EVCLIDIS ELEMENTORVM LIBRI XV.

ACCESSIT LIBER XVI. DE QVINque solidorum Regularium inter se comparatione.

AD EXEMPLARIA R.P. CHRISTOphori Clauij è Societ. IESV, & aliorum hac postrema editione collati, emendati & aucti.

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IMMORTALITAS ACQVIRITVR EX LITERARVM STVDIIS

COLONIAE
Apud Petrum Cholinum
M. DC. XXVII.
Cum gratia & Priuilegio.

MATHESEOS ET GEOMETRIAE DIVISIO.

Mathematicæ diſciplinæ, quæ omnes circa quantitatem verſantur, nomen acceperunt à Græca dictione μάθημα ſeu μάθησις, quæ diſciplinam, & doctrinam ſignificat; eò quod tum gradatim aſcendendo doceantur & addiſcantur; tùm ſolæ ſemper ex præcognitis quibuſdam, conceſſis, & probatis, principijs, (Haud ex hypotheſibus nondum explicatis) ad concluſiones demonſtrandas, quod proprium eſt doctrinarum & diſciplinarum officium, teſte Ariſtotele libr. 1. Poſter. procedant.

Pythagoras, & Mathematici vniuerſis Mathematicas diſciplinas in quatuor partes principes diſtribuunt, nempe in Arithmeticam, Muſicam, Geometriam, & Aſtronomiam. Cùm enim omnis quantitas, circa quam hæ diſciplinæ verſantur, ſit vel diſcreta, ſub qua omnes numeri continentur; vel continua, ſub qua omnes magnitudines comprehenduntur; & vtraque tam ſecundum

dum se, quam comparatione alterius possit considerari; Visum est illis has quatuor partes instituere, quæ vtramque quantitatem pro duplici consideratione, diligenter contemplarentur. Itaque Arithmetica agit de quantitate discreta secundum se; omnesque numerorum proprietates ac passiones inquirit, & accuratè explicat. Musica tractat eandem quantitatem discretam, seu numerum cum alio comparatum; quatenus sonorum Harmoniam, & concentum respicit. Geometria de magnitudine, seu quantitate continua, secundum se quoque, vt immobilis existit, disputat. Astronomia denique magnitudinem, vt est mobilis, considerat; qualia sunt corpora cœlestia continuè mota. Ad has autem quatuor scientias Mathematicas, tum puras, tum mixtas, omnes aliæ de quantitate tractant, vti Perspectiua Geographia, Stereometri , & cæteræ, facili negotio, tanquam ad sua capita, & fontes, ex quibus emanant. reducuntur.

Geometria apud Euclidem diuiditur in planorum (superficierum planarum,) contemplationem, seu Geometriam propriè dictam; quæ libris sex primis absoluitur; Et in solidorum (corporum solidorum:) speculationem, seu Stereometriam; quæ libris postremis pertractatur. Prior quidem pars, nempe Geometria, in tres partes subdiuidiuiditur. Nam in prioribus quatuor libris agitur de planis absolutè, ita vt eorum æqualitas, & inæqualitas inuestigetur. In quinto verò libro de rationalibus magnitudinum proportionibus in genere disputatur. In sexto denique libro proportiones figurarum planarum discutiuntur. Posterior verò pars, nempe Stereometrica, in tres quoque partes subdiuiditur. In quarum prima, videlicet libris tribus, septimo, octauo, & nono, agitur de numerorum proprietatibus, passionibusque ad linearum (& aliarum magnitudinum:) symetrarum seu commensurabilium, & asymetrarum, seu incommensurabilium tractationem necessarijs. In secunda, nimirum libro decimo, satis prolixo, de lineis commensurabilibus, & incommẽsurabilibus, sine quarum notitia, corpora illa quinque solida, regularia, seu platonica perfectè tractari nequeunt, differitur. In tertia denique, nempe libris sex postremis (qui & ipsi subdiuidi poterunt) de solidis illis acutissimè disputatur, eorumque proprietates inuestigantur. De punctis autem, & lineis in hoc opere nulla ex professo extat cõtemplatio: quoniam Geometria potissimum circa figuras, quibus plana, & solida duntaxat (non puncta, & lineæ:) afficiuntur versatur.

Demonstratio omnis Mathematicorum ab antiquis scriptoribus diuiditur in Problema,& Theorema. Problema quidem vocatur ea demonſtratio, quæ iubet atque docet aliquid conſtituere, facere, inuenire, deſcribere, &c. vt ſuper data linei recta terminata triangulum æquilaterum conſtituere. Theorema verò appellatur ea demonſtratio, quæ ſolum aliquam proprietatem, ſeu paſſionem vnius, vel plurium ſimul quantitatum (multarum vel magnarum iam inuentarum:) perſcrutatur, vt in omni triangulo tres angulos eſſe æquales duobus rectis. Cæterùm tam problema, quàm Theorema dicitur apud Mathematicos propoſitio; eò quod vtrumque aliquid nobis proponit, vti in exemplis adductis conſtat, Demonſtrationes problematum concluduntur his ferè verbis; Quod faciendum erat: Theorematum verò Demonſtrationes, his verbis; Quod oſtendendum, vel demonſtrandū erat; habita nimirum ratione vtriuſque. Adhæc problemata per modum infinitum, ſed Theoremata per modum finitum ferè proferuntur. Lemma (ſumptio, vel aſſumptum latinè) appellatur ea minus principalis, & aliqua tantum declaratione indigens, demōſtratio, quæ ad demonſtrationem alicuius Problematis, vel Theorematis principalis aſſumitur, vt illa demonſtratio expeditior, ac breuior fiat; vt videre licet libr. 6. propoſ. 22. & paſ-

ſim

sim Corollarium, seu porisma; denique est, quod protenus ex facta demonstratione tanquam lucrum aliquod additum, seu consectarium accipitur; vt videre est lib. 2. propos. 4. & passim.

Cùm omnis autem doctrina, omnisque disciplina ex præexistente cognitione gignatur, atque ex assumptis, & concessis quibusdam principijs suas demonstrat conclusiones: Nulla autem scientia, teste Aristotele, sua principia demonstret; habebunt & Mathematicæ disciplinæ sua principia, ex quibus positis, & concessis sua Problemata, & Theoremata confirment. Horum autem tria solum genera apud Mathematicos reperiuntur. Quorum primum genus continet definitiones, quas nonnulli Hypotheses appellant, vt punctum est, cuius pars nulla est. Secundum genus complectitur petitiones, seu postulata, quæ per se adeò perspicua sunt, vt nulla confirmatione indigeant, sed auditoris duntaxat assensum exposcant, vt postuletur, vt à quouis puncto in quoduis punctum, rectam lineam ducere concedatur Tertium denique genus comprehendit. Axiomata, seu communes animi notiones, quæ non solum in scientia proposita, sed etiam in alijs omnibus vsque adeo euidentes sunt, vt ab eis nulla ratione dissentire queat is, qui ipsa vocabula rectè perceperit vt:

Quæ eidem æqualia, inter se sunt æqualia. Verum secundum nonnullos principium formale duplex existit, nempe indemonstrabile seu facilè, vt definitio, postulatum, & axioma; demonstrabile, vt Problema, Theorema, & omnis propositio. Principium verò materiale est punctum, linea, &c. Vid. & P. Christophorus Clauius, nobilissimus Elementorum Euclidis Interpres. Coloniæ Agrippinæ, anno 1607. 8. Iunij.

EVCLI-

EVCLIDIS ELEMENTVM PRIMVM.

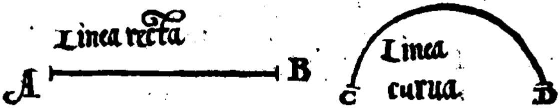
DEFINITIONES

1.

PVnctum est, cuius pars nulla est.

2.

Linea verò, longitudo latitudinis expers.



3.

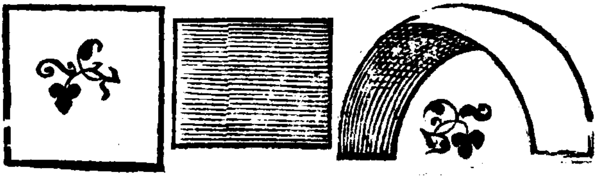
Lineæ autem termini, sunt puncta.

4.

Recta linea est, quæ ex æquo sua interiacet puncta.

5.

Superficies est, quæ longitudinem latitudinemque tantum habet.



6.

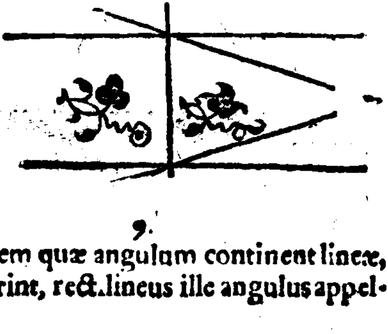
Superficiei extrema ſunt lineæ.

7.

Plana ſuperficies eſt, quæ ex æquo ſuas interiacet lineas.

8.

Planus angulus eſt duarum linearum in planò ſe mutuò tangentium, & non in directū iacentium alterius ad alterā inclinatio.



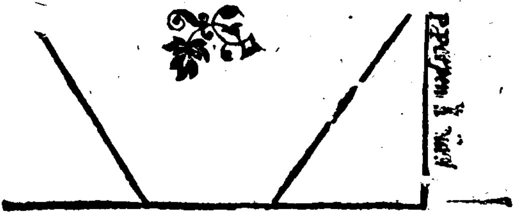
9.

Cùm autem quæ angulum continent lineæ, rectæ fuerint, rectilineus ille angulus appellatur.

10.

Cùm verò recta linea ſuper rectam conſiſtens lineam, eos qui ſunt deinceps, angulos æquales inter ſe fecerit; rectus eſt vterque æqua-

æqualium angulorum: quæ insistit recta linea, perpendicularis vocatur eius, cui insistit.



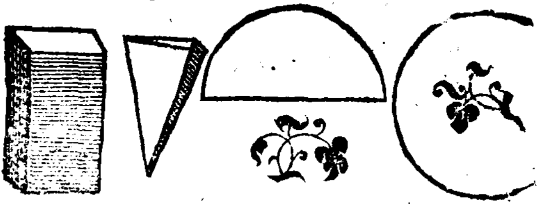
Obtusus angulus est qui recto maior est.

12.

Acutus verò, qui minor est recto.

13.

Terminus est, quod alicuius extremum est.



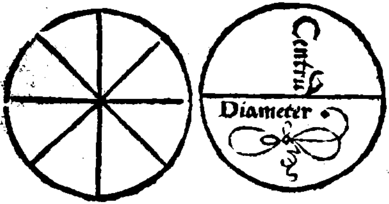
14.

Figura est, quæ sub aliquo, vel aliquibus terminis comprehenditur.

15.

Circulus est, figura plana sub vna linea comprehensa, quæ peripheria appellatur; ad quá ab vno puncto eorum, quæ intra figuram sunt

ſunt poſita, cadentes omnes rectæ lineæ inter ſe ſunt æquales.



16.

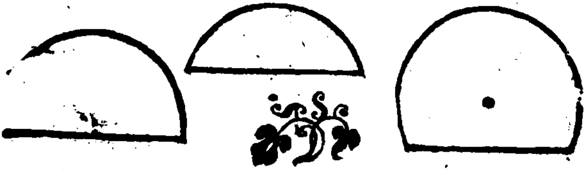
Hoc verò punctum, centrum circuli appellatur.

17.

Diameter autem circuli, eſt recta quædam linea per centrum ducta, & ex vtraque parte in circuli peripheriam terminata, quæ circulum bifariam ſecat.

18.

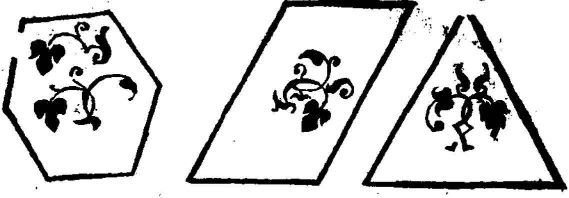
Semicirculus verò eſt figura, quæ continetur ſub diametro. & ſub ea linea, quæ de circuli peripheria aufertur.



19.

Rectæ lineæ figuræ ſunt, quæ ſub rectis lineis continentur.

20. Tri-



20. Trilateræ quidem, quæ sub tribus.

21.

Quadrilateræ, quæ sub quatuor.

22.

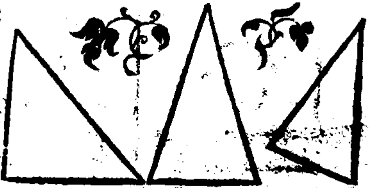
Multilateræ verò quæ sub pluribus, quàm quatuor, rectis lineis comprehenduntur.

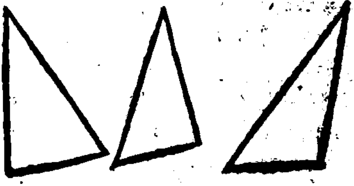
23.

Trilaterarum autem figurarum, æquilaterum est triangulum, quod tria latera habet æqualia.

24.

Isosceles autem est, quod duo tantùm æqualia habet latera.





25.

Scalenum verò est, quod tria inæ-

inæqualia habet látera.

26.

Ad hæc etiam, trilaterarum figurarum, rectangulum quidem triangulum est, quod rectum angulum habet.

27.

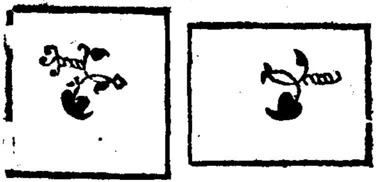
Amblygonium autem, quod obtusum angulum habet.

28.

Oxygonium verò, quod tres habet acutos angulos.

29.

Quadrilaterarum autem figurarum, quadratum quidem est, quod & æquilaterum, & rectangulum est.

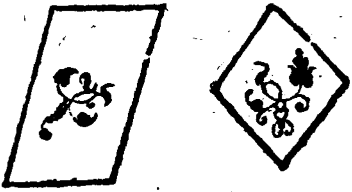


30.

Altera verò parte longior figura est, quæ rectangula quidem, at æquilatera non est

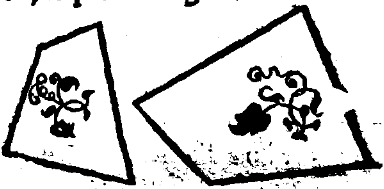
31 Rhom-

31.
Rhombus autem, quæ æquilatera, sed rectangula non est.

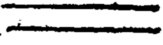


32.
Rhomboides verò, quæ aduersa & latera, & angulos habens inter se æquales, neque æquilatera est, neque rectangula.

33.
Præter has autem, reliquæ quadrilateræ figuræ, trapezia appellentur.

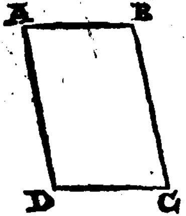


34.
Parallele rectæ lineæ sunt, quæ cùm in eodem sint plano & ex vtraque parte in infinitum producantur, in neutram sibi mutuò incidunt.



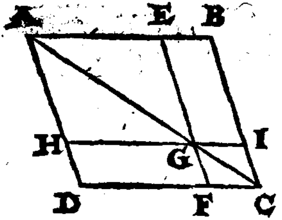
35.
Parallelogrammum est figura, quadrilatera, cuius bina apposita latera sunt parallela, seu æqui distantia.

36. Cùm



36.

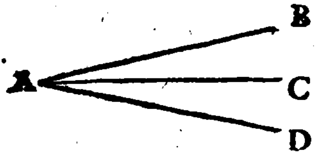
Cùm verò in parallelogrammo diameter ducta fuerit, duæq; lineæ lateribus parallelæ secantes diametrum in vno eodemque puncto, ita vt parallelogrammum ab hisce parallelis in quatuor distribuatur parallelogramma; appellantur duo illa, per quæ diameter non transit, complementa; duo verò reliqua, per quæ diameter incedit, circa diametrum consistere dicuntur.



Petitiones siue Postulata.

I.

Postuletur, vt à quouis puncto in quoduis pun-



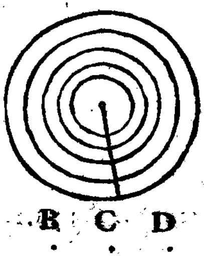
punctum, rectam lineam ducere conceda-
tur.

2.

Et rectam lineam terminatam in continu-
um recta producere.

3.

Item quouis centro, & interuallo circulum de-ſcribere.



4.

Item quacunque magnitudine data, ſu-
mi poſſe aliam magnitudinem vel maio-
rem, vel minorem.

Communes notiones ſiue axioma.

1.

Quæ eidem æqualia, & inter ſe ſunt æqua-
lia.

2.

Et, ſi æqualibus æqualia adiecta ſint, tota
ſunt æqualia.

3.

Et, ſi æqualibus æqualia adiecta ſint, quæ
relinquuntur ſunt æqualia.

4.

Et, si inæqualibus æqualia adiecta sint, tota sunt inæqualia.

5.

Et, si ab inæqualibus æqualia ablata sint, reliqua sunt inæqualia.

6.

Et, quæ eiusdem duplicia sunt, inter se sunt æqualia.

7.

Et quæ eiusdem sunt dimidia, inter se æqualia sunt.

8.

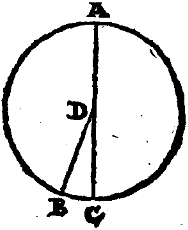
Et quæ sibi mutuò congruunt, ea inter se sunt æqualia.

9.

Et totum sua parte maius est.

10.

Duæ lineæ rectę non habent vnum, & idem segmentum commune.



11.

Duæ lineæ rectæ in vno puncto concurrentes,

rentes, si producantur ambæ, necessario se mutuò in eo puncto intersecabunt.

12.

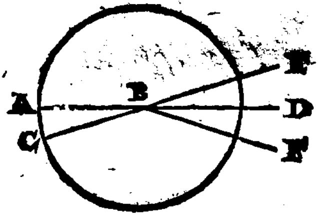
Item, omnes recti anguli sunt inter se æquales.

13.

Et, si in duas rectas lineas altera recta incidens, internos, ad easdemque partes angulos, duobus rectis minores faciat; duæ illæ rectæ lineæ in infinitum productę sibi mutuò incident ad eas partes, vbi sunt anguli duobus rectis minores.

14.

Duæ rectæ lineæ spatium non comprehendunt.



15.

Si æqualibus inæqualia adijciantur, erit totorum excessus, adiectorũ excessui æqualis.

16.

Si inæqualibus æqualia adiungantur, erit totorum excessus, excessui eorum, quæ à principio erant, æqualis.

17.

Si ab æqualibus inæqualia demantur, erit residuorum excessus, excessui ablatorum æqualis.

18.

Si ab inæqualibus æqualia demantur, erit residuorum excessus, excessui totorum æqualis.

19.

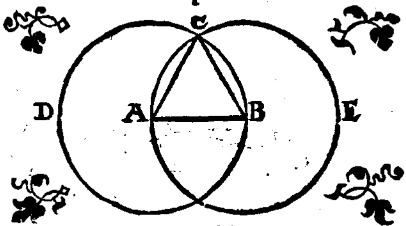
Omne totum æquale est omnibus suis partibus simul sumptus.

20.

Si totum totius est duplum, & ablatum ablati; erit & reliquum reliqui duplum.

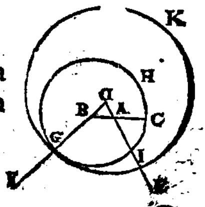
Problema 1. Propositio 1.

Super data recta linea terminata, triangulum æquilaterum constituere.



Problema 2. Propositio 2.

Ad datum punctum, datæ rectæ lineæ, æqualem rectam lineam ponere.



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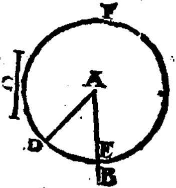
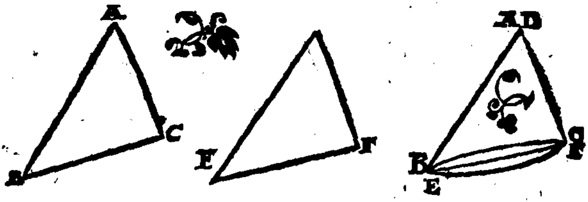
Problema 3. Propositio 3.



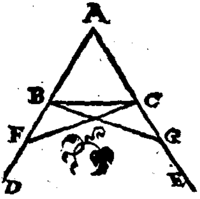
Duabus datis rectis lineis inæqualibus, de maiore æqualem minori rectam lineam detrahere.

Theorema 1. Propositio 4.

Si duo triangula duo latera duobus lateribus æqualia habeant, vtrunq; vtrique; habeant verò & angulum angulo æqualem sub æqualibus rectis lineis contentum: & basin basi æqualem habebunt; eritq; triangulum triangulo æquale, ac reliqui anguli reliquis angulis æquales erunt, vterque vtrique, sub quibus æqualia latera subtenduntur.



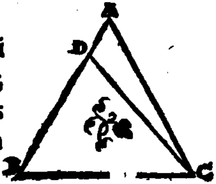
Theorema 2. Propositio 5.



Isosceleum triangulorum, qui ad basim sunt anguli, inter se sũt ęquales; & si vlterius productæ sint æquales illæ rectæ lineæ, qui sub basi sunt anguli, inter se æquales erunt.

Theorema
~~Problema 5~~. Propositio 6.

Si trianguli duo anguli æquales inter se fuerint; & sub æqualibus angulis subtensa latera æqualia inter se erunt.



Theorema 4. Propositio 7.

Super eadem recta linea, duabus eisdem rectis lineis aliæ duæ rectæ lineæ æquales, vtraque vtrique, non constituentur, ad aliud atque aliud punctum, ad easdem partes, eosdemque terminos cum duabus initio ductis rectis lineis habentes.

Theorema 5. Propositio 8.

Si duo triangula duo latera habuerunt duobus lateribus, vtrunque vtrique, æqualia: habuerunt verò & basim basi æqualem: angulum quoque sub æqualibus rectis lineis contentum angulo æqualem habebunt.

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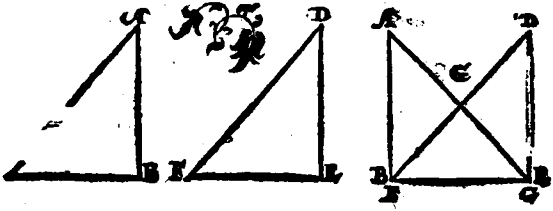
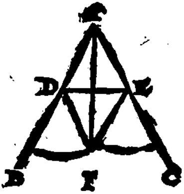


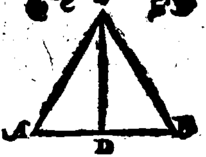
Problema 4. Propositio 9.

Datum angulum rectilineum bifariam secare.



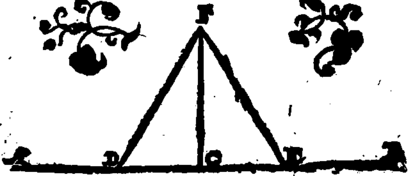
Problema 5. Propositio 10.

Datam rectaml ineam finitam bifariam secare.



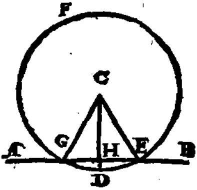
Problema 6. Propositio 11.

Data recta linea, à puncto in eo dato, rectam lineam ad angulos rectos excitare.



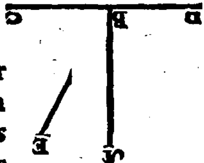
Problema 7. Propoſitio 12.

Super datam rectam lineam infinitam, à dato puncto, quod in ea non eſt, perpendicularem rectam deducere.



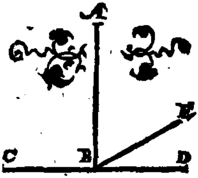
Theorema 6. Propoſitio 13.

Cùm recta linea ſuper rectam cóſiſtens lineam angulos facit, aut duos rectos, aut duobus rectis æquales efficiet.



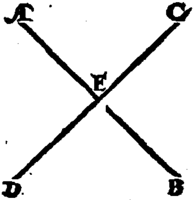
Theorema 7. Propoſitio 14.

Si ad aliquam rectam lineam, atq; ad eius punctum, duæ rectæ lineæ non ad eaſdem partes ductæ, eos, qui ſunt deinceps, angulos duobus rectis æquales fecerint, indirectum erũt inter ſe ipſe rectæ lineæ.



Theorema 8. Propoſitio 15.

Si duę rectæ lineę ſe mutuò ſecuerint, angulos, qui ad verticem ſunt, æquales in ter ſe efficient.



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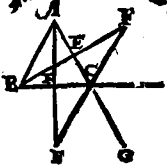
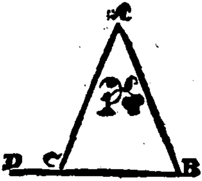
Theorema 9. Propoſitio 16.

Cuiuſcunque trianguli vno latere producto, externus angulus vtroque interno & oppoſito maior eſt.



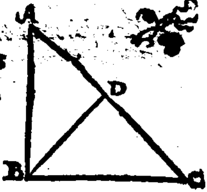
Theorema 10. Propoſitio 17.

Cuiuſcunque trianguli duo anguli duobus rectis ſunt minores, omnifariam ſumpti.



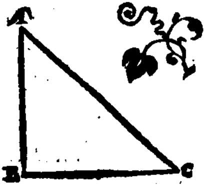
Theorema 11. Propoſitio. 18.

Omnis trianguli maius latus maiorem angulum ſubtendit.

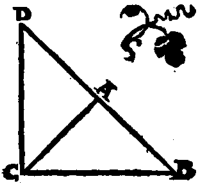


Theorema 12. Propoſitio 19.

Omnis trianguli maior angulus maiori lateri ſubtenditur.

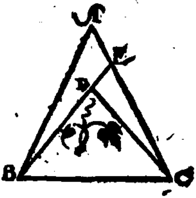


Theorema 13. Propositio 20.



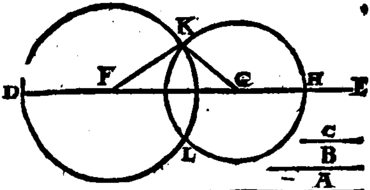
Omnis trianguli duo latera reliquo ſunt maiora, quomodocunque aſſumpta.

Theorema 14: Propositio 21.



Si ſuper trianguli vno latere, ab extremitatibus duæ rectæ lineæ, interius conſtitutæ fuerint; hæ conſtitutæ reliquis trianguli duobus lateribus minores quidem erunt; maiorem verò angulum continebũt.

Problema 8. Propoſitio 22.



Ex tribus rectis lineis, quæ ſunt trib. datis rectis lineis æquales, triangulum conſtituere. Oportet autem duas reliqua eſſe maiores omnifariam ſumptas: quoniam vnuſcuiuſque trianguli duo latera omnifariam ſumpta, reliquo ſunt maiora.

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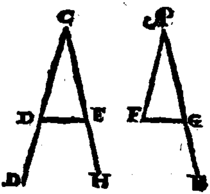
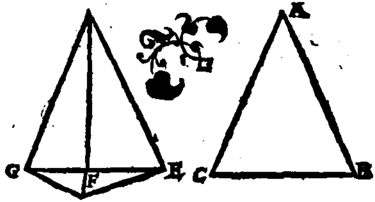
Problema 9. Propositio 23.



Ad datam rectam lineã, datumq; in ea punctum, dato angulo rectilineo æqualem angulum rectilineum constituere.

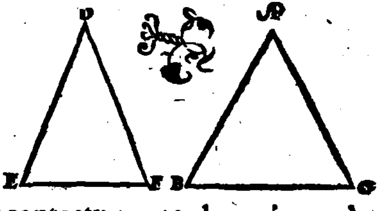
Theorema 15. Propositio 24.



Si duo triangula duo latera duobus lateribus ęqualia habuerint, vtrumq; vtriq;, angulum verò angulo maiorem sub ęqualib. rectis lineis cõtentũ: & basin basi maiorẽ habebũt.

Theorema 16. Propositio 25.

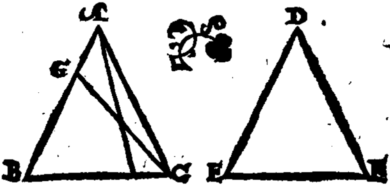
Si duo triangula duo latera duobus lateribus æqualia habuerint, vtrunque vtrique; basin verò basi maiorẽ; & angulum sub æqualibus rectis lineis contentum angulo maiorem habebunt.



Theo-

Theorema 17. Propositio 26.

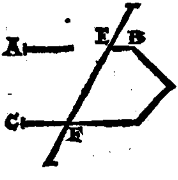
Si duo triangula duos angulos duobus angulis æquales habuerint, vtrunque vtrique; vnumque latus vni lateri æquale, siue quod æqualibus adiacet angulis, seu quod vni æqualium angulorum subtenditur: & reliqua latera reliquis lateribus æqualia, vtrunq; vtriq;



& reliquum angulum reliquo angulo ęqualem habebunt.

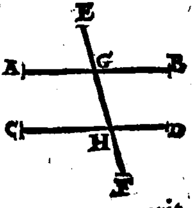
Theorema 18. Propositio 27.

Si in duas rectas lineas recta incidens linea alternatim angulos æquales inter se fecerit; parallelæ erunt inter se illæ rectæ lineæ.



Theorema 19. Propropositio 28.

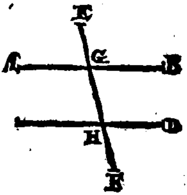
Si in duas rectas lineas recta incidens linea, externum angulum interno & opposito, & ad easdem partes, æqualem fe-



cerit,

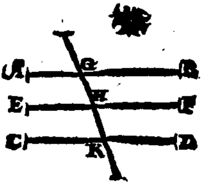
cerit, aut internos, & ad eaſdem partes duobus rectis æquales: parallelæ erunt inter ſe ipſæ rectæ lineæ.

Theorema 20. Propoſitio 29.



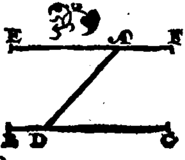
In parallelas rectas lineas recta incidens linea; & alternatim angulos inter ſe æquales efficit, & externum interno & oppoſito, & ad eaſdem partes ęqualem, & internos, & ad eaſdem partes duobus rectis æquales facit.

Theorema 21. Propoſitio 30.



Quæ eidem rectæ lineæ, parallelæ, & inter ſe ſunt parallelæ.

Problema 10. Propoſitio 31.

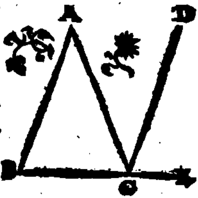


A dato puncto datæ rectæ lineæ parallelam rectam lineam ducere.

Theorema 22. Propoſitio 31.

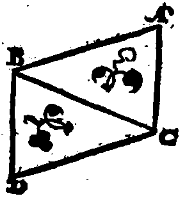
Cuiuſcunque trianguli vno latere alterius pro-

producto: externus angulus duobus internis & oppoſitis eſt æqualis. Et trianguli tres interni anguli duobus ſunt rectis æquales.



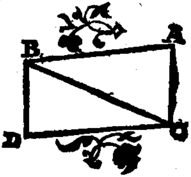
Theorema 23. Propoſitio 33.

Rectæ lineæ quæ æquales & parallelas lineas ad partes eaſdem coniungunt, & ipſæ æquales & parallelæ ſunt.



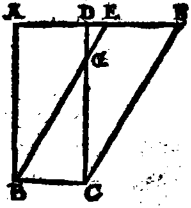
Theorema 24. Propoſitio 34.

