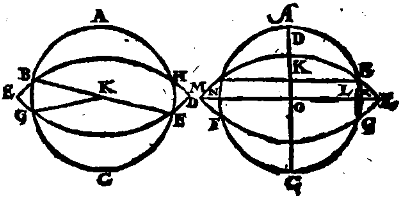
Si in circulo acceptum fuerit punctum aliquod, & ab eo puncto ad circulum cadant plures, quam- duæ, re- ctæ li- neæ æ- quales; acceptũ



punctum centrum eſt ipſius circuli.

Theorema 9. Propoſitio 10.

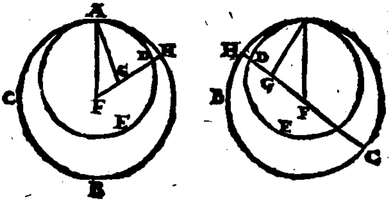
Circu- lus cir- culum in plu- ribus, quàm duob.



punctis non ſecat.

Theorema 10. Propoſitio 11.

Si duo circuli ſeſe in- tus cō- tingāt, atque acce- pta,

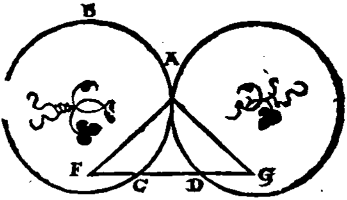


fuerint

fuerint eorum contra; ad eorum centra adiuncta recta linea, & producta, incōtactum circulorum cadet.

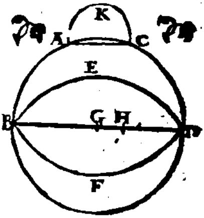
Theorema 11. Propoſitio 12.

Si duo circuli ſeſe exterius contingant, linea recta, quæ ad centra eorum adiungitur, per contactum illum tranſibit.



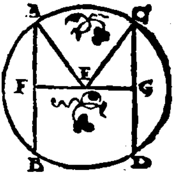
Theorema 12. Propoſitio 13.

Circulus circulum non tangit in pluribus punctis, quam vno, ſiue intus ſiue extra tangat.



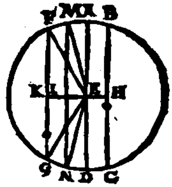
Theorema 13. Propoſitio 14.

In circulo æquales rectæ lineæ æqualiter diſtant à centro. Et quæ æqualiter diſtant à centro, æquales ſunt inter ſe.



Theo-

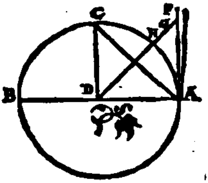
Theorema 14. Propoſitio 15.



In circulo maxima quidem linea eſt diameter: aliarum autem propinquior centro, remotiore ſemper maior.

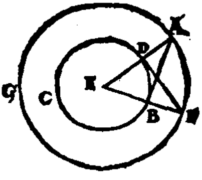
Theorema 15. Propoſitio 16.

Quæ ab extremitate diametri cuiuſque circuli ad angulos rectos ducitur, extra ipſum circulum cadet; & in locum inter ipſam rectam lineam, & peripheriam comprehẽſum, altera recta linea non cadet. Et ſemicirculi quidem angulus quouis angulo acuto rectilineo maior eſt, reliquus autem minor.



Problema 2. Propoſitio 17.

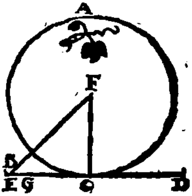
A dato puncto rectam lineam ducere, quæ datum tangat circulum.



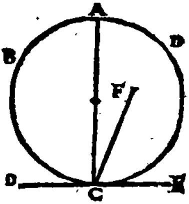
Theorema 16. Propoſitio 18.

Si circulum tangat recta quæpiam linea, à centro

centro autem ad contactum adiungatur recta quædam linea: quæ adiuncta fuerit, ad ipsam contingentem, perpendicularis erit.

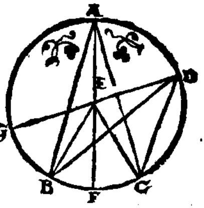


Theorema 17. Propositio 19.



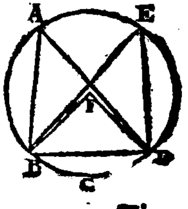
Si circulum tetigerit recta quæpiam linea, à contactu autem recta linea ad angulos rectos ipsi tangenti excitetur; in excitata erit centrum circuli.

Theorema 18. Propositio 20.



In circulo, angulus ad centrum duplex est anguli ad peripheriam, cùm fuerit eadem peripheria basis angulorum.

Theorema 19. Propositio 21.

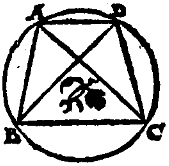


In circulo, qui in eodem segméto sunt, anguli, sunt inter se æquales.

Theo-

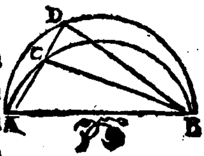
Theorema 20. Propositio 22.

Quadrilaterorum in circulis descriptorum anguli, qui ex aduerso, duob. rectis sunt æquales.

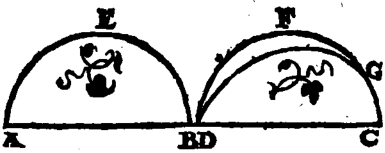


Theorema 21. Propositio 23.

Super eadem recta linea duo segmenta circulorum similia, & inæqualia non constituentur ad easdem partes.



Super æqualibus rectis lineis, similia circulorum segmenta, sunt inter se æqualia.

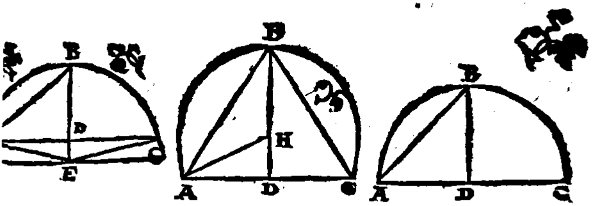


Problema 3. Propositio 25.

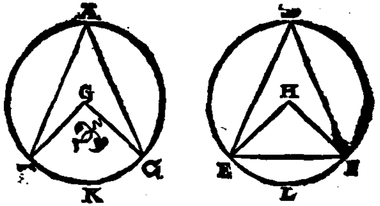
Circuli segmento dato, describere circulũ,

 cuius

cuius eſt ſegmentum.

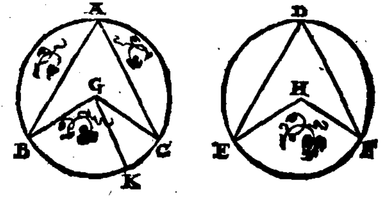


Theorema 23. Propoſitio 26.



In æqualibus circulis, æquales anguli æqualibus peripherijs inſiſtunt, ſiue ad cẽtra, ſiue ad peripherias conſtituti, inſiſtant.

Theorema 24. Propoſitio 27.

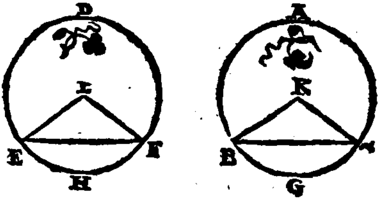


In æqualibus circulis, anguli, qui æqualibus peripherijs inſiſtunt, ſunt inter ſe æquales, ſiue ad centra, ſiue ad peripherias conſtituti, inſiſtant.

Theo-

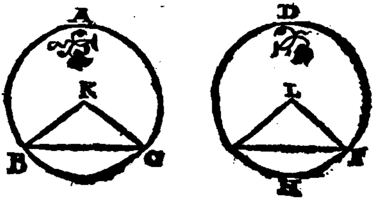
Theorema 25. Propositio 28.

In æqualibus circulis, æquales rectæ lineæ, æquales periphe rias auferunt; maiorē quidem maiori, minorem autem minori.



Theorema 26. Propositio 29.

In æqualib. circulis æquales periphe rias, æquales rectæ lineæ subtendunt.

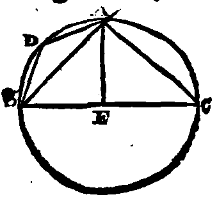


Problema 4. Propositio 30.

Datam peripheriam bifariam secare.

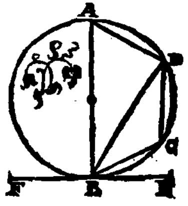
Theorema 27. Propositio 31.

In circulo angulus, qui in semicirculo, rectus eſt: qui autem in maiore ſegmento, minor recto: qui verò in minore ſegmento, maior eſt recto. Et inſuper angulus maioris ſegmenti, recto quidem maior eſt, minoris autem ſegmenti angulus, minor eſt recto.



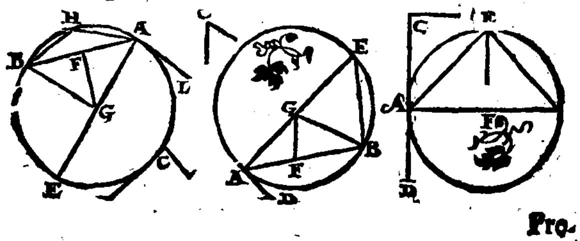
Theorema 28. Propoſitio 32.

Si circulum tetigerit aliqua recta linea, à cõtactu autem producatur quædam recta linea circulum ſecãs: anguli, quos ad contingentem facit, æquales ſunt ijs, qui alternis circuli ſegmentis conſiſtunt, angulis.



Problema 5. Propoſitio 33.

Super data recta linea deſcribere ſegmentũ circuli, quod capiat angulum æqualem dato angulo rectilineo

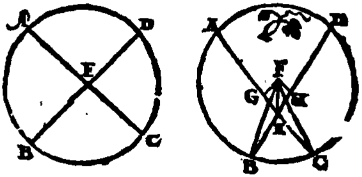


Problema 6. Propositio 34.

A dato circulo segmentum abscindere, capiés angulum ęqualem dato angulo rectilineo.

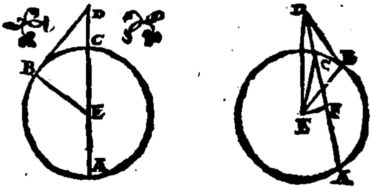
Theorema 29. Propositio 35.

Si in circulo duæ rectæ lineæ sese mutuò secuerint; rectangulum comprehensum sub segmentis vnius, æquale est ei, quod sub segmentis alterius comprehenditur, rectangulo.



Theorema 30. Propositio 36.

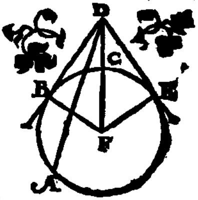
Si extra circulum sumatur pũctũ aliquod, ab eoq; in circulum cadant duæ rectæ lineæ; quarum altera quidem circulum secet, altera verè



verò tangat: quod ſub tota ſecante,& exterius inter punctum & conuexam peripheriam aſſumpta comprehenditur; rectangulum; æquale erit ei, quod à tangente deſcribitur, quadrato.

Theorema 31. Propoſitio 37.

Si extra circulum ſumatur punctum aliquod, ab eoque puncto in circulum cadant duæ rectæ lineæ, quarum altera circulum ſecet, altera in eum incidat; ſit autem, quod ſub tota ſecante, & exterius inter punctum, & conuexam peripheriam aſſumpta, comprehenditur rectangulum, æquale ei, quod ab incidente deſcribitur, quadrato; incidens ipſa circulum tanget.



FINIS ELEMENTI III.

EVCLI.

EVCLIDIS ELEMENTVM QVARTVM.

DEFINITIONES.

1.

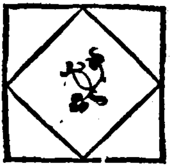
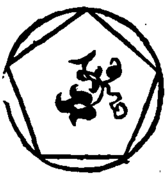
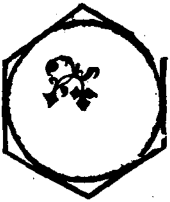


Figura rectilinea in figura rectilinea inſcribi dicitur, cum ſinguli eius figuræ; quæ inſcribitur, angali, ſingula latera eius, in qua inſcribitur, tangunt.

2.

Similiter & figura circum figuram deſcribi dicitur, quum ſingula eius, quę circumſcribitur, latera ſingulos eius figuræ angulos tetigerint, circum quam illa deſcribitur.





3.

Figura rectilinea in circulo inſcribi dicitur, quum ſinguli eius figuræ, quæ inſcribitur, anguli tetigerint circuli peripheriam.

4.

Figura verò rectilinea circa circulum describi dicitur, quùm singula latera eius, quæ circumscribitur, circuli peripheriam tangunt.

5.

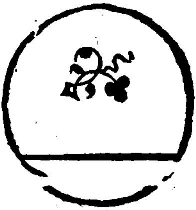
Similiter & circulus in figura rectilinea inscribi dicitur, quũ circuli peripheria singula latera tangit eius figuræ, cui inscribitur.

6.

Circulus autem circum figuræ describi dicitur, quum circuli peripheria singulos tangit eius figuræ, quam circumscribit, angulos.

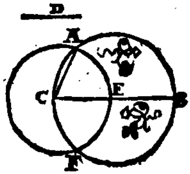
7.

Recta linea in circulo accommodari, seu coaptari dicitur, quum eius extrema in circuli peripheria fuerint.



Problema 1. Propositio 1.

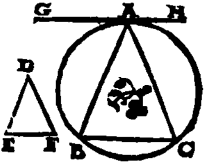
In dato circulo, rectam lineam accommodare æqualem datæ rectæ lineæ, quæ circuli diametro non sit maior.



Pro-

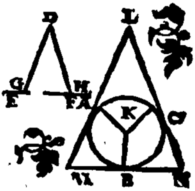
Problema 2. Propositio 2.

In dato circulo, triangulum deſcribere, dato triangulo æquiangulum.



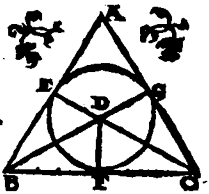
Problema 3. Propoſitio 3.

Circa datum circulum triangulum deſcribere, dato triangulo æquiangulum.



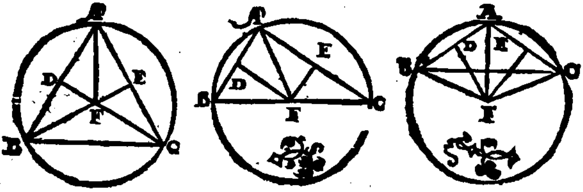
Problema 4. Propoſitio 4.

In dato triangulo circulum inſcribere.



Problema 5. Propoſitio 5.

Circa datum triangulum, circulum deſcribere.



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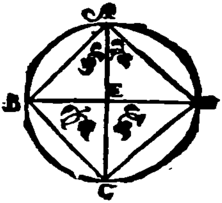
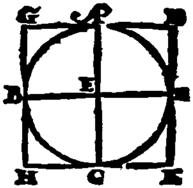
Problema 6. Propositio 6.



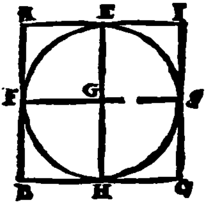
In dato circulo quadratum describere.

Problema 7. Propositio 7.



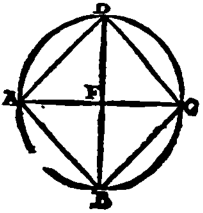
Circa datum circulum, quadratum describere.

Problema 8. Propositio 8.



In dato quadrato circulum inscribere.

Problema 9. Propositio 9.

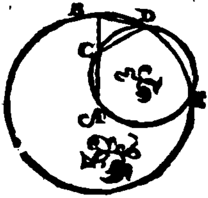


Circa datum quadratũ, circulum describere.

Pro-

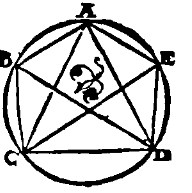
Problema 10. Propoſitio 10.

Iſoſceles triangulum cõſtituere; quod habeat vtrunque eorum, qui ad baſin ſunt, angulorum, duplum reliqui.



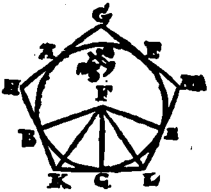
Problema 11. Propoſitio 11.

In dato circulo, pentagonũ æquilaterum, & æquiangulum inſcribere.



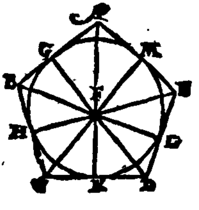
Problema 12. Propoſitio 12.

Circa datum circulum, pentagonum, æquilaterum & æquiangulum deſcribere.

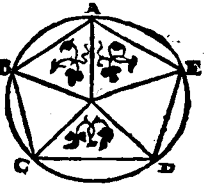


Problema 13. Propoſitio 13.

In dato pentagono ęquilatero, & ęquiangulo circulum inſcribere.



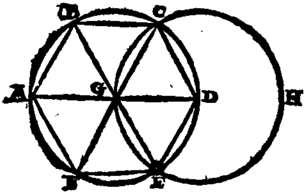
Problema 12. Propositio 14.

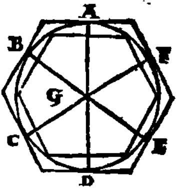


Circa datum pentagonum æquilaterum & æquiangulum, circulum describere.

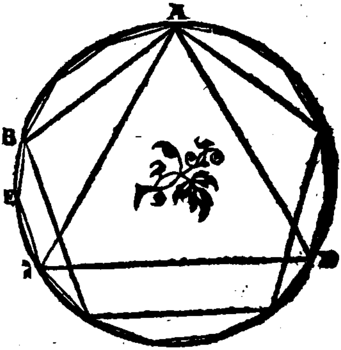
Problema 15. Propositio 15.

In dato circulo hexagonum & æquilaterum, & æquiangulum inscribere.





Problema 16. Propositio 16.



In dato circulo quinti decagonum & æquilaterũ, & æquiangulum describere.

FINIS ELEMENTI IV.

EVCLIDIS ELEMENTVM QVINTVM.

DEFINITIONES.

1.

PArs est magnitudo magnitudinis minor maioris, quum minor metitur maiorem.

2.

Multiplex autem est maior minoris, cùm minor metitur maiorem.

3.

Proportio, est duarum magnitudinum eiusdem generis mutua quędam secundum quanti talem habitudo.

4.

Proportionalitas verò est proportionum similitudo.

5.

Proportionem habere inter se magnitudines dicuntur, quæ possunt multiplicatæ sese mutuò superare.

6.

In eadē proportione magnitudines dicūtur esse, prima ad secundam, & tertia ad quartam, cùm primæ tertiæ æquè multiplicia, à secundæ & quartæ æquè multiplicibus, qua-

qualiſcunque ſit hæc multiplicatio, vtrunque ab vtroque; vel vnà deficiunt, vel vnà æqualia ſunt, vel vnà excedunt; ſi ea ſumantur, quæ inter ſe reſpondent.

7.

Eandem autem proportionem habentes magnitudines, proportionales vocentur.

8.

Cùm verò æquè multiplicium, multiplex primæ magnitudinis exceſſerit multiplicẽ ſecundæ; at multiplex tertiæ non exceſſerit multiplicem quartæ: tunc prima ad ſecundam, maiorem proportionem habere dicetur, quàm tertia ad quartam.

9.

Proportionalitas autem in tribus terminis pauciſſimis conſiſtit.

10.

Cùm autem tres magnitudines proportionales fuerint, prima ad tertiam, duplicatam proportionem habere dicitur eius, quam habet ad ſecundam. At cùm quatuor magnitudines proportionales fuerint, prima ad quartam, triplicatam proportionem habere dicitur eius, quam habet ad ſecundam: & ſemper deinceps, vno amplius, quamdiu proportio extiterit.

11.

Homologæ, ſeu ſimiles proportione magnitudines dicuntur, antecedentes quidem ante-

antecedentibus, consequentes verò consequentibus.

12.

Alterna proportio, est sumptio antecedentis ad antecedentem, & consequentis ad consequentem.

13.

Inuersa proportio, est sumptio consequentis, ceu antecedentis, ad antecedentem, velut ad ipsam consequentem.

14.

Compositio proportionis, est sumptio antecedentis cum consequente, ceu vnius ad ipsam consequentem.

15.

Diuisio proportionis, est sumptio excessus, quo consequentem superat antecedens, ad ipsam consequentem.

16.

Conuersio proportionis, est sumptio antecedentis ad excessum, quo superat antecedens ipsam consequentem.

17.

Ex æqualitate proportio est, si plures duabus sint magnitudines, & his aliæ multitudine pares, quæ binæ sumantur, & in eadem proportione: quum vt in primis magnitudinibus prima ad vltimam, sic & in secundis magnitudinibus prima ad vltimam sese habuerit.

Vel aliter, ſumptio extremorum per ſubdu-ctionem mediorum.

18.

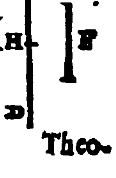
Ordinata proportio eſt, cùm fuerit, quem-admodum antecedens ad conſequentem, ita antecedens ad conſequentem: fuerit etiam vt conſequens ad aliud quidpiam, ita conſe-quens ad aliud quidpiam.

19.

Perturbata autem proportio eſt, cùm tribus poſitis magnitudinibus, & alijs quæ ſint his multitudine pares, vt in primis quidem ma-gnitudinibus ſe habet antecedens ad conſe-quentem, ita in ſecundis magnitudinibus antecedens ad conſequentem: vt autem ali-ud quidpiam ſic in ſecundis magnitudini-bus aliud quidpiam ad antecedentem.

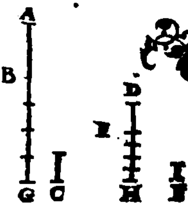
Theorema 1. Propoſitio 1.

Si ſint quotcunq; magnitudines, quotcunq; magnitudinum æqua-lium numero, ſingulæ ſingula-rum, æquè multiplices; quàm multiplex eſt vnius vna magnitu-do, tam multiplices erunt & om-nes omnium.



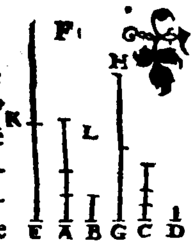
Theorema 2. Propositio 2.

Si prima secundæ æquè fuerit multiplex, atque tertia quartæ; fuerit autem & quinta secũdæ æquè multiplex, atq; sexta quartæ: erit & composita prima cum quinta, secundæ æquè multiplex; atque tertia cum sexta, quartæ.



Theorema 3. Propositio 3.

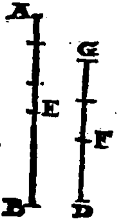
Si sit prima secundæ æquè multiplex, atq; tertia quartæ, sumantur autem æquè multiplices primæ, & tertiæ; erit & ex æquo, sumptarum vtraque vtriusque æquè multiplex; altera quidem secundæ, altera autem quartæ.



Theorema 4. Propositio 4.

Si prima ad secundã, eandem habuerit proportionem, & tertia ad quartam: etiam æquè multiplices primæ & tertiæ, ad æquè multiplices secundæ & quartæ, iuxta quamuis multiplicationẽ, ean-

dem habebunt propofitionem; fi, prout inter fe refpondent, ita fumptæ fuerint.



Theorema 5. Propofitio 5.

Si magnitudo magnitudinis æquè fuerit multiplex, atque ablata ablatæ: etiam reliqua reliquæ ita multiplex erit, vt tota totius.

Theorema 6. Propofitio 6.

Si duæ magnitudines, duarum magnitudinum fint æquè multiplices, & detractæ quædam fint earundem æquè multiplices: & reliquæ eifdem aut æquales funt, aut æquè ipfarum multiplices.

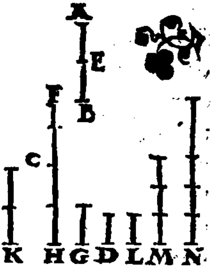
Theorema 7. Propofitio 7.

Aequales ad eandem, eandem habent rationem: & eadem ad æquales.

Theorema 8. Propofitio 8.

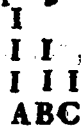
Inæqualium magnitudinum maior ad eandem

dem, maiorem proportionem habet: quàm minor: & eadem ad minorem, maiorem proportionem habet, quàm ad maiorem.



Theorema 9. Propositio 9.

Quæ ad eandem, eandem habent proportionem, æquales sunt inter se: & ad quas eadem, eandem habet proportionem, eæ quoque sunt inter se æquales.

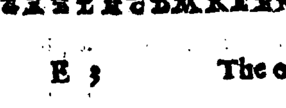


Theorema 10. Propositio 10.

Ad eandem magnitudinem proportionem habentium, quę maiorem proportionem habet, illa maior est, ad quam autem eadem maiorem proportionem habet, illa minor est.

Theorema 11. Propositio 11.

Quæ eidem sunt eædem proportiones, & inter se sunt eædem.



Theorema 12. Propositio 12.

Si sint magnitudines quotcunque proportionales; quemadmodũ se habuerit vna antecedentium ad vnā

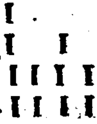
consequentium; ita se habebunt omnes antecedentes ad omnes consequentes.

Theorema 13. Propositio 13.

Si prima ad secundam, eādem habuerit proportionem, quam tertia ad quartam; tertia verò ad quartam maiorem proportionem habuerit, quam quinta ad sextam: prima quoque ad secũdam maiorē proportionem habebit, quam quinta ad sextam.

Theorema 14. Propositio 14.

Si prima ad secundam eandem habuerit proportionem, quā tertia ad quartā: prima verò quam tertia maior fuerit, erit & secunda maior, quam quarta.