Parallelogrammorum ſpatiorum æqualia ſunt inter ſe, quæ ex aduerſo, & latera, & anguli: atque illa bifariam ſecat diameter.



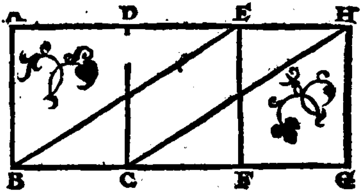
Theorema 25. Propoſitio 35.

Parallelogramma ſuper eadem baſi, & in eiſdem parallelis conſtituta inter ſe ſunt æqualia.



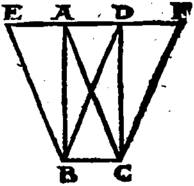
Theorema 26. Propositio 36.

Parallelogramma super æqualibus basibus, in eisdé parallelis cóstituta inter se sunt æqualia.



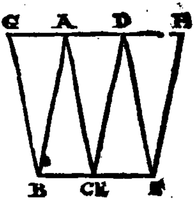
Theorema 27. Propositio 37.

Triangula super eadem basi constituta, & in eisdem parallelis, inter se sunt æqualia.



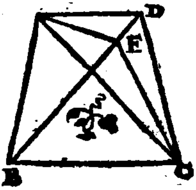
Theorema 28. Propositio 38.

Triangula super æqualibus basibus constituta, & in eisdem parallelis, inter se sunt æqualia.



Theorema 22. Propositio 39.

Triangula æqualia super eadem basi, & ad easdem partes constituta: & in eisdem sunt parallelis.



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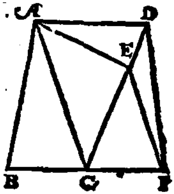
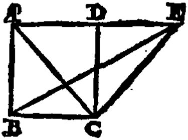
Theorema 30. Propoſitio 40.

Triangula ęqualia ſuper æqualibus baſibus,& ad eaſdem partes conſtituta, & in eiſdem ſunt parallelis.



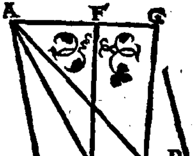
Theorema 31. Propoſitio 41.

Si parallelogrammum cum triangulo eandem baſin habuerit, in eiſdemque fuerit parallelis, duplum erit parallelo grammum ipſius trianguli.



Problema 11. Propoſitio 42.

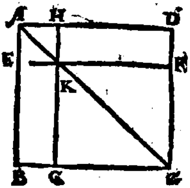
Dato triangulo æquale parallelogrammum conſtituere in dato angulo rectilineo.



Theorema 32. Propoſitio 34.

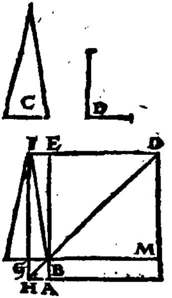
In omni parallelogrammo, com plimenta

eorum, quæ circa diametrum sunt, parallelogrammorum, inter se sunt æqualia.



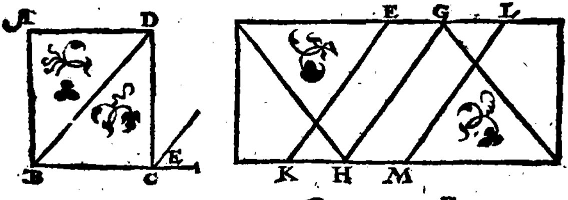
Problema 13. Propositio 44.

Ad datam rectam lineam, dato, triangulo, æquale parallelogrammum applicare, in dato angulo rectilineo.



Problema 13. Propositio 45.

Dato rectilineo, æquale parallelogrammum constituere in dato angulo rectilineo.



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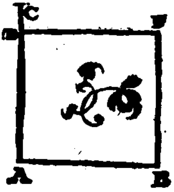
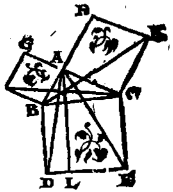
Problema 14. Propositio 46.

A data recta linea quadratum describere.



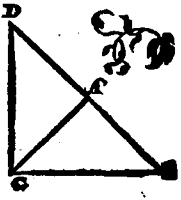
Theorema 33. Propositio 47.

In rectangulis triangulis, quadratum, quod à latere rectum angulum subtendente describitur, æquale est eis, quæ à lateribus rectum angulum continentib. describuntur, quadratis.



Theorema 34. Propositio 48.

Si quadratum, quod ab vno laterum trianguli describitur, æquale sit eis, qui à reliquis trianguli lateribus describuntur, quadratis: angulus comprahensus sub reliquis duobus trianguli lateribus, rectus est.



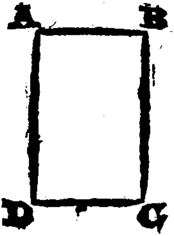
FINIS ELEMENTI I.

EVCLIDIS ELEMENTVM SECVNDVM.

DEFINITIONES.

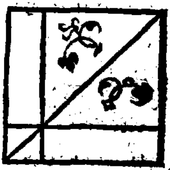
1.

Omne parallelogrammum rectangulum contineri dicitur sub rectis duabus lineis, quæ rectum comprehendunt angulum.



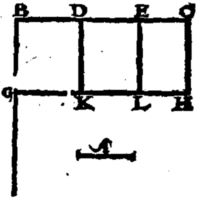
2.

In omni parallelogrammo spatio, vnum quodlibet eorum, quæ circa diametrum illius sunt, parallelogrammorum, cum duobus complementis, Gnomon vocetur.

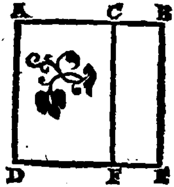


Theorema 1. Propositio 1.

Si fuerint duæ rectæ lineæ, seceturq; ipsarũ altera in quotcunque segmenta rectangulũ comprehensum sub illis duabus rectis lineis, equale est eis, quæ sub insecta & quolibet segmẽtorum comprænenduntur, rectangulis.



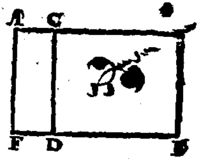
Theorema 2. Propositio 2.



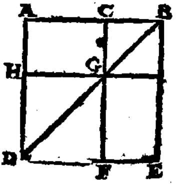
Si recta linea secta sit vtcunque: rectangula, quæ sub tota, & quolibet segmentorum comprænenduntur, æqualia sunt ei, quod à toto fit, quadrato.

Theorema 3. Propositio 3.

Si recta linea secta sit vtcunque rectangulum sub tota, & vno segmentorum comprænensum, æquale est illi, quod sub segmentis cõprehenditur triangulo, & illi, quòd à prædicto segmento describitur, quadrato.



Theorema 4. Propositio 4.



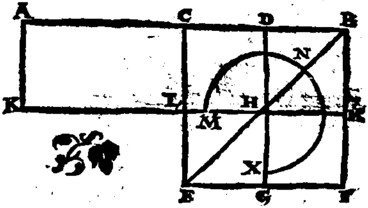
Si recta linea secta sit vtcunq; quadratum, quod à tota describitur, æquale est & illis, quæ à segmen-

tis

tis deſcribuntur, quadratis; & ei quod bis ſub ſegmentis comprehenditur, rectangulo.

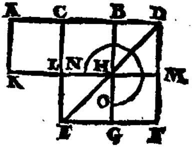
Theorema 5. Propoſitio 5.

Si recta linea ſecetur inæqualia, & non æqualia: rectangulum ſub inæqualibus ſegmentis totius comprehenſum, vnà cũ quadrato, quod ab intermedia ſectionum, æquale eſt ei, quod à dimidia deſcribitur, quadrato.



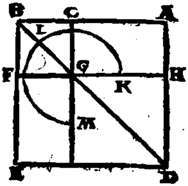
Theorema 6. Propoſitio 6.

Si recta linea bifariam ſecetur, & illi recta quædam linea in rectum adijciatur: rectangulum comprehenſum ſub tota cum adiecta, & adiecta, vnà cum quadrato à dimidia, æquale eſt quadrato à linea, quæ tum ex dimidia, tum ex adiecta componitur, tanquam ab vna deſcripto.



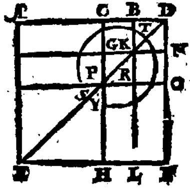
Theorema 7. Propoſitio 7.

Si recta linea ſecetur vtcunque; quod à tota quodque ab vno ſegmentorum, vtraq; ſimul quadrata, æqualia ſunt & illi, quod bis ſub tota, & dicto ſegmento comprehenditur, rectãgulo; & illi quod à reliquo ſegmento fit, quadrato.



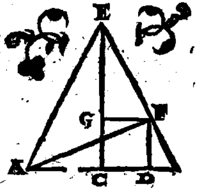
Theorema 8. Propoſitio 8.

Si recta linea ſecetur vtcunque; rectangulũ quater comprehenſum ſub tota, & vno ſegmentorum, cum eo quod à reliquo ſegmẽto fit, quadrato, ęquale eſt ei, quod à tota, & dicto ſegmento, tanquam ab vna linea deſcribitur, quadrato.



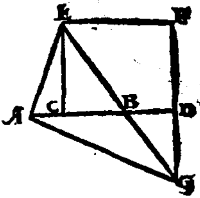
Theorema 9. Propoſitio 9.

Si recta linea ſecetur in æqualia & non æqualia: quadrata, quæ ab inæqualibus totius ſegmentis fiunt, ſimul duplicia ſunt, & eius, quod à dimidia, & eius, quod ab inter media ſectionum fit, quadratorum.



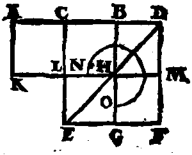
Theorema 10. Propoſitio 10.

Si recta linea ſecetur bifariã; adijciatur aũt ei in ei in rectū quæpiam recta linea: quod à tota cū adiuncta, & quod ab adiuncta, vtraque ſimul quadrata; duplicia ſunt, & eius, quod à dimidia, & eius, quod à compoſita ex dimidia & adiuncta, tanquam ab vna, deſcriptum ſit, quadratorum.



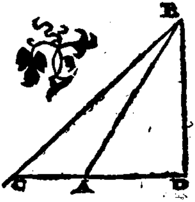
Problema 1. Propoſitio 11.

Datam rectam lineā ſecare, vt compræhēſum ſub tota, & altero ſegmentorum rectangulum, æquale ſit ei, quod à reliquo ſegmento fit, quadrato.



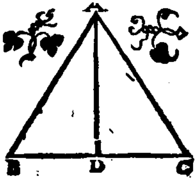
Theorema 11. Propoſitio 12.

In amblygonijs triangulis, quadratū, quod fit à latere angulum obtuſum ſubtendente, maius eſt quadratis, quæ fiunt à lateribus obtuſum angulum comprehendentibus; rectangulo bis compræhenſo, & ab vno laterum, quæ ſunt circa obtuſum angulum, in quod eum protractum fuerit, cadit perpendicularis, & ab aſſumpta exterius linea ſub perpendiculari prope angulum obtuſum.



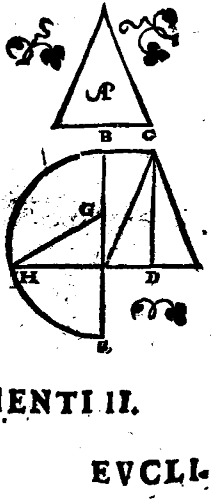
Theorema 12. Propositio 28.

In oxygonijs triangulis, quadratum à latere angulum acutum subtendente, minus est quadratis, quæ fiunt à lateribus acutum angulum comprehendentibus; rectangulo bis comprehenso; & vno laterum, quæ sunt circa acutum angulum, in quod perpendicularis cadit, & ab assumpta interius linea sub perpendiculari prope acutum angulum.



Problema 2. Propositio 14.

Dato rectilineo æquale quadratum constituere.



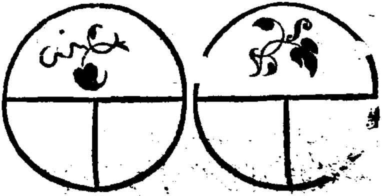
FINIS ELEMENTI II.

EVCLIDIS ELEMENTVM TERTIVM.

DEFINITIONES.

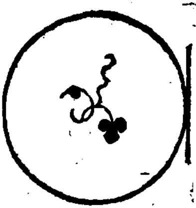
I.

Aequales circuli ſunt, quorum diametri ſunt æquales; vel quorum, quæ ex centris, rectæ lineæ ſunt æquales.



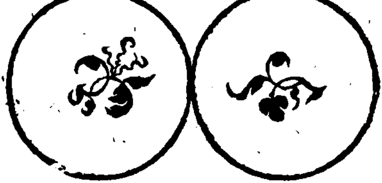
2.

Recta linea circulum tangere dicitur, quæ cùm circulum tangat; ſi producatur, circulum non ſecat.



3.

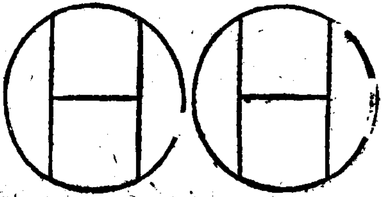
Circuli ſeſe mu tuò tangere dicuntur: qui ſeſe



mutuò tangentes, ſeſe mutuò non ſecant.

4.

In circulo æqualiter diſtare à centro rectæ lineæ dicuntur, cùm perpendiculares, quæ à centro in ipſas ducuntur, ſunt æquales. Lógius autem abeſſe illa dicitur, in quã maior



perpendicularis cadit.

5.

Segmentum circuli eſt figura, quæ ſub recta linea, & circuli peripheria cōprehenditur.

6.

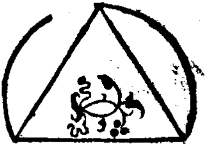
Segmenti autem angulus eſt, qui ſua recta

linea

linea, & circuli peripheria comprehenditur.

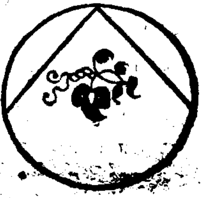
7.

In ſegmento autem angulus eſt, cùm in ſegmenti peripheria ſumptum fuerit quodpiam punctum, & ab illo in terminos rectæ eius lineæ quæ ſegmenti baſis eſt, adiunctæ fuerint, rectæ lineæ: is, inquam, angulus ab adiunctis illis lineis comprehenſus.



8.

Cùm verò comprehendentes angulum rectæ lineæ aliquam aſſumāt peripheriam, illi angulus inſiſtere dicitur.



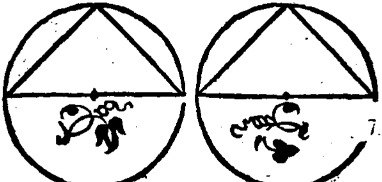
9.

Sector autem circuli eſt, cùm ad ipſius circuli cētrum conſtitutus fuerit angulus, comprehenſa nimirum figura, & à rectis lineis angulum continentibus, & à peripheria ab illis aſſumpta.

10.

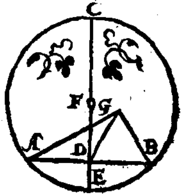
Similia circuli ſegmenta ſunt, quæ angulos capiunt

capiunt æquales aut in quibus anguli inter ſe ſunt æquales.



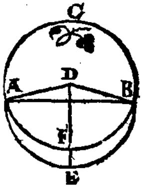
Problema 1. Propoſitio 1.

Dati circuli centrum reperire.



Theorema 1. Propoſitio 2.

Si in circuli peripheria duo quælibet puncta accepta fuerint; recta linea, quæ ad ipſa puncta adiungitur, intra circulum cadet.

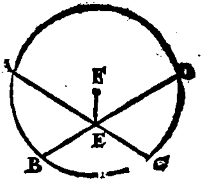


Theorema 2. Propoſitio 3.

Si in circulo recta quædam linea per centrum extenſa quandam non per centrum extenſam bifariam ſecet: & ad angulos rectos ipſam ſecabit: Et ſi ad angulos rectos eam ſecet, bifariam quoque eam ſecabit.

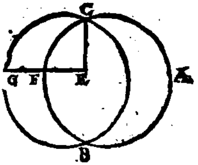
Theo-

Theorema 3. Propoſitio 4.



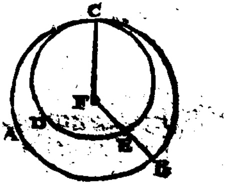
Si in circulo duæ rectæ lineæ ſeſe mutuò ſecent, nõ per centrum extenſæ; ſeſe mutuò bifariam non ſecabunt.

Theorema 4. Propoſitio 5.



Si duo circuli ſeſe mutuò ſecent; non erit illorum idem centrum.

Theorema 5. Propoſitio 6.



Si duo circuli ſeſe mutuò interius tangent, eorum non erit idem centrum.

Theorema 6. Propoſitio 7.

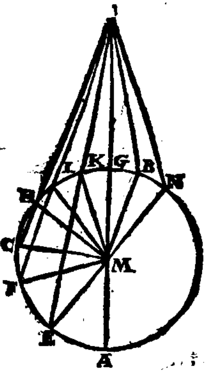
Si in diametro circuli quodpiam ſumatur punctum, quod circuli centrum non ſit, ab eoq́; puncto in circulum quædam rectæ lineæ cadant; maxima quidem erit ea in qua centrũ; minima verò reliqua; aliarum verò propinquior illi, quæ per centrum du-

citur

citur, remotiore semper maior est. Duæ autem solùm rectæ lineæ æquales ab eodem puncto in circulum cadunt, ad vtrasq; partes minimæ, vel maximæ.

Theorema 7. Propositio 8.

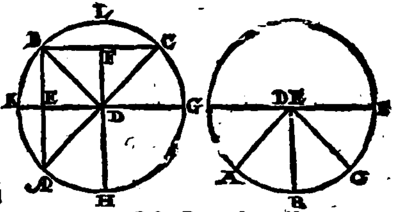
Si extra circulum sumatur punctum quodpiam, ab eoque puncto ad circulum deducantur rectæ quædam lineæ, quarum vna quidem per centrum protendatur, reliquæ verò vt libet: in cauam peripheriam cadentium rectarum linearum minima quidem est illa, quæ per centrum ducitur: aliarum autem propinquior ei, quæ per centrum transit, remotiore semper maior est: in conuexam verò peripheriã cadentium rectarum linearũ minima quidem est illa, quæ inter punctum, & diametrum interponitur: aliarum autem, ea, quæ propinquior est minimæ, remotiore semper minor est. Duæ autem tantùm rectæ lineæ æquales ab eo puncto in ipsum circulum cadunt, ad vtrasque partes minimæ, vel maximæ.



Theo-

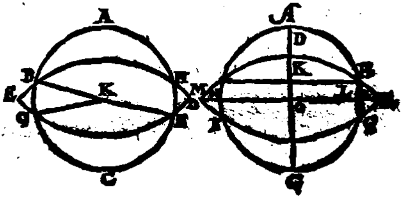
Theorema 8. Propositio 9.

Si in circulo acceptum fuerit punctum aliquod, & ab eo puncto ad circulum cadant plures, quam duæ, rectæ lineæ æquales; acceptũ punctum centrum est ipsius circuli.



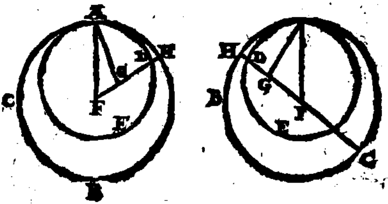
Theorema 9. Propositio 10.

Circulus circulum in pluribus, quàm duob. punctis non secat.



Theorema 10. Propositio 11.

Si duo circuli sese intus cõtingãt, atque accepta,

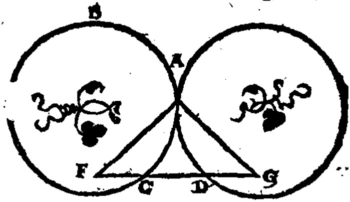


fuerint

fuerint eorum contra; ad eorum centra adiuncta recta linea, & producta, incótactum circulorum cadet.

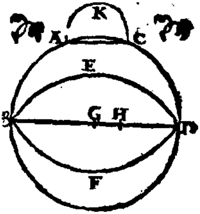
Theorema 11. Propoſitio 12.

Si duo circuli ſeſe exterius contingant, linea recta, quæ ad centra eorum adiungitur, per contactum illum tranſibit.



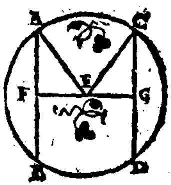
Theorema 12. Propoſitio 13.

Circulus circulum non tangit in pluribus punctis, quam vno, ſiue intus ſiue extra tangat.



Theorema 13. Propoſitio 14.

In circulo æquales rectæ lineæ æqualiter diſtant à centro. Et quæ æqualiter diſtant à centro, æquales ſunt inter ſe.



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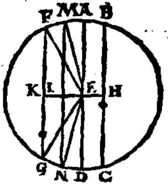
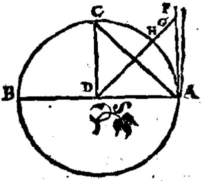
Theorema 14. Propositio 15.



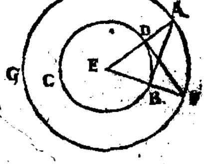
In circulo maxima quidem linea est diameter: aliarum autem propinquior centro, remotiore semper maior.

Theorema 15. Propositio 16.

Quæ ab extremitate diametri cuiusque circuli ad angulos rectos ducitur, extra ipsum circulum cadet; & in locum inter ipsam rectam lineam, & peripheriam comprehēsum, altera recta linea non cadet. Et semicirculi quidem angulus quouis angulo acuto rectilineo maior est, reliquus autem minor.



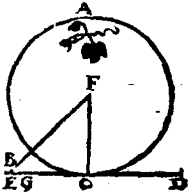
Problema 2. Propositio 17.



A dato puncto rectam lineam ducere, quæ datum tangat circulum.

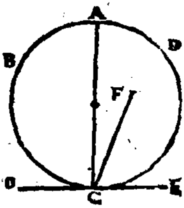
Theorema 16. Propositio 18.

Si circulum tangat recta quæpiam linea, à centro autem ad contactum adiungatur recta quædam linea: quæ adiuncta fuerit, ad ipsam contingentem, perpendicularis erit.



Theorema 17. Propositio 19.

Si circulum tetigerit recta quæpiam linea, à contactu autem recta linea ad angulos rectos ipsi tangenti excitetur; in excitata erit centrum circuli.

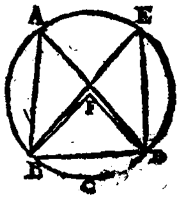


Theorema 18. Propositio 20.

In circulo, angulus ad centrum duplex est anguli ad peripheriam, cùm fuerit eadem peripheria basis angulorum.

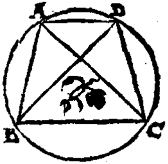
Theorema 19. Propositio 21.

In circulo, qui in eodem segmēto sunt, anguli, sunt inter se æquales.



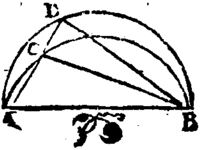
Theorema 20. Propoſitio 22.

Quadrilaterorum in circulis deſcriptorum anguli, qui ex aduerſo, duob. rectis ſunt æquales.

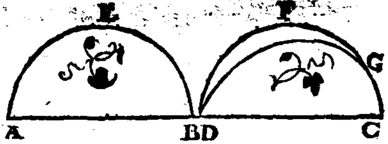


Theorema 21. Propoſitio 23.

Super eadem recta linea duo ſegmenta circulorum ſimilia, & inæqualia non conſtituentur ad eaſdem partes.

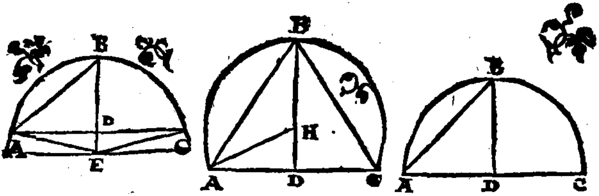


Super æqualibus rectis lineis, ſimilia circulorum ſegmenta, ſunt inter ſe æqualia.



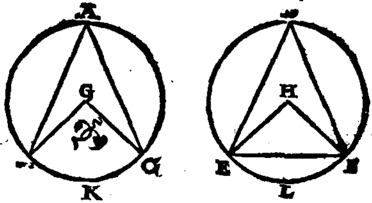
Problema 3. Propoſitio 25.

Circuli ſegmento dato, deſcribere circulū, cuius eſt ſegmentum.



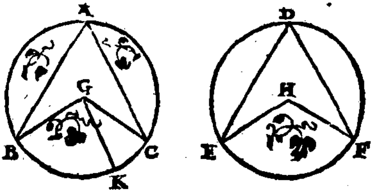
Theorema 23. Propoſitio 26.

In æqualibus circulis, æquales anguli æqualibus peripheriis inſiſtunt, ſiue ad cẽtra, ſiue ad peripherias conſtituti, inſiſtant.



Theorema 24. Propoſitio 27.

In æqualibus circulis, anguli, qui æqualibus peripherijs inſiſtunt, ſunt inter ſe æquales, ſiue ad centra, ſiue ad peripherias conſtituti, inſiſtant.



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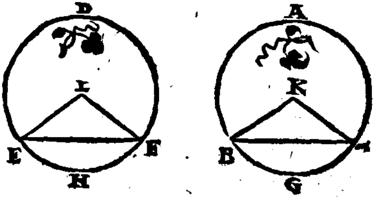
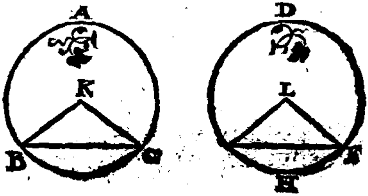
Theorema 25. Propositio 28.

In æqualibus circulis, æquales rectæ lineæ, æquales peripherias auferunt; maiore quidem maiori, minorem autem minori.



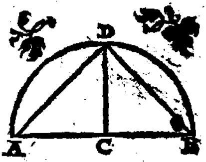
Theorema 26. Propositio 29.

In ęqualib. circulis æquales peripherias, æquales rectæ lineæ subtendunt.



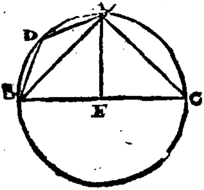
Problema 4. Propositio 30.

Datam peripheriam bifariam secare.



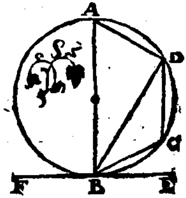
Theorema 27. Propositio 31.

n circulo angulus, qui in semicirculo, rectus eſt: qui autem in maiore ſegmento, minor recto: qui verò in minore ſegmento, maior eſt recto. Et inſuper angulus maioris ſegmenti, recto quidem maior eſt, minoris autem ſegmenti angulus, minor eſt recto.



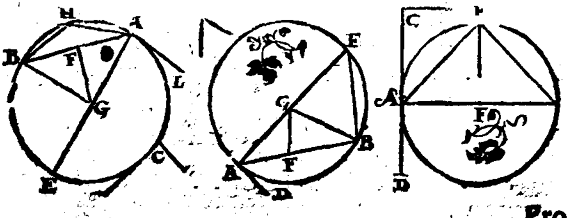
Theorema 28. Propoſitio 32.

Si circulum tetigerit aliqua recta linea, à cõtactu autem producatur quædam recta linea circulum ſecãs: anguli, quos ad contingentem facit, æquales ſunt ijs, qui alternis circuli ſegmentis conſiſtunt, angulis.



Problema 5. Propoſitio 33.

Super data recta linea deſcribere ſegmentũ circuli, quod capiat angulum æqualem dato angulo rectilineo

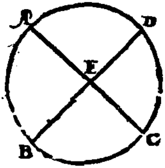


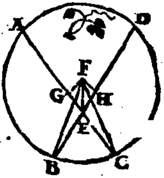
Problema 6. Propo-
ſitio 34.

A dato circulo ſegmentum abſcindere, capiés angulum ęqualem dato angulo rectilineo.

Theorema 29. Propoſitio 35.

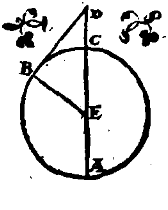
Si in circulo duæ rectæ lineæ ſeſe mutuò ſecuerint; rectangulum comprehenſum ſub ſegmentis vnius, æquale eſt ei, quod ſub ſegmentis alterius comprehenditur, rectangulo.

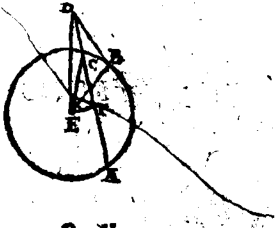




Theorema 30. Propoſitio 36.

Si extra circulum ſumatur pũctũ aliquod, ab eoq; in circulum cadant duæ rectæ lineæ; quarum altera quidem circulum ſecet, altera

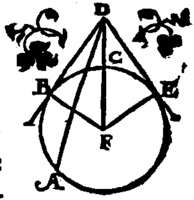




verò tangat: quod ſub tota ſecante,& exterius inter punctum & conuexam peripheriam aſſumpta comprehenditur; rectangulum; æquale erit ei, quod à tangente deſcribitur, quadrato.

Theorema 31. Propoſitio 37.

Si extra circulum ſumatur punctum aliquod, ab eoque puncto in circulum cadant duæ rectæ lineæ, quarum altera circulum ſecet, altera in eum incidat; ſit autem, quod ſub tota ſecante, & exterius inter punctum, & conuexam peripheriam aſſumpta, comprehenditur rectangulum, æquale ei, quod ab incidente deſcribitur quadrato; incidens ipſa circulum tanget.



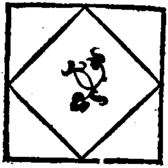
FINIS ELEMENTI III.

EVCLIDIS ELEMENTVM QVARTVM.

DEFINITIONES.

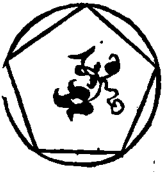
1.

Figura rectilinea in figura rectilinea inſcribi dicitur, cum ſinguli eius figuræ; quæ inſcribitur, anguli, ſingula latera eius, in qua inſcribitur, tangunt.



2.

Similiter & figura circum figuram deſcribi dicitur, quum ſingula eius, quę circumſcribitur, latera ſingulos eius figuræ angulos tetigerint, circum quam illa deſcribitur.



3.

Figura rectilinea in circulo inſcribi dicitur, quum ſinguli eius figuræ, quæ inſcribitur, anguli tetigeriat circuli peripheriam.

4.

Figura verò rectilinea circa circulum describi dicitur, quùm singula latera eius, quæ circumscribitur, circuli peripheriam tangunt.

5.

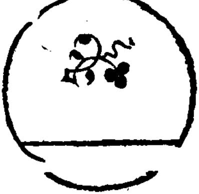
Similiter & circulus in figura rectilinea inscribi dicitur, quũ circuli peripheria singula latera tangit eius figurã, cui inscribitur.

6.

Circulus autem circum figuræ describi dicitur, quum circuli peripheria singulos tangit eius figuræ, quam circumscribit, angulos.

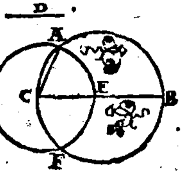
7.

Recta linea in circulo accommodari, seu coaptari dicitur, quum eius extrema in circuli peripheria fuerint.



Problema 1. Propositio 1.

In dato circulo, rectam lineam accommodare æqualem datæ rectæ lineæ, quæ circuli diametro non sit maior.



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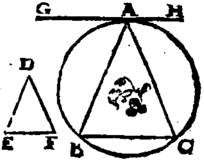
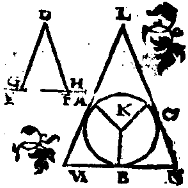
Problema 2. Propositio 2.

In dato circulo, triangulum describere, dato triangulo æquiangulum.



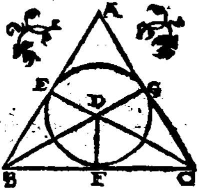
Problema 3. Propositio 3.

Circa datum circulum triangulum describere, dato triangulo æquiangulum.



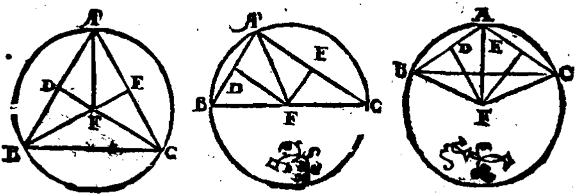
Problema 4. Propositio 4.

In dato triangulo circulum inscribere.



Problema 5. Propositio 5.

Circa datum triangulum, circulum describere.



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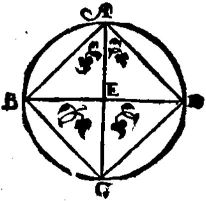
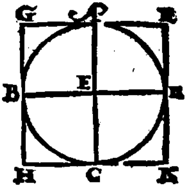
Problema 6. Propositio 6.



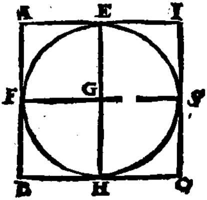
In dato circulo quadratum deſcribere.

Problema 7. Propoſitio 7.



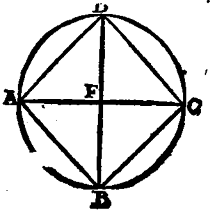
Circa datum circulum, quadratum deſcribere.

Problema 8. Propoſitio 8.



In dato quadrato circulum inſcribere.

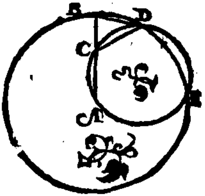
Problema 9. Propoſitio 9.



Circa datum quadratũ, circulum deſcribere.

Pro-

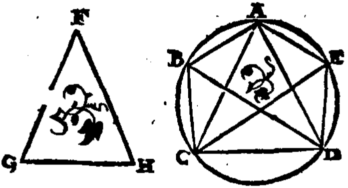
Problema 10. Propoſitio 10.



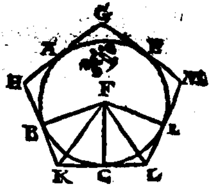
Iſoſceles triangulum cõſtituere; quod habeat vtrunque eorum, qui ad baſin ſunt, angulorum, duplum reliqui.

Problema 11. Propoſitio 11.

In dato circulo, pentagonũ æquilaterum, & æquiangulum inſcribere.

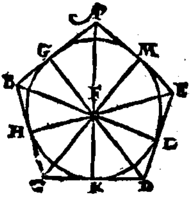


Problema 12. Propoſitio 12.



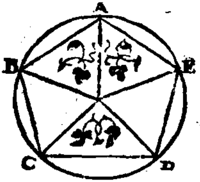
Circa datum circulum, pentagonum, æquilaterum & æquiangulum deſcribere.

Problema 13. Propoſitio 13.



In dato pentagono ęquilatero, & ęquiangulo circulum inſcribere.

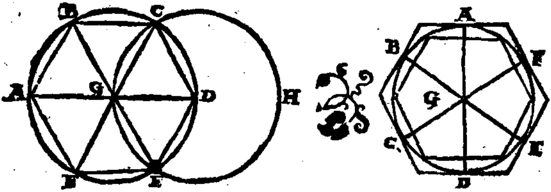
Problema 12. Propositio 14.



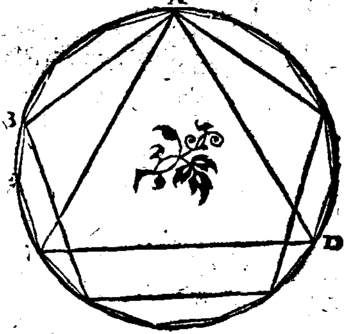
Circa datum pentagonum æquilaterum & æquiangulum, circulum deſcribere.

Problema 15. Propoſitio 15.

In dato circulo hexagonum & æquilaterum, & æquiangulum inſcribere.



Problema 16. Propoſitio 16.



In dato circulo quinti decagonum & æquilaterũ, & æquiangulum deſcribere.

FINIS ELEMENTI IV.

EVCLIDIS ELEMENTVM QVINTVM.

DEFINITIONES.

1.

PArs est magnitudo magnitudinis minor maioris, quum minor metitur maiorem.

2.

Multiplex autem est maior minoris, cùm minor metitur maiorem.

3.

Proportio, est duarum magnitudinum eiusdem generis mutua quędam secundum quãti tãlem habitudo.

4.

Proportionalitas verò est proportionum similitudo.

5.

Proportionem habere inter se magnitudines dicuntur, quæ possunt multiplicatæ sese mutuò superare.

6.

In eadẽ proportione magnitudines dicũtur esse, prima ad secundam, & tertia ad quartam, cùm primæ tertiæ æquè multiplicia, à secundæ & quartæ æquè multiplicibus, qua-

qualiſcunque ſit hæc multiplicatio, vtrunque ab vtroque; vel vnà deficiunt, vel vnà æqualia ſunt, vel vnà excedunt; ſi ea ſumantur, quæ inter ſe reſpondent.

7.

Eandem autem proportionem habentes magnitudines, proportionales vocentur.

8.

Cùm verò æquè multiplicium, multiplex primæ magnitudinis excesserit multiplicē ſecundæ; at multiplex tertiæ non exceſſerit multiplicem quartæ: tunc prima ad ſecundam, maiorem proportionem habere dicetur, quàm tertia ad quartam.

9.

Proportionalitas autem in tribus terminis pauciſſimis conſiſtit.

10.

Cùm autem tres magnitudines proportionales fuerint, prima ad tertiam, duplicatam proportionem habere dicitur eius, quam habet ad ſecundam. At cùm quatuor magnitudines proportionales fuerint, prima ad quartam, triplicatam proportionem habere dicitur eius, quam habet ad ſecundam: & ſemper deinceps, vno amplius, quamdiu proportio extiterit.

11.

Homologæ, ſeu ſimiles proportione magnitudines dicuntur, antecedentes quidem ante-

antecedentibus, consequentes verò consequentibus.

12.

Alterna proportio, est sumptio antecedentis ad antecedentem, & consequentis ad consequentem.

13.

Inuersa proportio, est sumptio consequentis, ceu antecedentis, ad antecedentem, velut ad ipsam consequentem.

14.

Compositio proportionis, est sumptio antecedentis cum consequente, ceu vnius ad ipsam consequentem.

15.

Diuisio proportionis, est sumptio excessus, quo consequentem superat antecedens, ad ipsam consequentem.

16.

Conuersio proportionis, est sumptio antecedentis ad excessum, quo superat antecedens ipsam consequentem.

17.

Ex æqualitate proportio est, si plures duabus sint magnitudines, & his aliæ multitudine pares, quæ binæ sumantur, & in eadem proportione: quum vt in primis magnitudinibus prima ad vltimam, sic & in secundis magnitudinibus prima ad vltimam sese habuerit.

Vel aliter, ſumptio extremorum per ſubdu-
ctionem mediorum.

18.

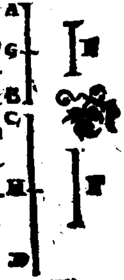
Ordinata proportio eſt, cùm fuerit, quemadmodum antecedens ad conſequentem, ita antecedens ad conſequentem: fuerit etiam vt conſequens ad aliud quidpiam, ita conſequens ad aliud quidpiam.

19.

Perturbata autem proportio eſt, cùm tribus poſitis magnitudinibus, & alijs quæ ſint his multitudine pares, vt in primis quidem magnitudinibus ſe habet antecedens ad conſequentem, ita in ſecundis magnitudinibus antecedens ad conſequentem: vt autem aliud quidpiam ſic in ſecundis magnitudinibus aliud quidpiam ad antecedentem.

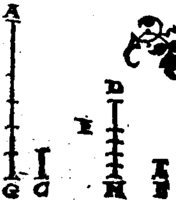
Theorema 1. Propoſitio 1.

Si ſint quotcunq; magnitudines, quotcunq; magnitudinum æqualium numero, ſingulæ ſingularum, æquè multiplices; quàm multiplex eſt vnius vna magnitudo, tam multiplices erunt & omnes omnium.



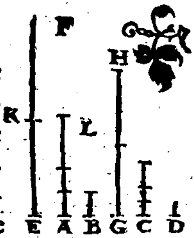
Theorema 2. Propositio 2.

Si prima ſecundæ æquè fuerit multiplex, atque tertia quartæ; fuerit autem & quinta ſecūdæ æquè multiplex, atq; ſexta quartæ: erit & compoſita prima cum quinta, ſecundæ æquè multiplex; atque tertia cum ſexta, quartæ.



Theorema 3. Propoſitio 3.

Si ſit prima ſecundæ æquè multiplex, atq; tertia quartæ, ſumantur autem æquè multiplices primæ, & tertiæ; erit & ex æquo, ſumptarum vtraque vtriuſque æquè multiplex; altera quidem ſecundæ, altera autem quartæ.

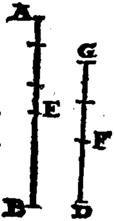


Theorema 4. Propoſitio 4.

Si prima ad ſecundã, eandem habuerit proportionem, & tertia ad quartam; etiam æquè multiplices primæ & tertiæ, ad æquè multiplices ſecundæ & quartæ, iuxta quamuis multiplicationē, ean-

dem habebunt propositionem; si, prout inter se respondent, ita sumptæ fuerint.

Theorema 5. Propositio 5.



Si magnitudo magnitudinis æquè fuerit multiplex, atque ablata ablatæ: etiam reliqua reliquæ ita multiplex erit, vt tota totius.

Theorema 6. Propositio 6.

Si duæ magnitudines, duarum magnitudinum sint æquè multiplices, & detractæ quædam sint earundem æquè multiplices: & reliquæ eisdem aut æquales sunt, aut æquè ipsarum multiplices.

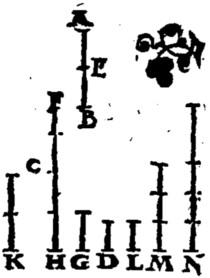
Theorema 7. Propositio 7.

Aequales ad eandem, eandem habent rationem: & eadem ad æquales.

Theorema 8. Propositio 8.

Inæqualium magnitudinum maior ad eandem

dem, maiorem proportionem habet: quàm minor: & eadem ad minorem, maiorem proportionem habet, quàm ad maiorem.



Theorema 9. Propositio 9.

Quæ ad eandem, eandem habent proportionem, æquales sunt inter se: & ad quas eadem, eandem habet proportionem, eæ quoque sunt inter se æquales.

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Theorema 10. Propositio 10.

Ad eandem magnitudinem proportionem habentium, quæ maiorem proportionem habet, illa maior est, ad quam autem eadem maiorem proportionem habet, illa minor est.

Theorema 11. Propositio 11.

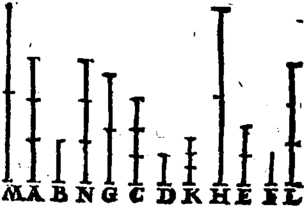
Quæ eidem sunt eædem proportiones, & inter se sunt eædem.

Theorema 12. Propoſitio 12.

Si ſint magnitudines quotcunque proportionales; quemadmodũ ſe habuerit vna antecedentium ad vnã conſequentium; ita ſe habebunt omnes antecedentes ad omnes conſequentes.

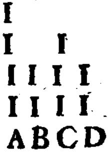
Theorema 13. Propoſitio 13.

Si prima ad ſecundam, eãdem habuerit proportionem, quam tertia ad quartam; tertia verò ad quartam maiorem proportionem habuerit, quam quinta ad ſextam: prima quoque ad ſecũdam maiorẽ proportionem habebit, quam quinta ad ſextam.



Theorema 14. Propoſitio 14.

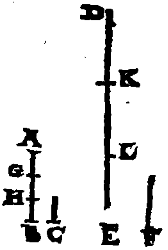
Si prima ad ſecundam eandem habuerit proportionem, quã tertia ad quartã: prima verò quam tertia maior fuerit, erit & ſecunda maior, quam quarta. Quod ſi prima fuerit æqualis tertiæ,



erit ſecunda æqualis quartæ: ſi verò minor, & minor erit.

Theorema 15. Propoſitio 15.

Partes cum pariter multiplicibus in eadem ſunt proportione, ſi prout ſibi mutuò reſpondent, ita ſumantur.

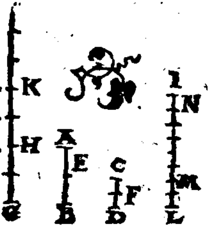


Theorema 16. Propoſitio 16.

Si quatuor magnitudines proportionales fuerint, & viciſſim proportionales erunt.

Theorema 17. Propoſitio 17.

Si compoſitæ magnitudines proportionales fuerint, hæ quoq; diuiſæ proportionales erunt.



Theorema 18. Propositio 18.

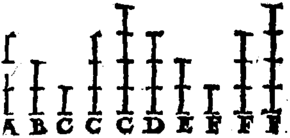
Si diuisæ magnitudines sint proportionales, hæ quoque compositæ proportionales erunt.

Theorema 19. Propositio 19.

Si quemadmodum totum ad totum, ita ablatum se habuerit ad ablatum: & reliquum ad reliquum, vt totum ad totum, se habebit.

Theorema 20. Propositio 20.

Si sint tres magnitudines, & aliæ ipsis æquales numero, quæ binæ, & in eadem proportione sumantur;



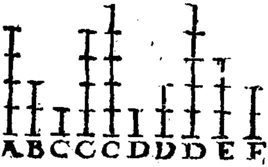
ex æquo autem prima quàm tertia maior fuerit; erit & quarta, quã sexta, maior.

Quod si prima tertiæ fuerit æqualis, erit & quarta æqualis sextæ: sin illa minor, quoque minor erit.

Theo-

Theorema 21. Propositio 21.

Si sint tres magnitudines, & aliæ ipsis æquales numero, quæ binæ, & in eadem proportione sumantur, fueritque perturbata earum proportio : ex æquo auté prima, quam tertia, maior fuerit; erit & quarta, quàm sexta, maior : quod si prima tertiæ fuerit æqualis, erit & quarta æqualis sextæ : sin illa minor, hæc quoque minor erit.



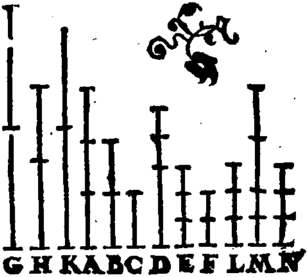
Problema 22. Propositio 22.

Si sint quodcunque magnitudines, & aliæ ipsis æquales numero, quæ binæ in eadem proportione sumantur : & ex æqualitate in eadem proportione erunt.

Theo ema 23. Propositio 23.

Si sint tres magnitudines, aliæq; ipsis æquales

les numero, quę binæ in eadem proportione ſumantur ; fuerit autem perturbata earum proportio : Etiam ex æqualitate in eadem proportione erunt.



Theorema 24. Propoſitio 24.

Si prima ad ſecundam, eandem habuerit proportionem, quam tertia ad quartam ; habuerit autem & quinta ad ſecundam, eandem proportionem, quàm ſexta ad quartam : Etiam compoſita prima cum quinta, ad ſecundam eandem habebit proportionem, quam tertia cum ſexta, ad quartam.

Theorema 25. Propoſitio 25.

Si quatuor magnitudines proportionales fuerint ; maxima, & minima reliquis duabus maiores erunt.

Theorema 26. Propositio 26.

Si prima ad ſecundam, maiorem habuerit proportionem, quàm tertia ad quartam: habebit conuertendo ſecunda ad primam, minorem proportionem, quàm quarta ad tertiam.

Theorema 27. Propositio 27.

Si prima ad ſecundam, habuerit maiorem proportionem, quàm tertia ad quartam: habebit quoque viciſſim prima ad tertiam, maiorem proportionem, quàm ſecunda ad quartam.

Theorema 28. Propoſitio 28.

Si prima ad ſecundam habuerit maiorem proportionem, quàm tertia ad quartam: habebit quoque compoſita prima cum ſecunda, ad ſecundam, maiorem proportionem, quàm compoſita tertia cum quarta, ad quartam.

Theorema 29. Propoſitio 29.

Si compoſita prima cum ſecunda, ad ſecundam, maiorem habuerit proportionem quā com-

composita tertia cum quarta, ad quartam: Habebit quoq; diuidendo prima ad secundam, maiorem proportionem, quam tertia, ad quartam.

Theorema 30. Propositio 30.

Si composita prima cum secunda, ad secundam habuerit maiorem proportionem, quá composita tertia cum quarta, ad quartam: Habebit quoq; per conuersionem proportionis, prima cum secunda, ad primam, minorem proportionem, quàm tertia cum quarta, ad tertiam.

Theorema 31. Propositio 31.

Si sint tres magnitudines, & aliæ ipsis æquales numero; sitque maior proportio primæ priorum ad secundam, quàm primæ posteriorum ad secundam; Item secundæ priorum ad tertiam maior, quàm secundæ posteriorum ad tertiam: Erit quoque ex æqualitate maior proportio primæ priorum ad tertiam, quàm primæ posteriorum ad tertiam.

Theorema 32. Propositio 32.

Si sint tres magnitudines, & aliæ ipsis æquales numero; sitque maior proportio primæ priorum ad secundã, quàm secundæ posteriorum

riorum ad tertiam; item secundæ priorum ad tertiam maior, quàm primæ posteriorum ad secundam: Erit quoq; ex æqualitate, maior proportio primæ priorum ad tertiam, quàm primæ posteriorum ad tertiam.

Problema 33. Propositio 33.

Si fuerit maior proportio totius ad totum, quàm ablati ad ablatum: Erit & reliqui ad reliquum, maior proportio, quàm totius ad totum.

Theorema 34. Propositio 34.

Si sint quotcunque magnitudines, & aliæ ipsis æquales numero; sitque maior proportio primę priorum ad primam posteriorum, quàm secundæ ad secundam; & hæc maior, quàm tertiæ ad quartam, & sic deinceps: Habebunt omnes priores simul, ad omnes posteriores simul, maiorem proportionem quàm omnes priores, relicta prima, ad omnes posteriores, relicta quoque prima; minorem autem, quàm prima priorum ad primam posteriorum; maiorem denique etiam, quàm vltima priorum ad vltimam posteriorum.

FINIS ELEMENTI V.

EVCLIDIS ELEMENTVM SEXTVM.

DEFINITIONES.

1.

Similes figuræ rectilineæ sunt, quæ & angulos singulos singulis æquales habent; atque etiam latera, quæ circum angulos æquales, proportionalia.

2.

Reciprocæ autem figuræ sunt, cùm in vtraque figura, antecedentes, & consequentes proportionum termini fuerint.

3.

Secundum extremam, & mediam proportionem recta linea secta esse dicitur, cùm vt tota ad maius segmentum, ita maius ad minus se habuerit.

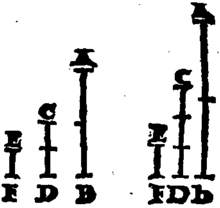
4.

Altitudo cuiusque figuræ est linea perpendicularis à vertice ad basim deducta.

5. Pro-

5.

Proportio ex proportionibus componi dicitur, cùm proportionum quantitates inter se multiplicatæ, aliquam effecerint proportionem.

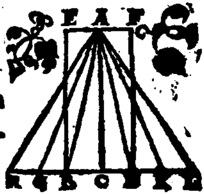


6.

Parallelogrammum secundum aliquam lineam applicatum, deficere dicitur parallelogrammo, quando non occupat totam lineam: Excedere vero, quando occupat maiorem lineam, quàm sit ea, secundum quam applicatur: Ita tamen, vt parallelogrammum deficiens, aut excedens, eandem habeat altitudinem cum parallelogrammo applicato, constituatque cum eo totum vnum parallelogrammum.

Theorema 1. Propositio 1.

Triangula, & parallelogramma, quorum eadem fuerit altitudo, ita se habent inter se, vt bases.

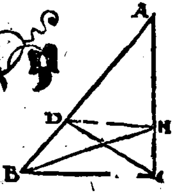


Theorema 2. Propositio 2.

Si ad vnum trianguli latus parallela ducta fuerit recta quædam linea.

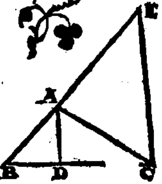
Hæc proportionaliter secabit ipsius trianguli

anguli latera. Et si trianguli latera proportionaliter secta fuerint; quæ ad sectiones adiuncta fuerit recta linea, erit ad reliquum ipsius trianguli latus parallela.



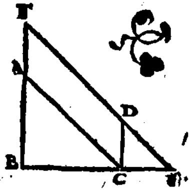
Theorema 3. Propositio 3.

Si trianguli angulus bifariam sectus sit; secans autem angulum recta linea secuerit & basin: basis segmenta, eandem habebunt proportionem, quã reliqua ipsius trianguli latera Et si basis segmenta eandem habeant proportionem quam reliqua ipsius trianguli latera: recta linea, quæ à vertice ad sectionem producitur, bifariam secat trianguli ipsius angulum.



Theorema 4. Propositio 4.

Aequiangulorum triangulorum proportionalia sunt latera, quæ circum ęquales angulos, & homologa sunt latera, quę æqualibus angulis subtenduntur.



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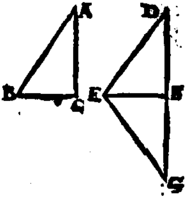
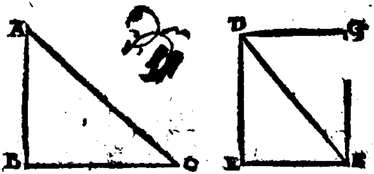
Theorema 5. Propositio 5.

Si duo triangula, latera proportionalia habeant, æquiangula erunt triangula, & æquales habebunt eos angulos, sub quibus homologa latera subtenduntur.



Theorema 6. Propositio 6.

Si duo triangula vnum angulum vni angulo æqualem, & circum æquales angulos latera proportionalia habuerint: æquiangula erunt triangula, æqualesque habebunt angulos, sub quibus homologa latera subtenduntur.



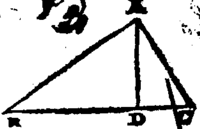
Theorema 7. Propositio 7.

Si duo triangula vnum angulum vni angulo æqualẽ, circũ autem alios angulos latera proportionalia habent; reliquorum ve-

rò simul vtrunque aut minorem, aut non minorem recto: æquiangula erunt triangula, & æquales habebunt eos angulos, circum quos proportionalia sunt latera.

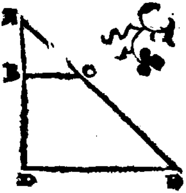
Theorema 8 Propositio 8.

Si in triangulo rectangulo, ab angulo recto in basin perpendicularis ducta sit; quæ ad perpendicularem triangula, tum toti triangulo, tum ipsa inter se similia sunt.



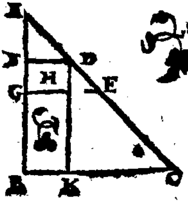
Probleme 1. Propositio 9.

A data recta linea imperatam partem auferre.



Problema 2. Propositio 10.

Datam rectam lineam insectam similiter secare, vt data altera recta secta fuerit.



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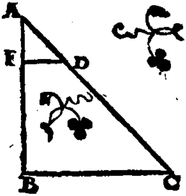
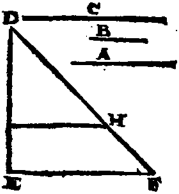
Problema 3. Propositio 11.

Duabus datus rectis lineis, tertiam proportionalem adinueniri.



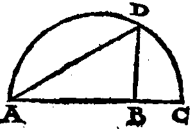
Problema 4. Propositio 12.

Tribus datis rectis lineis quartam proportionalem adinuenire.



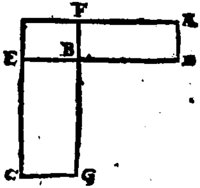
Problema 5. Propositio 13.

Duabus datis rectis lineis, mediam proportionalem adinuenire.



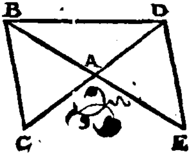
Theorema 9. Propositio 14.

Aequalium, & vnum vni æqualem habentium angulum, parallelogrammorum reciproca sunt latera, quæ circum ęquales angulos; Et quorum parallelogrammorum vnum angulum vni angulo æqualem habentium reciproca ſunt latera, quæ circum æquales angulos, illa ſunt æqualia.



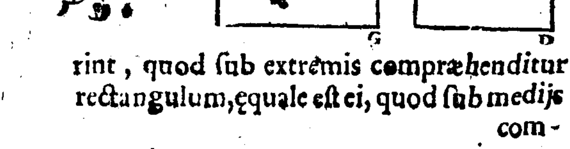
Theorema 10. Propoſitio 15.

Aequalium, & vnum angulum vni æqualem habentium triangulorum reciproca ſunt latera, quæ circum ęquales angulos: Et quorum triangulorum vnum angulum vni ęqualem habentium, reciproca ſunt latera, quæ circum æquales angulos, illa ſunt ęqualia.



Theorema 11. Propoſitio 16.

Si quatuor rectæ lineæ proportionales fue-

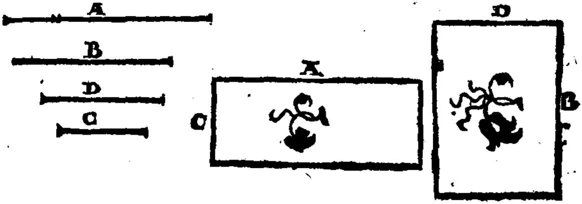


rint, quod ſub extremis compræhenditur rectangulum, ęquale eſt ei, quod ſub medijs com-

comprehenditur, rectangulo. Etſi ſub extremis comprehenſum rectangulum æquale fuerit ei, quod ſub medijs continetur, rectangulo, illæ quatuor rectæ lineæ proportionales erunt.

Theorema 12. Propoſitio 17.

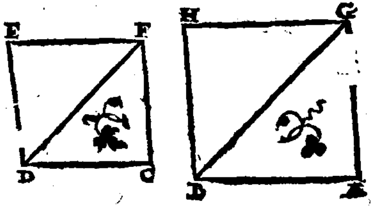
Si tres rectæ lineæ ſint proportionales; quod ſub extremis comprehenditur rectangulum



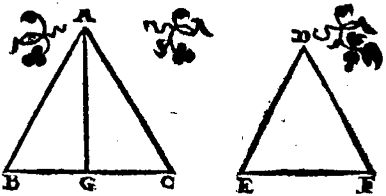
æquale ſit ei, quod à media deſcribitur, quadrato: Et ſi ſub extremis comprehenſum rectangulum æquale ſit ei, quòd à media deſcribitur, quadrato; illæ tres rectę lineæ proportionales erunt.

Problema 16. Propoſitio 16.

A data recta linea, dato rectilineo ſimile, ſimiliterq; poſitum rectilineum deſcribere.

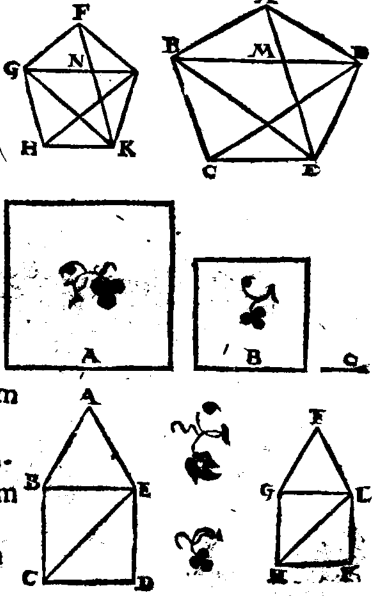


Theorema 13. Propositio 19.



Similia triangula inter se sunt in duplicata proportione laterum homologorum.

Theorema 14. Propositio 20.



Similia polygona in similia triangula diuiduntur, & numero æqualia, & homologa totis. Et polygona duplicatam habent eam inter se proportionem, quam latus homologum ad homologum latus.

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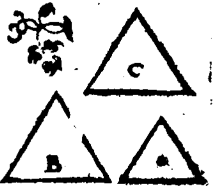
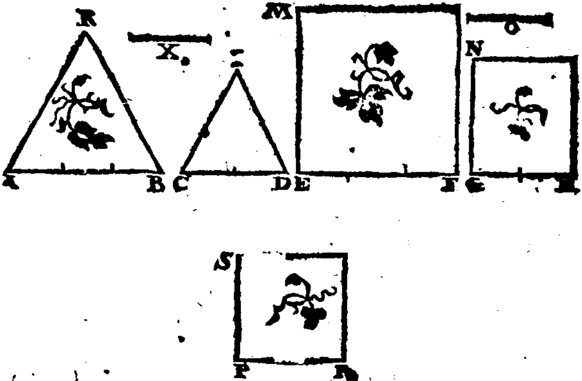
Theorema 15. Propositio 21.



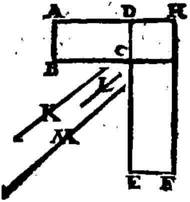
Quæ eidem rectilineo ſunt ſimilia, & inter ſe ſunt ſimilia.

Theorema 16. Propoſitio 22.

Si quatuor rectæ lineæ proportionales fuerint: & ab eis rectilinea ſimilia ſimiliterque deſcripta proportionalia erunt. Et ſi à rectis lineis ſimilia ſimiliterque deſcripta rectilinea proportionalia fuerint; ipſæ etiam rectæ lineæ proportionales erunt.

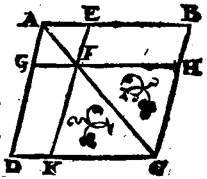


Theorema 17. Propoſitio 23.



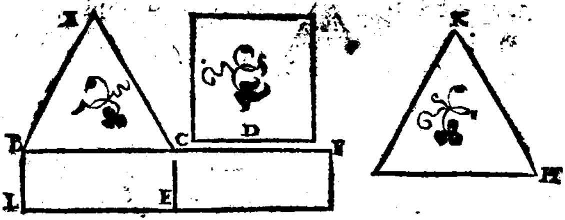
Aequiangula parallelogramma inter ſe proportionem habent eam, quæ ex lateribus componitur.

Theorema 18. Propoſitio 24.



In omni parallelogrammo, quæ circa diametrū ſunt parallelogramma, & toti, & inter ſe ſunt ſimilia.

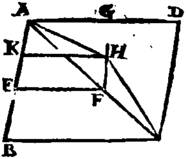
Problema 7. Propoſitio 25.



Dato recto lineo ſimile: ſimiliterque poſitum, & alteri dato æquale idem conſtituere.

Theo-

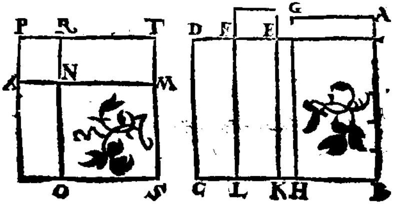
Theorema 19. Propositio 26.



Si à parallelogrammo parallelogrammum ablatũ sit, & simile toti, & similiter positum, commu nem cum eo habens angulum; hoc circum eandem cum toto diametrum consistit.