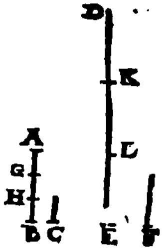
Quod A B C D si prima fuerit æqualis tertiæ,

erit

erit secunda æqualis quartæ: si verò minor, & minor erit.

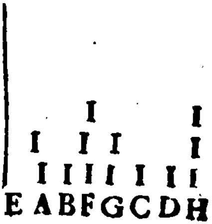
Theorema 15. Propositio 15.

Partes cum pariter multiplicibus in eadem sunt proportione, si prout sibi mutuò respondent, ita sumantur.



Theorema 16. Propositio 16.

Si quatuor magnitudines proportionales fuerint, & vicissim proportionales erunt.



Theorema 17. Propositio 17.

Si compositæ magnitudines proportionales fuerint, hæ quoq; diuisæ proportionales erunt.

Theorema 18. Propoſitio 18.

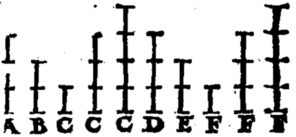
Si diuiſæ magnitudines ſint proportionales, hæ quoque compoſitæ proportionales erunt.

Theorema 19. Propoſitio 19.

Si quemadmodum totum ad totum, ita ablatum ſe habuerit ad ablatum: & reliquum ad reliquum, vt totum ad totum, ſe habebit.

Theorema 20. Propoſitio 20.

Si ſint tres magnitudines, & aliæ ipſis æquales numero, quæ binæ, & in eadem proportione ſumantur; ex æquo autem prima quàm tertia maior fuerit; erit & quarta, quã ſexta, maior.

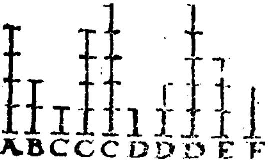


Quod ſi prima tertiæ fuerit æqualis, erit & quarta æqualis ſextæ: ſin illa minor, quoque minor erit.

Theo-

Theorema 21. Propoſitio 21.

Si ſint tres magnitudines, & aliæ ipſis æquales numero, quæ binæ, & in eadem proportione ſumantur, fueritque perturbata earum proportio : ex æquo autẽ prima, quam tertia, maior fuerit; erit & quarta, quàm ſexta, maior : quod ſi prima tertiæ fuerit æqualis, erit & quarta æqualis ſextæ : ſin illa minor, hæc quoque minor erit.

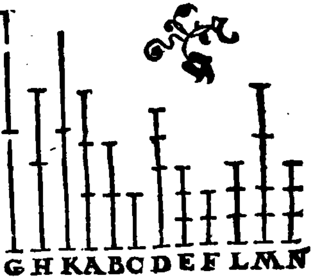


Problema 22. Propoſitio 22.

Si ſint quodcunque magnitudines, & aliæ ipſis æquales numero, quæ binæ in eadem proportione ſumantur : & ex æqualitate in eadem proportione erunt.

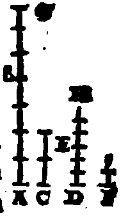
Theorema 23. Propoſitio 23.

Si ſint tres magnitudines, aliæq; ipſis æquales numero, quę binæ in eadem proportione ſumantur; fuerit autem perturbata earum proportio: Etiam ex æqualitate in eadem proportione erunt.



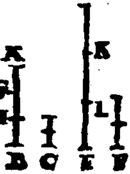
Theorema 24. Propoſitio 24.

Si prima ad ſecundam, eandem habuerit proportionem, quam tertia ad quartam; habuerit autem & quinta ad ſecundam, eandem proportionem, quàm ſexta ad quartam: Etiam compoſita prima cum quinta, ad ſecundam eandem habebit proportionem, quam tertia cum ſexta, ad quartam.



Theorema 25. Propoſitio 25.

Si quatuor magnitudines proportionales fuerint; maxima, & minima reliquis duabus maiores erunt.



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Theorema 26. Propositio 26.

Si prima ad ſecundam, maiorem habuerit proportionem, quàm tertia ad quartam: habebit conuertendo ſecunda ad primam, minorem proportionem, quàm quarta ad tertiam.

Theorema 27. Propositio 27.

Si prima ad ſecundam, habuerit maiorem proportionem, quàm tertia ad quartam: habebit quoque viciſſim prima ad tertiam, maiorem proportionem, quàm ſecunda ad quartam.

Theorema 28. Propositio 28.

Si prima ad ſecundam habuerit maiorem proportionem, quàm tertia ad quartam: habebit quoque compoſita prima cum ſecunda, ad ſecundam, maiorem proportionem, quàm compoſita tertia cum quarta, ad quartam.

Theorema 29. Propositio 29.

Si compoſita prima cum ſecunda, ad ſecundam, maiorem habuerit proportionem quã com-

composita tertia cum quarta, ad quartam: Habebit quoq; diuidendo prima ad secundam, maiorem proportionem, quam tertia, ad quartam.

Theorema 30. Propositio 30.

Si composita prima cum secunda, ad secundam habuerit maiorem proportionem, quã composita tertia cum quarta, ad quartam: Habebit quoq; per conuersionem proportionis, prima cum secunda, ad primam, minorem proportionem, quàm tertia cum quarta, ad tertiam.

Theorema 31. Propositio 31.

Si sint tres magnitudines, & aliæ ipsis æquales numero; sitque maior proportio primæ priorum ad secundam, quàm primæ posteriorum ad secundam; Item secundæ priorum ad tertiam maior, quàm secundæ posteriorum ad tertiam: Erit quoque ex æqualitate maior proportio primæ priorum ad tertiam, quàm primæ posteriorum ad tertiam.

Theorema 32. Propositio 32.

Si sint tres magnitudines, & aliæ ipsis æquales numero; sitque maior proportio primæ priorum ad secundã, quàm secundæ posteriorum

riorum ad tertiam; Item secundæ priorum ad tertiam maior, quàm primæ posteriorum ad secundam: Erit quoq; ex æqualitate, maior proportio primæ priorum ad tertiam, quàm primæ posteriorum ad tertiam.

Problema 33. Propositio 33.

Si fuerit maior proportio totius ad totum, quàm ablati ad ablatum: Erit & reliqui ad reliquum, maior proportio, quàm totius ad totum.

Theorema 34. Propositio 34.

Si sint quotcunque magnitudines, & aliæ ipsis æquales numero; sitque maior proportio primę priorum ad primam posteriorum, quàm secundæ ad secundam; & hæc maior, quàm tertiæ ad quartam, & sic deinceps: Habebunt omnes priores simul, ad omnes posteriores simul, maiorem proportionem quàm omnes priores, relicta prima, ad omnes posteriores, relicta quoque prima; minorem autem, quàm prima priorum ad primam posteriorum; maiorem denique etiam, quàm vltima priorum ad vltimam posteriorum.

FINIS ELEMENTI V.

EVCLIDIS ELEMENTVM SEXTVM.

DEFINITIONES.

1.

Similes figuræ rectilineæ sunt, quæ & angulos singulos singulis æquales habent; atque etiam latera, quæ circum angulos æquales, proportionalia.

2.

Reciprocæ autem figuræ sunt, cùm in vtraque figura, antecedentes, & consequentes proportionum termini fuerint.

3.

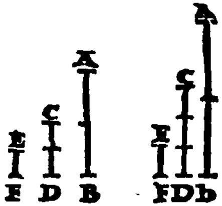
Secundum extremam, & mediam proportionem recta linea secta esse dicitur, cùm vt tota ad maius segmentum, ita maius ad minus se habuerit.

4.

Altitudo cuiusque figuræ est linea perpendicularis à vertice ad basin deducta.

5.

Proportio ex proportionibus componi dicitur, cùm proportionum quantitates inter se multiplicatæ, aliquam effecerint proportionem.

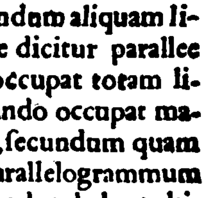


6.

Parallelogrammum secundum aliquam lineam applicatum, deficere dicitur parallelogrammo, quando non occupat totam lineam: Excedere vero, quando occupat maiorem lineam, quàm sit ea, secundum quam applicatur: Ita tamen, vt parallelogrammum deficiens, aut excedens, eandem habeat altitudinem cum parallelogrammo applicato, constituatq́ue cum eo totum vnum parallelogrammum.

Theorema 1. Propositio 1.

Triangula, & parallelogramma, quorum eadem fuerit altitudo, ita se habent inter se, vt bases.

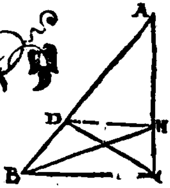


Theorema 2. Propositio 2.

Si ad vnum trianguli latus parallela ducta fuerit recta quædam linea.

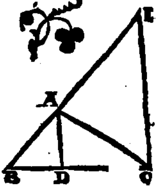
Hæc proportionaliter secabit ipsius trianguli

anguli latera. Etsi trianguli latera proportionaliter secta fuerint; quæ ad sectiones adiuncta fuerit recta linea, erit ad reliquum ipsius trianguli latus parallela.



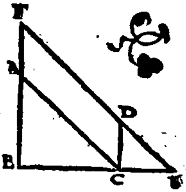
Theorema 3. Propositio 3.

Si trianguli angulus bifariam sectus sit; secans autem angulum recta linea secuerit & basin: basis segmenta, eandem habebunt proportionem, quã reliqua ipsius trianguli latera Et si basis segmenta eandem habeant proportionem quam reliqua ipsius trianguli latera; recta linea, quæ à vertice ad sectionem producitur, bifariam secat trianguli ipsius angulum.



Theorema 4. Propositio 4.

Aequiangulorum triangulorum proportionalia sunt latera, quæ circum ęquales angulos, & homologa sunt latera, quę æqualibus angulis subtenduntur.



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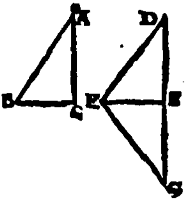
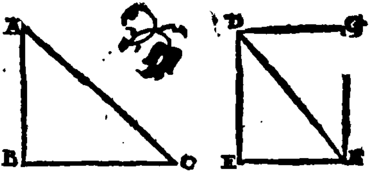
Theorema 5. Propositio 5.

Si duo triangula, latera proportionalia habeant, æquiangula erunt triangula, & æquales habebunt eos angulos, sub quibus homologa latera subtenduntur.



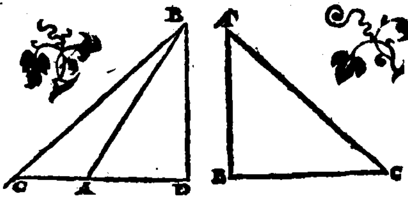
Theorema 6. Propositio 6.

Si duo triangula vnum angulum vni angulo æqualem, & circum æquales angulos latera proportionalia habuerint: æquiangula erunt triangula, æqualesque habebunt angulos, sub quibus homologa latera subtenduntur.



Theorema 7. Propositio 7.

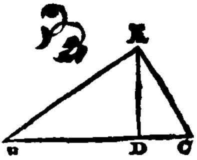
Si duo triangula vnum angulum vni angulo æqualẽ, circũ autem alios angulos latera proportionalia habent; reliquorum verò



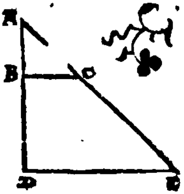
rò simul vtrunque aut minorem, aut non minorem recto: æquiangula erunt triangula, & æquales habebunt eos angulos, circum quos proportionalia sunt latera.

Theorema 8 Propositio 8.

Si in triangulo rectangulo, ab angulo recto in basin perpendicularis ducta sit; quæ ad perpendicularem triangula, tum toti triangulo, tum ipsa inter se similia sunt.

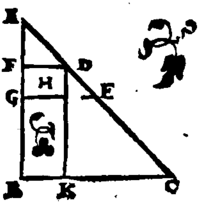


Problema 1. Propositio 9.



A data recta linea imperatam partem auferre.

Problema 2. Propositio 10.



Datam rectam lineam insectam similiter secare, vt data altera recta secta fuerit.

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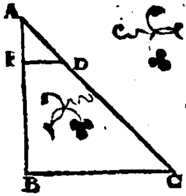
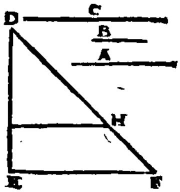
Problem a 3. Propositio 11.

Duabus datus rectis lineis, tertiam proportionalem adinueniri.



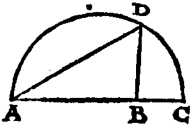
Problema 4. Propositio 12.

Tribus datis rectis lineis quartam proportionalem adinuenire.



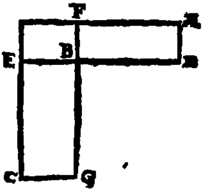
Problema 5. Propositio 13.

Duabus datis rectis lineis, mediam proportionalem adinuenire.



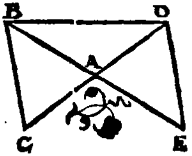
Theorema 9. Propositio 14.

Aequalium, & vnum vni æqualem habentium angulum, parallelogrammorum reciproca sunt latera, quæ circum ęquales angulos; Et quorum parallelogrammorum vnum angulum vni angulo æqualem habentium reciproca sunt latera, quæ circum æquales angulos, illa sunt æqualia.



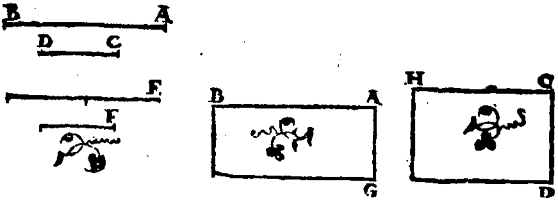
Theorema 10. Propositio 15.

Aequalium, & vnum angulum vni æqualem habentium triangulorum reciproca sunt latera, quæ circum ęquales angulos: Et quorum triangulorum vnum angulum vni ęqualem habentium, reciproca sunt latera, quæ circum æquales angulos, illa sunt ęqualia.



Theorema 11. Propositio 16.

Si quatuor rectæ lineæ proportionales fue-

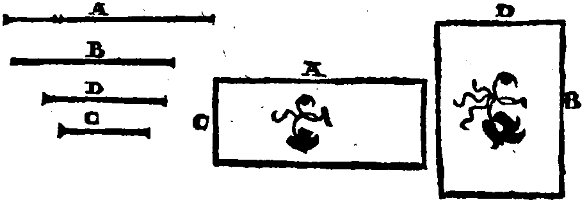


rint, quod sub extremis compræhenditur rectangulum, ęquale est ei, quod sub medijs com-

comprehenditur, rectangulo. Etsi sub extremis comprehensum rectangulum æquale fuerit ei, quod sub medijs continetur, rectangulo, illæ quatuor rectæ lineæ proportionales erunt.

Theorema 12. Propositio 17.

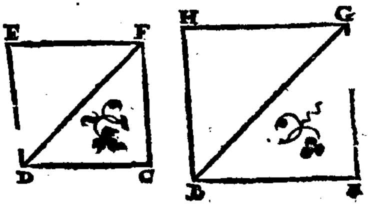
Si tres rectæ lineæ sint proportionales; quod sub extremis comprehenditur rectangulum



æquale sit ei, quod à media describitur, quadrato: Et si sub extremis comprehensum rectangulum æquale sit ei, quòd à media describitur, quadrato, illæ tres rectę lineæ proportionales erunt.

Problema 16. Propositio 16.

A data recta linea, dato rectilineo simile, similiterq; positum rectilineum describere.



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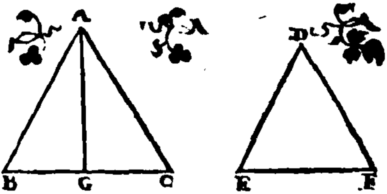
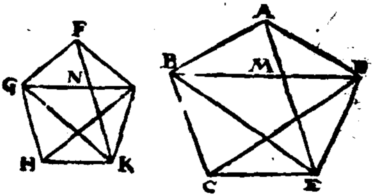
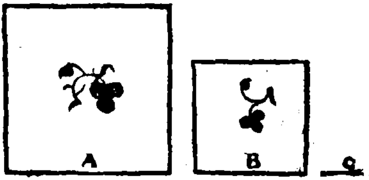
Theorema 13. Propositio 19.

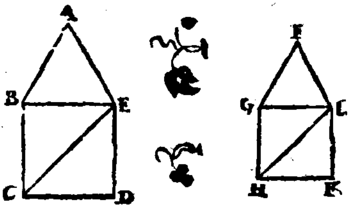


Similia triangula inter se sunt in duplicata proportione laterum homologorum.

Theorema 14. Propositio 20.







Similia polygona in similia triangula diuiduntur & numero æqualia, & homologa totis. Et polygona duplicatam habent eam inter se proportionem, quam latus homologum ad homologum latus.

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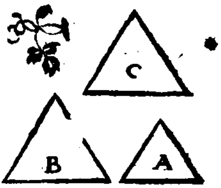
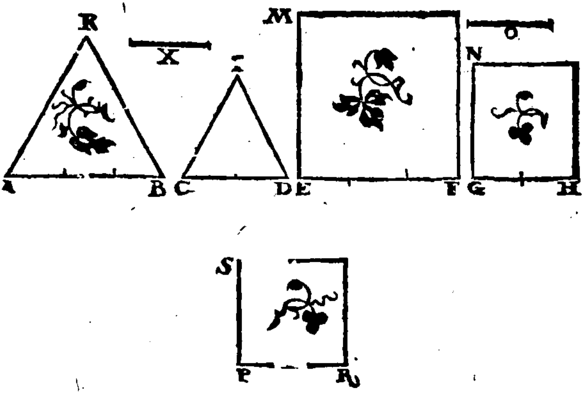
Theorema 15. Propoſitio 21.

Quæ eidem rectilineo ſunt ſimilia, & inter ſe ſunt ſimilia.

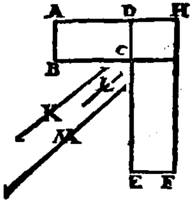


Theorema 16. Propoſitio 22.

Si quatuor rectæ lineæ proportionales fuerint: & ab eis rectilinea ſimilia ſimiliterque deſcripta proportionalia erunt. Et ſi à rectis lineis ſimilia ſimiliterque deſcripta rectilinea proportionalia fuerint; ipſæ etiam rectæ lineæ proportionales erunt.

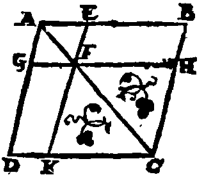


Theorema 17. Propositio 23.



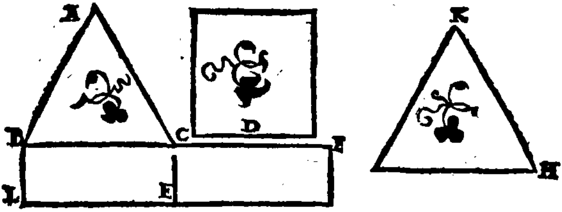
Aequiangula parallelogramma inter se proportionem habent eam, quæ ex lateribus componitur.

Theorema 18. Propositio 24.



In omni parallelogrammo, quæ circa diametrũ sunt parallelogramma, & toti, & inter se sunt similia.

Problema 7. Propositio 25.



Dato recto lineo simile: similiterq́ue positum; & alteri dato æquale idem constituere.

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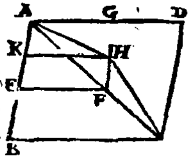
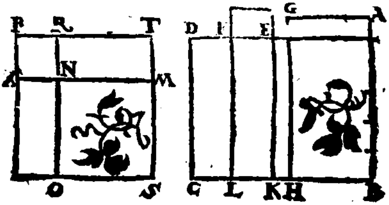
Theorema 19. Propositio 26.



Si à parallelogrammo parallelogrammum ablatũ sit, & simile toti, & similiter positum, communem cum eo habens angulum; hoc circum eandem cum toto diametrum consistit.

Theorema 20. Propositio 27.

Omnium parallelogrammorum secundum eandem rectam lineam applicatorum, deficientiumq; figuris parallelogrammis similibus,



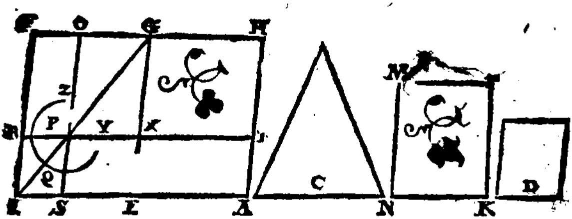
similiterque positis, ei, quod à dimidia describitur; maximum, id est, quod ad dimidiam applicatur, parallelogrammum simile existens defectui.

Problema 8. Propositio 28.

Ad datam lineam rectam, dato rectilineo æquale parallelogrammum applicare, deficiens figura parallelogramma, quæ similis

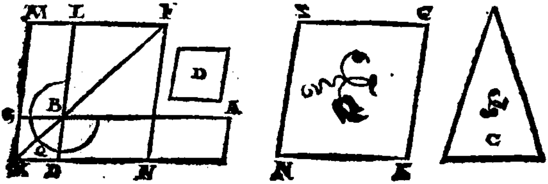
 sit

ſit alteri rectilineo dato. Oportet autem datum rectilineum, cui æquale applicandum eſt, non maius eſſe eo, quod ad dimidiam applicatur, cùm ſimiles ſint defectus & eius, quod à dimidia deſcribitur, & eius, cui ſimile deeſſe debet.



Problema 9. Propoſitio 29.

Ad datam rectam lineam, dato rectilineo æquale parallelogrammum applicare, excedens figura parallelogramma, quæ ſimilis ſit parallelogrammo alteri dato.



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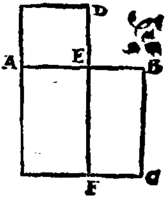
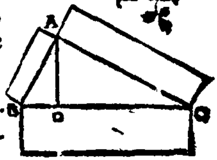
Problema 10. Propositio 30.



Propositam rectam lineam terminatam; extrema ac media ratione (proportione.) secare.

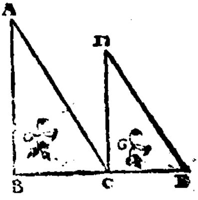
Theorema 21. Propositio 31.



In rectangulis triangulis, figura quæuis à latere rectum angulum subtendente descripta, æqualis est figuris, quæ priori illi similes; & similiter positæ, à laterib. rectum angulum continentibus describuntur.

Theorema 22. Propositio 32.

Si duo triangula, quæ duo latera duobus lateribus proportionalia habeant, secundum vnum angulum composita fuerint, ita vt homologa eorum latera sint etiam parallela; tum reliqua illorum triangulorum latera in rectam lineam collocata reperientur.



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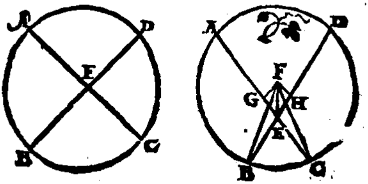
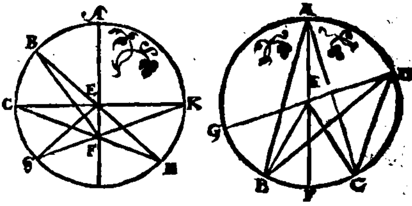
Theorema 23. Propoſitio 33.

In æqualibus circulis, anguli eandem habent rationem, cum ipſis peripherijs, in quibus inſiſtunt, ſiue ad centra, ſiue ad peripherias conſtituti, illis inſiſtant peripherijs. Inſuper verò & ſectores, quippe qui ad centra conſiſtunt.





FINIS ELEMENTI VI.

EVCLI-

EVCLIDIS ELEMENTVM SEPTIMVM.

DEFINITIONES.

1.

VNitas est, secũdum quam vnumquodque eorum, quæ sunt, vnum dicitur.

2.

Numerus autem, ex vnitatibus composita multitudo.

3.

Pars est numerus numeri, minor maioris, cum minor metitur maiorem.

4.

Partes autem, cùm non metitur.

5.

Multiplex verò, maior minoris, cùm maiorem metitur minor.

6.

Par numerus est, qui bifariam diuiditur.

7.

Impar verò, qui bifariam non diuiditur: vel qui vnitate differt à pari.

8.

Pariter par numerus est, quem par numerus metitur per numerum parem.

9.

Pariter autem impar est, quem par numerus metitur per numerum imparem.

10.

Impariter verò impar numerus èst. quem impar numerus metitur per numerum imparem.

11.

Primus numerus est, quem vnitas sola metitur.

12.

Primi inter se numeri sunt, quos sola vnitas, mensura communis, metitur.

13.

Compositus numerus est, quem numerus quispiam metitur.

14.

Compositi autem inter se numeri sunt, quos numerus aliquis, mensura communis, metitur.

15.

Numerus numerum multiplicare dicitur, cùm toties compositus fuerit is, qui multiplicatur, quot sunt in ipso multiplicante vnitates; & procreatus fuerit aliquis.

16.

Cùm autem duo numeri mutuò sese multiplicantes quempiam faciunt, qui factus erit, planus appellabitur. Qui verò numeri mutuò sese multiplicarint, illius latera dicentur.

Cùm

17.

Cùm verò tres numeri mutuò sese multiplicantes quempiam faciunt, qui procreatus erit, solidus appellabitur, qui autem numeri mutuò sese multiplicarint, illius latera dicentur.

18.

Quadratus numerus est, qui æqualiter æqualis: vel, qui à duabus æqualibus numeris continetur.

19.

Cubus verò, qui æqualiter æquali æqualiter: vel, qui à tribus æqualibus numeris continetur.

20.