Theorema 20. Propositio 27.

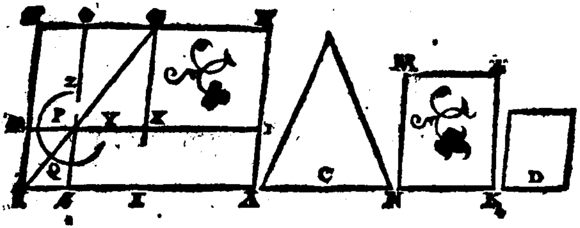
Omnium parallelogrammorum secundum eandem rectam lineam applicatorum, deficientiumq́; figuris parallelogrammis similibus,



similiterque positis, ei, quod a dimidia describitur; maximum, id est, quod ad dimidiam applicatur, parallelogrammum simile existens defectui.

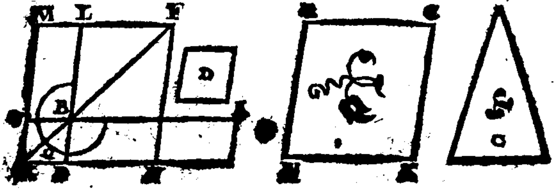
Problema 8. Propositio 28.

Ad datam lineam rectam, dato rectilineo æquale parallelogrammum applicare, deficiens figura parallelogramma, quæ similis ſit alteri rectilineo dato. Oportet autem datum rectilineum, cui æquale applicandum eſt, non maius eſſe eo, quod ad dimidiam applicatur, cùm ſimiles ſint defectus & eius, quod à dimidia deſcribitur, & eius, cui ſimile deeſſe debet.



Problema 9. Propoſitio 29.

Ad datam rectam lineam, dato rectilineo æquale parallelogrammum applicare, excedens figura parallelogramma, quæ ſimilis ſit parallelogrammo alteri dato.



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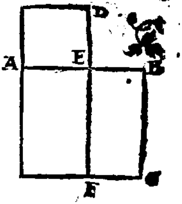
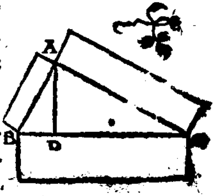
Problema 10. Propositio 30.



Propositam rectam lineam terminatam, extrema ac media ratione (proportione:) secare.

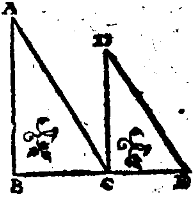
Theorema 21. Propositio 31.

In rectangulis triangulis, figura quæuis à latere rectum angulum subtendente descripta, æqualis est figuris, quæ priori illi similes, & similiter positæ, à laterib. rectum angulum continentibus describuntur.



Theorema 22. Propositio 32.

Si duo triangula, quæ duo latera duobus lateribus proportionalia habeant, secundum vnum angulum composita fuerint, ita vt homologa eorum latera sint etiam parallela; tum reliqua illorum triangulorum latera in rectam lineam collocata reperientur.



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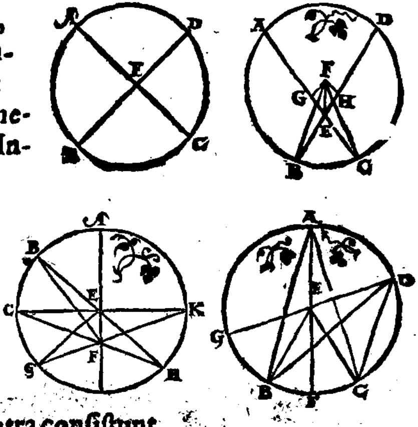
Theorema 23. Propositio 33.

In æqualibus circulis, anguli eandem habent rationem, cum ipsis peripherijs, in quibus insistunt, siue ad centra, siue ad peripherias constituti, illis insistant peripherijs. Insuper verò & sectosre, quippe qui ad centra consistunt.



FINIS ELEMENTI VI.

EVCLI-

EVCLIDIS ELEMENTVM SEPTIMVM.

DEFINITIONES.

1.

VNitas est, secũdum quam vnumquodque eorum, quæ sunt, vnum dicitur.

2.

Numerus autem, ex vnitatibus composita multitudo.

3.

Pars est numerus numeri, minor maioris, cum minor metitur maiorem.

4.

Partes autem, cùm non metitur.

5.

Multiplex verò, maior minoris, cùm maiorem metitur minor.

6.

Par numerus est, qui bifariam diuiditur.

7.

Impar verò, qui bifariam non diuiditur: vel qui vnitate differt à pari.

8.

Pariter par numerus est, quem par numerus metitur per numerum parem.

9.

Pariter autem impar est, quem par numerus metitur per numerum imparem.

10.

Impariter verò impar numerus est, quem impar numerus metitur per numerum imparem.

11.

Primus numerus est, quem vnitas sola metitur.

12.

Primi inter se numeri sunt, quos sola vnitas, mensura communis, metitur.

13.

Compositus numerus est, quem numerus quispiam metitur.

14.

Compositi autem inter se numeri sunt, quos numerus aliquis, mensura communis, metitur.

15.

Numerus numerum multiplicare dicitur, cùm toties compositus fuerit is, qui multiplicatur, quot sunt in ipso multiplicante vnitates; & procreatus fuerit aliquis.

16.

Cùm autem duo numeri mutuò sese multiplicantes quempiam faciunt, qui factus erit, planus appellabitur. Qui verò numeri mutuò sese multiplicarint, illius latera dicentur.

Cùm

17.

Cùm verò tres numeri mutuò sese multiplicantes quempiam faciunt, qui procreatus erit, solidus appellabitur, qui autem numeri mutuò sese multiplicarint, illius latera dicentur.

18.

Quadratus numerus est, qui æqualiter æqualis: vel, qui à duabus æqualibus numeris continetur.

19.

Cubus verò, qui æqualiter æquali æqualiter: vel, qui à tribus æqualibus numeris continetur.

20.

Numeri proportionales sunt, cùm primus secundi, & tertius quarti, æquè multiplex est; vel eadem pars, vel eædem partes.

21.

Similes plani, & solidi numeri sunt, qui proportionalia habent latera.

22.

Perfectus numerus est, qui suis ipsius partibus est æqualis.

23.

Numerus numerum metiri dicitur per illum numerum, quem multiplicans, vel à quo multiplicatus, illum producit.

24.

Proportio numerorum eſt habitudo quædam vnius numeri & alterum, ſecundum quod illius eſt multiplex, vel pars, parteſue.

25.

Termini, ſiue radices proportionis dicuntur duo numero, quibus in eadem proportione minores ſumi nequeunt.

26.

Cùm tres numeri proportionales fuerint; primus ad tertium, duplicatam proportionem dicitur habere eius, quam habet ad ſecundum. At cùm quatuor numeri proportionales fuerint; primus ad quartum, triplicatam proportionem habere dicitur eius, quam habet ad ſecundum. Et ſuper deinceps vno amplius, quam diu proportio extiterit.

27.

Quotlibet numerus ordine poſitis, proportio, prima ad vltimam componi dicitur ex proportionibus primi ad ſecundum, & ſecundi ad tertium, & tertij ad quartum, & ita deinceps, donec extiterit proportio.

Poſtulata, ſiue petitiones.

1.

Po[illegible] cuilibet numero, quotlibet poſſe [illegible]ales, vel multiplices.

2.

Q[illegible] umero, ſumi poſſe maiorem.

Axio-

Axiomata, siue pronunciata.

1.

Qui numeri æqualium numerorum, vel eiusdem, æquè multiplices sunt, inter se sunt æquales.

2.

Quorum idem numerus æquè multiplex est, vel æquè multiplices sunt æquales, inter se æquales sunt.

3.

Qui numeri æqualium numerorum, vel eiusdem, eadem pars, partes fuerint, inter se æquales sunt.

4.

Quorum idem numerus, vel æquales, eadem pars, vel eædem partes fuerint, æquales inter se sunt.

5.

Vnitas omnem numerum per vnitates, quæ in ipso sunt, hoc est, per ipsummet numerum, metitur.

6.

Omnis numerus seipsum metitur per vnitatem.

7.

Si numerus numerũ multiplicans, aliquem produxerit, metietur multiplicans productum per multiplicatum; multiplicatus autem eundem per multiplicantem.

8.

Si numerus numerum metiatur, & ille, per quem metitur, eundem metietur per eas, quæ in metiente sunt, vnitates, hoc est, per ipsum numerum metientem.

9.

Si numerus numerum metiens, multiplicet eum, per quem metitur; vel ab eo multiplicatur, illum, quem metitur, producat.

10.

Numerus quotcunque numeros metiens, compositum quoque ex ipsis metitur.

11.

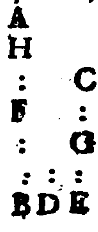
Numerus quemcunq; numerum metiens, metitur quoque omnem numerum, quem ille metitur.

12.

Numerus metiens totam, & ablatum, metitur & reliquum.

Theorema 1. Propositio 1.

Si duobus numeris inæqualibus propositis, detrahatur semper minor de maiore, alterna quadam detractione; neque reliquus vnquam metiatur præcedentem, quo ad assumpta sit vnitas: qui principiò propositi sunt numeri, primi inter se erunt.



Pro-

Problema 1. Propositio 2.

Duobus numeris datis non primis inter se, maximam eorum communem mensuram reperire.

A
A : C
E
: E E
: : : :
B D B D
: : : : :

Problema 2. Propositio 3.

A B C D E
8 6 4 2 3

Tribus numeris datis non primis inter se, maximam eorum communem mensuram reperire.

: : : : : :
A B C D E F
18 13 8 6 2 3

Theorema 2. Propositio 4.

Omnis numerus cuiusq; numeri, minor maioris, aut pars est, aut partes.

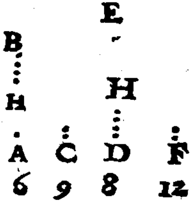
C
:
F
C C :
E
: :
: : : : :
A B B B D
12 7 6 9 3

Theorema 3. Propositio 5.

Si numerus numeri pars fuerit, & alter alterius eadem pars; & simul vterque vtriusque simul eadem pars erit, quæ vnus est vnius.

C
: F
: :
G H
: : : :
A B D C
62 1 5 8

Theorema 4. Propoſitio 6.

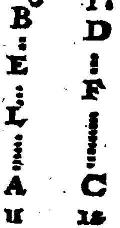


Si numerus ſit numeri partes, & alter alterius eædem partes; & ſimul vterque vtriuſque ſimul eædem partes erunt, quæ ſunt vnus vnius.

Theorema 5. Propoſitio 7.

Si numerus numeri eadem ſit pars quæ detractus detracti; & reliquus reliqui eadē pars erit, quæ totus eſt totius.

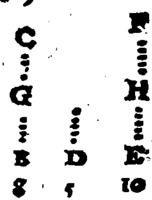
Theorema 6 Propoſitio 8.



Si numerus numeri eædem ſint partes quæ detractus detracti, & reliquus reliqui eædem partes erunt, quæ ſunt totus totius.

G.M.K.N.H.

Theorema 7. Propoſitio 9.



Si numerus numeri pars ſit, & altera alterius eadem pars: & viciſſim, quæ pars eſt, vel partes primus tertij, ! eadem pars, erit vel eædem A partes, & ſecundus quarti. 4

Theo-

Theorema 8. Propositio 10.

Si numerus numeri partes sint, & alter alterius eadem p rtes: etiam vicissim, q æ sunt partes, aut pars primus tertij, eædem partes erunt, vel pars & secundus quarti.

H		E	
G		H	
A	C	D	F
4	6	10	18

Theorema 9. Propositio 11.

Si quemadmodum se habet totus ad totum, ita detractus ad detractum: & reliquus ad reliquum ita se habebit, vt totus ad totum.

	D
B	F
E	
A	C
6	8

Theorema 10. Propositio 12.

Si sint quotcunque numeri proportionales; quemadmodum se habet vnus antecedentium ad vnum consequentium; ita se habebunt omnes antecedentes ad omnes consequentes.

A	B	C	D.
9	6	3	2

Theorema 11. Propositio 13.

Si quatuor numeri sint proportionales; & vicissim proportionales erunt.

A	B	C	D
12	4	9	3

Theorema 12. Propositio 13.

Si sint quotcunque numeri, & alij illis æquales multitudi-

A	B	C	D	E	F
12	6	3	8	4	2

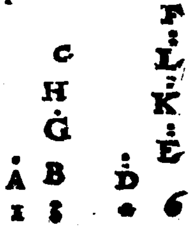
G 3

ne,

ne, qui bini ſumantur, & in eadem proportione, etiam ex æqualitate in eadem proportione erunt.

Theorema 13. Propoſitio 15.

Si vnitas numerum quempiam metiatur, alter verò numerus alium quendam numerum æquè metiatur; & viciſſim vnitas tertium numerum æqu metietur, atque ſecundus quartum.



Theorema 14 Propoſitio 16.

Si duo numeri mutuò ſeſe multiplicantes faciant aliquos; qui ex illis geniti fuerint, inter ſe æquales erunt.

E	A	B	C	D
1	2	4	8	8

Theorema 15. Propoſitio 17.

Si numerus duos numeros multiplicans faciat aliquos; qui ex illis procreati erunt, eandem proportioné inter ſe habebunt, quam multiplicati.

I	A	B	G	D	E
1	3	4	5	12	15

Theorema 16. Propoſitio 18.

Si duo numeri numerū quempiam multiplicantes faciant aliquos; geniti ex illis eandem habebunt proportionem, quam qui illum multiplicarunt.

A	B	C	D	E
4	5	3	12	15

Theo-

Theorema 17. Propositio 19.

Si quatuor numeri sint proportionales; quod ex primo, & quarto fit, numerus, ęqualis erit ei, qui ex secundo & tertio, fit, numero: Et si, qui ex primo & quarto fit numerus, æqualis sit ei, qui ex secundo & tertio, fit, numero; illi quatuor numeri proportionales erunt.

: : : : : : :
A B C D E F G
6 4 3 2 12 12 18

Theorema 18. Propositio 20.

Si tres numeri sint proportionales; qui ab extremis continetur, æqualis est ei, qui à medio efficitur: Et si, qui ab extremis continetur, æqualis sit ei, qui à medio describitur, illi tres numeri proportionales erunt.

: : :
A B C
9 6 4
:
D
6

Problema 19. Propositio 21.

Minimi numeri omnium qui eandem cum eis proportionem habent, æqualiter metiuntur numeros eandem cum eis proportionem habentes; maior quidem maiorem, minor verò minorem.

D L
G H
: : : :
C E A B
4 3 8 6

Theorema 20. Propositio 22

Si tres sunt numeri, & alij multitudine illis æquales, qui bini sumantur & in eadem ratione; sit autem perturbata eorum proportio; etiam ex æqualitate in eadem proportione erunt.

:	:	:	:	:	:
A	B	C	D	E	F
6	4	3	12	8	6

Theorema 21. Propositio 23.

Primi inter se numeri, minimi sunt omnium eandem cum eis proportionem habentium.

:	:	:	:	:
A	B	E	C	D
5	6	2	4	3

Theorema 22. Propositio 24.

Minimi numeri omnium eandem cum eis proportionem habentium, primi sunt inter se.

:	:	:	:	:
A	B	C	D	E
7	6	4	3	4

Theorema 23. Propositio 25.

Si duo numeri sint primi inter se, qui alterutrum eorum metitur, numerus, is ad reliquum primus erit.

:	:	:	:
A	B	C	D
6	7	3	4

Theorema 24. Propositio 26.

Si duo numeri ad quempiam numerum primi sint, ad eundem primus is quoque futurus est, qui ab illis productus fuerit.

3
B

A	C	D	E	F
5	5	5	3	2

Theo-

Theorema 25. Propositio 27.

Si duo numeri primi sint inter se, qui ab vno eorum gignitur, ad reliquum primus erit.

B		
A	C	D
7	6	3

Theorema 26 Propositio 28.

Si duo numeri ad duos numeros ambo ad vtrunque primi sint; & qui ex eis gignentur, primi inter se erunt.

A	B	E	C	D	F
3	5	5	2	4	8

Theorema 27. Propositio 29.

Si duo numeri primi sint inter se, & multiplicans vterque seipsum procreet aliquem; qui ex ijs producti fuerint, primi inter se erunt. Quòd si numeri initio propositi multiplicantes eos, qui producti sunt, effecerint aliquos; hi quoque inter se primi erunt; & circa extremos idem hoc semper eueniet.

A	C	E	B	D	F
3	6	27	4	16	36

Theorema 28. Propositio 30.

Si duo numeri primi sint inter se, etiam simul vterque ad vtrunque illorum primus erit. Et simul vterque ad vnum aliquem eorum primus sit, etiam qui initio positi sunt numeri, primi inter se erunt.

		C
A	B	D
7	5	4

 Theo-

Theorema 29. Propositio 31.

Omnis primus numerus ad omnem numerum, quem non metitur, primus est.

:	:	:
A	B	C
7	10	5

Theorema 30. Propositio 32.

Si duo numeri sese mutuò multiplicantes faciant aliquem; hunc autem ex ipsis productum metiatur primus quidam numerus: is alterum etiam eorum, qui initio positi erant, metietur.

:	:	:	:	:
A	B	C	D	E
3	6	12	3	4

Theorema 31. Propositio 33.

Omnem compositum numerum aliquis primus metietur.

:	:	:
A	B	C
27	9	3

Theorema 32. Propositio 34.

Omnis numerus aut primus est, aut eum aliquis primus metitur.

:	:	:
A	A	
3	6	3

Problema 3. Propositio 35.

Numeris datis quotcunque, reperire minimos omnium, qui eandem cum illis proportionem habeant.

:	:	:	:	:	:	:	:	:	:	:
A	B	C	D	E	F	G	H	K	I	M
6	8	12	2	3	4	6	2	3	4	3

Pro-

Problema 4. Propositio 36.

B

A C D E F

7 12 8 4 5

Duobus numeris datis, reperire, quē illi minimum metiantur, numerum.

A

F E C D G H

6 9 12 a 2 3

Theorema 33 Propositio 37.

Si duo numeri numerum quempiam metiantur; & minimus quem ille metiuntur, eundem metietur.

F

A B E C

2 3 6 12

Problema 5. Propositio 32.

Tribus numeris datis, reperire quem minimum numerum illi metiantur.

A B C D E

3 4 6 12 8

A B C D E F

3 6 8 12 24 16

Theorema 34. Propositio 39.

Si numerum quispiam numerus metiatur, mensus partem habebit metienti cognominem.

A B C D

12 4 3 1

Theo-

Theorema 35. Propositio 40.

Si numerus partem habuerit quamlibet, illum metietur numerus parti cognominis.

:	:	:	:
A	B	C	D
8	4	2	1

Problema 6. Propositio 41.

Numerum reperire, qui minimus cùm sit, datas habeat partes.

:	:	:	:	:
A	B	C	G	H
2	3	4	12	10

FINIS ELEMENTI VII.

EVCLIDIS ELEMENTVM OCTAVVM.

Theorema 1. Propositio 1.

SI sint quotcunque numeri deinceps proportionales, quorum extremi sint inter se, primi; ipsi minimi sunt omnium eandem cum eis proportionem habentium.

A	B	C	D	E	F	G	H
8	12	18	27	6	8	12	18

Problema 1. Propositio 2.

Numeros reperire deinceps proportionales minimos, quotcunque iusserit quispiam in data proportione.

A	B	C	D	E	F	G	H	K
3	4	9	12	16	27	36	49	64

Theorema 2. Propositio 3.
Conuersa primæ.

Si sint quotcunque numeri deinceps proportionales minimi habentium eandem cũ

eis

eis proportionem; illorum extremi ſunt inter ſe primi.

A	B	C	D	E	F	G	H	K	L	M	N	O
27	36	48	64	3	4	9	12	16	27	36	48	64

Problema 2. Propoſitio 4.

Proportionibus datis quotcunque in minimis numeris, reperire numeros deinceps minimos in datis proportionibus.

A	B	C	D	E	F	H	G	K	L	N	X	M	O
3	4	2	3	4	5	6	8	12	15	4	6	10	12

Theorema 3. Propoſitio 5.

Plani numeri proportionem inter ſe habent ex lateribus compoſitam.

A	L	B	C	D	E	F	G	H	K
18	22	32	3	6	4	8	9	12	10

Theorema 4. Propoſitio 6.

Si ſint quotlibet numeri deinceps,

A	B	C	D	E	F	G	H
16	24	36	54	81	4	6	9

inceps, proportionales; primus autem secundum non metiatur; neque alius quispiam vllum metietur.

Theorema 5. Propositio 7.

A B C D
4 6 12 24

Si sint quotcunque numeri deinceps proportionales; primus autem extremum metiatur; is etiam secundum metietur.

Theorema 6. Propositio 8.

Si inter duos numeros medij continua proportione indicant numeri; quot inter eos medij continua proportione incidunt numeri, totidem & inter alios eandem cum illis habentes proportionem medij continua proportione incident.

A C D B G H K L C M N F
4 9 27 81 1 3 9 27 2 6 18 54

Theorema 7. Propositio 9.

Si duo numeri sunt inter se primi, & inter eos medij continua proportione incidant numeri, quot inter eos medij continua proportione incidunt numeri, totidem & inter vtrunque eorum, ac vnitatem deinceps medij continua proportione incident.

A	M	H	E	F	N	C	K	X	G	D	L	O	B
27	27	9	36	3	36	1	12	48	4	48	16	64	64

Theorema 8. Propoſitio 10.

Si inter duos numeros, & vnitatem, continuè proportionales incidant numeri; quot inter vtrunque ipſorum, & vnitatem, deinceps medij continua proportione incidunt numeri; totidẽ & inter illos medij continua proportione incident.

A 27 E 9 D 3 K 36 H 12 C 1 L 48 F 4 G 16 B 64

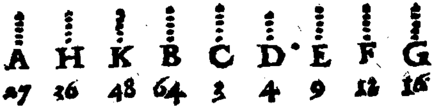
Theorema 9. Propoſitio 11.

Duorum quadratorum numerorum vnus medius proportionalis eſt numerus: & quadratus ad quadratum duplicatam habet lateris ad latus proportionem.

A	C	E	D	B
9	3	12	4	16

Theorema 10. Propoſitio 12.

Duorum cuborum numerorum duo medij proportionales ſunt numerorum; Et cubus ad ad cubum triplicatam habet lateris ad latus proportionem.



Theorema 11. Propoſitio 13.

Si ſint quotlibet numeri deinceps proportionales, & multiplicans quiſque ſeipſum faciat aliquos; qui ab illis producti fuerint, proportionales erunt. Et ſi numeri primum poſiti, ex ſuo id procreatos ductu, faciant aliquos; ipſi quoque proportionales erunt.

Theoreme 12. Propoſitio 4.

Si quadratus numerus quadratum numerũ metiatur, & latus vnius metietur latus alterius. Et si vnius quadrati latus metiatur latus alterius, & quadratus quadratum metietur.

A	E	B	C	D
9	12	16	8	4

Theorema 13. Propositio 15.

Si cubus numerus cubum numerus metiatur, & latus vnius metietur alterius latus. Et si latus vnius cubi latus alterius metiatur, tum cubus cubum metietur.

A	H	K	B	C	D	E	F	G
8	16	28	64	2	4	4	8	16

Theorema 14. Propositio 16.

Si quadratus numerus quadratum numerũ non metiatur, neque latus vnius metietur alterius latus. Et si latus vnius quadrati non metiatur latus alterius, neque quadratus quadratum metietur.

A	B	C	D
9	16	3	4

Theorema 15. Propositio 17.

Si cubus numerus cubum numerum non metiatur, neq; latus vnius latus alterius metietur, Et si latus cubi vnius latus alterius non metiatur, neque cubus cubum metietur.

A	B	C	D
8	27	9	11

Theo-

Theorema 16. Propositio 18.

Duorum similium planorum numerorum vnus medius proportionalis est numerus; & planus ad planũ duplicatam habet lateris homologi ad latus homologum proportionem.

A	G	B	C	D	E	F
12	18	27	2	6	3	9

Theorema 17. Propositio 19.

Duorum similium numerorum solidorum duo medij proportionales sunt numeri: Et solidus ad similem solidum triplicatam, habet lateris homologi ad latus homologum proportionem.

A	N	X	B	C	D	E	F	G	H	K	M	L
8	12	18	27	2	2	2	3	3	3	4	6	9

Theorema 18 Propositio 20.

Si inter duos numeros vnus medius proportionalis incidat numerus; similes plani erunt illi numeri.

A	C	B	D	E	F	G
18	24	33	3	4	6	8

Theorema 19. Propositio 21.

Si inter duos numeros duo medij proportionales incidant numeri; similes solidi sunt illi numeri.

A	C	D	B	E	F	G	H	K	L	M
27	36	44	64	9	12	16	3	3	3	4

Theorema 20. Propositio 22.

Si tres numeri deinceps sint proportionales; primus autem sit quadratus, & tertius quadratus erit.

A	B	D
9	15	25

Theorema 21. Propositio 23.

Si quatuor numeri deinceps sint proportionales; primus autem si cubus, & quartus cubus erit.

A	B	C	D
8	12	18	27

Theorema 22. Propositio 24.

Si duo numeri eam proportionem habeant inter se, quam quadratus numerus ad quadratum numerum, primus autem sit quadratus, & secundus quadratus erit.

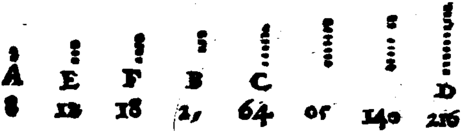
A B C D

4 9 9 16 34 36

Theorema 23. Propositio 25.

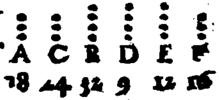
Si numeri duo proportionem inter se habeant,

beant, quam cubus numerus ad cubum numerum; primus autem cubus sit, & secundus cubus erit.



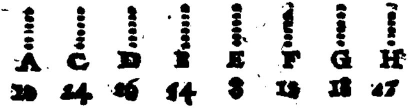
Theorema 24. Propositio 26.

Similes plani numeri proportionem inter se habent, quam quadratus numerus ad quadratum numerum.



Theorema 25. Propositio 27.

Similes solidi numeri proportionem habent inter se, quam cubus numerus ad cubum numerum.



FINIS ELEMENTI VIII.

EVCLIDIS ELEMENTVM NONVM.

Theorema 1. Propoſitio 1.

Si duo ſimiles plani numeri mutuò ſeſe multiplicantes, quendam procreent, productus quadratus erit.

A	E	B	D		C
4	6	9	16	24	36

Theorema 2. Propoſitio 2.

Si duo numeri mutuò ſeſe multiplicantes, quadratum faciant, illi ſimiles ſunt plani.

A		B	D		C
4	6	9		18	36

Theorema 3. Propoſitio 3.

Si cubus numerus ſeipſum multiplicás pro-

creet

creet aliquem, productus cubus erit.

vnitas	D	D	A			B
	3	4	8	16	32	64

Theorema 4. Propositio 4.

Si cubus numerus cubum numerum multiplicans quendam procreet, procreatus cubus erit.

A	B	D	C
8	27	64	216

Theorema 5. Propositio 5.

Si cubus numerus numerum quẽdam multiplicans cubum procreet; & multiplicatus cubus erit.

A	B	C	D
27	64	729	18

Theorema 6. Propositio 6.

Si numerus seipsum multiplicãs cubum procreet; & ipse cubus erit.

A	B	C
27	729	19683

Theorema 7. Propositio 7.

Si compositus numerus quendam numerũ multiplicans, quempiam procreet, productus solidus erit.

A	B	C	D	E
6	8	48	2	3

Theorema 8. Propositio 8.

Si ab vnitate quotlibet numeri deinceps proportionales sint: Tertius ab vnitate quadratus est, & vnũ intermittentes oẽs: Quartus aũt cubus est, & duobus intermissis omnes: Septimus vero cubus simul & quadratus est, & quinque intermissis omnes.

Vnitas	A	B	C	D	E	F
	3	9	27	81	243	729

Theorema 9. Propositio 9.

Si ab vnitate sint quotcunque numeri deinceps proportionales; sit autem quadratis is, qui vnitatem sequitur, & reliqui omnes quadrati erunt. Quòd si, qui vnitatem sequitur, cubus sit; & reliqui omnes cubi erunt.

quadrati		cubi
531441	F	732969
59049	E	531441
6561	D	6561
729	C	6561
81	B	729
9	A	81
	0	
	vnitas.	

Theorema 10. Propositio 10.

Si ab vnitate numeri quotcunque proportionales sint; non sit autem quadratus is, qui vnitatem sequitur, neque alius vllus quadratus

Vnitas	A	B	C	D	E	F
	3	9	36	182	43	729

tus erit; demptis, tertio ab vnitate ac omnibus vnum intermittentibus. Quòd si, qui vnitatem sequitur, cubus non sit: neque alius vllus cubus erit; demptis, quarto ab vnitate, ac omnibus duos intermittentibus.

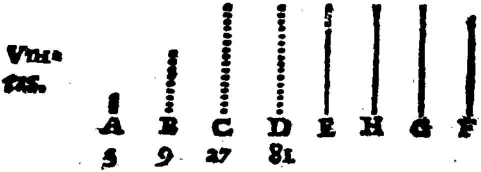
Theorema 11. Propositio 11.

Si ab vnitate numeri quotlibet deinceps proportionales sint; minor maiorem metitur per quempiam eorum, qui in proportionalibus sunt.

:	:	:	:	:
A	B	C	D	E
1	2	4	8	16

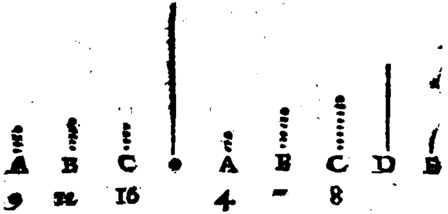
Theorema 12. Propositio 12.

Si ab vnitate quotlibet numeri sint proportionales; quot primorum numerorum vltimum metiuntur, totidem & eum, qui vnitati proximus est, metientur.



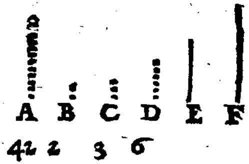
Teorema 13. Propositio 13.

Si ab vnitate sint quotcunque numeri deinceps proportionales; primus autem sit, qui vnitatem sequitur; maximum nullus alius metietur; ijs exceptis, qui in proportionalibus sunt numeris.



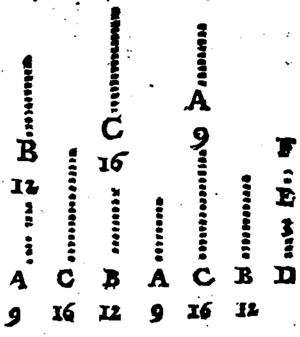
Theorema 14. Propositio 14.

Si minimum numerum primi aliquot numeri metiantur; nullus alius numerus primus illum metietur; ijs exceptis, qui primò metiuntur.



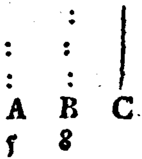
Theorema 15. Propositio 15.

Si tres numeri deinceps proportionales sint minimi omnium, eandem cum ipsis proportionem habentium, duo quilibet compositi, ad tertium primi erunt.



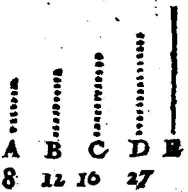
Theorema 16. Propositio 16.