Numeri proportionales sunt, cùm primus secundi, & tertius quarti, æquè multiplex est; vel eadem pars, vel eædem partes.

21.

Similes plani, & solidi numeri sunt, qui proportionalia habent latera.

22.

Perfectus numerus est, qui suis ipsius partibus est æqualis.

23.

Numerus numerum metiri dicitur per illum numerum, quem multiplicans, vel à quo multiplicatus, illum producit.

Pro-

24.

Proportio numerorum est habitudo quædam vnius numeri & alterum, secundum quod illius est multiplex, vel pars, partesue.

25.

Termini, siue radices proportionis dicuntur duo numero, quibus in eadem proportione minores sumi nequeunt.

26.

Cùm tres numeri proportionales fuerint; primus ad tertium, duplicatam proportionem dicitur habere eius, quam habet ad secundum. At cùm quatuor numeri proportionales fuerint; primus ad quartum, triplicatam proportionem habere dicitur eius, quam habet ad secundum. Et super deinceps vno amplius, quam diu proportio extiterit.

27.

Quotlibet numerus ordine positis, proportio, prima ad vltimam componi dicitur ex proportionibus primi ad secundum, & secundi ad tertium, & tertij ad quartum, & ita deinceps, donec extiterit proportio.

Postulata, siue petitiones.

1.

Postuletur, cuilibet numero, quotlibet posse sumi æquales, vel multiplices.

2.

Quolibet numero, sumi posse maiorem.

Axio-

Axiomata, siue pronunciata.

1.

Qui numeri æqualium numerorum, vel eiusdem, æquè multiplices sunt, inter se sunt æquales.

2.

Quorum idem numerus æquè multiplex est, vel æquè multiplices sunt æquales, inter se æquales sunt.

3.

Qui numeri æqualium numerorum, vel eiusdem, eadem pars, partes fuerint, inter se æquales sunt.

4.

Quorum idem numerus, vel æquales, eadem pars, vel eædem partes fuerint, æquales inter se sunt.

5.

Vnitas omnem numerum per vnitates, quæ in ipso sunt, hoc est, per ipsummet numerum, metitur.

6.

Omnis numerus seipsum metitur per vnitatem.

7.

Si numerus numerū multiplicans aliquem produxerit, metietur multiplicans productum per multiplicatum; multiplicatus autem eundem per multiplicantem.

8.

Si numerus numerum metiatur, & ille, per quem metitur, eundem metietur per eas, quæ in metiente sunt, vnitates, hoc est, per ipsum numerum metientem.

9.

Si numerus numerum metiens, multiplicet eum, per quem metitur; vel ab eo multiplicatur, illum, quem metitur, producat.

10.

Numerus quotcunque numeros metiens, compositum quoque ex ipsis metitur.

11.

Numerus quemcunq; numerum metiens, metitur quoque omnem numerum, quem ille metitur.

12.

Numerus metiens totum, & ablatum, metitur & reliquum.

Theorema 1. Propositio 1.

Si duobus numeris inæqualibus propositis, detrahatur semper minor de maiore, alterna quadam detractione; neque reliquus vnquam metiatur præcedentem, quoad assumpta sit vnitas: qui principiò propositi sunt numeri, primi inter se erunt.

A		
H		
:		C
F		:
:		G
:	:	:
B	D	E

Pro-

Problema 1. Propositio 2.

Duobus numeris datis non primis inter se, maximam eorum communem mensuram reperire.

A
A : C
E
: E E
: : : :
B D B D
: : : : :
A B C D E
8 6 4 2 3

Problema 2. Propositio 3.

Tribus numeris datis non primis inter se, maximam eorum communem mensuram reperire.

: : : : : :
A B C D E F
18 13 8 6 2 3

Theorema 2. Propositio 4.

Omnis numerus cuiusq; numeri, minor maioris, aut pars est, aut partes.

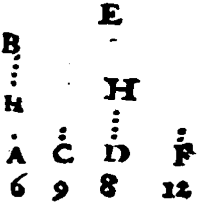
C
:
F
C C :
E
: :
: : : : :
A B B B D
12 7 6 9 3

Theorema 3. Propositio 5.

Si numerus numeri pars fuerit, & alter alterius eadem pars; & simul vterque vtriusque simul eadem pars erit, quæ vnus est vnius.

C
: F
: :
G H
: : : :
A B D C
62 1 5 8

Theorema 4 Propositio 6.

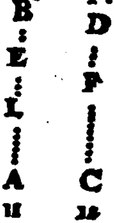


Si numerus sit numeri partes, & alter alterius eædem partes; & simul vterque vtriusque simul eædem partes erunt, quæ sunt vnus vnius.

Theorema 5 Propositio 7.

Si numerus numeri eadem sit pars quæ detractus detracti; & reliquus reliqui eadē pars erit, quæ totus est totius.

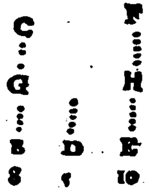
Theorema 6 Propositio 8.



Si numerus numeri eædem sint partes quæ detractus detracti, & reliquus reliqui eædem partes erunt, quæ sunt totus totius

G.M K.N.H.

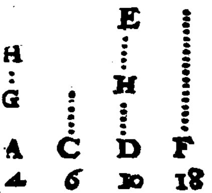
Theorema 7. Propositio 9.



Si numerus numeri pars sit & altera alterius eadem pars: & vicissim, quæ pars est vel partes primus tertij, eadem pars erit vel eædem A partes, & secundus quarti. 4

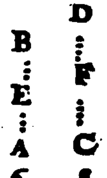
Theorema 8. Propositio 10.

Si numerus numeri partes sint, & alter alterius eadem partes: etiam viciſſim, quæ ſunt partes, aut pars primus tertij, eædem partes erunt, vel pars & ſecundus quarti.



Theorema 9. Propoſitio 11.

Si quemadmodum ſe habet totus ad totum, ita detractus ad detractum: & reliquus ad reliquum ita ſe habebit, vt totus ad totum.



Theorema 10 Propoſitio 12.

Si ſint quotcunque numeri proportionales; quemadmodum ſe habet vnus antecedentium ad vnum conſequentium; ita ſe habebunt omnes antecedentes ad omnes conſequentes.

A	B	C	D
9	6	3	2

Theorema 11. Propoſitio 13.

Si quatuor numeri ſint proportionales; & viciſſim proportionales erunt.

A	B	C	D
12	4	9	3

Theorema 12. Propoſitio 13.

Si ſint quotcunque numeri, & alij illis æquales multitudi-

A	B	C	D	E	F
12	6	3	8	4	2

G 3

ne;

ne, qui bini ſumantur, & in eadem proportione, etiam ex æqualitate in eadem proportione erunt.

Theorema 13. Propoſitio 15.

Si vnitas numerum quempiam metiatur, alter verò numerus alium quendam numerum æquè metiatur; & viciſſim vnitas tertium numerum æquè metietur, atque ſecundus quartum.

			F
	C		L
	H		K
	G		E
A	B	D	
1	3	4	6

Theorema 14 Propoſitio 16.

Si duo numeri mutuò ſeſe multiplicantes faciant aliquos; qui ex illis geniti fuerint, inter ſe æquales erunt.

E	A	B	C	D
1	2	4	8	8

Theorema 15. Propoſitio 17.

Si numerus duos numeros multiplicans faciat aliquos; qui ex illis procreati erunt, eandem proportionẽ inter ſe habebunt, quam multiplicati.

I	A	B	C	D	E
1	3	4	5	12	15

Theorema 16. Propoſitio 18.

Si duo numeri numerũ quempiam multiplicantes faciant aliquos; geniti ex illis eandem habebunt proportionem, quam qui illum multiplicarunt.

A	B	C	D	E
4	5	3	12	15

Theo-

Theorema 17. Propositio 19.

Si quatuor numeri sint proportionales; quod ex primo & quarto fit, numerus, æqualis erit ei, qui ex secundo & tertio, fit, numero: Et si, qui ex primo & quarto sit numerus, æqualis sit ei, qui ex secundo & tertio, fit, numero; illi quatuor numeri proportionales erunt.

A	B	C	D	E	F	G
6	4	3	2	12	12	18

Theorema 18. Propositio 20.

Si tres numeri sint proportionales; qui ab extremis continetur, æqualis est ei, qui à medio efficitur: Et si, qui ab extremis continetur, æqualis sit ei, qui à medio describitur, illi tres numeri proportionales erunt.

A	B	C	D
9	6	4	6

Problema 19. Propositio 21.

Minimi numeri omnium qui eandem cum eis proportionem habent, æqualiter metiuntur numeros eandem cum eis proportionem habentes; maior quidem maiorem, minor verò minorem.

D	L		
G	H		
C	E	A	B
4	3	8	6

Theorema 20. Propositio 22.

Si tres sunt numeri, & alij multitudine illis æquales, qui bini sumantur & in eadem ratione; sit autem perturbata eorum proportio: etiam ex æqualitate in eadem proportione erunt.

A	B	C	D	E	F
6	4	3	12	8	6

Theorema 21. Propositio 23.

Primi inter se numeri, minimi sunt omnium eandem cum eis proportionem habentium.

A	B	E	C	D
5	6	2	4	3

Theorema 22. Propositio 24.

Minimi numeri omnium eandem cum eis proportionem habentium, primi sunt inter se.

A	B	C	D	E
7	6	4	3	4

Theorema 23. Propositio 25.

Si duo numeri sint primi inter se, qui alterutrum eorum metitur, numerus, is ad reliquum primus erit.

A	B	C	D
6	7	3	4

Theorema 24. Propositio 26.

Si duo numeri ad quēpiam numerum primi sint, ad eundem primus is quoque futurus est, qui ab illis productus fuerit.

5 B

A	C	D	E	F
5	5	5	3	2

Theo-

Theorema 25 Propositio 27.

Si duo numeri primi sint inter se, qui ab vno eorum gignitur, ad reliquum primus erit.

B	C	D
A		
7	6	3

Theorema 26 Propositio 28.

Si duo numeri ad duos numeros ambo ad vtrunque primi sint; & qui ex eis gignentur, primi inter se erunt.

ABECDF

3 5 15 2 4 8

Theorema 27 Propositio 29.

Si duo numeri primi sint inter se, & multiplicans vterque seipsum procreet aliquem; qui ex ijs producti fuerint, primi inter se erunt. Quòd si numeri initio propositi multiplicantes eos, qui producti sunt, effecerint aliquos; hi quoque inter se primi erunt; & circa extremos idem hoc semper eueniet.

A C E B D F

3 6 27 4 16 36

Theorema 28. Propositió 30.

Si duo numeri primi sint inter se, etiam simul vterque ad vtrunque illorum primus erit. Et simul vterque ad vnum aliquem eorum primus sit, etiam qui initio positi sunt numeri, primi inter se erunt.

C

A B D

7 5 4

Theorema 29. Propositio 31.

Omnis primus numerus ad omnem numerum, quem non metitur, primus est.

A	B	C
7	10	5

Theorema 30. Propositio 32.

Si duo numeri sese mutuò multiplicantes faciant aliquem; hunc autem ex ipsis productum metiatur primus quidam numerus: is alterum etiam eorum, qui initio positi erant, metietur.

A	B	C	D	E
3	6	12	3	4

Theorema 31. Propositio 33.

Omnem compositum numerum aliquis primus metietur.

A	B	C
27	9	3

Theorema 32. Propositio 34.

Omnis numerus aut primus est, aut eum aliquis primus metitur.

A	A	
3	6	3

Problema 3. Propositio 35.

Numeris datis quotcunque, reperire minimos omnium, qui eandem cum illis proportionem habeant.

A	B	C	D	E	F	G	H	K	I	M
6	8	12	2	3	4	6	2	3	4	3

Pro-

Problema 4. Propositio 36.

B
A C D E F
7 12 8 4 5

Duobus numeris datis, reperire, quẽ illi minimum metiantur, numerum.

A
F E C D G H
6 9 12 a 2 3

Theorema 33. Propositio 37.

Si duo numeri numerum quempiam metiantur; & minimus, quem ille metiuntur, eundem metietur.

F
A B E C
2 3 6 12

Problema 5. Propositio 32.

Tribus numeris datis, reperire quem minimum numerum illi metiantur.

A B C D E
3 4 6 12 8

A B C D E F
3 6 8 12 24 16

Theorema 34. Propositio 39.

Si numerum quispiam numerus metiatur, mensus partem habebit metienti cognominem.

A B C D
12 4 3 1

Theo-

Theorema 35. Propositio 40.

Si numerus partem habuerit quamlibet, illum metietur numerus parti cognominis.

A	B	C	D
8	4	2	1

Problema 6. Propositio 41.

Numerum reperire, qui minimus cùm sit, datas habeat partes.

A	B	C	G	H
2	3	4	12	10

FINIS ELEMENTI VII.

EVCLIDIS ELEMENTVM OCTAVVM.

Theorema 1. Propositio 1.

SI sint quotcunque numeri deinceps proportionales, quorum extremi sint inter se, primi; ipsi minimi sunt omnium eandem cum eis proportionem habentium.

A	B	C	D	E	F	G	H
8	12	18	27	6	8	12	18

Problema 1. Propositio 2.

Numeros reperire deinceps proportionales minimos, quotcunque iusserit quispiam in data proportione.

A	B	C	D	E	F	G	H	K
3	4	9	12	16	27	36	49	64

Theorema 2. Propositio 3. Conuersa primæ.

Si sint quotcunque numeri deinceps proportionales minimi habentium eandem cũ

eis

çis proportionem; illorum extremi sunt inter se primi.

A	B	C	D	E	F	G	H	K	L	M	N	O
27	16	48	64	5	4	9	12	16	27	36	48	64

Problema 2. Propositio 4.

Proportionibus datis quotcunque in minimis numeris, reperire numeros deinceps minimos in datis proportionibus.

A	B	C	D	E	F	H	G	K	L	N	X	M	O
3	4	2	3	4	5	6	8	12	15	4	6	10	12

Theorema 3 Propositio 5.

Plani numeri proportionem inter se habent ex lateribus compositam.

A	L	B	C	D	E	F	G	H	K
18	22	32	3	6	4	8	9	12	16

Theorema 4. Propositio 6.

Si sint quotlibet numeri de-

A	B	C	D	E	F	G	H
16	24	36	54	81	4	6	9

inceps,

inceps, proportionales; primus autem secundum non metiatur; neque alius quispiam vllum metietur.

Theorema 5. Propositio 7.

Si sint quotcunque numeri deinceps proportionales; primus autem extremum metiatur; is etiam secundum metietur.

A	B	C	D
4	6	12	24

Theorema 6. Propositio 8.

Si inter duos numeros medij continua proportione indicant numeri; quot inter eos medij continua proportione incidunt numeri, totidem & inter alios eandem cum illis habentes proportionem medij continua proportione incident.

A	C	D	B	G	H	K	L	C	M	N	F
4	9	27	81	1	3	9	27	2	6	18	54

Theorema 7. Propositio 9.

Si duo numeri sunt inter se primi, & inter eos medij continua proportione incidant numeri, quot inter eos medij continua proportione incidunt numeri, totidem & inter vtrunque eorum, ac vnitatem deinceps medij continua proportione incident,

A	M	H	E	F	N	C	K	X	G	D	L	O	B
27	27	9	16	3	36	1	12	48	4	48	16	64	64

Theorema 8. Propositio 10

Si inter duos numeros, & vnitatem, continuè proportionales inciderint numeri; quot inter vtrunque ipsorum, & vnitatem, deinceps medij continua proportione incidunt numeri; totidē & inter illos medij continua proportione incident.

A 27 E 9 K 36 D 3 H 12 C 1 L 48 F 4 G 16 B 64

Theorema 9 Propositio 11.

Duorum quadratorum numerorum vnus medius proportionalis est numerus: & quadratus ad quadratum duplicatam habet lateris ad latus proportionem.

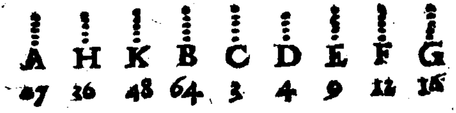
A C E D B
9 3 12 4 16

Theorema 10. Propositio 12.

Duorum cuborum numerorum duo medij proportionales sunt numerorum; Et cubus

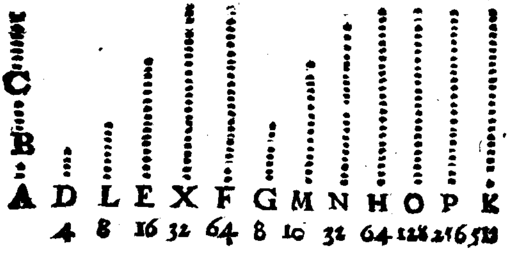
ad

ad cubum triplicatam habet lateris ad latus proportionem.



Theorema 11. Propositio 13.

Si sint quotlibet numeri deinceps proportionales, & multiplicans quisque seipsum faciat aliquos; qui ab illis producti fuerint, proportionales erunt Et, si numeri primum positi, ex suo id procreatos ductu, faciant aliquos; ipsi quoque proportionales erunt.



Theoreme 12. Propositio 14.

Si quadratus numerus quadratum numerū metiatur, & latus vnius metietur latus alterius.

rius Et si vnius quadrati latus metiatur latus alterius, & quadratus quadratum metietur.

A	E	B	C	D
9	12	16	8	4

Theorema 13. Propositio 15.

Si cubus numerus cubum numerus metiatur, & latus vnius metietur alterius latus. Et si latus vnius cubi latus alterius metiatur, tum cubus cubum metietur.

A	H	K	B	C	D	E	F	G
8	16	28	64	2	4	4	8	16

Theorema 14. Propositio 16.

Si quadratus numerus quadratum numerũ non metiatur, neque latus vnius metietur alterius latus. Et si latus vnius quadrati non metiatur latus alterius, neque quadratus quadratum metietur.

A	B	C	D
9	16	3	4

Theorema 15. Propositio 17.

Si cubus numerus cubum numerum non metiatur, neq; latus vnius latus alterius metietur, Et si latus cubi vnius latus alterius non metiatur, neque cubus cubum metietur.

A	B	C	D
8	27	9	11

Theo-

Theorema 16. Propositio 18.

Duorum similium planorum numerorum vnus medius proportionalis est numerus; & planus ad planũ duplicatam habet lateris homologi ad latus homologum proportionem.

A G B C D E F
12 18 27 2 6 3 9

Theorema 17. Propositio 19.

Duorum similium numerorum solidorum duo medij proportionales sunt numeri: Et solidus ad similem solidum triplicatam, habet lateris homologi ad latus homologum proportionem.

A N X B C D E F G H K M L
8 12 18 27 2 2 2 3 3 3 4 6 9

Theorema 18. Propositio 20.

Si inter duos numeros vnus medius proportionalis incidat numerus; similes plani erunt numeri.

A C B D E F G
18 24 33 3 4 6 8

Theorema 19. Propositio 21.

Si inter duos numeros duo medij proportionales incidant numeri; similes solidi sunt illi numeri.

A	C	D	B	E	F	G	H	K	L	M
27	36	44	64	9	12	16	3	3	3	4

Theorema 20. Propositio 22.

Si tres numeri deinceps sint proportionales; primus autem sit quadratus, & tertius quadratus erit.

A	B	D
9	15	25

Theorema 21. Propositio 23.

Si quatuor numeri deinceps sint proportionales; primus autem si cubus, & quartus cubus erit.

A	B	C	D
8	12	18	27

Theorema 22. Propositio 24.

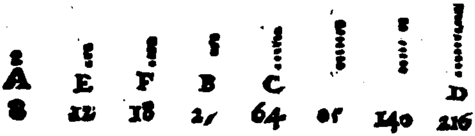
Si duo numeri eam proportionem habeant inter se, quam quadratus numerus ad quadratum numerum, primus autem sit quadratus, & secundus quadratus erit.

A		B	C		D
4	9	9	16	24	36

Theorema 23. Propositio 25.

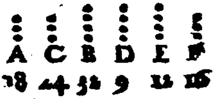
Si numeri duo proportionem inter se habeant,

beant, quam cubus numerus ad cubum numerum; primus autem cubus sit, & secundus cubus erit.



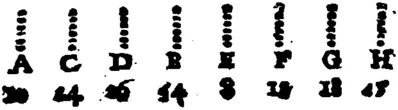
Theorema 24. Propositio 26.

Similes plani numeri proportionem inter se habent, quam quadratus numerus ad quadratum numerum.



Theorema 25. Propositio 27.

Similes solidi numeri proportionem habent inter se, quam cubus numerus ad cubum numerum.



FINIS ELEMENTI VIII.

EVCLIDIS ELEMENTVM NONVM.

Theorema 1. Propositio 1.

Si duo similes plani numeri mutuò sese multiplicantes, quendam procreent, productus quadratus erit.

A	E	B	D		C
4	6	9	16	24	36

Theorema 2. Propositio 2.

Si duo numeri mutuò sese multiplicantes, quadratum faciant, illi similes sunt plani.

A		B D		C
4	6	9	18	36

Theorema 3. Propositio 3.

Si cubus numerus seipsum multiplicās procreet

ereet ali-
quem, pro- vni- D D A B
ductus cu- tas 5 4 8 16 32 64
bus erit.

Theorema 4. Propositio 4.

Si cubus numerus cubum	:	:	:	:
numerum multiplicans	:	:	:	:
quendam procreet, pro-	A	B	D	C
creatus cubus erit.	8	27	64	216

Theorema 5. Propositio 5.

Si cubus numerus numerum quēdam mul-

tiplicans cubum pro-	:	:	:	:
creet; & multiplica-	A	B	C	D
tus cubus erit.	27	64	729	18

Theorema 6. Propositio 6.

Si numerus seipsum	:	:	:
multiplicās cubum	:	:	:
procreet; & ipse cu-	A	B	C
bus erit.	27	729	19683

Theorema 7. Propositio 7.

Si compositus numerus quendam numerū

multiplicans, quem-	:	:	:	:	:
piam procreet, pro-	A	B	C	D	E
ductus solidus erit.	6	8	48	2	3

Theorema 8. Propositio 8.

Si ab vnitate quotlibet numeri deinceps proportionales sint: Tertius ab vnitate quadratus est, & vnū intermittentes oēs: Quartus aūt cubus est, & duobus intermissis omnes:

nes: Septimus verò cubus simul & æquadratus est, & quinque intermissis omnes.

Vnitas	A	B	C	D	E	F
	3	9	27	81	243	729

Theorema 9. Propositio 9.

Si ab vnitate sint quotcunque numeri deinceps proportionales; sit autem quadratis is, qui vnitatem sequitur, & reliqui omnes quadrati erunt. Quòd si, qui vnitatem sequitur, cubus sit; & reliqui omnes cubi erunt.

quadrati		cubi
531441	F	732969
59040	E	531441
6561	D	6561
723	C	6561
81	B	729
9	A	81
	o	
	vnitas.	

Theorema 10. Propositio 10.

Si ab vnitate numeri quotcunque proportionales sint; non sit autem quadratus is, qui vnitatem sequitur, neque alius vllus quadratus

Vnitas.	A	B	C	D	E	F
	3	9	36	18	243	729

tus

tus erit; demptis, tertio ab vnitate ac omnibus vnum intermittentibus. Quòd si, qui vnitatem sequitur, cubus non sit: neque alius vllus cubus erit; demptis, quarto ab vnitate, ac omnibus duos intermittentibus.

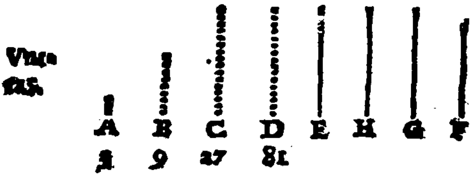
Theorema 11. Propositio 11.

Si ab vnitate numeri quotlibet deinceps proportionales sint; minor maiorem metitur per quempiam eorum, qui in proportionalibus sunt.

A	B	C	D	E
1	2	4	8	16

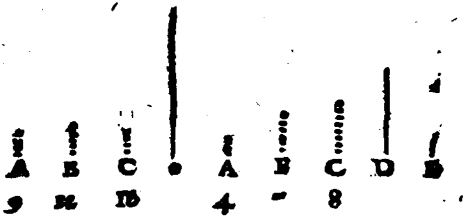
Theorema 12. Propositio 12.

Si ab vnitate quotlibet numeri sint proportionales; quot primorum numerorum vltimum metiuntur, totidem & eum, qui vnitati proximus est, metientur.



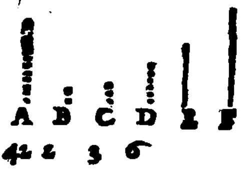
Teorema 13. Propositio 13.

Si ab vnitate sint quotcunque numeri deinceps proportionales; primus autem sit, qui vnitatem sequitur; maximum nullus alius metietur; ijs exceptis, qui in proportionalibus sunt numeris.



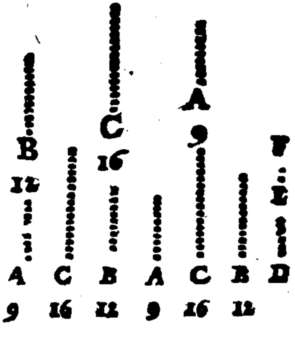
Theorema 14. Propositio 14.

Si minimum numerum primi aliquot numeri metiantur; nullus alius numerus primus illum metietur; ijs exceptis, qui primò metiuntur.



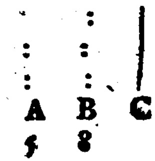
Theorema 15. Propositio 15.

Si tres numeri deinceps proportionales sint minimi omnium, eandem cum ipsis proportionem habentium, duo quilibet compositi, ad tertium primi erunt.



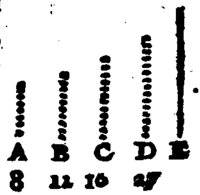
Theorema 16. Propositio 16.

Si duo numeri sint inter se primi; non se habebit quémadmodum primus ad secundum, ita secundus ad quempiam alium.



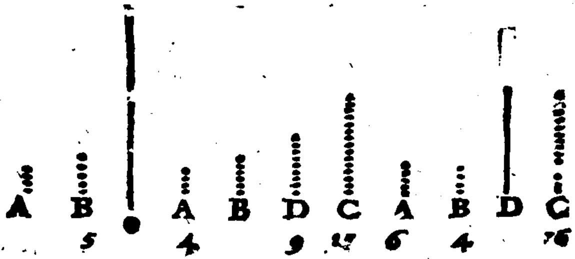
Theorema 17. Propositio 17.

Si sint quotlibet numeri deinceps proportionales, quorum extremi sint inter se primi; non erit quémadmodum primus ad secundum, ita vltimus ad quempiam alium.



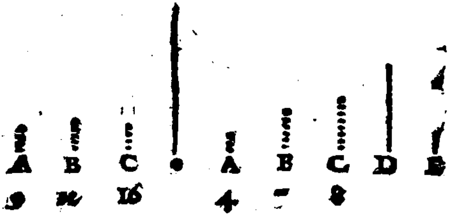
Problema 1. Propositio 18.

Duobus numeris datis, considerare an possit ipsis tertius inueniri proportionalis.



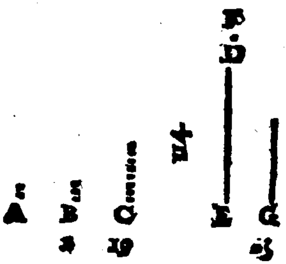
Problema 2. Propositio 19.

Tribus numeris datis, considerare, an possit ipsis quartus reperiri proportionalis.



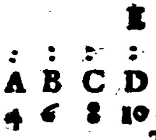
Theorema 18. Propositio 20.

Primi numeri plures sunt, quacunque proposita multitudine primorum numerorum.



Theorema 19. Propositio 21.

Si pares numeri quotlibet compositi sint, totus est par.



Theo-

Theorema 20. Propositio 22.

Si impares numeri quotlibet compositi sint; sit autem par illorum multitudo; totus par erit.

			E
A	B	C	D
1	9	7	3

Theorema 21. Propositio 23.

Si impares numeri quotcunque compositi sint; sit autem impar illorum multitudo; & totus impar erit.

A	B	C	E
5	7	8	1

Theorema 22. Propositio 24.

Si à pari numero par detractus sit; & reliquus par erit.

B	
A	C
6	4

Theorema 23. Propositio 25.

Si à pari numero impar detractus sit; & reliquus impar erit.

A	C	D
8	1	4

Theorema 24. Propositio 26.

Si ab impari numero impar detractus sit; & reliquus par erit.

		B
A	C	D
4	6	

Theo-

Theorema 25. Propositio 27.

Si ab impari numero par ablatus sit, reliquis impar erit.

A D C B
2 4 4

Theorema 27. Propopositio 28.

Si impar numerus parem multiplicans, procreet quempiam; procreatus par erit.

A B C
3 4 28

Theorema 27. Propositio 29.

Si impar numerus imparem numerum multiplicans, quendam procreet; procreatus impar erit.

A C D
3 5 15

Theorema 28. Propositio 30.

Si impar numerus parem numerum metiatur, & illius dimidium metietur.

A C B
3 6 18

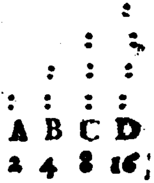
Theorema 29. Propositio 31.

Si impar numerus ad numerum quempiam primus sit: & illius duplum primus erit.

A B C D
7 8 16

Theorema 30. Propositio 32.

Numerorum, qui à binario dupli sunt, Vnusquisque pariter tantum par est tantum.



Theorema 31. Propositio 33.

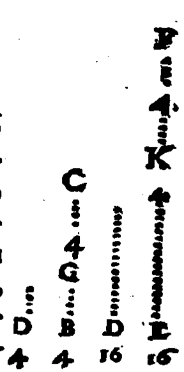
Si numerus dimidium habeat imparem: pariter impar est tantum.

Theorema 32. Propositio 34.

Si par numerus neque à binario duplus sit, neque dimidium habeat imparem: pariter par est, & pariter impar.

Theorema 33 Propositio 35.

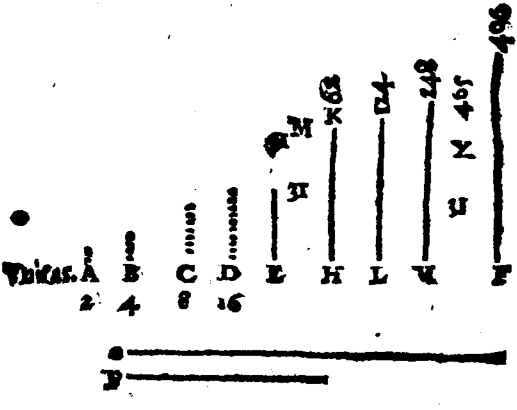
Si sint quotlibet numeri deinceps proportionales; detrahantur autem à secundo & vltimo æquales ipsi primo: Erit quemadmodum secundi excessus ad primũ, ita vltimi excessus ad omnes, qui vltimum antecedunt.



Theo.

Theorema 34 Propositio 36.

Si ab vnitate numeri quotlibet deinceps expositi sint in dupla proportione, quoad totus compositus primus factus sit; isq; totus in vltimum multiplicatus, quempiam procreatus perfectus erit.



FINIS ELEMENTI IX.

EVCLI-

EVCLIDIS ELEMENTVM DECIMVM.

DEFINITIONES.

PRIMAE, NEMPE MAGNITVDInem symmetrarum.

1.

Commensurabiles magnitudines dicuntur illæ, quas eadem mensura metitur.

2.

Incommensurabiles verò magnitudines dicuntur, quarum nullam mensuram communem contingit reperiri.

3.

Lineæ rectæ potentia commensurabiles sunt, quarum quadrata vna eadem superficies, siue area metitur.

4.

Incommensurabiles verò lineæ sunt, quarum quadrata quæ metiatur area communis, reperiri nulla potest.

5.

Hæc cum ita sint, ostendi potest, quòd quátacunque linea recta nobis proponatur, existunt etiam aliæ lineæ innumerabiles eidem commensurabiles, aliæ item incommensurabiles; hæ quidem longitudine & potentia; illæ verò potentia tantùm. Vocetur igitur linea recta, quantacunq; proponatur, ῥητὴ, id est, rationalis.

6.

Lineæ quoque illi ῥητῇ commensurabiles siue longitudine & potentia, & siue potentia tantùm, vocentur & ipsæ ῥηταί, id est, rationales.

7.

Quæ verò lineæ sunt incommensurabiles illi τῇ ῥητῇ, id est, primo loco rationali, vocentur ἄλογοι, id est, irrationales.

8.

Et quadratum, quod à linea proposita describitur, quam ῥητὴν vocari volumus, vocetur ῥητὸν, id est, rationale.

9.

Et, quæ sunt huic commensurabilia, vocentur ῥητά, id est, rationalia.

10.

Quæ verò sunt illi quadrato, ῥητῷ scilicet, incommensurabilia, vocentur, ἄλογα, id est, surda, siue irrationalia.

11.

Et lineæ, quæ illa incommensurabilia describunt, vocentur ἄλογοι. Et quidam si illa incommensurabilia fuerint quadrata, ipsa eorum altera vocabuntur ἄλογοι lineæ, quòd

si quæ

si quadrata quidem non fuerint, verum aliæ quæpiam superficies siue figuræ rectilineæ, tunc vero lineæ illæ quæ describunt quadrata æqualia figuris rectilineis, vocentur ἄλογοι.

Postulatum, siue petitio.

Postulatur quemlibet magnitudinem toties posse multiplicari, donec quamlibet magnitudinem eiusdem generis excedat.

Axiomata, siue pronunciata.

1.

Magnitudo quotcunque magnitudines metiens, compositam quoque ex ipsis metitur.

2.

Magnitudo quamcũq; magnitudinem metiens, metitur quoque omnem magnitudinem, quam illa metitur.

3.

Magnitudo metiens totam magnitudinem, & ablatam; metitur, & reliquam.

Problema 1. Propositio 1.

Duabus magnitudinibus inæqualibus propositis, si de maiore detrahatur plus dimidio; & rursus de residuo iterum detrahatur plus dimidio; idque semper fiat: relinquetur tãdem quędam magnitudo minor altera minore ex duabus propositis.

Theorema 2. Propoſitio 2.

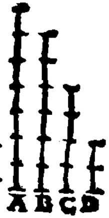
Duabus magnitudinib. propoſitis inæqualibus, ſi detrahatur ſemper minor de maiore, alterna quadam detractione; neque reſiduum vnquam metiatur id quod ante ſe metiebatur: incommenſurabiles ſunt illæ magnitudines.

Problema 1. Propoſitio 3.

Duabus magnitudinibus commenſurabilibus datis, maximam ipſarum communem menſuram reperire.

Problema 2. Propoſitio 4.

Tribus magnitudinibus commenſurabilibus datis, maximam ipſarum communem menſuram reperire.

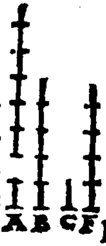


Theorema 3. Propoſitio 5.

Commenſurabiles magnitudines inter ſe proportionem eam habent, quam habet numerus ad numerum.

Theorema 4. Propositio 6.

Si duæ magnitudines proportionem eam habent inter se, quam numerus ad numerum: commensurabiles sunt illæ magnitudines.



Theorema 5. Propositio 7.

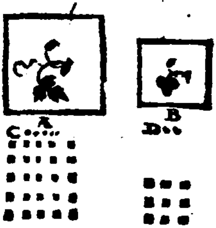
Incommensurabiles magnitudines inter se proportionem non habent, quam numerus ad numerum.

Theorema 6. Propositio 8.

Si duæ magnitudines inter se proportionem non habent, quam numerus ad numerum: incommensurabiles illæ sunt magnitudines.

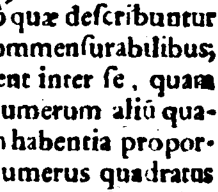
Theorema 7. Propositio 9.

Quadrata, quæ describitur à rectis lineis lõgitudine cõmensurabilibus: inter se proportionem habent, quam numerus quadratus ad alium numerum quadratum. Et quadrata habentia proportionem inter ſe, quam quadratus numerus ad numerum quadratum; habent quoque latera longitudine commenſurabilia. Quadrata verò quæ deſcribuntur à lineis longitudine incommenſurabilibus; proportionem non habent inter ſe, quam quadratus numerus ad numerum aliũ quadratum. Et quadrata non habentia proportionem inter ſe, quam numerus quadratus ad numerum quadratum, neque latera habebunt longitudine commenſurabilia.



Theorema 8. Propoſitio 10.

Si quatuor magnitudines fuerint proportionales; prima verò ſecundæ fuerit commenſurabilis; tertia quoque quartæ commenſurabilis erit; quòd ſi prima ſecundæ fuerit incõmenſurabilis; tertia quoque quartæ incommenſurabilis erit.



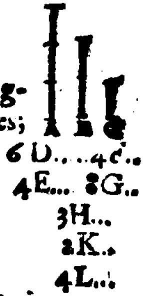
Problema 3. Propoſitio 11.

Propoſitæ lineæ rectæ (quam ῥητὴν vocari

dixi-

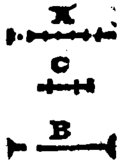
diximus) reperire duas lineas rectas incommẽſurabiles, alteram quidem longitudine tantùm, alteram verò non longitudine tãtùm, ſed etiam potentia incommẽſurabilem.

Theorema 9. Propoſitio 12.



Magnitudines, quæ eidem magnitudini ſunt commenſurabiles; inter ſe quoque ſunt commenſurabiles.

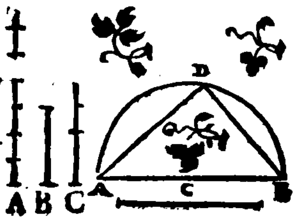
Theorema 10. Propoſitio 13.



Si ex duabus magnitudinibus hæc quidem commenſurabilis ſit tertiæ magnitudini, illa verò eidem incommenſurabilis, incommenſurabiles ſunt illæ duæ magnitudines.

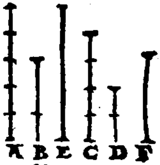
Theorema 11. Propoſitio 14.

Si duarum magnitudinum commenſurabilium, altera fuerit commenſurabilis magnitunitudini alteri cuipiam tertiæ: reliquá quoq; magnitudo eidem tertiæ incommensurabilis erit.



Theorema 12. Propositio 15.

Si quatuor rectæ lineæ proportionales fuerint; possit autem prima plusquam secunda tanto, quantum est quadratum rectæ lineæ sibi commensurabilis longitudine: *tertia* quoque potuerit plusquam quarta tanto, quantum est quadratum rectæ lineæ sibi commensurabilis longitudine. Quòd si prima possit plusquam secũda quadrato rectæ lineæ sibi longitudine incommensurabilis: tertia quoque poterit plusquã quarta quadrato rectæ lineæ sibi incommensurabilis longitudine.

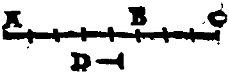


Theorema 13. Propositio 16.

Si duæ magnitudines commensurabiles componantur; tota magnitudo composita singularis partibus commensurabilis erit; Quòd si tota magnitudo composita alterutri parti commensurabilis fuerit; illæ duæ

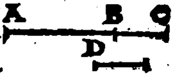
quo-

quoque partes cómenſurabiles erunt.



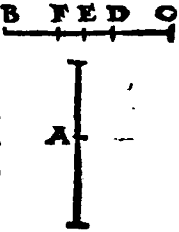
Theorema 14. Propoſitio 17.

Si duæ magnitudines incommenſurabiles componantur, ipſa quoque tota magnitudo ſingulis partibus componentibus incommenſurabilis erit. Quòd ſi tota alteri parti incommenſurabilis fuerit, illæ quoque primæ magnitudines inter ſe incommenſurabiles erunt.



Theorema 15. Propoſitio 18.

Si fuerint duæ rectæ lineæ inæquales, & quartæ parti quadrati, quod deſcribitur à minore, æquale parallelogrammum applicetur ad maiorem, ex qua maiore tantum excurrat extra latus parallelogrammi, quantum eſt alterum latus ipſius parallelogrammi: ſi præterea parallelogrammum ſui applicatione diuidat lineam illam in partes inter ſe commenſurabiles longitudine, illa maior linea tanto plus poteſt quam minor, quantum eſt quadratum lineæ ſibi commẽſurabilis longitudine. Quòd ſi maior plus poſſit quam minor, tanto, quantum eſt quadratum lineæ ſibi commenſurabilis longitudine; & præterea quartæ parti quadrati lineæ minoris, ęquale parallelogrammum applicetur ad maiorem, ex qua maiore tantum excurrat extra latus parallelogrammi, quantum est alterum latus ipsius parallelogrammi: parallelogrammum sui applicatione diuidit maiorem in partes inter se longitudine commensurabiles.

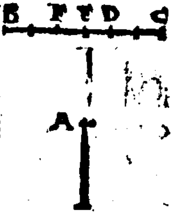


Theorema 16. Propositio 19.

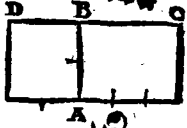
Si fuerint duæ rectæ lineæ inæquales; quartæ autem parti quadrati lineæ minoris, æquale parallelogrammorum ad lineam maiorem applicetur, ex qua linea tantum excurrat extra latus parallelogrammi, quantum est alterum latus eiusdem parallelogrammi: si parallelogrammum pręterea sui applicatione diuidat lineam in partes inter se longitudine incommensurabiles, maior illa linea tantò plus potest quàm minor quantum est quadratum lineæ sibi minori commēsurabiles longitudine. Quòd si maior linea tantò plus possit quàm minor, quantum est quadratum lineæ incommensurabilis sibi longitudine: & pręterea quartæ parti quadrati lineæ minoris æquale parallelogrammum applicetur ad maiorem

ex

ex qua tantum excurrat extra latus parallelogrammi, quantum est alterum latus ipsius: parallelogrammum sui applicatione diuidit maiorẽ in partes inter se incommensurabiles longitudine.



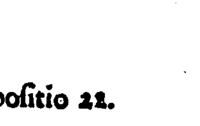
Theorema 17. Propositio 20.



Superficies rectanguli contenta ex lineis rectis rationalibus longitudine commensurabilibus secundum vnum aliquem modum ex antedictis, rationalis est.

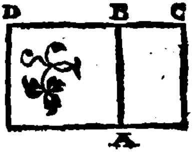
Theorema 18. Propositio 21.

Si rationale ad lineam rationalem applicetur; habebit alterum latus lineam rationalem & commensurabilem longitudine lineæ cui rationale parallelogrammum applicatur.



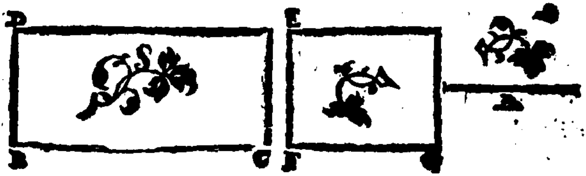
Theorema 19. Propositio 22.

Superficies rectangula contenta duabus lineis rectis rationalib. potentia tantũ commensuraſurabilibus, irrationalis eſt. Linea autem quæ illam ſuperficiem poteſt, irrationalis & ipſa eſt; vocetur verò media.

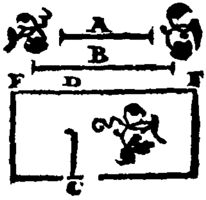


Theorema 20. Propoſitio 23.

Quadrati lineæ mediæ ad lineam rationalem applicanti, alterum latus eſt linea rationalis, & incommenſurabilis longitudine lineæ, ad quam applicatur.



Theorema 21. Propoſitio 24.

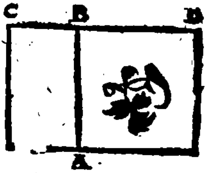


Linea recta mediæ commenſurabilis, eſt ipſa quoque media.

Theo-

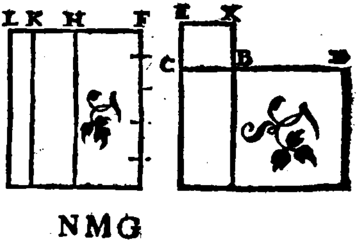
Theorema 22. Propoſitio 25.

Parallelogrammum rectangulum contentum ſub rectis lineis medijs longitudine commenſurabilibus, medium eſt.



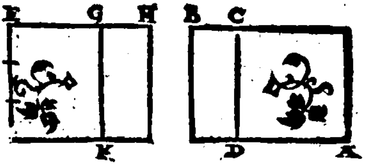
Theorema 23. Propoſitio 26.

Parallelogrammum rectangulum comprehenſum ſub duabus lineis medijs potentia tantum commenſuralibus; vel rationale eſt, vel medium.



Theorema 24. Propoſitio 27.

Medium non eſt maius, quàm medium ſuperficie rationali.



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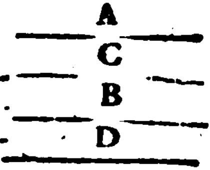
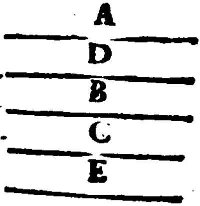
Problema 4. Propositio 28.

Medias lineas inuenire potentia tantùm commensurabiles ratione comprehendentes.



Problema 5. Propositio 29.

Medias lineas inuenire potentia tantùm commensurabiles, medium comprehendentes.



Problema 6. Propositio 30.

Reperire duas rationales potentia tantum cõmensurabiles huiusmodi, vt maior ex illis possit plus, quàm minor, quadrato rectæ lineæ sibi commensurabilis longitudine.

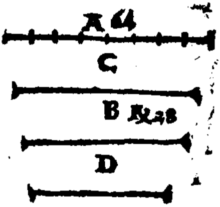
Problema 7. Propositio 31.

Inuenire duas rationales potentia tantùm commensurabiles: ita vt maior, quam minor plus possit, quadrato rectæ lineæ sibi longitudine incommensurabilis.

Pro-

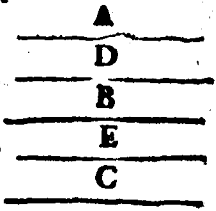
Problema 8. Propositio 32.

Reperire duas lineas medias potentia tantum commensurabiles rationalem superficiem continentes tales in quā vt maior possit plus, quàm minor, quadrato rectæ lineæ sibi commensurabilis longitudine.



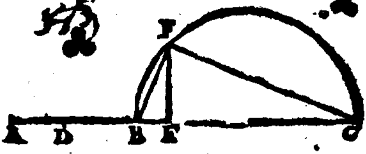
Problema 9. Propositio 33.

Inuenire duas lineas medias potentia tantùm cōmensurabiles, quæ mediam superficiem contineant, ita vt maior plus possit, quam minor, quadrato rectæ lineæ sibi longitudine commensurabilis.



Problema 10. Propositio 34.

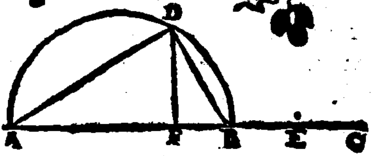
Inuenire duas rectas lineas potentia incommensurabiles; quarum quadrata simul composita faciant superficiem rationalem: Rectangulum verò sub ipsis contentum, faciant medium.



Pro-

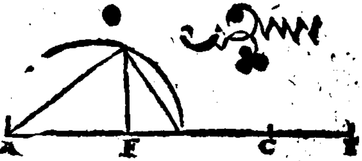
Problema 11. Propositio 35.

Repe rire lineas duas rectas potentia incommensurabiles, conficientes compositum ex ipsarum quadratis medium parallelogrammũ verò ex ipsis contentum rationale.



Problema 12. Propositio 36.

Reperire duas lineas rectas potentia incommensurabiles, conficientes id, quod ex ipsarum quadratis componitur, mediũ, parallelogrammũ ex ipsis cōtentum, mediũ; quod præterea parallelogrammum sit incommensurabile composito ex quadratis ipsarum.



PRINCIPIVM SENARIORVM per compositionem, & Synthesin.

Theorema 25. Propositio 37.

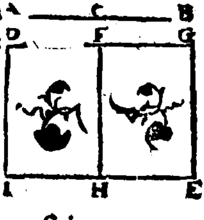
Si duæ rationales potentia tantùm commensurabiles componantur; tota irrationalis erit. Vocetur autem Binomium, vel ex binis nominibus.

Theorema 26 Propositio 38.

Si duæ mediæ potentia tantùm commensurabiles, rationale continentes, componantur; tota linea est irrationalis, vocetur autem ex binis mediis prima.

Theorema 27 Propositio 39.

Si duæ mediæ potentia tantùm commensurabiles medium continentes cõponantur; tota linea est irrationalis: vocetur autem ex binis medijs secunda.



Theorema 28 Propositio 40.

Si duæ rectæ lineæ potentia incommensurabiles componantur, conficientes compositum ex quadratis ipsarum, rationale parallelolelogrammum verò ex ipsis contentum, medium.

tota linea recta est irrationalis. Vocetur autem linea maior.

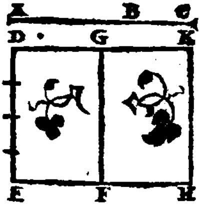
Theorema 29 Propositio 41.

Si duæ rectæ lineæ potentia incommensurabiles componantur, conficientes compositum ex ipsarum quadratis, medium: id verò, quod fit ex ipsis, rationale; tota recta linea &

irrationalis erit. Vocetur autem potens rationale & medium.

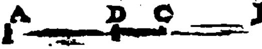
Theorema 30. Propositio 42.

Si duæ rectæ lineæ potentia incommensurabiles componantur, conficientes compositum ex ipsarum quadratis medium; & quod continetur ex ipsis, medium; & præterea incommensurabile composito ex quadratis ipsarum: tota recta linea est irrationalis. Vocetur autem bina media potens.



Theorema 31 Propositio 43.

Quæ linea ex binis nominibus vocata, in vnico tantum puncto diuiditur in sua nomina.

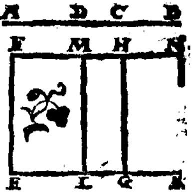


Theo-

Theorema 32. Propositio 44.

Quæ ex binis medijs prima, in vnica tantum puncto diuiditur in sua nomina.

Theorema 33. Propositio 45.



Quæ ex binis medijs secunda, in vnico tantum puncto diuiditur in sua nomina.

Theorema 34. Propositio 46.

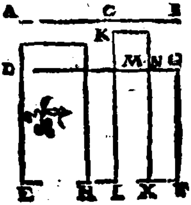
Linea maior in vnico tantum puncto diuiditur in sua nomina.

Theorema 35. Propositio 47.

Linea potens rationale & medium, in vnico tantum puncto, diuiditur in sua nomina.

Theorema 36. Propositio 48.

Linea potens duo media in vnico tantum puncto diuiditur in sua nomina.



DEFINITIONES secundæ, nempe binorum nominum.