Si duo numeri sint inter se primi; non se habebit quemadmodum primus ad secundum, ita secundus ad quempiam alium.



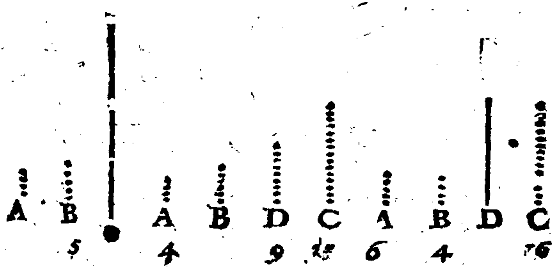
Theorema 17. Propositio 17.

Si sint quotlibet numeri deinceps proportionales, quorum extremi sint inter se primi; non erit quemadmodum primus ad secundum, ita vltimus ad quempiam alium.



Problema 1. Propositio 18.

Duobus numeris datis, considerare an possit ipsis tertius inueniri proportionalis.



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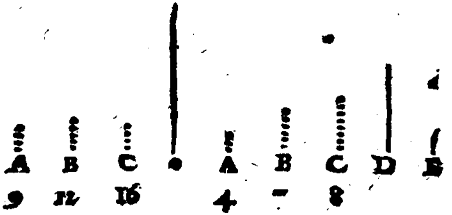
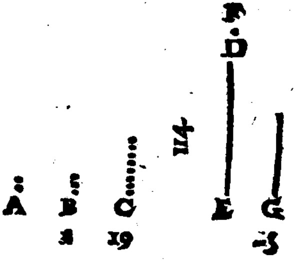
Problema 2. Propositio 19.

Tribus numeris datis, considerare, an possit ipsis quartus reperiri proportionalis.



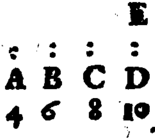
Theorema 18. Propositio 20.

Primi numeri plures sunt, quacunque proposita multitudine primorum numerorum.



Theorema 19. Propositio 21.

Si pares numeri quotlibet compositi sint, totus est par.



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Theorema 20. Propositio 22.

Si impares numeri quotlibet compositi sint; sit autem par illorum multitudo; totus par erit.

E

A B C D

5 9 7 3

Theorema 21. Propositio 23.

Si impares numeri quotcunque compositi sint; sit autem impar illorum multitudo; & totus impar erit.

A B C E

5 7 8 I

Theorema 22. Propositio 24.

Si à pari numero par detractus sit; & reliquus par erit.

B

A C

6 4

Theorema 23. Propositio 25.

Si à pari numero impar detractus sit; & reliquus impar erit.

A C D

8 1 4

Theorema 24. Propositio 26.

Si ab impari numero impar detractus sit; & reliquus par erit.

B

A C D

4 6

Theo-

Theorema 25. Propositio 27.

Si ab impari numero par ablatus sit; reliquis impar erit.

A D C
1 4 4

B

Theorema 27. Propositio 28.

Si impar numerus parem multiplicans, procreet quempiam; procreatus par erit.

A B C
3 4 21

Theorema 27. Propositio 29.

Si impar numerus imparem numerum multiplicans, quendam procreet; procreatus impar erit.

A C D
3 5 15

Theorema 28. Propositio 30.

Si impar numerus parem numerum metiatur; & illius dimidium metietur.

A C B
3 6 18

Theorema 29. Propositio 31.

Si impar numerus ad numerum quempiam primus sit: & illius duplum primus erit.

A B C D
7 8 16

Theo-

Theorema 30. Propositio 32.

Numerorum, qui à binario dupli sunt, *Vnivnusquisque* pariter *tantum* par est tantum.

A B C D
2 4 8 16

Theorema 31. Propositio 33.

Si numerus dimidium habeat imparem: pariter impar est tantùm.

A
20

Theorema 32. Propositio 34.

Si par numerus neque à binario duplus sit, neque dimidium habeat imparem: pariter par est, & pariter impar.

A
20

Theorema 33. Propositio 35.

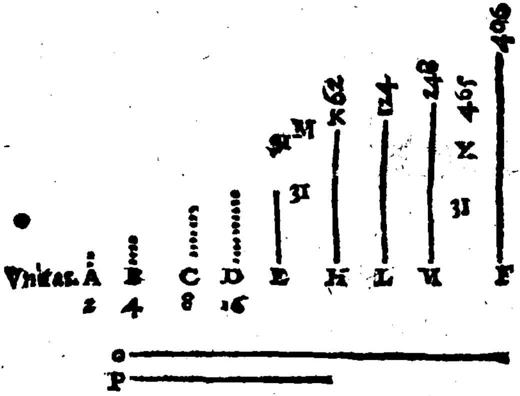
Si sint quotlibet numeri deinceps proportionales; detrahantur autem à secundo & vltimo æquales ipsi primo: Erit quemadmodum secundi excessus ad primũ, ita vltimi excessus ad omnes, qui vltimum antecedunt.

F 4 K 4
C 4 G
D B D E
4 4 16 16

Theo-

Theorema 34 Propositio 36.

Si ab vnitate numeri quotlibet deinceps expositi sint in dupla proportione, quoad totus compositus primus factus sit; isq; totus in vltimum multiplicatus, quempiam procreatus perfectus erit.



FINIS ELEMENTI IX.

EVCLIDIS ELEMENTVM DECIMVM.

DEFINITIONES.

PRIMAE, NEMPE MAGNITVDInem symmetratum.

1.

Commensurabiles magnitudines dicuntur illæ, quas eadem mensura metitur.

2.

Incommensurabiles verò magnitudines dicuntur, quarum nullam mensuram communem contingit reperiri.

3.

Lineæ rectæ potentia commensurabiles sunt, quarum quadrata vna eadem superficies, siue area metitur.

4.

Incommensurabiles verò lineæ sunt, quarum quadrata, quæ metiatur area communis, reperiri nulla potest.

5.

Hæc cum ita sint, ostendi potest, quòd quãtacunque linea recta nobis proponatur, existunt etiam aliæ lineæ innumerabiles eidem commenſurabiles, aliæ item incommenſurabiles; hæ quidem longitudine & potentia; illæ verò potentia tantùm. Vocetur igitur linea recta, quantacunq; proponatur, ῥητὴ, id eſt, rationalis.

6.

Lineæ quoque illi ῥητῇ commenſurabiles ſiue longitudine & potentia, & ſiue potentia tantùm, vocentur & ipſe ῥηταὶ, id eſt, rationales.

7.

Quæ verò lineæ ſunt incommenſurabiles illi τῇ ῥητῇ, id eſt, primo loco rationali, vocentur ἄλογοι, id eſt, irrationales.

8.

Et quadratum, quod à linea propoſita deſcribitur, quam ῥητὴν vocari volumus, vocetur ῥητὸν, id eſt, rationale.

9.

Et, quæ ſunt huic commenſurabilia, vocentur ῥητά, id eſt, rationalia.

10.

Quæ verò ſunt illi quadrato, ῥητῷ ſcilicet, incommenſurabilia, vocentur, ἄλογα, id eſt, ſurda, ſiue irrationalia.

11.

Et lineæ, quæ illa incommenſurabilia deſcribunt, vocentur ἄλογοι. Et quidam ſi illa incommenſurabilia fuerint quadrata, ipſa eorum altera vocabuntur ἄλογοι lineæ, quòd

ſi qua-si quadrata quidem non fuerint, verum aliæ quæpiam superficies siue figuræ rectilineæ, tunc vero lineæ illæ quæ describunt quadrata æqualia figuris rectilineis, vocentur ἄλογοι.

Postulatum, siue petitio.

Postulatur quemlibet magnitudinem totius posse multiplicari, donec quamlibet magnitudinem eiusdem generis excedat.

Axiomata, siue pronunciata.

1.

Magnitudo quotcunque magnitudines metiens, compositam quoque ex ipsis metitur.

2.

Magnitudo quamcũq; magnitudinem metiens, metitur quoque omnem magnitudinem, quam illa metitur.

3.

Magnitudo metiens totam magnitudinem, & ablatam, metitur, & reliquam.

Problema 1. Propositio 1.

Duabus magnitudinibus inæqualibus propositis, si de maiore detrahatur plus dimidio; & rursus de residuo iterum detrahatur plus dimidio; idque semper fiat: relinquetur tãdem quędam magnitudo minor altera minore ex duabus propositis.

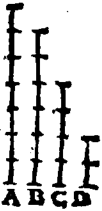
Theorema 2. Propositio 2.

Duabus magnitudinib. propositis inæqualibus, si detrahatur semper minor de maiore, alterna quadam detractione; neque residuum vnquam metiatur id quod ante se metiebatur : incommensurabiles sunt illæ magnitudines.

Problema 1. Propositio 3.

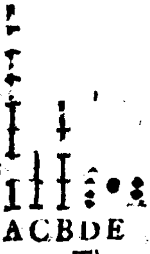
Duabus magnitudinibus commensurabilibus datis, maximam ipsarum communem mensuram reperire.

Problema 2. Propositio 4.

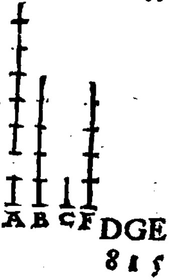


Tribus magnitudinibus commensurabilibus datis, maximam ipsarum communem mensuram reperire.

Theorema 3. Propositio 5.

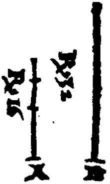


Commensurabiles magnitudines inter se proportionem eam habent, quam habet numerus ad numerum.



Theorema 4. Propositio 6.

Si duæ magnitudines proportionem eam habent inter se, quam numerus ad numerum : commensurabiles sunt illæ magnitudines.



Theorema 5. Propositio 7.

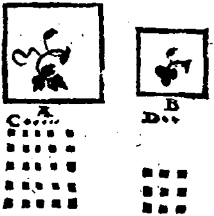
Incommensurabiles magnitudines inter se proportionem non habent, quam numerus ad numerum.

Theorema 6. Propositio 8.

Si duæ magnitudines inter se proportionem non habent, quam numerus ad numerum: incommensurabiles illæ sunt magnitudines.

Theorema 7. Propositio 9.

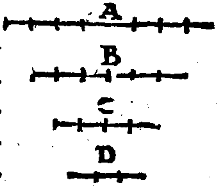
Quadrata, quæ describitur à rectis lineis lõgitudine cõmensurabilibus: inter se proportionem habent, quam numerus quadratus ad alium numerum quadratum. Et quadrata habentia proportionem inter se, quam qua



dratus numerus ad numerum quadratum; habent quoque latera longitudine commensurabilia. Quadrata verò quæ describuntur à lineis longitudine incommensurabilibus; proportionem non habent inter se, quam quadratus numerus ad numerum aliū quadratum. Et quadrata non habentia proportionem inter se, quam numerus quadratus ad numerum quadratum, neque latera habebunt longitudine commensurabilia.

Theorema 8. Propositio 10.

Si quatuor magnitudines fuerint proportionales; prima verò secundæ fuerit commensurabilis; tertia quoque quartæ commensurabilis erit; quòd si prima secundæ fuerit incōmensurabilis; tertia quoque quartæ incommensurabilis erit.



Problema 3. Propositio 11.

Propositæ lineæ rectæ (quam ῥητὴν vocari dixi-

diximus) reperire duas lineas rectas incommensurabiles, alteram quidem longitudine tantùm, alteram verò non longitudine tãtùm, sed etiam potentia incommẽsurabilem.

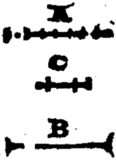
Theorema 9. Propositio 12.

Magnitudines, quæ eidem magnitudini sunt commensurabiles; inter se quoque sunt commensurabiles.

6 D.. ..4 F..
4 E.... 8 G..
3 H...
2 K..
4 L...

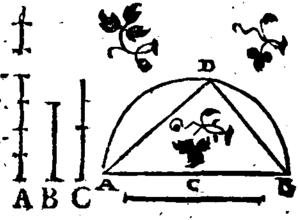
Theorema 10. Propositio 13.

Si ex duabus magnitudinibus hæc quidem commensurabilis sit tertiæ magnitudini, illa verò eidem incommensurabilis, incommensurabiles sunt illæ duæ magnitudines.



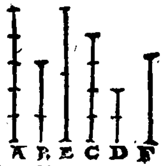
Theorema 11. Propositio 14.

Si duarum magnitudinum commensurabilium, altera fuerit commensurabilis magninitudini alteri cuipiam tertię; reliqua quoq; magnitudo eidem tertiæ incommensurabilis erit.



Theorema 12. Propositio 15.

Si quatuor rectæ lineæ proportionales fuerint; possit autem prima plusquam secunda tanto, quantum est quadratum rectæ lineæ sibi commensurabilis longitudine: tertia quoque potuerit plusquam quarta tanto, quantum est quadratum rectæ lineæ sibi commensurabilis longitudine. Quòd si prima possit plusquam secũda quadrato rectæ lineæ sibi longitudine incommensurabilis: tertia quoque poterit plusquã quarta quadrato rectæ lineæ sibi incommensurabilis longitudine.

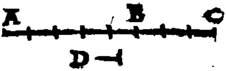


Theorema 13. Propositio 16.

Si duæ magnitudines commensurabiles componantur; tota magnitudo composita singularis partibus commensurabilis erit; Quòd si tota magnitudo composita alterutri parti commensurabilis fuerit; illæ duæ

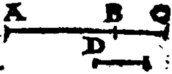
quo-

quoque partes cómensurabiles erunt.



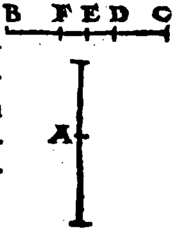
Theorema 14. Propositio 17.

Si duæ magnitudines incommensurabiles componantur, ipsa quoque tota magnitudo singulis partibus componentibus incommensurabilis erit. Quòd si tota alteri parti incommensurabilis fuerit, illæ quoque primæ magnitudines inter se incommensurabiles erunt.



Theorema 15. Propositio 18.

Si fuerint duæ rectæ lineæ inæquales, & quartæ parti quadrati, quod describitur à minore, æquale parallelogrammum applicetur ad maiorem, ex qua maiore tantum excurrat extra latus parallelogrammi, quantum est alterum latus ipsius parallelogrammi: si præterea parallelogrammum sui applicatione diuidat lineam illam in partes inter se commensurabiles longitudine, illa maior linea tanto plus potest quam minor, quantum est quadratum lineæ sibi commẽsurabilis longitudine. Quòd si maior plus possit quam minor, tanto, quantum est quadratum lineæ sibi commensurabilis longitudine; & præterea quartæ parti quadrati lineæ minoris, ęquale parallelogrammum applicetur ad maiorem, ex qua maiore tantum excurrat extra latus parallelogrammi, quantum eſt alterum latus ipſius parallelogrammi: parallelogrammum ſui applicatione diuidit maiorem in partes inter ſe longitudine commenſurabiles.

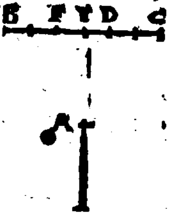


Theorema 16. Propoſitio 19.

Si fuerint duæ rectæ lineæ inæquales; quartæ autem parti quadrati lineæ minoris, æquale parallelogrammorum ad lineam maiorem applicetur, ex qua linea tantum excurrat extra latus parallelogrammi, quantum eſt alterum latus eiuſdem parallelogrammi: ſi parallelogrammum pręterea ſui applicatione diuidat lineam in partes inter ſe longitudine incommenſurabiles, maior illa linea tantò plus poteſt quàm minor quantum eſt quadratum lineæ ſibi minori commẽſurabiles longitudine. Quòd ſi maior linea tantò plus poſſit quàm minor, quantum eſt quadratum lineæ incommenſurabilis ſibi longitudine: & pręterea quartæ parti quadrati lineæ minoris æquale parallelogrammum applicetur ad maiorem

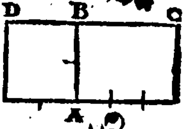
ex

ex qua tantum excurrat extra latus parallelogrammi, quantum est alterum latus ipsius: parallelogrammum sui applicatione diuidit maiorē in partes inter se incommensurabiles longitudine.



Theorema 17. Propositio 20.

Superficies rectanguli contenta ex lineis rectis rationalibus longitudine commensurabilibus secundum vnum aliquem modum ex antedictis, rationalis est.

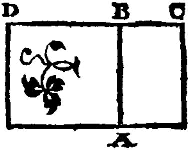


Theorema 18. Propositio 21.

Si rationale ad lineam rationalem applicetur; habebit alterum latus lineam rationalem & commensurabilem longitudine lineæ cui rationale parallelogrammum applicatur.

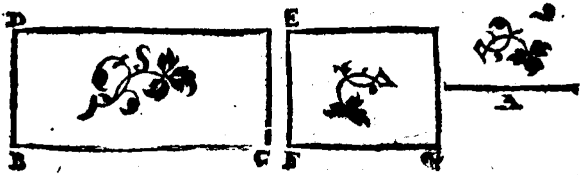
Theorema 19. Propositio 22.

Superficies rectangula contenta duabus lineis rectis rationalib. potentia tantū cōmensuraſurabilibus, irrationalis eſt. Linea autem quæ illam ſuperficiem poteſt, irrationalis & ipſa eſt; vocetur verò media.



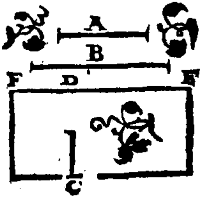
Theorema 20. Propoſitio 23.

Quadrati lineæ mediæ ad lineam rationalem applicanti, alterum latus eſt linea rationalis, & incommenſurabilis longitudine lineæ, ad quam applicatur.



Theorema 21. Propoſitio 24.

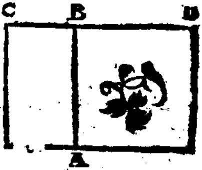
Linea recta mediæ commenſurabilis, eſt ipſa quoque media.



Theo-

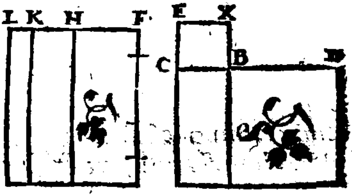
Theorema 22. Propositio 25.

Parallelogrammum rectangulum contentum sub rectis lineis medijs longitudine commensurabilibus, medium est.



Theorema 23. Propositio 26.

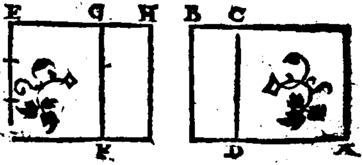
Parallelogrammum rectangulum comprehensum sub duabus lineis medijs potentia tantum commensuralibus;



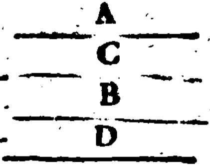
vel rationale est, vel medium.

Theorema 24. Propositio 27.

Medium non est maius, quàm medium superficie rationali.

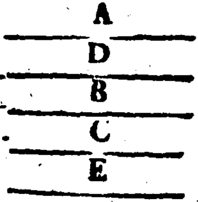


Problema 4. Propoſitio 28.



Medias lineas inuenire potentia tantùm commenſurabiles ratione comprehendentes.

Problema 5. Propoſitio 29.



Medias lineas inuenire potentia tantùm commenſurabiles, medium comprehendentes.

Problema 6. Propoſitio 30.

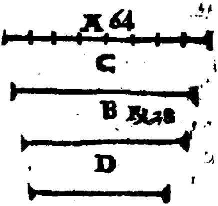
Reperire duas rationales potentia tantum cõmenſurabiles huiuſmodi, vt maior ex illis poſſit plus, quàm minor, quadrato rectæ lineæ ſibi commenſurabilis longitudine.

Problema 7. Propoſitio 31.

Inuenire duas rationales potentia tantùm commenſurabiles; ita vt maior, quam minor plus poſſit, quadrato rectæ lineæ ſibi longitudine incommenſurabilis.

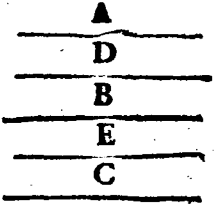
Problema 8. Propoſitio 32.

Reperire duas lineas medias potentia tantùm commenſurabiles rationalem ſuperficiem continentes tales in quã vt maior poſſit plus, quàm minor, quadrato rectæ lineæ ſibi commenſurabilis longitudine.



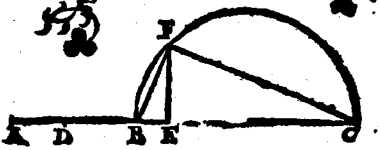
Problema 9. Propoſitio 33.

Inuenire duas lineas medias potentia tantùm cõmenſurabiles, quæ mediam ſuperficiem contineant, ita vt maior plus poſſit, quam minor, quadrato rectæ lineæ ſibi longitudine commenſurabilis.



Problema 10. Propoſitio 34.

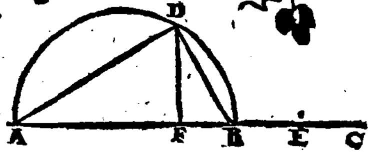
Inuenire duas rectas lineas potentia incommenſurabiles; quarum quadrata ſimul compoſita faciant ſuperficiem rationalem: Rectangulum verò ſub ipſis contentum, faciant medium.



Pro-

Problema 11. Propositio 35.

Reperire lineas duas rectas potentia incommensurabiles, conficientes compositum ex ipsarum quadratis medium, parallelogrammũ verò ex ipsis contentum rationale.



Problema 12. Propositio 36.

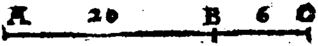
Reperire duas lineas rectas potentia incommensurabiles, conficientes id, quod ex ipsarum quadratis componitur, mediũ, parallelogrammũ ex ipsis cõtentum, mediũ; quod præterea parallelogrammum sit incommensurabile composito ex quadratis ipsarum.

PRIN-

PRINCIPIVM SENARIORVM per compositionem, & Synthesin.

Theorema 25. Propositio 37.

Si duæ rationales potentia tantùm commensurabiles componantur; tota irrationalis erit. Vocetur autem Binomium, vel ex binis nominibus.

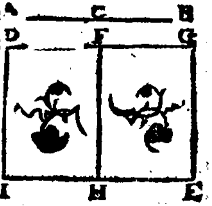


Theorema 26. Propositio 38.

Si duæ mediæ potentia tantùm commensurabiles, rationale continentes, componantur; tota linea est irrationalis, vocetur autem ex binis medijs prima.

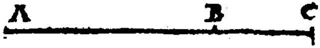
Theorema 27. Propositio 39.

Si duæ mediæ potentia tantùm commensurabiles medium continentes cōponantur; tota linea est irrationalis: vocetur autem ex binis medijs secunda.



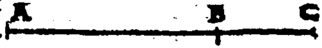
Theorema 28. Propositio 40.

Si duæ rectæ lineæ potentia incommensurabiles componantur; conficientes compositum ex quadratis ipsarum, rationale parallelollogrammum verò ex ipsis contentum, medium. tota linea recta est irrationalis. Vocetur autem linea maior.



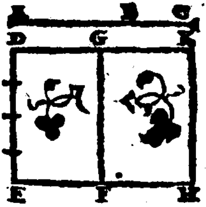
Theorema 29. Propositio 41.

Si duæ rectæ lineæ potentia incommensurabiles componantur, conficientes compositum ex ipsarum quadratis, medium: id verò, quod fit ex ipsis, rationale; tota recta linea & irrationalis erit. Vocetur autem potens rationale & medium.



Theorema 30. Propositio 42.

Si duæ rectæ lineæ potentia incommensurabiles componantur, conficientes compositum ex ipsarum quadratis medium; & quod continetur ex ipsis, medium; & præterea incommensurabile composito ex quadratis ipsarum: tota recta linea est irrationalis. Vocetur autem bina media potens.



Theorema 31 Propositio 43.

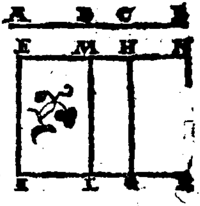
Quæ linea ex binis nominibus vocata, in vnico tantum puncto diuiditur in sua nomina.

Theo-

Theorema 32. Propositio 44.

Quæ ex binis medijs prima, in vnica tantum puncto diuiditur in sua nomina.

Theorema 33. Propositio 45.



Quæ ex binis medijs secunda, in vnico tantum puncto diuiditur in sua nomina.

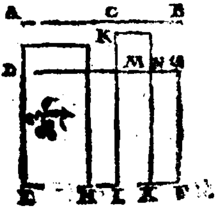
Theorema 34. Propositio 46.

Linea maior in vnico tantum puncto diuiditur in sua nomina.

Theorema 35. Propositio 47.

Linea potens rationale & medium, in vnico tantum puncto, diuiditur in sua nomina.

Theorema 36. Propositio 48.



Linea potens duo media in vnico tantum puncto diuiditur in sua nomina.

DEFINITIONES secundæ, nempe binorum nominum.

Proposita linea rationali; & linea ex binis nominibus vocata, diuisa, in sua nomina, cuius maius nomen, id est, maior portio; possit plusquam minus nomen; quadrato lineæ sibi, maiori inquam nomini, commensurabilis longitudine.

1.

Si quidem maius nomen fuerit commensurabile longitudine propositæ lineæ rationali; Vocetur tota linea composita ex binis nominibus, prima.

2.

Si verò minus nomen, id est, minor portio, fuerit commensurabile longitudine propositæ lineæ rationali; Vocetur tota linea ex binis nominibus secunda.

3.

Si verò neutrum: ipsorum nominum fuerit commensurabile longitudine propositæ lineæ rationali, Vocetur tota ex binis nominibus tertia.

Rursus si maius nomen possit plusquam minus nomen, quadrato lineæ sibi incommensurabiles longitudine.

4.

Si quidem maius nomen sit commensurabile

bile longitudine propositæ lineæ rationali; Vocetur tota linea ex binis nominibus quarta.

5.

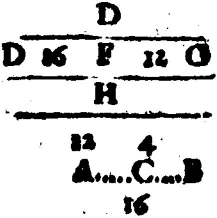
Si verò minus nomen fuerit commensurabile longitudine lineę rationali; Vocetur tota ex binis nominibus quinta.

6.

Si verò neutrum ipsorum nominum fuerit longitudine commensurabile: propositæ lineæ rationali; Vocetur tota ex binis nominibus sexta.

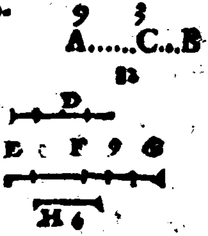
Problema 13. Propositio 49.

Reperire lineam ex binis nominibus primam.

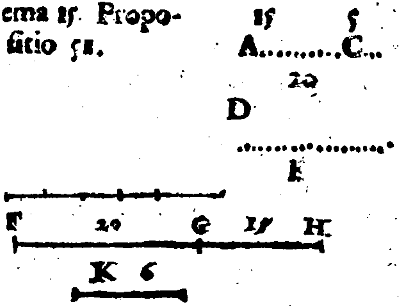


Problema 14. Propositio 50.

Reperire ex binis nominibus secundam.

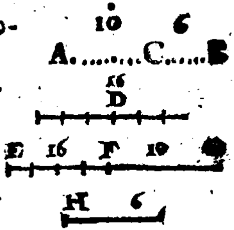


Problema 15. Propositio 51.



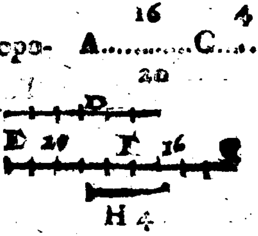
Reperire ex binis nominibus tertiam.

Problema 16. Propositio 52.



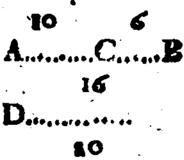
Reperire ex binis nominibus quartam.

Problema 17. Propositio 53.



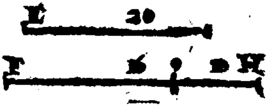
Reperire ex binis nominibus quintam.

Problema 18. Propositio 54.



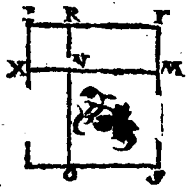
Repe-

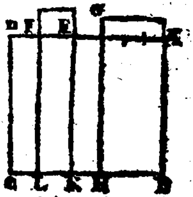
Reperire ex binis nominibus sextam.



Theorema 37. Propositio 55.

Si superficies contenta fuerit sub rationali, & ex binis nominibus prima recta linea,

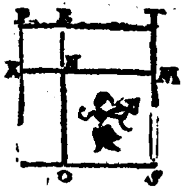


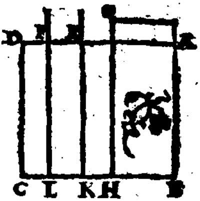


quæ illam superficiem potest, est irrationalis; quæ ex binis nominibus vocatur.

Theorema 38. Propositio 56.

Si superficies contenta fuerit sub linea rationali, & ex binis nominibus secunda; Recta linea po-

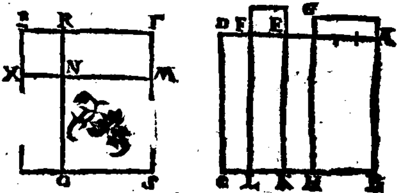




tens illam superficiem, est irrationalis; quæ ex binis medijs prima vocatur.

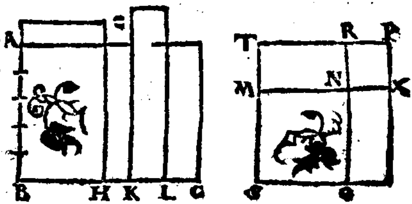
Theorema 39. Propositio 57.

Si superficies contineatur sub rationali, & ex binis nominibus tertia; recta linea qua illã superficiem potest, est irrationalis; quæ ex binis medijs dicitur tertia.



Theorema 40. Propositio 58.

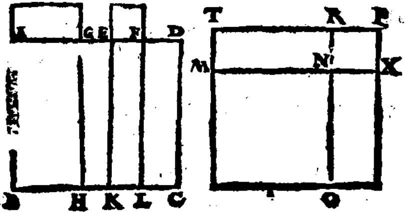
Si superficies contineatur sub rationali, & ex binis nominibus quarta; recta linea potens, superficiem illam, est irrationalis; quæ dicitur maior.



Theorema 41. Propositio 59.

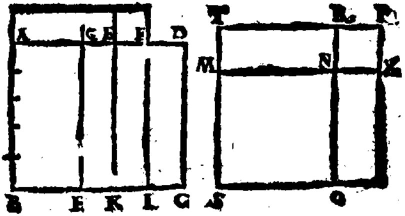
Si superficies cõtineatur sub rationali, & ex binis

binis nominibus quarta; recta linea, quæ illam superficiem potest, est irrationalis; quæ dicitur potens rationale, & medium.



Theorema 42. Propositio 60.

Si superficies contineatur sub rationali, & ex binis nominibus sexta; Recta linea, quæ illam superficiem potest, est irrationalis; quæ dicitur potens, bina media.

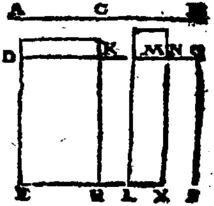


Theorema 43. Propositio 61.

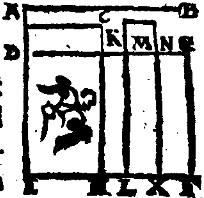
Quadratum eius lineæ, quæ est ex binis no-

 mini-

minibus, ad lineam rationalem applicatum, facit latitudinem ex binis nominibus primam.