Proposita linea rationali; & linea ex binis nominibus vocata, diuisa, in sua nomina, cuius maius nomen, id est, maior portio; possit plusquam minus nomen; quadrato lineæ sibi, maiori inquam nomini, commensurabilis longitudine.

1.

Si quidem maius nomen fuerit commensurabile longitudine propositæ lineæ rationali; Vocetur tota linea composita ex binis nominibus prima.

2.

Si verò minus nomen, id est, minor portio, fuerit commensurabile longitudine propositæ lineæ rationali; Vocetur tota linea ex binis nominibus secunda.

3.

Si verò neutrum: ipsorum nominum fuerit commensurabile longitudine propositæ lineæ rationali, Vocetur tota ex binis nominibus tertia.

Rursus si maius nomen possit plusquam minus nomen, quadrato lineæ sibi incommensurabiles longitudine.

4.

Si quidem maius nomen sit commensurabile

bile longitudine propositæ lineæ rationali; Vocetur tota linea ex binis nominibus quarta.

5.

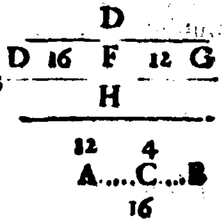
Si verò minus nomen fuerit commensurabile longitudine lineę rationali; Vocetur tota ex binis nominibus quinta.

6.

Si verò neutrum ipsorum nominum fuerit longitudine commensurabile: propositæ lineæ rationali; Vocetur tota ex binis nominibus sexta.

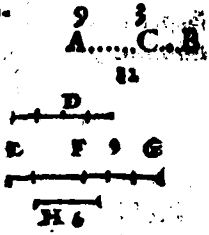
Problema 13. Propositio 49.

Reperire lineam ex binis nominibus primam.

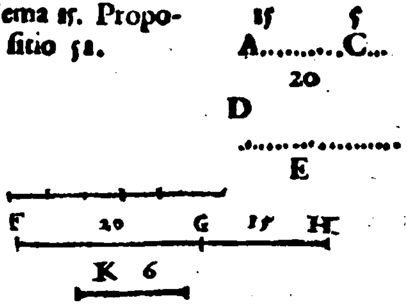


Problema 14. Propositio 50.

Reperire ex binis nominibus secundam.

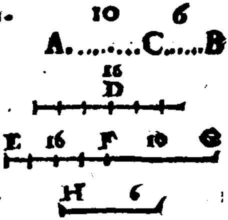


Problema 15. Propositio 51.



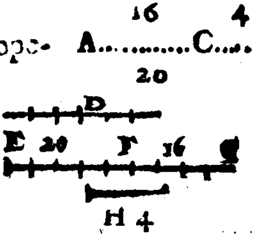
Reperire ex binis nominibus tertiam.

Problema 16. Propositio 52.



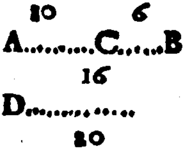
Reperire ex binis nominibus quartam.

Problema 17. Propositio 53.



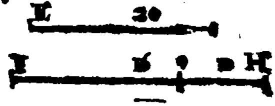
Reperire ex binis nominibus quintam.

Problema 18. Propositio 54.



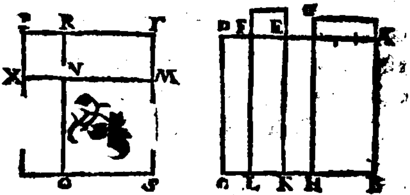
Repe-

Reperire ex binis nominibus sextam.



Theorema 37. Propositio 55.

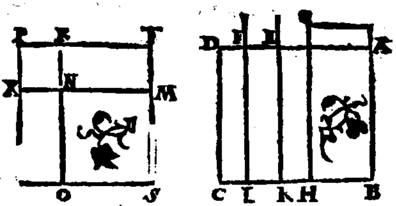
Si superficies contenta fuerit sub rationali, & ex binis nominibus prima recta linea,



quæ illam superficiem potest, est irrationalis; quæ ex binis nominibns vocatur.

Theorema 38. Propositio 56.

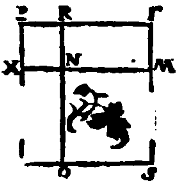
Si superficies contenta fuerit sub linea rationali, & ex binis nominibus secunda; Recta linea po-

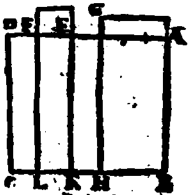


tens illam superficiem, est irrationalis; quæ ex binis medijs prima vocatur.

Theorema 39. Propoſitio 57

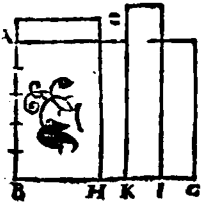
Si ſuperficies contineatur ſub rationali, & ex binis nominibus tertia; recta linea quæ illã ſuperficiem poteſt, eſt irrationalis, quæ ex binis medijs dicitur tertia.

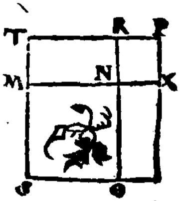




Theorema 40. Propoſitio 58.

Si ſuperficies contineatur ſub rationali, & ex binis nominibus quarta; recta linea potens ſuperficiem illam, eſt irrationalis; quæ dicitur maior.

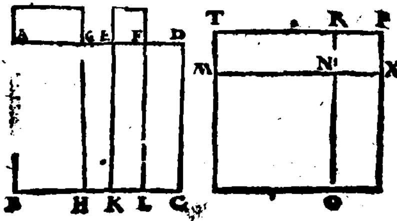




Theorema 41. Propoſitio 59.

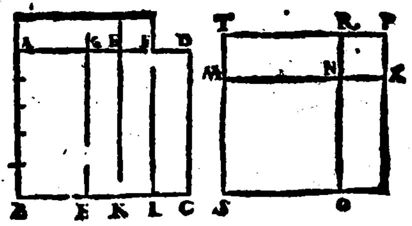
Si ſuperficies cõtineatur ſub rationali, & ex binis

binis nominibus quarta; recta linea, quæ illam superficiem potest, est irrationalis; quæ dicitur potens rationale, & medium.



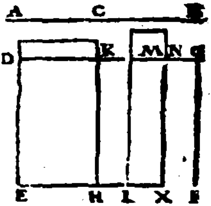
Theorema 42. Propositio 60.

Si superficies continetur sub rationali, & ex binis nominibus sexta; Recta linea, quæ illam superficiem potest, est irrationalis; quæ dicitur potens, bina media.



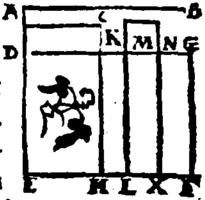
Theorema 43. Propositio 61.

Quadratum eius lineæ, quæ est ex binis nomini-minibus, ad lineam rationalem applicatum, facit latitudinem ex binis minibus primam.



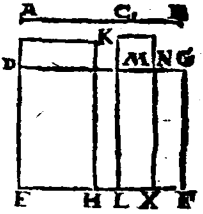
Theorema 44 Propositio 62.

Quadratum, eius quæ est ex binis medijs prima, ad lineam rationalem applicatum, facit latitudinem ex binis nominibus secundam.



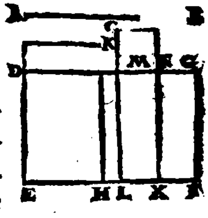
Theorema 45. Propositio 63.

Quadratum eius, quæ est ex binis medijs secunda, ad lineam rationalem applicatum, facit latitudinē ex binis nominibus tertiam.



Theorema 46. Propositio 64.

Quadratum lineæ maioris secundum lineam rationalem applicatum, facit latitudinem ex binis nominibus quartam.



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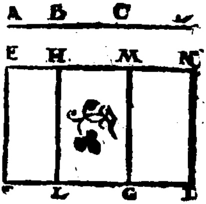
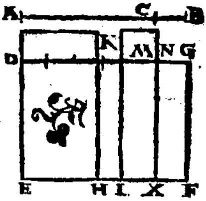
Theorema 47. Propositio 65.



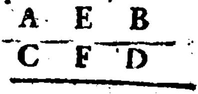
Quadratum lineæ potentis rationale, & medium secundum rationalem applicatum, facit latitudinem ex binis nominibus quintam.

Theorema 48. Propositio 66.



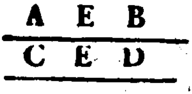
Quadratum lineæ potentis duo media, secundum rationalem applicatum, facit latitudinem ex binis nominibus sextam.

Theorema 49. Propositio 67.



Linea longitudine commensurabilis, ei lineæ quę est ex binis nominibus; & ipsa ex binis nominibus est, atque in ordine eadem.

Theorema 50. Propositio 68.



Linea longitudine commensurabilis alteri lineæ quæ est ex binis medijs; & ipsa ex binis medijs est, atque in ordine eadem.

Theorema 51. Propositio 69.

Linea commensurabilis lineæ maiori, & ipsa maior est.

Theorema 52. Propositio 70.

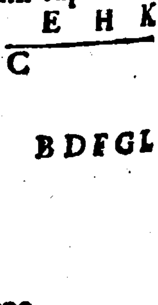
Linea commenſurabilis lineæ potenti rationale & medium, eſt & ipſa linea potens rationale & medium.

Theorema 53. Propoſitio 71.

Linea commenſurabilis lineæ potenti duo media, eſt & ipſa linea potens duo media.

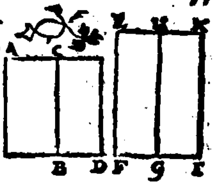
Theorema 54. Propoſitio 72.

Si duæ ſuperficies, rationalis, & media ſimul componantur, linea quæ totam ſuperficiem compoſitam poteſt, eſt vna ex quatuor irrationalibus; vel ea, quæ dicitur ex binis nominibus, vel ea, quæ ex binis medijs prima, vel linea maior, vel linea potens, rationale & medium.



Theorema 55. Propoſitio 73.

Si duæ ſuperficies mediæ inter ſe incómenſurasurabiles simul componantur; fiunt reliquæ duæ lineæ irrationales; & ex binis medijs secunda, vt bina media potens.



SCHOLIVM EX THEONE, ZAMberto, Campano, & P.Christ. Clauio.

Ex his omnibus facilè colligitur, quod linea ea, quæ est ex binis nominibus, & cætera ipsam subsequentes, lineæ irrationales, neque sunt eadem cum linea media, neque ipsa inter se sunt eadem.

Nam quadratum lineæ mediæ, ad lineam rationalem comparatum & applicatum, efficit alterum latus, lineam rationalem, seu latitudinem rationalem ipsi lineæ rationali (hoc est, lineæ, ad quam applicatur) longitudine incommensurabilem: per propos. 23. libri decimi.

Quadratum verò eius lineæ, quæ est ex binis nominibus, ad rationalem, applicatum, efficit alterũ latus, & lineam, seu latitudinem ex binis nominibus primam: per 61.

Quadratum verò eius, quæ est ex binis medijs prima ad Rationalem applicatum, latitudinem efficit ex binis nominibus secundam: per 26.

Quadratũ verò eius, quæ est ex binis medijs secunda, ad rationalem applicatum, latitudinem efficit

ficit ex binis nominibus tertiam: per 63.
Quadratum lineæ maioris, ad rationalem applicatum, latitudinem efficit ex binis nominibus quartam: per 64.
Quadratum vero eius, quæ rationale, & medium potest, ad rationalem applicatum; efficit alterum latus, seu latitudinem ex binis nominibus quintam: per 65.

Quadratum denique lineæ eius, quæ bina media potest, ad rationalem applicatum, efficit alterum latus, seu latitudinem ex binis nominibus sextam: per 66.

Cùm igitur hæ latitudines (quæ à nonnullis latera dicuntur) differant, & à latitudine media & inter se, à latitudine quidem media, quod hæc rationalis sit, illæ verò irrationales, inter se autem, quod in ordine non sint eadem cum iis ex binis nominibus: manifestum est omnes ipsas irrationales lineas, de quibus hactenus dictum est, inter se differentes esse.

PRINCIPIVM SENARIORVM per detractionem, & aphæresin.

Theorema 56 Propositio 74.

Si de linea rationali detrahatur rationalis, potentia tantum commensurabilis ipsi toti: Residua est irrationalis: Vocetur autem Residuum, hoc est, apotome.

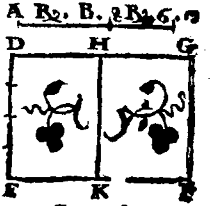
Theo-

Theorema 57. Propositio 75.

Si de linea media detrahatur media, potentia tantum commensurabilis toti lineæ; quæ verò detracta est, cum tota contineat superficiem rationalem; Residua est irrationalis. Vocetur autem Residuũ medium primum: hoc est, mediæ apotome prima.

Theorema 58. Propositio 76.

Si de linea media detrahatur media, potentia tantum commensurabilis toti; quæ verò detracta est, cum tota contineat superficiem mediam: Reliqua est irrationalis. Vocetur autem Residuum medium secundum, hoc est, mediæ apotome secunda.



Theorema 59. Propositio 77.

Si de linea recta detrahatur recta, potentia incommensurabilis toti; compositum autem ex quadratis totius lineæ, & lineæ detractæ, sit rationale; parallelogrammum verò ex rò ex eiſdem contentum, ſit medium: Reliqua linea erit irrationalis. Vocetur autem linea minor.

A C B

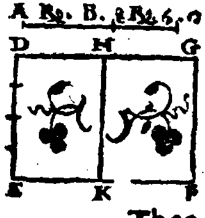
Theorema 60. Propoſitio 78.

Si de linea recta detrahatur recta, potentia incommenſurabilis toti lineæ; compoſitum autem ex quadratis totius, & lineæ detractæ ſit medium; parallelogrammum verò bis ex eiſdem contentum, ſit rationale: Reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie rationali totam ſuperficiem mediam.

A C B

Theorema 61. Propoſitio 79.

Si de linea recta detrahatur recta potentia incommenſurabilis toti lineæ: compoſitum autem ex quadratis totius, & lineæ detractæ, ſit medium: Parallelogrammum verò bis ex ijſdem ſit etiam medium: præterea ſint quadrata ipſarum incommenſurabilia parallelogrammo bis ex ijſdem contento, Reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie media totam ſuperficiem mediam.



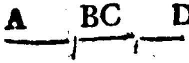
Theo-

Theorema 62. Propoſitio 80.

Reſiduo vnica tantum linea recta coniungitur rationalis, potentia tantum cõmenſurabilis toti lineæ.

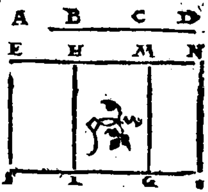
Theorema 63. Propoſitio 81.

Reſiduo medio primo vnica tantum linea coniungitur media, potentia tantum commenſurabilis toti, ipſa cum tota rationale continens.



Theorema 64. Propoſitio 82.

Reſiduo medio ſecundo vnica tantum coniungitur recta linea media, potentia tantum commenſurabilis toti, ipſa cum tota medium continens.



Theorema 65. Propoſitio 83.

Lineæ minori vnica tantum recta linea coniungitur, potentia incommenſurabilis toti, faciens cum tota cempoſitum ex quadratis

L ipſa-

ipsarum rationale: id verò parallelogrammum, quod bis ex ipsis fit, medium.

A B C D

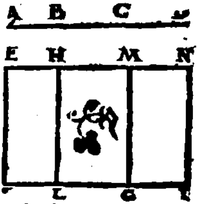
Theorema 66. Propositio 84.

Lineæ facienti cum superficie rationali totam superficiem mediam, vnica tantum cõiungitur linea recta, potentia incommensurabilis toti; faciens autem cum tota compositum ex quadratis ipsarum, medium; Id verò, quod fit bis ex ipsis, rationale.

A B C D

Theorema 67. Propositio 85.

Lineæ cum media superficie facienti totam superficiẽ mediam, vnica tantum coniungitur linea, potentia toti incommensurabilis, faciens cum tota compositum ex quadratis ipsarum, medium, id verò, quod bis ex ipsis etiam medium: & præterea faciens compositum ex quadratis ipsarum incommensurabile ei, quod fit bis ex ipsis.



DEFINITIONES.

TERTIAE, NEMPE APOTOMArum, seu residuorum.

Proposita linea rationali, & Residuo, si tota nempe composita ex ipso Residuo, & linea illi coniuncta, seu congruente, plus possit, quam coniuncta, quadrato rectæ lineæ sibi longitudine commensurabilis.

1.

Si quidem tota lineæ propositæ rationali sit longitudine commensurabilis; Vocetur Residuum primum, seu Apotome prima.

2.

Si verò coniuncta fuerit longitudine commensurabilis propositæ rationali; ipsa autem tota plus possit, quam coniuncta, quadrato lineæ, sibi longitudine commensurabilis; Vocetur Residuum secundum, seu Apotome secunda.

3.

Si verò neutra linearum fuerit longitudine commensurabilis propositæ rationali; possit autem ipsa tota plusquam cõiuncta, quadrato lineæ sibi longitudine commensurabilis; Vocetur Residuum tertium, seu Apotome tertia.

Rursus si tota possit plus, quam coniuncta seu congruens, quadrato rectæ lineæ sibi longitudine incommensurabilis.

4.

Et quidem si tota fuerit longitudine commensurabilis ipsi rationali; Vocetur Residuum quartum, seu Apotome quarta.

5.

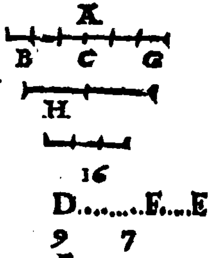
Si verò coniuncta fuerit longitudine commensurabilis ipsi rationali; & tota plus possit, quam coniuncta, quadrato lineę sibi longitudine incommensurabilis; Vocetur Residuum quintum, seu Apotome quinta.

6.

Si denique neutra linearum fuerit commensurabilis longitudine ipsi rationali; fueritque tota potentior, quam coniuncta; quadrato lineæ sibi longitudine incommensurabilis; Vocetur Residuum sextum, seu Apotome sexta.

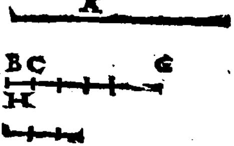
Problema 19. Propositio 86.

Reperire primum Residuum, seu Apotomen.



Problema 20. Propositio 87.

Reperire secundum Residuum.



D.........F....E

27 9

E.........

12

Problema 21. Propositio 88.

B.........E....C

9 7

A

Reperire tertium Residuum.

F H G

K

Problema 22. Propositio 89.

A

Reperire quartum Residuum.

B C G

H

D.........F....E

16 4

Problema 23. Propositio 90.

A

B C G

Reperire quintum Residuum.

H

D.........F....E

25 7

Problema 24. Propositio 91.

F H G

Reperire sextum Residuum.

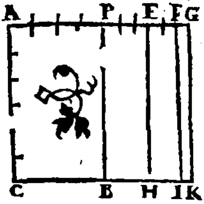
K

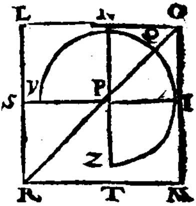
E.........13

B....D....C

Theorema 68. Propoſitio 92.

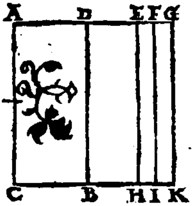
Si ſuperficies contineatur ſub linea rationali, & reſiduo primo; recta linea, quę illam ſuperficiem poteſt, eſt reſiduum.

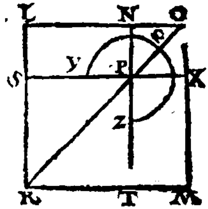




Theorema 69. Propoſitio 93.

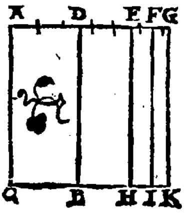
Si ſuperficies contineatur ſub linea rationali, & reſiduo ſecundo; recta linea, quæ illam ſuperficiem poteſt, eſt reſiduum mediale primum, ſeu mediæ Apotome prima.

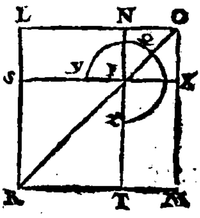




Theorema 70. Propoſitio 94.

Si ſuperficies cōtineatur ſub linea rationali, &

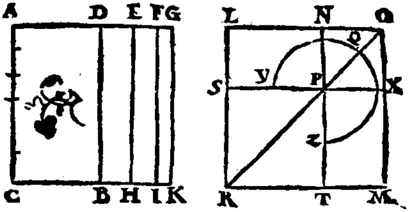




residuo tertio; recta linea, quæ illam superficiem potest, est residuum mediale secundum, seu mediæ, est apotome secunda.

Theorema 71. Propositio 95.

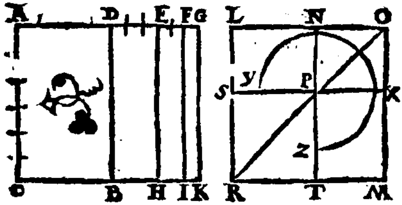
Si superficies contineatur sub linea rationali, & residuo quarto; recta linea, quæ



illam superficiem potest, est linea minor.

Theorema 72. Propositio 96.

Si superficies contineatur sub linea rationali, & residuo quinto; recta linea, quæ illam superficiem potest, est ea, quæ dicitur cum

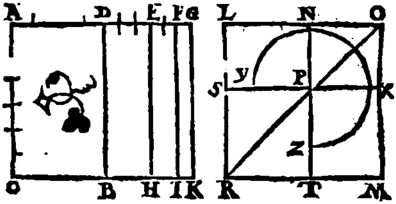


rationali superficie faciens totam medialem.

Theorema 73. Propositio 97.

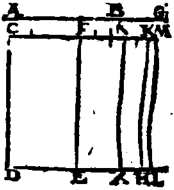
Si superficies cõtineatur sub linea rationali, & Re-

& Reſiduo ſexto; recta linea, quæ illam ſuperficiẽ poteſt, eſt ea, quæ dicitur faciens cum mediali ſuperficie totam medialem.



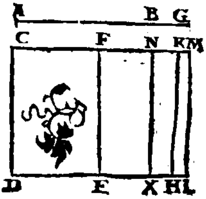
Theorema 74. Propoſitio 98.

Quadratum reſidui ad lineam rationalem applicatum, facit alterum latus reſiduum primum.



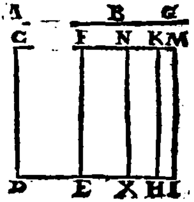
Theorema 75. Propoſitio 99.

Quadratum reſidui medialis primi ad rationalem applicatum, facit alterum latus, reſiduum ſecundum.



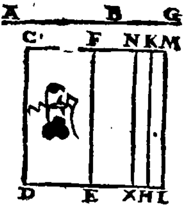
Theorema 76. Propoſitio 100.

Quadratum reſidui medialis ſecundi ad rationalem applicatum, facit alterum latus reſiduũ tertium.



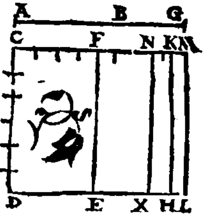
Theo-

Theorema 77. Propositio 101.



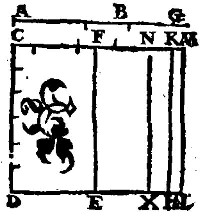
Quadratum lineæ minoris ad rationalem applicatum, facit alterum latus residuum quartum.

Theorema 78. Propositio 102.



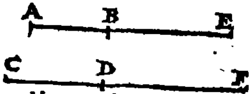
Quadratum lineæ cum rationali superficie facientis totam medialem, ad rationalem applicatum, facit alterum latus residuum quintum.

Theorema 79. Propositio 103.



Quadratum lineæ cum mediali superficie facientis totam medialem, ad rationalem applicatum, facit alterum latus residuum sextum.

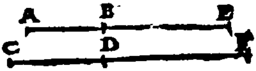
Theorema 80. Propositio 104.



Recta linea residuo commensurabilis longitudine; est & ipsa residuum, seu in ordine eadem.

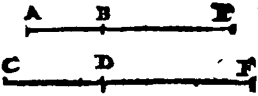
Theorema 81. Propositio 105.

Recta linea commensurabilis residuo mediali, eſt & ipſa reſiduum mediale, & eiuſdem ordinis; ſeu in ordine eadem.



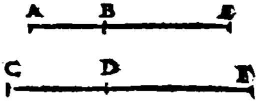
Theorema 82. Propoſitio 106.

Recta linea cõmenſurabilis lineæ minori: eſt & ipſa linea minor.



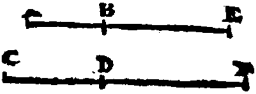
Theorema 83. Propoſitio 107.

Recta linea commenſurabilis lineæ cum rationali ſuperficie facienti totam medialem; eſt & ipſa linea cum rationali ſuperficie faciens totam medialem.



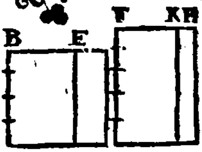
Theorema 84. Propoſitio 108.

Recta linea commenſurabilis lineæ cum mediali ſuperficie facienti totam medialem; eſt & ipſa cum mediali ſuperficie faciens totam medialem.



Theorema 85 Propoſitio 109.

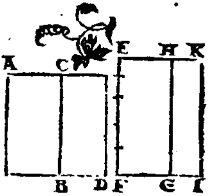
Si de ſuperficie rationali detrahatur ſuperficies medialis; recta linea, quæ reliquam ſuperficiem poteſt, eſt alterutra ex duabus irrationalibus, aut reſiduum, aut linea minor.



Theo-

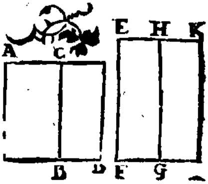
Theorema 86. Propoſitio 110.

Si de ſuperficie mediali detrahatur ſuperficies rationalis; aliæ duæ irrationales fiunt, aut reſiduum mediale primum, aut cum rationali ſuperficies totam medialem.



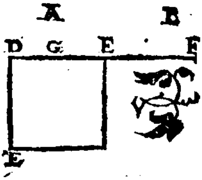
Theorema 87. Propoſitio 111.

Si de ſuperficie mediali detrahatur ſuperficies medialis, quæ ſit incommenſurabilis toti; reliquæ duæ fiunt irrationales, aut reſiduum mediale ſecundum, aut cum mediali ſuperficie faciens totam medialem.



Theorema 88. Propoſitio 112.

Linea, quæ reſiduum dicitur, non eſt eadem cum ea, quæ dicitur Binomium.



SCHO-

SCHOLIVM EX THEONE, ZAMberto, Campano, & P. Christ. Clauio.

Ex his demonstratis facilè intelligitur, quod recta linea, quæ residuum dicitur, & cætera quinque eam consequentes irrationales, neque lineæ mediali, neque sibi ipsæ inter se sunt eadem.

Nam quadratum lineæ medialis secundum rationalem applicatum, facit alterum latus, rationalem lineam, longitudine incommensurabilem ei, seu ad quam applicatur, per propos. 23. libri decimi.

Quadratum verò residui secundum rationalem applicatum, facit alterum latus, residuum primam: per 98.

Quadratum verò residui medialis primi secundum rationalem applicatum, facit alterum latus, residuum secundum, per 99.

Quadratum verò residui medialis secundi, facit alterum latus, residuum tertium, per 100.

Quadratum verò lineæ minoris, facit alterum latus residuum quartum, per 101.

Quadratum verò lineæ cum rationali superficie facientis totam medialem, facit alterum latus residuum quintum, per 102.

Quadratum verò lineæ cum mediali superficie facientis totam medialem, secundum rationalem applicatum, facit alterum latus residuum sextum, per 103.

Cùm

Cùm igitur dicta latera, quæ sunt latitudines cuiusq; parallelogrammi vnicuiq; quadrato æqualis, & ad rationalem, applicati, differant & à primo latere, & ipsa inter se, (nam à primo differunt: quoniam sunt residua non eiusdem ordinis) constat ipsas quoq; lineas irrationales inter se differentes esse. Et quoniam demonstratum est, residuum non esse idem quod Binomium: quadrata autem residui, & quinq; linearum irrationalium illud consequentium, ad rationalem applicata, faciunt altera latera ex residuis eiusdem ordinis, cuius sunt & residua, quorum quadrata applicantur rationali, similiter & quadrata Binomij, & quinq; linearum irrationalium illud consequentium, ad rationalem applicata, faciunt altera latera ex Binomijs eiusdem ordinis, cuius sunt & Binomia, quorum quadrata applicantur rationali. Ergo lineæ irrationales, quæ consequuntur Binomium, & quæ consequuntur residuum, sunt inter se differentes. Quare dictæ lineæ omnes irrationales sunt numero, 13. subsequentes.

1. Media linea; quæ vulgò medialis appellatur: propos. 22.

2. Linea ex binis nominibus (vulgò Binomium:) cuius sex sunt species inuentæ: propos. 37.

3. Ex binis medijs prima; vulgò Bimediale primū: propos. 38.

4. Ex binis medijs secunda; vulgò Bimediale secundum: propos. 39.

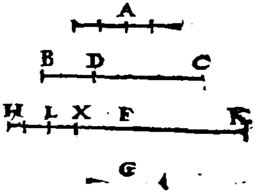
5. Maior: propos. 40.

6. Li-

6. Linea rationale, ac medium potens: propos. 41.
7. Bina media potens: propos. 42.
8. Apotome (vulgò residuum:) cuius etiam species sex sunt reperta: propos. 74.
9. Media Apotome prima; vulgò residuum: propos. 75.
10. Media Apotome secunda, vulgò residuum mediale secundum: propos. 76.
11. Minor: propos. 77.
12. Linea cum rationali medium totum efficiens; vulgò linea cum rationali superficie totam medialem faciens: propos. 78.
13 Cum medio medium totum efficiens; vulgo linea cum mediali superficie totam medialem faciens: propos 79.

Theorema 89. Propositio 113.

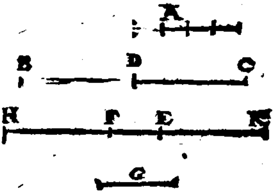
Quadratum lineæ rationalis ad Binomium applicatum, facit alterum latus residuum, cuius nomina sunt commensurabilia Binomij nominibus, & in eadem proportione, præterea id, quod sit residuum, eundem ordinem retinet, quem Binomium.



Theorema 90. Propositio 114.

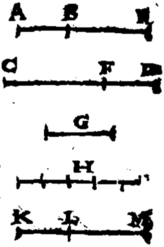
Quadratum lineæ rationalis ad residuum applicatum, facit alterum latus Binomium, cuius

cuius nomina sunt commensurabilia, nominibus residui, & in eadem proportione; præterea id quod fit Binomium, est eiusdem ordinis, cuius & residuum.



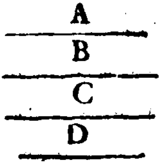
Theorema 91. Propositio 115.

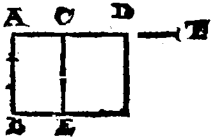
Si parallelogrammum contineatur ex residuo, & Binomio, cuius nomina sunt commensurabilia nominibus residui, & in eadem proportione; recta linea, quæ illam superficiem potest, est rationalis.



Theorema 92. Propositio 116.

Ex linea media nascuntur lineæ irrationales innumerabiles, quarum nulla vlli antedictarum eadem sit:



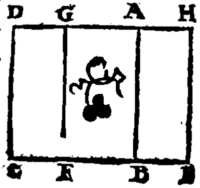


Theo-

Theorema 103. Propositio 117.

E...H..E
E...

Propositum sit nobis demonstrare, in figuris quadratis diametrum esse lõgitudine incommensurabilem ipsi lateri.



FINIS ELEMENTI X.

EVCLI-

EVCLIDIS ELEMENTVM VNDECIMVM.

ET SOLIDORVM primum.

DEFINITIONES

1.

Solidum, est quod longitudinem, latitudinem, & crassitudinem habet.

2.

Solidi autem extremum est superficies.

3.

Linea recta est ad planum recta, cùm ad rectas omnes lineas, à quibus illa tangitur, quæque in proposito sunt plano, rectos angulos efficit.

4.

Planum ad planum rectũ est, cùm rectæ lineæ, quæ communi planorum sectioni ad rectos angulos in vno planorum ducuntur, alteri plano ad rectos sunt angulos.

5.

Rectæ lineæ ad planum inclinatio est angulus acutus, ipsa insistente linea, & adiuncta altera comprehensus, cùm à sublimi rectæ illius lineæ termino deducta fuerit perpendicularis in ipso plano fecerit ad propositæ illius lineæ extremum quod in eodem est plano, altera recta linea fuerit adiuncta.

6.

Plani ad planum inclinatio, est angulus acutus rectis lineis contentus, quæ in vtroque planorum ad idem cõmunis sectionis punctum ductæ, rectos ipsi sectioni angulos efficiunt.

7.

Planum similiter inclinatum esse ad planum, atque alterum ad alterum dicitur, cũ dicti inclinationum anguli inter se sunt æquales.

8.

Parallela plana sunt, quæ inter se non incidunt, nec concurrunt.

9.

Similes figuræ solidæ sunt, quæ similibus planis, multitudine æqualibus continentur.

10.

Aequales, & similes figuræ solidæ sunt, quæ similibus planis, multitudine & magnitudine æqualibus continentur.

11.

Solidus angulus est plurimum, quàm duarum linearum, quæ se mutuò contingant, nec in eadem sint superficie, ad omnes lineas inclinatio.

Aliter.

Solidus angulus est, qui pluribus, quã duobus planis angulis, in eodem plano non consistentibus, sed ad vnum punctum collectis continetur.

12. Py-

12.

Pyramis est figura solida, quæ planis continetur, ab vno plano ad vnum punctum collecta.

13.

Prima figura est solida, quæ planis continetur, quorum aduersa duo sunt, & æqualia, similia, & parallela; alia verò parallelogramma.

14.

Sphæra est figura, quę conuerso circum quiescentem diametrum semicirculo continetur, cùm in eundem rursus locum restitutus fuerit, vnde moueri cœperat.

Aliter ex Theodosio.

Sphæra est figura solida, sub vna superficie comprehensa, ad quam ab vno puncto eorum, quæ intra figuram sunt posita, cadentes omnes rectæ lineæ, inter se sunt æquales.

15.

Axis autem sphæræ est, quiescens illa linea recta, per centrum ducta, circum quam semicirculus conuertitur.

16.

Centrum verò Sphæræ est idem, quod & semicirculi.

17.

Diameter autem sphæræ est, recta quædam linea per centrum ducta, & vtrinque à sphęræ superficie terminata.

18.

Conus est figura, quæ sub cõuerso circumquiescens alterum latus eorum, quæ rectum angulum continent, orthogonio triangulo continetur, cum in eundem rursus locum illud triangulum restitutum fuerit, vnde moueri cœperat.

Atque si quiescens recta linea æqualis sit alteri, quæ circum rectum angulum cõuertitur, orthogonius erit Conus: sin minor, amblygonius: si verò maior, oxygonius.

19.

Axis autem Coni est, quiescens illa recta linea, circum quam triangulum vertitur.

20.

Basis verò Coni est, circulus qui à circumducta linea recta describitur.

21.

Cylindrus est figura, quæ sub conuerso circumquiescens alterum latus eorum, quæ rectum angulum continent, parallelogrammo orthogonio compræhenditur, cum in eundem rursus locum restitutum fuerit illud parallelogrammum, vnde moueri cœperat.

22.

Axis autem Cylindri est quiescens illa recta linea, circum quam parallelogrammum vertitur.

23. Ba-

23.

Bases verò cylindri sunt circuli, à duobus aduersis lateribus, quæ circum aguntur, descripti.

24.

Similes coni, & cylindri sunt, quorum & axes, & basium diametri proportionales sunt.

25.

Cubus seu hexaëdrum est figura solida, quę sub sex quadratis ęqualibus continetur.

26.

Tetraëdrum est figura solida, quæ sub triangulis quatuor æqualibus, & æquilateris continetur.

27.

Octaëdrum figura est solida, quæ sub octo triangulis æqualibus, & æquilateris continetur.

28.

Dodecaëdrum figura est solida, quę sub duodecim pentagonis æqualibus, æquilateris, & æquiangulis continetur.

29.

Eicosaëdrum figura est solida, quæ sub triangulis viginti æqualibus, & æquilateris continetur.

30.

Parallelepipedum est figura solida, quæ sub sex figuris quadrilateris, quarum quæ ex aduerso, parallelæ sunt, continetur.

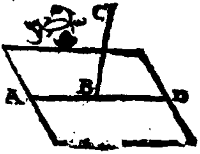
31.

Solida figura in ſolida dicitur inſcribi, quádo omnes anguli figuræ inſcriptæ conſtituuntur, vel in angulis, vel in lateribus, vel denique in planis figuræ, cui inſcribitur.

32.

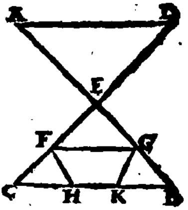
Solida figura ſolidæ figuræ viciſſim circũſcribi dicitur, quando vel anguli, vel latera, vel denique plana figuræ, circumſcriptæ tangunt omnes angulos figuræ, circum quá deſcribitur.

Theorema 1. Propoſitio 1.



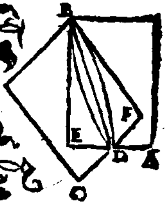
Quædam rectę lineę pars in ſubiecto quidem non eſt plano; quædam verò in ſublimi.

Theorema 2. Propoſitio 2.



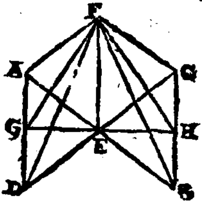
Si duæ rectæ lineæ ſe mutuo ſecent, in vno ſunt plano: atque triangulum omne in vno eſt plano.

Theorema 3. Propoſitio 3.



Si duo plana ſe mutuò ſecent, communis eorum ſectio eſt recta linea.

Theorema 4. Propositio 4.

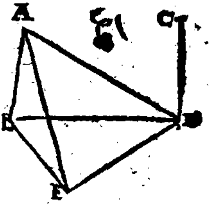


Si recta linea, rectis duabus lineis se mutuò secantibus, in communi sectione ad rectos angulos insistat: illa, ducto etiam per ipsas plano, ad angulos rectos erit.

Theorema 5. Propositio 5.

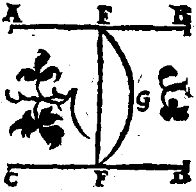
Si recta linea, rectis tribus lineis se mutuò tangentibus, in communi sectione ad rectos angulos insistat: illæ tres rectæ in vno sunt plano.

Theorema 6. Propositio 6.



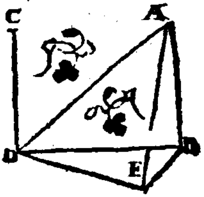
Si duæ rectæ lineæ eidem plano ad rectos sint angulos: parallelæ erunt illæ rectæ lineæ.

Theorema 7. Propositio 7.



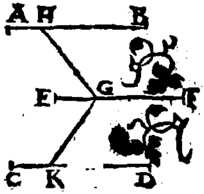
Si duæ sint parallelæ rectæ lineæ, in quarum vtraq; sumpta sint quælibet puncta: illa linea, quæ adhęc puncta adiungitur, in eodem est cum parallelis plano.

Theorema 8 Propoſitio 8.



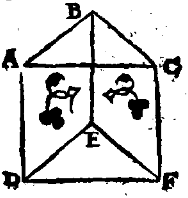
Si duæ ſint parallelæ rectæ lineæ, quarum altera ad rectos cuidam plano ſit angulos: & reliqua eidem plano ad rectos angulos erit.

Theorema 9 Propoſitio 9.



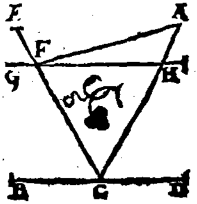
Quæ eidem rectæ lineæ ſunt parallelæ, ſed non in eodem cum illa plano: hæ quoque ſunt inter ſe parallelæ.

Theorema 10. Propoſitio 10.



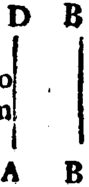
Si duę rectæ lineæ ſe mutuò tangentes ad duas rectas ſe mutuò tangentes ſint parallelæ, non autem in eodem plano; illæ angulos æquales comprehendent.

Problema 1. Propoſitio 11.



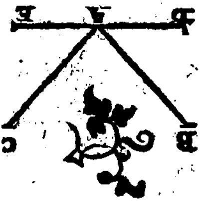
A dato puncto in ſublimi, ad ſubiectum planũ perpendicularem rectam lineam ducere.

Problema 2. Propo-
ſitio 12.



Dato plano, à puncto, quod in illo datum eſt ad rectos angulos rectam lineam excitare.

Theorema 11. Propo-
ſitio 13.

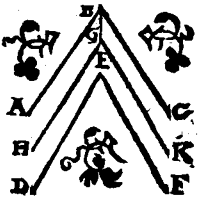


Dato plano à pũcto quod in illo datum eſt, duæ rectæ lineæ ad rectos angulos non excitabuntur, ad eaſdem partes.

Theorema 12. Propo-
ſitio 14.

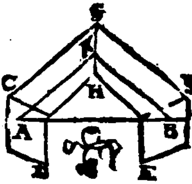
Ad quæ plana, eadem recta linea recta eſt; illa ſunt parallela.

Theorema 13. Propoſitio 15.



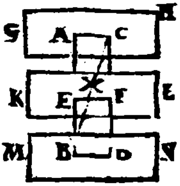
Si duæ rectæ lineæ ſe mutuo tangentes, ad duas rectas ſe mutuò tangentes ſint parallelæ, non in eodem conſiſtentes plano: paralela ſunt, quæ per illas ducuntur, plana.

Theorema 14. Propositio 16.



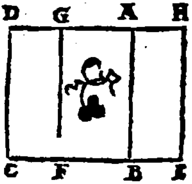
Si duo plana parallela plano quopiam secentur; communes illorum sectiones sunt parallelæ.

Theorema 15. Propositio 17.



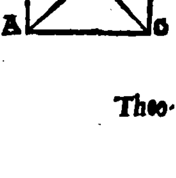
Si duæ rectæ lineæ parallelis planis secentur; in easdem proportiones secabuntur.

Theorema 16. Propositio 18.



Si recta linea plano cuipiam ad rectos sit angulos; illa etiam omnia, quæ per ipsam plana, ad rectos eidem plano angulos erunt.

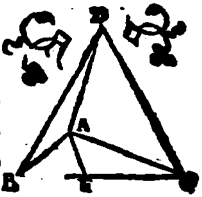
Theorema 17. Propositio 19.



Si duo plana se mutuò secantia, plano cuidam ad rectos sint angulos; communis etiam illorum sectio ad rectos eidem plano angulos erit.

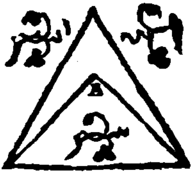
Theo-

Theorema 18. Propositio 20.



Si angulus solidus sub planis tribus angulis contineatur: ex his duo quilibet, vtut assumpti, tertio sunt maiores.

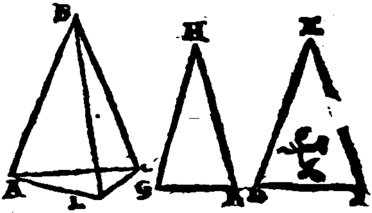
Theorema 19. Propositio 21.



Solidus omnis angulus sub minoribus quàm rectis quatuor angulis planis, continetur.

Theorema 20. Propositio 22.

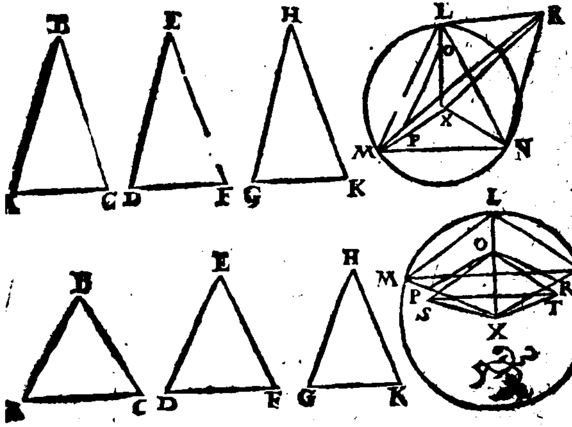
Si plani tres anguli æqualibus rectis contineantur lineis, quorum duo vt libet assumpti, tertio sint maiores; triangulum constitui potest ex lineis æquales, illas rectas coniungentibus.



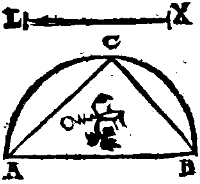
Problema 3. Propositio 23.

Ex planis tribus angulis, quorum duo, vt libet assumpti, tertio sint maiores, solidum angu-

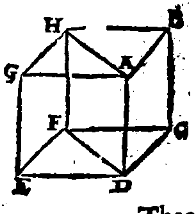
angulum constituere. Oportet autem illos tres angulos rectis quatuor esse minores.



Theorema 21. Propositio 24.



Si solidum sub parallelis planis contineatur; aduersa illius plana, sunt parallelogramma, similia, & æqualia.



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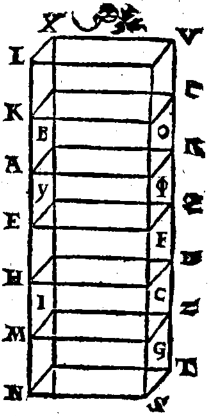
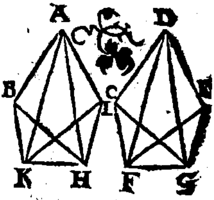


Theorema 22. Propositio 25.

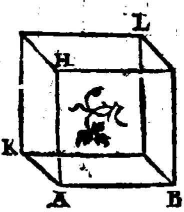
Si ſolidum parallelopipedum, ſub parallelis planis contentum plano ſecetur, aduerſis planis parallelo: erit quemadmodum baſis ad baſim, ita ſolidum ad ſolidum.

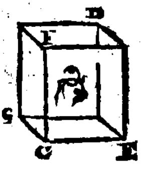


Problema 4. Propoſitio 26.

Ad datam rectam lineã, eiuſque punctum, angulum ſolidum conſtituere, ſolido angulo dato æqualem.

Problema 5. Propoſitio 27.

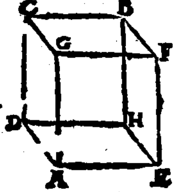




A data recta linea, dato ſolido parallelis planis comprehenſo, ſimile, & ſimiliter poſitum ſolidum parallelis planis contentum deſcribere.

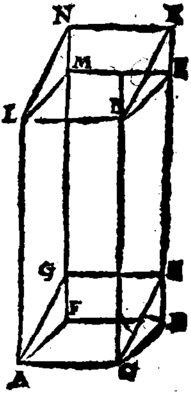
Theorema 23. Propoſitio 28.

Si ſolidum parallelis planis comprehenſum ducto per aduerſorum planorum diagonios plano, ſectum ſit: illud ſolidum ab hoc plano bifariam ſecabitur.



Theorema 24. Propoſitio 29.

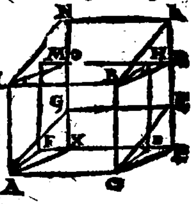
Solida parallelapipeda, ſeu parallelis planis cōprehenſa, quæ ſuper eandem baſim, & in eadem ſunt altitudine, quorum inſiſtentes lineæ in iiſdem collocantur rectis lineis: illa ſunt inter ſe æqualia.



Theo-

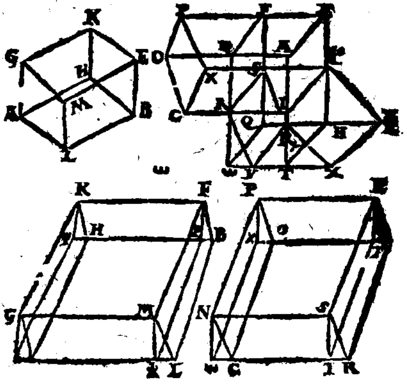
Theorema 25. Propositio 30.

Solida parallelis planis circumscripta, quæ super eandem basim, & in eadem sunt altitudine, quorum insistentes lineæ non in ijsdem reperiuntur rectis lineis; illa sunt inter se æqualia.



Theorema 26. Propositio 31.

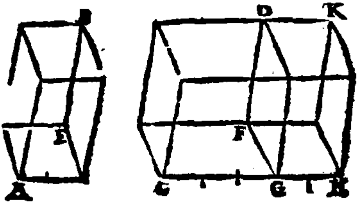
Solida parallelis planis, circumscripta, quæ super æqualibus basibus, & in eadem sunt altitudine: æqualia sunt inter se.



Theo-

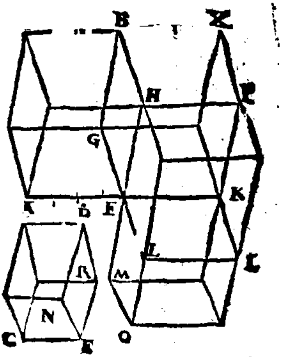
Theorema 27. Propoſitio 32.

Solida parallelis planis circumſcripta, quæ eiuſdem ſunt altitudinis; eam habent inter ſe proportionē, quam baſes.



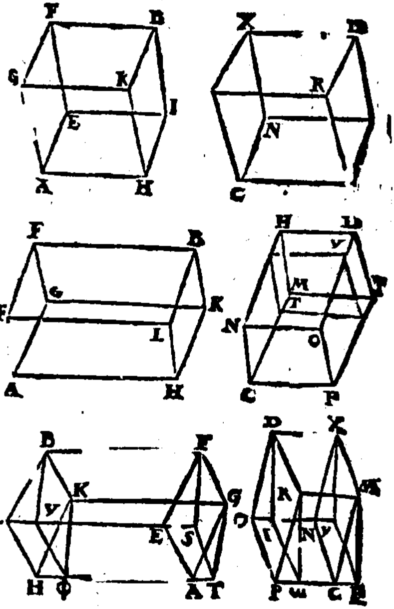
Theorema 28. Propoſitio 33.

Similia ſolida parallelis planis circumſcripta habent inter ſe proportionem homologorum laterum triplicatam.



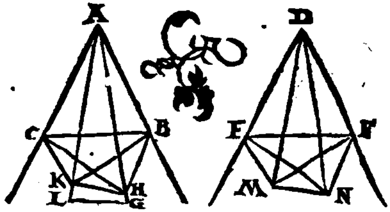
Theorema 29. Propoſitio 34.

Aequalium ſolidorum parallelis planis contentorum baſes, cũ altitudinibus reciprocãtur. Et ſolida parallelis planis contenta, quorum baſes cum altitudinibus reciprocantur; illa ſunt æqualia.



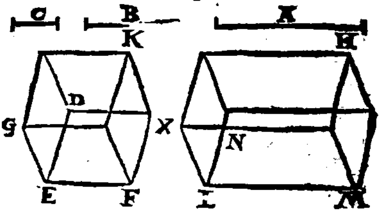
Theorema 30. Propoſitio 35.