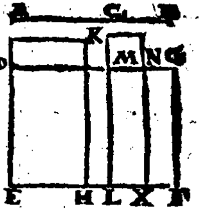


Theorema 44. Propo-
positio 62.

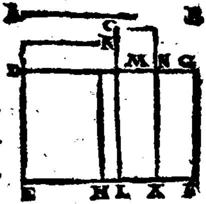


Quadratum, eius quæ est ex binis medijs prima, ad lineam rationalem applicatum, facit latitudinem ex binis nominibus secundam.

Theorema 45. Propo-
sitio 63.



Quadratum eius, quæ est ex binis medijs secunda, ad lineam rationalem applicatum, facit latitudiné ex binis nominibus tertiam.

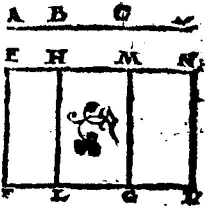


Theorema 46. Propo-
sitio 64.

Quadratum lineæ maioris secundum lineam rationalem applicatum, facit latitudinem ex binis nominibus quartam.

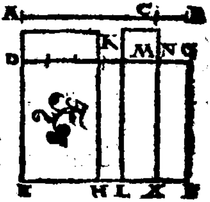
Theo-

Theorema 47. Propositio 65.



Quadratum lineæ potentis rationale, & medium secundum rationalem applicatum, facit latitudinem ex binis nominibus quintam.

Theorema 48. Propositio 66.



Quadratum lineæ potentis duo media, secundum rationalem applicatum, facit latitudinem ex binis nominibus sextam.

Theorema 49. Propositio 67.

Linea longitudine commensurabilis, ei lineæ quę est ex binis nominibus; & ipsa ex binis nominibus est, atque in ordine eadem.

Theorema 50. Propositio 68.

Linea longitudine commensurabilis alteri lineæ quæ est ex binis medijs; & ipsa ex binis medijs est, atque in ordine eadem.

Theorema 51. Propositio 69.

Linea commensurabilis lineæ maiori, & ipsa maior est.

Theorema 52. Propositio 70.

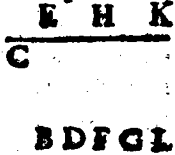
Linea commensurabilis lineæ potenti rationale & medium, est & ipsa linea potens rationale & medium.

Theorema 53. Propositio 71.

Linea commensurabilis lineæ potenti duo media, est & ipsa linea potens duo media.

Theorema 54. Propositio 72.

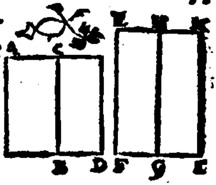
Si duæ superficies, rationalis, & media simul componantur, linea quæ totam superficiem compositam potest, est vna ex quatuor irrationalibus; vel ea, quæ dicitur ex binis nominibus, vel ea, quæ ex binis medijs prima, vel linea maior, vel linea potens, rationale & medium.



Theorema 55. Propositio 73.

Si duæ superficies mediæ inter se incommen-
sura-

surabiles simul componantur; fiunt reliquę duę lineæ irrationales; & ex binis medijs secunda, vt bina media potens.



SCHOLIVM EX THEONE, ZAMberto, Campano, & P. Christ. Clauio.

Ex his omnibus facilè colligitur, quod linea ea, quæ est ex binis nominibus, & cætera ipsam subsequentes, lineæ irrationales, neque sunt eadem cum linea media, neque ipsæ inter se sunt eadem.

Nam quadratum lineæ mediæ, ad lineam rationalem comparatum & applicatum, efficit alterum latus, lineam rationalem, seu latitudinem rationalem, ipsi lineæ rationali (hoc est, lineæ, ad quam applicatur) longitudine incommensurabilem: per propos. 23. libri decimi.

Quadratum verò eius lineæ, quæ est ex binis nominibus, ad rationalem, applicatum, efficit alterū latus, & lineam, seu latitudinem ex binis nominibus primam: per 61.

Quadratum verò eius, quæ est ex binis medijs prima ad Rationalem applicatum, latitudinem efficit ex binis nominibus secundam: per 26.

Quadratū verò eius, quæ est ex binis medijs secunda, ad rationalem applicatum, latitudinem efficit

ficit ex binis nominibus tertiam: per 63.

Quadratum lineæ maioris, ad rationalem applicatum, latitudinem efficit ex binis nominibus quartam: per 64.

Quadratum vero eius, quæ rationale, & medium potest, ad rationalem applicatum; efficit alterum latus, seu latitudinem ex binis nominibus quintam: per 65.

Quadratum denique lineæ eius, quæ bina media potest, ad rationalem applicatum, efficit alterum latus, seu latitudinem ex binis nominibus sextam: per 66.

Cùm igitur hæ latitudines (quæ à nonnullis latera dicuntur) differant, & à latitudine media & inter se, à latitudine quidem media, quod hæc rationalis sit, illæ verò irrationales, inter se autem, quod in ordine non sint eadem cum ijs ex binis nominibus: manifestum est omnes ipsas irrationales lineas, de quibus hactenus dictum est, inter se differentes esse.

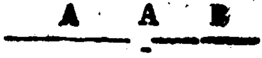
PRINCIPIVM SENARIORVM per detractionem, & aphæresin.

Theorema 56. Propositio 74.

Si de linea rationali detrahatur rationalis, potentia tantum commensurabilis ipsi toti: Residua est irrationalis: Vocetur autem Residuum, hoc est, apotome.

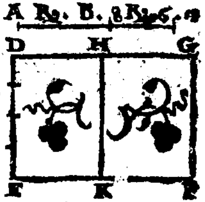
Theorema 57. Propositio 75.

Si de linea media detrahatur media, potentia tantum commensurabilis toti lineæ; quæ verò detracta est, cum tota contineat superficiem rationalem; Residua est irrationalis. Vocetur autem Residuū medium primum: hoc est, mediæ apotome prima.



Theorema 58. Propositio 76.

Si de linea media detrahatur media; potentia tantum commensurabilis toti; quæ verò detracta est, cum tota contineat superficiem mediam: Reliqua est irrationalis. Vocetur autem Residuum medium secundum, hoc est, mediæ apotome secunda.



Theorema 59. Propositio 77.

Si de linea recta detrahatur recta, potentia incommensurabilis toti; compositum autem ex quadratis totius lineæ, & lineæ detractæ, sit rationale; parallelogrammum verò ex rò ex eisdem contentum, sit medium: Reliqua linea erit irrationalis. Vocetur autem linea minor.

A C B

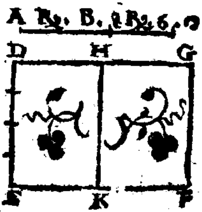
Theorema 60. Propositio 78.

Si de linea recta detrahatur recta, potentia incommensurabilis toti lineæ; compositum autem ex quadratis totius, & lineæ detractæ sit medium; parallelogrammum verò bis ex eisdem contentum, sit rationale: Reliqua linea est irrationalis. Vocetur autem linea faciens cum superficie rationali totam superficiem mediam.

A C B

Theorema 61. Propositio 79.

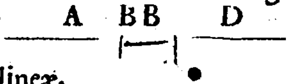
Si de linea recta detrahatur recta potentia incommensurabilis toti lineæ: compositum autem ex quadratis totius, & lineæ detractæ, sit medium: Parallelogrammum verò bis ex ijsdem sit etiam medium: præterea sint quadrata ipsarum incommensurabilia parallelogrammo bis ex ijsdem contento, Reliqua linea est irrationalis. Vocetur autem linea faciens cum superficie media totam superficiem mediam.



Theo-

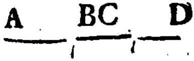
Theorema 62. Propositio 80.

Residuo vnica tantum linea recta coniungitur rationalis, potentia tantum commensurabilis toti lineæ.



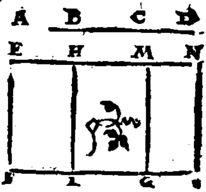
Theorema 63. Propositio 81.

Residuo medio primo vnica tantum linea coniungitur media, potentia tantum commensurabilis toti, ipsa cum tota rationale continens.



Theorema 64. Propositio 82.

Residuo medio secundo vnica tantum coniungitur recta linea media, potentia tantum commensurabilis toti, ipsa cum tota medium continens.



Theorema 65. Propositio 83.

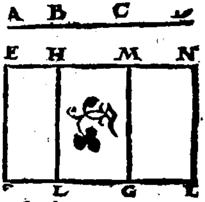
Lineæ minori vnica tantum recta linea coniungitur potentia incommensurabilis toti, faciens cum tota compositum ex quadratis ipsarum rationale: id vero parallelogrammum, quod bis ex ipsis fit, medium.

Theorema 66. Propositio 84.

Lineæ facienti cum superficie rationali totam superficiem mediam, vnica tantum cõiungitur linea recta, potentia incommensurabilis toti; faciens autem cum tota compositum ex quadratis ipsarum, medium; Id verò, quod fit bis ex ipsis, rationale.

Theorema 67. Propositio 85.

Lineæ cum media superficie facienti totam superficiẽ mediam, vnica tantum coniungitur linea, potentia toti incommensurabilis, faciens cum tota compositum ex quadratis ipsarum, medium, id verò, quod bis ex ipsis etiam medium: & præterea faciens compositum ex quadratis ipsarum incommensurabile ei, quod fit bis ex ipsis.



DE-

DEFINITIONES.

TERTIAE, NEMPE APOTOMArum, seu residuorum.

Proposita linea rationali, & Residuo, si tota nempe composita ex ipso Residuo, & linea illi coniuncta, seu congruente, plus possit, quam coniuncta, quadrato rectæ lineæ sibi longitudine commensurabilis.

1.

Si quidem tota lineæ propositæ rationali sit longitudine commensurabilis; Vocetur Residuum primum, seu Apotome prima.

2.

Si verò coniuncta fuerit longitudine commensurabilis propositæ rationali; ipsa autem tota plus possit, quam coniuncta, quadrato lineæ, sibi longitudine commensurabilis; Vocetur Residuum secundum, seu Apotome secunda.

3.

Si verò neutra linearum fuerit longitudine commensurabilis propositæ rationali; possit autem ipsa tota plusquam cõiuncta, quadrato lineæ sibi longitudine commensurabilis; Vocetur Residuum tertium, seu Apotome tertia.

Rursus si tota possit plus, quam coniuncta seu congruens, quadrato rectæ lineæ sibi longitudine incommensurabilis.

4.

Et quidem si tota fuerit longitudine commensurabilis ipsi rationali; Vocetur Residuum quartum, seu Apotome quarta.

5.

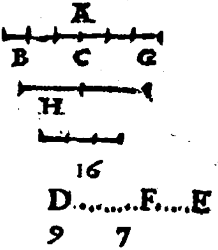
Si verò coniuncta fuerit longitudine commensurabilis ipsi rationali; & tota plus possit, quam coniuncta quadrato lineę sibi longitudine incommensurabilis; Vocetur Residuum quintum, seu Apotome quinta.

6.

Si denique neutra linearum fuerit commensurabilis longitudine ipsi rationali; fueritque tota potentior, quam coniuncta; quadrato lineæ sibi longitudine incommensurabilis; Vocetur Residuum sextum, seu Apotome sexta.

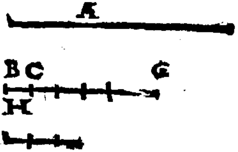
Problema 19. Propositio 86.

Reperire primum Residuum, seu Apotomen.



Problema 20. Propositio 87.

Reperire secundum Residuum.



Pro-

D........F....E

27 9

E.........

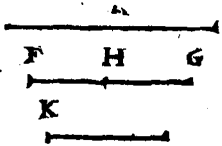
12

B.........E.....C

9 7

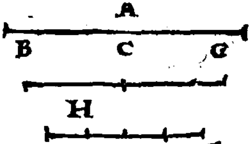
Problema 21. Propositio 88.

Reperire tertium Residuum.



Problema 22. Propositio 89.

Reperire quartum Residuum.

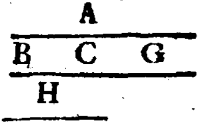


D..........F....E

16 4

Problema 23. Propositio 90.

Reperire quintum Residuum.



D..........F.....E

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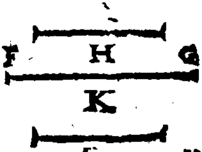
Problema 24. Propositio 91.

Reperire sextum Residuum.

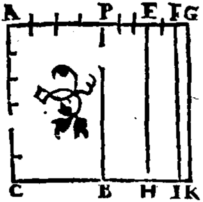


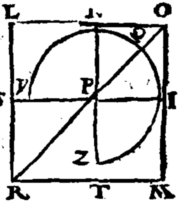
E..........13

B.....D....C

Theorema 68. Propoſitio 92.

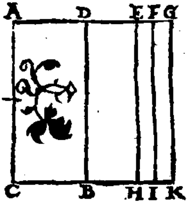
Si ſuperficies contineatur ſub linea rationali, & reſiduo primo; recta linea, quę illam ſuperficiem poteſt, eſt reſiduum.

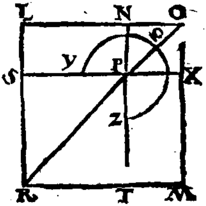




Theorema 69. Propoſitio 93.

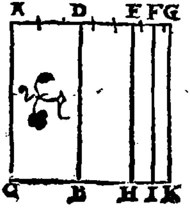
Si ſuperficies contineatur ſub linea rationali, & reſiduo ſecundo; recta linea, quæ illam ſuperficiem poteſt, eſt reſiduum mediale primum, ſeu mediæ Apotome prima.

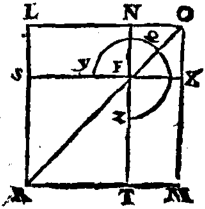




Theorema 70. Propoſitio 94.

Si ſuperficies cōtineatur ſub linea rationali, &

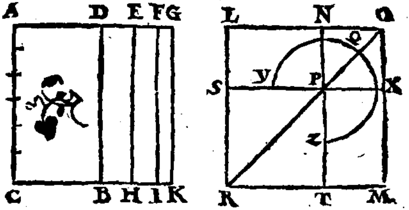




residuo tertio; recta linea, quæ illam superficiem potest, est residuum mediale secundum, seu mediæ, est Apotome secunda.

Theorema 71. Propositio 95.

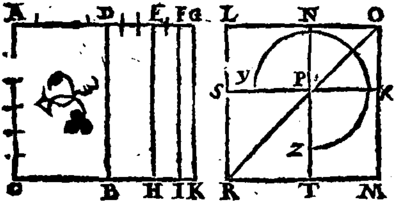
Si superficies contineatur sub linea rationali, & r[illegible] duo quarto; recta linea, quæ



illam superficiem potest, est linea minor.

Theorema 72. Propositio 96.

Si superficies contineatur sub linea rationali, & residuo quinto; recta linea, quæ illam superficiem potest, est ea, quæ dicitur cum



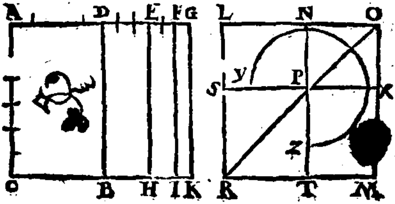
rationali superficie faciens totam medialem.

Theorema 73. Propositio 97.

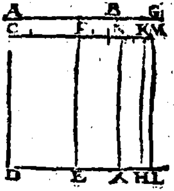
Si superficies cõtineatur sub linea rationali,

 & Re-

& Residuo sexto; recta linea, quæ illam superficiē potest, est ea, quæ dicitur faciens cum mediali superficie totam medialem.

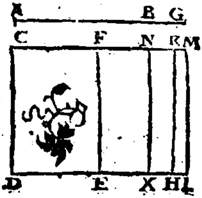


Theorema 74. Propositio 98.



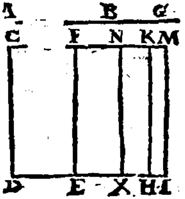
Quadratum residui ad lineam rationalem applicatum, facit alterum latus residuum primum.

Theorema 75. Propositio 99.



Quadratum residui medialis primi ad rationalem applicatum, facit alterum latus, residuum secundum.

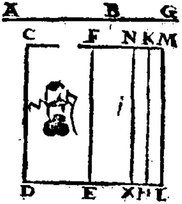
Theorema 76. Propositio 100.



Quadratum residui medialis secundi ad rationalem applicatum, facit alterum latus residuũ tertium.

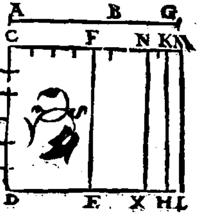
Theo-

Theorema 77. Propoſitio 101.



Quadratum lineæ minoris ad rationalem applicatum, facit alterum latus reſiduum quartum.

Theorema 78, Propoſitio 102.

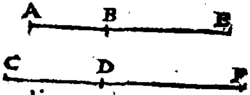


Quadratum lineæ cum rationali ſuperficie facientis totam medialem, ad rationalem applicatum, facit alterum latus reſiduum quintum.

Theorema 79. Propoſitio 103.

Quadratum lineæ cum mediali ſuperficie facientis totam medialem, ad rationalem applicatum, facit alterum latus reſiduum ſextum.

Theorema 80. Propoſitio 104.

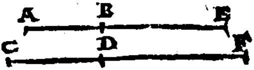


Recta linea reſiduo commenſurabilis longitudine; eſt & ipſa reſiduum, ſeu in ordine eadem.

Theorema 81. Propoſitio 105.

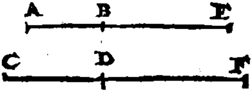
Recta linea commenſurabilis reſiduo mediali,

diali, eſt & ipſa reſiduum mediale, & eiuſdem ordinis; ſeu in ordine eadem.



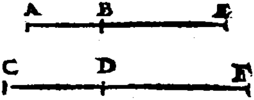
Theorema 82. Propoſitio 106.

Recta linea cómenſurabilis lineæ minori: eſt & ipſa linea minor.



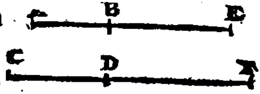
Theorema 83. Propoſitio 107.

Recta linea commenſurabilis lineæ cum rationali ſuperficie facienti totam medialem; eſt & ipſa linea cum rationali ſuperficie faciens totam medialem.



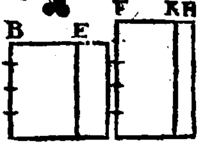
Theorema 84. Propoſitio 108.

Recta linea commenſurabilis lineæ cum mediali ſuperficie facienti totam medialem; eſt & ipſa cum mediali ſuperficie faciens totam medialem.



Theorema 85. Propoſitio 109.

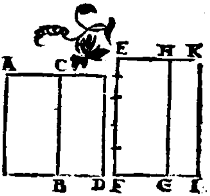
Si de ſuperficie rationali detrahatur ſuperficies medialis; recta linea, quæ reliquam ſuperficiem poteſt, eſt alterutra ex duabus irrationalibus, aut reſiduum, aut linea minor.



Theo-

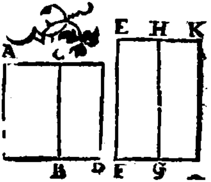
Theorema 86. Propoſitio 110.

Si de ſuperficie mediali detrahatur ſuperficies rationalis; aliæ duæ irrationales fiunt, aut reſiduum mediale primum, aut cum rationali ſuperficies totam medialem.



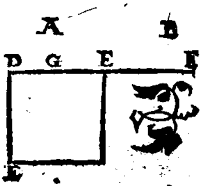
Theorema 87. Propoſitio 111.

Si de ſuperficie mediali detrahatur ſuperficies medialis, quæ ſit incommenſurabilis toti; reliquæ duæ fiunt irrationales, aut reſiduum mediale ſecundum, aut cum mediali ſuperficie faciens totam medialem.



Theorema 88 Propoſitio 112.

Linea, quæ reſiduum dicitur, non eſt eadem cum ea, quæ dicitur Binomium.



SCHO-

SCHOLIVM EX THEONE, ZAMberto, Campano, & P. Christ. Clauio.

Ex his demonstratis facilè intelligitur, quod recta linea, quæ residuum dicitur, & cæteræ quinque eam consequentes irrationales, neque lineæ mediali, neque sibi ipsæ inter se sunt eadem.

Nam quadratum lineæ medialis secundum rationalem applicatum, facit alterum latus, rationalem lineam, longitudine incommensurabilem ei, seu ad quam applicatur, per propos. 23. libri decimi.

Quadratum verò residui secundum rationalem applicatum, facit alterum latus, residuum primam: per 98.

Quadratum verò residui medialis primi secundum rationalem applicatum, facit alterum latus, residuum secundum, per 99.

Quadratum verò residui medialis secundi, facit alterum latus, residuum tertium, per 100.

Quadratum verò lineæ minoris, facit alterum latus residuum quartum, per 101.

Quadratum verò lineæ cum rationali superficie facientis totam medialem, facit alterum latus residuum quintum, per 102.

Quadratum verò lineæ cum mediali superficie facientis totam medialem, secundum rationalem applicatum, facit alterum latus residuum sextum, per 103.

Cùm

Cùm igitur dicta latera, quæ sunt latitudines cuiusq; parallelogrammi vnicuiq; quadrato æqualis & ad rationalem, applicati, differant & à primo latere, & ipsa inter se, (nam à primo differunt: quoniam sunt residua non eiusdem ordinis) constat ipsas quoq; lineas irrationales inter se differentes esse. Et quoniam demonstratum est, residuum non esse idem quod Binomium: quadrata autem residui, & quinq; linearum irrationalium illud consequentium, ad rationalem applicata, faciunt altera latera ex residuis eiusdem ordinis, cuius sunt & residua, quorum quadrata applicantur rationali, similiter & quadrata Binomij, & quinq; linearum irrationalium illud consequentium, ad rationalem applicata, faciunt altera latera ex Binomijs eiusdem ordinis, cuius sunt & Binomia, quorum quadrata applicantur rationali. Ergo lineæ irrationales, quæ consequuntur Binomium, & quæ consequuntur residuum, sunt inter se differentes. Quàre dictæ lineæ omnes irrationales sunt numero, 13. subsequentes.

1. *Media linea; quæ vulgò medialis appellatur: propos. 22.*
2. *Linea ex binis nominibus (vulgò Binomium:) cuius sex sunt species inuentæ: propos. 37.*
3. *Ex binis medijs prima; vulgò Bimediale primũ: propos. 38.*
4. *Ex binis medijs secunda; vulgò Bimediale secundum: propos. 39.*
5. *Maior: propos. 40.*

6. Li-

6. Linea rationale, ac medium potens: propos. 41.

7. Bina media potens: propos. 42.

8. Apotome (vulgò residuum:) cuius etiam species sex sunt reperta: propos. 74.

9. Media Apotome prima; vulgò residuum: propos. 75.

10. Media Apotome secunda, vulgò residuum mediale secundum: propos 76.

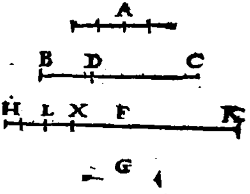
11. Minor: propos. 77.

12. Linea cum rationali medium totum efficiens; vulgò linea cum rationali superficie totam medialem faciens: propos. 78.

13 Cum medio medium totum efficiens; vulgo linea cum mediali superficie totam medialem faciens: propos. 79.

Theorema 89. Propositio 113.

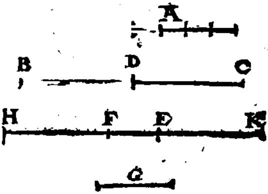
Quadratum lineæ rationalis ad Binomium applicatum, facit alterum latus residuum, cuius nomina sunt commensurabilia Binomij nominibus, & in eadem proportione, præterea id, quod sit residuum, eundem ordinem retinet, quem Binomium.



Theorema 90. Propositio 114.

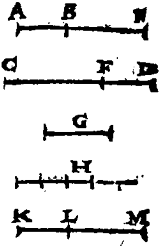
Quadratum lineæ rationalis ad residuum applicatum, facit alterum latus Binomium, cuius

cuius nomina sunt commensurabilia, nominibus residui, & in eadem proportione; præterea id qùod fit Binomium, est eiusdem ordinis, cuius & residuum.



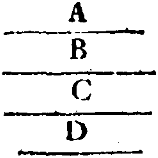
Theorema 91. Propositio 115.

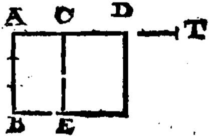
Si parallelogrammum contineatur ex residuo, & Binomio, cuius nomina sunt commensurabilia nominibus residui, & in eadem proportione; recta linea, quæ illam superficiem potest, est rationalis.



Theorema 92. Propositio 116.

Ex linea media nascuntur lineæ irrationales innumerabiles, quarum nulla vlli antedictarum eadem sit:



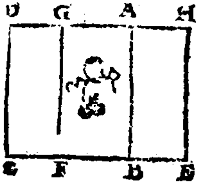


Theo-

Theorema 103. Propositio 117.

E. H. E
E ..

Propositum sit nobis demonstrare, in figuris quadratis diametrum esse lõgitudine incommensurabilem ipsi lateri.



FINIS ELEMENTI X.

EVCLIDIS ELEMENTVM VNDECIMVM.

ET SOLIDORVM primum.

DEFINITIONES

1.

Solidum, est quod longitudinem, latitudinem, & crassitudinem habet.

2.

Solidi autem extremum est superficies.

3.

Linea recta est ad planum recta, cùm ad rectas omnes lineas, à quibus illa tangitur, quæque in proposito sunt plano, rectos angulos efficit.

4.

Planum ad planum rectũ est, cùm rectæ lineæ, quæ communi planorum sectioni ad rectos angulos in vno planorum ducuntur, alteri plano ad rectos sunt angulos.

5.

Rectæ lineæ ad planum inclinatio est angulus acutus, ipsa insistente linea, & adiuncta altera comprehensus, cùm à sublimi rectæ illius lineæ termino deducta fuerit perpendicularis in ipso plano fecerit ad propositæ illius lineæ extremum, quod in eodem est plano, altera recta linea fuerit adiuncta.

6.

Plani ad planum inclinatio, est angulus acutus rectis lineis contentus, quæ in vtroque planorum ad idem cōmunis sectionis punctum ductæ, rectos ipsi sectioni angulos efficiunt.

7.

Planum similiter inclinatum esse ad planum, atque alterum ad alterum dicitur, cū dicti inclinationum anguli inter se sunt æquales.

8.

Parallela plana sunt, quæ inter se non incidunt, nec concurrunt.

9.

Similes figuræ solidæ sunt, quæ similibus planis, multitudine æqualibus continentur.

10.

Aequales, & similes figuræ solidæ sunt, quæ similibus planis, multitudine & magnitudine æqualibus continentur.

11.

Solidus angulus est plurimum, quàm duarum linearum, quæ se mutuò contingant, nec in eadem sint superficie, ad omnes lineas inclinatio.

Aliter.

Solidus angulus est, qui pluribus, quā duobus planis angulis, in eodem plano non consistentibus, sed ad vnum punctum collectis continetur.

12. Py-

Pyra[illegible] quæ planis [illegible] netur, [illegible] plano ad [illegible] punctum [illegible] lecta [illegible]

13.

Prima figura est solida, quæ planis continetur, quorum aduersa duo sunt, & æqualia, similia, & parallela; alia verò parallelogramma.

14.

Sphæra est figura, quę conuerso circum quiescentem diametrum semicirculo continetur, cùm in eundem rursus locum restitutus fuerit, vnde moueri cœperat.

[illegible]

Sphæ[illegible] con[illegible] rum[illegible] res omnes [illegible] æquales.

[illegible]

Axis autem sphæræ est, quiescens illa [illegible]

[illegible]

Ce[illegible] mic[illegible]

Diam[illegible] linea per[illegible] perfin[illegible]

18.

Conus est figura, quæ sub côuerso circumquiescens alterum latus eorum, quæ rectum angulum continent, orthogonio triangulo continetur, cum in eundem rursus locum illud triangulum restitutum fuerit, vnde moueri cœperat.

Atque si quiescens recta linea æqualis sit alteri, quæ circum rectum angulum cõuertitur, orthogonius erit Conus: sin minor, amblygonius: si verò maior, oxygonius.

19.

Axis autem Coni est, quiescens illa recta linea, circum quam triangulum vertitur.

20.

Basis verò Coni est, circulus qui à circumducta linea recta describitur.

21.

Cylindrus est figura, quæ sub conuerso circumquiescens alterum latus eorum, quæ rectum angulum continent, parallelogrammo orthogonio compræhenditur, cum in eundem rursus locum restitutum fuerit illud parallelogrammum, vnde moueri cœperat.

22.

Axis autem Cylindri est quiescens illa recta linea, circum quam parallelogrammum vertitur.

23. Ba-

23

Baſes verò cylindri ſunt circuli, à duobus aduerſis lateribus, quæ circum aguntur, deſcripti.

24.

Similes coni & cylindri ſunt, quorum & axes, & baſium diametri proportionales ſunt.

25.

Cubus ſeu hexaëdrum eſt figura ſolida, quę ſub ſex quadratis ęqualibus continetur.

26.

Tetraëdrum eſt figura ſolida, quæ ſub triangulis quatuor æqualibus, & æquilateris continetur.

27.

Octaëdrum figura eſt ſolida, quæ ſub octo triangulis æqualibus, & æquilateris continetur.

28.

Dodecaëdrum figura eſt ſolida quę ſub duodecim pentagonis æqualibus, æquilateris, & æquiangulis continetur.

29

Eicoſaëdrum figura eſt ſolida, quæ ſub triangulis viginti æqualibus, & æquilateris continetur.

30.

Parallelepipedum eſt figura ſolida quæ ſub ſex figuris quadrilateris, quarum quæ ex aduerſo, parallelæ ſunt, continetur.

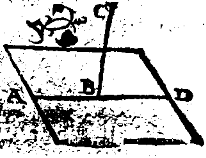
31.

Solida figura in solida dicitur inscribi, quādo omnes anguli figuræ inscriptæ constituuntur, vel in angulis, vel in lateribus, vel denique in planis figuræ, cui inscribitur.

32.

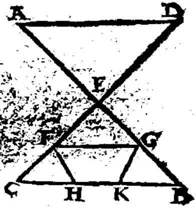
Solida figura solidæ figuræ vicissim circūscribi dicitur, quando vel anguli, vel latera, vel denique plana figuræ, circumscriptæ tangunt omnes angulos figuræ, circum quā describitur.

Theorema 1. Propositio 1.



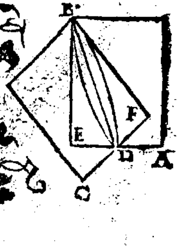
Quædam rectę lineę pars in subiecto quidem non est plano, quædam vero in sublimi.

Theorema 2. Propositio 2.



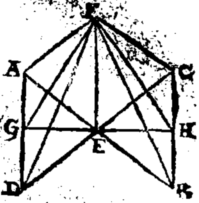
Si duæ rectæ lineæ se mutuo secent, in vno sunt plano: atque triangulum omne in vno est plano.

Theorema 3. Propositio 3.



Si duo plana se mutuò secent, communis eorum sectio est recta linea.

Theorema 4. Propoſitio 4.

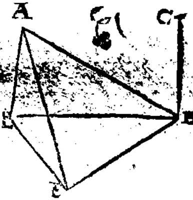


Si recta linea, rectis duabus lineis ſe mutuò ſecantibus, in communi ſectione ad rectos angulos inſiſtat: illa, ducto etiam per ipſas plano, ad angulos rectos erit.

Theorema 5. Propoſitio 5.