Si duo plani ſunt anguli æquales, quorum verticibus ſublimes rectæ lineæ inſiſtant, quæ cum lineis primò poſitis angulos contineant æquales, vtrunque vtrique: in ſublimibus autem lineis quælibet ſumpta ſint puncta, & ab his ad plana, in quibus conſiſtunt anguli primum poſiti, ductæ ſint perpendiculares; ab earum verò punctis, quæ in planis ſignata fuerint, ad angulos primum poſitos adiunctæ ſint rectæ lineæ; hę cum ſublimibus æquales angulos comprehendent.



Theorema 31. Propoſitio 36.

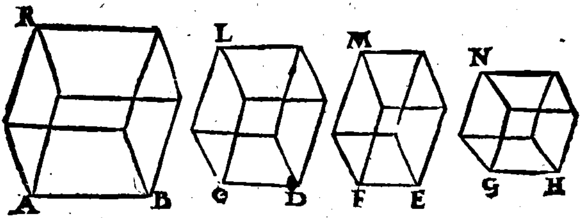
Si rectæ tres lineæ ſint proportionales; quod ex his tribus fit ſolidum parallelis planis contentum, æquale eſt deſcripto à media linea ſolido parallelis planis comprehenſo, quod æquilaterum quidem ſit, ſed antedicto æquiangulum.



Theo-

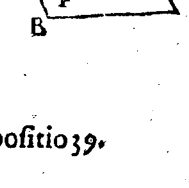
Theorema 32. Propositio 37.

Si rectæ quatuor lineæ sint proportionales, illa quoque solida parallelis planis contenta, quæ ab ipsis lineis & similia & similiter describuntur, proportionalia erunt. Etsi solida parallelis planis comprehensa, quæ & similia & similiter describuntur, sint proportionalia; illæ quoque rectæ lineæ proportionales erunt.



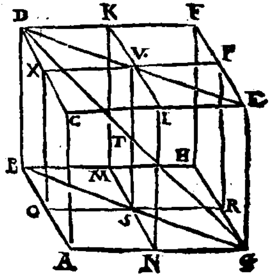
Theorema 33. Propositio 38.

Si planum ad planum rectum sit; & à quodam puncto eorum, quæ in vno sunt planorum, perpendicularis ad alterum planum ducta sit: illa, quæ ducitur perpendicularis, in communem cadet ipsorum planorum sectionem.



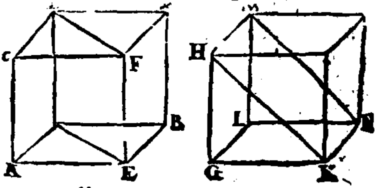
Theorema 34. Propositio 39.

Si in solido parallelis planis circumscripto, aduerſorum planorum lateribus bifariam ſectis, educta ſint per ſectiones plana; communis illorum planorum ſectio, & ſolidi parallelis planis circumſcripti diameter, ſe mutuò bifariam ſecabunt.



Theorema 35. Propoſitio 40.

Si duo ſint æqualis altitudinis priſmata; quorum hoc quidem baſim habeat parallelogrammum: illud verò triangulum ſit autem parallelogrammũ trianguli duplum: illa priſmata erunt æqualia.



FINIS ELEMENTI XI.

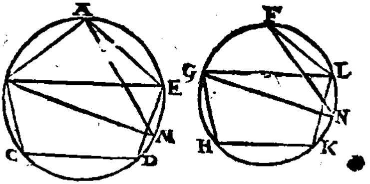
EVCLI-

EVCLIDIS ELEMENTVM DVODECIMVM.

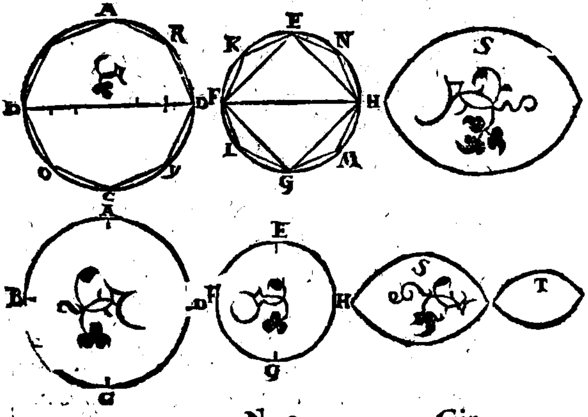
ET SOLIDORVM ſecundum.

Theorema 1. Propoſitio 1.

Similia, quæ ſunt in circulis polygona, proportionem habent inter ſe; quã deſcripta à diametris quadrata.



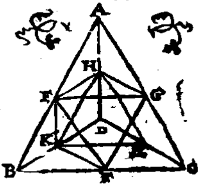
Theorema 2. Propoſitio 2.



Circuli eam inter ſe proportionem habent, quàm deſcripta à diametris quadrata.

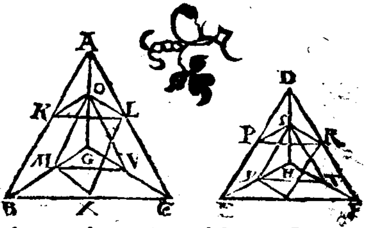
Theorema 3. Propoſitio 3.

Omnis pyramis trigonam habens baſim, in duas diuiditur pyramides nõ tantùm æquales & ſimiles inter ſe, ſed toti etiam pyramidi ſimiles, quarum trigonæ ſunt baſes; atq; in duo priſmata æqualia, quę duo priſmata dimidio pyramidis totius ſunt maiora.



Theorema 4. Propoſitio 4.

Si duę eiuſdem altitudinis pyramides trigonas habeant baſes; ſit autem illarum vtraque diuiſa & in duas pyramides inter ſe ęquales, totique ſimiles; & in duo priſmata æqualia; Ac eodem modo diuidatur vtraque pyramidum, quæ ex ſuperiore diuiſione natæ ſunt; idque perpetuo fiat: quemadmodum

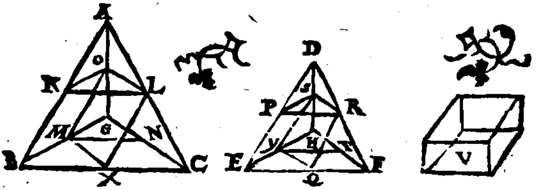


ſe habet vnius pyramidis baſis ad alterius pyramidis baſim; ita & omnia, quæ in vna pyra-

pyramide, priſmata ad omnia, quæ in altera pyramide, priſmata multitudine æqualia.

Theorema 5. Propoſitio 5.

Pyramides eiuſdem altitudinis, quarum trigonæ ſunt baſes; eam inter ſe proportionem habent, quàm ipſæ baſes.

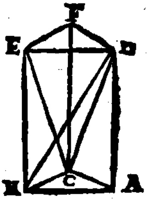


Theorema 6. Propoſitio 6.

Pyramides eiuſdem altitudinis, quarum polygonæ ſunt baſes, eam interſe proportionem habent quam ipſæ baſes.

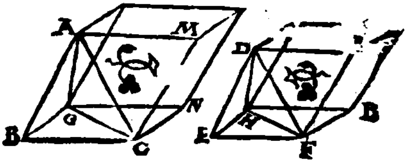
Theorema 7. Propoſitio 7.

Omne priſma trigonam habens baſim, diuiditur in tres pyramides inter ſe æquales, quarum trigonę ſunt baſes.



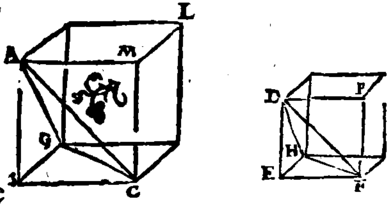
Theorema 8. Propositio 8.

Similes pyramides, quæ trigonas habent bases; in triplicata sunt homologorũ laterũ proportione.



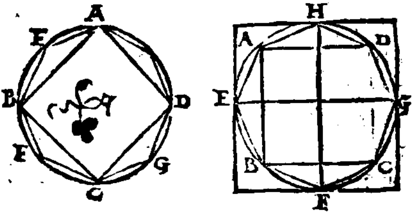
Theorema 9 Propositio 9.

Aequalium pyramidum, & trigonas bases habentium, reciprocantur bases cum altitudinibus. Et quarum pyramidum trigonas bases habentium reciprocantur bases cũ altitudinibus; illę sunt æquales.



Theorema 10. Propositio 10.

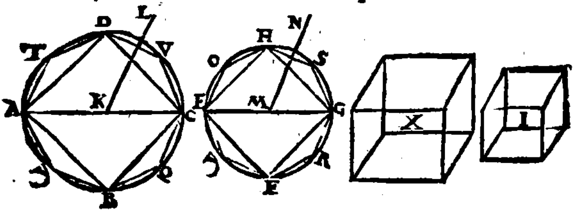
Omnis conus tertia pars est cylindri,



eandem cum ipso cono basim habentis, & altitudinem æqualem.

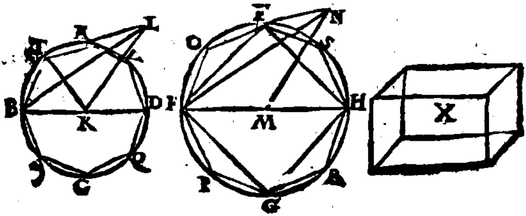
Theorema 11. Propositio 11.

Coni, & cylindri eiusdem altitudinis, eam inter se proportionem habent, quam bases.

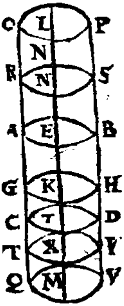


Theorema 12. Propositio 12.

Similes coni, & cylindri, triplicatam habent, inter se proportionem diametrorum, quæ sunt in basibus.



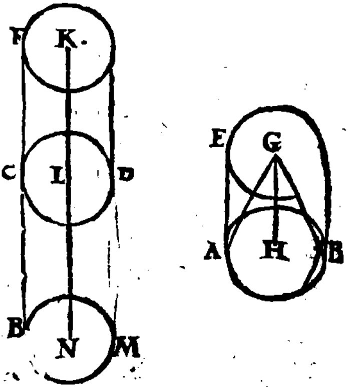
Theorema 13. Propositio 13.



Si cylindrus plano ſectus ſit aduerſis planis parallelo: Erit quéadmodum cylindrus ad cylindrum, ita axis ad axem.

Theorema 14. Propoſitio 14.

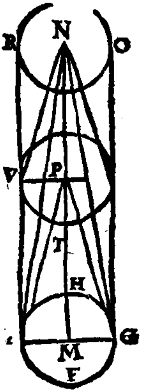
Coni, & cylindri, qui in æqualibus ſunt baſibus; eam habent interſe proportioné, quam altitudines.

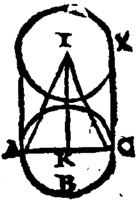


Theo-

Theorema 15. Propositio 15.

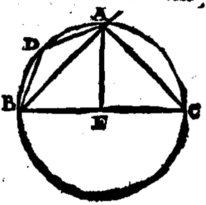
Aequalium conorum, & cylindrorum baſes cum altitudinibus reciprocãtur. Et quorum conorum & cylindrorum baſes cum altitudinibus reciprocantur; illi ſunt æquales.





Problema 1. Propoſitio 16.

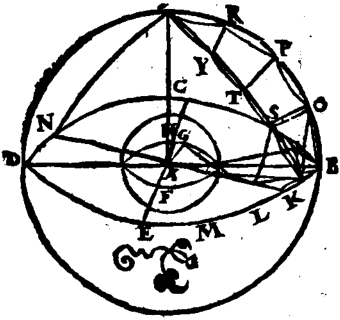
Duobus circulis circa idem centrum exiſtentibus, in maiore circulo polygonum æqualium, pariumq́ue laterum inſcribere, quod minorem circulum non tangat.

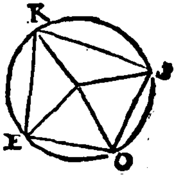


Pro-

Problema 2. Propositio 17.

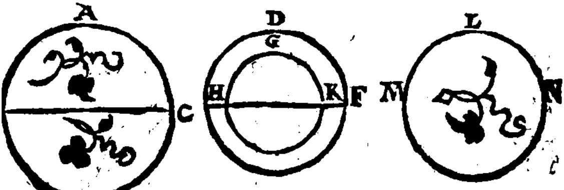
Duabus ſphæris circa idem centrum exſiſtentibus, in maiori ſphæra ſolidum polyedrum inſcribere, quod minoris ſphæræ ſuperficiem non tangat.





Theorema 16. Propoſitio 18.

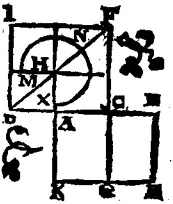
Sphæræ inter ſe proportionem habent ſuarum diametrorum triplicatam.



FINIS ELEMENTI XII.

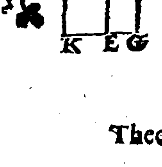
EVCLIDIS ELEMENTVM DECIMVM TERTIVM, ET SOLIDORVM tertium.

Theorema 1. Propositio 1.



Si recta linea per extremã & mediam proportionem secta sit; maius segmentũ, quod totius lineæ dimidium assumpserit, quintuplum potest eius, quòd à totius dimidia describitur, quadrati.

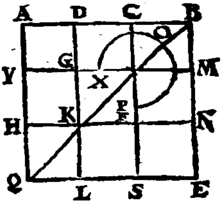
Theorema 2. Propositio 2.



Si recta linea sui ipsius segmenti quintuplum possit, & dupla segmenti huius linea per extremam & mediam proportionem secetur, maius segmentum reliqua pars est lineæ primùm positæ.

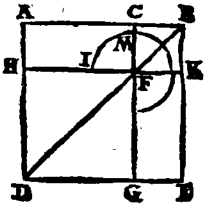
Theo-

Theorema 3. Propoſitio 3.



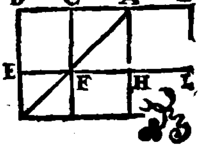
Si recta linea per extremam & mediam proportionem ſecta ſit; minus ſegmentum, quod maioris ſegmenti dimidium aſſumpſerit, quintuplum poteſt eius, quòd à maioris ſegmenti dimidio deſcribitur quadrati.

Theorema 4. Propoſitio 4.



Si recta linea per extremam & mediam proportionem ſecta ſit: quòd à tota, quodque à minore ſegmento, ſimul vtraque quadrata, tripla ſunt eius, quod à maiore ſegmento deſcribitur, quadrati.

Theorema 5. Propoſitio 5.



Si ad rectam lineam, quæ per extremam & mediam proportionē ſecatur, adiuncta ſit altera ſegmento maiori æqualss: tota hæc linea recta per extremam & mediam proportionem

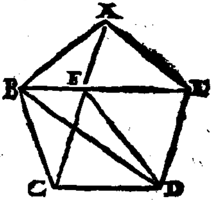
nem secatur; estque maius segmentum recta linea primùm posita.

Theorema 6. Propositio 6.

Si recta linea ῥητὴ, siue rationalis, per extremam & mediam proportionem secta sit; vtrunque segmentorum ἄλογος, siue irrationalis, est linea, quæ dicitur Residuum, seu Apotome.

Theorema 7. Propositio 7.

Si pentagoni æquilaterî tres sint æquales anguli, siue qui deinceps, siue qui non deinceps sequuntur: illud pentagonum erit æquiangulum.



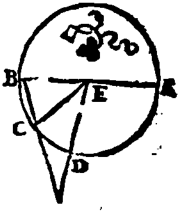
Theorema 8. Propositio 8.

Si pentagoni æquilateri, & ęquianguli duos qui deinceps sequuntur, angulos rectę subtendant lineæ; illæ per extremam & mediam proportionem se mutuò secant; earumque maiora segmenta, ipsius pentagoni lateri sunt æqualia.

Theo-

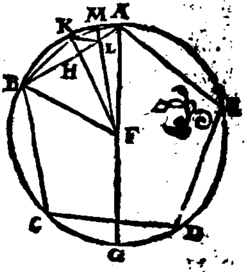
Theorema 9. Propositio 9.

Si latus hexagoni, & latus decagoni eidem circulo inscriptorum composita sint; tota recta linea per extremam & mediam proportionem secta est; eiusque segmentum maius, est hexagoni latus.



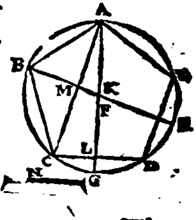
Theorema 10. Propositio 10.

Si in circulo pentagonum æquilaterũ inscriptum sit; pentagoni latus potest & latus hexagoni, & latus decagoni, eidem circulo inscriptorum.



Theorema 11. Propositio 11.

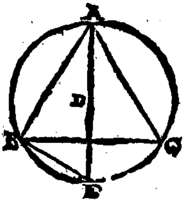
Si in circulo ρητὴν, seu rationalem diametrum habente, inscriptum sit pentagonum æquilaterum; pentagoni latus irrationalis est linea, quæ vocatur minor.



Theo-

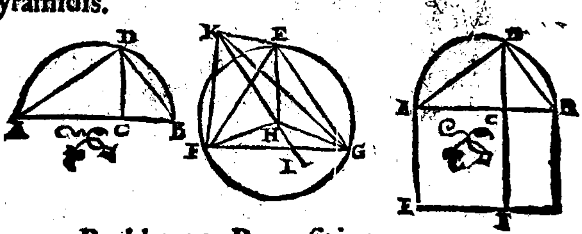
Theorema 12, Propositio 12.

Si in circulo inſcriptum ſit triangulum æquilaterum; huius trianguli latus potentia triplum eſt eius lineæ, quæ ex circuli centro ducitur.



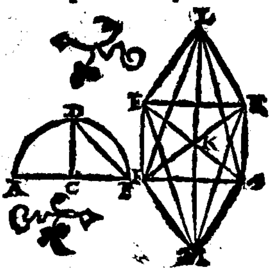
Problema 1. Propoſitio 13.

Pyramidem conſtituere; & data ſphæra cōplecti: atque docere quod illius ſphæræ diameter potentia ſeſquialtera ſit lateris ipſius pyramidis.



Problema 2. Propoſitio 14.

Octaëdrum conſtituere, eaq; ſphęra, qua pyramidē complecti; atque probare, quod illius ſphęræ diameter potentia dupla ſit lateris ipſius octaëdri.



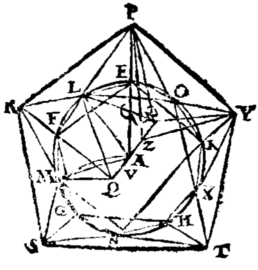
Pro-

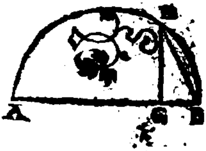
Problema 3. Propositio 15.

Cubum constituere; eaque sphæra, qua & superiores figuras complecti; atque docere quod illius sphæræ diameter potentia tripla sit lateris ipsius cubi.

Problema 4. Propositio 16.

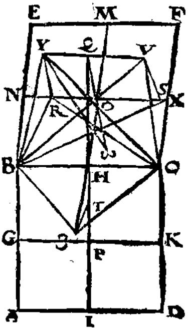
Icosaedrum constituere, eademque sphæra, qua & antedictas figuras, complecti; atque probare, quod illius Icosaedri latus irrationalis sit linea, quæ vocatur Minor.





Pro-

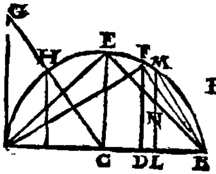
Problema 5. Propositio 17.

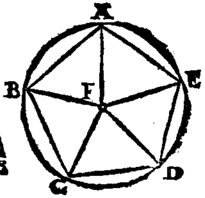


Dodecaedrum constituere; eademq́ue sphæra, qua & antedictas figuras, cõplecti; atq; probare quod illius dodecaedri latus irrationalis sit linea, quæ vocatur Residuum, seu Apotome.

Problema 6. Propositio 18.

Quinque figurarũ latera proponere, & inter se comparare.





SCHOLIVM.

Interpretes hoc in loco demonstrant, præter dictas quinque figuras, non posse aliam constitui figuram solidam, quæ ex planis & æquilateris & æquiangulis contineatur, inter se æqualibus.

Non enim ex duobus triangulis, [illegible] duabus figuris, solidus cōstituitur angulus. [illegible] tem tres anguli plani requirantur ad solidi [illegible] li constitutionem.

Ex tribus autem triangulis æquilateris, [illegible] pyramidis angulus.

Ex quatuor angulis, Octaëdri.

Ex quinque angulis, Icosaëdri.

Nam ex triangulis, sex autem triangulis æquilateris & æquiangulis ad idem punctum coeuntibus, non fiet angulus solidus: cum n. trianguli æquilateri angulus, recti vnius bessem (hoc est duas tertias partes:) contineat, erunt eiusmodi sex anguli rectis quatuor æquales. Quod fieri non potest. Nam solidus omnis angulus minoribus, quam rectis quatuor angulis, continetur; per propos. 21. l. 11.

Multò ergo minus ex pluribus, quam sex planis eiusmodi angulis, solidus angulus constabit.

Sed ex tribus quadratis, Cubi angulus continetur.

Ex quatuor autem quadratis, nullus angulus solidus constitui potest. Rursus enim recti quatuor erunt. Multo ergo minus ex pluribus, quàm quatuor eiusmodi angulis solidus angulus constabit.

Ex tribus autem pentagonis æquilateris, & æquiangulis, Dodecaedri angulus continetur.

Sed ex quatuor huiusmodi angulis, nullus solidus angulus confici potest. Cùm enim pentagoni æquilateri angulus rectus sit, & quinta recti pars erunt quatuor anguli rectis quatuor maiores. Quod

fieri

fieri nequit. Nec sanè ex aliis polygonis figuris solidus angulus continebitur, quod hinc quoque absurdum sequatur.

Quamobrem perspicuum est, præter dictas quinque figuras aliam figuram solidam non posse constitui, quæ ex planis æquilateris, & æquiangulis inter se æqualibus, contineatur.

Vide Theon. p. 244. Et p. Clauius p. 277.

FINIS ELEMENTI XIII.

EVCLIDIS ELEMENTVM DECIMVMQVARTVM ET SOlidorum quartum, vt quidam arbitratur; Vt alij verò, Hypſiclis Alexandrini, de quinque corporibus.

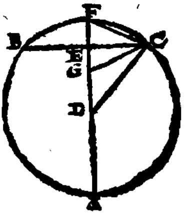
LIBER PRIMVS.

Proœmium Hypſiclis Alexandrini ad Protarchum.

Baſilides Tyrius, Protarche, Alexandriam profectus, patriq; noſtro ob diſciplinæ ſocietatem commēdatus, longiſſimo peregrinationis tempore cum eo verſatus eſt. Cumq; differerent aliquando de ſcripta ab Apollonio comparatione Dodecaedri, & Icoſaedri eidem ſphæræ inſcriptorum; quam hæc inter ſe habeant rationem, cenſuerunt ea non rectè tradidiſſe Apollonium; quæ à ſe emendata, vt de patre audire erat, literis prodiderunt. Ego autem poſtea incidi in alterum librum ab Apollonio editum, qui demonſtrationem accuratè complecteretur de re propoſita, ex eiuſq; problematis indagatione magnam equidem cœpi voluptatem. Illud certè ab omnibus perſpici poteſt quod ſcripſit Apollonius, cum ſit in omnium manibus. Quod autem diligenti, quantum conijcere licet, ſtudio nos poſtea ſcri-

pisse videmur, id monumentis consignatum tibi dedicandum duximus, vt qui feliciter cum in omnibus disciplinis, tum vel maximè in Geometria versatus, scitè ac prudenter iudices ea, quæ dicturi sumus: Ob eam verò, quæ tibi cum patre fuit, vitæ consuetudinem, quaq; nos complecteris, beneuolentiam, tractationem ipsam libenter audias. Sed iam tempus est, vt procemio finem facientes, hanc sytaxim aggrediamur.

Theorema 1. Propositio 1.



Perpendicularis linea, quæ ex circuli cuiuspiam centro in latus pentagoni ipsi circulo inscripti ducitur; dimidia est vtriusque simul lineæ, & eius, quæ ex centro, & lateris decagoni in eodem circulo inscripti.

Theorema 2. Propositio 2.

Si binæ rectæ lineæ extrema, & media proportione secentur; ipsæ similiter secabuntur, in easdem scilicet proportiones.

Theorema 3. Propositio 3.

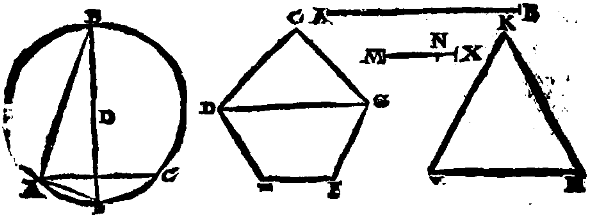
Si in circulo pentagonum æquilaterum inscribatur; quod ex latere pentagoni; & quod ex ea, quæ binis lateribus pentagonis subtéditur, recta linea; vtraque simul quadrata, quintupla sunt eius, quod ex se midiametro describitur, quadrati.

Theorema 4. Propoſitio 4.

Si latus hexagoni alicuius circuli ſecetur extrema, & media proportione; maius illius ſegmentum erit latus decagoni eiuſdem circuli.

Theorema 5. Propoſitio 5.

Idem circulus comprehendit, & dodecaë-dri pentagonum, & icoſaëdri triangulum, eidem ſphæræ inſcriptorum.



Theorema 6. Propoſitio 6.

Si pentagono, & æquilatero, & æquiangulo circumſcribatur circulus; ex cuius centro ad vnum pentagoni latus ducatur linea perpendicularis: Erit, quod ſub dicto latere, & perpendiculari continetur, rectangulum trigeſies ſumptum, dodecaëdri ſuperficiei æquale.

Theorema 7. Propoſitio 7.

Si ex centro circuli triangulum icoſaedri cir-

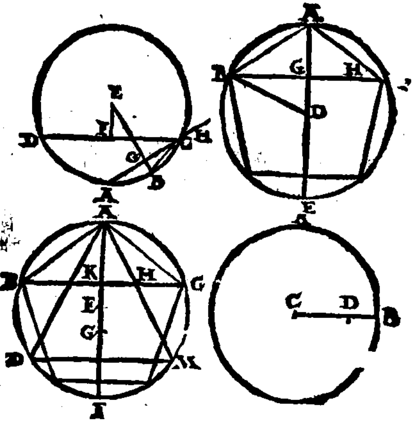
circumscribentis, linea perpendicularis ducatur ad vnum latus trianguli: Erit, quod sub dicto latere, & perpendiculari comprehenditur, rectangulum trigesies sumptum, Icosaedri superficiei æquale.

Theorema 8. Propositio 8.

Rectangulum contentum sub tribus quartis partibus diametri alicuius circuli; & sub quinque sextis partibus lineæ subtendentis, angulum pentagoni æquilateri in eodẽ circulo descripti; æquale est dicto pentagono.

Theorema 9. Propositio 9.

Superficies Dodecaedri ad superficiem Icosaedri, in eadem sphæra descripti, eandem proportionem habet, quàm latus cubi ad latus Icosaedri.



Cubi latus.

E ————————

Dodecaedri, latus.

E ————————

Icoſaedri, latus.

C ————

Theorema 10. Propoſitio 10.

Si recta linea ſecetur extrema, & media proportione; Erit, vt recta potens id, quod à tota; & id, quod maiori ſegmento, ad rectam potentem id, quod à tota, & id, quod à minori ſegmento; Ita latus cubi ad latus icoſaedri, in eadem ſphæra cum cubo inſcripti.

Theorema 11. Propoſitio 11.

Dodecaedrum, ad Icoſaedrum in eadem cũ ipſo ſphæra inſcriptum eſt, vt cubi latus, ad Icoſaedri latus, in vna eademq; ſphæra.

Theorema 12. Propoſitio 12.

Latus trianguli æquilateri potentia ſeſquitertium eſt lineæ perpendicularis ab vno angulo ad latus oppoſitum, ſeu baſim, deductæ.

Theorema 13. Propoſitio 13.

Si ſphæræ (duo ſolida corpora, Tetraedrum & Octaedrum circumſcribentis:) diameter fuerit Rationalis: Erit tam ſuperficies Tetraedri, quam Octaedri in ea ſphæra media.

Theo-

Theorema 14. Propositio 14.

Si Tetraedrum, atque octaedrum in eadem sphæra inscribantur: Erit basis Tetraedri sesquitertiā baseos Octaedri: Superficies autem Octaedri sesquialtera superficiei Tetraedri.

Theorema 15. Propositio 15.

Recta linea ex angulo quouis tetraedri in sphæra inscripti, per centrum sphærę ducta; cadit in centrum baseos oppositæ; estq; perpendicularis ad ductam basin.

Theorema 16. Propositio 16.

Octaedrum in sphæra inscriptum, diuiditur in duas pyramides æquales, & similes, æqualium altitudinum, basis verò vtriusq; pyramidis est quadratum sub duplum quadrati diametri sphæræ.

Theorema 17. Propositio 17.

Tetraedrum sphæræ impositum ad Octaedrum in eadem sphæra descriptum, se habet, vt rectangulum sub linea potente viginti septē sexagesimas quartas partes quadrati lateris tetraedri; & sub linea continente octo nonas partes eiusdem lateris, comprehensum, ad quadratum diametri sphæræ.

Theorema 18. Propositio 18.

Linea perpendicularis ex quolibet angulo trianguli æquilateri ad basin oppositam demissa; tripla est eius perpendicularis, quæ ex centro trianguli ad eandem basin deducitur.

Theo.

Theorema 19. Propoſitio 19.

Si Octaedrum ſphæræ inſcribatur; erit ſemidiameter ſphæræ potentia tripla eius perpendicularis, quæ ex centro ſphæræ ad baſin quamcunque Octaedri ducitur.

Theorema 20. Propoſitio 20.

Duplum quadrati ex diametro cuiuslibet ſphæræ deſcripti, æquale eſt ſuperficiei cubi in illa ſphæra collocati: perpendicularis autem à centro ſphæræ in aliquam baſin cubi demiſſa, æqualis eſt dimidio lateris cubi.

Theorema 21. Propoſitio 21.

Idem circulus comprehendit, & cubi quadratum, & Octaedri triangulum, eiuſdem ſphæræ.

Theorema 22. Propoſitio 22.

Si Octaedrum, atque Tetraedrum eidem ſphæræ inſcribantur; Erit Octaedrum ad triplum Tetraedri, vt latus Octaedri ad latus Tetraedri.

Theorema 23. Propoſitio 23.

Si recta linea propoſita potuerit totam aliquam lineam ſectam extrema, & media ratione, & maius eius ſegmentum; Item totam aliam ſimiliter ſectam, & minus eius ſegmentum: Erit maius ſegmentum prioris lineæ latus Icoſaedri; minus autem ſegmentum poſterioris lineæ latus Dodecaedri, eius ſphæræ cuius recta linea propoſita diameter exiſtit.

Theo-

Theorema 24. Propositio 24.

Si latus Octaedri potuerit maius, & minus segmentum rectæ lineæ extrema, ac media ratione sectæ; poterit latus Icosaedri in eadem sphæra descripti, duplum minoris segmenti.

Theorema 25. Propositio 25.

Si recta linea diuisa extrema ac media ratione cum minore segmento angulum rectum constituat, cui recta subtendatur: Erit recta linea, quæ potentia sit sub dupla ipsius rectę subtensæ, latus Octaedri eius sphæræ, in qua dictum minus segmentum latus existit dodecaedri.

Theorema 26. Propositio 26.

Si latus Tetraedri possit maius, & minus segmentum lineæ rectæ extrema ac media ratione sectæ: latus Icosaedri eidem sphæræ inscripti potentia sesquialterum est minoris segmenti.

Theorema 27. Propositio 27.

Cubus ad Octaedrum in eadem cum ipso sphæra descriptum, est, vt superficies cubui ad Octaedri superficiem: Item vt latus cubui ad semidiametrum sphæræ.

Theorema 28. Propositio 28.

Si sint quatuor lineæ rectæ continuæ proportionalis, nec non & aliæ quatuor, ita vt sit eadem antecedens omnium: Erit proportio tertiæ ad tertiam proportionis secundæ

ad se-

ad secundam duplicata; & proportio quar-
tæ ad quartam eiusdem proportionis secun-
dæ ad secundam triplicata.

Theorema 29. Propositio 29.

Qradratum lateris trianguli æquilateri ad ipsum triangulum habet proportionem duplicatam proportionis lateris trianguli ad lineam medio loco proportionalem inter perpendicularem ab vno angulo ad latus oppositum ductam, & dimidium ipsius lateris.

Theorema 30. Propositio 30.

Si cubus, & Tetraedrum in eadem sphæra describantur; Erit quadratum cubi ad triangulum Tetraedri, vt latus Tetraedri ad lineam perpendicularem, quæ ex vno angulo trianguli Tetraedri ad latus oppositum protrahatur.

Theorema 31. Propositio 31.

Latus Tetraedri potentia sesquialterum est axis, seu altitudinis ipsius; Axis verò, siue altitudo Tetraedri potentia sesquialtera est lateris cubi in eadem sphæra descripti.

Theorema 32. Propositio 32.

Cubus triplus est Tetraedri eidem sphæræ inscripti.

FINIS ELEMENTI XIV.

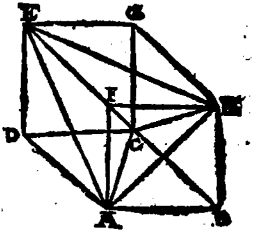
EVCLIDIS ELEMENTVM

DECIMVMQVINTVM ET SOlidorum quintum, vt nonnulli putant. Vt autem alij. Hypsiclis Alexandrini, de quinque corporibus.

LIBER II.

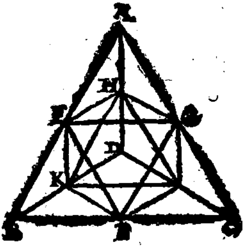
Problema 1. Propositio 1.

In dato cubo pyramidem, Tetraedrũ inſcribere.



Problema 2. Propositio 2.

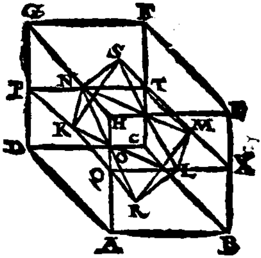
In data pyramide Octaedrum inſcribere.



Pro-

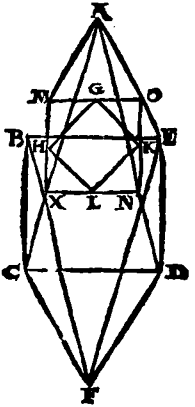
Problema 3. Propositio 3.

In dato cubo (hexaedro) Octaedrum inscribere.



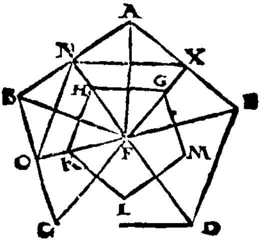
Problema 4. Propositio 4.

In dato octaedro cubum inscribere.

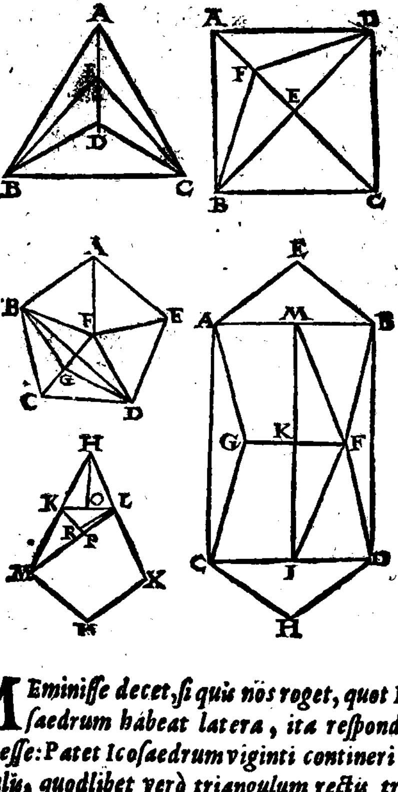


Problema 5. Propositio 5.

In dato Icosaedro dodecaedrum inscribere.



SCHOLIVM EX ZAMBERTO lib. 15. propos. 5. p. 260.



Meminisse decet, si quis nos roget, quot Icosaedrum habeat latera, ita respondendum esse: Patet Icosaedrum viginti contineri triangulis, quodlibet verò triangulum rectis tribus *constare lineis. Quare multiplicanda sunt nobis viginti triangula in trianguli vnius latera, fiuntq́ sexaginta, quorum dimidium est triginta. Ad eundem modum & in dodecaedro. Cum enim rursus duodecim pentagona dodecaedrum comprehendant, itemq́ pentagonum quodues rectis quinque constet lineis; quinque duodecies multiplicamus, fiunt sexaginta, quorum rursus dimidium est triginta. Sed cur dimidium capimus? Quoniam vnumquodque latus siue sit trianguli, siue pentagoni, siue quadrati, vt in cubo, iterato sumitur. Similiter autem eadem via & in cubo, & in tetraedro & in octaedro latera inuenies.*

Latera figurarum inuenienda.

Quòd si item velis singularum quoque figurarum angulos reperire, facta eadem multiplicatione, numerum procreatum partire in numerum planorum, quæ vnum solidum angulum includunt: vt quoniam triangula quinque vnum Icosaedri angulum continet, partire 60. in quinque nascuntur duodecim anguli Icosaedri. In dodecaedro autem tria pentagona angulum comprehendunt, partire ergo 60. in tria, & habebis dodecaedri angulos viginti. Atque simili ratione in reliquis figuris angulos reperies, &c.

Anguli figurarum inueniendi.

Problema 6. Propositio 6.

In dato octaedro pyramidem, seu tetraedrũ describere.

Pro-

Problema 7. Propositio 7.

In dato Dodecaedro Icosaedrum describere.

Problema 8 Propositio 8.

In dato Decaedro Cubum describere.

Problema 9. Propositio 9.

In dato Dodecaedro Octaedrum describere.

Problema 10. Propositio 10.

In dato Dodecaedro pyramidem describere.

Problema 11. Propositio 11.

In dato Icosaedro Cubum describere.

Problema 12. Propositio 12.

In dato Icosaedro pyramidem describere.

Problema 13. Propositio 13.

In dato cubo Dodecaedrum describere.

Problema 14 Propositio 14.

In dato cubo Icosaedrum describere.

Problema 15. Propositio 15.

In dato Icosaedro Octaedrum describere.

Problema 16. Propositio 16.

In dato Octaedro Icosaedrum describere.

Problema 17. Propoſitio 17.

In dato Octaedro Dodecaedrum deſcribere.

Problema 18. Propoſitio 18.

In data pyramide Cubum deſcribere: hoc eſt, in propoſito Tetraedro hexaedrum delineare.

Problema 19. Propoſitio 19.

Iu data pyramide Icoſaedrum deſcribere.

Problema 20. Propoſitio 20.

In data pyramide Dodecaedrum deſcribere.

Problema 21. Propoſitio 21.

In dato ſolido regulari ſphæram deſcribere: hoc eſt, Interprete Campano; in fabricato quouis quinque corporum regularium, ſphæram fabricare.

FINIS ELEMENTI XV.

EVCLI-

EVCLIDIS ELEMENTVM DECIMVM SEXTVM, ET Solidorum sextum.

Quo varia solidorum regularium sibi mutuò inscriptorum, & laterum eorundem comparationes explicantur, à Francisco Flussate Candalla, & P. Christophoro Clauio adiectam, & de quinque corporibus.

LIBER TERTIVS.

Theorema 1. Propositio 1.

SI in Dodecaedro Cubus describatur, & in hoc Cubo aliud Dodecaedrum: Erit proportio Dodecaedri exterioris ad Dodecaedrum interius proportionis eius, quam habet maius segmentum ad minus rectæ lineæ diuisæ extrema, ac media ratione triplicata.

Theorema 2. Propositio 2.

Linea perpendicularis ex quouis angulo pentagoni æquilateri, & æquianguli in latus oppositum demissa; secatur à linea recta illum angulum subtendente, extrema ac media ratione.

Theorema 3. Propositio 3.

Si ab angulis trianguli pyramidis ducantur tres lineæ rectæ, opposita latera secantes extrema ac media ratione; ita vt prope quemuis angulum sit maius segmentum vnius lateris, & minus alterius: Hæ tres sectionibus suis in medio producent basin Icosaedri in dicta pyramide descripti, inscriptam quidem alij triangulo, cuius anguli latera trianguli pyramidis secant extrema ac media ratione; & latera ipsa bifariam secantur ab angulis basis Icosaedri.

Theorema 4. Propositio 4.

Minus segmentum lateris pyramidis extrema ac media ratione secti, duplum est potentia lateris Icosaedri in ea pyramide descripti.

Theorema 5. Propositio 5.

Latus Cubi potentia dimidium est lateris pyramidis in eo descriptæ: Latus verò pyramidis duplum est longitudine lateris Octaedri sibi inscripti: latus denique Cubi duplum est potentia lateris sibi inscripti Octaedri.

Theorema 6. Propositio 6.

Latus Dodecaedri maius segmentum est rectæ lineæ quæ potentia est dimidia lateris pyramidis sibi inscriptæ.

Theorema 7. Propositio 7.

Si in Cubo describatur & Icosaedrum, & Dode-

Dodecaedrũ: Latus Icoſaedri medium proportionale erit inter latus Cubi, & Dodecaedri.

Theorema 8. Propoſitio 8.

Latus pyramidi potentia Octaedecuplum eſt lateris Cubi in ea deſcripti.

Theorema 9. Propoſitio 9.

Latus pyramidis potentia Octaedecuplum eſt rectæ lineæ extrema ac media ratione ſectæ, cuius maius ſegmentum eſt latus Dodecaedri in pyramide deſcripti.

Theorema 10. Propoſitio 10.

Si in Octaedro Icoſaedrum deſcribatur: Erit latus Icoſaedri potentia duplum minoris ſegmenti lateris Octaedri extrema ac media ratione diuiſi.

Theorema 11. Propoſitio 11.

Latus Octaedri potentia quadtuplum eſt ſeſquialterum lateris Cubi in ipſo deſcripti.

Theorema 12. Propoſitio 12.

Latus Icoſaedri maius ſegmentum eſt eius rectæ lineæ extrema ac media ratione ſectæ, quæ potentia dupla eſt lateris Octaedri in eo Icoſaedro deſcripti.

Theorema 13. Propoſitio 13.

Latus cubi ad latus Dodecaedri in ipſo deſcripti proportionẽ habet duplicatam eius, quam habet maius ſegmentum ad minus, rectæ lineæ diuiſæ extrema ac media ratione. Latus verò Dodecaedri ad latus cubi in ipſo descripti, proportionem habet, quam minus segmentum ad maius, eiusdem rectæ lineæ.

Theorema 14. Propositio 14.

Latus octaedri sesquialterum est lateris pyramidis sibi inscriptæ.

Theorema 15. Propositio 15.

Si ex quadrato diametri Icosaedri auferatur triplum quadrati lateris cubi in eo descripti; relinquitur quadratum sesquitertium quadrati lateris Icosaedri.

Theorema 16. Propositio 16.

Latus Dodecaedri minus segmentum est rectæ lineæ extrema ac media ratione diuisæ, quæ duplum potest lateris octaedri in eo descripti.

Theorema 17. Propositio 17.

Diameter Icosaedri potest, & sui ipsius lateris sesquitertium; & lateris pyramidis in eo descriptæ sesquialterum.

Theorema 18. Propositio 18.

Latus Dodecaedri ad Icosaedri sibi inscripti latus; se habet, vt minus segmentum lineæ perpendicularis ab vno angulo pentagoni ad latus oppositum ductæ, atque extrema ac media ratione diuisæ, ad partem eiusdem lineæ inter centrum pentagoni, & latus eiusdem positæ.

Theorema 19. Propositio 19.

Si dimidium lateris Icosaedri extrema ac

media

media ratione sectum fuerit, minusque eius segmentum à toto latere Icosaedri sublatum; à reliqua quoque recta linea pars rursum tertia detracta: Relinquetur latus Dodecaedri in Icosaedro descripti.

Theorema 20. Propositio 20.

Cubus sibi inscriptæ pyramidis triplus est.

Theorema 21. Propositio 21.

Pyramis sibi inscripti Octaedri dupla est.

Theorema 22. Propositio 22.

Cubus sibi inscripti Octaedri sextuplus est.

Theorema 23. Propositio 23.

Octaedrum sibi inscripti Cubi quadruplū sesquialterum est.

Theorema 24. Propositio 24.

Octaedrum sibi inscriptæ pyramidis tredecuplum sesquialterum est.

Theorema 25. Propositio 25.

Pyramis sibi inscripti Cubi non cupla est.

Theorema 26. Propositio 26.

Octaedrum ad Icosaedrum sibi inscriptum proportionem habet, quam duæ bases octaedri ad quinque bases Icosaedri.

Theorema 27. Propositio 27.

Icosaedrum ad Dodecaedrum sibi inscriptum, proportionem habet compositam ex proportione lateris Icosaedri ad latus cubi in eadem cum Icosaedro sphæra descripti; & ex proportione triplicata eius quam habet diameter Icosaedri ad rectam lineam contra basium Icosaedri oppositarum coniungentem.

Theorema 28. Propositio 28.

Dodecaedrum excedit Cubum sibi inscriptum parallelopipedo, cuius quidem basis à quadrato Cubi deficit rectangulo contento sub latere Cubi, tertiaque parte, minoris segmenti eiusdem lateris Cubi: At verò altitudo ab altitudine, siue latere Cubi deficit, minore segmento eius lineæ, quæ dimidiati lateris Cubi segmentum minus existit.

Theorema 29. Propositio 29.

Dodecaedrum ad Icosaedrum sibi inscriptum, proportionem habet compositam ex proportione triplicata eius, quam habet diameter Dodecaedri ad rectam lineam contra basium Dodecaedri oppositarum copulantem; & ex proportione lateris Cubi ad latus Icosaedri in eadem sphæra cum Cubo descripti.

Theorema 30. Propositio 30.

Dodecaedrum pyramidis, in qua describitur, duas nonas partes continet, minus duobus parallelepipedis; quorum vnius longitudo lateri Cubi in eadem pyramide descripti, æqualis est; latitudo verò tertiæ parti miti minoris ſegmenti lateris eiuſdem Cubi; altitudo denique à latere eiuſdem Cubi deficit minore ſegmento eius lineæ, quæ dimidiati lateris Cubi eiuſdem minus ſegmētum exiſtit: Alterius autem & longitudo, & latitudo lateri Cubi prædicti, eſt æqualis; altitudo verò minus ſegmentum eius lineæ quæ dimidiati lateris Cubi eiuſdem ſegmētum minus exiſtit; ita vt amborum altitudines ſimul altitudini, ſiue lateri eiuſdem Cubi ſint æquales.

Theorema 31. Propoſitio 31.

Octaedrum excedit ſibi inſcriptum Icoſaedrum parallelepipedo, cuius baſis eſt quadratum lateris Icoſaedri; altitudo verò maius ſegmentum ſemidiametri Octaedri extrema ac media ratione ſectæ.

DE QVINQVE CORPORVM regularium deſcriptione in data ſphæra, ex Pappo Alexandrino.

Lemma I.

Datis duobus circulis in ſphæra parallelis, dataque in vno eorum linea recta; ducare in altero diametrum huius rectæ datæ parallelam.

Lem-

Lemma II.

Si in planis parallelis descripti sint duo circuli, in quibus duæ rectæ parallelæ abscindant arcus similes: Erunt duæ rectæ coniungentes extrema vnius rectæ cum centro parallelæ duabus rectis, quæ extrema alterius rectæ cum centro coniungant.

Lemma III.

Si in sphæra sint duo circuli paralleli, & æquales, in quibus ductæ sint duæ rectę parallelæ, & æquales, ad eaidem partes centrorum: Rectæ harum parallelarum extrema puncta ad easdem partes coniungentes, æquales quoque, & parallelæ sunt, & ad plana circulorum perpendiculares.

Lemma IV.

Si in sphæra sint duo circuli paralleli, & æquales, in quibus ductæ sint duæ rectæ parallelæ, & æquales, non ad easdem partes cétrorum: Rectæ lineæ harum parallelarum extrema puncta non ad easdem partes coniungentes, in centro sphærę sese intersecant; ac proinde diametri sphæræ erunt, & inter se æquales: Rectæ verò lineæ earundem parallelarum extrema puncta ad easdem partes connectentes, & æquales, & parallelæ inter se ter ſe ſunt, & cum parallelis rectos angulos conſtituunt.

Lemma V.

Si in ſphæra ſint duæ rectæ parallelæ; rectæ earum puncta extrema ad eaſdem partes coniungentes, æquales inter ſe erunt: Et ſi parallelæ ſint æquales, coniungentes non ſolum æquales, ſed & parallelæ erunt, rectoſque cum ipſis angulos conficient.

Lemma VI.

In data ſphæra duos circulos æquales, ac parallelos deſcribere; ita vt diameter ſphæræ ſit vtriuſque diametri potentia ſeſquialtera.

Problema 1. Propoſitio 1.

In data ſphæra pyramidem trigonam deſcribere.

Problema 2. Propoſitio 2.

In data ſphæra Octaedrum deſcribere.

Problema 3. Propoſitio 3.

In data ſphæra Cubum deſcribere.

Problema 4. Propoſitio 4.

In data ſphæra Icoſaedrum deſcribere.

Problema 5. Propoſitio 5.

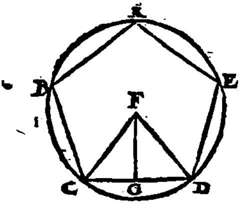
In data ſphæra Dodecaedrum deſcribere.

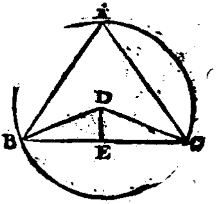
SCHO-

SCHOLIVM EX P. CLAVIO.

EX his, quæ hoc in loco & lib. 13 & lib. 14. demonstrata sunt, facile etiam ostenditur, si omnia quinque corpora regularia in vna eademque sphæra describantur, maximum omnium esse Dodecaedrum: Deinde Icosaedrum maius reliquis tribus: Tertio Cubum maiorem reliquis duobus: Octaedrum denique Tetraedro esse maius. Ex quo euidenter constabit, Euclidem recto ordine quinque hæc corpora cõstruxisse, cum post Tetraedrum statim Octaedrum non autem Cubum constituerit. Ita enim à minoribus ad maiora progressus est.

FINIS ELEMENTI XVI. & Vltimi.





Pagina 216. Theoremate 6. desunt hæ duæ Figuræ.