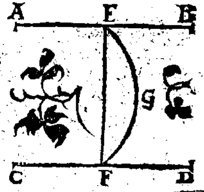
Si recta linea, rectis tribus lineis ſe mutuò tangentibus, in communi ſectione ad rectos angulos inſiſtat: illæ tres rectæ in vno ſunt plano.

Theorema 6. Propoſitio 6.



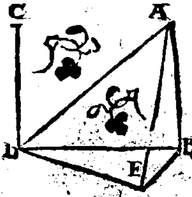
Si duæ rectæ lineæ eidem plano ad rectos ſint angulos: parallelæ erunt illæ rectæ lineæ.

Theorema 7. Propoſitio 7.



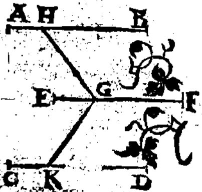
Si duæ ſint parallelæ rectæ lineæ, in quarum vtraq; ſumpta ſint quælibet puncta: illa linea, quæ ad hæc puncta adiungitur, in eodem eſt cum parallelis plano.

Theorema 8 Propoſitio 8.



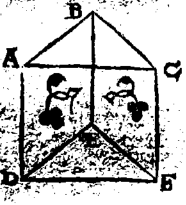
Si duæ ſint parallelæ rectæ lineæ, quarum altera ad rectos cuidam plano ſit angulos: & reliqua eidem plano ad rectos angulos erit.

Theorema 9 Propoſitio 9.



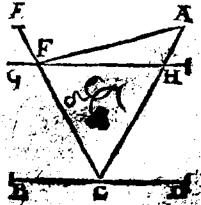
Quæ eidem rectæ lineæ ſunt parallelæ, ſed non in eodem cum illa plano: hæ quoque ſunt inter ſe parallelæ.

Theorema 10. Propoſitio 10.



Si duę rectæ lineæ ſe mutuo tangentes ad duas rectas ſe mutuò tangentes ſint parallelę, non autem in eodem plano, illæ angulos æquales comprehendent.

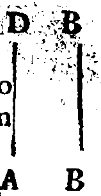
Problema 1. Propoſitio 11.



A dato puncto in ſublimi, ad ſubiectum planũ perpendicularem rectam lineam ducere.

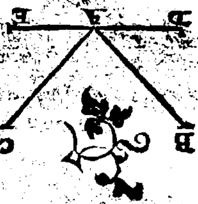
Pro-

Problema 2. Propoſitio 12.



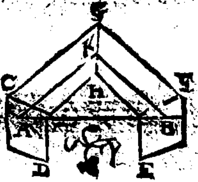
Dato plano, à puncto, quod in illo datum eſt, ad rectos angulos rectam lineam excitare.

Theorema 11. Propoſitio 13.



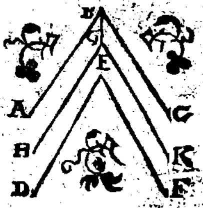
Dato plano à puncto quod in illo datum eſt, duæ rectæ lineæ ad rectos angulos non excitabuntur, ad eaſdem partes.

Theorema 12. Propoſitio 14.



Ad quæ plana, eadem recta linea recta eſt, illa ſunt parallela.

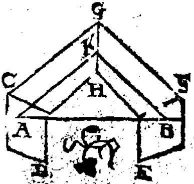
Theorema 13. Propoſitio 15.



Si duæ rectæ lineæ ſe mutuo tangentes, ad duas rectas ſe mutuò tangentes ſint parallelæ, non in eodem conſiſtentes plano: parallela ſunt, quæ per illas ducuntur, plana.

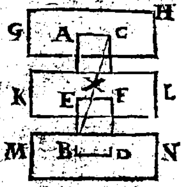
Theorema 14. Propoſitio 16.

Si duo plana parallela plano quopiam ſecentur; communes illorum ſectiones ſunt parallelæ.



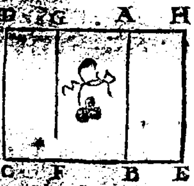
Theorema 15. Propoſitio 17.

Si duæ rectæ lineæ parallelis planis ſecentur; in eaſdem proportiones ſecabuntur.



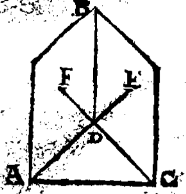
Theorema 16. Propoſitio 18.

Si recta linea plano cuipiam ad rectos ſit angulos; illa etiam omnia, quæ per ipſam plana, ad rectos eidem plano angulos erunt.



Theorema 17. Propoſitio 19.

Si duo plana ſe mutuò ſecantia, plano cuidam ad rectos ſint angulos; communis etiam illorum ſectio ad rectos eidem plano angulos erit.



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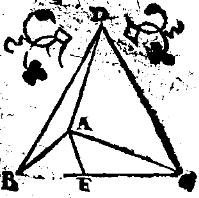
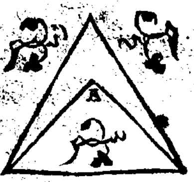
Theorema 18. Propositio 20.



Si angulus solidus sub planis tribus angulis contineatur: ex his duo quilibet, vtut assumpti, tertio sunt maiores.

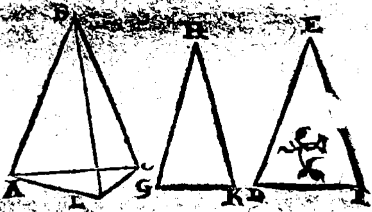
Theorema 19. Propositio 21.



Solidus omnis angulus sub minoribus quam rectis quatuor angulis planis, continetur.

Theorema 20. Propositio 22.

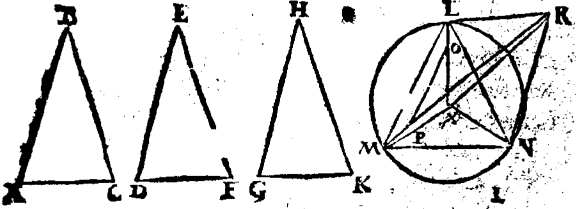
Si plani tres anguli æqualibus rectis contineantur lineis, quorum duo vt libet assumpti, tertio sint maiores; triangulum constitui potest, ex lineis æquales, illas rectas coniungentibus.

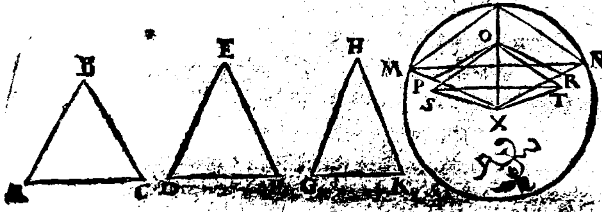


Problema 3. Propositio 23.

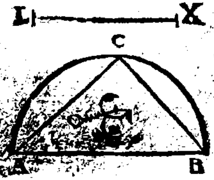
Ex planis tribus angulis, quorum duo, vt libet assumpti, tertio sint maiores, solidum angu-

angulum constituere. Oportet autem illos tres angulos rectis quatuor esse minores.

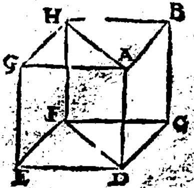




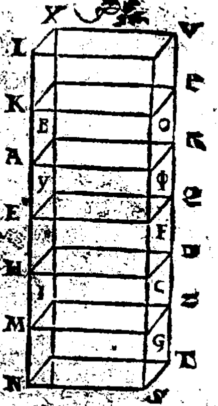
Theorema 21. Propositio 24.



Si solidum sub parallelis planis contineatur; aduersa illius plana, sunt parallelogramma, similia, & æqualia.

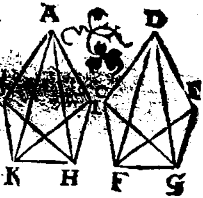


Theorema 22. Propositio 25.



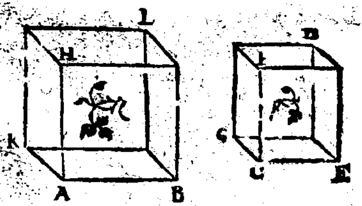
Si solidum parallelopipedum, sub parallelis planis contentum plano secetur, aduersis planis parallelo; erit quemadmodum basis ad basim, ita solidum ad solidum

Problema 4. Propositio 26.



Ad datam rectam lineã, eiusque punctum, angulum solidum constituere, solido angulo dato æqualem.

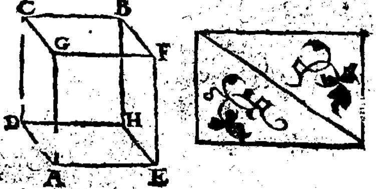
Problema 5. Propositio 27.



A data recta linea, dato solido parallelis planis comprehenso simile, & similiter positum solidum parallelis planis contentum describere.

Theorema 23. Propoſitio 28.

Si ſolidum parallelis planis comprehenſum ducto per aduerſorum planorum diagonios plano, ſectum ſit: illud ſolidum ab hoc plano bifariam ſecabitur.

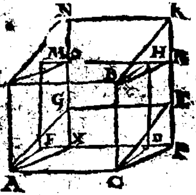


Theorema 24. Propoſitio 29.

Solida parallelapipeda, ſeu parallelis planis comprehenſa, quæ ſuper eandem baſim, & in eadem ſunt altitudine, quorum inſiſtentes lineæ in iiſdem collocantur rectis lineis: illa ſunt inter ſe æqualia.

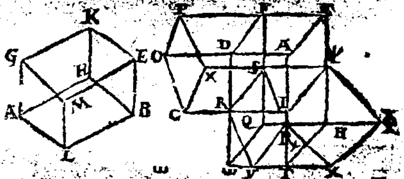
Theorema 25. Propositio 30.

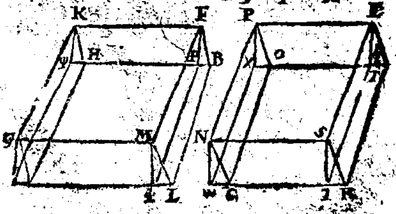
Solida parallelis planis circumscripta, quæ super eandem basim, & in eadem sunt altitudine, quorum insistentes lineæ non in ijsdem reperiuntur rectis lineis; illa sunt inter se æqualia.



Theorema 26. Propositio 31.

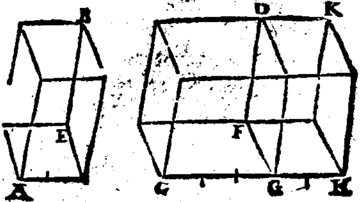
Solida parallelis planis, circumscripta, quæ super æqualibus basibus, & in eadem sunt altitudine: æqualia sunt inter se.





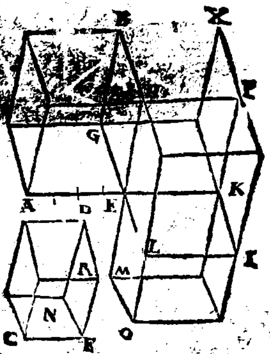
Theorema 27 Propoſitio 32.

Solida parallelis planis circumſcripta, quæ eiuſdem ſunt altitudinis; eam habent inter ſe proportioné, quam baſes.



Theorema 28 Propoſitio 33.

Similia ſolida parallelis planis circumſcripta habent inter ſe proportionem homologorum laterum triplicatam.



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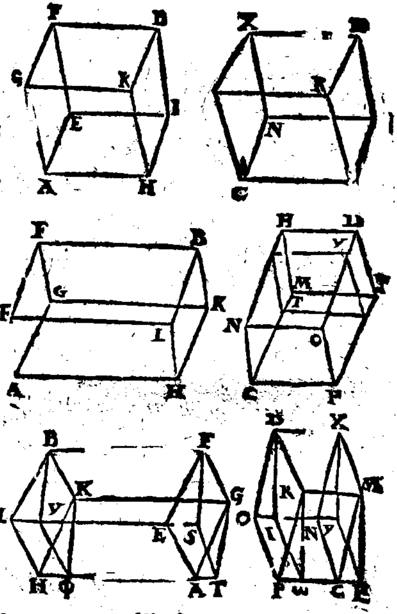
Theorema 29. Propositio 34.



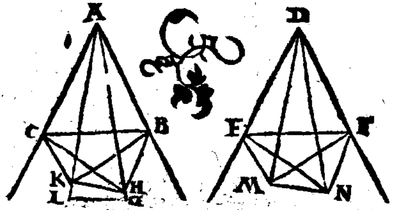
Aequalium solidorum parallelis planis contentorum bases, cũ altitudinibus reciprocãtur. Et solida parallelis planis contenta, quorum bases cum altitudinibus reciprocantur; illa sunt æqualia.

Theorema 30. Propositio 35.

Si duo plani sunt anguli æquales, quorum

N ver-

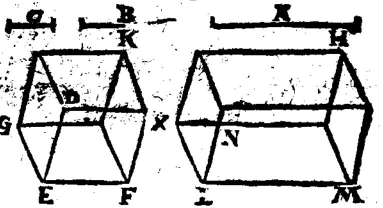
verticibus ſublimes rectæ lineæ inſiſtant, quæ cum lineis primò poſitis angulos contineant æquales, vtrunque vtrique: in ſublimibus autem lineis quælibet ſumpta ſint puncta, & ab his ad plana, in quibus conſiſtunt anguli primum poſiti, ductæ ſint perpendiculares; ab earum verò punctis, quæ in planis ſignata fuerint, ad angulos primum poſitos adiunctæ ſint rectæ lineæ, hę cum



ſublimibus æquales angulos comprehendent.

Theorema 31. Propoſitio 36.

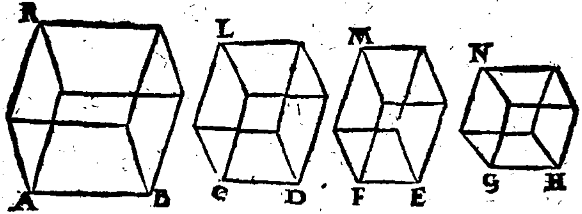
Si rectæ tres lineæ ſint proportionales; quod ex his tribus fit ſolidum parallelis planis contentum, æquale eſt deſcripto à media linea ſolido parallelis planis comprehenſo, quod æquilaterum quidem ſit, ſed antedicto æquiangulum.



Theo-

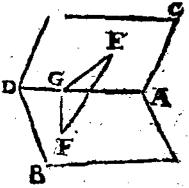
Theorema 32. Propoſitio 37.

Si rectæ quatuor lineæ ſint proportionales, illa quoque ſolida parallelis planis contenta, quæ ab ipſis lineis & ſimilia & ſimiliter deſcribuntur, proportionalia erunt. Et ſi ſolida parallelis planis comprehenſa, quæ & ſimilia & ſimiliter deſcribuntur, ſint proportionalia; illæ quoque rectæ lineæ proportionales erunt.



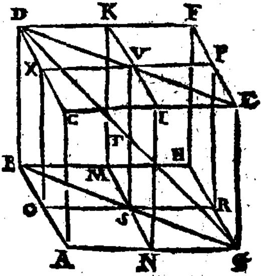
Theorema 33 Propoſitio 38.

Si planum ad planum rectum ſit; & à quodam puncto eorum, quæ in vno ſunt planorum, perpendicularis ad alterum planum ducta ſit: illa, quæ ducitur perpendicularis, in communem cadet ipſorum planorum ſectionem.



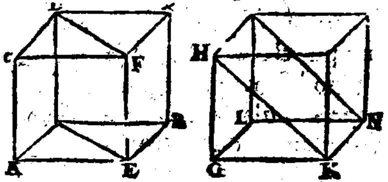
Theorema 34. Propoſitio 39.

Si in ſolido parallelis planis circumſcripto, aduerſorum planorum lateribus bifariam ſectis, educta ſint per ſectiones plana; communis illorum planorum ſectio, & ſolidi parallelis planis circumſcripti diameter, ſe mutuò bifariam ſecabunt.



Theorema 35. Propoſitio 40.

Si duo ſint æqualis altitudinis priſmata; quorum hoc quidem baſim habeat parallelogrammum: illud verò triangulum ſit autem parallelogrammũ trianguli duplum: illa priſmata erunt æqualia.

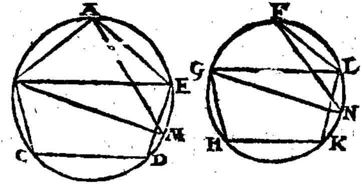


FINIS ELEMENTI XI.

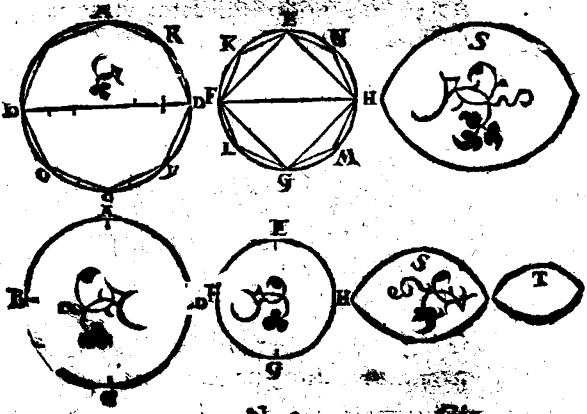
EVCLIDIS ELEMENTVM DVODECIMVM, ET SOLIDORVM secundum.

Theorema 1. Propoſitio 1.

Similia, quæ ſunt in circulis polygona, proportionem habent inter ſe, quã deſcripta à diametris quadrata.



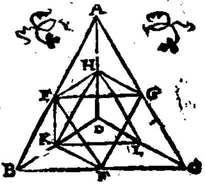
Theorema 2. Propoſitio 2.



Circuli eam inter ſe proportionem habent, quàm deſcripta à diametris quadrata.

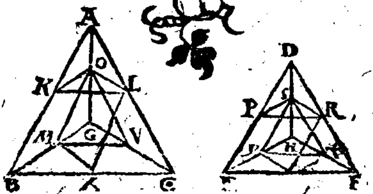
Theorema 3. Propoſitio 3.

Omnis pyramis trigonam habens baſim, in duas diuiditur pyramides nõ tantùm æquales & ſimiles inter ſe, ſed toti etiam pyramidi ſimiles, quarum trigonæ ſunt baſes; atq; in duo priſmata æqualia, quę duo priſmata dimidio pyramidis totius ſunt maiora.



Theorema 4. Propoſitio 4.

Si duę eiuſdem altitudinis pyramides trigonas habeant baſes; ſit autem illarum vtraque diuiſa & in duas pyramides inter ſe ęquales, totique ſimiles; & in duo priſmata æqualia; Ac eodem modo diuidatur vtraque pyramidum, quæ ex ſuperiore diuiſione natæ ſunt; idque perpetuo fiat: quemadmodum

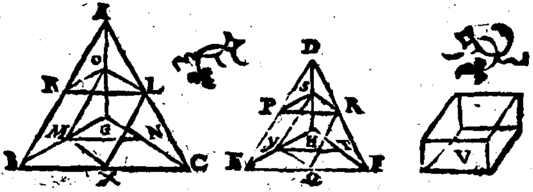


ſe habet vnius pyramidis baſis ad alterius pyramidis baſim; ita & omnia, quæ in vna pyra-

pyramide, priſmata ad omnia, quæ in altera pyramide priſmata multitudine æqualia.

Theorema 5. Propoſitio 5.

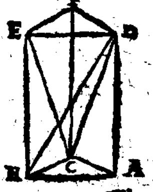
Pyramides eiuſdem altitudinis, quarum trigonæ ſunt baſes; eam inter ſe proportionem habent, quàm ipſæ baſes.



Theorema 6. Propoſitio 6.

Pyramides eiuſdem altitudinis, quarum polygonæ ſunt baſes, eam interſe proportionem habent quam ipſæ baſes.

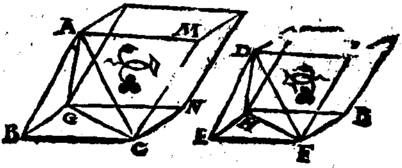
Theorema 7. Propoſitio 7.



Omne priſma trigonam habens baſim, diuiditur in tres pyramides inter ſe æquales, quarum trigonę ſunt baſes.

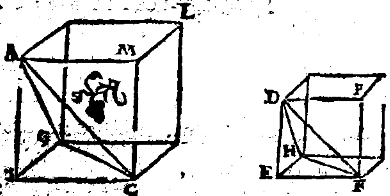
Theorema 8. Propoſitio 8.

Similes pyramides, quæ trigonas habent baſes; in triplicata ſunt homologorú laterú proportione.



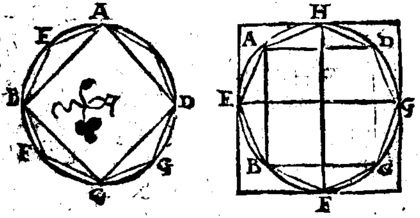
Theorema 9. Propoſitio 9.

Aequalium pyramidum, & trigonas baſes habentium, reciprocantur baſes cum altitudinibus. Et quarum pyramidum trigonas baſes habentium reciprocantur baſes cú altitudinibus; illę ſunt æquales.



Theorema 10. Propoſitio 10.

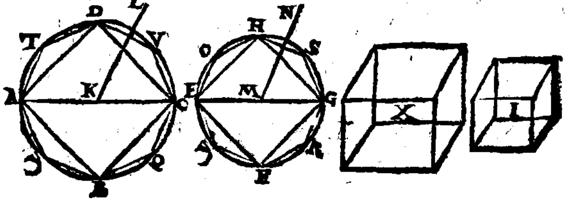
Omnis conus tertia pars eſt cylindri,



eandem cum ipso cono basim habentis, & altitudinem æqualem.

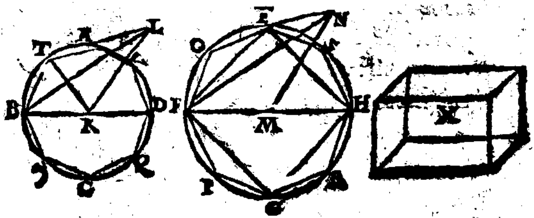
Theorema 11. Propositio 11.

Coni, & cylindri eiusdem altitudinis, eam inter se proportionem habent, quam bases.

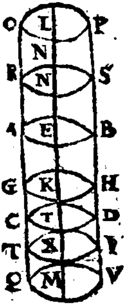


Theorema 12. Propositio 12.

Similes coni, & cylindri, triplicatam habent, inter se proportionem diametrorum, quæ sunt in basibus.



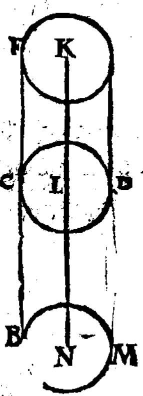
Theorema 13 Propositio 13.

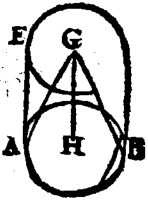


Si cylindrus plano sectus sit aduersis planis parallélo: Erit quémadmodum cylindrus ad cylindrum, ita axis ad axem.

Theorema 14. Propositio 14.

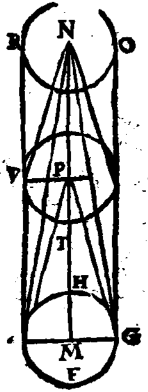
Coni, & cylindri, qui in æqualibus sunt basibus; eam habent inter se proportioné, quam altitudines.

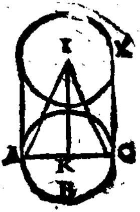




Theorema 15. Propositio 15.

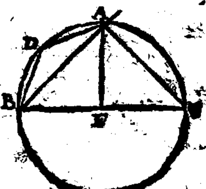
Aequalium conorum, & cylindrorum bases cum altitudinibus reciprocãtur. Et quorum conorum & cylindrorum bases cum altitudinibus reciprocantur; illi sunt aequales.





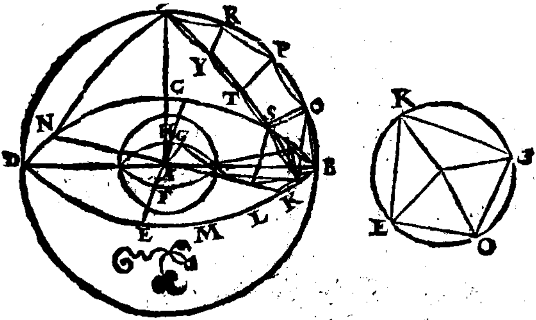
Problema 1. Propositio 16.

Duobus circulis circa idem centrum existentibus, in maiore circulo polygonum aequalium, pariumque laterum inscribere, quod minorem circulum non tangat.



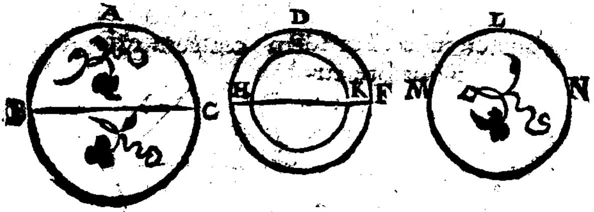
Problema 2, Propoſitio 17.

Duabus ſphæris circa idem centrum exſiſtentibus, in maiori ſphæra ſolidum polyedrum inſeribere, quod minoris ſphæræ ſuperficiem non tangat.



Theorema 16 Propoſitio 18.

Sphæræ inter ſe proportionem habent ſuarum diametrorum triplicatam.

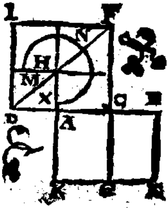


FINIS ELEMENTI XII.

EVCLIDIS ELEMENTVM DECIMVM TERTIVM, ET SOLIDORVM tertium.

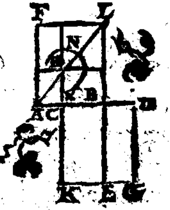
Theorema 1. Propositio 1.

Si recta linea per extremã & mediam proportionem secta sit; maius segmentũ, quod totius lineæ dimidium assumpserit, quintuplum potest eius, quòd à totius dimidia describitur, quadrati.



Theorema 2. Propositio 2.

Si recta linea sui ipsius segmenti quintuplum possit, & dupla segmenti huius linea per extremam & mediam proportionem secetur, maius segmentum reliqua pars est lineæ primùm positæ.



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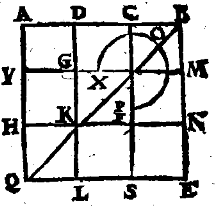
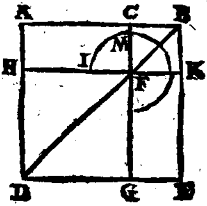
Theorema 3. Propoſitio 3.



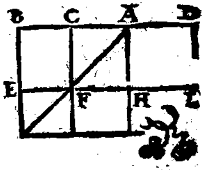
Si recta linea per extremam & mediam proportionem ſecta ſit; minus ſegmentum, quod maioris ſegmenti dimidium aſſumpſerit, quintuplum poteſt eius, quòd à maioris ſegmenti dimidio deſcribitur quadrati.

Theorema 4. Propoſitio 4.



Si recta linea per extremam & mediam proportionem ſecta ſit: quòd à tota, quodque à minore ſegmento, ſimul vtraque quadrata, tripla ſunt eius quòd à maiore ſegmento deſcribitur, quadrati.

Theorema 5. Propoſitio 5.



Si ad rectam lineam, quæ per extremam & mediam proportioné ſecatur, adiuncta ſit altera ſegmento maiori æqualss: tota hæc linea recta per extremam & mediam proportionem

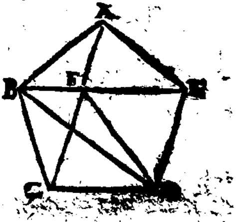
nem ſecatur; eſtque maius ſegmentum recta linea primùm poſita.

Theorema 6. Propoſitio 6.

Si recta linea ῥητὴ, ſiue rationalis, per extremam & mediam proportionem ſecta ſit; vtrunque ſegmentorum ἄλογος, ſiue irrationalis, eſt linea, quæ dicitur Reſiduum, ſeu Apotome.

Theorema 7. Propoſitio 7.

Si pentagoni æquilateri tres ſint æquales anguli, ſiue qui deinceps, ſiue qui non deinceps ſequuntur: illud pentagonum erit æquiangulum.



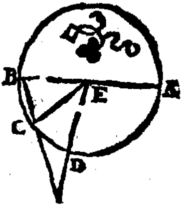
Theorema 8. Propoſitio 8.

Si pentagoni æquilateri, & æquianguli duos qui deinceps ſequuntur, angulos rectę ſubtendant lineæ; illæ per extremam & mediam proportionem ſe mutuò ſecant; earumque maiora ſegmenta, ipſius pentagoni lateri ſunt æqualia.

Theo-

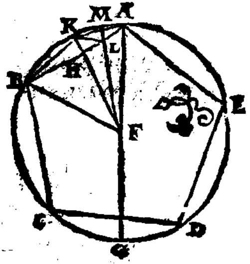
Theorema 9. Propositio 9.

Si latus hexagoni, & latus decagoni eidem circulo inscriptorum composita sint; tota recta linea per extremam & mediam proportionem secta est; eiusque segmentum maius, est hexagoni latus.



Theorema 10. Propositio 10.

Si in circulo pentagonum æquilaterũ inscriptum sit; pentagoni latus potest & latus hexagoni, & latus decagoni, eidem circulo inscriptorum.



Theorema 11. Propositio 11.

Si in circulo ῥητὸν, seu rationalem diametrum habente, inscriptum sit pentagonum æquilaterum; pentagoni latus irrationalis est linea, quæ vocatur minor.

Theo-

Theorema 12. Propositio 12.

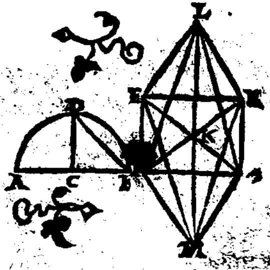
Si in circulo inscriptum sit triangulum æquilaterum; huius trianguli latus potentia triplum est eius lineæ, quæ ex circuli centro ducitur.

Problema 1. Propositio 13.

Pyramidem constituere; & data sphæra cõplecti: atque docere quod illius sphæræ diameter potentia sesquialtera sit lateris ipsius pyramidis.

Problema 2. Propositio 14.

Octaëdrum constituere, eaq; sphęra, qua pyramidẽ complecti; atque probare, quod illius sphęræ diameter potentia dupla sit lateris ipsius octaëdri.

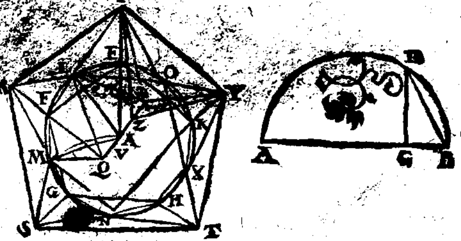


Problema 3. Propositio 15.

Cubum constituere; eaque sphæra qua & superiores figuras complecti; atque docere quod illius sphæræ diameter potentia tripla sit lateris ipsius cubi.

Problema 4. Propopositio 16.

Icosaedrum constituere, eademque sphæra, qua & antedictas figuras, complecti; atque probare, quod illius Icosaedri latus irrationalis sit linea, quæ vocatur Minor.



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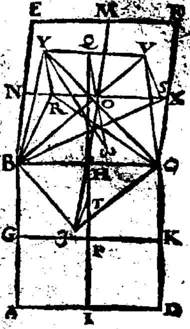
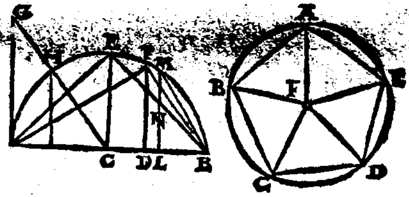
Problema 5. Propositio 17.



Dodecaedrum constituere; eademque sphæra, qua & antedictas figuras, cōplecti; atq; probare quod illius dodecaedri latus irrationalis sit linea, quæ vocatur Residuum, seu Apotome.

Problema 6. Propositio 18.

Quinque figurarū latera proponere, & inter se comparare.



SCHOLIVM.

Interpretes hoc in loco demonstrant, præter dictas quinque figuras, non posse aliam constitui figuram solidam, quæ ex planis & æquilateris & æquiangulis contineatur, inter se æqualibus.

Non enim ex duobus triangulis, neque ex alijs duabus figuris, solidus cōstituitur angulus; cùm saltem tres anguli plani requirantur ad solidi anguli constitutionem.

Ex tribus autem triangulis æquilateris, constat pyramidis angulus.

Ex quatuor angulis, Octaëdri.

Ex quinque angulis, Icosaëdri.

Nam ex triangulis, sex autem triangulis & æquilateris & æquiangulis ad idem punctum coëuntibus, non fiet angulus solidus: cum n. trianguli æquilateri angulus, recti vnius bessem (hoc est duas tertias partes:) contineat, erunt eiusmodi sex anguli rectis quatuor æquales. Quod fieri non potest. Nam solidus omnis angulus minoribus, quam rectis quatuor angulis, continetur, per propos. 21. l. 11.

Multò ergo minus ex pluribus, quam sex planis eiusmodi angulis, solidus angulus constabit.

Soli ex tribus quadratis, Cubi angulus continetur.

Ex quatuor autem quadratis, nullus angulus solidus constitui potest. Anguli enim recti quatuor erunt. Multò ergo minus ex pluribus, quàm quatuor eiusmodi angulis solidus angulus constabit.

Ex tribus autem pentagonis æquilateris, & æquiangulis, Dodecaedri angulus continetur.

Sed ex quatuor huiusmodi angulis, nullus solidus angulus confici potest. Cùm enim pentagoni æquilateri angulus rectus sit, & quinta recti pars, erunt quatuor anguli rectis quatuor maiores. Quod

fieri

fieri nequeunt, quae sunt ex alijs polygonis [illegible] solidis angulis continebuntur, quod hinc quoque [illegible] sequitur.

Quamobrem perspicuum est praeter dictas quinque figuras aliam figuram solidam non posse constitui, quae ex planis aequilateris [illegible] angulis inter se aequalibus, contineatur.

Vide Theon. p. [illegible]

FINIS ELEMENTI XIII.

[illegible] IOB [illegible] VOLL[illegible]

EVCLIDIS ELEMENTVM DECIMVMQVARTVM ET SOlidorum quartum, vt quidam arbitrantur; Vt alij verò, Hypsiclis Alexandrini, de quinque corporibus.

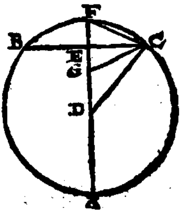
LIBER PRIMVS.

Prooemium Hypsiclis Alexandrini ad Protarchum.

Basilides Tyrius, Protarche, Alexandriam profectus, patriq́ nostro ob disciplinæ societatem commendatus, longissimo peregrinationis tempore cum eo versatus est. Cumq́ disserent aliquando de scripta ab Apollonio comparatione Dodecaedri, & Icosaedri eidem sphæræ inscriptorum, quam hæc inter se habeant rationem, censuerunt ea non rectè tradidisse Apollonium, quæ à se emendata, vt de patre audire erat, literis prodiderunt. Ego autem postea incidi in alterum librum ab Apollonio editum, qui demonstrationem accuratè complecteretur de re proposita, ex eiusq́ problematis indagatione magnam equidem cœpi voluptatem. Illud certè ab omnibus perspici potest quod scripsit Apollonius, cum sit in omnium manibus. Quod autem diligenti, quantum conijcere licet, studio nos postea scripsisse *pßiſſe videmur, id monumentis conſignatum tibi dedicandum duximus, vt qui feliciter cum in omnibus diſciplinis, tum vel maximè in Geometria verſatus, ſcitè ac prudenter iudices ea, quæ dicturi ſumus: Ob eam verò, quæ tibi cum patre fuit, vitæ conſuetudinem, quaq́; nos complecteris, beneuolentiam, tractationem ipſam libenter audias. Sed iam tempus eſt, vt proœmio finem facientes, hanc ſytaxim aggrediamur.*

Theorema 1. Propoſitio 1.

Perpendicularis linea, quæ ex circuli cuiuſpiam centro in latus pentagoni ipſi circulo inſcripti ducitur; dimidia eſt vtriuſque ſimul lineæ, & eius, quæ ex centro, & lateris decagoni in eodem circulo inſcripti.



Theorema 2. Propoſitio 2.

Si binæ rectæ lineæ extrema, & media proportione ſecentur; ipſæ ſimiliter ſecabuntur, in eaſdem ſcilicet proportiones.

Theorema 3. Propoſitio 3.

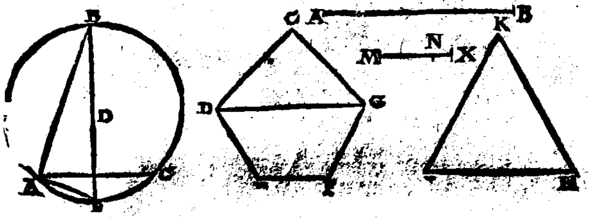
Si in circulo pentagonum æquilaterum inſcribatur; quod ex latere pentagoni; & quod ex ea, quæ binis lateribus pentagonis ſubtēditur, recta linea; vtraque ſimul quadrata, quintupla ſunt eius, quod ex ſe midiametro deſcribitur, quadrati.

Theorema 4. Propositio 4.

Si latus hexagoni alicuius circuli secetur extrema, & media proportione; maius illius segmentum erit latus decagoni eiusdem circuli.

Theorema 5. Propositio 5.

Idem circulus comprehendit, & dodecaê-dri pentagonum, & icosaëdri triangulum, eidem sphæræ inscriptorum.



Theorema 6. Propositio 6.

Si pentagono, & æquilatero, & æquiangulo circumscribatur circulus; ex cuius centro ad vnum pentagoni latus ducatur linea perpendicularis: Erit, quod sub dicto latere, & perpendiculari continetur, rectangulum trigesies sumptum, dodecaëdri superficiei æquale.

Theorema 7. Propositio 7.

Si ex centro circuli triangulum icosaedri cir-

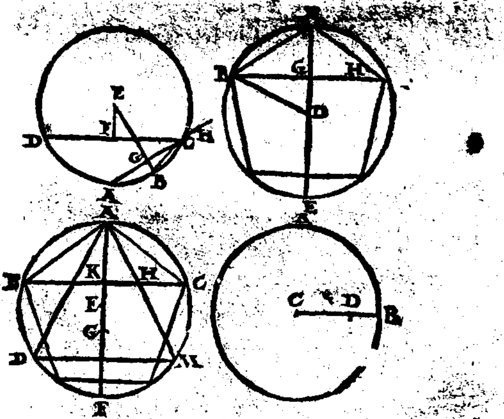
circumſcribentis, linea perpendicularis ducatur ad vnum latus trianguli: Erit, quod ſub dicto latere, & perpendiculari comprehenditur rectangulum trigeſies ſumptum, Icoſaedri ſuperficiei æquale.

Theorema 8. Propoſitio 8.

Rectangulum contentum ſub tribus quartis partibus diametri alicuius circuli, & ſub quinque ſextis partibus lineæ ſubtendentis angulum pentagoni æquilateri in eodé circulo deſcripti; æquale eſt dicto pentagono.

Theorema 9. Propoſitio 9.

Superficies Dodecaedri ad ſuperficiem Icoſaedri, in eadem ſphæra deſcripti, eandem proportionem habet, quàm latus cubi ad latus Icoſaedri.



Cubi latus.

E

Dodecaedri, latus.

E

Icoſaedri, latus.

C

Theorema 10. Propoſitio 10.

Si recta linea ſecetur extrema, & media proportione; Erit; vt recta potens id, quod à tota; & id, quod maiori ſegmento, ad rectam potentem id, quod à tota, & id, quod à minori ſegmento; Ita latus cubi ad latus icoſaedri, in eadem ſphæra cum cubo inſcripti.

Theorema 11. Propoſitio 11.

Dodecaedrum, ad Icoſaedrum in eadem cũ ipſo ſphæra inſcriptum eſt, vt cubi latus, ad Icoſaedri latus, in vna eademq; ſphæra.

Theorema 12. Propoſitio 12.

Latus trianguli æquilateri potentia ſeſquitertium eſt lineæ perpendicularis ab vno angulo ad latus oppoſitum, ſeu baſim, deductæ.

Theorema 13. Propoſitio 13.

Si ſphæræ (duo ſolida corpora, Tetraedrum & Octaedrum circumſcribentis:) diameter fuerit Rationalis: Erit tam ſuperficies Tetraedri, quam Octaedri in ea ſphæra media.

Theo-

Theorema 14. Propositio 14.

Si Tetraedrum, atque octaedrum in eadem sphæra inscribantur: Erit basis Tetraedri sesquitertia baseos Octaedri: Superficies autem Octaedri sesquialtera superficiei Tetraedri.

Theorema 15. Propositio 15.

Recta linea ex angulo quouis tetraedri in sphæra inscripti, per centrum sphærę ducta; cadit in centrum baseos oppositæ; estq; perpendicularis ad ductam basin.

Theorema 16. Propositio 16.

Octaedrum in sphæra inscriptum, diuiditur in duas pyramides æquales, & similes, æqualium altitudinum, basis verò vtriusq; pyramidis est quadratum sub duplum quadrati diametri sphæræ.

Theorema 17. Propositio 17.

Tetraedrum sphæræ impositum ad Octaedrum in eadem sphæra descriptum, se habet, vt rectangulum sub linea potente viginti septē sexagesimas quartas partes quadrati lateris tetraedri; & sub linea continente octo nonas partes eiusdem lateris, comprehensum, ad quadratum diametri sphæræ.

Theorema 18. Propositio 18.

Linea perpendicularis ex quolibet angulo triangulo æquilateri ad basin oppositam demissa; tripla est eius perpendicularis, quæ ex centro trianguli ad eandem basin deducitur.

Theo-

Theorema 19. Propositio 19.

Si Octaedrum sphæræ inscribatur; erit semidiameter sphæræ potentia tripla eius perpendicularis, quæ ex centro sphæræ ad basin quamcunque Octaedri ducitur.

Theorema 20. Propositio 20.

Duplum quadrati ex diametro cuiuslibet sphæræ descripti, æquale est superficiei cubi in illa sphæra collocati: perpendicularis autem à centro sphæræ in aliquam basin cubi demissa, æqualis est dimidio lateris cubi.

Theorema 21. Propositio 21.

Idem circulus comprehendit, & cubi quadratum, & Octaedri triangulum, eiusdem sphæræ.

Theorema 22. Propositio 22.

Si Octaedrum, atque Tetraedrum eidem sphæræ inscribantur; Erit Octaedrum ad triplum Tetraedri, vt latus Octaedri ad latus Tetraedri.

Theorema 23. Propositio 23.

Si recta linea [illegible] totam aliquam lineam sectam extrema, & media ratione, & maius eius segmentum; Item totam aliam similiter sectam, & minus eius segmentum: Erit maius segmentum prioris lineæ latus Icosaedri; minus autem segmentum posterioris lineæ latus Dodecaedri, eius sphæræ cuius recta linea proposita diameter existit.

Theo-

Theorema 24. Propoſitio 24.

Si latus Octaedri potuerit maius, & minus ſegmentum rectæ lineæ extrema, ac media ratione ſectæ; poterit latus Icoſaedri in eadem ſphæra deſcripti, duplum minoris ſegmenti.

Theorema 25. Propoſitio 25.

Si recta linea diuiſa extrema ac media ratione cum minore ſegmento angulum rectum conſtituat, cui recta ſubtendatur: Erit recta linea, quæ potentia ſit ſub dupla ipſius rectę ſubtenſæ, latus Octaedri eius ſphæræ, in qua dictum minus ſegmentum latus exiſtit dodecaedri.

Theorema 26. Propoſitio 26.

Si latus Tetraedri poſſit maius, & minus ſegmentum lineæ rectæ extrema ac media ratione ſectæ: latus Icoſaedri eidem ſphæræ inſcripti potentia ſeſquialterum eſt minoris ſegmenti.

Theorema 27. Propoſitio 27.

Cubus ad Octaedrum in eadem cum ipſo ſphæra deſcriptum, eſt, vt ſuperficies cubi ad Octaedri ſuperficiem: Item vt latus cubi ad ſemidiametrum ſphæræ.

Theorema 28. Propoſitio 28.

Si ſint quatuor lineæ rectæ continuæ proportionalis, nec non & aliæ quatuor, ita vt ſit eadem antecedens omnium: Erit proportio tertiæ ad tertiam proportionis ſecundæ

ad ſe-ad ſecundam duplicata; & proportio quartę ad quartam eiuſdem proportionis ſecundæ ad ſecundam triplicata.

Theorema 29 Propoſitio 29.

Qradratum lateris trianguli æquilateri ad ipſum triangulum habet proportionem duplicatam proportionis lateris trianguli ad lineam medio loco proportionalem inter perpendicularem ab vno angulo ad latus oppoſitum ductam, & dimidium ipſius lateris.

Theorema 30. Propoſitio 30.

Si cubus, & Tetraedrum in eadem ſphæra deſcribantur; Erit quadratum cubi ad triangulum Tetraedri, vt latus Tetraedri ad lineam perpendicularem, quæ ex vno angulo trianguli Tetraedri ad latus oppoſitum protrahatur.

Theorema 31. Propoſitio 31.

Latus Tetraedri potentia ſeſquialterum eſt axis, ſeu altitudinis ipſius; Axis verò, ſiue altitudo Tetraedri potentia ſeſquialtera eſt lateris cubi in eadem ſphæra deſcripti.

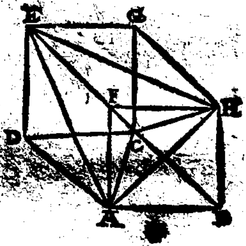
Theorema 32. Propoſitio 32.

Cubus triplus eſt Tetraedri eidem ſphæræ inſcripti.

FINIS ELEMENTI XIV.

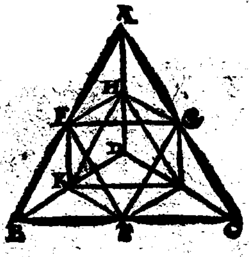
EVCLIDIS ELEMENTVM DECIMVMQVINTVM ET SOliderum quintum, vt nonnulli putant; Vt autem alij, Hypsiclis Alexandrini, de quinque corporibus.

LIBER II.



Problema 1. Propositio 1.

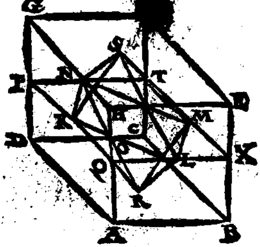
In dato cubo pyramidem, Tetraedrū, inscribere.



Problema 2. Propositio 2.

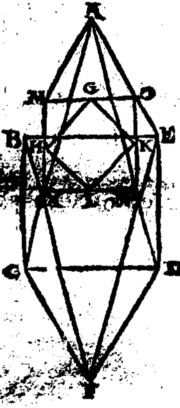
In data pyramide Octaedrum inscribere.

Pro-



Problema 3. Propositio 3.

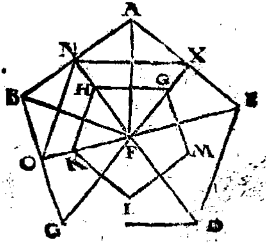
In dato cubo (hexaedro) Octaedrum inscribere.



Problema 4. Propositio 4.

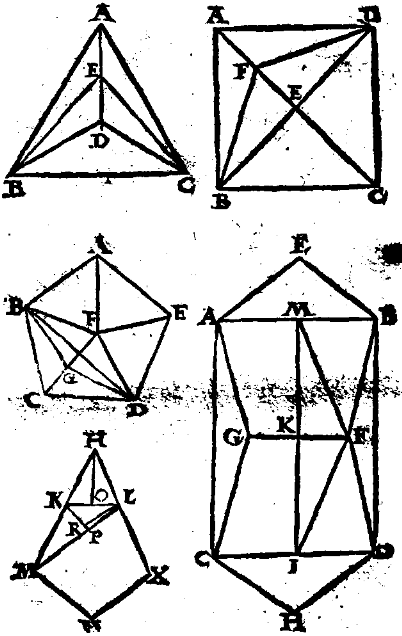
In dato octaedro cubum inscribere.

Problema 5. Propositio 5.



In dato Icosaedro dodecaedrum inscribere.

SCHOLIVM EX ZAMBERTO lib.15.propoſ.5.p.260.



MEminiſſe decet, ſi quis nōs roget, quot Icoſaedrum habeat latera, ita reſpondendum eſſe: Patet Icoſaedrum viginti contineri triangulis, quodlibet verò triangulum rectis tribus *constare lineis. Quare multiplicanda sunt nobis viginti triangula in trianguli vnius latera, fiunt q; sexaginta, quorum dimidium est triginta. Ad eundem modum & in dodecaedro. Cum enim rursus duodecim pentagona dodecaedrum comprehendant, itemq; pentagonum quoduis rectis quinque constet lineis; quinque duodecies multiplicamus, fiunt sexaginta, quorum rursus dimidium est triginta. Sed cur dimidium capimus? Quoniam vnūquodque latus siue sit trianguli, siue pentagoni, siue quadrati, vt in cubo, iterato sumitur. Similiter autem eadem via & in cubo, & in tetraedro & in octaedro latera inuenies.*

Latera figurarum inuenienda.

Quòd si item velis singularum quoque figurarum angulos reperire, facta eadem multiplicatione, numerum procreatum partire in numerum planorum, quæ vnum solidum angulum includunt: vt quoniam triangula quinque vnum Icosaedri angulum continet, partire 60. in quinque nascuntur duodecim anguli Icosaedri. In dodecaedro autem tria pentagona angulum comprehendunt, partire ergo 60. in tria, & habebis dodecaedri angulos viginti. Atque simili ratione in reliquis figuris angulos reperies, &c.

Anguli figurarum inueniendi.

Problema 6. Propositio 6.

In dato octaedro pyramidem, seu tetraedrū describere.

Pro-

Problema 7. Propositio 7.

In dato Dodecaedro Icosaedrum describere.

Problema 8. Propositio 8.

In dato Decaedro Cubum describere.

Problema 9. Propositio 9.

In dato Dodecaedro Octaedrum describere.

Problema 10. Propositio 10.

In dato Dodecaedro pyramidem describere.

Problema 11. Propositio 11.

In dato Icosaedro Cubum describere.

Problema 12. Propositio 12.

In dato Icosaedro pyramidem describere.

Problema 13. Propositio 13.

In dato cubo Dodecaedrum describere.

Problema 14. Propositio 14.

In dato cubo Icosaedrum describere.

Problema 15. Propositio 15.

In dato Icosaedro Octaedrum describere.

Problema 16. Propositio 16.

In dato Octaedro Icosaedrum describere.

Problema 17. Propositio 17.

In dato Octaedro Dodecaedrum describere.

Problema 18. Propositio 18.

In data pyramide Cubum describere: hoc est, in proposito Tetraedro hexaedrum delineare.

Problema 19. Propositio 19.

In data pyramide Icosaedrum describere.

Problema 20. Propositio 20.

In data pyramide Dodecaedrum describere.

Problema 21. Propositio 21.

In dato solido regulari sphæram describere: hoc est, Interprete Campano; In fabricato quouis quinque corporum regularium, sphæram fabricare.

FINIS ELEMENTI XV.

EVCLI-

EVCLIDIS ELEMENTVM DECIMVM SEXTVM, ET Solidorum sextum.

Quo varia solidorum regularium sibi mutuò inscriptorum, & laterum eorundem comparationes explicantur, à Francisco Flussate Candalla, & P. Christophoro Clauio adiectam, & de quinque corporibus.

LIBER TERTIVS.

Theorema 1. Propositio 1.

SI in Dodecaedro Cubus describatur, & in hoc Cubo aliud Dodecaedrum: Erit proportio Dodecaedri exterioris ad Dodecaedrum interius proportionis eius, quam habet maius segmentum ad minus rectæ lineæ diuisæ extrema, ac media ratione triplicata.

Theorema 2. Propositio 2.

Linea perpendicularis ex quouis angulo pentagoni æquilateri, & æquianguli in latus oppositum demissa; secatur à linea recta illum angulum subtendente, extrema ac media ratione.

Theorema 3. Propositio 3.

Si ab angulis trianguli pyramidis ducantur tres lineæ rectæ, opposita latera secantes extrema ac media ratione; ita vt prope quemuis angulum sit maius segmentum vnius lateris, & minus alterius: Hæ tres sectionibus suis in medio producent basin Icosaedri in dicta pyramide descripti, inscriptam quidem alij triangulo, cuius anguli latera trianguli pyramidis secant extrema ac media ratione; & latera ipsa bifariam secantur ab angulis basis Icosaedri.

Theorema 4. Propositio 4.

Minus segmentum lateris pyramidis extrema ac media ratione secti, duplum est potentia lateris Icosaedri in ea pyramide descripti.

Theorema 5. Propositio 5.

Latus Cubi potentia dimidium est lateris pyramidis in eo descriptæ: Latus verò pyramidis duplum est longitudine lateris Octaedri sibi inscripti: latus denique Cubi duplum est potentia lateris sibi inscripti Octaedri.

Theorema 6. Propositio 6.

Latus Dodecaedri maius segmentum est rectæ lineæ quæ potentia est dimidia lateris pyramidis sibi inscriptæ.

Theorema 7 Propositio 7.

Si in Cubo describatur & Icosaedrum, & Dode-

Dodecaedrũ: Latus Icosaedri medium proportionale erit inter latus Cubi, & Dodecaedri.

Theorema 8. Propositio 8.

Latus pyramidi potentia Octaedecuplum est lateris Cubi in ea descripti.

Theorema 9. Propositio 9.

Latus pyramidis potentia Octaedecuplum est rectæ lineæ extrema ac media ratione sectæ, cuius maius segmentum est latus Dodecaedri in pyramide descripti.

Theorema 10. Propositio 10.

Si in Octaedro Icosaedrum describatur: Erit latus Icosaedri potentia duplum minoris segmenti lateris Octaedri extrema ac media ratione diuisi.

Theorema 11. Propositio 11.

Latus Octaedri potentia quadruplum est sesquialterum lateris Cubi in ipso descripti.

Theorema 12. Propositio 12.

Latus Icosaedri maius segmentum est eius rectæ lineæ extrema ac media ratione sectæ, quæ potentia dupla est lateris Octaedri in eo Icosaedro descripti.

Theorema 13. Propositio 13.

Latus cubi ad latus Dodecaedri in ipso descripti proportionẽ habet duplicatam eius, quam habet maius segmentum ad minus, rectæ lineæ diuisæ extrema ac media ratione. Latus verò Dodecaedri ad latus cubi in ipso descri-

descripti, proportionem habet, quam minus segmentum ad maius, eiusdem rectæ lineæ.

Theorema 14. Propositio 14.

Latus octaedri sesquialterum est lateris pyramidis sibi inscriptæ.

Theorema 15. Propositio 15.

Si ex quadrato diametri Icosaedri auferatur triplum quadrati lateris cubi in eo descripti; relinquitur quadratum sesquitertium quadrati lateris Icosaedri.

Theorema 16. Propositio 16.

Latus Dodecaedri minus segmentum est rectæ lineæ extrema ac media ratione diuisæ, quæ duplum potest lateris octaedri in eo descripti.

Theorema 17. Propositio 17.

Diameter Icosaedri potest, & sui ipsius lateris sesquitertium; & lateris pyramidis in eo descriptæ sesquialterum.

Theorema 18. Propositio 18.

Latus Dodecaedri ad Icosaedri sibi inscripti latus; se habet, vt minus segmentum lineæ perpendicularis ab vno angulo pentagoni ad latus oppositum ductæ, atque extrema ac media ratione diuisæ, ad partem eiusdem lineæ inter centrum pentagoni, & latus eiusdem positæ.

Theorema 19. Propositio 19.

Si dimidium lateris Icosaedri extrema ac

media

media ratione ſectum fuerit, minuſque eius ſegmentum à toto latere Icoſaedri ſublatum; à reliqua quoque recta linea pars rurſum tertia detracta; Relinquetur latus Dodecaedri in Icoſaedro deſcripti.

Theorema 20. Propoſitio 20.

Cubus ſibi inſcriptæ pyramidis triplus eſt.

Theorema 21. Propoſitio 21.

Pyramis ſibi inſcripti Octaedri dupla eſt.

Theorema 22. Propoſitio 22.

Cubus ſibi inſcripti Octaedri ſextuplus eſt.

Theorema 23. Propoſitio 23.

Octaedrum ſibi inſcripti Cubi quadruplũ ſeſquialterum eſt.

Theorema 24. Propoſitio 24.

Octaedrum ſibi inſcriptæ pyramidis tredecuplum ſeſquialterum eſt.

Theorema 25. Propoſitio 25.

Pyramis ſibi inſcripti Cubi non cupla eſt.

Theorema 26. Propoſitio 26.

Octaedrum ad Icoſaedrum ſibi inſcriptum proportionem habet, quam duæ baſes octaedri ad quinque baſes Icoſaedri.

Theorema 27. Propoſitio 27.

Icoſaedrum ad Dodecaedrum ſibi inſcriptum, proportionem habet compoſitam ex proportione lateris Icoſaedri ad latus cubi in eadem cum Icoſaedro ſphæra deſcripti; & ex proportione triplicata eius quam habet diameter Icoſaedri ad rectam lineam contra basium Icosaedri oppositarum coniungentem.

Theorema 28. Propositio 28.

Dodecaedrum excedit Cubum sibi inscriptum parallelopipedo, cuius quidem basis à quadrato Cubi deficit rectangulo contento sub latere Cubi, tertiaque parte, minoris segmenti eiusdem lateris Cubi: At verò altitudo ab altitudine, siue latere Cubi deficit, minore segmento eius lineæ, quæ dimidiati lateris Cubi segmentum minus existit.

Theorema 29. Propositio 29.

Dodecaedrum ad Icosaedrum sibi inscriptum, proportionem habet compositam ex proportione triplicata eius, quam habet diameter Dodecaedri ad rectam lineam contra basium Dodecaedri oppositarum copulantem; & ex proportione lateris Cubi ad latus Icosaedri *in eadem sphæra cum Cubo descripti.*

Theorema 30. Propositio 30.

Dodecaedrum pyramidis, in qua describitur, duas nonas partes continet, minus duobus parallelepipedis; quorum vnius longitudo lateri Cubi in eadem pyramide descripti, æqualis est; latitudo verò tertiæ parti miti minoris ſegmenti lateris eiuſdem Cubi; altitudo denique à latere eiuſdem Cubi deficit minore ſegmento eius lineæ, quæ dimidiati lateris Cubi eiuſdem minus ſegmẽtum exiſtit: Alterius autem & longitudo, & latitudo lateri Cubi prædicti, eſt æqualis; altitudo verò minus ſegmentum eius lineæ quæ dimidiati lateris Cubi eiuſdem ſegmẽtum minus exiſtit; ita vt amborum altitudines ſimul altitudini, ſiue lateri eiuſdem Cubi ſint æquales.

Theorema 31. Propoſitio 31.

Octaedrum excedit ſibi inſcriptum Icoſaedrum parallelepipedo, cuius baſis eſt quadratum lateris Icoſaedri; altitudo verò maius ſegmentum ſemidiametri Octaedri extrema ac media ratione ſectæ.

DE QVINQVE CORPORVM regularium deſcriptione in data ſphæra, ex Pappo Alexandrino.

Lemma I.

Datis duobus circulis in ſphæra parallelis, dataque in vno eorum linea recta; ducare in altero diametrum huius rectæ datæ parallelam.

Lem-

Lemma II.

Si in planis parallelis deſcripti ſint duo circuli, in quibus duæ rectæ parallelæ abſcindant arcus ſimiles: Erunt duæ rectæ coniungentes extrema vnius rectæ cum centro parallelæ duabus rectis, quæ extrema alterius rectæ cum centro coniungant.

Lemma III.

Si in ſphæra ſint duo circuli paralleli, & æquales, in quibus ductæ ſint duæ rectę parallelæ, & æquales, ad eaſdem partes centrorum: Rectæ harum parallelarum extrema puncta ad eaſdem partes coniungentes, æquales quoque, & parallelæ ſunt, & ad plana circulorum perpendiculares.

Lemma IV.

Si in ſphæra ſint duo circuli paralleli, & æquales, in quibus ductæ ſint duæ rectæ parallelæ, & æquales, non ad eaſdem partes cẽtrorum: Rectæ lineæ harum parallelarum extrema puncta non ad eaſdem partes coniungentes, in centro ſphærę ſeſe interſecant; ac proinde diametri ſphæræ erunt, & inter ſe æquales: Rectæ verò lineæ earundem parallelarum extrema puncta ad eaſdem partes connectentes, & æquales, & parallelæ inter ſe

tet ſe

ter ſe ſunt, & cum parallelis rectos angulos conſtituunt.

Lemma V.

Si in ſphæra ſint duæ rectæ parallelæ; rectæ earum puncta extrema ad eaſdem partes coniungentes, æquales inter ſe erunt: Et ſi parallelæ ſint æquales, coniungentes non ſolum æquales, ſed & parallelæ erunt, rectoſque cum ipſis angulos conficient.

Lemma VI.

In data ſphæra duos circulos æquales, ac parallelos deſcribere; ita vt diameter ſphæræ ſit vtriuſque diametri potentia ſeſquialtera.

Problema 1. Propoſitio 1.

In data ſphæra pyramidem trigonam deſcribere.

Problema 2. Propoſitio 2.

In data ſphæra Octaedrum deſcribere.

Problema 3. Propoſitio 3.

In data ſphæra Cubum deſcribere.

Problema 4. Propoſitio 4.

In data ſphæra Icoſaedrum deſcribere.

Problema 5. Propoſitio 5.

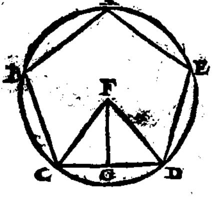
In data ſphæra Dodecaedrum deſcribere.

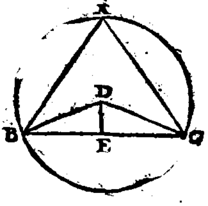
SCHO-

SCHOLIVM EX P. CLAVIO.

Ex his, quæ hoc in loco & lib. 13 & lib. 14. demonstrata sunt, facile etiam ostenditur, si omnia quinque corpora regularia in vna eademque sphæra describantur, maximum omnium esse Dodecaedrum: Deinde Icosaedrum maius reliquis tribus: Tertio Cubum maiorem reliquis duobus: Octaedrum denique Tetraedro esse maius. Ex quo euidenter constabit, Euclidem recto ordine quinque hæc corpora cōstruxisse, cum post Tetraedrum statim Octaedrum non autem Cubum constituerit. Ita enim à minoribus ad maiora progressus est.

FINIS ELEMENTI XVI. & Vltimi.





Pagina 216. Theoremate 6. desunt hæ duæ Figuræ.